Computer Algebra
Independent Integration Tests

Summer 2024

6-Hyperbolic-functions/6.1-Hyperbolic-sine/293-6.1.1

Nasser M. Abbasi

May 18, 2024 Compiled on May 18, 2024 at 2:19am

Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**502**]. This is test number [293].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	98.80 (496)	1.20 (6)
Fricas	92.63 (465)	7.37 (37)
Rubi	89.24 (448)	10.76 (54)
Maple	67.93 (341)	32.07 (161)
Maxima	57.57 (289)	42.43 (213)
Mupad	41.63 (209)	58.37 (293)
Giac	40.84 (205)	59.16 (297)
Reduce	36.45 (183)	63.55 (319)
Sympy	23.51 (118)	76.49 (384)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

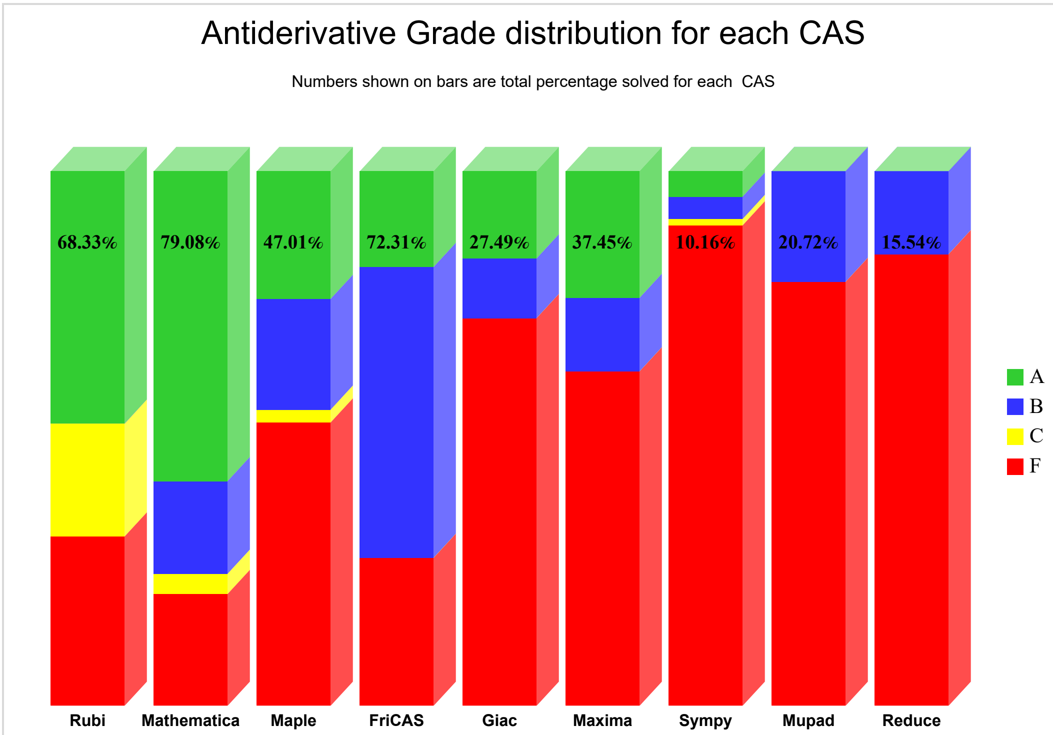
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

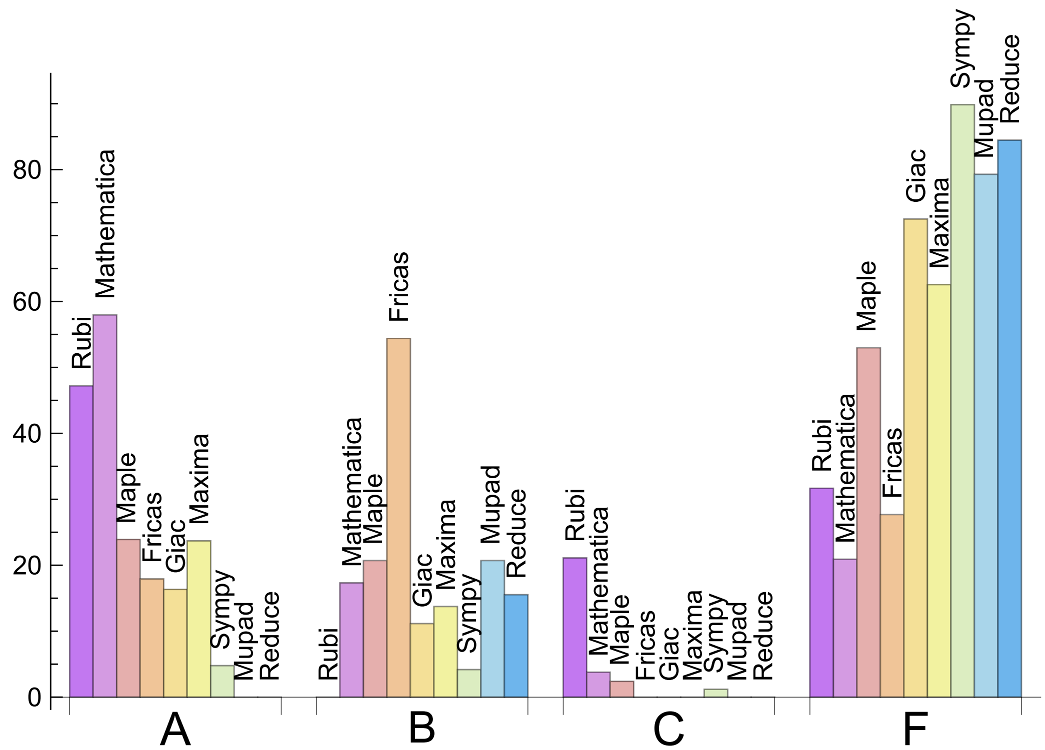
System	% A grade	% B grade	% C grade	% F grade
Mathematica	57.968	17.331	3.785	20.916
Rubi	47.211	0.000	21.116	31.673
Maple	23.904	20.717	2.390	52.988
Maxima	23.705	13.745	0.000	62.550
Fricas	17.928	54.382	0.000	27.689
Giac	16.335	11.155	0.000	72.510
Sympy	4.781	4.183	1.195	89.841
Mupad	0.000	20.717	0.000	79.283
Reduce	0.000	15.538	0.000	84.462

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	6	0.00	100.00	0.00
Fricas	37	24.32	2.70	72.97
Rubi	54	100.00	0.00	0.00
Maple	161	100.00	0.00	0.00
Maxima	213	91.55	0.00	8.45
Giac	297	60.61	39.06	0.34
Mupad	293	0.00	100.00	0.00
Reduce	319	100.00	0.00	0.00
Sympy	384	53.65	42.71	3.65

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.28
Fricas	0.64
Rubi	1.10
Reduce	1.41
Giac	1.86
Mupad	2.12
Maple	6.73
Sympy	9.10
Mathematica	9.84

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	130.37	1.77	38.00	1.12
Giac	165.39	1.73	102.00	1.30
Sympy	168.88	2.76	41.00	1.16
Rubi	209.83	1.02	125.50	1.00
Maxima	268.32	5.75	167.00	1.77
Maple	363.26	1.69	112.00	1.24
Reduce	591.72	22.08	101.00	2.26
Mathematica	598.12	1.54	144.00	1.07
Fricas	1827.51	5.16	454.00	2.87

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

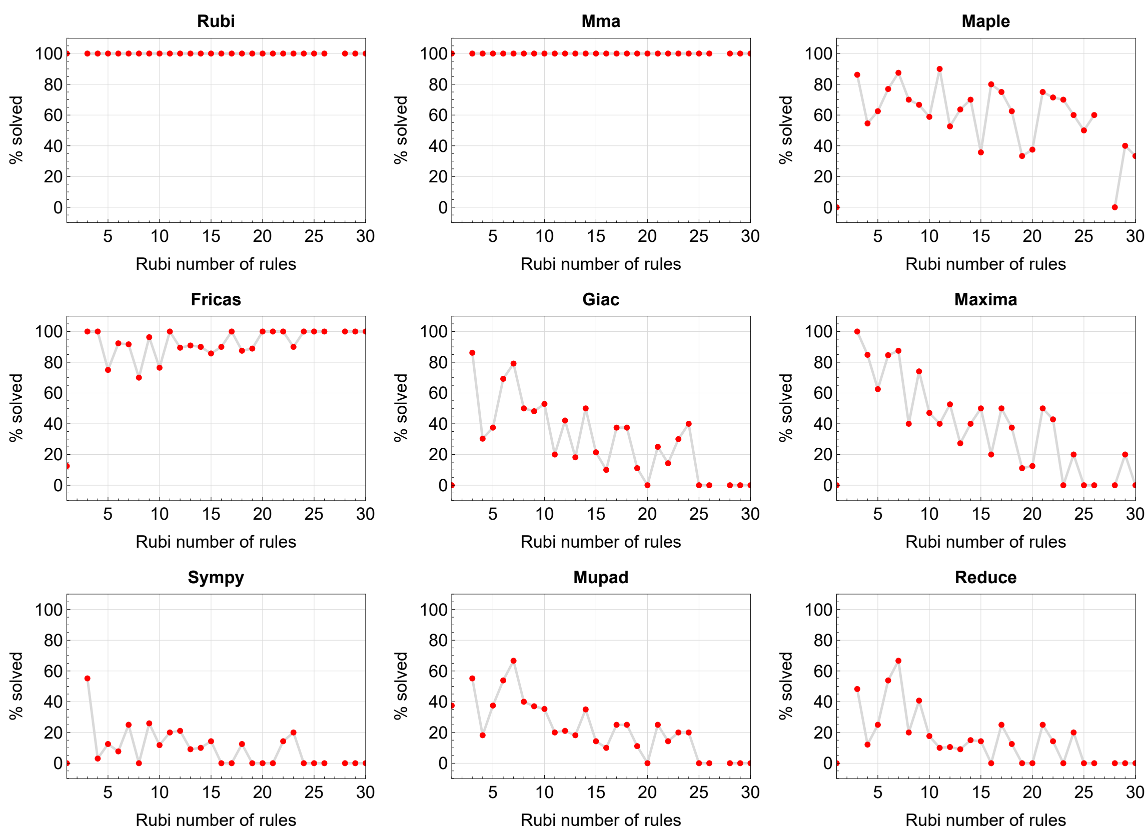


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

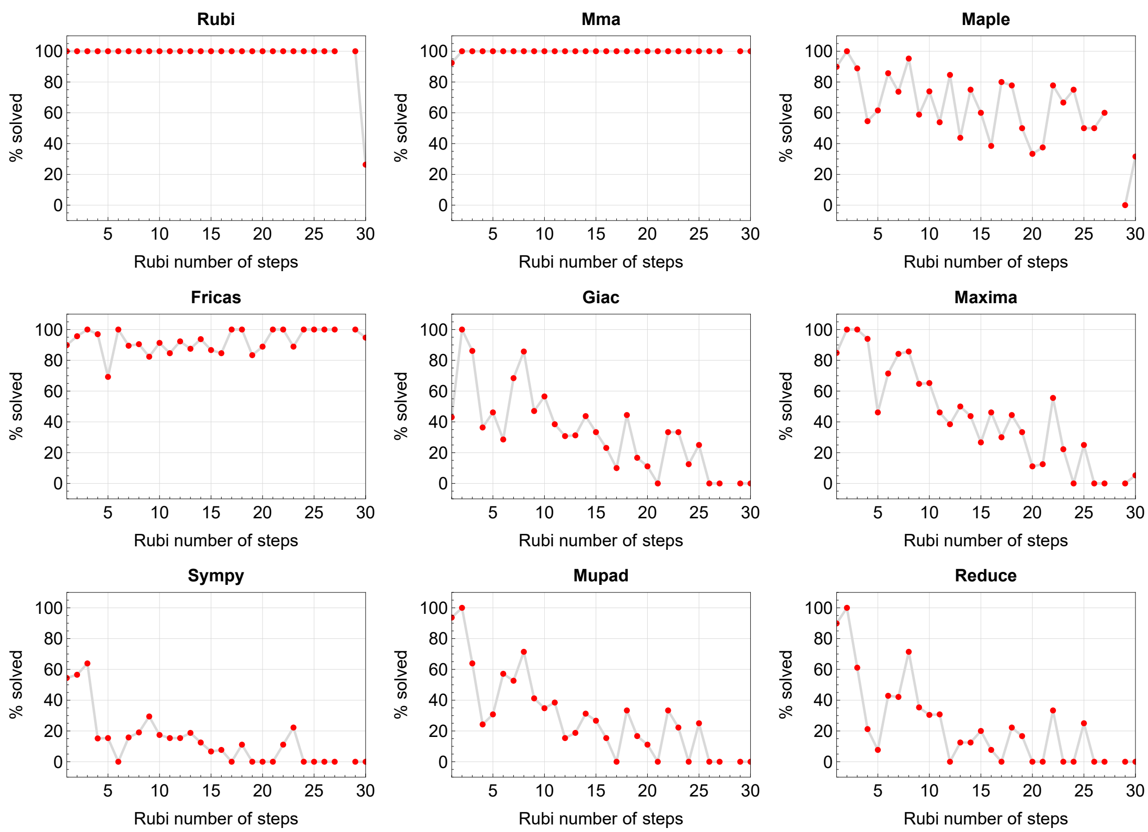


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

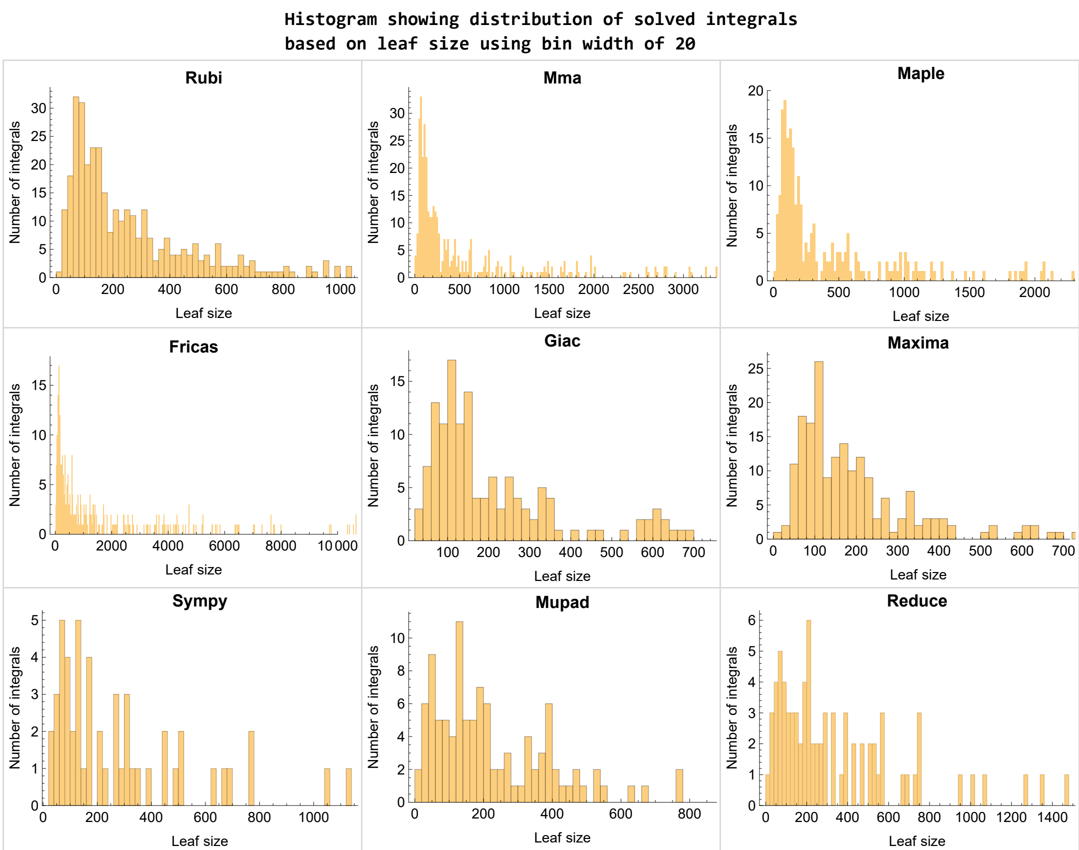


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

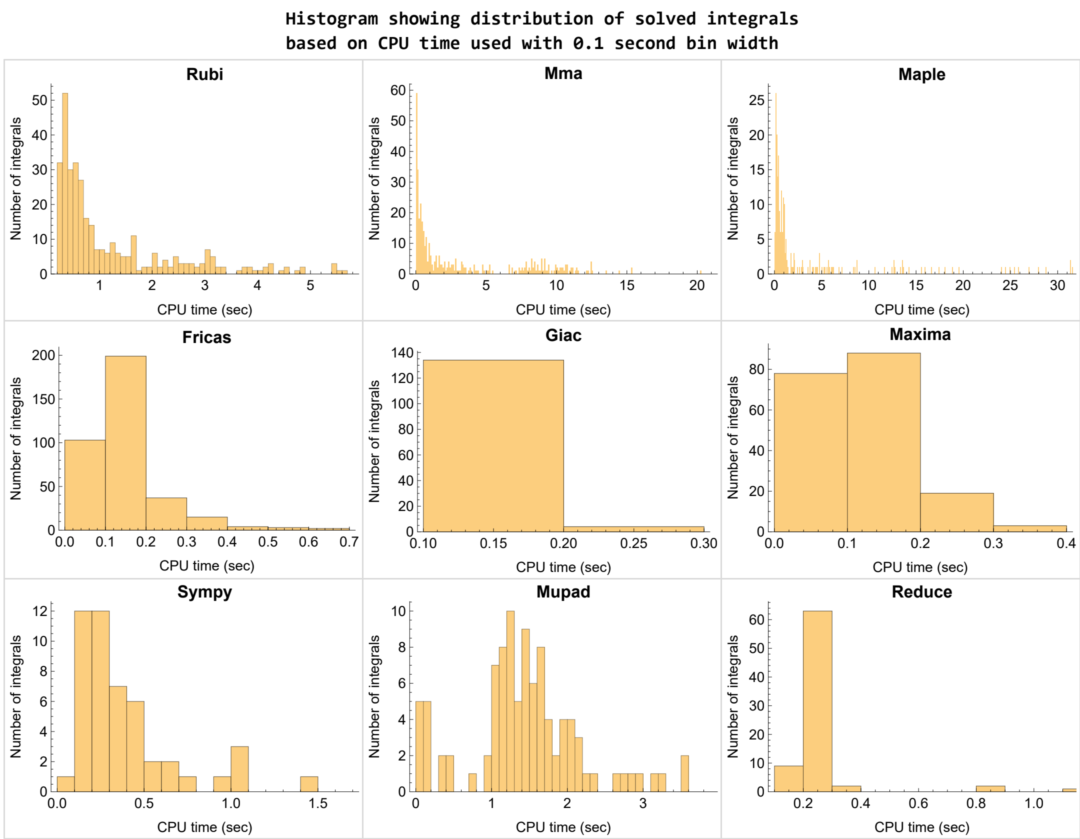


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

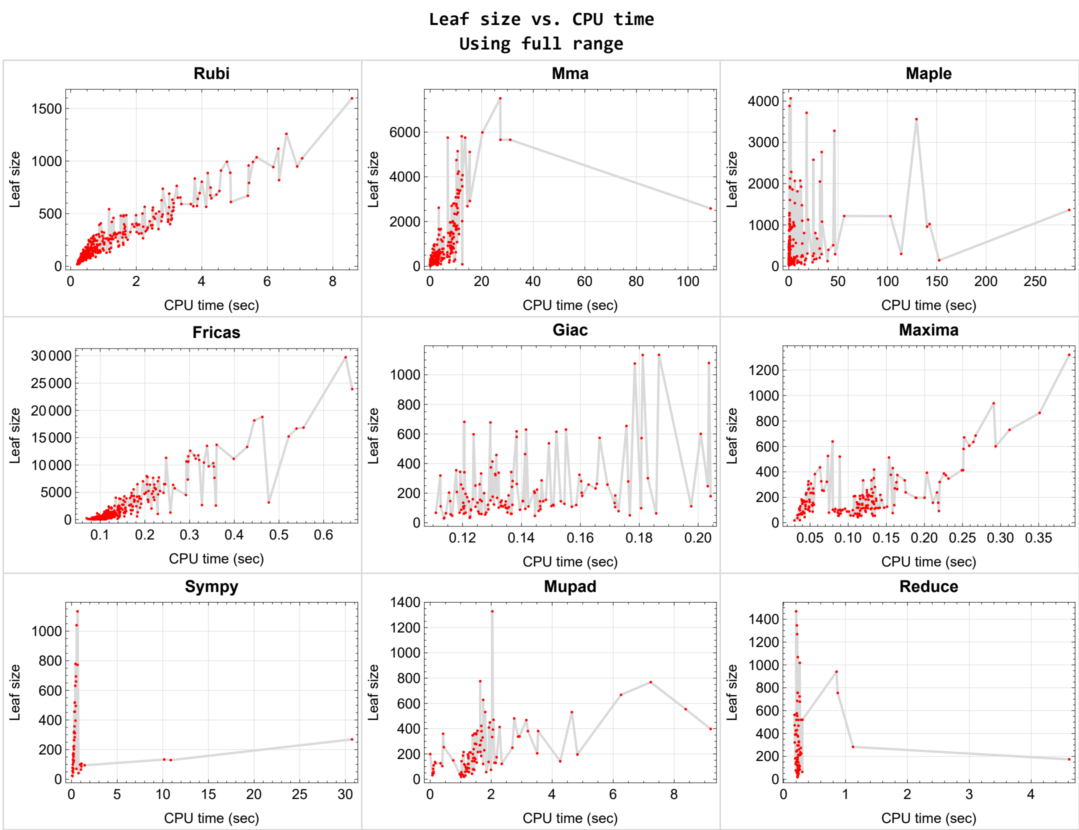


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{26, 27, 31, 32, 36, 37, 65, 66, 67, 72, 76, 77, 111, 112, 116, 117, 139, 140, 144, 145, 149, 150, 151, 155, 156, 172, 173, 176, 177, 179, 180, 181, 185, 186, 191, 192, 197, 198, 203, 204, 209, 210, 215, 216, 221, 222, 227, 232, 237, 242, 247, 252, 257, 258, 275, 276, 281, 282, 287, 288, 293, 298, 303, 308, 313, 317, 318, 319, 320, 337, 342, 347, 352, 357, 361, 366, 371, 376, 381, 386, 390, 395, 400, 405, 410, 415, 419, 424, 429, 434, 439, 444, 448, 453, 458, 463, 468, 472, 475, 480, 485, 490, 495, 499, 502}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {226, 231, 236, 241, 246, 251, 297, 312, 321, 324, 327, 330, 341, 356, 370, 385, 399, 414, 428, 457, 484}

Mathematica {141, 142, 143, 147, 148, 178, 211, 212, 217, 218, 243, 244, 248, 249, 250, 277, 283, 284, 301, 306, 309, 310, 311, 314, 315, 338, 339, 340, 343, 344, 350, 353, 354, 355, 358, 359, 362, 363, 372, 373, 374, 379, 382, 383, 384, 387, 388, 391, 392, 393, 396, 397, 398, 401, 403, 408, 411, 412, 413, 416, 417, 422, 432, 440, 441, 442, 445, 446, 451, 454, 455, 461, 466, 469, 470, 473, 478, 481, 482, 483, 488, 491, 492, 493, 496, 497, 500}

Maple {96, 97, 98}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
```

```
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
```

```
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

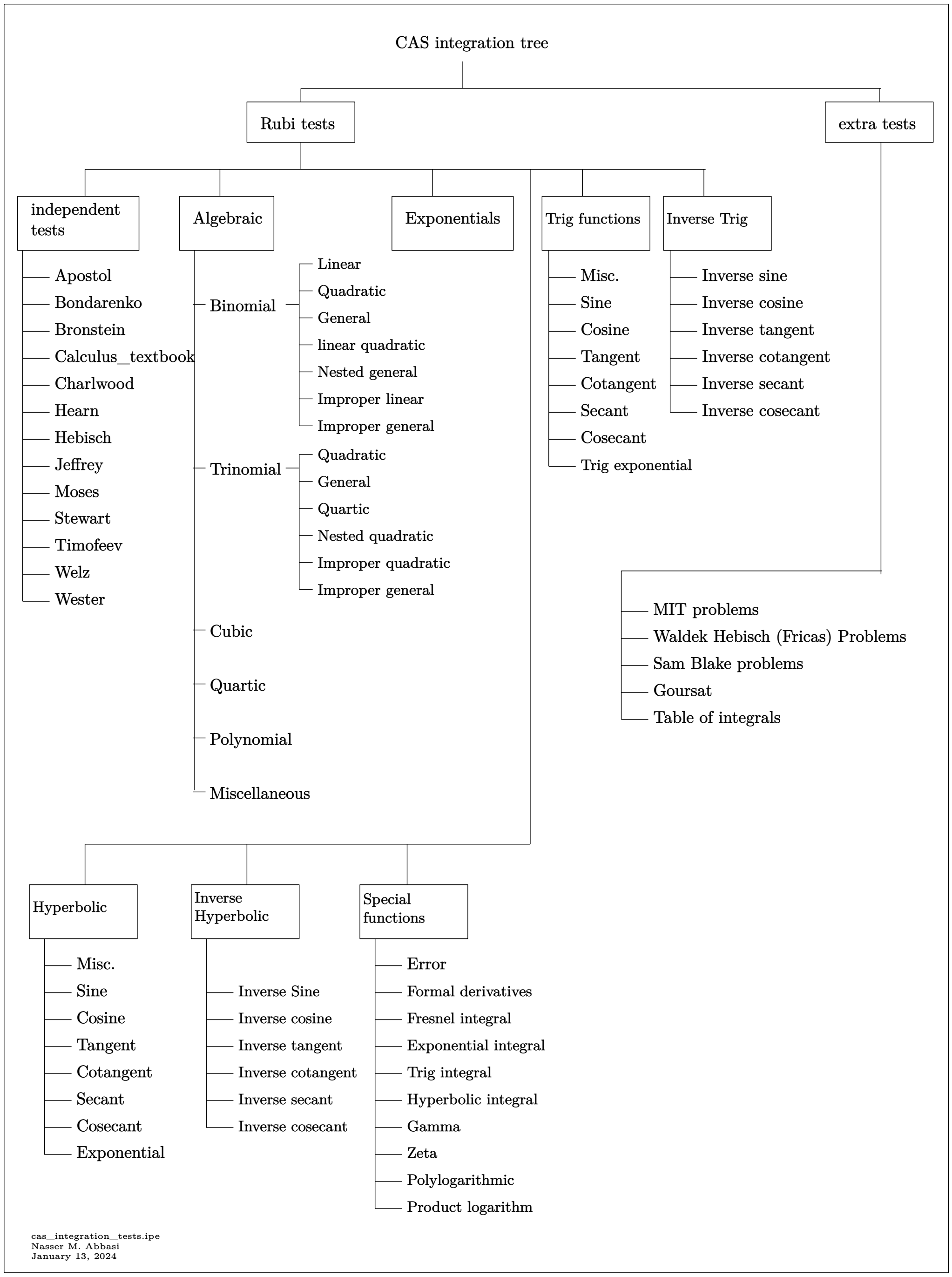
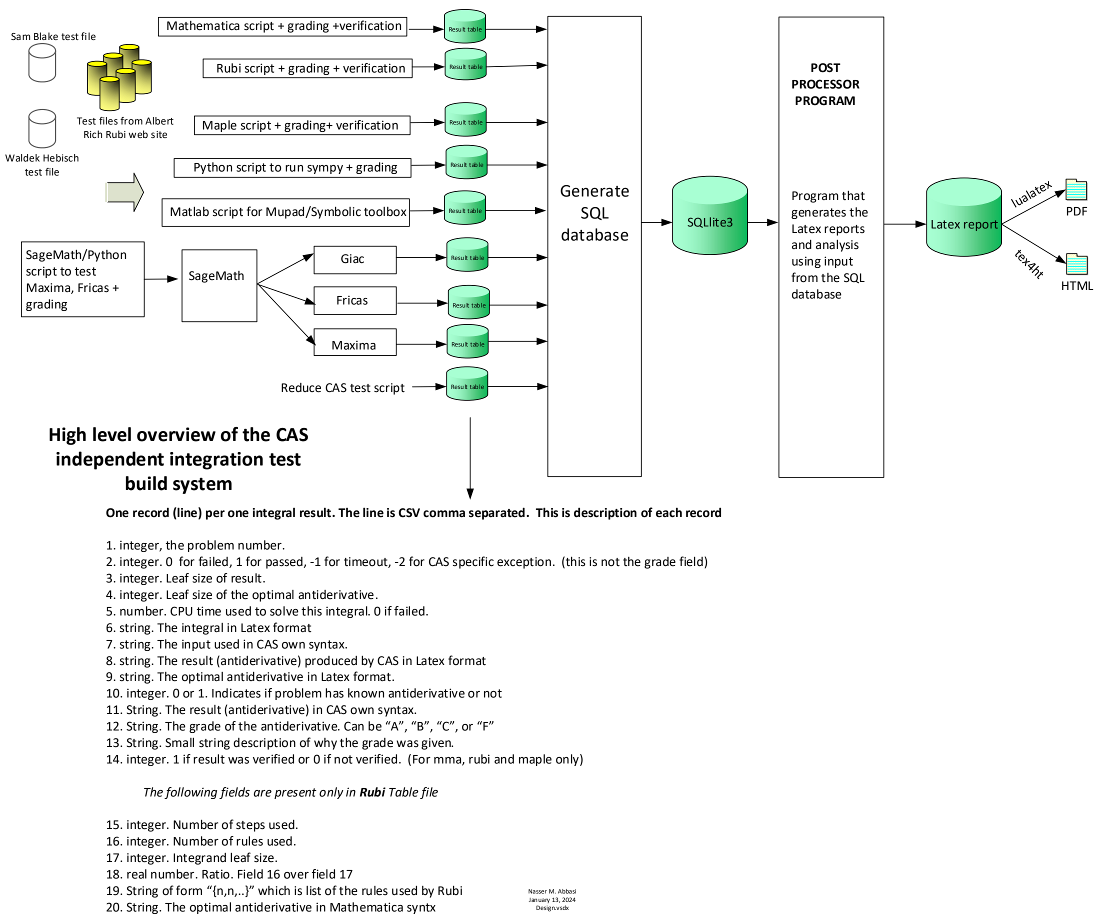


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 8, 9, 10, 11, 12, 14, 45, 46, 47, 48, 50, 52, 68, 69, 70, 71, 74, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 146, 147, 148, 152, 153, 154, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 174, 175, 178, 182, 183, 184, 187, 188, 189, 190, 193, 194, 195, 196, 201, 202, 205, 206, 207, 208, 213, 214, 220, 223, 224, 225, 253, 254, 255, 256, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 277, 278, 279, 280, 284, 285, 286, 289, 290, 291, 292, 297, 301, 302, 304, 305, 306, 307, 309, 310, 311, 312, 314, 315, 316, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 335, 336, 345, 346, 348, 349, 350, 351, 353, 354, 355, 358, 359, 360, 364, 365, 375, 377, 378, 379, 380, 382, 383, 384, 385, 387, 388, 389, 393, 394, 404, 408, 409, 418, 423, 433, 435, 436, 437, 438, 440, 441, 442, 445, 446, 447, 452, 462, 464, 465, 466, 467, 469, 471, 473, 474, 479, 489, 491, 492, 493, 494, 500, 501 }

B grade { }

C grade { 1, 2, 3, 4, 5, 6, 7, 13, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 28, 29, 30, 33, 34, 35, 38, 39, 40, 41, 42, 43, 44, 49, 51, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 73, 75, 78, 79, 80, 81, 82, 83, 84, 226, 228, 229, 230, 231, 235, 236, 238, 239, 240, 241, 243, 244, 245, 246, 251, 294, 295, 296, 299, 300, 333, 334, 338, 339, 340, 341, 356, 362, 363, 369, 370, 398, 399, 414, 420, 421, 422, 427, 428, 432, 443, 449, 450, 451, 457, 470, 477, 478, 484, 497, 498 }

F normal fail { 16, 130, 199, 200, 211, 212, 217, 218, 219, 233, 234, 248, 249, 250, 283, 343, 344, 367, 368, 372, 373, 374, 391, 392, 396, 397, 401, 402, 403, 406, 407, 411, 412, 413, 416, 417, 425, 426, 430, 431, 454, 455, 456, 459, 460, 461, 476, 481, 482, 483, 486, 487, 488, 496

}

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 28, 29, 30, 33, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 68, 69, 70, 73, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 136, 137, 138, 141, 142, 143, 146, 147, 148, 152, 153, 154, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 174, 175, 178, 182, 183, 184, 187, 188, 190, 194, 195, 196, 199, 201, 202, 205, 206, 208, 214, 220, 223, 224, 225, 226, 228, 229, 230, 231, 234, 235, 236, 238, 239, 240, 241, 244, 245, 246, 250, 251, 253, 254, 255, 256, 259, 260, 261, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 274, 278, 279, 280, 286, 289, 290, 291, 292, 294, 295, 296, 301, 302, 306, 307, 309, 310, 312, 315, 321, 322, 323, 324, 325, 326, 327, 330, 335, 336, 340, 341, 345, 346, 350, 353, 354, 356, 359, 364, 365, 367, 368, 369, 370, 374, 375, 379, 382, 383, 385, 388, 393, 394, 399, 402, 404, 408, 411, 412, 414, 417, 423, 425, 426, 427, 428, 432, 433, 437, 440, 441, 446, 447, 452, 455, 456, 457, 461, 462, 466, 467, 471, 473, 479, 483, 484, 488, 489, 493, 494, 496, 498, 500 }

B grade { 34, 35, 110, 130, 135, 189, 193, 200, 207, 211, 212, 213, 217, 218, 219, 233, 243, 248, 249, 262, 273, 277, 283, 284, 285, 299, 300, 304, 305, 314, 328, 329, 331, 332, 333, 334, 338, 339, 343, 344, 348, 349, 358, 362, 363, 372, 373, 377, 378, 387, 391, 392, 396, 397, 398, 401, 403, 406, 407, 416, 420, 421, 422, 430, 431, 435, 436, 443, 445, 449, 450, 451, 454, 459, 460, 464, 465, 469, 476, 477, 478, 481, 482, 486, 487, 491, 492 }

C grade { 71, 297, 311, 316, 351, 355, 360, 380, 384, 389, 409, 413, 418, 438, 442, 470, 474, 497, 501 }

F normal fail { }

F(-1) timedout fail { 221, 222, 276, 281, 282, 288 }

F(-2) exception fail { }

Maple

A grade { 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 16, 17, 18, 19, 20, 21, 25, 30, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 110, 114, 115, 118, 119, 120, 121, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 189, 190, 195, 196, 201, 202, 207, 208, 214, 220, 226, 231, 235, 236, 241, 246, 251, 256, 261, 262, 263, 264, 266, 267, 268, 269, 270, 273, 274, 278, 279, 280, 285, 286, 292, 297, 302, 307, 312, 316, 336, 341, 346, 351, 356, 360, 365, 370, 375, 380, 385, 389, 394, 399, 404, 409, 414, 418, 423, 428, 438, 443, 447, 452, 457, 467, 471, 474, 479, 484, 489, 494, 498, 501 }

B grade { 1, 7, 14, 15, 22, 23, 24, 28, 29, 33, 34, 35, 101, 107, 108, 109, 113, 162, 168, 171, 175, 178, 187, 188, 193, 194, 199, 200, 205, 206, 211, 212, 213, 217, 218, 219, 225, 230, 240, 245, 250, 253, 254, 255, 259, 260, 265, 271, 272, 277, 283, 284, 291, 296, 301, 306, 311, 315, 321, 322, 324, 325, 327, 328, 330, 331, 335, 340, 345, 350, 355, 359, 364, 369, 374, 379, 384, 388, 393, 398, 403, 408, 413, 417, 422, 427, 432, 433, 437, 442, 446, 451, 456, 461, 462, 466, 470, 473, 478, 483, 488, 493, 497, 500 }

C grade { 60, 61, 62, 63, 64, 78, 79, 80, 81, 82, 83, 84 }

F normal fail { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 68, 69, 70, 71, 73, 74, 75, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 146, 147, 148, 152, 153, 154, 169, 170, 174, 182, 183, 184, 223, 224, 228, 229, 233, 234, 238, 239, 243, 244, 248, 249, 289, 290, 294, 295, 299, 300, 304, 305, 309, 310, 314, 323, 326, 329, 332, 333, 334, 338, 339, 343, 344, 348, 349, 353, 354, 358, 362, 363, 367, 368, 372, 373, 377, 378, 382, 383, 387, 391, 392, 396, 397, 401, 402, 406, 407, 411, 412, 416, 420, 421, 425, 426, 430, 431, 435, 436, 440, 441, 445, 449, 450, 454, 455, 459, 460, 464, 465, 469, 476, 477, 481, 482, 486, 487, 491, 492, 496 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 9, 10, 11, 12, 18, 19, 20, 41, 48, 56, 62, 73, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 98, 99, 100, 101, 104, 105, 106, 110, 115, 152, 153, 154, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 182, 183, 184, 189, 190, 195, 196, 201, 202, 208, 255, 256, 261, 262, 263, 264, 265, 266, 267, 269, 270, 279, 280, 306, 307, 348, 349, 350, 351, 377, 378, 379, 380, 423, 438 }

B grade { 6, 7, 8, 13, 14, 15, 16, 17, 21, 22, 23, 24, 25, 28, 29, 30, 33, 34, 35, 38, 39, 40, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 63, 64, 69, 96, 97, 102, 103, 107, 108, 109, 113, 114, 162, 168, 169, 170, 171, 174, 175, 178, 187, 188, 193, 194, 199, 200, 205, 206, 207, 211, 212, 213, 214, 217, 218, 219, 220, 223, 224, 225, 226, 228, 229, 230, 231, 233, 234, 235, 236, 238, 239, 240, 241, 243, 244, 245, 246, 248, 249, 250, 251, 253, 254, 259, 260, 268, 271, 272, 273, 274, 277, 278, 283, 284, 285, 286, 289, 290, 291, 292, 294, 295, 296, 297, 299, 300, 301, 302, 304, 305, 309, 310, 311, 312, 314, 315, 316, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 338, 339, 340, 341, 343, 344, 345, 346, 353, 354, 355, 356, 358, 359, 360, 362, 363, 364, 365, 367, 368, 369, 370, 372, 373, 374, 375, 382, 383, 384, 385, 387, 388, 389, 391, 392, 393, 394, 396, 397, 398, 399, 401, 402, 403, 404, 406, 407, 408, 409, 411, 412, 413, 414, 416, 417, 418, 420, 421, 422, 425, 426, 427, 428, 430, 431, 432, 433, 435, 436, 437, 440, 441, 442, 443, 445, 446, 447, 449, 450, 451, 452, 454, 455, 456, 457, 459, 460, 461, 462, 464, 465, 466, 467, 469, 470, 471, 473, 474, 476, 477, 478, 479, 481, 482, 483, 484, 486, 487, 488, 489, 491, 492, 493, 494, 496, 497, 498, 500, 501 }

C grade { }

F normal fail { 136, 137, 138, 141, 142, 143, 146, 147, 148 }

F(-1) timedout fail { 502 }

F(-2) exception fail { 67, 68, 70, 71, 92, 93, 94, 95, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 150 }

Maxima

A grade { 5, 6, 7, 10, 11, 12, 13, 14, 15, 20, 21, 22, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 63, 64, 73, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 98, 99, 100, 101, 104, 105, 106, 107, 110, 152, 153, 154, 159, 160, 161, 162, 164, 165, 166, 167, 168, 182, 183, 184, 189, 190, 196, 202, 206, 208, 220, 226, 231, 236, 241, 246, 251, 254, 256, 263, 264, 271, 272, 277, 279, 292, 297, 307, 312, 316, 341, 351, 356, 360, 370, 380, 385, 389, 399, 409, 414, 418, 428, 438, 443, 447, 457, 467, 471, 474, 494, 498 }

B grade { 1, 2, 3, 4, 8, 9, 16, 17, 18, 19, 23, 24, 28, 30, 33, 34, 38, 39, 40, 41, 60, 61, 62, 96, 97, 102, 103, 108, 113, 115, 157, 158, 163, 187, 193, 195, 205, 211, 212, 214, 217, 218, 253, 259, 260, 261, 262, 268, 274, 280, 302, 321, 324, 327, 330, 336, 346, 365, 375, 394, 404, 423, 433, 452, 462, 479, 484, 489, 501 }

C grade { }

F normal fail { 25, 29, 35, 68, 69, 70, 71, 92, 93, 94, 95, 109, 114, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 146, 147, 148, 169, 170, 171, 174, 175, 178, 188, 194, 207, 213, 219, 223, 224, 225, 228, 229, 230, 233, 234, 235, 238, 239, 240, 243, 244, 245, 248, 249, 250, 255, 273, 278, 289, 290, 291, 294, 295, 296, 299, 300, 301, 304, 305, 306, 309, 310, 311, 314, 315, 322, 323, 325, 326, 328, 329, 331, 332, 333, 334, 335, 338, 339, 340, 343, 344, 345, 348, 349, 350, 353, 354, 355, 358, 359, 362, 363, 364, 367, 368, 369, 372, 373, 374, 377, 378, 379, 382, 383, 384, 387, 388, 391, 392, 393, 396, 397, 398, 401, 402, 403, 406, 407, 408, 411, 412, 413, 416, 417, 420, 421, 422, 425, 426, 427, 430, 431, 432, 435, 436, 437, 440, 441, 442, 445, 446, 449, 450, 451, 454, 455, 456, 459, 460, 461, 464, 465, 466, 469, 470, 473, 476, 477, 478, 481, 482, 483, 486, 487, 488, 491, 492, 493, 496, 497, 500 }

F(-1) timedout fail { }

F(-2) exception fail { 90, 91, 199, 200, 201, 203, 204, 265, 266, 267, 269, 270, 283, 284, 285, 286, 287, 288 }

Giac

A grade { 4, 5, 10, 11, 12, 19, 20, 38, 39, 40, 41, 48, 60, 61, 62, 98, 99, 104, 105, 110, 115, 118, 119, 120, 121, 125, 126, 127, 159, 160, 165, 166, 189, 190, 196, 202, 208, 214, 220, 226, 231, 236, 241, 246, 251, 256, 261, 263, 267, 269, 279, 280, 292, 297, 302, 307, 312, 336, 341, 346, 351, 356, 365, 370, 375, 380, 385, 394, 399, 404, 409, 414, 423, 428, 433, 438, 443, 457, 467, 471, 484, 498 }

B grade { 1, 2, 3, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 21, 22, 30, 96, 97, 100, 101, 102, 103, 106, 107, 157, 158, 161, 162, 163, 164, 167, 168, 195, 201, 259, 260, 262, 264, 265, 266, 268, 270, 274, 286, 316, 360, 389, 418, 447, 452, 462, 474, 479, 489, 494, 501 }

C grade { }

F normal fail { 23, 24, 25, 28, 29, 33, 34, 35, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 63, 64, 68, 69, 70, 71, 73, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 108, 109, 113, 114, 122, 123, 124, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 146, 147, 148, 152, 153, 154, 169, 170, 171, 174, 175, 178, 182, 183, 184, 187, 188, 193, 194, 199, 200, 205, 206, 207, 212, 213, 218, 219, 223, 224, 225, 228, 229, 230, 233, 234, 235, 253, 254, 255, 271, 272, 273, 277, 278, 283, 284, 285, 289, 290, 291, 294, 295, 296, 299, 300, 301, 304, 305, 306, 311, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 338, 339, 340, 343, 344, 345, 362, 363, 364, 367, 368, 369, 372, 373, 374, 391, 392, 393, 396, 397, 398, 401, 402, 403, 420, 421, 422, 435, 436, 437, 482 }

F(-1) timedout fail { 37, 210, 211, 215, 216, 217, 221, 222, 238, 239, 240, 242, 243, 244, 245, 247, 248, 249, 250, 252, 282, 288, 309, 310, 313, 314, 315, 317, 348, 349, 350, 352, 353, 354, 355, 357, 358, 359, 361, 377, 378, 379, 381, 382, 383, 384, 386, 387, 388, 390, 406, 407, 408, 410, 411, 412, 413, 415, 416, 417, 419, 425, 426, 427, 429, 430, 431, 432, 434, 440, 441, 442, 444, 445, 446, 448, 449, 450, 451, 453, 454, 455, 456, 458, 459, 460, 461, 463, 464, 465, 466, 468, 469, 470, 472, 473, 475, 476, 477, 478, 480, 481, 483, 485, 486, 487, 488, 490, 491, 492, 493, 495, 497, 499, 500, 502 }

F(-2) exception fail { 496 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 8, 9, 10, 11, 16, 17, 18, 19, 30, 68, 69, 70, 96, 97, 98, 102, 103, 104, 110, 115, 118, 119, 120, 121, 157, 158, 159, 163, 164, 165, 189, 190, 195, 196, 201, 202, 208, 214, 220, 226, 231, 236, 241, 246, 251, 256, 259, 260, 261, 262, 265, 266, 267, 268, 274, 279, 280, 286, 292, 297, 302, 307, 312, 316, 321, 324, 336, 341, 346, 351, 356, 360, 365, 370, 375, 380, 385, 389, 394, 399, 404, 409, 414, 418, 423, 428, 433, 443, 452, 457, 462, 467, 471, 474, 479, 484, 489, 494, 498, 501 }

C grade { }

F normal fail { }

F(-1) timedout fail { 5, 6, 7, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25, 28, 29, 33, 34, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 71, 73, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 105, 106, 107, 108, 109, 113, 114, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 146, 147, 148, 152, 153, 154, 160, 161, 162, 166, 167, 168, 169, 170, 171, 174, 175, 178, 182, 183, 184, 187, 188, 193, 194, 199, 200, 205, 206, 207, 211, 212, 213, 217, 218, 219, 223, 224, 225, 228, 229, 230, 233, 234, 235, 238, 239, 240, 243, 244, 245, 248, 249, 250, 253, 254, 255, 263, 264, 269, 270, 271, 272, 273, 277, 278, 283, 284, 285, 289, 290, 291, 294, 295, 296, 299, 300, 301, 304, 305, 306, 309, 310, 311, 314, 315, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 338, 339, 340, 343, 344, 345, 348, 349, 350, 353, 354, 355, 358, 359, 362, 363, 364, 367, 368, 369, 372, 373, 374, 377, 378, 379, 382, 383, 384, 387, 388, 391, 392, 393, 396, 397, 398, 401, 402, 403, 406, 407, 408, 411, 412, 413, 416, 417, 420, 421, 422, 425, 426, 427, 430, 431, 432, 435, 436, 437, 438, 440, 441, 442, 445, 446, 447, 449, 450, 451, 454, 455, 456, 459, 460, 461, 464, 465, 466, 469, 470, 473, 476, 477, 478, 481, 482, 483, 486, 487, 488, 491, 492, 493, 496, 497, 500 }

F(-2) exception fail { }

Sympy

A grade { 4, 19, 96, 97, 98, 102, 103, 104, 110, 115, 159, 189, 190, 195, 196, 201, 202, 256, 259, 260, 261, 262, 365, 394 }

B grade { 1, 2, 3, 8, 9, 10, 11, 16, 17, 18, 157, 158, 163, 164, 165, 265, 266, 267, 268, 292, 336 }

C grade { 60, 61, 62, 63, 64, 226 }

F normal fail { 5, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25, 28, 29, 30, 33, 34, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 54, 55, 56, 57, 58, 59, 68, 69, 70, 71, 73, 74, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 105, 106, 107, 108, 109, 113, 114, 118, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 136, 137, 138, 141, 142, 143, 160, 166, 167, 168, 169, 170, 171, 187, 188, 193, 194, 199, 200, 205, 206, 207, 208, 212, 213, 214, 219, 220, 224, 225, 238, 239, 240, 241, 244, 245, 246, 250, 251, 253, 254, 255, 263, 269, 271, 272, 273, 274, 277, 278, 279, 280, 283, 284, 285, 286, 291, 304, 305, 306, 307, 309, 310, 311, 312, 314, 315, 316, 348, 349, 350, 351, 353, 354, 355, 356, 358, 359, 360, 377, 378, 379, 380, 382, 383, 384, 385, 387, 388, 389, 406, 407, 408, 409, 411, 412, 413, 414, 416, 417, 418, 420, 421, 422, 423, 425, 426, 427, 428, 430, 431, 432, 433, 438, 450, 451, 452, 454, 455, 456, 457, 459, 460, 461, 462, 478, 479, 483, 484, 486, 487, 488, 489 }

F(-1) timedout fail { 6, 7, 52, 53, 100, 101, 116, 117, 125, 130, 131, 132, 133, 134, 135, 146, 147, 148, 149, 151, 156, 161, 162, 174, 175, 176, 177, 178, 179, 180, 181, 198, 204, 210, 211, 216, 217, 218, 222, 223, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 243, 248, 249, 264, 270, 289, 290, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 362, 363, 364, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 435, 436, 437, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 481, 482, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502 }

F(-2) exception fail { 75, 78, 79, 80, 81, 82, 83, 84, 152, 153, 154, 182, 183, 184 }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 8, 9, 10, 11, 16, 17, 18, 19, 30, 96, 97, 98, 102, 103, 104, 157, 158, 159, 163, 164, 165, 226, 231, 236, 241, 246, 251, 256, 292, 297, 302, 307, 312, 316, 321, 324, 327, 330, 336, 341, 346, 351, 356, 360, 365, 370, 375, 380, 385, 389, 394, 399, 404, 409, 414, 418, 423, 428, 433, 438, 443, 447, 452, 457, 462, 467, 471, 474, 479, 484, 489, 494, 498, 501 }

C grade { }

F normal fail { 5, 6, 7, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25, 28, 29, 33, 34, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 68, 69, 70, 71, 73, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 105, 106, 107, 108, 109, 110, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 146, 147, 148, 152, 153, 154, 160, 161, 162, 166, 167, 168, 169, 170, 171, 174, 175, 178, 182, 183, 184, 187, 188, 189, 190, 193, 194, 195, 196, 199, 200, 201, 202, 205, 206, 207, 208, 211, 212, 213, 214, 217, 218, 219, 220, 223, 224, 225, 228, 229, 230, 233, 234, 235, 238, 239, 240, 243, 244, 245, 248, 249, 250, 253, 254, 255, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 277, 278, 279, 280, 283, 284, 285, 286, 289, 290, 291, 294, 295, 296, 299, 300, 301, 304, 305, 306, 309, 310, 311, 314, 315, 322, 323, 325, 326, 328, 329, 331, 332, 333, 334, 335, 338, 339, 340, 343, 344, 345, 348, 349, 350, 353, 354, 355, 358, 359, 362, 363, 364, 367, 368, 369, 372, 373, 374, 377, 378, 379, 382, 383, 384, 387, 388, 391, 392, 393, 396, 397, 398, 401, 402, 403, 406, 407, 408, 411, 412, 413, 416, 417, 420, 421, 422, 425, 426, 427, 430, 431, 432, 435, 436, 437, 440, 441, 442, 445, 446, 449, 450, 451, 454, 455, 456, 459, 460, 461, 464, 465, 466, 469, 470, 473, 476, 477, 478, 481, 482, 483, 486, 487, 488, 491, 492, 493, 496, 497, 500 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	116	76	185	326	169	311	324	243	215
N.S.	1	1.27	0.84	2.03	3.58	1.86	3.42	3.56	2.67	2.36
time (sec)	N/A	0.604	0.182	0.314	0.048	0.093	0.370	0.160	0.206	1.097

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	92	61	119	222	109	202	204	153	143
N.S.	1	1.31	0.87	1.70	3.17	1.56	2.89	2.91	2.19	2.04
time (sec)	N/A	0.462	0.116	0.229	0.045	0.092	0.266	0.115	0.203	1.029

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	61	44	91	134	62	112	112	83	82
N.S.	1	1.24	0.90	1.86	2.73	1.27	2.29	2.29	1.69	1.67
time (sec)	N/A	0.352	0.091	0.182	0.053	0.088	0.204	0.146	0.259	0.098

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	37	27	37	68	29	46	46	33	35
N.S.	1	1.32	0.96	1.32	2.43	1.04	1.64	1.64	1.18	1.25
time (sec)	N/A	0.241	0.080	0.132	0.038	0.085	0.173	0.145	0.226	0.995

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	61	49	82	57	94	0	57	16	0
N.S.	1	1.20	0.96	1.61	1.12	1.84	0.00	1.12	0.31	0.00
time (sec)	N/A	0.403	0.058	0.142	0.103	0.073	0.000	0.116	0.218	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	84	65	133	80	148	0	615	78	0
N.S.	1	1.18	0.92	1.87	1.13	2.08	0.00	8.66	1.10	0.00
time (sec)	N/A	0.535	0.158	0.181	0.086	0.086	0.000	0.152	0.251	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	A	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	121	88	277	94	254	0	301	0	0
N.S.	1	1.16	0.85	2.66	0.90	2.44	0.00	2.89	0.00	0.00
time (sec)	N/A	0.673	0.320	0.237	0.090	0.070	0.000	0.183	0.233	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	167	132	145	382	312	660	374	576	334
N.S.	1	1.03	0.81	0.90	2.36	1.93	4.07	2.31	3.56	2.06
time (sec)	N/A	0.536	0.345	0.449	0.056	0.087	0.491	0.130	0.212	2.034

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	129	104	121	263	209	456	243	382	229
N.S.	1	1.04	0.84	0.98	2.12	1.69	3.68	1.96	3.08	1.85
time (sec)	N/A	0.374	0.246	0.362	0.048	0.085	0.369	0.137	0.223	1.774

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	97	75	79	165	123	264	136	226	127
N.S.	1	1.02	0.79	0.83	1.74	1.29	2.78	1.43	2.38	1.34
time (sec)	N/A	0.333	0.191	0.284	0.044	0.087	0.275	0.125	0.284	0.174

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	52	64	88	64	126	63	108	60
N.S.	1	1.00	0.95	1.16	1.60	1.16	2.29	1.15	1.96	1.09
time (sec)	N/A	0.229	0.170	0.178	0.041	0.087	0.187	0.186	0.228	0.111

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	66	97	72	104	0	68	18	0
N.S.	1	1.00	0.85	1.24	0.92	1.33	0.00	0.87	0.23	0.00
time (sec)	N/A	0.397	0.159	0.557	0.090	0.103	0.000	0.121	0.207	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	94	75	152	88	166	0	574	198	0
N.S.	1	1.16	0.93	1.88	1.09	2.05	0.00	7.09	2.44	0.00
time (sec)	N/A	0.550	0.265	0.588	0.101	0.095	0.000	0.167	0.203	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	147	102	299	99	280	0	330	408	0
N.S.	1	1.31	0.91	2.67	0.88	2.50	0.00	2.95	3.64	0.00
time (sec)	N/A	0.509	0.574	0.690	0.087	0.084	0.000	0.131	0.239	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	170	123	555	110	411	0	537	688	0
N.S.	1	1.05	0.76	3.43	0.68	2.54	0.00	3.31	4.25	0.00
time (sec)	N/A	0.799	0.519	0.846	0.096	0.091	0.000	0.149	0.233	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	B	B	B	B	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	0	150	181	639	528	772	654	1068	532
N.S.	1	0.00	0.67	0.80	2.84	2.35	3.43	2.91	4.75	2.36
time (sec)	N/A	0.000	0.583	1.007	0.080	0.091	0.649	0.176	0.228	1.804

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	246	127	142	435	345	495	414	678	364
N.S.	1	1.41	0.73	0.81	2.49	1.97	2.83	2.37	3.87	2.08
time (sec)	N/A	1.156	0.594	0.748	0.063	0.106	0.470	0.130	0.262	1.509

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	150	86	106	269	199	284	230	374	184
N.S.	1	1.22	0.70	0.86	2.19	1.62	2.31	1.87	3.04	1.50
time (sec)	N/A	0.655	0.246	0.590	0.054	0.095	0.342	0.120	0.231	1.674

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	88	59	63	141	97	126	98	154	79
N.S.	1	1.17	0.79	0.84	1.88	1.29	1.68	1.31	2.05	1.05
time (sec)	N/A	0.383	0.196	0.454	0.044	0.082	0.255	0.181	0.219	1.406

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	133	102	166	117	188	0	113	18	0
N.S.	1	1.10	0.84	1.37	0.97	1.55	0.00	0.93	0.15	0.00
time (sec)	N/A	0.523	0.249	0.677	0.113	0.090	0.000	0.121	0.217	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	156	160	271	145	301	0	1076	168	0
N.S.	1	1.08	1.10	1.87	1.00	2.08	0.00	7.42	1.16	0.00
time (sec)	N/A	0.493	0.703	0.766	0.115	0.085	0.000	0.178	0.251	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	264	220	562	145	529	0	601	227	0
N.S.	1	1.43	1.20	3.05	0.79	2.88	0.00	3.27	1.23	0.00
time (sec)	N/A	1.000	0.519	0.927	0.113	0.094	0.000	0.201	0.247	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	173	168	541	333	396	0	0	87	0
N.S.	1	1.16	1.13	3.63	2.23	2.66	0.00	0.00	0.58	0.00
time (sec)	N/A	0.733	0.186	0.234	0.120	0.090	0.000	0.000	0.251	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	115	118	306	195	242	0	0	66	0
N.S.	1	1.16	1.19	3.09	1.97	2.44	0.00	0.00	0.67	0.00
time (sec)	N/A	0.472	0.128	0.169	0.120	0.081	0.000	0.000	0.258	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	61	90	89	0	119	0	0	43	0
N.S.	1	1.22	1.80	1.78	0.00	2.38	0.00	0.00	0.86	0.00
time (sec)	N/A	0.293	0.039	0.060	0.000	0.085	0.000	0.000	0.248	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	12	16	16	18
N.S.	1	1.00	1.14	1.00	1.14	1.14	0.86	1.14	1.14	1.29
time (sec)	N/A	0.214	14.339	0.045	0.401	0.073	0.366	0.261	0.222	1.266

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	27	14	16	27	18
N.S.	1	1.00	1.14	1.00	1.14	1.93	1.00	1.14	1.93	1.29
time (sec)	N/A	0.213	14.552	0.043	0.323	0.080	0.530	2.080	0.225	1.317

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	146	185	473	320	1159	0	0	657	0
N.S.	1	1.42	1.80	4.59	3.11	11.25	0.00	0.00	6.38	0.00
time (sec)	N/A	0.717	0.665	0.131	0.220	0.093	0.000	0.000	0.249	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	106	137	240	0	623	0	0	327	0
N.S.	1	1.43	1.85	3.24	0.00	8.42	0.00	0.00	4.42	0.00
time (sec)	N/A	0.517	0.470	0.096	0.000	0.098	0.000	0.000	0.256	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	33	52	56	91	166	0	80	118	49
N.S.	1	1.14	1.79	1.93	3.14	5.72	0.00	2.76	4.07	1.69
time (sec)	N/A	0.282	0.145	0.129	0.049	0.089	0.000	0.146	0.237	0.081

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	157	18	14	18	18	18
N.S.	1	1.00	1.12	1.00	9.81	1.12	0.88	1.12	1.12	1.12
time (sec)	N/A	0.227	15.508	0.046	0.190	0.085	0.380	0.149	0.221	1.200

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	237	29	15	18	29	18
N.S.	1	1.00	1.12	1.00	14.81	1.81	0.94	1.12	1.81	1.12
time (sec)	N/A	0.229	15.611	0.046	0.217	0.081	0.533	0.180	0.268	1.275

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	294	440	876	605	4008	0	0	0	0
N.S.	1	1.15	1.72	3.42	2.36	15.66	0.00	0.00	0.00	0.00
time (sec)	N/A	1.206	2.559	0.247	0.259	0.135	0.000	0.000	0.241	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	184	420	444	393	2218	0	0	1141	0
N.S.	1	1.19	2.73	2.88	2.55	14.40	0.00	0.00	7.41	0.00
time (sec)	N/A	0.732	6.666	0.161	0.203	0.120	0.000	0.000	0.237	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	108	271	197	0	1026	0	0	516	0
N.S.	1	1.17	2.95	2.14	0.00	11.15	0.00	0.00	5.61	0.00
time (sec)	N/A	0.443	0.202	0.115	0.000	0.115	0.000	0.000	0.233	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	433	18	14	18	18	18
N.S.	1	1.00	1.12	1.00	27.06	1.12	0.88	1.12	1.12	1.12
time (sec)	N/A	0.233	61.701	0.049	0.404	0.088	0.384	2.494	0.239	1.260

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	545	29	15	0	29	18
N.S.	1	1.00	1.12	1.00	34.06	1.81	0.94	0.00	1.81	1.12
time (sec)	N/A	0.234	65.903	0.050	0.543	0.114	0.533	0.000	0.218	1.302

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	B	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	204	108	0	308	521	0	232	62	0
N.S.	1	1.19	0.63	0.00	1.80	3.05	0.00	1.36	0.36	0.00
time (sec)	N/A	0.872	0.041	0.000	0.051	0.113	0.000	0.165	0.243	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	B	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	146	175	106	0	268	385	0	202	36	0
N.S.	1	1.20	0.73	0.00	1.84	2.64	0.00	1.38	0.25	0.00
time (sec)	N/A	0.671	0.059	0.000	0.049	0.112	0.000	0.144	0.230	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	B	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	141	104	0	230	301	0	168	15	0
N.S.	1	1.15	0.85	0.00	1.87	2.45	0.00	1.37	0.12	0.00
time (sec)	N/A	0.535	0.052	0.000	0.054	0.125	0.000	0.132	0.225	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	112	104	0	181	122	0	90	17	0
N.S.	1	1.08	1.00	0.00	1.74	1.17	0.00	0.87	0.16	0.00
time (sec)	N/A	0.394	0.025	0.000	0.049	0.087	0.000	0.129	0.231	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	139	120	0	103	339	0	0	29	0
N.S.	1	1.18	1.02	0.00	0.87	2.87	0.00	0.00	0.25	0.00
time (sec)	N/A	0.516	0.095	0.000	0.047	0.100	0.000	0.000	0.285	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	176	161	0	114	532	0	0	46	0
N.S.	1	1.18	1.08	0.00	0.77	3.57	0.00	0.00	0.31	0.00
time (sec)	N/A	0.653	0.441	0.000	0.126	0.100	0.000	0.000	0.250	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	207	168	0	114	855	0	0	63	0
N.S.	1	1.19	0.97	0.00	0.66	4.91	0.00	0.00	0.36	0.00
time (sec)	N/A	0.802	0.353	0.000	0.131	0.137	0.000	0.000	0.222	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	239	246	137	0	281	1001	0	0	68	0
N.S.	1	1.03	0.57	0.00	1.18	4.19	0.00	0.00	0.28	0.00
time (sec)	N/A	0.748	0.364	0.000	0.135	0.108	0.000	0.000	0.264	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	211	218	137	0	239	755	0	0	40	0
N.S.	1	1.03	0.65	0.00	1.13	3.58	0.00	0.00	0.19	0.00
time (sec)	N/A	0.647	0.209	0.000	0.175	0.117	0.000	0.000	0.249	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	129	0	189	590	0	0	17	0
N.S.	1	1.00	0.78	0.00	1.14	3.55	0.00	0.00	0.10	0.00
time (sec)	N/A	0.504	0.360	0.000	0.137	0.101	0.000	0.000	0.229	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	142	0	107	155	0	115	19	0
N.S.	1	1.00	1.02	0.00	0.77	1.12	0.00	0.83	0.14	0.00
time (sec)	N/A	0.446	0.080	0.000	0.130	0.092	0.000	0.133	0.277	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	160	152	0	116	571	0	0	31	0
N.S.	1	1.13	1.07	0.00	0.82	4.02	0.00	0.00	0.22	0.00
time (sec)	N/A	0.588	0.356	0.000	0.123	0.099	0.000	0.000	0.265	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	221	156	0	118	864	0	0	48	0
N.S.	1	1.27	0.90	0.00	0.68	4.97	0.00	0.00	0.28	0.00
time (sec)	N/A	0.640	0.701	0.000	0.112	0.110	0.000	0.000	0.235	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	220	243	204	0	118	1352	0	0	65	0
N.S.	1	1.10	0.93	0.00	0.54	6.15	0.00	0.00	0.30	0.00
time (sec)	N/A	0.803	0.530	0.000	0.141	0.133	0.000	0.000	0.243	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	251	304	222	0	118	1827	0	0	82	0
N.S.	1	1.21	0.88	0.00	0.47	7.28	0.00	0.00	0.33	0.00
time (sec)	N/A	0.852	0.348	0.000	0.147	0.139	0.000	0.000	0.258	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	A	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	381	565	194	0	513	2090	0	0	68	0
N.S.	1	1.48	0.51	0.00	1.35	5.49	0.00	0.00	0.18	0.00
time (sec)	N/A	2.256	0.430	0.000	0.154	0.141	0.000	0.000	0.278	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	325	485	211	0	430	1543	0	0	40	0
N.S.	1	1.49	0.65	0.00	1.32	4.75	0.00	0.00	0.12	0.00
time (sec)	N/A	1.696	0.371	0.000	0.159	0.115	0.000	0.000	0.282	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	275	291	209	0	333	1216	0	0	17	0
N.S.	1	1.06	0.76	0.00	1.21	4.42	0.00	0.00	0.06	0.00
time (sec)	N/A	0.704	0.212	0.000	0.174	0.116	0.000	0.000	0.239	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	228	240	191	0	178	252	0	0	19	0
N.S.	1	1.05	0.84	0.00	0.78	1.11	0.00	0.00	0.08	0.00
time (sec)	N/A	0.600	0.146	0.000	0.144	0.124	0.000	0.000	0.245	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	246	265	245	0	197	1346	0	0	31	0
N.S.	1	1.08	1.00	0.00	0.80	5.47	0.00	0.00	0.13	0.00
time (sec)	N/A	0.627	0.724	0.000	0.200	0.120	0.000	0.000	0.235	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	277	422	253	0	196	2059	0	0	48	0
N.S.	1	1.52	0.91	0.00	0.71	7.43	0.00	0.00	0.17	0.00
time (sec)	N/A	1.255	1.857	0.000	0.189	0.125	0.000	0.000	0.224	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	331	480	376	0	197	3286	0	0	65	0
N.S.	1	1.45	1.14	0.00	0.60	9.93	0.00	0.00	0.20	0.00
time (sec)	N/A	1.528	1.122	0.000	0.189	0.176	0.000	0.000	0.236	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	B	B	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	140	50	132	175	189	133	146	40	0
N.S.	1	1.26	0.45	1.19	1.58	1.70	1.20	1.32	0.36	0.00
time (sec)	N/A	0.556	0.011	0.115	0.040	0.095	10.151	0.127	0.243	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	B	B	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	110	49	120	149	137	99	108	77	0
N.S.	1	1.20	0.53	1.30	1.62	1.49	1.08	1.17	0.84	0.00
time (sec)	N/A	0.431	0.012	0.082	0.040	0.094	0.939	0.124	0.227	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	B	A	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	85	47	71	116	58	70	61	16	0
N.S.	1	1.10	0.61	0.92	1.51	0.75	0.91	0.79	0.21	0.00
time (sec)	N/A	0.361	0.007	0.086	0.036	0.094	0.529	0.120	0.227	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	A	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	108	49	120	74	137	94	0	22	0
N.S.	1	1.24	0.56	1.38	0.85	1.57	1.08	0.00	0.25	0.00
time (sec)	N/A	0.459	0.015	0.089	0.038	0.095	1.436	0.000	0.230	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	A	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	141	84	132	57	178	129	0	22	0
N.S.	1	1.24	0.74	1.16	0.50	1.56	1.13	0.00	0.19	0.00
time (sec)	N/A	0.573	0.054	0.088	0.115	0.093	10.873	0.000	0.242	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	16	15	16	15	18
N.S.	1	1.00	1.12	0.88	1.00	1.00	0.94	1.00	0.94	1.12
time (sec)	N/A	0.230	24.057	0.058	0.830	0.096	0.610	0.163	0.215	0.941

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	16	15	16	17	18
N.S.	1	1.00	1.12	0.88	1.00	1.00	0.94	1.00	1.06	1.12
time (sec)	N/A	0.221	30.424	0.053	0.981	0.084	0.472	0.131	0.258	0.992

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	8	10	0	10	10	11	10
N.S.	1	1.00	1.20	0.80	1.00	0.00	1.00	1.00	1.10	1.00
time (sec)	N/A	0.334	4.531	0.018	0.217	0.000	4.290	0.132	0.239	0.913

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	17	0	0	0	0	0	21	38
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	1.05	1.90
time (sec)	N/A	0.200	0.125	0.000	0.000	0.000	0.000	0.000	0.209	1.009

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	0	0	108	0	0	25	40
N.S.	1	1.00	0.92	0.00	0.00	4.50	0.00	0.00	1.04	1.67
time (sec)	N/A	0.207	0.066	0.000	0.000	0.102	0.000	0.000	0.227	0.971

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	33	0	0	0	0	0	21	111
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	0.45	2.36
time (sec)	N/A	0.233	0.109	0.000	0.000	0.000	0.000	0.000	0.253	1.137

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	68	0	0	0	0	0	25	0
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.273	0.920	0.000	0.000	0.000	0.000	0.000	0.234	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	17	20	22	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.94	1.11	1.22	1.11
time (sec)	N/A	0.238	2.098	0.034	0.240	0.092	11.876	0.338	0.245	1.039

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	237	249	206	0	161	340	0	0	248	0
N.S.	1	1.05	0.87	0.00	0.68	1.43	0.00	0.00	1.05	0.00
time (sec)	N/A	0.550	0.136	0.000	0.112	0.096	0.000	0.000	0.261	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	131	0	102	241	0	0	317	0
N.S.	1	1.00	0.91	0.00	0.71	1.67	0.00	0.00	2.20	0.00
time (sec)	N/A	0.424	0.115	0.000	0.080	0.084	0.000	0.000	0.227	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	118	101	0	79	168	0	0	112	0
N.S.	1	1.07	0.92	0.00	0.72	1.53	0.00	0.00	1.02	0.00
time (sec)	N/A	0.315	0.036	0.000	0.075	0.090	0.000	0.000	0.225	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	14	16	16	18
N.S.	1	1.00	1.14	1.00	1.14	1.14	1.00	1.14	1.14	1.29
time (sec)	N/A	0.208	11.087	0.020	0.147	0.074	0.314	0.114	0.215	0.956

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	159	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	9.94	1.12
time (sec)	N/A	0.222	2.879	0.024	0.169	0.089	0.320	0.125	0.250	0.992

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	67	54	73	55	86	0	0	226	0
N.S.	1	1.14	0.92	1.24	0.93	1.46	0.00	0.00	3.83	0.00
time (sec)	N/A	0.272	0.019	0.138	0.107	0.089	0.000	0.000	0.234	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	67	53	73	55	86	0	0	318	0
N.S.	1	1.14	0.90	1.24	0.93	1.46	0.00	0.00	5.39	0.00
time (sec)	N/A	0.289	0.017	0.132	0.087	0.098	0.000	0.000	0.225	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	67	54	73	55	86	0	0	76	0
N.S.	1	1.14	0.92	1.24	0.93	1.46	0.00	0.00	1.29	0.00
time (sec)	N/A	0.279	0.017	0.119	0.117	0.101	0.000	0.000	0.244	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	67	53	73	55	78	0	0	82	0
N.S.	1	1.14	0.90	1.24	0.93	1.32	0.00	0.00	1.39	0.00
time (sec)	N/A	0.273	0.010	0.114	0.088	0.090	0.000	0.000	0.225	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	57	49	67	43	78	0	0	15	0
N.S.	1	1.16	1.00	1.37	0.88	1.59	0.00	0.00	0.31	0.00
time (sec)	N/A	0.270	0.016	0.129	0.113	0.108	0.000	0.000	0.223	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	63	51	67	55	86	0	0	15	0
N.S.	1	1.15	0.93	1.22	1.00	1.56	0.00	0.00	0.27	0.00
time (sec)	N/A	0.278	0.014	0.138	0.112	0.114	0.000	0.000	0.217	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	67	54	71	55	86	0	0	15	0
N.S.	1	1.14	0.92	1.20	0.93	1.46	0.00	0.00	0.25	0.00
time (sec)	N/A	0.285	0.017	0.122	0.135	0.091	0.000	0.000	0.248	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	79	0	71	136	0	0	728	0
N.S.	1	1.00	0.92	0.00	0.83	1.58	0.00	0.00	8.47	0.00
time (sec)	N/A	0.363	0.085	0.000	0.109	0.082	0.000	0.000	0.239	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	78	0	71	136	0	0	510	0
N.S.	1	1.00	0.92	0.00	0.84	1.60	0.00	0.00	6.00	0.00
time (sec)	N/A	0.352	0.085	0.000	0.131	0.113	0.000	0.000	0.240	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	79	0	71	136	0	0	330	0
N.S.	1	1.00	0.92	0.00	0.83	1.58	0.00	0.00	3.84	0.00
time (sec)	N/A	0.347	0.085	0.000	0.114	0.088	0.000	0.000	0.222	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	76	0	71	122	0	0	190	0
N.S.	1	1.00	0.89	0.00	0.84	1.44	0.00	0.00	2.24	0.00
time (sec)	N/A	0.334	0.071	0.000	0.112	0.093	0.000	0.000	0.244	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	63	0	55	117	0	0	17	0
N.S.	1	1.00	0.88	0.00	0.76	1.62	0.00	0.00	0.24	0.00
time (sec)	N/A	0.328	0.057	0.000	0.133	0.104	0.000	0.000	0.224	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	72	0	0	136	0	0	17	0
N.S.	1	1.00	0.87	0.00	0.00	1.64	0.00	0.00	0.20	0.00
time (sec)	N/A	0.350	0.071	0.000	0.000	0.101	0.000	0.000	0.241	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	77	0	0	136	0	0	17	0
N.S.	1	1.00	0.92	0.00	0.00	1.62	0.00	0.00	0.20	0.00
time (sec)	N/A	0.358	0.075	0.000	0.000	0.083	0.000	0.000	0.261	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	0	0	0	21	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.88	0.00
time (sec)	N/A	0.258	0.143	0.000	0.000	0.000	0.000	0.000	0.236	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	0	0	0	25	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	1.04	0.00
time (sec)	N/A	0.262	0.139	0.000	0.000	0.000	0.000	0.000	0.234	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	45	0	0	0	0	0	21	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.282	0.090	0.000	0.000	0.000	0.000	0.000	0.230	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	67	0	0	0	0	0	25	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.353	0.292	0.000	0.000	0.000	0.000	0.000	0.222	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	128	112	235	284	517	262	210	196
N.S.	1	1.00	1.31	1.14	2.40	2.90	5.28	2.67	2.14	2.00
time (sec)	N/A	0.377	0.361	0.456	0.053	0.087	0.344	0.138	0.252	1.237

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	88	85	141	172	314	150	122	118
N.S.	1	1.00	1.19	1.15	1.91	2.32	4.24	2.03	1.65	1.59
time (sec)	N/A	0.315	0.283	0.378	0.055	0.102	0.269	0.133	0.249	1.118

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	48	48	66	81	162	69	55	56
N.S.	1	1.00	0.96	0.96	1.32	1.62	3.24	1.38	1.10	1.12
time (sec)	N/A	0.254	0.109	0.270	0.036	0.094	0.184	0.128	0.228	1.026

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	60	96	71	79	0	69	31	0
N.S.	1	1.00	0.86	1.37	1.01	1.13	0.00	0.99	0.44	0.00
time (sec)	N/A	0.367	0.431	0.372	0.110	0.088	0.000	0.135	0.205	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	83	153	88	134	0	630	181	0
N.S.	1	1.00	0.87	1.61	0.93	1.41	0.00	6.63	1.91	0.00
time (sec)	N/A	0.411	0.429	0.428	0.082	0.111	0.000	0.155	0.231	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	109	303	99	222	0	322	0	0
N.S.	1	1.00	0.83	2.31	0.76	1.69	0.00	2.46	0.00	0.00
time (sec)	N/A	0.457	0.547	0.478	0.092	0.095	0.000	0.131	0.257	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	220	191	525	596	1134	580	755	393
N.S.	1	1.00	0.95	0.82	2.26	2.57	4.89	2.50	3.25	1.69
time (sec)	N/A	0.577	0.879	1.240	0.074	0.099	0.658	0.138	0.873	2.059

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	189	141	326	351	694	333	429	217
N.S.	1	1.00	1.09	0.81	1.87	2.02	3.99	1.91	2.47	1.25
time (sec)	N/A	0.490	0.488	0.931	0.065	0.095	0.483	0.126	0.215	1.647

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	86	84	167	164	357	155	189	104
N.S.	1	1.00	0.84	0.82	1.64	1.61	3.50	1.52	1.85	1.02
time (sec)	N/A	0.352	12.496	0.659	0.046	0.100	0.333	0.126	0.226	0.398

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	143	117	193	150	149	0	135	55	0
N.S.	1	0.96	0.79	1.30	1.01	1.00	0.00	0.91	0.37	0.00
time (sec)	N/A	0.680	0.726	1.040	0.112	0.110	0.000	0.120	0.262	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	173	214	313	183	266	0	1134	377	0
N.S.	1	1.02	1.26	1.84	1.08	1.56	0.00	6.67	2.22	0.00
time (sec)	N/A	0.649	0.727	1.098	0.123	0.108	0.000	0.181	0.234	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	327	198	625	205	454	0	682	763	0
N.S.	1	1.39	0.84	2.65	0.87	1.92	0.00	2.89	3.23	0.00
time (sec)	N/A	0.952	1.927	1.292	0.120	0.110	0.000	0.121	0.253	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	146	206	435	238	364	0	0	91	0
N.S.	1	1.11	1.56	3.30	1.80	2.76	0.00	0.00	0.69	0.00
time (sec)	N/A	0.838	0.928	0.398	0.216	0.105	0.000	0.000	0.241	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	115	152	227	0	201	0	0	65	0
N.S.	1	1.14	1.50	2.25	0.00	1.99	0.00	0.00	0.64	0.00
time (sec)	N/A	0.637	0.669	0.274	0.000	0.098	0.000	0.000	0.252	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	65	185	66	75	60	56	67	39	56
N.S.	1	1.03	2.94	1.05	1.19	0.95	0.89	1.06	0.62	0.89
time (sec)	N/A	0.370	0.348	0.381	0.042	0.100	0.110	0.111	0.229	1.270

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	102	126	117	23	32	24
N.S.	1	1.00	1.09	0.91	4.43	5.48	5.09	1.00	1.39	1.04
time (sec)	N/A	0.246	26.135	0.112	0.156	0.090	3.581	0.112	0.246	1.160

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	158	204	187	23	67	24
N.S.	1	1.00	1.09	0.91	6.87	8.87	8.13	1.00	2.91	1.04
time (sec)	N/A	0.247	24.807	0.107	0.231	0.074	11.161	0.167	0.263	1.149

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	308	473	723	635	917	0	0	129	0
N.S.	1	1.01	1.55	2.37	2.08	3.01	0.00	0.00	0.42	0.00
time (sec)	N/A	1.322	2.657	1.010	0.264	0.097	0.000	0.000	0.224	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	269	374	0	482	0	0	94	0
N.S.	1	1.00	1.12	1.55	0.00	2.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.973	1.558	0.793	0.000	0.099	0.000	0.000	0.224	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	154	241	113	255	160	167	195	59	160
N.S.	1	0.97	1.53	0.72	1.61	1.01	1.06	1.23	0.37	1.01
time (sec)	N/A	0.522	1.319	1.038	0.066	0.092	0.236	0.121	0.213	1.483

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	606	793	0	23	59	24
N.S.	1	1.00	1.09	0.91	26.35	34.48	0.00	1.00	2.57	1.04
time (sec)	N/A	0.238	24.422	0.208	0.459	0.109	0.000	0.126	0.245	1.222

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	723	965	0	23	107	24
N.S.	1	1.00	1.09	0.91	31.43	41.96	0.00	1.00	4.65	1.04
time (sec)	N/A	0.242	25.231	0.198	0.668	0.102	0.000	0.239	0.227	1.338

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-2)	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	204	141	174	0	0	0	134	20	149
N.S.	1	1.13	0.78	0.96	0.00	0.00	0.00	0.74	0.11	0.82
time (sec)	N/A	0.807	0.253	0.248	0.000	0.000	0.000	0.172	0.224	0.758

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-2)	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	164	125	151	0	0	0	106	20	126
N.S.	1	1.21	0.92	1.11	0.00	0.00	0.00	0.78	0.15	0.93
time (sec)	N/A	0.648	2.063	0.174	0.000	0.000	0.000	0.172	0.230	0.343

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-2)	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	130	105	128	0	0	0	78	20	92
N.S.	1	1.17	0.95	1.15	0.00	0.00	0.00	0.70	0.18	0.83
time (sec)	N/A	0.536	0.707	0.187	0.000	0.000	0.000	0.173	0.245	1.209

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-2)	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	90	87	105	0	0	0	50	18	80
N.S.	1	1.36	1.32	1.59	0.00	0.00	0.00	0.76	0.27	1.21
time (sec)	N/A	0.382	0.342	0.175	0.000	0.000	0.000	0.177	0.216	1.190

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	89	96	0	0	0	0	0	20	0
N.S.	1	0.71	0.77	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.516	0.299	0.000	0.000	0.000	0.000	0.000	0.245	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	122	133	0	0	0	0	0	20	0
N.S.	1	0.82	0.89	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.631	0.444	0.000	0.000	0.000	0.000	0.000	0.247	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	204	159	170	0	0	0	0	0	20	0
N.S.	1	0.78	0.83	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.735	0.549	0.000	0.000	0.000	0.000	0.000	0.241	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	377	398	269	0	0	0	0	247	47	0
N.S.	1	1.06	0.71	0.00	0.00	0.00	0.00	0.66	0.12	0.00
time (sec)	N/A	1.631	7.558	0.000	0.000	0.000	0.000	0.203	0.239	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	303	281	173	0	0	0	0	179	47	0
N.S.	1	0.93	0.57	0.00	0.00	0.00	0.00	0.59	0.16	0.00
time (sec)	N/A	0.878	7.370	0.000	0.000	0.000	0.000	0.204	0.247	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	185	176	138	0	0	0	0	111	43	0
N.S.	1	0.95	0.75	0.00	0.00	0.00	0.00	0.60	0.23	0.00
time (sec)	N/A	0.606	3.248	0.000	0.000	0.000	0.000	0.198	0.236	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	261	149	146	0	0	0	0	0	47	0
N.S.	1	0.57	0.56	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.532	1.610	0.000	0.000	0.000	0.000	0.000	0.255	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	302	184	243	0	0	0	0	0	47	0
N.S.	1	0.61	0.80	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.525	1.869	0.000	0.000	0.000	0.000	0.000	0.220	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	638	0	2918	0	0	0	0	0	77	0
N.S.	1	0.00	4.57	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.000	15.385	0.000	0.000	0.000	0.000	0.000	0.282	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	506	460	300	0	0	0	0	0	77	0
N.S.	1	0.91	0.59	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	1.305	9.670	0.000	0.000	0.000	0.000	0.000	0.249	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	312	262	218	0	0	0	0	0	71	0
N.S.	1	0.84	0.70	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.725	9.246	0.000	0.000	0.000	0.000	0.000	0.241	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	403	205	242	0	0	0	0	0	77	0
N.S.	1	0.51	0.60	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.640	4.318	0.000	0.000	0.000	0.000	0.000	0.253	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	444	240	347	0	0	0	0	0	77	0
N.S.	1	0.54	0.78	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.709	5.009	0.000	0.000	0.000	0.000	0.000	0.230	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	536	408	4751	0	0	0	0	0	77	0
N.S.	1	0.76	8.86	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.919	10.207	0.000	0.000	0.000	0.000	0.000	0.248	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	493	288	331	0	0	0	0	0	74	0
N.S.	1	0.58	0.67	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.970	0.767	0.000	0.000	0.000	0.000	0.000	0.240	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	349	210	276	0	0	0	0	0	74	0
N.S.	1	0.60	0.79	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.700	0.615	0.000	0.000	0.000	0.000	0.000	0.250	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	207	134	178	0	0	0	0	0	70	0
N.S.	1	0.65	0.86	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.433	0.456	0.000	0.000	0.000	0.000	0.000	0.225	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	17	19	40	19	19	72	20
N.S.	1	1.00	1.10	0.81	0.90	1.90	0.90	0.90	3.43	0.95
time (sec)	N/A	0.259	3.505	0.075	0.223	0.117	2.817	0.243	0.247	1.268

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	17	19	44	20	19	80	20
N.S.	1	1.00	1.10	0.81	0.90	2.10	0.95	0.90	3.81	0.95
time (sec)	N/A	0.256	3.574	0.079	0.228	0.083	5.391	0.264	0.236	1.284

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	807	475	546	0	0	0	0	0	110	0
N.S.	1	0.59	0.68	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	1.610	2.023	0.000	0.000	0.000	0.000	0.000	2.857	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	506	320	384	0	0	0	0	0	110	0
N.S.	1	0.63	0.76	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	1.015	1.082	0.000	0.000	0.000	0.000	0.000	0.251	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	288	215	258	0	0	0	0	0	106	0
N.S.	1	0.75	0.90	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.649	0.727	0.000	0.000	0.000	0.000	0.000	0.265	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	17	19	209	19	19	112	20
N.S.	1	1.00	1.10	0.81	0.90	9.95	0.90	0.90	5.33	0.95
time (sec)	N/A	0.266	20.459	0.082	0.239	0.128	29.202	0.273	0.233	1.736

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	17	19	209	20	19	128	20
N.S.	1	1.00	1.10	0.81	0.90	9.95	0.95	0.90	6.10	0.95
time (sec)	N/A	0.254	22.177	0.079	0.213	0.091	53.897	0.272	0.249	1.774

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1016	737	1200	0	0	0	0	0	71	0
N.S.	1	0.73	1.18	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	2.810	2.898	0.000	0.000	0.000	0.000	0.000	0.248	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	689	484	482	0	0	0	0	0	71	0
N.S.	1	0.70	0.70	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.615	1.409	0.000	0.000	0.000	0.000	0.000	0.221	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	416	299	337	0	0	0	0	0	69	0
N.S.	1	0.72	0.81	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.786	1.050	0.000	0.000	0.000	0.000	0.000	0.219	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	17	19	396	0	19	70	20
N.S.	1	1.00	1.10	0.81	0.90	18.86	0.00	0.90	3.33	0.95
time (sec)	N/A	0.265	35.359	0.080	0.215	0.103	0.000	0.304	0.226	2.087

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	17	19	0	17	19	22	20
N.S.	1	1.00	1.10	0.81	0.90	0.00	0.81	0.90	1.05	0.95
time (sec)	N/A	0.253	3.960	0.076	0.204	0.000	2.389	0.207	0.264	1.031

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	48	0	23	23	24
N.S.	1	1.00	1.09	0.91	1.00	2.09	0.00	1.00	1.00	1.04
time (sec)	N/A	0.239	2.825	0.052	0.219	0.107	0.000	0.199	0.276	1.216

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	410	394	410	0	375	374	0	0	832	0
N.S.	1	0.96	1.00	0.00	0.91	0.91	0.00	0.00	2.03	0.00
time (sec)	N/A	0.875	1.057	0.000	0.165	0.124	0.000	0.000	0.255	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	262	229	0	210	259	0	0	568	0
N.S.	1	0.98	0.85	0.00	0.78	0.97	0.00	0.00	2.12	0.00
time (sec)	N/A	0.691	0.496	0.000	0.109	0.101	0.000	0.000	0.257	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	207	0	101	134	0	0	283	0
N.S.	1	1.00	1.53	0.00	0.75	0.99	0.00	0.00	2.10	0.00
time (sec)	N/A	0.400	0.373	0.000	0.093	0.104	0.000	0.000	0.232	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	86	20	23	26	24
N.S.	1	1.00	1.09	0.91	1.00	3.74	0.87	1.00	1.13	1.04
time (sec)	N/A	0.256	3.359	0.056	0.151	0.085	12.392	0.114	0.253	1.097

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	696	0	23	36	24
N.S.	1	1.00	1.09	0.91	1.00	30.26	0.00	1.00	1.57	1.04
time (sec)	N/A	0.247	12.869	0.127	0.171	0.114	0.000	0.123	0.260	1.304

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	123	110	234	168	264	258	213	187
N.S.	1	1.00	1.38	1.24	2.63	1.89	2.97	2.90	2.39	2.10
time (sec)	N/A	0.360	0.272	0.323	0.051	0.094	0.277	0.119	0.235	1.078

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	83	83	139	102	151	146	124	110
N.S.	1	1.00	1.24	1.24	2.07	1.52	2.25	2.18	1.85	1.64
time (sec)	N/A	0.308	0.184	0.264	0.052	0.097	0.211	0.116	0.215	0.115

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	43	46	65	51	68	64	56	49
N.S.	1	1.00	0.96	1.02	1.44	1.13	1.51	1.42	1.24	1.09
time (sec)	N/A	0.240	0.123	0.210	0.039	0.084	0.177	0.114	0.212	0.089

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	57	94	71	111	0	68	32	0
N.S.	1	1.00	0.89	1.47	1.11	1.73	0.00	1.06	0.50	0.00
time (sec)	N/A	0.367	0.097	0.257	0.088	0.092	0.000	0.119	0.248	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	71	149	88	162	0	630	181	0
N.S.	1	1.00	0.82	1.71	1.01	1.86	0.00	7.24	2.08	0.00
time (sec)	N/A	0.387	0.246	0.299	0.098	0.088	0.000	0.142	0.243	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	95	296	99	274	0	319	0	0
N.S.	1	1.00	0.77	2.41	0.80	2.23	0.00	2.59	0.00	0.00
time (sec)	N/A	0.460	0.383	0.351	0.106	0.093	0.000	0.112	0.252	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	235	213	520	418	779	598	940	481
N.S.	1	1.00	0.99	0.90	2.19	1.76	3.29	2.52	3.97	2.03
time (sec)	N/A	0.565	0.648	0.928	0.089	0.098	0.437	0.124	0.854	2.756

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	249	157	322	247	456	344	554	281
N.S.	1	1.00	1.37	0.86	1.77	1.36	2.51	1.89	3.04	1.54
time (sec)	N/A	0.453	0.397	0.761	0.072	0.092	0.316	0.119	0.220	1.467

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	98	111	164	128	219	159	261	135
N.S.	1	1.00	0.87	0.98	1.45	1.13	1.94	1.41	2.31	1.19
time (sec)	N/A	0.329	4.168	0.559	0.050	0.084	0.218	0.123	0.249	0.162

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	134	201	148	232	0	144	59	0
N.S.	1	1.00	0.86	1.29	0.95	1.49	0.00	0.92	0.38	0.00
time (sec)	N/A	0.573	0.219	0.971	0.119	0.085	0.000	0.122	0.229	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	232	319	181	357	0	1135	398	0
N.S.	1	1.00	1.27	1.74	0.99	1.95	0.00	6.20	2.17	0.00
time (sec)	N/A	0.612	0.425	1.065	0.127	0.099	0.000	0.187	0.222	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	395	626	203	590	0	678	790	0
N.S.	1	1.00	1.63	2.59	0.84	2.44	0.00	2.80	3.26	0.00
time (sec)	N/A	0.729	0.581	1.182	0.126	0.124	0.000	0.129	0.237	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	404	377	318	0	0	1004	0	0	358	0
N.S.	1	0.93	0.79	0.00	0.00	2.49	0.00	0.00	0.89	0.00
time (sec)	N/A	1.660	0.168	0.000	0.000	0.137	0.000	0.000	0.262	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	296	283	233	0	0	708	0	0	250	0
N.S.	1	0.96	0.79	0.00	0.00	2.39	0.00	0.00	0.84	0.00
time (sec)	N/A	1.264	0.093	0.000	0.000	0.111	0.000	0.000	0.231	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	189	142	393	0	455	0	0	144	0
N.S.	1	1.01	0.76	2.10	0.00	2.43	0.00	0.00	0.77	0.00
time (sec)	N/A	0.805	0.025	0.189	0.000	0.088	0.000	0.000	0.266	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	27	17	22	31	22
N.S.	1	1.00	1.10	1.00	1.10	1.35	0.85	1.10	1.55	1.10
time (sec)	N/A	0.247	0.838	0.065	0.201	0.076	10.033	0.135	0.255	1.050

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	51	19	22	128	22
N.S.	1	1.00	1.10	1.00	1.10	2.55	0.95	1.10	6.40	1.10
time (sec)	N/A	0.251	0.772	0.063	0.304	0.095	57.061	0.310	0.250	1.059

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	549	520	428	0	0	3957	0	0	0	0
N.S.	1	0.95	0.78	0.00	0.00	7.21	0.00	0.00	0.00	0.00
time (sec)	N/A	2.696	0.961	0.000	0.000	0.167	0.000	0.000	0.272	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	263	194	519	0	1717	0	0	0	0
N.S.	1	1.04	0.76	2.04	0.00	6.76	0.00	0.00	0.00	0.00
time (sec)	N/A	1.076	0.598	0.325	0.000	0.118	0.000	0.000	0.260	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	406	55	0	22	66	22
N.S.	1	1.00	1.10	1.00	20.30	2.75	0.00	1.10	3.30	1.10
time (sec)	N/A	0.243	25.143	0.138	0.585	0.099	0.000	0.400	0.248	1.162

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	606	96	0	22	121	22
N.S.	1	1.00	1.10	1.00	30.30	4.80	0.00	1.10	6.05	1.10
time (sec)	N/A	0.243	25.590	0.135	0.857	0.109	0.000	1.369	0.253	1.267

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	544	834	773	1232	0	6396	0	0	0	0
N.S.	1	1.53	1.42	2.26	0.00	11.76	0.00	0.00	0.00	0.00
time (sec)	N/A	3.786	4.728	0.591	0.000	0.211	0.000	0.000	0.281	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	1651	83	0	22	101	22
N.S.	1	1.00	1.10	1.00	82.55	4.15	0.00	1.10	5.05	1.10
time (sec)	N/A	0.252	44.022	0.220	2.467	0.105	0.000	1.269	0.243	1.123

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	2122	141	0	22	181	22
N.S.	1	1.00	1.10	1.00	106.10	7.05	0.00	1.10	9.05	1.10
time (sec)	N/A	0.261	43.551	0.237	3.828	0.153	0.000	51.999	0.278	1.248

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	0	22	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.00	1.10	1.10	1.10
time (sec)	N/A	0.233	2.898	0.046	0.215	0.097	0.000	0.226	0.238	1.141

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	448	0	377	829	0	0	1232	0
N.S.	1	1.00	0.83	0.00	0.69	1.53	0.00	0.00	2.27	0.00
time (sec)	N/A	1.170	1.041	0.000	0.156	0.130	0.000	0.000	0.266	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	254	0	208	517	0	0	652	0
N.S.	1	1.00	0.90	0.00	0.74	1.84	0.00	0.00	2.32	0.00
time (sec)	N/A	0.679	0.443	0.000	0.107	0.109	0.000	0.000	0.251	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	118	0	101	249	0	0	284	0
N.S.	1	1.00	0.90	0.00	0.77	1.90	0.00	0.00	2.17	0.00
time (sec)	N/A	0.374	0.118	0.000	0.090	0.093	0.000	0.000	0.228	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	17	22	48	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.85	1.10	2.40	1.10
time (sec)	N/A	0.236	1.062	0.033	0.159	0.083	1.283	0.115	0.339	0.990

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	38	19	22	62433	22
N.S.	1	1.00	1.10	1.00	1.10	1.90	0.95	1.10	3121.65	1.10
time (sec)	N/A	0.232	4.420	0.115	0.166	0.101	30.748	0.149	1.393	1.086

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	168	214	579	318	456	0	0	130	0
N.S.	1	1.03	1.31	3.55	1.95	2.80	0.00	0.00	0.80	0.00
time (sec)	N/A	0.979	1.479	0.415	0.163	0.106	0.000	0.000	0.236	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	137	187	321	0	261	0	0	92	0
N.S.	1	1.05	1.44	2.47	0.00	2.01	0.00	0.00	0.71	0.00
time (sec)	N/A	0.755	0.990	0.314	0.000	0.094	0.000	0.000	0.239	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	239	86	108	96	73	111	54	74
N.S.	1	1.00	2.75	0.99	1.24	1.10	0.84	1.28	0.62	0.85
time (sec)	N/A	0.479	0.695	0.492	0.083	0.087	0.176	0.113	0.227	1.448

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	38	61	28	36	33	24	33	23	27
N.S.	1	1.09	1.74	0.80	1.03	0.94	0.69	0.94	0.66	0.77
time (sec)	N/A	0.268	0.142	0.157	0.034	0.089	0.079	0.114	0.239	1.109

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	116	154	425	29	47	30
N.S.	1	1.00	1.07	0.93	4.00	5.31	14.66	1.00	1.62	1.03
time (sec)	N/A	0.231	40.179	0.163	0.135	0.103	12.592	0.133	0.242	1.158

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	194	232	631	29	68	30
N.S.	1	1.00	1.07	0.93	6.69	8.00	21.76	1.00	2.34	1.03
time (sec)	N/A	0.239	28.790	0.133	0.192	0.091	24.156	0.201	0.217	1.199

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	266	2619	699	671	823	0	0	312	0
N.S.	1	1.10	10.87	2.90	2.78	3.41	0.00	0.00	1.29	0.00
time (sec)	N/A	2.207	3.341	0.836	0.252	0.116	0.000	0.000	0.228	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	204	260	385	0	474	0	0	195	0
N.S.	1	1.11	1.41	2.09	0.00	2.58	0.00	0.00	1.06	0.00
time (sec)	N/A	1.606	1.808	0.648	0.000	0.119	0.000	0.000	0.230	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	130	238	134	238	172	224	251	99	143
N.S.	1	1.09	2.00	1.13	2.00	1.45	1.88	2.11	0.83	1.20
time (sec)	N/A	0.897	2.826	0.899	0.125	0.096	0.268	0.124	0.220	1.450

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	50	59	60	74	69	99	61	37	59
N.S.	1	0.96	1.13	1.15	1.42	1.33	1.90	1.17	0.71	1.13
time (sec)	N/A	0.378	0.154	0.306	0.039	0.098	0.141	0.126	0.238	1.152

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	29	177	203	950	31	41	32
N.S.	1	1.00	1.06	0.94	5.71	6.55	30.65	1.00	1.32	1.03
time (sec)	N/A	0.268	24.146	0.369	0.211	0.115	110.618	0.143	0.223	1.143

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	29	270	281	0	31	70	32
N.S.	1	1.00	1.06	0.94	8.71	9.06	0.00	1.00	2.26	1.03
time (sec)	N/A	0.275	14.330	0.269	0.249	0.093	0.000	0.247	0.252	1.268

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	B	F(-2)	B	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	0	376	1006	0	1044	0	0	462	0
N.S.	1	0.00	0.99	2.66	0.00	2.76	0.00	0.00	1.22	0.00
time (sec)	N/A	0.000	4.424	1.102	0.000	0.124	0.000	0.000	0.253	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	F(-2)	B	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	0	1661	560	0	595	0	0	199	0
N.S.	1	0.00	5.79	1.95	0.00	2.07	0.00	0.00	0.69	0.00
time (sec)	N/A	0.000	3.346	0.803	0.000	0.092	0.000	0.000	0.227	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	197	325	197	0	230	396	341	155	215
N.S.	1	1.16	1.91	1.16	0.00	1.35	2.33	2.01	0.91	1.26
time (sec)	N/A	1.479	3.422	1.081	0.000	0.101	0.429	0.121	0.226	1.563

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	87	109	95	98	96	175	87	27	94
N.S.	1	1.05	1.31	1.14	1.18	1.16	2.11	1.05	0.33	1.13
time (sec)	N/A	0.364	0.118	0.426	0.037	0.092	0.199	0.120	0.242	1.169

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	29	0	249	1452	31	41	32
N.S.	1	1.00	1.06	0.94	0.00	8.03	46.84	1.00	1.32	1.03
time (sec)	N/A	0.256	34.420	0.366	0.000	0.087	111.008	0.157	0.226	1.215

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	29	0	327	0	31	70	32
N.S.	1	1.00	1.06	0.94	0.00	10.55	0.00	1.00	2.26	1.03
time (sec)	N/A	0.266	15.171	0.305	0.000	0.089	0.000	0.324	0.241	1.346

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	325	341	1034	580	1000	0	0	116	0
N.S.	1	1.04	1.09	3.30	1.85	3.19	0.00	0.00	0.37	0.00
time (sec)	N/A	2.011	2.378	0.704	0.251	0.144	0.000	0.000	0.237	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	236	275	573	347	559	0	0	84	0
N.S.	1	1.05	1.23	2.56	1.55	2.50	0.00	0.00	0.38	0.00
time (sec)	N/A	1.439	1.757	0.549	0.232	0.110	0.000	0.000	0.224	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	132	257	211	0	211	0	0	52	0
N.S.	1	1.05	2.04	1.67	0.00	1.67	0.00	0.00	0.41	0.00
time (sec)	N/A	0.762	2.371	0.608	0.000	0.110	0.000	0.000	0.216	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	50	52	36	62	57	0	48	25	56
N.S.	1	1.22	1.27	0.88	1.51	1.39	0.00	1.17	0.61	1.37
time (sec)	N/A	0.334	0.050	0.270	0.034	0.099	0.000	0.116	0.213	1.840

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	169	235	39	29	39	32
N.S.	1	1.00	1.07	0.93	5.83	8.10	1.34	1.00	1.34	1.10
time (sec)	N/A	0.235	34.676	0.234	0.271	0.102	19.325	20.482	0.342	1.573

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	277	341	0	0	68	32
N.S.	1	1.00	1.07	0.93	9.55	11.76	0.00	0.00	2.34	1.10
time (sec)	N/A	0.237	41.983	0.198	0.361	0.103	0.000	0.000	0.292	1.860

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	B	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	419	0	1205	1604	939	2562	0	0	124	0
N.S.	1	0.00	2.88	3.83	2.24	6.11	0.00	0.00	0.30	0.00
time (sec)	N/A	0.000	8.466	1.101	0.291	0.142	0.000	0.000	0.233	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	B	B	F	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	0	803	889	601	1355	0	0	90	0
N.S.	1	0.00	2.71	3.00	2.03	4.58	0.00	0.00	0.30	0.00
time (sec)	N/A	0.000	7.959	0.779	0.293	0.123	0.000	0.000	0.245	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	175	366	334	0	506	0	0	56	0
N.S.	1	1.07	2.25	2.05	0.00	3.10	0.00	0.00	0.34	0.00
time (sec)	N/A	1.233	3.095	0.723	0.000	0.129	0.000	0.000	0.233	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	59	61	63	109	146	0	90	27	122
N.S.	1	1.04	1.07	1.11	1.91	2.56	0.00	1.58	0.47	2.14
time (sec)	N/A	0.468	0.145	0.405	0.036	0.099	0.000	0.123	0.246	2.347

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	29	330	342	41	0	41	32
N.S.	1	1.00	1.06	0.94	10.65	11.03	1.32	0.00	1.32	1.03
time (sec)	N/A	0.268	67.504	0.263	0.395	0.109	20.208	0.000	0.264	1.434

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	29	477	500	0	0	70	32
N.S.	1	1.00	1.06	0.94	15.39	16.13	0.00	0.00	2.26	1.03
time (sec)	N/A	0.261	78.312	0.224	0.641	0.108	0.000	0.000	0.266	1.638

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	B	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	546	0	2585	2124	1320	4252	0	0	124	0
N.S.	1	0.00	4.73	3.89	2.42	7.79	0.00	0.00	0.23	0.00
time (sec)	N/A	0.000	108.769	1.399	0.390	0.187	0.000	0.000	0.228	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	B	B	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	0	1496	1147	863	2215	0	0	90	0
N.S.	1	0.00	4.07	3.12	2.35	6.02	0.00	0.00	0.24	0.00
time (sec)	N/A	0.000	8.565	0.995	0.351	0.125	0.000	0.000	0.253	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	F	B	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	0	461	441	0	817	0	0	56	0
N.S.	1	0.00	2.15	2.06	0.00	3.82	0.00	0.00	0.26	0.00
time (sec)	N/A	0.000	3.431	0.926	0.000	0.119	0.000	0.000	0.221	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	97	90	91	156	234	0	97	27	132
N.S.	1	1.11	1.03	1.05	1.79	2.69	0.00	1.11	0.31	1.52
time (sec)	N/A	0.632	0.313	0.493	0.054	0.111	0.000	0.135	0.240	1.526

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	0	29	775	903	41	0	41	32
N.S.	1	1.00	0.00	0.94	25.00	29.13	1.32	0.00	1.32	1.03
time (sec)	N/A	0.270	0.000	0.282	0.809	0.147	57.188	0.000	0.281	4.750

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	0	29	975	1144	0	0	70	32
N.S.	1	1.00	0.00	0.94	31.45	36.90	0.00	0.00	2.26	1.03
time (sec)	N/A	0.295	0.000	0.227	1.255	0.168	0.000	0.000	0.286	7.167

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	453	399	608	0	0	1112	0	0	462	0
N.S.	1	0.88	1.34	0.00	0.00	2.45	0.00	0.00	1.02	0.00
time (sec)	N/A	1.879	0.974	0.000	0.000	0.130	0.000	0.000	0.240	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	337	305	367	0	0	782	0	0	323	0
N.S.	1	0.91	1.09	0.00	0.00	2.32	0.00	0.00	0.96	0.00
time (sec)	N/A	1.404	0.734	0.000	0.000	0.117	0.000	0.000	0.266	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	211	189	440	0	500	0	0	186	0
N.S.	1	0.96	0.86	2.00	0.00	2.27	0.00	0.00	0.85	0.00
time (sec)	N/A	0.897	0.570	0.217	0.000	0.101	0.000	0.000	0.233	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	C	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	61	64	82	85	186	269	84	67	121
N.S.	1	1.13	1.19	1.52	1.57	3.44	4.98	1.56	1.24	2.24
time (sec)	N/A	0.317	0.272	0.103	0.123	0.097	30.723	0.123	0.236	1.736

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	88	34	0	28	49	28
N.S.	1	1.00	1.08	1.00	3.38	1.31	0.00	1.08	1.88	1.08
time (sec)	N/A	0.241	2.174	0.082	0.172	0.085	0.000	0.192	0.290	1.079

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	551	501	979	0	0	2612	0	0	877	0
N.S.	1	0.91	1.78	0.00	0.00	4.74	0.00	0.00	1.59	0.00
time (sec)	N/A	2.971	1.636	0.000	0.000	0.166	0.000	0.000	0.249	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	407	376	453	0	0	1689	0	0	559	0
N.S.	1	0.92	1.11	0.00	0.00	4.15	0.00	0.00	1.37	0.00
time (sec)	N/A	2.215	1.689	0.000	0.000	0.114	0.000	0.000	0.234	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	258	261	510	0	946	0	0	288	0
N.S.	1	0.98	1.00	1.95	0.00	3.61	0.00	0.00	1.10	0.00
time (sec)	N/A	1.296	2.453	0.355	0.000	0.114	0.000	0.000	0.270	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F(-1)	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	79	74	121	119	331	0	111	93	166
N.S.	1	1.11	1.04	1.70	1.68	4.66	0.00	1.56	1.31	2.34
time (sec)	N/A	0.438	1.034	0.186	0.113	0.111	0.000	0.133	0.229	1.256

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	158	36	0	30	781	30
N.S.	1	1.00	1.07	1.00	5.64	1.29	0.00	1.07	27.89	1.07
time (sec)	N/A	0.264	8.649	0.188	0.222	0.094	0.000	0.257	0.327	1.139

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	699	0	1407	0	0	5191	0	0	30	0
N.S.	1	0.00	2.01	0.00	0.00	7.43	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	3.381	0.000	0.000	0.218	0.000	0.000	200.033	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	522	0	740	0	0	3247	0	0	0	0
N.S.	1	0.00	1.42	0.00	0.00	6.22	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	3.536	0.000	0.000	0.155	0.000	0.000	0.281	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	325	306	589	0	1727	0	0	962	0
N.S.	1	0.99	0.93	1.79	0.00	5.25	0.00	0.00	2.92	0.00
time (sec)	N/A	1.819	4.713	0.500	0.000	0.139	0.000	0.000	0.288	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F(-1)	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	124	101	191	164	601	0	151	236	212
N.S.	1	1.16	0.94	1.79	1.53	5.62	0.00	1.41	2.21	1.98
time (sec)	N/A	0.664	2.849	0.277	0.133	0.104	0.000	0.129	0.269	1.294

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	236	36	0	30	267	30
N.S.	1	1.00	1.07	1.00	8.43	1.29	0.00	1.07	9.54	1.07
time (sec)	N/A	0.266	10.831	0.197	0.278	0.088	0.000	0.362	0.404	1.212

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	605	558	734	0	0	1645	0	0	599	0
N.S.	1	0.92	1.21	0.00	0.00	2.72	0.00	0.00	0.99	0.00
time (sec)	N/A	2.510	1.550	0.000	0.000	0.131	0.000	0.000	0.406	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	433	406	454	0	0	1096	0	0	436	0
N.S.	1	0.94	1.05	0.00	0.00	2.53	0.00	0.00	1.01	0.00
time (sec)	N/A	2.006	1.174	0.000	0.000	0.134	0.000	0.000	0.362	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	258	267	532	0	649	0	0	267	0
N.S.	1	0.99	1.02	2.04	0.00	2.49	0.00	0.00	1.02	0.00
time (sec)	N/A	1.249	2.008	0.326	0.000	0.109	0.000	0.000	0.315	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	70	82	63	112	223	0	102	113	347
N.S.	1	1.09	1.28	0.98	1.75	3.48	0.00	1.59	1.77	5.42
time (sec)	N/A	0.397	0.870	0.174	0.117	0.118	0.000	0.132	0.259	1.392

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	28	34	22	0	38	30
N.S.	1	1.00	1.08	1.00	1.08	1.31	0.85	0.00	1.46	1.15
time (sec)	N/A	0.244	4.112	0.122	0.300	0.102	6.628	0.000	0.314	1.169

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	F	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	745	715	1493	0	0	6416	0	0	1686	0
N.S.	1	0.96	2.00	0.00	0.00	8.61	0.00	0.00	2.26	0.00
time (sec)	N/A	4.539	8.203	0.000	0.000	0.221	0.000	0.000	0.368	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	535	523	920	0	0	3805	0	0	1234	0
N.S.	1	0.98	1.72	0.00	0.00	7.11	0.00	0.00	2.31	0.00
time (sec)	N/A	3.101	7.695	0.000	0.000	0.191	0.000	0.000	0.326	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	303	350	626	0	1830	0	0	766	0
N.S.	1	0.99	1.14	2.05	0.00	5.98	0.00	0.00	2.50	0.00
time (sec)	N/A	1.663	5.238	0.441	0.000	0.150	0.000	0.000	0.363	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	89	113	97	137	479	0	123	330	360
N.S.	1	1.11	1.41	1.21	1.71	5.99	0.00	1.54	4.12	4.50
time (sec)	N/A	0.516	1.712	0.217	0.126	0.132	0.000	0.131	0.245	0.425

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	304	36	24	0	40	30
N.S.	1	1.00	1.07	1.00	10.86	1.29	0.86	0.00	1.43	1.07
time (sec)	N/A	0.270	84.351	0.126	0.454	0.109	6.895	0.000	0.320	1.250

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1053	0	2801	0	0	18159	0	0	0	0
N.S.	1	0.00	2.66	0.00	0.00	17.25	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	8.997	0.000	0.000	0.445	0.000	0.000	0.863	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	725	0	1532	0	0	10341	0	0	0	0
N.S.	1	0.00	2.11	0.00	0.00	14.26	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	8.075	0.000	0.000	0.353	0.000	0.000	0.556	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	0	617	861	0	4720	0	0	0	0
N.S.	1	0.00	1.47	2.05	0.00	11.24	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	8.216	0.586	0.000	0.199	0.000	0.000	0.368	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	138	167	142	211	1203	0	176	724	776
N.S.	1	1.22	1.48	1.26	1.87	10.65	0.00	1.56	6.41	6.87
time (sec)	N/A	0.910	2.136	0.291	0.115	0.156	0.000	0.130	0.257	1.643

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	757	36	24	0	40	30
N.S.	1	1.00	1.07	1.00	27.04	1.29	0.86	0.00	1.43	1.07
time (sec)	N/A	0.269	72.762	0.125	0.996	0.236	19.689	0.000	0.351	1.590

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	118	647	264	299	0	0	114	0
N.S.	1	1.00	0.85	4.65	1.90	2.15	0.00	0.00	0.82	0.00
time (sec)	N/A	0.756	0.048	4.703	0.164	0.085	0.000	0.000	0.282	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	107	94	405	164	184	0	0	81	0
N.S.	1	1.01	0.89	3.82	1.55	1.74	0.00	0.00	0.76	0.00
time (sec)	N/A	0.581	0.030	1.739	0.133	0.091	0.000	0.000	0.266	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	76	66	188	0	91	0	0	48	0
N.S.	1	1.04	0.90	2.58	0.00	1.25	0.00	0.00	0.66	0.00
time (sec)	N/A	0.407	0.018	0.825	0.000	0.087	0.000	0.000	0.256	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	24	23	38	20	23	22	30	20	19
N.S.	1	1.04	1.00	1.65	0.87	1.00	0.96	1.30	0.87	0.83
time (sec)	N/A	0.210	0.012	0.497	0.037	0.111	0.102	0.114	0.224	1.108

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	48	40	39	29	39	30
N.S.	1	1.00	1.07	0.93	1.66	1.38	1.34	1.00	1.34	1.03
time (sec)	N/A	0.239	5.750	0.253	0.123	0.075	5.161	0.125	0.296	1.147

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	75	64	71	29	68	30
N.S.	1	1.00	1.07	0.93	2.59	2.21	2.45	1.00	2.34	1.03
time (sec)	N/A	0.238	25.794	0.169	0.187	0.080	24.954	0.183	0.236	1.204

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	111	106	266	373	263	518	355	124	269
N.S.	1	1.03	0.98	2.46	3.45	2.44	4.80	3.29	1.15	2.49
time (sec)	N/A	0.663	1.294	13.595	0.228	0.092	0.354	0.118	0.240	1.628

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	80	78	160	270	160	318	208	90	167
N.S.	1	0.98	0.95	1.95	3.29	1.95	3.88	2.54	1.10	2.04
time (sec)	N/A	0.520	0.833	5.504	0.161	0.108	0.294	0.118	0.295	1.413

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	70	188	76	167	96	56	87
N.S.	1	1.00	1.02	1.25	3.36	1.36	2.98	1.71	1.00	1.55
time (sec)	N/A	0.385	2.564	2.152	0.116	0.104	0.183	0.120	0.252	1.273

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	139	40	44	40	78	41	27	36
N.S.	1	1.00	6.32	1.82	2.00	1.82	3.55	1.86	1.23	1.64
time (sec)	N/A	0.222	0.108	2.020	0.043	0.101	0.113	0.123	0.255	1.060

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	62	103	76	79	0	76	41	0
N.S.	1	1.00	0.82	1.36	1.00	1.04	0.00	1.00	0.54	0.00
time (sec)	N/A	0.597	0.390	10.611	0.158	0.094	0.000	0.125	0.264	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	101	85	164	92	129	0	572	70	0
N.S.	1	0.98	0.83	1.59	0.89	1.25	0.00	5.55	0.68	0.00
time (sec)	N/A	0.682	0.481	25.810	0.219	0.094	0.000	0.181	0.324	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	223	134	429	0	405	1040	618	124	449
N.S.	1	0.97	0.58	1.86	0.00	1.75	4.50	2.68	0.54	1.94
time (sec)	N/A	1.206	1.671	31.387	0.000	0.103	0.557	0.138	0.242	1.960

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	154	99	241	0	227	631	338	90	271
N.S.	1	0.98	0.63	1.54	0.00	1.45	4.02	2.15	0.57	1.73
time (sec)	N/A	0.817	1.163	14.284	0.000	0.104	0.408	0.132	0.246	1.641

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	94	60	113	0	92	321	138	56	144
N.S.	1	0.96	0.61	1.15	0.00	0.94	3.28	1.41	0.57	1.47
time (sec)	N/A	0.565	2.949	5.763	0.000	0.104	0.282	0.133	0.272	1.381

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	28	30	60	49	133	55	27	29
N.S.	1	1.00	1.04	1.11	2.22	1.81	4.93	2.04	1.00	1.07
time (sec)	N/A	0.217	0.037	5.705	0.033	0.101	0.158	0.128	0.245	1.155

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	128	112	180	0	127	0	144	41	0
N.S.	1	0.98	0.85	1.37	0.00	0.97	0.00	1.10	0.31	0.00
time (sec)	N/A	1.092	0.454	27.976	0.000	0.076	0.000	0.135	0.246	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	187	212	299	0	226	0	1080	32	0
N.S.	1	1.04	1.18	1.66	0.00	1.26	0.00	6.00	0.18	0.00
time (sec)	N/A	1.458	0.624	113.990	0.000	0.089	0.000	0.204	200.028	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	442	828	1080	685	1461	0	0	116	0
N.S.	1	0.95	1.79	2.33	1.48	3.16	0.00	0.00	0.25	0.00
time (sec)	N/A	3.069	7.815	33.682	0.267	0.109	0.000	0.000	0.248	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	252	530	550	387	805	0	0	84	0
N.S.	1	0.94	1.98	2.05	1.44	3.00	0.00	0.00	0.31	0.00
time (sec)	N/A	1.738	5.281	13.677	0.226	0.128	0.000	0.000	0.231	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	151	400	218	0	352	0	0	52	0
N.S.	1	0.94	2.48	1.35	0.00	2.19	0.00	0.00	0.32	0.00
time (sec)	N/A	0.889	2.325	5.376	0.000	0.108	0.000	0.000	0.249	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	47	30	64	87	102	0	102	25	74
N.S.	1	1.12	0.71	1.52	2.07	2.43	0.00	2.43	0.60	1.76
time (sec)	N/A	0.254	0.034	4.756	0.053	0.094	0.000	0.135	0.239	1.993

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	327	391	39	29	39	32
N.S.	1	1.00	1.07	0.93	11.28	13.48	1.34	1.00	1.34	1.10
time (sec)	N/A	0.233	85.154	0.503	0.340	0.112	4.277	2.166	0.253	1.301

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	0	27	436	511	71	29	68	32
N.S.	1	1.00	0.00	0.93	15.03	17.62	2.45	1.00	2.34	1.10
time (sec)	N/A	0.238	0.000	0.386	0.556	0.114	39.413	42.269	0.272	1.382

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	450	443	1078	1021	730	1405	0	0	124	0
N.S.	1	0.98	2.40	2.27	1.62	3.12	0.00	0.00	0.28	0.00
time (sec)	N/A	2.976	8.143	142.721	0.312	0.114	0.000	0.000	0.248	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	310	575	510	0	714	0	0	90	0
N.S.	1	0.95	1.77	1.57	0.00	2.20	0.00	0.00	0.28	0.00
time (sec)	N/A	2.083	3.923	44.989	0.000	0.115	0.000	0.000	0.255	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	151	194	143	251	201	0	260	56	205
N.S.	1	0.96	1.23	0.91	1.59	1.27	0.00	1.65	0.35	1.30
time (sec)	N/A	1.006	2.872	16.602	0.068	0.108	0.000	0.126	0.225	3.510

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	43	104	54	0	59	27	43
N.S.	1	1.00	1.00	0.91	2.21	1.15	0.00	1.26	0.57	0.91
time (sec)	N/A	0.315	0.034	15.971	0.048	0.078	0.000	0.126	0.217	1.371

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	0	29	632	768	41	31	41	32
N.S.	1	1.00	0.00	0.94	20.39	24.77	1.32	1.00	1.32	1.03
time (sec)	N/A	0.268	0.000	0.527	0.500	0.110	4.220	59.304	0.246	1.520

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	0	29	762	919	73	0	70	32
N.S.	1	1.00	0.00	0.94	24.58	29.65	2.35	0.00	2.26	1.03
time (sec)	N/A	0.266	0.000	0.421	0.758	0.116	40.013	0.000	0.266	1.544

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	F(-2)	B	F	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	667	0	2008	1909	0	3854	0	0	124	0
N.S.	1	0.00	3.01	2.86	0.00	5.78	0.00	0.00	0.19	0.00
time (sec)	N/A	0.000	8.826	1.412	0.000	0.143	0.000	0.000	0.267	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	400	1363	961	0	2066	0	0	90	0
N.S.	1	0.95	3.22	2.27	0.00	4.88	0.00	0.00	0.21	0.00
time (sec)	N/A	3.034	7.813	140.191	0.000	0.125	0.000	0.000	0.243	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	219	617	396	0	917	0	0	56	0
N.S.	1	0.94	2.65	1.70	0.00	3.94	0.00	0.00	0.24	0.00
time (sec)	N/A	1.224	4.195	40.152	0.000	0.108	0.000	0.000	0.229	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	93	101	125	0	287	0	173	27	137
N.S.	1	0.96	1.04	1.29	0.00	2.96	0.00	1.78	0.28	1.41
time (sec)	N/A	0.305	0.071	39.331	0.000	0.112	0.000	0.122	0.240	1.916

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	29	0	1783	41	31	41	32
N.S.	1	1.00	1.06	0.94	0.00	57.52	1.32	1.00	1.32	1.03
time (sec)	N/A	0.304	49.466	0.547	0.000	0.185	9.310	153.666	0.268	1.983

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	F(-1)	N/A	F(-2)	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	0	29	0	2026	73	0	70	32
N.S.	1	1.00	0.00	0.94	0.00	65.35	2.35	0.00	2.26	1.03
time (sec)	N/A	0.303	0.000	0.438	0.000	0.213	40.800	0.000	0.275	2.127

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	356	358	329	0	0	882	0	0	323	0
N.S.	1	1.01	0.92	0.00	0.00	2.48	0.00	0.00	0.91	0.00
time (sec)	N/A	1.644	0.104	0.000	0.000	0.125	0.000	0.000	0.305	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	244	0	0	609	0	0	224	0
N.S.	1	1.00	0.92	0.00	0.00	2.31	0.00	0.00	0.85	0.00
time (sec)	N/A	1.115	0.094	0.000	0.000	0.127	0.000	0.000	0.278	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	157	412	0	380	0	0	125	0
N.S.	1	1.00	0.92	2.42	0.00	2.24	0.00	0.00	0.74	0.00
time (sec)	N/A	0.645	0.028	0.750	0.000	0.127	0.000	0.000	0.226	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	44	41	33	18	18
N.S.	1	1.00	1.00	1.06	1.00	2.44	2.28	1.83	1.00	1.00
time (sec)	N/A	0.209	0.007	0.408	0.030	0.106	0.771	0.122	0.222	1.014

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	93	34	0	28	159	28
N.S.	1	1.00	1.08	1.00	3.58	1.31	0.00	1.08	6.12	1.08
time (sec)	N/A	0.232	4.915	0.165	0.180	0.121	0.000	0.195	0.262	1.133

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	527	501	933	0	0	2020	0	0	1252	0
N.S.	1	0.95	1.77	0.00	0.00	3.83	0.00	0.00	2.38	0.00
time (sec)	N/A	2.603	1.418	0.000	0.000	0.215	0.000	0.000	0.293	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	389	376	447	0	0	1313	0	0	785	0
N.S.	1	0.97	1.15	0.00	0.00	3.38	0.00	0.00	2.02	0.00
time (sec)	N/A	1.939	1.331	0.000	0.000	0.125	0.000	0.000	0.344	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	258	258	901	0	742	0	0	384	0
N.S.	1	1.03	1.03	3.60	0.00	2.97	0.00	0.00	1.54	0.00
time (sec)	N/A	1.145	1.443	2.134	0.000	0.138	0.000	0.000	0.267	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	71	458	122	116	259	0	110	87	121
N.S.	1	1.04	6.74	1.79	1.71	3.81	0.00	1.62	1.28	1.78
time (sec)	N/A	0.467	0.986	1.787	0.137	0.120	0.000	0.142	0.231	1.259

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	166	36	0	30	238	30
N.S.	1	1.00	1.07	1.00	5.93	1.29	0.00	1.07	8.50	1.07
time (sec)	N/A	0.272	9.111	0.360	0.201	0.084	0.000	0.311	0.321	1.123

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	642	591	1977	0	0	4371	0	0	30	0
N.S.	1	0.92	3.08	0.00	0.00	6.81	0.00	0.00	0.05	0.00
time (sec)	N/A	3.666	8.336	0.000	0.000	0.171	0.000	0.000	200.027	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	465	428	1253	0	0	2726	0	0	1657	0
N.S.	1	0.92	2.69	0.00	0.00	5.86	0.00	0.00	3.56	0.00
time (sec)	N/A	2.412	8.692	0.000	0.000	0.149	0.000	0.000	9.491	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	274	418	975	0	1416	0	0	802	0
N.S.	1	0.92	1.40	3.27	0.00	4.75	0.00	0.00	2.69	0.00
time (sec)	N/A	1.369	3.287	5.815	0.000	0.126	0.000	0.000	0.238	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	53	53	53	127	327	0	92	181	120
N.S.	1	0.90	0.90	0.90	2.15	5.54	0.00	1.56	3.07	2.03
time (sec)	N/A	0.267	0.043	5.298	0.049	0.105	0.000	0.146	0.219	1.235

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	251	36	0	30	334	30
N.S.	1	1.00	1.07	1.00	8.96	1.29	0.00	1.07	11.93	1.07
time (sec)	N/A	0.253	28.961	0.468	0.339	0.076	0.000	0.393	0.306	1.243

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	786	690	3078	0	0	1718	0	0	481	0
N.S.	1	0.88	3.92	0.00	0.00	2.19	0.00	0.00	0.61	0.00
time (sec)	N/A	3.006	10.971	0.000	0.000	0.172	0.000	0.000	0.274	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	558	501	1639	0	0	1094	0	0	341	0
N.S.	1	0.90	2.94	0.00	0.00	1.96	0.00	0.00	0.61	0.00
time (sec)	N/A	2.191	10.354	0.000	0.000	0.129	0.000	0.000	0.282	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	318	520	954	0	588	0	0	199	0
N.S.	1	0.95	1.56	2.86	0.00	1.76	0.00	0.00	0.60	0.00
time (sec)	N/A	1.344	2.554	3.717	0.000	0.154	0.000	0.000	0.256	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	74	114	88	95	92	0	121	70	129
N.S.	1	1.07	1.65	1.28	1.38	1.33	0.00	1.75	1.01	1.87
time (sec)	N/A	0.270	0.074	3.597	0.122	0.098	0.000	0.134	0.215	2.148

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	28	34	22	28	38	30
N.S.	1	1.00	1.08	1.00	1.08	1.31	0.85	1.08	1.46	1.15
time (sec)	N/A	0.230	11.286	0.279	0.266	0.113	1.551	1.358	0.254	1.228

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	780	654	1072	0	0	6367	0	0	0	0
N.S.	1	0.84	1.37	0.00	0.00	8.16	0.00	0.00	0.00	0.00
time (sec)	N/A	3.346	7.123	0.000	0.000	0.263	0.000	0.000	0.283	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	548	469	633	0	0	3582	0	0	1511	0
N.S.	1	0.86	1.16	0.00	0.00	6.54	0.00	0.00	2.76	0.00
time (sec)	N/A	2.528	3.875	0.000	0.000	0.207	0.000	0.000	0.296	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	273	333	1928	0	1296	0	0	842	0
N.S.	1	0.93	1.13	6.54	0.00	4.39	0.00	0.00	2.85	0.00
time (sec)	N/A	1.576	2.276	12.434	0.000	0.133	0.000	0.000	0.298	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	75	104	90	115	353	0	108	209	413
N.S.	1	0.97	1.35	1.17	1.49	4.58	0.00	1.40	2.71	5.36
time (sec)	N/A	0.386	0.310	12.635	0.139	0.084	0.000	0.120	0.244	2.277

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	391	36	24	0	40	30
N.S.	1	1.00	1.07	1.00	13.96	1.29	0.86	0.00	1.43	1.07
time (sec)	N/A	0.255	61.964	0.151	0.407	0.120	1.551	0.000	0.302	1.533

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	928	764	3368	0	0	10642	0	0	0	0
N.S.	1	0.82	3.63	0.00	0.00	11.47	0.00	0.00	0.00	0.00
time (sec)	N/A	3.236	11.300	0.000	0.000	0.297	0.000	0.000	0.457	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	560	485	832	2051	0	4729	0	0	0	0
N.S.	1	0.87	1.49	3.66	0.00	8.44	0.00	0.00	0.00	0.00
time (sec)	N/A	1.995	8.484	31.575	0.000	0.200	0.000	0.000	0.330	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	158	131	205	216	893	0	282	519	381
N.S.	1	1.33	1.10	1.72	1.82	7.50	0.00	2.37	4.36	3.20
time (sec)	N/A	0.384	0.321	31.322	0.119	0.129	0.000	0.137	0.239	3.206

