Computer Algebra Independent Integration Tests

Summer 2024

5-Inverse-trig-functions/5.5-Inverse-secant/287-5.5.1

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CHAPTER 1

Introduction

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**174**]. This is test number [287].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (174)	0.00 (0)
Mathematica	100.00 (174)	0.00 (0)
Maple	80.46 (140)	19.54 (34)
Fricas	71.26 (124)	28.74 (50)
Giac	55.17 (96)	44.83 (78)
Sympy	38.51 (67)	61.49 (107)
Maxima	35.63 (62)	64.37 (112)
Mupad	30.46 (53)	69.54 (121)
Reduce	25.29 (44)	74.71 (130)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

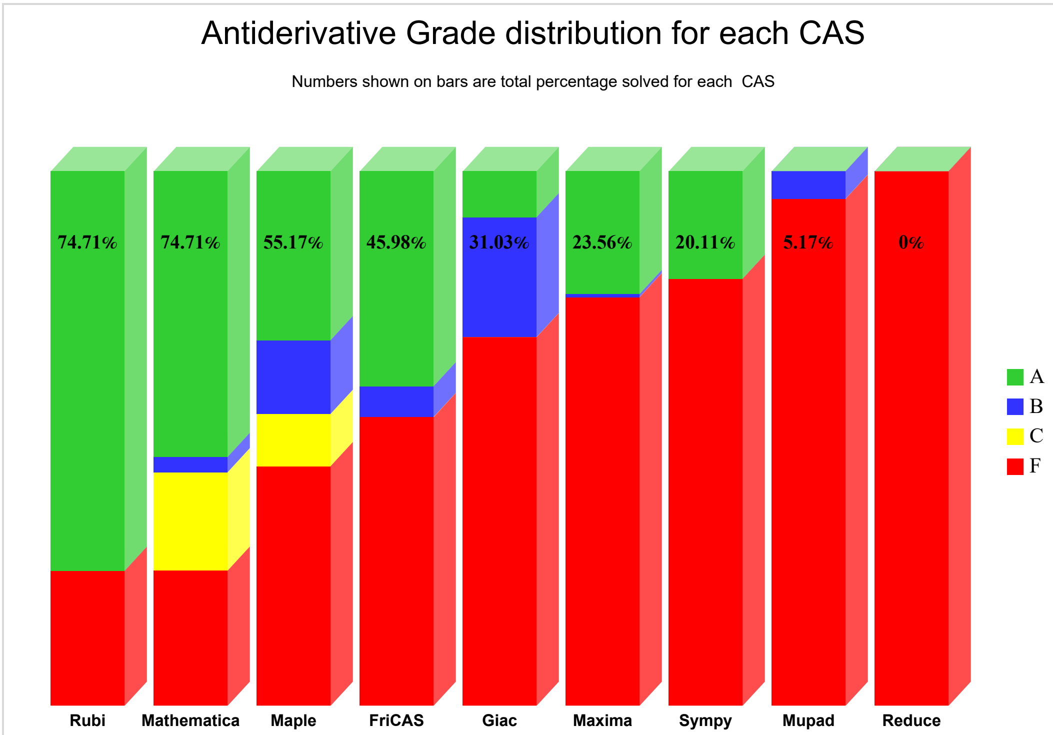
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

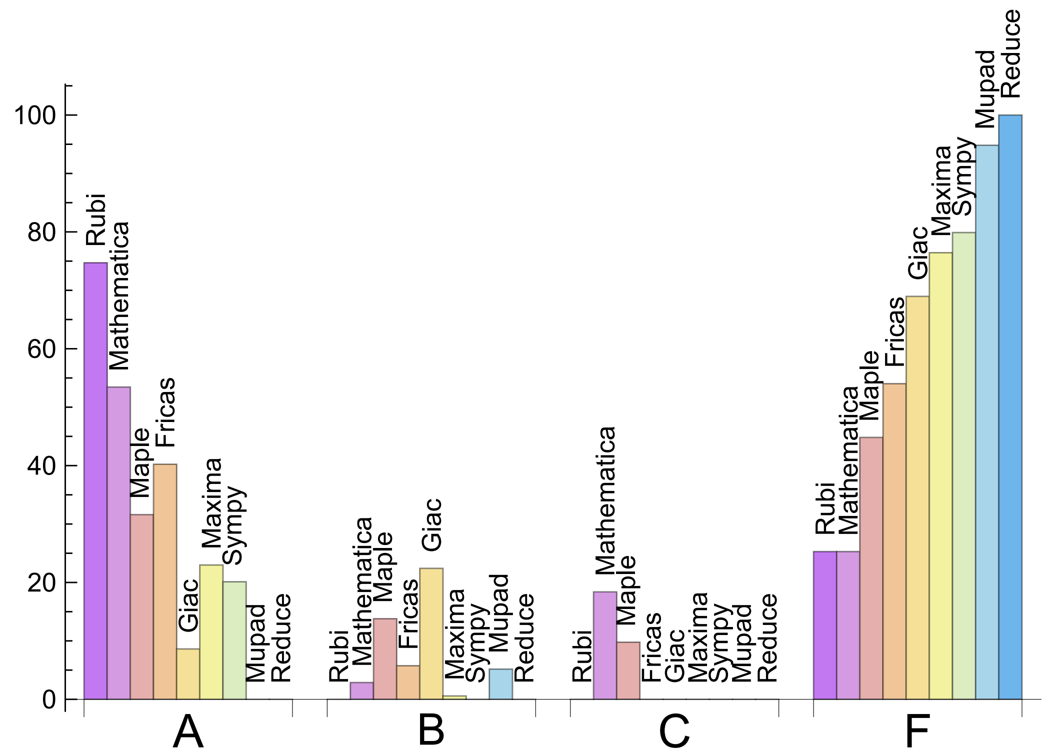
System	% A grade	% B grade	% C grade	% F grade
Rubi	74.713	0.000	0.000	25.287
Mathematica	53.448	2.874	18.391	25.287
Fricas	40.230	5.747	0.000	54.023
Maple	31.609	13.793	9.770	44.828
Maxima	22.989	0.575	0.000	76.437
Sympy	20.115	0.000	0.000	79.885
Giac	8.621	22.414	0.000	68.966
Mupad	0.000	5.172	0.000	94.828
Reduce	0.000	0.000	0.000	100.000

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	34	100.00	0.00	0.00
Fricas	50	92.00	8.00	0.00
Giac	78	48.72	6.41	44.87
Maxima	112	46.43	0.00	53.57
Sympy	107	62.62	37.38	0.00
Mupad	121	0.00	100.00	0.00
Reduce	130	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.21
Reduce	0.30
Rubi	0.66
Mupad	1.11
Maxima	2.10
Mathematica	3.49
Giac	3.71
Maple	4.55
Sympy	18.27

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	31.77	1.25	27.00	1.17
Reduce	112.59	5.24	79.50	3.51
Sympy	127.97	1.37	36.00	1.20
Rubi	201.26	1.00	129.00	1.00
Mathematica	249.16	1.14	124.00	1.09
Maxima	256.45	12.72	140.50	1.42
Fricas	257.24	1.91	93.50	1.23
Maple	294.86	1.46	138.50	1.14
Giac	1805.35	12.76	74.00	1.18

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

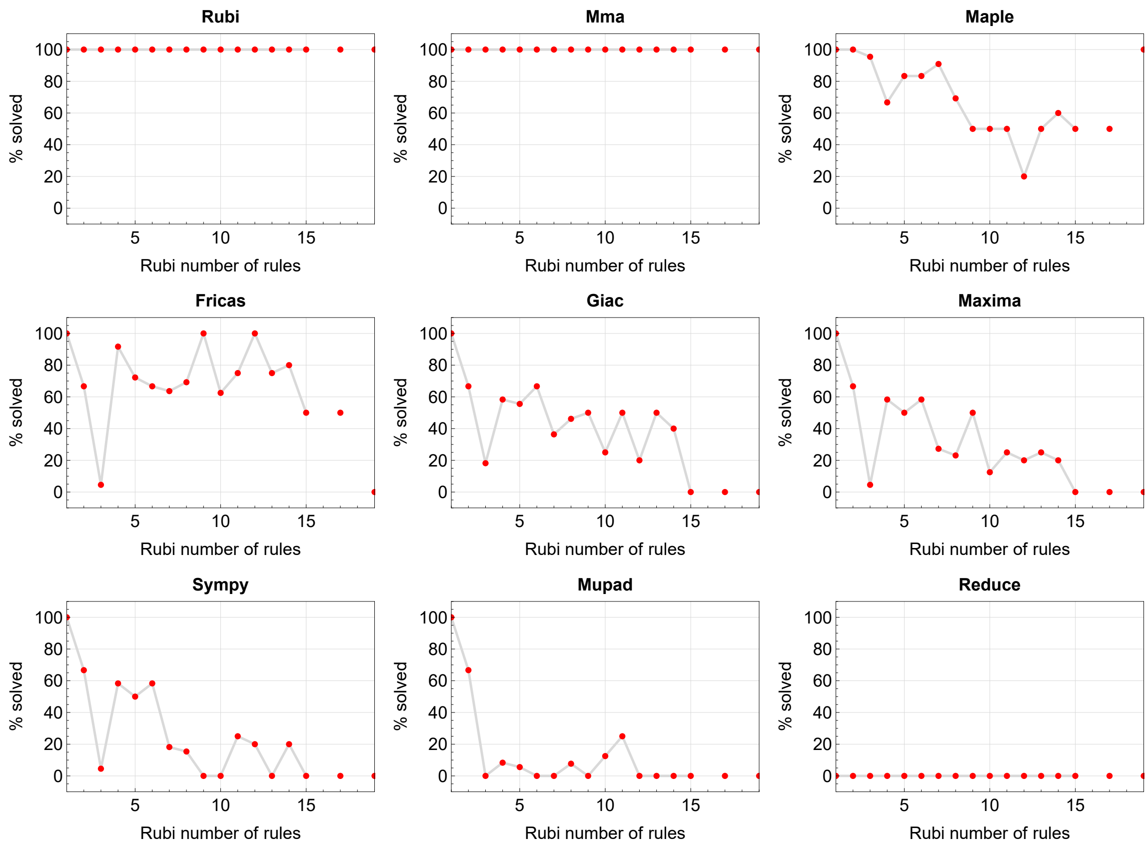


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

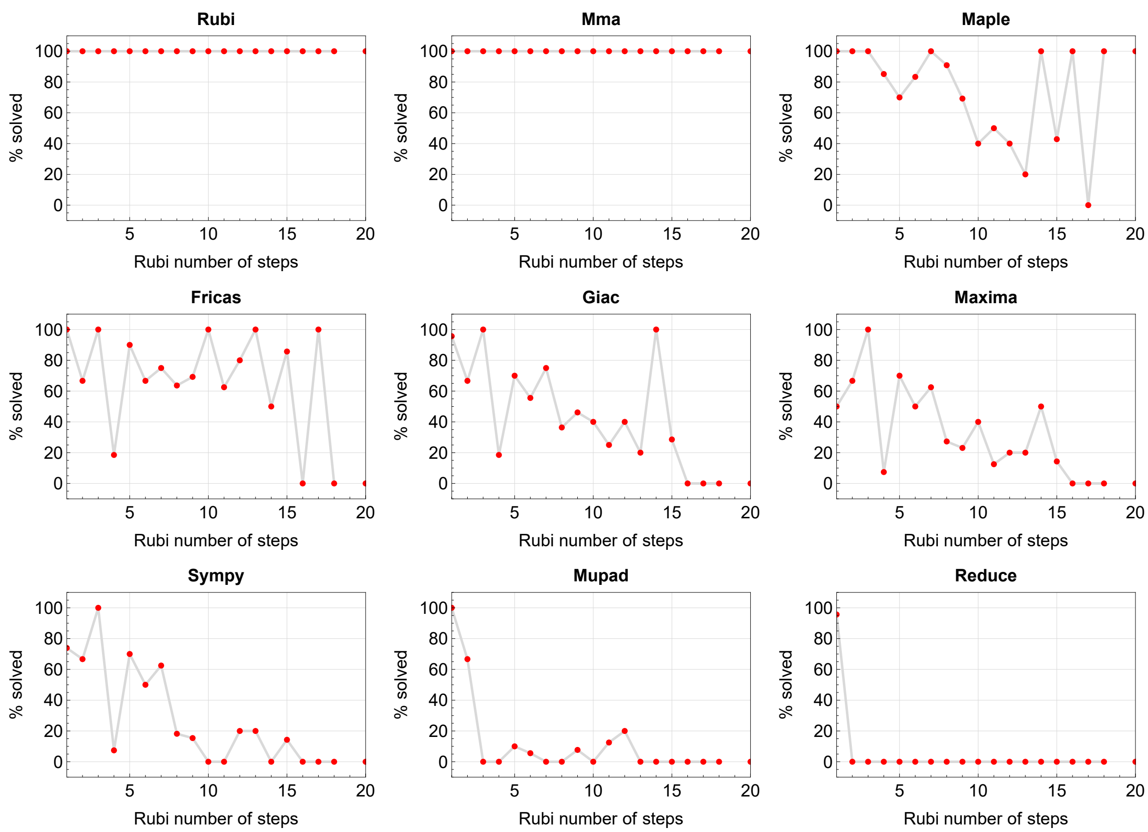


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

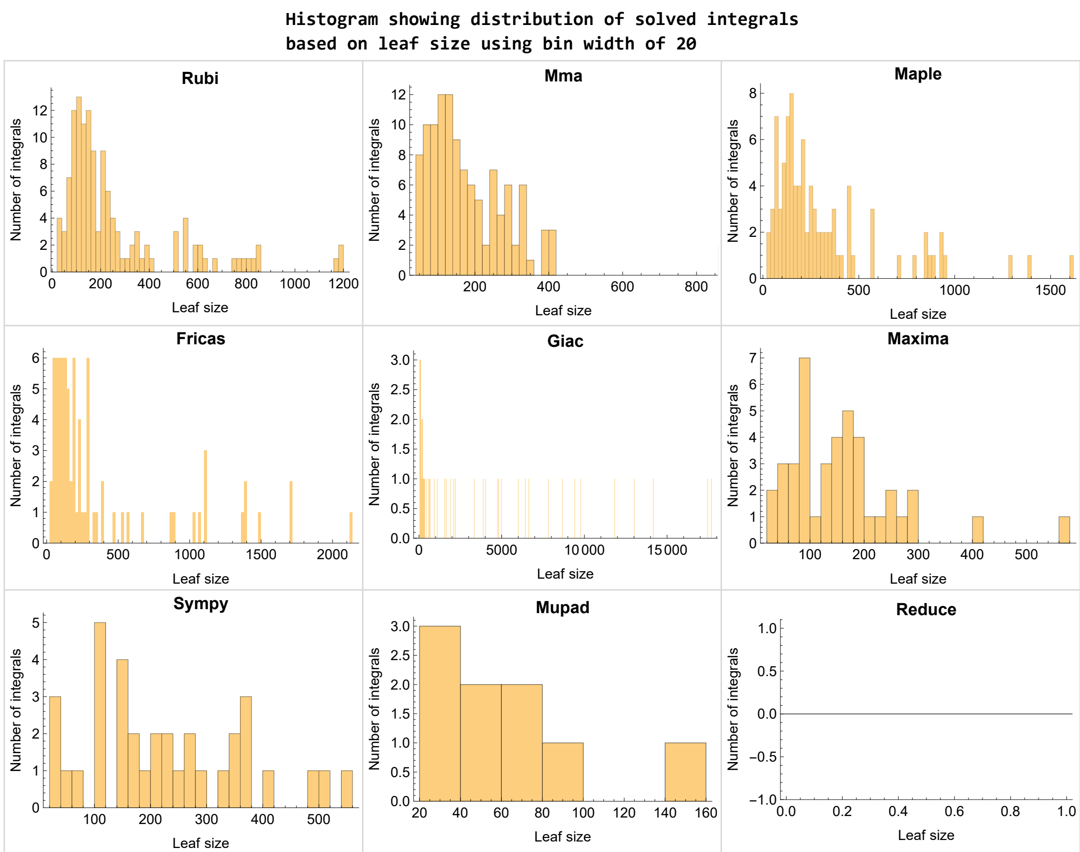


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

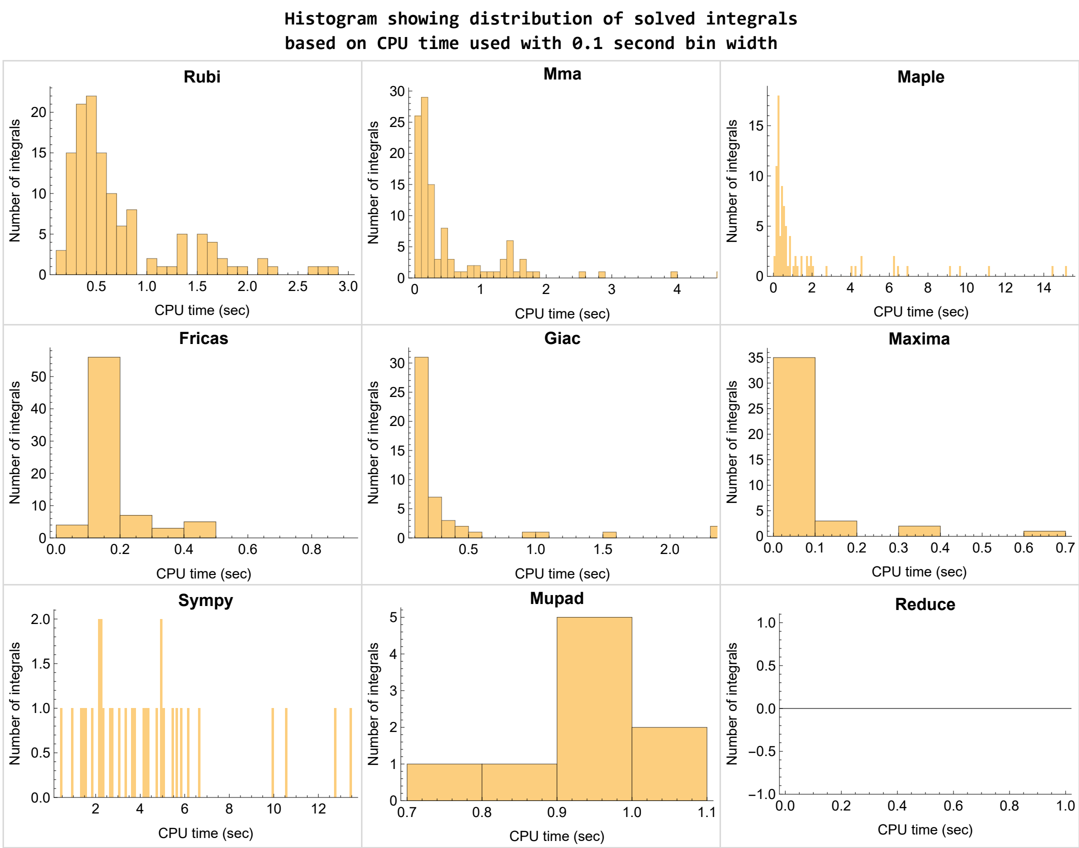


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

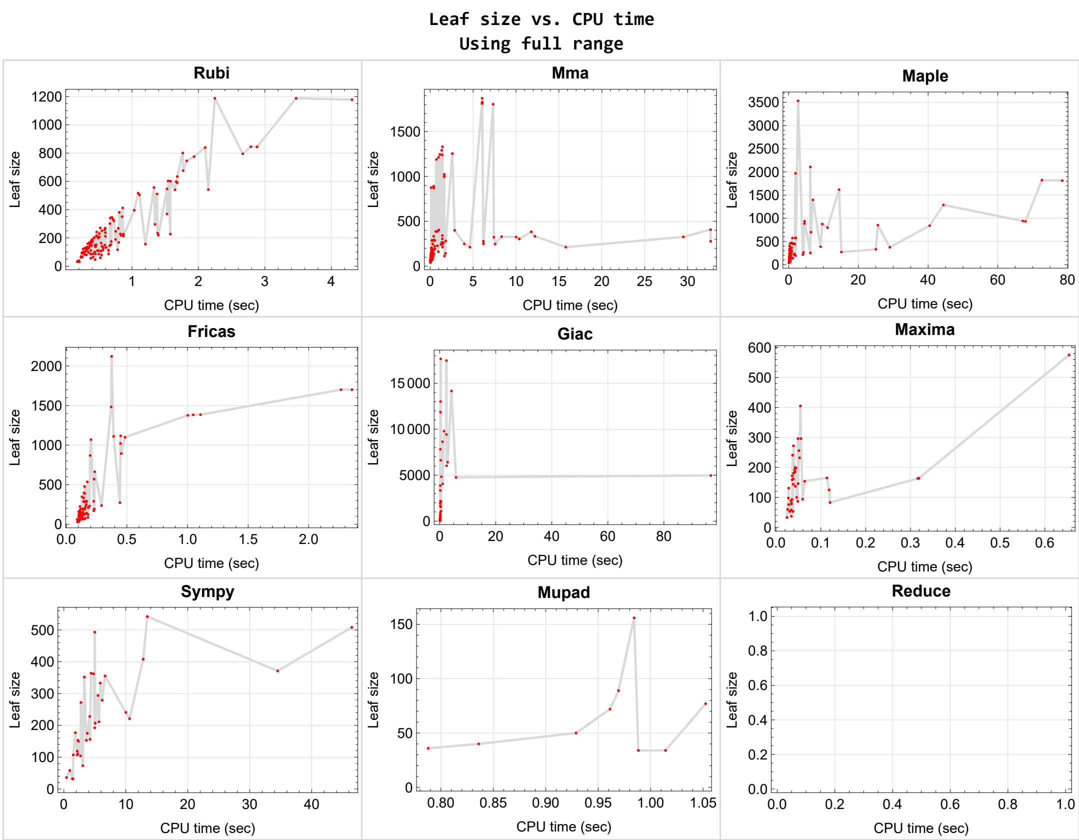


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{33, 34, 35, 39, 40, 41, 45, 46, 47, 51, 52, 54, 55, 114, 115, 116, 117, 118, 123, 124, 125, 126, 127, 128, 134, 135, 136, 137, 144, 145, 146, 147, 154, 155, 156, 157, 164, 165, 166, 167, 168, 169, 173, 174}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {171}

Mathematica {91, 92, 96, 97, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110, 111, 112, 113, 121, 122, 131, 132, 141, 151, 152, 153}

Maple {91, 92, 94, 96, 97, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

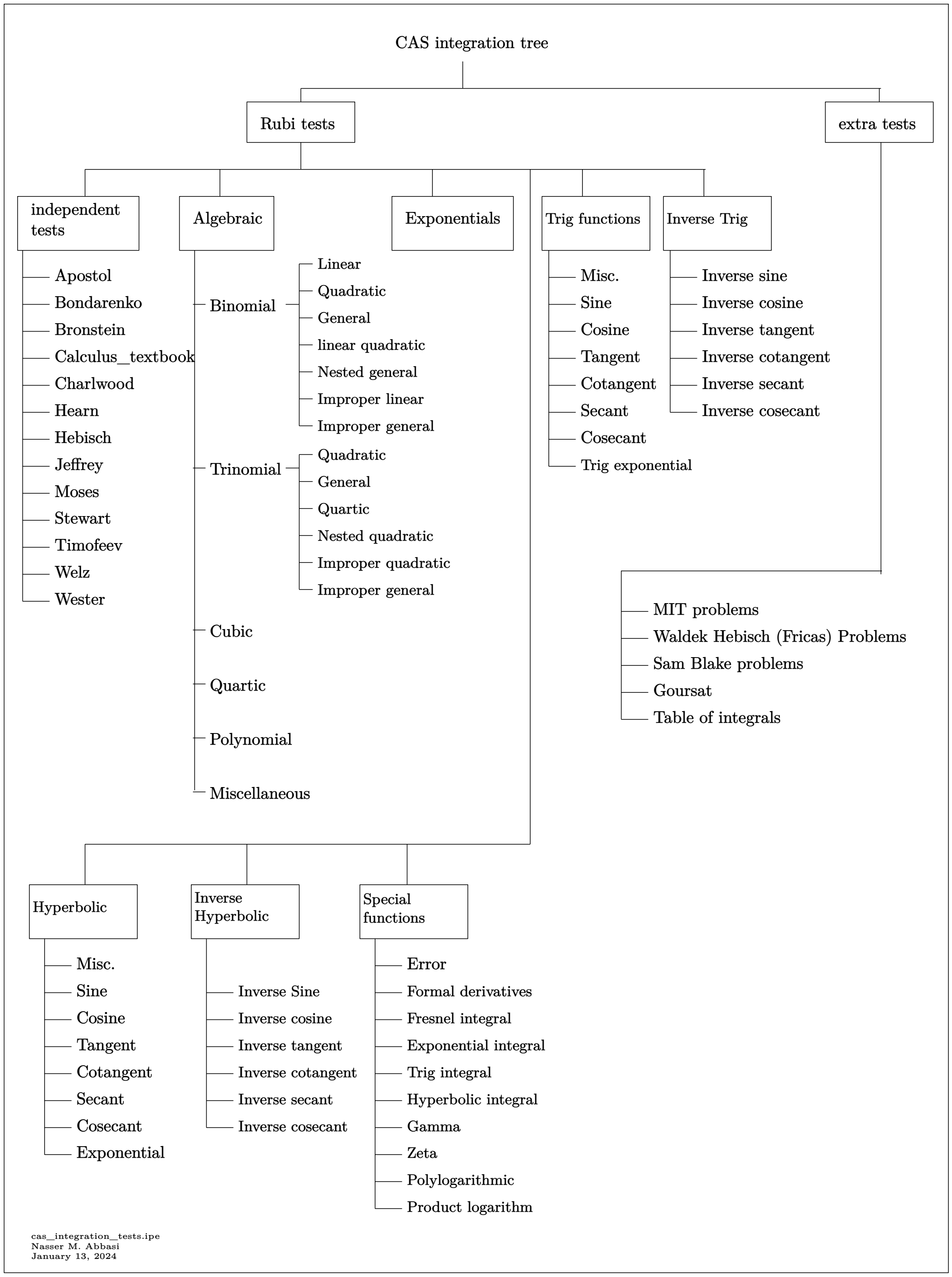
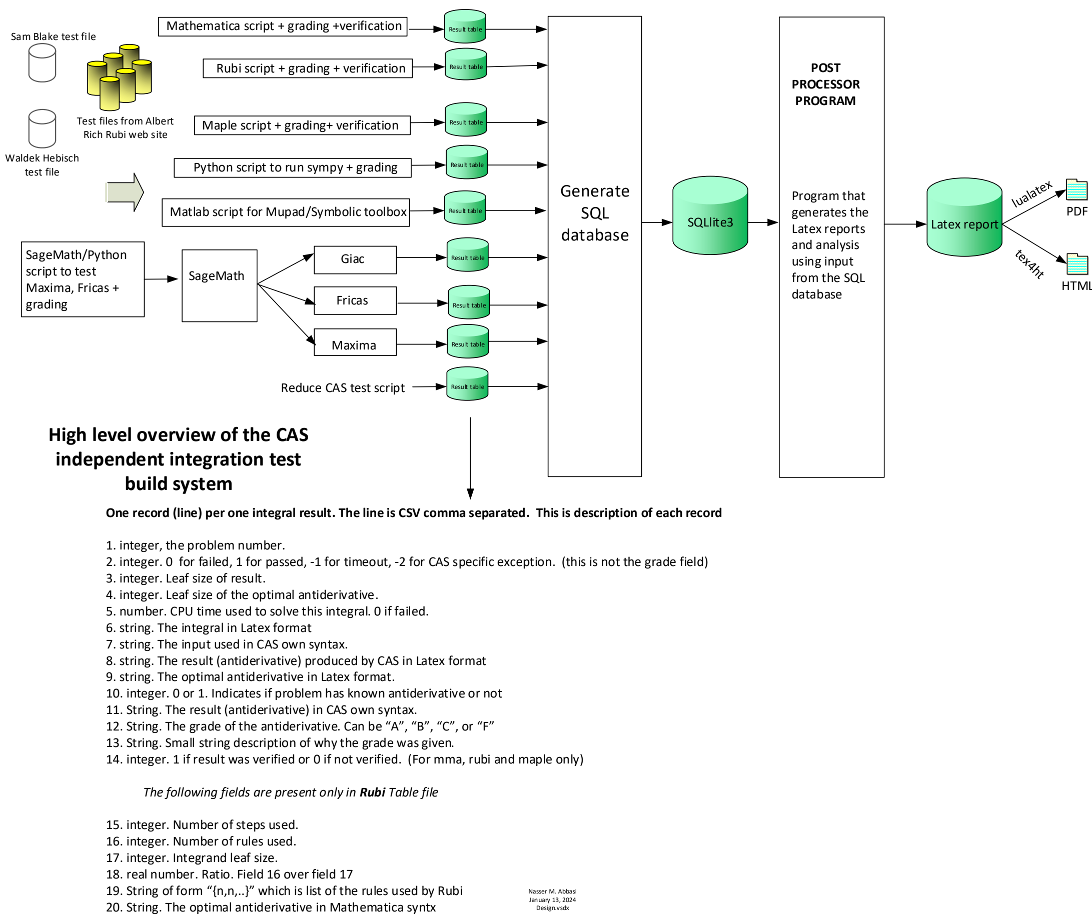


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS 29
2.2 Detailed conclusion table per each integral for all CAS systems 34
2.3 Detailed conclusion table specific for Rubi results 78

2.1 List of integrals sorted by grade for each CAS

Rubi 29
Mma 29
Maple 30
Fricas 30
Maxima 31
Giac 31
Mupad 32
Sympy 32
Reduce 33

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 42, 43, 44, 48, 49, 50, 53, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 119, 120, 121, 122, 129, 130, 131, 132, 133, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 158, 159, 160, 161, 162, 163, 170, 171, 172 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 42, 43, 44, 48, 49, 50, 53, 56, 57, 58, 59, 60, 61, 62, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 100, 101, 102, 103, 108, 109, 110, 138, 148, 158, 161, 162, 163, 170, 171, 172 }

B grade { 96, 97, 99, 104, 107 }

C grade { 63, 64, 65, 67, 68, 98, 105, 106, 111, 112, 113, 119, 120, 121, 122, 129, 130, 131, 132, 133, 139, 140, 141, 142, 143, 149, 150, 151, 152, 153, 159, 160 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 15, 16, 18, 19, 21, 22, 24, 26, 30, 36, 37, 38, 42, 43, 44, 48, 49, 50, 58, 59, 60, 61, 64, 65, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 83, 84, 85, 86, 87, 89, 90 }

B grade { 10, 12, 14, 17, 20, 23, 25, 27, 28, 29, 31, 32, 56, 57, 62, 63, 67, 68, 81, 82, 88, 98, 105, 106 }

C grade { 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110 }

F normal fail { 53, 111, 112, 113, 119, 120, 121, 122, 129, 130, 131, 132, 133, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 158, 159, 160, 161, 162, 163, 170, 171, 172 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 20, 21, 22, 23, 29, 30, 31, 32, 56, 57, 58, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88, 98, 111, 112, 113, 119, 120, 121, 122, 129, 130, 131, 132, 133, 138, 139, 140, 141, 142, 143, 148, 149, 150, 158, 159, 160, 170, 171, 172 }

B grade { 7, 17, 59, 61, 62, 105, 106, 151, 152, 153 }

C grade { }

F normal fail { 8, 16, 18, 19, 24, 25, 26, 27, 28, 36, 37, 38, 42, 43, 44, 48, 49, 50, 53, 60, 64, 68, 79, 80, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110, 161, 162, 163 }

F(-1) timedout fail { 63, 65, 66, 67 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 17, 20, 22, 29, 56, 57, 58, 59, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88 }

B grade { 31 }

C grade { }

F normal fail { 8, 16, 18, 19, 21, 23, 24, 25, 26, 27, 28, 30, 32, 36, 37, 38, 42, 43, 44, 48, 49, 50, 53, 60, 61, 62, 79, 80, 89, 90, 92, 94, 96, 97, 98, 99, 104, 105, 106, 107, 113, 122, 133, 143, 158, 159, 161, 162, 163, 170, 171, 172 }

F(-1) timedout fail { }

F(-2) exception fail { 63, 64, 65, 66, 67, 68, 91, 93, 95, 100, 101, 102, 103, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 160 }

Giac

A grade { 9, 10, 11, 12, 13, 14, 22, 36, 37, 38, 73, 74, 75, 85, 86 }

B grade { 1, 2, 3, 4, 5, 6, 7, 15, 17, 20, 21, 23, 29, 30, 31, 32, 42, 43, 44, 48, 49, 50, 56, 57, 58, 59, 69, 70, 71, 72, 76, 77, 78, 81, 82, 83, 84, 87, 88 }

C grade { }

F normal fail { 53, 63, 64, 65, 66, 67, 68, 111, 112, 113, 119, 120, 121, 122, 129, 130, 131, 132, 133, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 158, 159, 160, 161, 162, 163, 172 }

F(-1) timedout fail { 45, 96, 97, 100, 104 }

F(-2) exception fail { 8, 16, 18, 19, 24, 25, 26, 27, 28, 40, 60, 61, 62, 79, 80, 89, 90, 91, 92, 93, 94, 95, 98, 99, 101, 102, 103, 105, 106, 107, 108, 109, 110, 170, 171 }

Mupad

A grade { }

B grade { 6, 7, 9, 10, 20, 29, 58, 59, 72 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 8, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 36, 37, 38, 42, 43, 44, 48, 49, 50, 53, 56, 57, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 119, 120, 121, 122, 129, 130, 131, 132, 133, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 158, 159, 160, 161, 162, 163, 170, 171, 172 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 56, 57, 58, 59, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88 }

B grade { }

C grade { }

F normal fail { 8, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 42, 43, 44, 48, 49, 50, 53, 60, 61, 62, 64, 65, 66, 67, 79, 80, 89, 90, 91, 92, 93, 94, 95, 98, 100, 101, 102, 112, 113, 119, 120, 131, 132, 133, 138, 139, 141, 142, 143, 148, 152, 153, 162, 163, 172 }

F(-1) timedout fail { 63, 68, 96, 97, 99, 103, 104, 105, 106, 107, 108, 109, 110, 111, 116, 121, 122, 125, 126, 129, 130, 140, 145, 146, 149, 150, 151, 154, 155, 156, 157, 158, 159, 160, 161, 165, 166, 169, 170, 171 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 42, 43, 44, 48, 49, 50, 53, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 119, 120, 121, 122, 129, 130, 131, 132, 133, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 158, 159, 160, 161, 162, 163, 170, 171, 172 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	122	107	177	162	116	221	8644	19	0
N.S.	1	1.07	0.94	1.55	1.42	1.02	1.94	75.82	0.17	0.00
time (sec)	N/A	0.264	0.099	0.254	0.038	0.136	10.597	0.983	0.256	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	97	72	79	81	63	153	3862	19	0
N.S.	1	1.09	0.81	0.89	0.91	0.71	1.72	43.39	0.21	0.00
time (sec)	N/A	0.255	0.063	0.174	0.034	0.109	2.267	0.170	0.263	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	93	97	141	131	107	175	4828	19	0
N.S.	1	1.04	1.09	1.58	1.47	1.20	1.97	54.25	0.21	0.00
time (sec)	N/A	0.244	0.053	0.176	0.028	0.131	3.777	0.583	0.259	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	68	62	70	60	53	107	1926	19	0
N.S.	1	1.06	0.97	1.09	0.94	0.83	1.67	30.09	0.30	0.00
time (sec)	N/A	0.233	0.073	0.174	0.026	0.110	1.518	0.159	0.232	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	85	94	98	94	107	2101	19	0
N.S.	1	1.00	1.33	1.47	1.53	1.47	1.67	32.83	0.30	0.00
time (sec)	N/A	0.236	0.043	0.172	0.027	0.131	2.135	0.441	0.260	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	50	61	37	40	58	634	17	40
N.S.	1	1.00	1.28	1.56	0.95	1.03	1.49	16.26	0.44	1.03
time (sec)	N/A	0.196	0.021	0.168	0.034	0.107	0.956	0.142	0.227	0.836

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	59	38	53	63	32	63	12	34
N.S.	1	1.00	1.84	1.19	1.66	1.97	1.00	1.97	0.38	1.06
time (sec)	N/A	0.176	0.041	0.070	0.030	0.113	1.374	0.124	0.253	1.014

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	87	59	84	0	0	0	0	17	0
N.S.	1	1.36	0.92	1.31	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.450	0.020	0.466	0.000	0.000	0.000	0.000	0.275	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	40	58	33	27	36	43	21	36
N.S.	1	1.00	1.29	1.87	1.06	0.87	1.16	1.39	0.68	1.16
time (sec)	N/A	0.207	0.026	0.176	0.024	0.093	0.419	0.132	0.242	0.788

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	64	66	96	83	39	119	58	25	50
N.S.	1	1.25	1.29	1.88	1.63	0.76	2.33	1.14	0.49	0.98
time (sec)	N/A	0.234	0.031	0.172	0.121	0.104	2.151	0.129	0.241	0.929

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	65	59	71	58	40	110	65	25	0
N.S.	1	1.08	0.98	1.18	0.97	0.67	1.83	1.08	0.42	0.00
time (sec)	N/A	0.247	0.039	0.181	0.034	0.099	2.249	0.136	0.261	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	96	78	138	125	52	192	83	25	0
N.S.	1	1.26	1.03	1.82	1.64	0.68	2.53	1.09	0.33	0.00
time (sec)	N/A	0.257	0.046	0.183	0.118	0.094	4.974	0.135	0.251	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	86	69	79	76	51	156	87	25	0
N.S.	1	1.05	0.84	0.96	0.93	0.62	1.90	1.06	0.30	0.00
time (sec)	N/A	0.278	0.048	0.177	0.028	0.109	4.220	0.133	0.238	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	128	88	174	165	62	241	104	25	0
N.S.	1	1.27	0.87	1.72	1.63	0.61	2.39	1.03	0.25	0.00
time (sec)	N/A	0.290	0.056	0.176	0.114	0.091	9.989	0.133	0.218	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	114	124	170	163	146	0	6625	39	0
N.S.	1	1.07	1.16	1.59	1.52	1.36	0.00	61.92	0.36	0.00
time (sec)	N/A	0.563	0.138	0.527	0.316	0.124	0.000	0.388	0.240	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	140	226	285	0	0	0	0	39	0
N.S.	1	0.95	1.54	1.94	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.548	0.852	0.850	0.000	0.000	0.000	0.000	0.228	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	64	90	123	87	111	0	2181	35	0
N.S.	1	1.14	1.61	2.20	1.55	1.98	0.00	38.95	0.62	0.00
time (sec)	N/A	0.413	0.165	0.507	0.049	0.123	0.000	0.246	0.255	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	87	163	194	0	0	0	0	28	0
N.S.	1	0.95	1.77	2.11	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.411	0.158	0.357	0.000	0.000	0.000	0.000	0.269	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	107	129	206	0	0	0	0	37	0
N.S.	1	1.15	1.39	2.22	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.530	0.085	0.512	0.000	0.000	0.000	0.000	0.228	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	56	75	114	78	57	0	105	42	89
N.S.	1	1.12	1.50	2.28	1.56	1.14	0.00	2.10	0.84	1.78
time (sec)	N/A	0.379	0.083	0.377	0.034	0.110	0.000	0.145	0.237	0.969

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	97	102	145	0	82	0	147	48	0
N.S.	1	1.18	1.24	1.77	0.00	1.00	0.00	1.79	0.59	0.00
time (sec)	N/A	0.344	0.075	0.502	0.000	0.115	0.000	0.144	0.273	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	110	108	153	164	93	0	168	48	0
N.S.	1	1.08	1.06	1.50	1.61	0.91	0.00	1.65	0.47	0.00
time (sec)	N/A	0.494	0.122	0.680	0.319	0.108	0.000	0.173	0.261	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	139	148	265	0	120	0	215	48	0
N.S.	1	1.14	1.21	2.17	0.00	0.98	0.00	1.76	0.39	0.00
time (sec)	N/A	0.453	0.112	0.624	0.000	0.115	0.000	0.141	0.280	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	213	288	365	0	0	0	0	59	0
N.S.	1	1.03	1.39	1.76	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.871	0.539	0.890	0.000	0.000	0.000	0.000	0.288	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	227	403	574	0	0	0	0	59	0
N.S.	1	0.96	1.71	2.43	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.869	1.009	1.076	0.000	0.000	0.000	0.000	0.291	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	132	184	252	0	0	0	0	53	0
N.S.	1	1.05	1.46	2.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.611	0.403	0.769	0.000	0.000	0.000	0.000	0.270	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	149	289	446	0	0	0	0	44	0
N.S.	1	0.94	1.83	2.82	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.590	0.221	1.153	0.000	0.000	0.000	0.000	0.263	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	145	204	364	0	0	0	0	57	0
N.S.	1	1.13	1.59	2.84	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.661	0.118	0.608	0.000	0.000	0.000	0.000	0.263	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	86	141	196	146	98	0	196	63	156
N.S.	1	1.08	1.76	2.45	1.82	1.22	0.00	2.45	0.79	1.95
time (sec)	N/A	0.499	0.118	0.451	0.049	0.103	0.000	0.145	0.256	0.984

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	149	185	207	0	150	0	278	71	0
N.S.	1	1.09	1.35	1.51	0.00	1.09	0.00	2.03	0.52	0.00
time (sec)	N/A	0.445	0.144	0.561	0.000	0.111	0.000	0.163	0.243	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	189	204	299	575	172	0	336	71	0
N.S.	1	1.11	1.20	1.76	3.38	1.01	0.00	1.98	0.42	0.00
time (sec)	N/A	0.752	0.180	0.886	0.654	0.112	0.000	0.161	0.258	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	268	283	470	0	225	0	427	71	0
N.S.	1	1.29	1.36	2.26	0.00	1.08	0.00	2.05	0.34	0.00
time (sec)	N/A	0.793	0.207	0.885	0.000	0.113	0.000	0.155	0.250	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	10	14	14	18
N.S.	1	1.00	1.17	1.00	1.17	1.17	0.83	1.17	1.17	1.50
time (sec)	N/A	0.189	2.391	0.604	0.147	0.085	0.475	46.736	0.235	0.759

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12	16
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20	1.60
time (sec)	N/A	0.171	0.021	0.431	0.146	0.091	0.478	7.276	0.238	0.759

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	15	12	16	15	20
N.S.	1	1.00	1.14	1.00	1.14	1.07	0.86	1.14	1.07	1.43
time (sec)	N/A	0.197	0.214	0.348	0.172	0.090	1.101	1.818	0.251	0.825

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	47	0	0	0	55	19	0
N.S.	1	1.00	0.93	1.02	0.00	0.00	0.00	1.20	0.41	0.00
time (sec)	N/A	0.444	0.060	0.291	0.000	0.000	0.000	0.138	0.237	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	59	56	58	0	0	0	95	19	0
N.S.	1	0.94	0.89	0.92	0.00	0.00	0.00	1.51	0.30	0.00
time (sec)	N/A	0.523	0.055	0.420	0.000	0.000	0.000	0.143	0.256	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	109	91	102	0	0	0	199	19	0
N.S.	1	0.93	0.78	0.87	0.00	0.00	0.00	1.70	0.16	0.00
time (sec)	N/A	0.479	0.125	0.254	0.000	0.000	0.000	0.147	0.251	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	583	28	12	14	28	18
N.S.	1	1.00	1.17	1.00	48.58	2.33	1.00	1.17	2.33	1.50
time (sec)	N/A	0.178	10.362	0.515	0.872	0.087	1.091	98.444	0.262	0.793

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	577	26	12	0	26	16
N.S.	1	1.00	1.20	1.00	57.70	2.60	1.20	0.00	2.60	1.60
time (sec)	N/A	0.169	20.706	0.450	0.859	0.086	1.183	0.000	0.259	0.791

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	560	30	14	16	30	20
N.S.	1	1.00	1.14	1.00	40.00	2.14	1.00	1.14	2.14	1.43
time (sec)	N/A	0.188	2.753	0.368	0.630	0.085	2.147	5.398	0.261	0.865

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	79	69	78	0	0	0	226	36	0
N.S.	1	1.05	0.92	1.04	0.00	0.00	0.00	3.01	0.48	0.00
time (sec)	N/A	0.541	0.243	0.331	0.000	0.000	0.000	0.171	0.280	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	86	80	77	0	0	0	357	36	0
N.S.	1	1.02	0.95	0.92	0.00	0.00	0.00	4.25	0.43	0.00
time (sec)	N/A	0.646	0.286	0.466	0.000	0.000	0.000	0.175	0.239	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	164	223	153	0	0	0	694	36	0
N.S.	1	0.92	1.25	0.86	0.00	0.00	0.00	3.90	0.20	0.00
time (sec)	N/A	0.535	0.520	0.296	0.000	0.000	0.000	0.184	0.245	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	1790	42	12	0	42	18
N.S.	1	1.00	1.17	1.00	149.17	3.50	1.00	0.00	3.50	1.50
time (sec)	N/A	0.173	2.401	0.477	34.090	0.089	2.169	0.000	0.229	0.812

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	1744	40	12	12	40	16
N.S.	1	1.00	1.20	1.00	174.40	4.00	1.20	1.20	4.00	1.60
time (sec)	N/A	0.168	8.510	0.525	34.687	0.097	2.279	48.664	0.282	0.835

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	1568	45	14	16	45	20
N.S.	1	1.00	1.14	1.00	112.00	3.21	1.00	1.14	3.21	1.43
time (sec)	N/A	0.185	1.198	0.350	25.694	0.089	4.386	9.590	0.287	1.008

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	112	88	154	0	0	0	580	53	0
N.S.	1	1.09	0.85	1.50	0.00	0.00	0.00	5.63	0.51	0.00
time (sec)	N/A	0.645	0.270	0.307	0.000	0.000	0.000	0.210	0.255	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	116	114	157	0	0	0	929	53	0
N.S.	1	1.04	1.02	1.40	0.00	0.00	0.00	8.29	0.47	0.00
time (sec)	N/A	0.783	0.275	0.431	0.000	0.000	0.000	0.213	0.250	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	211	169	307	0	0	0	1640	53	0
N.S.	1	0.93	0.74	1.35	0.00	0.00	0.00	7.19	0.23	0.00
time (sec)	N/A	0.600	0.312	0.288	0.000	0.000	0.000	0.228	0.239	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	991	44	15	18	121	22
N.S.	1	1.00	1.12	1.00	61.94	2.75	0.94	1.12	7.56	1.38
time (sec)	N/A	0.196	3.879	0.524	17.407	0.120	26.310	0.562	0.284	1.093

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	512	30	15	18	80	22
N.S.	1	1.00	1.12	1.00	32.00	1.88	0.94	1.12	5.00	1.38
time (sec)	N/A	0.187	2.494	0.505	7.296	0.135	12.049	0.404	0.285	1.079

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	82	0	0	0	0	0	41	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.254	0.176	0.000	0.000	0.000	0.000	0.000	0.275	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	14	18	20	22
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.88	1.12	1.25	1.38
time (sec)	N/A	0.204	0.612	1.835	0.203	0.098	1.323	0.741	0.283	0.702

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	643	32	15	18	34	22
N.S.	1	1.00	1.12	1.00	40.19	2.00	0.94	1.12	2.12	1.38
time (sec)	N/A	0.201	1.248	1.144	1.942	0.091	6.412	1.509	0.232	0.732

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	168	166	402	272	291	362	9430	93	0
N.S.	1	1.01	0.99	2.41	1.63	1.74	2.17	56.47	0.56	0.00
time (sec)	N/A	0.801	0.165	0.240	0.039	0.226	4.794	2.376	0.253	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	130	124	306	200	209	228	6416	62	0
N.S.	1	1.05	1.00	2.47	1.61	1.69	1.84	51.74	0.50	0.00
time (sec)	N/A	0.568	0.108	0.237	0.043	0.182	4.172	2.814	0.239	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	87	114	110	93	130	104	1547	32	77
N.S.	1	1.04	1.36	1.31	1.11	1.55	1.24	18.42	0.38	0.92
time (sec)	N/A	0.377	0.135	0.209	0.036	0.154	2.689	0.315	0.261	1.052

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	59	38	53	63	32	63	12	34
N.S.	1	1.00	1.84	1.19	1.66	1.97	1.00	1.97	0.38	1.06
time (sec)	N/A	0.181	0.027	0.071	0.038	0.134	1.467	0.121	0.271	0.988

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	333	453	0	0	0	0	30	0
N.S.	1	1.00	1.35	1.83	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.754	0.413	1.483	0.000	0.000	0.000	0.000	0.288	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	111	142	193	0	477	0	0	75	0
N.S.	1	1.07	1.37	1.86	0.00	4.59	0.00	0.00	0.72	0.00
time (sec)	N/A	0.380	0.132	1.971	0.000	0.151	0.000	0.000	0.276	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	199	247	574	0	1117	0	0	159	0
N.S.	1	1.16	1.44	3.34	0.00	6.49	0.00	0.00	0.92	0.00
time (sec)	N/A	0.492	0.286	1.888	0.000	0.449	0.000	0.000	0.307	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	396	333	798	0	0	0	0	83	0
N.S.	1	1.06	0.90	2.15	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	1.033	12.204	11.148	0.000	0.000	0.000	0.000	0.359	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	350	277	386	0	0	0	0	44	0
N.S.	1	1.11	0.88	1.23	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.849	32.672	9.150	0.000	0.000	0.000	0.000	0.329	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	237	212	252	0	0	0	0	32	0
N.S.	1	1.12	1.00	1.19	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.599	15.807	6.211	0.000	0.000	0.000	0.000	0.328	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	137	124	215	0	0	0	0	52	0
N.S.	1	1.15	1.04	1.81	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.451	0.197	4.076	0.000	0.000	0.000	0.000	0.353	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	320	326	875	0	0	0	0	136	0
N.S.	1	1.07	1.09	2.94	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.727	29.494	9.631	0.000	0.000	0.000	0.000	0.523	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	498	516	407	1618	0	0	0	0	260	0
N.S.	1	1.04	0.82	3.25	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	1.092	32.667	14.467	0.000	0.000	0.000	0.000	0.867	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	167	141	328	296	192	408	17474	41	0
N.S.	1	0.81	0.68	1.59	1.44	0.93	1.98	84.83	0.20	0.00
time (sec)	N/A	0.359	0.164	0.436	0.056	0.232	12.798	2.397	0.255	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	136	123	254	232	170	294	9792	41	0
N.S.	1	0.84	0.76	1.58	1.44	1.06	1.83	60.82	0.25	0.00
time (sec)	N/A	0.325	0.122	0.429	0.053	0.227	5.498	1.519	0.257	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	101	150	132	154	141	153	4051	34	0
N.S.	1	0.93	1.38	1.21	1.41	1.29	1.40	37.17	0.31	0.00
time (sec)	N/A	0.270	0.166	0.209	0.064	0.167	3.626	1.039	0.241	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	83	104	116	89	123	73	1088	39	72
N.S.	1	0.95	1.20	1.33	1.02	1.41	0.84	12.51	0.45	0.83
time (sec)	N/A	0.286	0.091	0.213	0.033	0.180	3.059	0.450	0.214	0.961

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	101	69	108	94	67	150	113	51	0
N.S.	1	0.96	0.66	1.03	0.90	0.64	1.43	1.08	0.49	0.00
time (sec)	N/A	0.292	0.072	0.223	0.059	0.149	2.354	0.128	0.236	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	131	94	127	137	89	279	158	51	0
N.S.	1	0.86	0.62	0.84	0.90	0.59	1.84	1.04	0.34	0.00
time (sec)	N/A	0.309	0.096	0.216	0.043	0.115	6.191	0.135	0.245	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	159	110	145	172	110	371	202	51	0
N.S.	1	0.81	0.56	0.74	0.87	0.56	1.88	1.03	0.26	0.00
time (sec)	N/A	0.335	0.112	0.216	0.036	0.135	34.505	0.134	0.241	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	161	118	139	185	128	364	13018	41	0
N.S.	1	0.82	0.60	0.71	0.94	0.65	1.86	66.42	0.21	0.00
time (sec)	N/A	0.377	0.169	0.401	0.042	0.144	4.386	0.263	0.238	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	131	98	121	144	107	272	7820	41	0
N.S.	1	0.86	0.64	0.79	0.94	0.70	1.78	51.11	0.27	0.00
time (sec)	N/A	0.352	0.203	0.568	0.038	0.129	2.756	0.211	0.236	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	116	79	217	100	86	177	3346	39	0
N.S.	1	0.84	0.57	1.57	0.72	0.62	1.28	24.25	0.28	0.00
time (sec)	N/A	0.327	0.083	0.573	0.046	0.179	1.848	0.178	0.250	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	152	115	137	0	0	0	0	37	0
N.S.	1	1.23	0.93	1.10	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.683	0.071	1.224	0.000	0.000	0.000	0.000	0.251	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	175	132	143	0	0	0	0	53	0
N.S.	1	1.28	0.96	1.04	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.687	0.087	1.101	0.000	0.000	0.000	0.000	0.267	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	215	186	459	405	273	542	4760	72	0
N.S.	1	0.85	0.74	1.82	1.61	1.08	2.15	18.89	0.29	0.00
time (sec)	N/A	0.464	0.217	0.450	0.054	0.443	13.468	5.800	0.243	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	179	153	340	296	237	355	14166	65	0
N.S.	1	0.94	0.80	1.78	1.55	1.24	1.86	74.17	0.34	0.00
time (sec)	N/A	0.386	0.138	0.240	0.049	0.291	6.647	4.168	0.244	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	146	136	249	198	230	207	6018	74	0
N.S.	1	0.90	0.84	1.54	1.22	1.42	1.28	37.15	0.46	0.00
time (sec)	N/A	0.393	0.124	0.266	0.045	0.192	5.063	2.423	0.233	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	147	127	231	159	221	211	4968	81	0
N.S.	1	0.93	0.80	1.46	1.01	1.40	1.34	31.44	0.51	0.00
time (sec)	N/A	0.379	0.132	0.247	0.036	0.158	5.668	96.738	0.249	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	171	127	175	181	127	333	222	85	0
N.S.	1	0.93	0.69	0.96	0.99	0.69	1.82	1.21	0.46	0.00
time (sec)	N/A	0.401	0.139	0.247	0.039	0.104	5.873	0.132	0.244	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	201	153	207	241	159	508	293	85	0
N.S.	1	0.83	0.63	0.86	1.00	0.66	2.11	1.22	0.35	0.00
time (sec)	N/A	0.430	0.147	0.259	0.037	0.102	46.474	0.138	0.267	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	207	162	198	256	187	493	17666	72	0
N.S.	1	0.86	0.67	0.82	1.06	0.77	2.04	73.00	0.30	0.00
time (sec)	N/A	0.484	0.165	0.625	0.051	0.138	4.998	0.335	0.281	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	160	125	352	192	153	352	11858	70	0
N.S.	1	0.82	0.64	1.81	0.98	0.78	1.81	60.81	0.36	0.00
time (sec)	N/A	0.371	0.173	0.616	0.042	0.137	3.306	0.250	0.263	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	217	160	231	0	0	0	0	67	0
N.S.	1	1.17	0.86	1.24	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.837	0.222	1.702	0.000	0.000	0.000	0.000	0.233	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	228	194	216	0	0	0	0	84	0
N.S.	1	1.21	1.03	1.14	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.823	0.565	1.482	0.000	0.000	0.000	0.000	0.253	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	546	598	1023	374	0	0	0	0	52	0
N.S.	1	1.10	1.87	0.68	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.665	1.640	29.015	0.000	0.000	0.000	0.000	0.253	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	499	547	891	446	0	0	0	0	37	0
N.S.	1	1.10	1.79	0.89	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.527	0.411	1.791	0.000	0.000	0.000	0.000	0.250	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	509	557	871	272	0	0	0	0	46	0
N.S.	1	1.09	1.71	0.53	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.331	0.428	15.122	0.000	0.000	0.000	0.000	0.252	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	459	511	876	1970	0	0	0	0	44	0
N.S.	1	1.11	1.91	4.29	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.377	0.118	1.966	0.000	0.000	0.000	0.000	0.275	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	551	603	997	331	0	0	0	0	58	0
N.S.	1	1.09	1.81	0.60	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	1.544	1.635	25.023	0.000	0.000	0.000	0.000	0.251	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	617	676	1255	701	0	0	0	0	137	0
N.S.	1	1.10	2.03	1.14	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	1.770	2.576	6.415	0.000	0.000	0.000	0.000	0.271	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	582	634	1213	577	0	0	0	0	123	0
N.S.	1	1.09	2.08	0.99	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	1.682	0.974	2.094	0.000	0.000	0.000	0.000	0.258	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	118	286	263	0	390	0	0	90	0
N.S.	1	0.90	2.18	2.01	0.00	2.98	0.00	0.00	0.69	0.00
time (sec)	N/A	0.315	0.423	4.262	0.000	0.152	0.000	0.000	0.272	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	546	602	1190	2108	0	0	0	0	138	0
N.S.	1	1.10	2.18	3.86	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	1.578	0.726	6.215	0.000	0.000	0.000	0.000	0.278	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	784	844	1331	941	0	0	0	0	149	0
N.S.	1	1.08	1.70	1.20	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	2.883	1.434	67.161	0.000	0.000	0.000	0.000	0.260	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	745	801	1245	852	0	0	0	0	144	0
N.S.	1	1.08	1.67	1.14	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	1.764	1.124	25.597	0.000	0.000	0.000	0.000	0.255	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	739	795	1239	840	0	0	0	0	137	0
N.S.	1	1.08	1.68	1.14	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	2.668	1.405	40.434	0.000	0.000	0.000	0.000	0.275	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	785	845	1291	933	0	0	0	0	155	0
N.S.	1	1.08	1.64	1.19	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	2.789	1.342	67.926	0.000	0.000	0.000	0.000	0.267	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	719	775	1805	1396	0	0	0	0	238	0
N.S.	1	1.08	2.51	1.94	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	1.934	7.334	6.962	0.000	0.000	0.000	0.000	0.258	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	144	389	937	0	1021	0	0	180	0
N.S.	1	0.92	2.48	5.97	0.00	6.50	0.00	0.00	1.15	0.00
time (sec)	N/A	0.364	0.856	4.574	0.000	0.447	0.000	0.000	0.281	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	189	386	886	0	894	0	0	172	0
N.S.	1	0.98	2.00	4.59	0.00	4.63	0.00	0.00	0.89	0.00
time (sec)	N/A	0.401	0.644	4.505	0.000	0.454	0.000	0.000	0.280	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	685	745	1871	3533	0	0	0	0	265	0
N.S.	1	1.09	2.73	5.16	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	1.824	6.053	2.709	0.000	0.000	0.000	0.000	0.358	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1124	1188	1819	1822	0	0	0	0	272	0
N.S.	1	1.06	1.62	1.62	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	2.247	6.069	72.634	0.000	0.000	0.000	0.000	0.300	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1124	1188	1827	1288	0	0	0	0	271	0
N.S.	1	1.06	1.63	1.15	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	3.470	6.057	44.393	0.000	0.000	0.000	0.000	0.273	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1114	1178	1812	1812	0	0	0	0	263	0
N.S.	1	1.06	1.63	1.63	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	4.312	6.038	78.478	0.000	0.000	0.000	0.000	0.295	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	403	369	342	0	0	1701	0	0	97	0
N.S.	1	0.92	0.85	0.00	0.00	4.22	0.00	0.00	0.24	0.00
time (sec)	N/A	1.527	1.415	0.000	0.000	2.357	0.000	0.000	0.408	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	294	269	263	0	0	1383	0	0	76	0
N.S.	1	0.91	0.89	0.00	0.00	4.70	0.00	0.00	0.26	0.00
time (sec)	N/A	0.597	1.535	0.000	0.000	1.047	0.000	0.000	0.358	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	195	179	213	0	0	1100	0	0	51	0
N.S.	1	0.92	1.09	0.00	0.00	5.64	0.00	0.00	0.26	0.00
time (sec)	N/A	0.406	1.449	0.000	0.000	0.484	0.000	0.000	0.320	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	20	23	84	27
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.87	1.00	3.65	1.17
time (sec)	N/A	0.275	6.970	0.310	0.000	0.120	15.186	0.193	0.305	1.162

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	22	23	107	27
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.96	1.00	4.65	1.17
time (sec)	N/A	0.280	4.370	2.474	0.000	0.098	20.597	0.202	0.296	1.337

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	27	0	23	87	27
N.S.	1	1.00	1.09	0.91	0.00	1.17	0.00	1.00	3.78	1.17
time (sec)	N/A	0.286	11.403	0.265	0.000	0.103	0.000	0.182	0.292	1.299

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	20	19	20	61	24
N.S.	1	1.00	1.10	0.90	0.00	1.00	0.95	1.00	3.05	1.20
time (sec)	N/A	0.216	16.045	0.299	0.000	0.111	55.454	0.188	0.265	1.211

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	22	23	67	27
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.96	1.00	2.91	1.17
time (sec)	N/A	0.265	1.378	0.301	0.000	0.091	12.088	0.198	0.282	1.459

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	328	301	247	0	0	194	0	0	70	0
N.S.	1	0.92	0.75	0.00	0.00	0.59	0.00	0.00	0.21	0.00
time (sec)	N/A	0.668	7.568	0.000	0.000	0.153	0.000	0.000	0.298	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	455	412	325	0	0	290	0	0	94	0
N.S.	1	0.91	0.71	0.00	0.00	0.64	0.00	0.00	0.21	0.00
time (sec)	N/A	0.859	10.007	0.000	0.000	0.159	0.000	0.000	0.312	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	374	341	305	0	0	1701	0	0	121	0
N.S.	1	0.91	0.82	0.00	0.00	4.55	0.00	0.00	0.32	0.00
time (sec)	N/A	0.673	1.424	0.000	0.000	2.266	0.000	0.000	0.462	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	262	242	247	0	0	1377	0	0	96	0
N.S.	1	0.92	0.94	0.00	0.00	5.26	0.00	0.00	0.37	0.00
time (sec)	N/A	0.467	1.442	0.000	0.000	1.002	0.000	0.000	0.394	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	20	23	123	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.87	1.00	5.35	1.17
time (sec)	N/A	0.298	7.588	0.298	0.000	0.101	87.351	0.315	0.333	1.135

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	23	145	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	1.00	6.30	1.17
time (sec)	N/A	0.302	5.085	2.811	0.000	0.105	76.888	0.202	0.364	1.410

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	43	0	23	133	27
N.S.	1	1.00	1.09	0.91	0.00	1.87	0.00	1.00	5.78	1.17
time (sec)	N/A	0.298	11.501	0.283	0.000	0.097	0.000	0.184	0.345	1.399

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	37	0	20	108	24
N.S.	1	1.00	1.10	0.90	0.00	1.85	0.00	1.00	5.40	1.20
time (sec)	N/A	0.240	16.559	0.287	0.000	0.093	0.000	0.217	0.316	1.180

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	23	109	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	1.00	4.74	1.17
time (sec)	N/A	0.302	32.239	0.329	0.000	0.105	112.588	0.238	0.315	1.501

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	23	111	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	1.00	4.83	1.17
time (sec)	N/A	0.300	7.627	0.599	0.000	0.099	83.104	0.216	0.316	1.322

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	416	381	303	0	0	288	0	0	120	0
N.S.	1	0.92	0.73	0.00	0.00	0.69	0.00	0.00	0.29	0.00
time (sec)	N/A	0.809	10.377	0.000	0.000	0.142	0.000	0.000	0.360	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	554	504	383	0	0	395	0	0	142	0
N.S.	1	0.91	0.69	0.00	0.00	0.71	0.00	0.00	0.26	0.00
time (sec)	N/A	1.112	11.774	0.000	0.000	0.146	0.000	0.000	0.419	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	321	296	281	0	0	1385	0	0	79	0
N.S.	1	0.92	0.88	0.00	0.00	4.31	0.00	0.00	0.25	0.00
time (sec)	N/A	1.345	1.815	0.000	0.000	1.107	0.000	0.000	0.372	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	225	207	242	0	0	1111	0	0	58	0
N.S.	1	0.92	1.08	0.00	0.00	4.94	0.00	0.00	0.26	0.00
time (sec)	N/A	0.510	1.353	0.000	0.000	0.390	0.000	0.000	0.340	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	128	108	0	0	869	0	0	36	0
N.S.	1	0.97	0.82	0.00	0.00	6.58	0.00	0.00	0.27	0.00
time (sec)	N/A	0.341	0.348	0.000	0.000	0.197	0.000	0.000	0.303	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	31	20	23	81	27
N.S.	1	1.00	1.09	0.91	0.00	1.35	0.87	1.00	3.52	1.17
time (sec)	N/A	0.279	0.998	0.326	0.000	0.105	9.694	0.203	0.279	1.251

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	33	22	23	111	27
N.S.	1	1.00	1.09	0.91	0.00	1.43	0.96	1.00	4.83	1.17
time (sec)	N/A	0.284	4.199	1.431	0.000	0.117	32.957	0.206	0.294	1.337

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	27	22	23	69	27
N.S.	1	1.00	1.09	0.91	0.00	1.17	0.96	1.00	3.00	1.17
time (sec)	N/A	0.271	31.600	0.349	0.000	0.101	59.678	0.193	0.281	1.351

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	20	19	20	48	24
N.S.	1	1.00	1.10	0.90	0.00	1.00	0.95	1.00	2.40	1.20
time (sec)	N/A	0.212	0.586	0.311	0.000	0.098	11.636	0.204	0.292	1.105

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	243	231	143	0	0	107	0	0	49	0
N.S.	1	0.95	0.59	0.00	0.00	0.44	0.00	0.00	0.20	0.00
time (sec)	N/A	0.506	1.787	0.000	0.000	0.127	0.000	0.000	0.297	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	362	332	249	0	0	196	0	0	74	0
N.S.	1	0.92	0.69	0.00	0.00	0.54	0.00	0.00	0.20	0.00
time (sec)	N/A	0.715	6.206	0.000	0.000	0.135	0.000	0.000	0.305	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1006	839	329	0	0	291	0	0	96	0
N.S.	1	0.83	0.33	0.00	0.00	0.29	0.00	0.00	0.10	0.00
time (sec)	N/A	2.102	8.315	0.000	0.000	0.167	0.000	0.000	0.293	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	252	234	265	0	0	1483	0	0	149	0
N.S.	1	0.93	1.05	0.00	0.00	5.88	0.00	0.00	0.59	0.00
time (sec)	N/A	1.385	1.354	0.000	0.000	0.370	0.000	0.000	0.426	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	152	161	0	0	1070	0	0	126	0
N.S.	1	0.97	1.03	0.00	0.00	6.82	0.00	0.00	0.80	0.00
time (sec)	N/A	0.460	0.957	0.000	0.000	0.203	0.000	0.000	0.421	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	79	0	0	283	0	0	105	0
N.S.	1	1.00	0.99	0.00	0.00	3.54	0.00	0.00	1.31	0.00
time (sec)	N/A	0.312	0.216	0.000	0.000	0.152	0.000	0.000	0.396	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	42	20	23	223	27
N.S.	1	1.00	1.09	0.91	0.00	1.83	0.87	1.00	9.70	1.17
time (sec)	N/A	0.297	7.640	0.335	0.000	0.099	81.140	0.205	0.324	1.258

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	44	0	23	269	27
N.S.	1	1.00	1.09	0.91	0.00	1.91	0.00	1.00	11.70	1.17
time (sec)	N/A	0.321	9.815	2.308	0.000	0.106	0.000	0.206	0.330	1.485

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	47	0	23	208	27
N.S.	1	1.00	1.09	0.91	0.00	2.04	0.00	1.00	9.04	1.17
time (sec)	N/A	0.292	13.835	0.354	0.000	0.104	0.000	0.222	0.316	1.395

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	47	22	23	179	27
N.S.	1	1.00	1.09	0.91	0.00	2.04	0.96	1.00	7.78	1.17
time (sec)	N/A	0.278	4.996	0.329	0.000	0.109	35.335	0.266	0.303	1.166

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	113	0	0	74	0	0	123	0
N.S.	1	1.00	1.04	0.00	0.00	0.68	0.00	0.00	1.13	0.00
time (sec)	N/A	0.307	1.656	0.000	0.000	0.156	0.000	0.000	0.282	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	274	258	212	0	0	190	0	0	147	0
N.S.	1	0.94	0.77	0.00	0.00	0.69	0.00	0.00	0.54	0.00
time (sec)	N/A	0.587	4.630	0.000	0.000	0.153	0.000	0.000	0.302	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	697	591	277	0	0	331	0	0	177	0
N.S.	1	0.85	0.40	0.00	0.00	0.47	0.00	0.00	0.25	0.00
time (sec)	N/A	1.680	6.193	0.000	0.000	0.143	0.000	0.000	0.299	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	244	219	242	0	0	2123	0	0	266	0
N.S.	1	0.90	0.99	0.00	0.00	8.70	0.00	0.00	1.09	0.00
time (sec)	N/A	1.396	1.247	0.000	0.000	0.374	0.000	0.000	0.419	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	148	137	0	0	664	0	0	246	0
N.S.	1	0.91	0.84	0.00	0.00	4.07	0.00	0.00	1.51	0.00
time (sec)	N/A	0.448	0.446	0.000	0.000	0.235	0.000	0.000	0.411	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	126	132	0	0	571	0	0	222	0
N.S.	1	0.91	0.96	0.00	0.00	4.14	0.00	0.00	1.61	0.00
time (sec)	N/A	0.335	0.411	0.000	0.000	0.229	0.000	0.000	0.425	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	53	0	23	433	27
N.S.	1	1.00	1.09	0.91	0.00	2.30	0.00	1.00	18.83	1.17
time (sec)	N/A	0.310	13.223	0.309	0.000	0.117	0.000	0.208	0.326	1.265

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	55	0	23	477	27
N.S.	1	1.00	1.09	0.91	0.00	2.39	0.00	1.00	20.74	1.17
time (sec)	N/A	0.325	14.274	3.411	0.000	0.111	0.000	0.212	0.350	1.502

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	58	0	23	391	27
N.S.	1	1.00	1.09	0.91	0.00	2.52	0.00	1.00	17.00	1.17
time (sec)	N/A	0.297	13.749	0.311	0.000	0.119	0.000	0.232	0.304	1.470

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	58	0	23	336	27
N.S.	1	1.00	1.09	0.91	0.00	2.52	0.00	1.00	14.61	1.17
time (sec)	N/A	0.291	13.062	0.321	0.000	0.099	0.000	0.281	0.319	1.344

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	273	251	186	0	0	286	0	0	268	0
N.S.	1	0.92	0.68	0.00	0.00	1.05	0.00	0.00	0.98	0.00
time (sec)	N/A	0.548	0.286	0.000	0.000	0.167	0.000	0.000	0.301	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	296	271	248	0	0	348	0	0	276	0
N.S.	1	0.92	0.84	0.00	0.00	1.18	0.00	0.00	0.93	0.00
time (sec)	N/A	0.522	3.980	0.000	0.000	0.130	0.000	0.000	0.277	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	452	540	323	0	0	536	0	0	300	0
N.S.	1	1.19	0.71	0.00	0.00	1.19	0.00	0.00	0.66	0.00
time (sec)	N/A	1.649	7.418	0.000	0.000	0.173	0.000	0.000	0.293	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	589	542	400	0	0	0	0	0	656	0
N.S.	1	0.92	0.68	0.00	0.00	0.00	0.00	0.00	1.11	0.00
time (sec)	N/A	2.148	2.852	0.000	0.000	0.000	0.000	0.000	0.279	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	374	345	291	0	0	0	0	0	360	0
N.S.	1	0.92	0.78	0.00	0.00	0.00	0.00	0.00	0.96	0.00
time (sec)	N/A	0.698	0.429	0.000	0.000	0.000	0.000	0.000	0.256	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	178	203	169	0	0	0	0	0	149	0
N.S.	1	1.14	0.95	0.00	0.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	0.429	0.303	0.000	0.000	0.000	0.000	0.000	0.259	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	20	25	43	29
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.87	1.09	1.87	1.26
time (sec)	N/A	0.261	2.121	2.122	0.461	0.125	44.952	0.168	0.261	0.893

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	36	0	25	65	29
N.S.	1	1.00	1.09	1.00	1.09	1.57	0.00	1.09	2.83	1.26
time (sec)	N/A	0.260	3.738	1.698	0.451	0.111	0.000	0.171	0.293	0.914

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	42	0	25	87	29
N.S.	1	1.00	1.08	0.92	1.00	1.68	0.00	1.00	3.48	1.16
time (sec)	N/A	0.290	0.862	0.986	0.409	0.127	0.000	0.209	0.490	0.979

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	41	29
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.64	1.16
time (sec)	N/A	0.276	0.124	0.961	0.304	0.113	57.846	0.189	0.341	0.917

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	45	29
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.80	1.16
time (sec)	N/A	0.292	0.839	0.967	0.280	0.163	25.557	0.193	0.281	0.980

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	45	0	25	77	29
N.S.	1	1.00	1.08	0.92	1.00	1.80	0.00	1.00	3.08	1.16
time (sec)	N/A	0.290	1.059	0.989	0.298	0.136	0.000	0.189	0.347	1.058

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	401	227	194	0	0	249	0	0	98	0
N.S.	1	0.57	0.48	0.00	0.00	0.62	0.00	0.00	0.24	0.00
time (sec)	N/A	1.577	0.229	0.000	0.000	0.165	0.000	0.000	0.444	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	156	159	0	0	192	0	0	78	0
N.S.	1	0.58	0.59	0.00	0.00	0.72	0.00	0.00	0.29	0.00
time (sec)	N/A	1.201	0.266	0.000	0.000	0.119	0.000	0.000	0.401	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	101	118	0	0	136	0	0	58	0
N.S.	1	0.80	0.94	0.00	0.00	1.08	0.00	0.00	0.46	0.00
time (sec)	N/A	0.438	0.213	0.000	0.000	0.110	0.000	0.000	0.309	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	87	37	34	26	50	30
N.S.	1	1.00	1.08	0.92	3.35	1.42	1.31	1.00	1.92	1.15
time (sec)	N/A	0.274	0.585	0.208	0.393	0.106	12.186	0.203	0.240	1.799

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	111	39	36	26	79	30
N.S.	1	1.00	1.08	0.92	4.27	1.50	1.38	1.00	3.04	1.15
time (sec)	N/A	0.288	9.344	4.072	0.392	0.126	99.998	0.197	0.269	1.483

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [68] had the largest ratio of [1.05556000000000005]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	7	1.07	12	0.583
2	A	4	4	1.09	12	0.333
3	A	7	6	1.04	12	0.500
4	A	3	3	1.06	12	0.250
5	A	6	5	1.00	12	0.417
6	A	2	2	1.00	10	0.200
7	A	1	1	1.00	8	0.125
8	A	8	7	1.36	12	0.583
9	A	2	2	1.00	12	0.167
10	A	5	4	1.25	12	0.333
11	A	5	4	1.08	12	0.333
12	A	6	5	1.26	12	0.417
13	A	5	4	1.05	12	0.333
14	A	7	6	1.27	12	0.500
15	A	10	9	1.07	14	0.643
16	A	9	8	0.95	14	0.571
17	A	8	7	1.14	12	0.583
18	A	7	6	0.95	10	0.600
19	A	8	7	1.15	14	0.500
20	A	9	8	1.12	14	0.571
21	A	6	5	1.18	14	0.357

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	10	9	1.08	14	0.643
23	A	8	7	1.14	14	0.500
24	A	15	14	1.03	14	1.000
25	A	11	10	0.96	14	0.714
26	A	11	10	1.05	12	0.833
27	A	8	7	0.94	10	0.700
28	A	9	8	1.13	14	0.571
29	A	11	10	1.08	14	0.714
30	A	9	8	1.09	14	0.571
31	A	14	13	1.11	14	0.929
32	A	15	14	1.29	14	1.000
33	N/A	1	0	1.00	12	0.000
34	N/A	1	0	1.00	10	0.000
35	N/A	1	0	1.00	14	0.000
36	A	7	6	1.00	14	0.429
37	A	9	8	0.94	14	0.571
38	A	4	3	0.93	14	0.214
39	N/A	1	0	1.00	12	0.000
40	N/A	1	0	1.00	10	0.000
41	N/A	1	0	1.00	14	0.000
42	A	9	8	1.05	14	0.571
43	A	11	10	1.02	14	0.714
44	A	4	3	0.92	14	0.214
45	N/A	1	0	1.00	12	0.000
46	N/A	1	0	1.00	10	0.000
47	N/A	1	0	1.00	14	0.000
48	A	12	11	1.09	14	0.786
49	A	14	13	1.04	14	0.929
50	A	4	3	0.93	14	0.214
51	N/A	1	0	1.00	16	0.000
52	N/A	1	0	1.00	16	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
53	A	4	3	1.00	14	0.214
54	N/A	1	0	1.00	16	0.000
55	N/A	1	0	1.00	16	0.000
56	A	15	14	1.01	16	0.875
57	A	13	12	1.05	16	0.750
58	A	12	11	1.04	14	0.786
59	A	1	1	1.00	8	0.125
60	A	2	2	1.00	16	0.125
61	A	8	7	1.07	16	0.438
62	A	9	8	1.16	16	0.500
63	A	18	17	1.06	18	0.944
64	A	16	15	1.11	18	0.833
65	A	4	4	1.12	18	0.222
66	A	8	7	1.15	18	0.389
67	A	16	15	1.07	18	0.833
68	A	20	19	1.04	18	1.056
69	A	8	7	0.81	19	0.368
70	A	7	6	0.84	19	0.316
71	A	6	5	0.93	16	0.312
72	A	6	5	0.95	19	0.263
73	A	4	4	0.96	19	0.211
74	A	5	5	0.86	19	0.263
75	A	6	6	0.81	19	0.316
76	A	6	5	0.82	19	0.263
77	A	6	5	0.86	19	0.263
78	A	5	4	0.84	17	0.235
79	A	6	5	1.23	19	0.263
80	A	6	5	1.28	19	0.263
81	A	9	8	0.85	21	0.381
82	A	7	6	0.94	18	0.333
83	A	9	8	0.90	21	0.381
84	A	7	6	0.93	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
85	A	5	5	0.93	21	0.238
86	A	6	6	0.83	21	0.286
87	A	6	5	0.86	21	0.238
88	A	5	4	0.82	19	0.211
89	A	6	5	1.17	21	0.238
90	A	6	5	1.21	21	0.238
91	A	4	3	1.10	21	0.143
92	A	4	3	1.10	19	0.158
93	A	4	3	1.09	18	0.167
94	A	4	3	1.11	21	0.143
95	A	4	3	1.09	21	0.143
96	A	4	3	1.10	21	0.143
97	A	4	3	1.09	21	0.143
98	A	6	5	0.90	19	0.263
99	A	4	3	1.10	21	0.143
100	A	4	3	1.08	21	0.143
101	A	4	3	1.08	21	0.143
102	A	4	3	1.08	18	0.167
103	A	4	3	1.08	21	0.143
104	A	4	3	1.08	21	0.143
105	A	7	6	0.92	21	0.286
106	A	8	7	0.98	19	0.368
107	A	4	3	1.09	21	0.143
108	A	4	3	1.06	21	0.143
109	A	4	3	1.06	21	0.143
110	A	4	3	1.06	18	0.167
111	A	15	14	0.92	23	0.609
112	A	13	12	0.91	23	0.522
113	A	10	9	0.92	21	0.429
114	N/A	1	0	1.00	23	0.000
115	N/A	1	0	1.00	23	0.000
116	N/A	1	0	1.00	23	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
117	N/A	1	0	1.00	20	0.000
118	N/A	1	0	1.00	23	0.000
119	A	13	13	0.92	23	0.565
120	A	15	15	0.91	23	0.652
121	A	15	14	0.91	23	0.609
122	A	12	11	0.92	21	0.524
123	N/A	1	0	1.00	23	0.000
124	N/A	1	0	1.00	23	0.000
125	N/A	1	0	1.00	23	0.000
126	N/A	1	0	1.00	20	0.000
127	N/A	1	0	1.00	23	0.000
128	N/A	1	0	1.00	23	0.000
129	A	15	15	0.92	23	0.652
130	A	17	17	0.91	23	0.739
131	A	13	12	0.92	23	0.522
132	A	11	10	0.92	23	0.435
133	A	9	8	0.97	21	0.381
134	N/A	1	0	1.00	23	0.000
135	N/A	1	0	1.00	23	0.000
136	N/A	1	0	1.00	23	0.000
137	N/A	1	0	1.00	20	0.000
138	A	12	12	0.95	23	0.522
139	A	13	13	0.92	23	0.565
140	A	4	4	0.83	23	0.174
141	A	11	10	0.93	23	0.435
142	A	9	8	0.97	23	0.348
143	A	5	4	1.00	21	0.190
144	N/A	1	0	1.00	23	0.000
145	N/A	1	0	1.00	23	0.000
146	N/A	1	0	1.00	23	0.000

Continued on next page

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
147	N/A	1	0	1.00	23	0.000
148	A	5	5	1.00	20	0.250
149	A	12	12	0.94	23	0.522
150	A	4	4	0.85	23	0.174
151	A	11	10	0.90	23	0.435
152	A	9	8	0.91	23	0.348
153	A	6	5	0.91	21	0.238
154	N/A	1	0	1.00	23	0.000
155	N/A	1	0	1.00	23	0.000
156	N/A	1	0	1.00	23	0.000
157	N/A	1	0	1.00	23	0.000
158	A	10	10	0.92	23	0.435
159	A	11	11	0.92	20	0.550
160	A	4	4	1.19	23	0.174
161	A	6	6	0.92	23	0.261
162	A	6	6	0.92	23	0.261
163	A	5	5	1.14	21	0.238
164	N/A	1	0	1.00	23	0.000
165	N/A	1	0	1.00	23	0.000
166	N/A	1	0	1.00	25	0.000
167	N/A	1	0	1.00	25	0.000
168	N/A	1	0	1.00	25	0.000
169	N/A	1	0	1.00	25	0.000
170	A	8	7	0.57	26	0.269
171	A	10	9	0.58	26	0.346
172	A	9	8	0.80	26	0.308
173	N/A	1	0	1.00	26	0.000
174	N/A	1	0	1.00	26	0.000

CHAPTER 3

LISTING OF INTEGRALS

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3.1 $\int x^6(a+b\sec^{-1}(cx))\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 114

$$\int x^6\left(a+b\sec^{-1}(cx)\right)\,dx=-\frac{5b\sqrt{1-\frac{1}{c^2x^2}}x^2}{112c^5}-\frac{5b\sqrt{1-\frac{1}{c^2x^2}}x^4}{168c^3}-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^6}{42c}+\frac{1}{7}x^7\left(a+b\sec^{-1}(cx)\right)-\frac{5b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{112c^7}$$

output
```
-5/112*b*(1-1/c^2/x^2)^(1/2)*x^2/c^5-5/168*b*(1-1/c^2/x^2)^(1/2)*x^4/c^3-1
/42*b*(1-1/c^2/x^2)^(1/2)*x^6/c+1/7*x^7*(a+b*arcsec(c*x))-5/112*b*arctanh(
(1-1/c^2/x^2)^(1/2))/c^7
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.94

$$\int x^6\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{ax^7}{7}+b\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\left(-\frac{5x^2}{112c^5}-\frac{5x^4}{168c^3}-\frac{x^6}{42c}\right)+\frac{1}{7}bx^7\sec^{-1}(cx)-\frac{5b\log\left(x\left(1+\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\right)\right)}{112c^7}$$

input
```
Integrate[x^6*(a + b*ArcSec[c*x]),x]
```

output
```
(a*x^7)/7 + b*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]*((-5*x^2)/(112*c^5) - (5*x^4)
/(168*c^3) - x^6/(42*c)) + (b*x^7*ArcSec[c*x])/7 - (5*b*Log[x*(1 + Sqrt[(-
1 + c^2*x^2)/(c^2*x^2)])])/(112*c^7)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5743, 798, 52, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \left(a+b\sec^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{5743}$$

$$\frac{1}{7}x^7\left(a+b\sec^{-1}(cx)\right)-\frac{b\int \frac{x^5}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{7c}$$

$$\Big\downarrow \text{798}$$

$$\frac{b\int \frac{x^8}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{14c}+\frac{1}{7}x^7\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{b\left(\frac{5\int \frac{x^6}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{6c^2}-\frac{1}{3}x^6\sqrt{1-\frac{1}{c^2x^2}}\right)}{14c}+\frac{1}{7}x^7\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{b\left(\frac{5\left(\frac{3\int \frac{x^4}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{4c^2}-\frac{1}{2}x^4\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^2}-\frac{1}{3}x^6\sqrt{1-\frac{1}{c^2x^2}}\right)}{14c}+\frac{1}{7}x^7\left(a+b\sec^{-1}(cx)\right)$$

$\downarrow$ 52

$$\frac{b\left(\frac{5\left(\frac{3\left(\frac{\int \frac{x^2}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{2c^2}-x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{4c^2}-\frac{1}{2}x^4\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^2}-\frac{1}{3}x^6\sqrt{1-\frac{1}{c^2x^2}}\right)}{14c}+\frac{1}{7}x^7\left(a+b\sec^{-1}(cx)\right)$$

$\downarrow$ 73

$$\frac{b\left(\frac{5\left(\frac{3\left(x^2\left(-\sqrt{1-\frac{1}{c^2x^2}}\right)-\int \frac{1}{c^2-\frac{c^2}{x^4}}d\sqrt{1-\frac{1}{c^2x^2}}\right)}{4c^2}-\frac{1}{2}x^4\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^2}-\frac{1}{3}x^6\sqrt{1-\frac{1}{c^2x^2}}\right)}{14c}+\frac{1}{7}x^7\left(a+b\sec^{-1}(cx)\right)$$

$\downarrow$ 221

$$\frac{1}{7}x^7\left(a+b\sec^{-1}(cx)\right)+\frac{b\left(\frac{5\left(\frac{3\left(x^2\left(-\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^2}\right)}{4c^2}-\frac{1}{2}x^4\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^2}-\frac{1}{3}x^6\sqrt{1-\frac{1}{c^2x^2}}\right)}{14c}$$

input
```
Int[x^6*(a + b*ArcSec[c*x]),x]
```

output
```
(x^7*(a + b*ArcSec[c*x]))/7 + (b*(-1/3*(Sqrt[1 - 1/(c^2*x^2)]*x^6) + (5*(-
1/2*(Sqrt[1 - 1/(c^2*x^2)]*x^4) + (3*(-(Sqrt[1 - 1/(c^2*x^2)]*x^2) - ArcTa
nh[Sqrt[1 - 1/(c^2*x^2)]]/c^2))/(4*c^2)))/(6*c^2)))/(14*c)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.55

method	result
parts	$\frac{a\,x^{7}}{7}+\frac{b\,x^{7}\operatorname{arcsec}(cx)}{7}-\frac{b(c^{2}x^{2}-1)x^{4}}{42c^{3}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}-\frac{5b(c^{2}x^{2}-1)x^{2}}{168c^{5}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}-\frac{5b(c^{2}x^{2}-1)}{112c^{7}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}-\frac{5b\sqrt{c^{2}x^{2}-1}\,\ln\left(cx+\sqrt{c^{2}x^{2}-\right.}}{112c^{8}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,x}$
derivativedivides	$\frac{\frac{a\,c^{7}x^{7}}{7}+\frac{b\,c^{7}x^{7}\operatorname{arcsec}(cx)}{7}-\frac{b\left(c^{2}x^{2}-1\right)c^{4}x^{4}}{42\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}-\frac{5b\left(c^{2}x^{2}-1\right)c^{2}x^{2}}{168\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}-\frac{5b\left(c^{2}x^{2}-1\right)}{112\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}-\frac{5b\sqrt{c^{2}x^{2}-1}\,\ln\left(cx+\sqrt{c^{2}x^{2}-1}\right)}{112\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}}{c^{7}}$
default	$\frac{\frac{a\,c^{7}x^{7}}{7}+\frac{b\,c^{7}x^{7}\operatorname{arcsec}(cx)}{7}-\frac{b\left(c^{2}x^{2}-1\right)c^{4}x^{4}}{42\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}-\frac{5b\left(c^{2}x^{2}-1\right)c^{2}x^{2}}{168\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}-\frac{5b\left(c^{2}x^{2}-1\right)}{112\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}-\frac{5b\sqrt{c^{2}x^{2}-1}\,\ln\left(cx+\sqrt{c^{2}x^{2}-1}\right)}{112\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}}{c^{7}}$

input
```
int(x^6*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/7*a*x^7+1/7*b*x^7*arcsec(c*x)-1/42*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^
2)^(1/2)*x^4-5/168*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2-5/112
*b/c^7*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)-5/112*b/c^8*(c^2*x^2-1)^(1/
2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*ln(c*x+(c^2*x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02

$$\int x^6 \left(a+b\sec^{-1}(cx)\right)\,dx = \frac{48\,ac^7x^7+96\,bc^7\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+48\left(bc^7x^7-bc^7\right)\operatorname{arcsec}\left(cx\right)+15\,b\log\left(-cx+\sqrt{c^2x^2-1}\right)}{336\,c^7}$$

input
```
integrate(x^6*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/336*(48*a*c^7*x^7 + 96*b*c^7*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 48*(b*c^
7*x^7 - b*c^7)*arcsec(c*x) + 15*b*log(-c*x + sqrt(c^2*x^2 - 1)) - (8*b*c^5
*x^5 + 10*b*c^3*x^3 + 15*b*c*x)*sqrt(c^2*x^2 - 1))/c^7
```

Sympy [A] (verification not implemented)

Time = 10.60 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.94

$$\int x^6 \left(a+b\sec^{-1}(cx)\right)\,dx = \frac{ax^7}{7}+\frac{bx^7\operatorname{asec}\left(cx\right)}{7} - \frac{b\left(\begin{cases}\frac{cx^7}{6\sqrt{c^2x^2-1}}+\frac{x^5}{24c\sqrt{c^2x^2-1}}+\frac{5x^3}{48c^3\sqrt{c^2x^2-1}}-\frac{5x}{16c^5\sqrt{c^2x^2-1}}+\frac{5\operatorname{acosh}\left(cx\right)}{16c^6} & \text{for}\ |c^2x^2|>1\\ -\frac{icx^7}{6\sqrt{-c^2x^2+1}}-\frac{ix^5}{24c\sqrt{-c^2x^2+1}}-\frac{5ix^3}{48c^3\sqrt{-c^2x^2+1}}+\frac{5ix}{16c^5\sqrt{-c^2x^2+1}}-\frac{5i\operatorname{asin}\left(cx\right)}{16c^6} & \text{otherwise}\end{cases}\right)}{7c}$$

input
```
integrate(x**6*(a+b*asec(c*x)),x)
```

output
```
a*x**7/7 + b*x**7*asec(c*x)/7 - b*Piecewise((c*x**7/(6*sqrt(c**2*x**2 - 1)
) + x**5/(24*c*sqrt(c**2*x**2 - 1)) + 5*x**3/(48*c**3*sqrt(c**2*x**2 - 1))
 - 5*x/(16*c**5*sqrt(c**2*x**2 - 1)) + 5*acosh(c*x)/(16*c**6), Abs(c**2*x*
*2) > 1), (-I*c*x**7/(6*sqrt(-c**2*x**2 + 1)) - I*x**5/(24*c*sqrt(-c**2*x*
*2 + 1)) - 5*I*x**3/(48*c**3*sqrt(-c**2*x**2 + 1)) + 5*I*x/(16*c**5*sqrt(-
c**2*x**2 + 1)) - 5*I*asin(c*x)/(16*c**6), True))/(7*c)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.42

$$\int x^6\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{1}{7}ax^7+\frac{1}{672}\left(96\,x^7\,\mathrm{arcsec}\,(cx)-\frac{\frac{2\left(15\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-40\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^6\left(\frac{1}{c^2x^2}-1\right)^3+3\,c^6\left(\frac{1}{c^2x^2}-1\right)^2+3\,c^6\left(\frac{1}{c^2x^2}-1\right)+c^6}+\frac{15\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^6}-\frac{15\,\log\left(\sqrt{-\cdots}\right.}{c\cdots}}{c}\right.$$

input
```
integrate(x^6*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*x^7 + 1/672*(96*x^7*arcsec(c*x) - (2*(15*(-1/(c^2*x^2) + 1)^(5/2) -
40*(-1/(c^2*x^2) + 1)^(3/2) + 33*sqrt(-1/(c^2*x^2) + 1))/(c^6*(1/(c^2*x^2)
 - 1)^3 + 3*c^6*(1/(c^2*x^2) - 1)^2 + 3*c^6*(1/(c^2*x^2) - 1) + c^6) + 15*
log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 - 15*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c
^6)/c)*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8644 vs. 2(96) = 192.

Time = 0.98 (sec) , antiderivative size = 8644, normalized size of antiderivative = 75.82

$$\int x^6\left(a+b\sec^{-1}(cx)\right)\,dx=\text{Too large to display}$$

input
```
integrate(x^6*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/336*c*(48*b*arccos(1/(c*x))/(c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)
^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)
^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(
1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) +
 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14) - 15*b*log(abs(sqrt(-1/
(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1
)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1
)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*
(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x)
+ 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14) + 15*b*log(abs(sqrt(-1
/(c^2*x^2) + 1) - 1/(c*x) - 1))/(c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) +
1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) -
1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8
*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x)
 + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14) + 48*a/(c^8 + 7*c^8*(
1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1
)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1
)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*
(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) +
1)^14) - 336*b*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^8 + 7*c^8*(1/(c^2*...
```

Mupad [F(-1)]

Timed out.

$$\int x^6 \left(a + b\sec^{-1}(cx)\right) \, dx = \int x^6 \left(a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right) \, dx$$

input
```
int(x^6*(a + b*acos(1/(c*x))),x)
```

output
```
int(x^6*(a + b*acos(1/(c*x))), x)
```

Reduce [F]

$$\int x^6\left(a+b\sec^{-1}(cx)\right)\,dx=\left(\int asec(cx)\,x^6dx\right)b+\frac{a\,x^7}{7}$$

input
```
int(x^6*(a+b*asec(c*x)),x)
```

output
```
(7*int(asec(c*x)*x**6,x)*b + a*x**7)/7
```

3.2 $\int x^5(a+b\sec^{-1}(cx))\,dx$

Optimal result . . . 99
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Optimal result

Integrand size = 12, antiderivative size = 89

$$\int x^5\left(a+b\sec^{-1}(cx)\right)\,dx=-\frac{4b\sqrt{1-\frac{1}{c^2x^2}}\,x}{45c^5}-\frac{2b\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{45c^3}-\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^5}{30c}+\frac{1}{6}x^6\left(a+b\sec^{-1}(cx)\right)$$

output
```
-4/45*b*(1-1/c^2/x^2)^(1/2)*x/c^5-2/45*b*(1-1/c^2/x^2)^(1/2)*x^3/c^3-1/30*
b*(1-1/c^2/x^2)^(1/2)*x^5/c+1/6*x^6*(a+b*arcsec(c*x))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.81

$$\int x^5\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{ax^6}{6}+b\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\left(-\frac{4x}{45c^5}-\frac{2x^3}{45c^3}-\frac{x^5}{30c}\right)+\frac{1}{6}bx^6\sec^{-1}(cx)$$

input
```
Integrate[x^5*(a + b*ArcSec[c*x]),x]
```

output
```
(a*x^6)/6 + b*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]*((-4*x)/(45*c^5) - (2*x^3)/(4
5*c^3) - x^5/(30*c)) + (b*x^6*ArcSec[c*x])/6
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5743, 803, 803, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+b \sec^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{ 5743}$$

$$\frac{1}{6} x^6 \left(a+b \sec^{-1}(cx)\right)-\frac{b \int \frac{x^4}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{6c}$$

$$\Big\downarrow \text{ 803}$$

$$\frac{1}{6} x^6 \left(a+b \sec^{-1}(cx)\right)-\frac{b \left(\frac{4 \int \frac{x^2}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{5c^2}+\frac{1}{5} x^5 \sqrt{1-\frac{1}{c^2x^2}}\right)}{6c}$$

$$\Big\downarrow \text{ 803}$$

$$\frac{1}{6} x^6 \left(a+b \sec^{-1}(cx)\right)-\frac{b \left(\frac{4 \left(\frac{2 \int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{3c^2}+\frac{1}{3} x^3 \sqrt{1-\frac{1}{c^2x^2}}\right)}{5c^2}+\frac{1}{5} x^5 \sqrt{1-\frac{1}{c^2x^2}}\right)}{6c}$$

$$\Big\downarrow \text{ 746}$$

$$\frac{1}{6} x^6 \left(a+b \sec^{-1}(cx)\right)-\frac{b \left(\frac{1}{5} x^5 \sqrt{1-\frac{1}{c^2x^2}}+\frac{4 \left(\frac{2x \sqrt{1-\frac{1}{c^2x^2}}}{3c^2}+\frac{1}{3} x^3 \sqrt{1-\frac{1}{c^2x^2}}\right)}{5c^2}\right)}{6c}$$

input
```
Int[x^5*(a + b*ArcSec[c*x]),x]
```

output
```
-1/6*(b*((Sqrt[1 - 1/(c^2*x^2)]*x^5)/5 + (4*((2*Sqrt[1 - 1/(c^2*x^2)]*x)/(
3*c^2) + (Sqrt[1 - 1/(c^2*x^2)]*x^3)/3))/(5*c^2)))/c + (x^6*(a + b*ArcSec[
c*x]))/6
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

method	result	size
parts	$\frac{x^{6} a}{6}+\frac{b \left(\frac{c^{6} x^{6} \operatorname{arcsec}\left(c x \right)}{6}-\frac{\left(c^{2} x^{2}-1\right) \left(3 c^{4} x^{4}+4 c^{2} x^{2}+8\right)}{90 \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}\, c x}\right)}{c^{6}}$	79
derivativedivides	$\frac{\frac{a \,c^{6} x^{6}}{6}+b \left(\frac{c^{6} x^{6} \operatorname{arcsec}\left(c x \right)}{6}-\frac{\left(c^{2} x^{2}-1\right) \left(3 c^{4} x^{4}+4 c^{2} x^{2}+8\right)}{90 \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}\, c x}\right)}{c^{6}}$	83
default	$\frac{\frac{a \,c^{6} x^{6}}{6}+b \left(\frac{c^{6} x^{6} \operatorname{arcsec}\left(c x \right)}{6}-\frac{\left(c^{2} x^{2}-1\right) \left(3 c^{4} x^{4}+4 c^{2} x^{2}+8\right)}{90 \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}\, c x}\right)}{c^{6}}$	83

input
```
int(x^5*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^6*a+b/c^6*(1/6*c^6*x^6*arcsec(c*x)-1/90*(c^2*x^2-1)*(3*c^4*x^4+4*c^2
*x^2+8)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.71

$$\int x^5\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{15\,bc^6x^6\,\mathrm{arcsec}\,(cx)+15\,ac^6x^6-(3\,bc^4x^4+4\,bc^2x^2+8\,b)\sqrt{c^2x^2-1}}{90\,c^6}$$

input
```
integrate(x^5*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/90*(15*b*c^6*x^6*arcsec(c*x) + 15*a*c^6*x^6 - (3*b*c^4*x^4 + 4*b*c^2*x^2
 + 8*b)*sqrt(c^2*x^2 - 1))/c^6
```

Sympy [A] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.72

$$\int x^5\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{ax^6}{6}+\frac{bx^6\,\mathrm{asec}\,(cx)}{6}-\frac{b\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c}+\frac{4x^2\sqrt{c^2x^2-1}}{15c^3}+\frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for } |c^2x^2|>1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c}+\frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3}+\frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{6c}$$

input
```
integrate(x**5*(a+b*asec(c*x)),x)
```

output
```
a*x**6/6 + b*x**6*asec(c*x)/6 - b*Piecewise((x**4*sqrt(c**2*x**2 - 1)/(5*c
) + 4*x**2*sqrt(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**2*x**2 - 1)/(15*c**5)
, Abs(c**2*x**2) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(5*c) + 4*I*x**2*sqrt(
-c**2*x**2 + 1)/(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/(15*c**5), True))/(6*
c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int x^5\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{1}{6}\,ax^6+\frac{1}{90}\left(15\,x^6\,\mathrm{arcsec}\,(cx)-\frac{3\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+10\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^5}\right)b$$

input
```
integrate(x^5*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*x^6 + 1/90*(15*x^6*arcsec(c*x) - (3*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2)
 + 10*c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(-1/(c^2*x^2) + 1))/c^5)
*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3862 vs. 2(75) = 150.

Time = 0.17 (sec) , antiderivative size = 3862, normalized size of antiderivative = 43.39

$$\int x^5\left(a+b\sec^{-1}(cx)\right)\,dx=\text{Too large to display}$$

input
```
integrate(x^5*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/90*c*(15*b*arccos(1/(c*x))/(c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^
2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^
3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/
(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^
12) + 15*a/(c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2
*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6
+ 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(
1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12) - 90*b*(1/(c^2
*x^2) - 1)*arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3
/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(
c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^1
2)*(1/(c*x) + 1)^2) - 30*b*sqrt(-1/(c^2*x^2) + 1)/((c^7 + 6*c^7*(1/(c^2*x^
2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*
c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c
*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2)
 - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)) - 90*a*(1/(c^2*x^2) - 1)/((c^7 +
6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c
*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x
^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10...
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \left(a + b\sec^{-1}(cx)\right) dx = \int x^5 \left(a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right) dx$$

input
```
int(x^5*(a + b*acos(1/(c*x))),x)
```

output
```
int(x^5*(a + b*acos(1/(c*x))), x)
```

Reduce [F]

$$\int x^5\left(a+b\sec^{-1}(cx)\right)\,dx=\left(\int asec(cx)\,x^5dx\right)b+\frac{a\,x^6}{6}$$

input
```
int(x^5*(a+b*asec(c*x)),x)
```

output
```
(6*int(asec(c*x)*x**5,x)*b + a*x**6)/6
```

3.3 $\int x^4(a + b\sec^{-1}(cx))\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 89

$$\int x^4\left(a+b\sec^{-1}(cx)\right)\,dx=-\frac{3b\sqrt{1-\frac{1}{c^2x^2}}x^2}{40c^3}-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^4}{20c}+\frac{1}{5}x^5\left(a+b\sec^{-1}(cx)\right)-\frac{3b\,\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{40c^5}$$

output
```
-3/40*b*(1-1/c^2/x^2)^(1/2)*x^2/c^3-1/20*b*(1-1/c^2/x^2)^(1/2)*x^4/c+1/5*x
^5*(a+b*arcsec(c*x))-3/40*b*arctanh((1-1/c^2/x^2)^(1/2))/c^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09

$$\int x^4\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{ax^5}{5}+b\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\left(-\frac{3x^2}{40c^3}-\frac{x^4}{20c}\right)+\frac{1}{5}bx^5\sec^{-1}(cx)-\frac{3b\log\left(x\left(1+\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\right)\right)}{40c^5}$$

input
```
Integrate[x^4*(a + b*ArcSec[c*x]),x]
```

output
```
(a*x^5)/5 + b*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]*((-3*x^2)/(40*c^3) - x^4/(20*
c)) + (b*x^5*ArcSec[c*x])/5 - (3*b*Log[x*(1 + Sqrt[(-1 + c^2*x^2)/(c^2*x^2
)])])/(40*c^5)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5743, 798, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+b \sec^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{5743}$$

$$\frac{1}{5} x^5 \left(a+b \sec^{-1}(cx)\right)-\frac{b \int \frac{x^3}{\sqrt{1-\frac{1}{c^2 x^2}}}dx}{5 c}$$

$$\Big\downarrow \text{798}$$

$$\frac{b \int \frac{x^6}{\sqrt{1-\frac{1}{c^2 x^2}}}d\frac{1}{x^2}}{10 c}+\frac{1}{5} x^5 \left(a+b \sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{b \left(\frac{3 \int \frac{x^4}{\sqrt{1-\frac{1}{c^2 x^2}}}d\frac{1}{x^2}}{4 c^2}-\frac{1}{2} x^4 \sqrt{1-\frac{1}{c^2 x^2}}\right)}{10 c}+\frac{1}{5} x^5 \left(a+b \sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{b \left(\frac{3 \left(\frac{\int \frac{x^2}{\sqrt{1-\frac{1}{c^2 x^2}}}d\frac{1}{x^2}}{2 c^2}-x^2 \sqrt{1-\frac{1}{c^2 x^2}}\right)}{4 c^2}-\frac{1}{2} x^4 \sqrt{1-\frac{1}{c^2 x^2}}\right)}{10 c}+\frac{1}{5} x^5 \left(a+b \sec^{-1}(cx)\right)$$

$$\Big\downarrow \; 73$$

$$\frac{b\left(\frac{3\left(x^2\left(-\sqrt{1-\frac{1}{c^2x^2}}\right)-\int \frac{1}{c^2-\frac{c^2}{x^4}}d\sqrt{1-\frac{1}{c^2x^2}}\right)}{4c^2}-\frac{1}{2}x^4\sqrt{1-\frac{1}{c^2x^2}}\right)}{10c}+\frac{1}{5}x^5\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{5}x^5\left(a+b\sec^{-1}(cx)\right)+\frac{b\left(\frac{3\left(x^2\left(-\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^2}\right)}{4c^2}-\frac{1}{2}x^4\sqrt{1-\frac{1}{c^2x^2}}\right)}{10c}$$

input
```
Int[x^4*(a + b*ArcSec[c*x]),x]
```

output
```
(x^5*(a + b*ArcSec[c*x]))/5 + (b*(-1/2*(Sqrt[1 - 1/(c^2*x^2)]*x^4) + (3*(-
(Sqrt[1 - 1/(c^2*x^2)]*x^2) - ArcTanh[Sqrt[1 - 1/(c^2*x^2)]]/c^2))/(4*c^2)
))/(10*c)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.58

method	result	size
parts	$\frac{a\,x^5}{5}+\frac{\operatorname{arcsec}(cx)b\,x^5}{5}-\frac{b(c^2x^2-1)x^2}{20c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{3b(c^2x^2-1)}{40c^5\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{3b\sqrt{c^2x^2-1}\,\ln\left(cx+\sqrt{c^2x^2-1}\right)}{40c^6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}$	141
derivativedivides	$\frac{\frac{a\,c^5x^5}{5}+\frac{b\,c^5x^5\operatorname{arcsec}(cx)}{5}-\frac{b\left(c^2x^2-1\right)c^2x^2}{20\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{3b\left(c^2x^2-1\right)}{40\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{3b\sqrt{c^2x^2-1}\,\ln\left(cx+\sqrt{c^2x^2-1}\right)}{40\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c^5}$	148
default	$\frac{\frac{a\,c^5x^5}{5}+\frac{b\,c^5x^5\operatorname{arcsec}(cx)}{5}-\frac{b\left(c^2x^2-1\right)c^2x^2}{20\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{3b\left(c^2x^2-1\right)}{40\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{3b\sqrt{c^2x^2-1}\,\ln\left(cx+\sqrt{c^2x^2-1}\right)}{40\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c^5}$	148

input
```
int(x^4*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/5*a*x^5+1/5*arcsec(c*x)*b*x^5-1/20*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^
2)^(1/2)*x^2-3/40*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)-3/40*b/c^6
*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*ln(c*x+(c^2*x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.20

$$\int x^4 \left(a+b\sec^{-1}(cx)\right) dx = \frac{8\,ac^5x^5+16\,bc^5\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+8\left(bc^5x^5-bc^5\right)\operatorname{arcsec}(cx)+3\,b\log\left(-cx+\sqrt{c^2x^2-1}\right)-}{40\,c^5}$$

input
```
integrate(x^4*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/40*(8*a*c^5*x^5 + 16*b*c^5*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 8*(b*c^5*x
^5 - b*c^5)*arcsec(c*x) + 3*b*log(-c*x + sqrt(c^2*x^2 - 1)) - (2*b*c^3*x^3
 + 3*b*c*x)*sqrt(c^2*x^2 - 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 3.78 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.97

$$\int x^4 \left(a+b\sec^{-1}(cx)\right) dx = \frac{ax^5}{5}+\frac{bx^5\operatorname{asec}(cx)}{5} - \frac{b\left(\begin{cases}\frac{cx^5}{4\sqrt{c^2x^2-1}}+\frac{x^3}{8c\sqrt{c^2x^2-1}}-\frac{3x}{8c^3\sqrt{c^2x^2-1}}+\frac{3\operatorname{acosh}(cx)}{8c^4} & \text{for } |c^2x^2|>1\\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}}-\frac{ix^3}{8c\sqrt{-c^2x^2+1}}+\frac{3ix}{8c^3\sqrt{-c^2x^2+1}}-\frac{3i\operatorname{asin}(cx)}{8c^4} & \text{otherwise}\end{cases}\right)}{5c}$$

input
```
integrate(x**4*(a+b*asec(c*x)),x)
```

output
```
a*x**5/5 + b*x**5*asec(c*x)/5 - b*Piecewise((c*x**5/(4*sqrt(c**2*x**2 - 1)
) + x**3/(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*x**2 - 1)) + 3*
acosh(c*x)/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqrt(-c**2*x**2 +
1)) - I*x**3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**2 +
1)) - 3*I*asin(c*x)/(8*c**4), True))/(5*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.47

$$\int x^4 \left(a+b \sec^{-1}(cx)\right) \, dx = \frac{1}{5} a x^5 + \frac{1}{80} \left(16 x^5 \operatorname{arcsec}(cx) + \frac{\frac{2 \left(3 \left(-\frac{1}{c^2 x^2}+1\right)^{\frac{3}{2}} - 5 \sqrt{-\frac{1}{c^2 x^2}+1}\right)}{c^4 \left(\frac{1}{c^2 x^2}-1\right)^2 + 2 c^4 \left(\frac{1}{c^2 x^2}-1\right) + c^4} - \frac{3 \log\left(\sqrt{-\frac{1}{c^2 x^2}+1}+1\right)}{c^4} + \frac{3 \log\left(\sqrt{-\frac{1}{c^2 x^2}+1}-1\right)}{c^4}}{c}\right) b$$

input
```
integrate(x^4*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*x^5 + 1/80*(16*x^5*arcsec(c*x) + (2*(3*(-1/(c^2*x^2) + 1)^(3/2) - 5*
sqrt(-1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1)
 + c^4) - 3*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(-1/(c^2*x^2)
+ 1) - 1)/c^4)/c)*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4828 vs. 2(75) = 150.

Time = 0.58 (sec) , antiderivative size = 4828, normalized size of antiderivative = 54.25

$$\int x^4 \left(a+b \sec^{-1}(cx)\right) \, dx = \text{Too large to display}$$

input
```
integrate(x^4*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/40*c*(8*b*arccos(1/(c*x))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3
/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2
*x^2) - 1)^5/(1/(c*x) + 1)^10) - 3*b*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c
*x) + 1))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*
x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 +
 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c
*x) + 1)^10) + 3*b*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(c^6 + 5
*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*
x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2
) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10) + 8*a
/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)
^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1
/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^
10) - 40*b*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) -
1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(
1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) +
1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^2) - 15*b*(
1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*
c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(...
```

Mupad [F(-1)]

Timed out.

$$\int x^4\left(a+b\sec^{-1}(cx)\right)\,dx=\int x^4\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^4*(a + b*acos(1/(c*x))),x)
```

output
```
int(x^4*(a + b*acos(1/(c*x))), x)
```

Reduce [F]

$$\int x^4\left(a+b\sec^{-1}(cx)\right)\,dx=\left(\int asec(cx)\,x^4dx\right)b+\frac{a\,x^5}{5}$$

input
```
int(x^4*(a+b*asec(c*x)),x)
```

output
```
(5*int(asec(c*x)*x**4,x)*b + a*x**5)/5
```

3.4 $\int x^3(a+b\sec^{-1}(cx))\,dx$

Optimal result 114
Mathematica [A] (verified) 114
Rubi [A] (verified) 115
Maple [A] (verified) 116
Fricas [A] (verification not implemented) 117
Sympy [A] (verification not implemented) 117
Maxima [A] (verification not implemented) 118
Giac [B] (verification not implemented) 118
Mupad [F(-1)] 119
Reduce [F] 120

Optimal result

Integrand size = 12, antiderivative size = 64

$$\int x^3(a+b\sec^{-1}(cx))\,dx=-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x}{6c^3}-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c}+\frac{1}{4}x^4(a+b\sec^{-1}(cx))$$

output
```
-1/6*b*(1-1/c^2/x^2)^(1/2)*x/c^3-1/12*b*(1-1/c^2/x^2)^(1/2)*x^3/c+1/4*x^4*
(a+b*arcsec(c*x))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int x^3(a+b\sec^{-1}(cx))\,dx=\frac{ax^4}{4}+b\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\left(-\frac{x}{6c^3}-\frac{x^3}{12c}\right)+\frac{1}{4}bx^4\sec^{-1}(cx)$$

input
```
Integrate[x^3*(a + b*ArcSec[c*x]),x]
```

output
```
(a*x^4)/4 + b*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]*(-1/6*x/c^3 - x^3/(12*c)) + (
b*x^4*ArcSec[c*x])/4
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5743, 803, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b \sec^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{ 5743}$$

$$\frac{1}{4}x^4\left(a+b\sec^{-1}(cx)\right)-\frac{b\int \frac{x^2}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{4c}$$

$$\Big\downarrow \text{ 803}$$

$$\frac{1}{4}x^4\left(a+b\sec^{-1}(cx)\right)-\frac{b\left(\frac{2\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{3c^2}+\frac{1}{3}x^3\sqrt{1-\frac{1}{c^2x^2}}\right)}{4c}$$

$$\Big\downarrow \text{ 746}$$

$$\frac{1}{4}x^4\left(a+b\sec^{-1}(cx)\right)-\frac{b\left(\frac{2x\sqrt{1-\frac{1}{c^2x^2}}}{3c^2}+\frac{1}{3}x^3\sqrt{1-\frac{1}{c^2x^2}}\right)}{4c}$$

input
```
Int[x^3*(a + b*ArcSec[c*x]),x]
```

output
```
-1/4*(b*((2*Sqrt[1 - 1/(c^2*x^2)]*x)/(3*c^2) + (Sqrt[1 - 1/(c^2*x^2)]*x^3)
/3))/c + (x^4*(a + b*ArcSec[c*x]))/4
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

method	result	size
parts	$\frac{a\,x^{4}}{4}+\frac{b\left(\frac{c^{4}x^{4}\operatorname{arcsec}\left(cx\right)}{4}-\frac{\left(c^{2}x^{2}-1\right)\left(c^{2}x^{2}+2\right)}{12\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}\right)}{c^{4}}$	70
derivativedivides	$\frac{\frac{a\,c^{4}x^{4}}{4}+b\left(\frac{c^{4}x^{4}\operatorname{arcsec}\left(cx\right)}{4}-\frac{\left(c^{2}x^{2}-1\right)\left(c^{2}x^{2}+2\right)}{12\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}\right)}{c^{4}}$	74
default	$\frac{\frac{a\,c^{4}x^{4}}{4}+b\left(\frac{c^{4}x^{4}\operatorname{arcsec}\left(cx\right)}{4}-\frac{\left(c^{2}x^{2}-1\right)\left(c^{2}x^{2}+2\right)}{12\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}\right)}{c^{4}}$	74

input
```
int(x^3*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x^4+b/c^4*(1/4*c^4*x^4*arcsec(c*x)-1/12*(c^2*x^2-1)*(c^2*x^2+2)/((c^
2*x^2-1)/c^2/x^2)^(1/2)/c/x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int x^3(a+b\sec^{-1}(cx))\,dx=\frac{3\,bc^4x^4\,\mathrm{arcsec}\,(cx)+3\,ac^4x^4-(bc^2x^2+2\,b)\sqrt{c^2x^2-1}}{12\,c^4}$$

input
```
integrate(x^3*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/12*(3*b*c^4*x^4*arcsec(c*x) + 3*a*c^4*x^4 - (b*c^2*x^2 + 2*b)*sqrt(c^2*x
^2 - 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.67

$$\int x^3(a+b\sec^{-1}(cx))\,dx=\frac{ax^4}{4}+\frac{bx^4\,\mathrm{asec}\,(cx)}{4}-\frac{b\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{4c}$$

input
```
integrate(x**3*(a+b*asec(c*x)),x)
```

output
```
a*x**4/4 + b*x**4*asec(c*x)/4 - b*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c
) + 2*sqrt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**
2*x**2 + 1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**3), True))/(4*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int x^3\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{1}{4}\,ax^4+\frac{1}{12}\left(3\,x^4\,\mathrm{arcsec}\,(cx)-\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)b$$

input
```
integrate(x^3*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/4*a*x^4 + 1/12*(3*x^4*arcsec(c*x) - (c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) +
3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1926 vs. 2(54) = 108.

Time = 0.16 (sec) , antiderivative size = 1926, normalized size of antiderivative = 30.09

$$\int x^3\left(a+b\sec^{-1}(cx)\right)\,dx=\text{Too large to display}$$

input
```
integrate(x^3*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/12*c*(3*b*arccos(1/(c*x))/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(
1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) + 3*a/(c^5 + 4*c
^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x)
+ 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)
^4/(1/(c*x) + 1)^8) - 12*b*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^5 + 4*c^5
*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4
/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) - 6*b*sqrt(-1/(c^2*x^2) + 1)/((c^5 + 4*
c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x)
 + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1
)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)) - 12*a*(1/(c^2*x^2) - 1)/((c^5 + 4*c^5
*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4
/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) + 18*b*(1/(c^2*x^2) - 1)^2*arccos(1/(c*
x))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) -
 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1
/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) + 10*b*(-1/(c^2*x^2) +
 1)^(3/2)/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*
x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6...
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\sec^{-1}(cx)\right)\,dx=\int x^3\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^3*(a + b*acos(1/(c*x))),x)
```

output
```
int(x^3*(a + b*acos(1/(c*x))), x)
```

Reduce [F]

$$\int x^3\left(a+b\sec^{-1}(cx)\right)\,dx=\left(\int asec(cx)\,x^3dx\right)b+\frac{a\,x^4}{4}$$

input
```
int(x^3*(a+b*asec(c*x)),x)
```

output
```
(4*int(asec(c*x)*x**3,x)*b + a*x**4)/4
```

3.5 $\int x^2(a + b\sec^{-1}(cx))\,dx$

Optimal result 121
Mathematica [A] (verified) 121
Rubi [A] (verified) 122
Maple [A] (verified) 124
Fricas [A] (verification not implemented) 124
Sympy [A] (verification not implemented) 125
Maxima [A] (verification not implemented) 125
Giac [B] (verification not implemented) 126
Mupad [F(-1)] 127
Reduce [F] 127

Optimal result

Integrand size = 12, antiderivative size = 64

$$\int x^2\left(a+b\sec^{-1}(cx)\right)\,dx=-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2}{6c}+\frac{1}{3}x^3\left(a+b\sec^{-1}(cx)\right)-\frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^3}$$

output
```
-1/6*b*(1-1/c^2/x^2)^(1/2)*x^2/c+1/3*x^3*(a+b*arcsec(c*x))-1/6*b*arctanh((
1-1/c^2/x^2)^(1/2))/c^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.33

$$\int x^2\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{ax^3}{3}-\frac{bx^2\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{6c}+\frac{1}{3}bx^3\sec^{-1}(cx)-\frac{b\log\left(x\left(1+\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\right)\right)}{6c^3}$$

input
```
Integrate[x^2*(a + b*ArcSec[c*x]),x]
```

output
```
(a*x^3)/3 - (b*x^2*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(6*c) + (b*x^3*ArcSec[c
*x])/3 - (b*Log[x*(1 + Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])])/(6*c^3)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5743, 798, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b\sec^{-1}(cx)\right) dx$$

$$\Big\downarrow \text{5743}$$

$$\frac{1}{3}x^3\left(a+b\sec^{-1}(cx)\right)-\frac{b\int \frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{3c}$$

$$\Big\downarrow \text{798}$$

$$\frac{b\int \frac{x^4}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{6c}+\frac{1}{3}x^3\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{b\left(\frac{\int \frac{x^2}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{2c^2}-x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c}+\frac{1}{3}x^3\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{b\left(x^2\left(-\sqrt{1-\frac{1}{c^2x^2}}\right)-\int \frac{1}{c^2-\frac{c^2}{x^4}}d\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c}+\frac{1}{3}x^3\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}x^3\left(a+b\sec^{-1}(cx)\right)+\frac{b\left(x^2\left(-\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^2}\right)}{6c}$$

input
```
Int[x^2*(a + b*ArcSec[c*x]),x]
```

output
```
(x^3*(a + b*ArcSec[c*x]))/3 + (b*(-(Sqrt[1 - 1/(c^2*x^2)]*x^2) - ArcTanh[S
qrt[1 - 1/(c^2*x^2)]]/c^2))/(6*c)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.47

method	result	size
parts	$\frac{a x^3}{3}+\frac{b\left(\frac{c^3x^3\,\mathrm{arcsec}(cx)}{3}-\frac{\sqrt{c^2x^2-1}\,\left(cx\sqrt{c^2x^2-1}+\ln\left(cx+\sqrt{c^2x^2-1}\right)\right)}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^3}$	94
derivativedivides	$\frac{\frac{a\,c^3x^3}{3}+b\left(\frac{c^3x^3\,\mathrm{arcsec}(cx)}{3}-\frac{\sqrt{c^2x^2-1}\,\left(cx\sqrt{c^2x^2-1}+\ln\left(cx+\sqrt{c^2x^2-1}\right)\right)}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^3}$	98
default	$\frac{\frac{a\,c^3x^3}{3}+b\left(\frac{c^3x^3\,\mathrm{arcsec}(cx)}{3}-\frac{\sqrt{c^2x^2-1}\,\left(cx\sqrt{c^2x^2-1}+\ln\left(cx+\sqrt{c^2x^2-1}\right)\right)}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^3}$	98

input
```
int(x^2*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*x^3+b/c^3*(1/3*c^3*x^3*arcsec(c*x)-1/6*(c^2*x^2-1)^(1/2)*(c*x*(c^2*x
^2-1)^(1/2)+ln(c*x+(c^2*x^2-1)^(1/2)))/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.47

$$\int x^2\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{2\,ac^3x^3+4\,bc^3\arctan\left(-cx+\sqrt{c^2x^2-1}\right)-\sqrt{c^2x^2-1}bcx+2\left(bc^3x^3-bc^3\right)\mathrm{arcsec}\left(cx\right)+b\log\left(-cx+\right.}{6\,c^3}$$

input
```
integrate(x^2*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/6*(2*a*c^3*x^3 + 4*b*c^3*arctan(-c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2
 - 1)*b*c*x + 2*(b*c^3*x^3 - b*c^3)*arcsec(c*x) + b*log(-c*x + sqrt(c^2*x^
2 - 1)))/c^3
```

Sympy [A] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.67

$$\int x^2\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{ax^3}{3}+\frac{bx^3\operatorname{asec}(cx)}{3}-\frac{b\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$

input
```
integrate(x**2*(a+b*asec(c*x)),x)
```

output
```
a*x**3/3 + b*x**3*asec(c*x)/3 - b*Piecewise((x*sqrt(c**2*x**2 - 1)/(2*c) +
 acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-I*c*x**3/(2*sqrt(-c**2*x**2 +
 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin(c*x)/(2*c**2), True))/(3*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.53

$$\int x^2\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{1}{3}ax^3+\frac{1}{12}\left(4x^3\operatorname{arcsec}(cx)-\frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)b$$

input
```
integrate(x^2*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*x^3 + 1/12*(4*x^3*arcsec(c*x) - (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c
^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c
^2*x^2) + 1) - 1)/c^2)/c)*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2101 vs. $2(54) = 108$.

Time = 0.44 (sec) , antiderivative size = 2101, normalized size of antiderivative = 32.83

$$\int x^2 (a + b \sec^{-1}(cx)) \, dx = \text{Too large to display}$$

input
```
integrate(x^2*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/6*c*(2*b*arccos(1/(c*x))/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
+ 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(
c*x) + 1)^6) - b*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^4 + 3*c
^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x)
+ 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) + b*log(abs(sqrt(-1/(c^2
*x^2) + 1) - 1/(c*x) - 1))/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
+ 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(
c*x) + 1)^6) + 2*a/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*
(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1
)^6) - 6*b*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^4 + 3*c^4*(1/(c^2*x^2) -
1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c
^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) - 3*b*(1/(c^2*x^2) - 1)*l
og(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^4 + 3*c^4*(1/(c^2*x^2) -
 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(
c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) + 3*b*(1/(c^2*x^2) - 1)*
log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^4 + 3*c^4*(1/(c^2*x^2)
- 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/
(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) - 2*b*sqrt(-1/(c^2*x^2)
 + 1)/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2)
 - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(...
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\sec^{-1}(cx)\right)\,dx=\int x^2\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^2*(a + b*acos(1/(c*x))),x)
```

output
```
int(x^2*(a + b*acos(1/(c*x))), x)
```

Reduce [F]

$$\int x^2\left(a+b\sec^{-1}(cx)\right)\,dx=\left(\int asec(cx)\,x^2dx\right)b+\frac{a\,x^3}{3}$$

input
```
int(x^2*(a+b*asec(c*x)),x)
```

output
```
(3*int(asec(c*x)*x**2,x)*b + a*x**3)/3
```

3.6 $\int x(a+b\sec^{-1}(cx))\,dx$

Optimal result 128
Mathematica [A] (verified) 128
Rubi [A] (verified) 129
Maple [A] (verified) 130
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Giac [B] (verification not implemented) 131
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Reduce [F] 132

Optimal result

Integrand size = 10, antiderivative size = 39

$$\int x\left(a+b\sec^{-1}(cx)\right)\,dx=-\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c}+\frac{1}{2}x^2\left(a+b\sec^{-1}(cx)\right)$$

output
```
-1/2*b*(1-1/c^2/x^2)^(1/2)*x/c+1/2*x^2*(a+b*arcsec(c*x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28

$$\int x\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{ax^2}{2}-\frac{bx\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{2c}+\frac{1}{2}bx^2\sec^{-1}(cx)$$

input
```
Integrate[x*(a + b*ArcSec[c*x]),x]
```

output
```
(a*x^2)/2 - (b*x*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(2*c) + (b*x^2*ArcSec[c*x
])/2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5743, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{5743}$$

$$\frac{1}{2}x^2\left(a+b\sec^{-1}(cx)\right)-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{2c}$$

$$\Big\downarrow \text{746}$$

$$\frac{1}{2}x^2\left(a+b\sec^{-1}(cx)\right)-\frac{bx\sqrt{1-\frac{1}{c^2x^2}}}{2c}$$

input
```
Int[x*(a + b*ArcSec[c*x]),x]
```

output
```
-1/2*(b*Sqrt[1 - 1/(c^2*x^2)]*x)/c + (x^2*(a + b*ArcSec[c*x]))/2
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.56

method	result	size
parts	$\frac{a\,x^{2}}{2}+\frac{b\left(\frac{c^{2}x^{2}\,\mathrm{arcsec}(cx)}{2}-\frac{c^{2}x^{2}-1}{2\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}\right)}{c^{2}}$	61
derivativedivides	$\frac{\frac{a\,c^{2}x^{2}}{2}+b\left(\frac{c^{2}x^{2}\,\mathrm{arcsec}(cx)}{2}-\frac{c^{2}x^{2}-1}{2\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}\right)}{c^{2}}$	65
default	$\frac{\frac{a\,c^{2}x^{2}}{2}+b\left(\frac{c^{2}x^{2}\,\mathrm{arcsec}(cx)}{2}-\frac{c^{2}x^{2}-1}{2\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}\right)}{c^{2}}$	65

input
```
int(x*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+b/c^2*(1/2*c^2*x^2*arcsec(c*x)-1/2/((c^2*x^2-1)/c^2/x^2)^(1/2)/c
/x*(c^2*x^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int x\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{bc^{2}x^{2}\,\mathrm{arcsec}\,(cx)+ac^{2}x^{2}-\sqrt{c^{2}x^{2}-1}b}{2\,c^{2}}$$

input
```
integrate(x*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/2*(b*c^2*x^2*arcsec(c*x) + a*c^2*x^2 - sqrt(c^2*x^2 - 1)*b)/c^2
```

Sympy [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.49

$$\int x\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{ax^2}{2}+\frac{bx^2\operatorname{asec}(cx)}{2}-\frac{b\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c}$$

input
```
integrate(x*(a+b*asec(c*x)),x)
```

output
```
a*x**2/2 + b*x**2*asec(c*x)/2 - b*Piecewise((sqrt(c**2*x**2 - 1)/c, Abs(c*
*2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/(2*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int x\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{1}{2}\,ax^2+\frac{1}{2}\left(x^2\operatorname{arcsec}(cx)-\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)b$$

input
```
integrate(x*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/2*a*x^2 + 1/2*(x^2*arcsec(c*x) - x*sqrt(-1/(c^2*x^2) + 1)/c)*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 634 vs. 2(33) = 66.

Time = 0.14 (sec) , antiderivative size = 634, normalized size of antiderivative = 16.26

$$\int x\left(a+b\sec^{-1}(cx)\right)\,dx=\text{Too large to display}$$

input
```
integrate(x*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/2*c*(b*arccos(1/(c*x))/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 +
c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + a/(c^3 + 2*c^3*(1/(c^2*x^2) - 1
)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) - 2*b*(1/(c^2
*x^2) - 1)*arccos(1/(c*x))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) - 2*b*sqrt(-1
/(c^2*x^2) + 1)/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(
c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)) - 2*a*(1/(c^2*x^2) - 1)/((
c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1
/(c*x) + 1)^4)*(1/(c*x) + 1)^2) + b*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((
c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1
/(c*x) + 1)^4)*(1/(c*x) + 1)^4) + 2*b*(-1/(c^2*x^2) + 1)^(3/2)/((c^3 + 2*c
^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
1)^4)*(1/(c*x) + 1)^3) + a*(1/(c^2*x^2) - 1)^2/((c^3 + 2*c^3*(1/(c^2*x^2)
- 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) +
 1)^4))
```

Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int x\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{a\,x^2}{2}+\frac{b\,x^2\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{2}-\frac{b\,x\,\sqrt{1-\frac{1}{c^2\,x^2}}}{2\,c}$$

input
```
int(x*(a + b*acos(1/(c*x))),x)
```

output
```
(a*x^2)/2 + (b*x^2*acos(1/(c*x)))/2 - (b*x*(1 - 1/(c^2*x^2))^(1/2))/(2*c)
```

Reduce [F]

$$\int x\left(a+b\sec^{-1}(cx)\right)\,dx=\left(\int asec(cx)\,xdx\right)b+\frac{a\,x^2}{2}$$

input
```
int(x*(a+b*asec(c*x)),x)
```

output
```
(2*int(asec(c*x)*x,x)*b + a*x**2)/2
```

3.7 $\int \left(a+b\sec^{-1}(cx)\right)\,dx$

Optimal result 134
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Rubi [A] (verified) 135
Maple [A] (verified) 135
Fricas [B] (verification not implemented) 136
Sympy [A] (verification not implemented) 136
Maxima [A] (verification not implemented) 137
Giac [B] (verification not implemented) 137
Mupad [B] (verification not implemented) 138
Reduce [F] 138

Optimal result

Integrand size = 8, antiderivative size = 32

$$\int \left(a+b\sec^{-1}(cx)\right)\,dx=ax+bx\sec^{-1}(cx)-\frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$$

output
```
a*x+b*x*arcsec(c*x)-b*arctanh((1-1/c^2/x^2)^(1/2))/c
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.84

$$\int \left(a+b\sec^{-1}(cx)\right)\,dx=ax+bx\sec^{-1}(cx)-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{-1+c^2x^2}}$$

input
```
Integrate[a + b*ArcSec[c*x],x]
```

output
```
a*x + b*x*ArcSec[c*x] - (b*Sqrt[1 - 1/(c^2*x^2)]*x*ArcTanh[(c*x)/Sqrt[-1 +
 c^2*x^2]])/Sqrt[-1 + c^2*x^2]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \; 2009$$

$$ax-\frac{b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}+bx\sec^{-1}(cx)$$

input
```
Int[a + b*ArcSec[c*x],x]
```

output
```
a*x + b*x*ArcSec[c*x] - (b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/c
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

method	result	size
default	$ax+bx\ \text{arcsec}\left(cx\right)-\frac{b\ln\left(cx+cx\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$	38
parts	$ax+bx\ \text{arcsec}\left(cx\right)-\frac{b\ln\left(cx+cx\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$	38
derivativedivides	$\frac{acx+b\left(cx\ \text{arcsec}(cx)-\ln\left(cx+cx\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{c}$	42

input
```
int(a+b*arcsec(c*x),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*x*arcsec(c*x)-b/c*ln(c*x+c*x*(1-1/c^2/x^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(30) = 60$.

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.97

$$\int \left(a+b\sec^{-1}(cx)\right)\,dx = \frac{acx + 2\,bc\arctan\left(-cx+\sqrt{c^2x^2-1}\right) + (bcx-bc)\,\mathrm{arcsec}\,(cx) + b\log\left(-cx+\sqrt{c^2x^2-1}\right)}{c}$$

input
```
integrate(a+b*arcsec(c*x),x, algorithm="fricas")
```

output
```
(a*c*x + 2*b*c*arctan(-c*x + sqrt(c^2*x^2 - 1)) + (b*c*x - b*c)*arcsec(c*x
) + b*log(-c*x + sqrt(c^2*x^2 - 1)))/c
```

Sympy [A] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \left(a+b\sec^{-1}(cx)\right)\,dx = ax + b\left(x\,\mathrm{asec}\,(cx) - \frac{\begin{cases}\mathrm{acosh}\,(cx) & \text{for } |c^2x^2| > 1 \\ -i\,\mathrm{asin}\,(cx) & \text{otherwise}\end{cases}}{c}\right)$$

input
```
integrate(a+b*asec(c*x),x)
```

output
```
a*x + b*(x*asec(c*x) - Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asi
n(c*x), True))/c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.66

$$\int \left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=ax+\frac{\left(2\,cx\,\mathrm{arcsec}\,(cx)-\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)+\log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)b}{2\,c}$$

input
```
integrate(a+b*arcsec(c*x),x, algorithm="maxima")
```

output
```
a*x + 1/2*(2*c*x*arcsec(c*x) - log(sqrt(-1/(c^2*x^2) + 1) + 1) + log(-sqrt
(-1/(c^2*x^2) + 1) + 1))*b/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(30) = 60$.

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.97

$$\int \left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{1}{2}\,bc\left(\frac{2\,x\arccos\left(\frac{1}{cx}\right)}{c}-\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}\right)+ax$$

input
```
integrate(a+b*arcsec(c*x),x, algorithm="giac")
```

output
```
1/2*b*c*(2*x*arccos(1/(c*x))/c - (log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-s
qrt(-1/(c^2*x^2) + 1) + 1))/c^2) + a*x
```

Mupad [B] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \left(a+b\sec^{-1}(cx)\right)\,dx = a\,x+b\,x\,\mathrm{acos}\left(\frac{1}{c\,x}\right)-\frac{b\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{c^2\,x^2}}}\right)}{c}$$

input
```
int(a + b*acos(1/(c*x)),x)
```

output
```
a*x + b*x*acos(1/(c*x)) - (b*atanh(1/(1 - 1/(c^2*x^2))^(1/2)))/c
```

Reduce [F]

$$\int \left(a+b\sec^{-1}(cx)\right)\,dx = \left(\int asec(cx)\,dx\right)b+ax$$

input
```
int(a+b*asec(c*x),x)
```

output
```
int(asec(c*x),x)*b + a*x
```

3.8 $\int \frac{a+b\sec^{-1}(cx)}{x}\,dx$

Optimal result 139
Mathematica [A] (verified) 139
Rubi [A] (verified) 140
Maple [A] (verified) 142
Fricas [F] 143
Sympy [F] 143
Maxima [F] 143
Giac [F(-2)] 144
Mupad [F(-1)] 144
Reduce [F] 144

Optimal result

Integrand size = 12, antiderivative size = 64

$$\int \frac{a+b\sec^{-1}(cx)}{x}\,dx=\frac{i\left(a+b\sec^{-1}(cx)\right)^2}{2b}-\left(a+b\sec^{-1}(cx)\right)\log\left(1+e^{2i\sec^{-1}(cx)}\right)+\frac{1}{2}ib\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)$$

output
```
1/2*I*(a+b*arcsec(c*x))^2/b-(a+b*arcsec(c*x))*ln(1+(1/c/x+I*(1-1/c^2/x^2)^
(1/2))^2)+1/2*I*b*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int \frac{a+b\sec^{-1}(cx)}{x}\,dx=\frac{1}{2}ib\sec^{-1}(cx)^2-b\sec^{-1}(cx)\log\left(1+e^{2i\sec^{-1}(cx)}\right)+a\log(x)+\frac{1}{2}ib\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)$$

input
```
Integrate[(a + b*ArcSec[c*x])/x,x]
```

output
```
(I/2)*b*ArcSec[c*x]^2 - b*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] + a*L
og[x] + (I/2)*b*PolyLog[2, -E^((2*I)*ArcSec[c*x])]
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.36, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5741, 5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x}dx$$

$$\Big\downarrow \text{5741}$$

$$-\int x\left(a+b\arccos\left(\frac{1}{cx}\right)\right)d\frac{1}{x}$$

$$\Big\downarrow \text{5137}$$

$$\int cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)d\arccos\left(\frac{1}{cx}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(\arccos\left(\frac{1}{cx}\right)\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)d\arccos\left(\frac{1}{cx}\right)$$

$$\Big\downarrow \text{4202}$$

$$\frac{i\left(a+b\arccos\left(\frac{1}{cx}\right)\right)^2}{2b}-2i\int \frac{e^{2i\arccos\left(\frac{1}{cx}\right)}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{1+e^{2i\arccos\left(\frac{1}{cx}\right)}}d\arccos\left(\frac{1}{cx}\right)$$

$$\Big\downarrow \text{2620}$$

$$\frac{i\left(a+b\arccos\left(\frac{1}{cx}\right)\right)^2}{2b}-2i\left(\frac{1}{2}ib\int \log\left(1+e^{2i\arccos\left(\frac{1}{cx}\right)}\right)d\arccos\left(\frac{1}{cx}\right)-\frac{1}{2}i\log\left(1+e^{2i\arccos\left(\frac{1}{cx}\right)}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\right)$$

$$\Big\downarrow \text{2715}$$

$$\frac{i\left(a+b\arccos\left(\frac{1}{cx}\right)\right)^2}{2b}-$$
$$2i\left(\frac{1}{4}b\int x\log\left(1+e^{2i\arccos\left(\frac{1}{cx}\right)}\right)de^{2i\arccos\left(\frac{1}{cx}\right)}-\frac{1}{2}i\log\left(1+e^{2i\arccos\left(\frac{1}{cx}\right)}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\right)$$

$$\Big\downarrow \text{2838}$$

$$\frac{i\left(a+b\arccos\left(\frac{1}{cx}\right)\right)^2}{2b}-$$
$$2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos\left(\frac{1}{cx}\right)}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)-\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2i\arccos\left(\frac{1}{cx}\right)}\right)\right)$$

input
```
Int[(a + b*ArcSec[c*x])/x,x]
```

output
```
((I/2)*(a + b*ArcCos[1/(c*x)])^2)/b - (2*I)*((-1/2*I)*(a + b*ArcCos[1/(c*x
)])*Log[1 + E^((2*I)*ArcCos[1/(c*x)])] - (b*PolyLog[2, -E^((2*I)*ArcCos[1/
(c*x)])])/4)
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5741
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a + b
*ArcCos[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.31

method	result
parts	$a\ln(x)+b\left(\frac{i\,\mathrm{arcsec}(cx)^2}{2}-\mathrm{arcsec}(cx)\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right)+\frac{i\,\mathrm{polylog}\left(2,-\left(\frac{1}{cx}+i\right.\right.}{2}\right.$
derivativedivides	$a\ln(cx)+b\left(\frac{i\,\mathrm{arcsec}(cx)^2}{2}-\mathrm{arcsec}(cx)\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right)+\frac{i\,\mathrm{polylog}\left(2,-\left(\frac{1}{cx}+\right.\right.}{2}\right.$
default	$a\ln(cx)+b\left(\frac{i\,\mathrm{arcsec}(cx)^2}{2}-\mathrm{arcsec}(cx)\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right)+\frac{i\,\mathrm{polylog}\left(2,-\left(\frac{1}{cx}+\right.\right.}{2}\right.$

input
```
int((a+b*arcsec(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
a*ln(x)+b*(1/2*I*arcsec(c*x)^2-arcsec(c*x)*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/
2))^2)+1/2*I*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2))
```

Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x,x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)/x, x)
```

Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x}\,dx=\int \frac{a+b\,\mathrm{asec}\,(cx)}{x}\,dx$$

input
```
integrate((a+b*asec(c*x))/x,x)
```

output
```
Integral((a + b*asec(c*x))/x, x)
```

Maxima [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x,x, algorithm="maxima")
```

output
```
-(c^2*integrate(sqrt(c*x + 1)*sqrt(c*x - 1)*log(x)/(c^4*x^3 - c^2*x), x) -
 arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(x))*b + a*log(x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsec(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x}\,dx = \int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/x,x)
```

output
```
int((a + b*acos(1/(c*x)))/x, x)
```

Reduce [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x}\,dx = \left(\int \frac{asec(cx)}{x}dx\right)b+\log(x)\,a$$

input
```
int((a+b*asec(c*x))/x,x)
```

output
```
int(asec(c*x)/x,x)*b + log(x)*a
```

3.9 $\int \frac{a+b\sec^{-1}(cx)}{x^2} dx$

Optimal result . . . 145
Mathematica [A] (verified) . . . 145
Rubi [A] (verified) . . . 146
Maple [A] (verified) . . . 147
Fricas [A] (verification not implemented) . . . 147
Sympy [A] (verification not implemented) . . . 148
Maxima [A] (verification not implemented) . . . 148
Giac [A] (verification not implemented) . . . 148
Mupad [B] (verification not implemented) . . . 149
Reduce [F] . . . 149

Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{a+b\sec^{-1}(cx)}{x^2} dx = bc\sqrt{1-\frac{1}{c^2x^2}} - \frac{a+b\sec^{-1}(cx)}{x}$$

output
```
b*c*(1-1/c^2/x^2)^(1/2)-(a+b*arcsec(c*x))/x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{a+b\sec^{-1}(cx)}{x^2} dx = -\frac{a}{x} + bc\sqrt{\frac{-1+c^2x^2}{c^2x^2}} - \frac{b\sec^{-1}(cx)}{x}$$

input
```
Integrate[(a + b*ArcSec[c*x])/x^2,x]
```

output
```
-(a/x) + b*c*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)] - (b*ArcSec[c*x])/x
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5743, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2}\,dx$$

$$\Big\downarrow \ 5743$$

$$\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^3}dx}{c}-\frac{a+b\sec^{-1}(cx)}{x}$$

$$\Big\downarrow \ 793$$

$$bc\sqrt{1-\frac{1}{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{x}$$

input
```
Int[(a + b*ArcSec[c*x])/x^2,x]
```

output
```
b*c*Sqrt[1 - 1/(c^2*x^2)] - (a + b*ArcSec[c*x])/x
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.87

method	result	size
parts	$-\frac{a}{x}+bc\left(-\frac{\operatorname{arcsec}(cx)}{cx}+\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x^2}\right)$	58
derivativedivides	$c\left(-\frac{a}{cx}+b\left(-\frac{\operatorname{arcsec}(cx)}{cx}+\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x^2}\right)\right)$	62
default	$c\left(-\frac{a}{cx}+b\left(-\frac{\operatorname{arcsec}(cx)}{cx}+\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x^2}\right)\right)$	62

input
```
int((a+b*arcsec(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/x+b*c*(-1/c/x*arcsec(c*x)+1/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^2/x^2*(c^2*x^
2-1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{a+b\sec^{-1}(cx)}{x^2}\,dx=-\frac{b\operatorname{arcsec}(cx)-\sqrt{c^2x^2-1}b+a}{x}$$

input
```
integrate((a+b*arcsec(c*x))/x^2,x, algorithm="fricas")
```

output
```
-(b*arcsec(c*x) - sqrt(c^2*x^2 - 1)*b + a)/x
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{a+b\sec^{-1}(cx)}{x^2}\,dx = \begin{cases} -\frac{a}{x} + bc\sqrt{1-\frac{1}{c^2x^2}} - \frac{b\,\mathrm{asec}\,(cx)}{x} & \text{for } c \neq 0 \\ -\frac{a+\tilde{\infty}b}{x} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*asec(c*x))/x**2,x)
```

output
```
Piecewise((-a/x + b*c*sqrt(1 - 1/(c**2*x**2)) - b*asec(c*x)/x, Ne(c, 0)),
(-(a + zoo*b)/x, True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{a+b\sec^{-1}(cx)}{x^2}\,dx = \left(c\sqrt{-\frac{1}{c^2x^2}+1} - \frac{\mathrm{arcsec}\,(cx)}{x}\right)b - \frac{a}{x}$$

input
```
integrate((a+b*arcsec(c*x))/x^2,x, algorithm="maxima")
```

output
```
(c*sqrt(-1/(c^2*x^2) + 1) - arcsec(c*x)/x)*b - a/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.39

$$\int \frac{a+b\sec^{-1}(cx)}{x^2}\,dx = \left(b\sqrt{-\frac{1}{c^2x^2}+1} - \frac{b\arccos\left(\frac{1}{cx}\right)}{cx} - \frac{a}{cx}\right)c$$

input
```
integrate((a+b*arcsec(c*x))/x^2,x, algorithm="giac")
```

output
```
(b*sqrt(-1/(c^2*x^2) + 1) - b*arccos(1/(c*x))/(c*x) - a/(c*x))*c
```

Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{a + b\sec^{-1}(cx)}{x^2}\,dx = b\,c\,\sqrt{1 - \frac{1}{c^2\,x^2}} - \frac{a}{x} - \frac{b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x}$$

input
```
int((a + b*acos(1/(c*x)))/x^2,x)
```

output
```
b*c*(1 - 1/(c^2*x^2))^(1/2) - a/x - (b*acos(1/(c*x)))/x
```

Reduce [F]

$$\int \frac{a + b\sec^{-1}(cx)}{x^2}\,dx = \frac{\left(\int \frac{asec(cx)}{x^2}dx\right)bx - a}{x}$$

input
```
int((a+b*asec(c*x))/x^2,x)
```

output
```
(int(asec(c*x)/x**2,x)*b*x - a)/x
```

3.10 $\int \frac{a+b\sec^{-1}(cx)}{x^3}\,dx$

Optimal result 150
Mathematica [A] (verified) 150
Rubi [A] (verified) 151
Maple [B] (verified) 152
Fricas [A] (verification not implemented) 153
Sympy [A] (verification not implemented) 153
Maxima [A] (verification not implemented) 154
Giac [A] (verification not implemented) 154
Mupad [B] (verification not implemented) 155
Reduce [F] 155

Optimal result

Integrand size = 12, antiderivative size = 51

$$\int \frac{a+b\sec^{-1}(cx)}{x^3}\,dx=\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{4x}-\frac{1}{4}bc^2\csc^{-1}(cx)-\frac{a+b\sec^{-1}(cx)}{2x^2}$$

output
```
1/4*b*c*(1-1/c^2/x^2)^(1/2)/x-1/4*b*c^2*arccsc(c*x)-1/2*(a+b*arcsec(c*x))/
x^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int \frac{a+b\sec^{-1}(cx)}{x^3}\,dx=-\frac{a}{2x^2}+\frac{bc\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{4x}-\frac{b\sec^{-1}(cx)}{2x^2}-\frac{1}{4}bc^2\arcsin\left(\frac{1}{cx}\right)$$

input
```
Integrate[(a + b*ArcSec[c*x])/x^3,x]
```

output
```
-1/2*a/x^2 + (b*c*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(4*x) - (b*ArcSec[c*x])/
(2*x^2) - (b*c^2*ArcSin[1/(c*x)])/4
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5743, 858, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x^3}dx$$

$$\Big\downarrow \text{5743}$$

$$\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^4}dx}{2c}-\frac{a+b\sec^{-1}(cx)}{2x^2}$$

$$\Big\downarrow \text{858}$$

$$-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2}d\frac{1}{x}}{2c}-\frac{a+b\sec^{-1}(cx)}{2x^2}$$

$$\Big\downarrow \text{262}$$

$$-\frac{b\left(\frac{1}{2}c^2\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{2x}\right)}{2c}-\frac{a+b\sec^{-1}(cx)}{2x^2}$$

$$\Big\downarrow \text{223}$$

$$-\frac{a+b\sec^{-1}(cx)}{2x^2}-\frac{b\left(\frac{1}{2}c^3\arcsin\left(\frac{1}{cx}\right)-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{2x}\right)}{2c}$$

input
```
Int[(a + b*ArcSec[c*x])/x^3,x]
```

output
```
-1/2*(a + b*ArcSec[c*x])/x^2 - (b*(-1/2*(c^2*Sqrt[1 - 1/(c^2*x^2)])/x + (c
^3*ArcSin[1/(c*x)])/2))/(2*c)
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(43) = 86$.

Time = 0.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.88

method	result	size
parts	$-\frac{a}{2x^{2}}+b\,c^{2}\left(-\frac{\operatorname{arcsec}\left(cx\right)}{2c^{2}x^{2}}-\frac{\sqrt{c^{2}x^{2}-1}\,\left(\arctan\left(\frac{1}{\sqrt{c^{2}x^{2}-1}}\right)c^{2}x^{2}-\sqrt{c^{2}x^{2}-1}\right)}{4\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,c^{3}x^{3}}\right)$	96
derivativedivides	$c^{2}\left(-\frac{a}{2c^{2}x^{2}}+b\left(-\frac{\operatorname{arcsec}\left(cx\right)}{2c^{2}x^{2}}-\frac{\sqrt{c^{2}x^{2}-1}\,\left(\arctan\left(\frac{1}{\sqrt{c^{2}x^{2}-1}}\right)c^{2}x^{2}-\sqrt{c^{2}x^{2}-1}\right)}{4\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,c^{3}x^{3}}\right)\right)$	100
default	$c^{2}\left(-\frac{a}{2c^{2}x^{2}}+b\left(-\frac{\operatorname{arcsec}\left(cx\right)}{2c^{2}x^{2}}-\frac{\sqrt{c^{2}x^{2}-1}\,\left(\arctan\left(\frac{1}{\sqrt{c^{2}x^{2}-1}}\right)c^{2}x^{2}-\sqrt{c^{2}x^{2}-1}\right)}{4\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,c^{3}x^{3}}\right)\right)$	100

input
```
int((a+b*arcsec(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/x^2+b*c^2*(-1/2/c^2/x^2*arcsec(c*x)-1/4*(c^2*x^2-1)^(1/2)*(arctan(1
/(c^2*x^2-1)^(1/2))*c^2*x^2-(c^2*x^2-1)^(1/2))/((c^2*x^2-1)/c^2/x^2)^(1/2)
/c^3/x^3)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{a+b\sec^{-1}(cx)}{x^3}\,dx = \frac{(bc^2x^2-2\,b)\,\mathrm{arcsec}\,(cx)+\sqrt{c^2x^2-1}b-2\,a}{4\,x^2}$$

input
```
integrate((a+b*arcsec(c*x))/x^3,x, algorithm="fricas")
```

output
```
1/4*((b*c^2*x^2 - 2*b)*arcsec(c*x) + sqrt(c^2*x^2 - 1)*b - 2*a)/x^2
```

Sympy [A] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.33

$$\int \frac{a+b\sec^{-1}(cx)}{x^3}\,dx = -\frac{a}{2x^2}-\frac{b\,\mathrm{asec}\,(cx)}{2x^2}+\frac{b\left(\begin{cases}\frac{ic^3\,\mathrm{acosh}\left(\frac{1}{cx}\right)}{2}-\frac{ic^2}{2x\sqrt{-1+\frac{1}{c^2x^2}}}+\frac{i}{2x^3\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for } \frac{1}{|c^2x^2|}>1\\ -\frac{c^3\,\mathrm{asin}\left(\frac{1}{cx}\right)}{2}+\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{2x} & \text{otherwise}\end{cases}\right)}{2c}$$

input
```
integrate((a+b*asec(c*x))/x**3,x)
```

output
```
-a/(2*x**2) - b*asec(c*x)/(2*x**2) + b*Piecewise((I*c**3*acosh(1/(c*x))/2
- I*c**2/(2*x**sqrt(-1 + 1/(c**2*x**2))) + I/(2*x**3*sqrt(-1 + 1/(c**2*x**2
))), 1/Abs(c**2*x**2) > 1), (-c**3*asin(1/(c*x))/2 + c**2*sqrt(1 - 1/(c**2
*x**2))/(2*x), True))/(2*c)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.63

$$\int \frac{a + b\sec^{-1}(cx)}{x^3}\,dx$$
$$= -\frac{1}{4}\,b\left(\frac{\frac{c^4 x\sqrt{-\frac{1}{c^2x^2}+1}}{c^2x^2\left(\frac{1}{c^2x^2}-1\right)-1} - c^3\arctan\left(cx\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c} + \frac{2\,\mathrm{arcsec}\,(cx)}{x^2}\right) - \frac{a}{2\,x^2}$$

input
```
integrate((a+b*arcsec(c*x))/x^3,x, algorithm="maxima")
```

output
```
-1/4*b*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c^
3*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c + 2*arcsec(c*x)/x^2) - 1/2*a/x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14

$$\int \frac{a + b\sec^{-1}(cx)}{x^3}\,dx = \frac{1}{4}\left(bc\arccos\left(\frac{1}{cx}\right) + \frac{b\sqrt{-\frac{1}{c^2x^2}+1}}{x} - \frac{2\,b\arccos\left(\frac{1}{cx}\right)}{cx^2} - \frac{2\,a}{cx^2}\right)c$$

input
```
integrate((a+b*arcsec(c*x))/x^3,x, algorithm="giac")
```

output
```
1/4*(b*c*arccos(1/(c*x)) + b*sqrt(-1/(c^2*x^2) + 1)/x - 2*b*arccos(1/(c*x)
)/(c*x^2) - 2*a/(c*x^2))*c
```

Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{a+b\sec^{-1}(cx)}{x^3}\,dx = \frac{b\,c\,\sqrt{1-\frac{1}{c^2\,x^2}}}{4\,x} - \frac{b\,c^2\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\,\left(\frac{2}{c^2\,x^2}-1\right)}{4} - \frac{a}{2\,x^2}$$

input
```
int((a + b*acos(1/(c*x)))/x^3,x)
```

output
```
(b*c*(1 - 1/(c^2*x^2))^(1/2))/(4*x) - (b*c^2*acos(1/(c*x))*(2/(c^2*x^2) -
1))/4 - a/(2*x^2)
```

Reduce [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^3}\,dx = \frac{2\left(\int \frac{asec(cx)}{x^3}dx\right)b\,x^2-a}{2x^2}$$

input
```
int((a+b*asec(c*x))/x^3,x)
```

output
```
(2*int(asec(c*x)/x**3,x)*b*x**2 - a)/(2*x**2)
```

3.11 $\int \frac{a+b\sec^{-1}(cx)}{x^4}\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 60

$$\int \frac{a+b\sec^{-1}(cx)}{x^4}\,dx=\frac{1}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}-\frac{1}{9}bc^3\left(1-\frac{1}{c^2x^2}\right)^{3/2}-\frac{a+b\sec^{-1}(cx)}{3x^3}$$

output
```
1/3*b*c^3*(1-1/c^2/x^2)^(1/2)-1/9*b*c^3*(1-1/c^2/x^2)^(3/2)-1/3*(a+b*arcse
c(c*x))/x^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

$$\int \frac{a+b\sec^{-1}(cx)}{x^4}\,dx=-\frac{a}{3x^3}+b\left(\frac{2c^3}{9}+\frac{c}{9x^2}\right)\sqrt{\frac{-1+c^2x^2}{c^2x^2}}-\frac{b\sec^{-1}(cx)}{3x^3}$$

input
```
Integrate[(a + b*ArcSec[c*x])/x^4,x]
```

output
```
-1/3*a/x^3 + b*((2*c^3)/9 + c/(9*x^2))*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)] - (b
*ArcSec[c*x])/(3*x^3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5743, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x^4}dx$$

$$\Big\downarrow \text{5743}$$

$$\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^5}dx}{3c}-\frac{a+b\sec^{-1}(cx)}{3x^3}$$

$$\Big\downarrow \text{798}$$

$$-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2}d\frac{1}{x^2}}{6c}-\frac{a+b\sec^{-1}(cx)}{3x^3}$$

$$\Big\downarrow \text{53}$$

$$-\frac{b\int \left(\frac{c^2}{\sqrt{1-\frac{1}{c^2x^2}}}-c^2\sqrt{1-\frac{1}{c^2x^2}}\right)d\frac{1}{x^2}}{6c}-\frac{a+b\sec^{-1}(cx)}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a+b\sec^{-1}(cx)}{3x^3}-\frac{b\left(\frac{2}{3}c^4\left(1-\frac{1}{c^2x^2}\right)^{3/2}-2c^4\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c}$$

input
```
Int[(a + b*ArcSec[c*x])/x^4,x]
```

output
```
-1/6*(b*(-2*c^4*Sqrt[1 - 1/(c^2*x^2)] + (2*c^4*(1 - 1/(c^2*x^2))^(3/2))/3)
)/c - (a + b*ArcSec[c*x])/(3*x^3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

method	result	size
parts	$-\frac{a}{3x^3}+b\,c^3\left(-\frac{\operatorname{arcsec}(cx)}{3c^3x^3}+\frac{(c^2x^2-1)\,(2c^2x^2+1)}{9\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^4x^4}\right)$	71
derivativedivides	$c^3\left(-\frac{a}{3c^3x^3}+b\left(-\frac{\operatorname{arcsec}(cx)}{3c^3x^3}+\frac{(c^2x^2-1)\,(2c^2x^2+1)}{9\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^4x^4}\right)\right)$	75
default	$c^3\left(-\frac{a}{3c^3x^3}+b\left(-\frac{\operatorname{arcsec}(cx)}{3c^3x^3}+\frac{(c^2x^2-1)\,(2c^2x^2+1)}{9\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^4x^4}\right)\right)$	75

input
```
int((a+b*arcsec(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/x^3+b*c^3*(-1/3/c^3/x^3*arcsec(c*x)+1/9*(c^2*x^2-1)*(2*c^2*x^2+1)/(
(c^2*x^2-1)/c^2/x^2)^(1/2)/c^4/x^4)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.67

$$\int \frac{a+b\sec^{-1}(cx)}{x^4}\,dx = -\frac{3\,b\,\text{arcsec}\,(cx)-(2\,bc^2x^2+b)\sqrt{c^2x^2-1}+3\,a}{9\,x^3}$$

input
```
integrate((a+b*arcsec(c*x))/x^4,x, algorithm="fricas")
```

output
```
-1/9*(3*b*arcsec(c*x) - (2*b*c^2*x^2 + b)*sqrt(c^2*x^2 - 1) + 3*a)/x^3
```

Sympy [A] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.83

$$\int \frac{a+b\sec^{-1}(cx)}{x^4}\,dx = -\frac{a}{3x^3}-\frac{b\,\text{asec}\,(cx)}{3x^3} + \frac{b\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x}+\frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2|>1\\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x}+\frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c}$$

input
```
integrate((a+b*asec(c*x))/x**4,x)
```

output
```
-a/(3*x**3) - b*asec(c*x)/(3*x**3) + b*Piecewise((2*c**3*sqrt(c**2*x**2 -
1)/(3*x) + c*sqrt(c**2*x**2 - 1)/(3*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*
sqrt(-c**2*x**2 + 1)/(3*x) + I*c*sqrt(-c**2*x**2 + 1)/(3*x**3), True))/(3*
c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int \frac{a+b\sec^{-1}(cx)}{x^4}\,dx = -\frac{1}{9}\,b\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{3\,\text{arcsec}\,(cx)}{x^3}\right) - \frac{a}{3\,x^3}$$

input
```
integrate((a+b*arcsec(c*x))/x^4,x, algorithm="maxima")
```

output
```
-1/9*b*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c +
3*arcsec(c*x)/x^3) - 1/3*a/x^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int \frac{a+b\sec^{-1}(cx)}{x^4}\,dx = \frac{1}{9}\left(2\,bc^2\sqrt{-\frac{1}{c^2x^2}+1}+\frac{b\sqrt{-\frac{1}{c^2x^2}+1}}{x^2}-\frac{3\,b\arccos\left(\frac{1}{cx}\right)}{cx^3}-\frac{3\,a}{cx^3}\right)c$$

input
```
integrate((a+b*arcsec(c*x))/x^4,x, algorithm="giac")
```

output
```
1/9*(2*b*c^2*sqrt(-1/(c^2*x^2) + 1) + b*sqrt(-1/(c^2*x^2) + 1)/x^2 - 3*b*a
rccos(1/(c*x))/(c*x^3) - 3*a/(c*x^3))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^4}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^4}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/x^4,x)
```

output
```
int((a + b*acos(1/(c*x)))/x^4, x)
```

Reduce [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^4}\,dx=\frac{3\left(\int \frac{asec(cx)}{x^4}dx\right)b\,x^3-a}{3x^3}$$

input
```
int((a+b*asec(c*x))/x^4,x)
```

output
```
(3*int(asec(c*x)/x**4,x)*b*x**3 - a)/(3*x**3)
```

3.12 $\int \frac{a+b\sec^{-1}(cx)}{x^5}\,dx$

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Mupad [F(-1)] 167
Reduce [F] 168

Optimal result

Integrand size = 12, antiderivative size = 76

$$\int \frac{a+b\sec^{-1}(cx)}{x^5}\,dx=\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16x^3}+\frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}}{32x}-\frac{3}{32}bc^4\csc^{-1}(cx)-\frac{a+b\sec^{-1}(cx)}{4x^4}$$

output
```
1/16*b*c*(1-1/c^2/x^2)^(1/2)/x^3+3/32*b*c^3*(1-1/c^2/x^2)^(1/2)/x-3/32*b*c
^4*arccsc(c*x)-1/4*(a+b*arcsec(c*x))/x^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int \frac{a+b\sec^{-1}(cx)}{x^5}\,dx=-\frac{a}{4x^4}+b\left(\frac{c}{16x^3}+\frac{3c^3}{32x}\right)\sqrt{\frac{-1+c^2x^2}{c^2x^2}}-\frac{b\sec^{-1}(cx)}{4x^4}-\frac{3}{32}bc^4\arcsin\left(\frac{1}{cx}\right)$$

input
```
Integrate[(a + b*ArcSec[c*x])/x^5,x]
```

output
```
-1/4*a/x^4 + b*(c/(16*x^3) + (3*c^3)/(32*x))*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)
] - (b*ArcSec[c*x])/(4*x^4) - (3*b*c^4*ArcSin[1/(c*x)])/32
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5743, 858, 262, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x^5}\,dx$$

$$\Big\downarrow \text{5743}$$

$$\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^6}dx}{4c}-\frac{a+b\sec^{-1}(cx)}{4x^4}$$

$$\Big\downarrow \text{858}$$

$$-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^4}d\frac{1}{x}}{4c}-\frac{a+b\sec^{-1}(cx)}{4x^4}$$

$$\Big\downarrow \text{262}$$

$$-\frac{b\left(\frac{3}{4}c^2\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2}d\frac{1}{x}-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{4x^3}\right)}{4c}-\frac{a+b\sec^{-1}(cx)}{4x^4}$$

$$\Big\downarrow \text{262}$$

$$-\frac{b\left(\frac{3}{4}c^2\left(\frac{1}{2}c^2\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{2x}\right)-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{4x^3}\right)}{4c}-\frac{a+b\sec^{-1}(cx)}{4x^4}$$

$$\Big\downarrow \text{223}$$

$$-\frac{a+b\sec^{-1}(cx)}{4x^4}-\frac{b\left(\frac{3}{4}c^2\left(\frac{1}{2}c^3\arcsin\left(\frac{1}{cx}\right)-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{2x}\right)-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{4x^3}\right)}{4c}$$

input
```
Int[(a + b*ArcSec[c*x])/x^5,x]
```

output
```
-1/4*(a + b*ArcSec[c*x])/x^4 - (b*(-1/4*(c^2*Sqrt[1 - 1/(c^2*x^2)])/x^3 +
(3*c^2*(-1/2*(c^2*Sqrt[1 - 1/(c^2*x^2)])/x + (c^3*ArcSin[1/(c*x)])/2))/4))
/(4*c)
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(64) = 128$.

Time = 0.18 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.82

method	result
parts	$-\frac{a}{4x^4}-\frac{\operatorname{arcsec}(cx)b}{4x^4}-\frac{3b\,c^3\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}+\frac{3bc(c^2x^2-1)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^3}+\frac{b(c^2x^2-1)}{16c\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^5}$
derivativedivides	$c^4\left(-\frac{a}{4c^4x^4}-\frac{b\operatorname{arcsec}(cx)}{4c^4x^4}-\frac{3b\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}+\frac{3b(c^2x^2-1)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}+\frac{b(c^2x^2-1)}{16\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^5x^5}\right)$
default	$c^4\left(-\frac{a}{4c^4x^4}-\frac{b\operatorname{arcsec}(cx)}{4c^4x^4}-\frac{3b\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}+\frac{3b(c^2x^2-1)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}+\frac{b(c^2x^2-1)}{16\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^5x^5}\right)$

input
```
int((a+b*arcsec(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/x^4-1/4*arcsec(c*x)*b/x^4-3/32*b*c^3*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)
/c^2/x^2)^(1/2)/x*arctan(1/(c^2*x^2-1)^(1/2))+3/32*b*c*(c^2*x^2-1)/((c^2*x
^2-1)/c^2/x^2)^(1/2)/x^3+1/16*b/c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/
x^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68

$$\int \frac{a+b\sec^{-1}(cx)}{x^5}\,dx=\frac{(3\,bc^4x^4-8\,b)\operatorname{arcsec}(cx)+(3\,bc^2x^2+2\,b)\sqrt{c^2x^2-1}-8\,a}{32\,x^4}$$

input
```
integrate((a+b*arcsec(c*x))/x^5,x, algorithm="fricas")
```

output
```
1/32*((3*b*c^4*x^4 - 8*b)*arcsec(c*x) + (3*b*c^2*x^2 + 2*b)*sqrt(c^2*x^2 -
 1) - 8*a)/x^4
```

Sympy [A] (verification not implemented)

Time = 4.97 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.53

$$\int \frac{a+b\sec^{-1}(cx)}{x^5}\,dx$$
$$= -\frac{a}{4x^4} - \frac{b\,\mathrm{asec}\,(cx)}{4x^4} + \frac{b\left(\begin{cases} \frac{3ic^5\,\mathrm{acosh}\,\left(\frac{1}{cx}\right)}{8} - \frac{3ic^4}{8x\sqrt{-1+\frac{1}{c^2x^2}}} + \frac{ic^2}{8x^3\sqrt{-1+\frac{1}{c^2x^2}}} + \frac{i}{4x^5\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for } \frac{1}{|c^2x^2|} > 1 \\ -\frac{3c^5\,\mathrm{asin}\,\left(\frac{1}{cx}\right)}{8} + \frac{3c^4}{8x\sqrt{1-\frac{1}{c^2x^2}}} - \frac{c^2}{8x^3\sqrt{1-\frac{1}{c^2x^2}}} - \frac{1}{4x^5\sqrt{1-\frac{1}{c^2x^2}}} & \text{otherwise} \end{cases}\right)}{4c}$$

input
```
integrate((a+b*asec(c*x))/x**5,x)
```

output
```
-a/(4*x**4) - b*asec(c*x)/(4*x**4) + b*Piecewise((3*I*c**5*acosh(1/(c*x))/
8 - 3*I*c**4/(8*x*sqrt(-1 + 1/(c**2*x**2))) + I*c**2/(8*x**3*sqrt(-1 + 1/(
c**2*x**2))) + I/(4*x**5*sqrt(-1 + 1/(c**2*x**2))), 1/Abs(c**2*x**2) > 1),
 (-3*c**5*asin(1/(c*x))/8 + 3*c**4/(8*x*sqrt(1 - 1/(c**2*x**2))) - c**2/(8
*x**3*sqrt(1 - 1/(c**2*x**2))) - 1/(4*x**5*sqrt(1 - 1/(c**2*x**2))), True)
)/(4*c)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.64

$$\int \frac{a+b\sec^{-1}(cx)}{x^5}\,dx$$
$$= \frac{1}{32}\,b\left(\frac{3\,c^5\arctan\left(cx\sqrt{-\frac{1}{c^2x^2}+1}\right) + \frac{3\,c^8x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+5\,c^6x\sqrt{-\frac{1}{c^2x^2}+1}}{c^4x^4\left(\frac{1}{c^2x^2}-1\right)^2-2\,c^2x^2\left(\frac{1}{c^2x^2}-1\right)+1}}{c} - \frac{8\,\mathrm{arcsec}\,(cx)}{x^4}\right) - \frac{a}{4\,x^4}$$

input
```
integrate((a+b*arcsec(c*x))/x^5,x, algorithm="maxima")
```

output
```
1/32*b*((3*c^5*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)) + (3*c^8*x^3*(-1/(c^2*x^
2) + 1)^(3/2) + 5*c^6*x*sqrt(-1/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) - 1)
^2 - 2*c^2*x^2*(1/(c^2*x^2) - 1) + 1))/c - 8*arcsec(c*x)/x^4) - 1/4*a/x^4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09

$$\int \frac{a+b\sec^{-1}(cx)}{x^5}\,dx$$
$$=\frac{1}{32}\left(3\,bc^3\arccos\left(\frac{1}{cx}\right)+\frac{3\,bc^2\sqrt{-\frac{1}{c^2x^2}+1}}{x}+\frac{2\,b\sqrt{-\frac{1}{c^2x^2}+1}}{x^3}-\frac{8\,b\arccos\left(\frac{1}{cx}\right)}{cx^4}-\frac{8\,a}{cx^4}\right)c$$

input
```
integrate((a+b*arcsec(c*x))/x^5,x, algorithm="giac")
```

output
```
1/32*(3*b*c^3*arccos(1/(c*x)) + 3*b*c^2*sqrt(-1/(c^2*x^2) + 1)/x + 2*b*sqr
t(-1/(c^2*x^2) + 1)/x^3 - 8*b*arccos(1/(c*x))/(c*x^4) - 8*a/(c*x^4))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^5}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^5}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/x^5,x)
```

output
```
int((a + b*acos(1/(c*x)))/x^5, x)
```

Reduce [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^5}\,dx = \frac{4\left(\int \frac{asec(cx)}{x^5}dx\right)b\,x^4-a}{4x^4}$$

input
```
int((a+b*asec(c*x))/x^5,x)
```

output
```
(4*int(asec(c*x)/x**5,x)*b*x**4 - a)/(4*x**4)
```

3.13 $\int \frac{a+b\sec^{-1}(cx)}{x^6}\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 82

$$\int \frac{a+b\sec^{-1}(cx)}{x^6}\,dx = \frac{1}{5}bc^5\sqrt{1-\frac{1}{c^2x^2}}-\frac{2}{15}bc^5\left(1-\frac{1}{c^2x^2}\right)^{3/2}+\frac{1}{25}bc^5\left(1-\frac{1}{c^2x^2}\right)^{5/2}-\frac{a+b\sec^{-1}(cx)}{5x^5}$$

output
```
1/5*b*c^5*(1-1/c^2/x^2)^(1/2)-2/15*b*c^5*(1-1/c^2/x^2)^(3/2)+1/25*b*c^5*(1
-1/c^2/x^2)^(5/2)-1/5*(a+b*arcsec(c*x))/x^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$\int \frac{a+b\sec^{-1}(cx)}{x^6}\,dx = -\frac{a}{5x^5}+b\left(\frac{8c^5}{75}+\frac{c}{25x^4}+\frac{4c^3}{75x^2}\right)\sqrt{\frac{-1+c^2x^2}{c^2x^2}}-\frac{b\sec^{-1}(cx)}{5x^5}$$

input
```
Integrate[(a + b*ArcSec[c*x])/x^6,x]
```

output
```
-1/5*a/x^5 + b*((8*c^5)/75 + c/(25*x^4) + (4*c^3)/(75*x^2))*Sqrt[(-1 + c^2
*x^2)/(c^2*x^2)] - (b*ArcSec[c*x])/(5*x^5)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5743, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x^6}dx$$

$$\Big\downarrow \text{5743}$$

$$\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^7}dx}{5c}-\frac{a+b\sec^{-1}(cx)}{5x^5}$$

$$\Big\downarrow \text{798}$$

$$-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^4}d\frac{1}{x^2}}{10c}-\frac{a+b\sec^{-1}(cx)}{5x^5}$$

$$\Big\downarrow \text{53}$$

$$-\frac{b\int \left(\left(1-\frac{1}{c^2x^2}\right)^{3/2}c^4-2\sqrt{1-\frac{1}{c^2x^2}}c^4+\frac{c^4}{\sqrt{1-\frac{1}{c^2x^2}}}\right)d\frac{1}{x^2}}{10c}-\frac{a+b\sec^{-1}(cx)}{5x^5}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a+b\sec^{-1}(cx)}{5x^5}-\frac{b\left(-\frac{2}{5}c^6\left(1-\frac{1}{c^2x^2}\right)^{5/2}+\frac{4}{3}c^6\left(1-\frac{1}{c^2x^2}\right)^{3/2}-2c^6\sqrt{1-\frac{1}{c^2x^2}}\right)}{10c}$$

input
```
Int[(a + b*ArcSec[c*x])/x^6,x]
```

output
```
-1/10*(b*(-2*c^6*Sqrt[1 - 1/(c^2*x^2)] + (4*c^6*(1 - 1/(c^2*x^2))^(3/2))/3
 - (2*c^6*(1 - 1/(c^2*x^2))^(5/2))/5))/c - (a + b*ArcSec[c*x])/(5*x^5)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

method	result	size
parts	$-\frac{a}{5x^5}+b\,c^5\left(-\frac{\operatorname{arcsec}(cx)}{5c^5x^5}+\frac{(c^2x^2-1)\,(8c^4x^4+4c^2x^2+3)}{75\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)$	79
derivativedivides	$c^5\left(-\frac{a}{5c^5x^5}+b\left(-\frac{\operatorname{arcsec}(cx)}{5c^5x^5}+\frac{(c^2x^2-1)\,(8c^4x^4+4c^2x^2+3)}{75\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)\right)$	83
default	$c^5\left(-\frac{a}{5c^5x^5}+b\left(-\frac{\operatorname{arcsec}(cx)}{5c^5x^5}+\frac{(c^2x^2-1)\,(8c^4x^4+4c^2x^2+3)}{75\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)\right)$	83

input
```
int((a+b*arcsec(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a/x^5+b*c^5*(-1/5/c^5/x^5*arcsec(c*x)+1/75*(c^2*x^2-1)*(8*c^4*x^4+4*c
^2*x^2+3)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^6/x^6)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.62

$$\int \frac{a+b\sec^{-1}(cx)}{x^6}\,dx = -\frac{15\,b\,\mathrm{arcsec}\,(cx)-(8\,bc^4x^4+4\,bc^2x^2+3\,b)\sqrt{c^2x^2-1}+15\,a}{75\,x^5}$$

input
```
integrate((a+b*arcsec(c*x))/x^6,x, algorithm="fricas")
```

output
```
-1/75*(15*b*arcsec(c*x) - (8*b*c^4*x^4 + 4*b*c^2*x^2 + 3*b)*sqrt(c^2*x^2 -
 1) + 15*a)/x^5
```

Sympy [A] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.90

$$\int \frac{a+b\sec^{-1}(cx)}{x^6}\,dx = -\frac{a}{5x^5}-\frac{b\,\mathrm{asec}\,(cx)}{5x^5}+\frac{b\left(\begin{cases}\frac{8c^5\sqrt{c^2x^2-1}}{15x}+\frac{4c^3\sqrt{c^2x^2-1}}{15x^3}+\frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for } |c^2x^2|>1\\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x}+\frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3}+\frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise}\end{cases}\right)}{5c}$$

input
```
integrate((a+b*asec(c*x))/x**6,x)
```

output
```
-a/(5*x**5) - b*asec(c*x)/(5*x**5) + b*Piecewise((8*c**5*sqrt(c**2*x**2 -
1)/(15*x) + 4*c**3*sqrt(c**2*x**2 - 1)/(15*x**3) + c*sqrt(c**2*x**2 - 1)/(
5*x**5), Abs(c**2*x**2) > 1), (8*I*c**5*sqrt(-c**2*x**2 + 1)/(15*x) + 4*I*
c**3*sqrt(-c**2*x**2 + 1)/(15*x**3) + I*c*sqrt(-c**2*x**2 + 1)/(5*x**5), T
rue))/(5*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

$$\int \frac{a+b\sec^{-1}(cx)}{x^6}\,dx$$
$$=\frac{1}{75}\,b\left(\frac{3\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{15\,\operatorname{arcsec}\left(cx\right)}{x^5}\right)$$
$$-\frac{a}{5\,x^5}$$

input
```
integrate((a+b*arcsec(c*x))/x^6,x, algorithm="maxima")
```

output
```
1/75*b*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(-1/(c^2*x^2) + 1)^(3/2)
+ 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c - 15*arcsec(c*x)/x^5) - 1/5*a/x^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.06

$$\int \frac{a+b\sec^{-1}(cx)}{x^6}\,dx$$
$$=\frac{1}{75}\left(8\,bc^4\sqrt{-\frac{1}{c^2x^2}+1}+\frac{4\,bc^2\sqrt{-\frac{1}{c^2x^2}+1}}{x^2}+\frac{3\,b\sqrt{-\frac{1}{c^2x^2}+1}}{x^4}-\frac{15\,b\arccos\left(\frac{1}{cx}\right)}{cx^5}-\frac{15\,a}{cx^5}\right)c$$

input
```
integrate((a+b*arcsec(c*x))/x^6,x, algorithm="giac")
```

output
```
1/75*(8*b*c^4*sqrt(-1/(c^2*x^2) + 1) + 4*b*c^2*sqrt(-1/(c^2*x^2) + 1)/x^2
+ 3*b*sqrt(-1/(c^2*x^2) + 1)/x^4 - 15*b*arccos(1/(c*x))/(c*x^5) - 15*a/(c*
x^5))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^6}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^6}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/x^6,x)
```

output
```
int((a + b*acos(1/(c*x)))/x^6, x)
```

Reduce [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^6}\,dx=\frac{5\left(\int \frac{asec(cx)}{x^6}dx\right)b\,x^5-a}{5x^5}$$

input
```
int((a+b*asec(c*x))/x^6,x)
```

output
```
(5*int(asec(c*x)/x**6,x)*b*x**5 - a)/(5*x**5)
```

3.14 $\int \frac{a+b\sec^{-1}(cx)}{x^7}\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 101

$$\int \frac{a+b\sec^{-1}(cx)}{x^7}\,dx = \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{36x^5}+\frac{5bc^3\sqrt{1-\frac{1}{c^2x^2}}}{144x^3}+\frac{5bc^5\sqrt{1-\frac{1}{c^2x^2}}}{96x}-\frac{5}{96}bc^6\csc^{-1}(cx)-\frac{a+b\sec^{-1}(cx)}{6x^6}$$

output
```
1/36*b*c*(1-1/c^2/x^2)^(1/2)/x^5+5/144*b*c^3*(1-1/c^2/x^2)^(1/2)/x^3+5/96*
b*c^5*(1-1/c^2/x^2)^(1/2)/x-5/96*b*c^6*arccsc(c*x)-1/6*(a+b*arcsec(c*x))/x
^6
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

$$\int \frac{a+b\sec^{-1}(cx)}{x^7}\,dx = -\frac{a}{6x^6}+b\left(\frac{c}{36x^5}+\frac{5c^3}{144x^3}+\frac{5c^5}{96x}\right)\sqrt{\frac{-1+c^2x^2}{c^2x^2}}-\frac{b\sec^{-1}(cx)}{6x^6}-\frac{5}{96}bc^6\arcsin\left(\frac{1}{cx}\right)$$

input
```
Integrate[(a + b*ArcSec[c*x])/x^7,x]
```

output
```
-1/6*a/x^6 + b*(c/(36*x^5) + (5*c^3)/(144*x^3) + (5*c^5)/(96*x))*Sqrt[(-1
+ c^2*x^2)/(c^2*x^2)] - (b*ArcSec[c*x])/(6*x^6) - (5*b*c^6*ArcSin[1/(c*x)]
)/96
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.27, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5743, 858, 262, 262, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x^7}\,dx$$

$$\Big\downarrow \text{5743}$$

$$\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^8}dx}{6c}-\frac{a+b\sec^{-1}(cx)}{6x^6}$$

$$\Big\downarrow \text{858}$$

$$-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^6}d\frac{1}{x}}{6c}-\frac{a+b\sec^{-1}(cx)}{6x^6}$$

$$\Big\downarrow \text{262}$$

$$-\frac{b\left(\frac{5}{6}c^2\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^4}d\frac{1}{x}-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{6x^5}\right)}{6c}-\frac{a+b\sec^{-1}(cx)}{6x^6}$$

$$\Big\downarrow \text{262}$$

$$-\frac{b\left(\frac{5}{6}c^2\left(\frac{3}{4}c^2\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2}d\frac{1}{x}-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{4x^3}\right)-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{6x^5}\right)}{6c}-\frac{a+b\sec^{-1}(cx)}{6x^6}$$

$$\Big\downarrow \text{262}$$

$$-\frac{b\left(\frac{5}{6}c^2\left(\frac{3}{4}c^2\left(\frac{1}{2}c^2\int\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{2x}\right)-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{4x^3}\right)-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{6x^5}\right)}{6c}-\frac{a+b\sec^{-1}(cx)}{6x^6}$$

$$\Big\downarrow \text{223}$$

$$-\frac{a+b\sec^{-1}(cx)}{6x^6}-\frac{b\left(\frac{5}{6}c^2\left(\frac{3}{4}c^2\left(\frac{1}{2}c^3\arcsin\left(\frac{1}{cx}\right)-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{2x}\right)-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{4x^3}\right)-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{6x^5}\right)}{6c}$$

input
```
Int[(a + b*ArcSec[c*x])/x^7,x]
```

output
```
-1/6*(a + b*ArcSec[c*x])/x^6 - (b*(-1/6*(c^2*Sqrt[1 - 1/(c^2*x^2)])/x^5 +
(5*c^2*(-1/4*(c^2*Sqrt[1 - 1/(c^2*x^2)])/x^3 + (3*c^2*(-1/2*(c^2*Sqrt[1 -
1/(c^2*x^2)])/x + (c^3*ArcSin[1/(c*x)])/2))/4))/6))/(6*c)
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(85) = 170$.

Time = 0.18 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.72

method	result
parts	$-\frac{a}{6x^6} - \frac{\operatorname{arcsec}(cx)b}{6x^6} - \frac{5b\,c^5\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x} + \frac{5b\,c^3\left(c^2x^2-1\right)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^3} + \frac{5bc\left(c^2x^2-1\right)}{144\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^5} + \frac{b(c^2x^2}{36c\sqrt{\frac{c^2x}{c^2}}}$
derivativedivides	$c^6\left(-\frac{a}{6c^6x^6} - \frac{b\operatorname{arcsec}(cx)}{6c^6x^6} - \frac{5b\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx} + \frac{5b\left(c^2x^2-1\right)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3} + \frac{5b\left(c^2x^2-1\right)}{144\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^5x^5} +\right.$
default	$c^6\left(-\frac{a}{6c^6x^6} - \frac{b\operatorname{arcsec}(cx)}{6c^6x^6} - \frac{5b\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx} + \frac{5b\left(c^2x^2-1\right)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3} + \frac{5b\left(c^2x^2-1\right)}{144\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^5x^5} +\right.$

input
```
int((a+b*arcsec(c*x))/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a/x^6-1/6*arcsec(c*x)*b/x^6-5/96*b*c^5*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)
/c^2/x^2)^(1/2)/x*arctan(1/(c^2*x^2-1)^(1/2))+5/96*b*c^3*(c^2*x^2-1)/((c^2
*x^2-1)/c^2/x^2)^(1/2)/x^3+5/144*b*c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/
2)/x^5+1/36*b/c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.61

$$\int \frac{a + b\sec^{-1}(cx)}{x^7}\,dx = \frac{3\,(5\,bc^6x^6 - 16\,b)\operatorname{arcsec}(cx) + (15\,bc^4x^4 + 10\,bc^2x^2 + 8\,b)\sqrt{c^2x^2-1} - 48\,a}{288\,x^6}$$

input
```
integrate((a+b*arcsec(c*x))/x^7,x, algorithm="fricas")
```

output
```
1/288*(3*(5*b*c^6*x^6 - 16*b)*arcsec(c*x) + (15*b*c^4*x^4 + 10*b*c^2*x^2 +
 8*b)*sqrt(c^2*x^2 - 1) - 48*a)/x^6
```

Sympy [A] (verification not implemented)

Time = 9.99 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.39

$$\int \frac{a+b\sec^{-1}(cx)}{x^7}\,dx = -\frac{a}{6x^6} - \frac{b\operatorname{asec}(cx)}{6x^6} + \frac{b\left(\begin{cases} \frac{5ic^7\operatorname{acosh}\left(\frac{1}{cx}\right)}{16} - \frac{5ic^6}{16x\sqrt{-1+\frac{1}{c^2x^2}}} + \frac{5ic^4}{48x^3\sqrt{-1+\frac{1}{c^2x^2}}} + \frac{ic^2}{24x^5\sqrt{-1+\frac{1}{c^2x^2}}} + \frac{i}{6x^7\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for}\ \frac{1}{|c^2x^2|} > 1 \\ -\frac{5c^7\operatorname{asin}\left(\frac{1}{cx}\right)}{16} + \frac{5c^6}{16x\sqrt{1-\frac{1}{c^2x^2}}} - \frac{5c^4}{48x^3\sqrt{1-\frac{1}{c^2x^2}}} - \frac{c^2}{24x^5\sqrt{1-\frac{1}{c^2x^2}}} - \frac{1}{6x^7\sqrt{1-\frac{1}{c^2x^2}}} & \text{otherwise} \end{cases}\right)}{6c}$$

input
```
integrate((a+b*asec(c*x))/x**7,x)
```

output
```
-a/(6*x**6) - b*asec(c*x)/(6*x**6) + b*Piecewise((5*I*c**7*acosh(1/(c*x))/
16 - 5*I*c**6/(16*x*sqrt(-1 + 1/(c**2*x**2))) + 5*I*c**4/(48*x**3*sqrt(-1
+ 1/(c**2*x**2))) + I*c**2/(24*x**5*sqrt(-1 + 1/(c**2*x**2))) + I/(6*x**7*
sqrt(-1 + 1/(c**2*x**2))), 1/Abs(c**2*x**2) > 1), (-5*c**7*asin(1/(c*x))/1
6 + 5*c**6/(16*x*sqrt(1 - 1/(c**2*x**2))) - 5*c**4/(48*x**3*sqrt(1 - 1/(c*
*2*x**2))) - c**2/(24*x**5*sqrt(1 - 1/(c**2*x**2))) - 1/(6*x**7*sqrt(1 - 1
/(c**2*x**2))), True))/(6*c)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.63

$$\int \frac{a+b\sec^{-1}(cx)}{x^7}\,dx$$

$$= \frac{1}{288}b\left(\frac{15\,c^7\arctan\left(cx\sqrt{-\frac{1}{c^2x^2}+1}\right)-\frac{15\,c^{12}x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+40\,c^{10}x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\,c^8x\sqrt{-\frac{1}{c^2x^2}+1}}{c^6x^6\left(\frac{1}{c^2x^2}-1\right)^3-3\,c^4x^4\left(\frac{1}{c^2x^2}-1\right)^2+3\,c^2x^2\left(\frac{1}{c^2x^2}-1\right)-1}}{c}-\frac{48\,\mathrm{arcsec}\,(}{x^6}\right.$$

$$-\frac{a}{6\,x^6}$$

input
```
integrate((a+b*arcsec(c*x))/x^7,x, algorithm="maxima")
```

output
```
1/288*b*((15*c^7*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)) - (15*c^12*x^5*(-1/(c^
2*x^2) + 1)^(5/2) + 40*c^10*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 33*c^8*x*sqrt(-
1/(c^2*x^2) + 1))/(c^6*x^6*(1/(c^2*x^2) - 1)^3 - 3*c^4*x^4*(1/(c^2*x^2) -
1)^2 + 3*c^2*x^2*(1/(c^2*x^2) - 1) - 1))/c - 48*arcsec(c*x)/x^6) - 1/6*a/x
^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03

$$\int \frac{a+b\sec^{-1}(cx)}{x^7}\,dx$$

$$= \frac{1}{288}\left(15\,bc^5\arccos\left(\frac{1}{cx}\right)+\frac{15\,bc^4\sqrt{-\frac{1}{c^2x^2}+1}}{x}+\frac{10\,bc^2\sqrt{-\frac{1}{c^2x^2}+1}}{x^3}+\frac{8\,b\sqrt{-\frac{1}{c^2x^2}+1}}{x^5}-\frac{48\,b\arccos\left(\frac{1}{cx}\right.}{cx^6}\right.$$

input
```
integrate((a+b*arcsec(c*x))/x^7,x, algorithm="giac")
```

output
```
1/288*(15*b*c^5*arccos(1/(c*x)) + 15*b*c^4*sqrt(-1/(c^2*x^2) + 1)/x + 10*b
*c^2*sqrt(-1/(c^2*x^2) + 1)/x^3 + 8*b*sqrt(-1/(c^2*x^2) + 1)/x^5 - 48*b*ar
ccos(1/(c*x))/(c*x^6) - 48*a/(c*x^6))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^7}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^7}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/x^7,x)
```

output
```
int((a + b*acos(1/(c*x)))/x^7, x)
```

Reduce [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^7}\,dx=\frac{6\left(\int \frac{asec(cx)}{x^7}dx\right)b\,x^6-a}{6x^6}$$

input
```
int((a+b*asec(c*x))/x^7,x)
```

output
```
(6*int(asec(c*x)/x**7,x)*b*x**6 - a)/(6*x**6)
```

3.15 $\int x^3(a+b\sec^{-1}(cx))^2\,dx$

Optimal result 182
Mathematica [A] (verified) 182
Rubi [A] (verified) 183
Maple [A] (verified) 186
Fricas [A] (verification not implemented) 186
Sympy [F] 187
Maxima [A] (verification not implemented) 187
Giac [B] (verification not implemented) 188
Mupad [F(-1)] 189
Reduce [F] 189

Optimal result

Integrand size = 14, antiderivative size = 107

$$\int x^3\left(a+b\sec^{-1}(cx)\right)^2\,dx=\frac{b^2x^2}{12c^2}-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x(a+b\sec^{-1}(cx))}{3c^3}-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^3(a+b\sec^{-1}(cx))}{6c}+\frac{1}{4}x^4\left(a+b\sec^{-1}(cx)\right)^2+\frac{b^2\log(x)}{3c^4}$$

output
```
1/12*b^2*x^2/c^2-1/3*b*(1-1/c^2/x^2)^(1/2)*x*(a+b*arcsec(c*x))/c^3-1/6*b*(
1-1/c^2/x^2)^(1/2)*x^3*(a+b*arcsec(c*x))/c+1/4*x^4*(a+b*arcsec(c*x))^2+1/3
*b^2*ln(x)/c^4
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.16

$$\int x^3\left(a+b\sec^{-1}(cx)\right)^2\,dx$$
$$=\frac{cx\left(b^2cx+3a^2c^3x^3-2ab\sqrt{1-\frac{1}{c^2x^2}}(2+c^2x^2)\right)-2bcx\left(-3ac^3x^3+b\sqrt{1-\frac{1}{c^2x^2}}(2+c^2x^2)\right)\sec^{-1}(cx)+}{12c^4}$$

input
```
Integrate[x^3*(a + b*ArcSec[c*x])^2,x]
```

output
```
(c*x*(b^2*c*x + 3*a^2*c^3*x^3 - 2*a*b*Sqrt[1 - 1/(c^2*x^2)]*(2 + c^2*x^2))
 - 2*b*c*x*(-3*a*c^3*x^3 + b*Sqrt[1 - 1/(c^2*x^2)]*(2 + c^2*x^2))*ArcSec[c
*x] + 3*b^2*c^4*x^4*ArcSec[c*x]^2 + 4*b^2*Log[x])/(12*c^4)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {5745, 4909, 3042, 4673, 3042, 4672, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b \sec^{-1}(cx)\right)^2 \, dx$$

$$\Big\downarrow \text{ 5745}$$

$$\frac{\int c^5 \sqrt{1-\frac{1}{c^2x^2}} x^5 \left(a+b \sec^{-1}(cx)\right)^2 d\sec^{-1}(cx)}{c^4}$$

$$\Big\downarrow \text{ 4909}$$

$$\frac{\frac{1}{4} c^4 x^4 \left(a+b \sec^{-1}(cx)\right)^2-\frac{1}{2} b \int c^4 x^4 \left(a+b \sec^{-1}(cx)\right) d\sec^{-1}(cx)}{c^4}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{1}{4} c^4 x^4 \left(a+b \sec^{-1}(cx)\right)^2-\frac{1}{2} b \int \left(a+b \sec^{-1}(cx)\right) \csc \left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^4 d\sec^{-1}(cx)}{c^4}$$

$$\Big\downarrow \text{ 4673}$$

$$\frac{\frac{1}{4} c^4 x^4 \left(a+b \sec^{-1}(cx)\right)^2-\frac{1}{2} b \left(\frac{2}{3} \int c^2 x^2 \left(a+b \sec^{-1}(cx)\right) d\sec^{-1}(cx)+\frac{1}{3} c^3 x^3 \sqrt{1-\frac{1}{c^2x^2}} \left(a+b \sec^{-1}(cx)\right)-\frac{1}{6} b c^2}{c^4}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^2-\frac{1}{2}b\left(\frac{2}{3}\int\left(a+b\sec^{-1}(cx)\right)\csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^2d\sec^{-1}(cx)+\frac{1}{3}c^3x^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\,\mathrm{se}\right.}{c^4}$$

$\downarrow$ 4672

$$\frac{\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^2-\frac{1}{2}b\left(\frac{2}{3}\left(b\int-c\sqrt{1-\frac{1}{c^2x^2}}xd\sec^{-1}(cx)+cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)\right)+\frac{1}{3}c^3x^3\sqrt{1-\right.}{c^4}$$

$\downarrow$ 25

$$\frac{\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^2-\frac{1}{2}b\left(\frac{2}{3}\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)-b\int c\sqrt{1-\frac{1}{c^2x^2}}xd\sec^{-1}(cx)\right)+\frac{1}{3}c^3x^3\sqrt{1-\frac{}{c}\right.}{c^4}$$

$\downarrow$ 3042

$$\frac{\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^2-\frac{1}{2}b\left(\frac{2}{3}\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)-b\int\tan\left(\sec^{-1}(cx)\right)d\sec^{-1}(cx)\right)+\frac{1}{3}c^3x^3\sqrt{1\right.}{c^4}$$

$\downarrow$ 3956

$$\frac{\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^2-\frac{1}{2}b\left(\frac{2}{3}\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)+b\log\left(\frac{1}{cx}\right)\right)+\frac{1}{3}c^3x^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx\right.\right.}{c^4}$$

input
```
Int[x^3*(a + b*ArcSec[c*x])^2,x]
```

output
```
((c^4*x^4*(a + b*ArcSec[c*x])^2)/4 - (b*(-1/6*(b*c^2*x^2) + (c^3*Sqrt[1 -
1/(c^2*x^2)]*x^3*(a + b*ArcSec[c*x]))/3 + (2*(c*Sqrt[1 - 1/(c^2*x^2)]*x*(a
 + b*ArcSec[c*x]) + b*Log[1/(c*x)]))/3))/2)/c^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.59

method	result
parts	$\frac{a^2x^4}{4}+\frac{b^2\left(\frac{\mathrm{arcsec}(cx)^2c^4x^4}{4}-\frac{\mathrm{arcsec}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}{6}+\frac{c^2x^2}{12}-\frac{\mathrm{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3}-\frac{\ln\left(\frac{1}{cx}\right)}{3}\right)}{c^4}+\frac{2ab\left(\frac{c^4x^4\,\mathrm{arcsec}}{4}\right.}{}$
derivativedivides	$\frac{\frac{a^2c^4x^4}{4}+b^2\left(\frac{\mathrm{arcsec}(cx)^2c^4x^4}{4}-\frac{\mathrm{arcsec}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}{6}+\frac{c^2x^2}{12}-\frac{\mathrm{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3}-\frac{\ln\left(\frac{1}{cx}\right)}{3}\right)+2ab\left(\frac{c^4x^4\,\mathrm{arcsec}(cx)}{4}\right.}{c^4}$
default	$\frac{\frac{a^2c^4x^4}{4}+b^2\left(\frac{\mathrm{arcsec}(cx)^2c^4x^4}{4}-\frac{\mathrm{arcsec}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}{6}+\frac{c^2x^2}{12}-\frac{\mathrm{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3}-\frac{\ln\left(\frac{1}{cx}\right)}{3}\right)+2ab\left(\frac{c^4x^4\,\mathrm{arcsec}(cx)}{4}\right.}{c^4}$

input
```
int(x^3*(a+b*arcsec(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x^4+b^2/c^4*(1/4*arcsec(c*x)^2*c^4*x^4-1/6*arcsec(c*x)*((c^2*x^2-1
)/c^2/x^2)^(1/2)*c^3*x^3+1/12*c^2*x^2-1/3*arcsec(c*x)*c*x*((c^2*x^2-1)/c^2
/x^2)^(1/2)-1/3*ln(1/c/x))+2*a*b/c^4*(1/4*c^4*x^4*arcsec(c*x)-1/12*(c^2*x^
2-1)*(c^2*x^2+2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.36

$$\int x^3\left(a+b\sec^{-1}(cx)\right)^2\,dx$$
$$=\frac{3\,b^2c^4x^4\,\mathrm{arcsec}\,(cx)^2+3\,a^2c^4x^4+12\,abc^4\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+b^2c^2x^2+4\,b^2\log\left(x\right)+6\,(abc^4x^4\ -}{12\,c^4}$$

input
```
integrate(x^3*(a+b*arcsec(c*x))^2,x, algorithm="fricas")
```

output
```
1/12*(3*b^2*c^4*x^4*arcsec(c*x)^2 + 3*a^2*c^4*x^4 + 12*a*b*c^4*arctan(-c*x
 + sqrt(c^2*x^2 - 1)) + b^2*c^2*x^2 + 4*b^2*log(x) + 6*(a*b*c^4*x^4 - a*b*
c^4)*arcsec(c*x) - 2*(a*b*c^2*x^2 + 2*a*b + (b^2*c^2*x^2 + 2*b^2)*arcsec(c
*x))*sqrt(c^2*x^2 - 1))/c^4
```

Sympy [F]

$$\int x^3\left(a+b\sec^{-1}(cx)\right)^2\,dx=\int x^3(a+b\,\mathrm{asec}\,(cx))^2\,dx$$

input
```
integrate(x**3*(a+b*asec(c*x))**2,x)
```

output
```
Integral(x**3*(a + b*asec(c*x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.52

$$\int x^3\left(a+b\sec^{-1}(cx)\right)^2\,dx=\frac{1}{4}\,b^2x^4\,\mathrm{arcsec}\,(cx)^2+\frac{1}{4}\,a^2x^4+\frac{1}{6}\left(3\,x^4\,\mathrm{arcsec}\,(cx)-\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)ab+\frac{\left((c^2x^2+2\,\log\left(x^2\right))\sqrt{cx+1}\sqrt{cx-1}-2\left(c^4x^4+c^2x^2-2\right)\arctan\left(\sqrt{cx+1}\sqrt{cx-1}\right)\right)b^2}{12\,\sqrt{cx+1}\sqrt{cx-1}c^4}$$

input
```
integrate(x^3*(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

output
```
1/4*b^2*x^4*arcsec(c*x)^2 + 1/4*a^2*x^4 + 1/6*(3*x^4*arcsec(c*x) - (c^2*x^
3*(-1/(c^2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*a*b + 1/12*(
(c^2*x^2 + 2*log(x^2))*sqrt(c*x + 1)*sqrt(c*x - 1) - 2*(c^4*x^4 + c^2*x^2
- 2)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)))*b^2/(sqrt(c*x + 1)*sqrt(c*x - 1)
*c^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6625 vs. $2(93) = 186$.

