Computer Algebra Independent Integration Tests

Summer 2024

5-Inverse-trig-functions/5.4-Inverse-cotangent/285-5.4.5

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**53**]. This is test number [285].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	98.11 (52)	1.89 (1)
Maple	98.11 (52)	1.89 (1)
Mathematica	92.45 (49)	7.55 (4)
Maxima	56.60 (30)	43.40 (23)
Giac	49.06 (26)	50.94 (27)
Mupad	45.28 (24)	54.72 (29)
Reduce	43.40 (23)	56.60 (30)
Fricas	41.51 (22)	58.49 (31)
Sympy	35.85 (19)	64.15 (34)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

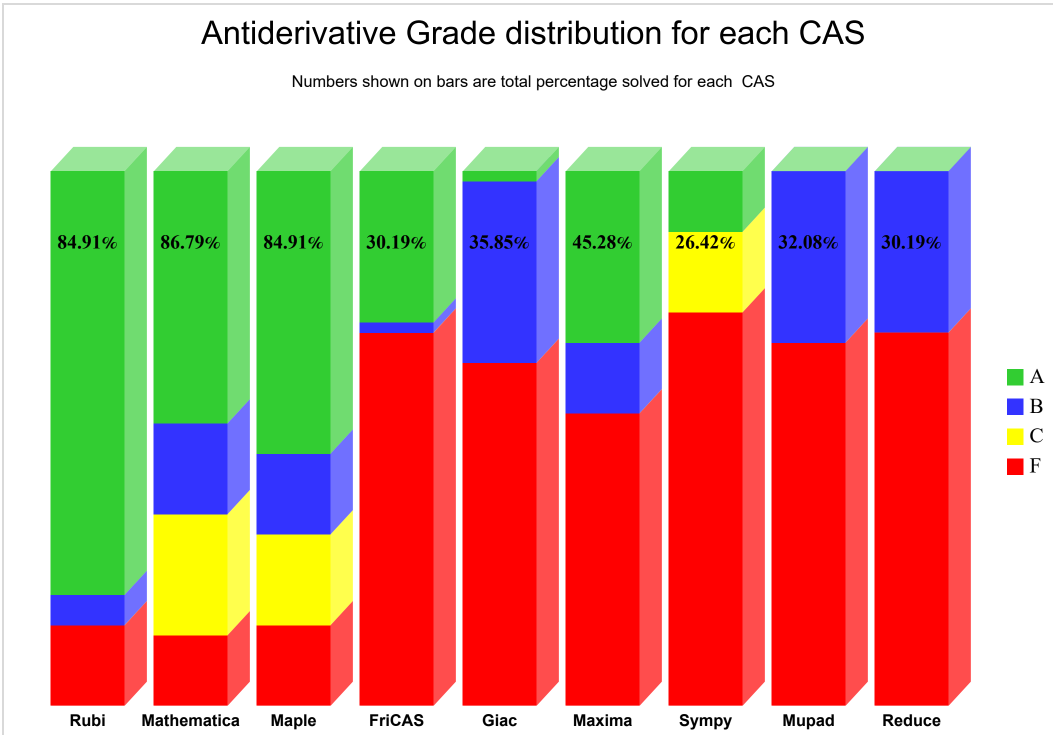
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

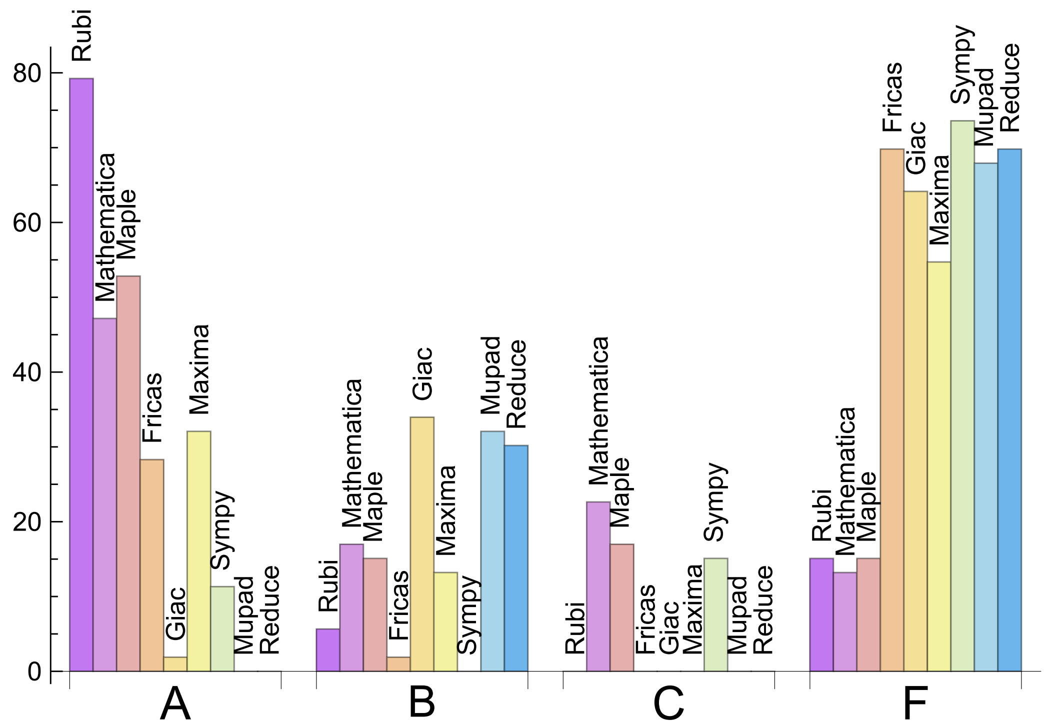
System	% A grade	% B grade	% C grade	% F grade
Rubi	79.245	5.660	0.000	15.094
Maple	52.830	15.094	16.981	15.094
Mathematica	47.170	16.981	22.642	13.208
Maxima	32.075	13.208	0.000	54.717
Fricas	28.302	1.887	0.000	69.811
Sympy	11.321	0.000	15.094	73.585
Giac	1.887	33.962	0.000	64.151
Mupad	0.000	32.075	0.000	67.925
Reduce	0.000	30.189	0.000	69.811

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	4	100.00	0.00	0.00
Rubi	1	100.00	0.00	0.00
Maple	1	100.00	0.00	0.00
Maxima	23	91.30	0.00	8.70
Giac	27	62.96	22.22	14.81
Mupad	29	0.00	100.00	0.00
Fricas	31	96.77	0.00	3.23
Reduce	30	100.00	0.00	0.00
Sympy	34	29.41	70.59	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.19
Reduce	0.26
Giac	0.44
Maxima	0.81
Rubi	0.83
Mupad	1.41
Mathematica	1.79
Maple	3.37
Sympy	6.83

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Fricas	119.41	1.35	61.50	1.24
Mupad	138.71	1.43	61.50	1.23
Sympy	179.26	1.99	99.00	1.46
Rubi	338.08	1.15	145.00	1.00
Mathematica	353.47	1.69	163.00	1.11
Maple	563.56	1.80	160.50	1.02
Maxima	726.47	4.93	112.00	1.22
Giac	731.35	5.81	159.50	3.43
Reduce	2820.78	133.71	111.00	1.63

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

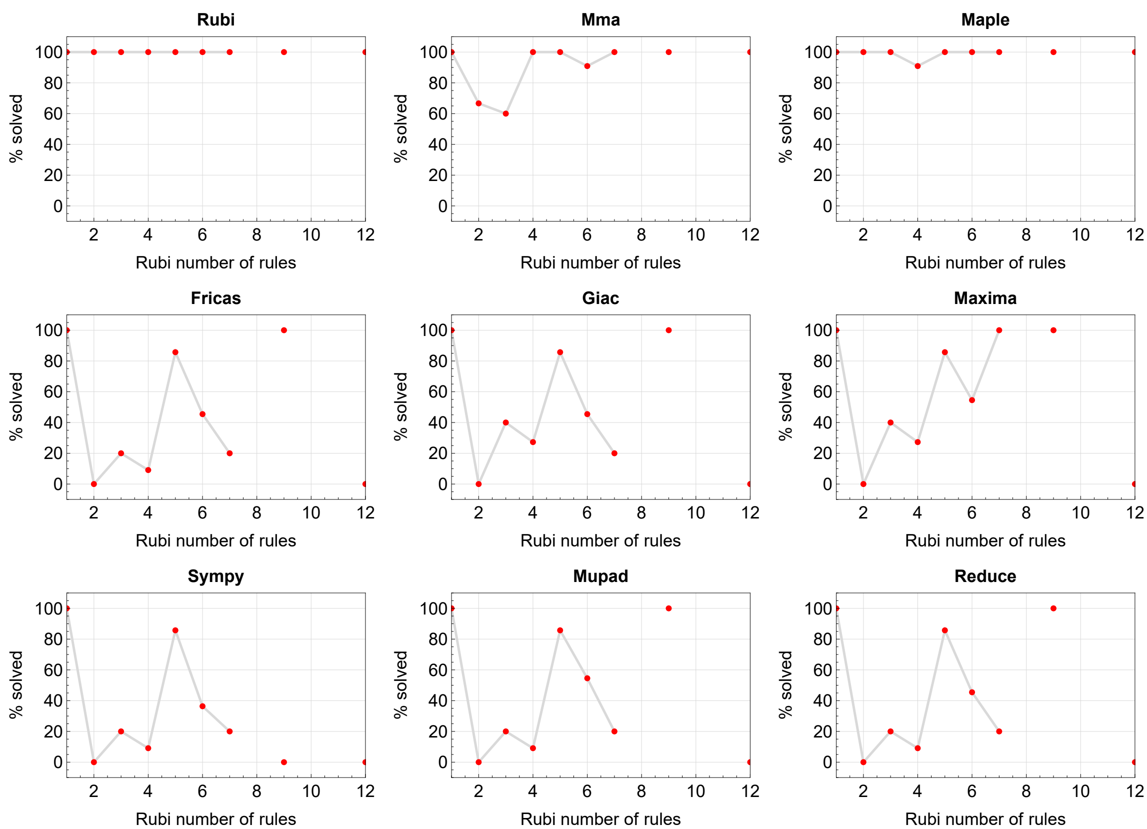


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

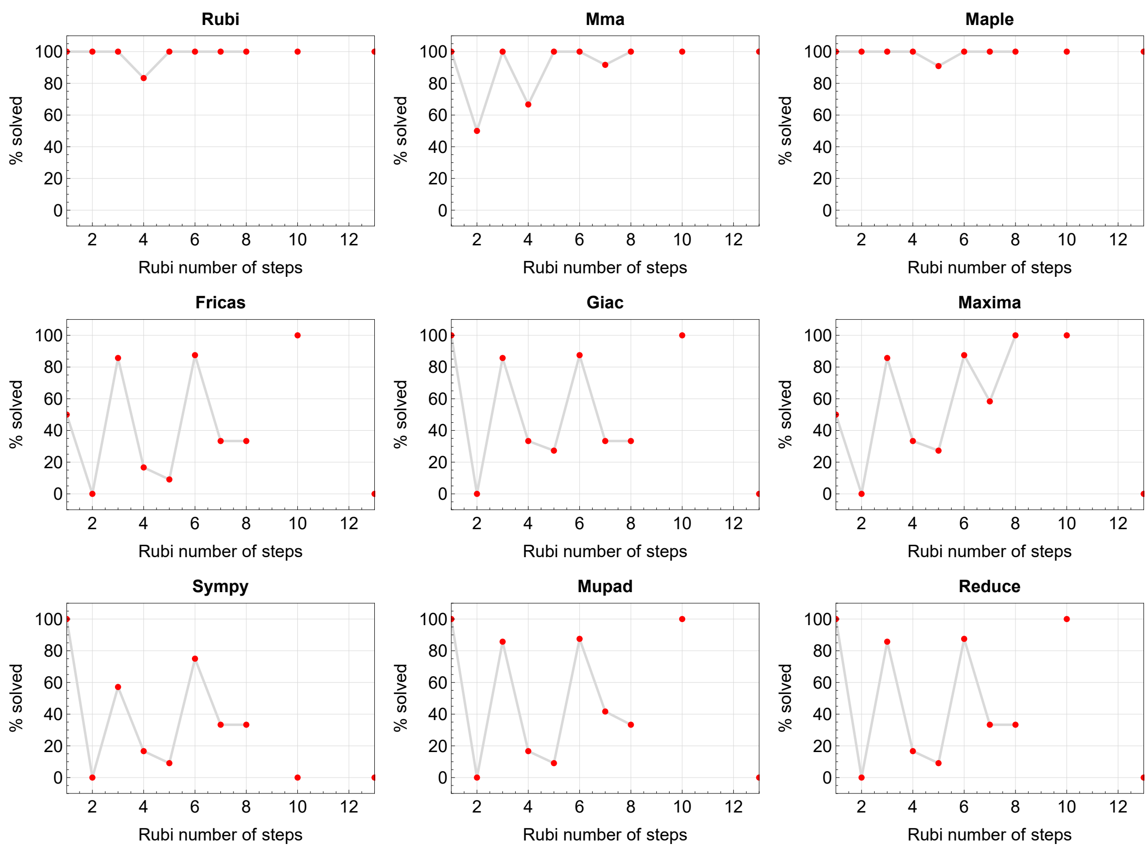


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

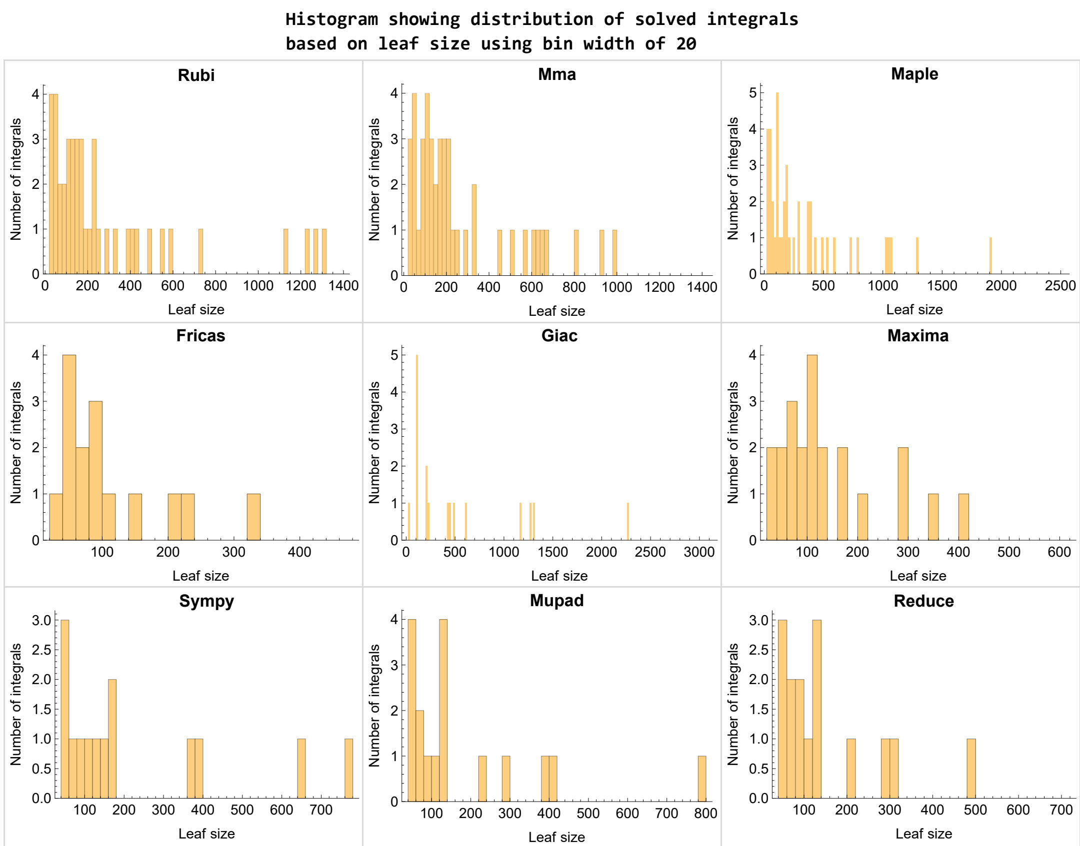


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

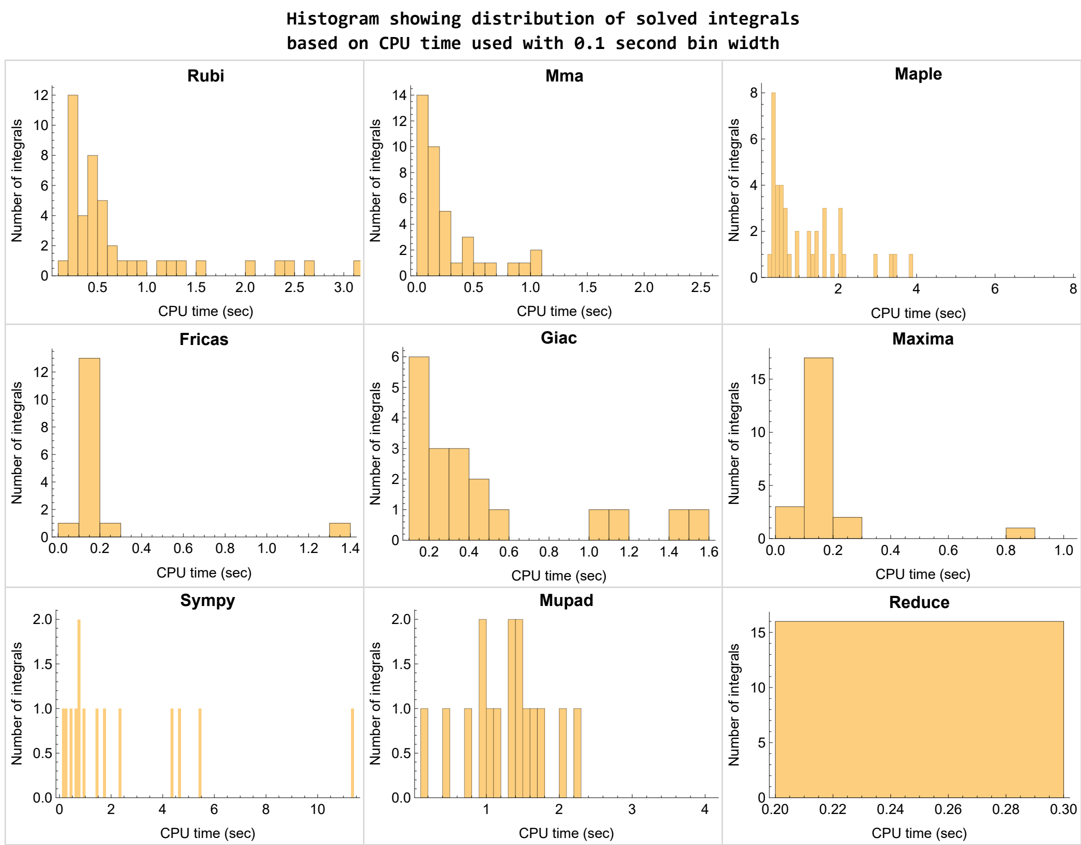


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

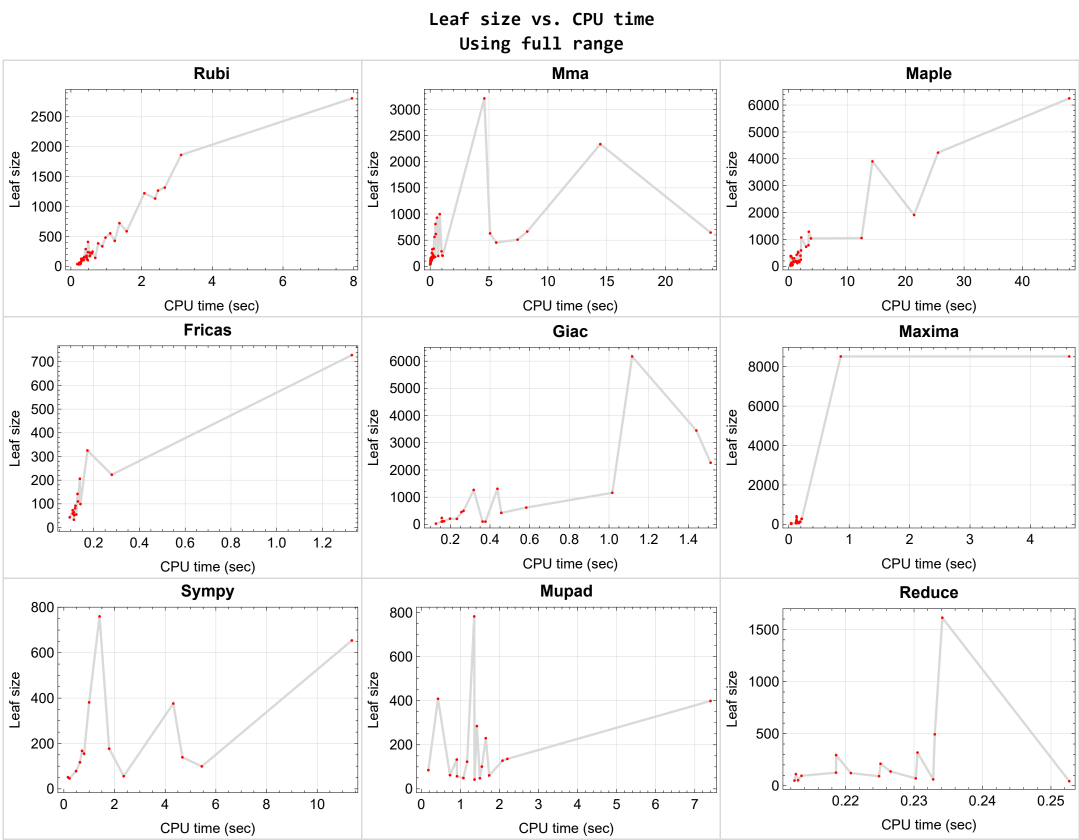


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{15, 34, 35, 47, 48, 52, 53\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{47, 48, 52, 53\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Reduce $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{9, 49\}$

Mathematica $\{23, 27, 28, 39, 47, 48, 49, 50, 51, 52, 53\}$

Maple $\{26, 28, 31, 32, 36, 43, 49\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

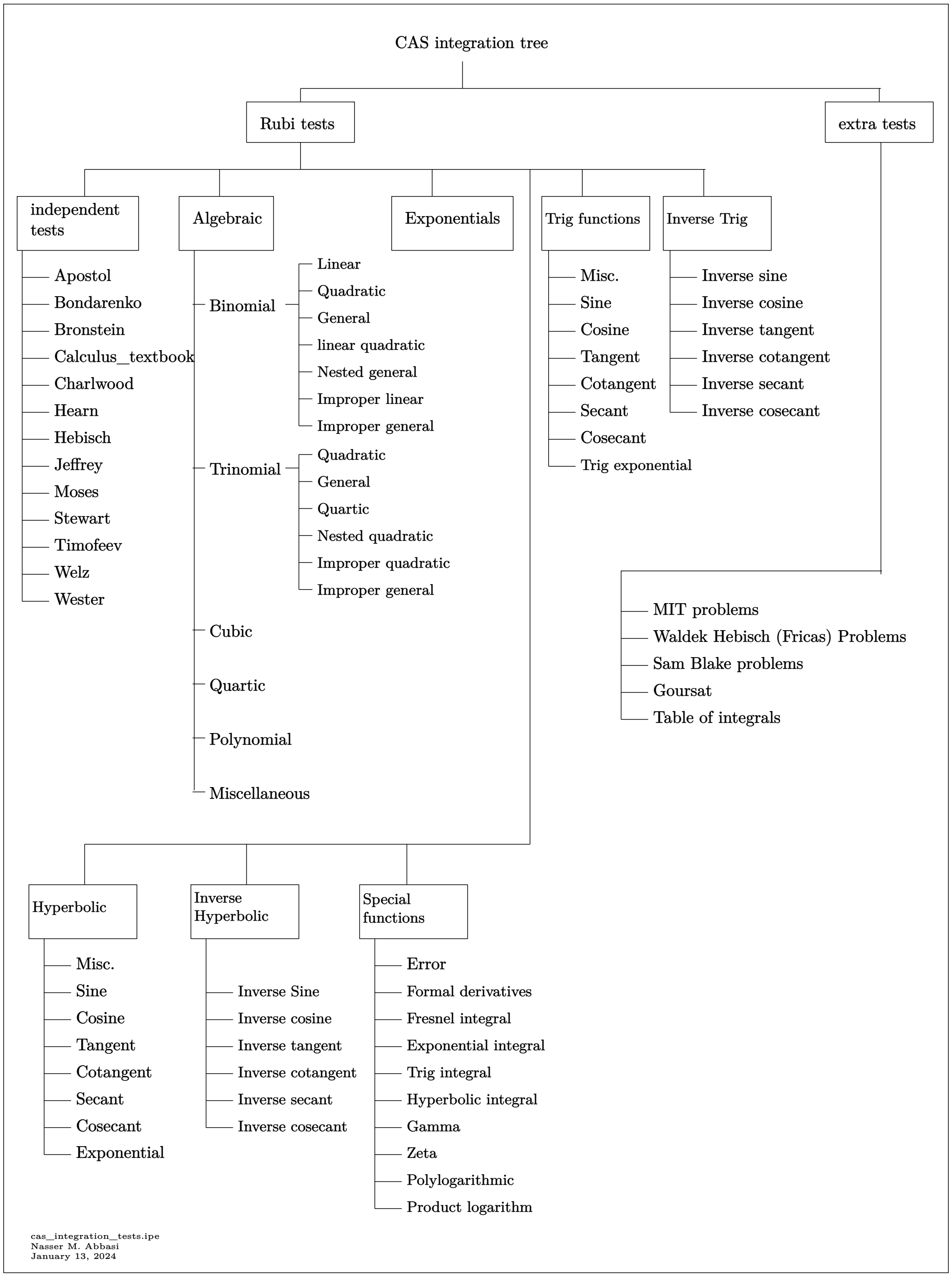
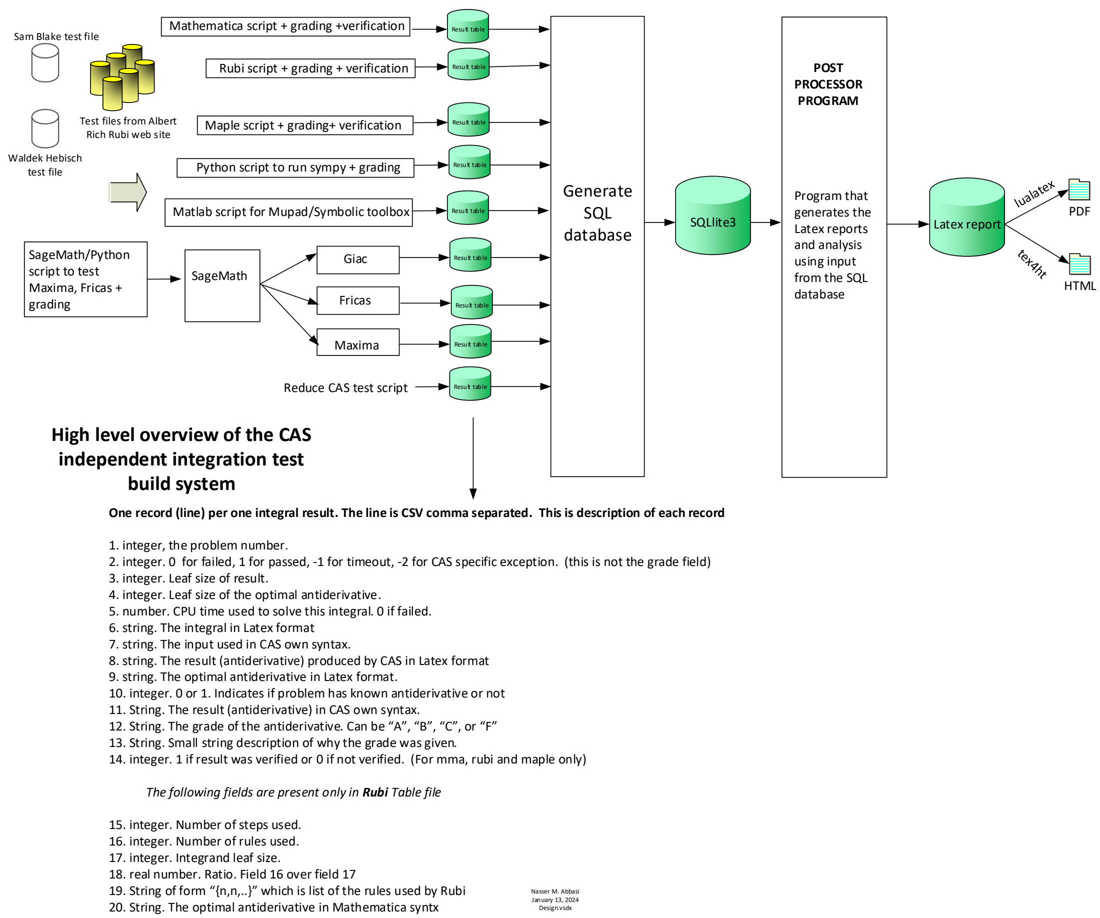


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 38, 39, 40, 42, 43, 44, 45, 46, 50, 51 }

B grade { 36, 37, 49 }

C grade { }

F normal fail { 41 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 4, 9, 11, 12, 13, 14, 19, 23, 24, 25, 27, 29, 30, 33, 36, 37, 39, 40, 41, 42, 43, 45, 46, 50, 51 }

B grade { 5, 20, 28, 38, 47, 48, 49, 52, 53 }

C grade { 1, 2, 3, 6, 7, 8, 10, 16, 17, 18, 21, 22 }

F normal fail { 26, 31, 32, 44 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 18, 19, 20, 21, 22, 25, 27, 38, 39, 40, 45, 46, 50, 51 }

B grade { 16, 17, 23, 24, 29, 30, 37, 44 }

C grade { 26, 28, 31, 32, 36, 41, 42, 43, 49 }

F normal fail { 33 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 6, 7, 8, 9, 10, 12, 16, 17, 18, 19, 21 }

B grade { 22 }

C grade { }

F normal fail { 5, 11, 13, 14, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 51 }

F(-1) timedout fail { }

F(-2) exception fail { 15 }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 16, 17, 18, 19, 21, 22, 39 }

B grade { 9, 11, 13, 14, 37, 38, 40 }

C grade { }

F normal fail { 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 41, 42, 43, 45, 46, 49, 50, 51 }

F(-1) timedout fail { }

F(-2) exception fail { 15, 44 }

Giac

A grade { 13 }

B grade { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 14, 16, 17, 18, 19, 21, 22 }

C grade { }

F normal fail { 5, 20, 23, 24, 25, 26, 28, 29, 30, 33, 38, 39, 41, 45, 46, 50, 51 }

F(-1) timedout fail { 27, 31, 36, 37, 40, 49 }

F(-2) exception fail { 32, 42, 43, 44 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 6, 7, 8, 9, 10, 12, 16, 17, 18, 19, 21, 22, 25 }

C grade { }

F normal fail { }

F(-1) timedout fail { 5, 11, 13, 14, 20, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 51 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 10, 19 }

B grade { }

C grade { 6, 7, 8, 9, 12, 16, 17, 18 }

F normal fail { 11, 13, 14, 23, 24, 25, 28, 29, 30, 45 }

F(-1) timedout fail { 5, 20, 21, 22, 26, 27, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 49, 50, 51 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 6, 7, 8, 9, 10, 12, 16, 17, 18, 19, 21, 22 }

C grade { }

F normal fail { 5, 11, 13, 14, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 51 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	99	95	131	104	92	155	617	121	133
N.S.	1	0.93	0.90	1.24	0.98	0.87	1.46	5.82	1.14	1.25
time (sec)	N/A	0.308	0.048	0.462	0.131	0.120	0.798	0.583	0.221	0.907

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	81	114	102	85	73	117	423	95	101
N.S.	1	1.01	1.42	1.28	1.06	0.91	1.46	5.29	1.19	1.26
time (sec)	N/A	0.296	0.031	0.381	0.118	0.108	0.636	0.457	0.214	1.550

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	90	63	68	55	78	210	60	61
N.S.	1	1.00	1.50	1.05	1.13	0.92	1.30	3.50	1.00	1.02
time (sec)	N/A	0.275	0.023	0.375	0.134	0.124	0.481	0.198	0.233	1.737

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	31	44	30	29	43	46	111	43	42
N.S.	1	0.94	1.33	0.91	0.88	1.30	1.39	3.36	1.30	1.27
time (sec)	N/A	0.212	0.009	0.267	0.036	0.095	0.223	0.160	0.253	1.361

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	256	103	133	0	0	0	12	0
N.S.	1	1.00	2.13	0.86	1.11	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.448	0.142	0.547	0.192	0.000	0.000	0.000	0.246	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	61	66	61	77	64	167	498	70	62
N.S.	1	0.98	1.06	0.98	1.24	1.03	2.69	8.03	1.13	1.00
time (sec)	N/A	0.237	0.043	0.331	0.130	0.113	0.717	0.267	0.230	0.732

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	102	92	83	112	99	381	1309	125	230
N.S.	1	1.07	0.97	0.87	1.18	1.04	4.01	13.78	1.32	2.42
time (sec)	N/A	0.299	0.071	0.427	0.134	0.142	0.998	0.438	0.219	1.650

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	136	126	115	165	142	760	3449	210	285
N.S.	1	1.05	0.98	0.89	1.28	1.10	5.89	26.74	1.63	2.21
time (sec)	N/A	0.352	0.103	0.465	0.135	0.129	1.409	1.440	0.225	1.421

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	43	42	42	93	81	99	203	91	85
N.S.	1	0.83	0.81	0.81	1.79	1.56	1.90	3.90	1.75	1.63
time (sec)	N/A	0.258	0.010	0.366	0.138	0.119	5.446	0.233	0.225	0.179

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	141	36	52	33	56	100	49	49
N.S.	1	1.00	3.62	0.92	1.33	0.85	1.44	2.56	1.26	1.26
time (sec)	N/A	0.227	0.034	0.348	0.120	0.113	2.362	0.157	0.213	1.077

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	43	38	55	112	0	0	100	16	0
N.S.	1	0.96	0.84	1.22	2.49	0.00	0.00	2.22	0.36	0.00
time (sec)	N/A	0.271	0.006	0.607	0.181	0.000	0.000	0.377	0.212	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	45	42	41	53	59	139	238	111	57
N.S.	1	0.96	0.89	0.87	1.13	1.26	2.96	5.06	2.36	1.21
time (sec)	N/A	0.233	0.012	0.530	0.039	0.108	4.679	0.157	0.213	0.913

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	39	35	37	64	0	0	26	14	0
N.S.	1	1.11	1.00	1.06	1.83	0.00	0.00	0.74	0.40	0.00
time (sec)	N/A	0.256	0.005	0.565	0.167	0.000	0.000	0.127	0.215	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	43	38	55	122	0	0	103	21	0
N.S.	1	0.96	0.84	1.22	2.71	0.00	0.00	2.29	0.47	0.00
time (sec)	N/A	0.277	0.006	0.662	0.179	0.000	0.000	0.363	0.238	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	0	0	17	18	224	18
N.S.	1	1.00	1.11	0.89	0.00	0.00	0.94	1.00	12.44	1.00
time (sec)	N/A	0.185	3.505	0.834	0.000	0.000	9.431	0.144	0.259	1.118

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	230	157	496	341	325	654	2265	493	783
N.S.	1	0.99	0.67	2.13	1.46	1.39	2.81	9.72	2.12	3.36
time (sec)	N/A	0.529	0.184	1.609	0.130	0.173	11.368	1.512	0.233	1.354

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	163	118	294	216	206	376	1161	293	409
N.S.	1	1.06	0.77	1.91	1.40	1.34	2.44	7.54	1.90	2.66
time (sec)	N/A	0.422	0.105	0.920	0.128	0.139	4.319	1.017	0.219	0.426

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	103	163	113	113	110	177	451	136	136
N.S.	1	1.10	1.73	1.20	1.20	1.17	1.88	4.80	1.45	1.45
time (sec)	N/A	0.362	0.061	0.598	0.125	0.131	1.787	0.256	0.227	2.202

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	49	35	34	52	51	116	52	48
N.S.	1	1.00	1.29	0.92	0.89	1.37	1.34	3.05	1.37	1.26
time (sec)	N/A	0.180	0.008	0.382	0.045	0.115	0.162	0.168	0.213	1.499

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	180	336	197	0	0	0	0	32	0
N.S.	1	1.11	2.07	1.22	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.540	0.297	1.200	0.000	0.000	0.000	0.000	0.215	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	149	118	161	177	223	0	1264	318	128
N.S.	1	0.97	0.77	1.05	1.16	1.46	0.00	8.26	2.08	0.84
time (sec)	N/A	0.446	0.146	0.985	0.124	0.279	0.000	0.318	0.231	2.077

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	245	180	245	410	728	0	6173	1612	399
N.S.	1	1.07	0.79	1.07	1.80	3.19	0.00	27.07	7.07	1.75
time (sec)	N/A	0.612	0.421	2.063	0.128	1.329	0.000	1.117	0.234	7.402

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	383	665	1072	0	0	0	0	769	0
N.S.	1	1.00	1.74	2.81	0.00	0.00	0.00	0.00	2.01	0.00
time (sec)	N/A	0.771	8.248	2.126	0.000	0.000	0.000	0.000	0.228	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	227	286	427	0	0	0	0	345	0
N.S.	1	1.03	1.30	1.94	0.00	0.00	0.00	0.00	1.57	0.00
time (sec)	N/A	0.596	0.966	1.347	0.000	0.000	0.000	0.000	0.254	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	118	184	0	0	0	0	106	123
N.S.	1	1.00	1.16	1.80	0.00	0.00	0.00	0.00	1.04	1.21
time (sec)	N/A	0.478	0.125	1.288	0.000	0.000	0.000	0.000	0.221	1.171

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	261	288	0	1911	0	0	0	0	59	0
N.S.	1	1.10	0.00	7.32	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.418	0.000	21.449	0.000	0.000	0.000	0.000	0.246	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	480	586	454	784	0	0	0	0	0	0
N.S.	1	1.22	0.95	1.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.577	5.610	3.376	0.000	0.000	0.000	0.000	0.412	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	565	551	2336	6248	0	0	0	0	1362	0
N.S.	1	0.98	4.13	11.06	0.00	0.00	0.00	0.00	2.41	0.00
time (sec)	N/A	1.112	14.455	48.004	0.000	0.000	0.000	0.000	0.469	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	334	630	1051	0	0	0	0	575	0
N.S.	1	0.99	1.87	3.12	0.00	0.00	0.00	0.00	1.71	0.00
time (sec)	N/A	0.888	5.087	12.459	0.000	0.000	0.000	0.000	0.222	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	141	228	395	0	0	0	0	172	0
N.S.	1	0.99	1.59	2.76	0.00	0.00	0.00	0.00	1.20	0.00
time (sec)	N/A	0.690	0.233	2.000	0.000	0.000	0.000	0.000	0.219	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	372	408	0	3903	0	0	0	0	86	0
N.S.	1	1.10	0.00	10.49	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.483	0.000	14.321	0.000	0.000	0.000	0.000	0.267	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	693	1266	0	4229	0	0	0	0	0	0
N.S.	1	1.83	0.00	6.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.464	0.000	25.537	0.000	0.000	0.000	0.000	0.800	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	177	236	162	0	0	0	0	0	1310	0
N.S.	1	1.33	0.92	0.00	0.00	0.00	0.00	0.00	7.40	0.00
time (sec)	N/A	0.479	0.280	0.000	0.000	0.000	0.000	0.000	0.282	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	618	36	0	22	17519	22
N.S.	1	1.00	1.10	1.00	30.90	1.80	0.00	1.10	875.95	1.10
time (sec)	N/A	0.294	4.498	0.614	5.751	0.135	0.000	0.361	0.419	0.814

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	880	52	0	22	42990	22
N.S.	1	1.00	1.10	1.00	44.00	2.60	0.00	1.10	2149.50	1.10
time (sec)	N/A	0.296	0.463	0.647	9.259	0.130	0.000	0.404	0.647	0.854

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	C	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	725	1862	998	536	0	0	0	0	18	0
N.S.	1	2.57	1.38	0.74	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	3.117	0.813	1.601	0.000	0.000	0.000	0.000	102.148	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	B	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	721	563	591	8519	0	0	0	18	0
N.S.	1	2.26	1.76	1.85	26.71	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.374	0.367	2.091	4.643	0.000	0.000	0.000	0.252	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	170	325	187	283	0	0	0	16	0
N.S.	1	1.12	2.14	1.23	1.86	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.533	0.181	0.798	0.211	0.000	0.000	0.000	0.220	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	481	509	296	280	0	0	0	17	0
N.S.	1	1.42	1.51	0.88	0.83	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.979	7.427	0.644	0.216	0.000	0.000	0.000	0.220	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	735	1222	930	728	8518	0	0	0	21	0
N.S.	1	1.66	1.27	0.99	11.59	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	2.078	0.582	2.968	0.860	0.000	0.000	0.000	0.235	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	726	0	645	372	0	0	0	0	44	0
N.S.	1	0.00	0.89	0.51	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	23.823	0.434	0.000	0.000	0.000	0.000	0.272	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	693	1133	618	364	0	0	0	0	17	0
N.S.	1	1.63	0.89	0.53	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	2.382	0.480	0.320	0.000	0.000	0.000	0.000	0.230	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	811	1316	809	388	0	0	0	0	19	0
N.S.	1	1.62	1.00	0.48	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	2.655	0.452	0.323	0.000	0.000	0.000	0.000	0.256	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	427	0	1037	0	0	0	0	1056	0
N.S.	1	1.07	0.00	2.61	0.00	0.00	0.00	0.00	2.65	0.00
time (sec)	N/A	1.241	0.000	3.802	0.000	0.000	0.000	0.000	0.291	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	127	127	118	0	0	0	0	29	0
N.S.	1	0.96	0.96	0.89	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.295	0.125	1.458	0.000	0.000	0.000	0.000	0.223	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	155	138	156	0	0	0	0	34	0
N.S.	1	0.72	0.64	0.72	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.377	0.104	1.406	0.000	0.000	0.000	0.000	0.217	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	177	26	28	28	29	28	28	28
N.S.	1	1.00	6.32	0.93	1.00	1.00	1.04	1.00	1.00	1.00
time (sec)	N/A	0.260	0.275	1.129	0.147	0.150	0.861	0.261	0.225	0.865

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	180	31	33	33	31	33	32	33
N.S.	1	1.00	5.45	0.94	1.00	1.00	0.94	1.00	0.97	1.00
time (sec)	N/A	0.276	0.139	1.030	0.163	0.117	7.451	0.281	0.242	0.851

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	C	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1199	2808	3211	1287	0	0	0	0	419	0
N.S.	1	2.34	2.68	1.07	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	7.952	4.602	3.425	0.000	0.000	0.000	0.000	0.464	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	177	202	160	0	0	0	0	104	0
N.S.	1	0.95	1.08	0.86	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.442	1.063	1.834	0.000	0.000	0.000	0.000	0.250	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	215	207	202	0	0	0	0	109	0
N.S.	1	0.77	0.74	0.72	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.588	1.026	1.693	0.000	0.000	0.000	0.000	0.249	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	198	33	35	44	36	35	101	35
N.S.	1	1.00	5.66	0.94	1.00	1.26	1.03	1.00	2.89	1.00
time (sec)	N/A	0.331	0.684	1.102	0.270	0.123	4.110	0.308	0.292	0.881

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	200	38	40	49	37	40	105	40
N.S.	1	1.00	5.00	0.95	1.00	1.22	0.92	1.00	2.62	1.00
time (sec)	N/A	0.386	0.284	1.102	0.337	0.115	72.528	0.316	0.273	0.909

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [3] had the largest ratio of [.7500000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	6	0.93	10	0.600
2	A	6	5	1.01	10	0.500
3	A	7	6	1.00	8	0.750
4	A	4	3	0.94	6	0.500
5	A	8	7	1.00	10	0.700
6	A	8	7	0.98	10	0.700
7	A	6	5	1.07	10	0.500
8	A	7	6	1.05	10	0.600
9	A	6	5	0.83	14	0.357
10	A	5	4	1.00	12	0.333
11	A	4	3	0.96	14	0.214
12	A	7	6	0.96	14	0.429
13	A	5	4	1.11	12	0.333
14	A	5	4	0.96	19	0.211
15	N/A	1	0	1.00	18	0.000
16	A	6	5	0.99	18	0.278
17	A	6	5	1.06	18	0.278
18	A	6	5	1.10	16	0.312
19	A	1	1	1.00	10	0.100
20	A	7	6	1.11	18	0.333
21	A	10	9	0.97	18	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	6	6	1.07	18	0.333
23	A	5	4	1.00	20	0.200
24	A	5	4	1.03	18	0.222
25	A	7	6	1.00	12	0.500
26	A	4	3	1.10	20	0.150
27	A	7	6	1.22	20	0.300
28	A	5	4	0.98	20	0.200
29	A	5	4	0.99	18	0.222
30	A	7	6	0.99	12	0.500
31	A	4	3	1.10	20	0.150
32	A	7	6	1.83	20	0.300
33	A	5	4	1.33	18	0.222
34	N/A	3	0	1.00	20	0.000
35	N/A	3	0	1.00	20	0.000
36	B	13	12	2.57	16	0.750
37	B	8	7	2.26	16	0.438
38	A	7	6	1.12	14	0.429
39	A	7	7	1.42	16	0.438
40	A	7	7	1.66	16	0.438
41	F	0	0	N/A	0.000	N/A
42	A	5	4	1.63	18	0.222
43	A	5	4	1.62	18	0.222
44	A	2	2	1.07	23	0.087
45	A	3	2	0.96	28	0.071
46	A	4	3	0.72	33	0.091
47	N/A	3	0	1.00	28	0.000
48	N/A	3	0	1.00	33	0.000
49	B	2	2	2.34	25	0.080
50	A	5	4	0.95	35	0.114
51	A	6	5	0.77	40	0.125
52	N/A	3	0	1.00	35	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
53	N/A	3	0	1.00	40	0.000

CHAPTER 3

Listing of integrals

3.1 $\int x^3 \cot^{-1}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 106

$$\int x^3 \cot^{-1}(a+bx)\,dx = -\frac{(1-6a^2)\,x}{4b^3} - \frac{a(a+bx)^2}{2b^4} + \frac{(a+bx)^3}{12b^4} + \frac{1}{4}x^4\cot^{-1}(a+bx) + \frac{(1-6a^2+a^4)\arctan(a+bx)}{4b^4} + \frac{a(1-a^2)\log\left(1+(a+bx)^2\right)}{2b^4}$$

output
```
-1/4*(-6*a^2+1)*x/b^3-1/2*a*(b*x+a)^2/b^4+1/12*(b*x+a)^3/b^4+1/4*x^4*arcco
t(b*x+a)+1/4*(a^4-6*a^2+1)*arctan(b*x+a)/b^4+1/2*a*(-a^2+1)*ln(1+(b*x+a)^2
)/b^4
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

$$\int x^3 \cot^{-1}(a+bx)\,dx = \frac{6(-1+6a^2)\,bx - 12a(a+bx)^2 + 2(a+bx)^3 + 6b^4x^4\cot^{-1}(a+bx) - 3i(-i+a)^4\log(i-a-bx) + 3i(i}{24b^4}$$

input
```
Integrate[x^3*ArcCot[a + b*x],x]
```

output
```
(6*(-1 + 6*a^2)*b*x - 12*a*(a + b*x)^2 + 2*(a + b*x)^3 + 6*b^4*x^4*ArcCot[
a + b*x] - (3*I)*(-I + a)^4*Log[I - a - b*x] + (3*I)*(I + a)^4*Log[I + a +
 b*x])/(24*b^4)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5571, 25, 27, 5388, 478, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cot^{-1}(a+bx)\,dx$$

$$\Big\downarrow \text{5571}$$

$$\frac{\int x^3 \cot^{-1}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -x^3 \cot^{-1}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int -b^3x^3 \cot^{-1}(a+bx)d(a+bx)}{b^4}$$

$$\Big\downarrow \text{5388}$$

$$-\frac{-\frac{1}{4}\int \frac{b^4x^4}{(a+bx)^2+1}d(a+bx)-\frac{1}{4}b^4x^4\cot^{-1}(a+bx)}{b^4}$$

$$\Big\downarrow \text{478}$$

$$-\frac{-\frac{1}{4}\int \left(6a^2-4(a+bx)a+(a+bx)^2+\frac{a^4-6a^2+4(1-a^2)(a+bx)a+1}{(a+bx)^2+1}-1\right)d(a+bx)-\frac{1}{4}b^4x^4\cot^{-1}(a+bx)}{b^4}$$

$\Big\downarrow$ 2009

$$-\frac{\frac{1}{4}\big(\left(1-6a^2\right)(a+bx)-2a\left(1-a^2\right)\log\left((a+bx)^2+1\right)-\left(a^4-6a^2+1\right)\arctan(a+bx)-\frac{1}{3}(a+bx)^3+2a(a+}{b^4}$$

input
```
Int[x^3*ArcCot[a + b*x],x]
```

output
```
-((-1/4*(b^4*x^4*ArcCot[a + b*x]) + ((1 - 6*a^2)*(a + b*x) + 2*a*(a + b*x)
^2 - (a + b*x)^3/3 - (1 - 6*a^2 + a^4)*ArcTan[a + b*x] - 2*a*(1 - a^2)*Log
[1 + (a + b*x)^2])/4)/b^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 478
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[n, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5388
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])/(e*(q + 1))), x] + Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.24

method	result
parallelrisch	$-\frac{-3\operatorname{arccot}(bx+a)x^4b^4-b^3x^3+3a\,b^2x^2+3\operatorname{arccot}(bx+a)a^4+6a^3\ln(b^2x^2+2abx+a^2+1)-9a^2bx-18\operatorname{arccot}(bx+a)a^2\cdots}{12b^4}$
parts	$\frac{x^4\operatorname{arccot}(bx+a)}{4}+\frac{b\left(\frac{\frac{1}{3}b^2x^3-ab\,x^2+3a^2x-x}{b^4}+\frac{\left(-4a^3b+4ab\right)\ln\left(b^2x^2+2abx+a^2+1\right)}{2b^2}+\frac{\left(-3a^4-2a^2+1-\frac{\left(-4a^3b+4ab\right)a}{b}\right)\cdots}{b^4}\cdots\right)}{4}$
derivativedivides	$\frac{\frac{\operatorname{arccot}(bx+a)a^4}{4}-\operatorname{arccot}(bx+a)a^3(bx+a)+\frac{3\operatorname{arccot}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arccot}(bx+a)a(bx+a)^3+\frac{\operatorname{arccot}(bx+a)(bx+a)^4}{4}+3a\cdots}{b^4}$
default	$\frac{\frac{\operatorname{arccot}(bx+a)a^4}{4}-\operatorname{arccot}(bx+a)a^3(bx+a)+\frac{3\operatorname{arccot}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arccot}(bx+a)a(bx+a)^3+\frac{\operatorname{arccot}(bx+a)(bx+a)^4}{4}+3a\cdots}{b^4}$
risch	$\frac{ix^4\ln(1+i(bx+a))}{8}-\frac{ix^4\ln(1-i(bx+a))}{8}+\frac{\pi x^4}{8}+\frac{x^3}{12b}+\frac{a^4\arctan(bx+a)}{4b^4}-\frac{a\,x^2}{4b^2}-\frac{a^3\ln(b^2x^2+2abx+a^2+\cdots}{2b^4}$

input
```
int(x^3*arccot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-3*arccot(b*x+a)*x^4*b^4-b^3*x^3+3*a*b^2*x^2+3*arccot(b*x+a)*a^4+6*
a^3*ln(b^2*x^2+2*a*b*x+a^2+1)-9*a^2*b*x-18*arccot(b*x+a)*a^2+15*a^3-6*a*ln
(b^2*x^2+2*a*b*x+a^2+1)+3*b*x+3*arccot(b*x+a)-9*a)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int x^3 \cot^{-1}(a + bx) \, dx$$
$$= \frac{3\, b^4 x^4 \operatorname{arccot}\left(bx + a\right) + b^3 x^3 - 3\, a b^2 x^2 + 3\left(3\, a^2 - 1\right) b x + 3\left(a^4 - 6\, a^2 + 1\right) \arctan\left(bx + a\right) - 6\left(a^3 - a\right) \mathrm{l}}{12\, b^4}$$

input
```
integrate(x^3*arccot(b*x+a),x, algorithm="fricas")
```

output
```
1/12*(3*b^4*x^4*arccot(b*x + a) + b^3*x^3 - 3*a*b^2*x^2 + 3*(3*a^2 - 1)*b*
x + 3*(a^4 - 6*a^2 + 1)*arctan(b*x + a) - 6*(a^3 - a)*log(b^2*x^2 + 2*a*b*
x + a^2 + 1))/b^4
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.46

$$\int x^3 \cot^{-1}(a + bx) \, dx$$
$$= \begin{cases} -\frac{a^4 \operatorname{acot}(a+bx)}{4b^4} - \frac{a^3 \log(a^2+2abx+b^2x^2+1)}{2b^4} + \frac{3a^2x}{4b^3} + \frac{3a^2 \operatorname{acot}(a+bx)}{2b^4} - \frac{ax^2}{4b^2} + \frac{a \log(a^2+2abx+b^2x^2+1)}{2b^4} + \frac{x^4 \operatorname{acot}(a+bx)}{4} + \\ \frac{x^4 \operatorname{acot}(a)}{4} \end{cases}$$

input
```
integrate(x**3*acot(b*x+a),x)
```

output
```
Piecewise((-a**4*acot(a + b*x)/(4*b**4) - a**3*log(a**2 + 2*a*b*x + b**2*x
**2 + 1)/(2*b**4) + 3*a**2*x/(4*b**3) + 3*a**2*acot(a + b*x)/(2*b**4) - a*
x**2/(4*b**2) + a*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(2*b**4) + x**4*acot
(a + b*x)/4 + x**3/(12*b) - x/(4*b**3) - acot(a + b*x)/(4*b**4), Ne(b, 0))
, (x**4*acot(a)/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.98

$$\int x^3 \cot^{-1}(a+bx)\,dx = \frac{1}{4}\,x^4 \operatorname{arccot}(bx+a) + \frac{1}{12}\,b\left(\frac{b^2x^3 - 3\,abx^2 + 3\,(3\,a^2-1)x}{b^4} + \frac{3\,(a^4-6\,a^2+1)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^5} - \frac{6\,(a^3-a)\log\left(b^2x^2+2\,abx\right.}{b^5}\right.$$

input
```
integrate(x^3*arccot(b*x+a),x, algorithm="maxima")
```

output
```
1/4*x^4*arccot(b*x + a) + 1/12*b*((b^2*x^3 - 3*a*b*x^2 + 3*(3*a^2 - 1)*x)/
b^4 + 3*(a^4 - 6*a^2 + 1)*arctan((b^2*x + a*b)/b)/b^5 - 6*(a^3 - a)*log(b^
2*x^2 + 2*a*b*x + a^2 + 1)/b^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. 2(92) = 184.

Time = 0.58 (sec) , antiderivative size = 617, normalized size of antiderivative = 5.82

$$\int x^3 \cot^{-1}(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^3*arccot(b*x+a),x, algorithm="giac")
```

output
```
1/192*(96*a^3*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^5 + 72*a^2*
arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^6 + 24*a*arctan(1/(b*x +
a))*tan(1/2*arctan(1/(b*x + a)))^7 + 3*arctan(1/(b*x + a))*tan(1/2*arctan(
1/(b*x + a)))^8 + 96*a^3*log(16*tan(1/2*arctan(1/(b*x + a)))^2/(tan(1/2*ar
ctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arct
an(1/(b*x + a)))^4 - 96*a^3*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)
))^3 + 144*a^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 - 144*a^
2*tan(1/2*arctan(1/(b*x + a)))^5 - 72*a*arctan(1/(b*x + a))*tan(1/2*arctan
(1/(b*x + a)))^5 - 24*a*tan(1/2*arctan(1/(b*x + a)))^6 - 12*arctan(1/(b*x
+ a))*tan(1/2*arctan(1/(b*x + a)))^6 - 2*tan(1/2*arctan(1/(b*x + a)))^7 -
96*a*log(16*tan(1/2*arctan(1/(b*x + a)))^2/(tan(1/2*arctan(1/(b*x + a)))^4
 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^4 +
 72*a^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 + 144*a^2*tan(1
/2*arctan(1/(b*x + a)))^3 + 72*a*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x
 + a)))^3 - 48*a*tan(1/2*arctan(1/(b*x + a)))^4 - 30*arctan(1/(b*x + a))*t
an(1/2*arctan(1/(b*x + a)))^4 + 30*tan(1/2*arctan(1/(b*x + a)))^5 - 24*a*a
rctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a))) - 24*a*tan(1/2*arctan(1/(b
*x + a)))^2 - 12*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 - 30*t
an(1/2*arctan(1/(b*x + a)))^3 + 3*arctan(1/(b*x + a)) + 2*tan(1/2*arctan(1
/(b*x + a))))/(b^4*tan(1/2*arctan(1/(b*x + a)))^4)
```

Mupad [B] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.25

$$\int x^3 \cot^{-1}(a+bx)\,dx = \frac{\mathrm{atan}(a+b\,x)}{4\,b^4} + \frac{x^4\,\mathrm{acot}(a+b\,x)}{4} - \frac{x}{4\,b^3} + \frac{x^3}{12\,b} - \frac{a^3\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2\,b^4} - \frac{3\,a^2\,\mathrm{atan}(a+b\,x)}{2\,b^4} + \frac{a^4\,\mathrm{atan}(a+b\,x)}{4\,b^4} - \frac{a\,x^2}{4\,b^2} + \frac{3\,a^2\,x}{4\,b^3} + \frac{a\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2\,b^4}$$

input
```
int(x^3*acot(a + b*x),x)
```

output
```
atan(a + b*x)/(4*b^4) + (x^4*acot(a + b*x))/4 - x/(4*b^3) + x^3/(12*b) - (
a^3*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/(2*b^4) - (3*a^2*atan(a + b*x))/(2*b
^4) + (a^4*atan(a + b*x))/(4*b^4) - (a*x^2)/(4*b^2) + (3*a^2*x)/(4*b^3) +
(a*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/(2*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.14

$$\int x^3 \cot^{-1}(a+bx)\,dx$$
$$= \frac{-3acot(bx+a)\,a^4+18acot(bx+a)\,a^2+3acot(bx+a)\,b^4x^4-3acot(bx+a)-6\log(b^2x^2+2abx+a^2}{12b^4}$$

input
```
int(x^3*acot(b*x+a),x)
```

output
```
( - 3*acot(a + b*x)*a**4 + 18*acot(a + b*x)*a**2 + 3*acot(a + b*x)*b**4*x*
*4 - 3*acot(a + b*x) - 6*log(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**3 + 6*log(
a**2 + 2*a*b*x + b**2*x**2 + 1)*a + 9*a**2*b*x - 3*a*b**2*x**2 + b**3*x**3
 - 3*b*x)/(12*b**4)
```

3.2 $\int x^2 \cot^{-1}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 80

$$\int x^2 \cot^{-1}(a+bx)\,dx = -\frac{ax}{b^2}+\frac{(a+bx)^2}{6b^3}+\frac{1}{3}x^3\cot^{-1}(a+bx)+\frac{a(3-a^2)\arctan(a+bx)}{3b^3}-\frac{(1-3a^2)\log\left(1+(a+bx)^2\right)}{6b^3}$$

output
```
-a*x/b^2+1/6*(b*x+a)^2/b^3+1/3*x^3*arccot(b*x+a)+1/3*a*(-a^2+3)*arctan(b*x
+a)/b^3-1/6*(-3*a^2+1)*ln(1+(b*x+a)^2)/b^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.42

$$\int x^2 \cot^{-1}(a+bx)\,dx = \frac{\frac{1}{3}b\left(-\frac{a}{b}+\frac{a+bx}{b}\right)^3\cot^{-1}(a+bx)+\frac{1}{3}b\left(-\frac{3ax}{b^2}+\frac{(a+bx)^2}{2b^3}-\frac{(1+ia)^3\log(i-a-bx)}{2b^3}-\frac{(1-ia)^3\log(i+a+bx)}{2b^3}\right)}{b}$$

input
```
Integrate[x^2*ArcCot[a + b*x],x]
```

output
```
((b*(-(a/b) + (a + b*x)/b)^3*ArcCot[a + b*x])/3 + (b*((-3*a*x)/b^2 + (a +
b*x)^2/(2*b^3) - ((1 + I*a)^3*Log[I - a - b*x])/(2*b^3) - ((1 - I*a)^3*Log
[I + a + b*x])/(2*b^3)))/3)/b
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5571, 27, 5388, 478, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot^{-1}(a+bx)\,dx$$

$$\Big\downarrow \; 5571$$

$$\frac{\int x^2 \cot^{-1}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{\int b^2x^2 \cot^{-1}(a+bx)d(a+bx)}{b^3}$$

$$\Big\downarrow \; 5388$$

$$\frac{\frac{1}{3}b^3x^3\cot^{-1}(a+bx)-\frac{1}{3}\int -\frac{b^3x^3}{(a+bx)^2+1}d(a+bx)}{b^3}$$

$$\Big\downarrow \; 478$$

$$\frac{\frac{1}{3}b^3x^3\cot^{-1}(a+bx)-\frac{1}{3}\int \left(2a-bx-\frac{a(3-a^2)-(1-3a^2)(a+bx)}{(a+bx)^2+1}\right)d(a+bx)}{b^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{1}{3}\left(a(3-a^2)\arctan(a+bx)-\frac{1}{2}(1-3a^2)\log\left((a+bx)^2+1\right)+\frac{1}{2}(a+bx)^2-3a(a+bx)\right)+\frac{1}{3}b^3x^3\cot^{-1}(a+bx)}{b^3}$$

input
```
Int[x^2*ArcCot[a + b*x],x]
```

output
```
((b^3*x^3*ArcCot[a + b*x])/3 + (-3*a*(a + b*x) + (a + b*x)^2/2 + a*(3 - a^
2)*ArcTan[a + b*x] - ((1 - 3*a^2)*Log[1 + (a + b*x)^2])/2)/3)/b^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 478
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[n, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5388
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])/(e*(q + 1))), x] + Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.28

method	result
parallelrisch	$\frac{2x^3 \operatorname{arccot}(bx+a)b^3+b^2x^2+2 \operatorname{arccot}(bx+a)a^3+3a^2 \ln(b^2x^2+2abx+a^2+1)-4abx-6a \operatorname{arccot}(bx+a)+7a^2-1-\ln(b^2x^2 \cdots}{6b^3}$
derivativedivides	$\frac{-\frac{\operatorname{arccot}(bx+a)a^3}{3}+\operatorname{arccot}(bx+a)a^2(bx+a)-\operatorname{arccot}(bx+a)a(bx+a)^2+\frac{\operatorname{arccot}(bx+a)(bx+a)^3}{3}-(bx+a)a+\frac{(bx+a)^2}{6}-\frac{(-3a^2 \cdots}{\cdots}}{b^3}$
default	$\frac{-\frac{\operatorname{arccot}(bx+a)a^3}{3}+\operatorname{arccot}(bx+a)a^2(bx+a)-\operatorname{arccot}(bx+a)a(bx+a)^2+\frac{\operatorname{arccot}(bx+a)(bx+a)^3}{3}-(bx+a)a+\frac{(bx+a)^2}{6}-\frac{(-3a^2 \cdots}{\cdots}}{b^3}$
parts	$\frac{x^3 \operatorname{arccot}(bx+a)}{3}+\frac{b\left(-\frac{-\frac{1}{2}x^2b+2ax}{b^3}+\frac{\frac{(3a^2b-b)\ln\left(b^2x^2+2abx+a^2+1\right)}{2b^2}+\frac{\left(2a^3+2a-\frac{(3a^2b-b)a}{b}\right)\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b}}{b^3}\right)}{3}$
risch	$\frac{ix^3 \ln(1+i(bx+a))}{6}-\frac{ix^3 \ln(1-i(bx+a))}{6}+\frac{\pi x^3}{6}-\frac{a^3 \arctan(bx+a)}{3b^3}+\frac{x^2}{6b}+\frac{a^2 \ln(b^2x^2+2abx+a^2+1)}{2b^3}-\frac{2a \cdots}{3b^2}$

input
```
int(x^2*arccot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*x^3*arccot(b*x+a)*b^3+b^2*x^2+2*arccot(b*x+a)*a^3+3*a^2*ln(b^2*x^2+
2*a*b*x+a^2+1)-4*a*b*x-6*a*arccot(b*x+a)+7*a^2-1-ln(b^2*x^2+2*a*b*x+a^2+1)
)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

$$\int x^2 \cot^{-1}(a+bx)\,dx$$
$$= \frac{2\,b^3x^3 \operatorname{arccot}(bx+a)+b^2x^2-4\,abx-2\,(a^3-3\,a)\arctan(bx+a)+(3\,a^2-1)\log(b^2x^2+2\,abx+a^2+\cdots}{6\,b^3}$$

input
```
integrate(x^2*arccot(b*x+a),x, algorithm="fricas")
```

output
```
1/6*(2*b^3*x^3*arccot(b*x + a) + b^2*x^2 - 4*a*b*x - 2*(a^3 - 3*a)*arctan(
b*x + a) + (3*a^2 - 1)*log(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.46

$$\int x^2 \cot^{-1}(a+bx)\,dx = \begin{cases} \frac{a^3 \operatorname{acot}(a+bx)}{3b^3} + \frac{a^2 \log(a^2+2abx+b^2x^2+1)}{2b^3} - \frac{2ax}{3b^2} - \frac{a \operatorname{acot}(a+bx)}{b^3} + \frac{x^3 \operatorname{acot}(a+bx)}{3} + \frac{x^2}{6b} - \frac{\log(a^2+2abx+b^2x^2+1)}{6b^3} & \text{for } b \ne \\ \frac{x^3 \operatorname{acot}(a)}{3} & \text{other} \end{cases}$$

input
```
integrate(x**2*acot(b*x+a),x)
```

output
```
Piecewise((a**3*acot(a + b*x)/(3*b**3) + a**2*log(a**2 + 2*a*b*x + b**2*x*
*2 + 1)/(2*b**3) - 2*a*x/(3*b**2) - a*acot(a + b*x)/b**3 + x**3*acot(a + b
*x)/3 + x**2/(6*b) - log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(6*b**3), Ne(b, 0
)), (x**3*acot(a)/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.06

$$\int x^2 \cot^{-1}(a+bx)\,dx = \frac{1}{3}x^3 \operatorname{arccot}(bx+a) + \frac{1}{6}b\left(\frac{bx^2-4ax}{b^3} - \frac{2(a^3-3a)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^4} + \frac{(3a^2-1)\log(b^2x^2+2abx+a^2+1)}{b^4}\right)$$

input
```
integrate(x^2*arccot(b*x+a),x, algorithm="maxima")
```

output
```
1/3*x^3*arccot(b*x + a) + 1/6*b*((b*x^2 - 4*a*x)/b^3 - 2*(a^3 - 3*a)*arcta
n((b^2*x + a*b)/b)/b^4 + (3*a^2 - 1)*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(70) = 140$.

Time = 0.46 (sec) , antiderivative size = 423, normalized size of antiderivative = 5.29

$$\int x^2 \cot^{-1}(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^2*arccot(b*x+a),x, algorithm="giac")
```

output
```
-1/24*(12*a^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 + 6*a*arc
tan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^5 + arctan(1/(b*x + a))*tan(
1/2*arctan(1/(b*x + a)))^6 + 12*a^2*log(16*tan(1/2*arctan(1/(b*x + a)))^2/
(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*t
an(1/2*arctan(1/(b*x + a)))^3 - 12*a^2*arctan(1/(b*x + a))*tan(1/2*arctan(
1/(b*x + a)))^2 + 12*a*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^3
- 12*a*tan(1/2*arctan(1/(b*x + a)))^4 - 3*arctan(1/(b*x + a))*tan(1/2*arct
an(1/(b*x + a)))^4 - tan(1/2*arctan(1/(b*x + a)))^5 - 4*log(16*tan(1/2*arc
tan(1/(b*x + a)))^2/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(
b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^3 + 6*a*arctan(1/(b*x + a)
)*tan(1/2*arctan(1/(b*x + a))) + 12*a*tan(1/2*arctan(1/(b*x + a)))^2 + 3*a
rctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 - 2*tan(1/2*arctan(1/(b*
x + a)))^3 - arctan(1/(b*x + a)) - tan(1/2*arctan(1/(b*x + a))))/(b^3*tan(
1/2*arctan(1/(b*x + a)))^3)
```

Mupad [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.26

$$\int x^2 \cot^{-1}(a+bx)\,dx = \frac{x^3\,\mathrm{acot}(a+b\,x)}{3} - \frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{6\,b^3} + \frac{x^2}{6\,b} + \frac{a^2\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2\,b^3} - \frac{a^3\,\mathrm{atan}(a+b\,x)}{3\,b^3} + \frac{a\,\mathrm{atan}(a+b\,x)}{b^3} - \frac{2\,a\,x}{3\,b^2}$$

input
```
int(x^2*acot(a + b*x),x)
```

output
```
(x^3*acot(a + b*x))/3 - log(a^2 + b^2*x^2 + 2*a*b*x + 1)/(6*b^3) + x^2/(6*
b) + (a^2*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/(2*b^3) - (a^3*atan(a + b*x))/
(3*b^3) + (a*atan(a + b*x))/b^3 - (2*a*x)/(3*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.19

$$\int x^2 \cot^{-1}(a+bx)\,dx$$
$$= \frac{2acot(bx+a)\,a^3 - 6acot(bx+a)\,a + 2acot(bx+a)\,b^3x^3 + 3\log(b^2x^2+2abx+a^2+1)\,a^2 - \log(b^2x^2+}{6b^3}$$

input
```
int(x^2*acot(b*x+a),x)
```

output
```
(2*acot(a + b*x)*a**3 - 6*acot(a + b*x)*a + 2*acot(a + b*x)*b**3*x**3 + 3*
log(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**2 - log(a**2 + 2*a*b*x + b**2*x**2
+ 1) - 4*a*b*x + b**2*x**2)/(6*b**3)
```

3.3 $\int x \cot^{-1}(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 60

$$\int x \cot^{-1}(a+bx)\,dx = \frac{x}{2b}+\frac{1}{2}x^2\cot^{-1}(a+bx)-\frac{(1-a^2)\arctan(a+bx)}{2b^2} -\frac{a\log\left(1+(a+bx)^2\right)}{2b^2}$$

output
```
1/2*x/b+1/2*x^2*arccot(b*x+a)-1/2*(-a^2+1)*arctan(b*x+a)/b^2-1/2*a*ln(1+(b
*x+a)^2)/b^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.50

$$\int x \cot^{-1}(a+bx)\,dx = \frac{2bx+2b^2x^2\cot^{-1}(a+bx)-i(-i+a)^2\log(i-a-bx)-i\log(i+a+bx)-2a\log(i+a+bx)+ia^2\log}{4b^2}$$

input
```
Integrate[x*ArcCot[a + b*x],x]
```

output
```
(2*b*x + 2*b^2*x^2*ArcCot[a + b*x] - I*(-I + a)^2*Log[I - a - b*x] - I*Log
[I + a + b*x] - 2*a*Log[I + a + b*x] + I*a^2*Log[I + a + b*x])/(4*b^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5571, 25, 27, 5388, 478, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cot^{-1}(a+bx)\,dx$$

$$\Big\downarrow \; 5571$$

$$\frac{\int x\cot^{-1}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -x\cot^{-1}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int -bx\cot^{-1}(a+bx)d(a+bx)}{b^2}$$

$$\Big\downarrow \; 5388$$

$$-\frac{-\frac{1}{2}\int \frac{b^2x^2}{(a+bx)^2+1}d(a+bx)-\frac{1}{2}b^2x^2\cot^{-1}(a+bx)}{b^2}$$

$$\Big\downarrow \; 478$$

$$-\frac{-\frac{1}{2}\int \left(1-\frac{-a^2+2(a+bx)a+1}{(a+bx)^2+1}\right)d(a+bx)-\frac{1}{2}b^2x^2\cot^{-1}(a+bx)}{b^2}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\frac{1}{2}\left(\left(1-a^2\right)\arctan(a+bx)+a\log\left((a+bx)^2+1\right)-a-bx\right)-\frac{1}{2}b^2x^2\cot^{-1}(a+bx)}{b^2}$$

input
```
Int[x*ArcCot[a + b*x],x]
```

output
```
-((-1/2*(b^2*x^2*ArcCot[a + b*x]) + (-a - b*x + (1 - a^2)*ArcTan[a + b*x]
+ a*Log[1 + (a + b*x)^2])/2)/b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 478
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[n, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5388
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])/(e*(q + 1))), x] + Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{2}-\operatorname{arccot}(bx+a)a(bx+a)+\frac{bx}{2}+\frac{a}{2}-\frac{a\ln\left(1+(bx+a)^2\right)}{2}-\frac{\arctan(bx+a)}{2}}{b^2}$
default	$\frac{\frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{2}-\operatorname{arccot}(bx+a)a(bx+a)+\frac{bx}{2}+\frac{a}{2}-\frac{a\ln\left(1+(bx+a)^2\right)}{2}-\frac{\arctan(bx+a)}{2}}{b^2}$
parallelrisch	$-\frac{-\operatorname{arccot}(bx+a)b^2x^2+\operatorname{arccot}(bx+a)a^2+a\ln(b^2x^2+2abx+a^2+1)-bx-\operatorname{arccot}(bx+a)+2a}{2b^2}$
parts	$\frac{x^2\operatorname{arccot}(bx+a)}{2}+\frac{b\left(\frac{x}{b^2}+\frac{-\frac{a\ln\left(b^2x^2+2abx+a^2+1\right)}{b}+\frac{\left(a^2-1\right)\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b}}{b^2}\right)}{2}$
risch	$\frac{ix^2\ln(1+i(bx+a))}{4}-\frac{ix^2\ln(1-i(bx+a))}{4}+\frac{\pi x^2}{4}+\frac{a^2\arctan(bx+a)}{2b^2}-\frac{a\ln(b^2x^2+2abx+a^2+1)}{2b^2}+\frac{x}{2b}-$ arcta

input
```
int(x*arccot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/2*(b*x+a)^2*arccot(b*x+a)-arccot(b*x+a)*a*(b*x+a)+1/2*b*x+1/2*a-1
/2*a*ln(1+(b*x+a)^2)-1/2*arctan(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int x\cot^{-1}(a+bx)\,dx = \frac{b^2x^2\operatorname{arccot}(bx+a)+bx+(a^2-1)\arctan(bx+a)-a\log(b^2x^2+2\,abx+a^2+1)}{2\,b^2}$$

input
```
integrate(x*arccot(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2*arccot(b*x + a) + b*x + (a^2 - 1)*arctan(b*x + a) - a*log(b^2
*x^2 + 2*a*b*x + a^2 + 1))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.30

$$\int x\cot^{-1}(a+bx)\,dx = \begin{cases} -\frac{a^2\operatorname{acot}(a+bx)}{2b^2} - \frac{a\log(a^2+2abx+b^2x^2+1)}{2b^2} + \frac{x^2\operatorname{acot}(a+bx)}{2} + \frac{x}{2b} + \frac{\operatorname{acot}(a+bx)}{2b^2} & \text{for } b\neq 0 \\ \frac{x^2\operatorname{acot}(a)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*acot(b*x+a),x)
```

output
```
Piecewise((-a**2*acot(a + b*x)/(2*b**2) - a*log(a**2 + 2*a*b*x + b**2*x**2
 + 1)/(2*b**2) + x**2*acot(a + b*x)/2 + x/(2*b) + acot(a + b*x)/(2*b**2),
Ne(b, 0)), (x**2*acot(a)/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13

$$\int x\cot^{-1}(a+bx)\,dx = \frac{1}{2}x^2\operatorname{arccot}(bx+a) + \frac{1}{2}b\left(\frac{x}{b^2} + \frac{(a^2-1)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^3} - \frac{a\log(b^2x^2+2abx+a^2+1)}{b^3}\right)$$

input
```
integrate(x*arccot(b*x+a),x, algorithm="maxima")
```

output
```
1/2*x^2*arccot(b*x + a) + 1/2*b*(x/b^2 + (a^2 - 1)*arctan((b^2*x + a*b)/b)
/b^3 - a*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(50) = 100$.

Time = 0.20 (sec) , antiderivative size = 210, normalized size of antiderivative = 3.50

$$\int x \cot^{-1}(a+bx)\,dx$$
$$= \frac{4\,a \arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^3 + \arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4 + 4\,a\log\left(\frac{16}{\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx}\right.\right.}\right.}{}$$

input
```
integrate(x*arccot(b*x+a),x, algorithm="giac")
```

output
```
1/8*(4*a*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^3 + arctan(1/(b*
x + a))*tan(1/2*arctan(1/(b*x + a)))^4 + 4*a*log(16*tan(1/2*arctan(1/(b*x
+ a)))^2/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^
2 + 1))*tan(1/2*arctan(1/(b*x + a)))^2 - 4*a*arctan(1/(b*x + a))*tan(1/2*a
rctan(1/(b*x + a))) + 2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2
 - 2*tan(1/2*arctan(1/(b*x + a)))^3 + arctan(1/(b*x + a)) + 2*tan(1/2*arct
an(1/(b*x + a))))/(b^2*tan(1/2*arctan(1/(b*x + a)))^2)
```

Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int x \cot^{-1}(a+bx)\,dx = \frac{x^2\,\mathrm{acot}(a+b\,x)}{2} + \frac{\frac{\mathrm{acot}(a+b\,x)}{2} + \frac{b\,x}{2} - \frac{a^2\,\mathrm{acot}(a+b\,x)}{2} - \frac{a\,\ln(a^2+2\,a\,b\,x+b^2\,x^2+1)}{2}}{b^2}$$

input
```
int(x*acot(a + b*x),x)
```

output
```
(x^2*acot(a + b*x))/2 + (acot(a + b*x)/2 + (b*x)/2 - (a^2*acot(a + b*x))/2
 - (a*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/2)/b^2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int x\cot^{-1}(a+bx)\,dx$$
$$= \frac{-acot(bx+a)\,a^2 + acot(bx+a)\,b^2x^2 + acot(bx+a) - \log(b^2x^2+2abx+a^2+1)\,a + bx}{2b^2}$$

input
```
int(x*acot(b*x+a),x)
```

output
```
( - acot(a + b*x)*a**2 + acot(a + b*x)*b**2*x**2 + acot(a + b*x) - log(a**
2 + 2*a*b*x + b**2*x**2 + 1)*a + b*x)/(2*b**2)
```

3.4 $\int \cot^{-1}(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 33

$$\int \cot^{-1}(a+bx)\,dx=\frac{(a+bx)\cot^{-1}(a+bx)}{b}+\frac{\log\left(1+(a+bx)^2\right)}{2b}$$

output
```
(b*x+a)*arccot(b*x+a)/b+1/2*ln(1+(b*x+a)^2)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \cot^{-1}(a+bx)\,dx=x\cot^{-1}(a+bx)+\frac{-2a\arctan(a+bx)+\log\left(1+a^2+2abx+b^2x^2\right)}{2b}$$

input
```
Integrate[ArcCot[a + b*x],x]
```

output
```
x*ArcCot[a + b*x] + (-2*a*ArcTan[a + b*x] + Log[1 + a^2 + 2*a*b*x + b^2*x^
2])/(2*b)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5563, 5346, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(a+bx)\,dx$$

$$\Big\downarrow \text{ 5563}$$

$$\frac{\int \cot^{-1}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{ 5346}$$

$$\frac{\int \frac{a+bx}{(a+bx)^2+1}d(a+bx)+(a+bx)\cot^{-1}(a+bx)}{b}$$

$$\Big\downarrow \text{ 240}$$

$$\frac{\frac{1}{2}\log\left((a+bx)^2+1\right)+(a+bx)\cot^{-1}(a+bx)}{b}$$

input
```
Int[ArcCot[a + b*x],x]
```

output
```
((a + b*x)*ArcCot[a + b*x] + Log[1 + (a + b*x)^2]/2)/b
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5563
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
 x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{(bx+a)\,\mathrm{arccot}(bx+a)+\frac{\ln\left(1+(bx+a)^2\right)}{2}}{b}$	30
default	$\frac{(bx+a)\,\mathrm{arccot}(bx+a)+\frac{\ln\left(1+(bx+a)^2\right)}{2}}{b}$	30
parallelrisch	$\frac{2x\ \mathrm{arccot}(bx+a)b^2+2\ \mathrm{arccot}(bx+a)ab+\ln(b^2x^2+2abx+a^2+1)b}{2b^2}$	49
parts	$x\ \mathrm{arccot}\left(bx+a\right)+b\left(\frac{\ln(b^2x^2+2abx+a^2+1)}{2b^2}-\frac{a\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b^2}\right)$	59
risch	$\frac{ix\ln(1+i(bx+a))}{2}-\frac{ix\ln(1-i(bx+a))}{2}+\frac{\pi x}{2}-\frac{a\arctan(bx+a)}{b}+\frac{\ln(b^2x^2+2abx+a^2+1)}{2b}$	71

input
```
int(arccot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*((b*x+a)*arccot(b*x+a)+1/2*ln(1+(b*x+a)^2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30

$$\int \cot^{-1}(a+bx)\,dx$$
$$=\frac{2\,bx\,\mathrm{arccot}\left(bx+a\right)-2\,a\arctan\left(bx+a\right)+\log\left(b^2x^2+2\,abx+a^2+1\right)}{2\,b}$$

input
```
integrate(arccot(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*b*x*arccot(b*x + a) - 2*a*arctan(b*x + a) + log(b^2*x^2 + 2*a*b*x +
 a^2 + 1))/b
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \cot^{-1}(a+bx)\,dx = \begin{cases} \frac{a\,\mathrm{acot}\,(a+bx)}{b} + x\,\mathrm{acot}\,(a+bx) + \frac{\log\left(a^2+2abx+b^2x^2+1\right)}{2b} & \text{for } b \neq 0 \\ x\,\mathrm{acot}\,(a) & \text{otherwise} \end{cases}$$

input
```
integrate(acot(b*x+a),x)
```

output
```
Piecewise((a*acot(a + b*x)/b + x*acot(a + b*x) + log(a**2 + 2*a*b*x + b**2
*x**2 + 1)/(2*b), Ne(b, 0)), (x*acot(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \cot^{-1}(a+bx)\,dx = \frac{2\,(bx+a)\,\mathrm{arccot}\,(bx+a) + \log\left((bx+a)^2+1\right)}{2\,b}$$

input
```
integrate(arccot(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(2*(b*x + a)*arccot(b*x + a) + log((b*x + a)^2 + 1))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(31) = 62$.

Time = 0.16 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.36

$$\int \cot^{-1}(a+bx)\,dx =$$

$$-\frac{\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2+\log\left(\frac{16\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2}{\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4+2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2+1}\right)\tan\left(\frac{1}{2}\,\text{arcta}\right.}{2\,b\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)}$$

input
```
integrate(arccot(b*x+a),x, algorithm="giac")
```

output
```
-1/2*(arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 + log(16*tan(1/2*
arctan(1/(b*x + a)))^2/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(
1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a))) - arctan(1/(b*x + a)))/
(b*tan(1/2*arctan(1/(b*x + a))))
```

Mupad [B] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \cot^{-1}(a+bx)\,dx = \frac{\frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2}+a\,\mathrm{acot}(a+b\,x)}{b}+x\,\mathrm{acot}(a+b\,x)$$

input
```
int(acot(a + b*x),x)
```

output
```
(log(a^2 + b^2*x^2 + 2*a*b*x + 1)/2 + a*acot(a + b*x))/b + x*acot(a + b*x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30

$$\int \cot^{-1}(a+bx)\,dx = \frac{2acot(bx+a)\,a + 2acot(bx+a)\,bx + \log(b^2x^2 + 2abx + a^2 + 1)}{2b}$$

input
```
int(acot(b*x+a),x)
```

output
```
(2*acot(a + b*x)*a + 2*acot(a + b*x)*b*x + log(a**2 + 2*a*b*x + b**2*x**2
+ 1))/(2*b)
```

3.5 $\int \frac{\cot^{-1}(a+bx)}{x}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 120

$$
\begin{aligned}
\int \frac{\cot^{-1}(a+bx)}{x}\,dx = & -\cot^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right) \\
& +\cot^{-1}(a+bx)\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right) \\
& -\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right) \\
& +\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2bx}{(i-a)(1-i(a+bx))}\right)
\end{aligned}
$$

output

```
-arccot(b*x+a)*ln(2/(1-I*(b*x+a)))+arccot(b*x+a)*ln(2*b*x/(I-a)/(1-I*(b*x+
a)))-1/2*I*polylog(2,1-2/(1-I*(b*x+a)))+1/2*I*polylog(2,1-2*b*x/(I-a)/(1-I
*(b*x+a)))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 256 vs. $2(120) = 240$.