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	1100	36	24	0	40	30
N.S.	1	1.00	1.07	1.00	39.29	1.29	0.86	0.00	1.43	1.07
time (sec)	N/A	0.269	62.072	0.253	1.748	4.676	2.942	0.000	0.317	3.813

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	22	26	2652	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.92	1.08	110.50	1.08
time (sec)	N/A	0.247	9.496	0.388	0.280	0.084	2.545	0.152	0.430	1.146

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	22	26	1317	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.92	1.08	54.88	1.08
time (sec)	N/A	0.245	8.571	0.249	0.249	0.096	1.320	0.141	0.315	1.065

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	205	24	20	24	181	24
N.S.	1	1.00	1.09	1.00	9.32	1.09	0.91	1.09	8.23	1.09
time (sec)	N/A	0.222	2.682	0.079	0.978	0.100	0.771	0.153	0.228	1.026

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	72	78	164	157	411	0	0	296	199
N.S.	1	0.97	1.05	2.22	2.12	5.55	0.00	0.00	4.00	2.69
time (sec)	N/A	0.346	0.925	4.588	0.211	0.082	0.000	0.000	0.223	1.399

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	224	202	491	0	1378	0	0	0	0
N.S.	1	0.96	0.86	2.10	0.00	5.89	0.00	0.00	0.00	0.00
time (sec)	N/A	1.024	1.008	4.385	0.000	0.118	0.000	0.000	0.259	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	348	318	368	0	0	2420	0	0	0	0
N.S.	1	0.91	1.06	0.00	0.00	6.95	0.00	0.00	0.00	0.00
time (sec)	N/A	1.510	0.952	0.000	0.000	0.173	0.000	0.000	0.323	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	72	78	164	157	411	0	0	296	199
N.S.	1	0.97	1.05	2.22	2.12	5.55	0.00	0.00	4.00	2.69
time (sec)	N/A	0.338	0.174	4.193	0.218	0.090	0.000	0.000	0.242	0.002

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	224	202	491	0	1378	0	0	0	0
N.S.	1	0.96	0.86	2.10	0.00	5.89	0.00	0.00	0.00	0.00
time (sec)	N/A	1.003	0.794	4.089	0.000	0.131	0.000	0.000	0.245	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	348	318	368	0	0	2420	0	0	0	0
N.S.	1	0.91	1.06	0.00	0.00	6.95	0.00	0.00	0.00	0.00
time (sec)	N/A	1.462	0.694	0.000	0.000	0.119	0.000	0.000	0.280	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	116	112	308	413	1230	0	0	756	0
N.S.	1	1.04	1.00	2.75	3.69	10.98	0.00	0.00	6.75	0.00
time (sec)	N/A	0.465	1.354	25.447	0.251	0.121	0.000	0.000	0.223	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	301	623	805	0	5233	0	0	0	0
N.S.	1	0.98	2.04	2.63	0.00	17.10	0.00	0.00	0.00	0.00
time (sec)	N/A	1.387	9.936	26.975	0.000	0.208	0.000	0.000	0.311	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	631	561	5753	0	0	11757	0	0	0	0
N.S.	1	0.89	9.12	0.00	0.00	18.63	0.00	0.00	0.00	0.00
time (sec)	N/A	3.138	13.583	0.000	0.000	0.319	0.000	0.000	0.375	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	116	112	308	413	1230	0	0	756	0
N.S.	1	1.04	1.00	2.75	3.69	10.98	0.00	0.00	6.75	0.00
time (sec)	N/A	0.464	0.586	18.869	0.249	0.133	0.000	0.000	0.228	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	301	623	805	0	5233	0	0	0	0
N.S.	1	0.98	2.04	2.63	0.00	17.10	0.00	0.00	0.00	0.00
time (sec)	N/A	1.320	6.647	15.522	0.000	0.234	0.000	0.000	0.285	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	631	561	5753	0	0	11757	0	0	0	0
N.S.	1	0.89	9.12	0.00	0.00	18.63	0.00	0.00	0.00	0.00
time (sec)	N/A	3.001	6.826	0.000	0.000	0.310	0.000	0.000	0.416	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	448	452	2809	0	0	1976	0	0	483	0
N.S.	1	1.01	6.27	0.00	0.00	4.41	0.00	0.00	1.08	0.00
time (sec)	N/A	2.567	9.373	0.000	0.000	0.144	0.000	0.000	0.330	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	330	334	1301	0	0	1265	0	0	304	0
N.S.	1	1.01	3.94	0.00	0.00	3.83	0.00	0.00	0.92	0.00
time (sec)	N/A	1.669	9.147	0.000	0.000	0.111	0.000	0.000	0.245	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	209	377	483	0	692	0	0	146	0
N.S.	1	0.99	1.78	2.28	0.00	3.26	0.00	0.00	0.69	0.00
time (sec)	N/A	1.024	2.395	1.764	0.000	0.104	0.000	0.000	0.247	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	30	33	33	83	132	65	60	30	31
N.S.	1	0.88	0.97	0.97	2.44	3.88	1.91	1.76	0.88	0.91
time (sec)	N/A	0.264	0.027	0.935	0.041	0.094	1.033	0.137	0.228	0.075

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	164	40	0	34	163	34
N.S.	1	1.00	1.06	1.00	5.12	1.25	0.00	1.06	5.09	1.06
time (sec)	N/A	0.261	20.258	0.176	0.235	0.081	0.000	0.274	0.331	1.158

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	683	641	1971	0	0	3847	0	0	36	0
N.S.	1	0.94	2.89	0.00	0.00	5.63	0.00	0.00	0.05	0.00
time (sec)	N/A	3.877	8.903	0.000	0.000	0.188	0.000	0.000	200.023	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	510	484	1188	0	0	2410	0	0	1482	0
N.S.	1	0.95	2.33	0.00	0.00	4.73	0.00	0.00	2.91	0.00
time (sec)	N/A	3.071	4.268	0.000	0.000	0.172	0.000	0.000	0.335	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	324	582	1012	0	1284	0	0	700	0
N.S.	1	1.01	1.81	3.15	0.00	4.00	0.00	0.00	2.18	0.00
time (sec)	N/A	1.606	1.791	5.045	0.000	0.134	0.000	0.000	0.306	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F(-1)	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	107	109	167	160	446	0	155	147	212
N.S.	1	1.13	1.15	1.76	1.68	4.69	0.00	1.63	1.55	2.23
time (sec)	N/A	0.556	0.448	3.059	0.132	0.102	0.000	0.148	0.237	1.324

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	243	42	0	36	265	36
N.S.	1	1.00	1.06	1.00	7.15	1.24	0.00	1.06	7.79	1.06
time (sec)	N/A	0.301	11.702	0.243	0.242	0.105	0.000	0.464	0.310	1.480

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	864	0	5656	0	0	7980	0	0	36	0
N.S.	1	0.00	6.55	0.00	0.00	9.24	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	31.073	0.000	0.000	0.204	0.000	0.000	200.028	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	623	0	1961	0	0	4887	0	0	0	0
N.S.	1	0.00	3.15	0.00	0.00	7.84	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	7.722	0.000	0.000	0.170	0.000	0.000	7.441	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	360	604	1102	0	2465	0	0	1186	0
N.S.	1	0.90	1.51	2.76	0.00	6.16	0.00	0.00	2.96	0.00
time (sec)	N/A	2.487	1.493	13.586	0.000	0.143	0.000	0.000	0.272	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	75	75	80	183	652	0	145	251	180
N.S.	1	0.88	0.88	0.94	2.15	7.67	0.00	1.71	2.95	2.12
time (sec)	N/A	0.348	0.134	8.595	0.039	0.101	0.000	0.153	0.216	1.253

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	348	42	0	36	336	36
N.S.	1	1.00	1.06	1.00	10.24	1.24	0.00	1.06	9.88	1.06
time (sec)	N/A	0.323	36.461	0.276	0.381	0.079	0.000	0.411	0.385	2.767

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1021	886	3078	0	0	1715	0	0	865	0
N.S.	1	0.87	3.01	0.00	0.00	1.68	0.00	0.00	0.85	0.00
time (sec)	N/A	4.182	10.742	0.000	0.000	0.168	0.000	0.000	0.377	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	716	633	1640	0	0	1083	0	0	591	0
N.S.	1	0.88	2.29	0.00	0.00	1.51	0.00	0.00	0.83	0.00
time (sec)	N/A	3.104	9.986	0.000	0.000	0.135	0.000	0.000	0.319	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	390	521	1287	0	589	0	0	319	0
N.S.	1	0.93	1.24	3.06	0.00	1.40	0.00	0.00	0.76	0.00
time (sec)	N/A	2.027	2.680	0.684	0.000	0.119	0.000	0.000	0.329	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	71	71	97	95	92	0	121	70	130
N.S.	1	1.03	1.03	1.41	1.38	1.33	0.00	1.75	1.01	1.88
time (sec)	N/A	0.280	0.065	0.237	0.116	0.107	0.000	0.129	0.227	2.108

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	28	34	22	0	38	28
N.S.	1	1.00	1.08	1.00	1.08	1.31	0.85	0.00	1.46	1.08
time (sec)	N/A	0.252	10.254	0.157	0.320	0.101	1.581	0.000	0.262	1.311

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	917	792	1070	0	0	6503	0	0	0	0
N.S.	1	0.86	1.17	0.00	0.00	7.09	0.00	0.00	0.00	0.00
time (sec)	N/A	5.441	7.355	0.000	0.000	0.243	0.000	0.000	0.526	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	648	573	633	0	0	3662	0	0	0	0
N.S.	1	0.88	0.98	0.00	0.00	5.65	0.00	0.00	0.00	0.00
time (sec)	N/A	3.853	3.881	0.000	0.000	0.206	0.000	0.000	0.367	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	313	331	1858	0	1338	0	0	1082	0
N.S.	1	0.93	0.99	5.55	0.00	3.99	0.00	0.00	3.23	0.00
time (sec)	N/A	2.163	2.468	3.770	0.000	0.184	0.000	0.000	0.294	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	84	104	101	117	350	0	106	211	170
N.S.	1	1.08	1.33	1.29	1.50	4.49	0.00	1.36	2.71	2.18
time (sec)	N/A	0.407	0.352	2.056	0.125	0.084	0.000	0.136	0.240	1.442

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	386	40	29	0	44	36
N.S.	1	1.00	1.06	1.00	12.06	1.25	0.91	0.00	1.38	1.12
time (sec)	N/A	0.270	68.724	0.158	0.418	0.122	2.783	0.000	0.317	1.731

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1176	958	3390	0	0	11164	0	0	0	0
N.S.	1	0.81	2.88	0.00	0.00	9.49	0.00	0.00	0.00	0.00
time (sec)	N/A	5.429	11.499	0.000	0.000	0.314	0.000	0.000	0.641	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	711	600	821	2074	0	4993	0	0	0	0
N.S.	1	0.84	1.15	2.92	0.00	7.02	0.00	0.00	0.00	0.00
time (sec)	N/A	3.142	8.651	11.626	0.000	0.203	0.000	0.000	0.401	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	145	130	212	218	926	0	286	520	337
N.S.	1	1.19	1.07	1.74	1.79	7.59	0.00	2.34	4.26	2.76
time (sec)	N/A	0.395	0.410	6.881	0.127	0.119	0.000	0.147	0.303	2.884

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	1104	42	31	0	46	36
N.S.	1	1.00	1.06	1.00	32.47	1.24	0.91	0.00	1.35	1.06
time (sec)	N/A	0.292	69.926	0.186	1.811	4.038	4.797	0.000	0.340	4.129

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	606	591	2684	0	0	3891	0	0	810	0
N.S.	1	0.98	4.43	0.00	0.00	6.42	0.00	0.00	1.34	0.00
time (sec)	N/A	3.378	14.443	0.000	0.000	0.159	0.000	0.000	0.280	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	437	428	1453	0	0	2414	0	0	479	0
N.S.	1	0.98	3.32	0.00	0.00	5.52	0.00	0.00	1.10	0.00
time (sec)	N/A	2.766	9.018	0.000	0.000	0.140	0.000	0.000	0.315	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	274	483	565	0	1248	0	0	221	0
N.S.	1	0.99	1.74	2.03	0.00	4.49	0.00	0.00	0.79	0.00
time (sec)	N/A	1.687	0.305	7.197	0.000	0.126	0.000	0.000	0.256	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	48	49	54	119	309	87	88	47	46
N.S.	1	0.87	0.89	0.98	2.16	5.62	1.58	1.60	0.85	0.84
time (sec)	N/A	0.312	0.048	2.852	0.049	0.099	1.077	0.139	0.241	0.111

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	235	42	0	36	334	36
N.S.	1	1.00	1.06	1.00	6.91	1.24	0.00	1.06	9.82	1.06
time (sec)	N/A	0.314	25.909	0.289	0.306	0.095	0.000	0.372	0.395	1.517

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	883	0	1667	0	0	7042	0	0	38	0
N.S.	1	0.00	1.89	0.00	0.00	7.98	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	4.144	0.000	0.000	0.229	0.000	0.000	200.025	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	649	0	966	0	0	4311	0	0	0	0
N.S.	1	0.00	1.49	0.00	0.00	6.64	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	2.677	0.000	0.000	0.175	0.000	0.000	7.091	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	391	626	1128	0	2195	0	0	695	0
N.S.	1	0.99	1.58	2.86	0.00	5.56	0.00	0.00	1.76	0.00
time (sec)	N/A	2.493	2.267	19.551	0.000	0.156	0.000	0.000	0.289	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F(-1)	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	163	123	244	209	745	0	211	214	278
N.S.	1	1.16	0.87	1.73	1.48	5.28	0.00	1.50	1.52	1.97
time (sec)	N/A	1.177	0.451	8.355	0.130	0.114	0.000	0.144	0.247	1.508

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	36	334	44	0	38	246	38
N.S.	1	1.00	1.06	1.00	9.28	1.22	0.00	1.06	6.83	1.06
time (sec)	N/A	0.346	10.362	0.234	0.272	0.078	0.000	0.390	0.367	1.246

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1123	0	7510	0	0	12603	0	0	38	0
N.S.	1	0.00	6.69	0.00	0.00	11.22	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	27.203	0.000	0.000	0.301	0.000	0.000	200.029	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	792	0	5113	0	0	7645	0	0	38	0
N.S.	1	0.00	6.46	0.00	0.00	9.65	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	15.355	0.000	0.000	0.218	0.000	0.000	200.021	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	499	0	904	1217	0	3795	0	0	0	0
N.S.	1	0.00	1.81	2.44	0.00	7.61	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	1.601	56.189	0.000	0.145	0.000	0.000	0.331	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	100	98	97	234	1069	0	202	324	238
N.S.	1	0.88	0.87	0.86	2.07	9.46	0.00	1.79	2.87	2.11
time (sec)	N/A	0.391	0.161	24.097	0.045	0.108	0.000	0.160	0.245	1.428

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	36	442	44	0	38	511	38
N.S.	1	1.00	1.06	1.00	12.28	1.22	0.00	1.06	14.19	1.06
time (sec)	N/A	0.350	29.556	0.368	0.417	0.090	0.000	0.542	0.452	1.285

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1218	1036	3251	0	0	1962	0	0	1251	0
N.S.	1	0.85	2.67	0.00	0.00	1.61	0.00	0.00	1.03	0.00
time (sec)	N/A	5.673	10.614	0.000	0.000	0.156	0.000	0.000	0.402	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	861	748	1759	0	0	1248	0	0	850	0
N.S.	1	0.87	2.04	0.00	0.00	1.45	0.00	0.00	0.99	0.00
time (sec)	N/A	4.269	10.060	0.000	0.000	0.153	0.000	0.000	0.371	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	516	471	578	3882	0	682	0	0	451	0
N.S.	1	0.91	1.12	7.52	0.00	1.32	0.00	0.00	0.87	0.00
time (sec)	N/A	2.784	3.290	0.727	0.000	0.144	0.000	0.000	0.354	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	85	78	132	110	111	0	121	91	174
N.S.	1	1.15	1.05	1.78	1.49	1.50	0.00	1.64	1.23	2.35
time (sec)	N/A	0.393	0.071	0.285	0.135	0.110	0.000	0.137	0.253	2.179

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	252	40	29	0	435	34
N.S.	1	1.00	1.06	1.00	7.88	1.25	0.91	0.00	13.59	1.06
time (sec)	N/A	0.287	13.729	0.178	0.423	0.161	1.671	0.000	0.520	1.560

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1118	949	1071	0	0	6494	0	0	0	0
N.S.	1	0.85	0.96	0.00	0.00	5.81	0.00	0.00	0.00	0.00
time (sec)	N/A	6.911	7.531	0.000	0.000	0.245	0.000	0.000	0.582	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	772	670	633	0	0	3661	0	0	0	0
N.S.	1	0.87	0.82	0.00	0.00	4.74	0.00	0.00	0.00	0.00
time (sec)	N/A	5.406	4.827	0.000	0.000	0.176	0.000	0.000	0.439	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	353	331	1928	0	1337	0	0	1262	0
N.S.	1	0.92	0.86	5.01	0.00	3.47	0.00	0.00	3.28	0.00
time (sec)	N/A	2.680	2.760	1.045	0.000	0.142	0.000	0.000	0.293	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	88	106	103	115	351	0	108	213	422
N.S.	1	0.98	1.18	1.14	1.28	3.90	0.00	1.20	2.37	4.69
time (sec)	N/A	0.552	0.377	0.341	0.120	0.101	0.000	0.157	0.269	1.677

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	391	36	24	0	40	30
N.S.	1	1.00	1.07	1.00	13.96	1.29	0.86	0.00	1.43	1.07
time (sec)	N/A	0.299	50.771	0.164	0.474	0.123	1.551	0.000	0.323	1.481

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1256	1026	3390	0	0	10934	0	0	0	0
N.S.	1	0.82	2.70	0.00	0.00	8.71	0.00	0.00	0.00	0.00
time (sec)	N/A	7.062	11.559	0.000	0.000	0.321	0.000	0.000	0.683	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	760	646	823	2068	0	4903	0	0	0	0
N.S.	1	0.85	1.08	2.72	0.00	6.45	0.00	0.00	0.00	0.00
time (sec)	N/A	4.293	8.949	6.060	0.000	0.242	0.000	0.000	0.411	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	146	130	209	219	917	0	281	520	339
N.S.	1	1.21	1.07	1.73	1.81	7.58	0.00	2.32	4.30	2.80
time (sec)	N/A	0.436	0.256	2.069	0.142	0.112	0.000	0.160	0.280	2.930

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	1102	42	31	0	46	38
N.S.	1	1.00	1.06	1.00	32.41	1.24	0.91	0.00	1.35	1.12
time (sec)	N/A	0.328	64.645	0.182	1.967	4.259	3.857	0.000	0.420	3.497

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	792	0	5656	0	0	7020	0	0	36	0
N.S.	1	0.00	7.14	0.00	0.00	8.86	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	27.255	0.000	0.000	0.196	0.000	0.000	200.030	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	565	0	1961	0	0	4263	0	0	0	0
N.S.	1	0.00	3.47	0.00	0.00	7.55	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	7.883	0.000	0.000	0.154	0.000	0.000	7.621	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	333	452	671	0	2129	0	0	953	0
N.S.	1	0.96	1.30	1.93	0.00	6.12	0.00	0.00	2.74	0.00
time (sec)	N/A	2.287	0.812	28.707	0.000	0.139	0.000	0.000	0.286	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	66	66	65	171	602	105	117	64	63
N.S.	1	0.87	0.87	0.86	2.25	7.92	1.38	1.54	0.84	0.83
time (sec)	N/A	0.304	0.115	12.635	0.040	0.133	1.086	0.151	0.301	1.216

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	326	42	0	36	336	36
N.S.	1	1.00	1.06	1.00	9.59	1.24	0.00	1.06	9.88	1.06
time (sec)	N/A	0.291	34.893	0.286	0.339	0.091	0.000	0.421	0.391	2.956

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1022	0	5984	0	0	10658	0	0	38	0
N.S.	1	0.00	5.86	0.00	0.00	10.43	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	20.259	0.000	0.000	0.293	0.000	0.000	200.027	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	755	0	3579	0	0	6459	0	0	38	0
N.S.	1	0.00	4.74	0.00	0.00	8.55	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	12.467	0.000	0.000	0.205	0.000	0.000	200.025	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	465	455	1627	1213	0	3228	0	0	1437	0
N.S.	1	0.98	3.50	2.61	0.00	6.94	0.00	0.00	3.09	0.00
time (sec)	N/A	3.085	3.745	103.053	0.000	0.208	0.000	0.000	0.362	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F(-1)	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	222	153	293	257	1134	0	258	274	330
N.S.	1	1.21	0.83	1.59	1.40	6.16	0.00	1.40	1.49	1.79
time (sec)	N/A	1.598	1.449	33.224	0.124	0.121	0.000	0.169	0.277	1.720

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	36	417	44	0	38	445	38
N.S.	1	1.00	1.06	1.00	11.58	1.22	0.00	1.06	12.36	1.06
time (sec)	N/A	0.351	15.471	0.322	0.326	0.085	0.000	0.686	1.463	1.394

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1443	0	5147	0	0	18801	0	0	38	0
N.S.	1	0.00	3.57	0.00	0.00	13.03	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	10.651	0.000	0.000	0.463	0.000	0.000	200.019	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1021	0	1811	0	0	11318	0	0	38	0
N.S.	1	0.00	1.77	0.00	0.00	11.09	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	9.116	0.000	0.000	0.247	0.000	0.000	200.018	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	641	0	2704	1363	0	5548	0	0	0	0
N.S.	1	0.00	4.22	2.13	0.00	8.66	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	8.325	284.300	0.000	0.175	0.000	0.000	0.494	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	125	123	144	300	1660	0	258	426	307
N.S.	1	0.89	0.87	1.02	2.13	11.77	0.00	1.83	3.02	2.18
time (sec)	N/A	0.450	0.261	152.475	0.048	0.114	0.000	0.163	0.195	1.669

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	36	551	44	0	38	336	38
N.S.	1	1.00	1.06	1.00	15.31	1.22	0.00	1.06	9.33	1.06
time (sec)	N/A	0.369	30.588	0.486	0.468	0.080	0.000	0.401	0.440	1.442

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1519	0	4239	0	0	4522	0	0	0	0
N.S.	1	0.00	2.79	0.00	0.00	2.98	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	11.065	0.000	0.000	0.292	0.000	0.000	0.600	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1067	0	3418	0	0	2775	0	0	0	0
N.S.	1	0.00	3.20	0.00	0.00	2.60	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	10.574	0.000	0.000	0.169	0.000	0.000	0.471	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	631	570	621	4066	0	1410	0	0	1104	0
N.S.	1	0.90	0.98	6.44	0.00	2.23	0.00	0.00	1.75	0.00
time (sec)	N/A	3.741	8.413	1.865	0.000	0.145	0.000	0.000	0.309	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	97	91	169	147	288	0	145	181	249
N.S.	1	1.09	1.02	1.90	1.65	3.24	0.00	1.63	2.03	2.80
time (sec)	N/A	0.432	0.132	1.090	0.134	0.101	0.000	0.146	0.187	2.696

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	322	42	31	0	46	36
N.S.	1	1.00	1.06	1.00	9.47	1.24	0.91	0.00	1.35	1.06
time (sec)	N/A	0.301	38.179	0.602	0.393	0.172	2.452	0.000	0.247	2.377

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1294	0	1111	0	0	7331	0	0	0	0
N.S.	1	0.00	0.86	0.00	0.00	5.67	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	4.777	0.000	0.000	0.296	0.000	0.000	0.583	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	904	0	665	0	0	4196	0	0	0	0
N.S.	1	0.00	0.74	0.00	0.00	4.64	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	2.887	0.000	0.000	0.182	0.000	0.000	0.342	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	454	0	358	1897	0	1571	0	0	1554	0
N.S.	1	0.00	0.79	4.18	0.00	3.46	0.00	0.00	3.42	0.00
time (sec)	N/A	0.000	5.455	1.254	0.000	0.143	0.000	0.000	0.235	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	127	96	125	141	459	0	128	396	468
N.S.	1	1.05	0.79	1.03	1.17	3.79	0.00	1.06	3.27	3.87
time (sec)	N/A	0.828	0.762	0.415	0.118	0.090	0.000	0.155	0.201	3.154

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	411	42	31	0	981	36
N.S.	1	1.00	1.06	1.00	12.09	1.24	0.91	0.00	28.85	1.06
time (sec)	N/A	0.293	36.679	0.254	0.347	0.137	2.536	0.000	0.592	3.869

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1479	0	3368	0	0	10574	0	0	0	0
N.S.	1	0.00	2.28	0.00	0.00	7.15	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	11.468	0.000	0.000	0.296	0.000	0.000	0.743	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	894	0	834	2284	0	4729	0	0	0	0
N.S.	1	0.00	0.93	2.55	0.00	5.29	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	8.950	2.466	0.000	0.216	0.000	0.000	0.406	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	146	152	210	217	896	0	279	519	381
N.S.	1	1.22	1.27	1.75	1.81	7.47	0.00	2.32	4.32	3.18
time (sec)	N/A	0.405	0.326	0.966	0.125	0.126	0.000	0.176	0.205	3.539

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	1095	36	24	0	2848	30
N.S.	1	1.00	1.07	1.00	39.11	1.29	0.86	0.00	101.71	1.07
time (sec)	N/A	0.266	58.902	0.258	1.761	4.106	2.112	0.000	0.702	4.099

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	451	532	1914	0	0	1228	0	0	515	0
N.S.	1	1.18	4.24	0.00	0.00	2.72	0.00	0.00	1.14	0.00
time (sec)	N/A	2.497	9.464	0.000	0.000	0.126	0.000	0.000	0.456	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	325	397	1296	0	0	813	0	0	360	0
N.S.	1	1.22	3.99	0.00	0.00	2.50	0.00	0.00	1.11	0.00
time (sec)	N/A	2.088	5.288	0.000	0.000	0.105	0.000	0.000	0.355	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	263	431	451	0	476	0	0	203	0
N.S.	1	1.28	2.10	2.20	0.00	2.32	0.00	0.00	0.99	0.00
time (sec)	N/A	1.203	2.049	0.941	0.000	0.113	0.000	0.000	0.318	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	53	75	67	0	61	57	254
N.S.	1	1.00	0.82	1.56	2.21	1.97	0.00	1.79	1.68	7.47
time (sec)	N/A	0.231	0.020	0.434	0.039	0.099	0.000	0.136	0.216	0.450

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	28	34	22	28	38	28
N.S.	1	1.00	1.08	1.00	1.08	1.31	0.85	1.08	1.46	1.08
time (sec)	N/A	0.239	9.667	0.254	0.308	0.096	3.373	0.434	0.281	1.409

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	638	0	781	0	0	1470	0	0	34	0
N.S.	1	0.00	1.22	0.00	0.00	2.30	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.965	0.000	0.000	0.155	0.000	0.000	200.022	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	462	0	490	0	0	992	0	0	34	0
N.S.	1	0.00	1.06	0.00	0.00	2.15	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	0.663	0.000	0.000	0.123	0.000	0.000	200.031	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	368	298	970	0	598	0	0	32	0
N.S.	1	1.29	1.04	3.39	0.00	2.09	0.00	0.00	0.11	0.00
time (sec)	N/A	2.352	1.303	1.230	0.000	0.141	0.000	0.000	200.029	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	80	94	109	126	209	0	113	77	384
N.S.	1	1.13	1.32	1.54	1.77	2.94	0.00	1.59	1.08	5.41
time (sec)	N/A	0.583	0.308	0.986	0.151	0.103	0.000	0.150	0.196	1.552

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	167	40	29	0	419	34
N.S.	1	1.00	1.06	1.00	5.22	1.25	0.91	0.00	13.09	1.06
time (sec)	N/A	0.270	7.856	0.299	0.272	0.108	4.750	0.000	0.462	1.586

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	656	0	3089	0	0	3344	0	0	36	0
N.S.	1	0.00	4.71	0.00	0.00	5.10	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	9.937	0.000	0.000	0.184	0.000	0.000	200.016	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	486	0	1462	0	0	2101	0	0	36	0
N.S.	1	0.00	3.01	0.00	0.00	4.32	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	7.719	0.000	0.000	0.146	0.000	0.000	200.027	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	422	487	932	0	1108	0	0	34	0
N.S.	1	1.31	1.51	2.89	0.00	3.44	0.00	0.00	0.11	0.00
time (sec)	N/A	2.970	3.558	2.973	0.000	0.124	0.000	0.000	200.023	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	55	48	133	130	203	0	94	174	360
N.S.	1	0.96	0.84	2.33	2.28	3.56	0.00	1.65	3.05	6.32
time (sec)	N/A	0.328	0.065	2.938	0.037	0.110	0.000	0.141	4.616	1.446

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	249	42	31	0	36	36
N.S.	1	1.00	1.06	1.00	7.32	1.24	0.91	0.00	1.06	1.06
time (sec)	N/A	0.301	42.312	0.669	0.406	0.111	7.833	0.000	200.050	2.309

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1049	910	3572	0	0	2448	0	0	705	0
N.S.	1	0.87	3.41	0.00	0.00	2.33	0.00	0.00	0.67	0.00
time (sec)	N/A	4.585	12.447	0.000	0.000	0.181	0.000	0.000	0.369	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	734	651	2019	0	0	1535	0	0	516	0
N.S.	1	0.89	2.75	0.00	0.00	2.09	0.00	0.00	0.70	0.00
time (sec)	N/A	3.267	12.416	0.000	0.000	0.158	0.000	0.000	0.325	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	439	402	784	1065	0	808	0	0	317	0
N.S.	1	0.92	1.79	2.43	0.00	1.84	0.00	0.00	0.72	0.00
time (sec)	N/A	2.003	5.141	4.754	0.000	0.132	0.000	0.000	0.274	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	104	92	108	138	134	0	147	135	0
N.S.	1	1.16	1.02	1.20	1.53	1.49	0.00	1.63	1.50	0.00
time (sec)	N/A	0.427	0.101	1.955	0.160	0.139	0.000	0.140	0.193	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	40	29	34	44	38
N.S.	1	1.00	1.06	1.00	1.06	1.25	0.91	1.06	1.38	1.19
time (sec)	N/A	0.299	18.623	0.318	0.425	0.718	91.856	9.239	0.320	1.667

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1164	991	1441	0	0	9707	0	0	0	0
N.S.	1	0.85	1.24	0.00	0.00	8.34	0.00	0.00	0.00	0.00
time (sec)	N/A	5.561	8.928	0.000	0.000	0.354	0.000	0.000	0.429	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	795	695	928	0	0	5562	0	0	0	0
N.S.	1	0.87	1.17	0.00	0.00	7.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.934	8.081	0.000	0.000	0.224	0.000	0.000	0.375	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	442	388	428	1815	0	2176	0	0	1109	0
N.S.	1	0.88	0.97	4.11	0.00	4.92	0.00	0.00	2.51	0.00
time (sec)	N/A	2.453	7.449	8.778	0.000	0.189	0.000	0.000	0.349	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	126	233	106	168	581	0	146	468	668
N.S.	1	1.12	2.06	0.94	1.49	5.14	0.00	1.29	4.14	5.91
time (sec)	N/A	0.535	0.967	4.473	0.132	0.182	0.000	0.140	0.238	6.258

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	463	42	0	0	46	38
N.S.	1	1.00	1.06	1.00	13.62	1.24	0.00	0.00	1.35	1.12
time (sec)	N/A	0.283	48.069	0.179	0.597	0.451	0.000	0.000	0.437	5.430

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1176	993	4072	0	0	16670	0	0	0	0
N.S.	1	0.84	3.46	0.00	0.00	14.18	0.00	0.00	0.00	0.00
time (sec)	N/A	4.767	12.567	0.000	0.000	0.539	0.000	0.000	0.503	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	746	627	1080	2580	0	7645	0	0	0	0
N.S.	1	0.84	1.45	3.46	0.00	10.25	0.00	0.00	0.00	0.00
time (sec)	N/A	2.772	10.440	24.970	0.000	0.356	0.000	0.000	0.394	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	181	196	230	265	1279	0	343	1018	0
N.S.	1	1.13	1.22	1.44	1.66	7.99	0.00	2.14	6.36	0.00
time (sec)	N/A	0.458	0.512	12.767	0.131	0.257	0.000	0.137	0.261	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	1214	42	0	0	46	38
N.S.	1	1.00	1.06	1.00	35.71	1.24	0.00	0.00	1.35	1.12
time (sec)	N/A	0.297	96.791	0.185	2.398	19.729	0.000	0.000	0.471	11.462

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	F	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	601	681	2598	0	0	4313	0	0	0	0
N.S.	1	1.13	4.32	0.00	0.00	7.18	0.00	0.00	0.00	0.00
time (sec)	N/A	4.447	10.602	0.000	0.000	0.170	0.000	0.000	0.595	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	419	492	1735	0	0	2528	0	0	0	0
N.S.	1	1.17	4.14	0.00	0.00	6.03	0.00	0.00	0.00	0.00
time (sec)	N/A	2.806	10.059	0.000	0.000	0.132	0.000	0.000	0.415	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	304	561	528	0	1221	0	0	729	0
N.S.	1	1.25	2.31	2.17	0.00	5.02	0.00	0.00	3.00	0.00
time (sec)	N/A	1.597	7.886	0.768	0.000	0.120	0.000	0.000	0.286	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	49	50	35	110	211	0	110	171	409
N.S.	1	0.98	1.00	0.70	2.20	4.22	0.00	2.20	3.42	8.18
time (sec)	N/A	0.291	0.033	0.292	0.044	0.091	0.000	0.134	0.224	1.926

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	316	40	29	0	44	36
N.S.	1	1.00	1.06	1.00	9.88	1.25	0.91	0.00	1.38	1.12
time (sec)	N/A	0.266	72.590	0.151	0.401	0.158	7.482	0.000	0.300	1.713

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	721	0	1490	0	0	4612	0	0	0	0
N.S.	1	0.00	2.07	0.00	0.00	6.40	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	7.028	0.000	0.000	0.230	0.000	0.000	0.856	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	517	0	917	0	0	2729	0	0	0	0
N.S.	1	0.00	1.77	0.00	0.00	5.28	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	6.824	0.000	0.000	0.152	0.000	0.000	0.523	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	0	347	1017	0	1338	0	0	1287	0
N.S.	1	0.00	1.18	3.46	0.00	4.55	0.00	0.00	4.38	0.00
time (sec)	N/A	0.000	3.108	0.720	0.000	0.126	0.000	0.000	0.311	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	88	112	105	134	360	0	120	189	380
N.S.	1	1.14	1.45	1.36	1.74	4.68	0.00	1.56	2.45	4.94
time (sec)	N/A	0.762	0.536	0.494	0.124	0.115	0.000	0.159	0.210	1.682

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	311	36	24	0	40	30
N.S.	1	1.00	1.07	1.00	11.11	1.29	0.86	0.00	1.43	1.07
time (sec)	N/A	0.257	64.717	0.153	0.469	0.131	3.239	0.000	0.314	1.537

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	718	0	2696	0	0	5829	0	0	36	0
N.S.	1	0.00	3.75	0.00	0.00	8.12	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	10.180	0.000	0.000	0.222	0.000	0.000	200.020	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	518	0	1806	0	0	3506	0	0	36	0
N.S.	1	0.00	3.49	0.00	0.00	6.77	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	9.802	0.000	0.000	0.177	0.000	0.000	200.026	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	0	504	938	0	1735	0	0	34	0
N.S.	1	0.00	1.56	2.90	0.00	5.35	0.00	0.00	0.10	0.00
time (sec)	N/A	0.000	6.680	1.219	0.000	0.118	0.000	0.000	200.025	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	52	128	131	299	0	121	283	356
N.S.	1	1.00	0.88	2.17	2.22	5.07	0.00	2.05	4.80	6.03
time (sec)	N/A	0.336	0.064	0.812	0.044	0.098	0.000	0.151	1.121	1.511

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	353	42	31	0	979	36
N.S.	1	1.00	1.06	1.00	10.38	1.24	0.91	0.00	28.79	1.06
time (sec)	N/A	0.289	51.100	0.190	0.448	0.314	6.393	0.000	0.607	3.894

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1428	1259	4139	0	0	9763	0	0	0	0
N.S.	1	0.88	2.90	0.00	0.00	6.84	0.00	0.00	0.00	0.00
time (sec)	N/A	6.582	10.722	0.000	0.000	0.343	0.000	0.000	0.382	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	982	889	2323	0	0	5664	0	0	1410	0
N.S.	1	0.91	2.37	0.00	0.00	5.77	0.00	0.00	1.44	0.00
time (sec)	N/A	4.870	10.300	0.000	0.000	0.224	0.000	0.000	0.325	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	591	529	864	1529	0	2593	0	0	882	0
N.S.	1	0.90	1.46	2.59	0.00	4.39	0.00	0.00	1.49	0.00
time (sec)	N/A	3.134	8.750	6.148	0.000	0.165	0.000	0.000	0.275	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	118	160	140	173	441	0	200	390	142
N.S.	1	1.13	1.54	1.35	1.66	4.24	0.00	1.92	3.75	1.37
time (sec)	N/A	0.393	0.404	2.682	0.137	0.143	0.000	0.127	0.185	4.264

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	472	42	0	0	46	38
N.S.	1	1.00	1.06	1.00	13.88	1.24	0.00	0.00	1.35	1.12
time (sec)	N/A	0.289	40.036	0.180	0.664	6.692	0.000	0.000	0.346	4.037

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	914	818	1829	0	0	10432	0	0	0	0
N.S.	1	0.89	2.00	0.00	0.00	11.41	0.00	0.00	0.00	0.00
time (sec)	N/A	6.358	9.452	0.000	0.000	0.332	0.000	0.000	0.355	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	499	444	780	3717	0	4086	0	0	1374	0
N.S.	1	0.89	1.56	7.45	0.00	8.19	0.00	0.00	2.75	0.00
time (sec)	N/A	3.076	8.248	18.052	0.000	0.219	0.000	0.000	0.283	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	152	139	208	1040	0	185	562	768
N.S.	1	1.00	1.06	0.97	1.44	7.22	0.00	1.28	3.90	5.33
time (sec)	N/A	0.663	1.610	8.791	0.131	0.169	0.000	0.136	0.181	7.230

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	36	626	44	0	0	48	38
N.S.	1	1.00	1.06	1.00	17.39	1.22	0.00	0.00	1.33	1.06
time (sec)	N/A	0.348	50.593	0.192	0.668	0.645	0.000	0.000	0.331	7.187

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	974	803	1437	3280	0	15223	0	0	0	0
N.S.	1	0.82	1.48	3.37	0.00	15.63	0.00	0.00	0.00	0.00
time (sec)	N/A	4.001	11.171	46.244	0.000	0.522	0.000	0.000	0.578	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	207	227	249	350	2568	0	458	1469	398
N.S.	1	1.15	1.26	1.38	1.94	14.27	0.00	2.54	8.16	2.21
time (sec)	N/A	0.529	0.702	24.440	0.134	0.359	0.000	0.131	0.201	9.195

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	36	1586	44	0	0	48	38
N.S.	1	1.00	1.06	1.00	44.06	1.22	0.00	0.00	1.33	1.06
time (sec)	N/A	0.350	104.443	0.205	2.649	147.079	0.000	0.000	0.419	21.312

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	752	0	3254	0	0	11595	0	0	0	0
N.S.	1	0.00	4.33	0.00	0.00	15.42	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	11.178	0.000	0.000	0.297	0.000	0.000	0.988	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	F	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	502	565	1816	0	0	6479	0	0	0	0
N.S.	1	1.13	3.62	0.00	0.00	12.91	0.00	0.00	0.00	0.00
time (sec)	N/A	4.135	10.450	0.000	0.000	0.183	0.000	0.000	0.567	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	350	713	649	0	2899	0	0	0	0
N.S.	1	1.17	2.39	2.18	0.00	9.73	0.00	0.00	0.00	0.00
time (sec)	N/A	2.334	8.218	1.334	0.000	0.192	0.000	0.000	0.309	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	68	60	54	161	545	0	145	321	470
N.S.	1	0.94	0.83	0.75	2.24	7.57	0.00	2.01	4.46	6.53
time (sec)	N/A	0.309	0.080	0.433	0.041	0.110	0.000	0.147	0.178	2.077

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	684	42	31	0	46	36
N.S.	1	1.00	1.06	1.00	20.12	1.24	0.91	0.00	1.35	1.06
time (sec)	N/A	0.293	134.924	0.152	0.737	0.662	21.627	0.000	0.269	2.175

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1038	0	2799	0	0	13504	0	0	0	0
N.S.	1	0.00	2.70	0.00	0.00	13.01	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	9.177	0.000	0.000	0.339	0.000	0.000	3.083	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	714	0	1530	0	0	7726	0	0	0	0
N.S.	1	0.00	2.14	0.00	0.00	10.82	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	8.201	0.000	0.000	0.231	0.000	0.000	0.661	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	0	615	1284	0	3585	0	0	0	0
N.S.	1	0.00	1.49	3.11	0.00	8.68	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	8.638	0.908	0.000	0.221	0.000	0.000	0.330	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	136	167	140	217	892	0	182	472	628
N.S.	1	1.23	1.50	1.26	1.95	8.04	0.00	1.64	4.25	5.66
time (sec)	N/A	1.196	1.921	0.560	0.127	0.121	0.000	0.161	0.177	1.739

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	762	42	31	0	46	38
N.S.	1	1.00	1.06	1.00	22.41	1.24	0.91	0.00	1.35	1.12
time (sec)	N/A	0.337	67.129	0.184	1.159	0.200	19.431	0.000	0.293	2.039

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	972	0	3868	0	0	13683	0	0	0	0
N.S.	1	0.00	3.98	0.00	0.00	14.08	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	12.165	0.000	0.000	0.361	0.000	0.000	7.037	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	680	0	2403	0	0	7775	0	0	0	0
N.S.	1	0.00	3.53	0.00	0.00	11.43	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	10.748	0.000	0.000	0.206	0.000	0.000	0.869	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	435	0	766	1098	0	3547	0	0	0	0
N.S.	1	0.00	1.76	2.52	0.00	8.15	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	8.410	1.021	0.000	0.152	0.000	0.000	0.412	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	76	64	143	173	617	0	184	563	1329
N.S.	1	0.95	0.80	1.79	2.16	7.71	0.00	2.30	7.04	16.61
time (sec)	N/A	0.305	0.080	0.720	0.042	0.129	0.000	0.172	0.175	2.040

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	751	36	24	0	40	30
N.S.	1	1.00	1.07	1.00	26.82	1.29	0.86	0.00	1.43	1.07
time (sec)	N/A	0.256	133.723	0.163	0.983	0.798	6.393	0.000	0.303	2.235

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1795	1596	5813	0	0	23903	0	0	0	0
N.S.	1	0.89	3.24	0.00	0.00	13.32	0.00	0.00	0.00	0.00
time (sec)	N/A	8.583	12.244	0.000	0.000	0.664	0.000	0.000	0.458	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1210	1118	2784	0	0	13309	0	0	0	0
N.S.	1	0.92	2.30	0.00	0.00	11.00	0.00	0.00	0.00	0.00
time (sec)	N/A	6.336	10.202	0.000	0.000	0.429	0.000	0.000	0.373	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	762	673	1009	1478	0	5731	0	0	1577	0
N.S.	1	0.88	1.32	1.94	0.00	7.52	0.00	0.00	2.07	0.00
time (sec)	N/A	4.259	9.154	13.306	0.000	0.267	0.000	0.000	0.311	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	144	164	186	236	1035	0	263	685	196
N.S.	1	1.11	1.26	1.43	1.82	7.96	0.00	2.02	5.27	1.51
time (sec)	N/A	0.452	0.262	6.297	0.133	0.229	0.000	0.166	0.220	4.828

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	912	42	0	0	46	38
N.S.	1	1.00	1.06	1.00	26.82	1.24	0.00	0.00	1.35	1.12
time (sec)	N/A	0.307	104.521	0.201	1.109	38.486	0.000	0.000	0.381	11.823

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1212	0	2346	0	0	29722	0	0	0	0
N.S.	1	0.00	1.94	0.00	0.00	24.52	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	9.429	0.000	0.000	0.649	0.000	0.000	1.192	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	695	611	798	2767	0	11126	0	0	0	0
N.S.	1	0.88	1.15	3.98	0.00	16.01	0.00	0.00	0.00	0.00
time (sec)	N/A	4.888	9.133	33.374	0.000	0.399	0.000	0.000	0.551	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	231	212	183	334	2653	0	224	1346	531
N.S.	1	1.14	1.05	0.91	1.65	13.13	0.00	1.11	6.66	2.63
time (sec)	N/A	0.665	3.200	17.404	0.131	0.328	0.000	0.145	0.213	4.646

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	36	1426	44	0	0	48	38
N.S.	1	1.00	1.06	1.00	39.61	1.22	0.00	0.00	1.33	1.06
time (sec)	N/A	0.343	75.865	0.203	1.307	2.096	0.000	0.000	0.425	19.059

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1117	943	1675	3563	0	16848	0	0	0	0
N.S.	1	0.84	1.50	3.19	0.00	15.08	0.00	0.00	0.00	0.00
time (sec)	N/A	6.182	11.097	129.460	0.000	0.555	0.000	0.000	0.698	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	234	237	292	418	3148	0	464	1269	554
N.S.	1	1.11	1.12	1.38	1.98	14.92	0.00	2.20	6.01	2.63
time (sec)	N/A	0.606	0.669	47.012	0.134	0.477	0.000	0.141	0.217	8.378

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	36	1950	0	0	0	48	38
N.S.	1	1.00	1.06	1.00	54.17	0.00	0.00	0.00	1.33	1.06
time (sec)	N/A	0.344	129.107	0.221	3.124	0.000	0.000	0.000	0.418	19.748

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [17] had the largest ratio of [1.3750000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	C	15	15	1.27	14	1.071
2	C	11	11	1.31	14	0.786
3	C	9	9	1.24	14	0.643
4	C	5	5	1.32	12	0.417
5	C	8	8	1.20	14	0.571
6	C	10	10	1.18	14	0.714
7	C	14	14	1.16	14	1.000
8	A	14	14	1.03	16	0.875
9	A	9	9	1.04	16	0.562
10	A	9	9	1.02	16	0.562
11	A	4	4	1.00	14	0.286
12	A	4	4	1.00	16	0.250
13	C	12	12	1.16	16	0.750
14	A	9	9	1.31	16	0.562
15	C	17	17	1.05	16	1.062
16	F	0	0	N/A	0.000	N/A
17	C	22	22	1.41	16	1.375
18	C	16	15	1.22	16	0.938
19	C	9	9	1.17	14	0.643
20	C	4	4	1.10	16	0.250
21	C	4	4	1.08	16	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	C	14	14	1.43	16	0.875
23	C	8	7	1.16	14	0.500
24	C	7	6	1.16	14	0.429
25	C	6	5	1.22	12	0.417
26	N/A	3	0	1.00	14	0.000
27	N/A	3	0	1.00	14	0.000
28	C	12	11	1.42	16	0.688
29	C	11	10	1.43	16	0.625
30	C	7	7	1.14	14	0.500
31	N/A	3	0	1.00	16	0.000
32	N/A	3	0	1.00	16	0.000
33	C	14	13	1.15	16	0.812
34	C	12	11	1.19	16	0.688
35	C	10	9	1.17	14	0.643
36	N/A	3	0	1.00	16	0.000
37	N/A	3	0	1.00	16	0.000
38	C	16	15	1.19	16	0.938
39	C	13	12	1.20	16	0.750
40	C	10	9	1.15	16	0.562
41	C	7	6	1.08	16	0.375
42	C	10	9	1.18	16	0.562
43	C	13	12	1.18	16	0.750
44	C	16	15	1.19	16	0.938
45	A	9	9	1.03	18	0.500
46	A	9	9	1.03	18	0.500
47	A	4	4	1.00	18	0.222
48	A	4	4	1.00	18	0.222
49	C	11	10	1.13	18	0.556
50	A	9	9	1.27	18	0.500
51	C	16	15	1.10	18	0.833
52	A	14	14	1.21	18	0.778
53	C	22	21	1.48	18	1.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	C	19	18	1.49	18	1.000
55	C	4	4	1.06	18	0.222
56	C	4	4	1.05	18	0.222
57	C	4	4	1.08	18	0.222
58	C	13	12	1.52	18	0.667
59	C	16	15	1.45	18	0.833
60	C	13	12	1.26	12	1.000
61	C	10	9	1.20	12	0.750
62	C	7	6	1.10	12	0.500
63	C	10	9	1.24	12	0.750
64	C	13	12	1.24	12	1.000
65	N/A	3	0	1.00	16	0.000
66	N/A	3	0	1.00	16	0.000
67	N/A	4	0	1.00	10	0.000
68	A	1	1	1.00	18	0.056
69	A	1	1	1.00	20	0.050
70	A	1	1	1.00	20	0.050
71	A	1	1	1.00	22	0.045
72	N/A	2	0	1.00	18	0.000
73	C	4	4	1.05	16	0.250
74	A	4	4	1.00	16	0.250
75	C	4	4	1.07	14	0.286
76	N/A	3	0	1.00	14	0.000
77	N/A	3	0	1.00	16	0.000
78	C	4	4	1.14	12	0.333
79	C	4	4	1.14	12	0.333
80	C	4	4	1.14	12	0.333
81	C	4	4	1.14	10	0.400
82	C	4	4	1.16	12	0.333
83	C	4	4	1.15	12	0.333
84	C	4	4	1.14	12	0.333
85	A	4	4	1.00	14	0.286

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	4	4	1.00	14	0.286
87	A	4	4	1.00	14	0.286
88	A	4	4	1.00	12	0.333
89	A	4	4	1.00	14	0.286
90	A	4	4	1.00	14	0.286
91	A	4	4	1.00	14	0.286
92	A	1	1	1.00	20	0.050
93	A	1	1	1.00	20	0.050
94	A	1	1	1.00	20	0.050
95	A	1	1	1.00	24	0.042
96	A	3	3	1.00	21	0.143
97	A	3	3	1.00	21	0.143
98	A	3	3	1.00	19	0.158
99	A	3	3	1.00	21	0.143
100	A	3	3	1.00	21	0.143
101	A	3	3	1.00	21	0.143
102	A	3	3	1.00	23	0.130
103	A	3	3	1.00	23	0.130
104	A	3	3	1.00	21	0.143
105	A	5	5	0.96	23	0.217
106	A	5	5	1.02	23	0.217
107	A	7	7	1.39	23	0.304
108	A	16	15	1.11	23	0.652
109	A	15	14	1.14	23	0.609
110	A	10	10	1.03	21	0.476
111	N/A	2	0	1.00	23	0.000
112	N/A	2	0	1.00	23	0.000
113	A	17	16	1.01	23	0.696
114	A	17	16	1.00	23	0.696
115	A	10	10	0.97	21	0.476
116	N/A	2	0	1.00	23	0.000
117	N/A	2	0	1.00	23	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	16	16	1.13	21	0.762
119	A	14	14	1.21	21	0.667
120	A	10	10	1.17	21	0.476
121	A	8	8	1.36	19	0.421
122	A	9	9	0.71	21	0.429
123	A	13	13	0.82	21	0.619
124	A	15	15	0.78	21	0.714
125	A	23	23	1.06	21	1.095
126	A	15	14	0.93	21	0.667
127	A	10	10	0.95	19	0.526
128	A	5	5	0.57	21	0.238
129	A	5	5	0.61	21	0.238
130	F	0	0	N/A	0.000	N/A
131	A	19	18	0.91	21	0.857
132	A	12	12	0.84	19	0.632
133	A	5	5	0.51	21	0.238
134	A	5	5	0.54	21	0.238
135	A	7	7	0.76	21	0.333
136	A	9	8	0.58	21	0.381
137	A	8	7	0.60	21	0.333
138	A	7	6	0.65	19	0.316
139	N/A	2	0	1.00	21	0.000
140	N/A	2	0	1.00	21	0.000
141	A	13	12	0.59	21	0.571
142	A	11	10	0.63	21	0.476
143	A	9	8	0.75	19	0.421
144	N/A	2	0	1.00	21	0.000
145	N/A	2	0	1.00	21	0.000
146	A	20	19	0.73	21	0.905
147	A	16	15	0.70	21	0.714
148	A	11	10	0.72	19	0.526
149	N/A	2	0	1.00	21	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	N/A	2	0	1.00	21	0.000
151	N/A	2	0	1.00	23	0.000
152	A	7	7	0.96	23	0.304
153	A	5	5	0.98	23	0.217
154	A	3	3	1.00	21	0.143
155	N/A	2	0	1.00	23	0.000
156	N/A	2	0	1.00	23	0.000
157	A	3	3	1.00	18	0.167
158	A	3	3	1.00	18	0.167
159	A	3	3	1.00	16	0.188
160	A	3	3	1.00	18	0.167
161	A	3	3	1.00	18	0.167
162	A	3	3	1.00	18	0.167
163	A	3	3	1.00	20	0.150
164	A	3	3	1.00	20	0.150
165	A	3	3	1.00	18	0.167
166	A	3	3	1.00	20	0.150
167	A	3	3	1.00	20	0.150
168	A	3	3	1.00	20	0.150
169	A	11	10	0.93	20	0.500
170	A	10	9	0.96	20	0.450
171	A	9	8	1.01	18	0.444
172	N/A	2	0	1.00	20	0.000
173	N/A	2	0	1.00	20	0.000
174	A	16	15	0.95	20	0.750
175	A	13	12	1.04	18	0.667
176	N/A	2	0	1.00	20	0.000
177	N/A	2	0	1.00	20	0.000
178	A	20	19	1.53	18	1.056
179	N/A	2	0	1.00	20	0.000
180	N/A	2	0	1.00	20	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
181	N/A	2	0	1.00	20	0.000
182	A	3	3	1.00	20	0.150
183	A	3	3	1.00	20	0.150
184	A	3	3	1.00	18	0.167
185	N/A	2	0	1.00	20	0.000
186	N/A	2	0	1.00	20	0.000
187	A	18	17	1.03	29	0.586
188	A	17	16	1.05	29	0.552
189	A	12	12	1.00	27	0.444
190	A	5	5	1.09	22	0.227
191	N/A	1	0	1.00	29	0.000
192	N/A	1	0	1.00	29	0.000
193	A	30	29	1.10	31	0.935
194	A	27	26	1.11	31	0.839
195	A	18	18	1.09	29	0.621
196	A	9	9	0.96	24	0.375
197	N/A	1	0	1.00	31	0.000
198	N/A	1	0	1.00	31	0.000
199	F	0	0	N/A	0.000	N/A
200	F	0	0	N/A	0.000	N/A
201	A	23	23	1.16	29	0.793
202	A	7	7	1.05	24	0.292
203	N/A	1	0	1.00	31	0.000
204	N/A	1	0	1.00	31	0.000
205	A	23	22	1.04	29	0.759
206	A	21	20	1.05	29	0.690
207	A	16	15	1.05	27	0.556
208	A	8	8	1.22	22	0.364
209	N/A	1	0	1.00	29	0.000
210	N/A	1	0	1.00	29	0.000
211	F	0	0	N/A	0.000	N/A

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
212	F	0	0	N/A	0.000	N/A
213	A	24	23	1.07	29	0.793
214	A	15	14	1.04	24	0.583
215	N/A	1	0	1.00	31	0.000
216	N/A	1	0	1.00	31	0.000
217	F	0	0	N/A	0.000	N/A
218	F	0	0	N/A	0.000	N/A
219	F	0	0	N/A	0.000	N/A
220	A	20	19	1.11	24	0.792
221	N/A	1	0	1.00	31	0.000
222	N/A	1	0	1.00	31	0.000
223	A	13	12	0.88	26	0.462
224	A	12	11	0.91	26	0.423
225	A	11	10	0.96	24	0.417
226	C	8	7	1.13	19	0.368
227	N/A	1	0	1.00	26	0.000
228	C	25	24	0.91	28	0.857
229	C	22	21	0.92	28	0.750
230	C	17	16	0.98	26	0.615
231	C	13	12	1.11	21	0.571
232	N/A	1	0	1.00	28	0.000
233	F	0	0	N/A	0.000	N/A
234	F	0	0	N/A	0.000	N/A
235	C	22	21	0.99	26	0.808
236	C	13	12	1.16	21	0.571
237	N/A	1	0	1.00	28	0.000
238	C	15	14	0.92	26	0.538
239	C	15	14	0.94	26	0.538
240	C	14	13	0.99	24	0.542
241	C	11	10	1.09	19	0.526
242	N/A	1	0	1.00	26	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
243	C	27	26	0.96	28	0.929
244	C	26	25	0.98	28	0.893
245	C	22	21	0.99	26	0.808
246	C	15	14	1.11	21	0.667
247	N/A	1	0	1.00	28	0.000
248	F	0	0	N/A	0.000	N/A
249	F	0	0	N/A	0.000	N/A
250	F	0	0	N/A	0.000	N/A
251	C	19	18	1.22	21	0.857
252	N/A	1	0	1.00	28	0.000
253	A	7	6	1.00	29	0.207
254	A	6	5	1.01	29	0.172
255	A	5	4	1.04	27	0.148
256	A	4	3	1.04	22	0.136
257	N/A	1	0	1.00	29	0.000
258	N/A	1	0	1.00	29	0.000
259	A	13	13	1.03	31	0.419
260	A	11	11	0.98	31	0.355
261	A	7	7	1.00	29	0.241
262	A	3	3	1.00	24	0.125
263	A	10	10	1.00	31	0.323
264	A	12	12	0.98	31	0.387
265	A	23	23	0.97	31	0.742
266	A	14	14	0.98	31	0.452
267	A	12	12	0.96	29	0.414
268	A	4	3	1.00	24	0.125
269	A	18	18	0.98	31	0.581
270	A	24	24	1.04	31	0.774
271	A	22	21	0.95	29	0.724
272	A	17	16	0.94	29	0.552
273	A	12	11	0.94	27	0.407

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
274	A	5	4	1.12	22	0.182
275	N/A	1	0	1.00	29	0.000
276	N/A	1	0	1.00	29	0.000
277	A	23	22	0.98	31	0.710
278	A	22	21	0.95	31	0.677
279	A	14	14	0.96	29	0.483
280	A	6	5	1.00	24	0.208
281	N/A	1	0	1.00	31	0.000
282	N/A	1	0	1.00	31	0.000
283	F	0	0	N/A	0.000	N/A
284	A	24	23	0.95	31	0.742
285	A	14	13	0.94	29	0.448
286	A	5	4	0.96	24	0.167
287	N/A	1	0	1.00	31	0.000
288	N/A	1	0	1.00	31	0.000
289	A	7	6	1.01	26	0.231
290	A	6	5	1.00	26	0.192
291	A	5	4	1.00	24	0.167
292	A	4	3	1.00	19	0.158
293	N/A	1	0	1.00	26	0.000
294	C	23	22	0.95	28	0.786
295	C	20	19	0.97	28	0.679
296	C	15	14	1.03	26	0.538
297	A	10	9	1.04	21	0.429
298	N/A	1	0	1.00	28	0.000
299	C	30	29	0.92	28	1.036
300	C	20	19	0.92	28	0.679
301	A	17	16	0.92	26	0.615
302	A	6	5	0.90	21	0.238
303	N/A	1	0	1.00	28	0.000
304	A	10	9	0.88	26	0.346

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	A	9	8	0.90	26	0.308
306	A	8	7	0.95	24	0.292
307	A	8	7	1.07	19	0.368
308	N/A	1	0	1.00	26	0.000
309	A	14	13	0.84	28	0.464
310	A	13	12	0.86	28	0.429
311	A	12	11	0.93	26	0.423
312	A	9	8	0.97	21	0.381
313	N/A	1	0	1.00	28	0.000
314	A	10	9	0.82	28	0.321
315	A	9	8	0.87	26	0.308
316	A	7	6	1.33	21	0.286
317	N/A	1	0	1.00	28	0.000
318	N/A	1	0	1.00	24	0.000
319	N/A	1	0	1.00	24	0.000
320	N/A	1	0	1.00	22	0.000
321	A	6	5	0.97	24	0.208
322	A	10	9	0.96	26	0.346
323	A	11	10	0.91	26	0.385
324	A	6	5	0.97	24	0.208
325	A	10	9	0.96	26	0.346
326	A	11	10	0.91	26	0.385
327	A	10	9	1.04	24	0.375
328	A	14	13	0.98	26	0.500
329	A	17	16	0.89	26	0.615
330	A	10	9	1.04	24	0.375
331	A	14	13	0.98	26	0.500
332	A	17	16	0.89	26	0.615
333	C	20	19	1.01	32	0.594
334	C	15	14	1.01	32	0.438
335	A	12	11	0.99	30	0.367
336	A	8	7	0.88	25	0.280

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
337	N/A	1	0	1.00	32	0.000
338	C	30	29	0.94	34	0.853
339	C	27	26	0.95	34	0.765
340	C	19	18	1.01	32	0.562
341	C	11	10	1.13	27	0.370
342	N/A	1	0	1.00	34	0.000
343	F	0	0	N/A	0.000	N/A
344	F	0	0	N/A	0.000	N/A
345	A	26	25	0.90	32	0.781
346	A	8	7	0.88	27	0.259
347	N/A	1	0	1.00	34	0.000
348	A	14	13	0.87	26	0.500
349	A	14	13	0.88	26	0.500
350	A	13	12	0.93	24	0.500
351	A	10	9	1.03	19	0.474
352	N/A	1	0	1.00	26	0.000
353	A	25	24	0.86	32	0.750
354	A	23	22	0.88	32	0.688
355	A	19	18	0.93	30	0.600
356	C	10	9	1.08	25	0.360
357	N/A	1	0	1.00	32	0.000
358	A	18	17	0.81	34	0.500
359	A	16	15	0.84	32	0.469
360	A	10	9	1.19	27	0.333
361	N/A	1	0	1.00	34	0.000
362	C	31	30	0.98	34	0.882
363	C	21	20	0.98	34	0.588
364	A	18	17	0.99	32	0.531
365	A	8	7	0.87	27	0.259
366	N/A	1	0	1.00	34	0.000
367	F	0	0	N/A	0.000	N/A

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
368	F	0	0	N/A	0.000	N/A
369	C	24	23	0.99	34	0.676
370	C	22	21	1.16	29	0.724
371	N/A	1	0	1.00	36	0.000
372	F	0	0	N/A	0.000	N/A
373	F	0	0	N/A	0.000	N/A
374	F	0	0	N/A	0.000	N/A
375	A	7	6	0.88	29	0.207
376	N/A	1	0	1.00	36	0.000
377	A	20	19	0.85	32	0.594
378	A	21	20	0.87	32	0.625
379	A	20	19	0.91	30	0.633
380	A	7	6	1.15	25	0.240
381	N/A	1	0	1.00	32	0.000
382	A	31	30	0.85	28	1.071
383	A	29	28	0.87	28	1.000
384	A	23	22	0.92	26	0.846
385	A	14	13	0.98	21	0.619
386	N/A	1	0	1.00	28	0.000
387	A	26	25	0.82	34	0.735
388	A	21	20	0.85	32	0.625
389	A	11	10	1.21	27	0.370
390	N/A	1	0	1.00	34	0.000
391	F	0	0	N/A	0.000	N/A
392	F	0	0	N/A	0.000	N/A
393	A	24	23	0.96	32	0.719
394	A	8	7	0.87	27	0.259
395	N/A	1	0	1.00	34	0.000
396	F	0	0	N/A	0.000	N/A
397	F	0	0	N/A	0.000	N/A
398	C	27	26	0.98	34	0.765

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
399	C	25	24	1.21	29	0.828
400	N/A	1	0	1.00	36	0.000
401	F	0	0	N/A	0.000	N/A
402	F	0	0	N/A	0.000	N/A
403	F	0	0	N/A	0.000	N/A
404	A	8	7	0.89	29	0.241
405	N/A	1	0	1.00	36	0.000
406	F	0	0	N/A	0.000	N/A
407	F	0	0	N/A	0.000	N/A
408	A	31	30	0.90	32	0.938
409	A	10	9	1.09	27	0.333
410	N/A	1	0	1.00	34	0.000
411	F	0	0	N/A	0.000	N/A
412	F	0	0	N/A	0.000	N/A
413	F	0	0	N/A	0.000	N/A
414	C	18	17	1.05	27	0.630
415	N/A	1	0	1.00	34	0.000
416	F	0	0	N/A	0.000	N/A
417	F	0	0	N/A	0.000	N/A
418	A	9	8	1.22	21	0.381
419	N/A	1	0	1.00	28	0.000
420	C	13	12	1.18	26	0.462
421	C	13	12	1.22	26	0.462
422	C	12	11	1.28	24	0.458
423	A	7	6	1.00	19	0.316
424	N/A	1	0	1.00	26	0.000
425	F	0	0	N/A	0.000	N/A
426	F	0	0	N/A	0.000	N/A
427	C	25	24	1.29	30	0.800
428	C	15	14	1.13	25	0.560

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
429	N/A	1	0	1.00	32	0.000
430	F	0	0	N/A	0.000	N/A
431	F	0	0	N/A	0.000	N/A
432	C	30	29	1.31	32	0.906
433	A	8	7	0.96	27	0.259
434	N/A	1	0	1.00	34	0.000
435	A	16	15	0.87	32	0.469
436	A	16	15	0.89	32	0.469
437	A	15	14	0.92	30	0.467
438	A	8	7	1.16	25	0.280
439	N/A	1	0	1.00	32	0.000
440	A	19	18	0.85	34	0.529
441	A	18	17	0.87	34	0.500
442	A	15	14	0.88	32	0.438
443	C	4	4	1.12	27	0.148
444	N/A	1	0	1.00	34	0.000
445	A	15	14	0.84	34	0.412
446	A	12	11	0.84	32	0.344
447	A	8	7	1.13	27	0.259
448	N/A	1	0	1.00	34	0.000
449	C	21	20	1.13	32	0.625
450	C	20	19	1.17	32	0.594
451	C	17	16	1.25	30	0.533
452	A	8	7	0.98	25	0.280
453	N/A	1	0	1.00	32	0.000
454	F	0	0	N/A	0.000	N/A
455	F	0	0	N/A	0.000	N/A
456	F	0	0	N/A	0.000	N/A
457	C	18	17	1.14	21	0.810
458	N/A	1	0	1.00	28	0.000
459	F	0	0	N/A	0.000	N/A

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
460	F	0	0	N/A	0.000	N/A
461	F	0	0	N/A	0.000	N/A
462	A	7	6	1.00	27	0.222
463	N/A	1	0	1.00	34	0.000
464	A	21	20	0.88	34	0.588
465	A	21	20	0.91	34	0.588
466	A	18	17	0.90	32	0.531
467	A	7	6	1.13	27	0.222
468	N/A	1	0	1.00	34	0.000
469	A	30	29	0.89	36	0.806
470	C	24	23	0.89	34	0.676
471	A	4	4	1.00	29	0.138
472	N/A	1	0	1.00	36	0.000
473	A	15	14	0.82	34	0.412
474	A	8	7	1.15	29	0.241
475	N/A	1	0	1.00	36	0.000
476	F	0	0	N/A	0.000	N/A
477	C	29	28	1.13	34	0.824
478	C	23	22	1.17	32	0.688
479	A	8	7	0.94	27	0.259
480	N/A	1	0	1.00	34	0.000
481	F	0	0	N/A	0.000	N/A
482	F	0	0	N/A	0.000	N/A
483	F	0	0	N/A	0.000	N/A
484	C	22	21	1.23	27	0.778
485	N/A	1	0	1.00	34	0.000
486	F	0	0	N/A	0.000	N/A
487	F	0	0	N/A	0.000	N/A
488	F	0	0	N/A	0.000	N/A
489	A	7	6	0.95	21	0.286

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
490	N/A	1	0	1.00	28	0.000
491	A	24	23	0.89	34	0.676
492	A	24	23	0.92	34	0.676
493	A	21	20	0.88	32	0.625
494	A	8	7	1.11	27	0.259
495	N/A	1	0	1.00	34	0.000
496	F	0	0	N/A	0.000	N/A
497	C	27	26	0.88	34	0.765
498	C	4	4	1.14	29	0.138
499	N/A	1	0	1.00	36	0.000
500	A	26	25	0.84	34	0.735
501	A	8	7	1.11	29	0.241
502	N/A	1	0	1.00	36	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (c+dx)^4 \sinh(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 91

$$\int (c+dx)^4 \sinh(a+bx)\,dx = \frac{24d^4\cosh(a+bx)}{b^5}+\frac{12d^2(c+dx)^2\cosh(a+bx)}{b^3}+\frac{(c+dx)^4\cosh(a+bx)}{b}-\frac{24d^3(c+dx)\sinh(a+bx)}{b^4}-\frac{4d(c+dx)^3\sinh(a+bx)}{b^2}$$

output
```
24*d^4*cosh(b*x+a)/b^5+12*d^2*(d*x+c)^2*cosh(b*x+a)/b^3+(d*x+c)^4*cosh(b*x
+a)/b-24*d^3*(d*x+c)*sinh(b*x+a)/b^4-4*d*(d*x+c)^3*sinh(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int (c+dx)^4 \sinh(a+bx)\,dx = \frac{(24d^4+12b^2d^2(c+dx)^2+b^4(c+dx)^4)\cosh(a+bx)-4bd(c+dx)\left(6d^2+b^2(c+dx)^2\right)\sinh(a+bx)}{b^5}$$

input
```
Integrate[(c + d*x)^4*Sinh[a + b*x],x]
```

output
```
((24*d^4 + 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cosh[a + b*x] - 4*b*d
*(c + d*x)*(6*d^2 + b^2*(c + d*x)^2)*Sinh[a + b*x])/b^5
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.27, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.071$, Rules used = {3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \sinh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i(c+dx)^4 \sin(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int (c+dx)^4 \sin(ia+ibx)dx$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\int (c+dx)^3\cosh(a+bx)dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\int (c+dx)^3\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}-\frac{3id\int -i(c+dx)^2\sinh(a+bx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{26}$$

$$-i \left(\frac{i (c+dx)^4 \cosh(a+bx)}{b}-\frac{4 i d \left(\frac{(c+dx)^3 \sinh(a+bx)}{b}-\frac{3 d \int (c+dx)^2 \sinh(a+bx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$-i \left(\frac{i (c+dx)^4 \cosh(a+bx)}{b}-\frac{4 i d \left(\frac{(c+dx)^3 \sinh(a+bx)}{b}-\frac{3 d \int -i (c+dx)^2 \sin(i a+i b x)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \textbf{26}$$

$$-i \left(\frac{i (c+dx)^4 \cosh(a+bx)}{b}-\frac{4 i d \left(\frac{(c+dx)^3 \sinh(a+bx)}{b}+\frac{3 i d \int (c+dx)^2 \sin(i a+i b x)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \textbf{3777}$$

$$-i \left(\frac{i (c+dx)^4 \cosh(a+bx)}{b}-\frac{4 i d \left(\frac{(c+dx)^3 \sinh(a+bx)}{b}+\frac{3 i d \left(\frac{i (c+dx)^2 \cosh(a+bx)}{b}-\frac{2 i d \int (c+dx) \cosh(a+bx)dx}{b}\right)}{b}\right)}{b}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$-i \left(\frac{i (c+dx)^4 \cosh(a+bx)}{b}-\frac{4 i d \left(\frac{(c+dx)^3 \sinh(a+bx)}{b}+\frac{3 i d \left(\frac{i (c+dx)^2 \cosh(a+bx)}{b}-\frac{2 i d \int (c+dx) \sin\left(i a+i b x+\frac{\pi}{2}\right)dx}{b}\right)}{b}\right)}{b}\right)$$

$$\Big\downarrow \textbf{3777}$$

$$-i \left(\frac{i (c+dx)^4 \cosh(a+bx)}{b}-\frac{4 i d \left(\frac{(c+dx)^3 \sinh(a+bx)}{b}+\frac{3 i d \left(\frac{i (c+dx)^2 \cosh(a+bx)}{b}-\frac{2 i d \left(\frac{(c+dx) \sinh(a+bx)}{b}-\frac{i d \int -i \sinh(a+bx)dx}{b}\right)}{b}\right)}{b}\right)}{b}\right)$$

$$\Big\downarrow \; 26$$

$$-i\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\int\sinh(a+bx)dx}{b}\right)}{b}\right)}{b}\right)}{b}\right.$$

$$\Big\downarrow \; 3042$$

$$-i\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\int-i\sin(ia+ibx)dx}{b}\right)}{b}\right)}{b}\right.}{b}\right.$$

$$\Big\downarrow \; 26$$

$$-i\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}+\frac{id\int\sin(ia+ibx)dx}{b}\right)}{b}\right)}{b}\right.}{b}\right.$$

$$\Big\downarrow \; 3118$$

$$-i\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\cosh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)}{b}\right)$$

input
```
Int[(c + d*x)^4*Sinh[a + b*x],x]
```

output
```
(-I)*((I*(c + d*x)^4*Cosh[a + b*x])/b - ((4*I)*d*(((c + d*x)^3*Sinh[a + b*
x])/b + ((3*I)*d*((I*(c + d*x)^2*Cosh[a + b*x])/b - ((2*I)*d*(-((d*Cosh[a
+ b*x])/b^2) + ((c + d*x)*Sinh[a + b*x])/b))/b))/b))/b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(91) = 182$.

Time = 0.31 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.03

method	result
parallelrisch	$\frac{-4\left(\frac{dx}{2}+c\right)b^{2}dx\left(\left(\frac{1}{2}x^{2}d^{2}+cdx+c^{2}\right)b^{2}+6d^{2}\right)\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+8b\left((dx+c)^{2}b^{2}+6d^{2}\right)d(dx+c)\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+\left(-d^{4}x^{4}-4\right.}{b^{5}\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)}$
orering	$-\frac{8d\left(d^{4}x^{4}b^{4}+4b^{4}c\,d^{3}x^{3}+6b^{4}c^{2}d^{2}x^{2}+4b^{4}c^{3}dx+b^{4}c^{4}+9b^{2}d^{4}x^{2}+18b^{2}c\,d^{3}x+9b^{2}c^{2}d^{2}+12d^{4}\right)\sinh(bx+a)}{b^{6}(dx+c)}+\frac{(d^{4}x^{4}b^{4}-}{}$
risch	$\frac{(d^{4}x^{4}b^{4}+4b^{4}c\,d^{3}x^{3}+6b^{4}c^{2}d^{2}x^{2}-4b^{3}d^{4}x^{3}+4b^{4}c^{3}dx-12b^{3}c\,d^{3}x^{2}+b^{4}c^{4}-12b^{3}c^{2}d^{2}x+12b^{2}d^{4}x^{2}-4b^{3}c^{3}d+24b^{2}c\,d^{3}x+1}{2b^{5}}$
parts	$\frac{\cosh(bx+a)d^{4}x^{4}}{b}+\frac{4\cosh(bx+a)c\,d^{3}x^{3}}{b}+\frac{6\cosh(bx+a)c^{2}d^{2}x^{2}}{b}+\frac{4\cosh(bx+a)c^{3}dx}{b}+\frac{\cosh(bx+a)c^{4}}{b}-\frac{4d\left(\right.}{}$
meijerg	$-\frac{16d^{4}\sqrt{\pi}\,\cosh(a)\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8}x^{4}b^{4}+\frac{9}{2}x^{2}b^{2}+9\right)\cosh(bx)}{6\sqrt{\pi}}+\frac{xb\left(\frac{3x^{2}b^{2}}{2}+9\right)\sinh(bx)}{6\sqrt{\pi}}\right)}{b^{5}}-\frac{16id^{4}\sqrt{\pi}\,\sinh(a)\left(-\frac{ixb\left(\frac{5x^{2}b^{2}}{2}\right.}{10}\right.}{}$
derivativedivides	$\frac{c^{4}\cosh(bx+a)-\frac{12d^{3}ac\left((bx+a)^{2}\cosh(bx+a)-2(bx+a)\sinh(bx+a)+2\cosh(bx+a)\right)}{b^{3}}+\frac{12d^{3}a^{2}c((bx+a)\cosh(bx+a)-\sinh(bx+a))}{b^{3}}-}{}$
default	$\frac{c^{4}\cosh(bx+a)-\frac{12d^{3}ac\left((bx+a)^{2}\cosh(bx+a)-2(bx+a)\sinh(bx+a)+2\cosh(bx+a)\right)}{b^{3}}+\frac{12d^{3}a^{2}c((bx+a)\cosh(bx+a)-\sinh(bx+a))}{b^{3}}-}{}$

input
```
int((d*x+c)^4*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(-4*(1/2*d*x+c)*b^2*d*x*((1/2*x^2*d^2+c*d*x+c^2)*b^2+6*d^2)*tanh(1/2*b*x+1
/2*a)^2+8*b*((d*x+c)^2*b^2+6*d^2)*d*(d*x+c)*tanh(1/2*b*x+1/2*a)+(-d^4*x^4-
4*c*d^3*x^3-6*c^2*d^2*x^2-4*c^3*d*x-2*c^4)*b^4+12*(-d^4*x^2-2*c*d^3*x-2*c^
2*d^2)*b^2-48*d^4)/b^5/(tanh(1/2*b*x+1/2*a)^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.86

$$\int (c+dx)^4\sinh(a+bx)\,dx$$
$$=\frac{\left(b^{4}d^{4}x^{4}+4\,b^{4}cd^{3}x^{3}+b^{4}c^{4}+12\,b^{2}c^{2}d^{2}+24\,d^{4}+6\,(b^{4}c^{2}d^{2}+2\,b^{2}d^{4})x^{2}+4\,(b^{4}c^{3}d+6\,b^{2}cd^{3})x\right)\cosh\left(bx+\right.}{b^{5}}$$

input
```
integrate((d*x+c)^4*sinh(b*x+a),x, algorithm="fricas")
```

output
```
((b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 + 12*b^2*c^2*d^2 + 24*d^4 + 6*(b
^4*c^2*d^2 + 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d + 6*b^2*c*d^3)*x)*cosh(b*x + a)
 - 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + b^3*c^3*d + 6*b*c*d^3 + 3*(b^3*c^2*d
^2 + 2*b*d^4)*x)*sinh(b*x + a))/b^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(92) = 184$.

Time = 0.37 (sec) , antiderivative size = 311, normalized size of antiderivative = 3.42

$$\int (c+dx)^4 \sinh(a+bx)\,dx$$
$$= \begin{cases} \frac{c^4\cosh(a+bx)}{b} + \frac{4c^3dx\cosh(a+bx)}{b} + \frac{6c^2d^2x^2\cosh(a+bx)}{b} + \frac{4cd^3x^3\cosh(a+bx)}{b} + \frac{d^4x^4\cosh(a+bx)}{b} - \frac{4c^3d\sinh(a+bx)}{b^2} - \frac{12c^2 \\ \left(c^4x + 2c^3dx^2 + 2c^2d^2x^3 + cd^3x^4 + \frac{d^4x^5}{5}\right)\sinh(a) \end{cases}$$

input
```
integrate((d*x+c)**4*sinh(b*x+a),x)
```

output
```
Piecewise((c**4*cosh(a + b*x)/b + 4*c**3*d*x*cosh(a + b*x)/b + 6*c**2*d**2
*x**2*cosh(a + b*x)/b + 4*c*d**3*x**3*cosh(a + b*x)/b + d**4*x**4*cosh(a +
 b*x)/b - 4*c**3*d*sinh(a + b*x)/b**2 - 12*c**2*d**2*x*sinh(a + b*x)/b**2
- 12*c*d**3*x**2*sinh(a + b*x)/b**2 - 4*d**4*x**3*sinh(a + b*x)/b**2 + 12*
c**2*d**2*cosh(a + b*x)/b**3 + 24*c*d**3*x*cosh(a + b*x)/b**3 + 12*d**4*x*
*2*cosh(a + b*x)/b**3 - 24*c*d**3*sinh(a + b*x)/b**4 - 24*d**4*x*sinh(a +
b*x)/b**4 + 24*d**4*cosh(a + b*x)/b**5, Ne(b, 0)), ((c**4*x + 2*c**3*d*x**
2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sinh(a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(91) = 182$.

Time = 0.05 (sec) , antiderivative size = 326, normalized size of antiderivative = 3.58

$$\int (c+dx)^4 \sinh(a+bx)\,dx = \frac{c^4e^{(bx+a)}}{2\,b} + \frac{2\,(bxe^a - e^a)c^3de^{(bx)}}{b^2} + \frac{c^4e^{(-bx-a)}}{2\,b} + \frac{2\,(bx+1)c^3de^{(-bx-a)}}{b^2} + \frac{3\,(b^2x^2e^a - 2\,bxe^a + 2\,e^a)c^2d^2e^{(bx)}}{b^3} + \frac{3\,(b^2x^2 + 2\,bx + 2)c^2d^2e^{(-bx-a)}}{b^3} + \frac{2\,(b^3x^3e^a - 3\,b^2x^2e^a + 6\,bxe^a - 6\,e^a)cd^3e^{(bx)}}{b^4} + \frac{2\,(b^3x^3 + 3\,b^2x^2 + 6\,bx + 6)cd^3e^{(-bx-a)}}{b^4} + \frac{(b^4x^4e^a - 4\,b^3x^3e^a + 12\,b^2x^2e^a - 24\,bxe^a + 24\,e^a)d^4e^{(bx)}}{2\,b^5} + \frac{(b^4x^4 + 4\,b^3x^3 + 12\,b^2x^2 + 24\,bx + 24)d^4e^{(-bx-a)}}{2\,b^5}$$

input
```
integrate((d*x+c)^4*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*c^4*e^(b*x + a)/b + 2*(b*x*e^a - e^a)*c^3*d*e^(b*x)/b^2 + 1/2*c^4*e^(-
b*x - a)/b + 2*(b*x + 1)*c^3*d*e^(-b*x - a)/b^2 + 3*(b^2*x^2*e^a - 2*b*x*e
^a + 2*e^a)*c^2*d^2*e^(b*x)/b^3 + 3*(b^2*x^2 + 2*b*x + 2)*c^2*d^2*e^(-b*x
- a)/b^3 + 2*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x*e^a - 6*e^a)*c*d^3*e^(b*
x)/b^4 + 2*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*c*d^3*e^(-b*x - a)/b^4 + 1/2*
(b^4*x^4*e^a - 4*b^3*x^3*e^a + 12*b^2*x^2*e^a - 24*b*x*e^a + 24*e^a)*d^4*e
^(b*x)/b^5 + 1/2*(b^4*x^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*d^4*e^(-
b*x - a)/b^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(91) = 182$.

Time = 0.16 (sec) , antiderivative size = 324, normalized size of antiderivative = 3.56

$$\int (c+dx)^4 \sinh(a+bx)\,dx = \frac{(b^4d^4x^4 + 4\,b^4cd^3x^3 + 6\,b^4c^2d^2x^2 - 4\,b^3d^4x^3 + 4\,b^4c^3dx - 12\,b^3cd^3x^2 + b^4c^4 - 12\,b^3c^2d^2x + 12\,b^2d^4x^2 - 4}{2\,b^5} + \frac{(b^4d^4x^4 + 4\,b^4cd^3x^3 + 6\,b^4c^2d^2x^2 + 4\,b^3d^4x^3 + 4\,b^4c^3dx + 12\,b^3cd^3x^2 + b^4c^4 + 12\,b^3c^2d^2x + 12\,b^2d^4x^2 +}{2\,b^5}$$

input
```
integrate((d*x+c)^4*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 - 4*b^3*d^4*x^3 + 4
*b^4*c^3*d*x - 12*b^3*c*d^3*x^2 + b^4*c^4 - 12*b^3*c^2*d^2*x + 12*b^2*d^4*
x^2 - 4*b^3*c^3*d + 24*b^2*c*d^3*x + 12*b^2*c^2*d^2 - 24*b*d^4*x - 24*b*c*
d^3 + 24*d^4)*e^(b*x + a)/b^5 + 1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4
*c^2*d^2*x^2 + 4*b^3*d^4*x^3 + 4*b^4*c^3*d*x + 12*b^3*c*d^3*x^2 + b^4*c^4
+ 12*b^3*c^2*d^2*x + 12*b^2*d^4*x^2 + 4*b^3*c^3*d + 24*b^2*c*d^3*x + 12*b^
2*c^2*d^2 + 24*b*d^4*x + 24*b*c*d^3 + 24*d^4)*e^(-b*x - a)/b^5
```

Mupad [B] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.36

$$\int (c+dx)^4 \sinh(a+bx)\,dx = \frac{\cosh(a+b\,x)\,\left(b^4\,c^4+12\,b^2\,c^2\,d^2+24\,d^4\right)}{b^5} - \frac{4\sinh(a+b\,x)\,\left(b^2\,c^3\,d+6\,c\,d^3\right)}{b^4} + \frac{d^4\,x^4\cosh(a+b\,x)}{b} + \frac{4\,x\cosh(a+b\,x)\,\left(b^2\,c^3\,d+6\,c\,d^3\right)}{b^3} - \frac{4\,d^4\,x^3\sinh(a+b\,x)}{b^2} - \frac{12\,x\sinh(a+b\,x)\,\left(b^2\,c^2\,d^2+2\,d^4\right)}{b^4} + \frac{6\,x^2\cosh(a+b\,x)\,\left(b^2\,c^2\,d^2+2\,d^4\right)}{b^3} + \frac{4\,c\,d^3\,x^3\cosh(a+b\,x)}{b} - \frac{12\,c\,d^3\,x^2\sinh(a+b\,x)}{b^2}$$

input
```
int(sinh(a + b*x)*(c + d*x)^4,x)
```

output
```
(cosh(a + b*x)*(24*d^4 + b^4*c^4 + 12*b^2*c^2*d^2))/b^5 - (4*sinh(a + b*x)
*(6*c*d^3 + b^2*c^3*d))/b^4 + (d^4*x^4*cosh(a + b*x))/b + (4*x*cosh(a + b*
x)*(6*c*d^3 + b^2*c^3*d))/b^3 - (4*d^4*x^3*sinh(a + b*x))/b^2 - (12*x*sinh
(a + b*x)*(2*d^4 + b^2*c^2*d^2))/b^4 + (6*x^2*cosh(a + b*x)*(2*d^4 + b^2*c
^2*d^2))/b^3 + (4*c*d^3*x^3*cosh(a + b*x))/b - (12*c*d^3*x^2*sinh(a + b*x)
)/b^2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.67

$$\int (c+dx)^4 \sinh(a+bx)\,dx = \frac{\cosh(bx+a)\,b^4c^4 + 4\cosh(bx+a)\,b^4c^3dx + 6\cosh(bx+a)\,b^4c^2d^2x^2 + 4\cosh(bx+a)\,b^4c\,d^3x^3 + \cosh(\cdots}{\cdots}$$

input
```
int((d*x+c)^4*sinh(b*x+a),x)
```

output
```
(cosh(a + b*x)*b**4*c**4 + 4*cosh(a + b*x)*b**4*c**3*d*x + 6*cosh(a + b*x)
*b**4*c**2*d**2*x**2 + 4*cosh(a + b*x)*b**4*c*d**3*x**3 + cosh(a + b*x)*b*
*4*d**4*x**4 + 12*cosh(a + b*x)*b**2*c**2*d**2 + 24*cosh(a + b*x)*b**2*c*d
**3*x + 12*cosh(a + b*x)*b**2*d**4*x**2 + 24*cosh(a + b*x)*d**4 - 4*sinh(a
 + b*x)*b**3*c**3*d - 12*sinh(a + b*x)*b**3*c**2*d**2*x - 12*sinh(a + b*x)
*b**3*c*d**3*x**2 - 4*sinh(a + b*x)*b**3*d**4*x**3 - 24*sinh(a + b*x)*b*c*
d**3 - 24*sinh(a + b*x)*b*d**4*x)/b**5
```

3.2 $\int (c+dx)^3 \sinh(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 70

$$\int (c+dx)^3 \sinh(a+bx)\,dx = \frac{6d^2(c+dx)\cosh(a+bx)}{b^3} + \frac{(c+dx)^3\cosh(a+bx)}{b} - \frac{6d^3\sinh(a+bx)}{b^4} - \frac{3d(c+dx)^2\sinh(a+bx)}{b^2}$$

output
```
6*d^2*(d*x+c)*cosh(b*x+a)/b^3+(d*x+c)^3*cosh(b*x+a)/b-6*d^3*sinh(b*x+a)/b^
4-3*d*(d*x+c)^2*sinh(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int (c+dx)^3 \sinh(a+bx)\,dx = \frac{b(c+dx)\left(6d^2+b^2(c+dx)^2\right)\cosh(a+bx) - 3d\left(2d^2+b^2(c+dx)^2\right)\sinh(a+bx)}{b^4}$$

input
```
Integrate[(c + d*x)^3*Sinh[a + b*x],x]
```

output
```
(b*(c + d*x)*(6*d^2 + b^2*(c + d*x)^2)*Cosh[a + b*x] - 3*d*(2*d^2 + b^2*(c
 + d*x)^2)*Sinh[a + b*x])/b^4
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.31, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\sinh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i(c+dx)^3\sin(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int (c+dx)^3\sin(ia+ibx)dx$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\int (c+dx)^2\cosh(a+bx)dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\int (c+dx)^2\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}-\frac{2id\int -i(c+dx)\sinh(a+bx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}-\frac{2d\int(c+dx)\sinh(a+bx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}-\frac{2d\int-i(c+dx)\sin(ia+ibx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\int(c+dx)\sin(ia+ibx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\int\cosh(a+bx)dx}{b}\right)}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\int\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{3117}$$

$$-i\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\sinh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)$$

input
```
Int[(c + d*x)^3*Sinh[a + b*x],x]
```

output
```
(-I)*((I*(c + d*x)^3*Cosh[a + b*x])/b - ((3*I)*d*(((c + d*x)^2*Sinh[a + b*
x])/b + ((2*I)*d*((I*(c + d*x)*Cosh[a + b*x])/b - (I*d*Sinh[a + b*x])/b^2)
)/b))/b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.70

method	result
orering	$-\frac{6d(d^2x^2b^2+2b^2cdx+b^2c^2+4d^2)\sinh(bx+a)}{b^4}+\frac{(d^2x^2b^2+2b^2cdx+b^2c^2+6d^2)\left(3(dx+c)^2\sinh(bx+a)d+(dx+c)^3b\cosh\right.}{(dx+c)^2b^4}$
parallelrisch	$\frac{-3bd\left(\left(\frac{1}{3}x^2d^2+cdx+c^2\right)b^2+2d^2\right)x\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2+6d\left((dx+c)^2b^2+2d^2\right)\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-2\left(\frac{dx}{2}+c\right)b\left((x^2d^2+cdx+c^2)\right.}{b^4\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)}$
risch	$\frac{(d^3x^3b^3+3b^3cd^2x^2+3b^3c^2dx-3b^2d^3x^2+b^3c^3-6b^2cd^2x-3b^2c^2d+6bd^3x+6bcd^2-6d^3)e^{bx+a}}{2b^4}+\frac{(d^3x^3b^3+3b^3cd^2x^2}{}$
parts	$\frac{\cosh(bx+a)d^3x^3}{b}+\frac{3\cosh(bx+a)cd^2x^2}{b}+\frac{3\cosh(bx+a)c^2dx}{b}+\frac{\cosh(bx+a)c^3}{b}-\frac{3d\left(\frac{d^2\left((bx+a)^2\sinh(bx+a)-2(b\right.}{}\right.}{}$
derivativedivides	$\frac{d^3\left((bx+a)^3\cosh(bx+a)-3(bx+a)^2\sinh(bx+a)+6(bx+a)\cosh(bx+a)-6\sinh(bx+a)\right)}{b^3}-\frac{3d^3a\left((bx+a)^2\cosh(bx+a)-2(bx+a)\sinh(b\right.}{b^3}$
default	$\frac{d^3\left((bx+a)^3\cosh(bx+a)-3(bx+a)^2\sinh(bx+a)+6(bx+a)\cosh(bx+a)-6\sinh(bx+a)\right)}{b^3}-\frac{3d^3a\left((bx+a)^2\cosh(bx+a)-2(bx+a)\sinh(b\right.}{b^3}$
meijerg	$-\frac{8id^3\sqrt{\pi}\cosh(a)\left(\frac{ixb\left(\frac{5x^2b^2}{2}+15\right)\cosh(bx)}{20\sqrt{\pi}}-\frac{i\left(\frac{15x^2b^2}{2}+15\right)\sinh(bx)}{20\sqrt{\pi}}\right)}{b^4}+\frac{8d^3\sqrt{\pi}\sinh(a)\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(\frac{3x^2b^2}{2}+3\right)\cosh(b\right.}{4\sqrt{\pi}}\right.}{b^4}$

input
```
int((d*x+c)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-6*d*(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2+4*d^2)/b^4*sinh(b*x+a)+(b^2*d^2*x^2+
2*b^2*c*d*x+b^2*c^2+6*d^2)/(d*x+c)^2/b^4*(3*(d*x+c)^2*sinh(b*x+a)*d+(d*x+c
)^3*b*cosh(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.56

$$\int (c+dx)^3\sinh(a+bx)\,dx = \frac{(b^3d^3x^3+3\,b^3cd^2x^2+b^3c^3+6\,bcd^2+3\,(b^3c^2d+2\,bd^3)x)\cosh(bx+a)-3\,(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d+}{b^4}$$

input
```
integrate((d*x+c)^3*sinh(b*x+a),x, algorithm="fricas")
```

output
```
((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 6*b*c*d^2 + 3*(b^3*c^2*d + 2*b
*d^3)*x)*cosh(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d + 2*d^
3)*sinh(b*x + a))/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(70) = 140$.

Time = 0.27 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.89

$$\int (c+dx)^3 \sinh(a+bx)\,dx$$

$$= \begin{cases} \frac{c^3\cosh(a+bx)}{b} + \frac{3c^2dx\cosh(a+bx)}{b} + \frac{3cd^2x^2\cosh(a+bx)}{b} + \frac{d^3x^3\cosh(a+bx)}{b} - \frac{3c^2d\sinh(a+bx)}{b^2} - \frac{6cd^2x\sinh(a+bx)}{b^2} - \frac{3d^3x^2 s}{} \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sinh(a) \end{cases}$$

input
```
integrate((d*x+c)**3*sinh(b*x+a),x)
```

output
```
Piecewise((c**3*cosh(a + b*x)/b + 3*c**2*d*x*cosh(a + b*x)/b + 3*c*d**2*x*
*2*cosh(a + b*x)/b + d**3*x**3*cosh(a + b*x)/b - 3*c**2*d*sinh(a + b*x)/b*
*2 - 6*c*d**2*x*sinh(a + b*x)/b**2 - 3*d**3*x**2*sinh(a + b*x)/b**2 + 6*c*
d**2*cosh(a + b*x)/b**3 + 6*d**3*x*cosh(a + b*x)/b**3 - 6*d**3*sinh(a + b*
x)/b**4, Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4
)*sinh(a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(70) = 140$.

Time = 0.04 (sec) , antiderivative size = 222, normalized size of antiderivative = 3.17

$$\int (c+dx)^3\sinh(a+bx)\,dx = \frac{c^3e^{(bx+a)}}{2\,b}+\frac{3\,(bxe^a-e^a)c^2de^{(bx)}}{2\,b^2}+\frac{c^3e^{(-bx-a)}}{2\,b}+\frac{3\,(bx+1)c^2de^{(-bx-a)}}{2\,b^2}+\frac{3\,(b^2x^2e^a-2\,bxe^a+2\,e^a)cd^2e^{(bx)}}{2\,b^3}+\frac{3\,(b^2x^2+2\,bx+2)cd^2e^{(-bx-a)}}{2\,b^3}+\frac{(b^3x^3e^a-3\,b^2x^2e^a+6\,bxe^a-6\,e^a)d^3e^{(bx)}}{2\,b^4}+\frac{(b^3x^3+3\,b^2x^2+6\,bx+6)d^3e^{(-bx-a)}}{2\,b^4}$$

input
```
integrate((d*x+c)^3*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*c^3*e^(b*x + a)/b + 3/2*(b*x*e^a - e^a)*c^2*d*e^(b*x)/b^2 + 1/2*c^3*e^
(-b*x - a)/b + 3/2*(b*x + 1)*c^2*d*e^(-b*x - a)/b^2 + 3/2*(b^2*x^2*e^a - 2
*b*x*e^a + 2*e^a)*c*d^2*e^(b*x)/b^3 + 3/2*(b^2*x^2 + 2*b*x + 2)*c*d^2*e^(-
b*x - a)/b^3 + 1/2*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x*e^a - 6*e^a)*d^3*e
^(b*x)/b^4 + 1/2*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*d^3*e^(-b*x - a)/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(70) = 140$.

Time = 0.12 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.91

$$\int (c+dx)^3\sinh(a+bx)\,dx = \frac{(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx-3\,b^2d^3x^2+b^3c^3-6\,b^2cd^2x-3\,b^2c^2d+6\,bd^3x+6\,bcd^2-6\,d^3)e^{(bx+a)}}{2\,b^4}+\frac{(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+3\,b^2d^3x^2+b^3c^3+6\,b^2cd^2x+3\,b^2c^2d+6\,bd^3x+6\,bcd^2+6\,d^3)e^{(-bx-a)}}{2\,b^4}$$

input
```
integrate((d*x+c)^3*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x - 3*b^2*d^3*x^2 + b^3*c
^3 - 6*b^2*c*d^2*x - 3*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 - 6*d^3)*e^(b*x +
 a)/b^4 + 1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*b^2*d^3*x
^2 + b^3*c^3 + 6*b^2*c*d^2*x + 3*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 + 6*d^3
)*e^(-b*x - a)/b^4
```

Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.04

$$\int (c+dx)^3 \sinh(a+bx)\,dx = \frac{\cosh(a+b\,x)\,\left(b^2\,c^3+6\,c\,d^2\right)}{b^3} - \frac{3\sinh(a+b\,x)\,\left(b^2\,c^2\,d+2\,d^3\right)}{b^4} + \frac{d^3\,x^3\cosh(a+b\,x)}{b} - \frac{3\,d^3\,x^2\sinh(a+b\,x)}{b^2} + \frac{3\,x\cosh(a+b\,x)\,\left(b^2\,c^2\,d+2\,d^3\right)}{b^3} - \frac{6\,c\,d^2\,x\sinh(a+b\,x)}{b^2} + \frac{3\,c\,d^2\,x^2\cosh(a+b\,x)}{b}$$

input
```
int(sinh(a + b*x)*(c + d*x)^3,x)
```

output
```
(cosh(a + b*x)*(6*c*d^2 + b^2*c^3))/b^3 - (3*sinh(a + b*x)*(2*d^3 + b^2*c^
2*d))/b^4 + (d^3*x^3*cosh(a + b*x))/b - (3*d^3*x^2*sinh(a + b*x))/b^2 + (3
*x*cosh(a + b*x)*(2*d^3 + b^2*c^2*d))/b^3 - (6*c*d^2*x*sinh(a + b*x))/b^2
+ (3*c*d^2*x^2*cosh(a + b*x))/b
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.19

$$\int (c+dx)^3 \sinh(a+bx)\,dx$$

$$= \frac{\cosh(bx+a)\,b^3c^3 + 3\cosh(bx+a)\,b^3c^2dx + 3\cosh(bx+a)\,b^3c\,d^2x^2 + \cosh(bx+a)\,b^3d^3x^3 + 6\cosh(bx}{}$$

input
```
int((d*x+c)^3*sinh(b*x+a),x)
```

output
```
(cosh(a + b*x)*b**3*c**3 + 3*cosh(a + b*x)*b**3*c**2*d*x + 3*cosh(a + b*x)
*b**3*c*d**2*x**2 + cosh(a + b*x)*b**3*d**3*x**3 + 6*cosh(a + b*x)*b*c*d**
2 + 6*cosh(a + b*x)*b*d**3*x - 3*sinh(a + b*x)*b**2*c**2*d - 6*sinh(a + b*
x)*b**2*c*d**2*x - 3*sinh(a + b*x)*b**2*d**3*x**2 - 6*sinh(a + b*x)*d**3)/
b**4
```

3.3 $\int (c+dx)^2 \sinh(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 49

$$\int (c+dx)^2 \sinh(a+bx)\,dx = \frac{2d^2\cosh(a+bx)}{b^3} + \frac{(c+dx)^2\cosh(a+bx)}{b} - \frac{2d(c+dx)\sinh(a+bx)}{b^2}$$

output
```
2*d^2*cosh(b*x+a)/b^3+(d*x+c)^2*cosh(b*x+a)/b-2*d*(d*x+c)*sinh(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int (c+dx)^2 \sinh(a+bx)\,dx = \frac{(2d^2+b^2(c+dx)^2)\cosh(a+bx) - 2bd(c+dx)\sinh(a+bx)}{b^3}$$

input
```
Integrate[(c + d*x)^2*Sinh[a + b*x],x]
```

output
```
((2*d^2 + b^2*(c + d*x)^2)*Cosh[a + b*x] - 2*b*d*(c + d*x)*Sinh[a + b*x])/
b^3
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\sinh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i(c+dx)^2\sin(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int (c+dx)^2\sin(ia+ibx)dx$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\int (c+dx)\cosh(a+bx)dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\int (c+dx)\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{id\int -i\sinh(a+bx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\int \sinh(a+bx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\int -i\sin(ia+ibx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}+\frac{id\int \sin(ia+ibx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{3118}$$

$$-i\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\cosh(a+bx)}{b^2}\right)}{b}\right)$$

input
```
Int[(c + d*x)^2*Sinh[a + b*x],x]
```

output
```
(-I)*((I*(c + d*x)^2*Cosh[a + b*x])/b - ((2*I)*d*(-((d*Cosh[a + b*x])/b^2)
 + ((c + d*x)*Sinh[a + b*x])/b))/b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.86

method	result
parallelrisch	$\frac{-2\left(\frac{dx}{2}+c\right)b^2dx\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2+4bd(dx+c)\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+(-x^2d^2-2cdx-2c^2)b^2-4d^2}{b^3\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)}$
parts	$\frac{\cosh(bx+a)x^2d^2}{b}+\frac{2\cosh(bx+a)cdx}{b}+\frac{\cosh(bx+a)c^2}{b}-\frac{2d\left(\frac{d((bx+a)\sinh(bx+a)-\cosh(bx+a))}{b}-\frac{da\sinh(bx+a)}{b}+c\,\mathrm{si}\right.}{b^2}$
risch	$\frac{(d^2x^2b^2+2b^2cdx+b^2c^2-2b\,d^2x-2bcd+2d^2)\,\mathrm{e}^{bx+a}}{2b^3}+\frac{(d^2x^2b^2+2b^2cdx+b^2c^2+2b\,d^2x+2bcd+2d^2)\,\mathrm{e}^{-bx-a}}{2b^3}$
orering	$-\frac{4d(d^2x^2b^2+2b^2cdx+b^2c^2+d^2)\sinh(bx+a)}{b^4(dx+c)}+\frac{(d^2x^2b^2+2b^2cdx+b^2c^2+2d^2)\left(2(dx+c)\sinh(bx+a)d+(dx+c)^2b\cos\right.}{b^4(dx+c)^2}$
derivativedivides	$\frac{\frac{d^2\left((bx+a)^2\cosh(bx+a)-2(bx+a)\sinh(bx+a)+2\cosh(bx+a)\right)}{b^2}-\frac{2d^2a((bx+a)\cosh(bx+a)-\sinh(bx+a))}{b^2}+\frac{2dc((bx+a)\cosh(bx+a)-}{b}}{b}$
default	$\frac{\frac{d^2\left((bx+a)^2\cosh(bx+a)-2(bx+a)\sinh(bx+a)+2\cosh(bx+a)\right)}{b^2}-\frac{2d^2a((bx+a)\cosh(bx+a)-\sinh(bx+a))}{b^2}+\frac{2dc((bx+a)\cosh(bx+a)-}{b}}{b}$
meijerg	$\frac{4d^2\sqrt{\pi}\cosh(a)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(\frac{x^2b^2}{2}+1\right)\cosh(bx)}{2\sqrt{\pi}}-\frac{xb\sinh(bx)}{2\sqrt{\pi}}\right)}{b^3}+\frac{4id^2\sqrt{\pi}\sinh(a)\left(\frac{ixb\cosh(bx)}{2\sqrt{\pi}}-\frac{i\left(\frac{3x^2b^2}{2}+3\right)\sinh(bx)}{6\sqrt{\pi}}\right)}{b^3}$

input
```
int((d*x+c)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(-2*(1/2*d*x+c)*b^2*d*x*tanh(1/2*b*x+1/2*a)^2+4*b*d*(d*x+c)*tanh(1/2*b*x+1
/2*a)+(-d^2*x^2-2*c*d*x-2*c^2)*b^2-4*d^2)/b^3/(tanh(1/2*b*x+1/2*a)^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.27

$$\int (c+dx)^2\sinh(a+bx)\,dx = \frac{(b^2d^2x^2+2\,b^2cdx+b^2c^2+2\,d^2)\cosh\left(bx+a\right)-2\left(bd^2x+bcd\right)\sinh\left(bx+a\right)}{b^3}$$

input
```
integrate((d*x+c)^2*sinh(b*x+a),x, algorithm="fricas")
```

output
```
((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*d^2)*cosh(b*x + a) - 2*(b*d^2*x
+ b*c*d)*sinh(b*x + a))/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(48) = 96$.

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.29

$$\int (c+dx)^2 \sinh(a+bx)\, dx = \begin{cases} \frac{c^2\cosh(a+bx)}{b} + \frac{2cdx\cosh(a+bx)}{b} + \frac{d^2x^2\cosh(a+bx)}{b} - \frac{2cd\sinh(a+bx)}{b^2} - \frac{2d^2x\sinh(a+bx)}{b^2} + \frac{2d^2\cosh(a+bx)}{b^3} & \text{for } b \neq 0 \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sinh(a) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)**2*sinh(b*x+a),x)
```

output
```
Piecewise((c**2*cosh(a + b*x)/b + 2*c*d*x*cosh(a + b*x)/b + d**2*x**2*cosh
(a + b*x)/b - 2*c*d*sinh(a + b*x)/b**2 - 2*d**2*x*sinh(a + b*x)/b**2 + 2*d
**2*cosh(a + b*x)/b**3, Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sinh
(a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(49) = 98$.

Time = 0.05 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.73

$$\begin{aligned}\int (c+dx)^2 \sinh(a+bx)\, dx = {} & \frac{c^2e^{(bx+a)}}{2\,b} + \frac{(bxe^a - e^a)cde^{(bx)}}{b^2} + \frac{c^2e^{(-bx-a)}}{2\,b} \\ & + \frac{(bx+1)cde^{(-bx-a)}}{b^2} + \frac{(b^2x^2e^a - 2\,bxe^a + 2\,e^a)d^2e^{(bx)}}{2\,b^3} \\ & + \frac{(b^2x^2 + 2\,bx + 2)d^2e^{(-bx-a)}}{2\,b^3}\end{aligned}$$

input
```
integrate((d*x+c)^2*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*c^2*e^(b*x + a)/b + (b*x*e^a - e^a)*c*d*e^(b*x)/b^2 + 1/2*c^2*e^(-b*x
- a)/b + (b*x + 1)*c*d*e^(-b*x - a)/b^2 + 1/2*(b^2*x^2*e^a - 2*b*x*e^a + 2
*e^a)*d^2*e^(b*x)/b^3 + 1/2*(b^2*x^2 + 2*b*x + 2)*d^2*e^(-b*x - a)/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(49) = 98.

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.29

$$\int (c+dx)^2 \sinh(a+bx)\,dx = \frac{(b^2d^2x^2+2\,b^2cdx+b^2c^2-2\,bd^2x-2\,bcd+2\,d^2)e^{(bx+a)}}{2\,b^3} + \frac{(b^2d^2x^2+2\,b^2cdx+b^2c^2+2\,bd^2x+2\,bcd+2\,d^2)e^{(-bx-a)}}{2\,b^3}$$

input
```
integrate((d*x+c)^2*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*b*d^2*x - 2*b*c*d + 2*d^2)*e^
(b*x + a)/b^3 + 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*b*d^2*x + 2*b
*c*d + 2*d^2)*e^(-b*x - a)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.67

$$\int (c+dx)^2 \sinh(a+bx)\,dx = \frac{\cosh(a+b\,x)\,\left(b^2\,c^2+2\,d^2\right)}{b^3} + \frac{d^2\,x^2\cosh(a+b\,x)}{b} - \frac{2\,c\,d\sinh(a+b\,x)}{b^2} - \frac{2\,d^2\,x\sinh(a+b\,x)}{b^2} + \frac{2\,c\,d\,x\cosh(a+b\,x)}{b}$$

input
```
int(sinh(a + b*x)*(c + d*x)^2,x)
```

output
```
(cosh(a + b*x)*(2*d^2 + b^2*c^2))/b^3 + (d^2*x^2*cosh(a + b*x))/b - (2*c*d
*sinh(a + b*x))/b^2 - (2*d^2*x*sinh(a + b*x))/b^2 + (2*c*d*x*cosh(a + b*x)
)/b
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.69

$$\int (c+dx)^2 \sinh(a+bx)\,dx$$
$$= \frac{\cosh(bx+a)\,b^2c^2 + 2\cosh(bx+a)\,b^2cdx + \cosh(bx+a)\,b^2d^2x^2 + 2\cosh(bx+a)\,d^2 - 2\sinh(bx+a)\,b}{b^3}$$

input
```
int((d*x+c)^2*sinh(b*x+a),x)
```

output
```
(cosh(a + b*x)*b**2*c**2 + 2*cosh(a + b*x)*b**2*c*d*x + cosh(a + b*x)*b**2
*d**2*x**2 + 2*cosh(a + b*x)*d**2 - 2*sinh(a + b*x)*b*c*d - 2*sinh(a + b*x
)*b*d**2*x)/b**3
```

3.4 $\int (c+dx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 28

$$\int (c+dx)\sinh(a+bx)\,dx=\frac{(c+dx)\cosh(a+bx)}{b}-\frac{d\sinh(a+bx)}{b^2}$$

output
```
(d*x+c)*cosh(b*x+a)/b-d*sinh(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int (c+dx)\sinh(a+bx)\,dx=\frac{b(c+dx)\cosh(a+bx)-d\sinh(a+bx)}{b^2}$$

input
```
Integrate[(c + d*x)*Sinh[a + b*x],x]
```

output
```
(b*(c + d*x)*Cosh[a + b*x] - d*Sinh[a + b*x])/b^2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sinh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i(c+dx)\sin(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int (c+dx)\sin(ia+ibx)dx$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\int \cosh(a+bx)dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\int \sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)$$

$$\Big\downarrow \text{3117}$$

$$-i\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\sinh(a+bx)}{b^2}\right)$$

input
```
Int[(c + d*x)*Sinh[a + b*x],x]
```

output
```
(-I)*((I*(c + d*x)*Cosh[a + b*x])/b - (I*d*Sinh[a + b*x])/b^2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

method	result
parts	$\frac{\cosh(bx+a)dx}{b}+\frac{\cosh(bx+a)c}{b}-\frac{d\sinh(bx+a)}{b^2}$
orering	$-\frac{2d\sinh(bx+a)}{b^2}+\frac{\sinh(bx+a)d+(dx+c)b\cosh(bx+a)}{b^2}$
risch	$\frac{(dxb+bc-d)\mathrm{e}^{bx+a}}{2b^2}+\frac{(dxb+bc+d)\mathrm{e}^{-bx-a}}{2b^2}$
derivativedivides	$\frac{\frac{d((bx+a)\cosh(bx+a)-\sinh(bx+a))}{b}-\frac{da\cosh(bx+a)}{b}+\cosh(bx+a)c}{b}$
default	$\frac{\frac{d((bx+a)\cosh(bx+a)-\sinh(bx+a))}{b}-\frac{da\cosh(bx+a)}{b}+\cosh(bx+a)c}{b}$
parallelrisch	$\frac{-x\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2bd+2\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)d-2\left(\frac{dx}{2}+c\right)b}{b^2\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)}$
meijerg	$-\frac{d\cosh(a)(-\cosh(bx)xb+\sinh(bx))}{b^2}-\frac{2d\sqrt{\pi}\sinh(a)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cosh(bx)}{2\sqrt{\pi}}-\frac{xb\sinh(bx)}{2\sqrt{\pi}}\right)}{b^2}-\frac{c\sqrt{\pi}\cosh(a)\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(b}{\sqrt{\pi}}\right.}{b}$

input
```
int((d*x+c)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
cosh(b*x+a)/b*d*x+cosh(b*x+a)/b*c-d*sinh(b*x+a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int (c+dx)\sinh(a+bx)\,dx = \frac{(bdx+bc)\cosh(bx+a)-d\sinh(bx+a)}{b^2}$$

input
```
integrate((d*x+c)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
((b*d*x + b*c)*cosh(b*x + a) - d*sinh(b*x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int (c+dx)\sinh(a+bx)\,dx = \begin{cases} \frac{c\cosh(a+bx)}{b} + \frac{dx\cosh(a+bx)}{b} - \frac{d\sinh(a+bx)}{b^2} & \text{for } b \neq 0 \\ \left(cx+\frac{dx^2}{2}\right)\sinh(a) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*sinh(b*x+a),x)
```

output
```
Piecewise((c*cosh(a + b*x)/b + d*x*cosh(a + b*x)/b - d*sinh(a + b*x)/b**2,
 Ne(b, 0)), ((c*x + d*x**2/2)*sinh(a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(28) = 56$.

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.43

$$\int (c+dx)\sinh(a+bx)\,dx = \frac{ce^{(bx+a)}}{2\,b} + \frac{(bxe^a - e^a)de^{(bx)}}{2\,b^2} + \frac{ce^{(-bx-a)}}{2\,b} + \frac{(bx+1)de^{(-bx-a)}}{2\,b^2}$$

input
```
integrate((d*x+c)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*c*e^(b*x + a)/b + 1/2*(b*x*e^a - e^a)*d*e^(b*x)/b^2 + 1/2*c*e^(-b*x -
a)/b + 1/2*(b*x + 1)*d*e^(-b*x - a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int (c+dx)\sinh(a+bx)\,dx = \frac{(bdx + bc - d)e^{(bx+a)}}{2\,b^2} + \frac{(bdx + bc + d)e^{(-bx-a)}}{2\,b^2}$$

input
```
integrate((d*x+c)*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/2*(b*d*x + b*c - d)*e^(b*x + a)/b^2 + 1/2*(b*d*x + b*c + d)*e^(-b*x - a)
/b^2
```

Mupad [B] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int (c+dx)\sinh(a+bx)\,dx = \frac{c\cosh(a+b\,x) + d\,x\cosh(a+b\,x)}{b} - \frac{d\sinh(a+b\,x)}{b^2}$$

input
```
int(sinh(a + b*x)*(c + d*x),x)
```

output
```
(c*cosh(a + b*x) + d*x*cosh(a + b*x))/b - (d*sinh(a + b*x))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

$$\int (c+dx)\sinh(a+bx)\,dx = \frac{\cosh(bx+a)\,bc+\cosh(bx+a)\,bdx-\sinh(bx+a)\,d}{b^2}$$

input
```
int((d*x+c)*sinh(b*x+a),x)
```

output
```
(cosh(a + b*x)*b*c + cosh(a + b*x)*b*d*x - sinh(a + b*x)*d)/b**2
```

3.5 $\int \frac{\sinh(a+bx)}{c+dx}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 51

$$\int \frac{\sinh(a+bx)}{c+dx}\,dx=\frac{\text{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{d}+\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}$$

output
```
Chi(b*c/d+b*x)*sinh(a-b*c/d)/d+cosh(a-b*c/d)*Shi(b*c/d+b*x)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{\sinh(a+bx)}{c+dx}\,dx=\frac{\text{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)+\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}$$

input
```
Integrate[Sinh[a + b*x]/(c + d*x),x]
```

output
```
(CoshIntegral[(b*c)/d + b*x]*Sinh[a - (b*c)/d] + Cosh[a - (b*c)/d]*SinhInt
egral[(b*c)/d + b*x])/d
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)}{c+dx} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ia+ibx)}{c+dx}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ia+ibx)}{c+dx}dx$$

$$\Big\downarrow \text{3784}$$

$$-i\left(i\sinh\left(a-\frac{bc}{d}\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}dx+\cosh\left(a-\frac{bc}{d}\right)\int \frac{i\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(i\sinh\left(a-\frac{bc}{d}\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}dx+i\cosh\left(a-\frac{bc}{d}\right)\int \frac{\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(i\sinh\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx+i\cosh\left(a-\frac{bc}{d}\right)\int -\frac{i\sin\left(\frac{ibc}{d}+ibx\right)}{c+dx}dx\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(i\sinh\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx+\cosh\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{ibc}{d}+ibx\right)}{c+dx}dx\right)$$

$$\Big\downarrow \text{3779}$$

$$-i\left(i\sinh\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx+\frac{i\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}\right)$$

$$\Big\downarrow \text{3782}$$

$$-i\left(\frac{i\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d}+\frac{i\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}\right)$$

input
```
Int[Sinh[a + b*x]/(c + d*x),x]
```

output
```
(-I)*((I*CoshIntegral[(b*c)/d + b*x]*Sinh[a - (b*c)/d])/d + (I*Cosh[a - (b
*c)/d]*SinhIntegral[(b*c)/d + b*x])/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.61

method	result	size
risch	$\frac{\mathrm{e}^{-\frac{da-bc}{d}} \operatorname{expIntegral}_1\left(bx+a-\frac{da-bc}{d}\right)}{2d} - \frac{\mathrm{e}^{\frac{da-bc}{d}} \operatorname{expIntegral}_1\left(-bx-a-\frac{-da+bc}{d}\right)}{2d}$	82

input
```
int(sinh(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)-1/2/d*exp((a*d-b*c)/d)*Ei(
1,-b*x-a-(-a*d+b*c)/d)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.84

$$\int \frac{\sinh(a+bx)}{c+dx}\,dx = \frac{\left(\operatorname{Ei}\left(\frac{bdx+bc}{d}\right)-\operatorname{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cosh\left(-\frac{bc-ad}{d}\right)+\left(\operatorname{Ei}\left(\frac{bdx+bc}{d}\right)+\operatorname{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\sinh\left(-\frac{bc-ad}{d}\right)}{2\,d}$$

input
```
integrate(sinh(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
1/2*((Ei((b*d*x + b*c)/d) - Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d) + (
Ei((b*d*x + b*c)/d) + Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d))/d
```

Sympy [F]

$$\int \frac{\sinh(a+bx)}{c+dx}\,dx = \int \frac{\sinh\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(sinh(b*x+a)/(d*x+c),x)
```

output
```
Integral(sinh(a + b*x)/(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12

$$\int \frac{\sinh(a+bx)}{c+dx}\,dx = \frac{e^{\left(-a+\frac{bc}{d}\right)}E_1\left(\frac{(dx+c)b}{d}\right)}{2\,d} - \frac{e^{\left(a-\frac{bc}{d}\right)}E_1\left(-\frac{(dx+c)b}{d}\right)}{2\,d}$$

input
```
integrate(sinh(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
1/2*e^(-a + b*c/d)*exp_integral_e(1, (d*x + c)*b/d)/d - 1/2*e^(a - b*c/d)*
exp_integral_e(1, -(d*x + c)*b/d)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12

$$\int \frac{\sinh(a+bx)}{c+dx}\,dx = \frac{\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)} - \mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}}{2\,d}$$

input
```
integrate(sinh(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
1/2*(Ei((b*d*x + b*c)/d)*e^(a - b*c/d) - Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/
d))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(a+bx)}{c+dx}\,dx = \int \frac{\sinh(a+b\,x)}{c+d\,x}\,dx$$

input
```
int(sinh(a + b*x)/(c + d*x),x)
```

output
```
int(sinh(a + b*x)/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\sinh(a+bx)}{c+dx}\,dx = \int \frac{\sinh(bx+a)}{dx+c}dx$$

input
```
int(sinh(b*x+a)/(d*x+c),x)
```

output
```
int(sinh(a + b*x)/(c + d*x),x)
```

3.6 $\int \frac{\sinh(a+bx)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 71

$$\int \frac{\sinh(a+bx)}{(c+dx)^2}\,dx = \frac{b\cosh\left(a-\frac{bc}{d}\right)\operatorname{Chi}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{\sinh(a+bx)}{d(c+dx)} + \frac{b\sinh\left(a-\frac{bc}{d}\right)\operatorname{Shi}\left(\frac{bc}{d}+bx\right)}{d^2}$$

output
```
b*cosh(a-b*c/d)*Chi(b*c/d+b*x)/d^2-sinh(b*x+a)/d/(d*x+c)+b*sinh(a-b*c/d)*S
hi(b*c/d+b*x)/d^2
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{\sinh(a+bx)}{(c+dx)^2}\,dx = \frac{b\cosh\left(a-\frac{bc}{d}\right)\operatorname{Chi}\left(b\left(\frac{c}{d}+x\right)\right)-\frac{d\sinh(a+bx)}{c+dx}+b\sinh\left(a-\frac{bc}{d}\right)\operatorname{Shi}\left(b\left(\frac{c}{d}+x\right)\right)}{d^2}$$

input
```
Integrate[Sinh[a + b*x]/(c + d*x)^2,x]
```

output
```
(b*Cosh[a - (b*c)/d]*CoshIntegral[b*(c/d + x)] - (d*Sinh[a + b*x])/(c + d*
x) + b*Sinh[a - (b*c)/d]*SinhIntegral[b*(c/d + x)])/d^2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 26, 3778, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ia+ibx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ia+ibx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3778}$$

$$-i\left(\frac{ib\int \frac{\cosh(a+bx)}{c+dx}dx}{d}-\frac{i\sinh(a+bx)}{d(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{ib\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{c+dx}dx}{d}-\frac{i\sinh(a+bx)}{d(c+dx)}\right)$$

$$\Big\downarrow \text{3784}$$

$$-i\left(\frac{ib\left(\cosh\left(a-\frac{bc}{d}\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}dx-i\sinh\left(a-\frac{bc}{d}\right)\int \frac{i\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}-\frac{i\sinh(a+bx)}{d(c+dx)}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{ib\left(\sinh\left(a-\frac{bc}{d}\right)\int\frac{\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}dx+\cosh\left(a-\frac{bc}{d}\right)\int\frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}-\frac{i\sinh(a+bx)}{d(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{ib\left(\sinh\left(a-\frac{bc}{d}\right)\int-\frac{i\sin\left(\frac{ibc}{d}+ibx\right)}{c+dx}dx+\cosh\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx\right)}{d}-\frac{i\sinh(a+bx)}{d(c+dx)}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{ib\left(\cosh\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx-i\sinh\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{ibc}{d}+ibx\right)}{c+dx}dx\right)}{d}-\frac{i\sinh(a+bx)}{d(c+dx)}\right)$$

$$\Big\downarrow \text{3779}$$

$$-i\left(\frac{ib\left(\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}+\cosh\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx\right)}{d}-\frac{i\sinh(a+bx)}{d(c+dx)}\right)$$

$$\Big\downarrow \text{3782}$$

$$-i\left(\frac{ib\left(\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}-\frac{i\sinh(a+bx)}{d(c+dx)}\right)$$

input
```
Int[Sinh[a + b*x]/(c + d*x)^2,x]
```

output
```
(-I)*(((-I)*Sinh[a + b*x])/(d*(c + d*x)) + (I*b*((Cosh[a - (b*c)/d]*CoshIn
tegral[(b*c)/d + b*x])/d + (Sinh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])
/d))/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.87

method	result	size
risch	$\frac{b\,\mathrm{e}^{-bx-a}}{2d(dxb+bc)}-\frac{b\,\mathrm{e}^{-\frac{da-bc}{d}}\,\mathrm{expIntegral}_1\left(bx+a-\frac{da-bc}{d}\right)}{2d^2}-\frac{b\,\mathrm{e}^{bx+a}}{2d^2\left(\frac{bc}{d}+bx\right)}-\frac{b\,\mathrm{e}^{\frac{da-bc}{d}}\,\mathrm{expIntegral}_1\left(-bx-a-\frac{-da+bc}{d}\right)}{2d^2}$	133

input
```
int(sinh(b*x+a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*b*exp(-b*x-a)/d/(b*d*x+b*c)-1/2*b/d^2*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*
d-b*c)/d)-1/2*b/d^2*exp(b*x+a)/(b*c/d+b*x)-1/2*b/d^2*exp((a*d-b*c)/d)*Ei(1
,-b*x-a-(-a*d+b*c)/d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(71) = 142.

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.08

$$\int \frac{\sinh(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{\left((bdx+bc)\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)+(bdx+bc)\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cosh\left(-\frac{bc-ad}{d}\right)-2\,d\sinh\left(bx+a\right)+\left((bdx+bc)\mathrm{Ei}\left(\frac{bdx+}{d}\right.\right.}{2\left(d^3x+cd^2\right)}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*(((b*d*x + b*c)*Ei((b*d*x + b*c)/d) + (b*d*x + b*c)*Ei(-(b*d*x + b*c)/
d))*cosh(-(b*c - a*d)/d) - 2*d*sinh(b*x + a) + ((b*d*x + b*c)*Ei((b*d*x +
b*c)/d) - (b*d*x + b*c)*Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d))/(d^3*x
 + c*d^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(a+bx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13

$$\int \frac{\sinh(a+bx)}{(c+dx)^2}\,dx = -\frac{b\left(\frac{e^{\left(-a+\frac{bc}{d}\right)}E_1\left(\frac{(dx+c)b}{d}\right)}{d}+\frac{e^{\left(a-\frac{bc}{d}\right)}E_1\left(-\frac{(dx+c)b}{d}\right)}{d}\right)}{2\,d}-\frac{\sinh\left(bx+a\right)}{(dx+c)d}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/2*b*(e^(-a + b*c/d)*exp_integral_e(1, (d*x + c)*b/d)/d + e^(a - b*c/d)*
exp_integral_e(1, -(d*x + c)*b/d)/d)/d - sinh(b*x + a)/((d*x + c)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 615 vs. $2(71) = 142$.