Time = 0.39 (sec) , antiderivative size = 6625, normalized size of antiderivative = 61.92

$$\int x^3 \left(a + b\sec^{-1}(cx)\right)^2 dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

output
```
1/12*(3*b^2*arccos(1/(c*x))^2/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)
^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3
/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) + 6*a*b*arccos
(1/(c*x))/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x
^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c
^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) - 12*b^2*(1/(c^2*x^2) - 1)*arccos(
1/(c*x))^2/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2
*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 +
 c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) - 4*b^2*log(2)/
(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2
/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2
*x^2) - 1)^4/(1/(c*x) + 1)^8) + 4*b^2*log(2/(c*x) + 2)/(c^5 + 4*c^5*(1/(c^
2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*
x) + 1)^8) - 4*b^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^5 + 4
*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x
) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) -
1)^4/(1/(c*x) + 1)^8) - 4*b^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1
))/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1
)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(...
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a + b \sec^{-1}(cx)\right)^2 \, dx = \int x^3 \left(a + b \operatorname{acos}\left(\frac{1}{c\,x}\right)\right)^2 \, dx$$

input
```
int(x^3*(a + b*acos(1/(c*x)))^2,x)
```

output
```
int(x^3*(a + b*acos(1/(c*x)))^2, x)
```

Reduce [F]

$$\int x^3 \left(a + b \sec^{-1}(cx)\right)^2 \, dx = 2\left(\int asec(cx)\, x^3 dx\right) ab + \left(\int asec(cx)^2\, x^3 dx\right) b^2 + \frac{a^2 x^4}{4}$$

input
```
int(x^3*(a+b*asec(c*x))^2,x)
```

output
```
(8*int(asec(c*x)*x**3,x)*a*b + 4*int(asec(c*x)**2*x**3,x)*b**2 + a**2*x**4
)/4
```

3.16 $\int x^2(a+b\sec^{-1}(cx))^2\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 147

$$\int x^2(a+b\sec^{-1}(cx))^2\,dx = \frac{b^2x}{3c^2}-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2(a+b\sec^{-1}(cx))}{3c}+\frac{1}{3}x^3(a+b\sec^{-1}(cx))^2+\frac{2ib(a+b\sec^{-1}(cx))\arctan\left(e^{i\sec^{-1}(cx)}\right)}{3c^3}-\frac{ib^2\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)}{3c^3}+\frac{ib^2\,\text{PolyLog}\left(2,ie^{i\sec^{-1}(cx)}\right)}{3c^3}$$

output
```
1/3*b^2*x/c^2-1/3*b*(1-1/c^2/x^2)^(1/2)*x^2*(a+b*arcsec(c*x))/c+1/3*x^3*(a
+b*arcsec(c*x))^2+2/3*I*b*(a+b*arcsec(c*x))*arctan(1/c/x+I*(1-1/c^2/x^2)^(
1/2))/c^3-1/3*I*b^2*polylog(2,-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/c^3+1/3*I*
b^2*polylog(2,I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/c^3
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.54

$$\int x^2\left(a+b\sec^{-1}(cx)\right)^2\,dx$$
$$=\frac{1}{3}\left(a^2x^3+\frac{ab\left(2x^4\sec^{-1}(cx)+\frac{cx-c^3x^3+\sqrt{-1+c^2x^2}\log\left(-cx+\sqrt{-1+c^2x^2}\right)}{c^4\sqrt{1-\frac{1}{c^2x^2}}}\right)}{x}\right.$$
$$+\frac{b^2\left(cx-c^2\sqrt{1-\frac{1}{c^2x^2}}x^2\sec^{-1}(cx)+c^3x^3\sec^{-1}(cx)^2-\sec^{-1}(cx)\log\left(1-ie^{i\sec^{-1}(cx)}\right)+\sec^{-1}(cx)\log\right.}{c^3}$$

input
```
Integrate[x^2*(a + b*ArcSec[c*x])^2,x]
```

output
```
(a^2*x^3 + (a*b*(2*x^4*ArcSec[c*x] + (c*x - c^3*x^3 + Sqrt[-1 + c^2*x^2]*L
og[-(c*x) + Sqrt[-1 + c^2*x^2]])/(c^4*Sqrt[1 - 1/(c^2*x^2)])))/x + (b^2*(c
*x - c^2*Sqrt[1 - 1/(c^2*x^2)]*x^2*ArcSec[c*x] + c^3*x^3*ArcSec[c*x]^2 - A
rcSec[c*x]*Log[1 - I*E^(I*ArcSec[c*x])] + ArcSec[c*x]*Log[1 + I*E^(I*ArcSe
c[c*x])] - I*PolyLog[2, (-I)*E^(I*ArcSec[c*x])] + I*PolyLog[2, I*E^(I*ArcS
ec[c*x])]))/c^3)/3
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5745, 4909, 3042, 4673, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\sec^{-1}(cx)\right)^2\,dx$$
$$\Big\downarrow \text{5745}$$

$$\frac{\int c^4\sqrt{1-\frac{1}{c^2x^2}}x^4\left(a+b\sec^{-1}(cx)\right)^2d\sec^{-1}(cx)}{c^3}$$

$\big\downarrow$ 4909

$$\frac{\frac{1}{3}c^3x^3\left(a+b\sec^{-1}(cx)\right)^2-\frac{2}{3}b\int c^3x^3\left(a+b\sec^{-1}(cx)\right)d\sec^{-1}(cx)}{c^3}$$

$\big\downarrow$ 3042

$$\frac{\frac{1}{3}c^3x^3\left(a+b\sec^{-1}(cx)\right)^2-\frac{2}{3}b\int\left(a+b\sec^{-1}(cx)\right)\csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^3d\sec^{-1}(cx)}{c^3}$$

$\big\downarrow$ 4673

$$\frac{\frac{1}{3}c^3x^3\left(a+b\sec^{-1}(cx)\right)^2-\frac{2}{3}b\left(\frac{1}{2}\int cx\left(a+b\sec^{-1}(cx)\right)d\sec^{-1}(cx)+\frac{1}{2}c^2x^2\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)-\frac{bcx}{2}\right)}{c^3}$$

$\big\downarrow$ 3042

$$\frac{\frac{1}{3}c^3x^3\left(a+b\sec^{-1}(cx)\right)^2-\frac{2}{3}b\left(\frac{1}{2}\int\left(a+b\sec^{-1}(cx)\right)\csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)d\sec^{-1}(cx)+\frac{1}{2}c^2x^2\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec\right.\right.}{c^3}$$

$\big\downarrow$ 4669

$$\frac{\frac{1}{3}c^3x^3\left(a+b\sec^{-1}(cx)\right)^2-\frac{2}{3}b\left(\frac{1}{2}\left(-b\int\log\left(1-ie^{i\sec^{-1}(cx)}\right)d\sec^{-1}(cx)+b\int\log\left(1+ie^{i\sec^{-1}(cx)}\right)d\sec^{-1}(cx)-\right.\right.}{c^3}$$

$\big\downarrow$ 2715

$$\frac{\frac{1}{3}c^3x^3\left(a+b\sec^{-1}(cx)\right)^2-\frac{2}{3}b\left(\frac{1}{2}\left(ib\int e^{-i\sec^{-1}(cx)}\log\left(1-ie^{i\sec^{-1}(cx)}\right)de^{i\sec^{-1}(cx)}-ib\int e^{-i\sec^{-1}(cx)}\log\left(1+ie\right.\right.\right.}{c^3}$$

$\big\downarrow$ 2838

$$\frac{\frac{1}{3}c^3x^3\left(a+b\sec^{-1}(cx)\right)^2-\frac{2}{3}b\left(\frac{1}{2}\left(-2i\arctan\left(e^{i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)+ib\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)-ib\right.\right.}{c^3}$$

input
```
Int[x^2*(a + b*ArcSec[c*x])^2,x]
```

output
```
((c^3*x^3*(a + b*ArcSec[c*x])^2)/3 - (2*b*(-1/2*(b*c*x) + (c^2*Sqrt[1 - 1/
(c^2*x^2)]*x^2*(a + b*ArcSec[c*x]))/2 + ((-2*I)*(a + b*ArcSec[c*x])*ArcTan
[E^(I*ArcSec[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcSec[c*x])] - I*b*PolyLog
[2, I*E^(I*ArcSec[c*x])])/2))/3)/c^3
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.94

method	result
parts	$\frac{a^2x^3}{3}+\frac{b^2\left(\frac{\left(c^2x^2\operatorname{arcsec}(cx)^2-\operatorname{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+1\right)cx}{3}+\frac{\operatorname{arcsec}(cx)\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{3}-\frac{\operatorname{arcsec}(cx)\ln\left(1-i\left(\frac{}{}\right.\right.}{3}\right.}{c^3}$
derivativedivides	$\frac{\frac{a^2c^3x^3}{3}+b^2\left(\frac{\left(c^2x^2\operatorname{arcsec}(cx)^2-\operatorname{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+1\right)cx}{3}+\frac{\operatorname{arcsec}(cx)\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{3}-\frac{\operatorname{arcsec}(cx)\ln\left(1-i\left(\frac{1}{c}\right.\right.}{3}\right.}{}$
default	$\frac{\frac{a^2c^3x^3}{3}+b^2\left(\frac{\left(c^2x^2\operatorname{arcsec}(cx)^2-\operatorname{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+1\right)cx}{3}+\frac{\operatorname{arcsec}(cx)\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{3}-\frac{\operatorname{arcsec}(cx)\ln\left(1-i\left(\frac{1}{c}\right.\right.}{3}\right.}{}$

input
```
int(x^2*(a+b*arcsec(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*x^3+b^2/c^3*(1/3*(c^2*x^2*arcsec(c*x)^2-arcsec(c*x)*c*x*((c^2*x^2-
1)/c^2/x^2)^(1/2)+1)*c*x+1/3*arcsec(c*x)*ln(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/
2)))-1/3*arcsec(c*x)*ln(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-1/3*I*dilog(1+I
*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))+1/3*I*dilog(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/
2))))+2*a*b/c^3*(1/3*c^3*x^3*arcsec(c*x)-1/6*(c^2*x^2-1)^(1/2)*(c*x*(c^2*x
^2-1)^(1/2)+ln(c*x+(c^2*x^2-1)^(1/2)))/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x)
```

Fricas [F]

$$\int x^2 \left(a+b\sec^{-1}(cx)\right)^2 \, dx=\int (b\operatorname{arcsec}(cx)+a)^2x^2 \, dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^2*arcsec(c*x)^2 + 2*a*b*x^2*arcsec(c*x) + a^2*x^2, x)
```

Sympy [F]

$$\int x^2 \left(a+b\sec^{-1}(cx)\right)^2 \, dx=\int x^2(a+b\operatorname{asec}(cx))^2 \, dx$$

input
```
integrate(x**2*(a+b*asec(c*x))**2,x)
```

output
```
Integral(x**2*(a + b*asec(c*x))**2, x)
```

Maxima [F]

$$\int x^2 \left(a+b\sec^{-1}(cx)\right)^2 \, dx=\int (b\operatorname{arcsec}(cx)+a)^2x^2 \, dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*x^3 + 1/6*(4*x^3*arcsec(c*x) - (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(
c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(
c^2*x^2) + 1) - 1)/c^2)/c)*a*b + 1/12*(4*x^3*arctan(sqrt(c*x + 1)*sqrt(c*x
 - 1))^2 - x^3*log(c^2*x^2)^2 - 2*c^2*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x +
 1)/c^5 + 3*log(c*x - 1)/c^5)*log(c)^2 + 36*c^2*integrate(1/3*x^4*log(c^2*
x^2)/(c^2*x^2 - 1), x)*log(c) - 72*c^2*integrate(1/3*x^4*log(x)/(c^2*x^2 -
 1), x)*log(c) + 36*c^2*integrate(1/3*x^4*log(c^2*x^2)*log(x)/(c^2*x^2 - 1
), x) - 36*c^2*integrate(1/3*x^4*log(x)^2/(c^2*x^2 - 1), x) + 12*c^2*integ
rate(1/3*x^4*log(c^2*x^2)/(c^2*x^2 - 1), x) + 6*(2*x/c^2 - log(c*x + 1)/c^
3 + log(c*x - 1)/c^3)*log(c)^2 - 36*integrate(1/3*x^2*log(c^2*x^2)/(c^2*x^
2 - 1), x)*log(c) + 72*integrate(1/3*x^2*log(x)/(c^2*x^2 - 1), x)*log(c) -
 24*integrate(1/3*sqrt(c*x + 1)*sqrt(c*x - 1)*x^2*arctan(sqrt(c*x + 1)*sqr
t(c*x - 1))/(c^2*x^2 - 1), x) - 36*integrate(1/3*x^2*log(c^2*x^2)*log(x)/(
c^2*x^2 - 1), x) + 36*integrate(1/3*x^2*log(x)^2/(c^2*x^2 - 1), x) - 12*in
tegrate(1/3*x^2*log(c^2*x^2)/(c^2*x^2 - 1), x))*b^2
```

Giac [F(-2)]

Exception generated.

$$\int x^2 \left(a+b\sec^{-1}(cx)\right)^2 \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+b\sec^{-1}(cx)\right)^2 \, dx = \int x^2 \left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^2 \,dx$$

input
```
int(x^2*(a + b*acos(1/(c*x)))^2,x)
```

output
```
int(x^2*(a + b*acos(1/(c*x)))^2, x)
```

Reduce [F]

$$\int x^2 \left(a+b \sec^{-1}(cx)\right)^2 \, dx = 2\left(\int asec(cx)\, x^2 dx\right) ab + \left(\int asec(cx)^2\, x^2 dx\right) b^2 + \frac{a^2 x^3}{3}$$

input
```
int(x^2*(a+b*asec(c*x))^2,x)
```

output
```
(6*int(asec(c*x)*x**2,x)*a*b + 3*int(asec(c*x)**2*x**2,x)*b**2 + a**2*x**3
)/3
```

3.17 $\int x(a+b\sec^{-1}(cx))^2\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 56

$$\int x\left(a+b\sec^{-1}(cx)\right)^2\,dx=-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x\left(a+b\sec^{-1}(cx)\right)}{c}+\frac{1}{2}x^2\left(a+b\sec^{-1}(cx)\right)^2+\frac{b^2\log(x)}{c^2}$$

output
```
-b*(1-1/c^2/x^2)^(1/2)*x*(a+b*arcsec(c*x))/c+1/2*x^2*(a+b*arcsec(c*x))^2+b
^2*ln(x)/c^2
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.61

$$\int x\left(a+b\sec^{-1}(cx)\right)^2\,dx=\frac{acx\left(-2b\sqrt{1-\frac{1}{c^2x^2}}+acx\right)+2bcx\left(-b\sqrt{1-\frac{1}{c^2x^2}}+acx\right)\sec^{-1}(cx)+b^2c^2x^2\sec^{-1}(cx)^2+2b^2\log(cx)}{2c^2}$$

input
```
Integrate[x*(a + b*ArcSec[c*x])^2,x]
```

output
```
(a*c*x*(-2*b*Sqrt[1 - 1/(c^2*x^2)] + a*c*x) + 2*b*c*x*(-(b*Sqrt[1 - 1/(c^2
*x^2)]) + a*c*x)*ArcSec[c*x] + b^2*c^2*x^2*ArcSec[c*x]^2 + 2*b^2*Log[c*x])
/(2*c^2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5745, 4909, 3042, 4672, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sec^{-1}(cx)\right)^2\,dx$$

$$\Big\downarrow \text{5745}$$

$$\frac{\int c^3\sqrt{1-\frac{1}{c^2x^2}}x^3\left(a+b\sec^{-1}(cx)\right)^2d\sec^{-1}(cx)}{c^2}$$

$$\Big\downarrow \text{4909}$$

$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^2-b\int c^2x^2\left(a+b\sec^{-1}(cx)\right)d\sec^{-1}(cx)}{c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^2-b\int \left(a+b\sec^{-1}(cx)\right)\csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^2d\sec^{-1}(cx)}{c^2}$$

$$\Big\downarrow \text{4672}$$

$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^2-b\left(b\int -c\sqrt{1-\frac{1}{c^2x^2}}xd\sec^{-1}(cx)+cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)\right)}{c^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^2-b\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)-b\int c\sqrt{1-\frac{1}{c^2x^2}}xd\sec^{-1}(cx)\right)}{c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^2-b\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)-b\int\tan\left(\sec^{-1}(cx)\right)d\sec^{-1}(cx)\right)}{c^2}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^2-b\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)+b\log\left(\frac{1}{cx}\right)\right)}{c^2}$$

input
```
Int[x*(a + b*ArcSec[c*x])^2,x]
```

output
```
((c^2*x^2*(a + b*ArcSec[c*x])^2)/2 - b*(c*Sqrt[1 - 1/(c^2*x^2)]*x*(a + b*A
rcSec[c*x]) + b*Log[1/(c*x)]))/c^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(52) = 104$.

Time = 0.51 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.20

method	result	size
parts	$\frac{a^{2} x^{2}}{2}+\frac{b^{2} \left(\frac{c^{2} x^{2} \operatorname{arcsec}\left(c x \right)^{2}}{2}-\operatorname{arcsec}\left(c x \right) c x \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}-\ln \left(\frac{1}{c x}\right)\right)}{c^{2}}+\frac{2 a b \left(\frac{c^{2} x^{2} \operatorname{arcsec}\left(c x \right)}{2}-\frac{c^{2} x^{2}-1}{2 \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}\, c x}\right)}{c^{2}}$	123
derivativedivides	$\frac{\frac{a^{2} c^{2} x^{2}}{2}+b^{2} \left(\frac{c^{2} x^{2} \operatorname{arcsec}\left(c x \right)^{2}}{2}-\operatorname{arcsec}\left(c x \right) c x \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}-\ln \left(\frac{1}{c x}\right)\right)+2 a b \left(\frac{c^{2} x^{2} \operatorname{arcsec}\left(c x \right)}{2}-\frac{c^{2} x^{2}-1}{2 \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}\, c x}\right)}{c^{2}}$	124
default	$\frac{\frac{a^{2} c^{2} x^{2}}{2}+b^{2} \left(\frac{c^{2} x^{2} \operatorname{arcsec}\left(c x \right)^{2}}{2}-\operatorname{arcsec}\left(c x \right) c x \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}-\ln \left(\frac{1}{c x}\right)\right)+2 a b \left(\frac{c^{2} x^{2} \operatorname{arcsec}\left(c x \right)}{2}-\frac{c^{2} x^{2}-1}{2 \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}\, c x}\right)}{c^{2}}$	124

input
```
int(x*(a+b*arcsec(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x^2+b^2/c^2*(1/2*c^2*x^2*arcsec(c*x)^2-arcsec(c*x)*c*x*((c^2*x^2-1
)/c^2/x^2)^(1/2)-ln(1/c/x))+2*a*b/c^2*(1/2*c^2*x^2*arcsec(c*x)-1/2/((c^2*x
^2-1)/c^2/x^2)^(1/2)/c/x*(c^2*x^2-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(52) = 104$.

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.98

$$\int x\left(a+b\sec^{-1}(cx)\right)^2\,dx$$
$$=\frac{b^2c^2x^2\,\mathrm{arcsec}\,(cx)^2+a^2c^2x^2+4\,abc^2\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+2\,b^2\log\left(x\right)+2\left(abc^2x^2-abc^2\right)\mathrm{arcsec}}{2\,c^2}$$

input
```
integrate(x*(a+b*arcsec(c*x))^2,x, algorithm="fricas")
```

output
```
1/2*(b^2*c^2*x^2*arcsec(c*x)^2 + a^2*c^2*x^2 + 4*a*b*c^2*arctan(-c*x + sqr
t(c^2*x^2 - 1)) + 2*b^2*log(x) + 2*(a*b*c^2*x^2 - a*b*c^2)*arcsec(c*x) - 2
*sqrt(c^2*x^2 - 1)*(b^2*arcsec(c*x) + a*b))/c^2
```

Sympy [F]

$$\int x\left(a+b\sec^{-1}(cx)\right)^2\,dx=\int x(a+b\operatorname{asec}(cx))^2\,dx$$

input
```
integrate(x*(a+b*asec(c*x))**2,x)
```

output
```
Integral(x*(a + b*asec(c*x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.55

$$\int x\left(a+b\sec^{-1}(cx)\right)^2\,dx=\frac{1}{2}\,b^2x^2\operatorname{arcsec}(cx)^2+\frac{1}{2}\,a^2x^2+\left(x^2\operatorname{arcsec}(cx)-\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)ab-\left(\frac{x\sqrt{-\frac{1}{c^2x^2}+1}\operatorname{arcsec}(cx)}{c}-\frac{\log(x)}{c^2}\right)b^2$$

input
```
integrate(x*(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2*arcsec(c*x)^2 + 1/2*a^2*x^2 + (x^2*arcsec(c*x) - x*sqrt(-1/(c^
2*x^2) + 1)/c)*a*b - (x*sqrt(-1/(c^2*x^2) + 1)*arcsec(c*x)/c - log(x)/c^2)
*b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2181 vs. $2(52) = 104$.

Time = 0.25 (sec) , antiderivative size = 2181, normalized size of antiderivative = 38.95

$$\int x\left(a+b\sec^{-1}(cx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(x*(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

output
```
1/2*(b^2*arccos(1/(c*x))^2/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
+ c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + 2*a*b*arccos(1/(c*x))/(c^3 +
2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x)
 + 1)^4) - 2*b^2*(1/(c^2*x^2) - 1)*arccos(1/(c*x))^2/((c^3 + 2*c^3*(1/(c^2
*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(
c*x) + 1)^2) - 2*b^2*log(2)/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + 2*b^2*log(2/(c*x) + 2)/(c^3
+ 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*
x) + 1)^4) - 2*b^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^3 + 2
*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x)
+ 1)^4) - 2*b^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(c^3 + 2*c^
3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1
)^4) - 4*b^2*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x))/((c^3 + 2*c^3*(1/(c^2*
x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c
*x) + 1)) + a^2/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c
^2*x^2) - 1)^2/(1/(c*x) + 1)^4) - 4*a*b*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/
((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/
(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) + b^2*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x)
)^2/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1
)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4) - 4*b^2*(1/(c^2*x^2) - 1)*log(2)/...
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\sec^{-1}(cx)\right)^2\,dx=\int x\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^2\,dx$$

input
```
int(x*(a + b*acos(1/(c*x)))^2,x)
```

output
```
int(x*(a + b*acos(1/(c*x)))^2, x)
```

Reduce [F]

$$\int x\left(a+b\sec^{-1}(cx)\right)^2\,dx=2\left(\int asec(cx)\,xdx\right)ab+\left(\int asec(cx)^2\,xdx\right)b^2+\frac{a^2x^2}{2}$$

input
```
int(x*(a+b*asec(c*x))^2,x)
```

output
```
(4*int(asec(c*x)*x,x)*a*b + 2*int(asec(c*x)**2*x,x)*b**2 + a**2*x**2)/2
```

3.18 $\int \left(a+b\sec^{-1}(cx)\right)^2\,dx$

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Optimal result

Integrand size = 10, antiderivative size = 92

$$\int \left(a+b\sec^{-1}(cx)\right)^2\,dx=x\left(a+b\sec^{-1}(cx)\right)^2+\frac{4ib\left(a+b\sec^{-1}(cx)\right)\arctan\left(e^{i\sec^{-1}(cx)}\right)}{c}-\frac{2ib^2\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)}{c}+\frac{2ib^2\,\text{PolyLog}\left(2,ie^{i\sec^{-1}(cx)}\right)}{c}$$

output
```
x*(a+b*arcsec(c*x))^2+4*I*b*(a+b*arcsec(c*x))*arctan(1/c/x+I*(1-1/c^2/x^2)
^(1/2))/c-2*I*b^2*polylog(2,-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/c+2*I*b^2*po
lylog(2,I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/c
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.77

$$\int \left(a+b\sec^{-1}(cx)\right)^2\,dx$$
$$= \frac{a^2cx+2ab\left(cx\sec^{-1}(cx)+\log\left(\cos\left(\frac{1}{2}\sec^{-1}(cx)\right)-\sin\left(\frac{1}{2}\sec^{-1}(cx)\right)\right)-\log\left(\cos\left(\frac{1}{2}\sec^{-1}(cx)\right)+\sin\left(\frac{1}{2}\right.\right.\right.}{}$$

input
```
Integrate[(a + b*ArcSec[c*x])^2,x]
```

output
```
(a^2*c*x + 2*a*b*(c*x*ArcSec[c*x] + Log[Cos[ArcSec[c*x]/2] - Sin[ArcSec[c*
x]/2]] - Log[Cos[ArcSec[c*x]/2] + Sin[ArcSec[c*x]/2]]) + b^2*(ArcSec[c*x]*
(c*x*ArcSec[c*x] - 2*Log[1 - I*E^(I*ArcSec[c*x])] + 2*Log[1 + I*E^(I*ArcSe
c[c*x])]) - (2*I)*PolyLog[2, (-I)*E^(I*ArcSec[c*x])] + (2*I)*PolyLog[2, I*
E^(I*ArcSec[c*x])]))/c
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5739, 4909, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sec^{-1}(cx)\right)^2\,dx$$
$$\Big\downarrow \ 5739$$
$$\frac{\int c^2\sqrt{1-\frac{1}{c^2x^2}}x^2\left(a+b\sec^{-1}(cx)\right)^2d\sec^{-1}(cx)}{c}$$
$$\Big\downarrow \ 4909$$
$$\frac{cx\left(a+b\sec^{-1}(cx)\right)^2-2b\int cx\left(a+b\sec^{-1}(cx)\right)d\sec^{-1}(cx)}{c}$$
$$\Big\downarrow \ 3042$$

$$\frac{cx\left(a+b\sec^{-1}(cx)\right)^2-2b\int \left(a+b\sec^{-1}(cx)\right)\csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)d\sec^{-1}(cx)}{c}$$

$$\Big\downarrow \text{ 4669}$$

$$\frac{cx\left(a+b\sec^{-1}(cx)\right)^2-2b\left(-b\int \log\left(1-ie^{i\sec^{-1}(cx)}\right)d\sec^{-1}(cx)+b\int \log\left(1+ie^{i\sec^{-1}(cx)}\right)d\sec^{-1}(cx)-2i\,\text{arc}\right.}{c}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{cx\left(a+b\sec^{-1}(cx)\right)^2-2b\left(ib\int e^{-i\sec^{-1}(cx)}\log\left(1-ie^{i\sec^{-1}(cx)}\right)de^{i\sec^{-1}(cx)}-ib\int e^{-i\sec^{-1}(cx)}\log\left(1+ie^{i\sec^{-1}(c}\right.\right.}{c}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{cx\left(a+b\sec^{-1}(cx)\right)^2-2b\left(-2i\arctan\left(e^{i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)+ib\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)-ib\,\text{PolyLo}\right.}{c}$$

input
```
Int[(a + b*ArcSec[c*x])^2,x]
```

output
```
(c*x*(a + b*ArcSec[c*x])^2 - 2*b*((-2*I)*(a + b*ArcSec[c*x])*ArcTan[E^(I*A
rcSec[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcSec[c*x])] - I*b*PolyLog[2, I*E
^(I*ArcSec[c*x])]))/c
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5739
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/c   Subst[
Int[(a + b*x)^n*Sec[x]*Tan[x], x], x, ArcSec[c*x]], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.11

method	result
derivativedivides	$\frac{a^2cx+b^2\left(\operatorname{arcsec}(cx)^2cx+2\operatorname{arcsec}(cx)\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)-2\operatorname{arcsec}(cx)\ln\left(1-i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)-2i\operatorname{dilog}\right.}{c}$
default	$\frac{a^2cx+b^2\left(\operatorname{arcsec}(cx)^2cx+2\operatorname{arcsec}(cx)\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)-2\operatorname{arcsec}(cx)\ln\left(1-i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)-2i\operatorname{dilog}\right.}{c}$
parts	$a^2x+\frac{b^2\left(\operatorname{arcsec}(cx)^2cx+2\operatorname{arcsec}(cx)\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)-2\operatorname{arcsec}(cx)\ln\left(1-i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)-2i\operatorname{dil}\right.}{c}$

input
```
int((a+b*arcsec(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(a^2*c*x+b^2*(arcsec(c*x)^2*c*x+2*arcsec(c*x)*ln(1+I*(1/c/x+I*(1-1/c^2
/x^2)^(1/2)))-2*arcsec(c*x)*ln(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-2*I*dilo
g(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))+2*I*dilog(1-I*(1/c/x+I*(1-1/c^2/x^2)^
(1/2))))+2*a*b*(c*x*arcsec(c*x)-ln(c*x+c*x*(1-1/c^2/x^2)^(1/2))))
```

Fricas [F]

$$\int \left(a + b \sec^{-1}(cx)\right)^2 \, dx = \int (b \operatorname{arcsec}(cx) + a)^2 \, dx$$

input
```
integrate((a+b*arcsec(c*x))^2,x, algorithm="fricas")
```

output
```
integral(b^2*arcsec(c*x)^2 + 2*a*b*arcsec(c*x) + a^2, x)
```

Sympy [F]

$$\int \left(a + b \sec^{-1}(cx)\right)^2 \, dx = \int (a + b \operatorname{asec}(cx))^2 \, dx$$

input
```
integrate((a+b*asec(c*x))**2,x)
```

output
```
Integral((a + b*asec(c*x))**2, x)
```

Maxima [F]

$$\int \left(a + b \sec^{-1}(cx)\right)^2 \, dx = \int (b \operatorname{arcsec}(cx) + a)^2 \, dx$$

input
```
integrate((a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

output
```
-1/4*(2*c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*log(c)^2 - 4*c
^2*integrate(x^2*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) + 8*c^2*integrate(x
^2*log(x)/(c^2*x^2 - 1), x)*log(c) - 4*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1
))^2 - 4*c^2*integrate(x^2*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) + 4*c^2*i
ntegrate(x^2*log(x)^2/(c^2*x^2 - 1), x) - 4*c^2*integrate(x^2*log(c^2*x^2)
/(c^2*x^2 - 1), x) + x*log(c^2*x^2)^2 + 2*(log(c*x + 1)/c - log(c*x - 1)/c
)*log(c)^2 + 4*integrate(log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 8*integra
te(log(x)/(c^2*x^2 - 1), x)*log(c) + 8*integrate(sqrt(c*x + 1)*sqrt(c*x -
1)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^2 - 1), x) + 4*integrate(log
(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) - 4*integrate(log(x)^2/(c^2*x^2 - 1), x
) + 4*integrate(log(c^2*x^2)/(c^2*x^2 - 1), x))*b^2 + a^2*x + (2*c*x*arcse
c(c*x) - log(sqrt(-1/(c^2*x^2) + 1) + 1) + log(-sqrt(-1/(c^2*x^2) + 1) + 1
))*a*b/c
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+b\sec^{-1}(cx)\right)^2\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsec(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^{-1}(cx)\right)^2\,dx=\int \left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^2\,dx$$

input
```
int((a + b*acos(1/(c*x)))^2,x)
```

output
```
int((a + b*acos(1/(c*x)))^2, x)
```

Reduce [F]

$$\int \left(a+b\sec^{-1}(cx)\right)^2\,dx=2\left(\int asec(cx)\,dx\right)ab+\left(\int asec(cx)^2\,dx\right)b^2+a^2x$$

input
```
int((a+b*asec(c*x))^2,x)
```

output
```
2*int(asec(c*x),x)*a*b + int(asec(c*x)**2,x)*b**2 + a**2*x
```

3.19 $\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x}\,dx$

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Sympy [F] 217
Maxima [F] 217
Giac [F(-2)] 218
Mupad [F(-1)] 218
Reduce [F] 218

Optimal result

Integrand size = 14, antiderivative size = 93

$$\begin{aligned}\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x}\,dx &= \frac{i\left(a+b\sec^{-1}(cx)\right)^3}{3b}-\left(a+b\sec^{-1}(cx)\right)^2\log\left(1+e^{2i\sec^{-1}(cx)}\right)\\ &\quad+ib\left(a+b\sec^{-1}(cx)\right)\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)\\ &\quad-\frac{1}{2}b^2\operatorname{PolyLog}\left(3,-e^{2i\sec^{-1}(cx)}\right)\end{aligned}$$

output
```
1/3*I*(a+b*arcsec(c*x))^3/b-(a+b*arcsec(c*x))^2*ln(1+(1/c/x+I*(1-1/c^2/x^2
)^(1/2))^2)+I*b*(a+b*arcsec(c*x))*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))
^2)-1/2*b^2*polylog(3,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.39

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x}\,dx = iab\sec^{-1}(cx)^2+\frac{1}{3}ib^2\sec^{-1}(cx)^3 - 2ab\sec^{-1}(cx)\log\left(1+e^{2i\sec^{-1}(cx)}\right) - b^2\sec^{-1}(cx)^2\log\left(1+e^{2i\sec^{-1}(cx)}\right)+a^2\log(cx) + ib(a+b\sec^{-1}(cx))\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right) - \frac{1}{2}b^2\,\text{PolyLog}\left(3,-e^{2i\sec^{-1}(cx)}\right)$$

input
```
Integrate[(a + b*ArcSec[c*x])^2/x,x]
```

output
```
I*a*b*ArcSec[c*x]^2 + (I/3)*b^2*ArcSec[c*x]^3 - 2*a*b*ArcSec[c*x]*Log[1 +
E^((2*I)*ArcSec[c*x])] - b^2*ArcSec[c*x]^2*Log[1 + E^((2*I)*ArcSec[c*x])]
+ a^2*Log[c*x] + I*b*(a + b*ArcSec[c*x])*PolyLog[2, -E^((2*I)*ArcSec[c*x])
] - (b^2*PolyLog[3, -E^((2*I)*ArcSec[c*x])])/2
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5745, 3042, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x}\,dx$$

$$\Big\downarrow \text{5745}$$

$$\int cx\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2\,d\sec^{-1}(cx)$$

$$\Big\downarrow \text{3042}$$

$$\int \tan \left(\sec^{-1}(cx)\right) \left(a+b\sec^{-1}(cx)\right)^2 d\sec^{-1}(cx)$$

$$\Big\downarrow \text{4202}$$

$$\frac{i\left(a+b\sec^{-1}(cx)\right)^3}{3b}-2i\int \frac{e^{2i\sec^{-1}(cx)}\left(a+b\sec^{-1}(cx)\right)^2}{1+e^{2i\sec^{-1}(cx)}}d\sec^{-1}(cx)$$

$$\Big\downarrow \text{2620}$$

$$\frac{i\left(a+b\sec^{-1}(cx)\right)^3}{3b}-$$
$$2i\left(ib\int \left(a+b\sec^{-1}(cx)\right)\log\left(1+e^{2i\sec^{-1}(cx)}\right)d\sec^{-1}(cx)-\frac{1}{2}i\log\left(1+e^{2i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)^2\right)$$

$$\Big\downarrow \text{3011}$$

$$\frac{i\left(a+b\sec^{-1}(cx)\right)^3}{3b}-$$
$$2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)-\frac{1}{2}ib\int \text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)d\sec^{-1}(cx)\right)-\frac{1}{2}i\log$$

$$\Big\downarrow \text{2720}$$

$$\frac{i\left(a+b\sec^{-1}(cx)\right)^3}{3b}-$$
$$2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)-\frac{1}{4}b\int e^{-2i\sec^{-1}(cx)}\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)de^{2i\sec^{-1}(c}\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$\frac{i\left(a+b\sec^{-1}(cx)\right)^3}{3b}-$$
$$2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)-\frac{1}{4}b\,\text{PolyLog}\left(3,-e^{2i\sec^{-1}(cx)}\right)\right)-\frac{1}{2}i\log\left(1+e^{2i\sec^{-1}(c}\right.\right.$$

input
```
Int[(a + b*ArcSec[c*x])^2/x,x]
```

output
```
((I/3)*(a + b*ArcSec[c*x])^3)/b - (2*I)*((-1/2*I)*(a + b*ArcSec[c*x])^2*Lo
g[1 + E^((2*I)*ArcSec[c*x])] + I*b*((I/2)*(a + b*ArcSec[c*x])*PolyLog[2, -
E^((2*I)*ArcSec[c*x])] - (b*PolyLog[3, -E^((2*I)*ArcSec[c*x])])/4))
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.22

method	result
parts	$a^2 \ln(x) + b^2 \left(\frac{i \operatorname{arcsec}(cx)^3}{3} - \operatorname{arcsec}(cx)^2 \ln\left(1 + \left(\frac{1}{cx} + i\sqrt{1 - \frac{1}{c^2x^2}}\right)^2\right) + i \operatorname{arcsec}(cx)\right.$
derivativedivides	$a^2 \ln(cx) + b^2 \left(\frac{i \operatorname{arcsec}(cx)^3}{3} - \operatorname{arcsec}(cx)^2 \ln\left(1 + \left(\frac{1}{cx} + i\sqrt{1 - \frac{1}{c^2x^2}}\right)^2\right) + i \operatorname{arcsec}(cx\right.$
default	$a^2 \ln(cx) + b^2 \left(\frac{i \operatorname{arcsec}(cx)^3}{3} - \operatorname{arcsec}(cx)^2 \ln\left(1 + \left(\frac{1}{cx} + i\sqrt{1 - \frac{1}{c^2x^2}}\right)^2\right) + i \operatorname{arcsec}(cx\right.$

input
```
int((a+b*arcsec(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
a^2*ln(x)+b^2*(1/3*I*arcsec(c*x)^3-arcsec(c*x)^2*ln(1+(1/c/x+I*(1-1/c^2/x^
2)^(1/2))^2)+I*arcsec(c*x)*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)-1/2
*polylog(3,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2))+I*a*b*arcsec(c*x)^2+I*a*b*po
lylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)-2*a*b*arcsec(c*x)*ln(1+(1/c/x+I*
(1-1/c^2/x^2)^(1/2))^2)
```

Fricas [F]

$$\int \frac{(a + b \sec^{-1}(cx))^2}{x} \, dx = \int \frac{(b \operatorname{arcsec}(cx) + a)^2}{x} \, dx$$

input
```
integrate((a+b*arcsec(c*x))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*arcsec(c*x)^2 + 2*a*b*arcsec(c*x) + a^2)/x, x)
```

Sympy [F]

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x}\,dx=\int \frac{\left(a+b\operatorname{asec}\left(cx\right)\right)^2}{x}\,dx$$

input
```
integrate((a+b*asec(c*x))**2/x,x)
```

output
```
Integral((a + b*asec(c*x))**2/x, x)
```

Maxima [F]

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x}\,dx=\int \frac{\left(b\operatorname{arcsec}\left(cx\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))^2/x,x, algorithm="maxima")
```

output
```
-1/2*b^2*c^2*(log(c*x + 1)/c^2 + log(c*x - 1)/c^2)*log(c)^2 + b^2*c^2*inte
grate(x^2*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) - 2*b^2*c^2*integrate(x^2*
log(x)/(c^2*x^3 - x), x)*log(c) + 2*b^2*c^2*integrate(x^2*log(c^2*x^2)*log
(x)/(c^2*x^3 - x), x) - b^2*c^2*integrate(x^2*log(x)^2/(c^2*x^3 - x), x) +
 2*a*b*c^2*integrate(x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^3 - x)
, x) + 1/2*b^2*(log(c*x + 1) + log(c*x - 1) - 2*log(x))*log(c)^2 + b^2*arc
tan(sqrt(c*x + 1)*sqrt(c*x - 1))^2*log(x) - 1/4*b^2*log(c^2*x^2)^2*log(x)
- b^2*integrate(log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) + 2*b^2*integrate(lo
g(x)/(c^2*x^3 - x), x)*log(c) - 2*b^2*integrate(sqrt(c*x + 1)*sqrt(c*x - 1
)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(x)/(c^2*x^3 - x), x) - 2*b^2*int
egrate(log(c^2*x^2)*log(x)/(c^2*x^3 - x), x) + b^2*integrate(log(x)^2/(c^2
*x^3 - x), x) - 2*a*b*integrate(arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x
^3 - x), x) + a^2*log(x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsec(c*x))^2/x,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x}\,dx = \int \frac{\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^2}{x}\,dx$$

input
```
int((a + b*acos(1/(c*x)))^2/x,x)
```

output
```
int((a + b*acos(1/(c*x)))^2/x, x)
```

Reduce [F]

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x}\,dx = 2\left(\int \frac{asec(cx)}{x}dx\right)ab + \left(\int \frac{asec(cx)^2}{x}dx\right)b^2 + \log(x)\,a^2$$

input
```
int((a+b*asec(c*x))^2/x,x)
```

output
```
2*int(asec(c*x)/x,x)*a*b + int(asec(c*x)**2/x,x)*b**2 + log(x)*a**2
```

3.20 $\int \frac{(a+b\sec^{-1}(cx))^2}{x^2}\,dx$

Optimal result 219
Mathematica [A] (verified) 219
Rubi [A] (verified) 220
Maple [B] (verified) 222
Fricas [A] (verification not implemented) 222
Sympy [F] 223
Maxima [A] (verification not implemented) 223
Giac [B] (verification not implemented) 223
Mupad [B] (verification not implemented) 224
Reduce [F] 224

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^2}\,dx = \frac{2b^2}{x}+2bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))-\frac{(a+b\sec^{-1}(cx))^2}{x}$$

output
```
2*b^2/x+2*b*c*(1-1/c^2/x^2)^(1/2)*(a+b*arcsec(c*x))-(a+b*arcsec(c*x))^2/x
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.50

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^2}\,dx = \frac{-a^2+2b^2+2abc\sqrt{1-\frac{1}{c^2x^2}}x+2b\left(-a+bc\sqrt{1-\frac{1}{c^2x^2}}x\right)\sec^{-1}(cx)-b^2\sec^{-1}(cx)^2}{x}$$

input
```
Integrate[(a + b*ArcSec[c*x])^2/x^2,x]
```

output
```
(-a^2 + 2*b^2 + 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + 2*b*(-a + b*c*Sqrt[1 - 1
/(c^2*x^2)]*x)*ArcSec[c*x] - b^2*ArcSec[c*x]^2)/x
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5745, 3042, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^2}dx$$

$$\Big\downarrow \text{5745}$$

$$c\int \sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2d\sec^{-1}(cx)$$

$$\Big\downarrow \text{3042}$$

$$c\int (a+b\sec^{-1}(cx))^2\sin\left(\sec^{-1}(cx)\right)d\sec^{-1}(cx)$$

$$\Big\downarrow \text{3777}$$

$$c\left(2b\int \frac{a+b\sec^{-1}(cx)}{cx}d\sec^{-1}(cx)-\frac{(a+b\sec^{-1}(cx))^2}{cx}\right)$$

$$\Big\downarrow \text{3042}$$

$$c\left(2b\int (a+b\sec^{-1}(cx))\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)d\sec^{-1}(cx)-\frac{(a+b\sec^{-1}(cx))^2}{cx}\right)$$

$$\Big\downarrow \text{3777}$$

$$c\left(2b\left(b\int -\sqrt{1-\frac{1}{c^2x^2}}d\sec^{-1}(cx)+\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))\right)-\frac{(a+b\sec^{-1}(cx))^2}{cx}\right)$$

$$\Big\downarrow \text{25}$$

$$c\left(2b\left(\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))-b\int \sqrt{1-\frac{1}{c^2x^2}}d\sec^{-1}(cx)\right)-\frac{(a+b\sec^{-1}(cx))^2}{cx}\right)$$

$$\Big\downarrow \text{3042}$$

$$c\left(2b\left(\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))-b\int \sin\left(\sec^{-1}(cx)\right)d\sec^{-1}(cx)\right)-\frac{(a+b\sec^{-1}(cx))^2}{cx}\right)$$

$$\Big\downarrow \text{3118}$$

$$c\left(2b\left(\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))+\frac{b}{cx}\right)-\frac{(a+b\sec^{-1}(cx))^2}{cx}\right)$$

input
```
Int[(a + b*ArcSec[c*x])^2/x^2,x]
```

output
```
c*(-((a + b*ArcSec[c*x])^2/(c*x)) + 2*b*(b/(c*x) + Sqrt[1 - 1/(c^2*x^2)]*(
a + b*ArcSec[c*x])))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(48) = 96$.

Time = 0.38 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.28

method	result
parts	$-\frac{a^2}{x} + b^2 c\left(-\frac{\operatorname{arcsec}(cx)^2}{cx} + \frac{2}{cx} + 2\operatorname{arcsec}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right) + 2abc\left(-\frac{\operatorname{arcsec}(cx)}{cx} + \frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\, c^2}\right.$
derivativedivides	$c\left(-\frac{a^2}{cx} + b^2\left(-\frac{\operatorname{arcsec}(cx)^2}{cx} + \frac{2}{cx} + 2\operatorname{arcsec}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right) + 2ab\left(-\frac{\operatorname{arcsec}(cx)}{cx} + \frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\, c}\right.\right.$
default	$c\left(-\frac{a^2}{cx} + b^2\left(-\frac{\operatorname{arcsec}(cx)^2}{cx} + \frac{2}{cx} + 2\operatorname{arcsec}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right) + 2ab\left(-\frac{\operatorname{arcsec}(cx)}{cx} + \frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\, c}\right.\right.$

input
```
int((a+b*arcsec(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/x+b^2*c*(-1/c/x*arcsec(c*x)^2+2/c/x+2*arcsec(c*x)*((c^2*x^2-1)/c^2/x^
2)^(1/2))+2*a*b*c*(-1/c/x*arcsec(c*x)+1/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^2/x^
2*(c^2*x^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.14

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^2}\,dx = -\frac{b^2\operatorname{arcsec}(cx)^2 + 2\,ab\operatorname{arcsec}(cx) + a^2 - 2\,b^2 - 2\,\sqrt{c^2x^2-1}(b^2\operatorname{arcsec}(cx) + ab)}{x}$$

input
```
integrate((a+b*arcsec(c*x))^2/x^2,x, algorithm="fricas")
```

output
```
-(b^2*arcsec(c*x)^2 + 2*a*b*arcsec(c*x) + a^2 - 2*b^2 - 2*sqrt(c^2*x^2 - 1
)*(b^2*arcsec(c*x) + a*b))/x
```

Sympy [F]

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^2}\,dx = \int \frac{(a+b\operatorname{asec}(cx))^2}{x^2}\,dx$$

input
```
integrate((a+b*asec(c*x))**2/x**2,x)
```

output
```
Integral((a + b*asec(c*x))**2/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.56

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^2}\,dx = 2\left(c\sqrt{-\frac{1}{c^2x^2}+1}-\frac{\operatorname{arcsec}(cx)}{x}\right)ab + 2\left(c\sqrt{-\frac{1}{c^2x^2}+1}\operatorname{arcsec}(cx)+\frac{1}{x}\right)b^2 - \frac{b^2\operatorname{arcsec}(cx)^2}{x} - \frac{a^2}{x}$$

input
```
integrate((a+b*arcsec(c*x))^2/x^2,x, algorithm="maxima")
```

output
```
2*(c*sqrt(-1/(c^2*x^2) + 1) - arcsec(c*x)/x)*a*b + 2*(c*sqrt(-1/(c^2*x^2)
+ 1)*arcsec(c*x) + 1/x)*b^2 - b^2*arcsec(c*x)^2/x - a^2/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(48) = 96.

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.10

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^2}\,dx = \left(2\,b^2\sqrt{-\frac{1}{c^2x^2}+1}\arccos\left(\frac{1}{cx}\right)+2\,ab\sqrt{-\frac{1}{c^2x^2}+1}-\frac{b^2\arccos\left(\frac{1}{cx}\right)^2}{cx}-\frac{2\,ab\arccos\left(\frac{1}{cx}\right)}{cx}-\frac{a^2}{cx}+\frac{2\,b^2}{cx}\right)c$$

input
```
integrate((a+b*arcsec(c*x))^2/x^2,x, algorithm="giac")
```

output
```
(2*b^2*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x)) + 2*a*b*sqrt(-1/(c^2*x^2) +
1) - b^2*arccos(1/(c*x))^2/(c*x) - 2*a*b*arccos(1/(c*x))/(c*x) - a^2/(c*x)
 + 2*b^2/(c*x))*c
```

Mupad [B] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.78

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^2}\,dx = 2\,b^2\,c\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\,\sqrt{1-\frac{1}{c^2\,x^2}}-\frac{b^2\,\left({\mathrm{acos}\left(\frac{1}{c\,x}\right)}^2-2\right)}{x}-\frac{a^2}{x}+2\,a\,b\,c\,\left(\sqrt{1-\frac{1}{c^2\,x^2}}-\frac{\mathrm{acos}\left(\frac{1}{c\,x}\right)}{c\,x}\right)$$

input
```
int((a + b*acos(1/(c*x)))^2/x^2,x)
```

output
```
2*b^2*c*acos(1/(c*x))*(1 - 1/(c^2*x^2))^(1/2) - (b^2*(acos(1/(c*x))^2 - 2)
)/x - a^2/x + 2*a*b*c*((1 - 1/(c^2*x^2))^(1/2) - acos(1/(c*x))/(c*x))
```

Reduce [F]

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^2}\,dx = \frac{2\left(\int \frac{asec(cx)}{x^2}dx\right)abx+\left(\int \frac{asec(cx)^2}{x^2}dx\right)b^2x-a^2}{x}$$

input
```
int((a+b*asec(c*x))^2/x^2,x)
```

output
```
(2*int(asec(c*x)/x**2,x)*a*b*x + int(asec(c*x)**2/x**2,x)*b**2*x - a**2)/x
```

3.21 $\int \frac{(a+b\sec^{-1}(cx))^2}{x^3}\,dx$

Optimal result 225
Mathematica [A] (verified) 225
Rubi [A] (verified) 226
Maple [A] (verified) 228
Fricas [A] (verification not implemented) 228
Sympy [F] 229
Maxima [F] 229
Giac [B] (verification not implemented) 230
Mupad [F(-1)] 230
Reduce [F] 231

Optimal result

Integrand size = 14, antiderivative size = 82

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^3}\,dx = \frac{b^2}{4x^2}+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))}{2x} -\frac{1}{4}c^2(a+b\sec^{-1}(cx))^2+\frac{1}{2}\left(c^2-\frac{1}{x^2}\right)(a+b\sec^{-1}(cx))^2$$

output
```
1/4*b^2/x^2+1/2*b*c*(1-1/c^2/x^2)^(1/2)*(a+b*arcsec(c*x))/x-1/4*c^2*(a+b*a
rcsec(c*x))^2+1/2*(c^2-1/x^2)*(a+b*arcsec(c*x))^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.24

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^3}\,dx = \frac{-2a^2+b^2+2abc\sqrt{1-\frac{1}{c^2x^2}}x+2b\left(-2a+bc\sqrt{1-\frac{1}{c^2x^2}}x\right)\sec^{-1}(cx)+b^2(-2+c^2x^2)\sec^{-1}(cx)^2-2abc^2}{4x^2}$$

input
```
Integrate[(a + b*ArcSec[c*x])^2/x^3,x]
```

output
```
(-2*a^2 + b^2 + 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + 2*b*(-2*a + b*c*Sqrt[1 -
 1/(c^2*x^2)]*x)*ArcSec[c*x] + b^2*(-2 + c^2*x^2)*ArcSec[c*x]^2 - 2*a*b*c^
2*x^2*ArcSin[1/(c*x)])/(4*x^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5745, 4904, 3042, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^3}dx$$

$$\Big\downarrow \text{5745}$$

$$c^2\int \frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{cx}d\sec^{-1}(cx)$$

$$\Big\downarrow \text{4904}$$

$$c^2\left(\frac{1}{2}\left(1-\frac{1}{c^2x^2}\right)\left(a+b\sec^{-1}(cx)\right)^2-b\int \left(1-\frac{1}{c^2x^2}\right)\left(a+b\sec^{-1}(cx)\right)d\sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{3042}$$

$$c^2\left(\frac{1}{2}\left(1-\frac{1}{c^2x^2}\right)\left(a+b\sec^{-1}(cx)\right)^2-b\int \left(a+b\sec^{-1}(cx)\right)\sin\left(\sec^{-1}(cx)\right)^2d\sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{3791}$$

$$c^2\left(\frac{1}{2}\left(1-\frac{1}{c^2x^2}\right)\left(a+b\sec^{-1}(cx)\right)^2-b\left(\frac{1}{2}\int \left(a+b\sec^{-1}(cx)\right)d\sec^{-1}(cx)-\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{2cx}+\frac{1}{4}\right.\right.$$

$$\Big\downarrow \text{17}$$

$$c^2\left(\frac{1}{2}\left(1-\frac{1}{c^2x^2}\right)\left(a+b\sec^{-1}(cx)\right)^2-b\left(-\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{2cx}+\frac{\left(a+b\sec^{-1}(cx)\right)^2}{4b}+\frac{1}{4}b\left(1-\frac{1}{c^2x^2}\right.\right.\right.$$

input
```
Int[(a + b*ArcSec[c*x])^2/x^3,x]
```

output
```
c^2*(((1 - 1/(c^2*x^2))*(a + b*ArcSec[c*x])^2)/2 - b*((b*(1 - 1/(c^2*x^2))
)/4 - (Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcSec[c*x]))/(2*c*x) + (a + b*ArcSec[
c*x])^2/(4*b)))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.77

method	result
parts	$-\frac{a^2}{2x^2}+b^2c^2\left(-\frac{\cos(2\operatorname{arcsec}(cx))\operatorname{arcsec}(cx)^2}{4}+\frac{\cos(2\operatorname{arcsec}(cx))}{8}+\frac{\sin(2\operatorname{arcsec}(cx))\operatorname{arcsec}(cx)}{4}\right)+2ab\,c$
derivativedivides	$c^2\left(-\frac{a^2}{2c^2x^2}+b^2\left(-\frac{\cos(2\operatorname{arcsec}(cx))\operatorname{arcsec}(cx)^2}{4}+\frac{\cos(2\operatorname{arcsec}(cx))}{8}+\frac{\sin(2\operatorname{arcsec}(cx))\operatorname{arcsec}(cx)}{4}\right)+2\right.$
default	$c^2\left(-\frac{a^2}{2c^2x^2}+b^2\left(-\frac{\cos(2\operatorname{arcsec}(cx))\operatorname{arcsec}(cx)^2}{4}+\frac{\cos(2\operatorname{arcsec}(cx))}{8}+\frac{\sin(2\operatorname{arcsec}(cx))\operatorname{arcsec}(cx)}{4}\right)+2\right.$

input
```
int((a+b*arcsec(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/x^2+b^2*c^2*(-1/4*cos(2*arcsec(c*x))*arcsec(c*x)^2+1/8*cos(2*arcs
ec(c*x))+1/4*sin(2*arcsec(c*x))*arcsec(c*x))+2*a*b*c^2*(-1/2/c^2/x^2*arcse
c(c*x)-1/4*(c^2*x^2-1)^(1/2)*(arctan(1/(c^2*x^2-1)^(1/2))*c^2*x^2-(c^2*x^2
-1)^(1/2))/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^3/x^3)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int\frac{(a+b\sec^{-1}(cx))^2}{x^3}\,dx$$
$$=\frac{(b^2c^2x^2-2\,b^2)\operatorname{arcsec}(cx)^2-2\,a^2+b^2+2\,(abc^2x^2-2\,ab)\operatorname{arcsec}(cx)+2\sqrt{c^2x^2-1}(b^2\operatorname{arcsec}(cx)+ab)}{4\,x^2}$$

input
```
integrate((a+b*arcsec(c*x))^2/x^3,x, algorithm="fricas")
```

output
```
1/4*((b^2*c^2*x^2 - 2*b^2)*arcsec(c*x)^2 - 2*a^2 + b^2 + 2*(a*b*c^2*x^2 -
2*a*b)*arcsec(c*x) + 2*sqrt(c^2*x^2 - 1)*(b^2*arcsec(c*x) + a*b))/x^2
```

Sympy [F]

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^3}\,dx = \int \frac{(a+b\,\mathrm{asec}\,(cx))^2}{x^3}\,dx$$

input
```
integrate((a+b*asec(c*x))**2/x**3,x)
```

output
```
Integral((a + b*asec(c*x))**2/x**3, x)
```

Maxima [F]

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^3}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((a+b*arcsec(c*x))^2/x^3,x, algorithm="maxima")
```

output
```
-1/2*a*b*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) -
c^3*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c + 2*arcsec(c*x)/x^2) - 1/8*(4*(c
^2*(log(c*x + 1) + log(c*x - 1) - 2*log(x))*log(c)^2 - 4*c^2*integrate(1/2
*x^2*log(c^2*x^2)/(c^2*x^5 - x^3), x)*log(c) + 8*c^2*integrate(1/2*x^2*log
(x)/(c^2*x^5 - x^3), x)*log(c) - 4*c^2*integrate(1/2*x^2*log(c^2*x^2)*log(
x)/(c^2*x^5 - x^3), x) + 4*c^2*integrate(1/2*x^2*log(x)^2/(c^2*x^5 - x^3),
 x) + 2*c^2*integrate(1/2*x^2*log(c^2*x^2)/(c^2*x^5 - x^3), x) - (c^2*log(
c*x + 1) + c^2*log(c*x - 1) - 2*c^2*log(x) + 1/x^2)*log(c)^2 + 4*integrate
(1/2*log(c^2*x^2)/(c^2*x^5 - x^3), x)*log(c) - 8*integrate(1/2*log(x)/(c^2
*x^5 - x^3), x)*log(c) - 4*integrate(1/2*sqrt(c*x + 1)*sqrt(c*x - 1)*arcta
n(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^5 - x^3), x) + 4*integrate(1/2*log(c
^2*x^2)*log(x)/(c^2*x^5 - x^3), x) - 4*integrate(1/2*log(x)^2/(c^2*x^5 - x
^3), x) - 2*integrate(1/2*log(c^2*x^2)/(c^2*x^5 - x^3), x))*x^2 + 4*arctan
(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - log(c^2*x^2)^2)*b^2/x^2 - 1/2*a^2/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(72) = 144.

Time = 0.14 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.79

$$\int \frac{(a + b\sec^{-1}(cx))^2}{x^3}\,dx$$
$$= \frac{1}{8}\left(2\,b^2c\arccos\left(\frac{1}{cx}\right)^2 + 4\,abc\arccos\left(\frac{1}{cx}\right) - b^2c + \frac{4\,b^2\sqrt{-\frac{1}{c^2x^2}+1}\arccos\left(\frac{1}{cx}\right)}{x} + \frac{4\,ab\sqrt{-\frac{1}{c^2x^2}+1}}{x} - \right.$$

input
```
integrate((a+b*arcsec(c*x))^2/x^3,x, algorithm="giac")
```

output
```
1/8*(2*b^2*c*arccos(1/(c*x))^2 + 4*a*b*c*arccos(1/(c*x)) - b^2*c + 4*b^2*s
qrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x))/x + 4*a*b*sqrt(-1/(c^2*x^2) + 1)/x -
 4*b^2*arccos(1/(c*x))^2/(c*x^2) - 8*a*b*arccos(1/(c*x))/(c*x^2) - 4*a^2/(
c*x^2) + 2*b^2/(c*x^2))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\sec^{-1}(cx))^2}{x^3}\,dx = \int \frac{\left(a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^2}{x^3}\,dx$$

input
```
int((a + b*acos(1/(c*x)))^2/x^3,x)
```

output
```
int((a + b*acos(1/(c*x)))^2/x^3, x)
```

Reduce [F]

$$\int \frac{(a + b\sec^{-1}(cx))^2}{x^3}\,dx = \frac{4\left(\int \frac{asec(cx)}{x^3}dx\right)ab\,x^2 + 2\left(\int \frac{asec(cx)^2}{x^3}dx\right)b^2x^2 - a^2}{2x^2}$$

input
```
int((a+b*asec(c*x))^2/x^3,x)
```

output
```
(4*int(asec(c*x)/x**3,x)*a*b*x**2 + 2*int(asec(c*x)**2/x**3,x)*b**2*x**2 -
 a**2)/(2*x**2)
```

3.22 $\int \frac{(a+b\sec^{-1}(cx))^2}{x^4}\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 102

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^4}\,dx = \frac{2b^2}{27x^3}+\frac{4b^2c^2}{9x}+\frac{4}{9}bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx)) + \frac{2bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))}{9x^2}-\frac{(a+b\sec^{-1}(cx))^2}{3x^3}$$

output
```
2/27*b^2/x^3+4/9*b^2*c^2/x+4/9*b*c^3*(1-1/c^2/x^2)^(1/2)*(a+b*arcsec(c*x))
+2/9*b*c*(1-1/c^2/x^2)^(1/2)*(a+b*arcsec(c*x))/x^2-1/3*(a+b*arcsec(c*x))^2
/x^3
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^4}\,dx = \frac{-9a^2+6abc\sqrt{1-\frac{1}{c^2x^2}}x(1+2c^2x^2)+2b^2(1+6c^2x^2)+6b\left(-3a+bc\sqrt{1-\frac{1}{c^2x^2}}x(1+2c^2x^2)\right)\sec^{-1}(cx)}{27x^3}$$

input
```
Integrate[(a + b*ArcSec[c*x])^2/x^4,x]
```

output
```
(-9*a^2 + 6*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2) + 2*b^2*(1 + 6*c
^2*x^2) + 6*b*(-3*a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2))*ArcSec[
c*x] - 9*b^2*ArcSec[c*x]^2)/(27*x^3)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5745, 4905, 3042, 3791, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^4}dx$$

$$\Big\downarrow \text{5745}$$

$$c^3\int \frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{c^2x^2}d\sec^{-1}(cx)$$

$$\Big\downarrow \text{4905}$$

$$c^3\left(\frac{2}{3}b\int \frac{a+b\sec^{-1}(cx)}{c^3x^3}d\sec^{-1}(cx)-\frac{\left(a+b\sec^{-1}(cx)\right)^2}{3c^3x^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$c^3\left(\frac{2}{3}b\int \left(a+b\sec^{-1}(cx)\right)\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^3d\sec^{-1}(cx)-\frac{\left(a+b\sec^{-1}(cx)\right)^2}{3c^3x^3}\right)$$

$$\Big\downarrow \text{3791}$$

$$c^3\left(\frac{2}{3}b\left(\frac{2}{3}\int \frac{a+b\sec^{-1}(cx)}{cx}d\sec^{-1}(cx)+\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{3c^2x^2}+\frac{b}{9c^3x^3}\right)-\frac{\left(a+b\sec^{-1}(cx)\right)^2}{3c^3x^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$c^3\left(\frac{2}{3}b\left(\frac{2}{3}\int \left(a+b\sec^{-1}(cx)\right)\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)d\sec^{-1}(cx)+\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{3c^2x^2}+\frac{b}{9c^3x^3}\right)-\frac{(a}{}$$

$$\Big\downarrow \text{3777}$$

$$c^3\left(\frac{2}{3}b\left(\frac{2}{3}\left(b\int -\sqrt{1-\frac{1}{c^2x^2}}d\sec^{-1}(cx)+\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))\right)+\frac{\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))}{3c^2x^2}+\frac{b}{9c^3}\right.\right.$$

$$\Big\downarrow \text{25}$$

$$c^3\left(\frac{2}{3}b\left(\frac{2}{3}\left(\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))-b\int \sqrt{1-\frac{1}{c^2x^2}}d\sec^{-1}(cx)\right)+\frac{\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))}{3c^2x^2}+\frac{b}{9c^3x^3}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$c^3\left(\frac{2}{3}b\left(\frac{2}{3}\left(\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))-b\int \sin\left(\sec^{-1}(cx)\right)d\sec^{-1}(cx)\right)+\frac{\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))}{3c^2x^2}+\frac{}{9}\right.\right.$$

$$\Big\downarrow \text{3118}$$

$$c^3\left(\frac{2}{3}b\left(\frac{\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))}{3c^2x^2}+\frac{2}{3}\left(\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))+\frac{b}{cx}\right)+\frac{b}{9c^3x^3}\right)-\frac{(a+b\sec^{-1}(cx))^2}{3c^3x^3}\right.$$

input
```
Int[(a + b*ArcSec[c*x])^2/x^4,x]
```

output
```
c^3*(-1/3*(a + b*ArcSec[c*x])^2/(c^3*x^3) + (2*b*(b/(9*c^3*x^3) + (Sqrt[1
- 1/(c^2*x^2)]*(a + b*ArcSec[c*x]))/(3*c^2*x^2) + (2*(b/(c*x) + Sqrt[1 - 1
/(c^2*x^2)]*(a + b*ArcSec[c*x])))/3))/3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4905
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.50

method	result
parts	$-\frac{a^2}{3x^3}+b^2c^3\left(-\frac{\operatorname{arcsec}(cx)^2}{3c^3x^3}+\frac{2\operatorname{arcsec}(cx)(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{9c^2x^2}+\frac{2}{27c^3x^3}+\frac{4}{9cx}\right)+2ab\,c^3\left(-\frac{\operatorname{arcsec}}{3c^3}\right.$
derivativedivides	$c^3\left(-\frac{a^2}{3c^3x^3}+b^2\left(-\frac{\operatorname{arcsec}(cx)^2}{3c^3x^3}+\frac{2\operatorname{arcsec}(cx)(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{9c^2x^2}+\frac{2}{27c^3x^3}+\frac{4}{9cx}\right)+2ab\left(-\frac{\operatorname{arcse}}{3c}\right.\right.$
default	$c^3\left(-\frac{a^2}{3c^3x^3}+b^2\left(-\frac{\operatorname{arcsec}(cx)^2}{3c^3x^3}+\frac{2\operatorname{arcsec}(cx)(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{9c^2x^2}+\frac{2}{27c^3x^3}+\frac{4}{9cx}\right)+2ab\left(-\frac{\operatorname{arcse}}{3c}\right.\right.$

input
```
int((a+b*arcsec(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2/x^3+b^2*c^3*(-1/3/c^3/x^3*arcsec(c*x)^2+2/9*arcsec(c*x)*(2*c^2*x^
2+1)/c^2/x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)+2/27/c^3/x^3+4/9/c/x)+2*a*b*c^3*(
-1/3/c^3/x^3*arcsec(c*x)+1/9*(c^2*x^2-1)*(2*c^2*x^2+1)/((c^2*x^2-1)/c^2/x^
2)^(1/2)/c^4/x^4)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^4}\,dx$$
$$= \frac{12\,b^2c^2x^2 - 9\,b^2\,\mathrm{arcsec}\,(cx)^2 - 18\,ab\,\mathrm{arcsec}\,(cx) - 9\,a^2 + 2\,b^2 + 6\,(2\,abc^2x^2 + ab + (2\,b^2c^2x^2 + b^2)\,\mathrm{arcsec}\,(}{27\,x^3}$$

input
```
integrate((a+b*arcsec(c*x))^2/x^4,x, algorithm="fricas")
```

output
```
1/27*(12*b^2*c^2*x^2 - 9*b^2*arcsec(c*x)^2 - 18*a*b*arcsec(c*x) - 9*a^2 +
2*b^2 + 6*(2*a*b*c^2*x^2 + a*b + (2*b^2*c^2*x^2 + b^2)*arcsec(c*x))*sqrt(c
^2*x^2 - 1))/x^3
```

Sympy [F]

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^4}\,dx = \int \frac{(a+b\,\mathrm{asec}\,(cx))^2}{x^4}\,dx$$

input
```
integrate((a+b*asec(c*x))**2/x**4,x)
```

output
```
Integral((a + b*asec(c*x))**2/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.61

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^4}\,dx$$
$$= -\frac{2}{9}\,ab\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{3\,\mathrm{arcsec}\,(cx)}{x^3}\right)-\frac{b^2\,\mathrm{arcsec}\,(cx)^2}{3\,x^3}-\frac{a^2}{3\,x^3}$$
$$+\frac{2\left(\left(6\,c^3x^2+c\right)\sqrt{cx+1}\sqrt{cx-1}+3\left(2\,c^5x^4-c^3x^2-c\right)\arctan\left(\sqrt{cx+1}\sqrt{cx-1}\right)\right)b^2}{27\,\sqrt{cx+1}\sqrt{cx-1}cx^3}$$

input
```
integrate((a+b*arcsec(c*x))^2/x^4,x, algorithm="maxima")
```

output
```
-2/9*a*b*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c
+ 3*arcsec(c*x)/x^3) - 1/3*b^2*arcsec(c*x)^2/x^3 - 1/3*a^2/x^3 + 2/27*((6*
c^3*x^2 + c)*sqrt(c*x + 1)*sqrt(c*x - 1) + 3*(2*c^5*x^4 - c^3*x^2 - c)*arc
tan(sqrt(c*x + 1)*sqrt(c*x - 1)))*b^2/(sqrt(c*x + 1)*sqrt(c*x - 1)*c*x^3)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.65

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^4}\,dx$$
$$= \frac{1}{27}\left(12\,b^2c^2\sqrt{-\frac{1}{c^2x^2}+1}\arccos\left(\frac{1}{cx}\right)+12\,abc^2\sqrt{-\frac{1}{c^2x^2}+1}+\frac{12\,b^2c}{x}+\frac{6\,b^2\sqrt{-\frac{1}{c^2x^2}+1}\arccos\left(\frac{1}{cx}\right)}{x^2}+\right.$$

input
```
integrate((a+b*arcsec(c*x))^2/x^4,x, algorithm="giac")
```

output
```
1/27*(12*b^2*c^2*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x)) + 12*a*b*c^2*sqrt(
-1/(c^2*x^2) + 1) + 12*b^2*c/x + 6*b^2*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*
x))/x^2 + 6*a*b*sqrt(-1/(c^2*x^2) + 1)/x^2 - 9*b^2*arccos(1/(c*x))^2/(c*x^
3) - 18*a*b*arccos(1/(c*x))/(c*x^3) - 9*a^2/(c*x^3) + 2*b^2/(c*x^3))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^4}\,dx=\int \frac{\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^2}{x^4}\,dx$$

input
```
int((a + b*acos(1/(c*x)))^2/x^4,x)
```

output
```
int((a + b*acos(1/(c*x)))^2/x^4, x)
```

Reduce [F]

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^4}\,dx=\frac{6\left(\int \frac{asec(cx)}{x^4}dx\right)ab\,x^3+3\left(\int \frac{asec(cx)^2}{x^4}dx\right)b^2x^3-a^2}{3x^3}$$

input
```
int((a+b*asec(c*x))^2/x^4,x)
```

output
```
(6*int(asec(c*x)/x**4,x)*a*b*x**3 + 3*int(asec(c*x)**2/x**4,x)*b**2*x**3 -
 a**2)/(3*x**3)
```

3.23 $\int \frac{(a+b\sec^{-1}(cx))^2}{x^5}\,dx$

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Sympy [F] 244
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Mupad [F(-1)] 246
Reduce [F] 246

Optimal result

Integrand size = 14, antiderivative size = 122

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^5}\,dx = \frac{b^2}{32x^4}+\frac{3b^2c^2}{32x^2}+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))}{8x^3} + \frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))}{16x} + \frac{3}{32}c^4(a+b\sec^{-1}(cx))^2-\frac{(a+b\sec^{-1}(cx))^2}{4x^4}$$

output
```
1/32*b^2/x^4+3/32*b^2*c^2/x^2+1/8*b*c*(1-1/c^2/x^2)^(1/2)*(a+b*arcsec(c*x)
)/x^3+3/16*b*c^3*(1-1/c^2/x^2)^(1/2)*(a+b*arcsec(c*x))/x+3/32*c^4*(a+b*arc
sec(c*x))^2-1/4*(a+b*arcsec(c*x))^2/x^4
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.21

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^5}\,dx$$
$$= \frac{-8a^2+b^2+4abc\sqrt{1-\frac{1}{c^2x^2}}x+3b^2c^2x^2+6abc^3\sqrt{1-\frac{1}{c^2x^2}}x^3+2b\left(-8a+bc\sqrt{1-\frac{1}{c^2x^2}}x(2+3c^2x^2)\right)\sec}{32x^4}$$

input
```
Integrate[(a + b*ArcSec[c*x])^2/x^5,x]
```

output
```
(-8*a^2 + b^2 + 4*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + 3*b^2*c^2*x^2 + 6*a*b*c^
3*Sqrt[1 - 1/(c^2*x^2)]*x^3 + 2*b*(-8*a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(2 +
 3*c^2*x^2))*ArcSec[c*x] + b^2*(-8 + 3*c^4*x^4)*ArcSec[c*x]^2 - 6*a*b*c^4*
x^4*ArcSin[1/(c*x)])/(32*x^4)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5745, 4905, 3042, 3791, 3042, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^5}\,dx$$

$$\Big\downarrow \; 5745$$

$$c^4\int \frac{\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2}{c^3x^3}d\sec^{-1}(cx)$$

$$\Big\downarrow \; 4905$$

$$c^4\left(\frac{1}{2}b\int \frac{a+b\sec^{-1}(cx)}{c^4x^4}d\sec^{-1}(cx)-\frac{(a+b\sec^{-1}(cx))^2}{4c^4x^4}\right)$$

$$\Big\downarrow \; 3042$$

$$c^4\left(\frac{1}{2}b\int\left(a+b\sec^{-1}(cx)\right)\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^4d\sec^{-1}(cx)-\frac{\left(a+b\sec^{-1}(cx)\right)^2}{4c^4x^4}\right)$$

$$\Big\downarrow \text{3791}$$

$$c^4\left(\frac{1}{2}b\left(\frac{3}{4}\int\frac{a+b\sec^{-1}(cx)}{c^2x^2}d\sec^{-1}(cx)+\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{4c^3x^3}+\frac{b}{16c^4x^4}\right)-\frac{\left(a+b\sec^{-1}(cx)\right)^2}{4c^4x^4}\right)$$

$$\Big\downarrow \text{3042}$$

$$c^4\left(\frac{1}{2}b\left(\frac{3}{4}\int\left(a+b\sec^{-1}(cx)\right)\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^2d\sec^{-1}(cx)+\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{4c^3x^3}+\frac{b}{16c^4x^4}\right)-\right.$$

$$\Big\downarrow \text{3791}$$

$$c^4\left(\frac{1}{2}b\left(\frac{3}{4}\left(\frac{1}{2}\int\left(a+b\sec^{-1}(cx)\right)d\sec^{-1}(cx)+\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{2cx}+\frac{b}{4c^2x^2}\right)+\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec\right.}{4c^3x^3}\right.\right.$$

$$\Big\downarrow \text{17}$$

$$c^4\left(\frac{1}{2}b\left(\frac{3}{4}\left(\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{2cx}+\frac{\left(a+b\sec^{-1}(cx)\right)^2}{4b}+\frac{b}{4c^2x^2}\right)+\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{4c^3x^3}+\frac{b}{16c^4}\right.\right.$$

input
```
Int[(a + b*ArcSec[c*x])^2/x^5,x]
```

output
```
c^4*(-1/4*(a + b*ArcSec[c*x])^2/(c^4*x^4) + (b*(b/(16*c^4*x^4) + (Sqrt[1 -
 1/(c^2*x^2)]*(a + b*ArcSec[c*x]))/(4*c^3*x^3) + (3*(b/(4*c^2*x^2) + (Sqrt
[1 - 1/(c^2*x^2)]*(a + b*ArcSec[c*x]))/(2*c*x) + (a + b*ArcSec[c*x])^2/(4*
b)))/4))/2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4905
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(106) = 212.

Time = 0.62 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.17

method	result
parts	$-\frac{a^2}{4x^4}+b^2c^4\left(-\frac{\operatorname{arcsec}(cx)^2}{4c^4x^4}+\frac{\operatorname{arcsec}(cx)\left(3c^3x^3\operatorname{arcsec}(cx)+3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}+2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{16c^3x^3}-\frac{3\operatorname{arcsec}(cx)^2}{32}\right.$
derivativedivides	$c^4\left(-\frac{a^2}{4c^4x^4}+b^2\left(-\frac{\operatorname{arcsec}(cx)^2}{4c^4x^4}+\frac{\operatorname{arcsec}(cx)\left(3c^3x^3\operatorname{arcsec}(cx)+3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}+2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{16c^3x^3}-\frac{3\operatorname{arcsec}(}{32}\right.\right.$
default	$c^4\left(-\frac{a^2}{4c^4x^4}+b^2\left(-\frac{\operatorname{arcsec}(cx)^2}{4c^4x^4}+\frac{\operatorname{arcsec}(cx)\left(3c^3x^3\operatorname{arcsec}(cx)+3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}+2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{16c^3x^3}-\frac{3\operatorname{arcsec}(}{32}\right.\right.$

input
```
int((a+b*arcsec(c*x))^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^2/x^4+b^2*c^4*(-1/4/c^4/x^4*arcsec(c*x)^2+1/16*arcsec(c*x)*(3*c^3*x
^3*arcsec(c*x)+3*c^2*x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)+2*((c^2*x^2-1)/c^2/x^
2)^(1/2))/c^3/x^3-3/32*arcsec(c*x)^2+1/128*(3*c^2*x^2+2)^2/c^4/x^4)-1/2*ar
csec(c*x)*a*b/x^4-3/16*a*b*c^3*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/
2)/x*arctan(1/(c^2*x^2-1)^(1/2))+3/16*a*b*c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x
^2)^(1/2)/x^3+1/8*a*b/c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^5
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.98

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^5}\,dx$$
$$=\frac{3\,b^2c^2x^2+(3\,b^2c^4x^4-8\,b^2)\operatorname{arcsec}(cx)^2-8\,a^2+b^2+2\,(3\,abc^4x^4-8\,ab)\operatorname{arcsec}(cx)+2\,(3\,abc^2x^2+2\,ab}{32\,x^4}$$

input
```
integrate((a+b*arcsec(c*x))^2/x^5,x, algorithm="fricas")
```

output
```
1/32*(3*b^2*c^2*x^2 + (3*b^2*c^4*x^4 - 8*b^2)*arcsec(c*x)^2 - 8*a^2 + b^2
+ 2*(3*a*b*c^4*x^4 - 8*a*b)*arcsec(c*x) + 2*(3*a*b*c^2*x^2 + 2*a*b + (3*b^
2*c^2*x^2 + 2*b^2)*arcsec(c*x))*sqrt(c^2*x^2 - 1))/x^4
```

Sympy [F]

$$\int \frac{(a + b\sec^{-1}(cx))^2}{x^5}\,dx = \int \frac{(a + b\,\mathrm{asec}\,(cx))^2}{x^5}\,dx$$

input
```
integrate((a+b*asec(c*x))**2/x**5,x)
```

output
```
Integral((a + b*asec(c*x))**2/x**5, x)
```

Maxima [F]

$$\int \frac{(a + b\sec^{-1}(cx))^2}{x^5}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx) + a)^2}{x^5}\,dx$$

input
```
integrate((a+b*arcsec(c*x))^2/x^5,x, algorithm="maxima")
```

output
```
1/16*a*b*((3*c^5*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)) + (3*c^8*x^3*(-1/(c^2*
x^2) + 1)^(3/2) + 5*c^6*x*sqrt(-1/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) -
1)^2 - 2*c^2*x^2*(1/(c^2*x^2) - 1) + 1))/c - 8*arcsec(c*x)/x^4) - 1/16*(4*
(2*(c^2*log(c*x + 1) + c^2*log(c*x - 1) - 2*c^2*log(x) + 1/x^2)*c^2*log(c)
^2 - 16*c^2*integrate(1/4*x^2*log(c^2*x^2)/(c^2*x^7 - x^5), x)*log(c) + 32
*c^2*integrate(1/4*x^2*log(x)/(c^2*x^7 - x^5), x)*log(c) - 16*c^2*integrat
e(1/4*x^2*log(c^2*x^2)*log(x)/(c^2*x^7 - x^5), x) + 16*c^2*integrate(1/4*x
^2*log(x)^2/(c^2*x^7 - x^5), x) + 4*c^2*integrate(1/4*x^2*log(c^2*x^2)/(c^
2*x^7 - x^5), x) - (2*c^4*log(c*x + 1) + 2*c^4*log(c*x - 1) - 4*c^4*log(x)
 + (2*c^2*x^2 + 1)/x^4)*log(c)^2 + 16*integrate(1/4*log(c^2*x^2)/(c^2*x^7
- x^5), x)*log(c) - 32*integrate(1/4*log(x)/(c^2*x^7 - x^5), x)*log(c) - 8
*integrate(1/4*sqrt(c*x + 1)*sqrt(c*x - 1)*arctan(sqrt(c*x + 1)*sqrt(c*x -
 1))/(c^2*x^7 - x^5), x) + 16*integrate(1/4*log(c^2*x^2)*log(x)/(c^2*x^7 -
 x^5), x) - 16*integrate(1/4*log(x)^2/(c^2*x^7 - x^5), x) - 4*integrate(1/
4*log(c^2*x^2)/(c^2*x^7 - x^5), x))*x^4 + 4*arctan(sqrt(c*x + 1)*sqrt(c*x
- 1))^2 - log(c^2*x^2)^2)*b^2/x^4 - 1/4*a^2/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(106) = 212$.

Time = 0.14 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.76

$$\int \frac{(a + b\sec^{-1}(cx))^2}{x^5}\,dx$$

$$= \frac{1}{256}\left(24\,b^2c^3\arccos\left(\frac{1}{cx}\right)^2 + 48\,abc^3\arccos\left(\frac{1}{cx}\right) - 15\,b^2c^3 + \frac{48\,b^2c^2\sqrt{-\frac{1}{c^2x^2}+1}\arccos\left(\frac{1}{cx}\right)}{x} + \frac{48\,abc\right.$$

input
```
integrate((a+b*arcsec(c*x))^2/x^5,x, algorithm="giac")
```

output
```
1/256*(24*b^2*c^3*arccos(1/(c*x))^2 + 48*a*b*c^3*arccos(1/(c*x)) - 15*b^2*
c^3 + 48*b^2*c^2*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x))/x + 48*a*b*c^2*sqr
t(-1/(c^2*x^2) + 1)/x + 24*b^2*c/x^2 + 32*b^2*sqrt(-1/(c^2*x^2) + 1)*arcco
s(1/(c*x))/x^3 + 32*a*b*sqrt(-1/(c^2*x^2) + 1)/x^3 - 64*b^2*arccos(1/(c*x)
)^2/(c*x^4) - 128*a*b*arccos(1/(c*x))/(c*x^4) - 64*a^2/(c*x^4) + 8*b^2/(c*
x^4))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^5}\,dx = \int \frac{\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^2}{x^5}\,dx$$

input
```
int((a + b*acos(1/(c*x)))^2/x^5,x)
```

output
```
int((a + b*acos(1/(c*x)))^2/x^5, x)
```

Reduce [F]

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^5}\,dx = \frac{8\left(\int \frac{asec(cx)}{x^5}dx\right)ab\,x^4+4\left(\int \frac{asec(cx)^2}{x^5}dx\right)b^2x^4-a^2}{4x^4}$$

input
```
int((a+b*asec(c*x))^2/x^5,x)
```

output
```
(8*int(asec(c*x)/x**5,x)*a*b*x**4 + 4*int(asec(c*x)**2/x**5,x)*b**2*x**4 -
 a**2)/(4*x**4)
```

3.24 $\int x^3(a+b\sec^{-1}(cx))^3\,dx$

Optimal result 247
Mathematica [A] (verified) 248
Rubi [A] (verified) 248
Maple [A] (verified) 252
Fricas [F] 253
Sympy [F] 254
Maxima [F] 254
Giac [F(-2)] 255
Mupad [F(-1)] 255
Reduce [F] 255

Optimal result

Integrand size = 14, antiderivative size = 207

$$\begin{aligned}\int x^3(a+b\sec^{-1}(cx))^3\,dx = &-\frac{b^3\sqrt{1-\frac{1}{c^2x^2}}\,x}{4c^3}+\frac{b^2x^2(a+b\sec^{-1}(cx))}{4c^2}\\&+\frac{ib(a+b\sec^{-1}(cx))^2}{2c^4}-\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x(a+b\sec^{-1}(cx))^2}{2c^3}\\&-\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3(a+b\sec^{-1}(cx))^2}{4c}+\frac{1}{4}x^4(a+b\sec^{-1}(cx))^3\\&-\frac{b^2(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{c^4}\\&+\frac{ib^3\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2c^4}\end{aligned}$$

output

```
-1/4*b^3*(1-1/c^2/x^2)^(1/2)*x/c^3+1/4*b^2*x^2*(a+b*arcsec(c*x))/c^2+1/2*I
*b*(a+b*arcsec(c*x))^2/c^4-1/2*b*(1-1/c^2/x^2)^(1/2)*x*(a+b*arcsec(c*x))^2
/c^3-1/4*b*(1-1/c^2/x^2)^(1/2)*x^3*(a+b*arcsec(c*x))^2/c+1/4*x^4*(a+b*arcs
ec(c*x))^3-b^2*(a+b*arcsec(c*x))*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)/c^4
+1/2*I*b^3*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)/c^4
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.39

$$\int x^3\left(a+b\sec^{-1}(cx)\right)^3\,dx$$

$$=\frac{-2a^2bc\sqrt{1-\frac{1}{c^2x^2}}x-b^3c\sqrt{1-\frac{1}{c^2x^2}}x+ab^2c^2x^2-a^2bc^3\sqrt{1-\frac{1}{c^2x^2}}x^3+a^3c^4x^4-b^2\Big(-3ac^4x^4+b\Big(-2i+}{}$$

input
```
Integrate[x^3*(a + b*ArcSec[c*x])^3,x]
```

output
```
(-2*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x - b^3*c*Sqrt[1 - 1/(c^2*x^2)]*x + a*b^
2*c^2*x^2 - a^2*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*x^3 + a^3*c^4*x^4 - b^2*(-3*a*
c^4*x^4 + b*(-2*I + 2*c*Sqrt[1 - 1/(c^2*x^2)]*x + c^3*Sqrt[1 - 1/(c^2*x^2)
]*x^3))*ArcSec[c*x]^2 + b^3*c^4*x^4*ArcSec[c*x]^3 + b*ArcSec[c*x]*(c*x*(b^
2*c*x + 3*a^2*c^3*x^3 - 2*a*b*Sqrt[1 - 1/(c^2*x^2)]*(2 + c^2*x^2)) - 4*b^2
*Log[1 + E^((2*I)*ArcSec[c*x])]) - 4*a*b^2*Log[1/(c*x)] + (2*I)*b^3*PolyLo
g[2, -E^((2*I)*ArcSec[c*x])])/(4*c^4)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5745, 4909, 3042, 4674, 3042, 4254, 24, 4672, 25, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\sec^{-1}(cx)\right)^3\,dx$$

$$\Big\downarrow \text{ 5745}$$

$$\frac{\int c^5\sqrt{1-\frac{1}{c^2x^2}}x^5\left(a+b\sec^{-1}(cx)\right)^3d\sec^{-1}(cx)}{c^4}$$

$$\Big\downarrow \text{ 4909}$$

$$\frac{\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{4}b\int c^4x^4\left(a+b\sec^{-1}(cx)\right)^2d\sec^{-1}(cx)}{c^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{4}b\int \left(a+b\sec^{-1}(cx)\right)^2\csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^4d\sec^{-1}(cx)}{c^4}$$

$$\Big\downarrow \; 4674$$

$$\frac{\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{4}b\Big(\frac{2}{3}\int c^2x^2\left(a+b\sec^{-1}(cx)\right)^2d\sec^{-1}(cx)+\frac{1}{3}b^2\int c^2x^2d\sec^{-1}(cx)-\frac{1}{3}bc^2x^2\left(a+b\,\mathrm{se}\right.}{c^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{4}b\Big(\frac{2}{3}\int \left(a+b\sec^{-1}(cx)\right)^2\csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^2d\sec^{-1}(cx)+\frac{1}{3}b^2\int \csc\left(\sec^{-1}(cx)+\right.}{c^4}$$

$$\Big\downarrow \; 4254$$

$$\frac{\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{4}b\Big(\frac{2}{3}\int \left(a+b\sec^{-1}(cx)\right)^2\csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^2d\sec^{-1}(cx)-\frac{1}{3}b^2\int 1d\Big(-c\sqrt{1-\frac{1}{c^2x^2}}}{c^4}$$

$$\Big\downarrow \; 24$$

$$\frac{\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{4}b\Big(\frac{2}{3}\int \left(a+b\sec^{-1}(cx)\right)^2\csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^2d\sec^{-1}(cx)-\frac{1}{3}bc^2x^2\left(a+b\sec^{-1}(cx)\right)}{c^4}$$

$$\Big\downarrow \; 4672$$

$$\frac{\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{4}b\Big(\frac{2}{3}\Big(2b\int -c\sqrt{1-\frac{1}{c^2x^2}}x\left(a+b\sec^{-1}(cx)\right)d\sec^{-1}(cx)+cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(c\right.}{c^4}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{4}b\Big(\frac{2}{3}\Big(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\int c\sqrt{1-\frac{1}{c^2x^2}}x\left(a+b\sec^{-1}(cx)\right)d\sec^{-1}(c}{c^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{4}b\Big(\frac{2}{3}\Big(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\int \left(a+b\sec^{-1}(cx)\right)\tan\left(\sec^{-1}(cx)\right)d\sec^{-}}{c^4}$$

$\Big\downarrow$ 4202

$$\frac{\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{4}b\left(\frac{2}{3}\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\left(\frac{i\left(a+b\sec^{-1}(cx)\right)^2}{2b}-2i\int\frac{e^{2i\sec^{-1}(cx)}\left(a+b\sec^{-1}\right.}{1+e^{2i\sec^{-1}(cx)}}\right.\right.\right.}{c^4}$$

$\Big\downarrow$ 2620

$$\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{4}b\left(\frac{2}{3}\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\left(\frac{i\left(a+b\sec^{-1}(cx)\right)^2}{2b}-2i\left(\frac{1}{2}ib\int\log\left(1+e^{2i\sec}\right.\right.\right.\right.\right.$$

$\Big\downarrow$ 2715

$$\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{4}b\left(\frac{2}{3}\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\left(\frac{i\left(a+b\sec^{-1}(cx)\right)^2}{2b}-2i\left(\frac{1}{4}b\int e^{-2i\sec^{-1}(cx)}\,\mathrm{l}\right.\right.\right.\right.$$

$\Big\downarrow$ 2838

$$\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{4}b\left(\frac{2}{3}\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\left(\frac{i\left(a+b\sec^{-1}(cx)\right)^2}{2b}-2i\left(-\frac{1}{2}i\log\left(1+e^{2i\sec}\right.\right.\right.\right.\right.$$

input

```
Int[x^3*(a + b*ArcSec[c*x])^3,x]
```

output

```
((c^4*x^4*(a + b*ArcSec[c*x])^3)/4 - (3*b*((b^2*c*Sqrt[1 - 1/(c^2*x^2)]*x)
/3 - (b*c^2*x^2*(a + b*ArcSec[c*x]))/3 + (c^3*Sqrt[1 - 1/(c^2*x^2)]*x^3*(a
 + b*ArcSec[c*x])^2)/3 + (2*(c*Sqrt[1 - 1/(c^2*x^2)]*x*(a + b*ArcSec[c*x])
^2 - 2*b*(((I/2)*(a + b*ArcSec[c*x])^2)/b - (2*I)*((-1/2*I)*(a + b*ArcSec[
c*x])*Log[1 + E^((2*I)*ArcSec[c*x])] - (b*PolyLog[2, -E^((2*I)*ArcSec[c*x]
)])/4))))/3))/4)/c^4
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.76

method	result
derivativedivides	$\frac{\frac{a^3c^4x^4}{4}+b^3\left(\frac{\operatorname{arcsec}(cx)^3c^4x^4}{4}-\frac{\operatorname{arcsec}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}{4}-\frac{\operatorname{arcsec}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}{2}+\frac{i\operatorname{arcsec}(cx)^2}{2}+\frac{c^2x^2\operatorname{arcsec}(cx)}{4}-\frac{xc\sqrt{}}{}\right.}{}$
default	$\frac{\frac{a^3c^4x^4}{4}+b^3\left(\frac{\operatorname{arcsec}(cx)^3c^4x^4}{4}-\frac{\operatorname{arcsec}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}{4}-\frac{\operatorname{arcsec}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}{2}+\frac{i\operatorname{arcsec}(cx)^2}{2}+\frac{c^2x^2\operatorname{arcsec}(cx)}{4}-\frac{xc\sqrt{}}{}\right.}{}$
parts	$\frac{a^3x^4}{4}+\frac{b^3\left(\frac{\operatorname{arcsec}(cx)^3c^4x^4}{4}-\frac{\operatorname{arcsec}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}{4}-\frac{\operatorname{arcsec}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}{2}+\frac{i\operatorname{arcsec}(cx)^2}{2}+\frac{c^2x^2\operatorname{arcsec}(cx)}{4}-\frac{xc\sqrt{}}{}\right.}{c^4}$

input
```
int(x^3*(a+b*arcsec(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(1/4*a^3*c^4*x^4+b^3*(1/4*arcsec(c*x)^3*c^4*x^4-1/4*arcsec(c*x)^2*((
c^2*x^2-1)/c^2/x^2)^(1/2)*c^3*x^3-1/2*arcsec(c*x)^2*((c^2*x^2-1)/c^2/x^2)^
(1/2)*c*x+1/2*I*arcsec(c*x)^2+1/4*c^2*x^2*arcsec(c*x)-1/4*x*c*((c^2*x^2-1)
/c^2/x^2)^(1/2)-1/4*I-arcsec(c*x)*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)+1/
2*I*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2))+3*a*b^2*(1/4*arcsec(c*x)^
2*c^4*x^4-1/6*arcsec(c*x)*((c^2*x^2-1)/c^2/x^2)^(1/2)*c^3*x^3+1/12*c^2*x^2
-1/3*arcsec(c*x)*c*x*((c^2*x^2-1)/c^2/x^2)^(1/2)-1/3*ln(1/c/x))+3*a^2*b*(1
/4*c^4*x^4*arcsec(c*x)-1/12*(c^2*x^2-1)*(c^2*x^2+2)/((c^2*x^2-1)/c^2/x^2)^
(1/2)/c/x))
```

Fricas [F]

$$\int x^3\left(a+b\sec^{-1}(cx)\right)^3\,dx=\int\left(b\operatorname{arcsec}(cx)+a\right)^3x^3\,dx$$

input
```
integrate(x^3*(a+b*arcsec(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^3*arcsec(c*x)^3 + 3*a*b^2*x^3*arcsec(c*x)^2 + 3*a^2*b*x^3*a
rcsec(c*x) + a^3*x^3, x)
```

Sympy [F]

$$\int x^3 \left(a+b\sec^{-1}(cx)\right)^3 \, dx = \int x^3 (a+b\operatorname{asec}(cx))^3 \, dx$$

input
```
integrate(x**3*(a+b*asec(c*x))**3,x)
```

output
```
Integral(x**3*(a + b*asec(c*x))**3, x)
```

Maxima [F]

$$\int x^3 \left(a+b\sec^{-1}(cx)\right)^3 \, dx = \int (b\operatorname{arcsec}(cx)+a)^3 x^3 \, dx$$

input
```
integrate(x^3*(a+b*arcsec(c*x))^3,x, algorithm="maxima")
```

output
```
3/4*a*b^2*x^4*arcsec(c*x)^2 + 1/4*a^3*x^4 + 1/4*(3*x^4*arcsec(c*x) - (c^2*
x^3*(-1/(c^2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*a^2*b + 1/
16*(4*x^4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3*x^4*arctan(sqrt(c*x +
1)*sqrt(c*x - 1))*log(c^2*x^2)^2 - 16*integrate(3/16*((4*x^3*arctan(sqrt(c
*x + 1)*sqrt(c*x - 1))^2 - x^3*log(c^2*x^2)^2)*sqrt(c*x + 1)*sqrt(c*x - 1)
 + 4*(4*c^2*x^5*log(c)^2 - 4*x^3*log(c)^2 + 4*(c^2*x^5 - x^3)*log(x)^2 - (
(4*c^2*log(c) + c^2)*x^5 - x^3*(4*log(c) + 1) + 4*(c^2*x^5 - x^3)*log(x))*
log(c^2*x^2) + 8*(c^2*x^5*log(c) - x^3*log(c))*log(x))*arctan(sqrt(c*x + 1
)*sqrt(c*x - 1)))/(c^2*x^2 - 1), x))*b^3 + 1/4*((c^2*x^2 + 2*log(x^2))*sqr
t(c*x + 1)*sqrt(c*x - 1) - 2*(c^4*x^4 + c^2*x^2 - 2)*arctan(sqrt(c*x + 1)*
sqrt(c*x - 1)))*a*b^2/(sqrt(c*x + 1)*sqrt(c*x - 1)*c^4)
```

Giac [F(-2)]

Exception generated.

$$\int x^3 (a + b \sec^{-1}(cx))^3 \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*(a+b*arcsec(c*x))^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (a + b \sec^{-1}(cx))^3 \, dx = \int x^3 \left(a + b \operatorname{acos}\left(\frac{1}{c\,x}\right)\right)^3 \, dx$$

input
```
int(x^3*(a + b*acos(1/(c*x)))^3,x)
```

output
```
int(x^3*(a + b*acos(1/(c*x)))^3, x)
```

Reduce [F]

$$\int x^3 (a + b \sec^{-1}(cx))^3 \, dx = 3\left(\int asec(cx)\, x^3 dx\right) a^2 b + \left(\int asec(cx)^3\, x^3 dx\right) b^3 + 3\left(\int asec(cx)^2\, x^3 dx\right) a\, b^2 + \frac{a^3 x^4}{4}$$

input
```
int(x^3*(a+b*asec(c*x))^3,x)
```

output
```
(12*int(asec(c*x)*x**3,x)*a**2*b + 4*int(asec(c*x)**3*x**3,x)*b**3 + 12*in
t(asec(c*x)**2*x**3,x)*a*b**2 + a**3*x**4)/4
```

3.25 $\int x^2(a+b\sec^{-1}(cx))^3\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 236

$$\begin{aligned}\int x^2(a+b\sec^{-1}(cx))^3\,dx &= \frac{b^2x(a+b\sec^{-1}(cx))}{c^2}-\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2(a+b\sec^{-1}(cx))^2}{2c}\\
&\quad+\frac{1}{3}x^3(a+b\sec^{-1}(cx))^3\\
&\quad+\frac{ib(a+b\sec^{-1}(cx))^2\arctan\left(e^{i\sec^{-1}(cx)}\right)}{c^3}\\
&\quad-\frac{b^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^3}\\
&\quad-\frac{ib^2(a+b\sec^{-1}(cx))\operatorname{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)}{c^3}\\
&\quad+\frac{ib^2(a+b\sec^{-1}(cx))\operatorname{PolyLog}\left(2,ie^{i\sec^{-1}(cx)}\right)}{c^3}\\
&\quad+\frac{b^3\operatorname{PolyLog}\left(3,-ie^{i\sec^{-1}(cx)}\right)}{c^3}\\
&\quad-\frac{b^3\operatorname{PolyLog}\left(3,ie^{i\sec^{-1}(cx)}\right)}{c^3}\end{aligned}$$

output
```
b^2*x*(a+b*arcsec(c*x))/c^2-1/2*b*(1-1/c^2/x^2)^(1/2)*x^2*(a+b*arcsec(c*x)
)^2/c+1/3*x^3*(a+b*arcsec(c*x))^3+I*b*(a+b*arcsec(c*x))^2*arctan(1/c/x+I*(
1-1/c^2/x^2)^(1/2))/c^3-b^3*arctanh((1-1/c^2/x^2)^(1/2))/c^3-I*b^2*(a+b*ar
csec(c*x))*polylog(2,-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/c^3+I*b^2*(a+b*arcs
ec(c*x))*polylog(2,I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/c^3+b^3*polylog(3,-I*(
1/c/x+I*(1-1/c^2/x^2)^(1/2)))/c^3-b^3*polylog(3,I*(1/c/x+I*(1-1/c^2/x^2)^(
1/2)))/c^3
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.71

$$\int x^2\left(a+b\sec^{-1}(cx)\right)^3\,dx$$
$$=\frac{6ab^2cx-3a^2bc^2\sqrt{1-\frac{1}{c^2x^2}}x^2+2a^3c^3x^3-6b^3\coth^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)+6b^3cx\sec^{-1}(cx)-6ab^2c^2\sqrt{1-\frac{1}{c^2x^2}}}{}$$

input
```
Integrate[x^2*(a + b*ArcSec[c*x])^3,x]
```

output
```
(6*a*b^2*c*x - 3*a^2*b*c^2*Sqrt[1 - 1/(c^2*x^2)]*x^2 + 2*a^3*c^3*x^3 - 6*b
^3*ArcCoth[Sqrt[1 - 1/(c^2*x^2)]] + 6*b^3*c*x*ArcSec[c*x] - 6*a*b^2*c^2*Sq
rt[1 - 1/(c^2*x^2)]*x^2*ArcSec[c*x] + 6*a^2*b*c^3*x^3*ArcSec[c*x] - 3*b^3*
c^2*Sqrt[1 - 1/(c^2*x^2)]*x^2*ArcSec[c*x]^2 + 6*a*b^2*c^3*x^3*ArcSec[c*x]^
2 + 2*b^3*c^3*x^3*ArcSec[c*x]^3 + (6*I)*b^3*ArcSec[c*x]^2*ArcTan[E^(I*ArcS
ec[c*x])] - 6*a*b^2*ArcSec[c*x]*Log[1 - I*E^(I*ArcSec[c*x])] + 6*a*b^2*Arc
Sec[c*x]*Log[1 + I*E^(I*ArcSec[c*x])] - 3*a^2*b*Log[(1 + Sqrt[1 - 1/(c^2*x
^2)])*x] - (6*I)*b^2*(a + b*ArcSec[c*x])*PolyLog[2, (-I)*E^(I*ArcSec[c*x])
] + (6*I)*b^2*(a + b*ArcSec[c*x])*PolyLog[2, I*E^(I*ArcSec[c*x])] + 6*b^3*
PolyLog[3, (-I)*E^(I*ArcSec[c*x])] - 6*b^3*PolyLog[3, I*E^(I*ArcSec[c*x])]
)/(6*c^3)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5745, 4909, 3042, 4674, 3042, 4257, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b \sec^{-1}(cx)\right)^3 \, dx$$

$$\Big\downarrow \text{5745}$$

$$\frac{\int c^4 \sqrt{1-\frac{1}{c^2x^2}} x^4 \left(a+b \sec^{-1}(cx)\right)^3 d\sec^{-1}(cx)}{c^3}$$

$$\Big\downarrow \text{4909}$$

$$\frac{\frac{1}{3}c^3x^3\left(a+b\sec^{-1}(cx)\right)^3 - b\int c^3x^3\left(a+b\sec^{-1}(cx)\right)^2 d\sec^{-1}(cx)}{c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}c^3x^3\left(a+b\sec^{-1}(cx)\right)^3 - b\int \left(a+b\sec^{-1}(cx)\right)^2 \csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^3 d\sec^{-1}(cx)}{c^3}$$

$$\Big\downarrow \text{4674}$$

$$\frac{\frac{1}{3}c^3x^3\left(a+b\sec^{-1}(cx)\right)^3 - b\left(\frac{1}{2}\int cx\left(a+b\sec^{-1}(cx)\right)^2 d\sec^{-1}(cx) + b^2\int cxd\sec^{-1}(cx) + \frac{1}{2}c^2x^2\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\right.\right.}{c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}c^3x^3\left(a+b\sec^{-1}(cx)\right)^3 - b\left(\frac{1}{2}\int \left(a+b\sec^{-1}(cx)\right)^2 \csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right) d\sec^{-1}(cx) + b^2\int \csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right) d\right.}{c^3}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{1}{3}c^3x^3\left(a+b\sec^{-1}(cx)\right)^3 - b\left(\frac{1}{2}\int \left(a+b\sec^{-1}(cx)\right)^2 \csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right) d\sec^{-1}(cx) + \frac{1}{2}c^2x^2\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec\right.\right.}{c^3}$$

$$\Big\downarrow \text{4669}$$

$$\frac{1}{3}c^3x^3\left(a+b\sec^{-1}(cx)\right)^3-b\left(\frac{1}{2}\left(-2b\int\left(a+b\sec^{-1}(cx)\right)\log\left(1-ie^{i\sec^{-1}(cx)}\right)d\sec^{-1}(cx)+2b\int\left(a+b\sec^{-1}(cx\right.\right.\right.$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{3}c^3x^3\left(a+b\sec^{-1}(cx)\right)^3-b\left(\frac{1}{2}\left(2b\left(i\operatorname{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)-ib\int\operatorname{PolyLog}\left(2,-ie^{i\sec^{-1}(c\right.\right.\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{3}c^3x^3\left(a+b\sec^{-1}(cx)\right)^3-b\left(\frac{1}{2}\left(2b\left(i\operatorname{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)-b\int e^{-i\sec^{-1}(cx)}\operatorname{PolyLog}\left(2,\right.\right.\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{3}c^3x^3\left(a+b\sec^{-1}(cx)\right)^3-b\left(\frac{1}{2}\left(-2i\arctan\left(e^{i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)^2+2b\left(i\operatorname{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)(a\right.\right.\right.$$

input
```
Int[x^2*(a + b*ArcSec[c*x])^3,x]
```

output
```
((c^3*x^3*(a + b*ArcSec[c*x])^3)/3 - b*(-(b*c*x*(a + b*ArcSec[c*x])) + (c^
2*Sqrt[1 - 1/(c^2*x^2)]*x^2*(a + b*ArcSec[c*x])^2)/2 + b^2*ArcTanh[Sqrt[1
- 1/(c^2*x^2)]] + ((-2*I)*(a + b*ArcSec[c*x])^2*ArcTan[E^(I*ArcSec[c*x])]
+ 2*b*(I*(a + b*ArcSec[c*x])*PolyLog[2, (-I)*E^(I*ArcSec[c*x])] - b*PolyLo
g[3, (-I)*E^(I*ArcSec[c*x])]) - 2*b*(I*(a + b*ArcSec[c*x])*PolyLog[2, I*E^
(I*ArcSec[c*x])] - b*PolyLog[3, I*E^(I*ArcSec[c*x])]))/2))/c^3
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(286) = 572$.

Time = 1.08 (sec) , antiderivative size = 574, normalized size of antiderivative = 2.43

method	result
derivativedivides	$\frac{\frac{a^3c^3x^3}{3}+b^3\left(\frac{\operatorname{arcsec}(cx)\left(2c^2x^2\operatorname{arcsec}(cx)^2-3\operatorname{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+6\right)cx}{6}-\frac{\operatorname{arcsec}(cx)^2\ln\left(1-i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{2}+i\operatorname{arcse}$
default	$\frac{\frac{a^3c^3x^3}{3}+b^3\left(\frac{\operatorname{arcsec}(cx)\left(2c^2x^2\operatorname{arcsec}(cx)^2-3\operatorname{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+6\right)cx}{6}-\frac{\operatorname{arcsec}(cx)^2\ln\left(1-i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{2}+i\operatorname{arcse}$
parts	$\frac{a^3x^3}{3}+\frac{b^3\left(\frac{\operatorname{arcsec}(cx)\left(2c^2x^2\operatorname{arcsec}(cx)^2-3\operatorname{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+6\right)cx}{6}-\frac{\operatorname{arcsec}(cx)^2\ln\left(1-i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{2}+i\operatorname{arcse}$

input
```
int(x^2*(a+b*arcsec(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(1/3*a^3*c^3*x^3+b^3*(1/6*arcsec(c*x)*(2*c^2*x^2*arcsec(c*x)^2-3*arc
sec(c*x)*c*x*((c^2*x^2-1)/c^2/x^2)^(1/2)+6)*c*x-1/2*arcsec(c*x)^2*ln(1-I*(
1/c/x+I*(1-1/c^2/x^2)^(1/2)))+I*arcsec(c*x)*polylog(2,I*(1/c/x+I*(1-1/c^2/
x^2)^(1/2)))-polylog(3,I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))+1/2*arcsec(c*x)^2*
ln(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-I*arcsec(c*x)*polylog(2,-I*(1/c/x+I*
(1-1/c^2/x^2)^(1/2)))+polylog(3,-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))+2*I*arct
an(1/c/x+I*(1-1/c^2/x^2)^(1/2)))+3*a*b^2*(1/3*(c^2*x^2*arcsec(c*x)^2-arcse
c(c*x)*c*x*((c^2*x^2-1)/c^2/x^2)^(1/2)+1)*c*x+1/3*arcsec(c*x)*ln(1+I*(1/c/
x+I*(1-1/c^2/x^2)^(1/2)))-1/3*arcsec(c*x)*ln(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1
/2)))-1/3*I*dilog(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))+1/3*I*dilog(1-I*(1/c/
x+I*(1-1/c^2/x^2)^(1/2))))+3*a^2*b*(1/3*c^3*x^3*arcsec(c*x)-1/6*(c^2*x^2-1
)^(1/2)*(c*x*(c^2*x^2-1)^(1/2)+ln(c*x+(c^2*x^2-1)^(1/2)))/((c^2*x^2-1)/c^2
/x^2)^(1/2)/c/x))
```

Fricas [F]

$$\int x^2 \left(a+b\sec^{-1}(cx)\right)^3 \, dx = \int (b\,\mathrm{arcsec}\,(cx)+a)^3 x^2 \, dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^2*arcsec(c*x)^3 + 3*a*b^2*x^2*arcsec(c*x)^2 + 3*a^2*b*x^2*a
rcsec(c*x) + a^3*x^2, x)
```

Sympy [F]

$$\int x^2 \left(a+b\sec^{-1}(cx)\right)^3 \, dx = \int x^2 (a+b\,\mathrm{asec}\,(cx))^3 \, dx$$

input
```
integrate(x**2*(a+b*asec(c*x))**3,x)
```

output
```
Integral(x**2*(a + b*asec(c*x))**3, x)
```

Maxima [F]

$$\int x^2 \left(a + b\sec^{-1}(cx)\right)^3 \, dx = \int (b \operatorname{arcsec}(cx) + a)^3 x^2 \, dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))^3,x, algorithm="maxima")
```

output
```
1/3*b^3*x^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 1/4*b^3*x^3*arctan(sqr
t(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 - 1/2*a*b^2*c^2*(2*(c^2*x^3 + 3*x
)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5)*log(c)^2 - 12*b^3*c^2*int
egrate(1/4*x^4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^2 - 1), x)*log(c
)^2 + 12*b^3*c^2*integrate(1/4*x^4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log
(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 24*b^3*c^2*integrate(1/4*x^4*arctan(s
qrt(c*x + 1)*sqrt(c*x - 1))*log(x)/(c^2*x^2 - 1), x)*log(c) + 12*a*b^2*c^2
*integrate(1/4*x^4*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 24*a*b^2*c^2*in
tegrate(1/4*x^4*log(x)/(c^2*x^2 - 1), x)*log(c) + 1/3*a^3*x^3 + 12*b^3*c^2
*integrate(1/4*x^4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)*log(x)
/(c^2*x^2 - 1), x) - 12*b^3*c^2*integrate(1/4*x^4*arctan(sqrt(c*x + 1)*sqr
t(c*x - 1))*log(x)^2/(c^2*x^2 - 1), x) + 12*a*b^2*c^2*integrate(1/4*x^4*ar
ctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2/(c^2*x^2 - 1), x) + 4*b^3*c^2*integrat
e(1/4*x^4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)/(c^2*x^2 - 1),
x) - 3*a*b^2*c^2*integrate(1/4*x^4*log(c^2*x^2)^2/(c^2*x^2 - 1), x) + 12*a
*b^2*c^2*integrate(1/4*x^4*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) - 12*a*b^
2*c^2*integrate(1/4*x^4*log(x)^2/(c^2*x^2 - 1), x) + 3/2*a*b^2*(2*x/c^2 -
log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*log(c)^2 + 12*b^3*integrate(1/4*x^2*a
rctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^2 - 1), x)*log(c)^2 - 12*b^3*int
egrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)/(c^2*x^...
```

Giac [F(-2)]

Exception generated.

$$\int x^2 \left(a + b\sec^{-1}(cx)\right)^3 \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a+b*arcsec(c*x))^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (a + b \sec^{-1}(cx))^3 \, dx = \int x^2 \left(a + b \operatorname{acos}\left(\frac{1}{c\,x}\right)\right)^3 \, dx$$

input
```
int(x^2*(a + b*acos(1/(c*x)))^3,x)
```

output
```
int(x^2*(a + b*acos(1/(c*x)))^3, x)
```

Reduce [F]

$$\int x^2 (a + b \sec^{-1}(cx))^3 \, dx = 3\left(\int asec(cx)\, x^2 dx\right) a^2 b + \left(\int asec(cx)^3\, x^2 dx\right) b^3 + 3\left(\int asec(cx)^2\, x^2 dx\right) a\, b^2 + \frac{a^3 x^3}{3}$$

input
```
int(x^2*(a+b*asec(c*x))^3,x)
```

output
```
(9*int(asec(c*x)*x**2,x)*a**2*b + 3*int(asec(c*x)**3*x**2,x)*b**3 + 9*int(
asec(c*x)**2*x**2,x)*a*b**2 + a**3*x**3)/3
```

3.26 $\int x\left(a+b\sec^{-1}(cx)\right)^3\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 126

$$\int x\left(a+b\sec^{-1}(cx)\right)^3\,dx=\frac{3ib\left(a+b\sec^{-1}(cx)\right)^2}{2c^2}-\frac{3b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(a+b\sec^{-1}(cx)\right)^2}{2c}+\frac{1}{2}x^2\left(a+b\sec^{-1}(cx)\right)^3-\frac{3b^2\left(a+b\sec^{-1}(cx)\right)\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{c^2}+\frac{3ib^3\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2c^2}$$

output
```
3/2*I*b*(a+b*arcsec(c*x))^2/c^2-3/2*b*(1-1/c^2/x^2)^(1/2)*x*(a+b*arcsec(c*
x))^2/c+1/2*x^2*(a+b*arcsec(c*x))^3-3*b^2*(a+b*arcsec(c*x))*ln(1+(1/c/x+I*
(1-1/c^2/x^2)^(1/2))^2)/c^2+3/2*I*b^3*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1
/2))^2)/c^2
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.46

$$\int x\left(a+b\sec^{-1}(cx)\right)^3\,dx$$
$$= \frac{-3b^2\left(-ac^2x^2+b\left(-i+c\sqrt{1-\frac{1}{c^2x^2}}x\right)\right)\sec^{-1}(cx)^2+b^3c^2x^2\sec^{-1}(cx)^3-3b\sec^{-1}(cx)\left(acx\left(2b\sqrt{1-\frac{1}{c^2}}\right.\right.}{}$$

input
```
Integrate[x*(a + b*ArcSec[c*x])^3,x]
```

output
```
(-3*b^2*(-(a*c^2*x^2) + b*(-I + c*Sqrt[1 - 1/(c^2*x^2)]*x))*ArcSec[c*x]^2
+ b^3*c^2*x^2*ArcSec[c*x]^3 - 3*b*ArcSec[c*x]*(a*c*x*(2*b*Sqrt[1 - 1/(c^2*
x^2)] - a*c*x) + 2*b^2*Log[1 + E^((2*I)*ArcSec[c*x])]) + a*(a*c*x*(-3*b*Sq
rt[1 - 1/(c^2*x^2)] + a*c*x) - 6*b^2*Log[1/(c*x)]) + (3*I)*b^3*PolyLog[2,
-E^((2*I)*ArcSec[c*x])])/(2*c^2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5745, 4909, 3042, 4672, 25, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sec^{-1}(cx)\right)^3\,dx$$

$$\Big\downarrow \text{5745}$$

$$\frac{\int c^3\sqrt{1-\frac{1}{c^2x^2}}x^3\left(a+b\sec^{-1}(cx)\right)^3d\sec^{-1}(cx)}{c^2}$$

$$\Big\downarrow \text{4909}$$

$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{2}b\int c^2x^2\left(a+b\sec^{-1}(cx)\right)^2d\sec^{-1}(cx)}{c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{2}b\int\left(a+b\sec^{-1}(cx)\right)^2\csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^2d\sec^{-1}(cx)}{c^2}$$

$$\Big\downarrow \text{4672}$$

$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{2}b\Big(2b\int-c\sqrt{1-\frac{1}{c^2x^2}}x\left(a+b\sec^{-1}(cx)\right)d\sec^{-1}(cx)+cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{c^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{2}b\Big(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\int c\sqrt{1-\frac{1}{c^2x^2}}x\left(a+b\sec^{-1}(cx)\right)d\sec^{-1}(cx)}{c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{2}b\Big(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\int\left(a+b\sec^{-1}(cx)\right)\tan\left(\sec^{-1}(cx)\right)d\sec^{-1}(}{c^2}$$

$$\Big\downarrow \text{4202}$$

$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{2}b\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\left(\frac{i\left(a+b\sec^{-1}(cx)\right)^2}{2b}-2i\int\frac{e^{2i\sec^{-1}(cx)}\left(a+b\sec^{-1}(c\right.}{1+e^{2i\sec^{-1}(cx)}}\right.\right.}{c^2}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{2}b\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\left(\frac{i\left(a+b\sec^{-1}(cx)\right)^2}{2b}-2i\left(\frac{1}{2}ib\int\log\left(1+e^{2i\sec^{-1}}\right.\right.\right.\right.}{c^2}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{2}b\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\left(\frac{i\left(a+b\sec^{-1}(cx)\right)^2}{2b}-2i\left(\frac{1}{4}b\int e^{-2i\sec^{-1}(cx)}\log\left(\right.\right.\right.\right.}{c^2}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{2}b\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\left(\frac{i\left(a+b\sec^{-1}(cx)\right)^2}{2b}-2i\left(-\frac{1}{2}i\log\left(1+e^{2i\sec^{-1}(}\right.\right.\right.\right.}{c^2}$$

input
```
Int[x*(a + b*ArcSec[c*x])^3,x]
```

output
```
((c^2*x^2*(a + b*ArcSec[c*x])^3)/2 - (3*b*(c*Sqrt[1 - 1/(c^2*x^2)]*x*(a +
b*ArcSec[c*x])^2 - 2*b*(((I/2)*(a + b*ArcSec[c*x])^2)/b - (2*I)*((-1/2*I)*
(a + b*ArcSec[c*x])*Log[1 + E^((2*I)*ArcSec[c*x])] - (b*PolyLog[2, -E^((2*
I)*ArcSec[c*x])])/4))))/2)/c^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.00

method	result
derivativedivides	$\frac{\frac{a^3 c^2 x^2}{2}+b^3\left(\frac{\operatorname{arcsec}(cx)^2\left(c^2x^2\operatorname{arcsec}(cx)-3xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-3i\right)}{2}+3i\operatorname{arcsec}(cx)^2-3\operatorname{arcsec}(cx)\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right)\right.}{}$
default	$\frac{\frac{a^3 c^2 x^2}{2}+b^3\left(\frac{\operatorname{arcsec}(cx)^2\left(c^2x^2\operatorname{arcsec}(cx)-3xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-3i\right)}{2}+3i\operatorname{arcsec}(cx)^2-3\operatorname{arcsec}(cx)\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right)\right.}{}$
parts	$\frac{a^3x^2}{2}+\frac{b^3\left(\frac{\operatorname{arcsec}(cx)^2\left(c^2x^2\operatorname{arcsec}(cx)-3xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-3i\right)}{2}+3i\operatorname{arcsec}(cx)^2-3\operatorname{arcsec}(cx)\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right)\right.}{c^2}$

input
```
int(x*(a+b*arcsec(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(1/2*a^3*c^2*x^2+b^3*(1/2*arcsec(c*x)^2*(c^2*x^2*arcsec(c*x)-3*x*c*(
(c^2*x^2-1)/c^2/x^2)^(1/2)-3*I)+3*I*arcsec(c*x)^2-3*arcsec(c*x)*ln(1+(1/c/
x+I*(1-1/c^2/x^2)^(1/2))^2)+3/2*I*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))
^2))+3*a*b^2*(1/2*c^2*x^2*arcsec(c*x)^2-arcsec(c*x)*c*x*((c^2*x^2-1)/c^2/x
^2)^(1/2)-ln(1/c/x))+3*a^2*b*(1/2*c^2*x^2*arcsec(c*x)-1/2/((c^2*x^2-1)/c^2
/x^2)^(1/2)/c/x*(c^2*x^2-1)))
```

Fricas [F]

$$\int x\left(a+b\sec^{-1}(cx)\right)^3\,dx=\int (b\,\mathrm{arcsec}\,(cx)+a)^3x\,dx$$

input
```
integrate(x*(a+b*arcsec(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x*arcsec(c*x)^3 + 3*a*b^2*x*arcsec(c*x)^2 + 3*a^2*b*x*arcsec(
c*x) + a^3*x, x)
```

Sympy [F]

$$\int x\left(a+b\sec^{-1}(cx)\right)^3\,dx=\int x(a+b\,\mathrm{asec}\,(cx))^3\,dx$$

input
```
integrate(x*(a+b*asec(c*x))**3,x)
```

output
```
Integral(x*(a + b*asec(c*x))**3, x)
```

Maxima [F]

$$\int x\left(a+b\sec^{-1}(cx)\right)^3\,dx=\int (b\,\mathrm{arcsec}\,(cx)+a)^3x\,dx$$

input
```
integrate(x*(a+b*arcsec(c*x))^3,x, algorithm="maxima")
```

output
```
3/2*a*b^2*x^2*arcsec(c*x)^2 + 1/2*a^3*x^2 + 3/2*(x^2*arcsec(c*x) - x*sqrt(
-1/(c^2*x^2) + 1)/c)*a^2*b - 3*(x*sqrt(-1/(c^2*x^2) + 1)*arcsec(c*x)/c - l
og(x)/c^2)*a*b^2 + 1/8*(4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3*x^
2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 - 8*integrate(3/8*((4
*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - x*log(c^2*x^2)^2)*sqrt(c*x + 1)
*sqrt(c*x - 1) + 4*(2*c^2*x^3*log(c)^2 - 2*x*log(c)^2 + 2*(c^2*x^3 - x)*lo
g(x)^2 - ((2*c^2*log(c) + c^2)*x^3 - x*(2*log(c) + 1) + 2*(c^2*x^3 - x)*lo
g(x))*log(c^2*x^2) + 4*(c^2*x^3*log(c) - x*log(c))*log(x))*arctan(sqrt(c*x
 + 1)*sqrt(c*x - 1)))/(c^2*x^2 - 1), x))*b^3
```

Giac [F(-2)]

Exception generated.

$$\int x\left(a+b\sec^{-1}(cx)\right)^3\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a+b*arcsec(c*x))^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\sec^{-1}(cx)\right)^3\,dx=\int x\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^3\,dx$$

input
```
int(x*(a + b*acos(1/(c*x)))^3,x)
```

output
```
int(x*(a + b*acos(1/(c*x)))^3, x)
```

Reduce [F]

$$\int x\left(a+b\sec^{-1}(cx)\right)^3\,dx=3\left(\int asec(cx)\,xdx\right)a^2b+\left(\int asec(cx)^3\,xdx\right)b^3+3\left(\int asec(cx)^2\,xdx\right)a\,b^2+\frac{a^3x^2}{2}$$

input
```
int(x*(a+b*asec(c*x))^3,x)
```

output
```
(6*int(asec(c*x)*x,x)*a**2*b + 2*int(asec(c*x)**3*x,x)*b**3 + 6*int(asec(c
*x)**2*x,x)*a*b**2 + a**3*x**2)/2
```

3.27 $\int (a + b\sec^{-1}(cx))^3 \, dx$

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Optimal result

Integrand size = 10, antiderivative size = 158

$$\int (a+b\sec^{-1}(cx))^3 \, dx = x(a+b\sec^{-1}(cx))^3 + \frac{6ib(a+b\sec^{-1}(cx))^2 \arctan\left(e^{i\sec^{-1}(cx)}\right)}{c} - \frac{6ib^2(a+b\sec^{-1}(cx))\operatorname{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)}{c} + \frac{6ib^2(a+b\sec^{-1}(cx))\operatorname{PolyLog}\left(2,ie^{i\sec^{-1}(cx)}\right)}{c} + \frac{6b^3\operatorname{PolyLog}\left(3,-ie^{i\sec^{-1}(cx)}\right)}{c} - \frac{6b^3\operatorname{PolyLog}\left(3,ie^{i\sec^{-1}(cx)}\right)}{c}$$

output

```
x*(a+b*arcsec(c*x))^3+6*I*b*(a+b*arcsec(c*x))^2*arctan(1/c/x+I*(1-1/c^2/x^
2)^(1/2))/c-6*I*b^2*(a+b*arcsec(c*x))*polylog(2,-I*(1/c/x+I*(1-1/c^2/x^2)^
(1/2)))/c+6*I*b^2*(a+b*arcsec(c*x))*polylog(2,I*(1/c/x+I*(1-1/c^2/x^2)^(1/
2)))/c+6*b^3*polylog(3,-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/c-6*b^3*polylog(3
,I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/c
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.83

$$\int \left(a+b\sec^{-1}(cx)\right)^3\,dx$$

$$=\frac{a^3cx+3a^2bcx\sec^{-1}(cx)+3ab^2cx\sec^{-1}(cx)^2+b^3cx\sec^{-1}(cx)^3-6ab^2\sec^{-1}(cx)\log\left(1-ie^{i\sec^{-1}(cx)}\right)-}{}$$

input
```
Integrate[(a + b*ArcSec[c*x])^3,x]
```

output
```
(a^3*c*x + 3*a^2*b*c*x*ArcSec[c*x] + 3*a*b^2*c*x*ArcSec[c*x]^2 + b^3*c*x*A
rcSec[c*x]^3 - 6*a*b^2*ArcSec[c*x]*Log[1 - I*E^(I*ArcSec[c*x])] - 3*b^3*Ar
cSec[c*x]^2*Log[1 - I*E^(I*ArcSec[c*x])] + 6*a*b^2*ArcSec[c*x]*Log[1 + I*E
^(I*ArcSec[c*x])] + 3*b^3*ArcSec[c*x]^2*Log[1 + I*E^(I*ArcSec[c*x])] - 3*a
^2*b*Log[c*(1 + Sqrt[1 - 1/(c^2*x^2)])*x] - (6*I)*b^2*(a + b*ArcSec[c*x])*
PolyLog[2, (-I)*E^(I*ArcSec[c*x])] + (6*I)*b^2*(a + b*ArcSec[c*x])*PolyLog
[2, I*E^(I*ArcSec[c*x])] + 6*b^3*PolyLog[3, (-I)*E^(I*ArcSec[c*x])] - 6*b^
3*PolyLog[3, I*E^(I*ArcSec[c*x])])/c
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5739, 4909, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sec^{-1}(cx)\right)^3\,dx$$

$$\Big\downarrow \ 5739$$

$$\frac{\int c^2\sqrt{1-\frac{1}{c^2x^2}}x^2\left(a+b\sec^{-1}(cx)\right)^3d\sec^{-1}(cx)}{c}$$

$$\Big\downarrow \ 4909$$

$$\frac{cx\left(a+b\sec^{-1}(cx)\right)^3-3b\int cx\left(a+b\sec^{-1}(cx)\right)^2d\sec^{-1}(cx)}{c}$$

$\downarrow$ 3042

$$\frac{cx\left(a+b\sec^{-1}(cx)\right)^3-3b\int \left(a+b\sec^{-1}(cx)\right)^2\csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)d\sec^{-1}(cx)}{c}$$

$\downarrow$ 4669

$$\frac{cx\left(a+b\sec^{-1}(cx)\right)^3-3b\left(-2b\int \left(a+b\sec^{-1}(cx)\right)\log\left(1-ie^{i\sec^{-1}(cx)}\right)d\sec^{-1}(cx)+2b\int \left(a+b\sec^{-1}(cx)\right)\log\right.}{c}$$

$\downarrow$ 3011

$$cx\left(a+b\sec^{-1}(cx)\right)^3-3b\left(2b\left(i\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)-ib\int \text{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)d\right.\right.$$

$\downarrow$ 2720

$$cx\left(a+b\sec^{-1}(cx)\right)^3-3b\left(2b\left(i\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)-b\int e^{-i\sec^{-1}(cx)}\text{PolyLog}\left(2,-ie^{i\right.\right.\right.$$

$\downarrow$ 7143

$$cx\left(a+b\sec^{-1}(cx)\right)^3-3b\left(-2i\arctan\left(e^{i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)^2+2b\left(i\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)\left(a+b\right.\right.\right.$$

input
```
Int[(a + b*ArcSec[c*x])^3,x]
```

output
```
(c*x*(a + b*ArcSec[c*x])^3 - 3*b*((-2*I)*(a + b*ArcSec[c*x])^2*ArcTan[E^(I
*ArcSec[c*x])] + 2*b*(I*(a + b*ArcSec[c*x])*PolyLog[2, (-I)*E^(I*ArcSec[c*
x])] - b*PolyLog[3, (-I)*E^(I*ArcSec[c*x])]) - 2*b*(I*(a + b*ArcSec[c*x])*
PolyLog[2, I*E^(I*ArcSec[c*x])] - b*PolyLog[3, I*E^(I*ArcSec[c*x])])))/c
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5739
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/c   Subst[
Int[(a + b*x)^n*Sec[x]*Tan[x], x], x, ArcSec[c*x]], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 445 vs. 2(216) = 432.

Time = 1.15 (sec) , antiderivative size = 446, normalized size of antiderivative = 2.82

method	result
derivativedivides	$cx\left(b^3 \operatorname{arcsec}(cx)^3+3a\,b^2 \operatorname{arcsec}(cx)^2+3 \operatorname{arcsec}(cx)a^2b+a^3\right)-3\ln\left(1-i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)b^3 \operatorname{arcsec}(cx)^2+3\ln\Big(1+i\Big($
default	$cx\left(b^3 \operatorname{arcsec}(cx)^3+3a\,b^2 \operatorname{arcsec}(cx)^2+3 \operatorname{arcsec}(cx)a^2b+a^3\right)-3\ln\left(1-i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)b^3 \operatorname{arcsec}(cx)^2+3\ln\Big(1+i\Big($

input
```
int((a+b*arcsec(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(c*x*(b^3*arcsec(c*x)^3+3*a*b^2*arcsec(c*x)^2+3*arcsec(c*x)*a^2*b+a^3)
-3*ln(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))*b^3*arcsec(c*x)^2+3*ln(1+I*(1/c/x
+I*(1-1/c^2/x^2)^(1/2)))*b^3*arcsec(c*x)^2-6*ln(1-I*(1/c/x+I*(1-1/c^2/x^2)
^(1/2)))*arcsec(c*x)*a*b^2+6*arcsec(c*x)*ln(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/
2)))*a*b^2-6*polylog(3,I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))*b^3+6*polylog(3,-I
*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))*b^3+6*I*b^3*arcsec(c*x)*polylog(2,I*(1/c/x
+I*(1-1/c^2/x^2)^(1/2)))-6*I*b^3*arcsec(c*x)*polylog(2,-I*(1/c/x+I*(1-1/c^
2/x^2)^(1/2)))+6*I*arctan(1/c/x+I*(1-1/c^2/x^2)^(1/2))*a^2*b+6*I*polylog(2
,I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))*a*b^2-6*I*polylog(2,-I*(1/c/x+I*(1-1/c^2
/x^2)^(1/2)))*a*b^2)
```

Fricas [F]

$$\int \left(a+b\sec^{-1}(cx)\right)^3\,dx=\int \left(b\,\mathrm{arcsec}\left(cx\right)+a\right)^3\,dx$$

input
```
integrate((a+b*arcsec(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*arcsec(c*x)^3 + 3*a*b^2*arcsec(c*x)^2 + 3*a^2*b*arcsec(c*x) +
 a^3, x)
```

Sympy [F]

$$\int \left(a+b\sec^{-1}(cx)\right)^3\,dx=\int \left(a+b\,\mathrm{asec}\left(cx\right)\right)^3\,dx$$

input
```
integrate((a+b*asec(c*x))**3,x)
```

output
```
Integral((a + b*asec(c*x))**3, x)
```

Maxima [F]

$$\int \left(a+b\sec^{-1}(cx)\right)^3\,dx=\int \left(b\,\mathrm{arcsec}\left(cx\right)+a\right)^3\,dx$$

input
```
integrate((a+b*arcsec(c*x))^3,x, algorithm="maxima")
```

output
```
-3/2*a*b^2*c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*log(c)^2 -
12*b^3*c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^2
- 1), x)*log(c)^2 + b^3*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3/4*b^3*
x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 + 12*b^3*c^2*integrat
e(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)/(c^2*x^2 - 1),
x)*log(c) - 24*b^3*c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1
))*log(x)/(c^2*x^2 - 1), x)*log(c) + 12*a*b^2*c^2*integrate(1/4*x^2*log(c^
2*x^2)/(c^2*x^2 - 1), x)*log(c) - 24*a*b^2*c^2*integrate(1/4*x^2*log(x)/(c
^2*x^2 - 1), x)*log(c) + 12*b^3*c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)
*sqrt(c*x - 1))*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) - 12*b^3*c^2*integra
te(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(x)^2/(c^2*x^2 - 1), x)
+ 12*a*b^2*c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2/(c^
2*x^2 - 1), x) + 12*b^3*c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*
x - 1))*log(c^2*x^2)/(c^2*x^2 - 1), x) - 3*a*b^2*c^2*integrate(1/4*x^2*log
(c^2*x^2)^2/(c^2*x^2 - 1), x) + 12*a*b^2*c^2*integrate(1/4*x^2*log(c^2*x^2
)*log(x)/(c^2*x^2 - 1), x) - 12*a*b^2*c^2*integrate(1/4*x^2*log(x)^2/(c^2*
x^2 - 1), x) - 3/2*a*b^2*(log(c*x + 1)/c - log(c*x - 1)/c)*log(c)^2 + 12*b
^3*integrate(1/4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^2 - 1), x)*log
(c)^2 - 12*b^3*integrate(1/4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x
^2)/(c^2*x^2 - 1), x)*log(c) + 24*b^3*integrate(1/4*arctan(sqrt(c*x + 1...
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+b\sec^{-1}(cx)\right)^3\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsec(c*x))^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^{-1}(cx)\right)^3\,dx=\int \left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^3\,dx$$

input
```
int((a + b*acos(1/(c*x)))^3,x)
```

output
```
int((a + b*acos(1/(c*x)))^3, x)
```

Reduce [F]

$$\int \left(a+b\sec^{-1}(cx)\right)^3\,dx=3\left(\int asec(cx)\,dx\right)a^2b+\left(\int asec(cx)^3\,dx\right)b^3+3\left(\int asec(cx)^2\,dx\right)a\,b^2+a^3x$$

input
```
int((a+b*asec(c*x))^3,x)
```

output
```
3*int(asec(c*x),x)*a**2*b + int(asec(c*x)**3,x)*b**3 + 3*int(asec(c*x)**2,
x)*a*b**2 + a**3*x
```

3.28 $\int \frac{(a+b\sec^{-1}(cx))^3}{x}\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 128

$$\begin{aligned}\int \frac{(a+b\sec^{-1}(cx))^3}{x}\,dx &= \frac{i(a+b\sec^{-1}(cx))^4}{4b} - (a+b\sec^{-1}(cx))^3\log\left(1+e^{2i\sec^{-1}(cx)}\right)\\ &\quad+\frac{3}{2}ib(a+b\sec^{-1}(cx))^2\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)\\ &\quad-\frac{3}{2}b^2(a+b\sec^{-1}(cx))\operatorname{PolyLog}\left(3,-e^{2i\sec^{-1}(cx)}\right)\\ &\quad-\frac{3}{4}ib^3\operatorname{PolyLog}\left(4,-e^{2i\sec^{-1}(cx)}\right)\end{aligned}$$

output

```
1/4*I*(a+b*arcsec(c*x))^4/b-(a+b*arcsec(c*x))^3*ln(1+(1/c/x+I*(1-1/c^2/x^2
)^(1/2))^2)+3/2*I*b*(a+b*arcsec(c*x))^2*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^
(1/2))^2)-3/2*b^2*(a+b*arcsec(c*x))*polylog(3,-(1/c/x+I*(1-1/c^2/x^2)^(1/2
))^2)-3/4*I*b^3*polylog(4,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.59

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x}\,dx = \frac{1}{4}\Big(6ia^2b\sec^{-1}(cx)^2+4iab^2\sec^{-1}(cx)^3+ib^3\sec^{-1}(cx)^4 - 12a^2b\sec^{-1}(cx)\log\left(1+e^{2i\sec^{-1}(cx)}\right) - 12ab^2\sec^{-1}(cx)^2\log\left(1+e^{2i\sec^{-1}(cx)}\right) - 4b^3\sec^{-1}(cx)^3\log\left(1+e^{2i\sec^{-1}(cx)}\right)+4a^3\log(cx) + 6ib(a+b\sec^{-1}(cx))^2\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right) - 6b^2(a+b\sec^{-1}(cx))\operatorname{PolyLog}\left(3,-e^{2i\sec^{-1}(cx)}\right) - 3ib^3\operatorname{PolyLog}\left(4,-e^{2i\sec^{-1}(cx)}\right)\Big)$$

input
```
Integrate[(a + b*ArcSec[c*x])^3/x,x]
```

output
```
((6*I)*a^2*b*ArcSec[c*x]^2 + (4*I)*a*b^2*ArcSec[c*x]^3 + I*b^3*ArcSec[c*x]
^4 - 12*a^2*b*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] - 12*a*b^2*ArcSec
[c*x]^2*Log[1 + E^((2*I)*ArcSec[c*x])] - 4*b^3*ArcSec[c*x]^3*Log[1 + E^((2
*I)*ArcSec[c*x])] + 4*a^3*Log[c*x] + (6*I)*b*(a + b*ArcSec[c*x])^2*PolyLog
[2, -E^((2*I)*ArcSec[c*x])] - 6*b^2*(a + b*ArcSec[c*x])*PolyLog[3, -E^((2*
I)*ArcSec[c*x])] - (3*I)*b^3*PolyLog[4, -E^((2*I)*ArcSec[c*x])])/4
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5745, 3042, 4202, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x}\,dx$$

$$\Big\downarrow \text{5745}$$

$$\int cx\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^3d\sec^{-1}(cx)$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(\sec^{-1}(cx)\right)\left(a+b\sec^{-1}(cx)\right)^3d\sec^{-1}(cx)$$

$$\Big\downarrow \text{4202}$$

$$\frac{i(a+b\sec^{-1}(cx))^4}{4b}-2i\int \frac{e^{2i\sec^{-1}(cx)}(a+b\sec^{-1}(cx))^3}{1+e^{2i\sec^{-1}(cx)}}d\sec^{-1}(cx)$$

$$\Big\downarrow \text{2620}$$

$$\frac{i(a+b\sec^{-1}(cx))^4}{4b}-2i\left(\frac{3}{2}ib\int (a+b\sec^{-1}(cx))^2\log\left(1+e^{2i\sec^{-1}(cx)}\right)d\sec^{-1}(cx)-\frac{1}{2}i\log\left(1+e^{2i\sec^{-1}(cx)}\right)(a+b\sec^{-1}(cx))^3\right)$$

$$\Big\downarrow \text{3011}$$

$$\frac{i(a+b\sec^{-1}(cx))^4}{4b}-2i\left(\frac{3}{2}ib\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)(a+b\sec^{-1}(cx))^2-ib\int (a+b\sec^{-1}(cx))\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)d\,\mathrm{s}$$

$$\Big\downarrow \text{7163}$$

$$\frac{i(a+b\sec^{-1}(cx))^4}{4b}-2i\left(\frac{3}{2}ib\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)(a+b\sec^{-1}(cx))^2-ib\left(\frac{1}{2}ib\int \operatorname{PolyLog}\left(3,-e^{2i\sec^{-1}(cx)}\right)d\sec^{-1}(cx)-\frac{1}{2}\right.\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$\frac{i(a+b\sec^{-1}(cx))^4}{4b}-2i\left(\frac{3}{2}ib\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)(a+b\sec^{-1}(cx))^2-ib\left(\frac{1}{4}b\int e^{-2i\sec^{-1}(cx)}\operatorname{PolyLog}\left(3,-e^{2i\sec^{-1}(cx)}\right)de^{2}\right.\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$\frac{i(a+b\sec^{-1}(cx))^4}{4b}-2i\left(\frac{3}{2}ib\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)(a+b\sec^{-1}(cx))^2-ib\left(\frac{1}{4}b\operatorname{PolyLog}\left(4,-e^{2i\sec^{-1}(cx)}\right)-\frac{1}{2}i\operatorname{PolyLog}\left(3,-\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcSec[c*x])^3/x,x]
```

output
```
((I/4)*(a + b*ArcSec[c*x])^4)/b - (2*I)*((-1/2*I)*(a + b*ArcSec[c*x])^3*Lo
g[1 + E^((2*I)*ArcSec[c*x])] + ((3*I)/2)*b*((I/2)*(a + b*ArcSec[c*x])^2*Po
lyLog[2, -E^((2*I)*ArcSec[c*x])] - I*b*((-1/2*I)*(a + b*ArcSec[c*x])*PolyL
og[3, -E^((2*I)*ArcSec[c*x])] + (b*PolyLog[4, -E^((2*I)*ArcSec[c*x])])/4))
)
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 363 vs. 2(177) = 354.

Time = 0.61 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.84

method	result
parts	$a^3\ln(x)+b^3\left(\frac{i\,\mathrm{arcsec}(cx)^4}{4}-\mathrm{arcsec}(cx)^3\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right)+\frac{3i\,\mathrm{arcsec}(cx)^2\,\mathrm{po}}{}\right.$
derivativedivides	$a^3\ln(cx)+b^3\left(\frac{i\,\mathrm{arcsec}(cx)^4}{4}-\mathrm{arcsec}(cx)^3\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right)+\frac{3i\,\mathrm{arcsec}(cx)^2\,\mathrm{p}}{}\right.$
default	$a^3\ln(cx)+b^3\left(\frac{i\,\mathrm{arcsec}(cx)^4}{4}-\mathrm{arcsec}(cx)^3\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right)+\frac{3i\,\mathrm{arcsec}(cx)^2\,\mathrm{p}}{}\right.$

input
```
int((a+b*arcsec(c*x))^3/x,x,method=_RETURNVERBOSE)
```

output
```
a^3*ln(x)+b^3*(1/4*I*arcsec(c*x)^4-arcsec(c*x)^3*ln(1+(1/c/x+I*(1-1/c^2/x^
2)^(1/2))^2)+3/2*I*arcsec(c*x)^2*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^
2)-3/2*arcsec(c*x)*polylog(3,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)-3/4*I*polyl
og(4,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2))+3*a*b^2*(1/3*I*arcsec(c*x)^3-arcse
c(c*x)^2*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)+I*arcsec(c*x)*polylog(2,-(1
/c/x+I*(1-1/c^2/x^2)^(1/2))^2)-1/2*polylog(3,-(1/c/x+I*(1-1/c^2/x^2)^(1/2)
)^2))+3*a^2*b*(1/2*I*arcsec(c*x)^2-arcsec(c*x)*ln(1+(1/c/x+I*(1-1/c^2/x^2)
^(1/2))^2)+1/2*I*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2))
```

Fricas [F]

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x}\,dx=\int \frac{(b\,\mathrm{arcsec}\,(cx)+a)^3}{x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))^3/x,x, algorithm="fricas")
```

output
```
integral((b^3*arcsec(c*x)^3 + 3*a*b^2*arcsec(c*x)^2 + 3*a^2*b*arcsec(c*x)
+ a^3)/x, x)
```

Sympy [F]

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x}\,dx=\int \frac{(a+b\,\mathrm{asec}\,(cx))^3}{x}\,dx$$

input
```
integrate((a+b*asec(c*x))**3/x,x)
```

output
```
Integral((a + b*asec(c*x))**3/x, x)
```

Maxima [F]

$$\int \frac{(a + b\sec^{-1}(cx))^3}{x}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx) + a)^3}{x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))^3/x,x, algorithm="maxima")
```

output
```
-3/2*a*b^2*c^2*(log(c*x + 1)/c^2 + log(c*x - 1)/c^2)*log(c)^2 - 12*b^3*c^2
*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^3 - x), x)*l
og(c)^2 + 12*b^3*c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))
*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) - 24*b^3*c^2*integrate(1/4*x^2*arct
an(sqrt(c*x + 1)*sqrt(c*x - 1))*log(x)/(c^2*x^3 - x), x)*log(c) + 12*a*b^2
*c^2*integrate(1/4*x^2*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) - 24*a*b^2*c^
2*integrate(1/4*x^2*log(x)/(c^2*x^3 - x), x)*log(c) + b^3*arctan(sqrt(c*x
+ 1)*sqrt(c*x - 1))^3*log(x) - 3/4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))
*log(c^2*x^2)^2*log(x) + 24*b^3*c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)
*sqrt(c*x - 1))*log(c^2*x^2)*log(x)/(c^2*x^3 - x), x) - 12*b^3*c^2*integra
te(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(x)^2/(c^2*x^3 - x), x)
+ 12*a*b^2*c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2/(c^
2*x^3 - x), x) - 3*a*b^2*c^2*integrate(1/4*x^2*log(c^2*x^2)^2/(c^2*x^3 - x
), x) + 12*a*b^2*c^2*integrate(1/4*x^2*log(c^2*x^2)*log(x)/(c^2*x^3 - x),
x) - 12*a*b^2*c^2*integrate(1/4*x^2*log(x)^2/(c^2*x^3 - x), x) + 12*a^2*b*
c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^3 - x), x
) + 3/2*a*b^2*(log(c*x + 1) + log(c*x - 1) - 2*log(x))*log(c)^2 + 12*b^3*i
ntegrate(1/4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^3 - x), x)*log(c)^
2 - 12*b^3*integrate(1/4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)/
(c^2*x^3 - x), x)*log(c) + 24*b^3*integrate(1/4*arctan(sqrt(c*x + 1)*sq...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b\sec^{-1}(cx))^3}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsec(c*x))^3/x,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x}\,dx = \int \frac{\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^3}{x}\,dx$$

input
```
int((a + b*acos(1/(c*x)))^3/x,x)
```

output
```
int((a + b*acos(1/(c*x)))^3/x, x)
```

Reduce [F]

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x}\,dx = 3\left(\int \frac{asec(cx)}{x}dx\right)a^2b + \left(\int \frac{asec(cx)^3}{x}dx\right)b^3 + 3\left(\int \frac{asec(cx)^2}{x}dx\right)a\,b^2 + \log(x)\,a^3$$

input
```
int((a+b*asec(c*x))^3/x,x)
```

output
```
3*int(asec(c*x)/x,x)*a**2*b + int(asec(c*x)**3/x,x)*b**3 + 3*int(asec(c*x)
**2/x,x)*a*b**2 + log(x)*a**3
```

3.29 $\int \frac{(a+b\sec^{-1}(cx))^3}{x^2}\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 80

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^2}\,dx = -6b^3c\sqrt{1-\frac{1}{c^2x^2}} + \frac{6b^2(a+b\sec^{-1}(cx))}{x} + 3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2 - \frac{(a+b\sec^{-1}(cx))^3}{x}$$

output
```
-6*b^3*c*(1-1/c^2/x^2)^(1/2)+6*b^2*(a+b*arcsec(c*x))/x+3*b*c*(1-1/c^2/x^2)
^(1/2)*(a+b*arcsec(c*x))^2-(a+b*arcsec(c*x))^3/x
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.76

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^2}\,dx = \frac{-a^3+6ab^2+3a^2bc\sqrt{1-\frac{1}{c^2x^2}}x-6b^3c\sqrt{1-\frac{1}{c^2x^2}}x+3b\left(-a^2+2b^2+2abc\sqrt{1-\frac{1}{c^2x^2}}x\right)\sec^{-1}(cx)+3b^2}{x}$$

input
```
Integrate[(a + b*ArcSec[c*x])^3/x^2,x]
```

output
```
(-a^3 + 6*a*b^2 + 3*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x - 6*b^3*c*Sqrt[1 - 1/(
c^2*x^2)]*x + 3*b*(-a^2 + 2*b^2 + 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x)*ArcSec[
c*x] + 3*b^2*(-a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x)*ArcSec[c*x]^2 - b^3*ArcSec
[c*x]^3)/x
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5745, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x^2}dx$$

$$\Big\downarrow \text{5745}$$

$$c\int \sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^3d\sec^{-1}(cx)$$

$$\Big\downarrow \text{3042}$$

$$c\int \left(a+b\sec^{-1}(cx)\right)^3\sin\left(\sec^{-1}(cx)\right)d\sec^{-1}(cx)$$

$$\Big\downarrow \text{3777}$$

$$c\left(3b\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{cx}d\sec^{-1}(cx)-\frac{\left(a+b\sec^{-1}(cx)\right)^3}{cx}\right)$$

$$\Big\downarrow \text{3042}$$

$$c\left(3b\int \left(a+b\sec^{-1}(cx)\right)^2\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)d\sec^{-1}(cx)-\frac{\left(a+b\sec^{-1}(cx)\right)^3}{cx}\right)$$

$$\Big\downarrow \text{3777}$$

$$c\left(3b\left(2b\int -\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)d\sec^{-1}(cx)+\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2\right)-\frac{\left(a+b\sec^{-1}(cx)\right)^3}{cx}\right)$$

$$\Big\downarrow \text{25}$$

$$c\left(3b\left(\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2-2b\int \sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))\,d\sec^{-1}(cx)\right)-\frac{(a+b\sec^{-1}(cx))^3}{cx}\right)$$

$$\Big\downarrow \ 3042$$

$$c\left(3b\left(\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2-2b\int (a+b\sec^{-1}(cx))\sin\left(\sec^{-1}(cx)\right)d\sec^{-1}(cx)\right)-\frac{(a+b\sec^{-1}(cx))^3}{cx}$$

$$\Big\downarrow \ 3777$$

$$c\left(3b\left(\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2-2b\left(b\int \frac{1}{cx}d\sec^{-1}(cx)-\frac{a+b\sec^{-1}(cx)}{cx}\right)\right)-\frac{(a+b\sec^{-1}(cx))^3}{cx}\right)$$

$$\Big\downarrow \ 3042$$

$$c\left(3b\left(\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2-2b\left(b\int \sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)d\sec^{-1}(cx)-\frac{a+b\sec^{-1}(cx)}{cx}\right)\right)-(a+b$$

$$\Big\downarrow \ 3117$$

$$c\left(3b\left(\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2-2b\left(b\sqrt{1-\frac{1}{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{cx}\right)\right)-\frac{(a+b\sec^{-1}(cx))^3}{cx}\right)$$

input
```
Int[(a + b*ArcSec[c*x])^3/x^2,x]
```

output
```
c*(-((a + b*ArcSec[c*x])^3/(c*x)) + 3*b*(Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcS
ec[c*x])^2 - 2*b*(b*Sqrt[1 - 1/(c^2*x^2)] - (a + b*ArcSec[c*x])/(c*x))))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(76) = 152$.

Time = 0.45 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.45

method	result
parts	$-\frac{a^3}{x}+b^3c\left(-\frac{\operatorname{arcsec}(cx)^3}{cx}+3\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,\operatorname{arcsec}(cx)^2-6\sqrt{\frac{c^2x^2-1}{c^2x^2}}+\frac{6\operatorname{arcsec}(cx)}{cx}\right)+3a\,b^2c\Bigg($
derivativedivides	$c\left(-\frac{a^3}{cx}+b^3\left(-\frac{\operatorname{arcsec}(cx)^3}{cx}+3\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,\operatorname{arcsec}(cx)^2-6\sqrt{\frac{c^2x^2-1}{c^2x^2}}+\frac{6\operatorname{arcsec}(cx)}{cx}\right)+3a\,b^2\Bigg(\right.$
default	$c\left(-\frac{a^3}{cx}+b^3\left(-\frac{\operatorname{arcsec}(cx)^3}{cx}+3\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,\operatorname{arcsec}(cx)^2-6\sqrt{\frac{c^2x^2-1}{c^2x^2}}+\frac{6\operatorname{arcsec}(cx)}{cx}\right)+3a\,b^2\Bigg(\right.$

input
```
int((a+b*arcsec(c*x))^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^3/x+b^3*c*(-1/c/x*arcsec(c*x)^3+3*((c^2*x^2-1)/c^2/x^2)^(1/2)*arcsec(c*
x)^2-6*((c^2*x^2-1)/c^2/x^2)^(1/2)+6/c/x*arcsec(c*x))+3*a*b^2*c*(-1/c/x*ar
csec(c*x)^2+2/c/x+2*arcsec(c*x)*((c^2*x^2-1)/c^2/x^2)^(1/2))+3*a^2*b*c*(-1
/c/x*arcsec(c*x)+1/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^2/x^2*(c^2*x^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.22

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^2}\,dx =$$
$$-\frac{b^3\operatorname{arcsec}(cx)^3+3\,ab^2\operatorname{arcsec}(cx)^2+a^3-6\,ab^2+3\,(a^2b-2\,b^3)\operatorname{arcsec}(cx)-3\,\left(b^3\operatorname{arcsec}(cx)^2+2\,ab^2\right.}{x}$$

input
```
integrate((a+b*arcsec(c*x))^3/x^2,x, algorithm="fricas")
```

output
```
-(b^3*arcsec(c*x)^3 + 3*a*b^2*arcsec(c*x)^2 + a^3 - 6*a*b^2 + 3*(a^2*b - 2
*b^3)*arcsec(c*x) - 3*(b^3*arcsec(c*x)^2 + 2*a*b^2*arcsec(c*x) + a^2*b - 2
*b^3)*sqrt(c^2*x^2 - 1))/x
```

Sympy [F]

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^2}\,dx = \int \frac{(a+b\operatorname{asec}(cx))^3}{x^2}\,dx$$

input
```
integrate((a+b*asec(c*x))**3/x**2,x)
```

output
```
Integral((a + b*asec(c*x))**3/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.82

$$\int \frac{(a + b\sec^{-1}(cx))^3}{x^2}\,dx$$
$$= -\frac{b^3 \operatorname{arcsec}(cx)^3}{x} + 3\left(c\sqrt{-\frac{1}{c^2x^2} + 1} - \frac{\operatorname{arcsec}(cx)}{x}\right)a^2b$$
$$+ 6\left(c\sqrt{-\frac{1}{c^2x^2} + 1}\operatorname{arcsec}(cx) + \frac{1}{x}\right)ab^2$$
$$+ 3\left(c\sqrt{-\frac{1}{c^2x^2} + 1}\operatorname{arcsec}(cx)^2 - 2c\sqrt{-\frac{1}{c^2x^2} + 1} + \frac{2\operatorname{arcsec}(cx)}{x}\right)b^3$$
$$- \frac{3ab^2 \operatorname{arcsec}(cx)^2}{x} - \frac{a^3}{x}$$

input
```
integrate((a+b*arcsec(c*x))^3/x^2,x, algorithm="maxima")
```

output
```
-b^3*arcsec(c*x)^3/x + 3*(c*sqrt(-1/(c^2*x^2) + 1) - arcsec(c*x)/x)*a^2*b
+ 6*(c*sqrt(-1/(c^2*x^2) + 1)*arcsec(c*x) + 1/x)*a*b^2 + 3*(c*sqrt(-1/(c^2
*x^2) + 1)*arcsec(c*x)^2 - 2*c*sqrt(-1/(c^2*x^2) + 1) + 2*arcsec(c*x)/x)*b
^3 - 3*a*b^2*arcsec(c*x)^2/x - a^3/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(76) = 152.

Time = 0.15 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.45

$$\int \frac{(a + b\sec^{-1}(cx))^3}{x^2}\,dx$$
$$= \left(3b^3\sqrt{-\frac{1}{c^2x^2} + 1}\arccos\left(\frac{1}{cx}\right)^2 + 6ab^2\sqrt{-\frac{1}{c^2x^2} + 1}\arccos\left(\frac{1}{cx}\right) - \frac{b^3\arccos\left(\frac{1}{cx}\right)^3}{cx} + 3a^2b\sqrt{-\frac{1}{c^2x^2} + \right.$$

input
```
integrate((a+b*arcsec(c*x))^3/x^2,x, algorithm="giac")
```

output
```
(3*b^3*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x))^2 + 6*a*b^2*sqrt(-1/(c^2*x^2
) + 1)*arccos(1/(c*x)) - b^3*arccos(1/(c*x))^3/(c*x) + 3*a^2*b*sqrt(-1/(c^
2*x^2) + 1) - 6*b^3*sqrt(-1/(c^2*x^2) + 1) - 3*a*b^2*arccos(1/(c*x))^2/(c*
x) - 3*a^2*b*arccos(1/(c*x))/(c*x) + 6*b^3*arccos(1/(c*x))/(c*x) - a^3/(c*
x) + 6*a*b^2/(c*x))*c
```

Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.95

$$\int \frac{(a + b\sec^{-1}(cx))^3}{x^2}\,dx = \frac{b^3\left(6\,\mathrm{acos}\left(\frac{1}{c\,x}\right) - \mathrm{acos}\left(\frac{1}{c\,x}\right)^3\right)}{x} - \frac{a^3}{x} + 3\,a^2\,b\,c\left(\sqrt{1 - \frac{1}{c^2\,x^2}} - \frac{\mathrm{acos}\left(\frac{1}{c\,x}\right)}{c\,x}\right) + b^3\,c\,\sqrt{1 - \frac{1}{c^2\,x^2}}\left(3\,\mathrm{acos}\left(\frac{1}{c\,x}\right)^2 - 6\right) + 3\,a\,b^2\,c\left(2\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\sqrt{1 - \frac{1}{c^2\,x^2}} - \frac{\mathrm{acos}\left(\frac{1}{c\,x}\right)^2 - 2}{c\,x}\right)$$

input
```
int((a + b*acos(1/(c*x)))^3/x^2,x)
```

output
```
(b^3*(6*acos(1/(c*x)) - acos(1/(c*x))^3))/x - a^3/x + 3*a^2*b*c*((1 - 1/(c
^2*x^2))^(1/2) - acos(1/(c*x))/(c*x)) + b^3*c*(1 - 1/(c^2*x^2))^(1/2)*(3*a
cos(1/(c*x))^2 - 6) + 3*a*b^2*c*(2*acos(1/(c*x))*(1 - 1/(c^2*x^2))^(1/2) -
 (acos(1/(c*x))^2 - 2)/(c*x))
```

Reduce [F]

$$\int \frac{(a + b\sec^{-1}(cx))^3}{x^2}\,dx = \frac{3\left(\int \frac{asec(cx)}{x^2}dx\right)a^2bx + \left(\int \frac{asec(cx)^3}{x^2}dx\right)b^3x + 3\left(\int \frac{asec(cx)^2}{x^2}dx\right)a\,b^2x - a^3}{x}$$

input
```
int((a+b*asec(c*x))^3/x^2,x)
```

output
```
(3*int(asec(c*x)/x**2,x)*a**2*b*x + int(asec(c*x)**3/x**2,x)*b**3*x + 3*in
t(asec(c*x)**2/x**2,x)*a*b**2*x - a**3)/x
```

3.30 $\int \frac{(a+b\sec^{-1}(cx))^3}{x^3} dx$

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Reduce [F] . . . 305

Optimal result

Integrand size = 14, antiderivative size = 137

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^3}\,dx = -\frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{8x}+\frac{3}{8}b^3c^2\sec^{-1}(cx) -\frac{3}{4}b^2\left(c^2-\frac{1}{x^2}\right)(a+b\sec^{-1}(cx)) +\frac{3bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2}{4x} -\frac{1}{4}c^2(a+b\sec^{-1}(cx))^3+\frac{1}{2}\left(c^2-\frac{1}{x^2}\right)(a+b\sec^{-1}(cx))^3$$

output
```
-3/8*b^3*c*(1-1/c^2/x^2)^(1/2)/x+3/8*b^3*c^2*arcsec(c*x)-3/4*b^2*(c^2-1/x^
2)*(a+b*arcsec(c*x))+3/4*b*c*(1-1/c^2/x^2)^(1/2)*(a+b*arcsec(c*x))^2/x-1/4
*c^2*(a+b*arcsec(c*x))^3+1/2*(c^2-1/x^2)*(a+b*arcsec(c*x))^3
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.35

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^3}\,dx$$
$$= \frac{-4a^3+6ab^2+6a^2bc\sqrt{1-\frac{1}{c^2x^2}}x-3b^3c\sqrt{1-\frac{1}{c^2x^2}}x+6b\left(-2a^2+b^2+2abc\sqrt{1-\frac{1}{c^2x^2}}x\right)\sec^{-1}(cx)+6b}{8}$$

input
```
Integrate[(a + b*ArcSec[c*x])^3/x^3,x]
```

output
```
(-4*a^3 + 6*a*b^2 + 6*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x - 3*b^3*c*Sqrt[1 - 1
/(c^2*x^2)]*x + 6*b*(-2*a^2 + b^2 + 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x)*ArcSe
c[c*x] + 6*b^2*(b*c*Sqrt[1 - 1/(c^2*x^2)]*x + a*(-2 + c^2*x^2))*ArcSec[c*x
]^2 + 2*b^3*(-2 + c^2*x^2)*ArcSec[c*x]^3 + 3*b*(-2*a^2 + b^2)*c^2*x^2*ArcS
in[1/(c*x)])/(8*x^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5745, 4904, 3042, 3792, 17, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^3}\,dx$$
$$\Big\downarrow \text{5745}$$
$$c^2\int \frac{\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\sec^{-1}(cx))^3}{cx}\,d\sec^{-1}(cx)$$
$$\Big\downarrow \text{4904}$$
$$c^2\left(\frac{1}{2}\left(1-\frac{1}{c^2x^2}\right)(a+b\sec^{-1}(cx))^3-\frac{3}{2}b\int\left(1-\frac{1}{c^2x^2}\right)(a+b\sec^{-1}(cx))^2\,d\sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{3042}$$

$$c^2\left(\frac{1}{2}\left(1-\frac{1}{c^2x^2}\right)\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{2}b\int\left(a+b\sec^{-1}(cx)\right)^2\sin\left(\sec^{-1}(cx)\right)^2d\sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{3792}$$

$$c^2\left(\frac{1}{2}\left(1-\frac{1}{c^2x^2}\right)\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{2}b\left(\frac{1}{2}\int\left(a+b\sec^{-1}(cx)\right)^2d\sec^{-1}(cx)-\frac{1}{2}b^2\int\left(1-\frac{1}{c^2x^2}\right)d\sec^{-1}(cx\right.\right.$$

$$\Big\downarrow \text{17}$$

$$c^2\left(\frac{1}{2}\left(1-\frac{1}{c^2x^2}\right)\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{2}b\left(-\frac{1}{2}b^2\int\left(1-\frac{1}{c^2x^2}\right)d\sec^{-1}(cx)-\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{2cx}+\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$c^2\left(\frac{1}{2}\left(1-\frac{1}{c^2x^2}\right)\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{2}b\left(-\frac{1}{2}b^2\int\sin\left(\sec^{-1}(cx)\right)^2d\sec^{-1}(cx)-\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{2cx}\right.\right.$$

$$\Big\downarrow \text{3115}$$

$$c^2\left(\frac{1}{2}\left(1-\frac{1}{c^2x^2}\right)\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{2}b\left(-\frac{1}{2}b^2\left(\frac{1}{2}\int 1d\sec^{-1}(cx)-\frac{\sqrt{1-\frac{1}{c^2x^2}}}{2cx}\right)-\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(\right.}{2cx}\right.\right.$$

$$\Big\downarrow \text{24}$$

$$c^2\left(\frac{1}{2}\left(1-\frac{1}{c^2x^2}\right)\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{2}b\left(-\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{2cx}+\frac{1}{2}b\left(1-\frac{1}{c^2x^2}\right)\left(a+b\sec^{-1}(cx)\right)\right.\right.$$

input
```
Int[(a + b*ArcSec[c*x])^3/x^3,x]
```

output
```
c^2*(((1 - 1/(c^2*x^2))*(a + b*ArcSec[c*x])^3)/2 - (3*b*(-1/2*(b^2*(-1/2*S
qrt[1 - 1/(c^2*x^2)]/(c*x) + ArcSec[c*x]/2)) + (b*(1 - 1/(c^2*x^2))*(a + b
*ArcSec[c*x]))/2 - (Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcSec[c*x])^2)/(2*c*x) +
 (a + b*ArcSec[c*x])^3/(6*b)))/2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.51

method	result
derivativedivides	$c^2\left(-\frac{a^3}{2c^2x^2}+b^3\left(-\frac{\cos(2\,\mathrm{arcsec}(cx))\,\mathrm{arcsec}(cx)^3}{4}+\frac{3\sin(2\,\mathrm{arcsec}(cx))\,\mathrm{arcsec}(cx)^2}{8}-\frac{3\sin(2\,\mathrm{arcsec}(cx))}{16}+\right.\right.$
default	$c^2\left(-\frac{a^3}{2c^2x^2}+b^3\left(-\frac{\cos(2\,\mathrm{arcsec}(cx))\,\mathrm{arcsec}(cx)^3}{4}+\frac{3\sin(2\,\mathrm{arcsec}(cx))\,\mathrm{arcsec}(cx)^2}{8}-\frac{3\sin(2\,\mathrm{arcsec}(cx))}{16}+\right.\right.$
parts	$-\frac{a^3}{2x^2}+b^3c^2\left(-\frac{\cos(2\,\mathrm{arcsec}(cx))\,\mathrm{arcsec}(cx)^3}{4}+\frac{3\sin(2\,\mathrm{arcsec}(cx))\,\mathrm{arcsec}(cx)^2}{8}-\frac{3\sin(2\,\mathrm{arcsec}(cx))}{16}+\frac{3\,\mathrm{co}}{}\right.$

input
```
int((a+b*arcsec(c*x))^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(-1/2*a^3/c^2/x^2+b^3*(-1/4*cos(2*arcsec(c*x))*arcsec(c*x)^3+3/8*sin(2
*arcsec(c*x))*arcsec(c*x)^2-3/16*sin(2*arcsec(c*x))+3/8*cos(2*arcsec(c*x))
*arcsec(c*x))+3*a*b^2*(-1/4*cos(2*arcsec(c*x))*arcsec(c*x)^2+1/8*cos(2*arc
sec(c*x))+1/4*sin(2*arcsec(c*x))*arcsec(c*x))+3*a^2*b*(-1/2/c^2/x^2*arcsec
(c*x)-1/4*(c^2*x^2-1)^(1/2)*(arctan(1/(c^2*x^2-1)^(1/2))*c^2*x^2-(c^2*x^2-
1)^(1/2))/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^3/x^3))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x^3}\,dx = \frac{2\left(b^3c^2x^2-2\,b^3\right)\mathrm{arcsec}\,(cx)^3-4\,a^3+6\,ab^2+6\left(ab^2c^2x^2-2\,ab^2\right)\mathrm{arcsec}\,(cx)^2+3\left(\left(2\,a^2b-b^3\right)c^2x^2-4\,a^2\right.}{8\,x^2}$$

input
```
integrate((a+b*arcsec(c*x))^3/x^3,x, algorithm="fricas")
```

output
```
1/8*(2*(b^3*c^2*x^2 - 2*b^3)*arcsec(c*x)^3 - 4*a^3 + 6*a*b^2 + 6*(a*b^2*c^
2*x^2 - 2*a*b^2)*arcsec(c*x)^2 + 3*((2*a^2*b - b^3)*c^2*x^2 - 4*a^2*b + 2*
b^3)*arcsec(c*x) + 3*(2*b^3*arcsec(c*x)^2 + 4*a*b^2*arcsec(c*x) + 2*a^2*b
- b^3)*sqrt(c^2*x^2 - 1))/x^2
```

Sympy [F]

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x^3}\,dx=\int \frac{\left(a+b\operatorname{asec}\left(cx\right)\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*asec(c*x))**3/x**3,x)
```

output
```
Integral((a + b*asec(c*x))**3/x**3, x)
```

Maxima [F]

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x^3}\,dx=\int \frac{\left(b\,\mathrm{arcsec}\left(cx\right)+a\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*arcsec(c*x))^3/x^3,x, algorithm="maxima")
```

output
```
-3/4*a^2*b*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1)
- c^3*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c + 2*arcsec(c*x)/x^2) - 1/2*a^3
/x^2 - 1/8*(4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3*b^3*arctan(sqr
t(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 + 12*(a*b^2*c^2*(log(c*x + 1) + l
og(c*x - 1) - 2*log(x))*log(c)^2 + 16*b^3*c^2*integrate(1/8*x^2*arctan(sqr
t(c*x + 1)*sqrt(c*x - 1))/(c^2*x^5 - x^3), x)*log(c)^2 - 16*b^3*c^2*integr
ate(1/8*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)/(c^2*x^5 - x^
3), x)*log(c) + 32*b^3*c^2*integrate(1/8*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x
 - 1))*log(x)/(c^2*x^5 - x^3), x)*log(c) - 16*a*b^2*c^2*integrate(1/8*x^2*
log(c^2*x^2)/(c^2*x^5 - x^3), x)*log(c) + 32*a*b^2*c^2*integrate(1/8*x^2*l
og(x)/(c^2*x^5 - x^3), x)*log(c) - 16*b^3*c^2*integrate(1/8*x^2*arctan(sqr
t(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)*log(x)/(c^2*x^5 - x^3), x) + 16*b^3
*c^2*integrate(1/8*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(x)^2/(c^2*x
^5 - x^3), x) - 16*a*b^2*c^2*integrate(1/8*x^2*arctan(sqrt(c*x + 1)*sqrt(c
*x - 1))^2/(c^2*x^5 - x^3), x) + 8*b^3*c^2*integrate(1/8*x^2*arctan(sqrt(c
*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)/(c^2*x^5 - x^3), x) + 4*a*b^2*c^2*inte
grate(1/8*x^2*log(c^2*x^2)^2/(c^2*x^5 - x^3), x) - 16*a*b^2*c^2*integrate(
1/8*x^2*log(c^2*x^2)*log(x)/(c^2*x^5 - x^3), x) + 16*a*b^2*c^2*integrate(1
/8*x^2*log(x)^2/(c^2*x^5 - x^3), x) - (c^2*log(c*x + 1) + c^2*log(c*x - 1)
 - 2*c^2*log(x) + 1/x^2)*a*b^2*log(c)^2 - 16*b^3*integrate(1/8*arctan(s...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(121) = 242$.

Time = 0.16 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.03

$$\int \frac{(a + b\sec^{-1}(cx))^3}{x^3}\,dx$$

$$= \frac{1}{8}\left(2\,b^3c\arccos\left(\frac{1}{cx}\right)^3 + 6\,ab^2c\arccos\left(\frac{1}{cx}\right)^2 + 6\,a^2bc\arccos\left(\frac{1}{cx}\right) - 3\,b^3c\arccos\left(\frac{1}{cx}\right) + \frac{6\,b^3\sqrt{-\frac{1}{c^2x^2}}}{\ldots}\right.$$

input
```
integrate((a+b*arcsec(c*x))^3/x^3,x, algorithm="giac")
```

output
```
1/8*(2*b^3*c*arccos(1/(c*x))^3 + 6*a*b^2*c*arccos(1/(c*x))^2 + 6*a^2*b*c*a
rccos(1/(c*x)) - 3*b^3*c*arccos(1/(c*x)) + 6*b^3*sqrt(-1/(c^2*x^2) + 1)*ar
ccos(1/(c*x))^2/x - 3*a*b^2*c + 12*a*b^2*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(
c*x))/x - 4*b^3*arccos(1/(c*x))^3/(c*x^2) + 6*a^2*b*sqrt(-1/(c^2*x^2) + 1)
/x - 3*b^3*sqrt(-1/(c^2*x^2) + 1)/x - 12*a*b^2*arccos(1/(c*x))^2/(c*x^2) -
 12*a^2*b*arccos(1/(c*x))/(c*x^2) + 6*b^3*arccos(1/(c*x))/(c*x^2) - 4*a^3/
(c*x^2) + 6*a*b^2/(c*x^2))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^3}\,dx=\int \frac{\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^3}{x^3}\,dx$$

input
```
int((a + b*acos(1/(c*x)))^3/x^3,x)
```

output
```
int((a + b*acos(1/(c*x)))^3/x^3, x)
```

Reduce [F]

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^3}\,dx = \frac{6\left(\int \frac{asec(cx)}{x^3}dx\right)a^2b\,x^2+2\left(\int \frac{asec(cx)^3}{x^3}dx\right)b^3x^2+6\left(\int \frac{asec(cx)^2}{x^3}dx\right)a\,b^2x^2-a^3}{2x^2}$$

input
```
int((a+b*asec(c*x))^3/x^3,x)
```

output
```
(6*int(asec(c*x)/x**3,x)*a**2*b*x**2 + 2*int(asec(c*x)**3/x**3,x)*b**3*x**
2 + 6*int(asec(c*x)**2/x**3,x)*a*b**2*x**2 - a**3)/(2*x**2)
```

3.31 $\int \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x^4}\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 170

$$\begin{aligned}\int \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x^4}\,dx = &-\frac{14}{9}b^3c^3\sqrt{1-\frac{1}{c^2x^2}}+\frac{2}{27}b^3c^3\left(1-\frac{1}{c^2x^2}\right)^{3/2}\\&+\frac{2b^2\left(a+b\sec^{-1}(cx)\right)}{9x^3}+\frac{4b^2c^2\left(a+b\sec^{-1}(cx)\right)}{3x}\\&+\frac{2}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2\\&+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{3x^2}-\frac{\left(a+b\sec^{-1}(cx)\right)^3}{3x^3}\end{aligned}$$

output

```
-14/9*b^3*c^3*(1-1/c^2/x^2)^(1/2)+2/27*b^3*c^3*(1-1/c^2/x^2)^(3/2)+2/9*b^2
*(a+b*arcsec(c*x))/x^3+4/3*b^2*c^2*(a+b*arcsec(c*x))/x+2/3*b*c^3*(1-1/c^2/
x^2)^(1/2)*(a+b*arcsec(c*x))^2+1/3*b*c*(1-1/c^2/x^2)^(1/2)*(a+b*arcsec(c*x
))^2/x^2-1/3*(a+b*arcsec(c*x))^3/x^3
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.20

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^4}\,dx$$
$$= \frac{-9a^3+9a^2bc\sqrt{1-\frac{1}{c^2x^2}}x(1+2c^2x^2)+6ab^2(1+6c^2x^2)-2b^3c\sqrt{1-\frac{1}{c^2x^2}}x(1+20c^2x^2)+3b\left(-9a^2+6a\right.}{}$$

input
```
Integrate[(a + b*ArcSec[c*x])^3/x^4,x]
```

output
```
(-9*a^3 + 9*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2) + 6*a*b^2*(1 +
 6*c^2*x^2) - 2*b^3*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 20*c^2*x^2) + 3*b*(-9*a
^2 + 6*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2) + 2*b^2*(1 + 6*c^2*x^
2))*ArcSec[c*x] + 9*b^2*(-3*a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2
))*ArcSec[c*x]^2 - 9*b^3*ArcSec[c*x]^3)/(27*x^3)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {5745, 4905, 3042, 3792, 3042, 3113, 2009, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^4}\,dx$$

$$\Big\downarrow \text{ 5745}$$

$$c^3\int \frac{\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\sec^{-1}(cx))^3}{c^2x^2}d\sec^{-1}(cx)$$

$$\Big\downarrow \text{ 4905}$$

$$c^3\left(b\int \frac{(a+b\sec^{-1}(cx))^2}{c^3x^3}d\sec^{-1}(cx)-\frac{(a+b\sec^{-1}(cx))^3}{3c^3x^3}\right)$$

$\downarrow$ 3042

$$c^3\left(b\int\left(a+b\sec^{-1}(cx)\right)^2\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^3d\sec^{-1}(cx)-\frac{\left(a+b\sec^{-1}(cx)\right)^3}{3c^3x^3}\right)$$

$\downarrow$ 3792

$$c^3\left(b\left(\frac{2}{3}\int\frac{\left(a+b\sec^{-1}(cx)\right)^2}{cx}d\sec^{-1}(cx)-\frac{2}{9}b^2\int\frac{1}{c^3x^3}d\sec^{-1}(cx)+\frac{2b\left(a+b\sec^{-1}(cx)\right)}{9c^3x^3}+\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\,\text{s}\right.}{3c^2x^2}\right.\right.$$

$\downarrow$ 3042

$$c^3\left(b\left(\frac{2}{3}\int\left(a+b\sec^{-1}(cx)\right)^2\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)d\sec^{-1}(cx)-\frac{2}{9}b^2\int\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^3d\sec^{-1}(cx)+\frac{2b(}{}\right.\right.$$

$\downarrow$ 3113

$$c^3\left(b\left(\frac{2}{3}\int\left(a+b\sec^{-1}(cx)\right)^2\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)d\sec^{-1}(cx)+\frac{2}{9}b^2\int\frac{1}{c^2x^2}d\left(-\sqrt{1-\frac{1}{c^2x^2}}\right)+\frac{2b\left(a+b\sec^{-}\right.}{9c^3x^3}\right.\right.$$

$\downarrow$ 2009

$$c^3\left(b\left(\frac{2}{3}\int\left(a+b\sec^{-1}(cx)\right)^2\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)d\sec^{-1}(cx)+\frac{2b\left(a+b\sec^{-1}(cx)\right)}{9c^3x^3}+\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}\right.}{3c^2x^2}\right.\right.$$

$\downarrow$ 3777

$$c^3\left(b\left(\frac{2}{3}\left(2b\int-\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)d\sec^{-1}(cx)+\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2\right)+\frac{2b\left(a+b\sec^{-1}(c\right.}{9c^3x^3}\right.\right.$$

$\downarrow$ 25

$$c^3\left(b\left(\frac{2}{3}\left(\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\int\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)d\sec^{-1}(cx)\right)+\frac{2b\left(a+b\sec^{-1}(cx)\right.}{9c^3x^3}\right.\right.$$

$\downarrow$ 3042

$$c^3\left(b\left(\frac{2}{3}\left(\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2-2b\int(a+b\sec^{-1}(cx))\sin\left(\sec^{-1}(cx)\right)d\sec^{-1}(cx)\right)+\frac{2b(a+b\sec^{-1}}{9c^3x^3}\right.\right.$$

$$\Big\downarrow \text{3777}$$

$$c^3\left(b\left(\frac{2}{3}\left(\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2-2b\left(b\int\frac{1}{cx}d\sec^{-1}(cx)-\frac{a+b\sec^{-1}(cx)}{cx}\right)\right)+\frac{2b(a+b\sec^{-1}(cx))}{9c^3x^3}+\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$c^3\left(b\left(\frac{2}{3}\left(\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2-2b\left(b\int\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)d\sec^{-1}(cx)-\frac{a+b\sec^{-1}(cx)}{cx}\right)\right)+\frac{2b(}{}\right.\right.$$

$$\Big\downarrow \text{3117}$$

$$c^3\left(b\left(\frac{2b(a+b\sec^{-1}(cx))}{9c^3x^3}+\frac{\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2}{3c^2x^2}+\frac{2}{3}\left(\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2-2b\left(b\sqrt{1-\frac{}{c^2}}\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcSec[c*x])^3/x^4,x]
```

output
```
c^3*(-1/3*(a + b*ArcSec[c*x])^3/(c^3*x^3) + b*((2*b^2*(-Sqrt[1 - 1/(c^2*x^
2)] + (1 - 1/(c^2*x^2))^(3/2)/3))/9 + (2*b*(a + b*ArcSec[c*x]))/(9*c^3*x^3
) + (Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcSec[c*x])^2)/(3*c^2*x^2) + (2*(Sqrt[1
 - 1/(c^2*x^2)]*(a + b*ArcSec[c*x])^2 - 2*b*(b*Sqrt[1 - 1/(c^2*x^2)] - (a
+ b*ArcSec[c*x])/(c*x))))/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4905
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(148) = 296$.

Time = 0.89 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.76

method	result
derivativedivides	$c^3\left(-\frac{a^3}{3c^3x^3}+b^3\left(-\frac{\operatorname{arcsec}(cx)^3}{3c^3x^3}+\frac{\operatorname{arcsec}(cx)^2(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3c^2x^2}-\frac{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3}+\frac{4\operatorname{arcsec}(cx)}{3cx}+\frac{2\operatorname{arc}}{9}\right.\right.$
default	$c^3\left(-\frac{a^3}{3c^3x^3}+b^3\left(-\frac{\operatorname{arcsec}(cx)^3}{3c^3x^3}+\frac{\operatorname{arcsec}(cx)^2(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3c^2x^2}-\frac{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3}+\frac{4\operatorname{arcsec}(cx)}{3cx}+\frac{2\operatorname{arc}}{9}\right.\right.$
parts	$-\frac{a^3}{3x^3}+b^3c^3\left(-\frac{\operatorname{arcsec}(cx)^3}{3c^3x^3}+\frac{\operatorname{arcsec}(cx)^2(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3c^2x^2}-\frac{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3}+\frac{4\operatorname{arcsec}(cx)}{3cx}+\frac{2\operatorname{arcsec}(}{9c^3x^3}\right.$

input
```
int((a+b*arcsec(c*x))^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
c^3*(-1/3*a^3/c^3/x^3+b^3*(-1/3/c^3/x^3*arcsec(c*x)^3+1/3*arcsec(c*x)^2*(2
*c^2*x^2+1)/c^2/x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)-4/3*((c^2*x^2-1)/c^2/x^2)^
(1/2)+4/3/c/x*arcsec(c*x)+2/9/c^3/x^3*arcsec(c*x)-2/27*(2*c^2*x^2+1)/c^2/x
^2*((c^2*x^2-1)/c^2/x^2)^(1/2))+3*a*b^2*(-1/3/c^3/x^3*arcsec(c*x)^2+2/9*ar
csec(c*x)*(2*c^2*x^2+1)/c^2/x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)+2/27/c^3/x^3+4
/9/c/x)+3*a^2*b*(-1/3/c^3/x^3*arcsec(c*x)+1/9*(c^2*x^2-1)*(2*c^2*x^2+1)/((
c^2*x^2-1)/c^2/x^2)^(1/2)/c^4/x^4))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.01

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^4}\,dx$$
$$=\frac{36\,ab^2c^2x^2-9\,b^3\operatorname{arcsec}(cx)^3-27\,ab^2\operatorname{arcsec}(cx)^2-9\,a^3+6\,ab^2+3\,(12\,b^3c^2x^2-9\,a^2b+2\,b^3)\operatorname{arcsec}(cx}{}$$

input
```
integrate((a+b*arcsec(c*x))^3/x^4,x, algorithm="fricas")
```

output
```
1/27*(36*a*b^2*c^2*x^2 - 9*b^3*arcsec(c*x)^3 - 27*a*b^2*arcsec(c*x)^2 - 9*
a^3 + 6*a*b^2 + 3*(12*b^3*c^2*x^2 - 9*a^2*b + 2*b^3)*arcsec(c*x) + (2*(9*a
^2*b - 20*b^3)*c^2*x^2 + 9*a^2*b - 2*b^3 + 9*(2*b^3*c^2*x^2 + b^3)*arcsec(
c*x)^2 + 18*(2*a*b^2*c^2*x^2 + a*b^2)*arcsec(c*x))*sqrt(c^2*x^2 - 1))/x^3
```

Sympy [F]

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x^4}\,dx=\int \frac{\left(a+b\,\mathrm{asec}\,(cx)\right)^3}{x^4}\,dx$$

input
```
integrate((a+b*asec(c*x))**3/x**4,x)
```

output
```
Integral((a + b*asec(c*x))**3/x**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 575 vs. $2(148) = 296$.

Time = 0.65 (sec) , antiderivative size = 575, normalized size of antiderivative = 3.38

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x^4}\,dx =$$
$$-\frac{1}{216}\left(\frac{72\left(c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}\right)\mathrm{arcsec}\,(cx)^2}{c}+\frac{72\,c^4\left(\frac{c^2\arcsin\left(\frac{1}{c|x|}\right)+\frac{2\sqrt{c^2x^2-1}c}{x}-\frac{\sqrt{c^2x^2-1}}{x^2}}{c}-\right.}{}\right.$$
$$-\frac{1}{3}\,a^2b\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{3\,\mathrm{arcsec}\,(cx)}{x^3}\right)$$
$$-\frac{b^3\,\mathrm{arcsec}\,(cx)^3}{3\,x^3}-\frac{ab^2\,\mathrm{arcsec}\,(cx)^2}{x^3}-\frac{a^3}{3\,x^3}$$
$$+\frac{2\left(\left(6\,c^3x^2+c\right)\sqrt{cx+1}\sqrt{cx-1}+3\left(2\,c^5x^4-c^3x^2-c\right)\arctan\left(\sqrt{cx+1}\sqrt{cx-1}\right)\right)ab^2}{9\,\sqrt{cx+1}\sqrt{cx-1}cx^3}$$

input
```
integrate((a+b*arcsec(c*x))^3/x^4,x, algorithm="maxima")
```

output
```
-1/216*(72*(c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))*a
rcsec(c*x)^2/c + (72*c^4*((c^2*arcsin(1/(c*abs(x))) + 2*sqrt(c^2*x^2 - 1)*
c/x - sqrt(c^2*x^2 - 1)/x^2)/c - (c^2*arcsin(1/(c*abs(x))) - 2*sqrt(c^2*x^
2 - 1)*c/x - sqrt(c^2*x^2 - 1)/x^2)/c - 4*arctan(sqrt(c*x + 1)*sqrt(c*x -
1))/x) + c^2*((9*c^4*arcsin(1/(c*abs(x))) + 16*sqrt(c^2*x^2 - 1)*c^3/x - 9
*sqrt(c^2*x^2 - 1)*c^2/x^2 + 8*sqrt(c^2*x^2 - 1)*c/x^3 - 6*sqrt(c^2*x^2 -
1)/x^4)/c - (9*c^4*arcsin(1/(c*abs(x))) - 16*sqrt(c^2*x^2 - 1)*c^3/x - 9*s
qrt(c^2*x^2 - 1)*c^2/x^2 - 8*sqrt(c^2*x^2 - 1)*c/x^3 - 6*sqrt(c^2*x^2 - 1)
/x^4)/c - 48*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/x^3))/c^2)*b^3 - 1/3*a^2*
b*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c + 3*arc
sec(c*x)/x^3) - 1/3*b^3*arcsec(c*x)^3/x^3 - a*b^2*arcsec(c*x)^2/x^3 - 1/3*
a^3/x^3 + 2/9*((6*c^3*x^2 + c)*sqrt(c*x + 1)*sqrt(c*x - 1) + 3*(2*c^5*x^4
- c^3*x^2 - c)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)))*a*b^2/(sqrt(c*x + 1)*s
qrt(c*x - 1)*c*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(148) = 296$.