Time = 0.14 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.13

$$\int \frac{\cot^{-1}(a+bx)}{x}\,dx = \left(\cot^{-1}(a+bx)+\arctan(a+bx)\right)\log(x) + \arctan(a+bx)\left(\log\left(\frac{1}{\sqrt{1+(a+bx)^2}}\right) - \log(-\sin(\arctan(a)-\arctan(a+bx)))\right) + \frac{1}{2}\left(\frac{1}{4}i(\pi-2\arctan(a+bx))^2 + i(\arctan(a)-\arctan(a+bx))^2 - (\pi-2\arctan(a+bx))\log\left(1+e^{-2i\arctan(a+bx)}\right) + 2(\arctan(a)-\arctan(a+bx))\log\left(1-e^{2i(-\arctan(a)+\arctan(a+bx))}\right) + (\pi-2\arctan(a+bx))\log\left(\frac{2}{\sqrt{1+(a+bx)^2}}\right) + 2(-\arctan(a)+\arctan(a+bx))\log(-2\sin(\arctan(a)-\arctan(a+bx))) + i\,\mathrm{PolyLog}\left(2,-e^{-2i\arctan(a+bx)}\right) + i\,\mathrm{PolyLog}\left(2,e^{2i(-\arctan(a)+\arctan(a+bx))}\right)\right)$$

input
```
Integrate[ArcCot[a + b*x]/x,x]
```

output
```
(ArcCot[a + b*x] + ArcTan[a + b*x])*Log[x] + ArcTan[a + b*x]*(Log[1/Sqrt[1
 + (a + b*x)^2]] - Log[-Sin[ArcTan[a] - ArcTan[a + b*x]]]) + ((I/4)*(Pi -
2*ArcTan[a + b*x])^2 + I*(ArcTan[a] - ArcTan[a + b*x])^2 - (Pi - 2*ArcTan[
a + b*x])*Log[1 + E^((-2*I)*ArcTan[a + b*x])] + 2*(ArcTan[a] - ArcTan[a +
b*x])*Log[1 - E^((2*I)*(-ArcTan[a] + ArcTan[a + b*x]))] + (Pi - 2*ArcTan[a
 + b*x])*Log[2/Sqrt[1 + (a + b*x)^2]] + 2*(-ArcTan[a] + ArcTan[a + b*x])*L
og[-2*Sin[ArcTan[a] - ArcTan[a + b*x]]] + I*PolyLog[2, -E^((-2*I)*ArcTan[a
 + b*x])] + I*PolyLog[2, E^((2*I)*(-ArcTan[a] + ArcTan[a + b*x]))])/2
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5571, 25, 27, 5382, 2849, 2752, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{x} \, dx$$

$$\Big\downarrow \text{5571}$$

$$\frac{\int \frac{\cot^{-1}(a+bx)}{x}d(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{\cot^{-1}(a+bx)}{x}d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\int -\frac{\cot^{-1}(a+bx)}{bx}d(a+bx)$$

$$\Big\downarrow \text{5382}$$

$$-\int \frac{\log\left(\frac{2}{1-i(a+bx)}\right)}{(a+bx)^2+1}d(a+bx)+\int \frac{\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right)}{(a+bx)^2+1}d(a+bx)+\log\left(\frac{2}{1-i(a+bx)}\right)\left(-\cot^{-1}(a+bx)\right)+\log\left(\frac{2bx}{(-a+i)(1-i(a+bx))}\right)\cot^{-1}(a+bx)$$

$$\Big\downarrow \text{2849}$$

$$-i\int \frac{\log\left(\frac{2}{1-i(a+bx)}\right)}{1-\frac{2}{1-i(a+bx)}}d\frac{1}{1-i(a+bx)}+\int \frac{\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right)}{(a+bx)^2+1}d(a+bx)+\log\left(\frac{2}{1-i(a+bx)}\right)\left(-\cot^{-1}(a+bx)\right)+\log\left(\frac{2bx}{(-a+i)(1-i(a+bx))}\right)\cot^{-1}(a+bx)$$

$$\Big\downarrow \text{2752}$$

$$\int \frac{\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right)}{(a+bx)^2+1}d(a+bx)-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)+\log\left(\frac{2}{1-i(a+bx)}\right)\left(-\cot^{-1}(a+bx)\right)+\log\left(\frac{2bx}{(-a+i)(1-i(a+bx))}\right)\cot^{-1}(a+bx)$$

$$\Big\downarrow \text{2897}$$

$$-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2bx}{(i-a)(1-i(a+bx))}\right)+\log\left(\frac{2}{1-i(a+bx)}\right)\left(-\cot^{-1}(a+bx)\right)+\log\left(\frac{2bx}{(-a+i)(1-i(a+bx))}\right)\cot^{-1}(a+bx)$$

input
```
Int[ArcCot[a + b*x]/x,x]
```

output
```
-(ArcCot[a + b*x]*Log[2/(1 - I*(a + b*x))]) + ArcCot[a + b*x]*Log[(2*b*x)/
((I - a)*(1 - I*(a + b*x)))] - (I/2)*PolyLog[2, 1 - 2/(1 - I*(a + b*x))] +
 (I/2)*PolyLog[2, 1 - (2*b*x)/((I - a)*(1 - I*(a + b*x)))]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5382
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcCot[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*ArcCot[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] - Simp[b*(c/e)
   Int[Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] + Simp[b*(c/e)   Int[Log[2*
c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x]) /; FreeQ[{a
, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.86

method	result
risch	$\frac{\pi \ln(-bxi)}{2}-\frac{i\ln(-bxi-ai+1)\ln\left(-\frac{ixb}{ai-1}\right)}{2}-\frac{i\,\mathrm{dilog}\left(-\frac{ixb}{ai-1}\right)}{2}+\frac{i\ln(bxi+ai+1)\ln\left(\frac{ixb}{-ai-1}\right)}{2}+\frac{i\,\mathrm{dilog}\left(\frac{ixb}{-ai-1}\right)}{2}$
parts	$\ln(x)\,\mathrm{arccot}(bx+a)+b\left(-\frac{i\ln(x)\left(\ln\left(\frac{-bx-a+i}{i-a}\right)-\ln\left(\frac{bx+a+i}{i+a}\right)\right)}{2b}-\frac{i\left(\mathrm{dilog}\left(\frac{-bx-a+i}{i-a}\right)-\mathrm{dilog}\left(\frac{bx+a+i}{i+a}\right.\right.}{2b}\right.$
derivativedivides	$\ln(-bx)\,\mathrm{arccot}(bx+a)+\frac{i\ln(-bx)\ln\left(\frac{bx+a+i}{i+a}\right)}{2}-\frac{i\ln(-bx)\ln\left(\frac{-bx-a+i}{i-a}\right)}{2}+\frac{i\,\mathrm{dilog}\left(\frac{bx+a+i}{i+a}\right)}{2}-\frac{i\,\mathrm{dil}}{}$
default	$\ln(-bx)\,\mathrm{arccot}(bx+a)+\frac{i\ln(-bx)\ln\left(\frac{bx+a+i}{i+a}\right)}{2}-\frac{i\ln(-bx)\ln\left(\frac{-bx-a+i}{i-a}\right)}{2}+\frac{i\,\mathrm{dilog}\left(\frac{bx+a+i}{i+a}\right)}{2}-\frac{i\,\mathrm{dil}}{}$

input
```
int(arccot(b*x+a)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*Pi*ln(-I*b*x)-1/2*I*ln(1-I*a-I*b*x)*ln(-I*x*b/(I*a-1))-1/2*I*dilog(-I*
x*b/(I*a-1))+1/2*I*ln(1+I*a+I*b*x)*ln(I*x*b/(-I*a-1))+1/2*I*dilog(I*x*b/(-
I*a-1))
```

Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{x}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{x}\,dx$$

input
```
integrate(arccot(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(arccot(b*x + a)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{x}\,dx = \text{Timed out}$$

input
```
integrate(acot(b*x+a)/x,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.11

$$\int \frac{\cot^{-1}(a+bx)}{x}\,dx = \frac{1}{2}\arctan\left(\frac{bx}{a^2+1}, -\frac{abx}{a^2+1}\right)\log\left(b^2x^2+2\,abx+a^2+1\right) - \frac{1}{2}\arctan\left(bx+a\right)\log\left(\frac{b^2x^2}{a^2+1}\right) + \operatorname{arccot}\left(bx+a\right)\log\left(x\right) + \arctan\left(\frac{b^2x+ab}{b}\right)\log\left(x\right) + \frac{1}{2}i\,\mathrm{Li}_2\left(\frac{i\,bx+i\,a+1}{i\,a+1}\right) - \frac{1}{2}i\,\mathrm{Li}_2\left(\frac{i\,bx+i\,a-1}{i\,a-1}\right)$$

input
```
integrate(arccot(b*x+a)/x,x, algorithm="maxima")
```

output
```
1/2*arctan2(b*x/(a^2 + 1), -a*b*x/(a^2 + 1))*log(b^2*x^2 + 2*a*b*x + a^2 +
 1) - 1/2*arctan(b*x + a)*log(b^2*x^2/(a^2 + 1)) + arccot(b*x + a)*log(x)
+ arctan((b^2*x + a*b)/b)*log(x) + 1/2*I*dilog((I*b*x + I*a + 1)/(I*a + 1)
) - 1/2*I*dilog((I*b*x + I*a - 1)/(I*a - 1))
```

Giac [F]

$$\int \frac{\cot^{-1}(a+bx)}{x}\,dx = \int \frac{\operatorname{arccot}\left(bx+a\right)}{x}\,dx$$

input
```
integrate(arccot(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(arccot(b*x + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{x}\,dx = \int \frac{\operatorname{acot}(a+b\,x)}{x}\,dx$$

input
```
int(acot(a + b*x)/x,x)
```

output
```
int(acot(a + b*x)/x, x)
```

Reduce [F]

$$\int \frac{\cot^{-1}(a+bx)}{x}\,dx = \int \frac{acot(bx+a)}{x}dx$$

input
```
int(acot(b*x+a)/x,x)
```

output
```
int(acot(a + b*x)/x,x)
```

3.6 $\int \frac{\cot^{-1}(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 62

$$\int \frac{\cot^{-1}(a+bx)}{x^2}\,dx = -\frac{\cot^{-1}(a+bx)}{x}+\frac{ab\arctan(a+bx)}{1+a^2}-\frac{b\log(x)}{1+a^2}+\frac{b\log\left(1+(a+bx)^2\right)}{2\left(1+a^2\right)}$$

output
```
-arccot(b*x+a)/x+a*b*arctan(b*x+a)/(a^2+1)-b*ln(x)/(a^2+1)+b*ln(1+(b*x+a)^
2)/(2*a^2+2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06

$$\int \frac{\cot^{-1}(a+bx)}{x^2}\,dx = -\frac{\cot^{-1}(a+bx)}{x}+\frac{b(-2\log(x)+(1-ia)\log(i-a-bx)+(1+ia)\log(i+a+bx))}{2\left(1+a^2\right)}$$

input
```
Integrate[ArcCot[a + b*x]/x^2,x]
```

output
```
-(ArcCot[a + b*x]/x) + (b*(-2*Log[x] + (1 - I*a)*Log[I - a - b*x] + (1 + I
*a)*Log[I + a + b*x]))/(2*(1 + a^2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5569, 896, 25, 479, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{x^2}dx$$

$$\Big\downarrow \text{ 5569}$$

$$-b\int \frac{1}{x\left((a+bx)^2+1\right)}dx-\frac{\cot^{-1}(a+bx)}{x}$$

$$\Big\downarrow \text{ 896}$$

$$-b\int \frac{1}{bx\left((a+bx)^2+1\right)}d(a+bx)-\frac{\cot^{-1}(a+bx)}{x}$$

$$\Big\downarrow \text{ 25}$$

$$b\int -\frac{1}{bx\left((a+bx)^2+1\right)}d(a+bx)-\frac{\cot^{-1}(a+bx)}{x}$$

$$\Big\downarrow \text{ 479}$$

$$-b\left(\frac{\log(-bx)}{a^2+1}-\frac{\int \frac{2a+bx}{(a+bx)^2+1}d(a+bx)}{a^2+1}\right)-\frac{\cot^{-1}(a+bx)}{x}$$

$$\Big\downarrow \text{ 452}$$

$$-b\left(\frac{\log(-bx)}{a^2+1}-\frac{a\int \frac{1}{(a+bx)^2+1}d(a+bx)+\int \frac{a+bx}{(a+bx)^2+1}d(a+bx)}{a^2+1}\right)-\frac{\cot^{-1}(a+bx)}{x}$$

$$\Big\downarrow \text{ 216}$$

$$-b\left(\frac{\log(-bx)}{a^2+1}-\frac{\int \frac{a+bx}{(a+bx)^2+1}d(a+bx)+a\arctan(a+bx)}{a^2+1}\right)-\frac{\cot^{-1}(a+bx)}{x}$$

$$\downarrow \text{240}$$

$$-b\left(\frac{\log(-bx)}{a^2+1}-\frac{a\arctan(a+bx)+\frac{1}{2}\log\left((a+bx)^2+1\right)}{a^2+1}\right)-\frac{\cot^{-1}(a+bx)}{x}$$

input
```
Int[ArcCot[a + b*x]/x^2,x]
```

output
```
-(ArcCot[a + b*x]/x) - b*(Log[-(b*x)]/(1 + a^2) - (a*ArcTan[a + b*x] + Log
[1 + (a + b*x)^2]/2)/(1 + a^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 479
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[d*(Log
[RemoveContent[c + d*x, x]]/(b*c^2 + a*d^2)), x] + Simp[b/(b*c^2 + a*d^2)
 Int[(c - d*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 5569
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCot[c + d*x])^p/(f*(m +
1))), x] + Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcCot[
c + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[p, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

method	result
derivativedivides	$b\left(-\frac{\operatorname{arccot}(bx+a)}{bx}-\frac{\ln(-bx)}{a^2+1}+\frac{\frac{\ln\left(1+(bx+a)^2\right)}{2}+a\arctan(bx+a)}{a^2+1}\right)$
default	$b\left(-\frac{\operatorname{arccot}(bx+a)}{bx}-\frac{\ln(-bx)}{a^2+1}+\frac{\frac{\ln\left(1+(bx+a)^2\right)}{2}+a\arctan(bx+a)}{a^2+1}\right)$
parts	$-\frac{\operatorname{arccot}(bx+a)}{x}-b\left(-\frac{b\left(\frac{\ln\left(b^2x^2+2abx+a^2+1\right)}{2b}+\frac{a\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b}\right)}{a^2+1}+\frac{\ln(x)}{a^2+1}\right)$
parallelrisch	$-\frac{2x\operatorname{arccot}(bx+a)a^2b^2+2b^2\ln(x)ax-b^2\ln\left(b^2x^2+2abx+a^2+1\right)ax+2\operatorname{arccot}(bx+a)a^3b+2\operatorname{arccot}(bx+a)ab}{2xab(a^2+1)}$
risch	$-\frac{i\ln(1+i(bx+a))}{2x}-\frac{-ia^2\ln(1-i(bx+a))-i\ln(1-i(bx+a))+\pi a^2+\pi+2b\ln(x)x-xb\ln\left((iab+3b)x+ia^2+3i+2a\right)-i\cdots}{2x(i+\cdots}$

input
```
int(arccot(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
b*(-1/b/x*arccot(b*x+a)-1/(a^2+1)*ln(-b*x)+1/(a^2+1)*(1/2*ln(1+(b*x+a)^2)+
a*arctan(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

$$\int \frac{\cot^{-1}(a+bx)}{x^2}\,dx = \frac{2\,abx\arctan\left(bx+a\right)+bx\log\left(b^2x^2+2\,abx+a^2+1\right)-2\,bx\log\left(x\right)-2\left(a^2+1\right)\operatorname{arccot}\left(bx+a\right)}{2\left(a^2+1\right)x}$$

input
```
integrate(arccot(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
1/2*(2*a*b*x*arctan(b*x + a) + b*x*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*b*
x*log(x) - 2*(a^2 + 1)*arccot(b*x + a))/((a^2 + 1)*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.69

$$\int \frac{\cot^{-1}(a+bx)}{x^2}\,dx = \begin{cases} -\frac{ib\operatorname{acot}\left(bx-i\right)}{2}-\frac{\operatorname{acot}\left(bx-i\right)}{x}+\frac{i}{2x} & \text{for}\: a=-i \\ \frac{ib\operatorname{acot}\left(bx+i\right)}{2}-\frac{\operatorname{acot}\left(bx+i\right)}{x}-\frac{i}{2x} & \text{for}\: a=i \\ -\frac{2a^2\operatorname{acot}\left(a+bx\right)}{2a^2x+2x}-\frac{2abx\operatorname{acot}\left(a+bx\right)}{2a^2x+2x}-\frac{2bx\log\left(x\right)}{2a^2x+2x}+\frac{bx\log\left(a^2+2abx+b^2x^2+1\right)}{2a^2x+2x}-\frac{2\operatorname{acot}\left(a+bx\right)}{2a^2x+2x} & \text{otherwise} \end{cases}$$

input
```
integrate(acot(b*x+a)/x**2,x)
```

output
```
Piecewise(((-I*b*acot(b*x - I)/2 - acot(b*x - I)/x + I/(2*x), Eq(a, -I)), (
I*b*acot(b*x + I)/2 - acot(b*x + I)/x - I/(2*x), Eq(a, I)), (-2*a**2*acot(
a + b*x)/(2*a**2*x + 2*x) - 2*a*b*x*acot(a + b*x)/(2*a**2*x + 2*x) - 2*b*x
*log(x)/(2*a**2*x + 2*x) + b*x*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(2*a**2
*x + 2*x) - 2*acot(a + b*x)/(2*a**2*x + 2*x), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.24

$$\int \frac{\cot^{-1}(a+bx)}{x^2}\,dx = \frac{1}{2}\,b\left(\frac{2\,a\arctan\left(\frac{b^2x+ab}{b}\right)}{a^2+1}+\frac{\log\left(b^2x^2+2\,abx+a^2+1\right)}{a^2+1}-\frac{2\,\log\left(x\right)}{a^2+1}\right) - \frac{\operatorname{arccot}\left(bx+a\right)}{x}$$

input
```
integrate(arccot(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
1/2*b*(2*a*arctan((b^2*x + a*b)/b)/(a^2 + 1) + log(b^2*x^2 + 2*a*b*x + a^2
 + 1)/(a^2 + 1) - 2*log(x)/(a^2 + 1)) - arccot(b*x + a)/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. 2(60) = 120.

Time = 0.27 (sec) , antiderivative size = 498, normalized size of antiderivative = 8.03

$$\int \frac{\cot^{-1}(a+bx)}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate(arccot(b*x+a)/x^2,x, algorithm="giac")
```

output
```
-1/2*(2*a*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 + 2*a*log(4*(
4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3
+ tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*ta
n(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(
1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a))) + log(4*(4*a
^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + t
an(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1
/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2
*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^2 - 2*a*arctan(
1/(b*x + a)) - 4*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a))) - log(4*
(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3
 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*t
an(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan
(1/2*arctan(1/(b*x + a)))^2 + 1)))*b/(2*a^3*tan(1/2*arctan(1/(b*x + a))) +
 a^2*tan(1/2*arctan(1/(b*x + a)))^2 - a^2 + 2*a*tan(1/2*arctan(1/(b*x + a)
)) + tan(1/2*arctan(1/(b*x + a)))^2 - 1)
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(a+bx)}{x^2}\,dx = -\frac{\operatorname{acot}(a+b\,x)}{x} - \frac{b\,x\,\ln(x) - \frac{b\,x\,\ln(a^2+2\,a\,b\,x+b^2\,x^2+1)}{2} + a\,b\,x\,\operatorname{acot}(a+b\,x)}{x\,(a^2+1)}$$

input
```
int(acot(a + b*x)/x^2,x)
```

output
```
- acot(a + b*x)/x - (b*x*log(x) - (b*x*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/2
 + a*b*x*acot(a + b*x))/(x*(a^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(a+bx)}{x^2}\,dx = \frac{-2acot(bx+a)\,a^2-2acot(bx+a)\,abx-2acot(bx+a)+\log(b^2x^2+2abx+a^2+1)\,bx-2\log(x)\,bx}{2x\,(a^2+1)}$$

input
```
int(acot(b*x+a)/x^2,x)
```

output
```
( - 2*acot(a + b*x)*a**2 - 2*acot(a + b*x)*a*b*x - 2*acot(a + b*x) + log(a
**2 + 2*a*b*x + b**2*x**2 + 1)*b*x - 2*log(x)*b*x)/(2*x*(a**2 + 1))
```

3.7 $\int \frac{\cot^{-1}(a+bx)}{x^3} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 95

$$\int \frac{\cot^{-1}(a+bx)}{x^3} \, dx = \frac{b}{2(1+a^2)x} - \frac{\cot^{-1}(a+bx)}{2x^2} + \frac{(1-a^2)b^2 \arctan(a+bx)}{2(1+a^2)^2} + \frac{ab^2 \log(x)}{(1+a^2)^2} - \frac{ab^2 \log(1+(a+bx)^2)}{2(1+a^2)^2}$$

output
```
1/2*b/(a^2+1)/x-1/2*arccot(b*x+a)/x^2+1/2*(-a^2+1)*b^2*arctan(b*x+a)/(a^2+
1)^2+a*b^2*ln(x)/(a^2+1)^2-1/2*a*b^2*ln(1+(b*x+a)^2)/(a^2+1)^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int \frac{\cot^{-1}(a+bx)}{x^3} \, dx = \frac{-2\cot^{-1}(a+bx) + \frac{bx(4abx\log(x)+i(i+a)^2bx\log(i-a-bx)+(-i+a)(2(i+a)+(-1-ia)bx\log(i+a+bx)))}{(1+a^2)^2}}{4x^2}$$

input
```
Integrate[ArcCot[a + b*x]/x^3,x]
```

output
```
(-2*ArcCot[a + b*x] + (b*x*(4*a*b*x*Log[x] + I*(I + a)^2*b*x*Log[I - a - b
*x] + (-I + a)*(2*(I + a) + (-1 - I*a)*b*x*Log[I + a + b*x])))/(1 + a^2)^2
)/(4*x^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5569, 896, 480, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{x^3} \, dx$$

$$\Big\downarrow \text{5569}$$

$$-\frac{1}{2} b \int \frac{1}{x^2 \left((a+bx)^2+1\right)} dx - \frac{\cot^{-1}(a+bx)}{2x^2}$$

$$\Big\downarrow \text{896}$$

$$-\frac{1}{2} b^2 \int \frac{1}{b^2 x^2 \left((a+bx)^2+1\right)} d(a+bx) - \frac{\cot^{-1}(a+bx)}{2x^2}$$

$$\Big\downarrow \text{480}$$

$$-\frac{1}{2} b^2 \left(\frac{\int -\frac{2a+bx}{bx\left((a+bx)^2+1\right)} d(a+bx)}{a^2+1} - \frac{1}{\left(a^2+1\right) bx}\right) - \frac{\cot^{-1}(a+bx)}{2x^2}$$

$$\Big\downarrow \text{657}$$

$$-\frac{1}{2} b^2 \left(\frac{\int \left(\frac{a^2+2(a+bx)a-1}{\left(a^2+1\right)\left((a+bx)^2+1\right)} - \frac{2a}{\left(a^2+1\right) bx}\right) d(a+bx)}{a^2+1} - \frac{1}{\left(a^2+1\right) bx}\right) - \frac{\cot^{-1}(a+bx)}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2} b^2 \left(\frac{-\frac{\left(1-a^2\right) \arctan(a+bx)}{a^2+1} - \frac{2a \log(-bx)}{a^2+1} + \frac{a \log\left((a+bx)^2+1\right)}{a^2+1}}{a^2+1} - \frac{1}{\left(a^2+1\right) bx}\right) - \frac{\cot^{-1}(a+bx)}{2x^2}$$

input
```
Int[ArcCot[a + b*x]/x^3,x]
```

output
```
-1/2*ArcCot[a + b*x]/x^2 - (b^2*(-(1/((1 + a^2)*b*x)) + (-(((1 - a^2)*ArcT
an[a + b*x])/(1 + a^2)) - (2*a*Log[-(b*x)])/(1 + a^2) + (a*Log[1 + (a + b*
x)^2])/(1 + a^2))/(1 + a^2)))/2
```

Defintions of rubi rules used

rule 480
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + Simp[b/(b*c^2 + a*d^2)   I
nt[(c + d*x)^(n + 1)*((c - d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[n, -1]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5569
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCot[c + d*x])^p/(f*(m +
1))), x] + Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcCot[
c + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[p, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.87

method	result
derivativedivides	$b^2\left(-\frac{\operatorname{arccot}(bx+a)}{2b^2x^2}+\frac{1}{2(a^2+1)bx}+\frac{a\ln(-bx)}{(a^2+1)^2}-\frac{a\ln\left(1+(bx+a)^2\right)+(a^2-1)\arctan(bx+a)}{2(a^2+1)^2}\right)$
default	$b^2\left(-\frac{\operatorname{arccot}(bx+a)}{2b^2x^2}+\frac{1}{2(a^2+1)bx}+\frac{a\ln(-bx)}{(a^2+1)^2}-\frac{a\ln\left(1+(bx+a)^2\right)+(a^2-1)\arctan(bx+a)}{2(a^2+1)^2}\right)$
parts	$-\frac{\operatorname{arccot}(bx+a)}{2x^2}-\frac{b\left(\frac{b^2\left(\frac{a\ln\left(b^2x^2+2abx+a^2+1\right)}{b}+\frac{\left(a^2-1\right)\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b}\right)}{\left(a^2+1\right)^2}-\frac{1}{\left(a^2+1\right)x}-\frac{2ab\ln(x)}{\left(a^2+1\right)^2}\right)}{2}$
parallelrisch	$\frac{x^2\operatorname{arccot}(bx+a)a^2b^2+2a\,b^2\ln(x)x^2-a\,b^2\ln(b^2x^2+2abx+a^2+1)x^2-\operatorname{arccot}(bx+a)b^2x^2-2a\,b^2x^2-\operatorname{arccot}(bx+a)a^4+a}{2x^2(a^4+2a^2+1)}$
risch	$-\frac{i\ln(1+i(bx+a))}{4x^2}+\frac{ia^4\ln(1-i(bx+a))+2ia^2\ln(1-i(bx+a))+i\ln(1-i(bx+a))+4b^2a\ln(-x)x^2-\pi a^4+2a^2bx-2\pi a}{}$

input
```
int(arccot(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
b^2*(-1/2/b^2/x^2*arccot(b*x+a)+1/2/(a^2+1)/b/x+1/(a^2+1)^2*a*ln(-b*x)-1/2
/(a^2+1)^2*(a*ln(1+(b*x+a)^2)+(a^2-1)*arctan(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04

$$\int\frac{\cot^{-1}(a+bx)}{x^3}\,dx = -\frac{(a^2-1)b^2x^2\arctan(bx+a)+ab^2x^2\log(b^2x^2+2\,abx+a^2+1)-2\,ab^2x^2\log(x)-(a^2+1)bx+(a^4}{2\,(a^4+2\,a^2+1)x^2}$$

input
```
integrate(arccot(b*x+a)/x^3,x, algorithm="fricas")
```

output
```
-1/2*((a^2 - 1)*b^2*x^2*arctan(b*x + a) + a*b^2*x^2*log(b^2*x^2 + 2*a*b*x
+ a^2 + 1) - 2*a*b^2*x^2*log(x) - (a^2 + 1)*b*x + (a^4 + 2*a^2 + 1)*arccot
(b*x + a))/((a^4 + 2*a^2 + 1)*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 381, normalized size of antiderivative = 4.01

$$\int \frac{\cot^{-1}(a+bx)}{x^3}\,dx$$
$$= \begin{cases} -\frac{b^2 \operatorname{acot}(bx-i)}{8} + \frac{b}{8x} - \frac{\operatorname{acot}(bx-i)}{2x^2} + \frac{i}{8x^2} \\ -\frac{b^2 \operatorname{acot}(bx+i)}{8} + \frac{b}{8x} - \frac{\operatorname{acot}(bx+i)}{2x^2} - \frac{i}{8x^2} \\ -\frac{a^4 \operatorname{acot}(a+bx)}{2a^4x^2+4a^2x^2+2x^2} + \frac{a^2b^2x^2 \operatorname{acot}(a+bx)}{2a^4x^2+4a^2x^2+2x^2} + \frac{a^2bx}{2a^4x^2+4a^2x^2+2x^2} - \frac{2a^2 \operatorname{acot}(a+bx)}{2a^4x^2+4a^2x^2+2x^2} + \frac{2ab^2x^2 \log(x)}{2a^4x^2+4a^2x^2+2x^2} - \frac{ab^2x^2 \log(a^2+2a}{2a^4x^2+4a^2} \end{cases}$$

input
```
integrate(acot(b*x+a)/x**3,x)
```

output
```
Piecewise((-b**2*acot(b*x - I)/8 + b/(8*x) - acot(b*x - I)/(2*x**2) + I/(8
*x**2), Eq(a, -I)), (-b**2*acot(b*x + I)/8 + b/(8*x) - acot(b*x + I)/(2*x*
*2) - I/(8*x**2), Eq(a, I)), (-a**4*acot(a + b*x)/(2*a**4*x**2 + 4*a**2*x*
*2 + 2*x**2) + a**2*b**2*x**2*acot(a + b*x)/(2*a**4*x**2 + 4*a**2*x**2 + 2
*x**2) + a**2*b*x/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) - 2*a**2*acot(a + b
*x)/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) + 2*a*b**2*x**2*log(x)/(2*a**4*x*
*2 + 4*a**2*x**2 + 2*x**2) - a*b**2*x**2*log(a**2 + 2*a*b*x + b**2*x**2 +
1)/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) - b**2*x**2*acot(a + b*x)/(2*a**4*
x**2 + 4*a**2*x**2 + 2*x**2) + b*x/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) -
acot(a + b*x)/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.18

$$\int \frac{\cot^{-1}(a+bx)}{x^3}\,dx = -\frac{1}{2}\left(\frac{(a^2-1)b\arctan\left(\frac{b^2x+ab}{b}\right)}{a^4+2a^2+1}+\frac{ab\log\left(b^2x^2+2abx+a^2+1\right)}{a^4+2a^2+1}-\frac{2ab\log(x)}{a^4+2a^2+1}-\frac{1}{(a^2+1)x}\right)b - \frac{\operatorname{arccot}(bx+a)}{2x^2}$$

input
```
integrate(arccot(b*x+a)/x^3,x, algorithm="maxima")
```

output
```
-1/2*((a^2 - 1)*b*arctan((b^2*x + a*b)/b)/(a^4 + 2*a^2 + 1) + a*b*log(b^2*
x^2 + 2*a*b*x + a^2 + 1)/(a^4 + 2*a^2 + 1) - 2*a*b*log(x)/(a^4 + 2*a^2 + 1
) - 1/((a^2 + 1)*x))*b - 1/2*arccot(b*x + a)/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1309 vs. 2(85) = 170.

Time = 0.44 (sec) , antiderivative size = 1309, normalized size of antiderivative = 13.78

$$\int \frac{\cot^{-1}(a+bx)}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate(arccot(b*x+a)/x^3,x, algorithm="giac")
```

output
```
1/2*(4*a^3*b*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^3 + a^2*b*ar
ctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 + 4*a^3*b*log(4*(4*a^2*ta
n(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/
2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*ar
ctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arct
an(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a^2*b*log(4*(4
*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 +
 tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan
(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1
/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^3 + a*b*log(4
*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^
3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*
tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*ta
n(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^4 - 4*a^3*
b*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a))) - 14*a^2*b*arctan(1/(b*
x + a))*tan(1/2*arctan(1/(b*x + a)))^2 + 2*a^2*b*tan(1/2*arctan(1/(b*x + a
)))^3 - 4*a*b*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^3 + a*b*tan
(1/2*arctan(1/(b*x + a)))^4 - b*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x
+ a)))^4 - 4*a^2*b*log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1
/2*arctan(1/(b*x + a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/...
```

Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.42

$$\int \frac{\cot^{-1}(a+bx)}{x^3}\,dx$$
$$= \frac{\mathrm{atan}\left(\frac{2\,x\,b^2+2\,a\,b}{2\,\sqrt{b^2\,(a^2+1)-a^2\,b^2}}\right)\,\left(b^3-a^2\,b^3\right)}{\sqrt{b^2}\,\left(2\,a^4+4\,a^2+2\right)} - \frac{a\,b^2\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2\,\left(a^2+1\right)^2} - \frac{\mathrm{acot}(a+b\,x)\,\left(\frac{a^2}{2}+\frac{1}{2}\right)-\frac{b\,x}{2}+\frac{b^2\,x^2\,\mathrm{acot}(a+b\,x)}{2}-\frac{x^3\,\left(b^3-3\,a^2\,b^3\right)}{2\,\left(a^4+2\,a^2+1\right)}+\frac{a\,b^4\,x^4}{\left(a^2+1\right)^2}+a\,b\,x\,\mathrm{acot}(a+b\,x)}{a^2\,x^2+2\,a\,b\,x^3+b^2\,x^4+x^2} + \frac{a\,b^2\,\ln\left(x\right)}{\left(a^2+1\right)^2}$$

input
```
int(acot(a + b*x)/x^3,x)
```

output
```
(atan((2*a*b + 2*b^2*x)/(2*(b^2*(a^2 + 1) - a^2*b^2)^(1/2)))*(b^3 - a^2*b^
3))/((b^2)^(1/2)*(4*a^2 + 2*a^4 + 2)) - (a*b^2*log(a^2 + b^2*x^2 + 2*a*b*x
 + 1))/(2*(a^2 + 1)^2) - (acot(a + b*x)*(a^2/2 + 1/2) - (b*x)/2 + (b^2*x^2
*acot(a + b*x))/2 - (x^3*(b^3 - 3*a^2*b^3))/(2*(2*a^2 + a^4 + 1)) + (a*b^4
*x^4)/(a^2 + 1)^2 + a*b*x*acot(a + b*x))/(x^2 + a^2*x^2 + b^2*x^4 + 2*a*b*
x^3) + (a*b^2*log(x))/(a^2 + 1)^2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.32

$$\int \frac{\cot^{-1}(a+bx)}{x^3}\,dx = \frac{-acot(bx+a)\,a^4 + acot(bx+a)\,a^2b^2x^2 - 2acot(bx+a)\,a^2 - acot(bx+a)\,b^2x^2 - acot(bx+a) - \log(b^2}{2x^2\,(a^4+2a^2+1)}$$

input
```
int(acot(b*x+a)/x^3,x)
```

output
```
( - acot(a + b*x)*a**4 + acot(a + b*x)*a**2*b**2*x**2 - 2*acot(a + b*x)*a*
*2 - acot(a + b*x)*b**2*x**2 - acot(a + b*x) - log(a**2 + 2*a*b*x + b**2*x
**2 + 1)*a*b**2*x**2 + 2*log(x)*a*b**2*x**2 + a**2*b*x + b*x)/(2*x**2*(a**
4 + 2*a**2 + 1))
```

3.8 $\int \frac{\cot^{-1}(a+bx)}{x^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 129

$$\int \frac{\cot^{-1}(a+bx)}{x^4}\,dx = \frac{b}{6(1+a^2)x^2} - \frac{2ab^2}{3(1+a^2)^2 x} - \frac{\cot^{-1}(a+bx)}{3x^3} - \frac{a(3-a^2)b^3\arctan(a+bx)}{3(1+a^2)^3} + \frac{(1-3a^2)b^3\log(x)}{3(1+a^2)^3} - \frac{(1-3a^2)b^3\log(1+(a+bx)^2)}{6(1+a^2)^3}$$

output
```
1/6*b/(a^2+1)/x^2-2/3*a*b^2/(a^2+1)^2/x-1/3*arccot(b*x+a)/x^3-1/3*a*(-a^2+
3)*b^3*arctan(b*x+a)/(a^2+1)^3+1/3*(-3*a^2+1)*b^3*ln(x)/(a^2+1)^3-1/6*(-3*
a^2+1)*b^3*ln(1+(b*x+a)^2)/(a^2+1)^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98

$$\int \frac{\cot^{-1}(a+bx)}{x^4}\,dx$$
$$= \frac{-2(1+a^2)^3\cot^{-1}(a+bx) + 2(1-3a^2)b^3x^3\log(x) + (-1+ia)^3b^3x^3\log(i-a-bx) + (-i+a)bx((i+}{6(1+a^2)^3x^3}$$

input
```
Integrate[ArcCot[a + b*x]/x^4,x]
```

output
```
(-2*(1 + a^2)^3*ArcCot[a + b*x] + 2*(1 - 3*a^2)*b^3*x^3*Log[x] + (-1 + I*a
)^3*b^3*x^3*Log[I - a - b*x] + (-I + a)*b*x*((I + a)*(1 + a^2 - 4*a*b*x) +
 I*(-I + a)^2*b^2*x^2*Log[I + a + b*x]))/(6*(1 + a^2)^3*x^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5569, 896, 25, 480, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{x^4}\,dx$$

$$\Big\downarrow \text{5569}$$

$$-\frac{1}{3}b\int \frac{1}{x^3\left((a+bx)^2+1\right)}dx-\frac{\cot^{-1}(a+bx)}{3x^3}$$

$$\Big\downarrow \text{896}$$

$$-\frac{1}{3}b^3\int \frac{1}{b^3x^3\left((a+bx)^2+1\right)}d(a+bx)-\frac{\cot^{-1}(a+bx)}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}b^3\int -\frac{1}{b^3x^3\left((a+bx)^2+1\right)}d(a+bx)-\frac{\cot^{-1}(a+bx)}{3x^3}$$

$$\Big\downarrow \text{480}$$

$$-\frac{1}{3}b^3\left(-\frac{\int \frac{2a+bx}{b^2x^2\left((a+bx)^2+1\right)}d(a+bx)}{a^2+1}-\frac{1}{2\left(a^2+1\right)b^2x^2}\right)-\frac{\cot^{-1}(a+bx)}{3x^3}$$

$$\Big\downarrow \text{657}$$

$$-\frac{1}{3}b^3\left(-\frac{\int\left(\frac{2a}{(a^2+1)b^2x^2}-\frac{3a^2-1}{(a^2+1)^2bx}+\frac{-a(3-a^2)-(1-3a^2)(a+bx)}{(a^2+1)^2((a+bx)^2+1)}\right)d(a+bx)}{a^2+1}-\frac{1}{2(a^2+1)b^2x^2}\right)-\frac{\cot^{-1}(a+bx)}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{3}b^3\left(-\frac{-\frac{(3-a^2)a\arctan(a+bx)}{(a^2+1)^2}-\frac{2a}{(a^2+1)bx}+\frac{(1-3a^2)\log(-bx)}{(a^2+1)^2}-\frac{(1-3a^2)\log((a+bx)^2+1)}{2(a^2+1)^2}}{a^2+1}-\frac{1}{2(a^2+1)b^2x^2}\right)-\frac{\cot^{-1}(a+bx)}{3x^3}$$

input
```
Int[ArcCot[a + b*x]/x^4,x]
```

output
```
-1/3*ArcCot[a + b*x]/x^3 - (b^3*(-1/2*1/((1 + a^2)*b^2*x^2) - ((-2*a)/((1
+ a^2)*b*x) - (a*(3 - a^2)*ArcTan[a + b*x])/(1 + a^2)^2 + ((1 - 3*a^2)*Log
[-(b*x)])/(1 + a^2)^2 - ((1 - 3*a^2)*Log[1 + (a + b*x)^2])/(2*(1 + a^2)^2)
)/(1 + a^2)))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 480
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + Simp[b/(b*c^2 + a*d^2)   I
nt[(c + d*x)^(n + 1)*((c - d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[n, -1]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5569
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCot[c + d*x])^p/(f*(m +
1))), x] + Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcCot[
c + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[p, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.89

method	result
derivativedivides	$b^3\left(-\frac{\operatorname{arccot}(bx+a)}{3b^3x^3}+\frac{\frac{(3a^2-1)\ln\left(1+(bx+a)^2\right)}{2}+(a^3-3a)\arctan(bx+a)}{3(a^2+1)^3}+\frac{(-3a^2+1)\ln(-bx)}{3(a^2+1)^3}+\frac{1}{6(a^2+1)b^2x^2}\right.$
default	$b^3\left(-\frac{\operatorname{arccot}(bx+a)}{3b^3x^3}+\frac{\frac{(3a^2-1)\ln\left(1+(bx+a)^2\right)}{2}+(a^3-3a)\arctan(bx+a)}{3(a^2+1)^3}+\frac{(-3a^2+1)\ln(-bx)}{3(a^2+1)^3}+\frac{1}{6(a^2+1)b^2x^2}\right.$
parts	$-\frac{\operatorname{arccot}(bx+a)}{3x^3}-\frac{b\left(-\frac{b^3\left(\frac{(3a^2b-b)\ln\left(b^2x^2+2abx+a^2+1\right)}{2b^2}+\frac{\left(4a^3-4a-\frac{(3a^2b-b)a}{b}\right)\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b}\right)}{(a^2+1)^3}-\frac{1}{2(a^2+1)x}\right.}{3}$
parallelrisch	$-\frac{2x^3\operatorname{arccot}(bx+a)a^3b^3+6\ln(x)x^3a^2b^3-3\ln(b^2x^2+2abx+a^2+1)x^3a^2b^3-6x^3\operatorname{arccot}(bx+a)ab^3-7x^3a^2b^3-2b^3\ln(x)}{}$
risch	$-\frac{i\ln(1+i(bx+a))}{6x^3}-\frac{ix^3\ln((-a^7b-5ia^6b-27a^5b+41ia^4b+29a^3b-15ia^2b-9ab+3ib)x-a^8-32a^6-4ia^7+70a^4+68i}{}$

input
```
int(arccot(b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

output
```
b^3*(-1/3/b^3/x^3*arccot(b*x+a)+1/3/(a^2+1)^3*(1/2*(3*a^2-1)*ln(1+(b*x+a)^
2)+(a^3-3*a)*arctan(b*x+a))+1/3*(-3*a^2+1)/(a^2+1)^3*ln(-b*x)+1/6/(a^2+1)/
b^2/x^2-2/3/(a^2+1)^2*a/b/x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.10

$$\int \frac{\cot^{-1}(a+bx)}{x^4}\,dx = \frac{2\,(a^3-3\,a)b^3x^3\arctan\left(bx+a\right)+(3\,a^2-1)b^3x^3\log\left(b^2x^2+2\,abx+a^2+1\right)-2\,(3\,a^2-1)b^3x^3\log\left(x\right)-}{6\,(a^6+3\,a^4+3\,a^2+1)x^3}$$

input
```
integrate(arccot(b*x+a)/x^4,x, algorithm="fricas")
```

output
```
1/6*(2*(a^3 - 3*a)*b^3*x^3*arctan(b*x + a) + (3*a^2 - 1)*b^3*x^3*log(b^2*x
^2 + 2*a*b*x + a^2 + 1) - 2*(3*a^2 - 1)*b^3*x^3*log(x) - 4*(a^3 + a)*b^2*x
^2 + (a^4 + 2*a^2 + 1)*b*x - 2*(a^6 + 3*a^4 + 3*a^2 + 1)*arccot(b*x + a))/
((a^6 + 3*a^4 + 3*a^2 + 1)*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 760, normalized size of antiderivative = 5.89

$$\int \frac{\cot^{-1}(a+bx)}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate(acot(b*x+a)/x**4,x)
```

output
```
Piecewise((I*b**3*acot(b*x - I)/24 - I*b**2/(24*x) + b/(24*x**2) - acot(b*
x - I)/(3*x**3) + I/(18*x**3), Eq(a, -I)), (-I*b**3*acot(b*x + I)/24 + I*b
**2/(24*x) + b/(24*x**2) - acot(b*x + I)/(3*x**3) - I/(18*x**3), Eq(a, I))
, (-2*a**6*acot(a + b*x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x*
*3) + a**4*b*x/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 6*a*
*4*acot(a + b*x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 2*
a**3*b**3*x**3*acot(a + b*x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 +
6*x**3) - 4*a**3*b**2*x**2/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*
x**3) - 6*a**2*b**3*x**3*log(x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3
 + 6*x**3) + 3*a**2*b**3*x**3*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(6*a**6*
x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) + 2*a**2*b*x/(6*a**6*x**3 + 1
8*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 6*a**2*acot(a + b*x)/(6*a**6*x**3 +
 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) + 6*a*b**3*x**3*acot(a + b*x)/(6*a*
*6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 4*a*b**2*x**2/(6*a**6*x*
*3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) + 2*b**3*x**3*log(x)/(6*a**6*x*
*3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - b**3*x**3*log(a**2 + 2*a*b*x
+ b**2*x**2 + 1)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) + b*
x/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 2*acot(a + b*x)/(
6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.28

$$\int \frac{\cot^{-1}(a+bx)}{x^4}\,dx$$

$$= \frac{1}{6}\left(\frac{2\,(a^3-3\,a)b^2\arctan\left(\frac{b^2x+ab}{b}\right)}{a^6+3\,a^4+3\,a^2+1}+\frac{(3\,a^2-1)b^2\log\left(b^2x^2+2\,abx+a^2+1\right)}{a^6+3\,a^4+3\,a^2+1}-\frac{2\,(3\,a^2-1)b^2\log\left(x\right)}{a^6+3\,a^4+3\,a^2+1}-\right.$$

$$-\frac{\operatorname{arccot}\left(bx+a\right)}{3\,x^3}$$

input
```
integrate(arccot(b*x+a)/x^4,x, algorithm="maxima")
```

output
```
1/6*(2*(a^3 - 3*a)*b^2*arctan((b^2*x + a*b)/b)/(a^6 + 3*a^4 + 3*a^2 + 1) +
 (3*a^2 - 1)*b^2*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/(a^6 + 3*a^4 + 3*a^2 + 1
) - 2*(3*a^2 - 1)*b^2*log(x)/(a^6 + 3*a^4 + 3*a^2 + 1) - (4*a*b*x - a^2 -
1)/((a^4 + 2*a^2 + 1)*x^2))*b - 1/3*arccot(b*x + a)/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3449 vs. $2(115) = 230$.

Time = 1.44 (sec) , antiderivative size = 3449, normalized size of antiderivative = 26.74

$$\int \frac{\cot^{-1}(a+bx)}{x^4} \, dx = \text{Too large to display}$$

input
```
integrate(arccot(b*x+a)/x^4,x, algorithm="giac")
```

output
```
-1/6*(24*a^5*b^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 + 12*a
^4*b^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^5 + 2*a^3*b^2*arct
an(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^6 + 24*a^5*b^2*log(4*(4*a^2*t
an(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1
/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*a
rctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arc
tan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^3 + 36*a^4*b^2*log(
4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))
^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2
*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*t
an(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^4 + 18*a^
3*b^2*log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(
b*x + a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x
+ a))) - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)
))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))
^5 + 3*a^2*b^2*log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*a
rctan(1/(b*x + a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arcta
n(1/(b*x + a))) - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/
(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b
*x + a)))^6 - 24*a^5*b^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)...
```

Mupad [B] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.21

$$\int \frac{\cot^{-1}(a+bx)}{x^4}\,dx = \frac{\ln(x)\left(\frac{b^3}{3}-a^2\,b^3\right)}{a^6+3\,a^4+3\,a^2+1} - \frac{\mathrm{acot}(a+b\,x)\left(\frac{a^2}{3}+\frac{1}{3}\right)-\frac{b\,x}{6}+\frac{b^2\,x^2\,\mathrm{acot}(a+b\,x)}{3}-\frac{x^3\,(b^3-7\,a^2\,b^3)}{6\,(a^4+2\,a^2+1)}+\frac{a\,b^2\,x^2}{3\,(a^2+1)}+\frac{2\,a\,b^4\,x^4}{3\,(a^2+1)^2}+\frac{2\,a\,b\,x\,\mathrm{acot}(a+b\,x)}{3}}{a^2\,x^3+2\,a\,b\,x^4+b^2\,x^5+x^3} + \frac{b^3\,\ln(a^2+2\,a\,b\,x+b^2\,x^2+1)\,(3\,a^2-1)}{6\,(a^6+3\,a^4+3\,a^2+1)} + \frac{a\,\mathrm{atan}\left(\frac{2\,x\,b^2+2\,a\,b}{2\,\sqrt{b^2\,(a^2+1)-a^2\,b^2}}\right)\,(a^2-3)\,(b^2)^{3/2}}{3\,(a^6+3\,a^4+3\,a^2+1)}$$

input
```
int(acot(a + b*x)/x^4,x)
```

output
```
(log(x)*(b^3/3 - a^2*b^3))/(3*a^2 + 3*a^4 + a^6 + 1) - (acot(a + b*x)*(a^2
/3 + 1/3) - (b*x)/6 + (b^2*x^2*acot(a + b*x))/3 - (x^3*(b^3 - 7*a^2*b^3))/
(6*(2*a^2 + a^4 + 1)) + (a*b^2*x^2)/(3*(a^2 + 1)) + (2*a*b^4*x^4)/(3*(a^2
+ 1)^2) + (2*a*b*x*acot(a + b*x))/3)/(x^3 + a^2*x^3 + b^2*x^5 + 2*a*b*x^4)
 + (b^3*log(a^2 + b^2*x^2 + 2*a*b*x + 1)*(3*a^2 - 1))/(6*(3*a^2 + 3*a^4 +
a^6 + 1)) + (a*atan((2*a*b + 2*b^2*x)/(2*(b^2*(a^2 + 1) - a^2*b^2)^(1/2)))
*(a^2 - 3)*(b^2)^(3/2))/(3*(3*a^2 + 3*a^4 + a^6 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.63

$$\int \frac{\cot^{-1}(a+bx)}{x^4}\,dx = \frac{-2acot(bx+a)\,a^6-6acot(bx+a)\,a^4-2acot(bx+a)\,a^3b^3x^3-6acot(bx+a)\,a^2+6acot(bx+a)\,a\,b^3x^3}{}$$

input
```
int(acot(b*x+a)/x^4,x)
```

output
```
( - 2*acot(a + b*x)*a**6 - 6*acot(a + b*x)*a**4 - 2*acot(a + b*x)*a**3*b**
3*x**3 - 6*acot(a + b*x)*a**2 + 6*acot(a + b*x)*a*b**3*x**3 - 2*acot(a + b
*x) + 3*log(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**2*b**3*x**3 - log(a**2 + 2*
a*b*x + b**2*x**2 + 1)*b**3*x**3 - 6*log(x)*a**2*b**3*x**3 + 2*log(x)*b**3
*x**3 + a**4*b*x - 4*a**3*b**2*x**2 + 2*a**2*b*x - 4*a*b**2*x**2 + b*x)/(6
*x**3*(a**6 + 3*a**4 + 3*a**2 + 1))
```

3.9 $\int (a+bx)^2 \cot^{-1}(a+bx)\, dx$

Optimal result

Integrand size = 14, antiderivative size = 52

$$\int (a+bx)^2 \cot^{-1}(a+bx)\, dx = \frac{(a+bx)^2}{6b} + \frac{(a+bx)^3 \cot^{-1}(a+bx)}{3b} - \frac{\log\left(1+(a+bx)^2\right)}{6b}$$

output
```
1/6*(b*x+a)^2/b+1/3*(b*x+a)^3*arccot(b*x+a)/b-1/6*ln(1+(b*x+a)^2)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int (a+bx)^2 \cot^{-1}(a+bx)\, dx = \frac{(a+bx)^2 + 2(a+bx)^3 \cot^{-1}(a+bx) - \log\left(1+(a+bx)^2\right)}{6b}$$

input
```
Integrate[(a + b*x)^2*ArcCot[a + b*x],x]
```

output
```
((a + b*x)^2 + 2*(a + b*x)^3*ArcCot[a + b*x] - Log[1 + (a + b*x)^2])/(6*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5567, 5362, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2 \cot^{-1}(a+bx) \, dx$$

$$\Big\downarrow \text{5567}$$

$$\frac{\int (a+bx)^2 \cot^{-1}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{5362}$$

$$\frac{\frac{1}{3}\int \frac{(a+bx)^3}{(a+bx)^2+1}d(a+bx)+\frac{1}{3}(a+bx)^3 \cot^{-1}(a+bx)}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{6}\int \frac{(a+bx)^2}{(a+bx)^2+1}d(a+bx)^2+\frac{1}{3}(a+bx)^3 \cot^{-1}(a+bx)}{b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\frac{1}{6}\int \left(1+\frac{1}{-a-bx-1}\right)d(a+bx)^2+\frac{1}{3}(a+bx)^3 \cot^{-1}(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{6}\left((a+bx)^2-\log(a+bx+1)\right)+\frac{1}{3}(a+bx)^3 \cot^{-1}(a+bx)}{b}$$

input
```
Int[(a + b*x)^2*ArcCot[a + b*x],x]
```

output
```
(((a + b*x)^3*ArcCot[a + b*x])/3 + ((a + b*x)^2 - Log[1 + a + b*x])/6)/b
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5567
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcCot[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{\frac{\operatorname{arccot}(bx+a)(bx+a)^3}{3}+\frac{(bx+a)^2}{6}-\frac{\ln\left(1+(bx+a)^2\right)}{6}}{b}$
default	$\frac{\frac{\operatorname{arccot}(bx+a)(bx+a)^3}{3}+\frac{(bx+a)^2}{6}-\frac{\ln\left(1+(bx+a)^2\right)}{6}}{b}$
parts	$\frac{\operatorname{arccot}(bx+a)b^2x^3}{3}+\operatorname{arccot}(bx+a)\,ba\,x^2+\operatorname{arccot}(bx+a)\,a^2x+\frac{\operatorname{arccot}(bx+a)a^3}{3b}+\frac{x^2b}{6}+\frac{ax}{3}$
parallelrisch	$-\frac{-2b^4\operatorname{arccot}(bx+a)x^3-6a\,b^3x^2\operatorname{arccot}(bx+a)-6x\operatorname{arccot}(bx+a)a^2b^2-x^2b^3-2\operatorname{arccot}(bx+a)a^3b-2xa\,b^2+5a^2b+\ln(}{6b^2}$
risch	$\frac{i(bx+a)^3\ln(1+i(bx+a))}{6b}-\frac{ib^2x^3\ln(1-i(bx+a))}{6}+\frac{\pi b^2x^3}{6}-\frac{iba\,x^2\ln(1-i(bx+a))}{2}+\frac{b\pi a\,x^2}{2}-\frac{ia^2x\ln(1-i(b}{2}$

input
```
int((b*x+a)^2*arccot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/3*arccot(b*x+a)*(b*x+a)^3+1/6*(b*x+a)^2-1/6*ln(1+(b*x+a)^2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.56

$$\int (a+bx)^2\cot^{-1}(a+bx)\,dx$$
$$=\frac{b^2x^2-2\,a^3\arctan(bx+a)+2\,abx+2\,(b^3x^3+3\,ab^2x^2+3\,a^2bx)\operatorname{arccot}(bx+a)-\log(b^2x^2+2\,abx+a}{6\,b}$$

input
```
integrate((b*x+a)^2*arccot(b*x+a),x, algorithm="fricas")
```

output
```
1/6*(b^2*x^2 - 2*a^3*arctan(b*x + a) + 2*a*b*x + 2*(b^3*x^3 + 3*a*b^2*x^2
+ 3*a^2*b*x)*arccot(b*x + a) - log(b^2*x^2 + 2*a*b*x + a^2 + 1))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.45 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.90

$$\int (a+bx)^2\cot^{-1}(a+bx)\,dx$$
$$=\begin{cases} \frac{a^3\operatorname{acot}(a+bx)}{3b}+a^2x\operatorname{acot}(a+bx)+abx^2\operatorname{acot}(a+bx)+\frac{ax}{3}+\frac{b^2x^3\operatorname{acot}(a+bx)}{3}+\frac{bx^2}{6}-\frac{\log\left(\frac{a}{b}+x-\frac{i}{b}\right)}{3b}-\frac{i\operatorname{acot}(a\cdot}{3b} \\ a^2x\operatorname{acot}(a) \end{cases}$$

input
```
integrate((b*x+a)**2*acot(b*x+a),x)
```

output
```
Piecewise((a**3*acot(a + b*x)/(3*b) + a**2*x*acot(a + b*x) + a*b*x**2*acot
(a + b*x) + a*x/3 + b**2*x**3*acot(a + b*x)/3 + b*x**2/6 - log(a/b + x - I
/b)/(3*b) - I*acot(a + b*x)/(3*b), Ne(b, 0)), (a**2*x*acot(a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(46) = 92.

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.79

$$\int (a+bx)^2\cot^{-1}(a+bx)\,dx$$
$$=-\frac{1}{6}\left(\frac{2a^3\arctan\left(\frac{b^2x+ab}{b}\right)}{b^2}-\frac{bx^2+2ax}{b}+\frac{\log\left(b^2x^2+2abx+a^2+1\right)}{b^2}\right)b$$
$$+\frac{1}{3}\left(b^2x^3+3abx^2+3a^2x\right)\operatorname{arccot}(bx+a)$$

input
```
integrate((b*x+a)^2*arccot(b*x+a),x, algorithm="maxima")
```

output
```
-1/6*(2*a^3*arctan((b^2*x + a*b)/b)/b^2 - (b*x^2 + 2*a*x)/b + log(b^2*x^2
+ 2*a*b*x + a^2 + 1)/b^2)*b + 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)*arccot(b
*x + a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(46) = 92.

Time = 0.23 (sec) , antiderivative size = 203, normalized size of antiderivative = 3.90

$$\int (a+bx)^2 \cot^{-1}(a+bx)\,dx =$$

$$-\frac{\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^6 - 3\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4 - \tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right.\right.}{}$$

input
```
integrate((b*x+a)^2*arccot(b*x+a),x, algorithm="giac")
```

output
```
-1/24*(arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^6 - 3*arctan(1/(b*
x + a))*tan(1/2*arctan(1/(b*x + a)))^4 - tan(1/2*arctan(1/(b*x + a)))^5 -
4*log(16*tan(1/2*arctan(1/(b*x + a)))^2/(tan(1/2*arctan(1/(b*x + a)))^4 +
2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^3 + 3*
arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 - 2*tan(1/2*arctan(1/(b
*x + a)))^3 - arctan(1/(b*x + a)) - tan(1/2*arctan(1/(b*x + a))))/(b*tan(1
/2*arctan(1/(b*x + a)))^3)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.63

$$\int (a+bx)^2 \cot^{-1}(a+bx)\,dx = \frac{a\,x}{3} - \frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{6\,b} + \frac{b\,x^2}{6} - \frac{a^3\,\mathrm{atan}(a+b\,x)}{3\,b} + \frac{b^2\,x^3\,\mathrm{acot}(a+b\,x)}{3} + a^2\,x\,\mathrm{acot}(a+b\,x) + a\,b\,x^2\,\mathrm{acot}(a+b\,x)$$

input
```
int(acot(a + b*x)*(a + b*x)^2,x)
```

output
```
(a*x)/3 - log(a^2 + b^2*x^2 + 2*a*b*x + 1)/(6*b) + (b*x^2)/6 - (a^3*atan(a
 + b*x))/(3*b) + (b^2*x^3*acot(a + b*x))/3 + a^2*x*acot(a + b*x) + a*b*x^2
*acot(a + b*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.75

$$\int (a+bx)^2 \cot^{-1}(a+bx)\,dx$$
$$= \frac{2acot(bx+a)\,a^3 + 6acot(bx+a)\,a^2bx + 6acot(bx+a)\,a\,b^2x^2 + 2acot(bx+a)\,b^3x^3 - \log(b^2x^2 + 2abx +}{6b}$$

input
```
int((b*x+a)^2*acot(b*x+a),x)
```

output
```
(2*acot(a + b*x)*a**3 + 6*acot(a + b*x)*a**2*b*x + 6*acot(a + b*x)*a*b**2*
x**2 + 2*acot(a + b*x)*b**3*x**3 - log(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2
*a*b*x + b**2*x**2)/(6*b)
```

3.10 $\int (a+bx)\cot^{-1}(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 39

$$\int (a+bx)\cot^{-1}(a+bx)\,dx=\frac{x}{2}+\frac{(a+bx)^2\cot^{-1}(a+bx)}{2b}-\frac{\arctan(a+bx)}{2b}$$

output
```
1/2*x+1/2*(b*x+a)^2*arccot(b*x+a)/b-1/2*arctan(b*x+a)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.62

$$\begin{aligned}\int (a+bx)\cot^{-1}(a+bx)\,dx&=ax\cot^{-1}(a+bx)+\frac{1}{2}b\left(-\frac{a}{b}+\frac{a+bx}{b}\right)^2\cot^{-1}(a+bx)\\&+\frac{1}{2}b\left(\frac{x}{b}-\frac{i(i-a)^2\log(i-a-bx)}{2b^2}\right.\\&\left.+\frac{i(i+a)^2\log(i+a+bx)}{2b^2}\right)\\&+\frac{a(-2a\arctan(a+bx)+\log(1+a^2+2abx+b^2x^2))}{2b}\end{aligned}$$

input
```
Integrate[(a + b*x)*ArcCot[a + b*x],x]
```

output
```
a*x*ArcCot[a + b*x] + (b*(-(a/b) + (a + b*x)/b)^2*ArcCot[a + b*x])/2 + (b*
(x/b - ((I/2)*(I - a)^2*Log[I - a - b*x])/b^2 + ((I/2)*(I + a)^2*Log[I + a
 + b*x])/b^2))/2 + (a*(-2*a*ArcTan[a + b*x] + Log[1 + a^2 + 2*a*b*x + b^2*
x^2]))/(2*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5567, 5362, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\cot^{-1}(a+bx)\,dx$$

$$\Big\downarrow \text{ 5567}$$

$$\frac{\int (a+bx)\cot^{-1}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{ 5362}$$

$$\frac{\frac{1}{2}\int \frac{(a+bx)^2}{(a+bx)^2+1}d(a+bx)+\frac{1}{2}(a+bx)^2\cot^{-1}(a+bx)}{b}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{\frac{1}{2}\left(-\int \frac{1}{(a+bx)^2+1}d(a+bx)+a+bx\right)+\frac{1}{2}(a+bx)^2\cot^{-1}(a+bx)}{b}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\frac{1}{2}(-\arctan(a+bx)+a+bx)+\frac{1}{2}(a+bx)^2\cot^{-1}(a+bx)}{b}$$

input
```
Int[(a + b*x)*ArcCot[a + b*x],x]
```

output
```
(((a + b*x)^2*ArcCot[a + b*x])/2 + (a + b*x - ArcTan[a + b*x])/2)/b
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5567
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcCot[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{\frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}-\frac{\arctan(bx+a)}{2}}{b}$
default	$\frac{\frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}-\frac{\arctan(bx+a)}{2}}{b}$
parts	$\frac{\operatorname{arccot}(bx+a)x^2b}{2}+\operatorname{arccot}(bx+a)\,ax+\frac{b\left(\frac{x}{b}+\frac{(-a^2-1)\arctan(bx+a)}{b^2}\right)}{2}$
parallelrisch	$\frac{b^3x^2 \operatorname{arccot}(bx+a)+2a \operatorname{arccot}(bx+a)x\,b^2+\operatorname{arccot}(bx+a)a^2b+b^2x+b \operatorname{arccot}(bx+a)-2ab}{2b^2}$
risch	$\frac{i(x^2b+2ax)\ln(1+i(bx+a))}{4}-\frac{ib\,x^2\ln(1-i(bx+a))}{4}-\frac{iax\ln(1-i(bx+a))}{2}+\frac{\pi b\,x^2}{4}+\frac{\pi ax}{2}-\frac{\arctan(bx+a)a^2}{2b}-$
orering	$\frac{(2b^3x^3+5a\,b^2x^2+4a^2bx+a^3+2bx+a)\operatorname{arccot}(bx+a)}{2(bx+a)b}-\frac{x(b^2x^2+2abx+a^2+1)\left(b \operatorname{arccot}(bx+a)-\frac{(bx+a)b}{1+(bx+a)^2}\right)}{2b(bx+a)}$

input
```
int((b*x+a)*arccot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*(b*x+a)^2*arccot(b*x+a)+1/2*b*x+1/2*a-1/2*arctan(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int (a+bx)\cot^{-1}(a+bx)\,dx = \frac{bx+(b^2x^2+2\,abx+a^2+1)\operatorname{arccot}(bx+a)}{2\,b}$$

input
```
integrate((b*x+a)*arccot(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b*x + (b^2*x^2 + 2*a*b*x + a^2 + 1)*arccot(b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.44

$$\int (a+bx)\cot^{-1}(a+bx)\,dx = \begin{cases} \frac{a^2\operatorname{acot}(a+bx)}{2b} + ax\operatorname{acot}(a+bx) + \frac{bx^2\operatorname{acot}(a+bx)}{2} + \frac{x}{2} + \frac{\operatorname{acot}(a+bx)}{2b} & \text{for } b \neq 0 \\ ax\operatorname{acot}(a) & \text{otherwise} \end{cases}$$

input
```
integrate((b*x+a)*acot(b*x+a),x)
```

output
```
Piecewise((a**2*acot(a + b*x)/(2*b) + a*x*acot(a + b*x) + b*x**2*acot(a +
b*x)/2 + x/2 + acot(a + b*x)/(2*b), Ne(b, 0)), (a*x*acot(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.33

$$\int (a+bx)\cot^{-1}(a+bx)\,dx = \frac{1}{2}b\left(\frac{x}{b} - \frac{(a^2+1)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^2}\right) + \frac{1}{2}\left(bx^2+2\,ax\right)\operatorname{arccot}(bx+a)$$

input
```
integrate((b*x+a)*arccot(b*x+a),x, algorithm="maxima")
```

output
```
1/2*b*(x/b - (a^2 + 1)*arctan((b^2*x + a*b)/b)/b^2) + 1/2*(b*x^2 + 2*a*x)*
arccot(b*x + a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(33) = 66$.

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.56

$$\int (a+bx)\cot^{-1}(a+bx)\,dx = \frac{\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4+2\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2-2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right.}{8\,b\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2}$$

input
```
integrate((b*x+a)*arccot(b*x+a),x, algorithm="giac")
```

output
```
1/8*(arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 + 2*arctan(1/(b*x
+ a))*tan(1/2*arctan(1/(b*x + a)))^2 - 2*tan(1/2*arctan(1/(b*x + a)))^3 +
arctan(1/(b*x + a)) + 2*tan(1/2*arctan(1/(b*x + a))))/(b*tan(1/2*arctan(1/
(b*x + a)))^2)
```

Mupad [B] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int (a+bx)\cot^{-1}(a+bx)\,dx = \frac{x}{2}+\frac{\frac{\mathrm{acot}(a+b\,x)}{2}+\frac{a^2\,\mathrm{acot}(a+b\,x)}{2}}{b} + a\,x\,\mathrm{acot}(a+b\,x)+\frac{b\,x^2\,\mathrm{acot}(a+b\,x)}{2}$$

input
```
int(acot(a + b*x)*(a + b*x),x)
```

output
```
x/2 + (acot(a + b*x)/2 + (a^2*acot(a + b*x))/2)/b + a*x*acot(a + b*x) + (b
*x^2*acot(a + b*x))/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int (a+bx)\cot^{-1}(a+bx)\,dx = \frac{acot(bx+a)\,a^2+2acot(bx+a)\,abx+acot(bx+a)\,b^2x^2+acot(bx+a)+bx}{2b}$$

input
```
int((b*x+a)*acot(b*x+a),x)
```

output
```
(acot(a + b*x)*a**2 + 2*acot(a + b*x)*a*b*x + acot(a + b*x)*b**2*x**2 + ac
ot(a + b*x) + b*x)/(2*b)
```

3.11 $\int \frac{\cot^{-1}(a+bx)}{a+bx} dx$

Optimal result

Integrand size = 14, antiderivative size = 45

$$\int \frac{\cot^{-1}(a+bx)}{a+bx} dx = -\frac{i \operatorname{PolyLog}\left(2,-\frac{i}{a+bx}\right)}{2b} + \frac{i \operatorname{PolyLog}\left(2,\frac{i}{a+bx}\right)}{2b}$$

output
```
-1/2*I*polylog(2,-I/(b*x+a))/b+1/2*I*polylog(2,I/(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{\cot^{-1}(a+bx)}{a+bx} dx = -\frac{i\left(\operatorname{PolyLog}\left(2,-\frac{i}{a+bx}\right) - \operatorname{PolyLog}\left(2,\frac{i}{a+bx}\right)\right)}{2b}$$

input
```
Integrate[ArcCot[a + b*x]/(a + b*x),x]
```

output
```
((-1/2*I)*(PolyLog[2, (-I)/(a + b*x)] - PolyLog[2, I/(a + b*x)]))/b
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5567, 5356, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{a+bx} \, dx$$

$$\Big\downarrow \text{5567}$$

$$\frac{\int \frac{\cot^{-1}(a+bx)}{a+bx}d(a+bx)}{b}$$

$$\Big\downarrow \text{5356}$$

$$\frac{\frac{1}{2}i\int \frac{\log\left(1-\frac{i}{a+bx}\right)}{a+bx}d(a+bx)-\frac{1}{2}i\int \frac{\log\left(1+\frac{i}{a+bx}\right)}{a+bx}d(a+bx)}{b}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{i}{a+bx}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{i}{a+bx}\right)}{b}$$

input
```
Int[ArcCot[a + b*x]/(a + b*x),x]
```

output
```
((-1/2*I)*PolyLog[2, (-I)/(a + b*x)] + (I/2)*PolyLog[2, I/(a + b*x)])/b
```

Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5356
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Simp[I*(b/2)   Int[Log[1 + I/(c*x)]/x, x], x] + Simp[I*(b/2)   Int[Log
[1 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5567
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcCot[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

method	result
risch	$\frac{i\,\mathrm{dilog}(-bxi-ai+1)}{2b}+\frac{\pi\ln(-bxi-ai)}{2b}-\frac{i\,\mathrm{dilog}(bxi+ai+1)}{2b}$
derivativedivides	$\frac{\ln(bx+a)\,\mathrm{arccot}(bx+a)-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\,\mathrm{dilog}(1+i(bx+a))}{2}+\frac{i\,\mathrm{dilog}(1-i(bx+a))}{2}}{b}$
default	$\frac{\ln(bx+a)\,\mathrm{arccot}(bx+a)-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\,\mathrm{dilog}(1+i(bx+a))}{2}+\frac{i\,\mathrm{dilog}(1-i(bx+a))}{2}}{b}$
parts	$\frac{\ln(bx+a)\,\mathrm{arccot}(bx+a)}{b}+\frac{-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\,\mathrm{dilog}(1+i(bx+a))}{2}+\frac{i\,\mathrm{dilog}(1-i(bx+a))}{2}}{b}$

input
```
int(arccot(b*x+a)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/b*dilog(1-I*a-I*b*x)+1/2/b*Pi*ln(-I*b*x-I*a)-1/2*I/b*dilog(1+I*a+I*b
*x)
```

Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{a+bx}\,dx=\int \frac{\operatorname{arccot}(bx+a)}{bx+a}\,dx$$

input
```
integrate(arccot(b*x+a)/(b*x+a),x, algorithm="fricas")
```

output
```
integral(arccot(b*x + a)/(b*x + a), x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(a+bx)}{a+bx}\,dx = \int \frac{\operatorname{acot}(a+bx)}{a+bx}\,dx$$

input
```
integrate(acot(b*x+a)/(b*x+a),x)
```

output
```
Integral(acot(a + b*x)/(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(31) = 62$.

Time = 0.18 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.49

$$\int \frac{\cot^{-1}(a+bx)}{a+bx}\,dx = \frac{\operatorname{arccot}(bx+a)\log(bx+a)}{b} + \frac{\arctan\left(\frac{b^2x+ab}{b}\right)\log(bx+a)}{b} + \frac{\arctan(bx+a,0)\log(b^2x^2+2\,abx+a^2+1) - 2\,\arctan(bx+a)\log(|bx+a|) + i\,\mathrm{Li}_2(i\,bx+i\,a+1)}{2\,b}$$

input
```
integrate(arccot(b*x+a)/(b*x+a),x, algorithm="maxima")
```

output
```
arccot(b*x + a)*log(b*x + a)/b + arctan((b^2*x + a*b)/b)*log(b*x + a)/b +
1/2*(arctan2(b*x + a, 0)*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*arctan(b*x +
 a)*log(abs(b*x + a)) + I*dilog(I*b*x + I*a + 1) - I*dilog(-I*b*x - I*a +
1))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(31) = 62$.

Time = 0.38 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.22

$$\int \frac{\cot^{-1}(a+bx)}{a+bx}\,dx = -\frac{\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4 + 2\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2 - 2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right.\right.}{8\,b^2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2}$$

input
```
integrate(arccot(b*x+a)/(b*x+a),x, algorithm="giac")
```

output
```
-1/8*(arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 + 2*arctan(1/(b*x
 + a))*tan(1/2*arctan(1/(b*x + a)))^2 - 2*tan(1/2*arctan(1/(b*x + a)))^3 +
 arctan(1/(b*x + a)) + 2*tan(1/2*arctan(1/(b*x + a))))/(b^2*tan(1/2*arctan
(1/(b*x + a)))^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{a+bx}\,dx = \int \frac{\mathrm{acot}(a+b\,x)}{a+b\,x}\,dx$$

input
```
int(acot(a + b*x)/(a + b*x),x)
```

output
```
int(acot(a + b*x)/(a + b*x), x)
```

Reduce [F]

$$\int \frac{\cot^{-1}(a+bx)}{a+bx}\,dx = \int \frac{acot(bx+a)}{bx+a}dx$$

input
```
int(acot(b*x+a)/(b*x+a),x)
```

output
```
int(acot(a + b*x)/(a + b*x),x)
```

3.12 $\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 47

$$\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2}\,dx=-\frac{\cot^{-1}(a+bx)}{b(a+bx)}-\frac{\log(a+bx)}{b}+\frac{\log\left(1+(a+bx)^2\right)}{2b}$$

output
```
-arccot(b*x+a)/b/(b*x+a)-ln(b*x+a)/b+1/2*ln(1+(b*x+a)^2)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2}\,dx=\frac{-\frac{\cot^{-1}(a+bx)}{a+bx}-\log(a+bx)+\frac{1}{2}\log\left(1+(a+bx)^2\right)}{b}$$

input
```
Integrate[ArcCot[a + b*x]/(a + b*x)^2,x]
```

output
```
(-(ArcCot[a + b*x]/(a + b*x)) - Log[a + b*x] + Log[1 + (a + b*x)^2]/2)/b
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5567, 5362, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2}dx$$

$$\Big\downarrow \text{5567}$$

$$\frac{\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2}d(a+bx)}{b}$$

$$\Big\downarrow \text{5362}$$

$$\frac{-\int \frac{1}{(a+bx)((a+bx)^2+1)}d(a+bx)-\frac{\cot^{-1}(a+bx)}{a+bx}}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{-\frac{1}{2}\int \frac{1}{(a+bx)^2((a+bx)^2+1)}d(a+bx)^2-\frac{\cot^{-1}(a+bx)}{a+bx}}{b}$$

$$\Big\downarrow \text{47}$$

$$\frac{\frac{1}{2}\left(\int \frac{1}{(a+bx)^2+1}d(a+bx)^2-\int \frac{1}{(a+bx)^2}d(a+bx)^2\right)-\frac{\cot^{-1}(a+bx)}{a+bx}}{b}$$

$$\Big\downarrow \text{14}$$

$$\frac{\frac{1}{2}\left(\int \frac{1}{(a+bx)^2+1}d(a+bx)^2-\log\left((a+bx)^2\right)\right)-\frac{\cot^{-1}(a+bx)}{a+bx}}{b}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{1}{2}\left(\log\left((a+bx)^2+1\right)-\log\left((a+bx)^2\right)\right)-\frac{\cot^{-1}(a+bx)}{a+bx}}{b}$$

input
```
Int[ArcCot[a + b*x]/(a + b*x)^2,x]
```

output
```
(-(ArcCot[a + b*x]/(a + b*x)) + (-Log[(a + b*x)^2] + Log[1 + (a + b*x)^2])
/2)/b
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5567
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcCot[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{-\frac{\operatorname{arccot}(bx+a)}{bx+a}-\ln(bx+a)+\frac{\ln\left(1+(bx+a)^2\right)}{2}}{b}$
default	$\frac{-\frac{\operatorname{arccot}(bx+a)}{bx+a}-\ln(bx+a)+\frac{\ln\left(1+(bx+a)^2\right)}{2}}{b}$
parts	$-\frac{\operatorname{arccot}(bx+a)}{b(bx+a)}+\frac{\ln(b^2x^2+2abx+a^2+1)}{2b}-\frac{\ln(bx+a)}{b}$
parallelrisch	$-\frac{6\ln(bx+a)xa\,b^2-3b^2\ln(b^2x^2+2abx+a^2+1)ax+6\ln(bx+a)a^2b-3\ln(b^2x^2+2abx+a^2+1)a^2b+6\operatorname{arccot}(bx+a)ab}{6(bx+a)b^2a}$
risch	$-\frac{i\ln(1+i(bx+a))}{2b(bx+a)}-\frac{2\ln(-bx-a)bx-\ln(b^2x^2+2abx+a^2+1)bx+2\ln(-bx-a)a-a\ln(b^2x^2+2abx+a^2+1)-i\ln(1-i(}{2b(bx+a)}$

input
```
int(arccot(b*x+a)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-arccot(b*x+a)/(b*x+a)-ln(b*x+a)+1/2*ln(1+(b*x+a)^2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2}\,dx = \frac{(bx+a)\log\left(b^2x^2+2\,abx+a^2+1\right)-2\,(bx+a)\log\left(bx+a\right)-2\,\operatorname{arccot}\left(bx+a\right)}{2\,(b^2x+ab)}$$

input
```
integrate(arccot(b*x+a)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*((b*x + a)*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*(b*x + a)*log(b*x + a)
 - 2*arccot(b*x + a))/(b^2*x + a*b)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.68 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.96

$$\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2}\,dx$$
$$= \begin{cases} -\frac{a\log\left(\frac{a}{b}+x\right)}{ab+b^2x} + \frac{a\log\left(\frac{a}{b}+x-\frac{i}{b}\right)}{ab+b^2x} + \frac{ia\operatorname{acot}(a+bx)}{ab+b^2x} - \frac{bx\log\left(\frac{a}{b}+x\right)}{ab+b^2x} + \frac{bx\log\left(\frac{a}{b}+x-\frac{i}{b}\right)}{ab+b^2x} + \frac{ibx\operatorname{acot}(a+bx)}{ab+b^2x} - \frac{\operatorname{acot}(a+bx)}{ab+b^2x} & \text{for } b \\ \frac{x\operatorname{acot}(a)}{a^2} & \text{othe} \end{cases}$$

input
```
integrate(acot(b*x+a)/(b*x+a)**2,x)
```

output
```
Piecewise((-a*log(a/b + x)/(a*b + b**2*x) + a*log(a/b + x - I/b)/(a*b + b*
*2*x) + I*a*acot(a + b*x)/(a*b + b**2*x) - b*x*log(a/b + x)/(a*b + b**2*x)
 + b*x*log(a/b + x - I/b)/(a*b + b**2*x) + I*b*x*acot(a + b*x)/(a*b + b**2
*x) - acot(a + b*x)/(a*b + b**2*x), Ne(b, 0)), (x*acot(a)/a**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2}\,dx = \frac{\log(b^2x^2+2\,abx+a^2+1)}{2\,b} - \frac{\log(bx+a)}{b} - \frac{\operatorname{arccot}(bx+a)}{(bx+a)b}$$

input
```
integrate(arccot(b*x+a)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b - log(b*x + a)/b - arccot(b*x + a)/
((b*x + a)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(45) = 90$.

Time = 0.16 (sec) , antiderivative size = 238, normalized size of antiderivative = 5.06

$$\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2}\,dx = -\frac{\arctan\left(\frac{1}{bx+a}\right)^2 - \frac{\arctan\left(\frac{1}{bx+a}\right)^2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2 - \log\left(\frac{4\left(\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4 - 2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2 + 1\right)}{\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4 + 2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2 + 1}\right)\tan\left(\frac{1}{2}\,\mathrm{ar}\cdots\right.}{\mathrm{ta}\cdots}}{2\cdots}$$

input
```
integrate(arccot(b*x+a)/(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*(arctan(1/(b*x + a))^2 - (arctan(1/(b*x + a))^2*tan(1/2*arctan(1/(b*x
 + a)))^2 - log(4*(tan(1/2*arctan(1/(b*x + a)))^4 - 2*tan(1/2*arctan(1/(b*
x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x
+ a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^2 - arctan(1/(b*x + a))^2 + 4*
arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a))) + log(4*(tan(1/2*arctan(1
/(b*x + a)))^4 - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(
b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)))/(tan(1/2*arctan(1/(
b*x + a)))^2 - 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2}\,dx = \frac{\ln\left(-a^2-2\,a\,b\,x-b^2\,x^2-1\right)}{2\,b} - \frac{\ln\left(a+b\,x\right)}{b} - \frac{\mathrm{acot}(a+b\,x)}{x\,b^2+a\,b}$$

input
```
int(acot(a + b*x)/(a + b*x)^2,x)
```

output
```
log(- a^2 - b^2*x^2 - 2*a*b*x - 1)/(2*b) - log(a + b*x)/b - acot(a + b*x)/
(a*b + b^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.36

$$\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2}\,dx$$
$$= \frac{2acot(bx+a)\,bx+2atan(bx+a)\,a+2atan(bx+a)\,bx+\log(b^2x^2+2abx+a^2+1)\,a^2+\log(b^2x^2+2ab}{2ab\,(bx+a)}$$

input
```
int(acot(b*x+a)/(b*x+a)^2,x)
```

output
```
(2*acot(a + b*x)*b*x + 2*atan(a + b*x)*a + 2*atan(a + b*x)*b*x + log(a**2
+ 2*a*b*x + b**2*x**2 + 1)*a**2 + log(a**2 + 2*a*b*x + b**2*x**2 + 1)*a*b*
x - 2*log(a + b*x)*a**2 - 2*log(a + b*x)*a*b*x)/(2*a*b*(a + b*x))
```

3.13 $\int \frac{\cot^{-1}(1+x)}{2+2x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 35

$$\int \frac{\cot^{-1}(1+x)}{2+2x}\,dx=-\frac{1}{4}i\,\text{PolyLog}\left(2,-\frac{i}{1+x}\right)+\frac{1}{4}i\,\text{PolyLog}\left(2,\frac{i}{1+x}\right)$$

output
```
-1/4*I*polylog(2,-I/(1+x))+1/4*I*polylog(2,I/(1+x))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(1+x)}{2+2x}\,dx=-\frac{1}{4}i\,\text{PolyLog}\left(2,-\frac{i}{1+x}\right)+\frac{1}{4}i\,\text{PolyLog}\left(2,\frac{i}{1+x}\right)$$

input
```
Integrate[ArcCot[1 + x]/(2 + 2*x),x]
```

output
```
(-1/4*I)*PolyLog[2, (-I)/(1 + x)] + (I/4)*PolyLog[2, I/(1 + x)]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5567, 27, 5356, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(x+1)}{2x+2}dx$$

$$\Big\downarrow \text{5567}$$

$$\int \frac{\cot^{-1}(x+1)}{2(x+1)}d(x+1)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{\cot^{-1}(x+1)}{x+1}d(x+1)$$

$$\Big\downarrow \text{5356}$$

$$\frac{1}{2}\left(\frac{1}{2}i\int \frac{\log\left(1-\frac{i}{x+1}\right)}{x+1}d(x+1)-\frac{1}{2}i\int \frac{\log\left(1+\frac{i}{x+1}\right)}{x+1}d(x+1)\right)$$

$$\Big\downarrow \text{2838}$$

$$\frac{1}{2}\left(\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{i}{x+1}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{i}{x+1}\right)\right)$$

input
```
Int[ArcCot[1 + x]/(2 + 2*x),x]
```

output
```
((-1/2*I)*PolyLog[2, (-I)/(1 + x)] + (I/2)*PolyLog[2, I/(1 + x)])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5356
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Simp[I*(b/2)   Int[Log[1 + I/(c*x)]/x, x], x] + Simp[I*(b/2)   Int[Log
[1 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5567
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcCot[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

method	result
risch	$\frac{\pi \ln(-ix-i)}{4}+\frac{i\,\mathrm{dilog}(-ix-i+1)}{4}-\frac{i\,\mathrm{dilog}(ix+i+1)}{4}$
derivativedivides	$\frac{\ln(1+x)\,\mathrm{arccot}(1+x)}{2}-\frac{i\ln(1+x)\ln(1+i(1+x))}{4}+\frac{i\ln(1+x)\ln(1-i(1+x))}{4}-\frac{i\,\mathrm{dilog}(1+i(1+x))}{4}+\frac{i\,\mathrm{dilog}(1-i(}{4}$
default	$\frac{\ln(1+x)\,\mathrm{arccot}(1+x)}{2}-\frac{i\ln(1+x)\ln(1+i(1+x))}{4}+\frac{i\ln(1+x)\ln(1-i(1+x))}{4}-\frac{i\,\mathrm{dilog}(1+i(1+x))}{4}+\frac{i\,\mathrm{dilog}(1-i(}{4}$
parts	$\frac{\ln(1+x)\,\mathrm{arccot}(1+x)}{2}-\frac{i\ln(1+x)\ln(1+i(1+x))}{4}+\frac{i\ln(1+x)\ln(1-i(1+x))}{4}-\frac{i\,\mathrm{dilog}(1+i(1+x))}{4}+\frac{i\,\mathrm{dilog}(1-i(}{4}$

input
```
int(arccot(1+x)/(2+2*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*Pi*ln(-I*x-I)+1/4*I*dilog(-I*x+1-I)-1/4*I*dilog(I*x+1+I)
```

Fricas [F]

$$\int \frac{\cot^{-1}(1+x)}{2+2x}\,dx = \int \frac{\operatorname{arccot}(x+1)}{2\,(x+1)}\,dx$$

input
```
integrate(arccot(1+x)/(2+2*x),x, algorithm="fricas")
```

output
```
integral(1/2*arccot(x + 1)/(x + 1), x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(1+x)}{2+2x}\,dx = \frac{\int \frac{\operatorname{acot}(x+1)}{x+1}\,dx}{2}$$

input
```
integrate(acot(1+x)/(2+2*x),x)
```

output
```
Integral(acot(x + 1)/(x + 1), x)/2
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(21) = 42$.