Time = 0.15 (sec) , antiderivative size = 615, normalized size of antiderivative = 8.66

$$\int \frac{\sinh(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\mathrm{Ei}\left(-\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}+b^3c\mathrm{Ei}\left(-\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+b\cdots}{d}\right.\right.}{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right.\right.}$$
$$+\frac{\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\mathrm{Ei}\left(\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}+b^3c\mathrm{Ei}\left(\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+b\cdots}{d}\right.\right.}{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+\cdots}\right.\right.}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*Ei(-((d*x + c)*(b -
 b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c - a*d)/d) + b^3*c*
Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c
 - a*d)/d) - a*b^2*d*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) +
b*c - a*d)/d)*e^((b*c - a*d)/d) + b^2*d*e^(-(d*x + c)*(b - b*c/(d*x + c) +
 a*d/(d*x + c))/d))*d^2/(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*d^
4 + b*c*d^4 - a*d^5)*b) + 1/2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c
))*b^2*Ei(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e
^(-(b*c - a*d)/d) + b^3*c*Ei(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)
) + b*c - a*d)/d)*e^(-(b*c - a*d)/d) - a*b^2*d*Ei(((d*x + c)*(b - b*c/(d*x
 + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(-(b*c - a*d)/d) - b^2*d*e^((d*x
+ c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d))*d^2/(((d*x + c)*(b - b*c/(d*x
 + c) + a*d/(d*x + c))*d^4 + b*c*d^4 - a*d^5)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sinh(a+b\,x)}{{(c+d\,x)}^2}\,dx$$

input
```
int(sinh(a + b*x)/(c + d*x)^2,x)
```

output
```
int(sinh(a + b*x)/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{\sinh(a+bx)}{(c+dx)^2}\,dx = \frac{e^{2a}\left(\int \frac{e^{bx}}{d^2x^2+2cdx+c^2}dx\right)-\left(\int \frac{1}{e^{bx}c^2+2e^{bx}cdx+e^{bx}d^2x^2}dx\right)}{2e^a}$$

input
```
int(sinh(b*x+a)/(d*x+c)^2,x)
```

output
```
(e**(2*a)*int(e**(b*x)/(c**2 + 2*c*d*x + d**2*x**2),x) - int(1/(e**(b*x)*c
**2 + 2*e**(b*x)*c*d*x + e**(b*x)*d**2*x**2),x))/(2*e**a)
```

3.7 $\int \frac{\sinh(a+bx)}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 104

$$\int \frac{\sinh(a+bx)}{(c+dx)^3}\,dx = -\frac{b\cosh(a+bx)}{2d^2(c+dx)} + \frac{b^2\text{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{2d^3} - \frac{\sinh(a+bx)}{2d(c+dx)^2} + \frac{b^2\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{2d^3}$$

output
```
-1/2*b*cosh(b*x+a)/d^2/(d*x+c)+1/2*b^2*Chi(b*c/d+b*x)*sinh(a-b*c/d)/d^3-1/
2*sinh(b*x+a)/d/(d*x+c)^2+1/2*b^2*cosh(a-b*c/d)*Shi(b*c/d+b*x)/d^3
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{\sinh(a+bx)}{(c+dx)^3}\,dx = \frac{b^2\text{Chi}\left(b\left(\frac{c}{d}+x\right)\right)\sinh\left(a-\frac{bc}{d}\right) - \frac{d(b(c+dx)\cosh(a+bx)+d\sinh(a+bx))}{(c+dx)^2} + b^2\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(b\left(\frac{c}{d}+x\right)\right)}{2d^3}$$

input
```
Integrate[Sinh[a + b*x]/(c + d*x)^3,x]
```

output
```
(b^2*CoshIntegral[b*(c/d + x)]*Sinh[a - (b*c)/d] - (d*(b*(c + d*x)*Cosh[a
+ b*x] + d*Sinh[a + b*x]))/(c + d*x)^2 + b^2*Cosh[a - (b*c)/d]*SinhIntegra
l[b*(c/d + x)])/(2*d^3)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.16, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 26, 3778, 3042, 3778, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ia+ibx)}{(c+dx)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ia+ibx)}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3778}$$

$$-i\left(\frac{ib\int \frac{\cosh(a+bx)}{(c+dx)^2}dx}{2d}-\frac{i\sinh(a+bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{ib\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{(c+dx)^2}dx}{2d}-\frac{i\sinh(a+bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{3778}$$

$$-i\left(\frac{ib\left(-\frac{\cosh(a+bx)}{d(c+dx)}+\frac{ib\int -\frac{i\sinh(a+bx)}{c+dx}dx}{d}\right)}{2d}-\frac{i\sinh(a+bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{ib\left(\frac{b\int\frac{\sinh(a+bx)}{c+dx}dx}{d}-\frac{\cosh(a+bx)}{d(c+dx)}\right)}{2d}-\frac{i\sinh(a+bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{ib\left(-\frac{\cosh(a+bx)}{d(c+dx)}+\frac{b\int-\frac{i\sin(ia+ibx)}{c+dx}dx}{d}\right)}{2d}-\frac{i\sinh(a+bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{ib\left(-\frac{\cosh(a+bx)}{d(c+dx)}-\frac{ib\int\frac{\sin(ia+ibx)}{c+dx}dx}{d}\right)}{2d}-\frac{i\sinh(a+bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{3784}$$

$$-i\left(\frac{ib\left(-\frac{\cosh(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(a-\frac{bc}{d}\right)\int\frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}dx+\cosh\left(a-\frac{bc}{d}\right)\int\frac{i\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}\right)}{2d}-\frac{i\sinh(a+bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{ib\left(-\frac{\cosh(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(a-\frac{bc}{d}\right)\int\frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}dx+i\cosh\left(a-\frac{bc}{d}\right)\int\frac{\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}\right)}{2d}-\frac{i\sinh(a+bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{ib\left(-\frac{\cosh(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx+i\cosh\left(a-\frac{bc}{d}\right)\int-\frac{i\sin\left(\frac{ibc}{d}+ibx\right)}{c+dx}dx\right)}{d}\right)}{2d}-\frac{i\sinh(a+bx)}{2d(c+dx)^2}\right)$$

$\downarrow$ 26

$$-i\left(\frac{ib\left(-\frac{\cosh(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx+\cosh\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{ibc}{d}+ibx\right)}{c+dx}dx\right)}{d}\right)}{2d}-\frac{i\sinh(a+bx)}{2d(c+dx)^2}\right)$$

$\downarrow$ 3779

$$-i\left(\frac{ib\left(-\frac{\cosh(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx+\frac{i\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}\right)}{2d}-\frac{i\sinh(a+bx)}{2d(c+dx)^2}\right)$$

$\downarrow$ 3782

$$-i\left(\frac{ib\left(-\frac{\cosh(a+bx)}{d(c+dx)}-\frac{ib\left(\frac{i\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d}+\frac{i\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}\right)}{2d}-\frac{i\sinh(a+bx)}{2d(c+dx)^2}\right)$$

input
```
Int[Sinh[a + b*x]/(c + d*x)^3,x]
```

output
```
(-I)*(((-1/2*I)*Sinh[a + b*x])/(d*(c + d*x)^2) + ((I/2)*b*(-(Cosh[a + b*x]
/(d*(c + d*x))) - (I*b*((I*CoshIntegral[(b*c)/d + b*x]*Sinh[a - (b*c)/d])/
d + (I*Cosh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/d))/d))/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. 2(96) = 192.

Time = 0.24 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.66

method	result
risch	$-\frac{b^3 e^{-bx-a} x}{4d(d^2x^2b^2+2b^2cdx+b^2c^2)} - \frac{b^3 e^{-bx-a} c}{4d^2(d^2x^2b^2+2b^2cdx+b^2c^2)} + \frac{b^2 e^{-bx-a}}{4d(d^2x^2b^2+2b^2cdx+b^2c^2)} + \frac{b^2 e^{-\frac{da-bc}{d}} \operatorname{expIntegral}_1\left(bx+a-\right.}{4d^3}$

input
```
int(sinh(b*x+a)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*b^3*exp(-b*x-a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*x-1/4*b^3*exp(-b*
x-a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*c+1/4*b^2*exp(-b*x-a)/d/(b^2*d^
2*x^2+2*b^2*c*d*x+b^2*c^2)+1/4*b^2/d^3*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b
*c)/d)-1/4*b^2/d^3*exp(b*x+a)/(b*c/d+b*x)^2-1/4*b^2/d^3*exp(b*x+a)/(b*c/d+
b*x)-1/4*b^2/d^3*exp((a*d-b*c)/d)*Ei(1,-b*x-a-(-a*d+b*c)/d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(96) = 192.

Time = 0.07 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.44

$$\int \frac{\sinh(a+bx)}{(c+dx)^3}\,dx = -\frac{2\,d^2\sinh(bx+a) + 2\,(bd^2x+bcd)\cosh(bx+a) - \left((b^2d^2x^2+2\,b^2cdx+b^2c^2)\operatorname{Ei}\left(\frac{bdx+bc}{d}\right) - (b^2d^2x^2+2\right.}{}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*d^2*sinh(b*x + a) + 2*(b*d^2*x + b*c*d)*cosh(b*x + a) - ((b^2*d^2*
x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei((b*d*x + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*
d*x + b^2*c^2)*Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d) - ((b^2*d^2*x^2
+ 2*b^2*c*d*x + b^2*c^2)*Ei((b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2)*Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*
x + c^2*d^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(a+bx)}{(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{\sinh(a+bx)}{(c+dx)^3}\,dx = -\frac{b\left(\frac{e^{\left(-a+\frac{bc}{d}\right)}E_2\left(\frac{(dx+c)b}{d}\right)}{(dx+c)d}+\frac{e^{\left(a-\frac{bc}{d}\right)}E_2\left(-\frac{(dx+c)b}{d}\right)}{(dx+c)d}\right)}{4\,d}-\frac{\sinh\left(bx+a\right)}{2\left(dx+c\right)^2d}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/4*b*(e^(-a + b*c/d)*exp_integral_e(2, (d*x + c)*b/d)/((d*x + c)*d) + e^
(a - b*c/d)*exp_integral_e(2, -(d*x + c)*b/d)/((d*x + c)*d))/d - 1/2*sinh(
b*x + a)/((d*x + c)^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(96) = 192.

Time = 0.18 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.89

$$\int \frac{\sinh(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{b^2d^2x^2\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}-b^2d^2x^2\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}+2\,b^2cdx\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}-2\,b^2cdx\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right.}{}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/4*(b^2*d^2*x^2*Ei((b*d*x + b*c)/d)*e^(a - b*c/d) - b^2*d^2*x^2*Ei(-(b*d*
x + b*c)/d)*e^(-a + b*c/d) + 2*b^2*c*d*x*Ei((b*d*x + b*c)/d)*e^(a - b*c/d)
 - 2*b^2*c*d*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + b^2*c^2*Ei((b*d*x + b
*c)/d)*e^(a - b*c/d) - b^2*c^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - b*d^2
*x*e^(b*x + a) - b*d^2*x*e^(-b*x - a) - b*c*d*e^(b*x + a) - b*c*d*e^(-b*x
- a) - d^2*e^(b*x + a) + d^2*e^(-b*x - a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(a+bx)}{(c+dx)^3} \, dx = \int \frac{\sinh(a+b\,x)}{{(c+d\,x)}^3} \, dx$$

input
```
int(sinh(a + b*x)/(c + d*x)^3,x)
```

output
```
int(sinh(a + b*x)/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{\sinh(a+bx)}{(c+dx)^3} \, dx = \text{too large to display}$$

input
```
int(sinh(b*x+a)/(d*x+c)^3,x)
```

output

```
(2*e**(a + b*x)*cosh(a + b*x)*b**3*c**3 + 2*e**(a + b*x)*cosh(a + b*x)*b**
3*c**2*d*x - 8*e**(a + b*x)*cosh(a + b*x)*b*c*d**2 - 8*e**(a + b*x)*cosh(a
 + b*x)*b*d**3*x - e**(2*a + 2*b*x)*b**3*c**2*d*x + e**(2*a + 2*b*x)*b**2*
c**2*d + 2*e**(2*a + 2*b*x)*b*c*d**2 + 4*e**(2*a + 2*b*x)*b*d**3*x - e**(2
*a + b*x)*int((e**(b*x)*x)/(b**2*c**5 + 3*b**2*c**4*d*x + 3*b**2*c**3*d**2
*x**2 + b**2*c**2*d**3*x**3 - 4*c**3*d**2 - 12*c**2*d**3*x - 12*c*d**4*x**
2 - 4*d**5*x**3),x)*b**6*c**7*d - 2*e**(2*a + b*x)*int((e**(b*x)*x)/(b**2*
c**5 + 3*b**2*c**4*d*x + 3*b**2*c**3*d**2*x**2 + b**2*c**2*d**3*x**3 - 4*c
**3*d**2 - 12*c**2*d**3*x - 12*c*d**4*x**2 - 4*d**5*x**3),x)*b**6*c**6*d**
2*x - e**(2*a + b*x)*int((e**(b*x)*x)/(b**2*c**5 + 3*b**2*c**4*d*x + 3*b**
2*c**3*d**2*x**2 + b**2*c**2*d**3*x**3 - 4*c**3*d**2 - 12*c**2*d**3*x - 12
*c*d**4*x**2 - 4*d**5*x**3),x)*b**6*c**5*d**3*x**2 - 2*e**(2*a + b*x)*int(
(e**(b*x)*x)/(b**2*c**5 + 3*b**2*c**4*d*x + 3*b**2*c**3*d**2*x**2 + b**2*c
**2*d**3*x**3 - 4*c**3*d**2 - 12*c**2*d**3*x - 12*c*d**4*x**2 - 4*d**5*x**
3),x)*b**5*c**6*d**2 - 4*e**(2*a + b*x)*int((e**(b*x)*x)/(b**2*c**5 + 3*b*
*2*c**4*d*x + 3*b**2*c**3*d**2*x**2 + b**2*c**2*d**3*x**3 - 4*c**3*d**2 -
12*c**2*d**3*x - 12*c*d**4*x**2 - 4*d**5*x**3),x)*b**5*c**5*d**3*x - 2*e**
(2*a + b*x)*int((e**(b*x)*x)/(b**2*c**5 + 3*b**2*c**4*d*x + 3*b**2*c**3*d*
*2*x**2 + b**2*c**2*d**3*x**3 - 4*c**3*d**2 - 12*c**2*d**3*x - 12*c*d**4*x
**2 - 4*d**5*x**3),x)*b**5*c**4*d**4*x**2 + 6*e**(2*a + b*x)*int((e**(b...
```

3.8 $\int (c+dx)^4 \sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 162

$$\begin{aligned}\int (c+dx)^4 \sinh^2(a+bx)\,dx = &-\frac{3d^4x}{4b^4}-\frac{d(c+dx)^3}{2b^2}-\frac{(c+dx)^5}{10d}\\ &+\frac{3d^4\cosh(a+bx)\sinh(a+bx)}{4b^5}\\ &+\frac{3d^2(c+dx)^2\cosh(a+bx)\sinh(a+bx)}{2b^3}\\ &+\frac{(c+dx)^4\cosh(a+bx)\sinh(a+bx)}{2b}\\ &-\frac{3d^3(c+dx)\sinh^2(a+bx)}{2b^4}-\frac{d(c+dx)^3\sinh^2(a+bx)}{b^2}\end{aligned}$$

output
```
-3/4*d^4*x/b^4-1/2*d*(d*x+c)^3/b^2-1/10*(d*x+c)^5/d+3/4*d^4*cosh(b*x+a)*si
nh(b*x+a)/b^5+3/2*d^2*(d*x+c)^2*cosh(b*x+a)*sinh(b*x+a)/b^3+1/2*(d*x+c)^4*
cosh(b*x+a)*sinh(b*x+a)/b-3/2*d^3*(d*x+c)*sinh(b*x+a)^2/b^4-d*(d*x+c)^3*si
nh(b*x+a)^2/b^2
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.81

$$\int (c+dx)^4\sinh^2(a+bx)\,dx$$
$$=\frac{-8b^5x(5c^4+10c^3dx+10c^2d^2x^2+5cd^3x^3+d^4x^4)-20bd(c+dx)\left(3d^2+2b^2(c+dx)^2\right)\cosh(2(a+bx))}{80b^5}$$

input
```
Integrate[(c + d*x)^4*Sinh[a + b*x]^2,x]
```

output
```
(-8*b^5*x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4) -
20*b*d*(c + d*x)*(3*d^2 + 2*b^2*(c + d*x)^2)*Cosh[2*(a + b*x)] + 10*(3*d^4
 + 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Sinh[2*(a + b*x)])/(80*b^5)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {3042, 25, 3792, 17, 25, 3042, 25, 3792, 17, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4\sinh^2(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int -(c+dx)^4\sin(ia+ibx)^2dx$$
$$\Big\downarrow \text{25}$$
$$-\int (c+dx)^4\sin(ia+ibx)^2dx$$
$$\Big\downarrow \text{3792}$$

$$-\frac{3d^2\int -(c+dx)^2\sinh^2(a+bx)dx}{b^2}-\frac{1}{2}\int (c+dx)^4dx-\frac{d(c+dx)^3\sinh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}$$

$$\Big\downarrow \; 17$$

$$-\frac{3d^2\int -(c+dx)^2\sinh^2(a+bx)dx}{b^2}-\frac{d(c+dx)^3\sinh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow \; 25$$

$$\frac{3d^2\int (c+dx)^2\sinh^2(a+bx)dx}{b^2}-\frac{d(c+dx)^3\sinh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow \; 3042$$

$$\frac{3d^2\int -(c+dx)^2\sin(ia+ibx)^2dx}{b^2}-\frac{d(c+dx)^3\sinh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow \; 25$$

$$-\frac{3d^2\int (c+dx)^2\sin(ia+ibx)^2dx}{b^2}-\frac{d(c+dx)^3\sinh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow \; 3792$$

$$-\frac{3d^2\left(\frac{d^2\int -\sinh^2(a+bx)dx}{2b^2}+\frac{1}{2}\int (c+dx)^2dx+\frac{d(c+dx)\sinh^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}\right)}{b^2}-\frac{d(c+dx)^3\sinh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow \; 17$$

$$-\frac{3d^2\left(\frac{d^2\int -\sinh^2(a+bx)dx}{2b^2}+\frac{d(c+dx)\sinh^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{b^2}-\frac{d(c+dx)^3\sinh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow \; 25$$

$$-\frac{3d^2\left(-\frac{d^2\int \sinh^2(a+bx)dx}{2b^2}+\frac{d(c+dx)\sinh^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{b^2}-\frac{d(c+dx)^3\sinh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3d^2\left(-\frac{d^2\int -\sin(ia+ibx)^2dx}{2b^2}+\frac{d(c+dx)\sinh^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{b^2}-\frac{d(c+dx)^3\sinh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3d^2\left(\frac{d^2\int \sin(ia+ibx)^2dx}{2b^2}+\frac{d(c+dx)\sinh^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{b^2}-\frac{d(c+dx)^3\sinh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{3d^2\left(\frac{d^2\left(\frac{\int 1dx}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)}{2b^2}+\frac{d(c+dx)\sinh^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{b^2}-\frac{d(c+dx)^3\sinh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow \text{24}$$

$$-\frac{3d^2\left(\frac{d(c+dx)\sinh^2(a+bx)}{2b^2}+\frac{d^2\left(\frac{x}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)}{2b^2}-\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{b^2}-\frac{d(c+dx)^3\sinh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^5}{10d}$$

input
```
Int[(c + d*x)^4*Sinh[a + b*x]^2,x]
```

output
```
-1/10*(c + d*x)^5/d + ((c + d*x)^4*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) - (d
*(c + d*x)^3*Sinh[a + b*x]^2)/b^2 - (3*d^2*((c + d*x)^3/(6*d) - ((c + d*x)
^2*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) + (d*(c + d*x)*Sinh[a + b*x]^2)/(2*b
^2) + (d^2*(x/2 - (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)))/(2*b^2)))/b^2
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.90

method	result
parallelrisch	$\frac{\left(2(dx+c)^4b^4+6d^2(dx+c)^2b^2+3d^4\right)\sinh(2bx+2a)-4b\left(d\left((dx+c)^2b^2+\frac{3d^2}{2}\right)(dx+c)\cosh(2bx+2a)+x\left(\frac{1}{5}d^4x^4+c\,d^3x^3\right.\right.}{8b^5}$
risch	$-\frac{d^4x^5}{10}-\frac{d^3c\,x^4}{2}-d^2c^2x^3-d\,c^3x^2-\frac{c^4x}{2}-\frac{c^5}{10d}+\frac{(2d^4x^4b^4+8b^4c\,d^3x^3+12b^4c^2d^2x^2-4b^3d^4x^3+8b^4c^3d}{}$
orering	$\frac{(2b^6d^6x^7+14b^6c\,d^5x^6+42b^6c^2d^4x^5+70b^6c^3d^3x^4+70b^6c^4d^2x^3-20b^4d^6x^5+40b^6c^5d\,x^2-100b^4c\,d^5x^4+10b^6c^6x-200b^4}{10b}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((d*x+c)^4*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*((2*(d*x+c)^4*b^4+6*d^2*(d*x+c)^2*b^2+3*d^4)*sinh(2*b*x+2*a)-4*b*(d*((
d*x+c)^2*b^2+3/2*d^2)*(d*x+c)*cosh(2*b*x+2*a)+x*(1/5*d^4*x^4+c*d^3*x^3+2*c
^2*d^2*x^2+2*c^3*d*x+c^4)*b^4-b^2*c^3*d-3/2*d^3*c))/b^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 312 vs. 2(148) = 296.

Time = 0.09 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.93

$$\int (c+dx)^4\sinh^2(a+bx)\,dx =$$

$$-\frac{2\,b^5d^4x^5+10\,b^5cd^3x^4+20\,b^5c^2d^2x^3+20\,b^5c^3dx^2+10\,b^5c^4x+5\left(2\,b^3d^4x^3+6\,b^3cd^3x^2+2\,b^3c^3d+3\,bc\right.}{}$$

input
```
integrate((d*x+c)^4*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/20*(2*b^5*d^4*x^5 + 10*b^5*c*d^3*x^4 + 20*b^5*c^2*d^2*x^3 + 20*b^5*c^3*
d*x^2 + 10*b^5*c^4*x + 5*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d +
3*b*c*d^3 + 3*(2*b^3*c^2*d^2 + b*d^4)*x)*cosh(b*x + a)^2 - 5*(2*b^4*d^4*x^
4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 + 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2
 + b^2*d^4)*x^2 + 4*(2*b^4*c^3*d + 3*b^2*c*d^3)*x)*cosh(b*x + a)*sinh(b*x
+ a) + 5*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d + 3*b*c*d^3 + 3*(2
*b^3*c^2*d^2 + b*d^4)*x)*sinh(b*x + a)^2)/b^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(156) = 312$.

Time = 0.49 (sec) , antiderivative size = 660, normalized size of antiderivative = 4.07

$$\int (c + dx)^4 \sinh^2(a + bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**4*sinh(b*x+a)**2,x)
```

output
```
Piecewise((c**4*x*sinh(a + b*x)**2/2 - c**4*x*cosh(a + b*x)**2/2 + c**3*d*
x**2*sinh(a + b*x)**2 - c**3*d*x**2*cosh(a + b*x)**2 + c**2*d**2*x**3*sinh
(a + b*x)**2 - c**2*d**2*x**3*cosh(a + b*x)**2 + c*d**3*x**4*sinh(a + b*x)
**2/2 - c*d**3*x**4*cosh(a + b*x)**2/2 + d**4*x**5*sinh(a + b*x)**2/10 - d
**4*x**5*cosh(a + b*x)**2/10 + c**4*sinh(a + b*x)*cosh(a + b*x)/(2*b) + 2*
c**3*d*x*sinh(a + b*x)*cosh(a + b*x)/b + 3*c**2*d**2*x**2*sinh(a + b*x)*co
sh(a + b*x)/b + 2*c*d**3*x**3*sinh(a + b*x)*cosh(a + b*x)/b + d**4*x**4*si
nh(a + b*x)*cosh(a + b*x)/(2*b) - c**3*d*sinh(a + b*x)**2/b**2 - 3*c**2*d*
*2*x*sinh(a + b*x)**2/(2*b**2) - 3*c**2*d**2*x*cosh(a + b*x)**2/(2*b**2) -
 3*c*d**3*x**2*sinh(a + b*x)**2/(2*b**2) - 3*c*d**3*x**2*cosh(a + b*x)**2/
(2*b**2) - d**4*x**3*sinh(a + b*x)**2/(2*b**2) - d**4*x**3*cosh(a + b*x)**
2/(2*b**2) + 3*c**2*d**2*sinh(a + b*x)*cosh(a + b*x)/(2*b**3) + 3*c*d**3*x
*sinh(a + b*x)*cosh(a + b*x)/b**3 + 3*d**4*x**2*sinh(a + b*x)*cosh(a + b*x
)/(2*b**3) - 3*c*d**3*sinh(a + b*x)**2/(2*b**4) - 3*d**4*x*sinh(a + b*x)**
2/(4*b**4) - 3*d**4*x*cosh(a + b*x)**2/(4*b**4) + 3*d**4*sinh(a + b*x)*cos
h(a + b*x)/(4*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**
3 + c*d**3*x**4 + d**4*x**5/5)*sinh(a)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(148) = 296$.

Time = 0.06 (sec) , antiderivative size = 382, normalized size of antiderivative = 2.36

$$\int (c+dx)^4 \sinh^2(a+bx)\,dx$$
$$= -\frac{1}{4}\left(4\,x^2 - \frac{\left(2\,bxe^{(2\,a)} - e^{(2\,a)}\right)e^{(2\,bx)}}{b^2} + \frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{b^2}\right)c^3d$$
$$-\frac{1}{8}\left(8\,x^3 - \frac{3\left(2\,b^2x^2e^{(2\,a)} - 2\,bxe^{(2\,a)} + e^{(2\,a)}\right)e^{(2\,bx)}}{b^3} + \frac{3\,(2\,b^2x^2 + 2\,bx + 1)e^{(-2\,bx-2\,a)}}{b^3}\right)c^2d^2$$
$$-\frac{1}{8}\left(4\,x^4 - \frac{\left(4\,b^3x^3e^{(2\,a)} - 6\,b^2x^2e^{(2\,a)} + 6\,bxe^{(2\,a)} - 3\,e^{(2\,a)}\right)e^{(2\,bx)}}{b^4} + \frac{(4\,b^3x^3 + 6\,b^2x^2 + 6\,bx + 3)e^{(-2\,bx-2}}{b^4}\right.$$
$$-\frac{1}{80}\left(8\,x^5 - \frac{5\left(2\,b^4x^4e^{(2\,a)} - 4\,b^3x^3e^{(2\,a)} + 6\,b^2x^2e^{(2\,a)} - 6\,bxe^{(2\,a)} + 3\,e^{(2\,a)}\right)e^{(2\,bx)}}{b^5} + \frac{5\,(2\,b^4x^4 + 4\,b^3x^3 +}{}\right.$$
$$-\frac{1}{8}\,c^4\left(4\,x - \frac{e^{(2\,bx+2\,a)}}{b} + \frac{e^{(-2\,bx-2\,a)}}{b}\right)$$

input
```
integrate((d*x+c)^4*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(4*x^2 - (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 + (2*b*x + 1)*e^(-2*
b*x - 2*a)/b^2)*c^3*d - 1/8*(8*x^3 - 3*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a)
+ e^(2*a))*e^(2*b*x)/b^3 + 3*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3)
*c^2*d^2 - 1/8*(4*x^4 - (4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^(
2*a) - 3*e^(2*a))*e^(2*b*x)/b^4 + (4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-
2*b*x - 2*a)/b^4)*c*d^3 - 1/80*(8*x^5 - 5*(2*b^4*x^4*e^(2*a) - 4*b^3*x^3*e
^(2*a) + 6*b^2*x^2*e^(2*a) - 6*b*x*e^(2*a) + 3*e^(2*a))*e^(2*b*x)/b^5 + 5*
(2*b^4*x^4 + 4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b^5)*d^4
- 1/8*c^4*(4*x - e^(2*b*x + 2*a)/b + e^(-2*b*x - 2*a)/b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(148) = 296$.

Time = 0.13 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.31

$$\int (c+dx)^4 \sinh^2(a+bx)\,dx = -\frac{1}{10}\,d^4x^5 - \frac{1}{2}\,cd^3x^4 - c^2d^2x^3 - c^3dx^2 - \frac{1}{2}\,c^4x$$
$$+ \frac{(2\,b^4d^4x^4 + 8\,b^4cd^3x^3 + 12\,b^4c^2d^2x^2 - 4\,b^3d^4x^3 + 8\,b^4c^3dx - 12\,b^3cd^3x^2 + 2\,b^4c^4 - 12\,b^3c^2d^2x + 6\,b^2d^4x}{16\,b^5}$$
$$- \frac{(2\,b^4d^4x^4 + 8\,b^4cd^3x^3 + 12\,b^4c^2d^2x^2 + 4\,b^3d^4x^3 + 8\,b^4c^3dx + 12\,b^3cd^3x^2 + 2\,b^4c^4 + 12\,b^3c^2d^2x + 6\,b^2d^4x}{16\,b^5}$$

input
```
integrate((d*x+c)^4*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/10*d^4*x^5 - 1/2*c*d^3*x^4 - c^2*d^2*x^3 - c^3*d*x^2 - 1/2*c^4*x + 1/16
*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4*c^2*d^2*x^2 - 4*b^3*d^4*x^3 + 8
*b^4*c^3*d*x - 12*b^3*c*d^3*x^2 + 2*b^4*c^4 - 12*b^3*c^2*d^2*x + 6*b^2*d^4
*x^2 - 4*b^3*c^3*d + 12*b^2*c*d^3*x + 6*b^2*c^2*d^2 - 6*b*d^4*x - 6*b*c*d^
3 + 3*d^4)*e^(2*b*x + 2*a)/b^5 - 1/16*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 1
2*b^4*c^2*d^2*x^2 + 4*b^3*d^4*x^3 + 8*b^4*c^3*d*x + 12*b^3*c*d^3*x^2 + 2*b
^4*c^4 + 12*b^3*c^2*d^2*x + 6*b^2*d^4*x^2 + 4*b^3*c^3*d + 12*b^2*c*d^3*x +
 6*b^2*c^2*d^2 + 6*b*d^4*x + 6*b*c*d^3 + 3*d^4)*e^(-2*b*x - 2*a)/b^5
```

Mupad [B] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.06

$$\int (c+dx)^4 \sinh^2(a+bx)\,dx = \frac{c^4 \sinh(2a+2bx)}{4b} - \frac{d^4 x^5}{10} - c^3 d x^2 - \frac{c d^3 x^4}{2} - \frac{c^4 x}{2} + \frac{3 d^4 \sinh(2a+2bx)}{8 b^5} - c^2 d^2 x^3 - \frac{c^3 d \cosh(2a+2bx)}{2 b^2} - \frac{3 c d^3 \cosh(2a+2bx)}{4 b^4} - \frac{3 d^4 x \cosh(2a+2bx)}{4 b^4} + \frac{3 c^2 d^2 \sinh(2a+2bx)}{4 b^3} - \frac{d^4 x^3 \cosh(2a+2bx)}{2 b^2} + \frac{d^4 x^4 \sinh(2a+2bx)}{4b} + \frac{3 d^4 x^2 \sinh(2a+2bx)}{4 b^3} + \frac{3 c^2 d^2 x^2 \sinh(2a+2bx)}{2b} + \frac{c^3 d x \sinh(2a+2bx)}{b} + \frac{3 c d^3 x \sinh(2a+2bx)}{2 b^3} - \frac{3 c^2 d^2 x \cosh(2a+2bx)}{2 b^2} - \frac{3 c d^3 x^2 \cosh(2a+2bx)}{2 b^2} + \frac{c d^3 x^3 \sinh(2a+2bx)}{b}$$

input
```
int(sinh(a + b*x)^2*(c + d*x)^4,x)
```

output
```
(c^4*sinh(2*a + 2*b*x))/(4*b) - (d^4*x^5)/10 - c^3*d*x^2 - (c*d^3*x^4)/2 -
 (c^4*x)/2 + (3*d^4*sinh(2*a + 2*b*x))/(8*b^5) - c^2*d^2*x^3 - (c^3*d*cosh
(2*a + 2*b*x))/(2*b^2) - (3*c*d^3*cosh(2*a + 2*b*x))/(4*b^4) - (3*d^4*x*co
sh(2*a + 2*b*x))/(4*b^4) + (3*c^2*d^2*sinh(2*a + 2*b*x))/(4*b^3) - (d^4*x^
3*cosh(2*a + 2*b*x))/(2*b^2) + (d^4*x^4*sinh(2*a + 2*b*x))/(4*b) + (3*d^4*
x^2*sinh(2*a + 2*b*x))/(4*b^3) + (3*c^2*d^2*x^2*sinh(2*a + 2*b*x))/(2*b) +
 (c^3*d*x*sinh(2*a + 2*b*x))/b + (3*c*d^3*x*sinh(2*a + 2*b*x))/(2*b^3) - (
3*c^2*d^2*x*cosh(2*a + 2*b*x))/(2*b^2) - (3*c*d^3*x^2*cosh(2*a + 2*b*x))/(
2*b^2) + (c*d^3*x^3*sinh(2*a + 2*b*x))/b
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 576, normalized size of antiderivative = 3.56

$$\int (c+dx)^4 \sinh^2(a+bx)\,dx$$
$$= \frac{15e^{4bx+4a}d^4 - 10b^4c^4 - 15d^4 + 10e^{4bx+4a}b^4d^4x^4 - 20e^{4bx+4a}b^3c^3d - 20e^{4bx+4a}b^3d^4x^3 + 30e^{4bx+4a}b^2c^2d^2 +}{}$$

input
```
int((d*x+c)^4*sinh(b*x+a)^2,x)
```

output
```
(10*e**(4*a + 4*b*x)*b**4*c**4 + 40*e**(4*a + 4*b*x)*b**4*c**3*d*x + 60*e*
*(4*a + 4*b*x)*b**4*c**2*d**2*x**2 + 40*e**(4*a + 4*b*x)*b**4*c*d**3*x**3
+ 10*e**(4*a + 4*b*x)*b**4*d**4*x**4 - 20*e**(4*a + 4*b*x)*b**3*c**3*d - 6
0*e**(4*a + 4*b*x)*b**3*c**2*d**2*x - 60*e**(4*a + 4*b*x)*b**3*c*d**3*x**2
 - 20*e**(4*a + 4*b*x)*b**3*d**4*x**3 + 30*e**(4*a + 4*b*x)*b**2*c**2*d**2
 + 60*e**(4*a + 4*b*x)*b**2*c*d**3*x + 30*e**(4*a + 4*b*x)*b**2*d**4*x**2
- 30*e**(4*a + 4*b*x)*b*c*d**3 - 30*e**(4*a + 4*b*x)*b*d**4*x + 15*e**(4*a
 + 4*b*x)*d**4 - 40*e**(2*a + 2*b*x)*b**5*c**4*x - 80*e**(2*a + 2*b*x)*b**
5*c**3*d*x**2 - 80*e**(2*a + 2*b*x)*b**5*c**2*d**2*x**3 - 40*e**(2*a + 2*b
*x)*b**5*c*d**3*x**4 - 8*e**(2*a + 2*b*x)*b**5*d**4*x**5 - 10*b**4*c**4 -
40*b**4*c**3*d*x - 60*b**4*c**2*d**2*x**2 - 40*b**4*c*d**3*x**3 - 10*b**4*
d**4*x**4 - 20*b**3*c**3*d - 60*b**3*c**2*d**2*x - 60*b**3*c*d**3*x**2 - 2
0*b**3*d**4*x**3 - 30*b**2*c**2*d**2 - 60*b**2*c*d**3*x - 30*b**2*d**4*x**
2 - 30*b*c*d**3 - 30*b*d**4*x - 15*d**4)/(80*e**(2*a + 2*b*x)*b**5)
```

3.9 $\int (c+dx)^3 \sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 124

$$\int (c+dx)^3 \sinh^2(a+bx)\,dx = -\frac{3d(c+dx)^2}{8b^2} - \frac{(c+dx)^4}{8d} + \frac{3d^2(c+dx)\cosh(a+bx)\sinh(a+bx)}{4b^3} + \frac{(c+dx)^3\cosh(a+bx)\sinh(a+bx)}{2b} - \frac{3d^3\sinh^2(a+bx)}{8b^4} - \frac{3d(c+dx)^2\sinh^2(a+bx)}{4b^2}$$

output
```
-3/8*d*(d*x+c)^2/b^2-1/8*(d*x+c)^4/d+3/4*d^2*(d*x+c)*cosh(b*x+a)*sinh(b*x+
a)/b^3+1/2*(d*x+c)^3*cosh(b*x+a)*sinh(b*x+a)/b-3/8*d^3*sinh(b*x+a)^2/b^4-3
/4*d*(d*x+c)^2*sinh(b*x+a)^2/b^2
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.84

$$\int (c+dx)^3 \sinh^2(a+bx)\,dx$$
$$= \frac{-2b^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)-3d(d^2+2b^2(c+dx)^2)\cosh(2(a+bx))+2b(c+dx)\left(3d^2+2b^2(c\right.}{16b^4}$$

input
```
Integrate[(c + d*x)^3*Sinh[a + b*x]^2,x]
```

output
```
(-2*b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) - 3*d*(d^2 + 2*b^2*(
c + d*x)^2)*Cosh[2*(a + b*x)] + 2*b*(c + d*x)*(3*d^2 + 2*b^2*(c + d*x)^2)*
Sinh[2*(a + b*x)])/(16*b^4)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {3042, 25, 3792, 17, 25, 3042, 25, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sinh^2(a+bx)\,dx$$
$$\Big\downarrow \; 3042$$
$$\int -(c+dx)^3 \sin(ia+ibx)^2 dx$$
$$\Big\downarrow \; 25$$
$$-\int (c+dx)^3 \sin(ia+ibx)^2 dx$$
$$\Big\downarrow \; 3792$$

$$-\frac{3d^2\int -\left((c+dx)\sinh^2(a+bx)\right)dx}{2b^2}-\frac{1}{2}\int (c+dx)^3dx-\frac{3d(c+dx)^2\sinh^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh(a+bx)}{2b}$$

$\downarrow$ 17

$$-\frac{3d^2\int -\left((c+dx)\sinh^2(a+bx)\right)dx}{2b^2}-\frac{3d(c+dx)^2\sinh^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^4}{8d}$$

$\downarrow$ 25

$$\frac{3d^2\int (c+dx)\sinh^2(a+bx)dx}{2b^2}-\frac{3d(c+dx)^2\sinh^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^4}{8d}$$

$\downarrow$ 3042

$$\frac{3d^2\int -\left((c+dx)\sin(ia+ibx)^2\right)dx}{2b^2}-\frac{3d(c+dx)^2\sinh^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^4}{8d}$$

$\downarrow$ 25

$$-\frac{3d^2\int (c+dx)\sin(ia+ibx)^2dx}{2b^2}-\frac{3d(c+dx)^2\sinh^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^4}{8d}$$

$\downarrow$ 3791

$$-\frac{3d^2\left(\frac{1}{2}\int (c+dx)dx+\frac{d\sinh^2(a+bx)}{4b^2}-\frac{(c+dx)\sinh(a+bx)\cosh(a+bx)}{2b}\right)}{2b^2}-\frac{3d(c+dx)^2\sinh^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^4}{8d}$$

$\downarrow$ 17

$$-\frac{3d^2\left(\frac{d\sinh^2(a+bx)}{4b^2}-\frac{(c+dx)\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)}{2b^2}-\frac{3d(c+dx)^2\sinh^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^4}{8d}$$

input
```
Int[(c + d*x)^3*Sinh[a + b*x]^2,x]
```

output
```
-1/8*(c + d*x)^4/d + ((c + d*x)^3*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) - (3*
d*(c + d*x)^2*Sinh[a + b*x]^2)/(4*b^2) - (3*d^2*((c + d*x)^2/(4*d) - ((c +
 d*x)*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) + (d*Sinh[a + b*x]^2)/(4*b^2)))/(
2*b^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

method	result
parallelrisch	$\frac{4\left((dx+c)^2b^2+\frac{3d^2}{2}\right)b(dx+c)\sinh(2bx+2a)-6\left((dx+c)^2b^2+\frac{d^2}{2}\right)d\cosh(2bx+2a)+(-2d^3x^4-8d^2c\,x^3-12d\,c^2x^2-8c^3x)}{16b^4}$
risch	$-\frac{d^3x^4}{8}-\frac{d^2c\,x^3}{2}-\frac{3d\,c^2x^2}{4}-\frac{c^3x}{2}-\frac{c^4}{8d}+\frac{(4d^3x^3b^3+12b^3c\,d^2x^2+12b^3c^2dx-6b^2d^3x^2+4b^3c^3-12b^2c\,d^2x-6b^2}{32b^4}$
orering	$\frac{(b^4d^5x^6+6b^4c\,d^4x^5+15b^4c^2d^3x^4+20b^4c^3d^2x^3+14b^4c^4d\,x^2-6b^2d^5x^4+4b^4c^5x-24b^2c\,d^4x^3-39b^2c^2d^3x^2-30b^2c^3d^2x-}{4b^4(dx+c)^2}$
derivativedivides	$\frac{d^3\left(\frac{(bx+a)^3\cosh(bx+a)\sinh(bx+a)}{2}-\frac{(bx+a)^4}{8}-\frac{3(bx+a)^2\cosh(bx+a)^2}{4}+\frac{3(bx+a)\cosh(bx+a)\sinh(bx+a)}{4}+\frac{3(bx+a)^2}{8}-\frac{3\cosh(bx}{8}\right.}{b^3}$
default	$\frac{d^3\left(\frac{(bx+a)^3\cosh(bx+a)\sinh(bx+a)}{2}-\frac{(bx+a)^4}{8}-\frac{3(bx+a)^2\cosh(bx+a)^2}{4}+\frac{3(bx+a)\cosh(bx+a)\sinh(bx+a)}{4}+\frac{3(bx+a)^2}{8}-\frac{3\cosh(bx}{8}\right.}{b^3}$

input
```
int((d*x+c)^3*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*(4*((d*x+c)^2*b^2+3/2*d^2)*b*(d*x+c)*sinh(2*b*x+2*a)-6*((d*x+c)^2*b^2
+1/2*d^2)*d*cosh(2*b*x+2*a)+(-2*d^3*x^4-8*c*d^2*x^3-12*c^2*d*x^2-8*c^3*x)*
b^4+6*b^2*c^2*d+3*d^3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.69

$$\int (c+dx)^3\sinh^2(a+bx)\,dx =$$
$$-\frac{2\,b^4d^3x^4+8\,b^4cd^2x^3+12\,b^4c^2dx^2+8\,b^4c^3x+3\,(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d+d^3)\cosh{(bx+a)}^2-4}{}$$

input
```
integrate((d*x+c)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/16*(2*b^4*d^3*x^4 + 8*b^4*c*d^2*x^3 + 12*b^4*c^2*d*x^2 + 8*b^4*c^3*x +
3*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d + d^3)*cosh(b*x + a)^2 - 4*
(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 + 3*b*c*d^2 + 3*(2*b^3*c^2*d
+ b*d^3)*x)*cosh(b*x + a)*sinh(b*x + a) + 3*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x
 + 2*b^2*c^2*d + d^3)*sinh(b*x + a)^2)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. 2(119) = 238.

Time = 0.37 (sec) , antiderivative size = 456, normalized size of antiderivative = 3.68

$$\int (c+dx)^3 \sinh^2(a+bx)\, dx$$
$$= \begin{cases} \frac{c^3 x \sinh^2(a+bx)}{2} - \frac{c^3 x \cosh^2(a+bx)}{2} + \frac{3c^2 dx^2 \sinh^2(a+bx)}{4} - \frac{3c^2 dx^2 \cosh^2(a+bx)}{4} + \frac{cd^2 x^3 \sinh^2(a+bx)}{2} - \frac{cd^2 x^3 \cosh^2(a+bx)}{2} + \\ \left(c^3 x + \frac{3c^2 dx^2}{2} + cd^2 x^3 + \frac{d^3 x^4}{4}\right) \sinh^2(a) \end{cases}$$

input
```
integrate((d*x+c)**3*sinh(b*x+a)**2,x)
```

output
```
Piecewise((c**3*x*sinh(a + b*x)**2/2 - c**3*x*cosh(a + b*x)**2/2 + 3*c**2*
d*x**2*sinh(a + b*x)**2/4 - 3*c**2*d*x**2*cosh(a + b*x)**2/4 + c*d**2*x**3
*sinh(a + b*x)**2/2 - c*d**2*x**3*cosh(a + b*x)**2/2 + d**3*x**4*sinh(a +
b*x)**2/8 - d**3*x**4*cosh(a + b*x)**2/8 + c**3*sinh(a + b*x)*cosh(a + b*x
)/(2*b) + 3*c**2*d*x*sinh(a + b*x)*cosh(a + b*x)/(2*b) + 3*c*d**2*x**2*sin
h(a + b*x)*cosh(a + b*x)/(2*b) + d**3*x**3*sinh(a + b*x)*cosh(a + b*x)/(2*
b) - 3*c**2*d*sinh(a + b*x)**2/(4*b**2) - 3*c*d**2*x*sinh(a + b*x)**2/(4*b
**2) - 3*c*d**2*x*cosh(a + b*x)**2/(4*b**2) - 3*d**3*x**2*sinh(a + b*x)**2
/(8*b**2) - 3*d**3*x**2*cosh(a + b*x)**2/(8*b**2) + 3*c*d**2*sinh(a + b*x)
*cosh(a + b*x)/(4*b**3) + 3*d**3*x*sinh(a + b*x)*cosh(a + b*x)/(4*b**3) -
3*d**3*sinh(a + b*x)**2/(8*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 +
c*d**2*x**3 + d**3*x**4/4)*sinh(a)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(112) = 224$.

Time = 0.05 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.12

$$\int (c+dx)^3\sinh^2(a+bx)\,dx$$
$$= -\frac{3}{16}\left(4x^2 - \frac{(2bxe^{(2a)} - e^{(2a)})e^{(2bx)}}{b^2} + \frac{(2bx+1)e^{(-2bx-2a)}}{b^2}\right)c^2d$$
$$- \frac{1}{16}\left(8x^3 - \frac{3\left(2b^2x^2e^{(2a)} - 2bxe^{(2a)} + e^{(2a)}\right)e^{(2bx)}}{b^3} + \frac{3(2b^2x^2 + 2bx + 1)e^{(-2bx-2a)}}{b^3}\right)cd^2$$
$$- \frac{1}{32}\left(4x^4 - \frac{(4b^3x^3e^{(2a)} - 6b^2x^2e^{(2a)} + 6bxe^{(2a)} - 3e^{(2a)})e^{(2bx)}}{b^4} + \frac{(4b^3x^3 + 6b^2x^2 + 6bx + 3)e^{(-2bx-}}{b^4}\right.$$
$$- \frac{1}{8}c^3\left(4x - \frac{e^{(2bx+2a)}}{b} + \frac{e^{(-2bx-2a)}}{b}\right)$$

input
```
integrate((d*x+c)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-3/16*(4*x^2 - (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 + (2*b*x + 1)*e^(-2
*b*x - 2*a)/b^2)*c^2*d - 1/16*(8*x^3 - 3*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a
) + e^(2*a))*e^(2*b*x)/b^3 + 3*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^
3)*c*d^2 - 1/32*(4*x^4 - (4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^
(2*a) - 3*e^(2*a))*e^(2*b*x)/b^4 + (4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(
-2*b*x - 2*a)/b^4)*d^3 - 1/8*c^3*(4*x - e^(2*b*x + 2*a)/b + e^(-2*b*x - 2*
a)/b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(112) = 224$.

Time = 0.14 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.96

$$\int (c+dx)^3\sinh^2(a+bx)\,dx = -\frac{1}{8}d^3x^4 - \frac{1}{2}cd^2x^3 - \frac{3}{4}c^2dx^2 - \frac{1}{2}c^3x$$
$$+ \frac{(4b^3d^3x^3 + 12b^3cd^2x^2 + 12b^3c^2dx - 6b^2d^3x^2 + 4b^3c^3 - 12b^2cd^2x - 6b^2c^2d + 6bd^3x + 6bcd^2 - 3d^3)e}{32b^4}$$
$$- \frac{(4b^3d^3x^3 + 12b^3cd^2x^2 + 12b^3c^2dx + 6b^2d^3x^2 + 4b^3c^3 + 12b^2cd^2x + 6b^2c^2d + 6bd^3x + 6bcd^2 + 3d^3)e}{32b^4}$$

input
```
integrate((d*x+c)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/8*d^3*x^4 - 1/2*c*d^2*x^3 - 3/4*c^2*d*x^2 - 1/2*c^3*x + 1/32*(4*b^3*d^3
*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x - 6*b^2*d^3*x^2 + 4*b^3*c^3 - 12*
b^2*c*d^2*x - 6*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 - 3*d^3)*e^(2*b*x + 2*a)
/b^4 - 1/32*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x + 6*b^2*d^3
*x^2 + 4*b^3*c^3 + 12*b^2*c*d^2*x + 6*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 +
3*d^3)*e^(-2*b*x - 2*a)/b^4
```

Mupad [B] (verification not implemented)

Time = 1.77 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.85

$$\int (c+dx)^3 \sinh^2(a+bx)\,dx =$$
$$-\frac{\frac{3\,d^3\cosh(2\,a+2\,b\,x)}{2}+4\,b^4\,c^3\,x-2\,b^3\,c^3\sinh(2\,a+2\,b\,x)+b^4\,d^3\,x^4+3\,b^2\,c^2\,d\cosh(2\,a+2\,b\,x)+6\,b^4\,c^2\,d\,x^2}{}$$

input
```
int(sinh(a + b*x)^2*(c + d*x)^3,x)
```

output
```
-((3*d^3*cosh(2*a + 2*b*x))/2 + 4*b^4*c^3*x - 2*b^3*c^3*sinh(2*a + 2*b*x)
+ b^4*d^3*x^4 + 3*b^2*c^2*d*cosh(2*a + 2*b*x) + 6*b^4*c^2*d*x^2 + 4*b^4*c*
d^2*x^3 + 3*b^2*d^3*x^2*cosh(2*a + 2*b*x) - 2*b^3*d^3*x^3*sinh(2*a + 2*b*x
) - 3*b*c*d^2*sinh(2*a + 2*b*x) - 3*b*d^3*x*sinh(2*a + 2*b*x) + 6*b^2*c*d^
2*x*cosh(2*a + 2*b*x) - 6*b^3*c^2*d*x*sinh(2*a + 2*b*x) - 6*b^3*c*d^2*x^2*
sinh(2*a + 2*b*x))/(8*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 382, normalized size of antiderivative = 3.08

$$\int (c+dx)^3 \sinh^2(a+bx)\,dx$$
$$=\frac{4e^{4bx+4a}b^3c^3+12e^{4bx+4a}b^3c^2dx+12e^{4bx+4a}b^3c\,d^2x^2+4e^{4bx+4a}b^3d^3x^3-6e^{4bx+4a}b^2c^2d-12e^{4bx+4a}b^2c\,d^2x}{}$$

input
```
int((d*x+c)^3*sinh(b*x+a)^2,x)
```

output
```
(4*e**(4*a + 4*b*x)*b**3*c**3 + 12*e**(4*a + 4*b*x)*b**3*c**2*d*x + 12*e**
(4*a + 4*b*x)*b**3*c*d**2*x**2 + 4*e**(4*a + 4*b*x)*b**3*d**3*x**3 - 6*e**
(4*a + 4*b*x)*b**2*c**2*d - 12*e**(4*a + 4*b*x)*b**2*c*d**2*x - 6*e**(4*a
+ 4*b*x)*b**2*d**3*x**2 + 6*e**(4*a + 4*b*x)*b*c*d**2 + 6*e**(4*a + 4*b*x)
*b*d**3*x - 3*e**(4*a + 4*b*x)*d**3 - 16*e**(2*a + 2*b*x)*b**4*c**3*x - 24
*e**(2*a + 2*b*x)*b**4*c**2*d*x**2 - 16*e**(2*a + 2*b*x)*b**4*c*d**2*x**3
- 4*e**(2*a + 2*b*x)*b**4*d**3*x**4 - 4*b**3*c**3 - 12*b**3*c**2*d*x - 12*
b**3*c*d**2*x**2 - 4*b**3*d**3*x**3 - 6*b**2*c**2*d - 12*b**2*c*d**2*x - 6
*b**2*d**3*x**2 - 6*b*c*d**2 - 6*b*d**3*x - 3*d**3)/(32*e**(2*a + 2*b*x)*b
**4)
```

3.10 $\int (c+dx)^2 \sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 95

$$\int (c+dx)^2 \sinh^2(a+bx)\,dx = -\frac{d^2x}{4b^2} - \frac{(c+dx)^3}{6d} + \frac{d^2\cosh(a+bx)\sinh(a+bx)}{4b^3} + \frac{(c+dx)^2\cosh(a+bx)\sinh(a+bx)}{2b} - \frac{d(c+dx)\sinh^2(a+bx)}{2b^2}$$

output
```
-1/4*d^2*x/b^2-1/6*(d*x+c)^3/d+1/4*d^2*cosh(b*x+a)*sinh(b*x+a)/b^3+1/2*(d*
x+c)^2*cosh(b*x+a)*sinh(b*x+a)/b-1/2*d*(d*x+c)*sinh(b*x+a)^2/b^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.79

$$\int (c+dx)^2 \sinh^2(a+bx)\,dx = \frac{-4b^3x(3c^2+3cdx+d^2x^2) - 6bd(c+dx)\cosh(2(a+bx)) + 3(d^2+2b^2(c+dx)^2)\sinh(2(a+bx))}{24b^3}$$

input
```
Integrate[(c + d*x)^2*Sinh[a + b*x]^2,x]
```

output
```
(-4*b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) - 6*b*d*(c + d*x)*Cosh[2*(a + b*x)]
+ 3*(d^2 + 2*b^2*(c + d*x)^2)*Sinh[2*(a + b*x)])/(24*b^3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {3042, 25, 3792, 17, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sinh^2(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -(c+dx)^2 \sin(ia+ibx)^2 dx$$

$$\Big\downarrow \; 25$$

$$-\int (c+dx)^2 \sin(ia+ibx)^2 dx$$

$$\Big\downarrow \; 3792$$

$$-\frac{d^2 \int -\sinh^2(a+bx)dx}{2b^2}-\frac{1}{2}\int (c+dx)^2dx-\frac{d(c+dx)\sinh^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}$$

$$\Big\downarrow \; 17$$

$$-\frac{d^2 \int -\sinh^2(a+bx)dx}{2b^2}-\frac{d(c+dx)\sinh^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^3}{6d}$$

$$\Big\downarrow \; 25$$

$$\frac{d^2 \int \sinh^2(a+bx)dx}{2b^2}-\frac{d(c+dx)\sinh^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^3}{6d}$$

$$\Big\downarrow \; 3042$$

$$\frac{d^2 \int -\sin(ia+ibx)^2dx}{2b^2} - \frac{d(c+dx)\sinh^2(a+bx)}{2b^2} + \frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b} - \frac{(c+dx)^3}{6d}$$

$$\Big\downarrow \; 25$$

$$-\frac{d^2 \int \sin(ia+ibx)^2dx}{2b^2} - \frac{d(c+dx)\sinh^2(a+bx)}{2b^2} + \frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b} - \frac{(c+dx)^3}{6d}$$

$$\Big\downarrow \; 3115$$

$$-\frac{d^2\left(\frac{\int 1dx}{2} - \frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)}{2b^2} - \frac{d(c+dx)\sinh^2(a+bx)}{2b^2} + \frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b} - \frac{(c+dx)^3}{6d}$$

$$\Big\downarrow \; 24$$

$$-\frac{d(c+dx)\sinh^2(a+bx)}{2b^2} - \frac{d^2\left(\frac{x}{2} - \frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)}{2b^2} + \frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b} - \frac{(c+dx)^3}{6d}$$

input
```
Int[(c + d*x)^2*Sinh[a + b*x]^2,x]
```

output
```
-1/6*(c + d*x)^3/d + ((c + d*x)^2*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) - (d*
(c + d*x)*Sinh[a + b*x]^2)/(2*b^2) - (d^2*(x/2 - (Cosh[a + b*x]*Sinh[a + b
*x])/(2*b)))/(2*b^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

method	result
parallelrisch	$\frac{\left(2(dx+c)^2b^2+d^2\right)\sinh(2bx+2a)-4b\left(\frac{d(dx+c)\cosh(2bx+2a)}{2}+x\left(\frac{1}{3}x^2d^2+cdx+c^2\right)b^2-\frac{dc}{2}\right)}{8b^3}$
risch	$-\frac{d^2x^3}{6}-\frac{dc\,x^2}{2}-\frac{c^2x}{2}-\frac{c^3}{6d}+\frac{(2d^2x^2b^2+4b^2cdx+2b^2c^2-2b\,d^2x-2bcd+d^2)\,\mathrm{e}^{2bx+2a}}{16b^3}-\frac{(2d^2x^2b^2+4b^2cdx+2b}{}$
derivativedivides	$\frac{\frac{d^2\left(\frac{(bx+a)^2\cosh(bx+a)\sinh(bx+a)}{2}-\frac{(bx+a)^3}{6}-\frac{(bx+a)\cosh(bx+a)^2}{2}+\frac{\cosh(bx+a)\sinh(bx+a)}{4}+\frac{bx}{4}+\frac{a}{4}\right)}{b^2}-\frac{2d^2a\left(\frac{(bx+a)\cosh(bx}{}\right.}{}}{}$
default	$\frac{\frac{d^2\left(\frac{(bx+a)^2\cosh(bx+a)\sinh(bx+a)}{2}-\frac{(bx+a)^3}{6}-\frac{(bx+a)\cosh(bx+a)^2}{2}+\frac{\cosh(bx+a)\sinh(bx+a)}{4}+\frac{bx}{4}+\frac{a}{4}\right)}{b^2}-\frac{2d^2a\left(\frac{(bx+a)\cosh(bx}{}\right.}{}}{}$
orering	$\frac{(4b^4d^4x^5+20b^4c\,d^3x^4+40b^4c^2d^2x^3+36b^4c^3d\,x^2+12b^4c^4x-12b^2d^4x^3-42b^2c\,d^3x^2-48b^2c^2d^2x-12b^2c^3d-12d^4x-3d^3}{12(dx+c)^2b^4}$

input
```
int((d*x+c)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*((2*(d*x+c)^2*b^2+d^2)*sinh(2*b*x+2*a)-4*b*(1/2*d*(d*x+c)*cosh(2*b*x+2
*a)+x*(1/3*x^2*d^2+c*d*x+c^2)*b^2-1/2*d*c))/b^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.29

$$\int (c+dx)^2\sinh^2(a+bx)\,dx = -\frac{2\,b^3d^2x^3+6\,b^3cdx^2+6\,b^3c^2x+3\,(bd^2x+bcd)\cosh(bx+a)^2-3\,(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2+d^2)\cos}{12\,b^3}$$

input
```
integrate((d*x+c)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/12*(2*b^3*d^2*x^3 + 6*b^3*c*d*x^2 + 6*b^3*c^2*x + 3*(b*d^2*x + b*c*d)*c
osh(b*x + a)^2 - 3*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + d^2)*cosh(b*
x + a)*sinh(b*x + a) + 3*(b*d^2*x + b*c*d)*sinh(b*x + a)^2)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(85) = 170.

Time = 0.27 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.78

$$\int (c+dx)^2\sinh^2(a+bx)\,dx = \begin{cases} \frac{c^2x\sinh^2(a+bx)}{2} - \frac{c^2x\cosh^2(a+bx)}{2} + \frac{cdx^2\sinh^2(a+bx)}{2} - \frac{cdx^2\cosh^2(a+bx)}{2} + \frac{d^2x^3\sinh^2(a+bx)}{6} - \frac{d^2x^3\cosh^2(a+bx)}{6} + \frac{c^2\sin}{} \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\sinh^2(a) \end{cases}$$

input
```
integrate((d*x+c)**2*sinh(b*x+a)**2,x)
```

output
```
Piecewise((c**2*x*sinh(a + b*x)**2/2 - c**2*x*cosh(a + b*x)**2/2 + c*d*x**
2*sinh(a + b*x)**2/2 - c*d*x**2*cosh(a + b*x)**2/2 + d**2*x**3*sinh(a + b*
x)**2/6 - d**2*x**3*cosh(a + b*x)**2/6 + c**2*sinh(a + b*x)*cosh(a + b*x)/
(2*b) + c*d*x*sinh(a + b*x)*cosh(a + b*x)/b + d**2*x**2*sinh(a + b*x)*cosh
(a + b*x)/(2*b) - c*d*sinh(a + b*x)**2/(2*b**2) - d**2*x*sinh(a + b*x)**2/
(4*b**2) - d**2*x*cosh(a + b*x)**2/(4*b**2) + d**2*sinh(a + b*x)*cosh(a +
b*x)/(4*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sinh(a)**2, T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.74

$$\int (c+dx)^2 \sinh^2(a+bx)\,dx$$
$$= -\frac{1}{8}\left(4\,x^2 - \frac{\left(2\,bxe^{(2\,a)} - e^{(2\,a)}\right)e^{(2\,bx)}}{b^2} + \frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{b^2}\right)cd$$
$$-\frac{1}{48}\left(8\,x^3 - \frac{3\left(2\,b^2x^2e^{(2\,a)} - 2\,bxe^{(2\,a)} + e^{(2\,a)}\right)e^{(2\,bx)}}{b^3} + \frac{3\,(2\,b^2x^2 + 2\,bx + 1)e^{(-2\,bx-2\,a)}}{b^3}\right)d^2$$
$$-\frac{1}{8}\,c^2\left(4\,x - \frac{e^{(2\,bx+2\,a)}}{b} + \frac{e^{(-2\,bx-2\,a)}}{b}\right)$$

input
```
integrate((d*x+c)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/8*(4*x^2 - (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 + (2*b*x + 1)*e^(-2*
b*x - 2*a)/b^2)*c*d - 1/48*(8*x^3 - 3*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) +
 e^(2*a))*e^(2*b*x)/b^3 + 3*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3)*
d^2 - 1/8*c^2*(4*x - e^(2*b*x + 2*a)/b + e^(-2*b*x - 2*a)/b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.43

$$\int (c+dx)^2 \sinh^2(a+bx)\,dx$$
$$= -\frac{1}{6}\,d^2x^3 - \frac{1}{2}\,cdx^2 - \frac{1}{2}\,c^2x + \frac{(2\,b^2d^2x^2 + 4\,b^2cdx + 2\,b^2c^2 - 2\,bd^2x - 2\,bcd + d^2)e^{(2\,bx+2\,a)}}{16\,b^3} - \frac{(2\,b^2d^2x^2 + 4\,b^2cdx + 2\,b^2c^2 + 2\,bd^2x + 2\,bcd + d^2)e^{(-2\,bx-2\,a)}}{16\,b^3}$$

input
```
integrate((d*x+c)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/6*d^2*x^3 - 1/2*c*d*x^2 - 1/2*c^2*x + 1/16*(2*b^2*d^2*x^2 + 4*b^2*c*d*x
 + 2*b^2*c^2 - 2*b*d^2*x - 2*b*c*d + d^2)*e^(2*b*x + 2*a)/b^3 - 1/16*(2*b^
2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + 2*b*d^2*x + 2*b*c*d + d^2)*e^(-2*b*x
 - 2*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.34

$$\int (c+dx)^2 \sinh^2(a+bx)\,dx = \frac{c^2\sinh(2\,a+2\,b\,x)}{4\,b} - \frac{d^2\,x^3}{6} - \frac{c^2\,x}{2} + \frac{d^2\sinh(2\,a+2\,b\,x)}{8\,b^3} - \frac{c\,d\,x^2}{2} - \frac{d^2\,x\cosh(2\,a+2\,b\,x)}{4\,b^2} + \frac{d^2\,x^2\sinh(2\,a+2\,b\,x)}{4\,b} - \frac{c\,d\cosh(2\,a+2\,b\,x)}{4\,b^2} + \frac{c\,d\,x\sinh(2\,a+2\,b\,x)}{2\,b}$$

input
```
int(sinh(a + b*x)^2*(c + d*x)^2,x)
```

output
```
(c^2*sinh(2*a + 2*b*x))/(4*b) - (d^2*x^3)/6 - (c^2*x)/2 + (d^2*sinh(2*a +
2*b*x))/(8*b^3) - (c*d*x^2)/2 - (d^2*x*cosh(2*a + 2*b*x))/(4*b^2) + (d^2*x
^2*sinh(2*a + 2*b*x))/(4*b) - (c*d*cosh(2*a + 2*b*x))/(4*b^2) + (c*d*x*sin
h(2*a + 2*b*x))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.38

$$\int (c+dx)^2 \sinh^2(a+bx)\,dx$$
$$= \frac{6e^{4bx+4a}b^2c^2 + 12e^{4bx+4a}b^2cdx + 6e^{4bx+4a}b^2d^2x^2 - 6e^{4bx+4a}bcd - 6e^{4bx+4a}b\,d^2x + 3e^{4bx+4a}d^2 - 24e^{2bx+2a}b^{3}}{48e^{2bx+2a}b^3}$$

input
```
int((d*x+c)^2*sinh(b*x+a)^2,x)
```

output
```
(6*e**(4*a + 4*b*x)*b**2*c**2 + 12*e**(4*a + 4*b*x)*b**2*c*d*x + 6*e**(4*a
 + 4*b*x)*b**2*d**2*x**2 - 6*e**(4*a + 4*b*x)*b*c*d - 6*e**(4*a + 4*b*x)*b
*d**2*x + 3*e**(4*a + 4*b*x)*d**2 - 24*e**(2*a + 2*b*x)*b**3*c**2*x - 24*e
**(2*a + 2*b*x)*b**3*c*d*x**2 - 8*e**(2*a + 2*b*x)*b**3*d**2*x**3 - 6*b**2
*c**2 - 12*b**2*c*d*x - 6*b**2*d**2*x**2 - 6*b*c*d - 6*b*d**2*x - 3*d**2)/
(48*e**(2*a + 2*b*x)*b**3)
```

3.11 $\int (c+dx)\sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 55

$$\int (c+dx)\sinh^2(a+bx)\,dx=-\frac{(c+dx)^2}{4d}+\frac{(c+dx)\cosh(a+bx)\sinh(a+bx)}{2b}-\frac{d\sinh^2(a+bx)}{4b^2}$$

output
```
-1/4*(d*x+c)^2/d+1/2*(d*x+c)*cosh(b*x+a)*sinh(b*x+a)/b-1/4*d*sinh(b*x+a)^2
/b^2
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int (c+dx)\sinh^2(a+bx)\,dx=\frac{-d\cosh(2(a+bx))+2b(-2ac-bx(2c+dx)+(c+dx)\sinh(2(a+bx)))}{8b^2}$$

input
```
Integrate[(c + d*x)*Sinh[a + b*x]^2,x]
```

output
```
(-(d*Cosh[2*(a + b*x)]) + 2*b*(-2*a*c - b*x*(2*c + d*x) + (c + d*x)*Sinh[2
*(a + b*x)]))/(8*b^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sinh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\left((c+dx)\sin(ia+ibx)^2\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{3791}$$

$$-\frac{1}{2}\int (c+dx)dx-\frac{d\sinh^2(a+bx)}{4b^2}+\frac{(c+dx)\sinh(a+bx)\cosh(a+bx)}{2b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{d\sinh^2(a+bx)}{4b^2}+\frac{(c+dx)\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^2}{4d}$$

input
```
Int[(c + d*x)*Sinh[a + b*x]^2,x]
```

output
```
-1/4*(c + d*x)^2/d + ((c + d*x)*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) - (d*Si
nh[a + b*x]^2)/(4*b^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{d\,x^{2}}{4}-\frac{c x}{2}+\frac{\left(2 d x b +2 b c -d \right) {\mathrm e}^{2 b x +2 a}}{16 b^{2}}-\frac{\left(2 d x b +2 b c +d \right) {\mathrm e}^{-2 b x -2 a}}{16 b^{2}}$
derivativedivides	$\frac{\frac{d \left(\frac{\left(b x +a \right) \cosh \left(b x +a \right) \sinh \left(b x +a \right)}{2}-\frac{\left(b x +a \right)^{2}}{4}-\frac{\cosh \left(b x +a \right)^{2}}{4}\right)}{b}-\frac{d a \left(\frac{\cosh \left(b x +a \right) \sinh \left(b x +a \right)}{2}-\frac{b x}{2}-\frac{a}{2}\right)}{b}+c \left(\frac{\cosh \left(b x +a \right) \sinh \left(b x +a \right)}{2}\right.}{b}$
default	$\frac{\frac{d \left(\frac{\left(b x +a \right) \cosh \left(b x +a \right) \sinh \left(b x +a \right)}{2}-\frac{\left(b x +a \right)^{2}}{4}-\frac{\cosh \left(b x +a \right)^{2}}{4}\right)}{b}-\frac{d a \left(\frac{\cosh \left(b x +a \right) \sinh \left(b x +a \right)}{2}-\frac{b x}{2}-\frac{a}{2}\right)}{b}+c \left(\frac{\cosh \left(b x +a \right) \sinh \left(b x +a \right)}{2}\right.}{b}$
orering	$\frac{\left(2 b^{2} d^{3} x^{4}+8 b^{2} c\,d^{2} x^{3}+10 b^{2} c^{2} d\,x^{2}+4 b^{2} c^{3} x -3 d^{3} x^{2}-6 c\,d^{2} x -2 d\,c^{2}\right) \sinh \left(b x +a \right)^{2}}{4 b^{2} \left(d x +c \right)^{2}}+\frac{\left(2 x^{2} d^{2}+4 c d x +c^{2}\right) \left(d \sinh \left(b x +a \right)\right.}{4 (d}$

input
```
int((d*x+c)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*d*x^2-1/2*c*x+1/16*(2*b*d*x+2*b*c-d)/b^2*exp(2*b*x+2*a)-1/16*(2*b*d*x
+2*b*c+d)/b^2*exp(-2*b*x-2*a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16

$$\int (c+dx)\sinh^2(a+bx)\,dx = -\frac{2\,b^2dx^2+4\,b^2cx+d\cosh\left(bx+a\right)^2-4\left(bdx+bc\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)+d\sinh\left(bx+a\right)^2}{8\,b^2}$$

input
```
integrate((d*x+c)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/8*(2*b^2*d*x^2 + 4*b^2*c*x + d*cosh(b*x + a)^2 - 4*(b*d*x + b*c)*cosh(b
*x + a)*sinh(b*x + a) + d*sinh(b*x + a)^2)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(48) = 96.

Time = 0.19 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.29

$$\int (c+dx)\sinh^2(a+bx)\,dx = \begin{cases} \frac{cx\sinh^2\left(a+bx\right)}{2}-\frac{cx\cosh^2\left(a+bx\right)}{2}+\frac{dx^2\sinh^2\left(a+bx\right)}{4}-\frac{dx^2\cosh^2\left(a+bx\right)}{4}+\frac{c\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{2b}+\frac{dx\sinh\left(a+bx\right)\cosh\left(a+b\right.}{2b} \\ \left(cx+\frac{dx^2}{2}\right)\sinh^2\left(a\right) \end{cases}$$

input
```
integrate((d*x+c)*sinh(b*x+a)**2,x)
```

output
```
Piecewise((c*x*sinh(a + b*x)**2/2 - c*x*cosh(a + b*x)**2/2 + d*x**2*sinh(a
 + b*x)**2/4 - d*x**2*cosh(a + b*x)**2/4 + c*sinh(a + b*x)*cosh(a + b*x)/(
2*b) + d*x*sinh(a + b*x)*cosh(a + b*x)/(2*b) - d*sinh(a + b*x)**2/(4*b**2)
, Ne(b, 0)), ((c*x + d*x**2/2)*sinh(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.60

$$\int (c+dx)\sinh^2(a+bx)\,dx = -\frac{1}{16}\left(4x^2 - \frac{(2bxe^{(2a)} - e^{(2a)})e^{(2bx)}}{b^2} + \frac{(2bx+1)e^{(-2bx-2a)}}{b^2}\right)d - \frac{1}{8}c\left(4x - \frac{e^{(2bx+2a)}}{b} + \frac{e^{(-2bx-2a)}}{b}\right)$$

input
```
integrate((d*x+c)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/16*(4*x^2 - (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 + (2*b*x + 1)*e^(-2
*b*x - 2*a)/b^2)*d - 1/8*c*(4*x - e^(2*b*x + 2*a)/b + e^(-2*b*x - 2*a)/b)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int (c+dx)\sinh^2(a+bx)\,dx = -\frac{1}{4}dx^2 - \frac{1}{2}cx + \frac{(2bdx+2bc-d)e^{(2bx+2a)}}{16b^2} - \frac{(2bdx+2bc+d)e^{(-2bx-2a)}}{16b^2}$$

input
```
integrate((d*x+c)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/4*d*x^2 - 1/2*c*x + 1/16*(2*b*d*x + 2*b*c - d)*e^(2*b*x + 2*a)/b^2 - 1/
16*(2*b*d*x + 2*b*c + d)*e^(-2*b*x - 2*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int (c+dx)\sinh^2(a+bx)\,dx$$
$$= -\frac{\frac{d\cosh(2\,a+2\,b\,x)}{2}+b^2\,d\,x^2-b\,c\sinh(2\,a+2\,b\,x)+2\,b^2\,c\,x-b\,d\,x\sinh(2\,a+2\,b\,x)}{4\,b^2}$$

input
```
int(sinh(a + b*x)^2*(c + d*x),x)
```

output
```
-((d*cosh(2*a + 2*b*x))/2 + b^2*d*x^2 - b*c*sinh(2*a + 2*b*x) + 2*b^2*c*x
- b*d*x*sinh(2*a + 2*b*x))/(4*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.96

$$\int (c+dx)\sinh^2(a+bx)\,dx$$
$$= \frac{2e^{4bx+4a}bc+2e^{4bx+4a}bdx-e^{4bx+4a}d-8e^{2bx+2a}b^2cx-4e^{2bx+2a}b^2d\,x^2-2bc-2bdx-d}{16e^{2bx+2a}b^2}$$

input
```
int((d*x+c)*sinh(b*x+a)^2,x)
```

output
```
(2*e**(4*a + 4*b*x)*b*c + 2*e**(4*a + 4*b*x)*b*d*x - e**(4*a + 4*b*x)*d -
8*e**(2*a + 2*b*x)*b**2*c*x - 4*e**(2*a + 2*b*x)*b**2*d*x**2 - 2*b*c - 2*b
*d*x - d)/(16*e**(2*a + 2*b*x)*b**2)
```

3.12 $\int \frac{\sinh^2(a+bx)}{c+dx}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 78

$$\int \frac{\sinh^2(a+bx)}{c+dx}\,dx = \frac{\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{2d} - \frac{\log(c+dx)}{2d} + \frac{\sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

output
```
1/2*cosh(2*a-2*b*c/d)*Chi(2*b*c/d+2*b*x)/d-1/2*ln(d*x+c)/d+1/2*sinh(2*a-2*
b*c/d)*Shi(2*b*c/d+2*b*x)/d
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int \frac{\sinh^2(a+bx)}{c+dx}\,dx = \frac{\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2b(c+dx)}{d}\right) - \log(c+dx) + \sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2b(c+dx)}{d}\right)}{2d}$$

input
```
Integrate[Sinh[a + b*x]^2/(c + d*x),x]
```

output
```
(Cosh[2*a - (2*b*c)/d]*CoshIntegral[(2*b*(c + d*x))/d] - Log[c + d*x] + Si
nh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*(c + d*x))/d])/(2*d)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)}{c+dx}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int -\frac{\sin(ia+ibx)^2}{c+dx}dx$$
$$\Big\downarrow \; 25$$
$$-\int \frac{\sin(ia+ibx)^2}{c+dx}dx$$
$$\Big\downarrow \; 3793$$
$$-\int \left(\frac{1}{2(c+dx)}-\frac{\cosh(2a+2bx)}{2(c+dx)}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{2d}-\frac{\log(c+dx)}{2d}$$

input
```
Int[Sinh[a + b*x]^2/(c + d*x),x]
```

output
```
(Cosh[2*a - (2*b*c)/d]*CoshIntegral[(2*b*c)/d + 2*b*x])/(2*d) - Log[c + d*
x]/(2*d) + (Sinh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*x])/(2*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.24

method	result	size
risch	$-\frac{\ln(dx+c)}{2d}-\frac{\mathrm{e}^{-\frac{2(da-bc)}{d}}\,\mathrm{expIntegral}_1\left(2bx+2a-\frac{2(da-bc)}{d}\right)}{4d}-\frac{\mathrm{e}^{\frac{2da-2bc}{d}}\,\mathrm{expIntegral}_1\left(-2bx-2a-\frac{2(-da+bc)}{d}\right)}{4d}$	97

input
```
int(sinh(b*x+a)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(d*x+c)/d-1/4/d*exp(-2*(a*d-b*c)/d)*Ei(1,2*b*x+2*a-2*(a*d-b*c)/d)-1
/4/d*exp(2*(a*d-b*c)/d)*Ei(1,-2*b*x-2*a-2*(-a*d+b*c)/d)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.33

$$\int \frac{\sinh^2(a+bx)}{c+dx}\,dx = \frac{\left(\operatorname{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)+\operatorname{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cosh\left(-\frac{2\,(bc-ad)}{d}\right)+\left(\operatorname{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)-\operatorname{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sinh\left(-\frac{2\,(bc-a}{d}\right.}{4\,d}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
1/4*((Ei(2*(b*d*x + b*c)/d) + Ei(-2*(b*d*x + b*c)/d))*cosh(-2*(b*c - a*d)/
d) + (Ei(2*(b*d*x + b*c)/d) - Ei(-2*(b*d*x + b*c)/d))*sinh(-2*(b*c - a*d)/
d) - 2*log(d*x + c))/d
```

Sympy [F]

$$\int \frac{\sinh^2(a+bx)}{c+dx}\,dx = \int \frac{\sinh^2\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(sinh(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(sinh(a + b*x)**2/(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int \frac{\sinh^2(a+bx)}{c+dx}\,dx = -\frac{e^{\left(-2\,a+\frac{2\,bc}{d}\right)}E_1\left(\frac{2\,(dx+c)b}{d}\right)}{4\,d}-\frac{e^{\left(2\,a-\frac{2\,bc}{d}\right)}E_1\left(-\frac{2\,(dx+c)b}{d}\right)}{4\,d}-\frac{\log\left(dx+c\right)}{2\,d}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
-1/4*e^(-2*a + 2*b*c/d)*exp_integral_e(1, 2*(d*x + c)*b/d)/d - 1/4*e^(2*a
- 2*b*c/d)*exp_integral_e(1, -2*(d*x + c)*b/d)/d - 1/2*log(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.87

$$\int \frac{\sinh^2(a+bx)}{c+dx}\,dx = \frac{\operatorname{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}+\operatorname{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}-2\,\log\left(dx+c\right)}{4\,d}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
1/4*(Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) + Ei(-2*(b*d*x + b*c)/d)*e^(-
2*a + 2*b*c/d) - 2*log(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(a+bx)}{c+dx}\,dx = \int \frac{\sinh(a+b\,x)^2}{c+d\,x}\,dx$$

input
```
int(sinh(a + b*x)^2/(c + d*x),x)
```

output
```
int(sinh(a + b*x)^2/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\sinh^2(a+bx)}{c+dx}\,dx = \int \frac{\sinh{(bx+a)}^2}{dx+c}dx$$

input
```
int(sinh(b*x+a)^2/(d*x+c),x)
```

output
```
int(sinh(a + b*x)**2/(c + d*x),x)
```

3.13 $\int \frac{\sinh^2(a+bx)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 81

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^2}\,dx = \frac{b\text{Chi}\left(\frac{2bc}{d}+2bx\right)\sinh\left(2a-\frac{2bc}{d}\right)}{d^2} - \frac{\sinh^2(a+bx)}{d(c+dx)} + \frac{b\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{d^2}$$

output
```
b*Chi(2*b*c/d+2*b*x)*sinh(2*a-2*b*c/d)/d^2-sinh(b*x+a)^2/d/(d*x+c)+b*cosh(
2*a-2*b*c/d)*Shi(2*b*c/d+2*b*x)/d^2
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^2}\,dx = \frac{b\text{Chi}\left(\frac{2b(c+dx)}{d}\right)\sinh\left(2a-\frac{2bc}{d}\right) - \frac{d\sinh^2(a+bx)}{c+dx} + b\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2b(c+dx)}{d}\right)}{d^2}$$

input
```
Integrate[Sinh[a + b*x]^2/(c + d*x)^2,x]
```

output
```
(b*CoshIntegral[(2*b*(c + d*x))/d]*Sinh[2*a - (2*b*c)/d] - (d*Sinh[a + b*x
]^2)/(c + d*x) + b*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*(c + d*x))/d])/
d^2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 25, 3794, 27, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ia+ibx)^2}{(c+dx)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ia+ibx)^2}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3794}$$

$$-\frac{\sinh^2(a+bx)}{d(c+dx)}-\frac{2ib\int \frac{i\sinh(2a+2bx)}{2(c+dx)}dx}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{\sinh(2a+2bx)}{c+dx}dx}{d}-\frac{\sinh^2(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh^2(a+bx)}{d(c+dx)}+\frac{b\int -\frac{i\sin(2ia+2ibx)}{c+dx}dx}{d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\sinh^2(a+bx)}{d(c+dx)}-\frac{ib\int \frac{\sin(2ia+2ibx)}{c+dx}dx}{d}$$

$\Big\downarrow$ 3784

$$-\frac{\sinh^2(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int \frac{\cosh\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx+\cosh\left(2a-\frac{2bc}{d}\right)\int \frac{i\sinh\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}$$

$\Big\downarrow$ 26

$$-\frac{\sinh^2(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int \frac{\cosh\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx+i\cosh\left(2a-\frac{2bc}{d}\right)\int \frac{\sinh\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}$$

$\Big\downarrow$ 3042

$$-\frac{\sinh^2(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2ibc}{d}+2ibx+\frac{\pi}{2}\right)}{c+dx}dx+i\cosh\left(2a-\frac{2bc}{d}\right)\int -\frac{i\sin\left(\frac{2ibc}{d}+2ibx\right)}{c+dx}dx\right)}{d}$$

$\Big\downarrow$ 26

$$-\frac{\sinh^2(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2ibc}{d}+2ibx+\frac{\pi}{2}\right)}{c+dx}dx+\cosh\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2ibc}{d}+2ibx\right)}{c+dx}dx\right)}{d}$$

$\Big\downarrow$ 3779

$$-\frac{\sinh^2(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2ibc}{d}+2ibx+\frac{\pi}{2}\right)}{c+dx}dx+\frac{i\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}$$

$\Big\downarrow$ 3782

$$-\frac{\sinh^2(a+bx)}{d(c+dx)}-\frac{ib\left(\frac{i\sinh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{d}+\frac{i\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}$$

input
```
Int[Sinh[a + b*x]^2/(c + d*x)^2,x]
```

output
```
-(Sinh[a + b*x]^2/(d*(c + d*x))) - (I*b*((I*CoshIntegral[(2*b*c)/d + 2*b*x
]*Sinh[2*a - (2*b*c)/d])/d + (I*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)
/d + 2*b*x])/d))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.88

method	result
risch	$\frac{1}{2(dx+c)d}-\frac{b\,\mathrm{e}^{-2bx-2a}}{4d(dxb+bc)}+\frac{b\,\mathrm{e}^{-\frac{2(da-bc)}{d}}\operatorname{expIntegral}_1\left(2bx+2a-\frac{2(da-bc)}{d}\right)}{2d^2}-\frac{b\,\mathrm{e}^{2bx+2a}}{4d^2\left(\frac{bc}{d}+bx\right)}-\frac{b\,\mathrm{e}^{\frac{2da-2bc}{d}}\operatorname{expIntegral}_1\left(-2b\right.}{2d^2}$

input
```
int(sinh(b*x+a)^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/(d*x+c)/d-1/4*b*exp(-2*b*x-2*a)/d/(b*d*x+b*c)+1/2*b/d^2*exp(-2*(a*d-b*
c)/d)*Ei(1,2*b*x+2*a-2*(a*d-b*c)/d)-1/4*b/d^2*exp(2*b*x+2*a)/(b*c/d+b*x)-1
/2*b/d^2*exp(2*(a*d-b*c)/d)*Ei(1,-2*b*x-2*a-2*(-a*d+b*c)/d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(81) = 162.

Time = 0.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.05

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^2}\,dx =$$
$$-\frac{d\cosh\left(bx+a\right)^2+d\sinh\left(bx+a\right)^2-\left((bdx+bc)\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)-(bdx+bc)\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cosh\left(-\right.}{2\left(d^3x+cd^2\right.}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(d*cosh(b*x + a)^2 + d*sinh(b*x + a)^2 - ((b*d*x + b*c)*Ei(2*(b*d*x +
 b*c)/d) - (b*d*x + b*c)*Ei(-2*(b*d*x + b*c)/d))*cosh(-2*(b*c - a*d)/d) -
((b*d*x + b*c)*Ei(2*(b*d*x + b*c)/d) + (b*d*x + b*c)*Ei(-2*(b*d*x + b*c)/d
))*sinh(-2*(b*c - a*d)/d) - d)/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sinh^2{(a+bx)}}{(c+dx)^2}\,dx$$

input
```
integrate(sinh(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(sinh(a + b*x)**2/(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.09

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^2}\,dx = -\frac{e^{\left(-2\,a+\frac{2\,bc}{d}\right)}E_2\left(\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)d} - \frac{e^{\left(2\,a-\frac{2\,bc}{d}\right)}E_2\left(-\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)d} + \frac{1}{2\,(d^2x+cd)}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/4*e^(-2*a + 2*b*c/d)*exp_integral_e(2, 2*(d*x + c)*b/d)/((d*x + c)*d) -
 1/4*e^(2*a - 2*b*c/d)*exp_integral_e(2, -2*(d*x + c)*b/d)/((d*x + c)*d) +
 1/2/(d^2*x + c*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 574 vs. $2(81) = 162$.

Time = 0.17 (sec) , antiderivative size = 574, normalized size of antiderivative = 7.09

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^2}\,dx =$$

$$-\frac{\left(2\,(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\mathrm{Ei}\left(-\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)e^{\left(\frac{2\,(bc-ad)}{d}\right)}+2\,b^3c\mathrm{Ei}\left(-\frac{2\left((dx+c)\left(b-\right.\right.}{}\right.\right.}{}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/4*(2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*Ei(-2*((d*x + c)
*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(2*(b*c - a*d)/d) +
 2*b^3*c*Ei(-2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)
/d)*e^(2*(b*c - a*d)/d) - 2*a*b^2*d*Ei(-2*((d*x + c)*(b - b*c/(d*x + c) +
a*d/(d*x + c)) + b*c - a*d)/d)*e^(2*(b*c - a*d)/d) - 2*(d*x + c)*(b - b*c/
(d*x + c) + a*d/(d*x + c))*b^2*Ei(2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d
*x + c)) + b*c - a*d)/d)*e^(-2*(b*c - a*d)/d) - 2*b^3*c*Ei(2*((d*x + c)*(b
 - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(-2*(b*c - a*d)/d) + 2
*a*b^2*d*Ei(2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/
d)*e^(-2*(b*c - a*d)/d) + b^2*d*e^(2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d
*x + c))/d) + b^2*d*e^(-2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d)
 - 2*b^2*d)*d^2/(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*d^4 + b*c*
d^4 - a*d^5)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{{\sinh(a+b\,x)}^2}{{(c+d\,x)}^2}\,dx$$

input
```
int(sinh(a + b*x)^2/(c + d*x)^2,x)
```

output
```
int(sinh(a + b*x)^2/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{e^{2a}\left(\int \frac{e^{2bx}}{d^2x^2+2cdx+c^2}dx\right)c^2 + e^{2a}\left(\int \frac{e^{2bx}}{d^2x^2+2cdx+c^2}dx\right)cdx + \left(\int \frac{1}{e^{2bx+2a}c^2+2e^{2bx+2a}cdx+e^{2bx+2a}d^2x^2}dx\right)c^2 + \left(\int \frac{}{e^{2}}\right.}{4c\,(dx+c)}$$

input
```
int(sinh(b*x+a)^2/(d*x+c)^2,x)
```

output
```
(e**(2*a)*int(e**(2*b*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*c**2 + e**(2*a)*i
nt(e**(2*b*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*c*d*x + int(1/(e**(2*a + 2*b
*x)*c**2 + 2*e**(2*a + 2*b*x)*c*d*x + e**(2*a + 2*b*x)*d**2*x**2),x)*c**2
+ int(1/(e**(2*a + 2*b*x)*c**2 + 2*e**(2*a + 2*b*x)*c*d*x + e**(2*a + 2*b*
x)*d**2*x**2),x)*c*d*x - 2*x)/(4*c*(c + d*x))
```

3.14 $\int \frac{\sinh^2(a+bx)}{(c+dx)^3} dx$

Optimal result

Integrand size = 16, antiderivative size = 112

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^3} dx = \frac{b^2\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{d^3} - \frac{b\cosh(a+bx)\sinh(a+bx)}{d^2(c+dx)} - \frac{\sinh^2(a+bx)}{2d(c+dx)^2} + \frac{b^2\sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{d^3}$$

output
```
b^2*cosh(2*a-2*b*c/d)*Chi(2*b*c/d+2*b*x)/d^3-b*cosh(b*x+a)*sinh(b*x+a)/d^2
/(d*x+c)-1/2*sinh(b*x+a)^2/d/(d*x+c)^2+b^2*sinh(2*a-2*b*c/d)*Shi(2*b*c/d+2
*b*x)/d^3
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.91

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^3} dx = \frac{2b^2\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2b(c+dx)}{d}\right) - \frac{d(d\sinh^2(a+bx)+b(c+dx)\sinh(2(a+bx)))}{(c+dx)^2} + 2b^2\sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2b(c+dx)}{d}\right)}{2d^3}$$

input
```
Integrate[Sinh[a + b*x]^2/(c + d*x)^3,x]
```

output
```
(2*b^2*Cosh[2*a - (2*b*c)/d]*CoshIntegral[(2*b*(c + d*x))/d] - (d*(d*Sinh[
a + b*x]^2 + b*(c + d*x)*Sinh[2*(a + b*x)]))/(c + d*x)^2 + 2*b^2*Sinh[2*a
- (2*b*c)/d]*SinhIntegral[(2*b*(c + d*x))/d])/(2*d^3)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.31, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {3042, 25, 3795, 16, 25, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ia+ibx)^2}{(c+dx)^3}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ia+ibx)^2}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3795}$$

$$-\frac{2b^2\int -\frac{\sinh^2(a+bx)}{c+dx}dx}{d^2}+\frac{b^2\int \frac{1}{c+dx}dx}{d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{d^2(c+dx)}-\frac{\sinh^2(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{16}$$

$$-\frac{2b^2\int -\frac{\sinh^2(a+bx)}{c+dx}dx}{d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{d^2(c+dx)}-\frac{\sinh^2(a+bx)}{2d(c+dx)^2}+\frac{b^2\log(c+dx)}{d^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{2b^2\int \frac{\sinh^2(a+bx)}{c+dx}dx}{d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{d^2(c+dx)}-\frac{\sinh^2(a+bx)}{2d(c+dx)^2}+\frac{b^2\log(c+dx)}{d^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b^2\int -\frac{\sin(ia+ibx)^2}{c+dx}dx}{d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{d^2(c+dx)}-\frac{\sinh^2(a+bx)}{2d(c+dx)^2}+\frac{b^2\log(c+dx)}{d^3}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{2b^2\int \frac{\sin(ia+ibx)^2}{c+dx}dx}{d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{d^2(c+dx)}-\frac{\sinh^2(a+bx)}{2d(c+dx)^2}+\frac{b^2\log(c+dx)}{d^3}$$

$$\Big\downarrow \text{ 3793}$$

$$-\frac{2b^2\int \left(\frac{1}{2(c+dx)}-\frac{\cosh(2a+2bx)}{2(c+dx)}\right)dx}{d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{d^2(c+dx)}-\frac{\sinh^2(a+bx)}{2d(c+dx)^2}+\frac{b^2\log(c+dx)}{d^3}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2b^2\left(-\frac{\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{2d}-\frac{\sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\log(c+dx)}{2d}\right)}{d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{d^2(c+dx)}-\frac{\sinh^2(a+bx)}{2d(c+dx)^2}+\frac{b^2\log(c+dx)}{d^3}$$

input
```
Int[Sinh[a + b*x]^2/(c + d*x)^3,x]
```

output
```
(b^2*Log[c + d*x])/d^3 - (b*Cosh[a + b*x]*Sinh[a + b*x])/(d^2*(c + d*x)) -
 Sinh[a + b*x]^2/(2*d*(c + d*x)^2) - (2*b^2*(-1/2*(Cosh[2*a - (2*b*c)/d]*C
oshIntegral[(2*b*c)/d + 2*b*x])/d + Log[c + d*x]/(2*d) - (Sinh[2*a - (2*b*
c)/d]*SinhIntegral[(2*b*c)/d + 2*b*x])/(2*d)))/d^2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(110) = 220$.

Time = 0.69 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.67

method	result
risch	$\frac{1}{4(dx+c)^2 d} + \frac{b^3 \mathrm{e}^{-2bx-2a} x}{4d(d^2x^2b^2+2b^2cdx+b^2c^2)} + \frac{b^3 \mathrm{e}^{-2bx-2a} c}{4d^2(d^2x^2b^2+2b^2cdx+b^2c^2)} - \frac{b^2 \mathrm{e}^{-2bx-2a}}{8d(d^2x^2b^2+2b^2cdx+b^2c^2)} - \frac{b^2 \mathrm{e}^{-\frac{2(da-bc)}{d}} \mathrm{expInt}}{}$

input
```
int(sinh(b*x+a)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/(d*x+c)^2/d+1/4*b^3*exp(-2*b*x-2*a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2
)*x+1/4*b^3*exp(-2*b*x-2*a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*c-1/8*b^
2*exp(-2*b*x-2*a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)-1/2*b^2/d^3*exp(-2*(
a*d-b*c)/d)*Ei(1,2*b*x+2*a-2*(a*d-b*c)/d)-1/8*b^2/d^3*exp(2*b*x+2*a)/(b*c/
d+b*x)^2-1/4*b^2/d^3*exp(2*b*x+2*a)/(b*c/d+b*x)-1/2*b^2/d^3*exp(2*(a*d-b*c
)/d)*Ei(1,-2*b*x-2*a-2*(-a*d+b*c)/d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(110) = 220$.

Time = 0.08 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.50

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^3}\,dx = -\frac{d^2\cosh(bx+a)^2 + d^2\sinh(bx+a)^2 + 4(bd^2x+bcd)\cosh(bx+a)\sinh(bx+a) - d^2 - 2\left((b^2d^2x^2 + \cdots\right.}{\cdots}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(d^2*cosh(b*x + a)^2 + d^2*sinh(b*x + a)^2 + 4*(b*d^2*x + b*c*d)*cosh
(b*x + a)*sinh(b*x + a) - d^2 - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*E
i(2*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-2*(b*d*x
+ b*c)/d))*cosh(-2*(b*c - a*d)/d) - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^
2)*Ei(2*(b*d*x + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-2*(b*
d*x + b*c)/d))*sinh(-2*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sinh^2(a+bx)}{(c+dx)^3}\,dx$$

input
```
integrate(sinh(b*x+a)**2/(d*x+c)**3,x)
```

output
```
Integral(sinh(a + b*x)**2/(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^3}\,dx = \frac{1}{4\,(d^3x^2+2\,cd^2x+c^2d)} - \frac{e^{\left(-2\,a+\frac{2\,bc}{d}\right)}E_3\left(\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)^2d} - \frac{e^{\left(2\,a-\frac{2\,bc}{d}\right)}E_3\left(-\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)^2d}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/4/(d^3*x^2 + 2*c*d^2*x + c^2*d) - 1/4*e^(-2*a + 2*b*c/d)*exp_integral_e(
3, 2*(d*x + c)*b/d)/((d*x + c)^2*d) - 1/4*e^(2*a - 2*b*c/d)*exp_integral_e
(3, -2*(d*x + c)*b/d)/((d*x + c)^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. 2(110) = 220.

Time = 0.13 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.95

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^3}\,dx = \frac{4\,b^2d^2x^2\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)} + 4\,b^2d^2x^2\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)} + 8\,b^2cdx\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)} + \cdots}{\cdots}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/8*(4*b^2*d^2*x^2*Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) + 4*b^2*d^2*x^2
*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d) + 8*b^2*c*d*x*Ei(2*(b*d*x + b*c
)/d)*e^(2*a - 2*b*c/d) + 8*b^2*c*d*x*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*
c/d) + 4*b^2*c^2*Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) + 4*b^2*c^2*Ei(-2
*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d) - 2*b*d^2*x*e^(2*b*x + 2*a) + 2*b*d^2
*x*e^(-2*b*x - 2*a) - 2*b*c*d*e^(2*b*x + 2*a) + 2*b*c*d*e^(-2*b*x - 2*a) -
 d^2*e^(2*b*x + 2*a) - d^2*e^(-2*b*x - 2*a) + 2*d^2)/(d^5*x^2 + 2*c*d^4*x
+ c^2*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sinh(a+b\,x)^2}{{(c+d\,x)}^3}\,dx$$

input
```
int(sinh(a + b*x)^2/(c + d*x)^3,x)
```

output
```
int(sinh(a + b*x)^2/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{e^{2a}\left(\int \frac{e^{2bx}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)c^2d+2e^{2a}\left(\int \frac{e^{2bx}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)c\,d^2x+e^{2a}\left(\int \frac{e^{2bx}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right.}{}$$

input
```
int(sinh(b*x+a)^2/(d*x+c)^3,x)
```

output
```
(e**(2*a)*int(e**(2*b*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x
)*c**2*d + 2*e**(2*a)*int(e**(2*b*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 +
d**3*x**3),x)*c*d**2*x + e**(2*a)*int(e**(2*b*x)/(c**3 + 3*c**2*d*x + 3*c*
d**2*x**2 + d**3*x**3),x)*d**3*x**2 + int(1/(e**(2*a + 2*b*x)*c**3 + 3*e**
(2*a + 2*b*x)*c**2*d*x + 3*e**(2*a + 2*b*x)*c*d**2*x**2 + e**(2*a + 2*b*x)
*d**3*x**3),x)*c**2*d + 2*int(1/(e**(2*a + 2*b*x)*c**3 + 3*e**(2*a + 2*b*x
)*c**2*d*x + 3*e**(2*a + 2*b*x)*c*d**2*x**2 + e**(2*a + 2*b*x)*d**3*x**3),
x)*c*d**2*x + int(1/(e**(2*a + 2*b*x)*c**3 + 3*e**(2*a + 2*b*x)*c**2*d*x +
 3*e**(2*a + 2*b*x)*c*d**2*x**2 + e**(2*a + 2*b*x)*d**3*x**3),x)*d**3*x**2
 + 1)/(4*d*(c**2 + 2*c*d*x + d**2*x**2))
```

3.15 $\int \frac{\sinh^2(a+bx)}{(c+dx)^4} dx$

Optimal result

Integrand size = 16, antiderivative size = 162

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^4} dx = -\frac{b^2}{3d^3(c+dx)} + \frac{2b^3\text{Chi}\left(\frac{2bc}{d}+2bx\right)\sinh\left(2a-\frac{2bc}{d}\right)}{3d^4} - \frac{b\cosh(a+bx)\sinh(a+bx)}{3d^2(c+dx)^2} - \frac{\sinh^2(a+bx)}{3d(c+dx)^3} - \frac{2b^2\sinh^2(a+bx)}{3d^3(c+dx)} + \frac{2b^3\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{3d^4}$$

output
```
-1/3*b^2/d^3/(d*x+c)+2/3*b^3*Chi(2*b*c/d+2*b*x)*sinh(2*a-2*b*c/d)/d^4-1/3*
b*cosh(b*x+a)*sinh(b*x+a)/d^2/(d*x+c)^2-1/3*sinh(b*x+a)^2/d/(d*x+c)^3-2/3*
b^2*sinh(b*x+a)^2/d^3/(d*x+c)+2/3*b^3*cosh(2*a-2*b*c/d)*Shi(2*b*c/d+2*b*x)
/d^4
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.76

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^4}\,dx = \frac{4b^3\text{Chi}\left(\frac{2b(c+dx)}{d}\right)\sinh\left(2a-\frac{2bc}{d}\right)-\frac{d\left(\left(d^2+2b^2(c+dx)^2\right)\cosh(2(a+bx))+d(-d+b(c+dx)\sinh(2(a+bx)))\right)}{(c+dx)^3}+4b^3\cosh\left(2a-\right.}{6d^4}$$

input
```
Integrate[Sinh[a + b*x]^2/(c + d*x)^4,x]
```

output
```
(4*b^3*CoshIntegral[(2*b*(c + d*x))/d]*Sinh[2*a - (2*b*c)/d] - (d*((d^2 +
2*b^2*(c + d*x)^2)*Cosh[2*(a + b*x)] + d*(-d + b*(c + d*x)*Sinh[2*(a + b*x
)]))))/(c + d*x)^3 + 4*b^3*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*(c + d*x
))/d])/(6*d^4)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.05, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.062$, Rules used = {3042, 25, 3795, 17, 25, 3042, 25, 3794, 27, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^4}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\sin(ia+ibx)^2}{(c+dx)^4}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\sin(ia+ibx)^2}{(c+dx)^4}dx$$

$$\Big\downarrow \; 3795$$

$$-\frac{2b^2\int -\frac{\sinh^2(a+bx)}{(c+dx)^2}dx}{3d^2}+\frac{b^2\int \frac{1}{(c+dx)^2}dx}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3}$$

$$\Big\downarrow \; 17$$

$$-\frac{2b^2\int -\frac{\sinh^2(a+bx)}{(c+dx)^2}dx}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \; 25$$

$$\frac{2b^2\int \frac{\sinh^2(a+bx)}{(c+dx)^2}dx}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{2b^2\int -\frac{\sin(ia+ibx)^2}{(c+dx)^2}dx}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \; 25$$

$$-\frac{2b^2\int \frac{\sin(ia+ibx)^2}{(c+dx)^2}dx}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \; 3794$$

$$-\frac{2b^2\left(\frac{\sinh^2(a+bx)}{d(c+dx)}+\frac{2ib\int \frac{i\sinh(2a+2bx)}{2(c+dx)}dx}{d}\right)}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \; 27$$

$$-\frac{2b^2\left(\frac{\sinh^2(a+bx)}{d(c+dx)}-\frac{b\int \frac{\sinh(2a+2bx)}{c+dx}dx}{d}\right)}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2b^2\left(\frac{\sinh^2(a+bx)}{d(c+dx)}-\frac{b\int -\frac{i\sin(2ia+2ibx)}{c+dx}dx}{d}\right)}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \; 26$$

$$-\frac{2b^2\left(\frac{\sinh^2(a+bx)}{d(c+dx)}+\frac{ib\int\frac{\sin(2ia+2ibx)}{c+dx}dx}{d}\right)}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \; 3784$$

$$-\frac{2b^2\left(\frac{\sinh^2(a+bx)}{d(c+dx)}+\frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int\frac{\cosh\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx+\cosh\left(2a-\frac{2bc}{d}\right)\int\frac{i\sinh\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}\right)}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \; 26$$

$$-\frac{2b^2\left(\frac{\sinh^2(a+bx)}{d(c+dx)}+\frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int\frac{\cosh\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx+i\cosh\left(2a-\frac{2bc}{d}\right)\int\frac{\sinh\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}\right)}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2b^2\left(\frac{\sinh^2(a+bx)}{d(c+dx)}+\frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2ibc}{d}+2ibx+\frac{\pi}{2}\right)}{c+dx}dx+i\cosh\left(2a-\frac{2bc}{d}\right)\int-\frac{i\sin\left(\frac{2ibc}{d}+2ibx\right)}{c+dx}dx\right)}{d}\right)}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \; 26$$

$$-\frac{2b^2\left(\frac{\sinh^2(a+bx)}{d(c+dx)}+\frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2ibc}{d}+2ibx+\frac{\pi}{2}\right)}{c+dx}dx+\cosh\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2ibc}{d}+2ibx\right)}{c+dx}dx\right)}{d}\right)}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \; 3779$$

$$-\frac{2b^2\left(\frac{\sinh^2(a+bx)}{d(c+dx)}+\frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2ibc}{d}+2ibx+\frac{\pi}{2}\right)}{c+dx}dx+\frac{i\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}\right)}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \text{3782}$$

$$-\frac{2b^2\left(\frac{\sinh^2(a+bx)}{d(c+dx)}+\frac{ib\left(\frac{i\sinh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{d}+\frac{i\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}\right)}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

input
```
Int[Sinh[a + b*x]^2/(c + d*x)^4,x]
```

output
```
-1/3*b^2/(d^3*(c + d*x)) - (b*Cosh[a + b*x]*Sinh[a + b*x])/(3*d^2*(c + d*x
)^2) - Sinh[a + b*x]^2/(3*d*(c + d*x)^3) - (2*b^2*(Sinh[a + b*x]^2/(d*(c +
 d*x)) + (I*b*((I*CoshIntegral[(2*b*c)/d + 2*b*x]*Sinh[2*a - (2*b*c)/d])/d
 + (I*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*x])/d))/d))/(3*d^
2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(150) = 300$.

Time = 0.85 (sec) , antiderivative size = 555, normalized size of antiderivative = 3.43

method	result
risch	$\frac{1}{6(dx+c)^3 d} - \frac{b^5 e^{-2bx-2a} x^2}{6d(d^3x^3b^3+3b^3c\,d^2x^2+3b^3c^2dx+b^3c^3)} - \frac{b^5 e^{-2bx-2a} cx}{3d^2(d^3x^3b^3+3b^3c\,d^2x^2+3b^3c^2dx+b^3c^3)} - \frac{b^5 e^{-2bx-2a} c^2}{6d^3(d^3x^3b^3+3b^3c\,d^2x^2+3b}$

input
```
int(sinh(b*x+a)^2/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/6/(d*x+c)^3/d-1/6*b^5*exp(-2*b*x-2*a)/d/(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b
^3*c^2*d*x+b^3*c^3)*x^2-1/3*b^5*exp(-2*b*x-2*a)/d^2/(b^3*d^3*x^3+3*b^3*c*d
^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*c*x-1/6*b^5*exp(-2*b*x-2*a)/d^3/(b^3*d^3*x^3
+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*c^2+1/12*b^4*exp(-2*b*x-2*a)/d/(b^
3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*x+1/12*b^4*exp(-2*b*x-2*a
)/d^2/(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*c-1/12*b^3*exp(-
2*b*x-2*a)/d/(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)+1/3*b^3/d
^4*exp(-2*(a*d-b*c)/d)*Ei(1,2*b*x+2*a-2*(a*d-b*c)/d)-1/12*b^3/d^4*exp(2*b*
x+2*a)/(b*c/d+b*x)^3-1/12*b^3/d^4*exp(2*b*x+2*a)/(b*c/d+b*x)^2-1/6*b^3/d^4
*exp(2*b*x+2*a)/(b*c/d+b*x)-1/3*b^3/d^4*exp(2*(a*d-b*c)/d)*Ei(1,-2*b*x-2*a
-2*(-a*d+b*c)/d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(150) = 300$.

Time = 0.09 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.54

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^4}\,dx$$
$$= \frac{d^3 - (2\,b^2d^3x^2 + 4\,b^2cd^2x + 2\,b^2c^2d + d^3)\cosh(bx+a)^2 - 2\,(bd^3x + bcd^2)\cosh(bx+a)\sinh(bx+a) - (}{}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/6*(d^3 - (2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d + d^3)*cosh(b*x +
a)^2 - 2*(b*d^3*x + b*c*d^2)*cosh(b*x + a)*sinh(b*x + a) - (2*b^2*d^3*x^2
+ 4*b^2*c*d^2*x + 2*b^2*c^2*d + d^3)*sinh(b*x + a)^2 + 2*((b^3*d^3*x^3 + 3
*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*Ei(2*(b*d*x + b*c)/d) - (b^3*d^3
*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*Ei(-2*(b*d*x + b*c)/d))*
cosh(-2*(b*c - a*d)/d) + 2*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x
 + b^3*c^3)*Ei(2*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3
*c^2*d*x + b^3*c^3)*Ei(-2*(b*d*x + b*c)/d))*sinh(-2*(b*c - a*d)/d))/(d^7*x
^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Sympy [F]

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^4}\,dx = \int \frac{\sinh^2{(a+bx)}}{(c+dx)^4}\,dx$$

input
```
integrate(sinh(b*x+a)**2/(d*x+c)**4,x)
```

output
```
Integral(sinh(a + b*x)**2/(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.68

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^4}\,dx = \frac{1}{6\,(d^4x^3+3\,cd^3x^2+3\,c^2d^2x+c^3d)} - \frac{e^{\left(-2\,a+\frac{2\,bc}{d}\right)}E_4\left(\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)^3d} - \frac{e^{\left(2\,a-\frac{2\,bc}{d}\right)}E_4\left(-\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)^3d}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/6/(d^4*x^3 + 3*c*d^3*x^2 + 3*c^2*d^2*x + c^3*d) - 1/4*e^(-2*a + 2*b*c/d)
*exp_integral_e(4, 2*(d*x + c)*b/d)/((d*x + c)^3*d) - 1/4*e^(2*a - 2*b*c/d
)*exp_integral_e(4, -2*(d*x + c)*b/d)/((d*x + c)^3*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 537 vs. $2(150) = 300$.

Time = 0.15 (sec) , antiderivative size = 537, normalized size of antiderivative = 3.31

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^4}\,dx = \frac{4\,b^3d^3x^3\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)} - 4\,b^3d^3x^3\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)} + 12\,b^3cd^2x^2\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}}{}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")
```

output
```
1/12*(4*b^3*d^3*x^3*Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) - 4*b^3*d^3*x^
3*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d) + 12*b^3*c*d^2*x^2*Ei(2*(b*d*x
 + b*c)/d)*e^(2*a - 2*b*c/d) - 12*b^3*c*d^2*x^2*Ei(-2*(b*d*x + b*c)/d)*e^(
-2*a + 2*b*c/d) + 12*b^3*c^2*d*x*Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) -
 12*b^3*c^2*d*x*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d) - 2*b^2*d^3*x^2*
e^(2*b*x + 2*a) - 2*b^2*d^3*x^2*e^(-2*b*x - 2*a) + 4*b^3*c^3*Ei(2*(b*d*x +
 b*c)/d)*e^(2*a - 2*b*c/d) - 4*b^3*c^3*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*
b*c/d) - 4*b^2*c*d^2*x*e^(2*b*x + 2*a) - 4*b^2*c*d^2*x*e^(-2*b*x - 2*a) -
2*b^2*c^2*d*e^(2*b*x + 2*a) - b*d^3*x*e^(2*b*x + 2*a) - 2*b^2*c^2*d*e^(-2*
b*x - 2*a) + b*d^3*x*e^(-2*b*x - 2*a) - b*c*d^2*e^(2*b*x + 2*a) + b*c*d^2*
e^(-2*b*x - 2*a) - d^3*e^(2*b*x + 2*a) - d^3*e^(-2*b*x - 2*a) + 2*d^3)/(d^
7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^4}\,dx = \int \frac{{\mathrm{sinh}\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^4}\,dx$$

input
```
int(sinh(a + b*x)^2/(c + d*x)^4,x)
```

output
```
int(sinh(a + b*x)^2/(c + d*x)^4, x)
```

Reduce [F]

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^4}\,dx$$
$$= \frac{3e^{2a}\left(\int \frac{e^{2bx}}{d^4x^4+4c\,d^3x^3+6c^2d^2x^2+4c^3dx+c^4}dx\right)c^3d+9e^{2a}\left(\int \frac{e^{2bx}}{d^4x^4+4c\,d^3x^3+6c^2d^2x^2+4c^3dx+c^4}dx\right)c^2d^2x+9e^{2a}\left(\int \frac{}{d^4x^4+}\right.}{}$$

input
```
int(sinh(b*x+a)^2/(d*x+c)^4,x)
```

output
```
(3*e**(2*a)*int(e**(2*b*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**
3*x**3 + d**4*x**4),x)*c**3*d + 9*e**(2*a)*int(e**(2*b*x)/(c**4 + 4*c**3*d
*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*c**2*d**2*x + 9*e**(
2*a)*int(e**(2*b*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3
+ d**4*x**4),x)*c*d**3*x**2 + 3*e**(2*a)*int(e**(2*b*x)/(c**4 + 4*c**3*d*x
 + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*d**4*x**3 + 3*int(1/(e
**(2*a + 2*b*x)*c**4 + 4*e**(2*a + 2*b*x)*c**3*d*x + 6*e**(2*a + 2*b*x)*c*
*2*d**2*x**2 + 4*e**(2*a + 2*b*x)*c*d**3*x**3 + e**(2*a + 2*b*x)*d**4*x**4
),x)*c**3*d + 9*int(1/(e**(2*a + 2*b*x)*c**4 + 4*e**(2*a + 2*b*x)*c**3*d*x
 + 6*e**(2*a + 2*b*x)*c**2*d**2*x**2 + 4*e**(2*a + 2*b*x)*c*d**3*x**3 + e*
*(2*a + 2*b*x)*d**4*x**4),x)*c**2*d**2*x + 9*int(1/(e**(2*a + 2*b*x)*c**4
+ 4*e**(2*a + 2*b*x)*c**3*d*x + 6*e**(2*a + 2*b*x)*c**2*d**2*x**2 + 4*e**(
2*a + 2*b*x)*c*d**3*x**3 + e**(2*a + 2*b*x)*d**4*x**4),x)*c*d**3*x**2 + 3*
int(1/(e**(2*a + 2*b*x)*c**4 + 4*e**(2*a + 2*b*x)*c**3*d*x + 6*e**(2*a + 2
*b*x)*c**2*d**2*x**2 + 4*e**(2*a + 2*b*x)*c*d**3*x**3 + e**(2*a + 2*b*x)*d
**4*x**4),x)*d**4*x**3 + 2)/(12*d*(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**
3*x**3))
```

3.16 $\int (c+dx)^4 \sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 225

$$\int (c+dx)^4 \sinh^3(a+bx)\,dx = -\frac{488d^4\cosh(a+bx)}{27b^5} - \frac{80d^2(c+dx)^2\cosh(a+bx)}{9b^3} - \frac{2(c+dx)^4\cosh(a+bx)}{3b} + \frac{8d^4\cosh^3(a+bx)}{81b^5} + \frac{160d^3(c+dx)\sinh(a+bx)}{9b^4} + \frac{8d(c+dx)^3\sinh(a+bx)}{3b^2} + \frac{4d^2(c+dx)^2\cosh(a+bx)\sinh^2(a+bx)}{9b^3} + \frac{(c+dx)^4\cosh(a+bx)\sinh^2(a+bx)}{3b} - \frac{8d^3(c+dx)\sinh^3(a+bx)}{27b^4} - \frac{4d(c+dx)^3\sinh^3(a+bx)}{9b^2}$$

output
```
-488/27*d^4*cosh(b*x+a)/b^5-80/9*d^2*(d*x+c)^2*cosh(b*x+a)/b^3-2/3*(d*x+c)
^4*cosh(b*x+a)/b+8/81*d^4*cosh(b*x+a)^3/b^5+160/9*d^3*(d*x+c)*sinh(b*x+a)/
b^4+8/3*d*(d*x+c)^3*sinh(b*x+a)/b^2+4/9*d^2*(d*x+c)^2*cosh(b*x+a)*sinh(b*x
+a)^2/b^3+1/3*(d*x+c)^4*cosh(b*x+a)*sinh(b*x+a)^2/b-8/27*d^3*(d*x+c)*sinh(
b*x+a)^3/b^4-4/9*d*(d*x+c)^3*sinh(b*x+a)^3/b^2
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.67

$$\int (c+dx)^4 \sinh^3(a+bx)\,dx$$
$$= \frac{-243(24d^4+12b^2d^2(c+dx)^2+b^4(c+dx)^4)\cosh(a+bx)+(8d^4+36b^2d^2(c+dx)^2+27b^4(c+dx)^4)\,\mathrm{co}}{}$$

input
```
Integrate[(c + d*x)^4*Sinh[a + b*x]^3,x]
```

output
```
(-243*(24*d^4 + 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cosh[a + b*x] +
(8*d^4 + 36*b^2*d^2*(c + d*x)^2 + 27*b^4*(c + d*x)^4)*Cosh[3*(a + b*x)] -
24*b*d*(c + d*x)*(-242*d^2 - 39*b^2*(c + d*x)^2 + (2*d^2 + 3*b^2*(c + d*x)
^2)*Cosh[2*(a + b*x)])*Sinh[a + b*x])/(324*b^5)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \sinh^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i(c+dx)^4 \sin(ia+ibx)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int (c+dx)^4 \sin(ia+ibx)^3dx$$

$$\Big\downarrow \text{3792}$$

$$i\left(\frac{4d^2\int -i(c+dx)^2\sinh^3(a+bx)dx}{3b^2}+\frac{2}{3}\int i(c+dx)^4\sinh(a+bx)dx+\frac{4id(c+dx)^3\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx}{}\right.$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{4id^2\int(c+dx)^2\sinh^3(a+bx)dx}{3b^2}+\frac{2}{3}i\int(c+dx)^4\sinh(a+bx)dx+\frac{4id(c+dx)^3\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)^{\cdot}}{\ }\right.$$

$\downarrow$ 3042

$$i\left(-\frac{4id^2\int i(c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}i\int-i(c+dx)^4\sin(ia+ibx)dx+\frac{4id(c+dx)^3\sinh^3(a+bx)}{9b^2}-\frac{i(c+}{\ }\right.$$

$\downarrow$ 26

$$i\left(\frac{4d^2\int(c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\int(c+dx)^4\sin(ia+ibx)dx+\frac{4id(c+dx)^3\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)^4\mathrm{si}}{\ }\right.$$

$\downarrow$ 3777

$$i\left(\frac{4d^2\int(c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\int(c+dx)^3\cosh(a+bx)dx}{b}\right)+\frac{4id(c+d}{\ }\right.$$

$\downarrow$ 3042

$$i\left(\frac{4d^2\int(c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\int(c+dx)^3\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)+\frac{4id}{\ }\right.$$

$\downarrow$ 3777

$$i\left(\frac{4d^2\int(c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}-\frac{3id\int-i(c+dx)^2\sinh(a+}{b}\right.}{b}\right.\right.$$

$\downarrow$ 26

$$i\left(\frac{4d^2\int(c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}-\frac{3d\int(c+dx)^2\sinh(a+bx)a}{b}\right.}{b}\right.\right.$$

$\downarrow$ 3042

$$i\left(\frac{4d^2\int(c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}-\frac{3d\int-i(c+dx)^2\sin(ia+ib}{b}\right.}{b}\right.\right.$$

$\downarrow$ 26

$$i\left(\frac{4d^2\int(c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\int(c+dx)^2\sin(ia+ibx)}{b}\right.}{b}\right.\right.$$

$\downarrow$ 3777

$$i\left(\frac{4d^2\int(c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}\right.}{}\right.}{b}\right.\right.$$

$\downarrow$ 3042

$$i\left(\frac{4d^2\int(c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}\right.}{}\right.}{b}\right.\right.$$

$\downarrow$ 3777

$$i\left(\frac{4d^2\int(c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}\right.}{}\right.}{b}\right.\right.$$

$\downarrow$ 26

$$i\left(\frac{4d^2\int (c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}\right.}{}\right.}{b}\right.\right.$$

$\downarrow$ 3042

$$i\left(\frac{4d^2\int (c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}\right.}{}\right.}{b}\right.\right.$$

$\downarrow$ 26

$$i\left(\frac{4d^2\int (c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}\right.}{}\right.}{b}\right.\right.$$

$\downarrow$ 3118

$$i\left(\frac{4d^2\int(c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{4id(c+dx)^3\sinh^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sin}{b}\right.}{}\right.\right.$$

$\downarrow$ 3792

$$i\left(\frac{4d^2\left(\frac{2d^2\int -i\sinh^3(a+bx)dx}{9b^2}+\frac{2}{3}\int i(c+dx)^2\sinh(a+bx)dx+\frac{2id(c+dx)\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)^2\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{3b^2}\right.$$

$\downarrow$ 26

$$i\left(\frac{4d^2\left(-\frac{2id^2\int\sinh^3(a+bx)dx}{9b^2}+\frac{2}{3}i\int(c+dx)^2\sinh(a+bx)dx+\frac{2id(c+dx)\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)^2\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{3b^2}\right.$$

$\downarrow$ 3042

$$i\left(\frac{4d^2\left(-\frac{2id^2\int i\sin(ia+ibx)^3dx}{9b^2}+\frac{2}{3}i\int -i(c+dx)^2\sin(ia+ibx)dx+\frac{2id(c+dx)\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)^2\sinh^2(a+bx)\cosh(a+\right.}{3b^2}\right.$$

$\downarrow$ 26

$$i\left(\frac{4d^2\left(\frac{2d^2\int\sin(ia+ibx)^3dx}{9b^2}+\frac{2}{3}\int(c+dx)^2\sin(ia+ibx)dx+\frac{2id(c+dx)\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)^2\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{3b^2}+\right.$$

$\downarrow$ 3113

$$i\left(\frac{4d^2\left(\frac{2id^2\int(1-\cosh^2(a+bx))d\cosh(a+bx)}{9b^3}+\frac{2}{3}\int(c+dx)^2\sin(ia+ibx)dx+\frac{2id(c+dx)\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)^2\sinh^2(a+bx)\,c}{3b}\right.}{3b^2}\right.$$

$\downarrow$ 2009

$$i\left(\frac{4d^2\left(\frac{2}{3}\int (c+dx)^2\sin(ia+ibx)dx+\frac{2id^2\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)\right)}{9b^3}+\frac{2id(c+dx)\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)^2\sinh^2(a+bx)\cosh}{3b}\right.}{3b^2}\right.$$

$\downarrow$ 3777

$$i\left(\frac{4d^2\left(\frac{2}{3}\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\int (c+dx)\cosh(a+bx)dx}{b}\right)+\frac{2id^2\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)\right)}{9b^3}+\frac{2id(c+dx)\sinh^3(a+bx)}{9b^2}-\frac{i(c+}{}\right.}{3b^2}\right.$$

$\downarrow$ 3042

$$i\left(\frac{4d^2\left(\frac{2}{3}\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\int (c+dx)\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)+\frac{2id^2\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)\right)}{9b^3}+\frac{2id(c+dx)\sinh^3(a+bx)}{9b^2}-\right.}{3b^2}\right.$$

$\downarrow$ 3777

$$i\left(\frac{4d^2\left(\frac{2}{3}\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{id\int -i\sinh(a+bx)dx}{b}\right)}{b}\right)+\frac{2id^2\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)\right)}{9b^3}+\frac{2id(c+dx)\sin}{9b^2}\right.}{3b^2}\right.$$

$\downarrow$ 26

$$i\left(\frac{4d^2\left(\frac{2}{3}\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\int \sinh(a+bx)dx}{b}\right)}{b}\right)+\frac{2id^2\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)\right)}{9b^3}+\frac{2id(c+dx)\sinh^3}{9b^2}\right.}{3b^2}\right.$$

$\downarrow$ 3042

$$i\left(\frac{4d^2\left(\frac{2}{3}\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\int -i\sin(ia+ibx)dx}{b}\right)}{b}\right)+\frac{2id^2\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)\right)}{9b^3}+\frac{2id(c+dx)\sin}{9b^2}\right.}{3b^2}\right.$$

input
```
Int[(c + d*x)^4*Sinh[a + b*x]^3,x]
```

output
```
$Aborted
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.80

method	result
parallelrisch	$\frac{\left(27(dx+c)^4b^4+36d^2(dx+c)^2b^2+8d^4\right)\cosh(3bx+3a)-36bd\left((dx+c)^2b^2+\frac{2d^2}{3}\right)(dx+c)\sinh(3bx+3a)+\left(-243(dx+c)^4b\right.}{3}$
risch	$\frac{(27d^4x^4b^4+108b^4c\,d^3x^3+162b^4c^2d^2x^2-36b^3d^4x^3+108b^4c^3dx-108b^3c\,d^3x^2+27b^4c^4-108b^3c^2d^2x+36b^2d^4x^2-36b^3c^3}{648b^5}$
orering	$-\frac{16d(135b^6d^6x^6+810b^6c\,d^5x^5+2025b^6c^2d^4x^4+2700b^6c^3d^3x^3+2025b^6c^4d^2x^2+891b^4d^6x^4+810b^6c^5dx+3564b^4c\,d^5x}{243b^8}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((d*x+c)^4*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/324*((27*(d*x+c)^4*b^4+36*d^2*(d*x+c)^2*b^2+8*d^4)*cosh(3*b*x+3*a)-36*b*
d*((d*x+c)^2*b^2+2/3*d^2)*(d*x+c)*sinh(3*b*x+3*a)+(-243*(d*x+c)^4*b^4-2916
*d^2*(d*x+c)^2*b^2-5832*d^4)*cosh(b*x+a)+972*b*d*((d*x+c)^2*b^2+6*d^2)*(d*
x+c)*sinh(b*x+a)-216*b^4*c^4-2880*b^2*c^2*d^2-5824*d^4)/b^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. 2(205) = 410.