Time = 0.16 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.98

$$\int \frac{(a + b\sec^{-1}(cx))^3}{x^4}\,dx$$

$$= \frac{1}{27}\left(18\,b^3c^2\sqrt{-\frac{1}{c^2x^2}+1}\arccos\left(\frac{1}{cx}\right)^2 + 36\,ab^2c^2\sqrt{-\frac{1}{c^2x^2}+1}\arccos\left(\frac{1}{cx}\right) + 18\,a^2bc^2\sqrt{-\frac{1}{c^2x^2}+1} -\right.$$

input
```
integrate((a+b*arcsec(c*x))^3/x^4,x, algorithm="giac")
```

output
```
1/27*(18*b^3*c^2*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x))^2 + 36*a*b^2*c^2*s
qrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x)) + 18*a^2*b*c^2*sqrt(-1/(c^2*x^2) + 1
) - 40*b^3*c^2*sqrt(-1/(c^2*x^2) + 1) + 36*b^3*c*arccos(1/(c*x))/x + 9*b^3
*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x))^2/x^2 + 36*a*b^2*c/x + 18*a*b^2*sq
rt(-1/(c^2*x^2) + 1)*arccos(1/(c*x))/x^2 - 9*b^3*arccos(1/(c*x))^3/(c*x^3)
 + 9*a^2*b*sqrt(-1/(c^2*x^2) + 1)/x^2 - 2*b^3*sqrt(-1/(c^2*x^2) + 1)/x^2 -
 27*a*b^2*arccos(1/(c*x))^2/(c*x^3) - 27*a^2*b*arccos(1/(c*x))/(c*x^3) + 6
*b^3*arccos(1/(c*x))/(c*x^3) - 9*a^3/(c*x^3) + 6*a*b^2/(c*x^3))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\sec^{-1}(cx))^3}{x^4}\,dx = \int \frac{\left(a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^3}{x^4}\,dx$$

input
```
int((a + b*acos(1/(c*x)))^3/x^4,x)
```

output
```
int((a + b*acos(1/(c*x)))^3/x^4, x)
```

Reduce [F]

$$\int \frac{(a + b\sec^{-1}(cx))^3}{x^4}\,dx$$
$$= \frac{9\left(\int \frac{asec(cx)}{x^4}dx\right) a^2 b\,x^3 + 3\left(\int \frac{asec(cx)^3}{x^4}dx\right) b^3 x^3 + 9\left(\int \frac{asec(cx)^2}{x^4}dx\right) a\,b^2 x^3 - a^3}{3x^3}$$

input
```
int((a+b*asec(c*x))^3/x^4,x)
```

output
```
(9*int(asec(c*x)/x**4,x)*a**2*b*x**3 + 3*int(asec(c*x)**3/x**4,x)*b**3*x**
3 + 9*int(asec(c*x)**2/x**4,x)*a*b**2*x**3 - a**3)/(3*x**3)
```

3.32 $\int \frac{(a+b\sec^{-1}(cx))^3}{x^5}\,dx$

Optimal result 315
Mathematica [A] (verified) 316
Rubi [A] (verified) 316
Maple [B] (verified) 320
Fricas [A] (verification not implemented) 320
Sympy [F] 321
Maxima [F] 321
Giac [B] (verification not implemented) 322
Mupad [F(-1)] 323
Reduce [F] 323

Optimal result

Integrand size = 14, antiderivative size = 208

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^5}\,dx = -\frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{128x^3}-\frac{45b^3c^3\sqrt{1-\frac{1}{c^2x^2}}}{256x} - \frac{45}{256}b^3c^4\sec^{-1}(cx)+\frac{3b^2(a+b\sec^{-1}(cx))}{32x^4} + \frac{9b^2c^2(a+b\sec^{-1}(cx))}{32x^2}+\frac{3bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2}{16x^3} + \frac{9bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2}{32x} + \frac{3}{32}c^4(a+b\sec^{-1}(cx))^3-\frac{(a+b\sec^{-1}(cx))^3}{4x^4}$$

output
```
-3/128*b^3*c*(1-1/c^2/x^2)^(1/2)/x^3-45/256*b^3*c^3*(1-1/c^2/x^2)^(1/2)/x-
45/256*b^3*c^4*arcsec(c*x)+3/32*b^2*(a+b*arcsec(c*x))/x^4+9/32*b^2*c^2*(a+
b*arcsec(c*x))/x^2+3/16*b*c*(1-1/c^2/x^2)^(1/2)*(a+b*arcsec(c*x))^2/x^3+9/
32*b*c^3*(1-1/c^2/x^2)^(1/2)*(a+b*arcsec(c*x))^2/x+3/32*c^4*(a+b*arcsec(c*
x))^3-1/4*(a+b*arcsec(c*x))^3/x^4
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.36

$$\int \frac{(a + b\sec^{-1}(cx))^3}{x^5}\,dx$$
$$= \frac{-64a^3 + 24ab^2 + 48a^2bc\sqrt{1-\frac{1}{c^2x^2}}x - 6b^3c\sqrt{1-\frac{1}{c^2x^2}}x + 72ab^2c^2x^2 + 72a^2bc^3\sqrt{1-\frac{1}{c^2x^2}}x^3 - 45b^3c^3\sqrt{1}}{}$$

input
```
Integrate[(a + b*ArcSec[c*x])^3/x^5,x]
```

output
```
(-64*a^3 + 24*a*b^2 + 48*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x - 6*b^3*c*Sqrt[1
- 1/(c^2*x^2)]*x + 72*a*b^2*c^2*x^2 + 72*a^2*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*x
^3 - 45*b^3*c^3*Sqrt[1 - 1/(c^2*x^2)]*x^3 + 24*b*(-8*a^2 + b^2*(1 + 3*c^2*
x^2) + 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(2 + 3*c^2*x^2))*ArcSec[c*x] + 24*b
^2*(b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(2 + 3*c^2*x^2) + a*(-8 + 3*c^4*x^4))*ArcS
ec[c*x]^2 + 8*b^3*(-8 + 3*c^4*x^4)*ArcSec[c*x]^3 + 9*b*(-8*a^2 + 5*b^2)*c^
4*x^4*ArcSin[1/(c*x)])/(256*x^4)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.29, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5745, 4905, 3042, 3792, 3042, 3115, 3042, 3115, 24, 3792, 17, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b\sec^{-1}(cx))^3}{x^5}\,dx$$

$$\Big\downarrow \text{ 5745}$$

$$c^4 \int \frac{\sqrt{1-\frac{1}{c^2x^2}}\,(a + b\sec^{-1}(cx))^3}{c^3x^3}\,d\sec^{-1}(cx)$$

$$\Big\downarrow \text{ 4905}$$

$$c^4\left(\frac{3}{4}b\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{c^4x^4}d\sec^{-1}(cx)-\frac{\left(a+b\sec^{-1}(cx)\right)^3}{4c^4x^4}\right)$$

$$\Big\downarrow \text{3042}$$

$$c^4\left(\frac{3}{4}b\int \left(a+b\sec^{-1}(cx)\right)^2\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^4d\sec^{-1}(cx)-\frac{\left(a+b\sec^{-1}(cx)\right)^3}{4c^4x^4}\right)$$

$$\Big\downarrow \text{3792}$$

$$c^4\left(\frac{3}{4}b\left(\frac{3}{4}\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{c^2x^2}d\sec^{-1}(cx)-\frac{1}{8}b^2\int \frac{1}{c^4x^4}d\sec^{-1}(cx)+\frac{b\left(a+b\sec^{-1}(cx)\right)}{8c^4x^4}+\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\mathrm{s}\right.}{4c^3x^3}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$c^4\left(\frac{3}{4}b\left(\frac{3}{4}\int \left(a+b\sec^{-1}(cx)\right)^2\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^2d\sec^{-1}(cx)-\frac{1}{8}b^2\int \sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^4d\sec^{-1}(cx)+\right.\right.$$

$$\Big\downarrow \text{3115}$$

$$c^4\left(\frac{3}{4}b\left(\frac{3}{4}\int \left(a+b\sec^{-1}(cx)\right)^2\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^2d\sec^{-1}(cx)-\frac{1}{8}b^2\left(\frac{3}{4}\int \frac{1}{c^2x^2}d\sec^{-1}(cx)+\frac{\sqrt{1-\frac{1}{c^2x^2}}}{4c^3x^3}\right)\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$c^4\left(\frac{3}{4}b\left(\frac{3}{4}\int \left(a+b\sec^{-1}(cx)\right)^2\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^2d\sec^{-1}(cx)-\frac{1}{8}b^2\left(\frac{3}{4}\int \sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^2d\sec^{-1}(cx)\right.\right.\right.$$

$$\Big\downarrow \text{3115}$$

$$c^4\left(\frac{3}{4}b\left(\frac{3}{4}\int \left(a+b\sec^{-1}(cx)\right)^2\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^2d\sec^{-1}(cx)-\frac{1}{8}b^2\left(\frac{3}{4}\left(\frac{1}{2}\int 1d\sec^{-1}(cx)+\frac{\sqrt{1-\frac{1}{c^2x^2}}}{2cx}\right)\right.\right.\right.$$

$$\Big\downarrow \text{24}$$

$$c^4\left(\frac{3}{4}b\left(\frac{3}{4}\int \left(a+b\sec^{-1}(cx)\right)^2\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^2d\sec^{-1}(cx)+\frac{b\left(a+b\sec^{-1}(cx)\right)}{8c^4x^4}+\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-}\right.}{4c^3x^3}\right.\right.$$

$$\Big\downarrow \; 3792$$

$$c^4\left(\frac{3}{4}b\left(\frac{3}{4}\left(\frac{1}{2}\int (a+b\sec^{-1}(cx))^2 d\sec^{-1}(cx) - \frac{1}{2}b^2\int \frac{1}{c^2x^2}d\sec^{-1}(cx) + \frac{b(a+b\sec^{-1}(cx))}{2c^2x^2} + \frac{\sqrt{1-\frac{1}{c^2x^2}}(a\right.\right.\right.$$

$$\Big\downarrow \; 17$$

$$c^4\left(\frac{3}{4}b\left(\frac{3}{4}\left(-\frac{1}{2}b^2\int \frac{1}{c^2x^2}d\sec^{-1}(cx) + \frac{\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2}{2cx} + \frac{b(a+b\sec^{-1}(cx))}{2c^2x^2} + \frac{(a+b\sec^{-1}(cx))}{6b}\right.\right.\right.$$

$$\Big\downarrow \; 3042$$

$$c^4\left(\frac{3}{4}b\left(\frac{3}{4}\left(-\frac{1}{2}b^2\int \sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^2 d\sec^{-1}(cx) + \frac{\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2}{2cx} + \frac{b(a+b\sec^{-1}(cx))}{2c^2x^2} + \right.\right.\right.$$

$$\Big\downarrow \; 3115$$

$$c^4\left(\frac{3}{4}b\left(\frac{3}{4}\left(-\frac{1}{2}b^2\left(\frac{1}{2}\int 1d\sec^{-1}(cx) + \frac{\sqrt{1-\frac{1}{c^2x^2}}}{2cx}\right) + \frac{\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2}{2cx} + \frac{b(a+b\sec^{-1}(cx))}{2c^2x^2} + \frac{(a}{}\right.\right.\right.$$

$$\Big\downarrow \; 24$$

$$c^4\left(\frac{3}{4}b\left(\frac{3}{4}\left(\frac{\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2}{2cx} + \frac{b(a+b\sec^{-1}(cx))}{2c^2x^2} + \frac{(a+b\sec^{-1}(cx))^3}{6b} - \frac{1}{2}b^2\left(\frac{\sqrt{1-\frac{1}{c^2x^2}}}{2cx} + \frac{1}{2}\text{se}\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcSec[c*x])^3/x^5,x]
```

output
```
c^4*(-1/4*(a + b*ArcSec[c*x])^3/(c^4*x^4) + (3*b*(-1/8*(b^2*(Sqrt[1 - 1/(c
^2*x^2)]/(4*c^3*x^3) + (3*(Sqrt[1 - 1/(c^2*x^2)]/(2*c*x) + ArcSec[c*x]/2))
/4)) + (b*(a + b*ArcSec[c*x]))/(8*c^4*x^4) + (Sqrt[1 - 1/(c^2*x^2)]*(a + b
*ArcSec[c*x])^2)/(4*c^3*x^3) + (3*(-1/2*(b^2*(Sqrt[1 - 1/(c^2*x^2)]/(2*c*x
) + ArcSec[c*x]/2)) + (b*(a + b*ArcSec[c*x]))/(2*c^2*x^2) + (Sqrt[1 - 1/(c
^2*x^2)]*(a + b*ArcSec[c*x])^2)/(2*c*x) + (a + b*ArcSec[c*x])^3/(6*b)))/4)
)/4)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4905
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. 2(182) = 364.

Time = 0.88 (sec) , antiderivative size = 470, normalized size of antiderivative = 2.26

method	result
parts	$-\frac{a^3}{4x^4}+b^3c^4\left(-\frac{\mathrm{arcsec}(cx)^3}{4c^4x^4}+\frac{3\,\mathrm{arcsec}(cx)^2\left(3c^3x^3\,\mathrm{arcsec}(cx)+3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}+2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{32c^3x^3}+\frac{3\,\mathrm{arcsec}(cx}{32c^4x^4}\right.$
derivativedivides	$c^4\left(-\frac{a^3}{4c^4x^4}+b^3\left(-\frac{\mathrm{arcsec}(cx)^3}{4c^4x^4}+\frac{3\,\mathrm{arcsec}(cx)^2\left(3c^3x^3\,\mathrm{arcsec}(cx)+3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}+2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{32c^3x^3}+\frac{3\,\mathrm{arcs}}{32c}\right.\right.$
default	$c^4\left(-\frac{a^3}{4c^4x^4}+b^3\left(-\frac{\mathrm{arcsec}(cx)^3}{4c^4x^4}+\frac{3\,\mathrm{arcsec}(cx)^2\left(3c^3x^3\,\mathrm{arcsec}(cx)+3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}+2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{32c^3x^3}+\frac{3\,\mathrm{arcs}}{32c}\right.\right.$

input

```
int((a+b*arcsec(c*x))^3/x^5,x,method=_RETURNVERBOSE)
```

output

```
-1/4*a^3/x^4+b^3*c^4*(-1/4/c^4/x^4*arcsec(c*x)^3+3/32*arcsec(c*x)^2*(3*c^3
*x^3*arcsec(c*x)+3*c^2*x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)+2*((c^2*x^2-1)/c^2/
x^2)^(1/2))/c^3/x^3+3/32*arcsec(c*x)/c^4/x^4-3/256*(3*c^2*x^2+2)/c^3/x^3*(
(c^2*x^2-1)/c^2/x^2)^(1/2)-45/256*arcsec(c*x)+9/32/c^2/x^2*arcsec(c*x)-9/6
4*((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x-3/16*arcsec(c*x)^3)+3*a*b^2*c^4*(-1/4/c^
4/x^4*arcsec(c*x)^2+1/16*arcsec(c*x)*(3*c^3*x^3*arcsec(c*x)+3*c^2*x^2*((c^
2*x^2-1)/c^2/x^2)^(1/2)+2*((c^2*x^2-1)/c^2/x^2)^(1/2))/c^3/x^3-3/32*arcsec
(c*x)^2+1/128*(3*c^2*x^2+2)^2/c^4/x^4)-3/4*arcsec(c*x)*a^2*b/x^4-9/32*a^2*
b*c^3*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*arctan(1/(c^2*x^2-1)
^(1/2))+9/32*a^2*b*c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^3+3/16*a^2*
b/c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^5
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.08

$$\int\frac{(a+b\sec^{-1}(cx))^3}{x^5}\,dx$$
$$=\frac{72\,ab^2c^2x^2+8\left(3\,b^3c^4x^4-8\,b^3\right)\mathrm{arcsec}\left(cx\right)^3-64\,a^3+24\,ab^2+24\left(3\,ab^2c^4x^4-8\,ab^2\right)\mathrm{arcsec}\left(cx\right)^2+3\,(3\,(}{}$$

input
```
integrate((a+b*arcsec(c*x))^3/x^5,x, algorithm="fricas")
```

output
```
1/256*(72*a*b^2*c^2*x^2 + 8*(3*b^3*c^4*x^4 - 8*b^3)*arcsec(c*x)^3 - 64*a^3
 + 24*a*b^2 + 24*(3*a*b^2*c^4*x^4 - 8*a*b^2)*arcsec(c*x)^2 + 3*(3*(8*a^2*b
 - 5*b^3)*c^4*x^4 + 24*b^3*c^2*x^2 - 64*a^2*b + 8*b^3)*arcsec(c*x) + 3*(3*
(8*a^2*b - 5*b^3)*c^2*x^2 + 16*a^2*b - 2*b^3 + 8*(3*b^3*c^2*x^2 + 2*b^3)*a
rcsec(c*x)^2 + 16*(3*a*b^2*c^2*x^2 + 2*a*b^2)*arcsec(c*x))*sqrt(c^2*x^2 -
1))/x^4
```

Sympy [F]

$$\int \frac{(a + b\sec^{-1}(cx))^3}{x^5}\,dx = \int \frac{(a + b\operatorname{asec}(cx))^3}{x^5}\,dx$$

input
```
integrate((a+b*asec(c*x))**3/x**5,x)
```

output
```
Integral((a + b*asec(c*x))**3/x**5, x)
```

Maxima [F]

$$\int \frac{(a + b\sec^{-1}(cx))^3}{x^5}\,dx = \int \frac{(b\operatorname{arcsec}(cx) + a)^3}{x^5}\,dx$$

input
```
integrate((a+b*arcsec(c*x))^3/x^5,x, algorithm="maxima")
```

output
```
3/32*a^2*b*((3*c^5*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)) + (3*c^8*x^3*(-1/(c^
2*x^2) + 1)^(3/2) + 5*c^6*x*sqrt(-1/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2)
- 1)^2 - 2*c^2*x^2*(1/(c^2*x^2) - 1) + 1))/c - 8*arcsec(c*x)/x^4) - 1/4*a^
3/x^4 - 1/16*(4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3*b^3*arctan(s
qrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 + 12*(2*(c^2*log(c*x + 1) + c^2
*log(c*x - 1) - 2*c^2*log(x) + 1/x^2)*a*b^2*c^2*log(c)^2 + 64*b^3*c^2*inte
grate(1/16*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^7 - x^5), x)*log
(c)^2 - 64*b^3*c^2*integrate(1/16*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*
log(c^2*x^2)/(c^2*x^7 - x^5), x)*log(c) + 128*b^3*c^2*integrate(1/16*x^2*a
rctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(x)/(c^2*x^7 - x^5), x)*log(c) - 64*
a*b^2*c^2*integrate(1/16*x^2*log(c^2*x^2)/(c^2*x^7 - x^5), x)*log(c) + 128
*a*b^2*c^2*integrate(1/16*x^2*log(x)/(c^2*x^7 - x^5), x)*log(c) - 64*b^3*c
^2*integrate(1/16*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)*log
(x)/(c^2*x^7 - x^5), x) + 64*b^3*c^2*integrate(1/16*x^2*arctan(sqrt(c*x +
1)*sqrt(c*x - 1))*log(x)^2/(c^2*x^7 - x^5), x) - 64*a*b^2*c^2*integrate(1/
16*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2/(c^2*x^7 - x^5), x) + 16*b^3*
c^2*integrate(1/16*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)/(c
^2*x^7 - x^5), x) + 16*a*b^2*c^2*integrate(1/16*x^2*log(c^2*x^2)^2/(c^2*x^
7 - x^5), x) - 64*a*b^2*c^2*integrate(1/16*x^2*log(c^2*x^2)*log(x)/(c^2*x^
7 - x^5), x) + 64*a*b^2*c^2*integrate(1/16*x^2*log(x)^2/(c^2*x^7 - x^5)...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(182) = 364$.

Time = 0.16 (sec) , antiderivative size = 427, normalized size of antiderivative = 2.05

$$\int \frac{(a + b\sec^{-1}(cx))^3}{x^5}\,dx$$

$$= \frac{1}{256}\left(24\,b^3c^3\arccos\left(\frac{1}{cx}\right)^3 + 72\,ab^2c^3\arccos\left(\frac{1}{cx}\right)^2 + 72\,a^2bc^3\arccos\left(\frac{1}{cx}\right) - 45\,b^3c^3\arccos\left(\frac{1}{cx}\right) + \right.$$

input
```
integrate((a+b*arcsec(c*x))^3/x^5,x, algorithm="giac")
```

output
```
1/256*(24*b^3*c^3*arccos(1/(c*x))^3 + 72*a*b^2*c^3*arccos(1/(c*x))^2 + 72*
a^2*b*c^3*arccos(1/(c*x)) - 45*b^3*c^3*arccos(1/(c*x)) + 72*b^3*c^2*sqrt(-
1/(c^2*x^2) + 1)*arccos(1/(c*x))^2/x - 45*a*b^2*c^3 + 144*a*b^2*c^2*sqrt(-
1/(c^2*x^2) + 1)*arccos(1/(c*x))/x + 72*a^2*b*c^2*sqrt(-1/(c^2*x^2) + 1)/x
 - 45*b^3*c^2*sqrt(-1/(c^2*x^2) + 1)/x + 72*b^3*c*arccos(1/(c*x))/x^2 + 48
*b^3*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x))^2/x^3 + 72*a*b^2*c/x^2 + 96*a*
b^2*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x))/x^3 - 64*b^3*arccos(1/(c*x))^3/
(c*x^4) + 48*a^2*b*sqrt(-1/(c^2*x^2) + 1)/x^3 - 6*b^3*sqrt(-1/(c^2*x^2) +
1)/x^3 - 192*a*b^2*arccos(1/(c*x))^2/(c*x^4) - 192*a^2*b*arccos(1/(c*x))/(
c*x^4) + 24*b^3*arccos(1/(c*x))/(c*x^4) - 64*a^3/(c*x^4) + 24*a*b^2/(c*x^4
))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x^5}\,dx=\int \frac{\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^3}{x^5}\,dx$$

input
```
int((a + b*acos(1/(c*x)))^3/x^5,x)
```

output
```
int((a + b*acos(1/(c*x)))^3/x^5, x)
```

Reduce [F]

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x^5}\,dx = \frac{12\left(\int \frac{asec(cx)}{x^5}dx\right)a^2b\,x^4+4\left(\int \frac{asec(cx)^3}{x^5}dx\right)b^3x^4+12\left(\int \frac{asec(cx)^2}{x^5}dx\right)a\,b^2x^4-a^3}{4x^4}$$

input
```
int((a+b*asec(c*x))^3/x^5,x)
```

output
```
(12*int(asec(c*x)/x**5,x)*a**2*b*x**4 + 4*int(asec(c*x)**3/x**5,x)*b**3*x*
*4 + 12*int(asec(c*x)**2/x**5,x)*a*b**2*x**4 - a**3)/(4*x**4)
```

3.33 $\int \frac{x}{a+b\sec^{-1}(cx)}\,dx$

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Mathematica [N/A] 324
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Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{x}{a+b\sec^{-1}(cx)}\,dx = \text{Int}\left(\frac{x}{a+b\sec^{-1}(cx)},x\right)$$

output
```
Defer(Int)(x/(a+b*arcsec(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 2.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a+b\sec^{-1}(cx)}\,dx = \int \frac{x}{a+b\sec^{-1}(cx)}\,dx$$

input
```
Integrate[x/(a + b*ArcSec[c*x]),x]
```

output
```
Integrate[x/(a + b*ArcSec[c*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+b\sec^{-1}(cx)}dx$$

$$\Big\downarrow \text{5771}$$

$$\int \frac{x}{a+b\sec^{-1}(cx)}dx$$

input
```
Int[x/(a + b*ArcSec[c*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+b\ \mathrm{arcsec}\left(cx\right)}dx$$

input
```
int(x/(a+b*arcsec(c*x)),x)
```

output
```
int(x/(a+b*arcsec(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a + b\sec^{-1}(cx)}\,dx = \int \frac{x}{b\,\mathrm{arcsec}\,(cx) + a}\,dx$$

input
```
integrate(x/(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral(x/(b*arcsec(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{a + b\sec^{-1}(cx)}\,dx = \int \frac{x}{a + b\,\mathrm{asec}\,(cx)}\,dx$$

input
```
integrate(x/(a+b*asec(c*x)),x)
```

output
```
Integral(x/(a + b*asec(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a + b\sec^{-1}(cx)}\,dx = \int \frac{x}{b\,\mathrm{arcsec}\,(cx) + a}\,dx$$

input
```
integrate(x/(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
integrate(x/(b*arcsec(c*x) + a), x)
```

Giac [N/A]

Not integrable

Time = 46.74 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a + b\sec^{-1}(cx)}\,dx = \int \frac{x}{b\,\mathrm{arcsec}\,(cx) + a}\,dx$$

input
```
integrate(x/(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate(x/(b*arcsec(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{x}{a + b\sec^{-1}(cx)}\,dx = \int \frac{x}{a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}\,dx$$

input
```
int(x/(a + b*acos(1/(c*x))),x)
```

output
```
int(x/(a + b*acos(1/(c*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a + b\sec^{-1}(cx)}\,dx = \int \frac{x}{asec\,(cx)\,b + a}dx$$

input
```
int(x/(a+b*asec(c*x)),x)
```

output
```
int(x/(asec(c*x)*b + a),x)
```

3.34 $\int \frac{1}{a+b\sec^{-1}(cx)}\,dx$

Optimal result 329
Mathematica [N/A] 329
Rubi [N/A] 330
Maple [N/A] 330
Fricas [N/A] 331
Sympy [N/A] 331
Maxima [N/A] 331
Giac [N/A] 332
Mupad [N/A] 332
Reduce [N/A] 333

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{a+b\sec^{-1}(cx)}\,dx = \operatorname{Int}\left(\frac{1}{a+b\sec^{-1}(cx)},x\right)$$

output
```
Defer(Int)(1/(a+b*arcsec(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\sec^{-1}(cx)}\,dx = \int \frac{1}{a+b\sec^{-1}(cx)}\,dx$$

input
```
Integrate[(a + b*ArcSec[c*x])^(-1),x]
```

output
```
Integrate[(a + b*ArcSec[c*x])^(-1), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sec^{-1}(cx)}dx$$
$$\Big\downarrow \text{5771}$$
$$\int \frac{1}{a+b\sec^{-1}(cx)}dx$$

input
```
Int[(a + b*ArcSec[c*x])^(-1),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\,\mathrm{arcsec}\,(cx)}dx$$

input
```
int(1/(a+b*arcsec(c*x)),x)
```

output
```
int(1/(a+b*arcsec(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\sec^{-1}(cx)}\,dx=\int \frac{1}{b\,\text{arcsec}\,(cx)+a}\,dx$$

input
```
integrate(1/(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*arcsec(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\sec^{-1}(cx)}\,dx=\int \frac{1}{a+b\,\text{asec}\,(cx)}\,dx$$

input
```
integrate(1/(a+b*asec(c*x)),x)
```

output
```
Integral(1/(a + b*asec(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\sec^{-1}(cx)}\,dx=\int \frac{1}{b\,\text{arcsec}\,(cx)+a}\,dx$$

input
```
integrate(1/(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/(b*arcsec(c*x) + a), x)
```

Giac [N/A]

Not integrable

Time = 7.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\sec^{-1}(cx)}\,dx=\int \frac{1}{b\,\mathrm{arcsec}\,(cx)+a}\,dx$$

input
```
integrate(1/(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate(1/(b*arcsec(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1}{a+b\sec^{-1}(cx)}\,dx=\int \frac{1}{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}\,dx$$

input
```
int(1/(a + b*acos(1/(c*x))),x)
```

output
```
int(1/(a + b*acos(1/(c*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\sec^{-1}(cx)}\,dx=\int \frac{1}{asec\,(cx)\,b+a}dx$$

input
```
int(1/(a+b*asec(c*x)),x)
```

output
```
int(1/(asec(c*x)*b + a),x)
```

3.35 $\int \frac{1}{x(a+b\sec^{-1}(cx))}\,dx$

Optimal result . . . 334
Mathematica [N/A] . . . 334
Rubi [N/A] . . . 335
Maple [N/A] . . . 335
Fricas [N/A] . . . 336
Sympy [N/A] . . . 336
Maxima [N/A] . . . 336
Giac [N/A] . . . 337
Mupad [N/A] . . . 337
Reduce [N/A] . . . 338

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x\,(a+b\sec^{-1}(cx))}\,dx=\operatorname{Int}\left(\frac{1}{x\,(a+b\sec^{-1}(cx))},x\right)$$

output
```
Defer(Int)(1/x/(a+b*arcsec(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\,(a+b\sec^{-1}(cx))}\,dx=\int \frac{1}{x\,(a+b\sec^{-1}(cx))}\,dx$$

input
```
Integrate[1/(x*(a + b*ArcSec[c*x])),x]
```

output
```
Integrate[1/(x*(a + b*ArcSec[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a+b \sec^{-1}(cx))} \, dx$$
$$\Big\downarrow \text{5771}$$
$$\int \frac{1}{x (a+b \sec^{-1}(cx))}dx$$

input
```
Int[1/(x*(a + b*ArcSec[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x (a+b \,\mathrm{arcsec}\,(cx))}dx$$

input
```
int(1/x/(a+b*arcsec(c*x)),x)
```

output
```
int(1/x/(a+b*arcsec(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\,\mathrm{arcsec}\,(cx)+a)x}\,dx$$

input
```
integrate(1/x/(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*x*arcsec(c*x) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)}\,dx=\int \frac{1}{x\left(a+b\,\mathrm{asec}\,(cx)\right)}\,dx$$

input
```
integrate(1/x/(a+b*asec(c*x)),x)
```

output
```
Integral(1/(x*(a + b*asec(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\,\mathrm{arcsec}\,(cx)+a)x}\,dx$$

input
```
integrate(1/x/(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((b*arcsec(c*x) + a)*x), x)
```

Giac [N/A]

Not integrable

Time = 1.82 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)}\,dx = \int \frac{1}{(b\,\mathrm{arcsec}\,(cx)+a)x}\,dx$$

input
```
integrate(1/x/(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((b*arcsec(c*x) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)}\,dx = \int \frac{1}{x\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}\,dx$$

input
```
int(1/(x*(a + b*acos(1/(c*x)))),x)
```

output
```
int(1/(x*(a + b*acos(1/(c*x)))), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)}\,dx=\int \frac{1}{asec\left(cx\right)bx+ax}dx$$

input
```
int(1/x/(a+b*asec(c*x)),x)
```

output
```
int(1/(asec(c*x)*b*x + a*x),x)
```

3.36 $\int \frac{1}{x^2(a+b\sec^{-1}(cx))}\,dx$

Optimal result 339
Mathematica [A] (verified) 339
Rubi [A] (verified) 340
Maple [A] (verified) 342
Fricas [F] 342
Sympy [F] 342
Maxima [F] 343
Giac [A] (verification not implemented) 343
Mupad [F(-1)] 343
Reduce [F] 344

Optimal result

Integrand size = 14, antiderivative size = 46

$$\int \frac{1}{x^2(a+b\sec^{-1}(cx))}\,dx = -\frac{c\,\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\sin\left(\frac{a}{b}\right)}{b} + \frac{c\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b}$$

output
```
-c*Ci(a/b+arcsec(c*x))*sin(a/b)/b+c*cos(a/b)*Si(a/b+arcsec(c*x))/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2(a+b\sec^{-1}(cx))}\,dx = \frac{c\left(-\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\sin\left(\frac{a}{b}\right)+\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\right)}{b}$$

input
```
Integrate[1/(x^2*(a + b*ArcSec[c*x])),x]
```

output
```
(c*(-(CosIntegral[a/b + ArcSec[c*x]]*Sin[a/b]) + Cos[a/b]*SinIntegral[a/b
+ ArcSec[c*x]]))/b
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5745, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)}dx$$

$$\Big\downarrow \text{5745}$$

$$c\int \frac{\sqrt{1-\frac{1}{c^2x^2}}}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)$$

$$\Big\downarrow \text{3042}$$

$$c\int \frac{\sin\left(\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)$$

$$\Big\downarrow \text{3784}$$

$$c\left(\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)-\sin\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{3042}$$

$$c\left(\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)-\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a}{b}+\sec^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{3780}$$

$$c\left(\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b}-\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a}{b}+\sec^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{3783}$$

$$c\left(\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b}-\frac{\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b}\right)$$

input
```
Int[1/(x^2*(a + b*ArcSec[c*x])),x]
```

output
```
c*(-((CosIntegral[a/b + ArcSec[c*x]]*Sin[a/b])/b) + (Cos[a/b]*SinIntegral[
a/b + ArcSec[c*x]])/b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$c\left(\frac{\mathrm{Si}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)\cos\left(\frac{a}{b}\right)}{b}-\frac{\mathrm{Ci}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)\sin\left(\frac{a}{b}\right)}{b}\right)$	47
default	$c\left(\frac{\mathrm{Si}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)\cos\left(\frac{a}{b}\right)}{b}-\frac{\mathrm{Ci}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)\sin\left(\frac{a}{b}\right)}{b}\right)$	47

input
```
int(1/x^2/(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
c*(Si(a/b+arcsec(c*x))*cos(a/b)/b-Ci(a/b+arcsec(c*x))*sin(a/b)/b)
```

Fricas [F]

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\,\mathrm{arcsec}\,(cx)+a)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*x^2*arcsec(c*x) + a*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)}\,dx=\int \frac{1}{x^2\left(a+b\,\mathrm{asec}\,(cx)\right)}\,dx$$

input
```
integrate(1/x**2/(a+b*asec(c*x)),x)
```

output
```
Integral(1/(x**2*(a + b*asec(c*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\,\mathrm{arcsec}\,(cx)+a)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((b*arcsec(c*x) + a)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)}\,dx = -c\left(\frac{\mathrm{Ci}\left(\frac{a}{b}+\arccos\left(\frac{1}{cx}\right)\right)\sin\left(\frac{a}{b}\right)}{b}-\frac{\cos\left(\frac{a}{b}\right)\mathrm{Si}\left(\frac{a}{b}+\arccos\left(\frac{1}{cx}\right)\right)}{b}\right)$$

input
```
integrate(1/x^2/(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
-c*(cos_integral(a/b + arccos(1/(c*x)))*sin(a/b)/b - cos(a/b)*sin_integral
(a/b + arccos(1/(c*x)))/b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)}\,dx=\int \frac{1}{x^2\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}\,dx$$

input
```
int(1/(x^2*(a + b*acos(1/(c*x)))),x)
```

output
```
int(1/(x^2*(a + b*acos(1/(c*x)))), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a + b \sec^{-1}(cx))} \, dx = \int \frac{1}{asec\,(cx)\,b\,x^2 + a\,x^2} dx$$

input
```
int(1/x^2/(a+b*asec(c*x)),x)
```

output
```
int(1/(asec(c*x)*b*x**2 + a*x**2),x)
```

3.37 $\int \frac{1}{x^3(a+b\sec^{-1}(cx))}\,dx$

Optimal result . . . 345
Mathematica [A] (verified) . . . 345
Rubi [A] (verified) . . . 346
Maple [A] (verified) . . . 348
Fricas [F] . . . 349
Sympy [F] . . . 349
Maxima [F] . . . 349
Giac [A] (verification not implemented) . . . 350
Mupad [F(-1)] . . . 350
Reduce [F] . . . 350

Optimal result

Integrand size = 14, antiderivative size = 63

$$\int \frac{1}{x^3\left(a+b\sec^{-1}(cx)\right)}\,dx = -\frac{c^2\,\text{CosIntegral}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)\sin\left(\frac{2a}{b}\right)}{2b} + \frac{c^2\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{2b}$$

output
```
-1/2*c^2*Ci(2*a/b+2*arcsec(c*x))*sin(2*a/b)/b+1/2*c^2*cos(2*a/b)*Si(2*a/b+
2*arcsec(c*x))/b
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^3\left(a+b\sec^{-1}(cx)\right)}\,dx = \frac{c^2\left(-\text{CosIntegral}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)\sin\left(\frac{2a}{b}\right)+\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)\right)}{2b}$$

input
```
Integrate[1/(x^3*(a + b*ArcSec[c*x])),x]
```

output
```
(c^2*(-(CosIntegral[(2*a)/b + 2*ArcSec[c*x]]*Sin[(2*a)/b]) + Cos[(2*a)/b]*
SinIntegral[(2*a)/b + 2*ArcSec[c*x]]))/(2*b)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5745, 4906, 27, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+b\sec^{-1}(cx))}dx$$

$$\Big\downarrow \text{5745}$$

$$c^2\int \frac{\sqrt{1-\frac{1}{c^2x^2}}}{cx(a+b\sec^{-1}(cx))}d\sec^{-1}(cx)$$

$$\Big\downarrow \text{4906}$$

$$c^2\int \frac{\sin\left(2\sec^{-1}(cx)\right)}{2(a+b\sec^{-1}(cx))}d\sec^{-1}(cx)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}c^2\int \frac{\sin\left(2\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}c^2\int \frac{\sin\left(2\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)$$

$$\Big\downarrow \text{3784}$$

$$\frac{1}{2}c^2\left(\cos\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)-\sin\left(\frac{2a}{b}\right)\int \frac{\cos\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}c^2\left(\cos\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)-\sin\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2a}{b}+2\sec^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{3780}$$

$$\frac{1}{2}c^2\left(\frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b}-\sin\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2a}{b}+2\sec^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{3783}$$

$$\frac{1}{2}c^2\left(\frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b}-\frac{\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b}\right)$$

input
```
Int[1/(x^3*(a + b*ArcSec[c*x])),x]
```

output
```
(c^2*(-((CosIntegral[(2*a)/b + 2*ArcSec[c*x]]*Sin[(2*a)/b])/b) + (Cos[(2*a
)/b]*SinIntegral[(2*a)/b + 2*ArcSec[c*x]])/b))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$c^2\left(\frac{\mathrm{Si}\left(\frac{2a}{b}+2\,\mathrm{arcsec}(cx)\right)\cos\left(\frac{2a}{b}\right)}{2b}-\frac{\mathrm{Ci}\left(\frac{2a}{b}+2\,\mathrm{arcsec}(cx)\right)\sin\left(\frac{2a}{b}\right)}{2b}\right)$	58
default	$c^2\left(\frac{\mathrm{Si}\left(\frac{2a}{b}+2\,\mathrm{arcsec}(cx)\right)\cos\left(\frac{2a}{b}\right)}{2b}-\frac{\mathrm{Ci}\left(\frac{2a}{b}+2\,\mathrm{arcsec}(cx)\right)\sin\left(\frac{2a}{b}\right)}{2b}\right)$	58

input
```
int(1/x^3/(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
c^2*(1/2*Si(2*a/b+2*arcsec(c*x))*cos(2*a/b)/b-1/2*Ci(2*a/b+2*arcsec(c*x))*
sin(2*a/b)/b)
```

Fricas [F]

$$\int \frac{1}{x^3 \left(a + b \sec^{-1}(cx)\right)} \, dx = \int \frac{1}{(b \operatorname{arcsec}(cx) + a)x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*x^3*arcsec(c*x) + a*x^3), x)
```

Sympy [F]

$$\int \frac{1}{x^3 \left(a + b \sec^{-1}(cx)\right)} \, dx = \int \frac{1}{x^3 \left(a + b \operatorname{asec}(cx)\right)} \, dx$$

input
```
integrate(1/x**3/(a+b*asec(c*x)),x)
```

output
```
Integral(1/(x**3*(a + b*asec(c*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^3 \left(a + b \sec^{-1}(cx)\right)} \, dx = \int \frac{1}{(b \operatorname{arcsec}(cx) + a)x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((b*arcsec(c*x) + a)*x^3), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.51

$$\int \frac{1}{x^3 (a + b \sec^{-1}(cx))} dx =$$
$$-\frac{1}{2} \left(\frac{2 c \cos\left(\frac{a}{b}\right) \text{Ci}\left(\frac{2a}{b} + 2 \arccos\left(\frac{1}{cx}\right)\right) \sin\left(\frac{a}{b}\right)}{b} - \frac{2 c \cos\left(\frac{a}{b}\right)^2 \text{Si}\left(\frac{2a}{b} + 2 \arccos\left(\frac{1}{cx}\right)\right)}{b} + \frac{c \text{Si}\left(\frac{2a}{b} + 2 \text{ar}\right.}{b}\right.$$

input
```
integrate(1/x^3/(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
-1/2*(2*c*cos(a/b)*cos_integral(2*a/b + 2*arccos(1/(c*x)))*sin(a/b)/b - 2*
c*cos(a/b)^2*sin_integral(2*a/b + 2*arccos(1/(c*x)))/b + c*sin_integral(2*
a/b + 2*arccos(1/(c*x)))/b)*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + b \sec^{-1}(cx))} dx = \int \frac{1}{x^3 \left(a + b \,\text{acos}\left(\frac{1}{cx}\right)\right)} dx$$

input
```
int(1/(x^3*(a + b*acos(1/(c*x)))),x)
```

output
```
int(1/(x^3*(a + b*acos(1/(c*x)))), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (a + b \sec^{-1}(cx))} dx = \int \frac{1}{asec(cx) b x^3 + a x^3} dx$$

input
```
int(1/x^3/(a+b*asec(c*x)),x)
```

output
```
int(1/(asec(c*x)*b*x**3 + a*x**3),x)
```

3.38 $\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)}\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 117

$$\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)}\,dx = -\frac{c^3\,\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\sin\left(\frac{a}{b}\right)}{4b} - \frac{c^3\,\text{CosIntegral}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)\sin\left(\frac{3a}{b}\right)}{4b} + \frac{c^3\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{4b} + \frac{c^3\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)}{4b}$$

output
```
-1/4*c^3*Ci(a/b+arcsec(c*x))*sin(a/b)/b-1/4*c^3*Ci(3*a/b+3*arcsec(c*x))*si
n(3*a/b)/b+1/4*c^3*cos(a/b)*Si(a/b+arcsec(c*x))/b+1/4*c^3*cos(3*a/b)*Si(3*
a/b+3*arcsec(c*x))/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^4 (a + b\sec^{-1}(cx))} dx$$
$$= \frac{c^3\left(-\text{CosIntegral}\left(\frac{a}{b} + \sec^{-1}(cx)\right)\sin\left(\frac{a}{b}\right) - \text{CosIntegral}\left(3\left(\frac{a}{b} + \sec^{-1}(cx)\right)\right)\sin\left(\frac{3a}{b}\right) + \cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b} + \text{se}\right.\right.}{4b}$$

input
```
Integrate[1/(x^4*(a + b*ArcSec[c*x])),x]
```

output
```
(c^3*(-(CosIntegral[a/b + ArcSec[c*x]]*Sin[a/b]) - CosIntegral[3*(a/b + Ar
cSec[c*x])]*Sin[(3*a)/b] + Cos[a/b]*SinIntegral[a/b + ArcSec[c*x]] + Cos[(
3*a)/b]*SinIntegral[3*(a/b + ArcSec[c*x])]))/(4*b)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5745, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a + b\sec^{-1}(cx))} dx$$
$$\Big\downarrow \text{ 5745}$$
$$c^3 \int \frac{\sqrt{1 - \frac{1}{c^2x^2}}}{c^2x^2 (a + b\sec^{-1}(cx))} d\sec^{-1}(cx)$$
$$\Big\downarrow \text{ 4906}$$
$$c^3 \int \left(\frac{\sin(3\sec^{-1}(cx))}{4(a + b\sec^{-1}(cx))} + \frac{\sqrt{1 - \frac{1}{c^2x^2}}}{4(a + b\sec^{-1}(cx))}\right) d\sec^{-1}(cx)$$
$$\Big\downarrow \text{ 2009}$$

$$c^3\left(-\frac{\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{4b}-\frac{\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)}{4b}+\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{4b}\right.$$

input
```
Int[1/(x^4*(a + b*ArcSec[c*x])),x]
```

output
```
c^3*(-1/4*(CosIntegral[a/b + ArcSec[c*x]]*Sin[a/b])/b - (CosIntegral[(3*a)
/b + 3*ArcSec[c*x]]*Sin[(3*a)/b])/(4*b) + (Cos[a/b]*SinIntegral[a/b + ArcS
ec[c*x]])/(4*b) + (Cos[(3*a)/b]*SinIntegral[(3*a)/b + 3*ArcSec[c*x]])/(4*b
))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87

method	result
derivativedivides	$c^3\left(\frac{\text{Si}\left(\frac{3a}{b}+3\,\text{arcsec}(cx)\right)\cos\left(\frac{3a}{b}\right)}{4b}-\frac{\text{Ci}\left(\frac{3a}{b}+3\,\text{arcsec}(cx)\right)\sin\left(\frac{3a}{b}\right)}{4b}+\frac{\text{Si}\left(\frac{a}{b}+\text{arcsec}(cx)\right)\cos\left(\frac{a}{b}\right)}{4b}-\frac{\text{Ci}\left(\frac{a}{b}+\text{arcsec}\right.}{4\right.}$
default	$c^3\left(\frac{\text{Si}\left(\frac{3a}{b}+3\,\text{arcsec}(cx)\right)\cos\left(\frac{3a}{b}\right)}{4b}-\frac{\text{Ci}\left(\frac{3a}{b}+3\,\text{arcsec}(cx)\right)\sin\left(\frac{3a}{b}\right)}{4b}+\frac{\text{Si}\left(\frac{a}{b}+\text{arcsec}(cx)\right)\cos\left(\frac{a}{b}\right)}{4b}-\frac{\text{Ci}\left(\frac{a}{b}+\text{arcsec}\right.}{4\right.}$

input
```
int(1/x^4/(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
c^3*(1/4*Si(3*a/b+3*arcsec(c*x))*cos(3*a/b)/b-1/4*Ci(3*a/b+3*arcsec(c*x))*
sin(3*a/b)/b+1/4*Si(a/b+arcsec(c*x))*cos(a/b)/b-1/4*Ci(a/b+arcsec(c*x))*si
n(a/b)/b)
```

Fricas [F]

$$\int \frac{1}{x^4 (a + b \sec^{-1}(cx))} dx = \int \frac{1}{(b \operatorname{arcsec}(cx) + a)x^4} dx$$

input
```
integrate(1/x^4/(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*x^4*arcsec(c*x) + a*x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^4 (a + b \sec^{-1}(cx))} dx = \int \frac{1}{x^4 (a + b \operatorname{asec}(cx))} dx$$

input
```
integrate(1/x**4/(a+b*asec(c*x)),x)
```

output
```
Integral(1/(x**4*(a + b*asec(c*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^4 \left(a + b\sec^{-1}(cx)\right)}\,dx = \int \frac{1}{(b\,\mathrm{arcsec}\,(cx) + a)x^4}\,dx$$

input
```
integrate(1/x^4/(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((b*arcsec(c*x) + a)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.70

$$\int \frac{1}{x^4 \left(a + b\sec^{-1}(cx)\right)}\,dx = -\frac{1}{4}\left(\frac{4\,c^2\cos\left(\frac{a}{b}\right)^2 \mathrm{Ci}\left(\frac{3\,a}{b} + 3\,\arccos\left(\frac{1}{cx}\right)\right)\sin\left(\frac{a}{b}\right)}{b} - \frac{4\,c^2\cos\left(\frac{a}{b}\right)^3 \mathrm{Si}\left(\frac{3\,a}{b} + 3\,\arccos\left(\frac{1}{cx}\right)\right)}{b} - \frac{c^2\,\mathrm{Ci}\left(\frac{3\,a}{b} + \right.}{}\right.$$

input
```
integrate(1/x^4/(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
-1/4*(4*c^2*cos(a/b)^2*cos_integral(3*a/b + 3*arccos(1/(c*x)))*sin(a/b)/b
- 4*c^2*cos(a/b)^3*sin_integral(3*a/b + 3*arccos(1/(c*x)))/b - c^2*cos_int
egral(3*a/b + 3*arccos(1/(c*x)))*sin(a/b)/b + c^2*cos_integral(a/b + arcco
s(1/(c*x)))*sin(a/b)/b + 3*c^2*cos(a/b)*sin_integral(3*a/b + 3*arccos(1/(c
*x)))/b - c^2*cos(a/b)*sin_integral(a/b + arccos(1/(c*x)))/b)*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a + b \sec^{-1}(cx))} \, dx = \int \frac{1}{x^4 \left(a + b \operatorname{acos}\left(\frac{1}{c x}\right)\right)} \, dx$$

input
```
int(1/(x^4*(a + b*acos(1/(c*x)))),x)
```

output
```
int(1/(x^4*(a + b*acos(1/(c*x)))), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (a + b \sec^{-1}(cx))} \, dx = \int \frac{1}{asec(cx) \, b \, x^4 + a \, x^4} dx$$

input
```
int(1/x^4/(a+b*asec(c*x)),x)
```

output
```
int(1/(asec(c*x)*b*x**4 + a*x**4),x)
```

3.39 $\int \frac{x}{(a+b\sec^{-1}(cx))^2}\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{x}{(a+b\sec^{-1}(cx))^2}\,dx = \text{Int}\left(\frac{x}{(a+b\sec^{-1}(cx))^2},x\right)$$

output
```
Defer(Int)(x/(a+b*arcsec(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 10.36 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{(a+b\sec^{-1}(cx))^2}\,dx = \int \frac{x}{(a+b\sec^{-1}(cx))^2}\,dx$$

input
```
Integrate[x/(a + b*ArcSec[c*x])^2,x]
```

output
```
Integrate[x/(a + b*ArcSec[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+b\sec^{-1}(cx))^2} \, dx$$
$$\Big\downarrow \text{ 5771}$$
$$\int \frac{x}{(a+b\sec^{-1}(cx))^2}dx$$

input
```
Int[x/(a + b*ArcSec[c*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+b \operatorname{arcsec}(cx))^2}dx$$

input
```
int(x/(a+b*arcsec(c*x))^2,x)
```

output
```
int(x/(a+b*arcsec(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.33

$$\int \frac{x}{(a+b\sec^{-1}(cx))^2}\,dx=\int \frac{x}{(b\,\mathrm{arcsec}\,(cx)+a)^2}\,dx$$

input
```
integrate(x/(a+b*arcsec(c*x))^2,x, algorithm="fricas")
```

output
```
integral(x/(b^2*arcsec(c*x)^2 + 2*a*b*arcsec(c*x) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+b\sec^{-1}(cx))^2}\,dx=\int \frac{x}{(a+b\,\mathrm{asec}\,(cx))^2}\,dx$$

input
```
integrate(x/(a+b*asec(c*x))**2,x)
```

output
```
Integral(x/(a + b*asec(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 583, normalized size of antiderivative = 48.58

$$\int \frac{x}{(a+b\sec^{-1}(cx))^2}\,dx=\int \frac{x}{(b\,\mathrm{arcsec}\,(cx)+a)^2}\,dx$$

input
```
integrate(x/(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

output
```
-(4*(b*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + a*x^2)*sqrt(c*x + 1)*sqrt
(c*x - 1) - (4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*log(c^2*x^2
)^2 + 4*b^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2 + 8*a*b^2*arct
an(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*log(c) + b^3*log(x))*lo
g(c^2*x^2))*integrate(-4*(3*a*c^2*x^3 - 2*a*x + (3*b*c^2*x^3 - 2*b*x)*arct
an(sqrt(c*x + 1)*sqrt(c*x - 1)))*sqrt(c*x + 1)*sqrt(c*x - 1)/(4*b^3*log(c)
^2 + 4*a^2*b - 4*(b^3*c^2*log(c)^2 + a^2*b*c^2)*x^2 - 4*(b^3*c^2*x^2 - b^3
)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - (b^3*c^2*x^2 - b^3)*log(c^2*x^2)
^2 - 4*(b^3*c^2*x^2 - b^3)*log(x)^2 - 8*(a*b^2*c^2*x^2 - a*b^2)*arctan(sqr
t(c*x + 1)*sqrt(c*x - 1)) + 4*(b^3*c^2*x^2*log(c) - b^3*log(c) + (b^3*c^2*
x^2 - b^3)*log(x))*log(c^2*x^2) - 8*(b^3*c^2*x^2*log(c) - b^3*log(c))*log(
x)), x))/(4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*log(c^2*x^2)^2
 + 4*b^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2 + 8*a*b^2*arctan(
sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*log(c) + b^3*log(x))*log(c
^2*x^2))
```

Giac [N/A]

Not integrable

Time = 98.44 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{(a+b\sec^{-1}(cx))^2}\,dx = \int \frac{x}{(b\,\mathrm{arcsec}\,(cx)+a)^2}\,dx$$

input
```
integrate(x/(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

output
```
integrate(x/(b*arcsec(c*x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{x}{(a+b\sec^{-1}(cx))^2}\,dx = \int \frac{x}{\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^2}\,dx$$

input
```
int(x/(a + b*acos(1/(c*x)))^2,x)
```

output
```
int(x/(a + b*acos(1/(c*x)))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.33

$$\int \frac{x}{(a+b\sec^{-1}(cx))^2}\,dx = \int \frac{x}{asec(cx)^2 b^2 + 2asec(cx)\,ab + a^2}dx$$

input
```
int(x/(a+b*asec(c*x))^2,x)
```

output
```
int(x/(asec(c*x)**2*b**2 + 2*asec(c*x)*a*b + a**2),x)
```

3.40 $\int \frac{1}{(a+b\sec^{-1}(cx))^2}\,dx$

Optimal result 362
Mathematica [N/A] 362
Rubi [N/A] 363
Maple [N/A] 363
Fricas [N/A] 364
Sympy [N/A] 364
Maxima [N/A] 364
Giac [F(-2)] 365
Mupad [N/A] 365
Reduce [N/A] 366

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{(a+b\sec^{-1}(cx))^2}\,dx = \operatorname{Int}\left(\frac{1}{(a+b\sec^{-1}(cx))^2},x\right)$$

output

```
Defer(Int)(1/(a+b*arcsec(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 20.71 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{(a+b\sec^{-1}(cx))^2}\,dx = \int \frac{1}{(a+b\sec^{-1}(cx))^2}\,dx$$

input

```
Integrate[(a + b*ArcSec[c*x])^(-2),x]
```

output

```
Integrate[(a + b*ArcSec[c*x])^(-2), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec^{-1}(cx))^2} \, dx$$

$$\big\downarrow \text{ 5771}$$

$$\int \frac{1}{(a+b\sec^{-1}(cx))^2}dx$$

input
```
Int[(a + b*ArcSec[c*x])^(-2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+b\,\mathrm{arcsec}\,(cx))^2}dx$$

input
```
int(1/(a+b*arcsec(c*x))^2,x)
```

output
```
int(1/(a+b*arcsec(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.60

$$\int \frac{1}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(b\,\mathrm{arcsec}\left(cx\right)+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*arcsec(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*arcsec(c*x)^2 + 2*a*b*arcsec(c*x) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(a+b\,\mathrm{asec}\left(cx\right)\right)^2}\,dx$$

input
```
integrate(1/(a+b*asec(c*x))**2,x)
```

output
```
Integral((a + b*asec(c*x))**(-2), x)
```

Maxima [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 577, normalized size of antiderivative = 57.70

$$\int \frac{1}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(b\,\mathrm{arcsec}\left(cx\right)+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

output
```
-(4*(b*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + a*x)*sqrt(c*x + 1)*sqrt(c*x
 - 1) - (4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*log(c^2*x^2)^2
+ 4*b^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2 + 8*a*b^2*arctan(s
qrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*log(c) + b^3*log(x))*log(c^
2*x^2))*integrate(-4*(2*a*c^2*x^2 + (2*b*c^2*x^2 - b)*arctan(sqrt(c*x + 1)
*sqrt(c*x - 1)) - a)*sqrt(c*x + 1)*sqrt(c*x - 1)/(4*b^3*log(c)^2 + 4*a^2*b
 - 4*(b^3*c^2*log(c)^2 + a^2*b*c^2)*x^2 - 4*(b^3*c^2*x^2 - b^3)*arctan(sqr
t(c*x + 1)*sqrt(c*x - 1))^2 - (b^3*c^2*x^2 - b^3)*log(c^2*x^2)^2 - 4*(b^3*
c^2*x^2 - b^3)*log(x)^2 - 8*(a*b^2*c^2*x^2 - a*b^2)*arctan(sqrt(c*x + 1)*s
qrt(c*x - 1)) + 4*(b^3*c^2*x^2*log(c) - b^3*log(c) + (b^3*c^2*x^2 - b^3)*l
og(x))*log(c^2*x^2) - 8*(b^3*c^2*x^2*log(c) - b^3*log(c))*log(x)), x))/(4*
b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*log(c^2*x^2)^2 + 4*b^3*log
(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2 + 8*a*b^2*arctan(sqrt(c*x + 1
)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*log(c) + b^3*log(x))*log(c^2*x^2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\sec^{-1}(cx))^2}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(1/(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

Mupad [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1}{(a+b\sec^{-1}(cx))^2}\,dx = \int \frac{1}{\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^2}\,dx$$

input
```
int(1/(a + b*acos(1/(c*x)))^2,x)
```

output
```
int(1/(a + b*acos(1/(c*x)))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.60

$$\int \frac{1}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx=\int \frac{1}{asec\,(cx)^2\,b^2+2asec\,(cx)\,ab+a^2}dx$$

input
```
int(1/(a+b*asec(c*x))^2,x)
```

output
```
int(1/(asec(c*x)**2*b**2 + 2*asec(c*x)*a*b + a**2),x)
```

3.41 $\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}\,dx$

Optimal result 367
Mathematica [N/A] 367
Rubi [N/A] 368
Maple [N/A] 368
Fricas [N/A] 369
Sympy [N/A] 369
Maxima [N/A] 369
Giac [N/A] 370
Mupad [N/A] 370
Reduce [N/A] 371

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \text{Int}\left(\frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2},x\right)$$

output
```
Defer(Int)(1/x/(a+b*arcsec(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 2.75 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}\,dx$$

input
```
Integrate[1/(x*(a + b*ArcSec[c*x])^2),x]
```

output
```
Integrate[1/(x*(a + b*ArcSec[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}\,dx$$

$$\Big\downarrow \text{ 5771}$$

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}dx$$

input
```
Int[1/(x*(a + b*ArcSec[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\,\mathrm{arcsec}\left(cx\right)\right)^{2}}dx$$

input
```
int(1/x/(a+b*arcsec(c*x))^2,x)
```

output
```
int(1/x/(a+b*arcsec(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(b\,\mathrm{arcsec}\left(cx\right)+a\right)^2x}\,dx$$

input
```
integrate(1/x/(a+b*arcsec(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*x*arcsec(c*x)^2 + 2*a*b*x*arcsec(c*x) + a^2*x), x)
```

Sympy [N/A]

Not integrable

Time = 2.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}\,dx=\int \frac{1}{x\left(a+b\,\mathrm{asec}\left(cx\right)\right)^2}\,dx$$

input
```
integrate(1/x/(a+b*asec(c*x))**2,x)
```

output
```
Integral(1/(x*(a + b*asec(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 560, normalized size of antiderivative = 40.00

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(b\,\mathrm{arcsec}\left(cx\right)+a\right)^2x}\,dx$$

input
```
integrate(1/x/(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

output
```
-(4*sqrt(c*x + 1)*sqrt(c*x - 1)*(b*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + a
) - (4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*log(c^2*x^2)^2 + 4*
b^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2 + 8*a*b^2*arctan(sqrt(
c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*log(c) + b^3*log(x))*log(c^2*x^
2))*integrate(-4*(b*c^2*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + a*c^2*x)*s
qrt(c*x + 1)*sqrt(c*x - 1)/(4*b^3*log(c)^2 + 4*a^2*b - 4*(b^3*c^2*log(c)^2
 + a^2*b*c^2)*x^2 - 4*(b^3*c^2*x^2 - b^3)*arctan(sqrt(c*x + 1)*sqrt(c*x -
1))^2 - (b^3*c^2*x^2 - b^3)*log(c^2*x^2)^2 - 4*(b^3*c^2*x^2 - b^3)*log(x)^
2 - 8*(a*b^2*c^2*x^2 - a*b^2)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*(b^3
*c^2*x^2*log(c) - b^3*log(c) + (b^3*c^2*x^2 - b^3)*log(x))*log(c^2*x^2) -
8*(b^3*c^2*x^2*log(c) - b^3*log(c))*log(x)), x))/(4*b^3*arctan(sqrt(c*x +
1)*sqrt(c*x - 1))^2 + b^3*log(c^2*x^2)^2 + 4*b^3*log(c)^2 + 8*b^3*log(c)*l
og(x) + 4*b^3*log(x)^2 + 8*a*b^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a
^2*b - 4*(b^3*log(c) + b^3*log(x))*log(c^2*x^2))
```

Giac [N/A]

Not integrable

Time = 5.40 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(b\operatorname{arcsec}\left(cx\right)+a\right)^2x}\,dx$$

input
```
integrate(1/x/(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*arcsec(c*x) + a)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}\,dx=\int \frac{1}{x\left(a+b\operatorname{acos}\left(\frac{1}{c\,x}\right)\right)^2}\,dx$$

input
```
int(1/(x*(a + b*acos(1/(c*x)))^2),x)
```

output
```
int(1/(x*(a + b*acos(1/(c*x)))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}\,dx=\int \frac{1}{asec\left(cx\right)^2b^2x+2asec\left(cx\right)abx+a^2x}dx$$

input
```
int(1/x/(a+b*asec(c*x))^2,x)
```

output
```
int(1/(asec(c*x)**2*b**2*x + 2*asec(c*x)*a*b*x + a**2*x),x)
```

3.42 $\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)^2}\,dx$

Optimal result 372
Mathematica [A] (verified) 372
Rubi [A] (verified) 373
Maple [A] (verified) 375
Fricas [F] 376
Sympy [F] 376
Maxima [F] 376
Giac [B] (verification not implemented) 377
Mupad [F(-1)] 378
Reduce [F] 378

Optimal result

Integrand size = 14, antiderivative size = 75

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)^2}\,dx = -\frac{c\sqrt{1-\frac{1}{c^2x^2}}}{b\left(a+b\sec^{-1}(cx)\right)} + \frac{c\cos\left(\frac{a}{b}\right)\operatorname{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b^2} + \frac{c\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b^2}$$

output

```
-c*(1-1/c^2/x^2)^(1/2)/b/(a+b*arcsec(c*x))+c*cos(a/b)*Ci(a/b+arcsec(c*x))/
b^2+c*sin(a/b)*Si(a/b+arcsec(c*x))/b^2
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \frac{c\left(-\frac{b\sqrt{1-\frac{1}{c^2x^2}}}{a+b\sec^{-1}(cx)}+\cos\left(\frac{a}{b}\right)\operatorname{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)+\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\right)}{b^2}$$

input
```
Integrate[1/(x^2*(a + b*ArcSec[c*x])^2),x]
```

output
```
(c*(-((b*Sqrt[1 - 1/(c^2*x^2)])/(a + b*ArcSec[c*x])) + Cos[a/b]*CosIntegra
l[a/b + ArcSec[c*x]] + Sin[a/b]*SinIntegral[a/b + ArcSec[c*x]]))/b^2
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5745, 3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+b\sec^{-1}(cx)\right)^2}dx$$

$$\Big\downarrow \text{5745}$$

$$c\int \frac{\sqrt{1-\frac{1}{c^2x^2}}}{\left(a+b\sec^{-1}(cx)\right)^2}d\sec^{-1}(cx)$$

$$\Big\downarrow \text{3042}$$

$$c\int \frac{\sin\left(\sec^{-1}(cx)\right)}{\left(a+b\sec^{-1}(cx)\right)^2}d\sec^{-1}(cx)$$

$$\Big\downarrow \text{3778}$$

$$c\left(\frac{\int \frac{1}{cx\left(a+b\sec^{-1}(cx)\right)}d\sec^{-1}(cx)}{b}-\frac{\sqrt{1-\frac{1}{c^2x^2}}}{b\left(a+b\sec^{-1}(cx)\right)}\right)$$

$$\Big\downarrow \text{3042}$$

$$c\left(\frac{\int \frac{\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)}{b}-\frac{\sqrt{1-\frac{1}{c^2x^2}}}{b\left(a+b\sec^{-1}(cx)\right)}\right)$$

$$\Big\downarrow \text{3784}$$

$$c\left(\frac{\cos\left(\frac{a}{b}\right)\int\frac{\cos\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)+\sin\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)}{b}-\frac{\sqrt{1-\frac{1}{c^2x^2}}}{b\left(a+b\sec^{-1}(cx)\right)}\right)$$

$$\Big\downarrow \text{3042}$$

$$c\left(\frac{\sin\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)+\cos\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a}{b}+\sec^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)}{b}-\frac{\sqrt{1-\frac{1}{c^2x^2}}}{b\left(a+b\sec^{-1}(cx)\right)}\right)$$

$$\Big\downarrow \text{3780}$$

$$c\left(\frac{\cos\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a}{b}+\sec^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)+\frac{\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b}}{b}-\frac{\sqrt{1-\frac{1}{c^2x^2}}}{b\left(a+b\sec^{-1}(cx)\right)}\right)$$

$$\Big\downarrow \text{3783}$$

$$c\left(\frac{\frac{\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b}+\frac{\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b}}{b}-\frac{\sqrt{1-\frac{1}{c^2x^2}}}{b\left(a+b\sec^{-1}(cx)\right)}\right)$$

input
```
Int[1/(x^2*(a + b*ArcSec[c*x])^2),x]
```

output
```
c*(-(Sqrt[1 - 1/(c^2*x^2)]/(b*(a + b*ArcSec[c*x]))) + ((Cos[a/b]*CosIntegr
al[a/b + ArcSec[c*x]])/b + (Sin[a/b]*SinIntegral[a/b + ArcSec[c*x]])/b)/b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$c\left(-\frac{\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{(a+b\,\mathrm{arcsec}(cx))b}+\frac{\mathrm{Si}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)\sin\left(\frac{a}{b}\right)+\mathrm{Ci}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)\cos\left(\frac{a}{b}\right)}{b^2}\right)$	78
default	$c\left(-\frac{\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{(a+b\,\mathrm{arcsec}(cx))b}+\frac{\mathrm{Si}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)\sin\left(\frac{a}{b}\right)+\mathrm{Ci}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)\cos\left(\frac{a}{b}\right)}{b^2}\right)$	78

input
```
int(1/x^2/(a+b*arcsec(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
c*(-((c^2*x^2-1)/c^2/x^2)^(1/2)/(a+b*arcsec(c*x))/b+(Si(a/b+arcsec(c*x))*s
in(a/b)+Ci(a/b+arcsec(c*x))*cos(a/b))/b^2)
```

Fricas [F]

$$\int \frac{1}{x^2 \left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(b\,\mathrm{arcsec}\left(cx\right)+a\right)^2 x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arcsec(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*x^2*arcsec(c*x)^2 + 2*a*b*x^2*arcsec(c*x) + a^2*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2 \left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{x^2 \left(a+b\,\mathrm{asec}\left(cx\right)\right)^2}\,dx$$

input
```
integrate(1/x**2/(a+b*asec(c*x))**2,x)
```

output
```
Integral(1/(x**2*(a + b*asec(c*x))**2), x)
```

Maxima [F]

$$\int \frac{1}{x^2 \left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(b\,\mathrm{arcsec}\left(cx\right)+a\right)^2 x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

output
```
-(4*sqrt(c*x + 1)*sqrt(c*x - 1)*(b*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + a
) - (4*b^3*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*x*log(c^2*x^2)^2
+ 8*b^3*x*log(c)*log(x) + 4*b^3*x*log(x)^2 + 8*a*b^2*x*arctan(sqrt(c*x + 1
)*sqrt(c*x - 1)) + 4*(b^3*log(c)^2 + a^2*b)*x - 4*(b^3*x*log(c) + b^3*x*lo
g(x))*log(c^2*x^2))*integrate(4*sqrt(c*x + 1)*sqrt(c*x - 1)*(b*arctan(sqrt
(c*x + 1)*sqrt(c*x - 1)) + a)/(4*(b^3*c^2*log(c)^2 + a^2*b*c^2)*x^4 - 4*(b
^3*log(c)^2 + a^2*b)*x^2 + 4*(b^3*c^2*x^4 - b^3*x^2)*arctan(sqrt(c*x + 1)*
sqrt(c*x - 1))^2 + (b^3*c^2*x^4 - b^3*x^2)*log(c^2*x^2)^2 + 4*(b^3*c^2*x^4
 - b^3*x^2)*log(x)^2 + 8*(a*b^2*c^2*x^4 - a*b^2*x^2)*arctan(sqrt(c*x + 1)*
sqrt(c*x - 1)) - 4*(b^3*c^2*x^4*log(c) - b^3*x^2*log(c) + (b^3*c^2*x^4 - b
^3*x^2)*log(x))*log(c^2*x^2) + 8*(b^3*c^2*x^4*log(c) - b^3*x^2*log(c))*log
(x)), x))/(4*b^3*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*x*log(c^2*x
^2)^2 + 8*b^3*x*log(c)*log(x) + 4*b^3*x*log(x)^2 + 8*a*b^2*x*arctan(sqrt(c
*x + 1)*sqrt(c*x - 1)) + 4*(b^3*log(c)^2 + a^2*b)*x - 4*(b^3*x*log(c) + b^
3*x*log(x))*log(c^2*x^2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(73) = 146$.

Time = 0.17 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.01

$$\int \frac{1}{x^2 (a + b \sec^{-1}(cx))^2} \, dx$$
$$= \left(\frac{b \arccos\left(\frac{1}{cx}\right) \cos\left(\frac{a}{b}\right) \operatorname{Ci}\left(\frac{a}{b} + \arccos\left(\frac{1}{cx}\right)\right)}{b^3 \arccos\left(\frac{1}{cx}\right) + ab^2} + \frac{b \arccos\left(\frac{1}{cx}\right) \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a}{b} + \arccos\left(\frac{1}{cx}\right)\right)}{b^3 \arccos\left(\frac{1}{cx}\right) + ab^2} + \frac{a \cos\left(\frac{a}{b}\right) \operatorname{Ci}\left(\right.}{b^3 \arccos}\right.$$

input
```
integrate(1/x^2/(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

output
```
(b*arccos(1/(c*x))*cos(a/b)*cos_integral(a/b + arccos(1/(c*x)))/(b^3*arcco
s(1/(c*x)) + a*b^2) + b*arccos(1/(c*x))*sin(a/b)*sin_integral(a/b + arccos
(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) + a*cos(a/b)*cos_integral(a/b + a
rccos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) + a*sin(a/b)*sin_integral(a/
b + arccos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) - b*sqrt(-1/(c^2*x^2) +
 1)/(b^3*arccos(1/(c*x)) + a*b^2))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + b \sec^{-1}(cx))^2} \, dx = \int \frac{1}{x^2 \left(a + b \operatorname{acos}\left(\frac{1}{c x}\right)\right)^2} \, dx$$

input
```
int(1/(x^2*(a + b*acos(1/(c*x)))^2),x)
```

output
```
int(1/(x^2*(a + b*acos(1/(c*x)))^2), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a + b \sec^{-1}(cx))^2} \, dx = \int \frac{1}{asec(cx)^2 b^2 x^2 + 2asec(cx) ab x^2 + a^2 x^2} dx$$

input
```
int(1/x^2/(a+b*asec(c*x))^2,x)
```

output
```
int(1/(asec(c*x)**2*b**2*x**2 + 2*asec(c*x)*a*b*x**2 + a**2*x**2),x)
```

3.43 $\int \frac{1}{x^3\left(a+b\sec^{-1}(cx)\right)^2}\,dx$

Optimal result 379
Mathematica [A] (verified) 379
Rubi [A] (verified) 380
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Fricas [F] 383
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Giac [B] (verification not implemented) 385
Mupad [F(-1)] 385
Reduce [F] 386

Optimal result

Integrand size = 14, antiderivative size = 84

$$\int \frac{1}{x^3\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \frac{c^2\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b^2} - \frac{c^2\sin\left(2\sec^{-1}(cx)\right)}{2b\left(a+b\sec^{-1}(cx)\right)} + \frac{c^2\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b^2}$$

output
```
c^2*cos(2*a/b)*Ci(2*a/b+2*arcsec(c*x))/b^2-1/2*c^2*sin(2*arcsec(c*x))/b/(a
+b*arcsec(c*x))+c^2*sin(2*a/b)*Si(2*a/b+2*arcsec(c*x))/b^2
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \frac{c\left(-\frac{b\sqrt{1-\frac{1}{c^2x^2}}}{ax+bx\sec^{-1}(cx)}+c\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(2\left(\frac{a}{b}+\sec^{-1}(cx)\right)\right)+c\sin\left(\frac{2a}{b}\right)\text{Si}\left(2\left(\frac{a}{b}+\sec^{-1}(cx)\right)\right)\right)}{b^2}$$

input
```
Integrate[1/(x^3*(a + b*ArcSec[c*x])^2),x]
```

output
```
(c*(-((b*Sqrt[1 - 1/(c^2*x^2)])/(a*x + b*x*ArcSec[c*x])) + c*Cos[(2*a)/b]*
CosIntegral[2*(a/b + ArcSec[c*x])] + c*Sin[(2*a)/b]*SinIntegral[2*(a/b + A
rcSec[c*x])]))/b^2
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5745, 4906, 27, 3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+b \sec^{-1}(cx)\right)^2} \, dx$$

$$\Big\downarrow \text{5745}$$

$$c^2 \int \frac{\sqrt{1-\frac{1}{c^2x^2}}}{cx \left(a+b \sec^{-1}(cx)\right)^2}d\sec^{-1}(cx)$$

$$\Big\downarrow \text{4906}$$

$$c^2 \int \frac{\sin \left(2 \sec^{-1}(cx)\right)}{2 \left(a+b \sec^{-1}(cx)\right)^2}d\sec^{-1}(cx)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}c^2 \int \frac{\sin \left(2 \sec^{-1}(cx)\right)}{\left(a+b \sec^{-1}(cx)\right)^2}d\sec^{-1}(cx)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}c^2 \int \frac{\sin \left(2 \sec^{-1}(cx)\right)}{\left(a+b \sec^{-1}(cx)\right)^2}d\sec^{-1}(cx)$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{2}c^2 \left(\frac{2 \int \frac{\cos \left(2 \sec^{-1}(cx)\right)}{a+b \sec^{-1}(cx)}d\sec^{-1}(cx)}{b}-\frac{\sin \left(2 \sec^{-1}(cx)\right)}{b \left(a+b \sec^{-1}(cx)\right)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}c^2\left(\frac{2\int \frac{\sin\left(2\sec^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)}{b}-\frac{\sin\left(2\sec^{-1}(cx)\right)}{b\left(a+b\sec^{-1}(cx)\right)}\right)$$

$$\Big\downarrow \; 3784$$

$$\frac{1}{2}c^2\left(\frac{2\left(\cos\left(\frac{2a}{b}\right)\int \frac{\cos\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)+\sin\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)\right)}{b}-\frac{\sin\left(2\sec^{-1}(cx)\right)}{b\left(a+b\sec^{-1}(cx)\right)}\right.$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}c^2\left(\frac{2\left(\sin\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)+\cos\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2a}{b}+2\sec^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)\right)}{b}-\frac{\sin\left(2\sec^{-1}(cx\right.}{b\left(a+b\sec^{-1}(cx\right.}\right.$$

$$\Big\downarrow \; 3780$$

$$\frac{1}{2}c^2\left(\frac{2\left(\cos\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2a}{b}+2\sec^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)+\frac{\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b}\right)}{b}-\frac{\sin\left(2\sec^{-1}(cx)\right)}{b\left(a+b\sec^{-1}(cx)\right)}\right)$$

$$\Big\downarrow \; 3783$$

$$\frac{1}{2}c^2\left(\frac{2\left(\frac{\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b}+\frac{\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b}\right)}{b}-\frac{\sin\left(2\sec^{-1}(cx)\right)}{b\left(a+b\sec^{-1}(cx)\right)}\right)$$

input
```
Int[1/(x^3*(a + b*ArcSec[c*x])^2),x]
```

output
```
(c^2*(-(Sin[2*ArcSec[c*x]]/(b*(a + b*ArcSec[c*x]))) + (2*((Cos[(2*a)/b]*Co
sIntegral[(2*a)/b + 2*ArcSec[c*x]])/b + (Sin[(2*a)/b]*SinIntegral[(2*a)/b
+ 2*ArcSec[c*x]])/b))/b))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$c^2\left(-\frac{\sin(2\operatorname{arcsec}(cx))}{2(a+b\operatorname{arcsec}(cx))b}+\frac{\operatorname{Si}\left(\frac{2a}{b}+2\operatorname{arcsec}(cx)\right)\sin\left(\frac{2a}{b}\right)+\operatorname{Ci}\left(\frac{2a}{b}+2\operatorname{arcsec}(cx)\right)\cos\left(\frac{2a}{b}\right)}{b^2}\right)$	77
default	$c^2\left(-\frac{\sin(2\operatorname{arcsec}(cx))}{2(a+b\operatorname{arcsec}(cx))b}+\frac{\operatorname{Si}\left(\frac{2a}{b}+2\operatorname{arcsec}(cx)\right)\sin\left(\frac{2a}{b}\right)+\operatorname{Ci}\left(\frac{2a}{b}+2\operatorname{arcsec}(cx)\right)\cos\left(\frac{2a}{b}\right)}{b^2}\right)$	77

input
```
int(1/x^3/(a+b*arcsec(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
c^2*(-1/2*sin(2*arcsec(c*x))/(a+b*arcsec(c*x))/b+(Si(2*a/b+2*arcsec(c*x))*
sin(2*a/b)+Ci(2*a/b+2*arcsec(c*x))*cos(2*a/b))/b^2)
```

Fricas [F]

$$\int \frac{1}{x^3\left(a+b\sec^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(b\operatorname{arcsec}(cx)+a\right)^2x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*arcsec(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*x^3*arcsec(c*x)^2 + 2*a*b*x^3*arcsec(c*x) + a^2*x^3), x)
```

Sympy [F]

$$\int \frac{1}{x^3 (a + b\sec^{-1}(cx))^2}\,dx = \int \frac{1}{x^3 (a + b\,\mathrm{asec}\,(cx))^2}\,dx$$

input
```
integrate(1/x**3/(a+b*asec(c*x))**2,x)
```

output
```
Integral(1/(x**3*(a + b*asec(c*x))**2), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (a + b\sec^{-1}(cx))^2}\,dx = \int \frac{1}{(b\,\mathrm{arcsec}\,(cx) + a)^2 x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

output
```
-(4*sqrt(c*x + 1)*sqrt(c*x - 1)*(b*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + a
) + (4*b^3*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*x^2*log(c^2*x^2
)^2 + 8*b^3*x^2*log(c)*log(x) + 4*b^3*x^2*log(x)^2 + 8*a*b^2*x^2*arctan(sq
rt(c*x + 1)*sqrt(c*x - 1)) + 4*(b^3*log(c)^2 + a^2*b)*x^2 - 4*(b^3*x^2*log
(c) + b^3*x^2*log(x))*log(c^2*x^2))*integrate(4*(a*c^2*x^2 + (b*c^2*x^2 -
2*b)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 2*a)*sqrt(c*x + 1)*sqrt(c*x - 1
)/(4*(b^3*c^2*log(c)^2 + a^2*b*c^2)*x^5 - 4*(b^3*log(c)^2 + a^2*b)*x^3 + 4
*(b^3*c^2*x^5 - b^3*x^3)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + (b^3*c^2*
x^5 - b^3*x^3)*log(c^2*x^2)^2 + 4*(b^3*c^2*x^5 - b^3*x^3)*log(x)^2 + 8*(a*
b^2*c^2*x^5 - a*b^2*x^3)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 4*(b^3*c^2*
x^5*log(c) - b^3*x^3*log(c) + (b^3*c^2*x^5 - b^3*x^3)*log(x))*log(c^2*x^2)
 + 8*(b^3*c^2*x^5*log(c) - b^3*x^3*log(c))*log(x)), x))/(4*b^3*x^2*arctan(
sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*x^2*log(c^2*x^2)^2 + 8*b^3*x^2*log(c)
*log(x) + 4*b^3*x^2*log(x)^2 + 8*a*b^2*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x -
 1)) + 4*(b^3*log(c)^2 + a^2*b)*x^2 - 4*(b^3*x^2*log(c) + b^3*x^2*log(x))*
log(c^2*x^2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(82) = 164$.

Time = 0.18 (sec) , antiderivative size = 357, normalized size of antiderivative = 4.25

$$\int \frac{1}{x^3 (a + b\sec^{-1}(cx))^2} dx = \left(\frac{2\, bc \arccos\left(\frac{1}{cx}\right) \cos\left(\frac{a}{b}\right)^2 \operatorname{Ci}\left(\frac{2a}{b} + 2\arccos\left(\frac{1}{cx}\right)\right)}{b^3 \arccos\left(\frac{1}{cx}\right) + ab^2} + \frac{2\, bc \arccos\left(\frac{1}{cx}\right) \cos\left(\frac{a}{b}\right) \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{2a}{b} + 2\arccos\left(\frac{1}{cx}\right.\right.}{b^3 \arccos\left(\frac{1}{cx}\right) + ab^2}\right.$$

input
```
integrate(1/x^3/(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

output
```
(2*b*c*arccos(1/(c*x))*cos(a/b)^2*cos_integral(2*a/b + 2*arccos(1/(c*x)))/
(b^3*arccos(1/(c*x)) + a*b^2) + 2*b*c*arccos(1/(c*x))*cos(a/b)*sin(a/b)*si
n_integral(2*a/b + 2*arccos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) + 2*a*
c*cos(a/b)^2*cos_integral(2*a/b + 2*arccos(1/(c*x)))/(b^3*arccos(1/(c*x))
+ a*b^2) + 2*a*c*cos(a/b)*sin(a/b)*sin_integral(2*a/b + 2*arccos(1/(c*x)))
/(b^3*arccos(1/(c*x)) + a*b^2) - b*c*arccos(1/(c*x))*cos_integral(2*a/b +
2*arccos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) - a*c*cos_integral(2*a/b
+ 2*arccos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) - b*sqrt(-1/(c^2*x^2) +
 1)/((b^3*arccos(1/(c*x)) + a*b^2)*x))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + b\sec^{-1}(cx))^2} dx = \int \frac{1}{x^3 \left(a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^2} dx$$

input
```
int(1/(x^3*(a + b*acos(1/(c*x)))^2),x)
```

output
```
int(1/(x^3*(a + b*acos(1/(c*x)))^2), x)
```

Reduce [F]

$$\int \frac{1}{x^3 \left(a + b \sec^{-1}(cx)\right)^2} \, dx = \int \frac{1}{asec\left(cx\right)^2 b^2 x^3 + 2asec\left(cx\right) ab\, x^3 + a^2 x^3} dx$$

input
```
int(1/x^3/(a+b*asec(c*x))^2,x)
```

output
```
int(1/(asec(c*x)**2*b**2*x**3 + 2*asec(c*x)*a*b*x**3 + a**2*x**3),x)
```

3.44 $\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)^2}\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 178

$$\begin{aligned}\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)^2}\,dx =& -\frac{c^3\sqrt{1-\frac{1}{c^2x^2}}}{4b\left(a+b\sec^{-1}(cx)\right)} \\ &+ \frac{c^3\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{4b^2} \\ &+ \frac{3c^3\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)}{4b^2} \\ &- \frac{c^3\sin\left(3\sec^{-1}(cx)\right)}{4b\left(a+b\sec^{-1}(cx)\right)} + \frac{c^3\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{4b^2} \\ &+ \frac{3c^3\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)}{4b^2}\end{aligned}$$

output

```
-1/4*c^3*(1-1/c^2/x^2)^(1/2)/b/(a+b*arcsec(c*x))+1/4*c^3*cos(a/b)*Ci(a/b+a
rcsec(c*x))/b^2+3/4*c^3*cos(3*a/b)*Ci(3*a/b+3*arcsec(c*x))/b^2-1/4*c^3*sin
(3*arcsec(c*x))/b/(a+b*arcsec(c*x))+1/4*c^3*sin(a/b)*Si(a/b+arcsec(c*x))/b
^2+3/4*c^3*sin(3*a/b)*Si(3*a/b+3*arcsec(c*x))/b^2
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^4 \left(a+b \sec^{-1}(cx)\right)^2} \, dx = \frac{-4bc\sqrt{1-\frac{1}{c^2x^2}}+c^3x^2(a+b\sec^{-1}(cx))\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)+3c^3x^2(a+b\sec^{-1}(cx))\text{co}}{}$$

input
```
Integrate[1/(x^4*(a + b*ArcSec[c*x])^2),x]
```

output
```
(-4*b*c*Sqrt[1 - 1/(c^2*x^2)] + c^3*x^2*(a + b*ArcSec[c*x])*Cos[a/b]*CosIn
tegral[a/b + ArcSec[c*x]] + 3*c^3*x^2*(a + b*ArcSec[c*x])*Cos[(3*a)/b]*Cos
Integral[3*(a/b + ArcSec[c*x])] + a*c^3*x^2*Sin[a/b]*SinIntegral[a/b + Arc
Sec[c*x]] + b*c^3*x^2*ArcSec[c*x]*Sin[a/b]*SinIntegral[a/b + ArcSec[c*x]]
+ 3*a*c^3*x^2*Sin[(3*a)/b]*SinIntegral[3*(a/b + ArcSec[c*x])] + 3*b*c^3*x^
2*ArcSec[c*x]*Sin[(3*a)/b]*SinIntegral[3*(a/b + ArcSec[c*x])])/(4*b^2*x^2*
(a + b*ArcSec[c*x]))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5745, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+b \sec^{-1}(cx)\right)^2} \, dx$$

$$\Big\downarrow \; 5745$$

$$c^3 \int \frac{\sqrt{1-\frac{1}{c^2x^2}}}{c^2x^2 \left(a+b\sec^{-1}(cx)\right)^2} d\sec^{-1}(cx)$$

$$\Big\downarrow \; 4906$$

$$c^3\int\left(\frac{\sin\left(3\sec^{-1}(cx)\right)}{4\left(a+b\sec^{-1}(cx)\right)^2}+\frac{\sqrt{1-\frac{1}{c^2x^2}}}{4\left(a+b\sec^{-1}(cx)\right)^2}\right)d\sec^{-1}(cx)$$

$$\Big\downarrow \text{2009}$$

$$c^3\left(\frac{\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{4b^2}+\frac{3\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)}{4b^2}+\frac{\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx\right)}{4b^2}\right.$$

input
```
Int[1/(x^4*(a + b*ArcSec[c*x])^2),x]
```

output
```
c^3*(-1/4*Sqrt[1 - 1/(c^2*x^2)]/(b*(a + b*ArcSec[c*x])) + (Cos[a/b]*CosInt
egral[a/b + ArcSec[c*x]])/(4*b^2) + (3*Cos[(3*a)/b]*CosIntegral[(3*a)/b +
3*ArcSec[c*x]])/(4*b^2) - Sin[3*ArcSec[c*x]]/(4*b*(a + b*ArcSec[c*x])) + (
Sin[a/b]*SinIntegral[a/b + ArcSec[c*x]])/(4*b^2) + (3*Sin[(3*a)/b]*SinInte
gral[(3*a)/b + 3*ArcSec[c*x]])/(4*b^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.86

method	result
derivativedivides	$c^{3}\left(-\frac{\sin(3\,\mathrm{arcsec}(cx))}{4(a+b\,\mathrm{arcsec}(cx))b}+\frac{\frac{3\,\mathrm{Si}\left(\frac{3a}{b}+3\,\mathrm{arcsec}(cx)\right)\sin\left(\frac{3a}{b}\right)}{4}+\frac{3\,\mathrm{Ci}\left(\frac{3a}{b}+3\,\mathrm{arcsec}(cx)\right)\cos\left(\frac{3a}{b}\right)}{4}}{b^{2}}-\frac{\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}{4(a+b\,\mathrm{arcsec}(cx))b}\right.$
default	$c^{3}\left(-\frac{\sin(3\,\mathrm{arcsec}(cx))}{4(a+b\,\mathrm{arcsec}(cx))b}+\frac{\frac{3\,\mathrm{Si}\left(\frac{3a}{b}+3\,\mathrm{arcsec}(cx)\right)\sin\left(\frac{3a}{b}\right)}{4}+\frac{3\,\mathrm{Ci}\left(\frac{3a}{b}+3\,\mathrm{arcsec}(cx)\right)\cos\left(\frac{3a}{b}\right)}{4}}{b^{2}}-\frac{\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}{4(a+b\,\mathrm{arcsec}(cx))b}\right.$

input
```
int(1/x^4/(a+b*arcsec(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
c^3*(-1/4*sin(3*arcsec(c*x))/(a+b*arcsec(c*x))/b+3/4*(Si(3*a/b+3*arcsec(c*
x))*sin(3*a/b)+Ci(3*a/b+3*arcsec(c*x))*cos(3*a/b))/b^2-1/4*((c^2*x^2-1)/c^
2/x^2)^(1/2)/(a+b*arcsec(c*x))/b+1/4*(Si(a/b+arcsec(c*x))*sin(a/b)+Ci(a/b+
arcsec(c*x))*cos(a/b))/b^2)
```

Fricas [F]

$$\int \frac{1}{x^4 (a+b\sec^{-1}(cx))^2}\,dx = \int \frac{1}{(b\,\mathrm{arcsec}\,(cx)+a)^2 x^4}\,dx$$

input
```
integrate(1/x^4/(a+b*arcsec(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*x^4*arcsec(c*x)^2 + 2*a*b*x^4*arcsec(c*x) + a^2*x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^4 \left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{x^4 \left(a+b\operatorname{asec}(cx)\right)^2}\,dx$$

input
```
integrate(1/x**4/(a+b*asec(c*x))**2,x)
```

output
```
Integral(1/(x**4*(a + b*asec(c*x))**2), x)
```

Maxima [F]

$$\int \frac{1}{x^4 \left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(b\operatorname{arcsec}(cx)+a\right)^2 x^4}\,dx$$

input
```
integrate(1/x^4/(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

output
```
-(4*sqrt(c*x + 1)*sqrt(c*x - 1)*(b*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + a
) + (4*b^3*x^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*x^3*log(c^2*x^2
)^2 + 8*b^3*x^3*log(c)*log(x) + 4*b^3*x^3*log(x)^2 + 8*a*b^2*x^3*arctan(sq
rt(c*x + 1)*sqrt(c*x - 1)) + 4*(b^3*log(c)^2 + a^2*b)*x^3 - 4*(b^3*x^3*log
(c) + b^3*x^3*log(x))*log(c^2*x^2))*integrate(4*(2*a*c^2*x^2 + (2*b*c^2*x^
2 - 3*b)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 3*a)*sqrt(c*x + 1)*sqrt(c*x
 - 1)/(4*(b^3*c^2*log(c)^2 + a^2*b*c^2)*x^6 - 4*(b^3*log(c)^2 + a^2*b)*x^4
 + 4*(b^3*c^2*x^6 - b^3*x^4)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + (b^3*
c^2*x^6 - b^3*x^4)*log(c^2*x^2)^2 + 4*(b^3*c^2*x^6 - b^3*x^4)*log(x)^2 + 8
*(a*b^2*c^2*x^6 - a*b^2*x^4)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 4*(b^3*
c^2*x^6*log(c) - b^3*x^4*log(c) + (b^3*c^2*x^6 - b^3*x^4)*log(x))*log(c^2*
x^2) + 8*(b^3*c^2*x^6*log(c) - b^3*x^4*log(c))*log(x)), x))/(4*b^3*x^3*arc
tan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*x^3*log(c^2*x^2)^2 + 8*b^3*x^3*lo
g(c)*log(x) + 4*b^3*x^3*log(x)^2 + 8*a*b^2*x^3*arctan(sqrt(c*x + 1)*sqrt(c
*x - 1)) + 4*(b^3*log(c)^2 + a^2*b)*x^3 - 4*(b^3*x^3*log(c) + b^3*x^3*log(
x))*log(c^2*x^2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 694 vs. $2(164) = 328$.

Time = 0.18 (sec) , antiderivative size = 694, normalized size of antiderivative = 3.90

$$\int \frac{1}{x^4 (a + b\sec^{-1}(cx))^2}\, dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

output
```
1/4*(12*b*c^2*arccos(1/(c*x))*cos(a/b)^3*cos_integral(3*a/b + 3*arccos(1/(
c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) + 12*b*c^2*arccos(1/(c*x))*cos(a/b)^2
*sin(a/b)*sin_integral(3*a/b + 3*arccos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a
*b^2) + 12*a*c^2*cos(a/b)^3*cos_integral(3*a/b + 3*arccos(1/(c*x)))/(b^3*a
rccos(1/(c*x)) + a*b^2) + 12*a*c^2*cos(a/b)^2*sin(a/b)*sin_integral(3*a/b
+ 3*arccos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) - 9*b*c^2*arccos(1/(c*x
))*cos(a/b)*cos_integral(3*a/b + 3*arccos(1/(c*x)))/(b^3*arccos(1/(c*x)) +
 a*b^2) + b*c^2*arccos(1/(c*x))*cos(a/b)*cos_integral(a/b + arccos(1/(c*x)
))/(b^3*arccos(1/(c*x)) + a*b^2) - 3*b*c^2*arccos(1/(c*x))*sin(a/b)*sin_in
tegral(3*a/b + 3*arccos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) + b*c^2*ar
ccos(1/(c*x))*sin(a/b)*sin_integral(a/b + arccos(1/(c*x)))/(b^3*arccos(1/(
c*x)) + a*b^2) - 9*a*c^2*cos(a/b)*cos_integral(3*a/b + 3*arccos(1/(c*x)))/
(b^3*arccos(1/(c*x)) + a*b^2) + a*c^2*cos(a/b)*cos_integral(a/b + arccos(1
/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) - 3*a*c^2*sin(a/b)*sin_integral(3*a
/b + 3*arccos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) + a*c^2*sin(a/b)*sin
_integral(a/b + arccos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) - 4*b*sqrt(
-1/(c^2*x^2) + 1)/((b^3*arccos(1/(c*x)) + a*b^2)*x^2))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a + b\sec^{-1}(cx))^2}\, dx = \int \frac{1}{x^4 \left(a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^2}\, dx$$

input
```
int(1/(x^4*(a + b*acos(1/(c*x)))^2),x)
```

output
```
int(1/(x^4*(a + b*acos(1/(c*x)))^2), x)
```

Reduce [F]

$$\int \frac{1}{x^4 \left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{asec\,(cx)^2\,b^2x^4+2asec\,(cx)\,ab\,x^4+a^2x^4}dx$$

input
```
int(1/x^4/(a+b*asec(c*x))^2,x)
```

output
```
int(1/(asec(c*x)**2*b**2*x**4 + 2*asec(c*x)*a*b*x**4 + a**2*x**4),x)
```

3.45 $\int \frac{x}{\left(a+b\sec^{-1}(cx)\right)^3} dx$

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Rubi [N/A] 395
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Giac [F(-1)] 397
Mupad [N/A] 398
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Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{x}{\left(a+b\sec^{-1}(cx)\right)^3} dx = \text{Int}\left(\frac{x}{\left(a+b\sec^{-1}(cx)\right)^3}, x\right)$$

output
```
Defer(Int)(x/(a+b*arcsec(c*x))^3,x)
```

Mathematica [N/A]

Not integrable

Time = 2.40 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{\left(a+b\sec^{-1}(cx)\right)^3} dx = \int \frac{x}{\left(a+b\sec^{-1}(cx)\right)^3} dx$$

input
```
Integrate[x/(a + b*ArcSec[c*x])^3,x]
```

output
```
Integrate[x/(a + b*ArcSec[c*x])^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+b\sec^{-1}(cx))^3} \, dx$$
$$\Big\downarrow \ 5771$$
$$\int \frac{x}{(a+b\sec^{-1}(cx))^3}dx$$

input
```
Int[x/(a + b*ArcSec[c*x])^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+b\,\mathrm{arcsec}\,(cx))^3}dx$$

input
```
int(x/(a+b*arcsec(c*x))^3,x)
```

output
```
int(x/(a+b*arcsec(c*x))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.50

$$\int \frac{x}{(a+b\sec^{-1}(cx))^3}\,dx = \int \frac{x}{(b\,\mathrm{arcsec}\,(cx)+a)^3}\,dx$$

input
```
integrate(x/(a+b*arcsec(c*x))^3,x, algorithm="fricas")
```

output
```
integral(x/(b^3*arcsec(c*x)^3 + 3*a*b^2*arcsec(c*x)^2 + 3*a^2*b*arcsec(c*x
) + a^3), x)
```

Sympy [N/A]

Not integrable

Time = 2.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+b\sec^{-1}(cx))^3}\,dx = \int \frac{x}{(a+b\,\mathrm{asec}\,(cx))^3}\,dx$$

input
```
integrate(x/(a+b*asec(c*x))**3,x)
```

output
```
Integral(x/(a + b*asec(c*x))**3, x)
```

Maxima [N/A]

Not integrable

Time = 34.09 (sec) , antiderivative size = 1790, normalized size of antiderivative = 149.17

$$\int \frac{x}{(a+b\sec^{-1}(cx))^3}\,dx = \int \frac{x}{(b\,\mathrm{arcsec}\,(cx)+a)^3}\,dx$$

input
```
integrate(x/(a+b*arcsec(c*x))^3,x, algorithm="maxima")
```

output
```
-(24*(a*b^2*c^2*log(c)^2 + a^3*c^2)*x^4 + 8*(3*b^3*c^2*x^4 - 2*b^3*x^2)*ar
ctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 16*(a*b^2*log(c)^2 + a^3)*x^2 + 24*(
3*a*b^2*c^2*x^4 - 2*a*b^2*x^2)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 2*(
3*a*b^2*c^2*x^4 - 2*a*b^2*x^2)*log(c^2*x^2)^2 + 8*(3*a*b^2*c^2*x^4 - 2*a*b
^2*x^2)*log(x)^2 + 2*(4*b^3*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - b^
3*x^2*log(c^2*x^2)^2 - 8*b^3*x^2*log(c)*log(x) - 4*b^3*x^2*log(x)^2 + 8*a*
b^2*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 4*(b^3*log(c)^2 - a^2*b)*x^2
 + 4*(b^3*x^2*log(c) + b^3*x^2*log(x))*log(c^2*x^2))*sqrt(c*x + 1)*sqrt(c*
x - 1) + 2*(12*(b^3*c^2*log(c)^2 + 3*a^2*b*c^2)*x^4 - 8*(b^3*log(c)^2 + 3*
a^2*b)*x^2 + (3*b^3*c^2*x^4 - 2*b^3*x^2)*log(c^2*x^2)^2 + 4*(3*b^3*c^2*x^4
 - 2*b^3*x^2)*log(x)^2 - 4*(3*b^3*c^2*x^4*log(c) - 2*b^3*x^2*log(c) + (3*b
^3*c^2*x^4 - 2*b^3*x^2)*log(x))*log(c^2*x^2) + 8*(3*b^3*c^2*x^4*log(c) - 2
*b^3*x^2*log(c))*log(x))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - (16*b^6*arc
tan(sqrt(c*x + 1)*sqrt(c*x - 1))^4 + b^6*log(c^2*x^2)^4 + 16*b^6*log(c)^4
+ 64*b^6*log(c)*log(x)^3 + 16*b^6*log(x)^4 + 64*a*b^5*arctan(sqrt(c*x + 1)
*sqrt(c*x - 1))^3 + 32*a^2*b^4*log(c)^2 + 16*a^4*b^2 - 8*(b^6*log(c) + b^6
*log(x))*log(c^2*x^2)^3 + 8*(b^6*log(c^2*x^2)^2 + 4*b^6*log(c)^2 + 8*b^6*l
og(c)*log(x) + 4*b^6*log(x)^2 + 12*a^2*b^4 - 4*(b^6*log(c) + b^6*log(x))*l
og(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 8*(3*b^6*log(c)^2 + 6
*b^6*log(c)*log(x) + 3*b^6*log(x)^2 + a^2*b^4)*log(c^2*x^2)^2 + 32*(3*b...
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{(a+b\sec^{-1}(cx))^3}\,dx = \text{Timed out}$$

input
```
integrate(x/(a+b*arcsec(c*x))^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{x}{(a+b\sec^{-1}(cx))^3}\,dx=\int \frac{x}{\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^3}\,dx$$

input
```
int(x/(a + b*acos(1/(c*x)))^3,x)
```

output
```
int(x/(a + b*acos(1/(c*x)))^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.50

$$\int \frac{x}{(a+b\sec^{-1}(cx))^3}\,dx=\int \frac{x}{asec\,(cx)^3\,b^3+3asec\,(cx)^2\,a\,b^2+3asec\,(cx)\,a^2b+a^3}dx$$

input
```
int(x/(a+b*asec(c*x))^3,x)
```

output
```
int(x/(asec(c*x)**3*b**3 + 3*asec(c*x)**2*a*b**2 + 3*asec(c*x)*a**2*b + a*
*3),x)
```

3.46 $\int \frac{1}{(a+b\sec^{-1}(cx))^3}\,dx$

Optimal result 399
Mathematica [N/A] 399
Rubi [N/A] 400
Maple [N/A] 400
Fricas [N/A] 401
Sympy [N/A] 401
Maxima [N/A] 401
Giac [N/A] 402
Mupad [N/A] 403
Reduce [N/A] 403

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{(a+b\sec^{-1}(cx))^3}\,dx = \text{Int}\left(\frac{1}{(a+b\sec^{-1}(cx))^3},x\right)$$

output
```
Defer(Int)(1/(a+b*arcsec(c*x))^3,x)
```

Mathematica [N/A]

Not integrable

Time = 8.51 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{(a+b\sec^{-1}(cx))^3}\,dx = \int \frac{1}{(a+b\sec^{-1}(cx))^3}\,dx$$

input
```
Integrate[(a + b*ArcSec[c*x])^(-3),x]
```

output
```
Integrate[(a + b*ArcSec[c*x])^(-3), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec^{-1}(cx))^3}\,dx$$
$$\Big\downarrow \text{5771}$$
$$\int \frac{1}{(a+b\sec^{-1}(cx))^3}dx$$

input
```
Int[(a + b*ArcSec[c*x])^(-3),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+b\,\mathrm{arcsec}\,(cx))^3}dx$$

input
```
int(1/(a+b*arcsec(c*x))^3,x)
```

output
```
int(1/(a+b*arcsec(c*x))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 4.00

$$\int \frac{1}{(a+b\sec^{-1}(cx))^3}\,dx = \int \frac{1}{(b\,\mathrm{arcsec}\,(cx)+a)^3}\,dx$$

input
```
integrate(1/(a+b*arcsec(c*x))^3,x, algorithm="fricas")
```

output
```
integral(1/(b^3*arcsec(c*x)^3 + 3*a*b^2*arcsec(c*x)^2 + 3*a^2*b*arcsec(c*x
) + a^3), x)
```

Sympy [N/A]

Not integrable

Time = 2.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{(a+b\sec^{-1}(cx))^3}\,dx = \int \frac{1}{(a+b\,\mathrm{asec}\,(cx))^3}\,dx$$

input
```
integrate(1/(a+b*asec(c*x))**3,x)
```

output
```
Integral((a + b*asec(c*x))**(-3), x)
```

Maxima [N/A]

Not integrable

Time = 34.69 (sec) , antiderivative size = 1744, normalized size of antiderivative = 174.40

$$\int \frac{1}{(a+b\sec^{-1}(cx))^3}\,dx = \int \frac{1}{(b\,\mathrm{arcsec}\,(cx)+a)^3}\,dx$$

input
```
integrate(1/(a+b*arcsec(c*x))^3,x, algorithm="maxima")
```

output
```
-(16*(a*b^2*c^2*log(c)^2 + a^3*c^2)*x^3 + 8*(2*b^3*c^2*x^3 - b^3*x)*arctan
(sqrt(c*x + 1)*sqrt(c*x - 1))^3 + 24*(2*a*b^2*c^2*x^3 - a*b^2*x)*arctan(sq
rt(c*x + 1)*sqrt(c*x - 1))^2 + 2*(2*a*b^2*c^2*x^3 - a*b^2*x)*log(c^2*x^2)^
2 + 8*(2*a*b^2*c^2*x^3 - a*b^2*x)*log(x)^2 + 2*(4*b^3*x*arctan(sqrt(c*x +
1)*sqrt(c*x - 1))^2 - b^3*x*log(c^2*x^2)^2 - 8*b^3*x*log(c)*log(x) - 4*b^3
*x*log(x)^2 + 8*a*b^2*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 4*(b^3*log(c
)^2 - a^2*b)*x + 4*(b^3*x*log(c) + b^3*x*log(x))*log(c^2*x^2))*sqrt(c*x +
1)*sqrt(c*x - 1) - 8*(a*b^2*log(c)^2 + a^3)*x + 2*(8*(b^3*c^2*log(c)^2 + 3
*a^2*b*c^2)*x^3 + (2*b^3*c^2*x^3 - b^3*x)*log(c^2*x^2)^2 + 4*(2*b^3*c^2*x^
3 - b^3*x)*log(x)^2 - 4*(b^3*log(c)^2 + 3*a^2*b)*x - 4*(2*b^3*c^2*x^3*log(
c) - b^3*x*log(c) + (2*b^3*c^2*x^3 - b^3*x)*log(x))*log(c^2*x^2) + 8*(2*b^
3*c^2*x^3*log(c) - b^3*x*log(c))*log(x))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1
)) - (16*b^6*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^4 + b^6*log(c^2*x^2)^4 +
16*b^6*log(c)^4 + 64*b^6*log(c)*log(x)^3 + 16*b^6*log(x)^4 + 64*a*b^5*arct
an(sqrt(c*x + 1)*sqrt(c*x - 1))^3 + 32*a^2*b^4*log(c)^2 + 16*a^4*b^2 - 8*(
b^6*log(c) + b^6*log(x))*log(c^2*x^2)^3 + 8*(b^6*log(c^2*x^2)^2 + 4*b^6*lo
g(c)^2 + 8*b^6*log(c)*log(x) + 4*b^6*log(x)^2 + 12*a^2*b^4 - 4*(b^6*log(c)
 + b^6*log(x))*log(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 8*(3*
b^6*log(c)^2 + 6*b^6*log(c)*log(x) + 3*b^6*log(x)^2 + a^2*b^4)*log(c^2*x^2
)^2 + 32*(3*b^6*log(c)^2 + a^2*b^4)*log(x)^2 + 16*(a*b^5*log(c^2*x^2)^2...
```

Giac [N/A]

Not integrable

Time = 48.66 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+b\sec^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(b\,\mathrm{arcsec}\,(cx)+a\right)^3}\,dx$$

input
```
integrate(1/(a+b*arcsec(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)^(-3), x)
```

Mupad [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1}{(a+b\sec^{-1}(cx))^3}\,dx = \int \frac{1}{\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^3}\,dx$$

input
```
int(1/(a + b*acos(1/(c*x)))^3,x)
```

output
```
int(1/(a + b*acos(1/(c*x)))^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 4.00

$$\int \frac{1}{(a+b\sec^{-1}(cx))^3}\,dx = \int \frac{1}{asec\,(cx)^3\,b^3+3asec\,(cx)^2\,a\,b^2+3asec\,(cx)\,a^2b+a^3}dx$$

input
```
int(1/(a+b*asec(c*x))^3,x)
```

output
```
int(1/(asec(c*x)**3*b**3 + 3*asec(c*x)**2*a*b**2 + 3*asec(c*x)*a**2*b + a*
*3),x)
```

3.47 $\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}\,dx$

Optimal result 404
Mathematica [N/A] 404
Rubi [N/A] 405
Maple [N/A] 405
Fricas [N/A] 406
Sympy [N/A] 406
Maxima [N/A] 406
Giac [N/A] 407
Mupad [N/A] 408
Reduce [N/A] 408

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}\,dx=\text{Int}\left(\frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3},x\right)$$

output
```
Defer(Int)(1/x/(a+b*arcsec(c*x))^3,x)
```

Mathematica [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}\,dx=\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}\,dx$$

input
```
Integrate[1/(x*(a + b*ArcSec[c*x])^3),x]
```

output
```
Integrate[1/(x*(a + b*ArcSec[c*x])^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}\,dx$$

$$\Big\downarrow \ 5771$$

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}dx$$

input
```
Int[1/(x*(a + b*ArcSec[c*x])^3),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\ \mathrm{arcsec}\left(cx\right)\right)^{3}}dx$$

input
```
int(1/x/(a+b*arcsec(c*x))^3,x)
```

output
```
int(1/x/(a+b*arcsec(c*x))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 3.21

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(b\,\mathrm{arcsec}\left(cx\right)+a\right)^3x}\,dx$$

input
```
integrate(1/x/(a+b*arcsec(c*x))^3,x, algorithm="fricas")
```

output
```
integral(1/(b^3*x*arcsec(c*x)^3 + 3*a*b^2*x*arcsec(c*x)^2 + 3*a^2*b*x*arcs
ec(c*x) + a^3*x), x)
```

Sympy [N/A]

Not integrable

Time = 4.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}\,dx=\int \frac{1}{x\left(a+b\,\mathrm{asec}\left(cx\right)\right)^3}\,dx$$

input
```
integrate(1/x/(a+b*asec(c*x))**3,x)
```

output
```
Integral(1/(x*(a + b*asec(c*x))**3), x)
```

Maxima [N/A]

Not integrable

Time = 25.69 (sec) , antiderivative size = 1568, normalized size of antiderivative = 112.00

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(b\,\mathrm{arcsec}\left(cx\right)+a\right)^3x}\,dx$$

input
```
integrate(1/x/(a+b*arcsec(c*x))^3,x, algorithm="maxima")
```

output
```
-(8*b^3*c^2*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 + 24*a*b^2*c^2*x^2*a
rctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 2*a*b^2*c^2*x^2*log(c^2*x^2)^2 + 16
*a*b^2*c^2*x^2*log(c)*log(x) + 8*a*b^2*c^2*x^2*log(x)^2 + 8*(a*b^2*c^2*log
(c)^2 + a^3*c^2)*x^2 + 2*(4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - b^
3*log(c^2*x^2)^2 - 4*b^3*log(c)^2 - 8*b^3*log(c)*log(x) - 4*b^3*log(x)^2 +
 8*a*b^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b + 4*(b^3*log(c) + b
^3*log(x))*log(c^2*x^2))*sqrt(c*x + 1)*sqrt(c*x - 1) + 2*(b^3*c^2*x^2*log(
c^2*x^2)^2 + 8*b^3*c^2*x^2*log(c)*log(x) + 4*b^3*c^2*x^2*log(x)^2 + 4*(b^3
*c^2*log(c)^2 + 3*a^2*b*c^2)*x^2 - 4*(b^3*c^2*x^2*log(c) + b^3*c^2*x^2*log
(x))*log(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - (16*b^6*arctan(sq
rt(c*x + 1)*sqrt(c*x - 1))^4 + b^6*log(c^2*x^2)^4 + 16*b^6*log(c)^4 + 64*b
^6*log(c)*log(x)^3 + 16*b^6*log(x)^4 + 64*a*b^5*arctan(sqrt(c*x + 1)*sqrt(
c*x - 1))^3 + 32*a^2*b^4*log(c)^2 + 16*a^4*b^2 - 8*(b^6*log(c) + b^6*log(x
))*log(c^2*x^2)^3 + 8*(b^6*log(c^2*x^2)^2 + 4*b^6*log(c)^2 + 8*b^6*log(c)*
log(x) + 4*b^6*log(x)^2 + 12*a^2*b^4 - 4*(b^6*log(c) + b^6*log(x))*log(c^2
*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 8*(3*b^6*log(c)^2 + 6*b^6*l
og(c)*log(x) + 3*b^6*log(x)^2 + a^2*b^4)*log(c^2*x^2)^2 + 32*(3*b^6*log(c)
^2 + a^2*b^4)*log(x)^2 + 16*(a*b^5*log(c^2*x^2)^2 + 4*a*b^5*log(c)^2 + 8*a
*b^5*log(c)*log(x) + 4*a*b^5*log(x)^2 + 4*a^3*b^3 - 4*(a*b^5*log(c) + a*b^
5*log(x))*log(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 32*(b^6*l...
```

Giac [N/A]

Not integrable

Time = 9.59 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(b\,\mathrm{arcsec}\,(cx)+a\right)^3x}\,dx$$

input
```
integrate(1/x/(a+b*arcsec(c*x))^3,x, algorithm="giac")
```

output
```
integrate(1/((b*arcsec(c*x) + a)^3*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}\,dx=\int \frac{1}{x\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^3}\,dx$$

input
```
int(1/(x*(a + b*acos(1/(c*x)))^3),x)
```

output
```
int(1/(x*(a + b*acos(1/(c*x)))^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 3.21

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}\,dx = \int \frac{1}{asec\left(cx\right)^3 b^3x+3asec\left(cx\right)^2 a\,b^2x+3asec\left(cx\right)a^2bx+a^3x}dx$$

input
```
int(1/x/(a+b*asec(c*x))^3,x)
```

output
```
int(1/(asec(c*x)**3*b**3*x + 3*asec(c*x)**2*a*b**2*x + 3*asec(c*x)*a**2*b*
x + a**3*x),x)
```

3.48 $\int \frac{1}{x^2(a+b\sec^{-1}(cx))^3}\,dx$

Optimal result 409
Mathematica [A] (verified) 410
Rubi [A] (verified) 410
Maple [A] (verified) 413
Fricas [F] 414
Sympy [F] 414
Maxima [F] 414
Giac [B] (verification not implemented) 415
Mupad [F(-1)] 416
Reduce [F] 416

Optimal result

Integrand size = 14, antiderivative size = 103

$$\int \frac{1}{x^2(a+b\sec^{-1}(cx))^3}\,dx = -\frac{c\sqrt{1-\frac{1}{c^2x^2}}}{2b(a+b\sec^{-1}(cx))^2} - \frac{1}{2b^2x(a+b\sec^{-1}(cx))} + \frac{c\,\mathrm{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\sin\left(\frac{a}{b}\right)}{2b^3} - \frac{c\cos\left(\frac{a}{b}\right)\mathrm{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{2b^3}$$

output
```
-1/2*c*(1-1/c^2/x^2)^(1/2)/b/(a+b*arcsec(c*x))^2-1/2/b^2/x/(a+b*arcsec(c*x
))+1/2*c*Ci(a/b+arcsec(c*x))*sin(a/b)/b^3-1/2*c*cos(a/b)*Si(a/b+arcsec(c*x
))/b^3
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^2 (a+b\sec^{-1}(cx))^3} dx = -\frac{\frac{b\left(a+bc\sqrt{1-\frac{1}{c^2x^2}}x+b\sec^{-1}(cx)\right)}{x(a+b\sec^{-1}(cx))^2} - c\,\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\sin\left(\frac{a}{b}\right) + c\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{2b^3}$$

input
```
Integrate[1/(x^2*(a + b*ArcSec[c*x])^3),x]
```

output
```
-1/2*((b*(a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x + b*ArcSec[c*x]))/(x*(a + b*ArcS
ec[c*x])^2) - c*CosIntegral[a/b + ArcSec[c*x]]*Sin[a/b] + c*Cos[a/b]*SinIn
tegral[a/b + ArcSec[c*x]])/b^3
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {5745, 3042, 3778, 3042, 3778, 25, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+b\sec^{-1}(cx))^3} dx$$

$$\Big\downarrow \text{5745}$$

$$c\int \frac{\sqrt{1-\frac{1}{c^2x^2}}}{(a+b\sec^{-1}(cx))^3} d\sec^{-1}(cx)$$

$$\Big\downarrow \text{3042}$$

$$c\int \frac{\sin\left(\sec^{-1}(cx)\right)}{(a+b\sec^{-1}(cx))^3} d\sec^{-1}(cx)$$

$$\Big\downarrow \text{3778}$$

$$c\left(\frac{\int \frac{1}{cx(a+b\sec^{-1}(cx))^2}d\sec^{-1}(cx)}{2b}-\frac{\sqrt{1-\frac{1}{c^2x^2}}}{2b\left(a+b\sec^{-1}(cx)\right)^2}\right)$$

$$\Big\downarrow \ 3042$$

$$c\left(\frac{\int \frac{\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)}{(a+b\sec^{-1}(cx))^2}d\sec^{-1}(cx)}{2b}-\frac{\sqrt{1-\frac{1}{c^2x^2}}}{2b\left(a+b\sec^{-1}(cx)\right)^2}\right)$$

$$\Big\downarrow \ 3778$$

$$c\left(\frac{\frac{\int -\frac{\sqrt{1-\frac{1}{c^2x^2}}}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)}{b}-\frac{1}{bcx(a+b\sec^{-1}(cx))}}{2b}-\frac{\sqrt{1-\frac{1}{c^2x^2}}}{2b\left(a+b\sec^{-1}(cx)\right)^2}\right)$$

$$\Big\downarrow \ 25$$

$$c\left(\frac{-\frac{\int \frac{\sqrt{1-\frac{1}{c^2x^2}}}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)}{b}-\frac{1}{bcx(a+b\sec^{-1}(cx))}}{2b}-\frac{\sqrt{1-\frac{1}{c^2x^2}}}{2b\left(a+b\sec^{-1}(cx)\right)^2}\right)$$

$$\Big\downarrow \ 3042$$

$$c\left(\frac{-\frac{\int \frac{\sin\left(\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)}{b}-\frac{1}{bcx(a+b\sec^{-1}(cx))}}{2b}-\frac{\sqrt{1-\frac{1}{c^2x^2}}}{2b\left(a+b\sec^{-1}(cx)\right)^2}\right)$$

$$\Big\downarrow \ 3784$$

$$c\left(\frac{-\frac{\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)-\sin\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)}{b}-\frac{1}{bcx(a+b\sec^{-1}(cx))}}{2b}-\frac{\sqrt{1-\frac{1}{c^2x^2}}}{2b\left(a+b\sec^{-1}(cx)\right)^2}\right)$$

$$\Big\downarrow \ 3042$$

$$c\left(\frac{-\frac{\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)-\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a}{b}+\sec^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)}{b}-\frac{1}{bcx(a+b\sec^{-1}(cx))}}{2b}-\frac{\sqrt{1-\frac{1}{c^2x^2}}}{2b\left(a+b\sec^{-1}(cx)\right)^2}\right.$$

$$\big\downarrow \text{3780}$$

$$c\left(-\frac{\frac{\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b}-\sin\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a}{b}+\sec^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)}{b}-\frac{1}{bcx(a+b\sec^{-1}(cx))}}{2b}-\frac{\sqrt{1-\frac{1}{c^2x^2}}}{2b\left(a+b\sec^{-1}(cx)\right)^2}\right)$$

$$\big\downarrow \text{3783}$$

$$c\left(-\frac{\frac{\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b}-\frac{\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b}}{b}-\frac{1}{bcx(a+b\sec^{-1}(cx))}}{2b}-\frac{\sqrt{1-\frac{1}{c^2x^2}}}{2b\left(a+b\sec^{-1}(cx)\right)^2}\right)$$

input
```
Int[1/(x^2*(a + b*ArcSec[c*x])^3),x]
```

output
```
c*(-1/2*Sqrt[1 - 1/(c^2*x^2)]/(b*(a + b*ArcSec[c*x])^2) + (-(1/(b*c*x*(a +
 b*ArcSec[c*x]))) - (-((CosIntegral[a/b + ArcSec[c*x]]*Sin[a/b])/b) + (Cos
[a/b]*SinIntegral[a/b + ArcSec[c*x]])/b)/b)/(2*b))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.50

method	result
derivativedivides	$c\left(-\frac{\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{2(a+b\ \mathrm{arcsec}(cx))^2 b}-\frac{\mathrm{arcsec}(cx)\cos\left(\frac{a}{b}\right)\mathrm{Si}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)bcx-\mathrm{arcsec}(cx)\sin\left(\frac{a}{b}\right)\mathrm{Ci}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)bcx+\mathrm{co}}{2cx(a+b\ \mathrm{arcsec}(cx))b^3}\right.$
default	$c\left(-\frac{\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{2(a+b\ \mathrm{arcsec}(cx))^2 b}-\frac{\mathrm{arcsec}(cx)\cos\left(\frac{a}{b}\right)\mathrm{Si}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)bcx-\mathrm{arcsec}(cx)\sin\left(\frac{a}{b}\right)\mathrm{Ci}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)bcx+\mathrm{co}}{2cx(a+b\ \mathrm{arcsec}(cx))b^3}\right.$

input
```
int(1/x^2/(a+b*arcsec(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
c*(-1/2*((c^2*x^2-1)/c^2/x^2)^(1/2)/(a+b*arcsec(c*x))^2/b-1/2*(arcsec(c*x)
*cos(a/b)*Si(a/b+arcsec(c*x))*b*c*x-arcsec(c*x)*sin(a/b)*Ci(a/b+arcsec(c*x
))*b*c*x+cos(a/b)*Si(a/b+arcsec(c*x))*a*c*x-sin(a/b)*Ci(a/b+arcsec(c*x))*a
*c*x+b)/c/x/(a+b*arcsec(c*x))/b^3)
```

Fricas [F]

$$\int \frac{1}{x^2 \left(a + b \sec^{-1}(cx)\right)^3} \, dx = \int \frac{1}{\left(b \operatorname{arcsec}\left(cx\right) + a\right)^3 x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*arcsec(c*x))^3,x, algorithm="fricas")
```

output
```
integral(1/(b^3*x^2*arcsec(c*x)^3 + 3*a*b^2*x^2*arcsec(c*x)^2 + 3*a^2*b*x^
2*arcsec(c*x) + a^3*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2 \left(a + b \sec^{-1}(cx)\right)^3} \, dx = \int \frac{1}{x^2 \left(a + b \operatorname{asec}\left(cx\right)\right)^3} \, dx$$

input
```
integrate(1/x**2/(a+b*asec(c*x))**3,x)
```

output
```
Integral(1/(x**2*(a + b*asec(c*x))**3), x)
```

Maxima [F]

$$\int \frac{1}{x^2 \left(a + b \sec^{-1}(cx)\right)^3} \, dx = \int \frac{1}{\left(b \operatorname{arcsec}\left(cx\right) + a\right)^3 x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*arcsec(c*x))^3,x, algorithm="maxima")
```

output
```
-(8*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 + 24*a*b^2*arctan(sqrt(c*x +
 1)*sqrt(c*x - 1))^2 + 2*a*b^2*log(c^2*x^2)^2 + 8*a*b^2*log(c)^2 + 16*a*b^
2*log(c)*log(x) + 8*a*b^2*log(x)^2 + 8*a^3 + 2*(4*b^3*arctan(sqrt(c*x + 1)
*sqrt(c*x - 1))^2 - b^3*log(c^2*x^2)^2 - 4*b^3*log(c)^2 - 8*b^3*log(c)*log
(x) - 4*b^3*log(x)^2 + 8*a*b^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2
*b + 4*(b^3*log(c) + b^3*log(x))*log(c^2*x^2))*sqrt(c*x + 1)*sqrt(c*x - 1)
 + 2*(b^3*log(c^2*x^2)^2 + 4*b^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*lo
g(x)^2 + 12*a^2*b - 4*(b^3*log(c) + b^3*log(x))*log(c^2*x^2))*arctan(sqrt(
c*x + 1)*sqrt(c*x - 1)) + (16*b^6*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^4
+ b^6*x*log(c^2*x^2)^4 + 64*b^6*x*log(c)*log(x)^3 + 16*b^6*x*log(x)^4 + 64
*a*b^5*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 8*(b^6*x*log(c) + b^6*x*l
og(x))*log(c^2*x^2)^3 + 32*(3*b^6*log(c)^2 + a^2*b^4)*x*log(x)^2 + 8*(b^6*
x*log(c^2*x^2)^2 + 8*b^6*x*log(c)*log(x) + 4*b^6*x*log(x)^2 + 4*(b^6*log(c
)^2 + 3*a^2*b^4)*x - 4*(b^6*x*log(c) + b^6*x*log(x))*log(c^2*x^2))*arctan(
sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 8*(6*b^6*x*log(c)*log(x) + 3*b^6*x*log(x)
^2 + (3*b^6*log(c)^2 + a^2*b^4)*x)*log(c^2*x^2)^2 + 64*(b^6*log(c)^3 + a^2
*b^4*log(c))*x*log(x) + 16*(b^6*log(c)^4 + 2*a^2*b^4*log(c)^2 + a^4*b^2)*x
 + 16*(a*b^5*x*log(c^2*x^2)^2 + 8*a*b^5*x*log(c)*log(x) + 4*a*b^5*x*log(x)
^2 + 4*(a*b^5*log(c)^2 + a^3*b^3)*x - 4*(a*b^5*x*log(c) + a*b^5*x*log(x))*
log(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 32*(3*b^6*x*log(c)*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(93) = 186$.

Time = 0.21 (sec) , antiderivative size = 580, normalized size of antiderivative = 5.63

$$\int \frac{1}{x^2 (a + b\sec^{-1}(cx))^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(a+b*arcsec(c*x))^3,x, algorithm="giac")
```

output
```
1/2*(b^2*arccos(1/(c*x))^2*cos_integral(a/b + arccos(1/(c*x)))*sin(a/b)/(b
^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) - b^2*arccos(1/(
c*x))^2*cos(a/b)*sin_integral(a/b + arccos(1/(c*x)))/(b^5*arccos(1/(c*x))^
2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) + 2*a*b*arccos(1/(c*x))*cos_integra
l(a/b + arccos(1/(c*x)))*sin(a/b)/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(
1/(c*x)) + a^2*b^3) - 2*a*b*arccos(1/(c*x))*cos(a/b)*sin_integral(a/b + ar
ccos(1/(c*x)))/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3)
 + a^2*cos_integral(a/b + arccos(1/(c*x)))*sin(a/b)/(b^5*arccos(1/(c*x))^2
 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) - a^2*cos(a/b)*sin_integral(a/b + ar
ccos(1/(c*x)))/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3)
 - b^2*sqrt(-1/(c^2*x^2) + 1)/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c
*x)) + a^2*b^3) - b^2*arccos(1/(c*x))/((b^5*arccos(1/(c*x))^2 + 2*a*b^4*ar
ccos(1/(c*x)) + a^2*b^3)*c*x) - a*b/((b^5*arccos(1/(c*x))^2 + 2*a*b^4*arcc
os(1/(c*x)) + a^2*b^3)*c*x))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+b\sec^{-1}(cx))^3}\, dx = \int \frac{1}{x^2 \left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^3}\, dx$$

input
```
int(1/(x^2*(a + b*acos(1/(c*x)))^3),x)
```

output
```
int(1/(x^2*(a + b*acos(1/(c*x)))^3), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a+b\sec^{-1}(cx))^3}\, dx = \int \frac{1}{\mathit{asec}(cx)^3 b^3x^2+3\mathit{asec}(cx)^2 a\,b^2x^2+3\mathit{asec}(cx)\,a^2b\,x^2+a^3x^2}dx$$

input
```
int(1/x^2/(a+b*asec(c*x))^3,x)
```

output
```
int(1/(asec(c*x)**3*b**3*x**2 + 3*asec(c*x)**2*a*b**2*x**2 + 3*asec(c*x)*a
**2*b*x**2 + a**3*x**2),x)
```

3.49 $\int \frac{1}{x^3(a+b\sec^{-1}(cx))^3}\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 112

$$\int \frac{1}{x^3\left(a+b\sec^{-1}(cx)\right)^3}\,dx = -\frac{c^2\cos\left(2\sec^{-1}(cx)\right)}{2b^2\left(a+b\sec^{-1}(cx)\right)} + \frac{c^2\,\text{CosIntegral}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)\sin\left(\frac{2a}{b}\right)}{b^3} - \frac{c^2\sin\left(2\sec^{-1}(cx)\right)}{4b\left(a+b\sec^{-1}(cx)\right)^2} - \frac{c^2\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b^3}$$

output
```
-1/2*c^2*cos(2*arcsec(c*x))/b^2/(a+b*arcsec(c*x))+c^2*Ci(2*a/b+2*arcsec(c*
x))*sin(2*a/b)/b^3-1/4*c^2*sin(2*arcsec(c*x))/b/(a+b*arcsec(c*x))^2-c^2*co
s(2*a/b)*Si(2*a/b+2*arcsec(c*x))/b^3
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^3\left(a+b\sec^{-1}(cx)\right)^3}\,dx$$
$$= \frac{-\frac{b^2c\sqrt{1-\frac{1}{c^2x^2}}}{x(a+b\sec^{-1}(cx))^2} + \frac{b(-2+c^2x^2)}{x^2(a+b\sec^{-1}(cx))} + 2c^2\left(\text{CosIntegral}\left(2\left(\frac{a}{b}+\sec^{-1}(cx)\right)\right)\sin\left(\frac{2a}{b}\right) - \cos\left(\frac{2a}{b}\right)\text{Si}\left(2\left(\frac{a}{b}+\sec\right.\right.\right.}{2b^3}$$

input
```
Integrate[1/(x^3*(a + b*ArcSec[c*x])^3),x]
```

output
```
(-((b^2*c*Sqrt[1 - 1/(c^2*x^2)])/(x*(a + b*ArcSec[c*x])^2)) + (b*(-2 + c^2
*x^2))/(x^2*(a + b*ArcSec[c*x])) + 2*c^2*(CosIntegral[2*(a/b + ArcSec[c*x]
)]*Sin[(2*a)/b] - Cos[(2*a)/b]*SinIntegral[2*(a/b + ArcSec[c*x])]))/(2*b^3
)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {5745, 4906, 27, 3042, 3778, 3042, 3778, 25, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+b\sec^{-1}(cx))^3}dx$$

$$\Big\downarrow \text{ 5745}$$

$$c^2 \int \frac{\sqrt{1-\frac{1}{c^2x^2}}}{cx (a+b\sec^{-1}(cx))^3}d\sec^{-1}(cx)$$

$$\Big\downarrow \text{ 4906}$$

$$c^2 \int \frac{\sin \left(2\sec^{-1}(cx)\right)}{2 (a+b\sec^{-1}(cx))^3}d\sec^{-1}(cx)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}c^2 \int \frac{\sin \left(2\sec^{-1}(cx)\right)}{(a+b\sec^{-1}(cx))^3}d\sec^{-1}(cx)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}c^2 \int \frac{\sin \left(2\sec^{-1}(cx)\right)}{(a+b\sec^{-1}(cx))^3}d\sec^{-1}(cx)$$

$$\Big\downarrow \text{ 3778}$$

$$\frac{1}{2}c^2\left(\frac{\int \frac{\cos(2\sec^{-1}(cx))}{(a+b\sec^{-1}(cx))^2}d\sec^{-1}(cx)}{b}-\frac{\sin\left(2\sec^{-1}(cx)\right)}{2b\left(a+b\sec^{-1}(cx)\right)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}c^2\left(\frac{\int \frac{\sin\left(2\sec^{-1}(cx)+\frac{\pi}{2}\right)}{(a+b\sec^{-1}(cx))^2}d\sec^{-1}(cx)}{b}-\frac{\sin\left(2\sec^{-1}(cx)\right)}{2b\left(a+b\sec^{-1}(cx)\right)^2}\right)$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{2}c^2\left(\frac{\frac{2\int -\frac{\sin\left(2\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)}{b}-\frac{\cos(2\sec^{-1}(cx))}{b(a+b\sec^{-1}(cx))}}{b}-\frac{\sin\left(2\sec^{-1}(cx)\right)}{2b\left(a+b\sec^{-1}(cx)\right)^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}c^2\left(\frac{-\frac{2\int \frac{\sin\left(2\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)}{b}-\frac{\cos(2\sec^{-1}(cx))}{b(a+b\sec^{-1}(cx))}}{b}-\frac{\sin\left(2\sec^{-1}(cx)\right)}{2b\left(a+b\sec^{-1}(cx)\right)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}c^2\left(\frac{-\frac{2\int \frac{\sin\left(2\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)}{b}-\frac{\cos(2\sec^{-1}(cx))}{b(a+b\sec^{-1}(cx))}}{b}-\frac{\sin\left(2\sec^{-1}(cx)\right)}{2b\left(a+b\sec^{-1}(cx)\right)^2}\right)$$

$$\Big\downarrow \text{3784}$$

$$\frac{1}{2}c^2\left(\frac{-\frac{2\left(\cos\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)-\sin\left(\frac{2a}{b}\right)\int \frac{\cos\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)\right)}{b}-\frac{\cos(2\sec^{-1}(cx))}{b(a+b\sec^{-1}(cx))}}{b}-\frac{\sin\left(2\sec^{-1}\right.}{2b\left(a+b\sec^{-}\right.}\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}c^2\left(\frac{-\frac{2\left(\cos\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)-\sin\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2a}{b}+2\sec^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)\right)}{b}-\frac{\cos(2\sec^{-1}(cx))}{b(a+b\sec^{-1}(cx))}}{b}-\frac{\sin\left(2\sec\right.}{2b\left(a+b\,s\right.}\right.$$

$$\Big\downarrow \text{ 3780}$$

$$\frac{1}{2}c^2\left(\frac{-\frac{2\left(\frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b}-\sin\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2a}{b}+2\sec^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)\right)}{b}-\frac{\cos\left(2\sec^{-1}(cx)\right)}{b\left(a+b\sec^{-1}(cx)\right)}}{b}-\frac{\sin\left(2\sec^{-1}(cx)\right)}{2b\left(a+b\sec^{-1}(cx)\right)^2}\right)$$

$$\Big\downarrow \text{ 3783}$$

$$\frac{1}{2}c^2\left(\frac{-\frac{2\left(\frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b}-\frac{\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b}\right)}{b}-\frac{\cos\left(2\sec^{-1}(cx)\right)}{b\left(a+b\sec^{-1}(cx)\right)}}{b}-\frac{\sin\left(2\sec^{-1}(cx)\right)}{2b\left(a+b\sec^{-1}(cx)\right)^2}\right)$$

input

```
Int[1/(x^3*(a + b*ArcSec[c*x])^3),x]
```

output

```
(c^2*(-1/2*Sin[2*ArcSec[c*x]]/(b*(a + b*ArcSec[c*x])^2) + (-(Cos[2*ArcSec[
c*x]]/(b*(a + b*ArcSec[c*x]))) - (2*(-((CosIntegral[(2*a)/b + 2*ArcSec[c*x
]]*Sin[(2*a)/b])/b) + (Cos[(2*a)/b]*SinIntegral[(2*a)/b + 2*ArcSec[c*x]])/
b))/b)/b))/2
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.40

method	result
derivativedivides	$c^2\left(-\frac{\sin(2\,\mathrm{arcsec}(cx))}{4(a+b\,\mathrm{arcsec}(cx))^2 b}-\frac{2\,\mathrm{arcsec}(cx)\cos\left(\frac{2a}{b}\right)\mathrm{Si}\left(\frac{2a}{b}+2\,\mathrm{arcsec}(cx)\right)b-2\,\mathrm{arcsec}(cx)\sin\left(\frac{2a}{b}\right)\mathrm{Ci}\left(\frac{2a}{b}+2\,\mathrm{arcsec}(cx\right.}{2(a+b}\right.$
default	$c^2\left(-\frac{\sin(2\,\mathrm{arcsec}(cx))}{4(a+b\,\mathrm{arcsec}(cx))^2 b}-\frac{2\,\mathrm{arcsec}(cx)\cos\left(\frac{2a}{b}\right)\mathrm{Si}\left(\frac{2a}{b}+2\,\mathrm{arcsec}(cx)\right)b-2\,\mathrm{arcsec}(cx)\sin\left(\frac{2a}{b}\right)\mathrm{Ci}\left(\frac{2a}{b}+2\,\mathrm{arcsec}(c\right.}{2(a+b}\right.$

input
```
int(1/x^3/(a+b*arcsec(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(-1/4*sin(2*arcsec(c*x))/(a+b*arcsec(c*x))^2/b-1/2*(2*arcsec(c*x)*cos(
2*a/b)*Si(2*a/b+2*arcsec(c*x))*b-2*arcsec(c*x)*sin(2*a/b)*Ci(2*a/b+2*arcse
c(c*x))*b+2*cos(2*a/b)*Si(2*a/b+2*arcsec(c*x))*a-2*sin(2*a/b)*Ci(2*a/b+2*a
rcsec(c*x))*a+cos(2*arcsec(c*x))*b)/(a+b*arcsec(c*x))/b^3)
```

Fricas [F]

$$\int \frac{1}{x^3\left(a+b\sec^{-1}(cx)\right)^3}\,dx=\int \frac{1}{(b\,\mathrm{arcsec}\,(cx)+a)^3x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*arcsec(c*x))^3,x, algorithm="fricas")
```

output
```
integral(1/(b^3*x^3*arcsec(c*x)^3 + 3*a*b^2*x^3*arcsec(c*x)^2 + 3*a^2*b*x^
3*arcsec(c*x) + a^3*x^3), x)
```

Sympy [F]

$$\int \frac{1}{x^3 (a + b\sec^{-1}(cx))^3} \, dx = \int \frac{1}{x^3 (a + b\operatorname{asec}(cx))^3} \, dx$$

input
```
integrate(1/x**3/(a+b*asec(c*x))**3,x)
```

output
```
Integral(1/(x**3*(a + b*asec(c*x))**3), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (a + b\sec^{-1}(cx))^3} \, dx = \int \frac{1}{(b\operatorname{arcsec}(cx) + a)^3 x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*arcsec(c*x))^3,x, algorithm="maxima")
```

output
```
-(16*a*b^2*log(c)^2 - 8*(b^3*c^2*x^2 - 2*b^3)*arctan(sqrt(c*x + 1)*sqrt(c*
x - 1))^3 + 16*a^3 - 8*(a*b^2*c^2*log(c)^2 + a^3*c^2)*x^2 - 24*(a*b^2*c^2*
x^2 - 2*a*b^2)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - 2*(a*b^2*c^2*x^2 -
2*a*b^2)*log(c^2*x^2)^2 - 8*(a*b^2*c^2*x^2 - 2*a*b^2)*log(x)^2 + 2*(4*b^3*
arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - b^3*log(c^2*x^2)^2 - 4*b^3*log(c)^
2 - 8*b^3*log(c)*log(x) - 4*b^3*log(x)^2 + 8*a*b^2*arctan(sqrt(c*x + 1)*sq
rt(c*x - 1)) + 4*a^2*b + 4*(b^3*log(c) + b^3*log(x))*log(c^2*x^2))*sqrt(c*
x + 1)*sqrt(c*x - 1) + 2*(8*b^3*log(c)^2 + 24*a^2*b - 4*(b^3*c^2*log(c)^2
+ 3*a^2*b*c^2)*x^2 - (b^3*c^2*x^2 - 2*b^3)*log(c^2*x^2)^2 - 4*(b^3*c^2*x^2
 - 2*b^3)*log(x)^2 + 4*(b^3*c^2*x^2*log(c) - 2*b^3*log(c) + (b^3*c^2*x^2 -
 2*b^3)*log(x))*log(c^2*x^2) - 8*(b^3*c^2*x^2*log(c) - 2*b^3*log(c))*log(x
))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + (16*b^6*x^2*arctan(sqrt(c*x + 1)*
sqrt(c*x - 1))^4 + b^6*x^2*log(c^2*x^2)^4 + 64*b^6*x^2*log(c)*log(x)^3 + 1
6*b^6*x^2*log(x)^4 + 64*a*b^5*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 +
32*(3*b^6*log(c)^2 + a^2*b^4)*x^2*log(x)^2 - 8*(b^6*x^2*log(c) + b^6*x^2*l
og(x))*log(c^2*x^2)^3 + 64*(b^6*log(c)^3 + a^2*b^4*log(c))*x^2*log(x) + 16
*(b^6*log(c)^4 + 2*a^2*b^4*log(c)^2 + a^4*b^2)*x^2 + 8*(b^6*x^2*log(c^2*x^
2)^2 + 8*b^6*x^2*log(c)*log(x) + 4*b^6*x^2*log(x)^2 + 4*(b^6*log(c)^2 + 3*
a^2*b^4)*x^2 - 4*(b^6*x^2*log(c) + b^6*x^2*log(x))*log(c^2*x^2))*arctan(sq
rt(c*x + 1)*sqrt(c*x - 1))^2 + 8*(6*b^6*x^2*log(c)*log(x) + 3*b^6*x^2*l...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 929 vs. $2(108) = 216$.

Time = 0.21 (sec) , antiderivative size = 929, normalized size of antiderivative = 8.29

$$\int \frac{1}{x^3 \left(a + b\sec^{-1}(cx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(a+b*arcsec(c*x))^3,x, algorithm="giac")
```

output
```
1/2*(4*b^2*c*arccos(1/(c*x))^2*cos(a/b)*cos_integral(2*a/b + 2*arccos(1/(c
*x)))*sin(a/b)/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3)
 - 4*b^2*c*arccos(1/(c*x))^2*cos(a/b)^2*sin_integral(2*a/b + 2*arccos(1/(c
*x)))/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) + 8*a*b*
c*arccos(1/(c*x))*cos(a/b)*cos_integral(2*a/b + 2*arccos(1/(c*x)))*sin(a/b
)/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) - 8*a*b*c*ar
ccos(1/(c*x))*cos(a/b)^2*sin_integral(2*a/b + 2*arccos(1/(c*x)))/(b^5*arcc
os(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) + 4*a^2*c*cos(a/b)*cos_
integral(2*a/b + 2*arccos(1/(c*x)))*sin(a/b)/(b^5*arccos(1/(c*x))^2 + 2*a*
b^4*arccos(1/(c*x)) + a^2*b^3) + 2*b^2*c*arccos(1/(c*x))^2*sin_integral(2*
a/b + 2*arccos(1/(c*x)))/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x))
+ a^2*b^3) - 4*a^2*c*cos(a/b)^2*sin_integral(2*a/b + 2*arccos(1/(c*x)))/(b
^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) + 4*a*b*c*arccos
(1/(c*x))*sin_integral(2*a/b + 2*arccos(1/(c*x)))/(b^5*arccos(1/(c*x))^2 +
 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) + b^2*c*arccos(1/(c*x))/(b^5*arccos(1/
(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) + 2*a^2*c*sin_integral(2*a/b
 + 2*arccos(1/(c*x)))/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a
^2*b^3) + a*b*c/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3
) - b^2*sqrt(-1/(c^2*x^2) + 1)/((b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/
(c*x)) + a^2*b^3)*x) - 2*b^2*arccos(1/(c*x))/((b^5*arccos(1/(c*x))^2 + ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(a+b\sec^{-1}(cx)\right)^3}\,dx=\int \frac{1}{x^3\,{\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}^3}\,dx$$

input
```
int(1/(x^3*(a + b*acos(1/(c*x)))^3),x)
```

output
```
int(1/(x^3*(a + b*acos(1/(c*x)))^3), x)
```

Reduce [F]

$$\int \frac{1}{x^3\left(a+b\sec^{-1}(cx)\right)^3}\,dx$$
$$=\int \frac{1}{asec\,(cx)^3\,b^3x^3+3asec\,(cx)^2\,a\,b^2x^3+3asec\,(cx)\,a^2b\,x^3+a^3x^3}dx$$

input
```
int(1/x^3/(a+b*asec(c*x))^3,x)
```

output
```
int(1/(asec(c*x)**3*b**3*x**3 + 3*asec(c*x)**2*a*b**2*x**3 + 3*asec(c*x)*a
**2*b*x**3 + a**3*x**3),x)
```

3.50 $\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)^3}\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 228

$$\begin{aligned}\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)^3}\,dx =& -\frac{c^3\sqrt{1-\frac{1}{c^2x^2}}}{8b\left(a+b\sec^{-1}(cx)\right)^2}-\frac{c^2}{8b^2x\left(a+b\sec^{-1}(cx)\right)}\\&-\frac{3c^3\cos\left(3\sec^{-1}(cx)\right)}{8b^2\left(a+b\sec^{-1}(cx)\right)}\\&+\frac{c^3\,\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\sin\left(\frac{a}{b}\right)}{8b^3}\\&+\frac{9c^3\,\text{CosIntegral}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)\sin\left(\frac{3a}{b}\right)}{8b^3}\\&-\frac{c^3\sin\left(3\sec^{-1}(cx)\right)}{8b\left(a+b\sec^{-1}(cx)\right)^2}-\frac{c^3\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{8b^3}\\&-\frac{9c^3\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)}{8b^3}\end{aligned}$$

output
```
-1/8*c^3*(1-1/c^2/x^2)^(1/2)/b/(a+b*arcsec(c*x))^2-1/8*c^2/b^2/x/(a+b*arcs
ec(c*x))-3/8*c^3*cos(3*arcsec(c*x))/b^2/(a+b*arcsec(c*x))+1/8*c^3*Ci(a/b+a
rcsec(c*x))*sin(a/b)/b^3+9/8*c^3*Ci(3*a/b+3*arcsec(c*x))*sin(3*a/b)/b^3-1/
8*c^3*sin(3*arcsec(c*x))/b/(a+b*arcsec(c*x))^2-1/8*c^3*cos(a/b)*Si(a/b+arc
sec(c*x))/b^3-9/8*c^3*cos(3*a/b)*Si(3*a/b+3*arcsec(c*x))/b^3
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^4 (a + b \sec^{-1}(cx))^3} \, dx$$
$$= \frac{-\frac{4b^2c\sqrt{1-\frac{1}{c^2x^2}}}{x^2(a+b\sec^{-1}(cx))^2} - \frac{12b}{x^3(a+b\sec^{-1}(cx))} + \frac{8bc^2}{ax+bx\sec^{-1}(cx)} + c^3 \operatorname{CosIntegral}\left(\frac{a}{b} + \sec^{-1}(cx)\right)\sin\left(\frac{a}{b}\right) + 9c^3 \operatorname{CosInt}}{8b^3}$$

input
```
Integrate[1/(x^4*(a + b*ArcSec[c*x])^3),x]
```

output
```
((-4*b^2*c*Sqrt[1 - 1/(c^2*x^2)])/(x^2*(a + b*ArcSec[c*x])^2) - (12*b)/(x^
3*(a + b*ArcSec[c*x])) + (8*b*c^2)/(a*x + b*x*ArcSec[c*x]) + c^3*CosIntegr
al[a/b + ArcSec[c*x]]*Sin[a/b] + 9*c^3*CosIntegral[3*(a/b + ArcSec[c*x])]*
Sin[(3*a)/b] - c^3*Cos[a/b]*SinIntegral[a/b + ArcSec[c*x]] - 9*c^3*Cos[(3*
a)/b]*SinIntegral[3*(a/b + ArcSec[c*x])])/(8*b^3)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5745, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a + b \sec^{-1}(cx))^3} \, dx$$

$$\Big\downarrow \ 5745$$

$$c^3 \int \frac{\sqrt{1 - \frac{1}{c^2x^2}}}{c^2x^2 (a + b\sec^{-1}(cx))^3} d\sec^{-1}(cx)$$

$$\Big\downarrow \ 4906$$

$$c^3 \int \left(\frac{\sin\left(3\sec^{-1}(cx)\right)}{4 (a + b\sec^{-1}(cx))^3} + \frac{\sqrt{1 - \frac{1}{c^2x^2}}}{4 (a + b\sec^{-1}(cx))^3} \right) d\sec^{-1}(cx)$$

$\downarrow$ 2009

$$c^3\left(\frac{\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{8b^3}+\frac{9\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)}{8b^3}-\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{8b^3}\right.$$

input
```
Int[1/(x^4*(a + b*ArcSec[c*x])^3),x]
```

output
```
c^3*(-1/8*Sqrt[1 - 1/(c^2*x^2)]/(b*(a + b*ArcSec[c*x])^2) - 1/(8*b^2*c*x*(
a + b*ArcSec[c*x])) - (3*Cos[3*ArcSec[c*x]])/(8*b^2*(a + b*ArcSec[c*x])) +
 (CosIntegral[a/b + ArcSec[c*x]]*Sin[a/b])/(8*b^3) + (9*CosIntegral[(3*a)/
b + 3*ArcSec[c*x]]*Sin[(3*a)/b])/(8*b^3) - Sin[3*ArcSec[c*x]]/(8*b*(a + b*
ArcSec[c*x])^2) - (Cos[a/b]*SinIntegral[a/b + ArcSec[c*x]])/(8*b^3) - (9*C
os[(3*a)/b]*SinIntegral[(3*a)/b + 3*ArcSec[c*x]])/(8*b^3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.35

method	result
derivativedivides	$c^3\left(-\frac{\sin(3\,\mathrm{arcsec}(cx))}{8(a+b\,\mathrm{arcsec}(cx))^2 b} - \frac{3\left(3\,\mathrm{arcsec}(cx)\cos\left(\frac{3a}{b}\right)\mathrm{Si}\left(\frac{3a}{b}+3\,\mathrm{arcsec}(cx)\right)b-3\,\mathrm{arcsec}(cx)\sin\left(\frac{3a}{b}\right)\mathrm{Ci}\left(\frac{3a}{b}+3\,\mathrm{arcsec}\right.\right.}{8(a\text{-}}\right.$
default	$c^3\left(-\frac{\sin(3\,\mathrm{arcsec}(cx))}{8(a+b\,\mathrm{arcsec}(cx))^2 b} - \frac{3\left(3\,\mathrm{arcsec}(cx)\cos\left(\frac{3a}{b}\right)\mathrm{Si}\left(\frac{3a}{b}+3\,\mathrm{arcsec}(cx)\right)b-3\,\mathrm{arcsec}(cx)\sin\left(\frac{3a}{b}\right)\mathrm{Ci}\left(\frac{3a}{b}+3\,\mathrm{arcsec}\right.\right.}{8(a\text{-}}\right.$

input
```
int(1/x^4/(a+b*arcsec(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
c^3*(-1/8*sin(3*arcsec(c*x))/(a+b*arcsec(c*x))^2/b-3/8*(3*arcsec(c*x)*cos(
3*a/b)*Si(3*a/b+3*arcsec(c*x))*b-3*arcsec(c*x)*sin(3*a/b)*Ci(3*a/b+3*arcse
c(c*x))*b+3*cos(3*a/b)*Si(3*a/b+3*arcsec(c*x))*a-3*sin(3*a/b)*Ci(3*a/b+3*a
rcsec(c*x))*a+cos(3*arcsec(c*x))*b)/(a+b*arcsec(c*x))/b^3-1/8*((c^2*x^2-1)
/c^2/x^2)^(1/2)/(a+b*arcsec(c*x))^2/b-1/8*(arcsec(c*x)*cos(a/b)*Si(a/b+arc
sec(c*x))*b*c*x-arcsec(c*x)*sin(a/b)*Ci(a/b+arcsec(c*x))*b*c*x+cos(a/b)*Si
(a/b+arcsec(c*x))*a*c*x-sin(a/b)*Ci(a/b+arcsec(c*x))*a*c*x+b)/c/x/(a+b*arc
sec(c*x))/b^3)
```

Fricas [F]

$$\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)^3}\,dx = \int \frac{1}{\left(b\,\mathrm{arcsec}\left(cx\right)+a\right)^3 x^4}\,dx$$

input
```
integrate(1/x^4/(a+b*arcsec(c*x))^3,x, algorithm="fricas")
```

output
```
integral(1/(b^3*x^4*arcsec(c*x)^3 + 3*a*b^2*x^4*arcsec(c*x)^2 + 3*a^2*b*x^
4*arcsec(c*x) + a^3*x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^4 (a + b \sec^{-1}(cx))^3} \, dx = \int \frac{1}{x^4 (a + b \operatorname{asec}(cx))^3} \, dx$$

input
```
integrate(1/x**4/(a+b*asec(c*x))**3,x)
```

output
```
Integral(1/(x**4*(a + b*asec(c*x))**3), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (a + b \sec^{-1}(cx))^3} \, dx = \int \frac{1}{(b \operatorname{arcsec}(cx) + a)^3 x^4} \, dx$$

input
```
integrate(1/x^4/(a+b*arcsec(c*x))^3,x, algorithm="maxima")
```

output
```
-(24*a*b^2*log(c)^2 - 8*(2*b^3*c^2*x^2 - 3*b^3)*arctan(sqrt(c*x + 1)*sqrt(
c*x - 1))^3 + 24*a^3 - 16*(a*b^2*c^2*log(c)^2 + a^3*c^2)*x^2 - 24*(2*a*b^2
*c^2*x^2 - 3*a*b^2)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - 2*(2*a*b^2*c^2
*x^2 - 3*a*b^2)*log(c^2*x^2)^2 - 8*(2*a*b^2*c^2*x^2 - 3*a*b^2)*log(x)^2 +
2*(4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - b^3*log(c^2*x^2)^2 - 4*b^
3*log(c)^2 - 8*b^3*log(c)*log(x) - 4*b^3*log(x)^2 + 8*a*b^2*arctan(sqrt(c*
x + 1)*sqrt(c*x - 1)) + 4*a^2*b + 4*(b^3*log(c) + b^3*log(x))*log(c^2*x^2)
)*sqrt(c*x + 1)*sqrt(c*x - 1) + 2*(12*b^3*log(c)^2 + 36*a^2*b - 8*(b^3*c^2
*log(c)^2 + 3*a^2*b*c^2)*x^2 - (2*b^3*c^2*x^2 - 3*b^3)*log(c^2*x^2)^2 - 4*
(2*b^3*c^2*x^2 - 3*b^3)*log(x)^2 + 4*(2*b^3*c^2*x^2*log(c) - 3*b^3*log(c)
+ (2*b^3*c^2*x^2 - 3*b^3)*log(x))*log(c^2*x^2) - 8*(2*b^3*c^2*x^2*log(c) -
 3*b^3*log(c))*log(x))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - (16*b^6*x^3*a
rctan(sqrt(c*x + 1)*sqrt(c*x - 1))^4 + b^6*x^3*log(c^2*x^2)^4 + 64*b^6*x^3
*log(c)*log(x)^3 + 16*b^6*x^3*log(x)^4 + 64*a*b^5*x^3*arctan(sqrt(c*x + 1)
*sqrt(c*x - 1))^3 + 32*(3*b^6*log(c)^2 + a^2*b^4)*x^3*log(x)^2 + 64*(b^6*l
og(c)^3 + a^2*b^4*log(c))*x^3*log(x) + 16*(b^6*log(c)^4 + 2*a^2*b^4*log(c)
^2 + a^4*b^2)*x^3 - 8*(b^6*x^3*log(c) + b^6*x^3*log(x))*log(c^2*x^2)^3 + 8
*(b^6*x^3*log(c^2*x^2)^2 + 8*b^6*x^3*log(c)*log(x) + 4*b^6*x^3*log(x)^2 +
4*(b^6*log(c)^2 + 3*a^2*b^4)*x^3 - 4*(b^6*x^3*log(c) + b^6*x^3*log(x))*log
(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 8*(6*b^6*x^3*log(c)*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1640 vs. $2(210) = 420$.

Time = 0.23 (sec) , antiderivative size = 1640, normalized size of antiderivative = 7.19

$$\int \frac{1}{x^4 \left(a + b\sec^{-1}(cx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(a+b*arcsec(c*x))^3,x, algorithm="giac")
```

output
```
1/8*(36*b^2*c^2*arccos(1/(c*x))^2*cos(a/b)^2*cos_integral(3*a/b + 3*arccos
(1/(c*x)))*sin(a/b)/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2
*b^3) - 36*b^2*c^2*arccos(1/(c*x))^2*cos(a/b)^3*sin_integral(3*a/b + 3*arc
cos(1/(c*x)))/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3)
+ 72*a*b*c^2*arccos(1/(c*x))*cos(a/b)^2*cos_integral(3*a/b + 3*arccos(1/(c
*x)))*sin(a/b)/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3)
 - 72*a*b*c^2*arccos(1/(c*x))*cos(a/b)^3*sin_integral(3*a/b + 3*arccos(1/(
c*x)))/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) - 9*b^2
*c^2*arccos(1/(c*x))^2*cos_integral(3*a/b + 3*arccos(1/(c*x)))*sin(a/b)/(b
^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) + 36*a^2*c^2*cos
(a/b)^2*cos_integral(3*a/b + 3*arccos(1/(c*x)))*sin(a/b)/(b^5*arccos(1/(c*
x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) + b^2*c^2*arccos(1/(c*x))^2*cos
_integral(a/b + arccos(1/(c*x)))*sin(a/b)/(b^5*arccos(1/(c*x))^2 + 2*a*b^4
*arccos(1/(c*x)) + a^2*b^3) + 27*b^2*c^2*arccos(1/(c*x))^2*cos(a/b)*sin_in
tegral(3*a/b + 3*arccos(1/(c*x)))/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(
1/(c*x)) + a^2*b^3) - 36*a^2*c^2*cos(a/b)^3*sin_integral(3*a/b + 3*arccos(
1/(c*x)))/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) - b^
2*c^2*arccos(1/(c*x))^2*cos(a/b)*sin_integral(a/b + arccos(1/(c*x)))/(b^5*
arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) - 18*a*b*c^2*arccos
(1/(c*x))*cos_integral(3*a/b + 3*arccos(1/(c*x)))*sin(a/b)/(b^5*arccos(...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(a+b\sec^{-1}(cx)\right)^3}\,dx=\int \frac{1}{x^4\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^3}\,dx$$

input
```
int(1/(x^4*(a + b*acos(1/(c*x)))^3),x)
```

output
```
int(1/(x^4*(a + b*acos(1/(c*x)))^3), x)
```

Reduce [F]

$$\int \frac{1}{x^4(a+b\sec^{-1}(cx))^3}\,dx$$
$$=\int \frac{1}{asec(cx)^3 b^3x^4+3asec(cx)^2 a b^2x^4+3asec(cx)a^2b x^4+a^3x^4}dx$$

input
```
int(1/x^4/(a+b*asec(c*x))^3,x)
```

output
```
int(1/(asec(c*x)**3*b**3*x**4 + 3*asec(c*x)**2*a*b**2*x**4 + 3*asec(c*x)*a
**2*b*x**4 + a**3*x**4),x)
```

3.51 $\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^3 \, dx$

Optimal result 436
Mathematica [N/A] 436
Rubi [N/A] 437
Maple [N/A] 437
Fricas [N/A] 438
Sympy [N/A] 438
Maxima [N/A] 438
Giac [N/A] 439
Mupad [N/A] 440
Reduce [N/A] 440

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^3 \, dx = \text{Int}\left((dx)^m \left(a+b\sec^{-1}(cx)\right)^3, x\right)$$

output
```
Defer(Int)((d*x)^m*(a+b*arcsec(c*x))^3,x)
```

Mathematica [N/A]

Not integrable

Time = 3.88 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^3 \, dx = \int (dx)^m \left(a+b\sec^{-1}(cx)\right)^3 \, dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcSec[c*x])^3,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcSec[c*x])^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^3 dx$$

$$\Big\downarrow \text{5771}$$

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^3 dx$$

input
```
Int[(d*x)^m*(a + b*ArcSec[c*x])^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m (a+b \operatorname{arcsec}(cx))^3 dx$$

input
```
int((d*x)^m*(a+b*arcsec(c*x))^3,x)
```

output
```
int((d*x)^m*(a+b*arcsec(c*x))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.75

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^3 \, dx=\int (b\,\mathrm{arcsec}\,(cx)+a)^3(dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arcsec(c*x))^3,x, algorithm="fricas")
```

output
```
integral((b^3*arcsec(c*x)^3 + 3*a*b^2*arcsec(c*x)^2 + 3*a^2*b*arcsec(c*x)
+ a^3)*(d*x)^m, x)
```

Sympy [N/A]

Not integrable

Time = 26.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^3 \, dx=\int (dx)^m\,(a+b\,\mathrm{asec}\,(cx))^3 \, dx$$

input
```
integrate((d*x)**m*(a+b*asec(c*x))**3,x)
```

output
```
Integral((d*x)**m*(a + b*asec(c*x))**3, x)
```

Maxima [N/A]

Not integrable

Time = 17.41 (sec) , antiderivative size = 991, normalized size of antiderivative = 61.94

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^3 \, dx=\int (b\,\mathrm{arcsec}\,(cx)+a)^3(dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arcsec(c*x))^3,x, algorithm="maxima")
```

output

```
(d*x)^(m + 1)*a^3/(d*(m + 1)) + 1/4*(4*b^3*d^m*x*x^m*arctan(sqrt(c*x + 1)*
sqrt(c*x - 1))^3 - 3*b^3*d^m*x*x^m*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log
(c^2*x^2)^2 - 4*(m + 1)*integrate(3/4*(4*(a*b^2*d^m*m + a*b^2*d^m - (a*b^2
*c^2*d^m*m + a*b^2*c^2*d^m)*x^2)*x^m*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2
 - (a*b^2*d^m*m + a*b^2*d^m - (a*b^2*c^2*d^m*m + a*b^2*c^2*d^m)*x^2)*x^m*l
og(c^2*x^2)^2 - 4*(a*b^2*d^m*m + a*b^2*d^m - (a*b^2*c^2*d^m*m + a*b^2*c^2*
d^m)*x^2)*x^m*log(x)^2 - 8*(a*b^2*d^m*m*log(c) + a*b^2*d^m*log(c) - (a*b^2
*c^2*d^m*m*log(c) + a*b^2*c^2*d^m*log(c))*x^2)*x^m*log(x) + (4*b^3*d^m*x^m
*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - b^3*d^m*x^m*log(c^2*x^2)^2)*sqrt(
c*x + 1)*sqrt(c*x - 1) - 4*(a*b^2*d^m*m*log(c)^2 + a*b^2*d^m*log(c)^2 - (a
*b^2*c^2*d^m*m*log(c)^2 + a*b^2*c^2*d^m*log(c)^2)*x^2)*x^m - 4*((b^3*d^m*m
 + b^3*d^m - (b^3*c^2*d^m*m + b^3*c^2*d^m)*x^2)*x^m*log(x)^2 + 2*(b^3*d^m*
m*log(c) + b^3*d^m*log(c) - (b^3*c^2*d^m*m*log(c) + b^3*c^2*d^m*log(c))*x^
2)*x^m*log(x) + ((b^3*log(c)^2 - a^2*b)*d^m*m - ((b^3*c^2*log(c)^2 - a^2*b
*c^2)*d^m*m + (b^3*c^2*log(c)^2 - a^2*b*c^2)*d^m)*x^2 + (b^3*log(c)^2 - a^
2*b)*d^m)*x^m - ((b^3*d^m*m + b^3*d^m - (b^3*c^2*d^m*m + b^3*c^2*d^m)*x^2)
*x^m*log(x) + (b^3*d^m*m*log(c) - (b^3*c^2*d^m*m*log(c) + (b^3*c^2*log(c)
+ b^3*c^2)*d^m)*x^2 + (b^3*log(c) + b^3)*d^m)*x^m)*log(c^2*x^2))*arctan(sq
rt(c*x + 1)*sqrt(c*x - 1)) + 4*((a*b^2*d^m*m + a*b^2*d^m - (a*b^2*c^2*d^m*
m + a*b^2*c^2*d^m)*x^2)*x^m*log(x) + (a*b^2*d^m*m*log(c) + a*b^2*d^m*lo...
```

Giac [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a + b\sec^{-1}(cx)\right)^3 \, dx = \int (b\,\text{arcsec}\,(cx) + a)^3 (dx)^m \, dx$$

input

```
integrate((d*x)^m*(a+b*arcsec(c*x))^3,x, algorithm="giac")
```

output

```
integrate((b*arcsec(c*x) + a)^3*(d*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^3 \, dx = \int (d\,x)^m \left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^3 dx$$

input
```
int((d*x)^m*(a + b*acos(1/(c*x)))^3,x)
```

output
```
int((d*x)^m*(a + b*acos(1/(c*x)))^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 7.56

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^3 \, dx$$
$$= \frac{d^m \left(x^m a^3 x + 3\left(\int x^m asec(cx)\, dx\right) a^2 bm + 3\left(\int x^m asec(cx)\, dx\right) a^2 b + \left(\int x^m asec(cx)^3\, dx\right) b^3 m + \left(\int x^m as\right.}{m+1}$$

input
```
int((d*x)^m*(a+b*asec(c*x))^3,x)
```

output
```
(d**m*(x**m*a**3*x + 3*int(x**m*asec(c*x),x)*a**2*b*m + 3*int(x**m*asec(c*
x),x)*a**2*b + int(x**m*asec(c*x)**3,x)*b**3*m + int(x**m*asec(c*x)**3,x)*
b**3 + 3*int(x**m*asec(c*x)**2,x)*a*b**2*m + 3*int(x**m*asec(c*x)**2,x)*a*
b**2))/(m + 1)
```

3.52 $\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^2\,dx$

Optimal result . . . 441
Mathematica [N/A] . . . 441
Rubi [N/A] . . . 442
Maple [N/A] . . . 442
Fricas [N/A] . . . 443
Sympy [N/A] . . . 443
Maxima [N/A] . . . 443
Giac [N/A] . . . 444
Mupad [N/A] . . . 444
Reduce [N/A] . . . 445

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^2\,dx=\mathrm{Int}\left((dx)^m \left(a+b\sec^{-1}(cx)\right)^2,x\right)$$

output
```
Defer(Int)((d*x)^m*(a+b*arcsec(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 2.49 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^2\,dx=\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^2\,dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcSec[c*x])^2,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcSec[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^2 dx$$

$$\Big\downarrow \text{5771}$$

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^2 dx$$

input
```
Int[(d*x)^m*(a + b*ArcSec[c*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b\,\mathrm{arcsec}\left(cx\right)\right)^2 dx$$

input
```
int((d*x)^m*(a+b*arcsec(c*x))^2,x)
```

output
```
int((d*x)^m*(a+b*arcsec(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^2 \, dx = \int (b\,\mathrm{arcsec}\,(cx)+a)^2 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arcsec(c*x))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arcsec(c*x)^2 + 2*a*b*arcsec(c*x) + a^2)*(d*x)^m, x)
```

Sympy [N/A]

Not integrable

Time = 12.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^2 \, dx = \int (dx)^m (a+b\,\mathrm{asec}\,(cx))^2 \, dx$$

input
```
integrate((d*x)**m*(a+b*asec(c*x))**2,x)
```

output
```
Integral((d*x)**m*(a + b*asec(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 7.30 (sec) , antiderivative size = 512, normalized size of antiderivative = 32.00

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^2 \, dx = \int (b\,\mathrm{arcsec}\,(cx)+a)^2 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

output
```
(d*x)^(m + 1)*a^2/(d*(m + 1)) + 1/4*(4*b^2*d^m*x*x^m*arctan(sqrt(c*x + 1)*
sqrt(c*x - 1))^2 - b^2*d^m*x*x^m*log(c^2*x^2)^2 - 4*(m + 1)*integrate((2*s
qrt(c*x + 1)*sqrt(c*x - 1)*b^2*d^m*x^m*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))
 - (b^2*d^m*m + b^2*d^m - (b^2*c^2*d^m*m + b^2*c^2*d^m)*x^2)*x^m*log(x)^2
+ 2*(a*b*d^m*m + a*b*d^m - (a*b*c^2*d^m*m + a*b*c^2*d^m)*x^2)*x^m*arctan(s
qrt(c*x + 1)*sqrt(c*x - 1)) - 2*(b^2*d^m*m*log(c) + b^2*d^m*log(c) - (b^2*
c^2*d^m*m*log(c) + b^2*c^2*d^m*log(c))*x^2)*x^m*log(x) - (b^2*d^m*m*log(c)
^2 + b^2*d^m*log(c)^2 - (b^2*c^2*d^m*m*log(c)^2 + b^2*c^2*d^m*log(c)^2)*x^
2)*x^m + ((b^2*d^m*m + b^2*d^m - (b^2*c^2*d^m*m + b^2*c^2*d^m)*x^2)*x^m*lo
g(x) + (b^2*d^m*m*log(c) - (b^2*c^2*d^m*m*log(c) + (b^2*c^2*log(c) + b^2*c
^2)*d^m)*x^2 + (b^2*log(c) + b^2)*d^m)*x^m)*log(c^2*x^2))/((c^2*m + c^2)*x
^2 - m - 1), x))/(m + 1)
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^2 dx = \int (b\operatorname{arcsec}(cx)+a)^2(dx)^m\, dx$$

input
```
integrate((d*x)^m*(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)^2*(d*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^2 dx = \int (d\,x)^m \left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^2 dx$$

input
```
int((d*x)^m*(a + b*acos(1/(c*x)))^2,x)
```

output
```
int((d*x)^m*(a + b*acos(1/(c*x)))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 5.00

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^2 dx$$
$$= \frac{d^m\left(x^m a^2 x + 2\left(\int x^m asec(cx)\,dx\right) abm + 2\left(\int x^m asec(cx)\,dx\right) ab + \left(\int x^m asec(cx)^2\,dx\right) b^2 m + \left(\int x^m asec\right.}{m+1}$$

input
```
int((d*x)^m*(a+b*asec(c*x))^2,x)
```

output
```
(d**m*(x**m*a**2*x + 2*int(x**m*asec(c*x),x)*a*b*m + 2*int(x**m*asec(c*x),
x)*a*b + int(x**m*asec(c*x)**2,x)*b**2*m + int(x**m*asec(c*x)**2,x)*b**2))
/(m + 1)
```

3.53 $\int (dx)^m \left(a+b\sec^{-1}(cx)\right)\,dx$

Optimal result 446
Mathematica [A] (verified) 446
Rubi [A] (verified) 447
Maple [F] 448
Fricas [F] 448
Sympy [F] 449
Maxima [F] 449
Giac [F] 449
Mupad [F(-1)] 450
Reduce [F] 450

Optimal result

Integrand size = 14, antiderivative size = 67

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)\,dx = \frac{(dx)^{1+m}\left(a+b\sec^{-1}(cx)\right)}{d(1+m)} - \frac{b(dx)^m \text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{c^2x^2}\right)}{cm(1+m)}$$

output
```
(d*x)^(1+m)*(a+b*arcsec(c*x))/d/(1+m)-b*(d*x)^m*hypergeom([1/2, -1/2*m],[1
-1/2*m],1/c^2/x^2)/c/m/(1+m)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.22

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)\,dx = \frac{x(dx)^m\left((1+m)\left(a+b\sec^{-1}(cx)\right)+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\,x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{\sqrt{1-c^2x^2}}\right)}{(1+m)^2}$$

input
```
Integrate[(d*x)^m*(a + b*ArcSec[c*x]),x]
```

output
```
(x*(d*x)^m*((1 + m)*(a + b*ArcSec[c*x]) + (b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Hyp
ergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/Sqrt[1 - c^2*x^2]))/(1
 + m)^2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5743, 862, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right) dx$$

$$\Big\downarrow \ 5743$$

$$\frac{(dx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{d(m+1)}-\frac{bd\int \frac{(dx)^{m-1}}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{c(m+1)}$$

$$\Big\downarrow \ 862$$

$$\frac{b\left(\frac{1}{x}\right)^m (dx)^m \int \frac{\left(\frac{1}{x}\right)^{-m-1}}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{c(m+1)}+\frac{(dx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{d(m+1)}$$

$$\Big\downarrow \ 278$$

$$\frac{(dx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{d(m+1)}-\frac{b(dx)^m \text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{c^2x^2}\right)}{cm(m+1)}$$

input
```
Int[(d*x)^m*(a + b*ArcSec[c*x]),x]
```

output
```
((d*x)^(1 + m)*(a + b*ArcSec[c*x]))/(d*(1 + m)) - (b*(d*x)^m*Hypergeometri
c2F1[1/2, -1/2*m, 1 - m/2, 1/(c^2*x^2)])/(c*m*(1 + m))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 862
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Maple [F]

$$\int (dx)^m (a + b \operatorname{arcsec}(cx))\, dx$$

input
```
int((d*x)^m*(a+b*arcsec(c*x)),x)
```

output
```
int((d*x)^m*(a+b*arcsec(c*x)),x)
```

Fricas [F]

$$\int (dx)^m \left(a + b\sec^{-1}(cx)\right)\, dx = \int (b\operatorname{arcsec}(cx) + a)(dx)^m\, dx$$

input
```
integrate((d*x)^m*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)*(d*x)^m, x)
```

Sympy [F]

$$\int (dx)^m \left(a + b\sec^{-1}(cx)\right) dx = \int (dx)^m \left(a + b\,\mathrm{asec}\,(cx)\right) dx$$

input
```
integrate((d*x)**m*(a+b*asec(c*x)),x)
```

output
```
Integral((d*x)**m*(a + b*asec(c*x)), x)
```

Maxima [F]

$$\int (dx)^m \left(a + b\sec^{-1}(cx)\right) dx = \int (b\,\mathrm{arcsec}\,(cx) + a)(dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
(d^m*x*x^m*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - (c^2*d^m*m + c^2*d^m)*int
egrate(-sqrt(c*x + 1)*sqrt(c*x - 1)*x^m/(c^2*m - (c^4*m + c^4)*x^2 + c^2),
 x))*b/(m + 1) + (d*x)^(m + 1)*a/(d*(m + 1))
```

Giac [F]

$$\int (dx)^m \left(a + b\sec^{-1}(cx)\right) dx = \int (b\,\mathrm{arcsec}\,(cx) + a)(dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right) \, dx = \int (d\,x)^m \left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right) \, dx$$

input
```
int((d*x)^m*(a + b*acos(1/(c*x))),x)
```

output
```
int((d*x)^m*(a + b*acos(1/(c*x))), x)
```

Reduce [F]

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right) \, dx = \frac{d^m \left(x^m a x + \left(\int x^m asec(cx)\, dx\right) b m + \left(\int x^m asec(cx)\, dx\right) b\right)}{m+1}$$

input
```
int((d*x)^m*(a+b*asec(c*x)),x)
```

output
```
(d**m*(x**m*a*x + int(x**m*asec(c*x),x)*b*m + int(x**m*asec(c*x),x)*b))/(m
 + 1)
```

3.54 $\int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx$

Optimal result 451
Mathematica [N/A] 451
Rubi [N/A] 452
Maple [N/A] 452
Fricas [N/A] 453
Sympy [N/A] 453
Maxima [N/A] 453
Giac [N/A] 454
Mupad [N/A] 454
Reduce [N/A] 455

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx = \text{Int}\left(\frac{(dx)^m}{a+b\sec^{-1}(cx)},x\right)$$

output
```
Defer(Int)((d*x)^m/(a+b*arcsec(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx = \int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcSec[c*x]),x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcSec[c*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx$$

$$\Big\downarrow \text{5771}$$

$$\int \frac{(dx)^m}{a+b\sec^{-1}(cx)}dx$$

input
```
Int[(d*x)^m/(a + b*ArcSec[c*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.84 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{a+b\ \mathrm{arcsec}\,(cx)}dx$$

input
```
int((d*x)^m/(a+b*arcsec(c*x)),x)
```

output
```
int((d*x)^m/(a+b*arcsec(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx=\int \frac{(dx)^m}{b\,\mathrm{arcsec}\,(cx)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b*arcsec(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx=\int \frac{(dx)^m}{a+b\,\mathrm{asec}\,(cx)}\,dx$$

input
```
integrate((d*x)**m/(a+b*asec(c*x)),x)
```

output
```
Integral((d*x)**m/(a + b*asec(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx=\int \frac{(dx)^m}{b\,\mathrm{arcsec}\,(cx)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(b*arcsec(c*x) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx=\int \frac{(dx)^m}{b\,\mathrm{arcsec}\,(cx)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b*arcsec(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx=\int \frac{(d\,x)^m}{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}\,dx$$

input
```
int((d*x)^m/(a + b*acos(1/(c*x))),x)
```

output
```
int((d*x)^m/(a + b*acos(1/(c*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx = d^m\left(\int \frac{x^m}{asec\,(cx)\,b+a}dx\right)$$

input
```
int((d*x)^m/(a+b*asec(c*x)),x)
```

output
```
d**m*int(x**m/(asec(c*x)*b + a),x)
```

3.55 $\int \frac{(dx)^m}{(a+b\sec^{-1}(cx))^2} dx$

Optimal result 456
Mathematica [N/A] 456
Rubi [N/A] 457
Maple [N/A] 457
Fricas [N/A] 458
Sympy [N/A] 458
Maxima [N/A] 458
Giac [N/A] 459
Mupad [N/A] 460
Reduce [N/A] 460

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(dx)^m}{(a+b\sec^{-1}(cx))^2} dx = \text{Int}\left(\frac{(dx)^m}{(a+b\sec^{-1}(cx))^2}, x\right)$$

output
```
Defer(Int)((d*x)^m/(a+b*arcsec(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{(a+b\sec^{-1}(cx))^2} dx = \int \frac{(dx)^m}{(a+b\sec^{-1}(cx))^2} dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcSec[c*x])^2,x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcSec[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(a+b\sec^{-1}(cx))^2}\,dx$$
$$\Big\downarrow \text{ 5771}$$
$$\int \frac{(dx)^m}{(a+b\sec^{-1}(cx))^2}dx$$

input
```
Int[(d*x)^m/(a + b*ArcSec[c*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{(a+b\,\mathrm{arcsec}\,(cx))^2}dx$$

input
```
int((d*x)^m/(a+b*arcsec(c*x))^2,x)
```

output
```
int((d*x)^m/(a+b*arcsec(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \frac{(dx)^m}{(a+b\sec^{-1}(cx))^2}\,dx = \int \frac{(dx)^m}{(b\,\mathrm{arcsec}\,(cx)+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arcsec(c*x))^2,x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b^2*arcsec(c*x)^2 + 2*a*b*arcsec(c*x) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 6.41 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{(dx)^m}{(a+b\sec^{-1}(cx))^2}\,dx = \int \frac{(dx)^m}{(a+b\,\mathrm{asec}\,(cx))^2}\,dx$$

input
```
integrate((d*x)**m/(a+b*asec(c*x))**2,x)
```

output
```
Integral((d*x)**m/(a + b*asec(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.94 (sec) , antiderivative size = 643, normalized size of antiderivative = 40.19

$$\int \frac{(dx)^m}{(a+b\sec^{-1}(cx))^2}\,dx = \int \frac{(dx)^m}{(b\,\mathrm{arcsec}\,(cx)+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

output
```
-(4*(b*d^m*x*x^m*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + a*d^m*x*x^m)*sqrt(c
*x + 1)*sqrt(c*x - 1) - (4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3
*log(c^2*x^2)^2 + 4*b^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2 +
8*a*b^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*log(c) + b^
3*log(x))*log(c^2*x^2))*integrate(4*((b*d^m*m - (b*c^2*d^m*m + 2*b*c^2*d^m
)*x^2 + b*d^m)*x^m*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + (a*d^m*m - (a*c^2
*d^m*m + 2*a*c^2*d^m)*x^2 + a*d^m)*x^m)*sqrt(c*x + 1)*sqrt(c*x - 1)/(4*b^3
*log(c)^2 + 4*a^2*b - 4*(b^3*c^2*log(c)^2 + a^2*b*c^2)*x^2 - 4*(b^3*c^2*x^
2 - b^3)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - (b^3*c^2*x^2 - b^3)*log(c
^2*x^2)^2 - 4*(b^3*c^2*x^2 - b^3)*log(x)^2 - 8*(a*b^2*c^2*x^2 - a*b^2)*arc
tan(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*(b^3*c^2*x^2*log(c) - b^3*log(c) + (b
^3*c^2*x^2 - b^3)*log(x))*log(c^2*x^2) - 8*(b^3*c^2*x^2*log(c) - b^3*log(c
))*log(x)), x))/(4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*log(c^2
*x^2)^2 + 4*b^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2 + 8*a*b^2*
arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*log(c) + b^3*log(x)
)*log(c^2*x^2))
```

Giac [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{(a+b\sec^{-1}(cx))^2}\,dx=\int \frac{(dx)^m}{(b\,\mathrm{arcsec}\,(cx)+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b*arcsec(c*x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{(dx)^m}{(a+b\sec^{-1}(cx))^2}\,dx=\int \frac{(d\,x)^m}{\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^2}\,dx$$

input
```
int((d*x)^m/(a + b*acos(1/(c*x)))^2,x)
```

output
```
int((d*x)^m/(a + b*acos(1/(c*x)))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.12

$$\int \frac{(dx)^m}{(a+b\sec^{-1}(cx))^2}\,dx=d^m\left(\int \frac{x^m}{asec\,(cx)^2\,b^2+2asec\,(cx)\,ab+a^2}dx\right)$$

input
```
int((d*x)^m/(a+b*asec(c*x))^2,x)
```

output
```
d**m*int(x**m/(asec(c*x)**2*b**2 + 2*asec(c*x)*a*b + a**2),x)
```

3.56 $\int (d+ex)^3 \left(a+b\sec^{-1}(cx)\right)\,dx$

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Optimal result

Integrand size = 16, antiderivative size = 167

$$\begin{aligned}\int (d+ex)^3 \left(a+b\sec^{-1}(cx)\right)\,dx = &-\frac{be(9c^2d^2+e^2)\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3} \\ &-\frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{2c}-\frac{be^3\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c} \\ &+\frac{bd^4\csc^{-1}(cx)}{4e}+\frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e} \\ &-\frac{bd(2c^2d^2+e^2)\,\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{2c^3}\end{aligned}$$

output
```
-1/6*b*e*(9*c^2*d^2+e^2)*(1-1/c^2/x^2)^(1/2)*x/c^3-1/2*b*d*e^2*(1-1/c^2/x^
2)^(1/2)*x^2/c-1/12*b*e^3*(1-1/c^2/x^2)^(1/2)*x^3/c+1/4*b*d^4*arccsc(c*x)/
e+1/4*(e*x+d)^4*(a+b*arcsec(c*x))/e-1/2*b*d*(2*c^2*d^2+e^2)*arctanh((1-1/c
^2/x^2)^(1/2))/c^3
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.99

$$\int (d+ex)^3 \left(a+b\sec^{-1}(cx)\right)\,dx$$
$$= \frac{3ac^3x(4d^3+6d^2ex+4de^2x^2+e^3x^3)-be\sqrt{1-\frac{1}{c^2x^2}}x(2e^2+c^2(18d^2+6dex+e^2x^2))+3bc^3x(4d^3+6d^2}{12c^3}$$

input
```
Integrate[(d + e*x)^3*(a + b*ArcSec[c*x]),x]
```

output
```
(3*a*c^3*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) - b*e*Sqrt[1 - 1/(c
^2*x^2)]*x*(2*e^2 + c^2*(18*d^2 + 6*d*e*x + e^2*x^2)) + 3*b*c^3*x*(4*d^3 +
 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)*ArcSec[c*x] - 6*b*d*(2*c^2*d^2 + e^2)*
Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(12*c^3)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {5749, 1892, 1803, 540, 25, 2338, 27, 2338, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3 \left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{5749}$$

$$\frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e}-\frac{b\int \frac{(d+ex)^4}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{4ce}$$

$$\Big\downarrow \text{1892}$$

$$\frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e}-\frac{b\int \frac{\left(\frac{d}{x}+e\right)^4x^2}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{4ce}$$

$$\Big\downarrow \text{1803}$$

$$\frac{b\int \frac{\left(\frac{d}{x}+e\right)^4x^4}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{4ce}+\frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \text{540}$$

$$\frac{b\left(-\frac{1}{3}\int -\frac{\left(\frac{3d^4}{x^3}+\frac{12ed^3}{x^2}+12e^3d+\frac{2e^2\left(9d^2+\frac{e^2}{c^2}\right)}{x}\right)x^3}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-\frac{1}{3}e^4x^3\sqrt{1-\frac{1}{c^2x^2}}\right)}{4ce}+\frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{1}{3}\int \frac{\left(\frac{3d^4}{x^3}+\frac{12ed^3}{x^2}+12e^3d+\frac{2e^2\left(9d^2+\frac{e^2}{c^2}\right)}{x}\right)x^3}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-\frac{1}{3}e^4x^3\sqrt{1-\frac{1}{c^2x^2}}\right)}{4ce}+\frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \text{2338}$$

$$\frac{b\left(\frac{1}{3}\left(-\frac{1}{2}\int -\frac{2\left(\frac{3d^4}{x^2}+\frac{6e\left(2d^2+\frac{e^2}{c^2}\right)d}{x}+2e^2\left(9d^2+\frac{e^2}{c^2}\right)\right)x^2}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-6de^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{3}e^4x^3\sqrt{1-\frac{1}{c^2x^2}}\right)}{4ce}+\frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\left(\frac{1}{3}\left(\int \frac{\left(\frac{3d^4}{x^2}+\frac{6e\left(2d^2+\frac{e^2}{c^2}\right)d}{x}+2e^2\left(9d^2+\frac{e^2}{c^2}\right)\right)x^2}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-6de^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{3}e^4x^3\sqrt{1-\frac{1}{c^2x^2}}\right)}{4ce}+\frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \text{2338}$$

$$\frac{b\left(\frac{1}{3}\left(-\int -\frac{3d\left(\frac{d^3}{x}+2e\left(2d^2+\frac{e^2}{c^2}\right)\right)x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-2e^2x\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{e^2}{c^2}+9d^2\right)-6de^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{3}e^4x^3\sqrt{1-\frac{1}{c^2x^2}}\right)}{4ce}+\frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{1}{3}\left(3d\int \frac{\left(\frac{d^3}{x}+2e\left(2d^2+\frac{e^2}{c^2}\right)\right)x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-2e^2x\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{e^2}{c^2}+9d^2\right)-6de^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{3}e^4x^3\sqrt{1-\frac{1}{c^2x^2}}\right)}{4ce}+\frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \; 538$$

$$\frac{b\left(\frac{1}{3}\left(3d\left(d^3\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}+2e\left(\frac{e^2}{c^2}+2d^2\right)\int \frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}\right)-2e^2x\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{e^2}{c^2}+9d^2\right)-6de^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)-}{4ce}$$
$$\frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \; 223$$

$$\frac{b\left(\frac{1}{3}\left(3d\left(2e\left(\frac{e^2}{c^2}+2d^2\right)\int \frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}+cd^3\arcsin\left(\frac{1}{cx}\right)\right)-2e^2x\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{e^2}{c^2}+9d^2\right)-6de^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)-}{4ce}$$
$$\frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \; 243$$

$$\frac{b\left(\frac{1}{3}\left(3d\left(e\left(\frac{e^2}{c^2}+2d^2\right)\int \frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}+cd^3\arcsin\left(\frac{1}{cx}\right)\right)-2e^2x\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{e^2}{c^2}+9d^2\right)-6de^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)-}{4ce}$$
$$\frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \; 73$$

$$\frac{b\left(\frac{1}{3}\left(3d\left(cd^3\arcsin\left(\frac{1}{cx}\right)-2c^2e\left(\frac{e^2}{c^2}+2d^2\right)\int\frac{1}{c^2-c^2\sqrt{1-\frac{1}{c^2x^2}}}d\sqrt{1-\frac{1}{c^2x^2}}\right)-2e^2x\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{e^2}{c^2}+9d^2\right)-6de^3x^2\right.\right.}{4ce}$$

$$\frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \ 221$$

$$\frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e}+$$

$$\frac{b\left(\frac{1}{3}\left(3d\left(cd^3\arcsin\left(\frac{1}{cx}\right)-2e\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)\left(\frac{e^2}{c^2}+2d^2\right)\right)-2e^2x\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{e^2}{c^2}+9d^2\right)-6de^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right.\right.}{4ce}$$

input
```
Int[(d + e*x)^3*(a + b*ArcSec[c*x]),x]
```

output
```
((d + e*x)^4*(a + b*ArcSec[c*x]))/(4*e) + (b*(-1/3*(e^4*Sqrt[1 - 1/(c^2*x^
2)]*x^3) + (-2*e^2*(9*d^2 + e^2/c^2)*Sqrt[1 - 1/(c^2*x^2)]*x - 6*d*e^3*Sqr
t[1 - 1/(c^2*x^2)]*x^2 + 3*d*(c*d^3*ArcSin[1/(c*x)] - 2*e*(2*d^2 + e^2/c^2
)*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]]))/3))/(4*c*e)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1892
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

rule 2338

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 5749

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(147) = 294$.

Time = 0.24 (sec) , antiderivative size = 402, normalized size of antiderivative = 2.41

method	result
parts	$\frac{a(ex+d)^4}{4e} + \frac{b e^3 \operatorname{arcsec}(cx) x^4}{4} + b e^2 \operatorname{arcsec}(cx) x^3 d + \frac{3be \operatorname{arcsec}(cx) x^2 d^2}{2} + b \operatorname{arcsec}(cx) x d^3 +$
derivativedivides	$\frac{\frac{a(cex+cd)^4}{4c^3e} + \frac{bc \operatorname{arcsec}(cx) d^4}{4e} + b \operatorname{arcsec}(cx) d^3 cx + \frac{3bce \operatorname{arcsec}(cx) d^2 x^2}{2} + bc e^2 \operatorname{arcsec}(cx) d x^3 + \frac{bc e^3 \operatorname{arcsec}(cx) x^4}{4} + \frac{b\sqrt{c^2x^2-}}{}}{}$
default	$\frac{\frac{a(cex+cd)^4}{4c^3e} + \frac{bc \operatorname{arcsec}(cx) d^4}{4e} + b \operatorname{arcsec}(cx) d^3 cx + \frac{3bce \operatorname{arcsec}(cx) d^2 x^2}{2} + bc e^2 \operatorname{arcsec}(cx) d x^3 + \frac{bc e^3 \operatorname{arcsec}(cx) x^4}{4} + \frac{b\sqrt{c^2x^2-}}{}}{}$

input

```
int((e*x+d)^3*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*(e*x+d)^4/e+1/4*b*e^3*arcsec(c*x)*x^4+b*e^2*arcsec(c*x)*x^3*d+3/2*b*
e*arcsec(c*x)*x^2*d^2+b*arcsec(c*x)*x*d^3+1/4*b/e*arcsec(c*x)*d^4-1/12*b/c
^3*e^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x+1/4*b/c/e*(c^2*x^2-1)^(1/
2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^4*arctan(1/(c^2*x^2-1)^(1/2))-1/2*b/c^3
*e^2*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d-b/c^2*(c^2*x^2-1)^(1/2)/((c
^2*x^2-1)/c^2/x^2)^(1/2)/x*d^3*ln(c*x+(c^2*x^2-1)^(1/2))-3/2*b/c^3*e*(c^2*
x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^2-1/2*b/c^4*e^2*(c^2*x^2-1)^(1/2)/(
(c^2*x^2-1)/c^2/x^2)^(1/2)/x*d*ln(c*x+(c^2*x^2-1)^(1/2))-1/6*b/c^5*e^3*(c^
2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.74

$$\int (d+ex)^3 \left(a+b\sec^{-1}(cx)\right)\,dx$$

$$= \frac{3\,ac^4e^3x^4 + 12\,ac^4de^2x^3 + 18\,ac^4d^2ex^2 + 12\,ac^4d^3x + 3\left(bc^4e^3x^4 + 4\,bc^4de^2x^3 + 6\,bc^4d^2ex^2 + 4\,bc^4d^3x - \cdots\right.}{}$$

input
```
integrate((e*x+d)^3*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/12*(3*a*c^4*e^3*x^4 + 12*a*c^4*d*e^2*x^3 + 18*a*c^4*d^2*e*x^2 + 12*a*c^4
*d^3*x + 3*(b*c^4*e^3*x^4 + 4*b*c^4*d*e^2*x^3 + 6*b*c^4*d^2*e*x^2 + 4*b*c^
4*d^3*x - 4*b*c^4*d^3 - 6*b*c^4*d^2*e - 4*b*c^4*d*e^2 - b*c^4*e^3)*arcsec(
c*x) + 6*(4*b*c^4*d^3 + 6*b*c^4*d^2*e + 4*b*c^4*d*e^2 + b*c^4*e^3)*arctan(
-c*x + sqrt(c^2*x^2 - 1)) + 6*(2*b*c^3*d^3 + b*c*d*e^2)*log(-c*x + sqrt(c^
2*x^2 - 1)) - (b*c^2*e^3*x^2 + 6*b*c^2*d*e^2*x + 18*b*c^2*d^2*e + 2*b*e^3)
*sqrt(c^2*x^2 - 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 4.79 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.17

$$\int (d+ex)^3 \left(a+b\sec^{-1}(cx)\right) dx$$
$$= ad^3x + \frac{3ad^2ex^2}{2} + ade^2x^3 + \frac{ae^3x^4}{4} + bd^3x\,\mathrm{asec}\,(cx) + \frac{3bd^2ex^2\,\mathrm{asec}\,(cx)}{2} + bde^2x^3\,\mathrm{asec}\,(cx) + \frac{be^3x^4\,\mathrm{asec}\,(cx)}{4} - \frac{bd^3\left(\begin{cases}\mathrm{acosh}\,(cx) & \text{for } |c^2x^2|>1\\ -i\,\mathrm{asin}\,(cx) & \text{otherwise}\end{cases}\right)}{c} - \frac{3bd^2e\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c} - \frac{bde^2\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\mathrm{acosh}\,(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\,\mathrm{asin}\,(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{c} - \frac{be^3\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{4c}$$

input
```
integrate((e*x+d)**3*(a+b*asec(c*x)),x)
```

output
```
a*d**3*x + 3*a*d**2*e*x**2/2 + a*d*e**2*x**3 + a*e**3*x**4/4 + b*d**3*x*as
ec(c*x) + 3*b*d**2*e*x**2*asec(c*x)/2 + b*d*e**2*x**3*asec(c*x) + b*e**3*x
**4*asec(c*x)/4 - b*d**3*Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*a
sin(c*x), True))/c - 3*b*d**2*e*Piecewise((sqrt(c**2*x**2 - 1)/c, Abs(c**2
*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/(2*c) - b*d*e**2*Piecewise(
(x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (
-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*as
in(c*x)/(2*c**2), True))/c - b*e**3*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3
*c) + 2*sqrt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c
**2*x**2 + 1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**3), True))/(4*c)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.63

$$\int (d+ex)^3 \left(a+b\sec^{-1}(cx)\right)\,dx$$
$$= \frac{1}{4}\,ae^3x^4 + ade^2x^3 + \frac{3}{2}\,ad^2ex^2 + \frac{3}{2}\left(x^2\,\mathrm{arcsec}\,(cx) - \frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)bd^2e$$
$$+ \frac{1}{4}\left(4\,x^3\,\mathrm{arcsec}\,(cx) - \frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bde^2$$
$$+ \frac{1}{12}\left(3\,x^4\,\mathrm{arcsec}\,(cx) - \frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)be^3 + ad^3x$$
$$+ \frac{\left(2\,cx\,\mathrm{arcsec}\,(cx) - \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) + \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd^3}{2\,c}$$

input
```
integrate((e*x+d)^3*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/4*a*e^3*x^4 + a*d*e^2*x^3 + 3/2*a*d^2*e*x^2 + 3/2*(x^2*arcsec(c*x) - x*s
qrt(-1/(c^2*x^2) + 1)/c)*b*d^2*e + 1/4*(4*x^3*arcsec(c*x) - (2*sqrt(-1/(c^
2*x^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1
)/c^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d*e^2 + 1/12*(3*x^4*arcs
ec(c*x) - (c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/
c^3)*b*e^3 + a*d^3*x + 1/2*(2*c*x*arcsec(c*x) - log(sqrt(-1/(c^2*x^2) + 1)
 + 1) + log(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*d^3/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9430 vs. $2(147) = 294$.