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.83

$$\begin{aligned}\int \frac{\cot^{-1}(1+x)}{2+2x}\,dx = {} & \frac{1}{4}\arctan\left(x+1,0\right)\log\left(x^2+2\,x+2\right)+\frac{1}{2}\operatorname{arccot}\left(x+1\right)\log\left(x+1\right)\\ & +\frac{1}{2}\arctan\left(x+1\right)\log\left(x+1\right)-\frac{1}{2}\arctan\left(x+1\right)\log\left(|x+1|\right)\\ & +\frac{1}{4}i\operatorname{Li}_2(i\,x+i+1)-\frac{1}{4}i\operatorname{Li}_2(-i\,x-i+1)\end{aligned}$$

input
```
integrate(arccot(1+x)/(2+2*x),x, algorithm="maxima")
```

output
```
1/4*arctan2(x + 1, 0)*log(x^2 + 2*x + 2) + 1/2*arccot(x + 1)*log(x + 1) +
1/2*arctan(x + 1)*log(x + 1) - 1/2*arctan(x + 1)*log(abs(x + 1)) + 1/4*I*d
ilog(I*x + I + 1) - 1/4*I*dilog(-I*x - I + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \frac{\cot^{-1}(1+x)}{2+2x}\,dx = -\frac{1}{4}\,(x+1)^2\arctan\left(\frac{1}{x+1}\right) - \frac{1}{4}\,x - \frac{1}{4}\,\arctan\left(\frac{1}{x+1}\right) - \frac{1}{4}$$

input
```
integrate(arccot(1+x)/(2+2*x),x, algorithm="giac")
```

output
```
-1/4*(x + 1)^2*arctan(1/(x + 1)) - 1/4*x - 1/4*arctan(1/(x + 1)) - 1/4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(1+x)}{2+2x}\,dx = \int \frac{\mathrm{acot}(x+1)}{2\,x+2}\,dx$$

input
```
int(acot(x + 1)/(2*x + 2),x)
```

output
```
int(acot(x + 1)/(2*x + 2), x)
```

Reduce [F]

$$\int \frac{\cot^{-1}(1+x)}{2+2x}\,dx = \frac{\left(\int \frac{acot(x+1)}{x+1}dx\right)}{2}$$

input
```
int(acot(1+x)/(2+2*x),x)
```

output
```
int(acot(x + 1)/(x + 1),x)/2
```

3.14 $\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 45

$$\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=-\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i}{a+bx}\right)}{2d}+\frac{i\,\mathrm{PolyLog}\left(2,\frac{i}{a+bx}\right)}{2d}$$

output
```
-1/2*I*polylog(2,-I/(b*x+a))/d+1/2*I*polylog(2,I/(b*x+a))/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=-\frac{i\left(\mathrm{PolyLog}\left(2,-\frac{i}{a+bx}\right)-\mathrm{PolyLog}\left(2,\frac{i}{a+bx}\right)\right)}{2d}$$

input
```
Integrate[ArcCot[a + b*x]/((a*d)/b + d*x),x]
```

output
```
((-1/2*I)*(PolyLog[2, (-I)/(a + b*x)] - PolyLog[2, I/(a + b*x)]))/d
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {5567, 27, 5356, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx} \, dx$$

$$\Big\downarrow \text{ 5567}$$

$$\frac{\int \frac{b\cot^{-1}(a+bx)}{d(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\cot^{-1}(a+bx)}{a+bx}d(a+bx)}{d}$$

$$\Big\downarrow \text{ 5356}$$

$$\frac{\frac{1}{2}i\int \frac{\log\left(1-\frac{i}{a+bx}\right)}{a+bx}d(a+bx)-\frac{1}{2}i\int \frac{\log\left(1+\frac{i}{a+bx}\right)}{a+bx}d(a+bx)}{d}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{i}{a+bx}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{i}{a+bx}\right)}{d}$$

input
```
Int[ArcCot[a + b*x]/((a*d)/b + d*x),x]
```

output
```
((-1/2*I)*PolyLog[2, (-I)/(a + b*x)] + (I/2)*PolyLog[2, I/(a + b*x)])/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5356
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Simp[I*(b/2)   Int[Log[1 + I/(c*x)]/x, x], x] + Simp[I*(b/2)   Int[Log
[1 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5567
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcCot[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

method	result
risch	$\frac{i\,\mathrm{dilog}(-bxi-ai+1)}{2d}+\frac{\pi\ln(-bxi-ai)}{2d}-\frac{i\,\mathrm{dilog}(bxi+ai+1)}{2d}$
parts	$\frac{\ln(bx+a)\,\mathrm{arccot}(bx+a)}{d}+\frac{-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\,\mathrm{dilog}(1+i(bx+a))}{2}+\frac{i\,\mathrm{dilog}(1-i(bx+a))}{2}}{d}$
derivativedivides	$\frac{\frac{b\ln(bx+a)\,\mathrm{arccot}(bx+a)}{d}+\frac{b\left(-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\,\mathrm{dilog}(1+i(bx+a))}{2}+\frac{i\,\mathrm{dilog}(1-i(bx+a))}{2}\right)}{d}}{b}$
default	$\frac{\frac{b\ln(bx+a)\,\mathrm{arccot}(bx+a)}{d}+\frac{b\left(-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\,\mathrm{dilog}(1+i(bx+a))}{2}+\frac{i\,\mathrm{dilog}(1-i(bx+a))}{2}\right)}{d}}{b}$

input
```
int(arccot(b*x+a)/(a*d/b+d*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/d*dilog(1-I*a-I*b*x)+1/2/d*Pi*ln(-I*b*x-I*a)-1/2*I/d*dilog(1+I*a+I*b
*x)
```

Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=\int \frac{\operatorname{arccot}(bx+a)}{dx+\frac{ad}{b}}\,dx$$

input
```
integrate(arccot(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

output
```
integral(b*arccot(b*x + a)/(b*d*x + a*d), x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=\frac{b\int \frac{\operatorname{acot}(a+bx)}{a+bx}\,dx}{d}$$

input
```
integrate(acot(b*x+a)/(a*d/b+d*x),x)
```

output
```
b*Integral(acot(a + b*x)/(a + b*x), x)/d
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(31)=62$.

Time = 0.18 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.71

$$\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=\frac{\operatorname{arccot}(bx+a)\log\left(dx+\frac{ad}{b}\right)}{d}+\frac{\arctan\left(\frac{b^2x+ab}{b}\right)\log\left(dx+\frac{ad}{b}\right)}{d}+\frac{\arctan(bx+a,0)\log\left(b^2x^2+2\,abx+a^2+1\right)-2\,\arctan(bx+a)\log\left(|bx+a|\right)+i\,\mathrm{Li}_2(i\,bx+i\,a+1)}{2\,d}$$

input
```
integrate(arccot(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

output
```
arccot(b*x + a)*log(d*x + a*d/b)/d + arctan((b^2*x + a*b)/b)*log(d*x + a*d
/b)/d + 1/2*(arctan2(b*x + a, 0)*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*arct
an(b*x + a)*log(abs(b*x + a)) + I*dilog(I*b*x + I*a + 1) - I*dilog(-I*b*x
- I*a + 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(31) = 62$.

Time = 0.36 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.29

$$\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = -\frac{\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4 + 2\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2 - 2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right.\right.\cdots}{8\,bd\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2}$$

input
```
integrate(arccot(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

output
```
-1/8*(arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 + 2*arctan(1/(b*x
 + a))*tan(1/2*arctan(1/(b*x + a)))^2 - 2*tan(1/2*arctan(1/(b*x + a)))^3 +
 arctan(1/(b*x + a)) + 2*tan(1/2*arctan(1/(b*x + a))))/(b*d*tan(1/2*arctan
(1/(b*x + a)))^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = \int \frac{\mathrm{acot}(a+b\,x)}{d\,x+\frac{a\,d}{b}}\,dx$$

input
```
int(acot(a + b*x)/(d*x + (a*d)/b),x)
```

output
```
int(acot(a + b*x)/(d*x + (a*d)/b), x)
```

Reduce [F]

$$\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = \frac{\left(\int \frac{acot(bx+a)}{bx+a}dx\right)b}{d}$$

input
```
int(acot(b*x+a)/(a*d/b+d*x),x)
```

output
```
(int(acot(a + b*x)/(a + b*x),x)*b)/d
```

3.15 $\int (a+bx)^2 \sqrt{\cot^{-1}(a+bx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (a+bx)^2 \sqrt{\cot^{-1}(a+bx)}\,dx = \text{Int}\left((a+bx)^2 \sqrt{\cot^{-1}(a+bx)},x\right)$$

output
```
Defer(Int)((b*x+a)^2*arccot(b*x+a)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 3.51 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (a+bx)^2 \sqrt{\cot^{-1}(a+bx)}\,dx = \int (a+bx)^2 \sqrt{\cot^{-1}(a+bx)}\,dx$$

input
```
Integrate[(a + b*x)^2*Sqrt[ArcCot[a + b*x]],x]
```

output
```
Integrate[(a + b*x)^2*Sqrt[ArcCot[a + b*x]], x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2\sqrt{\cot^{-1}(a+bx)}\,dx$$

$$\Big\downarrow \text{5573}$$

$$\int (a+bx)^2\sqrt{\cot^{-1}(a+bx)}dx$$

input
```
Int[(a + b*x)^2*Sqrt[ArcCot[a + b*x]],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int (bx+a)^2\sqrt{\operatorname{arccot}(bx+a)}dx$$

input
```
int((b*x+a)^2*arccot(b*x+a)^(1/2),x)
```

output
```
int((b*x+a)^2*arccot(b*x+a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (a+bx)^2\sqrt{\cot^{-1}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 9.43 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (a+bx)^2\sqrt{\cot^{-1}(a+bx)}\,dx = \int (a+bx)^2\,\sqrt{\operatorname{acot}(a+bx)}\,dx$$

input
```
integrate((b*x+a)**2*acot(b*x+a)**(1/2),x)
```

output
```
Integral((a + b*x)**2*sqrt(acot(a + b*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (a+bx)^2\sqrt{\cot^{-1}(a+bx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a+bx)^2\sqrt{\cot^{-1}(a+bx)}\,dx=\int (bx+a)^2\sqrt{\operatorname{arccot}\left(bx+a\right)}\,dx$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^2*sqrt(arccot(b*x + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a+bx)^2\sqrt{\cot^{-1}(a+bx)}\,dx=\int \sqrt{\operatorname{acot}\left(a+b\,x\right)}\,{\left(a+b\,x\right)}^2\,dx$$

input
```
int(acot(a + b*x)^(1/2)*(a + b*x)^2,x)
```

output
```
int(acot(a + b*x)^(1/2)*(a + b*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 224, normalized size of antiderivative = 12.44

$$\int (a+bx)^2\sqrt{\cot^{-1}(a+bx)}\,dx$$
$$= \sqrt{acot\,(bx+a)}\,a^2x + \sqrt{acot\,(bx+a)}\,ab\,x^2 + \frac{\sqrt{acot\,(bx+a)}\,b^2x^3}{3}$$
$$+ \frac{\left(\int \frac{\sqrt{acot(bx+a)}\,x^3}{acot(bx+a)a^2+2acot(bx+a)abx+acot(bx+a)b^2x^2+acot(bx+a)}dx\right)b^3}{6}$$
$$+ \frac{\left(\int \frac{\sqrt{acot(bx+a)}\,x^2}{acot(bx+a)a^2+2acot(bx+a)abx+acot(bx+a)b^2x^2+acot(bx+a)}dx\right)a\,b^2}{2}$$
$$+ \frac{\left(\int \frac{\sqrt{acot(bx+a)}\,x}{acot(bx+a)a^2+2acot(bx+a)abx+acot(bx+a)b^2x^2+acot(bx+a)}dx\right)a^2b}{2}$$

input

```
int((b*x+a)^2*acot(b*x+a)^(1/2),x)
```

output

```
(6*sqrt(acot(a + b*x))*a**2*x + 6*sqrt(acot(a + b*x))*a*b*x**2 + 2*sqrt(ac
ot(a + b*x))*b**2*x**3 + int((sqrt(acot(a + b*x))*x**3)/(acot(a + b*x)*a**
2 + 2*acot(a + b*x)*a*b*x + acot(a + b*x)*b**2*x**2 + acot(a + b*x)),x)*b*
*3 + 3*int((sqrt(acot(a + b*x))*x**2)/(acot(a + b*x)*a**2 + 2*acot(a + b*x
)*a*b*x + acot(a + b*x)*b**2*x**2 + acot(a + b*x)),x)*a*b**2 + 3*int((sqrt
(acot(a + b*x))*x)/(acot(a + b*x)*a**2 + 2*acot(a + b*x)*a*b*x + acot(a +
b*x)*b**2*x**2 + acot(a + b*x)),x)*a**2*b)/6
```

3.16 $\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 233

$$\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right)\,dx = \frac{bf\left(6d^2e^2-12cdef-(1-6c^2)\,f^2\right)x}{4d^3} + \frac{bf^2(de-cf)(c+dx)^2}{2d^4} + \frac{bf^3(c+dx)^3}{12d^4} + \frac{(e+fx)^4\left(a+b\cot^{-1}(c+dx)\right)}{4f} + \frac{b\left(d^4e^4-4cd^3e^3f-6(1-c^2)\,d^2e^2f^2+4c(3-c^2)\,def^3+(1-6c^2+c^4)\,f^4\right)\arctan(c+dx)}{4d^4f} + \frac{b(de-cf)(de+f-cf)(de-(1+c)f)\log\left(1+(c+dx)^2\right)}{2d^4}$$

output
```
1/4*b*f*(6*d^2*e^2-12*c*d*e*f-(-6*c^2+1)*f^2)*x/d^3+1/2*b*f^2*(-c*f+d*e)*(
d*x+c)^2/d^4+1/12*b*f^3*(d*x+c)^3/d^4+1/4*(f*x+e)^4*(a+b*arccot(d*x+c))/f+
1/4*b*(d^4*e^4-4*c*d^3*e^3*f-6*(-c^2+1)*d^2*e^2*f^2+4*c*(-c^2+3)*d*e*f^3+(
c^4-6*c^2+1)*f^4)*arctan(d*x+c)/d^4/f+1/2*b*(-c*f+d*e)*(-c*f+d*e+f)*(d*e-(
1+c)*f)*ln(1+(d*x+c)^2)/d^4
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.67

$$\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right)\,dx$$
$$=\frac{(e+fx)^4\left(a+b\cot^{-1}(c+dx)\right)+\frac{b\left(6df^2\left(6d^2e^2-12cdef+\left(-1+6c^2\right)f^2\right)x+12f^3(de-cf)(c+dx)^2+2f^4(c+dx)^3-3i(de-(-i+c)f)\right.}{6d^4}}{4f}$$

input
```
Integrate[(e + f*x)^3*(a + b*ArcCot[c + d*x]),x]
```

output
```
((e + f*x)^4*(a + b*ArcCot[c + d*x]) + (b*(6*d*f^2*(6*d^2*e^2 - 12*c*d*e*f
 + (-1 + 6*c^2)*f^2)*x + 12*f^3*(d*e - c*f)*(c + d*x)^2 + 2*f^4*(c + d*x)^
3 - (3*I)*(d*e - (-I + c)*f)^4*Log[I - c - d*x] + (3*I)*(d*e - (I + c)*f)^
4*Log[I + c + d*x]))/(6*d^4))/(4*f)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5571, 27, 5388, 478, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right)\,dx$$
$$\Big\downarrow \; 5571$$
$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)^3\left(a+b\cot^{-1}(c+dx)\right)}{d^3}d(c+dx)}{d}$$
$$\Big\downarrow \; 27$$
$$\frac{\int (de-cf+f(c+dx))^3\left(a+b\cot^{-1}(c+dx)\right)d(c+dx)}{d^4}$$

$$\Big\downarrow \text{5388}$$

$$\frac{\frac{b\int \frac{(de-cf+f(c+dx))^4}{(c+dx)^2+1}d(c+dx)}{4f}+\frac{(f(c+dx)-cf+de)^4\left(a+b\cot^{-1}(c+dx)\right)}{4f}}{d^4}$$

$$\Big\downarrow \text{478}$$

$$\frac{\frac{b\int \Bigg((c+dx)^2f^4+4(de-cf)(c+dx)f^3+\left(6d^2e^2-12cdfe-(1-6c^2)f^2\right)f^2+\frac{d^4e^4-4cd^3fe^3-6\left(1-c^2\right)d^2f^2e^2+4c\left(3-c^2\right)df^3e+\left(c^4-6c^2+1\right)f^4+4f(de-cf)}{(c+dx)^2+1}}{4f}}{d^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(f(c+dx)-cf+de)^4\left(a+b\cot^{-1}(c+dx)\right)}{4f}+\frac{b\left(\arctan(c+dx)\left(-6\left(1-c^2\right)d^2e^2f^2+4c\left(3-c^2\right)def^3+\left(c^4-6c^2+1\right)f^4-4cd^3e^3f+d^4e^4\right)+f^2(c+dx)(-}{}}{d^4}$$

input
```
Int[(e + f*x)^3*(a + b*ArcCot[c + d*x]),x]
```

output
```
(((d*e - c*f + f*(c + d*x))^4*(a + b*ArcCot[c + d*x]))/(4*f) + (b*(f^2*(6*
d^2*e^2 - 12*c*d*e*f - (1 - 6*c^2)*f^2)*(c + d*x) + 2*f^3*(d*e - c*f)*(c +
 d*x)^2 + (f^4*(c + d*x)^3)/3 + (d^4*e^4 - 4*c*d^3*e^3*f - 6*(1 - c^2)*d^2
*e^2*f^2 + 4*c*(3 - c^2)*d*e*f^3 + (1 - 6*c^2 + c^4)*f^4)*ArcTan[c + d*x]
+ 2*f*(d*e - c*f)*(d*e + f - c*f)*(d*e - (1 + c)*f)*Log[1 + (c + d*x)^2]))
/(4*f))/d^4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 478
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[n, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5388
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])/(e*(q + 1))), x] + Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(221) = 442$.

Time = 1.61 (sec) , antiderivative size = 496, normalized size of antiderivative = 2.13

method	result
parts	$-\frac{2f^2bcex}{d^2}+\frac{3f^2bce\arctan(dx+c)}{d^3}-\frac{f^2bc^3e\arctan(dx+c)}{d^3}+\frac{3fbc^2e^2\arctan(dx+c)}{2d^2}-\frac{bf^3c}{4d^4}+\frac{13bf^3c^3}{12d^4}+$
derivativedivides	$\frac{\frac{a(cf-de-f(dx+c))^4}{4d^3f}-\frac{b\left(-\frac{f^3\operatorname{arccot}(dx+c)c^4}{4}+f^2\operatorname{arccot}(dx+c)c^3de+f^3\operatorname{arccot}(dx+c)c^3(dx+c)-\frac{3f\operatorname{arccot}(dx+c)c^2d^2e^2}{2}-3f^2\operatorname{a}\right.}{}}{}$
default	$\frac{\frac{a(cf-de-f(dx+c))^4}{4d^3f}-\frac{b\left(-\frac{f^3\operatorname{arccot}(dx+c)c^4}{4}+f^2\operatorname{arccot}(dx+c)c^3de+f^3\operatorname{arccot}(dx+c)c^3(dx+c)-\frac{3f\operatorname{arccot}(dx+c)c^2d^2e^2}{2}-3f^2\operatorname{a}\right.}{}}{}$
parallelrisch	$-\frac{-3x^4ad^4f^3-12xad^4e^3+3xbdf^3-x^3bd^3f^3+6\ln(d^2x^2+2cdx+c^2+1)bc^3f^3-6\ln(d^2x^2+2cdx+c^2+1)bd^3e^3-6\ln(}{}$
risch	$-\frac{if^3bx^4\ln(1-i(dx+c))}{8}-\frac{ibe^4\ln(d^2x^2+2cdx+c^2+1)}{16f}+\frac{i(fx+e)^4b\ln(1+i(dx+c))}{8f}-\frac{2f^2bcex}{d^2}+\frac{3f^2bce\operatorname{arcta}}{d^3}$

input
```
int((f*x+e)^3*(a+b*arccot(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/d^2*f^2*b*c*e*x+3/d^3*f^2*b*c*e*arctan(d*x+c)-1/d^3*f^2*b*c^3*e*arctan(
d*x+c)+3/2/d^2*f*b*c^2*e^2*arctan(d*x+c)-1/4*b/d^4*f^3*c+13/12*b/d^4*f^3*c
^3+1/12/d*f^3*b*x^3-1/4/d^3*f^3*b*x+1/4/d^4*f^3*b*arctan(d*x+c)+1/4/f*b*e^
4*arctan(d*x+c)+1/4*a*(f*x+e)^4/f+b*f^2*arccot(d*x+c)*e*x^3+3/2*b*f*arccot
(d*x+c)*e^2*x^2-1/2*b/d^4*f^3*ln(1+(d*x+c)^2)*c^3+1/2*b/d^4*f^3*ln(1+(d*x+
c)^2)*c-1/2*b/d^3*f^2*ln(1+(d*x+c)^2)*e-5/2*b/d^3*f^2*c^2*e+3/2*b/d^2*f*c*
e^2+3/2*b/d^3*f^2*ln(1+(d*x+c)^2)*c^2*e-3/2*b/d^2*f*ln(1+(d*x+c)^2)*c*e^2+
1/2*b/d*ln(1+(d*x+c)^2)*e^3-1/4/d^2*f^3*b*c*x^2+1/2/d*f^2*b*e*x^2+3/4/d^3*
f^3*b*c^2*x+3/2/d*f*b*e^2*x-1/d*b*c*e^3*arctan(d*x+c)+1/4/d^4*f^3*b*c^4*ar
ctan(d*x+c)-3/2/d^4*f^3*b*c^2*arctan(d*x+c)-3/2/d^2*f*b*e^2*arctan(d*x+c)+
1/4*b/f*arccot(d*x+c)*e^4+1/4*b*f^3*arccot(d*x+c)*x^4+b*arccot(d*x+c)*x*e^
3
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.39

$$\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right)\,dx$$
$$= \frac{3\,ad^4f^3x^4 + (12\,ad^4ef^2 + bd^3f^3)x^3 + 3\,(6\,ad^4e^2f + 2\,bd^3ef^2 - bcd^2f^3)x^2 + 3\,(4\,ad^4e^3 + 6\,bd^3e^2f - 8\,bcd}{}$$

input
```
integrate((f*x+e)^3*(a+b*arccot(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(3*a*d^4*f^3*x^4 + (12*a*d^4*e*f^2 + b*d^3*f^3)*x^3 + 3*(6*a*d^4*e^2*
f + 2*b*d^3*e*f^2 - b*c*d^2*f^3)*x^2 + 3*(4*a*d^4*e^3 + 6*b*d^3*e^2*f - 8*
b*c*d^2*e*f^2 + (3*b*c^2 - b)*d*f^3)*x + 3*(b*d^4*f^3*x^4 + 4*b*d^4*e*f^2*
x^3 + 6*b*d^4*e^2*f*x^2 + 4*b*d^4*e^3*x)*arccot(d*x + c) - 3*(4*b*c*d^3*e^
3 - 6*(b*c^2 - b)*d^2*e^2*f + 4*(b*c^3 - 3*b*c)*d*e*f^2 - (b*c^4 - 6*b*c^2
 + b)*f^3)*arctan(d*x + c) + 6*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + (3*b*c^2 - b
)*d*e*f^2 - (b*c^3 - b*c)*f^3)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/d^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.37 (sec) , antiderivative size = 654, normalized size of antiderivative = 2.81

$$\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right) \, dx = \text{Too large to display}$$

input
```
integrate((f*x+e)**3*(a+b*acot(d*x+c)),x)
```

output
```
Piecewise((a*e**3*x + 3*a*e**2*f*x**2/2 + a*e*f**2*x**3 + a*f**3*x**4/4 -
b*c**4*f**3*acot(c + d*x)/(4*d**4) + b*c**3*e*f**2*acot(c + d*x)/d**3 - b*
c**3*f**3*log(c/d + x - I/d)/d**4 - I*b*c**3*f**3*acot(c + d*x)/d**4 - 3*b
*c**2*e**2*f*acot(c + d*x)/(2*d**2) + 3*b*c**2*e*f**2*log(c/d + x - I/d)/d
**3 + 3*I*b*c**2*e*f**2*acot(c + d*x)/d**3 + 3*b*c**2*f**3*x/(4*d**3) + 3*
b*c**2*f**3*acot(c + d*x)/(2*d**4) + b*c*e**3*acot(c + d*x)/d - 3*b*c*e**2
*f*log(c/d + x - I/d)/d**2 - 3*I*b*c*e**2*f*acot(c + d*x)/d**2 - 2*b*c*e*f
**2*x/d**2 - b*c*f**3*x**2/(4*d**2) - 3*b*c*e*f**2*acot(c + d*x)/d**3 + b*
c*f**3*log(c/d + x - I/d)/d**4 + I*b*c*f**3*acot(c + d*x)/d**4 + b*e**3*x*
acot(c + d*x) + 3*b*e**2*f*x**2*acot(c + d*x)/2 + b*e*f**2*x**3*acot(c + d
*x) + b*f**3*x**4*acot(c + d*x)/4 + b*e**3*log(c/d + x - I/d)/d + I*b*e**3
*acot(c + d*x)/d + 3*b*e**2*f*x/(2*d) + b*e*f**2*x**2/(2*d) + b*f**3*x**3/
(12*d) + 3*b*e**2*f*acot(c + d*x)/(2*d**2) - b*e*f**2*log(c/d + x - I/d)/d
**3 - I*b*e*f**2*acot(c + d*x)/d**3 - b*f**3*x/(4*d**3) - b*f**3*acot(c +
d*x)/(4*d**4), Ne(d, 0)), ((a + b*acot(c))*(e**3*x + 3*e**2*f*x**2/2 + e*f
**2*x**3 + f**3*x**4/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.46

$$\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right) \, dx = \frac{1}{4} a f^3 x^4 + a e f^2 x^3 + \frac{3}{2} a e^2 f x^2$$
$$+ \frac{3}{2} \left(x^2 \operatorname{arccot}(dx+c) + d \left(\frac{x}{d^2} + \frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3} - \frac{c\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^3} \right) \right) b e^2 f$$
$$+ \frac{1}{2} \left(2\,x^3 \operatorname{arccot}(dx+c) + d \left(\frac{dx^2 - 4\,cx}{d^3} - \frac{2\,(c^3-3\,c)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^4} + \frac{(3\,c^2-1)\log\left(d^2x^2+2\,cdx\right.}{d^4} \right.\right.$$
$$+ \frac{1}{12} \left(3\,x^4 \operatorname{arccot}(dx+c) + d \left(\frac{d^2x^3 - 3\,cdx^2 + 3\,(3\,c^2-1)x}{d^4} + \frac{3\,(c^4-6\,c^2+1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^5} - \frac{6\,(}{} \right.\right.$$
$$+ a e^3 x + \frac{\left(2\,(dx+c)\operatorname{arccot}(dx+c) + \log\left((dx+c)^2+1\right)\right) b e^3}{2\,d}$$

input
```
integrate((f*x+e)^3*(a+b*arccot(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*a*f^3*x^4 + a*e*f^2*x^3 + 3/2*a*e^2*f*x^2 + 3/2*(x^2*arccot(d*x + c) +
 d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*
x + c^2 + 1)/d^3))*b*e^2*f + 1/2*(2*x^3*arccot(d*x + c) + d*((d*x^2 - 4*c*
x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x
^2 + 2*c*d*x + c^2 + 1)/d^4))*b*e*f^2 + 1/12*(3*x^4*arccot(d*x + c) + d*((
d^2*x^3 - 3*c*d*x^2 + 3*(3*c^2 - 1)*x)/d^4 + 3*(c^4 - 6*c^2 + 1)*arctan((d
^2*x + c*d)/d)/d^5 - 6*(c^3 - c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^5))*b*
f^3 + a*e^3*x + 1/2*(2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*b
*e^3/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2265 vs. $2(216) = 432$.

Time = 1.51 (sec) , antiderivative size = 2265, normalized size of antiderivative = 9.72

$$\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right) \, dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*(a+b*arccot(d*x+c)),x, algorithm="giac")
```

output
```
-1/192*(96*b*d^3*e^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^5 -
288*b*c*d^2*e^2*f*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^5 + 288
*b*c^2*d*e*f^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^5 - 96*b*c
^3*f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^5 - 72*b*d^2*e^2*f
*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^6 + 144*b*c*d*e*f^2*arct
an(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^6 - 72*b*c^2*f^3*arctan(1/(d*
x + c))*tan(1/2*arctan(1/(d*x + c)))^6 + 24*b*d*e*f^2*arctan(1/(d*x + c))*
tan(1/2*arctan(1/(d*x + c)))^7 - 24*b*c*f^3*arctan(1/(d*x + c))*tan(1/2*ar
ctan(1/(d*x + c)))^7 - 3*b*f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x +
 c)))^8 + 96*b*d^3*e^3*log(16*tan(1/2*arctan(1/(d*x + c)))^2/(tan(1/2*arct
an(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan
(1/(d*x + c)))^4 - 288*b*c*d^2*e^2*f*log(16*tan(1/2*arctan(1/(d*x + c)))^2
/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*
tan(1/2*arctan(1/(d*x + c)))^4 + 288*b*c^2*d*e*f^2*log(16*tan(1/2*arctan(1
/(d*x + c)))^2/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x +
 c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c)))^4 - 96*b*c^3*f^3*log(16*tan(1/2
*arctan(1/(d*x + c)))^2/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan
(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c)))^4 + 96*a*d^3*e^3*tan(1
/2*arctan(1/(d*x + c)))^5 - 288*a*c*d^2*e^2*f*tan(1/2*arctan(1/(d*x + c)))
^5 + 288*a*c^2*d*e*f^2*tan(1/2*arctan(1/(d*x + c)))^5 - 96*a*c^3*f^3*ta...
```

Mupad [B] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 783, normalized size of antiderivative = 3.36

$$\int (e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)\,dx$$

$$= \operatorname{acot}(c+d\,x)\left(b\,e^3\,x+\frac{3\,b\,e^2\,f\,x^2}{2}+b\,e\,f^2\,x^3+\frac{b\,f^3\,x^4}{4}\right)$$

$$+x\left(\frac{e\,(6\,a\,c^2\,f^2+12\,a\,c\,d\,e\,f+2\,a\,d^2\,e^2+3\,b\,d\,e\,f+6\,a\,f^2)}{2\,d^2}-\frac{(4\,c^2+4)\,\left(\frac{f^2\,(b\,f+8\,a\,c\,f+12\,a\,d\,e)}{4\,d}-\frac{2\,a\,c\,f^3}{d}\right)}{4\,d^2}+\frac{2\,c\,\left(\frac{2\,c\,\left(\frac{f^2\,(b\,f+8\,a\,c\,f+12\,a\,d\,e)}{4\,d}-\frac{2\,a\,c\,f^3}{d}\right)}{d}-\frac{4\,a\,c^2\,f^3+24\,a\,c\,d\,e\,f^2+12\,a\,d^2\,e^2\,f+4\,b\,d\,e\,f^2+4\,a\,f^3}{4\,d^2}+\frac{a\,f^3\,(4\,c^2+4)}{4\,d^2}\right)}{d}\right)$$

$$-x^2\left(\frac{c\,\left(\frac{f^2\,(b\,f+8\,a\,c\,f+12\,a\,d\,e)}{4\,d}-\frac{2\,a\,c\,f^3}{d}\right)}{d}-\frac{4\,a\,c^2\,f^3+24\,a\,c\,d\,e\,f^2+12\,a\,d^2\,e^2\,f+4\,b\,d\,e\,f^2+4\,a\,f^3}{8\,d^2}+\frac{a\,f^3\,(4\,c^2+4)}{8\,d^2}\right)$$

$$+x^3\left(\frac{f^2\,(b\,f+8\,a\,c\,f+12\,a\,d\,e)}{12\,d}-\frac{2\,a\,c\,f^3}{3\,d}\right)+\frac{a\,f^3\,x^4}{4}$$

$$+\frac{\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)\,\left(-64\,b\,c^3\,d^4\,f^3+192\,b\,c^2\,d^5\,e\,f^2-192\,b\,c\,d^6\,e^2\,f+64\,b\,c\,d^4\,f^3+64\,b\,d^7\,e^3\,\cdots\right.}{128\,d^8}$$

$$+\frac{b\,\operatorname{atan}\left(\frac{4\,d^3\,\left(\frac{c\,\left(c^4\,f^3-4\,c^3\,d\,e\,f^2+6\,c^2\,d^2\,e^2\,f-6\,c^2\,f^3-4\,c\,d^3\,e^3+12\,c\,d\,e\,f^2-6\,d^2\,e^2\,f+f^3\right)}{4\,d^3}+\frac{x\,\left(c^4\,f^3-4\,c^3\,d\,e\,f^2+6\,c^2\,d^2\,e^2\,f-6\,c^2\,f^3-4\,c\,d^3\,e^3+1\cdots\right.}{4\,d^2}\right.}{c^4\,f^3-4\,c^3\,d\,e\,f^2+6\,c^2\,d^2\,e^2\,f-6\,c^2\,f^3-4\,c\,d^3\,e^3+12\,c\,d\,e\,f^2-6\,d^2\,e^2\,f+f^3}\right.}{4\cdots}$$

input
```
int((e + f*x)^3*(a + b*acot(c + d*x)),x)
```

output
```
acot(c + d*x)*((b*f^3*x^4)/4 + b*e^3*x + (3*b*e^2*f*x^2)/2 + b*e*f^2*x^3)
+ x*((e*(6*a*f^2 + 6*a*c^2*f^2 + 2*a*d^2*e^2 + 3*b*d*e*f + 12*a*c*d*e*f))/
(2*d^2) - ((4*c^2 + 4)*((f^2*(b*f + 8*a*c*f + 12*a*d*e))/(4*d) - (2*a*c*f^
3)/d))/(4*d^2) + (2*c*((2*c*((f^2*(b*f + 8*a*c*f + 12*a*d*e))/(4*d) - (2*a
*c*f^3)/d))/d - (4*a*f^3 + 4*a*c^2*f^3 + 4*b*d*e*f^2 + 12*a*d^2*e^2*f + 24
*a*c*d*e*f^2)/(4*d^2) + (a*f^3*(4*c^2 + 4))/(4*d^2)))/d) - x^2*((c*((f^2*(
b*f + 8*a*c*f + 12*a*d*e))/(4*d) - (2*a*c*f^3)/d))/d - (4*a*f^3 + 4*a*c^2*
f^3 + 4*b*d*e*f^2 + 12*a*d^2*e^2*f + 24*a*c*d*e*f^2)/(8*d^2) + (a*f^3*(4*c
^2 + 4))/(8*d^2)) + x^3*((f^2*(b*f + 8*a*c*f + 12*a*d*e))/(12*d) - (2*a*c*
f^3)/(3*d)) + (a*f^3*x^4)/4 + (log(c^2 + d^2*x^2 + 2*c*d*x + 1)*(64*b*d^7*
e^3 - 64*b*c^3*d^4*f^3 + 64*b*c*d^4*f^3 - 64*b*d^5*e*f^2 - 192*b*c*d^6*e^2
*f + 192*b*c^2*d^5*e*f^2))/(128*d^8) + (b*atan((4*d^3*((c*(f^3 - 6*c^2*f^3
 + c^4*f^3 - 4*c*d^3*e^3 - 6*d^2*e^2*f + 6*c^2*d^2*e^2*f + 12*c*d*e*f^2 -
4*c^3*d*e*f^2))/(4*d^3) + (x*(f^3 - 6*c^2*f^3 + c^4*f^3 - 4*c*d^3*e^3 - 6*
d^2*e^2*f + 6*c^2*d^2*e^2*f + 12*c*d*e*f^2 - 4*c^3*d*e*f^2))/(4*d^2)))/(f^
3 - 6*c^2*f^3 + c^4*f^3 - 4*c*d^3*e^3 - 6*d^2*e^2*f + 6*c^2*d^2*e^2*f + 12
*c*d*e*f^2 - 4*c^3*d*e*f^2))*(f^3 - 6*c^2*f^3 + c^4*f^3 - 4*c*d^3*e^3 - 6*
d^2*e^2*f + 6*c^2*d^2*e^2*f + 12*c*d*e*f^2 - 4*c^3*d*e*f^2))/(4*d^4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.12

$$\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right) \, dx$$
$$= \frac{12acot(dx+c)\,b\,d^4e^3x + 3acot(dx+c)\,b\,d^4f^3x^4 + 18acot(dx+c)\,b\,d^2e^2f - 6\log(d^2x^2+2cdx+c^2+1)}{}$$

input
```
int((f*x+e)^3*(a+b*acot(d*x+c)),x)
```

output

```
( - 3*acot(c + d*x)*b*c**4*f**3 + 12*acot(c + d*x)*b*c**3*d*e*f**2 - 18*ac
ot(c + d*x)*b*c**2*d**2*e**2*f + 18*acot(c + d*x)*b*c**2*f**3 + 12*acot(c
+ d*x)*b*c*d**3*e**3 - 36*acot(c + d*x)*b*c*d*e*f**2 + 12*acot(c + d*x)*b*
d**4*e**3*x + 18*acot(c + d*x)*b*d**4*e**2*f*x**2 + 12*acot(c + d*x)*b*d**
4*e*f**2*x**3 + 3*acot(c + d*x)*b*d**4*f**3*x**4 + 18*acot(c + d*x)*b*d**2
*e**2*f - 3*acot(c + d*x)*b*f**3 - 6*log(c**2 + 2*c*d*x + d**2*x**2 + 1)*b
*c**3*f**3 + 18*log(c**2 + 2*c*d*x + d**2*x**2 + 1)*b*c**2*d*e*f**2 - 18*l
og(c**2 + 2*c*d*x + d**2*x**2 + 1)*b*c*d**2*e**2*f + 6*log(c**2 + 2*c*d*x
+ d**2*x**2 + 1)*b*c*f**3 + 6*log(c**2 + 2*c*d*x + d**2*x**2 + 1)*b*d**3*e
**3 - 6*log(c**2 + 2*c*d*x + d**2*x**2 + 1)*b*d*e*f**2 + 12*a*d**4*e**3*x
+ 18*a*d**4*e**2*f*x**2 + 12*a*d**4*e*f**2*x**3 + 3*a*d**4*f**3*x**4 + 9*b
*c**2*d*f**3*x - 24*b*c*d**2*e*f**2*x - 3*b*c*d**2*f**3*x**2 + 18*b*d**3*e
**2*f*x + 6*b*d**3*e*f**2*x**2 + b*d**3*f**3*x**3 - 3*b*d*f**3*x)/(12*d**4
)
```

3.17 $\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 154

$$\begin{aligned}\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)\,dx &= \frac{bf(de-cf)x}{d^2}+\frac{bf^2(c+dx)^2}{6d^3}+\frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)}{3f}\\ &\quad+\frac{b(de-cf)\left(d^2e^2-2cdef-(3-c^2)f^2\right)\arctan(c+dx)}{3d^3f}\\ &\quad+\frac{b(3d^2e^2-6cdef-(1-3c^2)f^2)\log\left(1+(c+dx)^2\right)}{6d^3}\end{aligned}$$

output
```
b*f*(-c*f+d*e)*x/d^2+1/6*b*f^2*(d*x+c)^2/d^3+1/3*(f*x+e)^3*(a+b*arccot(d*x
+c))/f+1/3*b*(-c*f+d*e)*(d^2*e^2-2*c*d*e*f-(-c^2+3)*f^2)*arctan(d*x+c)/d^3
/f+1/6*b*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*ln(1+(d*x+c)^2)/d^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.77

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)\,dx$$
$$=\frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)+\frac{b\left(6df^2(de-cf)x+f^3(c+dx)^2-i(de-(-i+c)f)^3\log(i-c-dx)+i(de-(i+c)f)^3\log(i+c+dx)\right)}{2d^3}}{3f}$$

input
```
Integrate[(e + f*x)^2*(a + b*ArcCot[c + d*x]),x]
```

output
```
((e + f*x)^3*(a + b*ArcCot[c + d*x]) + (b*(6*d*f^2*(d*e - c*f)*x + f^3*(c
+ d*x)^2 - I*(d*e - (-I + c)*f)^3*Log[I - c - d*x] + I*(d*e - (I + c)*f)^3
*Log[I + c + d*x]))/(2*d^3))/(3*f)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5571, 27, 5388, 478, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)dx$$

$$\Big\downarrow \; 5571$$

$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)^2\left(a+b\cot^{-1}(c+dx)\right)}{d^2}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\int (de-cf+f(c+dx))^2\left(a+b\cot^{-1}(c+dx)\right)d(c+dx)}{d^3}$$

$$\Big\downarrow \; 5388$$

$$\frac{\frac{b\int \frac{(de-cf+f(c+dx))^3}{(c+dx)^2+1}d(c+dx)}{3f}+\frac{(f(c+dx)-cf+de)^3(a+b\cot^{-1}(c+dx))}{3f}}{d^3}$$

$$\Big\downarrow \; 478$$

$$\frac{\frac{b\int \left((c+dx)f^3+3(de-cf)f^2+\frac{(de-cf)\left(d^2e^2-2cdfe+c^2f^2-3f^2\right)+f\left(3d^2e^2-6cdfe-\left(1-3c^2\right)f^2\right)(c+dx)}{(c+dx)^2+1}\right)d(c+dx)}{3f}+\frac{(f(c+dx)-cf+de)^3(a+b\cot^{-1}}{3f}}{d^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{(f(c+dx)-cf+de)^3(a+b\cot^{-1}(c+dx))}{3f}+\frac{b(\arctan(c+dx)(de-cf)(-(3-c^2)f^2-2cdef+d^2e^2)+\frac{1}{2}f(-(1-3c^2)f^2-6cdef+3d^2e^2)\log((c+dx)^2}{3f}}{d^3}$$

input
```
Int[(e + f*x)^2*(a + b*ArcCot[c + d*x]),x]
```

output
```
(((d*e - c*f + f*(c + d*x))^3*(a + b*ArcCot[c + d*x]))/(3*f) + (b*(3*f^2*(
d*e - c*f)*(c + d*x) + (f^3*(c + d*x)^2)/2 + (d*e - c*f)*(d^2*e^2 - 2*c*d*
e*f - (3 - c^2)*f^2)*ArcTan[c + d*x] + (f*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*
c^2)*f^2)*Log[1 + (c + d*x)^2])/2))/(3*f))/d^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 478
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[n, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5388
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])/(e*(q + 1))), x] + Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(146) = 292.

Time = 0.92 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.91

method	result
parts	$\frac{a(fx+e)^3}{3f}-\frac{2f^2bcx}{3d^2}+\frac{fbex}{d}+\frac{b\,f^2\ln\left(1+(dx+c)^2\right)c^2}{2d^3}+\frac{b\ln\left(1+(dx+c)^2\right)e^2}{2d}-\frac{f^2b\,c^3\arctan(dx+c)}{3d^3}+$
parallelrisch	$-\ln(d^2x^2+2cdx+c^2+1)b\,f^2-f^2b-6a\,c^2efd-6efad+7b\,c^2f^2+6\operatorname{arccot}(dx+c)bdef+2x^3\operatorname{arccot}(dx+c)b\,d^3f^2+6x$
derivativedivides	$-\frac{a(cf-de-f(dx+c))^3}{3d^2f}+\frac{b\left(-\frac{f^2\operatorname{arccot}(dx+c)c^3}{3}+f\operatorname{arccot}(dx+c)c^2de+f^2\operatorname{arccot}(dx+c)c^2(dx+c)-\operatorname{arccot}(dx+c)c\,d^2e^2-2f\right.}{}$
default	$-\frac{a(cf-de-f(dx+c))^3}{3d^2f}+\frac{b\left(-\frac{f^2\operatorname{arccot}(dx+c)c^3}{3}+f\operatorname{arccot}(dx+c)c^2de+f^2\operatorname{arccot}(dx+c)c^2(dx+c)-\operatorname{arccot}(dx+c)c\,d^2e^2-2f\right.}{}$
risch	$-\frac{ib\,e^2x\ln(1-i(dx+c))}{2}-\frac{f^2b\ln\left(d^2x^2+2cdx+c^2+1\right)}{6d^3}+\frac{e^2b\ln\left(d^2x^2+2cdx+c^2+1\right)}{2d}-\frac{fbe\arctan(dx+c)}{d^2}-$

input
```
int((f*x+e)^2*(a+b*arccot(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*(f*x+e)^3/f-2/3/d^2*f^2*b*c*x+1/d*f*b*e*x+1/2*b/d^3*f^2*ln(1+(d*x+c)
^2)*c^2+1/2*b/d*ln(1+(d*x+c)^2)*e^2-1/3/d^3*f^2*b*c^3*arctan(d*x+c)+1/3/f*
b*e^3*arctan(d*x+c)+1/d^3*f^2*b*c*arctan(d*x+c)-1/d^2*f*b*e*arctan(d*x+c)+
1/3*b*f^2*arccot(d*x+c)*x^3+b*arccot(d*x+c)*x*e^2+b/d^2*f*c*e+1/3*b/f*arcc
ot(d*x+c)*e^3-b/d^2*f*ln(1+(d*x+c)^2)*c*e+1/d^2*f*b*c^2*e*arctan(d*x+c)-1/
d*b*c*e^2*arctan(d*x+c)+b*f*arccot(d*x+c)*e*x^2-5/6*b/d^3*f^2*c^2+1/6/d*f^
2*b*x^2-1/6*b/d^3*f^2*ln(1+(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.34

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)\,dx$$
$$= \frac{2\,ad^3f^2x^3 + (6\,ad^3ef + bd^2f^2)x^2 + 2\,(3\,ad^3e^2 + 3\,bd^2ef - 2\,bcdf^2)x + 2\,(bd^3f^2x^3 + 3\,bd^3efx^2 + 3\,bd^3e^2x}{}$$

input
```
integrate((f*x+e)^2*(a+b*arccot(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(2*a*d^3*f^2*x^3 + (6*a*d^3*e*f + b*d^2*f^2)*x^2 + 2*(3*a*d^3*e^2 + 3*
b*d^2*e*f - 2*b*c*d*f^2)*x + 2*(b*d^3*f^2*x^3 + 3*b*d^3*e*f*x^2 + 3*b*d^3*
e^2*x)*arccot(d*x + c) - 2*(3*b*c*d^2*e^2 - 3*(b*c^2 - b)*d*e*f + (b*c^3 -
 3*b*c)*f^2)*arctan(d*x + c) + (3*b*d^2*e^2 - 6*b*c*d*e*f + (3*b*c^2 - b)*
f^2)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/d^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.32 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.44

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)\,dx$$
$$= \begin{cases} ae^2x + aefx^2 + \frac{af^2x^3}{3} + \frac{bc^3f^2\operatorname{acot}(c+dx)}{3d^3} - \frac{bc^2ef\operatorname{acot}(c+dx)}{d^2} + \frac{bc^2f^2\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d^3} + \frac{ibc^2f^2\operatorname{acot}(c+dx)}{d^3} + \frac{bce^2\operatorname{acot}(c+}{d} \\ (a+b\operatorname{acot}(c))\left(e^2x + efx^2 + \frac{f^2x^3}{3}\right) \end{cases}$$

input
```
integrate((f*x+e)**2*(a+b*acot(d*x+c)),x)
```

output
```
Piecewise((a*e**2*x + a*e*f*x**2 + a*f**2*x**3/3 + b*c**3*f**2*acot(c + d*
x)/(3*d**3) - b*c**2*e*f*acot(c + d*x)/d**2 + b*c**2*f**2*log(c/d + x - I/
d)/d**3 + I*b*c**2*f**2*acot(c + d*x)/d**3 + b*c*e**2*acot(c + d*x)/d - 2*
b*c*e*f*log(c/d + x - I/d)/d**2 - 2*I*b*c*e*f*acot(c + d*x)/d**2 - 2*b*c*f
**2*x/(3*d**2) - b*c*f**2*acot(c + d*x)/d**3 + b*e**2*x*acot(c + d*x) + b*
e*f*x**2*acot(c + d*x) + b*f**2*x**3*acot(c + d*x)/3 + b*e**2*log(c/d + x
- I/d)/d + I*b*e**2*acot(c + d*x)/d + b*e*f*x/d + b*f**2*x**2/(6*d) + b*e*
f*acot(c + d*x)/d**2 - b*f**2*log(c/d + x - I/d)/(3*d**3) - I*b*f**2*acot(
c + d*x)/(3*d**3), Ne(d, 0)), ((a + b*acot(c))*(e**2*x + e*f*x**2 + f**2*x
**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.40

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)\,dx = \frac{1}{3}\,af^2x^3 + aefx^2 + \left(x^2\,\mathrm{arccot}\,(dx+c) + d\left(\frac{x}{d^2} + \frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3} - \frac{c\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^3}\right)\right)bef + \frac{1}{6}\left(2\,x^3\,\mathrm{arccot}\,(dx+c) + d\left(\frac{dx^2-4\,cx}{d^3} - \frac{2\,(c^3-3\,c)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^4} + \frac{(3\,c^2-1)\log\left(d^2x^2+2\,cdx\right.}{d^4} + ae^2x + \frac{\left(2\,(dx+c)\,\mathrm{arccot}\,(dx+c) + \log\left((dx+c)^2+1\right)\right)be^2}{2\,d}$$

input
```
integrate((f*x+e)^2*(a+b*arccot(d*x+c)),x, algorithm="maxima")
```

output
```
1/3*a*f^2*x^3 + a*e*f*x^2 + (x^2*arccot(d*x + c) + d*(x/d^2 + (c^2 - 1)*ar
ctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*b*e*f
 + 1/6*(2*x^3*arccot(d*x + c) + d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arc
tan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^
4))*b*f^2 + a*e^2*x + 1/2*(2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 +
 1))*b*e^2/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1161 vs. $2(142) = 284$.

Time = 1.02 (sec) , antiderivative size = 1161, normalized size of antiderivative = 7.54

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*(a+b*arccot(d*x+c)),x, algorithm="giac")
```

output
```
-1/24*(12*b*d^2*e^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^4 - 2
4*b*c*d*e*f*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^4 + 12*b*c^2*
f^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^4 - 6*b*d*e*f*arctan(
1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^5 + 6*b*c*f^2*arctan(1/(d*x + c)
)*tan(1/2*arctan(1/(d*x + c)))^5 + b*f^2*arctan(1/(d*x + c))*tan(1/2*arcta
n(1/(d*x + c)))^6 + 12*b*d^2*e^2*log(16*tan(1/2*arctan(1/(d*x + c)))^2/(ta
n(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(
1/2*arctan(1/(d*x + c)))^3 - 24*b*c*d*e*f*log(16*tan(1/2*arctan(1/(d*x + c
)))^2/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 +
 1))*tan(1/2*arctan(1/(d*x + c)))^3 + 12*b*c^2*f^2*log(16*tan(1/2*arctan(1
/(d*x + c)))^2/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x +
 c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c)))^3 + 12*a*d^2*e^2*tan(1/2*arctan
(1/(d*x + c)))^4 - 24*a*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^4 + 12*a*c^2*
f^2*tan(1/2*arctan(1/(d*x + c)))^4 - 6*a*d*e*f*tan(1/2*arctan(1/(d*x + c))
)^5 + 6*a*c*f^2*tan(1/2*arctan(1/(d*x + c)))^5 + a*f^2*tan(1/2*arctan(1/(d
*x + c)))^6 - 12*b*d^2*e^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c))
)^2 + 24*b*c*d*e*f*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 - 12
*b*c^2*f^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 - 12*b*d*e*f
*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 + 12*b*c*f^2*arctan(1/
(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 + 12*b*d*e*f*tan(1/2*arctan(1...
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.66

$$\int (e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)\,dx$$
$$= x^2\left(\frac{f\,(b\,f+6\,a\,c\,f+6\,a\,d\,e)}{6\,d}-\frac{a\,c\,f^2}{d}\right)-x\left(\frac{2\,c\left(\frac{f\,(b\,f+6\,a\,c\,f+6\,a\,d\,e)}{3\,d}-\frac{2\,a\,c\,f^2}{d}\right)}{d}-\frac{3\,a\,c^2\,f^2+12\,a\,c\,d\,e\,f+3\,a\,d^2\,e^2+3\,b\,d\,e\,f+3\,a\,f^2}{3\,d^2}+\frac{a\,f^2\,(3\,c^2+3)}{3\,d^2}\right)$$
$$+\mathrm{acot}(c+d\,x)\left(b\,e^2\,x+b\,e\,f\,x^2+\frac{b\,f^2\,x^3}{3}\right)+\frac{a\,f^2\,x^3}{3}$$
$$+\frac{\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)\left(36\,b\,c^2\,d^3\,f^2-72\,b\,c\,d^4\,e\,f+36\,b\,d^5\,e^2-12\,b\,d^3\,f^2\right)}{72\,d^6}$$
$$-\frac{b\,\mathrm{atan}\left(\frac{3\,d^2\left(\frac{c\,(c^3\,f^2-3\,c^2\,d\,e\,f+3\,c\,d^2\,e^2-3\,c\,f^2+3\,d\,e\,f)}{3\,d^2}+\frac{x\,(c^3\,f^2-3\,c^2\,d\,e\,f+3\,c\,d^2\,e^2-3\,c\,f^2+3\,d\,e\,f)}{3\,d}\right)}{c^3\,f^2-3\,c^2\,d\,e\,f+3\,c\,d^2\,e^2-3\,c\,f^2+3\,d\,e\,f}\right)(c^3\,f^2-3\,c^2\,d\,e\,f+3\,c\ldots}{3\,d^3}$$

input
```
int((e + f*x)^2*(a + b*acot(c + d*x)),x)
```

output
```
x^2*((f*(b*f + 6*a*c*f + 6*a*d*e))/(6*d) - (a*c*f^2)/d) - x*((2*c*((f*(b*f
 + 6*a*c*f + 6*a*d*e))/(3*d) - (2*a*c*f^2)/d))/d - (3*a*f^2 + 3*a*c^2*f^2
+ 3*a*d^2*e^2 + 3*b*d*e*f + 12*a*c*d*e*f)/(3*d^2) + (a*f^2*(3*c^2 + 3))/(3
*d^2)) + acot(c + d*x)*((b*f^2*x^3)/3 + b*e^2*x + b*e*f*x^2) + (a*f^2*x^3)
/3 + (log(c^2 + d^2*x^2 + 2*c*d*x + 1)*(36*b*d^5*e^2 - 12*b*d^3*f^2 + 36*b
*c^2*d^3*f^2 - 72*b*c*d^4*e*f))/(72*d^6) - (b*atan((3*d^2*((c*(c^3*f^2 - 3
*c*f^2 + 3*c*d^2*e^2 + 3*d*e*f - 3*c^2*d*e*f))/(3*d^2) + (x*(c^3*f^2 - 3*c
*f^2 + 3*c*d^2*e^2 + 3*d*e*f - 3*c^2*d*e*f))/(3*d)))/(c^3*f^2 - 3*c*f^2 +
3*c*d^2*e^2 + 3*d*e*f - 3*c^2*d*e*f))*(c^3*f^2 - 3*c*f^2 + 3*c*d^2*e^2 + 3
*d*e*f - 3*c^2*d*e*f))/(3*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.90

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)\,dx$$
$$= \frac{2acot(dx+c)\,b\,c^3f^2 - 6acot(dx+c)\,b\,c^2def + 6acot(dx+c)\,bc\,d^2e^2 - 6acot(dx+c)\,bc\,f^2 + 6acot(dx+}{}$$

input
```
int((f*x+e)^2*(a+b*acot(d*x+c)),x)
```

output
```
(2*acot(c + d*x)*b*c**3*f**2 - 6*acot(c + d*x)*b*c**2*d*e*f + 6*acot(c + d
*x)*b*c*d**2*e**2 - 6*acot(c + d*x)*b*c*f**2 + 6*acot(c + d*x)*b*d**3*e**2
*x + 6*acot(c + d*x)*b*d**3*e*f*x**2 + 2*acot(c + d*x)*b*d**3*f**2*x**3 +
6*acot(c + d*x)*b*d*e*f + 3*log(c**2 + 2*c*d*x + d**2*x**2 + 1)*b*c**2*f**
2 - 6*log(c**2 + 2*c*d*x + d**2*x**2 + 1)*b*c*d*e*f + 3*log(c**2 + 2*c*d*x
 + d**2*x**2 + 1)*b*d**2*e**2 - log(c**2 + 2*c*d*x + d**2*x**2 + 1)*b*f**2
 + 6*a*d**3*e**2*x + 6*a*d**3*e*f*x**2 + 2*a*d**3*f**2*x**3 - 4*b*c*d*f**2
*x + 6*b*d**2*e*f*x + b*d**2*f**2*x**2)/(6*d**3)
```

3.18 $\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 94

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)\,dx = \frac{bfx}{2d} + \frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)}{2f} - \frac{b(f^2-(de-cf)^2)\arctan(c+dx)}{2d^2f} + \frac{b(de-cf)\log\left(1+(c+dx)^2\right)}{2d^2}$$

output
```
1/2*b*f*x/d+1/2*(f*x+e)^2*(a+b*arccot(d*x+c))/f-1/2*b*(f^2-(-c*f+d*e)^2)*a
rctan(d*x+c)/d^2/f+1/2*b*(-c*f+d*e)*ln(1+(d*x+c)^2)/d^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.73

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)\,dx$$
$$=aex+\frac{1}{2}afx^2+bex\cot^{-1}(c+dx)$$
$$+\frac{bf\left(\frac{1}{2}d\left(-\frac{c}{d}+\frac{c+dx}{d}\right)^2\cot^{-1}(c+dx)+\frac{1}{2}d\left(\frac{x}{d}-\frac{i(i-c)^2\log(i-c-dx)}{2d^2}+\frac{i(i+c)^2\log(i+c+dx)}{2d^2}\right)\right)}{d}$$
$$+\frac{be(-2c\arctan(c+dx)+\log\left(1+c^2+2cdx+d^2x^2\right))}{2d}$$

input
```
Integrate[(e + f*x)*(a + b*ArcCot[c + d*x]),x]
```

output
```
a*e*x + (a*f*x^2)/2 + b*e*x*ArcCot[c + d*x] + (b*f*((d*(-(c/d) + (c + d*x)
/d)^2*ArcCot[c + d*x])/2 + (d*(x/d - ((I/2)*(I - c)^2*Log[I - c - d*x])/d^
2 + ((I/2)*(I + c)^2*Log[I + c + d*x])/d^2))/2))/d + (b*e*(-2*c*ArcTan[c +
 d*x] + Log[1 + c^2 + 2*c*d*x + d^2*x^2]))/(2*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5571, 27, 5388, 478, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)\,dx$$
$$\Big\downarrow \; 5571$$
$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)\left(a+b\cot^{-1}(c+dx)\right)}{d}d(c+dx)}{d}$$
$$\Big\downarrow \; 27$$
$$\frac{\int (de-cf+f(c+dx))\left(a+b\cot^{-1}(c+dx)\right)d(c+dx)}{d^2}$$

$$\Big\downarrow \; 5388$$

$$\frac{\frac{b\int \frac{(de-cf+f(c+dx))^2}{(c+dx)^2+1}d(c+dx)}{2f}+\frac{(f(c+dx)-cf+de)^2\left(a+b\cot^{-1}(c+dx)\right)}{2f}}{d^2}$$

$$\Big\downarrow \; 478$$

$$\frac{\frac{b\int \left(f^2+\frac{(de-cf-f)(de-cf+f)+2f(de-cf)(c+dx)}{(c+dx)^2+1}\right)d(c+dx)}{2f}+\frac{(f(c+dx)-cf+de)^2\left(a+b\cot^{-1}(c+dx)\right)}{2f}}{d^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{(f(c+dx)-cf+de)^2\left(a+b\cot^{-1}(c+dx)\right)}{2f}+\frac{b\left(\arctan(c+dx)(-cf+de+f)(de-(c+1)f)+f(de-cf)\log\left((c+dx)^2+1\right)+f^2(c+dx)\right)}{2f}}{d^2}$$

input
```
Int[(e + f*x)*(a + b*ArcCot[c + d*x]),x]
```

output
```
(((d*e - c*f + f*(c + d*x))^2*(a + b*ArcCot[c + d*x]))/(2*f) + (b*(f^2*(c
+ d*x) + (d*e + f - c*f)*(d*e - (1 + c)*f)*ArcTan[c + d*x] + f*(d*e - c*f)
*Log[1 + (c + d*x)^2]))/(2*f))/d^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 478
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[n, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5388
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])/(e*(q + 1))), x] + Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.20

method	result
parts	$a\left(\frac{1}{2} f\,x^{2}+e x\right)+\frac{b\left(\frac{\operatorname{arccot}(d x+c)(d x+c)^{2} f}{2 d}-\frac{\operatorname{arccot}(d x+c) c f(d x+c)}{d}+\operatorname{arccot}(d x+c) e(d x+c)+\frac{f(d x+c)+\frac{(-2 c f+2 d e)}{\cdots}}{\cdots}\right.}{d}$
derivativedivides	$\frac{-\frac{a\left(f c(d x+c)-e d(d x+c)-\frac{f(d x+c)^{2}}{2}\right)}{d}-\frac{b\left(\operatorname{arccot}(d x+c) f c(d x+c)-\operatorname{arccot}(d x+c) e d(d x+c)-\frac{\operatorname{arccot}(d x+c) f(d x+c)^{2}}{2}-\frac{f(d x+c)}{2}+\cdots\right.}{d}}{d}$
default	$\frac{-\frac{a\left(f c(d x+c)-e d(d x+c)-\frac{f(d x+c)^{2}}{2}\right)}{d}-\frac{b\left(\operatorname{arccot}(d x+c) f c(d x+c)-\operatorname{arccot}(d x+c) e d(d x+c)-\frac{\operatorname{arccot}(d x+c) f(d x+c)^{2}}{2}-\frac{f(d x+c)}{2}+\cdots\right.}{d}}{d}$
parallelrisch	$-\frac{-\operatorname{arccot}(d x+c) b\,d^{2} f\,x^{2}-a\,d^{2} f\,x^{2}-2 x \operatorname{arccot}(d x+c) b\,d^{2} e-2 a\,d^{2} e x+\operatorname{arccot}(d x+c) b\,c^{2} f-2 \operatorname{arccot}(d x+c) b c d e+b c f \cdots}{2 d^{2}}$
risch	$\frac{i b\left(f\,x^{2}+2 e x\right) \ln(1+i(d x+c))}{4}-\frac{i b f\,x^{2} \ln(1-i(d x+c))}{4}+\frac{\pi b f\,x^{2}}{4}-\frac{i b e x \ln(1-i(d x+c))}{2}+\frac{\pi b e x}{2}+\frac{a f\,x^{2}}{2}+\mathrm{ar}\cdots$

input
```
int((f*x+e)*(a+b*arccot(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/2*f*x^2+e*x)+b/d*(1/2/d*arccot(d*x+c)*(d*x+c)^2*f-1/d*arccot(d*x+c)*c
*f*(d*x+c)+arccot(d*x+c)*e*(d*x+c)+1/2/d*(f*(d*x+c)+1/2*(-2*c*f+2*d*e)*ln(
1+(d*x+c)^2)-f*arctan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.17

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)\,dx$$
$$=\frac{ad^2fx^2+(2\,ad^2e+bdf)x+(bd^2fx^2+2\,bd^2ex)\,\mathrm{arccot}\,(dx+c)-(2\,bcde-(bc^2-b)f)\arctan\,(dx+c)-}{2\,d^2}$$

input
```
integrate((f*x+e)*(a+b*arccot(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(a*d^2*f*x^2 + (2*a*d^2*e + b*d*f)*x + (b*d^2*f*x^2 + 2*b*d^2*e*x)*arc
cot(d*x + c) - (2*b*c*d*e - (b*c^2 - b)*f)*arctan(d*x + c) + (b*d*e - b*c*
f)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/d^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.79 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.88

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)\,dx$$
$$=\begin{cases} aex+\frac{afx^2}{2}-\frac{bc^2f\,\mathrm{acot}\,(c+dx)}{2d^2}+\frac{bce\,\mathrm{acot}\,(c+dx)}{d}-\frac{bcf\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d^2}-\frac{ibcf\,\mathrm{acot}\,(c+dx)}{d^2}+bex\,\mathrm{acot}\,(c+dx)+\frac{bfx^2\,\mathrm{acc}}{} \\ (a+b\,\mathrm{acot}\,(c))\left(ex+\frac{fx^2}{2}\right) \end{cases}$$

input
```
integrate((f*x+e)*(a+b*acot(d*x+c)),x)
```

output
```
Piecewise((a*e*x + a*f*x**2/2 - b*c**2*f*acot(c + d*x)/(2*d**2) + b*c*e*ac
ot(c + d*x)/d - b*c*f*log(c/d + x - I/d)/d**2 - I*b*c*f*acot(c + d*x)/d**2
 + b*e*x*acot(c + d*x) + b*f*x**2*acot(c + d*x)/2 + b*e*log(c/d + x - I/d)
/d + I*b*e*acot(c + d*x)/d + b*f*x/(2*d) + b*f*acot(c + d*x)/(2*d**2), Ne(
d, 0)), ((a + b*acot(c))*(e*x + f*x**2/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.20

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)\,dx = \frac{1}{2}\,afx^2 + \frac{1}{2}\left(x^2\,\mathrm{arccot}\left(dx+c\right)+d\left(\frac{x}{d^2}+\frac{\left(c^2-1\right)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3}-\frac{c\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^3}\right)\right)bf + aex + \frac{\left(2\left(dx+c\right)\mathrm{arccot}\left(dx+c\right)+\log\left(\left(dx+c\right)^2+1\right)\right)be}{2\,d}$$

input
```
integrate((f*x+e)*(a+b*arccot(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*a*f*x^2 + 1/2*(x^2*arccot(d*x + c) + d*(x/d^2 + (c^2 - 1)*arctan((d^2*
x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*b*f + a*e*x + 1
/2*(2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*b*e/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. 2(86) = 172.

Time = 0.26 (sec) , antiderivative size = 451, normalized size of antiderivative = 4.80

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*(a+b*arccot(d*x+c)),x, algorithm="giac")
```

output
```
-1/8*(4*b*d*e*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 - 4*b*c*f
*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 - b*f*arctan(1/(d*x +
c))*tan(1/2*arctan(1/(d*x + c)))^4 + 4*b*d*e*log(16*tan(1/2*arctan(1/(d*x
+ c)))^2/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^
2 + 1))*tan(1/2*arctan(1/(d*x + c)))^2 - 4*b*c*f*log(16*tan(1/2*arctan(1/(
d*x + c)))^2/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c
)))^2 + 1))*tan(1/2*arctan(1/(d*x + c)))^2 + 4*a*d*e*tan(1/2*arctan(1/(d*x
 + c)))^3 - 4*a*c*f*tan(1/2*arctan(1/(d*x + c)))^3 - a*f*tan(1/2*arctan(1/
(d*x + c)))^4 - 4*b*d*e*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c))) +
 4*b*c*f*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c))) - 2*b*f*arctan(1
/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 + 2*b*f*tan(1/2*arctan(1/(d*x +
 c)))^3 - 4*a*d*e*tan(1/2*arctan(1/(d*x + c))) + 4*a*c*f*tan(1/2*arctan(1/
(d*x + c))) - 2*a*f*tan(1/2*arctan(1/(d*x + c)))^2 - b*f*arctan(1/(d*x + c
)) - 2*b*f*tan(1/2*arctan(1/(d*x + c))) - a*f)/(d^2*tan(1/2*arctan(1/(d*x
+ c)))^2)
```

Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.45

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)\,dx = a\,e\,x+\frac{a\,f\,x^2}{2}+\frac{b\,e\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{2\,d} + \frac{b\,f\,\mathrm{acot}(c+d\,x)}{2\,d^2}+\frac{b\,f\,x^2\,\mathrm{acot}(c+d\,x)}{2}+\frac{b\,f\,x}{2\,d} + b\,e\,x\,\mathrm{acot}(c+d\,x)-\frac{b\,c^2\,f\,\mathrm{acot}(c+d\,x)}{2\,d^2} - \frac{b\,c\,f\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{2\,d^2} + \frac{b\,c\,e\,\mathrm{acot}(c+d\,x)}{d}$$

input
```
int((e + f*x)*(a + b*acot(c + d*x)),x)
```

output
```
a*e*x + (a*f*x^2)/2 + (b*e*log(c^2 + d^2*x^2 + 2*c*d*x + 1))/(2*d) + (b*f*
acot(c + d*x))/(2*d^2) + (b*f*x^2*acot(c + d*x))/2 + (b*f*x)/(2*d) + b*e*x
*acot(c + d*x) - (b*c^2*f*acot(c + d*x))/(2*d^2) - (b*c*f*log(c^2 + d^2*x^
2 + 2*c*d*x + 1))/(2*d^2) + (b*c*e*acot(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.45

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)\,dx$$
$$=\frac{-acot(dx+c)\,b\,c^2f+2acot(dx+c)\,bcde+2acot(dx+c)\,b\,d^2ex+acot(dx+c)\,b\,d^2f\,x^2+acot(dx+c)}{2d^2}$$

input
```
int((f*x+e)*(a+b*acot(d*x+c)),x)
```

output
```
( - acot(c + d*x)*b*c**2*f + 2*acot(c + d*x)*b*c*d*e + 2*acot(c + d*x)*b*d
**2*e*x + acot(c + d*x)*b*d**2*f*x**2 + acot(c + d*x)*b*f - log(c**2 + 2*c
*d*x + d**2*x**2 + 1)*b*c*f + log(c**2 + 2*c*d*x + d**2*x**2 + 1)*b*d*e +
2*a*d**2*e*x + a*d**2*f*x**2 + b*d*f*x)/(2*d**2)
```

3.19 $\int (a+b\cot^{-1}(c+dx))\ dx$

Optimal result

Integrand size = 10, antiderivative size = 38

$$\int (a+b\cot^{-1}(c+dx))\ dx=ax+\frac{b(c+dx)\cot^{-1}(c+dx)}{d}+\frac{b\log(1+(c+dx)^2)}{2d}$$

output
```
a*x+b*(d*x+c)*arccot(d*x+c)/d+1/2*b*ln(1+(d*x+c)^2)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int (a+b\cot^{-1}(c+dx))\ dx=ax+bx\cot^{-1}(c+dx)+\frac{b(-2c\arctan(c+dx)+\log(1+c^2+2cdx+d^2x^2))}{2d}$$

input
```
Integrate[a + b*ArcCot[c + d*x],x]
```

output
```
a*x + b*x*ArcCot[c + d*x] + (b*(-2*c*ArcTan[c + d*x] + Log[1 + c^2 + 2*c*d
*x + d^2*x^2]))/(2*d)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\cot^{-1}(c+dx)\right)\,dx$$

$$\Big\downarrow \text{2009}$$

$$ax+\frac{b\log\left((c+dx)^2+1\right)}{2d}+\frac{b(c+dx)\cot^{-1}(c+dx)}{d}$$

input
```
Int[a + b*ArcCot[c + d*x],x]
```

output
```
a*x + (b*(c + d*x)*ArcCot[c + d*x])/d + (b*Log[1 + (c + d*x)^2])/(2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
default	$ax+\frac{b\left(\operatorname{arccot}(dx+c)(dx+c)+\frac{\ln\left(1+(dx+c)^{2}\right)}{2}\right)}{d}$	35
parts	$ax+\frac{b\left(\operatorname{arccot}(dx+c)(dx+c)+\frac{\ln\left(1+(dx+c)^{2}\right)}{2}\right)}{d}$	35
derivativedivides	$\frac{(dx+c)a+b\left(\operatorname{arccot}(dx+c)(dx+c)+\frac{\ln\left(1+(dx+c)^{2}\right)}{2}\right)}{d}$	40
parallelrisch	$\frac{b(2x\ \operatorname{arccot}(dx+c)d^{2}+2c\ \operatorname{arccot}(dx+c)d+\ln(d^{2}x^{2}+2cdx+c^{2}+1)d)}{2d^{2}}+ax$	54
risch	$ax+\frac{ibx\ln(1+i(dx+c))}{2}-\frac{ibx\ln(1-i(dx+c))}{2}+\frac{\pi bx}{2}-\frac{bc\arctan(dx+c)}{d}+\frac{b\ln(d^{2}x^{2}+2cdx+c^{2}+1)}{2d}$	79

input
```
int(a+b*arccot(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*x+b/d*(arccot(d*x+c)*(d*x+c)+1/2*ln(1+(d*x+c)^2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

$$\int \left(a+b\cot^{-1}(c+dx)\right)\,dx=\frac{2\,bdx\operatorname{arccot}\left(dx+c\right)+2\,adx-2\,bc\arctan\left(dx+c\right)+b\log\left(d^{2}x^{2}+2\,cdx+c^{2}+1\right)}{2\,d}$$

input
```
integrate(a+b*arccot(d*x+c),x, algorithm="fricas")
```

output
```
1/2*(2*b*d*x*arccot(d*x + c) + 2*a*d*x - 2*b*c*arctan(d*x + c) + b*log(d^2
*x^2 + 2*c*d*x + c^2 + 1))/d
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int \left(a+b\cot^{-1}(c+dx)\right)\,dx = ax+b\left(\begin{cases}\frac{c\,\mathrm{acot}\,(c+dx)}{d}+x\,\mathrm{acot}\,(c+dx)+\frac{\log\left(c^2+2cdx+d^2x^2+1\right)}{2d} & \text{for } d\neq 0\\ x\,\mathrm{acot}\,(c) & \text{otherwise}\end{cases}\right)$$

input
```
integrate(a+b*acot(d*x+c),x)
```

output
```
a*x + b*Piecewise((c*acot(c + d*x)/d + x*acot(c + d*x) + log(c**2 + 2*c*d*
x + d**2*x**2 + 1)/(2*d), Ne(d, 0)), (x*acot(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \left(a+b\cot^{-1}(c+dx)\right)\,dx = ax+\frac{\left(2\,(dx+c)\,\mathrm{arccot}\,(dx+c)+\log\left((dx+c)^2+1\right)\right)b}{2\,d}$$

input
```
integrate(a+b*arccot(d*x+c),x, algorithm="maxima")
```

output
```
a*x + 1/2*(2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*b/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(36) = 72.

Time = 0.17 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.05

$$\int \left(a+b\cot^{-1}(c+dx)\right)\,dx = ax - \frac{\left(\arctan\left(\frac{1}{dx+c}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{dx+c}\right)\right)^2+\log\left(\frac{16\tan\left(\frac{1}{2}\arctan\left(\frac{1}{dx+c}\right)\right)^2}{\tan\left(\frac{1}{2}\arctan\left(\frac{1}{dx+c}\right)\right)^4+2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{dx+c}\right)\right)^2+1}\right)\tan\left(\frac{1}{2}\,\mathrm{arct}\right.}{2\,d\tan\left(\frac{1}{2}\arctan\left(\frac{1}{dx+c}\right)\right)}$$

input
```
integrate(a+b*arccot(d*x+c),x, algorithm="giac")
```

output
```
a*x - 1/2*(arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 + log(16*tan
(1/2*arctan(1/(d*x + c)))^2/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*ar
ctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c))) - arctan(1/(d*x +
c)))*b/(d*tan(1/2*arctan(1/(d*x + c))))
```

Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \left(a+b\cot^{-1}(c+dx)\right)\,dx = a\,x+\frac{\frac{b\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{2}+b\,c\,\mathrm{acot}(c+d\,x)}{d} + b\,x\,\mathrm{acot}(c+d\,x)$$

input
```
int(a + b*acot(c + d*x),x)
```

output
```
a*x + ((b*log(c^2 + d^2*x^2 + 2*c*d*x + 1))/2 + b*c*acot(c + d*x))/d + b*x
*acot(c + d*x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

$$\int \left(a+b\cot^{-1}(c+dx)\right)\,dx = \frac{2acot(dx+c)\,bc+2acot(dx+c)\,bdx+\log(d^2x^2+2cdx+c^2+1)\,b+2adx}{2d}$$

input
```
int(a+b*acot(d*x+c),x)
```

output
```
(2*acot(c + d*x)*b*c + 2*acot(c + d*x)*b*d*x + log(c**2 + 2*c*d*x + d**2*x
**2 + 1)*b + 2*a*d*x)/(2*d)
```

3.20 $\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 162

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx = -\frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2}{1-i(c+dx)}\right)}{f} + \frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2d(e+fx)}{(de+(i-c)f)(1-i(c+dx))}\right)}{f} - \frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{2f} + \frac{ib\,\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+(i-c)f)(1-i(c+dx))}\right)}{2f}$$

output
```
-(a+b*arccot(d*x+c))*ln(2/(1-I*(d*x+c)))/f+(a+b*arccot(d*x+c))*ln(2*d*(f*x
+e)/(d*e+(I-c)*f)/(1-I*(d*x+c)))/f-1/2*I*b*polylog(2,1-2/(1-I*(d*x+c)))/f+
1/2*I*b*polylog(2,1-2*d*(f*x+e)/(d*e+(I-c)*f)/(1-I*(d*x+c)))/f
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 336 vs. 2(162) = 324.

Time = 0.30 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.07

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx$$
$$= \frac{a\log(e+fx)+b\Big((\cot^{-1}(c+dx)+\arctan(c+dx))\log(e+fx)+\arctan(c+dx)\left(\log\left(\frac{1}{\sqrt{1+(c+dx)^2}}\right)-\right.}{}$$

input
```
Integrate[(a + b*ArcCot[c + d*x])/(e + f*x),x]
```

output
```
(a*Log[e + f*x] + b*((ArcCot[c + d*x] + ArcTan[c + d*x])*Log[e + f*x] + Ar
cTan[c + d*x]*(Log[1/Sqrt[1 + (c + d*x)^2]] - Log[Sin[ArcTan[(d*e - c*f)/f
] + ArcTan[c + d*x]]]) + ((I/4)*(Pi - 2*ArcTan[c + d*x])^2 + I*(ArcTan[(d*
e - c*f)/f] + ArcTan[c + d*x])^2 - (Pi - 2*ArcTan[c + d*x])*Log[1 + E^((-2
*I)*ArcTan[c + d*x])] - 2*(ArcTan[(d*e - c*f)/f] + ArcTan[c + d*x])*Log[1
- E^((2*I)*(ArcTan[(d*e - c*f)/f] + ArcTan[c + d*x]))] + (Pi - 2*ArcTan[c
+ d*x])*Log[2/Sqrt[1 + (c + d*x)^2]] + 2*(ArcTan[(d*e - c*f)/f] + ArcTan[c
 + d*x])*Log[2*Sin[ArcTan[(d*e - c*f)/f] + ArcTan[c + d*x]]] + I*PolyLog[2
, -E^((-2*I)*ArcTan[c + d*x])] + I*PolyLog[2, E^((2*I)*(ArcTan[(d*e - c*f)
/f] + ArcTan[c + d*x]))])/2))/f
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5571, 27, 5382, 2849, 2752, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx$$

$$\Big\downarrow \ 5571$$

$$\frac{\int \frac{d(a+b\cot^{-1}(c+dx))}{d\left(e-\frac{cf}{d}\right)+f(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \ 27$$

$$\int \frac{a+b\cot^{-1}(c+dx)}{f(c+dx)-cf+de}d(c+dx)$$

$$\Big\downarrow \ 5382$$

$$\frac{b\int \frac{\log\left(\frac{2(de-cf+f(c+dx))}{(de-cf+if)(1-i(c+dx))}\right)}{(c+dx)^2+1}d(c+dx)}{f}-\frac{b\int \frac{\log\left(\frac{2}{1-i(c+dx)}\right)}{(c+dx)^2+1}d(c+dx)}{f}+\frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2(f(c+dx)-cf+de)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}-\frac{\log\left(\frac{2}{1-i(c+dx)}\right)(a+b\cot^{-1}(c+dx))}{f}$$

$$\Big\downarrow \ 2849$$

$$\frac{b\int \frac{\log\left(\frac{2(de-cf+f(c+dx))}{(de-cf+if)(1-i(c+dx))}\right)}{(c+dx)^2+1}d(c+dx)}{f}-\frac{ib\int \frac{\log\left(\frac{2}{1-i(c+dx)}\right)}{1-\frac{2}{1-i(c+dx)}}d\frac{1}{1-i(c+dx)}}{f}+\frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2(f(c+dx)-cf+de)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}-\frac{\log\left(\frac{2}{1-i(c+dx)}\right)(a+b\cot^{-1}(c+dx))}{f}$$

$$\Big\downarrow \ 2752$$

$$\frac{b\int \frac{\log\left(\frac{2(de-cf+f(c+dx))}{(de-cf+if)(1-i(c+dx))}\right)}{(c+dx)^2+1}d(c+dx)}{f}+\frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2(f(c+dx)-cf+de)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}-\frac{\log\left(\frac{2}{1-i(c+dx)}\right)(a+b\cot^{-1}(c+dx))}{f}-\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{2f}$$

$$\Big\downarrow \ 2897$$

$$\frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2(f(c+dx)-cf+de)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}-\frac{\log\left(\frac{2}{1-i(c+dx)}\right)(a+b\cot^{-1}(c+dx))}{f}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2(de-cf+f(c+dx))}{(de-cf+if)(1-i(c+dx))}\right)}{2f}-\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{2f}$$

input
```
Int[(a + b*ArcCot[c + d*x])/(e + f*x),x]
```

output
```
-(((a + b*ArcCot[c + d*x])*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcCot[c
 + d*x])*Log[(2*(d*e - c*f + f*(c + d*x)))/((d*e + I*f - c*f)*(1 - I*(c +
d*x)))])/f - ((I/2)*b*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/f + ((I/2)*b*Po
lyLog[2, 1 - (2*(d*e - c*f + f*(c + d*x)))/((d*e + I*f - c*f)*(1 - I*(c +
d*x)))])/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5382
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcCot[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*ArcCot[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] - Simp[b*(c/e)
   Int[Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] + Simp[b*(c/e)   Int[Log[2*
c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x]) /; FreeQ[{a
, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.22

method	result
parts	$\frac{a\ln(fx+e)}{f}+\frac{b\left(\frac{d\ln(f(dx+c)-cf+de)\,\mathrm{arccot}(dx+c)}{f}+d\left(-\frac{i\ln(f(dx+c)-cf+de)\left(\ln\left(\frac{if-f(dx+c)}{-cf+de+if}\right)-\ln\left(\frac{if+f(dx+c)}{cf-de+if}\right)\right)}{2f}-\cdots\right)\right)}{d}$
derivativedivides	$\frac{\frac{ad\ln(cf-de-f(dx+c))}{f}-bd\left(-\frac{\ln(cf-de-f(dx+c))\,\mathrm{arccot}(dx+c)}{f}-\frac{i\ln(cf-de-f(dx+c))\left(\ln\left(\frac{if+f(dx+c)}{cf-de+if}\right)-\ln\left(\frac{if-f(dx+c)}{-cf+de+if}\right)\cdots\right)}{2f}\cdots\right)}{d}$
default	$\frac{\frac{ad\ln(cf-de-f(dx+c))}{f}-bd\left(-\frac{\ln(cf-de-f(dx+c))\,\mathrm{arccot}(dx+c)}{f}-\frac{i\ln(cf-de-f(dx+c))\left(\ln\left(\frac{if+f(dx+c)}{cf-de+if}\right)-\ln\left(\frac{if-f(dx+c)}{-cf+de+if}\right)\cdots\right)}{2f}\cdots\right)}{d}$
risch	$-\frac{ib\,\mathrm{dilog}\left(\frac{icf-ide+(-idx-ic+1)f-f}{icf-ide-f}\right)}{2f}-\frac{ib\ln(-idx-ic+1)\ln\left(\frac{icf-ide+(-idx-ic+1)f-f}{icf-ide-f}\right)}{2f}+\frac{\ln(icf-ide+(-idx-\cdots)}{2f}$

input
```
int((a+b*arccot(d*x+c))/(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
a*ln(f*x+e)/f+b/d*(d*ln(f*(d*x+c)-c*f+d*e)/f*arccot(d*x+c)+d*(-1/2*I*ln(f*
(d*x+c)-c*f+d*e)*(ln((I*f-f*(d*x+c))/(d*e+I*f-c*f))-ln((I*f+f*(d*x+c))/(c*
f-d*e+I*f)))/f-1/2*I*(dilog((I*f-f*(d*x+c))/(d*e+I*f-c*f))-dilog((I*f+f*(d
*x+c))/(c*f-d*e+I*f)))/f))
```

Fricas [F]

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx=\int \frac{b\,\mathrm{arccot}\,(dx+c)+a}{fx+e}\,dx$$

input
```
integrate((a+b*arccot(d*x+c))/(f*x+e),x, algorithm="fricas")
```

output
```
integral((b*arccot(d*x + c) + a)/(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acot(d*x+c))/(f*x+e),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx=\int \frac{b\operatorname{arccot}(dx+c)+a}{fx+e}\,dx$$

input
```
integrate((a+b*arccot(d*x+c))/(f*x+e),x, algorithm="maxima")
```

output
```
2*b*integrate(1/2*arctan2(1, d*x + c)/(f*x + e), x) + a*log(f*x + e)/f
```