Time = 0.09 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.35

$$\int (c+dx)^4 \sinh^3(a+bx)\,dx$$
$$= \frac{(27\,b^4d^4x^4+108\,b^4cd^3x^3+27\,b^4c^4+36\,b^2c^2d^2+8\,d^4+18\,(9\,b^4c^2d^2+2\,b^2d^4)x^2+36\,(3\,b^4c^3d+2\,b^2cd^3)x}{}$$

input
```
integrate((d*x+c)^4*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/324*((27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 + 36*b^2*c^2*d^2 +
 8*d^4 + 18*(9*b^4*c^2*d^2 + 2*b^2*d^4)*x^2 + 36*(3*b^4*c^3*d + 2*b^2*c*d^
3)*x)*cosh(b*x + a)^3 + 3*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4
 + 36*b^2*c^2*d^2 + 8*d^4 + 18*(9*b^4*c^2*d^2 + 2*b^2*d^4)*x^2 + 36*(3*b^4
*c^3*d + 2*b^2*c*d^3)*x)*cosh(b*x + a)*sinh(b*x + a)^2 - 12*(3*b^3*d^4*x^3
 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d + 2*b*c*d^3 + (9*b^3*c^2*d^2 + 2*b*d^4)*x
)*sinh(b*x + a)^3 - 243*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 + 12*b^2*
c^2*d^2 + 24*d^4 + 6*(b^4*c^2*d^2 + 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d + 6*b^2*
c*d^3)*x)*cosh(b*x + a) + 36*(27*b^3*d^4*x^3 + 81*b^3*c*d^3*x^2 + 27*b^3*c
^3*d + 162*b*c*d^3 - (3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d + 2*b*
c*d^3 + (9*b^3*c^2*d^2 + 2*b*d^4)*x)*cosh(b*x + a)^2 + 81*(b^3*c^2*d^2 + 2
*b*d^4)*x)*sinh(b*x + a))/b^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(226) = 452$.

Time = 0.65 (sec) , antiderivative size = 772, normalized size of antiderivative = 3.43

$$\int (c + dx)^4 \sinh^3(a + bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**4*sinh(b*x+a)**3,x)
```

output
```
Piecewise((c**4*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*c**4*cosh(a + b*x)**3
/(3*b) + 4*c**3*d*x*sinh(a + b*x)**2*cosh(a + b*x)/b - 8*c**3*d*x*cosh(a +
 b*x)**3/(3*b) + 6*c**2*d**2*x**2*sinh(a + b*x)**2*cosh(a + b*x)/b - 4*c**
2*d**2*x**2*cosh(a + b*x)**3/b + 4*c*d**3*x**3*sinh(a + b*x)**2*cosh(a + b
*x)/b - 8*c*d**3*x**3*cosh(a + b*x)**3/(3*b) + d**4*x**4*sinh(a + b*x)**2*
cosh(a + b*x)/b - 2*d**4*x**4*cosh(a + b*x)**3/(3*b) - 28*c**3*d*sinh(a +
b*x)**3/(9*b**2) + 8*c**3*d*sinh(a + b*x)*cosh(a + b*x)**2/(3*b**2) - 28*c
**2*d**2*x*sinh(a + b*x)**3/(3*b**2) + 8*c**2*d**2*x*sinh(a + b*x)*cosh(a
+ b*x)**2/b**2 - 28*c*d**3*x**2*sinh(a + b*x)**3/(3*b**2) + 8*c*d**3*x**2*
sinh(a + b*x)*cosh(a + b*x)**2/b**2 - 28*d**4*x**3*sinh(a + b*x)**3/(9*b**
2) + 8*d**4*x**3*sinh(a + b*x)*cosh(a + b*x)**2/(3*b**2) + 28*c**2*d**2*si
nh(a + b*x)**2*cosh(a + b*x)/(3*b**3) - 80*c**2*d**2*cosh(a + b*x)**3/(9*b
**3) + 56*c*d**3*x*sinh(a + b*x)**2*cosh(a + b*x)/(3*b**3) - 160*c*d**3*x*
cosh(a + b*x)**3/(9*b**3) + 28*d**4*x**2*sinh(a + b*x)**2*cosh(a + b*x)/(3
*b**3) - 80*d**4*x**2*cosh(a + b*x)**3/(9*b**3) - 488*c*d**3*sinh(a + b*x)
**3/(27*b**4) + 160*c*d**3*sinh(a + b*x)*cosh(a + b*x)**2/(9*b**4) - 488*d
**4*x*sinh(a + b*x)**3/(27*b**4) + 160*d**4*x*sinh(a + b*x)*cosh(a + b*x)*
*2/(9*b**4) + 488*d**4*sinh(a + b*x)**2*cosh(a + b*x)/(27*b**5) - 1456*d**
4*cosh(a + b*x)**3/(81*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2
*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sinh(a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 639 vs. 2(205) = 410.

Time = 0.08 (sec) , antiderivative size = 639, normalized size of antiderivative = 2.84

$$\int (c+dx)^4 \sinh^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/18*c^3*d*((3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 - 27*(b*x*e^a - e^a)*e
^(b*x)/b^2 - 27*(b*x + 1)*e^(-b*x - a)/b^2 + (3*b*x + 1)*e^(-3*b*x - 3*a)/
b^2) + 1/24*c^4*(e^(3*b*x + 3*a)/b - 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b +
e^(-3*b*x - 3*a)/b) + 1/36*c^2*d^2*((9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2
*e^(3*a))*e^(3*b*x)/b^3 - 81*(b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3
 - 81*(b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 + (9*b^2*x^2 + 6*b*x + 2)*e^(
-3*b*x - 3*a)/b^3) + 1/54*c*d^3*((9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) +
6*b*x*e^(3*a) - 2*e^(3*a))*e^(3*b*x)/b^4 - 81*(b^3*x^3*e^a - 3*b^2*x^2*e^a
 + 6*b*x*e^a - 6*e^a)*e^(b*x)/b^4 - 81*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e
^(-b*x - a)/b^4 + (9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^4
) + 1/648*d^4*((27*b^4*x^4*e^(3*a) - 36*b^3*x^3*e^(3*a) + 36*b^2*x^2*e^(3*
a) - 24*b*x*e^(3*a) + 8*e^(3*a))*e^(3*b*x)/b^5 - 243*(b^4*x^4*e^a - 4*b^3*
x^3*e^a + 12*b^2*x^2*e^a - 24*b*x*e^a + 24*e^a)*e^(b*x)/b^5 - 243*(b^4*x^4
 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*e^(-b*x - a)/b^5 + (27*b^4*x^4 +
36*b^3*x^3 + 36*b^2*x^2 + 24*b*x + 8)*e^(-3*b*x - 3*a)/b^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(205) = 410$.

Time = 0.18 (sec) , antiderivative size = 654, normalized size of antiderivative = 2.91

$$\int (c+dx)^4 \sinh^3(a+bx)\,dx$$

$$= \frac{(27\,b^4d^4x^4 + 108\,b^4cd^3x^3 + 162\,b^4c^2d^2x^2 - 36\,b^3d^4x^3 + 108\,b^4c^3dx - 108\,b^3cd^3x^2 + 27\,b^4c^4 - 108\,b^3c^2d^2x}{648\,b^5}$$

$$- \frac{3\,(b^4d^4x^4 + 4\,b^4cd^3x^3 + 6\,b^4c^2d^2x^2 - 4\,b^3d^4x^3 + 4\,b^4c^3dx - 12\,b^3cd^3x^2 + b^4c^4 - 12\,b^3c^2d^2x + 12\,b^2d^4x^2}{8\,b^5}$$

$$- \frac{3\,(b^4d^4x^4 + 4\,b^4cd^3x^3 + 6\,b^4c^2d^2x^2 + 4\,b^3d^4x^3 + 4\,b^4c^3dx + 12\,b^3cd^3x^2 + b^4c^4 + 12\,b^3c^2d^2x + 12\,b^2d^4x^2}{8\,b^5}$$

$$+ \frac{(27\,b^4d^4x^4 + 108\,b^4cd^3x^3 + 162\,b^4c^2d^2x^2 + 36\,b^3d^4x^3 + 108\,b^4c^3dx + 108\,b^3cd^3x^2 + 27\,b^4c^4 + 108\,b^3c^2d}{648\,b^5}$$

input
```
integrate((d*x+c)^4*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/648*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 - 36*b^3*d
^4*x^3 + 108*b^4*c^3*d*x - 108*b^3*c*d^3*x^2 + 27*b^4*c^4 - 108*b^3*c^2*d^
2*x + 36*b^2*d^4*x^2 - 36*b^3*c^3*d + 72*b^2*c*d^3*x + 36*b^2*c^2*d^2 - 24
*b*d^4*x - 24*b*c*d^3 + 8*d^4)*e^(3*b*x + 3*a)/b^5 - 3/8*(b^4*d^4*x^4 + 4*
b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 - 4*b^3*d^4*x^3 + 4*b^4*c^3*d*x - 12*b^3
*c*d^3*x^2 + b^4*c^4 - 12*b^3*c^2*d^2*x + 12*b^2*d^4*x^2 - 4*b^3*c^3*d + 2
4*b^2*c*d^3*x + 12*b^2*c^2*d^2 - 24*b*d^4*x - 24*b*c*d^3 + 24*d^4)*e^(b*x
+ a)/b^5 - 3/8*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^3*
d^4*x^3 + 4*b^4*c^3*d*x + 12*b^3*c*d^3*x^2 + b^4*c^4 + 12*b^3*c^2*d^2*x +
12*b^2*d^4*x^2 + 4*b^3*c^3*d + 24*b^2*c*d^3*x + 12*b^2*c^2*d^2 + 24*b*d^4*
x + 24*b*c*d^3 + 24*d^4)*e^(-b*x - a)/b^5 + 1/648*(27*b^4*d^4*x^4 + 108*b^
4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 + 36*b^3*d^4*x^3 + 108*b^4*c^3*d*x + 108
*b^3*c*d^3*x^2 + 27*b^4*c^4 + 108*b^3*c^2*d^2*x + 36*b^2*d^4*x^2 + 36*b^3*
c^3*d + 72*b^2*c*d^3*x + 36*b^2*c^2*d^2 + 24*b*d^4*x + 24*b*c*d^3 + 8*d^4)
*e^(-3*b*x - 3*a)/b^5
```

Mupad [B] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.36

$$\int (c+dx)^4 \sinh^3(a+bx)\,dx$$
$$= \frac{\cosh(a+b\,x)\,\sinh(a+b\,x)^2\,(27\,b^4\,c^4+252\,b^2\,c^2\,d^2+488\,d^4)}{27\,b^5} - \frac{2\cosh(a+b\,x)^3\,(27\,b^4\,c^4+360\,b^2\,c^2\,d^2+728\,d^4)}{81\,b^5} - \frac{4\sinh(a+b\,x)^3\,(21\,b^2\,c^3\,d+122\,c\,d^3)}{27\,b^4} + \frac{8\cosh(a+b\,x)^2\sinh(a+b\,x)\,(3\,b^2\,c^3\,d+20\,c\,d^3)}{9\,b^4} - \frac{2\,d^4\,x^4\cosh(a+b\,x)^3}{3\,b} - \frac{8\,x\cosh(a+b\,x)^3\,(3\,b^2\,c^3\,d+20\,c\,d^3)}{9\,b^3} - \frac{28\,d^4\,x^3\sinh(a+b\,x)^3}{9\,b^2} - \frac{4\,x\sinh(a+b\,x)^3\,(63\,b^2\,c^2\,d^2+122\,d^4)}{27\,b^4} - \frac{4\,x^2\cosh(a+b\,x)^3\,(9\,b^2\,c^2\,d^2+20\,d^4)}{9\,b^3} + \frac{2\,x^2\cosh(a+b\,x)\,\sinh(a+b\,x)^2\,(9\,b^2\,c^2\,d^2+14\,d^4)}{3\,b^3} - \frac{8\,c\,d^3\,x^3\cosh(a+b\,x)^3}{3\,b} + \frac{d^4\,x^4\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{b} + \frac{8\,d^4\,x^3\cosh(a+b\,x)^2\sinh(a+b\,x)}{3\,b^2} - \frac{28\,c\,d^3\,x^2\sinh(a+b\,x)^3}{3\,b^2} + \frac{8\,x\cosh(a+b\,x)^2\sinh(a+b\,x)\,(9\,b^2\,c^2\,d^2+20\,d^4)}{9\,b^4} + \frac{4\,x\cosh(a+b\,x)\,\sinh(a+b\,x)^2\,(3\,b^2\,c^3\,d+14\,c\,d^3)}{3\,b^3} + \frac{4\,c\,d^3\,x^3\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{b} + \frac{8\,c\,d^3\,x^2\cosh(a+b\,x)^2\sinh(a+b\,x)}{b^2}$$

input
```
int(sinh(a + b*x)^3*(c + d*x)^4,x)
```

output
```
(cosh(a + b*x)*sinh(a + b*x)^2*(488*d^4 + 27*b^4*c^4 + 252*b^2*c^2*d^2))/(
27*b^5) - (2*cosh(a + b*x)^3*(728*d^4 + 27*b^4*c^4 + 360*b^2*c^2*d^2))/(81
*b^5) - (4*sinh(a + b*x)^3*(122*c*d^3 + 21*b^2*c^3*d))/(27*b^4) + (8*cosh(
a + b*x)^2*sinh(a + b*x)*(20*c*d^3 + 3*b^2*c^3*d))/(9*b^4) - (2*d^4*x^4*co
sh(a + b*x)^3)/(3*b) - (8*x*cosh(a + b*x)^3*(20*c*d^3 + 3*b^2*c^3*d))/(9*b
^3) - (28*d^4*x^3*sinh(a + b*x)^3)/(9*b^2) - (4*x*sinh(a + b*x)^3*(122*d^4
 + 63*b^2*c^2*d^2))/(27*b^4) - (4*x^2*cosh(a + b*x)^3*(20*d^4 + 9*b^2*c^2*
d^2))/(9*b^3) + (2*x^2*cosh(a + b*x)*sinh(a + b*x)^2*(14*d^4 + 9*b^2*c^2*d
^2))/(3*b^3) - (8*c*d^3*x^3*cosh(a + b*x)^3)/(3*b) + (d^4*x^4*cosh(a + b*x
)*sinh(a + b*x)^2)/b + (8*d^4*x^3*cosh(a + b*x)^2*sinh(a + b*x))/(3*b^2) -
 (28*c*d^3*x^2*sinh(a + b*x)^3)/(3*b^2) + (8*x*cosh(a + b*x)^2*sinh(a + b*
x)*(20*d^4 + 9*b^2*c^2*d^2))/(9*b^4) + (4*x*cosh(a + b*x)*sinh(a + b*x)^2*
(14*c*d^3 + 3*b^2*c^3*d))/(3*b^3) + (4*c*d^3*x^3*cosh(a + b*x)*sinh(a + b*
x)^2)/b + (8*c*d^3*x^2*cosh(a + b*x)^2*sinh(a + b*x))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1068, normalized size of antiderivative = 4.75

$$\int (c+dx)^4 \sinh^3(a+bx)\,dx$$

$$= \frac{-5832e^{2bx+2a}d^4 - 5832e^{4bx+4a}d^4 + 27b^4c^4 + 108e^{6bx+6a}b^4c^3dx + 162e^{6bx+6a}b^4c^2d^2x^2 + 108e^{6bx+6a}b^4c\,d^3x^3}{}$$

input
```
int((d*x+c)^4*sinh(b*x+a)^3,x)
```

output
```
(27*e**(6*a + 6*b*x)*b**4*c**4 + 108*e**(6*a + 6*b*x)*b**4*c**3*d*x + 162*
e**(6*a + 6*b*x)*b**4*c**2*d**2*x**2 + 108*e**(6*a + 6*b*x)*b**4*c*d**3*x*
*3 + 27*e**(6*a + 6*b*x)*b**4*d**4*x**4 - 36*e**(6*a + 6*b*x)*b**3*c**3*d
- 108*e**(6*a + 6*b*x)*b**3*c**2*d**2*x - 108*e**(6*a + 6*b*x)*b**3*c*d**3
*x**2 - 36*e**(6*a + 6*b*x)*b**3*d**4*x**3 + 36*e**(6*a + 6*b*x)*b**2*c**2
*d**2 + 72*e**(6*a + 6*b*x)*b**2*c*d**3*x + 36*e**(6*a + 6*b*x)*b**2*d**4*
x**2 - 24*e**(6*a + 6*b*x)*b*c*d**3 - 24*e**(6*a + 6*b*x)*b*d**4*x + 8*e**
(6*a + 6*b*x)*d**4 - 243*e**(4*a + 4*b*x)*b**4*c**4 - 972*e**(4*a + 4*b*x)
*b**4*c**3*d*x - 1458*e**(4*a + 4*b*x)*b**4*c**2*d**2*x**2 - 972*e**(4*a +
 4*b*x)*b**4*c*d**3*x**3 - 243*e**(4*a + 4*b*x)*b**4*d**4*x**4 + 972*e**(4
*a + 4*b*x)*b**3*c**3*d + 2916*e**(4*a + 4*b*x)*b**3*c**2*d**2*x + 2916*e*
*(4*a + 4*b*x)*b**3*c*d**3*x**2 + 972*e**(4*a + 4*b*x)*b**3*d**4*x**3 - 29
16*e**(4*a + 4*b*x)*b**2*c**2*d**2 - 5832*e**(4*a + 4*b*x)*b**2*c*d**3*x -
 2916*e**(4*a + 4*b*x)*b**2*d**4*x**2 + 5832*e**(4*a + 4*b*x)*b*c*d**3 + 5
832*e**(4*a + 4*b*x)*b*d**4*x - 5832*e**(4*a + 4*b*x)*d**4 - 243*e**(2*a +
 2*b*x)*b**4*c**4 - 972*e**(2*a + 2*b*x)*b**4*c**3*d*x - 1458*e**(2*a + 2*
b*x)*b**4*c**2*d**2*x**2 - 972*e**(2*a + 2*b*x)*b**4*c*d**3*x**3 - 243*e**
(2*a + 2*b*x)*b**4*d**4*x**4 - 972*e**(2*a + 2*b*x)*b**3*c**3*d - 2916*e**
(2*a + 2*b*x)*b**3*c**2*d**2*x - 2916*e**(2*a + 2*b*x)*b**3*c*d**3*x**2 -
972*e**(2*a + 2*b*x)*b**3*d**4*x**3 - 2916*e**(2*a + 2*b*x)*b**2*c**2*d...
```

3.17 $\int (c+dx)^3 \sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 175

$$\begin{aligned}\int (c+dx)^3 \sinh^3(a+bx)\,dx = &-\frac{40d^2(c+dx)\cosh(a+bx)}{9b^3}-\frac{2(c+dx)^3\cosh(a+bx)}{3b}\\&+\frac{40d^3\sinh(a+bx)}{9b^4}+\frac{2d(c+dx)^2\sinh(a+bx)}{b^2}\\&+\frac{2d^2(c+dx)\cosh(a+bx)\sinh^2(a+bx)}{9b^3}\\&+\frac{(c+dx)^3\cosh(a+bx)\sinh^2(a+bx)}{3b}\\&-\frac{2d^3\sinh^3(a+bx)}{27b^4}-\frac{d(c+dx)^2\sinh^3(a+bx)}{3b^2}\end{aligned}$$

output
```
-40/9*d^2*(d*x+c)*cosh(b*x+a)/b^3-2/3*(d*x+c)^3*cosh(b*x+a)/b+40/9*d^3*sin
h(b*x+a)/b^4+2*d*(d*x+c)^2*sinh(b*x+a)/b^2+2/9*d^2*(d*x+c)*cosh(b*x+a)*sin
h(b*x+a)^2/b^3+1/3*(d*x+c)^3*cosh(b*x+a)*sinh(b*x+a)^2/b-2/27*d^3*sinh(b*x
+a)^3/b^4-1/3*d*(d*x+c)^2*sinh(b*x+a)^3/b^2
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.73

$$\int (c+dx)^3 \sinh^3(a+bx)\,dx$$
$$= \frac{-162b(c+dx)\left(6d^2+b^2(c+dx)^2\right)\cosh(a+bx)+6b(c+dx)\left(2d^2+3b^2(c+dx)^2\right)\cosh(3(a+bx))-4d}{216b^4}$$

input
```
Integrate[(c + d*x)^3*Sinh[a + b*x]^3,x]
```

output
```
(-162*b*(c + d*x)*(6*d^2 + b^2*(c + d*x)^2)*Cosh[a + b*x] + 6*b*(c + d*x)*
(2*d^2 + 3*b^2*(c + d*x)^2)*Cosh[3*(a + b*x)] - 4*d*(-242*d^2 - 117*b^2*(c
 + d*x)^2 + (2*d^2 + 9*b^2*(c + d*x)^2)*Cosh[2*(a + b*x)])*Sinh[a + b*x])/
(216*b^4)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.16 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.41, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 1.375$, Rules used = {3042, 26, 3792, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117, 3791, 26, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sinh^3(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int i(c+dx)^3 \sin(ia+ibx)^3 dx$$
$$\Big\downarrow \text{26}$$
$$i\int (c+dx)^3 \sin(ia+ibx)^3 dx$$

$$\Big\downarrow \ 3792$$

$$i\left(\frac{2d^2\int -i(c+dx)\sinh^3(a+bx)dx}{3b^2}+\frac{2}{3}\int i(c+dx)^3\sinh(a+bx)dx+\frac{id(c+dx)^2\sinh^3(a+bx)}{3b^2}-\frac{i(c+dx)^3 s}{}\right.$$

$$\Big\downarrow \ 26$$

$$i\left(-\frac{2id^2\int (c+dx)\sinh^3(a+bx)dx}{3b^2}+\frac{2}{3}i\int (c+dx)^3\sinh(a+bx)dx+\frac{id(c+dx)^2\sinh^3(a+bx)}{3b^2}-\frac{i(c+dx)^3 s}{}\right.$$

$$\Big\downarrow \ 3042$$

$$i\left(-\frac{2id^2\int i(c+dx)\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}i\int -i(c+dx)^3\sin(ia+ibx)dx+\frac{id(c+dx)^2\sinh^3(a+bx)}{3b^2}-\frac{i(c+d}{}\right.$$

$$\Big\downarrow \ 26$$

$$i\left(\frac{2d^2\int (c+dx)\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\int (c+dx)^3\sin(ia+ibx)dx+\frac{id(c+dx)^2\sinh^3(a+bx)}{3b^2}-\frac{i(c+dx)^3\sinh}{}\right.$$

$$\Big\downarrow \ 3777$$

$$i\left(\frac{2d^2\int (c+dx)\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\int (c+dx)^2\cosh(a+bx)dx}{b}\right)+\frac{id(c+dx)}{}\right.$$

$$\Big\downarrow \ 3042$$

$$i\left(\frac{2d^2\int (c+dx)\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\int (c+dx)^2\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)+\frac{id(c}{}\right.$$

$$\Big\downarrow \ 3777$$

$$i\left(\frac{2d^2\int (c+dx)\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}-\frac{2id\int -i(c+dx)\sinh(a+bx)}{b}\right.}{b}\right.\right.$$

$$\Big\downarrow \ 26$$

$$i\left(\frac{2d^2\int (c+dx)\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}-\frac{2d\int (c+dx)\sinh(a+bx)dx}{b}\right.}{b}\right.\right.$$

$$\Big\downarrow \ 3042$$

$$i\left(\frac{2d^2\int(c+dx)\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}-\frac{2d\int-i(c+dx)\sin(ia+ibx)}{b}\right.}{b}\right.\right.$$

$\downarrow$ 26

$$i\left(\frac{2d^2\int(c+dx)\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\int(c+dx)\sin(ia+ibx)dx}{b}\right.}{b}\right.\right.$$

$\downarrow$ 3777

$$i\left(\frac{2d^2\int(c+dx)\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\right.}{b}\right.}{b}\right.\right.$$

$\downarrow$ 3042

$$i\left(\frac{2d^2\int(c+dx)\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\right.}{b}\right.}{b}\right.\right.$$

$\downarrow$ 3117

$$i\left(\frac{2d^2\int(c+dx)\sin(ia+ibx)^3dx}{3b^2}+\frac{id(c+dx)^2\sinh^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(}{b}\right.}{}\right.\right.$$

$\downarrow$ 3791

$$i\left(\frac{2d^2\left(\frac{2}{3}\int i(c+dx)\sinh(a+bx)dx+\frac{id\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{3b^2}+\frac{id(c+dx)^2\sinh^3(a+bx)}{3b^2}-\right.$$

$$\Big\downarrow \ 26$$

$$i\left(\frac{2d^2\left(\frac{2}{3}i\int(c+dx)\sinh(a+bx)dx+\frac{id\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{3b^2}+\frac{id(c+dx)^2\sinh^3(a+bx)}{3b^2}+\right.$$

$$\Big\downarrow \ 3042$$

$$i\left(\frac{2d^2\left(\frac{2}{3}i\int-i(c+dx)\sin(ia+ibx)dx+\frac{id\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{3b^2}+\frac{id(c+dx)^2\sinh^3(a+bx}{3b^2}\right.$$

$$\Big\downarrow \ 26$$

$$i\left(\frac{2d^2\left(\frac{2}{3}\int(c+dx)\sin(ia+ibx)dx+\frac{id\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{3b^2}+\frac{id(c+dx)^2\sinh^3(a+bx)}{3b^2}+\right.$$

$$\Big\downarrow \ 3777$$

$$i\left(\frac{2d^2\left(\frac{2}{3}\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\int\cosh(a+bx)dx}{b}\right)+\frac{id\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{3b^2}+\frac{id(c+dx)^2\sinh^3}{3b^2}\right.$$

$$\Big\downarrow \ 3042$$

$$i\left(\frac{2d^2\left(\frac{2}{3}\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\int\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)+\frac{id\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{3b^2}+\frac{id(c+dx)^2\sin}{3b^2}\right.$$

$$\Big\downarrow \ 3117$$

$$i\left(\frac{2d^2\left(\frac{2}{3}\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\sinh(a+bx)}{b^2}\right)+\frac{id\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{3b^2}+\frac{id(c+dx)^2\sinh^3(a+\right.$$

input
```
Int[(c + d*x)^3*Sinh[a + b*x]^3,x]
```

output
```
I*(((-1/3*I)*(c + d*x)^3*Cosh[a + b*x]*Sinh[a + b*x]^2)/b + ((I/3)*d*(c +
d*x)^2*Sinh[a + b*x]^3)/b^2 + (2*d^2*(((-1/3*I)*(c + d*x)*Cosh[a + b*x]*Si
nh[a + b*x]^2)/b + ((I/9)*d*Sinh[a + b*x]^3)/b^2 + (2*((I*(c + d*x)*Cosh[a
 + b*x])/b - (I*d*Sinh[a + b*x])/b^2))/3))/(3*b^2) + (2*((I*(c + d*x)^3*Co
sh[a + b*x])/b - ((3*I)*d*(((c + d*x)^2*Sinh[a + b*x])/b + ((2*I)*d*((I*(c
 + d*x)*Cosh[a + b*x])/b - (I*d*Sinh[a + b*x])/b^2))/b))/b))/3)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.81

method	result
parallelrisch	$\frac{3b\left((dx+c)^2b^2+\frac{2d^2}{3}\right)(dx+c)\cosh(3bx+3a)-3d\left((dx+c)^2b^2+\frac{2d^2}{9}\right)\sinh(3bx+3a)-27b\left((dx+c)^2b^2+6d^2\right)(dx+c)\cosh(}{36b^4}$
risch	$\frac{(9d^3x^3b^3+27b^3c\,d^2x^2+27b^3c^2dx-9b^2d^3x^2+9b^3c^3-18b^2c\,d^2x-9b^2c^2d+6b\,d^3x+6bc\,d^2-2d^3)\mathrm{e}^{3bx+3a}}{216b^4}-\frac{3(d^3x^3b^3+3}{}$
orering	$-\frac{20d(9d^4x^4b^4+36b^4c\,d^3x^3+54b^4c^2d^2x^2+36b^4c^3dx+9b^4c^4+22b^2d^4x^2+44b^2c\,d^3x+22b^2c^2d^2-72d^4)\sinh(bx+a)^3}{27b^6(dx+c)^2}+$
derivativedivides	$\frac{\frac{d^3\left(-\frac{2(bx+a)^3\cosh(bx+a)}{3}+\frac{(bx+a)^3\cosh(bx+a)\sinh(bx+a)^2}{3}+2(bx+a)^2\sinh(bx+a)-\frac{40(bx+a)\cosh(bx+a)}{9}+\frac{40\sinh(bx+a)}{9}-\frac{(b}{}\right.}{b^3}}{}$
default	$\frac{\frac{d^3\left(-\frac{2(bx+a)^3\cosh(bx+a)}{3}+\frac{(bx+a)^3\cosh(bx+a)\sinh(bx+a)^2}{3}+2(bx+a)^2\sinh(bx+a)-\frac{40(bx+a)\cosh(bx+a)}{9}+\frac{40\sinh(bx+a)}{9}-\frac{(b}{}\right.}{b^3}}{}$

input
```
int((d*x+c)^3*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/36*(3*b*((d*x+c)^2*b^2+2/3*d^2)*(d*x+c)*cosh(3*b*x+3*a)-3*d*((d*x+c)^2*b
^2+2/9*d^2)*sinh(3*b*x+3*a)-27*b*((d*x+c)^2*b^2+6*d^2)*(d*x+c)*cosh(b*x+a)
+81*((d*x+c)^2*b^2+2*d^2)*d*sinh(b*x+a)-24*b^3*c^3-160*b*c*d^2)/b^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(161) = 322$.

Time = 0.11 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.97

$$\int (c+dx)^3 \sinh^3(a+bx)\,dx$$
$$= \frac{3\,(3\,b^3d^3x^3 + 9\,b^3cd^2x^2 + 3\,b^3c^3 + 2\,bcd^2 + (9\,b^3c^2d + 2\,bd^3)x)\cosh(bx+a)^3 + 9\,(3\,b^3d^3x^3 + 9\,b^3cd^2x^2 + \cdots}{\cdots}$$

input
```
integrate((d*x+c)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/108*(3*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 + 2*b*c*d^2 + (9*b^3
*c^2*d + 2*b*d^3)*x)*cosh(b*x + a)^3 + 9*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2
+ 3*b^3*c^3 + 2*b*c*d^2 + (9*b^3*c^2*d + 2*b*d^3)*x)*cosh(b*x + a)*sinh(b*
x + a)^2 - (9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d + 2*d^3)*sinh(b*x
 + a)^3 - 81*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 6*b*c*d^2 + 3*(b^3
*c^2*d + 2*b*d^3)*x)*cosh(b*x + a) + 3*(81*b^2*d^3*x^2 + 162*b^2*c*d^2*x +
 81*b^2*c^2*d + 162*d^3 - (9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d +
2*d^3)*cosh(b*x + a)^2)*sinh(b*x + a))/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(173) = 346$.

Time = 0.47 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.83

$$\int (c+dx)^3 \sinh^3(a+bx)\,dx$$
$$= \begin{cases} \frac{c^3\sinh^2(a+bx)\cosh(a+bx)}{b} - \frac{2c^3\cosh^3(a+bx)}{3b} + \frac{3c^2dx\sinh^2(a+bx)\cosh(a+bx)}{b} - \frac{2c^2dx\cosh^3(a+bx)}{b} + \frac{3cd^2x^2\sinh^2(a+bx)\cosh\cdots}{b} \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sinh^3(a) \end{cases}$$

input
```
integrate((d*x+c)**3*sinh(b*x+a)**3,x)
```

output
```
Piecewise((c**3*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*c**3*cosh(a + b*x)**3
/(3*b) + 3*c**2*d*x*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*c**2*d*x*cosh(a +
 b*x)**3/b + 3*c*d**2*x**2*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*c*d**2*x**
2*cosh(a + b*x)**3/b + d**3*x**3*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*d**3
*x**3*cosh(a + b*x)**3/(3*b) - 7*c**2*d*sinh(a + b*x)**3/(3*b**2) + 2*c**2
*d*sinh(a + b*x)*cosh(a + b*x)**2/b**2 - 14*c*d**2*x*sinh(a + b*x)**3/(3*b
**2) + 4*c*d**2*x*sinh(a + b*x)*cosh(a + b*x)**2/b**2 - 7*d**3*x**2*sinh(a
 + b*x)**3/(3*b**2) + 2*d**3*x**2*sinh(a + b*x)*cosh(a + b*x)**2/b**2 + 14
*c*d**2*sinh(a + b*x)**2*cosh(a + b*x)/(3*b**3) - 40*c*d**2*cosh(a + b*x)*
*3/(9*b**3) + 14*d**3*x*sinh(a + b*x)**2*cosh(a + b*x)/(3*b**3) - 40*d**3*
x*cosh(a + b*x)**3/(9*b**3) - 122*d**3*sinh(a + b*x)**3/(27*b**4) + 40*d**
3*sinh(a + b*x)*cosh(a + b*x)**2/(9*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*
x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sinh(a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(161) = 322$.

Time = 0.06 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.49

$$\int (c+dx)^3 \sinh^3(a+bx)\,dx$$

$$= \frac{1}{24}c^2d\left(\frac{(3\,bxe^{(3\,a)}-e^{(3\,a)})e^{(3\,bx)}}{b^2}-\frac{27\,(bxe^a-e^a)e^{(bx)}}{b^2}-\frac{27\,(bx+1)e^{(-bx-a)}}{b^2}+\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{b^2}\right)$$
$$+\frac{1}{24}c^3\left(\frac{e^{(3\,bx+3\,a)}}{b}-\frac{9\,e^{(bx+a)}}{b}-\frac{9\,e^{(-bx-a)}}{b}+\frac{e^{(-3\,bx-3\,a)}}{b}\right)$$
$$+\frac{1}{72}cd^2\left(\frac{(9\,b^2x^2e^{(3\,a)}-6\,bxe^{(3\,a)}+2\,e^{(3\,a)})e^{(3\,bx)}}{b^3}-\frac{81\,(b^2x^2e^a-2\,bxe^a+2\,e^a)e^{(bx)}}{b^3}-\frac{81\,(b^2x^2+2\,bx+}{b^3}\right.$$
$$+\frac{1}{216}d^3\left(\frac{(9\,b^3x^3e^{(3\,a)}-9\,b^2x^2e^{(3\,a)}+6\,bxe^{(3\,a)}-2\,e^{(3\,a)})e^{(3\,bx)}}{b^4}-\frac{81\,(b^3x^3e^a-3\,b^2x^2e^a+6\,bxe^a-6\,e^a)}{b^4}\right.$$

input
```
integrate((d*x+c)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/24*c^2*d*((3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 - 27*(b*x*e^a - e^a)*e
^(b*x)/b^2 - 27*(b*x + 1)*e^(-b*x - a)/b^2 + (3*b*x + 1)*e^(-3*b*x - 3*a)/
b^2) + 1/24*c^3*(e^(3*b*x + 3*a)/b - 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b +
e^(-3*b*x - 3*a)/b) + 1/72*c*d^2*((9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e
^(3*a))*e^(3*b*x)/b^3 - 81*(b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 -
 81*(b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 + (9*b^2*x^2 + 6*b*x + 2)*e^(-3
*b*x - 3*a)/b^3) + 1/216*d^3*((9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) + 6*b
*x*e^(3*a) - 2*e^(3*a))*e^(3*b*x)/b^4 - 81*(b^3*x^3*e^a - 3*b^2*x^2*e^a +
6*b*x*e^a - 6*e^a)*e^(b*x)/b^4 - 81*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-
b*x - a)/b^4 + (9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(161) = 322$.

Time = 0.13 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.37

$$\int (c+dx)^3 \sinh^3(a+bx)\,dx$$
$$= \frac{(9\,b^3d^3x^3+27\,b^3cd^2x^2+27\,b^3c^2dx-9\,b^2d^3x^2+9\,b^3c^3-18\,b^2cd^2x-9\,b^2c^2d+6\,bd^3x+6\,bcd^2-2\,d^3)e^{(3}}{216\,b^4}$$
$$-\frac{3\,(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx-3\,b^2d^3x^2+b^3c^3-6\,b^2cd^2x-3\,b^2c^2d+6\,bd^3x+6\,bcd^2-6\,d^3)e^{(bx+a}}{8\,b^4}$$
$$-\frac{3\,(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+3\,b^2d^3x^2+b^3c^3+6\,b^2cd^2x+3\,b^2c^2d+6\,bd^3x+6\,bcd^2+6\,d^3)e^{(-bx-}}{8\,b^4}$$
$$+\frac{(9\,b^3d^3x^3+27\,b^3cd^2x^2+27\,b^3c^2dx+9\,b^2d^3x^2+9\,b^3c^3+18\,b^2cd^2x+9\,b^2c^2d+6\,bd^3x+6\,bcd^2+2\,d^3)e}{216\,b^4}$$

input
```
integrate((d*x+c)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/216*(9*b^3*d^3*x^3 + 27*b^3*c*d^2*x^2 + 27*b^3*c^2*d*x - 9*b^2*d^3*x^2 +
 9*b^3*c^3 - 18*b^2*c*d^2*x - 9*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 - 2*d^3)
*e^(3*b*x + 3*a)/b^4 - 3/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x
- 3*b^2*d^3*x^2 + b^3*c^3 - 6*b^2*c*d^2*x - 3*b^2*c^2*d + 6*b*d^3*x + 6*b*
c*d^2 - 6*d^3)*e^(b*x + a)/b^4 - 3/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^
3*c^2*d*x + 3*b^2*d^3*x^2 + b^3*c^3 + 6*b^2*c*d^2*x + 3*b^2*c^2*d + 6*b*d^
3*x + 6*b*c*d^2 + 6*d^3)*e^(-b*x - a)/b^4 + 1/216*(9*b^3*d^3*x^3 + 27*b^3*
c*d^2*x^2 + 27*b^3*c^2*d*x + 9*b^2*d^3*x^2 + 9*b^3*c^3 + 18*b^2*c*d^2*x +
9*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 + 2*d^3)*e^(-3*b*x - 3*a)/b^4
```

Mupad [B] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.08

$$\int (c+dx)^3 \sinh^3(a+bx)\,dx = \frac{\cosh(a+b\,x)\,\sinh(a+b\,x)^2\,(3\,b^2\,c^3+14\,c\,d^2)}{3\,b^3} - \frac{\sinh(a+b\,x)^3\,(63\,b^2\,c^2\,d+122\,d^3)}{27\,b^4} - \frac{2\cosh(a+b\,x)^3\,(3\,b^2\,c^3+20\,c\,d^2)}{9\,b^3} + \frac{2\cosh(a+b\,x)^2\sinh(a+b\,x)\,(9\,b^2\,c^2\,d+20\,d^3)}{9\,b^4} - \frac{2\,x\cosh(a+b\,x)^3\,(9\,b^2\,c^2\,d+20\,d^3)}{9\,b^3} - \frac{2\,d^3\,x^3\cosh(a+b\,x)^3}{3\,b} - \frac{7\,d^3\,x^2\sinh(a+b\,x)^3}{3\,b^2} - \frac{14\,c\,d^2\,x\sinh(a+b\,x)^3}{3\,b^2} + \frac{x\cosh(a+b\,x)\,\sinh(a+b\,x)^2\,(9\,b^2\,c^2\,d+14\,d^3)}{3\,b^3} - \frac{2\,c\,d^2\,x^2\cosh(a+b\,x)^3}{b} + \frac{d^3\,x^3\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{b} + \frac{2\,d^3\,x^2\cosh(a+b\,x)^2\sinh(a+b\,x)}{b^2} + \frac{3\,c\,d^2\,x^2\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{b} + \frac{4\,c\,d^2\,x\cosh(a+b\,x)^2\sinh(a+b\,x)}{b^2}$$

input
```
int(sinh(a + b*x)^3*(c + d*x)^3,x)
```

output
```
(cosh(a + b*x)*sinh(a + b*x)^2*(14*c*d^2 + 3*b^2*c^3))/(3*b^3) - (sinh(a +
 b*x)^3*(122*d^3 + 63*b^2*c^2*d))/(27*b^4) - (2*cosh(a + b*x)^3*(20*c*d^2
+ 3*b^2*c^3))/(9*b^3) + (2*cosh(a + b*x)^2*sinh(a + b*x)*(20*d^3 + 9*b^2*c
^2*d))/(9*b^4) - (2*x*cosh(a + b*x)^3*(20*d^3 + 9*b^2*c^2*d))/(9*b^3) - (2
*d^3*x^3*cosh(a + b*x)^3)/(3*b) - (7*d^3*x^2*sinh(a + b*x)^3)/(3*b^2) - (1
4*c*d^2*x*sinh(a + b*x)^3)/(3*b^2) + (x*cosh(a + b*x)*sinh(a + b*x)^2*(14*
d^3 + 9*b^2*c^2*d))/(3*b^3) - (2*c*d^2*x^2*cosh(a + b*x)^3)/b + (d^3*x^3*c
osh(a + b*x)*sinh(a + b*x)^2)/b + (2*d^3*x^2*cosh(a + b*x)^2*sinh(a + b*x)
)/b^2 + (3*c*d^2*x^2*cosh(a + b*x)*sinh(a + b*x)^2)/b + (4*c*d^2*x*cosh(a
+ b*x)^2*sinh(a + b*x))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 678, normalized size of antiderivative = 3.87

$$\int (c+dx)^3 \sinh^3(a+bx)\,dx$$

$$= \frac{9e^{6bx+6a}b^3d^3x^3 - 9e^{6bx+6a}b^2c^2d - 9e^{6bx+6a}b^2d^3x^2 + 6e^{6bx+6a}bc\,d^2 + 6e^{6bx+6a}b\,d^3x - 81e^{2bx+2a}b^3d^3x^3 - 243}{}$$

input
```
int((d*x+c)^3*sinh(b*x+a)^3,x)
```

output
```
(9*e**(6*a + 6*b*x)*b**3*c**3 + 27*e**(6*a + 6*b*x)*b**3*c**2*d*x + 27*e**
(6*a + 6*b*x)*b**3*c*d**2*x**2 + 9*e**(6*a + 6*b*x)*b**3*d**3*x**3 - 9*e**
(6*a + 6*b*x)*b**2*c**2*d - 18*e**(6*a + 6*b*x)*b**2*c*d**2*x - 9*e**(6*a
+ 6*b*x)*b**2*d**3*x**2 + 6*e**(6*a + 6*b*x)*b*c*d**2 + 6*e**(6*a + 6*b*x)
*b*d**3*x - 2*e**(6*a + 6*b*x)*d**3 - 81*e**(4*a + 4*b*x)*b**3*c**3 - 243*
e**(4*a + 4*b*x)*b**3*c**2*d*x - 243*e**(4*a + 4*b*x)*b**3*c*d**2*x**2 - 8
1*e**(4*a + 4*b*x)*b**3*d**3*x**3 + 243*e**(4*a + 4*b*x)*b**2*c**2*d + 486
*e**(4*a + 4*b*x)*b**2*c*d**2*x + 243*e**(4*a + 4*b*x)*b**2*d**3*x**2 - 48
6*e**(4*a + 4*b*x)*b*c*d**2 - 486*e**(4*a + 4*b*x)*b*d**3*x + 486*e**(4*a
+ 4*b*x)*d**3 - 81*e**(2*a + 2*b*x)*b**3*c**3 - 243*e**(2*a + 2*b*x)*b**3*
c**2*d*x - 243*e**(2*a + 2*b*x)*b**3*c*d**2*x**2 - 81*e**(2*a + 2*b*x)*b**
3*d**3*x**3 - 243*e**(2*a + 2*b*x)*b**2*c**2*d - 486*e**(2*a + 2*b*x)*b**2
*c*d**2*x - 243*e**(2*a + 2*b*x)*b**2*d**3*x**2 - 486*e**(2*a + 2*b*x)*b*c
*d**2 - 486*e**(2*a + 2*b*x)*b*d**3*x - 486*e**(2*a + 2*b*x)*d**3 + 9*b**3
*c**3 + 27*b**3*c**2*d*x + 27*b**3*c*d**2*x**2 + 9*b**3*d**3*x**3 + 9*b**2
*c**2*d + 18*b**2*c*d**2*x + 9*b**2*d**3*x**2 + 6*b*c*d**2 + 6*b*d**3*x +
2*d**3)/(216*e**(3*a + 3*b*x)*b**4)
```

3.18 $\int (c+dx)^2 \sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 123

$$\int (c+dx)^2 \sinh^3(a+bx)\,dx = -\frac{14d^2\cosh(a+bx)}{9b^3} - \frac{2(c+dx)^2\cosh(a+bx)}{3b} + \frac{2d^2\cosh^3(a+bx)}{27b^3} + \frac{4d(c+dx)\sinh(a+bx)}{3b^2} + \frac{(c+dx)^2\cosh(a+bx)\sinh^2(a+bx)}{3b} - \frac{2d(c+dx)\sinh^3(a+bx)}{9b^2}$$

output
```
-14/9*d^2*cosh(b*x+a)/b^3-2/3*(d*x+c)^2*cosh(b*x+a)/b+2/27*d^2*cosh(b*x+a)
^3/b^3+4/3*d*(d*x+c)*sinh(b*x+a)/b^2+1/3*(d*x+c)^2*cosh(b*x+a)*sinh(b*x+a)
^2/b-2/9*d*(d*x+c)*sinh(b*x+a)^3/b^2
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.70

$$\int (c+dx)^2 \sinh^3(a+bx)\,dx$$
$$= \frac{-81(2d^2+b^2(c+dx)^2)\cosh(a+bx)+(2d^2+9b^2(c+dx)^2)\cosh(3(a+bx))-6bd(c+dx)(-27\sinh(a}{108b^3}$$

input
```
Integrate[(c + d*x)^2*Sinh[a + b*x]^3,x]
```

output
```
(-81*(2*d^2 + b^2*(c + d*x)^2)*Cosh[a + b*x] + (2*d^2 + 9*b^2*(c + d*x)^2)
*Cosh[3*(a + b*x)] - 6*b*d*(c + d*x)*(-27*Sinh[a + b*x] + Sinh[3*(a + b*x)
]))/(108*b^3)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.22, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.938$, Rules used = {3042, 26, 3792, 26, 3042, 26, 3113, 2009, 3777, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sinh^3(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int i(c+dx)^2 \sin(ia+ibx)^3 dx$$
$$\Big\downarrow \text{26}$$
$$i\int (c+dx)^2 \sin(ia+ibx)^3 dx$$
$$\Big\downarrow \text{3792}$$

$$i\left(\frac{2d^2\int -i\sinh^3(a+bx)dx}{9b^2}+\frac{2}{3}\int i(c+dx)^2\sinh(a+bx)dx+\frac{2id(c+dx)\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)^2\sinh^2(a+\right.$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{2id^2\int \sinh^3(a+bx)dx}{9b^2}+\frac{2}{3}i\int (c+dx)^2\sinh(a+bx)dx+\frac{2id(c+dx)\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)^2\sinh^2(a+\right.$$

$$\Big\downarrow \text{3042}$$

$$i\left(-\frac{2id^2\int i\sin(ia+ibx)^3dx}{9b^2}+\frac{2}{3}i\int -i(c+dx)^2\sin(ia+ibx)dx+\frac{2id(c+dx)\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)^2\sinh\right.$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{2d^2\int \sin(ia+ibx)^3dx}{9b^2}+\frac{2}{3}\int (c+dx)^2\sin(ia+ibx)dx+\frac{2id(c+dx)\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)^2\sinh^2(a+bx}{3b}\right.$$

$$\Big\downarrow \text{3113}$$

$$i\left(\frac{2id^2\int \left(1-\cosh^2(a+bx)\right)d\cosh(a+bx)}{9b^3}+\frac{2}{3}\int (c+dx)^2\sin(ia+ibx)dx+\frac{2id(c+dx)\sinh^3(a+bx)}{9b^2}-\frac{i(c}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$i\left(\frac{2}{3}\int (c+dx)^2\sin(ia+ibx)dx+\frac{2id^2\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)\right)}{9b^3}+\frac{2id(c+dx)\sinh^3(a+bx)}{9b^2}-\frac{i(c+d}{}\right.$$

$$\Big\downarrow \text{3777}$$

$$i\left(\frac{2}{3}\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\int (c+dx)\cosh(a+bx)dx}{b}\right)+\frac{2id^2\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)\right)}{9b^3}+\frac{2id(}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{2}{3}\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\int (c+dx)\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)+\frac{2id^2\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)\right)}{9b^3}\right.$$

$$\Big\downarrow \text{3777}$$

$$i\left(\frac{2}{3}\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{id\int -i\sinh(a+bx)dx}{b}\right)}{b}\right)+\frac{2id^2\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a-\right.}{9b^3}\right.$$

$\Big\downarrow$ 26

$$i\left(\frac{2}{3}\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\int \sinh(a+bx)dx}{b}\right)}{b}\right)+\frac{2id^2\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+b\right.}{9b^3}\right.$$

$\Big\downarrow$ 3042

$$i\left(\frac{2}{3}\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\int -i\sin(ia+ibx)dx}{b}\right)}{b}\right)+\frac{2id^2\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+\right.}{9b^3}\right.$$

$\Big\downarrow$ 26

$$i\left(\frac{2}{3}\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}+\frac{id\int \sin(ia+ibx)dx}{b}\right)}{b}\right)+\frac{2id^2\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+b\right.}{9b^3}\right.$$

$\Big\downarrow$ 3118

$$i\left(\frac{2id^2\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)\right)}{9b^3}+\frac{2id(c+dx)\sinh^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+d}{}\right.}{}\right.\right.$$

input
```
Int[(c + d*x)^2*Sinh[a + b*x]^3,x]
```

output
```
I*((((2*I)/9)*d^2*(Cosh[a + b*x] - Cosh[a + b*x]^3/3))/b^3 - ((I/3)*(c + d
*x)^2*Cosh[a + b*x]*Sinh[a + b*x]^2)/b + (((2*I)/9)*d*(c + d*x)*Sinh[a + b
*x]^3)/b^2 + (2*((I*(c + d*x)^2*Cosh[a + b*x])/b - ((2*I)*d*(-((d*Cosh[a +
 b*x])/b^2) + ((c + d*x)*Sinh[a + b*x])/b))/b))/3)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.86

method	result
parallelrisch	$\frac{\left(9(dx+c)^{2}b^{2}+2d^{2}\right)\cosh(3bx+3a)-6bd(dx+c)\sinh(3bx+3a)+\left(-81(dx+c)^{2}b^{2}-162d^{2}\right)\cosh(bx+a)+162bd(dx+c)\sinh}{108b^{3}}$
risch	$\frac{(9d^{2}x^{2}b^{2}+18b^{2}cdx+9b^{2}c^{2}-6b\,d^{2}x-6bcd+2d^{2})\,\mathrm{e}^{3bx+3a}}{216b^{3}}-\frac{3(d^{2}x^{2}b^{2}+2b^{2}cdx+b^{2}c^{2}-2b\,d^{2}x-2bcd+2d^{2})\,\mathrm{e}^{bx+a}}{8b^{3}}-\frac{3}{}$
derivativedivides	$\frac{\frac{d^{2}\left(-\frac{2(bx+a)^{2}\cosh(bx+a)}{3}+\frac{(bx+a)^{2}\cosh(bx+a)\sinh(bx+a)^{2}}{3}+\frac{4(bx+a)\sinh(bx+a)}{3}-\frac{40\cosh(bx+a)}{27}-\frac{2(bx+a)\sinh(bx+a)^{3}}{9}+\frac{2}{}\right.}{b^{2}}}{}$
default	$\frac{\frac{d^{2}\left(-\frac{2(bx+a)^{2}\cosh(bx+a)}{3}+\frac{(bx+a)^{2}\cosh(bx+a)\sinh(bx+a)^{2}}{3}+\frac{4(bx+a)\sinh(bx+a)}{3}-\frac{40\cosh(bx+a)}{27}-\frac{2(bx+a)\sinh(bx+a)^{3}}{9}+\frac{2}{}\right.}{b^{2}}}{}$
orering	$-\frac{40d(9d^{4}x^{4}b^{4}+36b^{4}c\,d^{3}x^{3}+54b^{4}c^{2}d^{2}x^{2}+36b^{4}c^{3}dx+9b^{4}c^{4}+b^{2}d^{4}x^{2}+2b^{2}c\,d^{3}x+b^{2}c^{2}d^{2}-12d^{4})\sinh(bx+a)^{3}}{81b^{6}(dx+c)^{3}}+\frac{2(45}{}$

input
```
int((d*x+c)^2*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/108*((9*(d*x+c)^2*b^2+2*d^2)*cosh(3*b*x+3*a)-6*b*d*(d*x+c)*sinh(3*b*x+3*
a)+(-81*(d*x+c)^2*b^2-162*d^2)*cosh(b*x+a)+162*b*d*(d*x+c)*sinh(b*x+a)-72*
b^2*c^2-160*d^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.62

$$\int (c+dx)^{2}\sinh^{3}(a+bx)\,dx$$
$$=\frac{(9\,b^{2}d^{2}x^{2}+18\,b^{2}cdx+9\,b^{2}c^{2}+2\,d^{2})\cosh{(bx+a)}^{3}+3\,(9\,b^{2}d^{2}x^{2}+18\,b^{2}cdx+9\,b^{2}c^{2}+2\,d^{2})\cosh{(bx+}}{}$$

input
```
integrate((d*x+c)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/108*((9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 + 2*d^2)*cosh(b*x + a)^3
+ 3*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 + 2*d^2)*cosh(b*x + a)*sinh(
b*x + a)^2 - 6*(b*d^2*x + b*c*d)*sinh(b*x + a)^3 - 81*(b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2 + 2*d^2)*cosh(b*x + a) + 18*(9*b*d^2*x + 9*b*c*d - (b*d^2
*x + b*c*d)*cosh(b*x + a)^2)*sinh(b*x + a))/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(121) = 242$.

Time = 0.34 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.31

$$\int (c+dx)^2 \sinh^3(a+bx)\,dx$$

$$= \begin{cases} \frac{c^2 \sinh^2(a+bx)\cosh(a+bx)}{b} - \frac{2c^2\cosh^3(a+bx)}{3b} + \frac{2cdx\sinh^2(a+bx)\cosh(a+bx)}{b} - \frac{4cdx\cosh^3(a+bx)}{3b} + \frac{d^2x^2\sinh^2(a+bx)\cosh(a+ \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sinh^3(a) \end{cases}$$

input
```
integrate((d*x+c)**2*sinh(b*x+a)**3,x)
```

output
```
Piecewise((c**2*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*c**2*cosh(a + b*x)**3
/(3*b) + 2*c*d*x*sinh(a + b*x)**2*cosh(a + b*x)/b - 4*c*d*x*cosh(a + b*x)*
*3/(3*b) + d**2*x**2*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*d**2*x**2*cosh(a
 + b*x)**3/(3*b) - 14*c*d*sinh(a + b*x)**3/(9*b**2) + 4*c*d*sinh(a + b*x)*
cosh(a + b*x)**2/(3*b**2) - 14*d**2*x*sinh(a + b*x)**3/(9*b**2) + 4*d**2*x
*sinh(a + b*x)*cosh(a + b*x)**2/(3*b**2) + 14*d**2*sinh(a + b*x)**2*cosh(a
 + b*x)/(9*b**3) - 40*d**2*cosh(a + b*x)**3/(27*b**3), Ne(b, 0)), ((c**2*x
 + c*d*x**2 + d**2*x**3/3)*sinh(a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(111) = 222$.

Time = 0.05 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.19

$$\int (c+dx)^2 \sinh^3(a+bx)\,dx$$
$$= \frac{1}{36}cd\left(\frac{(3\,bxe^{(3\,a)} - e^{(3\,a)})e^{(3\,bx)}}{b^2} - \frac{27\,(bxe^a - e^a)e^{(bx)}}{b^2} - \frac{27\,(bx+1)e^{(-bx-a)}}{b^2} + \frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{b^2}\right) + \frac{1}{24}c^2\left(\frac{e^{(3\,bx+3\,a)}}{b} - \frac{9\,e^{(bx+a)}}{b} - \frac{9\,e^{(-bx-a)}}{b} + \frac{e^{(-3\,bx-3\,a)}}{b}\right) + \frac{1}{216}d^2\left(\frac{(9\,b^2x^2e^{(3\,a)} - 6\,bxe^{(3\,a)} + 2\,e^{(3\,a)})e^{(3\,bx)}}{b^3} - \frac{81\,(b^2x^2e^a - 2\,bxe^a + 2\,e^a)e^{(bx)}}{b^3} - \frac{81\,(b^2x^2 + 2\,bx +}{b^3}\right.$$

input
```
integrate((d*x+c)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/36*c*d*((3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 - 27*(b*x*e^a - e^a)*e^(
b*x)/b^2 - 27*(b*x + 1)*e^(-b*x - a)/b^2 + (3*b*x + 1)*e^(-3*b*x - 3*a)/b^
2) + 1/24*c^2*(e^(3*b*x + 3*a)/b - 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b + e^
(-3*b*x - 3*a)/b) + 1/216*d^2*((9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(3
*a))*e^(3*b*x)/b^3 - 81*(b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 - 81
*(b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 + (9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*
x - 3*a)/b^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(111) = 222$.

Time = 0.12 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.87

$$\int (c+dx)^2 \sinh^3(a+bx)\,dx = \frac{(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-6\,bd^2x-6\,bcd+2\,d^2)e^{(3\,bx+3\,a)}}{216\,b^3} - \frac{3\,(b^2d^2x^2+2\,b^2cdx+b^2c^2-2\,bd^2x-2\,bcd+2\,d^2)e^{(bx+a)}}{8\,b^3} - \frac{3\,(b^2d^2x^2+2\,b^2cdx+b^2c^2+2\,bd^2x+2\,bcd+2\,d^2)e^{(-bx-a)}}{8\,b^3} + \frac{(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2+6\,bd^2x+6\,bcd+2\,d^2)e^{(-3\,bx-3\,a)}}{216\,b^3}$$

input
```
integrate((d*x+c)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/216*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 6*b*d^2*x - 6*b*c*d + 2*
d^2)*e^(3*b*x + 3*a)/b^3 - 3/8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*b*
d^2*x - 2*b*c*d + 2*d^2)*e^(b*x + a)/b^3 - 3/8*(b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2 + 2*b*d^2*x + 2*b*c*d + 2*d^2)*e^(-b*x - a)/b^3 + 1/216*(9*b^2*d
^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 + 6*b*d^2*x + 6*b*c*d + 2*d^2)*e^(-3*b*x
 - 3*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.50

$$\int (c+dx)^2 \sinh^3(a+bx)\,dx = -\frac{\frac{3\,d^2\cosh(a+b\,x)}{2} - \frac{d^2\cosh(3\,a+3\,b\,x)}{54} + \frac{3\,b^2\,c^2\cosh(a+b\,x)}{4} - \frac{b^2\,c^2\cosh(3\,a+3\,b\,x)}{12} + \frac{3\,b^2\,d^2\,x^2\cosh(a+b\,x)}{4} + \frac{b\,c\,d\sinh(3\,a+3\,b\,x)}{18}}{}$$

input
```
int(sinh(a + b*x)^3*(c + d*x)^2,x)
```

output
```
-((3*d^2*cosh(a + b*x))/2 - (d^2*cosh(3*a + 3*b*x))/54 + (3*b^2*c^2*cosh(a
 + b*x))/4 - (b^2*c^2*cosh(3*a + 3*b*x))/12 + (3*b^2*d^2*x^2*cosh(a + b*x)
)/4 + (b*c*d*sinh(3*a + 3*b*x))/18 - (3*b*d^2*x*sinh(a + b*x))/2 - (b^2*d^
2*x^2*cosh(3*a + 3*b*x))/12 + (b*d^2*x*sinh(3*a + 3*b*x))/18 - (3*b*c*d*si
nh(a + b*x))/2 - (b^2*c*d*x*cosh(3*a + 3*b*x))/6 + (3*b^2*c*d*x*cosh(a + b
*x))/2)/b^3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 374, normalized size of antiderivative = 3.04

$$\int (c+dx)^2 \sinh^3(a+bx)\,dx$$
$$= \frac{9e^{6bx+6a}b^2c^2 + 18e^{6bx+6a}b^2cdx + 9e^{6bx+6a}b^2d^2x^2 - 6e^{6bx+6a}bcd - 6e^{6bx+6a}b\,d^2x + 2e^{6bx+6a}d^2 - 81e^{4bx+4a}b^2}{}$$

input
```
int((d*x+c)^2*sinh(b*x+a)^3,x)
```

output
```
(9*e**(6*a + 6*b*x)*b**2*c**2 + 18*e**(6*a + 6*b*x)*b**2*c*d*x + 9*e**(6*a
 + 6*b*x)*b**2*d**2*x**2 - 6*e**(6*a + 6*b*x)*b*c*d - 6*e**(6*a + 6*b*x)*b
*d**2*x + 2*e**(6*a + 6*b*x)*d**2 - 81*e**(4*a + 4*b*x)*b**2*c**2 - 162*e*
*(4*a + 4*b*x)*b**2*c*d*x - 81*e**(4*a + 4*b*x)*b**2*d**2*x**2 + 162*e**(4
*a + 4*b*x)*b*c*d + 162*e**(4*a + 4*b*x)*b*d**2*x - 162*e**(4*a + 4*b*x)*d
**2 - 81*e**(2*a + 2*b*x)*b**2*c**2 - 162*e**(2*a + 2*b*x)*b**2*c*d*x - 81
*e**(2*a + 2*b*x)*b**2*d**2*x**2 - 162*e**(2*a + 2*b*x)*b*c*d - 162*e**(2*
a + 2*b*x)*b*d**2*x - 162*e**(2*a + 2*b*x)*d**2 + 9*b**2*c**2 + 18*b**2*c*
d*x + 9*b**2*d**2*x**2 + 6*b*c*d + 6*b*d**2*x + 2*d**2)/(216*e**(3*a + 3*b
*x)*b**3)
```

3.19 $\int (c+dx)\sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 75

$$\int (c+dx)\sinh^3(a+bx)\,dx = -\frac{2(c+dx)\cosh(a+bx)}{3b}+\frac{2d\sinh(a+bx)}{3b^2}+\frac{(c+dx)\cosh(a+bx)\sinh^2(a+bx)}{3b}-\frac{d\sinh^3(a+bx)}{9b^2}$$

output
```
-2/3*(d*x+c)*cosh(b*x+a)/b+2/3*d*sinh(b*x+a)/b^2+1/3*(d*x+c)*cosh(b*x+a)*s
inh(b*x+a)^2/b-1/9*d*sinh(b*x+a)^3/b^2
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

$$\int (c+dx)\sinh^3(a+bx)\,dx = \frac{-27b(c+dx)\cosh(a+bx)+3b(c+dx)\cosh(3(a+bx))+d(27\sinh(a+bx)-\sinh(3(a+bx)))}{36b^2}$$

input
```
Integrate[(c + d*x)*Sinh[a + b*x]^3,x]
```

output
```
(-27*b*(c + d*x)*Cosh[a + b*x] + 3*b*(c + d*x)*Cosh[3*(a + b*x)] + d*(27*S
inh[a + b*x] - Sinh[3*(a + b*x)]))/(36*b^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3042, 26, 3791, 26, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sinh^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i(c+dx)\sin(ia+ibx)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int (c+dx)\sin(ia+ibx)^3dx$$

$$\Big\downarrow \text{3791}$$

$$i\left(\frac{2}{3}\int i(c+dx)\sinh(a+bx)dx+\frac{id\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{2}{3}i\int (c+dx)\sinh(a+bx)dx+\frac{id\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{2}{3}i\int -i(c+dx)\sin(ia+ibx)dx+\frac{id\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{2}{3}\int (c+dx)\sin(ia+ibx)dx+\frac{id\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)$$

$$\Big\downarrow \text{3777}$$

$$i\left(\frac{2}{3}\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\int\cosh(a+bx)dx}{b}\right)+\frac{id\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{3b}\right.$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{2}{3}\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\int\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)+\frac{id\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)\sinh^2(a+bx)\cosh(a}{3b}\right.$$

$$\Big\downarrow \text{3117}$$

$$i\left(\frac{2}{3}\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\sinh(a+bx)}{b^2}\right)+\frac{id\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)$$

input
```
Int[(c + d*x)*Sinh[a + b*x]^3,x]
```

output
```
I*(((-1/3*I)*(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x]^2)/b + ((I/9)*d*Sinh[a
+ b*x]^3)/b^2 + (2*((I*(c + d*x)*Cosh[a + b*x])/b - (I*d*Sinh[a + b*x])/b^
2))/3)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

method	result
parallelrisch	$\frac{3b(dx+c)\cosh(3bx+3a)-d\sinh(3bx+3a)-27(dx+c)b\cosh(bx+a)-24bc+27\sinh(bx+a)d}{36b^2}$
risch	$\frac{(3dxb+3bc-d)\mathrm{e}^{3bx+3a}}{72b^2}-\frac{3(dxb+bc-d)\mathrm{e}^{bx+a}}{8b^2}-\frac{3(dxb+bc+d)\mathrm{e}^{-bx-a}}{8b^2}+\frac{(3dxb+3bc+d)\mathrm{e}^{-3bx-3a}}{72b^2}$
derivativedivides	$\frac{\frac{d\left(-\frac{2(bx+a)\cosh(bx+a)}{3}+\frac{(bx+a)\cosh(bx+a)\sinh(bx+a)^2}{3}+\frac{2\sinh(bx+a)}{3}-\frac{\sinh(bx+a)^3}{9}\right)}{b}-\frac{da\left(-\frac{2}{3}+\frac{\sinh(bx+a)^2}{3}\right)\cosh(bx+a)}{b}}{b}$
default	$\frac{\frac{d\left(-\frac{2(bx+a)\cosh(bx+a)}{3}+\frac{(bx+a)\cosh(bx+a)\sinh(bx+a)^2}{3}+\frac{2\sinh(bx+a)}{3}-\frac{\sinh(bx+a)^3}{9}\right)}{b}-\frac{da\left(-\frac{2}{3}+\frac{\sinh(bx+a)^2}{3}\right)\cosh(bx+a)}{b}}{b}$
orering	$-\frac{4d(5d^2x^2b^2+10b^2cdx+5b^2c^2-2d^2)\sinh(bx+a)^3}{9b^4(dx+c)^2}+\frac{2(5d^2x^2b^2+10b^2cdx+5b^2c^2-4d^2)\left(d\sinh(bx+a)^3+3(dx+c)\right.}{9b^4(dx+c)^2}$

input
```
int((d*x+c)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/36*(3*b*(d*x+c)*cosh(3*b*x+3*a)-d*sinh(3*b*x+3*a)-27*(d*x+c)*b*cosh(b*x+
a)-24*b*c+27*sinh(b*x+a)*d)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.29

$$\int (c+dx)\sinh^3(a+bx)\,dx$$
$$=\frac{3\,(bdx+bc)\cosh\left(bx+a\right)^3+9\,(bdx+bc)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2-d\sinh\left(bx+a\right)^3-27\,(bdx+bc)}{36\,b^2}$$

input
```
integrate((d*x+c)*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/36*(3*(b*d*x + b*c)*cosh(b*x + a)^3 + 9*(b*d*x + b*c)*cosh(b*x + a)*sinh
(b*x + a)^2 - d*sinh(b*x + a)^3 - 27*(b*d*x + b*c)*cosh(b*x + a) - 3*(d*co
sh(b*x + a)^2 - 9*d)*sinh(b*x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.68

$$\int (c+dx)\sinh^3(a+bx)\,dx$$
$$=\begin{cases}\frac{c\sinh^2(a+bx)\cosh(a+bx)}{b}-\frac{2c\cosh^3(a+bx)}{3b}+\frac{dx\sinh^2(a+bx)\cosh(a+bx)}{b}-\frac{2dx\cosh^3(a+bx)}{3b}-\frac{7d\sinh^3(a+bx)}{9b^2}+\frac{2d\sinh(a+b}{}\\ \left(cx+\frac{dx^2}{2}\right)\sinh^3(a)\end{cases}$$

input
```
integrate((d*x+c)*sinh(b*x+a)**3,x)
```

output
```
Piecewise((c*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*c*cosh(a + b*x)**3/(3*b)
 + d*x*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*d*x*cosh(a + b*x)**3/(3*b) - 7
*d*sinh(a + b*x)**3/(9*b**2) + 2*d*sinh(a + b*x)*cosh(a + b*x)**2/(3*b**2)
, Ne(b, 0)), ((c*x + d*x**2/2)*sinh(a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(67) = 134$.

Time = 0.04 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.88

$$\int (c+dx)\sinh^3(a+bx)\,dx = \frac{1}{72}\,d\left(\frac{(3\,bxe^{(3\,a)}-e^{(3\,a)})e^{(3\,bx)}}{b^2}-\frac{27\,(bxe^{a}-e^{a})e^{(bx)}}{b^2}-\frac{27\,(bx+1)e^{(-bx-a)}}{b^2}+\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{b^2}\right) + \frac{1}{24}\,c\left(\frac{e^{(3\,bx+3\,a)}}{b}-\frac{9\,e^{(bx+a)}}{b}-\frac{9\,e^{(-bx-a)}}{b}+\frac{e^{(-3\,bx-3\,a)}}{b}\right)$$

input
```
integrate((d*x+c)*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/72*d*((3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 - 27*(b*x*e^a - e^a)*e^(b*
x)/b^2 - 27*(b*x + 1)*e^(-b*x - a)/b^2 + (3*b*x + 1)*e^(-3*b*x - 3*a)/b^2)
 + 1/24*c*(e^(3*b*x + 3*a)/b - 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b + e^(-3*
b*x - 3*a)/b)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.31

$$\int (c+dx)\sinh^3(a+bx)\,dx = \frac{(3\,bdx+3\,bc-d)e^{(3\,bx+3\,a)}}{72\,b^2}-\frac{3\,(bdx+bc-d)e^{(bx+a)}}{8\,b^2} - \frac{3\,(bdx+bc+d)e^{(-bx-a)}}{8\,b^2}+\frac{(3\,bdx+3\,bc+d)e^{(-3\,bx-3\,a)}}{72\,b^2}$$

input
```
integrate((d*x+c)*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/72*(3*b*d*x + 3*b*c - d)*e^(3*b*x + 3*a)/b^2 - 3/8*(b*d*x + b*c - d)*e^(
b*x + a)/b^2 - 3/8*(b*d*x + b*c + d)*e^(-b*x - a)/b^2 + 1/72*(3*b*d*x + 3*
b*c + d)*e^(-3*b*x - 3*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int (c+dx)\sinh^3(a+bx)\,dx$$
$$= \frac{7\,d\sinh(a+b\,x)}{9\,b^2} - \frac{c\cosh(a+b\,x) - \frac{c\cosh(a+b\,x)^3}{3} + d\,x\cosh(a+b\,x) - \frac{d\,x\cosh(a+b\,x)^3}{3}}{b} - \frac{d\cosh(a+b\,x)^2\sinh(a+b\,x)}{9\,b^2}$$

input
```
int(sinh(a + b*x)^3*(c + d*x),x)
```

output
```
(7*d*sinh(a + b*x))/(9*b^2) - (c*cosh(a + b*x) - (c*cosh(a + b*x)^3)/3 + d
*x*cosh(a + b*x) - (d*x*cosh(a + b*x)^3)/3)/b - (d*cosh(a + b*x)^2*sinh(a
+ b*x))/(9*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.05

$$\int (c+dx)\sinh^3(a+bx)\,dx$$
$$= \frac{3e^{6bx+6a}bc + 3e^{6bx+6a}bdx - e^{6bx+6a}d - 27e^{4bx+4a}bc - 27e^{4bx+4a}bdx + 27e^{4bx+4a}d - 27e^{2bx+2a}bc - 27e^{2bx+2}}{72e^{3bx+3a}b^2}$$

input
```
int((d*x+c)*sinh(b*x+a)^3,x)
```

output
```
(3*e**(6*a + 6*b*x)*b*c + 3*e**(6*a + 6*b*x)*b*d*x - e**(6*a + 6*b*x)*d -
27*e**(4*a + 4*b*x)*b*c - 27*e**(4*a + 4*b*x)*b*d*x + 27*e**(4*a + 4*b*x)*
d - 27*e**(2*a + 2*b*x)*b*c - 27*e**(2*a + 2*b*x)*b*d*x - 27*e**(2*a + 2*b
*x)*d + 3*b*c + 3*b*d*x + d)/(72*e**(3*a + 3*b*x)*b**2)
```

3.20 $\int \frac{\sinh^3(a+bx)}{c+dx}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 121

$$\int \frac{\sinh^3(a+bx)}{c+dx}\,dx = \frac{\operatorname{Chi}\left(\frac{3bc}{d}+3bx\right)\sinh\left(3a-\frac{3bc}{d}\right)}{4d} - \frac{3\operatorname{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{4d} - \frac{3\cosh\left(a-\frac{bc}{d}\right)\operatorname{Shi}\left(\frac{bc}{d}+bx\right)}{4d} + \frac{\cosh\left(3a-\frac{3bc}{d}\right)\operatorname{Shi}\left(\frac{3bc}{d}+3bx\right)}{4d}$$

output
```
1/4*Chi(3*b*c/d+3*b*x)*sinh(3*a-3*b*c/d)/d-3/4*Chi(b*c/d+b*x)*sinh(a-b*c/d
)/d-3/4*cosh(a-b*c/d)*Shi(b*c/d+b*x)/d+1/4*cosh(3*a-3*b*c/d)*Shi(3*b*c/d+3
*b*x)/d
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

$$\int \frac{\sinh^3(a+bx)}{c+dx}\,dx = \frac{\operatorname{Chi}\left(\frac{3b(c+dx)}{d}\right)\sinh\left(3a-\frac{3bc}{d}\right) - 3\operatorname{Chi}\left(b\left(\frac{c}{d}+x\right)\right)\sinh\left(a-\frac{bc}{d}\right) - 3\cosh\left(a-\frac{bc}{d}\right)\operatorname{Shi}\left(b\left(\frac{c}{d}+x\right)\right) + \cosh\left(3\right.}{4d}$$

input
```
Integrate[Sinh[a + b*x]^3/(c + d*x),x]
```

output
```
(CoshIntegral[(3*b*(c + d*x))/d]*Sinh[3*a - (3*b*c)/d] - 3*CoshIntegral[b*
(c/d + x)]*Sinh[a - (b*c)/d] - 3*Cosh[a - (b*c)/d]*SinhIntegral[b*(c/d + x
)] + Cosh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*(c + d*x))/d])/(4*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 26, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ia+ibx)^3}{c+dx}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ia+ibx)^3}{c+dx}dx$$

$$\Big\downarrow \text{3793}$$

$$i\int \left(\frac{3i\sinh(a+bx)}{4(c+dx)}-\frac{i\sinh(3a+3bx)}{4(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$i\left(-\frac{i\sinh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d}+\frac{3i\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{3i\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{i\cosh\left(3\right.}{}\right.$$

input
```
Int[Sinh[a + b*x]^3/(c + d*x),x]
```

output
```
I*(((-1/4*I)*CoshIntegral[(3*b*c)/d + 3*b*x]*Sinh[3*a - (3*b*c)/d])/d + ((
(3*I)/4)*CoshIntegral[(b*c)/d + b*x]*Sinh[a - (b*c)/d])/d + (((3*I)/4)*Cos
h[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/d - ((I/4)*Cosh[3*a - (3*b*c)/
d]*SinhIntegral[(3*b*c)/d + 3*b*x])/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.37

method	result
risch	$\frac{e^{-\frac{3(da-bc)}{d}} \text{expIntegral}_1\left(3bx+3a-\frac{3(da-bc)}{d}\right)}{8d} - \frac{3\,e^{-\frac{da-bc}{d}} \text{expIntegral}_1\left(bx+a-\frac{da-bc}{d}\right)}{8d} + \frac{3\,e^{\frac{da-bc}{d}} \text{expIntegral}_1\left(-bx-a-\right.}{8d}$

input
```
int(sinh(b*x+a)^3/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*exp(-3*(a*d-b*c)/d)*Ei(1,3*b*x+3*a-3*(a*d-b*c)/d)-3/8/d*exp(-(a*d-b*
c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)+3/8/d*exp((a*d-b*c)/d)*Ei(1,-b*x-a-(-a*d+b*c
)/d)-1/8/d*exp(3*(a*d-b*c)/d)*Ei(1,-3*b*x-3*a-3*(-a*d+b*c)/d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.55

$$\int \frac{\sinh^3(a+bx)}{c+dx}\,dx =$$
$$-\frac{3\left(\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)-\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cosh\left(-\frac{bc-ad}{d}\right)-\left(\mathrm{Ei}\left(\frac{3\,(bdx+bc)}{d}\right)-\mathrm{Ei}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\cosh\left(-\frac{3\,(bc-ad)}{d}\right)+}{8\,d}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output
```
-1/8*(3*(Ei((b*d*x + b*c)/d) - Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d)
- (Ei(3*(b*d*x + b*c)/d) - Ei(-3*(b*d*x + b*c)/d))*cosh(-3*(b*c - a*d)/d)
+ 3*(Ei((b*d*x + b*c)/d) + Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d) - (E
i(3*(b*d*x + b*c)/d) + Ei(-3*(b*d*x + b*c)/d))*sinh(-3*(b*c - a*d)/d))/d
```

Sympy [F]

$$\int \frac{\sinh^3(a+bx)}{c+dx}\,dx = \int \frac{\sinh^3\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(sinh(b*x+a)**3/(d*x+c),x)
```

output
```
Integral(sinh(a + b*x)**3/(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.97

$$\int \frac{\sinh^3(a+bx)}{c+dx}\,dx = \frac{e^{\left(-3\,a+\frac{3\,bc}{d}\right)}E_1\left(\frac{3\,(dx+c)b}{d}\right)}{8\,d}-\frac{3\,e^{\left(-a+\frac{bc}{d}\right)}E_1\left(\frac{(dx+c)b}{d}\right)}{8\,d}+\frac{3\,e^{\left(a-\frac{bc}{d}\right)}E_1\left(-\frac{(dx+c)b}{d}\right)}{8\,d}-\frac{e^{\left(3\,a-\frac{3\,bc}{d}\right)}E_1\left(-\frac{3\,(dx+c)b}{d}\right)}{8\,d}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output
```
1/8*e^(-3*a + 3*b*c/d)*exp_integral_e(1, 3*(d*x + c)*b/d)/d - 3/8*e^(-a +
b*c/d)*exp_integral_e(1, (d*x + c)*b/d)/d + 3/8*e^(a - b*c/d)*exp_integral
_e(1, -(d*x + c)*b/d)/d - 1/8*e^(3*a - 3*b*c/d)*exp_integral_e(1, -3*(d*x
+ c)*b/d)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.93

$$\int \frac{\sinh^3(a+bx)}{c+dx}\,dx$$
$$= \frac{\text{Ei}\left(\frac{3\,(bdx+bc)}{d}\right)e^{\left(3\,a-\frac{3\,bc}{d}\right)} - 3\,\text{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)} + 3\,\text{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)} - \text{Ei}\left(-\frac{3\,(bdx+bc)}{d}\right)e^{\left(-3\,a+\frac{3\,bc}{d}\right)}}{8\,d}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output
```
1/8*(Ei(3*(b*d*x + b*c)/d)*e^(3*a - 3*b*c/d) - 3*Ei((b*d*x + b*c)/d)*e^(a
- b*c/d) + 3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - Ei(-3*(b*d*x + b*c)/d)*
e^(-3*a + 3*b*c/d))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(a+bx)}{c+dx}\,dx = \int \frac{\sinh(a+b\,x)^3}{c+d\,x}\,dx$$

input
```
int(sinh(a + b*x)^3/(c + d*x),x)
```

output
```
int(sinh(a + b*x)^3/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\sinh^3(a+bx)}{c+dx}\,dx = \int \frac{\sinh(bx+a)^3}{dx+c}dx$$

input
```
int(sinh(b*x+a)^3/(d*x+c),x)
```

output
```
int(sinh(a + b*x)**3/(c + d*x),x)
```

3.21 $\int \frac{\sinh^3(a+bx)}{(c+dx)^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 145

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^2} dx = -\frac{3b\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{4d^2} + \frac{3b\cosh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d^2} - \frac{\sinh^3(a+bx)}{d(c+dx)} - \frac{3b\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{4d^2} + \frac{3b\sinh\left(3a-\frac{3bc}{d}\right)\text{Shi}\left(\frac{3bc}{d}+3bx\right)}{4d^2}$$

output

```
-3/4*b*cosh(a-b*c/d)*Chi(b*c/d+b*x)/d^2+3/4*b*cosh(3*a-3*b*c/d)*Chi(3*b*c/
d+3*b*x)/d^2-sinh(b*x+a)^3/d/(d*x+c)-3/4*b*sinh(a-b*c/d)*Shi(b*c/d+b*x)/d^
2+3/4*b*sinh(3*a-3*b*c/d)*Shi(3*b*c/d+3*b*x)/d^2
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.10

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{6d\cosh(bx)\sinh(a) - 2d\cosh(3bx)\sinh(3a) + 6d\cosh(a)\sinh(bx) - 2d\cosh(3a)\sinh(3bx) + 6b(c+dx)}{}$$

input
```
Integrate[Sinh[a + b*x]^3/(c + d*x)^2,x]
```

output
```
(6*d*Cosh[b*x]*Sinh[a] - 2*d*Cosh[3*b*x]*Sinh[3*a] + 6*d*Cosh[a]*Sinh[b*x]
 - 2*d*Cosh[3*a]*Sinh[3*b*x] + 6*b*(c + d*x)*(-(Cosh[a - (b*c)/d]*CoshInte
gral[b*(c/d + x)]) + Cosh[3*a - (3*b*c)/d]*CoshIntegral[(3*b*(c + d*x))/d]
 - Sinh[a - (b*c)/d]*SinhIntegral[b*(c/d + x)] + Sinh[3*a - (3*b*c)/d]*Sin
hIntegral[(3*b*(c + d*x))/d]))/(8*d^2*(c + d*x))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 26, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i\sin(ia+ibx)^3}{(c+dx)^2}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{\sin(ia+ibx)^3}{(c+dx)^2}dx$$

$$\downarrow \text{3794}$$

$$i\left(\frac{3ib\int\left(\frac{\cosh(a+bx)}{4(c+dx)}-\frac{\cosh(3a+3bx)}{4(c+dx)}\right)dx}{d}+\frac{i\sinh^3(a+bx)}{d(c+dx)}\right)$$

$$\downarrow \text{2009}$$

$$i\left(\frac{3ib\left(\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{\cosh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d}+\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{\sinh\left(3a-\frac{3bc}{d}\right)\text{Shi}\left(\frac{3bc}{d}+3bx\right)}{4d}\right)}{d}+\right.$$

input
```
Int[Sinh[a + b*x]^3/(c + d*x)^2,x]
```

output
```
I*((I*Sinh[a + b*x]^3)/(d*(c + d*x)) + ((3*I)*b*((Cosh[a - (b*c)/d]*CoshIn
tegral[(b*c)/d + b*x])/(4*d) - (Cosh[3*a - (3*b*c)/d]*CoshIntegral[(3*b*c)
/d + 3*b*x])/(4*d) + (Sinh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/(4*d)
 - (Sinh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*c)/d + 3*b*x])/(4*d)))/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.87

method	result
risch	$\frac{b\,\mathrm{e}^{-3bx-3a}}{8d(dxb+bc)}-\frac{3b\,\mathrm{e}^{-\frac{3(da-bc)}{d}}\,\mathrm{expIntegral}_1\left(3bx+3a-\frac{3(da-bc)}{d}\right)}{8d^2}-\frac{3b\,\mathrm{e}^{-bx-a}}{8d(dxb+bc)}+\frac{3b\,\mathrm{e}^{-\frac{da-bc}{d}}\,\mathrm{expIntegral}_1\left(bx+a-\frac{da-bc}{d}\right)}{8d^2}+$

input
```
int(sinh(b*x+a)^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*b*exp(-3*b*x-3*a)/d/(b*d*x+b*c)-3/8*b/d^2*exp(-3*(a*d-b*c)/d)*Ei(1,3*b
*x+3*a-3*(a*d-b*c)/d)-3/8*b*exp(-b*x-a)/d/(b*d*x+b*c)+3/8*b/d^2*exp(-(a*d-
b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)+3/8*b/d^2*exp(b*x+a)/(b*c/d+b*x)+3/8*b/d^2
*exp((a*d-b*c)/d)*Ei(1,-b*x-a-(-a*d+b*c)/d)-1/8*b/d^2*exp(3*b*x+3*a)/(b*c/
d+b*x)-3/8*b/d^2*exp(3*(a*d-b*c)/d)*Ei(1,-3*b*x-3*a-3*(-a*d+b*c)/d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(137) = 274.

Time = 0.08 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.08

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^2}\,dx =$$

$$-\frac{2\,d\sinh\left(bx+a\right)^3+3\left((bdx+bc)\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)+(bdx+bc)\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cosh\left(-\frac{bc-ad}{d}\right)-3\left((bdx+bc)\mathrm{I}\right.}{}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/8*(2*d*sinh(b*x + a)^3 + 3*((b*d*x + b*c)*Ei((b*d*x + b*c)/d) + (b*d*x
+ b*c)*Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d) - 3*((b*d*x + b*c)*Ei(3*
(b*d*x + b*c)/d) + (b*d*x + b*c)*Ei(-3*(b*d*x + b*c)/d))*cosh(-3*(b*c - a*
d)/d) + 6*(d*cosh(b*x + a)^2 - d)*sinh(b*x + a) + 3*((b*d*x + b*c)*Ei((b*d
*x + b*c)/d) - (b*d*x + b*c)*Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d) -
3*((b*d*x + b*c)*Ei(3*(b*d*x + b*c)/d) - (b*d*x + b*c)*Ei(-3*(b*d*x + b*c)
/d))*sinh(-3*(b*c - a*d)/d))/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sinh^3{(a+bx)}}{(c+dx)^2}\,dx$$

input
```
integrate(sinh(b*x+a)**3/(d*x+c)**2,x)
```

output
```
Integral(sinh(a + b*x)**3/(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^2}\,dx = \frac{e^{\left(-3\,a+\frac{3\,bc}{d}\right)}E_2\left(\frac{3\,(dx+c)b}{d}\right)}{8\,(dx+c)d} - \frac{3\,e^{\left(-a+\frac{bc}{d}\right)}E_2\left(\frac{(dx+c)b}{d}\right)}{8\,(dx+c)d} + \frac{3\,e^{\left(a-\frac{bc}{d}\right)}E_2\left(-\frac{(dx+c)b}{d}\right)}{8\,(dx+c)d} - \frac{e^{\left(3\,a-\frac{3\,bc}{d}\right)}E_2\left(-\frac{3\,(dx+c)b}{d}\right)}{8\,(dx+c)d}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/8*e^(-3*a + 3*b*c/d)*exp_integral_e(2, 3*(d*x + c)*b/d)/((d*x + c)*d) -
3/8*e^(-a + b*c/d)*exp_integral_e(2, (d*x + c)*b/d)/((d*x + c)*d) + 3/8*e^
(a - b*c/d)*exp_integral_e(2, -(d*x + c)*b/d)/((d*x + c)*d) - 1/8*e^(3*a -
 3*b*c/d)*exp_integral_e(2, -3*(d*x + c)*b/d)/((d*x + c)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1076 vs. $2(137) = 274$.

Time = 0.18 (sec) , antiderivative size = 1076, normalized size of antiderivative = 7.42

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/8*(3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*Ei(-3*((d*x + c)*
(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(3*(b*c - a*d)/d) +
3*b^3*c*Ei(-3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/
d)*e^(3*(b*c - a*d)/d) - 3*a*b^2*d*Ei(-3*((d*x + c)*(b - b*c/(d*x + c) + a
*d/(d*x + c)) + b*c - a*d)/d)*e^(3*(b*c - a*d)/d) - 3*(d*x + c)*(b - b*c/(
d*x + c) + a*d/(d*x + c))*b^2*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x
 + c)) + b*c - a*d)/d)*e^((b*c - a*d)/d) - 3*b^3*c*Ei(-((d*x + c)*(b - b*c
/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c - a*d)/d) + 3*a*b^2*d*
Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c
 - a*d)/d) - 3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*Ei(((d*x
+ c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(-(b*c - a*d)/d
) - 3*b^3*c*Ei(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)
/d)*e^(-(b*c - a*d)/d) + 3*a*b^2*d*Ei(((d*x + c)*(b - b*c/(d*x + c) + a*d/
(d*x + c)) + b*c - a*d)/d)*e^(-(b*c - a*d)/d) + 3*(d*x + c)*(b - b*c/(d*x
+ c) + a*d/(d*x + c))*b^2*Ei(3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x +
c)) + b*c - a*d)/d)*e^(-3*(b*c - a*d)/d) + 3*b^3*c*Ei(3*((d*x + c)*(b - b*
c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(-3*(b*c - a*d)/d) - 3*a*b^
2*d*Ei(3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^
(-3*(b*c - a*d)/d) - b^2*d*e^(3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x +
c))/d) + 3*b^2*d*e^((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d) - ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sinh(a+b\,x)^3}{(c+d\,x)^2}\,dx$$

input
```
int(sinh(a + b*x)^3/(c + d*x)^2,x)
```

output
```
int(sinh(a + b*x)^3/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^2}\,dx = \frac{e^{4a}\left(\int \frac{e^{3bx}}{d^2x^2+2cdx+c^2}dx\right) - 3e^{2a}\left(\int \frac{e^{bx}}{d^2x^2+2cdx+c^2}dx\right) - e^{a}\left(\int \frac{1}{e^{3bx+3a}c^2+2e^{3bx+3a}cdx+e^{3bx+3a}d^2x^2}dx\right) + 3\left(\int \frac{1}{e^{bx}c^2+2}\right.}{8e^{a}}$$

input
```
int(sinh(b*x+a)^3/(d*x+c)^2,x)
```

output
```
(e**(4*a)*int(e**(3*b*x)/(c**2 + 2*c*d*x + d**2*x**2),x) - 3*e**(2*a)*int(
e**(b*x)/(c**2 + 2*c*d*x + d**2*x**2),x) - e**a*int(1/(e**(3*a + 3*b*x)*c*
*2 + 2*e**(3*a + 3*b*x)*c*d*x + e**(3*a + 3*b*x)*d**2*x**2),x) + 3*int(1/(
e**(b*x)*c**2 + 2*e**(b*x)*c*d*x + e**(b*x)*d**2*x**2),x))/(8*e**a)
```

3.22 $\int \frac{\sinh^3(a+bx)}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 184

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^3}\,dx = \frac{9b^2\text{Chi}\left(\frac{3bc}{d}+3bx\right)\sinh\left(3a-\frac{3bc}{d}\right)}{8d^3} - \frac{3b^2\text{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{8d^3} - \frac{3b\cosh(a+bx)\sinh^2(a+bx)}{2d^2(c+dx)} - \frac{\sinh^3(a+bx)}{2d(c+dx)^2} - \frac{3b^2\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{8d^3} + \frac{9b^2\cosh\left(3a-\frac{3bc}{d}\right)\text{Shi}\left(\frac{3bc}{d}+3bx\right)}{8d^3}$$

output
```
9/8*b^2*Chi(3*b*c/d+3*b*x)*sinh(3*a-3*b*c/d)/d^3-3/8*b^2*Chi(b*c/d+b*x)*si
nh(a-b*c/d)/d^3-3/2*b*cosh(b*x+a)*sinh(b*x+a)^2/d^2/(d*x+c)-1/2*sinh(b*x+a
)^3/d/(d*x+c)^2-3/8*b^2*cosh(a-b*c/d)*Shi(b*c/d+b*x)/d^3+9/8*b^2*cosh(3*a-
3*b*c/d)*Shi(3*b*c/d+3*b*x)/d^3
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.20

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{6d\cosh(bx)(b(c+dx)\cosh(a)+d\sinh(a))-2d\cosh(3bx)(3b(c+dx)\cosh(3a)+d\sinh(3a))+6d(d\cos}{}$$

input
```
Integrate[Sinh[a + b*x]^3/(c + d*x)^3,x]
```

output
```
(6*d*Cosh[b*x]*(b*(c + d*x)*Cosh[a] + d*Sinh[a]) - 2*d*Cosh[3*b*x]*(3*b*(c
 + d*x)*Cosh[3*a] + d*Sinh[3*a]) + 6*d*(d*Cosh[a] + b*(c + d*x)*Sinh[a])*S
inh[b*x] - 2*d*(d*Cosh[3*a] + 3*b*(c + d*x)*Sinh[3*a])*Sinh[3*b*x] + 6*b^2
*(c + d*x)^2*(3*CoshIntegral[(3*b*(c + d*x))/d]*Sinh[3*a - (3*b*c)/d] - Co
shIntegral[b*(c/d + x)]*Sinh[a - (b*c)/d] - Cosh[a - (b*c)/d]*SinhIntegral
[b*(c/d + x)] + 3*Cosh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*(c + d*x))/d]))/
(16*d^3*(c + d*x)^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.43, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {3042, 26, 3795, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ia+ibx)^3}{(c+dx)^3}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ia+ibx)^3}{(c+dx)^3}dx$$

$\big\downarrow$ 3795

$$i\left(\frac{9b^2\int -\frac{i\sinh^3(a+bx)}{c+dx}dx}{2d^2}-\frac{3b^2\int \frac{i\sinh(a+bx)}{c+dx}dx}{d^2}+\frac{3ib\sinh^2(a+bx)\cosh(a+bx)}{2d^2(c+dx)}+\frac{i\sinh^3(a+bx)}{2d(c+dx)^2}\right)$$

$\big\downarrow$ 26

$$i\left(-\frac{9ib^2\int \frac{\sinh^3(a+bx)}{c+dx}dx}{2d^2}-\frac{3ib^2\int \frac{\sinh(a+bx)}{c+dx}dx}{d^2}+\frac{3ib\sinh^2(a+bx)\cosh(a+bx)}{2d^2(c+dx)}+\frac{i\sinh^3(a+bx)}{2d(c+dx)^2}\right)$$

$\big\downarrow$ 3042

$$i\left(-\frac{3ib^2\int -\frac{i\sin(ia+ibx)}{c+dx}dx}{d^2}-\frac{9ib^2\int \frac{i\sin(ia+ibx)^3}{c+dx}dx}{2d^2}+\frac{3ib\sinh^2(a+bx)\cosh(a+bx)}{2d^2(c+dx)}+\frac{i\sinh^3(a+bx)}{2d(c+dx)^2}\right)$$

$\big\downarrow$ 26

$$i\left(-\frac{3b^2\int \frac{\sin(ia+ibx)}{c+dx}dx}{d^2}+\frac{9b^2\int \frac{\sin(ia+ibx)^3}{c+dx}dx}{2d^2}+\frac{3ib\sinh^2(a+bx)\cosh(a+bx)}{2d^2(c+dx)}+\frac{i\sinh^3(a+bx)}{2d(c+dx)^2}\right)$$

$\big\downarrow$ 3784

$$i\left(\frac{9b^2\int \frac{\sin(ia+ibx)^3}{c+dx}dx}{2d^2}-\frac{3b^2\left(i\sinh\left(a-\frac{bc}{d}\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}dx+\cosh\left(a-\frac{bc}{d}\right)\int \frac{i\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d^2}+\frac{3ib\sinh^2(a+\right.$$

$\big\downarrow$ 26

$$i\left(\frac{9b^2\int \frac{\sin(ia+ibx)^3}{c+dx}dx}{2d^2}-\frac{3b^2\left(i\sinh\left(a-\frac{bc}{d}\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}dx+i\cosh\left(a-\frac{bc}{d}\right)\int \frac{\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d^2}+\frac{3ib\sinh^2(a+\right.$$

$\big\downarrow$ 3042

$$i\left(\frac{9b^2\int\frac{\sin(ia+ibx)^3}{c+dx}dx}{2d^2}-\frac{3b^2\left(i\sinh\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx+i\cosh\left(a-\frac{bc}{d}\right)\int-\frac{i\sin\left(\frac{ibc}{d}+ibx\right)}{c+dx}dx\right)}{d^2}+\frac{3ib\sinh\right.$$

$\downarrow$ 26

$$i\left(\frac{9b^2\int\frac{\sin(ia+ibx)^3}{c+dx}dx}{2d^2}-\frac{3b^2\left(i\sinh\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx+\cosh\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{ibc}{d}+ibx\right)}{c+dx}dx\right)}{d^2}+\frac{3ib\sinh^2(a}{2d}\right.$$

$\downarrow$ 3779

$$i\left(-\frac{3b^2\left(i\sinh\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx+\frac{i\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d^2}+\frac{9b^2\int\frac{\sin(ia+ibx)^3}{c+dx}dx}{2d^2}+\frac{3ib\sinh^2(a+bx)}{2d^2(c+}\right.$$

$\downarrow$ 3782

$$i\left(\frac{9b^2\int\frac{\sin(ia+ibx)^3}{c+dx}dx}{2d^2}-\frac{3b^2\left(\frac{i\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d}+\frac{i\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d^2}+\frac{3ib\sinh^2(a+bx)\cosh(a+bx)}{2d^2(c+dx)}\right.$$

$\downarrow$ 3793

$$i\left(\frac{9b^2\int\left(\frac{3i\sinh(a+bx)}{4(c+dx)}-\frac{i\sinh(3a+3bx)}{4(c+dx)}\right)dx}{2d^2}-\frac{3b^2\left(\frac{i\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d}+\frac{i\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d^2}+\frac{3ib\sinh^2(a}{2}\right.$$

$\downarrow$ 2009

$$i\left(-\frac{3b^2\left(\frac{i\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d}+\frac{i\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d^2}+\frac{9b^2\left(-\frac{i\sinh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d}+\frac{3i\sinh\left(a-\frac{bc}{d}\right)\text{Ch}}{4d}\right.}{}\right.$$

input
```
Int[Sinh[a + b*x]^3/(c + d*x)^3,x]
```

output
```
I*((((3*I)/2)*b*Cosh[a + b*x]*Sinh[a + b*x]^2)/(d^2*(c + d*x)) + ((I/2)*Si
nh[a + b*x]^3)/(d*(c + d*x)^2) - (3*b^2*((I*CoshIntegral[(b*c)/d + b*x]*Si
nh[a - (b*c)/d])/d + (I*Cosh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/d))
/d^2 + (9*b^2*(((-1/4*I)*CoshIntegral[(3*b*c)/d + 3*b*x]*Sinh[3*a - (3*b*c
)/d])/d + (((3*I)/4)*CoshIntegral[(b*c)/d + b*x]*Sinh[a - (b*c)/d])/d + ((
(3*I)/4)*Cosh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/d - ((I/4)*Cosh[3*
a - (3*b*c)/d]*SinhIntegral[(3*b*c)/d + 3*b*x])/d))/(2*d^2))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(172) = 344$.

Time = 0.93 (sec) , antiderivative size = 562, normalized size of antiderivative = 3.05

method	result
risch	$-\frac{3b^3 \mathrm{e}^{-3bx-3a} x}{16d(d^2x^2b^2+2b^2cdx+b^2c^2)} - \frac{3b^3 \mathrm{e}^{-3bx-3a} c}{16d^2(d^2x^2b^2+2b^2cdx+b^2c^2)} + \frac{b^2 \mathrm{e}^{-3bx-3a}}{16d(d^2x^2b^2+2b^2cdx+b^2c^2)} + \frac{9b^2 \mathrm{e}^{-\frac{3(da-bc)}{d}} \operatorname{expIntegral}_1\left(3\right.}{16d^3}$

input
```
int(sinh(b*x+a)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/16*b^3*exp(-3*b*x-3*a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*x-3/16*b^3*e
xp(-3*b*x-3*a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*c+1/16*b^2*exp(-3*b*x
-3*a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)+9/16*b^2/d^3*exp(-3*(a*d-b*c)/d)
*Ei(1,3*b*x+3*a-3*(a*d-b*c)/d)+3/16*b^3*exp(-b*x-a)/d/(b^2*d^2*x^2+2*b^2*c
*d*x+b^2*c^2)*x+3/16*b^3*exp(-b*x-a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)
*c-3/16*b^2*exp(-b*x-a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)-3/16*b^2/d^3*e
xp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)+3/16*b^2/d^3*exp(b*x+a)/(b*c/d+b*
x)^2+3/16*b^2/d^3*exp(b*x+a)/(b*c/d+b*x)+3/16*b^2/d^3*exp((a*d-b*c)/d)*Ei(
1,-b*x-a-(-a*d+b*c)/d)-1/16*b^2/d^3*exp(3*b*x+3*a)/(b*c/d+b*x)^2-3/16*b^2/
d^3*exp(3*b*x+3*a)/(b*c/d+b*x)-9/16*b^2/d^3*exp(3*(a*d-b*c)/d)*Ei(1,-3*b*x
-3*a-3*(-a*d+b*c)/d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(172) = 344$.

Time = 0.09 (sec) , antiderivative size = 529, normalized size of antiderivative = 2.88

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^3}\,dx =$$

$$-\frac{2\,d^2\sinh\left(bx+a\right)^3+6\left(bd^2x+bcd\right)\cosh\left(bx+a\right)^3+18\left(bd^2x+bcd\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2-6\,(}{}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/16*(2*d^2*sinh(b*x + a)^3 + 6*(b*d^2*x + b*c*d)*cosh(b*x + a)^3 + 18*(b
*d^2*x + b*c*d)*cosh(b*x + a)*sinh(b*x + a)^2 - 6*(b*d^2*x + b*c*d)*cosh(b
*x + a) + 3*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei((b*d*x + b*c)/d) - (
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*
d)/d) - 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(3*(b*d*x + b*c)/d) - (
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-3*(b*d*x + b*c)/d))*cosh(-3*(b*c
- a*d)/d) + 6*(d^2*cosh(b*x + a)^2 - d^2)*sinh(b*x + a) + 3*((b^2*d^2*x^2
+ 2*b^2*c*d*x + b^2*c^2)*Ei((b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2)*Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d) - 9*((b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*Ei(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2)*Ei(-3*(b*d*x + b*c)/d))*sinh(-3*(b*c - a*d)/d))/(d^5*x^2 + 2*c*
d^4*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sinh^3(a+bx)}{(c+dx)^3}\,dx$$

input
```
integrate(sinh(b*x+a)**3/(d*x+c)**3,x)
```

output
```
Integral(sinh(a + b*x)**3/(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.79

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^3}\,dx = \frac{e^{\left(-3a+\frac{3bc}{d}\right)}E_3\left(\frac{3(dx+c)b}{d}\right)}{8(dx+c)^2d} - \frac{3e^{\left(-a+\frac{bc}{d}\right)}E_3\left(\frac{(dx+c)b}{d}\right)}{8(dx+c)^2d} + \frac{3e^{\left(a-\frac{bc}{d}\right)}E_3\left(-\frac{(dx+c)b}{d}\right)}{8(dx+c)^2d} - \frac{e^{\left(3a-\frac{3bc}{d}\right)}E_3\left(-\frac{3(dx+c)b}{d}\right)}{8(dx+c)^2d}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/8*e^(-3*a + 3*b*c/d)*exp_integral_e(3, 3*(d*x + c)*b/d)/((d*x + c)^2*d)
- 3/8*e^(-a + b*c/d)*exp_integral_e(3, (d*x + c)*b/d)/((d*x + c)^2*d) + 3/
8*e^(a - b*c/d)*exp_integral_e(3, -(d*x + c)*b/d)/((d*x + c)^2*d) - 1/8*e^
(3*a - 3*b*c/d)*exp_integral_e(3, -3*(d*x + c)*b/d)/((d*x + c)^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(172) = 344$.

Time = 0.20 (sec) , antiderivative size = 601, normalized size of antiderivative = 3.27

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^3}\,dx = \frac{9\,b^2d^2x^2\mathrm{Ei}\left(\frac{3\,(bdx+bc)}{d}\right)e^{\left(3\,a-\frac{3\,bc}{d}\right)} - 3\,b^2d^2x^2\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)} + 3\,b^2d^2x^2\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)} - 9\,b^2d^2x^2}{}$$

input

```
integrate(sinh(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

output

```
1/16*(9*b^2*d^2*x^2*Ei(3*(b*d*x + b*c)/d)*e^(3*a - 3*b*c/d) - 3*b^2*d^2*x^
2*Ei((b*d*x + b*c)/d)*e^(a - b*c/d) + 3*b^2*d^2*x^2*Ei(-(b*d*x + b*c)/d)*e
^(-a + b*c/d) - 9*b^2*d^2*x^2*Ei(-3*(b*d*x + b*c)/d)*e^(-3*a + 3*b*c/d) +
18*b^2*c*d*x*Ei(3*(b*d*x + b*c)/d)*e^(3*a - 3*b*c/d) - 6*b^2*c*d*x*Ei((b*d
*x + b*c)/d)*e^(a - b*c/d) + 6*b^2*c*d*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/
d) - 18*b^2*c*d*x*Ei(-3*(b*d*x + b*c)/d)*e^(-3*a + 3*b*c/d) + 9*b^2*c^2*Ei
(3*(b*d*x + b*c)/d)*e^(3*a - 3*b*c/d) - 3*b^2*c^2*Ei((b*d*x + b*c)/d)*e^(a
 - b*c/d) + 3*b^2*c^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 9*b^2*c^2*Ei(-
3*(b*d*x + b*c)/d)*e^(-3*a + 3*b*c/d) - 3*b*d^2*x*e^(3*b*x + 3*a) + 3*b*d^
2*x*e^(b*x + a) + 3*b*d^2*x*e^(-b*x - a) - 3*b*d^2*x*e^(-3*b*x - 3*a) - 3*
b*c*d*e^(3*b*x + 3*a) + 3*b*c*d*e^(b*x + a) + 3*b*c*d*e^(-b*x - a) - 3*b*c
*d*e^(-3*b*x - 3*a) - d^2*e^(3*b*x + 3*a) + 3*d^2*e^(b*x + a) - 3*d^2*e^(-
b*x - a) + d^2*e^(-3*b*x - 3*a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sinh(a+b\,x)^3}{(c+d\,x)^3}\,dx$$

input

```
int(sinh(a + b*x)^3/(c + d*x)^3,x)
```

output

```
int(sinh(a + b*x)^3/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{e^{4a}\left(\int \frac{e^{3bx}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right) - 3e^{2a}\left(\int \frac{e^{bx}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right) - e^{a}\left(\int \frac{1}{e^{3bx+3a}c^3+3e^{3bx+3a}c^2dx+3e^{3bx+3a}c\,d^2x^2}\right.}{8e^{a}}$$

input
```
int(sinh(b*x+a)^3/(d*x+c)^3,x)
```

output
```
(e**(4*a)*int(e**(3*b*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x
) - 3*e**(2*a)*int(e**(b*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3
),x) - e**a*int(1/(e**(3*a + 3*b*x)*c**3 + 3*e**(3*a + 3*b*x)*c**2*d*x + 3
*e**(3*a + 3*b*x)*c*d**2*x**2 + e**(3*a + 3*b*x)*d**3*x**3),x) + 3*int(1/(
e**(b*x)*c**3 + 3*e**(b*x)*c**2*d*x + 3*e**(b*x)*c*d**2*x**2 + e**(b*x)*d*
*3*x**3),x))/(8*e**a)
```

3.23 $\int (c+dx)^3 \mathrm{csch}(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 149

$$\int (c+dx)^3 \mathrm{csch}(a+bx)\,dx = -\frac{2(c+dx)^3 \mathrm{arctanh}\left(e^{a+bx}\right)}{b} - \frac{3d(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{b^2} + \frac{3d(c+dx)^2 \operatorname{PolyLog}\left(2,e^{a+bx}\right)}{b^2} + \frac{6d^2(c+dx) \operatorname{PolyLog}\left(3,-e^{a+bx}\right)}{b^3} - \frac{6d^2(c+dx) \operatorname{PolyLog}\left(3,e^{a+bx}\right)}{b^3} - \frac{6d^3 \operatorname{PolyLog}\left(4,-e^{a+bx}\right)}{b^4} + \frac{6d^3 \operatorname{PolyLog}\left(4,e^{a+bx}\right)}{b^4}$$

output
```
-2*(d*x+c)^3*arctanh(exp(b*x+a))/b-3*d*(d*x+c)^2*polylog(2,-exp(b*x+a))/b^
2+3*d*(d*x+c)^2*polylog(2,exp(b*x+a))/b^2+6*d^2*(d*x+c)*polylog(3,-exp(b*x
+a))/b^3-6*d^2*(d*x+c)*polylog(3,exp(b*x+a))/b^3-6*d^3*polylog(4,-exp(b*x+
a))/b^4+6*d^3*polylog(4,exp(b*x+a))/b^4
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.13

$$\int (c+dx)^3\text{csch}(a+bx)\,dx$$
$$=\frac{(c+dx)^3\log\left(1-e^{a+bx}\right)-(c+dx)^3\log\left(1+e^{a+bx}\right)-\frac{3d\left(b^2(c+dx)^2\operatorname{PolyLog}\left(2,-e^{a+bx}\right)-2bd(c+dx)\operatorname{PolyLog}\left(3,-e^{a+bx}\right.}{b^3}}{b}$$

input
```
Integrate[(c + d*x)^3*Csch[a + b*x],x]
```

output
```
((c + d*x)^3*Log[1 - E^(a + b*x)] - (c + d*x)^3*Log[1 + E^(a + b*x)] - (3*
d*(b^2*(c + d*x)^2*PolyLog[2, -E^(a + b*x)] - 2*b*d*(c + d*x)*PolyLog[3, -
E^(a + b*x)] + 2*d^2*PolyLog[4, -E^(a + b*x)]))/b^3 + (3*d*(b^2*(c + d*x)^
2*PolyLog[2, E^(a + b*x)] - 2*b*d*(c + d*x)*PolyLog[3, E^(a + b*x)] + 2*d^
2*PolyLog[4, E^(a + b*x)]))/b^3)/b
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 26, 4670, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\text{csch}(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int i(c+dx)^3\csc(ia+ibx)dx$$
$$\Big\downarrow \text{26}$$
$$i\int (c+dx)^3\csc(ia+ibx)dx$$

$$\Big\downarrow \; 4670$$

$$i\left(\frac{3id\int (c+dx)^2\log\left(1-e^{a+bx}\right)dx}{b}-\frac{3id\int (c+dx)^2\log\left(1+e^{a+bx}\right)dx}{b}+\frac{2i(c+dx)^3\operatorname{arctanh}\left(e^{a+bx}\right)}{b}\right)$$

$$\Big\downarrow \; 3011$$

$$i\left(-\frac{3id\left(\frac{2d\int (c+dx)\operatorname{PolyLog}(2,-e^{a+bx})dx}{b}-\frac{(c+dx)^2\operatorname{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{3id\left(\frac{2d\int (c+dx)\operatorname{PolyLog}(2,e^{a+bx})dx}{b}-\frac{(c+dx)^2\operatorname{Poly}}{b}\right.}{b}\right.$$

$$\Big\downarrow \; 7163$$

$$i\left(-\frac{3id\left(\frac{2d\left(\frac{(c+dx)\operatorname{PolyLog}\left(3,-e^{a+bx}\right)}{b}-\frac{d\int \operatorname{PolyLog}\left(3,-e^{a+bx}\right)dx}{b}\right)}{b}-\frac{(c+dx)^2\operatorname{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{3id\left(\frac{2d\left(\frac{(c+dx)\operatorname{PolyLog}\left(3,e^{a+}\right.}{b}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \; 2720$$

$$i\left(-\frac{3id\left(\frac{2d\left(\frac{(c+dx)\operatorname{PolyLog}\left(3,-e^{a+bx}\right)}{b}-\frac{d\int e^{-a-bx}\operatorname{PolyLog}\left(3,-e^{a+bx}\right)de^{a+bx}}{b^2}\right)}{b}-\frac{(c+dx)^2\operatorname{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{3id\left(\frac{2d\left(\frac{(c+dx)\operatorname{P}}{}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \; 7143$$

$$i\left(\frac{2i(c+dx)^3\operatorname{arctanh}\left(e^{a+bx}\right)}{b}-\frac{3id\left(\frac{2d\left(\frac{(c+dx)\operatorname{PolyLog}\left(3,-e^{a+bx}\right)}{b}-\frac{d\operatorname{PolyLog}\left(4,-e^{a+bx}\right)}{b^2}\right)}{b}-\frac{(c+dx)^2\operatorname{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\right.$$

input
```
Int[(c + d*x)^3*Csch[a + b*x],x]
```

output
```
I*(((2*I)*(c + d*x)^3*ArcTanh[E^(a + b*x)])/b - ((3*I)*d*(-(((c + d*x)^2*P
olyLog[2, -E^(a + b*x)])/b) + (2*d*(((c + d*x)*PolyLog[3, -E^(a + b*x)])/b
 - (d*PolyLog[4, -E^(a + b*x)])/b^2))/b))/b + ((3*I)*d*(-(((c + d*x)^2*Pol
yLog[2, E^(a + b*x)])/b) + (2*d*(((c + d*x)*PolyLog[3, E^(a + b*x)])/b - (
d*PolyLog[4, E^(a + b*x)])/b^2))/b))/b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(142) = 284$.

Time = 0.23 (sec) , antiderivative size = 541, normalized size of antiderivative = 3.63

method	result
risch	$-\frac{6d^3 \operatorname{polylog}(4,-e^{bx+a})}{b^4}+\frac{6d^3 \operatorname{polylog}(4,e^{bx+a})}{b^4}+\frac{3c\,d^2 \ln(1-e^{bx+a})x^2}{b}-\frac{3c\,d^2 \ln(1-e^{bx+a})a^2}{b^3}+\frac{6c\,d^2 \operatorname{polylog}(2,e^{bx+a})}{b^2}$

input
```
int((d*x+c)^3*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-6*d^3*polylog(4,-exp(b*x+a))/b^4+6*d^3*polylog(4,exp(b*x+a))/b^4+3/b*c*d^
2*ln(1-exp(b*x+a))*x^2-3/b^3*c*d^2*ln(1-exp(b*x+a))*a^2+6/b^2*c*d^2*polylo
g(2,exp(b*x+a))*x-3/b*c*d^2*ln(exp(b*x+a)+1)*x^2+3/b^3*c*d^2*ln(exp(b*x+a)
+1)*a^2-6/b^2*c*d^2*polylog(2,-exp(b*x+a))*x-3/b*c^2*d*ln(exp(b*x+a)+1)*x-
3/b^2*c^2*d*ln(exp(b*x+a)+1)*a+3/b*c^2*d*ln(1-exp(b*x+a))*x+3/b^2*c^2*d*ln
(1-exp(b*x+a))*a+6/b^2*d*a*c^2*arctanh(exp(b*x+a))-6/b^3*d^2*a^2*c*arctanh
(exp(b*x+a))-2/b*c^3*arctanh(exp(b*x+a))+1/b*d^3*ln(1-exp(b*x+a))*x^3+1/b^
4*d^3*ln(1-exp(b*x+a))*a^3+3/b^2*d^3*polylog(2,exp(b*x+a))*x^2-6/b^3*d^3*p
olylog(3,exp(b*x+a))*x-1/b*d^3*ln(exp(b*x+a)+1)*x^3-1/b^4*d^3*ln(exp(b*x+a
)+1)*a^3-3/b^2*d^3*polylog(2,-exp(b*x+a))*x^2+6/b^3*d^3*polylog(3,-exp(b*x
+a))*x+3/b^2*c^2*d*polylog(2,exp(b*x+a))-3/b^2*c^2*d*polylog(2,-exp(b*x+a)
)-6/b^3*c*d^2*polylog(3,exp(b*x+a))+6/b^3*c*d^2*polylog(3,-exp(b*x+a))+2/b
^4*d^3*a^3*arctanh(exp(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(140) = 280$.

Time = 0.09 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.66

$$\int (c+dx)^3 \operatorname{csch}(a+bx)\,dx$$
$$= \frac{6\,d^3 \mathrm{polylog}(4, \cosh(bx+a)+\sinh(bx+a)) - 6\,d^3 \mathrm{polylog}(4, -\cosh(bx+a)-\sinh(bx+a)) + 3\,(b^2d^3}{}$$

input
```
integrate((d*x+c)^3*csch(b*x+a),x, algorithm="fricas")
```

output
```
(6*d^3*polylog(4, cosh(b*x + a) + sinh(b*x + a)) - 6*d^3*polylog(4, -cosh(
b*x + a) - sinh(b*x + a)) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*di
log(cosh(b*x + a) + sinh(b*x + a)) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*
c^2*d)*dilog(-cosh(b*x + a) - sinh(b*x + a)) - (b^3*d^3*x^3 + 3*b^3*c*d^2*
x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + (b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cosh(b*x + a) + sinh
(b*x + a) - 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*
c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) -
 6*(b*d^3*x + b*c*d^2)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 6*(b*d^
3*x + b*c*d^2)*polylog(3, -cosh(b*x + a) - sinh(b*x + a)))/b^4
```

Sympy [F]

$$\int (c+dx)^3 \operatorname{csch}(a+bx)\,dx = \int (c+dx)^3 \operatorname{csch}(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*csch(b*x+a),x)
```

output
```
Integral((c + d*x)**3*csch(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(140) = 280$.

Time = 0.12 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.23

$$\int (c+dx)^3 \operatorname{csch}(a+bx)\,dx = -c^3\left(\frac{\log\left(e^{(-bx-a)}+1\right)}{b} - \frac{\log\left(e^{(-bx-a)}-1\right)}{b}\right) - \frac{3\left(bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)c^2d}{b^2} + \frac{3\left(bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(e^{(bx+a)}\right)\right)c^2d}{b^2} - \frac{3\left(b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)-2\,\mathrm{Li}_3\left(-e^{(bx+a)}\right)\right)cd^2}{b^3} + \frac{3\left(b^2x^2\log\left(-e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(e^{(bx+a)}\right)-2\,\mathrm{Li}_3\left(e^{(bx+a)}\right)\right)cd^2}{b^3} - \frac{\left(b^3x^3\log\left(e^{(bx+a)}+1\right)+3\,b^2x^2\mathrm{Li}_2\left(-e^{(bx+a)}\right)-6\,bx\mathrm{Li}_3\left(-e^{(bx+a)}\right)+6\,\mathrm{Li}_4\left(-e^{(bx+a)}\right)\right)d^3}{b^4} + \frac{\left(b^3x^3\log\left(-e^{(bx+a)}+1\right)+3\,b^2x^2\mathrm{Li}_2\left(e^{(bx+a)}\right)-6\,bx\mathrm{Li}_3\left(e^{(bx+a)}\right)+6\,\mathrm{Li}_4\left(e^{(bx+a)}\right)\right)d^3}{b^4}$$

input
```
integrate((d*x+c)^3*csch(b*x+a),x, algorithm="maxima")
```

output
```
-c^3*(log(e^(-b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b) - 3*(b*x*log(e^(b
*x + a) + 1) + dilog(-e^(b*x + a)))*c^2*d/b^2 + 3*(b*x*log(-e^(b*x + a) +
1) + dilog(e^(b*x + a)))*c^2*d/b^2 - 3*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b
*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))*c*d^2/b^3 + 3*(b^2*x^
2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x +
 a)))*c*d^2/b^3 - (b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x
+ a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))*d^3/b
^4 + (b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x
*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))*d^3/b^4
```

Giac [F]

$$\int (c+dx)^3\text{csch}(a+bx)\,dx=\int (dx+c)^3\,\text{csch}\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)^3*csch(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*csch(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3\text{csch}(a+bx)\,dx=\int \frac{(c+d\,x)^3}{\sinh\left(a+b\,x\right)}\,dx$$

input
```
int((c + d*x)^3/sinh(a + b*x),x)
```

output
```
int((c + d*x)^3/sinh(a + b*x), x)
```

Reduce [F]

$$\int (c+dx)^3\text{csch}(a+bx)\,dx$$
$$=\frac{\left(\int \text{csch}(bx+a)\,x^3dx\right)b\,d^3+3\left(\int \text{csch}(bx+a)\,x^2dx\right)bc\,d^2+3\left(\int \text{csch}(bx+a)\,xdx\right)b\,c^2d+\log\left(e^{bx+a}-1\right.}{b}$$

input
```
int((d*x+c)^3*csch(b*x+a),x)
```

output
```
(int(csch(a + b*x)*x**3,x)*b*d**3 + 3*int(csch(a + b*x)*x**2,x)*b*c*d**2 +
 3*int(csch(a + b*x)*x,x)*b*c**2*d + log(e**(a + b*x) - 1)*c**3 - log(e**(
a + b*x) + 1)*c**3)/b
```

3.24 $\int (c+dx)^2 \operatorname{csch}(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 99

$$\int (c+dx)^2 \operatorname{csch}(a+bx)\,dx = -\frac{2(c+dx)^2 \operatorname{arctanh}\left(e^{a+bx}\right)}{b} - \frac{2d(c+dx)\operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{b^2} + \frac{2d(c+dx)\operatorname{PolyLog}\left(2,e^{a+bx}\right)}{b^2} + \frac{2d^2\operatorname{PolyLog}\left(3,-e^{a+bx}\right)}{b^3} - \frac{2d^2\operatorname{PolyLog}\left(3,e^{a+bx}\right)}{b^3}$$

output
```
-2*(d*x+c)^2*arctanh(exp(b*x+a))/b-2*d*(d*x+c)*polylog(2,-exp(b*x+a))/b^2+
2*d*(d*x+c)*polylog(2,exp(b*x+a))/b^2+2*d^2*polylog(3,-exp(b*x+a))/b^3-2*d
^2*polylog(3,exp(b*x+a))/b^3
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.19

$$\int (c+dx)^2 \text{csch}(a+bx)\,dx$$
$$= \frac{(c+dx)^2\log\left(1-e^{a+bx}\right)-(c+dx)^2\log\left(1+e^{a+bx}\right)-\frac{2d\left(b(c+dx)\text{PolyLog}\left(2,-e^{a+bx}\right)-d\,\text{PolyLog}\left(3,-e^{a+bx}\right)\right)}{b^2}+\frac{2d(b}{}}{b}$$

input
```
Integrate[(c + d*x)^2*Csch[a + b*x],x]
```

output
```
((c + d*x)^2*Log[1 - E^(a + b*x)] - (c + d*x)^2*Log[1 + E^(a + b*x)] - (2*
d*(b*(c + d*x)*PolyLog[2, -E^(a + b*x)] - d*PolyLog[3, -E^(a + b*x)]))/b^2
 + (2*d*(b*(c + d*x)*PolyLog[2, E^(a + b*x)] - d*PolyLog[3, E^(a + b*x)]))
/b^2)/b
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \text{csch}(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int i(c+dx)^2\csc(ia+ibx)dx$$
$$\Big\downarrow \text{26}$$
$$i\int (c+dx)^2\csc(ia+ibx)dx$$
$$\Big\downarrow \text{4670}$$

$$i\left(\frac{2id\int(c+dx)\log\left(1-e^{a+bx}\right)dx}{b}-\frac{2id\int(c+dx)\log\left(1+e^{a+bx}\right)dx}{b}+\frac{2i(c+dx)^2\text{arctanh}\left(e^{a+bx}\right)}{b}\right)$$

$\Big\downarrow$ 3011

$$i\left(-\frac{2id\left(\frac{d\int\text{PolyLog}(2,-e^{a+bx})dx}{b}-\frac{(c+dx)\text{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{2id\left(\frac{d\int\text{PolyLog}(2,e^{a+bx})dx}{b}-\frac{(c+dx)\text{PolyLog}(2,e^{a+bx})}{b}\right)}{b}+\right.$$

$\Big\downarrow$ 2720

$$i\left(-\frac{2id\left(\frac{d\int e^{-a-bx}\text{PolyLog}(2,-e^{a+bx})de^{a+bx}}{b^2}-\frac{(c+dx)\text{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{2id\left(\frac{d\int e^{-a-bx}\text{PolyLog}(2,e^{a+bx})de^{a+bx}}{b^2}-\frac{(c+dx}{}\right.}{b}\right.$$

$\Big\downarrow$ 7143

$$i\left(\frac{2i(c+dx)^2\text{arctanh}\left(e^{a+bx}\right)}{b}-\frac{2id\left(\frac{d\text{PolyLog}(3,-e^{a+bx})}{b^2}-\frac{(c+dx)\text{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{2id\left(\frac{d\text{PolyLog}(3,e^{a+bx})}{b^2}-\frac{(c+}{}\right.}{b}\right.$$

input
```
Int[(c + d*x)^2*Csch[a + b*x],x]
```

output
```
I*(((2*I)*(c + d*x)^2*ArcTanh[E^(a + b*x)])/b - ((2*I)*d*(-(((c + d*x)*Pol
yLog[2, -E^(a + b*x)])/b) + (d*PolyLog[3, -E^(a + b*x)])/b^2))/b + ((2*I)*
d*(-(((c + d*x)*PolyLog[2, E^(a + b*x)])/b) + (d*PolyLog[3, E^(a + b*x)])/
b^2))/b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(94) = 188.

Time = 0.17 (sec) , antiderivative size = 306, normalized size of antiderivative = 3.09

method	result
risch	$-\frac{d^{2} \ln \left(1-\mathrm{e}^{b x +a}\right) a^{2}}{b^{3}}+\frac{2 d^{2} \operatorname{polylog}\left(2, \mathrm{e}^{b x +a}\right) x}{b^{2}}+\frac{d^{2} \ln \left(\mathrm{e}^{b x +a}+1\right) a^{2}}{b^{3}}-\frac{2 d^{2} \operatorname{polylog}\left(2, -\mathrm{e}^{b x +a}\right) x}{b^{2}}-\frac{2 c^{2} \operatorname{arctanh}\left(\mathrm{e}^{b x +a}\right)}{b}+$

input
```
int((d*x+c)^2*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/b^3*d^2*ln(1-exp(b*x+a))*a^2+2/b^2*d^2*polylog(2,exp(b*x+a))*x+1/b^3*d^
2*ln(exp(b*x+a)+1)*a^2-2/b^2*d^2*polylog(2,-exp(b*x+a))*x-2/b*c^2*arctanh(
exp(b*x+a))+1/b*d^2*ln(1-exp(b*x+a))*x^2-2*d^2*polylog(3,exp(b*x+a))/b^3-1
/b*d^2*ln(exp(b*x+a)+1)*x^2+2*d^2*polylog(3,-exp(b*x+a))/b^3+2/b*c*d*ln(1-
exp(b*x+a))*x+2/b^2*c*d*polylog(2,exp(b*x+a))-2/b*c*d*ln(exp(b*x+a)+1)*x-2
/b^2*c*d*polylog(2,-exp(b*x+a))-2/b^3*d^2*a^2*arctanh(exp(b*x+a))+2/b^2*c*
d*ln(1-exp(b*x+a))*a-2/b^2*c*d*ln(exp(b*x+a)+1)*a+4/b^2*d*a*c*arctanh(exp(
b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(92) = 184.

Time = 0.08 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.44

$$\int (c+dx)^2 \mathrm{csch}(a+bx)\,dx =$$
$$-\frac{2\,d^2\mathrm{polylog}(3,\cosh{(bx+a)}+\sinh{(bx+a)})-2\,d^2\mathrm{polylog}(3,-\cosh{(bx+a)}-\sinh{(bx+a)})-2\,(bc}{}$$

input
```
integrate((d*x+c)^2*csch(b*x+a),x, algorithm="fricas")
```

output
```
-(2*d^2*polylog(3, cosh(b*x + a) + sinh(b*x + a)) - 2*d^2*polylog(3, -cosh
(b*x + a) - sinh(b*x + a)) - 2*(b*d^2*x + b*c*d)*dilog(cosh(b*x + a) + sin
h(b*x + a)) + 2*(b*d^2*x + b*c*d)*dilog(-cosh(b*x + a) - sinh(b*x + a)) +
(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cosh(b*x + a) + sinh(b*x + a) +
1) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1
) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cosh(b*x + a) -
 sinh(b*x + a) + 1))/b^3
```

Sympy [F]

$$\int (c+dx)^2\text{csch}(a+bx)\,dx=\int (c+dx)^2\,\text{csch}\,(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*csch(b*x+a),x)
```

output
```
Integral((c + d*x)**2*csch(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(92) = 184$.

Time = 0.12 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.97

$$\begin{aligned}
&\int (c+dx)^2\text{csch}(a+bx)\,dx \\
&= -c^2\left(\frac{\log\left(e^{(-bx-a)}+1\right)}{b}-\frac{\log\left(e^{(-bx-a)}-1\right)}{b}\right) \\
&\quad-\frac{2\left(bx\log\left(e^{(bx+a)}+1\right)+\text{Li}_2\left(-e^{(bx+a)}\right)\right)cd}{b^2} \\
&\quad+\frac{2\left(bx\log\left(-e^{(bx+a)}+1\right)+\text{Li}_2\left(e^{(bx+a)}\right)\right)cd}{b^2} \\
&\quad-\frac{\left(b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\text{Li}_2\left(-e^{(bx+a)}\right)-2\,\text{Li}_3\left(-e^{(bx+a)}\right)\right)d^2}{b^3} \\
&\quad+\frac{\left(b^2x^2\log\left(-e^{(bx+a)}+1\right)+2\,bx\text{Li}_2\left(e^{(bx+a)}\right)-2\,\text{Li}_3\left(e^{(bx+a)}\right)\right)d^2}{b^3}
\end{aligned}$$

input
```
integrate((d*x+c)^2*csch(b*x+a),x, algorithm="maxima")
```

output
```
-c^2*(log(e^(-b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b) - 2*(b*x*log(e^(b
*x + a) + 1) + dilog(-e^(b*x + a)))*c*d/b^2 + 2*(b*x*log(-e^(b*x + a) + 1)
 + dilog(e^(b*x + a)))*c*d/b^2 - (b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dil
og(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))*d^2/b^3 + (b^2*x^2*log(-e^(
b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))*d^2/
b^3
```

Giac [F]

$$\int (c+dx)^2 \mathrm{csch}(a+bx)\,dx = \int (dx+c)^2 \operatorname{csch}(bx+a)\,dx$$

input
```
integrate((d*x+c)^2*csch(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*csch(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \mathrm{csch}(a+bx)\,dx = \int \frac{(c+d\,x)^2}{\sinh(a+b\,x)}\,dx$$

input
```
int((c + d*x)^2/sinh(a + b*x),x)
```

output
```
int((c + d*x)^2/sinh(a + b*x), x)
```

Reduce [F]

$$\int (c+dx)^2 \operatorname{csch}(a+bx)\,dx$$
$$= \frac{\left(\int \operatorname{csch}(bx+a)\,x^2dx\right) b\,d^2 + 2\left(\int \operatorname{csch}(bx+a)\,xdx\right) bcd + \log\left(e^{bx+a}-1\right)c^2 - \log\left(e^{bx+a}+1\right)c^2}{b}$$

input
```
int((d*x+c)^2*csch(b*x+a),x)
```

output
```
(int(csch(a + b*x)*x**2,x)*b*d**2 + 2*int(csch(a + b*x)*x,x)*b*c*d + log(e
**(a + b*x) - 1)*c**2 - log(e**(a + b*x) + 1)*c**2)/b
```

3.25 $\int (c+dx)\text{csch}(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 50

$$\int (c+dx)\text{csch}(a+bx)\,dx=-\frac{2(c+dx)\text{arctanh}\left(e^{a+bx}\right)}{b}-\frac{d\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{d\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}$$

output
```
-2*(d*x+c)*arctanh(exp(b*x+a))/b-d*polylog(2,-exp(b*x+a))/b^2+d*polylog(2,
exp(b*x+a))/b^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.80

$$\int (c+dx)\text{csch}(a+bx)\,dx=-\frac{c\text{arctanh}(\cosh(a+bx))}{b}+2d\left(\frac{x\log\left(1-e^{a+bx}\right)}{2b}-\frac{x\log\left(1+e^{a+bx}\right)}{2b}-\frac{\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2}+\frac{\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2}\right)$$

input
```
Integrate[(c + d*x)*Csch[a + b*x],x]
```

output
```
-((c*ArcTanh[Cosh[a + b*x]])/b) + 2*d*((x*Log[1 - E^(a + b*x)])/(2*b) - (x
*Log[1 + E^(a + b*x)])/(2*b) - PolyLog[2, -E^(a + b*x)]/(2*b^2) + PolyLog[
2, E^(a + b*x)]/(2*b^2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\text{csch}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i(c+dx)\csc(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$i\int (c+dx)\csc(ia+ibx)dx$$

$$\Big\downarrow \text{4670}$$

$$i\left(\frac{id\int \log\left(1-e^{a+bx}\right)dx}{b}-\frac{id\int \log\left(1+e^{a+bx}\right)dx}{b}+\frac{2i(c+dx)\text{arctanh}\left(e^{a+bx}\right)}{b}\right)$$

$$\Big\downarrow \text{2715}$$

$$i\left(\frac{id\int e^{-a-bx}\log\left(1-e^{a+bx}\right)de^{a+bx}}{b^2}-\frac{id\int e^{-a-bx}\log\left(1+e^{a+bx}\right)de^{a+bx}}{b^2}+\frac{2i(c+dx)\text{arctanh}\left(e^{a+bx}\right)}{b}\right)$$

$$\Big\downarrow \text{2838}$$

$$i\left(\frac{2i(c+dx)\text{arctanh}(e^{a+bx})}{b}+\frac{id\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}-\frac{id\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}\right)$$

input
```
Int[(c + d*x)*Csch[a + b*x],x]
```

output
```
I*(((2*I)*(c + d*x)*ArcTanh[E^(a + b*x)])/b + (I*d*PolyLog[2, -E^(a + b*x)
])/b^2 - (I*d*PolyLog[2, E^(a + b*x)])/b^2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.78

method	result
derivativedivides	$\frac{\frac{d\left((bx+a)\ln\left(1-e^{bx+a}\right)+\operatorname{polylog}\left(2,e^{bx+a}\right)-(bx+a)\ln\left(e^{bx+a}+1\right)-\operatorname{polylog}\left(2,-e^{bx+a}\right)\right)}{b}+\frac{2da\,\operatorname{arctanh}\left(e^{bx+a}\right)}{b}-2c\,\operatorname{arctanh}(e}{b}$
default	$\frac{\frac{d\left((bx+a)\ln\left(1-e^{bx+a}\right)+\operatorname{polylog}\left(2,e^{bx+a}\right)-(bx+a)\ln\left(e^{bx+a}+1\right)-\operatorname{polylog}\left(2,-e^{bx+a}\right)\right)}{b}+\frac{2da\,\operatorname{arctanh}\left(e^{bx+a}\right)}{b}-2c\,\operatorname{arctanh}(e}{b}$
parts	$\frac{\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)dx}{b}+\frac{\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)c}{b}+\frac{2d\left(-\frac{\operatorname{dilog}\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}-\frac{\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\ln\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}\right.}{b^2}$
risch	$-\frac{2c\,\operatorname{arctanh}(e^{bx+a})}{b}+\frac{d\ln(1-e^{bx+a})x}{b}+\frac{d\ln(1-e^{bx+a})a}{b^2}+\frac{d\,\operatorname{polylog}(2,e^{bx+a})}{b^2}-\frac{d\ln(e^{bx+a}+1)x}{b}-\frac{d\ln(}{}$

input
```
int((d*x+c)*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(d/b*((b*x+a)*ln(1-exp(b*x+a))+polylog(2,exp(b*x+a))-(b*x+a)*ln(exp(b*
x+a)+1)-polylog(2,-exp(b*x+a)))+2*d/b*a*arctanh(exp(b*x+a))-2*c*arctanh(ex
p(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(45) = 90.