Time = 2.38 (sec) , antiderivative size = 9430, normalized size of antiderivative = 56.47

$$\int (d+ex)^3 \left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/12*(12*b*c^3*d^3*arccos(1/(c*x))/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x)
 + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) -
 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) - 12*b*c^
3*d^3*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^5 + 4*c^5*(1/(c^2*
x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*
c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x)
 + 1)^8) + 12*b*c^3*d^3*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(c^
5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1
/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^
2) - 1)^4/(1/(c*x) + 1)^8) + 12*a*c^3*d^3/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(
1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2
*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) +
18*b*c^2*d^2*e*arccos(1/(c*x))/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1
)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^
3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) + 24*b*c^3*d^
3*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*
x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2)
 - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x
) + 1)^2) - 48*b*c^3*d^3*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1)
+ 1/(c*x) + 1))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5...
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3 \left(a+b\sec^{-1}(cx)\right) \, dx = \int \left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right) (d+e\,x)^3 \, dx$$

input
```
int((a + b*acos(1/(c*x)))*(d + e*x)^3,x)
```

output
```
int((a + b*acos(1/(c*x)))*(d + e*x)^3, x)
```

Reduce [F]

$$\int (d+ex)^3 \left(a+b\sec^{-1}(cx)\right) \, dx = \left(\int asec(cx)\,dx\right) b\,d^3 + \left(\int asec(cx)\,x^3 dx\right) b\,e^3 + 3\left(\int asec(cx)\,x^2 dx\right) bd\,e^2 + 3\left(\int asec(cx)\,x dx\right) b\,d^2 e + a\,d^3 x + \frac{3a\,d^2 e\,x^2}{2} + ad\,e^2 x^3 + \frac{a\,e^3 x^4}{4}$$

input
```
int((e*x+d)^3*(a+b*asec(c*x)),x)
```

output
```
(4*int(asec(c*x),x)*b*d**3 + 4*int(asec(c*x)*x**3,x)*b*e**3 + 12*int(asec(
c*x)*x**2,x)*b*d*e**2 + 12*int(asec(c*x)*x,x)*b*d**2*e + 4*a*d**3*x + 6*a*
d**2*e*x**2 + 4*a*d*e**2*x**3 + a*e**3*x**4)/4
```

3.57 $\int (d+ex)^2 \left(a+b\sec^{-1}(cx)\right) \, dx$

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Optimal result

Integrand size = 16, antiderivative size = 124

$$\int (d+ex)^2 \left(a+b\sec^{-1}(cx)\right) \, dx = -\frac{bde\sqrt{1-\frac{1}{c^2x^2}}x}{c} - \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{6c} + \frac{bd^3\csc^{-1}(cx)}{3e} + \frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e} - \frac{b(6c^2d^2+e^2)\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^3}$$

output

```
-b*d*e*(1-1/c^2/x^2)^(1/2)*x/c-1/6*b*e^2*(1-1/c^2/x^2)^(1/2)*x^2/c+1/3*b*d
^3*arccsc(c*x)/e+1/3*(e*x+d)^3*(a+b*arcsec(c*x))/e-1/6*b*(6*c^2*d^2+e^2)*a
rctanh((1-1/c^2/x^2)^(1/2))/c^3
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00

$$\int (d+ex)^2 \left(a+b\sec^{-1}(cx)\right) \, dx$$

$$= \frac{c^2x\left(-be\sqrt{1-\frac{1}{c^2x^2}}(6d+ex)+2ac(3d^2+3dex+e^2x^2)\right)+2bc^3x(3d^2+3dex+e^2x^2)\sec^{-1}(cx)-b(6c^2}{6c^3}$$

input
```
Integrate[(d + e*x)^2*(a + b*ArcSec[c*x]),x]
```

output
```
(c^2*x*(-(b*e*Sqrt[1 - 1/(c^2*x^2)]*(6*d + e*x)) + 2*a*c*(3*d^2 + 3*d*e*x
+ e^2*x^2)) + 2*b*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2)*ArcSec[c*x] - b*(6*c^2
*d^2 + e^2)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(6*c^3)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5749, 1892, 1803, 540, 25, 2338, 25, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2 \left(a+b\sec^{-1}(cx)\right) dx$$

$$\Big\downarrow \text{5749}$$

$$\frac{(d+ex)^3 \left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{b\int \frac{(d+ex)^3}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{3ce}$$

$$\Big\downarrow \text{1892}$$

$$\frac{(d+ex)^3 \left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{b\int \frac{\left(\frac{d}{x}+e\right)^3x}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{3ce}$$

$$\Big\downarrow \text{1803}$$

$$\frac{b\int \frac{\left(\frac{d}{x}+e\right)^3x^3}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{3ce}+\frac{(d+ex)^3 \left(a+b\sec^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{540}$$

$$\frac{b\left(-\frac{1}{2}\int -\frac{\left(\frac{2d^3}{x^2}+6e^2d+\frac{e\left(6d^2+\frac{e^2}{c^2}\right)}{x}\right)x^2}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-\frac{1}{2}e^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{3ce}+\frac{(d+ex)^3 \left(a+b\sec^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{1}{2}\int \frac{\left(\frac{2d^3}{x^2}+6e^2d+\frac{e\left(6d^2+\frac{e^2}{c^2}\right)}{x}\right)x^2}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-\frac{1}{2}e^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{3ce}+\frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{2338}$$

$$\frac{b\left(\frac{1}{2}\left(-\int -\frac{\left(\frac{2d^3}{x}+e\left(6d^2+\frac{e^2}{c^2}\right)\right)x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-6de^2x\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{2}e^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{3ce}+\frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{1}{2}\left(\int \frac{\left(\frac{2d^3}{x}+e\left(6d^2+\frac{e^2}{c^2}\right)\right)x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-6de^2x\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{2}e^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{3ce}+\frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{538}$$

$$\frac{b\left(\frac{1}{2}\left(2d^3\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}+e\left(\frac{e^2}{c^2}+6d^2\right)\int \frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-6de^2x\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{2}e^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{3ce}+\frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{223}$$

$$\frac{b\left(\frac{1}{2}\left(e\left(\frac{e^2}{c^2}+6d^2\right)\int \frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}+2cd^3\arcsin\left(\frac{1}{cx}\right)-6de^2x\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{2}e^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{3ce}+\frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{243}$$

$$\frac{b\left(\frac{1}{2}\left(\frac{1}{2}e\left(\frac{e^2}{c^2}+6d^2\right)\int\frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}+2cd^3\arcsin\left(\frac{1}{cx}\right)-6de^2x\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{2}e^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{3ce}+\frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \; 73$$

$$\frac{b\left(\frac{1}{2}\left(-c^2e\left(\frac{e^2}{c^2}+6d^2\right)\int\frac{1}{c^2-c^2\sqrt{1-\frac{1}{c^2x^2}}}d\sqrt{1-\frac{1}{c^2x^2}}+2cd^3\arcsin\left(\frac{1}{cx}\right)-6de^2x\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{2}e^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{3ce}+\frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \; 221$$

$$\frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e}+\frac{b\left(\frac{1}{2}\left(2cd^3\arcsin\left(\frac{1}{cx}\right)-e\,\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)\left(\frac{e^2}{c^2}+6d^2\right)-6de^2x\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{2}e^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{3ce}$$

input

```
Int[(d + e*x)^2*(a + b*ArcSec[c*x]),x]
```

output

```
((d + e*x)^3*(a + b*ArcSec[c*x]))/(3*e) + (b*(-1/2*(e^3*Sqrt[1 - 1/(c^2*x^
2)]*x^2) + (-6*d*e^2*Sqrt[1 - 1/(c^2*x^2)]*x + 2*c*d^3*ArcSin[1/(c*x)] - e
*(6*d^2 + e^2/c^2)*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/2))/(3*c*e)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1892
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 5749
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(110) = 220$.

Time = 0.24 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.47

method	result
parts	$\frac{a(ex+d)^3}{3e} + \frac{b\,e^2 \operatorname{arcsec}(cx)x^3}{3} + be \operatorname{arcsec}(cx)\, x^2 d + b \operatorname{arcsec}(cx)\, x\, d^2 + \frac{b \operatorname{arcsec}(cx) d^3}{3e} + \frac{b\sqrt{c^2x}}{}$
derivativedivides	$\frac{\frac{a(cex+cd)^3}{3c^2e} + \frac{bc \operatorname{arcsec}(cx) d^3}{3e} + b \operatorname{arcsec}(cx) d^2 cx + bce \operatorname{arcsec}(cx) d\, x^2 + \frac{bc\,e^2 \operatorname{arcsec}(cx)x^3}{3} + \frac{b\sqrt{c^2x^2-1}\, d^3 \arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{3e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\, x}}{c}$
default	$\frac{\frac{a(cex+cd)^3}{3c^2e} + \frac{bc \operatorname{arcsec}(cx) d^3}{3e} + b \operatorname{arcsec}(cx) d^2 cx + bce \operatorname{arcsec}(cx) d\, x^2 + \frac{bc\,e^2 \operatorname{arcsec}(cx)x^3}{3} + \frac{b\sqrt{c^2x^2-1}\, d^3 \arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{3e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\, x}}{c}$

input
```
int((e*x+d)^2*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*(e*x+d)^3/e+1/3*b*e^2*arcsec(c*x)*x^3+b*e*arcsec(c*x)*x^2*d+b*arcsec
(c*x)*x*d^2+1/3*b/e*arcsec(c*x)*d^3+1/3*b/c/e*(c^2*x^2-1)^(1/2)/((c^2*x^2-
1)/c^2/x^2)^(1/2)/x*d^3*arctan(1/(c^2*x^2-1)^(1/2))-1/6*b/c^3*e^2*(c^2*x^2
-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)-b/c^2*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x
^2)^(1/2)/x*d^2*ln(c*x+(c^2*x^2-1)^(1/2))-b/c^3*e*(c^2*x^2-1)/((c^2*x^2-1)
/c^2/x^2)^(1/2)/x*d-1/6*b/c^4*e^2*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^
(1/2)/x*ln(c*x+(c^2*x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.69

$$\int (d+ex)^2 \left(a+b\sec^{-1}(cx)\right) dx$$
$$= \frac{2\,ac^3e^2x^3 + 6\,ac^3dex^2 + 6\,ac^3d^2x + 2\,(bc^3e^2x^3 + 3\,bc^3dex^2 + 3\,bc^3d^2x - 3\,bc^3d^2 - 3\,bc^3de - bc^3e^2)\,\mathrm{arcsec}}{}$$

input
```
integrate((e*x+d)^2*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/6*(2*a*c^3*e^2*x^3 + 6*a*c^3*d*e*x^2 + 6*a*c^3*d^2*x + 2*(b*c^3*e^2*x^3
+ 3*b*c^3*d*e*x^2 + 3*b*c^3*d^2*x - 3*b*c^3*d^2 - 3*b*c^3*d*e - b*c^3*e^2)
*arcsec(c*x) + 4*(3*b*c^3*d^2 + 3*b*c^3*d*e + b*c^3*e^2)*arctan(-c*x + sqr
t(c^2*x^2 - 1)) + (6*b*c^2*d^2 + b*e^2)*log(-c*x + sqrt(c^2*x^2 - 1)) - (b
*c*e^2*x + 6*b*c*d*e)*sqrt(c^2*x^2 - 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 4.17 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.84

$$\int (d+ex)^2 \left(a+b\sec^{-1}(cx)\right) dx$$
$$= ad^2x + adex^2 + \frac{ae^2x^3}{3} + bd^2x\operatorname{asec}(cx) + bdex^2\operatorname{asec}(cx) + \frac{be^2x^3\operatorname{asec}(cx)}{3} - \frac{bd^2\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c} - \frac{bde\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{c} - \frac{be^2\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$

input
```
integrate((e*x+d)**2*(a+b*asec(c*x)),x)
```

output
```
a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 + b*d**2*x*asec(c*x) + b*d*e*x**2*as
ec(c*x) + b*e**2*x**3*asec(c*x)/3 - b*d**2*Piecewise((acosh(c*x), Abs(c**2
*x**2) > 1), (-I*asin(c*x), True))/c - b*d*e*Piecewise((sqrt(c**2*x**2 - 1
)/c, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/c - b*e**2*Pie
cewise((x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2)
> 1), (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1))
 - I*asin(c*x)/(2*c**2), True))/(3*c)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.61

$$\int (d+ex)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$= \frac{1}{3}ae^2x^3 + adex^2 + \left(x^2\,\mathrm{arcsec}\,(cx) - \frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)bde$$
$$+ \frac{1}{12}\left(4\,x^3\,\mathrm{arcsec}\,(cx) - \frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)be^2$$
$$+ ad^2x + \frac{\left(2\,cx\,\mathrm{arcsec}\,(cx) - \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) + \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd^2}{2\,c}$$

input
```
integrate((e*x+d)^2*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*e^2*x^3 + a*d*e*x^2 + (x^2*arcsec(c*x) - x*sqrt(-1/(c^2*x^2) + 1)/c)
*b*d*e + 1/12*(4*x^3*arcsec(c*x) - (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*
x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2*
x^2) + 1) - 1)/c^2)/c)*b*e^2 + a*d^2*x + 1/2*(2*c*x*arcsec(c*x) - log(sqrt
(-1/(c^2*x^2) + 1) + 1) + log(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*d^2/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6416 vs. 2(110) = 220.

Time = 2.81 (sec) , antiderivative size = 6416, normalized size of antiderivative = 51.74

$$\int (d+ex)^2\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
-1/6*(6*b*c^3*d*e*x^2*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/(c^4 + 3*c^4*(1/(c
^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
 c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) + 6*a*c^3*d*e*x^2*(1/(c^2*x^2) -
 1)/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) -
1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) + 18*b*c^3
*d*e*x^2*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^4 + 3*c^4*(1/(c^2*x^2) -
1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c
^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) + 18*a*c^3*d*e*x^2*(1/(c^
2*x^2) - 1)^2/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(
c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)
*(1/(c*x) + 1)^2) + 18*b*c^3*d*e*x^2*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/(
(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2
/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^
4) + 18*a*c^3*d*e*x^2*(1/(c^2*x^2) - 1)^3/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/
(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*
x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^4) + 6*b*c^3*d*e*x^2*(1/(c^2*x^
2) - 1)^4*arccos(1/(c*x))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
+ 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(
c*x) + 1)^6)*(1/(c*x) + 1)^6) + 6*b*c^2*d*e*x*sqrt(-1/(c^2*x^2) + 1)/(c^4
+ 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(...
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2 \left(a+b\sec^{-1}(cx)\right) \, dx = \int \left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right) \left(d+e\,x\right)^2 \, dx$$

input
```
int((a + b*acos(1/(c*x)))*(d + e*x)^2,x)
```

output
```
int((a + b*acos(1/(c*x)))*(d + e*x)^2, x)
```

Reduce [F]

$$\int (d+ex)^2 \left(a+b\sec^{-1}(cx)\right)\,dx = \left(\int asec(cx)\,dx\right)b\,d^2 + \left(\int asec(cx)\,x^2dx\right)b\,e^2 + 2\left(\int asec(cx)\,xdx\right)bde + a\,d^2x + ade\,x^2 + \frac{a\,e^2x^3}{3}$$

input
```
int((e*x+d)^2*(a+b*asec(c*x)),x)
```

output
```
(3*int(asec(c*x),x)*b*d**2 + 3*int(asec(c*x)*x**2,x)*b*e**2 + 6*int(asec(c
*x)*x,x)*b*d*e + 3*a*d**2*x + 3*a*d*e*x**2 + a*e**2*x**3)/3
```

3.58 $\int (d+ex)\left(a+b\sec^{-1}(cx)\right)\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 84

$$\int (d+ex)\left(a+b\sec^{-1}(cx)\right)\,dx=-\frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c}+\frac{bd^2\csc^{-1}(cx)}{2e}+\frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e}-\frac{bd\,\mathrm{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$$

output
```
-1/2*b*e*(1-1/c^2/x^2)^(1/2)*x/c+1/2*b*d^2*arccsc(c*x)/e+1/2*(e*x+d)^2*(a+
b*arcsec(c*x))/e-b*d*arctanh((1-1/c^2/x^2)^(1/2))/c
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.36

$$\int (d+ex)\left(a+b\sec^{-1}(cx)\right)\,dx=adx+\frac{1}{2}aex^2-\frac{bex\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{2c}+bdx\sec^{-1}(cx)+\frac{1}{2}bex^2\sec^{-1}(cx)-\frac{bd\sqrt{1-\frac{1}{c^2x^2}}\,x\,\mathrm{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{-1+c^2x^2}}$$

input
```
Integrate[(d + e*x)*(a + b*ArcSec[c*x]),x]
```

output
```
a*d*x + (a*e*x^2)/2 - (b*e*x*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(2*c) + b*d*x
*ArcSec[c*x] + (b*e*x^2*ArcSec[c*x])/2 - (b*d*Sqrt[1 - 1/(c^2*x^2)]*x*ArcT
anh[(c*x)/Sqrt[-1 + c^2*x^2]])/Sqrt[-1 + c^2*x^2]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {5749, 1892, 1730, 540, 25, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{ 5749}$$

$$\frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e}-\frac{b\int \frac{(d+ex)^2}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{2ce}$$

$$\Big\downarrow \text{ 1892}$$

$$\frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e}-\frac{b\int \frac{\left(\frac{d}{x}+e\right)^2}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{2ce}$$

$$\Big\downarrow \text{ 1730}$$

$$\frac{b\int \frac{\left(\frac{d}{x}+e\right)^2x^2}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{2ce}+\frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e}$$

$$\Big\downarrow \text{ 540}$$

$$\frac{b\left(e^2x\left(-\sqrt{1-\frac{1}{c^2x^2}}\right)-\int -\frac{d\left(\frac{d}{x}+2e\right)x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}\right)}{2ce}+\frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{b\left(\int \frac{d\left(\frac{d}{x}+2e\right)x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-e^2x\sqrt{1-\frac{1}{c^2x^2}}\right)}{2ce}+\frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e}$$

$\Big\downarrow$ 27

$$\frac{b\left(d\int \frac{\left(\frac{d}{x}+2e\right)x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-e^2x\sqrt{1-\frac{1}{c^2x^2}}\right)}{2ce}+\frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e}$$

$\Big\downarrow$ 538

$$\frac{b\left(d\left(d\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}+2e\int \frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}\right)-e^2x\sqrt{1-\frac{1}{c^2x^2}}\right)}{2ce}+\frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e}$$

$\Big\downarrow$ 223

$$\frac{b\left(d\left(2e\int \frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}+cd\arcsin\left(\frac{1}{cx}\right)\right)-e^2x\sqrt{1-\frac{1}{c^2x^2}}\right)}{2ce}+\frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e}$$

$\Big\downarrow$ 243

$$\frac{b\left(d\left(e\int \frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}+cd\arcsin\left(\frac{1}{cx}\right)\right)-e^2x\sqrt{1-\frac{1}{c^2x^2}}\right)}{2ce}+\frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e}$$

$\Big\downarrow$ 73

$$\frac{b\left(d\left(cd\arcsin\left(\frac{1}{cx}\right)-2c^2e\int \frac{1}{c^2-c^2\sqrt{1-\frac{1}{c^2x^2}}}d\sqrt{1-\frac{1}{c^2x^2}}\right)-e^2x\sqrt{1-\frac{1}{c^2x^2}}\right)}{2ce}+\frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e}$$

$\Big\downarrow$ 221

$$\frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e}+\frac{b\left(d\left(cd\arcsin\left(\frac{1}{cx}\right)-2e\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)\right)-e^2x\sqrt{1-\frac{1}{c^2x^2}}\right)}{2ce}$$

input
```
Int[(d + e*x)*(a + b*ArcSec[c*x]),x]
```

output
```
((d + e*x)^2*(a + b*ArcSec[c*x]))/(2*e) + (b*(-(e^2*Sqrt[1 - 1/(c^2*x^2)]*
x) + d*(c*d*ArcSin[1/(c*x)] - 2*e*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])))/(2*c*e
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 1730
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol
] :> -Subst[Int[(d + e/x^n)^q*((a + c/x^(2*n))^p/x^2), x], x, 1/x] /; FreeQ
[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && ILtQ[n, 0]
```

rule 1892
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

rule 5749
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.31

method	result	size
parts	$a\left(\frac{1}{2} e\,x^{2}+dx\right)+\frac{b\left(\frac{c\ \operatorname{arcsec}\left(cx\right) x^{2} e}{2}+\operatorname{arcsec}\left(cx\right) x c d-\frac{\sqrt{c^{2} x^{2}-1}\,\left(e\sqrt{c^{2} x^{2}-1}+2 d c \ln\left(cx+\sqrt{c^{2} x^{2}-1}\right)\right)}{2 c^{2} \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}\, x}\right)}{c}$	110
derivativedivides	$\frac{\frac{a\left(d\,c^{2} x+\frac{1}{2} e\,c^{2} x^{2}\right)}{c}+\frac{b\left(\operatorname{arcsec}\left(cx\right) d\,c^{2} x+\frac{\operatorname{arcsec}\left(cx\right) e\,c^{2} x^{2}}{2}-\frac{\sqrt{c^{2} x^{2}-1}\,\left(e\sqrt{c^{2} x^{2}-1}+2 d c \ln\left(cx+\sqrt{c^{2} x^{2}-1}\right)\right)}{2 \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}\, c x}\right)}{c}}{c}$	127
default	$\frac{\frac{a\left(d\,c^{2} x+\frac{1}{2} e\,c^{2} x^{2}\right)}{c}+\frac{b\left(\operatorname{arcsec}\left(cx\right) d\,c^{2} x+\frac{\operatorname{arcsec}\left(cx\right) e\,c^{2} x^{2}}{2}-\frac{\sqrt{c^{2} x^{2}-1}\,\left(e\sqrt{c^{2} x^{2}-1}+2 d c \ln\left(cx+\sqrt{c^{2} x^{2}-1}\right)\right)}{2 \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}\, c x}\right)}{c}}{c}$	127

input
```
int((e*x+d)*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/2*e*x^2+d*x)+b/c*(1/2*c*arcsec(c*x)*x^2*e+arcsec(c*x)*x*c*d-1/2/c^2/(
(c^2*x^2-1)/c^2/x^2)^(1/2)/x*(c^2*x^2-1)^(1/2)*(e*(c^2*x^2-1)^(1/2)+2*d*c*
ln(c*x+(c^2*x^2-1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.55

$$\int (d+ex)\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{ac^2ex^2+2\,ac^2dx+2\,bcd\log\left(-cx+\sqrt{c^2x^2-1}\right)-\sqrt{c^2x^2-1}be+\left(bc^2ex^2+2\,bc^2dx-2\,bc^2d-bc^2e\right)\mathrm{arc}}{2\,c^2}$$

input
```
integrate((e*x+d)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/2*(a*c^2*e*x^2 + 2*a*c^2*d*x + 2*b*c*d*log(-c*x + sqrt(c^2*x^2 - 1)) - s
qrt(c^2*x^2 - 1)*b*e + (b*c^2*e*x^2 + 2*b*c^2*d*x - 2*b*c^2*d - b*c^2*e)*a
rcsec(c*x) + 2*(2*b*c^2*d + b*c^2*e)*arctan(-c*x + sqrt(c^2*x^2 - 1)))/c^2
```

Sympy [A] (verification not implemented)

Time = 2.69 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.24

$$\int (d+ex)\left(a+b\sec^{-1}(cx)\right)\,dx = adx+\frac{aex^2}{2}+bdx\,\mathrm{asec}\,(cx)+\frac{bex^2\,\mathrm{asec}\,(cx)}{2} - \frac{bd\left(\begin{cases}\mathrm{acosh}\,(cx) & \text{for } |c^2x^2|>1\\ -i\,\mathrm{asin}\,(cx) & \text{otherwise}\end{cases}\right)}{c} - \frac{be\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c}$$

input
```
integrate((e*x+d)*(a+b*asec(c*x)),x)
```

output
```
a*d*x + a*e*x**2/2 + b*d*x*asec(c*x) + b*e*x**2*asec(c*x)/2 - b*d*Piecewis
e((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c - b*e*Piecewis
e((sqrt(c**2*x**2 - 1)/c, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c,
True))/(2*c)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.11

$$\int (d+ex)\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{2}\,aex^2+\frac{1}{2}\left(x^2\,\mathrm{arcsec}\left(cx\right)-\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)be+adx + \frac{\left(2\,cx\,\mathrm{arcsec}\left(cx\right)-\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)+\log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd}{2\,c}$$

input
```
integrate((e*x+d)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/2*a*e*x^2 + 1/2*(x^2*arcsec(c*x) - x*sqrt(-1/(c^2*x^2) + 1)/c)*b*e + a*d
*x + 1/2*(2*c*x*arcsec(c*x) - log(sqrt(-1/(c^2*x^2) + 1) + 1) + log(-sqrt(
-1/(c^2*x^2) + 1) + 1))*b*d/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1547 vs. 2(74) = 148.

Time = 0.32 (sec) , antiderivative size = 1547, normalized size of antiderivative = 18.42

$$\int (d+ex)\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/2*(2*b*c*d*arccos(1/(c*x))/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^
2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) - 2*b*c*d*log(abs(sqrt(-1/(c^
2*x^2) + 1) + 1/(c*x) + 1))/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + 2*b*c*d*log(abs(sqrt(-1/(c^2
*x^2) + 1) - 1/(c*x) - 1))/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
+ c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + 2*a*c*d/(c^3 + 2*c^3*(1/(c^2*
x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + b*e
*arccos(1/(c*x))/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(
c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) - 4*b*c*d*(1/(c^2*x^2) - 1)*log(abs(sqrt(
-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x)
 + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) + 4*b*
c*d*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^3
 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c
*x) + 1)^4)*(1/(c*x) + 1)^2) + a*e/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x)
 + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) - 2*b*e*(1/(c^2*x^2) -
1)*arccos(1/(c*x))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(
1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) - 2*b*c*d*(1/(c^2*x^2
) - 1)^2*arccos(1/(c*x))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 +
 c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4) - 2*b*c*d*(1/(c
^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^3 + 2...
```

Mupad [B] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

$$\int (d+ex)\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{a\,x\,(2\,d+e\,x)}{2}-\frac{b\,d\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{c^2\,x^2}}}\right)}{c}+b\,d\,x\,\mathrm{acos}\left(\frac{1}{c\,x}\right)-\frac{b\,e\,x\,\left(\sqrt{1-\frac{1}{c^2\,x^2}}-c\,x\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{2\,c}$$

input
```
int((a + b*acos(1/(c*x)))*(d + e*x),x)
```

output
```
(a*x*(2*d + e*x))/2 - (b*d*atanh(1/(1 - 1/(c^2*x^2))^(1/2)))/c + b*d*x*aco
s(1/(c*x)) - (b*e*x*((1 - 1/(c^2*x^2))^(1/2) - c*x*acos(1/(c*x))))/(2*c)
```

Reduce [F]

$$\int (d+ex)\left(a+b\sec^{-1}(cx)\right)\,dx = \left(\int asec(cx)\,dx\right)bd + \left(\int asec(cx)\,xdx\right)be + adx + \frac{ae\,x^2}{2}$$

input
```
int((e*x+d)*(a+b*asec(c*x)),x)
```

output
```
(2*int(asec(c*x),x)*b*d + 2*int(asec(c*x)*x,x)*b*e + 2*a*d*x + a*e*x**2)/2
```

3.59 $\int (a+b\sec^{-1}(cx))\,dx$

Optimal result 493
Mathematica [A] (verified) 493
Rubi [A] (verified) 494
Maple [A] (verified) 494
Fricas [B] (verification not implemented) 495
Sympy [A] (verification not implemented) 495
Maxima [A] (verification not implemented) 496
Giac [B] (verification not implemented) 496
Mupad [B] (verification not implemented) 497
Reduce [F] 497

Optimal result

Integrand size = 8, antiderivative size = 32

$$\int (a+b\sec^{-1}(cx))\,dx = ax+bx\sec^{-1}(cx)-\frac{b\,\mathrm{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$$

output
```
a*x+b*x*arcsec(c*x)-b*arctanh((1-1/c^2/x^2)^(1/2))/c
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.84

$$\int (a+b\sec^{-1}(cx))\,dx = ax+bx\sec^{-1}(cx)-\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\,\mathrm{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{-1+c^2x^2}}$$

input
```
Integrate[a + b*ArcSec[c*x],x]
```

output
```
a*x + b*x*ArcSec[c*x] - (b*Sqrt[1 - 1/(c^2*x^2)]*x*ArcTanh[(c*x)/Sqrt[-1 +
 c^2*x^2]])/Sqrt[-1 + c^2*x^2]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a + b\sec^{-1}(cx)\right) dx$$

$$\Big\downarrow \; 2009$$

$$ax - \frac{b\text{arctanh}\left(\sqrt{1 - \frac{1}{c^2x^2}}\right)}{c} + bx\sec^{-1}(cx)$$

input
```
Int[a + b*ArcSec[c*x],x]
```

output
```
a*x + b*x*ArcSec[c*x] - (b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/c
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

method	result	size
default	$ax + bx\ \text{arcsec}\left(cx\right) - \frac{b\ln\left(cx + cx\sqrt{1 - \frac{1}{c^2x^2}}\right)}{c}$	38
parts	$ax + bx\ \text{arcsec}\left(cx\right) - \frac{b\ln\left(cx + cx\sqrt{1 - \frac{1}{c^2x^2}}\right)}{c}$	38
derivativedivides	$\frac{acx + b\left(cx\ \text{arcsec}(cx) - \ln\left(cx + cx\sqrt{1 - \frac{1}{c^2x^2}}\right)\right)}{c}$	42

input
```
int(a+b*arcsec(c*x),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*x*arcsec(c*x)-b/c*ln(c*x+c*x*(1-1/c^2/x^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(30) = 60$.

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.97

$$\int \left(a+b\sec^{-1}(cx)\right)\,dx = \frac{acx+2\,bc\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+(bcx-bc)\operatorname{arcsec}(cx)+b\log\left(-cx+\sqrt{c^2x^2-1}\right)}{c}$$

input
```
integrate(a+b*arcsec(c*x),x, algorithm="fricas")
```

output
```
(a*c*x + 2*b*c*arctan(-c*x + sqrt(c^2*x^2 - 1)) + (b*c*x - b*c)*arcsec(c*x
) + b*log(-c*x + sqrt(c^2*x^2 - 1)))/c
```

Sympy [A] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \left(a+b\sec^{-1}(cx)\right)\,dx = ax+b\left(x\operatorname{asec}(cx)-\frac{\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}}{c}\right)$$

input
```
integrate(a+b*asec(c*x),x)
```

output
```
a*x + b*(x*asec(c*x) - Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asi
n(c*x), True))/c)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.66

$$\int \left(a+b\sec^{-1}(cx)\right)\,dx = ax + \frac{\left(2\,cx\,\mathrm{arcsec}\left(cx\right) - \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) + \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)b}{2\,c}$$

input
```
integrate(a+b*arcsec(c*x),x, algorithm="maxima")
```

output
```
a*x + 1/2*(2*c*x*arcsec(c*x) - log(sqrt(-1/(c^2*x^2) + 1) + 1) + log(-sqrt
(-1/(c^2*x^2) + 1) + 1))*b/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(30) = 60$.

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.97

$$\int \left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{2}\,bc\left(\frac{2\,x\arccos\left(\frac{1}{cx}\right)}{c} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) - \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}\right) + ax$$

input
```
integrate(a+b*arcsec(c*x),x, algorithm="giac")
```

output
```
1/2*b*c*(2*x*arccos(1/(c*x))/c - (log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-s
qrt(-1/(c^2*x^2) + 1) + 1))/c^2) + a*x
```

Mupad [B] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \left(a+b\sec^{-1}(cx)\right)\,dx=a\,x+b\,x\,\mathrm{acos}\left(\frac{1}{c\,x}\right)-\frac{b\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{c^2\,x^2}}}\right)}{c}$$

input
```
int(a + b*acos(1/(c*x)),x)
```

output
```
a*x + b*x*acos(1/(c*x)) - (b*atanh(1/(1 - 1/(c^2*x^2))^(1/2)))/c
```

Reduce [F]

$$\int \left(a+b\sec^{-1}(cx)\right)\,dx=\left(\int asec(cx)\,dx\right)b+ax$$

input
```
int(a+b*asec(c*x),x)
```

output
```
int(asec(c*x),x)*b + a*x
```

3.60 $\int \frac{a+b\sec^{-1}(cx)}{d+ex}\,dx$

Optimal result . . . 498
Mathematica [A] (verified) . . . 499
Rubi [A] (verified) . . . 500
Maple [A] (verified) . . . 501
Fricas [F] . . . 502
Sympy [F] . . . 503
Maxima [F] . . . 503
Giac [F(-2)] . . . 503
Mupad [F(-1)] . . . 504
Reduce [F] . . . 504

Optimal result

Integrand size = 16, antiderivative size = 247

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex}\,dx = \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{\left(e-\sqrt{-c^2d^2+e^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e} + \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{\left(e+\sqrt{-c^2d^2+e^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e} - \frac{(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{\left(e-\sqrt{-c^2d^2+e^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{\left(e+\sqrt{-c^2d^2+e^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e} + \frac{ib\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2e}$$

output
```
(a+b*arcsec(c*x))*ln(1+(e-(-c^2*d^2+e^2)^(1/2))*(1/c/x+I*(1-1/c^2/x^2)^(1/
2))/c/d)/e+(a+b*arcsec(c*x))*ln(1+(e+(-c^2*d^2+e^2)^(1/2))*(1/c/x+I*(1-1/c
^2/x^2)^(1/2))/c/d)/e-(a+b*arcsec(c*x))*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))
^2)/e-I*b*polylog(2,-(e-(-c^2*d^2+e^2)^(1/2))*(1/c/x+I*(1-1/c^2/x^2)^(1/2)
)/c/d)/e-I*b*polylog(2,-(e+(-c^2*d^2+e^2)^(1/2))*(1/c/x+I*(1-1/c^2/x^2)^(1
/2))/c/d)/e+1/2*I*b*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)/e
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.35

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex}\,dx = \frac{a\log(d+ex)}{e}$$
$$+ \frac{b\left(4i\arcsin\left(\frac{\sqrt{1+\frac{e}{cd}}}{\sqrt{2}}\right)\arctan\left(\frac{(-cd+e)\tan\left(\frac{1}{2}\sec^{-1}(cx)\right)}{\sqrt{-c^2d^2+e^2}}\right)+\left(\sec^{-1}(cx)+2\arcsin\left(\frac{\sqrt{1+\frac{e}{cd}}}{\sqrt{2}}\right)\right)\log\left(1+\frac{(e-}{}\right.\right.}{}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(d + e*x),x]
```

output
```
(a*Log[d + e*x])/e + (b*((4*I)*ArcSin[Sqrt[1 + e/(c*d)]/Sqrt[2]]*ArcTan[((
-(c*d) + e)*Tan[ArcSec[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] + (ArcSec[c*x] + 2
*ArcSin[Sqrt[1 + e/(c*d)]/Sqrt[2]])*Log[1 + ((e - Sqrt[-(c^2*d^2) + e^2])*
E^(I*ArcSec[c*x]))/(c*d)] + (ArcSec[c*x] - 2*ArcSin[Sqrt[1 + e/(c*d)]/Sqrt
[2]])*Log[1 + ((e + Sqrt[-(c^2*d^2) + e^2])*E^(I*ArcSec[c*x]))/(c*d)] - Ar
cSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] - I*(PolyLog[2, ((-e + Sqrt[-(c^2
*d^2) + e^2])*E^(I*ArcSec[c*x]))/(c*d)] + PolyLog[2, -(((e + Sqrt[-(c^2*d^
2) + e^2])*E^(I*ArcSec[c*x]))/(c*d))]) + (I/2)*PolyLog[2, -E^((2*I)*ArcSec
[c*x])]))/e
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5747, 2998}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex}dx$$

$$\Big\downarrow \text{5747}$$

$$-\frac{b\int \frac{\log\left(\frac{e^{i\sec^{-1}(cx)}\left(e-\sqrt{e^2-c^2d^2}\right)}{cd}+1\right)}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{ce}-\frac{b\int \frac{\log\left(\frac{e^{i\sec^{-1}(cx)}\left(e+\sqrt{e^2-c^2d^2}\right)}{cd}+1\right)}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{ce}+\frac{b\int \frac{\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{ce}+\frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{\left(e-\sqrt{e^2-c^2d^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e}+\frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{\left(\sqrt{e^2-c^2d^2}+e\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e}-\frac{\log\left(1+e^{2i\sec^{-1}(cx)}\right)(a+b\sec^{-1}(cx))}{e}$$

$$\Big\downarrow \text{2998}$$

$$\frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{\left(e-\sqrt{e^2-c^2d^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e}+\frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{\left(\sqrt{e^2-c^2d^2}+e\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e}-\frac{\log\left(1+e^{2i\sec^{-1}(cx)}\right)(a+b\sec^{-1}(cx))}{e}-\frac{ib\,\text{PolyLog}\left(2,-\frac{\left(e-\sqrt{e^2-c^2d^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e}-\frac{ib\,\text{PolyLog}\left(2,-\frac{\left(e+\sqrt{e^2-c^2d^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e}+\frac{ib\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2e}$$

input
```
Int[(a + b*ArcSec[c*x])/(d + e*x),x]
```

output
```
((a + b*ArcSec[c*x])*Log[1 + ((e - Sqrt[-(c^2*d^2) + e^2])*E^(I*ArcSec[c*x
]))/(c*d)])/e + ((a + b*ArcSec[c*x])*Log[1 + ((e + Sqrt[-(c^2*d^2) + e^2])
*E^(I*ArcSec[c*x]))/(c*d)])/e - ((a + b*ArcSec[c*x])*Log[1 + E^((2*I)*ArcS
ec[c*x])])/e - (I*b*PolyLog[2, -(((e - Sqrt[-(c^2*d^2) + e^2])*E^(I*ArcSec
[c*x]))/(c*d))])/e - (I*b*PolyLog[2, -(((e + Sqrt[-(c^2*d^2) + e^2])*E^(I*
ArcSec[c*x]))/(c*d))])/e + ((I/2)*b*PolyLog[2, -E^((2*I)*ArcSec[c*x])])/e
```

Defintions of rubi rules used

rule 2998
```
Int[Log[v_]*(u_), x_Symbol] :> With[{w = DerivativeDivides[v, u*(1 - v), x]
}, Simp[w*PolyLog[2, 1 - v], x] /;  !FalseQ[w]]
```

rule 5747
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))/((d_.) + (e_.)*(x_)), x_Symbol] :> S
imp[(a + b*ArcSec[c*x])*(Log[1 + (e - Sqrt[(-c^2)*d^2 + e^2])*(E^(I*ArcSec[
c*x])/(c*d))]/e), x] + (Simp[(a + b*ArcSec[c*x])*(Log[1 + (e + Sqrt[(-c^2)*
d^2 + e^2])*(E^(I*ArcSec[c*x])/(c*d))]/e), x] - Simp[(a + b*ArcSec[c*x])*(L
og[1 + E^(2*I*ArcSec[c*x])]/e), x] - Simp[b/(c*e)   Int[Log[1 + (e - Sqrt[(
-c^2)*d^2 + e^2])*(E^(I*ArcSec[c*x])/(c*d))]/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x
], x] - Simp[b/(c*e)   Int[Log[1 + (e + Sqrt[(-c^2)*d^2 + e^2])*(E^(I*ArcSe
c[c*x])/(c*d))]/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] + Simp[b/(c*e)   Int[Lo
g[1 + E^(2*I*ArcSec[c*x])]/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x]) /; FreeQ[{a
, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.83

method	result
parts	$\frac{a\ln(ex+d)}{e}+\frac{b\,\mathrm{arcsec}(cx)\ln\left(\frac{-cd\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)+\sqrt{-c^2d^2+e^2}-e}{-e+\sqrt{-c^2d^2+e^2}}\right)}{e}+\frac{b\,\mathrm{arcsec}(cx)\ln\left(\frac{cd\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)+\sqrt{-}}{e+\sqrt{-c^2d^2+e^2}}\right.}{e}$
derivativedivides	$\frac{\frac{ac\ln(cex+cd)}{e}+bc\left(-\frac{\mathrm{arcsec}(cx)\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{e}-\frac{\mathrm{arcsec}(cx)\ln\left(1-i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{e}+\frac{i\,\mathrm{dilog}\left(1+i\left(\frac{1}{cx}+i\sqrt{1-}\right.\right.}{e}\right.}{}$
default	$\frac{\frac{ac\ln(cex+cd)}{e}+bc\left(-\frac{\mathrm{arcsec}(cx)\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{e}-\frac{\mathrm{arcsec}(cx)\ln\left(1-i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{e}+\frac{i\,\mathrm{dilog}\left(1+i\left(\frac{1}{cx}+i\sqrt{1-}\right.\right.}{e}\right.}{}$

input
```
int((a+b*arcsec(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
a*ln(e*x+d)/e+b/e*arcsec(c*x)*ln((-c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))+(-c^2
*d^2+e^2)^(1/2)-e)/(-e+(-c^2*d^2+e^2)^(1/2)))+b/e*arcsec(c*x)*ln((c*d*(1/c
/x+I*(1-1/c^2/x^2)^(1/2))+(-c^2*d^2+e^2)^(1/2)+e)/(e+(-c^2*d^2+e^2)^(1/2))
)-I*b/e*dilog((-c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))+(-c^2*d^2+e^2)^(1/2)-e)/
(-e+(-c^2*d^2+e^2)^(1/2)))-I*b/e*dilog((c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))+
(-c^2*d^2+e^2)^(1/2)+e)/(e+(-c^2*d^2+e^2)^(1/2)))-b/e*arcsec(c*x)*ln(1+I*(
1/c/x+I*(1-1/c^2/x^2)^(1/2)))-b/e*arcsec(c*x)*ln(1-I*(1/c/x+I*(1-1/c^2/x^2
)^(1/2)))+I*b/e*dilog(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))+I*b/e*dilog(1-I*(
1/c/x+I*(1-1/c^2/x^2)^(1/2)))
```

Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{ex+d}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex}\,dx=\int \frac{a+b\operatorname{asec}(cx)}{d+ex}\,dx$$

input
```
integrate((a+b*asec(c*x))/(e*x+d),x)
```

output
```
Integral((a + b*asec(c*x))/(d + e*x), x)
```

Maxima [F]

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex}\,dx=\int \frac{b\operatorname{arcsec}(cx)+a}{ex+d}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d),x, algorithm="maxima")
```

output
```
b*integrate(arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x + d), x) + a*log(e*x
+ d)/e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{d+e\,x}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(d + e*x),x)
```

output
```
int((a + b*acos(1/(c*x)))/(d + e*x), x)
```

Reduce [F]

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex}\,dx=\frac{\left(\int \frac{asec(cx)}{ex+d}dx\right)be+\log(ex+d)\,a}{e}$$

input
```
int((a+b*asec(c*x))/(e*x+d),x)
```

output
```
(int(asec(c*x)/(d + e*x),x)*b*e + log(d + e*x)*a)/e
```

3.61 $\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^2}\,dx$

Optimal result 505
Mathematica [A] (verified) 505
Rubi [A] (verified) 506
Maple [A] (verified) 508
Fricas [B] (verification not implemented) 509
Sympy [F] 510
Maxima [F] 511
Giac [F(-2)] 511
Mupad [F(-1)] 511
Reduce [F] 512

Optimal result

Integrand size = 16, antiderivative size = 104

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^2}\,dx = -\frac{b\csc^{-1}(cx)}{de} - \frac{a+b\sec^{-1}(cx)}{e(d+ex)} - \frac{b\,\text{arctanh}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{d\sqrt{c^2d^2-e^2}}$$

output
```
-b*arccsc(c*x)/d/e-(a+b*arcsec(c*x))/e/(e*x+d)-b*arctanh((c^2*d+e/x)/c/(c^
2*d^2-e^2)^(1/2)/(1-1/c^2/x^2)^(1/2))/d/(c^2*d^2-e^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.37

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^2}\,dx = -\frac{a}{e(d+ex)} - \frac{b\sec^{-1}(cx)}{e(d+ex)} - \frac{b\arcsin\left(\frac{1}{cx}\right)}{de} - \frac{b\log(d+ex)}{d\sqrt{c^2d^2-e^2}} + \frac{b\log\left(e+c\left(cd-\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{d\sqrt{c^2d^2-e^2}}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(d + e*x)^2,x]
```

output
```
-(a/(e*(d + e*x))) - (b*ArcSec[c*x])/(e*(d + e*x)) - (b*ArcSin[1/(c*x)])/(
d*e) - (b*Log[d + e*x])/(d*Sqrt[c^2*d^2 - e^2]) + (b*Log[e + c*(c*d - Sqrt
[c^2*d^2 - e^2]*Sqrt[1 - 1/(c^2*x^2)])*x])/(d*Sqrt[c^2*d^2 - e^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5749, 1892, 1803, 605, 223, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^2}\,dx$$

$$\Big\downarrow \text{5749}$$

$$\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2(d+ex)}dx}{ce}-\frac{a+b\sec^{-1}(cx)}{e(d+ex)}$$

$$\Big\downarrow \text{1892}$$

$$\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)x^3}dx}{ce}-\frac{a+b\sec^{-1}(cx)}{e(d+ex)}$$

$$\Big\downarrow \text{1803}$$

$$-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)x}d\frac{1}{x}}{ce}-\frac{a+b\sec^{-1}(cx)}{e(d+ex)}$$

$$\Big\downarrow \text{605}$$

$$-\frac{b\left(\frac{\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{d}-\frac{e\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}}{d}\right)}{ce}-\frac{a+b\sec^{-1}(cx)}{e(d+ex)}$$

$$\Big\downarrow \text{223}$$

$$-\frac{b\left(\frac{c\arcsin\left(\frac{1}{cx}\right)}{d}-\frac{e\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}}{d}\right)}{ce}-\frac{a+b\sec^{-1}(cx)}{e(d+ex)}$$

$\Big\downarrow$ 488

$$-\frac{b\left(\frac{e\int \frac{1}{d^2-\frac{e^2}{c^2}-\frac{1}{x^2}}d\frac{d+\frac{e}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}}{d}+\frac{c\arcsin\left(\frac{1}{cx}\right)}{d}\right)}{ce}-\frac{a+b\sec^{-1}(cx)}{e(d+ex)}$$

$\Big\downarrow$ 219

$$-\frac{a+b\sec^{-1}(cx)}{e(d+ex)}-\frac{b\left(\frac{c\arcsin\left(\frac{1}{cx}\right)}{d}+\frac{ce\,\text{arctanh}\left(\frac{c\left(\frac{e}{c^2x}+d\right)}{\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d^2-e^2}}\right)}{d\sqrt{c^2d^2-e^2}}\right)}{ce}$$

input
```
Int[(a + b*ArcSec[c*x])/(d + e*x)^2,x]
```

output
```
-((a + b*ArcSec[c*x])/(e*(d + e*x))) - (b*((c*ArcSin[1/(c*x)])/d + (c*e*Ar
cTanh[(c*(d + e/(c^2*x)))/(Sqrt[c^2*d^2 - e^2]*Sqrt[1 - 1/(c^2*x^2)])])/(d
*Sqrt[c^2*d^2 - e^2])))/(c*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 605
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[1/d   Int[x^(m - 1)*(a + b*x^2)^p, x], x] - Simp[c/d   Int[x^(m - 1
)*((a + b*x^2)^p/(c + d*x)), x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m,
 0] && LtQ[-1, p, 0]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1892
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

rule 5749
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.86

method	result
parts	$-\frac{a}{(ex+d)e}+\frac{b\left(-\frac{c^{2}\operatorname{arcsec}(cx)}{(cex+cd)e}-\frac{\sqrt{c^{2}x^{2}-1}\left(\arctan\left(\frac{1}{\sqrt{c^{2}x^{2}-1}}\right)\sqrt{\frac{c^{2}d^{2}-e^{2}}{e^{2}}}-\ln\left(\frac{2\sqrt{c^{2}x^{2}-1}\sqrt{\frac{c^{2}d^{2}-e^{2}}{e^{2}}}\,e-2d\,c^{2}x-2e}{cex+cd}\right)\right)}{e\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,xd\sqrt{\frac{c^{2}d^{2}-e^{2}}{e^{2}}}}\right)}{c}$
derivativedivides	$\frac{-\frac{a\,c^{2}}{(cex+cd)e}+b\,c^{2}\left(-\frac{\operatorname{arcsec}(cx)}{(cex+cd)e}-\frac{\sqrt{c^{2}x^{2}-1}\left(\arctan\left(\frac{1}{\sqrt{c^{2}x^{2}-1}}\right)\sqrt{\frac{c^{2}d^{2}-e^{2}}{e^{2}}}-\ln\left(\frac{2\sqrt{c^{2}x^{2}-1}\sqrt{\frac{c^{2}d^{2}-e^{2}}{e^{2}}}\,e-2d\,c^{2}x-2e}{cex+cd}\right)\right)}{e\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,c^{2}xd\sqrt{\frac{c^{2}d^{2}-e^{2}}{e^{2}}}}\right)}{c}$
default	$\frac{-\frac{a\,c^{2}}{(cex+cd)e}+b\,c^{2}\left(-\frac{\operatorname{arcsec}(cx)}{(cex+cd)e}-\frac{\sqrt{c^{2}x^{2}-1}\left(\arctan\left(\frac{1}{\sqrt{c^{2}x^{2}-1}}\right)\sqrt{\frac{c^{2}d^{2}-e^{2}}{e^{2}}}-\ln\left(\frac{2\sqrt{c^{2}x^{2}-1}\sqrt{\frac{c^{2}d^{2}-e^{2}}{e^{2}}}\,e-2d\,c^{2}x-2e}{cex+cd}\right)\right)}{e\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,c^{2}xd\sqrt{\frac{c^{2}d^{2}-e^{2}}{e^{2}}}}\right)}{c}$

input
```
int((a+b*arcsec(c*x))/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-a/(e*x+d)/e+b/c*(-c^2/(c*e*x+c*d)/e*arcsec(c*x)-1/e*(c^2*x^2-1)^(1/2)*(ar
ctan(1/(c^2*x^2-1)^(1/2))*((c^2*d^2-e^2)/e^2)^(1/2)-ln(2*((c^2*x^2-1)^(1/2
)*((c^2*d^2-e^2)/e^2)^(1/2)*e-d*c^2*x-e)/(c*e*x+c*d)))/((c^2*x^2-1)/c^2/x^
2)^(1/2)/x/d/((c^2*d^2-e^2)/e^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(98) = 196$.

Time = 0.15 (sec) , antiderivative size = 477, normalized size of antiderivative = 4.59

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^2}\,dx$$

$$= \left[-\frac{ac^2d^3 - ade^2 - \sqrt{c^2d^2-e^2}(be^2x+bde)\log\left(\frac{c^3d^2x+cde-\sqrt{c^2d^2-e^2}(c^2dx+e)+\left(c^2d^2-\sqrt{c^2d^2-e^2}cd-e^2\right)\sqrt{c^2x^2-1}}{ex+d}\right) +}{c^2d^4e - d^2e^3 + (c^2d^3}\right.$$

$$-\frac{ac^2d^3 - ade^2 - 2\sqrt{-c^2d^2+e^2}(be^2x+bde)\arctan\left(-\frac{\sqrt{-c^2d^2+e^2}\sqrt{c^2x^2-1}e-\sqrt{-c^2d^2+e^2}(cex+cd)}{c^2d^2-e^2}\right) + (bc^2d^3 -}{c^2d^4e - d^2e^3 + (c^2d^3e^2 -}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^2,x, algorithm="fricas")
```

output
```
[-(a*c^2*d^3 - a*d*e^2 - sqrt(c^2*d^2 - e^2)*(b*e^2*x + b*d*e)*log((c^3*d^
2*x + c*d*e - sqrt(c^2*d^2 - e^2)*(c^2*d*x + e) + (c^2*d^2 - sqrt(c^2*d^2
- e^2)*c*d - e^2)*sqrt(c^2*x^2 - 1))/(e*x + d)) + (b*c^2*d^3 - b*d*e^2)*ar
csec(c*x) - 2*(b*c^2*d^3 - b*d*e^2 + (b*c^2*d^2*e - b*e^3)*x)*arctan(-c*x
+ sqrt(c^2*x^2 - 1)))/(c^2*d^4*e - d^2*e^3 + (c^2*d^3*e^2 - d*e^4)*x), -(a
*c^2*d^3 - a*d*e^2 - 2*sqrt(-c^2*d^2 + e^2)*(b*e^2*x + b*d*e)*arctan(-(sqr
t(-c^2*d^2 + e^2)*sqrt(c^2*x^2 - 1)*e - sqrt(-c^2*d^2 + e^2)*(c*e*x + c*d)
)/(c^2*d^2 - e^2)) + (b*c^2*d^3 - b*d*e^2)*arcsec(c*x) - 2*(b*c^2*d^3 - b*
d*e^2 + (b*c^2*d^2*e - b*e^3)*x)*arctan(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^
4*e - d^2*e^3 + (c^2*d^3*e^2 - d*e^4)*x)]
```

Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^2}\,dx = \int \frac{a+b\operatorname{asec}(cx)}{(d+ex)^2}\,dx$$

input
```
integrate((a+b*asec(c*x))/(e*x+d)**2,x)
```

output
```
Integral((a + b*asec(c*x))/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{a + b\sec^{-1}(cx)}{(d + ex)^2}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx) + a}{(ex + d)^2}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^2,x, algorithm="maxima")
```

output
```
((c^2*e^2*x + c^2*d*e)*integrate(x*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1))
/(c^2*e^2*x^3 + c^2*d*e*x^2 - e^2*x - d*e + (c^2*e^2*x^3 + c^2*d*e*x^2 - e
^2*x - d*e)*e^(log(c*x + 1) + log(c*x - 1))), x) - arctan(sqrt(c*x + 1)*sq
rt(c*x - 1))*b/(e^2*x + d*e) - a/(e^2*x + d*e)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\sec^{-1}(cx)}{(d + ex)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\sec^{-1}(cx)}{(d + ex)^2}\,dx = \int \frac{a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{(d + e\,x)^2}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(d + e*x)^2,x)
```

output
```
int((a + b*acos(1/(c*x)))/(d + e*x)^2, x)
```

Reduce [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^2}\,dx = \frac{\left(\int \frac{asec(cx)}{e^2x^2+2dex+d^2}dx\right)b\,d^2+\left(\int \frac{asec(cx)}{e^2x^2+2dex+d^2}dx\right)bdex+ax}{d\,(ex+d)}$$

input
```
int((a+b*asec(c*x))/(e*x+d)^2,x)
```

output
```
(int(asec(c*x)/(d**2 + 2*d*e*x + e**2*x**2),x)*b*d**2 + int(asec(c*x)/(d**
2 + 2*d*e*x + e**2*x**2),x)*b*d*e*x + a*x)/(d*(d + e*x))
```

3.62 $\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^3}\,dx$

Optimal result 513
Mathematica [A] (verified) 514
Rubi [A] (verified) 514
Maple [B] (verified) 518
Fricas [B] (verification not implemented) 519
Sympy [F] 520
Maxima [F] 520
Giac [F(-2)] 520
Mupad [F(-1)] 521
Reduce [F] 521

Optimal result

Integrand size = 16, antiderivative size = 172

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^3}\,dx = \frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d\left(c^2d^2-e^2\right)\left(e+\frac{d}{x}\right)} - \frac{b\csc^{-1}(cx)}{2d^2e} - \frac{a+b\sec^{-1}(cx)}{2e(d+ex)^2} - \frac{b(2c^2d^2-e^2)\,\text{arctanh}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2d^2\left(c^2d^2-e^2\right)^{3/2}}$$

output
```
1/2*b*c*e*(1-1/c^2/x^2)^(1/2)/d/(c^2*d^2-e^2)/(e+d/x)-1/2*b*arccsc(c*x)/d^
2/e-1/2*(a+b*arcsec(c*x))/e/(e*x+d)^2-1/2*b*(2*c^2*d^2-e^2)*arctanh((c^2*d
+e/x)/c/(c^2*d^2-e^2)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^2/(c^2*d^2-e^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.44

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^3}\,dx = \frac{1}{2}\left(-\frac{a}{e(d+ex)^2}+\frac{bce\sqrt{1-\frac{1}{c^2x^2}}x}{d(c^2d^2-e^2)(d+ex)}-\frac{b\sec^{-1}(cx)}{e(d+ex)^2}-\frac{b\arcsin\left(\frac{1}{cx}\right)}{d^2e}+\frac{b(-2c^2d^2+e^2)\log(d+ex)}{d^2(cd-e)(cd+e)\sqrt{c^2d^2-e^2}}+\frac{b(2c^2d^2-e^2)\log\left(e+c\left(cd-\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{d^2(cd-e)(cd+e)\sqrt{c^2d^2-e^2}}\right)$$

input
```
Integrate[(a + b*ArcSec[c*x])/(d + e*x)^3,x]
```

output
```
(-(a/(e*(d + e*x)^2)) + (b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x)/(d*(c^2*d^2 - e^2)
*(d + e*x)) - (b*ArcSec[c*x])/(e*(d + e*x)^2) - (b*ArcSin[1/(c*x)])/(d^2*e
) + (b*(-2*c^2*d^2 + e^2)*Log[d + e*x])/(d^2*(c*d - e)*(c*d + e)*Sqrt[c^2*
d^2 - e^2]) + (b*(2*c^2*d^2 - e^2)*Log[e + c*(c*d - Sqrt[c^2*d^2 - e^2]*Sq
rt[1 - 1/(c^2*x^2)])*x])/(d^2*(c*d - e)*(c*d + e)*Sqrt[c^2*d^2 - e^2]))/2
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5749, 1892, 1803, 603, 719, 223, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^3}dx$$

$$\Big\downarrow \text{5749}$$

$$\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2(d+ex)^2}dx}{2ce}-\frac{a+b\sec^{-1}(cx)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{1892}$$

$$\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)^2x^4}dx}{2ce}-\frac{a+b\sec^{-1}(cx)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{1803}$$

$$-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)^2x^2}d\frac{1}{x}}{2ce}-\frac{a+b\sec^{-1}(cx)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{603}$$

$$-\frac{b\left(-\frac{\int \frac{e-\frac{d-\frac{e^2}{c^2d}}{x}}{\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}}{d^2-\frac{e^2}{c^2}}-\frac{e^2\sqrt{1-\frac{1}{c^2x^2}}}{d\left(d^2-\frac{e^2}{c^2}\right)\left(\frac{d}{x}+e\right)}\right)}{2ce}-\frac{a+b\sec^{-1}(cx)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{719}$$

$$-\frac{b\left(-\frac{e\left(2-\frac{e^2}{c^2d^2}\right)\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}-\left(1-\frac{e^2}{c^2d^2}\right)\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{d^2-\frac{e^2}{c^2}}-\frac{e^2\sqrt{1-\frac{1}{c^2x^2}}}{d\left(d^2-\frac{e^2}{c^2}\right)\left(\frac{d}{x}+e\right)}\right)}{2ce}-\frac{a+b\sec^{-1}(cx)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{223}$$

$$-\frac{b\left(-\frac{e\left(2-\frac{e^2}{c^2d^2}\right)\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}-c\arcsin\left(\frac{1}{cx}\right)\left(1-\frac{e^2}{c^2d^2}\right)}{d^2-\frac{e^2}{c^2}}-\frac{e^2\sqrt{1-\frac{1}{c^2x^2}}}{d\left(d^2-\frac{e^2}{c^2}\right)\left(\frac{d}{x}+e\right)}\right)}{2ce}-\frac{a+b\sec^{-1}(cx)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{488}$$

$$-\frac{b\left(-\frac{-e\left(2-\frac{e^2}{c^2d^2}\right)\int \frac{1}{d^2-\frac{e^2}{c^2}-\frac{1}{x^2}}d\frac{d+\frac{e}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}-c\arcsin\left(\frac{1}{cx}\right)\left(1-\frac{e^2}{c^2d^2}\right)}{d^2-\frac{e^2}{c^2}}-\frac{e^2\sqrt{1-\frac{1}{c^2x^2}}}{d\left(d^2-\frac{e^2}{c^2}\right)\left(\frac{d}{x}+e\right)}\right)}{2ce}-\frac{a+b\sec^{-1}(cx)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a+b\sec^{-1}(cx)}{2e(d+ex)^2}-\frac{b\left(-\frac{-c\arcsin\left(\frac{1}{cx}\right)\left(1-\frac{e^2}{c^2d^2}\right)-\frac{ce\left(2-\frac{e^2}{c^2d^2}\right)\text{arctanh}\left(\frac{c\left(\frac{e}{c^2x}+d\right)}{\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d^2-e^2}}\right)}{\sqrt{c^2d^2-e^2}}}{d^2-\frac{e^2}{c^2}}-\frac{e^2\sqrt{1-\frac{1}{c^2x^2}}}{d\left(d^2-\frac{e^2}{c^2}\right)\left(\frac{d}{x}+e\right)}\right)}{2ce}$$

input
```
Int[(a + b*ArcSec[c*x])/(d + e*x)^3,x]
```

output
```
-1/2*(a + b*ArcSec[c*x])/(e*(d + e*x)^2) - (b*(-((e^2*Sqrt[1 - 1/(c^2*x^2)
])/(d*(d^2 - e^2/c^2)*(e + d/x))) - (-(c*(1 - e^2/(c^2*d^2))*ArcSin[1/(c*x
)]) - (c*e*(2 - e^2/(c^2*d^2))*ArcTanh[(c*(d + e/(c^2*x)))/(Sqrt[c^2*d^2 -
 e^2]*Sqrt[1 - 1/(c^2*x^2)])])/Sqrt[c^2*d^2 - e^2])/(d^2 - e^2/c^2)))/(2*c
*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 603
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[x^m, c + d*x, x], R = PolynomialRemainde
r[x^m, c + d*x, x]}, Simp[d*R*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n +
1)*(b*c^2 + a*d^2))), x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)
^(n + 1)*(a + b*x^2)^p*ExpandToSum[(n + 1)*(b*c^2 + a*d^2)*Qx + b*c*R*(n +
1) - b*d*R*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IGt
Q[m, 1] && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1892
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

rule 5749
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(156) = 312$.

Time = 1.89 (sec) , antiderivative size = 574, normalized size of antiderivative = 3.34

method	result
parts	$-\frac{a}{2(ex+d)^2 e}+\frac{b\left(-\frac{c^3\,\mathrm{arcsec}(cx)}{2(cex+cd)^2 e}+\frac{\sqrt{c^2x^2-1}\left(-\sqrt{\frac{c^2d^2-e^2}{e^2}}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)c^3d^3-\sqrt{\frac{c^2d^2-e^2}{e^2}}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)c^3 \cdots\right.}{\cdots}\right)}{\cdots}$
derivativedivides	$\frac{-\frac{a\,c^3}{2(cex+cd)^2 e}+b\,c^3\left(-\frac{\mathrm{arcsec}(cx)}{2(cex+cd)^2 e}+\frac{\sqrt{c^2x^2-1}\left(-\sqrt{\frac{c^2d^2-e^2}{e^2}}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)c^3d^3-\sqrt{\frac{c^2d^2-e^2}{e^2}}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)c^3 \cdots\right.}{\cdots}\right)}{\cdots}$
default	$\frac{-\frac{a\,c^3}{2(cex+cd)^2 e}+b\,c^3\left(-\frac{\mathrm{arcsec}(cx)}{2(cex+cd)^2 e}+\frac{\sqrt{c^2x^2-1}\left(-\sqrt{\frac{c^2d^2-e^2}{e^2}}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)c^3d^3-\sqrt{\frac{c^2d^2-e^2}{e^2}}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)c^3 \cdots\right.}{\cdots}\right)}{\cdots}$

input
```
int((a+b*arcsec(c*x))/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/(e*x+d)^2/e+b/c*(-1/2*c^3/(c*e*x+c*d)^2/e*arcsec(c*x)+1/2/e*(c^2*x^
2-1)^(1/2)*(-((c^2*d^2-e^2)/e^2)^(1/2)*arctan(1/(c^2*x^2-1)^(1/2))*c^3*d^3
-((c^2*d^2-e^2)/e^2)^(1/2)*arctan(1/(c^2*x^2-1)^(1/2))*c^3*d^2*e*x+2*ln(2*
((c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e-d*c^2*x-e)/(c*e*x+c*d))*c^3
*d^3+2*ln(2*((c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e-d*c^2*x-e)/(c*e
*x+c*d))*c^3*d^2*e*x+(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*c*d*e^2+(
(c^2*d^2-e^2)/e^2)^(1/2)*arctan(1/(c^2*x^2-1)^(1/2))*c*d*e^2+((c^2*d^2-e^2
)/e^2)^(1/2)*arctan(1/(c^2*x^2-1)^(1/2))*e^3*c*x-ln(2*((c^2*x^2-1)^(1/2)*(
(c^2*d^2-e^2)/e^2)^(1/2)*e-d*c^2*x-e)/(c*e*x+c*d))*c*d*e^2-ln(2*((c^2*x^2-
1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e-d*c^2*x-e)/(c*e*x+c*d))*e^3*c*x)/((c^
2*x^2-1)/c^2/x^2)^(1/2)/x/d^2/((c^2*d^2-e^2)/e^2)^(1/2)/(c^2*d^2-e^2)/(c*e
*x+c*d))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(156) = 312$.

Time = 0.45 (sec) , antiderivative size = 1117, normalized size of antiderivative = 6.49

$$\int \frac{a + b\sec^{-1}(cx)}{(d + ex)^3} dx = \text{Too large to display}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^3,x, algorithm="fricas")
```

output
```
[-1/2*(a*c^4*d^6 - b*c^3*d^5*e - 2*a*c^2*d^4*e^2 + b*c*d^3*e^3 + a*d^2*e^4
 - (b*c^3*d^3*e^3 - b*c*d*e^5)*x^2 - (2*b*c^2*d^4*e - b*d^2*e^3 + (2*b*c^2
*d^2*e^3 - b*e^5)*x^2 + 2*(2*b*c^2*d^3*e^2 - b*d*e^4)*x)*sqrt(c^2*d^2 - e^
2)*log((c^3*d^2*x + c*d*e - sqrt(c^2*d^2 - e^2)*(c^2*d*x + e) + (c^2*d^2 -
 sqrt(c^2*d^2 - e^2)*c*d - e^2)*sqrt(c^2*x^2 - 1))/(e*x + d)) - 2*(b*c^3*d
^4*e^2 - b*c*d^2*e^4)*x + (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4)*arcsec
(c*x) - 2*(b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2 - 2*b*
c^2*d^2*e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e - 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*
arctan(-c*x + sqrt(c^2*x^2 - 1)) - (b*c^2*d^4*e^2 - b*d^2*e^4 + (b*c^2*d^3
*e^3 - b*d*e^5)*x)*sqrt(c^2*x^2 - 1))/(c^4*d^8*e - 2*c^2*d^6*e^3 + d^4*e^5
 + (c^4*d^6*e^3 - 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 - 2*c^2*d^
5*e^4 + d^3*e^6)*x), -1/2*(a*c^4*d^6 - b*c^3*d^5*e - 2*a*c^2*d^4*e^2 + b*c
*d^3*e^3 + a*d^2*e^4 - (b*c^3*d^3*e^3 - b*c*d*e^5)*x^2 - 2*(2*b*c^2*d^4*e
- b*d^2*e^3 + (2*b*c^2*d^2*e^3 - b*e^5)*x^2 + 2*(2*b*c^2*d^3*e^2 - b*d*e^4
)*x)*sqrt(-c^2*d^2 + e^2)*arctan(-(sqrt(-c^2*d^2 + e^2)*sqrt(c^2*x^2 - 1)*
e - sqrt(-c^2*d^2 + e^2)*(c*e*x + c*d))/(c^2*d^2 - e^2)) - 2*(b*c^3*d^4*e^
2 - b*c*d^2*e^4)*x + (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4)*arcsec(c*x)
 - 2*(b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2 - 2*b*c^2*d
^2*e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e - 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*arcta
n(-c*x + sqrt(c^2*x^2 - 1)) - (b*c^2*d^4*e^2 - b*d^2*e^4 + (b*c^2*d^3*e...
```

Sympy [F]

$$\int \frac{a + b\sec^{-1}(cx)}{(d + ex)^3}\,dx = \int \frac{a + b\,\mathrm{asec}\,(cx)}{(d + ex)^3}\,dx$$

input
```
integrate((a+b*asec(c*x))/(e*x+d)**3,x)
```

output
```
Integral((a + b*asec(c*x))/(d + e*x)**3, x)
```

Maxima [F]

$$\int \frac{a + b\sec^{-1}(cx)}{(d + ex)^3}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx) + a}{(ex + d)^3}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(c^2*e^3*x^2 + 2*c^2*d*e^2*x + c^2*d^2*e)*integrate(1/2*x*e^(1/2*lo
g(c*x + 1) + 1/2*log(c*x - 1))/(c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 - 2*d*e^2*x
- d^2*e + (c^2*d^2*e - e^3)*x^2 + (c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 - 2*d*e^2
*x - d^2*e + (c^2*d^2*e - e^3)*x^2)*e^(log(c*x + 1) + log(c*x - 1))), x) -
 arctan(sqrt(c*x + 1)*sqrt(c*x - 1)))*b/(e^3*x^2 + 2*d*e^2*x + d^2*e) - 1/
2*a/(e^3*x^2 + 2*d*e^2*x + d^2*e)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\sec^{-1}(cx)}{(d + ex)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\sec^{-1}(cx)}{(d + ex)^3}\, dx = \int \frac{a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{(d + e\,x)^3}\, dx$$

input
```
int((a + b*acos(1/(c*x)))/(d + e*x)^3,x)
```

output
```
int((a + b*acos(1/(c*x)))/(d + e*x)^3, x)
```

Reduce [F]

$$\int \frac{a + b\sec^{-1}(cx)}{(d + ex)^3}\, dx = \frac{2\left(\int \frac{asec(cx)}{e^3x^3+3d\,e^2x^2+3d^2ex+d^3}dx\right) b\,d^2e + 4\left(\int \frac{asec(cx)}{e^3x^3+3d\,e^2x^2+3d^2ex+d^3}dx\right) bd\,e^2x + 2\left(\int \frac{asec(cx)}{e^3x^3+3d\,e^2x^2+3d^2ex+d^3}dx\right) b\,e}{2e\,(e^2x^2 + 2dex + d^2)}$$

input
```
int((a+b*asec(c*x))/(e*x+d)^3,x)
```

output
```
(2*int(asec(c*x)/(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3),x)*b*d**2
*e + 4*int(asec(c*x)/(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3),x)*b*
d*e**2*x + 2*int(asec(c*x)/(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3)
,x)*b*e**3*x**2 - a)/(2*e*(d**2 + 2*d*e*x + e**2*x**2))
```

3.63 $\int (d+ex)^{3/2} \left(a+b\sec^{-1}(cx)\right)\,dx$

Optimal result . . . 522
Mathematica [C] (verified) . . . 523
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Fricas [F(-1)] . . . 532
Sympy [F(-1)] . . . 532
Maxima [F(-2)] . . . 532
Giac [F] . . . 533
Mupad [F(-1)] . . . 533
Reduce [F] . . . 533

Optimal result

Integrand size = 18, antiderivative size = 372

$$\int (d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{4be\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}x}+\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}+\frac{28bd\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}+\frac{4b(2c^2d^2+e^2)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^4\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}+\frac{4bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5ce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

output
```
4/15*b*e*(e*x+d)^(1/2)*(-c^2*x^2+1)/c^3/(1-1/c^2/x^2)^(1/2)/x+2/5*(e*x+d)^
(5/2)*(a+b*arcsec(c*x))/e+28/15*b*d*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)*Ellip
ticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))/c^2/(1-1/c^2/x^
2)^(1/2)/x/(c*(e*x+d)/(c*d+e))^(1/2)+4/15*b*(2*c^2*d^2+e^2)*(c*(e*x+d)/(c*
d+e))^(1/2)*(-c^2*x^2+1)^(1/2)*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2
)*(e/(c*d+e))^(1/2))/c^4/(1-1/c^2/x^2)^(1/2)/x/(e*x+d)^(1/2)+4/5*b*d^3*(c*
(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)*EllipticPi(1/2*(-c*x+1)^(1/2)*2^
(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))/c/e/(1-1/c^2/x^2)^(1/2)/x/(e*x+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.20 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.90

$$\int (d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{1}{15}\left(-\frac{4be\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}{c}\right.$$
$$+\frac{6a(d+ex)^{5/2}}{e}+\frac{6b(d+ex)^{5/2}\sec^{-1}(cx)}{e}$$
$$+\frac{4ib\sqrt{\frac{e(1+cx)}{-cd+e}}\sqrt{\frac{e-cex}{cd+e}}\left(-7cd(cd-e)E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)|\frac{cd+e}{cd-e}\right)+\left(9c^2d^2-7cde+e^2\right)\text{EllipticF}}{c^3e\sqrt{-\frac{c}{cd+e}}\sqrt{}}$$

input
```
Integrate[(d + e*x)^(3/2)*(a + b*ArcSec[c*x]),x]
```

output
```
((-4*b*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])/c + (6*a*(d + e*x)^(5/2))/
e + (6*b*(d + e*x)^(5/2)*ArcSec[c*x])/e + ((4*I)*b*Sqrt[(e*(1 + c*x))/(-(c
*d) + e)]*Sqrt[(e - c*e*x)/(c*d + e)]*(-7*c*d*(c*d - e)*EllipticE[I*ArcSin
h[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] + (9*c^2*d^2 -
 7*c*d*e + e^2)*EllipticF[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (
c*d + e)/(c*d - e)] - 3*c^2*d^2*EllipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-(c
/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)]))/(c^3*e*Sqrt[-(c/(c*d +
 e))]*Sqrt[1 - 1/(c^2*x^2)]*x))/15
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.06, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.944$, Rules used = {5749, 1898, 634, 633, 632, 186, 413, 412, 2185, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \ 5749$$

$$\frac{2(d+ex)^{5/2} \left(a+b\sec^{-1}(cx)\right)}{5e}-\frac{2b\int \frac{(d+ex)^{5/2}}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{5ce}$$

$$\Big\downarrow \ 1898$$

$$\frac{2(d+ex)^{5/2} \left(a+b\sec^{-1}(cx)\right)}{5e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\int \frac{(d+ex)^{5/2}}{x\sqrt{x^2-\frac{1}{c^2}}}dx}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\Big\downarrow \ 634$$

$$\frac{2(d+ex)^{5/2} \left(a+b\sec^{-1}(cx)\right)}{5e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(d^3\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx-\int \frac{-x^2e^3-3dxe^2-3d^2e}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\Big\downarrow \ 633$$

$$\frac{2(d+ex)^{5/2} \left(a+b\sec^{-1}(cx)\right)}{5e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{d^3\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{\sqrt{x^2-\frac{1}{c^2}}}-\int \frac{-x^2e^3-3dxe^2-3d^2e}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\Big\downarrow \ 632$$

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{d^3\sqrt{1-c^2x^2}\int\frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx}{\sqrt{x^2-\frac{1}{c^2}}}-\int\frac{-x^2e^3-3dxe^2-3d^2e}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$\big\downarrow$ 186

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2d^3\sqrt{1-c^2x^2}\int\frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}}{\sqrt{x^2-\frac{1}{c^2}}}-\int\frac{-x^2e^3-3dxe^2-3d^2e}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$\big\downarrow$ 413

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\int\frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\int\frac{-x^2e^3-3dxe^2-3d^2e}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$\big\downarrow$ 412

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\int\frac{-x^2e^3-3dxe^2-3d^2e}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$\big\downarrow$ 2185

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2\int-\frac{e^3\left(9d^2c^2+7dexc^2+e^2\right)}{2c^2\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{3e^2}-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}+\frac{2}{3}e^2\sqrt{x^2-\frac{1}{c^2}}\sqrt{d}\cdots\right.}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$\big\downarrow$ 27

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{e\int\frac{9d^2c^2+7dexc^2+e^2}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{3c^2}-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}+\frac{2}{3}e^2\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$\big\downarrow$ 600

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\Bigg(\frac{e\left(\left(2c^2d^2+e^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx+7c^2d\int\frac{\sqrt{d+ex}}{\sqrt{x^2-\frac{1}{c^2}}}dx\right)}{3c^2}-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$\big\downarrow$ 509

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\Bigg(\frac{e\left(\left(2c^2d^2+e^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx+\frac{7c^2d\sqrt{1-c^2x^2}\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx}{\sqrt{x^2-\frac{1}{c^2}}}\right)}{3c^2}-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\Big(2,\arcsin\Big(\frac{\sqrt{1-cx}}{\sqrt{2}}}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$\big\downarrow$ 508

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\Bigg(\frac{e\left(\left(2c^2d^2+e^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{14cd\sqrt{1-c^2x^2}\sqrt{d+ex}\int\frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3c^2}-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\Big(2}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}}}}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$\big\downarrow$ 327

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{e\left(\left(2c^2d^2+e^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{14cd\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3c^2}-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{EllipticPi}}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-c}{c}}}\right.}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$\Big\downarrow$ 512

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{e\left(\frac{\sqrt{1-c^2x^2}\left(2c^2d^2+e^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{\sqrt{x^2-\frac{1}{c^2}}}-\frac{14cd\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3c^2}-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{E}}{\sqrt{x^2-\frac{1}{c^2}}}\right.}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$\Big\downarrow$ 511

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{e\left(-\frac{2\sqrt{1-c^2x^2}\left(2c^2d^2+e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}\int\frac{1}{\sqrt{1-\frac{e(1-cx)}{cd+e}}\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}-\frac{14cd\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3c^2}-\right.}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$\Big\downarrow$ 321

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}+\frac{e\left(-\frac{2\sqrt{1-c^2x^2}\left(2c^2d^2+e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-}}{\sqrt{}}\right.\right.}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}\right.}{}\right.}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

input
```
Int[(d + e*x)^(3/2)*(a + b*ArcSec[c*x]),x]
```

output
```
(2*(d + e*x)^(5/2)*(a + b*ArcSec[c*x]))/(5*e) - (2*b*Sqrt[-c^(-2) + x^2]*(
(2*e^2*Sqrt[d + e*x]*Sqrt[-c^(-2) + x^2])/3 + (e*((-14*c*d*Sqrt[d + e*x]*S
qrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]
)/(Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[-c^(-2) + x^2]) - (2*(2*c^2*d^2 + e^
2)*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1
 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[d + e*x]*Sqrt[-c^(-2) + x^2]))
)/(3*c^2) - (2*d^3*Sqrt[1 - c^2*x^2]*Sqrt[1 - (e*(1 - c*x))/(c*d + e)]*Ell
ipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[-c^(-2)
+ x^2]*Sqrt[d + e/c - (e*(1 - c*x))/c])))/(5*c*e*Sqrt[1 - 1/(c^2*x^2)]*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 634
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] - Int[(
1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]))*ExpandToSum[(c^(n + 1/2) - (c + d*x)^(n
+ 1/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n - 1/2, 0]
```

rule 1898
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p])   Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n
))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !I
ntegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

rule 5749
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 797 vs. $2(335) = 670$.

Time = 11.15 (sec) , antiderivative size = 798, normalized size of antiderivative = 2.15

method	result
derivativedivides	$\frac{\frac{2a(ex+d)^{\frac{5}{2}}}{5}+2b\left(\frac{(ex+d)^{\frac{5}{2}}\operatorname{arcsec}(cx)}{5}-\frac{2\left(\sqrt{\frac{c}{cd-e}}\,c^2(ex+d)^{\frac{5}{2}}-2\sqrt{\frac{c}{cd-e}}\,c^2d(ex+d)^{\frac{3}{2}}+9d^2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\,\sqrt{\frac{-c(ex+d)+cd}{cd+e}}\right.}{}\right.}{}$
default	$\frac{\frac{2a(ex+d)^{\frac{5}{2}}}{5}+2b\left(\frac{(ex+d)^{\frac{5}{2}}\operatorname{arcsec}(cx)}{5}-\frac{2\left(\sqrt{\frac{c}{cd-e}}\,c^2(ex+d)^{\frac{5}{2}}-2\sqrt{\frac{c}{cd-e}}\,c^2d(ex+d)^{\frac{3}{2}}+9d^2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\,\sqrt{\frac{-c(ex+d)+cd}{cd+e}}\right.}{}\right.}{}$
parts	$\frac{2a(ex+d)^{\frac{5}{2}}}{5e}+\frac{2b\left(\frac{(ex+d)^{\frac{5}{2}}\operatorname{arcsec}(cx)}{5}-\frac{2\left(\sqrt{\frac{c}{cd-e}}\,c^2(ex+d)^{\frac{5}{2}}-2\sqrt{\frac{c}{cd-e}}\,c^2d(ex+d)^{\frac{3}{2}}+9d^2\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\,\sqrt{-\frac{c(ex+d)}{cd+}}\right.}{}\right.}{}$

input
```
int((e*x+d)^(3/2)*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
2/e*(1/5*a*(e*x+d)^(5/2)+b*(1/5*(e*x+d)^(5/2)*arcsec(c*x)-2/15/c^3*((c/(c*
d-e))^(1/2)*c^2*(e*x+d)^(5/2)-2*(c/(c*d-e))^(1/2)*c^2*d*(e*x+d)^(3/2)+9*d^
2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*El
lipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2-7*((-
c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*Ellipti
cE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2-3*d^2*
((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*Elli
pticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/
(c*d-e))^(1/2))*c^2+(c/(c*d-e))^(1/2)*c^2*d^2*(e*x+d)^(1/2)+7*((-c*(e*x+d)
+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d
)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e-7*((-c*(e*x+d)+c*
d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(
1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e+((-c*(e*x+d)+c*d-e)/
(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*
(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^2-(c/(c*d-e))^(1/2)*e^2*(e*x+
d)^(1/2))/(c/(c*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)
/c^2/e^2/x^2)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int (d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)*(a+b*asec(c*x)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

Giac [F]

$$\int (d+ex)^{3/2} \left(a+b\sec^{-1}(cx)\right)\,dx = \int (ex+d)^{\frac{3}{2}}(b\,\mathrm{arcsec}\,(cx)+a)\,dx$$

input
```
integrate((e*x+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x + d)^(3/2)*(b*arcsec(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^{3/2} \left(a+b\sec^{-1}(cx)\right)\,dx = \int \left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)(d+e\,x)^{3/2}\,dx$$

input
```
int((a + b*acos(1/(c*x)))*(d + e*x)^(3/2),x)
```

output
```
int((a + b*acos(1/(c*x)))*(d + e*x)^(3/2), x)
```

Reduce [F]

$$\int (d+ex)^{3/2} \left(a+b\sec^{-1}(cx)\right)\,dx = \frac{2\sqrt{ex+d}\,a\,d^2+4\sqrt{ex+d}\,adex+2\sqrt{ex+d}\,a\,e^2x^2+5\left(\int \sqrt{ex+d}\,asec(cx)\,xdx\right)b\,e^2}{5e}$$

input
```
int((e*x+d)^(3/2)*(a+b*asec(c*x)),x)
```

output
```
(2*sqrt(d + e*x)*a*d**2 + 4*sqrt(d + e*x)*a*d*e*x + 2*sqrt(d + e*x)*a*e**2
*x**2 + 5*int(sqrt(d + e*x)*asec(c*x)*x,x)*b*e**2 + 5*int(sqrt(d + e*x)*as
ec(c*x),x)*b*d*e)/(5*e)
```

3.64 $\int \sqrt{d+ex}(a+b\sec^{-1}(cx))\,dx$

Optimal result 534
Mathematica [C] (verified) 535
Rubi [A] (verified) 535
Maple [A] (verified) 541
Fricas [F] 542
Sympy [F] 543
Maxima [F(-2)] 543
Giac [F] 543
Mupad [F(-1)] 544
Reduce [F] 544

Optimal result

Integrand size = 18, antiderivative size = 315

$$\int \sqrt{d+ex}(a+b\sec^{-1}(cx))\,dx = \frac{2(d+ex)^{3/2}(a+b\sec^{-1}(cx))}{3e} + \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{4bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{4bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3ce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

output

```
2/3*(e*x+d)^(3/2)*(a+b*arcsec(c*x))/e+4/3*b*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/
2)*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))/c^2/(1-
1/c^2/x^2)^(1/2)/x/(c*(e*x+d)/(c*d+e))^(1/2)+4/3*b*d*(c*(e*x+d)/(c*d+e))^(
1/2)*(-c^2*x^2+1)^(1/2)*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c
*d+e))^(1/2))/c^2/(1-1/c^2/x^2)^(1/2)/x/(e*x+d)^(1/2)+4/3*b*d^2*(c*(e*x+d)
/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2
,2^(1/2)*(e/(c*d+e))^(1/2))/c/e/(1-1/c^2/x^2)^(1/2)/x/(e*x+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.67 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.88

$$\int \sqrt{d+ex}(a+b\sec^{-1}(cx))\,dx$$
$$= \frac{2\left(a(d+ex)^{3/2}+b(d+ex)^{3/2}\sec^{-1}(cx)+\frac{2ib\sqrt{\frac{e(1+cx)}{-cd+e}}\sqrt{\frac{e-cex}{cd+e}}\left((-cd+e)E\left(i\text{arcsinh}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)|\frac{cd+e}{cd-e}\right)+(2cd-e)\right.}{}\right.}{3e}$$

input
```
Integrate[Sqrt[d + e*x]*(a + b*ArcSec[c*x]),x]
```

output
```
(2*(a*(d + e*x)^(3/2) + b*(d + e*x)^(3/2)*ArcSec[c*x] + ((2*I)*b*Sqrt[(e*(
1 + c*x))/(-(c*d) + e)]*Sqrt[(e - c*e*x)/(c*d + e)]*((-(c*d) + e)*Elliptic
E[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] + (2
*c*d - e)*EllipticF[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d +
e)/(c*d - e)] - c*d*EllipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-(c/(c*d + e))]
*Sqrt[d + e*x]], (c*d + e)/(c*d - e)]))/(c^2*Sqrt[-(c/(c*d + e))]*Sqrt[1 -
 1/(c^2*x^2)]*x)))/(3*e)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.11, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5749, 1898, 634, 600, 509, 508, 327, 512, 511, 321, 633, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex}(a+b\sec^{-1}(cx))\,dx$$

$$\Big\downarrow \text{5749}$$

$$\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{2b\int \frac{(d+ex)^{3/2}}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{3ce}$$

$$\Big\downarrow \text{ 1898}$$

$$\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\int\frac{(d+ex)^{3/2}}{x\sqrt{x^2-\frac{1}{c^2}}}dx}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\Big\downarrow \text{ 634}$$

$$\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(d^2\int\frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx-\int\frac{-xe^2-2de}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\Big\downarrow \text{ 600}$$

$$\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(d^2\int\frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx+de\int\frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx+e\int\frac{\sqrt{d+ex}}{\sqrt{x^2-\frac{1}{c^2}}}dx\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\Big\downarrow \text{ 509}$$

$$\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(d^2\int\frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx+de\int\frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx+\frac{e\sqrt{1-c^2x^2}\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx}{\sqrt{x^2-\frac{1}{c^2}}}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\Big\downarrow \text{ 508}$$

$$\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(d^2\int\frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx+de\int\frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{2e\sqrt{1-c^2x^2}\sqrt{d+ex}\int\frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(d^2\int\frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx+de\int\frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{2e\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$\downarrow$ 512

$$\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(d^2\int\frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx+\frac{de\sqrt{1-c^2x^2}\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{\sqrt{x^2-\frac{1}{c^2}}}-\frac{2e\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$\downarrow$ 511

$$\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(d^2\int\frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{2de\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\int\frac{1}{\sqrt{1-\frac{e(1-cx)}{cd+e}}\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}-\frac{2e\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\cdots\right.\right.}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+\cdots}{cd+\cdots}}}\cdots\right.}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$\downarrow$ 321

$$\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(d^2\int\frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{2de\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}-\frac{2e\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\cdots\right.\right.}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex\cdots}{cd+e}}}\cdots\right.}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$\downarrow$ 633

$$\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{d^2\sqrt{1-c^2x^2}\int\frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{\sqrt{x^2-\frac{1}{c^2}}}-\frac{2de\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}-\frac{2e\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\text{arc}\cdots\right.}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\cdots}}\cdots\right.}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$\downarrow$ 632

$$\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{d^2\sqrt{1-c^2x^2}\int\frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx}{\sqrt{x^2-\frac{1}{c^2}}}-\frac{2de\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}-\frac{2e\sqrt{1-c^2x^2}\sqrt{d+ex}E(}{c\sqrt{x^2-\frac{1}{c^2}}}\right.}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$\downarrow$ 186

$$\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2d^2\sqrt{1-c^2x^2}\int\frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}}{\sqrt{x^2-\frac{1}{c^2}}}-\frac{2de\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}-\frac{2e\sqrt{1-c}}{}\right.}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$\downarrow$ 413

$$\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2d^2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\int\frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{2de\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}-\right.}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$\downarrow$ 412

$$\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2d^2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{2de\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right.}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}\right.}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$

input
```
Int[Sqrt[d + e*x]*(a + b*ArcSec[c*x]),x]
```

output
```
(2*(d + e*x)^(3/2)*(a + b*ArcSec[c*x]))/(3*e) - (2*b*Sqrt[-c^(-2) + x^2]*(
(-2*e*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[
2]], (2*e)/(c*d + e)])/(c*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[-c^(-2) + x^2
]) - (2*d*e*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcS
in[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[d + e*x]*Sqrt[-c^(-2)
 + x^2]) - (2*d^2*Sqrt[1 - c^2*x^2]*Sqrt[1 - (e*(1 - c*x))/(c*d + e)]*Elli
pticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[-c^(-2) +
 x^2]*Sqrt[d + e/c - (e*(1 - c*x))/c])))/(3*c*e*Sqrt[1 - 1/(c^2*x^2)]*x)
```

Defintions of rubi rules used

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 634
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] - Int[(
1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]))*ExpandToSum[(c^(n + 1/2) - (c + d*x)^(n
+ 1/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n - 1/2, 0]
```

rule 1898
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p])   Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n
))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !I
ntegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

rule 5749
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 9.15 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{\frac{2a(ex+d)^{\frac{3}{2}}}{3}+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arcsec}(cx)}{3}-\frac{2\left(2d\,\mathrm{EllipticF}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c-\mathrm{EllipticE}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)cd-d\,\mathrm{El}\right.}{}\right.}{}$
default	$\frac{\frac{2a(ex+d)^{\frac{3}{2}}}{3}+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arcsec}(cx)}{3}-\frac{2\left(2d\,\mathrm{EllipticF}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c-\mathrm{EllipticE}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)cd-d\,\mathrm{El}\right.}{}\right.}{}$
parts	$\frac{2a(ex+d)^{\frac{3}{2}}}{3e}+\frac{2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arcsec}(cx)}{3}-\frac{2\left(2d\,\mathrm{EllipticF}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c-\mathrm{EllipticE}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)cd-\right.}{}\right.}{}$

input
```
int((e*x+d)^(1/2)*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
2/e*(1/3*a*(e*x+d)^(3/2)+b*(1/3*(e*x+d)^(3/2)*arcsec(c*x)-2/3/c^2*(2*d*Ell
ipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c-Elliptic
E((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d-d*EllipticP
i((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-
e))^(1/2))*c+EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(
1/2))*e-EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))
*e)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)/
(c/(c*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x
^2)^(1/2)))
```

Fricas [F]

$$\int \sqrt{d+ex}\left(a+b\sec^{-1}(cx)\right)\,dx=\int \sqrt{ex+d}(b\operatorname{arcsec}(cx)+a)\,dx$$

input
```
integrate((e*x+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arcsec(c*x) + a), x)
```

Sympy [F]

$$\int \sqrt{d+ex}\left(a+b\sec^{-1}(cx)\right)\,dx=\int \left(a+b\operatorname{asec}\left(cx\right)\right)\sqrt{d+ex}\,dx$$

input
```
integrate((e*x+d)**(1/2)*(a+b*asec(c*x)),x)
```

output
```
Integral((a + b*asec(c*x))*sqrt(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex}\left(a+b\sec^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

Giac [F]

$$\int \sqrt{d+ex}\left(a+b\sec^{-1}(cx)\right)\,dx=\int \sqrt{ex+d}(b\operatorname{arcsec}\left(cx\right)+a)\,dx$$

input
```
integrate((e*x+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)*(b*arcsec(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex}\left(a+b\sec^{-1}(cx)\right)\,dx=\int \left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d+e\,x}\,dx$$

input
```
int((a + b*acos(1/(c*x)))*(d + e*x)^(1/2),x)
```

output
```
int((a + b*acos(1/(c*x)))*(d + e*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d+ex}\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{2\sqrt{ex+d}\,ad+2\sqrt{ex+d}\,aex+3\left(\int \sqrt{ex+d}\,asec(cx)\,dx\right)be}{3e}$$

input
```
int((e*x+d)^(1/2)*(a+b*asec(c*x)),x)
```

output
```
(2*sqrt(d + e*x)*a*d + 2*sqrt(d + e*x)*a*e*x + 3*int(sqrt(d + e*x)*asec(c*
x),x)*b*e)/(3*e)
```

3.65 $\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex}}\,dx$

Optimal result 545
Mathematica [C] (verified) 546
Rubi [A] (verified) 546
Maple [A] (verified) 548
Fricas [F(-1)] 549
Sympy [F] 549
Maxima [F(-2)] 550
Giac [F] 550
Mupad [F(-1)] 550
Reduce [F] 551

Optimal result

Integrand size = 18, antiderivative size = 212

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{d+ex}(a+b\sec^{-1}(cx))}{e} + \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{4bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

output
```
2*(e*x+d)^(1/2)*(a+b*arcsec(c*x))/e+4*b*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^
2+1)^(1/2)*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))
/c^2/(1-1/c^2/x^2)^(1/2)/x/(e*x+d)^(1/2)+4*b*d*(c*(e*x+d)/(c*d+e))^(1/2)*(
-c^2*x^2+1)^(1/2)*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+
e))^(1/2))/c/e/(1-1/c^2/x^2)^(1/2)/x/(e*x+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 15.81 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex}}\,dx = \frac{2\left(a\sqrt{d+ex}+b\sqrt{d+ex}\sec^{-1}(cx)+\frac{2ib\sqrt{\frac{e(1+cx)}{-cd+e}}\sqrt{\frac{e-cex}{cd+e}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right),\frac{cd+e}{cd-e}\right)-\text{EllipticPi}\left(1+\frac{e}{cd}\right.\right.}{c\sqrt{-\frac{c}{cd+e}}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right.}{e}$$

input
```
Integrate[(a + b*ArcSec[c*x])/Sqrt[d + e*x],x]
```

output
```
(2*(a*Sqrt[d + e*x] + b*Sqrt[d + e*x]*ArcSec[c*x] + ((2*I)*b*Sqrt[(e*(1 +
c*x))/(-(c*d) + e)]*Sqrt[(e - c*e*x)/(c*d + e)]*(EllipticF[I*ArcSinh[Sqrt[
-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] - EllipticPi[1 + e/(c
*d), I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)]))
/(c*Sqrt[-(c/(c*d + e))]*Sqrt[1 - 1/(c^2*x^2)]*x)))/e
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5749, 1898, 637, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \text{ 5749}$$

$$\frac{2\sqrt{d+ex}\left(a+b\sec^{-1}(cx)\right)}{e}-\frac{2b\int \frac{\sqrt{d+ex}}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{ce}$$

$$\Big\downarrow \text{ 1898}$$

$$\frac{2\sqrt{d+ex}\left(a+b\sec^{-1}(cx)\right)}{e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\int\frac{\sqrt{d+ex}}{x\sqrt{x^2-\frac{1}{c^2}}}dx}{cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\Big\downarrow \text{637}$$

$$\frac{2\sqrt{d+ex}\left(a+b\sec^{-1}(cx)\right)}{e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\int\left(\frac{d}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}+\frac{e}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}\right)dx}{cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{d+ex}\left(a+b\sec^{-1}(cx)\right)}{e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2e\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}-\frac{2d\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}\right)}{cex\sqrt{1-\frac{1}{c^2x^2}}}$$

input
```
Int[(a + b*ArcSec[c*x])/Sqrt[d + e*x],x]
```

output
```
(2*Sqrt[d + e*x]*(a + b*ArcSec[c*x]))/e - (2*b*Sqrt[-c^(-2) + x^2]*((-2*e*
Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 -
c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[d + e*x]*Sqrt[-c^(-2) + x^2]) - (
2*d*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[S
qrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[d + e*x]*Sqrt[-c^(-2) + x^2
])))/(c*e*Sqrt[1 - 1/(c^2*x^2)]*x)
```

Defintions of rubi rules used

rule 637
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p/Sqrt[c + d*x], x^m*(c + d*x)^(n + 1
/2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[p + 1/2] && IntegerQ[n
 + 1/2] && IntegerQ[m]
```

rule 1898
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p])   Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n
))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !I
ntegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5749
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 6.21 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{2\sqrt{ex+d}\,a+2b\left(\sqrt{ex+d}\,\operatorname{arcsec}(cx)-\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\left(\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)-\operatorname{EllipticF}\right)}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,x\sqrt{\frac{c}{cd-e}}}\right)}{e}$
default	$\frac{2\sqrt{ex+d}\,a+2b\left(\sqrt{ex+d}\,\operatorname{arcsec}(cx)-\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\left(\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)-\operatorname{EllipticF}\right)}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,x\sqrt{\frac{c}{cd-e}}}\right)}{e}$
parts	$\frac{2a\sqrt{ex+d}}{e}+\frac{2b\left(\sqrt{ex+d}\,\operatorname{arcsec}(cx)-\frac{2\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\left(\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)-\operatorname{Ellipt}\right)}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,x\sqrt{\frac{c}{cd-}}}\right)}{e}$

input
```
int((a+b*arcsec(c*x))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/e*((e*x+d)^(1/2)*a+b*((e*x+d)^(1/2)*arcsec(c*x)-2/c*((-c*(e*x+d)+c*d-e)/
(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*(EllipticF((e*x+d)^(1/2)
*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))-EllipticPi((e*x+d)^(1/2)*(c/(c
*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2)))/((c^2*(e*
x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)/x/(c/(c*d-e))^(1/2)
))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{a + b\sec^{-1}(cx)}{\sqrt{d + ex}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{a + b\sec^{-1}(cx)}{\sqrt{d + ex}}\,dx = \int \frac{a + b\,\mathrm{asec}\,(cx)}{\sqrt{d + ex}}\,dx$$

input
```
integrate((a+b*asec(c*x))/(e*x+d)**(1/2),x)
```

output
```
Integral((a + b*asec(c*x))/sqrt(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

Giac [F]

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex}}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx)+a}{\sqrt{ex+d}}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/sqrt(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex}}\,dx = \int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{\sqrt{d+e\,x}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(d + e*x)^(1/2),x)
```

output
```
int((a + b*acos(1/(c*x)))/(d + e*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{ex+d}\,a+\left(\int \frac{asec(cx)}{\sqrt{ex+d}}dx\right)be}{e}$$

input
```
int((a+b*asec(c*x))/(e*x+d)^(1/2),x)
```

output
```
(2*sqrt(d + e*x)*a + int(asec(c*x)/sqrt(d + e*x),x)*b*e)/e
```

3.66 $\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{3/2}}\,dx$

Optimal result 552
Mathematica [A] (verified) 552
Rubi [A] (verified) 553
Maple [A] (verified) 556
Fricas [F(-1)] 556
Sympy [F] 557
Maxima [F(-2)] 557
Giac [F] 557
Mupad [F(-1)] 558
Reduce [F] 558

Optimal result

Integrand size = 18, antiderivative size = 119

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{3/2}}\,dx = -\frac{2(a+b\sec^{-1}(cx))}{e\sqrt{d+ex}} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

output
```
(-2*a-2*b*arcsec(c*x))/e/(e*x+d)^(1/2)-4*b*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2
*x^2+1)^(1/2)*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^
(1/2))/c/e/(1-1/c^2/x^2)^(1/2)/x/(e*x+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.04

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{3/2}}\,dx = -\frac{2\left(\left(-1+c^2x^2\right)\left(a+b\sec^{-1}(cx)\right)+2bc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right.\right.}{e\sqrt{d+ex}\left(-1+c^2x^2\right)}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(d + e*x)^(3/2),x]
```

output
```
(-2*((-1 + c^2*x^2)*(a + b*ArcSec[c*x]) + 2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sq
rt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1
- c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(e*Sqrt[d + e*x]*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5749, 1898, 633, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{5749}$$

$$\frac{2b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}dx}{ce}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

$$\Big\downarrow \text{1898}$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

$$\Big\downarrow \text{633}$$

$$\frac{2b\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

$$\Big\downarrow \text{632}$$

$$\frac{2b\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx}{cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

$$\Big\downarrow \text{186}$$

$$-\frac{4b\sqrt{1-c^2x^2}\int \frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}}{cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 413}$$

$$-\frac{4b\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\int \frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 412}$$

$$-\frac{2\left(a+b\sec^{-1}(cx)\right)}{e\sqrt{d+ex}}-\frac{4b\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}$$

input
```
Int[(a + b*ArcSec[c*x])/(d + e*x)^(3/2),x]
```

output
```
(-2*(a + b*ArcSec[c*x]))/(e*Sqrt[d + e*x]) - (4*b*Sqrt[1 - c^2*x^2]*Sqrt[1
 - (e*(1 - c*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (
2*e)/(c*d + e)])/(c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e/c - (e*(1 - c*x))
/c])
```

Defintions of rubi rules used

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 632

```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633

```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 1898

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p])   Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n
))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !I
ntegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

rule 5749

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 4.08 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.81

method	result
derivativedivides	$\frac{-\frac{2a}{\sqrt{ex+d}}+2b\left(-\frac{\operatorname{arcsec}(cx)}{\sqrt{ex+d}}-\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\operatorname{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\frac{cd-e}{cd},\frac{\sqrt{\frac{c}{cd+e}}}{\sqrt{\frac{c}{cd-e}}}\right)}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,xd\sqrt{\frac{c}{cd-e}}}\right)}{e}$
default	$\frac{-\frac{2a}{\sqrt{ex+d}}+2b\left(-\frac{\operatorname{arcsec}(cx)}{\sqrt{ex+d}}-\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\operatorname{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\frac{cd-e}{cd},\frac{\sqrt{\frac{c}{cd+e}}}{\sqrt{\frac{c}{cd-e}}}\right)}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,xd\sqrt{\frac{c}{cd-e}}}\right)}{e}$
parts	$-\frac{2a}{\sqrt{ex+d}\,e}+\frac{2b\left(-\frac{\operatorname{arcsec}(cx)}{\sqrt{ex+d}}-\frac{2\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\,\operatorname{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\frac{cd-e}{cd},\frac{\sqrt{\frac{c}{cd+e}}}{\sqrt{\frac{c}{cd-e}}}\right)}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,xd\sqrt{\frac{c}{cd-e}}}\right)}{e}$

input
```
int((a+b*arcsec(c*x))/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/e*(-a/(e*x+d)^(1/2)+b*(-1/(e*x+d)^(1/2)*arcsec(c*x)-2/c/((c^2*(e*x+d)^2-
2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)/x/d/(c/(c*d-e))^(1/2)*((-c
*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*Elliptic
Pi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d
-e))^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{3/2}}\,dx=\int \frac{a+b\operatorname{asec}(cx)}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asec(c*x))/(e*x+d)**(3/2),x)
```

output
```
Integral((a + b*asec(c*x))/(d + e*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

Giac [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{3/2}}\,dx=\int \frac{b\operatorname{arcsec}(cx)+a}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/(e*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{(d+e\,x)^{3/2}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(d + e*x)^(3/2),x)
```

output
```
int((a + b*acos(1/(c*x)))/(d + e*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{3/2}}\,dx=\frac{\sqrt{ex+d}\left(\int \frac{asec(cx)}{\sqrt{ex+d}\,d+\sqrt{ex+d}\,ex}dx\right)be-2a}{\sqrt{ex+d}\,e}$$

input
```
int((a+b*asec(c*x))/(e*x+d)^(3/2),x)
```

output
```
(sqrt(d + e*x)*int(asec(c*x)/(sqrt(d + e*x)*d + sqrt(d + e*x)*e*x),x)*b*e
- 2*a)/(sqrt(d + e*x)*e)
```

3.67 $\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{5/2}}\,dx$

Optimal result . . . 559
Mathematica [C] (verified) . . . 560
Rubi [A] (verified) . . . 560
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Fricas [F(-1)] . . . 568
Sympy [F] . . . 569
Maxima [F(-2)] . . . 569
Giac [F] . . . 569
Mupad [F(-1)] . . . 570
Reduce [F] . . . 570

Optimal result

Integrand size = 18, antiderivative size = 298

$$\begin{aligned}\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{5/2}}\,dx = & -\frac{4be(1-c^2x^2)}{3cd\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\ & -\frac{2(a+b\sec^{-1}(cx))}{3e(d+ex)^{3/2}}+\frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3d\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} \\ & -\frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\mathrm{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3cde\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}\end{aligned}$$

output

```
-4/3*b*e*(-c^2*x^2+1)/c/d/(c^2*d^2-e^2)/(1-1/c^2/x^2)^(1/2)/x/(e*x+d)^(1/2
)-2/3*(a+b*arcsec(c*x))/e/(e*x+d)^(3/2)+4/3*b*(e*x+d)^(1/2)*(-c^2*x^2+1)^(
1/2)*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))/d/(c^
2*d^2-e^2)/(1-1/c^2/x^2)^(1/2)/x/(c*(e*x+d)/(c*d+e))^(1/2)-4/3*b*(c*(e*x+d
)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),
2,2^(1/2)*(e/(c*d+e))^(1/2))/c/d/e/(1-1/c^2/x^2)^(1/2)/x/(e*x+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.49 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.09

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{5/2}}\,dx = \frac{2\left(-\frac{a}{(d+ex)^{3/2}}+\frac{2bce^2\sqrt{1-\frac{1}{c^2x^2}}\,x}{(c^2d^3-de^2)\sqrt{d+ex}}-\frac{b\sec^{-1}(cx)}{(d+ex)^{3/2}}-\frac{2ib\sqrt{\frac{e(1+cx)}{-cd+e}}\sqrt{\frac{e-cex}{cd+e}}\left(-cdE\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{c}{cd+\cdots}}\right.\right.\right.}{\cdots}\right)}{\cdots}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(d + e*x)^(5/2),x]
```

output
```
(2*(-(a/(d + e*x)^(3/2)) + (2*b*c*e^2*Sqrt[1 - 1/(c^2*x^2)]*x)/((c^2*d^3 -
 d*e^2)*Sqrt[d + e*x]) - (b*ArcSec[c*x])/(d + e*x)^(3/2) - ((2*I)*b*Sqrt[(
e*(1 + c*x))/(-(c*d) + e)]*Sqrt[(e - c*e*x)/(c*d + e)]*(-(c*d*EllipticE[I*
ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)]) + c*d*E
llipticF[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e
)] + (c*d + e)*EllipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt
[d + e*x]], (c*d + e)/(c*d - e)]))/(d^2*(-(c/(c*d + e)))^(3/2)*(c*d + e)^2
*Sqrt[1 - 1/(c^2*x^2)]*x)))/(3*e)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.07, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5749, 1898, 635, 25, 27, 498, 27, 509, 508, 327, 633, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{5/2}}\,dx$$

$$\Big\downarrow \; 5749$$

$$\frac{2b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2(d+ex)^{3/2}}dx}{3ce}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \text{1898}$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\int \frac{1}{x(d+ex)^{3/2}\sqrt{x^2-\frac{1}{c^2}}}dx}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \text{635}$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\int -\frac{e}{d(d+ex)^{3/2}\sqrt{x^2-\frac{1}{c^2}}}dx+\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d}-\int \frac{e}{d(d+ex)^{3/2}\sqrt{x^2-\frac{1}{c^2}}}dx\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d}-\frac{e\int \frac{1}{(d+ex)^{3/2}\sqrt{x^2-\frac{1}{c^2}}}dx}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \text{498}$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d}-\frac{e\left(-\frac{2\int -\frac{\sqrt{d+ex}}{2\sqrt{x^2-\frac{1}{c^2}}}dx}{d^2-\frac{e^2}{c^2}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+ex}}\right)}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d}-\frac{e\left(\frac{\int \frac{\sqrt{d+ex}}{\sqrt{x^2-\frac{1}{c^2}}}dx}{d^2-\frac{e^2}{c^2}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+ex}}\right)}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\downarrow$ 509

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d}-\frac{e\left(\frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx}{\sqrt{x^2-\frac{1}{c^2}}\left(d^2-\frac{e^2}{c^2}\right)}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+ex}}\right)}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\downarrow$ 508

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d}-\frac{e\left(-\frac{2\sqrt{1-c^2x^2}\sqrt{d+ex}\int \frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{x^2-\frac{1}{c^2}}\left(d^2-\frac{e^2}{c^2}\right)\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+ex}}\right)}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\downarrow$ 327

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d}-\frac{e\left(-\frac{2\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\left(d^2-\frac{e^2}{c^2}\right)\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+ex}}\right)}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\downarrow$ 633

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d\sqrt{x^2-\frac{1}{c^2}}}-\frac{e\left(-\frac{2\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\left(d^2-\frac{e^2}{c^2}\right)\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+ex}}\right)}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \text{632}$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx}{d\sqrt{x^2-\frac{1}{c^2}}}-\frac{e\left(-\frac{2\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\left(d^2-\frac{e^2}{c^2}\right)\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+ex}}\right)}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \text{186}$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2\sqrt{1-c^2x^2}\int \frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}}{d\sqrt{x^2-\frac{1}{c^2}}}-\frac{e\left(-\frac{2\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\left(d^2-\frac{e^2}{c^2}\right)\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+ex}}\right)}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \text{413}$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\int \frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{d\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{e\left(-\frac{2\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\left(d^2-\frac{e^2}{c^2}\right)\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+ex}}\right)}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\downarrow$ 412

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\left(-\frac{2\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\left(d^2-\frac{e^2}{c^2}\right)\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+ex}}\right)}{d}-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right.\right.}{d\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right.}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$
$$\frac{2(a+b\sec^{-1}(cx))}{3e(d+ex)^{3/2}}$$

input
```
Int[(a + b*ArcSec[c*x])/(d + e*x)^(5/2),x]
```

output
```
(-2*(a + b*ArcSec[c*x]))/(3*e*(d + e*x)^(3/2)) + (2*b*Sqrt[-c^(-2) + x^2]*
(-((e*((-2*e*Sqrt[-c^(-2) + x^2])/((d^2 - e^2/c^2)*Sqrt[d + e*x]) - (2*Sqr
t[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e
)/(c*d + e)])/(c*(d^2 - e^2/c^2)*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[-c^(-2
) + x^2])))/d) - (2*Sqrt[1 - c^2*x^2]*Sqrt[1 - (e*(1 - c*x))/(c*d + e)]*El
lipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(d*Sqrt[-c^(-
2) + x^2]*Sqrt[d + e/c - (e*(1 - c*x))/c])))/(3*c*e*Sqrt[1 - 1/(c^2*x^2)]*
x)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
 - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 635
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] + Int[(
(c + d*x)^n/Sqrt[a + b*x^2])*ExpandToSum[(1 - c^(n + 1/2)*(c + d*x)^(-n - 1
/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[n + 1/2, 0]
```

rule 1898
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p])   Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n
))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !I
ntegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

rule 5749
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 874 vs. $2(270) = 540$.

Time = 9.63 (sec) , antiderivative size = 875, normalized size of antiderivative = 2.94

method	result
derivativedivides	$-\frac{2a}{3(ex+d)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arcsec}(cx)}{3(ex+d)^{\frac{3}{2}}}-\frac{2\left(\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\mathrm{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c^2d^2\sqrt{ex+d}-\sqrt{\right.}{}\right.}{}\right.$
default	$-\frac{2a}{3(ex+d)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arcsec}(cx)}{3(ex+d)^{\frac{3}{2}}}-\frac{2\left(\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\mathrm{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c^2d^2\sqrt{ex+d}-\sqrt{\right.}{}\right.}{}\right.$
parts	$-\frac{2a}{3(ex+d)^{\frac{3}{2}}e}+\frac{2b\left(-\frac{\operatorname{arcsec}(cx)}{3(ex+d)^{\frac{3}{2}}}+\frac{2\sqrt{\frac{c}{cd-e}}\,c^2d(ex+d)^2}{3}-\frac{2\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\,\mathrm{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd}}\right.}{3}\right.}{}$

input
```
int((a+b*arcsec(c*x))/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/e*(-1/3*a/(e*x+d)^(3/2)+b*(-1/3/(e*x+d)^(3/2)*arcsec(c*x)-2/3/c*(((-c*(e
*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((
e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2*(e*x+d)^(1
/2)-((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*
EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2
*(e*x+d)^(1/2)+((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d
+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*
d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^2*d^2*(e*x+d)^(1/2)-(c/(c*d-e))^(1/2)*c^2
*d*(e*x+d)^2+((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e
))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2)
)*c*d*e*(e*x+d)^(1/2)-((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+
e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+
e))^(1/2))*c*d*e*(e*x+d)^(1/2)+2*(c/(c*d-e))^(1/2)*c^2*d^2*(e*x+d)-((-c*(e
*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi(
(e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e)
)^(1/2))*e^2*(e*x+d)^(1/2)-(c/(c*d-e))^(1/2)*c^2*d^3+(c/(c*d-e))^(1/2)*d*e
^2)/(c*d-e)/(c/(c*d-e))^(1/2)/(e*x+d)^(1/2)/(c*d+e)/d^2/x/((c^2*(e*x+d)^2-
2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{a + b\sec^{-1}(cx)}{(d + ex)^{5/2}}\,dx = \int \frac{a + b\,\mathrm{asec}\,(cx)}{(d + ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asec(c*x))/(e*x+d)**(5/2),x)
```

output
```
Integral((a + b*asec(c*x))/(d + e*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\sec^{-1}(cx)}{(d + ex)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

Giac [F]

$$\int \frac{a + b\sec^{-1}(cx)}{(d + ex)^{5/2}}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx) + a}{(ex + d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/(e*x + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{\left(d+e\,x\right)^{5/2}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(d + e*x)^(5/2),x)
```

output
```
int((a + b*acos(1/(c*x)))/(d + e*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{5/2}}\,dx=\frac{3\sqrt{ex+d}\left(\int \frac{asec(cx)}{\sqrt{ex+d}\,d^2+2\sqrt{ex+d}\,dex+\sqrt{ex+d}\,e^2x^2}dx\right)bde+3\sqrt{ex+d}\left(\int \frac{asec(}{\sqrt{ex+d}\,d^2+2\sqrt{ex+d}}\right.}{3\sqrt{ex+d}\,e\,(ex+d)}$$

input
```
int((a+b*asec(c*x))/(e*x+d)^(5/2),x)
```

output
```
(3*sqrt(d + e*x)*int(asec(c*x)/(sqrt(d + e*x)*d**2 + 2*sqrt(d + e*x)*d*e*x
 + sqrt(d + e*x)*e**2*x**2),x)*b*d*e + 3*sqrt(d + e*x)*int(asec(c*x)/(sqrt
(d + e*x)*d**2 + 2*sqrt(d + e*x)*d*e*x + sqrt(d + e*x)*e**2*x**2),x)*b*e**
2*x - 2*a)/(3*sqrt(d + e*x)*e*(d + e*x))
```

3.68 $\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

Optimal result 571
Mathematica [C] (verified) 572
Rubi [A] (verified) 573
Maple [B] (verified) 582
Fricas [F] 583
Sympy [F(-1)] 584
Maxima [F(-2)] 584
Giac [F] 584
Mupad [F(-1)] 585
Reduce [F] 585

Optimal result

Integrand size = 18, antiderivative size = 498

$$\begin{aligned}\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{7/2}}\,dx =& -\frac{4be(1-c^2x^2)}{15cd\,(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}\,x(d+ex)^{3/2}} \\ & - \frac{4be(7c^2d^2-3e^2)\,(1-c^2x^2)}{15cd^2\,(c^2d^2-e^2)^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2(a+b\sec^{-1}(cx))}{5e(d+ex)^{5/2}} \\ & + \frac{4b(7c^2d^2-3e^2)\sqrt{d+ex}\sqrt{1-c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15d^2\,(c^2d^2-e^2)^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{c(d+ex)}{cd+e}}} \\ & - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15d\,(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} \\ & - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5cd^2e\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}\end{aligned}$$

output
```
-4/15*b*e*(-c^2*x^2+1)/c/d/(c^2*d^2-e^2)/(1-1/c^2/x^2)^(1/2)/x/(e*x+d)^(3/
2)-4/15*b*e*(7*c^2*d^2-3*e^2)*(-c^2*x^2+1)/c/d^2/(c^2*d^2-e^2)^2/(1-1/c^2/
x^2)^(1/2)/x/(e*x+d)^(1/2)-2/5*(a+b*arcsec(c*x))/e/(e*x+d)^(5/2)+4/15*b*(7
*c^2*d^2-3*e^2)*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)*EllipticE(1/2*(-c*x+1)^(1
/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))/d^2/(c^2*d^2-e^2)^2/(1-1/c^2/x^2)^(
1/2)/x/(c*(e*x+d)/(c*d+e))^(1/2)-4/15*b*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^
2+1)^(1/2)*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))
/d/(c^2*d^2-e^2)/(1-1/c^2/x^2)^(1/2)/x/(e*x+d)^(1/2)-4/5*b*(c*(e*x+d)/(c*d
+e))^(1/2)*(-c^2*x^2+1)^(1/2)*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1
/2)*(e/(c*d+e))^(1/2))/c/d^2/e/(1-1/c^2/x^2)^(1/2)/x/(e*x+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.67 (sec) , antiderivative size = 407, normalized size of antiderivative = 0.82

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{7/2}}\,dx = \frac{2\left(-\frac{3a}{(d+ex)^{5/2}}+\frac{2bce^2\sqrt{1-\frac{1}{c^2x^2}}\,x\left(-e^2(4d+3ex)+c^2d^2(8d+7ex)\right)}{(c^2d^3-de^2)^2(d+ex)^{3/2}}-\frac{3b\sec^{-1}(cx)}{(d+ex)^{5/2}}+\frac{2ib\sqrt{\frac{e(1+cx)}{-cd+e}}\sqrt{\frac{e-c}{cd+}}}{}\right.}{}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(d + e*x)^(7/2),x]
```

output
```
(2*((-3*a)/(d + e*x)^(5/2) + (2*b*c*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*(-(e^2*(4*
d + 3*e*x)) + c^2*d^2*(8*d + 7*e*x)))/((c^2*d^3 - d*e^2)^2*(d + e*x)^(3/2)
) - (3*b*ArcSec[c*x])/(d + e*x)^(5/2) + ((2*I)*b*Sqrt[(e*(1 + c*x))/(-(c*d
) + e)]*Sqrt[(e - c*e*x)/(c*d + e)]*(c*d*(7*c^2*d^2 - 3*e^2)*EllipticE[I*A
rcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] - c*d*(6*
c^2*d^2 - c*d*e - 3*e^2)*EllipticF[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d +
 e*x]], (c*d + e)/(c*d - e)] - 3*(c*d - e)*(c*d + e)^2*EllipticPi[1 + e/(c
*d), I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)]))
/(d^3*(c*d - e)*(-(c/(c*d + e)))^(3/2)*(c*d + e)^3*Sqrt[1 - 1/(c^2*x^2)]*x
)))/(15*e)
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.04, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.056, Rules used = {5749, 1898, 635, 633, 632, 186, 413, 412, 688, 27, 688, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{7/2}}\,dx$$

$$\Big\downarrow \ 5749$$

$$\frac{2b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2(d+ex)^{5/2}}dx}{5ce}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \ 1898$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\int \frac{1}{x(d+ex)^{5/2}\sqrt{x^2-\frac{1}{c^2}}}dx}{5cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \ 635$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\int \frac{-\frac{xe^2}{d^2}-\frac{2e}{d}}{(d+ex)^{5/2}\sqrt{x^2-\frac{1}{c^2}}}dx+\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d^2}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \ 633$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\int \frac{-\frac{xe^2}{d^2}-\frac{2e}{d}}{(d+ex)^{5/2}\sqrt{x^2-\frac{1}{c^2}}}dx+\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d^2\sqrt{x^2-\frac{1}{c^2}}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \ 632$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\int \frac{-\frac{xe^2}{d^2}-\frac{2e}{d}}{(d+ex)^{5/2}\sqrt{x^2-\frac{1}{c^2}}}dx+\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx}{d^2\sqrt{x^2-\frac{1}{c^2}}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{186}$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\int \frac{-\frac{xe^2}{d^2}-\frac{2e}{d}}{(d+ex)^{5/2}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{2\sqrt{1-c^2x^2}\int \frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}}{d^2\sqrt{x^2-\frac{1}{c^2}}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{413}$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\int \frac{-\frac{xe^2}{d^2}-\frac{2e}{d}}{(d+ex)^{5/2}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\int \frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{d^2\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{412}$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\int \frac{-\frac{xe^2}{d^2}-\frac{2e}{d}}{(d+ex)^{5/2}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{d^2\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{688}$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2\int \frac{e\left(3d\left(2-\frac{e^2}{c^2d^2}\right)-ex\right)}{2d(d+ex)^{3/2}\sqrt{x^2-\frac{1}{c^2}}}dx}{3\left(d^2-\frac{e^2}{c^2}\right)}-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{d^2\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}+\frac{2e^2\sqrt{x^2-\frac{1}{c^2}}}{3d\left(d^2-\frac{e^2}{c^2}\right)(d+ex)^{3/2}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\int\frac{3\left(2d-\frac{e^2}{c^2d}\right)-ex}{(d+ex)^{3/2}\sqrt{x^2-\frac{1}{c^2}}}dx}{3d\left(d^2-\frac{e^2}{c^2}\right)}-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{d^2\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}+\frac{2e^2\sqrt{x^2-\frac{1}{c^2}}}{3d\left(d^2-\frac{e^2}{c^2}\right)(d+ex)^{3/2}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$\Big\downarrow$ 688

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\left(-\frac{2\int-\frac{6d^2-\frac{2e^2}{c^2}+e\left(7d-\frac{3e^2}{c^2d}\right)x}{2\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d^2-\frac{e^2}{c^2}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}\left(7c^2d^2-3e^2\right)}{d\left(c^2d^2-e^2\right)\sqrt{d+ex}}\right)}{3d\left(d^2-\frac{e^2}{c^2}\right)}-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{d^2\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$\Big\downarrow$ 27

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\left(\frac{\int\frac{2\left(3d^2-\frac{e^2}{c^2}\right)+e\left(7d-\frac{3e^2}{c^2d}\right)x}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d^2-\frac{e^2}{c^2}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}\left(7c^2d^2-3e^2\right)}{d\left(c^2d^2-e^2\right)\sqrt{d+ex}}\right)}{3d\left(d^2-\frac{e^2}{c^2}\right)}-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{d^2\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$\Big\downarrow$ 600

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\left(\frac{\left(7d-\frac{3e^2}{c^2d}\right)\int\frac{\sqrt{d+ex}}{\sqrt{x^2-\frac{1}{c^2}}}dx-\left(d^2-\frac{e^2}{c^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d^2-\frac{e^2}{c^2}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}\left(7c^2d^2-3e^2\right)}{d\left(c^2d^2-e^2\right)\sqrt{d+ex}}\right)}{3d\left(d^2-\frac{e^2}{c^2}\right)}-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticP}\cdots}{d^2\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(\cdots}{\cdots}}}\cdots\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$\downarrow$ 509

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\left(\frac{\frac{\sqrt{1-c^2x^2}\left(7d-\frac{3e^2}{c^2d}\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx}{\sqrt{x^2-\frac{1}{c^2}}}-\left(d^2-\frac{e^2}{c^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d^2-\frac{e^2}{c^2}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}\left(7c^2d^2-3e^2\right)}{d\left(c^2d^2-e^2\right)\sqrt{d+ex}}\right)}{3d\left(d^2-\frac{e^2}{c^2}\right)}-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\cdots}{d^2\sqrt{x^2-\cdots}}\cdots\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$\downarrow$ 508

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\left(\frac{-\left(\left(d^2-\frac{e^2}{c^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx\right)-\frac{2\sqrt{1-c^2x^2}\left(7d-\frac{3e^2}{c^2d}\right)\sqrt{d+ex}\int \frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}}{d^2-\frac{e^2}{c^2}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}\left(7c^2d^2-3e^2\right)}{d\left(c^2d^2-e^2\right)\sqrt{d+ex}}\right)}{3d\left(d^2-\frac{e^2}{c^2}\right)}-\frac{2\cdots}{\cdots}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$\downarrow$ 327

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\left(\frac{-\left(d^2-\frac{e^2}{c^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{2\sqrt{1-c^2x^2}\left(7d-\frac{3e^2}{c^2d}\right)\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}}{d^2-\frac{e^2}{c^2}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}\left(7c^2d^2-3e^2\right)}{d\left(c^2d^2-e^2\right)\sqrt{d+ex}}\right)}{3d\left(d^2-\frac{e^2}{c^2}\right)}-\frac{2\sqrt{\cdots}}{\cdots}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$\downarrow$ 512

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\left(\frac{-\frac{\sqrt{1-c^2x^2}\left(d^2-\frac{e^2}{c^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{\sqrt{x^2-\frac{1}{c^2}}}-\frac{2\sqrt{1-c^2x^2}\left(7d-\frac{3e^2}{c^2d}\right)\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}}{d^2-\frac{e^2}{c^2}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}\left(7c^2d^2-3e^2\right)}{d\left(c^2d^2-e^2\right)\sqrt{d+ex}}\right)}{3d\left(d^2-\frac{e^2}{c^2}\right)}\right.}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$\downarrow$ 511

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\left(\frac{\frac{2\sqrt{1-c^2x^2}\left(d^2-\frac{e^2}{c^2}\right)\sqrt{\frac{c(d+ex)}{cd+e}}\int\frac{1}{\sqrt{1-\frac{e(1-cx)}{cd+e}}\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}-\frac{2\sqrt{1-c^2x^2}\left(7d-\frac{3e^2}{c^2d}\right)\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}}{d^2-\frac{e^2}{c^2}}\right.}{3d\left(d^2-\frac{e^2}{c^2}\right)}\right.}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$\downarrow$ 321

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\left(\frac{\frac{2\sqrt{1-c^2x^2}\left(d^2-\frac{e^2}{c^2}\right)\sqrt{\frac{c(d+ex)}{cd+e}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}-\frac{2\sqrt{1-c^2x^2}\left(7d-\frac{3e^2}{c^2d}\right)\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}}{d^2-\frac{e^2}{c^2}}-\ldots}{3d\left(d^2-\frac{e^2}{c^2}\right)}\ldots\right.}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

input
```
Int[(a + b*ArcSec[c*x])/(d + e*x)^(7/2),x]
```

output
```
(-2*(a + b*ArcSec[c*x]))/(5*e*(d + e*x)^(5/2)) + (2*b*Sqrt[-c^(-2) + x^2]*
((2*e^2*Sqrt[-c^(-2) + x^2])/(3*d*(d^2 - e^2/c^2)*(d + e*x)^(3/2)) - (e*((
-2*e*(7*c^2*d^2 - 3*e^2)*Sqrt[-c^(-2) + x^2])/(d*(c^2*d^2 - e^2)*Sqrt[d +
e*x]) + ((-2*(7*d - (3*e^2)/(c^2*d))*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*Ellip
ticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[(c*(d + e*x)
)/(c*d + e)]*Sqrt[-c^(-2) + x^2]) + (2*(d^2 - e^2/c^2)*Sqrt[(c*(d + e*x))/
(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e
)/(c*d + e)])/(c*Sqrt[d + e*x]*Sqrt[-c^(-2) + x^2]))/(d^2 - e^2/c^2)))/(3*
d*(d^2 - e^2/c^2)) - (2*Sqrt[1 - c^2*x^2]*Sqrt[1 - (e*(1 - c*x))/(c*d + e)
]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(d^2*Sqrt
[-c^(-2) + x^2]*Sqrt[d + e/c - (e*(1 - c*x))/c])))/(5*c*e*Sqrt[1 - 1/(c^2*
x^2)]*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 635
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] + Int[(
(c + d*x)^n/Sqrt[a + b*x^2])*ExpandToSum[(1 - c^(n + 1/2)*(c + d*x)^(-n - 1
/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[n + 1/2, 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1898
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p])   Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n
))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !I
ntegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

rule 5749
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1617 vs. $2(455) = 910$.

Time = 14.47 (sec) , antiderivative size = 1618, normalized size of antiderivative = 3.25

method	result	size
derivativedivides	Expression too large to display	1618
default	Expression too large to display	1618
parts	Expression too large to display	1642

input
```
int((a+b*arcsec(c*x))/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/e*(-1/5*a/(e*x+d)^(5/2)+b*(-1/5/(e*x+d)^(5/2)*arcsec(c*x)+2/15/c*(7*(c/(
c*d-e))^(1/2)*c^4*d^3*(e*x+d)^3-6*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*
(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((
c*d-e)/(c*d+e))^(1/2))*c^4*d^4*(e*x+d)^(3/2)+7*((-c*(e*x+d)+c*d-e)/(c*d-e)
)^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d
-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^4*d^4*(e*x+d)^(3/2)-3*((-c*(e*x+d)+c
*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)
^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2)
)*c^4*d^4*(e*x+d)^(3/2)-13*(c/(c*d-e))^(1/2)*c^4*d^4*(e*x+d)^2-7*((-c*(e*x
+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*
x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^3*d^3*e*(e*x+d)^(3
/2)+7*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2
)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^3*d
^3*e*(e*x+d)^(3/2)+5*(c/(c*d-e))^(1/2)*c^4*d^5*(e*x+d)-3*(c/(c*d-e))^(1/2)
*c^2*d*e^2*(e*x+d)^3+2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d
+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d
+e))^(1/2))*c^2*d^2*e^2*(e*x+d)^(3/2)-3*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)
*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1
/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2*e^2*(e*x+d)^(3/2)+6*((-c*(e*x+d)+c*d-
e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)...
```

Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{7/2}}\,dx=\int \frac{b\,\mathrm{arcsec}\left(cx\right)+a}{(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arcsec(c*x) + a)/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*
e^2*x^2 + 4*d^3*e*x + d^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*asec(c*x))/(e*x+d)**(7/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

Giac [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{7/2}}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx)+a}{(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/(e*x + d)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{7/2}}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{(d+e\,x)^{7/2}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(d + e*x)^(7/2),x)
```

output
```
int((a + b*acos(1/(c*x)))/(d + e*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{7/2}}\,dx=\frac{5\sqrt{ex+d}\left(\int \frac{asec(cx)}{\sqrt{ex+d}\,d^3+3\sqrt{ex+d}\,d^2ex+3\sqrt{ex+d}\,d\,e^2x^2+\sqrt{ex+d}\,e^3x^3}dx\right)b\,d^2e+10\sqrt{ex+d}\left(\int\right.}{}$$

input
```
int((a+b*asec(c*x))/(e*x+d)^(7/2),x)
```

output
```
(5*sqrt(d + e*x)*int(asec(c*x)/(sqrt(d + e*x)*d**3 + 3*sqrt(d + e*x)*d**2*
e*x + 3*sqrt(d + e*x)*d*e**2*x**2 + sqrt(d + e*x)*e**3*x**3),x)*b*d**2*e +
 10*sqrt(d + e*x)*int(asec(c*x)/(sqrt(d + e*x)*d**3 + 3*sqrt(d + e*x)*d**2
*e*x + 3*sqrt(d + e*x)*d*e**2*x**2 + sqrt(d + e*x)*e**3*x**3),x)*b*d*e**2*
x + 5*sqrt(d + e*x)*int(asec(c*x)/(sqrt(d + e*x)*d**3 + 3*sqrt(d + e*x)*d*
*2*e*x + 3*sqrt(d + e*x)*d*e**2*x**2 + sqrt(d + e*x)*e**3*x**3),x)*b*e**3*
x**2 - 2*a)/(5*sqrt(d + e*x)*e*(d**2 + 2*d*e*x + e**2*x**2))
```

3.69 $\int x^4(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$

Optimal result . . . 586
Mathematica [A] (verified) . . . 587
Rubi [A] (verified) . . . 587
Maple [A] (verified) . . . 590
Fricas [A] (verification not implemented) . . . 591
Sympy [A] (verification not implemented) . . . 591
Maxima [A] (verification not implemented) . . . 592
Giac [B] (verification not implemented) . . . 593
Mupad [F(-1)] . . . 594
Reduce [F] . . . 595

Optimal result

Integrand size = 19, antiderivative size = 206

$$\int x^4(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = -\frac{b(42c^2d+25e)\,x^2\sqrt{-1+c^2x^2}}{560c^5\sqrt{c^2x^2}} - \frac{b(42c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} - \frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}} + \frac{1}{5}dx^5\left(a+b\sec^{-1}(cx)\right) + \frac{1}{7}ex^7\left(a+b\sec^{-1}(cx)\right) - \frac{b(42c^2d+25e)\,x\,\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{560c^6\sqrt{c^2x^2}}$$

output
```
-1/560*b*(42*c^2*d+25*e)*x^2*(c^2*x^2-1)^(1/2)/c^5/(c^2*x^2)^(1/2)-1/840*b
*(42*c^2*d+25*e)*x^4*(c^2*x^2-1)^(1/2)/c^3/(c^2*x^2)^(1/2)-1/42*b*e*x^6*(c
^2*x^2-1)^(1/2)/c/(c^2*x^2)^(1/2)+1/5*d*x^5*(a+b*arcsec(c*x))+1/7*e*x^7*(a
+b*arcsec(c*x))-1/560*b*(42*c^2*d+25*e)*x*arctanh(c*x/(c^2*x^2-1)^(1/2))/c
^6/(c^2*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.68

$$\int x^4(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{48ac^7x^5(7d+5ex^2)-bc^2\sqrt{1-\frac{1}{c^2x^2}}x^2(75e+2c^2(63d+25ex^2)+c^4(84dx^2+40ex^4))+48bc^7x^5(7d+5e}{1680c^7}$$

input
```
Integrate[x^4*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

output
```
(48*a*c^7*x^5*(7*d + 5*e*x^2) - b*c^2*Sqrt[1 - 1/(c^2*x^2)]*x^2*(75*e + 2*
c^2*(63*d + 25*e*x^2) + c^4*(84*d*x^2 + 40*e*x^4)) + 48*b*c^7*x^5*(7*d + 5
*e*x^2)*ArcSec[c*x] - 3*b*(42*c^2*d + 25*e)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)]
)*x])/(1680*c^7)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5761, 27, 363, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{ 5761}$$

$$-\frac{bcx\int\frac{x^4(5ex^2+7d)}{35\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{1}{5}dx^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{bcx\int\frac{x^4(5ex^2+7d)}{\sqrt{c^2x^2-1}}dx}{35\sqrt{c^2x^2}}+\frac{1}{5}dx^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 363}$$

$$-\frac{bcx\left(\frac{1}{6}\left(\frac{25e}{c^2}+42d\right)\int\frac{x^4}{\sqrt{c^2x^2-1}}dx+\frac{5ex^5\sqrt{c^2x^2-1}}{6c^2}\right)}{35\sqrt{c^2x^2}}+\frac{1}{5}dx^5(a+b\sec^{-1}(cx))+\frac{1}{7}ex^7(a+b\sec^{-1}(cx))$$

$$\Big\downarrow \text{262}$$

$$-\frac{bcx\left(\frac{1}{6}\left(\frac{25e}{c^2}+42d\right)\left(\frac{3\int\frac{x^2}{\sqrt{c^2x^2-1}}dx}{4c^2}+\frac{x^3\sqrt{c^2x^2-1}}{4c^2}\right)+\frac{5ex^5\sqrt{c^2x^2-1}}{6c^2}\right)}{35\sqrt{c^2x^2}}+\frac{1}{5}dx^5(a+b\sec^{-1}(cx))+\frac{1}{7}ex^7(a+b\sec^{-1}(cx))$$

$$\Big\downarrow \text{262}$$

$$-\frac{bcx\left(\frac{1}{6}\left(\frac{25e}{c^2}+42d\right)\left(\frac{3\left(\frac{\int\frac{1}{\sqrt{c^2x^2-1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{c^2x^2-1}}{4c^2}\right)+\frac{5ex^5\sqrt{c^2x^2-1}}{6c^2}\right)}{35\sqrt{c^2x^2}}+\frac{1}{5}dx^5(a+b\sec^{-1}(cx))+\frac{1}{7}ex^7(a+b\sec^{-1}(cx))$$

$$\Big\downarrow \text{224}$$

$$-\frac{bcx\left(\frac{1}{6}\left(\frac{25e}{c^2}+42d\right)\left(\frac{3\left(\frac{\int\frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}}{2c^2}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{c^2x^2-1}}{4c^2}\right)+\frac{5ex^5\sqrt{c^2x^2-1}}{6c^2}\right)}{35\sqrt{c^2x^2}}+\frac{1}{5}dx^5(a+b\sec^{-1}(cx))+\frac{1}{7}ex^7(a+b\sec^{-1}(cx))$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{5}dx^5(a+b\sec^{-1}(cx))+\frac{1}{7}ex^7(a+b\sec^{-1}(cx))-\frac{bcx\left(\frac{1}{6}\left(\frac{3\left(\frac{\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{2c^3}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{c^2x^2-1}}{4c^2}\right)\left(\frac{25e}{c^2}+42d\right)+\frac{5ex^5\sqrt{c^2x^2-1}}{6c^2}\right)}{35\sqrt{c^2x^2}}$$

input
```
Int[x^4*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

output
```
(d*x^5*(a + b*ArcSec[c*x]))/5 + (e*x^7*(a + b*ArcSec[c*x]))/7 - (b*c*x*((5
*e*x^5*Sqrt[-1 + c^2*x^2])/(6*c^2) + ((42*d + (25*e)/c^2)*((x^3*Sqrt[-1 +
c^2*x^2])/(4*c^2) + (3*((x*Sqrt[-1 + c^2*x^2])/(2*c^2) + ArcTanh[(c*x)/Sqr
t[-1 + c^2*x^2]]/(2*c^3)))/(4*c^2)))/6))/(35*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.59

method	result
parts	$a\left(\frac{1}{7}e\,x^{7}+\frac{1}{5}x^{5}d\right)+\frac{b\,\operatorname{arcsec}(cx)e\,x^{7}}{7}+\frac{b\,\operatorname{arcsec}(cx)d\,x^{5}}{5}-\frac{b(c^{2}x^{2}-1)x^{4}e}{42c^{3}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}-\frac{b(c^{2}x^{2}-1)x^{2}d}{20c^{3}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}-\cdots$
derivativedivides	$\frac{\frac{a\left(\frac{1}{5}d\,c^{7}x^{5}+\frac{1}{7}e\,c^{7}x^{7}\right)}{c^{2}}+\frac{b\,\operatorname{arcsec}(cx)d\,c^{5}x^{5}}{5}+\frac{b\,c^{5}\operatorname{arcsec}(cx)e\,x^{7}}{7}-\frac{b\left(c^{2}x^{2}-1\right)c^{2}x^{2}d}{20\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}-\frac{b\,c^{2}\left(c^{2}x^{2}-1\right)x^{4}e}{42\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}-\frac{3b\left(c^{2}x^{2}-1\right)d}{40\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}-\cdots}{c^{5}}$
default	$\frac{\frac{a\left(\frac{1}{5}d\,c^{7}x^{5}+\frac{1}{7}e\,c^{7}x^{7}\right)}{c^{2}}+\frac{b\,\operatorname{arcsec}(cx)d\,c^{5}x^{5}}{5}+\frac{b\,c^{5}\operatorname{arcsec}(cx)e\,x^{7}}{7}-\frac{b\left(c^{2}x^{2}-1\right)c^{2}x^{2}d}{20\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}-\frac{b\,c^{2}\left(c^{2}x^{2}-1\right)x^{4}e}{42\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}-\frac{3b\left(c^{2}x^{2}-1\right)d}{40\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}-\cdots}{c^{5}}$

input
```
int(x^4*(e*x^2+d)*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/7*e*x^7+1/5*x^5*d)+1/7*b*arcsec(c*x)*e*x^7+1/5*b*arcsec(c*x)*d*x^5-1/
42*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^4*e-1/20*b/c^3*(c^2*x^2
-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2*d-5/168*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)
/c^2/x^2)^(1/2)*x^2*e-3/40*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d
-5/112*b/c^7*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*e-3/40*b/c^6*(c^2*x^2
-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d*ln(c*x+(c^2*x^2-1)^(1/2))-5/112*
b/c^8*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*e*ln(c*x+(c^2*x^2-1)
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.93

$$\int x^4(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$= \frac{240\,ac^7ex^7+336\,ac^7dx^5+48\,(5\,bc^7ex^7+7\,bc^7dx^5-7\,bc^7d-5\,bc^7e)\,\text{arcsec}\,(cx)+96\,(7\,bc^7d+5\,bc^7e)\,\text{ar}}{}$$

input
```
integrate(x^4*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/1680*(240*a*c^7*e*x^7 + 336*a*c^7*d*x^5 + 48*(5*b*c^7*e*x^7 + 7*b*c^7*d*
x^5 - 7*b*c^7*d - 5*b*c^7*e)*arcsec(c*x) + 96*(7*b*c^7*d + 5*b*c^7*e)*arct
an(-c*x + sqrt(c^2*x^2 - 1)) + 3*(42*b*c^2*d + 25*b*e)*log(-c*x + sqrt(c^2
*x^2 - 1)) - (40*b*c^5*e*x^5 + 2*(42*b*c^5*d + 25*b*c^3*e)*x^3 + 3*(42*b*c
^3*d + 25*b*c*e)*x)*sqrt(c^2*x^2 - 1))/c^7
```

Sympy [A] (verification not implemented)

Time = 12.80 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.98

$$\int x^4(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{adx^5}{5}+\frac{aex^7}{7}+\frac{bdx^5\,\text{asec}\,(cx)}{5}+\frac{bex^7\,\text{asec}\,(cx)}{7}$$
$$-\frac{bd\left(\begin{cases}\frac{cx^5}{4\sqrt{c^2x^2-1}}+\frac{x^3}{8c\sqrt{c^2x^2-1}}-\frac{3x}{8c^3\sqrt{c^2x^2-1}}+\frac{3\,\text{acosh}\,(cx)}{8c^4} & \text{for } |c^2x^2|>1\\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}}-\frac{ix^3}{8c\sqrt{-c^2x^2+1}}+\frac{3ix}{8c^3\sqrt{-c^2x^2+1}}-\frac{3i\,\text{asin}\,(cx)}{8c^4} & \text{otherwise}\end{cases}\right)}{5c}$$
$$-\frac{be\left(\begin{cases}\frac{cx^7}{6\sqrt{c^2x^2-1}}+\frac{x^5}{24c\sqrt{c^2x^2-1}}+\frac{5x^3}{48c^3\sqrt{c^2x^2-1}}-\frac{5x}{16c^5\sqrt{c^2x^2-1}}+\frac{5\,\text{acosh}\,(cx)}{16c^6} & \text{for } |c^2x^2|>1\\ -\frac{icx^7}{6\sqrt{-c^2x^2+1}}-\frac{ix^5}{24c\sqrt{-c^2x^2+1}}-\frac{5ix^3}{48c^3\sqrt{-c^2x^2+1}}+\frac{5ix}{16c^5\sqrt{-c^2x^2+1}}-\frac{5i\,\text{asin}\,(cx)}{16c^6} & \text{otherwise}\end{cases}\right)}{7c}$$

input
```
integrate(x**4*(e*x**2+d)*(a+b*asec(c*x)),x)
```

output
```
a*d*x**5/5 + a*e*x**7/7 + b*d*x**5*asec(c*x)/5 + b*e*x**7*asec(c*x)/7 - b*
d*Piecewise((c*x**5/(4*sqrt(c**2*x**2 - 1)) + x**3/(8*c*sqrt(c**2*x**2 - 1
)) - 3*x/(8*c**3*sqrt(c**2*x**2 - 1)) + 3*acosh(c*x)/(8*c**4), Abs(c**2*x*
*2) > 1), (-I*c*x**5/(4*sqrt(-c**2*x**2 + 1)) - I*x**3/(8*c*sqrt(-c**2*x**
2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**2 + 1)) - 3*I*asin(c*x)/(8*c**4), Tr
ue))/(5*c) - b*e*Piecewise((c*x**7/(6*sqrt(c**2*x**2 - 1)) + x**5/(24*c*sq
rt(c**2*x**2 - 1)) + 5*x**3/(48*c**3*sqrt(c**2*x**2 - 1)) - 5*x/(16*c**5*s
qrt(c**2*x**2 - 1)) + 5*acosh(c*x)/(16*c**6), Abs(c**2*x**2) > 1), (-I*c*x
**7/(6*sqrt(-c**2*x**2 + 1)) - I*x**5/(24*c*sqrt(-c**2*x**2 + 1)) - 5*I*x*
*3/(48*c**3*sqrt(-c**2*x**2 + 1)) + 5*I*x/(16*c**5*sqrt(-c**2*x**2 + 1)) -
 5*I*asin(c*x)/(16*c**6), True))/(7*c)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.44

$$\int x^4(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{7}aex^7+\frac{1}{5}adx^5 + \frac{1}{80}\left(16\,x^5\operatorname{arcsec}(cx)+\frac{\frac{2\left(3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2\,c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}-\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)bd + \frac{1}{672}\left(96\,x^7\operatorname{arcsec}(cx)-\frac{\frac{2\left(15\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-40\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^6\left(\frac{1}{c^2x^2}-1\right)^3+3\,c^6\left(\frac{1}{c^2x^2}-1\right)^2+3\,c^6\left(\frac{1}{c^2x^2}-1\right)+c^6}+\frac{15\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^6}-\frac{15\log\left(\sqrt{-\right.}}{c}\right.$$

input
```
integrate(x^4*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*e*x^7 + 1/5*a*d*x^5 + 1/80*(16*x^5*arcsec(c*x) + (2*(3*(-1/(c^2*x^2)
 + 1)^(3/2) - 5*sqrt(-1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(
1/(c^2*x^2) - 1) + c^4) - 3*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sq
rt(-1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*d + 1/672*(96*x^7*arcsec(c*x) - (2*(15
*(-1/(c^2*x^2) + 1)^(5/2) - 40*(-1/(c^2*x^2) + 1)^(3/2) + 33*sqrt(-1/(c^2*
x^2) + 1))/(c^6*(1/(c^2*x^2) - 1)^3 + 3*c^6*(1/(c^2*x^2) - 1)^2 + 3*c^6*(1
/(c^2*x^2) - 1) + c^6) + 15*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 - 15*log(s
qrt(-1/(c^2*x^2) + 1) - 1)/c^6)/c)*b*e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17474 vs. $2(178) = 356$.

Time = 2.40 (sec) , antiderivative size = 17474, normalized size of antiderivative = 84.83

$$\int x^4(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx=\text{Too large to display}$$

input
```
integrate(x^4*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/1680*(336*b*c^2*d*arccos(1/(c*x))/(c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x
) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2
) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21
*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(
c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14) - 126*b*c^2*d*log
(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^8 + 7*c^8*(1/(c^2*x^2) - 1)
/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/
(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1
)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1
)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14) + 126*b*c
^2*d*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(c^8 + 7*c^8*(1/(c^2*x
^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35
*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(
c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*
x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14) +
 336*a*c^2*d/(c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c
^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^
6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^
5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(
c^2*x^2) - 1)^7/(1/(c*x) + 1)^14) - 1008*b*c^2*d*(1/(c^2*x^2) - 1)*arcc...
```

Mupad [F(-1)]

Timed out.

$$\int x^4\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx=\int x^4\,\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^4*(d + e*x^2)*(a + b*acos(1/(c*x))),x)
```

output
```
int(x^4*(d + e*x^2)*(a + b*acos(1/(c*x))), x)
```

Reduce [F]

$$\int x^4 (d+ex^2) (a+b\sec^{-1}(cx)) \, dx = \left(\int asec(cx)\, x^6 dx\right) be + \left(\int asec(cx)\, x^4 dx\right) bd + \frac{ad\, x^5}{5} + \frac{ae\, x^7}{7}$$

input
```
int(x^4*(e*x^2+d)*(a+b*asec(c*x)),x)
```

output
```
(35*int(asec(c*x)*x**6,x)*b*e + 35*int(asec(c*x)*x**4,x)*b*d + 7*a*d*x**5
+ 5*a*e*x**7)/35
```

3.70 $\int x^2(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$

Optimal result 596
Mathematica [A] (verified) 597
Rubi [A] (verified) 597
Maple [A] (verified) 600
Fricas [A] (verification not implemented) 600
Sympy [A] (verification not implemented) 601
Maxima [A] (verification not implemented) 602
Giac [B] (verification not implemented) 602
Mupad [F(-1)] 603
Reduce [F] 604

Optimal result

Integrand size = 19, antiderivative size = 161

$$\int x^2(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx=-\frac{b(20c^2d+9e)\,x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}}-\frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\sec^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\sec^{-1}(cx)\right)-\frac{b(20c^2d+9e)\,x\,\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{120c^4\sqrt{c^2x^2}}$$

output
```
-1/120*b*(20*c^2*d+9*e)*x^2*(c^2*x^2-1)^(1/2)/c^3/(c^2*x^2)^(1/2)-1/20*b*e
*x^4*(c^2*x^2-1)^(1/2)/c/(c^2*x^2)^(1/2)+1/3*d*x^3*(a+b*arcsec(c*x))+1/5*e
*x^5*(a+b*arcsec(c*x))-1/120*b*(20*c^2*d+9*e)*x*arctanh(c*x/(c^2*x^2-1)^(1
/2))/c^4/(c^2*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.76

$$\int x^2\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{c^2x^2\left(8ac^3x(5d+3ex^2)-b\sqrt{1-\frac{1}{c^2x^2}}(9e+c^2(20d+6ex^2))\right)+8bc^5x^3(5d+3ex^2)\sec^{-1}(cx)-b(20c^2d-}{120c^5}$$

input
```
Integrate[x^2*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

output
```
(c^2*x^2*(8*a*c^3*x*(5*d + 3*e*x^2) - b*Sqrt[1 - 1/(c^2*x^2)]*(9*e + c^2*(
20*d + 6*e*x^2))) + 8*b*c^5*x^3*(5*d + 3*e*x^2)*ArcSec[c*x] - b*(20*c^2*d
+ 9*e)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(120*c^5)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5761, 27, 363, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \ 5761$$

$$-\frac{bcx\int\frac{x^2(3ex^2+5d)}{15\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\sec^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \ 27$$

$$-\frac{bcx\int\frac{x^2(3ex^2+5d)}{\sqrt{c^2x^2-1}}dx}{15\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\sec^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \ 363$$

$$-\frac{bcx\left(\frac{1}{4}\left(\frac{9e}{c^2}+20d\right)\int \frac{x^2}{\sqrt{c^2x^2-1}}dx+\frac{3ex^3\sqrt{c^2x^2-1}}{4c^2}\right)}{15\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\sec^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{262}$$

$$-\frac{bcx\left(\frac{1}{4}\left(\frac{9e}{c^2}+20d\right)\left(\frac{\int \frac{1}{\sqrt{c^2x^2-1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)+\frac{3ex^3\sqrt{c^2x^2-1}}{4c^2}\right)}{15\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\sec^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{224}$$

$$-\frac{bcx\left(\frac{1}{4}\left(\frac{9e}{c^2}+20d\right)\left(\frac{\int \frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}}{2c^2}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)+\frac{3ex^3\sqrt{c^2x^2-1}}{4c^2}\right)}{15\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\sec^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}dx^3\left(a+b\sec^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\sec^{-1}(cx)\right)-\frac{bcx\left(\frac{1}{4}\left(\frac{\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{2c^3}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)\left(\frac{9e}{c^2}+20d\right)+\frac{3ex^3\sqrt{c^2x^2-1}}{4c^2}\right)}{15\sqrt{c^2x^2}}$$

input
```
Int[x^2*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

output
```
(d*x^3*(a + b*ArcSec[c*x]))/3 + (e*x^5*(a + b*ArcSec[c*x]))/5 - (b*c*x*((3
*e*x^3*Sqrt[-1 + c^2*x^2])/(4*c^2) + ((20*d + (9*e)/c^2)*((x*Sqrt[-1 + c^2
*x^2])/(2*c^2) + ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]]/(2*c^3)))/4))/(15*Sqrt[
c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.58

method	result
parts	$a\left(\frac{1}{5}e\,x^5+\frac{1}{3}d\,x^3\right)+\frac{\operatorname{arcsec}(cx)be\,x^5}{5}+\frac{b\operatorname{arcsec}(cx)x^3d}{3}-\frac{b(c^2x^2-1)x^2e}{20c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b(c^2x^2-1)d}{6c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{3b(c^2x^2-1}{40c^5\sqrt{\frac{c^2x^2}{c^2x}}}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b\operatorname{arcsec}(cx)d\,c^3x^3}{3}+\frac{b\,c^3\operatorname{arcsec}(cx)e\,x^5}{5}-\frac{b\left(c^2x^2-1\right)d}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b\left(c^2x^2-1\right)x^2e}{20\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b\sqrt{c^2x^2-1}\,d\ln\left(cx+\sqrt{c^2x^2-1}\right.}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c^3}$
default	$\frac{\frac{a\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b\operatorname{arcsec}(cx)d\,c^3x^3}{3}+\frac{b\,c^3\operatorname{arcsec}(cx)e\,x^5}{5}-\frac{b\left(c^2x^2-1\right)d}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b\left(c^2x^2-1\right)x^2e}{20\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b\sqrt{c^2x^2-1}\,d\ln\left(cx+\sqrt{c^2x^2-1}\right.}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c^3}$

input
```
int(x^2*(e*x^2+d)*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*e*x^5+1/3*d*x^3)+1/5*arcsec(c*x)*b*e*x^5+1/3*b*arcsec(c*x)*x^3*d-1/
20*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2*e-1/6*b/c^3*(c^2*x^2-
1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d-3/40*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x
^2)^(1/2)*e-1/6*b/c^4*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d*ln
(c*x+(c^2*x^2-1)^(1/2))-3/40*b/c^6*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)
^(1/2)/x*e*ln(c*x+(c^2*x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06

$$\int x^2\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{24\,ac^5ex^5+40\,ac^5dx^3+8\,(3\,bc^5ex^5+5\,bc^5dx^3-5\,bc^5d-3\,bc^5e)\operatorname{arcsec}(cx)+16\,(5\,bc^5d+3\,bc^5e)\operatorname{arctan}}{12}$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/120*(24*a*c^5*e*x^5 + 40*a*c^5*d*x^3 + 8*(3*b*c^5*e*x^5 + 5*b*c^5*d*x^3
- 5*b*c^5*d - 3*b*c^5*e)*arcsec(c*x) + 16*(5*b*c^5*d + 3*b*c^5*e)*arctan(-
c*x + sqrt(c^2*x^2 - 1)) + (20*b*c^2*d + 9*b*e)*log(-c*x + sqrt(c^2*x^2 -
1)) - (6*b*c^3*e*x^3 + (20*b*c^3*d + 9*b*c*e)*x)*sqrt(c^2*x^2 - 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 5.50 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.83

$$\int x^2(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{adx^3}{3}+\frac{aex^5}{5}+\frac{bdx^3\operatorname{asec}(cx)}{3}+\frac{bex^5\operatorname{asec}(cx)}{5} - \frac{bd\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c} - \frac{be\left(\begin{cases}\frac{cx^5}{4\sqrt{c^2x^2-1}}+\frac{x^3}{8c\sqrt{c^2x^2-1}}-\frac{3x}{8c^3\sqrt{c^2x^2-1}}+\frac{3\operatorname{acosh}(cx)}{8c^4} & \text{for } |c^2x^2|>1\\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}}-\frac{ix^3}{8c\sqrt{-c^2x^2+1}}+\frac{3ix}{8c^3\sqrt{-c^2x^2+1}}-\frac{3i\operatorname{asin}(cx)}{8c^4} & \text{otherwise}\end{cases}\right)}{5c}$$

input
```
integrate(x**2*(e*x**2+d)*(a+b*asec(c*x)),x)
```

output
```
a*d*x**3/3 + a*e*x**5/5 + b*d*x**3*asec(c*x)/3 + b*e*x**5*asec(c*x)/5 - b*
d*Piecewise((x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x
**2) > 1), (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2
+ 1)) - I*asin(c*x)/(2*c**2), True))/(3*c) - b*e*Piecewise((c*x**5/(4*sqrt
(c**2*x**2 - 1)) + x**3/(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*
x**2 - 1)) + 3*acosh(c*x)/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqr
t(-c**2*x**2 + 1)) - I*x**3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqr
t(-c**2*x**2 + 1)) - 3*I*asin(c*x)/(8*c**4), True))/(5*c)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.44

$$\int x^2(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{5}aex^5+\frac{1}{3}adx^3 + \frac{1}{12}\left(4x^3\,\mathrm{arcsec}\,(cx)-\frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bd + \frac{1}{80}\left(16x^5\,\mathrm{arcsec}\,(cx)+\frac{\frac{2\left(3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}-\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)be$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*e*x^5 + 1/3*a*d*x^3 + 1/12*(4*x^3*arcsec(c*x) - (2*sqrt(-1/(c^2*x^2)
 + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2
- log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d + 1/80*(16*x^5*arcsec(c*x) +
 (2*(3*(-1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(-1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x
^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4) - 3*log(sqrt(-1/(c^2*x^2) + 1)
 + 1)/c^4 + 3*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9792 vs. 2(139) = 278.

Time = 1.52 (sec) , antiderivative size = 9792, normalized size of antiderivative = 60.82

$$\int x^2(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/120*(40*b*c^2*d*arccos(1/(c*x))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x)
+ 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2)
- 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(
1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10) - 20*b*c^2*d*log(abs(sqrt(-1/(c^2*x^2
) + 1) + 1/(c*x) + 1))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10
*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(
c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2)
 - 1)^5/(1/(c*x) + 1)^10) + 20*b*c^2*d*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/
(c*x) - 1))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^
2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6
 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/
(c*x) + 1)^10) + 40*a*c^2*d/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3
/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2
*x^2) - 1)^5/(1/(c*x) + 1)^10) - 40*b*c^2*d*(1/(c^2*x^2) - 1)*arccos(1/(c*
x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2)
- 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^
6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) +
 1)^10)*(1/(c*x) + 1)^2) - 100*b*c^2*d*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(
c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) ...
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx=\int x^2\,\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^2*(d + e*x^2)*(a + b*acos(1/(c*x))),x)
```

output
```
int(x^2*(d + e*x^2)*(a + b*acos(1/(c*x))), x)
```

Reduce [F]

$$\int x^2(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \left(\int asec(cx)\,x^4dx\right)be + \left(\int asec(cx)\,x^2dx\right)bd + \frac{ad\,x^3}{3} + \frac{ae\,x^5}{5}$$

input
```
int(x^2*(e*x^2+d)*(a+b*asec(c*x)),x)
```

output
```
(15*int(asec(c*x)*x**4,x)*b*e + 15*int(asec(c*x)*x**2,x)*b*d + 5*a*d*x**3
+ 3*a*e*x**5)/15
```

3.71 $\int (d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$

Optimal result 605
Mathematica [A] (verified) 606
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Mupad [F(-1)] 612
Reduce [F] 613

Optimal result

Integrand size = 16, antiderivative size = 109

$$\int (d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = -\frac{bex^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}}+dx\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\sec^{-1}(cx)\right)-\frac{b(6c^2d+e)\,x\,\mathrm{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{6c^2\sqrt{c^2x^2}}$$

output
```
-1/6*b*e*x^2*(c^2*x^2-1)^(1/2)/c/(c^2*x^2)^(1/2)+d*x*(a+b*arcsec(c*x))+1/3
*e*x^3*(a+b*arcsec(c*x))-1/6*b*(6*c^2*d+e)*x*arctanh(c*x/(c^2*x^2-1)^(1/2)
)/c^2/(c^2*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.38

$$\int (d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = adx+\frac{1}{3}aex^3-\frac{bex^2\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{6c}+bdx\sec^{-1}(cx)+\frac{1}{3}bex^3\sec^{-1}(cx)-\frac{bd\sqrt{1-\frac{1}{c^2x^2}}x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{-1+c^2x^2}}-\frac{be\log\left(x\left(1+\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\right)\right)}{6c^3}$$

input
```
Integrate[(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

output
```
a*d*x + (a*e*x^3)/3 - (b*e*x^2*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(6*c) + b*d
*x*ArcSec[c*x] + (b*e*x^3*ArcSec[c*x])/3 - (b*d*Sqrt[1 - 1/(c^2*x^2)]*x*Ar
cTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/Sqrt[-1 + c^2*x^2] - (b*e*Log[x*(1 + Sqrt
[(-1 + c^2*x^2)/(c^2*x^2)])])/(6*c^3)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5751, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{ 5751}$$

$$-\frac{bcx\int \frac{ex^2+3d}{3\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+dx\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{bcx\int \frac{ex^2+3d}{\sqrt{c^2x^2-1}}dx}{3\sqrt{c^2x^2}}+dx\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \textbf{299}$$

$$-\frac{bcx\left(\frac{(6c^2d+e)\int \frac{1}{\sqrt{c^2x^2-1}}dx}{2c^2}+\frac{ex\sqrt{c^2x^2-1}}{2c^2}\right)}{3\sqrt{c^2x^2}}+dx\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \textbf{224}$$

$$-\frac{bcx\left(\frac{(6c^2d+e)\int \frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}}{2c^2}+\frac{ex\sqrt{c^2x^2-1}}{2c^2}\right)}{3\sqrt{c^2x^2}}+dx\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \textbf{219}$$

$$dx\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\sec^{-1}(cx)\right)-\frac{bcx\left(\frac{\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)(6c^2d+e)}{2c^3}+\frac{ex\sqrt{c^2x^2-1}}{2c^2}\right)}{3\sqrt{c^2x^2}}$$

input
```
Int[(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

output
```
d*x*(a + b*ArcSec[c*x]) + (e*x^3*(a + b*ArcSec[c*x]))/3 - (b*c*x*((e*x*Sqr
t[-1 + c^2*x^2])/(2*c^2) + ((6*c^2*d + e)*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]
])/(2*c^3)))/(3*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 5751
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSec[c*x])   u,
x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 -
 1]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1
/2, 0])
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.21

method	result
parts	$a\left(\frac{1}{3}x^3e+dx\right)+\frac{b\left(\frac{c\,\mathrm{arcsec}(cx)x^3e}{3}+\mathrm{arcsec}(cx)xcd-\frac{\sqrt{c^2x^2-1}\left(6d\,c^2\ln\left(cx+\sqrt{c^2x^2-1}\right)+ecx\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2\right.\right.}}{6c^3x\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right.}{c}$
derivativedivides	$\frac{\frac{a\left(c^3dx+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b\left(\mathrm{arcsec}(cx)d\,c^3x+\frac{\mathrm{arcsec}(cx)e\,c^3x^3}{3}-\frac{\sqrt{c^2x^2-1}\left(6d\,c^2\ln\left(cx+\sqrt{c^2x^2-1}\right)+ecx\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2x^2-\right.\right.}}{6cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right.}{c^2}}{c}$
default	$\frac{\frac{a\left(c^3dx+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b\left(\mathrm{arcsec}(cx)d\,c^3x+\frac{\mathrm{arcsec}(cx)e\,c^3x^3}{3}-\frac{\sqrt{c^2x^2-1}\left(6d\,c^2\ln\left(cx+\sqrt{c^2x^2-1}\right)+ecx\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2x^2-\right.\right.}}{6cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right.}{c^2}}{c}$

input
```
int((e*x^2+d)*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*x^3*e+d*x)+b/c*(1/3*c*arcsec(c*x)*x^3*e+arcsec(c*x)*x*c*d-1/6/c^3*(
c^2*x^2-1)^(1/2)*(6*d*c^2*ln(c*x+(c^2*x^2-1)^(1/2))+e*c*x*(c^2*x^2-1)^(1/2
)+e*ln(c*x+(c^2*x^2-1)^(1/2)))/x/((c^2*x^2-1)/c^2/x^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.29

$$\int\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{2\,ac^3ex^3+6\,ac^3dx-\sqrt{c^2x^2-1}bcex+2\left(bc^3ex^3+3\,bc^3dx-3\,bc^3d-bc^3e\right)\mathrm{arcsec}\left(cx\right)+4\left(3\,bc^3d+bc^3e\right.}{6\,c^3}$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/6*(2*a*c^3*e*x^3 + 6*a*c^3*d*x - sqrt(c^2*x^2 - 1)*b*c*e*x + 2*(b*c^3*e*
x^3 + 3*b*c^3*d*x - 3*b*c^3*d - b*c^3*e)*arcsec(c*x) + 4*(3*b*c^3*d + b*c^
3*e)*arctan(-c*x + sqrt(c^2*x^2 - 1)) + (6*b*c^2*d + b*e)*log(-c*x + sqrt(
c^2*x^2 - 1)))/c^3
```

Sympy [A] (verification not implemented)

Time = 3.63 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.40

$$\int (d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$= adx + \frac{aex^3}{3} + bdx\operatorname{asec}(cx) + \frac{bex^3\operatorname{asec}(cx)}{3} - \frac{bd\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2| > 1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c}$$
$$- \frac{be\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c} + \frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2| > 1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}} + \frac{ix}{2c\sqrt{-c^2x^2+1}} - \frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$

input
```
integrate((e*x**2+d)*(a+b*asec(c*x)),x)
```

output
```
a*d*x + a*e*x**3/3 + b*d*x*asec(c*x) + b*e*x**3*asec(c*x)/3 - b*d*Piecewis
e((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c - b*e*Piecewis
e((x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1),
 (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*
asin(c*x)/(2*c**2), True))/(3*c)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.41

$$\int (d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$= \frac{1}{3}\,aex^3$$
$$+ \frac{1}{12}\left(4\,x^3\operatorname{arcsec}(cx) - \frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)be$$
$$+ adx + \frac{\left(2\,cx\operatorname{arcsec}(cx) - \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) + \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd}{2\,c}$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*e*x^3 + 1/12*(4*x^3*arcsec(c*x) - (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/
(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/
(c^2*x^2) + 1) - 1)/c^2)/c)*b*e + a*d*x + 1/2*(2*c*x*arcsec(c*x) - log(sqr
t(-1/(c^2*x^2) + 1) + 1) + log(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*d/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4051 vs. $2(95) = 190$.

Time = 1.04 (sec) , antiderivative size = 4051, normalized size of antiderivative = 37.17

$$\int (d + ex^2) (a + b\sec^{-1}(cx)) \, dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/6*(6*b*c^2*d*arccos(1/(c*x))/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1
)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/
(1/(c*x) + 1)^6) - 6*b*c^2*d*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1)
)/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)
^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) + 6*b*c^2*d*
log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(c^4 + 3*c^4*(1/(c^2*x^2) -
 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(
c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) + 6*a*c^2*d/(c^4 + 3*c^4*(1/(c^2*x^2) - 1
)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^
2*x^2) - 1)^3/(1/(c*x) + 1)^6) + 6*b*c^2*d*(1/(c^2*x^2) - 1)*arccos(1/(c*x
))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) -
1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) +
 1)^2) - 18*b*c^2*d*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(
c*x) + 1))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2
*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1
/(c*x) + 1)^2) + 18*b*c^2*d*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) +
1) - 1/(c*x) - 1))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4
*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) +
1)^6)*(1/(c*x) + 1)^2) + 6*a*c^2*d*(1/(c^2*x^2) - 1)/((c^4 + 3*c^4*(1/(c^2
*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 ...
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx=\int (e\,x^2+d)\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int((d + e*x^2)*(a + b*acos(1/(c*x))),x)
```

output
```
int((d + e*x^2)*(a + b*acos(1/(c*x))), x)
```

Reduce [F]

$$\int (d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \left(\int asec(cx)\,dx\right)bd + \left(\int asec(cx)\,x^2dx\right)be + adx + \frac{ae\,x^3}{3}$$

input
```
int((e*x^2+d)*(a+b*asec(c*x)),x)
```

output
```
(3*int(asec(c*x),x)*b*d + 3*int(asec(c*x)*x**2,x)*b*e + 3*a*d*x + a*e*x**3
)/3
```

3.72 $\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^2}\,dx$

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Optimal result

Integrand size = 19, antiderivative size = 87

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^2}\,dx = \frac{bcd\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} - \frac{d(a+b\sec^{-1}(cx))}{x} + ex(a+b\sec^{-1}(cx)) - \frac{bex\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{c^2x^2}}$$

output
```
b*c*d*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)-d*(a+b*arcsec(c*x))/x+e*x*(a+b*arc
sec(c*x))-b*e*x*arctanh(c*x/(c^2*x^2-1)^(1/2))/(c^2*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^2}\,dx = -\frac{ad}{x} + aex + bcd\sqrt{\frac{-1+c^2x^2}{c^2x^2}} - \frac{bd\sec^{-1}(cx)}{x} + bex\sec^{-1}(cx) - \frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{-1+c^2x^2}}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^2,x]
```

output
```
-((a*d)/x) + a*e*x + b*c*d*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)] - (b*d*ArcSec[c*
x])/x + b*e*x*ArcSec[c*x] - (b*e*Sqrt[1 - 1/(c^2*x^2)]*x*ArcTanh[(c*x)/Sqr
t[-1 + c^2*x^2]])/Sqrt[-1 + c^2*x^2]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5761, 25, 358, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^2}\,dx$$

$$\Big\downarrow \text{5761}$$

$$-\frac{bcx\int -\frac{d-ex^2}{x^2\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}-\frac{d(a+b\sec^{-1}(cx))}{x}+ex(a+b\sec^{-1}(cx))$$

$$\Big\downarrow \text{25}$$

$$\frac{bcx\int \frac{d-ex^2}{x^2\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}-\frac{d(a+b\sec^{-1}(cx))}{x}+ex(a+b\sec^{-1}(cx))$$

$$\Big\downarrow \text{358}$$

$$\frac{bcx\left(\frac{d\sqrt{c^2x^2-1}}{x}-e\int \frac{1}{\sqrt{c^2x^2-1}}dx\right)}{\sqrt{c^2x^2}}-\frac{d(a+b\sec^{-1}(cx))}{x}+ex(a+b\sec^{-1}(cx))$$

$$\Big\downarrow \text{224}$$

$$\frac{bcx\left(\frac{d\sqrt{c^2x^2-1}}{x}-e\int \frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2}}-\frac{d(a+b\sec^{-1}(cx))}{x}+ex(a+b\sec^{-1}(cx))$$

$$\Big\downarrow \text{219}$$

$$-\frac{d(a+b\sec^{-1}(cx))}{x}+ex(a+b\sec^{-1}(cx))+\frac{bcx\left(\frac{d\sqrt{c^2x^2-1}}{x}-\frac{e\,\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{c}\right)}{\sqrt{c^2x^2}}$$

input
```
Int[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^2,x]
```

output
```
-((d*(a + b*ArcSec[c*x]))/x) + e*x*(a + b*ArcSec[c*x]) + (b*c*x*((d*Sqrt[-
1 + c^2*x^2])/x - (e*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/c))/Sqrt[c^2*x^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.33

method	result	s
parts	$a\left(ex-\frac{d}{x}\right)+bc\left(\frac{\operatorname{arcsec}(cx)ex}{c}-\frac{\operatorname{arcsec}(cx)d}{xc}-\frac{\sqrt{c^2x^2-1}\left(-d\,c^2\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2x^2-1}\right)cx\right)}{c^4x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right)$	1
derivativedivides	$c\left(\frac{a\left(cex-\frac{dc}{x}\right)}{c^2}+\frac{b\left(c\operatorname{arcsec}(cx)xe-\frac{\operatorname{arcsec}(cx)dc}{x}-\frac{\sqrt{c^2x^2-1}\left(-d\,c^2\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2x^2-1}\right)cx\right)}{c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right)}{c^2}\right)$	1
default	$c\left(\frac{a\left(cex-\frac{dc}{x}\right)}{c^2}+\frac{b\left(c\operatorname{arcsec}(cx)xe-\frac{\operatorname{arcsec}(cx)dc}{x}-\frac{\sqrt{c^2x^2-1}\left(-d\,c^2\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2x^2-1}\right)cx\right)}{c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right)}{c^2}\right)$	1

input
```
int((e*x^2+d)*(a+b*arcsec(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(e*x-d/x)+b*c*(1/c*arcsec(c*x)*e*x-arcsec(c*x)*d/x/c-1/c^4*(c^2*x^2-1)^(
1/2)*(-d*c^2*(c^2*x^2-1)^(1/2)+e*ln(c*x+(c^2*x^2-1)^(1/2))*c*x)/x^2/((c^2*
x^2-1)/c^2/x^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.41

$$\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^2}\,dx$$
$$=\frac{bc^2dx+acex^2+bex\log\left(-cx+\sqrt{c^2x^2-1}\right)+\sqrt{c^2x^2-1}bcd-acd-2\,(bcd-bce)x\arctan\left(-cx+\sqrt{c^2\right.}{cx}$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^2,x, algorithm="fricas")
```

output
```
(b*c^2*d*x + a*c*e*x^2 + b*e*x*log(-c*x + sqrt(c^2*x^2 - 1)) + sqrt(c^2*x^
2 - 1)*b*c*d - a*c*d - 2*(b*c*d - b*c*e)*x*arctan(-c*x + sqrt(c^2*x^2 - 1)
) + (b*c*e*x^2 - b*c*d + (b*c*d - b*c*e)*x)*arcsec(c*x))/(c*x)
```

Sympy [A] (verification not implemented)

Time = 3.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^2}\,dx = -\frac{ad}{x} + aex + bcd\sqrt{1-\frac{1}{c^2x^2}} - \frac{bd\operatorname{asec}(cx)}{x} + bex\operatorname{asec}(cx) - \frac{be\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c}$$

input
```
integrate((e*x**2+d)*(a+b*asec(c*x))/x**2,x)
```

output
```
-a*d/x + a*e*x + b*c*d*sqrt(1 - 1/(c**2*x**2)) - b*d*asec(c*x)/x + b*e*x*a
sec(c*x) - b*e*Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x),
True))/c
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^2}\,dx = \left(c\sqrt{-\frac{1}{c^2x^2}+1} - \frac{\operatorname{arcsec}(cx)}{x}\right)bd + aex + \frac{\left(2\,cx\operatorname{arcsec}(cx) - \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) + \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)be}{2\,c} - \frac{ad}{x}$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^2,x, algorithm="maxima")
```

output
```
(c*sqrt(-1/(c^2*x^2) + 1) - arcsec(c*x)/x)*b*d + a*e*x + 1/2*(2*c*x*arcsec
(c*x) - log(sqrt(-1/(c^2*x^2) + 1) + 1) + log(-sqrt(-1/(c^2*x^2) + 1) + 1)
)*b*e/c - a*d/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1088 vs. $2(79) = 158$.