Giac [F]

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx=\int \frac{b\operatorname{arccot}(dx+c)+a}{fx+e}\,dx$$

input
```
integrate((a+b*arccot(d*x+c))/(f*x+e),x, algorithm="giac")
```

output
```
integrate((b*arccot(d*x + c) + a)/(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx=\int \frac{a+b\,\mathrm{acot}(c+d\,x)}{e+f\,x}\,dx$$

input
```
int((a + b*acot(c + d*x))/(e + f*x),x)
```

output
```
int((a + b*acot(c + d*x))/(e + f*x), x)
```

Reduce [F]

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx=\frac{\left(\int \frac{acot(dx+c)}{fx+e}dx\right)bf+\log(fx+e)\,a}{f}$$

input
```
int((a+b*acot(d*x+c))/(f*x+e),x)
```

output
```
(int(acot(c + d*x)/(e + f*x),x)*b*f + log(e + f*x)*a)/f
```

3.21 $\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 153

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}\,dx = -\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)} - \frac{bd(de-cf)\arctan(c+dx)}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)} - \frac{bd\log(e+fx)}{d^2e^2-2cdef+(1+c^2)\,f^2} + \frac{bd\log\left(1+c^2+2cdx+d^2x^2\right)}{2\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)}$$

output
```
-(a+b*arccot(d*x+c))/f/(f*x+e)-b*d*(-c*f+d*e)*arctan(d*x+c)/f/(d^2*e^2-2*c
*d*e*f+(c^2+1)*f^2)-b*d*ln(f*x+e)/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+b*d*ln(d
^2*x^2+2*c*d*x+c^2+1)/(2*d^2*e^2-4*c*d*e*f+2*(c^2+1)*f^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.77

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}\,dx = \frac{-\frac{a+b\cot^{-1}(c+dx)}{e+fx} + \frac{bd((ide+f-icf)\log(i-c-dx)+(-ide+f+icf)\log(i+c+dx)-2f\log(d(e+fx)))}{2(d^2e^2-2cdef+(1+c^2)f^2)}}{f}$$

input
```
Integrate[(a + b*ArcCot[c + d*x])/(e + f*x)^2,x]
```

output
```
(-((a + b*ArcCot[c + d*x])/(e + f*x)) + (b*d*((I*d*e + f - I*c*f)*Log[I -
c - d*x] + ((-I)*d*e + f + I*c*f)*Log[I + c + d*x] - 2*f*Log[d*(e + f*x)])
)/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)))/f
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5569, 2081, 1144, 27, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}dx$$

$$\Big\downarrow \text{5569}$$

$$-\frac{bd\int \frac{1}{(e+fx)((c+dx)^2+1)}dx}{f}-\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)}$$

$$\Big\downarrow \text{2081}$$

$$-\frac{bd\int \frac{1}{(e+fx)(c^2+2dxc+d^2x^2+1)}dx}{f}-\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)}$$

$$\Big\downarrow \text{1144}$$

$$-\frac{bd\left(\frac{\int \frac{d(de-2cf-dfx)}{c^2+2dxc+d^2x^2+1}dx}{(c^2+1)f^2-2cdef+d^2e^2}+\frac{f\log(e+fx)}{(c^2+1)f^2-2cdef+d^2e^2}\right)}{f}-\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bd\left(\frac{d\int \frac{de-2cf-dfx}{c^2+2dxc+d^2x^2+1}dx}{(c^2+1)f^2-2cdef+d^2e^2}+\frac{f\log(e+fx)}{(c^2+1)f^2-2cdef+d^2e^2}\right)}{f}-\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)}$$

$$\Big\downarrow \text{1142}$$

$$-\frac{bd\left(\frac{d\left((de-cf)\int \frac{1}{c^2+2dxc+d^2x^2+1}dx-\frac{f\int \frac{2d(c+dx)}{c^2+2dxc+d^2x^2+1}dx}{2d}\right)}{(c^2+1)f^2-2cdef+d^2e^2}+\frac{f\log(e+fx)}{(c^2+1)f^2-2cdef+d^2e^2}\right)}{f}-\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)}$$

$\Big\downarrow$ 27

$$-\frac{bd\left(\frac{d\left((de-cf)\int \frac{1}{c^2+2dxc+d^2x^2+1}dx-f\int \frac{c+dx}{c^2+2dxc+d^2x^2+1}dx\right)}{(c^2+1)f^2-2cdef+d^2e^2}+\frac{f\log(e+fx)}{(c^2+1)f^2-2cdef+d^2e^2}\right)}{f}-\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)}$$

$\Big\downarrow$ 1083

$$-\frac{bd\left(\frac{d\left(-f\int \frac{c+dx}{c^2+2dxc+d^2x^2+1}dx-2(de-cf)\int \frac{1}{-4d^2-\left(2xd^2+2cd\right)^2}d(2xd^2+2cd)\right)}{(c^2+1)f^2-2cdef+d^2e^2}+\frac{f\log(e+fx)}{(c^2+1)f^2-2cdef+d^2e^2}\right)}{f}-\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)}$$

$\Big\downarrow$ 217

$$-\frac{bd\left(\frac{d\left(\frac{\arctan\left(\frac{2cd+2d^2x}{2d}\right)(de-cf)}{d}-f\int \frac{c+dx}{c^2+2dxc+d^2x^2+1}dx\right)}{(c^2+1)f^2-2cdef+d^2e^2}+\frac{f\log(e+fx)}{(c^2+1)f^2-2cdef+d^2e^2}\right)}{f}-\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)}$$

$\Big\downarrow$ 1103

$$-\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)}-\frac{bd\left(\frac{d\left(\frac{\arctan\left(\frac{2cd+2d^2x}{2d}\right)(de-cf)}{d}-\frac{f\log\left(c^2+2cdx+d^2x^2+1\right)}{2d}\right)}{(c^2+1)f^2-2cdef+d^2e^2}+\frac{f\log(e+fx)}{(c^2+1)f^2-2cdef+d^2e^2}\right)}{f}$$

input
```
Int[(a + b*ArcCot[c + d*x])/(e + f*x)^2,x]
```

output
```
-((a + b*ArcCot[c + d*x])/(f*(e + f*x))) - (b*d*((f*Log[e + f*x])/(d^2*e^2
 - 2*c*d*e*f + (1 + c^2)*f^2) + (d*(((d*e - c*f)*ArcTan[(2*c*d + 2*d^2*x)/
(2*d)])/d - (f*Log[1 + c^2 + 2*c*d*x + d^2*x^2])/(2*d)))/(d^2*e^2 - 2*c*d*
e*f + (1 + c^2)*f^2)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2081
```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

rule 5569
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCot[c + d*x])^p/(f*(m +
1))), x] + Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcCot[
c + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[p, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05

method	result
parts	$-\frac{a}{(fx+e)f}+\frac{b\left(-\frac{d^{2}\operatorname{arccot}(dx+c)}{(f(dx+c)-cf+de)f}-\frac{d^{2}\left(\frac{-\frac{f\ln\left(1+(dx+c)^{2}\right)}{2}+(-cf+de)\arctan(dx+c)}{c^{2}f^{2}-2cdef+d^{2}e^{2}+f^{2}}+\frac{f\ln(f(dx+c)-cf+de)}{c^{2}f^{2}-2cdef+d^{2}e^{2}+f^{2}}\right)}{f}\right)}{d}$
derivativedivides	$\frac{\frac{a\,d^{2}}{(cf-de-f(dx+c))f}+b\,d^{2}\left(\frac{\operatorname{arccot}(dx+c)}{(cf-de-f(dx+c))f}+\frac{-\frac{f\ln(cf-de-f(dx+c))}{c^{2}f^{2}-2cdef+d^{2}e^{2}+f^{2}}+\frac{\frac{f\ln\left(1+(dx+c)^{2}\right)}{2}+(cf-de)\arctan(dx+c)}{c^{2}f^{2}-2cdef+d^{2}e^{2}+f^{2}}}{f}\right)}{d}$
default	$\frac{\frac{a\,d^{2}}{(cf-de-f(dx+c))f}+b\,d^{2}\left(\frac{\operatorname{arccot}(dx+c)}{(cf-de-f(dx+c))f}+\frac{-\frac{f\ln(cf-de-f(dx+c))}{c^{2}f^{2}-2cdef+d^{2}e^{2}+f^{2}}+\frac{\frac{f\ln\left(1+(dx+c)^{2}\right)}{2}+(cf-de)\arctan(dx+c)}{c^{2}f^{2}-2cdef+d^{2}e^{2}+f^{2}}}{f}\right)}{d}$
parallelrisch	$-\frac{2x\ \operatorname{arccot}(dx+c)bc\,d^{3}f^{2}-2x\ \operatorname{arccot}(dx+c)b\,d^{4}ef+2\ln(fx+e)xb\,d^{3}f^{2}-\ln\left(d^{2}x^{2}+2cdx+c^{2}+1\right)xb\,d^{3}f^{2}+2\ \operatorname{arccot}(dx}{}$
risch	$-\frac{ib\ln(1+i(dx+c))}{2f(fx+e)}-\frac{\pi b\,c^{2}f^{2}+\pi b\,d^{2}e^{2}+2f^{2}a-4acdef+\pi b\,f^{2}+2e^{2}a\,d^{2}-ib\,c^{2}f^{2}\ln(1-i(dx+c))-ib\,d^{2}e^{2}\ln(1-i(dx}{}$

input
```
int((a+b*arccot(d*x+c))/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-a/(f*x+e)/f+b/d*(-d^2/(f*(d*x+c)-c*f+d*e)/f*arccot(d*x+c)-d^2/f*(1/(c^2*f
^2-2*c*d*e*f+d^2*e^2+f^2)*(-1/2*f*ln(1+(d*x+c)^2)+(-c*f+d*e)*arctan(d*x+c)
)+1/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f*ln(f*(d*x+c)-c*f+d*e)))
```

Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.46

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}\,dx = -\frac{2\,ad^2e^2-4\,acdef+2\,(ac^2+a)f^2+2\,(bd^2e^2-2\,bcdef+(bc^2+b)f^2)\,\mathrm{arccot}\,(dx+c)+2\,(bd^2e^2-bc\ldots}{2\,(d^2e^3f-2\,cde^2f^2+(c^2+\ldots}$$

input
```
integrate((a+b*arccot(d*x+c))/(f*x+e)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a*d^2*e^2 - 4*a*c*d*e*f + 2*(a*c^2 + a)*f^2 + 2*(b*d^2*e^2 - 2*b*c
*d*e*f + (b*c^2 + b)*f^2)*arccot(d*x + c) + 2*(b*d^2*e^2 - b*c*d*e*f + (b*
d^2*e*f - b*c*d*f^2)*x)*arctan(d*x + c) - (b*d*f^2*x + b*d*e*f)*log(d^2*x^
2 + 2*c*d*x + c^2 + 1) + 2*(b*d*f^2*x + b*d*e*f)*log(f*x + e))/(d^2*e^3*f
- 2*c*d*e^2*f^2 + (c^2 + 1)*e*f^3 + (d^2*e^2*f^2 - 2*c*d*e*f^3 + (c^2 + 1)
*f^4)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acot(d*x+c))/(f*x+e)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.16

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}\,dx = -\frac{1}{2}\left(d\left(\frac{2\,(d^2e-cdf)\arctan\left(\frac{d^2x+cd}{d}\right)}{(d^2e^2f-2\,cdef^2+(c^2+1)f^3)d}-\frac{\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^2e^2-2\,cdef+(c^2+1)f^2}+\frac{2\,\log\left(fx+e\right)}{d^2e^2-2\,cdef+(c^2+1)f^2}\right)\right. - \frac{a}{f^2x+ef}$$

input
```
integrate((a+b*arccot(d*x+c))/(f*x+e)^2,x, algorithm="maxima")
```

output
```
-1/2*(d*(2*(d^2*e - c*d*f)*arctan((d^2*x + c*d)/d)/((d^2*e^2*f - 2*c*d*e*f
^2 + (c^2 + 1)*f^3)*d) - log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2 - 2*c*d
*e*f + (c^2 + 1)*f^2) + 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 + 1)*f^
2)) + 2*arccot(d*x + c)/(f^2*x + e*f))*b - a/(f^2*x + e*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1264 vs. 2(151) = 302.

Time = 0.32 (sec) , antiderivative size = 1264, normalized size of antiderivative = 8.26

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*arccot(d*x+c))/(f*x+e)^2,x, algorithm="giac")
```

output
```
-1/2*(2*b*d*e*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 - 2*b*c*f
*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 + 2*b*d*e*log(4*(4*d^2
*e^2*tan(1/2*arctan(1/(d*x + c)))^2 - 8*c*d*e*f*tan(1/2*arctan(1/(d*x + c)
))^2 + 4*c^2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 - 4*d*e*f*tan(1/2*arctan(1
/(d*x + c)))^3 + 4*c*f^2*tan(1/2*arctan(1/(d*x + c)))^3 + f^2*tan(1/2*arct
an(1/(d*x + c)))^4 + 4*d*e*f*tan(1/2*arctan(1/(d*x + c))) - 4*c*f^2*tan(1/
2*arctan(1/(d*x + c))) - 2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 + f^2)/(tan(
1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/
2*arctan(1/(d*x + c))) - 2*b*c*f*log(4*(4*d^2*e^2*tan(1/2*arctan(1/(d*x +
c)))^2 - 8*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/2*arct
an(1/(d*x + c)))^2 - 4*d*e*f*tan(1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2*tan(
1/2*arctan(1/(d*x + c)))^3 + f^2*tan(1/2*arctan(1/(d*x + c)))^4 + 4*d*e*f*
tan(1/2*arctan(1/(d*x + c))) - 4*c*f^2*tan(1/2*arctan(1/(d*x + c))) - 2*f^
2*tan(1/2*arctan(1/(d*x + c)))^2 + f^2)/(tan(1/2*arctan(1/(d*x + c)))^4 +
2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c))) + 2*a*
d*e*tan(1/2*arctan(1/(d*x + c)))^2 - 2*a*c*f*tan(1/2*arctan(1/(d*x + c)))^
2 - b*f*log(4*(4*d^2*e^2*tan(1/2*arctan(1/(d*x + c)))^2 - 8*c*d*e*f*tan(1/
2*arctan(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 - 4*d*
e*f*tan(1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2*tan(1/2*arctan(1/(d*x + c)))^
3 + f^2*tan(1/2*arctan(1/(d*x + c)))^4 + 4*d*e*f*tan(1/2*arctan(1/(d*x ...
```

Mupad [B] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.84

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}\,dx = -\frac{a}{x\,f^2+e\,f}-\frac{b\,\mathrm{acot}(c+d\,x)}{f\,\left(e+f\,x\right)} - \frac{b\,d\,\ln\left(e+f\,x\right)}{d^2\,e^2-2\,c\,d\,e\,f+\left(c^2+1\right)\,f^2} + \frac{b\,d\,\ln\left(c+d\,x-\mathrm{i}\right)\,1\mathrm{i}}{2\,f\,\left(d\,e-c\,f+f\,1\mathrm{i}\right)}+\frac{b\,d\,\ln\left(c+d\,x+1\mathrm{i}\right)}{2\,f\,\left(f-c\,f\,1\mathrm{i}+d\,e\,1\mathrm{i}\right)}$$

input
```
int((a + b*acot(c + d*x))/(e + f*x)^2,x)
```

output
```
(b*d*log(c + d*x - 1i)*1i)/(2*f*(f*1i - c*f + d*e)) - (b*acot(c + d*x))/(f
*(e + f*x)) - (b*d*log(e + f*x))/(f^2*(c^2 + 1) + d^2*e^2 - 2*c*d*e*f) - a
/(e*f + f^2*x) + (b*d*log(c + d*x + 1i))/(2*f*(f - c*f*1i + d*e*1i))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.08

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}\,dx$$
$$= \frac{2acot(dx+c)\,b\,c^2f^2x-4acot(dx+c)\,bcdefx+2acot(dx+c)\,b\,d^2e^2x+2acot(dx+c)\,b\,f^2x+2atan(dx}{}$$

input
```
int((a+b*acot(d*x+c))/(f*x+e)^2,x)
```

output
```
(2*acot(c + d*x)*b*c**2*f**2*x - 4*acot(c + d*x)*b*c*d*e*f*x + 2*acot(c +
d*x)*b*d**2*e**2*x + 2*acot(c + d*x)*b*f**2*x + 2*atan(c + d*x)*b*c**2*e*f
 + 2*atan(c + d*x)*b*c**2*f**2*x - 2*atan(c + d*x)*b*c*d*e**2 - 2*atan(c +
 d*x)*b*c*d*e*f*x + 2*atan(c + d*x)*b*e*f + 2*atan(c + d*x)*b*f**2*x + log
(c**2 + 2*c*d*x + d**2*x**2 + 1)*b*d*e**2 + log(c**2 + 2*c*d*x + d**2*x**2
 + 1)*b*d*e*f*x - 2*log(e + f*x)*b*d*e**2 - 2*log(e + f*x)*b*d*e*f*x + 2*a
*c**2*f**2*x - 4*a*c*d*e*f*x + 2*a*d**2*e**2*x + 2*a*f**2*x)/(2*e*(c**2*e*
f**2 + c**2*f**3*x - 2*c*d*e**2*f - 2*c*d*e*f**2*x + d**2*e**3 + d**2*e**2
*f*x + e*f**2 + f**3*x))
```

3.22 $\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 228

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^3}\,dx = \frac{bd}{2\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)(e+fx)} - \frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2} - \frac{bd^2(de+f-cf)(de-(1+c)f)\arctan(c+dx)}{2f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)^2} - \frac{bd^2(de-cf)\log(e+fx)}{\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)^2} + \frac{bd^2(de-cf)\log\left(1+c^2+2cdx+d^2x^2\right)}{2\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)^2}$$

output
```
1/2*b*d/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)/(f*x+e)-1/2*(a+b*arccot(d*x+c))/f/
(f*x+e)^2-1/2*b*d^2*(-c*f+d*e+f)*(d*e-(1+c)*f)*arctan(d*x+c)/f/(d^2*e^2-2*
c*d*e*f+(c^2+1)*f^2)^2-b*d^2*(-c*f+d*e)*ln(f*x+e)/(d^2*e^2-2*c*d*e*f+(c^2+
1)*f^2)^2+1/2*b*d^2*(-c*f+d*e)*ln(d^2*x^2+2*c*d*x+c^2+1)/(d^2*e^2-2*c*d*e*
f+(c^2+1)*f^2)^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.79

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^3}\,dx$$
$$=\frac{\frac{bdf}{(d^2e^2-2cdef+(1+c^2)f^2)(e+fx)}-\frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}+\frac{ibd^2\log(i-c-dx)}{2(de-(-i+c)f)^2}-\frac{ibd^2\log(i+c+dx)}{2(de-(i+c)f)^2}-\frac{2bd^2f(de-cf)\log(d(e+fx))}{(d^2e^2-2cdef+(1+c^2)f^2)^2}}{2f}$$

input
```
Integrate[(a + b*ArcCot[c + d*x])/(e + f*x)^3,x]
```

output
```
((b*d*f)/((d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)*(e + f*x)) - (a + b*ArcCot
[c + d*x])/(e + f*x)^2 + ((I/2)*b*d^2*Log[I - c - d*x])/(d*e - (-I + c)*f)
^2 - ((I/2)*b*d^2*Log[I + c + d*x])/(d*e - (I + c)*f)^2 - (2*b*d^2*f*(d*e
- c*f)*Log[d*(e + f*x)])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^2)/(2*f)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {5569, 2081, 1145, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^3}\,dx$$
$$\Big\downarrow \; 5569$$
$$-\frac{bd\int \frac{1}{(e+fx)^2((c+dx)^2+1)}dx}{2f}-\frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2}$$
$$\Big\downarrow \; 2081$$
$$-\frac{bd\int \frac{1}{(e+fx)^2(c^2+2dxc+d^2x^2+1)}dx}{2f}-\frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2}$$

$$\Big\downarrow \text{1145}$$

$$-\frac{bd\left(\frac{\int \frac{d(de-2cf-dfx)}{(e+fx)\left(c^2+2dxc+d^2x^2+1\right)}dx}{(c^2+1)f^2-2cdef+d^2e^2}-\frac{f}{(e+fx)((c^2+1)f^2-2cdef+d^2e^2)}\right)}{2f}-\frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bd\left(\frac{d\int \frac{de-2cf-dfx}{(e+fx)\left(c^2+2dxc+d^2x^2+1\right)}dx}{(c^2+1)f^2-2cdef+d^2e^2}-\frac{f}{(e+fx)((c^2+1)f^2-2cdef+d^2e^2)}\right)}{2f}-\frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2}$$

$$\Big\downarrow \text{1200}$$

$$-\frac{bd\left(\frac{d\int \left(\frac{2(de-cf)f^2}{\left(d^2e^2-2cdfe+\left(c^2+1\right)f^2\right)(e+fx)}+\frac{d\left(d^2e^2-4cdfe-\left(1-3c^2\right)f^2-2df(de-cf)x\right)}{\left(d^2e^2-2cdfe+\left(c^2+1\right)f^2\right)\left(c^2+2dxc+d^2x^2+1\right)}\right)dx}{(c^2+1)f^2-2cdef+d^2e^2}-\frac{f}{(e+fx)((c^2+1)f^2-2cdef+d^2e^2)}\right)}{2f}-\frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2}-\frac{bd\left(\frac{d\left(\frac{\arctan(c+dx)(-cf+de+f)(de-(c+1)f)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}-\frac{f(de-cf)\log\left(c^2+2cdx+d^2x^2+1\right)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}+\frac{2f(de-cf)\log(e+fx)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}\right)}{(c^2+1)f^2-2cdef+d^2e^2}-\frac{f}{(e+fx)((c^2+1)f^2-2cdef+d^2e^2)}\right)}{2f}$$

input
```
Int[(a + b*ArcCot[c + d*x])/(e + f*x)^3,x]
```

output
```
-1/2*(a + b*ArcCot[c + d*x])/(f*(e + f*x)^2) - (b*d*(-(f/((d^2*e^2 - 2*c*d
*e*f + (1 + c^2)*f^2)*(e + f*x))) + (d*(((d*e + f - c*f)*(d*e - (1 + c)*f)
*ArcTan[c + d*x])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (2*f*(d*e - c*f)
*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (f*(d*e - c*f)*Log[
1 + c^2 + 2*c*d*x + d^2*x^2])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)))/(d^2
*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)))/(2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2081
```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

rule 5569
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCot[c + d*x])^p/(f*(m +
1))), x] + Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcCot[
c + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[p, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.07

method	result
parts	$-\frac{a}{2(fx+e)^2 f}+\frac{b\left(-\frac{d^3 \operatorname{arccot}(dx+c)}{2(f(dx+c)-cf+de)^2 f}-\frac{d^3\left(\frac{\frac{(2cf^2-2def)\ln\left(1+(dx+c)^2\right)}{2}+\left(c^2f^2-2cdef+d^2e^2-f^2\right)\arctan(dx+c)}{\left(c^2f^2-2cdef+d^2e^2+f^2\right)^2}-\right.}{}\right.}{d}$
derivativedivides	$\frac{-\frac{a\,d^3}{2(cf-de-f(dx+c))^2 f}-b\,d^3\left(\frac{\operatorname{arccot}(dx+c)}{2(cf-de-f(dx+c))^2 f}+\frac{\frac{f}{\left(c^2f^2-2cdef+d^2e^2+f^2\right)(cf-de-f(dx+c))}-\frac{2f(cf-de)\ln(cf-de-f(}{\left(c^2f^2-2cdef+d^2e^2+\right.}}{}\right.}{d}$
default	$\frac{-\frac{a\,d^3}{2(cf-de-f(dx+c))^2 f}-b\,d^3\left(\frac{\operatorname{arccot}(dx+c)}{2(cf-de-f(dx+c))^2 f}+\frac{\frac{f}{\left(c^2f^2-2cdef+d^2e^2+f^2\right)(cf-de-f(dx+c))}-\frac{2f(cf-de)\ln(cf-de-f(}{\left(c^2f^2-2cdef+d^2e^2+\right.}}{}\right.}{d}$
parallelrisch	$\frac{4\ln(fx+e)xbc\,d^4 e\,f^4-2\ln\left(d^2x^2+2cdx+c^2+1\right)xbc\,d^4 e\,f^4-2x^2 \operatorname{arccot}(dx+c)bc\,d^5 e\,f^4+2x \operatorname{arccot}(dx+c)b\,c^2 d^4 e\,f^4-4}{}$
risch	Expression too large to display

input
```
int((a+b*arccot(d*x+c))/(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/(f*x+e)^2/f+b/d*(-1/2*d^3/(f*(d*x+c)-c*f+d*e)^2/f*arccot(d*x+c)-1/2
*d^3/f*(1/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)^2*(1/2*(2*c*f^2-2*d*e*f)*ln(1+(d
*x+c)^2)+(c^2*f^2-2*c*d*e*f+d^2*e^2-f^2)*arctan(d*x+c))-f/(c^2*f^2-2*c*d*e
*f+d^2*e^2+f^2)/(f*(d*x+c)-c*f+d*e)-2*(c*f-d*e)*f/(c^2*f^2-2*c*d*e*f+d^2*e
^2+f^2)^2*ln(f*(d*x+c)-c*f+d*e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(220) = 440$.

Time = 1.33 (sec) , antiderivative size = 728, normalized size of antiderivative = 3.19

$$\int \frac{a + b\cot^{-1}(c + dx)}{(e + fx)^3}\, dx = \text{Too large to display}$$

input
```
integrate((a+b*arccot(d*x+c))/(f*x+e)^3,x, algorithm="fricas")
```

output
```
-1/2*(a*d^4*e^4 - (4*a*c + b)*d^3*e^3*f + 2*(3*a*c^2 + b*c + a)*d^2*e^2*f^
2 - (4*a*c^3 + b*c^2 + 4*a*c + b)*d*e*f^3 + (a*c^4 + 2*a*c^2 + a)*f^4 - (b
*d^3*e^2*f^2 - 2*b*c*d^2*e*f^3 + (b*c^2 + b)*d*f^4)*x + (b*d^4*e^4 - 4*b*c
*d^3*e^3*f + 2*(3*b*c^2 + b)*d^2*e^2*f^2 - 4*(b*c^3 + b*c)*d*e*f^3 + (b*c^
4 + 2*b*c^2 + b)*f^4)*arccot(d*x + c) + (b*d^4*e^4 - 2*b*c*d^3*e^3*f + (b*
c^2 - b)*d^2*e^2*f^2 + (b*d^4*e^2*f^2 - 2*b*c*d^3*e*f^3 + (b*c^2 - b)*d^2*
f^4)*x^2 + 2*(b*d^4*e^3*f - 2*b*c*d^3*e^2*f^2 + (b*c^2 - b)*d^2*e*f^3)*x)*
arctan(d*x + c) - (b*d^3*e^3*f - b*c*d^2*e^2*f^2 + (b*d^3*e*f^3 - b*c*d^2*
f^4)*x^2 + 2*(b*d^3*e^2*f^2 - b*c*d^2*e*f^3)*x)*log(d^2*x^2 + 2*c*d*x + c^
2 + 1) + 2*(b*d^3*e^3*f - b*c*d^2*e^2*f^2 + (b*d^3*e*f^3 - b*c*d^2*f^4)*x^
2 + 2*(b*d^3*e^2*f^2 - b*c*d^2*e*f^3)*x)*log(f*x + e))/(d^4*e^6*f - 4*c*d^
3*e^5*f^2 + 2*(3*c^2 + 1)*d^2*e^4*f^3 - 4*(c^3 + c)*d*e^3*f^4 + (c^4 + 2*c
^2 + 1)*e^2*f^5 + (d^4*e^4*f^3 - 4*c*d^3*e^3*f^4 + 2*(3*c^2 + 1)*d^2*e^2*f
^5 - 4*(c^3 + c)*d*e*f^6 + (c^4 + 2*c^2 + 1)*f^7)*x^2 + 2*(d^4*e^5*f^2 - 4
*c*d^3*e^4*f^3 + 2*(3*c^2 + 1)*d^2*e^3*f^4 - 4*(c^3 + c)*d*e^2*f^5 + (c^4
+ 2*c^2 + 1)*e*f^6)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\cot^{-1}(c + dx)}{(e + fx)^3}\, dx = \text{Timed out}$$

input
```
integrate((a+b*acot(d*x+c))/(f*x+e)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.80

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^3}\,dx$$
$$=\frac{1}{2}\left(d\left(\frac{(d^2e-cdf)\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^4e^4-4\,cd^3e^3f+2\,(3\,c^2+1)d^2e^2f^2-4\,(c^3+c)def^3+(c^4+2\,c^2+1)f^4}-\frac{}{d^4e^4-4\,cd^3e^3f+2}\right.\right.$$
$$-\frac{a}{2\,(f^3x^2+2\,ef^2x+e^2f)}$$

input
```
integrate((a+b*arccot(d*x+c))/(f*x+e)^3,x, algorithm="maxima")
```

output
```
1/2*(d*((d^2*e - c*d*f)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^4*e^4 - 4*c*d^
3*e^3*f + 2*(3*c^2 + 1)*d^2*e^2*f^2 - 4*(c^3 + c)*d*e*f^3 + (c^4 + 2*c^2 +
 1)*f^4) - 2*(d^2*e - c*d*f)*log(f*x + e)/(d^4*e^4 - 4*c*d^3*e^3*f + 2*(3*
c^2 + 1)*d^2*e^2*f^2 - 4*(c^3 + c)*d*e*f^3 + (c^4 + 2*c^2 + 1)*f^4) - (d^4
*e^2 - 2*c*d^3*e*f + (c^2 - 1)*d^2*f^2)*arctan((d^2*x + c*d)/d)/((d^4*e^4*
f - 4*c*d^3*e^3*f^2 + 2*(3*c^2 + 1)*d^2*e^2*f^3 - 4*(c^3 + c)*d*e*f^4 + (c
^4 + 2*c^2 + 1)*f^5)*d) + 1/(d^2*e^3 - 2*c*d*e^2*f + (c^2 + 1)*e*f^2 + (d^
2*e^2*f - 2*c*d*e*f^2 + (c^2 + 1)*f^3)*x)) - arccot(d*x + c)/(f^3*x^2 + 2*
e*f^2*x + e^2*f))*b - 1/2*a/(f^3*x^2 + 2*e*f^2*x + e^2*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6173 vs. $2(220) = 440$.

Time = 1.12 (sec) , antiderivative size = 6173, normalized size of antiderivative = 27.07

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*arccot(d*x+c))/(f*x+e)^3,x, algorithm="giac")
```

output
```
-1/2*(4*b*d^4*e^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 - 12*
b*c*d^3*e^2*f*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 + 12*b*c^
2*d^2*e*f^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 - 4*b*c^3*d
*f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 - b*d^3*e^2*f*arct
an(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^4 + 2*b*c*d^2*e*f^2*arctan(1/
(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^4 - b*c^2*d*f^3*arctan(1/(d*x + c)
)*tan(1/2*arctan(1/(d*x + c)))^4 + 4*b*d^4*e^3*log(4*(4*d^2*e^2*tan(1/2*ar
ctan(1/(d*x + c)))^2 - 8*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^2 + 4*c^2*f^
2*tan(1/2*arctan(1/(d*x + c)))^2 - 4*d*e*f*tan(1/2*arctan(1/(d*x + c)))^3
+ 4*c*f^2*tan(1/2*arctan(1/(d*x + c)))^3 + f^2*tan(1/2*arctan(1/(d*x + c))
)^4 + 4*d*e*f*tan(1/2*arctan(1/(d*x + c))) - 4*c*f^2*tan(1/2*arctan(1/(d*x
 + c))) - 2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 + f^2)/(tan(1/2*arctan(1/(d
*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x
 + c)))^2 - 12*b*c*d^3*e^2*f*log(4*(4*d^2*e^2*tan(1/2*arctan(1/(d*x + c)))
^2 - 8*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/2*arctan(1
/(d*x + c)))^2 - 4*d*e*f*tan(1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2*tan(1/2*
arctan(1/(d*x + c)))^3 + f^2*tan(1/2*arctan(1/(d*x + c)))^4 + 4*d*e*f*tan(
1/2*arctan(1/(d*x + c))) - 4*c*f^2*tan(1/2*arctan(1/(d*x + c))) - 2*f^2*ta
n(1/2*arctan(1/(d*x + c)))^2 + f^2)/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*ta
n(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c)))^2 + 12*...
```

Mupad [B] (verification not implemented)

Time = 7.40 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.75

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^3}\,dx = \frac{b\,d\,e}{2\,{\left(e+f\,x\right)}^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)} - \frac{a\,f}{2\,{\left(e+f\,x\right)}^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)} - \frac{b\,\mathrm{acot}(c+d\,x)}{2\,f\,{\left(e+f\,x\right)}^2} - \frac{a\,c^2\,f}{2\,{\left(e+f\,x\right)}^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)} - \frac{b\,d^3\,e\,\ln\left(e+f\,x\right)}{{\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)}^2} + \frac{b\,c\,d^2\,f\,\ln\left(e+f\,x\right)}{{\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)}^2} + \frac{a\,c\,d\,e}{{\left(e+f\,x\right)}^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)} + \frac{b\,d\,f\,x}{2\,{\left(e+f\,x\right)}^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)} - \frac{a\,d^2\,e^2}{2\,f\,{\left(e+f\,x\right)}^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)} + \frac{b\,d^2\,\ln\left(c+d\,x-\mathrm{i}\right)\,1{}\mathrm{i}}{4\,f\,{\left(d\,e-c\,f+f\,1{}\mathrm{i}\right)}^2} - \frac{b\,d^2\,\ln\left(c+d\,x+1{}\mathrm{i}\right)\,1{}\mathrm{i}}{4\,f\,{\left(c\,f-d\,e+f\,1{}\mathrm{i}\right)}^2}$$

input
```
int((a + b*acot(c + d*x))/(e + f*x)^3,x)
```

output
```
(b*d*e)/(2*(e + f*x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*e*f)) - (a*f)/(2*(
e + f*x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*e*f)) - (b*acot(c + d*x))/(2*f
*(e + f*x)^2) + (b*d^2*log(c + d*x - 1i)*1i)/(4*f*(f*1i - c*f + d*e)^2) -
(b*d^2*log(c + d*x + 1i)*1i)/(4*f*(f*1i + c*f - d*e)^2) - (a*c^2*f)/(2*(e
+ f*x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*e*f)) - (b*d^3*e*log(e + f*x))/(
f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*e*f)^2 + (b*c*d^2*f*log(e + f*x))/(f^2 + c
^2*f^2 + d^2*e^2 - 2*c*d*e*f)^2 + (a*c*d*e)/((e + f*x)^2*(f^2 + c^2*f^2 +
d^2*e^2 - 2*c*d*e*f)) + (b*d*f*x)/(2*(e + f*x)^2*(f^2 + c^2*f^2 + d^2*e^2
- 2*c*d*e*f)) - (a*d^2*e^2)/(2*f*(e + f*x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*
c*d*e*f))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1612, normalized size of antiderivative = 7.07

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^3}\,dx = \text{Too large to display}$$

input
```
int((a+b*acot(d*x+c))/(f*x+e)^3,x)
```

output
```
(4*acot(c + d*x)*b*c**4*e*f**5*x + 2*acot(c + d*x)*b*c**4*f**6*x**2 - 16*a
cot(c + d*x)*b*c**3*d*e**2*f**4*x - 8*acot(c + d*x)*b*c**3*d*e*f**5*x**2 +
 24*acot(c + d*x)*b*c**2*d**2*e**3*f**3*x + 12*acot(c + d*x)*b*c**2*d**2*e
**2*f**4*x**2 + 8*acot(c + d*x)*b*c**2*e*f**5*x + 4*acot(c + d*x)*b*c**2*f
**6*x**2 - 16*acot(c + d*x)*b*c*d**3*e**4*f**2*x - 8*acot(c + d*x)*b*c*d**
3*e**3*f**3*x**2 - 16*acot(c + d*x)*b*c*d*e**2*f**4*x - 8*acot(c + d*x)*b*
c*d*e*f**5*x**2 + 4*acot(c + d*x)*b*d**4*e**5*f*x + 2*acot(c + d*x)*b*d**4
*e**4*f**2*x**2 + 8*acot(c + d*x)*b*d**2*e**3*f**3*x + 4*acot(c + d*x)*b*d
**2*e**2*f**4*x**2 + 4*acot(c + d*x)*b*e*f**5*x + 2*acot(c + d*x)*b*f**6*x
**2 + 2*atan(c + d*x)*b*c**4*e**2*f**4 + 4*atan(c + d*x)*b*c**4*e*f**5*x +
 2*atan(c + d*x)*b*c**4*f**6*x**2 - 8*atan(c + d*x)*b*c**3*d*e**3*f**3 - 1
6*atan(c + d*x)*b*c**3*d*e**2*f**4*x - 8*atan(c + d*x)*b*c**3*d*e*f**5*x**
2 + 10*atan(c + d*x)*b*c**2*d**2*e**4*f**2 + 20*atan(c + d*x)*b*c**2*d**2*
e**3*f**3*x + 10*atan(c + d*x)*b*c**2*d**2*e**2*f**4*x**2 + 4*atan(c + d*x
)*b*c**2*e**2*f**4 + 8*atan(c + d*x)*b*c**2*e*f**5*x + 4*atan(c + d*x)*b*c
**2*f**6*x**2 - 4*atan(c + d*x)*b*c*d**3*e**5*f - 8*atan(c + d*x)*b*c*d**3
*e**4*f**2*x - 4*atan(c + d*x)*b*c*d**3*e**3*f**3*x**2 - 8*atan(c + d*x)*b
*c*d*e**3*f**3 - 16*atan(c + d*x)*b*c*d*e**2*f**4*x - 8*atan(c + d*x)*b*c*
d*e*f**5*x**2 + 6*atan(c + d*x)*b*d**2*e**4*f**2 + 12*atan(c + d*x)*b*d**2
*e**3*f**3*x + 6*atan(c + d*x)*b*d**2*e**2*f**4*x**2 + 2*atan(c + d*x)*...
```

3.23 $\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 382

$$\begin{aligned}
&\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx \\
&= \frac{b^2f^2x}{3d^2}+\frac{2abf(de-cf)x}{d^2}+\frac{2b^2f(de-cf)(c+dx)\cot^{-1}(c+dx)}{d^3} \\
&\quad +\frac{bf^2(c+dx)^2\left(a+b\cot^{-1}(c+dx)\right)}{3d^3} \\
&\quad +\frac{i(3d^2e^2-6cdef-(1-3c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)^2}{3d^3} \\
&\quad -\frac{(de-cf)\left(d^2e^2-2cdef-(3-c^2)f^2\right)\left(a+b\cot^{-1}(c+dx)\right)^2}{3d^3f} \\
&\quad +\frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)^2}{3f}-\frac{b^2f^2\arctan(c+dx)}{3d^3} \\
&\quad -\frac{2b(3d^2e^2-6cdef-(1-3c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)\log\left(\frac{2}{1+i(c+dx)}\right)}{3d^3} \\
&\quad +\frac{b^2f(de-cf)\log\left(1+(c+dx)^2\right)}{d^3} \\
&\quad +\frac{ib^2(3d^2e^2-6cdef-(1-3c^2)f^2)\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{3d^3}
\end{aligned}$$

output
```
1/3*b^2*f^2*x/d^2+2*a*b*f*(-c*f+d*e)*x/d^2+2*b^2*f*(-c*f+d*e)*(d*x+c)*arcc
ot(d*x+c)/d^3+1/3*b*f^2*(d*x+c)^2*(a+b*arccot(d*x+c))/d^3+1/3*I*(3*d^2*e^2
-6*c*d*e*f-(-3*c^2+1)*f^2)*(a+b*arccot(d*x+c))^2/d^3-1/3*(-c*f+d*e)*(d^2*e
^2-2*c*d*e*f-(-c^2+3)*f^2)*(a+b*arccot(d*x+c))^2/d^3/f+1/3*(f*x+e)^3*(a+b*
arccot(d*x+c))^2/f-1/3*b^2*f^2*arctan(d*x+c)/d^3-2/3*b*(3*d^2*e^2-6*c*d*e*
f-(-3*c^2+1)*f^2)*(a+b*arccot(d*x+c))*ln(2/(1+I*(d*x+c)))/d^3+b^2*f*(-c*f+
d*e)*ln(1+(d*x+c)^2)/d^3+1/3*I*b^2*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*po
lylog(2,1-2/(1+I*(d*x+c)))/d^3
```

Mathematica [A] (warning: unable to verify)

Time = 8.25 (sec) , antiderivative size = 665, normalized size of antiderivative = 1.74

$$\int (e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^2\,dx = a^2e^2x+a^2efx^2+\frac{1}{3}a^2f^2x^3$$
$$+\frac{ab(dfx(6de-4cf+dfx)+2d^3x(3e^2+3efx+f^2x^2)\cot^{-1}(c+dx)-2(3cd^2e^2+3def-3c^2def-3c\ldots}{3d^3}$$
$$+\frac{b^2e^2\left(\cot^{-1}(c+dx)\left((i+c+dx)\cot^{-1}(c+dx)-2\log\left(1-e^{2i\cot^{-1}(c+dx)}\right)\right)+i\,\mathrm{PolyLog}\left(2,e^{2i\cot^{-1}(c\ldots}\right.\right.}{d}$$
$$+\frac{b^2ef\left((1-2ic-c^2+d^2x^2)\cot^{-1}(c+dx)^2+2\cot^{-1}(c+dx)\left(c+dx+2c\log\left(1-e^{2i\cot^{-1}(c+dx)}\right)\right)-\ldots\right.}{d^2}$$
$$+\frac{b^2f^2\left((c+dx)\left(1+(c+dx)^2\right)\left(1-6c\cot^{-1}(c+dx)+3(1+c^2)\cot^{-1}(c+dx)^2\right)-(c+dx)\sqrt{1+\frac{1}{(c+\ldots}}\right.}{\ldots}$$

input
```
Integrate[(e + f*x)^2*(a + b*ArcCot[c + d*x])^2,x]
```

output
```
a^2*e^2*x + a^2*e*f*x^2 + (a^2*f^2*x^3)/3 + (a*b*(d*f*x*(6*d*e - 4*c*f + d
*f*x) + 2*d^3*x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcCot[c + d*x] - 2*(3*c*d^2*e
^2 + 3*d*e*f - 3*c^2*d*e*f - 3*c*f^2 + c^3*f^2)*ArcTan[c + d*x] + (3*d^2*e
^2 - 6*c*d*e*f + (-1 + 3*c^2)*f^2)*Log[1 + c^2 + 2*c*d*x + d^2*x^2]))/(3*d
^3) + (b^2*e^2*(ArcCot[c + d*x]*((I + c + d*x)*ArcCot[c + d*x] - 2*Log[1 -
 E^((2*I)*ArcCot[c + d*x])]) + I*PolyLog[2, E^((2*I)*ArcCot[c + d*x])]))/d
 + (b^2*e*f*((1 - (2*I)*c - c^2 + d^2*x^2)*ArcCot[c + d*x]^2 + 2*ArcCot[c
+ d*x]*(c + d*x + 2*c*Log[1 - E^((2*I)*ArcCot[c + d*x])]) - 2*Log[1/((c +
d*x)*Sqrt[1 + (c + d*x)^(-2)])] - (2*I)*c*PolyLog[2, E^((2*I)*ArcCot[c + d
*x])]))/d^2 + (b^2*f^2*((c + d*x)*(1 + (c + d*x)^2)*(1 - 6*c*ArcCot[c + d*
x] + 3*(1 + c^2)*ArcCot[c + d*x]^2) - (c + d*x)*Sqrt[1 + (c + d*x)^(-2)]*(
1 + (c + d*x)^2)*(1 - 6*c*ArcCot[c + d*x] + (-1 + 3*c^2)*ArcCot[c + d*x]^2
)*Cos[3*ArcCot[c + d*x]] + 2*(1 + (c + d*x)^2)*((-I)*ArcCot[c + d*x]^2*(1
- (6*I)*c - 3*c^2 + (-1 + 3*c^2)*Cos[2*ArcCot[c + d*x]]) + 2*ArcCot[c + d*
x]*(1 + (1 - 3*c^2)*Log[1 - E^((2*I)*ArcCot[c + d*x])] + (-1 + 3*c^2)*Cos[
2*ArcCot[c + d*x]]*Log[1 - E^((2*I)*ArcCot[c + d*x])]) - 6*c*(-1 + Cos[2*A
rcCot[c + d*x]])*Log[1/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)])]) + (4*I)*(-1
+ 3*c^2)*PolyLog[2, E^((2*I)*ArcCot[c + d*x])]))/(12*d^3)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5571, 27, 5390, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{5571}$$

$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)^2 \left(a+b\cot^{-1}(c+dx)\right)^2}{d^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (de-cf+f(c+dx))^2 \left(a+b\cot^{-1}(c+dx)\right)^2 d(c+dx)}{d^3}$$

$\downarrow$ 5390

$$\frac{\frac{2b\int\left((c+dx)(a+b\cot^{-1}(c+dx))f^3+3(de-cf)(a+b\cot^{-1}(c+dx))f^2+\frac{\left((de-cf)\left(d^2e^2-2cdfe-\left(3-c^2\right)f^2\right)+f\left(3d^2e^2-6cdfe-\left(1-3c^2\right)f^2\right)(c+dx)\right)(a+}{(c+dx)^2+1}}{3f}}{d^3}$$

$\downarrow$ 2009

$$\frac{(f(c+dx)-cf+de)^3(a+b\cot^{-1}(c+dx))^2}{3f}+\frac{2b\left(\frac{if\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)(a+b\cot^{-1}(c+dx))^2}{2b}-\frac{(de-cf)\left(-\left(3-c^2\right)f^2-2cdef+d^2e^2\right)(a+b\cot^{-}}{2b}\right.}{}$$

input
```
Int[(e + f*x)^2*(a + b*ArcCot[c + d*x])^2,x]
```

output
```
(((d*e - c*f + f*(c + d*x))^3*(a + b*ArcCot[c + d*x])^2)/(3*f) + (2*b*((b*
f^3*(c + d*x))/2 + 3*a*f^2*(d*e - c*f)*(c + d*x) + 3*b*f^2*(d*e - c*f)*(c
+ d*x)*ArcCot[c + d*x] + (f^3*(c + d*x)^2*(a + b*ArcCot[c + d*x]))/2 + ((I
/2)*f*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2)*f^2)*(a + b*ArcCot[c + d*x])^2)
/b - ((d*e - c*f)*(d^2*e^2 - 2*c*d*e*f - (3 - c^2)*f^2)*(a + b*ArcCot[c +
d*x])^2)/(2*b) - (b*f^3*ArcTan[c + d*x])/2 - f*(3*d^2*e^2 - 6*c*d*e*f - (1
 - 3*c^2)*f^2)*(a + b*ArcCot[c + d*x])*Log[2/(1 + I*(c + d*x))] + (3*b*f^2
*(d*e - c*f)*Log[1 + (c + d*x)^2])/2 + (I/2)*b*f*(3*d^2*e^2 - 6*c*d*e*f -
(1 - 3*c^2)*f^2)*PolyLog[2, 1 - 2/(1 + I*(c + d*x))]))/(3*f))/d^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5390
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])^p/(e*(q + 1))), x] + S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcCot[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1071 vs. $2(362) = 724$.

Time = 2.13 (sec), antiderivative size = 1072, normalized size of antiderivative = 2.81

method	result	size
parts	Expression too large to display	1072
derivativedivides	Expression too large to display	1087
default	Expression too large to display	1087
risch	Expression too large to display	3165

input
```
int((f*x+e)^2*(a+b*arccot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*(f*x+e)^3/f+b^2/d*(1/3/d^2*f^2*arccot(d*x+c)^2*(d*x+c)^3-1/d^2*f^2
*arccot(d*x+c)^2*(d*x+c)^2*c+1/d*f*arccot(d*x+c)^2*(d*x+c)^2*e+1/d^2*f^2*a
rccot(d*x+c)^2*(d*x+c)*c^2-2/d*f*arccot(d*x+c)^2*(d*x+c)*c*e+arccot(d*x+c)
^2*(d*x+c)*e^2-1/3/d^2*f^2*arccot(d*x+c)^2*c^3+1/d*f*arccot(d*x+c)^2*c^2*e
-arccot(d*x+c)^2*c*e^2+1/3*d/f*arccot(d*x+c)^2*e^3+2/3/d^2/f*(1/2*arccot(d
*x+c)*f^3*(d*x+c)^2-3*arccot(d*x+c)*c*f^3*(d*x+c)+3*arccot(d*x+c)*d*e*f^2*
(d*x+c)+3/2*arccot(d*x+c)*ln(1+(d*x+c)^2)*c^2*f^3-3*arccot(d*x+c)*ln(1+(d*
x+c)^2)*c*d*e*f^2+3/2*arccot(d*x+c)*ln(1+(d*x+c)^2)*d^2*e^2*f-1/2*arccot(d
*x+c)*ln(1+(d*x+c)^2)*f^3-arccot(d*x+c)*arctan(d*x+c)*c^3*f^3+3*arccot(d*x
+c)*arctan(d*x+c)*c^2*d*e*f^2-3*arccot(d*x+c)*arctan(d*x+c)*c*d^2*e^2*f+ar
ccot(d*x+c)*arctan(d*x+c)*d^3*e^3+3*arccot(d*x+c)*arctan(d*x+c)*c*f^3-3*ar
ccot(d*x+c)*arctan(d*x+c)*d*e*f^2+1/2*f^2*(f*(d*x+c)+1/2*(-6*c*f+6*d*e)*ln
(1+(d*x+c)^2)-f*arctan(d*x+c))+1/2*f*(3*c^2*f^2-6*c*d*e*f+3*d^2*e^2-f^2)*(
-1/2*I*(ln(d*x+c-I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c-I)^2-dilog(-1/2*I*(d*x+c+
I))-ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I)))+1/2*I*(ln(d*x+c+I)*ln(1+(d*x+c)^2)-1
/2*ln(d*x+c+I)^2-dilog(1/2*I*(d*x+c-I))-ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))))+
1/4*(-2*c^3*f^3+6*c^2*d*e*f^2-6*c*d^2*e^2*f+2*d^3*e^3+6*c*f^3-6*d*e*f^2)*a
rctan(d*x+c)^2))+2/d^2*c*f*e*b*a-1/3*a*b/d^3*f^2*ln(1+(d*x+c)^2)-2*a*b/d^2
*f*ln(1+(d*x+c)^2)*c*e+2*b/d^2*arctan(d*x+c)*a*c^2*e*f-2*b/d*arctan(d*x+c)
*a*c*e^2+1/3/d*f^2*b*a*x^2-5/3/d^3*c^2*f^2*b*a+2*a*b*f*arccot(d*x+c)*e*...
```

Fricas [F]

$$\int (e + fx)^2 \left(a + b\cot^{-1}(c + dx)\right)^2 \, dx = \int (fx + e)^2 (b\,\mathrm{arccot}\,(dx + c) + a)^2 \, dx$$

input
```
integrate((f*x+e)^2*(a+b*arccot(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(a^2*f^2*x^2 + 2*a^2*e*f*x + a^2*e^2 + (b^2*f^2*x^2 + 2*b^2*e*f*x
+ b^2*e^2)*arccot(d*x + c)^2 + 2*(a*b*f^2*x^2 + 2*a*b*e*f*x + a*b*e^2)*arc
cot(d*x + c), x)
```

Sympy [F]

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \int (a+b\,\mathrm{acot}\,(c+dx))^2 \,(e+fx)^2 \, dx$$

input
```
integrate((f*x+e)**2*(a+b*acot(d*x+c))**2,x)
```

output
```
Integral((a + b*acot(c + d*x))**2*(e + f*x)**2, x)
```

Maxima [F]

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \int (fx+e)^2 (b\,\mathrm{arccot}\,(dx+c)+a)^2 \, dx$$

input
```
integrate((f*x+e)^2*(a+b*arccot(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/12*b^2*f^2*x^3*arctan2(1, d*x + c)^2 + 1/4*b^2*e*f*x^2*arctan2(1, d*x +
c)^2 + 1/3*a^2*f^2*x^3 + 1/4*b^2*e^2*x*arctan2(1, d*x + c)^2 + a^2*e*f*x^2
 + 2*(x^2*arccot(d*x + c) + d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d
^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a*b*e*f + 1/3*(2*x^3*arccot(
d*x + c) + d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/
d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*a*b*f^2 + a^2*e^2
*x + (2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*a*b*e^2/d - 1/48
*(b^2*f^2*x^3 + 3*b^2*e*f*x^2 + 3*b^2*e^2*x)*log(d^2*x^2 + 2*c*d*x + c^2 +
 1)^2 + integrate(1/48*(36*b^2*d^2*f^2*x^4*arctan2(1, d*x + c)^2 + 8*(9*b^
2*d^2*e*f*arctan2(1, d*x + c)^2 + (9*b^2*c*arctan2(1, d*x + c)^2 + b^2*arc
tan2(1, d*x + c))*d*f^2)*x^3 + 36*(b^2*c^2*arctan2(1, d*x + c)^2 + b^2*arc
tan2(1, d*x + c)^2)*e^2 + 12*(3*b^2*d^2*e^2*arctan2(1, d*x + c)^2 + 2*(6*b
^2*c*arctan2(1, d*x + c)^2 + b^2*arctan2(1, d*x + c))*d*e*f + 3*(b^2*c^2*a
rctan2(1, d*x + c)^2 + b^2*arctan2(1, d*x + c)^2)*f^2)*x^2 + 3*(b^2*d^2*f^
2*x^4 + 2*(b^2*d^2*e*f + b^2*c*d*f^2)*x^3 + (b^2*c^2 + b^2)*e^2 + (b^2*d^2
*e^2 + 4*b^2*c*d*e*f + (b^2*c^2 + b^2)*f^2)*x^2 + 2*(b^2*c*d*e^2 + (b^2*c^
2 + b^2)*e*f)*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 24*((3*b^2*c*arctan2
(1, d*x + c)^2 + b^2*arctan2(1, d*x + c))*d*e^2 + 3*(b^2*c^2*arctan2(1, d*
x + c)^2 + b^2*arctan2(1, d*x + c)^2)*e*f)*x + 4*(b^2*d^2*f^2*x^4 + 3*b^2*
c*d*e^2*x + (3*b^2*d^2*e*f + b^2*c*d*f^2)*x^3 + 3*(b^2*d^2*e^2 + b^2*c*...
```

Giac [F]

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \int (fx+e)^2 (b \operatorname{arccot}(dx+c)+a)^2 \, dx$$

input
```
integrate((f*x+e)^2*(a+b*arccot(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*(b*arccot(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \int (e+f\,x)^2 \,(a+b\,\mathrm{acot}(c+d\,x))^2 \, dx$$

input
```
int((e + f*x)^2*(a + b*acot(c + d*x))^2,x)
```

output
```
int((e + f*x)^2*(a + b*acot(c + d*x))^2, x)
```

Reduce [F]

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \text{Too large to display}$$

input
```
int((f*x+e)^2*(a+b*acot(d*x+c))^2,x)
```

output
```
( - 2*acot(c + d*x)**2*b**2*c**3*f**2 + 3*acot(c + d*x)**2*b**2*c**2*d*e*f
 - 2*acot(c + d*x)**2*b**2*c*f**2 + 3*acot(c + d*x)**2*b**2*d**3*e**2*x +
3*acot(c + d*x)**2*b**2*d**3*e*f*x**2 + acot(c + d*x)**2*b**2*d**3*f**2*x*
*3 + 3*acot(c + d*x)**2*b**2*d*e*f + 2*acot(c + d*x)*a*b*c**3*f**2 - 6*aco
t(c + d*x)*a*b*c**2*d*e*f + 6*acot(c + d*x)*a*b*c*d**2*e**2 - 6*acot(c + d
*x)*a*b*c*f**2 + 6*acot(c + d*x)*a*b*d**3*e**2*x + 6*acot(c + d*x)*a*b*d**
3*e*f*x**2 + 2*acot(c + d*x)*a*b*d**3*f**2*x**3 + 6*acot(c + d*x)*a*b*d*e*
f - 5*acot(c + d*x)*b**2*c**2*f**2 + 6*acot(c + d*x)*b**2*c*d*e*f - 4*acot
(c + d*x)*b**2*c*d*f**2*x + 6*acot(c + d*x)*b**2*d**2*e*f*x + acot(c + d*x
)*b**2*d**2*f**2*x**2 + acot(c + d*x)*b**2*f**2 + 6*int((acot(c + d*x)*x)/
(c**2 + 2*c*d*x + d**2*x**2 + 1),x)*b**2*c**2*d**2*f**2 - 12*int((acot(c +
 d*x)*x)/(c**2 + 2*c*d*x + d**2*x**2 + 1),x)*b**2*c*d**3*e*f + 6*int((acot
(c + d*x)*x)/(c**2 + 2*c*d*x + d**2*x**2 + 1),x)*b**2*d**4*e**2 - 2*int((a
cot(c + d*x)*x)/(c**2 + 2*c*d*x + d**2*x**2 + 1),x)*b**2*d**2*f**2 + 3*log
(c**2 + 2*c*d*x + d**2*x**2 + 1)*a*b*c**2*f**2 - 6*log(c**2 + 2*c*d*x + d*
*2*x**2 + 1)*a*b*c*d*e*f + 3*log(c**2 + 2*c*d*x + d**2*x**2 + 1)*a*b*d**2*
e**2 - log(c**2 + 2*c*d*x + d**2*x**2 + 1)*a*b*f**2 - 3*log(c**2 + 2*c*d*x
 + d**2*x**2 + 1)*b**2*c*f**2 + 3*log(c**2 + 2*c*d*x + d**2*x**2 + 1)*b**2
*d*e*f + 3*a**2*d**3*e**2*x + 3*a**2*d**3*e*f*x**2 + a**2*d**3*f**2*x**3 -
 4*a*b*c*d*f**2*x + 6*a*b*d**2*e*f*x + a*b*d**2*f**2*x**2 + b**2*d*f**2...
```

3.24 $\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 220

$$\begin{aligned}\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^2\,dx &= \frac{abfx}{d}+\frac{b^2f(c+dx)\cot^{-1}(c+dx)}{d^2}+\frac{i(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^2}\\ &\quad-\frac{(de+f-cf)(de-(1+c)f)\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2f}\\ &\quad+\frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2f}\\ &\quad-\frac{2b(de-cf)\left(a+b\cot^{-1}(c+dx)\right)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2}\\ &\quad+\frac{b^2f\log\left(1+(c+dx)^2\right)}{2d^2}+\frac{ib^2(de-cf)\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2}\end{aligned}$$

output
```
a*b*f*x/d+b^2*f*(d*x+c)*arccot(d*x+c)/d^2+I*(-c*f+d*e)*(a+b*arccot(d*x+c))
^2/d^2-1/2*(-c*f+d*e+f)*(d*e-(1+c)*f)*(a+b*arccot(d*x+c))^2/d^2/f+1/2*(f*x
+e)^2*(a+b*arccot(d*x+c))^2/f-2*b*(-c*f+d*e)*(a+b*arccot(d*x+c))*ln(2/(1+I
*(d*x+c)))/d^2+1/2*b^2*f*ln(1+(d*x+c)^2)/d^2+I*b^2*(-c*f+d*e)*polylog(2,1-
2/(1+I*(d*x+c)))/d^2
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.30

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^2\,dx$$

$$= \frac{2a^2cde+2abcf-a^2c^2f+2a^2d^2ex+2abdfx+a^2d^2fx^2+b^2(i+c+dx)(-((i+c)f)+d(2e+fx))\cot}{}$$

input
```
Integrate[(e + f*x)*(a + b*ArcCot[c + d*x])^2,x]
```

output
```
(2*a^2*c*d*e + 2*a*b*c*f - a^2*c^2*f + 2*a^2*d^2*e*x + 2*a*b*d*f*x + a^2*d
^2*f*x^2 + b^2*(I + c + d*x)*(-((I + c)*f) + d*(2*e + f*x))*ArcCot[c + d*x
]^2 - 2*a*b*f*ArcTan[c + d*x] + 2*b*ArcCot[c + d*x]*(-((c + d*x)*(-(b*f) +
 a*c*f - a*d*(2*e + f*x))) - 2*b*(d*e - c*f)*Log[1 - E^((2*I)*ArcCot[c + d
*x])]) - 4*a*b*d*e*Log[1/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)])] - 2*b^2*f*L
og[1/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)])] + 4*a*b*c*f*Log[1/((c + d*x)*Sq
rt[1 + (c + d*x)^(-2)])] + (2*I)*b^2*(d*e - c*f)*PolyLog[2, E^((2*I)*ArcCo
t[c + d*x])])/(2*d^2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5571, 27, 5390, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{5571}$$

$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)\left(a+b\cot^{-1}(c+dx)\right)^2}{d}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (de-cf+f(c+dx))\left(a+b\cot^{-1}(c+dx)\right)^2d(c+dx)}{d^2}$$

$\Big\downarrow$ 5390

$$\frac{\frac{b\int \left((a+b\cot^{-1}(c+dx))f^2+\frac{((de-cf+f)(de-(c+1)f)+2f(de-cf)(c+dx))\left(a+b\cot^{-1}(c+dx)\right)}{(c+dx)^2+1}\right)d(c+dx)}{f}+\frac{(f(c+dx)-cf+de)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2f}}{d^2}$$

$\Big\downarrow$ 2009

$$\frac{\frac{(f(c+dx)-cf+de)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2f}+\frac{b\left(\frac{if(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2}{b}-\frac{(-cf+de+f)(de-(c+1)f)\left(a+b\cot^{-1}(c+dx)\right)^2}{2b}-2f(de-cf)\log\left(\frac{}{1+i(}\right.\right.}{}}{d^2}$$

input
```
Int[(e + f*x)*(a + b*ArcCot[c + d*x])^2,x]
```

output
```
(((d*e - c*f + f*(c + d*x))^2*(a + b*ArcCot[c + d*x])^2)/(2*f) + (b*(a*f^2
*(c + d*x) + b*f^2*(c + d*x)*ArcCot[c + d*x] + (I*f*(d*e - c*f)*(a + b*Arc
Cot[c + d*x])^2)/b - ((d*e + f - c*f)*(d*e - (1 + c)*f)*(a + b*ArcCot[c +
d*x])^2)/(2*b) - 2*f*(d*e - c*f)*(a + b*ArcCot[c + d*x])*Log[2/(1 + I*(c +
 d*x))] + (b*f^2*Log[1 + (c + d*x)^2])/2 + I*b*f*(d*e - c*f)*PolyLog[2, 1
- 2/(1 + I*(c + d*x))]))/f)/d^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5390
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])^p/(e*(q + 1))), x] + S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcCot[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(210) = 420$.

Time = 1.35 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.94

method	result
parts	$a^2\left(\frac{1}{2}f\,x^2+ex\right)+\frac{b^2\left(\frac{\operatorname{arccot}(dx+c)^2(dx+c)^2 f}{2d}-\frac{\operatorname{arccot}(dx+c)^2 cf(dx+c)}{d}+\operatorname{arccot}(dx+c)^2 e(dx+c)+\frac{-\ln\left(1+(dx+c)^2 \cdots\right.}{\cdots}\right.}{\cdots}$
derivativedivides	$\frac{-\frac{a^2\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^2}{2}\right)}{d}-\frac{b^2\left(\operatorname{arccot}(dx+c)^2 fc(dx+c)-\operatorname{arccot}(dx+c)^2 ed(dx+c)-\frac{\operatorname{arccot}(dx+c)^2 f(dx+c)^2}{2}+\ln\left(1 \cdots\right.\right.}{\cdots}}{\cdots}$
default	$\frac{-\frac{a^2\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^2}{2}\right)}{d}-\frac{b^2\left(\operatorname{arccot}(dx+c)^2 fc(dx+c)-\operatorname{arccot}(dx+c)^2 ed(dx+c)-\frac{\operatorname{arccot}(dx+c)^2 f(dx+c)^2}{2}+\ln\left(1 \cdots\right.\right.}{\cdots}}{\cdots}$
risch	Expression too large to display

input
```
int((f*x+e)*(a+b*arccot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/2*f*x^2+e*x)+b^2/d*(1/2/d*arccot(d*x+c)^2*(d*x+c)^2*f-1/d*arccot(d*
x+c)^2*c*f*(d*x+c)+arccot(d*x+c)^2*e*(d*x+c)+1/d*(-ln(1+(d*x+c)^2)*arccot(
d*x+c)*c*f+ln(1+(d*x+c)^2)*arccot(d*x+c)*d*e-arctan(d*x+c)*arccot(d*x+c)*f
+arccot(d*x+c)*f*(d*x+c)+1/2*f*ln(1+(d*x+c)^2)-1/2*arctan(d*x+c)^2*f+1/2*(
-2*c*f+2*d*e)*(-1/2*I*(ln(d*x+c-I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c-I)^2-dilog
(-1/2*I*(d*x+c+I))-ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I)))+1/2*I*(ln(d*x+c+I)*ln
(1+(d*x+c)^2)-1/2*ln(d*x+c+I)^2-dilog(1/2*I*(d*x+c-I))-ln(d*x+c+I)*ln(1/2*
I*(d*x+c-I))))))+2*a*b/d*(1/2/d*arccot(d*x+c)*(d*x+c)^2*f-1/d*arccot(d*x+c
)*c*f*(d*x+c)+arccot(d*x+c)*e*(d*x+c)+1/2/d*(f*(d*x+c)+1/2*(-2*c*f+2*d*e)*
ln(1+(d*x+c)^2)-f*arctan(d*x+c)))
```

Fricas [F]

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^2\,dx=\int (fx+e)(b\,\mathrm{arccot}\,(dx+c)+a)^2\,dx$$

input
```
integrate((f*x+e)*(a+b*arccot(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(a^2*f*x + a^2*e + (b^2*f*x + b^2*e)*arccot(d*x + c)^2 + 2*(a*b*f*
x + a*b*e)*arccot(d*x + c), x)
```

Sympy [F]

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^2\,dx=\int (a+b\,\mathrm{acot}\,(c+dx))^2\,(e+fx)\,\,dx$$

input
```
integrate((f*x+e)*(a+b*acot(d*x+c))**2,x)
```

output
```
Integral((a + b*acot(c + d*x))**2*(e + f*x), x)
```

Maxima [F]

$$\int (e+fx) \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \int (fx+e)(b \operatorname{arccot}(dx+c)+a)^2 \, dx$$

input
```
integrate((f*x+e)*(a+b*arccot(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/8*b^2*f*x^2*arctan2(1, d*x + c)^2 + 1/4*b^2*e*x*arctan2(1, d*x + c)^2 +
1/2*a^2*f*x^2 + (x^2*arccot(d*x + c) + d*(x/d^2 + (c^2 - 1)*arctan((d^2*x
+ c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a*b*f + a^2*e*x +
 (2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*a*b*e/d - 1/32*(b^2*
f*x^2 + 2*b^2*e*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + integrate(1/16*(12
*b^2*d^2*f*x^3*arctan2(1, d*x + c)^2 + 4*(3*b^2*d^2*e*arctan2(1, d*x + c)^
2 + (6*b^2*c*arctan2(1, d*x + c)^2 + b^2*arctan2(1, d*x + c))*d*f)*x^2 + (
b^2*d^2*f*x^3 + (b^2*d^2*e + 2*b^2*c*d*f)*x^2 + (b^2*c^2 + b^2)*e + (2*b^2
*c*d*e + (b^2*c^2 + b^2)*f)*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 12*(b^
2*c^2*arctan2(1, d*x + c)^2 + b^2*arctan2(1, d*x + c)^2)*e + 4*(2*(3*b^2*c
*arctan2(1, d*x + c)^2 + b^2*arctan2(1, d*x + c))*d*e + 3*(b^2*c^2*arctan2
(1, d*x + c)^2 + b^2*arctan2(1, d*x + c)^2)*f)*x + 2*(b^2*d^2*f*x^3 + 2*b^
2*c*d*e*x + (2*b^2*d^2*e + b^2*c*d*f)*x^2)*log(d^2*x^2 + 2*c*d*x + c^2 + 1
))/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
```

Giac [F]

$$\int (e+fx) \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \int (fx+e)(b \operatorname{arccot}(dx+c)+a)^2 \, dx$$

input
```
integrate((f*x+e)*(a+b*arccot(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)*(b*arccot(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^2\,dx=\int (e+f\,x)\,\left(a+b\,\mathrm{acot}(c+d\,x)\right)^2\,dx$$

input
```
int((e + f*x)*(a + b*acot(c + d*x))^2,x)
```

output
```
int((e + f*x)*(a + b*acot(c + d*x))^2, x)
```

Reduce [F]

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^2\,dx$$
$$=\frac{acot(dx+c)^2b^2c^2f+2acot(dx+c)^2b^2d^2ex+acot(dx+c)^2b^2d^2f\,x^2+acot(dx+c)^2b^2f-2acot(dx+}{}$$

input
```
int((f*x+e)*(a+b*acot(d*x+c))^2,x)
```

output
```
(acot(c + d*x)**2*b**2*c**2*f + 2*acot(c + d*x)**2*b**2*d**2*e*x + acot(c
+ d*x)**2*b**2*d**2*f*x**2 + acot(c + d*x)**2*b**2*f - 2*acot(c + d*x)*a*b
*c**2*f + 4*acot(c + d*x)*a*b*c*d*e + 4*acot(c + d*x)*a*b*d**2*e*x + 2*aco
t(c + d*x)*a*b*d**2*f*x**2 + 2*acot(c + d*x)*a*b*f + 2*acot(c + d*x)*b**2*
c*f + 2*acot(c + d*x)*b**2*d*f*x - 4*int((acot(c + d*x)*x)/(c**2 + 2*c*d*x
 + d**2*x**2 + 1),x)*b**2*c*d**2*f + 4*int((acot(c + d*x)*x)/(c**2 + 2*c*d
*x + d**2*x**2 + 1),x)*b**2*d**3*e - 2*log(c**2 + 2*c*d*x + d**2*x**2 + 1)
*a*b*c*f + 2*log(c**2 + 2*c*d*x + d**2*x**2 + 1)*a*b*d*e + log(c**2 + 2*c*
d*x + d**2*x**2 + 1)*b**2*f + 2*a**2*d**2*e*x + a**2*d**2*f*x**2 + 2*a*b*d
*f*x)/(2*d**2)
```

3.25 $\int \left(a+b\cot^{-1}(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 102

$$\int \left(a+b\cot^{-1}(c+dx)\right)^2\,dx=\frac{i\left(a+b\cot^{-1}(c+dx)\right)^2}{d}+\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{d}-\frac{2b\left(a+b\cot^{-1}(c+dx)\right)\log\left(\frac{2}{1+i(c+dx)}\right)}{d}+\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d}$$

output
```
I*(a+b*arccot(d*x+c))^2/d+(d*x+c)*(a+b*arccot(d*x+c))^2/d-2*b*(a+b*arccot(
d*x+c))*ln(2/(1+I*(d*x+c)))/d+I*b^2*polylog(2,1-2/(1+I*(d*x+c)))/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.16

$$\int \left(a+b\cot^{-1}(c+dx)\right)^2\,dx$$

$$=\frac{b^2(i+c+dx)\cot^{-1}(c+dx)^2+2b\cot^{-1}(c+dx)\left(ac+adx-b\log\left(1-e^{2i\cot^{-1}(c+dx)}\right)\right)+a\Big(ac+adx}{d}$$

input
```
Integrate[(a + b*ArcCot[c + d*x])^2,x]
```

output
```
(b^2*(I + c + d*x)*ArcCot[c + d*x]^2 + 2*b*ArcCot[c + d*x]*(a*c + a*d*x -
b*Log[1 - E^((2*I)*ArcCot[c + d*x])]) + a*(a*c + a*d*x - 2*b*Log[1/((c + d
*x)*Sqrt[1 + (c + d*x)^(-2)])]) + I*b^2*PolyLog[2, E^((2*I)*ArcCot[c + d*x
])])/d
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5563, 5346, 5456, 5380, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\cot^{-1}(c+dx)\right)^2dx$$

$$\Big\downarrow \text{5563}$$

$$\frac{\int \left(a+b\cot^{-1}(c+dx)\right)^2d(c+dx)}{d}$$

$$\Big\downarrow \text{5346}$$

$$\frac{2b\int \frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)}{(c+dx)^2+1}d(c+dx)+(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{d}$$

$$\Big\downarrow \text{5456}$$

$$\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2+2b\left(\frac{i\left(a+b\cot^{-1}(c+dx)\right)^2}{2b}-\int \frac{a+b\cot^{-1}(c+dx)}{-c-dx+i}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{5380}$$

$$\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2+2b\left(-b\int \frac{\log\left(\frac{2}{i(c+dx)+1}\right)}{(c+dx)^2+1}d(c+dx)+\frac{i\left(a+b\cot^{-1}(c+dx)\right)^2}{2b}-\log\left(\frac{2}{1+i(c+dx)}\right)\left(a+b\cot\right.\right.}{d}$$

$$\Big\downarrow \text{2849}$$

$$\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2+2b\left(ib\int\frac{\log\left(\frac{2}{i(c+dx)+1}\right)}{1-\frac{2}{i(c+dx)+1}}d\frac{1}{i(c+dx)+1}+\frac{i(a+b\cot^{-1}(c+dx))^2}{2b}-\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\,c}{d}$$

$$\Big\downarrow \text{2752}$$

$$\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2+2b\left(\frac{i(a+b\cot^{-1}(c+dx))^2}{2b}-\log\left(\frac{2}{1+i(c+dx)}\right)\left(a+b\cot^{-1}(c+dx)\right)+\frac{1}{2}ib\,\text{PolyLog}\left(\right.\right.}{d}$$

input
```
Int[(a + b*ArcCot[c + d*x])^2,x]
```

output
```
((c + d*x)*(a + b*ArcCot[c + d*x])^2 + 2*b*((((I/2)*(a + b*ArcCot[c + d*x])
^2)/b - (a + b*ArcCot[c + d*x])*Log[2/(1 + I*(c + d*x))] + (I/2)*b*PolyLog
[2, 1 - 2/(1 + I*(c + d*x))]))/d
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5380
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Simp[b*c*(
p/e)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5456
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Simp[
1/(c*d)   Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5563
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
 x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.80

method	result
parts	$a^2x + \frac{b^2\left(\operatorname{arccot}(dx+c)^2(dx+c-i)-2\operatorname{arccot}(dx+c)\ln\left(1-\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right)-2\operatorname{arccot}(dx+c)\ln\left(1+\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right)+\right.}{d}$
derivativedivides	$\frac{(dx+c)a^2+b^2\left(\operatorname{arccot}(dx+c)^2(dx+c-i)-2\operatorname{arccot}(dx+c)\ln\left(1-\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right)-2\operatorname{arccot}(dx+c)\ln\left(1+\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right.\right.}{}$
default	$\frac{(dx+c)a^2+b^2\left(\operatorname{arccot}(dx+c)^2(dx+c-i)-2\operatorname{arccot}(dx+c)\ln\left(1-\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right)-2\operatorname{arccot}(dx+c)\ln\left(1+\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right.\right.}{}$
risch	$a^2x+\frac{\pi^2b^2x}{4}+\pi abx-\frac{i\ln(-idx-ic+1)abc}{d}-\frac{i\pi\ln(-idx-ic+1)b^2c}{2d}+\frac{ib^2\pi\ln(d^2x^2+2cdx+c^2+1)c}{4d}+ib$

input
```
int((a+b*arccot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+b^2/d*(arccot(d*x+c)^2*(d*x+c-I)-2*arccot(d*x+c)*ln(1-(d*x+c+I)/(1+(
d*x+c)^2)^(1/2))-2*arccot(d*x+c)*ln(1+(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+2*I*a
rccot(d*x+c)^2+2*I*polylog(2,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+2*I*polylog(2,
-(d*x+c+I)/(1+(d*x+c)^2)^(1/2)))+2*a*b/d*(arccot(d*x+c)*(d*x+c)+1/2*ln(1+(
d*x+c)^2))
```

Fricas [F]

$$\int \left(a+b\cot^{-1}(c+dx)\right)^2\,dx=\int (b\,\mathrm{arccot}\,(dx+c)+a)^2\,dx$$

input
```
integrate((a+b*arccot(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(b^2*arccot(d*x + c)^2 + 2*a*b*arccot(d*x + c) + a^2, x)
```

Sympy [F]

$$\int \left(a+b\cot^{-1}(c+dx)\right)^2\,dx=\int (a+b\,\mathrm{acot}\,(c+dx))^2\,dx$$

input
```
integrate((a+b*acot(d*x+c))**2,x)
```

output
```
Integral((a + b*acot(c + d*x))**2, x)
```

Maxima [F]

$$\int \left(a+b\cot^{-1}(c+dx)\right)^2\,dx=\int (b\,\mathrm{arccot}\,(dx+c)+a)^2\,dx$$

input
```
integrate((a+b*arccot(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/16*(4*x*arctan2(1, d*x + c)^2 - x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 1
6*integrate(1/16*(12*d^2*x^2*arctan2(1, d*x + c)^2 + 12*c^2*arctan2(1, d*x
 + c)^2 + 8*(3*c*arctan2(1, d*x + c)^2 + arctan2(1, d*x + c))*d*x + (d^2*x
^2 + 2*c*d*x + c^2 + 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 12*arctan2(1,
 d*x + c)^2 + 4*(d^2*x^2 + c*d*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^2*x
^2 + 2*c*d*x + c^2 + 1), x))*b^2 + a^2*x + (2*(d*x + c)*arccot(d*x + c) +
log((d*x + c)^2 + 1))*a*b/d
```

Giac [F]

$$\int \left(a+b\cot^{-1}(c+dx)\right)^2\,dx=\int (b\,\mathrm{arccot}\,(dx+c)+a)^2\,dx$$

input
```
integrate((a+b*arccot(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*arccot(d*x + c) + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.21

$$\begin{aligned}\int \left(a+b\cot^{-1}(c+dx)\right)^2\,dx&=a^2\,x+\frac{a\,b\,\left(\ln\left({\left(c+d\,x\right)}^2+1\right)+2\,\mathrm{acot}(c+d\,x)\,\left(c+d\,x\right)\right)}{d}\\&\quad-\frac{2\,b^2\,\ln\left(1-{\mathrm{e}}^{\mathrm{acot}(c+d\,x)\,2{}\mathrm{i}}\right)\,\mathrm{acot}(c+d\,x)}{d}\\&\quad+\frac{b^2\,{\mathrm{acot}(c+d\,x)}^2\,\left(c+d\,x\right)}{d}\\&\quad+\frac{b^2\,\mathrm{polylog}\left(2,{\mathrm{e}}^{\mathrm{acot}(c+d\,x)\,2{}\mathrm{i}}\right)\,1{}\mathrm{i}}{d}+\frac{b^2\,{\mathrm{acot}(c+d\,x)}^2\,1{}\mathrm{i}}{d}\end{aligned}$$

input
```
int((a + b*acot(c + d*x))^2,x)
```

output
```
a^2*x + (b^2*polylog(2, exp(acot(c + d*x)*2i))*1i)/d + (b^2*acot(c + d*x)^
2*1i)/d + (a*b*(log((c + d*x)^2 + 1) + 2*acot(c + d*x)*(c + d*x)))/d - (2*
b^2*log(1 - exp(acot(c + d*x)*2i))*acot(c + d*x))/d + (b^2*acot(c + d*x)^2
*(c + d*x))/d
```

Reduce [F]

$$\int \left(a+b\cot^{-1}(c+dx)\right)^2\,dx$$
$$=\frac{acot(dx+c)^2 b^2dx+2acot(dx+c)\,abc+2acot(dx+c)\,abdx+2\left(\int \frac{acot(dx+c)x}{d^2x^2+2cdx+c^2+1}dx\right)b^2d^2+\log(d^2x^2 -}{d}$$

input
```
int((a+b*acot(d*x+c))^2,x)
```

output
```
(acot(c + d*x)**2*b**2*d*x + 2*acot(c + d*x)*a*b*c + 2*acot(c + d*x)*a*b*d
*x + 2*int((acot(c + d*x)*x)/(c**2 + 2*c*d*x + d**2*x**2 + 1),x)*b**2*d**2
 + log(c**2 + 2*c*d*x + d**2*x**2 + 1)*a*b + a**2*d*x)/d
```