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.38

$$\int (c+dx)\operatorname{csch}(a+bx)\,dx$$
$$=\frac{d\operatorname{Li}_2(\cosh(bx+a)+\sinh(bx+a))-d\operatorname{Li}_2(-\cosh(bx+a)-\sinh(bx+a))-(bdx+bc)\log(\cosh(bx+}{}$$

input
```
integrate((d*x+c)*csch(b*x+a),x, algorithm="fricas")
```

output
```
(d*dilog(cosh(b*x + a) + sinh(b*x + a)) - d*dilog(-cosh(b*x + a) - sinh(b*
x + a)) - (b*d*x + b*c)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + (b*c - a*
d)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + (b*d*x + a*d)*log(-cosh(b*x +
a) - sinh(b*x + a) + 1))/b^2
```

Sympy [F]

$$\int (c+dx)\text{csch}(a+bx)\,dx=\int (c+dx)\,\text{csch}\,(a+bx)\,dx$$

input
```
integrate((d*x+c)*csch(b*x+a),x)
```

output
```
Integral((c + d*x)*csch(a + b*x), x)
```

Maxima [F]

$$\int (c+dx)\text{csch}(a+bx)\,dx=\int (dx+c)\,\text{csch}\,(bx+a)\,\,dx$$

input
```
integrate((d*x+c)*csch(b*x+a),x, algorithm="maxima")
```

output
```
-c*(log(e^(-b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b) + 2*d*(integrate(1/
2*x/(e^(b*x + a) + 1), x) + integrate(1/2*x/(e^(b*x + a) - 1), x))
```

Giac [F]

$$\int (c+dx)\text{csch}(a+bx)\,dx=\int (dx+c)\,\text{csch}\,(bx+a)\,\,dx$$

input
```
integrate((d*x+c)*csch(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*csch(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\text{csch}(a+bx)\,dx = \int \frac{c+d\,x}{\sinh(a+b\,x)}\,dx$$

input
```
int((c + d*x)/sinh(a + b*x),x)
```

output
```
int((c + d*x)/sinh(a + b*x), x)
```

Reduce [F]

$$\int (c+dx)\text{csch}(a+bx)\,dx = \frac{\left(\int \text{csch}(bx+a)\,xdx\right)bd + \log\left(e^{bx+a}-1\right)c - \log\left(e^{bx+a}+1\right)c}{b}$$

input
```
int((d*x+c)*csch(b*x+a),x)
```

output
```
(int(csch(a + b*x)*x,x)*b*d + log(e**(a + b*x) - 1)*c - log(e**(a + b*x) +
 1)*c)/b
```

3.26 $\int \frac{\operatorname{csch}(a+bx)}{c+dx}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\operatorname{csch}(a+bx)}{c+dx}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}(a+bx)}{c+dx},x\right)$$

output
```
Defer(Int)(csch(b*x+a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 14.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{csch}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{csch}(a+bx)}{c+dx}\,dx$$

input
```
Integrate[Csch[a + b*x]/(c + d*x),x]
```

output
```
Integrate[Csch[a + b*x]/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(a+bx)}{c+dx} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{i\csc(ia+ibx)}{c+dx}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{\csc(ia+ibx)}{c+dx}dx$$

$$\Big\downarrow \; 4680$$

$$\int \frac{\operatorname{csch}(a+bx)}{c+dx}dx$$

input
```
Int[Csch[a + b*x]/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(bx+a)}{dx+c}dx$$

input
```
int(csch(b*x+a)/(d*x+c),x)
```

output
```
int(csch(b*x+a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{csch}(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)}{dx+c}\,dx$$

input
```
integrate(csch(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{csch}(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{csch}\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(csch(b*x+a)/(d*x+c),x)
```

output
```
Integral(csch(a + b*x)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{csch}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{csch}(bx+a)}{dx+c}\,dx$$

input
```
integrate(csch(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(csch(b*x + a)/(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{csch}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{csch}(bx+a)}{dx+c}\,dx$$

input
```
integrate(csch(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{\operatorname{csch}(a+bx)}{c+dx}\,dx = \int \frac{1}{\sinh(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(1/(sinh(a + b*x)*(c + d*x)),x)
```

output
```
int(1/(sinh(a + b*x)*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{csch}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{csch}(bx+a)}{dx+c}dx$$

input
```
int(csch(b*x+a)/(d*x+c),x)
```

output
```
int(csch(a + b*x)/(c + d*x),x)
```

3.27 $\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2} \, dx = \operatorname{Int}\left(\frac{\operatorname{csch}(a+bx)}{(c+dx)^2}, x\right)$$

output
```
Defer(Int)(csch(b*x+a)/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 14.55 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2} \, dx = \int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2} \, dx$$

input
```
Integrate[Csch[a + b*x]/(c + d*x)^2,x]
```

output
```
Integrate[Csch[a + b*x]/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{i \csc(ia+ibx)}{(c+dx)^2}dx$$

$$\Big\downarrow \; 26$$

$$i \int \frac{\csc(ia+ibx)}{(c+dx)^2}dx$$

$$\Big\downarrow \; 4680$$

$$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2}dx$$

input
```
Int[Csch[a + b*x]/(c + d*x)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(bx+a\right)}{\left(dx+c\right)^{2}}dx$$

input
```
int(csch(b*x+a)/(d*x+c)^2,x)
```

output
```
int(csch(b*x+a)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(csch(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{csch}\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(csch(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(csch(a + b*x)/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(csch(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(csch(b*x + a)/(d*x + c)^2, x)
```

Giac [N/A]

Not integrable

Time = 2.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(csch(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2}\,dx = \int \frac{1}{\sinh\left(a+b\,x\right)\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/(sinh(a + b*x)*(c + d*x)^2),x)
```

output
```
int(1/(sinh(a + b*x)*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{csch}(bx+a)}{d^2x^2+2cdx+c^2}dx$$

input
```
int(csch(b*x+a)/(d*x+c)^2,x)
```

output
```
int(csch(a + b*x)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.28 $\int (c+dx)^3 \text{csch}^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 103

$$\int (c+dx)^3 \text{csch}^2(a+bx)\,dx = -\frac{(c+dx)^3}{b}-\frac{(c+dx)^3\coth(a+bx)}{b}+\frac{3d(c+dx)^2\log\left(1-e^{2(a+bx)}\right)}{b^2}+\frac{3d^2(c+dx)\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^3}-\frac{3d^3\,\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^4}$$

output
```
-(d*x+c)^3/b-(d*x+c)^3*coth(b*x+a)/b+3*d*(d*x+c)^2*ln(1-exp(2*b*x+2*a))/b^
2+3*d^2*(d*x+c)*polylog(2,exp(2*b*x+2*a))/b^3-3/2*d^3*polylog(3,exp(2*b*x+
2*a))/b^4
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.80

$$\int (c+dx)^3\text{csch}^2(a+bx)\,dx$$
$$=\frac{-\frac{2(c+dx)^3}{-1+e^{2a}}+\frac{3d(c+dx)^2\log(1-e^{-a-bx})}{b}+\frac{3d(c+dx)^2\log(1+e^{-a-bx})}{b}-\frac{6d^2(b(c+dx)\,\text{PolyLog}(2,-e^{-a-bx})+d\,\text{PolyLog}(3,-e^{-a-bx}))}{b^3}}{b}$$

input
```
Integrate[(c + d*x)^3*Csch[a + b*x]^2,x]
```

output
```
((-2*(c + d*x)^3)/(-1 + E^(2*a)) + (3*d*(c + d*x)^2*Log[1 - E^(-a - b*x)])
/b + (3*d*(c + d*x)^2*Log[1 + E^(-a - b*x)])/b - (6*d^2*(b*(c + d*x)*PolyL
og[2, -E^(-a - b*x)] + d*PolyLog[3, -E^(-a - b*x)]))/b^3 - (6*d^2*(b*(c +
d*x)*PolyLog[2, E^(-a - b*x)] + d*PolyLog[3, E^(-a - b*x)]))/b^3 + (c + d*
x)^3*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.42, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {3042, 25, 4672, 26, 3042, 26, 4201, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\text{csch}^2(a+bx)\,dx$$
$$\Big\downarrow \; 3042$$
$$\int -(c+dx)^3\csc(ia+ibx)^2dx$$
$$\Big\downarrow \; 25$$
$$-\int (c+dx)^3\csc(ia+ibx)^2dx$$

$$\Big\downarrow \; 4672$$

$$-\frac{(c+dx)^3\coth(a+bx)}{b}+\frac{3id\int -i(c+dx)^2\coth(a+bx)dx}{b}$$

$$\Big\downarrow \; 26$$

$$\frac{3d\int (c+dx)^2\coth(a+bx)dx}{b}-\frac{(c+dx)^3\coth(a+bx)}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{(c+dx)^3\coth(a+bx)}{b}+\frac{3d\int -i(c+dx)^2\tan\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{(c+dx)^3\coth(a+bx)}{b}-\frac{3id\int (c+dx)^2\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx}{b}$$

$$\Big\downarrow \; 4201$$

$$-\frac{(c+dx)^3\coth(a+bx)}{b}-\frac{3id\left(2i\int \frac{e^{2a+2bx-i\pi}(c+dx)^2}{1+e^{2a+2bx-i\pi}}dx-\frac{i(c+dx)^3}{3d}\right)}{b}$$

$$\Big\downarrow \; 2620$$

$$-\frac{(c+dx)^3\coth(a+bx)}{b}-\frac{3id\left(2i\left(\frac{(c+dx)^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{d\int (c+dx)\log\left(1+e^{2a+2bx-i\pi}\right)dx}{b}\right)-\frac{i(c+dx)^3}{3d}\right)}{b}$$

$$\Big\downarrow \; 3011$$

$$-\frac{(c+dx)^3\coth(a+bx)}{b}-\frac{3id\left(2i\left(\frac{(c+dx)^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{d\left(\frac{d\int \text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{2b}-\frac{(c+dx)\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{b}\right)-\frac{i(c+dx)^3}{3d}\right)}{b}$$

$$\Big\downarrow \; 2720$$

$$-\frac{(c+dx)^3\coth(a+bx)}{b}-\frac{3id\left(2i\left(\frac{(c+dx)^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{d\left(\frac{d\int e^{-2a-2bx+i\pi}\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}-\frac{(c+dx)\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{b}\right)-\right.}{b}$$

$$\Big\downarrow \text{ 7143}$$

$$-\frac{(c+dx)^3\coth(a+bx)}{b}-\frac{3id\left(2i\left(\frac{(c+dx)^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{d\left(\frac{d\,\text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)}{4b^2}-\frac{(c+dx)\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{b}\right)-\frac{i(c+dx)^3}{3d}\right)}{b}$$

input
```
Int[(c + d*x)^3*Csch[a + b*x]^2,x]
```

output
```
-(((c + d*x)^3*Coth[a + b*x])/b) - ((3*I)*d*(((-1/3*I)*(c + d*x)^3)/d + (2
*I)*(((c + d*x)^2*Log[1 + E^(2*a - I*Pi + 2*b*x)])/(2*b) - (d*(-1/2*((c +
d*x)*PolyLog[2, -E^(2*a - I*Pi + 2*b*x)])/b + (d*PolyLog[3, -E^(2*a - I*Pi
 + 2*b*x)])/(4*b^2)))/b)))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(101) = 202$.

Time = 0.13 (sec) , antiderivative size = 473, normalized size of antiderivative = 4.59

method	result
risch	$-\frac{12 d^{2} c a x}{b^{2}}-\frac{6 d^{2} c a \ln \left(\mathrm{e}^{b x +a}-1\right)}{b^{3}}+\frac{12 d^{2} c a \ln \left(\mathrm{e}^{b x +a}\right)}{b^{3}}+\frac{6 d^{2} c \ln \left(1-\mathrm{e}^{b x +a}\right) x}{b^{2}}+\frac{6 d^{2} c \ln \left(1-\mathrm{e}^{b x +a}\right) a}{b^{3}}+\frac{6 d^{2} c \ln \left(\mathrm{e}^{b x +a}+1\right) x}{b^{2}}$

input
```
int((d*x+c)^3*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-12*d^2/b^2*c*a*x-6*d^2/b^3*c*a*ln(exp(b*x+a)-1)+12*d^2/b^3*c*a*ln(exp(b*x
+a))+6*d^2/b^2*c*ln(1-exp(b*x+a))*x+6*d^2/b^3*c*ln(1-exp(b*x+a))*a+6*d^2/b
^2*c*ln(exp(b*x+a)+1)*x-2*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/(exp(2*b*x+2
*a)-1)/b-6*d^3/b^4*polylog(3,exp(b*x+a))-6*d^3/b^4*polylog(3,-exp(b*x+a))-
2*d^3/b*x^3+4*d^3/b^4*a^3-6*d^2/b*c*x^2-6*d^2/b^3*c*a^2+6*d^3/b^3*a^2*x+3*
d/b^2*c^2*ln(exp(b*x+a)-1)-6*d/b^2*c^2*ln(exp(b*x+a))+3*d/b^2*c^2*ln(exp(b
*x+a)+1)+6*d^2/b^3*c*polylog(2,exp(b*x+a))+6*d^2/b^3*c*polylog(2,-exp(b*x+
a))+3*d^3/b^4*a^2*ln(exp(b*x+a)-1)-6*d^3/b^4*a^2*ln(exp(b*x+a))+3*d^3/b^2*
ln(1-exp(b*x+a))*x^2-3*d^3/b^4*ln(1-exp(b*x+a))*a^2+6*d^3/b^3*polylog(2,ex
p(b*x+a))*x+3*d^3/b^2*ln(exp(b*x+a)+1)*x^2+6*d^3/b^3*polylog(2,-exp(b*x+a)
)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1159 vs. $2(100) = 200$.

Time = 0.09 (sec) , antiderivative size = 1159, normalized size of antiderivative = 11.25

$$\int (c+dx)^3 \operatorname{csch}^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(2*b^3*c^3 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 2*a^3*d^3 + 2*(b^3*d^3*x^3 +
 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3
)*cosh(b*x + a)^2 + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a
*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*cosh(b*x + a)*sinh(b*x + a) + 2*(b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
 + a^3*d^3)*sinh(b*x + a)^2 + 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*c
osh(b*x + a)^2 - 2*(b*d^3*x + b*c*d^2)*cosh(b*x + a)*sinh(b*x + a) - (b*d^
3*x + b*c*d^2)*sinh(b*x + a)^2)*dilog(cosh(b*x + a) + sinh(b*x + a)) + 6*(
b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cosh(b*x + a)^2 - 2*(b*d^3*x + b*c
*d^2)*cosh(b*x + a)*sinh(b*x + a) - (b*d^3*x + b*c*d^2)*sinh(b*x + a)^2)*d
ilog(-cosh(b*x + a) - sinh(b*x + a)) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^
2*c^2*d - (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cosh(b*x + a)^2 - 2*(b
^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cosh(b*x + a)*sinh(b*x + a) - (b^2
*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*sinh(b*x + a)^2)*log(cosh(b*x + a) +
 sinh(b*x + a) + 1) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3 - (b^2*c^2*d -
2*a*b*c*d^2 + a^2*d^3)*cosh(b*x + a)^2 - 2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*
d^3)*cosh(b*x + a)*sinh(b*x + a) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sin
h(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 3*(b^2*d^3*x^2 + 2*
b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3 - (b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b
*c*d^2 - a^2*d^3)*cosh(b*x + a)^2 - 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*...
```

Sympy [F]

$$\int (c+dx)^3 \mathrm{csch}^2(a+bx)\,dx = \int (c+dx)^3 \,\mathrm{csch}^2\,(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*csch(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**3*csch(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(100) = 200$.

Time = 0.22 (sec) , antiderivative size = 320, normalized size of antiderivative = 3.11

$$\int (c+dx)^3\operatorname{csch}^2(a+bx)\,dx$$
$$= -3\,c^2d\left(\frac{2\,xe^{(2\,bx+2\,a)}}{be^{(2\,bx+2\,a)}-b} - \frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^2} - \frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^2}\right)$$
$$+ \frac{6\left(bx\log\left(e^{(bx+a)}+1\right)+\operatorname{Li}_2\left(-e^{(bx+a)}\right)\right)cd^2}{b^3}$$
$$+ \frac{6\left(bx\log\left(-e^{(bx+a)}+1\right)+\operatorname{Li}_2\left(e^{(bx+a)}\right)\right)cd^2}{b^3}$$
$$+ \frac{2\,c^3}{b(e^{(-2\,bx-2\,a)}-1)} - \frac{2\left(d^3x^3+3\,cd^2x^2\right)}{be^{(2\,bx+2\,a)}-b}$$
$$+ \frac{3\left(b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\operatorname{Li}_2\left(-e^{(bx+a)}\right)-2\operatorname{Li}_3(-e^{(bx+a)})\right)d^3}{b^4}$$
$$+ \frac{3\left(b^2x^2\log\left(-e^{(bx+a)}+1\right)+2\,bx\operatorname{Li}_2\left(e^{(bx+a)}\right)-2\operatorname{Li}_3(e^{(bx+a)})\right)d^3}{b^4}$$
$$- \frac{2\left(b^3d^3x^3+3\,b^3cd^2x^2\right)}{b^4}$$

input
```
integrate((d*x+c)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
-3*c^2*d*(2*x*e^(2*b*x + 2*a)/(b*e^(2*b*x + 2*a) - b) - log((e^(b*x + a) +
 1)*e^(-a))/b^2 - log((e^(b*x + a) - 1)*e^(-a))/b^2) + 6*(b*x*log(e^(b*x +
 a) + 1) + dilog(-e^(b*x + a)))*c*d^2/b^3 + 6*(b*x*log(-e^(b*x + a) + 1) +
 dilog(e^(b*x + a)))*c*d^2/b^3 + 2*c^3/(b*(e^(-2*b*x - 2*a) - 1)) - 2*(d^3
*x^3 + 3*c*d^2*x^2)/(b*e^(2*b*x + 2*a) - b) + 3*(b^2*x^2*log(e^(b*x + a) +
 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))*d^3/b^4 + 3*
(b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e
^(b*x + a)))*d^3/b^4 - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2)/b^4
```

Giac [F]

$$\int (c+dx)^3\text{csch}^2(a+bx)\,dx = \int (dx+c)^3\,\text{csch}\,(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^3*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*csch(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3\text{csch}^2(a+bx)\,dx = \int \frac{(c+d\,x)^3}{\sinh(a+b\,x)^2}\,dx$$

input
```
int((c + d*x)^3/sinh(a + b*x)^2,x)
```

output
```
int((c + d*x)^3/sinh(a + b*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^3\text{csch}^2(a+bx)\,dx$$
$$= \frac{-4b^3d^3x^3 - 6b^2d^3x^2 + 3e^{2bx+2a}\log\left(e^{bx+a}-1\right)d^3 + 3e^{2bx+2a}\log\left(e^{bx+a}+1\right)d^3 - 12e^{2bx+2a}\left(\int \frac{x^2}{e^{4bx+4a}-2e^{2bx+}}\right.}{}$$

input
```
int((d*x+c)^3*csch(b*x+a)^2,x)
```

output

```
( - 12*e**(2*a + 2*b*x)*int(x**2/(e**(4*a + 4*b*x) - 2*e**(2*a + 2*b*x) +
1),x)*b**3*d**3 - 24*e**(2*a + 2*b*x)*int(x/(e**(4*a + 4*b*x) - 2*e**(2*a
+ 2*b*x) + 1),x)*b**3*c*d**2 - 12*e**(2*a + 2*b*x)*int(x/(e**(4*a + 4*b*x)
 - 2*e**(2*a + 2*b*x) + 1),x)*b**2*d**3 + 6*e**(2*a + 2*b*x)*log(e**(a + b
*x) - 1)*b**2*c**2*d + 6*e**(2*a + 2*b*x)*log(e**(a + b*x) - 1)*b*c*d**2 +
 3*e**(2*a + 2*b*x)*log(e**(a + b*x) - 1)*d**3 + 6*e**(2*a + 2*b*x)*log(e*
*(a + b*x) + 1)*b**2*c**2*d + 6*e**(2*a + 2*b*x)*log(e**(a + b*x) + 1)*b*c
*d**2 + 3*e**(2*a + 2*b*x)*log(e**(a + b*x) + 1)*d**3 - 4*e**(2*a + 2*b*x)
*b**3*c**3 - 12*e**(2*a + 2*b*x)*b**3*c**2*d*x - 12*e**(2*a + 2*b*x)*b**2*
c*d**2*x - 6*e**(2*a + 2*b*x)*b*d**3*x + 12*int(x**2/(e**(4*a + 4*b*x) - 2
*e**(2*a + 2*b*x) + 1),x)*b**3*d**3 + 24*int(x/(e**(4*a + 4*b*x) - 2*e**(2
*a + 2*b*x) + 1),x)*b**3*c*d**2 + 12*int(x/(e**(4*a + 4*b*x) - 2*e**(2*a +
 2*b*x) + 1),x)*b**2*d**3 - 6*log(e**(a + b*x) - 1)*b**2*c**2*d - 6*log(e*
*(a + b*x) - 1)*b*c*d**2 - 3*log(e**(a + b*x) - 1)*d**3 - 6*log(e**(a + b*
x) + 1)*b**2*c**2*d - 6*log(e**(a + b*x) + 1)*b*c*d**2 - 3*log(e**(a + b*x
) + 1)*d**3 - 12*b**3*c*d**2*x**2 - 4*b**3*d**3*x**3 - 6*b**2*d**3*x**2)/(
2*b**4*(e**(2*a + 2*b*x) - 1))
```

3.29 $\int (c+dx)^2\mathbf{csch}^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 74

$$\int (c+dx)^2\text{csch}^2(a+bx)\,dx = -\frac{(c+dx)^2}{b}-\frac{(c+dx)^2\coth(a+bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{2(a+bx)}\right)}{b^2}+\frac{d^2\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^3}$$

output
```
-(d*x+c)^2/b-(d*x+c)^2*coth(b*x+a)/b+2*d*(d*x+c)*ln(1-exp(2*b*x+2*a))/b^2+
d^2*polylog(2,exp(2*b*x+2*a))/b^3
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.85

$$\int (c+dx)^2\text{csch}^2(a+bx)\,dx = \frac{-\frac{2b(c+dx)\left(b(c+dx)-d\left(-1+e^{2a}\right)\log\left(1-e^{-a-bx}\right)-d\left(-1+e^{2a}\right)\log\left(1+e^{-a-bx}\right)\right)}{-1+e^{2a}}-2d^2\,\text{PolyLog}\left(2,-e^{-a-bx}\right)-2d^2\,\text{PolyLog}}{b^3}$$

input
```
Integrate[(c + d*x)^2*Csch[a + b*x]^2,x]
```

output
```
((-2*b*(c + d*x)*(b*(c + d*x) - d*(-1 + E^(2*a))*Log[1 - E^(-a - b*x)] - d
*(-1 + E^(2*a))*Log[1 + E^(-a - b*x)]))/(-1 + E^(2*a)) - 2*d^2*PolyLog[2,
-E^(-a - b*x)] - 2*d^2*PolyLog[2, E^(-a - b*x)] + b^2*(c + d*x)^2*Csch[a]*
Csch[a + b*x]*Sinh[b*x])/b^3
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.43, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3042, 25, 4672, 26, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\text{csch}^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -(c+dx)^2\csc(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)^2\csc(ia+ibx)^2dx$$

$$\Big\downarrow \text{4672}$$

$$-\frac{(c+dx)^2\coth(a+bx)}{b}+\frac{2id\int -i(c+dx)\coth(a+bx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{2d\int (c+dx)\coth(a+bx)dx}{b}-\frac{(c+dx)^2\coth(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(c+dx)^2\coth(a+bx)}{b}+\frac{2d\int -i(c+dx)\tan\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{(c+dx)^2\coth(a+bx)}{b}-\frac{2id\int (c+dx)\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx}{b}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{(c+dx)^2\coth(a+bx)}{b}-\frac{2id\left(2i\int \frac{e^{2a+2bx-i\pi}(c+dx)}{1+e^{2a+2bx-i\pi}}dx-\frac{i(c+dx)^2}{2d}\right)}{b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{(c+dx)^2\coth(a+bx)}{b}-\frac{2id\left(2i\left(\frac{(c+dx)\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{d\int \log\left(1+e^{2a+2bx-i\pi}\right)dx}{2b}\right)-\frac{i(c+dx)^2}{2d}\right)}{b}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{(c+dx)^2\coth(a+bx)}{b}-\frac{2id\left(2i\left(\frac{(c+dx)\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{d\int e^{-2a-2bx+i\pi}\log\left(1+e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}\right)-\frac{i(c+dx)^2}{2d}\right)}{b}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{(c+dx)^2\coth(a+bx)}{b}-\frac{2id\left(2i\left(\frac{d\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{4b^2}+\frac{(c+dx)\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}\right)-\frac{i(c+dx)^2}{2d}\right)}{b}$$

input

```
Int[(c + d*x)^2*Csch[a + b*x]^2,x]
```

output

```
-(((c + d*x)^2*Coth[a + b*x])/b) - ((2*I)*d*(((-1/2*I)*(c + d*x)^2)/d + (2
*I)*(((c + d*x)*Log[1 + E^(2*a - I*Pi + 2*b*x)])/(2*b) + (d*PolyLog[2, -E^
(2*a - I*Pi + 2*b*x)])/(4*b^2))))/b
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(74) = 148$.

Time = 0.10 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.24

method	result
risch	$-\frac{2(x^2d^2+2cdx+c^2)}{(e^{2bx+2a}-1)b}+\frac{2dc\ln(e^{bx+a}-1)}{b^2}-\frac{4dc\ln(e^{bx+a})}{b^2}+\frac{2dc\ln(e^{bx+a}+1)}{b^2}-\frac{2d^2x^2}{b}-\frac{4d^2ax}{b^2}-\frac{2d^2a^2}{b^3}+\frac{2d^2\ln(1-}{b^2}$

input
```
int((d*x+c)^2*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(d^2*x^2+2*c*d*x+c^2)/(exp(2*b*x+2*a)-1)/b+2*d/b^2*c*ln(exp(b*x+a)-1)-4
*d/b^2*c*ln(exp(b*x+a))+2*d/b^2*c*ln(exp(b*x+a)+1)-2*d^2/b*x^2-4*d^2/b^2*a
*x-2*d^2/b^3*a^2+2*d^2/b^2*ln(1-exp(b*x+a))*x+2*d^2/b^3*ln(1-exp(b*x+a))*a
+2*d^2/b^3*polylog(2,exp(b*x+a))+2*d^2/b^2*ln(exp(b*x+a)+1)*x+2*d^2/b^3*po
lylog(2,-exp(b*x+a))-2*d^2/b^3*a*ln(exp(b*x+a)-1)+4*d^2/b^3*a*ln(exp(b*x+a
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 623 vs. $2(73) = 146$.

Time = 0.10 (sec) , antiderivative size = 623, normalized size of antiderivative = 8.42

$$\int (c+dx)^2 \text{csch}^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
-2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d
 - a^2*d^2)*cosh(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a
^2*d^2)*cosh(b*x + a)*sinh(b*x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c
*d - a^2*d^2)*sinh(b*x + a)^2 - (d^2*cosh(b*x + a)^2 + 2*d^2*cosh(b*x + a)
*sinh(b*x + a) + d^2*sinh(b*x + a)^2 - d^2)*dilog(cosh(b*x + a) + sinh(b*x
 + a)) - (d^2*cosh(b*x + a)^2 + 2*d^2*cosh(b*x + a)*sinh(b*x + a) + d^2*si
nh(b*x + a)^2 - d^2)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b*d^2*x + b*
c*d - (b*d^2*x + b*c*d)*cosh(b*x + a)^2 - 2*(b*d^2*x + b*c*d)*cosh(b*x + a
)*sinh(b*x + a) - (b*d^2*x + b*c*d)*sinh(b*x + a)^2)*log(cosh(b*x + a) + s
inh(b*x + a) + 1) + (b*c*d - a*d^2 - (b*c*d - a*d^2)*cosh(b*x + a)^2 - 2*(
b*c*d - a*d^2)*cosh(b*x + a)*sinh(b*x + a) - (b*c*d - a*d^2)*sinh(b*x + a)
^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + (b*d^2*x + a*d^2 - (b*d^2*x +
 a*d^2)*cosh(b*x + a)^2 - 2*(b*d^2*x + a*d^2)*cosh(b*x + a)*sinh(b*x + a)
- (b*d^2*x + a*d^2)*sinh(b*x + a)^2)*log(-cosh(b*x + a) - sinh(b*x + a) +
1))/(b^3*cosh(b*x + a)^2 + 2*b^3*cosh(b*x + a)*sinh(b*x + a) + b^3*sinh(b*
x + a)^2 - b^3)
```

Sympy [F]

$$\int (c+dx)^2\text{csch}^2(a+bx)\,dx = \int (c+dx)^2\,\text{csch}^2\,(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*csch(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**2*csch(a + b*x)**2, x)
```

Maxima [F]

$$\int (c+dx)^2\text{csch}^2(a+bx)\,dx = \int (dx+c)^2\,\text{csch}\,(bx+a)^2\;dx$$

input
```
integrate((d*x+c)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
-2*d^2*(x^2/(b*e^(2*b*x + 2*a) - b) + 2*integrate(1/2*x/(b*e^(b*x + a) + b
), x) - 2*integrate(1/2*x/(b*e^(b*x + a) - b), x)) - 2*c*d*(2*x*e^(2*b*x +
 2*a)/(b*e^(2*b*x + 2*a) - b) - log((e^(b*x + a) + 1)*e^(-a))/b^2 - log((e
^(b*x + a) - 1)*e^(-a))/b^2) + 2*c^2/(b*(e^(-2*b*x - 2*a) - 1))
```

Giac [F]

$$\int (c+dx)^2\text{csch}^2(a+bx)\,dx = \int (dx+c)^2\,\text{csch}\,(bx+a)^2\;dx$$

input
```
integrate((d*x+c)^2*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*csch(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2\text{csch}^2(a+bx)\,dx=\int \frac{(c+d\,x)^2}{\sinh{(a+b\,x)}^2}\,dx$$

input
```
int((c + d*x)^2/sinh(a + b*x)^2,x)
```

output
```
int((c + d*x)^2/sinh(a + b*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2\text{csch}^2(a+bx)\,dx = \frac{-4e^{2bx+2a}\left(\int \frac{x}{e^{4bx+4a}-2e^{2bx+2a}+1}dx\right)b^2d^2+2e^{2bx+2a}\log\left(e^{bx+a}-1\right)bcd+e^{2bx+2a}\log\left(e^{bx+a}-1\right)d^2+2e^{2bx+2}}{}$$

input
```
int((d*x+c)^2*csch(b*x+a)^2,x)
```

output
```
( - 4*e**(2*a + 2*b*x)*int(x/(e**(4*a + 4*b*x) - 2*e**(2*a + 2*b*x) + 1),x
)*b**2*d**2 + 2*e**(2*a + 2*b*x)*log(e**(a + b*x) - 1)*b*c*d + e**(2*a + 2
*b*x)*log(e**(a + b*x) - 1)*d**2 + 2*e**(2*a + 2*b*x)*log(e**(a + b*x) + 1
)*b*c*d + e**(2*a + 2*b*x)*log(e**(a + b*x) + 1)*d**2 - 2*e**(2*a + 2*b*x)
*b**2*c**2 - 4*e**(2*a + 2*b*x)*b**2*c*d*x - 2*e**(2*a + 2*b*x)*b*d**2*x +
 4*int(x/(e**(4*a + 4*b*x) - 2*e**(2*a + 2*b*x) + 1),x)*b**2*d**2 - 2*log(
e**(a + b*x) - 1)*b*c*d - log(e**(a + b*x) - 1)*d**2 - 2*log(e**(a + b*x)
+ 1)*b*c*d - log(e**(a + b*x) + 1)*d**2 - 2*b**2*d**2*x**2)/(b**3*(e**(2*a
 + 2*b*x) - 1))
```

3.30 $\int (c+dx)\mathrm{csch}^2(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 29

$$\int (c+dx)\mathrm{csch}^2(a+bx)\,dx=-\frac{(c+dx)\coth(a+bx)}{b}+\frac{d\log(\sinh(a+bx))}{b^2}$$

output
```
-(d*x+c)*coth(b*x+a)/b+d*ln(sinh(b*x+a))/b^2
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.79

$$\int (c+dx)\mathrm{csch}^2(a+bx)\,dx=-\frac{dx\coth(a)}{b}-\frac{c\coth(a+bx)}{b}+\frac{d\log(\sinh(a+bx))}{b^2}+\frac{dx\mathrm{csch}(a)\mathrm{csch}(a+bx)\sinh(bx)}{b}$$

input
```
Integrate[(c + d*x)*Csch[a + b*x]^2,x]
```

output
```
-((d*x*Coth[a])/b) - (c*Coth[a + b*x])/b + (d*Log[Sinh[a + b*x]])/b^2 + (d
*x*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 25, 4672, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\text{csch}^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\left((c+dx)\csc(ia+ibx)^2\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)\csc(ia+ibx)^2dx$$

$$\Big\downarrow \text{4672}$$

$$-\frac{(c+dx)\coth(a+bx)}{b}+\frac{id\int -i\coth(a+bx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{d\int \coth(a+bx)dx}{b}-\frac{(c+dx)\coth(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(c+dx)\coth(a+bx)}{b}+\frac{d\int -i\tan\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{(c+dx)\coth(a+bx)}{b}-\frac{id\int \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx}{b}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{(c+dx)\coth(a+bx)}{b}+\frac{d\log(-i\sinh(a+bx))}{b^2}$$

input
```
Int[(c + d*x)*Csch[a + b*x]^2,x]
```

output
```
-(((c + d*x)*Coth[a + b*x])/b) + (d*Log[(-I)*Sinh[a + b*x]])/b^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.93

method	result	size
risch	$-\frac{2dx}{b}-\frac{2da}{b^{2}}-\frac{2(dx+c)}{\left(\mathrm{e}^{2bx+2a}-1\right)b}+\frac{d\ln\left(\mathrm{e}^{2bx+2a}-1\right)}{b^{2}}$	56
parallelrisch	$\frac{-4\ln\left(1-\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+2\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d-b\left(\coth\left(\frac{bx}{2}+\frac{a}{2}\right)(dx+c)+(dx+c)\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+2dx\right)}{2b^{2}}$	75

input
```
int((d*x+c)*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*d/b*x-2*d/b^2*a-2*(d*x+c)/(exp(2*b*x+2*a)-1)/b+d/b^2*ln(exp(2*b*x+2*a)-
1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(29) = 58$.

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 5.72

$$\int (c+dx)\text{csch}^2(a+bx)\,dx = -\frac{2\,bdx\cosh\left(bx+a\right)^2+4\,bdx\cosh\left(bx+a\right)\sinh\left(bx+a\right)+2\,bdx\sinh\left(bx+a\right)^2+2\,bc-\left(d\cosh\left(bx+a\right.\right.}{b^2\cosh\left(bx+a\right)^2+2\,b^2\cosh\left(bx+a\right)\sinh\left(bx\right.}$$

input
```
integrate((d*x+c)*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(2*b*d*x*cosh(b*x + a)^2 + 4*b*d*x*cosh(b*x + a)*sinh(b*x + a) + 2*b*d*x*
sinh(b*x + a)^2 + 2*b*c - (d*cosh(b*x + a)^2 + 2*d*cosh(b*x + a)*sinh(b*x
+ a) + d*sinh(b*x + a)^2 - d)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*
x + a))))/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(b*x + a)*sinh(b*x + a) + b^2*s
inh(b*x + a)^2 - b^2)
```

Sympy [F]

$$\int (c+dx)\text{csch}^2(a+bx)\,dx = \int (c+dx)\,\text{csch}^2\left(a+bx\right)dx$$

input
```
integrate((d*x+c)*csch(b*x+a)**2,x)
```

output
```
Integral((c + d*x)*csch(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(29) = 58$.

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.14

$$\int (c+dx)\operatorname{csch}^2(a+bx)\,dx$$
$$= -d\left(\frac{2\,xe^{(2\,bx+2\,a)}}{be^{(2\,bx+2\,a)}-b} - \frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^2} - \frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^2}\right)$$
$$+ \frac{2\,c}{b(e^{(-2\,bx-2\,a)}-1)}$$

input
```
integrate((d*x+c)*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
-d*(2*x*e^(2*b*x + 2*a)/(b*e^(2*b*x + 2*a) - b) - log((e^(b*x + a) + 1)*e^
(-a))/b^2 - log((e^(b*x + a) - 1)*e^(-a))/b^2) + 2*c/(b*(e^(-2*b*x - 2*a)
- 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(29) = 58$.

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.76

$$\int (c+dx)\operatorname{csch}^2(a+bx)\,dx$$
$$= -\frac{2\,bdxe^{(2\,bx+2\,a)} - de^{(2\,bx+2\,a)}\log\left(e^{(2\,bx+2\,a)}-1\right) + 2\,bc + d\log\left(e^{(2\,bx+2\,a)}-1\right)}{b^2e^{(2\,bx+2\,a)} - b^2}$$

input
```
integrate((d*x+c)*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
-(2*b*d*x*e^(2*b*x + 2*a) - d*e^(2*b*x + 2*a)*log(e^(2*b*x + 2*a) - 1) + 2
*b*c + d*log(e^(2*b*x + 2*a) - 1))/(b^2*e^(2*b*x + 2*a) - b^2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int (c+dx)\mathrm{csch}^2(a+bx)\,dx=\frac{d\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1\right)}{b^2}-\frac{2\,(c+d\,x)}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}-\frac{2\,d\,x}{b}$$

input
```
int((c + d*x)/sinh(a + b*x)^2,x)
```

output
```
(d*log(exp(2*a)*exp(2*b*x) - 1))/b^2 - (2*(c + d*x))/(b*(exp(2*a + 2*b*x)
- 1)) - (2*d*x)/b
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 118, normalized size of antiderivative = 4.07

$$\int (c+dx)\mathrm{csch}^2(a+bx)\,dx$$
$$=\frac{e^{2bx+2a}\log\left(e^{bx+a}-1\right)d+e^{2bx+2a}\log\left(e^{bx+a}+1\right)d-2e^{2bx+2a}bc-2e^{2bx+2a}bdx-\log\left(e^{bx+a}-1\right)d-\log\left(e\right.}{b^2\left(e^{2bx+2a}-1\right)}$$

input
```
int((d*x+c)*csch(b*x+a)^2,x)
```

output
```
(e**(2*a + 2*b*x)*log(e**(a + b*x) - 1)*d + e**(2*a + 2*b*x)*log(e**(a + b
*x) + 1)*d - 2*e**(2*a + 2*b*x)*b*c - 2*e**(2*a + 2*b*x)*b*d*x - log(e**(a
 + b*x) - 1)*d - log(e**(a + b*x) + 1)*d)/(b**2*(e**(2*a + 2*b*x) - 1))
```

3.31 $\int \frac{\operatorname{csch}^2(a+bx)}{c+dx}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\operatorname{csch}^2(a+bx)}{c+dx}\,dx=\operatorname{Int}\left(\frac{\operatorname{csch}^2(a+bx)}{c+dx},x\right)$$

output
```
Defer(Int)(csch(b*x+a)^2/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 15.51 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^2(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{csch}^2(a+bx)}{c+dx}\,dx$$

input
```
Integrate[Csch[a + b*x]^2/(c + d*x),x]
```

output
```
Integrate[Csch[a + b*x]^2/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(a+bx)}{c+dx}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int -\frac{\csc(ia+ibx)^2}{c+dx}dx$$
$$\Big\downarrow \text{ 25}$$
$$-\int \frac{\csc(ia+ibx)^2}{c+dx}dx$$
$$\Big\downarrow \text{ 4680}$$
$$\int \frac{\operatorname{csch}^2(a+bx)}{c+dx}dx$$

input
```
Int[Csch[a + b*x]^2/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(bx+a\right)^{2}}{dx+c}dx$$

input
```
int(csch(b*x+a)^2/(d*x+c),x)
```

output
```
int(csch(b*x+a)^2/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^2(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^2}{dx+c}\,dx$$

input
```
integrate(csch(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)^2/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{csch}^2(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{csch}^2\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(csch(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(csch(a + b*x)**2/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 157, normalized size of antiderivative = 9.81

$$\int \frac{\operatorname{csch}^2(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{csch}(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(csch(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
4*d*integrate(1/4/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2*e^a + 2*b*c*
d*x*e^a + b*c^2*e^a)*e^(b*x)), x) - 4*d*integrate(-1/4/(b*d^2*x^2 + 2*b*c*
d*x + b*c^2 - (b*d^2*x^2*e^a + 2*b*c*d*x*e^a + b*c^2*e^a)*e^(b*x)), x) + 2
/(b*d*x + b*c - (b*d*x*e^(2*a) + b*c*e^(2*a))*e^(2*b*x))
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^2(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{csch}(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(csch(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)^2/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{csch}^2(a+bx)}{c+dx}\,dx = \int \frac{1}{\sinh\left(a+b\,x\right)^2\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/(sinh(a + b*x)^2*(c + d*x)),x)
```

output
```
int(1/(sinh(a + b*x)^2*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{csch}^2(a+bx)}{c+dx}\,dx = \int \frac{\text{csch}(bx+a)^2}{dx+c}dx$$

input
```
int(csch(b*x+a)^2/(d*x+c),x)
```

output
```
int(csch(a + b*x)**2/(c + d*x),x)
```

3.32 $\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2}\,dx=\operatorname{Int}\left(\frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2},x\right)$$

output
```
Defer(Int)(csch(b*x+a)^2/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 15.61 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2}\,dx=\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[Csch[a + b*x]^2/(c + d*x)^2,x]
```

output
```
Integrate[Csch[a + b*x]^2/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^2(a+bx)}{(c+dx)^2} \, dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int -\frac{\csc(ia+ibx)^2}{(c+dx)^2}dx$$
$$\Big\downarrow \text{ 25}$$
$$-\int \frac{\csc(ia+ibx)^2}{(c+dx)^2}dx$$
$$\Big\downarrow \text{ 4680}$$
$$\int \frac{\text{csch}^2(a+bx)}{(c+dx)^2}dx$$

input
```
Int[Csch[a + b*x]^2/(c + d*x)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\text{csch}\left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}dx$$

input
```
int(csch(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(csch(b*x+a)^2/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(csch(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2}\,dx=\int \frac{\operatorname{csch}^2\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(csch(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(csch(a + b*x)**2/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 237, normalized size of antiderivative = 14.81

$$\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{csch}(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate(csch(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
4*d*integrate(1/2/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^
3*x^3*e^a + 3*b*c*d^2*x^2*e^a + 3*b*c^2*d*x*e^a + b*c^3*e^a)*e^(b*x)), x)
- 4*d*integrate(-1/2/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 - (b
*d^3*x^3*e^a + 3*b*c*d^2*x^2*e^a + 3*b*c^2*d*x*e^a + b*c^3*e^a)*e^(b*x)),
x) + 2/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 - (b*d^2*x^2*e^(2*a) + 2*b*c*d*x*e^(
2*a) + b*c^2*e^(2*a))*e^(2*b*x))
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{csch}(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate(csch(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)^2/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2}\,dx=\int \frac{1}{\sinh\left(a+b\,x\right)^2\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/(sinh(a + b*x)^2*(c + d*x)^2),x)
```

output
```
int(1/(sinh(a + b*x)^2*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2}\,dx=\int \frac{\operatorname{csch}(bx+a)^2}{d^2x^2+2cdx+c^2}dx$$

input
```
int(csch(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(csch(a + b*x)**2/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.33 $\int (c+dx)^3 \mathrm{csch}^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 256

$$\begin{aligned}\int (c+dx)^3\mathrm{csch}^3(a+bx)\,dx = &-\frac{6d^2(c+dx)\mathrm{arctanh}\left(e^{a+bx}\right)}{b^3} \\ &+\frac{(c+dx)^3\mathrm{arctanh}\left(e^{a+bx}\right)}{b}-\frac{3d(c+dx)^2\mathrm{csch}(a+bx)}{2b^2} \\ &-\frac{(c+dx)^3\coth(a+bx)\mathrm{csch}(a+bx)}{2b} \\ &-\frac{3d^3\,\mathrm{PolyLog}\left(2,-e^{a+bx}\right)}{b^4} \\ &+\frac{3d(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2} \\ &+\frac{3d^3\,\mathrm{PolyLog}\left(2,e^{a+bx}\right)}{b^4} \\ &-\frac{3d(c+dx)^2\,\mathrm{PolyLog}\left(2,e^{a+bx}\right)}{2b^2} \\ &-\frac{3d^2(c+dx)\,\mathrm{PolyLog}\left(3,-e^{a+bx}\right)}{b^3} \\ &+\frac{3d^2(c+dx)\,\mathrm{PolyLog}\left(3,e^{a+bx}\right)}{b^3} \\ &+\frac{3d^3\,\mathrm{PolyLog}\left(4,-e^{a+bx}\right)}{b^4}-\frac{3d^3\,\mathrm{PolyLog}\left(4,e^{a+bx}\right)}{b^4}\end{aligned}$$

output
```
-6*d^2*(d*x+c)*arctanh(exp(b*x+a))/b^3+(d*x+c)^3*arctanh(exp(b*x+a))/b-3/2
*d*(d*x+c)^2*csch(b*x+a)/b^2-1/2*(d*x+c)^3*coth(b*x+a)*csch(b*x+a)/b-3*d^3
*polylog(2,-exp(b*x+a))/b^4+3/2*d*(d*x+c)^2*polylog(2,-exp(b*x+a))/b^2+3*d
^3*polylog(2,exp(b*x+a))/b^4-3/2*d*(d*x+c)^2*polylog(2,exp(b*x+a))/b^2-3*d
^2*(d*x+c)*polylog(3,-exp(b*x+a))/b^3+3*d^2*(d*x+c)*polylog(3,exp(b*x+a))/
b^3+3*d^3*polylog(4,-exp(b*x+a))/b^4-3*d^3*polylog(4,exp(b*x+a))/b^4
```

Mathematica [A] (verified)

Time = 2.56 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.72

$$\int (c+dx)^3\text{csch}^3(a+bx)\,dx = -\frac{b^2(c+dx)^2(3d+b(c+dx)\coth(a+bx))\text{csch}(a+bx)+b^3c^3\log\left(1-e^{a+bx}\right)-6bcd^2\log\left(1-e^{a+bx}\right)+}{}$$

input
```
Integrate[(c + d*x)^3*Csch[a + b*x]^3,x]
```

output
```
-1/2*(b^2*(c + d*x)^2*(3*d + b*(c + d*x)*Coth[a + b*x])*Csch[a + b*x] + b^
3*c^3*Log[1 - E^(a + b*x)] - 6*b*c*d^2*Log[1 - E^(a + b*x)] + 3*b^3*c^2*d*
x*Log[1 - E^(a + b*x)] - 6*b*d^3*x*Log[1 - E^(a + b*x)] + 3*b^3*c*d^2*x^2*
Log[1 - E^(a + b*x)] + b^3*d^3*x^3*Log[1 - E^(a + b*x)] - b^3*c^3*Log[1 +
E^(a + b*x)] + 6*b*c*d^2*Log[1 + E^(a + b*x)] - 3*b^3*c^2*d*x*Log[1 + E^(a
 + b*x)] + 6*b*d^3*x*Log[1 + E^(a + b*x)] - 3*b^3*c*d^2*x^2*Log[1 + E^(a +
 b*x)] - b^3*d^3*x^3*Log[1 + E^(a + b*x)] - 3*d*(-2*d^2 + b^2*(c + d*x)^2)
*PolyLog[2, -E^(a + b*x)] + 3*d*(-2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, E^(a
 + b*x)] + 6*b*c*d^2*PolyLog[3, -E^(a + b*x)] + 6*b*d^3*x*PolyLog[3, -E^(a
 + b*x)] - 6*b*c*d^2*PolyLog[3, E^(a + b*x)] - 6*b*d^3*x*PolyLog[3, E^(a +
 b*x)] - 6*d^3*PolyLog[4, -E^(a + b*x)] + 6*d^3*PolyLog[4, E^(a + b*x)])/b
^4
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.15, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {3042, 26, 4674, 26, 3042, 26, 4670, 2715, 2838, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\text{csch}^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i(c+dx)^3\csc(ia+ibx)^3dx$$

$$\Big\downarrow \text{26}$$

$$-i\int (c+dx)^3\csc(ia+ibx)^3dx$$

$$\Big\downarrow \text{4674}$$

$$-i\left(-\frac{3d^2\int -i(c+dx)\text{csch}(a+bx)dx}{b^2}+\frac{1}{2}\int -i(c+dx)^3\text{csch}(a+bx)dx-\frac{3id(c+dx)^2\text{csch}(a+bx)}{2b^2}-\frac{i(c+d\right.$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{3id^2\int (c+dx)\text{csch}(a+bx)dx}{b^2}-\frac{1}{2}i\int (c+dx)^3\text{csch}(a+bx)dx-\frac{3id(c+dx)^2\text{csch}(a+bx)}{2b^2}-\frac{i(c+dx)^3\,\text{cot}\right.$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{3id^2\int i(c+dx)\csc(ia+ibx)dx}{b^2}-\frac{1}{2}i\int i(c+dx)^3\csc(ia+ibx)dx-\frac{3id(c+dx)^2\text{csch}(a+bx)}{2b^2}-\frac{i(c+dx)^3}{}\right.$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{3d^2\int (c+dx)\csc(ia+ibx)dx}{b^2}+\frac{1}{2}\int (c+dx)^3\csc(ia+ibx)dx-\frac{3id(c+dx)^2\text{csch}(a+bx)}{2b^2}-\frac{i(c+dx)^3\,\text{c}}{}\right.$$

$$\Big\downarrow \text{4670}$$

$$-i\left(-\frac{3d^2\left(\frac{id\int\log(1-e^{a+bx})dx}{b}-\frac{id\int\log(1+e^{a+bx})dx}{b}+\frac{2i(c+dx)\text{arctanh}(e^{a+bx})}{b}\right)}{b^2}+\frac{1}{2}\left(\frac{3id\int(c+dx)^2\log\left(1-e^{a+bx}\right)}{b}\right.\right.$$

$\downarrow$ 2715

$$-i\left(-\frac{3d^2\left(\frac{id\int e^{-a-bx}\log(1-e^{a+bx})de^{a+bx}}{b^2}-\frac{id\int e^{-a-bx}\log(1+e^{a+bx})de^{a+bx}}{b^2}+\frac{2i(c+dx)\text{arctanh}(e^{a+bx})}{b}\right)}{b^2}+\frac{1}{2}\left(\frac{3id\int(c+\right.\right.$$

$\downarrow$ 2838

$$-i\left(\frac{1}{2}\left(\frac{3id\int(c+dx)^2\log\left(1-e^{a+bx}\right)dx}{b}-\frac{3id\int(c+dx)^2\log\left(1+e^{a+bx}\right)dx}{b}+\frac{2i(c+dx)^3\text{arctanh}(e^{a+bx})}{b}\right)-\right.$$

$\downarrow$ 3011

$$-i\left(\frac{1}{2}\left(-\frac{3id\left(\frac{2d\int(c+dx)\text{PolyLog}(2,-e^{a+bx})dx}{b}-\frac{(c+dx)^2\text{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{3id\left(\frac{2d\int(c+dx)\text{PolyLog}(2,e^{a+bx})dx}{b}-\frac{(c+dx}{}\right.}{b}\right.\right.$$

$\downarrow$ 7163

$$-i\left(\frac{1}{2}\left(-\frac{3id\left(\frac{2d\left(\frac{(c+dx)\text{PolyLog}\left(3,-e^{a+bx}\right)}{b}-\frac{d\int\text{PolyLog}\left(3,-e^{a+bx}\right)dx}{b}\right)}{b}-\frac{(c+dx)^2\text{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{3id\left(\frac{2d\left(\frac{(c+dx)\text{PolyLo}}{b}\right.}{}\right.}{}\right.\right.$$

$\downarrow$ 2720

$$-i\left(\frac{1}{2}\left(-\frac{3id\left(\frac{2d\left(\frac{(c+dx)\operatorname{PolyLog}\left(3,-e^{a+bx}\right)}{b}-\frac{d\int e^{-a-bx}\operatorname{PolyLog}\left(3,-e^{a+bx}\right)de^{a+bx}}{b^2}\right)}{b}-\frac{(c+dx)^2\operatorname{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{3id\left(\frac{2d\left(\right.}{}\right.}{}\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$-i\left(-\frac{3d^2\left(\frac{2i(c+dx)\operatorname{arctanh}(e^{a+bx})}{b}+\frac{id\operatorname{PolyLog}(2,-e^{a+bx})}{b^2}-\frac{id\operatorname{PolyLog}(2,e^{a+bx})}{b^2}\right)}{b^2}+\frac{1}{2}\left(\frac{2i(c+dx)^3\operatorname{arctanh}\left(e^{a+bx}\right)}{b}-\right.\right.$$

input
```
Int[(c + d*x)^3*Csch[a + b*x]^3,x]
```

output
```
(-I)*((((-3*I)/2)*d*(c + d*x)^2*Csch[a + b*x])/b^2 - ((I/2)*(c + d*x)^3*Co
th[a + b*x]*Csch[a + b*x])/b - (3*d^2*(((2*I)*(c + d*x)*ArcTanh[E^(a + b*x
)])/b + (I*d*PolyLog[2, -E^(a + b*x)])/b^2 - (I*d*PolyLog[2, E^(a + b*x)])
/b^2))/b^2 + (((2*I)*(c + d*x)^3*ArcTanh[E^(a + b*x)])/b - ((3*I)*d*(-(((c
 + d*x)^2*PolyLog[2, -E^(a + b*x)])/b) + (2*d*(((c + d*x)*PolyLog[3, -E^(a
 + b*x)])/b - (d*PolyLog[4, -E^(a + b*x)])/b^2))/b))/b + ((3*I)*d*(-(((c +
 d*x)^2*PolyLog[2, E^(a + b*x)])/b) + (2*d*(((c + d*x)*PolyLog[3, E^(a + b
*x)])/b - (d*PolyLog[4, E^(a + b*x)])/b^2))/b))/b)/2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 875 vs. $2(238) = 476$.

Time = 0.25 (sec) , antiderivative size = 876, normalized size of antiderivative = 3.42

method	result	size
risch	Expression too large to display	876

input
```
int((d*x+c)^3*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
3*d^3*polylog(4,-exp(b*x+a))/b^4-3*d^3*polylog(4,exp(b*x+a))/b^4-3*d^3*pol
ylog(2,-exp(b*x+a))/b^4+3*d^3*polylog(2,exp(b*x+a))/b^4+6/b^4*d^3*a*arctan
h(exp(b*x+a))-6/b^3*d^2*c*arctanh(exp(b*x+a))+3/b^3*d^3*ln(1-exp(b*x+a))*x
+3/b^4*d^3*ln(1-exp(b*x+a))*a-3/b^3*d^3*ln(exp(b*x+a)+1)*x-3/b^4*d^3*ln(ex
p(b*x+a)+1)*a-3/2/b*c*d^2*ln(1-exp(b*x+a))*x^2+3/2/b^3*c*d^2*ln(1-exp(b*x+
a))*a^2-3/b^2*c*d^2*polylog(2,exp(b*x+a))*x+3/2/b*c*d^2*ln(exp(b*x+a)+1)*x
^2-3/2/b^3*c*d^2*ln(exp(b*x+a)+1)*a^2+3/b^2*c*d^2*polylog(2,-exp(b*x+a))*x
+3/2/b*c^2*d*ln(exp(b*x+a)+1)*x+3/2/b^2*c^2*d*ln(exp(b*x+a)+1)*a-3/2/b*c^2
*d*ln(1-exp(b*x+a))*x-3/2/b^2*c^2*d*ln(1-exp(b*x+a))*a-3/b^2*d*a*c^2*arcta
nh(exp(b*x+a))+3/b^3*d^2*a^2*c*arctanh(exp(b*x+a))-exp(b*x+a)*(exp(2*b*x+2
*a)*b*d^3*x^3+3*exp(2*b*x+2*a)*b*c*d^2*x^2+3*exp(2*b*x+2*a)*b*c^2*d*x+b*d^
3*x^3+3*exp(2*b*x+2*a)*d^3*x^2+exp(2*b*x+2*a)*b*c^3+3*b*c*d^2*x^2+6*exp(2*
b*x+2*a)*c*d^2*x+3*b*c^2*d*x+3*exp(2*b*x+2*a)*c^2*d-3*d^3*x^2+b*c^3-6*c*d^
2*x-3*c^2*d)/b^2/(exp(2*b*x+2*a)-1)^2+1/b*c^3*arctanh(exp(b*x+a))-1/2/b*d^
3*ln(1-exp(b*x+a))*x^3-1/2/b^4*d^3*ln(1-exp(b*x+a))*a^3-3/2/b^2*d^3*polylo
g(2,exp(b*x+a))*x^2+3/b^3*d^3*polylog(3,exp(b*x+a))*x+1/2/b*d^3*ln(exp(b*x
+a)+1)*x^3+1/2/b^4*d^3*ln(exp(b*x+a)+1)*a^3+3/2/b^2*d^3*polylog(2,-exp(b*x
+a))*x^2-3/b^3*d^3*polylog(3,-exp(b*x+a))*x-3/2/b^2*c^2*d*polylog(2,exp(b*
x+a))+3/2/b^2*c^2*d*polylog(2,-exp(b*x+a))+3/b^3*c*d^2*polylog(3,exp(b*x+a
))-3/b^3*c*d^2*polylog(3,-exp(b*x+a))-1/b^4*d^3*a^3*arctanh(exp(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4008 vs. $2(234) = 468$.

Time = 0.13 (sec) , antiderivative size = 4008, normalized size of antiderivative = 15.66

$$\int (c + dx)^3 \operatorname{csch}^3(a + bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int (c+dx)^3 \mathrm{csch}^3(a+bx)\,dx = \int (c+dx)^3 \,\mathrm{csch}^3\,(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*csch(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**3*csch(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 605 vs. $2(234) = 468$.

Time = 0.26 (sec) , antiderivative size = 605, normalized size of antiderivative = 2.36

$$\int (c+dx)^3 \mathrm{csch}^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*c^3*(log(e^(-b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b + 2*(e^(-b*x -
a) + e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))) +
3/2*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(
3, -e^(b*x + a)))*c*d^2/b^3 - 3/2*(b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*d
ilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))*c*d^2/b^3 - 3*c*d^2*log(e^(
b*x + a) + 1)/b^3 + 3*c*d^2*log(e^(b*x + a) - 1)/b^3 - ((b*d^3*x^3*e^(3*a)
 + 3*c^2*d*e^(3*a) + 3*(b*c*d^2 + d^3)*x^2*e^(3*a) + 3*(b*c^2*d + 2*c*d^2)
*x*e^(3*a))*e^(3*b*x) + (b*d^3*x^3*e^a - 3*c^2*d*e^a + 3*(b*c*d^2 - d^3)*x
^2*e^a + 3*(b*c^2*d - 2*c*d^2)*x*e^a)*e^(b*x))/(b^2*e^(4*b*x + 4*a) - 2*b^
2*e^(2*b*x + 2*a) + b^2) + 1/2*(b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*d
ilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x
 + a)))*d^3/b^4 - 1/2*(b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(
b*x + a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))*d^3
/b^4 + 3/2*(b^2*c^2*d - 2*d^3)*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x +
 a)))/b^4 - 3/2*(b^2*c^2*d - 2*d^3)*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(
b*x + a)))/b^4
```

Giac [F]

$$\int (c+dx)^3\text{csch}^3(a+bx)\,dx = \int (dx+c)^3\,\text{csch}\,(bx+a)^3\;dx$$

input
```
integrate((d*x+c)^3*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*csch(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3\text{csch}^3(a+bx)\,dx = \int \frac{(c+d\,x)^3}{\sinh\,(a+b\,x)^3}\,dx$$

input
```
int((c + d*x)^3/sinh(a + b*x)^3,x)
```

output
```
int((c + d*x)^3/sinh(a + b*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3\text{csch}^3(a+bx)\,dx = \text{too large to display}$$

input
```
int((d*x+c)^3*csch(b*x+a)^3,x)
```

output

```
( - 48*e**(5*a + 4*b*x)*int((e**(b*x)*x**3)/(e**(6*a + 6*b*x) - 3*e**(4*a
+ 4*b*x) + 3*e**(2*a + 2*b*x) - 1),x)*b**4*d**3 - 144*e**(5*a + 4*b*x)*int
((e**(b*x)*x**2)/(e**(6*a + 6*b*x) - 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*
x) - 1),x)*b**4*c*d**2 - 192*e**(5*a + 4*b*x)*int((e**(b*x)*x**2)/(e**(6*a
 + 6*b*x) - 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) - 1),x)*b**3*d**3 - 14
4*e**(5*a + 4*b*x)*int((e**(b*x)*x)/(e**(6*a + 6*b*x) - 3*e**(4*a + 4*b*x)
 + 3*e**(2*a + 2*b*x) - 1),x)*b**4*c**2*d - 384*e**(5*a + 4*b*x)*int((e**(
b*x)*x)/(e**(6*a + 6*b*x) - 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) - 1),x
)*b**3*c*d**2 - 128*e**(5*a + 4*b*x)*int((e**(b*x)*x)/(e**(6*a + 6*b*x) -
3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) - 1),x)*b**2*d**3 - 9*e**(4*a + 4*
b*x)*log(e**(a + b*x) - 1)*b**3*c**3 - 36*e**(4*a + 4*b*x)*log(e**(a + b*x
) - 1)*b**2*c**2*d - 24*e**(4*a + 4*b*x)*log(e**(a + b*x) - 1)*b*c*d**2 -
8*e**(4*a + 4*b*x)*log(e**(a + b*x) - 1)*d**3 + 9*e**(4*a + 4*b*x)*log(e**
(a + b*x) + 1)*b**3*c**3 + 36*e**(4*a + 4*b*x)*log(e**(a + b*x) + 1)*b**2*
c**2*d + 24*e**(4*a + 4*b*x)*log(e**(a + b*x) + 1)*b*c*d**2 + 8*e**(4*a +
4*b*x)*log(e**(a + b*x) + 1)*d**3 - 18*e**(3*a + 3*b*x)*b**3*c**3 - 72*e**
(3*a + 3*b*x)*b**2*c**2*d - 48*e**(3*a + 3*b*x)*b*c*d**2 - 16*e**(3*a + 3*
b*x)*d**3 + 96*e**(3*a + 2*b*x)*int((e**(b*x)*x**3)/(e**(6*a + 6*b*x) - 3*
e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) - 1),x)*b**4*d**3 + 288*e**(3*a + 2*
b*x)*int((e**(b*x)*x**2)/(e**(6*a + 6*b*x) - 3*e**(4*a + 4*b*x) + 3*e**...
```

3.34 $\int (c+dx)^2\text{csch}^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 154

$$\int (c+dx)^2\text{csch}^3(a+bx)\,dx = \frac{(c+dx)^2\text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{d^2\text{arctanh}(\cosh(a+bx))}{b^3} - \frac{d(c+dx)\text{csch}(a+bx)}{b^2} - \frac{(c+dx)^2\coth(a+bx)\text{csch}(a+bx)}{2b} + \frac{d(c+dx)\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2} - \frac{d(c+dx)\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2} - \frac{d^2\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3} + \frac{d^2\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}$$

output
```
(d*x+c)^2*arctanh(exp(b*x+a))/b-d^2*arctanh(cosh(b*x+a))/b^3-d*(d*x+c)*csc
h(b*x+a)/b^2-1/2*(d*x+c)^2*coth(b*x+a)*csch(b*x+a)/b+d*(d*x+c)*polylog(2,-
exp(b*x+a))/b^2-d*(d*x+c)*polylog(2,exp(b*x+a))/b^2-d^2*polylog(3,-exp(b*x
+a))/b^3+d^2*polylog(3,exp(b*x+a))/b^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 420 vs. $2(154) = 308$.

Time = 6.67 (sec) , antiderivative size = 420, normalized size of antiderivative = 2.73

$$\int (c+dx)^2\text{csch}^3(a+bx)\,dx = -\frac{d(c+dx)\text{csch}(a)}{b^2} + \frac{(-c^2-2cdx-d^2x^2)\,\text{csch}^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}$$

$$+ \frac{-b^2c^2\log\left(1-e^{a+bx}\right)+2d^2\log\left(1-e^{a+bx}\right)-2b^2cdx\log\left(1-e^{a+bx}\right)-b^2d^2x^2\log\left(1-e^{a+bx}\right)+b^2c^2\text{lo}}{}$$

$$+ \frac{(-c^2-2cdx-d^2x^2)\,\text{sech}^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}$$

$$+ \frac{\text{csch}\left(\frac{a}{2}\right)\text{csch}\left(\frac{a}{2}+\frac{bx}{2}\right)\left(cd\sinh\left(\frac{bx}{2}\right)+d^2x\sinh\left(\frac{bx}{2}\right)\right)}{2b^2}$$

$$+ \frac{\text{sech}\left(\frac{a}{2}\right)\text{sech}\left(\frac{a}{2}+\frac{bx}{2}\right)\left(cd\sinh\left(\frac{bx}{2}\right)+d^2x\sinh\left(\frac{bx}{2}\right)\right)}{2b^2}$$

input
```
Integrate[(c + d*x)^2*Csch[a + b*x]^3,x]
```

output
```
-((d*(c + d*x)*Csch[a])/b^2) + ((-c^2 - 2*c*d*x - d^2*x^2)*Csch[a/2 + (b*x
)/2]^2)/(8*b) + (-(b^2*c^2*Log[1 - E^(a + b*x)]) + 2*d^2*Log[1 - E^(a + b*
x)] - 2*b^2*c*d*x*Log[1 - E^(a + b*x)] - b^2*d^2*x^2*Log[1 - E^(a + b*x)]
+ b^2*c^2*Log[1 + E^(a + b*x)] - 2*d^2*Log[1 + E^(a + b*x)] + 2*b^2*c*d*x*
Log[1 + E^(a + b*x)] + b^2*d^2*x^2*Log[1 + E^(a + b*x)] + 2*b*d*(c + d*x)*
PolyLog[2, -E^(a + b*x)] - 2*b*d*(c + d*x)*PolyLog[2, E^(a + b*x)] - 2*d^2
*PolyLog[3, -E^(a + b*x)] + 2*d^2*PolyLog[3, E^(a + b*x)])/(2*b^3) + ((-c^
2 - 2*c*d*x - d^2*x^2)*Sech[a/2 + (b*x)/2]^2)/(8*b) + (Csch[a/2]*Csch[a/2
+ (b*x)/2]*(c*d*Sinh[(b*x)/2] + d^2*x*Sinh[(b*x)/2]))/(2*b^2) + (Sech[a/2]
*Sech[a/2 + (b*x)/2]*(c*d*Sinh[(b*x)/2] + d^2*x*Sinh[(b*x)/2]))/(2*b^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.19, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {3042, 26, 4674, 26, 3042, 26, 4257, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\text{csch}^3(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -i(c+dx)^2\csc(ia+ibx)^3dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int (c+dx)^2\csc(ia+ibx)^3dx$$

$$\Big\downarrow \text{ 4674}$$

$$-i\Bigg(-\frac{d^2\int -i\text{csch}(a+bx)dx}{b^2}+\frac{1}{2}\int -i(c+dx)^2\text{csch}(a+bx)dx-\frac{id(c+dx)\text{csch}(a+bx)}{b^2}-\frac{i(c+dx)^2\coth(a+}{2b}$$

$$\Big\downarrow \text{ 26}$$

$$-i\Bigg(\frac{id^2\int \text{csch}(a+bx)dx}{b^2}-\frac{1}{2}i\int (c+dx)^2\text{csch}(a+bx)dx-\frac{id(c+dx)\text{csch}(a+bx)}{b^2}-\frac{i(c+dx)^2\coth(a+bx)\text{cs}}{2b}$$

$$\Big\downarrow \text{ 3042}$$

$$-i\Bigg(\frac{id^2\int i\csc(ia+ibx)dx}{b^2}-\frac{1}{2}i\int i(c+dx)^2\csc(ia+ibx)dx-\frac{id(c+dx)\text{csch}(a+bx)}{b^2}-\frac{i(c+dx)^2\coth(a+b}{2b}$$

$$\Big\downarrow \text{ 26}$$

$$-i\Bigg(-\frac{d^2\int \csc(ia+ibx)dx}{b^2}+\frac{1}{2}\int (c+dx)^2\csc(ia+ibx)dx-\frac{id(c+dx)\text{csch}(a+bx)}{b^2}-\frac{i(c+dx)^2\coth(a+bx)}{2b}$$

$$\Big\downarrow \text{ 4257}$$

$$-i\Bigg(\frac{1}{2}\int (c+dx)^2\csc(ia+ibx)dx-\frac{id^2\text{arctanh}(\cosh(a+bx))}{b^3}-\frac{id(c+dx)\text{csch}(a+bx)}{b^2}-\frac{i(c+dx)^2\coth(a+}{2b}$$

$$\Big\downarrow \text{ 4670}$$

$$-i\Bigg(\frac{1}{2}\Bigg(\frac{2id\int (c+dx)\log\left(1-e^{a+bx}\right)dx}{b}-\frac{2id\int (c+dx)\log\left(1+e^{a+bx}\right)dx}{b}+\frac{2i(c+dx)^2\text{arctanh}\left(e^{a+bx}\right)}{b}\Bigg)-\frac{i\text{c}}{}$$

$$\Big\downarrow \text{3011}$$

$$-i\left(\frac{1}{2}\left(-\frac{2id\left(\frac{d\int \text{PolyLog}(2,-e^{a+bx})dx}{b}-\frac{(c+dx)\,\text{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{2id\left(\frac{d\int \text{PolyLog}(2,e^{a+bx})dx}{b}-\frac{(c+dx)\,\text{PolyLog}(2,e^{a+bx}}{b}\right.}{b}\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$-i\left(\frac{1}{2}\left(-\frac{2id\left(\frac{d\int e^{-a-bx}\,\text{PolyLog}(2,-e^{a+bx})de^{a+bx}}{b^2}-\frac{(c+dx)\,\text{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{2id\left(\frac{d\int e^{-a-bx}\,\text{PolyLog}(2,e^{a+bx})de^{a+bx}}{b^2}-\right.}{b}\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$-i\left(-\frac{id^2\text{arctanh}(\cosh(a+bx))}{b^3}+\frac{1}{2}\left(\frac{2i(c+dx)^2\text{arctanh}(e^{a+bx})}{b}-\frac{2id\left(\frac{d\,\text{PolyLog}(3,-e^{a+bx})}{b^2}-\frac{(c+dx)\,\text{PolyLog}(2,-e^{a}}{b}\right.}{b}\right.\right.$$

input
```
Int[(c + d*x)^2*Csch[a + b*x]^3,x]
```

output
```
(-I)*(((-I)*d^2*ArcTanh[Cosh[a + b*x]])/b^3 - (I*d*(c + d*x)*Csch[a + b*x]
)/b^2 - ((I/2)*(c + d*x)^2*Coth[a + b*x]*Csch[a + b*x])/b + (((2*I)*(c + d
*x)^2*ArcTanh[E^(a + b*x)])/b - ((2*I)*d*(-(((c + d*x)*PolyLog[2, -E^(a +
b*x)])/b) + (d*PolyLog[3, -E^(a + b*x)])/b^2))/b + ((2*I)*d*(-(((c + d*x)*
PolyLog[2, E^(a + b*x)])/b) + (d*PolyLog[3, E^(a + b*x)])/b^2))/b)/2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(147) = 294$.

Time = 0.16 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.88

method	result
risch	$-\frac{e^{bx+a}\left(e^{2bx+2a} b\, d^2 x^2+2\, e^{2bx+2a} bcdx+e^{2bx+2a} b\, c^2+b\, d^2 x^2+2\, e^{2bx+2a} d^2 x+2bcdx+2\, e^{2bx+2a} cd+b\, c^2-2d^2 x-2dc\right)}{b^2\left(e^{2bx+2a}-1\right)^2}-\frac{2d^2 \operatorname{arcta}}{b}$

input
```
int((d*x+c)^2*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-exp(b*x+a)*(exp(2*b*x+2*a)*b*d^2*x^2+2*exp(2*b*x+2*a)*b*c*d*x+exp(2*b*x+2
*a)*b*c^2+b*d^2*x^2+2*exp(2*b*x+2*a)*d^2*x+2*b*c*d*x+2*exp(2*b*x+2*a)*c*d+
b*c^2-2*d^2*x-2*d*c)/b^2/(exp(2*b*x+2*a)-1)^2-2/b^3*d^2*arctanh(exp(b*x+a)
)+d^2*polylog(3,exp(b*x+a))/b^3-d^2*polylog(3,-exp(b*x+a))/b^3+1/2/b^3*d^2
*ln(1-exp(b*x+a))*a^2-1/2/b^3*d^2*ln(exp(b*x+a)+1)*a^2+1/b^3*d^2*a^2*arcta
nh(exp(b*x+a))+1/b*c^2*arctanh(exp(b*x+a))+1/b^2*c*d*ln(exp(b*x+a)+1)*a-2/
b^2*d*a*c*arctanh(exp(b*x+a))-1/b*c*d*ln(1-exp(b*x+a))*x-1/b^2*c*d*ln(1-ex
p(b*x+a))*a+1/b*c*d*ln(exp(b*x+a)+1)*x-1/2/b*d^2*ln(1-exp(b*x+a))*x^2-1/b^
2*d^2*polylog(2,exp(b*x+a))*x+1/2/b*d^2*ln(exp(b*x+a)+1)*x^2+1/b^2*d^2*pol
ylog(2,-exp(b*x+a))*x-1/b^2*c*d*polylog(2,exp(b*x+a))+1/b^2*c*d*polylog(2,
-exp(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2218 vs. $2(145) = 290$.

Time = 0.12 (sec) , antiderivative size = 2218, normalized size of antiderivative = 14.40

$$\int (c+dx)^2 \operatorname{csch}^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d + 2*(b^2*c*d + b*d^2)*x)*cosh(b*x
 + a)^3 + 6*(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d + 2*(b^2*c*d + b*d^2)*x)*cosh
(b*x + a)*sinh(b*x + a)^2 + 2*(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d + 2*(b^2*c*
d + b*d^2)*x)*sinh(b*x + a)^3 + 2*(b^2*d^2*x^2 + b^2*c^2 - 2*b*c*d + 2*(b^
2*c*d - b*d^2)*x)*cosh(b*x + a) + 2*((b*d^2*x + b*c*d)*cosh(b*x + a)^4 + 4
*(b*d^2*x + b*c*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*d^2*x + b*c*d)*sinh(
b*x + a)^4 + b*d^2*x + b*c*d - 2*(b*d^2*x + b*c*d)*cosh(b*x + a)^2 - 2*(b*
d^2*x + b*c*d - 3*(b*d^2*x + b*c*d)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 4*(
(b*d^2*x + b*c*d)*cosh(b*x + a)^3 - (b*d^2*x + b*c*d)*cosh(b*x + a))*sinh(
b*x + a))*dilog(cosh(b*x + a) + sinh(b*x + a)) - 2*((b*d^2*x + b*c*d)*cosh
(b*x + a)^4 + 4*(b*d^2*x + b*c*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*d^2*x
 + b*c*d)*sinh(b*x + a)^4 + b*d^2*x + b*c*d - 2*(b*d^2*x + b*c*d)*cosh(b*x
 + a)^2 - 2*(b*d^2*x + b*c*d - 3*(b*d^2*x + b*c*d)*cosh(b*x + a)^2)*sinh(b
*x + a)^2 + 4*((b*d^2*x + b*c*d)*cosh(b*x + a)^3 - (b*d^2*x + b*c*d)*cosh(
b*x + a))*sinh(b*x + a))*dilog(-cosh(b*x + a) - sinh(b*x + a)) - (b^2*d^2*
x^2 + 2*b^2*c*d*x + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cosh(b*x
 + a)^4 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cosh(b*x + a)*si
nh(b*x + a)^3 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sinh(b*x + a
)^4 + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cosh(b*x +
 a)^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 3*(b^2*d^2*x^2 + 2*b^2...
```

Sympy [F]

$$\int (c+dx)^2 \operatorname{csch}^3(a+bx)\,dx = \int (c+dx)^2 \operatorname{csch}^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*csch(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**2*csch(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(145) = 290$.

Time = 0.20 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.55

$$\int (c+dx)^2 \operatorname{csch}^3(a+bx)\,dx$$
$$= \frac{1}{2}c^2\left(\frac{\log\left(e^{(-bx-a)}+1\right)}{b} - \frac{\log\left(e^{(-bx-a)}-1\right)}{b} + \frac{2\left(e^{(-bx-a)}+e^{(-3\,bx-3\,a)}\right)}{b(2\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-1)}\right)$$
$$+ \frac{\left(bx\log\left(e^{(bx+a)}+1\right)+\operatorname{Li}_2\left(-e^{(bx+a)}\right)\right)cd}{b^2} - \frac{\left(bx\log\left(-e^{(bx+a)}+1\right)+\operatorname{Li}_2\left(e^{(bx+a)}\right)\right)cd}{b^2}$$
$$- \frac{\left(bd^2x^2e^{(3\,a)}+2\,cde^{(3\,a)}+2\,(bcd+d^2)xe^{(3\,a)}\right)e^{(3\,bx)}+\left(bd^2x^2e^a-2\,cde^a+2\,(bcd-d^2)xe^a\right)e^{(bx)}}{b^2e^{(4\,bx+4\,a)}-2\,b^2e^{(2\,bx+2\,a)}+b^2}$$
$$+ \frac{\left(b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\operatorname{Li}_2\left(-e^{(bx+a)}\right)-2\operatorname{Li}_3\left(-e^{(bx+a)}\right)\right)d^2}{2\,b^3}$$
$$- \frac{\left(b^2x^2\log\left(-e^{(bx+a)}+1\right)+2\,bx\operatorname{Li}_2\left(e^{(bx+a)}\right)-2\operatorname{Li}_3\left(e^{(bx+a)}\right)\right)d^2}{2\,b^3}$$
$$- \frac{d^2\log\left(e^{(bx+a)}+1\right)}{b^3} + \frac{d^2\log\left(e^{(bx+a)}-1\right)}{b^3}$$

input
```
integrate((d*x+c)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*c^2*(log(e^(-b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b + 2*(e^(-b*x -
a) + e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))) +
(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))*c*d/b^2 - (b*x*log(-e^(b*
x + a) + 1) + dilog(e^(b*x + a)))*c*d/b^2 - ((b*d^2*x^2*e^(3*a) + 2*c*d*e^
(3*a) + 2*(b*c*d + d^2)*x*e^(3*a))*e^(3*b*x) + (b*d^2*x^2*e^a - 2*c*d*e^a
+ 2*(b*c*d - d^2)*x*e^a)*e^(b*x))/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2*b*x +
2*a) + b^2) + 1/2*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)
) - 2*polylog(3, -e^(b*x + a)))*d^2/b^3 - 1/2*(b^2*x^2*log(-e^(b*x + a) +
1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))*d^2/b^3 - d^2*l
og(e^(b*x + a) + 1)/b^3 + d^2*log(e^(b*x + a) - 1)/b^3
```

Giac [F]

$$\int (c+dx)^2\text{csch}^3(a+bx)\,dx=\int (dx+c)^2\,\text{csch}\,(bx+a)^3\,\,dx$$

input
```
integrate((d*x+c)^2*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*csch(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2\text{csch}^3(a+bx)\,dx=\int \frac{(c+d\,x)^2}{\sinh\left(a+b\,x\right)^3}\,dx$$

input
```
int((c + d*x)^2/sinh(a + b*x)^3,x)
```

output
```
int((c + d*x)^2/sinh(a + b*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^2\text{csch}^3(a+bx)\,dx=\text{Too large to display}$$

input
```
int((d*x+c)^2*csch(b*x+a)^3,x)
```

output

```
( - 48*e**(5*a + 4*b*x)*int((e**(b*x)*x**2)/(e**(6*a + 6*b*x) - 3*e**(4*a
+ 4*b*x) + 3*e**(2*a + 2*b*x) - 1),x)*b**3*d**2 - 96*e**(5*a + 4*b*x)*int(
(e**(b*x)*x)/(e**(6*a + 6*b*x) - 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) -
 1),x)*b**3*c*d - 128*e**(5*a + 4*b*x)*int((e**(b*x)*x)/(e**(6*a + 6*b*x)
- 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) - 1),x)*b**2*d**2 - 9*e**(4*a +
4*b*x)*log(e**(a + b*x) - 1)*b**2*c**2 - 24*e**(4*a + 4*b*x)*log(e**(a + b
*x) - 1)*b*c*d - 8*e**(4*a + 4*b*x)*log(e**(a + b*x) - 1)*d**2 + 9*e**(4*a
 + 4*b*x)*log(e**(a + b*x) + 1)*b**2*c**2 + 24*e**(4*a + 4*b*x)*log(e**(a
+ b*x) + 1)*b*c*d + 8*e**(4*a + 4*b*x)*log(e**(a + b*x) + 1)*d**2 - 18*e**
(3*a + 3*b*x)*b**2*c**2 - 48*e**(3*a + 3*b*x)*b*c*d - 16*e**(3*a + 3*b*x)*
d**2 + 96*e**(3*a + 2*b*x)*int((e**(b*x)*x**2)/(e**(6*a + 6*b*x) - 3*e**(4
*a + 4*b*x) + 3*e**(2*a + 2*b*x) - 1),x)*b**3*d**2 + 192*e**(3*a + 2*b*x)*
int((e**(b*x)*x)/(e**(6*a + 6*b*x) - 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*
x) - 1),x)*b**3*c*d + 256*e**(3*a + 2*b*x)*int((e**(b*x)*x)/(e**(6*a + 6*b
*x) - 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) - 1),x)*b**2*d**2 + 18*e**(2
*a + 2*b*x)*log(e**(a + b*x) - 1)*b**2*c**2 + 48*e**(2*a + 2*b*x)*log(e**(
a + b*x) - 1)*b*c*d + 16*e**(2*a + 2*b*x)*log(e**(a + b*x) - 1)*d**2 - 18*
e**(2*a + 2*b*x)*log(e**(a + b*x) + 1)*b**2*c**2 - 48*e**(2*a + 2*b*x)*log
(e**(a + b*x) + 1)*b*c*d - 16*e**(2*a + 2*b*x)*log(e**(a + b*x) + 1)*d**2
- 18*e**(a + b*x)*b**2*c**2 - 96*e**(a + b*x)*b**2*c*d*x - 48*e**(a + b...
```

3.35 $\int (c+dx)\text{csch}^3(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 92

$$\int (c+dx)\text{csch}^3(a+bx)\,dx = \frac{(c+dx)\text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{d\text{csch}(a+bx)}{2b^2} - \frac{(c+dx)\coth(a+bx)\text{csch}(a+bx)}{2b} + \frac{d\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2} - \frac{d\,\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2}$$

output
```
(d*x+c)*arctanh(exp(b*x+a))/b-1/2*d*csch(b*x+a)/b^2-1/2*(d*x+c)*coth(b*x+a
)*csch(b*x+a)/b+1/2*d*polylog(2,-exp(b*x+a))/b^2-1/2*d*polylog(2,exp(b*x+a
))/b^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 271 vs. 2(92) = 184.

Time = 0.20 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.95

$$\int (c+dx)\operatorname{csch}^3(a+bx)\,dx = -\frac{dx\operatorname{csch}^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b} - \frac{c\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{dx\log\left(1-e^{a+bx}\right)}{2b} + \frac{dx\log\left(1+e^{a+bx}\right)}{2b} + \frac{c\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{2b} - \frac{c\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{2b} + \frac{d\operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2} - \frac{d\operatorname{PolyLog}\left(2,e^{a+bx}\right)}{2b^2} - \frac{dx\operatorname{sech}^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b} - \frac{c\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b} + \frac{d\operatorname{csch}\left(\frac{a}{2}\right)\operatorname{csch}\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{bx}{2}\right)}{4b^2} + \frac{d\operatorname{sech}\left(\frac{a}{2}\right)\operatorname{sech}\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{bx}{2}\right)}{4b^2}$$

input
```
Integrate[(c + d*x)*Csch[a + b*x]^3,x]
```

output
```
-1/8*(d*x*Csch[a/2 + (b*x)/2]^2)/b - (c*Csch[(a + b*x)/2]^2)/(8*b) - (d*x*
Log[1 - E^(a + b*x)])/(2*b) + (d*x*Log[1 + E^(a + b*x)])/(2*b) + (c*Log[Co
sh[(a + b*x)/2]])/(2*b) - (c*Log[Sinh[(a + b*x)/2]])/(2*b) + (d*PolyLog[2,
 -E^(a + b*x)])/(2*b^2) - (d*PolyLog[2, E^(a + b*x)])/(2*b^2) - (d*x*Sech[
a/2 + (b*x)/2]^2)/(8*b) - (c*Sech[(a + b*x)/2]^2)/(8*b) + (d*Csch[a/2]*Csc
h[a/2 + (b*x)/2]*Sinh[(b*x)/2])/(4*b^2) + (d*Sech[a/2]*Sech[a/2 + (b*x)/2]
*Sinh[(b*x)/2])/(4*b^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3042, 26, 4673, 26, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\text{csch}^3(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -i(c+dx)\csc(ia+ibx)^3dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int (c+dx)\csc(ia+ibx)^3dx$$

$$\Big\downarrow \text{ 4673}$$

$$-i\left(\frac{1}{2}\int -i(c+dx)\text{csch}(a+bx)dx-\frac{id\text{csch}(a+bx)}{2b^2}-\frac{i(c+dx)\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{ 26}$$

$$-i\left(-\frac{1}{2}i\int (c+dx)\text{csch}(a+bx)dx-\frac{id\text{csch}(a+bx)}{2b^2}-\frac{i(c+dx)\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-i\left(-\frac{1}{2}i\int i(c+dx)\csc(ia+ibx)dx-\frac{id\text{csch}(a+bx)}{2b^2}-\frac{i(c+dx)\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{ 26}$$

$$-i\left(\frac{1}{2}\int (c+dx)\csc(ia+ibx)dx-\frac{id\text{csch}(a+bx)}{2b^2}-\frac{i(c+dx)\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{ 4670}$$

$$-i\left(\frac{1}{2}\left(\frac{id\int \log\left(1-e^{a+bx}\right)dx}{b}-\frac{id\int \log\left(1+e^{a+bx}\right)dx}{b}+\frac{2i(c+dx)\text{arctanh}\left(e^{a+bx}\right)}{b}\right)-\frac{id\text{csch}(a+bx)}{2b^2}-\frac{i(c$$

$$\Big\downarrow \text{ 2715}$$

$$-i\left(\frac{1}{2}\left(\frac{id\int e^{-a-bx}\log\left(1-e^{a+bx}\right)de^{a+bx}}{b^2}-\frac{id\int e^{-a-bx}\log\left(1+e^{a+bx}\right)de^{a+bx}}{b^2}+\frac{2i(c+dx)\text{arctanh}\left(e^{a+bx}\right)}{b}\right)-$$

$$\Big\downarrow \text{ 2838}$$

$$-i\left(\frac{1}{2}\left(\frac{2i(c+dx)\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{id\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}-\frac{id\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}\right)-\frac{id\text{csch}(a+bx)}{2b^2}-\frac{i(c-$$

input
```
Int[(c + d*x)*Csch[a + b*x]^3,x]
```

output
```
(-I)*(((-1/2*I)*d*Csch[a + b*x])/b^2 - ((I/2)*(c + d*x)*Coth[a + b*x]*Csch
[a + b*x])/b + (((2*I)*(c + d*x)*ArcTanh[E^(a + b*x)])/b + (I*d*PolyLog[2,
 -E^(a + b*x)])/b^2 - (I*d*PolyLog[2, E^(a + b*x)])/b^2)/2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(81) = 162$.

Time = 0.12 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.14

method	result
risch	$-\frac{e^{bx+a}\left(bdx\,e^{2bx+2a}+bc\,e^{2bx+2a}+dxb+d\,e^{2bx+2a}+bc-d\right)}{b^{2}\left(e^{2bx+2a}-1\right)^{2}}+\frac{c\,\mathrm{arctanh}\left(e^{bx+a}\right)}{b}-\frac{d\ln\left(1-e^{bx+a}\right)x}{2b}-\frac{d\ln\left(1-e^{bx+a}\right)a}{2b^{2}}-\frac{d\,\mathrm{p}}{}$

input
```
int((d*x+c)*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-exp(b*x+a)*(b*d*x*exp(2*b*x+2*a)+b*c*exp(2*b*x+2*a)+d*x*b+d*exp(2*b*x+2*a
)+b*c-d)/b^2/(exp(2*b*x+2*a)-1)^2+1/b*c*arctanh(exp(b*x+a))-1/2/b*d*ln(1-e
xp(b*x+a))*x-1/2/b^2*d*ln(1-exp(b*x+a))*a-1/2*d*polylog(2,exp(b*x+a))/b^2+
1/2/b*d*ln(exp(b*x+a)+1)*x+1/2/b^2*d*ln(exp(b*x+a)+1)*a+1/2*d*polylog(2,-e
xp(b*x+a))/b^2-1/b^2*d*a*arctanh(exp(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1026 vs. $2(79) = 158$.

Time = 0.12 (sec) , antiderivative size = 1026, normalized size of antiderivative = 11.15

$$\int (c+dx)\text{csch}^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*(b*d*x + b*c + d)*cosh(b*x + a)^3 + 6*(b*d*x + b*c + d)*cosh(b*x +
 a)*sinh(b*x + a)^2 + 2*(b*d*x + b*c + d)*sinh(b*x + a)^3 + 2*(b*d*x + b*c
 - d)*cosh(b*x + a) + (d*cosh(b*x + a)^4 + 4*d*cosh(b*x + a)*sinh(b*x + a)
^3 + d*sinh(b*x + a)^4 - 2*d*cosh(b*x + a)^2 + 2*(3*d*cosh(b*x + a)^2 - d)
*sinh(b*x + a)^2 + 4*(d*cosh(b*x + a)^3 - d*cosh(b*x + a))*sinh(b*x + a) +
 d)*dilog(cosh(b*x + a) + sinh(b*x + a)) - (d*cosh(b*x + a)^4 + 4*d*cosh(b
*x + a)*sinh(b*x + a)^3 + d*sinh(b*x + a)^4 - 2*d*cosh(b*x + a)^2 + 2*(3*d
*cosh(b*x + a)^2 - d)*sinh(b*x + a)^2 + 4*(d*cosh(b*x + a)^3 - d*cosh(b*x
+ a))*sinh(b*x + a) + d)*dilog(-cosh(b*x + a) - sinh(b*x + a)) - ((b*d*x +
 b*c)*cosh(b*x + a)^4 + 4*(b*d*x + b*c)*cosh(b*x + a)*sinh(b*x + a)^3 + (b
*d*x + b*c)*sinh(b*x + a)^4 + b*d*x - 2*(b*d*x + b*c)*cosh(b*x + a)^2 - 2*
(b*d*x - 3*(b*d*x + b*c)*cosh(b*x + a)^2 + b*c)*sinh(b*x + a)^2 + b*c + 4*
((b*d*x + b*c)*cosh(b*x + a)^3 - (b*d*x + b*c)*cosh(b*x + a))*sinh(b*x + a
))*log(cosh(b*x + a) + sinh(b*x + a) + 1) + ((b*c - a*d)*cosh(b*x + a)^4 +
 4*(b*c - a*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*c - a*d)*sinh(b*x + a)^4
 - 2*(b*c - a*d)*cosh(b*x + a)^2 + 2*(3*(b*c - a*d)*cosh(b*x + a)^2 - b*c
+ a*d)*sinh(b*x + a)^2 + b*c - a*d + 4*((b*c - a*d)*cosh(b*x + a)^3 - (b*c
 - a*d)*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) -
1) + ((b*d*x + a*d)*cosh(b*x + a)^4 + 4*(b*d*x + a*d)*cosh(b*x + a)*sinh(b
*x + a)^3 + (b*d*x + a*d)*sinh(b*x + a)^4 + b*d*x - 2*(b*d*x + a*d)*cos...
```

Sympy [F]

$$\int (c+dx)\text{csch}^3(a+bx)\,dx=\int (c+dx)\,\text{csch}^3\,(a+bx)\,dx$$

input
```
integrate((d*x+c)*csch(b*x+a)**3,x)
```

output
```
Integral((c + d*x)*csch(a + b*x)**3, x)
```

Maxima [F]

$$\int (c+dx)\text{csch}^3(a+bx)\,dx=\int (dx+c)\,\text{csch}\,(bx+a)^3\,\,dx$$

input
```
integrate((d*x+c)*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
-d*(((b*x*e^(3*a) + e^(3*a))*e^(3*b*x) + (b*x*e^a - e^a)*e^(b*x))/(b^2*e^(
4*b*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2) + 8*integrate(1/16*x/(e^(b*x +
 a) + 1), x) + 8*integrate(1/16*x/(e^(b*x + a) - 1), x)) + 1/2*c*(log(e^(-
b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b + 2*(e^(-b*x - a) + e^(-3*b*x -
3*a))/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1)))
```

Giac [F]

$$\int (c+dx)\text{csch}^3(a+bx)\,dx=\int (dx+c)\,\text{csch}\,(bx+a)^3\,\,dx$$

input
```
integrate((d*x+c)*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)*csch(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\text{csch}^3(a+bx)\,dx=\int \frac{c+d\,x}{\sinh\left(a+b\,x\right)^3}\,dx$$

input
```
int((c + d*x)/sinh(a + b*x)^3,x)
```

output
```
int((c + d*x)/sinh(a + b*x)^3, x)
```

Reduce [F]

$$\int (c + dx)\text{csch}^3(a + bx)\, dx$$
$$= \frac{-16e^{4bx+5a}\left(\int \frac{e^{bx}x}{e^{6bx+6a}-3e^{4bx+4a}+3e^{2bx+2a}-1}dx\right) b^2d - 3e^{4bx+4a}\log\left(e^{bx+a} - 1\right) bc - 4e^{4bx+4a}\log\left(e^{bx+a} - 1\right) d +}{}$$

input
```
int((d*x+c)*csch(b*x+a)^3,x)
```

output
```
( - 16*e**(5*a + 4*b*x)*int((e**(b*x)*x)/(e**(6*a + 6*b*x) - 3*e**(4*a + 4
*b*x) + 3*e**(2*a + 2*b*x) - 1),x)*b**2*d - 3*e**(4*a + 4*b*x)*log(e**(a +
 b*x) - 1)*b*c - 4*e**(4*a + 4*b*x)*log(e**(a + b*x) - 1)*d + 3*e**(4*a +
4*b*x)*log(e**(a + b*x) + 1)*b*c + 4*e**(4*a + 4*b*x)*log(e**(a + b*x) + 1
)*d - 6*e**(3*a + 3*b*x)*b*c - 8*e**(3*a + 3*b*x)*d + 32*e**(3*a + 2*b*x)*
int((e**(b*x)*x)/(e**(6*a + 6*b*x) - 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*
x) - 1),x)*b**2*d + 6*e**(2*a + 2*b*x)*log(e**(a + b*x) - 1)*b*c + 8*e**(2
*a + 2*b*x)*log(e**(a + b*x) - 1)*d - 6*e**(2*a + 2*b*x)*log(e**(a + b*x)
+ 1)*b*c - 8*e**(2*a + 2*b*x)*log(e**(a + b*x) + 1)*d - 6*e**(a + b*x)*b*c
 - 16*e**(a + b*x)*b*d*x + 8*e**(a + b*x)*d - 16*e**a*int((e**(b*x)*x)/(e*
*(6*a + 6*b*x) - 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) - 1),x)*b**2*d -
3*log(e**(a + b*x) - 1)*b*c - 4*log(e**(a + b*x) - 1)*d + 3*log(e**(a + b*
x) + 1)*b*c + 4*log(e**(a + b*x) + 1)*d)/(6*b**2*(e**(4*a + 4*b*x) - 2*e**
(2*a + 2*b*x) + 1))
```

3.36 $\int \frac{\operatorname{csch}^3(a+bx)}{c+dx} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx} \, dx = \operatorname{Int}\left(\frac{\operatorname{csch}^3(a+bx)}{c+dx},x\right)$$

output
```
Defer(Int)(csch(b*x+a)^3/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 61.70 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx} \, dx = \int \frac{\operatorname{csch}^3(a+bx)}{c+dx} \, dx$$

input
```
Integrate[Csch[a + b*x]^3/(c + d*x),x]
```

output
```
Integrate[Csch[a + b*x]^3/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{i \csc(ia+ibx)^3}{c+dx}dx$$
$$\Big\downarrow \text{26}$$
$$-i\int \frac{\csc(ia+ibx)^3}{c+dx}dx$$
$$\Big\downarrow \text{4680}$$
$$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx}dx$$

input
```
Int[Csch[a + b*x]^3/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(bx+a\right)^{3}}{dx+c}dx$$

input
```
int(csch(b*x+a)^3/(d*x+c),x)
```

output
```
int(csch(b*x+a)^3/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^3}{dx+c}\,dx$$

input
```
integrate(csch(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)^3/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{csch}^3\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(csch(b*x+a)**3/(d*x+c),x)
```

output
```
Integral(csch(a + b*x)**3/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 433, normalized size of antiderivative = 27.06

$$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{csch}(bx+a)^3}{dx+c}\,dx$$

input
```
integrate(csch(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output
```
-((b*d*x*e^(3*a) + (b*c - d)*e^(3*a))*e^(3*b*x) + (b*d*x*e^a + (b*c + d)*e
^a)*e^(b*x))/(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2*e^(4*a) +
 2*b^2*c*d*x*e^(4*a) + b^2*c^2*e^(4*a))*e^(4*b*x) - 2*(b^2*d^2*x^2*e^(2*a)
 + 2*b^2*c*d*x*e^(2*a) + b^2*c^2*e^(2*a))*e^(2*b*x)) - 8*integrate(1/16*(b
^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3*e^a + 3*b^2*c*d^2*x^2*e^a + 3*b^
2*c^2*d*x*e^a + b^2*c^3*e^a)*e^(b*x)), x) - 8*integrate(-1/16*(b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3 - (b^2*d^3*x^3*e^a + 3*b^2*c*d^2*x^2*e^a + 3*b^2*c^2*d*x*
e^a + b^2*c^3*e^a)*e^(b*x)), x)
```

Giac [N/A]

Not integrable

Time = 2.49 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{csch}(bx+a)^3}{dx+c}\,dx$$

input
```
integrate(csch(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)^3/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx}\,dx=\int \frac{1}{{\mathrm{sinh}\left(a+b\,x\right)}^3\,\left(c+d\,x\right)} \,d x$$

input
```
int(1/(sinh(a + b*x)^3*(c + d*x)),x)
```

output
```
int(1/(sinh(a + b*x)^3*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{csch}\left(b x +a \right)^{3}}{d x +c}d x$$

input
```
int(csch(b*x+a)^3/(d*x+c),x)
```

output
```
int(csch(a + b*x)**3/(c + d*x),x)
```

3.37 $\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2} dx = \operatorname{Int}\left(\frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2}, x\right)$$

output
```
Defer(Int)(csch(b*x+a)^3/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 65.90 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2} dx = \int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2} dx$$

input
```
Integrate[Csch[a + b*x]^3/(c + d*x)^2,x]
```

output
```
Integrate[Csch[a + b*x]^3/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\csc(ia+ibx)^3}{(c+dx)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\csc(ia+ibx)^3}{(c+dx)^2}dx$$

$$\Big\downarrow \text{4680}$$

$$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2}dx$$

input
```
Int[Csch[a + b*x]^3/(c + d*x)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(bx+a)^3}{(dx+c)^2}dx$$

input
```
int(csch(b*x+a)^3/(d*x+c)^2,x)
```

output
```
int(csch(b*x+a)^3/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)^3}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(csch(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{csch}^3\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(csch(b*x+a)**3/(d*x+c)**2,x)
```

output
```
Integral(csch(a + b*x)**3/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 545, normalized size of antiderivative = 34.06

$$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)^3}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(csch(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-((b*d*x*e^(3*a) + (b*c - 2*d)*e^(3*a))*e^(3*b*x) + (b*d*x*e^a + (b*c + 2*
d)*e^a)*e^(b*x))/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3
+ (b^2*d^3*x^3*e^(4*a) + 3*b^2*c*d^2*x^2*e^(4*a) + 3*b^2*c^2*d*x*e^(4*a) +
 b^2*c^3*e^(4*a))*e^(4*b*x) - 2*(b^2*d^3*x^3*e^(2*a) + 3*b^2*c*d^2*x^2*e^(
2*a) + 3*b^2*c^2*d*x*e^(2*a) + b^2*c^3*e^(2*a))*e^(2*b*x)) - 8*integrate(1
/16*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 6*d^2)/(b^2*d^4*x^4 + 4*b^2*c*d
^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + (b^2*d^4*x^4*e^a +
4*b^2*c*d^3*x^3*e^a + 6*b^2*c^2*d^2*x^2*e^a + 4*b^2*c^3*d*x*e^a + b^2*c^4*
e^a)*e^(b*x)), x) - 8*integrate(-1/16*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
 - 6*d^2)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d
*x + b^2*c^4 - (b^2*d^4*x^4*e^a + 4*b^2*c*d^3*x^3*e^a + 6*b^2*c^2*d^2*x^2*
e^a + 4*b^2*c^3*d*x*e^a + b^2*c^4*e^a)*e^(b*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(csch(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{1}{\sinh\left(a+b\,x\right)^3\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(1/(sinh(a + b*x)^3*(c + d*x)^2),x)
```

output
```
int(1/(sinh(a + b*x)^3*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{csch}(bx+a)^3}{d^2x^2+2cdx+c^2}dx$$

input
```
int(csch(b*x+a)^3/(d*x+c)^2,x)
```

output
```
int(csch(a + b*x)**3/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.38 $\int (c+dx)^{5/2}\sinh(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 171

$$\int (c+dx)^{5/2}\sinh(a+bx)\,dx=\frac{15d^2\sqrt{c+dx}\cosh(a+bx)}{4b^3}+\frac{(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{15d^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{15d^{5/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{5d(c+dx)^{3/2}\sinh(a+bx)}{2b^2}$$

output
```
15/4*d^2*(d*x+c)^(1/2)*cosh(b*x+a)/b^3+(d*x+c)^(5/2)*cosh(b*x+a)/b-15/16*d
^(5/2)*exp(-a+b*c/d)*Pi^(1/2)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)-1
5/16*d^(5/2)*exp(a-b*c/d)*Pi^(1/2)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(
7/2)-5/2*d*(d*x+c)^(3/2)*sinh(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.63

$$\int (c+dx)^{5/2}\sinh(a+bx)\,dx=\frac{d^3e^{-a-\frac{bc}{d}}\left(-e^{2a}\sqrt{-\frac{b(c+dx)}{d}}\Gamma\left(\frac{7}{2},-\frac{b(c+dx)}{d}\right)+e^{\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\Gamma\left(\frac{7}{2},\frac{b(c+dx)}{d}\right)\right)}{2b^4\sqrt{c+dx}}$$

input
```
Integrate[(c + d*x)^(5/2)*Sinh[a + b*x],x]
```

output
```
(d^3*E^(-a - (b*c)/d)*(-(E^(2*a)*Sqrt[-((b*(c + d*x))/d)]*Gamma[7/2, -((b*
(c + d*x))/d)]) + E^((2*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[7/2, (b*(c + d
*x))/d]))/(2*b^4*Sqrt[c + d*x])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.19, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.938, Rules used = {3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sinh(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -i(c+dx)^{5/2}\sin(ia+ibx)dx$$

$$\Big\downarrow \; 26$$

$$-i\int (c+dx)^{5/2}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\int(c+dx)^{3/2}\cosh(a+bx)dx}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\int(c+dx)^{3/2}\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{2b}\right)$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}-\frac{3id\int-i\sqrt{c+dx}\sinh(a+bx)dx}{2b}\right)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}-\frac{3d\int\sqrt{c+dx}\sinh(a+bx)dx}{2b}\right)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}-\frac{3d\int-i\sqrt{c+dx}\sin(ia+ibx)dx}{2b}\right)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\int\sqrt{c+dx}\sin(ia+ibx)dx}{2b}\right)}{2b}\right)$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\int\frac{\cosh(a+bx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\int\frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)}{2b}\right)$$

$\downarrow$ 3788

$$-i\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{1}{2}i\int-\frac{ie^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int\frac{ie^{-a-bx}}{\sqrt{c+dx}}dx\right)}{2b}\right)}{2b}\right)}{2b}\right)$$

$\downarrow$ 26

$$-i\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{1}{2}\int\frac{e^{-a-bx}}{\sqrt{c+dx}}dx+\frac{1}{2}\int\frac{e^{a+bx}}{\sqrt{c+dx}}dx\right)}{2b}\right)}{2b}\right)}{2b}\right)$$

$\downarrow$ 2611

$$-i\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\int e^{a+\frac{b(c+\cdots}{\cdots}}}{\cdots}\right.}{2b}\right.}{2b}\right.}{2b}\right.$$

$\downarrow$ 2633

$$-i\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\cdots}{\cdots}\right.}{2b}\right.}{2b}\right.}{2b}\right.$$

$\downarrow$ 2634

$$-i\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{\sqrt{\pi}e^{\frac{bc}{d}-a}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\mathrm{e}}{2}\right.}{2b}\right.}{2b}\right.}{2b}\right.$$

input
```
Int[(c + d*x)^(5/2)*Sinh[a + b*x],x]
```

output
```
(-I)*((I*(c + d*x)^(5/2)*Cosh[a + b*x])/b - (((5*I)/2)*d*((((3*I)/2)*d*((I
*Sqrt[c + d*x]*Cosh[a + b*x])/b - ((I/2)*d*((E^(-a + (b*c)/d)*Sqrt[Pi]*Erf
[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) + (E^(a - (b*c)/d)*
Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d])))/b))/
b + ((c + d*x)^(3/2)*Sinh[a + b*x])/b))/b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

Maple [F]

$$\int (dx+c)^{\frac{5}{2}} \sinh (bx+a)\, dx$$

input
```
int((d*x+c)^(5/2)*sinh(b*x+a),x)
```

output
```
int((d*x+c)^(5/2)*sinh(b*x+a),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. 2(131) = 262.

Time = 0.11 (sec) , antiderivative size = 521, normalized size of antiderivative = 3.05

$$\int (c+dx)^{5/2}\sinh(a+bx)\,dx =$$

$$-\frac{15\,\sqrt{\pi}\left(d^3\cosh\left(bx+a\right)\cosh\left(-\frac{bc-ad}{d}\right)-d^3\cosh\left(bx+a\right)\sinh\left(-\frac{bc-ad}{d}\right)+\left(d^3\cosh\left(-\frac{bc-ad}{d}\right)-d^3\sinh\left(-\right.\right.\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
-1/16*(15*sqrt(pi)*(d^3*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d^3*cosh(b*x
+ a)*sinh(-(b*c - a*d)/d) + (d^3*cosh(-(b*c - a*d)/d) - d^3*sinh(-(b*c - a
*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) - 15*sqrt(pi
)*(d^3*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + d^3*cosh(b*x + a)*sinh(-(b*c -
 a*d)/d) + (d^3*cosh(-(b*c - a*d)/d) + d^3*sinh(-(b*c - a*d)/d))*sinh(b*x
+ a))*sqrt(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d)) - 2*(4*b^3*d^2*x^2 + 4*b^3*
c^2 + 10*b^2*c*d + 15*b*d^2 + (4*b^3*d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15
*b*d^2 + 2*(4*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)^2 + 2*(4*b^3*d^2*x^2 +
 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x
 + a)*sinh(b*x + a) + (4*b^3*d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 +
 2*(4*b^3*c*d - 5*b^2*d^2)*x)*sinh(b*x + a)^2 + 2*(4*b^3*c*d + 5*b^2*d^2)*
x)*sqrt(d*x + c))/(b^4*cosh(b*x + a) + b^4*sinh(b*x + a))
```

Sympy [F]

$$\int (c+dx)^{5/2}\sinh(a+bx)\,dx = \int (c+dx)^{\frac{5}{2}}\sinh\left(a+bx\right)dx$$

input
```
integrate((d*x+c)**(5/2)*sinh(b*x+a),x)
```

output
```
Integral((c + d*x)**(5/2)*sinh(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(131) = 262$.

Time = 0.05 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.80

$$\int (c+dx)^{5/2}\sinh(a+bx)\,dx = \frac{32\,(dx+c)^{\frac{7}{2}}\sinh(bx+a) - \frac{\left(\frac{105\,\sqrt{\pi}d^4\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^4\sqrt{-\frac{b}{d}}} + \frac{105\,\sqrt{\pi}d^4\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^4\sqrt{\frac{b}{d}}} - \frac{2\left(8\,(dx+c)^{\frac{7}{2}}b^3d\right.}{}}{}}{}$$

input

```
integrate((d*x+c)^(5/2)*sinh(b*x+a),x, algorithm="maxima")
```

output

```
1/112*(32*(d*x + c)^(7/2)*sinh(b*x + a) - (105*sqrt(pi)*d^4*erf(sqrt(d*x +
 c)*sqrt(-b/d))*e^(a - b*c/d)/(b^4*sqrt(-b/d)) + 105*sqrt(pi)*d^4*erf(sqrt
(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/(b^4*sqrt(b/d)) - 2*(8*(d*x + c)^(7/2)
*b^3*d*e^(b*c/d) + 28*(d*x + c)^(5/2)*b^2*d^2*e^(b*c/d) + 70*(d*x + c)^(3/
2)*b*d^3*e^(b*c/d) + 105*sqrt(d*x + c)*d^4*e^(b*c/d))*e^(-a - (d*x + c)*b/
d)/b^4 + 2*(8*(d*x + c)^(7/2)*b^3*d*e^a - 28*(d*x + c)^(5/2)*b^2*d^2*e^a +
 70*(d*x + c)^(3/2)*b*d^3*e^a - 105*sqrt(d*x + c)*d^4*e^a)*e^((d*x + c)*b/
d - b*c/d)/b^4)*b/d)/d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.36

$$\int (c+dx)^{5/2}\sinh(a+bx)\,dx = \frac{\frac{15\,\sqrt{\pi}d^4\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}}{\sqrt{bd}b^3} + \frac{15\,\sqrt{\pi}d^4\,\mathrm{erf}\left(-\frac{\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bd}b^3} + \frac{2\left(4\,(dx+c)^{\frac{5}{2}}b^2d-10\,(dx+c)^{\frac{3}{2}}bd^2+15\,\sqrt{dx+\right.}{b^3}}{16\,d}$$

input

```
integrate((d*x+c)^(5/2)*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/16*(15*sqrt(pi)*d^4*erf(-sqrt(b*d)*sqrt(d*x + c)/d)*e^((b*c - a*d)/d)/(s
qrt(b*d)*b^3) + 15*sqrt(pi)*d^4*erf(-sqrt(-b*d)*sqrt(d*x + c)/d)*e^(-(b*c
- a*d)/d)/(sqrt(-b*d)*b^3) + 2*(4*(d*x + c)^(5/2)*b^2*d - 10*(d*x + c)^(3/
2)*b*d^2 + 15*sqrt(d*x + c)*d^3)*e^(((d*x + c)*b - b*c + a*d)/d)/b^3 + 2*(
4*(d*x + c)^(5/2)*b^2*d + 10*(d*x + c)^(3/2)*b*d^2 + 15*sqrt(d*x + c)*d^3)
*e^(-((d*x + c)*b - b*c + a*d)/d)/b^3)/d
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\sinh(a+bx)\,dx=\int \sinh(a+b\,x)\,\left(c+d\,x\right)^{5/2}\,dx$$

input
```
int(sinh(a + b*x)*(c + d*x)^(5/2),x)
```

output
```
int(sinh(a + b*x)*(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int (c+dx)^{5/2}\sinh(a+bx)\,dx=\left(\int \sqrt{dx+c}\,\sinh\left(bx+a\right)x^2dx\right)d^2+2\left(\int \sqrt{dx+c}\,\sinh\left(bx+a\right)xdx\right)cd+\left(\int \sqrt{dx+c}\,\sinh\left(bx+a\right)dx\right)c^2$$

input
```
int((d*x+c)^(5/2)*sinh(b*x+a),x)
```

output
```
int(sqrt(c + d*x)*sinh(a + b*x)*x**2,x)*d**2 + 2*int(sqrt(c + d*x)*sinh(a
+ b*x)*x,x)*c*d + int(sqrt(c + d*x)*sinh(a + b*x),x)*c**2
```

3.39 $\int (c+dx)^{3/2}\sinh(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 146

$$\int (c+dx)^{3/2}\sinh(a+bx)\,dx=\frac{(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3d^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{3d^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{3d\sqrt{c+dx}\sinh(a+bx)}{2b^2}$$

output

```
(d*x+c)^(3/2)*cosh(b*x+a)/b-3/8*d^(3/2)*exp(-a+b*c/d)*Pi^(1/2)*erf(b^(1/2)
*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)+3/8*d^(3/2)*exp(a-b*c/d)*Pi^(1/2)*erfi(b^(
1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)-3/2*d*(d*x+c)^(1/2)*sinh(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.73

$$\int (c+dx)^{3/2}\sinh(a+bx)\,dx=\frac{de^{-a-\frac{bc}{d}}\sqrt{c+dx}\left(-\frac{e^{2a}\Gamma\left(\frac{5}{2},-\frac{b(c+dx)}{d}\right)}{\sqrt{-\frac{b(c+dx)}{d}}}+\frac{e^{\frac{2bc}{d}}\Gamma\left(\frac{5}{2},\frac{b(c+dx)}{d}\right)}{\sqrt{\frac{b(c+dx)}{d}}}\right)}{2b^2}$$

input
```
Integrate[(c + d*x)^(3/2)*Sinh[a + b*x],x]
```

output
```
(d*E^(-a - (b*c)/d)*Sqrt[c + d*x]*(-((E^(2*a)*Gamma[5/2, -((b*(c + d*x))/d
)])/Sqrt[-((b*(c + d*x))/d)]) + (E^((2*b*c)/d)*Gamma[5/2, (b*(c + d*x))/d]
)/Sqrt[(b*(c + d*x))/d]))/(2*b^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.20, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sinh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i(c+dx)^{3/2}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int (c+dx)^{3/2}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\int \sqrt{c+dx}\cosh(a+bx)dx}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\int \sqrt{c+dx}\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{2b}\right)$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}-\frac{id\int -\frac{i\sinh(a+bx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)$$

$\big\downarrow$ 26

$$-i\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}-\frac{d\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)$$

$\big\downarrow$ 3042

$$-i\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}-\frac{d\int -\frac{i\sin(ia+ibx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)$$

$\big\downarrow$ 26

$$-i\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\int \frac{\sin(ia+ibx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)$$

$\big\downarrow$ 3789

$$-i\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\left(\frac{1}{2}i\int \frac{e^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int \frac{e^{-a-bx}}{\sqrt{c+dx}}dx\right)}{2b}\right)}{2b}\right)$$

$\big\downarrow$ 2611

$$-i\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\left(\frac{i\int e^{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{d}-\frac{i\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{2b}\right)}{2b}\right)$$

$\big\downarrow$ 2633

$$-i\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{2b}\right)}{2b}\right)$$

$$\Big\downarrow \text{2634}$$

$$-i\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\pi}e^{\frac{bc}{d}-a}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{2b}\right)}{2b}\right)$$

input
```
Int[(c + d*x)^(3/2)*Sinh[a + b*x],x]
```

output
```
(-I)*((I*(c + d*x)^(3/2)*Cosh[a + b*x])/b - (((3*I)/2)*d*(((I/2)*d*(((-1/2
*I)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[
b]*Sqrt[d]) + ((I/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])
/Sqrt[d]])/(Sqrt[b]*Sqrt[d])))/b + (Sqrt[c + d*x]*Sinh[a + b*x])/b))/b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Maple [F]

$$\int (dx+c)^{\frac{3}{2}} \sinh (bx+a)\, dx$$

input
```
int((d*x+c)^(3/2)*sinh(b*x+a),x)
```

output
```
int((d*x+c)^(3/2)*sinh(b*x+a),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(110) = 220.

Time = 0.11 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.64

$$\int (c+dx)^{3/2} \sinh(a+bx)\, dx =$$

$$-\frac{3\sqrt{\pi}\left(d^2\cosh\left(bx+a\right)\cosh\left(-\frac{bc-ad}{d}\right)-d^2\cosh\left(bx+a\right)\sinh\left(-\frac{bc-ad}{d}\right)+\left(d^2\cosh\left(-\frac{bc-ad}{d}\right)-d^2\sinh\left(-\frac{b}{}\right.\right.\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
-1/8*(3*sqrt(pi)*(d^2*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d^2*cosh(b*x +
a)*sinh(-(b*c - a*d)/d) + (d^2*cosh(-(b*c - a*d)/d) - d^2*sinh(-(b*c - a*d
)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 3*sqrt(pi)*(
d^2*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + d^2*cosh(b*x + a)*sinh(-(b*c - a*
d)/d) + (d^2*cosh(-(b*c - a*d)/d) + d^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a
))*sqrt(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d)) - 2*(2*b^2*d*x + 2*b^2*c + (2*
b^2*d*x + 2*b^2*c - 3*b*d)*cosh(b*x + a)^2 + 2*(2*b^2*d*x + 2*b^2*c - 3*b*
d)*cosh(b*x + a)*sinh(b*x + a) + (2*b^2*d*x + 2*b^2*c - 3*b*d)*sinh(b*x +
a)^2 + 3*b*d)*sqrt(d*x + c))/(b^3*cosh(b*x + a) + b^3*sinh(b*x + a))
```

Sympy [F]

$$\int (c+dx)^{3/2}\sinh(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sinh(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*sinh(b*x+a),x)
```

output
```
Integral((c + d*x)**(3/2)*sinh(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(110) = 220.

Time = 0.05 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.84

$$\int (c+dx)^{3/2}\sinh(a+bx)\,dx = \frac{16\,(dx+c)^{\frac{5}{2}}\sinh(bx+a) + \frac{\left(\frac{15\sqrt{\pi}d^3\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}} - \frac{15\sqrt{\pi}d^3\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}} + \frac{2\left(4\,(dx+c)^{\frac{5}{2}}b^2de\left(\cdots\right.\right.}{\cdots}\cdots}{\cdots}}{4\cdots}$$

input
```
integrate((d*x+c)^(3/2)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/40*(16*(d*x + c)^(5/2)*sinh(b*x + a) + (15*sqrt(pi)*d^3*erf(sqrt(d*x + c
)*sqrt(-b/d))*e^(a - b*c/d)/(b^3*sqrt(-b/d)) - 15*sqrt(pi)*d^3*erf(sqrt(d*
x + c)*sqrt(b/d))*e^(-a + b*c/d)/(b^3*sqrt(b/d)) + 2*(4*(d*x + c)^(5/2)*b^
2*d*e^(b*c/d) + 10*(d*x + c)^(3/2)*b*d^2*e^(b*c/d) + 15*sqrt(d*x + c)*d^3*
e^(b*c/d))*e^(-a - (d*x + c)*b/d)/b^3 - 2*(4*(d*x + c)^(5/2)*b^2*d*e^a - 1
0*(d*x + c)^(3/2)*b*d^2*e^a + 15*sqrt(d*x + c)*d^3*e^a)*e^((d*x + c)*b/d -
 b*c/d)/b^3)*b/d)/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.38

$$\int (c+dx)^{3/2}\sinh(a+bx)\,dx = \frac{\frac{3\sqrt{\pi}d^3\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}}{\sqrt{bd}b^2} - \frac{3\sqrt{\pi}d^3\operatorname{erf}\left(-\frac{\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bd}b^2} + \frac{2\left(2(dx+c)^{\frac{3}{2}}bd-3\sqrt{dx+c}d^2\right)e^{\left(\frac{(dx+c)b-bc+ad}{d}\right)}}{b^2}}{8\,d}$$

input
```
integrate((d*x+c)^(3/2)*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/8*(3*sqrt(pi)*d^3*erf(-sqrt(b*d)*sqrt(d*x + c)/d)*e^((b*c - a*d)/d)/(sqr
t(b*d)*b^2) - 3*sqrt(pi)*d^3*erf(-sqrt(-b*d)*sqrt(d*x + c)/d)*e^(-(b*c - a
*d)/d)/(sqrt(-b*d)*b^2) + 2*(2*(d*x + c)^(3/2)*b*d - 3*sqrt(d*x + c)*d^2)*
e^((((d*x + c)*b - b*c + a*d)/d)/b^2 + 2*(2*(d*x + c)^(3/2)*b*d + 3*sqrt(d*
x + c)*d^2)*e^(-(((d*x + c)*b - b*c + a*d)/d)/b^2)/d
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\sinh(a+bx)\,dx = \int \sinh(a+b\,x)\,\left(c+d\,x\right)^{3/2}\,dx$$

input
```
int(sinh(a + b*x)*(c + d*x)^(3/2),x)
```

output
```
int(sinh(a + b*x)*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c+dx)^{3/2}\sinh(a+bx)\,dx = \left(\int \sqrt{dx+c}\,\sinh\left(bx+a\right)x dx\right)d + \left(\int \sqrt{dx+c}\,\sinh\left(bx+a\right)dx\right)c$$

input
```
int((d*x+c)^(3/2)*sinh(b*x+a),x)
```

output
```
int(sqrt(c + d*x)*sinh(a + b*x)*x,x)*d + int(sqrt(c + d*x)*sinh(a + b*x),x
)*c
```

3.40 $\int \sqrt{c+dx}\sinh(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 123

$$\int \sqrt{c+dx}\sinh(a+bx)\,dx=\frac{\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{\sqrt{d}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{d}e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}$$

output
```
(d*x+c)^(1/2)*cosh(b*x+a)/b-1/4*d^(1/2)*exp(-a+b*c/d)*Pi^(1/2)*erf(b^(1/2)
*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)-1/4*d^(1/2)*exp(a-b*c/d)*Pi^(1/2)*erfi(b^(
1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.85

$$\int \sqrt{c+dx}\sinh(a+bx)\,dx=\frac{e^{-a-\frac{bc}{d}}\sqrt{c+dx}\left(\frac{e^{2a}\Gamma\left(\frac{3}{2},-\frac{b(c+dx)}{d}\right)}{\sqrt{-\frac{b(c+dx)}{d}}}+\frac{e^{\frac{2bc}{d}}\Gamma\left(\frac{3}{2},\frac{b(c+dx)}{d}\right)}{\sqrt{\frac{b(c+dx)}{d}}}\right)}{2b}$$

input
```
Integrate[Sqrt[c + d*x]*Sinh[a + b*x],x]
```

output
```
(E^(-a - (b*c)/d)*Sqrt[c + d*x]*((E^(2*a)*Gamma[3/2, -((b*(c + d*x))/d)])/
Sqrt[-((b*(c + d*x))/d)] + (E^((2*b*c)/d)*Gamma[3/2, (b*(c + d*x))/d])/Sqr
t[(b*(c + d*x))/d]))/(2*b)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {3042, 26, 3777, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sinh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sqrt{c+dx}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sqrt{c+dx}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}dx}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{2b}\right)$$

$$\Big\downarrow \text{3788}$$

$$-i\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{1}{2}i\int -\frac{ie^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int \frac{ie^{-a-bx}}{\sqrt{c+dx}}dx\right)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{1}{2}\int\frac{e^{-a-bx}}{\sqrt{c+dx}}dx+\frac{1}{2}\int\frac{e^{a+bx}}{\sqrt{c+dx}}dx\right)}{2b}\right)$$

$$\Big\downarrow \text{2611}$$

$$-i\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\int e^{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{2b}\right)$$

$$\Big\downarrow \text{2633}$$

$$-i\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{2b}\right)$$

$$\Big\downarrow \text{2634}$$

$$-i\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{2b}\right)$$

input
```
Int[Sqrt[c + d*x]*Sinh[a + b*x],x]
```

output
```
(-I)*((I*Sqrt[c + d*x]*Cosh[a + b*x])/b - ((I/2)*d*((E^(-a + (b*c)/d)*Sqrt
[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) + (E^(a - (
b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]
)))/b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

Maple [F]

$$\int \sqrt{dx+c}\,\sinh(bx+a)\,dx$$

input
```
int((d*x+c)^(1/2)*sinh(b*x+a),x)
```

output
```
int((d*x+c)^(1/2)*sinh(b*x+a),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(91) = 182$.

Time = 0.12 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.45

$$\int \sqrt{c+dx}\sinh(a+bx)\,dx = -\frac{\sqrt{\pi}\left(d\cosh(bx+a)\cosh\left(-\frac{bc-ad}{d}\right)-d\cosh(bx+a)\sinh\left(-\frac{bc-ad}{d}\right)+\left(d\cosh\left(-\frac{bc-ad}{d}\right)-d\sinh\left(-\frac{bc-a}{d}\right.\right.\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*(d*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d*cosh(b*x + a)*sin
h(-(b*c - a*d)/d) + (d*cosh(-(b*c - a*d)/d) - d*sinh(-(b*c - a*d)/d))*sinh
(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) - sqrt(pi)*(d*cosh(b*x +
 a)*cosh(-(b*c - a*d)/d) + d*cosh(b*x + a)*sinh(-(b*c - a*d)/d) + (d*cosh(
-(b*c - a*d)/d) + d*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sq
rt(d*x + c)*sqrt(-b/d)) - 2*(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*
x + a) + b*sinh(b*x + a)^2 + b)*sqrt(d*x + c))/(b^2*cosh(b*x + a) + b^2*si
nh(b*x + a))
```

Sympy [F]

$$\int \sqrt{c+dx}\sinh(a+bx)\,dx = \int \sqrt{c+dx}\sinh{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**(1/2)*sinh(b*x+a),x)
```

output
```
Integral(sqrt(c + d*x)*sinh(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(91) = 182$.

Time = 0.05 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.87

$$\int \sqrt{c+dx}\sinh(a+bx)\,dx$$
$$= \frac{8\,(dx+c)^{\frac{3}{2}}\sinh{(bx+a)} - \frac{\left(\frac{3\sqrt{\pi}d^2\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}} + \frac{3\sqrt{\pi}d^2\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}} - \frac{2\left(2\,(dx+c)^{\frac{3}{2}}bde^{\left(\frac{bc}{d}\right)}+3\sqrt{dx+c}d^2e}{b^2}}{d}}{12\,d}$$

input
```
integrate((d*x+c)^(1/2)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/12*(8*(d*x + c)^(3/2)*sinh(b*x + a) - (3*sqrt(pi)*d^2*erf(sqrt(d*x + c)*
sqrt(-b/d))*e^(a - b*c/d)/(b^2*sqrt(-b/d)) + 3*sqrt(pi)*d^2*erf(sqrt(d*x +
 c)*sqrt(b/d))*e^(-a + b*c/d)/(b^2*sqrt(b/d)) - 2*(2*(d*x + c)^(3/2)*b*d*e
^(b*c/d) + 3*sqrt(d*x + c)*d^2*e^(b*c/d))*e^(-a - (d*x + c)*b/d)/b^2 + 2*(
2*(d*x + c)^(3/2)*b*d*e^a - 3*sqrt(d*x + c)*d^2*e^a)*e^((d*x + c)*b/d - b*
c/d)/b^2)*b/d)/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.37

$$\int \sqrt{c+dx}\sinh(a+bx)\,dx$$
$$=\frac{\frac{\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}}{\sqrt{bd}b}+\frac{\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bd}b}+\frac{2\sqrt{dx+c}de^{\left(\frac{(dx+c)b-bc+ad}{d}\right)}}{b}+\frac{2\sqrt{dx+c}de^{\left(-\frac{(dx+c)b-bc+a}{d}\right)}}{b}}{4\,d}$$

input
```
integrate((d*x+c)^(1/2)*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/4*(sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)/d)*e^((b*c - a*d)/d)/(sqrt(
b*d)*b) + sqrt(pi)*d^2*erf(-sqrt(-b*d)*sqrt(d*x + c)/d)*e^(-(b*c - a*d)/d)
/(sqrt(-b*d)*b) + 2*sqrt(d*x + c)*d*e^(((d*x + c)*b - b*c + a*d)/d)/b + 2*
sqrt(d*x + c)*d*e^(-((d*x + c)*b - b*c + a*d)/d)/b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\sinh(a+bx)\,dx=\int \sinh(a+b\,x)\,\sqrt{c+d\,x}\,dx$$

input
```
int(sinh(a + b*x)*(c + d*x)^(1/2),x)
```

output
```
int(sinh(a + b*x)*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\sinh(a+bx)\,dx=\int \sqrt{dx+c}\,\sinh(bx+a)\,dx$$

input
```
int((d*x+c)^(1/2)*sinh(b*x+a),x)
```

output
```
int(sqrt(c + d*x)*sinh(a + b*x),x)
```

3.41 $\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 104

$$\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx=-\frac{e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}$$

output
```
-1/2*exp(-a+b*c/d)*Pi^(1/2)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(1/2)/d^(
1/2)+1/2*exp(a-b*c/d)*Pi^(1/2)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(1/2)
/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx=\frac{e^{-a-\frac{bc}{d}}\left(e^{2a}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)+e^{\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},\frac{b(c+dx)}{d}\right)\right)}{2b\sqrt{c+dx}}$$

input
```
Integrate[Sinh[a + b*x]/Sqrt[c + d*x],x]
```

output
```
(E^(-a - (b*c)/d)*(E^(2*a)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, -((b*(c + d
*x))/d)] + E^((2*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (b*(c + d*x))/d]
))/(2*b*Sqrt[c + d*x])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ia+ibx)}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ia+ibx)}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{3789}$$

$$-i\left(\frac{1}{2}i\int \frac{e^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int \frac{e^{-a-bx}}{\sqrt{c+dx}}dx\right)$$

$$\Big\downarrow \text{2611}$$

$$-i\left(\frac{i\int e^{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{d}-\frac{i\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)$$

$$\Big\downarrow \text{2633}$$

$$-i\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)$$

$$\Big\downarrow \text{2634}$$

$$-i\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\pi}e^{\frac{bc}{d}-a}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)$$

input
```
Int[Sinh[a + b*x]/Sqrt[c + d*x],x]
```

output
```
(-I)*(((-1/2*I)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt
[d]])/(Sqrt[b]*Sqrt[d]) + ((I/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sq
rt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Maple [F]

$$\int \frac{\sinh(bx+a)}{\sqrt{dx+c}}dx$$

input
```
int(sinh(b*x+a)/(d*x+c)^(1/2),x)
```

output
```
int(sinh(b*x+a)/(d*x+c)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.17

$$\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx = -\frac{\sqrt{\pi}\sqrt{\frac{b}{d}}\left(\cosh\left(-\frac{bc-ad}{d}\right)-\sinh\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)+\sqrt{\pi}\sqrt{-\frac{b}{d}}\left(\cosh\left(-\frac{bc-ad}{d}\right)+\sinh\left(-\frac{bc-ad}{d}\right.\right.\cdots}{2\,b}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(pi)*sqrt(b/d)*(cosh(-(b*c - a*d)/d) - sinh(-(b*c - a*d)/d))*erf
(sqrt(d*x + c)*sqrt(b/d)) + sqrt(pi)*sqrt(-b/d)*(cosh(-(b*c - a*d)/d) + si
nh(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-b/d)))/b
```

Sympy [F]

$$\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\sinh{(a+bx)}}{\sqrt{c+dx}}\,dx$$

input
```
integrate(sinh(b*x+a)/(d*x+c)**(1/2),x)
```

output
```
Integral(sinh(a + b*x)/sqrt(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(74) = 148.