Time = 0.45 (sec) , antiderivative size = 1088, normalized size of antiderivative = 12.51

$$\int \frac{(d + ex^2)(a + b\sec^{-1}(cx))}{x^2} dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^2,x, algorithm="giac")
```

output
```
-(b*c^2*d*arccos(1/(c*x))/(c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)
+ a*c^2*d/(c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + 2*b*c^2*d*(1/(
c^2*x^2) - 1)*arccos(1/(c*x))/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1
)^4)*(1/(c*x) + 1)^2) - 2*b*c^2*d*sqrt(-1/(c^2*x^2) + 1)/((c^2 - c^2*(1/(c
^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)) + 2*a*c^2*d*(1/(c^2*x^2) -
1)/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) - b*e
*arccos(1/(c*x))/(c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + b*c^2*d
*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c
*x) + 1)^4)*(1/(c*x) + 1)^4) + b*e*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x
) + 1))/(c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) - b*e*log(abs(sqrt
(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x)
 + 1)^4) + 2*b*c^2*d*(-1/(c^2*x^2) + 1)^(3/2)/((c^2 - c^2*(1/(c^2*x^2) - 1
)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^3) - a*e/(c^2 - c^2*(1/(c^2*x^2) - 1)^2
/(1/(c*x) + 1)^4) + a*c^2*d*(1/(c^2*x^2) - 1)^2/((c^2 - c^2*(1/(c^2*x^2) -
 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4) + 2*b*e*(1/(c^2*x^2) - 1)*arccos(1
/(c*x))/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2)
+ 2*a*e*(1/(c^2*x^2) - 1)/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)
*(1/(c*x) + 1)^2) - b*e*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^2 - c^2*(1
/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4) - b*e*(1/(c^2*x^2) - 1
)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^2 - c^2*(1/(c^2*...
```

Mupad [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^2}\,dx = a\,e\,x - \frac{d\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)-b\,c\,x\,\sqrt{1-\frac{1}{c^2\,x^2}}\right)}{x} - \frac{b\,e\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{c^2\,x^2}}}\right)}{c} + b\,e\,x\,\mathrm{acos}\left(\frac{1}{c\,x}\right)$$

input
```
int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^2,x)
```

output
```
a*e*x - (d*(a + b*acos(1/(c*x)) - b*c*x*(1 - 1/(c^2*x^2))^(1/2)))/x - (b*e
*atanh(1/(1 - 1/(c^2*x^2))^(1/2)))/c + b*e*x*acos(1/(c*x))
```

Reduce [F]

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^2}\,dx = \frac{\left(\int asec(cx)\,dx\right)bex + \left(\int \frac{asec(cx)}{x^2}dx\right)bdx - ad + ae\,x^2}{x}$$

input
```
int((e*x^2+d)*(a+b*asec(c*x))/x^2,x)
```

output
```
(int(asec(c*x),x)*b*e*x + int(asec(c*x)/x**2,x)*b*d*x - a*d + a*e*x**2)/x
```

3.73 $\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^4}\,dx$

Optimal result 621
Mathematica [A] (verified) 621
Rubi [A] (verified) 622
Maple [A] (verified) 624
Fricas [A] (verification not implemented) 624
Sympy [A] (verification not implemented) 625
Maxima [A] (verification not implemented) 625
Giac [A] (verification not implemented) 626
Mupad [F(-1)] 626
Reduce [F] 626

Optimal result

Integrand size = 19, antiderivative size = 105

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^4}\,dx = \frac{bc(2c^2d+9e)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}} + \frac{bcd\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}} - \frac{d(a+b\sec^{-1}(cx))}{3x^3} - \frac{e(a+b\sec^{-1}(cx))}{x}$$

output

```
1/9*b*c*(2*c^2*d+9*e)*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)+1/9*b*c*d*(c^2*x^2
-1)^(1/2)/x^2/(c^2*x^2)^(1/2)-1/3*d*(a+b*arcsec(c*x))/x^3-e*(a+b*arcsec(c*
x))/x
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.66

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^4}\,dx = \frac{-3a(d+3ex^2)+bc\sqrt{1-\frac{1}{c^2x^2}}x(d+2c^2dx^2+9ex^2)-3b(d+3ex^2)\sec^{-1}(cx)}{9x^3}$$

input

```
Integrate[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^4,x]
```

output
```
(-3*a*(d + 3*e*x^2) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2*c^2*d*x^2 + 9*e*x
^2) - 3*b*(d + 3*e*x^2)*ArcSec[c*x])/(9*x^3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {5761, 27, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^4}dx$$

$$\Big\downarrow \ 5761$$

$$-\frac{bcx\int -\frac{3ex^2+d}{3x^4\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}-\frac{d(a+b\sec^{-1}(cx))}{3x^3}-\frac{e(a+b\sec^{-1}(cx))}{x}$$

$$\Big\downarrow \ 27$$

$$\frac{bcx\int \frac{3ex^2+d}{x^4\sqrt{c^2x^2-1}}dx}{3\sqrt{c^2x^2}}-\frac{d(a+b\sec^{-1}(cx))}{3x^3}-\frac{e(a+b\sec^{-1}(cx))}{x}$$

$$\Big\downarrow \ 359$$

$$\frac{bcx\left(\frac{1}{3}(2c^2d+9e)\int \frac{1}{x^2\sqrt{c^2x^2-1}}dx+\frac{d\sqrt{c^2x^2-1}}{3x^3}\right)}{3\sqrt{c^2x^2}}-\frac{d(a+b\sec^{-1}(cx))}{3x^3}-\frac{e(a+b\sec^{-1}(cx))}{x}$$

$$\Big\downarrow \ 242$$

$$-\frac{d(a+b\sec^{-1}(cx))}{3x^3}-\frac{e(a+b\sec^{-1}(cx))}{x}+\frac{bcx\left(\frac{\sqrt{c^2x^2-1}(2c^2d+9e)}{3x}+\frac{d\sqrt{c^2x^2-1}}{3x^3}\right)}{3\sqrt{c^2x^2}}$$

input
```
Int[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^4,x]
```

output
```
(b*c*x*((d*Sqrt[-1 + c^2*x^2])/(3*x^3) + ((2*c^2*d + 9*e)*Sqrt[-1 + c^2*x^
2])/(3*x)))/(3*Sqrt[c^2*x^2]) - (d*(a + b*ArcSec[c*x]))/(3*x^3) - (e*(a +
b*ArcSec[c*x]))/x
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.03

method	result	size
parts	$a\left(-\frac{e}{x}-\frac{d}{3x^{3}}\right)+b\,c^{3}\left(-\frac{\operatorname{arcsec}\left(cx\right)e}{c^{3}x}-\frac{\operatorname{arcsec}\left(cx\right)d}{3x^{3}c^{3}}+\frac{\left(c^{2}x^{2}-1\right)\left(2c^{4}d\,x^{2}+9e\,c^{2}x^{2}+c^{2}d\right)}{9c^{6}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,x^{4}}\right)$	108
derivativedivides	$c^{3}\left(\frac{a\left(-\frac{d}{3c\,x^{3}}-\frac{e}{cx}\right)}{c^{2}}+\frac{b\left(-\frac{\operatorname{arcsec}\left(cx\right)d}{3c\,x^{3}}-\frac{\operatorname{arcsec}\left(cx\right)e}{cx}+\frac{\left(c^{2}x^{2}-1\right)\left(2c^{4}d\,x^{2}+9e\,c^{2}x^{2}+c^{2}d\right)}{9\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,c^{4}x^{4}}\right)}{c^{2}}\right)$	121
default	$c^{3}\left(\frac{a\left(-\frac{d}{3c\,x^{3}}-\frac{e}{cx}\right)}{c^{2}}+\frac{b\left(-\frac{\operatorname{arcsec}\left(cx\right)d}{3c\,x^{3}}-\frac{\operatorname{arcsec}\left(cx\right)e}{cx}+\frac{\left(c^{2}x^{2}-1\right)\left(2c^{4}d\,x^{2}+9e\,c^{2}x^{2}+c^{2}d\right)}{9\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,c^{4}x^{4}}\right)}{c^{2}}\right)$	121

input
```
int((e*x^2+d)*(a+b*arcsec(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
a*(-e/x-1/3*d/x^3)+b*c^3*(-1/c^3*arcsec(c*x)*e/x-1/3*arcsec(c*x)*d/x^3/c^3
+1/9/c^6*(c^2*x^2-1)*(2*c^4*d*x^2+9*c^2*e*x^2+c^2*d)/((c^2*x^2-1)/c^2/x^2)
^(1/2)/x^4)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.64

$$\int\frac{(d+ex^{2})\,(a+b\sec^{-1}(cx))}{x^{4}}\,dx$$
$$=-\frac{9\,aex^{2}+3\,ad+3\,(3\,bex^{2}+bd)\operatorname{arcsec}(cx)-\sqrt{c^{2}x^{2}-1}((2\,bc^{2}d+9\,be)x^{2}+bd)}{9\,x^{3}}$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^4,x, algorithm="fricas")
```

output
```
-1/9*(9*a*e*x^2 + 3*a*d + 3*(3*b*e*x^2 + b*d)*arcsec(c*x) - sqrt(c^2*x^2 -
 1)*((2*b*c^2*d + 9*b*e)*x^2 + b*d))/x^3
```

Sympy [A] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.43

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^4}\,dx = -\frac{ad}{3x^3}-\frac{ae}{x}+bce\sqrt{1-\frac{1}{c^2x^2}}-\frac{bd\operatorname{asec}(cx)}{3x^3}-\frac{be\operatorname{asec}(cx)}{x}+\frac{bd\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x}+\frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2|>1\\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x}+\frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c}$$

input
```
integrate((e*x**2+d)*(a+b*asec(c*x))/x**4,x)
```

output
```
-a*d/(3*x**3) - a*e/x + b*c*e*sqrt(1 - 1/(c**2*x**2)) - b*d*asec(c*x)/(3*x
**3) - b*e*asec(c*x)/x + b*d*Piecewise((2*c**3*sqrt(c**2*x**2 - 1)/(3*x) +
 c*sqrt(c**2*x**2 - 1)/(3*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*sqrt(-c**2
*x**2 + 1)/(3*x) + I*c*sqrt(-c**2*x**2 + 1)/(3*x**3), True))/(3*c)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^4}\,dx = \left(c\sqrt{-\frac{1}{c^2x^2}+1}-\frac{\operatorname{arcsec}(cx)}{x}\right)be-\frac{1}{9}\,bd\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{3\,\operatorname{arcsec}(cx)}{x^3}\right)-\frac{ae}{x}-\frac{ad}{3\,x^3}$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^4,x, algorithm="maxima")
```

output
```
(c*sqrt(-1/(c^2*x^2) + 1) - arcsec(c*x)/x)*b*e - 1/9*b*d*((c^4*(-1/(c^2*x^
2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c + 3*arcsec(c*x)/x^3) - a*e
/x - 1/3*a*d/x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^4}\,dx$$
$$=\frac{1}{9}\left(2\,bc^2d\sqrt{-\frac{1}{c^2x^2}+1}+9\,be\sqrt{-\frac{1}{c^2x^2}+1}-\frac{9\,be\arccos\left(\frac{1}{cx}\right)}{cx}+\frac{bd\sqrt{-\frac{1}{c^2x^2}+1}}{x^2}-\frac{9\,ae}{cx}-\frac{3\,bd\arccos\left(\frac{1}{cx}\right)}{cx^3}\right.$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^4,x, algorithm="giac")
```

output
```
1/9*(2*b*c^2*d*sqrt(-1/(c^2*x^2) + 1) + 9*b*e*sqrt(-1/(c^2*x^2) + 1) - 9*b
*e*arccos(1/(c*x))/(c*x) + b*d*sqrt(-1/(c^2*x^2) + 1)/x^2 - 9*a*e/(c*x) -
3*b*d*arccos(1/(c*x))/(c*x^3) - 3*a*d/(c*x^3))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^4}\,dx=\int \frac{(e\,x^2+d)\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

input
```
int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^4,x)
```

output
```
int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^4, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^4}\,dx$$
$$=\frac{3\left(\int \frac{asec(cx)}{x^4}dx\right)bd\,x^3+3\left(\int \frac{asec(cx)}{x^2}dx\right)be\,x^3-ad-3ae\,x^2}{3x^3}$$

input
```
int((e*x^2+d)*(a+b*asec(c*x))/x^4,x)
```

output
```
(3*int(asec(c*x)/x**4,x)*b*d*x**3 + 3*int(asec(c*x)/x**2,x)*b*e*x**3 - a*d
 - 3*a*e*x**2)/(3*x**3)
```

3.74 $\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^6}\,dx$

Optimal result 628
Mathematica [A] (verified) 629
Rubi [A] (verified) 629
Maple [A] (verified) 631
Fricas [A] (verification not implemented) 632
Sympy [A] (verification not implemented) 632
Maxima [A] (verification not implemented) 633
Giac [A] (verification not implemented) 634
Mupad [F(-1)] 634
Reduce [F] 635

Optimal result

Integrand size = 19, antiderivative size = 152

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^6}\,dx = \frac{2bc^3(12c^2d+25e)\sqrt{-1+c^2x^2}}{225\sqrt{c^2x^2}} + \frac{bcd\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}} + \frac{bc(12c^2d+25e)\sqrt{-1+c^2x^2}}{225x^2\sqrt{c^2x^2}} - \frac{d(a+b\sec^{-1}(cx))}{5x^5} - \frac{e(a+b\sec^{-1}(cx))}{3x^3}$$

output
```
2/225*b*c^3*(12*c^2*d+25*e)*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)+1/25*b*c*d*(
c^2*x^2-1)^(1/2)/x^4/(c^2*x^2)^(1/2)+1/225*b*c*(12*c^2*d+25*e)*(c^2*x^2-1)
^(1/2)/x^2/(c^2*x^2)^(1/2)-1/5*d*(a+b*arcsec(c*x))/x^5-1/3*e*(a+b*arcsec(c
*x))/x^3
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.62

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^6}\,dx$$
$$= \frac{-15a(3d+5ex^2)+bc\sqrt{1-\frac{1}{c^2x^2}}x(25ex^2(1+2c^2x^2)+3d(3+4c^2x^2+8c^4x^4))-15b(3d+5ex^2)\sec^{-1}(c}{225x^5}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^6,x]
```

output
```
(-15*a*(3*d + 5*e*x^2) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(25*e*x^2*(1 + 2*c^2*
x^2) + 3*d*(3 + 4*c^2*x^2 + 8*c^4*x^4)) - 15*b*(3*d + 5*e*x^2)*ArcSec[c*x]
)/(225*x^5)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5761, 27, 359, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^6}\,dx$$
$$\Big\downarrow \text{5761}$$
$$-\frac{bcx\int -\frac{5ex^2+3d}{15x^6\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}-\frac{d(a+b\sec^{-1}(cx))}{5x^5}-\frac{e(a+b\sec^{-1}(cx))}{3x^3}$$
$$\Big\downarrow \text{27}$$
$$\frac{bcx\int \frac{5ex^2+3d}{x^6\sqrt{c^2x^2-1}}dx}{15\sqrt{c^2x^2}}-\frac{d(a+b\sec^{-1}(cx))}{5x^5}-\frac{e(a+b\sec^{-1}(cx))}{3x^3}$$
$$\Big\downarrow \text{359}$$

$$\frac{bcx\left(\frac{1}{5}\left(12c^2d+25e\right)\int \frac{1}{x^4\sqrt{c^2x^2-1}}dx+\frac{3d\sqrt{c^2x^2-1}}{5x^5}\right)}{15\sqrt{c^2x^2}}-\frac{d\left(a+b\sec^{-1}(cx)\right)}{5x^5}-\frac{e\left(a+b\sec^{-1}(cx)\right)}{3x^3}$$

$$\Big\downarrow \text{245}$$

$$\frac{bcx\left(\frac{1}{5}\left(12c^2d+25e\right)\left(\frac{2}{3}c^2\int \frac{1}{x^2\sqrt{c^2x^2-1}}dx+\frac{\sqrt{c^2x^2-1}}{3x^3}\right)+\frac{3d\sqrt{c^2x^2-1}}{5x^5}\right)}{15\sqrt{c^2x^2}}-\frac{d\left(a+b\sec^{-1}(cx)\right)}{5x^5}-\frac{e\left(a+b\sec^{-1}(cx)\right)}{3x^3}$$

$$\Big\downarrow \text{242}$$

$$-\frac{d\left(a+b\sec^{-1}(cx)\right)}{5x^5}-\frac{e\left(a+b\sec^{-1}(cx)\right)}{3x^3}+\frac{bcx\left(\frac{1}{5}\left(\frac{2c^2\sqrt{c^2x^2-1}}{3x}+\frac{\sqrt{c^2x^2-1}}{3x^3}\right)\left(12c^2d+25e\right)+\frac{3d\sqrt{c^2x^2-1}}{5x^5}\right)}{15\sqrt{c^2x^2}}$$

input
```
Int[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^6,x]
```

output
```
(b*c*x*((3*d*Sqrt[-1 + c^2*x^2])/(5*x^5) + ((12*c^2*d + 25*e)*(Sqrt[-1 + c
^2*x^2]/(3*x^3) + (2*c^2*Sqrt[-1 + c^2*x^2])/(3*x)))/5))/(15*Sqrt[c^2*x^2]
) - (d*(a + b*ArcSec[c*x]))/(5*x^5) - (e*(a + b*ArcSec[c*x]))/(3*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.84

method	result
parts	$a\left(-\frac{d}{5x^5}-\frac{e}{3x^3}\right)+b\,c^5\left(-\frac{\operatorname{arcsec}(cx)d}{5x^5c^5}-\frac{\operatorname{arcsec}(cx)e}{3c^5x^3}+\frac{(c^2x^2-1)(24c^6d\,x^4+50c^4e\,x^4+12c^4d\,x^2+25e\,c^2x^2}{225c^8\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^6}\right.$
derivativedivides	$c^5\left(\frac{a\left(-\frac{d}{5c^3x^5}-\frac{e}{3c^3x^3}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arcsec}(cx)d}{5c^3x^5}-\frac{\operatorname{arcsec}(cx)e}{3c^3x^3}+\frac{(c^2x^2-1)(24c^6d\,x^4+50c^4e\,x^4+12c^4d\,x^2+25e\,c^2x^2+9c^2d)}{225\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)}{c^2}\right)$
default	$c^5\left(\frac{a\left(-\frac{d}{5c^3x^5}-\frac{e}{3c^3x^3}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arcsec}(cx)d}{5c^3x^5}-\frac{\operatorname{arcsec}(cx)e}{3c^3x^3}+\frac{(c^2x^2-1)(24c^6d\,x^4+50c^4e\,x^4+12c^4d\,x^2+25e\,c^2x^2+9c^2d)}{225\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)}{c^2}\right)$

input
```
int((e*x^2+d)*(a+b*arcsec(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/5*d/x^5-1/3*e/x^3)+b*c^5*(-1/5*arcsec(c*x)*d/x^5/c^5-1/3/c^5*arcsec(
c*x)*e/x^3+1/225/c^8*(c^2*x^2-1)*(24*c^6*d*x^4+50*c^4*e*x^4+12*c^4*d*x^2+2
5*c^2*e*x^2+9*c^2*d)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^6)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.59

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^6}\,dx = -\frac{75\,aex^2+45\,ad+15\,(5\,bex^2+3\,bd)\,\mathrm{arcsec}\,(cx)-(2\,(12\,bc^4d+25\,bc^2e)x^4+(12\,bc^2d+25\,be)x^2+9\,bc}{225\,x^5}$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^6,x, algorithm="fricas")
```

output
```
-1/225*(75*a*e*x^2 + 45*a*d + 15*(5*b*e*x^2 + 3*b*d)*arcsec(c*x) - (2*(12*
b*c^4*d + 25*b*c^2*e)*x^4 + (12*b*c^2*d + 25*b*e)*x^2 + 9*b*d)*sqrt(c^2*x^
2 - 1))/x^5
```

Sympy [A] (verification not implemented)

Time = 6.19 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.84

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^6}\,dx = -\frac{ad}{5x^5}-\frac{ae}{3x^3}-\frac{bd\,\mathrm{asec}\,(cx)}{5x^5}-\frac{be\,\mathrm{asec}\,(cx)}{3x^3} + \frac{bd\left(\begin{cases}\frac{8c^5\sqrt{c^2x^2-1}}{15x}+\frac{4c^3\sqrt{c^2x^2-1}}{15x^3}+\frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for } |c^2x^2|>1\\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x}+\frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3}+\frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise}\end{cases}\right)}{5c} + \frac{be\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x}+\frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2|>1\\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x}+\frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c}$$

input
```
integrate((e*x**2+d)*(a+b*asec(c*x))/x**6,x)
```

output
```
-a*d/(5*x**5) - a*e/(3*x**3) - b*d*asec(c*x)/(5*x**5) - b*e*asec(c*x)/(3*x
**3) + b*d*Piecewise((8*c**5*sqrt(c**2*x**2 - 1)/(15*x) + 4*c**3*sqrt(c**2
*x**2 - 1)/(15*x**3) + c*sqrt(c**2*x**2 - 1)/(5*x**5), Abs(c**2*x**2) > 1)
, (8*I*c**5*sqrt(-c**2*x**2 + 1)/(15*x) + 4*I*c**3*sqrt(-c**2*x**2 + 1)/(1
5*x**3) + I*c*sqrt(-c**2*x**2 + 1)/(5*x**5), True))/(5*c) + b*e*Piecewise(
(2*c**3*sqrt(c**2*x**2 - 1)/(3*x) + c*sqrt(c**2*x**2 - 1)/(3*x**3), Abs(c*
*2*x**2) > 1), (2*I*c**3*sqrt(-c**2*x**2 + 1)/(3*x) + I*c*sqrt(-c**2*x**2
+ 1)/(3*x**3), True))/(3*c)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^6}\,dx$$
$$= \frac{1}{75}\,bd\left(\frac{3\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{15\,\mathrm{arcsec}\,(cx)}{x^5}\right) - \frac{1}{9}\,be\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{3\,\mathrm{arcsec}\,(cx)}{x^3}\right)-\frac{ae}{3\,x^3}-\frac{ad}{5\,x^5}$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^6,x, algorithm="maxima")
```

output
```
1/75*b*d*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(-1/(c^2*x^2) + 1)^(3/2
) + 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c - 15*arcsec(c*x)/x^5) - 1/9*b*e*((c^4
*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c + 3*arcsec(c*x
)/x^3) - 1/3*a*e/x^3 - 1/5*a*d/x^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^6}\,dx$$
$$= \frac{1}{225}\left(24\,bc^4d\sqrt{-\frac{1}{c^2x^2}+1}+50\,bc^2e\sqrt{-\frac{1}{c^2x^2}+1}+\frac{12\,bc^2d\sqrt{-\frac{1}{c^2x^2}+1}}{x^2}+\frac{25\,be\sqrt{-\frac{1}{c^2x^2}+1}}{x^2}-\frac{75\,be\,\mathrm{arc}}{cx}\right.$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^6,x, algorithm="giac")
```

output
```
1/225*(24*b*c^4*d*sqrt(-1/(c^2*x^2) + 1) + 50*b*c^2*e*sqrt(-1/(c^2*x^2) +
1) + 12*b*c^2*d*sqrt(-1/(c^2*x^2) + 1)/x^2 + 25*b*e*sqrt(-1/(c^2*x^2) + 1)
/x^2 - 75*b*e*arccos(1/(c*x))/(c*x^3) + 9*b*d*sqrt(-1/(c^2*x^2) + 1)/x^4 -
 75*a*e/(c*x^3) - 45*b*d*arccos(1/(c*x))/(c*x^5) - 45*a*d/(c*x^5))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^6}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

input
```
int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^6,x)
```

output
```
int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^6, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^6}\,dx = \frac{15\left(\int \frac{asec(cx)}{x^6}dx\right)bd\,x^5+15\left(\int \frac{asec(cx)}{x^4}dx\right)be\,x^5-3ad-5ae\,x^2}{15x^5}$$

input

```
int((e*x^2+d)*(a+b*asec(c*x))/x^6,x)
```

output

```
(15*int(asec(c*x)/x**6,x)*b*d*x**5 + 15*int(asec(c*x)/x**4,x)*b*e*x**5 - 3
*a*d - 5*a*e*x**2)/(15*x**5)
```

3.75 $\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^8}\,dx$

Optimal result . . . 636
Mathematica [A] (verified) . . . 637
Rubi [A] (verified) . . . 637
Maple [A] (verified) . . . 640
Fricas [A] (verification not implemented) . . . 640
Sympy [A] (verification not implemented) . . . 641
Maxima [A] (verification not implemented) . . . 642
Giac [A] (verification not implemented) . . . 642
Mupad [F(-1)] . . . 643
Reduce [F] . . . 643

Optimal result

Integrand size = 19, antiderivative size = 197

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^8}\,dx = \frac{8bc^5(30c^2d+49e)\sqrt{-1+c^2x^2}}{3675\sqrt{c^2x^2}} + \frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} + \frac{bc(30c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}} + \frac{4bc^3(30c^2d+49e)\sqrt{-1+c^2x^2}}{3675x^2\sqrt{c^2x^2}} - \frac{d(a+b\sec^{-1}(cx))}{7x^7} - \frac{e(a+b\sec^{-1}(cx))}{5x^5}$$

output
```
8/3675*b*c^5*(30*c^2*d+49*e)*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)+1/49*b*c*d*
(c^2*x^2-1)^(1/2)/x^6/(c^2*x^2)^(1/2)+1/1225*b*c*(30*c^2*d+49*e)*(c^2*x^2-
1)^(1/2)/x^4/(c^2*x^2)^(1/2)+4/3675*b*c^3*(30*c^2*d+49*e)*(c^2*x^2-1)^(1/2
)/x^2/(c^2*x^2)^(1/2)-1/7*d*(a+b*arcsec(c*x))/x^7-1/5*e*(a+b*arcsec(c*x))/
x^5
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.56

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^8}\,dx = \frac{-105a(5d+7ex^2)+bc\sqrt{1-\frac{1}{c^2x^2}}x(49ex^2(3+4c^2x^2+8c^4x^4)+15d(5+6c^2x^2+8c^4x^4+16c^6x^6))-10}{3675x^7}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^8,x]
```

output
```
(-105*a*(5*d + 7*e*x^2) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(49*e*x^2*(3 + 4*c^2
*x^2 + 8*c^4*x^4) + 15*d*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c^6*x^6)) - 105*b
*(5*d + 7*e*x^2)*ArcSec[c*x])/(3675*x^7)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5761, 27, 359, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^8}\,dx$$

$$\Big\downarrow \ 5761$$

$$-\frac{bcx\int -\frac{7ex^2+5d}{35x^8\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}-\frac{d(a+b\sec^{-1}(cx))}{7x^7}-\frac{e(a+b\sec^{-1}(cx))}{5x^5}$$

$$\Big\downarrow \ 27$$

$$\frac{bcx\int \frac{7ex^2+5d}{x^8\sqrt{c^2x^2-1}}dx}{35\sqrt{c^2x^2}}-\frac{d(a+b\sec^{-1}(cx))}{7x^7}-\frac{e(a+b\sec^{-1}(cx))}{5x^5}$$

$$\Big\downarrow \ 359$$

$$\frac{bcx\left(\frac{1}{7}(30c^2d+49e)\int \frac{1}{x^6\sqrt{c^2x^2-1}}dx+\frac{5d\sqrt{c^2x^2-1}}{7x^7}\right)}{35\sqrt{c^2x^2}}-\frac{d(a+b\sec^{-1}(cx))}{7x^7}-\frac{e(a+b\sec^{-1}(cx))}{5x^5}$$

$$\Big\downarrow \text{245}$$

$$\frac{bcx\left(\frac{1}{7}(30c^2d+49e)\left(\frac{4}{5}c^2\int \frac{1}{x^4\sqrt{c^2x^2-1}}dx+\frac{\sqrt{c^2x^2-1}}{5x^5}\right)+\frac{5d\sqrt{c^2x^2-1}}{7x^7}\right)}{35\sqrt{c^2x^2}}-\frac{d(a+b\sec^{-1}(cx))}{7x^7}-\frac{e(a+b\sec^{-1}(cx))}{5x^5}$$

$$\Big\downarrow \text{245}$$

$$\frac{bcx\left(\frac{1}{7}(30c^2d+49e)\left(\frac{4}{5}c^2\left(\frac{2}{3}c^2\int \frac{1}{x^2\sqrt{c^2x^2-1}}dx+\frac{\sqrt{c^2x^2-1}}{3x^3}\right)+\frac{\sqrt{c^2x^2-1}}{5x^5}\right)+\frac{5d\sqrt{c^2x^2-1}}{7x^7}\right)}{35\sqrt{c^2x^2}}-\frac{d(a+b\sec^{-1}(cx))}{7x^7}-\frac{e(a+b\sec^{-1}(cx))}{5x^5}$$

$$\Big\downarrow \text{242}$$

$$-\frac{d(a+b\sec^{-1}(cx))}{7x^7}-\frac{e(a+b\sec^{-1}(cx))}{5x^5}+\frac{bcx\left(\frac{1}{7}\left(\frac{\sqrt{c^2x^2-1}}{5x^5}+\frac{4}{5}c^2\left(\frac{2c^2\sqrt{c^2x^2-1}}{3x}+\frac{\sqrt{c^2x^2-1}}{3x^3}\right)\right)(30c^2d+49e)+\frac{5d\sqrt{c^2x^2-1}}{7x^7}\right)}{35\sqrt{c^2x^2}}$$

input
```
Int[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^8,x]
```

output
```
(b*c*x*((5*d*Sqrt[-1 + c^2*x^2])/(7*x^7) + ((30*c^2*d + 49*e)*(Sqrt[-1 + c
^2*x^2]/(5*x^5) + (4*c^2*(Sqrt[-1 + c^2*x^2]/(3*x^3) + (2*c^2*Sqrt[-1 + c^
2*x^2])/(3*x)))/5))/7))/(35*Sqrt[c^2*x^2]) - (d*(a + b*ArcSec[c*x]))/(7*x^
7) - (e*(a + b*ArcSec[c*x]))/(5*x^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.74

method	result
parts	$a\left(-\frac{d}{7x^7}-\frac{e}{5x^5}\right)+b\,c^7\left(-\frac{\mathrm{arcsec}(cx)d}{7x^7c^7}-\frac{\mathrm{arcsec}(cx)e}{5c^7x^5}+\frac{(c^2x^2-1)(240c^8d\,x^6+392c^6e\,x^6+120c^6d\,x^4+196c^4}{3675c^{10}\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right.$
derivativedivides	$c^7\left(\frac{a\left(-\frac{e}{5c^5x^5}-\frac{d}{7c^5x^7}\right)}{c^2}+\frac{b\left(-\frac{\mathrm{arcsec}(cx)e}{5c^5x^5}-\frac{\mathrm{arcsec}(cx)d}{7c^5x^7}+\frac{(c^2x^2-1)(240c^8d\,x^6+392c^6e\,x^6+120c^6d\,x^4+196c^4e\,x^4+90c^4}{3675\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^8x^8}\right.}{c^2}\right.$
default	$c^7\left(\frac{a\left(-\frac{e}{5c^5x^5}-\frac{d}{7c^5x^7}\right)}{c^2}+\frac{b\left(-\frac{\mathrm{arcsec}(cx)e}{5c^5x^5}-\frac{\mathrm{arcsec}(cx)d}{7c^5x^7}+\frac{(c^2x^2-1)(240c^8d\,x^6+392c^6e\,x^6+120c^6d\,x^4+196c^4e\,x^4+90c^4}{3675\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^8x^8}\right.}{c^2}\right.$

input
```
int((e*x^2+d)*(a+b*arcsec(c*x))/x^8,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/7*d/x^7-1/5*e/x^5)+b*c^7*(-1/7*arcsec(c*x)*d/x^7/c^7-1/5/c^7*arcsec(
c*x)*e/x^5+1/3675/c^10*(c^2*x^2-1)*(240*c^8*d*x^6+392*c^6*e*x^6+120*c^6*d*
x^4+196*c^4*e*x^4+90*c^4*d*x^2+147*c^2*e*x^2+75*c^2*d)/((c^2*x^2-1)/c^2/x^
2)^(1/2)/x^8)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.56

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^8}\,dx =$$
$$-\frac{735\,aex^2+525\,ad+105\,(7\,bex^2+5\,bd)\,\mathrm{arcsec}\,(cx)-(8\,(30\,bc^6d+49\,bc^4e)x^6+4\,(30\,bc^4d+49\,bc^2e)x}{3675\,x^7}$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^8,x, algorithm="fricas")
```

output
```
-1/3675*(735*a*e*x^2 + 525*a*d + 105*(7*b*e*x^2 + 5*b*d)*arcsec(c*x) - (8*
(30*b*c^6*d + 49*b*c^4*e)*x^6 + 4*(30*b*c^4*d + 49*b*c^2*e)*x^4 + 3*(30*b*
c^2*d + 49*b*e)*x^2 + 75*b*d)*sqrt(c^2*x^2 - 1))/x^7
```

Sympy [A] (verification not implemented)

Time = 34.50 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.88

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^8}\,dx = -\frac{ad}{7x^7} - \frac{ae}{5x^5} - \frac{bd\,\mathrm{asec}(cx)}{7x^7} - \frac{be\,\mathrm{asec}(cx)}{5x^5} + \frac{bd\left(\begin{cases}\frac{16c^7\sqrt{c^2x^2-1}}{35x} + \frac{8c^5\sqrt{c^2x^2-1}}{35x^3} + \frac{6c^3\sqrt{c^2x^2-1}}{35x^5} + \frac{c\sqrt{c^2x^2-1}}{7x^7} & \text{for } |c^2x^2| > 1\\ \frac{16ic^7\sqrt{-c^2x^2+1}}{35x} + \frac{8ic^5\sqrt{-c^2x^2+1}}{35x^3} + \frac{6ic^3\sqrt{-c^2x^2+1}}{35x^5} + \frac{ic\sqrt{-c^2x^2+1}}{7x^7} & \text{otherwise}\end{cases}\right)}{7c} + \frac{be\left(\begin{cases}\frac{8c^5\sqrt{c^2x^2-1}}{15x} + \frac{4c^3\sqrt{c^2x^2-1}}{15x^3} + \frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for } |c^2x^2| > 1\\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x} + \frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3} + \frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise}\end{cases}\right)}{5c}$$

input
```
integrate((e*x**2+d)*(a+b*asec(c*x))/x**8,x)
```

output
```
-a*d/(7*x**7) - a*e/(5*x**5) - b*d*asec(c*x)/(7*x**7) - b*e*asec(c*x)/(5*x
**5) + b*d*Piecewise((16*c**7*sqrt(c**2*x**2 - 1)/(35*x) + 8*c**5*sqrt(c**
2*x**2 - 1)/(35*x**3) + 6*c**3*sqrt(c**2*x**2 - 1)/(35*x**5) + c*sqrt(c**2
*x**2 - 1)/(7*x**7), Abs(c**2*x**2) > 1), (16*I*c**7*sqrt(-c**2*x**2 + 1)/
(35*x) + 8*I*c**5*sqrt(-c**2*x**2 + 1)/(35*x**3) + 6*I*c**3*sqrt(-c**2*x**
2 + 1)/(35*x**5) + I*c*sqrt(-c**2*x**2 + 1)/(7*x**7), True))/(7*c) + b*e*P
iecewise((8*c**5*sqrt(c**2*x**2 - 1)/(15*x) + 4*c**3*sqrt(c**2*x**2 - 1)/(
15*x**3) + c*sqrt(c**2*x**2 - 1)/(5*x**5), Abs(c**2*x**2) > 1), (8*I*c**5*
sqrt(-c**2*x**2 + 1)/(15*x) + 4*I*c**3*sqrt(-c**2*x**2 + 1)/(15*x**3) + I*
c*sqrt(-c**2*x**2 + 1)/(5*x**5), True))/(5*c)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^8}\,dx =$$
$$-\frac{1}{245}\,bd\left(\frac{5\,c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}}-21\,c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+35\,c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-35\,c^8\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{35\,\mathrm{arcsec}\,(\right.$$
$$+\frac{1}{75}\,be\left(\frac{3\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{15\,\mathrm{arcsec}\,(cx)}{x^5}\right)$$
$$-\frac{ae}{5\,x^5}-\frac{ad}{7\,x^7}$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^8,x, algorithm="maxima")
```

output
```
-1/245*b*d*((5*c^8*(-1/(c^2*x^2) + 1)^(7/2) - 21*c^8*(-1/(c^2*x^2) + 1)^(5
/2) + 35*c^8*(-1/(c^2*x^2) + 1)^(3/2) - 35*c^8*sqrt(-1/(c^2*x^2) + 1))/c +
 35*arcsec(c*x)/x^7) + 1/75*b*e*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*c^6*
(-1/(c^2*x^2) + 1)^(3/2) + 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c - 15*arcsec(c*
x)/x^5) - 1/5*a*e/x^5 - 1/7*a*d/x^7
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^8}\,dx$$
$$=\frac{1}{3675}\left(240\,bc^6d\sqrt{-\frac{1}{c^2x^2}+1}+392\,bc^4e\sqrt{-\frac{1}{c^2x^2}+1}+\frac{120\,bc^4d\sqrt{-\frac{1}{c^2x^2}+1}}{x^2}+\frac{196\,bc^2e\sqrt{-\frac{1}{c^2x^2}+1}}{x^2}+\right.$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^8,x, algorithm="giac")
```

output
```
1/3675*(240*b*c^6*d*sqrt(-1/(c^2*x^2) + 1) + 392*b*c^4*e*sqrt(-1/(c^2*x^2)
 + 1) + 120*b*c^4*d*sqrt(-1/(c^2*x^2) + 1)/x^2 + 196*b*c^2*e*sqrt(-1/(c^2*
x^2) + 1)/x^2 + 90*b*c^2*d*sqrt(-1/(c^2*x^2) + 1)/x^4 + 147*b*e*sqrt(-1/(c
^2*x^2) + 1)/x^4 - 735*b*e*arccos(1/(c*x))/(c*x^5) + 75*b*d*sqrt(-1/(c^2*x
^2) + 1)/x^6 - 735*a*e/(c*x^5) - 525*b*d*arccos(1/(c*x))/(c*x^7) - 525*a*d
/(c*x^7))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^8}\,dx=\int \frac{(e\,x^2+d)\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

input
```
int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^8,x)
```

output
```
int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^8, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^8}\,dx=\frac{35\left(\int \frac{asec(cx)}{x^8}dx\right)bd\,x^7+35\left(\int \frac{asec(cx)}{x^6}dx\right)be\,x^7-5ad-7ae\,x^2}{35x^7}$$

input
```
int((e*x^2+d)*(a+b*asec(c*x))/x^8,x)
```

output
```
(35*int(asec(c*x)/x**8,x)*b*d*x**7 + 35*int(asec(c*x)/x**6,x)*b*e*x**7 - 5
*a*d - 7*a*e*x**2)/(35*x**7)
```

3.76 $\int x^5(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$

Optimal result 644
Mathematica [A] (verified) 645
Rubi [A] (verified) 645
Maple [A] (verified) 648
Fricas [A] (verification not implemented) 648
Sympy [A] (verification not implemented) 649
Maxima [A] (verification not implemented) 650
Giac [B] (verification not implemented) 650
Mupad [F(-1)] 651
Reduce [F] 652

Optimal result

Integrand size = 19, antiderivative size = 196

$$\int x^5\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx=-\frac{b(4c^2d+3e)\,x\sqrt{-1+c^2x^2}}{24c^7\sqrt{c^2x^2}}-\frac{b(8c^2d+9e)\,x(-1+c^2x^2)^{3/2}}{72c^7\sqrt{c^2x^2}}-\frac{b(4c^2d+9e)\,x(-1+c^2x^2)^{5/2}}{120c^7\sqrt{c^2x^2}}-\frac{bex(-1+c^2x^2)^{7/2}}{56c^7\sqrt{c^2x^2}}+\frac{1}{6}dx^6\left(a+b\sec^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\sec^{-1}(cx)\right)$$

output
```
-1/24*b*(4*c^2*d+3*e)*x*(c^2*x^2-1)^(1/2)/c^7/(c^2*x^2)^(1/2)-1/72*b*(8*c^
2*d+9*e)*x*(c^2*x^2-1)^(3/2)/c^7/(c^2*x^2)^(1/2)-1/120*b*(4*c^2*d+9*e)*x*(
c^2*x^2-1)^(5/2)/c^7/(c^2*x^2)^(1/2)-1/56*b*e*x*(c^2*x^2-1)^(7/2)/c^7/(c^2
*x^2)^(1/2)+1/6*d*x^6*(a+b*arcsec(c*x))+1/8*e*x^8*(a+b*arcsec(c*x))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.60

$$\int x^5(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{1}{24}ax^6(4d+3ex^2)$$
$$-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x(144e+8c^2(28d+9ex^2)+2c^4(56dx^2+27ex^4)+c^6(84dx^4+45ex^6))}{2520c^7}$$
$$+\frac{1}{24}bx^6(4d+3ex^2)\sec^{-1}(cx)$$

input
```
Integrate[x^5*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

output
```
(a*x^6*(4*d + 3*e*x^2))/24 - (b*Sqrt[1 - 1/(c^2*x^2)]*x*(144*e + 8*c^2*(28
*d + 9*e*x^2) + 2*c^4*(56*d*x^2 + 27*e*x^4) + c^6*(84*d*x^4 + 45*e*x^6)))/
(2520*c^7) + (b*x^6*(4*d + 3*e*x^2)*ArcSec[c*x])/24
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5761, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{5761}$$

$$-\frac{bcx\int\frac{x^5(3ex^2+4d)}{24\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{1}{6}dx^6\left(a+b\sec^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{bcx\int \frac{x^5(3ex^2+4d)}{\sqrt{c^2x^2-1}}dx}{24\sqrt{c^2x^2}}+\frac{1}{6}dx^6\left(a+b\sec^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \; 354$$

$$-\frac{bcx\int \frac{x^4(3ex^2+4d)}{\sqrt{c^2x^2-1}}dx^2}{48\sqrt{c^2x^2}}+\frac{1}{6}dx^6\left(a+b\sec^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \; 86$$

$$-\frac{bcx\int \left(\frac{3e(c^2x^2-1)^{5/2}}{c^6}+\frac{(4dc^2+9e)(c^2x^2-1)^{3/2}}{c^6}+\frac{(8dc^2+9e)\sqrt{c^2x^2-1}}{c^6}+\frac{4dc^2+3e}{c^6\sqrt{c^2x^2-1}}\right)dx^2}{48\sqrt{c^2x^2}}+\frac{1}{6}dx^6\left(a+b\sec^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{6}dx^6\left(a+b\sec^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\sec^{-1}(cx)\right)-\frac{bcx\left(\frac{2(c^2x^2-1)^{5/2}(4c^2d+9e)}{5c^8}+\frac{2(c^2x^2-1)^{3/2}(8c^2d+9e)}{3c^8}+\frac{2\sqrt{c^2x^2-1}(4c^2d+3e)}{c^8}+\frac{6e(c^2x^2-1)^{7/2}}{7c^8}\right)}{48\sqrt{c^2x^2}}$$

input
```
Int[x^5*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

output
```
-1/48*(b*c*x*((2*(4*c^2*d + 3*e)*Sqrt[-1 + c^2*x^2])/c^8 + (2*(8*c^2*d + 9
*e)*(-1 + c^2*x^2)^(3/2))/(3*c^8) + (2*(4*c^2*d + 9*e)*(-1 + c^2*x^2)^(5/2
))/(5*c^8) + (6*e*(-1 + c^2*x^2)^(7/2))/(7*c^8)))/Sqrt[c^2*x^2] + (d*x^6*(
a + b*ArcSec[c*x]))/6 + (e*x^8*(a + b*ArcSec[c*x]))/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.71

method	result
parts	$a\left(\frac{1}{8}e\,x^{8}+\frac{1}{6}x^{6}d\right)+\frac{b\left(\frac{c^{6}\,\mathrm{arcsec}(cx)e\,x^{8}}{8}+\frac{\mathrm{arcsec}(cx)d\,x^{6}c^{6}}{6}-\frac{\left(c^{2}x^{2}-1\right)\left(45c^{6}e\,x^{6}+84c^{6}d\,x^{4}+54c^{4}e\,x^{4}+112c^{4}d\,x^{2}+72e \cdots\right.}{2520c^{3}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,x}\right.}{c^{6}}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{6}c^{8}d\,x^{6}+\frac{1}{8}e\,c^{8}x^{8}\right)}{c^{2}}+\frac{b\left(\frac{\mathrm{arcsec}(cx)d\,c^{8}x^{6}}{6}+\frac{\mathrm{arcsec}(cx)e\,c^{8}x^{8}}{8}-\frac{\left(c^{2}x^{2}-1\right)\left(45c^{6}e\,x^{6}+84c^{6}d\,x^{4}+54c^{4}e\,x^{4}+112c^{4}d\,x^{2}+72e\,c^{2}x^{2} \cdots\right.}{2520\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}\right.}{c^{2}}}{c^{6}}$
default	$\frac{\frac{a\left(\frac{1}{6}c^{8}d\,x^{6}+\frac{1}{8}e\,c^{8}x^{8}\right)}{c^{2}}+\frac{b\left(\frac{\mathrm{arcsec}(cx)d\,c^{8}x^{6}}{6}+\frac{\mathrm{arcsec}(cx)e\,c^{8}x^{8}}{8}-\frac{\left(c^{2}x^{2}-1\right)\left(45c^{6}e\,x^{6}+84c^{6}d\,x^{4}+54c^{4}e\,x^{4}+112c^{4}d\,x^{2}+72e\,c^{2}x^{2} \cdots\right.}{2520\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}\right.}{c^{2}}}{c^{6}}$

input
```
int(x^5*(e*x^2+d)*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/8*e*x^8+1/6*x^6*d)+b/c^6*(1/8*c^6*arcsec(c*x)*e*x^8+1/6*arcsec(c*x)*d
*x^6*c^6-1/2520/c^3*(c^2*x^2-1)*(45*c^6*e*x^6+84*c^6*d*x^4+54*c^4*e*x^4+11
2*c^4*d*x^2+72*c^2*e*x^2+224*c^2*d+144*e)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.65

$$\int x^5\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{315\,ac^8ex^8+420\,ac^8dx^6+105\,(3\,bc^8ex^8+4\,bc^8dx^6)\,\mathrm{arcsec}\,(cx)-(45\,bc^6ex^6+6\,(14\,bc^6d+9\,bc^4e)x^4+2 \cdots}{2520\,c^8}$$

input
```
integrate(x^5*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/2520*(315*a*c^8*e*x^8 + 420*a*c^8*d*x^6 + 105*(3*b*c^8*e*x^8 + 4*b*c^8*d
*x^6)*arcsec(c*x) - (45*b*c^6*e*x^6 + 6*(14*b*c^6*d + 9*b*c^4*e)*x^4 + 224
*b*c^2*d + 8*(14*b*c^4*d + 9*b*c^2*e)*x^2 + 144*b*e)*sqrt(c^2*x^2 - 1))/c^
8
```

Sympy [A] (verification not implemented)

Time = 4.39 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.86

$$\int x^5(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$= \frac{adx^6}{6}+\frac{aex^8}{8}+\frac{bdx^6\operatorname{asec}(cx)}{6}+\frac{bex^8\operatorname{asec}(cx)}{8} - \frac{bd\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c}+\frac{4x^2\sqrt{c^2x^2-1}}{15c^3}+\frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for } |c^2x^2|>1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c}+\frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3}+\frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{6c} - \frac{be\left(\begin{cases}\frac{x^6\sqrt{c^2x^2-1}}{7c}+\frac{6x^4\sqrt{c^2x^2-1}}{35c^3}+\frac{8x^2\sqrt{c^2x^2-1}}{35c^5}+\frac{16\sqrt{c^2x^2-1}}{35c^7} & \text{for } |c^2x^2|>1\\ \frac{ix^6\sqrt{-c^2x^2+1}}{7c}+\frac{6ix^4\sqrt{-c^2x^2+1}}{35c^3}+\frac{8ix^2\sqrt{-c^2x^2+1}}{35c^5}+\frac{16i\sqrt{-c^2x^2+1}}{35c^7} & \text{otherwise}\end{cases}\right)}{8c}$$

input
```
integrate(x**5*(e*x**2+d)*(a+b*asec(c*x)),x)
```

output
```
a*d*x**6/6 + a*e*x**8/8 + b*d*x**6*asec(c*x)/6 + b*e*x**8*asec(c*x)/8 - b*
d*Piecewise((x**4*sqrt(c**2*x**2 - 1)/(5*c) + 4*x**2*sqrt(c**2*x**2 - 1)/(
15*c**3) + 8*sqrt(c**2*x**2 - 1)/(15*c**5), Abs(c**2*x**2) > 1), (I*x**4*s
qrt(-c**2*x**2 + 1)/(5*c) + 4*I*x**2*sqrt(-c**2*x**2 + 1)/(15*c**3) + 8*I*
sqrt(-c**2*x**2 + 1)/(15*c**5), True))/(6*c) - b*e*Piecewise((x**6*sqrt(c*
*2*x**2 - 1)/(7*c) + 6*x**4*sqrt(c**2*x**2 - 1)/(35*c**3) + 8*x**2*sqrt(c*
*2*x**2 - 1)/(35*c**5) + 16*sqrt(c**2*x**2 - 1)/(35*c**7), Abs(c**2*x**2)
> 1), (I*x**6*sqrt(-c**2*x**2 + 1)/(7*c) + 6*I*x**4*sqrt(-c**2*x**2 + 1)/(
35*c**3) + 8*I*x**2*sqrt(-c**2*x**2 + 1)/(35*c**5) + 16*I*sqrt(-c**2*x**2
+ 1)/(35*c**7), True))/(8*c)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.94

$$\int x^5 (d + ex^2) (a + b\sec^{-1}(cx)) \, dx = \frac{1}{8} aex^8 + \frac{1}{6} adx^6 + \frac{1}{90} \left(15\, x^6 \operatorname{arcsec}(cx) - \frac{3\, c^4 x^5 \left(-\frac{1}{c^2x^2} + 1\right)^{\frac{5}{2}} + 10\, c^2 x^3 \left(-\frac{1}{c^2x^2} + 1\right)^{\frac{3}{2}} + 15\, x \sqrt{-\frac{1}{c^2x^2} + 1}}{c^5} \right) bd + \frac{1}{280} \left(35\, x^8 \operatorname{arcsec}(cx) - \frac{5\, c^6 x^7 \left(-\frac{1}{c^2x^2} + 1\right)^{\frac{7}{2}} + 21\, c^4 x^5 \left(-\frac{1}{c^2x^2} + 1\right)^{\frac{5}{2}} + 35\, c^2 x^3 \left(-\frac{1}{c^2x^2} + 1\right)^{\frac{3}{2}} + 35\, x \sqrt{-\frac{1}{c^2x^2} + 1}}{c^7} \right)$$

input
```
integrate(x^5*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/8*a*e*x^8 + 1/6*a*d*x^6 + 1/90*(15*x^6*arcsec(c*x) - (3*c^4*x^5*(-1/(c^2
*x^2) + 1)^(5/2) + 10*c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(-1/(c^2
*x^2) + 1))/c^5)*b*d + 1/280*(35*x^8*arcsec(c*x) - (5*c^6*x^7*(-1/(c^2*x^2
) + 1)^(7/2) + 21*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2) + 35*c^2*x^3*(-1/(c^2*x
^2) + 1)^(3/2) + 35*x*sqrt(-1/(c^2*x^2) + 1))/c^7)*b*e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13018 vs. $2(168) = 336$.

Time = 0.26 (sec) , antiderivative size = 13018, normalized size of antiderivative = 66.42

$$\int x^5 (d + ex^2) (a + b\sec^{-1}(cx)) \, dx = \text{Too large to display}$$

input
```
integrate(x^5*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/2520*(420*b*c^2*d*arccos(1/(c*x))/(c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x
) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2
) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56
*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/
(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x
^2) - 1)^8/(1/(c*x) + 1)^16) + 420*a*c^2*d/(c^9 + 8*c^9*(1/(c^2*x^2) - 1)/
(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(
c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)
^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1
)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1
/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16) - 1680*b*c^2*d*(1/(c^2*x^2) - 1)*arcco
s(1/(c*x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^
2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6
 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5
/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1
/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)
^16)*(1/(c*x) + 1)^2) - 840*b*c^2*d*sqrt(-1/(c^2*x^2) + 1)/((c^9 + 8*c^9*(
1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1
)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1
)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*...
```

Mupad [F(-1)]

Timed out.

$$\int x^5(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx=\int x^5\left(e\,x^2+d\right)\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^5*(d + e*x^2)*(a + b*acos(1/(c*x))),x)
```

output
```
int(x^5*(d + e*x^2)*(a + b*acos(1/(c*x))), x)
```

Reduce [F]

$$\int x^5(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx=\left(\int asec(cx)\,x^7dx\right)be+\left(\int asec(cx)\,x^5dx\right)bd+\frac{ad\,x^6}{6}+\frac{ae\,x^8}{8}$$

input
```
int(x^5*(e*x^2+d)*(a+b*asec(c*x)),x)
```

output
```
(24*int(asec(c*x)*x**7,x)*b*e + 24*int(asec(c*x)*x**5,x)*b*d + 4*a*d*x**6
+ 3*a*e*x**8)/24
```

3.77 $\int x^3(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$

Optimal result . . . 653
Mathematica [A] (verified) . . . 654
Rubi [A] (verified) . . . 654
Maple [A] (verified) . . . 657
Fricas [A] (verification not implemented) . . . 657
Sympy [A] (verification not implemented) . . . 658
Maxima [A] (verification not implemented) . . . 658
Giac [B] (verification not implemented) . . . 659
Mupad [F(-1)] . . . 660
Reduce [F] . . . 661

Optimal result

Integrand size = 19, antiderivative size = 153

$$\int x^3(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = -\frac{b(3c^2d+2e)\,x\sqrt{-1+c^2x^2}}{12c^5\sqrt{c^2x^2}} - \frac{b(3c^2d+4e)\,x(-1+c^2x^2)^{3/2}}{36c^5\sqrt{c^2x^2}} - \frac{bex(-1+c^2x^2)^{5/2}}{30c^5\sqrt{c^2x^2}} + \frac{1}{4}dx^4\left(a+b\sec^{-1}(cx)\right) + \frac{1}{6}ex^6\left(a+b\sec^{-1}(cx)\right)$$

output
```
-1/12*b*(3*c^2*d+2*e)*x*(c^2*x^2-1)^(1/2)/c^5/(c^2*x^2)^(1/2)-1/36*b*(3*c^
2*d+4*e)*x*(c^2*x^2-1)^(3/2)/c^5/(c^2*x^2)^(1/2)-1/30*b*e*x*(c^2*x^2-1)^(5
/2)/c^5/(c^2*x^2)^(1/2)+1/4*d*x^4*(a+b*arcsec(c*x))+1/6*e*x^6*(a+b*arcsec(
c*x))
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.64

$$\int x^3(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{1}{180}x\left(15ax^3(3d+2ex^2)-\frac{b\sqrt{1-\frac{1}{c^2x^2}}(16e+c^2(30d+8ex^2)+3c^4(5dx^2+2ex^4))}{c^5}+15bx^3(3d+2ex^2)\sec^{-1}(cx)\right)$$

input
```
Integrate[x^3*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

output
```
(x*(15*a*x^3*(3*d + 2*e*x^2) - (b*Sqrt[1 - 1/(c^2*x^2)]*(16*e + c^2*(30*d
+ 8*e*x^2) + 3*c^4*(5*d*x^2 + 2*e*x^4)))/c^5 + 15*b*x^3*(3*d + 2*e*x^2)*Ar
cSec[c*x]))/180
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5761, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{5761}$$

$$-\frac{bcx\int\frac{x^3(2ex^2+3d)}{12\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{1}{4}dx^4\left(a+b\sec^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{bcx\int \frac{x^3(2ex^2+3d)}{\sqrt{c^2x^2-1}}dx}{12\sqrt{c^2x^2}}+\frac{1}{4}dx^4\left(a+b\sec^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{354}$$

$$-\frac{bcx\int \frac{x^2(2ex^2+3d)}{\sqrt{c^2x^2-1}}dx^2}{24\sqrt{c^2x^2}}+\frac{1}{4}dx^4\left(a+b\sec^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{86}$$

$$-\frac{bcx\int \left(\frac{2e(c^2x^2-1)^{3/2}}{c^4}+\frac{(3dc^2+4e)\sqrt{c^2x^2-1}}{c^4}+\frac{3dc^2+2e}{c^4\sqrt{c^2x^2-1}}\right)dx^2}{24\sqrt{c^2x^2}}+\frac{1}{4}dx^4\left(a+b\sec^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}dx^4\left(a+b\sec^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\sec^{-1}(cx)\right)-\frac{bcx\left(\frac{2(c^2x^2-1)^{3/2}(3c^2d+4e)}{3c^6}+\frac{2\sqrt{c^2x^2-1}(3c^2d+2e)}{c^6}+\frac{4e(c^2x^2-1)^{5/2}}{5c^6}\right)}{24\sqrt{c^2x^2}}$$

input
```
Int[x^3*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

output
```
-1/24*(b*c*x*((2*(3*c^2*d + 2*e)*Sqrt[-1 + c^2*x^2])/c^6 + (2*(3*c^2*d + 4
*e)*(-1 + c^2*x^2)^(3/2))/(3*c^6) + (4*e*(-1 + c^2*x^2)^(5/2))/(5*c^6)))/S
qrt[c^2*x^2] + (d*x^4*(a + b*ArcSec[c*x]))/4 + (e*x^6*(a + b*ArcSec[c*x]))
/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.79

method	result
parts	$a\left(\frac{1}{6}e\,x^{6}+\frac{1}{4}d\,x^{4}\right)+\frac{b\left(\frac{c^{4}\operatorname{arcsec}(cx)e\,x^{6}}{6}+\frac{\operatorname{arcsec}(cx)d\,x^{4}c^{4}}{4}-\frac{\left(c^{2}x^{2}-1\right)\left(6c^{4}e\,x^{4}+15c^{4}d\,x^{2}+8e\,c^{2}x^{2}+30c^{2}d+16e\right)}{180c^{3}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,x}\right)}{c^{4}}$
derivativedivides	$\frac{-\frac{a\left(\frac{c^{2}d\left(e\,c^{2}x^{2}+c^{2}d\right)^{2}}{2}-\frac{\left(e\,c^{2}x^{2}+c^{2}d\right)^{3}}{3}\right)}{2c^{2}e^{2}}-\frac{b\,c^{4}\operatorname{arcsec}(cx)d^{3}}{12e^{2}}+\frac{b\operatorname{arcsec}(cx)d\,c^{4}x^{4}}{4}+\frac{b\,c^{4}e\operatorname{arcsec}(cx)x^{6}}{6}-\frac{b\,c^{3}\sqrt{c^{2}x^{2}-1}\,d^{3}\operatorname{arct}}{12e^{2}\sqrt{\frac{c^{2}x}{c^{2}}}}}{c^{4}}$
default	$\frac{-\frac{a\left(\frac{c^{2}d\left(e\,c^{2}x^{2}+c^{2}d\right)^{2}}{2}-\frac{\left(e\,c^{2}x^{2}+c^{2}d\right)^{3}}{3}\right)}{2c^{2}e^{2}}-\frac{b\,c^{4}\operatorname{arcsec}(cx)d^{3}}{12e^{2}}+\frac{b\operatorname{arcsec}(cx)d\,c^{4}x^{4}}{4}+\frac{b\,c^{4}e\operatorname{arcsec}(cx)x^{6}}{6}-\frac{b\,c^{3}\sqrt{c^{2}x^{2}-1}\,d^{3}\operatorname{arct}}{12e^{2}\sqrt{\frac{c^{2}x}{c^{2}}}}}{c^{4}}$

input
```
int(x^3*(e*x^2+d)*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/6*e*x^6+1/4*d*x^4)+b/c^4*(1/6*c^4*arcsec(c*x)*e*x^6+1/4*arcsec(c*x)*d
*x^4*c^4-1/180/c^3*(c^2*x^2-1)*(6*c^4*e*x^4+15*c^4*d*x^2+8*c^2*e*x^2+30*c^
2*d+16*e)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.70

$$\int x^{3}\left(d+ex^{2}\right)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{30\,ac^{6}ex^{6}+45\,ac^{6}dx^{4}+15\left(2\,bc^{6}ex^{6}+3\,bc^{6}dx^{4}\right)\operatorname{arcsec}\left(cx\right)-\left(6\,bc^{4}ex^{4}+30\,bc^{2}d+\left(15\,bc^{4}d+8\,bc^{2}e\right)x^{2}\right.}{180\,c^{6}}$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/180*(30*a*c^6*e*x^6 + 45*a*c^6*d*x^4 + 15*(2*b*c^6*e*x^6 + 3*b*c^6*d*x^4
)*arcsec(c*x) - (6*b*c^4*e*x^4 + 30*b*c^2*d + (15*b*c^4*d + 8*b*c^2*e)*x^2
 + 16*b*e)*sqrt(c^2*x^2 - 1))/c^6
```

Sympy [A] (verification not implemented)

Time = 2.76 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.78

$$\int x^3(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$= \frac{adx^4}{4}+\frac{aex^6}{6}+\frac{bdx^4\operatorname{asec}(cx)}{4}+\frac{bex^6\operatorname{asec}(cx)}{6} - \frac{bd\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{4c} - \frac{be\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c}+\frac{4x^2\sqrt{c^2x^2-1}}{15c^3}+\frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for } |c^2x^2|>1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c}+\frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3}+\frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{6c}$$

input
```
integrate(x**3*(e*x**2+d)*(a+b*asec(c*x)),x)
```

output
```
a*d*x**4/4 + a*e*x**6/6 + b*d*x**4*asec(c*x)/4 + b*e*x**6*asec(c*x)/6 - b*
d*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c) + 2*sqrt(c**2*x**2 - 1)/(3*c**
3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*x**2 + 1)/(3*c) + 2*I*sqrt(-c*
*2*x**2 + 1)/(3*c**3), True))/(4*c) - b*e*Piecewise((x**4*sqrt(c**2*x**2 -
 1)/(5*c) + 4*x**2*sqrt(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**2*x**2 - 1)/(
15*c**5), Abs(c**2*x**2) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(5*c) + 4*I*x*
*2*sqrt(-c**2*x**2 + 1)/(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/(15*c**5), Tr
ue))/(6*c)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.94

$$\int x^3(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$= \frac{1}{6}aex^6+\frac{1}{4}adx^4+\frac{1}{12}\left(3\,x^4\operatorname{arcsec}(cx)-\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)bd + \frac{1}{90}\left(15\,x^6\operatorname{arcsec}(cx)-\frac{3\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+10\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^5}\right)be$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*e*x^6 + 1/4*a*d*x^4 + 1/12*(3*x^4*arcsec(c*x) - (c^2*x^3*(-1/(c^2*x^
2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b*d + 1/90*(15*x^6*arcsec
(c*x) - (3*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2) + 10*c^2*x^3*(-1/(c^2*x^2) + 1
)^(3/2) + 15*x*sqrt(-1/(c^2*x^2) + 1))/c^5)*b*e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7820 vs. $2(131) = 262$.

Time = 0.21 (sec) , antiderivative size = 7820, normalized size of antiderivative = 51.11

$$\int x^3(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/180*(45*b*c^2*d*arccos(1/(c*x))/(c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x)
+ 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2)
- 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^
7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x)
+ 1)^12) + 45*a*c^2*d/(c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*
c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c
*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^
2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12) - 9
0*b*c^2*d*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2) - 1
)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1
/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) +
1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^
6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^2) - 90*b*c^2*d*sqrt(-1/(c^2*x^2) + 1)/(
(c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^
2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1
/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1
)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)) - 90*a*c^2
*d*(1/(c^2*x^2) - 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*
c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c
*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2...
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx=\int x^3\,(e\,x^2+d)\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^3*(d + e*x^2)*(a + b*acos(1/(c*x))),x)
```

output
```
int(x^3*(d + e*x^2)*(a + b*acos(1/(c*x))), x)
```

Reduce [F]

$$\int x^3 (d+ex^2) (a+b\sec^{-1}(cx)) \, dx = \left(\int asec(cx)\, x^5 dx\right) be + \left(\int asec(cx)\, x^3 dx\right) bd + \frac{ad\, x^4}{4} + \frac{ae\, x^6}{6}$$

input
```
int(x^3*(e*x^2+d)*(a+b*asec(c*x)),x)
```

output
```
(12*int(asec(c*x)*x**5,x)*b*e + 12*int(asec(c*x)*x**3,x)*b*d + 3*a*d*x**4
+ 2*a*e*x**6)/12
```

3.78 $\int x(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$

Optimal result 662
Mathematica [A] (verified) 662
Rubi [A] (verified) 663
Maple [A] (verified) 665
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Maxima [A] (verification not implemented) 666
Giac [B] (verification not implemented) 667
Mupad [F(-1)] 668
Reduce [F] 668

Optimal result

Integrand size = 17, antiderivative size = 138

$$\int x\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx=-\frac{b(2c^2d+e)\,x\sqrt{-1+c^2x^2}}{4c^3\sqrt{c^2x^2}}-\frac{bex\left(-1+c^2x^2\right)^{3/2}}{12c^3\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{4e}-\frac{bcd^2x\arctan\left(\sqrt{-1+c^2x^2}\right)}{4e\sqrt{c^2x^2}}$$

output
```
-1/4*b*(2*c^2*d+e)*x*(c^2*x^2-1)^(1/2)/c^3/(c^2*x^2)^(1/2)-1/12*b*e*x*(c^2
*x^2-1)^(3/2)/c^3/(c^2*x^2)^(1/2)+1/4*(e*x^2+d)^2*(a+b*arcsec(c*x))/e-1/4*
b*c*d^2*x*arctan((c^2*x^2-1)^(1/2))/e/(c^2*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.57

$$\int x\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{x\left(3ac^3x(2d+ex^2)-b\sqrt{1-\frac{1}{c^2x^2}}(2e+c^2(6d+ex^2))+3bc^3x(2d+ex^2)\sec^{-1}(cx)\right)}{12c^3}$$

input
```
Integrate[x*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

output
```
(x*(3*a*c^3*x*(2*d + e*x^2) - b*Sqrt[1 - 1/(c^2*x^2)]*(2*e + c^2*(6*d + e*
x^2)) + 3*b*c^3*x*(2*d + e*x^2)*ArcSec[c*x]))/(12*c^3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {5759, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)dx$$

$$\Big\downarrow \text{ 5759}$$

$$\frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{4e}-\frac{bcx\int \frac{\left(ex^2+d\right)^2}{x\sqrt{c^2x^2-1}}dx}{4e\sqrt{c^2x^2}}$$

$$\Big\downarrow \text{ 354}$$

$$\frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{4e}-\frac{bcx\int \frac{\left(ex^2+d\right)^2}{x^2\sqrt{c^2x^2-1}}dx^2}{8e\sqrt{c^2x^2}}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{4e}-\frac{bcx\int \left(\frac{d^2}{x^2\sqrt{c^2x^2-1}}+\frac{e^2\sqrt{c^2x^2-1}}{c^2}+\frac{e\left(2dc^2+e\right)}{c^2\sqrt{c^2x^2-1}}\right)dx^2}{8e\sqrt{c^2x^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{4e}-\frac{bcx\left(2d^2\arctan\left(\sqrt{c^2x^2-1}\right)+\frac{2e\sqrt{c^2x^2-1}\left(2c^2d+e\right)}{c^4}+\frac{2e^2\left(c^2x^2-1\right)^{3/2}}{3c^4}\right)}{8e\sqrt{c^2x^2}}$$

input
```
Int[x*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

output
```
((d + e*x^2)^2*(a + b*ArcSec[c*x]))/(4*e) - (b*c*x*((2*e*(2*c^2*d + e)*Sqr
t[-1 + c^2*x^2])/c^4 + (2*e^2*(-1 + c^2*x^2)^(3/2))/(3*c^4) + 2*d^2*ArcTan
[Sqrt[-1 + c^2*x^2]]))/(8*e*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5759
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSec[c*x])/(2*e*(p + 1))), x
] - Simp[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2]))   Int[(d + e*x^2)^(p + 1)/(x*S
qrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.57

method	result
parts	$\frac{a(e\,x^2+d)^2}{4e}+\frac{\operatorname{arcsec}(cx)be\,x^4}{4}+\frac{b\operatorname{arcsec}(cx)x^2d}{2}+\frac{b\operatorname{arcsec}(cx)d^2}{4e}-\frac{b(c^2x^2-1)xe}{12c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\sqrt{c^2x^2-1}\,d^2\arctan\left(\right.}{4ce\sqrt{\frac{c^2x^2-1}{c^2x^2}}}$
derivativedivides	$\frac{\frac{a\left(e\,c^2x^2+c^2d\right)^2}{4c^2e}+\frac{b\,c^2\operatorname{arcsec}(cx)d^2}{4e}+\frac{b\operatorname{arcsec}(cx)d\,c^2x^2}{2}+\frac{b\,c^2e\operatorname{arcsec}(cx)x^4}{4}+\frac{bc\sqrt{c^2x^2-1}\,d^2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{4e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}-\frac{b\left(c^2x^2-1\right.}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}}{c^2}$
default	$\frac{\frac{a\left(e\,c^2x^2+c^2d\right)^2}{4c^2e}+\frac{b\,c^2\operatorname{arcsec}(cx)d^2}{4e}+\frac{b\operatorname{arcsec}(cx)d\,c^2x^2}{2}+\frac{b\,c^2e\operatorname{arcsec}(cx)x^4}{4}+\frac{bc\sqrt{c^2x^2-1}\,d^2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{4e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}-\frac{b\left(c^2x^2-1\right.}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}}{c^2}$

input
```
int(x*(e*x^2+d)*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*(e*x^2+d)^2/e+1/4*arcsec(c*x)*b*e*x^4+1/2*b*arcsec(c*x)*x^2*d+1/4*b/
e*arcsec(c*x)*d^2-1/12*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x*e+1
/4*b/c/e*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^2*arctan(1/(c^2
*x^2-1)^(1/2))-1/2*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d-1/6*b
/c^5*e*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.62

$$\int x(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{3\,ac^4ex^4+6\,ac^4dx^2+3\,(bc^4ex^4+2\,bc^4dx^2)\operatorname{arcsec}(cx)-(bc^2ex^2+6\,bc^2d+2\,be)\sqrt{c^2x^2-1}}{12\,c^4}$$

input
```
integrate(x*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/12*(3*a*c^4*e*x^4 + 6*a*c^4*d*x^2 + 3*(b*c^4*e*x^4 + 2*b*c^4*d*x^2)*arcs
ec(c*x) - (b*c^2*e*x^2 + 6*b*c^2*d + 2*b*e)*sqrt(c^2*x^2 - 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.28

$$\int x(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{adx^2}{2}+\frac{aex^4}{4}+\frac{bdx^2\operatorname{asec}(cx)}{2}+\frac{bex^4\operatorname{asec}(cx)}{4} - \frac{bd\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c} - \frac{be\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{4c}$$

input
```
integrate(x*(e*x**2+d)*(a+b*asec(c*x)),x)
```

output
```
a*d*x**2/2 + a*e*x**4/4 + b*d*x**2*asec(c*x)/2 + b*e*x**4*asec(c*x)/4 - b*
d*Piecewise((sqrt(c**2*x**2 - 1)/c, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**
2 + 1)/c, True))/(2*c) - b*e*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c) + 2
*sqrt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*x**
2 + 1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**3), True))/(4*c)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.72

$$\int x(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{4}aex^4+\frac{1}{2}adx^2+\frac{1}{2}\left(x^2\operatorname{arcsec}(cx)-\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)bd + \frac{1}{12}\left(3x^4\operatorname{arcsec}(cx)-\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)be$$

input
```
integrate(x*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/4*a*e*x^4 + 1/2*a*d*x^2 + 1/2*(x^2*arcsec(c*x) - x*sqrt(-1/(c^2*x^2) + 1
)/c)*b*d + 1/12*(3*x^4*arcsec(c*x) - (c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 3
*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b*e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3346 vs. $2(118) = 236$.

Time = 0.18 (sec) , antiderivative size = 3346, normalized size of antiderivative = 24.25

$$\int x(d + ex^2) (a + b\sec^{-1}(cx)) \, dx = \text{Too large to display}$$

input
```
integrate(x*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/12*(6*b*c^2*d*arccos(1/(c*x))/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) +
1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)
^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) + 6*a*c^2*d/
(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2
/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2
*x^2) - 1)^4/(1/(c*x) + 1)^8) - 12*b*c^2*d*sqrt(-1/(c^2*x^2) + 1)/((c^5 +
4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*
x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) -
 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)) + 3*b*e*arccos(1/(c*x))/(c^5 + 4*c^5
*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4
/(1/(c*x) + 1)^8) - 12*b*c^2*d*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^5 +
 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c
*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2)
- 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) + 36*b*c^2*d*(-1/(c^2*x^2) + 1)^(
3/2)/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2)
- 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(
1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^3) + 3*a*e/(c^5 + 4*c^5*
(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1
)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)...
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx=\int x\left(e\,x^2+d\right)\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x*(d + e*x^2)*(a + b*acos(1/(c*x))),x)
```

output
```
int(x*(d + e*x^2)*(a + b*acos(1/(c*x))), x)
```

Reduce [F]

$$\int x(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx=\left(\int asec(cx)\,x^3dx\right)be+\left(\int asec(cx)\,xdx\right)bd+\frac{ad\,x^2}{2}+\frac{ae\,x^4}{4}$$

input
```
int(x*(e*x^2+d)*(a+b*asec(c*x)),x)
```

output
```
(4*int(asec(c*x)*x**3,x)*b*e + 4*int(asec(c*x)*x,x)*b*d + 2*a*d*x**2 + a*e
*x**4)/4
```

3.79 $\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x}\,dx$

Optimal result 669
Mathematica [A] (verified) 670
Rubi [A] (verified) 670
Maple [A] (verified) 672
Fricas [F] 673
Sympy [F] 673
Maxima [F] 673
Giac [F(-2)] 674
Mupad [F(-1)] 674
Reduce [F] 675

Optimal result

Integrand size = 19, antiderivative size = 124

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x}\,dx = -\frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} - \frac{1}{2}ibd\csc^{-1}(cx)^2 + \frac{1}{2}ex^2(a+b\sec^{-1}(cx)) + bd\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) - bd\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - d(a+b\sec^{-1}(cx))\log\left(\frac{1}{x}\right) - \frac{1}{2}ibd\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)$$

output
```
-1/2*b*e*(1-1/c^2/x^2)^(1/2)*x/c-1/2*I*b*d*arccsc(c*x)^2+1/2*e*x^2*(a+b*ar
csec(c*x))+b*d*arccsc(c*x)*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)-b*d*arccsc(
c*x)*ln(1/x)-d*(a+b*arcsec(c*x))*ln(1/x)-1/2*I*b*d*polylog(2,(I/c/x+(1-1/c
^2/x^2)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x}\,dx = \frac{1}{2}aex^2 - \frac{bex\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{2c} + \frac{1}{2}bex^2\sec^{-1}(cx) + \frac{1}{2}ibd\sec^{-1}(cx)^2 - bd\sec^{-1}(cx)\log\left(1+e^{2i\sec^{-1}(cx)}\right) + ad\log(x) + \frac{1}{2}ibd\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcSec[c*x]))/x,x]
```

output
```
(a*e*x^2)/2 - (b*e*x*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(2*c) + (b*e*x^2*ArcS
ec[c*x])/2 + (I/2)*b*d*ArcSec[c*x]^2 - b*d*ArcSec[c*x]*Log[1 + E^((2*I)*Ar
cSec[c*x])] + a*d*Log[x] + (I/2)*b*d*PolyLog[2, -E^((2*I)*ArcSec[c*x])]
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5763, 5231, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x}\,dx$$

$$\Big\downarrow \text{5763}$$

$$-\int \left(\frac{d}{x^2}+e\right)x^3\left(a+b\arccos\left(\frac{1}{cx}\right)\right)d\frac{1}{x}$$

$$\Big\downarrow \text{5231}$$

$$-\frac{b\int -\frac{ex^2-2d\log\left(\frac{1}{x}\right)}{2\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{c} - d\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right) + \frac{1}{2}ex^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \; 27$$

$$\frac{b\int \frac{ex^2-2d\log\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{2c}-d\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+\frac{1}{2}ex^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \; 7293$$

$$\frac{b\int \left(\frac{ex^2}{\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2d\log\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{c^2x^2}}}\right)d\frac{1}{x}}{2c}-d\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+\frac{1}{2}ex^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \; 2009$$

$$-d\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+\frac{1}{2}ex^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+$$
$$\frac{b\left(-icd\operatorname{PolyLog}\left(2,e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)-icd\arcsin\left(\frac{1}{cx}\right)^2+2cd\arcsin\left(\frac{1}{cx}\right)\log\left(1-e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)-2cd\log\left(\frac{1}{x}\right)\arcsin\right.}{2c}$$

input
```
Int[((d + e*x^2)*(a + b*ArcSec[c*x]))/x,x]
```

output
```
(e*x^2*(a + b*ArcCos[1/(c*x)]))/2 - d*(a + b*ArcCos[1/(c*x)])*Log[x^(-1)]
+ (b*(-(e*Sqrt[1 - 1/(c^2*x^2)]*x) - I*c*d*ArcSin[1/(c*x)]^2 + 2*c*d*ArcSi
n[1/(c*x)]*Log[1 - E^((2*I)*ArcSin[1/(c*x)])] - 2*c*d*ArcSin[1/(c*x)]*Log[
x^(-1)] - I*c*d*PolyLog[2, E^((2*I)*ArcSin[1/(c*x)])]))/(2*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10

method	result
parts	$\frac{ae\,x^2}{2}+ad\ln(x)+b\left(\frac{i\,\mathrm{arcsec}(cx)^2 d}{2}+\frac{e\left(c^2x^2\,\mathrm{arcsec}(cx)-xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-i\right)}{2c^2}-d\,\mathrm{arcsec}(cx)\ln\left(1+\right.\right.$
derivativedivides	$\frac{ae\,x^2}{2}+ad\ln(cx)+\frac{b\left(\frac{ic^2d\,\mathrm{arcsec}(cx)^2}{2}+\frac{e\left(c^2x^2\,\mathrm{arcsec}(cx)-xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-i\right)}{2}-\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right)c^2d\,\mathrm{ar}\right.}{c^2}$
default	$\frac{ae\,x^2}{2}+ad\ln(cx)+\frac{b\left(\frac{ic^2d\,\mathrm{arcsec}(cx)^2}{2}+\frac{e\left(c^2x^2\,\mathrm{arcsec}(cx)-xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-i\right)}{2}-\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right)c^2d\,\mathrm{ar}\right.}{c^2}$

input
```
int((e*x^2+d)*(a+b*arcsec(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*e*x^2+a*d*ln(x)+b*(1/2*I*arcsec(c*x)^2*d+1/2*e*(c^2*x^2*arcsec(c*x)-
x*c*((c^2*x^2-1)/c^2/x^2)^(1/2)-I)/c^2-d*arcsec(c*x)*ln(1+(1/c/x+I*(1-1/c^
2/x^2)^(1/2))^2)+1/2*I*d*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2))
```

Fricas [F]

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{(ex^2+d)(b\,\mathrm{arcsec}\,(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsec(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{(a+b\,\mathrm{asec}\,(cx))(d+ex^2)}{x}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*asec(c*x))/x,x)
```

output
```
Integral((a + b*asec(c*x))*(d + e*x**2)/x, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{(ex^2+d)(b\,\mathrm{arcsec}\,(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x,x, algorithm="maxima")
```

output
```
1/2*a*e*x^2 + a*d*log(x) - 1/4*(-2*I*b*c^2*e*x^2*log(c) - 2*I*b*c^2*d*log(
-c*x + 1)*log(x) - 2*I*b*c^2*d*log(x)^2 - 2*I*b*c^2*d*dilog(c*x) - 2*I*b*c
^2*d*dilog(-c*x) + I*(b*e*(log(c*x + 1)/c^2 + log(c*x - 1)/c^2) + 8*b*d*in
tegrate(1/2*log(x)/(c^2*x^3 - x), x))*c^2 + 4*c^2*integrate(1/2*(b*e*x^2 +
 2*b*d*log(x))*sqrt(c*x + 1)*sqrt(c*x - 1)/(c^2*x^3 - x), x) - I*b*e*log(c
*x - 1) - 2*(b*c^2*e*x^2 + 2*b*c^2*d*log(x))*arctan(sqrt(c*x + 1)*sqrt(c*x
 - 1)) + (I*b*c^2*e*x^2 + 2*I*b*c^2*d*log(x))*log(c^2*x^2) + (-2*I*b*c^2*d
*log(x) - I*b*e)*log(c*x + 1) - 2*(I*b*c^2*e*x^2 + 2*I*b*c^2*d*log(c))*log
(x))/c^2
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

input
```
int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x,x)
```

output
```
int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x}\,dx = \left(\int \frac{asec(cx)}{x}dx\right)bd + \left(\int asec(cx)\,xdx\right)be + \log(x)\,ad + \frac{ae\,x^2}{2}$$

input
```
int((e*x^2+d)*(a+b*asec(c*x))/x,x)
```

output
```
(2*int(asec(c*x)/x,x)*b*d + 2*int(asec(c*x)*x,x)*b*e + 2*log(x)*a*d + a*e*
x**2)/2
```

3.80 $\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^3}\,dx$

Optimal result 676
Mathematica [A] (verified) 677
Rubi [A] (verified) 677
Maple [A] (verified) 679
Fricas [F] 680
Sympy [F] 680
Maxima [F] 680
Giac [F(-2)] 681
Mupad [F(-1)] 681
Reduce [F] 681

Optimal result

Integrand size = 19, antiderivative size = 137

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^3}\,dx = \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d\csc^{-1}(cx) - \frac{1}{2}ibe\csc^{-1}(cx)^2 - \frac{d(a+b\sec^{-1}(cx))}{2x^2} + be\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) - be\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - e(a+b\sec^{-1}(cx))\log\left(\frac{1}{x}\right) - \frac{1}{2}ibe\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)$$

output
```
1/4*b*c*d*(1-1/c^2/x^2)^(1/2)/x-1/4*b*c^2*d*arccsc(c*x)-1/2*I*b*e*arccsc(c
*x)^2-1/2*d*(a+b*arcsec(c*x))/x^2+b*e*arccsc(c*x)*ln(1-(I/c/x+(1-1/c^2/x^2
)^(1/2))^2)-b*e*arccsc(c*x)*ln(1/x)-e*(a+b*arcsec(c*x))*ln(1/x)-1/2*I*b*e*
polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^3}\,dx = -\frac{ad}{2x^2}+\frac{bcd\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{4x}-\frac{bd\sec^{-1}(cx)}{2x^2} +\frac{1}{2}ibe\sec^{-1}(cx)^2-\frac{1}{4}bc^2d\arcsin\left(\frac{1}{cx}\right) - be\sec^{-1}(cx)\log\left(1+e^{2i\sec^{-1}(cx)}\right) + ae\log(x)+\frac{1}{2}ibe\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^3,x]
```

output
```
-1/2*(a*d)/x^2 + (b*c*d*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(4*x) - (b*d*ArcSe
c[c*x])/(2*x^2) + (I/2)*b*e*ArcSec[c*x]^2 - (b*c^2*d*ArcSin[1/(c*x)])/4 -
b*e*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] + a*e*Log[x] + (I/2)*b*e*Po
lyLog[2, -E^((2*I)*ArcSec[c*x])]
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.28, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5763, 5231, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^3}\,dx$$

$$\Big\downarrow \text{ 5763}$$

$$-\int \left(\frac{d}{x^2}+e\right)x\left(a+b\arccos\left(\frac{1}{cx}\right)\right)d\frac{1}{x}$$

$$\Big\downarrow \text{ 5231}$$

$$-\frac{b\int \frac{\frac{d}{x^2}+2e\log\left(\frac{1}{x}\right)}{2\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{c}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{2x^2}-e\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{\frac{d}{x^2}+2e\log\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{2c}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{2x^2}-e\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{7293}$$

$$-\frac{b\int \left(\frac{d}{\sqrt{1-\frac{1}{c^2x^2}}x^2}+\frac{2e\log\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{c^2x^2}}}\right)d\frac{1}{x}}{2c}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{2x^2}-e\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{2x^2}-e\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)-$$
$$\frac{b\left(\frac{1}{2}c^3d\arcsin\left(\frac{1}{cx}\right)+ice\,\text{PolyLog}\left(2,e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)+ice\arcsin\left(\frac{1}{cx}\right)^2-2ce\arcsin\left(\frac{1}{cx}\right)\log\left(1-e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)+2\right.}{2c}$$

input
```
Int[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^3,x]
```

output
```
-1/2*(d*(a + b*ArcCos[1/(c*x)]))/x^2 - e*(a + b*ArcCos[1/(c*x)])*Log[x^(-1
)] - (b*(-1/2*(c^2*d*Sqrt[1 - 1/(c^2*x^2)])/x + (c^3*d*ArcSin[1/(c*x)])/2
+ I*c*e*ArcSin[1/(c*x)]^2 - 2*c*e*ArcSin[1/(c*x)]*Log[1 - E^((2*I)*ArcSin[
1/(c*x)])] + 2*c*e*ArcSin[1/(c*x)]*Log[x^(-1)] + I*c*e*PolyLog[2, E^((2*I)
*ArcSin[1/(c*x)])]))/(2*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04

method	result
parts	$ae\ln(x) - \frac{ad}{2x^2} + \frac{ib\,\mathrm{arcsec}(cx)^2 e}{2} + \frac{b\,c^2 d\,\mathrm{arcsec}(cx)}{4} + \frac{bcd\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4x} - \frac{bd\,\mathrm{arcsec}(cx)}{2x^2} - be\,\mathrm{arcsec}\,(cx$
derivativedivides	$c^2\left(\frac{ae\ln(cx)}{c^2} - \frac{ad}{2c^2x^2} + \frac{ibe\,\mathrm{arcsec}(cx)^2}{2c^2} + \frac{bd\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4cx} + \frac{bd\,\mathrm{arcsec}(cx)}{4} - \frac{b\,\mathrm{arcsec}(cx)d}{2c^2x^2} - \frac{be\,\mathrm{arcsec}(cx)\ln}{}\right.$
default	$c^2\left(\frac{ae\ln(cx)}{c^2} - \frac{ad}{2c^2x^2} + \frac{ibe\,\mathrm{arcsec}(cx)^2}{2c^2} + \frac{bd\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4cx} + \frac{bd\,\mathrm{arcsec}(cx)}{4} - \frac{b\,\mathrm{arcsec}(cx)d}{2c^2x^2} - \frac{be\,\mathrm{arcsec}(cx)\ln}{}\right.$

input
```
int((e*x^2+d)*(a+b*arcsec(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*e*ln(x)-1/2*a*d/x^2+1/2*I*b*arcsec(c*x)^2*e+1/4*b*c^2*d*arcsec(c*x)+1/4*
b*c*d/x*((c^2*x^2-1)/c^2/x^2)^(1/2)-1/2*b*d/x^2*arcsec(c*x)-b*e*arcsec(c*x
)*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)+1/2*I*b*e*polylog(2,-(1/c/x+I*(1-1
/c^2/x^2)^(1/2))^2)
```

Fricas [F]

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)(b\,\mathrm{arcsec}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsec(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{asec}\,(cx))\,(d+ex^2)}{x^3}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*asec(c*x))/x**3,x)
```

output
```
Integral((a + b*asec(c*x))*(d + e*x**2)/x**3, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)(b\,\mathrm{arcsec}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^3,x, algorithm="maxima")
```

output
```
-(c^2*integrate(sqrt(c*x + 1)*sqrt(c*x - 1)*log(x)/(c^4*x^3 - c^2*x), x) -
 arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(x))*b*e - 1/4*b*d*((c^4*x*sqrt(-1
/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c^3*arctan(c*x*sqrt(-1/(
c^2*x^2) + 1)))/c + 2*arcsec(c*x)/x^2) + a*e*log(x) - 1/2*a*d/x^2
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

input
```
int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^3,x)
```

output
```
int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^3, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^3}\,dx = \frac{2\left(\int \frac{asec(cx)}{x^3}dx\right)bd\,x^2+2\left(\int \frac{asec(cx)}{x}dx\right)be\,x^2+2\log(x)\,ae\,x^2-ad}{2x^2}$$

input
```
int((e*x^2+d)*(a+b*asec(c*x))/x^3,x)
```

output
```
(2*int(asec(c*x)/x**3,x)*b*d*x**2 + 2*int(asec(c*x)/x,x)*b*e*x**2 + 2*log(
x)*a*e*x**2 - a*d)/(2*x**2)
```

3.81 $\int x^2(d+ex^2)^2 (a+b\sec^{-1}(cx))\,dx$

Optimal result 683
Mathematica [A] (verified) 684
Rubi [A] (verified) 684
Maple [B] (verified) 688
Fricas [A] (verification not implemented) 688
Sympy [A] (verification not implemented) 689
Maxima [A] (verification not implemented) 690
Giac [B] (verification not implemented) 691
Mupad [F(-1)] 692
Reduce [F] 693

Optimal result

Integrand size = 21, antiderivative size = 252

$$\int x^2(d+ex^2)^2 (a+b\sec^{-1}(cx))\,dx = -\frac{b(280c^4d^2+252c^2de+75e^2)\,x^2\sqrt{-1+c^2x^2}}{1680c^5\sqrt{c^2x^2}} - \frac{be(84c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} - \frac{be^2x^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}} + \frac{1}{3}d^2x^3(a+b\sec^{-1}(cx)) + \frac{2}{5}dex^5(a+b\sec^{-1}(cx)) + \frac{1}{7}e^2x^7(a+b\sec^{-1}(cx)) - \frac{b(280c^4d^2+252c^2de+75e^2)\,x\,\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{1680c^6\sqrt{c^2x^2}}$$

output
```
-1/1680*b*(280*c^4*d^2+252*c^2*d*e+75*e^2)*x^2*(c^2*x^2-1)^(1/2)/c^5/(c^2*
x^2)^(1/2)-1/840*b*e*(84*c^2*d+25*e)*x^4*(c^2*x^2-1)^(1/2)/c^3/(c^2*x^2)^(
1/2)-1/42*b*e^2*x^6*(c^2*x^2-1)^(1/2)/c/(c^2*x^2)^(1/2)+1/3*d^2*x^3*(a+b*a
rcsec(c*x))+2/5*d*e*x^5*(a+b*arcsec(c*x))+1/7*e^2*x^7*(a+b*arcsec(c*x))-1/
1680*b*(280*c^4*d^2+252*c^2*d*e+75*e^2)*x*arctanh(c*x/(c^2*x^2-1)^(1/2))/c
^6/(c^2*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.74

$$\int x^2 (d + ex^2)^2 (a + b \sec^{-1}(cx)) \, dx$$
$$= \frac{c^2x^2\Big(16ac^5x(35d^2 + 42dex^2 + 15e^2x^4) - b\sqrt{1 - \frac{1}{c^2x^2}}(75e^2 + 2c^2e(126d + 25ex^2) + 8c^4(35d^2 + 21dex^2 +}{}$$

input
```
Integrate[x^2*(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

output
```
(c^2*x^2*(16*a*c^5*x*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4) - b*Sqrt[1 - 1/(c^
2*x^2)]*(75*e^2 + 2*c^2*e*(126*d + 25*e*x^2) + 8*c^4*(35*d^2 + 21*d*e*x^2
+ 5*e^2*x^4))) + 16*b*c^7*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)*ArcSec[c*
x] - b*(280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)]
)*x])/(1680*c^7)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.85, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {5761, 27, 1590, 27, 363, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (d + ex^2)^2 (a + b \sec^{-1}(cx)) \, dx$$

$$\Big\downarrow \text{5761}$$

$$-\frac{bcx \int \frac{x^2(15e^2x^4 + 42dex^2 + 35d^2)}{105\sqrt{c^2x^2 - 1}}dx}{\sqrt{c^2x^2}} + \frac{1}{3}d^2x^3(a + b\sec^{-1}(cx)) + \frac{2}{5}dex^5(a + b\sec^{-1}(cx)) + \frac{1}{7}e^2x^7(a + b\sec^{-1}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{bcx\int \frac{x^2\left(15e^2x^4+42dex^2+35d^2\right)}{\sqrt{c^2x^2-1}}dx}{105\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\sec^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\sec^{-1}(cx)\right)$$

$\Big\downarrow$ 1590

$$-\frac{bcx\left(\frac{\int \frac{3x^2\left(70c^2d^2+e\left(84dc^2+25e\right)x^2\right)}{\sqrt{c^2x^2-1}}dx}{6c^2}+\frac{5e^2x^5\sqrt{c^2x^2-1}}{2c^2}\right)}{105\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\sec^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\sec^{-1}(cx)\right)$$

$\Big\downarrow$ 27

$$-\frac{bcx\left(\frac{\int \frac{x^2\left(70c^2d^2+e\left(84dc^2+25e\right)x^2\right)}{\sqrt{c^2x^2-1}}dx}{2c^2}+\frac{5e^2x^5\sqrt{c^2x^2-1}}{2c^2}\right)}{105\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\sec^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\sec^{-1}(cx)\right)$$

$\Big\downarrow$ 363

$$-\frac{bcx\left(\frac{\frac{\left(280c^4d^2+252c^2de+75e^2\right)\int \frac{x^2}{\sqrt{c^2x^2-1}}dx}{4c^2}+\frac{ex^3\sqrt{c^2x^2-1}\left(84c^2d+25e\right)}{4c^2}}{2c^2}+\frac{5e^2x^5\sqrt{c^2x^2-1}}{2c^2}\right)}{105\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\sec^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\sec^{-1}(cx)\right)$$

$\Big\downarrow$ 262

$$-\frac{bcx\left(\frac{\frac{\left(280c^4d^2+252c^2de+75e^2\right)\left(\frac{\int \frac{1}{\sqrt{c^2x^2-1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)}{4c^2}+\frac{ex^3\sqrt{c^2x^2-1}\left(84c^2d+25e\right)}{4c^2}}{2c^2}+\frac{5e^2x^5\sqrt{c^2x^2-1}}{2c^2}\right)}{105\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\sec^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\sec^{-1}(cx)\right)$$

$\Big\downarrow$ 224

$$-\frac{bcx\left(\frac{\frac{\left(280c^4d^2+252c^2de+75e^2\right)\left(\frac{\int \frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}}{2c^2}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)}{4c^2}+\frac{ex^3\sqrt{c^2x^2-1}\left(84c^2d+25e\right)}{4c^2}}{2c^2}+\frac{5e^2x^5\sqrt{c^2x^2-1}}{2c^2}\right)}{105\sqrt{c^2x^2}}+$$
$$\frac{1}{3}d^2x^3\left(a+b\sec^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{3}d^2x^3\left(a+b\sec^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\sec^{-1}(cx)\right)-$$
$$\frac{bcx\left(\frac{\frac{\left(\frac{\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{2c^3}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)\left(280c^4d^2+252c^2de+75e^2\right)}{4c^2}+\frac{ex^3\sqrt{c^2x^2-1}\left(84c^2d+25e\right)}{4c^2}}{2c^2}+\frac{5e^2x^5\sqrt{c^2x^2-1}}{2c^2}\right)}{105\sqrt{c^2x^2}}$$

input
```
Int[x^2*(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

output
```
(d^2*x^3*(a + b*ArcSec[c*x]))/3 + (2*d*e*x^5*(a + b*ArcSec[c*x]))/5 + (e^2
*x^7*(a + b*ArcSec[c*x]))/7 - (b*c*x*((5*e^2*x^5*Sqrt[-1 + c^2*x^2])/(2*c^
2) + ((e*(84*c^2*d + 25*e)*x^3*Sqrt[-1 + c^2*x^2])/(4*c^2) + ((280*c^4*d^2
 + 252*c^2*d*e + 75*e^2)*((x*Sqrt[-1 + c^2*x^2])/(2*c^2) + ArcTanh[(c*x)/S
qrt[-1 + c^2*x^2]]/(2*c^3)))/(4*c^2))/(2*c^2)))/(105*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. 2(222) = 444.

Time = 0.45 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.82

method	result
parts	$a\left(\frac{1}{7}e^2x^7+\frac{2}{5}de\,x^5+\frac{1}{3}x^3d^2\right)+\frac{b\,\mathrm{arcsec}(cx)e^2x^7}{7}+\frac{2b\,\mathrm{arcsec}(cx)de\,x^5}{5}+\frac{b\,\mathrm{arcsec}(cx)d^2x^3}{3}-\frac{b(c^2x^2-1}{42c^3\sqrt{c^2}}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}dc^7ex^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\,\mathrm{arcsec}(cx)d^2c^3x^3}{3}+\frac{2bc^3\,\mathrm{arcsec}(cx)de\,x^5}{5}+\frac{bc^3\,\mathrm{arcsec}(cx)e^2x^7}{7}-\frac{b\left(c^2x^2-1\right)d^2}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b\left(c^2x^2\right.}{10\sqrt{c}}}{}$
default	$\frac{\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}dc^7ex^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\,\mathrm{arcsec}(cx)d^2c^3x^3}{3}+\frac{2bc^3\,\mathrm{arcsec}(cx)de\,x^5}{5}+\frac{bc^3\,\mathrm{arcsec}(cx)e^2x^7}{7}-\frac{b\left(c^2x^2-1\right)d^2}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b\left(c^2x^2\right.}{10\sqrt{c}}}{}$

input
```
int(x^2*(e*x^2+d)^2*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/7*e^2*x^7+2/5*d*e*x^5+1/3*x^3*d^2)+1/7*b*arcsec(c*x)*e^2*x^7+2/5*b*ar
csec(c*x)*d*e*x^5+1/3*b*arcsec(c*x)*d^2*x^3-1/42*b/c^3*(c^2*x^2-1)/((c^2*x
^2-1)/c^2/x^2)^(1/2)*x^4*e^2-1/10*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^
(1/2)*x^2*d*e-5/168*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2*e^2-
1/6*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d^2-3/20*b/c^5*(c^2*x^2-
1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d*e-1/6*b/c^4*(c^2*x^2-1)^(1/2)/((c^2*x^2-1
)/c^2/x^2)^(1/2)/x*d^2*ln(c*x+(c^2*x^2-1)^(1/2))-5/112*b/c^7*(c^2*x^2-1)/(
(c^2*x^2-1)/c^2/x^2)^(1/2)*e^2-3/20*b/c^6*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c
^2/x^2)^(1/2)/x*d*e*ln(c*x+(c^2*x^2-1)^(1/2))-5/112*b/c^8*(c^2*x^2-1)^(1/2
)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*e^2*ln(c*x+(c^2*x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.08

$$\int x^2\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{240\,ac^7e^2x^7+672\,ac^7dex^5+560\,ac^7d^2x^3+16\,(15\,bc^7e^2x^7+42\,bc^7dex^5+35\,bc^7d^2x^3-35\,bc^7d^2-42\,bc^7}{}$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/1680*(240*a*c^7*e^2*x^7 + 672*a*c^7*d*e*x^5 + 560*a*c^7*d^2*x^3 + 16*(15
*b*c^7*e^2*x^7 + 42*b*c^7*d*e*x^5 + 35*b*c^7*d^2*x^3 - 35*b*c^7*d^2 - 42*b
*c^7*d*e - 15*b*c^7*e^2)*arcsec(c*x) + 32*(35*b*c^7*d^2 + 42*b*c^7*d*e + 1
5*b*c^7*e^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)) + (280*b*c^4*d^2 + 252*b*c^2
*d*e + 75*b*e^2)*log(-c*x + sqrt(c^2*x^2 - 1)) - (40*b*c^5*e^2*x^5 + 2*(84
*b*c^5*d*e + 25*b*c^3*e^2)*x^3 + (280*b*c^5*d^2 + 252*b*c^3*d*e + 75*b*c*e
^2)*x)*sqrt(c^2*x^2 - 1))/c^7
```

Sympy [A] (verification not implemented)

Time = 13.47 (sec) , antiderivative size = 542, normalized size of antiderivative = 2.15

$$\int x^2(d+ex^2)^2\left(a+b\sec^{-1}(cx)\right)dx$$
$$= \frac{ad^2x^3}{3} + \frac{2adex^5}{5} + \frac{ae^2x^7}{7} + \frac{bd^2x^3\operatorname{asec}(cx)}{3} + \frac{2bdex^5\operatorname{asec}(cx)}{5} + \frac{be^2x^7\operatorname{asec}(cx)}{7} - \frac{bd^2\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c} - \frac{2bde\left(\begin{cases}\frac{cx^5}{4\sqrt{c^2x^2-1}}+\frac{x^3}{8c\sqrt{c^2x^2-1}}-\frac{3x}{8c^3\sqrt{c^2x^2-1}}+\frac{3\operatorname{acosh}(cx)}{8c^4} & \text{for } |c^2x^2|>1\\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}}-\frac{ix^3}{8c\sqrt{-c^2x^2+1}}+\frac{3ix}{8c^3\sqrt{-c^2x^2+1}}-\frac{3i\operatorname{asin}(cx)}{8c^4} & \text{otherwise}\end{cases}\right)}{5c} - \frac{be^2\left(\begin{cases}\frac{cx^7}{6\sqrt{c^2x^2-1}}+\frac{x^5}{24c\sqrt{c^2x^2-1}}+\frac{5x^3}{48c^3\sqrt{c^2x^2-1}}-\frac{5x}{16c^5\sqrt{c^2x^2-1}}+\frac{5\operatorname{acosh}(cx)}{16c^6} & \text{for } |c^2x^2|>1\\ -\frac{icx^7}{6\sqrt{-c^2x^2+1}}-\frac{ix^5}{24c\sqrt{-c^2x^2+1}}-\frac{5ix^3}{48c^3\sqrt{-c^2x^2+1}}+\frac{5ix}{16c^5\sqrt{-c^2x^2+1}}-\frac{5i\operatorname{asin}(cx)}{16c^6} & \text{otherwise}\end{cases}\right)}{7c}$$

input
```
integrate(x**2*(e*x**2+d)**2*(a+b*asec(c*x)),x)
```

output
```
a*d**2*x**3/3 + 2*a*d*e*x**5/5 + a*e**2*x**7/7 + b*d**2*x**3*asec(c*x)/3 +
 2*b*d*e*x**5*asec(c*x)/5 + b*e**2*x**7*asec(c*x)/7 - b*d**2*Piecewise((x*
sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-I*
c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin(
c*x)/(2*c**2), True))/(3*c) - 2*b*d*e*Piecewise((c*x**5/(4*sqrt(c**2*x**2
- 1)) + x**3/(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*x**2 - 1))
+ 3*acosh(c*x)/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqrt(-c**2*x**
2 + 1)) - I*x**3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**
2 + 1)) - 3*I*asin(c*x)/(8*c**4), True))/(5*c) - b*e**2*Piecewise((c*x**7/
(6*sqrt(c**2*x**2 - 1)) + x**5/(24*c*sqrt(c**2*x**2 - 1)) + 5*x**3/(48*c**
3*sqrt(c**2*x**2 - 1)) - 5*x/(16*c**5*sqrt(c**2*x**2 - 1)) + 5*acosh(c*x)/
(16*c**6), Abs(c**2*x**2) > 1), (-I*c*x**7/(6*sqrt(-c**2*x**2 + 1)) - I*x*
*5/(24*c*sqrt(-c**2*x**2 + 1)) - 5*I*x**3/(48*c**3*sqrt(-c**2*x**2 + 1)) +
 5*I*x/(16*c**5*sqrt(-c**2*x**2 + 1)) - 5*I*asin(c*x)/(16*c**6), True))/(7
*c)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.61

$$\int x^2 (d+ex^2)^2 \left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{7}\,ae^2x^7 + \frac{2}{5}\,adex^5 + \frac{1}{3}\,ad^2x^3$$
$$+ \frac{1}{12}\left(4\,x^3\,\mathrm{arcsec}\,(cx) - \frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bd^2$$
$$+ \frac{1}{40}\left(16\,x^5\,\mathrm{arcsec}\,(cx) + \frac{\frac{2\left(3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2\,c^4\left(\frac{1}{c^2x^2}-1\right)+c^4} - \frac{3\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4} + \frac{3\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)bde$$
$$+ \frac{1}{672}\left(96\,x^7\,\mathrm{arcsec}\,(cx) - \frac{\frac{2\left(15\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-40\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^6\left(\frac{1}{c^2x^2}-1\right)^3+3\,c^6\left(\frac{1}{c^2x^2}-1\right)^2+3\,c^6\left(\frac{1}{c^2x^2}-1\right)+c^6} + \frac{15\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^6} - \frac{15\,\log\left(\sqrt{-\right.}}{c}\right.$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*e^2*x^7 + 2/5*a*d*e*x^5 + 1/3*a*d^2*x^3 + 1/12*(4*x^3*arcsec(c*x) -
(2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2
*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d^2 + 1/40
*(16*x^5*arcsec(c*x) + (2*(3*(-1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(-1/(c^2*x^2
) + 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4) - 3*log(
sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^4)/c
)*b*d*e + 1/672*(96*x^7*arcsec(c*x) - (2*(15*(-1/(c^2*x^2) + 1)^(5/2) - 40
*(-1/(c^2*x^2) + 1)^(3/2) + 33*sqrt(-1/(c^2*x^2) + 1))/(c^6*(1/(c^2*x^2) -
 1)^3 + 3*c^6*(1/(c^2*x^2) - 1)^2 + 3*c^6*(1/(c^2*x^2) - 1) + c^6) + 15*lo
g(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 - 15*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^6
)/c)*b*e^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4760 vs. $2(222) = 444$.

Time = 5.80 (sec) , antiderivative size = 4760, normalized size of antiderivative = 18.89

$$\int x^2\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/1680*(560*b*c^4*d^2*arccos(1/(c*x)) - 280*b*c^4*d^2*log(abs(sqrt(-1/(c^2
*x^2) + 1) + 1/(c*x) + 1)) + 280*b*c^4*d^2*log(abs(sqrt(-1/(c^2*x^2) + 1)
- 1/(c*x) - 1)) + 560*a*c^4*d^2 + 560*b*c^4*d^2*(1/(c^2*x^2) - 1)*arccos(1
/(c*x))/(1/(c*x) + 1)^2 - 1960*b*c^4*d^2*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1
/(c^2*x^2) + 1) + 1/(c*x) + 1))/(1/(c*x) + 1)^2 + 1960*b*c^4*d^2*(1/(c^2*x
^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(1/(c*x) + 1)^2 -
560*b*c^4*d^2*sqrt(-1/(c^2*x^2) + 1)/(1/(c*x) + 1) + 560*a*c^4*d^2*(1/(c^2
*x^2) - 1)/(1/(c*x) + 1)^2 + 672*b*c^2*d*e*arccos(1/(c*x)) - 1680*b*c^4*d^
2*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/(1/(c*x) + 1)^4 - 252*b*c^2*d*e*log(
abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1)) - 5880*b*c^4*d^2*(1/(c^2*x^2) -
 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(1/(c*x) + 1)^4 + 252
*b*c^2*d*e*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1)) + 5880*b*c^4*d^2
*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(1/(c*
x) + 1)^4 + 2240*b*c^4*d^2*(-1/(c^2*x^2) + 1)^(3/2)/(1/(c*x) + 1)^3 + 672*
a*c^2*d*e - 1680*a*c^4*d^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 - 2016*b*c^
2*d*e*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/(1/(c*x) + 1)^2 - 1680*b*c^4*d^2*(
1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/(1/(c*x) + 1)^6 - 1764*b*c^2*d*e*(1/(c^
2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(1/(c*x) + 1)^2
 - 9800*b*c^4*d^2*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(
c*x) + 1))/(1/(c*x) + 1)^6 + 1764*b*c^2*d*e*(1/(c^2*x^2) - 1)*log(abs(s...
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx=\int x^2\left(e\,x^2+d\right)^2\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^2*(d + e*x^2)^2*(a + b*acos(1/(c*x))),x)
```

output
```
int(x^2*(d + e*x^2)^2*(a + b*acos(1/(c*x))), x)
```

Reduce [F]

$$\int x^2(d+ex^2)^2(a+b\sec^{-1}(cx))\,dx = \left(\int asec(cx)\,x^6dx\right)b\,e^2 + 2\left(\int asec(cx)\,x^4dx\right)bde + \left(\int asec(cx)\,x^2dx\right)b\,d^2 + \frac{a\,d^2x^3}{3} + \frac{2ade\,x^5}{5} + \frac{a\,e^2x^7}{7}$$

input
```
int(x^2*(e*x^2+d)^2*(a+b*asec(c*x)),x)
```

output
```
(105*int(asec(c*x)*x**6,x)*b*e**2 + 210*int(asec(c*x)*x**4,x)*b*d*e + 105*
int(asec(c*x)*x**2,x)*b*d**2 + 35*a*d**2*x**3 + 42*a*d*e*x**5 + 15*a*e**2*
x**7)/105
```

3.82 $\int (d+ex^2)^2 (a+b\sec^{-1}(cx))\, dx$

Optimal result 694
Mathematica [A] (verified) 695
Rubi [A] (verified) 695
Maple [B] (verified) 698
Fricas [A] (verification not implemented) 699
Sympy [A] (verification not implemented) 699
Maxima [A] (verification not implemented) 700
Giac [B] (verification not implemented) 701
Mupad [F(-1)] 702
Reduce [F] 703

Optimal result

Integrand size = 18, antiderivative size = 191

$$\int (d+ex^2)^2 (a+b\sec^{-1}(cx))\, dx = -\frac{be(40c^2d+9e)x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}} - \frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}} + d^2x(a+b\sec^{-1}(cx)) + \frac{2}{3}dex^3(a+b\sec^{-1}(cx)) + \frac{1}{5}e^2x^5(a+b\sec^{-1}(cx)) - \frac{b(120c^4d^2+40c^2de+9e^2)\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{120c^4\sqrt{c^2x^2}}$$

output

```
-1/120*b*e*(40*c^2*d+9*e)*x^2*(c^2*x^2-1)^(1/2)/c^3/(c^2*x^2)^(1/2)-1/20*b
*e^2*x^4*(c^2*x^2-1)^(1/2)/c/(c^2*x^2)^(1/2)+d^2*x*(a+b*arcsec(c*x))+2/3*d
*e*x^3*(a+b*arcsec(c*x))+1/5*e^2*x^5*(a+b*arcsec(c*x))-1/120*b*(120*c^4*d^
2+40*c^2*d*e+9*e^2)*x*arctanh(c*x/(c^2*x^2-1)^(1/2))/c^4/(c^2*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.80

$$\int \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\, dx$$
$$= \frac{c^2x\left(8ac^3(15d^2+10dex^2+3e^2x^4)-be\sqrt{1-\frac{1}{c^2x^2}}x(9e+c^2(40d+6ex^2))\right)+8bc^5x(15d^2+10dex^2+3e^2}{120c^5}$$

input
```
Integrate[(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

output
```
(c^2*x*(8*a*c^3*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - b*e*Sqrt[1 - 1/(c^2*x^
2)]*x*(9*e + c^2*(40*d + 6*e*x^2))) + 8*b*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e
^2*x^4)*ArcSec[c*x] - b*(120*c^4*d^2 + 40*c^2*d*e + 9*e^2)*Log[(1 + Sqrt[1
 - 1/(c^2*x^2)])*x])/(120*c^5)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5751, 27, 1473, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\, dx$$

$$\Big\downarrow \ 5751$$

$$-\frac{bcx\int \frac{3e^2x^4+10dex^2+15d^2}{15\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+d^2x\left(a+b\sec^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sec^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \ 27$$

$$-\frac{bcx\int \frac{3e^2x^4+10dex^2+15d^2}{\sqrt{c^2x^2-1}}dx}{15\sqrt{c^2x^2}}+d^2x\left(a+b\sec^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sec^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \ 1473$$

$$-\frac{bcx\left(\frac{\int \frac{60c^2d^2+e\left(40dc^2+9e\right)x^2}{\sqrt{c^2x^2-1}}dx}{4c^2}+\frac{3e^2x^3\sqrt{c^2x^2-1}}{4c^2}\right)}{15\sqrt{c^2x^2}}+d^2x\left(a+b\sec^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sec^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \ 299$$

$$-\frac{bcx\left(\frac{\frac{\left(120c^4d^2+40c^2de+9e^2\right)\int \frac{1}{\sqrt{c^2x^2-1}}dx}{2c^2}+\frac{ex\sqrt{c^2x^2-1}\left(40c^2d+9e\right)}{2c^2}}{4c^2}+\frac{3e^2x^3\sqrt{c^2x^2-1}}{4c^2}\right)}{15\sqrt{c^2x^2}}+d^2x\left(a+b\sec^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sec^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \ 224$$

$$-\frac{bcx\left(\frac{\frac{\left(120c^4d^2+40c^2de+9e^2\right)\int \frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}}{2c^2}+\frac{ex\sqrt{c^2x^2-1}\left(40c^2d+9e\right)}{2c^2}}{4c^2}+\frac{3e^2x^3\sqrt{c^2x^2-1}}{4c^2}\right)}{15\sqrt{c^2x^2}}+d^2x\left(a+b\sec^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sec^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \ 219$$

$$d^2x\left(a+b\sec^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sec^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\sec^{-1}(cx)\right)-\frac{bcx\left(\frac{\frac{\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)\left(120c^4d^2+40c^2de+9e^2\right)}{2c^3}+\frac{ex\sqrt{c^2x^2-1}\left(40c^2d+9e\right)}{2c^2}}{4c^2}+\frac{3e^2x^3\sqrt{c^2x^2-1}}{4c^2}\right)}{15\sqrt{c^2x^2}}$$

input
```
Int[(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

output
```
d^2*x*(a + b*ArcSec[c*x]) + (2*d*e*x^3*(a + b*ArcSec[c*x]))/3 + (e^2*x^5*(
a + b*ArcSec[c*x]))/5 - (b*c*x*((3*e^2*x^3*Sqrt[-1 + c^2*x^2])/(4*c^2) + (
(e*(40*c^2*d + 9*e)*x*Sqrt[-1 + c^2*x^2])/(2*c^2) + ((120*c^4*d^2 + 40*c^2
*d*e + 9*e^2)*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(2*c^3))/(4*c^2)))/(15*Sq
rt[c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 5751
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSec[c*x])   u,
x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 -
 1]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1
/2, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(169) = 338$.

Time = 0.24 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.78

method	result
parts	$a\left(\frac{1}{5}e^2x^5+\frac{2}{3}x^3de+d^2x\right)+\frac{b\,\mathrm{arcsec}(cx)e^2x^5}{5}+\frac{2b\,\mathrm{arcsec}(cx)de\,x^3}{3}+b\,\mathrm{arcsec}\left(cx\right)x\,d^2-\frac{b(c^2x^2\cdots}{20c^3\sqrt{\cdots}}$
derivativedivides	$\frac{\frac{a\left(d^2c^5x+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+b\,\mathrm{arcsec}(cx)d^2cx+\frac{2bc\,\mathrm{arcsec}(cx)de\,x^3}{3}+\frac{bc\,\mathrm{arcsec}(cx)e^2x^5}{5}-\frac{b\sqrt{c^2x^2-1}\,d^2\ln\left(cx+\sqrt{c^2x^2-1}\cdots\right.}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\cdots}{c}$
default	$\frac{\frac{a\left(d^2c^5x+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+b\,\mathrm{arcsec}(cx)d^2cx+\frac{2bc\,\mathrm{arcsec}(cx)de\,x^3}{3}+\frac{bc\,\mathrm{arcsec}(cx)e^2x^5}{5}-\frac{b\sqrt{c^2x^2-1}\,d^2\ln\left(cx+\sqrt{c^2x^2-1}\cdots\right.}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\cdots}{c}$

input
```
int((e*x^2+d)^2*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*e^2*x^5+2/3*x^3*d*e+d^2*x)+1/5*b*arcsec(c*x)*e^2*x^5+2/3*b*arcsec(c
*x)*d*e*x^3+b*arcsec(c*x)*x*d^2-1/20*b/c^3*(c^2*x^2-1)*x^2/((c^2*x^2-1)/c^
2/x^2)^(1/2)*e^2-1/3*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d*e-b/c
^2*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^2*ln(c*x+(c^2*x^2-1)^
(1/2))-3/40*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*e^2-1/3*b/c^4*(c
^2*x^2-1)^(1/2)/x/((c^2*x^2-1)/c^2/x^2)^(1/2)*d*e*ln(c*x+(c^2*x^2-1)^(1/2)
)-3/40*b/c^6*(c^2*x^2-1)^(1/2)/x/((c^2*x^2-1)/c^2/x^2)^(1/2)*e^2*ln(c*x+(c
^2*x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.24

$$\int \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx = \frac{24\,ac^5e^2x^5+80\,ac^5dex^3+120\,ac^5d^2x+8\,(3\,bc^5e^2x^5+10\,bc^5dex^3+15\,bc^5d^2x-15\,bc^5d^2-10\,bc^5de-3}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/120*(24*a*c^5*e^2*x^5 + 80*a*c^5*d*e*x^3 + 120*a*c^5*d^2*x + 8*(3*b*c^5*
e^2*x^5 + 10*b*c^5*d*e*x^3 + 15*b*c^5*d^2*x - 15*b*c^5*d^2 - 10*b*c^5*d*e
- 3*b*c^5*e^2)*arcsec(c*x) + 16*(15*b*c^5*d^2 + 10*b*c^5*d*e + 3*b*c^5*e^2
)*arctan(-c*x + sqrt(c^2*x^2 - 1)) + (120*b*c^4*d^2 + 40*b*c^2*d*e + 9*b*e
^2)*log(-c*x + sqrt(c^2*x^2 - 1)) - (6*b*c^3*e^2*x^3 + (40*b*c^3*d*e + 9*b
*c*e^2)*x)*sqrt(c^2*x^2 - 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 6.65 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.86

$$\begin{aligned}
&\int \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx \\
&= ad^2x + \frac{2adex^3}{3} + \frac{ae^2x^5}{5} + bd^2x\operatorname{asec}(cx) + \frac{2bdex^3\operatorname{asec}(cx)}{3} \\
&+ \frac{be^2x^5\operatorname{asec}(cx)}{5} - \frac{bd^2\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c} \\
&- \frac{2bde\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c} \\
&- \frac{be^2\left(\begin{cases}\frac{cx^5}{4\sqrt{c^2x^2-1}}+\frac{x^3}{8c\sqrt{c^2x^2-1}}-\frac{3x}{8c^3\sqrt{c^2x^2-1}}+\frac{3\operatorname{acosh}(cx)}{8c^4} & \text{for } |c^2x^2|>1\\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}}-\frac{ix^3}{8c\sqrt{-c^2x^2+1}}+\frac{3ix}{8c^3\sqrt{-c^2x^2+1}}-\frac{3i\operatorname{asin}(cx)}{8c^4} & \text{otherwise}\end{cases}\right)}{5c}
\end{aligned}$$

input
```
integrate((e*x**2+d)**2*(a+b*asec(c*x)),x)
```

output
```
a*d**2*x + 2*a*d*e*x**3/3 + a*e**2*x**5/5 + b*d**2*x*asec(c*x) + 2*b*d*e*x
**3*asec(c*x)/3 + b*e**2*x**5*asec(c*x)/5 - b*d**2*Piecewise((acosh(c*x),
Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c - 2*b*d*e*Piecewise((x*sqrt(c
**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-I*c*x**3
/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin(c*x)/(
2*c**2), True))/(3*c) - b*e**2*Piecewise((c*x**5/(4*sqrt(c**2*x**2 - 1)) +
 x**3/(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*x**2 - 1)) + 3*aco
sh(c*x)/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqrt(-c**2*x**2 + 1))
 - I*x**3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**2 + 1))
 - 3*I*asin(c*x)/(8*c**4), True))/(5*c)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.55

$$\int \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{5}\,ae^2x^5 + \frac{2}{3}\,adex^3 + \frac{1}{6}\left(4\,x^3\,\mathrm{arcsec}\left(cx\right) - \frac{\frac{2\,\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bde + \frac{1}{80}\left(16\,x^5\,\mathrm{arcsec}\left(cx\right) + \frac{\frac{2\left(3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\,\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2\,c^4\left(\frac{1}{c^2x^2}-1\right)+c^4} - \frac{3\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4} + \frac{3\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)be^2 + ad^2x + \frac{\left(2\,cx\,\mathrm{arcsec}\left(cx\right) - \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) + \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd^2}{2\,c}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*e^2*x^5 + 2/3*a*d*e*x^3 + 1/6*(4*x^3*arcsec(c*x) - (2*sqrt(-1/(c^2*x
^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c
^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d*e + 1/80*(16*x^5*arcsec(c
*x) + (2*(3*(-1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(-1/(c^2*x^2) + 1))/(c^4*(1/(
c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4) - 3*log(sqrt(-1/(c^2*x^2)
 + 1) + 1)/c^4 + 3*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*e^2 + a*d^2*x
 + 1/2*(2*c*x*arcsec(c*x) - log(sqrt(-1/(c^2*x^2) + 1) + 1) + log(-sqrt(-1
/(c^2*x^2) + 1) + 1))*b*d^2/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14166 vs. $2(169) = 338$.

Time = 4.17 (sec) , antiderivative size = 14166, normalized size of antiderivative = 74.17

$$\int \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/120*(120*b*c^4*d^2*arccos(1/(c*x))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*
x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^
2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^
6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10) - 120*b*c^4*d^2*log(abs(sqrt(-1/(c
^2*x^2) + 1) + 1/(c*x) + 1))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^
2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^
3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^
2*x^2) - 1)^5/(1/(c*x) + 1)^10) + 120*b*c^4*d^2*log(abs(sqrt(-1/(c^2*x^2)
+ 1) - 1/(c*x) - 1))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c
^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*
x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) -
 1)^5/(1/(c*x) + 1)^10) + 120*a*c^4*d^2/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/
(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2
*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 +
 c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10) + 360*b*c^4*d^2*(1/(c^2*x^2) -
1)*arccos(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^
6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x
) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) -
1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^2) - 600*b*c^4*d^2*(1/(c^2*x^2) - 1)*
log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^...
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)^2 (a+b\sec^{-1}(cx)) \, dx = \int (e\,x^2+d)^2 \left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right) \, dx$$

input
```
int((d + e*x^2)^2*(a + b*acos(1/(c*x))),x)
```

output
```
int((d + e*x^2)^2*(a + b*acos(1/(c*x))), x)
```

Reduce [F]

$$\int (d+ex^2)^2 (a+b\sec^{-1}(cx))\ dx = \left(\int asec(cx)\,dx\right) b\,d^2 + \left(\int asec(cx)\,x^4dx\right) b\,e^2 + 2\left(\int asec(cx)\,x^2dx\right) bde + a\,d^2x + \frac{2ade\,x^3}{3} + \frac{a\,e^2x^5}{5}$$

input
```
int((e*x^2+d)^2*(a+b*asec(c*x)),x)
```

output
```
(15*int(asec(c*x),x)*b*d**2 + 15*int(asec(c*x)*x**4,x)*b*e**2 + 30*int(ase
c(c*x)*x**2,x)*b*d*e + 15*a*d**2*x + 10*a*d*e*x**3 + 3*a*e**2*x**5)/15
```

3.83 $\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^2}\,dx$

Optimal result . . . 704
Mathematica [A] (verified) . . . 705
Rubi [A] (verified) . . . 705
Maple [A] (verified) . . . 708
Fricas [A] (verification not implemented) . . . 709
Sympy [A] (verification not implemented) . . . 710
Maxima [A] (verification not implemented) . . . 711
Giac [B] (verification not implemented) . . . 711
Mupad [F(-1)] . . . 712
Reduce [F] . . . 713

Optimal result

Integrand size = 21, antiderivative size = 162

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^2}\,dx = \frac{bcd^2\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} - \frac{be^2x^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{x} + 2dex\left(a+b\sec^{-1}(cx)\right) + \frac{1}{3}e^2x^3\left(a+b\sec^{-1}(cx)\right) - \frac{be(12c^2d+e)\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{6c^2\sqrt{c^2x^2}}$$

output
```
b*c*d^2*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)-1/6*b*e^2*x^2*(c^2*x^2-1)^(1/2)/
c/(c^2*x^2)^(1/2)-d^2*(a+b*arcsec(c*x))/x+2*d*e*x*(a+b*arcsec(c*x))+1/3*e^
2*x^3*(a+b*arcsec(c*x))-1/6*b*e*(12*c^2*d+e)*x*arctanh(c*x/(c^2*x^2-1)^(1/
2))/c^2/(c^2*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^2} dx$$
$$= \frac{c^2\left(b\sqrt{1-\frac{1}{c^2x^2}}x(6c^2d^2-e^2x^2)+2ac(-3d^2+6dex^2+e^2x^4)\right)+2bc^3(-3d^2+6dex^2+e^2x^4)\sec^{-1}(cx)-}{6c^3x}$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^2,x]
```

output
```
(c^2*(b*Sqrt[1 - 1/(c^2*x^2)]*x*(6*c^2*d^2 - e^2*x^2) + 2*a*c*(-3*d^2 + 6*
d*e*x^2 + e^2*x^4)) + 2*b*c^3*(-3*d^2 + 6*d*e*x^2 + e^2*x^4)*ArcSec[c*x] -
 b*e*(12*c^2*d + e)*x*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(6*c^3*x)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.90, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {5761, 27, 1588, 25, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^2} dx$$

$$\Big\downarrow \text{5761}$$

$$-\frac{bcx\int -\frac{-e^2x^4-6dex^2+3d^2}{3x^2\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}-\frac{d^2(a+b\sec^{-1}(cx))}{x}+2dex(a+b\sec^{-1}(cx))+\frac{1}{3}e^2x^3(a+b\sec^{-1}(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int \frac{-e^2x^4-6dex^2+3d^2}{x^2\sqrt{c^2x^2-1}}dx}{3\sqrt{c^2x^2}}-\frac{d^2(a+b\sec^{-1}(cx))}{x}+2dex(a+b\sec^{-1}(cx))+\frac{1}{3}e^2x^3(a+b\sec^{-1}(cx))$$

$$\Big\downarrow \ 1588$$

$$\frac{bcx\left(\int -\frac{e(ex^2+6d)}{\sqrt{c^2x^2-1}}dx+\frac{3d^2\sqrt{c^2x^2-1}}{x}\right)}{3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{x}+2dex\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \ 25$$

$$\frac{bcx\left(\frac{3d^2\sqrt{c^2x^2-1}}{x}-\int \frac{e(ex^2+6d)}{\sqrt{c^2x^2-1}}dx\right)}{3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{x}+2dex\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \ 27$$

$$\frac{bcx\left(\frac{3d^2\sqrt{c^2x^2-1}}{x}-e\int \frac{ex^2+6d}{\sqrt{c^2x^2-1}}dx\right)}{3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{x}+2dex\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \ 299$$

$$\frac{bcx\left(\frac{3d^2\sqrt{c^2x^2-1}}{x}-e\left(\frac{(12c^2d+e)\int \frac{1}{\sqrt{c^2x^2-1}}dx}{2c^2}+\frac{ex\sqrt{c^2x^2-1}}{2c^2}\right)\right)}{3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{x}+2dex\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \ 224$$

$$\frac{bcx\left(\frac{3d^2\sqrt{c^2x^2-1}}{x}-e\left(\frac{(12c^2d+e)\int \frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}}{2c^2}+\frac{ex\sqrt{c^2x^2-1}}{2c^2}\right)\right)}{3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{x}+2dex\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \ 219$$

$$-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{x}+2dex\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\sec^{-1}(cx)\right)+\frac{bcx\left(\frac{3d^2\sqrt{c^2x^2-1}}{x}-e\left(\frac{\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)(12c^2d+e)}{2c^3}+\frac{ex\sqrt{c^2x^2-1}}{2c^2}\right)\right)}{3\sqrt{c^2x^2}}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^2,x]
```

output
```
-((d^2*(a + b*ArcSec[c*x]))/x) + 2*d*e*x*(a + b*ArcSec[c*x]) + (e^2*x^3*(a
 + b*ArcSec[c*x]))/3 + (b*c*x*((3*d^2*Sqrt[-1 + c^2*x^2])/x - e*((e*x*Sqrt
[-1 + c^2*x^2])/(2*c^2) + ((12*c^2*d + e)*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]
])/(2*c^3))))/(3*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.54

method	result
parts	$a\left(\frac{e^2x^3}{3}+2dex-\frac{d^2}{x}\right)+\frac{b\,e^2 \operatorname{arcsec}(cx)x^3}{3}+2be \operatorname{arcsec}(cx)\,xd-\frac{b \operatorname{arcsec}(cx)d^2}{x}-\frac{b\,e^2(c^2x^2-1)}{6c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}$
derivativedivides	$c\left(\frac{a\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{2b \operatorname{arcsec}(cx)dex}{c}+\frac{b \operatorname{arcsec}(cx)e^2x^3}{3c}-\frac{b \operatorname{arcsec}(cx)d^2}{cx}+\frac{b(c^2x^2-1)d^2}{c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\right.$
default	$c\left(\frac{a\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{2b \operatorname{arcsec}(cx)dex}{c}+\frac{b \operatorname{arcsec}(cx)e^2x^3}{3c}-\frac{b \operatorname{arcsec}(cx)d^2}{cx}+\frac{b(c^2x^2-1)d^2}{c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\right.$

input
```
int((e*x^2+d)^2*(a+b*arcsec(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*e^2*x^3+2*d*e*x-d^2/x)+1/3*b*e^2*arcsec(c*x)*x^3+2*b*e*arcsec(c*x)*
x*d-b*arcsec(c*x)*d^2/x-1/6*b/c^3*e^2*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1
/2)+b/c*(c^2*x^2-1)/x^2/((c^2*x^2-1)/c^2/x^2)^(1/2)*d^2-2*b/c^2*(c^2*x^2-1
)^(1/2)/x/((c^2*x^2-1)/c^2/x^2)^(1/2)*d*e*ln(c*x+(c^2*x^2-1)^(1/2))-1/6*b/
c^4*e^2*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*ln(c*x+(c^2*x^2-1)
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.42

$$\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^2}\,dx = \frac{2\,ac^3e^2x^4+6\,bc^4d^2x+12\,ac^3dex^2-6\,ac^3d^2-4\,(3\,bc^3d^2-6\,bc^3de-bc^3e^2)x\arctan\left(-cx+\sqrt{c^2x^2-1}\right)}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^2,x, algorithm="fricas")
```

output
```
1/6*(2*a*c^3*e^2*x^4 + 6*b*c^4*d^2*x + 12*a*c^3*d*e*x^2 - 6*a*c^3*d^2 - 4*
(3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x*arctan(-c*x + sqrt(c^2*x^2 - 1))
 + (12*b*c^2*d*e + b*e^2)*x*log(-c*x + sqrt(c^2*x^2 - 1)) + 2*(b*c^3*e^2*x
^4 + 6*b*c^3*d*e*x^2 - 3*b*c^3*d^2 + (3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^
2)*x)*arcsec(c*x) + (6*b*c^3*d^2 - b*c*e^2*x^2)*sqrt(c^2*x^2 - 1))/(c^3*x)
```

Sympy [A] (verification not implemented)

Time = 5.06 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^2}\,dx$$
$$= -\frac{ad^2}{x} + 2adex + \frac{ae^2x^3}{3} + bcd^2\sqrt{1-\frac{1}{c^2x^2}} - \frac{bd^2\operatorname{asec}(cx)}{x} + 2bdex\operatorname{asec}(cx) + \frac{be^2x^3\operatorname{asec}(cx)}{3} - \frac{2bde\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c} - \frac{be^2\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$

input

```
integrate((e*x**2+d)**2*(a+b*asec(c*x))/x**2,x)
```

output

```
-a*d**2/x + 2*a*d*e*x + a*e**2*x**3/3 + b*c*d**2*sqrt(1 - 1/(c**2*x**2)) -
 b*d**2*asec(c*x)/x + 2*b*d*e*x*asec(c*x) + b*e**2*x**3*asec(c*x)/3 - 2*b*
d*e*Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c -
b*e**2*Piecewise((x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c
**2*x**2) > 1), (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*
x**2 + 1)) - I*asin(c*x)/(2*c**2), True))/(3*c)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^2}\,dx$$
$$= \frac{1}{3}ae^2x^3 + \left(c\sqrt{-\frac{1}{c^2x^2}+1} - \frac{\operatorname{arcsec}(cx)}{x}\right)bd^2 + \frac{1}{12}\left(4\,x^3\operatorname{arcsec}(cx) - \frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)be^2 + 2\,adex + \frac{\left(2\,cx\operatorname{arcsec}(cx) - \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) + \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bde}{c} - \frac{ad^2}{x}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^2,x, algorithm="maxima")
```

output
```
1/3*a*e^2*x^3 + (c*sqrt(-1/(c^2*x^2) + 1) - arcsec(c*x)/x)*b*d^2 + 1/12*(4
*x^3*arcsec(c*x) - (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2)
 + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c
^2)/c)*b*e^2 + 2*a*d*e*x + (2*c*x*arcsec(c*x) - log(sqrt(-1/(c^2*x^2) + 1)
 + 1) + log(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*d*e/c - a*d^2/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6018 vs. 2(144) = 288.

Time = 2.42 (sec) , antiderivative size = 6018, normalized size of antiderivative = 37.15

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^2,x, algorithm="giac")
```

output
```
-1/6*(6*b*c^4*d^2*arccos(1/(c*x))/(c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x)
+ 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)
^4/(1/(c*x) + 1)^8) + 6*a*c^4*d^2/(c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x)
+ 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)
^4/(1/(c*x) + 1)^8) + 24*b*c^4*d^2*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^4
 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/
(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) -
 12*b*c^4*d^2*sqrt(-1/(c^2*x^2) + 1)/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c
*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2)
- 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)) + 24*a*c^4*d^2*(1/(c^2*x^2) - 1)/((
c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/
(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2
) - 12*b*c^2*d*e*arccos(1/(c*x))/(c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) +
 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^
4/(1/(c*x) + 1)^8) + 36*b*c^4*d^2*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^
4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1
/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4)
+ 12*b*c^2*d*e*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^4 + 2*c^4
*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) +
1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) - 12*b*c^2*d*e*log(abs(...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^2}\,dx = \int \frac{(e\,x^2+d)^2\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

input
```
int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^2,x)
```

output
```
int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^2, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^2}\,dx$$
$$= \frac{6\left(\int asec(cx)\,dx\right) bdex + 3\left(\int \frac{asec(cx)}{x^2}dx\right) b\,d^2x + 3\left(\int asec(cx)\,x^2dx\right) b\,e^2x - 3a\,d^2 + 6ade\,x^2 + a\,e^2x^4}{3x}$$

input
```
int((e*x^2+d)^2*(a+b*asec(c*x))/x^2,x)
```

output
```
(6*int(asec(c*x),x)*b*d*e*x + 3*int(asec(c*x)/x**2,x)*b*d**2*x + 3*int(ase
c(c*x)*x**2,x)*b*e**2*x - 3*a*d**2 + 6*a*d*e*x**2 + a*e**2*x**4)/(3*x)
```

3.84 $\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^4}\,dx$

Optimal result . . . 714
Mathematica [A] (verified) . . . 715
Rubi [A] (verified) . . . 715
Maple [A] (verified) . . . 718
Fricas [A] (verification not implemented) . . . 718
Sympy [A] (verification not implemented) . . . 719
Maxima [A] (verification not implemented) . . . 720
Giac [B] (verification not implemented) . . . 720
Mupad [F(-1)] . . . 721
Reduce [F] . . . 722

Optimal result

Integrand size = 21, antiderivative size = 158

$$\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^4}\,dx = \frac{2bcd(c^2d+9e)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}} + \frac{bcd^2\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{3x^3} - \frac{2de(a+b\sec^{-1}(cx))}{x} + e^2x(a+b\sec^{-1}(cx)) - \frac{be^2x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{c^2x^2}}$$

output

```
2/9*b*c*d*(c^2*d+9*e)*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)+1/9*b*c*d^2*(c^2*x
^2-1)^(1/2)/x^2/(c^2*x^2)^(1/2)-1/3*d^2*(a+b*arcsec(c*x))/x^3-2*d*e*(a+b*a
rcsec(c*x))/x+e^2*x*(a+b*arcsec(c*x))-b*e^2*x*arctanh(c*x/(c^2*x^2-1)^(1/2
))/(c^2*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^4}\,dx$$
$$= \frac{c\left(bcd\sqrt{1-\frac{1}{c^2x^2}}x(d+2c^2dx^2+18ex^2)-3a(d^2+6dex^2-3e^2x^4)\right)-3bc(d^2+6dex^2-3e^2x^4)\sec^{-1}(cx}{9cx^3}$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^4,x]
```

output
```
(c*(b*c*d*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2*c^2*d*x^2 + 18*e*x^2) - 3*a*(d^2
+ 6*d*e*x^2 - 3*e^2*x^4)) - 3*b*c*(d^2 + 6*d*e*x^2 - 3*e^2*x^4)*ArcSec[c*x
] - 9*b*e^2*x^3*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(9*c*x^3)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5761, 27, 1588, 358, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^4}\,dx$$

$$\Big\downarrow \text{5761}$$

$$-\frac{bcx\int -\frac{-3e^2x^4+6dex^2+d^2}{3x^4\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}-\frac{d^2(a+b\sec^{-1}(cx))}{3x^3}-\frac{2de(a+b\sec^{-1}(cx))}{x}+e^2x(a+b\sec^{-1}(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int \frac{-3e^2x^4+6dex^2+d^2}{x^4\sqrt{c^2x^2-1}}dx}{3\sqrt{c^2x^2}}-\frac{d^2(a+b\sec^{-1}(cx))}{3x^3}-\frac{2de(a+b\sec^{-1}(cx))}{x}+e^2x(a+b\sec^{-1}(cx))$$

$$\Big\downarrow \text{1588}$$

$$\frac{bcx\left(\frac{1}{3}\int \frac{2d(dc^2+9e)-9e^2x^2}{x^2\sqrt{c^2x^2-1}}dx+\frac{d^2\sqrt{c^2x^2-1}}{3x^3}\right)}{3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\sec^{-1}(cx)\right)}{x}+e^2x\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{358}$$

$$\frac{bcx\left(\frac{1}{3}\left(\frac{2d\sqrt{c^2x^2-1}(c^2d+9e)}{x}-9e^2\int \frac{1}{\sqrt{c^2x^2-1}}dx\right)+\frac{d^2\sqrt{c^2x^2-1}}{3x^3}\right)}{3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\sec^{-1}(cx)\right)}{x}+e^2x\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{bcx\left(\frac{1}{3}\left(\frac{2d\sqrt{c^2x^2-1}(c^2d+9e)}{x}-9e^2\int \frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}\right)+\frac{d^2\sqrt{c^2x^2-1}}{3x^3}\right)}{3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\sec^{-1}(cx)\right)}{x}+e^2x\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \text{219}$$

$$-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\sec^{-1}(cx)\right)}{x}+e^2x\left(a+b\sec^{-1}(cx)\right)+\frac{bcx\left(\frac{1}{3}\left(\frac{2d\sqrt{c^2x^2-1}(c^2d+9e)}{x}-\frac{9e^2\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{c}\right)+\frac{d^2\sqrt{c^2x^2-1}}{3x^3}\right)}{3\sqrt{c^2x^2}}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^4,x]
```

output
```
-1/3*(d^2*(a + b*ArcSec[c*x]))/x^3 - (2*d*e*(a + b*ArcSec[c*x]))/x + e^2*x
*(a + b*ArcSec[c*x]) + (b*c*x*((d^2*Sqrt[-1 + c^2*x^2])/(3*x^3) + ((2*d*(c
^2*d + 9*e)*Sqrt[-1 + c^2*x^2])/x - (9*e^2*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2
]])/c)/3))/(3*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.46

method	result
parts	$a\left(e^{2} x-\frac{2 d e}{x}-\frac{d^{2}}{3 x^{3}}\right)+b \,\mathrm{arcsec}\left(c x\right) e^{2} x-\frac{2 b \,\mathrm{arcsec}\left(c x\right) d e}{x}-\frac{b \,\mathrm{arcsec}\left(c x\right) d^{2}}{3 x^{3}}+\frac{2 b c \left(c^{2} x^{2}-1\right) d^{2}}{9 \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}\, x^{2}}+$
derivativedivides	$c^{3}\left(\frac{a\left(e^{2} c x-\frac{2 c d e}{x}-\frac{c\, d^{2}}{3 x^{3}}\right)}{c^{4}}+\frac{b \,\mathrm{arcsec}\left(c x\right) e^{2} x}{c^{3}}-\frac{2 b \,\mathrm{arcsec}\left(c x\right) d e}{c^{3} x}-\frac{b \,\mathrm{arcsec}\left(c x\right) d^{2}}{3 c^{3} x^{3}}+\frac{2 b \left(c^{2} x^{2}-1\right) d^{2}}{9 c^{2} x^{2} \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}}+\frac{b(c^2}{9\sqrt{c^2}}\right.$
default	$c^{3}\left(\frac{a\left(e^{2} c x-\frac{2 c d e}{x}-\frac{c\, d^{2}}{3 x^{3}}\right)}{c^{4}}+\frac{b \,\mathrm{arcsec}\left(c x\right) e^{2} x}{c^{3}}-\frac{2 b \,\mathrm{arcsec}\left(c x\right) d e}{c^{3} x}-\frac{b \,\mathrm{arcsec}\left(c x\right) d^{2}}{3 c^{3} x^{3}}+\frac{2 b \left(c^{2} x^{2}-1\right) d^{2}}{9 c^{2} x^{2} \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}}+\frac{b(c^2}{9\sqrt{c^2}}\right.$

input
```
int((e*x^2+d)^2*(a+b*arcsec(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
a*(e^2*x-2*d*e/x-1/3*d^2/x^3)+b*arcsec(c*x)*e^2*x-2*b*arcsec(c*x)*d*e/x-1/
3*b*arcsec(c*x)*d^2/x^3+2/9*b*c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^
2*d^2+2*b/c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^2*d*e-b/c^2*(c^2*x^2
-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*e^2*ln(c*x+(c^2*x^2-1)^(1/2))+1/9*
b/c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^4*d^2
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.40

$$\int \frac{\left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)}{x^4}\,dx$$
$$= \frac{9\,ace^2x^4 + 9\,be^2x^3\log\left(-cx+\sqrt{c^2x^2-1}\right) - 18\,acdex^2 - 6\,(bcd^2+6\,bcde-3\,bce^2)x^3\arctan\left(-cx+\sqrt{c^2}\right.}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^4,x, algorithm="fricas")
```

output
```
1/9*(9*a*c*e^2*x^4 + 9*b*e^2*x^3*log(-c*x + sqrt(c^2*x^2 - 1)) - 18*a*c*d*
e*x^2 - 6*(b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3*arctan(-c*x + sqrt(c^2*x^2
 - 1)) - 3*a*c*d^2 + 2*(b*c^4*d^2 + 9*b*c^2*d*e)*x^3 + 3*(3*b*c*e^2*x^4 -
6*b*c*d*e*x^2 - b*c*d^2 + (b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3)*arcsec(c*
x) + (b*c*d^2 + 2*(b*c^3*d^2 + 9*b*c*d*e)*x^2)*sqrt(c^2*x^2 - 1))/(c*x^3)
```

Sympy [A] (verification not implemented)

Time = 5.67 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.34

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^4}\,dx$$
$$= -\frac{ad^2}{3x^3} - \frac{2ade}{x} + ae^2x + 2bcde\sqrt{1-\frac{1}{c^2x^2}} - \frac{bd^2\operatorname{asec}(cx)}{3x^3} - \frac{2bde\operatorname{asec}(cx)}{x} + be^2x\operatorname{asec}(cx) + \frac{bd^2\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x}+\frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2|>1\\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x}+\frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c} - \frac{be^2\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c}$$

input
```
integrate((e*x**2+d)**2*(a+b*asec(c*x))/x**4,x)
```

output
```
-a*d**2/(3*x**3) - 2*a*d*e/x + a*e**2*x + 2*b*c*d*e*sqrt(1 - 1/(c**2*x**2)
) - b*d**2*asec(c*x)/(3*x**3) - 2*b*d*e*asec(c*x)/x + b*e**2*x*asec(c*x) +
 b*d**2*Piecewise((2*c**3*sqrt(c**2*x**2 - 1)/(3*x) + c*sqrt(c**2*x**2 - 1
)/(3*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*sqrt(-c**2*x**2 + 1)/(3*x) + I*
c*sqrt(-c**2*x**2 + 1)/(3*x**3), True))/(3*c) - b*e**2*Piecewise((acosh(c*
x), Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^4}\,dx$$
$$= 2\left(c\sqrt{-\frac{1}{c^2x^2}+1}-\frac{\operatorname{arcsec}(cx)}{x}\right)bde+ae^2x$$
$$-\frac{1}{9}\,bd^2\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{3\,\operatorname{arcsec}(cx)}{x^3}\right)$$
$$+\frac{\left(2\,cx\operatorname{arcsec}(cx)-\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)+\log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)be^2}{2\,c}$$
$$-\frac{2\,ade}{x}-\frac{ad^2}{3\,x^3}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^4,x, algorithm="maxima")
```

output
```
2*(c*sqrt(-1/(c^2*x^2) + 1) - arcsec(c*x)/x)*b*d*e + a*e^2*x - 1/9*b*d^2*(
(c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c + 3*arcsec
(c*x)/x^3) + 1/2*(2*c*x*arcsec(c*x) - log(sqrt(-1/(c^2*x^2) + 1) + 1) + lo
g(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*e^2/c - 2*a*d*e/x - 1/3*a*d^2/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4968 vs. 2(140) = 280.

Time = 96.74 (sec) , antiderivative size = 4968, normalized size of antiderivative = 31.44

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^4,x, algorithm="giac")
```

output
```
-1/9*(3*b*c^4*d^2*arccos(1/(c*x))/(c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x)
+ 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)
^4/(1/(c*x) + 1)^8) + 3*a*c^4*d^2/(c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x)
+ 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)
^4/(1/(c*x) + 1)^8) + 12*b*c^4*d^2*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^2
 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/
(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) -
 6*b*c^4*d^2*sqrt(-1/(c^2*x^2) + 1)/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*
x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) -
 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)) + 12*a*c^4*d^2*(1/(c^2*x^2) - 1)/((c
^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(
1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2)
 + 18*b*c^2*d*e*arccos(1/(c*x))/(c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) +
1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4
/(1/(c*x) + 1)^8) + 18*b*c^4*d^2*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^2
 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/
(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) +
 2*b*c^4*d^2*(-1/(c^2*x^2) + 1)^(3/2)/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(
c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2)
 - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^3) + 18*a*c^2*d*e/(c^2 - 2*c^2*(...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{x^4}\,dx=\int \frac{\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

input
```
int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^4,x)
```

output
```
int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^4, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^4}\,dx$$
$$= \frac{3\left(\int asec(cx)\,dx\right) b\,e^2x^3 + 3\left(\int \frac{asec(cx)}{x^4}dx\right) b\,d^2x^3 + 6\left(\int \frac{asec(cx)}{x^2}dx\right) bde\,x^3 - a\,d^2 - 6ade\,x^2 + 3a\,e^2x^4}{3x^3}$$

input
```
int((e*x^2+d)^2*(a+b*asec(c*x))/x^4,x)
```

output
```
(3*int(asec(c*x),x)*b*e**2*x**3 + 3*int(asec(c*x)/x**4,x)*b*d**2*x**3 + 6*
int(asec(c*x)/x**2,x)*b*d*e*x**3 - a*d**2 - 6*a*d*e*x**2 + 3*a*e**2*x**4)/
(3*x**3)
```

3.85 $\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^6}\,dx$

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Optimal result

Integrand size = 21, antiderivative size = 183

$$\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^6}\,dx = \frac{bc(24c^4d^2+100c^2de+225e^2)\sqrt{-1+c^2x^2}}{225\sqrt{c^2x^2}} + \frac{bcd^2\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}} + \frac{2bcd(6c^2d+25e)\sqrt{-1+c^2x^2}}{225x^2\sqrt{c^2x^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{5x^5} - \frac{2de(a+b\sec^{-1}(cx))}{3x^3} - \frac{e^2(a+b\sec^{-1}(cx))}{x}$$

output
```
1/225*b*c*(24*c^4*d^2+100*c^2*d*e+225*e^2)*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/
2)+1/25*b*c*d^2*(c^2*x^2-1)^(1/2)/x^4/(c^2*x^2)^(1/2)+2/225*b*c*d*(6*c^2*d
+25*e)*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^2)^(1/2)-1/5*d^2*(a+b*arcsec(c*x))/x^5
-2/3*d*e*(a+b*arcsec(c*x))/x^3-e^2*(a+b*arcsec(c*x))/x
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^6}\,dx = \frac{-15a(3d^2+10dex^2+15e^2x^4)+bc\sqrt{1-\frac{1}{c^2x^2}}x(225e^2x^4+50dex^2(1+2c^2x^2)+3d^2(3+4c^2x^2+8c^4x^4))}{225x^5}$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^6,x]
```

output
```
(-15*a*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(22
5*e^2*x^4 + 50*d*e*x^2*(1 + 2*c^2*x^2) + 3*d^2*(3 + 4*c^2*x^2 + 8*c^4*x^4)
) - 15*b*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4)*ArcSec[c*x])/(225*x^5)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5761, 27, 1588, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^6}\,dx$$

$$\Big\downarrow \text{5761}$$

$$-\frac{bcx\int -\frac{15e^2x^4+10dex^2+3d^2}{15x^6\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}-\frac{d^2(a+b\sec^{-1}(cx))}{5x^5}-\frac{2de(a+b\sec^{-1}(cx))}{3x^3}-\frac{e^2(a+b\sec^{-1}(cx))}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int \frac{15e^2x^4+10dex^2+3d^2}{x^6\sqrt{c^2x^2-1}}dx}{15\sqrt{c^2x^2}}-\frac{d^2(a+b\sec^{-1}(cx))}{5x^5}-\frac{2de(a+b\sec^{-1}(cx))}{3x^3}-\frac{e^2(a+b\sec^{-1}(cx))}{x}$$

$$\Big\downarrow \text{1588}$$

$$\frac{bcx\left(\frac{1}{5}\int \frac{75e^2x^2+2d(6dc^2+25e)}{x^4\sqrt{c^2x^2-1}}dx+\frac{3d^2\sqrt{c^2x^2-1}}{5x^5}\right)}{15\sqrt{c^2x^2}}-\frac{d^2(a+b\sec^{-1}(cx))}{5x^5}-\frac{2de(a+b\sec^{-1}(cx))}{3x^3}-\frac{e^2(a+b\sec^{-1}(cx))}{x}$$

$$\Big\downarrow \text{359}$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}(24c^4d^2+100c^2de+225e^2)\int \frac{1}{x^2\sqrt{c^2x^2-1}}dx+\frac{2d\sqrt{c^2x^2-1}(6c^2d+25e)}{3x^3}\right)+\frac{3d^2\sqrt{c^2x^2-1}}{5x^5}\right)}{15\sqrt{c^2x^2}}-\frac{d^2(a+b\sec^{-1}(cx))}{5x^5}-\frac{2de(a+b\sec^{-1}(cx))}{3x^3}-\frac{e^2(a+b\sec^{-1}(cx))}{x}$$

$$\Big\downarrow \text{242}$$

$$-\frac{d^2(a+b\sec^{-1}(cx))}{5x^5}-\frac{2de(a+b\sec^{-1}(cx))}{3x^3}-\frac{e^2(a+b\sec^{-1}(cx))}{x}+\frac{bcx\left(\frac{3d^2\sqrt{c^2x^2-1}}{5x^5}+\frac{1}{5}\left(\frac{2d\sqrt{c^2x^2-1}(6c^2d+25e)}{3x^3}+\frac{\sqrt{c^2x^2-1}(24c^4d^2+100c^2de+225e^2)}{3x}\right)\right)}{15\sqrt{c^2x^2}}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^6,x]
```

output
```
(b*c*x*((3*d^2*Sqrt[-1 + c^2*x^2])/(5*x^5) + ((2*d*(6*c^2*d + 25*e)*Sqrt[-
1 + c^2*x^2])/(3*x^3) + ((24*c^4*d^2 + 100*c^2*d*e + 225*e^2)*Sqrt[-1 + c^
2*x^2])/(3*x))/5))/(15*Sqrt[c^2*x^2]) - (d^2*(a + b*ArcSec[c*x]))/(5*x^5)
- (2*d*e*(a + b*ArcSec[c*x]))/(3*x^3) - (e^2*(a + b*ArcSec[c*x]))/x
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.96

method	result
parts	$a\left(-\frac{e^2}{x}-\frac{d^2}{5x^5}-\frac{2de}{3x^3}\right)+b\,c^5\left(-\frac{\operatorname{arcsec}(cx)e^2}{c^5x}-\frac{\operatorname{arcsec}(cx)d^2}{5x^5c^5}-\frac{2\operatorname{arcsec}(cx)de}{3c^5x^3}+\frac{(c^2x^2-1)(24c^8d^2x^4+\ldots}{}\right.$
derivativedivides	$c^5\left(\frac{a\left(-\frac{d^2}{5c\,x^5}-\frac{e^2}{cx}-\frac{2de}{3c\,x^3}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arcsec}(cx)d^2}{5c\,x^5}-\frac{\operatorname{arcsec}(cx)e^2}{cx}-\frac{2\operatorname{arcsec}(cx)de}{3c\,x^3}+\frac{(c^2x^2-1)(24c^8d^2x^4+100c^6de\,x^4+12c^6\ldots}{225\sqrt{\frac{c^2x^2\ldots}{c^2x\ldots}}}\right.}{c^4}\right.$
default	$c^5\left(\frac{a\left(-\frac{d^2}{5c\,x^5}-\frac{e^2}{cx}-\frac{2de}{3c\,x^3}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arcsec}(cx)d^2}{5c\,x^5}-\frac{\operatorname{arcsec}(cx)e^2}{cx}-\frac{2\operatorname{arcsec}(cx)de}{3c\,x^3}+\frac{(c^2x^2-1)(24c^8d^2x^4+100c^6de\,x^4+12c^6\ldots}{225\sqrt{\frac{c^2x^2\ldots}{c^2x\ldots}}}\right.}{c^4}\right.$

input
```
int((e*x^2+d)^2*(a+b*arcsec(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
a*(-e^2/x-1/5*d^2/x^5-2/3*d*e/x^3)+b*c^5*(-1/c^5*arcsec(c*x)*e^2/x-1/5*arc
sec(c*x)*d^2/x^5/c^5-2/3/c^5*arcsec(c*x)*d*e/x^3+1/225/c^10*(c^2*x^2-1)*(2
4*c^8*d^2*x^4+100*c^6*d*e*x^4+12*c^6*d^2*x^2+225*c^4*e^2*x^4+50*c^4*d*e*x^
2+9*c^4*d^2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^6)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^6}\,dx =$$
$$-\frac{225\,ae^2x^4+150\,adex^2+45\,ad^2+15\,(15\,be^2x^4+10\,bdex^2+3\,bd^2)\operatorname{arcsec}(cx)-((24\,bc^4d^2+100\,bc^2d\ldots}{225\,x^5}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^6,x, algorithm="fricas")
```

output
```
-1/225*(225*a*e^2*x^4 + 150*a*d*e*x^2 + 45*a*d^2 + 15*(15*b*e^2*x^4 + 10*b
*d*e*x^2 + 3*b*d^2)*arcsec(c*x) - ((24*b*c^4*d^2 + 100*b*c^2*d*e + 225*b*e
^2)*x^4 + 9*b*d^2 + 2*(6*b*c^2*d^2 + 25*b*d*e)*x^2)*sqrt(c^2*x^2 - 1))/x^5
```

Sympy [A] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.82

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^6}\,dx$$

$$= -\frac{ad^2}{5x^5} - \frac{2ade}{3x^3} - \frac{ae^2}{x} + bce^2\sqrt{1-\frac{1}{c^2x^2}} - \frac{bd^2\operatorname{asec}(cx)}{5x^5} - \frac{2bde\operatorname{asec}(cx)}{3x^3} - \frac{be^2\operatorname{asec}(cx)}{x} + \frac{bd^2\left(\begin{cases}\frac{8c^5\sqrt{c^2x^2-1}}{15x} + \frac{4c^3\sqrt{c^2x^2-1}}{15x^3} + \frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for } |c^2x^2|>1\\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x} + \frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3} + \frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise}\end{cases}\right)}{5c} + \frac{2bde\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x} + \frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2|>1\\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x} + \frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c}$$

input
```
integrate((e*x**2+d)**2*(a+b*asec(c*x))/x**6,x)
```

output
```
-a*d**2/(5*x**5) - 2*a*d*e/(3*x**3) - a*e**2/x + b*c*e**2*sqrt(1 - 1/(c**2
*x**2)) - b*d**2*asec(c*x)/(5*x**5) - 2*b*d*e*asec(c*x)/(3*x**3) - b*e**2*
asec(c*x)/x + b*d**2*Piecewise((8*c**5*sqrt(c**2*x**2 - 1)/(15*x) + 4*c**3
*sqrt(c**2*x**2 - 1)/(15*x**3) + c*sqrt(c**2*x**2 - 1)/(5*x**5), Abs(c**2*
x**2) > 1), (8*I*c**5*sqrt(-c**2*x**2 + 1)/(15*x) + 4*I*c**3*sqrt(-c**2*x*
*2 + 1)/(15*x**3) + I*c*sqrt(-c**2*x**2 + 1)/(5*x**5), True))/(5*c) + 2*b*
d*e*Piecewise((2*c**3*sqrt(c**2*x**2 - 1)/(3*x) + c*sqrt(c**2*x**2 - 1)/(3
*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*sqrt(-c**2*x**2 + 1)/(3*x) + I*c*sq
rt(-c**2*x**2 + 1)/(3*x**3), True))/(3*c)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^6}\,dx = \left(c\sqrt{-\frac{1}{c^2x^2}+1} - \frac{\operatorname{arcsec}(cx)}{x}\right)be^2 + \frac{1}{75}bd^2\left(\frac{3c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} - 10c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 15c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c} - \frac{15\operatorname{arcsec}(cx)}{x^5}\right) - \frac{2}{9}bde\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c} + \frac{3\operatorname{arcsec}(cx)}{x^3}\right) - \frac{ae^2}{x} - \frac{2ade}{3x^3} - \frac{ad^2}{5x^5}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^6,x, algorithm="maxima")
```

output
```
(c*sqrt(-1/(c^2*x^2) + 1) - arcsec(c*x)/x)*b*e^2 + 1/75*b*d^2*((3*c^6*(-1/
(c^2*x^2) + 1)^(5/2) - 10*c^6*(-1/(c^2*x^2) + 1)^(3/2) + 15*c^6*sqrt(-1/(c
^2*x^2) + 1))/c - 15*arcsec(c*x)/x^5) - 2/9*b*d*e*((c^4*(-1/(c^2*x^2) + 1)
^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c + 3*arcsec(c*x)/x^3) - a*e^2/x -
2/3*a*d*e/x^3 - 1/5*a*d^2/x^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^6}\,dx = \frac{1}{225}\left(24bc^4d^2\sqrt{-\frac{1}{c^2x^2}+1} + 100bc^2de\sqrt{-\frac{1}{c^2x^2}+1} + 225be^2\sqrt{-\frac{1}{c^2x^2}+1} + \frac{12bc^2d^2\sqrt{-\frac{1}{c^2x^2}+1}}{x^2} - \frac{2}{}\right.$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^6,x, algorithm="giac")
```

output
```
1/225*(24*b*c^4*d^2*sqrt(-1/(c^2*x^2) + 1) + 100*b*c^2*d*e*sqrt(-1/(c^2*x^
2) + 1) + 225*b*e^2*sqrt(-1/(c^2*x^2) + 1) + 12*b*c^2*d^2*sqrt(-1/(c^2*x^2
) + 1)/x^2 - 225*b*e^2*arccos(1/(c*x))/(c*x) + 50*b*d*e*sqrt(-1/(c^2*x^2)
+ 1)/x^2 - 225*a*e^2/(c*x) - 150*b*d*e*arccos(1/(c*x))/(c*x^3) + 9*b*d^2*s
qrt(-1/(c^2*x^2) + 1)/x^4 - 150*a*d*e/(c*x^3) - 45*b*d^2*arccos(1/(c*x))/(
c*x^5) - 45*a*d^2/(c*x^5))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^6}\,dx = \int \frac{(e\,x^2+d)^2 \left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

input
```
int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^6,x)
```

output
```
int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^6, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^6}\,dx = \frac{15\left(\int \frac{asec(cx)}{x^6}dx\right) b\,d^2x^5 + 30\left(\int \frac{asec(cx)}{x^4}dx\right) bde\,x^5 + 15\left(\int \frac{asec(cx)}{x^2}dx\right) b\,e^2x^5 - 3a\,d^2 - 10ade\,x^2 - 15a\,e^2x}{15x^5}$$

input
```
int((e*x^2+d)^2*(a+b*asec(c*x))/x^6,x)
```

output
```
(15*int(asec(c*x)/x**6,x)*b*d**2*x**5 + 30*int(asec(c*x)/x**4,x)*b*d*e*x**
5 + 15*int(asec(c*x)/x**2,x)*b*e**2*x**5 - 3*a*d**2 - 10*a*d*e*x**2 - 15*a
*e**2*x**4)/(15*x**5)
```

3.86 $\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^8}\,dx$

Optimal result 731
Mathematica [A] (verified) 732
Rubi [A] (verified) 732
Maple [A] (verified) 735
Fricas [A] (verification not implemented) 735
Sympy [A] (verification not implemented) 736
Maxima [A] (verification not implemented) 737
Giac [A] (verification not implemented) 738
Mupad [F(-1)] 739
Reduce [F] 739

Optimal result

Integrand size = 21, antiderivative size = 241

$$\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^8}\,dx = \frac{2bc^3(360c^4d^2+1176c^2de+1225e^2)\sqrt{-1+c^2x^2}}{11025\sqrt{c^2x^2}} + \frac{bcd^2\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} + \frac{2bcd(15c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}} + \frac{bc(360c^4d^2+1176c^2de+1225e^2)\sqrt{-1+c^2x^2}}{11025x^2\sqrt{c^2x^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{7x^7} - \frac{2de(a+b\sec^{-1}(cx))}{5x^5} - \frac{e^2(a+b\sec^{-1}(cx))}{3x^3}$$

output
```
2/11025*b*c^3*(360*c^4*d^2+1176*c^2*d*e+1225*e^2)*(c^2*x^2-1)^(1/2)/(c^2*x
^2)^(1/2)+1/49*b*c*d^2*(c^2*x^2-1)^(1/2)/x^6/(c^2*x^2)^(1/2)+2/1225*b*c*d*
(15*c^2*d+49*e)*(c^2*x^2-1)^(1/2)/x^4/(c^2*x^2)^(1/2)+1/11025*b*c*(360*c^4
*d^2+1176*c^2*d*e+1225*e^2)*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^2)^(1/2)-1/7*d^2*
(a+b*arcsec(c*x))/x^7-2/5*d*e*(a+b*arcsec(c*x))/x^5-1/3*e^2*(a+b*arcsec(c*
x))/x^3
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.63

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^8}\,dx$$
$$= \frac{-105a(15d^2+42dex^2+35e^2x^4)+bc\sqrt{1-\frac{1}{c^2x^2}}x(1225e^2x^4(1+2c^2x^2)+294dex^2(3+4c^2x^2+8c^4x^4)+}{11025x^7}$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^8,x]
```

output
```
(-105*a*(15*d^2 + 42*d*e*x^2 + 35*e^2*x^4) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(
1225*e^2*x^4*(1 + 2*c^2*x^2) + 294*d*e*x^2*(3 + 4*c^2*x^2 + 8*c^4*x^4) + 4
5*d^2*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c^6*x^6)) - 105*b*(15*d^2 + 42*d*e*x
^2 + 35*e^2*x^4)*ArcSec[c*x])/(11025*x^7)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5761, 27, 1588, 359, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^8}\,dx$$
$$\Big\downarrow \text{5761}$$
$$-\frac{bcx\int -\frac{35e^2x^4+42dex^2+15d^2}{105x^8\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}-\frac{d^2(a+b\sec^{-1}(cx))}{7x^7}-\frac{2de(a+b\sec^{-1}(cx))}{5x^5}-\frac{e^2(a+b\sec^{-1}(cx))}{3x^3}$$
$$\Big\downarrow \text{27}$$
$$\frac{bcx\int \frac{35e^2x^4+42dex^2+15d^2}{x^8\sqrt{c^2x^2-1}}dx}{105\sqrt{c^2x^2}}-\frac{d^2(a+b\sec^{-1}(cx))}{7x^7}-\frac{2de(a+b\sec^{-1}(cx))}{5x^5}-\frac{e^2(a+b\sec^{-1}(cx))}{3x^3}$$

$$\Big\downarrow \ 1588$$

$$\frac{bcx\left(\frac{1}{7}\int \frac{245e^2x^2+6d(15dc^2+49e)}{x^6\sqrt{c^2x^2-1}}dx+\frac{15d^2\sqrt{c^2x^2-1}}{7x^7}\right)}{105\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\sec^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\sec^{-1}(cx)\right)}{3x^3}$$

$$\Big\downarrow \ 359$$

$$\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(360c^4d^2+1176c^2de+1225e^2\right)\int \frac{1}{x^4\sqrt{c^2x^2-1}}dx+\frac{6d\sqrt{c^2x^2-1}(15c^2d+49e)}{5x^5}\right)+\frac{15d^2\sqrt{c^2x^2-1}}{7x^7}\right)}{105\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\sec^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\sec^{-1}(cx)\right)}{3x^3}$$

$$\Big\downarrow \ 245$$

$$\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(360c^4d^2+1176c^2de+1225e^2\right)\left(\frac{2}{3}c^2\int \frac{1}{x^2\sqrt{c^2x^2-1}}dx+\frac{\sqrt{c^2x^2-1}}{3x^3}\right)+\frac{6d\sqrt{c^2x^2-1}(15c^2d+49e)}{5x^5}\right)+\frac{15d^2\sqrt{c^2x^2-1}}{7x^7}}{105\sqrt{c^2x^2}}$$
$$\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\sec^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\sec^{-1}(cx)\right)}{3x^3}$$

$$\Big\downarrow \ 242$$

$$-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\sec^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\sec^{-1}(cx)\right)}{3x^3}+\frac{bcx\left(\frac{15d^2\sqrt{c^2x^2-1}}{7x^7}+\frac{1}{7}\left(\frac{6d\sqrt{c^2x^2-1}(15c^2d+49e)}{5x^5}+\frac{1}{5}\left(\frac{2c^2\sqrt{c^2x^2-1}}{3x}+\frac{\sqrt{c^2x^2-1}}{3x^3}\right)\left(360c^4d^2+1176c^2de+1225e^2\right)\right)\right)}{105\sqrt{c^2x^2}}$$

input

```
Int[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^8,x]
```

output

```
(b*c*x*((15*d^2*Sqrt[-1 + c^2*x^2])/(7*x^7) + ((6*d*(15*c^2*d + 49*e)*Sqrt
[-1 + c^2*x^2])/(5*x^5) + ((360*c^4*d^2 + 1176*c^2*d*e + 1225*e^2)*(Sqrt[-
1 + c^2*x^2]/(3*x^3) + (2*c^2*Sqrt[-1 + c^2*x^2])/(3*x)))/5)/7))/(105*Sqrt
[c^2*x^2]) - (d^2*(a + b*ArcSec[c*x]))/(7*x^7) - (2*d*e*(a + b*ArcSec[c*x]
))/(5*x^5) - (e^2*(a + b*ArcSec[c*x]))/(3*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.86

method	result
parts	$a\left(-\frac{d^2}{7x^7}-\frac{2de}{5x^5}-\frac{e^2}{3x^3}\right)+b\,c^7\left(-\frac{\operatorname{arcsec}(cx)d^2}{7x^7c^7}-\frac{2\operatorname{arcsec}(cx)de}{5c^7x^5}-\frac{\operatorname{arcsec}(cx)e^2}{3c^7x^3}+\frac{(c^2x^2-1)\left(720c^{10}d^2\right.}{}\right.$
derivativedivides	$c^7\left(\frac{a\left(-\frac{e^2}{3c^3x^3}-\frac{d^2}{7c^3x^7}-\frac{2de}{5c^3x^5}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arcsec}(cx)e^2}{3c^3x^3}-\frac{\operatorname{arcsec}(cx)d^2}{7c^3x^7}-\frac{2\operatorname{arcsec}(cx)de}{5c^3x^5}+\frac{(c^2x^2-1)\left(720c^{10}d^2x^6+2352c^8de\right.}{}\right.}{}\right.$
default	$c^7\left(\frac{a\left(-\frac{e^2}{3c^3x^3}-\frac{d^2}{7c^3x^7}-\frac{2de}{5c^3x^5}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arcsec}(cx)e^2}{3c^3x^3}-\frac{\operatorname{arcsec}(cx)d^2}{7c^3x^7}-\frac{2\operatorname{arcsec}(cx)de}{5c^3x^5}+\frac{(c^2x^2-1)\left(720c^{10}d^2x^6+2352c^8de\right.}{}\right.}{}\right.$

input
```
int((e*x^2+d)^2*(a+b*arcsec(c*x))/x^8,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/7*d^2/x^7-2/5*d*e/x^5-1/3*e^2/x^3)+b*c^7*(-1/7*arcsec(c*x)*d^2/x^7/c
^7-2/5/c^7*arcsec(c*x)*d*e/x^5-1/3/c^7*arcsec(c*x)*e^2/x^3+1/11025/c^12*(c
^2*x^2-1)*(720*c^10*d^2*x^6+2352*c^8*d*e*x^6+360*c^8*d^2*x^4+2450*c^6*e^2*
x^6+1176*c^6*d*e*x^4+270*c^6*d^2*x^2+1225*c^4*e^2*x^4+882*c^4*d*e*x^2+225*
c^4*d^2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^8)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.66

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^8}\,dx =$$
$$-\frac{3675\,ae^2x^4+4410\,adex^2+1575\,ad^2+105\,(35\,be^2x^4+42\,bdex^2+15\,bd^2)\operatorname{arcsec}(cx)-(2\,(360\,bc^6d^2+}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^8,x, algorithm="fricas")
```

output
```
-1/11025*(3675*a*e^2*x^4 + 4410*a*d*e*x^2 + 1575*a*d^2 + 105*(35*b*e^2*x^4
 + 42*b*d*e*x^2 + 15*b*d^2)*arcsec(c*x) - (2*(360*b*c^6*d^2 + 1176*b*c^4*d
*e + 1225*b*c^2*e^2)*x^6 + (360*b*c^4*d^2 + 1176*b*c^2*d*e + 1225*b*e^2)*x
^4 + 225*b*d^2 + 18*(15*b*c^2*d^2 + 49*b*d*e)*x^2)*sqrt(c^2*x^2 - 1))/x^7
```

Sympy [A] (verification not implemented)

Time = 46.47 (sec) , antiderivative size = 508, normalized size of antiderivative = 2.11

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^8}\,dx = -\frac{ad^2}{7x^7} - \frac{2ade}{5x^5} - \frac{ae^2}{3x^3} - \frac{bd^2\operatorname{asec}(cx)}{7x^7} - \frac{2bde\operatorname{asec}(cx)}{5x^5} - \frac{be^2\operatorname{asec}(cx)}{3x^3} + \frac{bd^2\left(\begin{cases}\frac{16c^7\sqrt{c^2x^2-1}}{35x} + \frac{8c^5\sqrt{c^2x^2-1}}{35x^3} + \frac{6c^3\sqrt{c^2x^2-1}}{35x^5} + \frac{c\sqrt{c^2x^2-1}}{7x^7} & \text{for } |c^2x^2| > 1 \\ \frac{16ic^7\sqrt{-c^2x^2+1}}{35x} + \frac{8ic^5\sqrt{-c^2x^2+1}}{35x^3} + \frac{6ic^3\sqrt{-c^2x^2+1}}{35x^5} + \frac{ic\sqrt{-c^2x^2+1}}{7x^7} & \text{otherwise}\end{cases}\right)}{7c} + \frac{2bde\left(\begin{cases}\frac{8c^5\sqrt{c^2x^2-1}}{15x} + \frac{4c^3\sqrt{c^2x^2-1}}{15x^3} + \frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for } |c^2x^2| > 1 \\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x} + \frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3} + \frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise}\end{cases}\right)}{5c} + \frac{be^2\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x} + \frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2| > 1 \\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x} + \frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c}$$

input
```
integrate((e*x**2+d)**2*(a+b*asec(c*x))/x**8,x)
```

output
```
-a*d**2/(7*x**7) - 2*a*d*e/(5*x**5) - a*e**2/(3*x**3) - b*d**2*asec(c*x)/(
7*x**7) - 2*b*d*e*asec(c*x)/(5*x**5) - b*e**2*asec(c*x)/(3*x**3) + b*d**2*
Piecewise((16*c**7*sqrt(c**2*x**2 - 1)/(35*x) + 8*c**5*sqrt(c**2*x**2 - 1)
/(35*x**3) + 6*c**3*sqrt(c**2*x**2 - 1)/(35*x**5) + c*sqrt(c**2*x**2 - 1)/
(7*x**7), Abs(c**2*x**2) > 1), (16*I*c**7*sqrt(-c**2*x**2 + 1)/(35*x) + 8*
I*c**5*sqrt(-c**2*x**2 + 1)/(35*x**3) + 6*I*c**3*sqrt(-c**2*x**2 + 1)/(35*
x**5) + I*c*sqrt(-c**2*x**2 + 1)/(7*x**7), True))/(7*c) + 2*b*d*e*Piecewis
e((8*c**5*sqrt(c**2*x**2 - 1)/(15*x) + 4*c**3*sqrt(c**2*x**2 - 1)/(15*x**3
) + c*sqrt(c**2*x**2 - 1)/(5*x**5), Abs(c**2*x**2) > 1), (8*I*c**5*sqrt(-c
**2*x**2 + 1)/(15*x) + 4*I*c**3*sqrt(-c**2*x**2 + 1)/(15*x**3) + I*c*sqrt(
-c**2*x**2 + 1)/(5*x**5), True))/(5*c) + b*e**2*Piecewise((2*c**3*sqrt(c**
2*x**2 - 1)/(3*x) + c*sqrt(c**2*x**2 - 1)/(3*x**3), Abs(c**2*x**2) > 1), (
2*I*c**3*sqrt(-c**2*x**2 + 1)/(3*x) + I*c*sqrt(-c**2*x**2 + 1)/(3*x**3), T
rue))/(3*c)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{x^8}\,dx =$$

$$-\frac{1}{245}\,bd^2\left(\frac{5\,c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}}-21\,c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+35\,c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-35\,c^8\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{35\,\mathrm{arcsec}}{x^7}\right.$$
$$+\frac{2}{75}\,bde\left(\frac{3\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{15\,\mathrm{arcsec}\,(cx)}{x^5}\right)$$
$$-\frac{1}{9}\,be^2\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{3\,\mathrm{arcsec}\,(cx)}{x^3}\right)-\frac{ae^2}{3\,x^3}-\frac{2\,ade}{5\,x^5}-\frac{ad^2}{7\,x^7}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^8,x, algorithm="maxima")
```

output
```
-1/245*b*d^2*((5*c^8*(-1/(c^2*x^2) + 1)^(7/2) - 21*c^8*(-1/(c^2*x^2) + 1)^
(5/2) + 35*c^8*(-1/(c^2*x^2) + 1)^(3/2) - 35*c^8*sqrt(-1/(c^2*x^2) + 1))/c
 + 35*arcsec(c*x)/x^7) + 2/75*b*d*e*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*
c^6*(-1/(c^2*x^2) + 1)^(3/2) + 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c - 15*arcse
c(c*x)/x^5) - 1/9*b*e^2*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^
2*x^2) + 1))/c + 3*arcsec(c*x)/x^3) - 1/3*a*e^2/x^3 - 2/5*a*d*e/x^5 - 1/7*
a*d^2/x^7
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^8}\,dx$$
$$= \frac{1}{11025}\left(720\,bc^6d^2\sqrt{-\frac{1}{c^2x^2}+1}+2352\,bc^4de\sqrt{-\frac{1}{c^2x^2}+1}+2450\,bc^2e^2\sqrt{-\frac{1}{c^2x^2}+1}+\frac{360\,bc^4d^2\sqrt{-\frac{1}{c^2x^2}}}{x^2}\right.$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^8,x, algorithm="giac")
```

output
```
1/11025*(720*b*c^6*d^2*sqrt(-1/(c^2*x^2) + 1) + 2352*b*c^4*d*e*sqrt(-1/(c^
2*x^2) + 1) + 2450*b*c^2*e^2*sqrt(-1/(c^2*x^2) + 1) + 360*b*c^4*d^2*sqrt(-
1/(c^2*x^2) + 1)/x^2 + 1176*b*c^2*d*e*sqrt(-1/(c^2*x^2) + 1)/x^2 + 270*b*c
^2*d^2*sqrt(-1/(c^2*x^2) + 1)/x^4 + 1225*b*e^2*sqrt(-1/(c^2*x^2) + 1)/x^2
- 3675*b*e^2*arccos(1/(c*x))/(c*x^3) + 882*b*d*e*sqrt(-1/(c^2*x^2) + 1)/x^
4 - 3675*a*e^2/(c*x^3) - 4410*b*d*e*arccos(1/(c*x))/(c*x^5) + 225*b*d^2*sq
rt(-1/(c^2*x^2) + 1)/x^6 - 4410*a*d*e/(c*x^5) - 1575*b*d^2*arccos(1/(c*x))
/(c*x^7) - 1575*a*d^2/(c*x^7))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^8}\,dx = \int \frac{(e\,x^2+d)^2\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

input
```
int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^8,x)
```

output
```
int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^8, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^8}\,dx$$
$$= \frac{105\left(\int \frac{asec(cx)}{x^8}dx\right) b\,d^2x^7 + 210\left(\int \frac{asec(cx)}{x^6}dx\right) bde\,x^7 + 105\left(\int \frac{asec(cx)}{x^4}dx\right) b\,e^2x^7 - 15a\,d^2 - 42ade\,x^2 - 35}{105x^7}$$

input
```
int((e*x^2+d)^2*(a+b*asec(c*x))/x^8,x)
```

output
```
(105*int(asec(c*x)/x**8,x)*b*d**2*x**7 + 210*int(asec(c*x)/x**6,x)*b*d*e*x
**7 + 105*int(asec(c*x)/x**4,x)*b*e**2*x**7 - 15*a*d**2 - 42*a*d*e*x**2 -
35*a*e**2*x**4)/(105*x**7)
```

3.87 $\int x^3(d+ex^2)^2 (a+b\sec^{-1}(cx))\,dx$

Optimal result 740
Mathematica [A] (verified) 741
Rubi [A] (verified) 741
Maple [A] (verified) 744
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Maxima [A] (verification not implemented) 746
Giac [B] (verification not implemented) 747
Mupad [F(-1)] 748
Reduce [F] 749

Optimal result

Integrand size = 21, antiderivative size = 242

$$\begin{aligned}\int x^3(d+ex^2)^2 (a+b\sec^{-1}(cx))\,dx &= -\frac{b(6c^4d^2+8c^2de+3e^2)\,x\sqrt{-1+c^2x^2}}{24c^7\sqrt{c^2x^2}} - \frac{b(6c^4d^2+16c^2de+9e^2)\,x(-1+c^2x^2)^{3/2}}{72c^7\sqrt{c^2x^2}} \\ &\quad - \frac{be(8c^2d+9e)\,x(-1+c^2x^2)^{5/2}}{120c^7\sqrt{c^2x^2}} - \frac{be^2x(-1+c^2x^2)^{7/2}}{56c^7\sqrt{c^2x^2}} \\ &\quad + \frac{1}{4}d^2x^4(a+b\sec^{-1}(cx)) + \frac{1}{3}dex^6(a+b\sec^{-1}(cx)) + \frac{1}{8}e^2x^8(a+b\sec^{-1}(cx))\end{aligned}$$

output
```
-1/24*b*(6*c^4*d^2+8*c^2*d*e+3*e^2)*x*(c^2*x^2-1)^(1/2)/c^7/(c^2*x^2)^(1/2
)-1/72*b*(6*c^4*d^2+16*c^2*d*e+9*e^2)*x*(c^2*x^2-1)^(3/2)/c^7/(c^2*x^2)^(1
/2)-1/120*b*e*(8*c^2*d+9*e)*x*(c^2*x^2-1)^(5/2)/c^7/(c^2*x^2)^(1/2)-1/56*b
*e^2*x*(c^2*x^2-1)^(7/2)/c^7/(c^2*x^2)^(1/2)+1/4*d^2*x^4*(a+b*arcsec(c*x))
+1/3*d*e*x^6*(a+b*arcsec(c*x))+1/8*e^2*x^8*(a+b*arcsec(c*x))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.67

$$\int x^3\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{1}{24}ax^4\left(6d^2+8dex^2+3e^2x^4\right)$$
$$-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x(144e^2+8c^2e(56d+9ex^2)+c^4(420d^2+224dex^2+54e^2x^4)+3c^6(70d^2x^2+56dex^4+15}{2520c^7}$$
$$+\frac{1}{24}bx^4\left(6d^2+8dex^2+3e^2x^4\right)\sec^{-1}(cx)$$

input
```
Integrate[x^3*(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

output
```
(a*x^4*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4))/24 - (b*Sqrt[1 - 1/(c^2*x^2)]*x*(1
44*e^2 + 8*c^2*e*(56*d + 9*e*x^2) + c^4*(420*d^2 + 224*d*e*x^2 + 54*e^2*x^
4) + 3*c^6*(70*d^2*x^2 + 56*d*e*x^4 + 15*e^2*x^6)))/(2520*c^7) + (b*x^4*(6
*d^2 + 8*d*e*x^2 + 3*e^2*x^4)*ArcSec[c*x])/24
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5761, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \ 5761$$

$$-\frac{bcx\int\frac{x^3\left(3e^2x^4+8dex^2+6d^2\right)}{24\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{1}{4}d^2x^4\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\sec^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\sec^{-1}(cx)\right)$$

$$\Big\downarrow \ 27$$

$$-\frac{bcx\int \frac{x^3(3e^2x^4+8dex^2+6d^2)}{\sqrt{c^2x^2-1}}dx}{24\sqrt{c^2x^2}}+\frac{1}{4}d^2x^4(a+b\sec^{-1}(cx))+\frac{1}{3}dex^6(a+b\sec^{-1}(cx))+\frac{1}{8}e^2x^8(a+b\sec^{-1}(cx))$$

$$\Big\downarrow \text{1578}$$

$$-\frac{bcx\int \frac{x^2(3e^2x^4+8dex^2+6d^2)}{\sqrt{c^2x^2-1}}dx^2}{48\sqrt{c^2x^2}}+\frac{1}{4}d^2x^4(a+b\sec^{-1}(cx))+\frac{1}{3}dex^6(a+b\sec^{-1}(cx))+\frac{1}{8}e^2x^8(a+b\sec^{-1}(cx))$$

$$\Big\downarrow \text{1195}$$

$$-\frac{bcx\int \left(\frac{3e^2(c^2x^2-1)^{5/2}}{c^6}+\frac{e(8dc^2+9e)(c^2x^2-1)^{3/2}}{c^6}+\frac{(6d^2c^4+16dec^2+9e^2)\sqrt{c^2x^2-1}}{c^6}+\frac{6d^2c^4+8dec^2+3e^2}{c^6\sqrt{c^2x^2-1}}\right)dx^2}{48\sqrt{c^2x^2}}+\frac{1}{4}d^2x^4(a+b\sec^{-1}(cx))+\frac{1}{3}dex^6(a+b\sec^{-1}(cx))+\frac{1}{8}e^2x^8(a+b\sec^{-1}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}d^2x^4(a+b\sec^{-1}(cx))+\frac{1}{3}dex^6(a+b\sec^{-1}(cx))+\frac{1}{8}e^2x^8(a+b\sec^{-1}(cx))-\frac{bcx\left(\frac{2e(c^2x^2-1)^{5/2}(8c^2d+9e)}{5c^8}+\frac{6e^2(c^2x^2-1)^{7/2}}{7c^8}+\frac{2(c^2x^2-1)^{3/2}(6c^4d^2+16c^2de+9e^2)}{3c^8}+\frac{2\sqrt{c^2x^2-1}(6c^4d^2+8c^2de+3e^2)}{c^8}\right)}{48\sqrt{c^2x^2}}$$

input
```
Int[x^3*(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

output
```
-1/48*(b*c*x*((2*(6*c^4*d^2 + 8*c^2*d*e + 3*e^2)*Sqrt[-1 + c^2*x^2])/c^8 +
 (2*(6*c^4*d^2 + 16*c^2*d*e + 9*e^2)*(-1 + c^2*x^2)^(3/2))/(3*c^8) + (2*e*
(8*c^2*d + 9*e)*(-1 + c^2*x^2)^(5/2))/(5*c^8) + (6*e^2*(-1 + c^2*x^2)^(7/2
))/(7*c^8)))/Sqrt[c^2*x^2] + (d^2*x^4*(a + b*ArcSec[c*x]))/4 + (d*e*x^6*(a
 + b*ArcSec[c*x]))/3 + (e^2*x^8*(a + b*ArcSec[c*x]))/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.82

method	result
parts	$a\left(\frac{1}{8}e^2x^8+\frac{1}{3}de\,x^6+\frac{1}{4}d^2x^4\right)+\frac{b\left(\frac{c^4\,\mathrm{arcsec}(cx)e^2x^8}{8}+\frac{c^4\,\mathrm{arcsec}(cx)de\,x^6}{3}+\frac{\mathrm{arcsec}(cx)d^2x^4c^4}{4}-\frac{(c^2x^2-1)(45c^6e^2}{}\right.}{}$
derivativedivides	$\frac{-\frac{a\left(\frac{c^2d\left(e\,c^2x^2+c^2d\right)^3}{3}-\frac{\left(e\,c^2x^2+c^2d\right)^4}{4}\right)}{2c^4e^2}-\frac{b\,c^4\,\mathrm{arcsec}(cx)d^4}{24e^2}+\frac{b\,\mathrm{arcsec}(cx)d^2c^4x^4}{4}+\frac{b\,c^4e\,\mathrm{arcsec}(cx)d\,x^6}{3}+\frac{b\,c^4e^2\,\mathrm{arcsec}(cx)x^8}{8}}{}$
default	$\frac{-\frac{a\left(\frac{c^2d\left(e\,c^2x^2+c^2d\right)^3}{3}-\frac{\left(e\,c^2x^2+c^2d\right)^4}{4}\right)}{2c^4e^2}-\frac{b\,c^4\,\mathrm{arcsec}(cx)d^4}{24e^2}+\frac{b\,\mathrm{arcsec}(cx)d^2c^4x^4}{4}+\frac{b\,c^4e\,\mathrm{arcsec}(cx)d\,x^6}{3}+\frac{b\,c^4e^2\,\mathrm{arcsec}(cx)x^8}{8}}{}$

input
```
int(x^3*(e*x^2+d)^2*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/8*e^2*x^8+1/3*d*e*x^6+1/4*d^2*x^4)+b/c^4*(1/8*c^4*arcsec(c*x)*e^2*x^8
+1/3*c^4*arcsec(c*x)*d*e*x^6+1/4*arcsec(c*x)*d^2*x^4*c^4-1/2520/c^5*(c^2*x
^2-1)*(45*c^6*e^2*x^6+168*c^6*d*e*x^4+210*c^6*d^2*x^2+54*c^4*e^2*x^4+224*c
^4*d*e*x^2+420*c^4*d^2+72*c^2*e^2*x^2+448*c^2*d*e+144*e^2)/((c^2*x^2-1)/c^
2/x^2)^(1/2)/x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.77

$$\int x^3(d+ex^2)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{315\,ac^8e^2x^8+840\,ac^8dex^6+630\,ac^8d^2x^4+105\left(3\,bc^8e^2x^8+8\,bc^8dex^6+6\,bc^8d^2x^4\right)\mathrm{arcsec}\,(cx)-(45\,bc^6e}{}$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/2520*(315*a*c^8*e^2*x^8 + 840*a*c^8*d*e*x^6 + 630*a*c^8*d^2*x^4 + 105*(3
*b*c^8*e^2*x^8 + 8*b*c^8*d*e*x^6 + 6*b*c^8*d^2*x^4)*arcsec(c*x) - (45*b*c^
6*e^2*x^6 + 420*b*c^4*d^2 + 448*b*c^2*d*e + 6*(28*b*c^6*d*e + 9*b*c^4*e^2)
*x^4 + 144*b*e^2 + 2*(105*b*c^6*d^2 + 112*b*c^4*d*e + 36*b*c^2*e^2)*x^2)*s
qrt(c^2*x^2 - 1))/c^8
```

Sympy [A] (verification not implemented)

Time = 5.00 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.04

$$\int x^3(d+ex^2)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$= \frac{ad^2x^4}{4} + \frac{adex^6}{3} + \frac{ae^2x^8}{8} + \frac{bd^2x^4\operatorname{asec}(cx)}{4} + \frac{bdex^6\operatorname{asec}(cx)}{3} + \frac{be^2x^8\operatorname{asec}(cx)}{8} - \frac{bd^2\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{4c} - \frac{bde\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c}+\frac{4x^2\sqrt{c^2x^2-1}}{15c^3}+\frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for } |c^2x^2|>1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c}+\frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3}+\frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{3c} - \frac{be^2\left(\begin{cases}\frac{x^6\sqrt{c^2x^2-1}}{7c}+\frac{6x^4\sqrt{c^2x^2-1}}{35c^3}+\frac{8x^2\sqrt{c^2x^2-1}}{35c^5}+\frac{16\sqrt{c^2x^2-1}}{35c^7} & \text{for } |c^2x^2|>1\\ \frac{ix^6\sqrt{-c^2x^2+1}}{7c}+\frac{6ix^4\sqrt{-c^2x^2+1}}{35c^3}+\frac{8ix^2\sqrt{-c^2x^2+1}}{35c^5}+\frac{16i\sqrt{-c^2x^2+1}}{35c^7} & \text{otherwise}\end{cases}\right)}{8c}$$

input
```
integrate(x**3*(e*x**2+d)**2*(a+b*asec(c*x)),x)
```

output
```
a*d**2*x**4/4 + a*d*e*x**6/3 + a*e**2*x**8/8 + b*d**2*x**4*asec(c*x)/4 + b
*d*e*x**6*asec(c*x)/3 + b*e**2*x**8*asec(c*x)/8 - b*d**2*Piecewise((x**2*s
qrt(c**2*x**2 - 1)/(3*c) + 2*sqrt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2)
> 1), (I*x**2*sqrt(-c**2*x**2 + 1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**
3), True))/(4*c) - b*d*e*Piecewise((x**4*sqrt(c**2*x**2 - 1)/(5*c) + 4*x**
2*sqrt(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**2*x**2 - 1)/(15*c**5), Abs(c**
2*x**2) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(5*c) + 4*I*x**2*sqrt(-c**2*x**
2 + 1)/(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/(15*c**5), True))/(3*c) - b*e*
*2*Piecewise((x**6*sqrt(c**2*x**2 - 1)/(7*c) + 6*x**4*sqrt(c**2*x**2 - 1)/
(35*c**3) + 8*x**2*sqrt(c**2*x**2 - 1)/(35*c**5) + 16*sqrt(c**2*x**2 - 1)/
(35*c**7), Abs(c**2*x**2) > 1), (I*x**6*sqrt(-c**2*x**2 + 1)/(7*c) + 6*I*x
**4*sqrt(-c**2*x**2 + 1)/(35*c**3) + 8*I*x**2*sqrt(-c**2*x**2 + 1)/(35*c**
5) + 16*I*sqrt(-c**2*x**2 + 1)/(35*c**7), True))/(8*c)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.06

$$\int x^3\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{8}\,ae^2x^8+\frac{1}{3}\,adex^6+\frac{1}{4}\,ad^2x^4$$
$$+\frac{1}{12}\left(3\,x^4\,\mathrm{arcsec}\left(cx\right)-\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)bd^2$$
$$+\frac{1}{45}\left(15\,x^6\,\mathrm{arcsec}\left(cx\right)-\frac{3\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+10\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^5}\right)bde$$
$$+\frac{1}{280}\left(35\,x^8\,\mathrm{arcsec}\left(cx\right)-\frac{5\,c^6x^7\left(-\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}}+21\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+35\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+35\,x\sqrt{\cdot}}{c^7}\right.$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/8*a*e^2*x^8 + 1/3*a*d*e*x^6 + 1/4*a*d^2*x^4 + 1/12*(3*x^4*arcsec(c*x) -
(c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b*d^2
 + 1/45*(15*x^6*arcsec(c*x) - (3*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2) + 10*c^2
*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(-1/(c^2*x^2) + 1))/c^5)*b*d*e +
1/280*(35*x^8*arcsec(c*x) - (5*c^6*x^7*(-1/(c^2*x^2) + 1)^(7/2) + 21*c^4*x
^5*(-1/(c^2*x^2) + 1)^(5/2) + 35*c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 35*x*s
qrt(-1/(c^2*x^2) + 1))/c^7)*b*e^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17666 vs. $2(212) = 424$.

Time = 0.34 (sec) , antiderivative size = 17666, normalized size of antiderivative = 73.00

$$\int x^3 \left(d + ex^2\right)^2 \left(a + b \sec^{-1}(cx)\right) \, dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/2520*(630*b*c^4*d^2*arccos(1/(c*x))/(c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c
*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x
^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 +
56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(
1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2
*x^2) - 1)^8/(1/(c*x) + 1)^16) + 630*a*c^4*d^2/(c^9 + 8*c^9*(1/(c^2*x^2) -
 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*
(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x)
+ 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2)
 - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^
9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16) - 1260*b*c^4*d^2*sqrt(-1/(c^2*x^2)
 + 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2
) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70
*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(
c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2
*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*
(1/(c*x) + 1)) + 840*b*c^2*d*e*arccos(1/(c*x))/(c^9 + 8*c^9*(1/(c^2*x^2) -
 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*
(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x)
+ 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x...
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex^2)^2\left(a+b\sec^{-1}(cx)\right)\,dx=\int x^3\,\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^3*(d + e*x^2)^2*(a + b*acos(1/(c*x))),x)
```

output
```
int(x^3*(d + e*x^2)^2*(a + b*acos(1/(c*x))), x)
```

Reduce [F]

$$\int x^3 (d+ex^2)^2 (a+b\sec^{-1}(cx))\,dx = \left(\int asec(cx)\,x^7dx\right) b\,e^2 + 2\left(\int asec(cx)\,x^5dx\right) bde + \left(\int asec(cx)\,x^3dx\right) b\,d^2 + \frac{a\,d^2x^4}{4} + \frac{ade\,x^6}{3} + \frac{a\,e^2x^8}{8}$$

input
```
int(x^3*(e*x^2+d)^2*(a+b*asec(c*x)),x)
```

output
```
(24*int(asec(c*x)*x**7,x)*b*e**2 + 48*int(asec(c*x)*x**5,x)*b*d*e + 24*int
(asec(c*x)*x**3,x)*b*d**2 + 6*a*d**2*x**4 + 8*a*d*e*x**6 + 3*a*e**2*x**8)/
24
```

3.88 $\int x(d+ex^2)^2\left(a+b\sec^{-1}(cx)\right)\,dx$

Optimal result . . . 750
Mathematica [A] (verified) . . . 751
Rubi [A] (verified) . . . 751
Maple [B] (verified) . . . 753
Fricas [A] (verification not implemented) . . . 754
Sympy [A] (verification not implemented) . . . 754
Maxima [A] (verification not implemented) . . . 755
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Reduce [F] . . . 757

Optimal result

Integrand size = 19, antiderivative size = 195

$$\int x\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx=-\frac{b(3c^4d^2+3c^2de+e^2)\,x\sqrt{-1+c^2x^2}}{6c^5\sqrt{c^2x^2}}-\frac{be(3c^2d+2e)\,x(-1+c^2x^2)^{3/2}}{18c^5\sqrt{c^2x^2}}-\frac{be^2x(-1+c^2x^2)^{5/2}}{30c^5\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^3\left(a+b\sec^{-1}(cx)\right)}{6e}-\frac{bcd^3x\arctan\left(\sqrt{-1+c^2x^2}\right)}{6e\sqrt{c^2x^2}}$$

output
```
-1/6*b*(3*c^4*d^2+3*c^2*d*e+e^2)*x*(c^2*x^2-1)^(1/2)/c^5/(c^2*x^2)^(1/2)-1
/18*b*e*(3*c^2*d+2*e)*x*(c^2*x^2-1)^(3/2)/c^5/(c^2*x^2)^(1/2)-1/30*b*e^2*x
*(c^2*x^2-1)^(5/2)/c^5/(c^2*x^2)^(1/2)+1/6*(e*x^2+d)^3*(a+b*arcsec(c*x))/e
-1/6*b*c*d^3*x*arctan((c^2*x^2-1)^(1/2))/e/(c^2*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.64

$$\int x\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{1}{90}x\left(15ax\left(3d^2+3dex^2+e^2x^4\right)-\frac{b\sqrt{1-\frac{1}{c^2x^2}}\left(8e^2+2c^2e\left(15d+2ex^2\right)+3c^4\left(15d^2+5dex^2+e^2x^4\right)\right)}{c^5}+15bx\left(3d^2+3dex^2+e^2x^4\right)\sec^{-1}(cx)\right)$$

input
```
Integrate[x*(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

output
```
(x*(15*a*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4) - (b*Sqrt[1 - 1/(c^2*x^2)]*(8*e^2
 + 2*c^2*e*(15*d + 2*e*x^2) + 3*c^4*(15*d^2 + 5*d*e*x^2 + e^2*x^4)))/c^5 +
 15*b*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4)*ArcSec[c*x]))/90
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.82, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {5759, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$\Big\downarrow \text{5759}$$
$$\frac{\left(d+ex^2\right)^3\left(a+b\sec^{-1}(cx)\right)}{6e}-\frac{bcx\int\frac{\left(ex^2+d\right)^3}{x\sqrt{c^2x^2-1}}dx}{6e\sqrt{c^2x^2}}$$

$$\Big\downarrow \; 354$$

$$\frac{(d+ex^2)^3(a+b\sec^{-1}(cx))}{6e}-\frac{bcx\int\frac{(ex^2+d)^3}{x^2\sqrt{c^2x^2-1}}dx^2}{12e\sqrt{c^2x^2}}$$

$$\Big\downarrow \; 99$$

$$\frac{(d+ex^2)^3(a+b\sec^{-1}(cx))}{6e}-\frac{bcx\int\left(\frac{d^3}{x^2\sqrt{c^2x^2-1}}+\frac{e^3(c^2x^2-1)^{3/2}}{c^4}+\frac{e^2(3dc^2+2e)\sqrt{c^2x^2-1}}{c^4}+\frac{e(3d^2c^4+3dec^2+e^2)}{c^4\sqrt{c^2x^2-1}}\right)dx^2}{12e\sqrt{c^2x^2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{(d+ex^2)^3(a+b\sec^{-1}(cx))}{6e}-\frac{bcx\left(2d^3\arctan\left(\sqrt{c^2x^2-1}\right)+\frac{2e^2(c^2x^2-1)^{3/2}(3c^2d+2e)}{3c^6}+\frac{2e^3(c^2x^2-1)^{5/2}}{5c^6}+\frac{2e\sqrt{c^2x^2-1}(3c^4d^2+3c^2de+e^2)}{c^6}\right)}{12e\sqrt{c^2x^2}}$$

input
```
Int[x*(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

output
```
((d + e*x^2)^3*(a + b*ArcSec[c*x]))/(6*e) - (b*c*x*((2*e*(3*c^4*d^2 + 3*c^
2*d*e + e^2)*Sqrt[-1 + c^2*x^2])/c^6 + (2*e^2*(3*c^2*d + 2*e)*(-1 + c^2*x^
2)^(3/2))/(3*c^6) + (2*e^3*(-1 + c^2*x^2)^(5/2))/(5*c^6) + 2*d^3*ArcTan[Sq
rt[-1 + c^2*x^2]]))/(12*e*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5759
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSec[c*x])/(2*e*(p + 1))), x
] - Simp[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2]))   Int[(d + e*x^2)^(p + 1)/(x*S
qrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(169) = 338$.