3.26 $\int \frac{(a+b\cot^{-1}(c+dx))^2}{e+fx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 261

$$\begin{aligned}\int \frac{(a+b\cot^{-1}(c+dx))^2}{e+fx}\,dx &= -\frac{(a+b\cot^{-1}(c+dx))^2\log\left(\frac{2}{1-i(c+dx)}\right)}{f} \\ &+\frac{(a+b\cot^{-1}(c+dx))^2\log\left(\frac{2d(e+fx)}{(de+(i-c)f)(1-i(c+dx))}\right)}{f} \\ &-\frac{ib(a+b\cot^{-1}(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{f} \\ &+\frac{ib(a+b\cot^{-1}(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+(i-c)f)(1-i(c+dx))}\right)}{f} \\ &-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2f}+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+(i-c)f)(1-i(c+dx))}\right)}{2f}\end{aligned}$$

output
```
-(a+b*arccot(d*x+c))^2*ln(2/(1-I*(d*x+c)))/f+(a+b*arccot(d*x+c))^2*ln(2*d*
(f*x+e)/(d*e+(I-c)*f)/(1-I*(d*x+c)))/f-I*b*(a+b*arccot(d*x+c))*polylog(2,1
-2/(1-I*(d*x+c)))/f+I*b*(a+b*arccot(d*x+c))*polylog(2,1-2*d*(f*x+e)/(d*e+(
I-c)*f)/(1-I*(d*x+c)))/f-1/2*b^2*polylog(3,1-2/(1-I*(d*x+c)))/f+1/2*b^2*po
lylog(3,1-2*d*(f*x+e)/(d*e+(I-c)*f)/(1-I*(d*x+c)))/f
```

Mathematica [F]

$$\int \frac{(a + b\cot^{-1}(c + dx))^2}{e + fx}\,dx = \int \frac{(a + b\cot^{-1}(c + dx))^2}{e + fx}\,dx$$

input
```
Integrate[(a + b*ArcCot[c + d*x])^2/(e + f*x),x]
```

output
```
Integrate[(a + b*ArcCot[c + d*x])^2/(e + f*x), x]
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {5571, 27, 5384}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b\cot^{-1}(c + dx))^2}{e + fx}\,dx$$

$$\Big\downarrow \; 5571$$

$$\frac{\int \frac{d(a+b\cot^{-1}(c+dx))^2}{d\left(e-\frac{cf}{d}\right)+f(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\int \frac{(a + b\cot^{-1}(c + dx))^2}{f(c + dx) - cf + de}d(c + dx)$$

$$\Big\downarrow \text{5384}$$

$$\frac{ib(a+b\cot^{-1}(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2(de-cf+f(c+dx))}{(de-cf+if)(1-i(c+dx))}\right)}{f}+\frac{(a+b\cot^{-1}(c+dx))^2\log\left(\frac{2(f(c+dx)-cf+de)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}-\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)(a+b\cot^{-1}(c+dx))}{f}-\frac{\log\left(\frac{2}{1-i(c+dx)}\right)(a+b\cot^{-1}(c+dx))^2}{f}+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2(de-cf+f(c+dx))}{(de-cf+if)(1-i(c+dx))}\right)}{2f}-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2f}$$

input
```
Int[(a + b*ArcCot[c + d*x])^2/(e + f*x),x]
```

output
```
-(((a + b*ArcCot[c + d*x])^2*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcCot
[c + d*x])^2*Log[(2*(d*e - c*f + f*(c + d*x)))/((d*e + I*f - c*f)*(1 - I*(
c + d*x)))])/f - (I*b*(a + b*ArcCot[c + d*x])*PolyLog[2, 1 - 2/(1 - I*(c +
 d*x))])/f + (I*b*(a + b*ArcCot[c + d*x])*PolyLog[2, 1 - (2*(d*e - c*f + f
*(c + d*x)))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f - (b^2*PolyLog[3, 1
 - 2/(1 - I*(c + d*x))])/(2*f) + (b^2*PolyLog[3, 1 - (2*(d*e - c*f + f*(c
+ d*x)))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5384
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcCot[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Cot[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] - Simp[I
*b*(a + b*ArcCot[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] + Simp[I*b*(a
+ b*ArcCot[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 21.45 (sec) , antiderivative size = 1911, normalized size of antiderivative = 7.32

method	result	size
derivativedivides	Expression too large to display	1911
default	Expression too large to display	1911
parts	Expression too large to display	2019

input
```
int((a+b*arccot(d*x+c))^2/(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*d*ln(c*f-d*e-f*(d*x+c))/f-b^2*d*(-ln(c*f-d*e-f*(d*x+c))/f*arccot(
d*x+c)^2-2/f*(-1/2*arccot(d*x+c)^2*ln(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(
d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f-c*f+d*e)-1/2*I*
c*f/(-I*f+c*f-d*e)*arccot(d*x+c)*polylog(2,(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d
*x+c+I)^2/(1+(d*x+c)^2))+1/2*arccot(d*x+c)^2*ln((d*x+c+I)^2/(1+(d*x+c)^2)-
1)-1/2*arccot(d*x+c)^2*ln(1-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+1/4*I*Pi*csgn(I
*(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+
I)^2/(1+(d*x+c)^2)-I*f-c*f+d*e)/((d*x+c+I)^2/(1+(d*x+c)^2)-1))*(csgn(I*(-I
*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2
/(1+(d*x+c)^2)-I*f-c*f+d*e))*csgn(I/((d*x+c+I)^2/(1+(d*x+c)^2)-1))-csgn(I*
(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I
)^2/(1+(d*x+c)^2)-I*f-c*f+d*e)/((d*x+c+I)^2/(1+(d*x+c)^2)-1))*csgn(I/((d*x
+c+I)^2/(1+(d*x+c)^2)-1))-csgn(I*(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(d*x+
c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f-c*f+d*e))*csgn(I*(-
I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^
2/(1+(d*x+c)^2)-I*f-c*f+d*e)/((d*x+c+I)^2/(1+(d*x+c)^2)-1))+csgn(I*(-I*f*(
d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+
(d*x+c)^2)-I*f-c*f+d*e)/((d*x+c+I)^2/(1+(d*x+c)^2)-1))^2)*arccot(d*x+c)^2-
polylog(3,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-1/2*arccot(d*x+c)^2*ln(1+(d*x+c+I
)/(1+(d*x+c)^2)^(1/2))-1/2*I*f/(-I*f+c*f-d*e)*arccot(d*x+c)^2*ln(1-(d*e...
```

Fricas [F]

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{e+fx}\,dx=\int \frac{(b\operatorname{arccot}(dx+c)+a)^2}{fx+e}\,dx$$

input
```
integrate((a+b*arccot(d*x+c))^2/(f*x+e),x, algorithm="fricas")
```

output
```
integral((b^2*arccot(d*x + c)^2 + 2*a*b*arccot(d*x + c) + a^2)/(f*x + e),
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{e+fx}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acot(d*x+c))**2/(f*x+e),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{e+fx}\,dx=\int \frac{(b\operatorname{arccot}(dx+c)+a)^2}{fx+e}\,dx$$

input
```
integrate((a+b*arccot(d*x+c))^2/(f*x+e),x, algorithm="maxima")
```

output
```
a^2*log(f*x + e)/f + integrate(1/16*(12*b^2*arctan2(1, d*x + c)^2 + b^2*lo
g(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 32*a*b*arctan2(1, d*x + c))/(f*x + e),
x)
```

Giac [F]

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{e+fx}\,dx=\int \frac{(b\,\mathrm{arccot}\,(dx+c)+a)^2}{fx+e}\,dx$$

input
```
integrate((a+b*arccot(d*x+c))^2/(f*x+e),x, algorithm="giac")
```

output
```
integrate((b*arccot(d*x + c) + a)^2/(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{e+fx}\,dx=\int \frac{(a+b\,\mathrm{acot}(c+d\,x))^2}{e+f\,x}\,dx$$

input
```
int((a + b*acot(c + d*x))^2/(e + f*x),x)
```

output
```
int((a + b*acot(c + d*x))^2/(e + f*x), x)
```

Reduce [F]

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{e+fx}\,dx=\frac{2\left(\int \frac{acot(dx+c)}{fx+e}dx\right)abf+\left(\int \frac{acot(dx+c)^2}{fx+e}dx\right)b^2f+\log(fx+e)\,a^2}{f}$$

input
```
int((a+b*acot(d*x+c))^2/(f*x+e),x)
```

output
```
(2*int(acot(c + d*x)/(e + f*x),x)*a*b*f + int(acot(c + d*x)**2/(e + f*x),x
)*b**2*f + log(e + f*x)*a**2)/f
```

3.27 $\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 480

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{(e+fx)^2}\,dx = \frac{id(a+b\cot^{-1}(c+dx))^2}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{d(de-cf)(a+b\cot^{-1}(c+dx))^2}{f(d^2e^2-2cdef+(1+c^2)f^2)} - \frac{(a+b\cot^{-1}(c+dx))^2}{f(e+fx)} + \frac{2bd(a+b\cot^{-1}(c+dx))\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} - \frac{2bd(a+b\cot^{-1}(c+dx))\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2} - \frac{2bd(a+b\cot^{-1}(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{ib^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} - \frac{ib^2d\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{ib^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

output
```
I*d*(a+b*arccot(d*x+c))^2/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+d*(-c*f+d*e)*(a+
b*arccot(d*x+c))^2/f/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-(a+b*arccot(d*x+c))^2
/f/(f*x+e)+2*b*d*(a+b*arccot(d*x+c))*ln(2/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*
f+(c^2+1)*f^2)-2*b*d*(a+b*arccot(d*x+c))*ln(2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I
*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-2*b*d*(a+b*arccot(d*x+c))*ln(2/
(1+I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+I*b^2*d*polylog(2,1-2/(1-I*
(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-I*b^2*d*polylog(2,1-2*d*(f*x+e)/
(d*e+I*f-c*f)/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+I*b^2*d*polyl
og(2,1-2/(1+I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)
```

Mathematica [A] (warning: unable to verify)

Time = 5.61 (sec) , antiderivative size = 454, normalized size of antiderivative = 0.95

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{(e+fx)^2}\,dx =$$

$$-\frac{a^2+\frac{2abf\left(\left(-cde+f+c^2f-d^2ex+cdfx\right)\cot^{-1}(c+dx)+d(e+fx)\log\left(-\frac{d(e+fx)}{(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}}\right)\right)}{d^2e^2-2cdef+(1+c^2)f^2}+\frac{b^2d(e+fx)(1+(c+dx)^2)\left(\frac{e^{i\arctan\left(\cdots\right)}}{(-de+\cdots}\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(a + b*ArcCot[c + d*x])^2/(e + f*x)^2,x]
```

output
```
-((a^2 + (2*a*b*f*((-(c*d*e) + f + c^2*f - d^2*e*x + c*d*f*x)*ArcCot[c + d
*x] + d*(e + f*x)*Log[-((d*(e + f*x))/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)])
)]))/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (b^2*d*(e + f*x)*(1 + (c + d*
x)^2)*((E^(I*ArcTan[f/(d*e - c*f)])*ArcCot[c + d*x]^2)/((-(d*e) + c*f)*Sqr
t[1 + f^2/(d*e - c*f)^2]) + ArcCot[c + d*x]^2/(d*e + d*f*x) + (f*(I*Pi*Arc
Cot[c + d*x] + Pi*Log[1 + E^((-2*I)*ArcCot[c + d*x])] + 2*ArcCot[c + d*x]*
Log[1 - E^((2*I)*(ArcCot[c + d*x] + ArcTan[f/(d*e - c*f)]))] - Pi*Log[1/Sq
rt[1 + (c + d*x)^(-2)]] + 2*ArcTan[f/(-(d*e) + c*f)]*(I*ArcCot[c + d*x] -
Log[1 - E^((2*I)*(ArcCot[c + d*x] + ArcTan[f/(d*e - c*f)]))] + Log[Sin[Arc
Cot[c + d*x] + ArcTan[f/(d*e - c*f)]]]) - I*PolyLog[2, E^((2*I)*(ArcCot[c
+ d*x] + ArcTan[f/(d*e - c*f)])]))/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2))
)/((c + d*x)^2*(1 + (c + d*x)^(-2))))/(f*(e + f*x)))
```

Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5569, 7292, 5581, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{(e+fx)^2}dx$$

$$\Big\downarrow \text{5569}$$

$$-\frac{2bd\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)((c+dx)^2+1)}dx}{f}-\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)}$$

$$\Big\downarrow \text{7292}$$

$$-\frac{2bd\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)(c^2+2dxc+d^2x^2+1)}dx}{f}-\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)}$$

$$\Big\downarrow \text{5581}$$

$$-\frac{2b\int \frac{d\left(a+b\cot^{-1}(c+dx)\right)}{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)((c+dx)^2+1)}d(c+dx)}{f}-\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2bd\int \frac{a+b\cot^{-1}(c+dx)}{(de-cf+f(c+dx))((c+dx)^2+1)}d(c+dx)}{f}-\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)}$$

$$\Big\downarrow \text{7276}$$

$$-\frac{2bd\int \left(\frac{a}{(de-cf+f(c+dx))((c+dx)^2+1)}+\frac{b\cot^{-1}(c+dx)}{(de-cf+f(c+dx))((c+dx)^2+1)}\right)d(c+dx)}{f}-\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)}-$$

$$2bd\left(\frac{a\arctan(c+dx)(de-cf)}{(de-cf)^2+f^2}+\frac{af\log(f(c+dx)-cf+de)}{(de-cf)^2+f^2}-\frac{af\log((c+dx)^2+1)}{2((de-cf)^2+f^2)}-\frac{ibf\,\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{2((c^2+1)f^2-2cdef+d^2e^2)}-\frac{ibf\,\text{PolyLog}\left(2,1-\frac{}{i(c\right.}}{2((c^2+1)f^2-2cdef}\right.$$

input
```
Int[(a + b*ArcCot[c + d*x])^2/(e + f*x)^2,x]
```

output
```
-((a + b*ArcCot[c + d*x])^2/(f*(e + f*x))) - (2*b*d*(((-1/2*I)*b*f*ArcCot[
c + d*x]^2)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (b*(d*e - c*f)*ArcCot[
c + d*x]^2)/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)) + (a*(d*e - c*f)*Arc
Tan[c + d*x])/(f^2 + (d*e - c*f)^2) - (b*f*ArcCot[c + d*x]*Log[2/(1 - I*(c
 + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (b*f*ArcCot[c + d*x]*Lo
g[2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (a*f*Log[d
*e - c*f + f*(c + d*x)])/(f^2 + (d*e - c*f)^2) + (b*f*ArcCot[c + d*x]*Log[
(2*(d*e - c*f + f*(c + d*x)))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(d^2
*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (a*f*Log[1 + (c + d*x)^2])/(2*(f^2 + (
d*e - c*f)^2)) - ((I/2)*b*f*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/(d^2*e^2
 - 2*c*d*e*f + (1 + c^2)*f^2) - ((I/2)*b*f*PolyLog[2, 1 - 2/(1 + I*(c + d*x
))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((I/2)*b*f*PolyLog[2, 1 - (2*
(d*e - c*f + f*(c + d*x)))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(d^2*e^
2 - 2*c*d*e*f + (1 + c^2)*f^2)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5569
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCot[c + d*x])^p/(f*(m +
1))), x] + Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcCot[
c + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[p, 0] && ILtQ[m, -1]
```

rule 5581
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Subs
t[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcCot[x])
^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &
& EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 784, normalized size of antiderivative = 1.63

method	result
parts	$-\frac{a^2}{(fx+e)f}+\frac{b^2\left(-\frac{d^2\operatorname{arccot}(dx+c)^2}{(f(dx+c)-cf+de)f}-\frac{2d^2\left(-\frac{\operatorname{arccot}(dx+c)f\ln\left(1+(dx+c)^2\right)}{2\left(c^2f^2-2cdef+d^2e^2+f^2\right)}-\frac{\operatorname{arccot}(dx+c)\arctan(dx+c)cf}{c^2f^2-2cdef+d^2e^2+f^2}+\frac{\operatorname{arccot}(dx+}{c^2f^2-2}\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{a^2d^2}{(cf-de-f(dx+c))f}+b^2d^2\left(\frac{\operatorname{arccot}(dx+c)^2}{(cf-de-f(dx+c))f}+\frac{-\frac{2\operatorname{arccot}(dx+c)f\ln(cf-de-f(dx+c))}{c^2f^2-2cdef+d^2e^2+f^2}+\frac{2\operatorname{arccot}(dx+c)f\ln\left(1+(dx+c)^2\right)}{2c^2f^2-4cdef+2d^2e^2+2f^2}+2}{}\right.}{}$
default	$\frac{\frac{a^2d^2}{(cf-de-f(dx+c))f}+b^2d^2\left(\frac{\operatorname{arccot}(dx+c)^2}{(cf-de-f(dx+c))f}+\frac{-\frac{2\operatorname{arccot}(dx+c)f\ln(cf-de-f(dx+c))}{c^2f^2-2cdef+d^2e^2+f^2}+\frac{2\operatorname{arccot}(dx+c)f\ln\left(1+(dx+c)^2\right)}{2c^2f^2-4cdef+2d^2e^2+2f^2}+2}{}\right.}{}$

input
```
int((a+b*arccot(d*x+c))^2/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/(f*x+e)/f+b^2/d*(-d^2/(f*(d*x+c)-c*f+d*e)/f*arccot(d*x+c)^2-2*d^2/f*(
-1/2*arccot(d*x+c)/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f*ln(1+(d*x+c)^2)-arcco
t(d*x+c)/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)*c*f+arccot(d*x+c)/(
c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)*d*e+arccot(d*x+c)/(c^2*f^2-2*
c*d*e*f+d^2*e^2+f^2)*f*ln(f*(d*x+c)-c*f+d*e)+f^2/(c^2*f^2-2*c*d*e*f+d^2*e^
2+f^2)*(-1/2*I*ln(f*(d*x+c)-c*f+d*e)*(ln((I*f-f*(d*x+c))/(d*e+I*f-c*f))-ln
((I*f+f*(d*x+c))/(c*f-d*e+I*f)))/f-1/2*I*(dilog((I*f-f*(d*x+c))/(d*e+I*f-c
*f))-dilog((I*f+f*(d*x+c))/(c*f-d*e+I*f)))/f)-1/2/(c^2*f^2-2*c*d*e*f+d^2*e
^2+f^2)*f*(-1/2*I*(ln(d*x+c-I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c-I)^2-dilog(-1/
2*I*(d*x+c+I))-ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I)))+1/2*I*(ln(d*x+c+I)*ln(1+(
d*x+c)^2)-1/2*ln(d*x+c+I)^2-dilog(1/2*I*(d*x+c-I))-ln(d*x+c+I)*ln(1/2*I*(d
*x+c-I))))-1/2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*(c*f-d*e)*arctan(d*x+c)^2))
+2*a*b/d*(-d^2/(f*(d*x+c)-c*f+d*e)/f*arccot(d*x+c)-d^2/f*(1/(c^2*f^2-2*c*d
*e*f+d^2*e^2+f^2)*(-1/2*f*ln(1+(d*x+c)^2)+(-c*f+d*e)*arctan(d*x+c))+1/(c^2
*f^2-2*c*d*e*f+d^2*e^2+f^2)*f*ln(f*(d*x+c)-c*f+d*e)))
```

Fricas [F]

$$\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{(e+fx)^2}\,dx=\int \frac{\left(b\,\mathrm{arccot}\left(dx+c\right)+a\right)^2}{\left(fx+e\right)^2}\,dx$$

input
```
integrate((a+b*arccot(d*x+c))^2/(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arccot(d*x + c)^2 + 2*a*b*arccot(d*x + c) + a^2)/(f^2*x^2 +
2*e*f*x + e^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{(e+fx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acot(d*x+c))**2/(f*x+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{(e+fx)^2}\,dx = \int \frac{(b\,\text{arccot}\,(dx+c)+a)^2}{(fx+e)^2}\,dx$$

input
```
integrate((a+b*arccot(d*x+c))^2/(f*x+e)^2,x, algorithm="maxima")
```

output
```
-(d*(2*(d^2*e - c*d*f)*arctan((d^2*x + c*d)/d)/((d^2*e^2*f - 2*c*d*e*f^2 +
 (c^2 + 1)*f^3)*d) - log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2 - 2*c*d*e*f
 + (c^2 + 1)*f^2) + 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 + 1)*f^2))
+ 2*arccot(d*x + c)/(f^2*x + e*f))*a*b - 1/16*(4*arctan2(1, d*x + c)^2 - 1
6*(f^2*x + e*f)*integrate(1/16*(12*d^2*f*x^2*arctan2(1, d*x + c)^2 + 8*(3*
c*arctan2(1, d*x + c)^2 - arctan2(1, d*x + c))*d*f*x - 8*d*e*arctan2(1, d*
x + c) + (d^2*f*x^2 + 2*c*d*f*x + (c^2 + 1)*f)*log(d^2*x^2 + 2*c*d*x + c^2
 + 1)^2 + 12*(c^2*arctan2(1, d*x + c)^2 + arctan2(1, d*x + c)^2)*f - 4*(d^
2*f*x^2 + c*d*e + (d^2*e + c*d*f)*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^
2*f^3*x^4 + (c^2 + 1)*e^2*f + 2*(d^2*e*f^2 + c*d*f^3)*x^3 + (d^2*e^2*f + 4
*c*d*e*f^2 + (c^2 + 1)*f^3)*x^2 + 2*(c*d*e^2*f + (c^2 + 1)*e*f^2)*x), x) -
 log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2)*b^2/(f^2*x + e*f) - a^2/(f^2*x + e*f)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{(e+fx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arccot(d*x+c))^2/(f*x+e)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{(e+fx)^2}\,dx = \int \frac{(a+b\,\mathrm{acot}(c+d\,x))^2}{(e+f\,x)^2}\,dx$$

input
```
int((a + b*acot(c + d*x))^2/(e + f*x)^2,x)
```

output
```
int((a + b*acot(c + d*x))^2/(e + f*x)^2, x)
```

Reduce [F]

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{(e+fx)^2}\,dx = \text{too large to display}$$

input
```
int((a+b*acot(d*x+c))^2/(f*x+e)^2,x)
```

output
```
( - 2*acot(c + d*x)**2*b**2*c**3*e*f**3 + 5*acot(c + d*x)**2*b**2*c**2*d*e
**2*f**2 - acot(c + d*x)**2*b**2*c**2*d*e*f**3*x - 4*acot(c + d*x)**2*b**2
*c*d**2*e**3*f + 2*acot(c + d*x)**2*b**2*c*d**2*e**2*f**2*x - 2*acot(c + d
*x)**2*b**2*c*e*f**3 + acot(c + d*x)**2*b**2*d**3*e**4 - acot(c + d*x)**2*
b**2*d**3*e**3*f*x + acot(c + d*x)**2*b**2*d*e**2*f**2 - acot(c + d*x)**2*
b**2*d*e*f**3*x + 4*acot(c + d*x)*a*b*c**3*f**4*x - 12*acot(c + d*x)*a*b*c
**2*d*e*f**3*x + 12*acot(c + d*x)*a*b*c*d**2*e**2*f**2*x + 4*acot(c + d*x)
*a*b*c*f**4*x - 4*acot(c + d*x)*a*b*d**3*e**3*f*x - 4*acot(c + d*x)*a*b*d*
e*f**3*x - 2*acot(c + d*x)*b**2*c**2*f**4*x + 4*acot(c + d*x)*b**2*c*d*e*f
**3*x - 2*acot(c + d*x)*b**2*d**2*e**2*f**2*x - 2*acot(c + d*x)*b**2*f**4*
x + 4*atan(c + d*x)*a*b*c**3*e*f**3 + 4*atan(c + d*x)*a*b*c**3*f**4*x - 8*
atan(c + d*x)*a*b*c**2*d*e**2*f**2 - 8*atan(c + d*x)*a*b*c**2*d*e*f**3*x +
 4*atan(c + d*x)*a*b*c*d**2*e**3*f + 4*atan(c + d*x)*a*b*c*d**2*e**2*f**2*
x + 4*atan(c + d*x)*a*b*c*e*f**3 + 4*atan(c + d*x)*a*b*c*f**4*x - 4*atan(c
 + d*x)*a*b*d*e**2*f**2 - 4*atan(c + d*x)*a*b*d*e*f**3*x - 2*atan(c + d*x)
*b**2*c**2*e*f**3 - 2*atan(c + d*x)*b**2*c**2*f**4*x + 2*atan(c + d*x)*b**
2*c*d*e**2*f**2 + 2*atan(c + d*x)*b**2*c*d*e*f**3*x - 2*atan(c + d*x)*b**2
*e*f**3 - 2*atan(c + d*x)*b**2*f**4*x + 2*int(atan(1/(c + d*x))/(c**3*e**2
*f + 2*c**3*e*f**2*x + c**3*f**3*x**2 - c**2*d*e**3 + 3*c**2*d*e*f**2*x**2
 + 2*c**2*d*f**3*x**3 - 2*c*d**2*e**3*x - 3*c*d**2*e**2*f*x**2 + c*d**2...
```

3.28 $\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 565

$$
\begin{aligned}
&\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx \\
&= \frac{ab^2f^2x}{d^2}+\frac{b^3f^2(c+dx)\cot^{-1}(c+dx)}{d^3} \\
&+\frac{bf^2(a+b\cot^{-1}(c+dx))^2}{2d^3}+\frac{3ibf(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^3} \\
&+\frac{3bf(de-cf)(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^3}+\frac{bf^2(c+dx)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^3} \\
&+\frac{i(3d^2e^2-6cdef-(1-3c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)^3}{3d^3} \\
&-\frac{(de-cf)\left(d^2e^2-2cdef-(3-c^2)f^2\right)\left(a+b\cot^{-1}(c+dx)\right)^3}{3d^3f} \\
&+\frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)^3}{3f} \\
&-\frac{6b^2f(de-cf)\left(a+b\cot^{-1}(c+dx)\right)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&-\frac{b(3d^2e^2-6cdef-(1-3c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&+\frac{b^3f^2\log\left(1+(c+dx)^2\right)}{2d^3}+\frac{3ib^3f(de-cf)\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&+\frac{ib^2(3d^2e^2-6cdef-(1-3c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&-\frac{b^3(3d^2e^2-6cdef-(1-3c^2)f^2)\operatorname{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d^3}
\end{aligned}
$$

output
```
a*b^2*f^2*x/d^2+b^3*f^2*(d*x+c)*arccot(d*x+c)/d^3+1/2*b*f^2*(a+b*arccot(d*
x+c))^2/d^3+3*I*b*f*(-c*f+d*e)*(a+b*arccot(d*x+c))^2/d^3+3*b*f*(-c*f+d*e)*
(d*x+c)*(a+b*arccot(d*x+c))^2/d^3+1/2*b*f^2*(d*x+c)^2*(a+b*arccot(d*x+c))^
2/d^3+1/3*I*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*(a+b*arccot(d*x+c))^3/d^3
-1/3*(-c*f+d*e)*(d^2*e^2-2*c*d*e*f-(-c^2+3)*f^2)*(a+b*arccot(d*x+c))^3/d^3
/f+1/3*(f*x+e)^3*(a+b*arccot(d*x+c))^3/f-6*b^2*f*(-c*f+d*e)*(a+b*arccot(d*
x+c))*ln(2/(1+I*(d*x+c)))/d^3-b*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*(a+b*
arccot(d*x+c))^2*ln(2/(1+I*(d*x+c)))/d^3+1/2*b^3*f^2*ln(1+(d*x+c)^2)/d^3+3
*I*b^3*f*(-c*f+d*e)*polylog(2,1-2/(1+I*(d*x+c)))/d^3+I*b^2*(3*d^2*e^2-6*c*
d*e*f-(-3*c^2+1)*f^2)*(a+b*arccot(d*x+c))*polylog(2,1-2/(1+I*(d*x+c)))/d^3
-1/2*b^3*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*polylog(3,1-2/(1+I*(d*x+c)))
/d^3
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2336 vs. 2(565) = 1130.

Time = 14.45 (sec) , antiderivative size = 2336, normalized size of antiderivative = 4.13

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^3 dx = \text{Result too large to show}$$

input
```
Integrate[(e + f*x)^2*(a + b*ArcCot[c + d*x])^3,x]
```

output
```
(a^2*(a*d^2*e^2 + 3*b*d*e*f - 2*b*c*f^2)*x)/d^2 + (a^2*f*(2*a*d*e + b*f)*x
^2)/(2*d) + (a^3*f^2*x^3)/3 + a^2*b*x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcCot[c
 + d*x] + ((-3*a^2*b*c*d^2*e^2 - 3*a^2*b*d*e*f + 3*a^2*b*c^2*d*e*f + 3*a^2
*b*c*f^2 - a^2*b*c^3*f^2)*ArcTan[c + d*x])/d^3 + ((3*a^2*b*d^2*e^2 - 6*a^2
*b*c*d*e*f - a^2*b*f^2 + 3*a^2*b*c^2*f^2)*Log[1 + c^2 + 2*c*d*x + d^2*x^2]
)/(2*d^3) + (a*b^2*f^2*x^2*(1 + (c + d*x)^2)*((c + d*x)*(1 - 6*c*ArcCot[c
+ d*x] + 3*ArcCot[c + d*x]^2 + 3*c^2*ArcCot[c + d*x]^2) - (c + d*x)*Sqrt[1
 + (c + d*x)^(-2)]*(1 - 6*c*ArcCot[c + d*x] - ArcCot[c + d*x]^2 + 3*c^2*Ar
cCot[c + d*x]^2)*Cos[3*ArcCot[c + d*x]] - 2*(-2*ArcCot[c + d*x] + I*ArcCot
[c + d*x]^2 + 6*c*ArcCot[c + d*x]^2 - (3*I)*c^2*ArcCot[c + d*x]^2 + 2*(-1
+ 3*c^2)*ArcCot[c + d*x]*Log[1 - E^((2*I)*ArcCot[c + d*x])] - 6*c*Log[1/((
c + d*x)*Sqrt[1 + (c + d*x)^(-2)])] + Cos[2*ArcCot[c + d*x]]*(I*(-1 + 3*c^
2)*ArcCot[c + d*x]^2 + (2 - 6*c^2)*ArcCot[c + d*x]*Log[1 - E^((2*I)*ArcCot
[c + d*x])] + 6*c*Log[1/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)])])) + ((4*I)*(
-1 + 3*c^2)*PolyLog[2, E^((2*I)*ArcCot[c + d*x])])/((c + d*x)^2*(1 + (c +
d*x)^(-2)))))/(4*d*(c + d*x)^2*(1 + (c + d*x)^(-2))*(1/Sqrt[1 + (c + d*x)^
(-2)] - c/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)]))^2) - (3*a*b^2*e^2*(1 + (c
+ d*x)^2)*(-((c + d*x)*ArcCot[c + d*x]^2) + 2*ArcCot[c + d*x]*Log[1 - E^((
2*I)*ArcCot[c + d*x])] - I*(ArcCot[c + d*x]^2 + PolyLog[2, E^((2*I)*ArcCot
[c + d*x])])))/(d*(c + d*x)^2*(1 + (c + d*x)^(-2))) + (6*a*b^2*e*f*(1 +...
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 551, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5571, 27, 5390, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{5571}$$

$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)^2 (a+b\cot^{-1}(c+dx))^3}{d^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (de-cf+f(c+dx))^2 \left(a+b\cot^{-1}(c+dx)\right)^3 d(c+dx)}{d^3}$$

$$\Big\downarrow \; 5390$$

$$\frac{\frac{b\int \left((c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2 f^3+3(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2 f^2+\frac{\left((de-cf)\left(d^2e^2-2cdfe-\left(3-c^2\right)f^2\right)+f\left(3d^2e^2-6cdfe-\left(1-3c^2\right)f^2\right)(c+dx)\right)\left(a\right.}{(c+dx)^2+1}}{f}}{d^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{(f(c+dx)-cf+de)^3\left(a+b\cot^{-1}(c+dx)\right)^3}{3f}+\frac{b\left(ibf\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\text{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)\left(a+b\cot^{-1}(c+dx)\right)+\frac{if\left(-\left(1-3c^2\right)f\right.}{}\right.}{}}{}$$

input

```
Int[(e + f*x)^2*(a + b*ArcCot[c + d*x])^3,x]
```

output

```
(((d*e - c*f + f*(c + d*x))^3*(a + b*ArcCot[c + d*x])^3)/(3*f) + (b*(a*b*f
^3*(c + d*x) + b^2*f^3*(c + d*x)*ArcCot[c + d*x] + (f^3*(a + b*ArcCot[c +
d*x])^2)/2 + (3*I)*f^2*(d*e - c*f)*(a + b*ArcCot[c + d*x])^2 + 3*f^2*(d*e
- c*f)*(c + d*x)*(a + b*ArcCot[c + d*x])^2 + (f^3*(c + d*x)^2*(a + b*ArcCo
t[c + d*x])^2)/2 + ((I/3)*f*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2)*f^2)*(a +
 b*ArcCot[c + d*x])^3)/b - ((d*e - c*f)*(d^2*e^2 - 2*c*d*e*f - (3 - c^2)*f
^2)*(a + b*ArcCot[c + d*x])^3)/(3*b) - 6*b*f^2*(d*e - c*f)*(a + b*ArcCot[c
 + d*x])*Log[2/(1 + I*(c + d*x))] - f*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2)
*f^2)*(a + b*ArcCot[c + d*x])^2*Log[2/(1 + I*(c + d*x))] + (b^2*f^3*Log[1
+ (c + d*x)^2])/2 + (3*I)*b^2*f^2*(d*e - c*f)*PolyLog[2, 1 - 2/(1 + I*(c +
 d*x))] + I*b*f*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2)*f^2)*(a + b*ArcCot[c
+ d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))] - (b^2*f*(3*d^2*e^2 - 6*c*d*e*
f - (1 - 3*c^2)*f^2)*PolyLog[3, 1 - 2/(1 + I*(c + d*x))])/2))/f)/d^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5390
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])^p/(e*(q + 1))), x] + S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcCot[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 48.00 (sec) , antiderivative size = 6248, normalized size of antiderivative = 11.06

method	result	size
parts	Expression too large to display	6248
derivativedivides	Expression too large to display	10834
default	Expression too large to display	10834

input
```
int((f*x+e)^2*(a+b*arccot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \int (fx+e)^2(b\,\mathrm{arccot}\,(dx+c)+a)^3 \, dx$$

input
```
integrate((f*x+e)^2*(a+b*arccot(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(a^3*f^2*x^2 + 2*a^3*e*f*x + a^3*e^2 + (b^3*f^2*x^2 + 2*b^3*e*f*x
+ b^3*e^2)*arccot(d*x + c)^3 + 3*(a*b^2*f^2*x^2 + 2*a*b^2*e*f*x + a*b^2*e^
2)*arccot(d*x + c)^2 + 3*(a^2*b*f^2*x^2 + 2*a^2*b*e*f*x + a^2*b*e^2)*arcco
t(d*x + c), x)
```

Sympy [F]

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \int (a+b\,\mathrm{acot}\,(c+dx))^3\,(e+fx)^2 \, dx$$

input
```
integrate((f*x+e)**2*(a+b*acot(d*x+c))**3,x)
```

output
```
Integral((a + b*acot(c + d*x))**3*(e + f*x)**2, x)
```

Maxima [F]

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \int (fx+e)^2(b\,\mathrm{arccot}\,(dx+c)+a)^3 \, dx$$

input
```
integrate((f*x+e)^2*(a+b*arccot(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/24*b^3*f^2*x^3*arctan2(1, d*x + c)^3 + 1/8*b^3*e*f*x^2*arctan2(1, d*x +
c)^3 + 1/8*b^3*e^2*x*arctan2(1, d*x + c)^3 + 1/3*a^3*f^2*x^3 + a^3*e*f*x^2
 + 3*(x^2*arccot(d*x + c) + d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d
^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a^2*b*e*f + 1/2*(2*x^3*arcco
t(d*x + c) + d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d
)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*a^2*b*f^2 + a^3
*e^2*x + 3/2*(2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*a^2*b*e^
2/d - 1/32*(b^3*f^2*x^3*arctan2(1, d*x + c) + 3*b^3*e*f*x^2*arctan2(1, d*x
 + c) + 3*b^3*e^2*x*arctan2(1, d*x + c))*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^
2 + integrate(1/32*(4*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d
*x + c)^2)*d^2*f^2*x^4 + 4*(2*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arct
an2(1, d*x + c)^2)*d^2*e*f + (b^3*arctan2(1, d*x + c)^2 + 2*(7*b^3*arctan2
(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c)*d*f^2)*x^3 + 4*(7*b^3*
arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2 + (7*b^3*arctan2(1,
 d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c^2)*e^2 + 4*((7*b^3*arctan2
(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*d^2*e^2 + (3*b^3*arctan2(
1, d*x + c)^2 + 4*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x +
 c)^2)*c)*d*e*f + (7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x +
 c)^2 + (7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c^2
)*f^2)*x^2 + (3*b^3*d^2*f^2*x^4*arctan2(1, d*x + c) + (6*b^3*d^2*e*f*ar...
```

Giac [F]

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int (fx+e)^2(b\,\mathrm{arccot}\,(dx+c)+a)^3\,dx$$

input
```
integrate((f*x+e)^2*(a+b*arccot(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*(b*arccot(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \int (e+f\,x)^2\,(a+b\,\mathrm{acot}(c+d\,x))^3 \,dx$$

input
```
int((e + f*x)^2*(a + b*acot(c + d*x))^3,x)
```

output
```
int((e + f*x)^2*(a + b*acot(c + d*x))^3, x)
```

Reduce [F]

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
int((f*x+e)^2*(a+b*acot(d*x+c))^3,x)
```

output

```
( - 4*acot(c + d*x)**3*b**3*c**3*f**2 + 6*acot(c + d*x)**3*b**3*c**2*d*e*f
 - 4*acot(c + d*x)**3*b**3*c*f**2 + 6*acot(c + d*x)**3*b**3*d**3*e**2*x +
6*acot(c + d*x)**3*b**3*d**3*e*f*x**2 + 2*acot(c + d*x)**3*b**3*d**3*f**2*
x**3 + 6*acot(c + d*x)**3*b**3*d*e*f - 12*acot(c + d*x)**2*a*b**2*c**3*f**
2 + 18*acot(c + d*x)**2*a*b**2*c**2*d*e*f - 12*acot(c + d*x)**2*a*b**2*c*f
**2 + 18*acot(c + d*x)**2*a*b**2*d**3*e**2*x + 18*acot(c + d*x)**2*a*b**2*
d**3*e*f*x**2 + 6*acot(c + d*x)**2*a*b**2*d**3*f**2*x**3 + 18*acot(c + d*x
)**2*a*b**2*d*e*f + 3*acot(c + d*x)**2*b**3*c**2*f**2 - 12*acot(c + d*x)**
2*b**3*c*d*f**2*x + 18*acot(c + d*x)**2*b**3*d**2*e*f*x + 3*acot(c + d*x)*
*2*b**3*d**2*f**2*x**2 + 3*acot(c + d*x)**2*b**3*f**2 + 6*acot(c + d*x)*a*
*2*b*c**3*f**2 - 18*acot(c + d*x)*a**2*b*c**2*d*e*f + 18*acot(c + d*x)*a**
2*b*c*d**2*e**2 - 18*acot(c + d*x)*a**2*b*c*f**2 + 18*acot(c + d*x)*a**2*b
*d**3*e**2*x + 18*acot(c + d*x)*a**2*b*d**3*e*f*x**2 + 6*acot(c + d*x)*a**
2*b*d**3*f**2*x**3 + 18*acot(c + d*x)*a**2*b*d*e*f - 30*acot(c + d*x)*a*b*
*2*c**2*f**2 + 36*acot(c + d*x)*a*b**2*c*d*e*f - 24*acot(c + d*x)*a*b**2*c
*d*f**2*x + 36*acot(c + d*x)*a*b**2*d**2*e*f*x + 6*acot(c + d*x)*a*b**2*d*
*2*f**2*x**2 + 6*acot(c + d*x)*a*b**2*f**2 + 6*acot(c + d*x)*b**3*c*f**2 +
 6*acot(c + d*x)*b**3*d*f**2*x + 36*int((acot(c + d*x)*x)/(c**2 + 2*c*d*x
+ d**2*x**2 + 1),x)*a*b**2*c**2*d**2*f**2 - 72*int((acot(c + d*x)*x)/(c**2
 + 2*c*d*x + d**2*x**2 + 1),x)*a*b**2*c*d**3*e*f + 36*int((acot(c + d*x...
```

3.29 $\int (e+fx) \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx$

Optimal result

Integrand size = 18, antiderivative size = 337

$$
\begin{aligned}
&\int (e+fx) \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx \\
&= \frac{3ibf\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2} + \frac{3bf(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2} \\
&\quad + \frac{i(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^3}{d^2} \\
&\quad - \frac{(de+f-cf)(de-(1+c)f)\left(a+b\cot^{-1}(c+dx)\right)^3}{2d^2f} \\
&\quad + \frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^3}{2f} - \frac{3b^2f\left(a+b\cot^{-1}(c+dx)\right)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2} \\
&\quad - \frac{3b(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2} \\
&\quad + \frac{3ib^3f\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{2d^2} \\
&\quad + \frac{3ib^2(de-cf)\left(a+b\cot^{-1}(c+dx)\right)\mathrm{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2} \\
&\quad - \frac{3b^3(de-cf)\,\mathrm{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d^2}
\end{aligned}
$$

output
```
3/2*I*b*f*(a+b*arccot(d*x+c))^2/d^2+3/2*b*f*(d*x+c)*(a+b*arccot(d*x+c))^2/
d^2+I*(-c*f+d*e)*(a+b*arccot(d*x+c))^3/d^2-1/2*(-c*f+d*e+f)*(d*e-(1+c)*f)*
(a+b*arccot(d*x+c))^3/d^2/f+1/2*(f*x+e)^2*(a+b*arccot(d*x+c))^3/f-3*b^2*f*
(a+b*arccot(d*x+c))*ln(2/(1+I*(d*x+c)))/d^2-3*b*(-c*f+d*e)*(a+b*arccot(d*x
+c))^2*ln(2/(1+I*(d*x+c)))/d^2+3/2*I*b^3*f*polylog(2,1-2/(1+I*(d*x+c)))/d^
2+3*I*b^2*(-c*f+d*e)*(a+b*arccot(d*x+c))*polylog(2,1-2/(1+I*(d*x+c)))/d^2-
3/2*b^3*(-c*f+d*e)*polylog(3,1-2/(1+I*(d*x+c)))/d^2
```

Mathematica [A] (verified)

Time = 5.09 (sec) , antiderivative size = 630, normalized size of antiderivative = 1.87

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^3\,dx$$

$$= \frac{a^2(2ade+3bf-2acf)(c+dx)+a^3f(c+dx)^2-3a^2b(c+dx)(cf-d(2e+fx))\cot^{-1}(c+dx)-3a^2bf}{}$$

input
```
Integrate[(e + f*x)*(a + b*ArcCot[c + d*x])^3,x]
```

output
```
(a^2*(2*a*d*e + 3*b*f - 2*a*c*f)*(c + d*x) + a^3*f*(c + d*x)^2 - 3*a^2*b*(
c + d*x)*(c*f - d*(2*e + f*x))*ArcCot[c + d*x] - 3*a^2*b*f*ArcTan[c + d*x]
 + 6*a*b^2*f*((c + d*x)*ArcCot[c + d*x] + ((1 + (c + d*x)^2)*ArcCot[c + d*
x]^2)/2 - Log[1/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)])]) + 3*a^2*b*(d*e - c*
f)*Log[1 + (c + d*x)^2] + 6*a*b^2*d*e*(ArcCot[c + d*x]*((I + c + d*x)*ArcC
ot[c + d*x] - 2*Log[1 - E^((2*I)*ArcCot[c + d*x])]) + I*PolyLog[2, E^((2*I
)*ArcCot[c + d*x])]) - 6*a*b^2*c*f*(ArcCot[c + d*x]*((I + c + d*x)*ArcCot[
c + d*x] - 2*Log[1 - E^((2*I)*ArcCot[c + d*x])]) + I*PolyLog[2, E^((2*I)*A
rcCot[c + d*x])]) + b^3*f*(3*(c + d*x)*ArcCot[c + d*x]^2 + (1 + (c + d*x)^
2)*ArcCot[c + d*x]^3 - 6*ArcCot[c + d*x]*Log[1 - E^((2*I)*ArcCot[c + d*x])
] + (3*I)*(ArcCot[c + d*x]^2 + PolyLog[2, E^((2*I)*ArcCot[c + d*x])])) + 2
*b^3*d*e*((I/8)*Pi^3 - I*ArcCot[c + d*x]^3 + (c + d*x)*ArcCot[c + d*x]^3 -
 3*ArcCot[c + d*x]^2*Log[1 - E^((-2*I)*ArcCot[c + d*x])] - (3*I)*ArcCot[c
+ d*x]*PolyLog[2, E^((-2*I)*ArcCot[c + d*x])] - (3*PolyLog[3, E^((-2*I)*Ar
cCot[c + d*x])])/2) - 2*b^3*c*f*((I/8)*Pi^3 - I*ArcCot[c + d*x]^3 + (c + d
*x)*ArcCot[c + d*x]^3 - 3*ArcCot[c + d*x]^2*Log[1 - E^((-2*I)*ArcCot[c + d
*x])] - (3*I)*ArcCot[c + d*x]*PolyLog[2, E^((-2*I)*ArcCot[c + d*x])] - (3*
PolyLog[3, E^((-2*I)*ArcCot[c + d*x])])/2))/(2*d^2)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5571, 27, 5390, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx) \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx$$

$$\Big\downarrow \text{5571}$$

$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)\left(a+b\cot^{-1}(c+dx)\right)^3}{d}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (de-cf+f(c+dx))\left(a+b\cot^{-1}(c+dx)\right)^3 d(c+dx)}{d^2}$$

$$\Big\downarrow \text{5390}$$

$$\frac{\frac{3b\int \left(f^2\left(a+b\cot^{-1}(c+dx)\right)^2+\frac{((de-cf+f)(de-(c+1)f)+2f(de-cf)(c+dx))\left(a+b\cot^{-1}(c+dx)\right)^2}{(c+dx)^2+1}\right)d(c+dx)}{2f}+\frac{(f(c+dx)-cf+de)^2\left(a+b\cot^{-1}(c+dx\right.}{2f}}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(f(c+dx)-cf+de)^2\left(a+b\cot^{-1}(c+dx)\right)^3}{2f}+\frac{3b\left(2ibf(de-cf)\text{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)\left(a+b\cot^{-1}(c+dx)\right)+\frac{2if(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^3}{3b}-\frac{(\cdots}{}\right.}{}}{}$$

input
```
Int[(e + f*x)*(a + b*ArcCot[c + d*x])^3,x]
```

output
```
(((d*e - c*f + f*(c + d*x))^2*(a + b*ArcCot[c + d*x])^3)/(2*f) + (3*b*(I*f
^2*(a + b*ArcCot[c + d*x])^2 + f^2*(c + d*x)*(a + b*ArcCot[c + d*x])^2 + (
((2*I)/3)*f*(d*e - c*f)*(a + b*ArcCot[c + d*x])^3)/b - ((d*e + f - c*f)*(d
*e - (1 + c)*f)*(a + b*ArcCot[c + d*x])^3)/(3*b) - 2*b*f^2*(a + b*ArcCot[c
 + d*x])*Log[2/(1 + I*(c + d*x))] - 2*f*(d*e - c*f)*(a + b*ArcCot[c + d*x]
)^2*Log[2/(1 + I*(c + d*x))] + I*b^2*f^2*PolyLog[2, 1 - 2/(1 + I*(c + d*x)
)] + (2*I)*b*f*(d*e - c*f)*(a + b*ArcCot[c + d*x])*PolyLog[2, 1 - 2/(1 + I
*(c + d*x))] - b^2*f*(d*e - c*f)*PolyLog[3, 1 - 2/(1 + I*(c + d*x))]))/(2*
f))/d^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5390
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])^p/(e*(q + 1))), x] + S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcCot[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1050 vs. 2(316) = 632.

Time = 12.46 (sec) , antiderivative size = 1051, normalized size of antiderivative = 3.12

method	result	size
parts	Expression too large to display	1051
derivativedivides	Expression too large to display	17369
default	Expression too large to display	17369

input
```
int((f*x+e)*(a+b*arccot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*(1/2*f*x^2+e*x)+b^3/d*(1/2/d*arccot(d*x+c)^3*(d*x+c)^2*f-1/d*arccot(d*
x+c)^3*c*f*(d*x+c)+arccot(d*x+c)^3*e*(d*x+c)+3/2/d*(1/3*f*arccot(d*x+c)^3+
4*I*d*e*arccot(d*x+c)*polylog(2,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+arccot(d*x
+c)^2*f*(d*x+c-I)-4*I*c*f*arccot(d*x+c)*polylog(2,(d*x+c+I)/(1+(d*x+c)^2)^
(1/2))-4*I*c*f*arccot(d*x+c)*polylog(2,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+4*p
olylog(3,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))*c*f-2/3*I*arccot(d*x+c)^3*c*f+4*p
olylog(3,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))*c*f-2*f*arccot(d*x+c)*ln(1-(d*x+c+
I)/(1+(d*x+c)^2)^(1/2))+2*I*f*polylog(2,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-4*p
olylog(3,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))*d*e+2*I*f*arccot(d*x+c)^2-2*ln(1-(
d*x+c+I)/(1+(d*x+c)^2)^(1/2))*d*e*arccot(d*x+c)^2+2/3*I*arccot(d*x+c)^3*d*
e+2*ln(1+(d*x+c+I)/(1+(d*x+c)^2)^(1/2))*c*f*arccot(d*x+c)^2+2*I*f*polylog(
2,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+4*I*d*e*arccot(d*x+c)*polylog(2,(d*x+c+I
)/(1+(d*x+c)^2)^(1/2))+2*ln(1-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))*c*f*arccot(d*
x+c)^2-2*ln(1+(d*x+c+I)/(1+(d*x+c)^2)^(1/2))*d*e*arccot(d*x+c)^2-2*f*arcco
t(d*x+c)*ln(1+(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-4*polylog(3,-(d*x+c+I)/(1+(d*
x+c)^2)^(1/2))*d*e))+3*a*b^2/d*(1/2/d*arccot(d*x+c)^2*(d*x+c)^2*f-1/d*arcc
ot(d*x+c)^2*c*f*(d*x+c)+arccot(d*x+c)^2*e*(d*x+c)+1/d*(-ln(1+(d*x+c)^2)*ar
ccot(d*x+c)*c*f+ln(1+(d*x+c)^2)*arccot(d*x+c)*d*e-arctan(d*x+c)*arccot(d*x
+c)*f+arccot(d*x+c)*f*(d*x+c)+1/2*f*ln(1+(d*x+c)^2)-1/2*arctan(d*x+c)^2*f+
1/2*(-2*c*f+2*d*e)*(-1/2*I*(ln(d*x+c-I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c-I)...
```

Fricas [F]

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int (fx+e)(b\,\mathrm{arccot}\,(dx+c)+a)^3\,dx$$

input
```
integrate((f*x+e)*(a+b*arccot(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(a^3*f*x + a^3*e + (b^3*f*x + b^3*e)*arccot(d*x + c)^3 + 3*(a*b^2*
f*x + a*b^2*e)*arccot(d*x + c)^2 + 3*(a^2*b*f*x + a^2*b*e)*arccot(d*x + c)
, x)
```

Sympy [F]

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int (a+b\,\mathrm{acot}\,(c+dx))^3\,(e+fx)\,dx$$

input
```
integrate((f*x+e)*(a+b*acot(d*x+c))**3,x)
```

output
```
Integral((a + b*acot(c + d*x))**3*(e + f*x), x)
```

Maxima [F]

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int (fx+e)(b\,\mathrm{arccot}\,(dx+c)+a)^3\,dx$$

input
```
integrate((f*x+e)*(a+b*arccot(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/16*b^3*f*x^2*arctan2(1, d*x + c)^3 + 1/8*b^3*e*x*arctan2(1, d*x + c)^3 +
 1/2*a^3*f*x^2 + 3/2*(x^2*arccot(d*x + c) + d*(x/d^2 + (c^2 - 1)*arctan((d
^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a^2*b*f + a^
3*e*x + 3/2*(2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*a^2*b*e/d
 - 3/64*(b^3*f*x^2*arctan2(1, d*x + c) + 2*b^3*e*x*arctan2(1, d*x + c))*lo
g(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + integrate(1/64*(8*(7*b^3*arctan2(1, d*x
 + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*d^2*f*x^3 + 4*(2*(7*b^3*arctan2(
1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*d^2*e + (3*b^3*arctan2(1,
d*x + c)^2 + 4*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)
^2)*c)*d*f)*x^2 + 3*(2*b^3*d^2*f*x^3*arctan2(1, d*x + c) + (2*b^3*d^2*e*ar
ctan2(1, d*x + c) + (4*b^3*c*arctan2(1, d*x + c) - b^3)*d*f)*x^2 + 2*(b^3*
c^2*arctan2(1, d*x + c) + b^3*arctan2(1, d*x + c))*e + 2*((2*b^3*c*arctan2
(1, d*x + c) - b^3)*d*e + (b^3*c^2*arctan2(1, d*x + c) + b^3*arctan2(1, d*
x + c))*f)*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 8*(7*b^3*arctan2(1, d*x
 + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2 + (7*b^3*arctan2(1, d*x + c)^3 +
24*a*b^2*arctan2(1, d*x + c)^2)*c^2)*e + 8*((3*b^3*arctan2(1, d*x + c)^2 +
 2*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c)*d*e +
 (7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2 + (7*b^3*ar
ctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c^2)*f)*x + 12*(b^3*
d^2*f*x^3*arctan2(1, d*x + c) + 2*b^3*c*d*e*x*arctan2(1, d*x + c) + (2*...
```

Giac [F]

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int (fx+e)(b\operatorname{arccot}(dx+c)+a)^3\,dx$$

input
```
integrate((f*x+e)*(a+b*arccot(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)*(b*arccot(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int (e+f\,x)\,\left(a+b\,\mathrm{acot}(c+d\,x)\right)^3\,dx$$

input
```
int((e + f*x)*(a + b*acot(c + d*x))^3,x)
```

output
```
int((e + f*x)*(a + b*acot(c + d*x))^3, x)
```

Reduce [F]

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^3\,dx$$
$$=\frac{acot(dx+c)^3\,b^3d^2f\,x^2+6acot(dx+c)^2\,a\,b^2d^2ex+3acot(dx+c)^2\,a\,b^2d^2f\,x^2+6acot(dx+c)\,a^2bcde+6}{}$$

input
```
int((f*x+e)*(a+b*acot(d*x+c))^3,x)
```

output
```
(acot(c + d*x)**3*b**3*c**2*f + 2*acot(c + d*x)**3*b**3*d**2*e*x + acot(c
+ d*x)**3*b**3*d**2*f*x**2 + acot(c + d*x)**3*b**3*f + 3*acot(c + d*x)**2*
a*b**2*c**2*f + 6*acot(c + d*x)**2*a*b**2*d**2*e*x + 3*acot(c + d*x)**2*a*
b**2*d**2*f*x**2 + 3*acot(c + d*x)**2*a*b**2*f + 3*acot(c + d*x)**2*b**3*d
*f*x - 3*acot(c + d*x)*a**2*b*c**2*f + 6*acot(c + d*x)*a**2*b*c*d*e + 6*ac
ot(c + d*x)*a**2*b*d**2*e*x + 3*acot(c + d*x)*a**2*b*d**2*f*x**2 + 3*acot(
c + d*x)*a**2*b*f + 6*acot(c + d*x)*a*b**2*c*f + 6*acot(c + d*x)*a*b**2*d*
f*x - 12*int((acot(c + d*x)*x)/(c**2 + 2*c*d*x + d**2*x**2 + 1),x)*a*b**2*
c*d**2*f + 12*int((acot(c + d*x)*x)/(c**2 + 2*c*d*x + d**2*x**2 + 1),x)*a*
b**2*d**3*e + 6*int((acot(c + d*x)*x)/(c**2 + 2*c*d*x + d**2*x**2 + 1),x)*
b**3*d**2*f - 6*int((acot(c + d*x)**2*x)/(c**2 + 2*c*d*x + d**2*x**2 + 1),
x)*b**3*c*d**2*f + 6*int((acot(c + d*x)**2*x)/(c**2 + 2*c*d*x + d**2*x**2
+ 1),x)*b**3*d**3*e - 3*log(c**2 + 2*c*d*x + d**2*x**2 + 1)*a**2*b*c*f + 3
*log(c**2 + 2*c*d*x + d**2*x**2 + 1)*a**2*b*d*e + 3*log(c**2 + 2*c*d*x + d
**2*x**2 + 1)*a*b**2*f + 2*a**3*d**2*e*x + a**3*d**2*f*x**2 + 3*a**2*b*d*f
*x)/(2*d**2)
```

3.30 $\int (a+b\cot^{-1}(c+dx))^3 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 143

$$\int (a+b\cot^{-1}(c+dx))^3 \, dx = \frac{i(a+b\cot^{-1}(c+dx))^3}{d} + \frac{(c+dx)(a+b\cot^{-1}(c+dx))^3}{d} - \frac{3b(a+b\cot^{-1}(c+dx))^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d} + \frac{3ib^2(a+b\cot^{-1}(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d} - \frac{3b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d}$$

output

```
I*(a+b*arccot(d*x+c))^3/d+(d*x+c)*(a+b*arccot(d*x+c))^3/d-3*b*(a+b*arccot(
d*x+c))^2*ln(2/(1+I*(d*x+c)))/d+3*I*b^2*(a+b*arccot(d*x+c))*polylog(2,1-2/
(1+I*(d*x+c)))/d-3/2*b^3*polylog(3,1-2/(1+I*(d*x+c)))/d
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.59

$$\int \left(a+b\cot^{-1}(c+dx)\right)^3 dx$$
$$= \frac{2a^3(c+dx)+6a^2b(c+dx)\cot^{-1}(c+dx)+3a^2b\log\left(1+(c+dx)^2\right)+6ab^2\left(\cot^{-1}(c+dx)\left((i+c+dx\right.\right.}{}$$

input
```
Integrate[(a + b*ArcCot[c + d*x])^3,x]
```

output
```
(2*a^3*(c + d*x) + 6*a^2*b*(c + d*x)*ArcCot[c + d*x] + 3*a^2*b*Log[1 + (c
+ d*x)^2] + 6*a*b^2*(ArcCot[c + d*x]*((I + c + d*x)*ArcCot[c + d*x] - 2*Lo
g[1 - E^((2*I)*ArcCot[c + d*x])]) + I*PolyLog[2, E^((2*I)*ArcCot[c + d*x])
]) + 2*b^3*((I/8)*Pi^3 - I*ArcCot[c + d*x]^3 + (c + d*x)*ArcCot[c + d*x]^3
 - 3*ArcCot[c + d*x]^2*Log[1 - E^((-2*I)*ArcCot[c + d*x])] - (3*I)*ArcCot[
c + d*x]*PolyLog[2, E^((-2*I)*ArcCot[c + d*x])] - (3*PolyLog[3, E^((-2*I)*
ArcCot[c + d*x])])/2))/(2*d)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5563, 5346, 5456, 5380, 5530, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\cot^{-1}(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{5563}$$

$$\frac{\int \left(a+b\cot^{-1}(c+dx)\right)^3 d(c+dx)}{d}$$

$$\Big\downarrow \text{5346}$$

$$\frac{3b\int \frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{(c+dx)^2+1}d(c+dx)+(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^3}{d}$$

$\Big\downarrow$ 5456

$$\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^3+3b\left(\frac{i\left(a+b\cot^{-1}(c+dx)\right)^3}{3b}-\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{-c-dx+i}d(c+dx)\right)}{d}$$

$\Big\downarrow$ 5380

$$\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^3+3b\left(-2b\int \frac{\left(a+b\cot^{-1}(c+dx)\right)\log\left(\frac{2}{i(c+dx)+1}\right)}{(c+dx)^2+1}d(c+dx)+\frac{i\left(a+b\cot^{-1}(c+dx)\right)^3}{3b}-\log\left(\frac{}{1}\right.\right.}{d}$$

$\Big\downarrow$ 5530

$$\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^3+3b\left(-2b\left(-\frac{1}{2}ib\int \frac{\text{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)}{(c+dx)^2+1}d(c+dx)-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{i(c+dx)+}\right.\right.\right.}{d}$$

$\Big\downarrow$ 7164

$$\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^3+3b\left(-2b\left(\frac{1}{4}b\,\text{PolyLog}\left(3,1-\frac{2}{i(c+dx)+1}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)(a+\right.\right.}{d}$$

input
```
Int[(a + b*ArcCot[c + d*x])^3,x]
```

output
```
((c + d*x)*(a + b*ArcCot[c + d*x])^3 + 3*b*(((I/3)*(a + b*ArcCot[c + d*x])
^3)/b - (a + b*ArcCot[c + d*x])^2*Log[2/(1 + I*(c + d*x))] - 2*b*((-1/2*I)
*(a + b*ArcCot[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))] + (b*PolyLog[
3, 1 - 2/(1 + I*(c + d*x))])/4)))/d
```

Defintions of rubi rules used

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5380
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Simp[b*c*(
p/e)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5456
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Simp[
1/(c*d)   Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5530
```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] - Simp[b*p*(I/2)   Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5563
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
 x] && IGtQ[p, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 394 vs. 2(136) = 272.

Time = 2.00 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.76

method	result
derivativedivides	$\frac{(dx+c)a^3+b^3\left(\operatorname{arccot}(dx+c)^3(dx+c-i)+2i\operatorname{arccot}(dx+c)^3-3\operatorname{arccot}(dx+c)^2\ln\left(1+\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right)+6i\operatorname{arccot}(dx+c)\ \mathrm{p}\right.}{}$
default	$\frac{(dx+c)a^3+b^3\left(\operatorname{arccot}(dx+c)^3(dx+c-i)+2i\operatorname{arccot}(dx+c)^3-3\operatorname{arccot}(dx+c)^2\ln\left(1+\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right)+6i\operatorname{arccot}(dx+c)\ \mathrm{p}\right.}{}$
parts	$a^3x+\frac{b^3\left(\operatorname{arccot}(dx+c)^3(dx+c-i)+2i\operatorname{arccot}(dx+c)^3-3\operatorname{arccot}(dx+c)^2\ln\left(1+\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right)+6i\operatorname{arccot}(dx+c)\ \mathrm{pol}\right.}{}$

input
```
int((a+b*arccot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*((d*x+c)*a^3+b^3*(arccot(d*x+c)^3*(d*x+c-I)+2*I*arccot(d*x+c)^3-3*arcc
ot(d*x+c)^2*ln(1+(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+6*I*arccot(d*x+c)*polylog(
2,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-6*polylog(3,-(d*x+c+I)/(1+(d*x+c)^2)^(1/
2))-3*arccot(d*x+c)^2*ln(1-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+6*I*arccot(d*x+c
)*polylog(2,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-6*polylog(3,(d*x+c+I)/(1+(d*x+c
)^2)^(1/2)))+3*a*b^2*(arccot(d*x+c)^2*(d*x+c-I)-2*arccot(d*x+c)*ln(1-(d*x+
c+I)/(1+(d*x+c)^2)^(1/2))-2*arccot(d*x+c)*ln(1+(d*x+c+I)/(1+(d*x+c)^2)^(1/
2))+2*I*arccot(d*x+c)^2+2*I*polylog(2,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+2*I*p
olylog(2,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2)))+3*a^2*b*(arccot(d*x+c)*(d*x+c)+1
/2*ln(1+(d*x+c)^2)))
```

Fricas [F]

$$\int \left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int (b\operatorname{arccot}(dx+c)+a)^3\,dx$$

input
```
integrate((a+b*arccot(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(b^3*arccot(d*x + c)^3 + 3*a*b^2*arccot(d*x + c)^2 + 3*a^2*b*arcco
t(d*x + c) + a^3, x)
```

Sympy [F]

$$\int \left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int \left(a+b\,\mathrm{acot}\,(c+dx)\right)^3\,dx$$

input
```
integrate((a+b*acot(d*x+c))**3,x)
```

output
```
Integral((a + b*acot(c + d*x))**3, x)
```

Maxima [F]

$$\int \left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int \left(b\,\mathrm{arccot}\,(dx+c)+a\right)^3\,dx$$

input
```
integrate((a+b*arccot(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/8*b^3*x*arctan2(1, d*x + c)^3 - 3/32*b^3*x*arctan2(1, d*x + c)*log(d^2*x
^2 + 2*c*d*x + c^2 + 1)^2 + a^3*x + 3/2*(2*(d*x + c)*arccot(d*x + c) + log
((d*x + c)^2 + 1))*a^2*b/d + integrate(1/32*(28*b^3*arctan2(1, d*x + c)^3
+ 4*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*d^2*x^2
 + 96*a*b^2*arctan2(1, d*x + c)^2 + 4*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*
b^2*arctan2(1, d*x + c)^2)*c^2 + 4*(3*b^3*arctan2(1, d*x + c)^2 + 2*(7*b^3
*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c)*d*x + 3*(b^3*d
^2*x^2*arctan2(1, d*x + c) + b^3*c^2*arctan2(1, d*x + c) + b^3*arctan2(1,
d*x + c) + (2*b^3*c*arctan2(1, d*x + c) - b^3)*d*x)*log(d^2*x^2 + 2*c*d*x
+ c^2 + 1)^2 + 12*(b^3*d^2*x^2*arctan2(1, d*x + c) + b^3*c*d*x*arctan2(1,
d*x + c))*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^2*x^2 + 2*c*d*x + c^2 + 1),
 x)
```

Giac [F]

$$\int \left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int (b\,\mathrm{arccot}\,(dx+c)+a)^3\,dx$$

input
```
integrate((a+b*arccot(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*arccot(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int \left(a+b\,\mathrm{acot}(c+d\,x)\right)^3\,dx$$

input
```
int((a + b*acot(c + d*x))^3,x)
```

output
```
int((a + b*acot(c + d*x))^3, x)
```

Reduce [F]

$$\int \left(a+b\cot^{-1}(c+dx)\right)^3\,dx$$
$$=\frac{2acot(dx+c)^3\,b^3dx+6acot(dx+c)^2\,a\,b^2dx+6acot(dx+c)\,a^2bc+6acot(dx+c)\,a^2bdx+12\Big(\int\frac{acot(c}{d^2x^2+2c}}{2d}$$

input
```
int((a+b*acot(d*x+c))^3,x)
```

output
```
(2*acot(c + d*x)**3*b**3*d*x + 6*acot(c + d*x)**2*a*b**2*d*x + 6*acot(c +
d*x)*a**2*b*c + 6*acot(c + d*x)*a**2*b*d*x + 12*int((acot(c + d*x)*x)/(c**
2 + 2*c*d*x + d**2*x**2 + 1),x)*a*b**2*d**2 + 6*int((acot(c + d*x)**2*x)/(
c**2 + 2*c*d*x + d**2*x**2 + 1),x)*b**3*d**2 + 3*log(c**2 + 2*c*d*x + d**2
*x**2 + 1)*a**2*b + 2*a**3*d*x)/(2*d)
```

3.31 $\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{e+fx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 372

$$\begin{aligned}
\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{e+fx}\,dx
&= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^3\log\left(\frac{2}{1-i(c+dx)}\right)}{f} \\
&\quad+\frac{\left(a+b\cot^{-1}(c+dx)\right)^3\log\left(\frac{2d(e+fx)}{(de+(i-c)f)(1-i(c+dx))}\right)}{f} \\
&\quad-\frac{3ib\left(a+b\cot^{-1}(c+dx)\right)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{2f} \\
&\quad+\frac{3ib\left(a+b\cot^{-1}(c+dx)\right)^2\operatorname{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+(i-c)f)(1-i(c+dx))}\right)}{2f} \\
&\quad-\frac{3b^2\left(a+b\cot^{-1}(c+dx)\right)\operatorname{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2f} \\
&\quad+\frac{3b^2\left(a+b\cot^{-1}(c+dx)\right)\operatorname{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+(i-c)f)(1-i(c+dx))}\right)}{2f} \\
&\quad+\frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1-i(c+dx)}\right)}{4f}-\frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2d(e+fx)}{(de+(i-c)f)(1-i(c+dx))}\right)}{4f}
\end{aligned}$$

output
```
-(a+b*arccot(d*x+c))^3*ln(2/(1-I*(d*x+c)))/f+(a+b*arccot(d*x+c))^3*ln(2*d*
(f*x+e)/(d*e+(I-c)*f)/(1-I*(d*x+c)))/f-3/2*I*b*(a+b*arccot(d*x+c))^2*polyl
og(2,1-2/(1-I*(d*x+c)))/f+3/2*I*b*(a+b*arccot(d*x+c))^2*polylog(2,1-2*d*(f
*x+e)/(d*e+(I-c)*f)/(1-I*(d*x+c)))/f-3/2*b^2*(a+b*arccot(d*x+c))*polylog(3
,1-2/(1-I*(d*x+c)))/f+3/2*b^2*(a+b*arccot(d*x+c))*polylog(3,1-2*d*(f*x+e)/
(d*e+(I-c)*f)/(1-I*(d*x+c)))/f+3/4*I*b^3*polylog(4,1-2/(1-I*(d*x+c)))/f-3/
4*I*b^3*polylog(4,1-2*d*(f*x+e)/(d*e+(I-c)*f)/(1-I*(d*x+c)))/f
```

Mathematica [F]

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{e+fx}\,dx = \int \frac{(a+b\cot^{-1}(c+dx))^3}{e+fx}\,dx$$

input
```
Integrate[(a + b*ArcCot[c + d*x])^3/(e + f*x),x]
```

output
```
Integrate[(a + b*ArcCot[c + d*x])^3/(e + f*x), x]
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {5571, 27, 5386}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{e+fx}\,dx$$
$$\Big\downarrow \text{5571}$$
$$\frac{\int \frac{d(a+b\cot^{-1}(c+dx))^3}{d\left(e-\frac{cf}{d}\right)+f(c+dx)}d(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$

$$\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(c+dx)-cf+de}d(c+dx)$$

$$\Big\downarrow \text{ 5386}$$

$$\frac{3b^2\left(a+b\cot^{-1}(c+dx)\right)\text{PolyLog}\left(3,1-\frac{2(de-cf+f(c+dx))}{(de-cf+if)(1-i(c+dx))}\right)}{2f}-\frac{3b^2\,\text{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)\left(a+b\cot^{-1}(c+dx)\right)}{2f}+\frac{3ib\left(a+b\cot^{-1}(c+dx)\right)^2\text{PolyLog}\left(2,1-\frac{2(de-cf+f(c+dx))}{(de-cf+if)(1-i(c+dx))}\right)}{2f}+\frac{\left(a+b\cot^{-1}(c+dx)\right)^3\log\left(\frac{2(f(c+dx)-cf+de)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}-\frac{3ib\,\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)\left(a+b\cot^{-1}(c+dx)\right)^2}{2f}-\frac{\log\left(\frac{2}{1-i(c+dx)}\right)\left(a+b\cot^{-1}(c+dx)\right)^3}{f}-\frac{3ib^3\,\text{PolyLog}\left(4,1-\frac{2(de-cf+f(c+dx))}{(de-cf+if)(1-i(c+dx))}\right)}{4f}+\frac{3ib^3\,\text{PolyLog}\left(4,1-\frac{2}{1-i(c+dx)}\right)}{4f}$$

input
```
Int[(a + b*ArcCot[c + d*x])^3/(e + f*x),x]
```

output
```
-(((a + b*ArcCot[c + d*x])^3*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcCot
[c + d*x])^3*Log[(2*(d*e - c*f + f*(c + d*x)))/((d*e + I*f - c*f)*(1 - I*(
c + d*x)))])/f - (((3*I)/2)*b*(a + b*ArcCot[c + d*x])^2*PolyLog[2, 1 - 2/(
1 - I*(c + d*x))])/f + (((3*I)/2)*b*(a + b*ArcCot[c + d*x])^2*PolyLog[2, 1
 - (2*(d*e - c*f + f*(c + d*x)))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f
 - (3*b^2*(a + b*ArcCot[c + d*x])*PolyLog[3, 1 - 2/(1 - I*(c + d*x))])/(2*
f) + (3*b^2*(a + b*ArcCot[c + d*x])*PolyLog[3, 1 - (2*(d*e - c*f + f*(c +
d*x)))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(2*f) + (((3*I)/4)*b^3*Poly
Log[4, 1 - 2/(1 - I*(c + d*x))])/f - (((3*I)/4)*b^3*PolyLog[4, 1 - (2*(d*e
 - c*f + f*(c + d*x)))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5386
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^3/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcCot[c*x])^3)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Cot[c*x])^3*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] - Simp[3
*I*b*(a + b*ArcCot[c*x])^2*(PolyLog[2, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp
[3*I*b*(a + b*ArcCot[c*x])^2*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1
 - I*c*x)))]/(2*e)), x] - Simp[3*b^2*(a + b*ArcCot[c*x])*(PolyLog[3, 1 - 2/
(1 - I*c*x)]/(2*e)), x] + Simp[3*b^2*(a + b*ArcCot[c*x])*(PolyLog[3, 1 - 2*
c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x] + Simp[3*I*b^3*(PolyLog
[4, 1 - 2/(1 - I*c*x)]/(4*e)), x] - Simp[3*I*b^3*(PolyLog[4, 1 - 2*c*((d +
e*x)/((c*d + I*e)*(1 - I*c*x)))]/(4*e)), x]) /; FreeQ[{a, b, c, d, e}, x] &
& NeQ[c^2*d^2 + e^2, 0]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 14.32 (sec) , antiderivative size = 3903, normalized size of antiderivative = 10.49

method	result	size
derivativedivides	Expression too large to display	3903
default	Expression too large to display	3903
parts	Expression too large to display	4133

input
```
int((a+b*arccot(d*x+c))^3/(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*d*ln(c*f-d*e-f*(d*x+c))/f-b^3*d*(-ln(c*f-d*e-f*(d*x+c))/f*arccot(
d*x+c)^3-3/f*(-1/3*arccot(d*x+c)^3*ln(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(
d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f-c*f+d*e)-1/2*I*
f/(-I*f+c*f-d*e)*arccot(d*x+c)*polylog(3,(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x
+c+I)^2/(1+(d*x+c)^2))+1/3*arccot(d*x+c)^3*ln((d*x+c+I)^2/(1+(d*x+c)^2)-1)
-1/3*arccot(d*x+c)^3*ln(1-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-1/2*I*c*f/(-I*f+c
*f-d*e)*arccot(d*x+c)^2*polylog(2,(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2
/(1+(d*x+c)^2))-2*arccot(d*x+c)*polylog(3,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+I
*arccot(d*x+c)^2*polylog(2,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-1/3*arccot(d*x+c
)^3*ln(1+(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+I*d*e*arccot(d*x+c)^2*polylog(2,(d
*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))/(-2*I*f+2*c*f-2*d*e)
-2*arccot(d*x+c)*polylog(3,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-2*I*polylog(4,-
(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+1/3*c*f/(-I*f+c*f-d*e)*arccot(d*x+c)^3*ln(1
-(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))+1/2*c*f/(-I*f+c*f
-d*e)*arccot(d*x+c)*polylog(3,(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+
(d*x+c)^2))-1/3*I*f/(-I*f+c*f-d*e)*arccot(d*x+c)^3*ln(1-(d*e+I*f-c*f)/(-c*
f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))+I*arccot(d*x+c)^2*polylog(2,-(d*x+c+
I)/(1+(d*x+c)^2)^(1/2))-I*d*e*polylog(4,(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+
c+I)^2/(1+(d*x+c)^2))/(-4*I*f+4*c*f-4*d*e)+1/4*I*c*f/(-I*f+c*f-d*e)*polylo
g(4,(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))-1/2*f/(-I*f...
```

Fricas [F]

$$\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{e+fx}\,dx=\int \frac{\left(b\operatorname{arccot}\left(dx+c\right)+a\right)^3}{fx+e}\,dx$$

input
```
integrate((a+b*arccot(d*x+c))^3/(f*x+e),x, algorithm="fricas")
```

output
```
integral((b^3*arccot(d*x + c)^3 + 3*a*b^2*arccot(d*x + c)^2 + 3*a^2*b*arcc
ot(d*x + c) + a^3)/(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{e+fx}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acot(d*x+c))**3/(f*x+e),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{e+fx}\,dx = \int \frac{(b\,\mathrm{arccot}\,(dx+c)+a)^3}{fx+e}\,dx$$

input
```
integrate((a+b*arccot(d*x+c))^3/(f*x+e),x, algorithm="maxima")
```

output
```
a^3*log(f*x + e)/f + integrate(1/32*(28*b^3*arctan2(1, d*x + c)^3 + 3*b^3*
arctan2(1, d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 96*a*b^2*arctan2(
1, d*x + c)^2 + 96*a^2*b*arctan2(1, d*x + c))/(f*x + e), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{e+fx}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arccot(d*x+c))^3/(f*x+e),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{e+fx}\,dx=\int \frac{\left(a+b\,\mathrm{acot}(c+d\,x)\right)^3}{e+f\,x}\,dx$$

input
```
int((a + b*acot(c + d*x))^3/(e + f*x),x)
```

output
```
int((a + b*acot(c + d*x))^3/(e + f*x), x)
```

Reduce [F]

$$\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{e+fx}\,dx$$
$$=\frac{3\left(\int \frac{acot(dx+c)}{fx+e}dx\right)a^2bf+\left(\int \frac{acot(dx+c)^3}{fx+e}dx\right)b^3f+3\left(\int \frac{acot(dx+c)^2}{fx+e}dx\right)a\,b^2f+\log(fx+e)\,a^3}{f}$$

input
```
int((a+b*acot(d*x+c))^3/(f*x+e),x)
```

output
```
(3*int(acot(c + d*x)/(e + f*x),x)*a**2*b*f + int(acot(c + d*x)**3/(e + f*x
),x)*b**3*f + 3*int(acot(c + d*x)**2/(e + f*x),x)*a*b**2*f + log(e + f*x)*
a**3)/f
```

3.32 $\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 693

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{(e+fx)^2}\,dx$$
$$= \frac{id(a+b\cot^{-1}(c+dx))^3}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{d(de-cf)(a+b\cot^{-1}(c+dx))^3}{f(d^2e^2-2cdef+(1+c^2)f^2)}$$
$$- \frac{(a+b\cot^{-1}(c+dx))^3}{f(e+fx)} + \frac{3bd(a+b\cot^{-1}(c+dx))^2\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$- \frac{3bd(a+b\cot^{-1}(c+dx))^2\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$- \frac{3bd(a+b\cot^{-1}(c+dx))^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$+ \frac{3ib^2d(a+b\cot^{-1}(c+dx))\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$- \frac{3ib^2d(a+b\cot^{-1}(c+dx))\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$+ \frac{3ib^2d(a+b\cot^{-1}(c+dx))\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$+ \frac{3b^3d\,\mathrm{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2(d^2e^2-2cdef+(1+c^2)f^2)} - \frac{3b^3d\,\mathrm{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2(d^2e^2-2cdef+(1+c^2)f^2)}$$
$$- \frac{3b^3d\,\mathrm{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2(d^2e^2-2cdef+(1+c^2)f^2)}$$

output
```
I*d*(a+b*arccot(d*x+c))^3/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+d*(-c*f+d*e)*(a+
b*arccot(d*x+c))^3/f/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-(a+b*arccot(d*x+c))^3
/f/(f*x+e)+3*b*d*(a+b*arccot(d*x+c))^2*ln(2/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*
e*f+(c^2+1)*f^2)-3*b*d*(a+b*arccot(d*x+c))^2*ln(2*d*(f*x+e)/(d*e+I*f-c*f)/
(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-3*b*d*(a+b*arccot(d*x+c))^2
*ln(2/(1+I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*I*b^2*d*(a+b*arccot
(d*x+c))*polylog(2,1-2/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-3*I*
b^2*d*(a+b*arccot(d*x+c))*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+
c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*I*b^2*d*(a+b*arccot(d*x+c))*polylog
(2,1-2/(1+I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*b^3*d*polylog(3,1-
2/(1-I*(d*x+c)))/(2*d^2*e^2-4*c*d*e*f+2*(c^2+1)*f^2)-3*b^3*d*polylog(3,1-2
*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/(2*d^2*e^2-4*c*d*e*f+2*(c^2+1)*f^2
)-3*b^3*d*polylog(3,1-2/(1+I*(d*x+c)))/(2*d^2*e^2-4*c*d*e*f+2*(c^2+1)*f^2)
```

Mathematica [F]

$$\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx=\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx$$

input
```
Integrate[(a + b*ArcCot[c + d*x])^3/(e + f*x)^2,x]
```

output
```
Integrate[(a + b*ArcCot[c + d*x])^3/(e + f*x)^2, x]
```

Rubi [A] (verified)

Time = 2.46 (sec) , antiderivative size = 1266, normalized size of antiderivative = 1.83, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5569, 7292, 5581, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx$$

$$\Big\downarrow \; 5569$$

$$-\frac{3bd\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{(e+fx)\left((c+dx)^2+1\right)}dx}{f}-\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)}$$

$$\Big\downarrow \; 7292$$

$$-\frac{3bd\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{(e+fx)\left(c^2+2dxc+d^2x^2+1\right)}dx}{f}-\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)}$$

$$\Big\downarrow \; 5581$$

$$-\frac{3b\int \frac{d\left(a+b\cot^{-1}(c+dx)\right)^2}{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)\left((c+dx)^2+1\right)}d(c+dx)}{f}-\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)}$$

$$\Big\downarrow \; 27$$

$$-\frac{3bd\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{(de-cf+f(c+dx))\left((c+dx)^2+1\right)}d(c+dx)}{f}-\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)}$$

$$\Big\downarrow \; 7276$$

$$-\frac{3bd\int \left(\frac{a^2}{(de-cf+f(c+dx))\left((c+dx)^2+1\right)}+\frac{2b\cot^{-1}(c+dx)a}{(de-cf+f(c+dx))\left((c+dx)^2+1\right)}+\frac{b^2\cot^{-1}(c+dx)^2}{(de-cf+f(c+dx))\left((c+dx)^2+1\right)}\right)d(c+dx)}{f}-\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)}-$$

$$3bd\left(-\frac{ib^2f\cot^{-1}(c+dx)^3}{3\left(d^2e^2-2cdfe+(c^2+1)f^2\right)}-\frac{b^2(de-cf)\cot^{-1}(c+dx)^3}{3\left(d^2e^2-2cdfe+(c^2+1)f^2\right)}-\frac{b^2f\log\left(\frac{2}{1-i(c+dx)}\right)\cot^{-1}(c+dx)^2}{d^2e^2-2cdfe+(c^2+1)f^2}+\frac{b^2f\log\left(\frac{2}{i(c+dx)+1}\right)\cot^{-1}(c+dx)^2}{d^2e^2-2cdfe+(c^2+1)f^2}\right.$$

input
```
Int[(a + b*ArcCot[c + d*x])^3/(e + f*x)^2,x]
```

output
```
-((a + b*ArcCot[c + d*x])^3/(f*(e + f*x))) - (3*b*d*(((-I)*a*b*f*ArcCot[c
+ d*x]^2)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (a*b*(d*e - c*f)*ArcCot[
c + d*x]^2)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - ((I/3)*b^2*f*ArcCot[c
+ d*x]^3)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (b^2*(d*e - c*f)*ArcCot[
c + d*x]^3)/(3*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)) + (a^2*(d*e - c*f)*A
rcTan[c + d*x])/(f^2 + (d*e - c*f)^2) - (2*a*b*f*ArcCot[c + d*x]*Log[2/(1
- I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (b^2*f*ArcCot[c +
 d*x]^2*Log[2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) +
(2*a*b*f*ArcCot[c + d*x]*Log[2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f +
(1 + c^2)*f^2) + (b^2*f*ArcCot[c + d*x]^2*Log[2/(1 + I*(c + d*x))])/(d^2*e
^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (a^2*f*Log[d*e - c*f + f*(c + d*x)])/(f^
2 + (d*e - c*f)^2) + (2*a*b*f*ArcCot[c + d*x]*Log[(2*(d*e - c*f + f*(c + d
*x)))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 +
c^2)*f^2) + (b^2*f*ArcCot[c + d*x]^2*Log[(2*(d*e - c*f + f*(c + d*x)))/((d
*e + I*f - c*f)*(1 - I*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)
 - (a^2*f*Log[1 + (c + d*x)^2])/(2*(f^2 + (d*e - c*f)^2)) - (I*a*b*f*PolyL
og[2, 1 - 2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (I
*b^2*f*ArcCot[c + d*x]*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c
*d*e*f + (1 + c^2)*f^2) - (I*a*b*f*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/(d
^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (I*b^2*f*ArcCot[c + d*x]*PolyLog[...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5569
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCot[c + d*x])^p/(f*(m +
1))), x] + Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcCot[
c + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[p, 0] && ILtQ[m, -1]
```

rule 5581
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Subs
t[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcCot[x])
^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &
& EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 25.54 (sec) , antiderivative size = 4229, normalized size of antiderivative = 6.10

method	result	size
parts	Expression too large to display	4229
derivativedivides	Expression too large to display	4722
default	Expression too large to display	4722

input
```
int((a+b*arccot(d*x+c))^3/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-a^3/(f*x+e)/f+b^3/d*(-d^2/(f*(d*x+c)-c*f+d*e)/f*arccot(d*x+c)^3-3*d^2/f*(
-1/2*arccot(d*x+c)^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f*ln(1+(d*x+c)^2)-arc
cot(d*x+c)^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)*c*f+arccot(d*x+
c)^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)*d*e+arccot(d*x+c)^2/(c^
2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f*ln(f*(d*x+c)-c*f+d*e)+1/(c^2*f^2-2*c*d*e*f+
d^2*e^2+f^2)*f*arctan(d*x+c)^2*ln((1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+1/2*I
/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f^2/(c*f-d*e+I*f)*polylog(3,(c*f-d*e+I*f)
*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+1/4/(c^2*f^2-2*c*d*e*f+d^2*e
^2+f^2)*(-I*f*Pi*csgn(I*(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))^2*csgn(I*(1+I*(
d*x+c))^2/(1+(d*x+c)^2))-2*I*f*Pi*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))
)*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^2-2*I*f*Pi*csgn(I*(I*f*(1+I*
(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c
))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)
+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c
*f-d*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2-2*Pi*c*f+2*Pi*d*e+I*f*Pi*csgn
(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+
c)^2))^2)+I*f*Pi*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)*csgn(I*(1+I*(
d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^2-2*I*f*Pi*cs
gn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-
d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/(1+(1+I*(d*x+c))^2/(1+(d...
```