Time = 0.05 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.74

$$\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx = \frac{4\sqrt{dx+c}\sinh{(bx+a)} + \frac{\left(\frac{\sqrt{\pi}d\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}} - \frac{\sqrt{\pi}d\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b\sqrt{\frac{b}{d}}} - \frac{2\sqrt{dx+c}de^{\left(a+\frac{(dx+c)b}{d}-\frac{bc}{d}\right)}}{b} + \frac{2\sqrt{dx+c}de^{\left(-a-\frac{(dx+c)b}{d}+\frac{bc}{d}\right)}}{b}\right)b}{d}}{2\,d}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(4*sqrt(d*x + c)*sinh(b*x + a) + (sqrt(pi)*d*erf(sqrt(d*x + c)*sqrt(-b
/d))*e^(a - b*c/d)/(b*sqrt(-b/d)) - sqrt(pi)*d*erf(sqrt(d*x + c)*sqrt(b/d)
)*e^(-a + b*c/d)/(b*sqrt(b/d)) - 2*sqrt(d*x + c)*d*e^(a + (d*x + c)*b/d -
b*c/d)/b + 2*sqrt(d*x + c)*d*e^(-a - (d*x + c)*b/d + b*c/d)/b)*b/d)/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx = \frac{\left(\frac{\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{bc}{d}\right)}}{\sqrt{bd}} - \frac{\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{bc-2\,ad}{d}\right)}}{\sqrt{-bd}}\right)e^{(-a)}}{2\,d}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)/d)*e^(b*c/d)/sqrt(b*d) - sqrt
(pi)*d*erf(-sqrt(-b*d)*sqrt(d*x + c)/d)*e^(-(b*c - 2*a*d)/d)/sqrt(-b*d))*e
^(-a)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\sinh(a+b\,x)}{\sqrt{c+d\,x}}\,dx$$

input
```
int(sinh(a + b*x)/(c + d*x)^(1/2),x)
```

output
```
int(sinh(a + b*x)/(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\sinh(bx+a)}{\sqrt{dx+c}}dx$$

input
```
int(sinh(b*x+a)/(d*x+c)^(1/2),x)
```

output
```
int(sinh(a + b*x)/sqrt(c + d*x),x)
```

3.42 $\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 118

$$\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{\sqrt{b}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{b}e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2\sinh(a+bx)}{d\sqrt{c+dx}}$$

output
```
b^(1/2)*exp(-a+b*c/d)*Pi^(1/2)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(3/2)+
b^(1/2)*exp(a-b*c/d)*Pi^(1/2)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(3/2)-
2*sinh(b*x+a)/d/(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.02

$$\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{e^{-a-\frac{bc}{d}}\left(e^{2a}\sqrt{-\frac{b(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right) - e^{\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\Gamma\left(\frac{1}{2},\frac{b(c+dx)}{d}\right) - 2e^{a+\frac{bc}{d}}\sinh(a+\right.}{d\sqrt{c+dx}}$$

input
```
Integrate[Sinh[a + b*x]/(c + d*x)^(3/2),x]
```

output
```
(E^(-a - (b*c)/d)*(E^(2*a)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, -((b*(c + d
*x))/d)] - E^((2*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (b*(c + d*x))/d]
 - 2*E^(a + (b*c)/d)*Sinh[a + b*x]))/(d*Sqrt[c + d*x])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {3042, 26, 3778, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\sin(ia+ibx)}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\sin(ia+ibx)}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{ 3778}$$

$$-i\left(\frac{2ib\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}dx}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-i\left(\frac{2ib\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)$$

$$\Big\downarrow \text{ 3788}$$

$$-i\left(\frac{2ib\left(\frac{1}{2}i\int -\frac{ie^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int \frac{ie^{-a-bx}}{\sqrt{c+dx}}dx\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)$$

$$\Big\downarrow 26$$

$$-i\left(\frac{2ib\left(\frac{1}{2}\int\frac{e^{-a-bx}}{\sqrt{c+dx}}dx+\frac{1}{2}\int\frac{e^{a+bx}}{\sqrt{c+dx}}dx\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)$$

$$\Big\downarrow 2611$$

$$-i\left(\frac{2ib\left(\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\int e^{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)$$

$$\Big\downarrow 2633$$

$$-i\left(\frac{2ib\left(\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)$$

$$\Big\downarrow 2634$$

$$-i\left(\frac{2ib\left(\frac{\sqrt{\pi}e^{\frac{bc}{d}-a}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)$$

input
```
Int[Sinh[a + b*x]/(c + d*x)^(3/2),x]
```

output
```
(-I)*(((2*I)*b*((E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqr
t[d]])/(2*Sqrt[b]*Sqrt[d]) + (E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[
c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d])))/d - ((2*I)*Sinh[a + b*x])/(d*Sqr
t[c + d*x]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

Maple [F]

$$\int \frac{\sinh (bx+a)}{(dx+c)^{\frac{3}{2}}}dx$$

input
```
int(sinh(b*x+a)/(d*x+c)^(3/2),x)
```

output
```
int(sinh(b*x+a)/(d*x+c)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(90) = 180$.

Time = 0.10 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.87

$$\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{\sqrt{\pi}\left((dx+c)\cosh(bx+a)\cosh\left(-\frac{bc-ad}{d}\right)-(dx+c)\cosh(bx+a)\sinh\left(-\frac{bc-ad}{d}\right)+(\right.}{}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
(sqrt(pi)*((d*x + c)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - (d*x + c)*cosh(b
*x + a)*sinh(-(b*c - a*d)/d) + ((d*x + c)*cosh(-(b*c - a*d)/d) - (d*x + c)
*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d
)) - sqrt(pi)*((d*x + c)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + (d*x + c)*co
sh(b*x + a)*sinh(-(b*c - a*d)/d) + ((d*x + c)*cosh(-(b*c - a*d)/d) + (d*x
+ c)*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(d*x + c)*sqr
t(-b/d)) - sqrt(d*x + c)*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a)
+ sinh(b*x + a)^2 - 1))/((d^2*x + c*d)*cosh(b*x + a) + (d^2*x + c*d)*sinh(
b*x + a))
```

Sympy [F]

$$\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sinh{(a+bx)}}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)/(d*x+c)**(3/2),x)
```

output
```
Integral(sinh(a + b*x)/(c + d*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.87

$$\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{\frac{\left(\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{\sqrt{-\frac{b}{d}}}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{\sqrt{\frac{b}{d}}}\right)b}{d}-\frac{2\sinh(bx+a)}{\sqrt{dx+c}}}{d}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
((sqrt(pi)*erf(sqrt(d*x + c)*sqrt(-b/d))*e^(a - b*c/d)/sqrt(-b/d) + sqrt(p
i)*erf(sqrt(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/sqrt(b/d))*b/d - 2*sinh(b*x
 + a)/sqrt(d*x + c))/d
```

Giac [F]

$$\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sinh{(bx+a)}}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)/(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sinh(a+b\,x)}{{(c+d\,x)}^{3/2}}\,dx$$

input
```
int(sinh(a + b*x)/(c + d*x)^(3/2),x)
```

output
```
int(sinh(a + b*x)/(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sinh(bx+a)}{\sqrt{dx+c}\,c+\sqrt{dx+c}\,dx}dx$$

input
```
int(sinh(b*x+a)/(d*x+c)^(3/2),x)
```

output
```
int(sinh(a + b*x)/(sqrt(c + d*x)*c + sqrt(c + d*x)*d*x),x)
```

3.43 $\int \frac{\sinh(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 149

$$\int \frac{\sinh(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{4b\cosh(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2b^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2b^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{2\sinh(a+bx)}{3d(c+dx)^{3/2}}$$

output
```
-4/3*b*cosh(b*x+a)/d^2/(d*x+c)^(1/2)-2/3*b^(3/2)*exp(-a+b*c/d)*Pi^(1/2)*er
f(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(5/2)+2/3*b^(3/2)*exp(a-b*c/d)*Pi^(1/2)
*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(5/2)-2/3*sinh(b*x+a)/d/(d*x+c)^(3/
2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.08

$$\int \frac{\sinh(a+bx)}{(c+dx)^{5/2}}\,dx = \frac{2b\left(\frac{e^a\left(-e^{bx}+e^{-\frac{bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)\right)}{d\sqrt{c+dx}}+\frac{e^{-a-bx}\left(-1+e^{b\left(\frac{c}{d}+x\right)}\sqrt{\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},\frac{b(c+dx)}{d}\right)\right)}{d\sqrt{c+dx}}\right)}{3d} - \frac{2\sinh(a+bx)}{3d(c+dx)^{3/2}}$$

input
```
Integrate[Sinh[a + b*x]/(c + d*x)^(5/2),x]
```

output
```
(2*b*((E^a*(-E^(b*x) + (Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, -((b*(c + d*x)
)/d)])/E^((b*c)/d)))/(d*Sqrt[c + d*x]) + (E^(-a - b*x)*(-1 + E^(b*(c/d + x
))*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (b*(c + d*x))/d]))/(d*Sqrt[c + d*x])))
/(3*d) - (2*Sinh[a + b*x])/(3*d*(c + d*x)^(3/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.18, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 26, 3778, 3042, 3778, 26, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)}{(c+dx)^{5/2}} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i \sin(ia+ibx)}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i \int \frac{\sin(ia+ibx)}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{3778}$$

$$-i \left(\frac{2ib \int \frac{\cosh(a+bx)}{(c+dx)^{3/2}}dx}{3d} - \frac{2i \sinh(a+bx)}{3d(c+dx)^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i \left(\frac{2ib \int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{(c+dx)^{3/2}}dx}{3d} - \frac{2i \sinh(a+bx)}{3d(c+dx)^{3/2}}\right)$$

$$\Big\downarrow \text{3778}$$

$$-i\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}+\frac{2ib\int -\frac{i\sinh(a+bx)}{\sqrt{c+dx}}dx}{d}\right)}{3d}-\frac{2i\sinh(a+bx)}{3d(c+dx)^{3/2}}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{2ib\left(\frac{2b\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2i\sinh(a+bx)}{3d(c+dx)^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}+\frac{2b\int -\frac{i\sin(ia+ibx)}{\sqrt{c+dx}}dx}{d}\right)}{3d}-\frac{2i\sinh(a+bx)}{3d(c+dx)^{3/2}}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\int \frac{\sin(ia+ibx)}{\sqrt{c+dx}}dx}{d}\right)}{3d}-\frac{2i\sinh(a+bx)}{3d(c+dx)^{3/2}}\right)$$

$$\Big\downarrow \text{3789}$$

$$-i\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{1}{2}i\int \frac{e^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int \frac{e^{-a-bx}}{\sqrt{c+dx}}dx\right)}{d}\right)}{3d}-\frac{2i\sinh(a+bx)}{3d(c+dx)^{3/2}}\right)$$

$$\Big\downarrow \text{2611}$$

$$-i\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{i\int e^{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{d}-\frac{i\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{d}\right)}{3d}-\frac{2i\sinh(a+bx)}{3d(c+dx)^{3/2}}\right)$$

$$\Big\downarrow \text{2633}$$

$$-i\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{d}\right)}{3d}-\frac{2i\sinh(a+bx)}{3d(c+dx)^{3/2}}\right)$$

$$\Big\downarrow \text{2634}$$

$$-i\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d}\right)}{3d}-\frac{2i\sinh(a+bx)}{3d(c+dx)^{3/2}}\right)$$

input
```
Int[Sinh[a + b*x]/(c + d*x)^(5/2),x]
```

output
```
(-I)*((((2*I)/3)*b*((-2*Cosh[a + b*x])/(d*Sqrt[c + d*x]) - ((2*I)*b*(((-1/
2*I)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt
[b]*Sqrt[d]) + ((I/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x]
)/Sqrt[d]])/(Sqrt[b]*Sqrt[d])))/d))/d - (((2*I)/3)*Sinh[a + b*x])/(d*(c +
d*x)^(3/2)))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Maple [F]

$$\int \frac{\sinh(bx+a)}{(dx+c)^{\frac{5}{2}}}dx$$

input
```
int(sinh(b*x+a)/(d*x+c)^(5/2),x)
```

output
```
int(sinh(b*x+a)/(d*x+c)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 532 vs. 2(111) = 222.

Time = 0.10 (sec) , antiderivative size = 532, normalized size of antiderivative = 3.57

$$\int \frac{\sinh(a+bx)}{(c+dx)^{5/2}}\,dx =$$

$$-\frac{2\sqrt{\pi}\left((bd^2x^2+2\,bcdx+bc^2)\cosh(bx+a)\cosh\left(-\frac{bc-ad}{d}\right)-(bd^2x^2+2\,bcdx+bc^2)\cosh(bx+a)\sinh\left(-\frac{b}{}\right.\right.}{}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(2*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-(b*c
 - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-(b*c - a*
d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-(b*c - a*d)/d) - (b*d^2*x^2
 + 2*b*c*d*x + b*c^2)*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(s
qrt(d*x + c)*sqrt(b/d)) + 2*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh
(b*x + a)*cosh(-(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x
+ a)*sinh(-(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-(b*c -
a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-(b*c - a*d)/d))*sinh(b*x +
 a))*sqrt(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d)) + (2*b*d*x + (2*b*d*x + 2*b*
c + d)*cosh(b*x + a)^2 + 2*(2*b*d*x + 2*b*c + d)*cosh(b*x + a)*sinh(b*x +
a) + (2*b*d*x + 2*b*c + d)*sinh(b*x + a)^2 + 2*b*c - d)*sqrt(d*x + c))/((d
^4*x^2 + 2*c*d^3*x + c^2*d^2)*cosh(b*x + a) + (d^4*x^2 + 2*c*d^3*x + c^2*d
^2)*sinh(b*x + a))
```

Sympy [F]

$$\int \frac{\sinh(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sinh{(a+bx)}}{(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)/(d*x+c)**(5/2),x)
```

output
```
Integral(sinh(a + b*x)/(c + d*x)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.77

$$\int \frac{\sinh(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{\frac{\left(\frac{\sqrt{\frac{(dx+c)b}{d}}e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\frac{1}{2},\frac{(dx+c)b}{d}\right)}{\sqrt{dx+c}}+\frac{\sqrt{-\frac{(dx+c)b}{d}}e^{\left(a-\frac{bc}{d}\right)}\Gamma\left(-\frac{1}{2},-\frac{(dx+c)b}{d}\right)}{\sqrt{dx+c}}\right)b}{d}+\frac{2\sinh(bx+a)}{(dx+c)^{\frac{3}{2}}}}{3\,d}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*((sqrt((d*x + c)*b/d)*e^(-a + b*c/d)*gamma(-1/2, (d*x + c)*b/d)/sqrt(
d*x + c) + sqrt(-(d*x + c)*b/d)*e^(a - b*c/d)*gamma(-1/2, -(d*x + c)*b/d)/
sqrt(d*x + c))*b/d + 2*sinh(b*x + a)/(d*x + c)^(3/2))/d
```

Giac [F]

$$\int \frac{\sinh(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sinh(bx+a)}{(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)/(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sinh(a+b\,x)}{(c+d\,x)^{5/2}}\,dx$$

input
```
int(sinh(a + b*x)/(c + d*x)^(5/2),x)
```

output
```
int(sinh(a + b*x)/(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sinh(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sinh(bx+a)}{\sqrt{dx+c}\,c^2+2\sqrt{dx+c}\,cdx+\sqrt{dx+c}\,d^2x^2}dx$$

input
```
int(sinh(b*x+a)/(d*x+c)^(5/2),x)
```

output
```
int(sinh(a + b*x)/(sqrt(c + d*x)*c**2 + 2*sqrt(c + d*x)*c*d*x + sqrt(c + d
*x)*d**2*x**2),x)
```

3.44 $\int \frac{\sinh(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 174

$$\int \frac{\sinh(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{4b\cosh(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{4b^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{4b^{5/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} - \frac{2\sinh(a+bx)}{5d(c+dx)^{5/2}} - \frac{8b^2\sinh(a+bx)}{15d^3\sqrt{c+dx}}$$

output
```
-4/15*b*cosh(b*x+a)/d^2/(d*x+c)^(3/2)+4/15*b^(5/2)*exp(-a+b*c/d)*Pi^(1/2)*
erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(7/2)+4/15*b^(5/2)*exp(a-b*c/d)*Pi^(1
/2)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(7/2)-2/5*sinh(b*x+a)/d/(d*x+c)^
(5/2)-8/15*b^2*sinh(b*x+a)/d^3/(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.97

$$\int \frac{\sinh(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{2\left(-b(c+dx)\left(e^{a-\frac{bc}{d}}\left(e^{b\left(\frac{c}{d}+x\right)}(d+2b(c+dx))+2d\left(-\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)\right) \cdots}{15d^3(c \cdots}$$

input
```
Integrate[Sinh[a + b*x]/(c + d*x)^(7/2),x]
```

output
```
(2*(-(b*(c + d*x)*(E^(a - (b*c)/d)*(E^(b*(c/d + x))*(d + 2*b*(c + d*x)) +
2*d*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, -((b*(c + d*x))/d)]) + E^(-a - b
*x)*(d - 2*b*(c + d*x) + 2*d*E^(b*(c/d + x))*((b*(c + d*x))/d)^(3/2)*Gamma
[1/2, (b*(c + d*x))/d]))) - 3*d^2*Sinh[a + b*x]))/(15*d^3*(c + d*x)^(5/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.19, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.938$, Rules used = {3042, 26, 3778, 3042, 3778, 26, 3042, 26, 3778, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)}{(c+dx)^{7/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ia+ibx)}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ia+ibx)}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{3778}$$

$$-i\left(\frac{2ib\int \frac{\cosh(a+bx)}{(c+dx)^{5/2}}dx}{5d}-\frac{2i\sinh(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{2ib\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{(c+dx)^{5/2}}dx}{5d}-\frac{2i\sinh(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$$\Big\downarrow \text{3778}$$

$$-i\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}+\frac{2ib\int -\frac{i\sinh(a+bx)}{(c+dx)^{3/2}}dx}{3d}\right)}{5d}-\frac{2i\sinh(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$$\Big\downarrow \ 26$$

$$-i\left(\frac{2ib\left(\frac{2b\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}dx}{3d}-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}-\frac{2i\sinh(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$$\Big\downarrow \ 3042$$

$$-i\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}+\frac{2b\int -\frac{i\sin(ia+ibx)}{(c+dx)^{3/2}}dx}{3d}\right)}{5d}-\frac{2i\sinh(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$$\Big\downarrow \ 26$$

$$-i\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}-\frac{2ib\int \frac{\sin(ia+ibx)}{(c+dx)^{3/2}}dx}{3d}\right)}{5d}-\frac{2i\sinh(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$$\Big\downarrow \ 3778$$

$$-i\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}-\frac{2ib\left(\frac{2ib\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}dx}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{3d}\right)}{5d}-\frac{2i\sinh(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$$\Big\downarrow \ 3042$$

$$-i\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}-\frac{2ib\left(\frac{2ib\int\frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{3d}\right)}{5d}-\frac{2i\sinh(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$$\Big\downarrow \ 3788$$

$$-i\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}-\frac{2ib\left(\frac{2ib\left(\frac{1}{2}i\int-\frac{ie^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int\frac{ie^{-a-bx}}{\sqrt{c+dx}}dx\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{3d}\right)}{5d}-\frac{2i\sinh(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$$\Big\downarrow \ 26$$

$$-i\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}-\frac{2ib\left(\frac{2ib\left(\frac{1}{2}\int\frac{e^{-a-bx}}{\sqrt{c+dx}}dx+\frac{1}{2}\int\frac{e^{a+bx}}{\sqrt{c+dx}}dx\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{3d}\right)}{5d}-\frac{2i\sinh(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$$\Big\downarrow \ 2611$$

$$-i\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}-\frac{2ib\left(\frac{2ib\left(\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\int e^{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{3d}\right)}{5d}-\frac{2i\sinh(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$\downarrow$ 2633

$$-i\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}-\frac{2ib\left(\frac{2ib\left(\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{3d}\right)}{5d}-\frac{2i\sinh(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$\downarrow$ 2634

$$-i\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}-\frac{2ib\left(\frac{2ib\left(\frac{\sqrt{\pi}e^{\frac{bc}{d}-a}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{3d}\right)}{5d}-\frac{2i\sinh(a+bx)}{5d(c+dx)^{5/2}}\right)$$

input
```
Int[Sinh[a + b*x]/(c + d*x)^(7/2),x]
```

output
```
(-I)*((((-2*I)/5)*Sinh[a + b*x])/(d*(c + d*x)^(5/2)) + (((2*I)/5)*b*((-2*C
osh[a + b*x])/(3*d*(c + d*x)^(3/2)) - (((2*I)/3)*b*(((2*I)*b*((E^(-a + (b*
c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) +
 (E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[
b]*Sqrt[d])))/d - ((2*I)*Sinh[a + b*x])/(d*Sqrt[c + d*x])))/d))/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

Maple [F]

$$\int \frac{\sinh (bx+a)}{(dx+c)^{\frac{7}{2}}}dx$$

input
```
int(sinh(b*x+a)/(d*x+c)^(7/2),x)
```

output
```
int(sinh(b*x+a)/(d*x+c)^(7/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 855 vs. $2(132) = 264$.

Time = 0.14 (sec) , antiderivative size = 855, normalized size of antiderivative = 4.91

$$\int \frac{\sinh(a+bx)}{(c+dx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(4*sqrt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3
)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*
b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-(b*c - a*d)/d) + ((b^2*d^3*x^3
+ 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-(b*c - a*d)/d) - (b^2*d
^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-(b*c - a*d)/d))*
sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) - 4*sqrt(pi)*((b^2*d
^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*cosh(-(b
*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*c
osh(b*x + a)*sinh(-(b*c - a*d)/d) + ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^
2*c^2*d*x + b^2*c^3)*cosh(-(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d
)*erf(sqrt(d*x + c)*sqrt(-b/d)) + (4*b^2*d^2*x^2 + 4*b^2*c^2 - 2*b*c*d - (
4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*cos
h(b*x + a)^2 - 2*(4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2*(4*b^2*c
*d + b*d^2)*x)*cosh(b*x + a)*sinh(b*x + a) - (4*b^2*d^2*x^2 + 4*b^2*c^2 +
2*b*c*d + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*sinh(b*x + a)^2 + 3*d^2 + 2*(4*
b^2*c*d - b*d^2)*x)*sqrt(d*x + c))/((d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x +
 c^3*d^3)*cosh(b*x + a) + (d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)*
sinh(b*x + a))
```

Sympy [F]

$$\int \frac{\sinh(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sinh\left(a+bx\right)}{\left(c+dx\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)/(d*x+c)**(7/2),x)
```

output
```
Integral(sinh(a + b*x)/(c + d*x)**(7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.66

$$\int \frac{\sinh(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{\frac{\left(\frac{\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\frac{3}{2},\frac{(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}+\frac{\left(-\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(a-\frac{bc}{d}\right)}\Gamma\left(-\frac{3}{2},-\frac{(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}\right)b}{d}+\frac{2\sinh(bx+a)}{(dx+c)^{\frac{5}{2}}}}{5\,d}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
-1/5*((((d*x + c)*b/d)^(3/2)*e^(-a + b*c/d)*gamma(-3/2, (d*x + c)*b/d)/(d*
x + c)^(3/2) + (-(d*x + c)*b/d)^(3/2)*e^(a - b*c/d)*gamma(-3/2, -(d*x + c)
*b/d)/(d*x + c)^(3/2))*b/d + 2*sinh(b*x + a)/(d*x + c)^(5/2))/d
```

Giac [F]

$$\int \frac{\sinh(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sinh(bx+a)}{(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)/(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sinh(a+b\,x)}{(c+d\,x)^{7/2}}\,dx$$

input
```
int(sinh(a + b*x)/(c + d*x)^(7/2),x)
```

output
```
int(sinh(a + b*x)/(c + d*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{\sinh(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sinh(bx+a)}{\sqrt{dx+c}\,c^3+3\sqrt{dx+c}\,c^2dx+3\sqrt{dx+c}\,c\,d^2x^2+\sqrt{dx+c}\,d^3x^3}dx$$

input
```
int(sinh(b*x+a)/(d*x+c)^(7/2),x)
```

output
```
int(sinh(a + b*x)/(sqrt(c + d*x)*c**3 + 3*sqrt(c + d*x)*c**2*d*x + 3*sqrt(
c + d*x)*c*d**2*x**2 + sqrt(c + d*x)*d**3*x**3),x)
```

3.45 $\int (c+dx)^{5/2} \sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 239

$$\int (c+dx)^{5/2}\sinh^2(a+bx)\,dx = -\frac{5d(c+dx)^{3/2}}{16b^2} - \frac{(c+dx)^{7/2}}{7d} + \frac{15d^{5/2}e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}} - \frac{15d^{5/2}e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}} + \frac{(c+dx)^{5/2}\cosh(a+bx)\sinh(a+bx)}{2b} - \frac{5d(c+dx)^{3/2}\sinh^2(a+bx)}{8b^2} + \frac{15d^2\sqrt{c+dx}\sinh(2a+2bx)}{64b^3}$$

output
```
-5/16*d*(d*x+c)^(3/2)/b^2-1/7*(d*x+c)^(7/2)/d+15/512*d^(5/2)*exp(-2*a+2*b*
c/d)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)-1
5/512*d^(5/2)*exp(2*a-2*b*c/d)*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*b^(1/2)*(d*x+
c)^(1/2)/d^(1/2))/b^(7/2)+1/2*(d*x+c)^(5/2)*cosh(b*x+a)*sinh(b*x+a)/b-5/8*
d*(d*x+c)^(3/2)*sinh(b*x+a)^2/b^2+15/64*d^2*(d*x+c)^(1/2)*sinh(2*b*x+2*a)/
b^3
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.57

$$\int (c+dx)^{5/2}\sinh^2(a+bx)\,dx=\frac{-\frac{64(c+dx)^4}{d}-\frac{7\sqrt{2}d^3e^{2a-\frac{2bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\Gamma\left(\frac{7}{2},-\frac{2b(c+dx)}{d}\right)}{b^4}-\frac{7\sqrt{2}d^3e^{-2a+\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\Gamma\left(\frac{7}{2},\frac{2b(c+dx)}{d}\right)}{b^4}}{448\sqrt{c+dx}}$$

input
```
Integrate[(c + d*x)^(5/2)*Sinh[a + b*x]^2,x]
```

output
```
((-64*(c + d*x)^4)/d - (7*Sqrt[2]*d^3*E^(2*a - (2*b*c)/d)*Sqrt[-((b*(c + d
*x))/d)]*Gamma[7/2, (-2*b*(c + d*x))/d])/b^4 - (7*Sqrt[2]*d^3*E^(-2*a + (2
*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[7/2, (2*b*(c + d*x))/d])/b^4)/(448*Sq
rt[c + d*x])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 25, 3792, 17, 25, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sinh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -(c+dx)^{5/2}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)^{5/2}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{3792}$$

$$-\frac{15d^2\int -\sqrt{c+dx}\sinh^2(a+bx)dx}{16b^2}-\frac{1}{2}\int (c+dx)^{5/2}dx-\frac{5d(c+dx)^{3/2}\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh(a+bx)}{2b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{15d^2\int -\sqrt{c+dx}\sinh^2(a+bx)dx}{16b^2}-\frac{5d(c+dx)^{3/2}\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^{7/2}}{7d}$$

$$\Big\downarrow \text{25}$$

$$\frac{15d^2\int \sqrt{c+dx}\sinh^2(a+bx)dx}{16b^2}-\frac{5d(c+dx)^{3/2}\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^{7/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{15d^2\int -\sqrt{c+dx}\sin(ia+ibx)^2dx}{16b^2}-\frac{5d(c+dx)^{3/2}\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^{7/2}}{7d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{15d^2\int \sqrt{c+dx}\sin(ia+ibx)^2dx}{16b^2}-\frac{5d(c+dx)^{3/2}\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^{7/2}}{7d}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{15d^2\int \left(\frac{1}{2}\sqrt{c+dx}-\frac{1}{2}\sqrt{c+dx}\cosh(2a+2bx)\right)dx}{16b^2}-\frac{5d(c+dx)^{3/2}\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^{7/2}}{7d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{5d(c+dx)^{3/2}\sinh^2(a+bx)}{8b^2}-\frac{15d^2\left(-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}e^{\frac{2bc}{d}-2a}\text{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}e^{2a-\frac{2bc}{d}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{c+dx}\sinh(2a+2bx)}{4b}+\frac{(c+dx)^{3/2}}{3d}\right)}{16b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^{7/2}}{7d}$$

input
```
Int[(c + d*x)^(5/2)*Sinh[a + b*x]^2,x]
```

output
```
-1/7*(c + d*x)^(7/2)/d + ((c + d*x)^(5/2)*Cosh[a + b*x]*Sinh[a + b*x])/(2*
b) - (5*d*(c + d*x)^(3/2)*Sinh[a + b*x]^2)/(8*b^2) - (15*d^2*((c + d*x)^(3
/2)/(3*d) - (Sqrt[d]*E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*
Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3/2)) + (Sqrt[d]*E^(2*a - (2*b*c)/d)*Sqrt[
Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3/2)) - (Sqrt[
c + d*x]*Sinh[2*a + 2*b*x])/(4*b)))/(16*b^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int (dx+c)^{\frac{5}{2}} \sinh(bx+a)^2\, dx$$

input
```
int((d*x+c)^(5/2)*sinh(b*x+a)^2,x)
```

output
```
int((d*x+c)^(5/2)*sinh(b*x+a)^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. $2(183) = 366$.

Time = 0.11 (sec) , antiderivative size = 1001, normalized size of antiderivative = 4.19

$$\int (c+dx)^{5/2} \sinh^2(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/3584*(105*sqrt(2)*sqrt(pi)*(d^4*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) -
 d^4*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + (d^4*cosh(-2*(b*c - a*d)/d)
- d^4*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*(d^4*cosh(b*x + a)*cosh(
-2*(b*c - a*d)/d) - d^4*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a
))*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) + 105*sqrt(2)*sqrt(pi)*(
d^4*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + d^4*cosh(b*x + a)^2*sinh(-2*(
b*c - a*d)/d) + (d^4*cosh(-2*(b*c - a*d)/d) + d^4*sinh(-2*(b*c - a*d)/d))*
sinh(b*x + a)^2 + 2*(d^4*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + d^4*cosh(b
*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt
(d*x + c)*sqrt(-b/d)) - 4*(112*b^3*d^3*x^2 + 112*b^3*c^2*d + 140*b^2*c*d^2
 - 7*(16*b^3*d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c
*d^2 - 5*b^2*d^3)*x)*cosh(b*x + a)^4 - 28*(16*b^3*d^3*x^2 + 16*b^3*c^2*d -
 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c*d^2 - 5*b^2*d^3)*x)*cosh(b*x + a)*si
nh(b*x + a)^3 - 7*(16*b^3*d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3
 + 4*(8*b^3*c*d^2 - 5*b^2*d^3)*x)*sinh(b*x + a)^4 + 105*b*d^3 + 128*(b^4*d
^3*x^3 + 3*b^4*c*d^2*x^2 + 3*b^4*c^2*d*x + b^4*c^3)*cosh(b*x + a)^2 + 2*(6
4*b^4*d^3*x^3 + 192*b^4*c*d^2*x^2 + 192*b^4*c^2*d*x + 64*b^4*c^3 - 21*(16*
b^3*d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c*d^2 - 5*
b^2*d^3)*x)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 28*(8*b^3*c*d^2 + 5*b^2*d^3
)*x - 4*(7*(16*b^3*d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4...
```

Sympy [F]

$$\int (c+dx)^{5/2}\sinh^2(a+bx)\,dx = \int (c+dx)^{\frac{5}{2}}\sinh^2{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**(5/2)*sinh(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**(5/2)*sinh(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.18

$$\int (c+dx)^{5/2}\sinh^2(a+bx)\,dx = -\frac{512\,(dx+c)^{\frac{7}{2}} + \frac{105\,\sqrt{2}\sqrt{\pi}d^3\,\mathrm{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}} - \frac{105\,\sqrt{2}\sqrt{\pi}d^3\,\mathrm{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}} + \frac{28\left(16\,(dx+c)^{\frac{5}{2}}\right.}{}}{}$$

input
```
integrate((d*x+c)^(5/2)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/3584*(512*(d*x + c)^(7/2) + 105*sqrt(2)*sqrt(pi)*d^3*erf(sqrt(2)*sqrt(d
*x + c)*sqrt(-b/d))*e^(2*a - 2*b*c/d)/(b^3*sqrt(-b/d)) - 105*sqrt(2)*sqrt(
pi)*d^3*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d))*e^(-2*a + 2*b*c/d)/(b^3*sqrt(
b/d)) + 28*(16*(d*x + c)^(5/2)*b^2*d*e^(2*b*c/d) + 20*(d*x + c)^(3/2)*b*d^
2*e^(2*b*c/d) + 15*sqrt(d*x + c)*d^3*e^(2*b*c/d))*e^(-2*a - 2*(d*x + c)*b/
d)/b^3 - 28*(16*(d*x + c)^(5/2)*b^2*d*e^(2*a) - 20*(d*x + c)^(3/2)*b*d^2*e
^(2*a) + 15*sqrt(d*x + c)*d^3*e^(2*a))*e^(2*(d*x + c)*b/d - 2*b*c/d)/b^3)/
d
```

Giac [F]

$$\int (c+dx)^{5/2}\sinh^2(a+bx)\,dx = \int (dx+c)^{\frac{5}{2}}\sinh\left(bx+a\right)^2\,dx$$

input
```
integrate((d*x+c)^(5/2)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^(5/2)*sinh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\sinh^2(a+bx)\,dx=\int \sinh(a+b\,x)^2\,(c+d\,x)^{5/2}\,dx$$

input
```
int(sinh(a + b*x)^2*(c + d*x)^(5/2),x)
```

output
```
int(sinh(a + b*x)^2*(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int (c+dx)^{5/2}\sinh^2(a+bx)\,dx=\left(\int \sqrt{dx+c}\,\sinh\left(bx+a\right)^2x^2dx\right)d^2+2\left(\int \sqrt{dx+c}\,\sinh\left(bx+a\right)^2xdx\right)cd+\left(\int \sqrt{dx+c}\,\sinh\left(bx+a\right)^2dx\right)c^2$$

input
```
int((d*x+c)^(5/2)*sinh(b*x+a)^2,x)
```

output
```
int(sqrt(c + d*x)*sinh(a + b*x)**2*x**2,x)*d**2 + 2*int(sqrt(c + d*x)*sinh
(a + b*x)**2*x,x)*c*d + int(sqrt(c + d*x)*sinh(a + b*x)**2,x)*c**2
```

3.46 $\int (c+dx)^{3/2}\sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 211

$$\int (c+dx)^{3/2}\sinh^2(a+bx)\,dx = -\frac{3d\sqrt{c+dx}}{16b^2}-\frac{(c+dx)^{5/2}}{5d}+\frac{3d^{3/2}e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}}+\frac{3d^{3/2}e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}}+\frac{(c+dx)^{3/2}\cosh(a+bx)\sinh(a+bx)}{2b}-\frac{3d\sqrt{c+dx}\sinh^2(a+bx)}{8b^2}$$

output
```
-3/16*d*(d*x+c)^(1/2)/b^2-1/5*(d*x+c)^(5/2)/d+3/128*d^(3/2)*exp(-2*a+2*b*c
/d)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)+3/
128*d^(3/2)*exp(2*a-2*b*c/d)*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*b^(1/2)*(d*x+c)
^(1/2)/d^(1/2))/b^(5/2)+1/2*(d*x+c)^(3/2)*cosh(b*x+a)*sinh(b*x+a)/b-3/8*d*
(d*x+c)^(1/2)*sinh(b*x+a)^2/b^2
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.65

$$\int (c+dx)^{3/2}\sinh^2(a+bx)\,dx=\frac{\sqrt{c+dx}\left(-32(c+dx)^2-\frac{5\sqrt{2}d^2e^{2a-\frac{2bc}{d}}\Gamma\left(\frac{5}{2},-\frac{2b(c+dx)}{d}\right)}{b^2\sqrt{-\frac{b(c+dx)}{d}}}-\frac{5\sqrt{2}d^2e^{-2a+\frac{2bc}{d}}\Gamma\left(\frac{5}{2},\frac{2b(c+dx)}{d}\right)}{b^2\sqrt{\frac{b(c+dx)}{d}}}\right)}{160d}$$

input
```
Integrate[(c + d*x)^(3/2)*Sinh[a + b*x]^2,x]
```

output
```
(Sqrt[c + d*x]*(-32*(c + d*x)^2 - (5*Sqrt[2]*d^2*E^(2*a - (2*b*c)/d)*Gamma
[5/2, (-2*b*(c + d*x))/d])/(b^2*Sqrt[-((b*(c + d*x))/d)]) - (5*Sqrt[2]*d^2
*E^(-2*a + (2*b*c)/d)*Gamma[5/2, (2*b*(c + d*x))/d])/(b^2*Sqrt[(b*(c + d*x
))/d])))/(160*d)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 25, 3792, 17, 25, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sinh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -(c+dx)^{3/2}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)^{3/2}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{3792}$$

$$-\frac{3d^2\int -\frac{\sinh^2(a+bx)}{\sqrt{c+dx}}dx}{16b^2}-\frac{1}{2}\int (c+dx)^{3/2}dx-\frac{3d\sqrt{c+dx}\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh(a+bx)}{2b}$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{3d^2\int -\frac{\sinh^2(a+bx)}{\sqrt{c+dx}}dx}{16b^2}-\frac{3d\sqrt{c+dx}\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^{5/2}}{5d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{3d^2\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}dx}{16b^2}-\frac{3d\sqrt{c+dx}\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^{5/2}}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3d^2\int -\frac{\sin(ia+ibx)^2}{\sqrt{c+dx}}dx}{16b^2}-\frac{3d\sqrt{c+dx}\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^{5/2}}{5d}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{3d^2\int \frac{\sin(ia+ibx)^2}{\sqrt{c+dx}}dx}{16b^2}-\frac{3d\sqrt{c+dx}\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^{5/2}}{5d}$$

$$\Big\downarrow \text{ 3793}$$

$$-\frac{3d^2\int \left(\frac{1}{2\sqrt{c+dx}}-\frac{\cosh(2a+2bx)}{2\sqrt{c+dx}}\right)dx}{16b^2}-\frac{3d\sqrt{c+dx}\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^{5/2}}{5d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{3d^2\left(-\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2bc}{d}-2a}\text{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{2}}e^{2a-\frac{2bc}{d}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{\sqrt{c+dx}}{d}\right)}{16b^2}-\frac{3d\sqrt{c+dx}\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^{5/2}}{5d}$$

input
```
Int[(c + d*x)^(3/2)*Sinh[a + b*x]^2,x]
```

output
```
-1/5*(c + d*x)^(5/2)/d - (3*d^2*(Sqrt[c + d*x]/d - (E^(-2*a + (2*b*c)/d)*S
qrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*Sqrt[b]*Sqrt[d]
) - (E^(2*a - (2*b*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/S
qrt[d]])/(4*Sqrt[b]*Sqrt[d])))/(16*b^2) + ((c + d*x)^(3/2)*Cosh[a + b*x]*S
inh[a + b*x])/(2*b) - (3*d*Sqrt[c + d*x]*Sinh[a + b*x]^2)/(8*b^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int (dx+c)^{\frac{3}{2}} \sinh(bx+a)^2\,dx$$

input
```
int((d*x+c)^(3/2)*sinh(b*x+a)^2,x)
```

output
```
int((d*x+c)^(3/2)*sinh(b*x+a)^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 755 vs. $2(159) = 318$.

Time = 0.12 (sec) , antiderivative size = 755, normalized size of antiderivative = 3.58

$$\int (c+dx)^{3/2}\sinh^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/640*(15*sqrt(2)*sqrt(pi)*(d^3*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - d
^3*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + (d^3*cosh(-2*(b*c - a*d)/d) -
d^3*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*(d^3*cosh(b*x + a)*cosh(-2
*(b*c - a*d)/d) - d^3*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))
*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - 15*sqrt(2)*sqrt(pi)*(d^3
*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + d^3*cosh(b*x + a)^2*sinh(-2*(b*c
 - a*d)/d) + (d^3*cosh(-2*(b*c - a*d)/d) + d^3*sinh(-2*(b*c - a*d)/d))*sin
h(b*x + a)^2 + 2*(d^3*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + d^3*cosh(b*x
+ a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*
x + c)*sqrt(-b/d)) - 4*(20*b^2*d^2*x - 5*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^
2)*cosh(b*x + a)^4 - 20*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^2)*cosh(b*x + a)*
sinh(b*x + a)^3 - 5*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^2)*sinh(b*x + a)^4 +
20*b^2*c*d + 15*b*d^2 + 32*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2)*cosh(b*x
+ a)^2 + 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*x + 16*b^3*c^2 - 15*(4*b^2*d^2*x +
 4*b^2*c*d - 3*b*d^2)*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 4*(5*(4*b^2*d^2*x
 + 4*b^2*c*d - 3*b*d^2)*cosh(b*x + a)^3 - 16*(b^3*d^2*x^2 + 2*b^3*c*d*x +
b^3*c^2)*cosh(b*x + a))*sinh(b*x + a))*sqrt(d*x + c))/(b^3*d*cosh(b*x + a)
^2 + 2*b^3*d*cosh(b*x + a)*sinh(b*x + a) + b^3*d*sinh(b*x + a)^2)
```

Sympy [F]

$$\int (c+dx)^{3/2}\sinh^2(a+bx)\,dx=\int (c+dx)^{\frac{3}{2}}\sinh^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*sinh(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**(3/2)*sinh(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.13

$$\int (c+dx)^{3/2}\sinh^2(a+bx)\,dx=$$
$$-\frac{128\,(dx+c)^{\frac{5}{2}}-\frac{15\,\sqrt{2}\sqrt{\pi}d^2\,\mathrm{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}}-\frac{15\,\sqrt{2}\sqrt{\pi}d^2\,\mathrm{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}}+\frac{20\left(4\,(dx+c)^{\frac{3}{2}}bde\right.}{}}{640\,d}$$

input
```
integrate((d*x+c)^(3/2)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/640*(128*(d*x + c)^(5/2) - 15*sqrt(2)*sqrt(pi)*d^2*erf(sqrt(2)*sqrt(d*x
 + c)*sqrt(-b/d))*e^(2*a - 2*b*c/d)/(b^2*sqrt(-b/d)) - 15*sqrt(2)*sqrt(pi)
*d^2*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d))*e^(-2*a + 2*b*c/d)/(b^2*sqrt(b/d
)) + 20*(4*(d*x + c)^(3/2)*b*d*e^(2*b*c/d) + 3*sqrt(d*x + c)*d^2*e^(2*b*c/
d))*e^(-2*a - 2*(d*x + c)*b/d)/b^2 - 20*(4*(d*x + c)^(3/2)*b*d*e^(2*a) - 3
*sqrt(d*x + c)*d^2*e^(2*a))*e^(2*(d*x + c)*b/d - 2*b*c/d)/b^2)/d
```

Giac [F]

$$\int (c+dx)^{3/2}\sinh^2(a+bx)\,dx=\int (dx+c)^{\frac{3}{2}}\sinh (bx+a)^2\,dx$$

input
```
integrate((d*x+c)^(3/2)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^(3/2)*sinh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\sinh^2(a+bx)\,dx=\int \sinh(a+b\,x)^2\,(c+d\,x)^{3/2}\,dx$$

input
```
int(sinh(a + b*x)^2*(c + d*x)^(3/2),x)
```

output
```
int(sinh(a + b*x)^2*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c+dx)^{3/2}\sinh^2(a+bx)\,dx=\left(\int \sqrt{dx+c}\,\sinh (bx+a)^2\,xdx\right)d+\left(\int \sqrt{dx+c}\,\sinh (bx+a)^2\,dx\right)c$$

input
```
int((d*x+c)^(3/2)*sinh(b*x+a)^2,x)
```

output
```
int(sqrt(c + d*x)*sinh(a + b*x)**2*x,x)*d + int(sqrt(c + d*x)*sinh(a + b*x
)**2,x)*c
```

3.47 $\int \sqrt{c+dx}\sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 166

$$\int \sqrt{c+dx}\sinh^2(a+bx)\,dx = -\frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{d}e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}} - \frac{\sqrt{d}e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}} + \frac{\sqrt{c+dx}\sinh(2a+2bx)}{4b}$$

output
```
-1/3*(d*x+c)^(3/2)/d+1/32*d^(1/2)*exp(-2*a+2*b*c/d)*2^(1/2)*Pi^(1/2)*erf(2
^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)-1/32*d^(1/2)*exp(2*a-2*b*c/d
)*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)+1/4
*(d*x+c)^(1/2)*sinh(2*b*x+2*a)/b
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.78

$$\int \sqrt{c+dx}\sinh^2(a+bx)\,dx = \frac{1}{48}\sqrt{c+dx}\left(-\frac{16(c+dx)}{d}+\frac{3\sqrt{2}e^{2a-\frac{2bc}{d}}\Gamma\left(\frac{3}{2},-\frac{2b(c+dx)}{d}\right)}{b\sqrt{-\frac{b(c+dx)}{d}}}-\frac{3\sqrt{2}e^{-2a+\frac{2bc}{d}}\Gamma\left(\frac{3}{2},\frac{2b(c+dx)}{d}\right)}{b\sqrt{\frac{b(c+dx)}{d}}}\right)$$

input
```
Integrate[Sqrt[c + d*x]*Sinh[a + b*x]^2,x]
```

output
```
(Sqrt[c + d*x]*((-16*(c + d*x))/d + (3*Sqrt[2]*E^(2*a - (2*b*c)/d)*Gamma[3
/2, (-2*b*(c + d*x))/d])/(b*Sqrt[-((b*(c + d*x))/d)]) - (3*Sqrt[2]*E^(-2*a
 + (2*b*c)/d)*Gamma[3/2, (2*b*(c + d*x))/d])/(b*Sqrt[(b*(c + d*x))/d])))/4
8
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sinh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sqrt{c+dx}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sqrt{c+dx}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{3793}$$

$$-\int \left(\frac{1}{2}\sqrt{c+dx}-\frac{1}{2}\sqrt{c+dx}\cosh(2a+2bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}e^{\frac{2bc}{d}-2a}\text{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}e^{2a-\frac{2bc}{d}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{c+dx}\sinh(2a+2bx)}{4b}-\frac{(c+dx)^{3/2}}{3d}$$

input
```
Int[Sqrt[c + d*x]*Sinh[a + b*x]^2,x]
```

output
```
-1/3*(c + d*x)^(3/2)/d + (Sqrt[d]*E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqr
t[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3/2)) - (Sqrt[d]*E^(2*a - (2*
b*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3
/2)) + (Sqrt[c + d*x]*Sinh[2*a + 2*b*x])/(4*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int \sqrt{dx+c}\,\sinh(bx+a)^2\,dx$$

input
```
int((d*x+c)^(1/2)*sinh(b*x+a)^2,x)
```

output
```
int((d*x+c)^(1/2)*sinh(b*x+a)^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(122) = 244$.

Time = 0.10 (sec) , antiderivative size = 590, normalized size of antiderivative = 3.55

$$\int \sqrt{c+dx}\sinh^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/96*(3*sqrt(2)*sqrt(pi)*(d^2*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - d^2
*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + (d^2*cosh(-2*(b*c - a*d)/d) - d^
2*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*(d^2*cosh(b*x + a)*cosh(-2*(
b*c - a*d)/d) - d^2*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*s
qrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) + 3*sqrt(2)*sqrt(pi)*(d^2*co
sh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + d^2*cosh(b*x + a)^2*sinh(-2*(b*c -
a*d)/d) + (d^2*cosh(-2*(b*c - a*d)/d) + d^2*sinh(-2*(b*c - a*d)/d))*sinh(b
*x + a)^2 + 2*(d^2*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + d^2*cosh(b*x + a
)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x +
 c)*sqrt(-b/d)) + 4*(3*b*d*cosh(b*x + a)^4 + 12*b*d*cosh(b*x + a)*sinh(b*x
 + a)^3 + 3*b*d*sinh(b*x + a)^4 - 8*(b^2*d*x + b^2*c)*cosh(b*x + a)^2 - 2*
(4*b^2*d*x - 9*b*d*cosh(b*x + a)^2 + 4*b^2*c)*sinh(b*x + a)^2 - 3*b*d + 4*
(3*b*d*cosh(b*x + a)^3 - 4*(b^2*d*x + b^2*c)*cosh(b*x + a))*sinh(b*x + a))
*sqrt(d*x + c))/(b^2*d*cosh(b*x + a)^2 + 2*b^2*d*cosh(b*x + a)*sinh(b*x +
a) + b^2*d*sinh(b*x + a)^2)
```

Sympy [F]

$$\int \sqrt{c+dx}\sinh^2(a+bx)\,dx=\int \sqrt{c+dx}\sinh^2{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**(1/2)*sinh(b*x+a)**2,x)
```

output
```
Integral(sqrt(c + d*x)*sinh(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.14

$$\int \sqrt{c+dx}\sinh^2(a+bx)\,dx=$$
$$-\frac{\frac{3\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(2a-\frac{2bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}}-\frac{3\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-2a+\frac{2bc}{d}\right)}}{b\sqrt{\frac{b}{d}}}+32\,(dx+c)^{\frac{3}{2}}-\frac{12\sqrt{dx+c}de^{\left(2a+\frac{2(c}{}\right.}}{b}}{96\,d}$$

input
```
integrate((d*x+c)^(1/2)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/96*(3*sqrt(2)*sqrt(pi)*d*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d))*e^(2*a -
 2*b*c/d)/(b*sqrt(-b/d)) - 3*sqrt(2)*sqrt(pi)*d*erf(sqrt(2)*sqrt(d*x + c)*
sqrt(b/d))*e^(-2*a + 2*b*c/d)/(b*sqrt(b/d)) + 32*(d*x + c)^(3/2) - 12*sqrt
(d*x + c)*d*e^(2*a + 2*(d*x + c)*b/d - 2*b*c/d)/b + 12*sqrt(d*x + c)*d*e^(
-2*a - 2*(d*x + c)*b/d + 2*b*c/d)/b)/d
```

Giac [F]

$$\int \sqrt{c+dx}\sinh^2(a+bx)\,dx = \int \sqrt{dx+c}\sinh\left(bx+a\right)^2\,dx$$

input
```
integrate((d*x+c)^(1/2)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)*sinh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\sinh^2(a+bx)\,dx = \int \sinh(a+b\,x)^2\,\sqrt{c+d\,x}\,dx$$

input
```
int(sinh(a + b*x)^2*(c + d*x)^(1/2),x)
```

output
```
int(sinh(a + b*x)^2*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\sinh^2(a+bx)\,dx = \int \sqrt{dx+c}\,\sinh\left(bx+a\right)^2dx$$

input
```
int((d*x+c)^(1/2)*sinh(b*x+a)^2,x)
```

output
```
int(sqrt(c + d*x)*sinh(a + b*x)**2,x)
```

3.48 $\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 139

$$\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}\,dx = -\frac{\sqrt{c+dx}}{d}+\frac{e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}$$

output
```
-(d*x+c)^(1/2)/d+1/8*exp(-2*a+2*b*c/d)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*b^(1/2
)*(d*x+c)^(1/2)/d^(1/2))/b^(1/2)/d^(1/2)+1/8*exp(2*a-2*b*c/d)*2^(1/2)*Pi^(
1/2)*erfi(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(1/2)/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.02

$$\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}\,dx = -\frac{\sqrt{c+dx}}{d}+\frac{e^{2a-\frac{2bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{2b(c+dx)}{d}\right)}{4\sqrt{2}b\sqrt{c+dx}}-\frac{e^{-2a+\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},\frac{2b(c+dx)}{d}\right)}{4\sqrt{2}b\sqrt{c+dx}}$$

input
```
Integrate[Sinh[a + b*x]^2/Sqrt[c + d*x],x]
```

output
```
-(Sqrt[c + d*x]/d) + (E^(2*a - (2*b*c)/d)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1
/2, (-2*b*(c + d*x))/d])/(4*Sqrt[2]*b*Sqrt[c + d*x]) - (E^(-2*a + (2*b*c)/
d)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (2*b*(c + d*x))/d])/(4*Sqrt[2]*b*Sqrt[
c + d*x])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\sin(ia+ibx)^2}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\sin(ia+ibx)^2}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{ 3793}$$

$$-\int \left(\frac{1}{2\sqrt{c+dx}}-\frac{\cosh(2a+2bx)}{2\sqrt{c+dx}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2bc}{d}-2a}\text{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{2}}e^{2a-\frac{2bc}{d}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}-\frac{\sqrt{c+dx}}{d}$$

input
```
Int[Sinh[a + b*x]^2/Sqrt[c + d*x],x]
```

output
```
-(Sqrt[c + d*x]/d) + (E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]
*Sqrt[c + d*x])/Sqrt[d]])/(4*Sqrt[b]*Sqrt[d]) + (E^(2*a - (2*b*c)/d)*Sqrt[
Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*Sqrt[b]*Sqrt[d])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int \frac{\sinh(bx+a)^2}{\sqrt{dx+c}}dx$$

input
```
int(sinh(b*x+a)^2/(d*x+c)^(1/2),x)
```

output
```
int(sinh(b*x+a)^2/(d*x+c)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.12

$$\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}\,dx$$
$$= \frac{\sqrt{2}\sqrt{\pi}\left(d\cosh\left(-\frac{2\,(bc-ad)}{d}\right) - d\sinh\left(-\frac{2\,(bc-ad)}{d}\right)\right)\sqrt{\frac{b}{d}}\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right) - \sqrt{2}\sqrt{\pi}\left(d\cosh\left(-\frac{2\,(bc-ad)}{d}\right.\right.}{8\,bd}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(sqrt(2)*sqrt(pi)*(d*cosh(-2*(b*c - a*d)/d) - d*sinh(-2*(b*c - a*d)/d)
)*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - sqrt(2)*sqrt(pi)*(d*cos
h(-2*(b*c - a*d)/d) + d*sinh(-2*(b*c - a*d)/d))*sqrt(-b/d)*erf(sqrt(2)*sqr
t(d*x + c)*sqrt(-b/d)) - 8*sqrt(d*x + c)*b)/(b*d)
```

Sympy [F]

$$\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\sinh^2\left(a+bx\right)}{\sqrt{c+dx}}\,dx$$

input
```
integrate(sinh(b*x+a)**2/(d*x+c)**(1/2),x)
```

output
```
Integral(sinh(a + b*x)**2/sqrt(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.77

$$\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}\,dx$$
$$= \frac{\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}}{\sqrt{-\frac{b}{d}}} + \frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}}{\sqrt{\frac{b}{d}}} - 8\sqrt{dx+c}}{8\,d}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/8*(sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d))*e^(2*a - 2*b*c
/d)/sqrt(-b/d) + sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d))*e^(
-2*a + 2*b*c/d)/sqrt(b/d) - 8*sqrt(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.83

$$\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}\,dx = -\frac{\left(\frac{\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{2\,bc}{d}\right)}}{\sqrt{bd}}+\frac{\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{2\,(bc-2\,ad)}{d}\right)}}{\sqrt{-bd}}+8\,\sqrt{dx+c}e^{(2\,a)}\right)e^{(-2\,a)}}{8\,d}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
-1/8*(sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)/d)*e^(2*b*c/
d)/sqrt(b*d) + sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(-b*d)*sqrt(d*x + c)/d)
*e^(-2*(b*c - 2*a*d)/d)/sqrt(-b*d) + 8*sqrt(d*x + c)*e^(2*a))*e^(-2*a)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{{\sinh(a+b\,x)}^2}{\sqrt{c+d\,x}}\,dx$$

input
```
int(sinh(a + b*x)^2/(c + d*x)^(1/2),x)
```

output
```
int(sinh(a + b*x)^2/(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\sinh(bx+a)^2}{\sqrt{dx+c}}dx$$

input
```
int(sinh(b*x+a)^2/(d*x+c)^(1/2),x)
```

output
```
int(sinh(a + b*x)**2/sqrt(c + d*x),x)
```

3.49 $\int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 142

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}\,dx = -\frac{\sqrt{b}e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{b}e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}$$

output

```
-1/2*b^(1/2)*exp(-2*a+2*b*c/d)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*b^(1/2)*(d*x+c
)^(1/2)/d^(1/2))/d^(3/2)+1/2*b^(1/2)*exp(2*a-2*b*c/d)*2^(1/2)*Pi^(1/2)*erf
i(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(3/2)-2*sinh(b*x+a)^2/d/(d*x+c)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.07

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{e^{-2\left(a+b\left(\frac{c}{d}+x\right)\right)}\left(\sqrt{2}e^{4a+2bx}\sqrt{-\frac{b(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\frac{2b(c+dx)}{d}\right)+e^{\frac{2bc}{d}}\left(-\left(-1+e^{2(a+bx)}\right)^2+\sqrt{\right.\right.}{2d\sqrt{c+dx}}$$

input
```
Integrate[Sinh[a + b*x]^2/(c + d*x)^(3/2),x]
```

output
```
(Sqrt[2]*E^(4*a + 2*b*x)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, (-2*b*(c + d*
x))/d] + E^((2*b*c)/d)*(-(-1 + E^(2*(a + b*x)))^2 + Sqrt[2]*E^((2*b*(c + d
*x))/d)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (2*b*(c + d*x))/d]))/(2*d*E^(2*(a
 + b*(c/d + x)))*Sqrt[c + d*x])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 25, 3794, 27, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ia+ibx)^2}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ia+ibx)^2}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{3794}$$

$$-\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}-\frac{4ib\int \frac{i\sinh(2a+2bx)}{2\sqrt{c+dx}}dx}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\int \frac{\sinh(2a+2bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}+\frac{2b\int -\frac{i\sin(2ia+2ibx)}{\sqrt{c+dx}}dx}{d}$$

$\Big\downarrow$ 26

$$-\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\int \frac{\sin(2ia+2ibx)}{\sqrt{c+dx}}dx}{d}$$

$\Big\downarrow$ 3789

$$-\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{1}{2}i\int \frac{e^{2(a+bx)}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int \frac{e^{-2(a+bx)}}{\sqrt{c+dx}}dx\right)}{d}$$

$\Big\downarrow$ 2611

$$-\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{i\int e^{2\left(a-\frac{bc}{d}\right)+\frac{2b(c+dx)}{d}}d\sqrt{c+dx}}{d}-\frac{i\int e^{-2\left(a-\frac{bc}{d}\right)-\frac{2b(c+dx)}{d}}d\sqrt{c+dx}}{d}\right)}{d}$$

$\Big\downarrow$ 2633

$$-\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{i\sqrt{\frac{\pi}{2}}e^{2a-\frac{2bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\int e^{-2\left(a-\frac{bc}{d}\right)-\frac{2b(c+dx)}{d}}d\sqrt{c+dx}}{d}\right)}{d}$$

$\Big\downarrow$ 2634

$$-\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{i\sqrt{\frac{\pi}{2}}e^{2a-\frac{2bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\frac{\pi}{2}}e^{\frac{2bc}{d}-2a}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d}$$

input
```
Int[Sinh[a + b*x]^2/(c + d*x)^(3/2),x]
```

output
```
((-2*I)*b*(((-1/2*I)*E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*
Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) + ((I/2)*E^(2*a - (2*b*c)/d)*Sq
rt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]))
)/d - (2*Sinh[a + b*x]^2)/(d*Sqrt[c + d*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

Maple [F]

$$\int \frac{\sinh(bx+a)^2}{(dx+c)^{\frac{3}{2}}}dx$$

input
```
int(sinh(b*x+a)^2/(d*x+c)^(3/2),x)
```

output
```
int(sinh(b*x+a)^2/(d*x+c)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(109) = 218$.

Time = 0.10 (sec) , antiderivative size = 571, normalized size of antiderivative = 4.02

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(2)*sqrt(pi)*((d*x + c)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) -
 (d*x + c)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + ((d*x + c)*cosh(-2*(b*
c - a*d)/d) - (d*x + c)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((d*x
+ c)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) - (d*x + c)*cosh(b*x + a)*sinh(-
2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(
b/d)) + sqrt(2)*sqrt(pi)*((d*x + c)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d)
 + (d*x + c)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + ((d*x + c)*cosh(-2*(
b*c - a*d)/d) + (d*x + c)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((d*
x + c)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + (d*x + c)*cosh(b*x + a)*sinh
(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sq
rt(-b/d)) + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x
+ a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4
*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*sqrt(d*x + c))/((d^2
*x + c*d)*cosh(b*x + a)^2 + 2*(d^2*x + c*d)*cosh(b*x + a)*sinh(b*x + a) +
(d^2*x + c*d)*sinh(b*x + a)^2)
```

Sympy [F]

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sinh^2{(a+bx)}}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)**2/(d*x+c)**(3/2),x)
```

output
```
Integral(sinh(a + b*x)**2/(c + d*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.82

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}\,dx = -\frac{\frac{\sqrt{2}\sqrt{\frac{(dx+c)b}{d}}e^{\left(\frac{2\,(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},\frac{2\,(dx+c)b}{d}\right)}{\sqrt{dx+c}}+\frac{\sqrt{2}\sqrt{-\frac{(dx+c)b}{d}}e^{\left(-\frac{2\,(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},-\frac{2\,(dx+c)b}{d}\right)}{\sqrt{dx+c}}-\frac{4}{\sqrt{dx+c}}}{4\,d}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(sqrt(2)*sqrt((d*x + c)*b/d)*e^(2*(b*c - a*d)/d)*gamma(-1/2, 2*(d*x +
 c)*b/d)/sqrt(d*x + c) + sqrt(2)*sqrt(-(d*x + c)*b/d)*e^(-2*(b*c - a*d)/d)
*gamma(-1/2, -2*(d*x + c)*b/d)/sqrt(d*x + c) - 4/sqrt(d*x + c))/d
```

Giac [F]

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sinh(bx+a)^2}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^2/(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sinh(a+b\,x)^2}{(c+d\,x)^{3/2}}\,dx$$

input
```
int(sinh(a + b*x)^2/(c + d*x)^(3/2),x)
```

output
```
int(sinh(a + b*x)^2/(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sinh{(bx+a)}^2}{\sqrt{dx+c}\,c+\sqrt{dx+c}\,dx}dx$$

input
```
int(sinh(b*x+a)^2/(d*x+c)^(3/2),x)
```

output
```
int(sinh(a + b*x)**2/(sqrt(c + d*x)*c + sqrt(c + d*x)*d*x),x)
```

3.50 $\int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 174

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}\,dx = \frac{2b^{3/2}e^{-2a+\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2b^{3/2}e^{2a-\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}$$

output
```
2/3*b^(3/2)*exp(-2*a+2*b*c/d)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*b^(1/2)*(d*x+c)
^(1/2)/d^(1/2))/d^(5/2)+2/3*b^(3/2)*exp(2*a-2*b*c/d)*2^(1/2)*Pi^(1/2)*erfi
(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(5/2)-8/3*b*cosh(b*x+a)*sinh(b*x
+a)/d^2/(d*x+c)^(1/2)-2/3*sinh(b*x+a)^2/d/(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.90

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{2e^{-2\left(a+\frac{bc}{d}\right)}\left(\sqrt{2}de^{4a}\left(-\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{2b(c+dx)}{d}\right)+\sqrt{2}de^{\frac{4bc}{d}}\left(\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},\frac{2b(c+dx)}{d}\right)+e^{2\left(a+\frac{bc}{d}\right)}\left(d\sin\right.\right.}{3d^2(c+dx)^{3/2}}$$

input
```
Integrate[Sinh[a + b*x]^2/(c + d*x)^(5/2),x]
```

output
```
(-2*(Sqrt[2]*d*E^(4*a)*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, (-2*b*(c + d*
x))/d] + Sqrt[2]*d*E^((4*b*c)/d)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2, (2*b*(
c + d*x))/d] + E^(2*(a + (b*c)/d))*(d*Sinh[a + b*x]^2 + 2*b*(c + d*x)*Sinh
[2*(a + b*x)])))/(3*d^2*E^(2*(a + (b*c)/d))*(c + d*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 25, 3795, 17, 25, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\sin(ia+ibx)^2}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\sin(ia+ibx)^2}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{ 3795}$$

$$-\frac{16b^2\int -\frac{\sinh^2(a+bx)}{\sqrt{c+dx}}dx}{3d^2}+\frac{8b^2\int \frac{1}{\sqrt{c+dx}}dx}{3d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{16b^2\int -\frac{\sinh^2(a+bx)}{\sqrt{c+dx}}dx}{3d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}+\frac{16b^2\sqrt{c+dx}}{3d^3}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{16b^2\int\frac{\sinh^2(a+bx)}{\sqrt{c+dx}}dx}{3d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}+\frac{16b^2\sqrt{c+dx}}{3d^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{16b^2\int-\frac{\sin(ia+ibx)^2}{\sqrt{c+dx}}dx}{3d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}+\frac{16b^2\sqrt{c+dx}}{3d^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{16b^2\int\frac{\sin(ia+ibx)^2}{\sqrt{c+dx}}dx}{3d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}+\frac{16b^2\sqrt{c+dx}}{3d^3}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{16b^2\int\left(\frac{1}{2\sqrt{c+dx}}-\frac{\cosh(2a+2bx)}{2\sqrt{c+dx}}\right)dx}{3d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}+\frac{16b^2\sqrt{c+dx}}{3d^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{16b^2\left(-\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2bc}{d}-2a}\text{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{2}}e^{2a-\frac{2bc}{d}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{\sqrt{c+dx}}{d}\right)}{3d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}+\frac{16b^2\sqrt{c+dx}}{3d^3}$$

input
```
Int[Sinh[a + b*x]^2/(c + d*x)^(5/2),x]
```

output
```
(16*b^2*Sqrt[c + d*x])/(3*d^3) - (16*b^2*(Sqrt[c + d*x]/d - (E^(-2*a + (2*
b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*Sqrt[b
]*Sqrt[d]) - (E^(2*a - (2*b*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c
+ d*x])/Sqrt[d]])/(4*Sqrt[b]*Sqrt[d])))/(3*d^2) - (8*b*Cosh[a + b*x]*Sinh[
a + b*x])/(3*d^2*Sqrt[c + d*x]) - (2*Sinh[a + b*x]^2)/(3*d*(c + d*x)^(3/2)
)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Maple [F]

$$\int \frac{\sinh\left(bx+a\right)^{2}}{\left(dx+c\right)^{\frac{5}{2}}}dx$$

input
```
int(sinh(b*x+a)^2/(d*x+c)^(5/2),x)
```

output
```
int(sinh(b*x+a)^2/(d*x+c)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 864 vs. $2(134) = 268$.

Time = 0.11 (sec) , antiderivative size = 864, normalized size of antiderivative = 4.97

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/6*(4*sqrt(2)*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*c
osh(-2*(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*si
nh(-2*(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-2*(b*c - a*d
)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-2*(b*c - a*d)/d))*sinh(b*x +
a)^2 + 2*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-2*(b*c - a*d
)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d
))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - 4*sqrt(
2)*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*cosh(-2*(b*c
- a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*sinh(-2*(b*c -
 a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-2*(b*c - a*d)/d) + (b*d^
2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((b
*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + (b*d^
2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x
+ a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d)) - ((4*b*d*x + 4*b*c
 + d)*cosh(b*x + a)^4 + 4*(4*b*d*x + 4*b*c + d)*cosh(b*x + a)*sinh(b*x + a
)^3 + (4*b*d*x + 4*b*c + d)*sinh(b*x + a)^4 - 4*b*d*x - 2*d*cosh(b*x + a)^
2 + 2*(3*(4*b*d*x + 4*b*c + d)*cosh(b*x + a)^2 - d)*sinh(b*x + a)^2 - 4*b*
c + 4*((4*b*d*x + 4*b*c + d)*cosh(b*x + a)^3 - d*cosh(b*x + a))*sinh(b*x +
 a) + d)*sqrt(d*x + c))/((d^4*x^2 + 2*c*d^3*x + c^2*d^2)*cosh(b*x + a)^2 +
 2*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*cosh(b*x + a)*sinh(b*x + a) + (d^4*x...
```

Sympy [F]

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sinh^2{(a+bx)}}{(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)**2/(d*x+c)**(5/2),x)
```

output
```
Integral(sinh(a + b*x)**2/(c + d*x)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.68

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{\frac{3\sqrt{2}\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2},\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}} + \frac{3\sqrt{2}\left(-\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2},-\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}} - \frac{2}{(dx+c)^{\frac{3}{2}}}}{6\,d}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
-1/6*(3*sqrt(2)*((d*x + c)*b/d)^(3/2)*e^(2*(b*c - a*d)/d)*gamma(-3/2, 2*(d
*x + c)*b/d)/(d*x + c)^(3/2) + 3*sqrt(2)*(-(d*x + c)*b/d)^(3/2)*e^(-2*(b*c
 - a*d)/d)*gamma(-3/2, -2*(d*x + c)*b/d)/(d*x + c)^(3/2) - 2/(d*x + c)^(3/
2))/d
```

Giac [F]

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sinh(bx+a)^2}{(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^2/(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sinh(a+b\,x)^2}{(c+d\,x)^{5/2}}\,dx$$

input
```
int(sinh(a + b*x)^2/(c + d*x)^(5/2),x)
```

output
```
int(sinh(a + b*x)^2/(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sinh(bx+a)^2}{\sqrt{dx+c}\,c^2+2\sqrt{dx+c}\,cdx+\sqrt{dx+c}\,d^2x^2}dx$$

input
```
int(sinh(b*x+a)^2/(d*x+c)^(5/2),x)
```

output
```
int(sinh(a + b*x)**2/(sqrt(c + d*x)*c**2 + 2*sqrt(c + d*x)*c*d*x + sqrt(c
+ d*x)*d**2*x**2),x)
```

3.51 $\int \frac{\sinh^2(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 220

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{8b^{5/2}e^{-2a+\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{8b^{5/2}e^{2a-\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2\sinh^2(a+bx)}{15d^3\sqrt{c+dx}}$$

output
```
-16/15*b^2/d^3/(d*x+c)^(1/2)-8/15*b^(5/2)*exp(-2*a+2*b*c/d)*2^(1/2)*Pi^(1/
2)*erf(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(7/2)+8/15*b^(5/2)*exp(2*a
-2*b*c/d)*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(
7/2)-8/15*b*cosh(b*x+a)*sinh(b*x+a)/d^2/(d*x+c)^(3/2)-2/5*sinh(b*x+a)^2/d/
(d*x+c)^(5/2)-32/15*b^2*sinh(b*x+a)^2/d^3/(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.93

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{6d^2 + e^{2a}\left(-3d^2e^{2bx} - 4be^{-\frac{2bc}{d}}(c+dx)\left(e^{\frac{2b(c+dx)}{d}}(d+4b(c+dx)) + 4\sqrt{2}d\left(-\frac{b(c+dx)}{d}\right.\right.\right.}{}$$

input
```
Integrate[Sinh[a + b*x]^2/(c + d*x)^(7/2),x]
```

output
```
(6*d^2 + E^(2*a)*(-3*d^2*E^(2*b*x) - (4*b*(c + d*x)*(E^((2*b*(c + d*x))/d)
*(d + 4*b*(c + d*x)) + 4*Sqrt[2]*d*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, (
-2*b*(c + d*x))/d]))/E^((2*b*c)/d)) + (-3*d^2 + 4*b*(c + d*x)*(d - 4*b*(c
+ d*x) + 4*Sqrt[2]*d*E^((2*b*(c + d*x))/d)*((b*(c + d*x))/d)^(3/2)*Gamma[1
/2, (2*b*(c + d*x))/d]))/E^(2*(a + b*x)))/(30*d^3*(c + d*x)^(5/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.10, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3042, 25, 3795, 17, 25, 3042, 25, 3794, 27, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\sin(ia+ibx)^2}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\sin(ia+ibx)^2}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{ 3795}$$

$$-\frac{16b^2\int -\frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}dx}{15d^2}+\frac{8b^2\int \frac{1}{(c+dx)^{3/2}}dx}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}}$$

$\Big\downarrow$ 17

$$-\frac{16b^2\int -\frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}dx}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$\Big\downarrow$ 25

$$\frac{16b^2\int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}dx}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$\Big\downarrow$ 3042

$$\frac{16b^2\int -\frac{\sin(ia+ibx)^2}{(c+dx)^{3/2}}dx}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$\Big\downarrow$ 25

$$-\frac{16b^2\int \frac{\sin(ia+ibx)^2}{(c+dx)^{3/2}}dx}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$\Big\downarrow$ 3794

$$-\frac{16b^2\left(\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}+\frac{4ib\int \frac{i\sinh(2a+2bx)}{2\sqrt{c+dx}}dx}{d}\right)}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$\Big\downarrow$ 27

$$-\frac{16b^2\left(\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}-\frac{2b\int \frac{\sinh(2a+2bx)}{\sqrt{c+dx}}dx}{d}\right)}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$\Big\downarrow$ 3042

$$-\frac{16b^2\left(\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}-\frac{2b\int -\frac{i\sin(2ia+2ibx)}{\sqrt{c+dx}}dx}{d}\right)}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \; 26$$

$$-\frac{16b^2\left(\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}+\frac{2ib\int \frac{\sin(2ia+2ibx)}{\sqrt{c+dx}}dx}{d}\right)}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \; 3789$$

$$-\frac{16b^2\left(\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}+\frac{2ib\left(\frac{1}{2}i\int \frac{e^{2(a+bx)}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int \frac{e^{-2(a+bx)}}{\sqrt{c+dx}}dx\right)}{d}\right)}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \; 2611$$

$$-\frac{16b^2\left(\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}+\frac{2ib\left(\frac{i\int e^{2\left(a-\frac{bc}{d}\right)+\frac{2b(c+dx)}{d}}d\sqrt{c+dx}}{d}-\frac{i\int e^{-2\left(a-\frac{bc}{d}\right)-\frac{2b(c+dx)}{d}}d\sqrt{c+dx}}{d}\right)}{d}\right)}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \; 2633$$

$$-\frac{16b^2\left(\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}+\frac{2ib\left(\frac{i\sqrt{\frac{\pi}{2}}e^{2a-\frac{2bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\int e^{-2\left(a-\frac{bc}{d}\right)-\frac{2b(c+dx)}{d}}d\sqrt{c+dx}}{d}\right)}{d}\right)}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \; 2634$$

$$-\frac{16b^2\left(\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}+\frac{2ib\left(\frac{i\sqrt{\frac{\pi}{2}}e^{2a-\frac{2bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\frac{\pi}{2}}e^{\frac{2bc}{d}-2a}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d}\right)}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

input
```
Int[Sinh[a + b*x]^2/(c + d*x)^(7/2),x]
```

output
```
(-16*b^2)/(15*d^3*Sqrt[c + d*x]) - (8*b*Cosh[a + b*x]*Sinh[a + b*x])/(15*d
^2*(c + d*x)^(3/2)) - (2*Sinh[a + b*x]^2)/(5*d*(c + d*x)^(5/2)) - (16*b^2*
(((2*I)*b*(((-1/2*I)*E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*
Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) + ((I/2)*E^(2*a - (2*b*c)/d)*Sq
rt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]))
)/d + (2*Sinh[a + b*x]^2)/(d*Sqrt[c + d*x])))/(15*d^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Maple [F]

$$\int \frac{\sinh \left(bx+a\right)^{2}}{\left(dx+c\right)^{\frac{7}{2}}}dx$$

input
```
int(sinh(b*x+a)^2/(d*x+c)^(7/2),x)
```

output
```
int(sinh(b*x+a)^2/(d*x+c)^(7/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1352 vs. $2(172) = 344$.

Time = 0.13 (sec) , antiderivative size = 1352, normalized size of antiderivative = 6.15

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-1/30*(16*sqrt(2)*sqrt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
 + b^2*c^3)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*
c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d
) + ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-2*(b*
c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*si
nh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
+ 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) - (b^2*d^3
*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-2*(b
*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)
) + 16*sqrt(2)*sqrt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
b^2*c^3)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d
^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) +
 ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-2*(b*c -
 a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(
-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3
*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + (b^2*d^3*x^
3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-2*(b*c
- a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d))
 + (16*b^2*d^2*x^2 + (16*b^2*d^2*x^2 + 16*b^2*c^2 + 4*b*c*d + 3*d^2 + 4*(8
*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^4 + 4*(16*b^2*d^2*x^2 + 16*b^2*c^2 +...
```

Sympy [F]

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sinh^2(a+bx)}{(c+dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)**2/(d*x+c)**(7/2),x)
```

output
```
Integral(sinh(a + b*x)**2/(c + d*x)**(7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.54

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{\frac{5\sqrt{2}\left(\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2},\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}}+\frac{5\sqrt{2}\left(-\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2},-\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}}-\frac{1}{(dx+c)^{\frac{5}{2}}}}{5d}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
-1/5*(5*sqrt(2)*((d*x + c)*b/d)^(5/2)*e^(2*(b*c - a*d)/d)*gamma(-5/2, 2*(d
*x + c)*b/d)/(d*x + c)^(5/2) + 5*sqrt(2)*(-(d*x + c)*b/d)^(5/2)*e^(-2*(b*c
 - a*d)/d)*gamma(-5/2, -2*(d*x + c)*b/d)/(d*x + c)^(5/2) - 1/(d*x + c)^(5/
2))/d
```

Giac [F]

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sinh(bx+a)^2}{(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^2/(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sinh(a+b\,x)^2}{(c+d\,x)^{7/2}}\,dx$$

input
```
int(sinh(a + b*x)^2/(c + d*x)^(7/2),x)
```

output
```
int(sinh(a + b*x)^2/(c + d*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sinh(bx+a)^2}{\sqrt{dx+c}\,c^3+3\sqrt{dx+c}\,c^2dx+3\sqrt{dx+c}\,c\,d^2x^2+\sqrt{dx+c}\,d^3x^3}dx$$

input
```
int(sinh(b*x+a)^2/(d*x+c)^(7/2),x)
```

output
```
int(sinh(a + b*x)**2/(sqrt(c + d*x)*c**3 + 3*sqrt(c + d*x)*c**2*d*x + 3*sq
rt(c + d*x)*c*d**2*x**2 + sqrt(c + d*x)*d**3*x**3),x)
```

3.52 $\int \frac{\sinh^2(a+bx)}{(c+dx)^{9/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 251

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{9/2}}\,dx = -\frac{16b^2}{105d^3(c+dx)^{3/2}} + \frac{32b^{7/2}e^{-2a+\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}} + \frac{32b^{7/2}e^{2a-\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{128b^3\cosh(a+bx)\sinh(a+bx)}{105d^4\sqrt{c+dx}} - \frac{2\sinh^2(a+bx)}{7d(c+dx)^{7/2}} - \frac{32b^2\sinh^2(a+bx)}{105d^3(c+dx)^{3/2}}$$

output

```
-16/105*b^2/d^3/(d*x+c)^(3/2)+32/105*b^(7/2)*exp(-2*a+2*b*c/d)*2^(1/2)*Pi^
(1/2)*erf(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(9/2)+32/105*b^(7/2)*ex
p(2*a-2*b*c/d)*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)
)/d^(9/2)-8/35*b*cosh(b*x+a)*sinh(b*x+a)/d^2/(d*x+c)^(5/2)-128/105*b^3*cos
h(b*x+a)*sinh(b*x+a)/d^4/(d*x+c)^(1/2)-2/7*sinh(b*x+a)^2/d/(d*x+c)^(7/2)-3
2/105*b^2*sinh(b*x+a)^2/d^3/(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.88

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{9/2}}\,dx = \frac{2\left(-8b^2d(c+dx)^2+16\sqrt{2}b^3e^{2a-\frac{2bc}{d}}(c+dx)^3\sqrt{-\frac{b(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\frac{2b(c+dx)}{d}\right)-16\sqrt{2}b^3e\right.}{}$$

input
```
Integrate[Sinh[a + b*x]^2/(c + d*x)^(9/2),x]
```

output
```
(2*(-8*b^2*d*(c + d*x)^2 + 16*Sqrt[2]*b^3*E^(2*a - (2*b*c)/d)*(c + d*x)^3*
Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, (-2*b*(c + d*x))/d] - 16*Sqrt[2]*b^3*E
^(-2*a + (2*b*c)/d)*(c + d*x)^3*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (2*b*(c +
 d*x))/d] - 15*d^3*Sinh[a + b*x]^2 - 16*b^2*d*(c + d*x)^2*Sinh[a + b*x]^2
- 6*b*d^2*(c + d*x)*Sinh[2*(a + b*x)] - 32*b^3*(c + d*x)^3*Sinh[2*(a + b*x
)]))/(105*d^4*(c + d*x)^(7/2))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.21, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {3042, 25, 3795, 17, 25, 3042, 25, 3795, 17, 25, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ia+ibx)^2}{(c+dx)^{9/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ia+ibx)^2}{(c+dx)^{9/2}}dx$$

$$\Big\downarrow \text{3795}$$

$$-\frac{16b^2\int -\frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}dx}{35d^2}+\frac{8b^2\int \frac{1}{(c+dx)^{5/2}}dx}{35d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sinh^2(a+bx)}{7d(c+dx)^{7/2}}$$

$$\Big\downarrow \ 17$$

$$-\frac{16b^2\int -\frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}dx}{35d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sinh^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{16b^2\int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}dx}{35d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sinh^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{16b^2\int -\frac{\sin(ia+ibx)^2}{(c+dx)^{5/2}}dx}{35d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sinh^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \ 25$$

$$-\frac{16b^2\int \frac{\sin(ia+ibx)^2}{(c+dx)^{5/2}}dx}{35d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sinh^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \ 3795$$

$$-\frac{16b^2\left(\frac{16b^2\int -\frac{\sinh^2(a+bx)}{\sqrt{c+dx}}dx}{3d^2}-\frac{8b^2\int \frac{1}{\sqrt{c+dx}}dx}{3d^2}+\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}+\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}\right)}{35d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sinh^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \ 17$$

$$-\frac{16b^2\left(\frac{16b^2\int -\frac{\sinh^2(a+bx)}{\sqrt{c+dx}}dx}{3d^2}+\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}+\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}-\frac{16b^2\sqrt{c+dx}}{3d^3}\right)}{35d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sinh^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \ 25$$

$$-\frac{16b^2\left(-\frac{16b^2\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}dx}{3d^2}+\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}+\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}-\frac{16b^2\sqrt{c+dx}}{3d^3}\right)}{35d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sinh^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$\Big\downarrow$ 3042

$$-\frac{16b^2\left(-\frac{16b^2\int -\frac{\sin(ia+ibx)^2}{\sqrt{c+dx}}dx}{3d^2}+\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}+\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}-\frac{16b^2\sqrt{c+dx}}{3d^3}\right)}{35d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sinh^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$\Big\downarrow$ 25

$$-\frac{16b^2\left(\frac{16b^2\int \frac{\sin(ia+ibx)^2}{\sqrt{c+dx}}dx}{3d^2}+\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}+\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}-\frac{16b^2\sqrt{c+dx}}{3d^3}\right)}{35d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sinh^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$\Big\downarrow$ 3793

$$-\frac{16b^2\left(\frac{16b^2\int \left(\frac{1}{2\sqrt{c+dx}}-\frac{\cosh(2a+2bx)}{2\sqrt{c+dx}}\right)dx}{3d^2}+\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}+\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}-\frac{16b^2\sqrt{c+dx}}{3d^3}\right)}{35d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sinh^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$\Big\downarrow$ 2009

$$-\frac{16b^2\left(\frac{16b^2\left(-\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2bc}{d}-2a}\text{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{2}}e^{2a-\frac{2bc}{d}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{\sqrt{c+dx}}{d}\right)}{3d^2}+\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}+\frac{2\sinh^2(a+bx}{3d(c+dx)^{3/2}}\right.}{35d^2}$$
$$\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sinh^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

input
```
Int[Sinh[a + b*x]^2/(c + d*x)^(9/2),x]
```

output
```
(-16*b^2)/(105*d^3*(c + d*x)^(3/2)) - (8*b*Cosh[a + b*x]*Sinh[a + b*x])/(3
5*d^2*(c + d*x)^(5/2)) - (2*Sinh[a + b*x]^2)/(7*d*(c + d*x)^(7/2)) - (16*b
^2*((-16*b^2*Sqrt[c + d*x])/(3*d^3) + (16*b^2*(Sqrt[c + d*x]/d - (E^(-2*a
+ (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*S
qrt[b]*Sqrt[d]) - (E^(2*a - (2*b*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sq
rt[c + d*x])/Sqrt[d]])/(4*Sqrt[b]*Sqrt[d])))/(3*d^2) + (8*b*Cosh[a + b*x]*
Sinh[a + b*x])/(3*d^2*Sqrt[c + d*x]) + (2*Sinh[a + b*x]^2)/(3*d*(c + d*x)^
(3/2))))/(35*d^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Maple [F]

$$\int \frac{\sinh\left(bx+a\right)^{2}}{\left(dx+c\right)^{\frac{9}{2}}}dx$$

input
```
int(sinh(b*x+a)^2/(d*x+c)^(9/2),x)
```

output
```
int(sinh(b*x+a)^2/(d*x+c)^(9/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1827 vs. $2(199) = 398$.

Time = 0.14 (sec) , antiderivative size = 1827, normalized size of antiderivative = 7.28

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
1/210*(64*sqrt(2)*sqrt(pi)*((b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2
*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - (
b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^
4)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + ((b^3*d^4*x^4 + 4*b^3*c*d^3*x^
3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(-2*(b*c - a*d)/d) -
(b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c
^4)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((b^3*d^4*x^4 + 4*b^3*c*d^
3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)*cosh(-2
*(b*c - a*d)/d) - (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b
^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))
*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - 64*sqrt(2)*sqrt(pi)*((b^
3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)
*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 +
 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)^2*sinh(-2*(b*c
 - a*d)/d) + ((b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c
^3*d*x + b^3*c^4)*cosh(-2*(b*c - a*d)/d) + (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3
+ 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*sinh(-2*(b*c - a*d)/d))*sin
h(b*x + a)^2 + 2*((b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b
^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + (b^3*d^4*x^4
+ 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(sinh(b*x+a)**2/(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.47

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{9/2}}\,dx = -\frac{\frac{14\sqrt{2}\left(\frac{(dx+c)b}{d}\right)^{\frac{7}{2}}e^{\left(\frac{2\,(bc-ad)}{d}\right)}\Gamma\left(-\frac{7}{2},\frac{2\,(dx+c)b}{d}\right)}{(dx+c)^{\frac{7}{2}}}+\frac{14\sqrt{2}\left(-\frac{(dx+c)b}{d}\right)^{\frac{7}{2}}e^{\left(-\frac{2\,(bc-ad)}{d}\right)}\Gamma\left(-\frac{7}{2},-\frac{2\,(dx+c)b}{d}\right)}{(dx+c)^{\frac{7}{2}}}-\frac{1}{(dx+c)^{\frac{7}{2}}}}{7\,d}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
-1/7*(14*sqrt(2)*((d*x + c)*b/d)^(7/2)*e^(2*(b*c - a*d)/d)*gamma(-7/2, 2*(
d*x + c)*b/d)/(d*x + c)^(7/2) + 14*sqrt(2)*(-(d*x + c)*b/d)^(7/2)*e^(-2*(b
*c - a*d)/d)*gamma(-7/2, -2*(d*x + c)*b/d)/(d*x + c)^(7/2) - 1/(d*x + c)^(
7/2))/d
```

Giac [F]

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{9/2}}\,dx = \int \frac{\sinh\left(bx+a\right)^2}{(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^2/(d*x + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{9/2}}\,dx = \int \frac{\sinh(a+b\,x)^2}{(c+d\,x)^{9/2}}\,dx$$

input
```
int(sinh(a + b*x)^2/(c + d*x)^(9/2),x)
```

output
```
int(sinh(a + b*x)^2/(c + d*x)^(9/2), x)
```

Reduce [F]

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{9/2}}\,dx = \int \frac{\sinh(bx+a)^2}{\sqrt{dx+c}\,c^4+4\sqrt{dx+c}\,c^3dx+6\sqrt{dx+c}\,c^2d^2x^2+4\sqrt{dx+c}\,c\,d^3x^3+\sqrt{dx+c}\,d^4}$$

input
```
int(sinh(b*x+a)^2/(d*x+c)^(9/2),x)
```

output
```
int(sinh(a + b*x)**2/(sqrt(c + d*x)*c**4 + 4*sqrt(c + d*x)*c**3*d*x + 6*sq
rt(c + d*x)*c**2*d**2*x**2 + 4*sqrt(c + d*x)*c*d**3*x**3 + sqrt(c + d*x)*d
**4*x**4),x)
```

3.53 $\int (c+dx)^{5/2}\sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 381

$$\begin{aligned}\int (c+dx)^{5/2}\sinh^3(a+bx)\,dx =&-\frac{45d^2\sqrt{c+dx}\cosh(a+bx)}{16b^3}-\frac{2(c+dx)^{5/2}\cosh(a+bx)}{3b}\\&+\frac{5d^2\sqrt{c+dx}\cosh(3a+3bx)}{144b^3}+\frac{45d^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{7/2}}\\&-\frac{5d^{5/2}e^{-3a+\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{45d^{5/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{7/2}}\\&-\frac{5d^{5/2}e^{3a-\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{5d(c+dx)^{3/2}\sinh(a+bx)}{3b^2}\\&+\frac{(c+dx)^{5/2}\cosh(a+bx)\sinh^2(a+bx)}{3b}-\frac{5d(c+dx)^{3/2}\sinh^3(a+bx)}{18b^2}\end{aligned}$$

output
```
-45/16*d^2*(d*x+c)^(1/2)*cosh(b*x+a)/b^3-2/3*(d*x+c)^(5/2)*cosh(b*x+a)/b+5
/144*d^2*(d*x+c)^(1/2)*cosh(3*b*x+3*a)/b^3+45/64*d^(5/2)*exp(-a+b*c/d)*Pi^
(1/2)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)-5/1728*d^(5/2)*exp(-3*a+3
*b*c/d)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2
)+45/64*d^(5/2)*exp(a-b*c/d)*Pi^(1/2)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/
b^(7/2)-5/1728*d^(5/2)*exp(3*a-3*b*c/d)*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*b^(1
/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)+5/3*d*(d*x+c)^(3/2)*sinh(b*x+a)/b^2+1/3
*(d*x+c)^(5/2)*cosh(b*x+a)*sinh(b*x+a)^2/b-5/18*d*(d*x+c)^(3/2)*sinh(b*x+a
)^3/b^2
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.51

$$\int (c+dx)^{5/2}\sinh^3(a+bx)\,dx = \frac{e^{-3\left(a+\frac{bc}{d}\right)}(c+dx)^{3/2}\left(\sqrt{3}be^{6a}(c+dx)\Gamma\left(\frac{7}{2},-\frac{3b(c+dx)}{d}\right)-243be^{4a+\frac{2bc}{d}}(c+dx)\Gamma\left(\frac{7}{2},-\frac{b(c+dx)}{d}\right)+a\right.}{648b^2\left(-\frac{b(c+dx)}{d}\right)^{5/2}}$$

input
```
Integrate[(c + d*x)^(5/2)*Sinh[a + b*x]^3,x]
```

output
```
((c + d*x)^(3/2)*(Sqrt[3]*b*E^(6*a)*(c + d*x)*Gamma[7/2, (-3*b*(c + d*x))/
d] - 243*b*E^(4*a + (2*b*c)/d)*(c + d*x)*Gamma[7/2, -((b*(c + d*x))/d)] +
d*E^((4*b*c)/d)*Sqrt[-((b^2*(c + d*x)^2)/d^2)]*(-243*E^(2*a)*Gamma[7/2, (b
*(c + d*x))/d] + Sqrt[3]*E^((2*b*c)/d)*Gamma[7/2, (3*b*(c + d*x))/d])))/(6
48*b^2*E^(3*(a + (b*c)/d))*(-((b*(c + d*x))/d))^(5/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.26 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.48, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.167, Rules used = {3042, 26, 3792, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3788, 26, 2611, 2633, 2634, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sinh^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i(c+dx)^{5/2}\sin(ia+ibx)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int (c+dx)^{5/2}\sin(ia+ibx)^3dx$$

$$\Big\downarrow \text{3792}$$

$$i\left(\frac{5d^2\int -i\sqrt{c+dx}\sinh^3(a+bx)dx}{12b^2}+\frac{2}{3}\int i(c+dx)^{5/2}\sinh(a+bx)dx+\frac{5id(c+dx)^{3/2}\sinh^3(a+bx)}{18b^2}-\frac{i(c+\right.$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{5id^2\int \sqrt{c+dx}\sinh^3(a+bx)dx}{12b^2}+\frac{2}{3}i\int (c+dx)^{5/2}\sinh(a+bx)dx+\frac{5id(c+dx)^{3/2}\sinh^3(a+bx)}{18b^2}-\frac{i(c+\right.$$

$$\Big\downarrow \text{3042}$$

$$i\left(-\frac{5id^2\int i\sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{2}{3}i\int -i(c+dx)^{5/2}\sin(ia+ibx)dx+\frac{5id(c+dx)^{3/2}\sinh^3(a+bx)}{18b^2}-\frac{i(\right.$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{5d^2\int \sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{2}{3}\int (c+dx)^{5/2}\sin(ia+ibx)dx+\frac{5id(c+dx)^{3/2}\sinh^3(a+bx)}{18b^2}-\frac{i(c+dx)\right.$$

$\downarrow$ 3777

$$i\left(\frac{5d^2\int\sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\int(c+dx)^{3/2}\cosh(a+bx)dx}{2b}\right)+\frac{5id(c$$

$\downarrow$ 3042

$$i\left(\frac{5d^2\int\sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\int(c+dx)^{3/2}\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{2b}\right)+$$

$\downarrow$ 3777

$$i\left(\frac{5d^2\int\sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}-\frac{3id\int-i\sqrt{c+dx}\sinh(a}{2b}\right.}{2b}\right.\right.$$

$\downarrow$ 26

$$i\left(\frac{5d^2\int\sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}-\frac{3d\int\sqrt{c+dx}\sinh(a+b}{2b}\right.}{2b}\right.\right.$$

$\downarrow$ 3042

$$i\left(\frac{5d^2\int\sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}-\frac{3d\int-i\sqrt{c+dx}\sin(ia-}{2b}\right.}{2b}\right.\right.$$

$\downarrow$ 26

$$i\left(\frac{5d^2\int\sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\int\sqrt{c+dx}\sin(ia+i}{2b}\right.}{2b}\right.\right.$$

$\downarrow$ 3777

$$i\left(\frac{5d^2\int\sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+}{b}\right.}{}\right.}{2b}\right.\right.$$

$\downarrow$ 3042

$$i\left(\frac{5d^2\int\sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+}{b}\right.}{}\right.}{2b}\right.\right.$$

$\downarrow$ 3788

$$i\left(\frac{5d^2\int\sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+}{b}\right.}{}\right.}{2b}\right.\right.$$

$\downarrow$ 26

$$i\left(\frac{5d^2\int\sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+b}{b}\right.}{}\right.}{2b}\right.\right.$$

$\downarrow$ 2611

$$i\left(\frac{5d^2\int\sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+b}{b}\right.}{}\right.}{}\right.\right.$$

$\downarrow$ 2633

$$i\left(\frac{5d^2\int\sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+b}{b}\right.}{}\right.}{}\right.\right.$$

$\downarrow$ 2634

$$i\left(\frac{5d^2\int\sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{5id(c+dx)^{3/2}\sinh^3(a+bx)}{18b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)}{}\right.}{}\right.\right.$$

$\downarrow$ 3793

$$i\left(\frac{5d^2\int\left(\frac{3}{4}i\sqrt{c+dx}\sinh(a+bx)-\frac{1}{4}i\sqrt{c+dx}\sinh(3a+3bx)\right)dx}{12b^2}+\frac{5id(c+dx)^{3/2}\sinh^3(a+bx)}{18b^2}+\frac{2}{3}\left(\frac{i(c+d\right.\right.$$

$$\downarrow \text{2009}$$

$$i\left(\frac{5id(c+dx)^{3/2}\sinh^3(a+bx)}{18b^2}+\frac{5d^2\left(-\frac{3i\sqrt{\pi}\sqrt{d}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{i\sqrt{\frac{\pi}{3}}\sqrt{d}e^{\frac{3bc}{d}-3a}\text{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{3i\sqrt{\pi}\sqrt{d}e^{a-\frac{bc}{d}}\ldots}{16\ldots}\right.}{\ldots}\right.$$

input
```
Int[(c + d*x)^(5/2)*Sinh[a + b*x]^3,x]
```

output
```
I*((5*d^2*((((3*I)/4)*Sqrt[c + d*x]*Cosh[a + b*x])/b - ((I/12)*Sqrt[c + d*
x]*Cosh[3*a + 3*b*x])/b - (((3*I)/16)*Sqrt[d]*E^(-a + (b*c)/d)*Sqrt[Pi]*Er
f[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/b^(3/2) + ((I/48)*Sqrt[d]*E^(-3*a + (3
*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/b^(3/2)
- (((3*I)/16)*Sqrt[d]*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x]
)/Sqrt[d]])/b^(3/2) + ((I/48)*Sqrt[d]*E^(3*a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[
(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/b^(3/2)))/(12*b^2) - ((I/3)*(c +
 d*x)^(5/2)*Cosh[a + b*x]*Sinh[a + b*x]^2)/b + (((5*I)/18)*d*(c + d*x)^(3/
2)*Sinh[a + b*x]^3)/b^2 + (2*((I*(c + d*x)^(5/2)*Cosh[a + b*x])/b - (((5*I
)/2)*d*((((3*I)/2)*d*((I*Sqrt[c + d*x]*Cosh[a + b*x])/b - ((I/2)*d*((E^(-a
 + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt
[d]) + (E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2
*Sqrt[b]*Sqrt[d])))/b))/b + ((c + d*x)^(3/2)*Sinh[a + b*x])/b))/b))/3)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int (dx + c)^{\frac{5}{2}} \sinh (bx + a)^3 \, dx$$

input
```
int((d*x+c)^(5/2)*sinh(b*x+a)^3,x)
```

output
```
int((d*x+c)^(5/2)*sinh(b*x+a)^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2090 vs. $2(291) = 582$.

Time = 0.14 (sec) , antiderivative size = 2090, normalized size of antiderivative = 5.49

$$\int (c + dx)^{5/2} \sinh^3(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/1728*(5*sqrt(3)*sqrt(pi)*(d^3*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) -
d^3*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^3*cosh(-3*(b*c - a*d)/d) -
 d^3*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^3*cosh(b*x + a)*cosh(-
3*(b*c - a*d)/d) - d^3*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)
^2 + 3*(d^3*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) - d^3*cosh(b*x + a)^2*s
inh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*
sqrt(b/d)) - 5*sqrt(3)*sqrt(pi)*(d^3*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d
) + d^3*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^3*cosh(-3*(b*c - a*d)/
d) + d^3*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^3*cosh(b*x + a)*co
sh(-3*(b*c - a*d)/d) + d^3*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x
+ a)^2 + 3*(d^3*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) + d^3*cosh(b*x + a)
^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*x
+ c)*sqrt(-b/d)) - 1215*sqrt(pi)*(d^3*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d)
 - d^3*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + (d^3*cosh(-(b*c - a*d)/d) -
d^3*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^3*cosh(b*x + a)*cosh(-(b*
c - a*d)/d) - d^3*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*
(d^3*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) - d^3*cosh(b*x + a)^2*sinh(-(b*c
 - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 1215*s
qrt(pi)*(d^3*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) + d^3*cosh(b*x + a)^3*si
nh(-(b*c - a*d)/d) + (d^3*cosh(-(b*c - a*d)/d) + d^3*sinh(-(b*c - a*d)/...
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\sinh^3(a+bx)\,dx=\text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*sinh(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.35

$$\int (c+dx)^{5/2}\sinh^3(a+bx)\,dx =$$

$$-\frac{\frac{5\sqrt{3}\sqrt{\pi}d^3\,\mathrm{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3a-\frac{3bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}}+\frac{5\sqrt{3}\sqrt{\pi}d^3\,\mathrm{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3a+\frac{3bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}}-\frac{1215\sqrt{\pi}d^3\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}}-}{}$$

input
```
integrate((d*x+c)^(5/2)*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/1728*(5*sqrt(3)*sqrt(pi)*d^3*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3
*a - 3*b*c/d)/(b^3*sqrt(-b/d)) + 5*sqrt(3)*sqrt(pi)*d^3*erf(sqrt(3)*sqrt(d
*x + c)*sqrt(b/d))*e^(-3*a + 3*b*c/d)/(b^3*sqrt(b/d)) - 1215*sqrt(pi)*d^3*
erf(sqrt(d*x + c)*sqrt(-b/d))*e^(a - b*c/d)/(b^3*sqrt(-b/d)) - 1215*sqrt(p
i)*d^3*erf(sqrt(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/(b^3*sqrt(b/d)) + 162*(
4*(d*x + c)^(5/2)*b^2*d*e^(b*c/d) + 10*(d*x + c)^(3/2)*b*d^2*e^(b*c/d) + 1
5*sqrt(d*x + c)*d^3*e^(b*c/d))*e^(-a - (d*x + c)*b/d)/b^3 - 6*(12*(d*x + c
)^(5/2)*b^2*d*e^(3*b*c/d) + 10*(d*x + c)^(3/2)*b*d^2*e^(3*b*c/d) + 5*sqrt(
d*x + c)*d^3*e^(3*b*c/d))*e^(-3*a - 3*(d*x + c)*b/d)/b^3 - 6*(12*(d*x + c)
^(5/2)*b^2*d*e^(3*a) - 10*(d*x + c)^(3/2)*b*d^2*e^(3*a) + 5*sqrt(d*x + c)*
d^3*e^(3*a))*e^(3*(d*x + c)*b/d - 3*b*c/d)/b^3 + 162*(4*(d*x + c)^(5/2)*b^
2*d*e^a - 10*(d*x + c)^(3/2)*b*d^2*e^a + 15*sqrt(d*x + c)*d^3*e^a)*e^((d*x
 + c)*b/d - b*c/d)/b^3)/d
```

Giac [F]

$$\int (c+dx)^{5/2}\sinh^3(a+bx)\,dx = \int (dx+c)^{\frac{5}{2}}\sinh(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^(5/2)*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^(5/2)*sinh(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\sinh^3(a+bx)\,dx=\int \sinh(a+b\,x)^3\,(c+d\,x)^{5/2}\,dx$$

input
```
int(sinh(a + b*x)^3*(c + d*x)^(5/2),x)
```

output
```
int(sinh(a + b*x)^3*(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int (c+dx)^{5/2}\sinh^3(a+bx)\,dx=\left(\int \sqrt{dx+c}\,\sinh(bx+a)^3\,x^2dx\right)d^2+2\left(\int \sqrt{dx+c}\,\sinh(bx+a)^3\,xdx\right)cd+\left(\int \sqrt{dx+c}\,\sinh(bx+a)^3\,dx\right)c^2$$

input
```
int((d*x+c)^(5/2)*sinh(b*x+a)^3,x)
```

output
```
int(sqrt(c + d*x)*sinh(a + b*x)**3*x**2,x)*d**2 + 2*int(sqrt(c + d*x)*sinh
(a + b*x)**3*x,x)*c*d + int(sqrt(c + d*x)*sinh(a + b*x)**3,x)*c**2
```

3.54 $\int (c+dx)^{3/2}\sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 325

$$\begin{aligned}\int (c+dx)^{3/2}\sinh^3(a+bx)\,dx =&-\frac{2(c+dx)^{3/2}\cosh(a+bx)}{3b}+\frac{9d^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}}\\&-\frac{d^{3/2}e^{-3a+\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}-\frac{9d^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}}\\&+\frac{d^{3/2}e^{3a-\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{d\sqrt{c+dx}\sinh(a+bx)}{b^2}\\&+\frac{(c+dx)^{3/2}\cosh(a+bx)\sinh^2(a+bx)}{3b}-\frac{d\sqrt{c+dx}\sinh^3(a+bx)}{6b^2}\end{aligned}$$

output

```
-2/3*(d*x+c)^(3/2)*cosh(b*x+a)/b+9/32*d^(3/2)*exp(-a+b*c/d)*Pi^(1/2)*erf(b
^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)-1/288*d^(3/2)*exp(-3*a+3*b*c/d)*3^(1
/2)*Pi^(1/2)*erf(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)-9/32*d^(3/
2)*exp(a-b*c/d)*Pi^(1/2)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)+1/288
*d^(3/2)*exp(3*a-3*b*c/d)*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*b^(1/2)*(d*x+c)^(1
/2)/d^(1/2))/b^(5/2)+d*(d*x+c)^(1/2)*sinh(b*x+a)/b^2+1/3*(d*x+c)^(3/2)*cos
h(b*x+a)*sinh(b*x+a)^2/b-1/6*d*(d*x+c)^(1/2)*sinh(b*x+a)^3/b^2
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.65

$$\int (c+dx)^{3/2}\sinh^3(a+bx)\,dx = \frac{de^{-3\left(a+\frac{bc}{d}\right)}\sqrt{c+dx}\left(-\sqrt{3}e^{6a}\sqrt{\frac{b(c+dx)}{d}}\Gamma\left(\frac{5}{2},-\frac{3b(c+dx)}{d}\right)+81e^{4a+\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\Gamma\left(\frac{5}{2},-\frac{b(c+dx)}{d}\right)+e^{\frac{4b}{d}}\cdots\right)}{216b^2\sqrt{-\frac{b^2(c+dx)^2}{d^2}}}$$

input
```
Integrate[(c + d*x)^(3/2)*Sinh[a + b*x]^3,x]
```

output
```
(d*Sqrt[c + d*x]*(-(Sqrt[3]*E^(6*a)*Sqrt[(b*(c + d*x))/d]*Gamma[5/2, (-3*b
*(c + d*x))/d]) + 81*E^(4*a + (2*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[5/2,
-((b*(c + d*x))/d)] + E^((4*b*c)/d)*Sqrt[-((b*(c + d*x))/d)]*(-81*E^(2*a)*
Gamma[5/2, (b*(c + d*x))/d] + Sqrt[3]*E^((2*b*c)/d)*Gamma[5/2, (3*b*(c + d
*x))/d])))/(216*b^2*E^(3*(a + (b*c)/d))*Sqrt[-((b^2*(c + d*x)^2)/d^2)])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.70 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.49, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 26, 3792, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3789, 2611, 2633, 2634, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sinh^3(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int i(c+dx)^{3/2}\sin(ia+ibx)^3dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int (c+dx)^{3/2}\sin(ia+ibx)^3dx$$

$$\Big\downarrow \; 3792$$

$$i\left(\frac{d^2\int -\frac{i\sinh^3(a+bx)}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\int i(c+dx)^{3/2}\sinh(a+bx)dx+\frac{id\sqrt{c+dx}\sinh^3(a+bx)}{6b^2}-\frac{i(c+dx)^{3/2}\sinh^2(a+}{3b}\right.$$

$$\Big\downarrow \; 26$$

$$i\left(-\frac{id^2\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}i\int (c+dx)^{3/2}\sinh(a+bx)dx+\frac{id\sqrt{c+dx}\sinh^3(a+bx)}{6b^2}-\frac{i(c+dx)^{3/2}\sinh^2(a+}{3b}\right.$$

$$\Big\downarrow \; 3042$$

$$i\left(-\frac{id^2\int \frac{i\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}i\int -i(c+dx)^{3/2}\sin(ia+ibx)dx+\frac{id\sqrt{c+dx}\sinh^3(a+bx)}{6b^2}-\frac{i(c+dx)^{3/2}\sinh^2(}{}\right.$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{d^2\int \frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\int (c+dx)^{3/2}\sin(ia+ibx)dx+\frac{id\sqrt{c+dx}\sinh^3(a+bx)}{6b^2}-\frac{i(c+dx)^{3/2}\sinh^2(a+bx)}{3b}\right.$$

$$\Big\downarrow \; 3777$$

$$i\left(\frac{d^2\int \frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\int \sqrt{c+dx}\cosh(a+bx)dx}{2b}\right)+\frac{id\sqrt{c+dx}\sinh^3(a+}{6b^2}\right.$$

$$\Big\downarrow \; 3042$$

$$i\left(\frac{d^2\int \frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\int \sqrt{c+dx}\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{2b}\right)+\frac{id\sqrt{c+dx}\sinh}{6b^2}\right.$$

$$\Big\downarrow \; 3777$$

$$i\left(\frac{d^2\int \frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}-\frac{id\int -\frac{i\sinh(a+bx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)+\frac{id\sqrt{c+a}}{}\right.$$

$\downarrow$ 26

$$i\left(\frac{d^2\int\frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}-\frac{d\int\frac{\sinh(a+bx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)+\frac{id\sqrt{c+dx}\,\mathrm{s}}{6}\right.$$

$\downarrow$ 3042

$$i\left(\frac{d^2\int\frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}-\frac{d\int-\frac{i\sin(ia+ibx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)+\frac{id\sqrt{c+d}}{}\right.$$

$\downarrow$ 26

$$i\left(\frac{d^2\int\frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\int\frac{\sin(ia+ibx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)+\frac{id\sqrt{c+dx}}{}\right.$$

$\downarrow$ 3789

$$i\left(\frac{d^2\int\frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\left(\frac{1}{2}i\int\frac{e^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int\frac{e^{-a-bx}}{\sqrt{c+dx}}dx\right)}{2b}\right)}{2b}\right)\right.$$

$\downarrow$ 2611

$$i\left(\frac{d^2\int\frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\left(\frac{i\int e^{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{d}-\frac{i\int e^{-a-}}{}\right.}{2b}\right.}{2b}\right.\right.$$

$\downarrow$ 2633

$$i\left(\frac{d^2\int\frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\int e^{-a-}\cdots}{}\right.}{2b}\right.}{2b}\right.\right.$$

$\downarrow$ 2634

$$i\left(\frac{d^2\int\frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{id\sqrt{c+dx}\sinh^3(a+bx)}{6b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\left(\cdots\right.}{}\right.}{}\right.\right.$$

$\downarrow$ 3793

$$i\left(\frac{d^2\int\left(\frac{3i\sinh(a+bx)}{4\sqrt{c+dx}}-\frac{i\sinh(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{12b^2}+\frac{id\sqrt{c+dx}\sinh^3(a+bx)}{6b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+}\cdots}{}\right.}{}\right.\right.$$

$\downarrow$ 2009

$$i\left(\frac{d^2\left(-\frac{3i\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{i\sqrt{\frac{\pi}{3}}e^{\frac{3bc}{d}-3a}\text{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{3i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\frac{\pi}{3}}e^{3a-\frac{3bc}{d}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}\right.}{12b^2}\right.$$

input
```
Int[(c + d*x)^(3/2)*Sinh[a + b*x]^3,x]
```

output
```
I*((d^2*((((-3*I)/8)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])
/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) + ((I/8)*E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[
(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) + (((3*I)/8)*E
^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sq
rt[d]) - ((I/8)*E^(3*a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[
c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d])))/(12*b^2) - ((I/3)*(c + d*x)^(3/2)*
Cosh[a + b*x]*Sinh[a + b*x]^2)/b + ((I/6)*d*Sqrt[c + d*x]*Sinh[a + b*x]^3)
/b^2 + (2*((I*(c + d*x)^(3/2)*Cosh[a + b*x])/b - (((3*I)/2)*d*(((I/2)*d*((
(-1/2*I)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(
Sqrt[b]*Sqrt[d]) + ((I/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c +
d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d])))/b + (Sqrt[c + d*x]*Sinh[a + b*x])/b))/
b))/3)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int (dx+c)^{\frac{3}{2}} \sinh(bx+a)^3 \, dx$$

input
```
int((d*x+c)^(3/2)*sinh(b*x+a)^3,x)
```

output
```
int((d*x+c)^(3/2)*sinh(b*x+a)^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1543 vs. $2(245) = 490$.

Time = 0.11 (sec) , antiderivative size = 1543, normalized size of antiderivative = 4.75

$$\int (c+dx)^{3/2} \sinh^3(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/288*(sqrt(3)*sqrt(pi)*(d^2*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) - d^2
*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^2*cosh(-3*(b*c - a*d)/d) - d^
2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^2*cosh(b*x + a)*cosh(-3*(
b*c - a*d)/d) - d^2*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2
+ 3*(d^2*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) - d^2*cosh(b*x + a)^2*sinh
(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqr
t(b/d)) + sqrt(3)*sqrt(pi)*(d^2*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) + d
^2*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^2*cosh(-3*(b*c - a*d)/d) +
d^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^2*cosh(b*x + a)*cosh(-3
*(b*c - a*d)/d) + d^2*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^
2 + 3*(d^2*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) + d^2*cosh(b*x + a)^2*si
nh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*x + c)*
sqrt(-b/d)) - 81*sqrt(pi)*(d^2*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) - d^2*
cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + (d^2*cosh(-(b*c - a*d)/d) - d^2*sin
h(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^2*cosh(b*x + a)*cosh(-(b*c - a*d
)/d) - d^2*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d^2*co
sh(b*x + a)^2*cosh(-(b*c - a*d)/d) - d^2*cosh(b*x + a)^2*sinh(-(b*c - a*d)
/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) - 81*sqrt(pi)*(
d^2*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) + d^2*cosh(b*x + a)^3*sinh(-(b*c
- a*d)/d) + (d^2*cosh(-(b*c - a*d)/d) + d^2*sinh(-(b*c - a*d)/d))*sinh(...
```

Sympy [F]

$$\int (c+dx)^{3/2}\sinh^3(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sinh^3{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**(3/2)*sinh(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**(3/2)*sinh(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.32

$$\int (c+dx)^{3/2}\sinh^3(a+bx)\,dx = \frac{\frac{\sqrt{3}\sqrt{\pi}d^2\,\mathrm{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3\,a-\frac{3\,bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}}-\frac{\sqrt{3}\sqrt{\pi}d^2\,\mathrm{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3\,a+\frac{3\,bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}}-\frac{81\,\sqrt{\pi}d^2\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}}\cdots}{\cdots}$$

input
```
integrate((d*x+c)^(3/2)*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/288*(sqrt(3)*sqrt(pi)*d^2*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3*a -
 3*b*c/d)/(b^2*sqrt(-b/d)) - sqrt(3)*sqrt(pi)*d^2*erf(sqrt(3)*sqrt(d*x + c
)*sqrt(b/d))*e^(-3*a + 3*b*c/d)/(b^2*sqrt(b/d)) - 81*sqrt(pi)*d^2*erf(sqrt
(d*x + c)*sqrt(-b/d))*e^(a - b*c/d)/(b^2*sqrt(-b/d)) + 81*sqrt(pi)*d^2*erf
(sqrt(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/(b^2*sqrt(b/d)) - 54*(2*(d*x + c)
^(3/2)*b*d*e^(b*c/d) + 3*sqrt(d*x + c)*d^2*e^(b*c/d))*e^(-a - (d*x + c)*b/
d)/b^2 + 6*(2*(d*x + c)^(3/2)*b*d*e^(3*b*c/d) + sqrt(d*x + c)*d^2*e^(3*b*c
/d))*e^(-3*a - 3*(d*x + c)*b/d)/b^2 + 6*(2*(d*x + c)^(3/2)*b*d*e^(3*a) - s
qrt(d*x + c)*d^2*e^(3*a))*e^(3*(d*x + c)*b/d - 3*b*c/d)/b^2 - 54*(2*(d*x +
 c)^(3/2)*b*d*e^a - 3*sqrt(d*x + c)*d^2*e^a)*e^((d*x + c)*b/d - b*c/d)/b^2
)/d
```

Giac [F]

$$\int (c+dx)^{3/2}\sinh^3(a+bx)\,dx=\int (dx+c)^{\frac{3}{2}}\sinh\left(bx+a\right)^3\,dx$$

input
```
integrate((d*x+c)^(3/2)*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^(3/2)*sinh(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\sinh^3(a+bx)\,dx=\int \sinh(a+b\,x)^3\,(c+d\,x)^{3/2}\,dx$$

input
```
int(sinh(a + b*x)^3*(c + d*x)^(3/2),x)
```

output
```
int(sinh(a + b*x)^3*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c+dx)^{3/2}\sinh^3(a+bx)\,dx=\left(\int \sqrt{dx+c}\,\sinh\left(bx+a\right)^3 x dx\right)d+\left(\int \sqrt{dx+c}\,\sinh\left(bx+a\right)^3 dx\right)c$$

input
```
int((d*x+c)^(3/2)*sinh(b*x+a)^3,x)
```

output
```
int(sqrt(c + d*x)*sinh(a + b*x)**3*x,x)*d + int(sqrt(c + d*x)*sinh(a + b*x
)**3,x)*c
```

3.55 $\int \sqrt{c+dx}\sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 275

$$\int \sqrt{c+dx}\sinh^3(a+bx)\,dx = -\frac{3\sqrt{c+dx}\cosh(a+bx)}{4b}+\frac{\sqrt{c+dx}\cosh(3a+3bx)}{12b}+\frac{3\sqrt{d}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{d}e^{-3a+\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}+\frac{3\sqrt{d}e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{d}e^{3a-\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}$$

output
```
-3/4*(d*x+c)^(1/2)*cosh(b*x+a)/b+1/12*(d*x+c)^(1/2)*cosh(3*b*x+3*a)/b+3/16
*d^(1/2)*exp(-a+b*c/d)*Pi^(1/2)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)
-1/144*d^(1/2)*exp(-3*a+3*b*c/d)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*b^(1/2)*(d*x
+c)^(1/2)/d^(1/2))/b^(3/2)+3/16*d^(1/2)*exp(a-b*c/d)*Pi^(1/2)*erfi(b^(1/2)
*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)-1/144*d^(1/2)*exp(3*a-3*b*c/d)*3^(1/2)*Pi^
(1/2)*erfi(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.76

$$\int \sqrt{c+dx}\sinh^3(a+bx)\,dx$$
$$= \frac{e^{-3\left(a+\frac{bc}{d}\right)}\sqrt{c+dx}\left(\sqrt{3}e^{6a}\sqrt{\frac{b(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{3b(c+dx)}{d}\right)-27e^{4a+\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{b(c+dx)}{d}\right)+e^{\frac{4bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\right.}{72b\sqrt{-\frac{b^2(c+dx)^2}{d^2}}}$$

input
```
Integrate[Sqrt[c + d*x]*Sinh[a + b*x]^3,x]
```

output
```
(Sqrt[c + d*x]*(Sqrt[3]*E^(6*a)*Sqrt[(b*(c + d*x))/d]*Gamma[3/2, (-3*b*(c
+ d*x))/d] - 27*E^(4*a + (2*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[3/2, -((b*
(c + d*x))/d)] + E^((4*b*c)/d)*Sqrt[-((b*(c + d*x))/d)]*(-27*E^(2*a)*Gamma
[3/2, (b*(c + d*x))/d] + Sqrt[3]*E^((2*b*c)/d)*Gamma[3/2, (3*b*(c + d*x))/
d])))/(72*b*E^(3*(a + (b*c)/d))*Sqrt[-((b^2*(c + d*x)^2)/d^2)])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 26, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sinh^3(a+bx)\,dx$$
$$\Big\downarrow \; 3042$$
$$\int i\sqrt{c+dx}\sin(ia+ibx)^3dx$$
$$\Big\downarrow \; 26$$

$$i \int \sqrt{c+dx}\sin(ia+ibx)^3dx$$

$\downarrow$ 3793

$$i \int \left(\frac{3}{4}i\sqrt{c+dx}\sinh(a+bx)-\frac{1}{4}i\sqrt{c+dx}\sinh(3a+3bx)\right)dx$$

$\downarrow$ 2009

$$i\left(-\frac{3i\sqrt{\pi}\sqrt{d}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{i\sqrt{\frac{\pi}{3}}\sqrt{d}e^{\frac{3bc}{d}-3a}\text{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{3i\sqrt{\pi}\sqrt{d}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{i\sqrt{\frac{\pi}{3}}\sqrt{de}}{}\right.$$

input
```
Int[Sqrt[c + d*x]*Sinh[a + b*x]^3,x]
```

output
```
I*((((3*I)/4)*Sqrt[c + d*x]*Cosh[a + b*x])/b - ((I/12)*Sqrt[c + d*x]*Cosh[
3*a + 3*b*x])/b - (((3*I)/16)*Sqrt[d]*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[
b]*Sqrt[c + d*x])/Sqrt[d]])/b^(3/2) + ((I/48)*Sqrt[d]*E^(-3*a + (3*b*c)/d)
*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/b^(3/2) - (((3*I
)/16)*Sqrt[d]*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d
]])/b^(3/2) + ((I/48)*Sqrt[d]*E^(3*a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]
*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/b^(3/2))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int \sqrt{dx + c}\,\sinh\left(bx + a\right)^{3} dx$$

input
```
int((d*x+c)^(1/2)*sinh(b*x+a)^3,x)
```

output
```
int((d*x+c)^(1/2)*sinh(b*x+a)^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1216 vs. $2(201) = 402$.