Time = 0.62 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.81

method	result
parts	$\frac{a\left(e\,x^{2}+d\right)^{3}}{6e}+\frac{b\,e^{2}\operatorname{arcsec}\left(cx\right)x^{6}}{6}+\frac{be\,\operatorname{arcsec}\left(cx\right)x^{4}d}{2}+\frac{b\,\operatorname{arcsec}\left(cx\right)x^{2}d^{2}}{2}+\frac{b\,\operatorname{arcsec}\left(cx\right)d^{3}}{6e}-\frac{b\,e^{2}\left(c^{2}x^{2}-1\right)x^{3}}{30c^{3}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}\cdots$
derivativedivides	$\frac{\frac{a\left(e\,c^{2}x^{2}+c^{2}d\right)^{3}}{6c^{4}e}+\frac{b\,c^{2}\operatorname{arcsec}\left(cx\right)d^{3}}{6e}+\frac{b\,\operatorname{arcsec}\left(cx\right)d^{2}c^{2}x^{2}}{2}+\frac{b\,c^{2}e\,\operatorname{arcsec}\left(cx\right)d\,x^{4}}{2}+\frac{b\,c^{2}e^{2}\operatorname{arcsec}\left(cx\right)x^{6}}{6}+\frac{bc\sqrt{c^{2}x^{2}-1}\,d^{3}\arctan\left(\cdots\right)}{6e\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\cdots}\cdots}{c^{2}}$
default	$\frac{\frac{a\left(e\,c^{2}x^{2}+c^{2}d\right)^{3}}{6c^{4}e}+\frac{b\,c^{2}\operatorname{arcsec}\left(cx\right)d^{3}}{6e}+\frac{b\,\operatorname{arcsec}\left(cx\right)d^{2}c^{2}x^{2}}{2}+\frac{b\,c^{2}e\,\operatorname{arcsec}\left(cx\right)d\,x^{4}}{2}+\frac{b\,c^{2}e^{2}\operatorname{arcsec}\left(cx\right)x^{6}}{6}+\frac{bc\sqrt{c^{2}x^{2}-1}\,d^{3}\arctan\left(\cdots\right)}{6e\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\cdots}\cdots}{c^{2}}$

input
```
int(x*(e*x^2+d)^2*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*a*(e*x^2+d)^3/e+1/6*b*e^2*arcsec(c*x)*x^6+1/2*b*e*arcsec(c*x)*x^4*d+1/
2*b*arcsec(c*x)*x^2*d^2+1/6*b/e*arcsec(c*x)*d^3-1/30*b/c^3*e^2*(c^2*x^2-1)
/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^3-1/6*b/c^3*e*(c^2*x^2-1)/((c^2*x^2-1)/c^2/
x^2)^(1/2)*x*d+1/6*b/c/e*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d
^3*arctan(1/(c^2*x^2-1)^(1/2))-2/45*b/c^5*e^2*(c^2*x^2-1)/((c^2*x^2-1)/c^2
/x^2)^(1/2)*x-1/2*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^2-1/3*
b/c^5*e*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d-4/45*b/c^7*e^2*(c^2*x^
2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.78

$$\int x\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{15\,ac^6e^2x^6+45\,ac^6dex^4+45\,ac^6d^2x^2+15\left(bc^6e^2x^6+3\,bc^6dex^4+3\,bc^6d^2x^2\right)\operatorname{arcsec}(cx)-\left(3\,bc^4e^2x^4+4\right.}{90\,c^6}$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/90*(15*a*c^6*e^2*x^6 + 45*a*c^6*d*e*x^4 + 45*a*c^6*d^2*x^2 + 15*(b*c^6*e
^2*x^6 + 3*b*c^6*d*e*x^4 + 3*b*c^6*d^2*x^2)*arcsec(c*x) - (3*b*c^4*e^2*x^4
 + 45*b*c^4*d^2 + 30*b*c^2*d*e + 8*b*e^2 + (15*b*c^4*d*e + 4*b*c^2*e^2)*x^
2)*sqrt(c^2*x^2 - 1))/c^6
```

Sympy [A] (verification not implemented)

Time = 3.31 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.81

$$\int x\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{ad^2x^2}{2}+\frac{adex^4}{2}+\frac{ae^2x^6}{6}+\frac{bd^2x^2\operatorname{asec}(cx)}{2}+\frac{bdex^4\operatorname{asec}(cx)}{2}$$
$$+\frac{be^2x^6\operatorname{asec}(cx)}{6}-\frac{bd^2\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c}$$
$$-\frac{bde\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{2c}$$
$$-\frac{be^2\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c}+\frac{4x^2\sqrt{c^2x^2-1}}{15c^3}+\frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for } |c^2x^2|>1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c}+\frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3}+\frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{6c}$$

input
```
integrate(x*(e*x**2+d)**2*(a+b*asec(c*x)),x)
```

output
```
a*d**2*x**2/2 + a*d*e*x**4/2 + a*e**2*x**6/6 + b*d**2*x**2*asec(c*x)/2 + b
*d*e*x**4*asec(c*x)/2 + b*e**2*x**6*asec(c*x)/6 - b*d**2*Piecewise((sqrt(c
**2*x**2 - 1)/c, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/(2
*c) - b*d*e*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c) + 2*sqrt(c**2*x**2 -
 1)/(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*x**2 + 1)/(3*c) + 2*
I*sqrt(-c**2*x**2 + 1)/(3*c**3), True))/(2*c) - b*e**2*Piecewise((x**4*sqr
t(c**2*x**2 - 1)/(5*c) + 4*x**2*sqrt(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**
2*x**2 - 1)/(15*c**5), Abs(c**2*x**2) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(
5*c) + 4*I*x**2*sqrt(-c**2*x**2 + 1)/(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/
(15*c**5), True))/(6*c)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.98

$$\int x\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{1}{6}ae^2x^6+\frac{1}{2}adex^4+\frac{1}{2}ad^2x^2+\frac{1}{2}\left(x^2\operatorname{arcsec}\left(cx\right)-\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)bd^2$$
$$+\frac{1}{6}\left(3\,x^4\operatorname{arcsec}\left(cx\right)-\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)bde$$
$$+\frac{1}{90}\left(15\,x^6\operatorname{arcsec}\left(cx\right)-\frac{3\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+10\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^5}\right)be^2$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*e^2*x^6 + 1/2*a*d*e*x^4 + 1/2*a*d^2*x^2 + 1/2*(x^2*arcsec(c*x) - x*s
qrt(-1/(c^2*x^2) + 1)/c)*b*d^2 + 1/6*(3*x^4*arcsec(c*x) - (c^2*x^3*(-1/(c^
2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b*d*e + 1/90*(15*x^6*
arcsec(c*x) - (3*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2) + 10*c^2*x^3*(-1/(c^2*x^
2) + 1)^(3/2) + 15*x*sqrt(-1/(c^2*x^2) + 1))/c^5)*b*e^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11858 vs. $2(169) = 338$.

Time = 0.25 (sec) , antiderivative size = 11858, normalized size of antiderivative = 60.81

$$\int x(d+ex^2)^2 (a+b\sec^{-1}(cx)) \, dx = \text{Too large to display}$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/90*(45*b*c^4*d^2*arccos(1/(c*x))/(c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x)
 + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2)
 - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c
^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x)
 + 1)^12) + 45*a*c^4*d^2/(c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 +
15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1
/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2
*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)
+ 90*b*c^4*d^2*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2
) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c
^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*
x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2)
- 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^2) - 90*b*c^4*d^2*sqrt(-1/(c^2*x^2)
 + 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2
) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15
*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c
*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)) + 9
0*a*c^4*d^2*(1/(c^2*x^2) - 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1
)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1
)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^...
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex^2)^2 (a+b\sec^{-1}(cx))\ dx = \int x\,(e\,x^2+d)^2\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x*(d + e*x^2)^2*(a + b*acos(1/(c*x))),x)
```

output
```
int(x*(d + e*x^2)^2*(a + b*acos(1/(c*x))), x)
```

Reduce [F]

$$\int x(d+ex^2)^2 (a+b\sec^{-1}(cx))\ dx = \left(\int asec(cx)\,x^5dx\right)b\,e^2+2\left(\int asec(cx)\,x^3dx\right)bde + \left(\int asec(cx)\,xdx\right)b\,d^2 + \frac{a\,d^2x^2}{2}+\frac{ade\,x^4}{2}+\frac{a\,e^2x^6}{6}$$

input
```
int(x*(e*x^2+d)^2*(a+b*asec(c*x)),x)
```

output
```
(6*int(asec(c*x)*x**5,x)*b*e**2 + 12*int(asec(c*x)*x**3,x)*b*d*e + 6*int(a
sec(c*x)*x,x)*b*d**2 + 3*a*d**2*x**2 + 3*a*d*e*x**4 + a*e**2*x**6)/6
```

3.89 $\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x}\,dx$

Optimal result 758
Mathematica [A] (verified) 759
Rubi [A] (verified) 759
Maple [A] (verified) 762
Fricas [F] 762
Sympy [F] 763
Maxima [F] 763
Giac [F(-2)] 764
Mupad [F(-1)] 764
Reduce [F] 764

Optimal result

Integrand size = 21, antiderivative size = 186

$$\begin{aligned}\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x}\,dx &= -\frac{be(6c^2d+e)\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3}-\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}\\&\quad-\frac{1}{2}ibd^2\csc^{-1}(cx)^2+dex^2(a+b\sec^{-1}(cx))\\&\quad+\frac{1}{4}e^2x^4(a+b\sec^{-1}(cx))\\&\quad+bd^2\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)\\&\quad-bd^2\csc^{-1}(cx)\log\left(\frac{1}{x}\right)\\&\quad-d^2(a+b\sec^{-1}(cx))\log\left(\frac{1}{x}\right)\\&\quad-\frac{1}{2}ibd^2\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\end{aligned}$$

output
```
-1/6*b*e*(6*c^2*d+e)*(1-1/c^2/x^2)^(1/2)*x/c^3-1/12*b*e^2*(1-1/c^2/x^2)^(1
/2)*x^3/c-1/2*I*b*d^2*arccsc(c*x)^2+d*e*x^2*(a+b*arcsec(c*x))+1/4*e^2*x^4*
(a+b*arcsec(c*x))+b*d^2*arccsc(c*x)*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)-b*
d^2*arccsc(c*x)*ln(1/x)-d^2*(a+b*arcsec(c*x))*ln(1/x)-1/2*I*b*d^2*polylog(
2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x} dx = adex^2 + \frac{1}{4}ae^2x^4 - \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x(2+c^2x^2)}{12c^3} + \frac{1}{4}be^2x^4\sec^{-1}(cx) + \frac{bdex\left(-\sqrt{1-\frac{1}{c^2x^2}}+cx\sec^{-1}(cx)\right)}{c} + ad^2\log(x) + \frac{1}{2}ibd^2\left(\sec^{-1}(cx)\left(\sec^{-1}(cx) + 2i\log\left(1+e^{2i\sec^{-1}(cx)}\right)\right) + \text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x,x]
```

output
```
a*d*e*x^2 + (a*e^2*x^4)/4 - (b*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*(2 + c^2*x^2))/
(12*c^3) + (b*e^2*x^4*ArcSec[c*x])/4 + (b*d*e*x*(-Sqrt[1 - 1/(c^2*x^2)] +
c*x*ArcSec[c*x]))/c + a*d^2*Log[x] + (I/2)*b*d^2*(ArcSec[c*x]*(ArcSec[c*x]
 + (2*I)*Log[1 + E^((2*I)*ArcSec[c*x])]) + PolyLog[2, -E^((2*I)*ArcSec[c*x
])])
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5763, 5231, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x} dx$$

$$\Big\downarrow \ 5763$$

$$-\int \left(\frac{d}{x^2}+e\right)^2 x^5 \left(a+b\arccos\left(\frac{1}{cx}\right)\right)d\frac{1}{x}$$

$$\Big\downarrow \text{5231}$$

$$-\frac{b\int -\frac{e\left(\frac{4d}{x^2}+e\right)x^4-4d^2\log\left(\frac{1}{x}\right)}{4\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{c}-d^2\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+ dex^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+\frac{1}{4}e^2x^4\left(a+b\arccos\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{e\left(\frac{4d}{x^2}+e\right)x^4-4d^2\log\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{4c}-d^2\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+dex^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+ \frac{1}{4}e^2x^4\left(a+b\arccos\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{7293}$$

$$\frac{b\int \left(\frac{e\left(\frac{4d}{x^2}+e\right)x^4}{\sqrt{1-\frac{1}{c^2x^2}}}-\frac{4d^2\log\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{c^2x^2}}}\right)d\frac{1}{x}}{4c}-d^2\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+ dex^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+\frac{1}{4}e^2x^4\left(a+b\arccos\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{2009}$$

$$-d^2\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+dex^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+ \frac{1}{4}e^2x^4\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+ \frac{b\left(-2icd^2\operatorname{PolyLog}\left(2,e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)-2icd^2\arcsin\left(\frac{1}{cx}\right)^2+4cd^2\arcsin\left(\frac{1}{cx}\right)\log\left(1-e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)-4cd^2\log\left(\frac{1}{x}\right)\right.}{4c}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x,x]
```

output
```
d*e*x^2*(a + b*ArcCos[1/(c*x)]) + (e^2*x^4*(a + b*ArcCos[1/(c*x)]))/4 - d^
2*(a + b*ArcCos[1/(c*x)])*Log[x^(-1)] + (b*((-2*e*(6*d + e/c^2)*Sqrt[1 - 1
/(c^2*x^2)]*x)/3 - (e^2*Sqrt[1 - 1/(c^2*x^2)]*x^3)/3 - (2*I)*c*d^2*ArcSin[
1/(c*x)]^2 + 4*c*d^2*ArcSin[1/(c*x)]*Log[1 - E^((2*I)*ArcSin[1/(c*x)])] -
4*c*d^2*ArcSin[1/(c*x)]*Log[x^(-1)] - (2*I)*c*d^2*PolyLog[2, E^((2*I)*ArcS
in[1/(c*x)])]))/(4*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.24

method	result
parts	$a\left(\frac{e^2x^4}{4}+x^2de+d^2\ln(x)\right)+b\left(\frac{id^2\operatorname{arcsec}(cx)^2}{2}+\frac{e\left(12c^4d\operatorname{arcsec}(cx)x^2+3\operatorname{arcsec}(cx)e\,c^4x^4-12\sqrt{\frac{c^2x}{c^2}}\right.}{}\right.$
derivativedivides	$ade\,x^2+\frac{a\,e^2x^4}{4}+a\,d^2\ln(cx)+\frac{b\left(\frac{ic^4d^2\operatorname{arcsec}(cx)^2}{2}+\frac{e\left(12c^4d\operatorname{arcsec}(cx)x^2+3\operatorname{arcsec}(cx)e\,c^4x^4-12\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3\right.}{12}\right.}{}$
default	$ade\,x^2+\frac{a\,e^2x^4}{4}+a\,d^2\ln(cx)+\frac{b\left(\frac{ic^4d^2\operatorname{arcsec}(cx)^2}{2}+\frac{e\left(12c^4d\operatorname{arcsec}(cx)x^2+3\operatorname{arcsec}(cx)e\,c^4x^4-12\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3\right.}{12}\right.}{}$

input
```
int((e*x^2+d)^2*(a+b*arcsec(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
a*(1/4*e^2*x^4+x^2*d*e+d^2*ln(x))+b*(1/2*I*d^2*arcsec(c*x)^2+1/12/c^4*e*(1
2*c^4*d*arcsec(c*x)*x^2+3*arcsec(c*x)*e*c^4*x^4-12*((c^2*x^2-1)/c^2/x^2)^(
1/2)*c^3*d*x-((c^2*x^2-1)/c^2/x^2)^(1/2)*e*c^3*x^3-12*I*c^2*d-2*((c^2*x^2-
1)/c^2/x^2)^(1/2)*e*c*x-2*I*e)-d^2*arcsec(c*x)*ln(1+(1/c/x+I*(1-1/c^2/x^2)
^(1/2))^2)+1/2*I*d^2*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2))
```

Fricas [F]

$$\int\frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x}\,dx=\int\frac{(ex^2+d)^2(b\operatorname{arcsec}(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arcsec(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{(a+b\,\mathrm{asec}\,(cx))\,(d+ex^2)^2}{x}\,dx$$

input
```
integrate((e*x**2+d)**2*(a+b*asec(c*x))/x,x)
```

output
```
Integral((a + b*asec(c*x))*(d + e*x**2)**2/x, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{(ex^2+d)^2(b\,\mathrm{arcsec}\,(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x,x, algorithm="maxima")
```

output
```
1/4*a*e^2*x^4 + a*d*e*x^2 + a*d^2*log(x) - 1/8*(-2*I*b*c^4*e^2*x^4*log(c)
- 4*I*b*c^4*d^2*log(-c*x + 1)*log(x) - 4*I*b*c^4*d^2*log(x)^2 - 4*I*b*c^4*
d^2*dilog(c*x) - 4*I*b*c^4*d^2*dilog(-c*x) + I*(b*e^2*(x^2/c^2 + log(c*x +
 1)/c^4 + log(c*x - 1)/c^4) + 4*b*d*e*(log(c*x + 1)/c^2 + log(c*x - 1)/c^2
) + 32*b*d^2*integrate(1/4*log(x)/(c^2*x^3 - x), x))*c^4 + 8*c^4*integrate
(1/4*(b*e^2*x^4 + 4*b*d*e*x^2 + 4*b*d^2*log(x))*sqrt(c*x + 1)*sqrt(c*x - 1
)/(c^2*x^3 - x), x) + (-8*I*b*c^4*d*e*log(c) - I*b*c^2*e^2)*x^2 - 2*(b*c^4
*e^2*x^4 + 4*b*c^4*d*e*x^2 + 4*b*c^4*d^2*log(x))*arctan(sqrt(c*x + 1)*sqrt
(c*x - 1)) + (I*b*c^4*e^2*x^4 + 4*I*b*c^4*d*e*x^2 + 4*I*b*c^4*d^2*log(x))*
log(c^2*x^2) + (-4*I*b*c^4*d^2*log(x) - 4*I*b*c^2*d*e - I*b*e^2)*log(c*x +
 1) + (-4*I*b*c^2*d*e - I*b*e^2)*log(c*x - 1) - 2*(I*b*c^4*e^2*x^4 + 4*I*b
*c^4*d*e*x^2 + 4*I*b*c^4*d^2*log(c))*log(x))/c^4
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{(e\,x^2+d)^2 \left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

input
```
int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x,x)
```

output
```
int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x}\,dx = \left(\int \frac{asec(cx)}{x}dx\right) b\,d^2 + \left(\int asec(cx)\,x^3dx\right) b\,e^2 + 2\left(\int asec(cx)\,xdx\right) bde + \log(x)\,a\,d^2 + ade\,x^2 + \frac{a\,e^2x^4}{4}$$

input
```
int((e*x^2+d)^2*(a+b*asec(c*x))/x,x)
```

output
```
(4*int(asec(c*x)/x,x)*b*d**2 + 4*int(asec(c*x)*x**3,x)*b*e**2 + 8*int(asec
(c*x)*x,x)*b*d*e + 4*log(x)*a*d**2 + 4*a*d*e*x**2 + a*e**2*x**4)/4
```

3.90 $\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^3}\,dx$

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Giac [F(-2)] . . . 772
Mupad [F(-1)] . . . 772
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Optimal result

Integrand size = 21, antiderivative size = 189

$$\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^3}\,dx = \frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} - \frac{1}{4}bc^2d^2\csc^{-1}(cx) - ibde\csc^{-1}(cx)^2 - \frac{d^2(a+b\sec^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2(a+b\sec^{-1}(cx)) + 2bde\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) - 2bde\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - 2de(a+b\sec^{-1}(cx))\log\left(\frac{1}{x}\right) - ibde\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)$$

output
```
1/4*b*c*d^2*(1-1/c^2/x^2)^(1/2)/x-1/2*b*e^2*(1-1/c^2/x^2)^(1/2)*x/c-1/4*b*
c^2*d^2*arccsc(c*x)-I*b*d*e*arccsc(c*x)^2-1/2*d^2*(a+b*arcsec(c*x))/x^2+1/
2*e^2*x^2*(a+b*arcsec(c*x))+2*b*d*e*arccsc(c*x)*ln(1-(I/c/x+(1-1/c^2/x^2)^
(1/2))^2)-2*b*d*e*arccsc(c*x)*ln(1/x)-2*d*e*(a+b*arcsec(c*x))*ln(1/x)-I*b*
d*e*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^3}\,dx$$
$$= \frac{1}{4}\left(-\frac{2ad^2}{x^2}+2ae^2x^2-\frac{2bd^2\sec^{-1}(cx)}{x^2}+\frac{2be^2x\left(-\sqrt{1-\frac{1}{c^2x^2}}+cx\sec^{-1}(cx)\right)}{c}\right.$$
$$+\frac{bd^2\left(-1+c^2x^2+c^2x^2\sqrt{-1+c^2x^2}\arctan\left(\sqrt{-1+c^2x^2}\right)\right)}{c\sqrt{1-\frac{1}{c^2x^2}}x^3}+8ade\log(x)$$
$$+4ibde\left(\sec^{-1}(cx)\left(\sec^{-1}(cx)+2i\log\left(1+e^{2i\sec^{-1}(cx)}\right)\right)\right.$$
$$\left.\left.+\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)\right)\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^3,x]
```

output
```
((-2*a*d^2)/x^2 + 2*a*e^2*x^2 - (2*b*d^2*ArcSec[c*x])/x^2 + (2*b*e^2*x*(-S
qrt[1 - 1/(c^2*x^2)] + c*x*ArcSec[c*x]))/c + (b*d^2*(-1 + c^2*x^2 + c^2*x^
2*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]]))/(c*Sqrt[1 - 1/(c^2*x^2)]
*x^3) + 8*a*d*e*Log[x] + (4*I)*b*d*e*(ArcSec[c*x]*(ArcSec[c*x] + (2*I)*Log
[1 + E^((2*I)*ArcSec[c*x])]) + PolyLog[2, -E^((2*I)*ArcSec[c*x])]))/4
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.21, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5763, 5231, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^3}\,dx$$

$$\Big\downarrow \; 5763$$

$$-\int \left(\frac{d}{x^2}+e\right)^2 x^3 \left(a+b\arccos\left(\frac{1}{cx}\right)\right) d\frac{1}{x}$$

$$\Big\downarrow \; 5231$$

$$-\frac{b\int -\frac{-\frac{d^2}{x^2}-4e\log\left(\frac{1}{x}\right)d+e^2x^2}{2\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{c}-\frac{d^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{2x^2}-2de\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+\frac{1}{2}e^2x^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \; 27$$

$$\frac{b\int \frac{-\frac{d^2}{x^2}-4e\log\left(\frac{1}{x}\right)d+e^2x^2}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{2c}-\frac{d^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{2x^2}-2de\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+\frac{1}{2}e^2x^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \; 7293$$

$$\frac{b\int \left(-\frac{d^2}{\sqrt{1-\frac{1}{c^2x^2}}x^2}-\frac{4e\log\left(\frac{1}{x}\right)d}{\sqrt{1-\frac{1}{c^2x^2}}}+\frac{e^2x^2}{\sqrt{1-\frac{1}{c^2x^2}}}\right)d\frac{1}{x}}{2c}-\frac{d^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{2x^2}-2de\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+\frac{1}{2}e^2x^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \; 2009$$

$$-\frac{d^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{2x^2}-2de\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+\frac{1}{2}e^2x^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+\frac{b\left(-\frac{1}{2}c^3d^2\arcsin\left(\frac{1}{cx}\right)-2icde\,\text{PolyLog}\left(2,e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)-2icde\arcsin\left(\frac{1}{cx}\right)^2+4cde\arcsin\left(\frac{1}{cx}\right)\log\left(1-e^{2i\arcsin\right.\right.}{2c}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^3,x]
```

output
```
-1/2*(d^2*(a + b*ArcCos[1/(c*x)]))/x^2 + (e^2*x^2*(a + b*ArcCos[1/(c*x)]))
/2 - 2*d*e*(a + b*ArcCos[1/(c*x)])*Log[x^(-1)] + (b*((c^2*d^2*Sqrt[1 - 1/(
c^2*x^2)])/(2*x) - e^2*Sqrt[1 - 1/(c^2*x^2)]*x - (c^3*d^2*ArcSin[1/(c*x)])
/2 - (2*I)*c*d*e*ArcSin[1/(c*x)]^2 + 4*c*d*e*ArcSin[1/(c*x)]*Log[1 - E^((2
*I)*ArcSin[1/(c*x)])] - 4*c*d*e*ArcSin[1/(c*x)]*Log[x^(-1)] - (2*I)*c*d*e*
PolyLog[2, E^((2*I)*ArcSin[1/(c*x)])]))/(2*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.14

method	result
parts	$a\left(\frac{e^2x^2}{2}+2de\ln\left(x\right)-\frac{d^2}{2x^2}\right)+ibde\,\mathrm{arcsec}\left(cx\right)^2+\frac{bc\,d^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4x}+\frac{b\,c^2d^2\,\mathrm{arcsec}(cx)}{4}-\frac{b\,\mathrm{arcsec}(}{2x^2}$
derivativedivides	$c^2\left(\frac{a\,x^2e^2}{2c^2}+\frac{2ade\ln(cx)}{c^2}-\frac{a\,d^2}{2c^2x^2}+\frac{ibde\,\mathrm{arcsec}(cx)^2}{c^2}+\frac{b\,d^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4cx}+\frac{b\,\mathrm{arcsec}(cx)d^2}{4}-\frac{b\,\mathrm{arcsec}(cx)d^2}{2c^2x^2}\right.$
default	$c^2\left(\frac{a\,x^2e^2}{2c^2}+\frac{2ade\ln(cx)}{c^2}-\frac{a\,d^2}{2c^2x^2}+\frac{ibde\,\mathrm{arcsec}(cx)^2}{c^2}+\frac{b\,d^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4cx}+\frac{b\,\mathrm{arcsec}(cx)d^2}{4}-\frac{b\,\mathrm{arcsec}(cx)d^2}{2c^2x^2}\right.$

input
```
int((e*x^2+d)^2*(a+b*arcsec(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*(1/2*e^2*x^2+2*d*e*ln(x)-1/2*d^2/x^2)+I*b*d*e*arcsec(c*x)^2+1/4*b*c*d^2/
x*((c^2*x^2-1)/c^2/x^2)^(1/2)+1/4*b*c^2*d^2*arcsec(c*x)-1/2*b*arcsec(c*x)*
d^2/x^2+1/2*b*e^2*arcsec(c*x)*x^2-1/2*b/c*e^2*((c^2*x^2-1)/c^2/x^2)^(1/2)*
x-1/2*I*b/c^2*e^2-2*b*d*e*arcsec(c*x)*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2
)+I*b*d*e*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)
```

Fricas [F]

$$\int \frac{(d+ex^2)^2\left(a+b\sec^{-1}(cx)\right)}{x^3}\,dx=\int \frac{(ex^2+d)^2(b\,\mathrm{arcsec}\left(cx\right)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arcsec(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{(a+b\operatorname{asec}(cx))(d+ex^2)^2}{x^3}\,dx$$

input
```
integrate((e*x**2+d)**2*(a+b*asec(c*x))/x**3,x)
```

output
```
Integral((a + b*asec(c*x))*(d + e*x**2)**2/x**3, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)^2(b\operatorname{arcsec}(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^3,x, algorithm="maxima")
```

output
```
1/2*a*e^2*x^2 - 1/4*b*d^2*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*
x^2) - 1) - 1) - c^3*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c + 2*arcsec(c*x)
/x^2) + 2*a*d*e*log(x) - 1/2*a*d^2/x^2 - 1/4*(-2*I*b*c^2*e^2*x^2*log(c) -
4*I*b*c^2*d*e*log(-c*x + 1)*log(x) - 4*I*b*c^2*d*e*log(x)^2 - 4*I*b*c^2*d*
e*dilog(c*x) - 4*I*b*c^2*d*e*dilog(-c*x) - I*b*e^2*log(c*x - 1) + I*(b*e^2
*(log(c*x + 1)/c^2 + log(c*x - 1)/c^2) + 16*b*d*e*integrate(1/2*log(x)/(c^
2*x^3 - x), x))*c^2 + 4*c^2*integrate(1/2*(b*e^2*x^2 + 4*b*d*e*log(x))*sqr
t(c*x + 1)*sqrt(c*x - 1)/(c^2*x^3 - x), x) - 2*(b*c^2*e^2*x^2 + 4*b*c^2*d*
e*log(x))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + (I*b*c^2*e^2*x^2 + 4*I*b*c
^2*d*e*log(x))*log(c^2*x^2) + (-4*I*b*c^2*d*e*log(x) - I*b*e^2)*log(c*x +
1) - 2*(I*b*c^2*e^2*x^2 + 4*I*b*c^2*d*e*log(c))*log(x))/c^2
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^3} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^3} \, dx = \int \frac{(e\,x^2+d)^2 \left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^3} \,dx$$

input
```
int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^3,x)
```

output
```
int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^3, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^3} \, dx$$
$$= \frac{2\left(\int \frac{asec(cx)}{x^3}dx\right) b\,d^2x^2 + 4\left(\int \frac{asec(cx)}{x}dx\right) bde\,x^2 + 2\left(\int asec(cx)\,xdx\right) b\,e^2x^2 + 4\log(x)\,ade\,x^2 - a\,d^2 + a\,e^{2}}{2x^2}$$

input
```
int((e*x^2+d)^2*(a+b*asec(c*x))/x^3,x)
```

output
```
(2*int(asec(c*x)/x**3,x)*b*d**2*x**2 + 4*int(asec(c*x)/x,x)*b*d*e*x**2 + 2
*int(asec(c*x)*x,x)*b*e**2*x**2 + 4*log(x)*a*d*e*x**2 - a*d**2 + a*e**2*x*
*4)/(2*x**2)
```

3.91 $\int \frac{x^2(a+b\sec^{-1}(cx))}{d+ex^2}\,dx$

Optimal result 774
Mathematica [A] (warning: unable to verify) 775
Rubi [A] (verified) 776
Maple [C] (warning: unable to verify) 779
Fricas [F] 780
Sympy [F] 780
Maxima [F(-2)] 781
Giac [F(-2)] 781
Mupad [F(-1)] 781
Reduce [F] 782

Optimal result

Integrand size = 21, antiderivative size = 546

$$
\begin{aligned}
\int \frac{x^2(a+b\sec^{-1}(cx))}{d+ex^2}\,dx &= \frac{x(a+b\sec^{-1}(cx))}{e} - \frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce} \\
&\quad + \frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&\quad - \frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&\quad + \frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&\quad - \frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&\quad + \frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&\quad - \frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&\quad + \frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&\quad - \frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}}
\end{aligned}
$$

output
```
x*(a+b*arcsec(c*x))/e-b*arctanh((1-1/c^2/x^2)^(1/2))/c/e+1/2*(-d)^(1/2)*(a
+b*arcsec(c*x))*ln(1-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(
c^2*d+e)^(1/2)))/e^(3/2)-1/2*(-d)^(1/2)*(a+b*arcsec(c*x))*ln(1+c*(-d)^(1/2
)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)+1/2*(-d
)^(1/2)*(a+b*arcsec(c*x))*ln(1-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/
(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3/2)-1/2*(-d)^(1/2)*(a+b*arcsec(c*x))*ln(1+c
*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3/
2)+1/2*I*b*(-d)^(1/2)*polylog(2,-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2)
)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)-1/2*I*b*(-d)^(1/2)*polylog(2,c*(-d)^(
1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)+1/2*
I*b*(-d)^(1/2)*polylog(2,-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1
/2)+(c^2*d+e)^(1/2)))/e^(3/2)-1/2*I*b*(-d)^(1/2)*polylog(2,c*(-d)^(1/2)*(1
/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.64 (sec) , antiderivative size = 1023, normalized size of antiderivative = 1.87

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2),x]
```

output
```
(a*x)/e - (a*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^(3/2) + b*((c*x*ArcSec
[c*x] + Log[Cos[ArcSec[c*x]/2] - Sin[ArcSec[c*x]/2]] - Log[Cos[ArcSec[c*x]
/2] + Sin[ArcSec[c*x]/2]])/(c*e) - (Sqrt[d]*(8*ArcSin[Sqrt[1 + (I*Sqrt[e])
/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])
/Sqrt[c^2*d + e]] - (2*I)*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e
])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*
Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]
))/(c*Sqrt[d])] - (2*I)*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])
*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sq
rt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))
/(c*Sqrt[d])] + (2*I)*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] - 2*PolyL
og[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 2*
PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d]
)] + PolyLog[2, -E^((2*I)*ArcSec[c*x])]))/(4*e^(3/2)) + (Sqrt[d]*(8*ArcSin
[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt
[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] - (2*I)*ArcSec[c*x]*Log[1 + (I*(
-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (4*I)*ArcSin
[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^
2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (2*I)*ArcSec[c*x]*Log[1 - (I*(
Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (4*I)*ArcS...
```

Rubi [A] (verified)

Time = 1.67 (sec) , antiderivative size = 598, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5763, 5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+b\sec^{-1}(cx)\right)}{d+ex^2}dx$$

$$\Big\downarrow \text{5763}$$

$$-\int \frac{x^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{\frac{d}{x^2}+e}d\frac{1}{x}$$

$$\Big\downarrow \text{5233}$$

$$-\int \left(\frac{x^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e\left(\frac{d}{x^2}+e\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{-d}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}}-\frac{\sqrt{-d}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}}+\frac{\sqrt{-d}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^{3/2}}-\frac{\sqrt{-d}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^{3/2}}+\frac{x\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e}+\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^{3/2}}-\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^{3/2}}+\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^{3/2}}-\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^{3/2}}-\frac{b\,\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce}$$

input
```
Int[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2),x]
```

output
```
(x*(a + b*ArcCos[1/(c*x)]))/e - (b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(c*e) +
 (Sqrt[-d]*(a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)
]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcCos[1/
(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d +
 e])])/(2*e^(3/2)) + (Sqrt[-d]*(a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]
*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^(3/2)) - (Sqrt[
-d]*(a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sq
rt[e] + Sqrt[c^2*d + e])])/(2*e^(3/2)) + ((I/2)*b*Sqrt[-d]*PolyLog[2, -((c
*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/e^(3/2) -
((I/2)*b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] -
 Sqrt[c^2*d + e])])/e^(3/2) + ((I/2)*b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E
^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e]))])/e^(3/2) - ((I/2)*b*Sq
rt[-d]*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d
 + e])])/e^(3/2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 29.02 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.68

method	result
parts	$\frac{ax}{e} - \frac{ad\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}} + \frac{b\,\mathrm{arcsec}(cx)x}{e} + \frac{2ib\arctan\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce} + \frac{ibcd\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4\right.\right.}\right.}{}$
derivativedivides	$\frac{\frac{a\,c^3x}{e} - \frac{a\,c^3d\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}} + b\,c^2\left(\frac{\mathrm{arcsec}(cx)cx}{e} + \frac{2i\arctan\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)}{e} + \frac{ic^2d\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z\right.}\right.}{}\right.}{}$
default	$\frac{\frac{a\,c^3x}{e} - \frac{a\,c^3d\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}} + b\,c^2\left(\frac{\mathrm{arcsec}(cx)cx}{e} + \frac{2i\arctan\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)}{e} + \frac{ic^2d\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z\right.}\right.}{}\right.}{}$

input
```
int(x^2*(a+b*arcsec(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
a/e*x-a*d/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b*arcsec(c*x)/e*x+2*I*b/c/
e*arctan(1/c/x+I*(1-1/c^2/x^2)^(1/2))+1/8*I*b*c/e^2*d*sum((_R1^2*c^2*d+c^2
*d+4*e)/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^
2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootO
f(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/8*I*b*c/e^2*d*sum((_R1^2*c^2*d+4
*_R1^2*e+c^2*d)/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I
*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_
R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
```

Fricas [F]

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^2}{ex^2+d}\,dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*x^2*arcsec(c*x) + a*x^2)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \int \frac{x^2(a+b\,\mathrm{asec}\,(cx))}{d+ex^2}\,dx$$

input
```
integrate(x**2*(a+b*asec(c*x))/(e*x**2+d),x)
```

output
```
Integral(x**2*(a + b*asec(c*x))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \int \frac{x^2\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2+d}\,dx$$

input
```
int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2),x)
```

output
```
int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \frac{-\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a+\left(\int \frac{asec(cx)x^2}{e\,x^2+d}dx\right)b\,e^2+aex}{e^2}$$

input
```
int(x^2*(a+b*asec(c*x))/(e*x^2+d),x)
```

output
```
( - sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a + int((asec(c*x)*x**2)
/(d + e*x**2),x)*b*e**2 + a*e*x)/e**2
```

3.92 $\int \frac{x\left(a+b\sec^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal result 784
Mathematica [A] (warning: unable to verify) 785
Rubi [A] (verified) 786
Maple [C] (warning: unable to verify) 788
Fricas [F] 790
Sympy [F] 790
Maxima [F] 790
Giac [F(-2)] 791
Mupad [F(-1)] 791
Reduce [F] 791

Optimal result

Integrand size = 19, antiderivative size = 499

$$\int \frac{x(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} + \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} - \frac{(2a+b\pi-b(\pi-2\sec^{-1}(cx)))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{2e} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} + \frac{ib\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2e}$$

output
```
1/2*(a+b*arcsec(c*x))*ln(1-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(
1/2)-(c^2*d+e)^(1/2)))/e+1/2*(a+b*arcsec(c*x))*ln(1+c*(-d)^(1/2)*(1/c/x+I*
(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/e+1/2*(a+b*arcsec(c*x))*ln
(1-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/e
+1/2*(a+b*arcsec(c*x))*ln(1+c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^
(1/2)+(c^2*d+e)^(1/2)))/e-1/2*(2*a+b*Pi-b*(Pi-2*arcsec(c*x)))*ln(1+(1/c/x+
I*(1-1/c^2/x^2)^(1/2))^2)/e-1/2*I*b*polylog(2,-c*(-d)^(1/2)*(1/c/x+I*(1-1/
c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/e-1/2*I*b*polylog(2,c*(-d)^(1/2
)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/e-1/2*I*b*polyl
og(2,-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2))
)/e-1/2*I*b*polylog(2,c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+
(c^2*d+e)^(1/2)))/e+1/2*I*b*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)/e
```

Mathematica [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 891, normalized size of antiderivative = 1.79

$$\int \frac{x(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x*(a + b*ArcSec[c*x]))/(d + e*x^2),x]
```

output
```
((4*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c
*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] + (4*I)*b*ArcSin[
Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])
*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] + b*ArcSec[c*x]*Log[1 + (I*(Sqrt[e]
- Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 2*b*ArcSin[Sqrt[1 + (
I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^
(I*ArcSec[c*x]))/(c*Sqrt[d])] + b*ArcSec[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[
c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 2*b*ArcSin[Sqrt[1 - (I*Sqrt[
e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*Arc
Sec[c*x]))/(c*Sqrt[d])] + b*ArcSec[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d +
 e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 2*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*
Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]
))/(c*Sqrt[d])] + b*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(
I*ArcSec[c*x]))/(c*Sqrt[d])] - 2*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])
]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sq
rt[d])] - 2*b*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] + a*Log[d + e*x^2
] - I*b*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(
c*Sqrt[d])] - I*b*PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c
*x]))/(c*Sqrt[d])] - I*b*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I
*ArcSec[c*x]))/(c*Sqrt[d])] - I*b*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d +...
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {5763, 5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\sec^{-1}(cx)\right)}{d+ex^2}dx$$

$$\Big\downarrow \text{ 5763}$$

$$-\int \frac{x\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{\frac{d}{x^2}+e}d\frac{1}{x}$$

$$\Big\downarrow \text{ 5233}$$

$$-\int \left(\frac{x\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e\left(\frac{d}{x^2}+e\right)x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e}+\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e}+\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e}+\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e}-\frac{\log\left(1+e^{2i\arccos\left(\frac{1}{cx}\right)}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e}-\frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e}-\frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e}-\frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e}-\frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e}+\frac{ib\,\text{PolyLog}\left(2,-e^{2i\arccos\left(\frac{1}{cx}\right)}\right)}{2e}$$

input
```
Int[(x*(a + b*ArcSec[c*x]))/(d + e*x^2),x]
```

output
```
((a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[
e] - Sqrt[c^2*d + e])])/(2*e) + ((a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-
d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e) + ((a + b*Ar
cCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[
c^2*d + e])])/(2*e) + ((a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*Ar
cCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e) - ((a + b*ArcCos[1/(c*
x)])*Log[1 + E^((2*I)*ArcCos[1/(c*x)])])/e - ((I/2)*b*PolyLog[2, -((c*Sqrt
[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/e - ((I/2)*b*Po
lyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/
e - ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sq
rt[c^2*d + e]))])/e - ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]
))/(Sqrt[e] + Sqrt[c^2*d + e])])/e + ((I/2)*b*PolyLog[2, -E^((2*I)*ArcCos[
1/(c*x)])])/e
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.79 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.89

method	result
parts	$\frac{a\ln(e\,x^2+d)}{2e}-\frac{i b\, c^2 d\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2 d\,\_Z^4+\left(2c^2 d+4e\right)\_Z^2+c^2 d\right)}\frac{\left(\_R1^2+1\right)\left(i\,\mathrm{arcsec}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-i\sqrt{1-\cdots}}{\_R1}\right)\cdots\right.}{\_R1^2 c^2 d+c\cdots}\right.}{4e}$
derivativedivides	$\frac{\frac{a\,c^2\ln\left(e\,c^2x^2+c^2 d\right)}{2e}+b\,c^2\left(-\frac{i c^2 d\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2 d\,\_Z^4+\left(2c^2 d+4e\right)\_Z^2+c^2 d\right)}\frac{\left(\_R1^2+1\right)\left(i\,\mathrm{arcsec}(cx)\ln\left(\frac{\_R1-\cdots}{\cdots}\right)\cdots\right.}{\cdots}\right.}{4e}\cdots\right.}{\cdots}$
default	$\frac{\frac{a\,c^2\ln\left(e\,c^2x^2+c^2 d\right)}{2e}+b\,c^2\left(-\frac{i c^2 d\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2 d\,\_Z^4+\left(2c^2 d+4e\right)\_Z^2+c^2 d\right)}\frac{\left(\_R1^2+1\right)\left(i\,\mathrm{arcsec}(cx)\ln\left(\frac{\_R1-\cdots}{\cdots}\right)\cdots\right.}{\cdots}\right.}{4e}\cdots\right.}{\cdots}$

input
```
int(x*(a+b*arcsec(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*a/e*ln(e*x^2+d)-1/4*I*b*c^2*d/e*sum((_R1^2+1)/(_R1^2*c^2*d+c^2*d+2*e)*
(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-
I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*
d))-b/e*arcsec(c*x)*ln(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-b/e*arcsec(c*x)*
ln(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))+I*b/e*dilog(1+I*(1/c/x+I*(1-1/c^2/x^
2)^(1/2)))+I*b/e*dilog(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-1/4*I*b/e*sum((_
R1^2*c^2*d+c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x
-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1))
,_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
```

Fricas [F]

$$\int \frac{x(a + b\sec^{-1}(cx))}{d + ex^2}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx) + a)x}{ex^2 + d}\,dx$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*x*arcsec(c*x) + a*x)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x(a + b\sec^{-1}(cx))}{d + ex^2}\,dx = \int \frac{x(a + b\,\mathrm{asec}\,(cx))}{d + ex^2}\,dx$$

input
```
integrate(x*(a+b*asec(c*x))/(e*x**2+d),x)
```

output
```
Integral(x*(a + b*asec(c*x))/(d + e*x**2), x)
```

Maxima [F]

$$\int \frac{x(a + b\sec^{-1}(cx))}{d + ex^2}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx) + a)x}{ex^2 + d}\,dx$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
b*integrate(x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x^2 + d), x) + 1/2*a*
log(e*x^2 + d)/e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \int \frac{x\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2+d}\,dx$$

input
```
int((x*(a + b*acos(1/(c*x))))/(d + e*x^2),x)
```

output
```
int((x*(a + b*acos(1/(c*x))))/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \frac{2\left(\int \frac{asec(cx)x}{e\,x^2+d}dx\right)be+\log(e\,x^2+d)\,a}{2e}$$

input
```
int(x*(a+b*asec(c*x))/(e*x^2+d),x)
```

output
```
(2*int((asec(c*x)*x)/(d + e*x**2),x)*b*e + log(d + e*x**2)*a)/(2*e)
```

3.93 $\int \frac{a+b\sec^{-1}(cx)}{d+ex^2}\,dx$

Optimal result . . . 792
Mathematica [A] (verified) . . . 793
Rubi [A] (verified) . . . 794
Maple [C] (verified) . . . 796
Fricas [F] . . . 798
Sympy [F] . . . 798
Maxima [F(-2)] . . . 798
Giac [F(-2)] . . . 799
Mupad [F(-1)] . . . 799
Reduce [F] . . . 799

Optimal result

Integrand size = 18, antiderivative size = 509

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex^2}\,dx = \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

output
```
1/2*(a+b*arcsec(c*x))*ln(1-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(
1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsec(c*x))*ln(1+c*(-d
)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2
)/e^(1/2)+1/2*(a+b*arcsec(c*x))*ln(1-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(
1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsec(c*x))*
ln(1+c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))
/(-d)^(1/2)/e^(1/2)+1/2*I*b*polylog(2,-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)
^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*I*b*polylog(2,c*
(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(
1/2)/e^(1/2)+1/2*I*b*polylog(2,-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))
/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*I*b*polylog(2,c*(-d)^(1
/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^
(1/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 871, normalized size of antiderivative = 1.71

$$\int \frac{a + b\sec^{-1}(cx)}{d + ex^2} \, dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(d + e*x^2),x]
```

output
```
(2*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - 4*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt
[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt
[c^2*d + e]] + 4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTa
n[((I*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] - I*b*ArcS
ec[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[
d])] - (2*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (
I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + I*b*ArcSec
[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d
])] + (2*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I
*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + I*b*ArcSec
[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d]
)] - (2*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*
(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - I*b*ArcSec[c
*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])]
 + (2*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(S
qrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + b*PolyLog[2, (
(-I)*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - b*Poly
Log[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - b
*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d
])] + b*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c...
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5753, 5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex^2}\,dx$$

$$\Big\downarrow \ 5753$$

$$-\int \frac{a+b\arccos\left(\frac{1}{cx}\right)}{\frac{d}{x^2}+e}d\frac{1}{x}$$

$$\Big\downarrow \ 5173$$

$$-\int \left(\frac{a+b\arccos\left(\frac{1}{cx}\right)}{2\sqrt{e}\left(\sqrt{e}-\frac{\sqrt{-d}}{x}\right)}+\frac{a+b\arccos\left(\frac{1}{cx}\right)}{2\sqrt{e}\left(\frac{\sqrt{-d}}{x}+\sqrt{e}\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

input
```
Int[(a + b*ArcSec[c*x])/(d + e*x^2),x]
```

output
```
((a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[
e] - Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCos[1/(c*x)])*Lo
g[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*
Sqrt[-d]*Sqrt[e]) + ((a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcC
os[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b
*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sq
rt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) + ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*
E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/(Sqrt[-d]*Sqrt[e]) -
 ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^
2*d + e])])/(Sqrt[-d]*Sqrt[e]) + ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*Ar
cCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e]))])/(Sqrt[-d]*Sqrt[e]) - ((I/2)
*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e
])])/(Sqrt[-d]*Sqrt[e])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

rule 5753
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(2*(p + 1)))
, x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 15.12 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.53

method	result
parts	$\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}+\frac{ibc\left(\sum_{\_R1=\mathrm{RootOf}\left(c^{2}d\_Z^{4}+\left(2c^{2}d+4e\right)\_Z^{2}+c^{2}d\right)}\frac{\_R1\left(i\,\mathrm{arcsec}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-i\sqrt{1-\frac{1}{c^{2}x^{2}}}}{\_R1}\right)\dots\right.}{\_R1^{2}c^{2}d+c^{2}d+\dots}\dots\right.}{2}\dots$
derivativedivides	$\frac{\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}+bc^{2}\left(\frac{i\left(\sum_{\_R1=\mathrm{RootOf}\left(c^{2}d\_Z^{4}+\left(2c^{2}d+4e\right)\_Z^{2}+c^{2}d\right)}\frac{\_R1\left(i\,\mathrm{arcsec}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-i\sqrt{1-\frac{1}{c^{2}x^{2}}}}{\_R1}\right)\dots\right.}{\_R1^{2}c^{2}d+c^{2}d\dots}\dots\right.}{2}\dots\right.}{\dots}$
default	$\frac{\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}+bc^{2}\left(\frac{i\left(\sum_{\_R1=\mathrm{RootOf}\left(c^{2}d\_Z^{4}+\left(2c^{2}d+4e\right)\_Z^{2}+c^{2}d\right)}\frac{\_R1\left(i\,\mathrm{arcsec}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-i\sqrt{1-\frac{1}{c^{2}x^{2}}}}{\_R1}\right)\dots\right.}{\_R1^{2}c^{2}d+c^{2}d\dots}\dots\right.}{2}\dots\right.}{\dots}$

input
```
int((a+b*arcsec(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
a/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+1/2*I*b*c*sum(_R1/(_R1^2*c^2*d+c^2*d
+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-
1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^
2+c^2*d))-1/2*I*b*c*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_
R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2
))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
```

Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex^2}\,dx = \int \frac{b\operatorname{arcsec}(cx)+a}{ex^2+d}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex^2}\,dx = \int \frac{a+b\operatorname{asec}(cx)}{d+ex^2}\,dx$$

input
```
integrate((a+b*asec(c*x))/(e*x**2+d),x)
```

output
```
Integral((a + b*asec(c*x))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\sec^{-1}(cx)}{d + ex^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\sec^{-1}(cx)}{d + ex^2}\,dx = \int \frac{a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{e\,x^2 + d}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(d + e*x^2),x)
```

output
```
int((a + b*acos(1/(c*x)))/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{a + b\sec^{-1}(cx)}{d + ex^2}\,dx = \frac{\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a + \left(\int \frac{asec(cx)}{e\,x^2+d}dx\right)bde}{de}$$

input
```
int((a+b*asec(c*x))/(e*x^2+d),x)
```

output
```
(sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a + int(asec(c*x)/(d + e*x*
*2),x)*b*d*e)/(d*e)
```

3.94 $\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)}\,dx$

Optimal result 800
Mathematica [A] (verified) 801
Rubi [A] (verified) 802
Maple [C] (warning: unable to verify) 804
Fricas [F] 805
Sympy [F] 806
Maxima [F] 806
Giac [F(-2)] 806
Mupad [F(-1)] 807
Reduce [F] 807

Optimal result

Integrand size = 21, antiderivative size = 459

$$\begin{aligned}\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)}\,dx &= \frac{i(a+b\sec^{-1}(cx))^2}{2bd} - \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}\\
&\quad - \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}\\
&\quad - \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}\\
&\quad - \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}\\
&\quad + \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}\\
&\quad + \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}\\
&\quad + \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}\\
&\quad + \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}\end{aligned}$$

output
```
1/2*I*(a+b*arcsec(c*x))^2/b/d-1/2*(a+b*arcsec(c*x))*ln(1-c*(-d)^(1/2)*(1/c
/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/d-1/2*(a+b*arcsec(c*x
))*ln(1+c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2
)))/d-1/2*(a+b*arcsec(c*x))*ln(1-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2)
)/(e^(1/2)+(c^2*d+e)^(1/2)))/d-1/2*(a+b*arcsec(c*x))*ln(1+c*(-d)^(1/2)*(1/
c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/d+1/2*I*b*polylog(2,
-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/d+1
/2*I*b*polylog(2,c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*
d+e)^(1/2)))/d+1/2*I*b*polylog(2,-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2
))/(e^(1/2)+(c^2*d+e)^(1/2)))/d+1/2*I*b*polylog(2,c*(-d)^(1/2)*(1/c/x+I*(1
-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 876, normalized size of antiderivative = 1.91

$$\int \frac{a+b\sec^{-1}(cx)}{x\,(d+ex^2)}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x*(d + e*x^2)),x]
```

output
```
((I/2)*(b*ArcSec[c*x]^2 - 4*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqr
t[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d +
e]] - 4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*S
qrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] + I*b*ArcSec[c*x]*L
og[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (2
*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e
] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + I*b*ArcSec[c*x]*Log
[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (2*
I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e
] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + I*b*ArcSec[c*x]*Log
[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (2*I
)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e]
+ Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + I*b*ArcSec[c*x]*Log[1
 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (2*I)*
b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] +
Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (2*I)*a*Log[x] + I*a*Lo
g[d + e*x^2] + b*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec
[c*x]))/(c*Sqrt[d])] + b*PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*A
rcSec[c*x]))/(c*Sqrt[d])] + b*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e])
*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + b*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*...
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5763, 5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x\,(d+ex^2)}\,dx$$

$$\Big\downarrow \text{5763}$$

$$-\int \frac{a+b\arccos\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)x}d\frac{1}{x}$$

$$\Big\downarrow \text{5233}$$

$$-\int \left(\frac{\sqrt{-d}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{2d\left(\frac{\sqrt{-d}}{x}+\sqrt{e}\right)}-\frac{\sqrt{-d}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{2d\left(\sqrt{e}-\frac{\sqrt{-d}}{x}\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d}-\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d}+\frac{i\left(a+b\arccos\left(\frac{1}{cx}\right)\right)^2}{2bd}+\frac{ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d}+\frac{ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d}+\frac{ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d}+\frac{ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d}$$

input
```
Int[(a + b*ArcSec[c*x])/(x*(d + e*x^2)),x]
```

output
```
((I/2)*(a + b*ArcCos[1/(c*x)])^2)/(b*d) - ((a + b*ArcCos[1/(c*x)])*Log[1 -
 (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d) -
((a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[
e] - Sqrt[c^2*d + e])])/(2*d) - ((a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-
d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d) - ((a + b*Ar
cCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[
c^2*d + e])])/(2*d) + ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcCos[1/(c*x
)]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/d + ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^
(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/d + ((I/2)*b*PolyLog[2,
 -((c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e]))])/d + (
(I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*
d + e])])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.97 (sec) , antiderivative size = 1970, normalized size of antiderivative = 4.29

method	result	size
parts	Expression too large to display	1970
derivativedivides	Expression too large to display	1997
default	Expression too large to display	1997

input
```
int((a+b*arcsec(c*x))/x/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
a/d*ln(x)-1/2*a/d*ln(e*x^2+d)+b*(1/2*I/d*sum((_R1^2*c^2*d+2*c^2*d+4*e)/(_R
1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_
R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2
*c^2*d+4*e)*_Z^2+c^2*d))-1/4*I*(e*(c^2*d+e))^(1/2)/e/(c^2*d+e)*arcsec(c*x)
^2*c^2-I*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e
^2)*arcsec(c*x)^2/(c^2*d+e)/c^2/d^2-1/2*I*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^
2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*polylog(2,d*c^2*(1/c/x+I*(1-1/c^2/x^2
)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))/(c^2*d+e)/c^2/d^2-1/2*I*(e*
(c^2*d+e))^(1/2)/d/(c^2*d+e)*arcsec(c*x)^2-1/4*I*(-(e*(c^2*d+e))^(1/2)*c^2
*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*arcsec(c*x)^2/d/e/(c^2*d+e)+1/
2*(e*(c^2*d+e))^(1/2)/d/(c^2*d+e)*arcsec(c*x)*ln(1-d*c^2*(1/c/x+I*(1-1/c^2
/x^2)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))+1/4*(e*(c^2*d+e))^(1/2)
/e/(c^2*d+e)*c^2*arcsec(c*x)*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-
c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))-1/4*I*(e*(c^2*d+e))^(1/2)/d/(c^2*d+e)*po
lylog(2,d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2
)-2*e))+1/2*I*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*arcsec(c*x)^2/c^2/d^2-(c^2
*d-2*(e*(c^2*d+e))^(1/2)+2*e)/c^4/d^3*e*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^
(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsec(c*x)-1/8*I*(e*(c^2*d+e
))^(1/2)/e/(c^2*d+e)*polylog(2,d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2
*d+2*(e*(c^2*d+e))^(1/2)-2*e))*c^2+(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*...
```

Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x\,(d+ex^2)}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{(ex^2+d)x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)/(e*x^3 + d*x), x)
```

Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)}\,dx=\int \frac{a+b\operatorname{asec}\left(cx\right)}{x\left(d+ex^2\right)}\,dx$$

input
```
integrate((a+b*asec(c*x))/x/(e*x**2+d),x)
```

output
```
Integral((a + b*asec(c*x))/(x*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)}\,dx=\int \frac{b\operatorname{arcsec}\left(cx\right)+a}{(ex^2+d)x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d),x, algorithm="maxima")
```

output
```
-1/2*a*(log(e*x^2 + d)/d - 2*log(x)/d) + b*integrate(arctan(sqrt(c*x + 1)*
sqrt(c*x - 1))/(e*x^3 + d*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x\,\left(e\,x^2+d\right)}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)}\,dx=\frac{2\left(\int \frac{asec(cx)}{e\,x^3+dx}dx\right)bd-\log\left(e\,x^2+d\right)a+2\log(x)\,a}{2d}$$

input
```
int((a+b*asec(c*x))/x/(e*x^2+d),x)
```

output
```
(2*int(asec(c*x)/(d*x + e*x**3),x)*b*d - log(d + e*x**2)*a + 2*log(x)*a)/(
2*d)
```

3.95 $\int \frac{a+b\sec^{-1}(cx)}{x^2(d+ex^2)}\,dx$

Optimal result 809
Mathematica [A] (verified) 810
Rubi [A] (verified) 811
Maple [C] (verified) 814
Fricas [F] 815
Sympy [F] 815
Maxima [F(-2)] 815
Giac [F(-2)] 816
Mupad [F(-1)] 816
Reduce [F] 816

Optimal result

Integrand size = 21, antiderivative size = 551

$$\int \frac{a+b\sec^{-1}(cx)}{x^2(d+ex^2)}\,dx = \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\sec^{-1}(cx)}{dx} + \frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} - \frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} + \frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} - \frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} + \frac{ib\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} - \frac{ib\sqrt{e}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} + \frac{ib\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} - \frac{ib\sqrt{e}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$

output
```
b*c*(1-1/c^2/x^2)^(1/2)/d-a/d/x-b*arcsec(c*x)/d/x+1/2*e^(1/2)*(a+b*arcsec(
c*x))*ln(1-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(
1/2)))/(-d)^(3/2)-1/2*e^(1/2)*(a+b*arcsec(c*x))*ln(1+c*(-d)^(1/2)*(1/c/x+I
*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)+1/2*e^(1/2)*(a
+b*arcsec(c*x))*ln(1-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(
c^2*d+e)^(1/2)))/(-d)^(3/2)-1/2*e^(1/2)*(a+b*arcsec(c*x))*ln(1+c*(-d)^(1/2
)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)+1/2*
I*b*e^(1/2)*polylog(2,-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)
-(c^2*d+e)^(1/2)))/(-d)^(3/2)-1/2*I*b*e^(1/2)*polylog(2,c*(-d)^(1/2)*(1/c/
x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)+1/2*I*b*e^(
1/2)*polylog(2,-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d
+e)^(1/2)))/(-d)^(3/2)-1/2*I*b*e^(1/2)*polylog(2,c*(-d)^(1/2)*(1/c/x+I*(1-
1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)
```

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 997, normalized size of antiderivative = 1.81

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x^2*(d + e*x^2)),x]
```

output
```
-(a/(d*x)) - (a*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + b*((c*Sqrt[
1 - 1/(c^2*x^2)] - ArcSec[c*x]/x)/d - (Sqrt[e]*(8*ArcSin[Sqrt[1 + (I*Sqrt[
e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/
2])/Sqrt[c^2*d + e]] - (2*I)*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d
+ e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/
(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c
*x]))/(c*Sqrt[d])] - (2*I)*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d +
e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c
*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x
]))/(c*Sqrt[d])] + (2*I)*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] - 2*Po
lyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] -
 2*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt
[d])] + PolyLog[2, -E^((2*I)*ArcSec[c*x])]))/(4*d^(3/2)) + (Sqrt[e]*(8*Arc
Sin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + S
qrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] - (2*I)*ArcSec[c*x]*Log[1 + (
I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (4*I)*Arc
Sin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt
[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (2*I)*ArcSec[c*x]*Log[1 - (
I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (4*I)*ArcS
in[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqr...
```

Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 603, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5763, 5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2(d+ex^2)}dx$$

$$\Big\downarrow \text{5763}$$

$$-\int \frac{a+b\arccos\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{5233}$$

$$-\int \left(\frac{a+b\arccos\left(\frac{1}{cx}\right)}{d}-\frac{e\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{d\left(\frac{d}{x^2}+e\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{e}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}-\frac{\sqrt{e}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}+\frac{\sqrt{e}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2(-d)^{3/2}}-\frac{\sqrt{e}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2(-d)^{3/2}}-\frac{a}{dx}+\frac{ib\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}}-\frac{ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}}+\frac{ib\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}}-\frac{ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}}-\frac{b\arccos\left(\frac{1}{cx}\right)}{dx}+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d}$$

input
```
Int[(a + b*ArcSec[c*x])/(x^2*(d + e*x^2)),x]
```

output
```
(b*c*Sqrt[1 - 1/(c^2*x^2)])/d - a/(d*x) - (b*ArcCos[1/(c*x)])/(d*x) + (Sqr
t[e]*(a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(S
qrt[e] - Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcCos[1/(c*x
)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])
])/(2*(-d)^(3/2)) + (Sqrt[e]*(a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E
^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) - (Sqrt
[e]*(a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sq
rt[e] + Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) + ((I/2)*b*Sqrt[e]*PolyLog[2, -(
(c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/(-d)^(3/
2) - ((I/2)*b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[
e] - Sqrt[c^2*d + e])])/(-d)^(3/2) + ((I/2)*b*Sqrt[e]*PolyLog[2, -((c*Sqrt
[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e]))])/(-d)^(3/2) - ((
I/2)*b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sq
rt[c^2*d + e])])/(-d)^(3/2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 25.02 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.60

method	result
parts	$-\frac{a}{dx}-\frac{ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{d\sqrt{de}}+\frac{bc\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{d}-\frac{b\,\mathrm{arcsec}(cx)}{dx}-\frac{ibce\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\right.}{}$
derivativedivides	$c\left(-\frac{a}{dcx}-\frac{ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{cd\sqrt{de}}+\frac{b\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{d}-\frac{b\,\mathrm{arcsec}(cx)}{dcx}-\frac{ibe\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\right.}{}\right.$
default	$c\left(-\frac{a}{dcx}-\frac{ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{cd\sqrt{de}}+\frac{b\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{d}-\frac{b\,\mathrm{arcsec}(cx)}{dcx}-\frac{ibe\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\right.}{}\right.$

input
```
int((a+b*arcsec(c*x))/x^2/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-a/d/x-a*e/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b*c/d*((c^2*x^2-1)/c^2/x^
2)^(1/2)-b*arcsec(c*x)/d/x-1/2*I*b*c*e/d*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(
I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I
*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d
))+1/2*I*b*c*e/d*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-
1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/
_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
```

Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\,(d+ex^2)}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx)+a}{(ex^2+d)x^2}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)/(e*x^4 + d*x^2), x)
```

Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\,(d+ex^2)}\,dx = \int \frac{a+b\,\mathrm{asec}\,(cx)}{x^2\,(d+ex^2)}\,dx$$

input
```
integrate((a+b*asec(c*x))/x**2/(e*x**2+d),x)
```

output
```
Integral((a + b*asec(c*x))/(x**2*(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\,(d+ex^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\sec^{-1}(cx)}{x^2(d + ex^2)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\sec^{-1}(cx)}{x^2(d + ex^2)}\,dx = \int \frac{a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^2\,(e\,x^2 + d)}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{a + b\sec^{-1}(cx)}{x^2(d + ex^2)}\,dx = \frac{-\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ax + \left(\int \frac{asec(cx)}{e\,x^4 + d\,x^2}dx\right)b\,d^2x - ad}{d^2x}$$

input
```
int((a+b*asec(c*x))/x^2/(e*x^2+d),x)
```

output
```
( - sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*x + int(asec(c*x)/(d*x
**2 + e*x**4),x)*b*d**2*x - a*d)/(d**2*x)
```

3.96 $\int \frac{x^5 (a+b \sec^{-1}(cx))}{(d+ex^2)^2} \, dx$

Optimal result 818
Mathematica [B] (warning: unable to verify) 819
Rubi [A] (verified) 820
Maple [C] (warning: unable to verify) 823
Fricas [F] 824
Sympy [F(-1)] 824
Maxima [F] 825
Giac [F(-1)] 825
Mupad [F(-1)] 825
Reduce [F] 826

Optimal result

Integrand size = 21, antiderivative size = 617

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = -\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{2ce^2}+\frac{d(a+b\sec^{-1}(cx))}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2(a+b\sec^{-1}(cx))}{2e^2}+\frac{bd\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2e^{5/2}\sqrt{c^2d+e}} - \frac{d(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3} - \frac{d(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3} - \frac{d(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3} - \frac{d(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3} + \frac{d(2a+b\pi-b(\pi-2\sec^{-1}(cx)))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e^3} + \frac{ibd\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3} + \frac{ibd\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3} + \frac{ibd\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3} + \frac{ibd\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3} - \frac{ibd\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{e^3}$$

output
```
-1/2*b*(1-1/c^2/x^2)^(1/2)*x/c/e^2+1/2*d*(a+b*arcsec(c*x))/e^2/(e+d/x^2)+1
/2*x^2*(a+b*arcsec(c*x))/e^2+1/2*b*d*arctan((c^2*d+e)^(1/2)/c/e^(1/2)/(1-1
/c^2/x^2)^(1/2)/x)/e^(5/2)/(c^2*d+e)^(1/2)-d*(a+b*arcsec(c*x))*ln(1-c*(-d)
^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3-d*(a+b
*arcsec(c*x))*ln(1+c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^
2*d+e)^(1/2)))/e^3-d*(a+b*arcsec(c*x))*ln(1-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2
/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3-d*(a+b*arcsec(c*x))*ln(1+c*(-d
)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3+d*(2*
a+b*Pi-b*(Pi-2*arcsec(c*x)))*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)/e^3+I*b
*d*polylog(2,-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e
)^(1/2)))/e^3+I*b*d*polylog(2,c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(
e^(1/2)-(c^2*d+e)^(1/2)))/e^3+I*b*d*polylog(2,-c*(-d)^(1/2)*(1/c/x+I*(1-1/
c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3+I*b*d*polylog(2,c*(-d)^(1/2
)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3-I*b*d*polyl
og(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)/e^3
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1255 vs. 2(617) = 1234.

Time = 2.58 (sec) , antiderivative size = 1255, normalized size of antiderivative = 2.03

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^5*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/4*(-2*a*e*x^2 + (2*a*d^2)/(d + e*x^2) + 4*a*d*Log[d + e*x^2] + b*((2*e*
Sqrt[1 - 1/(c^2*x^2)]*x)/c - 2*e*x^2*ArcSec[c*x] + (d^(3/2)*ArcSec[c*x])/(
Sqrt[d] - I*Sqrt[e]*x) + (d^(3/2)*ArcSec[c*x])/(Sqrt[d] + I*Sqrt[e]*x) + 2
*d*ArcSin[1/(c*x)] + (16*I)*d*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqr
t[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d +
e]] + (16*I)*d*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((
I*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] + 4*d*ArcSec[c
*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])]
 + 8*d*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[
e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 4*d*ArcSec[c*x]*Lo
g[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 8*
d*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] +
 Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 4*d*ArcSec[c*x]*Log[1
- (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 8*d*Arc
Sin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[
c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 4*d*ArcSec[c*x]*Log[1 + (I*(
Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 8*d*ArcSin[Sq
rt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d
+ e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 8*d*ArcSec[c*x]*Log[1 + E^((2*I)*A
rcSec[c*x])] - (d*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt...
```

Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 676, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5763, 5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \left(a+b \sec^{-1}(cx)\right)}{\left(d+ex^2\right)^2} \, dx$$

$$\Big\downarrow \text{5763}$$

$$-\int \frac{x^3 \left(a+b \arccos\left(\frac{1}{cx}\right)\right)}{\left(\frac{d}{x^2}+e\right)^2} d\frac{1}{x}$$

$$\Big\downarrow \text{5233}$$

$$-\int \left(\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)x^3}{e^2}-\frac{2d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)x}{e^3}+\frac{2d^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e^3\left(\frac{d}{x^2}+e\right)x}+\frac{d^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e^2\left(\frac{d}{x^2}+e\right)^2x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{e^3}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{e^3}+\frac{2d\log\left(1+e^{2i\arccos\left(\frac{1}{cx}\right)}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e^3}+\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{2e^2\left(\frac{d}{x^2}+e\right)}+\frac{x^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{2e^2}+\frac{ibd\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{e^3}+\frac{ibd\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{e^3}+\frac{ibd\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{e^3}+\frac{ibd\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{e^3}-\frac{ibd\,\text{PolyLog}\left(2,-e^{2i\arccos\left(\frac{1}{cx}\right)}\right)}{e^3}+\frac{bd\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2e^{5/2}\sqrt{c^2d+e}}-\frac{bx\sqrt{1-\frac{1}{c^2x^2}}}{2ce^2}$$

input
```
Int[(x^5*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/2*(b*Sqrt[1 - 1/(c^2*x^2)]*x)/(c*e^2) + (d*(a + b*ArcCos[1/(c*x)]))/(2*
e^2*(e + d/x^2)) + (x^2*(a + b*ArcCos[1/(c*x)]))/(2*e^2) + (b*d*ArcTan[Sqr
t[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(2*e^(5/2)*Sqrt[c^2*d +
 e]) - (d*(a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]
))/(Sqrt[e] - Sqrt[c^2*d + e])])/e^3 - (d*(a + b*ArcCos[1/(c*x)])*Log[1 +
(c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/e^3 - (d*
(a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e
] + Sqrt[c^2*d + e])])/e^3 - (d*(a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d
]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 + (2*d*(a + b*A
rcCos[1/(c*x)])*Log[1 + E^((2*I)*ArcCos[1/(c*x)])])/e^3 + (I*b*d*PolyLog[2
, -((c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/e^3
+ (I*b*d*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2
*d + e])])/e^3 + (I*b*d*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(S
qrt[e] + Sqrt[c^2*d + e]))])/e^3 + (I*b*d*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcC
os[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 - (I*b*d*PolyLog[2, -E^((2
*I)*ArcCos[1/(c*x)])])/e^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.42 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.14

method	result
parts	$\frac{a\,x^2}{2e^2}-\frac{ad\ln\left(e\,x^2+d\right)}{e^3}-\frac{a\,d^2}{2e^3\left(e\,x^2+d\right)}+\frac{b\left(\frac{c^4\left(2c^4d\,\mathrm{arcsec}(cx)x^2+\mathrm{arcsec}(cx)e\,c^4x^4-\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3dx-\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,e\,c^3x^3 \cdots\right)}{2\left(e\,c^2x^2+c^2d\right)e^2}\cdots\right)}{\cdots}$
derivativedivides	$\frac{\frac{a\,c^6x^2}{2e^2}-\frac{a\,c^8d^2}{2e^3\left(e\,c^2x^2+c^2d\right)}-\frac{a\,c^6d\ln\left(e\,c^2x^2+c^2d\right)}{e^3}+b\,c^4\left(\frac{2c^4d\,\mathrm{arcsec}(cx)x^2+\mathrm{arcsec}(cx)e\,c^4x^4-\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3dx-\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,e\,c \cdots}{2\left(e\,c^2x^2+c^2d\right)e^2}\cdots\right)}{\cdots}$
default	$\frac{\frac{a\,c^6x^2}{2e^2}-\frac{a\,c^8d^2}{2e^3\left(e\,c^2x^2+c^2d\right)}-\frac{a\,c^6d\ln\left(e\,c^2x^2+c^2d\right)}{e^3}+b\,c^4\left(\frac{2c^4d\,\mathrm{arcsec}(cx)x^2+\mathrm{arcsec}(cx)e\,c^4x^4-\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3dx-\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,e\,c \cdots}{2\left(e\,c^2x^2+c^2d\right)e^2}\cdots\right)}{\cdots}$

input
```
int(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2/e^2-a*d/e^3*ln(e*x^2+d)-1/2*a*d^2/e^3/(e*x^2+d)+b/c^6*(1/2*c^4*(
2*c^4*d*arcsec(c*x)*x^2+arcsec(c*x)*e*c^4*x^4-((c^2*x^2-1)/c^2/x^2)^(1/2)*
c^3*d*x-((c^2*x^2-1)/c^2/x^2)^(1/2)*e*c^3*x^3-I*c^2*d-I*e*c^2*x^2)/(c^2*e*
x^2+c^2*d)/e^2+1/2*I/e^3*d*c^6*sum((_R1^2*c^2*d+c^2*d+4*e)/(_R1^2*c^2*d+c^
2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_
R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*
_Z^2+c^2*d))-2*I/e^3*d*c^6*dilog(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-1/2*I*
(e*(c^2*d+e))^(1/2)/(c^2*d+e)/e^3*arctanh(1/4*(2*c^2*d*(1/c/x+I*(1-1/c^2/x
^2)^(1/2))^2+2*c^2*d+4*e)/(c^2*d*e+e^2)^(1/2))*d*c^6+1/2*I/e^3*d^2*c^8*sum
((_R1^2+1)/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2
/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf
(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-2*I/e^3*d*c^6*dilog(1+I*(1/c/x+I*(1
-1/c^2/x^2)^(1/2)))+2/e^3*d*c^6*arcsec(c*x)*ln(1+I*(1/c/x+I*(1-1/c^2/x^2)^
(1/2)))+2/e^3*d*c^6*arcsec(c*x)*ln(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2))))
```

Fricas [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^5}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^5*arcsec(c*x) + a*x^5)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(a+b*asec(c*x))/(e*x**2+d)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^5}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/2*a*(d^2/(e^4*x^2 + d*e^3) - x^2/e^2 + 2*d*log(e*x^2 + d)/e^3) + b*inte
grate(x^5*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^4 + 2*d*e*x^2 + d^2),
 x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^5\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^2}\,dx$$

input
```
int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx$$
$$= \frac{2\left(\int \frac{asec(cx)x^5}{e^2x^4+2de\,x^2+d^2}dx\right)bd\,e^3 + 2\left(\int \frac{asec(cx)x^5}{e^2x^4+2de\,x^2+d^2}dx\right)b\,e^4x^2 - 2\log(e\,x^2+d)\,a\,d^2 - 2\log(e\,x^2+d)\,ade\,x^2}{2e^3\,(e\,x^2+d)}$$

input
```
int(x^5*(a+b*asec(c*x))/(e*x^2+d)^2,x)
```

output
```
(2*int((asec(c*x)*x**5)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*b*d*e**3 + 2*in
t((asec(c*x)*x**5)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*b*e**4*x**2 - 2*log(
d + e*x**2)*a*d**2 - 2*log(d + e*x**2)*a*d*e*x**2 + 2*a*d*e*x**2 + a*e**2*
x**4)/(2*e**3*(d + e*x**2))
```

3.97 $\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx$

Optimal result 828
Mathematica [B] (warning: unable to verify) 829
Rubi [A] (verified) 830
Maple [C] (warning: unable to verify) 833
Fricas [F] 834
Sympy [F(-1)] 834
Maxima [F] 835
Giac [F(-1)] 835
Mupad [F(-1)] 835
Reduce [F] 836

Optimal result

Integrand size = 21, antiderivative size = 582

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = -\frac{a+b\sec^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} - \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{3/2}\sqrt{c^2d+e}}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}$$
$$- \frac{(2a+b\pi-b(\pi-2\sec^{-1}(cx)))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{2e^2}$$
$$- \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}$$
$$- \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}$$
$$- \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}$$
$$- \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+ \frac{ib\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2e^2}$$

output
```
-1/2*(a+b*arcsec(c*x))/e/(e+d/x^2)-1/2*b*arctan((c^2*d+e)^(1/2)/c/e^(1/2)/
(1-1/c^2/x^2)^(1/2)/x)/e^(3/2)/(c^2*d+e)^(1/2)+1/2*(a+b*arcsec(c*x))*ln(1-
c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2+
1/2*(a+b*arcsec(c*x))*ln(1+c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(
1/2)-(c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arcsec(c*x))*ln(1-c*(-d)^(1/2)*(1/c/x+
I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arcsec(c*x)
)*ln(1+c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)
))/e^2-1/2*(2*a+b*Pi-b*(Pi-2*arcsec(c*x)))*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/
2))^2)/e^2-1/2*I*b*polylog(2,-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(
e^(1/2)-(c^2*d+e)^(1/2)))/e^2-1/2*I*b*polylog(2,c*(-d)^(1/2)*(1/c/x+I*(1-1
/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2-1/2*I*b*polylog(2,-c*(-d)^
(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/e^2-1/2*I*b
*polylog(2,c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(
1/2)))/e^2+1/2*I*b*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)/e^2
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1213 vs. 2(582) = 1164.

Time = 0.97 (sec) , antiderivative size = 1213, normalized size of antiderivative = 2.08

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^3*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

output
```
((2*a*d)/(d + e*x^2) + (b*Sqrt[d]*ArcSec[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (
b*Sqrt[d]*ArcSec[c*x])/(Sqrt[d] + I*Sqrt[e]*x) + 2*b*ArcSin[1/(c*x)] + (8*
I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqr
t[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] + (8*I)*b*ArcSin[Sqrt
[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Tan
[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] + 2*b*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] -
Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 4*b*ArcSin[Sqrt[1 + (I*
Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I
*ArcSec[c*x]))/(c*Sqrt[d])] + 2*b*ArcSec[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[
c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 4*b*ArcSin[Sqrt[1 - (I*Sqrt[
e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*Arc
Sec[c*x]))/(c*Sqrt[d])] + 2*b*ArcSec[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d
 + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 4*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(
c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*
x]))/(c*Sqrt[d])] + 2*b*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])
*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt
[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(
c*Sqrt[d])] - 4*b*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] - (b*Sqrt[e]*
Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt
[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))])/S...
```

Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 634, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5763, 5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx$$

$$\Big\downarrow \text{ 5763}$$

$$-\int \frac{x\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{\left(\frac{d}{x^2}+e\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 5233}$$

$$-\int \left(\frac{x\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e^2}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e^2\left(\frac{d}{x^2}+e\right)x}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e\left(\frac{d}{x^2}+e\right)^2x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}+\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}+\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^2}+\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^2}-\frac{a+b\arccos\left(\frac{1}{cx}\right)}{2e\left(\frac{d}{x^2}+e\right)}-\frac{\log\left(1+e^{2i\arccos\left(\frac{1}{cx}\right)}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e^2}-\frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^2}-\frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^2}-\frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^2}-\frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^2}+\frac{ib\,\text{PolyLog}\left(2,-e^{2i\arccos\left(\frac{1}{cx}\right)}\right)}{2e^2}-\frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2e^{3/2}\sqrt{c^2d+e}}$$

input
```
Int[(x^3*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/2*(a + b*ArcCos[1/(c*x)])/(e*(e + d/x^2)) - (b*ArcTan[Sqrt[c^2*d + e]/(
c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(2*e^(3/2)*Sqrt[c^2*d + e]) + ((a + b
*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sq
rt[c^2*d + e])])/(2*e^2) + ((a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^
(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^2) + ((a + b*ArcCo
s[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2
*d + e])])/(2*e^2) + ((a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*Arc
Cos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^2) - ((a + b*ArcCos[1/(c
*x)])*Log[1 + E^((2*I)*ArcCos[1/(c*x)])])/e^2 - ((I/2)*b*PolyLog[2, -((c*S
qrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/e^2 - ((I/2)
*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e
])])/e^2 - ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[
e] + Sqrt[c^2*d + e]))])/e^2 - ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos
[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^2 + ((I/2)*b*PolyLog[2, -E^((2
*I)*ArcCos[1/(c*x)])])/e^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.09 (sec) , antiderivative size = 577, normalized size of antiderivative = 0.99

method	result
parts	$\frac{a\ln\left(e\,x^{2}+d\right)}{2e^{2}}+\frac{ad}{2e^{2}\left(e\,x^{2}+d\right)}-\frac{b\,c^{2}x^{2}\operatorname{arcsec}\left(cx\right)}{2\left(e\,c^{2}x^{2}+c^{2}d\right)e}-\frac{ib\left(\sum_{\_R1=\operatorname{RootOf}\left(c^{2}d\,\_Z^{4}+\left(2c^{2}d+4e\right)\_Z^{2}+c^{2}d\right)}\frac{\left(\_R1^{2}c^{2}d\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{a\,c^{6}d}{2e^{2}\left(e\,c^{2}x^{2}+c^{2}d\right)}+\frac{a\,c^{4}\ln\left(e\,c^{2}x^{2}+c^{2}d\right)}{2e^{2}}+b\,c^{4}\left(-\frac{c^{2}x^{2}\operatorname{arcsec}\left(cx\right)}{2\left(e\,c^{2}x^{2}+c^{2}d\right)e}+\frac{i\operatorname{dilog}\left(1-i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^{2}x^{2}}}\right)\right)}{e^{2}}+\frac{i\sqrt{e\left(c^{2}d+e\right)}\,\operatorname{arctanh}\left(\right.}{}\right.}{}$
default	$\frac{\frac{a\,c^{6}d}{2e^{2}\left(e\,c^{2}x^{2}+c^{2}d\right)}+\frac{a\,c^{4}\ln\left(e\,c^{2}x^{2}+c^{2}d\right)}{2e^{2}}+b\,c^{4}\left(-\frac{c^{2}x^{2}\operatorname{arcsec}\left(cx\right)}{2\left(e\,c^{2}x^{2}+c^{2}d\right)e}+\frac{i\operatorname{dilog}\left(1-i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^{2}x^{2}}}\right)\right)}{e^{2}}+\frac{i\sqrt{e\left(c^{2}d+e\right)}\,\operatorname{arctanh}\left(\right.}{}\right.}{}$

input
```
int(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a/e^2*ln(e*x^2+d)+1/2*a*d/e^2/(e*x^2+d)-1/2*b*c^2*x^2*arcsec(c*x)/(c^2
*e*x^2+c^2*d)/e-1/4*I*b/e^2*sum((_R1^2*c^2*d+c^2*d+4*e)/(_R1^2*c^2*d+c^2*d
+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-
1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^
2+c^2*d))+I*b/e^2*dilog(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))+1/2*I*b*(e*(c^2
*d+e))^(1/2)/(c^2*d+e)/e^2*arctanh(1/4*(2*c^2*d*(1/c/x+I*(1-1/c^2/x^2)^(1/
2))^2+2*c^2*d+4*e)/(c^2*d*e+e^2)^(1/2))-1/4*I*b*c^2/e^2*d*sum((_R1^2+1)/(_
R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/
_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(
2*c^2*d+4*e)*_Z^2+c^2*d))+I*b/e^2*dilog(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))
-b/e^2*arcsec(c*x)*ln(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-b/e^2*arcsec(c*x)
*ln(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))
```

Fricas [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^3}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^3*arcsec(c*x) + a*x^3)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*asec(c*x))/(e*x**2+d)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^3}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a*(d/(e^3*x^2 + d*e^2) + log(e*x^2 + d)/e^2) + b*integrate(x^3*arctan(
sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^3\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^2}\,dx$$

input
```
int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx$$
$$= \frac{2\left(\int \frac{asec(cx)x^3}{e^2x^4+2de\,x^2+d^2}dx\right)bd\,e^2+2\left(\int \frac{asec(cx)x^3}{e^2x^4+2de\,x^2+d^2}dx\right)b\,e^3x^2+\log(e\,x^2+d)\,ad+\log(e\,x^2+d)\,ae\,x^2-ae\,x}{2e^2\,(e\,x^2+d)}$$

input
```
int(x^3*(a+b*asec(c*x))/(e*x^2+d)^2,x)
```

output
```
(2*int((asec(c*x)*x**3)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*b*d*e**2 + 2*in
t((asec(c*x)*x**3)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*b*e**3*x**2 + log(d
+ e*x**2)*a*d + log(d + e*x**2)*a*e*x**2 - a*e*x**2)/(2*e**2*(d + e*x**2))
```

3.98 $\int \frac{x\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal result 837
Mathematica [C] (verified) 837
Rubi [A] (verified) 838
Maple [B] (verified) 840
Fricas [A] (verification not implemented) 841
Sympy [F] 842
Maxima [F] 842
Giac [F(-2)] 842
Mupad [F(-1)] 843
Reduce [F] 843

Optimal result

Integrand size = 19, antiderivative size = 131

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = -\frac{a+b\sec^{-1}(cx)}{2e\,(d+ex^2)} + \frac{bcx\arctan\left(\sqrt{-1+c^2x^2}\right)}{2de\sqrt{c^2x^2}} - \frac{bcx\arctan\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{\sqrt{c^2d+e}}\right)}{2d\sqrt{e}\sqrt{c^2d+e}\sqrt{c^2x^2}}$$

output
```
-1/2*(a+b*arcsec(c*x))/e/(e*x^2+d)+1/2*b*c*x*arctan((c^2*x^2-1)^(1/2))/d/e
/(c^2*x^2)^(1/2)-1/2*b*c*x*arctan(e^(1/2)*(c^2*x^2-1)^(1/2)/(c^2*d+e)^(1/2
))/d/e^(1/2)/(c^2*d+e)^(1/2)/(c^2*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.18

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx$$

$$= \frac{-\frac{2a}{d+ex^2} - \frac{2b\sec^{-1}(cx)}{d+ex^2} - \frac{2b\arcsin\left(\frac{1}{cx}\right)}{d} + \frac{b\sqrt{e}\log\left(\frac{4ide+4cd\sqrt{e}\left(c\sqrt{d}-i\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x}{b\sqrt{-c^2d-e}\left(\sqrt{d}+i\sqrt{e}x\right)}\right)}{d\sqrt{-c^2d-e}} + \frac{b\sqrt{e}\log\left(\frac{-4ide+4cd\sqrt{e}\left(c\sqrt{d}+i\sqrt{-c^2d\cdots}\right.}{b\sqrt{-c^2d-e}\left(\sqrt{d}-i\cdots\right.}\right.}{d\sqrt{-c^2d-e}}}{4e}$$

input
```
Integrate[(x*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

output
```
((-2*a)/(d + e*x^2) - (2*b*ArcSec[c*x])/(d + e*x^2) - (2*b*ArcSin[1/(c*x)]
)/d + (b*Sqrt[e]*Log[((4*I)*d*e + 4*c*d*Sqrt[e]*(c*Sqrt[d] - I*Sqrt[-(c^2*
d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x)/(b*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[
e]*x))])/(d*Sqrt[-(c^2*d) - e]) + (b*Sqrt[e]*Log[((-4*I)*d*e + 4*c*d*Sqrt[
e]*(c*Sqrt[d] + I*Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x)/(b*Sqrt[-(c
^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))])/(d*Sqrt[-(c^2*d) - e]))/(4*e)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5759, 354, 97, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{5759}$$

$$\frac{bcx\int \frac{1}{x\sqrt{c^2x^2-1}(ex^2+d)}dx}{2e\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{2e\left(d+ex^2\right)}$$

$$\Big\downarrow \text{354}$$

$$\frac{bcx\int \frac{1}{x^2\sqrt{c^2x^2-1}(ex^2+d)}dx^2}{4e\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{2e\left(d+ex^2\right)}$$

$$\Big\downarrow \text{97}$$

$$\frac{bcx\left(\frac{\int \frac{1}{x^2\sqrt{c^2x^2-1}}dx^2}{d}-\frac{e\int \frac{1}{\sqrt{c^2x^2-1}\left(ex^2+d\right)}dx^2}{d}\right)}{4e\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{2e\left(d+ex^2\right)}$$

$$\Big\downarrow \text{73}$$

$$\frac{bcx\left(\frac{2\int \frac{1}{\frac{x^4}{c^2}+\frac{1}{c^2}}d\sqrt{c^2x^2-1}}{c^2d}-\frac{2e\int \frac{1}{\frac{ex^4}{c^2}+d+\frac{e}{c^2}}d\sqrt{c^2x^2-1}}{c^2d}\right)}{4e\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{2e\left(d+ex^2\right)}$$

$$\Big\downarrow \; 218$$

$$\frac{bcx\left(\frac{2\arctan\left(\sqrt{c^2x^2-1}\right)}{d}-\frac{2\sqrt{e}\arctan\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{d\sqrt{c^2d+e}}\right)}{4e\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{2e\left(d+ex^2\right)}$$

input
```
Int[(x*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/2*(a + b*ArcSec[c*x])/(e*(d + e*x^2)) + (b*c*x*((2*ArcTan[Sqrt[-1 + c^2
*x^2]])/d - (2*Sqrt[e]*ArcTan[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/Sqrt[c^2*d + e]
])/(d*Sqrt[c^2*d + e])))/(4*e*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5759
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSec[c*x])/(2*e*(p + 1))), x
] - Simp[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2]))   Int[(d + e*x^2)^(p + 1)/(x*S
qrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(109) = 218$.

Time = 4.26 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.01

method	result
parts	$-\frac{a}{2e(e\,x^2+d)}+\frac{b\left(-\frac{c^4 \operatorname{arcsec}(cx)}{2e\left(e\,c^2x^2+c^2d\right)}-\frac{c\sqrt{c^2x^2-1}\left(2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}-\ln\left(-\frac{2\left(\sqrt{c^2x^2-1}\sqrt{-\frac{c^2d+e}{e}}\,e+\sqrt{-}\right.}{-cex+\sqrt{-c^2de}}\right.\right.}{4e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,xd\sqrt{-}}\right.}{c^2}$
derivativedivides	$\frac{-\frac{a\,c^4}{2e\left(e\,c^2x^2+c^2d\right)}+b\,c^4\left(-\frac{\operatorname{arcsec}(cx)}{2e\left(e\,c^2x^2+c^2d\right)}-\frac{\sqrt{c^2x^2-1}\left(2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}-\ln\left(-\frac{2\left(\sqrt{c^2x^2-1}\sqrt{-\frac{c^2d+e}{e}}\,e+\sqrt{}\right.}{-cex+\sqrt{-c^2de}}\right.\right.}{4e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3xd}\right.}{c^2}$
default	$\frac{-\frac{a\,c^4}{2e\left(e\,c^2x^2+c^2d\right)}+b\,c^4\left(-\frac{\operatorname{arcsec}(cx)}{2e\left(e\,c^2x^2+c^2d\right)}-\frac{\sqrt{c^2x^2-1}\left(2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}-\ln\left(-\frac{2\left(\sqrt{c^2x^2-1}\sqrt{-\frac{c^2d+e}{e}}\,e+\sqrt{}\right.}{-cex+\sqrt{-c^2de}}\right.\right.}{4e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3xd}\right.}{c^2}$

input
```
int(x*(a+b*arcsec(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/e/(e*x^2+d)+b/c^2*(-1/2*c^4/e/(c^2*e*x^2+c^2*d)*arcsec(c*x)-1/4*c/e
*(c^2*x^2-1)^(1/2)*(2*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)-ln(
-2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c*e
*x+(-c^2*d*e)^(1/2)))-ln(-2*(-(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c
^2*d*e)^(1/2)*c*x+e)/(c*e*x+(-c^2*d*e)^(1/2))))/((c^2*x^2-1)/c^2/x^2)^(1/2
)/x/d/(-(c^2*d+e)/e)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.98

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx$$

$$= \Bigg[-\frac{2\,ac^2d^2+2\,ade+\sqrt{-c^2de-e^2}(bex^2+bd)\log\left(\frac{c^2ex^2-c^2d+2\,\sqrt{-c^2de-e^2}\sqrt{c^2x^2-1}-2\,e}{ex^2+d}\right)+2\,(bc^2d^2+bde)\,\mathrm{arc}}{4\,(c^2d^3e+d^2e^2+(c^2d^2e^2+de^3)}$$

$$-\frac{ac^2d^2+ade-\sqrt{c^2de+e^2}(bex^2+bd)\arctan\left(\frac{\sqrt{c^2de+e^2}\sqrt{c^2x^2-1}}{c^2ex^2-e}\right)+(bc^2d^2+bde)\,\mathrm{arcsec}\,(cx)-2\,(bc^2d^2}{2\,(c^2d^3e+d^2e^2+(c^2d^2e^2+de^3)x^2)}$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*a*c^2*d^2 + 2*a*d*e + sqrt(-c^2*d*e - e^2)*(b*e*x^2 + b*d)*log((c
^2*e*x^2 - c^2*d + 2*sqrt(-c^2*d*e - e^2)*sqrt(c^2*x^2 - 1) - 2*e)/(e*x^2
+ d)) + 2*(b*c^2*d^2 + b*d*e)*arcsec(c*x) - 4*(b*c^2*d^2 + b*d*e + (b*c^2*
d*e + b*e^2)*x^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^3*e + d^2*e^2 +
 (c^2*d^2*e^2 + d*e^3)*x^2), -1/2*(a*c^2*d^2 + a*d*e - sqrt(c^2*d*e + e^2)
*(b*e*x^2 + b*d)*arctan(sqrt(c^2*d*e + e^2)*sqrt(c^2*x^2 - 1)/(c^2*e*x^2 -
 e)) + (b*c^2*d^2 + b*d*e)*arcsec(c*x) - 2*(b*c^2*d^2 + b*d*e + (b*c^2*d*e
 + b*e^2)*x^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^3*e + d^2*e^2 + (c
^2*d^2*e^2 + d*e^3)*x^2)]
```

Sympy [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x(a+b\operatorname{asec}(cx))}{(d+ex^2)^2}\,dx$$

input
```
integrate(x*(a+b*asec(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral(x*(a + b*asec(c*x))/(d + e*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x}{(ex^2+d)^2}\,dx$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*(2*(c^2*e^2*x^2 + c^2*d*e)*integrate(1/2*x*e^(1/2*log(c*x + 1) + 1/2*l
og(c*x - 1))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e + (c^2*e^2*x^4 + (c^
2*d*e - e^2)*x^2 - d*e)*e^(log(c*x + 1) + log(c*x - 1))), x) - arctan(sqrt
(c*x + 1)*sqrt(c*x - 1)))*b/(e^2*x^2 + d*e) - 1/2*a/(e^2*x^2 + d*e)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^2}\,dx$$

input
```
int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx$$
$$= \frac{2\left(\int \frac{asec(cx)x}{e^2x^4+2de\,x^2+d^2}dx\right)b\,d^2+2\left(\int \frac{asec(cx)x}{e^2x^4+2de\,x^2+d^2}dx\right)bde\,x^2+a\,x^2}{2d\,(e\,x^2+d)}$$

input
```
int(x*(a+b*asec(c*x))/(e*x^2+d)^2,x)
```

output
```
(2*int((asec(c*x)*x)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*b*d**2 + 2*int((as
ec(c*x)*x)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*b*d*e*x**2 + a*x**2)/(2*d*(d
 + e*x**2))
```

3.99 $\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx$

Optimal result 845
Mathematica [B] (warning: unable to verify) 846
Rubi [A] (verified) 847
Maple [C] (warning: unable to verify) 850
Fricas [F] 851
Sympy [F(-1)] 851
Maxima [F] 851
Giac [F(-2)] 852
Mupad [F(-1)] 852
Reduce [F] 852

Optimal result

Integrand size = 21, antiderivative size = 546

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = -\frac{e(a+b\sec^{-1}(cx))}{2d^2\left(e+\frac{d}{x^2}\right)}+\frac{i(a+b\sec^{-1}(cx))^2}{2bd^2} - \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2d^2\sqrt{c^2d+e}} - \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2} - \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2} - \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2} - \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2} + \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2} + \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2} + \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2} + \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}$$

output
```
-1/2*e*(a+b*arcsec(c*x))/d^2/(e+d/x^2)+1/2*I*(a+b*arcsec(c*x))^2/b/d^2-1/2
*b*e^(1/2)*arctan((c^2*d+e)^(1/2)/c/e^(1/2)/(1-1/c^2/x^2)^(1/2)/x)/d^2/(c^
2*d+e)^(1/2)-1/2*(a+b*arcsec(c*x))*ln(1-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2
)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2-1/2*(a+b*arcsec(c*x))*ln(1+c*(-d)^
(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2-1/2*(a+
b*arcsec(c*x))*ln(1-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c
^2*d+e)^(1/2)))/d^2-1/2*(a+b*arcsec(c*x))*ln(1+c*(-d)^(1/2)*(1/c/x+I*(1-1/
c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/d^2+1/2*I*b*polylog(2,-c*(-d)^(
1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2+1/2*I*b*
polylog(2,c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1
/2)))/d^2+1/2*I*b*polylog(2,-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e
^(1/2)+(c^2*d+e)^(1/2)))/d^2+1/2*I*b*polylog(2,c*(-d)^(1/2)*(1/c/x+I*(1-1/
c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/d^2
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1190 vs. 2(546) = 1092.

Time = 0.73 (sec) , antiderivative size = 1190, normalized size of antiderivative = 2.18

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^2),x]
```

output
```
((2*a*d)/(d + e*x^2) + (b*Sqrt[d]*ArcSec[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (
b*Sqrt[d]*ArcSec[c*x])/(Sqrt[d] + I*Sqrt[e]*x) + (2*I)*b*ArcSec[c*x]^2 + 2
*b*ArcSin[1/(c*x)] - (8*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt
[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e
]] - (8*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*
c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] - 2*b*ArcSec[c*x
]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] -
 4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e]
 - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 2*b*ArcSec[c*x]*Log[
1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 4*b*
ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + S
qrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 2*b*ArcSec[c*x]*Log[1 -
(I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 4*b*ArcSi
n[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^
2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 2*b*ArcSec[c*x]*Log[1 + (I*(Sq
rt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 4*b*ArcSin[Sqrt
[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d +
e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 4*a*Log[x] - (b*Sqrt[e]*Log[(2*Sqrt[
d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*
x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))])/Sqrt[-(c^2*d)...
```

Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5763, 5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{ 5763}$$

$$-\int \frac{a+b\arccos\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^2x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{ 5233}$$

$$-\int \left(\frac{a+b\arccos\left(\frac{1}{cx}\right)}{d\left(\frac{d}{x^2}+e\right)x}-\frac{e\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{d\left(\frac{d}{x^2}+e\right)^2x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}-\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}-\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^2}-\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^2}-\frac{e\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{2d^2\left(\frac{d}{x^2}+e\right)}+\frac{i\left(a+b\arccos\left(\frac{1}{cx}\right)\right)^2}{2bd^2}+\frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^2}+\frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^2}+\frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^2}+\frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^2}-\frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2d^2\sqrt{c^2d+e}}$$

input
```
Int[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^2),x]
```

output
```
-1/2*(e*(a + b*ArcCos[1/(c*x)]))/(d^2*(e + d/x^2)) + ((I/2)*(a + b*ArcCos[
1/(c*x)])^2)/(b*d^2) - (b*Sqrt[e]*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1
 - 1/(c^2*x^2)]*x)])/(2*d^2*Sqrt[c^2*d + e]) - ((a + b*ArcCos[1/(c*x)])*Lo
g[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*
d^2) - ((a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))
/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*ArcCos[1/(c*x)])*Log[1 -
(c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^2) -
 ((a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt
[e] + Sqrt[c^2*d + e])])/(2*d^2) + ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*
ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/d^2 + ((I/2)*b*PolyLog[2,
 (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/d^2 + ((
I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2
*d + e]))])/d^2 + ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(
Sqrt[e] + Sqrt[c^2*d + e])])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.22 (sec) , antiderivative size = 2108, normalized size of antiderivative = 3.86

method	result	size
parts	Expression too large to display	2108
derivativedivides	Expression too large to display	2157
default	Expression too large to display	2157

input
```
int((a+b*arcsec(c*x))/x/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
a/d^2*ln(x)-1/2*a/d^2*ln(e*x^2+d)+1/2*a/d/(e*x^2+d)+b*(-1/8*I*(-(e*(c^2*d+
e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*polylog(2,d*c^2*(
1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))/d^2/e/(
c^2*d+e)+1/4*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e
+2*e^2)/d^2/e/(c^2*d+e)*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d
-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsec(c*x)+1/4*(e*(c^2*d+e))^(1/2)/d/e/(c^2*
d+e)*c^2*arcsec(c*x)*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d+2*
(e*(c^2*d+e))^(1/2)-2*e))-1/4*I*(e*(c^2*d+e))^(1/2)/d/e/(c^2*d+e)*arcsec(c
*x)^2*c^2-1/8*I*(e*(c^2*d+e))^(1/2)/d/e/(c^2*d+e)*polylog(2,d*c^2*(1/c/x+I
*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))*c^2-I*(-(e*(c^
2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e*arcsec(c*x)
^2/d^4/(c^2*d+e)/c^4+(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))
^(1/2)*e+2*e^2)/d^4*e/(c^2*d+e)/c^4*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2
))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsec(c*x)-1/2*I*(-(e*(c^2*d+e))
^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e*polylog(2,d*c^2*(1
/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))/d^4/(c^2
*d+e)/c^4+I*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*arcsec(c*x)^2*e/d^4/c^4+(-(e
*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)/d^3/(c^2*
d+e)/c^2*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e)
)^(1/2)-2*e))*arcsec(c*x)-I*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*...
```

Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx)+a}{\left(ex^2+d\right)^2 x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*asec(c*x))/x/(e*x**2+d)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx)+a}{\left(ex^2+d\right)^2 x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a*(1/(d*e*x^2 + d^2) - log(e*x^2 + d)/d^2 + 2*log(x)/d^2) + b*integrat
e(arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)^2}\,dx = \int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x\,(e\,x^2+d)^2}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^2),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^2), x)
```

Reduce [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)^2}\,dx = \frac{2\left(\int \frac{asec(cx)}{e^2x^5+2de\,x^3+d^2x}dx\right)b\,d^3+2\left(\int \frac{asec(cx)}{e^2x^5+2de\,x^3+d^2x}dx\right)b\,d^2e\,x^2-\log(e\,x^2+d)\,ad-\log(e\,x^2+d)\,ae\,x^2+2}{2d^2\,(e\,x^2+d)}$$

input
```
int((a+b*asec(c*x))/x/(e*x^2+d)^2,x)
```

output

```
(2*int(asec(c*x)/(d**2*x + 2*d*e*x**3 + e**2*x**5),x)*b*d**3 + 2*int(asec(
c*x)/(d**2*x + 2*d*e*x**3 + e**2*x**5),x)*b*d**2*e*x**2 - log(d + e*x**2)*
a*d - log(d + e*x**2)*a*e*x**2 + 2*log(x)*a*d + 2*log(x)*a*e*x**2 - a*e*x*
*2)/(2*d**2*(d + e*x**2))
```

3.100 $\int \frac{x^4 (a+b \sec^{-1}(cx))}{(d+ex^2)^2} \, dx$

Optimal result 855
Mathematica [A] (warning: unable to verify) 856
Rubi [A] (verified) 857
Maple [C] (warning: unable to verify) 860
Fricas [F] 861
Sympy [F] 861
Maxima [F(-2)] 861
Giac [F(-1)] 862
Mupad [F(-1)] 862
Reduce [F] 862

Optimal result

Integrand size = 21, antiderivative size = 784

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = -\frac{d(a+b\sec^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{d(a+b\sec^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{x(a+b\sec^{-1}(cx))}{e^2}-\frac{b\,\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce^2}-\frac{b\sqrt{d}\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}}-\frac{b\sqrt{d}\,\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}}+\frac{3\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}-\frac{3\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}+\frac{3\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}-\frac{3\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}+\frac{3ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}-\frac{3ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}+\frac{3ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}-\frac{3ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$

output
```
-1/4*d*(a+b*arcsec(c*x))/e^2/((-d)^(1/2)*e^(1/2)-d/x)+1/4*d*(a+b*arcsec(c*
x))/e^2/((-d)^(1/2)*e^(1/2)+d/x)+x*(a+b*arcsec(c*x))/e^2-b*arctanh((1-1/c^
2/x^2)^(1/2))/c/e^2-1/4*b*d^(1/2)*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d
^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/e^2/(c^2*d+e)^(1/2)-1/4*b*d^(1
/2)*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^
2/x^2)^(1/2))/e^2/(c^2*d+e)^(1/2)+3/4*(-d)^(1/2)*(a+b*arcsec(c*x))*ln(1-c*
(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2
)-3/4*(-d)^(1/2)*(a+b*arcsec(c*x))*ln(1+c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2
)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)+3/4*(-d)^(1/2)*(a+b*arcsec(c*x
))*ln(1-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2
)))/e^(5/2)-3/4*(-d)^(1/2)*(a+b*arcsec(c*x))*ln(1+c*(-d)^(1/2)*(1/c/x+I*(1
-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)+3/4*I*b*(-d)^(1/2)*p
olylog(2,-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1
/2)))/e^(5/2)-3/4*I*b*(-d)^(1/2)*polylog(2,c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/
x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)+3/4*I*b*(-d)^(1/2)*polylog(
2,-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/e
^(5/2)-3/4*I*b*(-d)^(1/2)*polylog(2,c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1
/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.43 (sec) , antiderivative size = 1331, normalized size of antiderivative = 1.70

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

output
```
(4*a*Sqrt[e]*x + (2*a*d*Sqrt[e]*x)/(d + e*x^2) - 6*a*Sqrt[d]*ArcTan[(Sqrt[
e]*x)/Sqrt[d]] + b*(4*Sqrt[e]*x*ArcSec[c*x] + (d*ArcSec[c*x])/((-I)*Sqrt[d
] + Sqrt[e]*x) + (d*ArcSec[c*x])/(I*Sqrt[d] + Sqrt[e]*x) + 12*Sqrt[d]*ArcS
in[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sq
rt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] - 12*Sqrt[d]*ArcSin[Sqrt[1 + (
I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Tan[ArcSe
c[c*x]/2])/Sqrt[c^2*d + e]] + (3*I)*Sqrt[d]*ArcSec[c*x]*Log[1 + (I*(Sqrt[e
] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (6*I)*Sqrt[d]*ArcSi
n[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^
2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (3*I)*Sqrt[d]*ArcSec[c*x]*Log[
1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (6*I
)*Sqrt[d]*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-S
qrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (3*I)*Sqrt[d]*
ArcSec[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*S
qrt[d])] + (6*I)*Sqrt[d]*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]
*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] +
(3*I)*Sqrt[d]*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcS
ec[c*x]))/(c*Sqrt[d])] - (6*I)*Sqrt[d]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt
[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(
c*Sqrt[d])] + (I*Sqrt[d]*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I...
```

Rubi [A] (verified)

Time = 2.88 (sec) , antiderivative size = 844, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5763, 5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(a+b \sec^{-1}(cx)\right)}{\left(d+ex^2\right)^2} \, dx$$

$$\Big\downarrow \text{ 5763}$$

$$-\int \frac{x^2 \left(a+b \arccos\left(\frac{1}{cx}\right)\right)}{\left(\frac{d}{x^2}+e\right)^2} d\frac{1}{x}$$

$$\Big\downarrow \text{ 5233}$$

$$-\int \left(\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)x^2}{e^2}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e^2\left(\frac{d}{x^2}+e\right)}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e\left(\frac{d}{x^2}+e\right)^2}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e^2}+\frac{3\sqrt{-d}\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4e^{5/2}}-\frac{3\sqrt{-d}\log\left(\frac{\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4e^{5/2}}+\frac{3\sqrt{-d}\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4e^{5/2}}-\frac{3\sqrt{-d}\log\left(\frac{\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4e^{5/2}}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4e^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}-\frac{b\,\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce^2}-\frac{b\sqrt{d}\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{dc^2+e}}-\frac{b\sqrt{d}\,\text{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{dc^2+e}}+\frac{3ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4e^{5/2}}-\frac{3ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4e^{5/2}}+\frac{3ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4e^{5/2}}-\frac{3ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4e^{5/2}}$$

input
```
Int[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/4*(d*(a + b*ArcCos[1/(c*x)]))/(e^2*(Sqrt[-d]*Sqrt[e] - d/x)) + (d*(a +
b*ArcCos[1/(c*x)]))/(4*e^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (x*(a + b*ArcCos[1/
(c*x)]))/e^2 - (b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(c*e^2) - (b*Sqrt[d]*Arc
Tanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/
(c^2*x^2)])])/(4*e^2*Sqrt[c^2*d + e]) - (b*Sqrt[d]*ArcTanh[(c^2*d + (Sqrt[
-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*e^2
*Sqrt[c^2*d + e]) + (3*Sqrt[-d]*(a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d
]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*e^(5/2)) - (3*Sq
rt[-d]*(a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/
(Sqrt[e] - Sqrt[c^2*d + e])])/(4*e^(5/2)) + (3*Sqrt[-d]*(a + b*ArcCos[1/(c
*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e
])])/(4*e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]
*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*e^(5/2)) + (((3*I
)/4)*b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] -
 Sqrt[c^2*d + e]))])/e^(5/2) - (((3*I)/4)*b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d
]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/e^(5/2) + (((3*I)/4
)*b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sq
rt[c^2*d + e]))])/e^(5/2) - (((3*I)/4)*b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E
^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^(5/2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 67.16 (sec) , antiderivative size = 941, normalized size of antiderivative = 1.20

method	result	size
parts	Expression too large to display	941
derivativedivides	Expression too large to display	966
default	Expression too large to display	966

input
```
int(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
a*(1/e^2*x-1/e^2*d*(-1/2*x/(e*x^2+d)+3/2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2
))))+b/c^5*(1/2*x*c^5*arcsec(c*x)*(2*c^2*e*x^2+3*c^2*d)/(c^2*e*x^2+c^2*d)/
e^2-1/2*I*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-(e*(c^2*d+e))^(1/2
)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*c*arctan(c*d*(1/c/x+I*(1-
1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e^2/(c^2*d+
e)/d^2+2*I/e^2*c^4*arctan(1/c/x+I*(1-1/c^2/x^2)^(1/2))-1/2*I*(-(c^2*d-2*(e
*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*
(c^2*d+e))^(1/2)*e+2*e^2)*c*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((-c
^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e^2/(c^2*d+e)/d^2+1/2*I*(-(c^2*d
-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*c*a
rctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*
e)*d)^(1/2))/d^2/e^2+1/2*I*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^
2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*c*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/
((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^2/e^2+3/16*I/e^3*c^6*d*sum(
(_R1^2*c^2*d+c^2*d+4*e)/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1
-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))
/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-3/16*I/e^3*c^6*d*s
um((_R1^2*c^2*d+4*_R1^2*e+c^2*d)/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x
)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^
2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d)))
```

Fricas [F]

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx=\int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^4}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^4*arcsec(c*x) + a*x^4)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx=\int \frac{x^4(a+b\,\mathrm{asec}\,(cx))}{(d+ex^2)^2}\,dx$$

input
```
integrate(x**4*(a+b*asec(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral(x**4*(a + b*asec(c*x))/(d + e*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^4\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^2}\,dx$$

input
```
int((x^4*(a + b*acos(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int((x^4*(a + b*acos(1/(c*x))))/(d + e*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx$$
$$= \frac{-3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ad-3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ae\,x^2+2\left(\int \frac{asec(cx)x^4}{e^2x^4+2de\,x^2+d^2}dx\right)bd\,e^3+2\left(\int \frac{asec(cx)x^4}{e^2x^4+2de\,x^2+\cdots}\right.}{2e^3\left(e\,x^2+d\right)}$$

input
```
int(x^4*(a+b*asec(c*x))/(e*x^2+d)^2,x)
```

output
```
( - 3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d - 3*sqrt(e)*sqrt(d
)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*e*x**2 + 2*int((asec(c*x)*x**4)/(d**2 +
2*d*e*x**2 + e**2*x**4),x)*b*d*e**3 + 2*int((asec(c*x)*x**4)/(d**2 + 2*d*e
*x**2 + e**2*x**4),x)*b*e**4*x**2 + 3*a*d*e*x + 2*a*e**2*x**3)/(2*e**3*(d
+ e*x**2))
```

3.101 $\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx$

Optimal result 865
Mathematica [A] (warning: unable to verify) 866
Rubi [A] (verified) 867
Maple [C] (warning: unable to verify) 870
Fricas [F] 871
Sympy [F] 872
Maxima [F(-2)] 872
Giac [F(-2)] 872
Mupad [F(-1)] 873
Reduce [F] 873

Optimal result

Integrand size = 21, antiderivative size = 745

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \frac{a+b\sec^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\sec^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\,\mathrm{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}} + \frac{b\,\mathrm{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}} + \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$

output
```
1/4*(a+b*arcsec(c*x))/e/((-d)^(1/2)*e^(1/2)-d/x)-1/4*(a+b*arcsec(c*x))/e/(
(-d)^(1/2)*e^(1/2)+d/x)+1/4*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/
2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(1/2)/e/(c^2*d+e)^(1/2)+1/4*b*ar
ctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)
^(1/2))/d^(1/2)/e/(c^2*d+e)^(1/2)+1/4*(a+b*arcsec(c*x))*ln(1-c*(-d)^(1/2)*
(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(3/2
)-1/4*(a+b*arcsec(c*x))*ln(1+c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e
^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(3/2)+1/4*(a+b*arcsec(c*x))*ln(1-c*(
-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1
/2)/e^(3/2)-1/4*(a+b*arcsec(c*x))*ln(1+c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)
^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(3/2)+1/4*I*b*polylog(2,-c
*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^
(1/2)/e^(3/2)-1/4*I*b*polylog(2,c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))
/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(3/2)+1/4*I*b*polylog(2,-c*(-d)^(
1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e
^(3/2)-1/4*I*b*polylog(2,c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/
2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.12 (sec) , antiderivative size = 1245, normalized size of antiderivative = 1.67

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

output
```
((-2*a*Sqrt[e]*x)/(d + e*x^2) + (2*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d]
+ b*(ArcSec[c*x]/(I*Sqrt[d] - Sqrt[e]*x) - ArcSec[c*x]/(I*Sqrt[d] + Sqrt[e
]*x) - (4*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*
c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]])/Sqrt[d] + (4*Ar
cSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqr
t[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]])/Sqrt[d] - (I*ArcSec[c*x]*Log[1
 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])])/Sqrt[d]
 - ((2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sq
rt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])])/Sqrt[d] + (I*Arc
Sec[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqr
t[d])])/Sqrt[d] + ((2*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]
*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])])/
Sqrt[d] + (I*ArcSec[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSe
c[c*x]))/(c*Sqrt[d])])/Sqrt[d] - ((2*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqr
t[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/
(c*Sqrt[d])])/Sqrt[d] - (I*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d +
e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])])/Sqrt[d] + ((2*I)*ArcSin[Sqrt[1 + (I*S
qrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*
ArcSec[c*x]))/(c*Sqrt[d])])/Sqrt[d] - (I*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(S
qrt[e] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))...
```

Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 801, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5763, 5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{5763}$$

$$-\int \frac{a+b\arccos\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{5173}$$

$$-\int \left(-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{2e\left(-\frac{d^2}{x^2}-ed\right)}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4e\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4\sqrt{-d}e^{3/2}}-\frac{\log\left(\frac{\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4\sqrt{-d}e^{3/2}}+\frac{\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4\sqrt{-d}e^{3/2}}-\frac{\log\left(\frac{\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4\sqrt{-d}e^{3/2}}+\frac{a+b\arccos\left(\frac{1}{cx}\right)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\arccos\left(\frac{1}{cx}\right)}{4e\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{b\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{dc^2+e}}+\frac{b\,\text{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{dc^2+e}}+\frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}}$$

input
```
Int[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

output

```
(a + b*ArcCos[1/(c*x)])/(4*e*(Sqrt[-d]*Sqrt[e] - d/x)) - (a + b*ArcCos[1/(
c*x)])/(4*e*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt
[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*Sqrt[d]*e*S
qrt[c^2*d + e]) + (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqr
t[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*Sqrt[d]*e*Sqrt[c^2*d + e]) + ((a
+ b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] -
 Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcCos[1/(c*x)])*Log[1
+ (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*Sqrt
[-d]*e^(3/2)) + ((a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1
/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*Arc
Cos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c
^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) + ((I/4)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I
*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/(Sqrt[-d]*e^(3/2)) - ((I
/4)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d
+ e])])/(Sqrt[-d]*e^(3/2)) + ((I/4)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcCos
[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e]))])/(Sqrt[-d]*e^(3/2)) - ((I/4)*b*P
olyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])
/(Sqrt[-d]*e^(3/2))
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

rule 5763

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 25.60 (sec) , antiderivative size = 852, normalized size of antiderivative = 1.14

method	result
parts	$-\frac{ax}{2e(e\,x^2+d)}+\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}}+\frac{b\left(-\frac{c^5\,\mathrm{arcsec}(cx)x}{2e\left(e\,c^2x^2+c^2d\right)}-\frac{i\sqrt{-\left(c^2d-2\sqrt{e\left(c^2d+e\right)}+2e\right)d}\,\left(c^2d+2\sqrt{e\left(c^2d+e\right)}+2e\right)\mathrm{arcta}}{2c\,d^3e}\right.}{}$
derivativedivides	$\frac{-\frac{a\,c^5x}{2e\left(e\,c^2x^2+c^2d\right)}+\frac{a\,c^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}}+b\,c^4\left(-\frac{\mathrm{arcsec}(cx)cx}{2e\left(e\,c^2x^2+c^2d\right)}-\frac{i\sqrt{-\left(c^2d-2\sqrt{e\left(c^2d+e\right)}+2e\right)d}\,\left(c^2d+2\sqrt{e\left(c^2d+e\right)}+2e\right)\mathrm{ar}}{2e\,c^5d^3}\right.}{}$
default	$\frac{-\frac{a\,c^5x}{2e\left(e\,c^2x^2+c^2d\right)}+\frac{a\,c^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}}+b\,c^4\left(-\frac{\mathrm{arcsec}(cx)cx}{2e\left(e\,c^2x^2+c^2d\right)}-\frac{i\sqrt{-\left(c^2d-2\sqrt{e\left(c^2d+e\right)}+2e\right)d}\,\left(c^2d+2\sqrt{e\left(c^2d+e\right)}+2e\right)\mathrm{ar}}{2e\,c^5d^3}\right.}{}$

input
```
int(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/e*x/(e*x^2+d)+1/2*a/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b/c^3*(-1
/2*c^5*arcsec(c*x)/e*x/(c^2*e*x^2+c^2*d)-1/2*I*(-(c^2*d-2*(e*(c^2*d+e))^(1
/2)+2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*arctanh(c*d*(1/c/x+I*(
1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/c/d^3/e+
1/2*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2*d+e))^(1/2)*c^
2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*arctanh(c*d*(1/c/x+I*(1-1/c^2
/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e/(c^2*d+e)/d^3
/c-1/2*I*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e)
)^(1/2)+2*e)*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+
e))^(1/2)+2*e)*d)^(1/2))/c/d^3/e+1/2*I*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*
d)^(1/2)*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e
^2)*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)
+2*e)*d)^(1/2))/e/(c^2*d+e)/d^3/c+1/4*I/e*c^4*sum(_R1/(_R1^2*c^2*d+c^2*d+2
*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/
c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+
c^2*d))-1/4*I/e*c^4*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_
R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2
))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d)))
```

Fricas [F]

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx=\int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^2}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^2*arcsec(c*x) + a*x^2)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^2(a + b\sec^{-1}(cx))}{(d + ex^2)^2}\, dx = \int \frac{x^2(a + b\operatorname{asec}(cx))}{(d + ex^2)^2}\, dx$$

input
```
integrate(x**2*(a+b*asec(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral(x**2*(a + b*asec(c*x))/(d + e*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b\sec^{-1}(cx))}{(d + ex^2)^2}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b\sec^{-1}(cx))}{(d + ex^2)^2}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^2\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^2}\,dx$$

input
```
int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \frac{\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ad+\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ae\,x^2+2\left(\int \frac{asec(cx)x^2}{e^2x^4+2de\,x^2+d^2}dx\right)b\,d^2e^2+2\left(\int \frac{asec(cx)x^2}{e^2x^4+2de\,x^2+d^2}d\right.}{2d\,e^2\left(e\,x^2+d\right)}$$

input
```
int(x^2*(a+b*asec(c*x))/(e*x^2+d)^2,x)
```

output
```
(sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d + sqrt(e)*sqrt(d)*atan(
(e*x)/(sqrt(e)*sqrt(d)))*a*e*x**2 + 2*int((asec(c*x)*x**2)/(d**2 + 2*d*e*x
**2 + e**2*x**4),x)*b*d**2*e**2 + 2*int((asec(c*x)*x**2)/(d**2 + 2*d*e*x**
2 + e**2*x**4),x)*b*d*e**3*x**2 - a*d*e*x)/(2*d*e**2*(d + e*x**2))
```

3.102 $\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^2}\,dx$

Optimal result 875
Mathematica [A] (warning: unable to verify) 876
Rubi [A] (verified) 877
Maple [C] (warning: unable to verify) 880
Fricas [F] 881
Sympy [F] 882
Maxima [F(-2)] 882
Giac [F(-2)] 882
Mupad [F(-1)] 883
Reduce [F] 883

Optimal result

Integrand size = 18, antiderivative size = 739

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^2}\,dx = -\frac{a+b\sec^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\sec^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}} - \frac{b\,\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}} - \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} + \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} + \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} + \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} + \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

output
```
-1/4*(a+b*arcsec(c*x))/d/((-d)^(1/2)*e^(1/2)-d/x)+1/4*(a+b*arcsec(c*x))/d/
((-d)^(1/2)*e^(1/2)+d/x)-1/4*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1
/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(3/2)/(c^2*d+e)^(1/2)-1/4*b*arc
tanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^
(1/2))/d^(3/2)/(c^2*d+e)^(1/2)-1/4*(a+b*arcsec(c*x))*ln(1-c*(-d)^(1/2)*(1/
c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1
/4*(a+b*arcsec(c*x))*ln(1+c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1
/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*(a+b*arcsec(c*x))*ln(1-c*(-d)
^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)
/e^(1/2)+1/4*(a+b*arcsec(c*x))*ln(1+c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1
/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*I*b*polylog(2,-c*(-
d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/
2)/e^(1/2)+1/4*I*b*polylog(2,c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e
^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*I*b*polylog(2,-c*(-d)^(1/2
)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1
/2)+1/4*I*b*polylog(2,c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+
(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.40 (sec) , antiderivative size = 1239, normalized size of antiderivative = 1.68

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(d + e*x^2)^2,x]
```

output
```
((a*x)/(d^2 + d*e*x^2) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqrt[e])
 + (b*((Sqrt[d]*ArcSec[c*x])/((-I)*Sqrt[d]*Sqrt[e] + e*x) + (Sqrt[d]*ArcSe
c[c*x])/(I*Sqrt[d]*Sqrt[e] + e*x) - (4*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt
[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt
[c^2*d + e]])/Sqrt[e] + (4*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2
]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]])/S
qrt[e] - (I*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec
[c*x]))/(c*Sqrt[d])])/Sqrt[e] - ((2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt
[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(
c*Sqrt[d])])/Sqrt[e] + (I*ArcSec[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d +
e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])])/Sqrt[e] + ((2*I)*ArcSin[Sqrt[1 - (I*S
qrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I
*ArcSec[c*x]))/(c*Sqrt[d])])/Sqrt[e] + (I*ArcSec[c*x]*Log[1 - (I*(Sqrt[e]
+ Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])])/Sqrt[e] - ((2*I)*ArcSi
n[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^
2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])])/Sqrt[e] - (I*ArcSec[c*x]*Log[1
+ (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])])/Sqrt[e]
+ ((2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqr
t[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])])/Sqrt[e] + (I*Log[
(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[...
```

Rubi [A] (verified)

Time = 2.67 (sec) , antiderivative size = 795, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5753, 5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^2}\,dx$$

$$\Big\downarrow \text{5753}$$

$$-\int \frac{a+b\arccos\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^2x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{5233}$$

$$-\int \left(\frac{a+b\arccos\left(\frac{1}{cx}\right)}{d\left(\frac{d}{x^2}+e\right)}-\frac{e\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{d\left(\frac{d}{x^2}+e\right)^2}\right)d\frac{1}{x}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{\log\left(\frac{\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{\log\left(\frac{\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{a+b\arccos\left(\frac{1}{cx}\right)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\arccos\left(\frac{1}{cx}\right)}{4d\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}-\frac{b\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{dc^2+e}}-\frac{b\,\text{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{dc^2+e}}-\frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

input
```
Int[(a + b*ArcSec[c*x])/(d + e*x^2)^2,x]
```

output
```
-1/4*(a + b*ArcCos[1/(c*x)])/(d*(Sqrt[-d]*Sqrt[e] - d/x)) + (a + b*ArcCos[
1/(c*x)])/(4*d*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*S
qrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*d^(3/2)*
Sqrt[c^2*d + e]) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sq
rt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*d^(3/2)*Sqrt[c^2*d + e]) - ((a +
 b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] -
Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcCos[1/(c*x)])*Log[1
 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d
)^(3/2)*Sqrt[e]) - ((a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCo
s[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) + ((a +
b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + S
qrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) - ((I/4)*b*PolyLog[2, -((c*Sqrt[-
d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/((-d)^(3/2)*Sqrt[
e]) + ((I/4)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sq
rt[c^2*d + e])])/((-d)^(3/2)*Sqrt[e]) - ((I/4)*b*PolyLog[2, -((c*Sqrt[-d]*
E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e]))])/((-d)^(3/2)*Sqrt[e])
 + ((I/4)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[
c^2*d + e])])/((-d)^(3/2)*Sqrt[e])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5753
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(2*(p + 1)))
, x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 40.43 (sec) , antiderivative size = 840, normalized size of antiderivative = 1.14

method	result
parts	$\frac{ax}{2d(e x^2+d)}+\frac{a \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+\frac{b\left(\frac{c^3 \operatorname{arcsec}(cx)x}{2d\left(e c^2x^2+c^2d\right)}+\frac{ic^2\left(\sum_{\_R1=\operatorname{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{\_R1\left(i \operatorname{arc}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$
derivativedivides	$\frac{\frac{a c^3 x}{2d\left(e c^2x^2+c^2d\right)}+\frac{ac \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+b c^4\left(\frac{\operatorname{arcsec}(cx)x}{2cd\left(e c^2x^2+c^2d\right)}+\frac{i\left(\sum_{\_R1=\operatorname{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{\_R1\left(i \operatorname{ar}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$
default	$\frac{\frac{a c^3 x}{2d\left(e c^2x^2+c^2d\right)}+\frac{ac \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+b c^4\left(\frac{\operatorname{arcsec}(cx)x}{2cd\left(e c^2x^2+c^2d\right)}+\frac{i\left(\sum_{\_R1=\operatorname{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{\_R1\left(i \operatorname{ar}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$

input
```
int((a+b*arcsec(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x/d/(e*x^2+d)+1/2*a/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b/c*(1/2*c
^3*arcsec(c*x)*x/d/(c^2*e*x^2+c^2*d)+1/4*I/d*c^2*sum(_R1/(_R1^2*c^2*d+c^2*
d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1
-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z
^2+c^2*d))+1/2*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d+2*(e*
(c^2*d+e))^(1/2)+2*e)*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2*d+2
*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4/c^3-1/2*I*(-(c^2*d-2*(e*(c^2*d+e))
^(1/2)+2*e)*d)^(1/2)*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^
(1/2)*e+2*e^2)*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^
2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4/(c^2*d+e)/c^3+1/2*I*((c^2*d+2*(e*(c^2*d+e
))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*arctan(c*d*(1/c/x
+I*(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^4/c
^3-1/2*I*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-(e*(c^2*d+e))^(1/2)
*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*arctan(c*d*(1/c/x+I*(1-1/c
^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^4/(c^2*d+e)/
c^3-1/4*I/d*c^2*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1
/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_
R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d)))
```

Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^2}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{(ex^2+d)^2}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{a + b\sec^{-1}(cx)}{(d + ex^2)^2}\,dx = \int \frac{a + b\,\mathrm{asec}\,(cx)}{(d + ex^2)^2}\,dx$$

input
```
integrate((a+b*asec(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral((a + b*asec(c*x))/(d + e*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\sec^{-1}(cx)}{(d + ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\sec^{-1}(cx)}{(d + ex^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^2}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{(e\,x^2+d)^2}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(d + e*x^2)^2,x)
```

output
```
int((a + b*acos(1/(c*x)))/(d + e*x^2)^2, x)
```

Reduce [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^2}\,dx$$
$$=\frac{\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ad+\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ae\,x^2+2\left(\int\frac{asec(cx)}{e^2x^4+2de\,x^2+d^2}dx\right)b\,d^3e+2\left(\int\frac{asec(cx)}{e^2x^4+2de\,x^2+d^2}dx\right.}{2d^2e\,(e\,x^2+d)}$$

input
```
int((a+b*asec(c*x))/(e*x^2+d)^2,x)
```

output
```
(sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d + sqrt(e)*sqrt(d)*atan(
(e*x)/(sqrt(e)*sqrt(d)))*a*e*x**2 + 2*int(asec(c*x)/(d**2 + 2*d*e*x**2 + e
**2*x**4),x)*b*d**3*e + 2*int(asec(c*x)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)
*b*d**2*e**2*x**2 + a*d*e*x)/(2*d**2*e*(d + e*x**2))
```

3.103 $\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx$

Optimal result 885
Mathematica [A] (warning: unable to verify) 886
Rubi [A] (verified) 887
Maple [C] (warning: unable to verify) 890
Fricas [F] 891
Sympy [F(-1)] 891
Maxima [F(-2)] 891
Giac [F(-2)] 892
Mupad [F(-1)] 892
Reduce [F] 892

Optimal result

Integrand size = 21, antiderivative size = 785

$$\int \frac{a+b\sec^{-1}(cx)}{x^2(d+ex^2)^2}\,dx = \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\sec^{-1}(cx)}{d^2x} + \frac{e(a+b\sec^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e(a+b\sec^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\,e\,\mathrm{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}} + \frac{b\,e\,\mathrm{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}} - \frac{3\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} + \frac{3\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} - \frac{3\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} + \frac{3\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} - \frac{3ib\sqrt{e}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} + \frac{3ib\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} - \frac{3ib\sqrt{e}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} + \frac{3ib\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$

output
```
b*c*(1-1/c^2/x^2)^(1/2)/d^2-a/d^2/x-b*arcsec(c*x)/d^2/x+1/4*e*(a+b*arcsec(
c*x))/d^2/((-d)^(1/2)*e^(1/2)-d/x)-1/4*e*(a+b*arcsec(c*x))/d^2/((-d)^(1/2)
*e^(1/2)+d/x)+1/4*b*e*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*
d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d+e)^(1/2)+1/4*b*e*arctanh((c
^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/
d^(5/2)/(c^2*d+e)^(1/2)-3/4*e^(1/2)*(a+b*arcsec(c*x))*ln(1-c*(-d)^(1/2)*(1
/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)+3/4*e^(1
/2)*(a+b*arcsec(c*x))*ln(1+c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(
1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)-3/4*e^(1/2)*(a+b*arcsec(c*x))*ln(1-c*(-d
)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2
)+3/4*e^(1/2)*(a+b*arcsec(c*x))*ln(1+c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(
1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)-3/4*I*b*e^(1/2)*polylog(2,-c*(
-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5
/2)+3/4*I*b*e^(1/2)*polylog(2,c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(
e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)-3/4*I*b*e^(1/2)*polylog(2,-c*(-d)^(1/
2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)+3/4
*I*b*e^(1/2)*polylog(2,c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)
+(c^2*d+e)^(1/2)))/(-d)^(5/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.34 (sec) , antiderivative size = 1291, normalized size of antiderivative = 1.64

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x^2*(d + e*x^2)^2),x]
```

output
```
((-4*a*Sqrt[d])/x - (2*a*Sqrt[d]*e*x)/(d + e*x^2) - 6*a*Sqrt[e]*ArcTan[(Sq
rt[e]*x)/Sqrt[d]] + b*(4*c*Sqrt[d]*Sqrt[1 - 1/(c^2*x^2)] - (4*Sqrt[d]*ArcS
ec[c*x])/x - (Sqrt[d]*e*ArcSec[c*x])/((-I)*Sqrt[d]*Sqrt[e] + e*x) - (Sqrt[
d]*e*ArcSec[c*x])/(I*Sqrt[d]*Sqrt[e] + e*x) + 12*Sqrt[e]*ArcSin[Sqrt[1 - (
I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Tan[Ar
cSec[c*x]/2])/Sqrt[c^2*d + e]] - 12*Sqrt[e]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c
*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sq
rt[c^2*d + e]] + (3*I)*Sqrt[e]*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*
d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (6*I)*Sqrt[e]*ArcSin[Sqrt[1 + (I
*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(
I*ArcSec[c*x]))/(c*Sqrt[d])] - (3*I)*Sqrt[e]*ArcSec[c*x]*Log[1 + (I*(-Sqrt
[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (6*I)*Sqrt[e]*Arc
Sin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt
[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (3*I)*Sqrt[e]*ArcSec[c*x]*L
og[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (6
*I)*Sqrt[e]*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(
Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (3*I)*Sqrt[e]
*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*
Sqrt[d])] - (6*I)*Sqrt[e]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]
]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])...
```

Rubi [A] (verified)

Time = 2.79 (sec) , antiderivative size = 845, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5763, 5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2(d+ex^2)^2}\,dx$$

$$\Big\downarrow \text{5763}$$

$$-\int \frac{a+b\arccos\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^2x^4}d\frac{1}{x}$$

$$\Big\downarrow \text{5233}$$

$$-\int \left(\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)e^2}{d^2\left(\frac{d}{x^2}+e\right)^2}-\frac{2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)e}{d^2\left(\frac{d}{x^2}+e\right)}+\frac{a+b\arccos\left(\frac{1}{cx}\right)}{d^2}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a}{d^2x}-\frac{b\arccos\left(\frac{1}{cx}\right)}{d^2x}+\frac{e\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{e\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4d^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{b\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{dc^2+e}}+\frac{b\,\text{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{dc^2+e}}-\frac{3\sqrt{e}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}+\frac{3\sqrt{e}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{4(-d)^{5/2}}-\frac{3\sqrt{e}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}+\frac{3\sqrt{e}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{4(-d)^{5/2}}-\frac{3ib\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}+\frac{3ib\sqrt{e}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}-\frac{3ib\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}+\frac{3ib\sqrt{e}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2}$$

input
```
Int[(a + b*ArcSec[c*x])/(x^2*(d + e*x^2)^2),x]
```

output
```
(b*c*Sqrt[1 - 1/(c^2*x^2)])/d^2 - a/(d^2*x) - (b*ArcCos[1/(c*x)])/(d^2*x)
+ (e*(a + b*ArcCos[1/(c*x)]))/(4*d^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (e*(a + b
*ArcCos[1/(c*x)]))/(4*d^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*e*ArcTanh[(c^2*d
- (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])]
)/(4*d^(5/2)*Sqrt[c^2*d + e]) + (b*e*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x
)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*d^(5/2)*Sqrt[c^2*
d + e]) - (3*Sqrt[e]*(a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcC
os[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) + (3*Sqrt[e]*(a
 + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e]
- Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) - (3*Sqrt[e]*(a + b*ArcCos[1/(c*x)])*L
og[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4
*(-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I
*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) - (((3*I)/
4)*b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sq
rt[c^2*d + e]))])/(-d)^(5/2) + (((3*I)/4)*b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]
*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(-d)^(5/2) - (((3*I)
/4)*b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + S
qrt[c^2*d + e]))])/(-d)^(5/2) + (((3*I)/4)*b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d
]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(-d)^(5/2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 67.93 (sec) , antiderivative size = 933, normalized size of antiderivative = 1.19

method	result	size
parts	Expression too large to display	933
derivativedivides	Expression too large to display	960
default	Expression too large to display	960

input
```
int((a+b*arcsec(c*x))/x^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/d^2/x-e/d^2*(1/2*x/(e*x^2+d)+3/2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
))+b*c*(-1/2*(arcsec(c*x)+I)/d^2*(I*((c^2*x^2-1)/c^2/x^2)^(1/2)*c*x+1)/x/c
+1/2*(I*((c^2*x^2-1)/c^2/x^2)^(1/2)*c*x-1)*(arcsec(c*x)-I)/d^2/x/c-1/2*arc
sec(c*x)/d^2*e*x*c/(c^2*e*x^2+c^2*d)-1/2*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+
2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*arctanh(c*d*(1/c/x+I*(1-1/
c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))*e/d^5/c^5+1/
2*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2*d+e))^(1/2)*c^2*
d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e*arctanh(c*d*(1/c/x+I*(1-1/c^2
/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^5/c^5/(c^2*d+
e)-1/2*I*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e)
)^(1/2)+2*e)*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+
e))^(1/2)+2*e)*d)^(1/2))*e/d^5/c^5+1/2*I*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e
)*d)^(1/2)*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2
*e^2)*e*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(
1/2)+2*e)*d)^(1/2))/d^5/c^5/(c^2*d+e)+3/4*I*e/d^2*sum(1/_R1/(_R1^2*c^2*d+c
^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((
_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)
*_Z^2+c^2*d))-3/4*I*e/d^2*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*l
n((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^
(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d)))
```

Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx = \int \frac{b\,\mathrm{arcsec}\left(cx\right)+a}{\left(ex^2+d\right)^2x^2}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)/(e^2*x^6 + 2*d*e*x^4 + d^2*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*asec(c*x))/x**2/(e*x**2+d)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2(d+ex^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2(d+ex^2)^2}\,dx = \int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^2\,(e\,x^2+d)^2}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^2),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^2), x)
```

Reduce [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^2(d+ex^2)^2}\,dx$$
$$= \frac{-3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)adx - 3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ae\,x^3 + 2\left(\int \frac{asec(cx)}{e^2x^6+2de\,x^4+d^2x^2}dx\right)b\,d^4x + 2\left(\int \frac{asec(}{e^2x^6+2de}\right.}{2d^3x\,(e\,x^2+d)}$$

input
```
int((a+b*asec(c*x))/x^2/(e*x^2+d)^2,x)
```

output
```
( - 3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d*x - 3*sqrt(e)*sqrt
(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*e*x**3 + 2*int(asec(c*x)/(d**2*x**2 +
2*d*e*x**4 + e**2*x**6),x)*b*d**4*x + 2*int(asec(c*x)/(d**2*x**2 + 2*d*e*x
**4 + e**2*x**6),x)*b*d**3*e*x**3 - 2*a*d**2 - 3*a*d*e*x**2)/(2*d**3*x*(d
+ e*x**2))
```

3.104 $\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx$

Optimal result . . . 895
Mathematica [B] (warning: unable to verify) . . . 896
Rubi [A] (verified) . . . 897
Maple [C] (warning: unable to verify) . . . 900
Fricas [F] . . . 901
Sympy [F(-1)] . . . 901
Maxima [F] . . . 901
Giac [F(-1)] . . . 902
Mupad [F(-1)] . . . 902
Reduce [F] . . . 902

Optimal result

Integrand size = 21, antiderivative size = 719

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2(c^2d+e)\left(e+\frac{d}{x^2}\right)x}-\frac{a+b\sec^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2}-\frac{a+b\sec^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)}-\frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2e^{5/2}\sqrt{c^2d+e}}-\frac{b(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{8e^{5/2}(c^2d+e)^{3/2}}+\frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}+\frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}+\frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3}+\frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3}-\frac{(2a+b\pi-b(\pi-2\sec^{-1}(cx)))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{2e^3}-\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}-\frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}-\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3}-\frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3}+\frac{ib\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2e^3}$$

output
```
-1/8*b*c*d*(1-1/c^2/x^2)^(1/2)/e^2/(c^2*d+e)/(e+d/x^2)/x-1/4*(a+b*arcsec(c
*x))/e/(e+d/x^2)^2-1/2*(a+b*arcsec(c*x))/e^2/(e+d/x^2)-1/2*b*arctan((c^2*d
+e)^(1/2)/c/e^(1/2)/(1-1/c^2/x^2)^(1/2)/x)/e^(5/2)/(c^2*d+e)^(1/2)-1/8*b*(
c^2*d+2*e)*arctan((c^2*d+e)^(1/2)/c/e^(1/2)/(1-1/c^2/x^2)^(1/2)/x)/e^(5/2)
/(c^2*d+e)^(3/2)+1/2*(a+b*arcsec(c*x))*ln(1-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2
/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3+1/2*(a+b*arcsec(c*x))*ln(1+c*(
-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3+1/2
*(a+b*arcsec(c*x))*ln(1-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2
)+(c^2*d+e)^(1/2)))/e^3+1/2*(a+b*arcsec(c*x))*ln(1+c*(-d)^(1/2)*(1/c/x+I*(
1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3-1/2*(2*a+b*Pi-b*(Pi-2*a
rcsec(c*x)))*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)/e^3-1/2*I*b*polylog(2,-
c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3-
1/2*I*b*polylog(2,c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2
*d+e)^(1/2)))/e^3-1/2*I*b*polylog(2,-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(
1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3-1/2*I*b*polylog(2,c*(-d)^(1/2)*(1/c/x
+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3+1/2*I*b*polylog(2,-
(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)/e^3
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1805 vs. 2(719) = 1438.

Time = 7.33 (sec) , antiderivative size = 1805, normalized size of antiderivative = 2.51

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^5*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/4*(a*d^2)/(e^3*(d + e*x^2)^2) + (a*d)/(e^3*(d + e*x^2)) + (a*Log[d + e*
x^2])/(2*e^3) + b*((((-7*I)/16)*Sqrt[d]*(-(ArcSec[c*x]/(I*Sqrt[d]*Sqrt[e]
+ e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] +
c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2
*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/e^(5/2)
+ (((7*I)/16)*Sqrt[d]*(-(ArcSec[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x)) - (I*(A
rcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*(I*c*Sqrt[d]
 + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt
[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/e^(5/2) - (d*(-(ArcSec
[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2)) + (ArcSin[1/(c*x)]/Sqrt[e] -
 I*((c*Sqrt[d]*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/((c^2*d + e)*((-I)*Sqrt[d]
 + Sqrt[e]*x)) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sqrt[
e] + c*(c*Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d +
 e)*((-I)*Sqrt[d] + Sqrt[e]*x))]/(c^2*d + e)^(3/2)))/d))/(16*e^(5/2)) - (
d*((I*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] +
 Sqrt[e]*x)) - ArcSec[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) + ArcSin[1/
(c*x)]/(d*Sqrt[e]) - (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*((-
I)*Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2
*c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(16*e^(5/2)
) + ((I/4)*(8*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[...
```

Rubi [A] (verified)

Time = 1.93 (sec) , antiderivative size = 775, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5763, 5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \left(a+b \sec^{-1}(cx)\right)}{\left(d+ex^2\right)^3} \, dx$$

$$\Big\downarrow \; 5763$$

$$-\int \frac{x \left(a+b \arccos\left(\frac{1}{cx}\right)\right)}{\left(\frac{d}{x^2}+e\right)^3} d\frac{1}{x}$$

$$\Big\downarrow \; 5233$$

$$-\int \left(\frac{x\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e^3}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e^3\left(\frac{d}{x^2}+e\right)x}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e^2\left(\frac{d}{x^2}+e\right)^2x}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e\left(\frac{d}{x^2}+e\right)^3x}\right)d\frac{1}{x}$$

$\downarrow$ 2009

$$\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}+\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}+\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^3}+\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^3}-\frac{a+b\arccos\left(\frac{1}{cx}\right)}{2e^2\left(\frac{d}{x^2}+e\right)}-\frac{a+b\arccos\left(\frac{1}{cx}\right)}{4e\left(\frac{d}{x^2}+e\right)^2}-\frac{\log\left(1+e^{2i\arccos\left(\frac{1}{cx}\right)}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e^3}-\frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^3}-\frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^3}-\frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^3}-\frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^3}+\frac{ib\,\text{PolyLog}\left(2,-e^{2i\arccos\left(\frac{1}{cx}\right)}\right)}{2e^3}-\frac{b(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{8e^{5/2}\left(c^2d+e\right)^{3/2}}-\frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2e^{5/2}\sqrt{c^2d+e}}-\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2x\left(c^2d+e\right)\left(\frac{d}{x^2}+e\right)}$$

input
```
Int[(x^5*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/8*(b*c*d*Sqrt[1 - 1/(c^2*x^2)])/(e^2*(c^2*d + e)*(e + d/x^2)*x) - (a +
b*ArcCos[1/(c*x)])/(4*e*(e + d/x^2)^2) - (a + b*ArcCos[1/(c*x)])/(2*e^2*(e
 + d/x^2)) - (b*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)
])/(2*e^(5/2)*Sqrt[c^2*d + e]) - (b*(c^2*d + 2*e)*ArcTan[Sqrt[c^2*d + e]/(
c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(8*e^(5/2)*(c^2*d + e)^(3/2)) + ((a +
 b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] -
Sqrt[c^2*d + e])])/(2*e^3) + ((a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*
E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^3) + ((a + b*Arc
Cos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c
^2*d + e])])/(2*e^3) + ((a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*A
rcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^3) - ((a + b*ArcCos[1/
(c*x)])*Log[1 + E^((2*I)*ArcCos[1/(c*x)])])/e^3 - ((I/2)*b*PolyLog[2, -((c
*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/e^3 - ((I/
2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d +
 e])])/e^3 - ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqr
t[e] + Sqrt[c^2*d + e]))])/e^3 - ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcC
os[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 + ((I/2)*b*PolyLog[2, -E^(
(2*I)*ArcCos[1/(c*x)])])/e^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.96 (sec) , antiderivative size = 1396, normalized size of antiderivative = 1.94

method	result	size
parts	Expression too large to display	1396
derivativedivides	Expression too large to display	1409
default	Expression too large to display	1409

input
```
int(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a*(d/e^3/(e*x^2+d)+1/2/e^3*ln(e*x^2+d)-1/4*d^2/e^3/(e*x^2+d)^2)+b/c^6*(-1/
8*c^6*(4*c^6*d^2*arcsec(c*x)*x^2+6*c^6*d*e*arcsec(c*x)*x^4+((c^2*x^2-1)/c^
2/x^2)^(1/2)*c^5*d^2*x+((c^2*x^2-1)/c^2/x^2)^(1/2)*c^5*d*e*x^3+4*c^4*d*e*a
rcsec(c*x)*x^2+6*arcsec(c*x)*e^2*c^4*x^4+I*c^4*d^2+2*I*c^4*d*e*x^2+I*e^2*c
^4*x^4)/e^2/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2-1/(c^2*d+e)/e^2*c^6*arcsec(c*x)*
ln(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-1/(c^2*d+e)/e^2*c^6*arcsec(c*x)*ln(1
-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-1/4*I/(c^2*d+e)/e^3*c^10*d^2*sum((_R1^2+
1)/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1
/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_
Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+5/8*I*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/e^3*a
rctanh(1/4*(2*c^2*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2+2*c^2*d+4*e)/(c^2*d*e+
e^2)^(1/2))*c^8*d+I/(c^2*d+e)/e^3*c^8*d*dilog(1+I*(1/c/x+I*(1-1/c^2/x^2)^(
1/2)))+I/(c^2*d+e)/e^3*c^8*d*dilog(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-1/(c
^2*d+e)/e^3*c^8*d*arcsec(c*x)*ln(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-1/(c^2
*d+e)/e^3*c^8*d*arcsec(c*x)*ln(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-1/4*I/(c
^2*d+e)/e^3*c^8*d*sum((_R1^2*c^2*d+c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+2*e)*(I*a
rcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1
-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+
I/(c^2*d+e)/e^2*c^6*dilog(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-1/4*I/(c^2*d+
e)/e^2*c^8*d*sum((_R1^2+1)/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_...
```

Fricas [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx=\int \frac{(b\operatorname{arcsec}(cx)+a)x^5}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^5*arcsec(c*x) + a*x^5)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx=\text{Timed out}$$

input
```
integrate(x**5*(a+b*asec(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx=\int \frac{(b\operatorname{arcsec}(cx)+a)x^5}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/4*a*((4*d*e*x^2 + 3*d^2)/(e^5*x^4 + 2*d*e^4*x^2 + d^2*e^3) + 2*log(e*x^2
 + d)/e^3) + b*integrate(x^5*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(e^3*x^6
+ 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{x^5\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^3}\,dx$$

input
```
int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^3,x)
```

output
```
int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx$$
$$= \frac{4\left(\int \frac{asec(cx)x^5}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx\right)b\,d^2e^3+8\left(\int \frac{asec(cx)x^5}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx\right)bd\,e^4x^2+4\left(\int \frac{asec(cx)x^5}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}d\right.}{4e^3\left(e^2x^4+2de\right.}$$

input
```
int(x^5*(a+b*asec(c*x))/(e*x^2+d)^3,x)
```

output
```
(4*int((asec(c*x)*x**5)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6)
,x)*b*d**2*e**3 + 8*int((asec(c*x)*x**5)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*
x**4 + e**3*x**6),x)*b*d*e**4*x**2 + 4*int((asec(c*x)*x**5)/(d**3 + 3*d**2
*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*e**5*x**4 + 2*log(d + e*x**2)*a*
d**2 + 4*log(d + e*x**2)*a*d*e*x**2 + 2*log(d + e*x**2)*a*e**2*x**4 + a*d*
*2 - 2*a*e**2*x**4)/(4*e**3*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

3.105 $\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^3} dx$

Optimal result 904
Mathematica [C] (verified) 904
Rubi [A] (verified) 905
Maple [B] (verified) 908
Fricas [B] (verification not implemented) 909
Sympy [F(-1)] 910
Maxima [F] 911
Giac [F(-2)] 911
Mupad [F(-1)] 912
Reduce [F] 912

Optimal result

Integrand size = 21, antiderivative size = 157

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^3} dx = \frac{bcx\sqrt{-1+c^2x^2}}{8e(c^2d+e)\sqrt{c^2x^2}(d+ex^2)} + \frac{x^4(a+b\sec^{-1}(cx))}{4d(d+ex^2)^2} - \frac{bc(c^2d+2e)x\arctan\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{\sqrt{c^2d+e}}\right)}{8de^{3/2}(c^2d+e)^{3/2}\sqrt{c^2x^2}}$$

output
```
1/8*b*c*x*(c^2*x^2-1)^(1/2)/e/(c^2*d+e)/(c^2*x^2)^(1/2)/(e*x^2+d)+1/4*x^4*
(a+b*arcsec(c*x))/d/(e*x^2+d)^2-1/8*b*c*(c^2*d+2*e)*x*arctan(e^(1/2)*(c^2*
x^2-1)^(1/2)/(c^2*d+e)^(1/2))/d/e^(3/2)/(c^2*d+e)^(3/2)/(c^2*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.48

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx =$$

$$-\frac{-\frac{4ad}{(d+ex^2)^2}+\frac{8a}{d+ex^2}-\frac{2bce\sqrt{1-\frac{1}{c^2x^2}}x}{(c^2d+e)(d+ex^2)}+\frac{4b(d+2ex^2)\sec^{-1}(cx)}{(d+ex^2)^2}+\frac{4b\arcsin\left(\frac{1}{cx}\right)}{d}+\frac{b\sqrt{e}(c^2d+2e)\log\left(-\frac{16d\sqrt{-c^2d-e}e^{3/2}\left(\sqrt{e}+c\right.}{b\left(c^2d+\right.}\right.}{d(-c^2d-e)^{3/}}}{16e^2}$$

input
```
Integrate[(x^3*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/16*((-4*a*d)/(d + e*x^2)^2 + (8*a)/(d + e*x^2) - (2*b*c*e*Sqrt[1 - 1/(c
^2*x^2)]*x)/((c^2*d + e)*(d + e*x^2)) + (4*b*(d + 2*e*x^2)*ArcSec[c*x])/(d
 + e*x^2)^2 + (4*b*ArcSin[1/(c*x)])/d + (b*Sqrt[e]*(c^2*d + 2*e)*Log[(-16*
d*Sqrt[-(c^2*d) - e]*e^(3/2)*(Sqrt[e] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e
]*Sqrt[1 - 1/(c^2*x^2)])*x))/(b*(c^2*d + 2*e)*(I*Sqrt[d] + Sqrt[e]*x))])/(
d*(-(c^2*d) - e)^(3/2)) + (b*Sqrt[e]*(c^2*d + 2*e)*Log[((16*I)*d*Sqrt[-(c^
2*d) - e]*e^(3/2)*(-Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 -
 1/(c^2*x^2)])*x))/(b*(c^2*d + 2*e)*(Sqrt[d] + I*Sqrt[e]*x))])/(d*(-(c^2*d
) - e)^(3/2)))/e^2
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5761, 27, 354, 87, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(a+b\sec^{-1}(cx)\right)}{(d+ex^2)^3}\,dx$$

$$\Big\downarrow \text{ 5761}$$

$$\frac{x^4\left(a+b\sec^{-1}(cx)\right)}{4d\,(d+ex^2)^2}-\frac{bcx\int \frac{x^3}{4d\sqrt{c^2x^2-1}(ex^2+d)^2}dx}{\sqrt{c^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^4(a+b\sec^{-1}(cx))}{4d\,(d+ex^2)^2}-\frac{bcx\int\frac{x^3}{\sqrt{c^2x^2-1}(ex^2+d)^2}dx}{4d\sqrt{c^2x^2}}$$

$$\Big\downarrow \text{354}$$

$$\frac{x^4(a+b\sec^{-1}(cx))}{4d\,(d+ex^2)^2}-\frac{bcx\int\frac{x^2}{\sqrt{c^2x^2-1}(ex^2+d)^2}dx^2}{8d\sqrt{c^2x^2}}$$

$$\Big\downarrow \text{87}$$

$$\frac{x^4(a+b\sec^{-1}(cx))}{4d\,(d+ex^2)^2}-\frac{bcx\left(\frac{(c^2d+2e)\int\frac{1}{\sqrt{c^2x^2-1}(ex^2+d)}dx^2}{2e(c^2d+e)}-\frac{d\sqrt{c^2x^2-1}}{e(c^2d+e)(d+ex^2)}\right)}{8d\sqrt{c^2x^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{x^4(a+b\sec^{-1}(cx))}{4d\,(d+ex^2)^2}-\frac{bcx\left(\frac{(c^2d+2e)\int\frac{1}{\frac{ex^4}{c^2}+d+\frac{e}{c^2}}d\sqrt{c^2x^2-1}}{c^2e(c^2d+e)}-\frac{d\sqrt{c^2x^2-1}}{e(c^2d+e)(d+ex^2)}\right)}{8d\sqrt{c^2x^2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{x^4(a+b\sec^{-1}(cx))}{4d\,(d+ex^2)^2}-\frac{bcx\left(\frac{(c^2d+2e)\arctan\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{e^{3/2}(c^2d+e)^{3/2}}-\frac{d\sqrt{c^2x^2-1}}{e(c^2d+e)(d+ex^2)}\right)}{8d\sqrt{c^2x^2}}$$

input
```
Int[(x^3*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]
```

output
```
(x^4*(a + b*ArcSec[c*x]))/(4*d*(d + e*x^2)^2) - (b*c*x*(-((d*Sqrt[-1 + c^2
*x^2])/(e*(c^2*d + e)*(d + e*x^2))) + ((c^2*d + 2*e)*ArcTan[(Sqrt[e]*Sqrt[
-1 + c^2*x^2])/Sqrt[c^2*d + e]])/(e^(3/2)*(c^2*d + e)^(3/2))))/(8*d*Sqrt[c
^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 936 vs. $2(135) = 270$.

Time = 4.57 (sec) , antiderivative size = 937, normalized size of antiderivative = 5.97

method	result
parts	$a\left(-\frac{1}{2e^2(e x^2+d)}+\frac{d}{4e^2(e x^2+d)^2}\right)+\frac{b\left(-\frac{c^6 \operatorname{arcsec}(cx)}{2e^2\left(e c^2x^2+c^2d\right)}+\frac{c^8 \operatorname{arcsec}(cx) d}{4e^2\left(e c^2x^2+c^2d\right)^2}+\frac{c^3\sqrt{c^2x^2-1}\left(4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\cdots\right.\right.}{\cdots}\right.}{\cdots}$
derivativedivides	$\frac{a c^6\left(-\frac{1}{2e^2\left(e c^2x^2+c^2d\right)}+\frac{d c^2}{4e^2\left(e c^2x^2+c^2d\right)^2}\right)+b c^6\left(-\frac{\operatorname{arcsec}(cx)}{2e^2\left(e c^2x^2+c^2d\right)}+\frac{\operatorname{arcsec}(cx) d c^2}{4e^2\left(e c^2x^2+c^2d\right)^2}-\frac{\sqrt{c^2x^2-1}\left(-4\arctan\left(\frac{\cdots}{\sqrt{c\cdots}}\right.\right.}{\cdots}\right.}{\cdots}$
default	$\frac{a c^6\left(-\frac{1}{2e^2\left(e c^2x^2+c^2d\right)}+\frac{d c^2}{4e^2\left(e c^2x^2+c^2d\right)^2}\right)+b c^6\left(-\frac{\operatorname{arcsec}(cx)}{2e^2\left(e c^2x^2+c^2d\right)}+\frac{\operatorname{arcsec}(cx) d c^2}{4e^2\left(e c^2x^2+c^2d\right)^2}-\frac{\sqrt{c^2x^2-1}\left(-4\arctan\left(\frac{\cdots}{\sqrt{c\cdots}}\right.\right.}{\cdots}\right.}{\cdots}$

input

```
int(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2/e^2/(e*x^2+d)+1/4*d/e^2/(e*x^2+d)^2)+b/c^4*(-1/2*c^6*arcsec(c*x)/e
^2/(c^2*e*x^2+c^2*d)+1/4*c^8*arcsec(c*x)*d/e^2/(c^2*e*x^2+c^2*d)^2+1/16*c^
3*(c^2*x^2-1)^(1/2)/e*(4*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*
c^4*d*e*x^2+4*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*c^4*d^2-ln(
-2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c*e
*x+(-c^2*d*e)^(1/2)))*c^4*d*e*x^2-ln(-2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^
(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2-ln(-2*(
-(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*e*x+(
-c^2*d*e)^(1/2)))*c^4*d*e*x^2-ln(-2*(-(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/
2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2+4*arctan(1/
(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*e^2*c^2*x^2+4*arctan(1/(c^2*x^2-1)
^(1/2))*(-(c^2*d+e)/e)^(1/2)*c^2*d*e-2*(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1
/2)*c^2*d*e-2*ln(-2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(
1/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^(1/2)))*e^2*c^2*x^2-2*ln(-2*((c^2*x^2-1)^(1
/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^(1/2
)))*c^2*d*e-2*ln(-2*(-(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^
(1/2)*c*x+e)/(c*e*x+(-c^2*d*e)^(1/2)))*e^2*c^2*x^2-2*ln(-2*(-(c^2*x^2-1)^(
1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*e*x+(-c^2*d*e)^(1/2
)))*c^2*d*e)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/d/(c^2*d+e)/(-c*e*x+(-c^2*d*e)^
(1/2))/(-(c^2*d+e)/e)^(1/2)/(c*e*x+(-c^2*d*e)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(135) = 270$.

Time = 0.45 (sec) , antiderivative size = 1021, normalized size of antiderivative = 6.50

$$\int \frac{x^3(a + b\sec^{-1}(cx))}{(d + ex^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[-1/16*(4*a*c^4*d^4 + 8*a*c^2*d^3*e + 4*a*d^2*e^2 + 8*(a*c^4*d^3*e + 2*a*c
^2*d^2*e^2 + a*d*e^3)*x^2 + (b*c^2*d^3 + (b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b
*d^2*e + 2*(b*c^2*d^2*e + 2*b*d*e^2)*x^2)*sqrt(-c^2*d*e - e^2)*log((c^2*e*
x^2 - c^2*d + 2*sqrt(-c^2*d*e - e^2)*sqrt(c^2*x^2 - 1) - 2*e)/(e*x^2 + d))
 + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2
*e^2 + b*d*e^3)*x^2)*arcsec(c*x) - 8*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^
2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2
*d^2*e^2 + b*d*e^3)*x^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)) - 2*(b*c^2*d^3*e
 + b*d^2*e^2 + (b*c^2*d^2*e^2 + b*d*e^3)*x^2)*sqrt(c^2*x^2 - 1))/(c^4*d^5*
e^2 + 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 + 2*c^2*d^2*e^5 + d*e^6)*x^4
+ 2*(c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^2), -1/8*(2*a*c^4*d^4 + 4*a*
c^2*d^3*e + 2*a*d^2*e^2 + 4*(a*c^4*d^3*e + 2*a*c^2*d^2*e^2 + a*d*e^3)*x^2
- (b*c^2*d^3 + (b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(b*c^2*d^2*e +
2*b*d*e^2)*x^2)*sqrt(c^2*d*e + e^2)*arctan(sqrt(c^2*d*e + e^2)*sqrt(c^2*x^
2 - 1)/(c^2*e*x^2 - e)) + 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*
c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*arcsec(c*x) - 4*(b*c^4*d^4 + 2
*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2
*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*arctan(-c*x + sqrt(c^2*x^2
 - 1)) - (b*c^2*d^3*e + b*d^2*e^2 + (b*c^2*d^2*e^2 + b*d*e^3)*x^2)*sqrt(c^
2*x^2 - 1))/(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 + 2*c...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*asec(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^3(a + b \sec^{-1}(cx))}{(d + ex^2)^3} dx = \int \frac{(b \operatorname{arcsec}(cx) + a)x^3}{(ex^2 + d)^3} dx$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*e*x^2 + d)*a/(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2) - 1/4*((2*e*x^2 + d
)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 4*(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2
)*integrate(1/4*(2*c^2*e*x^3 + c^2*d*x)*e^(1/2*log(c*x + 1) + 1/2*log(c*x
- 1))/(c^2*e^4*x^6 + (2*c^2*d*e^3 - e^4)*x^4 - d^2*e^2 + (c^2*d^2*e^2 - 2*
d*e^3)*x^2 + (c^2*e^4*x^6 + (2*c^2*d*e^3 - e^4)*x^4 - d^2*e^2 + (c^2*d^2*e
^2 - 2*d*e^3)*x^2)*e^(log(c*x + 1) + log(c*x - 1))), x))*b/(e^4*x^4 + 2*d*
e^3*x^2 + d^2*e^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b \sec^{-1}(cx))}{(d + ex^2)^3} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx=\int \frac{x^3\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^3}\,dx$$

input
```
int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^3,x)
```

output
```
int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx$$
$$=\frac{4\left(\int \frac{asec(cx)x^3}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx\right)b\,d^3+8\left(\int \frac{asec(cx)x^3}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx\right)b\,d^2e\,x^2+4\left(\int \frac{asec(cx)x^3}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx\right.}{4d\left(e^2x^4+2de\,x^2+d^2\right)}$$

input
```
int(x^3*(a+b*asec(c*x))/(e*x^2+d)^3,x)
```

output
```
(4*int((asec(c*x)*x**3)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6)
,x)*b*d**3 + 8*int((asec(c*x)*x**3)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4
+ e**3*x**6),x)*b*d**2*e*x**2 + 4*int((asec(c*x)*x**3)/(d**3 + 3*d**2*e*x*
*2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*d*e**2*x**4 + a*x**4)/(4*d*(d**2 + 2*
d*e*x**2 + e**2*x**4))
```

3.106 $\int \frac{x\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal result 913
Mathematica [C] (verified) 914
Rubi [A] (verified) 915
Maple [B] (verified) 918
Fricas [B] (verification not implemented) 919
Sympy [F(-1)] 920
Maxima [F] 921
Giac [F(-2)] 921
Mupad [F(-1)] 922
Reduce [F] 922

Optimal result

Integrand size = 19, antiderivative size = 193

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = -\frac{bcx\sqrt{-1+c^2x^2}}{8d\,(c^2d+e)\,\sqrt{c^2x^2}\,(d+ex^2)} - \frac{a+b\sec^{-1}(cx)}{4e\,(d+ex^2)^2} + \frac{bcx\arctan\left(\sqrt{-1+c^2x^2}\right)}{4d^2e\sqrt{c^2x^2}} - \frac{bc(3c^2d+2e)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{\sqrt{c^2d+e}}\right)}{8d^2\sqrt{e}\,(c^2d+e)^{3/2}\,\sqrt{c^2x^2}}$$

output
```
-1/8*b*c*x*(c^2*x^2-1)^(1/2)/d/(c^2*d+e)/(c^2*x^2)^(1/2)/(e*x^2+d)-1/4*(a+
b*arcsec(c*x))/e/(e*x^2+d)^2+1/4*b*c*x*arctan((c^2*x^2-1)^(1/2))/d^2/e/(c^
2*x^2)^(1/2)-1/8*b*c*(3*c^2*d+2*e)*x*arctan(e^(1/2)*(c^2*x^2-1)^(1/2)/(c^2
*d+e)^(1/2))/d^2/e^(1/2)/(c^2*d+e)^(3/2)/(c^2*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.00

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx$$
$$= \frac{1}{16}\left(-\frac{4a}{e(d+ex^2)^2}-\frac{2bc\sqrt{1-\frac{1}{c^2x^2}}x}{d(c^2d+e)(d+ex^2)}-\frac{4b\sec^{-1}(cx)}{e(d+ex^2)^2}-\frac{4b\arcsin\left(\frac{1}{cx}\right)}{d^2e}\right.$$
$$-\frac{b(3c^2d+2e)\log\left(-\frac{16d^2\sqrt{-c^2d-e}\sqrt{e}\left(\sqrt{e}+c\left(ic\sqrt{d}-\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{b(3c^2d+2e)\left(i\sqrt{d}+\sqrt{e}x\right)}\right)}{d^2(-c^2d-e)^{3/2}\sqrt{e}}$$
$$\left.-\frac{b(3c^2d+2e)\log\left(\frac{16id^2\sqrt{-c^2d-e}\sqrt{e}\left(-\sqrt{e}+c\left(ic\sqrt{d}+\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{b(3c^2d+2e)\left(\sqrt{d}+i\sqrt{e}x\right)}\right)}{d^2(-c^2d-e)^{3/2}\sqrt{e}}\right)$$

input
```
Integrate[(x*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]
```

output
```
((-4*a)/(e*(d + e*x^2)^2) - (2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x)/(d*(c^2*d + e)
*(d + e*x^2)) - (4*b*ArcSec[c*x])/(e*(d + e*x^2)^2) - (4*b*ArcSin[1/(c*x)]
)/(d^2*e) - (b*(3*c^2*d + 2*e)*Log[(-16*d^2*Sqrt[-(c^2*d) - e]*Sqrt[e]*(Sq
rt[e] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(b*
(3*c^2*d + 2*e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d^2*(-(c^2*d) - e)^(3/2)*Sqrt[
e]) - (b*(3*c^2*d + 2*e)*Log[((16*I)*d^2*Sqrt[-(c^2*d) - e]*Sqrt[e]*(-Sqrt
[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(b*(3
*c^2*d + 2*e)*(Sqrt[d] + I*Sqrt[e]*x))])/(d^2*(-(c^2*d) - e)^(3/2)*Sqrt[e]
))/16
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5759, 354, 114, 27, 174, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\sec^{-1}(cx)\right)}{(d+ex^2)^3}dx$$

$$\Big\downarrow \text{5759}$$

$$\frac{bcx\int \frac{1}{x\sqrt{c^2x^2-1}(ex^2+d)^2}dx}{4e\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{354}$$

$$\frac{bcx\int \frac{1}{x^2\sqrt{c^2x^2-1}(ex^2+d)^2}dx^2}{8e\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{114}$$

$$\frac{bcx\left(-\frac{\int -\frac{-ex^2c^2+2dc^2+2e}{2x^2\sqrt{c^2x^2-1}\left(ex^2+d\right)}dx^2}{d(c^2d+e)}-\frac{e\sqrt{c^2x^2-1}}{d(c^2d+e)(d+ex^2)}\right)}{8e\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\left(\frac{\int \frac{2\left(dc^2+e\right)-c^2ex^2}{x^2\sqrt{c^2x^2-1}\left(ex^2+d\right)}dx^2}{2d(c^2d+e)}-\frac{e\sqrt{c^2x^2-1}}{d(c^2d+e)(d+ex^2)}\right)}{8e\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{174}$$

$$\frac{bcx\left(\frac{\frac{2\left(c^2d+e\right)\int \frac{1}{x^2\sqrt{c^2x^2-1}}dx^2}{d}-\frac{e\left(3c^2d+2e\right)\int \frac{1}{\sqrt{c^2x^2-1}\left(ex^2+d\right)}dx^2}{d}}{2d(c^2d+e)}-\frac{e\sqrt{c^2x^2-1}}{d(c^2d+e)(d+ex^2)}\right)}{8e\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{bcx\left(\frac{\frac{4(c^2d+e)\int \frac{1}{\frac{x^4}{c^2}+\frac{1}{c^2}}d\sqrt{c^2x^2-1}}{c^2d}-\frac{2e(3c^2d+2e)\int \frac{1}{\frac{ex^4}{c^2}+d+\frac{e}{c^2}}d\sqrt{c^2x^2-1}}{c^2d}}{2d(c^2d+e)}-\frac{e\sqrt{c^2x^2-1}}{d(c^2d+e)(d+ex^2)}\right)}{8e\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{bcx\left(\frac{\frac{4\arctan\left(\sqrt{c^2x^2-1}\right)(c^2d+e)}{d}-\frac{2\sqrt{e}(3c^2d+2e)\arctan\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{d\sqrt{c^2d+e}}}{2d(c^2d+e)}-\frac{e\sqrt{c^2x^2-1}}{d(c^2d+e)(d+ex^2)}\right)}{8e\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

input
```
Int[(x*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/4*(a + b*ArcSec[c*x])/(e*(d + e*x^2)^2) + (b*c*x*(-((e*Sqrt[-1 + c^2*x^
2])/(d*(c^2*d + e)*(d + e*x^2))) + ((4*(c^2*d + e)*ArcTan[Sqrt[-1 + c^2*x^
2]])/d - (2*Sqrt[e]*(3*c^2*d + 2*e)*ArcTan[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/Sq
rt[c^2*d + e]])/(d*Sqrt[c^2*d + e]))/(2*d*(c^2*d + e))))/(8*e*Sqrt[c^2*x^2
])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5759
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSec[c*x])/(2*e*(p + 1))), x
] - Simp[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2]))   Int[(d + e*x^2)^(p + 1)/(x*S
qrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 885 vs. $2(165) = 330$.

Time = 4.50 (sec) , antiderivative size = 886, normalized size of antiderivative = 4.59

method	result
parts	$-\frac{a}{4e(e\,x^2+d)^2}+\frac{b\left(-\frac{c^6\,\mathrm{arcsec}(cx)}{4e\left(e\,c^2x^2+c^2d\right)^2}+\frac{c\sqrt{c^2x^2-1}\left(4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}\,c^4d^2+4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d}{e}} \cdots\right.}{\cdots}\right.}{\cdots}$
derivativedivides	$\frac{-\frac{a\,c^6}{4e\left(e\,c^2x^2+c^2d\right)^2}+b\,c^6\left(-\frac{\mathrm{arcsec}(cx)}{4e\left(e\,c^2x^2+c^2d\right)^2}+\frac{\sqrt{c^2x^2-1}\left(4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}\,c^4d^2+4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c}{}} \cdots\right.}{\cdots}\right.}{\cdots}$
default	$\frac{-\frac{a\,c^6}{4e\left(e\,c^2x^2+c^2d\right)^2}+b\,c^6\left(-\frac{\mathrm{arcsec}(cx)}{4e\left(e\,c^2x^2+c^2d\right)^2}+\frac{\sqrt{c^2x^2-1}\left(4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}\,c^4d^2+4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c}{}} \cdots\right.}{\cdots}\right.}{\cdots}$

input
```
int(x*(a+b*arcsec(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/e/(e*x^2+d)^2+b/c^2*(-1/4*c^6/e/(c^2*e*x^2+c^2*d)^2*arcsec(c*x)+1/1
6*c*(c^2*x^2-1)^(1/2)*(4*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*
c^4*d^2+4*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*c^4*d*e*x^2-3*l
n(-2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c
*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2-3*ln(-2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^
(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^(1/2)))*c^4*d*e*x^2-3*l
n(-2*(-(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c
*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2-3*ln(-2*(-(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)
^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^4*d*e*x^2+2*(
c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*c^2*d*e+4*arctan(1/(c^2*x^2-1)^(1/2)
)*(-(c^2*d+e)/e)^(1/2)*c^2*d*e+4*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e
)^(1/2)*e^2*c^2*x^2-2*ln(-2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^
2*d*e)^(1/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^(1/2)))*c^2*d*e-2*ln(-2*((c^2*x^2-1
)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^
(1/2)))*e^2*c^2*x^2-2*ln(-2*(-(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c
^2*d*e)^(1/2)*c*x+e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^2*d*e-2*ln(-2*(-(c^2*x^2-
1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*e*x+(-c^2*d*e)^
(1/2)))*e^2*c^2*x^2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/d^2/(-(c^2*d+e)/e)^(1/2
)/(c^2*d+e)/(-c*e*x+(-c^2*d*e)^(1/2))/(c*e*x+(-c^2*d*e)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(165) = 330$.

Time = 0.45 (sec) , antiderivative size = 894, normalized size of antiderivative = 4.63

$$\int \frac{x(a + b\sec^{-1}(cx))}{(d + ex^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[-1/16*(4*a*c^4*d^4 + 8*a*c^2*d^3*e + 4*a*d^2*e^2 + (3*b*c^2*d^3 + (3*b*c^
2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(3*b*c^2*d^2*e + 2*b*d*e^2)*x^2)*sq
rt(-c^2*d*e - e^2)*log((c^2*e*x^2 - c^2*d + 2*sqrt(-c^2*d*e - e^2)*sqrt(c^
2*x^2 - 1) - 2*e)/(e*x^2 + d)) + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2)
*arcsec(c*x) - 8*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 +
 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*
x^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 2*(b*c^2*d^3*e + b*d^2*e^2 + (b*c^
2*d^2*e^2 + b*d*e^3)*x^2)*sqrt(c^2*x^2 - 1))/(c^4*d^6*e + 2*c^2*d^5*e^2 +
d^4*e^3 + (c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 + 2
*c^2*d^4*e^3 + d^3*e^4)*x^2), -1/8*(2*a*c^4*d^4 + 4*a*c^2*d^3*e + 2*a*d^2*
e^2 - (3*b*c^2*d^3 + (3*b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(3*b*c^
2*d^2*e + 2*b*d*e^2)*x^2)*sqrt(c^2*d*e + e^2)*arctan(sqrt(c^2*d*e + e^2)*s
qrt(c^2*x^2 - 1)/(c^2*e*x^2 - e)) + 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e
^2)*arcsec(c*x) - 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^
2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^
3)*x^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)) + (b*c^2*d^3*e + b*d^2*e^2 + (b*c
^2*d^2*e^2 + b*d*e^3)*x^2)*sqrt(c^2*x^2 - 1))/(c^4*d^6*e + 2*c^2*d^5*e^2 +
 d^4*e^3 + (c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 +
2*c^2*d^4*e^3 + d^3*e^4)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(a + b\sec^{-1}(cx))}{(d + ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x*(a+b*asec(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x(a + b\sec^{-1}(cx))}{(d + ex^2)^3} dx = \int \frac{(b\,\mathrm{arcsec}\,(cx) + a)x}{(ex^2 + d)^3} dx$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/4*(4*(c^2*e^3*x^4 + 2*c^2*d*e^2*x^2 + c^2*d^2*e)*integrate(1/4*x*e^(1/2*
log(c*x + 1) + 1/2*log(c*x - 1))/(c^2*e^3*x^6 + (2*c^2*d*e^2 - e^3)*x^4 -
d^2*e + (c^2*d^2*e - 2*d*e^2)*x^2 + (c^2*e^3*x^6 + (2*c^2*d*e^2 - e^3)*x^4
 - d^2*e + (c^2*d^2*e - 2*d*e^2)*x^2)*e^(log(c*x + 1) + log(c*x - 1))), x)
 - arctan(sqrt(c*x + 1)*sqrt(c*x - 1)))*b/(e^3*x^4 + 2*d*e^2*x^2 + d^2*e)
- 1/4*a/(e^3*x^4 + 2*d*e^2*x^2 + d^2*e)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a + b\sec^{-1}(cx))}{(d + ex^2)^3} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx=\int \frac{x\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^3}\,dx$$

input
```
int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^3,x)
```

output
```
int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx$$
$$=\frac{4\left(\int \frac{asec(cx)x}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx\right)b\,d^2e+8\left(\int \frac{asec(cx)x}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx\right)bd\,e^2x^2+4\left(\int \frac{asec(cx)x}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}d\right.}{4e\left(e^2x^4+2de\,x^2+d^2\right)}$$

input
```
int(x*(a+b*asec(c*x))/(e*x^2+d)^3,x)
```

output
```
(4*int((asec(c*x)*x)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)
*b*d**2*e + 8*int((asec(c*x)*x)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e*
*3*x**6),x)*b*d*e**2*x**2 + 4*int((asec(c*x)*x)/(d**3 + 3*d**2*e*x**2 + 3*
d*e**2*x**4 + e**3*x**6),x)*b*e**3*x**4 - a)/(4*e*(d**2 + 2*d*e*x**2 + e**
2*x**4))
```

3.107 $\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx$

Optimal result 924
Mathematica [B] (warning: unable to verify) 925
Rubi [A] (verified) 926
Maple [C] (warning: unable to verify) 929
Fricas [F] 930
Sympy [F(-1)] 930
Maxima [F] 930
Giac [F(-2)] 931
Mupad [F(-1)] 931
Reduce [F] 931

Optimal result

Integrand size = 21, antiderivative size = 685

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx = \frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2(a+b\sec^{-1}(cx))}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e(a+b\sec^{-1}(cx))}{d^3\left(e+\frac{d}{x^2}\right)} + \frac{i(a+b\sec^{-1}(cx))^2}{2bd^3} - \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{d^3\sqrt{c^2d+e}} + \frac{b\sqrt{e}(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{8d^3\left(c^2d+e\right)^{3/2}} - \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} - \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} - \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3} - \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3} + \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} + \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} + \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3} + \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}$$

output
```
1/8*b*c*e*(1-1/c^2/x^2)^(1/2)/d^2/(c^2*d+e)/(e+d/x^2)/x+1/4*e^2*(a+b*arcse
c(c*x))/d^3/(e+d/x^2)^2-e*(a+b*arcsec(c*x))/d^3/(e+d/x^2)+1/2*I*(a+b*arcse
c(c*x))^2/b/d^3-b*e^(1/2)*arctan((c^2*d+e)^(1/2)/c/e^(1/2)/(1-1/c^2/x^2)^(
1/2)/x)/d^3/(c^2*d+e)^(1/2)+1/8*b*e^(1/2)*(c^2*d+2*e)*arctan((c^2*d+e)^(1/
2)/c/e^(1/2)/(1-1/c^2/x^2)^(1/2)/x)/d^3/(c^2*d+e)^(3/2)-1/2*(a+b*arcsec(c*
x))*ln(1-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/
2)))/d^3-1/2*(a+b*arcsec(c*x))*ln(1+c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1
/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/d^3-1/2*(a+b*arcsec(c*x))*ln(1-c*(-d)^(1/2
)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3-1/2*(a+b*ar
csec(c*x))*ln(1+c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d
+e)^(1/2)))/d^3+1/2*I*b*polylog(2,-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/
2))/(e^(1/2)-(c^2*d+e)^(1/2)))/d^3+1/2*I*b*polylog(2,c*(-d)^(1/2)*(1/c/x+I
*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/d^3+1/2*I*b*polylog(2,-c*
(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3+1/
2*I*b*polylog(2,c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d
+e)^(1/2)))/d^3
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1871 vs. 2(685) = 1370.