Fricas [F]

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{(e+fx)^2}\,dx = \int \frac{(b\operatorname{arccot}(dx+c)+a)^3}{(fx+e)^2}\,dx$$

input
```
integrate((a+b*arccot(d*x+c))^3/(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((b^3*arccot(d*x + c)^3 + 3*a*b^2*arccot(d*x + c)^2 + 3*a^2*b*arcc
ot(d*x + c) + a^3)/(f^2*x^2 + 2*e*f*x + e^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{(e+fx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acot(d*x+c))**3/(f*x+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{(e+fx)^2}\,dx = \int \frac{(b\,\mathrm{arccot}\,(dx+c)+a)^3}{(fx+e)^2}\,dx$$

input
```
integrate((a+b*arccot(d*x+c))^3/(f*x+e)^2,x, algorithm="maxima")
```

output
```
-3/2*(d*(2*(d^2*e - c*d*f)*arctan((d^2*x + c*d)/d)/((d^2*e^2*f - 2*c*d*e*f
^2 + (c^2 + 1)*f^3)*d) - log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2 - 2*c*d
*e*f + (c^2 + 1)*f^2) + 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 + 1)*f^
2)) + 2*arccot(d*x + c)/(f^2*x + e*f))*a^2*b - a^3/(f^2*x + e*f) - 1/32*(4
*b^3*arctan2(1, d*x + c)^3 - 3*b^3*arctan2(1, d*x + c)*log(d^2*x^2 + 2*c*d
*x + c^2 + 1)^2 - 32*(f^2*x + e*f)*integrate(-1/32*(12*b^3*d*e*arctan2(1,
d*x + c)^2 - 4*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)
^2)*d^2*f*x^2 + 4*(3*b^3*arctan2(1, d*x + c)^2 - 2*(7*b^3*arctan2(1, d*x +
 c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c)*d*f*x - 3*(b^3*d^2*f*x^2*arctan
2(1, d*x + c) + b^3*d*e + (2*b^3*c*arctan2(1, d*x + c) + b^3)*d*f*x + (b^3
*c^2*arctan2(1, d*x + c) + b^3*arctan2(1, d*x + c))*f)*log(d^2*x^2 + 2*c*d
*x + c^2 + 1)^2 - 4*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x
 + c)^2 + (7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c
^2)*f + 12*(b^3*d^2*f*x^2*arctan2(1, d*x + c) + b^3*c*d*e*arctan2(1, d*x +
 c) + (b^3*d^2*e*arctan2(1, d*x + c) + b^3*c*d*f*arctan2(1, d*x + c))*x)*l
og(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^2*f^3*x^4 + (c^2 + 1)*e^2*f + 2*(d^2*e
*f^2 + c*d*f^3)*x^3 + (d^2*e^2*f + 4*c*d*e*f^2 + (c^2 + 1)*f^3)*x^2 + 2*(c
*d*e^2*f + (c^2 + 1)*e*f^2)*x), x))/(f^2*x + e*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccot(d*x+c))^3/(f*x+e)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Not invertible Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx=\int \frac{\left(a+b\,\mathrm{acot}(c+d\,x)\right)^3}{\left(e+f\,x\right)^2}\,dx$$

input
```
int((a + b*acot(c + d*x))^3/(e + f*x)^2,x)
```

output
```
int((a + b*acot(c + d*x))^3/(e + f*x)^2, x)
```

Reduce [F]

$$\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx=\text{too large to display}$$

input
```
int((a+b*acot(d*x+c))^3/(f*x+e)^2,x)
```

output
```
( - 4*acot(c + d*x)**3*b**3*c**4*e*f**4 + 14*acot(c + d*x)**3*b**3*c**3*d*
e**2*f**3 - 2*acot(c + d*x)**3*b**3*c**3*d*e*f**4*x - 18*acot(c + d*x)**3*
b**3*c**2*d**2*e**3*f**2 + 6*acot(c + d*x)**3*b**3*c**2*d**2*e**2*f**3*x -
 4*acot(c + d*x)**3*b**3*c**2*e*f**4 + 10*acot(c + d*x)**3*b**3*c*d**3*e**
4*f - 6*acot(c + d*x)**3*b**3*c*d**3*e**3*f**2*x + 6*acot(c + d*x)**3*b**3
*c*d*e**2*f**3 - 2*acot(c + d*x)**3*b**3*c*d*e*f**4*x - 2*acot(c + d*x)**3
*b**3*d**4*e**5 + 2*acot(c + d*x)**3*b**3*d**4*e**4*f*x - 2*acot(c + d*x)*
*3*b**3*d**2*e**3*f**2 + 2*acot(c + d*x)**3*b**3*d**2*e**2*f**3*x - 12*aco
t(c + d*x)**2*a*b**2*c**4*e*f**4 + 42*acot(c + d*x)**2*a*b**2*c**3*d*e**2*
f**3 - 6*acot(c + d*x)**2*a*b**2*c**3*d*e*f**4*x - 54*acot(c + d*x)**2*a*b
**2*c**2*d**2*e**3*f**2 + 18*acot(c + d*x)**2*a*b**2*c**2*d**2*e**2*f**3*x
 - 12*acot(c + d*x)**2*a*b**2*c**2*e*f**4 + 30*acot(c + d*x)**2*a*b**2*c*d
**3*e**4*f - 18*acot(c + d*x)**2*a*b**2*c*d**3*e**3*f**2*x + 18*acot(c + d
*x)**2*a*b**2*c*d*e**2*f**3 - 6*acot(c + d*x)**2*a*b**2*c*d*e*f**4*x - 6*a
cot(c + d*x)**2*a*b**2*d**4*e**5 + 6*acot(c + d*x)**2*a*b**2*d**4*e**4*f*x
 - 6*acot(c + d*x)**2*a*b**2*d**2*e**3*f**2 + 6*acot(c + d*x)**2*a*b**2*d*
*2*e**2*f**3*x + 6*acot(c + d*x)**2*b**3*c**3*e*f**4 - 15*acot(c + d*x)**2
*b**3*c**2*d*e**2*f**3 + 3*acot(c + d*x)**2*b**3*c**2*d*e*f**4*x + 12*acot
(c + d*x)**2*b**3*c*d**2*e**3*f**2 - 6*acot(c + d*x)**2*b**3*c*d**2*e**2*f
**3*x + 6*acot(c + d*x)**2*b**3*c*e*f**4 - 3*acot(c + d*x)**2*b**3*d**3...
```

3.33 $\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right) dx$

Optimal result

Integrand size = 18, antiderivative size = 177

$$\begin{aligned}
&\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right) dx \\
&= \frac{(e+fx)^{1+m}\left(a+b\cot^{-1}(c+dx)\right)}{f(1+m)} \\
&\quad + \frac{ibd(e+fx)^{2+m}\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{d(e+fx)}{de+(i-c)f}\right)}{2f(de+(i-c)f)(1+m)(2+m)} \\
&\quad - \frac{ibd(e+fx)^{2+m}\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{d(e+fx)}{de-(i+c)f}\right)}{2f(de-(i+c)f)(1+m)(2+m)}
\end{aligned}$$

output
```
(f*x+e)^(1+m)*(a+b*arccot(d*x+c))/f/(1+m)+1/2*I*b*d*(f*x+e)^(2+m)*hypergeo
m([1, 2+m],[3+m],d*(f*x+e)/(d*e+(I-c)*f))/f/(d*e+(I-c)*f)/(1+m)/(2+m)-1/2*
I*b*d*(f*x+e)^(2+m)*hypergeom([1, 2+m],[3+m],d*(f*x+e)/(d*e-(I+c)*f))/f/(d
*e-(I+c)*f)/(1+m)/(2+m)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.92

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)\,dx$$
$$=\frac{(e+fx)^{1+m}\left(2(a+b\cot^{-1}(c+dx))+\frac{bd(e+fx)\left((de-(i+c)f)\text{ Hypergeometric2F1}\left(1,2+m,3+m,\frac{d(e+fx)}{de-(-i+c)f}\right)+(-de+(-i+c\right.}{(-ide+f+icf)(de-(i+c)f)(2+m)}}{2f(1+m)}$$

input
```
Integrate[(e + f*x)^m*(a + b*ArcCot[c + d*x]),x]
```

output
```
((e + f*x)^(1 + m)*(2*(a + b*ArcCot[c + d*x]) + (b*d*(e + f*x)*((d*e - (I
+ c)*f)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e - (-I + c)*f
)] + (-(d*e) + (-I + c)*f)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x)
)/(d*e - (I + c)*f)]))/(((-I)*d*e + f + I*c*f)*(d*e - (I + c)*f)*(2 + m)))
)/(2*f*(1 + m))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.33, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5571, 5388, 485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)\,dx$$

$$\Big\downarrow \ 5571$$

$$\frac{\int \left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^m \left(a+b\cot^{-1}(c+dx)\right)d(c+dx)}{d}$$

$$\Big\downarrow \ 5388$$

$$\frac{\frac{bd\int \frac{\left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^{m+1}}{(c+dx)^2+1}d(c+dx)}{f(m+1)}+\frac{d(a+b\cot^{-1}(c+dx))\left(\frac{f(c+dx)}{d}-\frac{cf}{d}+e\right)^{m+1}}{f(m+1)}}{d}$$

$\downarrow$ 485

$$\frac{\frac{d(a+b\cot^{-1}(c+dx))\left(\frac{f(c+dx)}{d}-\frac{cf}{d}+e\right)^{m+1}}{f(m+1)}+\frac{bd\int\left(\frac{i\left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^{m+1}}{2(-c-dx+i)}+\frac{i\left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^{m+1}}{2(c+dx+i)}\right)d(c+dx)}{f(m+1)}}{d}$$

$\downarrow$ 2009

$$\frac{\frac{d(a+b\cot^{-1}(c+dx))\left(\frac{f(c+dx)}{d}-\frac{cf}{d}+e\right)^{m+1}}{f(m+1)}+\frac{bd\left(\frac{id\left(\frac{f(c+dx)}{d}-\frac{cf}{d}+e\right)^{m+2}\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{de-cf+f(c+dx)}{de-cf+if}\right)}{2(m+2)(de+(-c+i)f)}-\frac{id\left(\frac{f(c+dx)}{d}-\frac{cf}{d}\right.}{}\right.}{f(m+1)}}{d}$$

input
```
Int[(e + f*x)^m*(a + b*ArcCot[c + d*x]),x]
```

output
```
((d*(e - (c*f)/d + (f*(c + d*x))/d)^(1 + m)*(a + b*ArcCot[c + d*x]))/(f*(1
 + m)) + (b*d*(((I/2)*d*(e - (c*f)/d + (f*(c + d*x))/d)^(2 + m)*Hypergeome
tric2F1[1, 2 + m, 3 + m, (d*e - c*f + f*(c + d*x))/(d*e + I*f - c*f)])/((d
*e + (I - c)*f)*(2 + m)) - ((I/2)*d*(e - (c*f)/d + (f*(c + d*x))/d)^(2 + m
)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*e - c*f + f*(c + d*x))/(d*e - (I +
 c)*f)])/((d*e - (I + c)*f)*(2 + m))))/(f*(1 + m)))/d
```

Defintions of rubi rules used

rule 485
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n, 1/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, n}, x] &
&  !IntegerQ[2*n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5388
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])/(e*(q + 1))), x] + Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

Maple [F]

$$\int (fx+e)^{m} (a+b \operatorname{arccot}(dx+c))\, dx$$

input
```
int((f*x+e)^m*(a+b*arccot(d*x+c)),x)
```

output
```
int((f*x+e)^m*(a+b*arccot(d*x+c)),x)
```

Fricas [F]

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)\, dx = \int (b \operatorname{arccot}(dx+c)+a)(fx+e)^{m}\, dx$$

input
```
integrate((f*x+e)^m*(a+b*arccot(d*x+c)),x, algorithm="fricas")
```

output
```
integral((b*arccot(d*x + c) + a)*(f*x + e)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)\, dx = \text{Timed out}$$

input
```
integrate((f*x+e)**m*(a+b*acot(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right) dx = \int (b\operatorname{arccot}(dx+c)+a)(fx+e)^m dx$$

input
```
integrate((f*x+e)^m*(a+b*arccot(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*((f*x*arctan2(1, d*x + c) + e*arctan2(1, d*x + c))*(f*x + e)^m + 2*(f*
m + f)*integrate(1/2*((c^2*arctan2(1, d*x + c) + arctan2(1, d*x + c))*f*m
+ (d^2*f*m*arctan2(1, d*x + c) + d^2*f*arctan2(1, d*x + c))*x^2 + d*e + (c
^2*arctan2(1, d*x + c) + arctan2(1, d*x + c))*f + (2*c*d*f*m*arctan2(1, d*
x + c) + (2*c*arctan2(1, d*x + c) + 1)*d*f)*x)*(f*x + e)^m/((c^2 + 1)*f*m
+ (d^2*f*m + d^2*f)*x^2 + (c^2 + 1)*f + 2*(c*d*f*m + c*d*f)*x), x))*b/(f*m
 + f) + (f*x + e)^(m + 1)*a/(f*(m + 1))
```

Giac [F]

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right) dx = \int (b\operatorname{arccot}(dx+c)+a)(fx+e)^m dx$$

input
```
integrate((f*x+e)^m*(a+b*arccot(d*x+c)),x, algorithm="giac")
```

output
```
integrate((b*arccot(d*x + c) + a)*(f*x + e)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right) dx = \int (e+f\,x)^m\,\left(a+b\,\mathrm{acot}(c+d\,x)\right) dx$$

input
```
int((e + f*x)^m*(a + b*acot(c + d*x)),x)
```

output
```
int((e + f*x)^m*(a + b*acot(c + d*x)), x)
```

Reduce [F]

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^m*(a+b*acot(d*x+c)),x)
```

output
```
((e + f*x)**m*acot(c + d*x)*b*c*e*m + (e + f*x)**m*acot(c + d*x)*b*c*f*m*x
 + (e + f*x)**m*a*c*e*m + (e + f*x)**m*a*c*f*m*x + (e + f*x)**m*b*e - int(
(e + f*x)**m/(c**2*e*m + c**2*e + c**2*f*m*x + c**2*f*x + 2*c*d*e*m*x + 2*
c*d*e*x + 2*c*d*f*m*x**2 + 2*c*d*f*x**2 + d**2*e*m*x**2 + d**2*e*x**2 + d*
*2*f*m*x**3 + d**2*f*x**3 + e*m + e + f*m*x + f*x),x)*b*c**2*e*f*m**2 - in
t((e + f*x)**m/(c**2*e*m + c**2*e + c**2*f*m*x + c**2*f*x + 2*c*d*e*m*x +
2*c*d*e*x + 2*c*d*f*m*x**2 + 2*c*d*f*x**2 + d**2*e*m*x**2 + d**2*e*x**2 +
d**2*f*m*x**3 + d**2*f*x**3 + e*m + e + f*m*x + f*x),x)*b*c**2*e*f*m + int
((e + f*x)**m/(c**2*e*m + c**2*e + c**2*f*m*x + c**2*f*x + 2*c*d*e*m*x + 2
*c*d*e*x + 2*c*d*f*m*x**2 + 2*c*d*f*x**2 + d**2*e*m*x**2 + d**2*e*x**2 + d
**2*f*m*x**3 + d**2*f*x**3 + e*m + e + f*m*x + f*x),x)*b*c*d*e**2*m**2 + i
nt((e + f*x)**m/(c**2*e*m + c**2*e + c**2*f*m*x + c**2*f*x + 2*c*d*e*m*x +
 2*c*d*e*x + 2*c*d*f*m*x**2 + 2*c*d*f*x**2 + d**2*e*m*x**2 + d**2*e*x**2 +
 d**2*f*m*x**3 + d**2*f*x**3 + e*m + e + f*m*x + f*x),x)*b*c*d*e**2*m - in
t((e + f*x)**m/(c**2*e*m + c**2*e + c**2*f*m*x + c**2*f*x + 2*c*d*e*m*x +
2*c*d*e*x + 2*c*d*f*m*x**2 + 2*c*d*f*x**2 + d**2*e*m*x**2 + d**2*e*x**2 +
d**2*f*m*x**3 + d**2*f*x**3 + e*m + e + f*m*x + f*x),x)*b*e*f*m**2 - int((
e + f*x)**m/(c**2*e*m + c**2*e + c**2*f*m*x + c**2*f*x + 2*c*d*e*m*x + 2*c
*d*e*x + 2*c*d*f*m*x**2 + 2*c*d*f*x**2 + d**2*e*m*x**2 + d**2*e*x**2 + d**
2*f*m*x**3 + d**2*f*x**3 + e*m + e + f*m*x + f*x),x)*b*e*f*m + int(((e ...
```

3.34 $\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 dx = \text{Int}\left((e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2, x\right)$$

output
```
Defer(Int)((f*x+e)^m*(a+b*arccot(d*x+c))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 4.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 dx = \int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 dx$$

input
```
Integrate[(e + f*x)^m*(a + b*ArcCot[c + d*x])^2,x]
```

output
```
Integrate[(e + f*x)^m*(a + b*ArcCot[c + d*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 dx$$

$$\Big\downarrow \; 5571$$

$$\frac{\int \left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^m \left(a+b\cot^{-1}(c+dx)\right)^2 d(c+dx)}{d}$$

$$\Big\downarrow \; 5561$$

$$\frac{\int \left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^m \left(a+b\cot^{-1}(c+dx)\right)^2 d(c+dx)}{d}$$

input
```
Int[(e + f*x)^m*(a + b*ArcCot[c + d*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (fx+e)^m \left(a+b\,\mathrm{arccot}\left(dx+c\right)\right)^2 dx$$

input
```
int((f*x+e)^m*(a+b*arccot(d*x+c))^2,x)
```

output
```
int((f*x+e)^m*(a+b*arccot(d*x+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \int (b \operatorname{arccot}(dx+c)+a)^2 (fx+e)^m \, dx$$

input
```
integrate((f*x+e)^m*(a+b*arccot(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arccot(d*x + c)^2 + 2*a*b*arccot(d*x + c) + a^2)*(f*x + e)^m
, x)
```

Sympy [F(-1)]

Timed out.

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \text{Timed out}$$

input
```
integrate((f*x+e)**m*(a+b*acot(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 5.75 (sec) , antiderivative size = 618, normalized size of antiderivative = 30.90

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \int (b \operatorname{arccot}(dx+c)+a)^2 (fx+e)^m \, dx$$

input
```
integrate((f*x+e)^m*(a+b*arccot(d*x+c))^2,x, algorithm="maxima")
```

output
```
(f*x + e)^(m + 1)*a^2/(f*(m + 1)) - 1/16*((b^2*f*x + b^2*e)*(f*x + e)^m*lo
g(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 - 4*(b^2*f*x*arctan2(1, d*x + c)^2 + b^2*
e*arctan2(1, d*x + c)^2)*(f*x + e)^m - 16*(f*m + f)*integrate(1/16*(((b^2*
c^2 + b^2)*f*m + (b^2*d^2*f*m + b^2*d^2*f)*x^2 + (b^2*c^2 + b^2)*f + 2*(b^
2*c*d*f*m + b^2*c*d*f)*x)*(f*x + e)^m*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 +
 4*(b^2*d^2*f*x^2 + b^2*c*d*e + (b^2*d^2*e + b^2*c*d*f)*x)*(f*x + e)^m*log
(d^2*x^2 + 2*c*d*x + c^2 + 1) + 4*(2*b^2*d*e*arctan2(1, d*x + c) + (3*b^2*
arctan2(1, d*x + c)^2 + (3*b^2*arctan2(1, d*x + c)^2 + 8*a*b*arctan2(1, d*
x + c))*c^2 + 8*a*b*arctan2(1, d*x + c))*f*m + ((3*b^2*arctan2(1, d*x + c)
^2 + 8*a*b*arctan2(1, d*x + c))*d^2*f*m + (3*b^2*arctan2(1, d*x + c)^2 + 8
*a*b*arctan2(1, d*x + c))*d^2*f)*x^2 + (3*b^2*arctan2(1, d*x + c)^2 + (3*b
^2*arctan2(1, d*x + c)^2 + 8*a*b*arctan2(1, d*x + c))*c^2 + 8*a*b*arctan2(
1, d*x + c))*f + 2*((3*b^2*arctan2(1, d*x + c)^2 + 8*a*b*arctan2(1, d*x +
c))*c*d*f*m + (b^2*arctan2(1, d*x + c) + (3*b^2*arctan2(1, d*x + c)^2 + 8*
a*b*arctan2(1, d*x + c))*c)*d*f)*x)*(f*x + e)^m)/((c^2 + 1)*f*m + (d^2*f*m
 + d^2*f)*x^2 + (c^2 + 1)*f + 2*(c*d*f*m + c*d*f)*x), x))/(f*m + f)
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 dx = \int (b\,\mathrm{arccot}\,(dx+c)+a)^2(fx+e)^m\,dx$$

input
```
integrate((f*x+e)^m*(a+b*arccot(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*arccot(d*x + c) + a)^2*(f*x + e)^m, x)
```

Mupad [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \int (e+f\,x)^m\,(a+b\,\mathrm{acot}(c+d\,x))^2 \, dx$$

input
```
int((e + f*x)^m*(a + b*acot(c + d*x))^2,x)
```

output
```
int((e + f*x)^m*(a + b*acot(c + d*x))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 17519, normalized size of antiderivative = 875.95

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \text{Too large to display}$$

input
```
int((f*x+e)^m*(a+b*acot(d*x+c))^2,x)
```

output

```
((e + f*x)**m*acot(c + d*x)**2*b**2*c**4*d*e*f*m**2 + (e + f*x)**m*acot(c
+ d*x)**2*b**2*c**4*d*f**2*m**2*x + (e + f*x)**m*acot(c + d*x)**2*b**2*c**
2*d*e*f*m**2 + (e + f*x)**m*acot(c + d*x)**2*b**2*c**2*d*f**2*m**2*x + 2*(
e + f*x)**m*acot(c + d*x)*a*b*c**4*d*e*f*m**2 + 2*(e + f*x)**m*acot(c + d*
x)*a*b*c**4*d*f**2*m**2*x + 2*(e + f*x)**m*acot(c + d*x)*a*b*c**2*d*e*f*m*
*2 + 2*(e + f*x)**m*acot(c + d*x)*a*b*c**2*d*f**2*m**2*x + 2*(e + f*x)**m*
acot(c + d*x)*b**2*c**4*f**2*m + 2*(e + f*x)**m*acot(c + d*x)*b**2*c**2*f*
*2*m - 2*(e + f*x)**m*atan(1/(c + d*x))*b**2*c**4*f**2*m + 2*(e + f*x)**m*
atan(1/(c + d*x))*b**2*c**2*d**2*e**2*m - 2*(e + f*x)**m*atan(1/(c + d*x))
*b**2*c**2*f**2*m + (e + f*x)**m*a**2*c**4*d*e*f*m**2 + (e + f*x)**m*a**2*
c**4*d*f**2*m**2*x + (e + f*x)**m*a**2*c**2*d*e*f*m**2 + (e + f*x)**m*a**2
*c**2*d*f**2*m**2*x + 2*(e + f*x)**m*a*b*c**3*d*e*f*m + 2*(e + f*x)**m*a*b
*c*d*e*f*m + (e + f*x)**m*b**2*c*d**2*e**2 + int((e + f*x)**m/(c**4*e*m +
c**4*e + c**4*f*m*x + c**4*f*x + 2*c**3*d*e*m*x + 2*c**3*d*e*x + 2*c**3*d*
f*m*x**2 + 2*c**3*d*f*x**2 + c**2*d**2*e*m*x**2 + c**2*d**2*e*x**2 + c**2*
d**2*f*m*x**3 + c**2*d**2*f*x**3 + 2*c**2*e*m + 2*c**2*e + 2*c**2*f*m*x +
2*c**2*f*x + 2*c*d*e*m*x + 2*c*d*e*x + 2*c*d*f*m*x**2 + 2*c*d*f*x**2 + d**
2*e*m*x**2 + d**2*e*x**2 + d**2*f*m*x**3 + d**2*f*x**3 + e*m + e + f*m*x +
 f*x),x)*b**2*c**6*d*e*f**2*m**2 + int((e + f*x)**m/(c**4*e*m + c**4*e + c
**4*f*m*x + c**4*f*x + 2*c**3*d*e*m*x + 2*c**3*d*e*x + 2*c**3*d*f*m*x**...
```

3.35 $\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \text{Int}\left((e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3, x\right)$$

output
```
Defer(Int)((f*x+e)^m*(a+b*arccot(d*x+c))^3,x)
```

Mathematica [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx$$

input
```
Integrate[(e + f*x)^m*(a + b*ArcCot[c + d*x])^3,x]
```

output
```
Integrate[(e + f*x)^m*(a + b*ArcCot[c + d*x])^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx$$

$$\Big\downarrow \text{ 5571}$$

$$\frac{\int \left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^m \left(a+b\cot^{-1}(c+dx)\right)^3 d(c+dx)}{d}$$

$$\Big\downarrow \text{ 5561}$$

$$\frac{\int \left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^m \left(a+b\cot^{-1}(c+dx)\right)^3 d(c+dx)}{d}$$

input
```
Int[(e + f*x)^m*(a + b*ArcCot[c + d*x])^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (fx+e)^m (a+b\operatorname{arccot}(dx+c))^3 \, dx$$

input
```
int((f*x+e)^m*(a+b*arccot(d*x+c))^3,x)
```

output
```
int((f*x+e)^m*(a+b*arccot(d*x+c))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \int (b \operatorname{arccot}(dx+c)+a)^3 (fx+e)^m \, dx$$

input
```
integrate((f*x+e)^m*(a+b*arccot(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral((b^3*arccot(d*x + c)^3 + 3*a*b^2*arccot(d*x + c)^2 + 3*a^2*b*arcc
ot(d*x + c) + a^3)*(f*x + e)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \text{Timed out}$$

input
```
integrate((f*x+e)**m*(a+b*acot(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 9.26 (sec) , antiderivative size = 880, normalized size of antiderivative = 44.00

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \int (b \operatorname{arccot}(dx+c)+a)^3 (fx+e)^m \, dx$$

input
```
integrate((f*x+e)^m*(a+b*arccot(d*x+c))^3,x, algorithm="maxima")
```

output
```
(f*x + e)^(m + 1)*a^3/(f*(m + 1)) - 1/32*(3*(b^3*f*x*arctan2(1, d*x + c) +
 b^3*e*arctan2(1, d*x + c))*(f*x + e)^m*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2
 - 4*(b^3*f*x*arctan2(1, d*x + c)^3 + b^3*e*arctan2(1, d*x + c)^3)*(f*x +
e)^m - 32*(f*m + f)*integrate(-1/32*(3*(b^3*d*e - (b^3*c^2*arctan2(1, d*x
+ c) + b^3*arctan2(1, d*x + c))*f*m - (b^3*d^2*f*m*arctan2(1, d*x + c) + b
^3*d^2*f*arctan2(1, d*x + c))*x^2 - (b^3*c^2*arctan2(1, d*x + c) + b^3*arc
tan2(1, d*x + c))*f - (2*b^3*c*d*f*m*arctan2(1, d*x + c) + (2*b^3*c*arctan
2(1, d*x + c) - b^3)*d*f)*x)*(f*x + e)^m*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^
2 - 12*(b^3*d^2*f*x^2*arctan2(1, d*x + c) + b^3*c*d*e*arctan2(1, d*x + c)
+ (b^3*d^2*e*arctan2(1, d*x + c) + b^3*c*d*f*arctan2(1, d*x + c))*x)*(f*x
+ e)^m*log(d^2*x^2 + 2*c*d*x + c^2 + 1) - 4*(3*b^3*d*e*arctan2(1, d*x + c)
^2 + (7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2 + 24*a^
2*b*arctan2(1, d*x + c) + (7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(
1, d*x + c)^2 + 24*a^2*b*arctan2(1, d*x + c))*c^2)*f*m + ((7*b^3*arctan2(1
, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2 + 24*a^2*b*arctan2(1, d*x +
c))*d^2*f*m + (7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^
2 + 24*a^2*b*arctan2(1, d*x + c))*d^2*f)*x^2 + (7*b^3*arctan2(1, d*x + c)^
3 + 24*a*b^2*arctan2(1, d*x + c)^2 + 24*a^2*b*arctan2(1, d*x + c) + (7*b^3
*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2 + 24*a^2*b*arctan2
(1, d*x + c))*c^2)*f + (2*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arcta...
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \int (b \operatorname{arccot}(dx+c)+a)^3 (fx+e)^m \, dx$$

input
```
integrate((f*x+e)^m*(a+b*arccot(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*arccot(d*x + c) + a)^3*(f*x + e)^m, x)
```

Mupad [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \int (e+f\,x)^m\,(a+b\,\mathrm{acot}(c+d\,x))^3 \, dx$$

input
```
int((e + f*x)^m*(a + b*acot(c + d*x))^3,x)
```

output
```
int((e + f*x)^m*(a + b*acot(c + d*x))^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 42990, normalized size of antiderivative = 2149.50

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
int((f*x+e)^m*(a+b*acot(d*x+c))^3,x)
```

output

```
(2*(e + f*x)**m*acot(c + d*x)**3*b**3*c**5*d*e*f*m**3 + 2*(e + f*x)**m*aco
t(c + d*x)**3*b**3*c**5*d*f**2*m**3*x + 2*(e + f*x)**m*acot(c + d*x)**3*b*
*3*c**3*d*e*f*m**3 + 2*(e + f*x)**m*acot(c + d*x)**3*b**3*c**3*d*f**2*m**3
*x + 6*(e + f*x)**m*acot(c + d*x)**2*a*b**2*c**5*d*e*f*m**3 + 6*(e + f*x)*
*m*acot(c + d*x)**2*a*b**2*c**5*d*f**2*m**3*x + 6*(e + f*x)**m*acot(c + d*
x)**2*a*b**2*c**3*d*e*f*m**3 + 6*(e + f*x)**m*acot(c + d*x)**2*a*b**2*c**3
*d*f**2*m**3*x + 6*(e + f*x)**m*acot(c + d*x)**2*b**3*c**5*f**2*m**2 + 6*(
e + f*x)**m*acot(c + d*x)**2*b**3*c**3*f**2*m**2 + 6*(e + f*x)**m*acot(c +
 d*x)*a**2*b*c**5*d*e*f*m**3 + 6*(e + f*x)**m*acot(c + d*x)*a**2*b*c**5*d*
f**2*m**3*x + 6*(e + f*x)**m*acot(c + d*x)*a**2*b*c**3*d*e*f*m**3 + 6*(e +
 f*x)**m*acot(c + d*x)*a**2*b*c**3*d*f**2*m**3*x + 12*(e + f*x)**m*acot(c
+ d*x)*a*b**2*c**5*f**2*m**2 + 12*(e + f*x)**m*acot(c + d*x)*a*b**2*c**3*f
**2*m**2 - 6*(e + f*x)**m*atan(1/(c + d*x))**2*b**3*c**5*f**2*m**2 + 6*(e
+ f*x)**m*atan(1/(c + d*x))**2*b**3*c**3*d**2*e**2*m**2 - 6*(e + f*x)**m*a
tan(1/(c + d*x))**2*b**3*c**3*f**2*m**2 - 12*(e + f*x)**m*atan(1/(c + d*x)
)*a*b**2*c**5*f**2*m**2 + 12*(e + f*x)**m*atan(1/(c + d*x))*a*b**2*c**3*d*
*2*e**2*m**2 - 12*(e + f*x)**m*atan(1/(c + d*x))*a*b**2*c**3*f**2*m**2 - 6
*(e + f*x)**m*atan(1/(c + d*x))*b**3*c**3*d*e*f*m + 18*(e + f*x)**m*atan(1
/(c + d*x))*b**3*c**2*d**2*e**2*m - 6*(e + f*x)**m*atan(1/(c + d*x))*b**3*
c*d*e*f*m + 2*(e + f*x)**m*a**3*c**5*d*e*f*m**3 + 2*(e + f*x)**m*a**3*c...
```

3.36 $\int \frac{\cot^{-1}(a+bx)}{c+dx^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 725

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^3}\,dx = -\frac{\cot^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{3c^{2/3}\sqrt[3]{d}} + \frac{\sqrt[3]{-1}\cot^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{3c^{2/3}\sqrt[3]{d}} - \frac{(-1)^{2/3}\cot^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{3c^{2/3}\sqrt[3]{d}} + \frac{\cot^{-1}(a+bx)\log\left(\frac{2b\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(b\sqrt[3]{c}+(i-a)\sqrt[3]{d}\right)(1-i(a+bx))}\right)}{3c^{2/3}\sqrt[3]{d}} + \frac{(-1)^{2/3}\cot^{-1}(a+bx)\log\left(\frac{2b\left(\sqrt[3]{c}-\sqrt[3]{-1}\sqrt[3]{d}x\right)}{\left(b\sqrt[3]{c}-\sqrt[3]{-1}(i-a)\sqrt[3]{d}\right)(1-i(a+bx))}\right)}{3c^{2/3}\sqrt[3]{d}} - \frac{\sqrt[3]{-1}\cot^{-1}(a+bx)\log\left(\frac{2b\left(\sqrt[3]{c}+(-1)^{2/3}\sqrt[3]{d}x\right)}{\left(b\sqrt[3]{c}+(-1)^{2/3}(i-a)\sqrt[3]{d}\right)(1-i(a+bx))}\right)}{3c^{2/3}\sqrt[3]{d}} - \frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{\sqrt[6]{-1}\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{(-1)^{5/6}\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2b\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(b\sqrt[3]{c}+(i-a)\sqrt[3]{d}\right)(1-i(a+bx))}\right)}{6c^{2/3}\sqrt[3]{d}} - \frac{\sqrt[6]{-1}\,\mathrm{PolyLog}\left(2,1-\frac{2b\left(\sqrt[3]{c}-\sqrt[3]{-1}\sqrt[3]{d}x\right)}{\left(b\sqrt[3]{c}-\sqrt[3]{-1}(i-a)\sqrt[3]{d}\right)(1-i(a+bx))}\right)}{6c^{2/3}\sqrt[3]{d}} - \frac{(-1)^{5/6}\,\mathrm{PolyLog}\left(2,1-\frac{2b\left(\sqrt[3]{c}+(-1)^{2/3}\sqrt[3]{d}x\right)}{\left(b\sqrt[3]{c}+(-1)^{2/3}(i-a)\sqrt[3]{d}\right)(1-i(a+bx))}\right)}{6c^{2/3}\sqrt[3]{d}}$$

output
```
-1/3*arccot(b*x+a)*ln(2/(1-I*(b*x+a)))/c^(2/3)/d^(1/3)+1/3*(-1)^(1/3)*arcc
ot(b*x+a)*ln(2/(1-I*(b*x+a)))/c^(2/3)/d^(1/3)-1/3*(-1)^(2/3)*arccot(b*x+a)
*ln(2/(1-I*(b*x+a)))/c^(2/3)/d^(1/3)+1/3*arccot(b*x+a)*ln(2*b*(c^(1/3)+d^(
1/3)*x)/(b*c^(1/3)+(I-a)*d^(1/3))/(1-I*(b*x+a)))/c^(2/3)/d^(1/3)+1/3*(-1)^
(2/3)*arccot(b*x+a)*ln(2*b*(c^(1/3)-(-1)^(1/3)*d^(1/3)*x)/(b*c^(1/3)-(-1)^
(1/3)*(I-a)*d^(1/3))/(1-I*(b*x+a)))/c^(2/3)/d^(1/3)-1/3*(-1)^(1/3)*arccot(
b*x+a)*ln(2*b*(c^(1/3)+(-1)^(2/3)*d^(1/3)*x)/(b*c^(1/3)+(-1)^(2/3)*(I-a)*d
^(1/3))/(1-I*(b*x+a)))/c^(2/3)/d^(1/3)-1/6*I*polylog(2,1-2/(1-I*(b*x+a)))/
c^(2/3)/d^(1/3)+1/6*(-1)^(1/6)*polylog(2,1-2/(1-I*(b*x+a)))/c^(2/3)/d^(1/3
)+1/6*(-1)^(5/6)*polylog(2,1-2/(1-I*(b*x+a)))/c^(2/3)/d^(1/3)+1/6*I*polylo
g(2,1-2*b*(c^(1/3)+d^(1/3)*x)/(b*c^(1/3)+(I-a)*d^(1/3))/(1-I*(b*x+a)))/c^(
2/3)/d^(1/3)-1/6*(-1)^(1/6)*polylog(2,1-2*b*(c^(1/3)-(-1)^(1/3)*d^(1/3)*x)
/(b*c^(1/3)-(-1)^(1/3)*(I-a)*d^(1/3))/(1-I*(b*x+a)))/c^(2/3)/d^(1/3)-1/6*(
-1)^(5/6)*polylog(2,1-2*b*(c^(1/3)+(-1)^(2/3)*d^(1/3)*x)/(b*c^(1/3)+(-1)^(
2/3)*(I-a)*d^(1/3))/(1-I*(b*x+a)))/c^(2/3)/d^(1/3)
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 998, normalized size of antiderivative = 1.38

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^3}\,dx = \text{Too large to display}$$

input
```
Integrate[ArcCot[a + b*x]/(c + d*x^3),x]
```

output
```
((-I)*Log[-((d^(1/3)*(-I + a + b*x))/(b*c^(1/3) - (-I + a)*d^(1/3)))]*Log[
-c^(1/3) - d^(1/3)*x] + I*Log[(-I + a + b*x)/(a + b*x)]*Log[-c^(1/3) - d^(
1/3)*x] + I*Log[-((d^(1/3)*(I + a + b*x))/(b*c^(1/3) - (I + a)*d^(1/3)))]*
Log[-c^(1/3) - d^(1/3)*x] - I*Log[(I + a + b*x)/(a + b*x)]*Log[-c^(1/3) -
d^(1/3)*x] + (-1)^(1/6)*Log[((-1)^(1/3)*d^(1/3)*(-I + a + b*x))/(b*c^(1/3)
 + (-1)^(1/3)*(-I + a)*d^(1/3))]*Log[-c^(1/3) + (-1)^(1/3)*d^(1/3)*x] - (-
1)^(1/6)*Log[(-I + a + b*x)/(a + b*x)]*Log[-c^(1/3) + (-1)^(1/3)*d^(1/3)*x
] - (-1)^(1/6)*Log[((-1)^(1/3)*d^(1/3)*(I + a + b*x))/(b*c^(1/3) + (-1)^(1
/3)*(I + a)*d^(1/3))]*Log[-c^(1/3) + (-1)^(1/3)*d^(1/3)*x] + (-1)^(1/6)*Lo
g[(I + a + b*x)/(a + b*x)]*Log[-c^(1/3) + (-1)^(1/3)*d^(1/3)*x] + (-1)^(5/
6)*Log[((-1)^(2/3)*d^(1/3)*(-I + a + b*x))/(-(b*c^(1/3)) + (-1)^(1/6)*(1 +
 I*a)*d^(1/3))]*Log[-c^(1/3) - (-1)^(2/3)*d^(1/3)*x] - (-1)^(5/6)*Log[(-I
+ a + b*x)/(a + b*x)]*Log[-c^(1/3) - (-1)^(2/3)*d^(1/3)*x] - (-1)^(5/6)*Lo
g[((-1)^(2/3)*d^(1/3)*(I + a + b*x))/(-(b*c^(1/3)) + (-1)^(2/3)*(I + a)*d^
(1/3))]*Log[-c^(1/3) - (-1)^(2/3)*d^(1/3)*x] + (-1)^(5/6)*Log[(I + a + b*x
)/(a + b*x)]*Log[-c^(1/3) - (-1)^(2/3)*d^(1/3)*x] - I*PolyLog[2, (b*(c^(1/
3) + d^(1/3)*x))/(b*c^(1/3) - (-I + a)*d^(1/3))] + I*PolyLog[2, (b*(c^(1/3
) + d^(1/3)*x))/(b*c^(1/3) - (I + a)*d^(1/3))] + (-1)^(1/6)*PolyLog[2, (b*
(c^(1/3) - (-1)^(1/3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(1/3)*(-I + a)*d^(1/3)
)] - (-1)^(1/6)*PolyLog[2, (b*(c^(1/3) - (-1)^(1/3)*d^(1/3)*x))/(b*c^(1...
```

Rubi [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1862 vs. $2(725) = 1450$.

Time = 3.12 (sec) , antiderivative size = 1862, normalized size of antiderivative = 2.57, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5575, 2993, 750, 16, 1142, 25, 27, 1082, 217, 1103, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^3}\,dx$$

$$\Big\downarrow \; 5575$$

$$\frac{1}{2}i\int \frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)}{dx^3+c}dx-\frac{1}{2}i\int \frac{\log\left(\frac{a+bx+i}{a+bx}\right)}{dx^3+c}dx$$

$$\Big\downarrow \text{2993}$$

$$\frac{1}{2}i\left(-\left(\left(\log(-a-bx+i)-\log\left(-\frac{-a-bx+i}{a+bx}\right)-\log(a+bx)\right)\int\frac{1}{dx^3+c}dx\right)+\int\frac{\log(-a-bx+i)}{dx^3+c}dx-\int\right.$$
$$\frac{1}{2}i\left(\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)\int\frac{1}{dx^3+c}dx-\int\frac{\log(a+bx)}{dx^3+c}dx+\int\frac{\log(a+bx+i)}{dx^3+c}\right.$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{2}i\left(-\left(\left(\log(-a-bx+i)-\log\left(-\frac{-a-bx+i}{a+bx}\right)-\log(a+bx)\right)\left(\frac{\int\frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\int\frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}}\right.\right.\right.$$
$$\frac{1}{2}i\left(\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)\left(\frac{\int\frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\int\frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}}\right)-\int\frac{\mathrm{lo}}{}\right.$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}i\left(-\left(\left(\log(-a-bx+i)-\log\left(-\frac{-a-bx+i}{a+bx}\right)-\log(a+bx)\right)\left(\frac{\int\frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right.}{3c^{2/3}\sqrt[3]{d}}\right.\right.\right.$$
$$\frac{1}{2}i\left(\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)\left(\frac{\int\frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)-\int\right.$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}i\left(-\left(\left(\log(-a-bx+i)-\log\left(-\frac{-a-bx+i}{a+bx}\right)-\log(a+bx)\right)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{d}(\cdots}{d^{2/3}x^2-\cdots}}{\cdots}}{3c^{2/3}}\cdots\right.\right.\right.$$

$$\frac{1}{2}i\left(\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\cdots\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c\cdots}}{2\sqrt[3]{d}}}{3c^{2/3}}\cdots\right.\right.$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}i\left(-\left(\left(\log(-a-bx+i)-\log\left(-\frac{-a-bx+i}{a+bx}\right)-\log(a+bx)\right)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{\int\frac{\sqrt[3]{d}\left(\sqrt[3]{\cdots}\right.}{d^{2/3}x^2-\sqrt[3]{\cdots}}}{2\cdots}}{3c^{2/3}}\cdots\right.\right.\right.$$

$$\frac{1}{2}i\left(\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{\int\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/\cdots}}}{2\sqrt[3]{d}}}{3c^{2/3}}\cdots\right.\right.$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}i\left(-\left(\left(\log(-a-bx+i)-\log\left(-\frac{-a-bx+i}{a+bx}\right)-\log(a+bx)\right)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{\cdots}}{d^{2/3}x^2-\cdots}}{3c^{2/3}}\cdots\right.\right.\right.$$

$$\frac{1}{2}i\left(\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+\cdots}}{3c^{2/3}}\cdots\right.\right.$$

$$\Big\downarrow \; 1082$$

$$\frac{1}{2}i\left(-\left(\left(\log(-a-bx+i)-\log\left(-\frac{-a-bx+i}{a+bx}\right)-\log(a+bx)\right)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right.}}{}}{3c^{2/3}}\right.\right.\right.$$

$$\frac{1}{2}i\left(\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2-3}d\left(1-\right.}{\sqrt[3]{d}}}{3c^{2/3}}\right.\right.$$

$\Big\downarrow$ 217

$$\frac{1}{2}i\left(-\left(\left(\log(-a-bx+i)-\log\left(-\frac{-a-bx+i}{a+bx}\right)-\log(a+bx)\right)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2}{}}{\sqrt{}}\right.}{\sqrt[3]{d}}}{3c^{2/3}}\right.\right.\right.$$

$$\frac{1}{2}i\left(\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\right.\right.$$

$\Big\downarrow$ 1103

$$\frac{1}{2}i\left(\int\frac{\log(-a-bx+i)}{dx^3+c}dx-\int\frac{\log(a+bx)}{dx^3+c}dx-\left(\left(\log(-a-bx+i)-\log\left(-\frac{-a-bx+i}{a+bx}\right)-\log(a+bx)\right)\cdots\right.\right.$$

$$\frac{1}{2}i\left(-\int\frac{\log(a+bx)}{dx^3+c}dx+\int\frac{\log(a+bx+i)}{dx^3+c}dx+\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)\left(-\frac{\sqrt{3}\,\mathrm{a}\cdots}{\cdots}\right.\right.$$

$\downarrow$ 2856

$$\frac{1}{2}i\left(-\left(\left(\log(-a-bx+i)-\log\left(-\frac{-a-bx+i}{a+bx}\right)-\log(a+bx)\right)\left(\frac{\log\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{3c^{2/3}\sqrt[3]{d}}+\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}\cdots}{\cdots}\right.\right.\right.$$

$$\frac{1}{2}i\left(\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)\left(\frac{\log\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{3c^{2/3}\sqrt[3]{d}}+\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{\log\left(d^{2/3}\cdots\right.}{\cdots}}{3c^{2/3}\cdots}\right.\right.$$

$\downarrow$ 2009

$$\frac{1}{2}i\left(\frac{\log(-a-bx+i)\log\left(\frac{b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\sqrt[3]{d}(i-a)+b\sqrt[3]{c}}\right)}{3c^{2/3}\sqrt[3]{d}}-\frac{\log(a+bx)\log\left(\frac{b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{b\sqrt[3]{c}-a\sqrt[3]{d}}\right)}{3c^{2/3}\sqrt[3]{d}}+\frac{(-1)^{2/3}\log(-a-bx+i)\log}{3c^{2/3}\sqrt[3]{d}}\right.$$

$$\frac{1}{2}i\left(-\frac{\log(a+bx)\log\left(\frac{b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{b\sqrt[3]{c}-a\sqrt[3]{d}}\right)}{3c^{2/3}\sqrt[3]{d}}+\frac{\log(a+bx+i)\log\left(\frac{b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{b\sqrt[3]{c}-(a+i)\sqrt[3]{d}}\right)}{3c^{2/3}\sqrt[3]{d}}-\frac{(-1)^{2/3}\log(a+bx)\log\left(\frac{b\left(\sqrt[3]{}\right.}{\sqrt[3]{}}\right.}{3c^{2/3}\sqrt[3]{d}}\right.$$

input
```
Int[ArcCot[a + b*x]/(c + d*x^3),x]
```

output
```
(I/2)*((Log[I - a - b*x]*Log[(b*(c^(1/3) + d^(1/3)*x))/(b*c^(1/3) + (I - a
)*d^(1/3))])/(3*c^(2/3)*d^(1/3)) - (Log[a + b*x]*Log[(b*(c^(1/3) + d^(1/3)
*x))/(b*c^(1/3) - a*d^(1/3))])/(3*c^(2/3)*d^(1/3)) + ((-1)^(2/3)*Log[I - a
 - b*x]*Log[(b*(c^(1/3) - (-1)^(1/3)*d^(1/3)*x))/(b*c^(1/3) - (-1)^(1/3)*(
I - a)*d^(1/3))])/(3*c^(2/3)*d^(1/3)) - ((-1)^(2/3)*Log[a + b*x]*Log[(b*(c
^(1/3) - (-1)^(1/3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(1/3)*a*d^(1/3))])/(3*c^
(2/3)*d^(1/3)) - ((-1)^(1/3)*Log[I - a - b*x]*Log[(b*(c^(1/3) + (-1)^(2/3)
*d^(1/3)*x))/(b*c^(1/3) + (-1)^(2/3)*(I - a)*d^(1/3))])/(3*c^(2/3)*d^(1/3)
) + ((-1)^(1/3)*Log[a + b*x]*Log[(b*(c^(1/3) + (-1)^(2/3)*d^(1/3)*x))/(b*c
^(1/3) - (-1)^(2/3)*a*d^(1/3))])/(3*c^(2/3)*d^(1/3)) - (Log[I - a - b*x] -
 Log[-((I - a - b*x)/(a + b*x))] - Log[a + b*x])*(Log[c^(1/3) + d^(1/3)*x]
/(3*c^(2/3)*d^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt
[3]])/d^(1/3)) - Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]/(2*d^(1/3)
))/(3*c^(2/3))) + PolyLog[2, (d^(1/3)*(I - a - b*x))/(b*c^(1/3) + (I - a)*
d^(1/3))]/(3*c^(2/3)*d^(1/3)) + ((-1)^(2/3)*PolyLog[2, -(((-1)^(1/3)*d^(1/
3)*(I - a - b*x))/(b*c^(1/3) - (-1)^(1/3)*(I - a)*d^(1/3)))])/(3*c^(2/3)*d
^(1/3)) - ((-1)^(1/3)*PolyLog[2, ((-1)^(2/3)*d^(1/3)*(I - a - b*x))/(b*c^(
1/3) + (-1)^(2/3)*(I - a)*d^(1/3))])/(3*c^(2/3)*d^(1/3)) - PolyLog[2, -((d
^(1/3)*(a + b*x))/(b*c^(1/3) - a*d^(1/3)))]/(3*c^(2/3)*d^(1/3)) - ((-1)^(2
/3)*PolyLog[2, ((-1)^(1/3)*d^(1/3)*(a + b*x))/(b*c^(1/3) + (-1)^(1/3)*a...
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 2993
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Simp[p*r   Int[RFx*Log[a + b*x], x], x] + (Si
mp[q*r   Int[RFx*Log[c + d*x], x], x] - Simp[(p*r*Log[a + b*x] + q*r*Log[c
+ d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])   Int[RFx, x], x]) /; FreeQ[
{a, b, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a
*d, 0] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[
m, n]]
```

rule 5575
```
Int[ArcCot[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[
I/2   Int[Log[(-I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] - Simp[I/2   In
t[Log[(I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x
] && RationalQ[n]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.60 (sec) , antiderivative size = 536, normalized size of antiderivative = 0.74

method	result
risch	$\frac{ib^2\left(\sum_{\_R1=\mathrm{RootOf}\left(d\_Z^3+\left(3\,\mathrm{RootOf}\left(\_Z^2+1,\mathrm{index}=1\right)ad-3d\right)\_Z^2+\left(-6\,\mathrm{RootOf}\left(\_Z^2+1,\mathrm{index}=1\right)ad-3a^2d+3d\right)\_Z-\mathrm{RootOf}\ldots}\right.}{\ldots}$
derivativedivides	$-\frac{b^3\left(\sum_{\_R=\mathrm{RootOf}\left(d\_Z^3-3ad\_Z^2+3a^2d\_Z-a^3d+b^3c\right)}\frac{\ln\left(bx-\_R+a\right)}{-\_R^2+2\_Ra-a^2}\right)\mathrm{arccot}(bx+a)}{3d}-\frac{b^3\left(\mathrm{arctan}(bx+a)\left(\sum_{\_R=\mathrm{Roo}\ldots}\right.\right.}{\ldots}$
default	$-\frac{b^3\left(\sum_{\_R=\mathrm{RootOf}\left(d\_Z^3-3ad\_Z^2+3a^2d\_Z-a^3d+b^3c\right)}\frac{\ln\left(bx-\_R+a\right)}{-\_R^2+2\_Ra-a^2}\right)\mathrm{arccot}(bx+a)}{3d}-\frac{b^3\left(\mathrm{arctan}(bx+a)\left(\sum_{\_R=\mathrm{Roo}\ldots}\right.\right.}{\ldots}$

input
```
int(arccot(b*x+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6*I*b^2/d*sum(1/(1+2*I*a*_R1-2*I*a+_R1^2-a^2-2*_R1)*(ln(1-I*a-I*b*x)*ln(
(_R1+I*b*x+I*a-1)/_R1)+dilog((_R1+I*b*x+I*a-1)/_R1)),_R1=RootOf(d*_Z^3+(3*
RootOf(_Z^2+1,index=1)*a*d-3*d)*_Z^2+(-6*RootOf(_Z^2+1,index=1)*a*d-3*a^2*
d+3*d)*_Z-RootOf(_Z^2+1,index=1)*a^3*d+RootOf(_Z^2+1,index=1)*b^3*c+3*Root
Of(_Z^2+1,index=1)*a*d+3*a^2*d-d))-1/6*b^2*Pi/d*sum(1/(1+2*I*a*_R-2*I*a+_R
^2-a^2-2*_R)*ln(-I*b*x-I*a+1-_R),_R=RootOf(d*_Z^3+(3*RootOf(_Z^2+1,index=1
)*a*d-3*d)*_Z^2+(-6*RootOf(_Z^2+1,index=1)*a*d-3*a^2*d+3*d)*_Z-RootOf(_Z^2
+1,index=1)*a^3*d+RootOf(_Z^2+1,index=1)*b^3*c+3*RootOf(_Z^2+1,index=1)*a*
d+3*a^2*d-d))-1/6*I*b^2/d*sum(1/(1-2*I*a*_R1+2*I*a+_R1^2-a^2-2*_R1)*(ln(1+
I*a+I*b*x)*ln((_R1-I*b*x-I*a-1)/_R1)+dilog((_R1-I*b*x-I*a-1)/_R1)),_R1=Roo
tOf(d*_Z^3+(-3*RootOf(_Z^2+1,index=1)*a*d-3*d)*_Z^2+(6*RootOf(_Z^2+1,index
=1)*a*d-3*a^2*d+3*d)*_Z+RootOf(_Z^2+1,index=1)*a^3*d-RootOf(_Z^2+1,index=1
)*b^3*c-3*RootOf(_Z^2+1,index=1)*a*d+3*a^2*d-d))
```

Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^3}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{dx^3+c}\,dx$$

input
```
integrate(arccot(b*x+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
integral(arccot(b*x + a)/(d*x^3 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^3}\,dx = \text{Timed out}$$

input
```
integrate(acot(b*x+a)/(d*x**3+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^3}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{dx^3+c}\,dx$$

input
```
integrate(arccot(b*x+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(arccot(b*x + a)/(d*x^3 + c), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^3}\,dx = \text{Timed out}$$

input
```
integrate(arccot(b*x+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^3}\,dx = \int \frac{\operatorname{acot}(a+b\,x)}{d\,x^3+c}\,dx$$

input
```
int(acot(a + b*x)/(c + d*x^3),x)
```

output
```
int(acot(a + b*x)/(c + d*x^3), x)
```

Reduce [F]

$$\int \frac{\cot^{-1}(a + bx)}{c + dx^3} \, dx = \int \frac{acot(bx + a)}{d\,x^3 + c} dx$$

input

```
int(acot(b*x+a)/(d*x^3+c),x)
```

output

```
int(acot(a + b*x)/(c + d*x**3),x)
```

3.37 $\int \frac{\cot^{-1}(a+bx)}{c+dx^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 319

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^2}\,dx = \frac{\cot^{-1}(a+bx)\log\left(\frac{2b\left(\sqrt{-c}-\sqrt{d}x\right)}{\left(b\sqrt{-c}-(i-a)\sqrt{d}\right)(1-i(a+bx))}\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\cot^{-1}(a+bx)\log\left(\frac{2b\left(\sqrt{-c}+\sqrt{d}x\right)}{\left(b\sqrt{-c}+(i-a)\sqrt{d}\right)(1-i(a+bx))}\right)}{2\sqrt{-c}\sqrt{d}} + \frac{i\,\text{PolyLog}\left(2,1-\frac{2b\left(\sqrt{-c}-\sqrt{d}x\right)}{\left(b\sqrt{-c}-(i-a)\sqrt{d}\right)(1-i(a+bx))}\right)}{4\sqrt{-c}\sqrt{d}} - \frac{i\,\text{PolyLog}\left(2,1-\frac{2b\left(\sqrt{-c}+\sqrt{d}x\right)}{\left(b\sqrt{-c}+(i-a)\sqrt{d}\right)(1-i(a+bx))}\right)}{4\sqrt{-c}\sqrt{d}}$$

output
```
1/2*arccot(b*x+a)*ln(2*b*((-c)^(1/2)-d^(1/2)*x)/(b*(-c)^(1/2)-(I-a)*d^(1/2
))/(1-I*(b*x+a)))/(-c)^(1/2)/d^(1/2)-1/2*arccot(b*x+a)*ln(2*b*((-c)^(1/2)+
d^(1/2)*x)/(b*(-c)^(1/2)+(I-a)*d^(1/2))/(1-I*(b*x+a)))/(-c)^(1/2)/d^(1/2)+
1/4*I*polylog(2,1-2*b*((-c)^(1/2)-d^(1/2)*x)/(b*(-c)^(1/2)-(I-a)*d^(1/2))/
(1-I*(b*x+a)))/(-c)^(1/2)/d^(1/2)-1/4*I*polylog(2,1-2*b*((-c)^(1/2)+d^(1/2
)*x)/(b*(-c)^(1/2)+(I-a)*d^(1/2))/(1-I*(b*x+a)))/(-c)^(1/2)/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.76

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^2}\,dx = -\frac{i\left(\log\left(\frac{\sqrt{d}(-i+a+bx)}{b\sqrt{-c}+(-i+a)\sqrt{d}}\right)\log\left(\sqrt{-c}-\sqrt{d}x\right)-\log\left(\frac{-i+a+bx}{a+bx}\right)\log\left(\sqrt{-c}-\sqrt{d}x\right)-\log\left(\frac{\sqrt{d}(i+a+bx)}{b\sqrt{-c}+(i+a)\sqrt{d}}\right)\mathrm{lo}\right.}{}$$

input
```
Integrate[ArcCot[a + b*x]/(c + d*x^2),x]
```

output
```
((-1/4*I)*(Log[(Sqrt[d]*(-I + a + b*x))/(b*Sqrt[-c] + (-I + a)*Sqrt[d])]*L
og[Sqrt[-c] - Sqrt[d]*x] - Log[(-I + a + b*x)/(a + b*x)]*Log[Sqrt[-c] - Sq
rt[d]*x] - Log[(Sqrt[d]*(I + a + b*x))/(b*Sqrt[-c] + (I + a)*Sqrt[d])]*Log
[Sqrt[-c] - Sqrt[d]*x] + Log[(I + a + b*x)/(a + b*x)]*Log[Sqrt[-c] - Sqrt[
d]*x] - Log[-((Sqrt[d]*(-I + a + b*x))/(b*Sqrt[-c] - (-I + a)*Sqrt[d]))]*L
og[Sqrt[-c] + Sqrt[d]*x] + Log[(-I + a + b*x)/(a + b*x)]*Log[Sqrt[-c] + Sq
rt[d]*x] + Log[-((Sqrt[d]*(I + a + b*x))/(b*Sqrt[-c] - (I + a)*Sqrt[d]))]*
Log[Sqrt[-c] + Sqrt[d]*x] - Log[(I + a + b*x)/(a + b*x)]*Log[Sqrt[-c] + Sq
rt[d]*x] + PolyLog[2, (b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] + (-I + a)*Sq
rt[d])] - PolyLog[2, (b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] + (I + a)*Sqrt
[d])] - PolyLog[2, (b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] - (-I + a)*Sqrt[
d])] + PolyLog[2, (b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] - (I + a)*Sqrt[d]
)]))/(Sqrt[-c]*Sqrt[d])
```

Rubi [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 721 vs. 2(319) = 638.

Time = 1.37 (sec) , antiderivative size = 721, normalized size of antiderivative = 2.26, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5575, 2976, 2804, 2009, 2977, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^2}\,dx$$

$$\Big\downarrow \text{ 5575}$$

$$\frac{1}{2}i\int \frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)}{dx^2+c}dx-\frac{1}{2}i\int \frac{\log\left(\frac{a+bx+i}{a+bx}\right)}{dx^2+c}dx$$

$$\Big\downarrow \text{ 2976}$$

$$\frac{1}{2}i\int \frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)}{dx^2+c}dx-$$
$$\frac{1}{2}b\int \frac{\log\left(\frac{a+bx+i}{a+bx}\right)}{d(a+i)^2+\frac{(da^2+b^2c)(a+bx+i)^2}{(a+bx)^2}+b^2c-\frac{2(cb^2+a(a+i)d)(a+bx+i)}{a+bx}}d\frac{a+bx+i}{a+bx}$$

$$\Big\downarrow \text{ 2804}$$

$$\frac{1}{2}i\int \frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)}{dx^2+c}dx-$$
$$\frac{1}{2}b\int\left(\frac{(da^2+b^2c)\log\left(\frac{a+bx+i}{a+bx}\right)}{b\sqrt{c}\sqrt{d}\left(2da^2+2ida+2b^2c-\frac{2(da^2+b^2c)(a+bx+i)}{a+bx}-2b\sqrt{c}\sqrt{d}\right)}+\frac{(da^2+b^2c)\log\left(\frac{a+bx}{a+b}\right.}{b\sqrt{c}\sqrt{d}\left(-2da^2-2ida-2b^2c+\frac{2(da^2+b}{a}\right.}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}i\int \frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)}{dx^2+c}dx-$$
$$\frac{1}{2}b\left(\frac{\text{PolyLog}\left(2,\frac{\left(b\sqrt{c}-ia\sqrt{d}\right)(a+bx+i)}{\left(\sqrt{d}(1-ia)+b\sqrt{c}\right)(a+bx)}\right)}{2b\sqrt{c}\sqrt{d}}-\frac{\text{PolyLog}\left(2,\frac{\left(i\sqrt{d}a+b\sqrt{c}\right)(a+bx+i)}{\left(i\sqrt{d}(a+i)+b\sqrt{c}\right)(a+bx)}\right)}{2b\sqrt{c}\sqrt{d}}+\frac{\log\left(\frac{a+bx+i}{a+bx}\right)\log\left(1-\frac{(a+bx+}{(a+bx)(}\right.}{2b\sqrt{c}\sqrt{d}}\right.$$

$$\Big\downarrow \text{ 2977}$$

$$\frac{1}{2}b\int \frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)}{d(i-a)^2+b^2c+\frac{2(b^2c-(i-a)ad)(-a-bx+i)}{a+bx}+\frac{(da^2+b^2c)(-a-bx+i)^2}{(a+bx)^2}}d\frac{-a-bx+i}{a+bx}-$$
$$\frac{1}{2}b\left(\frac{\text{PolyLog}\left(2,\frac{\left(b\sqrt{c}-ia\sqrt{d}\right)(a+bx+i)}{\left(\sqrt{d}(1-ia)+b\sqrt{c}\right)(a+bx)}\right)}{2b\sqrt{c}\sqrt{d}}-\frac{\text{PolyLog}\left(2,\frac{\left(i\sqrt{d}a+b\sqrt{c}\right)(a+bx+i)}{\left(i\sqrt{d}(a+i)+b\sqrt{c}\right)(a+bx)}\right)}{2b\sqrt{c}\sqrt{d}}+\frac{\log\left(\frac{a+bx+i}{a+bx}\right)\log\left(1-\frac{(a+bx+}{(a+bx)(}\right.}{2b\sqrt{c}\sqrt{d}}\right.$$

$$\Big\downarrow \text{ 2804}$$

$$\frac{1}{2}b\int\left(\frac{\left(da^2+b^2c\right)\log\left(-\frac{-a-bx+i}{a+bx}\right)}{b\sqrt{c}\sqrt{d}\left(-2da^2+2ida-2b^2c-2b\sqrt{c}\sqrt{d}-\frac{2(da^2+b^2c)(-a-bx+i)}{a+bx}\right)}+\frac{\left(da^2+b^2c\right)\log\left(-\frac{-}{}\right.}{b\sqrt{c}\sqrt{d}\left(2da^2-2ida+2b^2c-2b\sqrt{c}\right.}\right.$$

$$\frac{1}{2}b\left(\frac{\text{PolyLog}\left(2,\frac{\left(b\sqrt{c}-ia\sqrt{d}\right)(a+bx+i)}{\left(\sqrt{d}(1-ia)+b\sqrt{c}\right)(a+bx)}\right)}{2b\sqrt{c}\sqrt{d}}-\frac{\text{PolyLog}\left(2,\frac{\left(i\sqrt{d}a+b\sqrt{c}\right)(a+bx+i)}{\left(i\sqrt{d}(a+i)+b\sqrt{c}\right)(a+bx)}\right)}{2b\sqrt{c}\sqrt{d}}+\frac{\log\left(\frac{a+bx+i}{a+bx}\right)\log\left(1-\frac{(a+bx+}{(a+bx)(}\right.}{2b\sqrt{c}\sqrt{d}}\right.$$

$\downarrow$ 2009

$$\frac{1}{2}b\left(\frac{\text{PolyLog}\left(2,-\frac{\left(b\sqrt{c}-ia\sqrt{d}\right)(-a-bx+i)}{\left(b\sqrt{c}-(ia+1)\sqrt{d}\right)(a+bx)}\right)}{2b\sqrt{c}\sqrt{d}}-\frac{\text{PolyLog}\left(2,-\frac{\left(i\sqrt{d}a+b\sqrt{c}\right)(-a-bx+i)}{\left(\sqrt{d}(ia+1)+b\sqrt{c}\right)(a+bx)}\right)}{2b\sqrt{c}\sqrt{d}}+\frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)\log\left(1+\right.}{2b\sqrt{c}}\right.$$

$$\frac{1}{2}b\left(\frac{\text{PolyLog}\left(2,\frac{\left(b\sqrt{c}-ia\sqrt{d}\right)(a+bx+i)}{\left(\sqrt{d}(1-ia)+b\sqrt{c}\right)(a+bx)}\right)}{2b\sqrt{c}\sqrt{d}}-\frac{\text{PolyLog}\left(2,\frac{\left(i\sqrt{d}a+b\sqrt{c}\right)(a+bx+i)}{\left(i\sqrt{d}(a+i)+b\sqrt{c}\right)(a+bx)}\right)}{2b\sqrt{c}\sqrt{d}}+\frac{\log\left(\frac{a+bx+i}{a+bx}\right)\log\left(1-\frac{(a+bx+}{(a+bx)(}\right.}{2b\sqrt{c}\sqrt{d}}\right.$$

input
```
Int[ArcCot[a + b*x]/(c + d*x^2),x]
```

output
```
(b*((Log[-((I - a - b*x)/(a + b*x))]*Log[1 + ((b*Sqrt[c] - I*a*Sqrt[d])*(I
 - a - b*x))/((b*Sqrt[c] - (1 + I*a)*Sqrt[d])*(a + b*x))]])/(2*b*Sqrt[c]*Sq
rt[d]) - (Log[-((I - a - b*x)/(a + b*x))]*Log[1 + ((b*Sqrt[c] + I*a*Sqrt[d
])*(I - a - b*x))/((b*Sqrt[c] + (1 + I*a)*Sqrt[d])*(a + b*x))]])/(2*b*Sqrt[
c]*Sqrt[d]) + PolyLog[2, -(((b*Sqrt[c] - I*a*Sqrt[d])*(I - a - b*x))/((b*S
qrt[c] - (1 + I*a)*Sqrt[d])*(a + b*x)))]/(2*b*Sqrt[c]*Sqrt[d]) - PolyLog[2
, -(((b*Sqrt[c] + I*a*Sqrt[d])*(I - a - b*x))/((b*Sqrt[c] + (1 + I*a)*Sqrt
[d])*(a + b*x)))]/(2*b*Sqrt[c]*Sqrt[d])))/2 - (b*((Log[(I + a + b*x)/(a +
b*x)]*Log[1 - ((b*Sqrt[c] - I*a*Sqrt[d])*(I + a + b*x))/((b*Sqrt[c] + (1 -
 I*a)*Sqrt[d])*(a + b*x))]])/(2*b*Sqrt[c]*Sqrt[d]) - (Log[(I + a + b*x)/(a
+ b*x)]*Log[1 - ((b*Sqrt[c] + I*a*Sqrt[d])*(I + a + b*x))/((b*Sqrt[c] + I*
(I + a)*Sqrt[d])*(a + b*x))]])/(2*b*Sqrt[c]*Sqrt[d]) + PolyLog[2, ((b*Sqrt[
c] - I*a*Sqrt[d])*(I + a + b*x))/((b*Sqrt[c] + (1 - I*a)*Sqrt[d])*(a + b*x
))]/(2*b*Sqrt[c]*Sqrt[d]) - PolyLog[2, ((b*Sqrt[c] + I*a*Sqrt[d])*(I + a +
 b*x))/((b*Sqrt[c] + I*(I + a)*Sqrt[d])*(a + b*x))]/(2*b*Sqrt[c]*Sqrt[d]))
)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2976
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Simp[(b*c - a*d)   Subst[Int[(b^2*f -
a*b*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^
2*h)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/
(c + d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] &
& NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

rule 2977
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g =
Coeff[P2x, x, 1], h = Coeff[P2x, x, 2]}, Simp[(b*c - a*d)   Subst[Int[(b^2*
f - a*b*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g
+ c^2*h)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b
*x)/(c + d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x,
2] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] && I
GtQ[p, 0]
```

rule 5575
```
Int[ArcCot[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[
I/2   Int[Log[(-I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] - Simp[I/2   In
t[Log[(I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x
] && RationalQ[n]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(253) = 506$.

Time = 2.09 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.85

method	result
risch	$-\frac{ib\pi \arctan\left(\frac{2iad+2(-bxi-ai+1)d-2d}{2\sqrt{-b^2cd}}\right)}{2\sqrt{-b^2cd}} - \frac{\ln(-bxi-ai+1)\ln\left(\frac{iad-b\sqrt{cd}+(-bxi-ai+1)d-d}{iad-b\sqrt{cd}-d}\right)\sqrt{cd}}{4cd} + \frac{\ln(-bxi-ai+1)}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(arccot(b*x+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*b*Pi/(-b^2*c*d)^(1/2)*arctan(1/2*(2*I*a*d+2*(1-I*a-I*b*x)*d-2*d)/(-
b^2*c*d)^(1/2))-1/4*ln(1-I*a-I*b*x)/c/d*ln((I*a*d-b*(c*d)^(1/2)+(1-I*a-I*b
*x)*d-d)/(I*a*d-b*(c*d)^(1/2)-d))*(c*d)^(1/2)+1/4*ln(1-I*a-I*b*x)/c/d*ln((
I*a*d+b*(c*d)^(1/2)+(1-I*a-I*b*x)*d-d)/(I*a*d+b*(c*d)^(1/2)-d))*(c*d)^(1/2
)-1/4/c/d*dilog((I*a*d-b*(c*d)^(1/2)+(1-I*a-I*b*x)*d-d)/(I*a*d-b*(c*d)^(1/
2)-d))*(c*d)^(1/2)+1/4/c/d*dilog((I*a*d+b*(c*d)^(1/2)+(1-I*a-I*b*x)*d-d)/(
I*a*d+b*(c*d)^(1/2)-d))*(c*d)^(1/2)-1/4*ln(1+I*a+I*b*x)/c/d*ln((I*a*d+b*(c
*d)^(1/2)-(1+I*a+I*b*x)*d+d)/(I*a*d+b*(c*d)^(1/2)+d))*(c*d)^(1/2)+1/4*ln(1
+I*a+I*b*x)/c/d*ln((I*a*d-b*(c*d)^(1/2)-(1+I*a+I*b*x)*d+d)/(I*a*d-b*(c*d)^
(1/2)+d))*(c*d)^(1/2)-1/4/c/d*dilog((I*a*d+b*(c*d)^(1/2)-(1+I*a+I*b*x)*d+d
)/(I*a*d+b*(c*d)^(1/2)+d))*(c*d)^(1/2)+1/4/c/d*dilog((I*a*d-b*(c*d)^(1/2)-
(1+I*a+I*b*x)*d+d)/(I*a*d-b*(c*d)^(1/2)+d))*(c*d)^(1/2)
```

Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^2}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{dx^2+c}\,dx$$

input
```
integrate(arccot(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
integral(arccot(b*x + a)/(d*x^2 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^2}\,dx = \text{Timed out}$$

input
```
integrate(acot(b*x+a)/(d*x**2+c),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8519 vs. $2(235) = 470$.