Time = 0.12 (sec) , antiderivative size = 1216, normalized size of antiderivative = 4.42

$$\int \sqrt{c + dx}\sinh^3(a + bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/144*(sqrt(3)*sqrt(pi)*(d*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) - d*cos
h(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d*cosh(-3*(b*c - a*d)/d) - d*sinh(-
3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d*cosh(b*x + a)*cosh(-3*(b*c - a*d)
/d) - d*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d*cosh(
b*x + a)^2*cosh(-3*(b*c - a*d)/d) - d*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/
d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)) - sqrt(3
)*sqrt(pi)*(d*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) + d*cosh(b*x + a)^3*s
inh(-3*(b*c - a*d)/d) + (d*cosh(-3*(b*c - a*d)/d) + d*sinh(-3*(b*c - a*d)/
d))*sinh(b*x + a)^3 + 3*(d*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) + d*cosh(b
*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d*cosh(b*x + a)^2*cos
h(-3*(b*c - a*d)/d) + d*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x +
 a))*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d)) - 27*sqrt(pi)*(d*cos
h(b*x + a)^3*cosh(-(b*c - a*d)/d) - d*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d)
 + (d*cosh(-(b*c - a*d)/d) - d*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(
d*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d*cosh(b*x + a)*sinh(-(b*c - a*d)/d
))*sinh(b*x + a)^2 + 3*(d*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) - d*cosh(b*
x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*
sqrt(b/d)) + 27*sqrt(pi)*(d*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) + d*cosh(
b*x + a)^3*sinh(-(b*c - a*d)/d) + (d*cosh(-(b*c - a*d)/d) + d*sinh(-(b*c -
 a*d)/d))*sinh(b*x + a)^3 + 3*(d*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + d...
```

Sympy [F]

$$\int \sqrt{c+dx}\sinh^3(a+bx)\,dx = \int \sqrt{c+dx}\sinh^3{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**(1/2)*sinh(b*x+a)**3,x)
```

output
```
Integral(sqrt(c + d*x)*sinh(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.21

$$\int \sqrt{c+dx}\sinh^3(a+bx)\,dx = -\frac{\frac{\sqrt{3}\sqrt{\pi}d\,\mathrm{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3\,a-\frac{3\,bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}}+\frac{\sqrt{3}\sqrt{\pi}d\,\mathrm{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3\,a+\frac{3\,bc}{d}\right)}}{b\sqrt{\frac{b}{d}}}-\frac{27\,\sqrt{\pi}d\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}}-\frac{27\,\sqrt{}}{}}{}$$

input
```
integrate((d*x+c)^(1/2)*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/144*(sqrt(3)*sqrt(pi)*d*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3*a -
3*b*c/d)/(b*sqrt(-b/d)) + sqrt(3)*sqrt(pi)*d*erf(sqrt(3)*sqrt(d*x + c)*sqr
t(b/d))*e^(-3*a + 3*b*c/d)/(b*sqrt(b/d)) - 27*sqrt(pi)*d*erf(sqrt(d*x + c)
*sqrt(-b/d))*e^(a - b*c/d)/(b*sqrt(-b/d)) - 27*sqrt(pi)*d*erf(sqrt(d*x + c
)*sqrt(b/d))*e^(-a + b*c/d)/(b*sqrt(b/d)) - 6*sqrt(d*x + c)*d*e^(3*a + 3*(
d*x + c)*b/d - 3*b*c/d)/b + 54*sqrt(d*x + c)*d*e^(a + (d*x + c)*b/d - b*c/
d)/b + 54*sqrt(d*x + c)*d*e^(-a - (d*x + c)*b/d + b*c/d)/b - 6*sqrt(d*x +
c)*d*e^(-3*a - 3*(d*x + c)*b/d + 3*b*c/d)/b)/d
```

Giac [F]

$$\int \sqrt{c+dx}\sinh^3(a+bx)\,dx = \int \sqrt{dx+c}\sinh\left(bx+a\right)^3\,dx$$

input
```
integrate((d*x+c)^(1/2)*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)*sinh(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\sinh^3(a+bx)\,dx=\int \sinh(a+b\,x)^3\,\sqrt{c+d\,x}\,dx$$

input
```
int(sinh(a + b*x)^3*(c + d*x)^(1/2),x)
```

output
```
int(sinh(a + b*x)^3*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\sinh^3(a+bx)\,dx=\int \sqrt{dx+c}\,\sinh{(bx+a)}^3\,dx$$

input
```
int((d*x+c)^(1/2)*sinh(b*x+a)^3,x)
```

output
```
int(sqrt(c + d*x)*sinh(a + b*x)**3,x)
```

3.56 $\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 228

$$\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}\,dx = \frac{3e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}} - \frac{e^{-3a+\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\,\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}} - \frac{3e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}} + \frac{e^{3a-\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\,\mathrm{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}$$

output
```
3/8*exp(-a+b*c/d)*Pi^(1/2)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(1/2)/d^(1
/2)-1/24*exp(-3*a+3*b*c/d)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*b^(1/2)*(d*x+c)^(1
/2)/d^(1/2))/b^(1/2)/d^(1/2)-3/8*exp(a-b*c/d)*Pi^(1/2)*erfi(b^(1/2)*(d*x+c
)^(1/2)/d^(1/2))/b^(1/2)/d^(1/2)+1/24*exp(3*a-3*b*c/d)*3^(1/2)*Pi^(1/2)*er
fi(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(1/2)/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.84

$$\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}\,dx = \frac{e^{-3\left(a+\frac{bc}{d}\right)}\left(\sqrt{3}e^{6a}\sqrt{-\frac{b(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\frac{3b(c+dx)}{d}\right)-9e^{4a+\frac{2bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)+e^{\frac{4bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\left(-9e^{2a}\right.\right.}{24b\sqrt{c+dx}}$$

input
```
Integrate[Sinh[a + b*x]^3/Sqrt[c + d*x],x]
```

output
```
(Sqrt[3]*E^(6*a)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, (-3*b*(c + d*x))/d] -
 9*E^(4*a + (2*b*c)/d)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, -((b*(c + d*x))
/d)] + E^((4*b*c)/d)*Sqrt[(b*(c + d*x))/d]*(-9*E^(2*a)*Gamma[1/2, (b*(c +
d*x))/d] + Sqrt[3]*E^((2*b*c)/d)*Gamma[1/2, (3*b*(c + d*x))/d]))/(24*b*E^(
3*(a + (b*c)/d))*Sqrt[c + d*x])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 26, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ia+ibx)^3}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{ 3793}$$

$$i\int \left(\frac{3i\sinh(a+bx)}{4\sqrt{c+dx}}-\frac{i\sinh(3a+3bx)}{4\sqrt{c+dx}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$i\left(-\frac{3i\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{i\sqrt{\frac{\pi}{3}}e^{\frac{3bc}{d}-3a}\text{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{3i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\frac{\pi}{3}}e^{3a-\frac{3bc}{d}}\text{erfi}\left(\frac{\sqrt{}}{}\right.}{8\sqrt{b}\sqrt{d}}\right.$$

input
```
Int[Sinh[a + b*x]^3/Sqrt[c + d*x],x]
```

output
```
I*((((-3*I)/8)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[
d]])/(Sqrt[b]*Sqrt[d]) + ((I/8)*E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[
3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) + (((3*I)/8)*E^(a -
(b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d])
 - ((I/8)*E^(3*a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*
x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int \frac{\sinh(bx+a)^3}{\sqrt{dx+c}}dx$$

input
```
int(sinh(b*x+a)^3/(d*x+c)^(1/2),x)
```

output
```
int(sinh(b*x+a)^3/(d*x+c)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.11

$$\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}\,dx = -\frac{\sqrt{3}\sqrt{\pi}\sqrt{\frac{b}{d}}\left(\cosh\left(-\frac{3(bc-ad)}{d}\right)-\sinh\left(-\frac{3(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)+\sqrt{3}\sqrt{\pi}\sqrt{-\frac{b}{d}}\left(\cosh\left(-\frac{3(bc-}{d}\right.\right.}{}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/24*(sqrt(3)*sqrt(pi)*sqrt(b/d)*(cosh(-3*(b*c - a*d)/d) - sinh(-3*(b*c -
 a*d)/d))*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)) + sqrt(3)*sqrt(pi)*sqrt(-b/
d)*(cosh(-3*(b*c - a*d)/d) + sinh(-3*(b*c - a*d)/d))*erf(sqrt(3)*sqrt(d*x
+ c)*sqrt(-b/d)) - 9*sqrt(pi)*sqrt(b/d)*(cosh(-(b*c - a*d)/d) - sinh(-(b*c
 - a*d)/d))*erf(sqrt(d*x + c)*sqrt(b/d)) - 9*sqrt(pi)*sqrt(-b/d)*(cosh(-(b
*c - a*d)/d) + sinh(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-b/d)))/b
```

Sympy [F]

$$\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}\,dx$$

input
```
integrate(sinh(b*x+a)**3/(d*x+c)**(1/2),x)
```

output
```
Integral(sinh(a + b*x)**3/sqrt(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.78

$$\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}\,dx = \frac{\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3\,a-\frac{3\,bc}{d}\right)}}{\sqrt{-\frac{b}{d}}} - \frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3\,a+\frac{3\,bc}{d}\right)}}{\sqrt{\frac{b}{d}}} - \frac{9\,\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{\sqrt{-\frac{b}{d}}} + \frac{9\,\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{c}\right.}{}}{24\,d}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/24*(sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3*a - 3*b*
c/d)/sqrt(-b/d) - sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d))*e^
(-3*a + 3*b*c/d)/sqrt(b/d) - 9*sqrt(pi)*erf(sqrt(d*x + c)*sqrt(-b/d))*e^(a
 - b*c/d)/sqrt(-b/d) + 9*sqrt(pi)*erf(sqrt(d*x + c)*sqrt(b/d))*e^(-a + b*c
/d)/sqrt(b/d))/d
```

Giac [F]

$$\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\sinh\left(bx+a\right)^3}{\sqrt{dx+c}}\,dx$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^3/sqrt(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\sinh(a+b\,x)^3}{\sqrt{c+d\,x}}\,dx$$

input
```
int(sinh(a + b*x)^3/(c + d*x)^(1/2),x)
```

output
```
int(sinh(a + b*x)^3/(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\sinh(bx+a)^3}{\sqrt{dx+c}}dx$$

input
```
int(sinh(b*x+a)^3/(d*x+c)^(1/2),x)
```

output
```
int(sinh(a + b*x)**3/sqrt(c + d*x),x)
```

3.57 $\int \frac{\sinh^3(a+bx)}{(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 246

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{3/2}}\,dx = -\frac{3\sqrt{b}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}} + \frac{\sqrt{b}e^{-3a+\frac{3bc}{d}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}} - \frac{3\sqrt{b}e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}} + \frac{\sqrt{b}e^{3a-\frac{3bc}{d}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}} - \frac{2\sinh^3(a+bx)}{d\sqrt{c+dx}}$$

output
```
-3/4*b^(1/2)*exp(-a+b*c/d)*Pi^(1/2)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(
3/2)+1/4*b^(1/2)*exp(-3*a+3*b*c/d)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*b^(1/2)*(d
*x+c)^(1/2)/d^(1/2))/d^(3/2)-3/4*b^(1/2)*exp(a-b*c/d)*Pi^(1/2)*erfi(b^(1/2
)*(d*x+c)^(1/2)/d^(1/2))/d^(3/2)+1/4*b^(1/2)*exp(3*a-3*b*c/d)*3^(1/2)*Pi^(
1/2)*erfi(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(3/2)-2*sinh(b*x+a)^3/d
/(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{e^{-3\left(a+b\left(\frac{c}{d}+x\right)\right)}\left(\sqrt{3}e^{6a+3bx}\sqrt{-\frac{b(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\frac{3b(c+dx)}{d}\right)-3e^{4a+\frac{2bc}{d}+3bx}\sqrt{-\frac{b(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\right.\right.}{}$$

input
```
Integrate[Sinh[a + b*x]^3/(c + d*x)^(3/2),x]
```

output
```
(Sqrt[3]*E^(6*a + 3*b*x)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, (-3*b*(c + d*
x))/d] - 3*E^(4*a + (2*b*c)/d + 3*b*x)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2,
 -((b*(c + d*x))/d)] - E^((3*b*c)/d)*((-1 + E^(2*(a + b*x)))^3 - 3*E^(2*a
+ (b*c)/d + 3*b*x)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, b*(c/d + x)] + Sqrt[3]
*E^((3*b*(c + d*x))/d)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (3*b*(c + d*x))/d]
))/(4*d*E^(3*(a + b*(c/d + x)))*Sqrt[c + d*x])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 26, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ia+ibx)^3}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ia+ibx)^3}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{3794}$$

$$i\left(\frac{6ib\int\left(\frac{\cosh(a+bx)}{4\sqrt{c+dx}}-\frac{\cosh(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{d}+\frac{2i\sinh^3(a+bx)}{d\sqrt{c+dx}}\right)$$

$$\Big\downarrow \text{2009}$$

$$i\left(\frac{6ib\left(\frac{\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{3}}e^{\frac{3bc}{d}-3a}\text{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{3}}e^{3a-\frac{3bc}{d}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}\right)}{d}+\right.$$

input
```
Int[Sinh[a + b*x]^3/(c + d*x)^(3/2),x]
```

output
```
I*(((6*I)*b*((E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d
]])/(8*Sqrt[b]*Sqrt[d]) - (E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sq
rt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d]) + (E^(a - (b*c)/d)*Sqrt
[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d]) - (E^(3*a
- (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*
Sqrt[b]*Sqrt[d])))/d + ((2*I)*Sinh[a + b*x]^3)/(d*Sqrt[c + d*x]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

Maple [F]

$$\int \frac{\sinh\left(bx+a\right)^{3}}{\left(dx+c\right)^{\frac{3}{2}}}dx$$

input
```
int(sinh(b*x+a)^3/(d*x+c)^(3/2),x)
```

output
```
int(sinh(b*x+a)^3/(d*x+c)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1346 vs. $2(182) = 364$.

Time $= 0.12$ (sec) , antiderivative size $= 1346$, normalized size of antiderivative $= 5.47$

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(sqrt(3)*sqrt(pi)*((d*x + c)*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) -
(d*x + c)*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + ((d*x + c)*cosh(-3*(b*c
 - a*d)/d) - (d*x + c)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((d*x +
 c)*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) - (d*x + c)*cosh(b*x + a)*sinh(-3
*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*((d*x + c)*cosh(b*x + a)^2*cosh(-3*(b
*c - a*d)/d) - (d*x + c)*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x
+ a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)) - sqrt(3)*sqrt(pi)*((
d*x + c)*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) + (d*x + c)*cosh(b*x + a)^
3*sinh(-3*(b*c - a*d)/d) + ((d*x + c)*cosh(-3*(b*c - a*d)/d) + (d*x + c)*s
inh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((d*x + c)*cosh(b*x + a)*cosh(-
3*(b*c - a*d)/d) + (d*x + c)*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*
x + a)^2 + 3*((d*x + c)*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) + (d*x + c)
*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqr
t(3)*sqrt(d*x + c)*sqrt(-b/d)) - 3*sqrt(pi)*((d*x + c)*cosh(b*x + a)^3*cos
h(-(b*c - a*d)/d) - (d*x + c)*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + ((d*x
 + c)*cosh(-(b*c - a*d)/d) - (d*x + c)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)
^3 + 3*((d*x + c)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - (d*x + c)*cosh(b*x
+ a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*((d*x + c)*cosh(b*x + a)^2*
cosh(-(b*c - a*d)/d) - (d*x + c)*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sin
h(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 3*sqrt(pi)*((d*x +...
```

Sympy [F]

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sinh^3{(a+bx)}}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)**3/(d*x+c)**(3/2),x)
```

output
```
Integral(sinh(a + b*x)**3/(c + d*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.80

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{\frac{\sqrt{3}\sqrt{\frac{(dx+c)b}{d}}e^{\left(\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},\frac{3(dx+c)b}{d}\right)}{\sqrt{dx+c}} - \frac{\sqrt{3}\sqrt{-\frac{(dx+c)b}{d}}e^{\left(-\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},-\frac{3(dx+c)b}{d}\right)}{\sqrt{dx+c}} - \frac{3\sqrt{\frac{(dx+c)b}{d}}\cdots}{\cdots}}{8\,d}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
1/8*(sqrt(3)*sqrt((d*x + c)*b/d)*e^(3*(b*c - a*d)/d)*gamma(-1/2, 3*(d*x +
c)*b/d)/sqrt(d*x + c) - sqrt(3)*sqrt(-(d*x + c)*b/d)*e^(-3*(b*c - a*d)/d)*
gamma(-1/2, -3*(d*x + c)*b/d)/sqrt(d*x + c) - 3*sqrt((d*x + c)*b/d)*e^(-a
+ b*c/d)*gamma(-1/2, (d*x + c)*b/d)/sqrt(d*x + c) + 3*sqrt(-(d*x + c)*b/d)
*e^(a - b*c/d)*gamma(-1/2, -(d*x + c)*b/d)/sqrt(d*x + c))/d
```

Giac [F]

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sinh(bx+a)^3}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^3/(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sinh(a+b\,x)^3}{(c+d\,x)^{3/2}}\,dx$$

input
```
int(sinh(a + b*x)^3/(c + d*x)^(3/2),x)
```

output
```
int(sinh(a + b*x)^3/(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sinh(bx+a)^3}{\sqrt{dx+c}\,c+\sqrt{dx+c}\,dx}dx$$

input
```
int(sinh(b*x+a)^3/(d*x+c)^(3/2),x)
```

output
```
int(sinh(a + b*x)**3/(sqrt(c + d*x)*c + sqrt(c + d*x)*d*x),x)
```

3.58 $\int \frac{\sinh^3(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 277

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{5/2}}\,dx = \frac{b^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} - \frac{b^{3/2}e^{-3a+\frac{3bc}{d}}\sqrt{3\pi}\,\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} - \frac{b^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} + \frac{b^{3/2}e^{3a-\frac{3bc}{d}}\sqrt{3\pi}\,\mathrm{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} - \frac{4b\cosh(a+bx)\sinh^2(a+bx)}{d^2\sqrt{c+dx}} - \frac{2\sinh^3(a+bx)}{3d(c+dx)^{3/2}}$$

output
```
1/2*b^(3/2)*exp(-a+b*c/d)*Pi^(1/2)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(5
/2)-1/2*b^(3/2)*exp(-3*a+3*b*c/d)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*b^(1/2)*(d*
x+c)^(1/2)/d^(1/2))/d^(5/2)-1/2*b^(3/2)*exp(a-b*c/d)*Pi^(1/2)*erfi(b^(1/2)
*(d*x+c)^(1/2)/d^(1/2))/d^(5/2)+1/2*b^(3/2)*exp(3*a-3*b*c/d)*3^(1/2)*Pi^(1
/2)*erfi(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(5/2)-4*b*cosh(b*x+a)*si
nh(b*x+a)^2/d^2/(d*x+c)^(1/2)-2/3*sinh(b*x+a)^3/d/(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 1.86 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.91

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{5/2}}\,dx = \frac{e^{-3\left(a+\frac{bc}{d}\right)}\left(-3\sqrt{3}de^{6a}\left(-\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{3b(c+dx)}{d}\right)+3de^{4a+\frac{2bc}{d}}\left(-\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},\right.\right.}{}$$

input
```
Integrate[Sinh[a + b*x]^3/(c + d*x)^(5/2),x]
```

output
```
(-3*Sqrt[3]*d*E^(6*a)*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, (-3*b*(c + d*x
))/d] + 3*d*E^(4*a + (2*b*c)/d)*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, -((b
*(c + d*x))/d)] - 3*d*E^(2*a + (4*b*c)/d)*((b*(c + d*x))/d)^(3/2)*Gamma[1/
2, (b*(c + d*x))/d] + 3*Sqrt[3]*d*E^((6*b*c)/d)*((b*(c + d*x))/d)^(3/2)*Ga
mma[1/2, (3*b*(c + d*x))/d] - 4*E^(3*(a + (b*c)/d))*Sinh[a + b*x]^2*(6*b*(
c + d*x)*Cosh[a + b*x] + d*Sinh[a + b*x]))/(6*d^2*E^(3*(a + (b*c)/d))*(c +
 d*x)^(3/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.26 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.52, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 26, 3795, 26, 3042, 26, 3789, 2611, 2633, 2634, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{i\sin(ia+ibx)^3}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{\sin(ia+ibx)^3}{(c+dx)^{5/2}}dx$$

$\Big\downarrow$ 3795

$$i\left(\frac{12b^2\int -\frac{i\sinh^3(a+bx)}{\sqrt{c+dx}}dx}{d^2}-\frac{8b^2\int \frac{i\sinh(a+bx)}{\sqrt{c+dx}}dx}{d^2}+\frac{4ib\sinh^2(a+bx)\cosh(a+bx)}{d^2\sqrt{c+dx}}+\frac{2i\sinh^3(a+bx)}{3d(c+dx)^{3/2}}\right)$$

$\Big\downarrow$ 26

$$i\left(-\frac{12ib^2\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}dx}{d^2}-\frac{8ib^2\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}dx}{d^2}+\frac{4ib\sinh^2(a+bx)\cosh(a+bx)}{d^2\sqrt{c+dx}}+\frac{2i\sinh^3(a+bx)}{3d(c+dx)^{3/2}}\right)$$

$\Big\downarrow$ 3042

$$i\left(-\frac{8ib^2\int -\frac{i\sin(ia+ibx)}{\sqrt{c+dx}}dx}{d^2}-\frac{12ib^2\int \frac{i\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{d^2}+\frac{4ib\sinh^2(a+bx)\cosh(a+bx)}{d^2\sqrt{c+dx}}+\frac{2i\sinh^3(a+bx)}{3d(c+dx)^{3/2}}\right)$$

$\Big\downarrow$ 26

$$i\left(-\frac{8b^2\int \frac{\sin(ia+ibx)}{\sqrt{c+dx}}dx}{d^2}+\frac{12b^2\int \frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{d^2}+\frac{4ib\sinh^2(a+bx)\cosh(a+bx)}{d^2\sqrt{c+dx}}+\frac{2i\sinh^3(a+bx)}{3d(c+dx)^{3/2}}\right)$$

$\Big\downarrow$ 3789

$$i\left(-\frac{8b^2\left(\frac{1}{2}i\int \frac{e^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int \frac{e^{-a-bx}}{\sqrt{c+dx}}dx\right)}{d^2}+\frac{12b^2\int \frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{d^2}+\frac{4ib\sinh^2(a+bx)\cosh(a+bx)}{d^2\sqrt{c+dx}}+\frac{2i\sinh^3(a+}{3d(c+dx}\right.$$

$\Big\downarrow$ 2611

$$i\left(-\frac{8b^2\left(\frac{i\int e^{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{d}-\frac{i\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{d^2}+\frac{12b^2\int \frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{d^2}+\frac{4ib\sinh^2(a+bx)\cosh(a+}{d^2\sqrt{c+dx}}\right.$$

$\Big\downarrow$ 2633

$$i\left(-\frac{8b^2\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{d^2}+\frac{12b^2\int\frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{d^2}+\frac{4ib\sinh^2(a+bx)\cosh(a+}{d^2\sqrt{c+dx}}\right.$$

$$\Big\downarrow \text{2634}$$

$$i\left(\frac{12b^2\int\frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{d^2}-\frac{8b^2\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d^2}+\frac{4ib\sinh^2(a+bx)\cosh(a+bx)}{d^2\sqrt{c+dx}}\right.$$

$$\Big\downarrow \text{3793}$$

$$i\left(\frac{12b^2\int\left(\frac{3i\sinh(a+bx)}{4\sqrt{c+dx}}-\frac{i\sinh(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{d^2}-\frac{8b^2\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d^2}+\frac{4ib\sinh^2(a}{d}\right.$$

$$\Big\downarrow \text{2009}$$

$$i\left(-\frac{8b^2\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d^2}+\frac{12b^2\left(-\frac{3i\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{i\sqrt{\frac{\pi}{3}}e^{\frac{3bc}{d}-3a}\text{erf}\left(\frac{\sqrt{3}\sqrt{}}{}\right.}{8\sqrt{b}\sqrt{d}}\right.}{}\right.$$

input
```
Int[Sinh[a + b*x]^3/(c + d*x)^(5/2),x]
```

output
```
I*((-8*b^2*(((-1/2*I)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x]
)/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) + ((I/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt
[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d])))/d^2 + (12*b^2*((((-3*I)/8
)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]
*Sqrt[d]) + ((I/8)*E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[b]*Sq
rt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) + (((3*I)/8)*E^(a - (b*c)/d)*Sqrt
[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) - ((I/8)*E^(
3*a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])
/(Sqrt[b]*Sqrt[d])))/d^2 + ((4*I)*b*Cosh[a + b*x]*Sinh[a + b*x]^2)/(d^2*Sq
rt[c + d*x]) + (((2*I)/3)*Sinh[a + b*x]^3)/(d*(c + d*x)^(3/2)))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Maple [F]

$$\int \frac{\sinh{(bx+a)}^3}{(dx+c)^{\frac{5}{2}}}dx$$

input
```
int(sinh(b*x+a)^3/(d*x+c)^(5/2),x)
```

output
```
int(sinh(b*x+a)^3/(d*x+c)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2059 vs. $2(209) = 418$.

Time = 0.13 (sec) , antiderivative size = 2059, normalized size of antiderivative = 7.43

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(6*sqrt(3)*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3
*cosh(-3*(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3*
sinh(-3*(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-3*(b*c - a
*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-3*(b*c - a*d)/d))*sinh(b*x
+ a)^3 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-3*(b*c - a
*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-3*(b*c - a*d)
/d))*sinh(b*x + a)^2 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*
cosh(-3*(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*s
inh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*
sqrt(b/d)) + 6*sqrt(3)*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x
+ a)^3*cosh(-3*(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x +
 a)^3*sinh(-3*(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-3*(b
*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-3*(b*c - a*d)/d))*sin
h(b*x + a)^3 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-3*(b
*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-3*(b*c
- a*d)/d))*sinh(b*x + a)^2 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x +
 a)^2*cosh(-3*(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x +
a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*
x + c)*sqrt(-b/d)) - 6*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x
+ a)^3*cosh(-(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x ...
```

Sympy [F]

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sinh^3{(a+bx)}}{(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)**3/(d*x+c)**(5/2),x)
```

output
```
Integral(sinh(a + b*x)**3/(c + d*x)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.71

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{5/2}}\,dx = \frac{3\left(\frac{\sqrt{3}\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2},\frac{3(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}-\frac{\sqrt{3}\left(-\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(-\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2},-\frac{3(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}-\left(\right.\right.}{8\,d}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
3/8*(sqrt(3)*((d*x + c)*b/d)^(3/2)*e^(3*(b*c - a*d)/d)*gamma(-3/2, 3*(d*x
+ c)*b/d)/(d*x + c)^(3/2) - sqrt(3)*(-(d*x + c)*b/d)^(3/2)*e^(-3*(b*c - a*
d)/d)*gamma(-3/2, -3*(d*x + c)*b/d)/(d*x + c)^(3/2) - ((d*x + c)*b/d)^(3/2
)*e^(-a + b*c/d)*gamma(-3/2, (d*x + c)*b/d)/(d*x + c)^(3/2) + (-(d*x + c)*
b/d)^(3/2)*e^(a - b*c/d)*gamma(-3/2, -(d*x + c)*b/d)/(d*x + c)^(3/2))/d
```

Giac [F]

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sinh\left(bx+a\right)^3}{(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^3/(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sinh(a+b\,x)^3}{(c+d\,x)^{5/2}}\,dx$$

input
```
int(sinh(a + b*x)^3/(c + d*x)^(5/2),x)
```

output
```
int(sinh(a + b*x)^3/(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{5/2}}\,dx=\int \frac{\sinh{(bx+a)}^3}{\sqrt{dx+c}\,c^2+2\sqrt{dx+c}\,cdx+\sqrt{dx+c}\,d^2x^2}dx$$

input
```
int(sinh(b*x+a)^3/(d*x+c)^(5/2),x)
```

output
```
int(sinh(a + b*x)**3/(sqrt(c + d*x)*c**2 + 2*sqrt(c + d*x)*c*d*x + sqrt(c
+ d*x)*d**2*x**2),x)
```

3.59 $\int \frac{\sinh^3(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 331

$$\begin{aligned}\int \frac{\sinh^3(a+bx)}{(c+dx)^{7/2}}\,dx =& -\frac{b^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} \\ &+ \frac{3b^{5/2}e^{-3a+\frac{3bc}{d}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{b^{5/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} \\ &+ \frac{3b^{5/2}e^{3a-\frac{3bc}{d}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{16b^2\sinh(a+bx)}{5d^3\sqrt{c+dx}} \\ &- \frac{4b\cosh(a+bx)\sinh^2(a+bx)}{5d^2(c+dx)^{3/2}} - \frac{2\sinh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{24b^2\sinh^3(a+bx)}{5d^3\sqrt{c+dx}}\end{aligned}$$

output
```
-1/5*b^(5/2)*exp(-a+b*c/d)*Pi^(1/2)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(
7/2)+3/5*b^(5/2)*exp(-3*a+3*b*c/d)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*b^(1/2)*(d
*x+c)^(1/2)/d^(1/2))/d^(7/2)-1/5*b^(5/2)*exp(a-b*c/d)*Pi^(1/2)*erfi(b^(1/2
)*(d*x+c)^(1/2)/d^(1/2))/d^(7/2)+3/5*b^(5/2)*exp(3*a-3*b*c/d)*3^(1/2)*Pi^(
1/2)*erfi(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(7/2)-16/5*b^2*sinh(b*x
+a)/d^3/(d*x+c)^(1/2)-4/5*b*cosh(b*x+a)*sinh(b*x+a)^2/d^2/(d*x+c)^(3/2)-2/
5*sinh(b*x+a)^3/d/(d*x+c)^(5/2)-24/5*b^2*sinh(b*x+a)^3/d^3/(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.14

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{e^{-3a}\left(2e^{6a}\left(-d^2e^{3bx}-2be^{-\frac{3bc}{d}}(c+dx)\left(e^{\frac{3b(c+dx)}{d}}(d+6b(c+dx))+6\sqrt{3}d\left(-\frac{b(c+dx)}{d}\right.\right.\right.\right.}{}$$

input
```
Integrate[Sinh[a + b*x]^3/(c + d*x)^(7/2),x]
```

output
```
(2*E^(6*a)*(-(d^2*E^(3*b*x)) - (2*b*(c + d*x)*(E^((3*b*(c + d*x))/d)*(d +
6*b*(c + d*x)) + 6*Sqrt[3]*d*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, (-3*b*(
c + d*x))/d]))/E^((3*b*c)/d)) + 2*E^(4*a)*(3*d^2*E^(b*x) + (2*b*(c + d*x)*
(E^(b*(c/d + x))*(d + 2*b*(c + d*x)) + 2*d*(-((b*(c + d*x))/d))^(3/2)*Gamm
a[1/2, -((b*(c + d*x))/d)]))/E^((b*c)/d)) + E^(2*a - b*x)*(-6*d^2 + 4*b*d*
(c + d*x) - 8*b^2*(c + d*x)^2 + 8*d^2*E^(b*(c/d + x))*((b*(c + d*x))/d)^(5
/2)*Gamma[1/2, (b*(c + d*x))/d]) - (2*(-d^2 + 2*b*(c + d*x)*(d - 6*b*(c +
d*x) + 6*Sqrt[3]*d*E^((3*b*(c + d*x))/d)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2
, (3*b*(c + d*x))/d])))/E^(3*b*x))/(40*d^3*E^(3*a)*(c + d*x)^(5/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.53 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.45, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3042, 26, 3795, 26, 3042, 26, 3778, 3042, 3788, 26, 2611, 2633, 2634, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{7/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ia+ibx)^3}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \; 26$$

$$i \int \frac{\sin(ia+ibx)^3}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \; 3795$$

$$i\left(\frac{12b^2\int -\frac{i\sinh^3(a+bx)}{(c+dx)^{3/2}}dx}{5d^2} - \frac{8b^2\int \frac{i\sinh(a+bx)}{(c+dx)^{3/2}}dx}{5d^2} + \frac{4ib\sinh^2(a+bx)\cosh(a+bx)}{5d^2(c+dx)^{3/2}} + \frac{2i\sinh^3(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$$\Big\downarrow \; 26$$

$$i\left(-\frac{12ib^2\int \frac{\sinh^3(a+bx)}{(c+dx)^{3/2}}dx}{5d^2} - \frac{8ib^2\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}dx}{5d^2} + \frac{4ib\sinh^2(a+bx)\cosh(a+bx)}{5d^2(c+dx)^{3/2}} + \frac{2i\sinh^3(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$i\left(-\frac{8ib^2\int -\frac{i\sin(ia+ibx)}{(c+dx)^{3/2}}dx}{5d^2} - \frac{12ib^2\int \frac{i\sin(ia+ibx)^3}{(c+dx)^{3/2}}dx}{5d^2} + \frac{4ib\sinh^2(a+bx)\cosh(a+bx)}{5d^2(c+dx)^{3/2}} + \frac{2i\sinh^3(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$$\Big\downarrow \; 26$$

$$i\left(-\frac{8b^2\int \frac{\sin(ia+ibx)}{(c+dx)^{3/2}}dx}{5d^2} + \frac{12b^2\int \frac{\sin(ia+ibx)^3}{(c+dx)^{3/2}}dx}{5d^2} + \frac{4ib\sinh^2(a+bx)\cosh(a+bx)}{5d^2(c+dx)^{3/2}} + \frac{2i\sinh^3(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$$\Big\downarrow \; 3778$$

$$i\left(\frac{12b^2\int \frac{\sin(ia+ibx)^3}{(c+dx)^{3/2}}dx}{5d^2} - \frac{8b^2\left(\frac{2ib\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}dx}{d} - \frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2} + \frac{4ib\sinh^2(a+bx)\cosh(a+bx)}{5d^2(c+dx)^{3/2}} + \frac{2i\sinh^3(a+}{5d(c+dx)^{5}}\right.$$

$$\Big\downarrow \; 3042$$

$$i\left(\frac{12b^2\int \frac{\sin(ia+ibx)^3}{(c+dx)^{3/2}}dx}{5d^2} - \frac{8b^2\left(\frac{2ib\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{d} - \frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2} + \frac{4ib\sinh^2(a+bx)\cosh(a+bx)}{5d^2(c+dx)^{3/2}} + \frac{2i\sinh^3(}{5d(c+}\right.$$

$\Big\downarrow$ 3788

$$i\left(\frac{12b^2\int \frac{\sin(ia+ibx)^3}{(c+dx)^{3/2}}dx}{5d^2}-\frac{8b^2\left(\frac{2ib\left(\frac{1}{2}i\int -\frac{ie^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int \frac{ie^{-a-bx}}{\sqrt{c+dx}}dx\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}+\frac{4ib\sinh^2(a+bx)\cosh(a+bx)}{5d^2(c+dx)^{3/2}}\right.$$

$\Big\downarrow$ 26

$$i\left(\frac{12b^2\int \frac{\sin(ia+ibx)^3}{(c+dx)^{3/2}}dx}{5d^2}-\frac{8b^2\left(\frac{2ib\left(\frac{1}{2}\int \frac{e^{-a-bx}}{\sqrt{c+dx}}dx+\frac{1}{2}\int \frac{e^{a+bx}}{\sqrt{c+dx}}dx\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}+\frac{4ib\sinh^2(a+bx)\cosh(a+bx)}{5d^2(c+dx)^{3/2}}+\right.$$

$\Big\downarrow$ 2611

$$i\left(\frac{12b^2\int \frac{\sin(ia+ibx)^3}{(c+dx)^{3/2}}dx}{5d^2}-\frac{8b^2\left(\frac{2ib\left(\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\int e^{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}+\frac{4ib\sinh^2(a+}{5d^2(}\right.$$

$\Big\downarrow$ 2633

$$i\left(-\frac{8b^2\left(\frac{2ib\left(\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}+\frac{12b^2\int \frac{\sin(ia+ibx)^3}{(c+dx)^{3/2}}dx}{5d^2}+\frac{4ib\sinh^2(}{5d}\right.$$

$\Big\downarrow$ 2634

$$i\left(\frac{12b^2\int\frac{\sin(ia+ibx)^3}{(c+dx)^{3/2}}dx}{5d^2}-\frac{8b^2\left(\frac{2ib\left(\frac{\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}+\frac{4ib\sinh^2(a+}{5d^2(c}\right.$$

$\downarrow$ 3794

$$i\left(\frac{12b^2\left(\frac{6ib\int\left(\frac{\cosh(a+bx)}{4\sqrt{c+dx}}-\frac{\cosh(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{d}+\frac{2i\sinh^3(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}-\frac{8b^2\left(\frac{2ib\left(\frac{\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d}\right.}{5d^2}\right.$$

$\downarrow$ 2009

$$i\left(\frac{12b^2\left(\frac{6ib\left(\frac{\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{3}}e^{\frac{3bc}{d}-3a}\text{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{3}}e^{3a-\frac{3bc}{d}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}\right)}{d}+\right.}{5d^2}\right.$$

input
```
Int[Sinh[a + b*x]^3/(c + d*x)^(7/2),x]
```

output
```
I*((((4*I)/5)*b*Cosh[a + b*x]*Sinh[a + b*x]^2)/(d^2*(c + d*x)^(3/2)) + (((
2*I)/5)*Sinh[a + b*x]^3)/(d*(c + d*x)^(5/2)) - (8*b^2*(((2*I)*b*((E^(-a +
(b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]
) + (E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sq
rt[b]*Sqrt[d])))/d - ((2*I)*Sinh[a + b*x])/(d*Sqrt[c + d*x])))/(5*d^2) + (
12*b^2*(((6*I)*b*((E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/S
qrt[d]])/(8*Sqrt[b]*Sqrt[d]) - (E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[
3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d]) + (E^(a - (b*c)/d)
*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d]) - (E^
(3*a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]
)/(8*Sqrt[b]*Sqrt[d])))/d + ((2*I)*Sinh[a + b*x]^3)/(d*Sqrt[c + d*x])))/(5
*d^2))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Maple [F]

$$\int \frac{\sinh (bx+a)^3}{(dx+c)^{\frac{7}{2}}}dx$$

input
```
int(sinh(b*x+a)^3/(d*x+c)^(7/2),x)
```

output
```
int(sinh(b*x+a)^3/(d*x+c)^(7/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3286 vs. 2(253) = 506.

Time = 0.18 (sec) , antiderivative size = 3286, normalized size of antiderivative = 9.93

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sinh^3{(a+bx)}}{(c+dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)**3/(d*x+c)**(7/2),x)
```

output
```
Integral(sinh(a + b*x)**3/(c + d*x)**(7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.60

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{3\left(\frac{3\sqrt{3}\left(\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2},\frac{3(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}} - \frac{3\sqrt{3}\left(-\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(-\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2},-\frac{3(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}} - \cdots}{8d}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
3/8*(3*sqrt(3)*((d*x + c)*b/d)^(5/2)*e^(3*(b*c - a*d)/d)*gamma(-5/2, 3*(d*
x + c)*b/d)/(d*x + c)^(5/2) - 3*sqrt(3)*(-(d*x + c)*b/d)^(5/2)*e^(-3*(b*c
- a*d)/d)*gamma(-5/2, -3*(d*x + c)*b/d)/(d*x + c)^(5/2) - ((d*x + c)*b/d)^
(5/2)*e^(-a + b*c/d)*gamma(-5/2, (d*x + c)*b/d)/(d*x + c)^(5/2) + (-(d*x +
 c)*b/d)^(5/2)*e^(a - b*c/d)*gamma(-5/2, -(d*x + c)*b/d)/(d*x + c)^(5/2))/
d
```

Giac [F]

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sinh\left(bx+a\right)^3}{(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^3/(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sinh(a+b\,x)^3}{(c+d\,x)^{7/2}}\,dx$$

input
```
int(sinh(a + b*x)^3/(c + d*x)^(7/2),x)
```

output
```
int(sinh(a + b*x)^3/(c + d*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sinh(bx+a)^3}{\sqrt{dx+c}\,c^3+3\sqrt{dx+c}\,c^2dx+3\sqrt{dx+c}\,c\,d^2x^2+\sqrt{dx+c}\,d^3x^3}dx$$

input
```
int(sinh(b*x+a)^3/(d*x+c)^(7/2),x)
```

output
```
int(sinh(a + b*x)**3/(sqrt(c + d*x)*c**3 + 3*sqrt(c + d*x)*c**2*d*x + 3*sq
rt(c + d*x)*c*d**2*x**2 + sqrt(c + d*x)*d**3*x**3),x)
```

3.60 $\int (dx)^{3/2} \sinh(fx)\, dx$

Optimal result

Integrand size = 12, antiderivative size = 111

$$\int (dx)^{3/2} \sinh(fx)\, dx = \frac{(dx)^{3/2}\cosh(fx)}{f} - \frac{3d^{3/2}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}} + \frac{3d^{3/2}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}} - \frac{3d\sqrt{dx}\sinh(fx)}{2f^2}$$

output
```
(d*x)^(3/2)*cosh(f*x)/f-3/8*d^(3/2)*Pi^(1/2)*erf(f^(1/2)*(d*x)^(1/2)/d^(1/
2))/f^(5/2)+3/8*d^(3/2)*Pi^(1/2)*erfi(f^(1/2)*(d*x)^(1/2)/d^(1/2))/f^(5/2)
-3/2*d*(d*x)^(1/2)*sinh(f*x)/f^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.45

$$\int (dx)^{3/2} \sinh(fx)\, dx = \frac{d^2\left(\sqrt{-fx}\Gamma\left(\frac{5}{2}, -fx\right) + \sqrt{fx}\Gamma\left(\frac{5}{2}, fx\right)\right)}{2f^3\sqrt{dx}}$$

input
```
Integrate[(d*x)^(3/2)*Sinh[f*x],x]
```

output
```
(d^2*(Sqrt[-(f*x)]*Gamma[5/2, -(f*x)] + Sqrt[f*x]*Gamma[5/2, f*x]))/(2*f^3
*Sqrt[d*x])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.26, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2}\sinh(fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i(dx)^{3/2}\sin(ifx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int (dx)^{3/2}\sin(ifx)dx$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i(dx)^{3/2}\cosh(fx)}{f}-\frac{3id\int \sqrt{dx}\cosh(fx)dx}{2f}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i(dx)^{3/2}\cosh(fx)}{f}-\frac{3id\int \sqrt{dx}\sin\left(ifx+\frac{\pi}{2}\right)dx}{2f}\right)$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i(dx)^{3/2}\cosh(fx)}{f}-\frac{3id\left(\frac{\sqrt{dx}\sinh(fx)}{f}-\frac{id\int -\frac{i\sinh(fx)}{\sqrt{dx}}dx}{2f}\right)}{2f}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i(dx)^{3/2}\cosh(fx)}{f}-\frac{3id\left(\frac{\sqrt{dx}\sinh(fx)}{f}-\frac{d\int\frac{\sinh(fx)}{\sqrt{dx}}dx}{2f}\right)}{2f}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-i\left(\frac{i(dx)^{3/2}\cosh(fx)}{f}-\frac{3id\left(\frac{\sqrt{dx}\sinh(fx)}{f}-\frac{d\int-\frac{i\sin(ifx)}{\sqrt{dx}}dx}{2f}\right)}{2f}\right)$$

$$\Big\downarrow \text{ 26}$$

$$-i\left(\frac{i(dx)^{3/2}\cosh(fx)}{f}-\frac{3id\left(\frac{\sqrt{dx}\sinh(fx)}{f}+\frac{id\int\frac{\sin(ifx)}{\sqrt{dx}}dx}{2f}\right)}{2f}\right)$$

$$\Big\downarrow \text{ 3789}$$

$$-i\left(\frac{i(dx)^{3/2}\cosh(fx)}{f}-\frac{3id\left(\frac{\sqrt{dx}\sinh(fx)}{f}+\frac{id\left(\frac{1}{2}i\int\frac{e^{fx}}{\sqrt{dx}}dx-\frac{1}{2}i\int\frac{e^{-fx}}{\sqrt{dx}}dx\right)}{2f}\right)}{2f}\right)$$

$$\Big\downarrow \text{ 2611}$$

$$-i\left(\frac{i(dx)^{3/2}\cosh(fx)}{f}-\frac{3id\left(\frac{\sqrt{dx}\sinh(fx)}{f}+\frac{id\left(\frac{i\int e^{fx}d\sqrt{dx}}{d}-\frac{i\int e^{-fx}d\sqrt{dx}}{d}\right)}{2f}\right)}{2f}\right)$$

$$\Big\downarrow \text{ 2633}$$

$$-i\left(\frac{i(dx)^{3/2}\cosh(fx)}{f}-\frac{3id\left(\frac{\sqrt{dx}\sinh(fx)}{f}+\frac{id\left(\frac{i\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}-\frac{i\int e^{-fx}d\sqrt{dx}}{d}\right)}{2f}\right)}{2f}\right)$$

$$\Big\downarrow \text{ 2634}$$

$$-i\left(\frac{i(dx)^{3/2}\cosh(fx)}{f}-\frac{3id\left(\frac{\sqrt{dx}\sinh(fx)}{f}+\frac{id\left(\frac{i\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}-\frac{i\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}\right)}{2f}\right)}{2f}\right)$$

input
```
Int[(d*x)^(3/2)*Sinh[f*x],x]
```

output
```
(-I)*((I*(d*x)^(3/2)*Cosh[f*x])/f - (((3*I)/2)*d*(((I/2)*d*(((-1/2*I)*Sqrt
[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[d]*Sqrt[f]) + ((I/2)*Sqrt[Pi]
*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[d]*Sqrt[f])))/f + (Sqrt[d*x]*Sin
h[f*x])/f))/f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.19

method	result	si
meijerg	$-\frac{2 (dx)^{\frac{3}{2}} \sqrt{2}\,\sqrt{\pi}\left(-\frac{\sqrt{x}\,\sqrt{2}\,(if)^{\frac{7}{2}}(-14fx+21){\mathrm e}^{fx}}{112\sqrt{\pi}\,f^{3}}+\frac{\sqrt{x}\,\sqrt{2}\,(if)^{\frac{7}{2}}(14fx+21){\mathrm e}^{-fx}}{112\sqrt{\pi}\,f^{3}}-\frac{3(if)^{\frac{7}{2}}\sqrt{2}\,\mathrm{erf}(\sqrt{x}\,\sqrt{f})}{32f^{\frac{7}{2}}}+\frac{3(if)^{\frac{7}{2}}\sqrt{2}\,\mathrm{erfi}(\sqrt{x}\,\sqrt{f})}{32f^{\frac{7}{2}}}\right)}{x^{\frac{3}{2}}(if)^{\frac{3}{2}}f}$	13

input
```
int((d*x)^(3/2)*sinh(f*x),x,method=_RETURNVERBOSE)
```

output
```
-2*(d*x)^(3/2)/x^(3/2)*2^(1/2)/(I*f)^(3/2)*Pi^(1/2)/f*(-1/112/Pi^(1/2)*x^(
1/2)*2^(1/2)*(I*f)^(7/2)*(-14*f*x+21)/f^3*exp(f*x)+1/112/Pi^(1/2)*x^(1/2)*
2^(1/2)*(I*f)^(7/2)*(14*f*x+21)/f^3*exp(-f*x)-3/32*(I*f)^(7/2)*2^(1/2)/f^(
7/2)*erf(x^(1/2)*f^(1/2))+3/32*(I*f)^(7/2)*2^(1/2)/f^(7/2)*erfi(x^(1/2)*f^
(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(77) = 154$.

Time = 0.09 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.70

$$\int (dx)^{3/2}\sinh(fx)\,dx =$$

$$-\frac{3\sqrt{\pi}(d^2\cosh(fx)+d^2\sinh(fx))\sqrt{\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)+3\sqrt{\pi}(d^2\cosh(fx)+d^2\sinh(fx))\sqrt{-\frac{f}{d}}\operatorname{erf}\left(\sqrt{d\ldots}\right.}{8(\ldots}$$

input
```
integrate((d*x)^(3/2)*sinh(f*x),x, algorithm="fricas")
```

output
```
-1/8*(3*sqrt(pi)*(d^2*cosh(f*x) + d^2*sinh(f*x))*sqrt(f/d)*erf(sqrt(d*x)*s
qrt(f/d)) + 3*sqrt(pi)*(d^2*cosh(f*x) + d^2*sinh(f*x))*sqrt(-f/d)*erf(sqrt
(d*x)*sqrt(-f/d)) - 2*(2*d*f^2*x + (2*d*f^2*x - 3*d*f)*cosh(f*x)^2 + 2*(2*
d*f^2*x - 3*d*f)*cosh(f*x)*sinh(f*x) + (2*d*f^2*x - 3*d*f)*sinh(f*x)^2 + 3
*d*f)*sqrt(d*x))/(f^3*cosh(f*x) + f^3*sinh(f*x))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.15 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.20

$$\int (dx)^{3/2}\sinh(fx)\,dx = \frac{7d^{\frac{3}{2}}x^{\frac{3}{2}}\cosh(fx)\Gamma\left(\frac{7}{4}\right)}{4f\Gamma\left(\frac{11}{4}\right)} - \frac{21d^{\frac{3}{2}}\sqrt{x}\sinh(fx)\Gamma\left(\frac{7}{4}\right)}{8f^2\Gamma\left(\frac{11}{4}\right)} + \frac{21\sqrt{2}\sqrt{\pi}d^{\frac{3}{2}}e^{-\frac{3i\pi}{4}}S\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{7}{4}\right)}{16f^{\frac{5}{2}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((d*x)**(3/2)*sinh(f*x),x)
```

output
```
7*d**(3/2)*x**(3/2)*cosh(f*x)*gamma(7/4)/(4*f*gamma(11/4)) - 21*d**(3/2)*s
qrt(x)*sinh(f*x)*gamma(7/4)/(8*f**2*gamma(11/4)) + 21*sqrt(2)*sqrt(pi)*d**
(3/2)*exp(-3*I*pi/4)*fresnels(sqrt(2)*sqrt(f)*sqrt(x)*exp(I*pi/4)/sqrt(pi)
)*gamma(7/4)/(16*f**(5/2)*gamma(11/4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(77) = 154$.

Time = 0.04 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.58

$$\int (dx)^{3/2}\sinh(fx)\,dx = \frac{16\,(dx)^{\frac{5}{2}}\sinh(fx) - \frac{f\left(\frac{15\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)}{f^3\sqrt{\frac{f}{d}}} - \frac{15\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{f^3\sqrt{-\frac{f}{d}}} + \frac{2\left(4\,(dx)^{\frac{5}{2}}df^2 - 10\,(dx)^{\frac{3}{2}}d^2f + 15\cdots\right.}{f^3}\cdots\right.}{d}}{40\,d}$$

input
```
integrate((d*x)^(3/2)*sinh(f*x),x, algorithm="maxima")
```

output
```
1/40*(16*(d*x)^(5/2)*sinh(f*x) - f*(15*sqrt(pi)*d^3*erf(sqrt(d*x)*sqrt(f/d
))/(f^3*sqrt(f/d)) - 15*sqrt(pi)*d^3*erf(sqrt(d*x)*sqrt(-f/d))/(f^3*sqrt(-
f/d)) + 2*(4*(d*x)^(5/2)*d*f^2 - 10*(d*x)^(3/2)*d^2*f + 15*sqrt(d*x)*d^3)*
e^(f*x)/f^3 - 2*(4*(d*x)^(5/2)*d*f^2 + 10*(d*x)^(3/2)*d^2*f + 15*sqrt(d*x)
*d^3)*e^(-f*x)/f^3)/d)/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.32

$$\int (dx)^{3/2}\sinh(fx)\,dx = \frac{1}{8}\,d\left(\frac{\frac{3\sqrt{\pi}d^3\operatorname{erf}\left(-\frac{\sqrt{df}\sqrt{dx}}{d}\right)}{\sqrt{df}f^2} + \frac{2\left(2\sqrt{dx}d^2fx + 3\sqrt{dx}d^2\right)e^{(-fx)}}{f^2}}{d^2} - \frac{\frac{3\sqrt{\pi}d^3\operatorname{erf}\left(-\frac{\sqrt{-df}\sqrt{dx}}{d}\right)}{\sqrt{-df}f^2} - \frac{2\left(2\sqrt{dx}\cdots\right.}{\cdots}}{d^2}\cdots\right.$$

input
```
integrate((d*x)^(3/2)*sinh(f*x),x, algorithm="giac")
```

output
```
1/8*d*((3*sqrt(pi)*d^3*erf(-sqrt(d*f)*sqrt(d*x)/d)/(sqrt(d*f)*f^2) + 2*(2*
sqrt(d*x)*d^2*f*x + 3*sqrt(d*x)*d^2)*e^(-f*x)/f^2)/d^2 - (3*sqrt(pi)*d^3*e
rf(-sqrt(-d*f)*sqrt(d*x)/d)/(sqrt(-d*f)*f^2) - 2*(2*sqrt(d*x)*d^2*f*x - 3*
sqrt(d*x)*d^2)*e^(f*x)/f^2)/d^2)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{3/2}\sinh(fx)\,dx = \int \sinh(f\,x)\,\left(d\,x\right)^{3/2}\,dx$$

input
```
int(sinh(f*x)*(d*x)^(3/2),x)
```

output
```
int(sinh(f*x)*(d*x)^(3/2), x)
```

Reduce [F]

$$\int (dx)^{3/2}\sinh(fx)\,dx = \frac{\sqrt{d}\,d\left(4\sqrt{x}\,\cosh\left(fx\right)fx-6\sqrt{x}\,\sinh\left(fx\right)+3\left(\int\frac{\sinh(fx)}{\sqrt{x}}dx\right)\right)}{4f^2}$$

input
```
int((d*x)^(3/2)*sinh(f*x),x)
```

output
```
(sqrt(d)*d*(4*sqrt(x)*cosh(f*x)*f*x - 6*sqrt(x)*sinh(f*x) + 3*int(sinh(f*x
)/sqrt(x),x)))/(4*f**2)
```

3.61 $\int \sqrt{dx}\sinh(fx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 92

$$\int \sqrt{dx}\sinh(fx)\,dx=\frac{\sqrt{dx}\cosh(fx)}{f}-\frac{\sqrt{d}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}}-\frac{\sqrt{d}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}}$$

output
```
(d*x)^(1/2)*cosh(f*x)/f-1/4*d^(1/2)*Pi^(1/2)*erf(f^(1/2)*(d*x)^(1/2)/d^(1/
2))/f^(3/2)-1/4*d^(1/2)*Pi^(1/2)*erfi(f^(1/2)*(d*x)^(1/2)/d^(1/2))/f^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.53

$$\int \sqrt{dx}\sinh(fx)\,dx=\frac{d\left(-\sqrt{-fx}\Gamma\left(\frac{3}{2},-fx\right)+\sqrt{fx}\Gamma\left(\frac{3}{2},fx\right)\right)}{2f^2\sqrt{dx}}$$

input
```
Integrate[Sqrt[d*x]*Sinh[f*x],x]
```

output
```
(d*(-(Sqrt[-(f*x)]*Gamma[3/2, -(f*x)]) + Sqrt[f*x]*Gamma[3/2, f*x]))/(2*f^
2*Sqrt[d*x])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 26, 3777, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}\sinh(fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sqrt{dx}\sin(ifx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sqrt{dx}\sin(ifx)dx$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i\sqrt{dx}\cosh(fx)}{f}-\frac{id\int \frac{\cosh(fx)}{\sqrt{dx}}dx}{2f}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i\sqrt{dx}\cosh(fx)}{f}-\frac{id\int \frac{\sin\left(ifx+\frac{\pi}{2}\right)}{\sqrt{dx}}dx}{2f}\right)$$

$$\Big\downarrow \text{3788}$$

$$-i\left(\frac{i\sqrt{dx}\cosh(fx)}{f}-\frac{id\left(\frac{1}{2}i\int -\frac{ie^{fx}}{\sqrt{dx}}dx-\frac{1}{2}i\int \frac{ie^{-fx}}{\sqrt{dx}}dx\right)}{2f}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i\sqrt{dx}\cosh(fx)}{f}-\frac{id\left(\frac{1}{2}\int \frac{e^{-fx}}{\sqrt{dx}}dx+\frac{1}{2}\int \frac{e^{fx}}{\sqrt{dx}}dx\right)}{2f}\right)$$

$$\Big\downarrow \text{2611}$$

$$-i\left(\frac{i\sqrt{dx}\cosh(fx)}{f}-\frac{id\left(\frac{\int e^{-fx}d\sqrt{dx}}{d}+\frac{\int e^{fx}d\sqrt{dx}}{d}\right)}{2f}\right)$$

$\downarrow$ 2633

$$-i\left(\frac{i\sqrt{dx}\cosh(fx)}{f}-\frac{id\left(\frac{\int e^{-fx}d\sqrt{dx}}{d}+\frac{\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}\right)}{2f}\right)$$

$\downarrow$ 2634

$$-i\left(\frac{i\sqrt{dx}\cosh(fx)}{f}-\frac{id\left(\frac{\sqrt{\pi}\text{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}+\frac{\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}\right)}{2f}\right)$$

input
```
Int[Sqrt[d*x]*Sinh[f*x],x]
```

output
```
(-I)*((I*Sqrt[d*x]*Cosh[f*x])/f - ((I/2)*d*((Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*
x])/Sqrt[d]])/(2*Sqrt[d]*Sqrt[f]) + (Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqr
t[d]])/(2*Sqrt[d]*Sqrt[f])))/f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.30

method	result	size
meijerg	$-\frac{\sqrt{dx}\,\sqrt{2}\,\sqrt{\pi}\left(\frac{\sqrt{x}\,\sqrt{2}\,(if)^{\frac{5}{2}}\,{\mathrm e}^{-fx}}{4\sqrt{\pi}\,f^{2}}+\frac{\sqrt{x}\,\sqrt{2}\,(if)^{\frac{5}{2}}\,{\mathrm e}^{fx}}{4\sqrt{\pi}\,f^{2}}-\frac{(if)^{\frac{5}{2}}\sqrt{2}\,\operatorname{erf}\left(\sqrt{x}\,\sqrt{f}\right)}{8f^{\frac{5}{2}}}-\frac{(if)^{\frac{5}{2}}\sqrt{2}\,\operatorname{erfi}\left(\sqrt{x}\,\sqrt{f}\right)}{8f^{\frac{5}{2}}}\right)}{\sqrt{x}\,\sqrt{if}\,f}$	120

input
```
int((d*x)^(1/2)*sinh(f*x),x,method=_RETURNVERBOSE)
```

output
```
-(d*x)^(1/2)/x^(1/2)*2^(1/2)/(I*f)^(1/2)*Pi^(1/2)/f*(1/4/Pi^(1/2)*x^(1/2)*
2^(1/2)*(I*f)^(5/2)/f^2*exp(-f*x)+1/4/Pi^(1/2)*x^(1/2)*2^(1/2)*(I*f)^(5/2)
/f^2*exp(f*x)-1/8*(I*f)^(5/2)*2^(1/2)/f^(5/2)*erf(x^(1/2)*f^(1/2))-1/8*(I*
f)^(5/2)*2^(1/2)/f^(5/2)*erfi(x^(1/2)*f^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(62) = 124.

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.49

$$\int \sqrt{dx}\sinh(fx)\,dx =$$

$$-\frac{\sqrt{\pi}(d\cosh(fx)+d\sinh(fx))\sqrt{\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)-\sqrt{\pi}(d\cosh(fx)+d\sinh(fx))\sqrt{-\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{\right.}{4\left(f^2\cosh(fx)+f^2\sinh(fx\right.}$$

input
```
integrate((d*x)^(1/2)*sinh(f*x),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*(d*cosh(f*x) + d*sinh(f*x))*sqrt(f/d)*erf(sqrt(d*x)*sqrt(f/
d)) - sqrt(pi)*(d*cosh(f*x) + d*sinh(f*x))*sqrt(-f/d)*erf(sqrt(d*x)*sqrt(-
f/d)) - 2*(f*cosh(f*x)^2 + 2*f*cosh(f*x)*sinh(f*x) + f*sinh(f*x)^2 + f)*sq
rt(d*x))/(f^2*cosh(f*x) + f^2*sinh(f*x))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.08

$$\int \sqrt{dx}\sinh(fx)\,dx = \frac{5\sqrt{d}\sqrt{x}\cosh(fx)\Gamma\left(\frac{5}{4}\right)}{4f\Gamma\left(\frac{9}{4}\right)} - \frac{5\sqrt{2}\sqrt{\pi}\sqrt{d}e^{-\frac{i\pi}{4}}C\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{5}{4}\right)}{8f^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x)**(1/2)*sinh(f*x),x)
```

output
```
5*sqrt(d)*sqrt(x)*cosh(f*x)*gamma(5/4)/(4*f*gamma(9/4)) - 5*sqrt(2)*sqrt(p
i)*sqrt(d)*exp(-I*pi/4)*fresnelc(sqrt(2)*sqrt(f)*sqrt(x)*exp(I*pi/4)/sqrt(
pi))*gamma(5/4)/(8*f**(3/2)*gamma(9/4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(62) = 124$.

Time = 0.04 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.62

$$\int \sqrt{dx}\sinh(fx)\,dx = \frac{8\,(dx)^{\frac{3}{2}}\sinh\left(fx\right) - \frac{f\left(\frac{3\,\sqrt{\pi}d^{2}\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)}{f^{2}\sqrt{\frac{f}{d}}} + \frac{3\,\sqrt{\pi}d^{2}\operatorname{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{f^{2}\sqrt{-\frac{f}{d}}} + \frac{2\left(2\,(dx)^{\frac{3}{2}}df - 3\,\sqrt{dx}d^{2}\right)e^{(fx)}}{f^{2}} - \frac{2\left(2\,(dx)^{\frac{3}{2}}df + 3\,\sqrt{dx}d^{2}\right)e^{(-fx)}}{f^{2}}\right)}{d}}{12\,d}$$

input
```
integrate((d*x)^(1/2)*sinh(f*x),x, algorithm="maxima")
```

output
```
1/12*(8*(d*x)^(3/2)*sinh(f*x) - f*(3*sqrt(pi)*d^2*erf(sqrt(d*x)*sqrt(f/d))
/(f^2*sqrt(f/d)) + 3*sqrt(pi)*d^2*erf(sqrt(d*x)*sqrt(-f/d))/(f^2*sqrt(-f/d
)) + 2*(2*(d*x)^(3/2)*d*f - 3*sqrt(d*x)*d^2)*e^(f*x)/f^2 - 2*(2*(d*x)^(3/2
)*d*f + 3*sqrt(d*x)*d^2)*e^(-f*x)/f^2)/d)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.17

$$\int \sqrt{dx}\sinh(fx)\,dx = \frac{\frac{\sqrt{\pi}d^{2}\operatorname{erf}\left(-\frac{\sqrt{df}\sqrt{dx}}{d}\right)}{\sqrt{df}f} + \frac{2\,\sqrt{dx}de^{(-fx)}}{f}}{4\,d} + \frac{\frac{\sqrt{\pi}d^{2}\operatorname{erf}\left(-\frac{\sqrt{-df}\sqrt{dx}}{d}\right)}{\sqrt{-df}f} + \frac{2\,\sqrt{dx}de^{(fx)}}{f}}{4\,d}$$

input
```
integrate((d*x)^(1/2)*sinh(f*x),x, algorithm="giac")
```

output
```
1/4*(sqrt(pi)*d^2*erf(-sqrt(d*f)*sqrt(d*x)/d)/(sqrt(d*f)*f) + 2*sqrt(d*x)*
d*e^(-f*x)/f)/d + 1/4*(sqrt(pi)*d^2*erf(-sqrt(-d*f)*sqrt(d*x)/d)/(sqrt(-d*
f)*f) + 2*sqrt(d*x)*d*e^(f*x)/f)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}\sinh(fx)\,dx = \int \sinh(f\,x)\,\sqrt{d\,x}\,dx$$

input
```
int(sinh(f*x)*(d*x)^(1/2),x)
```

output
```
int(sinh(f*x)*(d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{dx}\sinh(fx)\,dx = \frac{\sqrt{d}\left(\sqrt{\pi}\,e^{fx}\mathrm{erf}\left(\sqrt{x}\,\sqrt{f}\,i\right)i+2\sqrt{x}\,\sqrt{f}\,e^{2fx}-\sqrt{f}\,e^{fx}\left(\int \frac{\sqrt{x}}{e^{fx}x}dx\right)+2\sqrt{x}\,\sqrt{f}\right)}{4\sqrt{f}\,e^{fx}f}$$

input
```
int((d*x)^(1/2)*sinh(f*x),x)
```

output
```
(sqrt(d)*(sqrt(pi)*e**(f*x)*erf(sqrt(x)*sqrt(f)*i)*i + 2*sqrt(x)*sqrt(f)*e
**(2*f*x) - sqrt(f)*e**(f*x)*int(sqrt(x)/(e**(f*x)*x),x) + 2*sqrt(x)*sqrt(
f)))/(4*sqrt(f)*e**(f*x)*f)
```

3.62 $\int \frac{\sinh(fx)}{\sqrt{dx}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 77

$$\int \frac{\sinh(fx)}{\sqrt{dx}}\,dx=-\frac{\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}+\frac{\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}$$

output
```
-1/2*Pi^(1/2)*erf(f^(1/2)*(d*x)^(1/2)/d^(1/2))/d^(1/2)/f^(1/2)+1/2*Pi^(1/2
)*erfi(f^(1/2)*(d*x)^(1/2)/d^(1/2))/d^(1/2)/f^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.61

$$\int \frac{\sinh(fx)}{\sqrt{dx}}\,dx=\frac{\sqrt{-fx}\,\Gamma\left(\frac{1}{2},-fx\right)+\sqrt{fx}\,\Gamma\left(\frac{1}{2},fx\right)}{2f\sqrt{dx}}$$

input
```
Integrate[Sinh[f*x]/Sqrt[d*x],x]
```

output
```
(Sqrt[-(f*x)]*Gamma[1/2, -(f*x)] + Sqrt[f*x]*Gamma[1/2, f*x])/(2*f*Sqrt[d*
x])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(fx)}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ifx)}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ifx)}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{3789}$$

$$-i\left(\frac{1}{2}i\int \frac{e^{fx}}{\sqrt{dx}}dx-\frac{1}{2}i\int \frac{e^{-fx}}{\sqrt{dx}}dx\right)$$

$$\Big\downarrow \text{2611}$$

$$-i\left(\frac{i\int e^{fx}d\sqrt{dx}}{d}-\frac{i\int e^{-fx}d\sqrt{dx}}{d}\right)$$

$$\Big\downarrow \text{2633}$$

$$-i\left(\frac{i\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}-\frac{i\int e^{-fx}d\sqrt{dx}}{d}\right)$$

$$\Big\downarrow \text{2634}$$

$$-i\left(\frac{i\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}-\frac{i\sqrt{\pi}\text{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}\right)$$

input
```
Int[Sinh[f*x]/Sqrt[d*x],x]
```

output
```
(-I)*(((-1/2*I)*Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[d]*Sqrt[f
]) + ((I/2)*Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[d]*Sqrt[f]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

method	result	size
meijerg	$-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{if}\left(-\frac{(if)^{\frac{3}{2}}\sqrt{2}\,\operatorname{erf}(\sqrt{x}\,\sqrt{f})}{2f^{\frac{3}{2}}}+\frac{(if)^{\frac{3}{2}}\sqrt{2}\,\operatorname{erfi}(\sqrt{x}\,\sqrt{f})}{2f^{\frac{3}{2}}}\right)}{2\sqrt{dx}\,f}$	71

input
```
int(sinh(f*x)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*Pi^(1/2)/(d*x)^(1/2)*x^(1/2)*2^(1/2)*(I*f)^(1/2)/f*(-1/2*(I*f)^(3/2)*
2^(1/2)/f^(3/2)*erf(x^(1/2)*f^(1/2))+1/2*(I*f)^(3/2)*2^(1/2)/f^(3/2)*erfi(
x^(1/2)*f^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.75

$$\int \frac{\sinh(fx)}{\sqrt{dx}}\,dx = -\frac{\sqrt{\pi}\sqrt{\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)+\sqrt{\pi}\sqrt{-\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{2\,f}$$

input
```
integrate(sinh(f*x)/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(pi)*sqrt(f/d)*erf(sqrt(d*x)*sqrt(f/d)) + sqrt(pi)*sqrt(-f/d)*er
f(sqrt(d*x)*sqrt(-f/d)))/f
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

$$\int \frac{\sinh(fx)}{\sqrt{dx}}\,dx = \frac{3\sqrt{2}\sqrt{\pi}e^{-\frac{3i\pi}{4}}S\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{3}{4}\right)}{4\sqrt{d}\sqrt{f}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(sinh(f*x)/(d*x)**(1/2),x)
```

output
```
3*sqrt(2)*sqrt(pi)*exp(-3*I*pi/4)*fresnels(sqrt(2)*sqrt(f)*sqrt(x)*exp(I*p
i/4)/sqrt(pi))*gamma(3/4)/(4*sqrt(d)*sqrt(f)*gamma(7/4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(49) = 98.

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.51

$$\int \frac{\sinh(fx)}{\sqrt{dx}}\,dx$$
$$= \frac{4\sqrt{dx}\sinh(fx) - \frac{\left(\frac{2\sqrt{dx}de^{(fx)}}{f} - \frac{2\sqrt{dx}de^{(-fx)}}{f} + \frac{\sqrt{\pi}d\,\mathrm{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)}{f\sqrt{\frac{f}{d}}} - \frac{\sqrt{\pi}d\,\mathrm{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{f\sqrt{-\frac{f}{d}}}\right)f}{d}}{2d}$$

input
```
integrate(sinh(f*x)/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(4*sqrt(d*x)*sinh(f*x) - (2*sqrt(d*x)*d*e^(f*x)/f - 2*sqrt(d*x)*d*e^(-
f*x)/f + sqrt(pi)*d*erf(sqrt(d*x)*sqrt(f/d))/(f*sqrt(f/d)) - sqrt(pi)*d*er
f(sqrt(d*x)*sqrt(-f/d))/(f*sqrt(-f/d)))*f/d)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{\sinh(fx)}{\sqrt{dx}}\,dx = \frac{\frac{\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{df}\sqrt{dx}}{d}\right)}{\sqrt{df}} - \frac{\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{-df}\sqrt{dx}}{d}\right)}{\sqrt{-df}}}{2\,d}$$

input
```
integrate(sinh(f*x)/(d*x)^(1/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(pi)*d*erf(-sqrt(d*f)*sqrt(d*x)/d)/sqrt(d*f) - sqrt(pi)*d*erf(-sq
rt(-d*f)*sqrt(d*x)/d)/sqrt(-d*f))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(fx)}{\sqrt{dx}}\,dx = \int \frac{\sinh(f\,x)}{\sqrt{d\,x}}\,dx$$

input
```
int(sinh(f*x)/(d*x)^(1/2),x)
```

output
```
int(sinh(f*x)/(d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sinh(fx)}{\sqrt{dx}}\,dx = \frac{\int \frac{\sinh(fx)}{\sqrt{x}}dx}{\sqrt{d}}$$

input
```
int(sinh(f*x)/(d*x)^(1/2),x)
```

output
```
int(sinh(f*x)/sqrt(x),x)/sqrt(d)
```

3.63 $\int \frac{\sinh(fx)}{(dx)^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 87

$$\int \frac{\sinh(fx)}{(dx)^{3/2}}\,dx=\frac{\sqrt{f}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{f}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sinh(fx)}{d\sqrt{dx}}$$

output
```
f^(1/2)*Pi^(1/2)*erf(f^(1/2)*(d*x)^(1/2)/d^(1/2))/d^(3/2)+f^(1/2)*Pi^(1/2)
*erfi(f^(1/2)*(d*x)^(1/2)/d^(1/2))/d^(3/2)-2*sinh(f*x)/d/(d*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.56

$$\int \frac{\sinh(fx)}{(dx)^{3/2}}\,dx=\frac{x\left(\sqrt{-fx}\Gamma\left(\frac{1}{2},-fx\right)-\sqrt{fx}\Gamma\left(\frac{1}{2},fx\right)-2\sinh(fx)\right)}{(dx)^{3/2}}$$

input
```
Integrate[Sinh[f*x]/(d*x)^(3/2),x]
```

output
```
(x*(Sqrt[-(f*x)]*Gamma[1/2, -(f*x)] - Sqrt[f*x]*Gamma[1/2, f*x] - 2*Sinh[f
*x]))/(d*x)^(3/2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 26, 3778, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(fx)}{(dx)^{3/2}} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ifx)}{(dx)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ifx)}{(dx)^{3/2}}dx$$

$$\Big\downarrow \text{3778}$$

$$-i\left(\frac{2if\int \frac{\cosh(fx)}{\sqrt{dx}}dx}{d}-\frac{2i\sinh(fx)}{d\sqrt{dx}}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{2if\int \frac{\sin\left(ifx+\frac{\pi}{2}\right)}{\sqrt{dx}}dx}{d}-\frac{2i\sinh(fx)}{d\sqrt{dx}}\right)$$

$$\Big\downarrow \text{3788}$$

$$-i\left(\frac{2if\left(\frac{1}{2}i\int -\frac{ie^{fx}}{\sqrt{dx}}dx-\frac{1}{2}i\int \frac{ie^{-fx}}{\sqrt{dx}}dx\right)}{d}-\frac{2i\sinh(fx)}{d\sqrt{dx}}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{2if\left(\frac{1}{2}\int \frac{e^{-fx}}{\sqrt{dx}}dx+\frac{1}{2}\int \frac{e^{fx}}{\sqrt{dx}}dx\right)}{d}-\frac{2i\sinh(fx)}{d\sqrt{dx}}\right)$$

$$\Big\downarrow \text{2611}$$

$$-i\left(\frac{2if\left(\frac{\int e^{-fx}d\sqrt{dx}}{d}+\frac{\int e^{fx}d\sqrt{dx}}{d}\right)}{d}-\frac{2i\sinh(fx)}{d\sqrt{dx}}\right)$$

$$\Big\downarrow \text{2633}$$

$$-i\left(\frac{2if\left(\frac{\int e^{-fx}d\sqrt{dx}}{d}+\frac{\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}\right)}{d}-\frac{2i\sinh(fx)}{d\sqrt{dx}}\right)$$

$$\Big\downarrow \text{2634}$$

$$-i\left(\frac{2if\left(\frac{\sqrt{\pi}\text{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}+\frac{\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}\right)}{d}-\frac{2i\sinh(fx)}{d\sqrt{dx}}\right)$$

input
```
Int[Sinh[f*x]/(d*x)^(3/2),x]
```

output
```
(-I)*(((2*I)*f*((Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(2*Sqrt[d]*Sqr
t[f]) + (Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(2*Sqrt[d]*Sqrt[f])))
/d - ((2*I)*Sinh[f*x])/(d*Sqrt[d*x]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.38

method	result	size
meijerg	$-\frac{\sqrt{\pi}\, x^{\frac{3}{2}} \sqrt{2}\, (i f)^{\frac{3}{2}} \left(\frac{2\sqrt{2}\,\sqrt{i f}\, e^{-f x}}{\sqrt{\pi}\,\sqrt{x}\, f}-\frac{2\sqrt{2}\,\sqrt{i f}\, e^{f x}}{\sqrt{\pi}\,\sqrt{x}\, f}+\frac{2\sqrt{i f}\,\sqrt{2}\, \operatorname{erf}(\sqrt{x}\,\sqrt{f})}{\sqrt{f}}+\frac{2\sqrt{i f}\,\sqrt{2}\, \operatorname{erfi}(\sqrt{x}\,\sqrt{f})}{\sqrt{f}}\right)}{4 (d x)^{\frac{3}{2}} f}$	120

input
```
int(sinh(f*x)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*Pi^(1/2)/(d*x)^(3/2)*x^(3/2)*2^(1/2)*(I*f)^(3/2)/f*(2/Pi^(1/2)/x^(1/2
)*2^(1/2)*(I*f)^(1/2)/f*exp(-f*x)-2/Pi^(1/2)/x^(1/2)*2^(1/2)*(I*f)^(1/2)/f
*exp(f*x)+2*(I*f)^(1/2)*2^(1/2)/f^(1/2)*erf(x^(1/2)*f^(1/2))+2*(I*f)^(1/2)
*2^(1/2)/f^(1/2)*erfi(x^(1/2)*f^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(61) = 122.

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.57

$$\int \frac{\sinh(fx)}{(dx)^{3/2}}\,dx = \frac{\sqrt{\pi}(dx\cosh(fx) + dx\sinh(fx))\sqrt{\frac{f}{d}}\,\mathrm{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right) - \sqrt{\pi}(dx\cosh(fx) + dx\sinh(fx))}{d^2x\cosh(fx) +}$$

input
```
integrate(sinh(f*x)/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
(sqrt(pi)*(d*x*cosh(f*x) + d*x*sinh(f*x))*sqrt(f/d)*erf(sqrt(d*x)*sqrt(f/d
)) - sqrt(pi)*(d*x*cosh(f*x) + d*x*sinh(f*x))*sqrt(-f/d)*erf(sqrt(d*x)*sqr
t(-f/d)) - sqrt(d*x)*(cosh(f*x)^2 + 2*cosh(f*x)*sinh(f*x) + sinh(f*x)^2 -
1))/(d^2*x*cosh(f*x) + d^2*x*sinh(f*x))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.44 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.08

$$\int \frac{\sinh(fx)}{(dx)^{3/2}}\,dx = \frac{\sqrt{2}\sqrt{\pi}\sqrt{f}e^{-\frac{i\pi}{4}}C\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{1}{4}\right)}{2d^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)} - \frac{\sinh(fx)\Gamma\left(\frac{1}{4}\right)}{2d^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(sinh(f*x)/(d*x)**(3/2),x)
```

output
```
sqrt(2)*sqrt(pi)*sqrt(f)*exp(-I*pi/4)*fresnelc(sqrt(2)*sqrt(f)*sqrt(x)*exp
(I*pi/4)/sqrt(pi))*gamma(1/4)/(2*d**(3/2)*gamma(5/4)) - sinh(f*x)*gamma(1/
4)/(2*d**(3/2)*sqrt(x)*gamma(5/4))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

$$\int \frac{\sinh(fx)}{(dx)^{3/2}}\,dx = \frac{\frac{f\left(\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)}{\sqrt{\frac{f}{d}}}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{\sqrt{-\frac{f}{d}}}\right)}{d}-\frac{2\sinh(fx)}{\sqrt{dx}}}{d}$$

input
```
integrate(sinh(f*x)/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
(f*(sqrt(pi)*erf(sqrt(d*x)*sqrt(f/d))/sqrt(f/d) + sqrt(pi)*erf(sqrt(d*x)*s
qrt(-f/d))/sqrt(-f/d))/d - 2*sinh(f*x)/sqrt(d*x))/d
```

Giac [F]

$$\int \frac{\sinh(fx)}{(dx)^{3/2}}\,dx = \int \frac{\sinh(fx)}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(f*x)/(d*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sinh(f*x)/(d*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(fx)}{(dx)^{3/2}}\,dx = \int \frac{\sinh(f\,x)}{(d\,x)^{3/2}}\,dx$$

input
```
int(sinh(f*x)/(d*x)^(3/2),x)
```

output
```
int(sinh(f*x)/(d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sinh(fx)}{(dx)^{3/2}}\,dx = \frac{\int \frac{\sinh(fx)}{\sqrt{x}\,x}dx}{\sqrt{d}\,d}$$

input
```
int(sinh(f*x)/(d*x)^(3/2),x)
```

output
```
int(sinh(f*x)/(sqrt(x)*x),x)/(sqrt(d)*d)
```

3.64 $\int \frac{\sinh(fx)}{(dx)^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 114

$$\int \frac{\sinh(fx)}{(dx)^{5/2}}\,dx = -\frac{4f\cosh(fx)}{3d^2\sqrt{dx}} - \frac{2f^{3/2}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2f^{3/2}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{2\sinh(fx)}{3d(dx)^{3/2}}$$

output
```
-4/3*f*cosh(f*x)/d^2/(d*x)^(1/2)-2/3*f^(3/2)*Pi^(1/2)*erf(f^(1/2)*(d*x)^(1
/2)/d^(1/2))/d^(5/2)+2/3*f^(3/2)*Pi^(1/2)*erfi(f^(1/2)*(d*x)^(1/2)/d^(1/2)
)/d^(5/2)-2/3*sinh(f*x)/d/(d*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.74

$$\int \frac{\sinh(fx)}{(dx)^{5/2}}\,dx = -\frac{e^{-fx}x\left(-1+e^{2fx}+2fx+2e^{2fx}fx+2e^{fx}(-fx)^{3/2}\Gamma\left(\frac{1}{2},-fx\right)-2e^{fx}(fx)^{3/2}\Gamma\left(\frac{1}{2},fx\right)\right)}{3(dx)^{5/2}}$$

input
```
Integrate[Sinh[f*x]/(d*x)^(5/2),x]
```

output
```
-1/3*(x*(-1 + E^(2*f*x) + 2*f*x + 2*E^(2*f*x)*f*x + 2*E^(f*x)*(-(f*x))^(3/
2)*Gamma[1/2, -(f*x)] - 2*E^(f*x)*(f*x)^(3/2)*Gamma[1/2, f*x]))/(E^(f*x)*(
d*x)^(5/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.24, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 26, 3778, 3042, 3778, 26, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(fx)}{(dx)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\sin(ifx)}{(dx)^{5/2}}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\sin(ifx)}{(dx)^{5/2}}dx$$

$$\Big\downarrow \text{ 3778}$$

$$-i\left(\frac{2if\int \frac{\cosh(fx)}{(dx)^{3/2}}dx}{3d}-\frac{2i\sinh(fx)}{3d(dx)^{3/2}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-i\left(\frac{2if\int \frac{\sin\left(ifx+\frac{\pi}{2}\right)}{(dx)^{3/2}}dx}{3d}-\frac{2i\sinh(fx)}{3d(dx)^{3/2}}\right)$$

$$\Big\downarrow \text{ 3778}$$

$$-i\left(\frac{2if\left(-\frac{2\cosh(fx)}{d\sqrt{dx}}+\frac{2if\int -\frac{i\sinh(fx)}{\sqrt{dx}}dx}{d}\right)}{3d}-\frac{2i\sinh(fx)}{3d(dx)^{3/2}}\right)$$

$\downarrow$ 26

$$-i\left(\frac{2if\left(\frac{2f\int \frac{\sinh(fx)}{\sqrt{dx}}dx}{d}-\frac{2\cosh(fx)}{d\sqrt{dx}}\right)}{3d}-\frac{2i\sinh(fx)}{3d(dx)^{3/2}}\right)$$

$\downarrow$ 3042

$$-i\left(\frac{2if\left(-\frac{2\cosh(fx)}{d\sqrt{dx}}+\frac{2f\int -\frac{i\sin(ifx)}{\sqrt{dx}}dx}{d}\right)}{3d}-\frac{2i\sinh(fx)}{3d(dx)^{3/2}}\right)$$

$\downarrow$ 26

$$-i\left(\frac{2if\left(-\frac{2\cosh(fx)}{d\sqrt{dx}}-\frac{2if\int \frac{\sin(ifx)}{\sqrt{dx}}dx}{d}\right)}{3d}-\frac{2i\sinh(fx)}{3d(dx)^{3/2}}\right)$$

$\downarrow$ 3789

$$-i\left(\frac{2if\left(-\frac{2\cosh(fx)}{d\sqrt{dx}}-\frac{2if\left(\frac{1}{2}i\int \frac{e^{fx}}{\sqrt{dx}}dx-\frac{1}{2}i\int \frac{e^{-fx}}{\sqrt{dx}}dx\right)}{d}\right)}{3d}-\frac{2i\sinh(fx)}{3d(dx)^{3/2}}\right)$$

$\downarrow$ 2611

$$-i\left(\frac{2if\left(-\frac{2\cosh(fx)}{d\sqrt{dx}}-\frac{2if\left(\frac{i\int e^{fx}d\sqrt{dx}}{d}-\frac{i\int e^{-fx}d\sqrt{dx}}{d}\right)}{d}\right)}{3d}-\frac{2i\sinh(fx)}{3d(dx)^{3/2}}\right)$$

$\downarrow$ 2633

$$-i\left(\frac{2if\left(-\frac{2\cosh(fx)}{d\sqrt{dx}}-\frac{2if\left(\frac{i\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}-\frac{i\int e^{-fx}d\sqrt{dx}}{d}\right)}{d}\right)}{3d}-\frac{2i\sinh(fx)}{3d(dx)^{3/2}}\right)$$

$$\Big\downarrow \text{2634}$$

$$-i\left(\frac{2if\left(-\frac{2\cosh(fx)}{d\sqrt{dx}}-\frac{2if\left(\frac{i\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}-\frac{i\sqrt{\pi}\text{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}\right)}{d}\right)}{3d}-\frac{2i\sinh(fx)}{3d(dx)^{3/2}}\right)$$

input
```
Int[Sinh[f*x]/(d*x)^(5/2),x]
```

output
```
(-I)*((((2*I)/3)*f*((-2*Cosh[f*x])/(d*Sqrt[d*x]) - ((2*I)*f*(((-1/2*I)*Sqr
t[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[d]*Sqrt[f]) + ((I/2)*Sqrt[Pi
]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[d]*Sqrt[f])))/d))/d - (((2*I)/3
)*Sinh[f*x])/(d*(d*x)^(3/2)))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.16

method	result	size
meijerg	$-\frac{\sqrt{\pi}\, x^{\frac{5}{2}} \sqrt{2}\, (i f)^{\frac{5}{2}} \left(-\frac{4 \sqrt{2}\, (2 f x +1) {\mathrm e}^{f x}}{3 \sqrt{\pi}\, x^{\frac{3}{2}} \sqrt{i f}\, f}+\frac{4 \sqrt{2}\, (-2 f x +1) {\mathrm e}^{-f x}}{3 \sqrt{\pi}\, x^{\frac{3}{2}} \sqrt{i f}\, f}-\frac{8 \sqrt{2}\, \sqrt{f}\, \operatorname{erf}\left(\sqrt{x}\, \sqrt{f}\right)}{3 \sqrt{i f}}+\frac{8 \sqrt{2}\, \sqrt{f}\, \operatorname{erfi}\left(\sqrt{x}\, \sqrt{f}\right)}{3 \sqrt{i f}}\right)}{8 (d x)^{\frac{5}{2}} f}$	132

input
```
int(sinh(f*x)/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*Pi^(1/2)/(d*x)^(5/2)*x^(5/2)*2^(1/2)*(I*f)^(5/2)/f*(-4/3/Pi^(1/2)/x^(
3/2)*2^(1/2)/(I*f)^(1/2)*(2*f*x+1)/f*exp(f*x)+4/3/Pi^(1/2)/x^(3/2)*2^(1/2)
/(I*f)^(1/2)*(-2*f*x+1)/f*exp(-f*x)-8/3/(I*f)^(1/2)*2^(1/2)*f^(1/2)*erf(x^
(1/2)*f^(1/2))+8/3/(I*f)^(1/2)*2^(1/2)*f^(1/2)*erfi(x^(1/2)*f^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(78) = 156$.

Time = 0.09 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.56

$$\int \frac{\sinh(fx)}{(dx)^{5/2}}\,dx = -\frac{2\sqrt{\pi}(dfx^2\cosh(fx) + dfx^2\sinh(fx))\sqrt{\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right) + 2\sqrt{\pi}(dfx^2\cosh(fx) + dfx^2\sinh(fx))\sqrt{-\cdots}}{3\,(d^3x^2\cos\cdots}$$

input
```
integrate(sinh(f*x)/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(2*sqrt(pi)*(d*f*x^2*cosh(f*x) + d*f*x^2*sinh(f*x))*sqrt(f/d)*erf(sqr
t(d*x)*sqrt(f/d)) + 2*sqrt(pi)*(d*f*x^2*cosh(f*x) + d*f*x^2*sinh(f*x))*sqr
t(-f/d)*erf(sqrt(d*x)*sqrt(-f/d)) + ((2*f*x + 1)*cosh(f*x)^2 + 2*(2*f*x +
1)*cosh(f*x)*sinh(f*x) + (2*f*x + 1)*sinh(f*x)^2 + 2*f*x - 1)*sqrt(d*x))/(
d^3*x^2*cosh(f*x) + d^3*x^2*sinh(f*x))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.87 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.13

$$\int \frac{\sinh(fx)}{(dx)^{5/2}}\,dx = -\frac{\sqrt{2}\sqrt{\pi}f^{\frac{3}{2}}e^{-\frac{3i\pi}{4}}S\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(-\frac{1}{4}\right)}{3d^{\frac{5}{2}}\Gamma\left(\frac{3}{4}\right)} + \frac{f\cosh(fx)\Gamma\left(-\frac{1}{4}\right)}{3d^{\frac{5}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)} + \frac{\sinh(fx)\Gamma\left(-\frac{1}{4}\right)}{6d^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(sinh(f*x)/(d*x)**(5/2),x)
```

output
```
-sqrt(2)*sqrt(pi)*f**(3/2)*exp(-3*I*pi/4)*fresnels(sqrt(2)*sqrt(f)*sqrt(x)
*exp(I*pi/4)/sqrt(pi))*gamma(-1/4)/(3*d**(5/2)*gamma(3/4)) + f*cosh(f*x)*g
amma(-1/4)/(3*d**(5/2)*sqrt(x)*gamma(3/4)) + sinh(f*x)*gamma(-1/4)/(6*d**(
5/2)*x**(3/2)*gamma(3/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.50

$$\int \frac{\sinh(fx)}{(dx)^{5/2}}\,dx = -\frac{\frac{f\left(\frac{\sqrt{fx}\Gamma\left(-\frac{1}{2},fx\right)}{\sqrt{dx}}+\frac{\sqrt{-fx}\Gamma\left(-\frac{1}{2},-fx\right)}{\sqrt{dx}}\right)}{d}+\frac{2\sinh(fx)}{(dx)^{\frac{3}{2}}}}{3\,d}$$

input
```
integrate(sinh(f*x)/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(f*(sqrt(f*x)*gamma(-1/2, f*x)/sqrt(d*x) + sqrt(-f*x)*gamma(-1/2, -f*
x)/sqrt(d*x))/d + 2*sinh(f*x)/(d*x)^(3/2))/d
```

Giac [F]

$$\int \frac{\sinh(fx)}{(dx)^{5/2}}\,dx = \int \frac{\sinh(fx)}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sinh(f*x)/(d*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(sinh(f*x)/(d*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(fx)}{(dx)^{5/2}}\,dx = \int \frac{\sinh(f\,x)}{(d\,x)^{5/2}}\,dx$$

input
```
int(sinh(f*x)/(d*x)^(5/2),x)
```

output
```
int(sinh(f*x)/(d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sinh(fx)}{(dx)^{5/2}}\,dx = \frac{\int \frac{\sinh(fx)}{\sqrt{x}\,x^2}dx}{\sqrt{d}\,d^2}$$

input
```
int(sinh(f*x)/(d*x)^(5/2),x)
```

output
```
int(sinh(f*x)/(sqrt(x)*x**2),x)/(sqrt(d)*d**2)
```

3.65 $\int \sqrt{c+dx}\,\text{csch}(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \sqrt{c+dx}\,\text{csch}(a+bx)\,dx = \text{Int}\left(\sqrt{c+dx}\,\text{csch}(a+bx),x\right)$$

output
```
Defer(Int)((d*x+c)^(1/2)*csch(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 24.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \sqrt{c+dx}\,\text{csch}(a+bx)\,dx = \int \sqrt{c+dx}\,\text{csch}(a+bx)\,dx$$

input
```
Integrate[Sqrt[c + d*x]*Csch[a + b*x],x]
```

output
```
Integrate[Sqrt[c + d*x]*Csch[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\text{csch}(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int i\sqrt{c+dx}\csc(ia+ibx)dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \sqrt{c+dx}\csc(ia+ibx)dx$$

$$\Big\downarrow \text{ 4680}$$

$$\int \sqrt{c+dx}\text{csch}(a+bx)dx$$

input
```
Int[Sqrt[c + d*x]*Csch[a + b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \sqrt{dx+c}\ \text{csch}\left(bx+a\right)dx$$

input
```
int((d*x+c)^(1/2)*csch(b*x+a),x)
```

output
```
int((d*x+c)^(1/2)*csch(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx}\text{csch}(a+bx)\,dx = \int \sqrt{dx+c}\,\text{csch}\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)^(1/2)*csch(b*x+a),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x + c)*csch(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \sqrt{c+dx}\text{csch}(a+bx)\,dx = \int \sqrt{c+dx}\,\text{csch}\left(a+bx\right)dx$$

input
```
integrate((d*x+c)**(1/2)*csch(b*x+a),x)
```

output
```
Integral(sqrt(c + d*x)*csch(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx}\operatorname{csch}(a+bx)\,dx = \int \sqrt{dx+c}\operatorname{csch}(bx+a)\,dx$$

input
```
integrate((d*x+c)^(1/2)*csch(b*x+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x + c)*csch(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx}\operatorname{csch}(a+bx)\,dx = \int \sqrt{dx+c}\operatorname{csch}(bx+a)\,dx$$

input
```
integrate((d*x+c)^(1/2)*csch(b*x+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)*csch(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \sqrt{c+dx}\operatorname{csch}(a+bx)\,dx = \int \frac{\sqrt{c+d\,x}}{\sinh(a+b\,x)}\,dx$$

input
```
int((c + d*x)^(1/2)/sinh(a + b*x),x)
```

output
```
int((c + d*x)^(1/2)/sinh(a + b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \sqrt{c+dx}\,\mathrm{csch}(a+bx)\,dx=\int \sqrt{dx+c}\,\mathrm{csch}(bx+a)\,dx$$

input
```
int((d*x+c)^(1/2)*csch(b*x+a),x)
```

output
```
int(sqrt(c + d*x)*csch(a + b*x),x)
```

3.66 $\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}} \, dx = \operatorname{Int}\left(\frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}, x\right)$$

output
```
Defer(Int)(csch(b*x+a)/(d*x+c)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 30.42 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}} \, dx = \int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}} \, dx$$

input
```
Integrate[Csch[a + b*x]/Sqrt[c + d*x],x]
```

output
```
Integrate[Csch[a + b*x]/Sqrt[c + d*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\csc(ia+ibx)}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\csc(ia+ibx)}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{4680}$$

$$\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}dx$$

input
```
Int[Csch[a + b*x]/Sqrt[c + d*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{csch}\left(bx+a\right)}{\sqrt{dx+c}}dx$$

input
```
int(csch(b*x+a)/(d*x+c)^(1/2),x)
```

output
```
int(csch(b*x+a)/(d*x+c)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)}{\sqrt{dx+c}}\,dx$$

input
```
integrate(csch(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)/sqrt(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\operatorname{csch}\left(a+bx\right)}{\sqrt{c+dx}}\,dx$$

input
```
integrate(csch(b*x+a)/(d*x+c)**(1/2),x)
```

output
```
Integral(csch(a + b*x)/sqrt(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\operatorname{csch}(bx+a)}{\sqrt{dx+c}}\,dx$$

input
```
integrate(csch(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csch(b*x + a)/sqrt(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\operatorname{csch}(bx+a)}{\sqrt{dx+c}}\,dx$$

input
```
integrate(csch(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)/sqrt(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{1}{\sinh(a+b\,x)\,\sqrt{c+d\,x}}\,dx$$

input
```
int(1/(sinh(a + b*x)*(c + d*x)^(1/2)),x)
```

output
```
int(1/(sinh(a + b*x)*(c + d*x)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}\,dx=\int \frac{\operatorname{csch}(bx+a)}{\sqrt{dx+c}}dx$$

input
```
int(csch(b*x+a)/(d*x+c)^(1/2),x)
```

output
```
int(csch(a + b*x)/sqrt(c + d*x),x)
```

3.67 $\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3} dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3} dx = -\frac{3\cosh(x)\sqrt{\sinh(x)}}{4x} - \frac{\sinh^{\frac{3}{2}}(x)}{2x^2} + \frac{3}{8}\text{Int}\left(\frac{1}{x\sqrt{\sinh(x)}}, x\right) + \frac{9}{8}\text{Int}\left(\frac{\sinh^{\frac{3}{2}}(x)}{x}, x\right)$$

output
```
-3/4*cosh(x)*sinh(x)^(1/2)/x-1/2*sinh(x)^(3/2)/x^2+3/8*Defer(Int)(1/x/sinh
(x)^(1/2),x)+9/8*Defer(Int)(sinh(x)^(3/2)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 4.53 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3} dx = \int \frac{\sinh^{\frac{3}{2}}(x)}{x^3} dx$$

input
```
Integrate[Sinh[x]^(3/2)/x^3,x]
```

output
```
Integrate[Sinh[x]^(3/2)/x^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(-i\sin(ix))^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{3795}$$

$$\frac{9}{8}\int \frac{\sinh^{\frac{3}{2}}(x)}{x}dx+\frac{3}{8}\int \frac{1}{x\sqrt{\sinh(x)}}dx-\frac{\sinh^{\frac{3}{2}}(x)}{2x^2}-\frac{3\sqrt{\sinh(x)}\cosh(x)}{4x}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{8}\int \frac{1}{x\sqrt{-i\sin(ix)}}dx+\frac{9}{8}\int \frac{(-i\sin(ix))^{3/2}}{x}dx-\frac{\sinh^{\frac{3}{2}}(x)}{2x^2}-\frac{3\sqrt{\sinh(x)}\cosh(x)}{4x}$$

$$\Big\downarrow \text{3807}$$

$$\frac{9}{8}\int \frac{\sinh^{\frac{3}{2}}(x)}{x}dx+\frac{3}{8}\int \frac{1}{x\sqrt{\sinh(x)}}dx-\frac{\sinh^{\frac{3}{2}}(x)}{2x^2}-\frac{3\sqrt{\sinh(x)}\cosh(x)}{4x}$$

input
```
Int[Sinh[x]^(3/2)/x^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sinh(x)^{\frac{3}{2}}}{x^3}dx$$

input
```
int(sinh(x)^(3/2)/x^3,x)
```

output
```
int(sinh(x)^(3/2)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sinh(x)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [N/A]

Not integrable

Time = 4.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3}\,dx = \int \frac{\sinh^{\frac{3}{2}}(x)}{x^3}\,dx$$

input
```
integrate(sinh(x)**(3/2)/x**3,x)
```

output
```
Integral(sinh(x)**(3/2)/x**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3}\, dx = \int \frac{\sinh{(x)}^{\frac{3}{2}}}{x^3}\, dx$$

input
```
integrate(sinh(x)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sinh(x)^(3/2)/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3}\, dx = \int \frac{\sinh{(x)}^{\frac{3}{2}}}{x^3}\, dx$$

input
```
integrate(sinh(x)^(3/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sinh(x)^(3/2)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3}\,dx = \int \frac{\sinh(x)^{3/2}}{x^3}\,dx$$

input
```
int(sinh(x)^(3/2)/x^3,x)
```

output
```
int(sinh(x)^(3/2)/x^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3}\,dx = \int \frac{\sqrt{\sinh(x)}\,\sinh(x)}{x^3}dx$$

input
```
int(sinh(x)^(3/2)/x^3,x)
```

output
```
int((sqrt(sinh(x))*sinh(x))/x**3,x)
```

3.68 $\int \left(\frac{x}{\sinh^{\frac{3}{2}}(x)} - x\sqrt{\sinh(x)}\right) dx$

Optimal result

Integrand size = 18, antiderivative size = 20

$$\int \left(\frac{x}{\sinh^{\frac{3}{2}}(x)} - x\sqrt{\sinh(x)}\right) dx = -\frac{2x\cosh(x)}{\sqrt{\sinh(x)}} + 4\sqrt{\sinh(x)}$$

output
```
-2*x*cosh(x)/sinh(x)^(1/2)+4*sinh(x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \left(\frac{x}{\sinh^{\frac{3}{2}}(x)} - x\sqrt{\sinh(x)}\right) dx = \frac{-2x\cosh(x) + 4\sinh(x)}{\sqrt{\sinh(x)}}$$

input
```
Integrate[x/Sinh[x]^(3/2) - x*Sqrt[Sinh[x]],x]
```

output
```
(-2*x*Cosh[x] + 4*Sinh[x])/Sqrt[Sinh[x]]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\sinh^{\frac{3}{2}}(x)}-x\sqrt{\sinh(x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$4\sqrt{\sinh(x)}-\frac{2x\cosh(x)}{\sqrt{\sinh(x)}}$$

input
```
Int[x/Sinh[x]^(3/2) - x*Sqrt[Sinh[x]],x]
```

output
```
(-2*x*Cosh[x])/Sqrt[Sinh[x]] + 4*Sqrt[Sinh[x]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{x}{\sinh\left(x\right)^{\frac{3}{2}}}-x\sqrt{\sinh\left(x\right)}\right)dx$$

input
```
int(x/sinh(x)^(3/2)-x*sinh(x)^(1/2),x)
```

output
```
int(x/sinh(x)^(3/2)-x*sinh(x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\sinh^{\frac{3}{2}}(x)}-x\sqrt{\sinh(x)}\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x/sinh(x)^(3/2)-x*sinh(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{x}{\sinh^{\frac{3}{2}}(x)}-x\sqrt{\sinh(x)}\right)\,dx=-\int \left(-\frac{x}{\sinh^{\frac{3}{2}}(x)}\right)\,dx-\int x\sqrt{\sinh(x)}\,dx$$

input
```
integrate(x/sinh(x)**(3/2)-x*sinh(x)**(1/2),x)
```

output
```
-Integral(-x/sinh(x)**(3/2), x) - Integral(x*sqrt(sinh(x)), x)
```

Maxima [F]

$$\int \left(\frac{x}{\sinh^{\frac{3}{2}}(x)}-x\sqrt{\sinh(x)}\right)\,dx=\int -x\sqrt{\sinh(x)}+\frac{x}{\sinh(x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/sinh(x)^(3/2)-x*sinh(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(-x*sqrt(sinh(x)) + x/sinh(x)^(3/2), x)
```

Giac [F]

$$\int \left(\frac{x}{\sinh^{\frac{3}{2}}(x)}-x\sqrt{\sinh(x)}\right)\,dx=\int -x\sqrt{\sinh\left(x\right)}+\frac{x}{\sinh\left(x\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/sinh(x)^(3/2)-x*sinh(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(-x*sqrt(sinh(x)) + x/sinh(x)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \left(\frac{x}{\sinh^{\frac{3}{2}}(x)}-x\sqrt{\sinh(x)}\right)\,dx=-\frac{2\sqrt{\frac{\mathrm{e}^{x}}{2}-\frac{\mathrm{e}^{-x}}{2}}\,\left(x-2\,\mathrm{e}^{2\,x}+x\,\mathrm{e}^{2\,x}+2\right)}{\mathrm{e}^{2\,x}-1}$$

input
```
int(x/sinh(x)^(3/2) - x*sinh(x)^(1/2),x)
```

output
```
-(2*(exp(x)/2 - exp(-x)/2)^(1/2)*(x - 2*exp(2*x) + x*exp(2*x) + 2))/(exp(2
*x) - 1)
```

Reduce [F]

$$\int \left(\frac{x}{\sinh^{\frac{3}{2}}(x)}-x\sqrt{\sinh(x)}\right)\,dx=\int \frac{\sqrt{\sinh\left(x\right)}\,x}{\sinh\left(x\right)^{2}}dx-\left(\int \sqrt{\sinh\left(x\right)}\,x dx\right)$$

input
```
int(x/sinh(x)^(3/2)-x*sinh(x)^(1/2),x)
```

output
```
int((sqrt(sinh(x))*x)/sinh(x)**2,x) - int(sqrt(sinh(x))*x,x)
```

3.69 $\int \left(\frac{x}{\sinh^{\frac{5}{2}}(x)}+\frac{x}{3\sqrt{\sinh(x)}}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 24

$$\int \left(\frac{x}{\sinh^{\frac{5}{2}}(x)}+\frac{x}{3\sqrt{\sinh(x)}}\right)\,dx=-\frac{2x\cosh(x)}{3\sinh^{\frac{3}{2}}(x)}-\frac{4}{3\sqrt{\sinh(x)}}$$

output
```
-2/3*x*cosh(x)/sinh(x)^(3/2)-4/3/sinh(x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \left(\frac{x}{\sinh^{\frac{5}{2}}(x)}+\frac{x}{3\sqrt{\sinh(x)}}\right)\,dx=\frac{1}{6}(-8\text{csch}(x)-4x\coth(x)\text{csch}(x))\sqrt{\sinh(x)}$$

input
```
Integrate[x/Sinh[x]^(5/2) + x/(3*Sqrt[Sinh[x]]),x]
```

output
```
((-8*Csch[x] - 4*x*Coth[x]*Csch[x])*Sqrt[Sinh[x]])/6
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\sinh^{\frac{5}{2}}(x)}+\frac{x}{3\sqrt{\sinh(x)}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{4}{3\sqrt{\sinh(x)}}-\frac{2x\cosh(x)}{3\sinh^{\frac{3}{2}}(x)}$$

input
```
Int[x/Sinh[x]^(5/2) + x/(3*Sqrt[Sinh[x]]),x]
```

output
```
(-2*x*Cosh[x])/(3*Sinh[x]^(3/2)) - 4/(3*Sqrt[Sinh[x]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{x}{\sinh(x)^{\frac{5}{2}}}+\frac{x}{3\sqrt{\sinh(x)}}\right)dx$$

input
```
int(x/sinh(x)^(5/2)+1/3*x/sinh(x)^(1/2),x)
```

output
```
int(x/sinh(x)^(5/2)+1/3*x/sinh(x)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(16) = 32$.

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 4.50

$$\int \left(\frac{x}{\sinh^{\frac{5}{2}}(x)}+\frac{x}{3\sqrt{\sinh(x)}}\right)\,dx = -\frac{4\left((x+2)\cosh(x)^3+3(x+2)\cosh(x)\sinh(x)^2+(x+2)\sinh(x)^3+(x-2)\cosh(x)+\left(3(x+2)\mathrm{c}\right.\right.}{3\left(\cosh(x)^4+4\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\cosh(x)^2-1\right)\sinh(x)^2-2\cosh(x)^2+4\left(\mathrm{c}\right.\right.}$$

input
```
integrate(x/sinh(x)^(5/2)+1/3*x/sinh(x)^(1/2),x, algorithm="fricas")
```

output
```
-4/3*((x + 2)*cosh(x)^3 + 3*(x + 2)*cosh(x)*sinh(x)^2 + (x + 2)*sinh(x)^3
+ (x - 2)*cosh(x) + (3*(x + 2)*cosh(x)^2 + x - 2)*sinh(x))*sqrt(sinh(x))/(
cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^
2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int \left(\frac{x}{\sinh^{\frac{5}{2}}(x)}+\frac{x}{3\sqrt{\sinh(x)}}\right)\,dx = \frac{\int \frac{3x}{\sinh^{\frac{5}{2}}(x)}\,dx+\int \frac{x}{\sqrt{\sinh(x)}}\,dx}{3}$$

input
```
integrate(x/sinh(x)**(5/2)+1/3*x/sinh(x)**(1/2),x)
```

output
```
(Integral(3*x/sinh(x)**(5/2), x) + Integral(x/sqrt(sinh(x)), x))/3
```

Maxima [F]

$$\int \left(\frac{x}{\sinh^{\frac{5}{2}}(x)}+\frac{x}{3\sqrt{\sinh(x)}}\right)\,dx=\int \frac{x}{3\,\sqrt{\sinh\left(x\right)}}+\frac{x}{\sinh\left(x\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/sinh(x)^(5/2)+1/3*x/sinh(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/3*x/sqrt(sinh(x)) + x/sinh(x)^(5/2), x)
```

Giac [F]

$$\int \left(\frac{x}{\sinh^{\frac{5}{2}}(x)}+\frac{x}{3\sqrt{\sinh(x)}}\right)\,dx=\int \frac{x}{3\,\sqrt{\sinh\left(x\right)}}+\frac{x}{\sinh\left(x\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/sinh(x)^(5/2)+1/3*x/sinh(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/3*x/sqrt(sinh(x)) + x/sinh(x)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \left(\frac{x}{\sinh^{\frac{5}{2}}(x)}+\frac{x}{3\sqrt{\sinh(x)}}\right)\,dx=-\frac{4\,{\mathrm{e}}^x\,\sqrt{\frac{{\mathrm{e}}^x}{2}-\frac{{\mathrm{e}}^{-x}}{2}}\,\left(x+2\,{\mathrm{e}}^{2\,x}+x\,{\mathrm{e}}^{2\,x}-2\right)}{3\,{\left({\mathrm{e}}^{2\,x}-1\right)}^2}$$

input
```
int(x/(3*sinh(x)^(1/2)) + x/sinh(x)^(5/2),x)
```

output
```
-(4*exp(x)*(exp(x)/2 - exp(-x)/2)^(1/2)*(x + 2*exp(2*x) + x*exp(2*x) - 2))
/(3*(exp(2*x) - 1)^2)
```

Reduce [F]

$$\int \left(\frac{x}{\sinh^{\frac{5}{2}}(x)}+\frac{x}{3\sqrt{\sinh(x)}}\right)\,dx=\int \frac{\sqrt{\sinh\left(x\right)}\,x}{\sinh\left(x\right)^{3}}dx+\frac{\left(\int \frac{\sqrt{\sinh(x)}\,x}{\sinh(x)}dx\right)}{3}$$

input
```
int(x/sinh(x)^(5/2)+1/3*x/sinh(x)^(1/2),x)
```

output
```
(3*int((sqrt(sinh(x))*x)/sinh(x)**3,x) + int((sqrt(sinh(x))*x)/sinh(x),x))
/3
```

3.70 $\int \left(\frac{x}{\sinh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\sinh(x)}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \left(\frac{x}{\sinh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\sinh(x)}\right)\,dx=-\frac{2x\cosh(x)}{5\sinh^{\frac{5}{2}}(x)}-\frac{4}{15\sinh^{\frac{3}{2}}(x)}+\frac{6x\cosh(x)}{5\sqrt{\sinh(x)}}-\frac{12\sqrt{\sinh(x)}}{5}$$

output
```
-2/5*x*cosh(x)/sinh(x)^(5/2)-4/15/sinh(x)^(3/2)+6/5*x*cosh(x)/sinh(x)^(1/2
)-12/5*sinh(x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \left(\frac{x}{\sinh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\sinh(x)}\right)\,dx=\frac{-21x\cosh(x)+9x\cosh(3x)+46\sinh(x)-18\sinh(3x)}{30\sinh^{\frac{5}{2}}(x)}$$

input
```
Integrate[x/Sinh[x]^(7/2) + (3*x*Sqrt[Sinh[x]])/5,x]
```

output
```
(-21*x*Cosh[x] + 9*x*Cosh[3*x] + 46*Sinh[x] - 18*Sinh[3*x])/(30*Sinh[x]^(5
/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\sinh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\sinh(x)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{4}{15\sinh^{\frac{3}{2}}(x)}-\frac{12\sqrt{\sinh(x)}}{5}-\frac{2x\cosh(x)}{5\sinh^{\frac{5}{2}}(x)}+\frac{6x\cosh(x)}{5\sqrt{\sinh(x)}}$$

input
```
Int[x/Sinh[x]^(7/2) + (3*x*Sqrt[Sinh[x]])/5,x]
```

output
```
(-2*x*Cosh[x])/(5*Sinh[x]^(5/2)) - 4/(15*Sinh[x]^(3/2)) + (6*x*Cosh[x])/(5
*Sqrt[Sinh[x]]) - (12*Sqrt[Sinh[x]])/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{x}{\sinh(x)^{\frac{7}{2}}}+\frac{3x\sqrt{\sinh(x)}}{5}\right)dx$$

input
```
int(x/sinh(x)^(7/2)+3/5*x*sinh(x)^(1/2),x)
```

output
```
int(x/sinh(x)^(7/2)+3/5*x*sinh(x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\sinh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\sinh(x)}\right)dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/sinh(x)^(7/2)+3/5*x*sinh(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{x}{\sinh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\sinh(x)}\right)dx = \frac{\int \frac{5x}{\sinh^{\frac{7}{2}}(x)}\,dx+\int 3x\sqrt{\sinh(x)}\,dx}{5}$$

input
```
integrate(x/sinh(x)**(7/2)+3/5*x*sinh(x)**(1/2),x)
```

output
```
(Integral(5*x/sinh(x)**(7/2), x) + Integral(3*x*sqrt(sinh(x)), x))/5
```

Maxima [F]

$$\int \left(\frac{x}{\sinh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\sinh(x)}\right)\,dx=\int \frac{3}{5}\,x\sqrt{\sinh\left(x\right)}+\frac{x}{\sinh\left(x\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x/sinh(x)^(7/2)+3/5*x*sinh(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(3/5*x*sqrt(sinh(x)) + x/sinh(x)^(7/2), x)
```

Giac [F]

$$\int \left(\frac{x}{\sinh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\sinh(x)}\right)\,dx=\int \frac{3}{5}\,x\sqrt{\sinh\left(x\right)}+\frac{x}{\sinh\left(x\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x/sinh(x)^(7/2)+3/5*x*sinh(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(3/5*x*sqrt(sinh(x)) + x/sinh(x)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.36

$$\int \left(\frac{x}{\sinh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\sinh(x)}\right)\,dx=\frac{12\,x\,\mathrm{e}^{2\,x}\,\sqrt{\frac{\mathrm{e}^x}{2}-\frac{\mathrm{e}^{-x}}{2}}}{5\,\left(\mathrm{e}^{2\,x}-1\right)}-\frac{\mathrm{e}^{2\,x}\,\left(\frac{8\,x}{5}+\frac{16}{15}\right)\,\sqrt{\frac{\mathrm{e}^x}{2}-\frac{\mathrm{e}^{-x}}{2}}}{{\left(\mathrm{e}^{2\,x}-1\right)}^2}-\left(\frac{6\,x}{5}+\frac{12}{5}\right)\,\sqrt{\frac{\mathrm{e}^x}{2}-\frac{\mathrm{e}^{-x}}{2}}-\frac{16\,x\,\mathrm{e}^{2\,x}\,\sqrt{\frac{\mathrm{e}^x}{2}-\frac{\mathrm{e}^{-x}}{2}}}{5\,{\left(\mathrm{e}^{2\,x}-1\right)}^3}$$

input
```
int((3*x*sinh(x)^(1/2))/5 + x/sinh(x)^(7/2),x)
```

output
```
(12*x*exp(2*x)*(exp(x)/2 - exp(-x)/2)^(1/2))/(5*(exp(2*x) - 1)) - (exp(2*x
)*((8*x)/5 + 16/15)*(exp(x)/2 - exp(-x)/2)^(1/2))/(exp(2*x) - 1)^2 - ((6*x
)/5 + 12/5)*(exp(x)/2 - exp(-x)/2)^(1/2) - (16*x*exp(2*x)*(exp(x)/2 - exp(
-x)/2)^(1/2))/(5*(exp(2*x) - 1)^3)
```

Reduce [F]

$$\int \left(\frac{x}{\sinh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\sinh(x)}\right) dx = \int \frac{\sqrt{\sinh (x)}\, x}{\sinh (x)^4} dx + \frac{3\left(\int \sqrt{\sinh (x)}\, x dx\right)}{5}$$

input
```
int(x/sinh(x)^(7/2)+3/5*x*sinh(x)^(1/2),x)
```

output
```
(5*int((sqrt(sinh(x))*x)/sinh(x)**4,x) + 3*int(sqrt(sinh(x))*x,x))/5
```

3.71 $\int \left(\frac{x^2}{\sinh^{\frac{3}{2}}(x)}-x^2\sqrt{\sinh(x)}\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 58

$$\int \left(\frac{x^2}{\sinh^{\frac{3}{2}}(x)}-x^2\sqrt{\sinh(x)}\right)\,dx=-\frac{2x^2\cosh(x)}{\sqrt{\sinh(x)}}+8x\sqrt{\sinh(x)}-\frac{16iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{\sinh(x)}}{\sqrt{i\sinh(x)}}$$

output
```
-2*x^2*cosh(x)/sinh(x)^(1/2)+8*x*sinh(x)^(1/2)-16*I*EllipticE(cos(1/4*Pi+1
/2*I*x),2^(1/2))*sinh(x)^(1/2)/(I*sinh(x))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.92 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.17

$$\int \left(\frac{x^2}{\sinh^{\frac{3}{2}}(x)}-x^2\sqrt{\sinh(x)}\right)\,dx=$$
$$-\frac{2\Big(x^2\cosh(x)-4(-2+x)\sinh(x)-8\sqrt{2}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},\cosh(2x)+\sinh(2x)\right)(-\cos}{\sqrt{\sinh(x)}}$$

input
```
Integrate[x^2/Sinh[x]^(3/2) - x^2*Sqrt[Sinh[x]],x]
```

output
```
(-2*(x^2*Cosh[x] - 4*(-2 + x)*Sinh[x] - 8*Sqrt[2]*Hypergeometric2F1[-1/4,
1/2, 3/4, Cosh[2*x] + Sinh[2*x]]*(-Cosh[x] + Sinh[x])*Sqrt[-(Sinh[x]*(Cosh
[x] + Sinh[x]))]))/Sqrt[Sinh[x]]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x^2}{\sinh^{\frac{3}{2}}(x)}-x^2\sqrt{\sinh(x)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2x^2\cosh(x)}{\sqrt{\sinh(x)}}+8x\sqrt{\sinh(x)}-\frac{16i\sqrt{\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{\sqrt{i\sinh(x)}}$$

input
```
Int[x^2/Sinh[x]^(3/2) - x^2*Sqrt[Sinh[x]],x]
```

output
```
(-2*x^2*Cosh[x])/Sqrt[Sinh[x]] + 8*x*Sqrt[Sinh[x]] - ((16*I)*EllipticE[Pi/
4 - (I/2)*x, 2]*Sqrt[Sinh[x]])/Sqrt[I*Sinh[x]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{x^2}{\sinh(x)^{\frac{3}{2}}} - x^2\sqrt{\sinh(x)}\right) dx$$

input
```
int(x^2/sinh(x)^(3/2)-x^2*sinh(x)^(1/2),x)
```

output
```
int(x^2/sinh(x)^(3/2)-x^2*sinh(x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x^2}{\sinh^{\frac{3}{2}}(x)} - x^2\sqrt{\sinh(x)}\right) dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/sinh(x)^(3/2)-x^2*sinh(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{x^2}{\sinh^{\frac{3}{2}}(x)} - x^2\sqrt{\sinh(x)}\right) dx = -\int \left(-\frac{x^2}{\sinh^{\frac{3}{2}}(x)}\right) dx - \int x^2\sqrt{\sinh(x)}\, dx$$

input
```
integrate(x**2/sinh(x)**(3/2)-x**2*sinh(x)**(1/2),x)
```

output
```
-Integral(-x**2/sinh(x)**(3/2), x) - Integral(x**2*sqrt(sinh(x)), x)
```

Maxima [F]

$$\int \left(\frac{x^2}{\sinh^{\frac{3}{2}}(x)}-x^2\sqrt{\sinh(x)}\right)\,dx=\int -x^2\sqrt{\sinh\left(x\right)}+\frac{x^2}{\sinh\left(x\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/sinh(x)^(3/2)-x^2*sinh(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(-x^2*sqrt(sinh(x)) + x^2/sinh(x)^(3/2), x)
```

Giac [F]

$$\int \left(\frac{x^2}{\sinh^{\frac{3}{2}}(x)}-x^2\sqrt{\sinh(x)}\right)\,dx=\int -x^2\sqrt{\sinh\left(x\right)}+\frac{x^2}{\sinh\left(x\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/sinh(x)^(3/2)-x^2*sinh(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(-x^2*sqrt(sinh(x)) + x^2/sinh(x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x^2}{\sinh^{\frac{3}{2}}(x)}-x^2\sqrt{\sinh(x)}\right)\,dx=-\int x^2\,\sqrt{\mathrm{sinh}\left(x\right)}-\frac{x^2}{{\mathrm{sinh}\left(x\right)}^{3/2}}\,dx$$

input
```
int(x^2/sinh(x)^(3/2) - x^2*sinh(x)^(1/2),x)
```

output
```
-int(x^2*sinh(x)^(1/2) - x^2/sinh(x)^(3/2), x)
```

Reduce [F]

$$\int \left(\frac{x^2}{\sinh^{\frac{3}{2}}(x)} - x^2\sqrt{\sinh(x)}\right) dx = \int \frac{\sqrt{\sinh(x)}\, x^2}{\sinh(x)^2} dx - \left(\int \sqrt{\sinh(x)}\, x^2 dx\right)$$

input
```
int(x^2/sinh(x)^(3/2)-x^2*sinh(x)^(1/2),x)
```

output
```
int((sqrt(sinh(x))*x**2)/sinh(x)**2,x) - int(sqrt(sinh(x))*x**2,x)
```

3.72 $\int (c+dx)^m (b\sinh(e+fx))^n \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (c+dx)^m (b\sinh(e+fx))^n \, dx = \text{Int}((c+dx)^m (b\sinh(e+fx))^n, x)$$

output
```
Defer(Int)((d*x+c)^m*(b*sinh(f*x+e))^n,x)
```

Mathematica [N/A]

Not integrable

Time = 2.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m (b\sinh(e+fx))^n \, dx = \int (c+dx)^m (b\sinh(e+fx))^n \, dx$$

input
```
Integrate[(c + d*x)^m*(b*Sinh[e + f*x])^n,x]
```

output
```
Integrate[(c + d*x)^m*(b*Sinh[e + f*x])^n, x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(b\sinh(e+fx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)^m(-ib\sin(ie+ifx))^ndx$$

$$\Big\downarrow \text{ 3807}$$

$$\int (c+dx)^m(b\sinh(e+fx))^ndx$$

input
```
Int[(c + d*x)^m*(b*Sinh[e + f*x])^n,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx+c)^m\left(b\sinh\left(fx+e\right)\right)^n dx$$

input
```
int((d*x+c)^m*(b*sinh(f*x+e))^n,x)
```

output
```
int((d*x+c)^m*(b*sinh(f*x+e))^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m(b\sinh(e+fx))^n\,dx = \int (dx+c)^m(b\sinh(fx+e))^n\,dx$$

input
```
integrate((d*x+c)^m*(b*sinh(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*(b*sinh(f*x + e))^n, x)
```

Sympy [N/A]

Not integrable

Time = 11.88 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (c+dx)^m(b\sinh(e+fx))^n\,dx = \int (b\sinh(e+fx))^n\,(c+dx)^m\,dx$$

input
```
integrate((d*x+c)**m*(b*sinh(f*x+e))**n,x)
```

output
```
Integral((b*sinh(e + f*x))**n*(c + d*x)**m, x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m(b\sinh(e+fx))^n\,dx = \int (dx+c)^m(b\sinh(fx+e))^n\,dx$$

input
```
integrate((d*x+c)^m*(b*sinh(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*(b*sinh(f*x + e))^n, x)
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m(b\sinh(e+fx))^n\,dx=\int (dx+c)^m(b\sinh{(fx+e)})^n\,dx$$

input
```
integrate((d*x+c)^m*(b*sinh(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*(b*sinh(f*x + e))^n, x)
```

Mupad [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m(b\sinh(e+fx))^n\,dx=\int (b\sinh(e+f\,x))^n\,(c+d\,x)^m\,dx$$

input
```
int((b*sinh(e + f*x))^n*(c + d*x)^m,x)
```

output
```
int((b*sinh(e + f*x))^n*(c + d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int (c+dx)^m(b\sinh(e+fx))^n\,dx = b^n\left(\int \sinh\left(fx+e\right)^n\left(dx+c\right)^m dx\right)$$

input
```
int((d*x+c)^m*(b*sinh(f*x+e))^n,x)
```

output
```
b**n*int(sinh(e + f*x)**n*(c + d*x)**m,x)
```

3.73 $\int (c+dx)^m \sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 237

$$\begin{aligned}\int (c+dx)^m \sinh^3(a+bx)\,dx &= \frac{3^{-1-m}e^{3a-\frac{3bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3b(c+dx)}{d}\right)}{8b}\\ &\quad-\frac{3e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{b(c+dx)}{d}\right)}{8b}\\ &\quad-\frac{3e^{-a+\frac{bc}{d}}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{b(c+dx)}{d}\right)}{8b}\\ &\quad+\frac{3^{-1-m}e^{-3a+\frac{3bc}{d}}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{3b(c+dx)}{d}\right)}{8b}\end{aligned}$$

output

```
1/8*3^(-1-m)*exp(3*a-3*b*c/d)*(d*x+c)^m*GAMMA(1+m,-3*b*(d*x+c)/d)/b/((-b*(
d*x+c)/d)^m)-3/8*exp(a-b*c/d)*(d*x+c)^m*GAMMA(1+m,-b*(d*x+c)/d)/b/((-b*(d*
x+c)/d)^m)-3/8*exp(-a+b*c/d)*(d*x+c)^m*GAMMA(1+m,b*(d*x+c)/d)/b/((b*(d*x+c
)/d)^m)+1/8*3^(-1-m)*exp(-3*a+3*b*c/d)*(d*x+c)^m*GAMMA(1+m,3*b*(d*x+c)/d)/
b/((b*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.87

$$\int (c+dx)^m \sinh^3(a+bx)\,dx$$
$$= \frac{3^{-1-m}e^{-3\left(a+\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(e^{6a}\left(b\left(\frac{c}{d}+x\right)\right)^m\Gamma\left(1+m,-\frac{3b(c+dx)}{d}\right)-3^{2+m}e^{4a+\frac{2bc}{d}}\left(b\left(\frac{c}{d}+x\right.\right.\right.}{8b}$$

input
```
Integrate[(c + d*x)^m*Sinh[a + b*x]^3,x]
```

output
```
(3^(-1 - m)*(c + d*x)^m*(E^(6*a)*(b*(c/d + x))^m*Gamma[1 + m, (-3*b*(c + d
*x))/d] - 3^(2 + m)*E^(4*a + (2*b*c)/d)*(b*(c/d + x))^m*Gamma[1 + m, -((b*
(c + d*x))/d)] + E^((4*b*c)/d)*(-((b*(c + d*x))/d))^m*(-(3^(2 + m)*E^(2*a)
*Gamma[1 + m, (b*(c + d*x))/d]) + E^((2*b*c)/d)*Gamma[1 + m, (3*b*(c + d*x
))/d])))/(8*b*E^(3*(a + (b*c)/d))*(-((b^2*(c + d*x)^2)/d^2))^m)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 26, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(a+bx)(c+dx)^m\,dx$$
$$\Big\downarrow \; 3042$$
$$\int i\sin(ia+ibx)^3(c+dx)^m dx$$
$$\Big\downarrow \; 26$$
$$i\int (c+dx)^m \sin(ia+ibx)^3 dx$$

$$\Big\downarrow \text{3793}$$

$$i\int \left(\frac{3}{4}i(c+dx)^m\sinh(a+bx)-\frac{1}{4}i(c+dx)^m\sinh(3a+3bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$i\left(-\frac{i3^{-m-1}e^{3a-\frac{3bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3b(c+dx)}{d}\right)}{8b}+\frac{3ie^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\right.}{8b}\right.$$

input
```
Int[(c + d*x)^m*Sinh[a + b*x]^3,x]
```

output
```
I*(((-1/8*I)*3^(-1 - m)*E^(3*a - (3*b*c)/d)*(c + d*x)^m*Gamma[1 + m, (-3*b
*(c + d*x))/d])/(b*(-((b*(c + d*x))/d))^m) + (((3*I)/8)*E^(a - (b*c)/d)*(c
 + d*x)^m*Gamma[1 + m, -((b*(c + d*x))/d)])/(b*(-((b*(c + d*x))/d))^m) + (
((3*I)/8)*E^(-a + (b*c)/d)*(c + d*x)^m*Gamma[1 + m, (b*(c + d*x))/d])/(b*(
(b*(c + d*x))/d)^m) - ((I/8)*3^(-1 - m)*E^(-3*a + (3*b*c)/d)*(c + d*x)^m*G
amma[1 + m, (3*b*(c + d*x))/d])/(b*((b*(c + d*x))/d)^m))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int (dx+c)^m \sinh (bx+a)^3 \, dx$$

input
```
int((d*x+c)^m*sinh(b*x+a)^3,x)
```

output
```
int((d*x+c)^m*sinh(b*x+a)^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.43

$$\int (c+dx)^m \sinh^3(a+bx) \, dx = \frac{\cosh\left(\frac{dm\log\left(\frac{3b}{d}\right)-3bc+3ad}{d}\right)\Gamma\left(m+1, \frac{3(bdx+bc)}{d}\right) - 9\cosh\left(\frac{dm\log\left(\frac{b}{d}\right)-bc+ad}{d}\right)\Gamma\left(m+1, \frac{bdx+bc}{d}\right) - 9\cosh\left(\right.}{}$$

input
```
integrate((d*x+c)^m*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/24*(cosh((d*m*log(3*b/d) - 3*b*c + 3*a*d)/d)*gamma(m + 1, 3*(b*d*x + b*c
)/d) - 9*cosh((d*m*log(b/d) - b*c + a*d)/d)*gamma(m + 1, (b*d*x + b*c)/d)
- 9*cosh((d*m*log(-b/d) + b*c - a*d)/d)*gamma(m + 1, -(b*d*x + b*c)/d) + c
osh((d*m*log(-3*b/d) + 3*b*c - 3*a*d)/d)*gamma(m + 1, -3*(b*d*x + b*c)/d)
- gamma(m + 1, 3*(b*d*x + b*c)/d)*sinh((d*m*log(3*b/d) - 3*b*c + 3*a*d)/d)
 + 9*gamma(m + 1, (b*d*x + b*c)/d)*sinh((d*m*log(b/d) - b*c + a*d)/d) + 9*
gamma(m + 1, -(b*d*x + b*c)/d)*sinh((d*m*log(-b/d) + b*c - a*d)/d) - gamma
(m + 1, -3*(b*d*x + b*c)/d)*sinh((d*m*log(-3*b/d) + 3*b*c - 3*a*d)/d))/b
```

Sympy [F]

$$\int (c+dx)^m \sinh^3(a+bx)\,dx = \int (c+dx)^m \sinh^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*sinh(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**m*sinh(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.68

$$\int (c+dx)^m \sinh^3(a+bx)\,dx = \frac{(dx+c)^{m+1}e^{\left(-3\,a+\frac{3\,bc}{d}\right)}E_{-m}\left(\frac{3\,(dx+c)b}{d}\right)}{8\,d} - \frac{3\,(dx+c)^{m+1}e^{\left(-a+\frac{bc}{d}\right)}E_{-m}\left(\frac{(dx+c)b}{d}\right)}{8\,d} + \frac{3\,(dx+c)^{m+1}e^{\left(a-\frac{bc}{d}\right)}E_{-m}\left(-\frac{(dx+c)b}{d}\right)}{8\,d} - \frac{(dx+c)^{m+1}e^{\left(3\,a-\frac{3\,bc}{d}\right)}E_{-m}\left(-\frac{3\,(dx+c)b}{d}\right)}{8\,d}$$

input
```
integrate((d*x+c)^m*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/8*(d*x + c)^(m + 1)*e^(-3*a + 3*b*c/d)*exp_integral_e(-m, 3*(d*x + c)*b/
d)/d - 3/8*(d*x + c)^(m + 1)*e^(-a + b*c/d)*exp_integral_e(-m, (d*x + c)*b
/d)/d + 3/8*(d*x + c)^(m + 1)*e^(a - b*c/d)*exp_integral_e(-m, -(d*x + c)*
b/d)/d - 1/8*(d*x + c)^(m + 1)*e^(3*a - 3*b*c/d)*exp_integral_e(-m, -3*(d*
x + c)*b/d)/d
```

Giac [F]

$$\int (c+dx)^m \sinh^3(a+bx)\,dx = \int (dx+c)^m \sinh{(bx+a)}^3\,dx$$

input
```
integrate((d*x+c)^m*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*sinh(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \sinh^3(a+bx)\,dx = \int \sinh(a+b\,x)^3\,(c+d\,x)^m\,dx$$

input
```
int(sinh(a + b*x)^3*(c + d*x)^m,x)
```

output
```
int(sinh(a + b*x)^3*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m \sinh^3(a+bx)\,dx$$
$$= \frac{e^{6bx+6a}(dx+c)^m - 9e^{4bx+4a}(dx+c)^m - 9e^{2bx+2a}(dx+c)^m + (dx+c)^m - e^{3bx+6a}\left(\int \frac{e^{3bx}(dx+c)^m}{dx+c}dx\right)dm}{24e^{3bx+}}$$

input
```
int((d*x+c)^m*sinh(b*x+a)^3,x)
```

output

```
(e**(6*a + 6*b*x)*(c + d*x)**m - 9*e**(4*a + 4*b*x)*(c + d*x)**m - 9*e**(2
*a + 2*b*x)*(c + d*x)**m + (c + d*x)**m - e**(6*a + 3*b*x)*int((e**(3*b*x)
*(c + d*x)**m)/(c + d*x),x)*d*m + 9*e**(4*a + 3*b*x)*int((e**(b*x)*(c + d*
x)**m)/(c + d*x),x)*d*m - e**(3*a + 3*b*x)*int((c + d*x)**m/(e**(3*a + 3*b
*x)*c + e**(3*a + 3*b*x)*d*x),x)*d*m + 9*e**(2*a + 3*b*x)*int((c + d*x)**m
/(e**(b*x)*c + e**(b*x)*d*x),x)*d*m)/(24*e**(3*a + 3*b*x)*b)
```