Time = 6.05 (sec) , antiderivative size = 1871, normalized size of antiderivative = 2.73

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^3),x]
```

output
```
a/(4*d*(d + e*x^2)^2) + a/(2*d^2*(d + e*x^2)) + (a*Log[x])/d^3 - (a*Log[d
+ e*x^2])/(2*d^3) + b*((((-5*I)/16)*Sqrt[e]*(-(ArcSec[c*x]/(I*Sqrt[d]*Sqrt
[e] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e
] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-
(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/d^(5
/2) + (((5*I)/16)*Sqrt[e]*(-(ArcSec[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x)) - (
I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*(I*c*Sqr
t[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(
Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/d^(5/2) + (Sqrt[e]
*(-(ArcSec[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2)) + (ArcSin[1/(c*x)]
/Sqrt[e] - I*((c*Sqrt[d]*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/((c^2*d + e)*((-
I)*Sqrt[d] + Sqrt[e]*x)) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e
]*(I*Sqrt[e] + c*(c*Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(
(2*c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x))])/(c^2*d + e)^(3/2)))/d))/(16*d^
2) + (Sqrt[e]*((I*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*
(I*Sqrt[d] + Sqrt[e]*x)) - ArcSec[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2)
 + ArcSin[1/(c*x)]/(d*Sqrt[e]) - (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^
2*d + e]*((-I)*Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^
2)])*x))/((2*c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))
/(16*d^2) + ((I/2)*ArcSec[c*x]^2 - ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[...
```

Rubi [A] (verified)

Time = 1.82 (sec) , antiderivative size = 745, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5763, 5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^3}dx$$

$$\Big\downarrow \text{ 5763}$$

$$-\int \frac{a+b\arccos\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^3x^5}d\frac{1}{x}$$

$$\Big\downarrow \text{ 5233}$$

$$-\int \left(\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)e^2}{d^2\left(\frac{d}{x^2}+e\right)^3x}-\frac{2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)e}{d^2\left(\frac{d}{x^2}+e\right)^2x}+\frac{a+b\arccos\left(\frac{1}{cx}\right)}{d^2\left(\frac{d}{x^2}+e\right)x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}-\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}-\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^3}-\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^3}+\frac{e^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4d^3\left(\frac{d}{x^2}+e\right)^2}-\frac{e\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{d^3\left(\frac{d}{x^2}+e\right)}+\frac{i\left(a+b\arccos\left(\frac{1}{cx}\right)\right)^2}{2bd^3}+\frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^3}+\frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^3}+\frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^3}+\frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^3}+\frac{b\sqrt{e}(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{8d^3\left(c^2d+e\right)^{3/2}}-\frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{d^3\sqrt{c^2d+e}}+\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2x\left(c^2d+e\right)\left(\frac{d}{x^2}+e\right)}$$

input
```
Int[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^3),x]
```

output
```
(b*c*e*Sqrt[1 - 1/(c^2*x^2)])/(8*d^2*(c^2*d + e)*(e + d/x^2)*x) + (e^2*(a
+ b*ArcCos[1/(c*x)]))/(4*d^3*(e + d/x^2)^2) - (e*(a + b*ArcCos[1/(c*x)]))/
(d^3*(e + d/x^2)) + ((I/2)*(a + b*ArcCos[1/(c*x)])^2)/(b*d^3) - (b*Sqrt[e]
*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(d^3*Sqrt[c^
2*d + e]) + (b*Sqrt[e]*(c^2*d + 2*e)*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqr
t[1 - 1/(c^2*x^2)]*x)])/(8*d^3*(c^2*d + e)^(3/2)) - ((a + b*ArcCos[1/(c*x)
])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])]
)/(2*d^3) - ((a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*
x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^3) - ((a + b*ArcCos[1/(c*x)])*Log
[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d
^3) - ((a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/
(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^3) + ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*
E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/d^3 + ((I/2)*b*PolyL
og[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/d^3
 + ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqr
t[c^2*d + e]))])/d^3 + ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)
]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.71 (sec) , antiderivative size = 3533, normalized size of antiderivative = 5.16

method	result	size
parts	Expression too large to display	3533
derivativedivides	Expression too large to display	3607
default	Expression too large to display	3607

input
```
int((a+b*arcsec(c*x))/x/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a/d^3*ln(x)-1/2*a/d^3*ln(e*x^2+d)+1/2*a/d^2/(e*x^2+d)+1/4*a/d/(e*x^2+d)^2+
b*(3/4*I*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^3*e*arctanh(1/4*(2*c^2*d*(1/c/x
+I*(1-1/c^2/x^2)^(1/2))^2+2*c^2*d+4*e)/(c^2*d*e+e^2)^(1/2))+1/2*(e*(c^2*d+
e))^(1/2)/(c^2*d+e)^2/d^3*e*arcsec(c*x)*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^
(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))-1/2*I*(e*(c^2*d+e))^(1/2)/(c^
2*d+e)^2/d^3*e*arcsec(c*x)^2+3/4*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^2*c^2*a
rcsec(c*x)*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+
e))^(1/2)-2*e))-1/4*I*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^3*e*polylog(2,d*c^
2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))-3/4*
I*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^2*arcsec(c*x)^2*c^2-3/8*I*(e*(c^2*d+e)
)^(1/2)/(c^2*d+e)^2/d^2*polylog(2,d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-
c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))*c^2+7/8*I*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^
2/d^2*arctanh(1/4*(2*c^2*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2+2*c^2*d+4*e)/(c
^2*d*e+e^2)^(1/2))*c^2-1/8*I*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d/e*polylog(2
,d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))
*c^4+1/4*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e
^2)/e/(c^4*d^2+2*c^2*d*e+e^2)*c^2/d^2*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1
/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsec(c*x)-1/2*I*arcsec(c*x)^2
/d^3+1/2*I*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*polylog(2,d*c^2*(1/c/x+I*(1-1
/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*e^2/c^4/d^5/(c^2...
```

Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx=\int \frac{b\,\mathrm{arcsec}\left(cx\right)+a}{\left(ex^2+d\right)^3x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x)
, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((a+b*asec(c*x))/x/(e*x**2+d)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx=\int \frac{b\,\mathrm{arcsec}\left(cx\right)+a}{\left(ex^2+d\right)^3x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/4*a*((2*e*x^2 + 3*d)/(d^2*e^2*x^4 + 2*d^3*e*x^2 + d^4) - 2*log(e*x^2 + d
)/d^3 + 4*log(x)/d^3) + b*integrate(arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(e
^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\sec^{-1}(cx)}{x\,(d + ex^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\sec^{-1}(cx)}{x\,(d + ex^2)^3}\,dx = \int \frac{a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x\,(e\,x^2 + d)^3}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^3),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^3), x)
```

Reduce [F]

$$\int \frac{a + b\sec^{-1}(cx)}{x\,(d + ex^2)^3}\,dx = \frac{4\left(\int \frac{asec(cx)}{e^3x^7+3d\,e^2x^5+3d^2e\,x^3+d^3x}dx\right) b\,d^5 + 8\left(\int \frac{asec(cx)}{e^3x^7+3d\,e^2x^5+3d^2e\,x^3+d^3x}dx\right) b\,d^4e\,x^2 + 4\left(\int \frac{asec(cx)}{e^3x^7+3d\,e^2x^5+3d^2e\,x^3+d^3x}\right.}{}$$

input
```
int((a+b*asec(c*x))/x/(e*x^2+d)^3,x)
```

output
```
(4*int(asec(c*x)/(d**3*x + 3*d**2*e*x**3 + 3*d*e**2*x**5 + e**3*x**7),x)*b
*d**5 + 8*int(asec(c*x)/(d**3*x + 3*d**2*e*x**3 + 3*d*e**2*x**5 + e**3*x**
7),x)*b*d**4*e*x**2 + 4*int(asec(c*x)/(d**3*x + 3*d**2*e*x**3 + 3*d*e**2*x
**5 + e**3*x**7),x)*b*d**3*e**2*x**4 - 2*log(d + e*x**2)*a*d**2 - 4*log(d
+ e*x**2)*a*d*e*x**2 - 2*log(d + e*x**2)*a*e**2*x**4 + 4*log(x)*a*d**2 + 8
*log(x)*a*d*e*x**2 + 4*log(x)*a*e**2*x**4 + 2*a*d**2 - a*e**2*x**4)/(4*d**
3*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

3.108 $\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx$

Optimal result 933
Mathematica [A] (warning: unable to verify) 934
Rubi [A] (verified) 935
Maple [C] (warning: unable to verify) 938
Fricas [F] 939
Sympy [F(-1)] 939
Maxima [F(-2)] 939
Giac [F(-2)] 940
Mupad [F(-1)] 940
Reduce [F] 940

Optimal result

Integrand size = 21, antiderivative size = 1124

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

output
```
1/16*b*c*(-d)^(1/2)*(1-1/c^2/x^2)^(1/2)/e^(3/2)/(c^2*d+e)/((-d)^(1/2)*e^(1
/2)-d/x)+1/16*b*c*(-d)^(1/2)*(1-1/c^2/x^2)^(1/2)/e^(3/2)/(c^2*d+e)/((-d)^(
1/2)*e^(1/2)+d/x)+1/16*(-d)^(1/2)*(a+b*arcsec(c*x))/e^(3/2)/((-d)^(1/2)*e^
(1/2)-d/x)^2+3/16*(a+b*arcsec(c*x))/e^2/((-d)^(1/2)*e^(1/2)-d/x)-1/16*(-d)
^(1/2)*(a+b*arcsec(c*x))/e^(3/2)/((-d)^(1/2)*e^(1/2)+d/x)^2-3/16*(a+b*arcs
ec(c*x))/e^2/((-d)^(1/2)*e^(1/2)+d/x)+1/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(
1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(1/2)/e/(c^2*d+e)
^(3/2)+3/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/
2)/(1-1/c^2/x^2)^(1/2))/d^(1/2)/e^2/(c^2*d+e)^(1/2)+1/16*b*arctanh((c^2*d+
(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(1/
2)/e/(c^2*d+e)^(3/2)+3/16*b*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)
/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(1/2)/e^2/(c^2*d+e)^(1/2)+3/16*(a+
b*arcsec(c*x))*ln(1-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c
^2*d+e)^(1/2)))/(-d)^(1/2)/e^(5/2)-3/16*(a+b*arcsec(c*x))*ln(1+c*(-d)^(1/2
)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(5
/2)+3/16*(a+b*arcsec(c*x))*ln(1-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))
/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(5/2)-3/16*(a+b*arcsec(c*x))*ln(1
+c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d
)^(1/2)/e^(5/2)-3/16*I*b*polylog(2,c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/
2))/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(5/2)-3/16*I*b*polylog(2,c*...
```

Mathematica [A] (warning: unable to verify)

Time = 6.07 (sec) , antiderivative size = 1819, normalized size of antiderivative = 1.62

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]
```

output
```
(a*d*x)/(4*e^2*(d + e*x^2)^2) - (5*a*x)/(8*e^2*(d + e*x^2)) + (3*a*ArcTan[
(Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[d]*e^(5/2)) + b*((5*(-(ArcSec[c*x]/(I*Sqrt[d
]*Sqrt[e] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(
Sqrt[e] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(
Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d])
)/(16*e^2) + (5*(-(ArcSec[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSin[
1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*(I*c*Sqrt[d] + Sqr
t[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] +
I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/(16*e^2) + ((I/16)*Sqrt[d]*(
-(ArcSec[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2)) + (ArcSin[1/(c*x)]/S
qrt[e] - I*((c*Sqrt[d]*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/((c^2*d + e)*((-I)
*Sqrt[d] + Sqrt[e]*x)) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*
(I*Sqrt[e] + c*(c*Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2
*c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x))])/(c^2*d + e)^(3/2)))/d))/e^2 - ((
I/16)*Sqrt[d]*((I*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*
(I*Sqrt[d] + Sqrt[e]*x)) - ArcSec[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2)
 + ArcSin[1/(c*x)]/(d*Sqrt[e]) - (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^
2*d + e]*((-I)*Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^
2)])*x))/((2*c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))
/e^2 + (3*(8*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(...
```

Rubi [A] (verified)

Time = 2.25 (sec) , antiderivative size = 1188, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5763, 5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(a+b \sec^{-1}(cx)\right)}{\left(d+ex^2\right)^3} \, dx$$

$$\Big\downarrow \text{ 5763}$$

$$-\int \frac{a+b \arccos\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{ 5173}$$

$$-\int \left(-\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)d^3}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^3}-\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)d^3}{8(-d)^{3/2}e^{3/2}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^3}-\frac{3\left(a+b\arccos\left(\frac{1}{cx}\right)\right)d}{8e^2\left(-\frac{d^2}{x^2}-ed\right)}-\frac{3\left(a+b\arccos\right.}{16e^2\left(\sqrt{-d}\sqrt{\right.}}\right.$$

$\big\downarrow$ 2009

$$\frac{b\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}c}{16e^{3/2}\left(dc^2+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{b\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}c}{16e^{3/2}\left(dc^2+e\right)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{3\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{3\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{16e^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{\sqrt{-d}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}-\frac{\sqrt{-d}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{16e^{3/2}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2}+\frac{3b\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{dc^2+e}}+\frac{b\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e\left(dc^2+e\right)^{3/2}}+\frac{3b\,\text{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{dc^2+e}}+\frac{b\,\text{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e\left(dc^2+e\right)^{3/2}}+\frac{3\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}-\frac{3\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{16\sqrt{-d}e^{5/2}}+\frac{3\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}-\frac{3\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{16\sqrt{-d}e^{5/2}}+\frac{3ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}-\frac{3ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}+\frac{3ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}-\frac{3ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}$$

input
```
Int[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]
```

output
```
(b*c*Sqrt[-d]*Sqrt[1 - 1/(c^2*x^2)])/(16*e^(3/2)*(c^2*d + e)*(Sqrt[-d]*Sqr
t[e] - d/x)) + (b*c*Sqrt[-d]*Sqrt[1 - 1/(c^2*x^2)])/(16*e^(3/2)*(c^2*d + e
)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[-d]*(a + b*ArcCos[1/(c*x)]))/(16*e^(3/
2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) + (3*(a + b*ArcCos[1/(c*x)]))/(16*e^2*(Sqrt
[-d]*Sqrt[e] - d/x)) - (Sqrt[-d]*(a + b*ArcCos[1/(c*x)]))/(16*e^(3/2)*(Sqr
t[-d]*Sqrt[e] + d/x)^2) - (3*(a + b*ArcCos[1/(c*x)]))/(16*e^2*(Sqrt[-d]*Sq
rt[e] + d/x)) + (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[
c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*Sqrt[d]*e*(c^2*d + e)^(3/2)) + (3*
b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1
 - 1/(c^2*x^2)])])/(16*Sqrt[d]*e^2*Sqrt[c^2*d + e]) + (b*ArcTanh[(c^2*d +
(Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/
(16*Sqrt[d]*e*(c^2*d + e)^(3/2)) + (3*b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e]
)/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*Sqrt[d]*e^2*S
qrt[c^2*d + e]) + (3*(a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcC
os[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*(a
+ b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] -
 Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) + (3*(a + b*ArcCos[1/(c*x)])*Log
[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*
Sqrt[-d]*e^(5/2)) - (3*(a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*Ar
cCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) + (...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 72.63 (sec) , antiderivative size = 1822, normalized size of antiderivative = 1.62

method	result	size
parts	Expression too large to display	1822
derivativedivides	Expression too large to display	1845
default	Expression too large to display	1845

input
```
int(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a*((-5/8/e*x^3-3/8*d/e^2*x)/(e*x^2+d)^2+3/8/e^2/(d*e)^(1/2)*arctan(e*x/(d*
e)^(1/2)))+b/c^5*(-1/8*x*c^7*(3*d^2*c^4*arcsec(c*x)+5*c^4*d*e*arcsec(c*x)*
x^2-((c^2*x^2-1)/c^2/x^2)^(1/2)*c^3*d*e*x-((c^2*x^2-1)/c^2/x^2)^(1/2)*e^2*
c^3*x^3+3*c^2*d*e*arcsec(c*x)+5*e^2*arcsec(c*x)*c^2*x^2)/e^2/(c^2*d+e)/(c^
2*e*x^2+c^2*d)^2-3/16*I/(c^2*d+e)/e^2*c^8*d*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2
*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/
c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+
c^2*d))-1/2*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d+2*(e*(c^
2*d+e))^(1/2)+2*e)*c*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*
(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/e/d^3+3/8*I*(-(c^2*d-2*(e*(c^
2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2
*d+e))^(1/2)*e+2*e^2)*c^3*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2
*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/e^2/d^2-1/2*I*((c^2*d+
2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*c*ar
ctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*
d)^(1/2))/(c^2*d+e)/e/d^3+1/2*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/
2)*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*c*a
rctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*
e)*d)^(1/2))/(c^2*d+e)^2/e/d^3-3/8*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d
)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*c^3*arctanh(c*d*(1/c/x+I*(1-1...
```

Fricas [F]

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^4}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^4*arcsec(c*x) + a*x^4)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(a+b*asec(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{x^4\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^3}\,dx$$

input
```
int((x^4*(a + b*acos(1/(c*x))))/(d + e*x^2)^3,x)
```

output
```
int((x^4*(a + b*acos(1/(c*x))))/(d + e*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \frac{3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a\,d^2+6\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ade\,x^2+3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a\,e^2x^4+8\left(\int \frac{asec}{e^3x^6+3d\,e^2x}\right.}{8d}$$

input
```
int(x^4*(a+b*asec(c*x))/(e*x^2+d)^3,x)
```

output
```
(3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d**2 + 6*sqrt(e)*sqrt(d
)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d*e*x**2 + 3*sqrt(e)*sqrt(d)*atan((e*x)/
(sqrt(e)*sqrt(d)))*a*e**2*x**4 + 8*int((asec(c*x)*x**4)/(d**3 + 3*d**2*e*x
**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*d**3*e**3 + 16*int((asec(c*x)*x**4)/
(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*d**2*e**4*x**2 + 8
*int((asec(c*x)*x**4)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x
)*b*d*e**5*x**4 - 3*a*d**2*e*x - 5*a*d*e**2*x**3)/(8*d*e**3*(d**2 + 2*d*e*
x**2 + e**2*x**4))
```

3.109 $\int \frac{x^2\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal result 942
Mathematica [A] (warning: unable to verify) 943
Rubi [A] (verified) 944
Maple [C] (warning: unable to verify) 947
Fricas [F] 948
Sympy [F(-1)] 948
Maxima [F(-2)] 948
Giac [F(-2)] 949
Mupad [F(-1)] 949
Reduce [F] 949

Optimal result

Integrand size = 21, antiderivative size = 1124

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

output
```
1/16*b*c*(1-1/c^2/x^2)^(1/2)/(-d)^(1/2)/e^(1/2)/(c^2*d+e)/((-d)^(1/2)*e^(1
/2)-d/x)+1/16*b*c*(1-1/c^2/x^2)^(1/2)/(-d)^(1/2)/e^(1/2)/(c^2*d+e)/((-d)^(
1/2)*e^(1/2)+d/x)+1/16*(a+b*arcsec(c*x))/(-d)^(1/2)/e^(1/2)/((-d)^(1/2)*e^
(1/2)-d/x)^2+1/16*(a+b*arcsec(c*x))/d/e/((-d)^(1/2)*e^(1/2)-d/x)-1/16*(a+b
*arcsec(c*x))/(-d)^(1/2)/e^(1/2)/((-d)^(1/2)*e^(1/2)+d/x)^2-1/16*(a+b*arcs
ec(c*x))/d/e/((-d)^(1/2)*e^(1/2)+d/x)-1/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(
1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(3/2)/(c^2*d+e)^(
3/2)+1/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)
/(1-1/c^2/x^2)^(1/2))/d^(3/2)/e/(c^2*d+e)^(1/2)-1/16*b*arctanh((c^2*d+(-d)
^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(3/2)/(
c^2*d+e)^(3/2)+1/16*b*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*
d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(3/2)/e/(c^2*d+e)^(1/2)-1/16*(a+b*arcsec
(c*x))*ln(1-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^
(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*(a+b*arcsec(c*x))*ln(1+c*(-d)^(1/2)*(1/c/x
+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16
*(a+b*arcsec(c*x))*ln(1-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2
)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*(a+b*arcsec(c*x))*ln(1+c*(-d)^
(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/
e^(3/2)+1/16*I*b*polylog(2,c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(
1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*I*b*polylog(2,-c*(-d)^(1...
```

Mathematica [A] (warning: unable to verify)

Time = 6.06 (sec) , antiderivative size = 1827, normalized size of antiderivative = 1.63

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/4*(a*x)/(e*(d + e*x^2)^2) + (a*x)/(8*d*e*(d + e*x^2)) + (a*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(3/2)) + b*(-1/16*(-(ArcSec[c*x]/(I*Sqrt[d]*
Sqrt[e] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sq
rt[e] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sq
rt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d])/(
d*e) - (-(ArcSec[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSin[1/(c*x)]/
Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d
) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]
*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d])/(16*d*e) - ((I/16)*(-(ArcSec[c*x]/(Sqr
t[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2)) + (ArcSin[1/(c*x)]/Sqrt[e] - I*((c*Sqr
t[d]*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/((c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]
*x)) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sqrt[e] + c*(c*
Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)*((-I)*
Sqrt[d] + Sqrt[e]*x))])/(c^2*d + e)^(3/2)))/d))/(Sqrt[d]*e) + ((I/16)*((I*
c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[
e]*x)) - ArcSec[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) + ArcSin[1/(c*x)]
/(d*Sqrt[e]) - (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*((-I)*Sqr
t[e] + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d
 + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(Sqrt[d]*e) + (8*
ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] ...
```

Rubi [A] (verified)

Time = 3.47 (sec) , antiderivative size = 1188, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5763, 5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^3}dx$$

$$\Big\downarrow \; 5763$$

$$-\int \frac{a+b\arccos\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^3x^2}d\frac{1}{x}$$

$$\Big\downarrow \; 5233$$

$$-\int \left(\frac{a+b\arccos\left(\frac{1}{cx}\right)}{d\left(\frac{d}{x^2}+e\right)^2}-\frac{e\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{d\left(\frac{d}{x^2}+e\right)^3}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\sqrt{1-\frac{1}{c^2x^2}}c}{16\sqrt{-d}\sqrt{e}\left(dc^2+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}c}{16\sqrt{-d}\sqrt{e}\left(dc^2+e\right)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{a+b\arccos\left(\frac{1}{cx}\right)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\arccos\left(\frac{1}{cx}\right)}{16de\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{a+b\arccos\left(\frac{1}{cx}\right)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}-\frac{a+b\arccos\left(\frac{1}{cx}\right)}{16\sqrt{-d}\sqrt{e}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2}+\frac{b\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}e\sqrt{dc^2+e}}-\frac{b\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\left(dc^2+e\right)^{3/2}}+\frac{b\,\text{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}e\sqrt{dc^2+e}}-\frac{b\,\text{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\left(dc^2+e\right)^{3/2}}-\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}+\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{16(-d)^{3/2}e^{3/2}}-\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}+\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{16(-d)^{3/2}e^{3/2}}-\frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}+\frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}-\frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}+\frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}$$

input
```
Int[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]
```

output
```
(b*c*Sqrt[1 - 1/(c^2*x^2)])/(16*Sqrt[-d]*Sqrt[e]*(c^2*d + e)*(Sqrt[-d]*Sqr
t[e] - d/x)) + (b*c*Sqrt[1 - 1/(c^2*x^2)])/(16*Sqrt[-d]*Sqrt[e]*(c^2*d + e
)*(Sqrt[-d]*Sqrt[e] + d/x)) + (a + b*ArcCos[1/(c*x)])/(16*Sqrt[-d]*Sqrt[e]
*(Sqrt[-d]*Sqrt[e] - d/x)^2) + (a + b*ArcCos[1/(c*x)])/(16*d*e*(Sqrt[-d]*S
qrt[e] - d/x)) - (a + b*ArcCos[1/(c*x)])/(16*Sqrt[-d]*Sqrt[e]*(Sqrt[-d]*Sq
rt[e] + d/x)^2) - (a + b*ArcCos[1/(c*x)])/(16*d*e*(Sqrt[-d]*Sqrt[e] + d/x)
) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*S
qrt[1 - 1/(c^2*x^2)])])/(16*d^(3/2)*(c^2*d + e)^(3/2)) + (b*ArcTanh[(c^2*d
 - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])
])/(16*d^(3/2)*e*Sqrt[c^2*d + e]) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])
/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(3/2)*(c^2*d
 + e)^(3/2)) + (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c
^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(3/2)*e*Sqrt[c^2*d + e]) - ((a +
b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - S
qrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) + ((a + b*ArcCos[1/(c*x)])*Log[1
 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-
d)^(3/2)*e^(3/2)) - ((a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcC
os[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) + ((a
+ b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] +
 Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) - ((I/16)*b*PolyLog[2, -((c...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 44.39 (sec) , antiderivative size = 1288, normalized size of antiderivative = 1.15

method	result	size
parts	Expression too large to display	1288
derivativedivides	Expression too large to display	1307
default	Expression too large to display	1307

input
```
int(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a*((1/8/d*x^3-1/8/e*x)/(e*x^2+d)^2+1/8/e/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1
/2)))+b/c^3*(1/8*x*c^5*(c^4*d*e*arcsec(c*x)*x^2-d^2*c^4*arcsec(c*x)-((c^2*
x^2-1)/c^2/x^2)^(1/2)*e^2*c^3*x^3-((c^2*x^2-1)/c^2/x^2)^(1/2)*c^3*d*e*x+e^
2*arcsec(c*x)*c^2*x^2-c^2*d*e*arcsec(c*x))/d/e/(c^2*d+e)/(c^2*e*x^2+c^2*d)
^2+1/16*I/(c^2*d+e)/d*c^4*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*l
n((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^
(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/8*I*(-(c^2
*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*c
*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-
2*e)*d)^(1/2))/(c^2*d+e)/e/d^3+1/8*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d
)^(1/2)*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2
)*c*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/
2)-2*e)*d)^(1/2))/(c^2*d+e)^2/e/d^3-1/8*I*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*
e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*c*arctan(c*d*(1/c/x+I*(1-1/c
^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)/e/d^
3+1/8*I*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-(e*(c^2*d+e))^(1/2)*
c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*c*arctan(c*d*(1/c/x+I*(1-1/
c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2/e
/d^3+1/16*I/(c^2*d+e)/e*c^6*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)
*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/...
```

Fricas [F]

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx=\int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^2}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^2*arcsec(c*x) + a*x^2)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx=\text{Timed out}$$

input
```
integrate(x**2*(a+b*asec(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{x^2\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^3}\,dx$$

input
```
int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^3,x)
```

output
```
int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx$$
$$= \frac{\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a\,d^2+2\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ade\,x^2+\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a\,e^2x^4+8\left(\int \frac{asec(cx}{e^3x^6+3d\,e^2x^4+\cdots}\right.\cdots}{8d^2e^2\cdots}$$

input
```
int(x^2*(a+b*asec(c*x))/(e*x^2+d)^3,x)
```

output
```
(sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d**2 + 2*sqrt(e)*sqrt(d)*
atan((e*x)/(sqrt(e)*sqrt(d)))*a*d*e*x**2 + sqrt(e)*sqrt(d)*atan((e*x)/(sqr
t(e)*sqrt(d)))*a*e**2*x**4 + 8*int((asec(c*x)*x**2)/(d**3 + 3*d**2*e*x**2
+ 3*d*e**2*x**4 + e**3*x**6),x)*b*d**4*e**2 + 16*int((asec(c*x)*x**2)/(d**
3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*d**3*e**3*x**2 + 8*int
((asec(c*x)*x**2)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*
d**2*e**4*x**4 - a*d**2*e*x + a*d*e**2*x**3)/(8*d**2*e**2*(d**2 + 2*d*e*x*
*2 + e**2*x**4))
```

3.110 $\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^3}\,dx$

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Optimal result

Integrand size = 18, antiderivative size = 1114

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

output
```
1/16*b*c*e^(1/2)*(1-1/c^2/x^2)^(1/2)/(-d)^(3/2)/(c^2*d+e)/((-d)^(1/2)*e^(1
/2)-d/x)+1/16*b*c*e^(1/2)*(1-1/c^2/x^2)^(1/2)/(-d)^(3/2)/(c^2*d+e)/((-d)^(
1/2)*e^(1/2)+d/x)+1/16*e^(1/2)*(a+b*arcsec(c*x))/(-d)^(3/2)/((-d)^(1/2)*e^
(1/2)-d/x)^2-5/16*(a+b*arcsec(c*x))/d^2/((-d)^(1/2)*e^(1/2)-d/x)-1/16*e^(1
/2)*(a+b*arcsec(c*x))/(-d)^(3/2)/((-d)^(1/2)*e^(1/2)+d/x)^2+5/16*(a+b*arcs
ec(c*x))/d^2/((-d)^(1/2)*e^(1/2)+d/x)+1/16*b*e*arctanh((c^2*d-(-d)^(1/2)*e
^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d+e)
^(3/2)-5/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/
2)/(1-1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d+e)^(1/2)+1/16*b*e*arctanh((c^2*d+(-
d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(5/2)
/(c^2*d+e)^(3/2)-5/16*b*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^
2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d+e)^(1/2)+3/16*(a+b*arcsec
(c*x))*ln(1-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^
(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*(a+b*arcsec(c*x))*ln(1+c*(-d)^(1/2)*(1/c/x
+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16
*(a+b*arcsec(c*x))*ln(1-c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2
)+(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*(a+b*arcsec(c*x))*ln(1+c*(-d)^
(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)/
e^(1/2)-3/16*I*b*polylog(2,c*(-d)^(1/2)*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/(e^(
1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*I*b*polylog(2,-c*(-d)^(1...
```

Mathematica [A] (warning: unable to verify)

Time = 6.04 (sec) , antiderivative size = 1812, normalized size of antiderivative = 1.63

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(d + e*x^2)^3,x]
```

output
```
(a*x)/(4*d*(d + e*x^2)^2) + (3*a*x)/(8*d^2*(d + e*x^2)) + (3*a*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]) + b*((-3*(-(ArcSec[c*x]/(I*Sqrt[d]*S
qrt[e] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sqr
t[e] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqr
t[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/(
16*d^2) - (3*(-(ArcSec[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSin[1/(
c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-
(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*S
qrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/(16*d^2) + ((I/16)*(-(ArcSec[c*
x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2)) + (ArcSin[1/(c*x)]/Sqrt[e] - I*
((c*Sqrt[d]*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/((c^2*d + e)*((-I)*Sqrt[d] +
Sqrt[e]*x)) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sqrt[e]
+ c*(c*Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)
*((-I)*Sqrt[d] + Sqrt[e]*x))])/(c^2*d + e)^(3/2)))/d))/d^(3/2) - ((I/16)*(
(I*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sq
rt[e]*x)) - ArcSec[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) + ArcSin[1/(c*
x)]/(d*Sqrt[e]) - (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*((-I)*
Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^
2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/d^(3/2) + (3*(
8*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d...
```

Rubi [A] (verified)

Time = 4.31 (sec) , antiderivative size = 1178, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5753, 5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^3}\,dx$$

$$\Big\downarrow \text{ 5753}$$

$$-\int \frac{a+b\arccos\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^3x^4}d\frac{1}{x}$$

$$\Big\downarrow \text{ 5233}$$

$$-\int \left(\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)e^2}{d^2\left(\frac{d}{x^2}+e\right)^3}-\frac{2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)e}{d^2\left(\frac{d}{x^2}+e\right)^2}+\frac{a+b\arccos\left(\frac{1}{cx}\right)}{d^2\left(\frac{d}{x^2}+e\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}c}{16(-d)^{3/2}(dc^2+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{b\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}c}{16(-d)^{3/2}(dc^2+e)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}-\frac{5\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{5\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{16d^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{\sqrt{e}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}-\frac{\sqrt{e}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{16(-d)^{3/2}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2}-\frac{5b\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\sqrt{dc^2+e}}+\frac{be\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}(dc^2+e)^{3/2}}-\frac{5b\text{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\sqrt{dc^2+e}}+\frac{be\text{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}(dc^2+e)^{3/2}}+\frac{3\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{16(-d)^{5/2}\sqrt{e}}+\frac{3\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{16(-d)^{5/2}\sqrt{e}}+\frac{3ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}+\frac{3ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}$$

input
```
Int[(a + b*ArcSec[c*x])/(d + e*x^2)^3,x]
```

output
```
(b*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)])/(16*(-d)^(3/2)*(c^2*d + e)*(Sqrt[-d]*S
qrt[e] - d/x)) + (b*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)])/(16*(-d)^(3/2)*(c^2*d
 + e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[e]*(a + b*ArcCos[1/(c*x)]))/(16*(-
d)^(3/2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) - (5*(a + b*ArcCos[1/(c*x)]))/(16*d^2
*(Sqrt[-d]*Sqrt[e] - d/x)) - (Sqrt[e]*(a + b*ArcCos[1/(c*x)]))/(16*(-d)^(3
/2)*(Sqrt[-d]*Sqrt[e] + d/x)^2) + (5*(a + b*ArcCos[1/(c*x)]))/(16*d^2*(Sqr
t[-d]*Sqrt[e] + d/x)) + (b*e*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqr
t[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*(c^2*d + e)^(3/2
)) - (5*b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e
]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*Sqrt[c^2*d + e]) + (b*e*ArcTanh[(c^
2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2
)])])/(16*d^(5/2)*(c^2*d + e)^(3/2)) - (5*b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqr
t[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*S
qrt[c^2*d + e]) + (3*(a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcC
os[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*(
a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e]
 - Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) + (3*(a + b*ArcCos[1/(c*x)])
*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/
(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E
^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5753
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(2*(p + 1)))
, x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 78.48 (sec) , antiderivative size = 1812, normalized size of antiderivative = 1.63

method	result	size
parts	Expression too large to display	1812
derivativedivides	Expression too large to display	1837
default	Expression too large to display	1837

input
```
int((a+b*arcsec(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x/d/(e*x^2+d)^2+3/8*a/d^2*x/(e*x^2+d)+3/8*a/d^2/(d*e)^(1/2)*arctan(e
*x/(d*e)^(1/2))+b/c*(1/8*x*c^3*(5*d^2*c^4*arcsec(c*x)+3*c^4*d*e*arcsec(c*x
)*x^2+((c^2*x^2-1)/c^2/x^2)^(1/2)*c^3*d*e*x+((c^2*x^2-1)/c^2/x^2)^(1/2)*e^
2*c^3*x^3+5*c^2*d*e*arcsec(c*x)+3*e^2*arcsec(c*x)*c^2*x^2)/d^2/(c^2*d+e)/(
c^2*e*x^2+c^2*d)^2-3/16*I/(c^2*d+e)/d*c^4*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e
)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/
x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^
2*d))-1/2*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2*d+e))^(1
/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e*arctanh(c*d*(1/c/x+I*
(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+
e)^2/d^5/c^3+1/2*I*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e
*(c^2*d+e))^(1/2)+2*e)*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((c^2*d+2*
(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))*e/c^3/d^5/(c^2*d+e)-5/8*I*(-(c^2*d-2*(e
*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*
(c^2*d+e))^(1/2)*e+2*e^2)*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2
*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/d^4/c+3/16*I/(c^2*d+e)
/d*c^4*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1
/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=Ro
otOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-5/8*I*((c^2*d+2*(e*(c^2*d+e))^(
1/2)+2*e)*d)^(1/2)*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e)...
```

Fricas [F]

$$\int \frac{a + b\sec^{-1}(cx)}{(d + ex^2)^3} \, dx = \int \frac{b \operatorname{arcsec}(cx) + a}{(ex^2 + d)^3} \, dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3),
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\sec^{-1}(cx)}{(d + ex^2)^3} \, dx = \text{Timed out}$$

input
```
integrate((a+b*asec(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\sec^{-1}(cx)}{(d + ex^2)^3} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^3}\,dx = \int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{(e\,x^2+d)^3}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(d + e*x^2)^3,x)
```

output
```
int((a + b*acos(1/(c*x)))/(d + e*x^2)^3, x)
```

Reduce [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^3}\,dx$$
$$= \frac{3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a\,d^2+6\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ade\,x^2+3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a\,e^2x^4+8\left(\int \frac{ase}{e^3x^6+3d\,e^2x}\right.}{8d}$$

input
```
int((a+b*asec(c*x))/(e*x^2+d)^3,x)
```

output
```
(3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d**2 + 6*sqrt(e)*sqrt(d
)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d*e*x**2 + 3*sqrt(e)*sqrt(d)*atan((e*x)/
(sqrt(e)*sqrt(d)))*a*e**2*x**4 + 8*int(asec(c*x)/(d**3 + 3*d**2*e*x**2 + 3
*d*e**2*x**4 + e**3*x**6),x)*b*d**5*e + 16*int(asec(c*x)/(d**3 + 3*d**2*e*
x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*d**4*e**2*x**2 + 8*int(asec(c*x)/(d
**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*d**3*e**3*x**4 + 5*a
*d**2*e*x + 3*a*d*e**2*x**3)/(8*d**3*e*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

3.111 $\int x^5\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx$

Optimal result 960
Mathematica [C] (warning: unable to verify) 961
Rubi [A] (verified) 962
Maple [F] 968
Fricas [A] (verification not implemented) 969
Sympy [F(-1)] 969
Maxima [F(-2)] 970
Giac [F] 970
Mupad [F(-1)] 971
Reduce [F] 971

Optimal result

Integrand size = 23, antiderivative size = 403

$$\begin{aligned}\int x^5\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx &= \frac{b(23c^4d^2+12c^2de-75e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} \\ &+ \frac{b(29c^2d-25e)\,x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e^2\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce^2\sqrt{c^2x^2}} \\ &+ \frac{d^2(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{3e^3} - \frac{2d(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5e^3} \\ &+ \frac{(d+ex^2)^{7/2}\,(a+b\sec^{-1}(cx))}{7e^3} + \frac{8bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{105e^3\sqrt{c^2x^2}} \\ &- \frac{b(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3)\,x\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{1680c^6e^{5/2}\sqrt{c^2x^2}}\end{aligned}$$

output
```
1/1680*b*(23*c^4*d^2+12*c^2*d*e-75*e^2)*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2
)/c^5/e^2/(c^2*x^2)^(1/2)+1/840*b*(29*c^2*d-25*e)*x*(c^2*x^2-1)^(1/2)*(e*x
^2+d)^(3/2)/c^3/e^2/(c^2*x^2)^(1/2)-1/42*b*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(
5/2)/c/e^2/(c^2*x^2)^(1/2)+1/3*d^2*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/e^3-2
/5*d*(e*x^2+d)^(5/2)*(a+b*arcsec(c*x))/e^3+1/7*(e*x^2+d)^(7/2)*(a+b*arcsec
(c*x))/e^3+8/105*b*c*d^(7/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^
(1/2))/e^3/(c^2*x^2)^(1/2)-1/1680*b*(105*c^6*d^3-35*c^4*d^2*e+63*c^2*d*e^2
+75*e^3)*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^6/e^(5/2
)/(c^2*x^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.41 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.85

$$\int x^5\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$= \frac{32a(d+ex^2)\left(8d^3-4d^2ex^2+3de^2x^4+15e^3x^6\right)-\frac{2be\sqrt{1-\frac{1}{c^2x^2}}\,x(d+ex^2)\left(75e^2+2c^2e(19d+25ex^2)+c^4(-41d^2+22dex^2+4\right.}{c^5}}{}$$

input
```
Integrate[x^5*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

output
```
(32*a*(d + e*x^2)*(8*d^3 - 4*d^2*e*x^2 + 3*d*e^2*x^4 + 15*e^3*x^6) - (2*b*
e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2)*(75*e^2 + 2*c^2*e*(19*d + 25*e*x^2)
+ c^4*(-41*d^2 + 22*d*e*x^2 + 40*e^2*x^4)))/c^5 + (128*b*d^4*Sqrt[1 + d/(e
*x^2)]*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))])/(c*x) + (b*e*(
105*c^6*d^3 - 35*c^4*d^2*e + 63*c^2*d*e^2 + 75*e^3)*Sqrt[1 - 1/(c^2*x^2)]*
x^3*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/(
c^5*Sqrt[1 - c^2*x^2]) + 32*b*(d + e*x^2)*(8*d^3 - 4*d^2*e*x^2 + 3*d*e^2*x
^4 + 15*e^3*x^6)*ArcSec[c*x])/(3360*e^3*Sqrt[d + e*x^2])
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.92, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {5761, 27, 7282, 2118, 27, 171, 27, 171, 27, 175, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{5761}$$

$$-\frac{bcx\int \frac{\left(ex^2+d\right)^{3/2}\left(15e^2x^4-12dex^2+8d^2\right)}{105e^3x\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bcx\int \frac{\left(ex^2+d\right)^{3/2}\left(15e^2x^4-12dex^2+8d^2\right)}{x\sqrt{c^2x^2-1}}dx}{105e^3\sqrt{c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \text{7282}$$

$$-\frac{bcx\int \frac{\left(ex^2+d\right)^{3/2}\left(15e^2x^4-12dex^2+8d^2\right)}{x^2\sqrt{c^2x^2-1}}dx^2}{210e^3\sqrt{c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \text{2118}$$

$$-\frac{bcx\left(\frac{\int \frac{3e\left(ex^2+d\right)^{3/2}\left(16c^2d^2-\left(29c^2d-25e\right)ex^2\right)}{2x^2\sqrt{c^2x^2-1}}dx^2}{3c^2e}+\frac{5e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{5/2}}{c^2}\right)}{210e^3\sqrt{c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow 27$$

$$-\frac{bcx\left(\frac{\int \frac{\left(ex^2+d\right)^{3/2}\left(16c^2d^2-\left(29c^2d-25e\right)ex^2\right)}{x^2\sqrt{c^2x^2-1}}dx^2}{2c^2}+\frac{5e\sqrt{c^2x^2-1}(d+ex^2)^{5/2}}{c^2}\right)}{210e^3\sqrt{c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow 171$$

$$-\frac{bcx\left(\frac{\frac{\int \frac{\sqrt{ex^2+d}\left(64c^4d^3-e\left(23d^2c^4+12dec^2-75e^2\right)x^2\right)}{2x^2\sqrt{c^2x^2-1}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(29c^2d-25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}}{2c^2}+\frac{5e\sqrt{c^2x^2-1}(d+ex^2)^{5/2}}{c^2}\right)}{210e^3\sqrt{c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow 27$$

$$-\frac{bcx\left(\frac{\frac{\int \frac{\sqrt{ex^2+d}\left(64c^4d^3-e\left(23d^2c^4+12dec^2-75e^2\right)x^2\right)}{x^2\sqrt{c^2x^2-1}}dx^2}{4c^2}-\frac{e\sqrt{c^2x^2-1}\left(29c^2d-25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}}{2c^2}+\frac{5e\sqrt{c^2x^2-1}(d+ex^2)^{5/2}}{c^2}\right)}{210e^3\sqrt{c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow 171$$

$$-\frac{bcx\left(\frac{\frac{\frac{\int \frac{128d^4c^6+e\left(105d^3c^6-35d^2ec^4+63de^2c^2+75e^3\right)x^2}{2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2}-\frac{e\sqrt{c^2x^2-1}\left(23c^4d^2+12c^2de-75e^2\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{e\sqrt{c^2x^2-1}\left(29c^2d-25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}}{2c^2}+\cdots\right)}{210e^3\sqrt{c^2x^2}}$$

$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}$$

$\downarrow$ 27

$$-\frac{bcx\left(\frac{\frac{\frac{\int \frac{128d^4c^6+e\left(105d^3c^6-35d^2ec^4+63de^2c^2+75e^3\right)x^2}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(23c^4d^2+12c^2de-75e^2\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{e\sqrt{c^2x^2-1}\left(29c^2d-25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}}{2c^2}+\cdots\right)}{210e^3\sqrt{c^2x^2}}$$

$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}$$

$\downarrow$ 175

$$-\frac{bcx\left(\frac{\frac{\frac{128c^6d^4\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+e\left(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3\right)\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(23c^4d^2+12c^2de-75e^2\right)\sqrt{d+ex^2}}{c^2}}{4c^2}\cdots}{2c^2}\right)}{210e^3\sqrt{c^2x^2}}$$

$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}$$

$\downarrow$ 66

$$-\frac{bcx\left(\frac{\frac{\frac{128c^6d^4\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\left(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3\right)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(23c^4d^2+12c^2de-75e^2\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-}{2c^2}\right.}{210e^3\sqrt{c^2x^2}}$$

$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}$$

$\downarrow$ 104

$$-\frac{bcx\left(\frac{\frac{\frac{256c^6d^4\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e\left(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3\right)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(23c^4d^2+12c^2de-75e^2\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{e\sqrt{c^2}}{}}{2c^2}\right.}{210e^3\sqrt{c^2x^2}}$$

$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}$$

$\downarrow$ 217

$$-\frac{bcx\left(\frac{\frac{\frac{2e\left(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3\right)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-256c^6d^{7/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(23c^4d^2+12c^2de-75e^2\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{e}{}}{2c^2}\right.}{210e^3\sqrt{c^2x^2}}$$

$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}$$

$\downarrow$ 221

$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}-$$

$$\frac{bcx\left(\frac{\frac{\frac{\frac{2\sqrt{e}\left(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3\right)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-256c^6d^{7/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(23c^4d^2+12c^2de-75e^2\right)\sqrt{d+ex^2}}{c^2}}{4c^2}}{2c^2}\right)}{210e^3\sqrt{c^2x^2}}$$

input
```
Int[x^5*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

output
```
(d^2*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/(3*e^3) - (2*d*(d + e*x^2)^(5/
2)*(a + b*ArcSec[c*x]))/(5*e^3) + ((d + e*x^2)^(7/2)*(a + b*ArcSec[c*x]))/
(7*e^3) - (b*c*x*((5*e*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(5/2))/c^2 + (-1/2*(
(29*c^2*d - 25*e)*e*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/c^2 + (-((e*(23*
c^4*d^2 + 12*c^2*d*e - 75*e^2)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/c^2) +
(-256*c^6*d^(7/2)*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])] + (
2*Sqrt[e]*(105*c^6*d^3 - 35*c^4*d^2*e + 63*c^2*d*e^2 + 75*e^3)*ArcTanh[(Sq
rt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/c)/(2*c^2))/(4*c^2))/(2*c^
2)))/(210*e^3*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

Maple [F]

$$\int x^5\sqrt{e\,x^2+d}\,(a+b\,\mathrm{arcsec}\,(cx))\,dx$$

input
```
int(x^5*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x)
```

output
```
int(x^5*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x)
```

Fricas [A] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 1701, normalized size of antiderivative = 4.22

$$\int x^5\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
[1/6720*(128*b*c^7*sqrt(-d)*d^3*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(
c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^
2 + d)*sqrt(-d) + 8*d^2)/x^4) + (105*b*c^6*d^3 - 35*b*c^4*d^2*e + 63*b*c^2
*d*e^2 + 75*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^
4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqr
t(e*x^2 + d)*sqrt(e) + e^2) + 4*(240*a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 -
64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 + 16*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*
x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*arcsec(c*x) - (40*b*c^5*e^3*x^4 - 4
1*b*c^5*d^2*e + 38*b*c^3*d*e^2 + 75*b*c*e^3 + 2*(11*b*c^5*d*e^2 + 25*b*c^3
*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^7*e^3), 1/6720*(256*b*c^
7*d^(7/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2
 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (105*b*c^6*d^3
- 35*b*c^4*d^2*e + 63*b*c^2*d*e^2 + 75*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 +
c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d -
 c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(240*a*c^7*e^3*
x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 + 16*(15*b*c
^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*arcsec(c
*x) - (40*b*c^5*e^3*x^4 - 41*b*c^5*d^2*e + 38*b*c^3*d*e^2 + 75*b*c*e^3 + 2
*(11*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/
(c^7*e^3), 1/3360*(64*b*c^7*sqrt(-d)*d^3*log(((c^4*d^2 - 6*c^2*d*e + e^...
```

Sympy [F(-1)]

Timed out.

$$\int x^5\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Timed out}$$

input
```
integrate(x**5*(e*x**2+d)**(1/2)*(a+b*asec(c*x)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^5\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int x^5\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\,\mathrm{arcsec}\,(cx)+a)x^5\,dx$$

input
```
integrate(x^5*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int x^5\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^5*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))),x)
```

output
```
int(x^5*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))), x)
```

Reduce [F]

$$\int x^5\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{8\sqrt{e\,x^2+d}\,a\,d^3-4\sqrt{e\,x^2+d}\,a\,d^2e\,x^2+3\sqrt{e\,x^2+d}\,ad\,e^2x^4+15\sqrt{e\,x^2+d}\,a\,e^3x^6+105\left(\int\sqrt{e\,x^2+d}\,ase}{105e^3}$$

input
```
int(x^5*(e*x^2+d)^(1/2)*(a+b*asec(c*x)),x)
```

output
```
(8*sqrt(d + e*x**2)*a*d**3 - 4*sqrt(d + e*x**2)*a*d**2*e*x**2 + 3*sqrt(d +
 e*x**2)*a*d*e**2*x**4 + 15*sqrt(d + e*x**2)*a*e**3*x**6 + 105*int(sqrt(d
+ e*x**2)*asec(c*x)*x**5,x)*b*e**3)/(105*e**3)
```

3.112 $\int x^3\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx$

Optimal result 972
Mathematica [C] (warning: unable to verify) 973
Rubi [A] (verified) 973
Maple [F] 978
Fricas [A] (verification not implemented) 978
Sympy [F] 979
Maxima [F(-2)] 979
Giac [F] 979
Mupad [F(-1)] 980
Reduce [F] 980

Optimal result

Integrand size = 23, antiderivative size = 294

$$\int x^3\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx = -\frac{b(c^2d+9e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5e^2} - \frac{2bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{15e^2\sqrt{c^2x^2}} + \frac{b(15c^4d^2-10c^2de-9e^2)\,x\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{3/2}\sqrt{c^2x^2}}$$

output
```
-1/120*b*(c^2*d+9*e)*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c^3/e/(c^2*x^2)^(
1/2)-1/20*b*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(3/2)/c/e/(c^2*x^2)^(1/2)-1/3*d*
(e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/e^2+1/5*(e*x^2+d)^(5/2)*(a+b*arcsec(c*x)
)/e^2-2/15*b*c*d^(5/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))
/e^2/(c^2*x^2)^(1/2)+1/120*b*(15*c^4*d^2-10*c^2*d*e-9*e^2)*x*arctanh(e^(1/
2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^4/e^(3/2)/(c^2*x^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.53 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.89

$$\int x^3\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{16a(d+ex^2)^2(-2d+3ex^2)-\frac{2be\sqrt{1-\frac{1}{c^2x^2}}\,x(d+ex^2)(9e+c^2(7d+6ex^2))}{c^3}+\frac{b\left(-16c^2d^3\sqrt{1+\frac{d}{ex^2}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right.\right.}{}}{240e^2\sqrt{d}}$$

input
```
Integrate[x^3*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

output
```
(16*a*(d + e*x^2)^2*(-2*d + 3*e*x^2) - (2*b*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d +
 e*x^2)*(9*e + c^2*(7*d + 6*e*x^2)))/c^3 + (b*(-16*c^2*d^3*Sqrt[1 + d/(e*x
^2)]*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))] + (e*(-15*c^4*d^2
 + 10*c^2*d*e + 9*e^2)*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*Appel
lF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/Sqrt[1 - c^2*x^2]))/(c^3*x) +
16*b*(d + e*x^2)^2*(-2*d + 3*e*x^2)*ArcSec[c*x])/(240*e^2*Sqrt[d + e*x^2])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.91, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {5761, 27, 435, 171, 27, 171, 27, 175, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \; 5761$$

$$-\frac{bcx\int -\frac{(2d-3ex^2)(ex^2+d)^{3/2}}{15e^2x\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{(d+ex^2)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}-\frac{d(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow\ 27$$

$$\frac{bcx\int \frac{(2d-3ex^2)(ex^2+d)^{3/2}}{x\sqrt{c^2x^2-1}}dx}{15e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}-\frac{d(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow\ 435$$

$$\frac{bcx\int \frac{(2d-3ex^2)(ex^2+d)^{3/2}}{x^2\sqrt{c^2x^2-1}}dx^2}{30e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}-\frac{d(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow\ 171$$

$$\frac{bcx\left(\frac{\int \frac{\sqrt{ex^2+d}\left(8c^2d^2-e\left(dc^2+9e\right)x^2\right)}{2x^2\sqrt{c^2x^2-1}}dx^2}{2c^2}-\frac{3e\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{30e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}-\frac{d(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow\ 27$$

$$\frac{bcx\left(\frac{\int \frac{\sqrt{ex^2+d}\left(8c^2d^2-e\left(dc^2+9e\right)x^2\right)}{x^2\sqrt{c^2x^2-1}}dx^2}{4c^2}-\frac{3e\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{30e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}-\frac{d(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow\ 171$$

$$\frac{bcx\left(\frac{\frac{\int \frac{16d^3c^4+e\left(15d^2c^4-10dec^2-9e^2\right)x^2}{2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2}-\frac{e\sqrt{c^2x^2-1}\left(c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3e\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{30e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}-\frac{d(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow\ 27$$

$$\frac{bcx\left(\frac{\frac{\int \frac{16d^3c^4+e\left(15d^2c^4-10dec^2-9e^2\right)x^2}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \; 175$$

$$\frac{bcx\left(\frac{\frac{16c^4d^3\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+e\left(15c^4d^2-10c^2de-9e^2\right)\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \; 66$$

$$\frac{bcx\left(\frac{\frac{16c^4d^3\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\left(15c^4d^2-10c^2de-9e^2\right)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \; 104$$

$$\frac{bcx\left(\frac{\frac{32c^4d^3\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e\left(15c^4d^2-10c^2de-9e^2\right)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \; 217$$

$$\frac{bcx\left(\frac{\frac{2e\left(15c^4d^2-10c^2de-9e^2\right)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-32c^4d^{5/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \; 221$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}+\frac{bcx\left(\frac{\frac{\frac{2\sqrt{e}\left(15c^4d^2-10c^2de-9e^2\right)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-32c^4d^{5/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)}{2c^2}\right.}{30e^2\sqrt{c^2x^2}}$$

input
```
Int[x^3*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

output
```
-1/3*(d*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/e^2 + ((d + e*x^2)^(5/2)*(a
 + b*ArcSec[c*x]))/(5*e^2) + (b*c*x*((-3*e*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^
(3/2))/(2*c^2) + (-((e*(c^2*d + 9*e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/c
^2) + (-32*c^4*d^(5/2)*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])
] + (2*Sqrt[e]*(15*c^4*d^2 - 10*c^2*d*e - 9*e^2)*ArcTanh[(Sqrt[e]*Sqrt[-1
+ c^2*x^2])/(c*Sqrt[d + e*x^2])])/c)/(2*c^2))/(4*c^2)))/(30*e^2*Sqrt[c^2*x
^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [F]

$$\int x^3\sqrt{e\,x^2+d}\,(a+b\,\mathrm{arcsec}\,(cx))\,dx$$

input
```
int(x^3*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x)
```

output
```
int(x^3*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x)
```

Fricas [A] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 1383, normalized size of antiderivative = 4.70

$$\int x^3\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
[1/480*(16*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^
2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2
+ d)*sqrt(-d) + 8*d^2)/x^4) - (15*b*c^4*d^2 - 10*b*c^2*d*e - 9*b*e^2)*sqrt
(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 -
4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) +
e^2) + 4*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 + 8*(3*b*c^5*e
^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*arcsec(c*x) - (6*b*c^3*e^2*x^2 + 7*b
*c^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e^2), -1/48
0*(32*b*c^5*d^(5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*
sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (15*b
*c^4*d^2 - 10*b*c^2*d*e - 9*b*e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6
*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(
c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 4*(24*a*c^5*e^2*x^4 + 8*a*c^
5*d*e*x^2 - 16*a*c^5*d^2 + 8*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^
2)*arcsec(c*x) - (6*b*c^3*e^2*x^2 + 7*b*c^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2
- 1))*sqrt(e*x^2 + d))/(c^5*e^2), 1/240*(8*b*c^5*sqrt(-d)*d^2*log(((c^4*d^
2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c
^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) - (15*b*c^4*d^
2 - 10*b*c^2*d*e - 9*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*
sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*...
```

Sympy [F]

$$\int x^3\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int x^3(a+b\operatorname{asec}(cx))\sqrt{d+ex^2}\,dx$$

input
```
integrate(x**3*(e*x**2+d)**(1/2)*(a+b*asec(c*x)),x)
```

output
```
Integral(x**3*(a + b*asec(c*x))*sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^3\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^3*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int x^3\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int\sqrt{ex^2+d}(b\,\mathrm{arcsec}(cx)+a)x^3\,dx$$

input
```
integrate(x^3*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int x^3\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^3*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))),x)
```

output
```
int(x^3*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))), x)
```

Reduce [F]

$$\int x^3\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{-2\sqrt{e\,x^2+d}\,a\,d^2+\sqrt{e\,x^2+d}\,ade\,x^2+3\sqrt{e\,x^2+d}\,a\,e^2x^4+15\left(\int\sqrt{e\,x^2+d}\,asec(cx)\,x^3dx\right)b\,e^2}{15e^2}$$

input
```
int(x^3*(e*x^2+d)^(1/2)*(a+b*asec(c*x)),x)
```

output
```
( - 2*sqrt(d + e*x**2)*a*d**2 + sqrt(d + e*x**2)*a*d*e*x**2 + 3*sqrt(d + e
*x**2)*a*e**2*x**4 + 15*int(sqrt(d + e*x**2)*asec(c*x)*x**3,x)*b*e**2)/(15
*e**2)
```

3.113 $\int x\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx$

Optimal result 981
Mathematica [C] (warning: unable to verify) 982
Rubi [A] (verified) 982
Maple [F] 986
Fricas [A] (verification not implemented) 986
Sympy [F] 987
Maxima [F] 988
Giac [F] 988
Mupad [F(-1)] 988
Reduce [F] 989

Optimal result

Integrand size = 21, antiderivative size = 195

$$\begin{aligned}\int x\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx =& -\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}} \\ &+ \frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e} \\ &+ \frac{bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e\sqrt{c^2x^2}} \\ &- \frac{b\left(3c^2d+e\right)x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2\sqrt{e}\sqrt{c^2x^2}}\end{aligned}$$

output
```
-1/6*b*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c/(c^2*x^2)^(1/2)+1/3*(e*x^2+d)
^(3/2)*(a+b*arcsec(c*x))/e+1/3*b*c*d^(3/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2
)/(c^2*x^2-1)^(1/2))/e/(c^2*x^2)^(1/2)-1/6*b*(3*c^2*d+e)*x*arctanh(e^(1/2)
*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^2/e^(1/2)/(c^2*x^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.45 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.09

$$\int x\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{\frac{2bd^2\sqrt{1+\frac{d}{ex^2}}\operatorname{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)}{cex}+\frac{\frac{b\left(3c^2d+e\right)\sqrt{1-\frac{1}{c^2x^2}}x^3\sqrt{1+\frac{ex^2}{d}}\operatorname{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,c^2x^2,-\frac{ex^2}{d}\right)}{\sqrt{1-c^2x^2}}+\frac{2\left(d+ex^2\right)\left(-be\sqrt{1-\frac{1}{c^2x^2}}x+2a\right.}{c}}{12\sqrt{d+ex^2}}$$

input
```
Integrate[x*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

output
```
((2*b*d^2*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e
*x^2))])/(c*e*x) + ((b*(3*c^2*d + e)*Sqrt[1 - 1/(c^2*x^2)]*x^3*Sqrt[1 + (e
*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/Sqrt[1 - c^2*x^2
] + (2*(d + e*x^2)*(-(b*e*Sqrt[1 - 1/(c^2*x^2)]*x) + 2*a*c*(d + e*x^2) + 2
*b*c*(d + e*x^2)*ArcSec[c*x]))/e)/c)/(12*Sqrt[d + e*x^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5759, 354, 113, 27, 175, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{5759}$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{bcx\int\frac{\left(ex^2+d\right)^{3/2}}{x\sqrt{c^2x^2-1}}dx}{3e\sqrt{c^2x^2}}$$

$$\Big\downarrow \text{354}$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{bcx\int\frac{\left(ex^2+d\right)^{3/2}}{x^2\sqrt{c^2x^2-1}}dx^2}{6e\sqrt{c^2x^2}}$$

$$\Big\downarrow \text{113}$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{bcx\left(\frac{\int\frac{2c^2d^2+e\left(3dc^2+e\right)x^2}{2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2}+\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e\sqrt{c^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{bcx\left(\frac{\int\frac{2c^2d^2+e\left(3dc^2+e\right)x^2}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e\sqrt{c^2x^2}}$$

$$\Big\downarrow \text{175}$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{bcx\left(\frac{2c^2d^2\int\frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+e\left(3c^2d+e\right)\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e\sqrt{c^2x^2}}$$

$$\Big\downarrow \text{66}$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{bcx\left(\frac{2c^2d^2\int\frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\left(3c^2d+e\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e\sqrt{c^2x^2}}$$

$$\Big\downarrow \text{104}$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{bcx\left(\frac{4c^2d^2\int\frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e\left(3c^2d+e\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e\sqrt{c^2x^2}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{bcx\left(\frac{2e\left(3c^2d+e\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-4c^2d^{3/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}+\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e\sqrt{c^2x^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{bcx\left(\frac{\frac{2\sqrt{e}\left(3c^2d+e\right)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-4c^2d^{3/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}+\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e\sqrt{c^2x^2}}$$

input
```
Int[x*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

output
```
((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/(3*e) - (b*c*x*((e*Sqrt[-1 + c^2*x
^2]*Sqrt[d + e*x^2])/c^2 + (-4*c^2*d^(3/2)*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]
*Sqrt[-1 + c^2*x^2])] + (2*Sqrt[e]*(3*c^2*d + e)*ArcTanh[(Sqrt[e]*Sqrt[-1
+ c^2*x^2])/(c*Sqrt[d + e*x^2])])/c)/(2*c^2)))/(6*e*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 113

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 175

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5759
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSec[c*x])/(2*e*(p + 1))), x
] - Simp[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2]))   Int[(d + e*x^2)^(p + 1)/(x*S
qrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Maple [F]

$$\int x\sqrt{e\,x^{2}+d}\,(a+b\,\mathrm{arcsec}\,(cx))\,dx$$

input
```
int(x*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x)
```

output
```
int(x*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x)
```

Fricas [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 1100, normalized size of antiderivative = 5.64

$$\int x\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\text{Too large to display}$$

input
```
integrate(x*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
[1/24*(2*b*c^3*sqrt(-d)*d*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^
2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)
*sqrt(-d) + 8*d^2)/x^4) + (3*b*c^2*d + b*e)*sqrt(e)*log(8*c^4*e^2*x^4 + c^
4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c
*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(2*a*c^3*e*x^2 +
2*a*c^3*d - sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 + b*c^3*d)*arcsec(c*x
))*sqrt(e*x^2 + d))/(c^3*e), 1/24*(4*b*c^3*d^(3/2)*arctan(-1/2*sqrt(c^2*x^
2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2
*d^2 - d*e)*x^2 - d^2)) + (3*b*c^2*d + b*e)*sqrt(e)*log(8*c^4*e^2*x^4 + c^
4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c
*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(2*a*c^3*e*x^2 +
2*a*c^3*d - sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 + b*c^3*d)*arcsec(c*x
))*sqrt(e*x^2 + d))/(c^3*e), 1/12*(b*c^3*sqrt(-d)*d*log(((c^4*d^2 - 6*c^2*
d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*
x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + (3*b*c^2*d + b*e)*sqrt
(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d
)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(2*a*c^3*e*x
^2 + 2*a*c^3*d - sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 + b*c^3*d)*arcse
c(c*x))*sqrt(e*x^2 + d))/(c^3*e), 1/12*(2*b*c^3*d^(3/2)*arctan(-1/2*sqrt(c
^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^...
```

Sympy [F]

$$\int x\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int x(a+b\,\mathrm{asec}\,(cx))\,\sqrt{d+ex^2}\,dx$$

input
```
integrate(x*(e*x**2+d)**(1/2)*(a+b*asec(c*x)),x)
```

output
```
Integral(x*(a + b*asec(c*x))*sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int x\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\mathrm{arcsec}\,(cx)+a)x\,dx$$

input
```
integrate(x*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/3*(e*x^2 + d)^(3/2)*a/e + 1/3*((e*x^2 + d)^(3/2)*arctan(sqrt(c*x + 1)*sq
rt(c*x - 1)) - 3*e*integrate((3*(c^2*e*x^3 - e*x + (c^2*e*x^3 - e*x)*e^(lo
g(c*x + 1) + log(c*x - 1)))*sqrt(e*x^2 + d)*log(x) + (3*c^2*e*x^3*log(c) -
 3*e*x*log(c) + ((3*c^2*log(c) + c^2)*e*x^3 + (c^2*d - 3*e*log(c))*x)*e^(l
og(c*x + 1) + log(c*x - 1)))*sqrt(e*x^2 + d))/(c^2*e*x^2 + (c^2*e*x^2 - e)
*e^(log(c*x + 1) + log(c*x - 1)) - e), x))*b/e
```

Giac [F]

$$\int x\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\mathrm{arcsec}\,(cx)+a)x\,dx$$

input
```
integrate(x*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int x\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))),x)
```

output
```
int(x*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))), x)
```

Reduce [F]

$$\int x\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{\sqrt{e\,x^2+d}\,ad+\sqrt{e\,x^2+d}\,ae\,x^2+3\left(\int\sqrt{e\,x^2+d}\,asec(cx)\,xdx\right)be}{3e}$$

input
```
int(x*(e*x^2+d)^(1/2)*(a+b*asec(c*x)),x)
```

output
```
(sqrt(d + e*x**2)*a*d + sqrt(d + e*x**2)*a*e*x**2 + 3*int(sqrt(d + e*x**2)
*asec(c*x)*x,x)*b*e)/(3*e)
```

3.114 $\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x}\,dx$

Optimal result . . . 990
Mathematica [N/A] . . . 990
Rubi [N/A] . . . 991
Maple [N/A] . . . 991
Fricas [N/A] . . . 992
Sympy [N/A] . . . 992
Maxima [F(-2)] . . . 992
Giac [N/A] . . . 993
Mupad [N/A] . . . 993
Reduce [N/A] . . . 994

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x}\,dx = \text{Int}\left(\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x},x\right)$$

output
```
Defer(Int)((e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/x,x)
```

Mathematica [N/A]

Not integrable

Time = 6.97 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x}\,dx$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x,x]
```

output
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x} dx$$

$$\Big\downarrow \text{ 5771}$$

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x}dx$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{e\,x^2+d}\,(a+b\ \mathrm{arcsec}\,(cx))}{x}dx$$

input
```
int((e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/x,x)
```

output
```
int((e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\mathrm{arcsec}\,(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/x,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 15.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{(a+b\,\mathrm{asec}\,(cx))\sqrt{d+ex^2}}{x}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(a+b*asec(c*x))/x,x)
```

output
```
Integral((a + b*asec(c*x))*sqrt(d + e*x**2)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x}\,dx=\int \frac{\sqrt{ex^2+d}(b\,\mathrm{arcsec}\,(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/x,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x}\,dx=\int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x,x)
```

output
```
int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.65

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x}\,dx = \sqrt{e\,x^2+d}\,a+\sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)a - \sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)a + \left(\int \frac{\sqrt{e\,x^2+d}\,asec(cx)}{x}dx\right)b$$

input
```
int((e*x^2+d)^(1/2)*(a+b*asec(c*x))/x,x)
```

output
```
sqrt(d + e*x**2)*a + sqrt(d)*log((sqrt(d + e*x**2) - sqrt(d) + sqrt(e)*x)/
sqrt(d))*a - sqrt(d)*log((sqrt(d + e*x**2) + sqrt(d) + sqrt(e)*x)/sqrt(d))
*a + int((sqrt(d + e*x**2)*asec(c*x))/x,x)*b
```

3.115 $\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx$

Optimal result 995
Mathematica [N/A] 995
Rubi [N/A] 996
Maple [N/A] 996
Fricas [N/A] 997
Sympy [N/A] 997
Maxima [F(-2)] 997
Giac [N/A] 998
Mupad [N/A] 998
Reduce [N/A] 999

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx = \text{Int}\left(\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3},x\right)$$

output
```
Defer(Int)((e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/x^3,x)
```

Mathematica [N/A]

Not integrable

Time = 4.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^3,x]
```

output
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{x^3}\,dx$$

$$\Big\downarrow \text{5771}$$

$$\int \frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{x^3}dx$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 2.47 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{e\,x^2+d}\,(a+b\ \mathrm{arcsec}\,(cx))}{x^3}dx$$

input
```
int((e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/x^3,x)
```

output
```
int((e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\mathrm{arcsec}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 20.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{asec}\,(cx))\sqrt{d+ex^2}}{x^3}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(a+b*asec(c*x))/x**3,x)
```

output
```
Integral((a + b*asec(c*x))*sqrt(d + e*x**2)/x**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\mathrm{arcsec}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^3,x)
```

output
```
int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 4.65

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx$$
$$= \frac{-\sqrt{e\,x^2+d}\,ad+\sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)ae\,x^2-\sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)ae\,x^2+2\left(\int\frac{\sqrt{e\,x^2+d}\,asec(cx)}{x^3}d}{2d\,x^2}$$

input
```
int((e*x^2+d)^(1/2)*(a+b*asec(c*x))/x^3,x)
```

output
```
( - sqrt(d + e*x**2)*a*d + sqrt(d)*log((sqrt(d + e*x**2) - sqrt(d) + sqrt(
e)*x)/sqrt(d))*a*e*x**2 - sqrt(d)*log((sqrt(d + e*x**2) + sqrt(d) + sqrt(e
)*x)/sqrt(d))*a*e*x**2 + 2*int((sqrt(d + e*x**2)*asec(c*x))/x**3,x)*b*d*x*
*2)/(2*d*x**2)
```

3.116 $\int x^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx$

Optimal result 1000
Mathematica [N/A] 1000
Rubi [N/A] 1001
Maple [N/A] 1001
Fricas [N/A] 1002
Sympy [F(-1)] 1002
Maxima [F(-2)] 1002
Giac [N/A] 1003
Mupad [N/A] 1003
Reduce [N/A] 1003

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx=\text{Int}\left(x^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx)),x\right)$$

output
```
Defer(Int)(x^2*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 11.40 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx=\int x^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx$$

input
```
Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

output
```
Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{5771}$$

$$\int x^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)dx$$

input
```
Int[x^2*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^2\sqrt{e\,x^2+d}\,(a+b\,\operatorname{arcsec}(cx))\,dx$$

input
```
int(x^2*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x)
```

output
```
int(x^2*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\text{arcsec}\,(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral((b*x^2*arcsec(c*x) + a*x^2)*sqrt(e*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\text{Timed out}$$

input
```
integrate(x**2*(e*x**2+d)**(1/2)*(a+b*asec(c*x)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\operatorname{arcsec}(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int x^2\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^2*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))),x)
```

output
```
int(x^2*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.78

$$\int x^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{\sqrt{e\,x^2+d}\,adex+2\sqrt{e\,x^2+d}\,a\,e^2x^3-\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)a\,d^2+8\left(\int\sqrt{e\,x^2+d}\,asec(cx)\,x^2dx\right)b\,e^2}{8e^2}$$

input
```
int(x^2*(e*x^2+d)^(1/2)*(a+b*asec(c*x)),x)
```

output
```
(sqrt(d + e*x**2)*a*d*e*x + 2*sqrt(d + e*x**2)*a*e**2*x**3 - sqrt(e)*log((
sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*d**2 + 8*int(sqrt(d + e*x**2)*ase
c(c*x)*x**2,x)*b*e**2)/(8*e**2)
```

3.117 $\int \sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx$

Optimal result 1005
Mathematica [N/A] 1005
Rubi [N/A] 1006
Maple [N/A] 1006
Fricas [N/A] 1007
Sympy [N/A] 1007
Maxima [F(-2)] 1007
Giac [N/A] 1008
Mupad [N/A] 1008
Reduce [N/A] 1009

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx = \text{Int}\left(\sqrt{d+ex^2}(a+b\sec^{-1}(cx)),x\right)$$

output
```
Defer(Int)((e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 16.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx = \int \sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx$$

input
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

output
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex^2}(a+b\sec^{-1}(cx))\, dx$$

$$\Big\downarrow \text{5771}$$

$$\int \sqrt{d+ex^2}(a+b\sec^{-1}(cx))\, dx$$

input
```
Int[Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \sqrt{e\,x^2+d}\,(a+b\ \mathrm{arcsec}\,(cx))\,dx$$

input
```
int((e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x)
```

output
```
int((e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\mathrm{arcsec}\,(cx)+a)\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 55.45 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int (a+b\,\mathrm{asec}\,(cx))\sqrt{d+ex^2}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(a+b*asec(c*x)),x)
```

output
```
Integral((a + b*asec(c*x))*sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}\,(b\,\mathrm{arcsec}\,(cx)+a)\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int \sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))),x)
```

output
```
int((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.05

$$\int \sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{\sqrt{e\,x^2+d}\,aex+\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)ad+2\left(\int\sqrt{e\,x^2+d}\,asec(cx)\,dx\right)be}{2e}$$

input
```
int((e*x^2+d)^(1/2)*(a+b*asec(c*x)),x)
```

output
```
(sqrt(d + e*x**2)*a*e*x + sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(
d))*a*d + 2*int(sqrt(d + e*x**2)*asec(c*x),x)*b*e)/(2*e)
```

3.118 $\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx$

Optimal result 1010
Mathematica [N/A] 1010
Rubi [N/A] 1011
Maple [N/A] 1011
Fricas [N/A] 1012
Sympy [N/A] 1012
Maxima [F(-2)] 1012
Giac [N/A] 1013
Mupad [N/A] 1013
Reduce [N/A] 1014

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \text{Int}\left(\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2},x\right)$$

output
```
Defer(Int)((e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^2,x]
```

output
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}\,(a+b\sec^{-1}(cx))}{x^2}\,dx$$
$$\Big\downarrow \; 5771$$
$$\int \frac{\sqrt{d+ex^2}\,(a+b\sec^{-1}(cx))}{x^2}dx$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{e\,x^2+d}\,(a+b\,\operatorname{arcsec}(cx))}{x^2}dx$$

input
```
int((e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/x^2,x)
```

output
```
int((e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\mathrm{arcsec}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 12.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \int \frac{(a+b\,\mathrm{asec}\,(cx))\sqrt{d+ex^2}}{x^2}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(a+b*asec(c*x))/x**2,x)
```

output
```
Integral((a + b*asec(c*x))*sqrt(d + e*x**2)/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\mathrm{arcsec}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^2,x)
```

output
```
int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.91

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx$$
$$= \frac{-\sqrt{e\,x^2+d}\,a+\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)ax-\sqrt{e}\,ax+\left(\int\frac{\sqrt{e\,x^2+d}\,asec(cx)}{x^2}dx\right)bx}{x}$$

input
```
int((e*x^2+d)^(1/2)*(a+b*asec(c*x))/x^2,x)
```

output
```
( - sqrt(d + e*x**2)*a + sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d
))*a*x - sqrt(e)*a*x + int((sqrt(d + e*x**2)*asec(c*x))/x**2,x)*b*x)/x
```

3.119 $\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^4}\,dx$

Optimal result 1015
Mathematica [C] (verified) 1016
Rubi [A] (verified) 1016
Maple [F] 1021
Fricas [A] (verification not implemented) 1021
Sympy [F] 1022
Maxima [F(-2)] 1022
Giac [F] 1022
Mupad [F(-1)] 1023
Reduce [F] 1023

Optimal result

Integrand size = 23, antiderivative size = 328

$$\begin{aligned}\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^4}\,dx &= \frac{2bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} \\ &\quad+\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}}-\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3dx^3} \\ &\quad-\frac{2bc^2(c^2d+2e)\,x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\,E\left(\arcsin(cx)\,\middle|\,-\frac{e}{c^2d}\right)}{9d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\ &\quad+\frac{b(c^2d+e)(2c^2d+3e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\end{aligned}$$

output

```
2/9*b*c*(c^2*d+2*e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/(c^2*x^2)^(1/2)+1/
9*b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/x^2/(c^2*x^2)^(1/2)-1/3*(e*x^2+d)^
(3/2)*(a+b*arcsec(c*x))/d/x^3-2/9*b*c^2*(c^2*d+2*e)*x*(-c^2*x^2+1)^(1/2)*(
e*x^2+d)^(1/2)*EllipticE(c*x,(-e/c^2/d)^(1/2))/d/(c^2*x^2)^(1/2)/(c^2*x^2-
1)^(1/2)/(1+e*x^2/d)^(1/2)+1/9*b*(c^2*d+e)*(2*c^2*d+3*e)*x*(-c^2*x^2+1)^(1
/2)*(1+e*x^2/d)^(1/2)*EllipticF(c*x,(-e/c^2/d)^(1/2))/d/(c^2*x^2)^(1/2)/(c
^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.57 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^4}\,dx$$
$$=\frac{\sqrt{d+ex^2}\left(-3a(d+ex^2)+bc\sqrt{1-\frac{1}{c^2x^2}}x(d+2c^2dx^2+4ex^2)-3b(d+ex^2)\sec^{-1}(cx)\right)}{9dx^3}$$
$$-\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(2c^2d(c^2d+2e)\,E\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)\middle|-\frac{e}{c^2d}\right)-(2c^4d^2+5c^2de+3e^2)\,\text{EllipticF}\right.}{9\sqrt{-c^2}d\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^4,x]
```

output
```
(Sqrt[d + e*x^2]*(-3*a*(d + e*x^2) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2*c^
2*d*x^2 + 4*e*x^2) - 3*b*(d + e*x^2)*ArcSec[c*x]))/(9*d*x^3) - ((I/9)*b*c*
Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(2*c^2*d*(c^2*d + 2*e)*Ellipti
cE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] - (2*c^4*d^2 + 5*c^2*d*e + 3*e^2
)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d*Sqrt[1
- c^2*x^2]*Sqrt[d + e*x^2])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {5761, 27, 376, 25, 445, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^4}\,dx$$
$$\Big\downarrow \text{5761}$$

$$-\frac{bcx\int -\frac{\left(ex^2+d\right)^{3/2}}{3dx^4\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$\big\downarrow$ 27

$$\frac{bcx\int \frac{\left(ex^2+d\right)^{3/2}}{x^4\sqrt{c^2x^2-1}}dx}{3d\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$\big\downarrow$ 376

$$\frac{bcx\left(\frac{d\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{3x^3}-\frac{1}{3}\int -\frac{e\left(dc^2+3e\right)x^2+2d\left(dc^2+2e\right)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx\right)}{3d\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$\big\downarrow$ 25

$$\frac{bcx\left(\frac{1}{3}\int \frac{e\left(dc^2+3e\right)x^2+2d\left(dc^2+2e\right)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{d\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{3x^3}\right)}{3d\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$\big\downarrow$ 445

$$\frac{bcx\left(\frac{1}{3}\left(\frac{\int \frac{de\left(-2\left(dc^2+2e\right)x^2c^2+dc^2+3e\right)}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{d}+\frac{2\sqrt{c^2x^2-1}\left(c^2d+2e\right)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{3x^3}\right)}{3d\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$\big\downarrow$ 27

$$\frac{bcx\left(\frac{1}{3}\left(e\int \frac{-2\left(dc^2+2e\right)x^2c^2+dc^2+3e}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{2\sqrt{c^2x^2-1}\left(c^2d+2e\right)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{3x^3}\right)}{3d\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$\big\downarrow$ 399

$$\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{\left(c^2d+e\right)\left(2c^2d+3e\right)\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e}-\frac{2c^2\left(c^2d+2e\right)\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{2\sqrt{c^2x^2-1}\left(c^2d+2e\right)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{c^2x^2-1}\sqrt{\cdots}}{3x^3}}{3d\sqrt{c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$\big\downarrow$ 323

$$\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{(c^2d+e)(2c^2d+3e)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}-\frac{2c^2(c^2d+2e)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{2\sqrt{c^2x^2-1}(c^2d+2e)\sqrt{d+ex^2}}{x}\right)+d\sqrt{\ldots}}{3d\sqrt{c^2x^2}}$$

$$\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3dx^3}$$

$$\Big\downarrow \text{323}$$

$$\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+e)(2c^2d+3e)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{2c^2(c^2d+2e)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{2\sqrt{c^2x^2-1}(c^2d+2e)\sqrt{d+ex^2}}{x}\ldots}{3d\sqrt{c^2x^2}}$$

$$\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3dx^3}$$

$$\Big\downarrow \text{321}$$

$$\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+e)(2c^2d+3e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{2c^2(c^2d+2e)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{2\sqrt{c^2x^2-1}(c^2d+2e)\sqrt{d\ldots}}{x}\ldots}{3d\sqrt{c^2x^2}}$$

$$\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3dx^3}$$

$$\Big\downarrow \text{331}$$

$$\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+e)(2c^2d+3e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{2c^2\sqrt{1-c^2x^2}(c^2d+2e)\int\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}}\right)+\frac{2\sqrt{c^2x^2-1}(c^2\ldots}{\ldots}\ldots}{3d\sqrt{c^2x^2}}$$

$$\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3dx^3}$$

$$\Big\downarrow \text{330}$$

$$\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+e)(2c^2d+3e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{2c^2\sqrt{1-c^2x^2}(c^2d+2e)\sqrt{d+ex^2}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\right)+2\sqrt{c\ldots}\ldots}{3d\sqrt{c^2x^2}}$$

$$\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3dx^3}$$

$$\Big\downarrow \text{327}$$

$$\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+e)(2c^2d+3e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{2c\sqrt{1-c^2x^2}(c^2d+2e)\sqrt{d+ex^2}\,E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\right)+\right.\right.}{3d\sqrt{c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^4,x]
```

output
```
-1/3*((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/(d*x^3) + (b*c*x*((d*Sqrt[-1
+ c^2*x^2]*Sqrt[d + e*x^2])/(3*x^3) + ((2*(c^2*d + 2*e)*Sqrt[-1 + c^2*x^2]
*Sqrt[d + e*x^2])/x + e*((-2*c*(c^2*d + 2*e)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*
x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(e*Sqrt[-1 + c^2*x^2]*Sqrt[1 +
(e*x^2)/d]) + ((c^2*d + e)*(2*c^2*d + 3*e)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x
^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(c*e*Sqrt[-1 + c^2*x^2]*Sqrt[
d + e*x^2])))/3))/(3*d*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [F]

$$\int \frac{\sqrt{e\,x^2+d}\,(a+b\ \mathrm{arcsec}\,(cx))}{x^4}dx$$

input
```
int((e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/x^4,x)
```

output
```
int((e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^4}\,dx = -\frac{\left(3\,acdex^2+3\,acd^2+3\left(bcdex^2+bcd^2\right)\mathrm{arcsec}\,(cx)-\left(bcd^2+2\left(bc^3d^2+2\,bcde\right)x^2\right)\sqrt{c^2x^2-1}\right)\sqrt{ex^2+}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/x^4,x, algorithm="fricas")
```

output
```
-1/9*((3*a*c*d*e*x^2 + 3*a*c*d^2 + 3*(b*c*d*e*x^2 + b*c*d^2)*arcsec(c*x) -
 (b*c*d^2 + 2*(b*c^3*d^2 + 2*b*c*d*e)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 +
 d) - (2*(b*c^6*d^2 + 2*b*c^4*d*e)*x^3*elliptic_e(arcsin(c*x), -e/(c^2*d))
 - (2*b*c^6*d^2 + (4*b*c^4 + b*c^2)*d*e + 3*b*e^2)*x^3*elliptic_f(arcsin(c
*x), -e/(c^2*d)))*sqrt(-d))/(c*d^2*x^3)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^4}\,dx = \int \frac{(a+b\,\mathrm{asec}\,(cx))\,\sqrt{d+ex^2}}{x^4}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(a+b*asec(c*x))/x**4,x)
```

output
```
Integral((a + b*asec(c*x))*sqrt(d + e*x**2)/x**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^4}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\mathrm{arcsec}\,(cx)+a)}{x^4}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^4}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^4,x)
```

output
```
int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^4}\,dx = \frac{-\sqrt{e\,x^2+d}\,ad-\sqrt{e\,x^2+d}\,ae\,x^2-\sqrt{e}\,ae\,x^3+3\left(\int \frac{\sqrt{e\,x^2+d}\,asec(cx)}{x^4}dx\right)bd\,x^3}{3d\,x^3}$$

input
```
int((e*x^2+d)^(1/2)*(a+b*asec(c*x))/x^4,x)
```

output
```
( - sqrt(d + e*x**2)*a*d - sqrt(d + e*x**2)*a*e*x**2 - sqrt(e)*a*e*x**3 +
3*int((sqrt(d + e*x**2)*asec(c*x))/x**4,x)*b*d*x**3)/(3*d*x**3)
```

3.120 $\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^6}\,dx$

Optimal result 1024
Mathematica [C] (verified) 1025
Rubi [A] (verified) 1026
Maple [F] 1031
Fricas [A] (verification not implemented) 1031
Sympy [F] 1032
Maxima [F(-2)] 1032
Giac [F] 1033
Mupad [F(-1)] 1033
Reduce [F] 1033

Optimal result

Integrand size = 23, antiderivative size = 455

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^6}\,dx = \frac{bc(24c^4d^2+19c^2de-31e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} + \frac{bc(12c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25dx^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{15d^2x^3} - \frac{bc^2(24c^4d^2+19c^2de-31e^2)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|-\frac{e}{c^2d}\right)}{225d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{\frac{d+ex^2}{d}}} + \frac{b(c^2d+e)(24c^4d^2+7c^2de-30e^2)x\sqrt{1-c^2x^2}\sqrt{\frac{d+ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{225d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

output
```
1/225*b*c*(24*c^4*d^2+19*c^2*d*e-31*e^2)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)
/d^2/(c^2*x^2)^(1/2)+1/225*b*c*(12*c^2*d-e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1
/2)/d/x^2/(c^2*x^2)^(1/2)+1/25*b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(3/2)/d/x^4
/(c^2*x^2)^(1/2)-1/5*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/d/x^5+2/15*e*(e*x^2
+d)^(3/2)*(a+b*arcsec(c*x))/d^2/x^3-1/225*b*c^2*(24*c^4*d^2+19*c^2*d*e-31*
e^2)*x*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)*EllipticE(c*x,(-e/c^2/d)^(1/2))/
d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/((e*x^2+d)/d)^(1/2)+1/225*b*(c^2*d+e
)*(24*c^4*d^2+7*c^2*d*e-30*e^2)*x*(-c^2*x^2+1)^(1/2)*((e*x^2+d)/d)^(1/2)*E
llipticF(c*x,(-e/c^2/d)^(1/2))/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^
2+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.01 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^6}\,dx = \frac{\sqrt{d+ex^2}\left(-15a(3d^2+dex^2-2e^2x^4)+bc\sqrt{1-\frac{1}{c^2x^2}}x(-31e^2x^4+dex^2(8+19c^2x^2)+3d^2(3+4c^2x^2+\right.}{225d^2x^5} - \frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(24c^4d^2+19c^2de-31e^2)\,E\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)\middle|-\frac{e}{c^2d}\right)+(-24c^6d^3-31c^4\right.}{225\sqrt{-c^2}d^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^6,x]
```

output
```
(Sqrt[d + e*x^2]*(-15*a*(3*d^2 + d*e*x^2 - 2*e^2*x^4) + b*c*Sqrt[1 - 1/(c^
2*x^2)]*x*(-31*e^2*x^4 + d*e*x^2*(8 + 19*c^2*x^2) + 3*d^2*(3 + 4*c^2*x^2 +
 8*c^4*x^4)) - 15*b*(3*d^2 + d*e*x^2 - 2*e^2*x^4)*ArcSec[c*x]))/(225*d^2*x
^5) - ((I/225)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(24*
c^4*d^2 + 19*c^2*d*e - 31*e^2)*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2
*d))] + (-24*c^6*d^3 - 31*c^4*d^2*e + 23*c^2*d*e^2 + 30*e^3)*EllipticF[I*A
rcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d^2*Sqrt[1 - c^2*x^2]*Sq
rt[d + e*x^2])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 412, normalized size of antiderivative = 0.91, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {5761, 27, 442, 25, 442, 25, 445, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{x^6}dx$$

$$\Big\downarrow \text{5761}$$

$$-\frac{bcx\int -\frac{\left(3d-2ex^2\right)\left(ex^2+d\right)^{3/2}}{15d^2x^6\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int \frac{\left(3d-2ex^2\right)\left(ex^2+d\right)^{3/2}}{x^6\sqrt{c^2x^2-1}}dx}{15d^2\sqrt{c^2x^2}}+\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\Big\downarrow \text{442}$$

$$\frac{bcx\left(\frac{3d\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{5x^5}-\frac{1}{5}\int -\frac{\sqrt{ex^2+d}\left(\left(3c^2d-10e\right)ex^2+d\left(12c^2d-e\right)\right)}{x^4\sqrt{c^2x^2-1}}dx\right)}{15d^2\sqrt{c^2x^2}}+\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{bcx\left(\frac{1}{5}\int \frac{\sqrt{ex^2+d}\left(\left(3c^2d-10e\right)ex^2+d\left(12c^2d-e\right)\right)}{x^4\sqrt{c^2x^2-1}}dx+\frac{3d\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{5x^5}\right)}{15d^2\sqrt{c^2x^2}}+\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\Big\downarrow \text{442}$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{d\sqrt{c^2x^2-1}(12c^2d-e)\sqrt{d+ex^2}}{3x^3}-\frac{1}{3}\int -\frac{2e(6d^2c^4+4dec^2-15e^2)x^2+d(24d^2c^4+19dec^2-31e^2)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx\right)+\frac{3d\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{5x^5}\right)}{15d^2\sqrt{c^2x^2}}+\frac{2e(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{5dx^5}$$

$\downarrow$ 25

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{2e(6d^2c^4+4dec^2-15e^2)x^2+d(24d^2c^4+19dec^2-31e^2)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{d\sqrt{c^2x^2-1}(12c^2d-e)\sqrt{d+ex^2}}{3x^3}\right)+\frac{3d\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{5x^5}\right)}{15d^2\sqrt{c^2x^2}}+\frac{2e(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{5dx^5}$$

$\downarrow$ 445

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\int \frac{de\left(2\left(6d^2c^4+4dec^2-15e^2\right)-c^2\left(24d^2c^4+19dec^2-31e^2\right)x^2\right)}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{d}+\frac{\sqrt{c^2x^2-1}(24c^4d^2+19c^2de-31e^2)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{c^2x^2-1}(12c}{3x}\right.\right.}{15d^2\sqrt{c^2x^2}}$$
$$\frac{2e(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{5dx^5}$$

$\downarrow$ 27

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\int \frac{2(6d^2c^4+4dec^2-15e^2)-c^2(24d^2c^4+19dec^2-31e^2)x^2}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{\sqrt{c^2x^2-1}(24c^4d^2+19c^2de-31e^2)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{c^2x^2-1}(12c}{3x}\right.\right.}{15d^2\sqrt{c^2x^2}}$$
$$\frac{2e(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{5dx^5}$$

$\downarrow$ 399

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{(c^2d+e)(24c^4d^2+7c^2de-30e^2)\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e}-\frac{c^2(24c^4d^2+19c^2de-31e^2)\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\sqrt{c^2x^2-1}(24c^4d^2+\right.\right.\right.}{15d^2\sqrt{c^2x^2}}$$
$$\frac{2e(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{5dx^5}$$

$\downarrow$ 323

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{(c^2d+e)(24c^4d^2+7c^2de-30e^2)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}-\frac{c^2(24c^4d^2+19c^2de-31e^2)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-}}{}\right.\right.\right.}{15d^2\sqrt{c^2x^2}}$$

$$\frac{2e(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{5dx^5}$$

$$\Big\downarrow 323$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+e)(24c^4d^2+7c^2de-30e^2)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2(24c^4d^2+19c^2de-31e^2)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\right.\right.\right.}{15d^2\sqrt{c^2x^2}}$$

$$\frac{2e(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{5dx^5}$$

$$\Big\downarrow 321$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+e)(24c^4d^2+7c^2de-30e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2(24c^4d^2+19c^2de-31e^2)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right.\right.\right.\right.}{15d^2\sqrt{c^2x^2}}$$

$$\frac{2e(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{5dx^5}$$

$$\Big\downarrow 331$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+e)(24c^4d^2+7c^2de-30e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}(24c^4d^2+19c^2de-31e^2)\int\frac{\sqrt{}}{\sqrt{1}}}{e\sqrt{c^2x^2-1}}\right.\right.\right.\right.}{15d^2\sqrt{c^2x^2}}$$

$$\frac{2e(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{5dx^5}$$

$$\Big\downarrow 330$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+e)(24c^4d^2+7c^2de-30e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}(24c^4d^2+19c^2de-31e^2)\sqrt{d+}}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\right.\right.\right.\right.}{15d^2\sqrt{c}}$$

$$\frac{2e(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{5dx^5}$$

$\big\downarrow$ 327

$$\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}+$$
$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}\left(c^2d+e\right)\left(24c^4d^2+7c^2de-30e^2\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c\sqrt{1-c^2x^2}\left(24c^4d^2+19c^2de-31e^2\right)\sqrt{d+ex}}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+}}\right.\right.\right.\right.}{15d^2\sqrt{}}$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^6,x]
```

output
```
-1/5*((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/(d*x^5) + (2*e*(d + e*x^2)^(3
/2)*(a + b*ArcSec[c*x]))/(15*d^2*x^3) + (b*c*x*((3*d*Sqrt[-1 + c^2*x^2]*(d
 + e*x^2)^(3/2))/(5*x^5) + ((d*(12*c^2*d - e)*Sqrt[-1 + c^2*x^2]*Sqrt[d +
e*x^2])/(3*x^3) + (((24*c^4*d^2 + 19*c^2*d*e - 31*e^2)*Sqrt[-1 + c^2*x^2]*
Sqrt[d + e*x^2])/x + e*(-((c*(24*c^4*d^2 + 19*c^2*d*e - 31*e^2)*Sqrt[1 - c
^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(e*Sqrt[-1 +
 c^2*x^2]*Sqrt[1 + (e*x^2)/d])) + ((c^2*d + e)*(24*c^4*d^2 + 7*c^2*d*e - 3
0*e^2)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c
^2*d))])/(c*e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])))/3)/5))/(15*d^2*Sqrt[c^
2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [F]

$$\int \frac{\sqrt{e\,x^2+d}\,(a+b\,\mathrm{arcsec}\,(cx))}{x^6}dx$$

input
```
int((e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/x^6,x)
```

output
```
int((e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/x^6,x)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^6}\,dx = \frac{(30\,acde^2x^4 - 15\,acd^2ex^2 - 45\,acd^3 + 15\,(2\,bcde^2x^4 - bcd^2ex^2 - 3\,bcd^3)\,\mathrm{arcsec}\,(cx) + (9\,bcd^3 + (24\,bc^5d^3$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/x^6,x, algorithm="fricas")
```

output
```
1/225*((30*a*c*d*e^2*x^4 - 15*a*c*d^2*e*x^2 - 45*a*c*d^3 + 15*(2*b*c*d*e^2
*x^4 - b*c*d^2*e*x^2 - 3*b*c*d^3)*arcsec(c*x) + (9*b*c*d^3 + (24*b*c^5*d^3
 + 19*b*c^3*d^2*e - 31*b*c*d*e^2)*x^4 + 4*(3*b*c^3*d^3 + 2*b*c*d^2*e)*x^2)
*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d) + ((24*b*c^8*d^3 + 19*b*c^6*d^2*e - 31
*b*c^4*d*e^2)*x^5*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (24*b*c^8*d^3 + (1
9*b*c^6 + 12*b*c^4)*d^2*e - (31*b*c^4 - 8*b*c^2)*d*e^2 - 30*b*e^3)*x^5*ell
iptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(-d))/(c*d^3*x^5)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^6}\,dx=\int \frac{(a+b\,\mathrm{asec}\,(cx))\,\sqrt{d+ex^2}}{x^6}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(a+b*asec(c*x))/x**6,x)
```

output
```
Integral((a + b*asec(c*x))*sqrt(d + e*x**2)/x**6, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^6}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/x^6,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^6}\,dx = \int \frac{\sqrt{ex^2+d}(b\operatorname{arcsec}(cx)+a)}{x^6}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/x^6,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^6}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^6,x)
```

output
```
int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^6}\,dx = \frac{-3\sqrt{e\,x^2+d}\,a\,d^2-\sqrt{e\,x^2+d}\,ade\,x^2+2\sqrt{e\,x^2+d}\,a\,e^2x^4-2\sqrt{e}\,a\,e^2x^5+15\left(\int \frac{\sqrt{e\,x^2+d}\,asec(cx)}{x^6}dx\right)b\,d^2x^5}{15d^2x^5}$$

input
```
int((e*x^2+d)^(1/2)*(a+b*asec(c*x))/x^6,x)
```

output
```
( - 3*sqrt(d + e*x**2)*a*d**2 - sqrt(d + e*x**2)*a*d*e*x**2 + 2*sqrt(d + e
*x**2)*a*e**2*x**4 - 2*sqrt(e)*a*e**2*x**5 + 15*int((sqrt(d + e*x**2)*asec
(c*x))/x**6,x)*b*d**2*x**5)/(15*d**2*x**5)
```

3.121 $\int x^3(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))\,dx$

Optimal result 1034
Mathematica [C] (warning: unable to verify) 1035
Rubi [A] (verified) 1036
Maple [F] 1041
Fricas [A] (verification not implemented) 1042
Sympy [F(-1)] 1042
Maxima [F(-2)] 1043
Giac [F] 1043
Mupad [F(-1)] 1044
Reduce [F] 1044

Optimal result

Integrand size = 23, antiderivative size = 374

$$\begin{aligned}\int x^3(d+ex^2)^{3/2}(a&+b\sec^{-1}(cx))\,dx = \frac{b(3c^4d^2-38c^2de-25e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}} \\ &-\frac{b(13c^2d+25e)\,x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e\sqrt{c^2x^2}} \\ &-\frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce\sqrt{c^2x^2}}-\frac{d(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^2} \\ &+\frac{(d+ex^2)^{7/2}(a+b\sec^{-1}(cx))}{7e^2}-\frac{2bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{35e^2\sqrt{c^2x^2}} \\ &+\frac{b(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3)\,x\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{560c^6e^{3/2}\sqrt{c^2x^2}}\end{aligned}$$

output
```
1/560*b*(3*c^4*d^2-38*c^2*d*e-25*e^2)*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/
c^5/e/(c^2*x^2)^(1/2)-1/840*b*(13*c^2*d+25*e)*x*(c^2*x^2-1)^(1/2)*(e*x^2+d
)^(3/2)/c^3/e/(c^2*x^2)^(1/2)-1/42*b*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(5/2)/c
/e/(c^2*x^2)^(1/2)-1/5*d*(e*x^2+d)^(5/2)*(a+b*arcsec(c*x))/e^2+1/7*(e*x^2+
d)^(7/2)*(a+b*arcsec(c*x))/e^2-2/35*b*c*d^(7/2)*x*arctan((e*x^2+d)^(1/2)/d
^(1/2)/(c^2*x^2-1)^(1/2))/e^2/(c^2*x^2)^(1/2)+1/560*b*(35*c^6*d^3-35*c^4*d
^2*e-63*c^2*d*e^2-25*e^3)*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^
(1/2))/c^6/e^(3/2)/(c^2*x^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.42 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.82

$$\int x^3 (d + ex^2)^{3/2} (a + b \sec^{-1}(cx)) \, dx = \frac{96a(d+ex^2)^3 (-2d+5ex^2) - \frac{2be\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)(75e^2+2c^2e(82d+25ex^2)+c^4(57d^2+106dex^2+40e^2x}{c^5}}{}$$

input
```
Integrate[x^3*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
```

output
```
(96*a*(d + e*x^2)^3*(-2*d + 5*e*x^2) - (2*b*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d +
 e*x^2)*(75*e^2 + 2*c^2*e*(82*d + 25*e*x^2) + c^4*(57*d^2 + 106*d*e*x^2 +
40*e^2*x^4)))/c^5 + (3*b*(-32*c^4*d^4*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2,
 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))] - (e*(35*c^6*d^3 - 35*c^4*d^2*e - 63*c
^2*d*e^2 - 25*e^3)*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*AppellF1[
1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/Sqrt[1 - c^2*x^2]))/(c^5*x) + 96*b
*(d + e*x^2)^3*(-2*d + 5*e*x^2)*ArcSec[c*x])/(3360*e^2*Sqrt[d + e*x^2])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.91, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {5761, 27, 435, 171, 27, 171, 27, 171, 27, 175, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right) dx$$

$$\Big\downarrow \text{5761}$$

$$-\frac{bcx \int -\frac{(2d-5ex^2)(ex^2+d)^{5/2}}{35e^2x\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx \int \frac{(2d-5ex^2)(ex^2+d)^{5/2}}{x\sqrt{c^2x^2-1}}dx}{35e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}$$

$$\Big\downarrow \text{435}$$

$$\frac{bcx \int \frac{(2d-5ex^2)(ex^2+d)^{5/2}}{x^2\sqrt{c^2x^2-1}}dx^2}{70e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}$$

$$\Big\downarrow \text{171}$$

$$\frac{bcx\left(\frac{\int \frac{(ex^2+d)^{3/2}\left(12c^2d^2-e\left(13dc^2+25e\right)x^2\right)}{2x^2\sqrt{c^2x^2-1}}dx^2}{3c^2}-\frac{5e\sqrt{c^2x^2-1}(d+ex^2)^{5/2}}{3c^2}\right)}{70e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\left(\frac{\int \frac{\left(ex^2+d\right)^{3/2}\left(12c^2d^2-e\left(13dc^2+25e\right)x^2\right)}{x^2\sqrt{c^2x^2-1}}dx^2}{6c^2}-\frac{5e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{5/2}}{3c^2}\right)}{70e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}$$

$$\Big\downarrow \ 171$$

$$\frac{bcx\left(\frac{\frac{\int \frac{3\sqrt{ex^2+d}\left(16d^3c^4+e\left(3d^2c^4-38dec^2-25e^2\right)x^2\right)}{2x^2\sqrt{c^2x^2-1}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(13c^2d+25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}}{6c^2}-\frac{5e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{5/2}}{3c^2}\right)}{70e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}$$

$$\Big\downarrow \ 27$$

$$\frac{bcx\left(\frac{\frac{3\int \frac{\sqrt{ex^2+d}\left(16d^3c^4+e\left(3d^2c^4-38dec^2-25e^2\right)x^2\right)}{x^2\sqrt{c^2x^2-1}}dx^2}{4c^2}-\frac{e\sqrt{c^2x^2-1}\left(13c^2d+25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}}{6c^2}-\frac{5e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{5/2}}{3c^2}\right)}{70e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}$$

$$\Big\downarrow \ 171$$

$$\frac{bcx\left(\frac{\frac{3\left(\frac{\int \frac{32d^4c^6+e\left(35d^3c^6-35d^2ec^4-63de^2c^2-25e^3\right)x^2}{2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2}+\frac{e\sqrt{c^2x^2-1}\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{d+ex^2}}{c^2}\right)}{4c^2}-\frac{e\sqrt{c^2x^2-1}\left(13c^2d+25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}}{6c^2}-5\right.}{70e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}$$

$$\Big\downarrow \ 27$$

$$\frac{bcx\left(\frac{\frac{3\left(\frac{\int \frac{32d^4c^6+e\left(35d^3c^6-35d^2ec^4-63de^2c^2-25e^3\right)x^2}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{d+ex^2}}{c^2}\right)}{4c^2}-\frac{e\sqrt{c^2x^2-1}\left(13c^2d+25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}}{6c^2}-\frac{5}{}\right.}{70e^2\sqrt{c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}$$

$$\Big\downarrow \text{175}$$

$$\frac{bcx\left(\frac{\frac{3\left(\frac{32c^6d^4\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+e\left(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3\right)\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{d+ex^2}}{c^2}\right)}{4c^2}-}{6c^2}\right.}{70e^2\sqrt{c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}$$

$$\Big\downarrow \text{66}$$

$$\frac{bcx\left(\frac{\frac{3\left(\frac{32c^6d^4\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\left(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3\right)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{d+ex^2}}{c^2}\right)}{4c^2}-\frac{e}{}}{6c^2}\right.}{70e^2\sqrt{c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}$$

$$\Big\downarrow \text{104}$$

$$\frac{bcx\left(\frac{3\left(\frac{64c^6d^4\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e\left(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3\right)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{d+ex^2}}{c^2}\right)}{4c^2}}{6c^2}-\frac{e\sqrt{c^2x \cdots}{\cdots}\right)}{70e^2\sqrt{c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{bcx\left(\frac{3\left(\frac{2e\left(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3\right)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-64c^6d^{7/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{d+ex^2}}{c^2}\right)}{4c^2}}{6c^2}-\frac{e\sqrt{\cdots}}{\cdots}\right)}{70e^2\sqrt{c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}+$$

$$\frac{bcx\left(\frac{3\left(\frac{\frac{2\sqrt{e}\left(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3\right)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-64c^6d^{7/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{d+ex^2 \cdots}}{c^2}\cdots\right)}{4c^2}}{6c^2}\cdots\right)}{70e^2\sqrt{c^2x^2}}$$

input
```
Int[x^3*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
```

output
```
-1/5*(d*(d + e*x^2)^(5/2)*(a + b*ArcSec[c*x]))/e^2 + ((d + e*x^2)^(7/2)*(a
 + b*ArcSec[c*x]))/(7*e^2) + (b*c*x*((-5*e*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^
(5/2))/(3*c^2) + (-1/2*(e*(13*c^2*d + 25*e)*Sqrt[-1 + c^2*x^2]*(d + e*x^2)
^(3/2))/c^2 + (3*((e*(3*c^4*d^2 - 38*c^2*d*e - 25*e^2)*Sqrt[-1 + c^2*x^2]*
Sqrt[d + e*x^2])/c^2 + (-64*c^6*d^(7/2)*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sq
rt[-1 + c^2*x^2])] + (2*Sqrt[e]*(35*c^6*d^3 - 35*c^4*d^2*e - 63*c^2*d*e^2
- 25*e^3)*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/c)/(2
*c^2)))/(4*c^2))/(6*c^2)))/(70*e^2*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [F]

$$\int x^3 \left(e\,x^2+d\right)^{\frac{3}{2}} (a+b\,\mathrm{arcsec}\,(cx))\,dx$$

input
```
int(x^3*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

output
```
int(x^3*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

Fricas [A] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 1701, normalized size of antiderivative = 4.55

$$\int x^3 (d+ex^2)^{3/2} (a+b\sec^{-1}(cx)) \, dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
[1/6720*(96*b*c^7*sqrt(-d)*d^3*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c
^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2
 + d)*sqrt(-d) + 8*d^2)/x^4) - 3*(35*b*c^6*d^3 - 35*b*c^4*d^2*e - 63*b*c^2
*d*e^2 - 25*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^
4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqr
t(e*x^2 + d)*sqrt(e) + e^2) + 4*(240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 +
 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + 48*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x
^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*arcsec(c*x) - (40*b*c^5*e^3*x^4 + 57*b
*c^5*d^2*e + 164*b*c^3*d*e^2 + 75*b*c*e^3 + 2*(53*b*c^5*d*e^2 + 25*b*c^3*e
^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^7*e^2), -1/6720*(192*b*c^7
*d^(7/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2
+ d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + 3*(35*b*c^6*d^3
- 35*b*c^4*d^2*e - 63*b*c^2*d*e^2 - 25*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 +
c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d -
 c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 4*(240*a*c^7*e^3*
x^6 + 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + 48*(5*b*c^
7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*arcsec(c*x)
 - (40*b*c^5*e^3*x^4 + 57*b*c^5*d^2*e + 164*b*c^3*d*e^2 + 75*b*c*e^3 + 2*(
53*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c
^7*e^2), 1/3360*(48*b*c^7*sqrt(-d)*d^3*log(((c^4*d^2 - 6*c^2*d*e + e^2)...
```

Sympy [F(-1)]

Timed out.

$$\int x^3 (d+ex^2)^{3/2} (a+b\sec^{-1}(cx)) \, dx = \text{Timed out}$$

input
```
integrate(x**3*(e*x**2+d)**(3/2)*(a+b*asec(c*x)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^{\frac{3}{2}}(b\,\mathrm{arcsec}\,(cx)+a)x^3\,dx$$

input
```
integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\, dx = \int x^3 \left(e\,x^2+d\right)^{3/2} \left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\, dx$$

input
```
int(x^3*(d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))),x)
```

output
```
int(x^3*(d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))), x)
```

Reduce [F]

$$\int x^3 \left(d+ex^2\right)^{3/2} (a +b\sec^{-1}(cx))\, dx = \frac{-2\sqrt{e\,x^2+d}\,a\,d^3+\sqrt{e\,x^2+d}\,a\,d^2e\,x^2+8\sqrt{e\,x^2+d}\,ad\,e^2x^4+5\sqrt{e\,x^2+d}\,a\,e^3x^6+35(}{35e^2}$$

input
```
int(x^3*(e*x^2+d)^(3/2)*(a+b*asec(c*x)),x)
```

output
```
( - 2*sqrt(d + e*x**2)*a*d**3 + sqrt(d + e*x**2)*a*d**2*e*x**2 + 8*sqrt(d
+ e*x**2)*a*d*e**2*x**4 + 5*sqrt(d + e*x**2)*a*e**3*x**6 + 35*int(sqrt(d +
 e*x**2)*asec(c*x)*x**5,x)*b*e**3 + 35*int(sqrt(d + e*x**2)*asec(c*x)*x**3
,x)*b*d*e**2)/(35*e**2)
```

3.122 $\int x(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx$

Optimal result . . . 1045
Mathematica [C] (warning: unable to verify) . . . 1046
Rubi [A] (verified) . . . 1046
Maple [F] . . . 1051
Fricas [A] (verification not implemented) . . . 1051
Sympy [F(-1)] . . . 1052
Maxima [F] . . . 1052
Giac [F] . . . 1052
Mupad [F(-1)] . . . 1053
Reduce [F] . . . 1053

Optimal result

Integrand size = 21, antiderivative size = 262

$$\int x(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx = -\frac{b(7c^2d+3e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e} + \frac{bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{5e\sqrt{c^2x^2}} - \frac{b(15c^4d^2+10c^2de+3e^2)\,x\,\mathrm{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{40c^4\sqrt{e}\sqrt{c^2x^2}}$$

output
```
-1/40*b*(7*c^2*d+3*e)*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c^3/(c^2*x^2)^(1
/2)-1/20*b*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(3/2)/c/(c^2*x^2)^(1/2)+1/5*(e*x^
2+d)^(5/2)*(a+b*arcsec(c*x))/e+1/5*b*c*d^(5/2)*x*arctan((e*x^2+d)^(1/2)/d^
(1/2)/(c^2*x^2-1)^(1/2))/e/(c^2*x^2)^(1/2)-1/40*b*(15*c^4*d^2+10*c^2*d*e+3
*e^2)*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^4/e^(1/2)/(
c^2*x^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.44 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.94

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{\frac{16a\left(d+ex^2\right)^3}{e}-\frac{2b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(d+ex^2\right)\left(3e+c^2\left(9d+2ex^2\right)\right)}{c^3}+\frac{b\left(\frac{8c^2d^3\sqrt{1+\frac{d}{ex^2}}\operatorname{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)}{e}+\frac{(15\cdots}{\cdots}\right)}{\cdots}}{80\sqrt{d+ex^2}}$$

input
```
Integrate[x*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
```

output
```
((16*a*(d + e*x^2)^3)/e - (2*b*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2)*(3*e +
c^2*(9*d + 2*e*x^2)))/c^3 + (b*((8*c^2*d^3*Sqrt[1 + d/(e*x^2)]*AppellF1[1,
 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))])/e + ((15*c^4*d^2 + 10*c^2*d*e +
3*e^2)*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2,
 2, c^2*x^2, -((e*x^2)/d)])/Sqrt[1 - c^2*x^2]))/(c^3*x) + (16*b*(d + e*x^2
)^3*ArcSec[c*x])/e)/(80*Sqrt[d + e*x^2])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.92, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {5759, 354, 113, 27, 171, 27, 175, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \; 5759$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}-\frac{bcx\int\frac{\left(ex^2+d\right)^{5/2}}{x\sqrt{c^2x^2-1}}dx}{5e\sqrt{c^2x^2}}$$

$$\Big\downarrow \ 354$$

$$\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e}-\frac{bcx\int\frac{(ex^2+d)^{5/2}}{x^2\sqrt{c^2x^2-1}}dx^2}{10e\sqrt{c^2x^2}}$$

$$\Big\downarrow \ 113$$

$$\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e}-\frac{bcx\left(\frac{\int\frac{\sqrt{ex^2+d}\left(4c^2d^2+e\left(7dc^2+3e\right)x^2\right)}{2x^2\sqrt{c^2x^2-1}}dx^2}{2c^2}+\frac{e\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{10e\sqrt{c^2x^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e}-\frac{bcx\left(\frac{\int\frac{\sqrt{ex^2+d}\left(4c^2d^2+e\left(7dc^2+3e\right)x^2\right)}{x^2\sqrt{c^2x^2-1}}dx^2}{4c^2}+\frac{e\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{10e\sqrt{c^2x^2}}$$

$$\Big\downarrow \ 171$$

$$\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e}-\frac{bcx\left(\frac{\frac{\int\frac{8d^3c^4+e\left(15d^2c^4+10dec^2+3e^2\right)x^2}{2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2}+\frac{e\sqrt{c^2x^2-1}\left(7c^2d+3e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{e\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{10e\sqrt{c^2x^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e}-\frac{bcx\left(\frac{\frac{\int\frac{8d^3c^4+e\left(15d^2c^4+10dec^2+3e^2\right)x^2}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(7c^2d+3e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{e\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{10e\sqrt{c^2x^2}}$$

$$\Big\downarrow \ 175$$

$$\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e}-\frac{bcx\left(\frac{\frac{8c^4d^3\int\frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+e\left(15c^4d^2+10c^2de+3e^2\right)\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(7c^2d+3e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{e\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{10e\sqrt{c^2x^2}}$$

$$\Big\downarrow \ 66$$

$$\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e} - \frac{bcx\left(\frac{\frac{8c^4d^3\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e(15c^4d^2+10c^2de+3e^2)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{c^2x^2-1}(7c^2d+3e)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{e\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{10e\sqrt{c^2x^2}}$$

$$\Big\downarrow \ 104$$

$$\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e} - \frac{bcx\left(\frac{\frac{16c^4d^3\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e(15c^4d^2+10c^2de+3e^2)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{c^2x^2-1}(7c^2d+3e)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{e\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{10e\sqrt{c^2x^2}}$$

$$\Big\downarrow \ 217$$

$$\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e} - \frac{bcx\left(\frac{\frac{2e(15c^4d^2+10c^2de+3e^2)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-16c^4d^{5/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}+\frac{e\sqrt{c^2x^2-1}(7c^2d+3e)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{e\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{10e\sqrt{c^2x^2}}$$

$$\Big\downarrow \ 221$$

$$\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e} - \frac{bcx\left(\frac{\frac{\frac{2\sqrt{e}(15c^4d^2+10c^2de+3e^2)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-16c^4d^{5/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}+\frac{e\sqrt{c^2x^2-1}(7c^2d+3e)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{e\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{10e\sqrt{c^2x^2}}$$

input

```
Int[x*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
```

output
```
((d + e*x^2)^(5/2)*(a + b*ArcSec[c*x]))/(5*e) - (b*c*x*((e*Sqrt[-1 + c^2*x
^2]*(d + e*x^2)^(3/2))/(2*c^2) + ((e*(7*c^2*d + 3*e)*Sqrt[-1 + c^2*x^2]*Sq
rt[d + e*x^2])/c^2 + (-16*c^4*d^(5/2)*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt
[-1 + c^2*x^2])] + (2*Sqrt[e]*(15*c^4*d^2 + 10*c^2*d*e + 3*e^2)*ArcTanh[(S
qrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/c)/(2*c^2))/(4*c^2)))/(10
*e*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 113
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5759
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSec[c*x])/(2*e*(p + 1))), x
] - Simp[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2]))   Int[(d + e*x^2)^(p + 1)/(x*S
qrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Maple [F]

$$\int x(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\text{arcsec}\,(cx))\,dx$$

input
```
int(x*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

output
```
int(x*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

Fricas [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 1377, normalized size of antiderivative = 5.26

$$\int x(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))\;dx = \text{Too large to display}$$

input
```
integrate(x*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
[1/160*(8*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2
*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 +
 d)*sqrt(-d) + 8*d^2)/x^4) + (15*b*c^4*d^2 + 10*b*c^2*d*e + 3*b*e^2)*sqrt(
e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4
*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e
^2) + 4*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + 8*(b*c^5*e^2*x
^4 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*arcsec(c*x) - (2*b*c^3*e^2*x^2 + 9*b*c^3
*d*e + 3*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e), 1/160*(16*b
*c^5*d^(5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*
x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (15*b*c^4*d^
2 + 10*b*c^2*d*e + 3*b*e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*
e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2
 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x
^2 + 8*a*c^5*d^2 + 8*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*arcsec(
c*x) - (2*b*c^3*e^2*x^2 + 9*b*c^3*d*e + 3*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt
(e*x^2 + d))/(c^5*e), 1/80*(4*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e
 + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2
 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + (15*b*c^4*d^2 + 10*b*c^2*
d*e + 3*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2
- 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*...
```

Sympy [F(-1)]

Timed out.

$$\int x(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\text{Timed out}$$

input
```
integrate(x*(e*x**2+d)**(3/2)*(a+b*asec(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int (ex^2+d)^{\frac{3}{2}}(b\operatorname{arcsec}(cx)+a)x\,dx$$

input
```
integrate(x*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/5*(e*x^2 + d)^(5/2)*a/e + 1/5*((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e*x^2 +
d)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 5*e*integrate((5*(c^2*e^2*x^5 + (
c^2*d*e - e^2)*x^3 - d*e*x + (c^2*e^2*x^5 + (c^2*d*e - e^2)*x^3 - d*e*x)*e
^(log(c*x + 1) + log(c*x - 1)))*sqrt(e*x^2 + d)*log(x) + (5*c^2*e^2*x^5*lo
g(c) + 5*(c^2*d*e*log(c) - e^2*log(c))*x^3 - 5*d*e*x*log(c) + ((5*c^2*log(
c) + c^2)*e^2*x^5 + ((5*c^2*log(c) + 2*c^2)*d*e - 5*e^2*log(c))*x^3 + (c^2
*d^2 - 5*d*e*log(c))*x)*e^(log(c*x + 1) + log(c*x - 1)))*sqrt(e*x^2 + d))/
(c^2*e*x^2 + (c^2*e*x^2 - e)*e^(log(c*x + 1) + log(c*x - 1)) - e), x))*b/e
```

Giac [F]

$$\int x(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int (ex^2+d)^{\frac{3}{2}}(b\operatorname{arcsec}(cx)+a)x\,dx$$

input
```
integrate(x*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int x\,{\left(e\,x^2+d\right)}^{3/2}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x*(d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))),x)
```

output
```
int(x*(d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))), x)
```

Reduce [F]

$$\int x\left(d+ex^2\right)^{3/2}(a+b\sec^{-1}(cx))\,dx=\frac{\sqrt{e\,x^2+d}\,a\,d^2+2\sqrt{e\,x^2+d}\,ade\,x^2+\sqrt{e\,x^2+d}\,a\,e^2x^4+5\left(\int\sqrt{e\,x^2+d}\,asec(cx)\,x^3dx\right)}{5e}$$

input
```
int(x*(e*x^2+d)^(3/2)*(a+b*asec(c*x)),x)
```

output
```
(sqrt(d + e*x**2)*a*d**2 + 2*sqrt(d + e*x**2)*a*d*e*x**2 + sqrt(d + e*x**2
)*a*e**2*x**4 + 5*int(sqrt(d + e*x**2)*asec(c*x)*x**3,x)*b*e**2 + 5*int(sq
rt(d + e*x**2)*asec(c*x)*x,x)*b*d*e)/(5*e)
```

3.123 $\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x}\,dx$

Optimal result . . . 1054
Mathematica [N/A] . . . 1054
Rubi [N/A] . . . 1055
Maple [N/A] . . . 1055
Fricas [N/A] . . . 1056
Sympy [N/A] . . . 1056
Maxima [F(-2)] . . . 1056
Giac [N/A] . . . 1057
Mupad [N/A] . . . 1057
Reduce [N/A] . . . 1058

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x}\,dx = \text{Int}\left(\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x},x\right)$$

output
```
Defer(Int)((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x,x)
```

Mathematica [N/A]

Not integrable

Time = 7.59 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x}\,dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x}\,dx$$

$$\Big\downarrow \text{ 5771}$$

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x}dx$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\mathrm{arcsec}\,(cx))}{x}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arcsec}\,(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsec(c*x))*sqrt(e*x^2 + d)/x,
x)
```

Sympy [N/A]

Not integrable

Time = 87.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x}\,dx=\int \frac{(a+b\,\mathrm{asec}\,(cx))\,(d+ex^2)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*asec(c*x))/x,x)
```

output
```
Integral((a + b*asec(c*x))*(d + e*x**2)**(3/2)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arcsec}\,(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{(e\,x^2+d)^{3/2}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 123, normalized size of antiderivative = 5.35

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x}\,dx = \frac{4\sqrt{e\,x^2+d}\,ad}{3}+\frac{\sqrt{e\,x^2+d}\,ae\,x^2}{3}+\sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)ad-\sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)ad+\left(\int \frac{\sqrt{e\,x^2+d}\,asec(cx)}{x}dx\right)bd+\left(\int \sqrt{e\,x^2+d}\,asec(cx)\,xdx\right)be$$

input
```
int((e*x^2+d)^(3/2)*(a+b*asec(c*x))/x,x)
```

output
```
(4*sqrt(d + e*x**2)*a*d + sqrt(d + e*x**2)*a*e*x**2 + 3*sqrt(d)*log((sqrt(
d + e*x**2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*a*d - 3*sqrt(d)*log((sqrt(d +
e*x**2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*a*d + 3*int((sqrt(d + e*x**2)*asec
(c*x))/x,x)*b*d + 3*int(sqrt(d + e*x**2)*asec(c*x)*x,x)*b*e)/3
```

3.124 $\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^3}\,dx$

Optimal result . . . 1059
Mathematica [N/A] . . . 1059
Rubi [N/A] . . . 1060
Maple [N/A] . . . 1060
Fricas [N/A] . . . 1061
Sympy [N/A] . . . 1061
Maxima [F(-2)] . . . 1061
Giac [N/A] . . . 1062
Mupad [N/A] . . . 1062
Reduce [N/A] . . . 1063

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^3}\,dx = \operatorname{Int}\left(\frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^3},x\right)$$

output
```
Defer(Int)((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^3,x)
```

Mathematica [N/A]

Not integrable

Time = 5.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^3}\,dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^3,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^3}\,dx$$

$$\Big\downarrow \text{ 5771}$$

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^3}dx$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 2.81 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}(a+b\,\mathrm{arcsec}\,(cx))}{x^3}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^3,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arcsec}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsec(c*x))*sqrt(e*x^2 + d)/x^3
, x)
```

Sympy [N/A]

Not integrable

Time = 76.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{asec}\,(cx))\,(d+ex^2)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*asec(c*x))/x**3,x)
```

output
```
Integral((a + b*asec(c*x))*(d + e*x**2)**(3/2)/x**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^3}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arcsec}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^3}\,dx=\int \frac{(e\,x^2+d)^{3/2}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^3,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 145, normalized size of antiderivative = 6.30

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^3}\,dx = \frac{-\sqrt{e\,x^2+d}\,ad+2\sqrt{e\,x^2+d}\,ae\,x^2+3\sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)ae\,x}{}$$

input
```
int((e*x^2+d)^(3/2)*(a+b*asec(c*x))/x^3,x)
```

output
```
( - sqrt(d + e*x**2)*a*d + 2*sqrt(d + e*x**2)*a*e*x**2 + 3*sqrt(d)*log((sq
rt(d + e*x**2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*a*e*x**2 - 3*sqrt(d)*log((s
qrt(d + e*x**2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*a*e*x**2 + 2*int((sqrt(d +
 e*x**2)*asec(c*x))/x**3,x)*b*d*x**2 + 2*int((sqrt(d + e*x**2)*asec(c*x))/
x,x)*b*e*x**2)/(2*x**2)
```

3.125 $\int x^2(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))\,dx$

Optimal result 1064
Mathematica [N/A] 1064
Rubi [N/A] 1065
Maple [N/A] 1065
Fricas [N/A] 1066
Sympy [F(-1)] 1066
Maxima [F(-2)] 1066
Giac [N/A] 1067
Mupad [N/A] 1067
Reduce [N/A] 1068

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\operatorname{Int}\left(x^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right),x\right)$$

output
```
Defer(Int)(x^2*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 11.50 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx$$

input
```
Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
```

output
```
Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (d+ex^2)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{5771}$$

$$\int x^2 (d+ex^2)^{3/2} \left(a+b\sec^{-1}(cx)\right) dx$$

input
```
Int[x^2*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^2 \left(e\,x^2+d\right)^{\frac{3}{2}} \left(a+b \operatorname{arcsec}\left(cx\right)\right) dx$$

input
```
int(x^2*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

output
```
int(x^2*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int x^2(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int (ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arcsec}\,(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e*x^4 + a*d*x^2 + (b*e*x^4 + b*d*x^2)*arcsec(c*x))*sqrt(e*x^2
+ d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\text{Timed out}$$

input
```
integrate(x**2*(e*x**2+d)**(3/2)*(a+b*asec(c*x)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^2(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx = \int (ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arcsec}\,(cx)+a)x^2 \, dx$$

input
```
integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx = \int x^2\,{\left(e\,x^2+d\right)}^{3/2}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right) \,d x$$

input
```
int(x^2*(d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))),x)
```

output
```
int(x^2*(d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 133, normalized size of antiderivative = 5.78

$$\int x^2 (d+ex^2)^{3/2} (a+b\sec^{-1}(cx))\,dx = \frac{3\sqrt{e\,x^2+d}\,a\,d^2ex+14\sqrt{e\,x^2+d}\,ad\,e^2x^3+8\sqrt{e\,x^2+d}\,a\,e^3x^5-3\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)}{48e^2}$$

input
```
int(x^2*(e*x^2+d)^(3/2)*(a+b*asec(c*x)),x)
```

output
```
(3*sqrt(d + e*x**2)*a*d**2*e*x + 14*sqrt(d + e*x**2)*a*d*e**2*x**3 + 8*sqr
t(d + e*x**2)*a*e**3*x**5 - 3*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/s
qrt(d))*a*d**3 + 48*int(sqrt(d + e*x**2)*asec(c*x)*x**4,x)*b*e**3 + 48*int
(sqrt(d + e*x**2)*asec(c*x)*x**2,x)*b*d*e**2)/(48*e**2)
```

3.126 $\int (d+ex^2)^{3/2} (a+b\sec^{-1}(cx)) \, dx$

Optimal result 1069
Mathematica [N/A] 1069
Rubi [N/A] 1070
Maple [N/A] 1070
Fricas [N/A] 1071
Sympy [F(-1)] 1071
Maxima [F(-2)] 1071
Giac [N/A] 1072
Mupad [N/A] 1072
Reduce [N/A] 1072

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (d+ex^2)^{3/2} (a+b\sec^{-1}(cx)) \, dx = \text{Int}\Big((d+ex^2)^{3/2} (a+b\sec^{-1}(cx)), x\Big)$$

output
```
Defer(Int)((e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 16.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (d+ex^2)^{3/2} (a+b\sec^{-1}(cx)) \, dx = \int (d+ex^2)^{3/2} (a+b\sec^{-1}(cx)) \, dx$$

input
```
Integrate[(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
```

output
```
Integrate[(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{5771}$$

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)dx$$

input
```
Int[(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(e\,x^2+d\right)^{\frac{3}{2}} \left(a+b\ \operatorname{arcsec}\left(cx\right)\right)dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\,dx=\int (ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arcsec}\,(cx)+a)\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsec(c*x))*sqrt(e*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\,dx=\text{Timed out}$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*asec(c*x)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx = \int (ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arcsec}\,(cx)+a)\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx = \int \left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right) \,dx$$

input
```
int((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))),x)
```

output
```
int((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 108, normalized size of antiderivative = 5.40

$$\int \left(d+ex^2\right)^{3/2} (a +b\sec^{-1}(cx)) \, dx = \frac{5\sqrt{e\,x^2+d}\,adex+2\sqrt{e\,x^2+d}\,a\,e^2x^3+3\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)a\,d^2+8\left(\int\sqrt{e\,x^2+d}\,as\right.}{8e}$$

input
```
int((e*x^2+d)^(3/2)*(a+b*asec(c*x)),x)
```

output
```
(5*sqrt(d + e*x**2)*a*d*e*x + 2*sqrt(d + e*x**2)*a*e**2*x**3 + 3*sqrt(e)*l
og((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*d**2 + 8*int(sqrt(d + e*x**2)
*asec(c*x)*x**2,x)*b*e**2 + 8*int(sqrt(d + e*x**2)*asec(c*x),x)*b*d*e)/(8*
e)
```

3.127 $\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^2}\,dx$

Optimal result 1074
Mathematica [N/A] 1074
Rubi [N/A] 1075
Maple [N/A] 1075
Fricas [N/A] 1076
Sympy [N/A] 1076
Maxima [F(-2)] 1076
Giac [N/A] 1077
Mupad [N/A] 1077
Reduce [N/A] 1078

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^2}\,dx = \text{Int}\left(\frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^2},x\right)$$

output
```
Defer(Int)((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 32.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^2}\,dx = \int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^2}\,dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^2,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^2}dx$$
$$\Big\downarrow \text{ 5771}$$
$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^2}dx$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsec}\left(cx\right)\right)}{x^{2}}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^2,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arcsec}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^2,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsec(c*x))*sqrt(e*x^2 + d)/x^2
, x)
```

Sympy [N/A]

Not integrable

Time = 112.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \int \frac{(a+b\,\mathrm{asec}\,(cx))\,(d+ex^2)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*asec(c*x))/x**2,x)
```

output
```
Integral((a + b*asec(c*x))*(d + e*x**2)**(3/2)/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arcsec}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^2,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \int \frac{(e\,x^2+d)^{3/2}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^2,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 4.74

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \frac{-8\sqrt{e\,x^2+d}\,ad+4\sqrt{e\,x^2+d}\,ae\,x^2+12\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)adx - $$

input
```
int((e*x^2+d)^(3/2)*(a+b*asec(c*x))/x^2,x)
```

output
```
( - 8*sqrt(d + e*x**2)*a*d + 4*sqrt(d + e*x**2)*a*e*x**2 + 12*sqrt(e)*log(
(sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*d*x - 9*sqrt(e)*a*d*x + 8*int((s
qrt(d + e*x**2)*asec(c*x))/x**2,x)*b*d*x + 8*int(sqrt(d + e*x**2)*asec(c*x
),x)*b*e*x)/(8*x)
```

3.128 $\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^4}\,dx$

Optimal result 1079
Mathematica [N/A] 1079
Rubi [N/A] 1080
Maple [N/A] 1080
Fricas [N/A] 1081
Sympy [N/A] 1081
Maxima [F(-2)] 1081
Giac [N/A] 1082
Mupad [N/A] 1082
Reduce [N/A] 1083

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^4}\,dx = \operatorname{Int}\left(\frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^4},x\right)$$

output
```
Defer(Int)((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^4,x)
```

Mathematica [N/A]

Not integrable

Time = 7.63 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^4}\,dx = \int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^4}\,dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^4,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^4, x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^4}\,dx$$
$$\Big\downarrow \text{5771}$$
$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^4}dx$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^4,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}(a+b\,\mathrm{arcsec}\,(cx))}{x^4}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^4,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^4}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arcsec}\,(cx)+a)}{x^4}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^4,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsec(c*x))*sqrt(e*x^2 + d)/x^4
, x)
```

Sympy [N/A]

Not integrable

Time = 83.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^4}\,dx = \int \frac{(a+b\,\mathrm{asec}\,(cx))\,(d+ex^2)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*asec(c*x))/x**4,x)
```

output
```
Integral((a + b*asec(c*x))*(d + e*x**2)**(3/2)/x**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^4}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arcsec}\,(cx)+a)}{x^4}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^4,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)/x^4, x)
```

Mupad [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^4}\,dx = \int \frac{(e\,x^2+d)^{3/2}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^4,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^4, x)
```

Reduce [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 111, normalized size of antiderivative = 4.83

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^4}\,dx = \frac{-\sqrt{e\,x^2+d}\,ad-4\sqrt{e\,x^2+d}\,ae\,x^2+3\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)ae\,x^3+}{3x^3}$$

input
```
int((e*x^2+d)^(3/2)*(a+b*asec(c*x))/x^4,x)
```

output
```
( - sqrt(d + e*x**2)*a*d - 4*sqrt(d + e*x**2)*a*e*x**2 + 3*sqrt(e)*log((sq
rt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*e*x**3 + 3*int((sqrt(d + e*x**2)*as
ec(c*x))/x**4,x)*b*d*x**3 + 3*int((sqrt(d + e*x**2)*asec(c*x))/x**2,x)*b*e
*x**3)/(3*x**3)
```

3.129 $\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^6}\,dx$

Optimal result 1084
Mathematica [C] (verified) 1085
Rubi [A] (verified) 1086
Maple [F] 1091
Fricas [A] (verification not implemented) 1091
Sympy [F(-1)] 1092
Maxima [F(-2)] 1092
Giac [F] 1093
Mupad [F(-1)] 1093
Reduce [F] 1093

Optimal result

Integrand size = 23, antiderivative size = 416

$$\begin{aligned}
\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^6}\,dx &= \frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} \\
&+ \frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} \\
&+ \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25x^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5dx^5} \\
&- \frac{bc^2(8c^4d^2+23c^2de+23e^2)\,x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\,E\left(\arcsin(cx)\,\middle|-\frac{e}{c^2d}\right)}{75d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&+ \frac{b(c^2d+e)(8c^4d^2+19c^2de+15e^2)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{75d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}$$

output
```
1/75*b*c*(8*c^4*d^2+23*c^2*d*e+23*e^2)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d
/(c^2*x^2)^(1/2)+4/75*b*c*(c^2*d+2*e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/x^
2/(c^2*x^2)^(1/2)+1/25*b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(3/2)/x^4/(c^2*x^2)
^(1/2)-1/5*(e*x^2+d)^(5/2)*(a+b*arcsec(c*x))/d/x^5-1/75*b*c^2*(8*c^4*d^2+2
3*c^2*d*e+23*e^2)*x*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)*EllipticE(c*x,(-e/c
^2/d)^(1/2))/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)+1/75*b*
(c^2*d+e)*(8*c^4*d^2+19*c^2*d*e+15*e^2)*x*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(
1/2)*EllipticF(c*x,(-e/c^2/d)^(1/2))/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(
e*x^2+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.38 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.73

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^6}\,dx = \frac{\sqrt{d+ex^2}\Big(-15a(d+ex^2)^2+bc\sqrt{1-\frac{1}{c^2x^2}}x(23e^2x^4+dex^2(11+23c}{75dx^5}$$
$$-\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(8c^4d^2+23c^2de+23e^2)\,E\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)|-\frac{e}{c^2d}\right)-(8c^6d^3+27c^4d^2e+3\right.}{75\sqrt{-c^2}d\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^6,x]
```

output
```
(Sqrt[d + e*x^2]*(-15*a*(d + e*x^2)^2 + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(23*e^
2*x^4 + d*e*x^2*(11 + 23*c^2*x^2) + d^2*(3 + 4*c^2*x^2 + 8*c^4*x^4)) - 15*
b*(d + e*x^2)^2*ArcSec[c*x]))/(75*d*x^5) - ((I/75)*b*c*Sqrt[1 - 1/(c^2*x^2
)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*Elliptic
E[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] - (8*c^6*d^3 + 27*c^4*d^2*e + 34*
c^2*d*e^2 + 15*e^3)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sq
rt[-c^2]*d*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 381, normalized size of antiderivative = 0.92, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {5761, 27, 376, 25, 442, 25, 445, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^6}dx$$

$$\Big\downarrow \text{5761}$$

$$-\frac{bcx\int -\frac{(ex^2+d)^{5/2}}{5dx^6\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}-\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5dx^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int \frac{(ex^2+d)^{5/2}}{x^6\sqrt{c^2x^2-1}}dx}{5d\sqrt{c^2x^2}}-\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5dx^5}$$

$$\Big\downarrow \text{376}$$

$$\frac{bcx\left(\frac{d\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{5x^5}-\frac{1}{5}\int -\frac{\sqrt{ex^2+d}(e(dc^2+5e)x^2+4d(dc^2+2e))}{x^4\sqrt{c^2x^2-1}}dx\right)}{5d\sqrt{c^2x^2}}-\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5dx^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{bcx\left(\frac{1}{5}\int \frac{\sqrt{ex^2+d}(e(dc^2+5e)x^2+4d(dc^2+2e))}{x^4\sqrt{c^2x^2-1}}dx+\frac{d\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{5x^5}\right)}{5d\sqrt{c^2x^2}}-\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5dx^5}$$

$$\Big\downarrow \text{442}$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{4d\sqrt{c^2x^2-1}(c^2d+2e)\sqrt{d+ex^2}}{3x^3}-\frac{1}{3}\int -\frac{e(4d^2c^4+11dec^2+15e^2)x^2+d(8d^2c^4+23dec^2+23e^2)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx\right)+\frac{d\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{5x^5}\right)}{5d\sqrt{c^2x^2}}-\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5dx^5}$$

$\downarrow$ 25

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{e\left(4d^2c^4+11dec^2+15e^2\right)x^2+d\left(8d^2c^4+23dec^2+23e^2\right)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{4d\sqrt{c^2x^2-1}\left(c^2d+2e\right)\sqrt{d+ex^2}}{3x^3}\right)+\frac{d\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{5x^5}\right)}{5d\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$\downarrow$ 445

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\int \frac{de\left(4d^2c^4-\left(8d^2c^4+23dec^2+23e^2\right)x^2c^2+11dec^2+15e^2\right)}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{d}+\frac{\sqrt{c^2x^2-1}\left(8c^4d^2+23c^2de+23e^2\right)\sqrt{d+ex^2}}{x}\right)+\frac{4d\sqrt{c^2x^2-1}\left(c^2d+2e\cdots\right)}{3x^3}\cdots\right)\cdots\right)}{5d\sqrt{c^2x^2}}\cdots$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$\downarrow$ 27

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\int \frac{4d^2c^4-\left(8d^2c^4+23dec^2+23e^2\right)x^2c^2+11dec^2+15e^2}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{\sqrt{c^2x^2-1}\left(8c^4d^2+23c^2de+23e^2\right)\sqrt{d+ex^2}}{x}\right)+\frac{4d\sqrt{c^2x^2-1}\left(c^2d+2e\cdots\right)}{3x^3}\cdots\right)\cdots\right)}{5d\sqrt{c^2x^2}}\cdots$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$\downarrow$ 399

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\left(c^2d+e\right)\left(8c^4d^2+19c^2de+15e^2\right)\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e}-\frac{c^2\left(8c^4d^2+23c^2de+23e^2\right)\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}\left(8c^4d^2+23\cdots\right)}{\cdots}\right)\cdots\right)\cdots\right)}{5d\sqrt{c^2x^2}}\cdots$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$\downarrow$ 323

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\left(c^2d+e\right)\left(8c^4d^2+19c^2de+15e^2\right)\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}-\frac{c^2\left(8c^4d^2+23c^2de+23e^2\right)\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\sqrt{c^2x^2-1}\cdots\right)\cdots\right)\cdots\right)}{5d\sqrt{c^2x^2}}\cdots$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\Big\downarrow \; 323$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+e)(8c^4d^2+19c^2de+15e^2)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2(8c^4d^2+23c^2de+23e^2)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+}{5d\sqrt{c^2x^2}}$$

$$\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5dx^5}$$

$$\Big\downarrow \; 321$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+e)(8c^4d^2+19c^2de+15e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2(8c^4d^2+23c^2de+23e^2)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)\right.\right.\right.}{5d\sqrt{c^2x^2}}$$

$$\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5dx^5}$$

$$\Big\downarrow \; 331$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+e)(8c^4d^2+19c^2de+15e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}(8c^4d^2+23c^2de+23e^2)\int\frac{\sqrt{e}}{\sqrt{1-}}}{e\sqrt{c^2x^2-1}}\right.\right.\right.\right.}{5d\sqrt{c^2x^2}}$$

$$\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5dx^5}$$

$$\Big\downarrow \; 330$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+e)(8c^4d^2+19c^2de+15e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}(8c^4d^2+23c^2de+23e^2)\sqrt{d+e}}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\right.\right.\right.\right.}{5d\sqrt{c^2x}}$$

$$\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5dx^5}$$

$$\Big\downarrow \; 327$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+e)(8c^4d^2+19c^2de+15e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c\sqrt{1-c^2x^2}(8c^4d^2+23c^2de+23e^2)\sqrt{d+ex^2}}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\right.\right.\right.\right.}{5d\sqrt{c^2}}$$

$$\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5dx^5}$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^6,x]
```

output
```
-1/5*((d + e*x^2)^(5/2)*(a + b*ArcSec[c*x]))/(d*x^5) + (b*c*x*((d*Sqrt[-1
+ c^2*x^2]*(d + e*x^2)^(3/2))/(5*x^5) + ((4*d*(c^2*d + 2*e)*Sqrt[-1 + c^2*
x^2]*Sqrt[d + e*x^2])/(3*x^3) + (((8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*Sqrt[-
1 + c^2*x^2]*Sqrt[d + e*x^2])/x + e*(-((c*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2
)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/
(e*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d])) + ((c^2*d + e)*(8*c^4*d^2 + 19
*c^2*d*e + 15*e^2)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[
c*x], -(e/(c^2*d))])/(c*e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])))/3)/5))/(5*
d*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [F]

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\text{arcsec}\,(cx))}{x^6}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^6,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^6,x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^6}\,dx = -\frac{\left(15\,acde^2x^4+30\,acd^2ex^2+15\,acd^3+15\left(bcde^2x^4+2\,bcd^2ex^2+bcd^3\right)\text{arcsec}\,(cx)-\left(3\,bcd^3+\left(8\,bc^5d^3+\right.\right.\right.}{}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^6,x, algorithm="fricas")
```

output
```
-1/75*((15*a*c*d*e^2*x^4 + 30*a*c*d^2*e*x^2 + 15*a*c*d^3 + 15*(b*c*d*e^2*x
^4 + 2*b*c*d^2*e*x^2 + b*c*d^3)*arcsec(c*x) - (3*b*c*d^3 + (8*b*c^5*d^3 +
23*b*c^3*d^2*e + 23*b*c*d*e^2)*x^4 + (4*b*c^3*d^3 + 11*b*c*d^2*e)*x^2)*sqr
t(c^2*x^2 - 1))*sqrt(e*x^2 + d) - ((8*b*c^8*d^3 + 23*b*c^6*d^2*e + 23*b*c^
4*d*e^2)*x^5*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (8*b*c^8*d^3 + (23*b*c^
6 + 4*b*c^4)*d^2*e + (23*b*c^4 + 11*b*c^2)*d*e^2 + 15*b*e^3)*x^5*elliptic_
f(arcsin(c*x), -e/(c^2*d)))*sqrt(-d))/(c*d^2*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^6}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*asec(c*x))/x**6,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^6,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^6}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arcsec}\,(cx)+a)}{x^6}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^6,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^6}\,dx=\int \frac{(e\,x^2+d)^{3/2}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^6,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^6, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^6}\,dx=\frac{-\sqrt{e\,x^2+d}\,a\,d^2-2\sqrt{e\,x^2+d}\,ade\,x^2-\sqrt{e\,x^2+d}\,a\,e^2x^4-\sqrt{e}\,a\,e^2x}{5d\,x^5}$$

input
```
int((e*x^2+d)^(3/2)*(a+b*asec(c*x))/x^6,x)
```

output
```
( - sqrt(d + e*x**2)*a*d**2 - 2*sqrt(d + e*x**2)*a*d*e*x**2 - sqrt(d + e*x
**2)*a*e**2*x**4 - sqrt(e)*a*e**2*x**5 + 5*int((sqrt(d + e*x**2)*asec(c*x)
)/x**6,x)*b*d**2*x**5 + 5*int((sqrt(d + e*x**2)*asec(c*x))/x**4,x)*b*d*e*x
**5)/(5*d*x**5)
```

3.130 $\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^8}\,dx$

Optimal result . . . 1094
Mathematica [C] (verified) . . . 1095
Rubi [A] (verified) . . . 1096
Maple [F] . . . 1102
Fricas [A] (verification not implemented) . . . 1102
Sympy [F(-1)] . . . 1103
Maxima [F(-2)] . . . 1103
Giac [F] . . . 1103
Mupad [F(-1)] . . . 1104
Reduce [F] . . . 1104

Optimal result

Integrand size = 23, antiderivative size = 554

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^8}\,dx = \frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{c^2x^2}}$$
$$+\frac{bc(120c^4d^2+159c^2de-37e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675dx^2\sqrt{c^2x^2}}$$
$$+\frac{bc(30c^2d+11e)\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{1225dx^4\sqrt{c^2x^2}}+\frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{49dx^6\sqrt{c^2x^2}}$$
$$-\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{7dx^7}+\frac{2e(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{35d^2x^5}$$
$$-\frac{bc^2(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|\,-\frac{e}{c^2d}\right)}{3675d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}$$
$$+\frac{2b(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3675d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

output
```
1/3675*b*c*(240*c^6*d^3+528*c^4*d^2*e+193*c^2*d*e^2-247*e^3)*(c^2*x^2-1)^(
1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)+1/3675*b*c*(120*c^4*d^2+159*c^2*d
*e-37*e^2)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(c^2*x^2)^(1/2)+1/1225*
b*c*(30*c^2*d+11*e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(3/2)/d/x^4/(c^2*x^2)^(1/2
)+1/49*b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(5/2)/d/x^6/(c^2*x^2)^(1/2)-1/7*(e*
x^2+d)^(5/2)*(a+b*arcsec(c*x))/d/x^7+2/35*e*(e*x^2+d)^(5/2)*(a+b*arcsec(c*
x))/d^2/x^5-1/3675*b*c^2*(240*c^6*d^3+528*c^4*d^2*e+193*c^2*d*e^2-247*e^3)
*x*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)*EllipticE(c*x,(-e/c^2/d)^(1/2))/d^2/
(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)+2/3675*b*(c^2*d+e)*(12
0*c^6*d^3+204*c^4*d^2*e+17*c^2*d*e^2-105*e^3)*x*(-c^2*x^2+1)^(1/2)*(1+e*x^
2/d)^(1/2)*EllipticF(c*x,(-e/c^2/d)^(1/2))/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)
^(1/2)/(e*x^2+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.77 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^8}\,dx = \frac{\sqrt{d+ex^2}\left(-105a(5d-2ex^2)(d+ex^2)^2+bc\sqrt{1-\frac{1}{c^2x^2}}x(-247e^3x^6\right.}{}$$
$$-\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)E\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)|-\frac{e}{c^2d}\right)-2(1\right.}{3675\sqrt{-c^2}d^2\sqrt{1-c^2x^2}}$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^8,x]
```

output
```
(Sqrt[d + e*x^2]*(-105*a*(5*d - 2*e*x^2)*(d + e*x^2)^2 + b*c*Sqrt[1 - 1/(c
^2*x^2)]*x*(-247*e^3*x^6 + d*e^2*x^4*(71 + 193*c^2*x^2) + 3*d^2*e*x^2*(61
+ 83*c^2*x^2 + 176*c^4*x^4) + 15*d^3*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c^6*x
^6)) - 105*b*(5*d - 2*e*x^2)*(d + e*x^2)^2*ArcSec[c*x]))/(3675*d^2*x^7) -
((I/3675)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(240*c^6*
d^3 + 528*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*EllipticE[I*ArcSinh[Sqrt[-c
^2]*x], -(e/(c^2*d))] - 2*(120*c^8*d^4 + 324*c^6*d^3*e + 221*c^4*d^2*e^2 -
 88*c^2*d*e^3 - 105*e^4)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))])
)/(Sqrt[-c^2]*d^2*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.91, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {5761, 27, 442, 25, 442, 25, 442, 25, 445, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)}{x^8}dx$$

$$\Big\downarrow \text{5761}$$

$$-\frac{bcx\int -\frac{(5d-2ex^2)(ex^2+d)^{5/2}}{35d^2x^8\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{7dx^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int \frac{(5d-2ex^2)(ex^2+d)^{5/2}}{x^8\sqrt{c^2x^2-1}}dx}{35d^2\sqrt{c^2x^2}}+\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{7dx^7}$$

$$\Big\downarrow \text{442}$$

$$\frac{bcx\left(\frac{5d\sqrt{c^2x^2-1}(d+ex^2)^{5/2}}{7x^7}-\frac{1}{7}\int -\frac{(ex^2+d)^{3/2}\left((5c^2d-14e)ex^2+d(30dc^2+11e)\right)}{x^6\sqrt{c^2x^2-1}}dx\right)}{35d^2\sqrt{c^2x^2}}+\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{7dx^7}$$

$$\Big\downarrow \text{25}$$

$$\frac{bcx\left(\frac{1}{7}\int \frac{(ex^2+d)^{3/2}\left((5c^2d-14e)ex^2+d(30dc^2+11e)\right)}{x^6\sqrt{c^2x^2-1}}dx+\frac{5d\sqrt{c^2x^2-1}(d+ex^2)^{5/2}}{7x^7}\right)}{35d^2\sqrt{c^2x^2}}+\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{7dx^7}$$

$$\Big\downarrow \text{442}$$

$$\frac{bcx\left(\frac{1}{7}\left(\frac{d\sqrt{c^2x^2-1}(30c^2d+11e)(d+ex^2)^{3/2}}{5x^5}-\frac{1}{5}\int -\frac{\sqrt{ex^2+d}\left(2e(15d^2c^4+18dec^2-35e^2)x^2+d(120d^2c^4+159dec^2-37e^2)\right)}{x^4\sqrt{c^2x^2-1}}dx\right)+\frac{5d\sqrt{c^2}\cdots}{}\right.}{35d^2\sqrt{c^2x^2}}$$

$$\frac{2e(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{35d^2x^5}-\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{7dx^7}$$

$\downarrow$ 25

$$\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\int \frac{\sqrt{ex^2+d}\left(2e(15d^2c^4+18dec^2-35e^2)x^2+d(120d^2c^4+159dec^2-37e^2)\right)}{x^4\sqrt{c^2x^2-1}}dx+\frac{d\sqrt{c^2x^2-1}(30c^2d+11e)(d+ex^2)^{3/2}}{5x^5}\right)+\frac{5d\sqrt{c^2x^2}\cdots}{}\right.}{35d^2\sqrt{c^2x^2}}$$

$$\frac{2e(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{35d^2x^5}-\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{7dx^7}$$

$\downarrow$ 442

$$\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{d\sqrt{c^2x^2-1}(120c^4d^2+159c^2de-37e^2)\sqrt{d+ex^2}}{3x^3}-\frac{1}{3}\int -\frac{e(120d^3c^6+249d^2ec^4+71de^2c^2-210e^3)x^2+d(240d^3c^6+528d^2ec^4+193d\cdots}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}\right.\right.\right.}{35d^2\sqrt{c^2x^2}}$$

$$\frac{2e(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{35d^2x^5}-\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{7dx^7}$$

$\downarrow$ 25

$$\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{e(120d^3c^6+249d^2ec^4+71de^2c^2-210e^3)x^2+d(240d^3c^6+528d^2ec^4+193de^2c^2-247e^3)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{d\sqrt{c^2x^2-1}(120c^4d^2+159c^2de-\cdots}{3x^3}\right.\right.\right.}{35d^2\sqrt{c^2x^2}}$$

$$\frac{2e(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{35d^2x^5}-\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{7dx^7}$$

$\downarrow$ 445

$$\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\int \frac{de\left(120d^3c^6+249d^2ec^4+71de^2c^2-\left(240d^3c^6+528d^2ec^4+193de^2c^2-247e^3\right)x^2c^2-210e^3\right)}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{d}+\frac{\sqrt{c^2x^2-1}(240c^6d^3+528c^4d^2e+19\cdots}{x}\right.\right.\right.\right.}{35\cdots}$$

$$\frac{2e(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{35d^2x^5}-\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{7dx^7}$$

$\downarrow$ 27

$$bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\int\frac{120d^3c^6+249d^2ec^4+71de^2c^2-(240d^3c^6+528d^2ec^4+193de^2c^2-247e^3)x^2c^2-210e^3}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{\sqrt{c^2x^2-1}(240c^6d^3+528c^4d^2e+}{x}\right.\right.\right.\right.$$

$$\frac{2e(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{35d^2x^5}-\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{7dx^7}$$

$\downarrow$ 399

$$bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{2(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e}-\frac{c^2(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\int}{e}\right.\right.\right.\right.\right.$$

$$\frac{2e(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{35d^2x^5}-\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{7dx^7}$$

$\downarrow$ 323

$$bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{2(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}-\frac{c^2(240c^6d^3+528c^4d^2e+193c^2de^2-}{e}\right.\right.\right.\right.\right.$$

$$\frac{2e(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{35d^2x^5}-\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{7dx^7}$$

$\downarrow$ 323

$$bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2(240c^6d^3+528c^4d^2e+1}{}\right.\right.\right.\right.\right.$$

$$\frac{2e(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{35d^2x^5}-\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{7dx^7}$$

$\downarrow$ 321

$$bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2(240c^6d^3+528c^4d}{}\right.\right.\right.\right.\right.$$

$$\frac{2e(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{35d^2x^5}-\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{7dx^7}$$

$\downarrow$ 331

$$bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}\left(c^2d+e\right)\left(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}\left(240c^6d^3\right.}{}\right.\right.\right.\right.\right.$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{7dx^7}$$

$\Big\downarrow$ 330

$$bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}\left(c^2d+e\right)\left(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}\left(240c^6\right.}{}\right.\right.\right.\right.\right.$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{7dx^7}$$

$\Big\downarrow$ 327

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{7dx^7}+$$
$$bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}\left(c^2d+e\right)\left(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c\sqrt{1-c^2x^2}\left(240c^6d^3\right.}{}\right.\right.\right.\right.\right.$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^8,x]
```

output
```
-1/7*((d + e*x^2)^(5/2)*(a + b*ArcSec[c*x]))/(d*x^7) + (2*e*(d + e*x^2)^(5
/2)*(a + b*ArcSec[c*x]))/(35*d^2*x^5) + (b*c*x*((5*d*Sqrt[-1 + c^2*x^2]*(d
 + e*x^2)^(5/2))/(7*x^7) + ((d*(30*c^2*d + 11*e)*Sqrt[-1 + c^2*x^2]*(d + e
*x^2)^(3/2))/(5*x^5) + ((d*(120*c^4*d^2 + 159*c^2*d*e - 37*e^2)*Sqrt[-1 +
c^2*x^2]*Sqrt[d + e*x^2])/(3*x^3) + (((240*c^6*d^3 + 528*c^4*d^2*e + 193*c
^2*d*e^2 - 247*e^3)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/x + e*(-(((c*(240*c
^6*d^3 + 528*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*Sqrt[1 - c^2*x^2]*Sqrt[d
 + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(e*Sqrt[-1 + c^2*x^2]*Sqrt
[1 + (e*x^2)/d])) + (2*(c^2*d + e)*(120*c^6*d^3 + 204*c^4*d^2*e + 17*c^2*d
*e^2 - 105*e^3)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x
], -(e/(c^2*d))])/(c*e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])))/3)/5)/7))/(35
*d^2*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [F]

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\mathrm{arcsec}\,(cx))}{x^8}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^8,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^8,x)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 395, normalized size of antiderivative = 0.71

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^8}\,dx = \frac{(210\,acde^3x^6-105\,acd^2e^2x^4-840\,acd^3ex^2-525\,acd^4+105\,(2\,bcde$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^8,x, algorithm="fricas")
```

output
```
1/3675*((210*a*c*d*e^3*x^6 - 105*a*c*d^2*e^2*x^4 - 840*a*c*d^3*e*x^2 - 525
*a*c*d^4 + 105*(2*b*c*d*e^3*x^6 - b*c*d^2*e^2*x^4 - 8*b*c*d^3*e*x^2 - 5*b*
c*d^4)*arcsec(c*x) + ((240*b*c^7*d^4 + 528*b*c^5*d^3*e + 193*b*c^3*d^2*e^2
 - 247*b*c*d*e^3)*x^6 + 75*b*c*d^4 + (120*b*c^5*d^4 + 249*b*c^3*d^3*e + 71
*b*c*d^2*e^2)*x^4 + 3*(30*b*c^3*d^4 + 61*b*c*d^3*e)*x^2)*sqrt(c^2*x^2 - 1)
)*sqrt(e*x^2 + d) + ((240*b*c^10*d^4 + 528*b*c^8*d^3*e + 193*b*c^6*d^2*e^2
 - 247*b*c^4*d*e^3)*x^7*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (240*b*c^10*
d^4 + 24*(22*b*c^8 + 5*b*c^6)*d^3*e + (193*b*c^6 + 249*b*c^4)*d^2*e^2 - (2
47*b*c^4 - 71*b*c^2)*d*e^3 - 210*b*e^4)*x^7*elliptic_f(arcsin(c*x), -e/(c^
2*d)))*sqrt(-d))/(c*d^3*x^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^8}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*asec(c*x))/x**8,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^8}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^8,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^8}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arcsec}\,(cx)+a)}{x^8}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^8,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^8}\,dx = \int \frac{(e\,x^2+d)^{3/2}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^8,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^8, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^8}\,dx = \frac{-5\sqrt{e\,x^2+d}\,a\,d^3-8\sqrt{e\,x^2+d}\,a\,d^2e\,x^2-\sqrt{e\,x^2+d}\,ad\,e^2x^4+2\sqrt{e}}{}$$

input
```
int((e*x^2+d)^(3/2)*(a+b*asec(c*x))/x^8,x)
```

output
```
( - 5*sqrt(d + e*x**2)*a*d**3 - 8*sqrt(d + e*x**2)*a*d**2*e*x**2 - sqrt(d
+ e*x**2)*a*d*e**2*x**4 + 2*sqrt(d + e*x**2)*a*e**3*x**6 - 2*sqrt(e)*a*e**
3*x**7 + 35*int((sqrt(d + e*x**2)*asec(c*x))/x**8,x)*b*d**3*x**7 + 35*int(
(sqrt(d + e*x**2)*asec(c*x))/x**6,x)*b*d**2*e*x**7)/(35*d**2*x**7)
```

3.131 $\int \frac{x^5\left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal result . . . 1105
Mathematica [C] (warning: unable to verify) . . . 1106
Rubi [A] (verified) . . . 1107
Maple [F] . . . 1112
Fricas [A] (verification not implemented) . . . 1112
Sympy [F] . . . 1113
Maxima [F(-2)] . . . 1114
Giac [F] . . . 1114
Mupad [F(-1)] . . . 1114
Reduce [F] . . . 1115

Optimal result

Integrand size = 23, antiderivative size = 321

$$\begin{aligned}\int \frac{x^5(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx =& \frac{b(19c^2d-9e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{c^2x^2}}\\ &-\frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce^2\sqrt{c^2x^2}}\\ &+\frac{d^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3}\\ &-\frac{2d(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{3e^3}\\ &+\frac{(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5e^3}\\ &+\frac{8bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{15e^3\sqrt{c^2x^2}}\\ &-\frac{b(45c^4d^2-10c^2de+9e^2)\,x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{5/2}\sqrt{c^2x^2}}\end{aligned}$$

output
```
1/120*b*(19*c^2*d-9*e)*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c^3/e^2/(c^2*x^
2)^(1/2)-1/20*b*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(3/2)/c/e^2/(c^2*x^2)^(1/2)+
d^2*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/e^3-2/3*d*(e*x^2+d)^(3/2)*(a+b*arcse
c(c*x))/e^3+1/5*(e*x^2+d)^(5/2)*(a+b*arcsec(c*x))/e^3+8/15*b*c*d^(5/2)*x*a
rctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e^3/(c^2*x^2)^(1/2)-1/120
*b*(45*c^4*d^2-10*c^2*d*e+9*e^2)*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*
x^2+d)^(1/2))/c^4/e^(5/2)/(c^2*x^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.82 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.88

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$

$$= \frac{16a(d+ex^2)\left(8d^2-4dex^2+3e^2x^4\right)-\frac{2be\sqrt{1-\frac{1}{c^2x^2}}(d+ex^2)\left(9ex+c^2(-13dx+6ex^3)\right)}{c^3}+\frac{64bd^3\sqrt{1+\frac{d}{ex^2}}\operatorname{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{}{c^2}\right.}{cx}}{2}$$

input
```
Integrate[(x^5*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(16*a*(d + e*x^2)*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4) - (2*b*e*Sqrt[1 - 1/(c^2
*x^2)]*(d + e*x^2)*(9*e*x + c^2*(-13*d*x + 6*e*x^3)))/c^3 + (64*b*d^3*Sqrt
[1 + d/(e*x^2)]*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))])/(c*x)
 + (b*e*(45*c^4*d^2 - 10*c^2*d*e + 9*e^2)*Sqrt[1 - 1/(c^2*x^2)]*x^3*Sqrt[1
 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/(c^3*Sqrt[1
 - c^2*x^2]) + 16*b*(d + e*x^2)*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4)*ArcSec[c*x
])/(240*e^3*Sqrt[d + e*x^2])
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {5761, 27, 7282, 2118, 27, 171, 27, 175, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}dx$$

$$\Big\downarrow \text{5761}$$

$$-\frac{bcx\int \frac{\sqrt{ex^2+d}(3e^2x^4-4dex^2+8d^2)}{15e^3x\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{d^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3}+\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^3}-\frac{2d(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bcx\int \frac{\sqrt{ex^2+d}(3e^2x^4-4dex^2+8d^2)}{x\sqrt{c^2x^2-1}}dx}{15e^3\sqrt{c^2x^2}}+\frac{d^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3}+\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^3}-\frac{2d(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^3}$$

$$\Big\downarrow \text{7282}$$

$$-\frac{bcx\int \frac{\sqrt{ex^2+d}(3e^2x^4-4dex^2+8d^2)}{x^2\sqrt{c^2x^2-1}}dx^2}{30e^3\sqrt{c^2x^2}}+\frac{d^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3}+\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^3}-\frac{2d(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^3}$$

$$\Big\downarrow \text{2118}$$

$$-\frac{bcx\left(\frac{\int \frac{e\sqrt{ex^2+d}\left(32c^2d^2-\left(19c^2d-9e\right)ex^2\right)}{2x^2\sqrt{c^2x^2-1}}dx^2}{2c^2e}+\frac{3e\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{30e^3\sqrt{c^2x^2}}+\frac{d^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3}+\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^3}-\frac{2d(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{bcx\left(\frac{\int \frac{\sqrt{ex^2+d}\left(32c^2d^2-\left(19c^2d-9e\right)ex^2\right)}{x^2\sqrt{c^2x^2-1}}dx^2}{4c^2}+\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3\sqrt{c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \ 171$$

$$-\frac{bcx\left(\frac{\frac{\int \frac{64d^3c^4+e\left(45d^2c^4-10dec^2+9e^2\right)x^2}{2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2}-\frac{e\sqrt{c^2x^2-1}\left(19c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3\sqrt{c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{bcx\left(\frac{\frac{\int \frac{64d^3c^4+e\left(45d^2c^4-10dec^2+9e^2\right)x^2}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(19c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3\sqrt{c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \ 175$$

$$-\frac{bcx\left(\frac{\frac{64c^4d^3\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+e\left(45c^4d^2-10c^2de+9e^2\right)\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(19c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)}{2c^2}\right)}{30e^3\sqrt{c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$\Big\downarrow$ 66

$$-\frac{bcx\left(\frac{\frac{64c^4d^3\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\left(45c^4d^2-10c^2de+9e^2\right)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(19c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3\sqrt{c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$\Big\downarrow$ 104

$$-\frac{bcx\left(\frac{\frac{128c^4d^3\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e\left(45c^4d^2-10c^2de+9e^2\right)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(19c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3\sqrt{c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$\Big\downarrow$ 217

$$-\frac{bcx\left(\frac{\frac{2e\left(45c^4d^2-10c^2de+9e^2\right)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-128c^4d^{5/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(19c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3\sqrt{c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$\Big\downarrow$ 221

$$\frac{d^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3}+\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^3}-\frac{2d(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^3}-\frac{bcx\left(\frac{\frac{\frac{2\sqrt{e}(45c^4d^2-10c^2de+9e^2)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-128c^4d^{5/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}(19c^2d-9e)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{c^2x^2-1}(d+e}{2c^2}\right.}{30e^3\sqrt{c^2x^2}}$$

input
```
Int[(x^5*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(d^2*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/e^3 - (2*d*(d + e*x^2)^(3/2)*(a
+ b*ArcSec[c*x]))/(3*e^3) + ((d + e*x^2)^(5/2)*(a + b*ArcSec[c*x]))/(5*e^3
) - (b*c*x*((3*e*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(2*c^2) + (-(((19*c
^2*d - 9*e)*e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/c^2) + (-128*c^4*d^(5/2)
*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])] + (2*Sqrt[e]*(45*c^4
*d^2 - 10*c^2*d*e + 9*e^2)*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d
+ e*x^2])])/c)/(2*c^2))/(4*c^2)))/(30*e^3*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2118

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

Maple [F]

$$\int \frac{x^5(a+b\,\mathrm{arcsec}\,(cx))}{\sqrt{e\,x^2+d}}dx$$

input
```
int(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 1385, normalized size of antiderivative = 4.31

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/480*(64*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^
2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2
+ d)*sqrt(-d) + 8*d^2)/x^4) + (45*b*c^4*d^2 - 10*b*c^2*d*e + 9*b*e^2)*sqrt
(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 -
4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) +
e^2) + 4*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 + 8*(3*b*c^5*
e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b*c^5*d^2)*arcsec(c*x) - (6*b*c^3*e^2*x^2 -
13*b*c^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e^3), 1
/480*(128*b*c^5*d^(5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2
*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (
45*b*c^4*d^2 - 10*b*c^2*d*e + 9*b*e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2
 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*s
qrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(24*a*c^5*e^2*x^4 - 32
*a*c^5*d*e*x^2 + 64*a*c^5*d^2 + 8*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b
*c^5*d^2)*arcsec(c*x) - (6*b*c^3*e^2*x^2 - 13*b*c^3*d*e + 9*b*c*e^2)*sqrt(
c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e^3), 1/240*(32*b*c^5*sqrt(-d)*d^2*log
(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2
 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + (45
*b*c^4*d^2 - 10*b*c^2*d*e + 9*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^
2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*...
```

Sympy [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{x^5(a+b\,\mathrm{asec}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate(x**5*(a+b*asec(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**5*(a + b*asec(c*x))/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^5}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x^5/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^5\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

output
```
int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$
$$=\frac{8\sqrt{e\,x^2+d}\,a\,d^2-4\sqrt{e\,x^2+d}\,ade\,x^2+3\sqrt{e\,x^2+d}\,a\,e^2x^4+15\left(\int\frac{asec(cx)x^5}{\sqrt{e\,x^2+d}}dx\right)b\,e^3}{15e^3}$$

input
```
int(x^5*(a+b*asec(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
(8*sqrt(d + e*x**2)*a*d**2 - 4*sqrt(d + e*x**2)*a*d*e*x**2 + 3*sqrt(d + e*
x**2)*a*e**2*x**4 + 15*int((asec(c*x)*x**5)/sqrt(d + e*x**2),x)*b*e**3)/(1
5*e**3)
```

3.132 $\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$

Optimal result 1116
Mathematica [C] (warning: unable to verify) 1117
Rubi [A] (verified) 1117
Maple [F] 1121
Fricas [A] (verification not implemented) 1121
Sympy [F] 1122
Maxima [F(-2)] 1123
Giac [F] 1123
Mupad [F(-1)] 1123
Reduce [F] 1124

Optimal result

Integrand size = 23, antiderivative size = 225

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = -\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2} - \frac{2bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e^2\sqrt{c^2x^2}} + \frac{b(3c^2d-e)\,x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{3/2}\sqrt{c^2x^2}}$$

output

```
-1/6*b*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c/e/(c^2*x^2)^(1/2)-d*(e*x^2+d)
^(1/2)*(a+b*arcsec(c*x))/e^2+1/3*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/e^2-2/3
*b*c*d^(3/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e^2/(c^2*
x^2)^(1/2)+1/6*b*(3*c^2*d-e)*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+
d)^(1/2))/c^2/e^(3/2)/(c^2*x^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.35 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.08

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$
$$= \frac{-4bd^2\sqrt{1+\frac{d}{ex^2}}(-1+c^2x^2)\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)+b(3c^2d-e)\,e\sqrt{1-\frac{1}{c^2x^2}}x^4\sqrt{1-c^2x^2}\sqrt{1+}}{12c}$$

input
```
Integrate[(x^3*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(-4*b*d^2*Sqrt[1 + d/(e*x^2)]*(-1 + c^2*x^2)*AppellF1[1, 1/2, 1/2, 2, 1/(c
^2*x^2), -(d/(e*x^2))] + b*(3*c^2*d - e)*e*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt[
1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^
2)/d)] - 2*x*(-1 + c^2*x^2)*(d + e*x^2)*(4*a*c*d + b*e*Sqrt[1 - 1/(c^2*x^2
)]*x - 2*a*c*e*x^2 + 2*b*c*(2*d - e*x^2)*ArcSec[c*x]))/(12*c*e^2*x*(-1 + c
^2*x^2)*Sqrt[d + e*x^2])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5761, 27, 435, 171, 27, 175, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \; 5761$$

$$-\frac{bcx\int -\frac{(2d-ex^2)\sqrt{ex^2+d}}{3e^2x\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2}$$

$$\Big\downarrow \; 27$$

$$\frac{bcx\int \frac{(2d-ex^2)\sqrt{ex^2+d}}{x\sqrt{c^2x^2-1}}dx}{3e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2}$$

$\Big\downarrow$ 435

$$\frac{bcx\int \frac{(2d-ex^2)\sqrt{ex^2+d}}{x^2\sqrt{c^2x^2-1}}dx^2}{6e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2}$$

$\Big\downarrow$ 171

$$\frac{bcx\left(\frac{\int \frac{4c^2d^2+(3c^2d-e)ex^2}{2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2}$$

$\Big\downarrow$ 27

$$\frac{bcx\left(\frac{\int \frac{4c^2d^2+(3c^2d-e)ex^2}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2}$$

$\Big\downarrow$ 175

$$\frac{bcx\left(\frac{4c^2d^2\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+e(3c^2d-e)\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2}$$

$\Big\downarrow$ 66

$$\frac{bcx\left(\frac{4c^2d^2\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e(3c^2d-e)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2}$$

$\Big\downarrow$ 104

$$\frac{bcx\left(\frac{8c^2d^2\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e(3c^2d-e)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2}$$

$$\Big\downarrow \ 217$$

$$\frac{bcx\left(\frac{2e(3c^2d-e)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-8c^2d^{3/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2}$$

$$\Big\downarrow \ 221$$

$$\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2}+\frac{bcx\left(\frac{\frac{2\sqrt{e}(3c^2d-e)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-8c^2d^{3/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^2\sqrt{c^2x^2}}$$

input
```
Int[(x^3*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
-((d*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/e^2) + ((d + e*x^2)^(3/2)*(a + b
*ArcSec[c*x]))/(3*e^2) + (b*c*x*(-((e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/
c^2) + (-8*c^2*d^(3/2)*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])
] + (2*(3*c^2*d - e)*Sqrt[e]*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[
d + e*x^2])])/c)/(2*c^2)))/(6*e^2*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [F]

$$\int \frac{x^3(a + b\ \mathrm{arcsec}\left(cx\right))}{\sqrt{e\,x^2 + d}}dx$$

input
```
int(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 1111, normalized size of antiderivative = 4.94

$$\int \frac{x^3(a + b\sec^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/24*(4*b*c^3*sqrt(-d)*d*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^
2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)
*sqrt(-d) + 8*d^2)/x^4) - (3*b*c^2*d - b*e)*sqrt(e)*log(8*c^4*e^2*x^4 + c^
4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c
*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(2*a*c^3*e*x^2 -
4*a*c^3*d - sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arcsec(c
*x))*sqrt(e*x^2 + d))/(c^3*e^2), -1/24*(8*b*c^3*d^(3/2)*arctan(-1/2*sqrt(c
^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 +
 (c^2*d^2 - d*e)*x^2 - d^2)) + (3*b*c^2*d - b*e)*sqrt(e)*log(8*c^4*e^2*x^4
 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*
d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 4*(2*a*c^3*e*x
^2 - 4*a*c^3*d - sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arc
sec(c*x))*sqrt(e*x^2 + d))/(c^3*e^2), 1/12*(2*b*c^3*sqrt(-d)*d*log(((c^4*d
^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((
c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) - (3*b*c^2*d
- b*e)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqr
t(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(
2*a*c^3*e*x^2 - 4*a*c^3*d - sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 - 2*b
*c^3*d)*arcsec(c*x))*sqrt(e*x^2 + d))/(c^3*e^2), -1/12*(4*b*c^3*d^(3/2)*ar
ctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqr...
```

Sympy [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^3(a+b\operatorname{asec}(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate(x**3*(a+b*asec(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**3*(a + b*asec(c*x))/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^3}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x^3/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^3\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

output
```
int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \frac{-2\sqrt{e\,x^2+d}\,ad+\sqrt{e\,x^2+d}\,ae\,x^2+3\left(\int \frac{asec(cx)x^3}{\sqrt{e\,x^2+d}}dx\right)b\,e^2}{3e^2}$$

input
```
int(x^3*(a+b*asec(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
( - 2*sqrt(d + e*x**2)*a*d + sqrt(d + e*x**2)*a*e*x**2 + 3*int((asec(c*x)*
x**3)/sqrt(d + e*x**2),x)*b*e**2)/(3*e**2)
```

3.133 $\int \frac{x\left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal result . . . 1125
Mathematica [C] (verified) . . . 1125
Rubi [A] (verified) . . . 1126
Maple [F] . . . 1129
Fricas [A] (verification not implemented) . . . 1129
Sympy [F] . . . 1130
Maxima [F] . . . 1131
Giac [F] . . . 1131
Mupad [F(-1)] . . . 1131
Reduce [F] . . . 1132

Optimal result

Integrand size = 21, antiderivative size = 132

$$\int \frac{x(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e} + \frac{bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{e\sqrt{c^2x^2}} - \frac{bx\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{c^2x^2}}$$

output
```
(e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/e+b*c*d^(1/2)*x*arctan((e*x^2+d)^(1/2)/d
^(1/2)/(c^2*x^2-1)^(1/2))/e/(c^2*x^2)^(1/2)-b*x*arctanh(e^(1/2)*(c^2*x^2-1
)^(1/2)/c/(e*x^2+d)^(1/2))/e^(1/2)/(c^2*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.82

$$\int \frac{x(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d+ex^2}\left(a-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\,x\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},1,\frac{3}{2},\frac{e-c^2ex^2}{c^2d+e},1-c^2x^2\right)}{\sqrt{\frac{c^2(d+ex^2)}{c^2d+e}}}+b\sec^{-1}(cx)\right)}{e}$$

input
```
Integrate[(x*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(Sqrt[d + e*x^2]*(a - (b*c*Sqrt[1 - 1/(c^2*x^2)]*x*AppellF1[1/2, -1/2, 1,
3/2, (e - c^2*e*x^2)/(c^2*d + e), 1 - c^2*x^2])/Sqrt[(c^2*(d + e*x^2))/(c^
2*d + e)] + b*ArcSec[c*x]))/e
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {5759, 354, 140, 27, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{5759}$$

$$\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e}-\frac{bcx\int \frac{\sqrt{ex^2+d}}{x\sqrt{c^2x^2-1}}dx}{e\sqrt{c^2x^2}}$$

$$\Big\downarrow \text{354}$$

$$\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e}-\frac{bcx\int \frac{\sqrt{ex^2+d}}{x^2\sqrt{c^2x^2-1}}dx^2}{2e\sqrt{c^2x^2}}$$

$$\Big\downarrow \text{140}$$

$$\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e}-\frac{bcx\left(e\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+\int\frac{d}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2\right)}{2e\sqrt{c^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e}-\frac{bcx\left(e\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+d\int\frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2\right)}{2e\sqrt{c^2x^2}}$$

$$\Big\downarrow \text{66}$$

$$\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e}-\frac{bcx\left(d\int\frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}\right)}{2e\sqrt{c^2x^2}}$$

$$\Big\downarrow \text{104}$$

$$\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e}-\frac{bcx\left(2d\int\frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}\right)}{2e\sqrt{c^2x^2}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e}-\frac{bcx\left(2e\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-2\sqrt{d}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)\right)}{2e\sqrt{c^2x^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e}-\frac{bcx\left(\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-2\sqrt{d}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)\right)}{2e\sqrt{c^2x^2}}$$

input
```
Int[(x*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/e - (b*c*x*(-2*Sqrt[d]*ArcTan[Sqrt[d
 + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])] + (2*Sqrt[e]*ArcTanh[(Sqrt[e]*Sqrt
[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/c))/(2*e*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5759
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSec[c*x])/(2*e*(p + 1))), x
] - Simp[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2]))   Int[(d + e*x^2)^(p + 1)/(x*S
qrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Maple [F]

$$\int \frac{x(a+b\,\mathrm{arcsec}\,(cx))}{\sqrt{e\,x^2+d}}dx$$

input
```
int(x*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(x*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 869, normalized size of antiderivative = 6.58

$$\int \frac{x(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(b*c*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e
)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-
d) + 8*d^2)/x^4) + b*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(
c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*s
qrt(e*x^2 + d)*sqrt(e) + e^2) + 4*sqrt(e*x^2 + d)*(b*c*arcsec(c*x) + a*c))
/(c*e), 1/4*(2*b*c*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2
- 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2))
+ b*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2
)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sq
rt(e) + e^2) + 4*sqrt(e*x^2 + d)*(b*c*arcsec(c*x) + a*c))/(c*e), 1/4*(b*c*
sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*
sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2
)/x^4) + 2*b*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 -
1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2))
 + 4*sqrt(e*x^2 + d)*(b*c*arcsec(c*x) + a*c))/(c*e), 1/2*(b*c*sqrt(d)*arct
an(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/
(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + b*sqrt(-e)*arctan(1/2*(2*c^2*
e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4
 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*sqrt(e*x^2 + d)*(b*c*arcsec(c*x) +
a*c))/(c*e)]
```

Sympy [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{x(a+b\operatorname{asec}(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate(x*(a+b*asec(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x*(a + b*asec(c*x))/sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
-(e*integrate((c^2*e*x^3*log(c) - e*x*log(c) + ((c^2*log(c) + c^2)*e*x^3 +
 (c^2*d - e*log(c))*x)*e^(log(c*x + 1) + log(c*x - 1)) + (c^2*e*x^3 - e*x
+ (c^2*e*x^3 - e*x)*e^(log(c*x + 1) + log(c*x - 1)))*log(x))/((c^2*e*x^2 +
 (c^2*e*x^2 - e)*e^(log(c*x + 1) + log(c*x - 1)) - e)*sqrt(e*x^2 + d)), x)
 - sqrt(e*x^2 + d)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)))*b/e + sqrt(e*x^2 +
 d)*a/e
```

Giac [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{x\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

output
```
int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{e\,x^2+d}\,a+\left(\int \frac{asec(cx)x}{\sqrt{e\,x^2+d}}dx\right)be}{e}$$

input
```
int(x*(a+b*asec(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
(sqrt(d + e*x**2)*a + int((asec(c*x)*x)/sqrt(d + e*x**2),x)*b*e)/e
```

3.134 $\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$

Optimal result 1133
Mathematica [N/A] 1133
Rubi [N/A] 1134
Maple [N/A] 1134
Fricas [N/A] 1135
Sympy [N/A] 1135
Maxima [F(-2)] 1135
Giac [N/A] 1136
Mupad [N/A] 1136
Reduce [N/A] 1137

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx=\text{Int}\left(\frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}},x\right)$$

output
```
Defer(Int)((a+b*arcsec(c*x))/x/(e*x^2+d)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx=\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x*Sqrt[d + e*x^2]),x]
```

output
```
Integrate[(a + b*ArcSec[c*x])/(x*Sqrt[d + e*x^2]), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \text{ 5771}$$
$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}}dx$$

input
```
Int[(a + b*ArcSec[c*x])/(x*Sqrt[d + e*x^2]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\mathrm{arcsec}\left(cx\right)}{x\sqrt{e\,x^{2}+d}}dx$$

input
```
int((a+b*arcsec(c*x))/x/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arcsec(c*x))/x/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{a + b \sec^{-1}(cx)}{x\sqrt{d + ex^2}} \, dx = \int \frac{b \operatorname{arcsec}(cx) + a}{\sqrt{ex^2 + dx}} \, dx$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/(e*x^3 + d*x), x)
```

Sympy [N/A]

Not integrable

Time = 9.69 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{a + b \sec^{-1}(cx)}{x\sqrt{d + ex^2}} \, dx = \int \frac{a + b \operatorname{asec}(cx)}{x\sqrt{d + ex^2}} \, dx$$

input
```
integrate((a+b*asec(c*x))/x/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asec(c*x))/(x*sqrt(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b \sec^{-1}(cx)}{x\sqrt{d + ex^2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx=\int \frac{b\operatorname{arcsec}(cx)+a}{\sqrt{ex^2+dx}}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/(sqrt(e*x^2 + d)*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.52

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)a-\sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)a+\left(\int\frac{asec(cx)}{\sqrt{e\,x^2+d}\,x}dx\right)bd}{d}$$

input
```
int((a+b*asec(c*x))/x/(e*x^2+d)^(1/2),x)
```

output
```
(sqrt(d)*log((sqrt(d + e*x**2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*a - sqrt(d)
*log((sqrt(d + e*x**2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*a + int(asec(c*x)/(
sqrt(d + e*x**2)*x),x)*b*d)/d
```

3.135 $\int \frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$

Optimal result 1138
Mathematica [N/A] 1138
Rubi [N/A] 1139
Maple [N/A] 1139
Fricas [N/A] 1140
Sympy [N/A] 1140
Maxima [F(-2)] 1140
Giac [N/A] 1141
Mupad [N/A] 1141
Reduce [N/A] 1142

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \text{Int}\left(\frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}},x\right)$$

output
```
Defer(Int)((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 4.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x^3*Sqrt[d + e*x^2]),x]
```

output
```
Integrate[(a + b*ArcSec[c*x])/(x^3*Sqrt[d + e*x^2]), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \; 5771$$
$$\int \frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}}dx$$

input
```
Int[(a + b*ArcSec[c*x])/(x^3*Sqrt[d + e*x^2]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\mathrm{arcsec}\left(cx\right)}{x^3\sqrt{e\,x^2+d}}dx$$

input
```
int((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{\sqrt{ex^2+dx^3}}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/(e*x^5 + d*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 32.96 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{asec}\,(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

input
```
integrate((a+b*asec(c*x))/x**3/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asec(c*x))/(x**3*sqrt(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{\sqrt{ex^2+d}x^3}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/(sqrt(e*x^2 + d)*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^3\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x^3*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x^3*(d + e*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 4.83

$$\int \frac{a + b\sec^{-1}(cx)}{x^3\sqrt{d + ex^2}}\,dx$$
$$= \frac{-\sqrt{e\,x^2+d}\,ad - \sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)ae\,x^2 + \sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)ae\,x^2 + 2\left(\int \frac{asec(cx)}{\sqrt{e\,x^2+d}\,x^3}dx\right)b}{2d^2x^2}$$

input
```
int((a+b*asec(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

output
```
( - sqrt(d + e*x**2)*a*d - sqrt(d)*log((sqrt(d + e*x**2) - sqrt(d) + sqrt(
e)*x)/sqrt(d))*a*e*x**2 + sqrt(d)*log((sqrt(d + e*x**2) + sqrt(d) + sqrt(e
)*x)/sqrt(d))*a*e*x**2 + 2*int(asec(c*x)/(sqrt(d + e*x**2)*x**3),x)*b*d**2
*x**2)/(2*d**2*x**2)
```

3.136 $\int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$

Optimal result . . . 1143
Mathematica [N/A] . . . 1143
Rubi [N/A] . . . 1144
Maple [N/A] . . . 1144
Fricas [N/A] . . . 1145
Sympy [N/A] . . . 1145
Maxima [F(-2)] . . . 1145
Giac [N/A] . . . 1146
Mupad [N/A] . . . 1146
Reduce [N/A] . . . 1147

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \text{Int}\left(\frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}},x\right)$$

output
```
Defer(Int)(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 31.60 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(x^2*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
Integrate[(x^2*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2], x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (a + b\sec^{-1}(cx))}{\sqrt{d + ex^2}} \, dx$$

$$\Big\downarrow \; 5771$$

$$\int \frac{x^2 (a + b\sec^{-1}(cx))}{\sqrt{d + ex^2}} dx$$

input
```
Int[(x^2*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^2 (a + b \operatorname{arcsec}(cx))}{\sqrt{e\,x^2 + d}} dx$$

input
```
int(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^2}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x^2*arcsec(c*x) + a*x^2)/sqrt(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 59.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{x^2(a+b\,\mathrm{asec}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate(x**2*(a+b*asec(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**2*(a + b*asec(c*x))/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^2}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x^2/sqrt(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{x^2\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

output
```
int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.00

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{e\,x^2+d}\,aex-\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)ad+2\left(\int\frac{asec(cx)x^2}{\sqrt{e\,x^2+d}}dx\right)b\,e^2}{2e^2}$$

input
```
int(x^2*(a+b*asec(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
(sqrt(d + e*x**2)*a*e*x - sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(
d))*a*d + 2*int((asec(c*x)*x**2)/sqrt(d + e*x**2),x)*b*e**2)/(2*e**2)
```

3.137 $\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$

Optimal result . . . 1148
Mathematica [N/A] . . . 1148
Rubi [N/A] . . . 1149
Maple [N/A] . . . 1149
Fricas [N/A] . . . 1150
Sympy [N/A] . . . 1150
Maxima [F(-2)] . . . 1150
Giac [N/A] . . . 1151
Mupad [N/A] . . . 1151
Reduce [N/A] . . . 1152

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \text{Int}\left(\frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}},x\right)$$

output
```
Defer(Int)((a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(a + b*ArcSec[c*x])/Sqrt[d + e*x^2],x]
```

output
```
Integrate[(a + b*ArcSec[c*x])/Sqrt[d + e*x^2], x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{5771}$$

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}}dx$$

input
```
Int[(a + b*ArcSec[c*x])/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a+b\,\mathrm{arcsec}\left(cx\right)}{\sqrt{e\,x^2+d}}dx$$

input
```
int((a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)/sqrt(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 11.64 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{asec}\,(cx)}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate((a+b*asec(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asec(c*x))/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/sqrt(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(d + e*x^2)^(1/2),x)
```

output
```
int((a + b*acos(1/(c*x)))/(d + e*x^2)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.40

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)a+\left(\int \frac{asec(cx)}{\sqrt{e\,x^2+d}}dx\right)be}{e}$$

input

```
int((a+b*asec(c*x))/(e*x^2+d)^(1/2),x)
```

output

```
(sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a + int(asec(c*x)/sqr
t(d + e*x**2),x)*b*e)/e
```

3.138 $\int \frac{a+b\sec^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx$

Optimal result . . . 1153
Mathematica [A] (verified) . . . 1154
Rubi [A] (verified) . . . 1154
Maple [F] . . . 1158
Fricas [A] (verification not implemented) . . . 1158
Sympy [F] . . . 1159
Maxima [F(-2)] . . . 1159
Giac [F] . . . 1160
Mupad [F(-1)] . . . 1160
Reduce [F] . . . 1160

Optimal result

Integrand size = 23, antiderivative size = 243

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{dx} + \frac{bc^2x\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}E\left(\arcsin(cx)\,\middle|-\frac{e}{c^2d}\right)}{\sqrt{c^2x^2}\sqrt{1-c^2x^2}\sqrt{d+ex^2}} - \frac{b(c^2d+e)\,x\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

output

```
b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/(c^2*x^2)^(1/2)-(e*x^2+d)^(1/2)*(a
+b*arcsec(c*x))/d/x+b*c^2*x*(c^2*x^2-1)^(1/2)*(1+e*x^2/d)^(1/2)*EllipticE(
c*x,(-e/c^2/d)^(1/2))/(c^2*x^2)^(1/2)/(-c^2*x^2+1)^(1/2)/(e*x^2+d)^(1/2)-b
*(c^2*d+e)*x*(c^2*x^2-1)^(1/2)*(1+e*x^2/d)^(1/2)*EllipticF(c*x,(-e/c^2/d)^
(1/2))/d/(c^2*x^2)^(1/2)/(-c^2*x^2+1)^(1/2)/(e*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.59

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx=\frac{\sqrt{d+ex^2}\left(-a+bc\sqrt{1-\frac{1}{c^2x^2}}x-b\sec^{-1}(cx)\right)}{dx}-\frac{bce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}E\left(\arcsin\left(\sqrt{-\frac{e}{d}}x\right)|-\frac{c^2d}{e}\right)}{d\sqrt{-\frac{e}{d}}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

output
```
(Sqrt[d + e*x^2]*(-a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x - b*ArcSec[c*x]))/(d*x)
 - (b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*EllipticE[ArcSin[Sqr
t[-(e/d)]*x], -((c^2*d)/e)])/(d*Sqrt[-(e/d)]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*
x^2])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.522$, Rules used = {5761, 25, 27, 377, 27, 326, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \ 5761$$

$$-\frac{bcx\int -\frac{\sqrt{ex^2+d}}{dx^2\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow \ 25$$

$$\frac{bcx\int \frac{\sqrt{ex^2+d}}{dx^2\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{bcx\int \frac{\sqrt{ex^2+d}}{x^2\sqrt{c^2x^2-1}}dx}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow \text{ 377}$$

$$\frac{bcx\left(\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}-\int \frac{e\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}dx\right)}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{bcx\left(\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}-e\int \frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}dx\right)}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow \text{ 326}$$

$$\frac{bcx\left(\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}-e\left(\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}-\frac{(c^2d+e)\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e}\right)\right)}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow \text{ 323}$$

$$\frac{bcx\left(\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}-e\left(\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}-\frac{(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}\right)\right)}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow \text{ 323}$$

$$\frac{bcx\left(\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}-e\left(\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}-\frac{\sqrt{1-c^2x^2}(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}\right)\right)}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow \text{ 321}$$

$$\frac{bcx\left(\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}-e\left(\frac{c^2\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}-\frac{\sqrt{1-c^2x^2}(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}\right)\right)}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{dx}$$

$\Big\downarrow$ 331

$$\frac{bcx\left(\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}-e\left(\frac{c^2\sqrt{1-c^2x^2}\int\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}}-\frac{\sqrt{1-c^2x^2}(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}\right)\right)}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{dx}$$

$\Big\downarrow$ 330

$$\frac{bcx\left(\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}-e\left(\frac{c^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}-\frac{\sqrt{1-c^2x^2}(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}\right)\right)}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{dx}$$

$\Big\downarrow$ 327

$$\frac{bcx\left(\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}-e\left(\frac{c\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\middle|-\frac{e}{c^2d}\right)}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}-\frac{\sqrt{1-c^2x^2}(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}\right)\right)}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{dx}$$

input
```
Int[(a + b*ArcSec[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

output
```
-((Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/(d*x)) + (b*c*x*((Sqrt[-1 + c^2*x^
2]*Sqrt[d + e*x^2])/x - e*((c*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[
ArcSin[c*x], -(e/(c^2*d))])/(e*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) - (
(c^2*d + e)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -
(e/(c^2*d))])/(c*e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]))))/(d*Sqrt[c^2*x^2]
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 377

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 5761

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [F]

$$\int \frac{a + b\,\mathrm{arcsec}\,(cx)}{x^2\sqrt{e\,x^2 + d}}dx$$

input

```
int((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

output

```
int((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.44

$$\int \frac{a + b\sec^{-1}(cx)}{x^2\sqrt{d + ex^2}}\,dx = -\frac{\left(bcd\,\mathrm{arcsec}\,(cx) - \sqrt{c^2x^2 - 1}bcd + acd\right)\sqrt{ex^2 + d} - \left(bc^4dxE(\arcsin\,(cx)\,|-\frac{e}{c^2d}) - (bc^4d + be)xF(\mathrm{arc}\right.}{cd^2x}$$

input

```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
-((b*c*d*arcsec(c*x) - sqrt(c^2*x^2 - 1)*b*c*d + a*c*d)*sqrt(e*x^2 + d) -
(b*c^4*d*x*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (b*c^4*d + b*e)*x*ellipti
c_f(arcsin(c*x), -e/(c^2*d)))*sqrt(-d))/(c*d^2*x)
```

Sympy [F]

$$\int \frac{a + b\sec^{-1}(cx)}{x^2\sqrt{d + ex^2}}\,dx = \int \frac{a + b\,\mathrm{asec}\,(cx)}{x^2\sqrt{d + ex^2}}\,dx$$

input
```
integrate((a+b*asec(c*x))/x**2/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asec(c*x))/(x**2*sqrt(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\sec^{-1}(cx)}{x^2\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{\sqrt{ex^2+dx^2}}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/(sqrt(e*x^2 + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^2\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx=\frac{-\sqrt{e\,x^2+d}\,a-\sqrt{e}\,ax+\left(\int \frac{asec(cx)}{\sqrt{e\,x^2+d\,x^2}}dx\right)bdx}{dx}$$

input
```
int((a+b*asec(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

output
```
( - sqrt(d + e*x**2)*a - sqrt(e)*a*x + int(asec(c*x)/(sqrt(d + e*x**2)*x**
2),x)*b*d*x)/(d*x)
```

3.139 $\int \frac{a+b\sec^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$

Optimal result 1161
Mathematica [C] (verified) 1162
Rubi [A] (verified) 1162
Maple [F] 1167
Fricas [A] (verification not implemented) 1167
Sympy [F] 1168
Maxima [F(-2)] 1168
Giac [F] 1168
Mupad [F(-1)] 1169
Reduce [F] 1169

Optimal result

Integrand size = 23, antiderivative size = 362

$$\begin{aligned}
\int \frac{a+b\sec^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx
&= \frac{bc(2c^2d-5e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}} \\
&\quad - \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3dx^3} + \frac{2e\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3d^2x} \\
&\quad - \frac{bc^2(2c^2d-5e)\,x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\,E\left(\arcsin(cx)\,\middle|-\frac{e}{c^2d}\right)}{9d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&\quad + \frac{2b(c^2d-3e)\,(c^2d+e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}$$

output
```
1/9*b*c*(2*c^2*d-5*e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2
)+1/9*b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(c^2*x^2)^(1/2)-1/3*(e*x
^2+d)^(1/2)*(a+b*arcsec(c*x))/d/x^3+2/3*e*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)
)/d^2/x-1/9*b*c^2*(2*c^2*d-5*e)*x*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)*Ellip
ticE(c*x,(-e/c^2/d)^(1/2))/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/
d)^(1/2)+2/9*b*(c^2*d-3*e)*(c^2*d+e)*x*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2
)*EllipticF(c*x,(-e/c^2/d)^(1/2))/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e
*x^2+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.21 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.69

$$\int \frac{a+b\sec^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$$
$$= \frac{\sqrt{d+ex^2}\left(bc\sqrt{1-\frac{1}{c^2x^2}}x(d+2c^2dx^2-5ex^2)-3a(d-2ex^2)-3b(d-2ex^2)\sec^{-1}(cx)\right)}{9d^2x^3}$$
$$-\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(2c^2d-5e)\,E\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)|-\frac{e}{c^2d}\right)+2(-c^4d^2+2c^2de+3e^2)\,\text{Ellipti}\right.}{9\sqrt{-c^2}d^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

output
```
(Sqrt[d + e*x^2]*(b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2*c^2*d*x^2 - 5*e*x^2)
- 3*a*(d - 2*e*x^2) - 3*b*(d - 2*e*x^2)*ArcSec[c*x]))/(9*d^2*x^3) - ((I/9)
*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(2*c^2*d - 5*e)*El
lipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] + 2*(-(c^4*d^2) + 2*c^2*d*e
 + 3*e^2)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d
^2*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {5761, 27, 442, 25, 445, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \; 5761$$
$$-\frac{bcx\int -\frac{(d-2ex^2)\sqrt{ex^2+d}}{3d^2x^4\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{2e\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3dx^3}$$

$\Big\downarrow$ 27

$$\frac{bcx\int \frac{(d-2ex^2)\sqrt{ex^2+d}}{x^4\sqrt{c^2x^2-1}}dx}{3d^2\sqrt{c^2x^2}}+\frac{2e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$\Big\downarrow$ 442

$$\frac{bcx\left(\frac{d\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{3x^3}-\frac{1}{3}\int -\frac{(c^2d-6e)ex^2+d(2c^2d-5e)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx\right)}{3d^2\sqrt{c^2x^2}}+\frac{2e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$\Big\downarrow$ 25

$$\frac{bcx\left(\frac{1}{3}\int \frac{(c^2d-6e)ex^2+d(2c^2d-5e)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{d\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{3x^3}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$\Big\downarrow$ 445

$$\frac{bcx\left(\frac{1}{3}\left(\frac{\int \frac{de\left(-\left(\left(2c^2d-5e\right)x^2c^2\right)+dc^2-6e\right)}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{d}+\frac{\sqrt{c^2x^2-1}(2c^2d-5e)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{3x^3}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$\Big\downarrow$ 27

$$\frac{bcx\left(\frac{1}{3}\left(e\int \frac{-((2c^2d-5e)x^2c^2)+dc^2-6e}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{\sqrt{c^2x^2-1}(2c^2d-5e)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{3x^3}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$\Big\downarrow$ 399

$$\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{2(c^2d-3e)(c^2d+e)\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e}-\frac{c^2(2c^2d-5e)\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}(2c^2d-5e)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{c^2x^2-1}\sqrt{}}{3x^3}\right.}{3d^2\sqrt{c^2x^2}}+\frac{2e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$\Big\downarrow$ 323

$$\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{2\left(c^2d-3e\right)\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}-\frac{c^2\left(2c^2d-5e\right)\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}\left(2c^2d-5e\right)\sqrt{d+ex^2}}{x}\right)+d\sqrt{\cdots}}{3d^2\sqrt{c^2x^2}}$$

$$\frac{2e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$\Big\downarrow$ 323

$$\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}\left(c^2d-3e\right)\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\left(2c^2d-5e\right)\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}\left(2c^2d-5e\right)\sqrt{d+ex^2}}{x}\cdots\right)}{3d^2\sqrt{c^2x^2}}$$

$$\frac{2e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$\Big\downarrow$ 321

$$\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}\left(c^2d-3e\right)\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\left(2c^2d-5e\right)\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}\left(2c^2d-5e\right)\sqrt{d\cdots}}{x}\cdots\right)}{3d^2\sqrt{c^2x^2}}$$

$$\frac{2e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$\Big\downarrow$ 331

$$\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}\left(c^2d-3e\right)\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}\left(2c^2d-5e\right)\int \frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}}\right)+\frac{\sqrt{c^2x^2-1}\left(2c^2\cdots\right.}{\cdots}\right)}{3d^2\sqrt{c^2x^2}}$$

$$\frac{2e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$\Big\downarrow$ 330

$$\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}\left(c^2d-3e\right)\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}\left(2c^2d-5e\right)\sqrt{d+ex^2}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\right)+\sqrt{c^2\cdots}\right)}{3d^2\sqrt{c^2x^2}}$$

$$\frac{2e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$$\Big\downarrow \text{327}$$

$$\frac{2e\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3d^2x} - \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3dx^3} + \frac{bcx\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}(c^2d-3e)(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}} - \frac{c\sqrt{1-c^2x^2}(2c^2d-5e)\sqrt{d+ex^2}E\left(\arcsin(cx)\middle|-\frac{e}{c^2d}\right)}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\right)+\right.\right.}{3d^2\sqrt{c^2x^2}}$$

input
```
Int[(a + b*ArcSec[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

output
```
-1/3*(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/(d*x^3) + (2*e*Sqrt[d + e*x^2]*
(a + b*ArcSec[c*x]))/(3*d^2*x) + (b*c*x*((d*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*
x^2])/(3*x^3) + (((2*c^2*d - 5*e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/x +
e*(-((c*(2*c^2*d - 5*e)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin
[c*x], -(e/(c^2*d))])/(e*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d])) + (2*(c^
2*d - 3*e)*(c^2*d + e)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[Arc
Sin[c*x], -(e/(c^2*d))])/(c*e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])))/3))/(3
*d^2*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [F]

$$\int \frac{a + b\,\mathrm{arcsec}\,(cx)}{x^4\sqrt{e\,x^2 + d}}dx$$

input
```
int((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.54

$$\int \frac{a + b\sec^{-1}(cx)}{x^4\sqrt{d + ex^2}}\,dx = \frac{\left(6\,acdex^2 - 3\,acd^2 + 3\left(2\,bcdex^2 - bcd^2\right)\mathrm{arcsec}\,(cx) + \left(bcd^2 + \left(2\,bc^3d^2 - 5\,bcde\right)x^2\right)\sqrt{c^2x^2 - 1}\right)\sqrt{ex^2 + d}}{}$$

input
```
integrate((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
1/9*((6*a*c*d*e*x^2 - 3*a*c*d^2 + 3*(2*b*c*d*e*x^2 - b*c*d^2)*arcsec(c*x)
+ (b*c*d^2 + (2*b*c^3*d^2 - 5*b*c*d*e)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2
+ d) + ((2*b*c^6*d^2 - 5*b*c^4*d*e)*x^3*elliptic_e(arcsin(c*x), -e/(c^2*d)
) - (2*b*c^6*d^2 - (5*b*c^4 - b*c^2)*d*e - 6*b*e^2)*x^3*elliptic_f(arcsin(
c*x), -e/(c^2*d)))*sqrt(-d))/(c*d^3*x^3)
```

Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{asec}\,(cx)}{x^4\sqrt{d+ex^2}}\,dx$$

input
```
integrate((a+b*asec(c*x))/x**4/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asec(c*x))/(x**4*sqrt(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{\sqrt{ex^2+d}x^4}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/(sqrt(e*x^2 + d)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\sec^{-1}(cx)}{x^4\sqrt{d + ex^2}}\, dx = \int \frac{a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^4\,\sqrt{e\,x^2 + d}}\, dx$$

input
```
int((a + b*acos(1/(c*x)))/(x^4*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x^4*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a + b\sec^{-1}(cx)}{x^4\sqrt{d + ex^2}}\, dx = \frac{-\sqrt{e\,x^2 + d}\,ad + 2\sqrt{e\,x^2 + d}\,ae\,x^2 - 2\sqrt{e}\,ae\,x^3 + 3\left(\int \frac{asec(cx)}{\sqrt{e\,x^2+d}\,x^4}dx\right) b\,d^2x^3}{3d^2x^3}$$

input
```
int((a+b*asec(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

output
```
( - sqrt(d + e*x**2)*a*d + 2*sqrt(d + e*x**2)*a*e*x**2 - 2*sqrt(e)*a*e*x**
3 + 3*int(asec(c*x)/(sqrt(d + e*x**2)*x**4),x)*b*d**2*x**3)/(3*d**2*x**3)
```

3.140 $\int \frac{a+b\sec^{-1}(cx)}{x^6\sqrt{d+ex^2}}\,dx$

Optimal result 1171
Mathematica [C] (verified) 1172
Rubi [A] (verified) 1173
Maple [F] 1175
Fricas [A] (verification not implemented) 1176
Sympy [F(-1)] 1176
Maxima [F(-2)] 1177
Giac [F] 1177
Mupad [F(-1)] 1177
Reduce [F] 1178

Optimal result

Integrand size = 23, antiderivative size = 1006

$$\int \frac{a+b\sec^{-1}(cx)}{x^6\sqrt{d+ex^2}}\,dx = \frac{8bce^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{15d^3\sqrt{c^2x^2}} - \frac{4bce(2c^2d+e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{45d^3\sqrt{c^2x^2}} + \frac{bc(8c^4d^2+3c^2de-2e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d^3\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{25dx^4\sqrt{c^2x^2}} - \frac{4bce\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{45d^2x^2\sqrt{c^2x^2}} + \frac{bc(4c^2d+e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d^2x^2\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{5dx^5} + \frac{4e\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{15d^2x^3} - \frac{8e^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{15d^3x} + \frac{4bc^2e(2c^2d+e)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{45d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} - \frac{bc^2(8c^4d^2+3c^2de-2e^2)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{75d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} + \frac{8bc^2e^2x\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{15d^2\sqrt{c^2x^2}\sqrt{1-c^2x^2}\sqrt{d+ex^2}} + \frac{bc^2(8c^2d-e)(c^2d+e)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{75d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} - \frac{8bc^2e(c^2d+e)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{45d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} - \frac{8be^2(c^2d+e)x\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{15d^3\sqrt{c^2x^2}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

output
```
8/15*b*c*e^2*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^3/(c^2*x^2)^(1/2)-4/45*b*
c*e*(2*c^2*d+e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^3/(c^2*x^2)^(1/2)+1/75
*b*c*(8*c^4*d^2+3*c^2*d*e-2*e^2)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^3/(c^
2*x^2)^(1/2)+1/25*b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^4/(c^2*x^2)^(1
/2)-4/45*b*c*e*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/x^2/(c^2*x^2)^(1/2)+1
/75*b*c*(4*c^2*d+e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/x^2/(c^2*x^2)^(1
/2)-1/5*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/d/x^5+4/15*e*(e*x^2+d)^(1/2)*(a+
b*arcsec(c*x))/d^2/x^3-8/15*e^2*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/d^3/x+4/
45*b*c^2*e*(2*c^2*d+e)*x*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)*EllipticE(c*x,
(-e/c^2/d)^(1/2))/d^3/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)-
1/75*b*c^2*(8*c^4*d^2+3*c^2*d*e-2*e^2)*x*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2
)*EllipticE(c*x,(-e/c^2/d)^(1/2))/d^3/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1
+e*x^2/d)^(1/2)+8/15*b*c^2*e^2*x*(c^2*x^2-1)^(1/2)*(1+e*x^2/d)^(1/2)*Ellip
ticE(c*x,(-e/c^2/d)^(1/2))/d^2/(c^2*x^2)^(1/2)/(-c^2*x^2+1)^(1/2)/(e*x^2+d
)^(1/2)+1/75*b*c^2*(8*c^2*d-e)*(c^2*d+e)*x*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^
(1/2)*EllipticF(c*x,(-e/c^2/d)^(1/2))/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2
)/(e*x^2+d)^(1/2)-8/45*b*c^2*e*(c^2*d+e)*x*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^
(1/2)*EllipticF(c*x,(-e/c^2/d)^(1/2))/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2
)/(e*x^2+d)^(1/2)-8/15*b*e^2*(c^2*d+e)*x*(c^2*x^2-1)^(1/2)*(1+e*x^2/d)^(1/
2)*EllipticF(c*x,(-e/c^2/d)^(1/2))/d^3/(c^2*x^2)^(1/2)/(-c^2*x^2+1)^(1/...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.31 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.33

$$\int \frac{a+b\sec^{-1}(cx)}{x^6\sqrt{d+ex^2}}\,dx$$
$$= \frac{\sqrt{d+ex^2}\left(-15a(3d^2-4dex^2+8e^2x^4)+bc\sqrt{1-\frac{1}{c^2x^2}}x(94e^2x^4-dex^2(17+31c^2x^2)+3d^2(3+4c^2x^2+\right.}{225d^3x^5}$$
$$-\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(24c^4d^2-31c^2de+94e^2)\,E\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)|-\frac{e}{c^2d}\right)-(24c^6d^3-19c^4d^2\right.}{225\sqrt{-c^2}d^3\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x^6*Sqrt[d + e*x^2]),x]
```

output
```
(Sqrt[d + e*x^2]*(-15*a*(3*d^2 - 4*d*e*x^2 + 8*e^2*x^4) + b*c*Sqrt[1 - 1/(
c^2*x^2)]*x*(94*e^2*x^4 - d*e*x^2*(17 + 31*c^2*x^2) + 3*d^2*(3 + 4*c^2*x^2
 + 8*c^4*x^4)) - 15*b*(3*d^2 - 4*d*e*x^2 + 8*e^2*x^4)*ArcSec[c*x]))/(225*d
^3*x^5) - ((I/225)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*
(24*c^4*d^2 - 31*c^2*d*e + 94*e^2)*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/
(c^2*d))] - (24*c^6*d^3 - 19*c^4*d^2*e + 77*c^2*d*e^2 + 120*e^3)*EllipticF
[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d^3*Sqrt[1 - c^2*x^2
]*Sqrt[d + e*x^2])
```

Rubi [A] (verified)

Time = 2.10 (sec) , antiderivative size = 839, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5761, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b \sec^{-1}(cx)}{x^6\sqrt{d+ex^2}}dx$$

$$\Big\downarrow \text{5761}$$

$$-\frac{bcx\int -\frac{\sqrt{ex^2+d}\left(8e^2x^4-4dex^2+3d^2\right)}{15d^3x^6\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}-\frac{8e^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{15d^3x}+\frac{4e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{15d^2x^3}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int \frac{\sqrt{ex^2+d}\left(8e^2x^4-4dex^2+3d^2\right)}{x^6\sqrt{c^2x^2-1}}dx}{15d^3\sqrt{c^2x^2}}-\frac{8e^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{15d^3x}+\frac{4e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{15d^2x^3}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\Big\downarrow \text{7293}$$

$$\frac{bcx\int \left(\frac{3\sqrt{ex^2+d}d^2}{x^6\sqrt{c^2x^2-1}}-\frac{4e\sqrt{ex^2+d}d}{x^4\sqrt{c^2x^2-1}}+\frac{8e^2\sqrt{ex^2+d}}{x^2\sqrt{c^2x^2-1}}\right)dx}{15d^3\sqrt{c^2x^2}}-\frac{8e^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{15d^3x}+\frac{4e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{15d^2x^3}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$\downarrow$ 2009

$$-\frac{8\sqrt{ex^2+d}\left(a+b\sec^{-1}(cx)\right)e^2}{15d^3x}+\frac{4\sqrt{ex^2+d}\left(a+b\sec^{-1}(cx)\right)e}{15d^2x^3}-\frac{\sqrt{ex^2+d}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}+$$

$$bcx\left(\frac{3\sqrt{c^2x^2-1}\sqrt{ex^2+d}d^2}{5x^5}+\frac{c(8c^2d-e)(dc^2+e)\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)d}{5\sqrt{c^2x^2-1}\sqrt{ex^2+d}}-\frac{8ce(dc^2+e)\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,\text{Elliptic}}{3\sqrt{c^2x^2-1}\sqrt{ex^2+}}\right.$$

input
```
Int[(a + b*ArcSec[c*x])/(x^6*Sqrt[d + e*x^2]),x]
```

output
```
-1/5*(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/(d*x^5) + (4*e*Sqrt[d + e*x^2]*
(a + b*ArcSec[c*x]))/(15*d^2*x^3) - (8*e^2*Sqrt[d + e*x^2]*(a + b*ArcSec[c
*x]))/(15*d^3*x) + (b*c*x*((3*d^2*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(5*x
^5) - (4*d*e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(3*x^3) + (d*(4*c^2*d + e
)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(5*x^3) + (8*e^2*Sqrt[-1 + c^2*x^2]*
Sqrt[d + e*x^2])/x - (4*e*(2*c^2*d + e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]
)/(3*x) + ((8*c^4*d^2 + 3*c^2*d*e - 2*e^2)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x
^2])/(5*x) - (8*c*e^2*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c
*x], -(e/(c^2*d))])/(Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) + (4*c*e*(2*c
^2*d + e)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^
2*d))])/(3*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) - (c*(8*c^4*d^2 + 3*c^2
*d*e - 2*e^2)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e
/(c^2*d))])/(5*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) + (c*d*(8*c^2*d - e
)*(c^2*d + e)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x],
 -(e/(c^2*d))])/(5*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]) - (8*c*d*e*(c^2*d +
 e)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*
d))])/(3*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]) + (8*e^2*(c^2*d + e)*Sqrt[1 -
 c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(c*Sqr
t[-1 + c^2*x^2]*Sqrt[d + e*x^2])))/(15*d^3*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{a+b\,\mathrm{arcsec}\left(cx\right)}{x^{6}\sqrt{e\,x^{2}+d}}dx$$

input
```
int((a+b*arcsec(c*x))/x^6/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arcsec(c*x))/x^6/(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.29

$$\int \frac{a+b\sec^{-1}(cx)}{x^6\sqrt{d+ex^2}}\,dx = -\frac{\left(120\,acde^2x^4-60\,acd^2ex^2+45\,acd^3+15\left(8\,bcde^2x^4-4\,bcd^2ex^2+3\,bcd^3\right)\operatorname{arcsec}\left(cx\right)-\left(9\,bcd^3+\left(24\right.\right.\right.}{}$$

input
```
integrate((a+b*arcsec(c*x))/x^6/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
-1/225*((120*a*c*d*e^2*x^4 - 60*a*c*d^2*e*x^2 + 45*a*c*d^3 + 15*(8*b*c*d*e
^2*x^4 - 4*b*c*d^2*e*x^2 + 3*b*c*d^3)*arcsec(c*x) - (9*b*c*d^3 + (24*b*c^5
*d^3 - 31*b*c^3*d^2*e + 94*b*c*d*e^2)*x^4 + (12*b*c^3*d^3 - 17*b*c*d^2*e)*
x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d) - ((24*b*c^8*d^3 - 31*b*c^6*d^2*e
+ 94*b*c^4*d*e^2)*x^5*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (24*b*c^8*d^3
- (31*b*c^6 - 12*b*c^4)*d^2*e + (94*b*c^4 - 17*b*c^2)*d*e^2 + 120*b*e^3)*x
^5*elliptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(-d))/(c*d^4*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^6\sqrt{d+ex^2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*asec(c*x))/x**6/(e*x**2+d)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x^6\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/x^6/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^6\sqrt{d+ex^2}}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx)+a}{\sqrt{ex^2+dx^6}}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^6/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/(sqrt(e*x^2 + d)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^6\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^6\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x^6*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x^6*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^6\sqrt{d+ex^2}}\,dx$$
$$= \frac{-3\sqrt{e\,x^2+d}\,a\,d^2+4\sqrt{e\,x^2+d}\,ade\,x^2-8\sqrt{e\,x^2+d}\,a\,e^2x^4+8\sqrt{e}\,a\,e^2x^5+15\left(\int \frac{asec(cx)}{\sqrt{e\,x^2+d}\,x^6}dx\right)b\,d^3x^5}{15d^3x^5}$$

input
```
int((a+b*asec(c*x))/x^6/(e*x^2+d)^(1/2),x)
```

output
```
( - 3*sqrt(d + e*x**2)*a*d**2 + 4*sqrt(d + e*x**2)*a*d*e*x**2 - 8*sqrt(d +
 e*x**2)*a*e**2*x**4 + 8*sqrt(e)*a*e**2*x**5 + 15*int(asec(c*x)/(sqrt(d +
e*x**2)*x**6),x)*b*d**3*x**5)/(15*d**3*x**5)
```

3.141 $\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$

Optimal result 1179
Mathematica [C] (warning: unable to verify) 1180
Rubi [A] (verified) 1180
Maple [F] 1184
Fricas [A] (verification not implemented) 1185
Sympy [F] 1185
Maxima [F(-2)] 1186
Giac [F] 1186
Mupad [F(-1)] 1187
Reduce [F] 1187

Optimal result

Integrand size = 23, antiderivative size = 252

$$\begin{aligned}\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx &= -\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{e^3\sqrt{d+ex^2}} \\ &\quad - \frac{2d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} + \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^3} \\ &\quad - \frac{8bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e^3\sqrt{c^2x^2}} + \frac{b(9c^2d-e)\,x\,\mathrm{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{5/2}\sqrt{c^2x^2}}\end{aligned}$$

output
```
-1/6*b*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c/e^2/(c^2*x^2)^(1/2)-d^2*(a+b*
arcsec(c*x))/e^3/(e*x^2+d)^(1/2)-2*d*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/e^3
+1/3*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/e^3-8/3*b*c*d^(3/2)*x*arctan((e*x^2
+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e^3/(c^2*x^2)^(1/2)+1/6*b*(9*c^2*d-e)
*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^2/e^(5/2)/(c^2*x
^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.35 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.05

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{-16bd^2\sqrt{1+\frac{d}{ex^2}}(-1+c^2x^2)\operatorname{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)+b(9c^2d-e)e\sqrt{}}{}$$

input
```
Integrate[(x^5*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
(-16*b*d^2*Sqrt[1 + d/(e*x^2)]*(-1 + c^2*x^2)*AppellF1[1, 1/2, 1/2, 2, 1/(
c^2*x^2), -(d/(e*x^2))] + b*(9*c^2*d - e)*e*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt
[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x
^2)/d)] - 2*x*(-1 + c^2*x^2)*(b*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2) + 2*
a*c*(8*d^2 + 4*d*e*x^2 - e^2*x^4) + 2*b*c*(8*d^2 + 4*d*e*x^2 - e^2*x^4)*Ar
cSec[c*x]))/(12*c*e^3*x*(-1 + c^2*x^2)*Sqrt[d + e*x^2])
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5761, 27, 7282, 2118, 27, 175, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\left(a+b\sec^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 5761$$

$$-\frac{bcx\int -\frac{-e^2x^4+4dex^2+8d^2}{3e^3x\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \ 27$$

$$\frac{bcx\int \frac{-e^2x^4+4dex^2+8d^2}{x\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{3e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$\Big\downarrow$ 7282

$$\frac{bcx\int \frac{-e^2x^4+4dex^2+8d^2}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{6e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$\Big\downarrow$ 2118

$$\frac{bcx\left(\frac{\int \frac{e\left(16c^2d^2+\left(9c^2d-e\right)ex^2\right)}{2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2e}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$\Big\downarrow$ 27

$$\frac{bcx\left(\frac{\int \frac{16c^2d^2+\left(9c^2d-e\right)ex^2}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$\Big\downarrow$ 175

$$\frac{bcx\left(\frac{16c^2d^2\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+e\left(9c^2d-e\right)\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$\Big\downarrow$ 66

$$\frac{bcx\left(\frac{16c^2d^2\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\left(9c^2d-e\right)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{104}$$

$$\frac{bcx\left(\frac{32c^2d^2\int\frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e(9c^2d-e)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2(a+b\sec^{-1}(cx))}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3}+\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^3}$$

$$\Big\downarrow \text{217}$$

$$\frac{bcx\left(\frac{2e(9c^2d-e)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-32c^2d^{3/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2(a+b\sec^{-1}(cx))}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3}+\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^3}$$

$$\Big\downarrow \text{221}$$

$$-\frac{d^2(a+b\sec^{-1}(cx))}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3}+\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^3}+\frac{bcx\left(\frac{\frac{2\sqrt{e}(9c^2d-e)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-32c^2d^{3/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^3\sqrt{c^2x^2}}$$

input
```
Int[(x^5*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
-((d^2*(a + b*ArcSec[c*x]))/(e^3*Sqrt[d + e*x^2])) - (2*d*Sqrt[d + e*x^2]*
(a + b*ArcSec[c*x]))/e^3 + ((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/(3*e^3)
 + (b*c*x*(-((e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/c^2) + (-32*c^2*d^(3/2
)*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])] + (2*(9*c^2*d - e)*
Sqrt[e]*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/c)/(2*c
^2)))/(6*e^3*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

Maple [F]

$$\int \frac{x^5(a + b \operatorname{arcsec}(cx))}{(e\,x^2 + d)^{\frac{3}{2}}}dx$$

input
```
int(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 1483, normalized size of antiderivative = 5.88

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[-1/24*((9*b*c^2*d^2 - b*d*e + (9*b*c^2*d*e - b*e^2)*x^2)*sqrt(e)*log(8*c^
4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x
^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 16*(b
*c^3*d*e*x^2 + b*c^3*d^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 -
8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e
*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) - 4*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 -
16*a*c^3*d^2 + 2*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*arcsec(c*
x) - (b*c*e^2*x^2 + b*c*d*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^3*e^4*
x^2 + c^3*d*e^3), -1/24*(32*(b*c^3*d*e*x^2 + b*c^3*d^2)*sqrt(d)*arctan(-1/
2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d
*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (9*b*c^2*d^2 - b*d*e + (9*b*c^2*d*e
 - b*e^2)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*
e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*
x^2 + d)*sqrt(e) + e^2) - 4*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 - 16*a*c^3*
d^2 + 2*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*arcsec(c*x) - (b*c
*e^2*x^2 + b*c*d*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^3
*d*e^3), -1/12*((9*b*c^2*d^2 - b*d*e + (9*b*c^2*d*e - b*e^2)*x^2)*sqrt(-e)
*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sq
rt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) - 8*(b*c^3*d*e*x^2 +
 b*c^3*d^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 ...
```

Sympy [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^5(a+b\,\mathrm{asec}\,(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5*(a+b*asec(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x**5*(a + b*asec(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a + b\sec^{-1}(cx))}{(d + ex^2)^{3/2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{x^5(a + b\sec^{-1}(cx))}{(d + ex^2)^{3/2}} \, dx = \int \frac{(b \operatorname{arcsec}(cx) + a)x^5}{(ex^2 + d)^{\frac{3}{2}}} \, dx$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x^5/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{x^5\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

input
```
int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\frac{-8\sqrt{e\,x^2+d}\,a\,d^2-4\sqrt{e\,x^2+d}\,ade\,x^2+\sqrt{e\,x^2+d}\,a\,e^2x^4+3\left(\int \frac{asec(cx)x^5}{\sqrt{e\,x^2+d}\,d+\sqrt{e\,x^2}}\right.}{3e^3\left(e\,x^2+d\right)}$$

input
```
int(x^5*(a+b*asec(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
( - 8*sqrt(d + e*x**2)*a*d**2 - 4*sqrt(d + e*x**2)*a*d*e*x**2 + sqrt(d + e
*x**2)*a*e**2*x**4 + 3*int((asec(c*x)*x**5)/(sqrt(d + e*x**2)*d + sqrt(d +
 e*x**2)*e*x**2),x)*b*d*e**3 + 3*int((asec(c*x)*x**5)/(sqrt(d + e*x**2)*d
+ sqrt(d + e*x**2)*e*x**2),x)*b*e**4*x**2)/(3*e**3*(d + e*x**2))
```