Time = 4.64 (sec) , antiderivative size = 8519, normalized size of antiderivative = 26.71

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^2}\,dx = \text{Too large to display}$$

input
```
integrate(arccot(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
-1/8*b*(8*arctan(d*x/sqrt(c*d))*arctan((b^2*x + a*b)/b)/b - (4*arctan(sqrt
(d)*x/sqrt(c))*arctan2((2*a*b^2*c*d + (a*b^3*c + (a^3 + a)*b*d + (b^4*c +
(a^2 + 3)*b^2*d)*x)*sqrt(c)*sqrt(d) + (3*b^3*c*d + (a^2 + 1)*b*d^2)*x)/(b^
4*c^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 + 4*(b^3*c + (a^2 + 1)
*b*d)*sqrt(c)*sqrt(d)), ((a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 + (2*a*
b^2*d*x + b^3*c + 3*(a^2 + 1)*b*d)*sqrt(c)*sqrt(d) + (a*b^3*c*d + (a^3 + a
)*b*d^2)*x)/(b^4*c^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 + 4*(b^
3*c + (a^2 + 1)*b*d)*sqrt(c)*sqrt(d))) + 4*arctan(sqrt(d)*x/sqrt(c))*arcta
n2((2*a*b^2*c*d - (a*b^3*c + (a^3 + a)*b*d + (b^4*c + (a^2 + 3)*b^2*d)*x)*
sqrt(c)*sqrt(d) + (3*b^3*c*d + (a^2 + 1)*b*d^2)*x)/(b^4*c^2 + 2*(a^2 + 3)*
b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 - 4*(b^3*c + (a^2 + 1)*b*d)*sqrt(c)*sqrt(d
)), ((a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 - (2*a*b^2*d*x + b^3*c + 3*
(a^2 + 1)*b*d)*sqrt(c)*sqrt(d) + (a*b^3*c*d + (a^3 + a)*b*d^2)*x)/(b^4*c^2
 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 - 4*(b^3*c + (a^2 + 1)*b*d)
*sqrt(c)*sqrt(d))) + log(d*x^2 + c)*log(((a^2 + 1)*b^22*c^11*d + 11*(a^4 +
 22*a^2 + 21)*b^20*c^10*d^2 + 55*(a^6 + 39*a^4 + 171*a^2 + 133)*b^18*c^9*d
^3 + 33*(5*a^8 + 260*a^6 + 1870*a^4 + 3876*a^2 + 2261)*b^16*c^8*d^4 + 330*
(a^10 + 61*a^8 + 570*a^6 + 1802*a^4 + 2261*a^2 + 969)*b^14*c^7*d^5 + 22*(2
1*a^12 + 1386*a^10 + 15015*a^8 + 60060*a^6 + 109395*a^4 + 92378*a^2 + 2939
3)*b^12*c^6*d^6 + 22*(21*a^14 + 1407*a^12 + 16401*a^10 + 75075*a^8 + 16...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^2}\,dx = \text{Timed out}$$

input
```
integrate(arccot(b*x+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^2}\,dx = \int \frac{\mathrm{acot}(a+b\,x)}{d\,x^2+c}\,dx$$

input
```
int(acot(a + b*x)/(c + d*x^2),x)
```

output
```
int(acot(a + b*x)/(c + d*x^2), x)
```

Reduce [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^2}\,dx = \int \frac{acot(bx+a)}{d\,x^2+c}\,dx$$

input
```
int(acot(b*x+a)/(d*x^2+c),x)
```

output
```
int(acot(a + b*x)/(c + d*x**2),x)
```

3.38 $\int \frac{\cot^{-1}(a+bx)}{c+dx}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 152

$$\int \frac{\cot^{-1}(a+bx)}{c+dx}\,dx = -\frac{\cot^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d} + \frac{\cot^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+(i-a)d)(1-i(a+bx))}\right)}{d} - \frac{i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)}{2d} + \frac{i\,\text{PolyLog}\left(2,1-\frac{2b(c+dx)}{(bc+(i-a)d)(1-i(a+bx))}\right)}{2d}$$

output
```
-arccot(b*x+a)*ln(2/(1-I*(b*x+a)))/d+arccot(b*x+a)*ln(2*b*(d*x+c)/(b*c+(I-
a)*d)/(1-I*(b*x+a)))/d-1/2*I*polylog(2,1-2/(1-I*(b*x+a)))/d+1/2*I*polylog(
2,1-2*b*(d*x+c)/(b*c+(I-a)*d)/(1-I*(b*x+a)))/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 325 vs. 2(152) = 304.

Time = 0.18 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.14

$$\int \frac{\cot^{-1}(a+bx)}{c+dx}\,dx$$
$$= \frac{(\cot^{-1}(a+bx)+\arctan(a+bx))\log(c+dx)+\arctan(a+bx)\left(\log\left(\frac{1}{\sqrt{1+(a+bx)^2}}\right)-\log\left(\sin\left(\arctan\left(\frac{bc}{}\right.\right.\right.\right.}{}$$

input
```
Integrate[ArcCot[a + b*x]/(c + d*x),x]
```

output
```
((ArcCot[a + b*x] + ArcTan[a + b*x])*Log[c + d*x] + ArcTan[a + b*x]*(Log[1
/Sqrt[1 + (a + b*x)^2]] - Log[Sin[ArcTan[(b*c - a*d)/d] + ArcTan[a + b*x]]
]) + ((I/4)*(Pi - 2*ArcTan[a + b*x])^2 + I*(ArcTan[(b*c - a*d)/d] + ArcTan
[a + b*x])^2 - (Pi - 2*ArcTan[a + b*x])*Log[1 + E^((-2*I)*ArcTan[a + b*x])
] - 2*(ArcTan[(b*c - a*d)/d] + ArcTan[a + b*x])*Log[1 - E^((2*I)*(ArcTan[(
b*c - a*d)/d] + ArcTan[a + b*x]))] + (Pi - 2*ArcTan[a + b*x])*Log[2/Sqrt[1
 + (a + b*x)^2]] + 2*(ArcTan[(b*c - a*d)/d] + ArcTan[a + b*x])*Log[2*Sin[A
rcTan[(b*c - a*d)/d] + ArcTan[a + b*x]]] + I*PolyLog[2, -E^((-2*I)*ArcTan[
a + b*x])] + I*PolyLog[2, E^((2*I)*(ArcTan[(b*c - a*d)/d] + ArcTan[a + b*x
]))])/2)/d
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5571, 27, 5382, 2849, 2752, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{c+dx}\,dx$$
$$\Big\downarrow \; 5571$$

$$\frac{\int \frac{b\cot^{-1}(a+bx)}{b\left(c-\frac{ad}{b}\right)+d(a+bx)}d(a+bx)}{b}$$

$\Big\downarrow$ 27

$$\int \frac{\cot^{-1}(a+bx)}{d(a+bx)-ad+bc}d(a+bx)$$

$\Big\downarrow$ 5382

$$\frac{\int \frac{\log\left(\frac{2(bc-ad+d(a+bx))}{(bc-ad+id)(1-i(a+bx))}\right)}{(a+bx)^2+1}d(a+bx)}{d}-\frac{\int \frac{\log\left(\frac{2}{1-i(a+bx)}\right)}{(a+bx)^2+1}d(a+bx)}{d}+\frac{\cot^{-1}(a+bx)\log\left(\frac{2(d(a+bx)-ad+bc)}{(1-i(a+bx))(-ad+bc+id)}\right)}{d}-\frac{\log\left(\frac{2}{1-i(a+bx)}\right)\cot^{-1}(a+bx)}{d}$$

$\Big\downarrow$ 2849

$$\frac{\int \frac{\log\left(\frac{2(bc-ad+d(a+bx))}{(bc-ad+id)(1-i(a+bx))}\right)}{(a+bx)^2+1}d(a+bx)}{d}-\frac{i\int \frac{\log\left(\frac{2}{1-i(a+bx)}\right)}{1-\frac{2}{1-i(a+bx)}}d\frac{1}{1-i(a+bx)}}{d}+\frac{\cot^{-1}(a+bx)\log\left(\frac{2(d(a+bx)-ad+bc)}{(1-i(a+bx))(-ad+bc+id)}\right)}{d}-\frac{\log\left(\frac{2}{1-i(a+bx)}\right)\cot^{-1}(a+bx)}{d}$$

$\Big\downarrow$ 2752

$$\frac{\int \frac{\log\left(\frac{2(bc-ad+d(a+bx))}{(bc-ad+id)(1-i(a+bx))}\right)}{(a+bx)^2+1}d(a+bx)}{d}+\frac{\cot^{-1}(a+bx)\log\left(\frac{2(d(a+bx)-ad+bc)}{(1-i(a+bx))(-ad+bc+id)}\right)}{d}-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)}{2d}-\frac{\log\left(\frac{2}{1-i(a+bx)}\right)\cot^{-1}(a+bx)}{d}$$

$\Big\downarrow$ 2897

$$\frac{i\,\text{PolyLog}\left(2,1-\frac{2(bc-ad+d(a+bx))}{(bc-ad+id)(1-i(a+bx))}\right)}{2d}+\frac{\cot^{-1}(a+bx)\log\left(\frac{2(d(a+bx)-ad+bc)}{(1-i(a+bx))(-ad+bc+id)}\right)}{d}-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)}{2d}-\frac{\log\left(\frac{2}{1-i(a+bx)}\right)\cot^{-1}(a+bx)}{d}$$

input
```
Int[ArcCot[a + b*x]/(c + d*x),x]
```

output
```
-((ArcCot[a + b*x]*Log[2/(1 - I*(a + b*x))])/d) + (ArcCot[a + b*x]*Log[(2*
(b*c - a*d + d*(a + b*x)))/((b*c + I*d - a*d)*(1 - I*(a + b*x)))])/d - ((I
/2)*PolyLog[2, 1 - 2/(1 - I*(a + b*x))])/d + ((I/2)*PolyLog[2, 1 - (2*(b*c
 - a*d + d*(a + b*x)))/((b*c + I*d - a*d)*(1 - I*(a + b*x)))])/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5382
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcCot[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*ArcCot[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] - Simp[b*(c/e)
   Int[Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] + Simp[b*(c/e)   Int[Log[2*
c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x]) /; FreeQ[{a
, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\dfrac{\frac{b\ln(ad-bc-d(bx+a))\operatorname{arccot}(bx+a)}{d}-b\Bigg(-\frac{i\ln(ad-bc-d(bx+a))\left(\ln\left(\frac{id+d(bx+a)}{ad-bc+id}\right)-\ln\left(\frac{id-d(bx+a)}{-ad+bc+id}\right)\right)}{2d}-\frac{i\Big(\operatorname{dilog}\Big(\frac{id+d(bx+a)}{ad-bc+id}}{}}{b}$
default	$\dfrac{\frac{b\ln(ad-bc-d(bx+a))\operatorname{arccot}(bx+a)}{d}-b\Bigg(-\frac{i\ln(ad-bc-d(bx+a))\left(\ln\left(\frac{id+d(bx+a)}{ad-bc+id}\right)-\ln\left(\frac{id-d(bx+a)}{-ad+bc+id}\right)\right)}{2d}-\frac{i\Big(\operatorname{dilog}\Big(\frac{id+d(bx+a)}{ad-bc+id}}{}}{b}$
parts	$\frac{\ln(dx+c)\operatorname{arccot}(bx+a)}{d}+b\Bigg(-\frac{i\ln(dx+c)\left(\ln\left(\frac{id-ad+bc-b(dx+c)}{-ad+bc+id}\right)-\ln\left(\frac{id+ad-bc+b(dx+c)}{ad-bc+id}\right)\right)}{2bd}-\frac{i\Big(\operatorname{dilog}\Big(\frac{id-ad+b}{-ad}}{}$
risch	$-\frac{i\operatorname{dilog}\left(\frac{iad-ibc+(-bxi-ai+1)d-d}{iad-ibc-d}\right)}{2d}-\frac{i\ln(-bxi-ai+1)\ln\left(\frac{iad-ibc+(-bxi-ai+1)d-d}{iad-ibc-d}\right)}{2d}+\frac{\pi\ln(iad-ibc+(-bxi-a}{2d}$

input
```
int(arccot(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/b*(b*ln(a*d-b*c-d*(b*x+a))/d*arccot(b*x+a)-b*(-1/2*I*ln(a*d-b*c-d*(b*x+a
))*(ln((I*d+d*(b*x+a))/(a*d-b*c+I*d))-ln((I*d-d*(b*x+a))/(I*d-a*d+b*c)))/d
-1/2*I*(dilog((I*d+d*(b*x+a))/(a*d-b*c+I*d))-dilog((I*d-d*(b*x+a))/(I*d-a*
d+b*c)))/d))
```

Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{arccot}(bx+a)}{dx+c}\,dx$$

input
```
integrate(arccot(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(arccot(b*x + a)/(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+dx}\,dx = \text{Timed out}$$

input
```
integrate(acot(b*x+a)/(d*x+c),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(128) = 256$.

Time = 0.21 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.86

$$\int \frac{\cot^{-1}(a+bx)}{c+dx}\,dx = \frac{\operatorname{arccot}(bx+a)\log(dx+c)}{d} + \frac{\arctan\left(\frac{b^2x+ab}{b}\right)\log(dx+c)}{d} + \frac{\arctan\left(\frac{bd^2x+bcd}{b^2c^2-2\,abcd+(a^2+1)d^2}, \frac{b^2c^2-abcd+(b^2cd-abd^2)x}{b^2c^2-2\,abcd+(a^2+1)d^2}\right)\log(b^2x^2+2\,abx+a^2+1) - \arctan(bx+a)\log\left(\frac{b^2}{b^2}\right.}{2\,d}$$

input
```
integrate(arccot(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
arccot(b*x + a)*log(d*x + c)/d + arctan((b^2*x + a*b)/b)*log(d*x + c)/d +
1/2*(arctan2((b*d^2*x + b*c*d)/(b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2), (b^2
*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x)/(b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d
^2))*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - arctan(b*x + a)*log((b^2*d^2*x^2 +
 2*b^2*c*d*x + b^2*c^2)/(b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)) + I*dilog((
I*b*d*x + (I*a + 1)*d)/(-I*b*c + (I*a + 1)*d)) - I*dilog((I*b*d*x + (I*a -
 1)*d)/(-I*b*c + (I*a - 1)*d)))/d
```

Giac [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{dx+c}\,dx$$

input
```
integrate(arccot(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(arccot(b*x + a)/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{acot}(a+b\,x)}{c+d\,x}\,dx$$

input
```
int(acot(a + b*x)/(c + d*x),x)
```

output
```
int(acot(a + b*x)/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+dx}\,dx = \int \frac{acot(bx+a)}{dx+c}dx$$

input
```
int(acot(b*x+a)/(d*x+c),x)
```

output
```
int(acot(a + b*x)/(c + d*x),x)
```

3.39 $\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 338

$$\begin{aligned}\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx =& \frac{\log(i-a-bx)}{2bc}+\frac{i(a+bx)\log\left(-\frac{i-a-bx}{a+bx}\right)}{2bc}\\&+\frac{\log(i+a+bx)}{2bc}-\frac{i(a+bx)\log\left(\frac{i+a+bx}{a+bx}\right)}{2bc}\\&+\frac{id\log\left(\frac{c(i-a-bx)}{ic-ac+bd}\right)\log(d+cx)}{2c^2}-\frac{id\log\left(-\frac{i-a-bx}{a+bx}\right)\log(d+cx)}{2c^2}\\&-\frac{id\log\left(\frac{c(i+a+bx)}{(i+a)c-bd}\right)\log(d+cx)}{2c^2}+\frac{id\log\left(\frac{i+a+bx}{a+bx}\right)\log(d+cx)}{2c^2}\\&-\frac{id\,\text{PolyLog}\left(2,-\frac{b(d+cx)}{(i+a)c-bd}\right)}{2c^2}+\frac{id\,\text{PolyLog}\left(2,\frac{b(d+cx)}{ic-ac+bd}\right)}{2c^2}\end{aligned}$$

output
```
1/2*ln(I-a-b*x)/b/c+1/2*I*(b*x+a)*ln(-(I-a-b*x)/(b*x+a))/b/c+1/2*ln(I+a+b*
x)/b/c-1/2*I*(b*x+a)*ln((I+a+b*x)/(b*x+a))/b/c+1/2*I*d*ln(c*(I-a-b*x)/(I*c
-a*c+b*d))*ln(c*x+d)/c^2-1/2*I*d*ln(-(I-a-b*x)/(b*x+a))*ln(c*x+d)/c^2-1/2*
I*d*ln(c*(I+a+b*x)/((I+a)*c-b*d))*ln(c*x+d)/c^2+1/2*I*d*ln((I+a+b*x)/(b*x+
a))*ln(c*x+d)/c^2-1/2*I*d*polylog(2,-b*(c*x+d)/((I+a)*c-b*d))/c^2+1/2*I*d*
polylog(2,b*(c*x+d)/(I*c-a*c+b*d))/c^2
```

Mathematica [A] (warning: unable to verify)

Time = 7.43 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.51

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx$$
$$= \frac{2ac^2\cot^{-1}(a+bx) - ibcd\pi\cot^{-1}(a+bx) + 2bc^2x\cot^{-1}(a+bx) - ibcd\cot^{-1}(a+bx)^2 + abcd\cot^{-1}(a+}{}$$

input
```
Integrate[ArcCot[a + b*x]/(c + d/x),x]
```

output
```
(2*a*c^2*ArcCot[a + b*x] - I*b*c*d*Pi*ArcCot[a + b*x] + 2*b*c^2*x*ArcCot[a
 + b*x] - I*b*c*d*ArcCot[a + b*x]^2 + a*b*c*d*ArcCot[a + b*x]^2 - b^2*d^2*
ArcCot[a + b*x]^2 - a*b*c*d*Sqrt[1 + c^2/(a*c - b*d)^2]*E^(I*ArcTan[c/(-(a
*c) + b*d)])*ArcCot[a + b*x]^2 + b^2*d^2*Sqrt[1 + c^2/(a*c - b*d)^2]*E^(I*
ArcTan[c/(-(a*c) + b*d)])*ArcCot[a + b*x]^2 + (2*I)*b*c*d*ArcCot[a + b*x]*
ArcTan[c/(-(a*c) + b*d)] - b*c*d*Pi*Log[1 + E^((-2*I)*ArcCot[a + b*x])] +
2*b*c*d*ArcCot[a + b*x]*Log[1 - E^((2*I)*ArcCot[a + b*x])] - 2*b*c*d*ArcCo
t[a + b*x]*Log[1 - E^((2*I)*(ArcCot[a + b*x] + ArcTan[c/(-(a*c) + b*d)]))]
 - 2*b*c*d*ArcTan[c/(-(a*c) + b*d)]*Log[1 - E^((2*I)*(ArcCot[a + b*x] + Ar
cTan[c/(-(a*c) + b*d)]))] + b*c*d*Pi*Log[1/Sqrt[1 + (a + b*x)^(-2)]] - 2*c
^2*Log[1/((a + b*x)*Sqrt[1 + (a + b*x)^(-2)])] + 2*b*c*d*ArcTan[c/(-(a*c)
+ b*d)]*Log[Sin[ArcCot[a + b*x] + ArcTan[c/(-(a*c) + b*d)]]] - I*b*c*d*Pol
yLog[2, E^((2*I)*ArcCot[a + b*x])] + I*b*c*d*PolyLog[2, E^((2*I)*(ArcCot[a
 + b*x] + ArcTan[c/(-(a*c) + b*d)]))])/(2*b*c^3)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.42, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5575, 2993, 772, 49, 2009, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx$$

$$\Big\downarrow \text{5575}$$

$$\frac{1}{2}i\int \frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)}{c+\frac{d}{x}}dx-\frac{1}{2}i\int \frac{\log\left(\frac{a+bx+i}{a+bx}\right)}{c+\frac{d}{x}}dx$$

$$\Big\downarrow \text{2993}$$

$$\frac{1}{2}i\left(-\left(\left(\log(-a-bx+i)-\log\left(-\frac{-a-bx+i}{a+bx}\right)-\log(a+bx)\right)\int \frac{1}{c+\frac{d}{x}}dx\right)+\int \frac{\log(-a-bx+i)}{c+\frac{d}{x}}dx-\int \frac{1}{}\right.$$
$$\frac{1}{2}i\left(\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)\int \frac{1}{c+\frac{d}{x}}dx-\int \frac{\log(a+bx)}{c+\frac{d}{x}}dx+\int \frac{\log(a+bx+i)}{c+\frac{d}{x}}d\right.$$

$$\Big\downarrow \text{772}$$

$$\frac{1}{2}i\left(-\left(\left(\log(-a-bx+i)-\log\left(-\frac{-a-bx+i}{a+bx}\right)-\log(a+bx)\right)\int \frac{x}{d+cx}dx\right)+\int \frac{\log(-a-bx+i)}{c+\frac{d}{x}}dx-\int\right.$$
$$\frac{1}{2}i\left(\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)\int \frac{x}{d+cx}dx-\int \frac{\log(a+bx)}{c+\frac{d}{x}}dx+\int \frac{\log(a+bx+i)}{c+\frac{d}{x}}\right.$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}i\left(-\left(\left(\log(-a-bx+i)-\log\left(-\frac{-a-bx+i}{a+bx}\right)-\log(a+bx)\right)\int \left(\frac{1}{c}-\frac{d}{c(d+cx)}\right)dx\right)+\int \frac{\log(-a-bx+}{c+\frac{d}{x}}\right.$$
$$\frac{1}{2}i\left(\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)\int \left(\frac{1}{c}-\frac{d}{c(d+cx)}\right)dx-\int \frac{\log(a+bx)}{c+\frac{d}{x}}dx+\int \frac{\log(}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}i\left(\int \frac{\log(-a-bx+i)}{c+\frac{d}{x}}dx-\int \frac{\log(a+bx)}{c+\frac{d}{x}}dx-\left(\left(\log(-a-bx+i)-\log\left(-\frac{-a-bx+i}{a+bx}\right)-\log(a+bx)\right)\right.\right.$$
$$\frac{1}{2}i\left(-\int \frac{\log(a+bx)}{c+\frac{d}{x}}dx+\int \frac{\log(a+bx+i)}{c+\frac{d}{x}}dx+\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)\left(\frac{x}{c}-\frac{d}{}\right.\right.$$

$$\Big\downarrow \text{2856}$$

$$\frac{1}{2}i\left(\int\left(\frac{\log(-a-bx+i)}{c}-\frac{d\log(-a-bx+i)}{c(d+cx)}\right)dx-\int\left(\frac{\log(a+bx)}{c}-\frac{d\log(a+bx)}{c(d+cx)}\right)dx-\left(\left(\log(-a-bx\right.\right.\right.$$
$$\frac{1}{2}i\left(-\int\left(\frac{\log(a+bx)}{c}-\frac{d\log(a+bx)}{c(d+cx)}\right)dx+\int\left(\frac{\log(a+bx+i)}{c}-\frac{d\log(a+bx+i)}{c(d+cx)}\right)dx+\left(\log(a+bx)-\mathrm{lo}\right.\right.$$

$\downarrow$ 2009

$$\frac{1}{2}i\left(-\frac{d\,\mathrm{PolyLog}\left(2,\frac{c(-a-bx+i)}{-ac+ic+bd}\right)}{c^2}+\frac{d\,\mathrm{PolyLog}\left(2,\frac{c(a+bx)}{ac-bd}\right)}{c^2}-\frac{d\log(-a-bx+i)\log\left(\frac{b(cx+d)}{-ac+bd+ic}\right)}{c^2}-\left(\log(-a-b\right.\right.$$
$$\frac{1}{2}i\left(\frac{d\,\mathrm{PolyLog}\left(2,\frac{c(a+bx)}{ac-bd}\right)}{c^2}-\frac{d\,\mathrm{PolyLog}\left(2,\frac{c(a+bx+i)}{(a+i)c-bd}\right)}{c^2}+\frac{d\log(a+bx)\log\left(-\frac{b(cx+d)}{ac-bd}\right)}{c^2}+\left(\log(a+bx)-\log(a\right.\right.$$

input
```
Int[ArcCot[a + b*x]/(c + d/x),x]
```

output
```
(I/2)*(-(((I - a - b*x)*Log[I - a - b*x])/(b*c)) - ((a + b*x)*Log[a + b*x]
)/(b*c) - (Log[I - a - b*x] - Log[-((I - a - b*x)/(a + b*x))] - Log[a + b*
x])*(x/c - (d*Log[d + c*x])/c^2) + (d*Log[a + b*x]*Log[-((b*(d + c*x))/(a*
c - b*d))])/c^2 - (d*Log[I - a - b*x]*Log[(b*(d + c*x))/(I*c - a*c + b*d)]
)/c^2 - (d*PolyLog[2, (c*(I - a - b*x))/(I*c - a*c + b*d)])/c^2 + (d*PolyL
og[2, (c*(a + b*x))/(a*c - b*d)])/c^2) - (I/2)*(-(((a + b*x)*Log[a + b*x])
/(b*c)) + ((I + a + b*x)*Log[I + a + b*x])/(b*c) + (Log[a + b*x] - Log[I +
 a + b*x] + Log[(I + a + b*x)/(a + b*x)])*(x/c - (d*Log[d + c*x])/c^2) + (
d*Log[a + b*x]*Log[-((b*(d + c*x))/(a*c - b*d))])/c^2 - (d*Log[I + a + b*x
]*Log[-((b*(d + c*x))/((I + a)*c - b*d))])/c^2 + (d*PolyLog[2, (c*(a + b*x
))/(a*c - b*d)])/c^2 - (d*PolyLog[2, (c*(I + a + b*x))/((I + a)*c - b*d)])
/c^2)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 2993
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Simp[p*r   Int[RFx*Log[a + b*x], x], x] + (Si
mp[q*r   Int[RFx*Log[c + d*x], x], x] - Simp[(p*r*Log[a + b*x] + q*r*Log[c
+ d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])   Int[RFx, x], x]) /; FreeQ[
{a, b, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a
*d, 0] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[
m, n]]
```

rule 5575
```
Int[ArcCot[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[
I/2   Int[Log[(-I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] - Simp[I/2   In
t[Log[(I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x
] && RationalQ[n]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{\frac{\operatorname{arccot}(bx+a)(bx+a)}{c}-\frac{\operatorname{arccot}(bx+a)db\ln(ac-bd-c(bx+a))}{c^2}-\frac{-\frac{\ln\left(a^2c^2-2abcd+b^2d^2-2ac(ac-bd-c(bx+a))+2bd(ac-bd-c(bx+a\right.}{2}}{}}{}$
default	$\frac{\frac{\operatorname{arccot}(bx+a)(bx+a)}{c}-\frac{\operatorname{arccot}(bx+a)db\ln(ac-bd-c(bx+a))}{c^2}-\frac{-\frac{\ln\left(a^2c^2-2abcd+b^2d^2-2ac(ac-bd-c(bx+a))+2bd(ac-bd-c(bx+a\right.}{2}}{}}{}$
parts	$\frac{\operatorname{arccot}(bx+a)x}{c}-\frac{\operatorname{arccot}(bx+a)d\ln(cx+d)}{c^2}+\frac{b\left(\frac{\ln\left(a^2c^2-2abcd+2abc(cx+d)+b^2d^2-2b^2d(cx+d)+b^2(cx+d)^2+c^2\right)}{2b^2}-\frac{a\,\mathrm{ar}}{}\right.}{}$
risch	$\frac{i\pi}{2bc}+\frac{id\operatorname{dilog}\left(\frac{iac-ibd+(-bxi-ai+1)c-c}{iac-ibd-c}\right)}{2c^2}+\frac{id\ln(-bxi-ai+1)\ln\left(\frac{iac-ibd+(-bxi-ai+1)c-c}{iac-ibd-c}\right)}{2c^2}+\frac{i\ln(bxi+ai+1)a}{2bc}$

input
```
int(arccot(b*x+a)/(c+d/x),x,method=_RETURNVERBOSE)
```

output
```
1/b*(arccot(b*x+a)/c*(b*x+a)-arccot(b*x+a)*d*b/c^2*ln(a*c-b*d-c*(b*x+a))-1
/c*(-1/2*ln(a^2*c^2-2*a*b*c*d+b^2*d^2-2*a*c*(a*c-b*d-c*(b*x+a))+2*b*d*(a*c
-b*d-c*(b*x+a))+c^2+(a*c-b*d-c*(b*x+a))^2)-b*d*(-1/2*I*ln(a*c-b*d-c*(b*x+a
))*(ln((I*c+c*(b*x+a))/(a*c-b*d+I*c))-ln((I*c-c*(b*x+a))/(I*c-a*c+b*d)))/c
-1/2*I*(dilog((I*c+c*(b*x+a))/(a*c-b*d+I*c))-dilog((I*c-c*(b*x+a))/(I*c-a*
c+b*d)))/c)))
```

Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx=\int \frac{\operatorname{arccot}(bx+a)}{c+\frac{d}{x}}\,dx$$

input
```
integrate(arccot(b*x+a)/(c+d/x),x, algorithm="fricas")
```

output
```
integral(x*arccot(b*x + a)/(c*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx = \text{Timed out}$$

input
```
integrate(acot(b*x+a)/(c+d/x),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.83

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx = \frac{2\,bcx\arctan\left(1,bx+a\right)-bd\arctan\left(1,bx+a\right)\log\left(-\frac{b^2c^2x^2+2\,b^2cdx+b^2d^2}{2\,abcd-b^2d^2-(a^2+1)c^2}\right)-2\,ac\arctan\left(bx+a\right)+i\,bd\mathrm{Li}_2$$

input
```
integrate(arccot(b*x+a)/(c+d/x),x, algorithm="maxima")
```

output
```
1/2*(2*b*c*x*arctan2(1, b*x + a) - b*d*arctan2(1, b*x + a)*log(-(b^2*c^2*x
^2 + 2*b^2*c*d*x + b^2*d^2)/(2*a*b*c*d - b^2*d^2 - (a^2 + 1)*c^2)) - 2*a*c
*arctan(b*x + a) + I*b*d*dilog((b*c*x + (a + I)*c)/((a + I)*c - b*d)) - I*
b*d*dilog((b*c*x + (a - I)*c)/((a - I)*c - b*d)) - (b*d*arctan2(-(b*c^2*x
+ b*c*d)/(2*a*b*c*d - b^2*d^2 - (a^2 + 1)*c^2), (a*b*c*d - b^2*d^2 + (a*b*
c^2 - b^2*c*d)*x)/(2*a*b*c*d - b^2*d^2 - (a^2 + 1)*c^2)) - c)*log(b^2*x^2
+ 2*a*b*x + a^2 + 1))/(b*c^2)
```

Giac [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{c+\frac{d}{x}}\,dx$$

input
```
integrate(arccot(b*x+a)/(c+d/x),x, algorithm="giac")
```

output
```
integrate(arccot(b*x + a)/(c + d/x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx = \int \frac{\operatorname{acot}(a+b\,x)}{c+\frac{d}{x}}\,dx$$

input
```
int(acot(a + b*x)/(c + d/x),x)
```

output
```
int(acot(a + b*x)/(c + d/x), x)
```

Reduce [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx = \int \frac{acot(bx+a)\,x}{cx+d}dx$$

input
```
int(acot(b*x+a)/(c+d/x),x)
```

output
```
int((acot(a + b*x)*x)/(c*x + d),x)
```

3.40 $\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 735

$$
\begin{aligned}
\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx =\ & \frac{\log(i-a-bx)}{2bc}+\frac{i(a+bx)\log\left(-\frac{i-a-bx}{a+bx}\right)}{2bc} \\
& -\frac{i\sqrt{d}\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{2c^{3/2}}+\frac{\log(i+a+bx)}{2bc} \\
& -\frac{i(a+bx)\log\left(\frac{i+a+bx}{a+bx}\right)}{2bc}+\frac{i\sqrt{d}\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\log\left(\frac{i+a+bx}{a+bx}\right)}{2c^{3/2}} \\
& -\frac{\sqrt{d}\log\left(\frac{\sqrt{c}(i-a-bx)}{(i-a)\sqrt{c}+ib\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{c}x}{\sqrt{d}}\right)}{4c^{3/2}} \\
& +\frac{\sqrt{d}\log\left(\frac{\sqrt{c}(i+a+bx)}{(i+a)\sqrt{c}-ib\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{c}x}{\sqrt{d}}\right)}{4c^{3/2}} \\
& +\frac{\sqrt{d}\log\left(\frac{\sqrt{c}(i-a-bx)}{(i-a)\sqrt{c}-ib\sqrt{d}}\right)\log\left(1+\frac{i\sqrt{c}x}{\sqrt{d}}\right)}{4c^{3/2}} \\
& -\frac{\sqrt{d}\log\left(\frac{\sqrt{c}(i+a+bx)}{(i+a)\sqrt{c}+ib\sqrt{d}}\right)\log\left(1+\frac{i\sqrt{c}x}{\sqrt{d}}\right)}{4c^{3/2}} \\
& -\frac{\sqrt{d}\,\mathrm{PolyLog}\left(2,\frac{b\left(\sqrt{d}-i\sqrt{c}x\right)}{(1+ia)\sqrt{c}+b\sqrt{d}}\right)}{4c^{3/2}} \\
& +\frac{\sqrt{d}\,\mathrm{PolyLog}\left(2,\frac{b\left(\sqrt{d}-i\sqrt{c}x\right)}{i(i+a)\sqrt{c}+b\sqrt{d}}\right)}{4c^{3/2}} \\
& +\frac{\sqrt{d}\,\mathrm{PolyLog}\left(2,-\frac{b\left(\sqrt{d}+i\sqrt{c}x\right)}{(1+ia)\sqrt{c}-b\sqrt{d}}\right)}{4c^{3/2}} \\
& -\frac{\sqrt{d}\,\mathrm{PolyLog}\left(2,\frac{b\left(\sqrt{d}+i\sqrt{c}x\right)}{(1-ia)\sqrt{c}+b\sqrt{d}}\right)}{4c^{3/2}}
\end{aligned}
$$

output
```
1/2*ln(I-a-b*x)/b/c+1/2*I*(b*x+a)*ln(-(I-a-b*x)/(b*x+a))/b/c-1/2*I*d^(1/2)
*arctan(c^(1/2)*x/d^(1/2))*ln(-(I-a-b*x)/(b*x+a))/c^(3/2)+1/2*ln(I+a+b*x)/
b/c-1/2*I*(b*x+a)*ln((I+a+b*x)/(b*x+a))/b/c+1/2*I*d^(1/2)*arctan(c^(1/2)*x
/d^(1/2))*ln((I+a+b*x)/(b*x+a))/c^(3/2)-1/4*d^(1/2)*ln(c^(1/2)*(I-a-b*x)/(
(I-a)*c^(1/2)+I*b*d^(1/2)))*ln(1-I*c^(1/2)*x/d^(1/2))/c^(3/2)+1/4*d^(1/2)*
ln(c^(1/2)*(I+a+b*x)/((I+a)*c^(1/2)-I*b*d^(1/2)))*ln(1-I*c^(1/2)*x/d^(1/2)
)/c^(3/2)+1/4*d^(1/2)*ln(c^(1/2)*(I-a-b*x)/((I-a)*c^(1/2)-I*b*d^(1/2)))*ln
(1+I*c^(1/2)*x/d^(1/2))/c^(3/2)-1/4*d^(1/2)*ln(c^(1/2)*(I+a+b*x)/((I+a)*c^
(1/2)+I*b*d^(1/2)))*ln(1+I*c^(1/2)*x/d^(1/2))/c^(3/2)-1/4*d^(1/2)*polylog(
2,b*(d^(1/2)-I*c^(1/2)*x)/((1+I*a)*c^(1/2)+b*d^(1/2)))/c^(3/2)+1/4*d^(1/2)
*polylog(2,b*(d^(1/2)-I*c^(1/2)*x)/(I*(I+a)*c^(1/2)+b*d^(1/2)))/c^(3/2)+1/
4*d^(1/2)*polylog(2,-b*(d^(1/2)+I*c^(1/2)*x)/((1+I*a)*c^(1/2)-b*d^(1/2)))/
c^(3/2)-1/4*d^(1/2)*polylog(2,b*(d^(1/2)+I*c^(1/2)*x)/((1-I*a)*c^(1/2)+b*d
^(1/2)))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 930, normalized size of antiderivative = 1.27

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx$$

$$= \frac{4c\log(a+bx)+2c\log\left(\frac{-i+a+bx}{a+bx}\right)+2iac\log\left(\frac{-i+a+bx}{a+bx}\right)+2ibcx\log\left(\frac{-i+a+bx}{a+bx}\right)+2c\log\left(\frac{i+a+bx}{a+bx}\right)-2iac\log}{}$$

input
```
Integrate[ArcCot[a + b*x]/(c + d/x^2),x]
```

output
```
(4*c*Log[a + b*x] + 2*c*Log[(-I + a + b*x)/(a + b*x)] + (2*I)*a*c*Log[(-I
+ a + b*x)/(a + b*x)] + (2*I)*b*c*x*Log[(-I + a + b*x)/(a + b*x)] + 2*c*Lo
g[(I + a + b*x)/(a + b*x)] - (2*I)*a*c*Log[(I + a + b*x)/(a + b*x)] - (2*I
)*b*c*x*Log[(I + a + b*x)/(a + b*x)] + I*b*Sqrt[-c]*Sqrt[d]*Log[(Sqrt[-c]*
(-I + a + b*x))/((-I)*Sqrt[-c] + a*Sqrt[-c] + b*Sqrt[d])]*Log[Sqrt[d] - Sq
rt[-c]*x] - I*b*Sqrt[-c]*Sqrt[d]*Log[(-I + a + b*x)/(a + b*x)]*Log[Sqrt[d]
 - Sqrt[-c]*x] - I*b*Sqrt[-c]*Sqrt[d]*Log[(Sqrt[-c]*(I + a + b*x))/((I + a
)*Sqrt[-c] + b*Sqrt[d])]*Log[Sqrt[d] - Sqrt[-c]*x] + I*b*Sqrt[-c]*Sqrt[d]*
Log[(I + a + b*x)/(a + b*x)]*Log[Sqrt[d] - Sqrt[-c]*x] - I*b*Sqrt[-c]*Sqrt
[d]*Log[(Sqrt[-c]*(I - a - b*x))/(-((-I + a)*Sqrt[-c]) + b*Sqrt[d])]*Log[S
qrt[d] + Sqrt[-c]*x] + I*b*Sqrt[-c]*Sqrt[d]*Log[(-I + a + b*x)/(a + b*x)]*
Log[Sqrt[d] + Sqrt[-c]*x] + I*b*Sqrt[-c]*Sqrt[d]*Log[(Sqrt[-c]*(I + a + b*
x))/((I + a)*Sqrt[-c] - b*Sqrt[d])]*Log[Sqrt[d] + Sqrt[-c]*x] - I*b*Sqrt[-
c]*Sqrt[d]*Log[(I + a + b*x)/(a + b*x)]*Log[Sqrt[d] + Sqrt[-c]*x] + I*b*Sq
rt[-c]*Sqrt[d]*PolyLog[2, (b*(Sqrt[d] - Sqrt[-c]*x))/((-I)*Sqrt[-c] + a*Sq
rt[-c] + b*Sqrt[d])] - I*b*Sqrt[-c]*Sqrt[d]*PolyLog[2, (b*(Sqrt[d] - Sqrt[
-c]*x))/(I*Sqrt[-c] + a*Sqrt[-c] + b*Sqrt[d])] + I*b*Sqrt[-c]*Sqrt[d]*Poly
Log[2, (b*(Sqrt[d] + Sqrt[-c]*x))/((-I)*Sqrt[-c] - a*Sqrt[-c] + b*Sqrt[d])
] - I*b*Sqrt[-c]*Sqrt[d]*PolyLog[2, (b*(Sqrt[d] + Sqrt[-c]*x))/(I*Sqrt[-c]
 - a*Sqrt[-c] + b*Sqrt[d])])/(4*b*c^2)
```

Rubi [A] (verified)

Time = 2.08 (sec) , antiderivative size = 1222, normalized size of antiderivative = 1.66, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5575, 2993, 772, 262, 218, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}dx$$

$$\Big\downarrow \text{5575}$$

$$\frac{1}{2}i\int \frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)}{c+\frac{d}{x^2}}dx-\frac{1}{2}i\int \frac{\log\left(\frac{a+bx+i}{a+bx}\right)}{c+\frac{d}{x^2}}dx$$

$$\Big\downarrow \text{2993}$$

$$\frac{1}{2}i\left(-\left(\left(\log(-a-bx+i)-\log\left(-\frac{-a-bx+i}{a+bx}\right)-\log(a+bx)\right)\int\frac{1}{c+\frac{d}{x^2}}dx\right)+\int\frac{\log(-a-bx+i)}{c+\frac{d}{x^2}}dx-\int\right.$$
$$\frac{1}{2}i\left(\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)\int\frac{1}{c+\frac{d}{x^2}}dx-\int\frac{\log(a+bx)}{c+\frac{d}{x^2}}dx+\int\frac{\log(a+bx+i)}{c+\frac{d}{x^2}}\right.$$

$\downarrow$ 772

$$\frac{1}{2}i\left(-\left(\left(\log(-a-bx+i)-\log\left(-\frac{-a-bx+i}{a+bx}\right)-\log(a+bx)\right)\int\frac{x^2}{cx^2+d}dx\right)+\int\frac{\log(-a-bx+i)}{c+\frac{d}{x^2}}dx-\int\right.$$
$$\frac{1}{2}i\left(\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)\int\frac{x^2}{cx^2+d}dx-\int\frac{\log(a+bx)}{c+\frac{d}{x^2}}dx+\int\frac{\log(a+bx+i)}{c+\frac{d}{x^2}}\right.$$

$\downarrow$ 262

$$\frac{1}{2}i\left(-\left(\left(\log(-a-bx+i)-\log\left(-\frac{-a-bx+i}{a+bx}\right)-\log(a+bx)\right)\left(\frac{x}{c}-\frac{d\int\frac{1}{cx^2+d}dx}{c}\right)\right)+\int\frac{\log(-a-bx+i)}{c+\frac{d}{x^2}}\right.$$
$$\frac{1}{2}i\left(\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)\left(\frac{x}{c}-\frac{d\int\frac{1}{cx^2+d}dx}{c}\right)-\int\frac{\log(a+bx)}{c+\frac{d}{x^2}}dx+\int\frac{\log(a+}{c+}\right.$$

$\downarrow$ 218

$$\frac{1}{2}i\left(\int\frac{\log(-a-bx+i)}{c+\frac{d}{x^2}}dx-\int\frac{\log(a+bx)}{c+\frac{d}{x^2}}dx-\left(\left(\log(-a-bx+i)-\log\left(-\frac{-a-bx+i}{a+bx}\right)-\log(a+bx)\right)\right.\right.$$
$$\frac{1}{2}i\left(-\int\frac{\log(a+bx)}{c+\frac{d}{x^2}}dx+\int\frac{\log(a+bx+i)}{c+\frac{d}{x^2}}dx+\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)\left(\frac{x}{c}-\right.\right.$$

$\downarrow$ 2856

$$\frac{1}{2}i\left(\int\left(\frac{\log(-a-bx+i)}{c}-\frac{d\log(-a-bx+i)}{c\left(cx^2+d\right)}\right)dx-\int\left(\frac{\log(a+bx)}{c}-\frac{d\log(a+bx)}{c\left(cx^2+d\right)}\right)dx-\left(\left(\log(-a-bx\right.\right.\right.$$
$$\frac{1}{2}i\left(-\int\left(\frac{\log(a+bx)}{c}-\frac{d\log(a+bx)}{c\left(cx^2+d\right)}\right)dx+\int\left(\frac{\log(a+bx+i)}{c}-\frac{d\log(a+bx+i)}{c\left(cx^2+d\right)}\right)dx+\left(\log(a+bx)-\mathrm{lo}\right.\right.$$

$\downarrow$ 2009

$$\frac{1}{2}i\left(-\frac{(-a-bx+i)\log(-a-bx+i)}{bc}-\frac{\sqrt{d}\log\left(-\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{(i-a)\sqrt{-c}-b\sqrt{d}}\right)\log(-a-bx+i)}{2(-c)^{3/2}}+\frac{\sqrt{d}\log\left(\frac{b\left(\sqrt{-c}x+\sqrt{d}\right)}{\sqrt{-c}(i-a)+b\sqrt{d}}\right)}{2(-c)^{3/2}}\right.$$

$$\frac{1}{2}i\left(-\frac{(a+bx)\log(a+bx)}{bc}+\frac{\sqrt{d}\log\left(\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{\sqrt{-c}a+b\sqrt{d}}\right)\log(a+bx)}{2(-c)^{3/2}}-\frac{\sqrt{d}\log\left(-\frac{b\left(\sqrt{-c}x+\sqrt{d}\right)}{a\sqrt{-c}-b\sqrt{d}}\right)\log(a+bx)}{2(-c)^{3/2}}+\frac{(a+\right.$$

input
```
Int[ArcCot[a + b*x]/(c + d/x^2),x]
```

output
```
(I/2)*(-(((I - a - b*x)*Log[I - a - b*x])/(b*c)) - (x/c - (Sqrt[d]*ArcTan[
(Sqrt[c]*x)/Sqrt[d]])/c^(3/2))*(Log[I - a - b*x] - Log[-((I - a - b*x)/(a
+ b*x))] - Log[a + b*x]) - ((a + b*x)*Log[a + b*x])/(b*c) - (Sqrt[d]*Log[I
 - a - b*x]*Log[-((b*(Sqrt[d] - Sqrt[-c]*x))/((I - a)*Sqrt[-c] - b*Sqrt[d]
))])/(2*(-c)^(3/2)) + (Sqrt[d]*Log[a + b*x]*Log[(b*(Sqrt[d] - Sqrt[-c]*x))
/(a*Sqrt[-c] + b*Sqrt[d])])/(2*(-c)^(3/2)) - (Sqrt[d]*Log[a + b*x]*Log[-((
b*(Sqrt[d] + Sqrt[-c]*x))/(a*Sqrt[-c] - b*Sqrt[d]))])/(2*(-c)^(3/2)) + (Sq
rt[d]*Log[I - a - b*x]*Log[(b*(Sqrt[d] + Sqrt[-c]*x))/((I - a)*Sqrt[-c] +
b*Sqrt[d])])/(2*(-c)^(3/2)) - (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(I - a - b*x))
/(I*Sqrt[-c] - a*Sqrt[-c] - b*Sqrt[d])])/(2*(-c)^(3/2)) + (Sqrt[d]*PolyLog
[2, (Sqrt[-c]*(I - a - b*x))/((I - a)*Sqrt[-c] + b*Sqrt[d])])/(2*(-c)^(3/2
)) - (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(a + b*x))/(a*Sqrt[-c] - b*Sqrt[d])])/(
2*(-c)^(3/2)) + (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(a + b*x))/(a*Sqrt[-c] + b*S
qrt[d])])/(2*(-c)^(3/2))) - (I/2)*(-(((a + b*x)*Log[a + b*x])/(b*c)) + ((I
 + a + b*x)*Log[I + a + b*x])/(b*c) + (x/c - (Sqrt[d]*ArcTan[(Sqrt[c]*x)/S
qrt[d]])/c^(3/2))*(Log[a + b*x] - Log[I + a + b*x] + Log[(I + a + b*x)/(a
+ b*x)]) + (Sqrt[d]*Log[a + b*x]*Log[(b*(Sqrt[d] - Sqrt[-c]*x))/(a*Sqrt[-c
] + b*Sqrt[d])])/(2*(-c)^(3/2)) - (Sqrt[d]*Log[I + a + b*x]*Log[(b*(Sqrt[d
] - Sqrt[-c]*x))/((I + a)*Sqrt[-c] + b*Sqrt[d])])/(2*(-c)^(3/2)) - (Sqrt[d
]*Log[a + b*x]*Log[-((b*(Sqrt[d] + Sqrt[-c]*x))/(a*Sqrt[-c] - b*Sqrt[d]...
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 2993
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Simp[p*r   Int[RFx*Log[a + b*x], x], x] + (Si
mp[q*r   Int[RFx*Log[c + d*x], x], x] - Simp[(p*r*Log[a + b*x] + q*r*Log[c
+ d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])   Int[RFx, x], x]) /; FreeQ[
{a, b, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a
*d, 0] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[
m, n]]
```

rule 5575
```
Int[ArcCot[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[
I/2   Int[Log[(-I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] - Simp[I/2   In
t[Log[(I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x
] && RationalQ[n]
```

Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 728, normalized size of antiderivative = 0.99

method	result
risch	$\frac{\pi x}{2c}+\frac{\pi a}{2bc}+\frac{ib\pi d \arctan\left(\frac{2iac+2(-bxi-ai+1)c-2c}{2\sqrt{-b^2cd}}\right)}{2c\sqrt{-b^2cd}}+\frac{i\pi}{2bc}+\frac{i\ln(bxi+ai+1)a}{2bc}-\frac{i\ln(-bxi-ai+1)a}{2bc}+\frac{i\ln(bxi\cdots)}{\cdots}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(arccot(b*x+a)/(c+d/x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*Pi/c*x+1/2/b*Pi/c*a+1/2*I*b*Pi*d/c/(-b^2*c*d)^(1/2)*arctan(1/2*(2*I*a*
c+2*(1-I*a-I*b*x)*c-2*c)/(-b^2*c*d)^(1/2))+1/2*I/b*Pi/c+1/2*I/b/c*ln(1+I*a
+I*b*x)*a-1/2*I/b/c*ln(1-I*a-I*b*x)*a+1/2*I/c*ln(1+I*a+I*b*x)*x-1/2*I/c*ln
(1-I*a-I*b*x)*x+1/2/b/c*ln(1-I*a-I*b*x)-1/b/c+1/4/c^2*ln(1-I*a-I*b*x)*ln((
I*a*c-b*(c*d)^(1/2)+(1-I*a-I*b*x)*c-c)/(I*a*c-b*(c*d)^(1/2)-c))*(c*d)^(1/2
)-1/4/c^2*ln(1-I*a-I*b*x)*ln((I*a*c+b*(c*d)^(1/2)+(1-I*a-I*b*x)*c-c)/(I*a*
c+b*(c*d)^(1/2)-c))*(c*d)^(1/2)+1/4/c^2*dilog((I*a*c-b*(c*d)^(1/2)+(1-I*a-
I*b*x)*c-c)/(I*a*c-b*(c*d)^(1/2)-c))*(c*d)^(1/2)-1/4/c^2*dilog((I*a*c+b*(c
*d)^(1/2)+(1-I*a-I*b*x)*c-c)/(I*a*c+b*(c*d)^(1/2)-c))*(c*d)^(1/2)+1/2/b/c*
ln(1+I*a+I*b*x)+1/4/c^2*ln(1+I*a+I*b*x)*(c*d)^(1/2)*ln((I*a*c+b*(c*d)^(1/2
)-(1+I*a+I*b*x)*c+c)/(I*a*c+b*(c*d)^(1/2)+c))-1/4/c^2*ln(1+I*a+I*b*x)*(c*d
)^(1/2)*ln((I*a*c-b*(c*d)^(1/2)-(1+I*a+I*b*x)*c+c)/(I*a*c-b*(c*d)^(1/2)+c)
)-1/4/c^2*(c*d)^(1/2)*dilog((I*a*c-b*(c*d)^(1/2)-(1+I*a+I*b*x)*c+c)/(I*a*c
-b*(c*d)^(1/2)+c))+1/4/c^2*(c*d)^(1/2)*dilog((I*a*c+b*(c*d)^(1/2)-(1+I*a+I
*b*x)*c+c)/(I*a*c+b*(c*d)^(1/2)+c))
```

Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{c+\frac{d}{x^2}}\,dx$$

input
```
integrate(arccot(b*x+a)/(c+d/x^2),x, algorithm="fricas")
```

output
```
integral(x^2*arccot(b*x + a)/(c*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx = \text{Timed out}$$

input
```
integrate(acot(b*x+a)/(c+d/x**2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8518 vs. 2(502) = 1004.

Time = 0.86 (sec) , antiderivative size = 8518, normalized size of antiderivative = 11.59

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx = \text{Too large to display}$$

input
```
integrate(arccot(b*x+a)/(c+d/x^2),x, algorithm="maxima")
```

output
```
-(d*arctan(c*x/sqrt(c*d))/(sqrt(c*d)*c) - x/c)*arccot(b*x + a) - 1/8*(8*a*
c*arctan(b*x + a) + (4*b*arctan(sqrt(c)*x/sqrt(d))*arctan2((2*a*b^2*c*d +
(a*b^3*d + (a^3 + a)*b*c + (b^4*d + (a^2 + 3)*b^2*c)*x)*sqrt(c)*sqrt(d) +
(3*b^3*c*d + (a^2 + 1)*b*c^2)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2
*a^2 + 1)*c^2 + 4*(b^3*d + (a^2 + 1)*b*c)*sqrt(c)*sqrt(d)), ((a^2 + 3)*b^2
*c*d + (a^4 + 2*a^2 + 1)*c^2 + (2*a*b^2*c*x + b^3*d + 3*(a^2 + 1)*b*c)*sqr
t(c)*sqrt(d) + (a*b^3*c*d + (a^3 + a)*b*c^2)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2
*c*d + (a^4 + 2*a^2 + 1)*c^2 + 4*(b^3*d + (a^2 + 1)*b*c)*sqrt(c)*sqrt(d)))
 + 4*b*arctan(sqrt(c)*x/sqrt(d))*arctan2((2*a*b^2*c*d - (a*b^3*d + (a^3 +
a)*b*c + (b^4*d + (a^2 + 3)*b^2*c)*x)*sqrt(c)*sqrt(d) + (3*b^3*c*d + (a^2
+ 1)*b*c^2)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*c^2 - 4*
(b^3*d + (a^2 + 1)*b*c)*sqrt(c)*sqrt(d)), ((a^2 + 3)*b^2*c*d + (a^4 + 2*a^
2 + 1)*c^2 - (2*a*b^2*c*x + b^3*d + 3*(a^2 + 1)*b*c)*sqrt(c)*sqrt(d) + (a*
b^3*c*d + (a^3 + a)*b*c^2)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^
2 + 1)*c^2 - 4*(b^3*d + (a^2 + 1)*b*c)*sqrt(c)*sqrt(d))) + b*log(c*x^2 + d
)*log(((a^2 + 1)*b^22*c*d^11 + 11*(a^4 + 22*a^2 + 21)*b^20*c^2*d^10 + 55*(
a^6 + 39*a^4 + 171*a^2 + 133)*b^18*c^3*d^9 + 33*(5*a^8 + 260*a^6 + 1870*a^
4 + 3876*a^2 + 2261)*b^16*c^4*d^8 + 330*(a^10 + 61*a^8 + 570*a^6 + 1802*a^
4 + 2261*a^2 + 969)*b^14*c^5*d^7 + 22*(21*a^12 + 1386*a^10 + 15015*a^8 + 6
0060*a^6 + 109395*a^4 + 92378*a^2 + 29393)*b^12*c^6*d^6 + 22*(21*a^14 +...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx=\text{Timed out}$$

input
```
integrate(arccot(b*x+a)/(c+d/x^2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx = \int \frac{\operatorname{acot}(a+b\,x)}{c+\frac{d}{x^2}}\,dx$$

input
```
int(acot(a + b*x)/(c + d/x^2),x)
```

output
```
int(acot(a + b*x)/(c + d/x^2), x)
```

Reduce [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx = \int \frac{acot(bx+a)\,x^2}{c\,x^2+d}dx$$

input
```
int(acot(b*x+a)/(c+d/x^2),x)
```

output
```
int((acot(a + b*x)*x**2)/(c*x**2 + d),x)
```

3.41 $\int \frac{a+b\cot^{-1}(c+dx)}{e+f\sqrt{x}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 726

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+f\sqrt{x}}\,dx = \frac{2a\sqrt{x}}{f} - \frac{2ib\sqrt{i+c}\arctan\left(\frac{\sqrt{d}\sqrt{x}}{\sqrt{i+c}}\right)}{\sqrt{d}f} + \frac{2ib\sqrt{i-c}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{x}}{\sqrt{i-c}}\right)}{\sqrt{d}f} - \frac{ibe\log\left(\frac{f\left(\sqrt{-i-c}-\sqrt{d}\sqrt{x}\right)}{\sqrt{d}e+\sqrt{-i-c}f}\right)\log\left(e+f\sqrt{x}\right)}{f^2} + \frac{ibe\log\left(\frac{f\left(\sqrt{i-c}-\sqrt{d}\sqrt{x}\right)}{\sqrt{d}e+\sqrt{i-c}f}\right)\log\left(e+f\sqrt{x}\right)}{f^2} - \frac{ibe\log\left(-\frac{f\left(\sqrt{-i-c}+\sqrt{d}\sqrt{x}\right)}{\sqrt{d}e-\sqrt{-i-c}f}\right)\log\left(e+f\sqrt{x}\right)}{f^2} + \frac{ibe\log\left(-\frac{f\left(\sqrt{i-c}+\sqrt{d}\sqrt{x}\right)}{\sqrt{d}e-\sqrt{i-c}f}\right)\log\left(e+f\sqrt{x}\right)}{f^2} + \frac{ib\sqrt{x}\log\left(-\frac{i-c-dx}{c+dx}\right)}{f} - \frac{e\log\left(e+f\sqrt{x}\right)\left(a+ib\log\left(-\frac{i-c-dx}{c+dx}\right)\right)}{f^2} - \frac{ib\sqrt{x}\log\left(\frac{i+c+dx}{c+dx}\right)}{f} - \frac{e\log\left(e+f\sqrt{x}\right)\left(a-ib\log\left(\frac{i+c+dx}{c+dx}\right)\right)}{f^2} - \frac{ibe\,\text{PolyLog}\left(2,\frac{\sqrt{d}(e+f\sqrt{x})}{\sqrt{d}e-\sqrt{-i-c}f}\right)}{f^2} - \frac{ibe\,\text{PolyLog}\left(2,\frac{\sqrt{d}(e+f\sqrt{x})}{\sqrt{d}e+\sqrt{-i-c}f}\right)}{f^2} + \frac{ibe\,\text{PolyLog}\left(2,\frac{\sqrt{d}(e+f\sqrt{x})}{\sqrt{d}e-\sqrt{i-c}f}\right)}{f^2} + \frac{ibe\,\text{PolyLog}\left(2,\frac{\sqrt{d}(e+f\sqrt{x})}{\sqrt{d}e+\sqrt{i-c}f}\right)}{f^2}$$

output
```
2*a*x^(1/2)/f-2*I*b*(I+c)^(1/2)*arctan(d^(1/2)*x^(1/2)/(I+c)^(1/2))/d^(1/2
)/f+2*I*b*(I-c)^(1/2)*arctanh(d^(1/2)*x^(1/2)/(I-c)^(1/2))/d^(1/2)/f-I*b*e
*ln(f*((-I-c)^(1/2)-d^(1/2)*x^(1/2))/(d^(1/2)*e+(-I-c)^(1/2)*f))*ln(e+f*x^
(1/2))/f^2+I*b*e*ln(f*((I-c)^(1/2)-d^(1/2)*x^(1/2))/(d^(1/2)*e+(I-c)^(1/2)
*f))*ln(e+f*x^(1/2))/f^2-I*b*e*ln(-f*((-I-c)^(1/2)+d^(1/2)*x^(1/2))/(d^(1/
2)*e-(-I-c)^(1/2)*f))*ln(e+f*x^(1/2))/f^2+I*b*e*ln(-f*((I-c)^(1/2)+d^(1/2)
*x^(1/2))/(d^(1/2)*e-(I-c)^(1/2)*f))*ln(e+f*x^(1/2))/f^2+I*b*x^(1/2)*ln(-(
I-c-d*x)/(d*x+c))/f-e*ln(e+f*x^(1/2))*(a+I*b*ln(-(I-c-d*x)/(d*x+c)))/f^2-I
*b*x^(1/2)*ln((I+c+d*x)/(d*x+c))/f-e*ln(e+f*x^(1/2))*(a-I*b*ln((I+c+d*x)/(
d*x+c)))/f^2-I*b*e*polylog(2,d^(1/2)*(e+f*x^(1/2))/(d^(1/2)*e-(-I-c)^(1/2)
*f))/f^2-I*b*e*polylog(2,d^(1/2)*(e+f*x^(1/2))/(d^(1/2)*e+(-I-c)^(1/2)*f))
/f^2+I*b*e*polylog(2,d^(1/2)*(e+f*x^(1/2))/(d^(1/2)*e-(I-c)^(1/2)*f))/f^2+
I*b*e*polylog(2,d^(1/2)*(e+f*x^(1/2))/(d^(1/2)*e+(I-c)^(1/2)*f))/f^2
```

Mathematica [A] (verified)

Time = 23.82 (sec) , antiderivative size = 645, normalized size of antiderivative = 0.89

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+f\sqrt{x}}\,dx$$
$$= \frac{2a\left(f\sqrt{x}-e\log\left(e+f\sqrt{x}\right)\right)-ib\left(\frac{2\sqrt{i+c}f\arctan\left(\frac{\sqrt{d}\sqrt{x}}{\sqrt{i+c}}\right)}{\sqrt{d}}-\frac{2\sqrt{i-c}f\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{x}}{\sqrt{i-c}}\right)}{\sqrt{d}}+e\log\left(\frac{f\left(\sqrt{-i-c}-\sqrt{d}\sqrt{x}\right)}{\sqrt{d}e+\sqrt{-i-c}f}\right)\cdots\right.}{\cdots}$$

input
```
Integrate[(a + b*ArcCot[c + d*x])/(e + f*Sqrt[x]),x]
```

output
```
(2*a*(f*Sqrt[x] - e*Log[e + f*Sqrt[x]]) - I*b*((2*Sqrt[I + c]*f*ArcTan[(Sq
rt[d]*Sqrt[x])/Sqrt[I + c]])/Sqrt[d] - (2*Sqrt[I - c]*f*ArcTanh[(Sqrt[d]*S
qrt[x])/Sqrt[I - c]])/Sqrt[d] + e*Log[(f*(Sqrt[-I - c] - Sqrt[d]*Sqrt[x]))
/(Sqrt[d]*e + Sqrt[-I - c]*f)]*Log[e + f*Sqrt[x]] - e*Log[(f*(Sqrt[I - c]
 - Sqrt[d]*Sqrt[x]))/(Sqrt[d]*e + Sqrt[I - c]*f)]*Log[e + f*Sqrt[x]] + e*Lo
g[(f*(Sqrt[-I - c] + Sqrt[d]*Sqrt[x]))/(-(Sqrt[d]*e) + Sqrt[-I - c]*f)]*Lo
g[e + f*Sqrt[x]] - e*Log[(f*(Sqrt[I - c] + Sqrt[d]*Sqrt[x]))/(-(Sqrt[d]*e)
 + Sqrt[I - c]*f)]*Log[e + f*Sqrt[x]] - f*Sqrt[x]*Log[(-I + c + d*x)/(c +
d*x)] + e*Log[e + f*Sqrt[x]]*Log[(-I + c + d*x)/(c + d*x)] + f*Sqrt[x]*Log
[(I + c + d*x)/(c + d*x)] - e*Log[e + f*Sqrt[x]]*Log[(I + c + d*x)/(c + d*
x)] + e*PolyLog[2, (Sqrt[d]*(e + f*Sqrt[x]))/(Sqrt[d]*e - Sqrt[-I - c]*f)]
 + e*PolyLog[2, (Sqrt[d]*(e + f*Sqrt[x]))/(Sqrt[d]*e + Sqrt[-I - c]*f)] -
e*PolyLog[2, (Sqrt[d]*(e + f*Sqrt[x]))/(Sqrt[d]*e - Sqrt[I - c]*f)] - e*Po
lyLog[2, (Sqrt[d]*(e + f*Sqrt[x]))/(Sqrt[d]*e + Sqrt[I - c]*f)]))/f^2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+f\sqrt{x}}\,dx$$

$$\Big\downarrow \text{ 7267}$$

$$2\int \frac{\sqrt{x}\left(a+b\cot^{-1}(c+dx)\right)}{e+f\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{ 7293}$$

$$2\int \left(\frac{\sqrt{x}a}{e+f\sqrt{x}}+\frac{b\sqrt{x}\cot^{-1}(c+dx)}{e+f\sqrt{x}}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(-\frac{be\int \frac{\cot^{-1}(c+dx)}{e+f\sqrt{x}}d\sqrt{x}}{f}-\frac{ae\log\left(e+f\sqrt{x}\right)}{f^2}+\frac{a\sqrt{x}}{f}-\frac{b\arctan\left(\frac{\sqrt{\sqrt{c^2+1}-c}-\sqrt{2}\sqrt{d}\sqrt{x}}{\sqrt{\sqrt{c^2+1}+c}}\right)}{\sqrt{2}\sqrt{\sqrt{c^2+1}+c}\sqrt{d}f}+\frac{b\arctan\left(\frac{\sqrt{\sqrt{c^2+1}-c+}}{\sqrt{\sqrt{c^2+}}}\right.}{\sqrt{2}\sqrt{\sqrt{c^2+1}+c}}\right.$$

input
```
Int[(a + b*ArcCot[c + d*x])/(e + f*Sqrt[x]),x]
```

output
```
$Aborted
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.43 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.51

method	result
derivativedivides	$\frac{2a\sqrt{x}}{f} - \frac{2ae\ln(e+f\sqrt{x})}{f^2} + \frac{2b\,\operatorname{arccot}(dx+c)\sqrt{x}}{f} - \frac{2b\,\operatorname{arccot}(dx+c)e\ln(e+f\sqrt{x})}{f^2} + b\Big(\sum_{\_R=\mathrm{RootOf}(d^2\_Z^4-4}$
default	$\frac{2a\sqrt{x}}{f} - \frac{2ae\ln(e+f\sqrt{x})}{f^2} + \frac{2b\,\operatorname{arccot}(dx+c)\sqrt{x}}{f} - \frac{2b\,\operatorname{arccot}(dx+c)e\ln(e+f\sqrt{x})}{f^2} + b\Big(\sum_{\_R=\mathrm{RootOf}(d^2\_Z^4-4}$
parts	$a\left(\frac{2\sqrt{x}}{f} + \frac{e\ln(f\sqrt{x}-e)}{f^2} - \frac{e\ln(e+f\sqrt{x})}{f^2} - \frac{e\ln(f^2x-e^2)}{f^2}\right) + \frac{2b\,\operatorname{arccot}(dx+c)\sqrt{x}}{f} - \frac{2b\,\operatorname{arccot}(dx+c)e\ln(e+}{f^2}$

input
```
int((a+b*arccot(d*x+c))/(e+f*x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2*a*x^(1/2)/f-2*a*e/f^2*ln(e+f*x^(1/2))+2*b*arccot(d*x+c)/f*x^(1/2)-2*b*ar
ccot(d*x+c)*e/f^2*ln(e+f*x^(1/2))+b*sum((_R^2-2*_R*e+e^2)/(_R^3*d-3*_R^2*d
*e+_R*c*f^2+3*_R*d*e^2-c*e*f^2-d*e^3)*ln(f*x^(1/2)-_R+e),_R=RootOf(d^2*_Z^
4-4*d^2*e*_Z^3+(2*c*d*f^2+6*d^2*e^2)*_Z^2+(-4*c*d*e*f^2-4*d^2*e^3)*_Z+c^2*
f^4+2*c*d*e^2*f^2+d^2*e^4+f^4))-b*e*sum(1/(_R1^2*d-2*_R1*d*e+c*f^2+d*e^2)*
(ln(e+f*x^(1/2))*ln((-f*x^(1/2)+_R1-e)/_R1)+dilog((-f*x^(1/2)+_R1-e)/_R1))
,_R1=RootOf(d^2*_Z^4-4*d^2*e*_Z^3+(2*c*d*f^2+6*d^2*e^2)*_Z^2+(-4*c*d*e*f^2
-4*d^2*e^3)*_Z+c^2*f^4+2*c*d*e^2*f^2+d^2*e^4+f^4))
```

Fricas [F]

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+f\sqrt{x}}\,dx=\int \frac{b\,\mathrm{arccot}\,(dx+c)+a}{f\sqrt{x}+e}\,dx$$

input
```
integrate((a+b*arccot(d*x+c))/(e+f*x^(1/2)),x, algorithm="fricas")
```

output
```
integral(-(b*e*arccot(d*x + c) + a*e - (b*f*arccot(d*x + c) + a*f)*sqrt(x)
)/(f^2*x - e^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+f\sqrt{x}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acot(d*x+c))/(e+f*x**(1/2)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+f\sqrt{x}}\,dx=\int \frac{b\,\mathrm{arccot}\,(dx+c)+a}{f\sqrt{x}+e}\,dx$$

input
```
integrate((a+b*arccot(d*x+c))/(e+f*x^(1/2)),x, algorithm="maxima")
```

output
```
2*(b*f^2*integrate(1/2*arctan2(1, d*x + c)/(f*sqrt(x) + e), x) - a*e*log(f
*sqrt(x) + e) + a*f*sqrt(x))/f^2
```

Giac [F]

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+f\sqrt{x}}\,dx=\int \frac{b\,\mathrm{arccot}\,(dx+c)+a}{f\sqrt{x}+e}\,dx$$

input
```
integrate((a+b*arccot(d*x+c))/(e+f*x^(1/2)),x, algorithm="giac")
```

output
```
integrate((b*arccot(d*x + c) + a)/(f*sqrt(x) + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+f\sqrt{x}}\,dx=\int \frac{a+b\,\mathrm{acot}(c+d\,x)}{e+f\,\sqrt{x}}\,dx$$

input
```
int((a + b*acot(c + d*x))/(e + f*x^(1/2)),x)
```

output
```
int((a + b*acot(c + d*x))/(e + f*x^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+f\sqrt{x}}\,dx=\frac{2\sqrt{x}\,af+\left(\int \frac{acot(dx+c)}{\sqrt{x}\,f+e}dx\right)b\,f^2-2\log\left(\sqrt{x}\,f+e\right)ae}{f^2}$$

input
```
int((a+b*acot(d*x+c))/(e+f*x^(1/2)),x)
```

output
```
(2*sqrt(x)*a*f + int(acot(c + d*x)/(sqrt(x)*f + e),x)*b*f**2 - 2*log(sqrt(
x)*f + e)*a*e)/f**2
```

3.42 $\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 693

$$\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = -\frac{2i\sqrt{i+a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}d}+\frac{2i\sqrt{i-a}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}d}$$
$$-\frac{ic\log\left(\frac{d\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}$$
$$+\frac{ic\log\left(\frac{d\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}$$
$$-\frac{ic\log\left(-\frac{d\left(\sqrt{-i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{-i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}$$
$$+\frac{ic\log\left(-\frac{d\left(\sqrt{i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}$$
$$+\frac{i\sqrt{x}\log\left(-\frac{i-a-bx}{a+bx}\right)}{d}-\frac{ic\log\left(c+d\sqrt{x}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{d^2}$$
$$-\frac{i\sqrt{x}\log\left(\frac{i+a+bx}{a+bx}\right)}{d}+\frac{ic\log\left(c+d\sqrt{x}\right)\log\left(\frac{i+a+bx}{a+bx}\right)}{d^2}$$
$$-\frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{-i-a}d}\right)}{d^2}-\frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{-i-a}d}\right)}{d^2}$$
$$+\frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{i-a}d}\right)}{d^2}+\frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{i-a}d}\right)}{d^2}$$

output
```
-2*I*(I+a)^(1/2)*arctan(b^(1/2)*x^(1/2)/(I+a)^(1/2))/b^(1/2)/d+2*I*(I-a)^(
1/2)*arctanh(b^(1/2)*x^(1/2)/(I-a)^(1/2))/b^(1/2)/d-I*c*ln(d*((-I-a)^(1/2)
-b^(1/2)*x^(1/2))/(b^(1/2)*c+(-I-a)^(1/2)*d))*ln(c+d*x^(1/2))/d^2+I*c*ln(d
*((I-a)^(1/2)-b^(1/2)*x^(1/2))/(b^(1/2)*c+(I-a)^(1/2)*d))*ln(c+d*x^(1/2))/
d^2-I*c*ln(-d*((-I-a)^(1/2)+b^(1/2)*x^(1/2))/(b^(1/2)*c-(-I-a)^(1/2)*d))*l
n(c+d*x^(1/2))/d^2+I*c*ln(-d*((I-a)^(1/2)+b^(1/2)*x^(1/2))/(b^(1/2)*c-(I-a
)^(1/2)*d))*ln(c+d*x^(1/2))/d^2+I*x^(1/2)*ln(-(I-a-b*x)/(b*x+a))/d-I*c*ln(
c+d*x^(1/2))*ln(-(I-a-b*x)/(b*x+a))/d^2-I*x^(1/2)*ln((I+a+b*x)/(b*x+a))/d+
I*c*ln(c+d*x^(1/2))*ln((I+a+b*x)/(b*x+a))/d^2-I*c*polylog(2,b^(1/2)*(c+d*x
^(1/2))/(b^(1/2)*c-(-I-a)^(1/2)*d))/d^2-I*c*polylog(2,b^(1/2)*(c+d*x^(1/2)
)/(b^(1/2)*c+(-I-a)^(1/2)*d))/d^2+I*c*polylog(2,b^(1/2)*(c+d*x^(1/2))/(b^(
1/2)*c-(I-a)^(1/2)*d))/d^2+I*c*polylog(2,b^(1/2)*(c+d*x^(1/2))/(b^(1/2)*c+
(I-a)^(1/2)*d))/d^2
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 618, normalized size of antiderivative = 0.89

$$\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}\,dx =$$
$$-\frac{i\left(\frac{2\sqrt{i+a}d\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}}-\frac{2\sqrt{i-a}d\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}}+c\log\left(\frac{d\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-i-a}d}\right)\log\left(c+d\sqrt{x}\right)-c\log\left(\frac{d\left(\sqrt{i-a}\cdots\right.}{\sqrt{b}c+\cdots}\right.\cdots\right.}{\cdots}$$

input
```
Integrate[ArcCot[a + b*x]/(c + d*Sqrt[x]),x]
```

output
```
((-I)*((2*Sqrt[I + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/Sqrt[b] - (
2*Sqrt[I - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]])/Sqrt[b] + c*Log[(d
*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-I - a]*d)]*Log[c + d
*Sqrt[x]] - c*Log[(d*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[I
- a]*d)]*Log[c + d*Sqrt[x]] + c*Log[(d*(Sqrt[-I - a] + Sqrt[b]*Sqrt[x]))/(
-(Sqrt[b]*c) + Sqrt[-I - a]*d)]*Log[c + d*Sqrt[x]] - c*Log[(d*(Sqrt[I - a]
 + Sqrt[b]*Sqrt[x]))/(-(Sqrt[b]*c) + Sqrt[I - a]*d)]*Log[c + d*Sqrt[x]] -
d*Sqrt[x]*Log[(-I + a + b*x)/(a + b*x)] + c*Log[c + d*Sqrt[x]]*Log[(-I + a
 + b*x)/(a + b*x)] + d*Sqrt[x]*Log[(I + a + b*x)/(a + b*x)] - c*Log[c + d*
Sqrt[x]]*Log[(I + a + b*x)/(a + b*x)] + c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[
x]))/(Sqrt[b]*c - Sqrt[-I - a]*d)] + c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x])
)/(Sqrt[b]*c + Sqrt[-I - a]*d)] - c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(
Sqrt[b]*c - Sqrt[I - a]*d)] - c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt
[b]*c + Sqrt[I - a]*d)]))/d^2
```

Rubi [A] (verified)

Time = 2.38 (sec) , antiderivative size = 1133, normalized size of antiderivative = 1.63, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5575, 7267, 3008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}\,dx$$

$$\Big\downarrow \text{ 5575}$$

$$\frac{1}{2}i\int \frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)}{c+d\sqrt{x}}dx-\frac{1}{2}i\int \frac{\log\left(\frac{a+bx+i}{a+bx}\right)}{c+d\sqrt{x}}dx$$

$$\Big\downarrow \text{ 7267}$$

$$i\int \frac{\sqrt{x}\log\left(-\frac{-a-bx+i}{a+bx}\right)}{c+d\sqrt{x}}d\sqrt{x}-i\int \frac{\sqrt{x}\log\left(\frac{a+bx+i}{a+bx}\right)}{c+d\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{ 3008}$$

$$i\int\left(\frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)}{d}-\frac{c\log\left(-\frac{-a-bx+i}{a+bx}\right)}{d\left(c+d\sqrt{x}\right)}\right)d\sqrt{x}-$$
$$i\int\left(\frac{\log\left(\frac{a+bx+i}{a+bx}\right)}{d}-\frac{c\log\left(\frac{a+bx+i}{a+bx}\right)}{d\left(c+d\sqrt{x}\right)}\right)d\sqrt{x}$$

$$\Big\downarrow \; 2009$$

$$i\left(-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}d}+\frac{2\sqrt{i-a}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}d}+\frac{c\log\left(\frac{d\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}-\frac{c\log\left(\frac{d\left(\sqrt{-a}-\sqrt{\right.}\right.}{\sqrt{b}c+\sqrt{-}}\right.}{}\right.$$
$$i\left(-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}d}+\frac{2\sqrt{a+i}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+i}}\right)}{\sqrt{b}d}+\frac{c\log\left(\frac{d\left(\sqrt{-a-i}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-a-i}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}-\frac{c\log\left(\frac{d\left(\sqrt{-a}-\right.}{\sqrt{b}c+\sqrt{}}\right.}{}\right.$$

input
```
Int[ArcCot[a + b*x]/(c + d*Sqrt[x]),x]
```

output
```
(-I)*((-2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]*d) + (2*Sqrt
[I + a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/(Sqrt[b]*d) + (c*Log[(d*(Sq
rt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-I - a]*d)]*Log[c + d*Sqr
t[x]])/d^2 - (c*Log[(d*(Sqrt[-a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-a]
*d)]*Log[c + d*Sqrt[x]])/d^2 + (c*Log[-((d*(Sqrt[-I - a] + Sqrt[b]*Sqrt[x]
))/(Sqrt[b]*c - Sqrt[-I - a]*d))]*Log[c + d*Sqrt[x]])/d^2 - (c*Log[-((d*(S
qrt[-a] + Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-a]*d))]*Log[c + d*Sqrt[x]])
/d^2 + (Sqrt[x]*Log[(I + a + b*x)/(a + b*x)])/d - (c*Log[c + d*Sqrt[x]]*Lo
g[(I + a + b*x)/(a + b*x)])/d^2 + (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/
(Sqrt[b]*c - Sqrt[-I - a]*d)])/d^2 + (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]
))/(Sqrt[b]*c + Sqrt[-I - a]*d)])/d^2 - (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt
[x]))/(Sqrt[b]*c - Sqrt[-a]*d)])/d^2 - (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[
x]))/(Sqrt[b]*c + Sqrt[-a]*d)])/d^2) + I*((-2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt
[x])/Sqrt[a]])/(Sqrt[b]*d) + (2*Sqrt[I - a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt
[I - a]])/(Sqrt[b]*d) + (c*Log[(d*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b
]*c + Sqrt[I - a]*d)]*Log[c + d*Sqrt[x]])/d^2 - (c*Log[(d*(Sqrt[-a] - Sqrt
[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-a]*d)]*Log[c + d*Sqrt[x]])/d^2 + (c*Log[-
((d*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c - Sqrt[I - a]*d))]*Log[c +
 d*Sqrt[x]])/d^2 - (c*Log[-((d*(Sqrt[-a] + Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c -
Sqrt[-a]*d))]*Log[c + d*Sqrt[x]])/d^2 + (Sqrt[x]*Log[-((I - a - b*x)/(a...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3008
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

rule 5575
```
Int[ArcCot[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[
I/2   Int[Log[(-I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] - Simp[I/2   In
t[Log[(I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x
] && RationalQ[n]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.32 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.53

method	result
derivativedivides	$\frac{2\,\mathrm{arccot}(bx+a)\sqrt{x}}{d}-\frac{2\,\mathrm{arccot}(bx+a)c\ln(c+d\sqrt{x})}{d^2}+\frac{4b\left(d^2\left(\sum_{\_R=\mathrm{RootOf}\left(b^2\_Z^4-4b^2c\_Z^3+\left(2ab\,d^2+6b^2c^2\right)\_Z^2+\right.}\right.\right.}{}$
default	$\frac{2\,\mathrm{arccot}(bx+a)\sqrt{x}}{d}-\frac{2\,\mathrm{arccot}(bx+a)c\ln(c+d\sqrt{x})}{d^2}+\frac{4b\left(d^2\left(\sum_{\_R=\mathrm{RootOf}\left(b^2\_Z^4-4b^2c\_Z^3+\left(2ab\,d^2+6b^2c^2\right)\_Z^2+\right.}\right.\right.}{}$

input
```
int(arccot(b*x+a)/(c+d*x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2*arccot(b*x+a)/d*x^(1/2)-2*arccot(b*x+a)*c/d^2*ln(c+d*x^(1/2))+4*b/d^2*(1
/4*d^2/b*sum((_R^2-2*_R*c+c^2)/(_R^3*b-3*_R^2*b*c+_R*a*d^2+3*_R*b*c^2-a*c*
d^2-b*c^3)*ln(d*x^(1/2)-_R+c),_R=RootOf(b^2*_Z^4-4*b^2*c*_Z^3+(2*a*b*d^2+6
*b^2*c^2)*_Z^2+(-4*a*b*c*d^2-4*b^2*c^3)*_Z+a^2*d^4+2*a*b*c^2*d^2+b^2*c^4+d
^4))-1/4*c*d^2/b*sum(1/(_R1^2*b-2*_R1*b*c+a*d^2+b*c^2)*(ln(c+d*x^(1/2))*ln
((-d*x^(1/2)+_R1-c)/_R1)+dilog((-d*x^(1/2)+_R1-c)/_R1)),_R1=RootOf(b^2*_Z^
4-4*b^2*c*_Z^3+(2*a*b*d^2+6*b^2*c^2)*_Z^2+(-4*a*b*c*d^2-4*b^2*c^3)*_Z+a^2*
d^4+2*a*b*c^2*d^2+b^2*c^4+d^4))
```

Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{d\sqrt{x}+c}\,dx$$

input
```
integrate(arccot(b*x+a)/(c+d*x^(1/2)),x, algorithm="fricas")
```

output
```
integral((d*sqrt(x)*arccot(b*x + a) - c*arccot(b*x + a))/(d^2*x - c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = \text{Timed out}$$

input
```
integrate(acot(b*x+a)/(c+d*x**(1/2)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{d\sqrt{x}+c}\,dx$$

input
```
integrate(arccot(b*x+a)/(c+d*x^(1/2)),x, algorithm="maxima")
```

output
```
integrate(arccot(b*x + a)/(d*sqrt(x) + c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccot(b*x+a)/(c+d*x^(1/2)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = \int \frac{\mathrm{acot}(a+b\,x)}{c+d\,\sqrt{x}}\,dx$$

input
```
int(acot(a + b*x)/(c + d*x^(1/2)),x)
```

output
```
int(acot(a + b*x)/(c + d*x^(1/2)), x)
```

Reduce [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = \int \frac{acot(bx+a)}{\sqrt{x}\,d+c}dx$$

input
```
int(acot(b*x+a)/(c+d*x^(1/2)),x)
```

output
```
int(acot(a + b*x)/(sqrt(x)*d + c),x)
```

3.43 $\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 811

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \frac{2i\sqrt{i+a}\,d\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}c^2} - \frac{2i\sqrt{i-a}\,d\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}c^2}$$
$$+ \frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3}$$
$$- \frac{id^2\log\left(\frac{c\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3}$$
$$+ \frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c-\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3}$$
$$- \frac{id^2\log\left(\frac{c\left(\sqrt{i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}c-\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} - \frac{id\sqrt{x}\log\left(-\frac{i-a-bx}{a+bx}\right)}{c^2}$$
$$- \frac{i(i-a-bx)\log\left(-\frac{i-a-bx}{a+bx}\right)}{2bc} + \frac{id^2\log\left(d+c\sqrt{x}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{c^3}$$
$$+ \frac{\log(a+bx)}{bc} + \frac{id\sqrt{x}\log\left(\frac{i+a+bx}{a+bx}\right)}{c^2}$$
$$- \frac{i(i+a+bx)\log\left(\frac{i+a+bx}{a+bx}\right)}{2bc} - \frac{id^2\log\left(d+c\sqrt{x}\right)\log\left(\frac{i+a+bx}{a+bx}\right)}{c^3}$$
$$+ \frac{id^2\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{-i-a}c-\sqrt{b}d}\right)}{c^3} - \frac{id^2\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{i-a}c-\sqrt{b}d}\right)}{c^3}$$
$$+ \frac{id^2\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{-i-a}c+\sqrt{b}d}\right)}{c^3} - \frac{id^2\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{i-a}c+\sqrt{b}d}\right)}{c^3}$$

output
```
I*d*x^(1/2)*ln((I+a+b*x)/(b*x+a))/c^2+I*d^2*ln(c*((-I-a)^(1/2)-b^(1/2)*x^(
1/2))/((-I-a)^(1/2)*c+b^(1/2)*d))*ln(d+c*x^(1/2))/c^3-1/2*I*(I-a-b*x)*ln(-
(I-a-b*x)/(b*x+a))/b/c-I*d^2*ln(c*((I-a)^(1/2)-b^(1/2)*x^(1/2))/((I-a)^(1/
2)*c+b^(1/2)*d))*ln(d+c*x^(1/2))/c^3+I*d^2*polylog(2,b^(1/2)*(d+c*x^(1/2))
/((-I-a)^(1/2)*c+b^(1/2)*d))/c^3-I*d^2*ln(d+c*x^(1/2))*ln((I+a+b*x)/(b*x+a
))/c^3-I*d^2*ln(c*((I-a)^(1/2)+b^(1/2)*x^(1/2))/((I-a)^(1/2)*c-b^(1/2)*d))
*ln(d+c*x^(1/2))/c^3+I*d^2*polylog(2,-b^(1/2)*(d+c*x^(1/2))/((-I-a)^(1/2)*
c-b^(1/2)*d))/c^3-2*I*(I-a)^(1/2)*d*arctanh(b^(1/2)*x^(1/2)/(I-a)^(1/2))/b
^(1/2)/c^2+ln(b*x+a)/b/c-1/2*I*(I+a+b*x)*ln((I+a+b*x)/(b*x+a))/b/c-I*d*x^(
1/2)*ln(-(I-a-b*x)/(b*x+a))/c^2+2*I*(I+a)^(1/2)*d*arctan(b^(1/2)*x^(1/2)/(
I+a)^(1/2))/b^(1/2)/c^2+I*d^2*ln(c*((-I-a)^(1/2)+b^(1/2)*x^(1/2))/((-I-a)^
(1/2)*c-b^(1/2)*d))*ln(d+c*x^(1/2))/c^3-I*d^2*polylog(2,-b^(1/2)*(d+c*x^(1
/2))/((I-a)^(1/2)*c-b^(1/2)*d))/c^3-I*d^2*polylog(2,b^(1/2)*(d+c*x^(1/2))/
((I-a)^(1/2)*c+b^(1/2)*d))/c^3+I*d^2*ln(d+c*x^(1/2))*ln(-(I-a-b*x)/(b*x+a)
)/c^3
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 809, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx$$

$$= \frac{4i\sqrt{i+a}\sqrt{b}cd\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)-4i\sqrt{i-a}\sqrt{b}cd\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)+2ibd^2\log\left(\frac{c\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{\vphantom{x}}\right.}{}$$

input
```
Integrate[ArcCot[a + b*x]/(c + d/Sqrt[x]),x]
```

output
```
((4*I)*Sqrt[I + a]*Sqrt[b]*c*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]] - (4*
I)*Sqrt[I - a]*Sqrt[b]*c*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]] + (2*I)*
b*d^2*Log[(c*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[b]*d
)]*Log[d + c*Sqrt[x]] - (2*I)*b*d^2*Log[(c*(Sqrt[I - a] - Sqrt[b]*Sqrt[x])
)/(Sqrt[I - a]*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]] + (2*I)*b*d^2*Log[(c*(Sq
rt[-I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d)]*Log[d + c*Sqr
t[x]] - (2*I)*b*d^2*Log[(c*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[I - a]*c
 - Sqrt[b]*d)]*Log[d + c*Sqrt[x]] + c^2*Log[I - a - b*x] + I*a*c^2*Log[I -
 a - b*x] - (2*I)*b*c*d*Sqrt[x]*Log[(-I + a + b*x)/(a + b*x)] + I*b*c^2*x*
Log[(-I + a + b*x)/(a + b*x)] + (2*I)*b*d^2*Log[d + c*Sqrt[x]]*Log[(-I + a
 + b*x)/(a + b*x)] + c^2*Log[I + a + b*x] - I*a*c^2*Log[I + a + b*x] + (2*
I)*b*c*d*Sqrt[x]*Log[(I + a + b*x)/(a + b*x)] - I*b*c^2*x*Log[(I + a + b*x
)/(a + b*x)] - (2*I)*b*d^2*Log[d + c*Sqrt[x]]*Log[(I + a + b*x)/(a + b*x)]
 + (2*I)*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(-(Sqrt[-I - a]*c) + S
qrt[b]*d)] + (2*I)*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-I - a
]*c + Sqrt[b]*d)] - (2*I)*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(-(Sq
rt[I - a]*c) + Sqrt[b]*d)] - (2*I)*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x
]))/(Sqrt[I - a]*c + Sqrt[b]*d)])/(2*b*c^3)
```

Rubi [A] (verified)

Time = 2.66 (sec) , antiderivative size = 1316, normalized size of antiderivative = 1.62, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5575, 7267, 3008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}dx$$

$$\Big\downarrow \text{5575}$$

$$\frac{1}{2}i\int \frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)}{c+\frac{d}{\sqrt{x}}}dx-\frac{1}{2}i\int \frac{\log\left(\frac{a+bx+i}{a+bx}\right)}{c+\frac{d}{\sqrt{x}}}dx$$

$$\Big\downarrow \text{7267}$$

$$i\int \frac{x\log\left(-\frac{-a-bx+i}{a+bx}\right)}{\sqrt{x}c+d}d\sqrt{x}-i\int \frac{x\log\left(\frac{a+bx+i}{a+bx}\right)}{\sqrt{x}c+d}d\sqrt{x}$$

$$\Big\downarrow \; 3008$$

$$i\int \left(\frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)d^2}{c^2\left(\sqrt{x}c+d\right)}-\frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)d}{c^2}+\frac{\sqrt{x}\log\left(-\frac{-a-bx+i}{a+bx}\right)}{c}\right)d\sqrt{x}-$$
$$i\int \left(\frac{\log\left(\frac{a+bx+i}{a+bx}\right)d^2}{c^2\left(\sqrt{x}c+d\right)}-\frac{\log\left(\frac{a+bx+i}{a+bx}\right)d}{c^2}+\frac{\sqrt{x}\log\left(\frac{a+bx+i}{a+bx}\right)}{c}\right)d\sqrt{x}$$

$$\Big\downarrow \; 2009$$

$$i\left(-\frac{\log\left(\frac{c\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}c+\sqrt{b}d}\right)\log\left(\sqrt{x}c+d\right)d^2}{c^3}+\frac{\log\left(\frac{c\left(\sqrt{-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-a}c+\sqrt{b}d}\right)\log\left(\sqrt{x}c+d\right)d^2}{c^3}-\frac{\log\left(\frac{c\left(\sqrt{i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}c-\sqrt{b}d}\right)\log}{c^3}\right.$$

$$i\left(-\frac{\log\left(\frac{c\left(\sqrt{-a-i}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-a-i}c+\sqrt{b}d}\right)\log\left(\sqrt{x}c+d\right)d^2}{c^3}+\frac{\log\left(\frac{c\left(\sqrt{-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-a}c+\sqrt{b}d}\right)\log\left(\sqrt{x}c+d\right)d^2}{c^3}-\frac{\log\left(\frac{c\left(\sqrt{-a-i}+\sqrt{b}\sqrt{x}\right)}{\sqrt{-a-i}c-\sqrt{b}d}\right)}{c^3}\right.$$

input
```
Int[ArcCot[a + b*x]/(c + d/Sqrt[x]),x]
```

output
```
(-I)*((2*Sqrt[a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]*c^2) - (2*S
qrt[I + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/(Sqrt[b]*c^2) - (d^2*L
og[(c*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[b]*d)]*Log[
d + c*Sqrt[x]])/c^3 + (d^2*Log[(c*(Sqrt[-a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-a]*
c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 - (d^2*Log[(c*(Sqrt[-I - a] + Sqrt
[b]*Sqrt[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 + (d^2
*Log[(c*(Sqrt[-a] + Sqrt[b]*Sqrt[x]))/(Sqrt[-a]*c - Sqrt[b]*d)]*Log[d + c*
Sqrt[x]])/c^3 - (a*Log[a + b*x])/(2*b*c) + ((I + a)*Log[I + a + b*x])/(2*b
*c) - (d*Sqrt[x]*Log[(I + a + b*x)/(a + b*x)])/c^2 + (x*Log[(I + a + b*x)/
(a + b*x)])/(2*c) + (d^2*Log[d + c*Sqrt[x]]*Log[(I + a + b*x)/(a + b*x)])/
c^3 - (d^2*PolyLog[2, -((Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-I - a]*c - Sqrt[b
]*d))])/c^3 + (d^2*PolyLog[2, -((Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-a]*c - Sq
rt[b]*d))])/c^3 - (d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-I - a]*
c + Sqrt[b]*d)])/c^3 + (d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-a]
*c + Sqrt[b]*d)])/c^3) + I*((2*Sqrt[a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]
)/(Sqrt[b]*c^2) - (2*Sqrt[I - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]])
/(Sqrt[b]*c^2) - (d^2*Log[(c*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[I - a]
*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 + (d^2*Log[(c*(Sqrt[-a] - Sqrt[b]
*Sqrt[x]))/(Sqrt[-a]*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 - (d^2*Log[(c
*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[I - a]*c - Sqrt[b]*d)]*Log[d + ...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3008
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

rule 5575
```
Int[ArcCot[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[
I/2   Int[Log[(-I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] - Simp[I/2   In
t[Log[(I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x
] && RationalQ[n]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.32 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.48

method	result
derivativedivides	$\frac{\operatorname{arccot}(bx+a)x}{c}-\frac{2\operatorname{arccot}(bx+a)d\sqrt{x}}{c^2}+\frac{2\operatorname{arccot}(bx+a)d^2\ln\left(d+c\sqrt{x}\right)}{c^3}+\frac{4b\left(-\frac{c\left(\_R=\mathrm{RootOf}\left(b^2\_Z^4-4b^2d\_Z^3\right.\right.}{}\right.}{}$
default	$\frac{\operatorname{arccot}(bx+a)x}{c}-\frac{2\operatorname{arccot}(bx+a)d\sqrt{x}}{c^2}+\frac{2\operatorname{arccot}(bx+a)d^2\ln\left(d+c\sqrt{x}\right)}{c^3}+\frac{4b\left(-\frac{c\left(\_R=\mathrm{RootOf}\left(b^2\_Z^4-4b^2d\_Z^3\right.\right.}{}\right.}{}$

input
```
int(arccot(b*x+a)/(c+d/x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
arccot(b*x+a)/c*x-2*arccot(b*x+a)/c^2*d*x^(1/2)+2*arccot(b*x+a)*d^2/c^3*ln
(d+c*x^(1/2))+4*b/c^2*(-1/8*c/b*sum((-_R^3+5*_R^2*d-7*_R*d^2+3*d^3)/(_R^3*
b-3*_R^2*b*d+_R*a*c^2+3*_R*b*d^2-a*c^2*d-b*d^3)*ln(c*x^(1/2)-_R+d),_R=Root
Of(b^2*_Z^4-4*b^2*d*_Z^3+(2*a*b*c^2+6*b^2*d^2)*_Z^2+(-4*a*b*c^2*d-4*b^2*d^
3)*_Z+a^2*c^4+2*a*b*c^2*d^2+b^2*d^4+c^4))+1/4*c*d^2/b*sum(1/(_R1^2*b-2*_R1
*b*d+a*c^2+b*d^2)*(ln(d+c*x^(1/2))*ln((-c*x^(1/2)+_R1-d)/_R1)+dilog((-c*x^
(1/2)+_R1-d)/_R1)),_R1=RootOf(b^2*_Z^4-4*b^2*d*_Z^3+(2*a*b*c^2+6*b^2*d^2)*
_Z^2+(-4*a*b*c^2*d-4*b^2*d^3)*_Z+a^2*c^4+2*a*b*c^2*d^2+b^2*d^4+c^4)))
```

Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{c+\frac{d}{\sqrt{x}}}\,dx$$

input
```
integrate(arccot(b*x+a)/(c+d/x^(1/2)),x, algorithm="fricas")
```

output
```
integral((c*x*arccot(b*x + a) - d*sqrt(x)*arccot(b*x + a))/(c^2*x - d^2),
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \text{Timed out}$$

input
```
integrate(acot(b*x+a)/(c+d/x**(1/2)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{c+\frac{d}{\sqrt{x}}}\,dx$$

input
```
integrate(arccot(b*x+a)/(c+d/x^(1/2)),x, algorithm="maxima")
```

output
```
integrate(arccot(b*x + a)/(c + d/sqrt(x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccot(b*x+a)/(c+d/x^(1/2)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \int \frac{\mathrm{acot}(a+b\,x)}{c+\frac{d}{\sqrt{x}}}\,dx$$

input
```
int(acot(a + b*x)/(c + d/x^(1/2)),x)
```

output
```
int(acot(a + b*x)/(c + d/x^(1/2)), x)
```

Reduce [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \int \frac{\sqrt{x}\,acot(bx+a)}{\sqrt{x}\,c+d}dx$$

input
```
int(acot(b*x+a)/(c+d/x^(1/2)),x)
```

output
```
int((sqrt(x)*acot(a + b*x))/(sqrt(x)*c + d),x)
```

3.44 $\int \frac{a+b\cot^{-1}(c+dx)}{e+fx+gx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 398

$$\begin{aligned}\int \frac{a+b\cot^{-1}(c+dx)}{e+fx+gx^2}\,dx &= \frac{(a+b\cot^{-1}(c+dx))\log\left(-\frac{2\left(2cg-d\left(f-\sqrt{f^2-4eg}\right)-2g(c+dx)\right)}{\left(df+2ig-2cg-d\sqrt{f^2-4eg}\right)(1-i(c+dx))}\right)}{\sqrt{f^2-4eg}} \\ &\quad - \frac{(a+b\cot^{-1}(c+dx))\log\left(-\frac{2\left(2cg-d\left(f+\sqrt{f^2-4eg}\right)-2g(c+dx)\right)}{\left(2(i-c)g+d\left(f+\sqrt{f^2-4eg}\right)\right)(1-i(c+dx))}\right)}{\sqrt{f^2-4eg}} \\ &\quad + \frac{ib\,\text{PolyLog}\left(2,1+\frac{2\left(2cg-d\left(f-\sqrt{f^2-4eg}\right)-2g(c+dx)\right)}{\left(df+2ig-2cg-d\sqrt{f^2-4eg}\right)(1-i(c+dx))}\right)}{2\sqrt{f^2-4eg}} \\ &\quad - \frac{ib\,\text{PolyLog}\left(2,1+\frac{2\left(2cg-d\left(f+\sqrt{f^2-4eg}\right)-2g(c+dx)\right)}{\left(2(i-c)g+d\left(f+\sqrt{f^2-4eg}\right)\right)(1-i(c+dx))}\right)}{2\sqrt{f^2-4eg}}\end{aligned}$$

output
```
(a+b*arccot(d*x+c))*ln((-4*c*g+2*d*(f-(-4*e*g+f^2)^(1/2))+4*g*(d*x+c))/(d*
f+2*I*g-2*c*g-d*(-4*e*g+f^2)^(1/2))/(1-I*(d*x+c)))/(-4*e*g+f^2)^(1/2)-(a+b
*arccot(d*x+c))*ln((-4*c*g+2*d*(f+(-4*e*g+f^2)^(1/2))+4*g*(d*x+c))/(2*(I-c
)*g+d*(f+(-4*e*g+f^2)^(1/2)))/(1-I*(d*x+c)))/(-4*e*g+f^2)^(1/2)+1/2*I*b*po
lylog(2,1+2*(2*c*g-d*(f-(-4*e*g+f^2)^(1/2))-2*g*(d*x+c))/(d*f+2*I*g-2*c*g-
d*(-4*e*g+f^2)^(1/2))/(1-I*(d*x+c)))/(-4*e*g+f^2)^(1/2)-1/2*I*b*polylog(2,
1+2*(2*c*g-d*(f+(-4*e*g+f^2)^(1/2))-2*g*(d*x+c))/(2*(I-c)*g+d*(f+(-4*e*g+f
^2)^(1/2)))/(1-I*(d*x+c)))/(-4*e*g+f^2)^(1/2)
```

Mathematica [F]

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx+gx^2}\,dx=\int \frac{a+b\cot^{-1}(c+dx)}{e+fx+gx^2}\,dx$$

input
```
Integrate[(a + b*ArcCot[c + d*x])/(e + f*x + g*x^2),x]
```

output
```
Integrate[(a + b*ArcCot[c + d*x])/(e + f*x + g*x^2), x]
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx+gx^2}\,dx$$

$$\Big\downarrow \text{ 7279}$$

$$\int \left(\frac{a}{e+fx+gx^2}+\frac{b\cot^{-1}(c+dx)}{e+fx+gx^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2a\operatorname{arctanh}\left(\frac{f+2gx}{\sqrt{f^2-4eg}}\right)}{\sqrt{f^2-4eg}}+\frac{ib\operatorname{PolyLog}\left(2,\frac{2\left(2cg-2(c+dx)g-d\left(f-\sqrt{f^2-4eg}\right)\right)}{\left(fd-\sqrt{f^2-4eg}d-2cg+2ig\right)(1-i(c+dx))}+1\right)}{2\sqrt{f^2-4eg}}-\frac{ib\operatorname{PolyLog}\left(2,\frac{2\left(2cg-2(c+dx)g-d\left(f+\sqrt{f^2-4eg}\right)\right)}{\left(2(i-c)g+d\left(f+\sqrt{f^2-4eg}\right)\right)(1-i(c+dx))}+1\right)}{2\sqrt{f^2-4eg}}+\frac{b\cot^{-1}(c+dx)\log\left(-\frac{2\left(-2g(c+dx)+2cg-d\left(f-\sqrt{f^2-4eg}\right)\right)}{(1-i(c+dx))\left(-2cg-d\sqrt{f^2-4eg}+df+2ig\right)}\right)}{\sqrt{f^2-4eg}}-\frac{b\cot^{-1}(c+dx)\log\left(-\frac{2\left(-2g(c+dx)+2cg-d\left(\sqrt{f^2-4eg}+f\right)\right)}{(1-i(c+dx))\left(d\left(\sqrt{f^2-4eg}+f\right)+2(-c+i)g\right)}\right)}{\sqrt{f^2-4eg}}$$

input
```
Int[(a + b*ArcCot[c + d*x])/(e + f*x + g*x^2),x]
```

output
```
(-2*a*ArcTanh[(f + 2*g*x)/Sqrt[f^2 - 4*e*g]])/Sqrt[f^2 - 4*e*g] + (b*ArcCo
t[c + d*x]*Log[(-2*(2*c*g - d*(f - Sqrt[f^2 - 4*e*g]) - 2*g*(c + d*x)))/((
d*f + (2*I)*g - 2*c*g - d*Sqrt[f^2 - 4*e*g])*(1 - I*(c + d*x)))])/Sqrt[f^2
 - 4*e*g] - (b*ArcCot[c + d*x]*Log[(-2*(2*c*g - d*(f + Sqrt[f^2 - 4*e*g])
- 2*g*(c + d*x)))/((2*(I - c)*g + d*(f + Sqrt[f^2 - 4*e*g]))*(1 - I*(c + d
*x)))])/Sqrt[f^2 - 4*e*g] + ((I/2)*b*PolyLog[2, 1 + (2*(2*c*g - d*(f - Sqr
t[f^2 - 4*e*g]) - 2*g*(c + d*x)))/((d*f + (2*I)*g - 2*c*g - d*Sqrt[f^2 - 4
*e*g])*(1 - I*(c + d*x)))])/Sqrt[f^2 - 4*e*g] - ((I/2)*b*PolyLog[2, 1 + (2
*(2*c*g - d*(f + Sqrt[f^2 - 4*e*g]) - 2*g*(c + d*x)))/((2*(I - c)*g + d*(f
 + Sqrt[f^2 - 4*e*g]))*(1 - I*(c + d*x)))])/Sqrt[f^2 - 4*e*g]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1036 vs. 2(358) = 716.

Time = 3.80 (sec) , antiderivative size = 1037, normalized size of antiderivative = 2.61

method	result
risch	$-\frac{id\arctan\left(\frac{2icg-ifd+2(-idx-ic+1)g-2g}{\sqrt{-4d^2eg+d^2f^2}}\right)b\pi}{\sqrt{-4d^2eg+d^2f^2}}-\frac{2id\arctan\left(\frac{2icg-ifd+2(-idx-ic+1)g-2g}{\sqrt{-4d^2eg+d^2f^2}}\right)a}{\sqrt{-4d^2eg+d^2f^2}}-\frac{db\ln(-idx-ic+1)}{}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((a+b*arccot(d*x+c))/(g*x^2+f*x+e),x,method=_RETURNVERBOSE)
```

output
```
-I*d/(-4*d^2*e*g+d^2*f^2)^(1/2)*arctan((2*I*c*g-I*f*d+2*(1-I*c-I*d*x)*g-2*
g)/(-4*d^2*e*g+d^2*f^2)^(1/2))*b*Pi-2*I*d/(-4*d^2*e*g+d^2*f^2)^(1/2)*arcta
n((2*I*c*g-I*f*d+2*(1-I*c-I*d*x)*g-2*g)/(-4*d^2*e*g+d^2*f^2)^(1/2))*a-1/2*
d*b*ln(1-I*c-I*d*x)/(4*d^2*e*g-d^2*f^2)^(1/2)*ln((2*I*c*g-I*f*d+2*(1-I*c-I
*d*x)*g-(4*d^2*e*g-d^2*f^2)^(1/2)-2*g)/(2*I*c*g-I*f*d-(4*d^2*e*g-d^2*f^2)^
(1/2)-2*g))+1/2*d*b*ln(1-I*c-I*d*x)/(4*d^2*e*g-d^2*f^2)^(1/2)*ln((2*I*c*g-
I*f*d+2*(1-I*c-I*d*x)*g+(4*d^2*e*g-d^2*f^2)^(1/2)-2*g)/(2*I*c*g-I*f*d+(4*d
^2*e*g-d^2*f^2)^(1/2)-2*g))-1/2*d*b/(4*d^2*e*g-d^2*f^2)^(1/2)*dilog((2*I*c
*g-I*f*d+2*(1-I*c-I*d*x)*g-(4*d^2*e*g-d^2*f^2)^(1/2)-2*g)/(2*I*c*g-I*f*d-(
4*d^2*e*g-d^2*f^2)^(1/2)-2*g))+1/2*d*b/(4*d^2*e*g-d^2*f^2)^(1/2)*dilog((2*
I*c*g-I*f*d+2*(1-I*c-I*d*x)*g+(4*d^2*e*g-d^2*f^2)^(1/2)-2*g)/(2*I*c*g-I*f*
d+(4*d^2*e*g-d^2*f^2)^(1/2)-2*g))-1/2*b*d*ln(1+I*c+I*d*x)/(4*d^2*e*g-d^2*f
^2)^(1/2)*ln((2*I*c*g-I*f*d-2*(1+I*c+I*d*x)*g+(4*d^2*e*g-d^2*f^2)^(1/2)+2*
g)/(2*I*c*g-I*f*d+2*g+(4*d^2*e*g-d^2*f^2)^(1/2)))+1/2*b*d*ln(1+I*c+I*d*x)/
(4*d^2*e*g-d^2*f^2)^(1/2)*ln((2*I*c*g-I*f*d-2*(1+I*c+I*d*x)*g-(4*d^2*e*g-d
^2*f^2)^(1/2)+2*g)/(2*I*c*g-I*f*d+2*g-(4*d^2*e*g-d^2*f^2)^(1/2)))+1/2*b*d/
(4*d^2*e*g-d^2*f^2)^(1/2)*dilog((2*I*c*g-I*f*d-2*(1+I*c+I*d*x)*g-(4*d^2*e*
g-d^2*f^2)^(1/2)+2*g)/(2*I*c*g-I*f*d+2*g-(4*d^2*e*g-d^2*f^2)^(1/2)))-1/2*b
*d/(4*d^2*e*g-d^2*f^2)^(1/2)*dilog((2*I*c*g-I*f*d-2*(1+I*c+I*d*x)*g+(4*d^2
*e*g-d^2*f^2)^(1/2)+2*g)/(2*I*c*g-I*f*d+2*g+(4*d^2*e*g-d^2*f^2)^(1/2)))
```

Fricas [F]

$$\int \frac{a + b\cot^{-1}(c + dx)}{e + fx + gx^2}\,dx = \int \frac{b\,\mathrm{arccot}\,(dx + c) + a}{gx^2 + fx + e}\,dx$$

input
```
integrate((a+b*arccot(d*x+c))/(g*x^2+f*x+e),x, algorithm="fricas")
```

output
```
integral((b*arccot(d*x + c) + a)/(g*x^2 + f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\cot^{-1}(c + dx)}{e + fx + gx^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acot(d*x+c))/(g*x**2+f*x+e),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\cot^{-1}(c + dx)}{e + fx + gx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccot(d*x+c))/(g*x^2+f*x+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*e*g-f^2>0)', see `assume?` for
 more deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\cot^{-1}(c + dx)}{e + fx + gx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccot(d*x+c))/(g*x^2+f*x+e),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Modgcd: no suitable evaluation poin
tModgcd: no suitable evaluation point Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\cot^{-1}(c + dx)}{e + fx + gx^2}\,dx = \int \frac{a + b\,\mathrm{acot}(c + d\,x)}{g\,x^2 + f\,x + e}\,dx$$

input
```
int((a + b*acot(c + d*x))/(e + f*x + g*x^2),x)
```

output
```
int((a + b*acot(c + d*x))/(e + f*x + g*x^2), x)
```

Reduce [F]

$$\int \frac{a + b\cot^{-1}(c + dx)}{e + fx + gx^2}\,dx = \text{Too large to display}$$

input
```
int((a+b*acot(d*x+c))/(g*x^2+f*x+e),x)
```

output

```
( - 4*acot(c + d*x)**2*b*d*e*g + acot(c + d*x)**2*b*d*f**2 + 4*sqrt(4*e*g
- f**2)*atan((f + 2*g*x)/sqrt(4*e*g - f**2))*a*g + 8*int(atan(1/(c + d*x))
/(c**2*e + c**2*f*x + c**2*g*x**2 + 2*c*d*e*x + 2*c*d*f*x**2 + 2*c*d*g*x**
3 + d**2*e*x**2 + d**2*f*x**3 + d**2*g*x**4 + e + f*x + g*x**2),x)*b*c**2*
e*g**2 - 2*int(atan(1/(c + d*x))/(c**2*e + c**2*f*x + c**2*g*x**2 + 2*c*d*
e*x + 2*c*d*f*x**2 + 2*c*d*g*x**3 + d**2*e*x**2 + d**2*f*x**3 + d**2*g*x**
4 + e + f*x + g*x**2),x)*b*c**2*f**2*g - 8*int(atan(1/(c + d*x))/(c**2*e +
 c**2*f*x + c**2*g*x**2 + 2*c*d*e*x + 2*c*d*f*x**2 + 2*c*d*g*x**3 + d**2*e
*x**2 + d**2*f*x**3 + d**2*g*x**4 + e + f*x + g*x**2),x)*b*d**2*e**2*g + 2
*int(atan(1/(c + d*x))/(c**2*e + c**2*f*x + c**2*g*x**2 + 2*c*d*e*x + 2*c*
d*f*x**2 + 2*c*d*g*x**3 + d**2*e*x**2 + d**2*f*x**3 + d**2*g*x**4 + e + f*
x + g*x**2),x)*b*d**2*e*f**2 + 8*int(atan(1/(c + d*x))/(c**2*e + c**2*f*x
+ c**2*g*x**2 + 2*c*d*e*x + 2*c*d*f*x**2 + 2*c*d*g*x**3 + d**2*e*x**2 + d*
*2*f*x**3 + d**2*g*x**4 + e + f*x + g*x**2),x)*b*e*g**2 - 2*int(atan(1/(c
+ d*x))/(c**2*e + c**2*f*x + c**2*g*x**2 + 2*c*d*e*x + 2*c*d*f*x**2 + 2*c*
d*g*x**3 + d**2*e*x**2 + d**2*f*x**3 + d**2*g*x**4 + e + f*x + g*x**2),x)*
b*f**2*g + 16*int((atan(1/(c + d*x))*x)/(c**2*e + c**2*f*x + c**2*g*x**2 +
 2*c*d*e*x + 2*c*d*f*x**2 + 2*c*d*g*x**3 + d**2*e*x**2 + d**2*f*x**3 + d**
2*g*x**4 + e + f*x + g*x**2),x)*b*c*d*e*g**2 - 4*int((atan(1/(c + d*x))*x)
/(c**2*e + c**2*f*x + c**2*g*x**2 + 2*c*d*e*x + 2*c*d*f*x**2 + 2*c*d*g*...
```

3.45 $\int \frac{\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 132

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = -\frac{2i\cot^{-1}(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b} - \frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b} + \frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}$$

output
```
-2*I*arccot(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b-I*pol
ylog(2,-I*(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b+I*polylog(2,I*(1+I*(b
*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.96

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = -\frac{\sqrt{1+a^2+2abx+b^2x^2}\left(\cot^{-1}(a+bx)\left(\log\left(1-e^{i\cot^{-1}(a+bx)}\right)-\log\left(1+e^{i\cot^{-1}(a+bx)}\right)\right)+i\,\mathrm{PolyLog}\right.}{b(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}$$

input
```
Integrate[ArcCot[a + b*x]/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
-((Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(ArcCot[a + b*x]*(Log[1 - E^(I*ArcCot
[a + b*x])] - Log[1 + E^(I*ArcCot[a + b*x])]) + I*PolyLog[2, -E^(I*ArcCot[
a + b*x])] - I*PolyLog[2, E^(I*ArcCot[a + b*x])]))/(b*(a + b*x)*Sqrt[1 + (
a + b*x)^(-2)]))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {5579, 5422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{a^2+2abx+b^2x^2+1}}\,dx$$

$$\Big\downarrow \; 5579$$

$$\frac{\int \frac{\cot^{-1}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)}{b}$$

$$\Big\downarrow \; 5422$$

$$\frac{-2i\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)\cot^{-1}(a+bx)-i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)+i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{b}$$

input
```
Int[ArcCot[a + b*x]/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
((-2*I)*ArcCot[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]
] - I*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]] + I*P
olyLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/b
```

Defintions of rubi rules used

rule 5422
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcCot[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x])]/(c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 -
I*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 GtQ[d, 0]
```

rule 5579
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((A_.) + (B_.)*(x_) + (
C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2)
^q*(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
p, q}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.89

method	result
default	$-\frac{\operatorname{arccot}(bx+a)\ln\left(1-\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}\right)-\operatorname{arccot}(bx+a)\ln\left(\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}+1\right)+i\operatorname{dilog}\left(\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}+1\right)-i\operatorname{dilog}\left(1-\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}\right)}{b}$

input
```
int(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/b*(arccot(b*x+a)*ln(1-(I+a+b*x)/(1+(b*x+a)^2)^(1/2))-arccot(b*x+a)*ln((
I+a+b*x)/(1+(b*x+a)^2)^(1/2)+1)+I*dilog((I+a+b*x)/(1+(b*x+a)^2)^(1/2)+1)-I
*dilog(1-(I+a+b*x)/(1+(b*x+a)^2)^(1/2)))
```

Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx=\int \frac{\operatorname{arccot}\left(bx+a\right)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

input
```
integrate(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fricas
")
```

output
```
integral(arccot(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx=\int \frac{\operatorname{acot}\left(a+bx\right)}{\sqrt{a^2+2abx+b^2x^2+1}}\,dx$$

input
```
integrate(acot(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

output
```
Integral(acot(a + b*x)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx=\int \frac{\operatorname{arccot}\left(bx+a\right)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

input
```
integrate(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(arccot(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

Giac [F]

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

input
```
integrate(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccot(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{acot}(a+b\,x)}{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}\,dx$$

input
```
int(acot(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

output
```
int(acot(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{acot(bx+a)}{\sqrt{b^2x^2+2abx+a^2+1}}dx$$

input
```
int(acot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x)
```

output
```
int(acot(a + b*x)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1),x)
```

3.46 $\int \frac{\cot^{-1}(a+bx)}{\sqrt{(1+a^2)c+2abcx+b^2cx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 216

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = -\frac{2i\sqrt{1+(a+bx)^2}\cot^{-1}(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}} - \frac{i\sqrt{1+(a+bx)^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}} + \frac{i\sqrt{1+(a+bx)^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}}$$

output
```
-2*I*(1+(b*x+a)^2)^(1/2)*arccot(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*
x+a))^(1/2))/b/(c+c*(b*x+a)^2)^(1/2)-I*(1+(b*x+a)^2)^(1/2)*polylog(2,-I*(1
+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b/(c+c*(b*x+a)^2)^(1/2)+I*(1+(b*x+a
)^2)^(1/2)*polylog(2,I*(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b/(c+c*(b*
x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.64

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx =$$

$$-\frac{(1+(a+bx)^2)\left(\cot^{-1}(a+bx)\left(\log\left(1-e^{i\cot^{-1}(a+bx)}\right)-\log\left(1+e^{i\cot^{-1}(a+bx)}\right)\right)+i\,\text{PolyLog}\left(2,-e^{i\cdots}\right.\right.}{b(a+bx)\sqrt{c\,(1+a^2+2abx+b^2x^2)}\sqrt{1+\frac{1}{(a+bx)^2}}}$$

input
```
Integrate[ArcCot[a + b*x]/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2],x]
```

output
```
-(((1 + (a + b*x)^2)*(ArcCot[a + b*x]*(Log[1 - E^(I*ArcCot[a + b*x])] - Lo
g[1 + E^(I*ArcCot[a + b*x])]) + I*PolyLog[2, -E^(I*ArcCot[a + b*x])] - I*P
olyLog[2, E^(I*ArcCot[a + b*x])]))/(b*(a + b*x)*Sqrt[c*(1 + a^2 + 2*a*b*x
+ b^2*x^2)]*Sqrt[1 + (a + b*x)^(-2)]))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5579, 5426, 5422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{(a^2+1)\,c+2abcx+b^2cx^2}}dx$$

$$\Big\downarrow \text{5579}$$

$$\frac{\int \frac{\cot^{-1}(a+bx)}{\sqrt{c(a+bx)^2+c}}d(a+bx)}{b}$$

$$\Big\downarrow \text{5426}$$

$$\frac{\sqrt{(a+bx)^2+1}\int \frac{\cot^{-1}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)}{b\sqrt{c(a+bx)^2+c}}$$

$$\Big\downarrow \text{5422}$$

$$\frac{\sqrt{(a+bx)^2+1}\left(-2i\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)\cot^{-1}(a+bx)-i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)+i\operatorname{PolyLog}\left(2,\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)\right)}{b\sqrt{c(a+bx)^2+c}}$$

input
```
Int[ArcCot[a + b*x]/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2],x]
```

output
```
(Sqrt[1 + (a + b*x)^2]*((-2*I)*ArcCot[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)
]/Sqrt[1 - I*(a + b*x)]] - I*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[
1 - I*(a + b*x)]] + I*PolyLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a +
 b*x)]]))/(b*Sqrt[c + c*(a + b*x)^2])
```

Defintions of rubi rules used

rule 5422
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcCot[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x])]/(c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 -
I*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 GtQ[d, 0]
```

rule 5426
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcCot[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5579
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((A_.) + (B_.)*(x_) + (
C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2)
^q*(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
p, q}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.72

method	result
default	$\frac{i\left(i \operatorname{arccot}(bx+a) \ln\left(1-\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}\right)-i \operatorname{arccot}(bx+a) \ln\left(\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}+1\right)+\operatorname{polylog}\left(2,\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}\right)-\operatorname{polylog}\left(2,-\frac{bx+a+i}{\sqrt{1+(bx+a}\right.\right.}{\sqrt{b^2x^2+2abx+a^2+1}\, bc}$

input
```
int(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
I*(I*arccot(b*x+a)*ln(1-(I+a+b*x)/(1+(b*x+a)^2)^(1/2))-I*arccot(b*x+a)*ln(
(I+a+b*x)/(1+(b*x+a)^2)^(1/2)+1)+polylog(2,(I+a+b*x)/(1+(b*x+a)^2)^(1/2))-
polylog(2,-(I+a+b*x)/(1+(b*x+a)^2)^(1/2)))*(c*(b*x+a-I)*(I+a+b*x))^(1/2)/(
b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b/c
```

Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\operatorname{arccot}\left(bx+a\right)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

input
```
integrate(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x, algorithm
="fricas")
```

output
```
integral(arccot(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\text{Timed out}$$

input
```
integrate(acot(b*x+a)/((a**2+1)*c+2*a*b*c*x+b**2*c*x**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\operatorname{arccot}(bx+a)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

input
```
integrate(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(arccot(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c), x)
```

Giac [F]

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\operatorname{arccot}(bx+a)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

input
```
integrate(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x, algorithm
="giac")
```

output
```
integrate(arccot(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{\operatorname{acot}(a+b\,x)}{\sqrt{c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)}}\,dx$$

input
```
int(acot(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2),x)
```

output
```
int(acot(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \frac{\int \frac{acot(bx+a)}{\sqrt{b^2x^2+2abx+a^2+1}}dx}{\sqrt{c}}$$

input
```
int(acot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x)
```

output
```
int(acot(a + b*x)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1),x)/sqrt(c)
```

3.47 $\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \mathrm{Int}\left(\frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+(a+bx)^2}},x\right)$$

output
```
Defer(Int)(arccot(b*x+a)/(1+(b*x+a)^2)^(1/3),x)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 177 vs. 2(23) = 46.

Time = 0.28 (sec) , antiderivative size = 177, normalized size of antiderivative = 6.32

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$$
$$= \frac{6\,\mathrm{Gamma}\left(\frac{11}{6}\right)\mathrm{Gamma}\left(\frac{7}{3}\right)\left(5(1+a^2+2abx+b^2x^2)\left(-3+2(a+bx)\cot^{-1}(a+bx)\right)+4(a+bx)\cot^{-1}(\cdots\right.}{20b\left(1+a^2+2abx+b^2x\cdots\right.}$$

input
```
Integrate[ArcCot[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3),x]
```

output
```
(6*Gamma[11/6]*Gamma[7/3]*(5*(1 + a^2 + 2*a*b*x + b^2*x^2)*(-3 + 2*(a + b*
x)*ArcCot[a + b*x]) + 4*(a + b*x)*ArcCot[a + b*x]*Hypergeometric2F1[1, 4/3
, 11/6, (1 + a^2 + 2*a*b*x + b^2*x^2)^(-1)]) - 5*2^(1/3)*Sqrt[Pi]*Gamma[5/
3]*HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1 + a^2 + 2*a*b*x + b^2*
x^2)^(-1)])/(20*b*(1 + a^2 + 2*a*b*x + b^2*x^2)^(4/3)*Gamma[11/6]*Gamma[7/
3])
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{a^2+2abx+b^2x^2+1}}dx$$

$$\Big\downarrow \text{5579}$$

$$\frac{\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(a+bx)^2+1}}d(a+bx)}{b}$$

$$\Big\downarrow \text{5561}$$

$$\frac{\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(a+bx)^2+1}}d(a+bx)}{b}$$

input
```
Int[ArcCot[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{arccot}(bx+a)}{(b^2x^2+2abx+a^2+1)^{\frac{1}{3}}}dx$$

input
```
int(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

output
```
int(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

Fricas [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{(b^2x^2+2\,abx+a^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="fricas
")
```

output
```
integral(arccot(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3), x)
```

Sympy [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{acot}(a+bx)}{\sqrt[3]{a^2+2abx+b^2x^2+1}}\,dx$$

input
```
integrate(acot(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/3),x)
```

output
```
Integral(acot(a + b*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(1/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{(b^2x^2+2\,abx+a^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="maxima
")
```

output
```
integrate(arccot(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{(b^2x^2+2\,abx+a^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="giac")
```

output
```
integrate(arccot(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3), x)
```

Mupad [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\mathrm{acot}(a+b\,x)}{{\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}^{1/3}}\,dx$$

input
```
int(acot(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3),x)
```

output
```
int(acot(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{acot(bx+a)}{(b^2x^2+2abx+a^2+1)^{\frac{1}{3}}}dx$$

input
```
int(acot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

output
```
int(acot(a + b*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(1/3),x)
```

3.48 $\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 33

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\text{Int}\left(\frac{\cot^{-1}(a+bx)}{\sqrt[3]{c+c(a+bx)^2}},x\right)$$

output
```
Defer(Int)(arccot(b*x+a)/(c+c*(b*x+a)^2)^(1/3),x)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 180 vs. 2(25) = 50.

Time = 0.14 (sec) , antiderivative size = 180, normalized size of antiderivative = 5.45

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx$$
$$=\frac{c\Big(6\,\text{Gamma}\left(\frac{11}{6}\right)\text{Gamma}\left(\frac{7}{3}\right)\left(5(1+a^2+2abx+b^2x^2)\left(-3+2(a+bx)\cot^{-1}(a+bx)\right)+4(a+bx)\cot^{-}\right.}{20b\left(c\left(1+a^2+2abx+\right.\right.}$$

input
```
Integrate[ArcCot[a + b*x]/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1/3),x]
```

output
```
(c*(6*Gamma[11/6]*Gamma[7/3]*(5*(1 + a^2 + 2*a*b*x + b^2*x^2)*(-3 + 2*(a +
 b*x)*ArcCot[a + b*x]) + 4*(a + b*x)*ArcCot[a + b*x]*Hypergeometric2F1[1,
4/3, 11/6, (1 + a^2 + 2*a*b*x + b^2*x^2)^(-1)]) - 5*2^(1/3)*Sqrt[Pi]*Gamma
[5/3]*HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1 + a^2 + 2*a*b*x + b
^2*x^2)^(-1)]))/(20*b*(c*(1 + a^2 + 2*a*b*x + b^2*x^2))^(4/3)*Gamma[11/6]*
Gamma[7/3])
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(a^2+1)\,c+2abcx+b^2cx^2}}dx$$

$$\Big\downarrow \text{ 5579}$$

$$\frac{\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{c(a+bx)^2+c}}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 5561}$$

$$\frac{\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{c(a+bx)^2+c}}d(a+bx)}{b}$$

input
```
Int[ArcCot[a + b*x]/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1/3),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{arccot}(bx+a)}{\left((a^2+1)\,c+2abcx+b^2c\,x^2\right)^{\frac{1}{3}}}dx$$

input
```
int(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

output
```
int(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{(b^2cx^2+2\,abcx+(a^2+1)c)^{\frac{1}{3}}}\,dx$$

input
```
integrate(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x, algorithm
="fricas")
```

output
```
integral(arccot(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)^(1/3), x)
```

Sympy [N/A]

Not integrable

Time = 7.45 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{\operatorname{acot}(a+bx)}{\sqrt[3]{c\,(a^2+2abx+b^2x^2+1)}}\,dx$$

input
```
integrate(acot(b*x+a)/((a**2+1)*c+2*a*b*c*x+b**2*c*x**2)**(1/3),x)
```

output
```
Integral(acot(a + b*x)/(c*(a**2 + 2*a*b*x + b**2*x**2 + 1))**(1/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\operatorname{arccot}\left(bx+a\right)}{\left(b^2cx^2+2\,abcx+(a^2+1)c\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x, algorithm
="maxima")
```

output
```
integrate(arccot(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)^(1/3), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\operatorname{arccot}\left(bx+a\right)}{\left(b^2cx^2+2\,abcx+(a^2+1)c\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x, algorithm
="giac")
```

output
```
integrate(arccot(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)^(1/3), x)
```

Mupad [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\mathrm{acot}(a+b\,x)}{{\left(c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)\right)}^{1/3}}\,dx$$

input
```
int(acot(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3),x)
```

output
```
int(acot(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\frac{\int \frac{acot(bx+a)}{(b^2x^2+2abx+a^2+1)^{\frac{1}{3}}}dx}{c^{\frac{1}{3}}}$$

input
```
int(acot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

output
```
int(acot(a + b*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(1/3),x)/c**(1/3)
```

3.49 $\int \frac{a+b\cot^{-1}(c+dx)}{e+fx^2+gx^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 1199

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx^2+gx^4}\,dx = \text{Too large to display}$$

output
```
-1/2*g^(1/2)*(a+b*arccot(d*x+c))*ln(-2*d*((-f-(-4*e*g+f^2)^(1/2))^(1/2)-g^
(1/2)*x*2^(1/2))/(2^(1/2)*(I-c)*g^(1/2)-d*(-f-(-4*e*g+f^2)^(1/2))^(1/2))/(
1-I*(d*x+c)))*2^(1/2)/(-4*e*g+f^2)^(1/2)/(-f-(-4*e*g+f^2)^(1/2))^(1/2)+1/2
*g^(1/2)*(a+b*arccot(d*x+c))*ln(-2*d*((-f+(-4*e*g+f^2)^(1/2))^(1/2)-g^(1/2
)*x*2^(1/2))/(2^(1/2)*(I-c)*g^(1/2)-d*(-f+(-4*e*g+f^2)^(1/2))^(1/2))/(1-I*
(d*x+c)))*2^(1/2)/(-4*e*g+f^2)^(1/2)/(-f+(-4*e*g+f^2)^(1/2))^(1/2)+1/2*g^(
1/2)*(a+b*arccot(d*x+c))*ln(2*d*((-f-(-4*e*g+f^2)^(1/2))^(1/2)+g^(1/2)*x*2
^(1/2))/(2^(1/2)*(I-c)*g^(1/2)+d*(-f-(-4*e*g+f^2)^(1/2))^(1/2))/(1-I*(d*x+
c)))*2^(1/2)/(-4*e*g+f^2)^(1/2)/(-f-(-4*e*g+f^2)^(1/2))^(1/2)-1/2*g^(1/2)*
(a+b*arccot(d*x+c))*ln(2*d*((-f+(-4*e*g+f^2)^(1/2))^(1/2)+g^(1/2)*x*2^(1/2
))/(2^(1/2)*(I-c)*g^(1/2)+d*(-f+(-4*e*g+f^2)^(1/2))^(1/2))/(1-I*(d*x+c)))*
2^(1/2)/(-4*e*g+f^2)^(1/2)/(-f+(-4*e*g+f^2)^(1/2))^(1/2)-1/4*I*b*g^(1/2)*p
olylog(2,1+2*d*((-f-(-4*e*g+f^2)^(1/2))^(1/2)-g^(1/2)*x*2^(1/2))/(2^(1/2)*
(I-c)*g^(1/2)-d*(-f-(-4*e*g+f^2)^(1/2))^(1/2))/(1-I*(d*x+c)))*2^(1/2)/(-4*
e*g+f^2)^(1/2)/(-f-(-4*e*g+f^2)^(1/2))^(1/2)+1/4*I*b*g^(1/2)*polylog(2,1+2
*d*((-f+(-4*e*g+f^2)^(1/2))^(1/2)-g^(1/2)*x*2^(1/2))/(2^(1/2)*(I-c)*g^(1/2
)-d*(-f+(-4*e*g+f^2)^(1/2))^(1/2))/(1-I*(d*x+c)))*2^(1/2)/(-4*e*g+f^2)^(1/
2)/(-f+(-4*e*g+f^2)^(1/2))^(1/2)+1/4*I*b*g^(1/2)*polylog(2,1-2*d*((-f-(-4*
e*g+f^2)^(1/2))^(1/2)+g^(1/2)*x*2^(1/2))/(2^(1/2)*(I-c)*g^(1/2)+d*(-f-(-4*
e*g+f^2)^(1/2))^(1/2))/(1-I*(d*x+c)))*2^(1/2)/(-4*e*g+f^2)^(1/2)/(-f-(-...
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3211 vs. 2(1199) = 2398.

Time = 4.60 (sec) , antiderivative size = 3211, normalized size of antiderivative = 2.68

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx^2+gx^4}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*ArcCot[c + d*x])/(e + f*x^2 + g*x^4),x]
```

output
```
(Sqrt[g]*(4*a*Sqrt[-f + Sqrt[f^2 - 4*e*g]]*Sqrt[-(f + Sqrt[f^2 - 4*e*g])^2
]*ArcTan[(Sqrt[2]*Sqrt[g]*x)/Sqrt[f - Sqrt[f^2 - 4*e*g]]] - 4*a*Sqrt[-f -
Sqrt[f^2 - 4*e*g]]*Sqrt[-(f - Sqrt[f^2 - 4*e*g])^2]*ArcTan[(Sqrt[2]*Sqrt[g
]*x)/Sqrt[f + Sqrt[f^2 - 4*e*g]]] + I*b*Sqrt[-(f - Sqrt[f^2 - 4*e*g])^2]*S
qrt[f + Sqrt[f^2 - 4*e*g]]*Log[(Sqrt[2]*Sqrt[g]*(-I + c + d*x))/(Sqrt[2]*(
-I + c)*Sqrt[g] + d*Sqrt[-f - Sqrt[f^2 - 4*e*g]])]*Log[Sqrt[-f - Sqrt[f^2
- 4*e*g]] - Sqrt[2]*Sqrt[g]*x] - I*b*Sqrt[-(f - Sqrt[f^2 - 4*e*g])^2]*Sqrt
[f + Sqrt[f^2 - 4*e*g]]*Log[(-I + c + d*x)/(c + d*x)]*Log[Sqrt[-f - Sqrt[f
^2 - 4*e*g]] - Sqrt[2]*Sqrt[g]*x] - I*b*Sqrt[-(f - Sqrt[f^2 - 4*e*g])^2]*S
qrt[f + Sqrt[f^2 - 4*e*g]]*Log[(Sqrt[2]*Sqrt[g]*(I + c + d*x))/(Sqrt[2]*(I
 + c)*Sqrt[g] + d*Sqrt[-f - Sqrt[f^2 - 4*e*g]])]*Log[Sqrt[-f - Sqrt[f^2 -
4*e*g]] - Sqrt[2]*Sqrt[g]*x] + I*b*Sqrt[-(f - Sqrt[f^2 - 4*e*g])^2]*Sqrt[f
 + Sqrt[f^2 - 4*e*g]]*Log[(I + c + d*x)/(c + d*x)]*Log[Sqrt[-f - Sqrt[f^2
- 4*e*g]] - Sqrt[2]*Sqrt[g]*x] - I*b*Sqrt[f - Sqrt[f^2 - 4*e*g]]*Sqrt[-(f
+ Sqrt[f^2 - 4*e*g])^2]*Log[(Sqrt[2]*Sqrt[g]*(-I + c + d*x))/(Sqrt[2]*(-I
+ c)*Sqrt[g] + d*Sqrt[-f + Sqrt[f^2 - 4*e*g]])]*Log[Sqrt[-f + Sqrt[f^2 - 4
*e*g]] - Sqrt[2]*Sqrt[g]*x] + I*b*Sqrt[f - Sqrt[f^2 - 4*e*g]]*Sqrt[-(f + S
qrt[f^2 - 4*e*g])^2]*Log[(-I + c + d*x)/(c + d*x)]*Log[Sqrt[-f + Sqrt[f^2
- 4*e*g]] - Sqrt[2]*Sqrt[g]*x] + I*b*Sqrt[f - Sqrt[f^2 - 4*e*g]]*Sqrt[-(f
+ Sqrt[f^2 - 4*e*g])^2]*Log[(Sqrt[2]*Sqrt[g]*(I + c + d*x))/(Sqrt[2]*(I...
```

Rubi [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2808 vs. $2(1199) = 2398$.

Time = 7.95 (sec) , antiderivative size = 2808, normalized size of antiderivative = 2.34, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx^2+gx^4}\,dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{a}{e+fx^2+gx^4}+\frac{b\cot^{-1}(c+dx)}{e+fx^2+gx^4}\right)dx$$

↓ 2009

$$\frac{\sqrt{2}a\sqrt{g}\arctan\left(\frac{\sqrt{2}\sqrt{g}x}{\sqrt{f-\sqrt{f^2-4eg}}}\right)}{\sqrt{f^2-4eg}\sqrt{f-\sqrt{f^2-4eg}}}-\frac{\sqrt{2}a\sqrt{g}\arctan\left(\frac{\sqrt{2}\sqrt{g}x}{\sqrt{f+\sqrt{f^2-4eg}}}\right)}{\sqrt{f^2-4eg}\sqrt{f+\sqrt{f^2-4eg}}}-$$

$$\frac{b\sqrt{g}\log\left(-\frac{-c-dx+i}{c+dx}\right)\log\left(1-\frac{\left(2gc^2+d^2\left(f-\sqrt{f^2-4eg}\right)\right)(-c-dx+i)}{\left(-\left(\left(f-\sqrt{f^2-4eg}\right)d^2\right)-\sqrt{2}\sqrt{g}\sqrt{f-\sqrt{f^2-4eg}}d+2(i-c)cg\right)(c+dx)}\right)}{2\sqrt{2}\sqrt{f^2-4eg}\sqrt{f-\sqrt{f^2-4eg}}}+$$

$$\frac{b\sqrt{g}\log\left(-\frac{-c-dx+i}{c+dx}\right)\log\left(1-\frac{\left(2gc^2+d^2\left(f-\sqrt{f^2-4eg}\right)\right)(-c-dx+i)}{\left(-\left(\left(f-\sqrt{f^2-4eg}\right)d^2\right)+\sqrt{2}\sqrt{g}\sqrt{f-\sqrt{f^2-4eg}}d+2(i-c)cg\right)(c+dx)}\right)}{2\sqrt{2}\sqrt{f^2-4eg}\sqrt{f-\sqrt{f^2-4eg}}}+$$

$$\frac{b\sqrt{g}\log\left(-\frac{-c-dx+i}{c+dx}\right)\log\left(1-\frac{\left(2gc^2+d^2\left(f+\sqrt{f^2-4eg}\right)\right)(-c-dx+i)}{\left(-\left(\left(f+\sqrt{f^2-4eg}\right)d^2\right)-\sqrt{2}\sqrt{g}\sqrt{f+\sqrt{f^2-4eg}}d+2(i-c)cg\right)(c+dx)}\right)}{2\sqrt{2}\sqrt{f^2-4eg}\sqrt{f+\sqrt{f^2-4eg}}}-$$

$$\frac{b\sqrt{g}\log\left(-\frac{-c-dx+i}{c+dx}\right)\log\left(1-\frac{\left(2gc^2+d^2\left(f+\sqrt{f^2-4eg}\right)\right)(-c-dx+i)}{\left(-\left(\left(f+\sqrt{f^2-4eg}\right)d^2\right)+\sqrt{2}\sqrt{g}\sqrt{f+\sqrt{f^2-4eg}}d+2(i-c)cg\right)(c+dx)}\right)}{2\sqrt{2}\sqrt{f^2-4eg}\sqrt{f+\sqrt{f^2-4eg}}}+$$

$$\frac{b\sqrt{g}\log\left(\frac{c+dx+i}{c+dx}\right)\log\left(1-\frac{\left(2gc^2+d^2\left(f-\sqrt{f^2-4eg}\right)\right)(c+dx+i)}{\left(\left(f-\sqrt{f^2-4eg}\right)d^2-\sqrt{2}\sqrt{g}\sqrt{f-\sqrt{f^2-4eg}}d+2c(c+i)g\right)(c+dx)}\right)}{2\sqrt{2}\sqrt{f^2-4eg}\sqrt{f-\sqrt{f^2-4eg}}}-$$

$$\frac{b\sqrt{g}\log\left(\frac{c+dx+i}{c+dx}\right)\log\left(1-\frac{\left(2gc^2+d^2\left(f-\sqrt{f^2-4eg}\right)\right)(c+dx+i)}{\left(\left(f-\sqrt{f^2-4eg}\right)d^2+\sqrt{2}\sqrt{g}\sqrt{f-\sqrt{f^2-4eg}}d+2c(c+i)g\right)(c+dx)}\right)}{2\sqrt{2}\sqrt{f^2-4eg}\sqrt{f-\sqrt{f^2-4eg}}}-$$

$$\frac{b\sqrt{g}\log\left(\frac{c+dx+i}{c+dx}\right)\log\left(1-\frac{\left(2gc^2+d^2\left(f+\sqrt{f^2-4eg}\right)\right)(c+dx+i)}{\left(\left(f+\sqrt{f^2-4eg}\right)d^2-\sqrt{2}\sqrt{g}\sqrt{f+\sqrt{f^2-4eg}}d+2c(c+i)g\right)(c+dx)}\right)}{2\sqrt{2}\sqrt{f^2-4eg}\sqrt{f+\sqrt{f^2-4eg}}}+$$

$$\frac{b\sqrt{g}\log\left(\frac{c+dx+i}{c+dx}\right)\log\left(1-\frac{\left(2gc^2+d^2\left(f+\sqrt{f^2-4eg}\right)\right)(c+dx+i)}{\left(\left(f+\sqrt{f^2-4eg}\right)d^2+\sqrt{2}\sqrt{g}\sqrt{f+\sqrt{f^2-4eg}}d+2c(c+i)g\right)(c+dx)}\right)}{2\sqrt{2}\sqrt{f^2-4eg}\sqrt{f+\sqrt{f^2-4eg}}}-$$

$$\frac{b\sqrt{g}\,\mathrm{PolyLog}\left(2,\frac{\left(2gc^2+d^2\left(f-\sqrt{f^2-4eg}\right)\right)(-c-dx+i)}{\left(-\left(\left(f-\sqrt{f^2-4eg}\right)d^2\right)-\sqrt{2}\sqrt{g}\sqrt{f-\sqrt{f^2-4eg}}d+2(i-c)cg\right)(c+dx)}\right)}{2\sqrt{2}\sqrt{f^2-4eg}\sqrt{f-\sqrt{f^2-4eg}}}+$$

$$\frac{b\sqrt{g}\,\mathrm{PolyLog}\left(2,\frac{\left(2gc^2+d^2\left(f-\sqrt{f^2-4eg}\right)\right)(-c-dx+i)}{\left(-\left(\left(f-\sqrt{f^2-4eg}\right)d^2\right)+\sqrt{2}\sqrt{g}\sqrt{f-\sqrt{f^2-4eg}}d+2(i-c)cg\right)(c+dx)}\right)}{2\sqrt{2}\sqrt{f^2-4eg}\sqrt{f-\sqrt{f^2-4eg}}}+$$

input
```
Int[(a + b*ArcCot[c + d*x])/(e + f*x^2 + g*x^4),x]
```

output
```
(Sqrt[2]*a*Sqrt[g]*ArcTan[(Sqrt[2]*Sqrt[g]*x)/Sqrt[f - Sqrt[f^2 - 4*e*g]]]
)/(Sqrt[f^2 - 4*e*g]*Sqrt[f - Sqrt[f^2 - 4*e*g]]) - (Sqrt[2]*a*Sqrt[g]*Arc
Tan[(Sqrt[2]*Sqrt[g]*x)/Sqrt[f + Sqrt[f^2 - 4*e*g]]])/(Sqrt[f^2 - 4*e*g]*S
qrt[f + Sqrt[f^2 - 4*e*g]]) - (b*Sqrt[g]*Log[-((I - c - d*x)/(c + d*x))]*L
og[1 - ((2*c^2*g + d^2*(f - Sqrt[f^2 - 4*e*g]))*(I - c - d*x))/((2*(I - c)
*c*g - Sqrt[2]*d*Sqrt[g]*Sqrt[f - Sqrt[f^2 - 4*e*g]] - d^2*(f - Sqrt[f^2 -
 4*e*g]))*(c + d*x))])/(2*Sqrt[2]*Sqrt[f^2 - 4*e*g]*Sqrt[f - Sqrt[f^2 - 4*
e*g]]) + (b*Sqrt[g]*Log[-((I - c - d*x)/(c + d*x))]*Log[1 - ((2*c^2*g + d^
2*(f - Sqrt[f^2 - 4*e*g]))*(I - c - d*x))/((2*(I - c)*c*g + Sqrt[2]*d*Sqrt
[g]*Sqrt[f - Sqrt[f^2 - 4*e*g]] - d^2*(f - Sqrt[f^2 - 4*e*g]))*(c + d*x))]
)/(2*Sqrt[2]*Sqrt[f^2 - 4*e*g]*Sqrt[f - Sqrt[f^2 - 4*e*g]]) + (b*Sqrt[g]*L
og[-((I - c - d*x)/(c + d*x))]*Log[1 - ((2*c^2*g + d^2*(f + Sqrt[f^2 - 4*e
*g]))*(I - c - d*x))/((2*(I - c)*c*g - Sqrt[2]*d*Sqrt[g]*Sqrt[f + Sqrt[f^2
 - 4*e*g]] - d^2*(f + Sqrt[f^2 - 4*e*g]))*(c + d*x))])/(2*Sqrt[2]*Sqrt[f^2
 - 4*e*g]*Sqrt[f + Sqrt[f^2 - 4*e*g]]) - (b*Sqrt[g]*Log[-((I - c - d*x)/(c
 + d*x))]*Log[1 - ((2*c^2*g + d^2*(f + Sqrt[f^2 - 4*e*g]))*(I - c - d*x))/
((2*(I - c)*c*g + Sqrt[2]*d*Sqrt[g]*Sqrt[f + Sqrt[f^2 - 4*e*g]] - d^2*(f +
 Sqrt[f^2 - 4*e*g]))*(c + d*x))])/(2*Sqrt[2]*Sqrt[f^2 - 4*e*g]*Sqrt[f + Sq
rt[f^2 - 4*e*g]]) + (b*Sqrt[g]*Log[(I + c + d*x)/(c + d*x)]*Log[1 - ((2*c^
2*g + d^2*(f - Sqrt[f^2 - 4*e*g]))*(I + c + d*x))/((2*c*(I + c)*g - Sqr...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.42 (sec) , antiderivative size = 1287, normalized size of antiderivative = 1.07

method	result	size
risch	Expression too large to display	1287
parts	Expression too large to display	2083
derivativedivides	Expression too large to display	2090
default	Expression too large to display	2090

input
```
int((a+b*arccot(d*x+c))/(g*x^4+f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*d^3*Pi*sum(1/(6*I*_R^2*c*g-2*I*c^3*g-I*c*d^2*f-12*I*_R*c*g+2*_R^3*g-
6*_R*c^2*g-_R*d^2*f+6*I*c*g-6*g*_R^2+6*c^2*g+d^2*f+6*g*_R-2*g)*ln(-I*d*x-I
*c+1-_R),_R=RootOf(g*_Z^4+(4*RootOf(_Z^2+1,index=1)*c*g-4*g)*_Z^3+(-12*Roo
tOf(_Z^2+1,index=1)*c*g-6*c^2*g-d^2*f+6*g)*_Z^2+(-4*RootOf(_Z^2+1,index=1)
*c^3*g-2*RootOf(_Z^2+1,index=1)*c*d^2*f+12*RootOf(_Z^2+1,index=1)*c*g+12*c
^2*g+2*d^2*f-4*g)*_Z+4*RootOf(_Z^2+1,index=1)*c^3*g+2*RootOf(_Z^2+1,index=
1)*c*d^2*f+c^4*g+c^2*d^2*f+e*d^4-4*RootOf(_Z^2+1,index=1)*c*g-6*c^2*g-d^2*
f+g))*b+1/2*I*d^3*a*sum(1/(6*I*_R^2*c*g-2*I*c^3*g-I*c*d^2*f-12*I*_R*c*g+2*
_R^3*g-6*_R*c^2*g-_R*d^2*f+6*I*c*g-6*g*_R^2+6*c^2*g+d^2*f+6*g*_R-2*g)*ln(-
I*d*x-I*c+1-_R),_R=RootOf(g*_Z^4+(4*RootOf(_Z^2+1,index=1)*c*g-4*g)*_Z^3+(
-12*RootOf(_Z^2+1,index=1)*c*g-6*c^2*g-d^2*f+6*g)*_Z^2+(-4*RootOf(_Z^2+1,i
ndex=1)*c^3*g-2*RootOf(_Z^2+1,index=1)*c*d^2*f+12*RootOf(_Z^2+1,index=1)*c
*g+12*c^2*g+2*d^2*f-4*g)*_Z+4*RootOf(_Z^2+1,index=1)*c^3*g+2*RootOf(_Z^2+1
,index=1)*c*d^2*f+c^4*g+c^2*d^2*f+e*d^4-4*RootOf(_Z^2+1,index=1)*c*g-6*c^2
*g-d^2*f+g))+1/4*d^3*b*sum(1/(6*I*_R1^2*c*g-2*I*c^3*g-I*c*d^2*f-12*I*_R1*c
*g+2*_R1^3*g-6*_R1*c^2*g-_R1*d^2*f+6*I*c*g-6*_R1^2*g+6*c^2*g+d^2*f+6*_R1*g
-2*g)*(ln(1-I*c-I*d*x)*ln((_R1+I*d*x+I*c-1)/_R1)+dilog((_R1+I*d*x+I*c-1)/_
R1)),_R1=RootOf(g*_Z^4+(4*RootOf(_Z^2+1,index=1)*c*g-4*g)*_Z^3+(-12*RootOf
(_Z^2+1,index=1)*c*g-6*c^2*g-d^2*f+6*g)*_Z^2+(-4*RootOf(_Z^2+1,index=1)*c^
3*g-2*RootOf(_Z^2+1,index=1)*c*d^2*f+12*RootOf(_Z^2+1,index=1)*c*g+12*c...
```

Fricas [F]

$$\int \frac{a + b\cot^{-1}(c + dx)}{e + fx^2 + gx^4}\, dx = \int \frac{b\,\mathrm{arccot}\,(dx + c) + a}{gx^4 + fx^2 + e}\, dx$$

input
```
integrate((a+b*arccot(d*x+c))/(g*x^4+f*x^2+e),x, algorithm="fricas")
```

output
```
integral((b*arccot(d*x + c) + a)/(g*x^4 + f*x^2 + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\cot^{-1}(c + dx)}{e + fx^2 + gx^4}\, dx = \text{Timed out}$$

input
```
integrate((a+b*acot(d*x+c))/(g*x**4+f*x**2+e),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a + b\cot^{-1}(c + dx)}{e + fx^2 + gx^4}\, dx = \int \frac{b\,\mathrm{arccot}\,(dx + c) + a}{gx^4 + fx^2 + e}\, dx$$

input
```
integrate((a+b*arccot(d*x+c))/(g*x^4+f*x^2+e),x, algorithm="maxima")
```

output
```
integrate((b*arccot(d*x + c) + a)/(g*x^4 + f*x^2 + e), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx^2+gx^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arccot(d*x+c))/(g*x^4+f*x^2+e),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx^2+gx^4}\,dx = \int \frac{a+b\,\mathrm{acot}(c+d\,x)}{g\,x^4+f\,x^2+e}\,dx$$

input
```
int((a + b*acot(c + d*x))/(e + f*x^2 + g*x^4),x)
```

output
```
int((a + b*acot(c + d*x))/(e + f*x^2 + g*x^4), x)
```

Reduce [F]

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx^2+gx^4}\,dx$$
$$= \frac{2\sqrt{e}\,\sqrt{2\sqrt{g}\,\sqrt{e}+f}\,atan\left(\frac{\sqrt{2\sqrt{g}\,\sqrt{e}-f}-2\sqrt{g}\,x}{\sqrt{2\sqrt{g}\,\sqrt{e}+f}}\right)af-4\sqrt{g}\,\sqrt{2\sqrt{g}\,\sqrt{e}+f}\,atan\left(\frac{\sqrt{2\sqrt{g}\,\sqrt{e}-f}-2\sqrt{g}\,x}{\sqrt{2\sqrt{g}\,\sqrt{e}+f}}\right)ae-2\sqrt{e}}{}$$

input
```
int((a+b*acot(d*x+c))/(g*x^4+f*x^2+e),x)
```

output
```
(2*sqrt(e)*sqrt(2*sqrt(g)*sqrt(e) + f)*atan((sqrt(2*sqrt(g)*sqrt(e) - f) -
 2*sqrt(g)*x)/sqrt(2*sqrt(g)*sqrt(e) + f))*a*f - 4*sqrt(g)*sqrt(2*sqrt(g)*
sqrt(e) + f)*atan((sqrt(2*sqrt(g)*sqrt(e) - f) - 2*sqrt(g)*x)/sqrt(2*sqrt(
g)*sqrt(e) + f))*a*e - 2*sqrt(e)*sqrt(2*sqrt(g)*sqrt(e) + f)*atan((sqrt(2*
sqrt(g)*sqrt(e) - f) + 2*sqrt(g)*x)/sqrt(2*sqrt(g)*sqrt(e) + f))*a*f + 4*s
qrt(g)*sqrt(2*sqrt(g)*sqrt(e) + f)*atan((sqrt(2*sqrt(g)*sqrt(e) - f) + 2*s
qrt(g)*x)/sqrt(2*sqrt(g)*sqrt(e) + f))*a*e - sqrt(e)*sqrt(2*sqrt(g)*sqrt(e
) - f)*log( - sqrt(2*sqrt(g)*sqrt(e) - f)*x + sqrt(e) + sqrt(g)*x**2)*a*f
+ sqrt(e)*sqrt(2*sqrt(g)*sqrt(e) - f)*log(sqrt(2*sqrt(g)*sqrt(e) - f)*x +
sqrt(e) + sqrt(g)*x**2)*a*f - 2*sqrt(g)*sqrt(2*sqrt(g)*sqrt(e) - f)*log( -
 sqrt(2*sqrt(g)*sqrt(e) - f)*x + sqrt(e) + sqrt(g)*x**2)*a*e + 2*sqrt(g)*s
qrt(2*sqrt(g)*sqrt(e) - f)*log(sqrt(2*sqrt(g)*sqrt(e) - f)*x + sqrt(e) + s
qrt(g)*x**2)*a*e + 16*int(acot(c + d*x)/(e + f*x**2 + g*x**4),x)*b*e**2*g
- 4*int(acot(c + d*x)/(e + f*x**2 + g*x**4),x)*b*e*f**2)/(4*e*(4*e*g - f**
2))
```

3.50 $\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 187

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}} \, dx = \frac{\sqrt{1+(a+bx)^2}}{2b} + \frac{(a+bx)\sqrt{1+(a+bx)^2}\cot^{-1}(a+bx)}{2b} + \frac{i\cot^{-1}(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b} + \frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b} - \frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b}$$

output

```
1/2*(1+(b*x+a)^2)^(1/2)/b+1/2*(b*x+a)*(1+(b*x+a)^2)^(1/2)*arccot(b*x+a)/b+
I*arccot(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b+1/2*I*po
lylog(2,-I*(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b-1/2*I*polylog(2,I*(1
+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b
```

Mathematica [A] (warning: unable to verify)

Time = 1.06 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = -\frac{\sqrt{(a+bx)^2\left(1+\frac{1}{(a+bx)^2}\right)}\left(-2\cot\left(\frac{1}{2}\cot^{-1}(a+bx)\right)-\cot^{-1}(a+bx)\csc^2\left(\frac{1}{2}\cot^{-1}(a+bx)\right)-4\cot^{-1}\right.}{}$$

input
```
Integrate[((a + b*x)^2*ArcCot[a + b*x])/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],
x]
```

output
```
-1/8*(Sqrt[(a + b*x)^2*(1 + (a + b*x)^(-2))]*(-2*Cot[ArcCot[a + b*x]/2] -
ArcCot[a + b*x]*Csc[ArcCot[a + b*x]/2]^2 - 4*ArcCot[a + b*x]*Log[1 - E^(I*
ArcCot[a + b*x])] + 4*ArcCot[a + b*x]*Log[1 + E^(I*ArcCot[a + b*x])] - (4*
I)*PolyLog[2, -E^(I*ArcCot[a + b*x])] + (4*I)*PolyLog[2, E^(I*ArcCot[a + b
*x])] + ArcCot[a + b*x]*Sec[ArcCot[a + b*x]/2]^2 - 2*Tan[ArcCot[a + b*x]/2
]))/(b*(a + b*x)*Sqrt[1 + (a + b*x)^(-2)])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {5581, 5488, 241, 5422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{a^2+2abx+b^2x^2+1}}\,dx$$

$$\Big\downarrow \text{5581}$$

$$\frac{\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{(a+bx)^2+1}}\,d(a+bx)}{b}$$

$$\Big\downarrow \text{5488}$$

$$\frac{\frac{1}{2}\int \frac{a+bx}{\sqrt{(a+bx)^2+1}}d(a+bx)-\frac{1}{2}\int \frac{\cot^{-1}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)+\frac{1}{2}(a+bx)\sqrt{(a+bx)^2+1}\cot^{-1}(a+bx)}{b}$$

$$\Big\downarrow \text{241}$$

$$\frac{-\frac{1}{2}\int \frac{\cot^{-1}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)+\frac{1}{2}\sqrt{(a+bx)^2+1}+\frac{1}{2}(a+bx)\sqrt{(a+bx)^2+1}\cot^{-1}(a+bx)}{b}$$

$$\Big\downarrow \text{5422}$$

$$\frac{\frac{1}{2}\left(2i\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)\cot^{-1}(a+bx)+i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)-i\operatorname{PolyLog}\left(2,\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)\right)+\frac{1}{2}\sqrt{(a}{b}$$

input
```
Int[((a + b*x)^2*ArcCot[a + b*x])/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(Sqrt[1 + (a + b*x)^2]/2 + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcCot[a + b*x
])/2 + ((2*I)*ArcCot[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a +
 b*x)]] + I*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]]
 - I*PolyLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/2)/b
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5422
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcCot[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x])]/(c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 -
I*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 GtQ[d, 0]
```

rule 5488
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcCot[c*x])^p/(c^2*d*m)), x] + (Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((a
 + b*ArcCot[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^2
*m))   Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/Sqrt[d + e*x^2]), x], x]) /
; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5581
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Subs
t[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcCot[x])
^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &
& EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.86

method	result
default	$\frac{(\operatorname{arccot}(bx+a)bx+a\ \operatorname{arccot}(bx+a)+1)\sqrt{b^2x^2+2abx+a^2+1}}{2b} - \frac{i\left(i\operatorname{arccot}(bx+a)\ln\left(1-\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}\right)-i\operatorname{arccot}(bx+a)\ln\left(\frac{bx+}{\sqrt{1+(b}}\right.\right.}{}$

input
```
int((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
1/2*(arccot(b*x+a)*b*x+a*arccot(b*x+a)+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b-
1/2*I*(I*arccot(b*x+a)*ln(1-(I+a+b*x)/(1+(b*x+a)^2)^(1/2))-I*arccot(b*x+a)
*ln((I+a+b*x)/(1+(b*x+a)^2)^(1/2)+1)+polylog(2,(I+a+b*x)/(1+(b*x+a)^2)^(1/
2))-polylog(2,-(I+a+b*x)/(1+(b*x+a)^2)^(1/2)))/b
```

Fricas [F]

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorit
hm="fricas")
```

output
```
integral((b^2*x^2 + 2*a*b*x + a^2)*arccot(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**2*acot(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate((b*x + a)^2*arccot(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x
)
```

Giac [F]

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorit
hm="giac")
```

output
```
integrate((b*x + a)^2*arccot(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{acot}(a+b\,x)\,(a+b\,x)^2}{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}\,dx$$

input
```
int((acot(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

output
```
int((acot(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \left(\int \frac{acot(bx+a)}{\sqrt{b^2x^2+2abx+a^2+1}}dx\right) a^2 + \left(\int \frac{acot(bx+a)\,x^2}{\sqrt{b^2x^2+2abx+a^2+1}}dx\right) b^2 + 2\left(\int \frac{acot(bx+a)\,x}{\sqrt{b^2x^2+2abx+a^2+1}}dx\right) ab$$

input
```
int((b*x+a)^2*acot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x)
```

output
```
int(acot(a + b*x)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1),x)*a**2 + int((acot
(a + b*x)*x**2)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1),x)*b**2 + 2*int((acot
(a + b*x)*x)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1),x)*a*b
```

3.51 $\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{(1+a^2)c+2abcx+b^2cx^2}} dx$

Optimal result

Integrand size = 40, antiderivative size = 281

$$\begin{aligned}\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx &= \frac{\sqrt{c+c(a+bx)^2}}{2bc}+\frac{(a+bx)\sqrt{c+c(a+bx)^2}\cot^{-1}(a+bx)}{2bc} \\ &+ \frac{i\sqrt{1+(a+bx)^2}\cot^{-1}(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}} \\ &+ \frac{i\sqrt{1+(a+bx)^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c+c(a+bx)^2}} \\ &- \frac{i\sqrt{1+(a+bx)^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c+c(a+bx)^2}}\end{aligned}$$

output
```
1/2*(c+c*(b*x+a)^2)^(1/2)/b/c+1/2*(b*x+a)*(c+c*(b*x+a)^2)^(1/2)*arccot(b*x
+a)/b/c+I*(1+(b*x+a)^2)^(1/2)*arccot(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-
I*(b*x+a))^(1/2))/b/(c+c*(b*x+a)^2)^(1/2)+1/2*I*(1+(b*x+a)^2)^(1/2)*polylo
g(2,-I*(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b/(c+c*(b*x+a)^2)^(1/2)-1/
2*I*(1+(b*x+a)^2)^(1/2)*polylog(2,I*(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2
))/b/(c+c*(b*x+a)^2)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.03 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx)^2\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx =$$
$$-\frac{\sqrt{c\left(1+a^2+2abx+b^2x^2\right)}\left(-2\cot\left(\frac{1}{2}\cot^{-1}(a+bx)\right)-\cot^{-1}(a+bx)\csc^2\left(\frac{1}{2}\cot^{-1}(a+bx)\right)-4\cot^{-}\right.}{}$$

input
```
Integrate[((a + b*x)^2*ArcCot[a + b*x])/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2
*c*x^2],x]
```

output
```
-1/8*(Sqrt[c*(1 + a^2 + 2*a*b*x + b^2*x^2)]*(-2*Cot[ArcCot[a + b*x]/2] - A
rcCot[a + b*x]*Csc[ArcCot[a + b*x]/2]^2 - 4*ArcCot[a + b*x]*Log[1 - E^(I*A
rcCot[a + b*x])] + 4*ArcCot[a + b*x]*Log[1 + E^(I*ArcCot[a + b*x])] - (4*I
)*PolyLog[2, -E^(I*ArcCot[a + b*x])] + (4*I)*PolyLog[2, E^(I*ArcCot[a + b*
x])] + ArcCot[a + b*x]*Sec[ArcCot[a + b*x]/2]^2 - 2*Tan[ArcCot[a + b*x]/2]
))/(b*c*(a + b*x)*Sqrt[1 + (a + b*x)^(-2)])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5581, 5488, 241, 5426, 5422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2\cot^{-1}(a+bx)}{\sqrt{(a^2+1)\,c+2abcx+b^2cx^2}}dx$$
$$\Big\downarrow \; 5581$$
$$\frac{\int \frac{(a+bx)^2\cot^{-1}(a+bx)}{\sqrt{c(a+bx)^2+c}}d(a+bx)}{b}$$
$$\Big\downarrow \; 5488$$

$$\frac{\frac{1}{2}\int \frac{a+bx}{\sqrt{c(a+bx)^2+c}}d(a+bx)-\frac{1}{2}\int \frac{\cot^{-1}(a+bx)}{\sqrt{c(a+bx)^2+c}}d(a+bx)+\frac{(a+bx)\sqrt{c(a+bx)^2+c}\cot^{-1}(a+bx)}{2c}}{b}$$

$$\Big\downarrow \; 241$$

$$\frac{-\frac{1}{2}\int \frac{\cot^{-1}(a+bx)}{\sqrt{c(a+bx)^2+c}}d(a+bx)+\frac{\sqrt{c(a+bx)^2+c}}{2c}+\frac{(a+bx)\sqrt{c(a+bx)^2+c}\cot^{-1}(a+bx)}{2c}}{b}$$

$$\Big\downarrow \; 5426$$

$$\frac{-\frac{\sqrt{(a+bx)^2+1}\int \frac{\cot^{-1}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)}{2\sqrt{c(a+bx)^2+c}}+\frac{\sqrt{c(a+bx)^2+c}}{2c}+\frac{(a+bx)\sqrt{c(a+bx)^2+c}\cot^{-1}(a+bx)}{2c}}{b}$$

$$\Big\downarrow \; 5422$$

$$\frac{-\frac{\sqrt{(a+bx)^2+1}\left(-2i\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)\cot^{-1}(a+bx)-i\,\text{PolyLog}\left(2,-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)+i\,\text{PolyLog}\left(2,\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)\right)}{2\sqrt{c(a+bx)^2+c}}+\frac{\sqrt{c(a+bx)^2+c}}{2c}+}{b}$$

input
```
Int[((a + b*x)^2*ArcCot[a + b*x])/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2
],x]
```

output
```
(Sqrt[c + c*(a + b*x)^2]/(2*c) + ((a + b*x)*Sqrt[c + c*(a + b*x)^2]*ArcCot
[a + b*x])/(2*c) - (Sqrt[1 + (a + b*x)^2]*((-2*I)*ArcCot[a + b*x]*ArcTan[S
qrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]] - I*PolyLog[2, ((-I)*Sqrt[1 +
I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]] + I*PolyLog[2, (I*Sqrt[1 + I*(a + b*x
)])/Sqrt[1 - I*(a + b*x)]]))/(2*Sqrt[c + c*(a + b*x)^2]))/b
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5422
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcCot[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x])]/(c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 -
I*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 GtQ[d, 0]
```

rule 5426
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcCot[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5488
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcCot[c*x])^p/(c^2*d*m)), x] + (Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((a
 + b*ArcCot[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^2
*m))   Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/Sqrt[d + e*x^2]), x], x]) /
; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5581
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Subs
t[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcCot[x])
^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &
& EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.72

method	result
default	$\frac{(\operatorname{arccot}(bx+a)bx+a\ \operatorname{arccot}(bx+a)+1)\sqrt{c(bx+a-i)(bx+a+i)}}{2bc} - \frac{i\left(i\operatorname{arccot}(bx+a)\ln\left(1-\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}\right)-i\operatorname{arccot}(bx+a)\ln\left(\frac{b}{\sqrt{1+}}\right.\right.}{}$

input
```
int((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,method
=_RETURNVERBOSE)
```

output
```
1/2*(arccot(b*x+a)*b*x+a*arccot(b*x+a)+1)*(c*(b*x+a-I)*(I+a+b*x))^(1/2)/b/
c-1/2*I*(I*arccot(b*x+a)*ln(1-(I+a+b*x)/(1+(b*x+a)^2)^(1/2))-I*arccot(b*x+
a)*ln((I+a+b*x)/(1+(b*x+a)^2)^(1/2)+1)+polylog(2,(I+a+b*x)/(1+(b*x+a)^2)^(
1/2))-polylog(2,-(I+a+b*x)/(1+(b*x+a)^2)^(1/2)))*(c*(b*x+a-I)*(I+a+b*x))^(
1/2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b/c
```

Fricas [F]

$$\int \frac{(a+bx)^2\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{(bx+a)^2\operatorname{arccot}(bx+a)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,
 algorithm="fricas")
```

output
```
integral((b^2*x^2 + 2*a*b*x + a^2)*arccot(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*
c*x + (a^2 + 1)*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**2*acot(b*x+a)/((a**2+1)*c+2*a*b*c*x+b**2*c*x**2)**(1/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,
 algorithm="maxima")
```

output
```
integrate((b*x + a)^2*arccot(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 +
1)*c), x)
```

Giac [F]

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,
 algorithm="giac")
```

output
```
integrate((b*x + a)^2*arccot(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 +
1)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{\mathrm{acot}(a+b\,x)\,\left(a+b\,x\right)^2}{\sqrt{c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)}}\,dx$$

input
```
int((acot(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2
),x)
```

output
```
int((acot(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2
), x)
```

Reduce [F]

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \frac{\left(\int \frac{acot(bx+a)}{\sqrt{b^2x^2+2abx+a^2+1}}dx\right)a^2+\left(\int \frac{acot(bx+a)x^2}{\sqrt{b^2x^2+2abx+a^2+1}}dx\right)b^2+2\left(\int \frac{acot(bx+a)x}{\sqrt{b^2x^2+2abx+a^2+1}}dx\right)ab}{\sqrt{c}}$$

input
```
int((b*x+a)^2*acot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x)
```

output
```
(int(acot(a + b*x)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1),x)*a**2 + int((aco
t(a + b*x)*x**2)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1),x)*b**2 + 2*int((aco
t(a + b*x)*x)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1),x)*a*b)/sqrt(c)
```

3.52 $\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \operatorname{Int}\left(\frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+(a+bx)^2}},x\right)$$

output
```
Defer(Int)((b*x+a)^2*arccot(b*x+a)/(1+(b*x+a)^2)^(1/3),x)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 198 vs. 2(30) = 60.

Time = 0.68 (sec) , antiderivative size = 198, normalized size of antiderivative = 5.66

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$$
$$= \frac{3\left(\operatorname{Gamma}\left(\frac{11}{6}\right)\operatorname{Gamma}\left(\frac{7}{3}\right)\left(5(1+(a+bx)^2)\left(3(7+(a+bx)^2)+4(a+bx)\left(-2+(a+bx)^2\right)\cot^{-1}(a+\right.\right.\right.}{140b\sqrt[3]{1+a^2+2a}}$$

input
```
Integrate[((a + b*x)^2*ArcCot[a + b*x])/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3
),x]
```

output
```
(3*(Gamma[11/6]*Gamma[7/3]*(5*(1 + (a + b*x)^2)*(3*(7 + (a + b*x)^2) + 4*(
a + b*x)*(-2 + (a + b*x)^2)*ArcCot[a + b*x]) - 24*(a + b*x)*ArcCot[a + b*x
]*Hypergeometric2F1[1, 4/3, 11/6, (1 + a^2 + 2*a*b*x + b^2*x^2)^(-1)]) + 5
*2^(1/3)*Sqrt[Pi]*Gamma[5/3]*HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3},
 (1 + a^2 + 2*a*b*x + b^2*x^2)^(-1)]))/(140*b*(1 + a^2 + 2*a*b*x + b^2*x^2
)^(1/3)*(1 + (a + b*x)^2)*Gamma[11/6]*Gamma[7/3])
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{a^2+2abx+b^2x^2+1}}dx$$

$$\Big\downarrow \ 5581$$

$$\frac{\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(a+bx)^2+1}}d(a+bx)}{b}$$

$$\Big\downarrow \ 5561$$

$$\frac{\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(a+bx)^2+1}}d(a+bx)}{b}$$

input
```
Int[((a + b*x)^2*ArcCot[a + b*x])/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{(b^2x^2+2abx+a^2+1)^{\frac{1}{3}}}dx$$

input
```
int((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

output
```
int((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{(b^2x^2+2\,abx+a^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorit
hm="fricas")
```

output
```
integral((b^2*x^2 + 2*a*b*x + a^2)*arccot(b*x + a)/(b^2*x^2 + 2*a*b*x + a^
2 + 1)^(1/3), x)
```

Sympy [N/A]

Not integrable

Time = 4.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(a+bx)^2 \operatorname{acot}(a+bx)}{\sqrt[3]{a^2+2abx+b^2x^2+1}}\,dx$$

input
```
integrate((b*x+a)**2*acot(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/3),x)
```

output
```
Integral((a + b*x)**2*acot(a + b*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(1/3
), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{(b^2x^2+2\,abx+a^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorit
hm="maxima")
```

output
```
integrate((b*x + a)^2*arccot(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3),
 x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{(b^2x^2+2\,abx+a^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorit
hm="giac")
```

output
```
integrate((b*x + a)^2*arccot(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3),
 x)
```

Mupad [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}} \, dx = \int \frac{\mathrm{acot}(a+b\,x)\ (a+b\,x)^2}{(a^2+2\,a\,b\,x+b^2\,x^2+1)^{1/3}} \, dx$$

input
```
int((acot(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3),x)
```

output
```
int((acot(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.89

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}} \, dx = \left(\int \frac{acot(bx+a)}{(b^2x^2+2abx+a^2+1)^{\frac{1}{3}}}dx\right) a^2 + \left(\int \frac{acot(bx+a)\,x^2}{(b^2x^2+2abx+a^2+1)^{\frac{1}{3}}}dx\right) b^2 + 2\left(\int \frac{acot(bx+a)\,x}{(b^2x^2+2abx+a^2+1)^{\frac{1}{3}}}dx\right) ab$$

input
```
int((b*x+a)^2*acot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

output
```
int(acot(a + b*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(1/3),x)*a**2 + int((a
cot(a + b*x)*x**2)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(1/3),x)*b**2 + 2*int
((acot(a + b*x)*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(1/3),x)*a*b
```

3.53 $\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)c+2abcx+b^2cx^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)c+2abcx+b^2cx^2}}\,dx = \text{Int}\left(\frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{c+c(a+bx)^2}},x\right)$$

output
```
Defer(Int)((b*x+a)^2*arccot(b*x+a)/(c+c*(b*x+a)^2)^(1/3),x)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 200 vs. 2(32) = 64.

Time = 0.28 (sec) , antiderivative size = 200, normalized size of antiderivative = 5.00

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)c+2abcx+b^2cx^2}}\,dx$$
$$= \frac{3\Big(\text{Gamma}\left(\frac{11}{6}\right)\text{Gamma}\left(\frac{7}{3}\right)\left(5(1+(a+bx)^2\right)\left(3(7+(a+bx)^2\right)+4(a+bx)\left(-2+(a+bx)^2\right)\cot^{-1}(a+}{140b\sqrt[3]{c\,(1+a^2+2}}$$

input
```
Integrate[((a + b*x)^2*ArcCot[a + b*x])/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x
^2)^(1/3),x]
```

output
```
(3*(Gamma[11/6]*Gamma[7/3]*(5*(1 + (a + b*x)^2)*(3*(7 + (a + b*x)^2) + 4*(
a + b*x)*(-2 + (a + b*x)^2)*ArcCot[a + b*x]) - 24*(a + b*x)*ArcCot[a + b*x
]*Hypergeometric2F1[1, 4/3, 11/6, (1 + a^2 + 2*a*b*x + b^2*x^2)^(-1)]) + 5
*2^(1/3)*Sqrt[Pi]*Gamma[5/3]*HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3},
 (1 + a^2 + 2*a*b*x + b^2*x^2)^(-1)]))/(140*b*(c*(1 + a^2 + 2*a*b*x + b^2*
x^2))^(1/3)*(1 + (a + b*x)^2)*Gamma[11/6]*Gamma[7/3])
```

Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(a^2+1)\,c+2abcx+b^2cx^2}}dx$$
$$\Big\downarrow \text{5581}$$
$$\frac{\int \frac{(a+bx)^2\cot^{-1}(a+bx)}{\sqrt[3]{c(a+bx)^2+c}}d(a+bx)}{b}$$
$$\Big\downarrow \text{5561}$$
$$\frac{\int \frac{(a+bx)^2\cot^{-1}(a+bx)}{\sqrt[3]{c(a+bx)^2+c}}d(a+bx)}{b}$$

input
```
Int[((a + b*x)^2*ArcCot[a + b*x])/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1
/3),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{\left((a^2+1)c+2abcx+b^2c\,x^2\right)^{\frac{1}{3}}}dx$$

input
```
int((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

output
```
int((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{(b^2cx^2+2\,abcx+(a^2+1)c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x,
 algorithm="fricas")
```

output
```
integral((b^2*x^2 + 2*a*b*x + a^2)*arccot(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x
+ (a^2 + 1)*c)^(1/3), x)
```

Sympy [N/A]

Not integrable

Time = 72.53 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{(a+bx)^2 \operatorname{acot}(a+bx)}{\sqrt[3]{c\,(a^2+2abx+b^2x^2+1)}}\,dx$$

input
```
integrate((b*x+a)**2*acot(b*x+a)/((a**2+1)*c+2*a*b*c*x+b**2*c*x**2)**(1/3)
,x)
```

output
```
Integral((a + b*x)**2*acot(a + b*x)/(c*(a**2 + 2*a*b*x + b**2*x**2 + 1))**
(1/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{(b^2cx^2+2\,abcx+(a^2+1)c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x,
 algorithm="maxima")
```

output
```
integrate((b*x + a)^2*arccot(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c
)^(1/3), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{(b^2cx^2+2\,abcx+(a^2+1)c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x,
 algorithm="giac")
```

output
```
integrate((b*x + a)^2*arccot(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c
)^(1/3), x)
```

Mupad [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\mathrm{acot}(a+b\,x)\,\left(a+b\,x\right)^2}{{\left(c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)\right)}^{1/3}}\,dx$$

input
```
int((acot(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3
),x)
```

output
```
int((acot(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3
), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.62

$$\int \frac{(a+bx)^2\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx$$
$$=\frac{\left(\int \frac{acot(bx+a)}{(b^2x^2+2abx+a^2+1)^{\frac{1}{3}}}dx\right)a^2+\left(\int \frac{acot(bx+a)x^2}{(b^2x^2+2abx+a^2+1)^{\frac{1}{3}}}dx\right)b^2+2\left(\int \frac{acot(bx+a)x}{(b^2x^2+2abx+a^2+1)^{\frac{1}{3}}}dx\right)ab}{c^{\frac{1}{3}}}$$

input
```
int((b*x+a)^2*acot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

output
```
(int(acot(a + b*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(1/3),x)*a**2 + int((
acot(a + b*x)*x**2)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(1/3),x)*b**2 + 2*in
t((acot(a + b*x)*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(1/3),x)*a*b)/c**(1/
3)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                      convert(ExpnType_result,string)," vs. order ",
                      convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file