3.142 $\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$

Optimal result 1188
Mathematica [C] (verified) 1188
Rubi [A] (verified) 1189
Maple [F] 1192
Fricas [A] (verification not implemented) 1192
Sympy [F] 1193
Maxima [F(-2)] 1194
Giac [F] 1194
Mupad [F(-1)] 1194
Reduce [F] 1195

Optimal result

Integrand size = 23, antiderivative size = 157

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{d(a+b\sec^{-1}(cx))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2} + \frac{2bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{e^2\sqrt{c^2x^2}} - \frac{bx\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{e^{3/2}\sqrt{c^2x^2}}$$

output

```
d*(a+b*arcsec(c*x))/e^2/(e*x^2+d)^(1/2)+(e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/
e^2+2*b*c*d^(1/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e^2/
(c^2*x^2)^(1/2)-b*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/e
^(3/2)/(c^2*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.96 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.03

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{\frac{2bd\sqrt{1+\frac{d}{ex^2}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)}{cx} + \frac{bce\sqrt{1-\frac{1}{c^2x^2}}x^3\sqrt{1+\frac{ex^2}{d}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,c^2x^2,-\frac{ex^2}{d}\right)}{\sqrt{1-c^2x^2}}}{2e^2\sqrt{d+ex^2}}$$

input
```
Integrate[(x^3*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
((2*b*d*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x
^2))])/(c*x) + (b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x^3*Sqrt[1 + (e*x^2)/d]*Appell
F1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/Sqrt[1 - c^2*x^2] + 2*(2*d + e*
x^2)*(a + b*ArcSec[c*x]))/(2*e^2*Sqrt[d + e*x^2])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = $\{5761, 27, 435, 175, 66, 104, 217, 221\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}dx$$

$$\Big\downarrow \; 5761$$

$$-\frac{bcx\int \frac{ex^2+2d}{e^2x\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{\sqrt{c^2x^2}}+\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2}+\frac{d(a+b\sec^{-1}(cx))}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{bcx\int \frac{ex^2+2d}{x\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e^2\sqrt{c^2x^2}}+\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2}+\frac{d(a+b\sec^{-1}(cx))}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 435$$

$$-\frac{bcx\int \frac{ex^2+2d}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2e^2\sqrt{c^2x^2}}+\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2}+\frac{d(a+b\sec^{-1}(cx))}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 175$$

$$-\frac{bcx\left(e\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2d\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2\right)}{2e^2\sqrt{c^2x^2}}+\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2}+\frac{d(a+b\sec^{-1}(cx))}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 66$$

$$-\frac{bcx\left(2d\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}\right)}{2e^2\sqrt{c^2x^2}}+\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2}+\frac{d(a+b\sec^{-1}(cx))}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 104$$

$$-\frac{bcx\left(4d\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}\right)}{2e^2\sqrt{c^2x^2}}+\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2}+\frac{d(a+b\sec^{-1}(cx))}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 217$$

$$-\frac{bcx\left(2e\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-4\sqrt{d}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)\right)}{2e^2\sqrt{c^2x^2}}+\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2}+\frac{d(a+b\sec^{-1}(cx))}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 221$$

$$\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2}+\frac{d(a+b\sec^{-1}(cx))}{e^2\sqrt{d+ex^2}}-\frac{bcx\left(\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-4\sqrt{d}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)\right)}{2e^2\sqrt{c^2x^2}}$$

input
```
Int[(x^3*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
(d*(a + b*ArcSec[c*x]))/(e^2*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*Ar
cSec[c*x]))/e^2 - (b*c*x*(-4*Sqrt[d]*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[
-1 + c^2*x^2])] + (2*Sqrt[e]*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[
d + e*x^2])])/c))/(2*e^2*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 5761

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [F]

$$\int \frac{x^3(a+b\,\mathrm{arcsec}\,(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input

```
int(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

output

```
int(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1070, normalized size of antiderivative = 6.82

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Too large to display}$$

input

```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((b*e*x^2 + b*d)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*
(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*
sqrt(e*x^2 + d)*sqrt(e) + e^2) + 2*(b*c*e*x^2 + b*c*d)*sqrt(-d)*log(((c^4*
d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*(
(c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + 4*(a*c*e*x
^2 + 2*a*c*d + (b*c*e*x^2 + 2*b*c*d)*arcsec(c*x))*sqrt(e*x^2 + d))/(c*e^3*
x^2 + c*d*e^2), 1/4*(4*(b*c*e*x^2 + b*c*d)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^
2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2
*d^2 - d*e)*x^2 - d^2)) + (b*e*x^2 + b*d)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*
d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e
)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(a*c*e*x^2 + 2*a*c*
d + (b*c*e*x^2 + 2*b*c*d)*arcsec(c*x))*sqrt(e*x^2 + d))/(c*e^3*x^2 + c*d*e
^2), 1/2*((b*e*x^2 + b*d)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sq
rt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e -
 c*e^2)*x^2)) + (b*c*e*x^2 + b*c*d)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e
^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2
*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + 2*(a*c*e*x^2 + 2*a*c*d + (b*c
*e*x^2 + 2*b*c*d)*arcsec(c*x))*sqrt(e*x^2 + d))/(c*e^3*x^2 + c*d*e^2), 1/2
*(2*(b*c*e*x^2 + b*c*d)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)
*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 ...
```

Sympy [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^3(a+b\operatorname{asec}(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*(a+b*asec(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x**3*(a + b*asec(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^3}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x^3/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^3\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{2\sqrt{e\,x^2+d}\,ad+\sqrt{e\,x^2+d}\,ae\,x^2+\left(\int \frac{asec(cx)x^3}{\sqrt{e\,x^2+d}\,d+\sqrt{e\,x^2+d}\,e\,x^2}dx\right)bd\,e^2+\left(\int \frac{}{\sqrt{e\,x^2}}\right.}{e^2\,(e\,x^2+d)}$$

input
```
int(x^3*(a+b*asec(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
(2*sqrt(d + e*x**2)*a*d + sqrt(d + e*x**2)*a*e*x**2 + int((asec(c*x)*x**3)
/(sqrt(d + e*x**2)*d + sqrt(d + e*x**2)*e*x**2),x)*b*d*e**2 + int((asec(c*
x)*x**3)/(sqrt(d + e*x**2)*d + sqrt(d + e*x**2)*e*x**2),x)*b*e**3*x**2)/(e
**2*(d + e*x**2))
```

3.143 $\int \frac{x\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal result 1196
Mathematica [C] (verified) 1196
Rubi [A] (verified) 1197
Maple [F] 1198
Fricas [A] (verification not implemented) 1199
Sympy [F] 1199
Maxima [F] 1200
Giac [F] 1200
Mupad [F(-1)] 1200
Reduce [F] 1201

Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = -\frac{a+b\sec^{-1}(cx)}{e\sqrt{d+ex^2}} - \frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{\sqrt{d}e\sqrt{c^2x^2}}$$

output
```
-(a+b*arcsec(c*x))/e/(e*x^2+d)^(1/2)-b*c*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/
(c^2*x^2-1)^(1/2))/d^(1/2)/e/(c^2*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.22 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{-b\sqrt{1+\frac{d}{ex^2}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right) - 2cx(a+b\sec^{-1}(cx))}{2cex\sqrt{d+ex^2}}$$

input
```
Integrate[(x*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
(-(b*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2)
)]) - 2*c*x*(a + b*ArcSec[c*x]))/(2*c*e*x*Sqrt[d + e*x^2])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5759, 354, 104, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\sec^{-1}(cx)\right)}{(d+ex^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 5759}$$

$$\frac{bcx\int \frac{1}{x\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{e\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 354}$$

$$\frac{bcx\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2e\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{e\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 104}$$

$$\frac{bcx\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}}{e\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{e\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 217}$$

$$-\frac{a+b\sec^{-1}(cx)}{e\sqrt{d+ex^2}}-\frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{\sqrt{d}e\sqrt{c^2x^2}}$$

input
```
Int[(x*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
-((a + b*ArcSec[c*x])/(e*Sqrt[d + e*x^2])) - (b*c*x*ArcTan[Sqrt[d + e*x^2]
/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(Sqrt[d]*e*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5759
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSec[c*x])/(2*e*(p + 1))), x
] - Simp[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2]))   Int[(d + e*x^2)^(p + 1)/(x*S
qrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Maple [F]

$$\int \frac{x(a + b \operatorname{arcsec}(cx))}{(e\,x^2 + d)^{\frac{3}{2}}} dx$$

input
```
int(x*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.54

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \left[-\frac{(bex^2+bd)\sqrt{-d}\log\left(\frac{(c^4d^2-6\,c^2de+e^2)x^4-8\,(c^2d^2-de)x^2-4\,\sqrt{c^2x^2-1}\left((c^2d-e)x^2-2\,d\right)\sqrt{ex^2+d}\sqrt{-d}+8\,d^2}{x^4}\right)+4\,\sqrt{ex^2+d}(bd\,\mathrm{arcsec}\,(cx)+ad)}{4\,(de^2x^2+d^2e)}, -\frac{(bex^2+bd)\sqrt{d}\arctan\left(-\frac{\sqrt{c^2x^2-1}\left((c^2d-e)x^2-2\,d\right)\sqrt{ex^2+d}\sqrt{d}}{2\,(c^2dex^4+(c^2d^2-de)x^2-d^2)}\right)+2\,\sqrt{ex^2+d}(bd\,\mathrm{arcsec}\,(cx)+ad)}{2\,(de^2x^2+d^2e)}\right]$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((b*e*x^2 + b*d)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(
c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^
2 + d)*sqrt(-d) + 8*d^2)/x^4) + 4*sqrt(e*x^2 + d)*(b*d*arcsec(c*x) + a*d))
/(d*e^2*x^2 + d^2*e), -1/2*((b*e*x^2 + b*d)*sqrt(d)*arctan(-1/2*sqrt(c^2*x
^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^
2*d^2 - d*e)*x^2 - d^2)) + 2*sqrt(e*x^2 + d)*(b*d*arcsec(c*x) + a*d))/(d*e
^2*x^2 + d^2*e)]
```

Sympy [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x(a+b\,\mathrm{asec}\,(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*asec(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x*(a + b*asec(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
-(sqrt(e*x^2 + d)*e*integrate((c^2*e*x^3*log(c) - e*x*log(c) + ((c^2*log(c
) - c^2)*e*x^3 - (c^2*d + e*log(c))*x)*e^(log(c*x + 1) + log(c*x - 1)) + (
c^2*e*x^3 - e*x + (c^2*e*x^3 - e*x)*e^(log(c*x + 1) + log(c*x - 1)))*log(x
))/((c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e + (c^2*e^2*x^4 + (c^2*d*e - e
^2)*x^2 - d*e)*e^(log(c*x + 1) + log(c*x - 1)))*sqrt(e*x^2 + d)), x) + arc
tan(sqrt(c*x + 1)*sqrt(c*x - 1))*b/(sqrt(e*x^2 + d)*e) - a/(sqrt(e*x^2 +
d)*e)
```

Giac [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

input
```
int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{-\sqrt{e\,x^2+d}\,a+\left(\int \frac{asec(cx)x}{\sqrt{e\,x^2+d}\,d+\sqrt{e\,x^2+d}\,e\,x^2}dx\right)bde+\left(\int \frac{asec(cx)x}{\sqrt{e\,x^2+d}\,d+\sqrt{e\,x^2+d}\,e\,x^2}dx\right)b\,e}{e\,(e\,x^2+d)}$$

input
```
int(x*(a+b*asec(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
( - sqrt(d + e*x**2)*a + int((asec(c*x)*x)/(sqrt(d + e*x**2)*d + sqrt(d +
e*x**2)*e*x**2),x)*b*d*e + int((asec(c*x)*x)/(sqrt(d + e*x**2)*d + sqrt(d
+ e*x**2)*e*x**2),x)*b*e**2*x**2)/(e*(d + e*x**2))
```

3.144 $\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)^{3/2}}\,dx$

Optimal result 1202
Mathematica [N/A] 1202
Rubi [N/A] 1203
Maple [N/A] 1203
Fricas [N/A] 1204
Sympy [N/A] 1204
Maxima [F(-2)] 1204
Giac [N/A] 1205
Mupad [N/A] 1205
Reduce [N/A] 1206

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\text{Int}\left(\frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}},x\right)$$

output
```
Defer(Int)((a+b*arcsec(c*x))/x/(e*x^2+d)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 7.64 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^(3/2)),x]
```

output
```
Integrate[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{5771}$$

$$\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)^{3/2}}dx$$

input
```
Int[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\mathrm{arcsec}\,(cx)}{x\,(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arcsec(c*x))/x/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arcsec(c*x))/x/(e*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)^{3/2}}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{(ex^2+d)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x)
, x)
```

Sympy [N/A]

Not integrable

Time = 81.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{asec}\,(cx)}{x(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asec(c*x))/x/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*asec(c*x))/(x*(d + e*x**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)^{3/2}}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{(ex^2+d)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/((e*x^2 + d)^(3/2)*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x\,(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^(3/2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 223, normalized size of antiderivative = 9.70

$$\int \frac{a + b\sec^{-1}(cx)}{x\,(d+ex^2)^{3/2}}\,dx = \frac{\sqrt{e\,x^2+d}\,ad + \sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)ad + \sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)ae\,x^2 - \sqrt{}}{}$$

input
```
int((a+b*asec(c*x))/x/(e*x^2+d)^(3/2),x)
```

output
```
(sqrt(d + e*x**2)*a*d + sqrt(d)*log((sqrt(d + e*x**2) - sqrt(d) + sqrt(e)*
x)/sqrt(d))*a*d + sqrt(d)*log((sqrt(d + e*x**2) - sqrt(d) + sqrt(e)*x)/sqr
t(d))*a*e*x**2 - sqrt(d)*log((sqrt(d + e*x**2) + sqrt(d) + sqrt(e)*x)/sqrt
(d))*a*d - sqrt(d)*log((sqrt(d + e*x**2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*a
*e*x**2 + int(asec(c*x)/(sqrt(d + e*x**2)*d*x + sqrt(d + e*x**2)*e*x**3),x
)*b*d**3 + int(asec(c*x)/(sqrt(d + e*x**2)*d*x + sqrt(d + e*x**2)*e*x**3),
x)*b*d**2*e*x**2)/(d**2*(d + e*x**2))
```

3.145 $\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$

Optimal result . . . 1207
Mathematica [N/A] . . . 1207
Rubi [N/A] . . . 1208
Maple [N/A] . . . 1208
Fricas [N/A] . . . 1209
Sympy [F(-1)] . . . 1209
Maxima [F(-2)] . . . 1209
Giac [N/A] . . . 1210
Mupad [N/A] . . . 1210
Reduce [N/A] . . . 1211

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\text{Int}\left(\frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}},x\right)$$

output
```
Defer(Int)((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 9.82 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
```

output
```
Integrate[(a + b*ArcSec[c*x])/(x^3*(d + e*x^2)^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x^3 (d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 5771$$

$$\int \frac{a+b\sec^{-1}(cx)}{x^3 (d+ex^2)^{3/2}}dx$$

input
```
Int[(a + b*ArcSec[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 2.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\operatorname{arcsec}\left(cx\right)}{x^3 \left(e\,x^2+d\right)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

$$\int \frac{a+b\sec^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{(ex^2+d)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/(e^2*x^7 + 2*d*e*x^5 + d^2*x^
3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*asec(c*x))/x**3/(e*x**2+d)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{b\,\mathrm{arcsec}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/((e*x^2 + d)^(3/2)*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^3\,\left(e\,x^2+d\right)^{3/2}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x^3*(d + e*x^2)^(3/2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x^3*(d + e*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 269, normalized size of antiderivative = 11.70

$$\int \frac{a+b\sec^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx = \frac{-\sqrt{e\,x^2+d}\,a\,d^2-3\sqrt{e\,x^2+d}\,ade\,x^2-3\sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)ade\,x^2-3\sqrt{d}\,\mathrm{lo}}{}$$

input
```
int((a+b*asec(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

output
```
( - sqrt(d + e*x**2)*a*d**2 - 3*sqrt(d + e*x**2)*a*d*e*x**2 - 3*sqrt(d)*lo
g((sqrt(d + e*x**2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*a*d*e*x**2 - 3*sqrt(d)
*log((sqrt(d + e*x**2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*a*e**2*x**4 + 3*sqr
t(d)*log((sqrt(d + e*x**2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*a*d*e*x**2 + 3*
sqrt(d)*log((sqrt(d + e*x**2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*a*e**2*x**4
+ 2*int(asec(c*x)/(sqrt(d + e*x**2)*d*x**3 + sqrt(d + e*x**2)*e*x**5),x)*b
*d**4*x**2 + 2*int(asec(c*x)/(sqrt(d + e*x**2)*d*x**3 + sqrt(d + e*x**2)*e
*x**5),x)*b*d**3*e*x**4)/(2*d**3*x**2*(d + e*x**2))
```

3.146 $\int \frac{x^4\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal result . . . 1212
Mathematica [N/A] . . . 1212
Rubi [N/A] . . . 1213
Maple [N/A] . . . 1213
Fricas [N/A] . . . 1214
Sympy [F(-1)] . . . 1214
Maxima [F(-2)] . . . 1214
Giac [N/A] . . . 1215
Mupad [N/A] . . . 1215
Reduce [N/A] . . . 1216

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\text{Int}\left(\frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}},x\right)$$

output
```
Defer(Int)(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 13.84 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (a + b \sec^{-1}(cx))}{(d + ex^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{5771}$$

$$\int \frac{x^4 (a + b \sec^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

input
```
Int[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^4 (a + b \operatorname{arcsec}(cx))}{(e\,x^2 + d)^{\frac{3}{2}}} dx$$

input
```
int(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^4}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*x^4*arcsec(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2
+ d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(x**4*(a+b*asec(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^4}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x^4/(e*x^2 + d)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{x^4\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

input
```
int((x^4*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int((x^4*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 208, normalized size of antiderivative = 9.04

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{12\sqrt{e\,x^2+d}\,adex+4\sqrt{e\,x^2+d}\,a\,e^2x^3-12\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)a\,d^2-12\sqrt{e}}{}$$

input

```
int(x^4*(a+b*asec(c*x))/(e*x^2+d)^(3/2),x)
```

output

```
(12*sqrt(d + e*x**2)*a*d*e*x + 4*sqrt(d + e*x**2)*a*e**2*x**3 - 12*sqrt(e)
*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*d**2 - 12*sqrt(e)*log((sqrt
(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*d*e*x**2 + 9*sqrt(e)*a*d**2 + 9*sqrt(
e)*a*d*e*x**2 + 8*int((asec(c*x)*x**4)/(sqrt(d + e*x**2)*d + sqrt(d + e*x*
*2)*e*x**2),x)*b*d*e**3 + 8*int((asec(c*x)*x**4)/(sqrt(d + e*x**2)*d + sqr
t(d + e*x**2)*e*x**2),x)*b*e**4*x**2)/(8*e**3*(d + e*x**2))
```

3.147 $\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$

Optimal result 1217
Mathematica [N/A] 1217
Rubi [N/A] 1218
Maple [N/A] 1218
Fricas [N/A] 1219
Sympy [N/A] 1219
Maxima [F(-2)] 1219
Giac [N/A] 1220
Mupad [N/A] 1220
Reduce [N/A] 1221

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Int}\left(\frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}},x\right)$$

output
```
Defer(Int)(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 5.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}} \, dx$$

$$\Big\downarrow \; 5771$$

$$\int \frac{x^2 (a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}dx$$

input
```
Int[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^2(a+b\ \mathrm{arcsec}\,(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^2}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*x^2*arcsec(c*x) + a*x^2)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2
+ d^2), x)
```

Sympy [N/A]

Not integrable

Time = 35.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{x^2(a+b\,\mathrm{asec}\,(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(a+b*asec(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x**2*(a + b*asec(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^2}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x^2/(e*x^2 + d)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{x^2\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

input
```
int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 179, normalized size of antiderivative = 7.78

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{-\sqrt{e\,x^2+d}\,aex+\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)ad+\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)ae\,x^2-\sqrt{e}}{e^2}$$

input
```
int(x^2*(a+b*asec(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
( - sqrt(d + e*x**2)*a*e*x + sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sq
rt(d))*a*d + sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*e*x**2
- sqrt(e)*a*d - sqrt(e)*a*e*x**2 + int((asec(c*x)*x**2)/(sqrt(d + e*x**2)*
d + sqrt(d + e*x**2)*e*x**2),x)*b*d*e**2 + int((asec(c*x)*x**2)/(sqrt(d +
e*x**2)*d + sqrt(d + e*x**2)*e*x**2),x)*b*e**3*x**2)/(e**2*(d + e*x**2))
```

3.148 $\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx$

Optimal result . . . 1222
Mathematica [A] (verified) . . . 1222
Rubi [A] (verified) . . . 1223
Maple [F] . . . 1225
Fricas [A] (verification not implemented) . . . 1225
Sympy [F] . . . 1225
Maxima [F(-2)] . . . 1226
Giac [F] . . . 1226
Mupad [F(-1)] . . . 1226
Reduce [F] . . . 1227

Optimal result

Integrand size = 20, antiderivative size = 109

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \frac{x(a+b\sec^{-1}(cx))}{d\sqrt{d+ex^2}} - \frac{bx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

output
```
x*(a+b*arcsec(c*x))/d/(e*x^2+d)^(1/2)-b*x*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(
1/2)*EllipticF(c*x,(-e/c^2/d)^(1/2))/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(
e*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \frac{x(a+b\sec^{-1}(cx))}{d\sqrt{d+ex^2}} - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d\left(-c+c^3x^2\right)\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(d + e*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcSec[c*x]))/(d*Sqrt[d + e*x^2]) - (b*c*Sqrt[1 - 1/(c^2*x^2)]*x
*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))
])/(d*(-c + c^3*x^2)*Sqrt[d + e*x^2])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5751, 27, 323, 323, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{3/2}}dx$$

$$\Big\downarrow \text{5751}$$

$$\frac{x(a+b\sec^{-1}(cx))}{d\sqrt{d+ex^2}}-\frac{bcx\int \frac{1}{d\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{\sqrt{c^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{x(a+b\sec^{-1}(cx))}{d\sqrt{d+ex^2}}-\frac{bcx\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{d\sqrt{c^2x^2}}$$

$$\Big\downarrow \text{323}$$

$$\frac{x(a+b\sec^{-1}(cx))}{d\sqrt{d+ex^2}}-\frac{bcx\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{d\sqrt{c^2x^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{323}$$

$$\frac{x(a+b\sec^{-1}(cx))}{d\sqrt{d+ex^2}}-\frac{bcx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{x(a+b\sec^{-1}(cx))}{d\sqrt{d+ex^2}}-\frac{bx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}$$

input
```
Int[(a + b*ArcSec[c*x])/(d + e*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcSec[c*x]))/(d*Sqrt[d + e*x^2]) - (b*x*Sqrt[1 - c^2*x^2]*Sqrt[
1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(d*Sqrt[c^2*x^2]*Sqrt
[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 5751
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSec[c*x])   u,
x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 -
 1]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1
/2, 0])
```

Maple [F]

$$\int \frac{a+b\,\mathrm{arcsec}\,(cx)}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.68

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \frac{(bex^2+bd)\sqrt{-d}F\left(\arcsin{(cx)}\mid -\frac{e}{c^2d}\right)+(bcdx\,\mathrm{arcsec}\,(cx)+acdx)\sqrt{ex^2+d}}{cd^2ex^2+cd^3}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
((b*e*x^2 + b*d)*sqrt(-d)*elliptic_f(arcsin(c*x), -e/(c^2*d)) + (b*c*d*x*a
rcsec(c*x) + a*c*d*x)*sqrt(e*x^2 + d))/(c*d^2*e*x^2 + c*d^3)
```

Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\,\mathrm{asec}\,(cx)}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asec(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*asec(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx)+a}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(d + e*x^2)^(3/2),x)
```

output
```
int((a + b*acos(1/(c*x)))/(d + e*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \frac{\sqrt{e\,x^2+d}\,aex+\sqrt{e}\,ad+\sqrt{e}\,ae\,x^2+\left(\int \frac{asec(cx)}{\sqrt{e\,x^2+d}\,d+\sqrt{e\,x^2+d}\,e\,x^2}dx\right)b\,d^2e+\left(\int \frac{}{\sqrt{e\,x^2+d}}\right.}{de\,(e\,x^2+d)}$$

input
```
int((a+b*asec(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
(sqrt(d + e*x**2)*a*e*x + sqrt(e)*a*d + sqrt(e)*a*e*x**2 + int(asec(c*x)/(
sqrt(d + e*x**2)*d + sqrt(d + e*x**2)*e*x**2),x)*b*d**2*e + int(asec(c*x)/
(sqrt(d + e*x**2)*d + sqrt(d + e*x**2)*e*x**2),x)*b*d*e**2*x**2)/(d*e*(d +
 e*x**2))
```

3.149 $\int \frac{a+b\sec^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx$

Optimal result 1228
Mathematica [C] (verified) 1229
Rubi [A] (verified) 1229
Maple [F] 1233
Fricas [A] (verification not implemented) 1234
Sympy [F(-1)] 1234
Maxima [F(-2)] 1235
Giac [F] 1235
Mupad [F(-1)] 1235
Reduce [F] 1236

Optimal result

Integrand size = 23, antiderivative size = 274

$$\int \frac{a+b\sec^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx=\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}}+\frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{d^2x}-\frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}+\frac{b(c^2d+2e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

output
```
b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)+(a+b*arcsec(c*x)
)/d/x/(e*x^2+d)^(1/2)-2*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/d^2/x-b*c^2*x*(-
c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)*EllipticE(c*x,(-e/c^2/d)^(1/2))/d^2/(c^2*
x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)+b*(c^2*d+2*e)*x*(-c^2*x^2+1
)^(1/2)*(1+e*x^2/d)^(1/2)*EllipticF(c*x,(-e/c^2/d)^(1/2))/d^2/(c^2*x^2)^(1
/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.63 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.77

$$\int \frac{a+b\sec^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx = \frac{bc\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)-a(d+2ex^2)-b(d+2ex^2)\sec^{-1}(cx)}{d^2x\sqrt{d+ex^2}} - \frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2dE\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)\middle|-\frac{e}{c^2d}\right)-(c^2d+2e)\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right),-\frac{e}{c^2d}\right)\right)}{\sqrt{-c^2}d^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

output
```
(b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2) - a*(d + 2*e*x^2) - b*(d + 2*e*x^
2)*ArcSec[c*x])/(d^2*x*Sqrt[d + e*x^2]) - (I*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*S
qrt[1 + (e*x^2)/d]*(c^2*d*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]
 - (c^2*d + 2*e)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[
-c^2]*d^2*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {5761, 25, 27, 445, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 5761}$$

$$-\frac{bcx\int -\frac{2ex^2+d}{d^2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{\sqrt{c^2x^2}}-\frac{2ex(a+b\sec^{-1}(cx))}{d^2\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{bcx\int \frac{2ex^2+d}{d^2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{\sqrt{c^2x^2}}-\frac{2ex\left(a+b\sec^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int \frac{2ex^2+d}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{d^2\sqrt{c^2x^2}}-\frac{2ex\left(a+b\sec^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{445}$$

$$\frac{bcx\left(\frac{\int \frac{de\left(2-c^2x^2\right)}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{d}+\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{c^2x^2}}-\frac{2ex\left(a+b\sec^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\left(e\int \frac{2-c^2x^2}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{c^2x^2}}-\frac{2ex\left(a+b\sec^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{bcx\left(e\left(\frac{\left(c^2d+2e\right)\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e}-\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{c^2x^2}}-\frac{2ex\left(a+b\sec^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{323}$$

$$\frac{bcx\left(e\left(\frac{\left(c^2d+2e\right)\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}-\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{c^2x^2}}-\frac{2ex\left(a+b\sec^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{323}$$

$$\frac{bcx\left(e\left(\frac{\sqrt{1-c^2x^2}\left(c^2d+2e\right)\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{c^2x^2}}-\frac{2ex\left(a+b\sec^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 321$$

$$\frac{bcx\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+2e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{c^2x^2}}-\frac{2ex\left(a+b\sec^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 331$$

$$\frac{bcx\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+2e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}\int\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}}\right)+\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{c^2x^2}}-\frac{2ex\left(a+b\sec^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 330$$

$$\frac{bcx\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+2e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\right)+\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{c^2x^2}}-\frac{2ex\left(a+b\sec^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 327$$

$$-\frac{2ex\left(a+b\sec^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}+\frac{bcx\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+2e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\middle|-\frac{e}{c^2d}\right)}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\right)+\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{c^2x^2}}$$

input
```
Int[(a + b*ArcSec[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

output
```
-((a + b*ArcSec[c*x])/(d*x*Sqrt[d + e*x^2])) - (2*e*x*(a + b*ArcSec[c*x]))
/(d^2*Sqrt[d + e*x^2]) + (b*c*x*((Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/x +
e*(-((c*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*
d))])/(e*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d])) + ((c^2*d + 2*e)*Sqrt[1
- c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(c*e*
Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]))))/(d^2*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [F]

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x^2 (e x^2 + d)^{\frac{3}{2}}} dx$$

input
```
int((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.69

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx =$$

$$-\frac{\left(2\,acdex^2+acd^2+\left(2\,bcdex^2+bcd^2\right)\operatorname{arcsec}\left(cx\right)-\left(bcdex^2+bcd^2\right)\sqrt{c^2x^2-1}\right)\sqrt{ex^2+d}-\left(\left(bc^4dex^3+b\right.\right.}{cd^3ex^3+c}$$

input
```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
-((2*a*c*d*e*x^2 + a*c*d^2 + (2*b*c*d*e*x^2 + b*c*d^2)*arcsec(c*x) - (b*c*
d*e*x^2 + b*c*d^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d) - ((b*c^4*d*e*x^3 +
b*c^4*d^2*x)*elliptic_e(arcsin(c*x), -e/(c^2*d)) - ((b*c^4*d*e + 2*b*e^2)*
x^3 + (b*c^4*d^2 + 2*b*d*e)*x)*elliptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(-
d))/(c*d^3*e*x^3 + c*d^4*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*asec(c*x))/x**2/(e*x**2+d)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\,(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\,(d+ex^2)^{3/2}}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx)+a}{(ex^2+d)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/((e*x^2 + d)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\,(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^2\,(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^(3/2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx = \frac{-\sqrt{e\,x^2+d}\,ad-2\sqrt{e\,x^2+d}\,ae\,x^2-2\sqrt{e}\,adx-2\sqrt{e}\,ae\,x^3+\left(\int \frac{asec(cx)}{\sqrt{e\,x^2+d}\,d\,x^2+\sqrt{e\,x^2+}}\right.}{d^2x\left(e\,x^2+d\right)}$$

input
```
int((a+b*asec(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

output
```
( - sqrt(d + e*x**2)*a*d - 2*sqrt(d + e*x**2)*a*e*x**2 - 2*sqrt(e)*a*d*x -
 2*sqrt(e)*a*e*x**3 + int(asec(c*x)/(sqrt(d + e*x**2)*d*x**2 + sqrt(d + e*
x**2)*e*x**4),x)*b*d**3*x + int(asec(c*x)/(sqrt(d + e*x**2)*d*x**2 + sqrt(
d + e*x**2)*e*x**4),x)*b*d**2*e*x**3)/(d**2*x*(d + e*x**2))
```

3.150 $\int \frac{a+b\sec^{-1}(cx)}{x^4(d+ex^2)^{3/2}}\,dx$

Optimal result . . . 1237
Mathematica [C] (verified) . . . 1238
Rubi [A] (verified) . . . 1239
Maple [F] . . . 1241
Fricas [A] (verification not implemented) . . . 1241
Sympy [F(-1)] . . . 1241
Maxima [F(-2)] . . . 1242
Giac [F] . . . 1242
Mupad [F(-1)] . . . 1243
Reduce [F] . . . 1243

Optimal result

Integrand size = 23, antiderivative size = 697

$$\begin{aligned}\int \frac{a+b\sec^{-1}(cx)}{x^4(d+ex^2)^{3/2}}\,dx &= \frac{2bc(c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^3\sqrt{c^2x^2}} \\ &- \frac{4bce\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3d^3\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2x^2\sqrt{c^2x^2}} + \frac{a+b\sec^{-1}(cx)}{dx^3\sqrt{d+ex^2}} \\ &- \frac{4\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3d^2x^3} + \frac{8e\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3d^3x} \\ &- \frac{2bc^2(c^2d-e)\,x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|-\frac{e}{c^2d}\right)}{9d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\ &+ \frac{4bc^2ex\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|-\frac{e}{c^2d}\right)}{3d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\ &+ \frac{bc^2(2c^2d-e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} \\ &- \frac{4bc^2ex\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} \\ &- \frac{8be^2x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\end{aligned}$$

output
```
2/9*b*c*(c^2*d-e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^3/(c^2*x^2)^(1/2)-4/
3*b*c*e*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^3/(c^2*x^2)^(1/2)+1/9*b*c*(c^2
*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/x^2/(c^2*x^2)^(1/2)+(a+b*arcsec(c*x))/d/
x^3/(e*x^2+d)^(1/2)-4/3*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/d^2/x^3+8/3*e*(e
*x^2+d)^(1/2)*(a+b*arcsec(c*x))/d^3/x-2/9*b*c^2*(c^2*d-e)*x*(-c^2*x^2+1)^(
1/2)*(e*x^2+d)^(1/2)*EllipticE(c*x,(-e/c^2/d)^(1/2))/d^3/(c^2*x^2)^(1/2)/(
c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)+4/3*b*c^2*e*x*(-c^2*x^2+1)^(1/2)*(e*x^2
+d)^(1/2)*EllipticE(c*x,(-e/c^2/d)^(1/2))/d^3/(c^2*x^2)^(1/2)/(c^2*x^2-1)^
(1/2)/(1+e*x^2/d)^(1/2)+1/9*b*c^2*(2*c^2*d-e)*x*(-c^2*x^2+1)^(1/2)*(1+e*x^
2/d)^(1/2)*EllipticF(c*x,(-e/c^2/d)^(1/2))/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)
^(1/2)/(e*x^2+d)^(1/2)-4/3*b*c^2*e*x*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)*
EllipticF(c*x,(-e/c^2/d)^(1/2))/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x
^2+d)^(1/2)-8/3*b*e^2*x*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)*EllipticF(c*x
,(-e/c^2/d)^(1/2))/d^3/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.19 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.40

$$\int \frac{a+b\sec^{-1}(cx)}{x^4\,(d+ex^2)^{3/2}}\,dx = \frac{bc\sqrt{1-\frac{1}{c^2x^2}}x(d+2c^2dx^2-14ex^2)\,(d+ex^2)-3a(d^2-4dex^2-8e^2x^4)-3b(d^2-}{9d^3x^3\sqrt{d+ex^2}}$$
$$-\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(2c^2d(c^2d-7e)\,E\left(i\mathrm{arcsinh}\left(\sqrt{-c^2}x\right)|-\frac{e}{c^2d}\right)+(-2c^4d^2+13c^2de+24e^2)\,\mathrm{Ellipti}\right.}{9\sqrt{-c^2}d^3\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x^4*(d + e*x^2)^(3/2)),x]
```

output
```
(b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2*c^2*d*x^2 - 14*e*x^2)*(d + e*x^2) - 3*
a*(d^2 - 4*d*e*x^2 - 8*e^2*x^4) - 3*b*(d^2 - 4*d*e*x^2 - 8*e^2*x^4)*ArcSec
[c*x])/(9*d^3*x^3*Sqrt[d + e*x^2]) - ((I/9)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sq
rt[1 + (e*x^2)/d]*(2*c^2*d*(c^2*d - 7*e)*EllipticE[I*ArcSinh[Sqrt[-c^2]*x]
, -(e/(c^2*d))] + (-2*c^4*d^2 + 13*c^2*d*e + 24*e^2)*EllipticF[I*ArcSinh[S
qrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d^3*Sqrt[1 - c^2*x^2]*Sqrt[d + e
*x^2])
```

Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 591, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5761, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x^4(d+ex^2)^{3/2}}dx$$

$$\Big\downarrow \text{5761}$$

$$-\frac{bcx\int -\frac{-8e^2x^4-4dex^2+d^2}{3d^3x^4\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{\sqrt{c^2x^2}}+\frac{8e^2x\left(a+b\sec^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}}+\frac{4e\left(a+b\sec^{-1}(cx)\right)}{3d^2x\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{3dx^3\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int \frac{-8e^2x^4-4dex^2+d^2}{x^4\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{3d^3\sqrt{c^2x^2}}+\frac{8e^2x\left(a+b\sec^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}}+\frac{4e\left(a+b\sec^{-1}(cx)\right)}{3d^2x\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{3dx^3\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{7293}$$

$$\frac{bcx\int \left(\frac{d^2}{x^4\sqrt{c^2x^2-1}\sqrt{ex^2+d}}-\frac{4ed}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}-\frac{8e^2}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}\right)dx}{3d^3\sqrt{c^2x^2}}+\frac{8e^2x\left(a+b\sec^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}}+\frac{4e\left(a+b\sec^{-1}(cx)\right)}{3d^2x\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{3dx^3\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{8e^2x\left(a+b\sec^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}}+\frac{4e\left(a+b\sec^{-1}(cx)\right)}{3d^2x\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{3dx^3\sqrt{d+ex^2}}+$$
$$bcx\left(-\frac{8e^2\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{c\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{4cde\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}+\frac{cd\sqrt{1-c^2x^2}(2c^2d-e)\sqrt{}\cdots}{3\sqrt{}\cdots}\right.$$

input
```
Int[(a + b*ArcSec[c*x])/(x^4*(d + e*x^2)^(3/2)),x]
```

output

```
-1/3*(a + b*ArcSec[c*x])/(d*x^3*Sqrt[d + e*x^2]) + (4*e*(a + b*ArcSec[c*x]
))/(3*d^2*x*Sqrt[d + e*x^2]) + (8*e^2*x*(a + b*ArcSec[c*x]))/(3*d^3*Sqrt[d
 + e*x^2]) + (b*c*x*((d*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(3*x^3) + (2*(
c^2*d - e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(3*x) - (4*e*Sqrt[-1 + c^2*
x^2]*Sqrt[d + e*x^2])/x - (2*c*(c^2*d - e)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^
2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(3*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e
*x^2)/d]) + (4*c*e*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x]
, -(e/(c^2*d))])/(Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) + (c*d*(2*c^2*d
- e)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2
*d))])/(3*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]) - (4*c*d*e*Sqrt[1 - c^2*x^2]
*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(Sqrt[-1 + c^2*
x^2]*Sqrt[d + e*x^2]) - (8*e^2*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*Ellip
ticF[ArcSin[c*x], -(e/(c^2*d))])/(c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])))/
(3*d^3*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5761

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

rule 7293

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{a+b\,\mathrm{arcsec}\,(cx)}{x^4\,(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.47

$$\int \frac{a+b\sec^{-1}(cx)}{x^4\,(d+ex^2)^{3/2}}\,dx = \frac{(24\,acde^2x^4+12\,acd^2ex^2-3\,acd^3+3\,(8\,bcde^2x^4+4\,bcd^2ex^2-bcd^3)\,\mathrm{arcsec}\,(cx)+}{}$$

input
```
integrate((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
1/9*((24*a*c*d*e^2*x^4 + 12*a*c*d^2*e*x^2 - 3*a*c*d^3 + 3*(8*b*c*d*e^2*x^4
 + 4*b*c*d^2*e*x^2 - b*c*d^3)*arcsec(c*x) + (b*c*d^3 + 2*(b*c^3*d^2*e - 7*
b*c*d*e^2)*x^4 + (2*b*c^3*d^3 - 13*b*c*d^2*e)*x^2)*sqrt(c^2*x^2 - 1))*sqrt
(e*x^2 + d) + (2*((b*c^6*d^2*e - 7*b*c^4*d*e^2)*x^5 + (b*c^6*d^3 - 7*b*c^4
*d^2*e)*x^3)*elliptic_e(arcsin(c*x), -e/(c^2*d)) - ((2*b*c^6*d^2*e - (14*b
*c^4 - b*c^2)*d*e^2 - 24*b*e^3)*x^5 + (2*b*c^6*d^3 - (14*b*c^4 - b*c^2)*d^
2*e - 24*b*d*e^2)*x^3)*elliptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(-d))/(c*d
^4*e*x^5 + c*d^5*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^4\,(d+ex^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*asec(c*x))/x**4/(e*x**2+d)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x^4\left(d+ex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^4\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/((e*x^2 + d)^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^4(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^4\,(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x^4*(d + e*x^2)^(3/2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x^4*(d + e*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^4(d+ex^2)^{3/2}}\,dx = \frac{-\sqrt{e\,x^2+d}\,a\,d^2+4\sqrt{e\,x^2+d}\,ade\,x^2+8\sqrt{e\,x^2+d}\,a\,e^2x^4-8\sqrt{e}\,ade\,x^3-8\sqrt{e}\,a\,e^2}{3d^3x^3\,(e}$$

input
```
int((a+b*asec(c*x))/x^4/(e*x^2+d)^(3/2),x)
```

output
```
( - sqrt(d + e*x**2)*a*d**2 + 4*sqrt(d + e*x**2)*a*d*e*x**2 + 8*sqrt(d + e
*x**2)*a*e**2*x**4 - 8*sqrt(e)*a*d*e*x**3 - 8*sqrt(e)*a*e**2*x**5 + 3*int(
asec(c*x)/(sqrt(d + e*x**2)*d*x**4 + sqrt(d + e*x**2)*e*x**6),x)*b*d**4*x*
*3 + 3*int(asec(c*x)/(sqrt(d + e*x**2)*d*x**4 + sqrt(d + e*x**2)*e*x**6),x
)*b*d**3*e*x**5)/(3*d**3*x**3*(d + e*x**2))
```

3.151 $\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$

Optimal result . . . 1244
Mathematica [C] (warning: unable to verify) . . . 1245
Rubi [A] (verified) . . . 1245
Maple [F] . . . 1249
Fricas [B] (verification not implemented) . . . 1250
Sympy [F(-1)] . . . 1251
Maxima [F(-2)] . . . 1251
Giac [F] . . . 1251
Mupad [F(-1)] . . . 1252
Reduce [F] . . . 1252

Optimal result

Integrand size = 23, antiderivative size = 244

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = -\frac{bcdx\sqrt{-1+c^2x^2}}{3e^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{3e^3(d+ex^2)^{3/2}} + \frac{2d(a+b\sec^{-1}(cx))}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} + \frac{8bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e^3\sqrt{c^2x^2}} - \frac{bx\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{e^{5/2}\sqrt{c^2x^2}}$$

output
```
-1/3*b*c*d*x*(c^2*x^2-1)^(1/2)/e^2/(c^2*d+e)/(c^2*x^2)^(1/2)/(e*x^2+d)^(1/
2)-1/3*d^2*(a+b*arcsec(c*x))/e^3/(e*x^2+d)^(3/2)+2*d*(a+b*arcsec(c*x))/e^3
/(e*x^2+d)^(1/2)+(e*x^2+d)^(1/2)*(a+b*arcsec(c*x))/e^3+8/3*b*c*d^(1/2)*x*a
rctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e^3/(c^2*x^2)^(1/2)-b*x*a
rctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/e^(5/2)/(c^2*x^2)^(1/2
)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.25 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.99

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{-\frac{2bcde\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)}{c^2d+e}+2a(8d^2+12dex^2+3e^2x^4)+\frac{b(d+ex^2)\left(8d\sqrt{1+\frac{d}{ex^2}}\text{AppellF1}\right.}{}}{}$$

input
```
Integrate[(x^5*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
((-2*b*c*d*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2))/(c^2*d + e) + 2*a*(8*d^2
 + 12*d*e*x^2 + 3*e^2*x^4) + (b*(d + e*x^2)*(8*d*Sqrt[1 + d/(e*x^2)]*Appel
lF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))] + (3*c^2*e*Sqrt[1 - 1/(c^2*
x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/
d)])/Sqrt[1 - c^2*x^2]))/(c*x) + 2*b*(8*d^2 + 12*d*e*x^2 + 3*e^2*x^4)*ArcS
ec[c*x])/(6*e^3*(d + e*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.90, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5761, 27, 7282, 2117, 27, 175, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}dx$$

$$\Big\downarrow \ 5761$$

$$-\frac{bcx\int \frac{3e^2x^4+12dex^2+8d^2}{3e^3x\sqrt{c^2x^2-1}\left(ex^2+d\right)^{3/2}}dx}{\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{bcx\int \frac{3e^2x^4+12dex^2+8d^2}{x\sqrt{c^2x^2-1}(ex^2+d)^{3/2}}dx}{3e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \ 7282$$

$$-\frac{bcx\int \frac{3e^2x^4+12dex^2+8d^2}{x^2\sqrt{c^2x^2-1}(ex^2+d)^{3/2}}dx^2}{6e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \ 2117$$

$$-\frac{bcx\left(\frac{2de\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{2\int -\frac{d\left(dc^2+e\right)\left(3ex^2+8d\right)}{2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{d(c^2d+e)}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{bcx\left(\int \frac{3ex^2+8d}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+\frac{2de\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \ 175$$

$$-\frac{bcx\left(3e\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+8d\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+\frac{2de\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \ 66$$

$$-\frac{bcx\left(8d\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+6e\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}+\frac{2de\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \ 104$$

$$-\frac{bcx\left(6e\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}+16d\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+\frac{2de\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \; 217$$

$$-\frac{bcx\left(6e\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-16\sqrt{d}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)+\frac{2de\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \; 221$$

$$-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}-\frac{bcx\left(-16\sqrt{d}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)+\frac{6\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}+\frac{2de\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^3\sqrt{c^2x^2}}$$

input
```
Int[(x^5*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
-1/3*(d^2*(a + b*ArcSec[c*x]))/(e^3*(d + e*x^2)^(3/2)) + (2*d*(a + b*ArcSe
c[c*x]))/(e^3*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/e^3
 - (b*c*x*((2*d*e*Sqrt[-1 + c^2*x^2])/((c^2*d + e)*Sqrt[d + e*x^2]) - 16*S
qrt[d]*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])] + (6*Sqrt[e]*A
rcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/c))/(6*e^3*Sqrt[
c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2117
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

Maple [F]

$$\int \frac{x^5(a + b \operatorname{arcsec}(cx))}{(e\,x^2 + d)^{\frac{5}{2}}}dx$$

input
```
int(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 515 vs. $2(206) = 412$.

Time = 0.37 (sec) , antiderivative size = 2123, normalized size of antiderivative = 8.70

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e
 + b*d*e^2)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*
d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(
e*x^2 + d)*sqrt(e) + e^2) + 8*(b*c^3*d^3 + b*c*d^2*e + (b*c^3*d*e^2 + b*c*
e^3)*x^4 + 2*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2
*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)
*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + 4*(8*a*c^3*d^3 + 8*a*
c*d^2*e + 3*(a*c^3*d*e^2 + a*c*e^3)*x^4 + 12*(a*c^3*d^2*e + a*c*d*e^2)*x^2
 + (8*b*c^3*d^3 + 8*b*c*d^2*e + 3*(b*c^3*d*e^2 + b*c*e^3)*x^4 + 12*(b*c^3*
d^2*e + b*c*d*e^2)*x^2)*arcsec(c*x) - (b*c*d*e^2*x^2 + b*c*d^2*e)*sqrt(c^2
*x^2 - 1))*sqrt(e*x^2 + d))/(c^3*d^3*e^3 + c*d^2*e^4 + (c^3*d*e^5 + c*e^6)
*x^4 + 2*(c^3*d^2*e^4 + c*d*e^5)*x^2), 1/12*(16*(b*c^3*d^3 + b*c*d^2*e + (
b*c^3*d*e^2 + b*c*e^3)*x^4 + 2*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(d)*arct
an(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/
(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + 3*(b*c^2*d^3 + (b*c^2*d*e^2 +
 b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(e)*log(8*c^4*e
^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2
+ c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(8*a*c
^3*d^3 + 8*a*c*d^2*e + 3*(a*c^3*d*e^2 + a*c*e^3)*x^4 + 12*(a*c^3*d^2*e + a
*c*d*e^2)*x^2 + (8*b*c^3*d^3 + 8*b*c*d^2*e + 3*(b*c^3*d*e^2 + b*c*e^3)*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(x**5*(a+b*asec(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{(b\operatorname{arcsec}(cx)+a)x^5}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x^5/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{x^5\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

input
```
int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\frac{8\sqrt{e\,x^2+d}\,a\,d^2+12\sqrt{e\,x^2+d}\,a d e\,x^2+3\sqrt{e\,x^2+d}\,a\,e^2x^4+3\left(\int \frac{}{\sqrt{e\,x^2+d}\,d^2+2\sqrt{}}\right.}{}$$

input
```
int(x^5*(a+b*asec(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
(8*sqrt(d + e*x**2)*a*d**2 + 12*sqrt(d + e*x**2)*a*d*e*x**2 + 3*sqrt(d + e
*x**2)*a*e**2*x**4 + 3*int((asec(c*x)*x**5)/(sqrt(d + e*x**2)*d**2 + 2*sqr
t(d + e*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*b*d**2*e**3 + 6*in
t((asec(c*x)*x**5)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 +
sqrt(d + e*x**2)*e**2*x**4),x)*b*d*e**4*x**2 + 3*int((asec(c*x)*x**5)/(sqr
t(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x
**4),x)*b*e**5*x**4)/(3*e**3*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

3.152 $\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$

Optimal result . . . 1253
Mathematica [C] (warning: unable to verify) . . . 1253
Rubi [A] (verified) . . . 1254
Maple [F] . . . 1257
Fricas [B] (verification not implemented) . . . 1257
Sympy [F] . . . 1258
Maxima [F(-2)] . . . 1258
Giac [F] . . . 1259
Mupad [F(-1)] . . . 1259
Reduce [F] . . . 1260

Optimal result

Integrand size = 23, antiderivative size = 163

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{bcx\sqrt{-1+c^2x^2}}{3e\,(c^2d+e)\,\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{d(a+b\sec^{-1}(cx))}{3e^2\,(d+ex^2)^{3/2}} - \frac{a+b\sec^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{2bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3\sqrt{d}e^2\sqrt{c^2x^2}}$$

output
```
1/3*b*c*x*(c^2*x^2-1)^(1/2)/e/(c^2*d+e)/(c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)+1/
3*d*(a+b*arcsec(c*x))/e^2/(e*x^2+d)^(3/2)-(a+b*arcsec(c*x))/e^2/(e*x^2+d)^
(1/2)-2/3*b*c*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/d^(1/2)/
e^2/(c^2*x^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.45 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.84

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = -\frac{-\frac{bce\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)}{c^2d+e}+a(2d+3ex^2)+\frac{b\sqrt{1+\frac{d}{ex^2}}(d+ex^2)\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)}{cx}+b(2d+3ex^2)\sec^{-1}(cx)}{3e^2(d+ex^2)^{3/2}}$$

input
```
Integrate[(x^3*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
-1/3*(-((b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2))/(c^2*d + e)) + a*(2*d
+ 3*e*x^2) + (b*Sqrt[1 + d/(e*x^2)]*(d + e*x^2)*AppellF1[1, 1/2, 1/2, 2, 1
/(c^2*x^2), -(d/(e*x^2))])/(c*x) + b*(2*d + 3*e*x^2)*ArcSec[c*x])/(e^2*(d
+ e*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5761, 27, 435, 169, 25, 27, 104, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(a+b\sec^{-1}(cx)\right)}{(d+ex^2)^{5/2}}dx$$

$$\Big\downarrow \text{5761}$$

$$-\frac{bcx\int -\frac{3ex^2+2d}{3e^2x\sqrt{c^2x^2-1}(ex^2+d)^{3/2}}dx}{\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\sec^{-1}(cx)\right)}{3e^2(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int \frac{3ex^2+2d}{x\sqrt{c^2x^2-1}(ex^2+d)^{3/2}}dx}{3e^2\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\sec^{-1}(cx)\right)}{3e^2(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{435}$$

$$\frac{bcx\int \frac{3ex^2+2d}{x^2\sqrt{c^2x^2-1}(ex^2+d)^{3/2}}dx^2}{6e^2\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d(a+b\sec^{-1}(cx))}{3e^2(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{169}$$

$$\frac{bcx\left(\frac{2e\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{2\int -\frac{d(dc^2+e)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{d(c^2d+e)}\right)}{6e^2\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d(a+b\sec^{-1}(cx))}{3e^2(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{bcx\left(\frac{2\int \frac{d(dc^2+e)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{d(c^2d+e)}+\frac{2e\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^2\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d(a+b\sec^{-1}(cx))}{3e^2(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\left(2\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+\frac{2e\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^2\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d(a+b\sec^{-1}(cx))}{3e^2(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{104}$$

$$\frac{bcx\left(4\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+\frac{2e\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^2\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d(a+b\sec^{-1}(cx))}{3e^2(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{a+b\sec^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d(a+b\sec^{-1}(cx))}{3e^2(d+ex^2)^{3/2}}+\frac{bcx\left(\frac{2e\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{4\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{\sqrt{d}}\right)}{6e^2\sqrt{c^2x^2}}$$

input

```
Int[(x^3*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

output

```
(d*(a + b*ArcSec[c*x]))/(3*e^2*(d + e*x^2)^(3/2)) - (a + b*ArcSec[c*x])/(e
^2*Sqrt[d + e*x^2]) + (b*c*x*((2*e*Sqrt[-1 + c^2*x^2])/((c^2*d + e)*Sqrt[d
 + e*x^2]) - (4*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/Sqrt
[d]))/(6*e^2*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [F]

$$\int \frac{x^3(a+b\,\mathrm{arcsec}\,(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(137) = 274.

Time = 0.23 (sec) , antiderivative size = 664, normalized size of antiderivative = 4.07

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \Bigg[-\frac{(bc^2d^3+(bc^2de^2+be^3)x^4+bd^2e+2\,(bc^2d^2e+bde^2)x^2)\sqrt{-d}\log\Big(\frac{(c^4d^2-6\,c^2}{}}{}$$

$$-\frac{(bc^2d^3+(bc^2de^2+be^3)x^4+bd^2e+2\,(bc^2d^2e+bde^2)x^2)\sqrt{d}\arctan\Big(-\frac{\sqrt{c^2x^2-1}((c^2d-e)x^2-2\,d)\sqrt{ex^2+d}\sqrt{d}}{2\,(c^2dex^4+(c^2d^2-de)x^2-d^2)}\Big)+(}{3\,(c^2d^4e^2+d^3e^3}$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[-1/6*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e +
 b*d*e^2)*x^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2
- d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*s
qrt(-d) + 8*d^2)/x^4) + 2*(2*a*c^2*d^3 + 2*a*d^2*e + 3*(a*c^2*d^2*e + a*d*
e^2)*x^2 + (2*b*c^2*d^3 + 2*b*d^2*e + 3*(b*c^2*d^2*e + b*d*e^2)*x^2)*arcse
c(c*x) - (b*d*e^2*x^2 + b*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^2*
d^4*e^2 + d^3*e^3 + (c^2*d^2*e^4 + d*e^5)*x^4 + 2*(c^2*d^3*e^3 + d^2*e^4)*
x^2), -1/3*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^
2*e + b*d*e^2)*x^2)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2
 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2))
 + (2*a*c^2*d^3 + 2*a*d^2*e + 3*(a*c^2*d^2*e + a*d*e^2)*x^2 + (2*b*c^2*d^3
 + 2*b*d^2*e + 3*(b*c^2*d^2*e + b*d*e^2)*x^2)*arcsec(c*x) - (b*d*e^2*x^2 +
 b*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^2*d^4*e^2 + d^3*e^3 + (c^
2*d^2*e^4 + d*e^5)*x^4 + 2*(c^2*d^3*e^3 + d^2*e^4)*x^2)]
```

Sympy [F]

$$\int \frac{x^3(a + b\sec^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \int \frac{x^3(a + b\,\mathrm{asec}\,(cx))}{(d + ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3*(a+b*asec(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Integral(x**3*(a + b*asec(c*x))/(d + e*x**2)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\sec^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^3}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x^3/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^3\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{-2\sqrt{e\,x^2+d}\,ad-3\sqrt{e\,x^2+d}\,ae\,x^2+3\left(\int \frac{asec(cx)x^3}{\sqrt{e\,x^2+d}\,d^2+2\sqrt{e\,x^2+d}\,de\,x^2+\sqrt{e\,x^2+d}\,e^2x^4}dx$$

input
```
int(x^3*(a+b*asec(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
( - 2*sqrt(d + e*x**2)*a*d - 3*sqrt(d + e*x**2)*a*e*x**2 + 3*int((asec(c*x
)*x**3)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d + e*
x**2)*e**2*x**4),x)*b*d**2*e**2 + 6*int((asec(c*x)*x**3)/(sqrt(d + e*x**2)
*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*b*d*e
**3*x**2 + 3*int((asec(c*x)*x**3)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x*
*2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*b*e**4*x**4)/(3*e**2*(d**2 +
 2*d*e*x**2 + e**2*x**4))
```

3.153 $\int \frac{x\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result 1261
Mathematica [C] (warning: unable to verify) 1261
Rubi [A] (verified) 1262
Maple [F] 1264
Fricas [B] (verification not implemented) 1264
Sympy [F] 1265
Maxima [F(-2)] 1265
Giac [F] 1266
Mupad [F(-1)] 1266
Reduce [F] 1267

Optimal result

Integrand size = 21, antiderivative size = 138

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = -\frac{bcx\sqrt{-1+c^2x^2}}{3d\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{a+b\sec^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}} - \frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3d^{3/2}e\sqrt{c^2x^2}}$$

output

```
-1/3*b*c*x*(c^2*x^2-1)^(1/2)/d/(c^2*d+e)/(c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)-1
/3*(a+b*arcsec(c*x))/e/(e*x^2+d)^(3/2)-1/3*b*c*x*arctan((e*x^2+d)^(1/2)/d^
(1/2)/(c^2*x^2-1)^(1/2))/d^(3/2)/e/(c^2*x^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.41 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{-\frac{b\sqrt{1+\frac{d}{ex^2}}\left(d+ex^2\right)\operatorname{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)}{cdex}+\frac{2\left(-a-\frac{bce\sqrt{1-\frac{1}{c^2x^2}}\,x\left(d+ex^2\right)}{d\left(c^2d+e\right)}-b\sec^{-1}(cx)\right)}{e}}{6\left(d+ex^2\right)^{3/2}}$$

input
```
Integrate[(x*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
(-((b*Sqrt[1 + d/(e*x^2)]*(d + e*x^2)*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2)
, -(d/(e*x^2))])/(c*d*e*x)) + (2*(-a - (b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d +
 e*x^2))/(d*(c^2*d + e)) - b*ArcSec[c*x]))/e)/(6*(d + e*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5759, 354, 107, 104, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\sec^{-1}(cx)\right)}{(d+ex^2)^{5/2}}dx$$

$$\Big\downarrow \text{5759}$$

$$\frac{bcx\int \frac{1}{x\sqrt{c^2x^2-1}(ex^2+d)^{3/2}}dx}{3e\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{354}$$

$$\frac{bcx\int \frac{1}{x^2\sqrt{c^2x^2-1}(ex^2+d)^{3/2}}dx^2}{6e\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{107}$$

$$\frac{bcx\left(\frac{\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{d}-\frac{2e\sqrt{c^2x^2-1}}{d(c^2d+e)\sqrt{d+ex^2}}\right)}{6e\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{104}$$

$$\frac{bcx\left(\frac{2\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}}{d}-\frac{2e\sqrt{c^2x^2-1}}{d(c^2d+e)\sqrt{d+ex^2}}\right)}{6e\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{217}$$

$$\frac{bcx\left(-\frac{2\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{d^{3/2}}-\frac{2e\sqrt{c^2x^2-1}}{d(c^2d+e)\sqrt{d+ex^2}}\right)}{6e\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

input
```
Int[(x*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
-1/3*(a + b*ArcSec[c*x])/(e*(d + e*x^2)^(3/2)) + (b*c*x*((-2*e*Sqrt[-1 + c
^2*x^2])/(d*(c^2*d + e)*Sqrt[d + e*x^2]) - (2*ArcTan[Sqrt[d + e*x^2]/(Sqrt
[d]*Sqrt[-1 + c^2*x^2])])/d^(3/2)))/(6*e*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5759
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSec[c*x])/(2*e*(p + 1))), x
] - Simp[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2]))   Int[(d + e*x^2)^(p + 1)/(x*S
qrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Maple [F]

$$\int \frac{x(a+b\,\mathrm{arcsec}\,(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int(x*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(114) = 228.

Time = 0.23 (sec) , antiderivative size = 571, normalized size of antiderivative = 4.14

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \left[-\frac{(bc^2d^3+(bc^2de^2+be^3)x^4+bd^2e+2\,(bc^2d^2e+bde^2)x^2)\sqrt{-d}\log\left(\frac{(c^4d^2-6\,c^2d}{}\right.}{}\right.$$

$$-\frac{(bc^2d^3+(bc^2de^2+be^3)x^4+bd^2e+2\,(bc^2d^2e+bde^2)x^2)\sqrt{d}\arctan\left(-\frac{\sqrt{c^2x^2-1}((c^2d-e)x^2-2\,d)\sqrt{ex^2+d}\sqrt{d}}{2\,(c^2dex^4+(c^2d^2-de)x^2-d^2)}\right)+2}{6\,(c^2d^5e+d^4e^2+(c^2d^3e^3+d^2e^4)x^4+2}$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e
+ b*d*e^2)*x^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2
 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*
sqrt(-d) + 8*d^2)/x^4) + 4*(a*c^2*d^3 + a*d^2*e + (b*c^2*d^3 + b*d^2*e)*ar
csec(c*x) + (b*d*e^2*x^2 + b*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c
^2*d^5*e + d^4*e^2 + (c^2*d^3*e^3 + d^2*e^4)*x^4 + 2*(c^2*d^4*e^2 + d^3*e^
3)*x^2), -1/6*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2
*d^2*e + b*d*e^2)*x^2)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*
x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^
2)) + 2*(a*c^2*d^3 + a*d^2*e + (b*c^2*d^3 + b*d^2*e)*arcsec(c*x) + (b*d*e^
2*x^2 + b*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^2*d^5*e + d^4*e^2
+ (c^2*d^3*e^3 + d^2*e^4)*x^4 + 2*(c^2*d^4*e^2 + d^3*e^3)*x^2)]
```

Sympy [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x(a+b\operatorname{asec}(cx))}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*asec(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Integral(x*(a + b*asec(c*x))/(d + e*x**2)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

input
```
int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{-\sqrt{e\,x^2+d}\,a+3\left(\int \frac{asec(cx)x}{\sqrt{e\,x^2+d}\,d^2+2\sqrt{e\,x^2+d}\,d e\,x^2+\sqrt{e\,x^2+d}\,e^2x^4}dx\right)b\,d^2e+6\left(\int \frac{}{\sqrt{e\,x^2+d}\,d}\right.}{3e\,(e^2x}$$

input
```
int(x*(a+b*asec(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
( - sqrt(d + e*x**2)*a + 3*int((asec(c*x)*x)/(sqrt(d + e*x**2)*d**2 + 2*sq
rt(d + e*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*b*d**2*e + 6*int(
(asec(c*x)*x)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(
d + e*x**2)*e**2*x**4),x)*b*d*e**2*x**2 + 3*int((asec(c*x)*x)/(sqrt(d + e*
x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*
b*e**3*x**4)/(3*e*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

3.154 $\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)^{5/2}}\,dx$

Optimal result . . . 1268
Mathematica [N/A] . . . 1268
Rubi [N/A] . . . 1269
Maple [N/A] . . . 1269
Fricas [N/A] . . . 1270
Sympy [F(-1)] . . . 1270
Maxima [F(-2)] . . . 1270
Giac [N/A] . . . 1271
Mupad [N/A] . . . 1271
Reduce [N/A] . . . 1272

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\text{Int}\left(\frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}},x\right)$$

output
```
Defer(Int)((a+b*arcsec(c*x))/x/(e*x^2+d)^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 13.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^(5/2)),x]
```

output
```
Integrate[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^(5/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{5771}$$

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}dx$$

input
```
Int[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\ \mathrm{arcsec}\left(cx\right)}{x\left(e\,x^{2}+d\right)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arcsec(c*x))/x/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arcsec(c*x))/x/(e*x^2+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \frac{a+b\sec^{-1}(cx)}{x\,(d+ex^2)^{5/2}}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{(ex^2+d)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^
2*e*x^3 + d^3*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x\,(d+ex^2)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*asec(c*x))/x/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x\,(d+ex^2)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)^{5/2}}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{(ex^2+d)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/((e*x^2 + d)^(5/2)*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x\,(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^(5/2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 433, normalized size of antiderivative = 18.83

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx = \frac{4\sqrt{e\,x^2+d}\,a\,d^2+3\sqrt{e\,x^2+d}\,ade\,x^2+3\sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)a\,d^2+6\sqrt{d}\log\left(\right.}{}$$

input
```
int((a+b*asec(c*x))/x/(e*x^2+d)^(5/2),x)
```

output
```
(4*sqrt(d + e*x**2)*a*d**2 + 3*sqrt(d + e*x**2)*a*d*e*x**2 + 3*sqrt(d)*log
((sqrt(d + e*x**2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*a*d**2 + 6*sqrt(d)*log(
(sqrt(d + e*x**2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*a*d*e*x**2 + 3*sqrt(d)*l
og((sqrt(d + e*x**2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*a*e**2*x**4 - 3*sqrt(
d)*log((sqrt(d + e*x**2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*a*d**2 - 6*sqrt(d
)*log((sqrt(d + e*x**2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*a*d*e*x**2 - 3*sqr
t(d)*log((sqrt(d + e*x**2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*a*e**2*x**4 + 3
*int(asec(c*x)/(sqrt(d + e*x**2)*d**2*x + 2*sqrt(d + e*x**2)*d*e*x**3 + sq
rt(d + e*x**2)*e**2*x**5),x)*b*d**5 + 6*int(asec(c*x)/(sqrt(d + e*x**2)*d*
*2*x + 2*sqrt(d + e*x**2)*d*e*x**3 + sqrt(d + e*x**2)*e**2*x**5),x)*b*d**4
*e*x**2 + 3*int(asec(c*x)/(sqrt(d + e*x**2)*d**2*x + 2*sqrt(d + e*x**2)*d*
e*x**3 + sqrt(d + e*x**2)*e**2*x**5),x)*b*d**3*e**2*x**4)/(3*d**3*(d**2 +
2*d*e*x**2 + e**2*x**4))
```

3.155 $\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result 1273
Mathematica [N/A] 1273
Rubi [N/A] 1274
Maple [N/A] 1274
Fricas [N/A] 1275
Sympy [F(-1)] 1275
Maxima [F(-2)] 1275
Giac [N/A] 1276
Mupad [N/A] 1276
Reduce [N/A] 1277

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\text{Int}\left(\frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}},x\right)$$

output
```
Defer(Int)((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 14.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
```

output
```
Integrate[(a + b*ArcSec[c*x])/(x^3*(d + e*x^2)^(5/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x^3(d+ex^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 5771}$$

$$\int \frac{a+b\sec^{-1}(cx)}{x^3(d+ex^2)^{5/2}}dx$$

input
```
Int[(a + b*ArcSec[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 3.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\mathrm{arcsec}\,(cx)}{x^3\,(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.39

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^
2*e*x^5 + d^3*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*asec(c*x))/x**3/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{b\,\mathrm{arcsec}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/((e*x^2 + d)^(5/2)*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^3\,{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x^3*(d + e*x^2)^(5/2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x^3*(d + e*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 477, normalized size of antiderivative = 20.74

$$\int \frac{a+b\sec^{-1}(cx)}{x^3(d+ex^2)^{5/2}}\,dx = \frac{-3\sqrt{e\,x^2+d}\,a\,d^3-20\sqrt{e\,x^2+d}\,a\,d^2e\,x^2-15\sqrt{e\,x^2+d}\,ad\,e^2x^4-15\sqrt{d}\log\left(\frac{\sqrt{e\,x^2}}{}\right.}{}$$

input
```
int((a+b*asec(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

output
```
( - 3*sqrt(d + e*x**2)*a*d**3 - 20*sqrt(d + e*x**2)*a*d**2*e*x**2 - 15*sqr
t(d + e*x**2)*a*d*e**2*x**4 - 15*sqrt(d)*log((sqrt(d + e*x**2) - sqrt(d) +
 sqrt(e)*x)/sqrt(d))*a*d**2*e*x**2 - 30*sqrt(d)*log((sqrt(d + e*x**2) - sq
rt(d) + sqrt(e)*x)/sqrt(d))*a*d*e**2*x**4 - 15*sqrt(d)*log((sqrt(d + e*x**
2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*a*e**3*x**6 + 15*sqrt(d)*log((sqrt(d +
e*x**2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*a*d**2*e*x**2 + 30*sqrt(d)*log((sq
rt(d + e*x**2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*a*d*e**2*x**4 + 15*sqrt(d)*
log((sqrt(d + e*x**2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*a*e**3*x**6 + 6*int(
asec(c*x)/(sqrt(d + e*x**2)*d**2*x**3 + 2*sqrt(d + e*x**2)*d*e*x**5 + sqrt
(d + e*x**2)*e**2*x**7),x)*b*d**6*x**2 + 12*int(asec(c*x)/(sqrt(d + e*x**2
)*d**2*x**3 + 2*sqrt(d + e*x**2)*d*e*x**5 + sqrt(d + e*x**2)*e**2*x**7),x)
*b*d**5*e*x**4 + 6*int(asec(c*x)/(sqrt(d + e*x**2)*d**2*x**3 + 2*sqrt(d +
e*x**2)*d*e*x**5 + sqrt(d + e*x**2)*e**2*x**7),x)*b*d**4*e**2*x**6)/(6*d**
4*x**2*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

3.156 $\int \frac{x^6(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$

Optimal result 1278
Mathematica [N/A] 1278
Rubi [N/A] 1279
Maple [N/A] 1279
Fricas [N/A] 1280
Sympy [F(-1)] 1280
Maxima [F(-2)] 1280
Giac [N/A] 1281
Mupad [N/A] 1281
Reduce [N/A] 1282

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^6(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \mathrm{Int}\left(\frac{x^6(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}},x\right)$$

output
```
Defer(Int)(x^6*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 13.75 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^6(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^6(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$

input
```
Integrate[(x^6*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
Integrate[(x^6*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 (a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}} \, dx$$
$$\Big\downarrow \; 5771$$
$$\int \frac{x^6 (a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}} dx$$

input
```
Int[(x^6*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^6 (a+b \operatorname{arcsec}(cx))}{(e x^2+d)^{\frac{5}{2}}} dx$$

input
```
int(x^6*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^6*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.52

$$\int \frac{x^6(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^6}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral((b*x^6*arcsec(c*x) + a*x^6)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^
4 + 3*d^2*e*x^2 + d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**6*(a+b*asec(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^6(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^6*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^6(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^6}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x^6/(e*x^2 + d)^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 1.47 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^6(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^6\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

input
```
int((x^6*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^6*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 391, normalized size of antiderivative = 17.00

$$\int \frac{x^6(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{30\sqrt{e\,x^2+d}\,a\,d^2ex+40\sqrt{e\,x^2+d}\,ad\,e^2x^3+6\sqrt{e\,x^2+d}\,a\,e^3x^5-30\sqrt{e}\log\Big(\frac{\sqrt{e}}{}$$

input
```
int(x^6*(a+b*asec(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
(30*sqrt(d + e*x**2)*a*d**2*e*x + 40*sqrt(d + e*x**2)*a*d*e**2*x**3 + 6*sq
rt(d + e*x**2)*a*e**3*x**5 - 30*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)
/sqrt(d))*a*d**3 - 60*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*
a*d**2*e*x**2 - 30*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*d
*e**2*x**4 - 5*sqrt(e)*a*d**3 - 10*sqrt(e)*a*d**2*e*x**2 - 5*sqrt(e)*a*d*e
**2*x**4 + 12*int((asec(c*x)*x**6)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x
**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*b*d**2*e**4 + 24*int((asec(
c*x)*x**6)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d +
 e*x**2)*e**2*x**4),x)*b*d*e**5*x**2 + 12*int((asec(c*x)*x**6)/(sqrt(d + e
*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)
*b*e**6*x**4)/(12*e**4*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

3.157 $\int \frac{x^4\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result 1283
Mathematica [N/A] 1283
Rubi [N/A] 1284
Maple [N/A] 1284
Fricas [N/A] 1285
Sympy [F(-1)] 1285
Maxima [F(-2)] 1285
Giac [N/A] 1286
Mupad [N/A] 1286
Reduce [N/A] 1287

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^4\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\text{Int}\left(\frac{x^4\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}},x\right)$$

output
```
Defer(Int)(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 13.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^4\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{x^4\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

input
```
Integrate[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
Integrate[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$
$$\Big\downarrow \text{ 5771}$$
$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}dx$$

input
```
Int[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^4(a+b\,\mathrm{arcsec}\,(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.52

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^4}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral((b*x^4*arcsec(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^
4 + 3*d^2*e*x^2 + d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(a+b*asec(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^4}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x^4/(e*x^2 + d)^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{x^4\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

input
```
int((x^4*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^4*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 336, normalized size of antiderivative = 14.61

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{-3\sqrt{e\,x^2+d}\,adex-4\sqrt{e\,x^2+d}\,a\,e^2x^3+3\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)a\,d^2+6\sqrt{e}\,\mathrm{lo}}{}$$

input
```
int(x^4*(a+b*asec(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
( - 3*sqrt(d + e*x**2)*a*d*e*x - 4*sqrt(d + e*x**2)*a*e**2*x**3 + 3*sqrt(e
)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*d**2 + 6*sqrt(e)*log((sqrt
(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*d*e*x**2 + 3*sqrt(e)*log((sqrt(d + e*
x**2) + sqrt(e)*x)/sqrt(d))*a*e**2*x**4 + 3*int((asec(c*x)*x**4)/(sqrt(d +
 e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),
x)*b*d**2*e**3 + 6*int((asec(c*x)*x**4)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d
+ e*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*b*d*e**4*x**2 + 3*int(
(asec(c*x)*x**4)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sq
rt(d + e*x**2)*e**2*x**4),x)*b*e**5*x**4)/(3*e**3*(d**2 + 2*d*e*x**2 + e**
2*x**4))
```

3.158 $\int \frac{x^2\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result 1288
Mathematica [A] (verified) 1289
Rubi [A] (verified) 1289
Maple [F] 1293
Fricas [A] (verification not implemented) 1293
Sympy [F(-1)] 1294
Maxima [F] 1294
Giac [F] 1295
Mupad [F(-1)] 1295
Reduce [F] 1295

Optimal result

Integrand size = 23, antiderivative size = 273

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = -\frac{bcx^2\sqrt{-1+c^2x^2}}{3d\,(c^2d+e)\,\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3(a+b\sec^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} - \frac{bc^2x\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3e\,(c^2d+e)\,\sqrt{c^2x^2}\sqrt{1-c^2x^2}\sqrt{d+ex^2}} + \frac{bx\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3de\sqrt{c^2x^2}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

output
```
-1/3*b*c*x^2*(c^2*x^2-1)^(1/2)/d/(c^2*d+e)/(c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)
+1/3*x^3*(a+b*arcsec(c*x))/d/(e*x^2+d)^(3/2)-1/3*b*c^2*x*(c^2*x^2-1)^(1/2)
*(1+e*x^2/d)^(1/2)*EllipticE(c*x,(-e/c^2/d)^(1/2))/e/(c^2*d+e)/(c^2*x^2)^(
1/2)/(-c^2*x^2+1)^(1/2)/(e*x^2+d)^(1/2)+1/3*b*x*(c^2*x^2-1)^(1/2)*(1+e*x^2
/d)^(1/2)*EllipticF(c*x,(-e/c^2/d)^(1/2))/d/e/(c^2*x^2)^(1/2)/(-c^2*x^2+1)
^(1/2)/(e*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.68

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{x^2\left(a(c^2d+e)\,x-bc\sqrt{1-\frac{1}{c^2x^2}}(d+ex^2)+b(c^2d+e)\,x\sec^{-1}(cx)\right)}{3d\,(c^2d+e)\,(d+ex^2)^{3/2}} + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}E\left(\arcsin\left(\sqrt{-\frac{e}{d}}x\right)|-\frac{c^2d}{e}\right)}{3d\sqrt{-\frac{e}{d}}\,(c^2d+e)\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
(x^2*(a*(c^2*d + e)*x - b*c*Sqrt[1 - 1/(c^2*x^2)]*(d + e*x^2) + b*(c^2*d +
 e)*x*ArcSec[c*x]))/(3*d*(c^2*d + e)*(d + e*x^2)^(3/2)) + (b*c*Sqrt[1 - 1/
(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*EllipticE[ArcSin[Sqrt[-(e/d)]*x], -((c^2*
d)/e)])/(3*d*Sqrt[-(e/d)]*(c^2*d + e)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5761, 27, 373, 326, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{5761}$$

$$\frac{x^3\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bcx\int \frac{x^2}{3d\sqrt{c^2x^2-1}(ex^2+d)^{3/2}}dx}{\sqrt{c^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bcx\int \frac{x^2}{\sqrt{c^2x^2-1}(ex^2+d)^{3/2}}dx}{3d\sqrt{c^2x^2}}$$

$$\Big\downarrow \; 373$$

$$\frac{x^3\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bcx\left(\frac{x\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{\int\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}dx}{c^2d+e}\right)}{3d\sqrt{c^2x^2}}$$

$$\Big\downarrow \; 326$$

$$\frac{x^3\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bcx\left(\frac{x\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{\frac{c^2\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}-\frac{\left(c^2d+e\right)\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e}}{c^2d+e}\right)}{3d\sqrt{c^2x^2}}$$

$$\Big\downarrow \; 323$$

$$\frac{x^3\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bcx\left(\frac{x\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{\frac{c^2\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}-\frac{\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}}{c^2d+e}\right)}{3d\sqrt{c^2x^2}}$$

$$\Big\downarrow \; 323$$

$$\frac{x^3\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bcx\left(\frac{x\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{\frac{c^2\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}-\frac{\sqrt{1-c^2x^2}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}}{c^2d+e}\right)}{3d\sqrt{c^2x^2}}$$

$$\Big\downarrow \; 321$$

$$\frac{x^3\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bcx\left(\frac{x\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{\frac{c^2\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}-\frac{\sqrt{1-c^2x^2}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}}{c^2d+e}\right)}{3d\sqrt{c^2x^2}}$$

$$\Big\downarrow \; 331$$

$$\frac{x^3\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bcx\left(\frac{x\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{\frac{c^2\sqrt{1-c^2x^2}\int\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}}-\frac{\sqrt{1-c^2x^2}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}}{c^2d+e}\right)}{3d\sqrt{c^2x^2}}$$

$\Big\downarrow$ 330

$$\frac{x^3\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bcx\left(\frac{x\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{\frac{c^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}-\frac{\sqrt{1-c^2x^2}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}}{c^2d+e}\right)}{3d\sqrt{c^2x^2}}$$

$\Big\downarrow$ 327

$$\frac{x^3\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bcx\left(\frac{x\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{\frac{c\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}-\frac{\sqrt{1-c^2x^2}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}}{c^2d+e}\right)}{3d\sqrt{c^2x^2}}$$

input
```
Int[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
(x^3*(a + b*ArcSec[c*x]))/(3*d*(d + e*x^2)^(3/2)) - (b*c*x*((x*Sqrt[-1 + c
^2*x^2])/((c^2*d + e)*Sqrt[d + e*x^2]) - ((c*Sqrt[1 - c^2*x^2]*Sqrt[d + e*
x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(e*Sqrt[-1 + c^2*x^2]*Sqrt[1 +
(e*x^2)/d]) - ((c^2*d + e)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF
[ArcSin[c*x], -(e/(c^2*d))])/(c*e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]))/(c^
2*d + e)))/(3*d*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [F]

$$\int \frac{x^2(a + b\,\mathrm{arcsec}\,(cx))}{(e\,x^2 + d)^{\frac{5}{2}}}dx$$

input
```
int(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.05

$$\int \frac{x^2(a + b\sec^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \frac{((bc^3d^2e + bcde^2)x^3\,\mathrm{arcsec}\,(cx) + (ac^3d^2e + acde^2)x^3 - (bcde^2x^3 + bcd^2ex)\sqrt{c}}{}$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(((b*c^3*d^2*e + b*c*d*e^2)*x^3*arcsec(c*x) + (a*c^3*d^2*e + a*c*d*e^2
)*x^3 - (b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d) -
 ((b*c^4*d*e^2*x^4 + 2*b*c^4*d^2*e*x^2 + b*c^4*d^3)*elliptic_e(arcsin(c*x)
, -e/(c^2*d)) - (b*c^4*d^3 + (b*c^4*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^
4*d^2*e + b*d*e^2)*x^2)*elliptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(-d))/(c^
3*d^5*e + c*d^4*e^2 + (c^3*d^3*e^3 + c*d^2*e^4)*x^4 + 2*(c^3*d^4*e^2 + c*d
^3*e^3)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a + b\sec^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*asec(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2(a + b\sec^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx) + a)x^2}{(ex^2 + d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*a*(x/((e*x^2 + d)^(3/2)*e) - x/(sqrt(e*x^2 + d)*d*e)) + b*integrate(x
^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e
*x^2 + d)), x)
```

Giac [F]

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x^2/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^2\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

input
```
int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{\sqrt{e\,x^2+d}\,a\,e^2x^3+\sqrt{e}\,a\,d^2+2\sqrt{e}\,ade\,x^2+\sqrt{e}\,a\,e^2x^4+3\Big(\int \frac{asec(c}{\sqrt{e\,x^2+d}\,d^2+2\sqrt{e\,x^2+d}}}{}$$

input
```
int(x^2*(a+b*asec(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
(sqrt(d + e*x**2)*a*e**2*x**3 + sqrt(e)*a*d**2 + 2*sqrt(e)*a*d*e*x**2 + sq
rt(e)*a*e**2*x**4 + 3*int((asec(c*x)*x**2)/(sqrt(d + e*x**2)*d**2 + 2*sqrt
(d + e*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*b*d**3*e**2 + 6*int
((asec(c*x)*x**2)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + s
qrt(d + e*x**2)*e**2*x**4),x)*b*d**2*e**3*x**2 + 3*int((asec(c*x)*x**2)/(s
qrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2
*x**4),x)*b*d*e**4*x**4)/(3*d*e**2*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

3.159 $\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx$

Optimal result 1297
Mathematica [C] (verified) 1298
Rubi [A] (verified) 1298
Maple [F] 1303
Fricas [A] (verification not implemented) 1303
Sympy [F(-1)] 1303
Maxima [F] 1304
Giac [F] 1304
Mupad [F(-1)] 1304
Reduce [F] 1305

Optimal result

Integrand size = 20, antiderivative size = 296

$$\begin{aligned}\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx &= \frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}}+\frac{x(a+b\sec^{-1}(cx))}{3d(d+ex^2)^{3/2}} \\ &+\frac{2x(a+b\sec^{-1}(cx))}{3d^2\sqrt{d+ex^2}}-\frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3d^2(c^2d+e)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\ &-\frac{2bx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\end{aligned}$$

output
```
1/3*b*c*e*x^2*(c^2*x^2-1)^(1/2)/d^2/(c^2*d+e)/(c^2*x^2)^(1/2)/(e*x^2+d)^(1
/2)+1/3*x*(a+b*arcsec(c*x))/d/(e*x^2+d)^(3/2)+2/3*x*(a+b*arcsec(c*x))/d^2/
(e*x^2+d)^(1/2)-1/3*b*c^2*x*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)*EllipticE(c
*x,(-e/c^2/d)^(1/2))/d^2/(c^2*d+e)/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*
x^2/d)^(1/2)-2/3*b*x*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)*EllipticF(c*x,(-
e/c^2/d)^(1/2))/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.98 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.84

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{x\left(bce\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)+a(c^2d+e)\left(3d+2ex^2\right)+b(c^2d+e)\left(3d+2ex^2\right)\sec^{-}\right.}{3d^2\left(c^2d+e\right)\left(d+ex^2\right)^{3/2}}$$
$$-\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2dE\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)\left|-\frac{e}{c^2d}\right.\right)+2(c^2d+e)\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right),-\frac{e}{c^2d}\right.\right.}{3\sqrt{-c^2}d^2\left(c^2d+e\right)\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(d + e*x^2)^(5/2),x]
```

output
```
(x*(b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2) + a*(c^2*d + e)*(3*d + 2*e*x
^2) + b*(c^2*d + e)*(3*d + 2*e*x^2)*ArcSec[c*x]))/(3*d^2*(c^2*d + e)*(d +
e*x^2)^(3/2)) - ((I/3)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^
2*d*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] + 2*(c^2*d + e)*Ellip
ticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d^2*(c^2*d + e)*
Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {5751, 27, 402, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx$$
$$\Big\downarrow \text{ 5751}$$
$$-\frac{bcx\int \frac{2ex^2+3d}{3d^2\sqrt{c^2x^2-1}(ex^2+d)^{3/2}}dx}{\sqrt{c^2x^2}}+\frac{2x\left(a+b\sec^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{bcx\int \frac{2ex^2+3d}{\sqrt{c^2x^2-1}(ex^2+d)^{3/2}}dx}{3d^2\sqrt{c^2x^2}}+\frac{2x\left(a+b\sec^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \; 402$$

$$-\frac{bcx\left(\frac{\int \frac{d\left(ex^2c^2+3dc^2+2e\right)}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{d(c^2d+e)}-\frac{ex\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2x\left(a+b\sec^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{bcx\left(\frac{\int \frac{ex^2c^2+3dc^2+2e}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{c^2d+e}-\frac{ex\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2x\left(a+b\sec^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \; 399$$

$$-\frac{bcx\left(\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx+2(c^2d+e)\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{c^2d+e}-\frac{ex\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2x\left(a+b\sec^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \; 323$$

$$-\frac{bcx\left(\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx+\frac{2\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{\sqrt{d+ex^2}}}{c^2d+e}-\frac{ex\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2x\left(a+b\sec^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \; 323$$

$$-\frac{bcx\left(\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx+\frac{2\sqrt{1-c^2x^2}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}}{c^2d+e}-\frac{ex\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2x\left(a+b\sec^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \ 321$$

$$-\frac{bcx\left(\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx+\frac{2\sqrt{1-c^2x^2}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{c\sqrt{c^2x^2-1}\sqrt{d+ex^2}}}{c^2d+e}-\frac{ex\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2x\left(a+b\sec^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \ 331$$

$$-\frac{bcx\left(\frac{\frac{c^2\sqrt{1-c^2x^2}\int \frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{\sqrt{c^2x^2-1}}+\frac{2\sqrt{1-c^2x^2}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{c\sqrt{c^2x^2-1}\sqrt{d+ex^2}}}{c^2d+e}-\frac{ex\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2x\left(a+b\sec^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \ 330$$

$$-\frac{bcx\left(\frac{\frac{c^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-c^2x^2}}dx}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}+\frac{2\sqrt{1-c^2x^2}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{c\sqrt{c^2x^2-1}\sqrt{d+ex^2}}}{c^2d+e}-\frac{ex\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2x\left(a+b\sec^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \ 327$$

$$\frac{2x(a+b\sec^{-1}(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\sec^{-1}(cx))}{3d(d+ex^2)^{3/2}}-\frac{bcx\left(\frac{\frac{2\sqrt{1-c^2x^2}(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{c\sqrt{c^2x^2-1}\sqrt{d+ex^2}}+\frac{c\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\middle|-\frac{e}{c^2d}\right)}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}}{c^2d+e}-\frac{ex\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2\sqrt{c^2x^2}}$$

input
```
Int[(a + b*ArcSec[c*x])/(d + e*x^2)^(5/2),x]
```

output
```
(x*(a + b*ArcSec[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*ArcSec[c*x])
)/(3*d^2*Sqrt[d + e*x^2]) - (b*c*x*(-((e*x*Sqrt[-1 + c^2*x^2])/((c^2*d + e
)*Sqrt[d + e*x^2])) + ((c*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcS
in[c*x], -(e/(c^2*d))])/(Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) + (2*(c^2
*d + e)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(
c^2*d))])/(c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]))/(c^2*d + e)))/(3*d^2*Sqr
t[c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 5751
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSec[c*x])   u,
x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 -
 1]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1
/2, 0])
```

Maple [F]

$$\int \frac{a+b\,\mathrm{arcsec}\,(cx)}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.18

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{(2\,(ac^3d^2e+acde^2)x^3+3\,(ac^3d^3+acd^2e)x+(2\,(bc^3d^2e+bcde^2)x^3+3\,(bc^3d^3+b}{}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
1/3*((2*(a*c^3*d^2*e + a*c*d*e^2)*x^3 + 3*(a*c^3*d^3 + a*c*d^2*e)*x + (2*(
b*c^3*d^2*e + b*c*d*e^2)*x^3 + 3*(b*c^3*d^3 + b*c*d^2*e)*x)*arcsec(c*x) +
(b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d) + ((b*c^4
*d*e^2*x^4 + 2*b*c^4*d^2*e*x^2 + b*c^4*d^3)*elliptic_e(arcsin(c*x), -e/(c^
2*d)) - (((b*c^4 - 3*b*c^2)*d*e^2 - 2*b*e^3)*x^4 + (b*c^4 - 3*b*c^2)*d^3 -
 2*b*d^2*e + 2*((b*c^4 - 3*b*c^2)*d^2*e - 2*b*d*e^2)*x^2)*elliptic_f(arcsi
n(c*x), -e/(c^2*d)))*sqrt(-d))/(c^3*d^6 + c*d^5*e + (c^3*d^4*e^2 + c*d^3*e
^3)*x^4 + 2*(c^3*d^5*e + c*d^4*e^2)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*asec(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx)+a}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
1/3*a*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + b*integrate(
arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e*x^
2 + d)), x)
```

Giac [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx)+a}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(d + e*x^2)^(5/2),x)
```

output
```
int((a + b*acos(1/(c*x)))/(d + e*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{3\sqrt{e\,x^2+d}\,adex+2\sqrt{e\,x^2+d}\,a\,e^2x^3-2\sqrt{e}\,a\,d^2-4\sqrt{e}\,ade\,x^2-2\sqrt{e}\,a\,e^2x^4+3\Big(}{}$$

input
```
int((a+b*asec(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
(3*sqrt(d + e*x**2)*a*d*e*x + 2*sqrt(d + e*x**2)*a*e**2*x**3 - 2*sqrt(e)*a
*d**2 - 4*sqrt(e)*a*d*e*x**2 - 2*sqrt(e)*a*e**2*x**4 + 3*int(asec(c*x)/(sq
rt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*
x**4),x)*b*d**4*e + 6*int(asec(c*x)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*
x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*b*d**3*e**2*x**2 + 3*int(a
sec(c*x)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d + e
*x**2)*e**2*x**4),x)*b*d**2*e**3*x**4)/(3*d**2*e*(d**2 + 2*d*e*x**2 + e**2
*x**4))
```

3.160 $\int \frac{a+b\sec^{-1}(cx)}{x^2(d+ex^2)^{5/2}}\,dx$

Optimal result . . . 1306
Mathematica [C] (verified) . . . 1307
Rubi [A] (verified) . . . 1308
Maple [F] . . . 1310
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Sympy [F(-1)] . . . 1311
Maxima [F(-2)] . . . 1311
Giac [F] . . . 1311
Mupad [F(-1)] . . . 1312
Reduce [F] . . . 1312

Optimal result

Integrand size = 23, antiderivative size = 452

$$\begin{aligned}\int \frac{a+b\sec^{-1}(cx)}{x^2(d+ex^2)^{5/2}}\,dx =& -\frac{bce\sqrt{-1+c^2x^2}}{d^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} \\ &-\frac{4bce^2x^2\sqrt{-1+c^2x^2}}{3d^3(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}}+\frac{bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^3(c^2d+e)\sqrt{c^2x^2}} \\ &-\frac{a+b\sec^{-1}(cx)}{dx(d+ex^2)^{3/2}}-\frac{4ex(a+b\sec^{-1}(cx))}{3d^2(d+ex^2)^{3/2}}-\frac{8ex(a+b\sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}} \\ &-\frac{b(3c^2d+2e)\sqrt{d+ex^2}\sqrt{c^2x^2-c^4x^4}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3d^3(c^2d+e)x\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\ &+\frac{b(3c^2d+8e)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\operatorname{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\end{aligned}$$

output
```
-b*c*e*(c^2*x^2-1)^(1/2)/d^2/(c^2*d+e)/(c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)-4/3
*b*c*e^2*x^2*(c^2*x^2-1)^(1/2)/d^3/(c^2*d+e)/(c^2*x^2)^(1/2)/(e*x^2+d)^(1/
2)+b*c*(c^2*d+2*e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^3/(c^2*d+e)/(c^2*x^
2)^(1/2)-(a+b*arcsec(c*x))/d/x/(e*x^2+d)^(3/2)-4/3*e*x*(a+b*arcsec(c*x))/d
^2/(e*x^2+d)^(3/2)-8/3*e*x*(a+b*arcsec(c*x))/d^3/(e*x^2+d)^(1/2)-1/3*b*(3*
c^2*d+2*e)*(e*x^2+d)^(1/2)*(-c^4*x^4+c^2*x^2)^(1/2)*EllipticE(c*x,(-e/c^2/
d)^(1/2))/d^3/(c^2*d+e)/x/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)+1/3*b*(3*c^2
*d+8*e)*x*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)*EllipticF(c*x,(-e/c^2/d)^(1
/2))/d^3/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.42 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.71

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^{5/2}}\,dx = \frac{-a(c^2d+e)\left(3d^2+12dex^2+8e^2x^4\right)+bc\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)\left(3c^2d(d+ex^2)+e(\right.}{3d^3\left(c^2d+e\right)x\left(d+ex^2\right)^3} - \frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(3c^2d+2e)\,E\left(i\operatorname{arcsinh}\left(\sqrt{-c^2}x\right)\middle|-\frac{e}{c^2d}\right)-\left(3c^4d^2+11c^2de+8e^2\right)\operatorname{EllipticF}\right.}{3\sqrt{-c^2}d^3\left(c^2d+e\right)\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x^2*(d + e*x^2)^(5/2)),x]
```

output
```
(-(a*(c^2*d + e)*(3*d^2 + 12*d*e*x^2 + 8*e^2*x^4)) + b*c*Sqrt[1 - 1/(c^2*x
^2)]*x*(d + e*x^2)*(3*c^2*d*(d + e*x^2) + e*(3*d + 2*e*x^2)) - b*(c^2*d +
e)*(3*d^2 + 12*d*e*x^2 + 8*e^2*x^4)*ArcSec[c*x])/(3*d^3*(c^2*d + e)*x*(d +
 e*x^2)^(3/2)) - ((I/3)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c
^2*d*(3*c^2*d + 2*e)*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] - (3
*c^4*d^2 + 11*c^2*d*e + 8*e^2)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2
*d))]))/(Sqrt[-c^2]*d^3*(c^2*d + e)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5761, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{5761}$$

$$-\frac{bcx\int -\frac{8e^2x^4+12dex^2+3d^2}{3d^3x^2\sqrt{c^2x^2-1}\left(ex^2+d\right)^{3/2}}dx}{\sqrt{c^2x^2}}-\frac{8ex\left(a+b\sec^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}}-\frac{4ex\left(a+b\sec^{-1}(cx)\right)}{3d^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\sec^{-1}(cx)}{dx\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int \frac{8e^2x^4+12dex^2+3d^2}{x^2\sqrt{c^2x^2-1}\left(ex^2+d\right)^{3/2}}dx}{3d^3\sqrt{c^2x^2}}-\frac{8ex\left(a+b\sec^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}}-\frac{4ex\left(a+b\sec^{-1}(cx)\right)}{3d^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\sec^{-1}(cx)}{dx\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{7293}$$

$$\frac{bcx\int \left(\frac{3d^2}{x^2\sqrt{c^2x^2-1}\left(ex^2+d\right)^{3/2}}+\frac{12ed}{\sqrt{c^2x^2-1}\left(ex^2+d\right)^{3/2}}+\frac{8e^2x^2}{\sqrt{c^2x^2-1}\left(ex^2+d\right)^{3/2}}\right)dx}{3d^3\sqrt{c^2x^2}}-\frac{8ex\left(a+b\sec^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}}-\frac{4ex\left(a+b\sec^{-1}(cx)\right)}{3d^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\sec^{-1}(cx)}{dx\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{8ex\left(a+b\sec^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}}-\frac{4ex\left(a+b\sec^{-1}(cx)\right)}{3d^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\sec^{-1}(cx)}{dx\left(d+ex^2\right)^{3/2}}+\frac{bcx\left(\frac{8e\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{c\sqrt{c^2x^2-1}\sqrt{d+ex^2}}+\frac{3cd\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}+\frac{4ce\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\text{arc}\cdots\right.}{\sqrt{c^2x^2-1}\left(c^2d+e\right)\sqrt{\cdots}}\cdots\right)}{3d\cdots}$$

input
```
Int[(a + b*ArcSec[c*x])/(x^2*(d + e*x^2)^(5/2)),x]
```

output
```
-((a + b*ArcSec[c*x])/(d*x*(d + e*x^2)^(3/2))) - (4*e*x*(a + b*ArcSec[c*x]
))/(3*d^2*(d + e*x^2)^(3/2)) - (8*e*x*(a + b*ArcSec[c*x]))/(3*d^3*Sqrt[d +
 e*x^2]) + (b*c*x*((-3*d*e*Sqrt[-1 + c^2*x^2])/((c^2*d + e)*x*Sqrt[d + e*x
^2]) - (4*e^2*x*Sqrt[-1 + c^2*x^2])/((c^2*d + e)*Sqrt[d + e*x^2]) + (3*(c^
2*d + 2*e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/((c^2*d + e)*x) + (4*c*e*Sq
rt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/((c^
2*d + e)*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) - (3*c*(c^2*d + 2*e)*Sqrt
[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/((c^2*
d + e)*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) + (3*c*d*Sqrt[1 - c^2*x^2]*
Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(Sqrt[-1 + c^2*x
^2]*Sqrt[d + e*x^2]) + (8*e*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*Elliptic
F[ArcSin[c*x], -(e/(c^2*d))])/(c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])))/(3*
d^3*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{a+b\,\mathrm{arcsec}\,(cx)}{x^2\,(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(5/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.19

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\,(d+ex^2)^{5/2}}\,dx = -\frac{(3\,ac^3d^4+3\,acd^3e+8\,(ac^3d^2e^2+acde^3)x^4+12\,(ac^3d^3e+acd^2e^2)x^2+(3\,bc^3d^4+3\,bcd^3e+8\,(bc^3d^2e^2+$$

input
```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*((3*a*c^3*d^4 + 3*a*c*d^3*e + 8*(a*c^3*d^2*e^2 + a*c*d*e^3)*x^4 + 12*
(a*c^3*d^3*e + a*c*d^2*e^2)*x^2 + (3*b*c^3*d^4 + 3*b*c*d^3*e + 8*(b*c^3*d^
2*e^2 + b*c*d*e^3)*x^4 + 12*(b*c^3*d^3*e + b*c*d^2*e^2)*x^2)*arcsec(c*x) -
 (3*b*c^3*d^4 + 3*b*c*d^3*e + (3*b*c^3*d^2*e^2 + 2*b*c*d*e^3)*x^4 + (6*b*c
^3*d^3*e + 5*b*c*d^2*e^2)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d) - (((3*b
*c^6*d^2*e^2 + 2*b*c^4*d*e^3)*x^5 + 2*(3*b*c^6*d^3*e + 2*b*c^4*d^2*e^2)*x^
3 + (3*b*c^6*d^4 + 2*b*c^4*d^3*e)*x)*elliptic_e(arcsin(c*x), -e/(c^2*d)) -
 ((3*b*c^6*d^2*e^2 + (2*b*c^4 + 9*b*c^2)*d*e^3 + 8*b*e^4)*x^5 + 2*(3*b*c^6
*d^3*e + (2*b*c^4 + 9*b*c^2)*d^2*e^2 + 8*b*d*e^3)*x^3 + (3*b*c^6*d^4 + (2*
b*c^4 + 9*b*c^2)*d^3*e + 8*b*d^2*e^2)*x)*elliptic_f(arcsin(c*x), -e/(c^2*d
)))*sqrt(-d))/((c^3*d^5*e^2 + c*d^4*e^3)*x^5 + 2*(c^3*d^6*e + c*d^5*e^2)*x
^3 + (c^3*d^7 + c*d^6*e)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*asec(c*x))/x**2/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{b\,\text{arcsec}\,(cx)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/((e*x^2 + d)^(5/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^2\,\left(e\,x^2+d\right)^{5/2}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^(5/2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^{5/2}}\,dx=\frac{-3\sqrt{e\,x^2+d}\,a\,d^2-12\sqrt{e\,x^2+d}\,ade\,x^2-8\sqrt{e\,x^2+d}\,a\,e^2x^4+8\sqrt{e}\,a\,d^2x+16\sqrt{e}\,c$$

input
```
int((a+b*asec(c*x))/x^2/(e*x^2+d)^(5/2),x)
```

output
```
( - 3*sqrt(d + e*x**2)*a*d**2 - 12*sqrt(d + e*x**2)*a*d*e*x**2 - 8*sqrt(d
+ e*x**2)*a*e**2*x**4 + 8*sqrt(e)*a*d**2*x + 16*sqrt(e)*a*d*e*x**3 + 8*sqr
t(e)*a*e**2*x**5 + 3*int(asec(c*x)/(sqrt(d + e*x**2)*d**2*x**2 + 2*sqrt(d
+ e*x**2)*d*e*x**4 + sqrt(d + e*x**2)*e**2*x**6),x)*b*d**5*x + 6*int(asec(
c*x)/(sqrt(d + e*x**2)*d**2*x**2 + 2*sqrt(d + e*x**2)*d*e*x**4 + sqrt(d +
e*x**2)*e**2*x**6),x)*b*d**4*e*x**3 + 3*int(asec(c*x)/(sqrt(d + e*x**2)*d*
*2*x**2 + 2*sqrt(d + e*x**2)*d*e*x**4 + sqrt(d + e*x**2)*e**2*x**6),x)*b*d
**3*e**2*x**5)/(3*d**3*x*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

3.161 $\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\sec^{-1}(cx)\right)\,dx$

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Optimal result

Integrand size = 23, antiderivative size = 589

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\sec^{-1}(cx)\right)\,dx =$$

$$-\frac{be\Big(e^2(15+8m+m^2)^2+3c^2de(3+m)^2\left(42+13m+m^2\right)+3c^4d^2(840+638m+179m^2+22m^3+m}{c^5f(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)\sqrt{c^2x^2}}$$

$$-\frac{be^2(e(5+m)^2+3c^2d(42+13m+m^2))\,x(fx)^{3+m}\sqrt{-1+c^2x^2}}{c^3f^3(4+m)(5+m)(6+m)(7+m)\sqrt{c^2x^2}}$$

$$-\frac{be^3x(fx)^{5+m}\sqrt{-1+c^2x^2}}{cf^5(6+m)(7+m)\sqrt{c^2x^2}}+\frac{d^3(fx)^{1+m}\left(a+b\sec^{-1}(cx)\right)}{f(1+m)}$$

$$+\frac{3d^2e(fx)^{3+m}\left(a+b\sec^{-1}(cx)\right)}{f^3(3+m)}$$

$$+\frac{3de^2(fx)^{5+m}\left(a+b\sec^{-1}(cx)\right)}{f^5(5+m)}+\frac{e^3(fx)^{7+m}\left(a+b\sec^{-1}(cx)\right)}{f^7(7+m)}$$

$$-\frac{b\left(\frac{c^6d^3(2+m)(4+m)(6+m)}{1+m}+\frac{e(1+m)\left(e^2\left(15+8m+m^2\right)^2+3c^2de(3+m)^2\left(42+13m+m^2\right)+3c^4d^2\left(840+638m+179m^2+22m^3+m^4\right)\right)}{(3+m)(5+m)(7+m)}\right)}{c^5f(1+m)(2+m)(4+m)(6+m)\sqrt{c^2x^2}\sqrt{-1}}$$

output
```
-b*e*(e^2*(m^2+8*m+15)^2+3*c^2*d*e*(3+m)^2*(m^2+13*m+42)+3*c^4*d^2*(m^4+22
*m^3+179*m^2+638*m+840))*x*(f*x)^(1+m)*(c^2*x^2-1)^(1/2)/c^5/f/(2+m)/(3+m)
/(4+m)/(5+m)/(6+m)/(7+m)/(c^2*x^2)^(1/2)-b*e^2*(e*(5+m)^2+3*c^2*d*(m^2+13*
m+42))*x*(f*x)^(3+m)*(c^2*x^2-1)^(1/2)/c^3/f^3/(4+m)/(5+m)/(6+m)/(7+m)/(c^
2*x^2)^(1/2)-b*e^3*x*(f*x)^(5+m)*(c^2*x^2-1)^(1/2)/c/f^5/(6+m)/(7+m)/(c^2*
x^2)^(1/2)+d^3*(f*x)^(1+m)*(a+b*arcsec(c*x))/f/(1+m)+3*d^2*e*(f*x)^(3+m)*(
a+b*arcsec(c*x))/f^3/(3+m)+3*d*e^2*(f*x)^(5+m)*(a+b*arcsec(c*x))/f^5/(5+m)
+e^3*(f*x)^(7+m)*(a+b*arcsec(c*x))/f^7/(7+m)-b*(c^6*d^3*(2+m)*(4+m)*(6+m)/
(1+m)+e*(1+m)*(e^2*(m^2+8*m+15)^2+3*c^2*d*e*(3+m)^2*(m^2+13*m+42)+3*c^4*d^
2*(m^4+22*m^3+179*m^2+638*m+840))/(3+m)/(5+m)/(7+m))*x*(f*x)^(1+m)*(-c^2*x
^2+1)^(1/2)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)/c^5/f/(1+m)/(2
+m)/(4+m)/(6+m)/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)
```

Mathematica [A] (verified)

Time = 2.85 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.68

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\sec^{-1}(cx)\right)\,dx$$

$$= x(fx)^m \left(\frac{ad^3}{1+m}+\frac{3ad^2ex^2}{3+m}+\frac{3ade^2x^4}{5+m}+\frac{ae^3x^6}{7+m}+\frac{bd^3\sec^{-1}(cx)}{1+m} + \frac{3bd^2ex^2\sec^{-1}(cx)}{3+m}+\frac{3bde^2x^4\sec^{-1}(cx)}{5+m}+\frac{be^3x^6\sec^{-1}(cx)}{7+m} + \frac{bcd^3\sqrt{1-\frac{1}{c^2x^2}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{(1+m)^2\sqrt{1-c^2x^2}} + \frac{3bcd^2e\sqrt{1-\frac{1}{c^2x^2}}x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{(3+m)^2\sqrt{1-c^2x^2}} + \frac{3bcde^2\sqrt{1-\frac{1}{c^2x^2}}x^5\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5+m}{2},\frac{7+m}{2},c^2x^2\right)}{(5+m)^2\sqrt{1-c^2x^2}} + \frac{bce^3\sqrt{1-\frac{1}{c^2x^2}}x^7\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7+m}{2},\frac{9+m}{2},c^2x^2\right)}{(7+m)^2\sqrt{1-c^2x^2}}\right)$$

input
```
Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*ArcSec[c*x]),x]
```

output
```
x*(f*x)^m*((a*d^3)/(1 + m) + (3*a*d^2*e*x^2)/(3 + m) + (3*a*d*e^2*x^4)/(5
+ m) + (a*e^3*x^6)/(7 + m) + (b*d^3*ArcSec[c*x])/(1 + m) + (3*b*d^2*e*x^2*
ArcSec[c*x])/(3 + m) + (3*b*d*e^2*x^4*ArcSec[c*x])/(5 + m) + (b*e^3*x^6*Ar
cSec[c*x])/(7 + m) + (b*c*d^3*Sqrt[1 - 1/(c^2*x^2)]*x*Hypergeometric2F1[1/
2, (1 + m)/2, (3 + m)/2, c^2*x^2])/((1 + m)^2*Sqrt[1 - c^2*x^2]) + (3*b*c*
d^2*e*Sqrt[1 - 1/(c^2*x^2)]*x^3*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/
2, c^2*x^2])/((3 + m)^2*Sqrt[1 - c^2*x^2]) + (3*b*c*d*e^2*Sqrt[1 - 1/(c^2*
x^2)]*x^5*Hypergeometric2F1[1/2, (5 + m)/2, (7 + m)/2, c^2*x^2])/((5 + m)^
2*Sqrt[1 - c^2*x^2]) + (b*c*e^3*Sqrt[1 - 1/(c^2*x^2)]*x^7*Hypergeometric2F
1[1/2, (7 + m)/2, (9 + m)/2, c^2*x^2])/((7 + m)^2*Sqrt[1 - c^2*x^2]))
```

Rubi [A] (verified)

Time = 2.15 (sec) , antiderivative size = 542, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5761, 2340, 1590, 363, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^3 (fx)^m \left(a+b\sec^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{5761}$$

$$-\frac{bcx\int \frac{(fx)^m\left(\frac{e^3x^6}{m+7}+\frac{3de^2x^4}{m+5}+\frac{3d^2ex^2}{m+3}+\frac{d^3}{m+1}\right)}{\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{d^3(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\sec^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\sec^{-1}(cx)\right)}{f^7(m+7)}$$

$$\Big\downarrow \text{2340}$$

$$-\frac{bcx\left(\frac{\int \frac{(fx)^m\left(\frac{e^2\left(3d\left(m^2+13m+42\right)c^2+e(m+5)^2\right)x^4}{(m+5)(m+7)}+\frac{3c^2d^2e(m+6)x^2}{m+3}+\frac{c^2d^3(m+6)}{m+1}\right)}{\sqrt{c^2x^2-1}}dx}{c^2(m+6)}+\frac{e^3\sqrt{c^2x^2-1}(fx)^{m+5}}{c^2f^5(m+6)(m+7)}\right)}{\sqrt{c^2x^2}}+\frac{d^3(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\sec^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\sec^{-1}(cx)\right)}{f^7(m+7)}$$

$\big\downarrow$ 1590

$$-\frac{bcx\left(\frac{\frac{\int \frac{(fx)^m\left(\frac{d^3(m+4)(m+6)c^4}{m+1}+\frac{e\left(3d^2\left(m^4+22m^3+179m^2+638m+840\right)c^4+3de(m+3)^2\left(m^2+13m+42\right)c^2+e^2\left(m^2+8m+15\right)^2\right)x^2}{(m+3)(m+5)(m+7)}\right)}{\sqrt{c^2x^2-1}}dx}{c^2(m+4)}+\frac{e^2\sqrt{c^2x^2-1}\ldots}{\ldots}}{c^2(m+6)}\ldots\right)}{\sqrt{c^2x^2}}+\frac{d^3(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\sec^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\sec^{-1}(cx)\right)}{f^7(m+7)}$$

$\big\downarrow$ 363

$$-\frac{bcx\left(\frac{\frac{\left(\frac{c^4d^3(m+4)(m+6)}{m+1}+\frac{e(m+1)\left(3c^4d^2\left(m^4+22m^3+179m^2+638m+840\right)+3c^2de(m+3)^2\left(m^2+13m+42\right)+e^2\left(m^2+8m+15\right)^2\right)}{c^2(m+2)(m+3)(m+5)(m+7)}\right)\int \frac{(fx)^m}{\sqrt{c^2x^2-1}}dx+\frac{e\sqrt{c^2\ldots}}{\ldots}}{c^2(m+4)}\ldots}{\ldots}\right)}{\ldots}+\frac{d^3(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\sec^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\sec^{-1}(cx)\right)}{f^7(m+7)}$$

$\big\downarrow$ 279

$$-\frac{bcx\left(\frac{\frac{\sqrt{1-c^2x^2}\left(\frac{c^4d^3(m+4)(m+6)}{m+1}+\frac{e(m+1)\left(3c^4d^2\left(m^4+22m^3+179m^2+638m+840\right)+3c^2de(m+3)^2\left(m^2+13m+42\right)+e^2\left(m^2+8m+15\right)^2\right)}{c^2(m+2)(m+3)(m+5)(m+7)}\right)\int\frac{(fx)^m}{\sqrt{1-c^2x^2}}}{\sqrt{c^2x^2-1}}}{c^2(m+4)}\right.}{}$$

$$\frac{d^3(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\sec^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\sec^{-1}(cx)\right)}{f^7(m+7)}$$

$\downarrow$ 278

$$\frac{d^3(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\sec^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\sec^{-1}(cx)\right)}{f^7(m+7)}-$$

$$\frac{bcx\left(\frac{e^3\sqrt{c^2x^2-1}(fx)^{m+5}}{c^2f^5(m+6)(m+7)}+\frac{\frac{e^2\sqrt{c^2x^2-1}(fx)^{m+3}\left(3c^2d\left(m^2+13m+42\right)+e(m+5)^2\right)}{c^2f^3(m+4)(m+5)(m+7)}+\frac{e\sqrt{c^2x^2-1}(fx)^{m+1}\left(3c^4d^2\left(m^4+22m^3+179m^2+638m+840\right)+3c\right.}{c^2f(m+2)(m+3)(m+5}}{}\right.}{}$$

input
```
Int[(f*x)^m*(d + e*x^2)^3*(a + b*ArcSec[c*x]),x]
```

output
```
(d^3*(f*x)^(1 + m)*(a + b*ArcSec[c*x]))/(f*(1 + m)) + (3*d^2*e*(f*x)^(3 +
m)*(a + b*ArcSec[c*x]))/(f^3*(3 + m)) + (3*d*e^2*(f*x)^(5 + m)*(a + b*ArcS
ec[c*x]))/(f^5*(5 + m)) + (e^3*(f*x)^(7 + m)*(a + b*ArcSec[c*x]))/(f^7*(7
+ m)) - (b*c*x*((e^3*(f*x)^(5 + m)*Sqrt[-1 + c^2*x^2])/(c^2*f^5*(6 + m)*(7
 + m)) + ((e^2*(e*(5 + m)^2 + 3*c^2*d*(42 + 13*m + m^2))*(f*x)^(3 + m)*Sqr
t[-1 + c^2*x^2])/(c^2*f^3*(4 + m)*(5 + m)*(7 + m)) + ((e*(e^2*(15 + 8*m +
m^2)^2 + 3*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*c^4*d^2*(840 + 638*m +
179*m^2 + 22*m^3 + m^4))*(f*x)^(1 + m)*Sqrt[-1 + c^2*x^2])/(c^2*f*(2 + m)*
(3 + m)*(5 + m)*(7 + m)) + (((c^4*d^3*(4 + m)*(6 + m))/(1 + m) + (e*(1 + m
)*(e^2*(15 + 8*m + m^2)^2 + 3*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*c^4*
d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4)))/(c^2*(2 + m)*(3 + m)*(5 + m)*
(7 + m)))*(f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2
, (3 + m)/2, c^2*x^2])/(f*(1 + m)*Sqrt[-1 + c^2*x^2]))/(c^2*(4 + m)))/(c^2
*(6 + m))))/Sqrt[c^2*x^2]
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right)^3 (a+b\,\mathrm{arcsec}\,(cx))\,dx$$

input
```
int((f*x)^m*(e*x^2+d)^3*(a+b*arcsec(c*x)),x)
```

output
```
int((f*x)^m*(e*x^2+d)^3*(a+b*arcsec(c*x)),x)
```

Fricas [F]

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\sec^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^3 (b\operatorname{arcsec}(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^3*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arcsec(c*x))*(f*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\sec^{-1}(cx)\right) \, dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(e*x**2+d)**3*(a+b*asec(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\sec^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^3 (b\operatorname{arcsec}(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^3*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
a*e^3*f^m*x^7*x^m/(m + 7) + 3*a*d*e^2*f^m*x^5*x^m/(m + 5) + 3*a*d^2*e*f^m*
x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d^3/(f*(m + 1)) + (((b*e^3*f^m*m^3 + 9*b
*e^3*f^m*m^2 + 23*b*e^3*f^m*m + 15*b*e^3*f^m)*x^7 + 3*(b*d*e^2*f^m*m^3 + 1
1*b*d*e^2*f^m*m^2 + 31*b*d*e^2*f^m*m + 21*b*d*e^2*f^m)*x^5 + 3*(b*d^2*e*f^
m*m^3 + 13*b*d^2*e*f^m*m^2 + 47*b*d^2*e*f^m*m + 35*b*d^2*e*f^m)*x^3 + (b*d
^3*f^m*m^3 + 15*b*d^3*f^m*m^2 + 71*b*d^3*f^m*m + 105*b*d^3*f^m)*x)*x^m*arc
tan(sqrt(c*x + 1)*sqrt(c*x - 1)) - (m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*i
ntegrate(-(b*d^3*f^m*m^3 + 15*b*d^3*f^m*m^2 + (b*e^3*f^m*m^3 + 9*b*e^3*f^m
*m^2 + 23*b*e^3*f^m*m + 15*b*e^3*f^m)*x^6 + 71*b*d^3*f^m*m + 105*b*d^3*f^m
 + 3*(b*d*e^2*f^m*m^3 + 11*b*d*e^2*f^m*m^2 + 31*b*d*e^2*f^m*m + 21*b*d*e^2
*f^m)*x^4 + 3*(b*d^2*e*f^m*m^3 + 13*b*d^2*e*f^m*m^2 + 47*b*d^2*e*f^m*m + 3
5*b*d^2*e*f^m)*x^2)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m/(m^4 + 16*m^3 - (c^2*m
^4 + 16*c^2*m^3 + 86*c^2*m^2 + 176*c^2*m + 105*c^2)*x^2 + 86*m^2 + 176*m +
 105), x))/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

Giac [F]

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\sec^{-1}(cx)\right) \, dx = \int (ex^2+d)^3 (b \operatorname{arcsec}(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^3*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^3*(b*arcsec(c*x) + a)*(f*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\sec^{-1}(cx)\right) \, dx = \int (f\,x)^m\,\left(e\,x^2+d\right)^3\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right) \,d x$$

input
```
int((f*x)^m*(d + e*x^2)^3*(a + b*acos(1/(c*x))),x)
```

output
```
int((f*x)^m*(d + e*x^2)^3*(a + b*acos(1/(c*x))), x)
```

Reduce [F]

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
int((f*x)^m*(e*x^2+d)^3*(a+b*asec(c*x)),x)
```

output
```
(f**m*(x**m*a*d**3*m**3*x + 15*x**m*a*d**3*m**2*x + 71*x**m*a*d**3*m*x + 1
05*x**m*a*d**3*x + 3*x**m*a*d**2*e*m**3*x**3 + 39*x**m*a*d**2*e*m**2*x**3
+ 141*x**m*a*d**2*e*m*x**3 + 105*x**m*a*d**2*e*x**3 + 3*x**m*a*d*e**2*m**3
*x**5 + 33*x**m*a*d*e**2*m**2*x**5 + 93*x**m*a*d*e**2*m*x**5 + 63*x**m*a*d
*e**2*x**5 + x**m*a*e**3*m**3*x**7 + 9*x**m*a*e**3*m**2*x**7 + 23*x**m*a*e
**3*m*x**7 + 15*x**m*a*e**3*x**7 + int(x**m*asec(c*x)*x**6,x)*b*e**3*m**4
+ 16*int(x**m*asec(c*x)*x**6,x)*b*e**3*m**3 + 86*int(x**m*asec(c*x)*x**6,x
)*b*e**3*m**2 + 176*int(x**m*asec(c*x)*x**6,x)*b*e**3*m + 105*int(x**m*ase
c(c*x)*x**6,x)*b*e**3 + 3*int(x**m*asec(c*x)*x**4,x)*b*d*e**2*m**4 + 48*in
t(x**m*asec(c*x)*x**4,x)*b*d*e**2*m**3 + 258*int(x**m*asec(c*x)*x**4,x)*b*
d*e**2*m**2 + 528*int(x**m*asec(c*x)*x**4,x)*b*d*e**2*m + 315*int(x**m*ase
c(c*x)*x**4,x)*b*d*e**2 + 3*int(x**m*asec(c*x)*x**2,x)*b*d**2*e*m**4 + 48*
int(x**m*asec(c*x)*x**2,x)*b*d**2*e*m**3 + 258*int(x**m*asec(c*x)*x**2,x)*
b*d**2*e*m**2 + 528*int(x**m*asec(c*x)*x**2,x)*b*d**2*e*m + 315*int(x**m*a
sec(c*x)*x**2,x)*b*d**2*e + int(x**m*asec(c*x),x)*b*d**3*m**4 + 16*int(x**
m*asec(c*x),x)*b*d**3*m**3 + 86*int(x**m*asec(c*x),x)*b*d**3*m**2 + 176*in
t(x**m*asec(c*x),x)*b*d**3*m + 105*int(x**m*asec(c*x),x)*b*d**3))/(m**4 +
16*m**3 + 86*m**2 + 176*m + 105)
```

3.162 $\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx$

Optimal result 1323
Mathematica [A] (verified) 1324
Rubi [A] (verified) 1324
Maple [F] 1328
Fricas [F] 1328
Sympy [F] 1329
Maxima [F] 1329
Giac [F] 1330
Mupad [F(-1)] 1330
Reduce [F] 1330

Optimal result

Integrand size = 23, antiderivative size = 374

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx$$
$$= -\frac{be(e(3+m)^2+2c^2d(20+9m+m^2))\,x(fx)^{1+m}\sqrt{-1+c^2x^2}}{c^3f(2+m)(3+m)(4+m)(5+m)\sqrt{c^2x^2}}$$
$$-\frac{be^2x(fx)^{3+m}\sqrt{-1+c^2x^2}}{cf^3(4+m)(5+m)\sqrt{c^2x^2}}+\frac{d^2(fx)^{1+m}\left(a+b\sec^{-1}(cx)\right)}{f(1+m)}$$
$$+\frac{2de(fx)^{3+m}\left(a+b\sec^{-1}(cx)\right)}{f^3(3+m)}+\frac{e^2(fx)^{5+m}\left(a+b\sec^{-1}(cx)\right)}{f^5(5+m)}$$
$$-\frac{b(c^4d^2(2+m)(3+m)(4+m)(5+m)+e(1+m)^2\left(e(3+m)^2+2c^2d(20+9m+m^2)\right))\,x(fx)^{1+m}\sqrt{1}}{c^3f(1+m)^2(2+m)(3+m)(4+m)(5+m)\sqrt{c^2x^2}\sqrt{-1+}}$$

output
```
-b*e*(e*(3+m)^2+2*c^2*d*(m^2+9*m+20))*x*(f*x)^(1+m)*(c^2*x^2-1)^(1/2)/c^3/
f/(2+m)/(3+m)/(4+m)/(5+m)/(c^2*x^2)^(1/2)-b*e^2*x*(f*x)^(3+m)*(c^2*x^2-1)^
(1/2)/c/f^3/(4+m)/(5+m)/(c^2*x^2)^(1/2)+d^2*(f*x)^(1+m)*(a+b*arcsec(c*x))/
f/(1+m)+2*d*e*(f*x)^(3+m)*(a+b*arcsec(c*x))/f^3/(3+m)+e^2*(f*x)^(5+m)*(a+b
*arcsec(c*x))/f^5/(5+m)-b*(c^4*d^2*(2+m)*(3+m)*(4+m)*(5+m)+e*(1+m)^2*(e*(3
+m)^2+2*c^2*d*(m^2+9*m+20)))*x*(f*x)^(1+m)*(-c^2*x^2+1)^(1/2)*hypergeom([1
/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)/c^3/f/(1+m)^2/(2+m)/(3+m)/(4+m)/(5+m)/
(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.78

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx$$
$$= x(fx)^m \left(\frac{ad^2}{1+m}+\frac{2adex^2}{3+m}+\frac{ae^2x^4}{5+m}+\frac{bd^2\sec^{-1}(cx)}{1+m}+\frac{2bdex^2\sec^{-1}(cx)}{3+m}+\frac{be^2x^4\sec^{-1}(cx)}{5+m}+\frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}\,x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{(1+m)^2\sqrt{1-c^2x^2}}+\frac{2bcde\sqrt{1-\frac{1}{c^2x^2}}\,x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{(3+m)^2\sqrt{1-c^2x^2}}+\frac{bce^2\sqrt{1-\frac{1}{c^2x^2}}\,x^5\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5+m}{2},\frac{7+m}{2},c^2x^2\right)}{(5+m)^2\sqrt{1-c^2x^2}}\right)$$

input
```
Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

output
```
x*(f*x)^m*((a*d^2)/(1 + m) + (2*a*d*e*x^2)/(3 + m) + (a*e^2*x^4)/(5 + m) +
 (b*d^2*ArcSec[c*x])/(1 + m) + (2*b*d*e*x^2*ArcSec[c*x])/(3 + m) + (b*e^2*
x^4*ArcSec[c*x])/(5 + m) + (b*c*d^2*Sqrt[1 - 1/(c^2*x^2)]*x*Hypergeometric
2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/((1 + m)^2*Sqrt[1 - c^2*x^2]) + (
2*b*c*d*e*Sqrt[1 - 1/(c^2*x^2)]*x^3*Hypergeometric2F1[1/2, (3 + m)/2, (5 +
 m)/2, c^2*x^2])/((3 + m)^2*Sqrt[1 - c^2*x^2]) + (b*c*e^2*Sqrt[1 - 1/(c^2*
x^2)]*x^5*Hypergeometric2F1[1/2, (5 + m)/2, (7 + m)/2, c^2*x^2])/((5 + m)^
2*Sqrt[1 - c^2*x^2]))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5761, 27, 1590, 363, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^2 (fx)^m \left(a+b\sec^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \; 5761$$

$$-\frac{bcx\int \frac{(fx)^m\left(e^2(m+1)(m+3)x^4+2de(m+1)(m+5)x^2+d^2(m+3)(m+5)\right)}{\left(m^3+9m^2+23m+15\right)\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{d^2(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\sec^{-1}(cx)\right)}{f^5(m+5)}$$

$$\Big\downarrow \; 27$$

$$-\frac{bcx\int \frac{(fx)^m\left(e^2(m+1)(m+3)x^4+2de(m+1)(m+5)x^2+d^2(m+3)(m+5)\right)}{\sqrt{c^2x^2-1}}dx}{\left(m^3+9m^2+23m+15\right)\sqrt{c^2x^2}}+\frac{d^2(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\sec^{-1}(cx)\right)}{f^5(m+5)}$$

$$\Big\downarrow \; 1590$$

$$-\frac{bcx\left(\frac{\int \frac{(fx)^m\left(c^2(m+3)(m+4)(m+5)d^2+e(m+1)\left(2d\left(m^2+9m+20\right)c^2+e(m+3)^2\right)x^2\right)}{\sqrt{c^2x^2-1}}dx}{c^2(m+4)}+\frac{e^2(m+1)(m+3)\sqrt{c^2x^2-1}(fx)^{m+3}}{c^2f^3(m+4)}\right)}{\left(m^3+9m^2+23m+15\right)\sqrt{c^2x^2}}+\frac{d^2(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\sec^{-1}(cx)\right)}{f^5(m+5)}$$

$$\Big\downarrow \; 363$$

$$-\frac{bcx\left(\frac{\frac{\left(c^4d^2(m+3)(m+4)(m+5)+\frac{e(m+1)^2\left(2c^2d\left(m^2+9m+20\right)+e(m+3)^2\right)}{m+2}\right)\int \frac{(fx)^m}{\sqrt{c^2x^2-1}}dx}{c^2}+\frac{e(m+1)\sqrt{c^2x^2-1}(fx)^{m+1}\left(2c^2d\left(m^2+9m+20\right)+e(m+3)^2\right)}{c^2f(m+2)}}{c^2(m+4)}\right)}{\left(m^3+9m^2+23m+15\right)\sqrt{c^2x^2}}+\frac{d^2(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\sec^{-1}(cx)\right)}{f^5(m+5)}$$

$$\Big\downarrow \; 279$$

$$-\frac{bcx\left(\frac{\frac{\sqrt{1-c^2x^2}\left(c^4d^2(m+3)(m+4)(m+5)+\frac{e(m+1)^2\left(2c^2d\left(m^2+9m+20\right)+e(m+3)^2\right)}{m+2}\right)\int\frac{(fx)^m}{\sqrt{1-c^2x^2}}dx}{c^2\sqrt{c^2x^2-1}}+\frac{e(m+1)\sqrt{c^2x^2-1}(fx)^{m+1}\left(2c^2d\left(m^2+9m+20\right)+\right.}{c^2f(m+2)}}{c^2(m+4)}}{(m^3+9m^2+23m+15)\sqrt{c^2x^2}}$$

$$\frac{d^2(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\sec^{-1}(cx)\right)}{f^5(m+5)}$$

$\downarrow$ 278

$$\frac{d^2(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\sec^{-1}(cx)\right)}{f^5(m+5)}-$$

$$\frac{bcx\left(\frac{e^2(m+1)(m+3)\sqrt{c^2x^2-1}(fx)^{m+3}}{c^2f^3(m+4)}+\frac{\frac{e(m+1)\sqrt{c^2x^2-1}(fx)^{m+1}\left(2c^2d\left(m^2+9m+20\right)+e(m+3)^2\right)}{c^2f(m+2)}+\frac{\sqrt{1-c^2x^2}(fx)^{m+1}\left(c^4d^2(m+3)(m+4)(m+5)\right.}{}}{c^2(m+4)}\right.}{(m^3+9m^2+23m+15)\sqrt{c^2x^2}}$$

input
```
Int[(f*x)^m*(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

output
```
(d^2*(f*x)^(1 + m)*(a + b*ArcSec[c*x]))/(f*(1 + m)) + (2*d*e*(f*x)^(3 + m)
*(a + b*ArcSec[c*x]))/(f^3*(3 + m)) + (e^2*(f*x)^(5 + m)*(a + b*ArcSec[c*x
]))/(f^5*(5 + m)) - (b*c*x*((e^2*(1 + m)*(3 + m)*(f*x)^(3 + m)*Sqrt[-1 + c
^2*x^2])/(c^2*f^3*(4 + m)) + ((e*(1 + m)*(e*(3 + m)^2 + 2*c^2*d*(20 + 9*m
+ m^2))*(f*x)^(1 + m)*Sqrt[-1 + c^2*x^2])/(c^2*f*(2 + m)) + ((c^4*d^2*(3 +
 m)*(4 + m)*(5 + m) + (e*(1 + m)^2*(e*(3 + m)^2 + 2*c^2*d*(20 + 9*m + m^2)
))/(2 + m))*(f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)
/2, (3 + m)/2, c^2*x^2])/(c^2*f*(1 + m)*Sqrt[-1 + c^2*x^2]))/(c^2*(4 + m))
))/((15 + 23*m + 9*m^2 + m^3)*Sqrt[c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right)^2 \left(a+b \operatorname{arcsec}\left(cx\right)\right) dx$$

input
```
int((f*x)^m*(e*x^2+d)^2*(a+b*arcsec(c*x)),x)
```

output
```
int((f*x)^m*(e*x^2+d)^2*(a+b*arcsec(c*x)),x)
```

Fricas [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\, dx = \int \left(ex^2+d\right)^2 (b \operatorname{arcsec}\left(cx\right)+a)(fx)^m\, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arcsec(c*x))*(f*x)^m, x)
```

Sympy [F]

$$\int (fx)^m \left(d + ex^2\right)^2 \left(a + b\sec^{-1}(cx)\right) \, dx = \int (fx)^m \left(a + b\operatorname{asec}(cx)\right) \left(d + ex^2\right)^2 \, dx$$

input
```
integrate((f*x)**m*(e*x**2+d)**2*(a+b*asec(c*x)),x)
```

output
```
Integral((f*x)**m*(a + b*asec(c*x))*(d + e*x**2)**2, x)
```

Maxima [F]

$$\int (fx)^m \left(d + ex^2\right)^2 \left(a + b\sec^{-1}(cx)\right) \, dx = \int \left(ex^2 + d\right)^2 (b\operatorname{arcsec}(cx) + a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
a*e^2*f^m*x^5*x^m/(m + 5) + 2*a*d*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*
d^2/(f*(m + 1)) + (((b*e^2*f^m*m^2 + 4*b*e^2*f^m*m + 3*b*e^2*f^m)*x^5 + 2*
(b*d*e*f^m*m^2 + 6*b*d*e*f^m*m + 5*b*d*e*f^m)*x^3 + (b*d^2*f^m*m^2 + 8*b*d
^2*f^m*m + 15*b*d^2*f^m)*x)*x^m*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - (m^3
 + 9*m^2 + 23*m + 15)*integrate(-(b*d^2*f^m*m^2 + 8*b*d^2*f^m*m + (b*e^2*f
^m*m^2 + 4*b*e^2*f^m*m + 3*b*e^2*f^m)*x^4 + 15*b*d^2*f^m + 2*(b*d*e*f^m*m^
2 + 6*b*d*e*f^m*m + 5*b*d*e*f^m)*x^2)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m/(m^3
 - (c^2*m^3 + 9*c^2*m^2 + 23*c^2*m + 15*c^2)*x^2 + 9*m^2 + 23*m + 15), x))
/(m^3 + 9*m^2 + 23*m + 15)
```

Giac [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx=\int \left(ex^2+d\right)^2(b\,\mathrm{arcsec}\,(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arcsec(c*x) + a)*(f*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx=\int (f\,x)^m\,\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int((f*x)^m*(d + e*x^2)^2*(a + b*acos(1/(c*x))),x)
```

output
```
int((f*x)^m*(d + e*x^2)^2*(a + b*acos(1/(c*x))), x)
```

Reduce [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{f^m\left(x^m a\,d^2m^2x+8x^m a\,d^2mx+15x^m a\,d^2x+2x^m ade\,m^2x^3+12x^m adem\,x^3+10x^m ade\,x^3+x^m a\,e^2m^2x\cdots\right.}{\cdots}$$

input
```
int((f*x)^m*(e*x^2+d)^2*(a+b*asec(c*x)),x)
```

output
```
(f**m*(x**m*a*d**2*m**2*x + 8*x**m*a*d**2*m*x + 15*x**m*a*d**2*x + 2*x**m*
a*d*e*m**2*x**3 + 12*x**m*a*d*e*m*x**3 + 10*x**m*a*d*e*x**3 + x**m*a*e**2*
m**2*x**5 + 4*x**m*a*e**2*m*x**5 + 3*x**m*a*e**2*x**5 + int(x**m*asec(c*x)
*x**4,x)*b*e**2*m**3 + 9*int(x**m*asec(c*x)*x**4,x)*b*e**2*m**2 + 23*int(x
**m*asec(c*x)*x**4,x)*b*e**2*m + 15*int(x**m*asec(c*x)*x**4,x)*b*e**2 + 2*
int(x**m*asec(c*x)*x**2,x)*b*d*e*m**3 + 18*int(x**m*asec(c*x)*x**2,x)*b*d*
e*m**2 + 46*int(x**m*asec(c*x)*x**2,x)*b*d*e*m + 30*int(x**m*asec(c*x)*x**
2,x)*b*d*e + int(x**m*asec(c*x),x)*b*d**2*m**3 + 9*int(x**m*asec(c*x),x)*b
*d**2*m**2 + 23*int(x**m*asec(c*x),x)*b*d**2*m + 15*int(x**m*asec(c*x),x)*
b*d**2))/(m**3 + 9*m**2 + 23*m + 15)
```

3.163 $\int (fx)^m \left(d+ex^2\right) \left(a+b\sec^{-1}(cx)\right) \, dx$

Optimal result 1332
Mathematica [A] (verified) 1333
Rubi [A] (verified) 1333
Maple [F] 1336
Fricas [F] 1336
Sympy [F] 1336
Maxima [F] 1337
Giac [F] 1337
Mupad [F(-1)] 1337
Reduce [F] 1338

Optimal result

Integrand size = 21, antiderivative size = 178

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\sec^{-1}(cx)\right) \, dx = -\frac{bex^{2+m}\sqrt{-1+c^2x^2}}{c\left(6+5m+m^2\right)\sqrt{c^2x^2}}+\frac{dx^{1+m}\left(a+b\sec^{-1}(cx)\right)}{1+m}+\frac{ex^{3+m}\left(a+b\sec^{-1}(cx)\right)}{3+m}+\frac{b(e(1+m)^2+c^2d(2+m)(3+m))\,x^{2+m}\sqrt{-1+c^2x^2}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{3+m}{2},c^2x^2\right)}{c(1+m)^2(2+m)(3+m)\sqrt{c^2x^2}}$$

output

```
-b*e*x^(2+m)*(c^2*x^2-1)^(1/2)/c/(m^2+5*m+6)/(c^2*x^2)^(1/2)+d*x^(1+m)*(a+
b*arcsec(c*x))/(1+m)+e*x^(3+m)*(a+b*arcsec(c*x))/(3+m)+b*(e*(1+m)^2+c^2*d*
(2+m)*(3+m))*x^(2+m)*(c^2*x^2-1)^(1/2)*hypergeom([1, 1+1/2*m],[3/2+1/2*m],
c^2*x^2)/c/(1+m)^2/(2+m)/(3+m)/(c^2*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.95

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\sec^{-1}(cx)\right) \, dx = x(fx)^m \left(\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}x \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{(1+m)^2\sqrt{1-c^2x^2}} + \frac{\frac{(3+m)\left(d(3+m)+e(1+m)x^2\right)\left(a+b\sec^{-1}(cx)\right)}{1+m} + \frac{bce\sqrt{1-\frac{1}{c^2x^2}}x^3 \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{\sqrt{1-c^2x^2}}}{(3+m)^2} \right)$$

input
```
Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

output
```
x*(f*x)^m*((b*c*d*Sqrt[1 - 1/(c^2*x^2)]*x*Hypergeometric2F1[1/2, (1 + m)/2
, (3 + m)/2, c^2*x^2])/((1 + m)^2*Sqrt[1 - c^2*x^2]) + (((3 + m)*(d*(3 + m
) + e*(1 + m)*x^2)*(a + b*ArcSec[c*x]))/(1 + m) + (b*c*e*Sqrt[1 - 1/(c^2*x
^2)]*x^3*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, c^2*x^2])/Sqrt[1 - c
^2*x^2])/(3 + m)^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5761, 27, 363, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right) (fx)^m \left(a+b\sec^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{5761}$$

$$-\frac{bcx\int \frac{(fx)^m\left(e(m+1)x^2+d(m+3)\right)}{\left(m^2+4m+3\right)\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}} + \frac{d(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)} + \frac{e(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bcx\int \frac{(fx)^m\left(e(m+1)x^2+d(m+3)\right)}{\sqrt{c^2x^2-1}}dx}{\left(m^2+4m+3\right)\sqrt{c^2x^2}}+\frac{d(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}$$

$$\Big\downarrow \text{363}$$

$$-\frac{bcx\left(\left(\frac{e(m+1)^2}{c^2(m+2)}+d(m+3)\right)\int \frac{(fx)^m}{\sqrt{c^2x^2-1}}dx+\frac{e(m+1)\sqrt{c^2x^2-1}(fx)^{m+1}}{c^2f(m+2)}\right)}{\left(m^2+4m+3\right)\sqrt{c^2x^2}}+\frac{d(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}$$

$$\Big\downarrow \text{279}$$

$$-\frac{bcx\left(\frac{\sqrt{1-c^2x^2}\left(\frac{e(m+1)^2}{c^2(m+2)}+d(m+3)\right)\int \frac{(fx)^m}{\sqrt{1-c^2x^2}}dx}{\sqrt{c^2x^2-1}}+\frac{e(m+1)\sqrt{c^2x^2-1}(fx)^{m+1}}{c^2f(m+2)}\right)}{\left(m^2+4m+3\right)\sqrt{c^2x^2}}+\frac{d(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}$$

$$\Big\downarrow \text{278}$$

$$\frac{d(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}-\frac{bcx\left(\frac{\sqrt{1-c^2x^2}(fx)^{m+1}\left(\frac{e(m+1)^2}{c^2(m+2)}+d(m+3)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)}{f(m+1)\sqrt{c^2x^2-1}}+\frac{e(m+1)\sqrt{c^2x^2-1}(fx)^{m+1}}{c^2f(m+2)}\right)}{\left(m^2+4m+3\right)\sqrt{c^2x^2}}$$

input

```
Int[(f*x)^m*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

output

```
(d*(f*x)^(1 + m)*(a + b*ArcSec[c*x]))/(f*(1 + m)) + (e*(f*x)^(3 + m)*(a +
b*ArcSec[c*x]))/(f^3*(3 + m)) - (b*c*x*((e*(1 + m)*(f*x)^(1 + m)*Sqrt[-1 +
 c^2*x^2])/(c^2*f*(2 + m)) + (((e*(1 + m)^2)/(c^2*(2 + m)) + d*(3 + m))*(f
*x)^(1 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2,
 c^2*x^2])/(f*(1 + m)*Sqrt[-1 + c^2*x^2])))/((3 + 4*m + m^2)*Sqrt[c^2*x^2]
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right) \left(a+b\ \operatorname{arcsec}\left(cx\right)\right) dx$$

input
```
int((f*x)^m*(e*x^2+d)*(a+b*arcsec(c*x)),x)
```

output
```
int((f*x)^m*(e*x^2+d)*(a+b*arcsec(c*x)),x)
```

Fricas [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\sec^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)\left(b\operatorname{arcsec}\left(cx\right)+a\right)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsec(c*x))*(f*x)^m, x)
```

Sympy [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\sec^{-1}(cx)\right)\,dx = \int (fx)^m \left(a+b\operatorname{asec}\left(cx\right)\right) \left(d+ex^2\right)\,dx$$

input
```
integrate((f*x)**m*(e*x**2+d)*(a+b*asec(c*x)),x)
```

output
```
Integral((f*x)**m*(a + b*asec(c*x))*(d + e*x**2), x)
```

Maxima [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\sec^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)(b\,\mathrm{arcsec}\,(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
a*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d/(f*(m + 1)) + (((b*e*f^m*m + b
*e*f^m)*x^3 + (b*d*f^m*m + 3*b*d*f^m)*x)*x^m*arctan(sqrt(c*x + 1)*sqrt(c*x
 - 1)) - (m^2 + 4*m + 3)*integrate((b*d*f^m*m + 3*b*d*f^m + (b*e*f^m*m + b
*e*f^m)*x^2)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m/((c^2*m^2 + 4*c^2*m + 3*c^2)*
x^2 - m^2 - 4*m - 3), x))/(m^2 + 4*m + 3)
```

Giac [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\sec^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)(b\,\mathrm{arcsec}\,(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arcsec(c*x) + a)*(f*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\sec^{-1}(cx)\right) \, dx = \int (f\,x)^m\,\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right) \,dx$$

input
```
int((f*x)^m*(d + e*x^2)*(a + b*acos(1/(c*x))),x)
```

output
```
int((f*x)^m*(d + e*x^2)*(a + b*acos(1/(c*x))), x)
```

Reduce [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\sec^{-1}(cx)\right) \, dx$$
$$= \frac{f^m \left(x^m a d m x + 3x^m a d x + x^m a e m\, x^3 + x^m a e\, x^3 + \left(\int x^m asec(cx)\, x^2 dx\right) be\, m^2 + 4\left(\int x^m asec(cx)\, x^2 dx\right) b\right.}{m}$$

input
```
int((f*x)^m*(e*x^2+d)*(a+b*asec(c*x)),x)
```

output
```
(f**m*(x**m*a*d*m*x + 3*x**m*a*d*x + x**m*a*e*m*x**3 + x**m*a*e*x**3 + int
(x**m*asec(c*x)*x**2,x)*b*e*m**2 + 4*int(x**m*asec(c*x)*x**2,x)*b*e*m + 3*
int(x**m*asec(c*x)*x**2,x)*b*e + int(x**m*asec(c*x),x)*b*d*m**2 + 4*int(x*
*m*asec(c*x),x)*b*d*m + 3*int(x**m*asec(c*x),x)*b*d))/(m**2 + 4*m + 3)
```

3.164 $\int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal result 1339
Mathematica [N/A] 1339
Rubi [N/A] 1340
Maple [N/A] 1340
Fricas [N/A] 1341
Sympy [N/A] 1341
Maxima [N/A] 1341
Giac [N/A] 1342
Mupad [N/A] 1342
Reduce [N/A] 1343

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{d+ex^2}\,dx=\text{Int}\left(\frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{d+ex^2},x\right)$$

output
```
Defer(Int)((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d),x)
```

Mathematica [N/A]

Not integrable

Time = 2.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{d+ex^2}\,dx=\int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{d+ex^2}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2),x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m \left(a+b\sec^{-1}(cx)\right)}{d+ex^2} \, dx$$
$$\Big\downarrow \text{ 5771}$$
$$\int \frac{(fx)^m \left(a+b\sec^{-1}(cx)\right)}{d+ex^2}dx$$

input
```
Int[((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 2.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a+b\,\mathrm{arcsec}\left(cx\right)\right)}{e\,x^2+d}dx$$

input
```
int((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d),x)
```

output
```
int((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a+b\sec^{-1}(cx)\right)}{d+ex^2}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)(fx)^m}{ex^2+d}\,dx$$

input
```
integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 44.95 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(fx)^m \left(a+b\sec^{-1}(cx)\right)}{d+ex^2}\,dx = \int \frac{(fx)^m\,(a+b\,\mathrm{asec}\,(cx))}{d+ex^2}\,dx$$

input
```
integrate((f*x)**m*(a+b*asec(c*x))/(e*x**2+d),x)
```

output
```
Integral((f*x)**m*(a + b*asec(c*x))/(d + e*x**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a+b\sec^{-1}(cx)\right)}{d+ex^2}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)(fx)^m}{ex^2+d}\,dx$$

input
```
integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
integrate((b*arcsec(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \int \frac{(b\,\text{arcsec}\,(cx)+a)(fx)^m}{ex^2+d}\,dx$$

input
```
integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{(fx)^m (a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \int \frac{(f\,x)^m\,\left(a+b\,\text{acos}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2+d}\,dx$$

input
```
int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2),x)
```

output
```
int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int \frac{(fx)^m (a + b\sec^{-1}(cx))}{d + ex^2} \, dx = f^m \left(\left(\int \frac{x^m}{e\,x^2 + d} dx \right) a + \left(\int \frac{x^m\,asec(cx)}{e\,x^2 + d} dx \right) b \right)$$

input
```
int((f*x)^m*(a+b*asec(c*x))/(e*x^2+d),x)
```

output
```
f**m*(int(x**m/(d + e*x**2),x)*a + int((x**m*asec(c*x))/(d + e*x**2),x)*b)
```

3.165 $\int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{(d+ex^2)^2}\,dx$

Optimal result . . . 1344
Mathematica [N/A] . . . 1344
Rubi [N/A] . . . 1345
Maple [N/A] . . . 1345
Fricas [N/A] . . . 1346
Sympy [F(-1)] . . . 1346
Maxima [N/A] . . . 1346
Giac [N/A] . . . 1347
Mupad [N/A] . . . 1347
Reduce [N/A] . . . 1347

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{(d+ex^2)^2}\,dx = \text{Int}\left(\frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{(d+ex^2)^2},x\right)$$

output
```
Defer(Int)((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 3.74 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{(d+ex^2)^2}\,dx = \int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{(d+ex^2)^2}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m (a + b\sec^{-1}(cx))}{(d + ex^2)^2} dx$$

$$\Big\downarrow \text{5771}$$

$$\int \frac{(fx)^m (a + b\sec^{-1}(cx))}{(d + ex^2)^2} dx$$

input
```
Int[((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.70 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a + b \operatorname{arcsec}(cx))}{(e\,x^2 + d)^2} dx$$

input
```
int((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^2,x)
```

output
```
int((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(fx)^m (a + b\sec^{-1}(cx))}{(d + ex^2)^2} dx = \int \frac{(b\,\mathrm{arcsec}\,(cx) + a)(fx)^m}{(ex^2 + d)^2} dx$$

input
```
integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m (a + b\sec^{-1}(cx))}{(d + ex^2)^2} dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(a+b*asec(c*x))/(e*x**2+d)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a + b\sec^{-1}(cx))}{(d + ex^2)^2} dx = \int \frac{(b\,\mathrm{arcsec}\,(cx) + a)(fx)^m}{(ex^2 + d)^2} dx$$

input
```
integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
integrate((b*arcsec(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a + b\sec^{-1}(cx))}{(d + ex^2)^2}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx) + a)(fx)^m}{(ex^2 + d)^2}\,dx$$

input
```
integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{(fx)^m (a + b\sec^{-1}(cx))}{(d + ex^2)^2}\,dx = \int \frac{(f\,x)^m\,\left(a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2 + d)^2}\,dx$$

input
```
int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.83

$$\int \frac{(fx)^m (a + b\sec^{-1}(cx))}{(d + ex^2)^2}\,dx = f^m\left(\left(\int \frac{x^m}{e^2x^4 + 2de\,x^2 + d^2}dx\right)a + \left(\int \frac{x^m\,asec(cx)}{e^2x^4 + 2de\,x^2 + d^2}dx\right)b\right)$$

input
```
int((f*x)^m*(a+b*asec(c*x))/(e*x^2+d)^2,x)
```

output
```
f**m*(int(x**m/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*a + int((x**m*asec(c*x))
/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*b)
```

3.166 $\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx$

Optimal result 1349
Mathematica [N/A] 1349
Rubi [N/A] 1350
Maple [N/A] 1350
Fricas [N/A] 1351
Sympy [F(-1)] 1351
Maxima [N/A] 1351
Giac [N/A] 1352
Mupad [N/A] 1352
Reduce [N/A] 1352

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx=\text{Int}\left((fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right),x\right)$$

output
```
Defer(Int)((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx=\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx$$

input
```
Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
```

output
```
Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^{3/2} (fx)^m \left(a+b\sec^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{5771}$$

$$\int \left(d+ex^2\right)^{3/2} (fx)^m \left(a+b\sec^{-1}(cx)\right) dx$$

input
```
Int[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (fx)^m \left(e\,x^2+d\right)^{\frac{3}{2}} \left(a+b\ \mathrm{arcsec}\left(cx\right)\right) dx$$

input
```
int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

output
```
int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^{\frac{3}{2}} (b\,\mathrm{arcsec}\,(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsec(c*x))*sqrt(e*x^2 + d)*(f*
x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(e*x**2+d)**(3/2)*(a+b*asec(c*x)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^{\frac{3}{2}} (b\,\mathrm{arcsec}\,(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)*(f*x)^m, x)
```

Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\,dx=\int \left(ex^2+d\right)^{\frac{3}{2}}(b\operatorname{arcsec}(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)*(f*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\,dx=\int (f\,x)^m\,\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int((f*x)^m*(d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))),x)
```

output
```
int((f*x)^m*(d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.48

$$\int (fx)^m (d+ex^2)^{3/2} (a+b\sec^{-1}(cx))\,dx = f^m\left(\left(\left(\int x^m\sqrt{e\,x^2+d}\,asec(cx)\,x^2dx\right)be+\left(\int x^m\sqrt{e\,x^2+d}\,asec(cx)\,dx\right)bd+\left(\int x^m\sqrt{e\,x^2+d}\,x^2dx\right)ae+\left(\int x^m\sqrt{e\,x^2+d}dx\right)ad\right)$$

input
```
int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*asec(c*x)),x)
```

output
```
f**m*(int(x**m*sqrt(d + e*x**2)*asec(c*x)*x**2,x)*b*e + int(x**m*sqrt(d +
e*x**2)*asec(c*x),x)*b*d + int(x**m*sqrt(d + e*x**2)*x**2,x)*a*e + int(x**
m*sqrt(d + e*x**2),x)*a*d)
```

3.167 $\int (fx)^m \sqrt{d+ex^2}(a+b\sec^{-1}(cx))\ dx$

Optimal result 1354
Mathematica [N/A] 1354
Rubi [N/A] 1355
Maple [N/A] 1355
Fricas [N/A] 1356
Sympy [N/A] 1356
Maxima [N/A] 1356
Giac [N/A] 1357
Mupad [N/A] 1357
Reduce [N/A] 1358

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (fx)^m \sqrt{d+ex^2}(a+b\sec^{-1}(cx))\ dx = \text{Int}\Big((fx)^m \sqrt{d+ex^2}(a+b\sec^{-1}(cx)),x\Big)$$

output
```
Defer(Int)((f*x)^m*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (fx)^m \sqrt{d+ex^2}(a+b\sec^{-1}(cx))\ dx = \int (fx)^m \sqrt{d+ex^2}(a+b\sec^{-1}(cx))\ dx$$

input
```
Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

output
```
Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex^2}(fx)^m \left(a+b\sec^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{ 5771}$$

$$\int \sqrt{d+ex^2}(fx)^m \left(a+b\sec^{-1}(cx)\right)dx$$

input
```
Int[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \left(f x \right)^{m} \sqrt{e \,x^{2}+d}\, \left(a +b \,\operatorname{arcsec}\left(c x \right)\right)dx$$

input
```
int((f*x)^m*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x)
```

output
```
int((f*x)^m*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\mathrm{arcsec}\,(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)*(f*x)^m, x)
```

Sympy [N/A]

Not integrable

Time = 57.85 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int (fx)^m\,(a+b\,\mathrm{asec}\,(cx))\,\sqrt{d+ex^2}\,dx$$

input
```
integrate((f*x)**m*(e*x**2+d)**(1/2)*(a+b*asec(c*x)),x)
```

output
```
Integral((f*x)**m*(a + b*asec(c*x))*sqrt(d + e*x**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\mathrm{arcsec}\,(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)*(f*x)^m, x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\mathrm{arcsec}\,(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)*(f*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int (f\,x)^m\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int((f*x)^m*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))),x)
```

output
```
int((f*x)^m*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

$$\int (fx)^m \sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = f^m \left(\left(\int x^m \sqrt{e\,x^2+d}\,asec(cx)\,dx\right) b + \left(\int x^m \sqrt{e\,x^2+d}dx\right) a\right)$$

input
```
int((f*x)^m*(e*x^2+d)^(1/2)*(a+b*asec(c*x)),x)
```

output
```
f**m*(int(x**m*sqrt(d + e*x**2)*asec(c*x),x)*b + int(x**m*sqrt(d + e*x**2)
,x)*a)
```

3.168 $\int \frac{(fx)^m \left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal result 1359
Mathematica [N/A] 1359
Rubi [N/A] 1360
Maple [N/A] 1360
Fricas [N/A] 1361
Sympy [N/A] 1361
Maxima [N/A] 1361
Giac [N/A] 1362
Mupad [N/A] 1362
Reduce [N/A] 1363

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(fx)^m \left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\text{Int}\left(\frac{(fx)^m \left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}},x\right)$$

output
```
Defer(Int)((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m \left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(fx)^m \left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2], x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m \left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}}dx$$
$$\Big\downarrow \text{5771}$$
$$\int \frac{(fx)^m \left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}}dx$$

input
```
Int[((f*x)^m*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(fx)^m \left(a+b\,\mathrm{arcsec}\,(cx)\right)}{\sqrt{e\,x^2+d}}dx$$

input
```
int((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\,\mathrm{arcsec}\,(cx)+a)(fx)^m}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 25.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(fx)^m\left(a+b\,\mathrm{asec}\,(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate((f*x)**m*(a+b*asec(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral((f*x)**m*(a + b*asec(c*x))/sqrt(d + e*x**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\,\mathrm{arcsec}\,(cx)+a)(fx)^m}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsec(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a + b\sec^{-1}(cx))}{\sqrt{d + ex^2}} \, dx = \int \frac{(b \operatorname{arcsec}(cx) + a)(fx)^m}{\sqrt{ex^2 + d}} \, dx$$

input
```
integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{(fx)^m (a + b\sec^{-1}(cx))}{\sqrt{d + ex^2}} \, dx = \int \frac{(f\,x)^m \left(a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2 + d}} \, dx$$

input
```
int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

output
```
int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.80

$$\int \frac{(fx)^m \left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=f^m\left(\left(\int \frac{x^m}{\sqrt{e\,x^2+d}}dx\right)a+\left(\int \frac{x^m\,asec(cx)}{\sqrt{e\,x^2+d}}dx\right)b\right)$$

input

```
int((f*x)^m*(a+b*asec(c*x))/(e*x^2+d)^(1/2),x)
```

output

```
f**m*(int(x**m/sqrt(d + e*x**2),x)*a + int((x**m*asec(c*x))/sqrt(d + e*x**
2),x)*b)
```

3.169 $\int \frac{(fx)^m(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$

Optimal result . . . 1364
Mathematica [N/A] . . . 1364
Rubi [N/A] . . . 1365
Maple [N/A] . . . 1365
Fricas [N/A] . . . 1366
Sympy [F(-1)] . . . 1366
Maxima [N/A] . . . 1366
Giac [N/A] . . . 1367
Mupad [N/A] . . . 1367
Reduce [N/A] . . . 1368

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(fx)^m (a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \operatorname{Int}\left(\frac{(fx)^m (a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}},x\right)$$

output
```
Defer(Int)((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m (a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(fx)^m (a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m (a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}} dx$$
$$\Big\downarrow \; 5771$$
$$\int \frac{(fx)^m (a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}} dx$$

input
```
Int[((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(fx)^m (a+b\,\mathrm{arcsec}\,(cx))}{(e\,x^2+d)^{\frac{3}{2}}} dx$$

input
```
int((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.80

$$\int \frac{(fx)^m (a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)(fx)^m}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2
+ d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m (a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(a+b*asec(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)(fx)^m}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsec(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a + b\sec^{-1}(cx))}{(d + ex^2)^{3/2}} dx = \int \frac{(b\,\mathrm{arcsec}\,(cx) + a)(fx)^m}{(ex^2 + d)^{\frac{3}{2}}} dx$$

input
```
integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{(fx)^m (a + b\sec^{-1}(cx))}{(d + ex^2)^{3/2}} dx = \int \frac{(f\,x)^m \left(a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2 + d)^{3/2}} dx$$

input
```
int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 77, normalized size of antiderivative = 3.08

$$\int \frac{(fx)^m (a + b\sec^{-1}(cx))}{(d + ex^2)^{3/2}} dx = f^m \left(\left(\int \frac{x^m}{\sqrt{e\,x^2 + d}\,d + \sqrt{e\,x^2 + d}\,e\,x^2} dx \right) a + \left(\int \frac{x^m\,asec(cx)}{\sqrt{e\,x^2 + d}\,d + \sqrt{e\,x^2 + d}\,e\,x^2} dx \right) b \right)$$

input

```
int((f*x)^m*(a+b*asec(c*x))/(e*x^2+d)^(3/2),x)
```

output

```
f**m*(int(x**m/(sqrt(d + e*x**2)*d + sqrt(d + e*x**2)*e*x**2),x)*a + int((
x**m*asec(c*x))/(sqrt(d + e*x**2)*d + sqrt(d + e*x**2)*e*x**2),x)*b)
```

3.170 $\int \frac{x^{11}(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$

Optimal result 1369
Mathematica [A] (verified) 1370
Rubi [A] (verified) 1370
Maple [F] 1373
Fricas [A] (verification not implemented) 1374
Sympy [F(-1)] 1374
Maxima [F] 1375
Giac [F(-2)] 1375
Mupad [F(-1)] 1376
Reduce [F] 1376

Optimal result

Integrand size = 26, antiderivative size = 401

$$\begin{aligned}\int \frac{x^{11}(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx &= \frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{7b\sqrt{1-c^2x^2}(1+c^2x^2)^{3/2}}{90c^{13}\sqrt{1-\frac{1}{c^2x^2}}x}\\ &\quad + \frac{13b\sqrt{1-c^2x^2}(1+c^2x^2)^{5/2}}{150c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{3b\sqrt{1-c^2x^2}(1+c^2x^2)^{7/2}}{70c^{13}\sqrt{1-\frac{1}{c^2x^2}}x}\\ &\quad + \frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{9/2}}{90c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^{12}}\\ &\quad + \frac{(1-c^4x^4)^{3/2}(a+b\sec^{-1}(cx))}{3c^{12}}\\ &\quad - \frac{(1-c^4x^4)^{5/2}(a+b\sec^{-1}(cx))}{10c^{12}}\\ &\quad - \frac{4b\sqrt{1-c^2x^2}\,\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}x}\end{aligned}$$

output
```
4/15*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/c^13/(1-1/c^2/x^2)^(1/2)/x-7/9
0*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(3/2)/c^13/(1-1/c^2/x^2)^(1/2)/x+13/150
*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(5/2)/c^13/(1-1/c^2/x^2)^(1/2)/x-3/70*b*
(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(7/2)/c^13/(1-1/c^2/x^2)^(1/2)/x+1/90*b*(-c
^2*x^2+1)^(1/2)*(c^2*x^2+1)^(9/2)/c^13/(1-1/c^2/x^2)^(1/2)/x-1/2*(-c^4*x^4
+1)^(1/2)*(a+b*arcsec(c*x))/c^12+1/3*(-c^4*x^4+1)^(3/2)*(a+b*arcsec(c*x))/
c^12-1/10*(-c^4*x^4+1)^(5/2)*(a+b*arcsec(c*x))/c^12-4/15*b*(-c^2*x^2+1)^(1
/2)*arctanh((c^2*x^2+1)^(1/2))/c^13/(1-1/c^2/x^2)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.48

$$\int \frac{x^{11}(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \frac{-105a\sqrt{1-c^4x^4}(8+4c^4x^4+3c^8x^8)+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{1-c^4x^4}(768+36c^2x^2+78c^4x^4+5c^6x^6+35c^8x^8)}{-1+c^2x^2}-105b\sqrt{1-c^4x^4}}{3150c^{12}}$$

input
```
Integrate[(x^11*(a + b*ArcSec[c*x]))/Sqrt[1 - c^4*x^4],x]
```

output
```
(-105*a*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^4 + 3*c^8*x^8) + (b*c*Sqrt[1 - 1/(c
^2*x^2)]*x*Sqrt[1 - c^4*x^4]*(768 + 36*c^2*x^2 + 78*c^4*x^4 + 5*c^6*x^6 +
35*c^8*x^8))/(-1 + c^2*x^2) - 105*b*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^4 + 3*c
^8*x^8)*ArcSec[c*x] + 840*b*ArcTan[(c*Sqrt[1 - 1/(c^2*x^2)]*x)/Sqrt[1 - c^
4*x^4]])/(3150*c^12)
```

Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.57, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {5769, 27, 7272, 1388, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}\left(a+b\sec^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}dx$$

$$\Big\downarrow \; 5769$$

$$-\frac{b\int -\frac{\sqrt{1-c^4x^4}\left(3c^8x^8+4c^4x^4+8\right)}{30c^{12}\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{c}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^{12}}$$

$$\Big\downarrow \; 27$$

$$\frac{b\int \frac{\sqrt{1-c^4x^4}\left(3c^8x^8+4c^4x^4+8\right)}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{30c^{13}}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^{12}}$$

$$\Big\downarrow \; 7272$$

$$\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{1-c^4x^4}\left(3c^8x^8+4c^4x^4+8\right)}{x\sqrt{1-c^2x^2}}dx}{30c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^{12}}$$

$$\Big\downarrow \; 1388$$

$$\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{c^2x^2+1}\left(3c^8x^8+4c^4x^4+8\right)}{x}dx}{30c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^{12}}$$

$$\Big\downarrow \; 2331$$

$$\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{c^2x^2+1}\left(3c^8x^8+4c^4x^4+8\right)}{x^2}dx^2}{60c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^{12}}$$

$$\Big\downarrow \; 2123$$

$$\frac{b\sqrt{1-c^2x^2}\int\left(3c^2\left(c^2x^2+1\right)^{7/2}-9c^2\left(c^2x^2+1\right)^{5/2}+13c^2\left(c^2x^2+1\right)^{3/2}-7c^2\sqrt{c^2x^2+1}+\frac{8\sqrt{c^2x^2+1}}{x^2}\right)dx^2}{60c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^{12}}$$

$\Big\downarrow$ 2009

$$-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^{12}}+\frac{b\sqrt{1-c^2x^2}\left(-16\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{3}\left(c^2x^2+1\right)^{9/2}-\frac{18}{7}\left(c^2x^2+1\right)^{7/2}+\frac{26}{5}\left(c^2x^2+1\right)^{5/2}-\frac{14}{3}\left(c^2x^2+1\right)^{3/2}\right.}{60c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}$$

input
```
Int[(x^11*(a + b*ArcSec[c*x]))/Sqrt[1 - c^4*x^4],x]
```

output
```
-1/2*(Sqrt[1 - c^4*x^4]*(a + b*ArcSec[c*x]))/c^12 + ((1 - c^4*x^4)^(3/2)*(
a + b*ArcSec[c*x]))/(3*c^12) - ((1 - c^4*x^4)^(5/2)*(a + b*ArcSec[c*x]))/(
10*c^12) + (b*Sqrt[1 - c^2*x^2]*(16*Sqrt[1 + c^2*x^2] - (14*(1 + c^2*x^2)^
(3/2))/3 + (26*(1 + c^2*x^2)^(5/2))/5 - (18*(1 + c^2*x^2)^(7/2))/7 + (2*(1
 + c^2*x^2)^(9/2))/3 - 16*ArcTanh[Sqrt[1 + c^2*x^2]]))/(60*c^13*Sqrt[1 - 1
/(c^2*x^2)]*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 5769
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Simp[(a + b*ArcSec[c*x])   v, x] - Simp[b/c   Int[SimplifyIntegran
d[v/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]]
/; FreeQ[{a, b, c}, x]
```

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Maple [F]

$$\int \frac{x^{11}(a + b\,\mathrm{arcsec}\,(cx))}{\sqrt{-x^4c^4 + 1}}dx$$

input
```
int(x^11*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x)
```

output
```
int(x^11*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.62

$$\int \frac{x^{11}(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$$
$$= \frac{(35\,bc^8x^8+5\,bc^6x^6+78\,bc^4x^4+36\,bc^2x^2+768\,b)\sqrt{-c^4x^4+1}\sqrt{c^2x^2-1}-840\,(bc^2x^2-b)\arctan\left(\frac{\sqrt{-c^4x^4}}{}\right.}{}$$

input
```
integrate(x^11*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3150*((35*b*c^8*x^8 + 5*b*c^6*x^6 + 78*b*c^4*x^4 + 36*b*c^2*x^2 + 768*b)
*sqrt(-c^4*x^4 + 1)*sqrt(c^2*x^2 - 1) - 840*(b*c^2*x^2 - b)*arctan(sqrt(-c
^4*x^4 + 1)*sqrt(c^2*x^2 - 1)/(c^4*x^4 - 1)) - 105*(3*a*c^10*x^10 - 3*a*c^
8*x^8 + 4*a*c^6*x^6 - 4*a*c^4*x^4 + 8*a*c^2*x^2 + (3*b*c^10*x^10 - 3*b*c^8
*x^8 + 4*b*c^6*x^6 - 4*b*c^4*x^4 + 8*b*c^2*x^2 - 8*b)*arcsec(c*x) - 8*a)*s
qrt(-c^4*x^4 + 1))/(c^14*x^2 - c^12)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \text{Timed out}$$

input
```
integrate(x**11*(a+b*asec(c*x))/(-c**4*x**4+1)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{11}(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^{11}}{\sqrt{-c^4x^4+1}}\,dx$$

input
```
integrate(x^11*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/30*a*(3*(-c^4*x^4 + 1)^(5/2)/c^12 - 10*(-c^4*x^4 + 1)^(3/2)/c^12 + 15*s
qrt(-c^4*x^4 + 1)/c^12) - 1/30*(30*c^12*integrate((30*sqrt(c*x + 1)*c^10*x
^11*log(c) + (3*c^8*x^9 + 4*c^6*x^7 + 3*(10*c^10*log(c) + c^10)*x^11 + 4*c
^4*x^5 + 8*c^2*x^3 + 8*x)*e^(3/2*log(c*x + 1) + log(c*x - 1)) + 30*(c^10*x
^11*e^(3/2*log(c*x + 1) + log(c*x - 1)) + sqrt(c*x + 1)*c^10*x^11)*log(x))
/((c^10*e^(2*log(c*x + 1) + log(c*x - 1) + 1/2*log(-c*x + 1)) + c^10*e^(lo
g(c*x + 1) + 1/2*log(-c*x + 1)))*sqrt(c^2*x^2 + 1)), x) + (3*c^8*x^8 + 4*c
^4*x^4 + 8)*sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan(sqrt(c*x
 + 1)*sqrt(c*x - 1)))*b/c^12
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^{11}(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^11*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11}(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx=\int \frac{x^{11}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

input
```
int((x^11*(a + b*acos(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
```

output
```
int((x^11*(a + b*acos(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{11}(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \frac{-3\sqrt{-c^4x^4+1}\,a\,c^8x^8-4\sqrt{-c^4x^4+1}\,a\,c^4x^4-8\sqrt{-c^4x^4+1}\,a-30\left(\int \frac{\sqrt{-c^4x^4+1}\,asec(cx)\,x^{11}}{c^4x^4-1}dx\right)b\,c^{12}}{30c^{12}}$$

input
```
int(x^11*(a+b*asec(c*x))/(-c^4*x^4+1)^(1/2),x)
```

output
```
( - 3*sqrt( - c**4*x**4 + 1)*a*c**8*x**8 - 4*sqrt( - c**4*x**4 + 1)*a*c**4
*x**4 - 8*sqrt( - c**4*x**4 + 1)*a - 30*int((sqrt( - c**4*x**4 + 1)*asec(c
*x)*x**11)/(c**4*x**4 - 1),x)*b*c**12)/(30*c**12)
```

3.171 $\int \frac{x^7(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$

Optimal result 1377
Mathematica [A] (verified) 1378
Rubi [A] (warning: unable to verify) 1378
Maple [F] 1381
Fricas [A] (verification not implemented) 1382
Sympy [F(-1)] 1382
Maxima [F] 1383
Giac [F(-2)] 1383
Mupad [F(-1)] 1384
Reduce [F] 1384

Optimal result

Integrand size = 26, antiderivative size = 268

$$\int \frac{x^7(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{3/2}}{18c^9\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{5/2}}{30c^9\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^8} + \frac{(1-c^4x^4)^{3/2}(a+b\sec^{-1}(cx))}{6c^8} - \frac{b\sqrt{1-c^2x^2}\,\mathrm{arctanh}\left(\sqrt{1+c^2x^2}\right)}{3c^9\sqrt{1-\frac{1}{c^2x^2}}x}$$

output
```
1/3*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/c^9/(1-1/c^2/x^2)^(1/2)/x-1/18*
b*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(3/2)/c^9/(1-1/c^2/x^2)^(1/2)/x+1/30*b*(-
c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(5/2)/c^9/(1-1/c^2/x^2)^(1/2)/x-1/2*(-c^4*x^4
+1)^(1/2)*(a+b*arcsec(c*x))/c^8+1/6*(-c^4*x^4+1)^(3/2)*(a+b*arcsec(c*x))/c
^8-1/3*b*(-c^2*x^2+1)^(1/2)*arctanh((c^2*x^2+1)^(1/2))/c^9/(1-1/c^2/x^2)^(
1/2)/x
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.59

$$\int \frac{x^7(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$$
$$= \frac{-15a\sqrt{1-c^4x^4}(2+c^4x^4)+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1-c^4x^4}(28+c^2x^2+3c^4x^4)}{-1+c^2x^2}-15b\sqrt{1-c^4x^4}(2+c^4x^4)\sec^{-1}(cx)+30b}{90c^8}$$

input
```
Integrate[(x^7*(a + b*ArcSec[c*x]))/Sqrt[1 - c^4*x^4],x]
```

output
```
(-15*a*Sqrt[1 - c^4*x^4]*(2 + c^4*x^4) + (b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt
[1 - c^4*x^4]*(28 + c^2*x^2 + 3*c^4*x^4))/(-1 + c^2*x^2) - 15*b*Sqrt[1 - c
^4*x^4]*(2 + c^4*x^4)*ArcSec[c*x] + 30*b*ArcTan[(c*Sqrt[1 - 1/(c^2*x^2)]*x
)/Sqrt[1 - c^4*x^4]])/(90*c^8)
```

Rubi [A] (warning: unable to verify)

Time = 1.20 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.58, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5769, 27, 7272, 1388, 1579, 517, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$$
$$\Big\downarrow \ 5769$$
$$-\frac{b\int -\frac{\sqrt{1-c^4x^4}(c^4x^4+2)}{6c^8\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{c}+\frac{(1-c^4x^4)^{3/2}(a+b\sec^{-1}(cx))}{6c^8}-\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^8}$$
$$\Big\downarrow \ 27$$
$$\frac{b\int \frac{\sqrt{1-c^4x^4}(c^4x^4+2)}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{6c^9}+\frac{(1-c^4x^4)^{3/2}(a+b\sec^{-1}(cx))}{6c^8}-\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^8}$$

$$\Big\downarrow \text{7272}$$

$$\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{1-c^4x^4}(c^4x^4+2)}{x\sqrt{1-c^2x^2}}dx}{6c^9x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^8}$$

$$\Big\downarrow \text{1388}$$

$$\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{c^2x^2+1}(c^4x^4+2)}{x}dx}{6c^9x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^8}$$

$$\Big\downarrow \text{1579}$$

$$\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{c^2x^2+1}(c^4x^4+2)}{x^2}dx^2}{12c^9x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^8}$$

$$\Big\downarrow \text{517}$$

$$\frac{b\sqrt{1-c^2x^2}\int -\frac{x^4(c^4x^8-2c^4x^4+3c^4)}{1-x^4}d\sqrt{c^2x^2+1}}{6c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^8}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\sqrt{1-c^2x^2}\int \frac{x^4(c^4x^8-2c^4x^4+3c^4)}{1-x^4}d\sqrt{c^2x^2+1}}{6c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^8}$$

$$\Big\downarrow \text{1584}$$

$$-\frac{b\sqrt{1-c^2x^2}\int \left(-c^4x^8+c^4x^4-2c^4+\frac{2c^4}{1-x^4}\right)d\sqrt{c^2x^2+1}}{6c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^8}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(1-c^4x^4)^{3/2}\,(a+b\sec^{-1}(cx))}{6c^8}-\frac{\sqrt{1-c^4x^4}\,(a+b\sec^{-1}(cx))}{2c^8}+\frac{b\sqrt{1-c^2x^2}\left(-2c^4\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{c^4x^{10}}{5}-\frac{c^4x^6}{3}+2c^4\sqrt{c^2x^2+1}\right)}{6c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}$$

input
```
Int[(x^7*(a + b*ArcSec[c*x]))/Sqrt[1 - c^4*x^4],x]
```

output
```
-1/2*(Sqrt[1 - c^4*x^4]*(a + b*ArcSec[c*x]))/c^8 + ((1 - c^4*x^4)^(3/2)*(a
 + b*ArcSec[c*x]))/(6*c^8) + (b*Sqrt[1 - c^2*x^2]*(-1/3*(c^4*x^6) + (c^4*x
^10)/5 + 2*c^4*Sqrt[1 + c^2*x^2] - 2*c^4*ArcTanh[Sqrt[1 + c^2*x^2]]))/(6*c
^13*Sqrt[1 - 1/(c^2*x^2)]*x)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 517
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.),
 x_Symbol] :> Simp[2*(e^m/d^(m + 2*p + 1))   Subst[Int[x^(2*n + 1)*(-c + x^
2)^m*(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4)^p, x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && ILtQ[m, 0] && IntegerQ[n + 1/2]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5769
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Simp[(a + b*ArcSec[c*x])   v, x] - Simp[b/c   Int[SimplifyIntegran
d[v/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]]
/; FreeQ[{a, b, c}, x]
```

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Maple [F]

$$\int \frac{x^7(a + b \operatorname{arcsec}(cx))}{\sqrt{-x^4c^4 + 1}}dx$$

input
```
int(x^7*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x)
```

output
```
int(x^7*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.72

$$\int \frac{x^7(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$$
$$= \frac{(3\,bc^4x^4+bc^2x^2+28\,b)\sqrt{-c^4x^4+1}\sqrt{c^2x^2-1}-30\,(bc^2x^2-b)\arctan\left(\frac{\sqrt{-c^4x^4+1}\sqrt{c^2x^2-1}}{c^4x^4-1}\right)-15\,(ac^6x^6-}{90\,(c^{10}x^2-c^8)}$$

input
```
integrate(x^7*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/90*((3*b*c^4*x^4 + b*c^2*x^2 + 28*b)*sqrt(-c^4*x^4 + 1)*sqrt(c^2*x^2 - 1
) - 30*(b*c^2*x^2 - b)*arctan(sqrt(-c^4*x^4 + 1)*sqrt(c^2*x^2 - 1)/(c^4*x^
4 - 1)) - 15*(a*c^6*x^6 - a*c^4*x^4 + 2*a*c^2*x^2 + (b*c^6*x^6 - b*c^4*x^4
 + 2*b*c^2*x^2 - 2*b)*arcsec(c*x) - 2*a)*sqrt(-c^4*x^4 + 1))/(c^10*x^2 - c
^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \text{Timed out}$$

input
```
integrate(x**7*(a+b*asec(c*x))/(-c**4*x**4+1)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^7(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x^7}{\sqrt{-c^4x^4+1}}\,dx$$

input
```
integrate(x^7*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
1/6*a*((-c^4*x^4 + 1)^(3/2)/c^8 - 3*sqrt(-c^4*x^4 + 1)/c^8) - 1/6*(6*sqrt(
c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*c^8*integrate((6*sqrt(c*x + 1)*c
^6*x^7*log(c) + (c^4*x^5 + (6*c^6*log(c) + c^6)*x^7 + 2*c^2*x^3 + 2*x)*e^(
3/2*log(c*x + 1) + log(c*x - 1)) + 6*(c^6*x^7*e^(3/2*log(c*x + 1) + log(c*
x - 1)) + sqrt(c*x + 1)*c^6*x^7)*log(x))/((c^6*e^(2*log(c*x + 1) + log(c*x
 - 1) + 1/2*log(-c*x + 1)) + c^6*e^(log(c*x + 1) + 1/2*log(-c*x + 1)))*sqr
t(c^2*x^2 + 1)), x) - (c^8*x^8 + c^4*x^4 - 2)*arctan(sqrt(c*x + 1)*sqrt(c*
x - 1)))*b/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*c^8)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^7(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^7*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx=\int \frac{x^7\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

input
```
int((x^7*(a + b*acos(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
```

output
```
int((x^7*(a + b*acos(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^7(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \frac{-\sqrt{-c^4x^4+1}\,a\,c^4x^4-2\sqrt{-c^4x^4+1}\,a-6\left(\int \frac{\sqrt{-c^4x^4+1}\,asec(cx)\,x^7}{c^4x^4-1}dx\right)b\,c^8}{6c^8}$$

input
```
int(x^7*(a+b*asec(c*x))/(-c^4*x^4+1)^(1/2),x)
```

output
```
( - sqrt( - c**4*x**4 + 1)*a*c**4*x**4 - 2*sqrt( - c**4*x**4 + 1)*a - 6*in
t((sqrt( - c**4*x**4 + 1)*asec(c*x)*x**7)/(c**4*x**4 - 1),x)*b*c**8)/(6*c*
*8)
```

3.172 $\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$

Optimal result 1385
Mathematica [A] (verified) 1385
Rubi [A] (verified) 1386
Maple [F] 1389
Fricas [A] (verification not implemented) 1389
Sympy [F] 1390
Maxima [F] 1390
Giac [F] 1390
Mupad [F(-1)] 1391
Reduce [F] 1391

Optimal result

Integrand size = 26, antiderivative size = 126

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \frac{bx\sqrt{1-c^4x^4}}{2c^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}} - \frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^4} - \frac{bx\arctan\left(\frac{\sqrt{1-c^4x^4}}{\sqrt{-1+c^2x^2}}\right)}{2c^3\sqrt{c^2x^2}}$$

output
```
1/2*b*x*(-c^4*x^4+1)^(1/2)/c^3/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)-1/2*(-c^4
*x^4+1)^(1/2)*(a+b*arcsec(c*x))/c^4-1/2*b*x*arctan((-c^4*x^4+1)^(1/2)/(c^2
*x^2-1)^(1/2))/c^3/(c^2*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \frac{\frac{\left(a+bc\sqrt{1-\frac{1}{c^2x^2}}x-ac^2x^2\right)\sqrt{1-c^4x^4}}{-1+c^2x^2} - b\sqrt{1-c^4x^4}\sec^{-1}(cx) + b\arctan\left(\frac{c\sqrt{1-\frac{1}{c^2x^2}}x}{\sqrt{1-c^4x^4}}\right)}{2c^4}$$

input
```
Integrate[(x^3*(a + b*ArcSec[c*x]))/Sqrt[1 - c^4*x^4],x]
```

output
```
(((a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x - a*c^2*x^2)*Sqrt[1 - c^4*x^4])/(-1 + c
^2*x^2) - b*Sqrt[1 - c^4*x^4]*ArcSec[c*x] + b*ArcTan[(c*Sqrt[1 - 1/(c^2*x^
2)]*x)/Sqrt[1 - c^4*x^4]])/(2*c^4)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5769, 27, 1896, 1388, 243, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}dx$$

$$\Big\downarrow \text{ 5769}$$

$$-\frac{b\int -\frac{\sqrt{1-c^4x^4}}{2c^4\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{c}-\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b\int \frac{\sqrt{1-c^4x^4}}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{2c^5}-\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^4}$$

$$\Big\downarrow \text{ 1896}$$

$$\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{1-c^4x^4}}{x\sqrt{1-c^2x^2}}dx}{2c^5x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^4}$$

$$\Big\downarrow \text{ 1388}$$

$$\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{c^2x^2+1}}{x}dx}{2c^5x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^4}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{c^2x^2+1}}{x^2}dx^2}{4c^5x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^4}$$

$\Big\downarrow$ 60

$$\frac{b\sqrt{1-c^2x^2}\left(\int \frac{1}{x^2\sqrt{c^2x^2+1}}dx^2+2\sqrt{c^2x^2+1}\right)}{4c^5x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^4}$$

$\Big\downarrow$ 73

$$\frac{b\sqrt{1-c^2x^2}\left(\frac{2\int \frac{1}{\frac{x^4}{c^2}-\frac{1}{c^2}}d\sqrt{c^2x^2+1}}{c^2}+2\sqrt{c^2x^2+1}\right)}{4c^5x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^4}$$

$\Big\downarrow$ 221

$$\frac{b\sqrt{1-c^2x^2}\left(2\sqrt{c^2x^2+1}-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{4c^5x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^4}$$

input
```
Int[(x^3*(a + b*ArcSec[c*x]))/Sqrt[1 - c^4*x^4],x]
```

output
```
-1/2*(Sqrt[1 - c^4*x^4]*(a + b*ArcSec[c*x]))/c^4 + (b*Sqrt[1 - c^2*x^2]*(2
*Sqrt[1 + c^2*x^2] - 2*ArcTanh[Sqrt[1 + c^2*x^2]]))/(4*c^5*Sqrt[1 - 1/(c^2
*x^2)]*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 1896
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_)*((a_) + (c_.)*(x_)^(n2_.))^(
p_.), x_Symbol] :> Simp[(e^IntPart[q]*((d + e*x^mn)^FracPart[q]/(1 + d*(1/(
x^mn*e)))^FracPart[q]))/x^(mn*FracPart[q])   Int[x^(m + mn*q)*(1 + d*(1/(x^
mn*e)))^q*(a + c*x^n2)^p, x], x] /; FreeQ[{a, c, d, e, m, mn, p, q}, x] &&
EqQ[n2, -2*mn] &&  !IntegerQ[p] &&  !IntegerQ[q] && PosQ[n2]
```

rule 5769
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Simp[(a + b*ArcSec[c*x])   v, x] - Simp[b/c   Int[SimplifyIntegran
d[v/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]]
/; FreeQ[{a, b, c}, x]
```

Maple [F]

$$\int \frac{x^3(a+b \operatorname{arcsec}(cx))}{\sqrt{-x^4c^4+1}}dx$$

input
```
int(x^3*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x)
```

output
```
int(x^3*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.08

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \frac{\sqrt{-c^4x^4+1}\sqrt{c^2x^2-1}b-(bc^2x^2-b)\arctan\left(\frac{\sqrt{-c^4x^4+1}\sqrt{c^2x^2-1}}{c^4x^4-1}\right)-\sqrt{-c^4x^4+1}(ac^2x^2+(bc^2x^2-b)\operatorname{arcs}}{2(c^6x^2-c^4)}$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(-c^4*x^4 + 1)*sqrt(c^2*x^2 - 1)*b - (b*c^2*x^2 - b)*arctan(sqrt(
-c^4*x^4 + 1)*sqrt(c^2*x^2 - 1)/(c^4*x^4 - 1)) - sqrt(-c^4*x^4 + 1)*(a*c^2
*x^2 + (b*c^2*x^2 - b)*arcsec(c*x) - a))/(c^6*x^2 - c^4)
```

Sympy [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{x^3(a+b\,\mathrm{asec}\,(cx))}{\sqrt{-(cx-1)\,(cx+1)\,(c^2x^2+1)}}\,dx$$

input
```
integrate(x**3*(a+b*asec(c*x))/(-c**4*x***4+1)**(1/2),x)
```

output
```
Integral(x**3*(a + b*asec(c*x))/sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))
, x)
```

Maxima [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^3}{\sqrt{-c^4x^4+1}}\,dx$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(2*c^4*integrate((2*sqrt(c*x + 1)*c^2*x^3*log(c) + ((2*c^2*log(c) + c
^2)*x^3 + x)*e^(3/2*log(c*x + 1) + log(c*x - 1)) + 2*(c^2*x^3*e^(3/2*log(c
*x + 1) + log(c*x - 1)) + sqrt(c*x + 1)*c^2*x^3)*log(x))/(sqrt(c^2*x^2 + 1
)*(c^2*e^(2*log(c*x + 1) + log(c*x - 1) + 1/2*log(-c*x + 1)) + c^2*e^(log(
c*x + 1) + 1/2*log(-c*x + 1)))), x) + sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt
(-c*x + 1)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)))*b/c^4 - 1/2*sqrt(-c^4*x^4
+ 1)*a/c^4
```

Giac [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^3}{\sqrt{-c^4x^4+1}}\,dx$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x^3/sqrt(-c^4*x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx=\int \frac{x^3\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

input
```
int((x^3*(a + b*acos(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
```

output
```
int((x^3*(a + b*acos(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx=\frac{-\sqrt{-c^4x^4+1}\,a-2\left(\int \frac{\sqrt{-c^4x^4+1}\,asec(cx)\,x^3}{c^4x^4-1}dx\right)b\,c^4}{2c^4}$$

input
```
int(x^3*(a+b*asec(c*x))/(-c^4*x^4+1)^(1/2),x)
```

output
```
( - sqrt( - c**4*x**4 + 1)*a - 2*int((sqrt( - c**4*x**4 + 1)*asec(c*x)*x**
3)/(c**4*x**4 - 1),x)*b*c**4)/(2*c**4)
```

3.173 $\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$

Optimal result 1392
Mathematica [N/A] 1392
Rubi [N/A] 1393
Maple [N/A] 1393
Fricas [N/A] 1394
Sympy [N/A] 1394
Maxima [N/A] 1394
Giac [N/A] 1395
Mupad [N/A] 1395
Reduce [N/A] 1396

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \text{Int}\left(\frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}},x\right)$$

output
```
Defer(Int)((a+b*arcsec(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x*Sqrt[1 - c^4*x^4]),x]
```

output
```
Integrate[(a + b*ArcSec[c*x])/(x*Sqrt[1 - c^4*x^4]), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

$$\Big\downarrow \text{ 5771}$$

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}dx$$

input
```
Int[(a + b*ArcSec[c*x])/(x*Sqrt[1 - c^4*x^4]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{a+b\,\mathrm{arcsec}\,(cx)}{x\sqrt{-x^4c^4+1}}dx$$

input
```
int((a+b*arcsec(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

output
```
int((a+b*arcsec(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx)+a}{\sqrt{-c^4x^4+1}x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^4*x^4 + 1)*(b*arcsec(c*x) + a)/(c^4*x^5 - x), x)
```

Sympy [N/A]

Not integrable

Time = 12.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \int \frac{a+b\,\mathrm{asec}\,(cx)}{x\sqrt{-(cx-1)(cx+1)(c^2x^2+1)}}\,dx$$

input
```
integrate((a+b*asec(c*x))/x/(-c**4*x**4+1)**(1/2),x)
```

output
```
Integral((a + b*asec(c*x))/(x*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))),
 x)
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.35

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx)+a}{\sqrt{-c^4x^4+1}x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*a*(log(sqrt(-c^4*x^4 + 1) + 1) - log(sqrt(-c^4*x^4 + 1) - 1)) + b*int
egrate(arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1
)*sqrt(-c*x + 1)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{\sqrt{-c^4x^4+1}x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.80 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{1-c^4\,x^4}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x*(1 - c^4*x^4)^(1/2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x*(1 - c^4*x^4)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = -\left(\int \frac{\sqrt{-c^4x^4+1}\,asec(cx)}{c^4x^5-x}dx\right)b+\frac{\log\left(\tan\left(\frac{asin(c^2x^2)}{2}\right)\right)a}{2}$$

input
```
int((a+b*asec(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

output
```
( - 2*int((sqrt( - c**4*x**4 + 1)*asec(c*x))/(c**4*x**5 - x),x)*b + log(ta
n(asin(c**2*x**2)/2))*a)/2
```

3.174 $\int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$

Optimal result . . . 1397
Mathematica [N/A] . . . 1397
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Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\text{Int}\left(\frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}},x\right)$$

output
```
Defer(Int)((a+b*arcsec(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 9.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x^5*Sqrt[1 - c^4*x^4]),x]
```

output
```
Integrate[(a + b*ArcSec[c*x])/(x^5*Sqrt[1 - c^4*x^4]), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

$$\Big\downarrow \text{ 5771}$$

$$\int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}dx$$

input
```
Int[(a + b*ArcSec[c*x])/(x^5*Sqrt[1 - c^4*x^4]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 4.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{a+b\,\mathrm{arcsec}\,(cx)}{x^5\sqrt{-x^4c^4+1}}dx$$

input
```
int((a+b*arcsec(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

output
```
int((a+b*arcsec(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{\sqrt{-c^4x^4+1}x^5}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^4*x^4 + 1)*(b*arcsec(c*x) + a)/(c^4*x^9 - x^5), x)
```

Sympy [N/A]

Not integrable

Time = 100.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\,\mathrm{asec}\,(cx)}{x^5\sqrt{-(cx-1)(cx+1)(c^2x^2+1)}}\,dx$$

input
```
integrate((a+b*asec(c*x))/x**5/(-c**4*x**4+1)**(1/2),x)
```

output
```
Integral((a + b*asec(c*x))/(x**5*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1)
)), x)
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 111, normalized size of antiderivative = 4.27

$$\int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{\sqrt{-c^4x^4+1}x^5}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*(c^4*log(sqrt(-c^4*x^4 + 1) + 1) - c^4*log(sqrt(-c^4*x^4 + 1) - 1) +
2*sqrt(-c^4*x^4 + 1)/x^4)*a + b*integrate(arctan(sqrt(c*x + 1)*sqrt(c*x -
1))/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^5), x)
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{\sqrt{-c^4x^4+1}x^5}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x^5), x)
```

Mupad [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^5\,\sqrt{1-c^4\,x^4}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x^5*(1 - c^4*x^4)^(1/2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x^5*(1 - c^4*x^4)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.04

$$\int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx = \frac{-\sqrt{-c^4x^4+1}\,a-4\left(\int \frac{\sqrt{-c^4x^4+1}\,asec(cx)}{c^4x^9-x^5}dx\right)b\,x^4+\log\left(\tan\left(\frac{asin(c^2x^2)}{2}\right)\right)a\,c^4x^4}{4x^4}$$

input

```
int((a+b*asec(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

output

```
( - sqrt( - c**4*x**4 + 1)*a - 4*int((sqrt( - c**4*x**4 + 1)*asec(c*x))/(c
**4*x**9 - x**5),x)*b*x**4 + log(tan(asin(c**2*x**2)/2))*a*c**4*x**4)/(4*x
**4)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions 1402
4.2 Links to plain text integration problems used in this report for each CAS . 1420

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                    convert(leaf_count_result,string)," vs. $2 (",
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]

def is_atom(expn):
  try:
    if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
      return True
    else:
      return False

  except AttributeError as error:
    return False

def expnType(expn):
  debug=False
  if debug:
    print("expn=",expn,"type(expn)=",type(expn))

  if is_atom(expn):
   return 1
  elif isinstance(expn,list):
    return max(map(expnType, expn))  #apply(max,map(ExpnType,expn))
  elif  is_sqrt(expn):
    if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
      return 1
    else:
      return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
  elif isinstance(expn,Pow):  #type(expn,'`^`')
    if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
      return expnType(expn.args[0])  #ExpnType(op(1,expn))
    elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
        return 1
      else:
        return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    else:
      return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
  elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType(


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file