3.74 $\int (c+dx)^m \sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 144

$$\int (c+dx)^m \sinh^2(a+bx)\,dx = -\frac{(c+dx)^{1+m}}{2d(1+m)} + \frac{2^{-3-m}e^{2a-\frac{2bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2b(c+dx)}{d}\right)}{b} - \frac{2^{-3-m}e^{-2a+\frac{2bc}{d}}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2b(c+dx)}{d}\right)}{b}$$

output
```
-1/2*(d*x+c)^(1+m)/d/(1+m)+2^(-3-m)*exp(2*a-2*b*c/d)*(d*x+c)^m*GAMMA(1+m,-
2*b*(d*x+c)/d)/b/((-b*(d*x+c)/d)^m)-2^(-3-m)*exp(-2*a+2*b*c/d)*(d*x+c)^m*G
AMMA(1+m,2*b*(d*x+c)/d)/b/((b*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.91

$$\int (c+dx)^m \sinh^2(a+bx)\,dx = \frac{1}{8}(c+dx)^m \left(-\frac{4(c+dx)}{d(1+m)} + \frac{2^{-m}e^{2a-\frac{2bc}{d}}\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2b(c+dx)}{d}\right)}{b} - \frac{2^{-m}e^{-2a+\frac{2bc}{d}}\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2b(c+dx)}{d}\right)}{b}\right)$$

input
```
Integrate[(c + d*x)^m*Sinh[a + b*x]^2,x]
```

output
```
((c + d*x)^m*((-4*(c + d*x))/(d*(1 + m)) + (E^(2*a - (2*b*c)/d)*Gamma[1 +
m, (-2*b*(c + d*x))/d])/(2^m*b*(-((b*(c + d*x))/d))^m) - (E^(-2*a + (2*b*c
)/d)*Gamma[1 + m, (2*b*(c + d*x))/d])/(2^m*b*((b*(c + d*x))/d)^m)))/8
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ia+ibx)^2\left(-(c+dx)^m\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)^m \sin(ia+ibx)^2 dx$$

$$\Big\downarrow \text{3793}$$

$$-\int \left(\frac{1}{2}(c+dx)^m - \frac{1}{2}(c+dx)^m \cosh(2a+2bx)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2^{-m-3}e^{2a-\frac{2bc}{d}}(c+dx)^m \left(-\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,-\frac{2b(c+dx)}{d}\right)}{b} - \frac{2^{-m-3}e^{\frac{2bc}{d}-2a}(c+dx)^m \left(\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,\frac{2b(c+dx)}{d}\right)}{b} - \frac{(c+dx)^{m+1}}{2d(m+1)}$$

input
```
Int[(c + d*x)^m*Sinh[a + b*x]^2,x]
```

output
```
-1/2*(c + d*x)^(1 + m)/(d*(1 + m)) + (2^(-3 - m)*E^(2*a - (2*b*c)/d)*(c +
d*x)^m*Gamma[1 + m, (-2*b*(c + d*x))/d])/(b*(-((b*(c + d*x))/d))^m) - (2^(
-3 - m)*E^(-2*a + (2*b*c)/d)*(c + d*x)^m*Gamma[1 + m, (2*b*(c + d*x))/d])/
(b*((b*(c + d*x))/d)^m)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int (dx+c)^m \sinh(bx+a)^2\, dx$$

input
```
int((d*x+c)^m*sinh(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*sinh(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.67

$$\int (c+dx)^m \sinh^2(a+bx)\, dx = -\frac{(dm+d)\cosh\left(\frac{dm\log\left(\frac{2b}{d}\right)-2bc+2ad}{d}\right)\Gamma\left(m+1,\frac{2(bdx+bc)}{d}\right)-(dm+d)\cosh\left(\frac{dm\log\left(-\frac{2b}{d}\right)+2bc-2ad}{d}\right)\Gamma\left(m\right.}{}$$

input
```
integrate((d*x+c)^m*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/8*((d*m + d)*cosh((d*m*log(2*b/d) - 2*b*c + 2*a*d)/d)*gamma(m + 1, 2*(b
*d*x + b*c)/d) - (d*m + d)*cosh((d*m*log(-2*b/d) + 2*b*c - 2*a*d)/d)*gamma
(m + 1, -2*(b*d*x + b*c)/d) - (d*m + d)*gamma(m + 1, 2*(b*d*x + b*c)/d)*si
nh((d*m*log(2*b/d) - 2*b*c + 2*a*d)/d) + (d*m + d)*gamma(m + 1, -2*(b*d*x
+ b*c)/d)*sinh((d*m*log(-2*b/d) + 2*b*c - 2*a*d)/d) + 4*(b*d*x + b*c)*cosh
(m*log(d*x + c)) + 4*(b*d*x + b*c)*sinh(m*log(d*x + c)))/(b*d*m + b*d)
```

Sympy [F]

$$\int (c+dx)^m \sinh^2(a+bx)\,dx = \int (c+dx)^m \sinh^2{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**m*sinh(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**m*sinh(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.71

$$\int (c+dx)^m \sinh^2(a+bx)\,dx = -\frac{(dx+c)^{m+1}e^{\left(-2\,a+\frac{2\,bc}{d}\right)}E_{-m}\left(\frac{2\,(dx+c)b}{d}\right)}{4\,d} - \frac{(dx+c)^{m+1}e^{\left(2\,a-\frac{2\,bc}{d}\right)}E_{-m}\left(-\frac{2\,(dx+c)b}{d}\right)}{4\,d} - \frac{(dx+c)^{m+1}}{2\,d(m+1)}$$

input
```
integrate((d*x+c)^m*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(d*x + c)^(m + 1)*e^(-2*a + 2*b*c/d)*exp_integral_e(-m, 2*(d*x + c)*b
/d)/d - 1/4*(d*x + c)^(m + 1)*e^(2*a - 2*b*c/d)*exp_integral_e(-m, -2*(d*x
 + c)*b/d)/d - 1/2*(d*x + c)^(m + 1)/(d*(m + 1))
```

Giac [F]

$$\int (c+dx)^m \sinh^2(a+bx)\,dx = \int (dx+c)^m \sinh{(bx+a)}^2\,dx$$

input
```
integrate((d*x+c)^m*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*sinh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \sinh^2(a+bx)\,dx = \int \sinh(a+b\,x)^2\,(c+d\,x)^m\,dx$$

input
```
int(sinh(a + b*x)^2*(c + d*x)^m,x)
```

output
```
int(sinh(a + b*x)^2*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m \sinh^2(a+bx)\,dx$$
$$= \frac{e^{4bx+4a}(dx+c)^m\,dm + e^{4bx+4a}(dx+c)^m\,d - 4e^{2bx+2a}(dx+c)^m\,bc - 4e^{2bx+2a}(dx+c)^m\,bdx - (dx+c)^m\,d}{}$$

input
```
int((d*x+c)^m*sinh(b*x+a)^2,x)
```

output
```
(e**(4*a + 4*b*x)*(c + d*x)**m*d*m + e**(4*a + 4*b*x)*(c + d*x)**m*d - 4*e
**(2*a + 2*b*x)*(c + d*x)**m*b*c - 4*e**(2*a + 2*b*x)*(c + d*x)**m*b*d*x -
 (c + d*x)**m*d*m - (c + d*x)**m*d - e**(4*a + 2*b*x)*int((e**(2*b*x)*(c +
 d*x)**m)/(c + d*x),x)*d**2*m**2 - e**(4*a + 2*b*x)*int((e**(2*b*x)*(c + d
*x)**m)/(c + d*x),x)*d**2*m + e**(2*a + 2*b*x)*int((c + d*x)**m/(e**(2*a +
 2*b*x)*c + e**(2*a + 2*b*x)*d*x),x)*d**2*m**2 + e**(2*a + 2*b*x)*int((c +
 d*x)**m/(e**(2*a + 2*b*x)*c + e**(2*a + 2*b*x)*d*x),x)*d**2*m)/(8*e**(2*a
 + 2*b*x)*b*d*(m + 1))
```

3.75 $\int (c+dx)^m \sinh(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 110

$$\int (c+dx)^m \sinh(a+bx)\,dx = \frac{e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{b(c+dx)}{d}\right)}{2b} + \frac{e^{-a+\frac{bc}{d}}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{b(c+dx)}{d}\right)}{2b}$$

output
```
1/2*exp(a-b*c/d)*(d*x+c)^m*GAMMA(1+m,-b*(d*x+c)/d)/b/((-b*(d*x+c)/d)^m)+1/
2*exp(-a+b*c/d)*(d*x+c)^m*GAMMA(1+m,b*(d*x+c)/d)/b/((b*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.92

$$\int (c+dx)^m \sinh(a+bx)\,dx = \frac{e^{-a-\frac{bc}{d}}(c+dx)^m\left(e^{2a}\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{b(c+dx)}{d}\right)+e^{\frac{2bc}{d}}\left(b\left(\frac{c}{d}+x\right)\right)^{-m}\Gamma\left(1+m,\frac{b(c+dx)}{d}\right)\right)}{2b}$$

input
```
Integrate[(c + d*x)^m*Sinh[a + b*x],x]
```

output
```
(E^(-a - (b*c)/d)*(c + d*x)^m*((E^(2*a)*Gamma[1 + m, -((b*(c + d*x))/d)])/
(-((b*(c + d*x))/d))^m + (E^((2*b*c)/d)*Gamma[1 + m, (b*(c + d*x))/d])/(b*
(c/d + x))^m))/(2*b)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 26, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ia+ibx)(c+dx)^m dx$$

$$\Big\downarrow \text{26}$$

$$-i\int (c+dx)^m\sin(ia+ibx)dx$$

$$\Big\downarrow \text{3789}$$

$$-i\left(\frac{1}{2}i\int e^{a+bx}(c+dx)^m dx-\frac{1}{2}i\int e^{-a-bx}(c+dx)^m dx\right)$$

$$\Big\downarrow \text{2612}$$

$$-i\left(\frac{ie^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{b(c+dx)}{d}\right)}{2b}+\frac{ie^{\frac{bc}{d}-a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{b(c+dx)}{d}\right)}{2b}\right)$$

input
```
Int[(c + d*x)^m*Sinh[a + b*x],x]
```

output
```
(-I)*(((I/2)*E^(a - (b*c)/d)*(c + d*x)^m*Gamma[1 + m, -((b*(c + d*x))/d)])
/(b*(-((b*(c + d*x))/d))^m) + ((I/2)*E^(-a + (b*c)/d)*(c + d*x)^m*Gamma[1
+ m, (b*(c + d*x))/d])/(b*((b*(c + d*x))/d)^m))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Maple [F]

$$\int (dx + c)^m \sinh (bx + a) \, dx$$

input
```
int((d*x+c)^m*sinh(b*x+a),x)
```

output
```
int((d*x+c)^m*sinh(b*x+a),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.53

$$\int (c+dx)^m \sinh(a+bx)\,dx = \frac{\cosh\left(\frac{dm\log\left(\frac{b}{d}\right)-bc+ad}{d}\right)\Gamma\left(m+1,\frac{bdx+bc}{d}\right)+\cosh\left(\frac{dm\log\left(-\frac{b}{d}\right)+bc-ad}{d}\right)\Gamma\left(m+1,-\frac{bdx+bc}{d}\right)-\Gamma\left(m+1,\frac{bdx+b}{d}\right.}{2b}$$

input
```
integrate((d*x+c)^m*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(cosh((d*m*log(b/d) - b*c + a*d)/d)*gamma(m + 1, (b*d*x + b*c)/d) + co
sh((d*m*log(-b/d) + b*c - a*d)/d)*gamma(m + 1, -(b*d*x + b*c)/d) - gamma(m
 + 1, (b*d*x + b*c)/d)*sinh((d*m*log(b/d) - b*c + a*d)/d) - gamma(m + 1, -
(b*d*x + b*c)/d)*sinh((d*m*log(-b/d) + b*c - a*d)/d))/b
```

Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m \sinh(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)**m*sinh(b*x+a),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

$$\int (c+dx)^m \sinh(a+bx)\,dx = \frac{(dx+c)^{m+1}e^{\left(-a+\frac{bc}{d}\right)}E_{-m}\left(\frac{(dx+c)b}{d}\right)}{2\,d} - \frac{(dx+c)^{m+1}e^{\left(a-\frac{bc}{d}\right)}E_{-m}\left(-\frac{(dx+c)b}{d}\right)}{2\,d}$$

input
```
integrate((d*x+c)^m*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(d*x + c)^(m + 1)*e^(-a + b*c/d)*exp_integral_e(-m, (d*x + c)*b/d)/d -
 1/2*(d*x + c)^(m + 1)*e^(a - b*c/d)*exp_integral_e(-m, -(d*x + c)*b/d)/d
```

Giac [F]

$$\int (c+dx)^m \sinh(a+bx)\,dx = \int (dx+c)^m \sinh(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \sinh(a+bx)\,dx = \int \sinh(a+b\,x)\,(c+d\,x)^m\,dx$$

input
```
int(sinh(a + b*x)*(c + d*x)^m,x)
```

output
```
int(sinh(a + b*x)*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m \sinh(a+bx)\,dx$$
$$= \frac{e^{2bx+2a}(dx+c)^m + (dx+c)^m - e^{bx+2a}\left(\int \frac{e^{bx}(dx+c)^m}{dx+c}dx\right)dm - e^{bx}\left(\int \frac{(dx+c)^m}{e^{bx}c+e^{bx}dx}dx\right)dm}{2e^{bx+a}b}$$

input

```
int((d*x+c)^m*sinh(b*x+a),x)
```

output

```
(e**(2*a + 2*b*x)*(c + d*x)**m + (c + d*x)**m - e**(2*a + b*x)*int((e**(b*
x)*(c + d*x)**m)/(c + d*x),x)*d*m - e**(b*x)*int((c + d*x)**m/(e**(b*x)*c
+ e**(b*x)*d*x),x)*d*m)/(2*e**(a + b*x)*b)
```

3.76 $\int (c+dx)^m \textbf{csch}(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (c+dx)^m \text{csch}(a+bx)\,dx = \text{Int}((c+dx)^m \text{csch}(a+bx),x)$$

output
```
Defer(Int)((d*x+c)^m*csch(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 11.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \text{csch}(a+bx)\,dx = \int (c+dx)^m \text{csch}(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Csch[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Csch[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int i\csc(ia+ibx)(c+dx)^m dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int (c+dx)^m\csc(ia+ibx)dx$$

$$\Big\downarrow \text{ 4680}$$

$$\int \operatorname{csch}(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Csch[a + b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \operatorname{csch}(bx+a)\,dx$$

input
```
int((d*x+c)^m*csch(b*x+a),x)
```

output
```
int((d*x+c)^m*csch(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \text{csch}(a+bx)\,dx = \int (dx+c)^m \operatorname{csch}(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*csch(b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*csch(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \text{csch}(a+bx)\,dx = \int (c+dx)^m \operatorname{csch}(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*csch(b*x+a),x)
```

output
```
Integral((c + d*x)**m*csch(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \operatorname{csch}(a+bx)\,dx = \int (dx+c)^m \operatorname{csch}(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*csch(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*csch(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \operatorname{csch}(a+bx)\,dx = \int (dx+c)^m \operatorname{csch}(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*csch(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*csch(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int (c+dx)^m \operatorname{csch}(a+bx)\,dx = \int \frac{(c+d\,x)^m}{\sinh(a+b\,x)}\,dx$$

input
```
int((c + d*x)^m/sinh(a + b*x),x)
```

output
```
int((c + d*x)^m/sinh(a + b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \operatorname{csch}(a+bx)\,dx = \int (dx+c)^m \operatorname{csch}(bx+a)\,dx$$

input
```
int((d*x+c)^m*csch(b*x+a),x)
```

output
```
int((c + d*x)**m*csch(a + b*x),x)
```

3.77 $\int (c+dx)^m \operatorname{csch}^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (c+dx)^m \operatorname{csch}^2(a+bx)\,dx = \operatorname{Int}\left((c+dx)^m \operatorname{csch}^2(a+bx),x\right)$$

output
```
Defer(Int)((d*x+c)^m*csch(b*x+a)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 2.88 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \operatorname{csch}^2(a+bx)\,dx = \int (c+dx)^m \operatorname{csch}^2(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Csch[a + b*x]^2,x]
```

output
```
Integrate[(c + d*x)^m*Csch[a + b*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(ia+ibx)^2\left(-(c+dx)^m\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)^m\csc(ia+ibx)^2dx$$

$$\Big\downarrow \text{4680}$$

$$\int \operatorname{csch}^2(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Csch[a + b*x]^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \operatorname{csch}(bx+a)^2\,dx$$

input
```
int((d*x+c)^m*csch(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*csch(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \text{csch}^2(a+bx)\,dx = \int (dx+c)^m \,\text{csch}\,(bx+a)^2 \,dx$$

input
```
integrate((d*x+c)^m*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*csch(b*x + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (c+dx)^m \text{csch}^2(a+bx)\,dx = \int (c+dx)^m \,\text{csch}^2\,(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*csch(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**m*csch(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \operatorname{csch}^2(a+bx)\,dx = \int (dx+c)^m \operatorname{csch}(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*csch(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \operatorname{csch}^2(a+bx)\,dx = \int (dx+c)^m \operatorname{csch}(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*csch(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \operatorname{csch}^2(a+bx)\,dx = \int \frac{(c+d\,x)^m}{\sinh(a+b\,x)^2}\,dx$$

input
```
int((c + d*x)^m/sinh(a + b*x)^2,x)
```

output
```
int((c + d*x)^m/sinh(a + b*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 159, normalized size of antiderivative = 9.94

$$\int (c+dx)^m \text{csch}^2(a+bx)\,dx$$
$$= \frac{2e^{2a}\left(-e^{2bx}(dx+c)^m + e^{2bx+2a}\left(\int \frac{e^{2bx}(dx+c)^m}{e^{2bx+2a}c+e^{2bx+2a}dx-c-dx}dx\right)dm - \left(\int \frac{e^{2bx}(dx+c)^m}{e^{2bx+2a}c+e^{2bx+2a}dx-c-dx}dx\right)dm\right)}{b\left(e^{2bx+2a}-1\right)}$$

input
```
int((d*x+c)^m*csch(b*x+a)^2,x)
```

output
```
(2*e**(2*a)*( - e**(2*b*x)*(c + d*x)**m + e**(2*a + 2*b*x)*int((e**(2*b*x)
*(c + d*x)**m)/(e**(2*a + 2*b*x)*c + e**(2*a + 2*b*x)*d*x - c - d*x),x)*d*
m - int((e**(2*b*x)*(c + d*x)**m)/(e**(2*a + 2*b*x)*c + e**(2*a + 2*b*x)*d
*x - c - d*x),x)*d*m))/(b*(e**(2*a + 2*b*x) - 1))
```

3.78 $\int x^{3+m}\sinh(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 59

$$\int x^{3+m}\sinh(a+bx)\,dx=-\frac{e^{a}x^{m}(-bx)^{-m}\Gamma(4+m,-bx)}{2b^{4}}+\frac{e^{-a}x^{m}(bx)^{-m}\Gamma(4+m,bx)}{2b^{4}}$$

output
```
-1/2*exp(a)*x^m*GAMMA(4+m,-b*x)/b^4/((-b*x)^m)+1/2*x^m*GAMMA(4+m,b*x)/b^4/
exp(a)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int x^{3+m}\sinh(a+bx)\,dx=\frac{e^{-a}x^{m}(-e^{2a}(-bx)^{-m}\Gamma(4+m,-bx)+(bx)^{-m}\Gamma(4+m,bx))}{2b^{4}}$$

input
```
Integrate[x^(3 + m)*Sinh[a + b*x],x]
```

output
```
(x^m*(-((E^(2*a)*Gamma[4 + m, -(b*x)])/(-(b*x))^m) + Gamma[4 + m, b*x]/(b*
x)^m))/(2*b^4*E^a)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+3}\sinh(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -ix^{m+3}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int x^{m+3}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{ 3789}$$

$$-i\left(\frac{1}{2}i\int e^{a+bx}x^{m+3}dx-\frac{1}{2}i\int e^{-a-bx}x^{m+3}dx\right)$$

$$\Big\downarrow \text{ 2612}$$

$$-i\left(\frac{ie^{-a}x^m(bx)^{-m}\Gamma(m+4,bx)}{2b^4}-\frac{ie^ax^m(-bx)^{-m}\Gamma(m+4,-bx)}{2b^4}\right)$$

input
```
Int[x^(3 + m)*Sinh[a + b*x],x]
```

output
```
(-I)*(((-1/2*I)*E^a*x^m*Gamma[4 + m, -(b*x)])/(b^4*(-(b*x))^m) + ((I/2)*x^
m*Gamma[4 + m, b*x])/(b^4*E^a*(b*x)^m))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

method	result	size
meijerg	$\frac{b\,x^{5+m} \operatorname{hypergeom}\left(\left[\frac{5}{2}+\frac{m}{2}\right],\left[\frac{3}{2},\frac{7}{2}+\frac{m}{2}\right],\frac{x^{2} b^{2}}{4}\right) \cosh \left(a \right)}{5+m}+\frac{x^{4+m} \operatorname{hypergeom}\left(\left[2+\frac{m}{2}\right],\left[\frac{1}{2},3+\frac{m}{2}\right],\frac{x^{2} b^{2}}{4}\right) \sinh \left(a \right)}{4+m}$	73

input
```
int(x^(3+m)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
b/(5+m)*x^(5+m)*hypergeom([5/2+1/2*m],[3/2,7/2+1/2*m],1/4*x^2*b^2)*cosh(a)
+1/(4+m)*x^(4+m)*hypergeom([2+1/2*m],[1/2,3+1/2*m],1/4*x^2*b^2)*sinh(a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.46

$$\int x^{3+m}\sinh(a+bx)\,dx$$
$$=\frac{\cosh\left((m+3)\log\left(b\right)+a\right)\Gamma(m+4,bx)+\cosh\left((m+3)\log\left(-b\right)-a\right)\Gamma(m+4,-bx)-\Gamma(m+4,-bx)\,\mathrm{s}}{2\,b}$$

input
```
integrate(x^(3+m)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(cosh((m + 3)*log(b) + a)*gamma(m + 4, b*x) + cosh((m + 3)*log(-b) - a
)*gamma(m + 4, -b*x) - gamma(m + 4, -b*x)*sinh((m + 3)*log(-b) - a) - gamm
a(m + 4, b*x)*sinh((m + 3)*log(b) + a))/b
```

Sympy [F(-2)]

Exception generated.

$$\int x^{3+m}\sinh(a+bx)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x**(3+m)*sinh(b*x+a),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int x^{3+m}\sinh(a+bx)\,dx=\frac{1}{2}\left(bx\right)^{-m-4}x^{m+4}e^{(-a)}\Gamma(m+4,bx)-\frac{1}{2}\left(-bx\right)^{-m-4}x^{m+4}e^{a}\Gamma(m+4,-bx)$$

input
```
integrate(x^(3+m)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(b*x)^(-m - 4)*x^(m + 4)*e^(-a)*gamma(m + 4, b*x) - 1/2*(-b*x)^(-m - 4
)*x^(m + 4)*e^a*gamma(m + 4, -b*x)
```

Giac [F]

$$\int x^{3+m}\sinh(a+bx)\,dx=\int x^{m+3}\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x^(3+m)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m + 3)*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3+m}\sinh(a+bx)\,dx=\int x^{m+3}\sinh(a+b\,x)\,dx$$

input
```
int(x^(m + 3)*sinh(a + b*x),x)
```

output
```
int(x^(m + 3)*sinh(a + b*x), x)
```

Reduce [F]

$$\int x^{3+m}\sinh(a+bx)\,dx$$
$$=\frac{x^m\cosh\left(bx+a\right)b^3x^3+x^m\cosh\left(bx+a\right)b\,m^2x+5x^m\cosh\left(bx+a\right)bmx+6x^m\cosh\left(bx+a\right)bx-x^m\,\mathrm{si}}{}$$

input
```
int(x^(3+m)*sinh(b*x+a),x)
```

output
```
(x**m*cosh(a + b*x)*b**3*x**3 + x**m*cosh(a + b*x)*b*m**2*x + 5*x**m*cosh(
a + b*x)*b*m*x + 6*x**m*cosh(a + b*x)*b*x - x**m*sinh(a + b*x)*b**2*m*x**2
 - 3*x**m*sinh(a + b*x)*b**2*x**2 - x**m*sinh(a + b*x)*m**3 - 6*x**m*sinh(
a + b*x)*m**2 - 11*x**m*sinh(a + b*x)*m - 6*x**m*sinh(a + b*x) + int((x**m
*sinh(a + b*x))/x,x)*m**4 + 6*int((x**m*sinh(a + b*x))/x,x)*m**3 + 11*int(
(x**m*sinh(a + b*x))/x,x)*m**2 + 6*int((x**m*sinh(a + b*x))/x,x)*m)/b**4
```

3.79 $\int x^{2+m} \sinh(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 59

$$\int x^{2+m}\sinh(a+bx)\,dx=\frac{e^a x^m(-bx)^{-m}\Gamma(3+m,-bx)}{2b^3}+\frac{e^{-a}x^m(bx)^{-m}\Gamma(3+m,bx)}{2b^3}$$

output
```
1/2*exp(a)*x^m*GAMMA(3+m,-b*x)/b^3/((-b*x)^m)+1/2*x^m*GAMMA(3+m,b*x)/b^3/e
xp(a)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int x^{2+m}\sinh(a+bx)\,dx=\frac{e^{-a}x^m\left(e^{2a}(-bx)^{-m}\Gamma(3+m,-bx)+(bx)^{-m}\Gamma(3+m,bx)\right)}{2b^3}$$

input
```
Integrate[x^(2 + m)*Sinh[a + b*x],x]
```

output
```
(x^m*((E^(2*a)*Gamma[3 + m, -(b*x)])/(-(b*x))^m + Gamma[3 + m, b*x]/(b*x)^
m))/(2*b^3*E^a)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+2}\sinh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -ix^{m+2}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int x^{m+2}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{3789}$$

$$-i\left(\frac{1}{2}i\int e^{a+bx}x^{m+2}dx-\frac{1}{2}i\int e^{-a-bx}x^{m+2}dx\right)$$

$$\Big\downarrow \text{2612}$$

$$-i\left(\frac{ie^{a}x^{m}(-bx)^{-m}\Gamma(m+3,-bx)}{2b^3}+\frac{ie^{-a}x^{m}(bx)^{-m}\Gamma(m+3,bx)}{2b^3}\right)$$

input
```
Int[x^(2 + m)*Sinh[a + b*x],x]
```

output
```
(-I)*(((I/2)*E^a*x^m*Gamma[3 + m, -(b*x)])/(b^3*(-(b*x))^m) + ((I/2)*x^m*G
amma[3 + m, b*x])/(b^3*E^a*(b*x)^m))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

method	result	size
meijerg	$\frac{b\,x^{4+m}\,\mathrm{hypergeom}\left(\left[2+\frac{m}{2}\right],\left[\frac{3}{2},3+\frac{m}{2}\right],\frac{x^{2} b^{2}}{4}\right)\cosh\left(a\right)}{4+m}+\frac{x^{3+m}\,\mathrm{hypergeom}\left(\left[\frac{3}{2}+\frac{m}{2}\right],\left[\frac{1}{2},\frac{5}{2}+\frac{m}{2}\right],\frac{x^{2} b^{2}}{4}\right)\sinh\left(a\right)}{3+m}$	73

input
```
int(x^(2+m)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
b/(4+m)*x^(4+m)*hypergeom([2+1/2*m],[3/2,3+1/2*m],1/4*x^2*b^2)*cosh(a)+1/(
3+m)*x^(3+m)*hypergeom([3/2+1/2*m],[1/2,5/2+1/2*m],1/4*x^2*b^2)*sinh(a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.46

$$\int x^{2+m}\sinh(a+bx)\,dx = \frac{\cosh\left((m+2)\log\left(b\right)+a\right)\Gamma(m+3,bx)+\cosh\left((m+2)\log\left(-b\right)-a\right)\Gamma(m+3,-bx)-\Gamma(m+3,-bx)\,\mathrm{s}}{2\,b}$$

input
```
integrate(x^(2+m)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(cosh((m + 2)*log(b) + a)*gamma(m + 3, b*x) + cosh((m + 2)*log(-b) - a
)*gamma(m + 3, -b*x) - gamma(m + 3, -b*x)*sinh((m + 2)*log(-b) - a) - gamm
a(m + 3, b*x)*sinh((m + 2)*log(b) + a))/b
```

Sympy [F(-2)]

Exception generated.

$$\int x^{2+m}\sinh(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x**(2+m)*sinh(b*x+a),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int x^{2+m}\sinh(a+bx)\,dx = \frac{1}{2}\,(bx)^{-m-3}\,x^{m+3}e^{(-a)}\Gamma(m+3,bx) - \frac{1}{2}\,(-bx)^{-m-3}\,x^{m+3}e^{a}\Gamma(m+3,-bx)$$

input
```
integrate(x^(2+m)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(b*x)^(-m - 3)*x^(m + 3)*e^(-a)*gamma(m + 3, b*x) - 1/2*(-b*x)^(-m - 3
)*x^(m + 3)*e^a*gamma(m + 3, -b*x)
```

Giac [F]

$$\int x^{2+m}\sinh(a+bx)\,dx=\int x^{m+2}\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x^(2+m)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m + 2)*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{2+m}\sinh(a+bx)\,dx=\int x^{m+2}\sinh(a+b\,x)\,dx$$

input
```
int(x^(m + 2)*sinh(a + b*x),x)
```

output
```
int(x^(m + 2)*sinh(a + b*x), x)
```

Reduce [F]

$$\int x^{2+m}\sinh(a+bx)\,dx$$
$$=\frac{x^m e^{2bx+2a}b^2x^2-x^m e^{2bx+2a}bmx-2x^m e^{2bx+2a}bx+x^m e^{2bx+2a}m^2+3x^m e^{2bx+2a}m+2x^m e^{2bx+2a}-e^{bx+2a}\big(}{}$$

input
```
int(x^(2+m)*sinh(b*x+a),x)
```

output
```
(x**m*e**(2*a + 2*b*x)*b**2*x**2 - x**m*e**(2*a + 2*b*x)*b*m*x - 2*x**m*e*
*(2*a + 2*b*x)*b*x + x**m*e**(2*a + 2*b*x)*m**2 + 3*x**m*e**(2*a + 2*b*x)*
m + 2*x**m*e**(2*a + 2*b*x) - e**(2*a + b*x)*int((x**m*e**(b*x))/x,x)*m**3
 - 3*e**(2*a + b*x)*int((x**m*e**(b*x))/x,x)*m**2 - 2*e**(2*a + b*x)*int((
x**m*e**(b*x))/x,x)*m - e**(b*x)*int(x**m/(e**(b*x)*x),x)*m**3 - 3*e**(b*x
)*int(x**m/(e**(b*x)*x),x)*m**2 - 2*e**(b*x)*int(x**m/(e**(b*x)*x),x)*m +
x**m*b**2*x**2 + x**m*b*m*x + 2*x**m*b*x + x**m*m**2 + 3*x**m*m + 2*x**m)/
(2*e**(a + b*x)*b**3)
```

3.80 $\int x^{1+m} \sinh(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 59

$$\int x^{1+m} \sinh(a+bx)\,dx = -\frac{e^a x^m (-bx)^{-m} \Gamma(2+m,-bx)}{2b^2} + \frac{e^{-a} x^m (bx)^{-m} \Gamma(2+m,bx)}{2b^2}$$

output
```
-1/2*exp(a)*x^m*GAMMA(2+m,-b*x)/b^2/((-b*x)^m)+1/2*x^m*GAMMA(2+m,b*x)/b^2/
exp(a)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int x^{1+m} \sinh(a+bx)\,dx = \frac{e^{-a} x^m \left(-e^{2a} (-bx)^{-m} \Gamma(2+m,-bx) + (bx)^{-m} \Gamma(2+m,bx)\right)}{2b^2}$$

input
```
Integrate[x^(1 + m)*Sinh[a + b*x],x]
```

output
```
(x^m*(-((E^(2*a)*Gamma[2 + m, -(b*x)])/(-(b*x))^m) + Gamma[2 + m, b*x]/(b*
x)^m))/(2*b^2*E^a)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+1}\sinh(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -ix^{m+1}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int x^{m+1}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{ 3789}$$

$$-i\left(\frac{1}{2}i\int e^{a+bx}x^{m+1}dx-\frac{1}{2}i\int e^{-a-bx}x^{m+1}dx\right)$$

$$\Big\downarrow \text{ 2612}$$

$$-i\left(\frac{ie^{-a}x^m(bx)^{-m}\Gamma(m+2,bx)}{2b^2}-\frac{ie^{a}x^m(-bx)^{-m}\Gamma(m+2,-bx)}{2b^2}\right)$$

input
```
Int[x^(1 + m)*Sinh[a + b*x],x]
```

output
```
(-I)*(((-1/2*I)*E^a*x^m*Gamma[2 + m, -(b*x)])/(b^2*(-(b*x))^m) + ((I/2)*x^
m*Gamma[2 + m, b*x])/(b^2*E^a*(b*x)^m))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

method	result	size
meijerg	$\frac{b\,x^{3+m}\,\mathrm{hypergeom}\left(\left[\frac{3}{2}+\frac{m}{2}\right],\left[\frac{3}{2},\frac{5}{2}+\frac{m}{2}\right],\frac{x^{2} b^{2}}{4}\right)\cosh\left(a\right)}{3+m}+\frac{x^{2+m}\,\mathrm{hypergeom}\left(\left[1+\frac{m}{2}\right],\left[\frac{1}{2},2+\frac{m}{2}\right],\frac{x^{2} b^{2}}{4}\right)\sinh\left(a\right)}{2+m}$	73

input
```
int(x^(1+m)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
b/(3+m)*x^(3+m)*hypergeom([3/2+1/2*m],[3/2,5/2+1/2*m],1/4*x^2*b^2)*cosh(a)
+1/(2+m)*x^(2+m)*hypergeom([1+1/2*m],[1/2,2+1/2*m],1/4*x^2*b^2)*sinh(a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.46

$$\int x^{1+m}\sinh(a+bx)\,dx$$
$$= \frac{\cosh\left((m+1)\log\left(b\right)+a\right)\Gamma(m+2,bx)+\cosh\left((m+1)\log\left(-b\right)-a\right)\Gamma(m+2,-bx)-\Gamma(m+2,-bx)\,\mathrm{s}}{2\,b}$$

input
```
integrate(x^(1+m)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(cosh((m + 1)*log(b) + a)*gamma(m + 2, b*x) + cosh((m + 1)*log(-b) - a
)*gamma(m + 2, -b*x) - gamma(m + 2, -b*x)*sinh((m + 1)*log(-b) - a) - gamm
a(m + 2, b*x)*sinh((m + 1)*log(b) + a))/b
```

Sympy [F(-2)]

Exception generated.

$$\int x^{1+m}\sinh(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x**(1+m)*sinh(b*x+a),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int x^{1+m}\sinh(a+bx)\,dx = \frac{1}{2}\left(bx\right)^{-m-2}x^{m+2}e^{(-a)}\Gamma(m+2,bx) - \frac{1}{2}\left(-bx\right)^{-m-2}x^{m+2}e^{a}\Gamma(m+2,-bx)$$

input
```
integrate(x^(1+m)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(b*x)^(-m - 2)*x^(m + 2)*e^(-a)*gamma(m + 2, b*x) - 1/2*(-b*x)^(-m - 2
)*x^(m + 2)*e^a*gamma(m + 2, -b*x)
```

Giac [F]

$$\int x^{1+m}\sinh(a+bx)\,dx = \int x^{m+1}\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x^(1+m)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m + 1)*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{1+m}\sinh(a+bx)\,dx = \int x^{m+1}\sinh(a+b\,x)\,dx$$

input
```
int(x^(m + 1)*sinh(a + b*x),x)
```

output
```
int(x^(m + 1)*sinh(a + b*x), x)
```

Reduce [F]

$$\int x^{1+m}\sinh(a+bx)\,dx$$
$$= \frac{x^m\cosh\left(bx+a\right)bx - x^m\sinh\left(bx+a\right)m - x^m\sinh\left(bx+a\right) + \left(\int\frac{x^m\sinh(bx+a)}{x}dx\right)m^2 + \left(\int\frac{x^m\sinh(bx+a)}{x}\right.}{b^2}$$

input
```
int(x^(1+m)*sinh(b*x+a),x)
```

output
```
(x**m*cosh(a + b*x)*b*x - x**m*sinh(a + b*x)*m - x**m*sinh(a + b*x) + int(
(x**m*sinh(a + b*x))/x,x)*m**2 + int((x**m*sinh(a + b*x))/x,x)*m)/b**2
```

3.81 $\int x^m \sinh(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 59

$$\int x^m \sinh(a+bx)\,dx = \frac{e^a x^m (-bx)^{-m}\Gamma(1+m,-bx)}{2b} + \frac{e^{-a} x^m (bx)^{-m}\Gamma(1+m,bx)}{2b}$$

output
```
1/2*exp(a)*x^m*GAMMA(1+m,-b*x)/b/((-b*x)^m)+1/2*x^m*GAMMA(1+m,b*x)/b/exp(a
)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int x^m \sinh(a+bx)\,dx = \frac{e^{-a} x^m \left(e^{2a}(-bx)^{-m}\Gamma(1+m,-bx) + (bx)^{-m}\Gamma(1+m,bx)\right)}{2b}$$

input
```
Integrate[x^m*Sinh[a + b*x],x]
```

output
```
(x^m*((E^(2*a)*Gamma[1 + m, -(b*x)])/(-(b*x))^m + Gamma[1 + m, b*x]/(b*x)^
m))/(2*b*E^a)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 26, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sinh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -ix^m \sin(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int x^m \sin(ia+ibx)dx$$

$$\Big\downarrow \text{3789}$$

$$-i\left(\frac{1}{2}i\int e^{a+bx}x^m dx-\frac{1}{2}i\int e^{-a-bx}x^m dx\right)$$

$$\Big\downarrow \text{2612}$$

$$-i\left(\frac{ie^a x^m(-bx)^{-m}\Gamma(m+1,-bx)}{2b}+\frac{ie^{-a}x^m(bx)^{-m}\Gamma(m+1,bx)}{2b}\right)$$

input
```
Int[x^m*Sinh[a + b*x],x]
```

output
```
(-I)*(((I/2)*E^a*x^m*Gamma[1 + m, -(b*x)])/(b*(-(b*x))^m) + ((I/2)*x^m*Gam
ma[1 + m, b*x])/(b*E^a*(b*x)^m))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

method	result	size
meijerg	$\frac{b\,x^{2+m}\,\mathrm{hypergeom}\left(\left[1+\frac{m}{2}\right],\left[\frac{3}{2},2+\frac{m}{2}\right],\frac{x^{2} b^{2}}{4}\right)\cosh\left(a\right)}{2+m}+\frac{x^{1+m}\,\mathrm{hypergeom}\left(\left[\frac{1}{2}+\frac{m}{2}\right],\left[\frac{1}{2},\frac{3}{2}+\frac{m}{2}\right],\frac{x^{2} b^{2}}{4}\right)\sinh\left(a\right)}{1+m}$	73

input
```
int(x^m*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
b/(2+m)*x^(2+m)*hypergeom([1+1/2*m],[3/2,2+1/2*m],1/4*x^2*b^2)*cosh(a)+1/(
1+m)*x^(1+m)*hypergeom([1/2+1/2*m],[1/2,3/2+1/2*m],1/4*x^2*b^2)*sinh(a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.32

$$\int x^m \sinh(a+bx)\,dx$$
$$= \frac{\cosh\left(m\log\left(b\right)+a\right)\Gamma(m+1,bx)+\cosh\left(m\log\left(-b\right)-a\right)\Gamma(m+1,-bx)-\Gamma(m+1,-bx)\sinh\left(m\log\left(-\right.\right.}{2\,b}$$

input
```
integrate(x^m*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(cosh(m*log(b) + a)*gamma(m + 1, b*x) + cosh(m*log(-b) - a)*gamma(m +
1, -b*x) - gamma(m + 1, -b*x)*sinh(m*log(-b) - a) - gamma(m + 1, b*x)*sinh
(m*log(b) + a))/b
```

Sympy [F(-2)]

Exception generated.

$$\int x^m \sinh(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x**m*sinh(b*x+a),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int x^m \sinh(a+bx)\,dx = \frac{1}{2}\,(bx)^{-m-1}\,x^{m+1}e^{(-a)}\Gamma(m+1,bx) - \frac{1}{2}\,(-bx)^{-m-1}\,x^{m+1}e^{a}\Gamma(m+1,-bx)$$

input
```
integrate(x^m*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(b*x)^(-m - 1)*x^(m + 1)*e^(-a)*gamma(m + 1, b*x) - 1/2*(-b*x)^(-m - 1
)*x^(m + 1)*e^a*gamma(m + 1, -b*x)
```

Giac [F]

$$\int x^m \sinh(a+bx)\,dx = \int x^m \sinh(bx+a)\,dx$$

input
```
integrate(x^m*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^m*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \sinh(a+bx)\,dx = \int x^m \sinh(a+b\,x)\,dx$$

input
```
int(x^m*sinh(a + b*x),x)
```

output
```
int(x^m*sinh(a + b*x), x)
```

Reduce [F]

$$\int x^m \sinh(a+bx)\,dx = \frac{x^m e^{2bx+2a} - e^{bx+2a}\left(\int \frac{x^m e^{bx}}{x}dx\right)m - e^{bx}\left(\int \frac{x^m}{e^{bx}x}dx\right)m + x^m}{2e^{bx+a}b}$$

input
```
int(x^m*sinh(b*x+a),x)
```

output
```
(x**m*e**(2*a + 2*b*x) - e**(2*a + b*x)*int((x**m*e**(b*x))/x,x)*m - e**(b
*x)*int(x**m/(e**(b*x)*x),x)*m + x**m)/(2*e**(a + b*x)*b)
```

3.82 $\int x^{-1+m} \sinh(a+bx) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 49

$$\int x^{-1+m} \sinh(a+bx) \, dx = -\frac{1}{2} e^a x^m (-bx)^{-m} \Gamma(m,-bx) + \frac{1}{2} e^{-a} x^m (bx)^{-m} \Gamma(m,bx)$$

output
```
-1/2*exp(a)*x^m*GAMMA(m,-b*x)/((-b*x)^m)+1/2*x^m*GAMMA(m,b*x)/exp(a)/((b*x
)^m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int x^{-1+m} \sinh(a+bx) \, dx = -\frac{1}{2} e^a x^m (-bx)^{-m} \Gamma(m,-bx) + \frac{1}{2} e^{-a} x^m (bx)^{-m} \Gamma(m,bx)$$

input
```
Integrate[x^(-1 + m)*Sinh[a + b*x],x]
```

output
```
-1/2*(E^a*x^m*Gamma[m, -(b*x)])/(-(b*x))^m + (x^m*Gamma[m, b*x])/(2*E^a*(b
*x)^m)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-1}\sinh(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -ix^{m-1}\sin(ia+ibx)dx$$

$$\Big\downarrow \; 26$$

$$-i\int x^{m-1}\sin(ia+ibx)dx$$

$$\Big\downarrow \; 3789$$

$$-i\left(\frac{1}{2}i\int e^{a+bx}x^{m-1}dx-\frac{1}{2}i\int e^{-a-bx}x^{m-1}dx\right)$$

$$\Big\downarrow \; 2612$$

$$-i\left(\frac{1}{2}ie^{-a}x^m(bx)^{-m}\Gamma(m,bx)-\frac{1}{2}ie^ax^m(-bx)^{-m}\Gamma(m,-bx)\right)$$

input
```
Int[x^(-1 + m)*Sinh[a + b*x],x]
```

output
```
(-I)*(((-1/2*I)*E^a*x^m*Gamma[m, -(b*x)])/(-(b*x))^m + ((I/2)*x^m*Gamma[m,
 b*x])/(E^a*(b*x)^m))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.37

method	result	size
meijerg	$\frac{b\,x^{1+m}\,\text{hypergeom}\left(\left[\frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2},\frac{3}{2}+\frac{m}{2}\right],\frac{x^2b^2}{4}\right)\cosh(a)}{1+m}+\frac{x^m\,\text{hypergeom}\left(\left[\frac{m}{2}\right],\left[\frac{1}{2},1+\frac{m}{2}\right],\frac{x^2b^2}{4}\right)\sinh(a)}{m}$	67

input
```
int(x^(-1+m)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
b/(1+m)*x^(1+m)*hypergeom([1/2+1/2*m],[3/2,3/2+1/2*m],1/4*x^2*b^2)*cosh(a)
+1/m*x^m*hypergeom([1/2*m],[1/2,1+1/2*m],1/4*x^2*b^2)*sinh(a)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59

$$\int x^{-1+m}\sinh(a+bx)\,dx$$
$$= \frac{\cosh\left((m-1)\log\left(b\right)+a\right)\Gamma(m,bx)+\cosh\left((m-1)\log\left(-b\right)-a\right)\Gamma(m,-bx)-\Gamma(m,-bx)\sinh\left((m-1)\right.}{2\,b}$$

input
```
integrate(x^(-1+m)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(cosh((m - 1)*log(b) + a)*gamma(m, b*x) + cosh((m - 1)*log(-b) - a)*ga
mma(m, -b*x) - gamma(m, -b*x)*sinh((m - 1)*log(-b) - a) - gamma(m, b*x)*si
nh((m - 1)*log(b) + a))/b
```

Sympy [F(-2)]

Exception generated.

$$\int x^{-1+m}\sinh(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x**(-1+m)*sinh(b*x+a),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int x^{-1+m}\sinh(a+bx)\,dx = \frac{x^m e^{(-a)}\Gamma(m,bx)}{2\,(bx)^m} - \frac{x^m e^a\Gamma(m,-bx)}{2\,(-bx)^m}$$

input
```
integrate(x^(-1+m)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*x^m*e^(-a)*gamma(m, b*x)/(b*x)^m - 1/2*x^m*e^a*gamma(m, -b*x)/(-b*x)^m
```

Giac [F]

$$\int x^{-1+m}\sinh(a+bx)\,dx=\int x^{m-1}\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x^(-1+m)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m - 1)*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+m}\sinh(a+bx)\,dx=\int x^{m-1}\sinh(a+b\,x)\,dx$$

input
```
int(x^(m - 1)*sinh(a + b*x),x)
```

output
```
int(x^(m - 1)*sinh(a + b*x), x)
```

Reduce [F]

$$\int x^{-1+m}\sinh(a+bx)\,dx=\int\frac{x^m\sinh\left(bx+a\right)}{x}dx$$

input
```
int(x^(-1+m)*sinh(b*x+a),x)
```

output
```
int((x**m*sinh(a + b*x))/x,x)
```

3.83 $\int x^{-2+m}\sinh(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 55

$$\int x^{-2+m}\sinh(a+bx)\,dx=\frac{1}{2}be^{a}x^{m}(-bx)^{-m}\Gamma(-1+m,-bx)+\frac{1}{2}be^{-a}x^{m}(bx)^{-m}\Gamma(-1+m,bx)$$

output
```
1/2*b*exp(a)*x^m*GAMMA(-1+m,-b*x)/((-b*x)^m)+1/2*b*x^m*GAMMA(-1+m,b*x)/exp
(a)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^{-2+m}\sinh(a+bx)\,dx=\frac{1}{2}be^{-a}x^{m}\left(e^{2a}(-bx)^{-m}\Gamma(-1+m,-bx)+(bx)^{-m}\Gamma(-1+m,bx)\right)$$

input
```
Integrate[x^(-2 + m)*Sinh[a + b*x],x]
```

output
```
(b*x^m*((E^(2*a)*Gamma[-1 + m, -(b*x)])/(-(b*x))^m + Gamma[-1 + m, b*x]/(b
*x)^m))/(2*E^a)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-2}\sinh(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -ix^{m-2}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int x^{m-2}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{ 3789}$$

$$-i\left(\frac{1}{2}i\int e^{a+bx}x^{m-2}dx-\frac{1}{2}i\int e^{-a-bx}x^{m-2}dx\right)$$

$$\Big\downarrow \text{ 2612}$$

$$-i\left(\frac{1}{2}ie^abx^m(-bx)^{-m}\Gamma(m-1,-bx)+\frac{1}{2}ie^{-a}bx^m(bx)^{-m}\Gamma(m-1,bx)\right)$$

input
```
Int[x^(-2 + m)*Sinh[a + b*x],x]
```

output
```
(-I)*(((I/2)*b*E^a*x^m*Gamma[-1 + m, -(b*x)])/(-(b*x))^m + ((I/2)*b*x^m*Ga
mma[-1 + m, b*x])/(E^a*(b*x)^m))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

method	result	size
meijerg	$\frac{b\,x^{m} \operatorname{hypergeom}\left(\left[\frac{m}{2}\right],\left[\frac{3}{2},1+\frac{m}{2}\right],\frac{x^{2} b^{2}}{4}\right) \cosh \left(a \right)}{m}+\frac{x^{-1+m} \operatorname{hypergeom}\left(\left[-\frac{1}{2}+\frac{m}{2}\right],\left[\frac{1}{2},\frac{1}{2}+\frac{m}{2}\right],\frac{x^{2} b^{2}}{4}\right) \sinh \left(a \right)}{-1+m}$	67

input
```
int(x^(-2+m)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
b/m*x^m*hypergeom([1/2*m],[3/2,1+1/2*m],1/4*x^2*b^2)*cosh(a)+1/(-1+m)*x^(-
1+m)*hypergeom([-1/2+1/2*m],[1/2,1/2+1/2*m],1/4*x^2*b^2)*sinh(a)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.56

$$\int x^{-2+m}\sinh(a+bx)\,dx$$
$$=\frac{\cosh\left((m-2)\log\left(b\right)+a\right)\Gamma(m-1,bx)+\cosh\left((m-2)\log\left(-b\right)-a\right)\Gamma(m-1,-bx)-\Gamma(m-1,-bx)\,\mathrm{s}}{2\,b}$$

input
```
integrate(x^(-2+m)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(cosh((m - 2)*log(b) + a)*gamma(m - 1, b*x) + cosh((m - 2)*log(-b) - a
)*gamma(m - 1, -b*x) - gamma(m - 1, -b*x)*sinh((m - 2)*log(-b) - a) - gamm
a(m - 1, b*x)*sinh((m - 2)*log(b) + a))/b
```

Sympy [F(-2)]

Exception generated.

$$\int x^{-2+m}\sinh(a+bx)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x**(-2+m)*sinh(b*x+a),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int x^{-2+m}\sinh(a+bx)\,dx=\frac{1}{2}\left(bx\right)^{-m+1}x^{m-1}e^{(-a)}\Gamma(m-1,bx)-\frac{1}{2}\left(-bx\right)^{-m+1}x^{m-1}e^{a}\Gamma(m-1,-bx)$$

input
```
integrate(x^(-2+m)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(b*x)^(-m + 1)*x^(m - 1)*e^(-a)*gamma(m - 1, b*x) - 1/2*(-b*x)^(-m + 1
)*x^(m - 1)*e^a*gamma(m - 1, -b*x)
```

Giac [F]

$$\int x^{-2+m}\sinh(a+bx)\,dx = \int x^{m-2}\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x^(-2+m)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m - 2)*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-2+m}\sinh(a+bx)\,dx = \int x^{m-2}\sinh(a+b\,x)\,dx$$

input
```
int(x^(m - 2)*sinh(a + b*x),x)
```

output
```
int(x^(m - 2)*sinh(a + b*x), x)
```

Reduce [F]

$$\int x^{-2+m}\sinh(a+bx)\,dx = \int \frac{x^m\sinh\left(bx+a\right)}{x^2}dx$$

input
```
int(x^(-2+m)*sinh(b*x+a),x)
```

output
```
int((x**m*sinh(a + b*x))/x**2,x)
```

3.84 $\int x^{-3+m}\sinh(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 59

$$\int x^{-3+m}\sinh(a+bx)\,dx = -\frac{1}{2}b^2e^ax^m(-bx)^{-m}\Gamma(-2+m,-bx) + \frac{1}{2}b^2e^{-a}x^m(bx)^{-m}\Gamma(-2+m,bx)$$

output
```
-1/2*b^2*exp(a)*x^m*GAMMA(-2+m,-b*x)/((-b*x)^m)+1/2*b^2*x^m*GAMMA(-2+m,b*x
)/exp(a)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int x^{-3+m}\sinh(a+bx)\,dx = \frac{1}{2}b^2e^{-a}x^m\left(-e^{2a}(-bx)^{-m}\Gamma(-2+m,-bx) + (bx)^{-m}\Gamma(-2+m,bx)\right)$$

input
```
Integrate[x^(-3 + m)*Sinh[a + b*x],x]
```

output
```
(b^2*x^m*(-((E^(2*a)*Gamma[-2 + m, -(b*x)])/(-(b*x))^m) + Gamma[-2 + m, b*
x]/(b*x)^m))/(2*E^a)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-3}\sinh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -ix^{m-3}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int x^{m-3}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{3789}$$

$$-i\left(\frac{1}{2}i\int e^{a+bx}x^{m-3}dx-\frac{1}{2}i\int e^{-a-bx}x^{m-3}dx\right)$$

$$\Big\downarrow \text{2612}$$

$$-i\left(\frac{1}{2}ie^{-a}b^2x^m(bx)^{-m}\Gamma(m-2,bx)-\frac{1}{2}ie^{a}b^2x^m(-bx)^{-m}\Gamma(m-2,-bx)\right)$$

input
```
Int[x^(-3 + m)*Sinh[a + b*x],x]
```

output
```
(-I)*(((-1/2*I)*b^2*E^a*x^m*Gamma[-2 + m, -(b*x)])/(-(b*x))^m + ((I/2)*b^2
*x^m*Gamma[-2 + m, b*x])/(E^a*(b*x)^m))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20

method	result	size
meijerg	$\frac{b\,x^{-1+m}\,\mathrm{hypergeom}\left(\left[-\frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2},\frac{1}{2}+\frac{m}{2}\right],\frac{x^{2} b^{2}}{4}\right)\cosh\left(a\right)}{-1+m}+\frac{x^{-2+m}\,\mathrm{hypergeom}\left(\left[-1+\frac{m}{2}\right],\left[\frac{1}{2},\frac{m}{2}\right],\frac{x^{2} b^{2}}{4}\right)\sinh\left(a\right)}{-2+m}$	71

input
```
int(x^(-3+m)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
b/(-1+m)*x^(-1+m)*hypergeom([-1/2+1/2*m],[3/2,1/2+1/2*m],1/4*x^2*b^2)*cosh
(a)+1/(-2+m)*x^(-2+m)*hypergeom([-1+1/2*m],[1/2,1/2*m],1/4*x^2*b^2)*sinh(a
)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.46

$$\int x^{-3+m} \sinh(a+bx)\,dx$$
$$= \frac{\cosh\left((m-3)\log\left(b\right)+a\right)\Gamma(m-2,bx)+\cosh\left((m-3)\log\left(-b\right)-a\right)\Gamma(m-2,-bx)-\Gamma(m-2,-bx)\,\mathrm{s}}{2\,b}$$

input
```
integrate(x^(-3+m)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(cosh((m - 3)*log(b) + a)*gamma(m - 2, b*x) + cosh((m - 3)*log(-b) - a
)*gamma(m - 2, -b*x) - gamma(m - 2, -b*x)*sinh((m - 3)*log(-b) - a) - gamm
a(m - 2, b*x)*sinh((m - 3)*log(b) + a))/b
```

Sympy [F(-2)]

Exception generated.

$$\int x^{-3+m} \sinh(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x**(-3+m)*sinh(b*x+a),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int x^{-3+m} \sinh(a+bx)\,dx = \frac{1}{2}\,(bx)^{-m+2}\,x^{m-2}e^{(-a)}\Gamma(m-2,bx) - \frac{1}{2}\,(-bx)^{-m+2}\,x^{m-2}e^{a}\Gamma(m-2,-bx)$$

input
```
integrate(x^(-3+m)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(b*x)^(-m + 2)*x^(m - 2)*e^(-a)*gamma(m - 2, b*x) - 1/2*(-b*x)^(-m + 2
)*x^(m - 2)*e^a*gamma(m - 2, -b*x)
```

Giac [F]

$$\int x^{-3+m}\sinh(a+bx)\,dx = \int x^{m-3}\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x^(-3+m)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m - 3)*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-3+m}\sinh(a+bx)\,dx = \int x^{m-3}\sinh(a+b\,x)\,dx$$

input
```
int(x^(m - 3)*sinh(a + b*x),x)
```

output
```
int(x^(m - 3)*sinh(a + b*x), x)
```

Reduce [F]

$$\int x^{-3+m}\sinh(a+bx)\,dx = \int \frac{x^m\sinh\left(bx+a\right)}{x^3}dx$$

input
```
int(x^(-3+m)*sinh(b*x+a),x)
```

output
```
int((x**m*sinh(a + b*x))/x**3,x)
```

3.85 $\int x^{3+m} \sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 86

$$\int x^{3+m}\sinh^2(a+bx)\,dx = -\frac{x^{4+m}}{2(4+m)} - \frac{2^{-6-m}e^{2a}x^m(-bx)^{-m}\Gamma(4+m,-2bx)}{b^4} - \frac{2^{-6-m}e^{-2a}x^m(bx)^{-m}\Gamma(4+m,2bx)}{b^4}$$

output
```
-1/2*x^(4+m)/(4+m)-2^(-6-m)*exp(2*a)*x^m*GAMMA(4+m,-2*b*x)/b^4/((-b*x)^m)-
2^(-6-m)*x^m*GAMMA(4+m,2*b*x)/b^4/exp(2*a)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

$$\int x^{3+m}\sinh^2(a+bx)\,dx = \frac{1}{64}x^m\left(-\frac{32x^4}{4+m} - \frac{2^{-m}e^{2a}(-bx)^{-m}\Gamma(4+m,-2bx)}{b^4} - \frac{2^{-m}e^{-2a}(bx)^{-m}\Gamma(4+m,2bx)}{b^4}\right)$$

input
```
Integrate[x^(3 + m)*Sinh[a + b*x]^2,x]
```

output
```
(x^m*((-32*x^4)/(4 + m) - (E^(2*a)*Gamma[4 + m, -2*b*x])/(2^m*b^4*(-(b*x))
^m) - Gamma[4 + m, 2*b*x]/(2^m*b^4*E^(2*a)*(b*x)^m)))/64
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+3}\sinh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -x^{m+3}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int x^{m+3}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{3793}$$

$$-\int \left(\frac{x^{m+3}}{2}-\frac{1}{2}x^{m+3}\cosh(2a+2bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^{2a}2^{-m-6}x^m(-bx)^{-m}\Gamma(m+4,-2bx)}{b^4}-\frac{e^{-2a}2^{-m-6}x^m(bx)^{-m}\Gamma(m+4,2bx)}{b^4}-\frac{x^{m+4}}{2(m+4)}$$

input
```
Int[x^(3 + m)*Sinh[a + b*x]^2,x]
```

output
```
-1/2*x^(4 + m)/(4 + m) - (2^(-6 - m)*E^(2*a)*x^m*Gamma[4 + m, -2*b*x])/(b^
4*(-(b*x))^m) - (2^(-6 - m)*x^m*Gamma[4 + m, 2*b*x])/(b^4*E^(2*a)*(b*x)^m)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int x^{3+m}\sinh\left(bx+a\right)^{2}dx$$

input
```
int(x^(3+m)*sinh(b*x+a)^2,x)
```

output
```
int(x^(3+m)*sinh(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.58

$$\int x^{3+m}\sinh^2(a+bx)\,dx = -\frac{4\,bx\cosh\left((m+3)\log\left(x\right)\right)+(m+4)\cosh\left((m+3)\log\left(2\,b\right)+2\,a\right)\Gamma(m+4,2\,bx)-(m+4)\cosh\left((m\right.}{}$$

input
```
integrate(x^(3+m)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/8*(4*b*x*cosh((m + 3)*log(x)) + (m + 4)*cosh((m + 3)*log(2*b) + 2*a)*ga
mma(m + 4, 2*b*x) - (m + 4)*cosh((m + 3)*log(-2*b) - 2*a)*gamma(m + 4, -2*
b*x) - (m + 4)*gamma(m + 4, 2*b*x)*sinh((m + 3)*log(2*b) + 2*a) + (m + 4)*
gamma(m + 4, -2*b*x)*sinh((m + 3)*log(-2*b) - 2*a) + 4*b*x*sinh((m + 3)*lo
g(x)))/(b*m + 4*b)
```

Sympy [F]

$$\int x^{3+m}\sinh^2(a+bx)\,dx=\int x^{m+3}\sinh^2{(a+bx)}\,dx$$

input
```
integrate(x**(3+m)*sinh(b*x+a)**2,x)
```

output
```
Integral(x**(m + 3)*sinh(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

$$\int x^{3+m}\sinh^2(a+bx)\,dx=-\frac{1}{4}\,(2\,bx)^{-m-4}\,x^{m+4}e^{(-2\,a)}\Gamma(m+4,2\,bx)-\frac{1}{4}\,(-2\,bx)^{-m-4}\,x^{m+4}e^{(2\,a)}\Gamma(m+4,-2\,bx)-\frac{x^{m+4}}{2\,(m+4)}$$

input
```
integrate(x^(3+m)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*b*x)^(-m - 4)*x^(m + 4)*e^(-2*a)*gamma(m + 4, 2*b*x) - 1/4*(-2*b*x
)^(-m - 4)*x^(m + 4)*e^(2*a)*gamma(m + 4, -2*b*x) - 1/2*x^(m + 4)/(m + 4)
```

Giac [F]

$$\int x^{3+m}\sinh^2(a+bx)\,dx = \int x^{m+3}\sinh\left(bx+a\right)^2\,dx$$

input
```
integrate(x^(3+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m + 3)*sinh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3+m}\sinh^2(a+bx)\,dx = \int x^{m+3}\,\sinh(a+b\,x)^2\,dx$$

input
```
int(x^(m + 3)*sinh(a + b*x)^2,x)
```

output
```
int(x^(m + 3)*sinh(a + b*x)^2, x)
```

Reduce [F]

$$\int x^{3+m}\sinh^2(a+bx)\,dx$$
$$= \frac{24e^{2bx+4a}\left(\int\frac{x^m e^{2bx}}{x}dx\right)m+10e^{2bx}\left(\int\frac{x^m}{e^{2bx}x}dx\right)m^4+35e^{2bx}\left(\int\frac{x^m}{e^{2bx}x}dx\right)m^3+50e^{2bx}\left(\int\frac{x^m}{e^{2bx}x}dx\right)m^2+e^{2bx}}{}$$

input
```
int(x^(3+m)*sinh(b*x+a)^2,x)
```

output
```
(8*x**m*e**(4*a + 4*b*x)*b**3*m*x**3 + 32*x**m*e**(4*a + 4*b*x)*b**3*x**3
- 4*x**m*e**(4*a + 4*b*x)*b**2*m**2*x**2 - 28*x**m*e**(4*a + 4*b*x)*b**2*m
*x**2 - 48*x**m*e**(4*a + 4*b*x)*b**2*x**2 + 2*x**m*e**(4*a + 4*b*x)*b*m**
3*x + 18*x**m*e**(4*a + 4*b*x)*b*m**2*x + 52*x**m*e**(4*a + 4*b*x)*b*m*x +
 48*x**m*e**(4*a + 4*b*x)*b*x - x**m*e**(4*a + 4*b*x)*m**4 - 10*x**m*e**(4
*a + 4*b*x)*m**3 - 35*x**m*e**(4*a + 4*b*x)*m**2 - 50*x**m*e**(4*a + 4*b*x
)*m - 24*x**m*e**(4*a + 4*b*x) + e**(4*a + 2*b*x)*int((x**m*e**(2*b*x))/x,
x)*m**5 + 10*e**(4*a + 2*b*x)*int((x**m*e**(2*b*x))/x,x)*m**4 + 35*e**(4*a
 + 2*b*x)*int((x**m*e**(2*b*x))/x,x)*m**3 + 50*e**(4*a + 2*b*x)*int((x**m*
e**(2*b*x))/x,x)*m**2 + 24*e**(4*a + 2*b*x)*int((x**m*e**(2*b*x))/x,x)*m -
 32*x**m*e**(2*a + 2*b*x)*b**4*x**4 + e**(2*b*x)*int(x**m/(e**(2*b*x)*x),x
)*m**5 + 10*e**(2*b*x)*int(x**m/(e**(2*b*x)*x),x)*m**4 + 35*e**(2*b*x)*int
(x**m/(e**(2*b*x)*x),x)*m**3 + 50*e**(2*b*x)*int(x**m/(e**(2*b*x)*x),x)*m*
*2 + 24*e**(2*b*x)*int(x**m/(e**(2*b*x)*x),x)*m - 8*x**m*b**3*m*x**3 - 32*
x**m*b**3*x**3 - 4*x**m*b**2*m**2*x**2 - 28*x**m*b**2*m*x**2 - 48*x**m*b**
2*x**2 - 2*x**m*b*m**3*x - 18*x**m*b*m**2*x - 52*x**m*b*m*x - 48*x**m*b*x
- x**m*m**4 - 10*x**m*m**3 - 35*x**m*m**2 - 50*x**m*m - 24*x**m)/(64*e**(2
*a + 2*b*x)*b**4*(m + 4))
```

3.86 $\int x^{2+m} \sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 85

$$\int x^{2+m}\sinh^2(a+bx)\,dx=-\frac{x^{3+m}}{2(3+m)}+\frac{2^{-5-m}e^{2a}x^m(-bx)^{-m}\Gamma(3+m,-2bx)}{b^3}-\frac{2^{-5-m}e^{-2a}x^m(bx)^{-m}\Gamma(3+m,2bx)}{b^3}$$

output
```
-1/2*x^(3+m)/(3+m)+2^(-5-m)*exp(2*a)*x^m*GAMMA(3+m,-2*b*x)/b^3/((-b*x)^m)-
2^(-5-m)*x^m*GAMMA(3+m,2*b*x)/b^3/exp(2*a)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int x^{2+m}\sinh^2(a+bx)\,dx=\frac{1}{32}x^m\left(-\frac{16x^3}{3+m}+\frac{2^{-m}e^{2a}(-bx)^{-m}\Gamma(3+m,-2bx)}{b^3}-\frac{2^{-m}e^{-2a}(bx)^{-m}\Gamma(3+m,2bx)}{b^3}\right)$$

input
```
Integrate[x^(2 + m)*Sinh[a + b*x]^2,x]
```

output
```
(x^m*((-16*x^3)/(3 + m) + (E^(2*a)*Gamma[3 + m, -2*b*x])/(2^m*b^3*(-(b*x))
^m) - Gamma[3 + m, 2*b*x]/(2^m*b^3*E^(2*a)*(b*x)^m)))/32
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+2}\sinh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -x^{m+2}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int x^{m+2}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{3793}$$

$$-\int \left(\frac{x^{m+2}}{2}-\frac{1}{2}x^{m+2}\cosh(2a+2bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{2a}2^{-m-5}x^m(-bx)^{-m}\Gamma(m+3,-2bx)}{b^3}-\frac{e^{-2a}2^{-m-5}x^m(bx)^{-m}\Gamma(m+3,2bx)}{b^3}-\frac{x^{m+3}}{2(m+3)}$$

input
```
Int[x^(2 + m)*Sinh[a + b*x]^2,x]
```

output
```
-1/2*x^(3 + m)/(3 + m) + (2^(-5 - m)*E^(2*a)*x^m*Gamma[3 + m, -2*b*x])/(b^
3*(-(b*x))^m) - (2^(-5 - m)*x^m*Gamma[3 + m, 2*b*x])/(b^3*E^(2*a)*(b*x)^m)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int x^{2+m} \sinh (bx+a)^2 \, dx$$

input
```
int(x^(2+m)*sinh(b*x+a)^2,x)
```

output
```
int(x^(2+m)*sinh(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.60

$$\int x^{2+m} \sinh^2(a+bx)\, dx = -\frac{4\, bx \cosh\left((m+2)\log\left(x\right)\right) + (m+3)\cosh\left((m+2)\log\left(2\, b\right) + 2\, a\right)\Gamma(m+3, 2\, bx) - (m+3)\cosh\left((m\right.}{}$$

input
```
integrate(x^(2+m)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/8*(4*b*x*cosh((m + 2)*log(x)) + (m + 3)*cosh((m + 2)*log(2*b) + 2*a)*ga
mma(m + 3, 2*b*x) - (m + 3)*cosh((m + 2)*log(-2*b) - 2*a)*gamma(m + 3, -2*
b*x) - (m + 3)*gamma(m + 3, 2*b*x)*sinh((m + 2)*log(2*b) + 2*a) + (m + 3)*
gamma(m + 3, -2*b*x)*sinh((m + 2)*log(-2*b) - 2*a) + 4*b*x*sinh((m + 2)*lo
g(x)))/(b*m + 3*b)
```

Sympy [F]

$$\int x^{2+m} \sinh^2(a+bx)\,dx = \int x^{m+2} \sinh^2(a+bx)\,dx$$

input
```
integrate(x**(2+m)*sinh(b*x+a)**2,x)
```

output
```
Integral(x**(m + 2)*sinh(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int x^{2+m} \sinh^2(a+bx)\,dx = -\frac{1}{4}(2bx)^{-m-3}x^{m+3}e^{(-2a)}\Gamma(m+3,2bx) - \frac{1}{4}(-2bx)^{-m-3}x^{m+3}e^{(2a)}\Gamma(m+3,-2bx) - \frac{x^{m+3}}{2(m+3)}$$

input
```
integrate(x^(2+m)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*b*x)^(-m - 3)*x^(m + 3)*e^(-2*a)*gamma(m + 3, 2*b*x) - 1/4*(-2*b*x
)^(-m - 3)*x^(m + 3)*e^(2*a)*gamma(m + 3, -2*b*x) - 1/2*x^(m + 3)/(m + 3)
```

Giac [F]

$$\int x^{2+m} \sinh^2(a+bx)\,dx = \int x^{m+2} \sinh\left(bx+a\right)^2\,dx$$

input
```
integrate(x^(2+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m + 2)*sinh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{2+m} \sinh^2(a+bx)\,dx = \int x^{m+2}\,\sinh(a+b\,x)^2\,dx$$

input
```
int(x^(m + 2)*sinh(a + b*x)^2,x)
```

output
```
int(x^(m + 2)*sinh(a + b*x)^2, x)
```

Reduce [F]

$$\int x^{2+m} \sinh^2(a+bx)\,dx$$
$$= \frac{-6e^{2bx+4a}\left(\int \frac{x^m e^{2bx}}{x}dx\right)m + e^{2bx}\left(\int \frac{x^m}{e^{2bx}x}dx\right)m^4 + 6e^{2bx}\left(\int \frac{x^m}{e^{2bx}x}dx\right)m^3 + 11e^{2bx}\left(\int \frac{x^m}{e^{2bx}x}dx\right)m^2 - e^{2bx+4a}}{}$$

input
```
int(x^(2+m)*sinh(b*x+a)^2,x)
```

output

```
(4*x**m*e**(4*a + 4*b*x)*b**2*m*x**2 + 12*x**m*e**(4*a + 4*b*x)*b**2*x**2
- 2*x**m*e**(4*a + 4*b*x)*b*m**2*x - 10*x**m*e**(4*a + 4*b*x)*b*m*x - 12*x
**m*e**(4*a + 4*b*x)*b*x + x**m*e**(4*a + 4*b*x)*m**3 + 6*x**m*e**(4*a + 4
*b*x)*m**2 + 11*x**m*e**(4*a + 4*b*x)*m + 6*x**m*e**(4*a + 4*b*x) - e**(4*
a + 2*b*x)*int((x**m*e**(2*b*x))/x,x)*m**4 - 6*e**(4*a + 2*b*x)*int((x**m*
e**(2*b*x))/x,x)*m**3 - 11*e**(4*a + 2*b*x)*int((x**m*e**(2*b*x))/x,x)*m**
2 - 6*e**(4*a + 2*b*x)*int((x**m*e**(2*b*x))/x,x)*m - 16*x**m*e**(2*a + 2*
b*x)*b**3*x**3 + e**(2*b*x)*int(x**m/(e**(2*b*x)*x),x)*m**4 + 6*e**(2*b*x)
*int(x**m/(e**(2*b*x)*x),x)*m**3 + 11*e**(2*b*x)*int(x**m/(e**(2*b*x)*x),x
)*m**2 + 6*e**(2*b*x)*int(x**m/(e**(2*b*x)*x),x)*m - 4*x**m*b**2*m*x**2 -
12*x**m*b**2*x**2 - 2*x**m*b*m**2*x - 10*x**m*b*m*x - 12*x**m*b*x - x**m*m
**3 - 6*x**m*m**2 - 11*x**m*m - 6*x**m)/(32*e**(2*a + 2*b*x)*b**3*(m + 3))
```

3.87 $\int x^{1+m} \sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 86

$$\int x^{1+m}\sinh^2(a+bx)\,dx = -\frac{x^{2+m}}{2(2+m)} - \frac{2^{-4-m}e^{2a}x^m(-bx)^{-m}\Gamma(2+m,-2bx)}{b^2} - \frac{2^{-4-m}e^{-2a}x^m(bx)^{-m}\Gamma(2+m,2bx)}{b^2}$$

output
```
-1/2*x^(2+m)/(2+m)-2^(-4-m)*exp(2*a)*x^m*GAMMA(2+m,-2*b*x)/b^2/((-b*x)^m)-
2^(-4-m)*x^m*GAMMA(2+m,2*b*x)/b^2/exp(2*a)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

$$\int x^{1+m}\sinh^2(a+bx)\,dx = \frac{1}{16}x^m\left(-\frac{8x^2}{2+m} - \frac{2^{-m}e^{2a}(-bx)^{-m}\Gamma(2+m,-2bx)}{b^2} - \frac{2^{-m}e^{-2a}(bx)^{-m}\Gamma(2+m,2bx)}{b^2}\right)$$

input
```
Integrate[x^(1 + m)*Sinh[a + b*x]^2,x]
```

output
```
(x^m*((-8*x^2)/(2 + m) - (E^(2*a)*Gamma[2 + m, -2*b*x])/(2^m*b^2*(-(b*x))^
m) - Gamma[2 + m, 2*b*x]/(2^m*b^2*E^(2*a)*(b*x)^m)))/16
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+1}\sinh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -x^{m+1}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int x^{m+1}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{3793}$$

$$-\int\left(\frac{x^{m+1}}{2}-\frac{1}{2}x^{m+1}\cosh(2a+2bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^{2a}2^{-m-4}x^m(-bx)^{-m}\Gamma(m+2,-2bx)}{b^2}-\frac{e^{-2a}2^{-m-4}x^m(bx)^{-m}\Gamma(m+2,2bx)}{b^2}-\frac{x^{m+2}}{2(m+2)}$$

input
```
Int[x^(1 + m)*Sinh[a + b*x]^2,x]
```

output
```
-1/2*x^(2 + m)/(2 + m) - (2^(-4 - m)*E^(2*a)*x^m*Gamma[2 + m, -2*b*x])/(b^
2*(-(b*x))^m) - (2^(-4 - m)*x^m*Gamma[2 + m, 2*b*x])/(b^2*E^(2*a)*(b*x)^m)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int x^{1+m} \sinh\left(bx+a\right)^{2} dx$$

input
```
int(x^(1+m)*sinh(b*x+a)^2,x)
```

output
```
int(x^(1+m)*sinh(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.58

$$\int x^{1+m}\sinh^2(a+bx)\,dx = -\frac{4\,bx\cosh\left((m+1)\log\left(x\right)\right)+(m+2)\cosh\left((m+1)\log\left(2\,b\right)+2\,a\right)\Gamma(m+2,2\,bx)-(m+2)\cosh\left((m\right.}{}$$

input
```
integrate(x^(1+m)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/8*(4*b*x*cosh((m + 1)*log(x)) + (m + 2)*cosh((m + 1)*log(2*b) + 2*a)*ga
mma(m + 2, 2*b*x) - (m + 2)*cosh((m + 1)*log(-2*b) - 2*a)*gamma(m + 2, -2*
b*x) - (m + 2)*gamma(m + 2, 2*b*x)*sinh((m + 1)*log(2*b) + 2*a) + (m + 2)*
gamma(m + 2, -2*b*x)*sinh((m + 1)*log(-2*b) - 2*a) + 4*b*x*sinh((m + 1)*lo
g(x)))/(b*m + 2*b)
```

Sympy [F]

$$\int x^{1+m}\sinh^2(a+bx)\,dx=\int x^{m+1}\sinh^2{(a+bx)}\,dx$$

input
```
integrate(x**(1+m)*sinh(b*x+a)**2,x)
```

output
```
Integral(x**(m + 1)*sinh(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

$$\int x^{1+m}\sinh^2(a+bx)\,dx=-\frac{1}{4}\,(2\,bx)^{-m-2}\,x^{m+2}e^{(-2\,a)}\Gamma(m+2,2\,bx)-\frac{1}{4}\,(-2\,bx)^{-m-2}\,x^{m+2}e^{(2\,a)}\Gamma(m+2,-2\,bx)-\frac{x^{m+2}}{2\,(m+2)}$$

input
```
integrate(x^(1+m)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*b*x)^(-m - 2)*x^(m + 2)*e^(-2*a)*gamma(m + 2, 2*b*x) - 1/4*(-2*b*x
)^(-m - 2)*x^(m + 2)*e^(2*a)*gamma(m + 2, -2*b*x) - 1/2*x^(m + 2)/(m + 2)
```

Giac [F]

$$\int x^{1+m}\sinh^2(a+bx)\,dx=\int x^{m+1}\sinh\left(bx+a\right)^2\,dx$$

input
```
integrate(x^(1+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m + 1)*sinh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{1+m}\sinh^2(a+bx)\,dx=\int x^{m+1}\,\sinh(a+b\,x)^2\,dx$$

input
```
int(x^(m + 1)*sinh(a + b*x)^2,x)
```

output
```
int(x^(m + 1)*sinh(a + b*x)^2, x)
```

Reduce [F]

$$\int x^{1+m}\sinh^2(a+bx)\,dx$$
$$=\frac{2x^me^{4bx+4a}bmx+4x^me^{4bx+4a}bx-x^me^{4bx+4a}m^2-3x^me^{4bx+4a}m-2x^me^{4bx+4a}+e^{2bx+4a}\left(\int\frac{x^me^{2bx}}{x}dx\right)m^3}{}$$

input
```
int(x^(1+m)*sinh(b*x+a)^2,x)
```

output
```
(2*x**m*e**(4*a + 4*b*x)*b*m*x + 4*x**m*e**(4*a + 4*b*x)*b*x - x**m*e**(4*
a + 4*b*x)*m**2 - 3*x**m*e**(4*a + 4*b*x)*m - 2*x**m*e**(4*a + 4*b*x) + e*
*(4*a + 2*b*x)*int((x**m*e**(2*b*x))/x,x)*m**3 + 3*e**(4*a + 2*b*x)*int((x
**m*e**(2*b*x))/x,x)*m**2 + 2*e**(4*a + 2*b*x)*int((x**m*e**(2*b*x))/x,x)*
m - 8*x**m*e**(2*a + 2*b*x)*b**2*x**2 + e**(2*b*x)*int(x**m/(e**(2*b*x)*x)
,x)*m**3 + 3*e**(2*b*x)*int(x**m/(e**(2*b*x)*x),x)*m**2 + 2*e**(2*b*x)*int
(x**m/(e**(2*b*x)*x),x)*m - 2*x**m*b*m*x - 4*x**m*b*x - x**m*m**2 - 3*x**m
*m - 2*x**m)/(16*e**(2*a + 2*b*x)*b**2*(m + 2))
```

3.88 $\int x^m \sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 85

$$\int x^m \sinh^2(a+bx)\,dx = -\frac{x^{1+m}}{2(1+m)} + \frac{2^{-3-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b} - \frac{2^{-3-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b}$$

output
```
-1/2*x^(1+m)/(1+m)+2^(-3-m)*exp(2*a)*x^m*GAMMA(1+m,-2*b*x)/b/((-b*x)^m)-2^
(-3-m)*x^m*GAMMA(1+m,2*b*x)/b/exp(2*a)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int x^m \sinh^2(a+bx)\,dx = \frac{1}{8}x^m\left(-\frac{4x}{1+m} + \frac{2^{-m}e^{2a}(-bx)^{-m}\Gamma(1+m,-2bx)}{b} - \frac{2^{-m}e^{-2a}(bx)^{-m}\Gamma(1+m,2bx)}{b}\right)$$

input
```
Integrate[x^m*Sinh[a + b*x]^2,x]
```

output
```
(x^m*((-4*x)/(1 + m) + (E^(2*a)*Gamma[1 + m, -2*b*x])/(2^m*b*(-(b*x))^m) -
 Gamma[1 + m, 2*b*x]/(2^m*b*E^(2*a)*(b*x)^m)))/8
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sinh^2(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -x^m \sin(ia+ibx)^2 dx$$

$$\Big\downarrow \; 25$$

$$-\int x^m \sin(ia+ibx)^2 dx$$

$$\Big\downarrow \; 3793$$

$$-\int \left(\frac{x^m}{2}-\frac{1}{2}x^m \cosh(2a+2bx)\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{e^{2a}2^{-m-3}x^m(-bx)^{-m}\Gamma(m+1,-2bx)}{b}-\frac{e^{-2a}2^{-m-3}x^m(bx)^{-m}\Gamma(m+1,2bx)}{b}-\frac{x^{m+1}}{2(m+1)}$$

input
```
Int[x^m*Sinh[a + b*x]^2,x]
```

output
```
-1/2*x^(1 + m)/(1 + m) + (2^(-3 - m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*
(-(b*x))^m) - (2^(-3 - m)*x^m*Gamma[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int x^m \sinh(bx+a)^2 \, dx$$

input
```
int(x^m*sinh(b*x+a)^2,x)
```

output
```
int(x^m*sinh(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.44

$$\int x^m \sinh^2(a+bx) \, dx = -\frac{4\,bx\cosh(m\log(x)) + (m+1)\cosh(m\log(2\,b)+2\,a)\,\Gamma(m+1,2\,bx) - (m+1)\cosh(m\log(-2\,b) -$$

input
```
integrate(x^m*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/8*(4*b*x*cosh(m*log(x)) + (m + 1)*cosh(m*log(2*b) + 2*a)*gamma(m + 1, 2
*b*x) - (m + 1)*cosh(m*log(-2*b) - 2*a)*gamma(m + 1, -2*b*x) - (m + 1)*gam
ma(m + 1, 2*b*x)*sinh(m*log(2*b) + 2*a) + (m + 1)*gamma(m + 1, -2*b*x)*sin
h(m*log(-2*b) - 2*a) + 4*b*x*sinh(m*log(x)))/(b*m + b)
```

Sympy [F]

$$\int x^m \sinh^2(a+bx)\,dx = \int x^m \sinh^2\left(a+bx\right)dx$$

input
```
integrate(x**m*sinh(b*x+a)**2,x)
```

output
```
Integral(x**m*sinh(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int x^m \sinh^2(a+bx)\,dx = -\frac{1}{4}\left(2\,bx\right)^{-m-1}x^{m+1}e^{(-2\,a)}\Gamma(m+1,2\,bx) - \frac{1}{4}\left(-2\,bx\right)^{-m-1}x^{m+1}e^{(2\,a)}\Gamma(m+1,-2\,bx) - \frac{x^{m+1}}{2\,(m+1)}$$

input
```
integrate(x^m*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*b*x)^(-m - 1)*x^(m + 1)*e^(-2*a)*gamma(m + 1, 2*b*x) - 1/4*(-2*b*x
)^(-m - 1)*x^(m + 1)*e^(2*a)*gamma(m + 1, -2*b*x) - 1/2*x^(m + 1)/(m + 1)
```

Giac [F]

$$\int x^m \sinh^2(a+bx)\,dx = \int x^m \sinh\left(bx+a\right)^2\,dx$$

input
```
integrate(x^m*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^m*sinh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \sinh^2(a+bx)\,dx = \int x^m\,\sinh(a+b\,x)^2\,dx$$

input
```
int(x^m*sinh(a + b*x)^2,x)
```

output
```
int(x^m*sinh(a + b*x)^2, x)
```

Reduce [F]

$$\int x^m \sinh^2(a+bx)\,dx = \frac{x^m e^{4bx+4a}m + x^m e^{4bx+4a} - e^{2bx+4a}\left(\int \frac{x^m e^{2bx}}{x}dx\right)m^2 - e^{2bx+4a}\left(\int \frac{x^m e^{2bx}}{x}dx\right)m - 4x^m e^{2bx+2a}bx + e^{2bx}\left(\int \right.}{8e^{2bx+2a}b\left(m+1\right)}$$

input
```
int(x^m*sinh(b*x+a)^2,x)
```

output
```
(x**m*e**(4*a + 4*b*x)*m + x**m*e**(4*a + 4*b*x) - e**(4*a + 2*b*x)*int((x
**m*e**(2*b*x))/x,x)*m**2 - e**(4*a + 2*b*x)*int((x**m*e**(2*b*x))/x,x)*m
- 4*x**m*e**(2*a + 2*b*x)*b*x + e**(2*b*x)*int(x**m/(e**(2*b*x)*x),x)*m**2
 + e**(2*b*x)*int(x**m/(e**(2*b*x)*x),x)*m - x**m*m - x**m)/(8*e**(2*a + 2
*b*x)*b*(m + 1))
```

3.89 $\int x^{-1+m} \sinh^2(a+bx) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 72

$$\int x^{-1+m} \sinh^2(a+bx) \, dx = -\frac{x^m}{2m} - 2^{-2-m} e^{2a} x^m (-bx)^{-m} \Gamma(m,-2bx) - 2^{-2-m} e^{-2a} x^m (bx)^{-m} \Gamma(m,2bx)$$

output
```
-1/2*x^m/m-2^(-2-m)*exp(2*a)*x^m*GAMMA(m,-2*b*x)/((-b*x)^m)-2^(-2-m)*x^m*G
AMMA(m,2*b*x)/exp(2*a)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.88

$$\int x^{-1+m} \sinh^2(a+bx) \, dx = -\frac{x^m (2 + 2^{-m} e^{2a} m (-bx)^{-m} \Gamma(m,-2bx) + 2^{-m} e^{-2a} m (bx)^{-m} \Gamma(m,2bx))}{4m}$$

input
```
Integrate[x^(-1 + m)*Sinh[a + b*x]^2,x]
```

output
```
-1/4*(x^m*(2 + (E^(2*a)*m*Gamma[m, -2*b*x])/(2^m*(-(b*x))^m) + (m*Gamma[m,
 2*b*x])/(2^m*E^(2*a)*(b*x)^m)))/m
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-1}\sinh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -x^{m-1}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int x^{m-1}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{3793}$$

$$-\int \left(\frac{x^{m-1}}{2}-\frac{1}{2}x^{m-1}\cosh(2a+2bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$e^{2a}\left(-2^{-m-2}\right)x^m(-bx)^{-m}\Gamma(m,-2bx)-e^{-2a}2^{-m-2}x^m(bx)^{-m}\Gamma(m,2bx)-\frac{x^m}{2m}$$

input
```
Int[x^(-1 + m)*Sinh[a + b*x]^2,x]
```

output
```
-1/2*x^m/m - (2^(-2 - m)*E^(2*a)*x^m*Gamma[m, -2*b*x])/(-(b*x))^m - (2^(-2
 - m)*x^m*Gamma[m, 2*b*x])/(E^(2*a)*(b*x)^m)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int x^{-1+m} \sinh \left(bx+a\right)^{2} dx$$

input
```
int(x^(-1+m)*sinh(b*x+a)^2,x)
```

output
```
int(x^(-1+m)*sinh(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.62

$$\int x^{-1+m}\sinh^2(a+bx)\,dx = -\frac{4\,bx\cosh\left((m-1)\log\left(x\right)\right)+m\cosh\left((m-1)\log\left(2\,b\right)+2\,a\right)\Gamma(m,2\,bx)-m\cosh\left((m-1)\log\left(-2\,b\right)\right.}{}$$

input
```
integrate(x^(-1+m)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/8*(4*b*x*cosh((m - 1)*log(x)) + m*cosh((m - 1)*log(2*b) + 2*a)*gamma(m,
 2*b*x) - m*cosh((m - 1)*log(-2*b) - 2*a)*gamma(m, -2*b*x) - m*gamma(m, 2*
b*x)*sinh((m - 1)*log(2*b) + 2*a) + m*gamma(m, -2*b*x)*sinh((m - 1)*log(-2
*b) - 2*a) + 4*b*x*sinh((m - 1)*log(x)))/(b*m)
```

Sympy [F]

$$\int x^{-1+m}\sinh^2(a+bx)\,dx=\int x^{m-1}\sinh^2{(a+bx)}\,dx$$

input
```
integrate(x**(-1+m)*sinh(b*x+a)**2,x)
```

output
```
Integral(x**(m - 1)*sinh(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

$$\int x^{-1+m}\sinh^2(a+bx)\,dx=-\frac{x^m e^{(-2a)}\Gamma(m,2bx)}{4\,(2bx)^m}-\frac{x^m e^{(2a)}\Gamma(m,-2bx)}{4\,(-2bx)^m}-\frac{x^m}{2m}$$

input
```
integrate(x^(-1+m)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*x^m*e^(-2*a)*gamma(m, 2*b*x)/(2*b*x)^m - 1/4*x^m*e^(2*a)*gamma(m, -2*
b*x)/(-2*b*x)^m - 1/2*x^m/m
```

Giac [F]

$$\int x^{-1+m} \sinh^2(a+bx)\,dx = \int x^{m-1} \sinh{(bx+a)}^2\,dx$$

input
```
integrate(x^(-1+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m - 1)*sinh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+m} \sinh^2(a+bx)\,dx = \int x^{m-1} \sinh(a+b\,x)^2\,dx$$

input
```
int(x^(m - 1)*sinh(a + b*x)^2,x)
```

output
```
int(x^(m - 1)*sinh(a + b*x)^2, x)
```

Reduce [F]

$$\int x^{-1+m} \sinh^2(a+bx)\,dx = \int \frac{x^m \sinh{(bx+a)}^2}{x}dx$$

input
```
int(x^(-1+m)*sinh(b*x+a)^2,x)
```

output
```
int((x**m*sinh(a + b*x)**2)/x,x)
```

3.90 $\int x^{-2+m} \sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 83

$$\int x^{-2+m}\sinh^2(a+bx)\,dx=\frac{x^{-1+m}}{2(1-m)}+2^{-1-m}be^{2a}x^m(-bx)^{-m}\Gamma(-1+m,-2bx)-2^{-1-m}be^{-2a}x^m(bx)^{-m}\Gamma(-1+m,2bx)$$

output
```
x^(-1+m)/(2-2*m)+2^(-1-m)*b*exp(2*a)*x^m*GAMMA(-1+m,-2*b*x)/((-b*x)^m)-2^(
-1-m)*b*x^m*GAMMA(-1+m,2*b*x)/exp(2*a)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87

$$\int x^{-2+m}\sinh^2(a+bx)\,dx=\frac{1}{2}x^m\left(\frac{1}{x-mx}+2^{-m}be^{2a}(-bx)^{-m}\Gamma(-1+m,-2bx)-2^{-m}be^{-2a}(bx)^{-m}\Gamma(-1+m,2bx)\right)$$

input
```
Integrate[x^(-2 + m)*Sinh[a + b*x]^2,x]
```

output
```
(x^m*((x - m*x)^(-1) + (b*E^(2*a)*Gamma[-1 + m, -2*b*x])/(2^m*(-(b*x))^m)
- (b*Gamma[-1 + m, 2*b*x])/(2^m*E^(2*a)*(b*x)^m)))/2
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-2}\sinh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -x^{m-2}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int x^{m-2}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{3793}$$

$$-\int \left(\frac{x^{m-2}}{2}-\frac{1}{2}x^{m-2}\cosh(2a+2bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$e^{2a}b2^{-m-1}x^m(-bx)^{-m}\Gamma(m-1,-2bx)-e^{-2a}b2^{-m-1}x^m(bx)^{-m}\Gamma(m-1,2bx)+\frac{x^{m-1}}{2(1-m)}$$

input
```
Int[x^(-2 + m)*Sinh[a + b*x]^2,x]
```

output
```
x^(-1 + m)/(2*(1 - m)) + (2^(-1 - m)*b*E^(2*a)*x^m*Gamma[-1 + m, -2*b*x])/
(-(b*x))^m - (2^(-1 - m)*b*x^m*Gamma[-1 + m, 2*b*x])/(E^(2*a)*(b*x)^m)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int x^{-2+m} \sinh(bx+a)^2\,dx$$

input
```
int(x^(-2+m)*sinh(b*x+a)^2,x)
```

output
```
int(x^(-2+m)*sinh(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.64

$$\int x^{-2+m}\sinh^2(a+bx)\,dx = -\frac{4\,bx\cosh\left((m-2)\log(x)\right) + (m-1)\cosh\left((m-2)\log(2\,b)+2\,a\right)\Gamma(m-1,2\,bx) - (m-1)\cosh\left((m$$

input
```
integrate(x^(-2+m)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/8*(4*b*x*cosh((m - 2)*log(x)) + (m - 1)*cosh((m - 2)*log(2*b) + 2*a)*ga
mma(m - 1, 2*b*x) - (m - 1)*cosh((m - 2)*log(-2*b) - 2*a)*gamma(m - 1, -2*
b*x) - (m - 1)*gamma(m - 1, 2*b*x)*sinh((m - 2)*log(2*b) + 2*a) + (m - 1)*
gamma(m - 1, -2*b*x)*sinh((m - 2)*log(-2*b) - 2*a) + 4*b*x*sinh((m - 2)*lo
g(x)))/(b*m - b)
```

Sympy [F]

$$\int x^{-2+m}\sinh^2(a+bx)\,dx=\int x^{m-2}\sinh^2{(a+bx)}\,dx$$

input
```
integrate(x**(-2+m)*sinh(b*x+a)**2,x)
```

output
```
Integral(x**(m - 2)*sinh(a + b*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^{-2+m}\sinh^2(a+bx)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^(-2+m)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(m-2>0)', see `assume?` for more
details)Is
```

Giac [F]

$$\int x^{-2+m}\sinh^2(a+bx)\,dx=\int x^{m-2}\sinh\left(bx+a\right)^2\,dx$$

input
```
integrate(x^(-2+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m - 2)*sinh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-2+m}\sinh^2(a+bx)\,dx=\int x^{m-2}\,\mathrm{sinh}(a+b\,x)^2\,dx$$

input
```
int(x^(m - 2)*sinh(a + b*x)^2,x)
```

output
```
int(x^(m - 2)*sinh(a + b*x)^2, x)
```

Reduce [F]

$$\int x^{-2+m}\sinh^2(a+bx)\,dx=\int\frac{x^m\sinh\left(bx+a\right)^2}{x^2}dx$$

input
```
int(x^(-2+m)*sinh(b*x+a)^2,x)
```

output
```
int((x**m*sinh(a + b*x)**2)/x**2,x)
```

3.91 $\int x^{-3+m} \sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 84

$$\int x^{-3+m}\sinh^2(a+bx)\,dx=\frac{x^{-2+m}}{2(2-m)}-2^{-m}b^2e^{2a}x^m(-bx)^{-m}\Gamma(-2+m,-2bx)-2^{-m}b^2e^{-2a}x^m(bx)^{-m}\Gamma(-2+m,2bx)$$

output
```
x^(-2+m)/(4-2*m)-b^2*exp(2*a)*x^m*GAMMA(-2+m,-2*b*x)/(2^m)/((-b*x)^m)-b^2*
x^m*GAMMA(-2+m,2*b*x)/(2^m)/exp(2*a)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

$$\int x^{-3+m}\sinh^2(a+bx)\,dx=x^m\left(\frac{1}{(4-2m)x^2}-2^{-m}b^2e^{2a}(-bx)^{-m}\Gamma(-2+m,-2bx)-2^{-m}b^2e^{-2a}(bx)^{-m}\Gamma(-2+m,2bx)\right)$$

input
```
Integrate[x^(-3 + m)*Sinh[a + b*x]^2,x]
```

output
```
x^m*(1/((4 - 2*m)*x^2) - (b^2*E^(2*a)*Gamma[-2 + m, -2*b*x])/(2^m*(-(b*x))
^m) - (b^2*Gamma[-2 + m, 2*b*x])/(2^m*E^(2*a)*(b*x)^m))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-3}\sinh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -x^{m-3}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int x^{m-3}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{3793}$$

$$-\int \left(\frac{x^{m-3}}{2}-\frac{1}{2}x^{m-3}\cosh(2a+2bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-e^{2a}b^2 2^{-m}x^m(-bx)^{-m}\Gamma(m-2,-2bx)-e^{-2a}b^2 2^{-m}x^m(bx)^{-m}\Gamma(m-2,2bx)+\frac{x^{m-2}}{2(2-m)}$$

input
```
Int[x^(-3 + m)*Sinh[a + b*x]^2,x]
```

output
```
x^(-2 + m)/(2*(2 - m)) - (b^2*E^(2*a)*x^m*Gamma[-2 + m, -2*b*x])/(2^m*(-(b
*x))^m) - (b^2*x^m*Gamma[-2 + m, 2*b*x])/(2^m*E^(2*a)*(b*x)^m)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int x^{-3+m}\sinh\left(bx+a\right)^{2}dx$$

input
```
int(x^(-3+m)*sinh(b*x+a)^2,x)
```

output
```
int(x^(-3+m)*sinh(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.62

$$\int x^{-3+m}\sinh^2(a+bx)\,dx = -\frac{4\,bx\cosh\left((m-3)\log\left(x\right)\right)+(m-2)\cosh\left((m-3)\log\left(2\,b\right)+2\,a\right)\Gamma(m-2,2\,bx)-(m-2)\cosh\left((m\right.}{}$$

input
```
integrate(x^(-3+m)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/8*(4*b*x*cosh((m - 3)*log(x)) + (m - 2)*cosh((m - 3)*log(2*b) + 2*a)*ga
mma(m - 2, 2*b*x) - (m - 2)*cosh((m - 3)*log(-2*b) - 2*a)*gamma(m - 2, -2*
b*x) - (m - 2)*gamma(m - 2, 2*b*x)*sinh((m - 3)*log(2*b) + 2*a) + (m - 2)*
gamma(m - 2, -2*b*x)*sinh((m - 3)*log(-2*b) - 2*a) + 4*b*x*sinh((m - 3)*lo
g(x)))/(b*m - 2*b)
```

Sympy [F]

$$\int x^{-3+m}\sinh^2(a+bx)\,dx=\int x^{m-3}\sinh^2{(a+bx)}\,dx$$

input
```
integrate(x**(-3+m)*sinh(b*x+a)**2,x)
```

output
```
Integral(x**(m - 3)*sinh(a + b*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^{-3+m}\sinh^2(a+bx)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^(-3+m)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(m-3>0)', see `assume?` for more
details)Is
```

Giac [F]

$$\int x^{-3+m}\sinh^2(a+bx)\,dx=\int x^{m-3}\sinh\left(bx+a\right)^2\,dx$$

input
```
integrate(x^(-3+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m - 3)*sinh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-3+m}\sinh^2(a+bx)\,dx=\int x^{m-3}\sinh(a+b\,x)^2\,dx$$

input
```
int(x^(m - 3)*sinh(a + b*x)^2,x)
```

output
```
int(x^(m - 3)*sinh(a + b*x)^2, x)
```

Reduce [F]

$$\int x^{-3+m}\sinh^2(a+bx)\,dx=\int\frac{x^m\sinh\left(bx+a\right)^2}{x^3}dx$$

input
```
int(x^(-3+m)*sinh(b*x+a)^2,x)
```

output
```
int((x**m*sinh(a + b*x)**2)/x**3,x)
```

3.92 $\int \left(\frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3} x \sqrt{\operatorname{csch}(x)}\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 24

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3} x \sqrt{\operatorname{csch}(x)}\right) dx=-\frac{4}{9 \operatorname{csch}^{\frac{3}{2}}(x)}+\frac{2 x \cosh (x)}{3 \sqrt{\operatorname{csch}(x)}}$$

output
```
-4/9/csch(x)^(3/2)+2/3*x*cosh(x)/csch(x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3} x \sqrt{\operatorname{csch}(x)}\right) dx=\frac{2(-2+3 x \coth (x))}{9 \operatorname{csch}^{\frac{3}{2}}(x)}$$

input
```
Integrate[x/Csch[x]^(3/2) + (x*Sqrt[Csch[x]])/3,x]
```

output
```
(2*(-2 + 3*x*Coth[x]))/(9*Csch[x]^(3/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\text{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x\sqrt{\text{csch}(x)}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{2x\cosh(x)}{3\sqrt{\text{csch}(x)}}-\frac{4}{9\text{csch}^{\frac{3}{2}}(x)}$$

input
```
Int[x/Csch[x]^(3/2) + (x*Sqrt[Csch[x]])/3,x]
```

output
```
-4/(9*Csch[x]^(3/2)) + (2*x*Cosh[x])/(3*Sqrt[Csch[x]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{x}{\text{csch}\left(x\right)^{\frac{3}{2}}}+\frac{x\sqrt{\text{csch}\left(x\right)}}{3}\right)dx$$

input
```
int(x/csch(x)^(3/2)+1/3*x*csch(x)^(1/2),x)
```

output
```
int(x/csch(x)^(3/2)+1/3*x*csch(x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x\sqrt{\operatorname{csch}(x)}\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x/csch(x)^(3/2)+1/3*x*csch(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x\sqrt{\operatorname{csch}(x)}\right)\,dx=\frac{\int \frac{3x}{\operatorname{csch}^{\frac{3}{2}}(x)}\,dx+\int x\sqrt{\operatorname{csch}(x)}\,dx}{3}$$

input
```
integrate(x/csch(x)**(3/2)+1/3*x*csch(x)**(1/2),x)
```

output
```
(Integral(3*x/csch(x)**(3/2), x) + Integral(x*sqrt(csch(x)), x))/3
```

Maxima [F]

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x\sqrt{\operatorname{csch}(x)}\right)\,dx=\int \frac{1}{3}\,x\sqrt{\operatorname{csch}(x)}+\frac{x}{\operatorname{csch}(x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/csch(x)^(3/2)+1/3*x*csch(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/3*x*sqrt(csch(x)) + x/csch(x)^(3/2), x)
```

Giac [F]

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x\sqrt{\operatorname{csch}(x)}\right)\,dx=\int \frac{1}{3}\,x\sqrt{\operatorname{csch}\left(x\right)}+\frac{x}{\operatorname{csch}\left(x\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/csch(x)^(3/2)+1/3*x*csch(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/3*x*sqrt(csch(x)) + x/csch(x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x\sqrt{\operatorname{csch}(x)}\right)\,dx=\int \frac{x\,\sqrt{\frac{1}{\sinh(x)}}}{3}+\frac{x}{{\left(\frac{1}{\sinh(x)}\right)}^{3/2}}\,dx$$

input
```
int((x*(1/sinh(x))^(1/2))/3 + x/(1/sinh(x))^(3/2),x)
```

output
```
int((x*(1/sinh(x))^(1/2))/3 + x/(1/sinh(x))^(3/2), x)
```

Reduce [F]

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x\sqrt{\operatorname{csch}(x)}\right)\,dx=\int \frac{\sqrt{\operatorname{csch}\left(x\right)}\,x}{\operatorname{csch}\left(x\right)^{2}}dx+\frac{\left(\int \sqrt{\operatorname{csch}\left(x\right)}\,x dx\right)}{3}$$

input
```
int(x/csch(x)^(3/2)+1/3*x*csch(x)^(1/2),x)
```

output
```
(3*int((sqrt(csch(x))*x)/csch(x)**2,x) + int(sqrt(csch(x))*x,x))/3
```

3.93 $\int \left(\frac{x}{\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{3x}{5\sqrt{\operatorname{csch}(x)}}\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 24

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{3x}{5\sqrt{\operatorname{csch}(x)}}\right) dx = -\frac{4}{25\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{2x\cosh(x)}{5\operatorname{csch}^{\frac{3}{2}}(x)}$$

output
```
-4/25/csch(x)^(5/2)+2/5*x*cosh(x)/csch(x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{3x}{5\sqrt{\operatorname{csch}(x)}}\right) dx = \frac{2(-2+5x\coth(x))}{25\operatorname{csch}^{\frac{5}{2}}(x)}$$

input
```
Integrate[x/Csch[x]^(5/2) + (3*x)/(5*Sqrt[Csch[x]]),x]
```

output
```
(2*(-2 + 5*x*Coth[x]))/(25*Csch[x]^(5/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\text{csch}^{\frac{5}{2}}(x)}+\frac{3x}{5\sqrt{\text{csch}(x)}}\right) \, dx$$
$$\Big\downarrow \; 2009$$
$$\frac{2x\cosh(x)}{5\text{csch}^{\frac{3}{2}}(x)}-\frac{4}{25\text{csch}^{\frac{5}{2}}(x)}$$

input
```
Int[x/Csch[x]^(5/2) + (3*x)/(5*Sqrt[Csch[x]]),x]
```

output
```
-4/(25*Csch[x]^(5/2)) + (2*x*Cosh[x])/(5*Csch[x]^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{x}{\text{csch}\left(x\right)^{\frac{5}{2}}}+\frac{3x}{5\sqrt{\text{csch}\left(x\right)}}\right)dx$$

input
```
int(x/csch(x)^(5/2)+3/5*x/csch(x)^(1/2),x)
```

output
```
int(x/csch(x)^(5/2)+3/5*x/csch(x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{3x}{5\sqrt{\operatorname{csch}(x)}}\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x/csch(x)^(5/2)+3/5*x/csch(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{3x}{5\sqrt{\operatorname{csch}(x)}}\right)\,dx=\frac{\int \frac{5x}{\operatorname{csch}^{\frac{5}{2}}(x)}\,dx+\int \frac{3x}{\sqrt{\operatorname{csch}(x)}}\,dx}{5}$$

input
```
integrate(x/csch(x)**(5/2)+3/5*x/csch(x)**(1/2),x)
```

output
```
(Integral(5*x/csch(x)**(5/2), x) + Integral(3*x/sqrt(csch(x)), x))/5
```

Maxima [F]

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{3x}{5\sqrt{\operatorname{csch}(x)}}\right)\,dx=\int \frac{3\,x}{5\,\sqrt{\operatorname{csch}\left(x\right)}}+\frac{x}{\operatorname{csch}\left(x\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/csch(x)^(5/2)+3/5*x/csch(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(3/5*x/sqrt(csch(x)) + x/csch(x)^(5/2), x)
```

Giac [F]

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{3x}{5\sqrt{\operatorname{csch}(x)}}\right)\,dx=\int \frac{3\,x}{5\,\sqrt{\operatorname{csch}\left(x\right)}}+\frac{x}{\operatorname{csch}\left(x\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/csch(x)^(5/2)+3/5*x/csch(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(3/5*x/sqrt(csch(x)) + x/csch(x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{3x}{5\sqrt{\operatorname{csch}(x)}}\right)\,dx=\int \frac{3\,x}{5\,\sqrt{\frac{1}{\sinh(x)}}}+\frac{x}{{\left(\frac{1}{\sinh(x)}\right)}^{5/2}}\,dx$$

input
```
int((3*x)/(5*(1/sinh(x))^(1/2)) + x/(1/sinh(x))^(5/2),x)
```

output
```
int((3*x)/(5*(1/sinh(x))^(1/2)) + x/(1/sinh(x))^(5/2), x)
```

Reduce [F]

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{3x}{5\sqrt{\operatorname{csch}(x)}}\right)\,dx=\frac{3\left(\int \frac{\sqrt{\operatorname{csch}(x)}\,x}{\operatorname{csch}(x)}dx\right)}{5}+\int \frac{\sqrt{\operatorname{csch}\left(x\right)}\,x}{\operatorname{csch}\left(x\right)^{3}}dx$$

input
```
int(x/csch(x)^(5/2)+3/5*x/csch(x)^(1/2),x)
```

output
```
(3*int((sqrt(csch(x))*x)/csch(x),x) + 5*int((sqrt(csch(x))*x)/csch(x)**3,x
))/5
```

3.94 $\int \left(\frac{x}{\operatorname{csch}^{\frac{7}{2}}(x)}-\frac{5}{21} x \sqrt{\operatorname{csch}(x)}\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{7}{2}}(x)}-\frac{5}{21} x \sqrt{\operatorname{csch}(x)}\right) dx = -\frac{4}{49\operatorname{csch}^{\frac{7}{2}}(x)}+\frac{2x\cosh(x)}{7\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{20}{63\operatorname{csch}^{\frac{3}{2}}(x)}-\frac{10x\cosh(x)}{21\sqrt{\operatorname{csch}(x)}}$$

output
```
-4/49/csch(x)^(7/2)+2/7*x*cosh(x)/csch(x)^(5/2)+20/63/csch(x)^(3/2)-10/21*
x*cosh(x)/csch(x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{7}{2}}(x)}-\frac{5}{21} x \sqrt{\operatorname{csch}(x)}\right) dx = \sqrt{\operatorname{csch}(x)}\left(-\frac{167}{882}+\frac{88}{441}\cosh(2x)-\frac{1}{98}\cosh(4x)-\frac{13}{42}x\sinh(2x)+\frac{1}{28}x\sinh(4x)\right)$$

input
```
Integrate[x/Csch[x]^(7/2) - (5*x*Sqrt[Csch[x]])/21,x]
```

output
```
Sqrt[Csch[x]]*(-167/882 + (88*Cosh[2*x])/441 - Cosh[4*x]/98 - (13*x*Sinh[2
*x])/42 + (x*Sinh[4*x])/28)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{7}{2}}(x)}-\frac{5}{21} x \sqrt{\operatorname{csch}(x)}\right) dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{20}{63 \operatorname{csch}^{\frac{3}{2}}(x)}-\frac{4}{49 \operatorname{csch}^{\frac{7}{2}}(x)}+\frac{2 x \cosh(x)}{7 \operatorname{csch}^{\frac{5}{2}}(x)}-\frac{10 x \cosh(x)}{21 \sqrt{\operatorname{csch}(x)}}$$

input
```
Int[x/Csch[x]^(7/2) - (5*x*Sqrt[Csch[x]])/21,x]
```

output
```
-4/(49*Csch[x]^(7/2)) + (2*x*Cosh[x])/(7*Csch[x]^(5/2)) + 20/(63*Csch[x]^(
3/2)) - (10*x*Cosh[x])/(21*Sqrt[Csch[x]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{x}{\operatorname{csch}(x)^{\frac{7}{2}}} - \frac{5x\sqrt{\operatorname{csch}(x)}}{21}\right) dx$$

input
```
int(x/csch(x)^(7/2)-5/21*x*csch(x)^(1/2),x)
```

output
```
int(x/csch(x)^(7/2)-5/21*x*csch(x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{7}{2}}(x)} - \frac{5}{21}x\sqrt{\operatorname{csch}(x)}\right) dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/csch(x)^(7/2)-5/21*x*csch(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{7}{2}}(x)} - \frac{5}{21}x\sqrt{\operatorname{csch}(x)}\right) dx = -\frac{\int \left(-\frac{21x}{\operatorname{csch}^{\frac{7}{2}}(x)}\right) dx + \int 5x\sqrt{\operatorname{csch}(x)}\, dx}{21}$$

input
```
integrate(x/csch(x)**(7/2)-5/21*x*csch(x)**(1/2),x)
```

output
```
-(Integral(-21*x/csch(x)**(7/2), x) + Integral(5*x*sqrt(csch(x)), x))/21
```

Maxima [F]

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{7}{2}}(x)}-\frac{5}{21}x\sqrt{\operatorname{csch}(x)}\right)\,dx=\int -\frac{5}{21}\,x\sqrt{\operatorname{csch}\left(x\right)}+\frac{x}{\operatorname{csch}\left(x\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x/csch(x)^(7/2)-5/21*x*csch(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(-5/21*x*sqrt(csch(x)) + x/csch(x)^(7/2), x)
```

Giac [F]

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{7}{2}}(x)}-\frac{5}{21}x\sqrt{\operatorname{csch}(x)}\right)\,dx=\int -\frac{5}{21}\,x\sqrt{\operatorname{csch}\left(x\right)}+\frac{x}{\operatorname{csch}\left(x\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x/csch(x)^(7/2)-5/21*x*csch(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(-5/21*x*sqrt(csch(x)) + x/csch(x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{7}{2}}(x)}-\frac{5}{21}x\sqrt{\operatorname{csch}(x)}\right)\,dx=-\int \frac{5\,x\,\sqrt{\frac{1}{\mathrm{sinh}(x)}}}{21}-\frac{x}{{\left(\frac{1}{\mathrm{sinh}(x)}\right)}^{7/2}}\,dx$$

input
```
int(x/(1/sinh(x))^(7/2) - (5*x*(1/sinh(x))^(1/2))/21,x)
```

output
```
-int((5*x*(1/sinh(x))^(1/2))/21 - x/(1/sinh(x))^(7/2), x)
```

Reduce [F]

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{7}{2}}(x)} - \frac{5}{21} x \sqrt{\operatorname{csch}(x)}\right) dx = \int \frac{\sqrt{\operatorname{csch}(x)}\, x}{\operatorname{csch}(x)^4} dx - \frac{5\left(\int \sqrt{\operatorname{csch}(x)}\, x dx\right)}{21}$$

input
```
int(x/csch(x)^(7/2)-5/21*x*csch(x)^(1/2),x)
```

output
```
(21*int((sqrt(csch(x))*x)/csch(x)**4,x) - 5*int(sqrt(csch(x))*x,x))/21
```

3.95 $\int \left(\frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x^2\sqrt{\operatorname{csch}(x)}\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 76

$$\int \left(\frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x^2\sqrt{\operatorname{csch}(x)}\right)\,dx=-\frac{8x}{9\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{16\cosh(x)}{27\sqrt{\operatorname{csch}(x)}}+\frac{2x^2\cosh(x)}{3\sqrt{\operatorname{csch}(x)}}-\frac{16}{27}i\sqrt{\operatorname{csch}(x)}\operatorname{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)\sqrt{i\sinh(x)}$$

output
```
-8/9*x/csch(x)^(3/2)+16/27*cosh(x)/csch(x)^(1/2)+2/3*x^2*cosh(x)/csch(x)^(
1/2)+16/27*I*csch(x)^(1/2)*InverseJacobiAM(-1/4*Pi+1/2*I*x,2^(1/2))*(I*sin
h(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.88

$$\int \left(\frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x^2\sqrt{\operatorname{csch}(x)}\right)\,dx=\frac{4(3+\operatorname{csch}^2(x))\left(-12x+(8+9x^2)\coth(x)+\frac{8\operatorname{csch}(x)\operatorname{EllipticF}\left(\frac{1}{4}(\pi-2ix),2\right)}{\sqrt{i\sinh(x)}}\right)}{27(-1+3\cosh(2x))\operatorname{csch}^{\frac{7}{2}}(x)}$$

input
```
Integrate[x^2/Csch[x]^(3/2) + (x^2*Sqrt[Csch[x]])/3,x]
```

output
```
(4*(3 + Csch[x]^2)*(-12*x + (8 + 9*x^2)*Coth[x] + (8*Csch[x]*EllipticF[(Pi
 - (2*I)*x)/4, 2])/Sqrt[I*Sinh[x]]))/(27*(-1 + 3*Cosh[2*x])*Csch[x]^(7/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x^2\sqrt{\operatorname{csch}(x)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2x^2\cosh(x)}{3\sqrt{\operatorname{csch}(x)}}-\frac{8x}{9\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{16\cosh(x)}{27\sqrt{\operatorname{csch}(x)}}-\frac{16}{27}i\sqrt{i\sinh(x)}\sqrt{\operatorname{csch}(x)}\operatorname{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)$$

input
```
Int[x^2/Csch[x]^(3/2) + (x^2*Sqrt[Csch[x]])/3,x]
```

output
```
(-8*x)/(9*Csch[x]^(3/2)) + (16*Cosh[x])/(27*Sqrt[Csch[x]]) + (2*x^2*Cosh[x
])/(3*Sqrt[Csch[x]]) - ((16*I)/27)*Sqrt[Csch[x]]*EllipticF[Pi/4 - (I/2)*x,
 2]*Sqrt[I*Sinh[x]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{x^2}{\operatorname{csch}(x)^{\frac{3}{2}}}+\frac{x^2\sqrt{\operatorname{csch}(x)}}{3}\right)dx$$

input
```
int(x^2/csch(x)^(3/2)+1/3*x^2*csch(x)^(1/2),x)
```

output
```
int(x^2/csch(x)^(3/2)+1/3*x^2*csch(x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x^2\sqrt{\operatorname{csch}(x)}\right)dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/csch(x)^(3/2)+1/3*x^2*csch(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x^2\sqrt{\operatorname{csch}(x)}\right)dx = \frac{\int \frac{3x^2}{\operatorname{csch}^{\frac{3}{2}}(x)}\,dx+\int x^2\sqrt{\operatorname{csch}(x)}\,dx}{3}$$

input
```
integrate(x**2/csch(x)**(3/2)+1/3*x**2*csch(x)**(1/2),x)
```

output
```
(Integral(3*x**2/csch(x)**(3/2), x) + Integral(x**2*sqrt(csch(x)), x))/3
```

Maxima [F]

$$\int\left(\frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x^2\sqrt{\operatorname{csch}(x)}\right)dx=\int\frac{1}{3}x^2\sqrt{\operatorname{csch}(x)}+\frac{x^2}{\operatorname{csch}(x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/csch(x)^(3/2)+1/3*x^2*csch(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/3*x^2*sqrt(csch(x)) + x^2/csch(x)^(3/2), x)
```

Giac [F]

$$\int\left(\frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x^2\sqrt{\operatorname{csch}(x)}\right)dx=\int\frac{1}{3}x^2\sqrt{\operatorname{csch}(x)}+\frac{x^2}{\operatorname{csch}(x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/csch(x)^(3/2)+1/3*x^2*csch(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/3*x^2*sqrt(csch(x)) + x^2/csch(x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int\left(\frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x^2\sqrt{\operatorname{csch}(x)}\right)dx=\int\frac{x^2\sqrt{\frac{1}{\sinh(x)}}}{3}+\frac{x^2}{\left(\frac{1}{\sinh(x)}\right)^{3/2}}\,dx$$

input
```
int((x^2*(1/sinh(x))^(1/2))/3 + x^2/(1/sinh(x))^(3/2),x)
```

output
```
int((x^2*(1/sinh(x))^(1/2))/3 + x^2/(1/sinh(x))^(3/2), x)
```

Reduce [F]

$$\int \left(\frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x^2\sqrt{\operatorname{csch}(x)}\right)dx=\int \frac{\sqrt{\operatorname{csch}(x)}\,x^2}{\operatorname{csch}(x)^2}dx+\frac{\left(\int \sqrt{\operatorname{csch}(x)}\,x^2dx\right)}{3}$$

input
```
int(x^2/csch(x)^(3/2)+1/3*x^2*csch(x)^(1/2),x)
```

output
```
(3*int((sqrt(csch(x))*x**2)/csch(x)**2,x) + int(sqrt(csch(x))*x**2,x))/3
```

3.96 $\int (c+dx)^3(a+ia\sinh(e+fx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int (c+dx)^3(a+ia\sinh(e+fx))\,dx = \frac{a(c+dx)^4}{4d}+\frac{6iad^2(c+dx)\cosh(e+fx)}{f^3}+\frac{ia(c+dx)^3\cosh(e+fx)}{f}-\frac{6iad^3\sinh(e+fx)}{f^4}-\frac{3iad(c+dx)^2\sinh(e+fx)}{f^2}$$

output
```
1/4*a*(d*x+c)^4/d+6*I*a*d^2*(d*x+c)*cosh(f*x+e)/f^3+I*a*(d*x+c)^3*cosh(f*x
+e)/f-6*I*a*d^3*sinh(f*x+e)/f^4-3*I*a*d*(d*x+c)^2*sinh(f*x+e)/f^2
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.31

$$\int (c+dx)^3(a+ia\sinh(e+fx))\,dx = \frac{a(f^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)+4if(c+dx)\left(c^2f^2+2cdf^2x+d^2(6+f^2x^2)\right)\cosh(e+fx)-12ia}{4f^4}$$

input
```
Integrate[(c + d*x)^3*(a + I*a*Sinh[e + f*x]),x]
```

output
```
(a*(f^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + (4*I)*f*(c + d*x)*
(c^2*f^2 + 2*c*d*f^2*x + d^2*(6 + f^2*x^2))*Cosh[e + f*x] - (12*I)*d*(c^2*
f^2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Sinh[e + f*x]))/(4*f^4)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3(a+ia\sinh(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3(a+a\sin(ie+ifx))dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(a(c+dx)^3+ia(c+dx)^3\sinh(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{6iad^2(c+dx)\cosh(e+fx)}{f^3}-\frac{3iad(c+dx)^2\sinh(e+fx)}{f^2}+\frac{ia(c+dx)^3\cosh(e+fx)}{f}+\frac{a(c+dx)^4}{4d}-\frac{6iad^3\sinh(e+fx)}{f^4}$$

input
```
Int[(c + d*x)^3*(a + I*a*Sinh[e + f*x]),x]
```

output
```
(a*(c + d*x)^4)/(4*d) + ((6*I)*a*d^2*(c + d*x)*Cosh[e + f*x])/f^3 + (I*a*(
c + d*x)^3*Cosh[e + f*x])/f - ((6*I)*a*d^3*Sinh[e + f*x])/f^4 - ((3*I)*a*d
*(c + d*x)^2*Sinh[e + f*x])/f^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14

method	result
parallelrisch	$\frac{a\left(i(dx+c)\left((dx+c)^2f^2+6d^2\right)f\cosh(fx+e)-3i\left((dx+c)^2f^2+2d^2\right)d\sinh(fx+e)+\left(\left(\frac{dx}{2}+c\right)x\left(\frac{1}{2}x^2d^2+cdx+c^2\right)f^3+ic\right.\right.}{f^4}$
risch	$\frac{a\,d^3x^4}{4}+a\,d^2c\,x^3+\frac{3ad\,c^2x^2}{2}+a\,c^3x+\frac{a\,c^4}{4d}+\frac{ia\left(d^3x^3f^3+3c\,d^2f^3x^2+3c^2d\,f^3x-3d^3f^2x^2+c^3f^3-6c\,d^2f^2\right.}{2f^4}$
parts	$\frac{a(dx+c)^4}{4d}+\frac{ia\left(\frac{d^3\left((fx+e)^3\cosh(fx+e)-3(fx+e)^2\sinh(fx+e)+6(fx+e)\cosh(fx+e)-6\sinh(fx+e)\right)}{f^3}-\frac{3d^3e\left((fx+e)^2\cosh(f\right.}{}\right.}{}$
orering	$\frac{\left(d^5f^4x^6+6c\,d^4f^4x^5+15c^2d^3f^4x^4+20c^3d^2f^4x^3+14c^4d\,f^4x^2-24d^5f^2x^4+4c^5f^4x-96c\,d^4f^2x^3-156c^2d^3f^2x^2-120c^3\right.}{4f^4(dx+c)^2}$
derivativedivides	$\frac{\frac{d^3a(fx+e)^4}{4f^3}+ic^3a\cosh(fx+e)-\frac{d^3ea(fx+e)^3}{f^3}-\frac{id^3e^3a\cosh(fx+e)}{f^3}+\frac{d^2ca(fx+e)^3}{f^2}+\frac{3id\,c^2a((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}}{}$
default	$\frac{\frac{d^3a(fx+e)^4}{4f^3}+ic^3a\cosh(fx+e)-\frac{d^3ea(fx+e)^3}{f^3}-\frac{id^3e^3a\cosh(fx+e)}{f^3}+\frac{d^2ca(fx+e)^3}{f^2}+\frac{3id\,c^2a((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}}{}$

input
```
int((d*x+c)^3*(a+I*a*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
a*(I*(d*x+c)*((d*x+c)^2*f^2+6*d^2)*f*cosh(f*x+e)-3*I*((d*x+c)^2*f^2+2*d^2)
*d*sinh(f*x+e)+((1/2*d*x+c)*x*(1/2*x^2*d^2+c*d*x+c^2)*f^3+I*c^3*f^2+6*I*c*
d^2)*f)/f^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(88) = 176$.

Time = 0.09 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.90

$$\int (c+dx)^3(a+ia\sinh(e+fx))\,dx$$
$$= \frac{(2i\,ad^3f^3x^3 + 2i\,ac^3f^3 + 6i\,ac^2df^2 + 12i\,acd^2f + 12i\,ad^3 - 6\,(-i\,acd^2f^3 - i\,ad^3f^2)x^2 - 6\,(-i\,ac^2df^3 - 2}{}$$

input
```
integrate((d*x+c)^3*(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
1/4*(2*I*a*d^3*f^3*x^3 + 2*I*a*c^3*f^3 + 6*I*a*c^2*d*f^2 + 12*I*a*c*d^2*f
+ 12*I*a*d^3 - 6*(-I*a*c*d^2*f^3 - I*a*d^3*f^2)*x^2 - 6*(-I*a*c^2*d*f^3 -
2*I*a*c*d^2*f^2 - 2*I*a*d^3*f)*x - 2*(-I*a*d^3*f^3*x^3 - I*a*c^3*f^3 + 3*I
*a*c^2*d*f^2 - 6*I*a*c*d^2*f + 6*I*a*d^3 + 3*(-I*a*c*d^2*f^3 + I*a*d^3*f^2
)*x^2 + 3*(-I*a*c^2*d*f^3 + 2*I*a*c*d^2*f^2 - 2*I*a*d^3*f)*x)*e^(2*f*x + 2
*e) + (a*d^3*f^4*x^4 + 4*a*c*d^2*f^4*x^3 + 6*a*c^2*d*f^4*x^2 + 4*a*c^3*f^4
*x)*e^(f*x + e))*e^(-f*x - e)/f^4
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 517, normalized size of antiderivative = 5.28

$$\int (c+dx)^3(a+ia\sinh(e+fx))\,dx = ac^3x + \frac{3ac^2dx^2}{2} + acd^2x^3 + \frac{ad^3x^4}{4}$$
$$+ \begin{cases} \frac{((2iac^3f^7+6iac^2df^7x+6iac^2df^6+6iacd^2f^7x^2+12iacd^2f^6x+12iacd^2f^5+2iad^3f^7x^3+6iad^3f^6x^2+12iad^3f^5x+12iad^3f^4)e^{-fx}+(2iac^3f^7}{} \\ \frac{x^4(iad^3e^{2e}-iad^3)e^{-e}}{8} + \frac{x^3(iacd^2e^{2e}-iacd^2)e^{-e}}{2} + \frac{x^2\cdot(3iac^2de^{2e}-3iac^2d)e^{-e}}{4} + \frac{x(iac^3e^{2e}-iac^3)e^{-e}}{2} \end{cases}$$

input
```
integrate((d*x+c)**3*(a+I*a*sinh(f*x+e)),x)
```

output
```
a*c**3*x + 3*a*c**2*d*x**2/2 + a*c*d**2*x**3 + a*d**3*x**4/4 + Piecewise((
((2*I*a*c**3*f**7 + 6*I*a*c**2*d*f**7*x + 6*I*a*c**2*d*f**6 + 6*I*a*c*d**2
*f**7*x**2 + 12*I*a*c*d**2*f**6*x + 12*I*a*c*d**2*f**5 + 2*I*a*d**3*f**7*x
**3 + 6*I*a*d**3*f**6*x**2 + 12*I*a*d**3*f**5*x + 12*I*a*d**3*f**4)*exp(-f
*x) + (2*I*a*c**3*f**7*exp(2*e) + 6*I*a*c**2*d*f**7*x*exp(2*e) - 6*I*a*c**
2*d*f**6*exp(2*e) + 6*I*a*c*d**2*f**7*x**2*exp(2*e) - 12*I*a*c*d**2*f**6*x
*exp(2*e) + 12*I*a*c*d**2*f**5*exp(2*e) + 2*I*a*d**3*f**7*x**3*exp(2*e) -
6*I*a*d**3*f**6*x**2*exp(2*e) + 12*I*a*d**3*f**5*x*exp(2*e) - 12*I*a*d**3*
f**4*exp(2*e))*exp(f*x))*exp(-e)/(4*f**8), Ne(f**8*exp(e), 0)), (x**4*(I*a
*d**3*exp(2*e) - I*a*d**3)*exp(-e)/8 + x**3*(I*a*c*d**2*exp(2*e) - I*a*c*d
**2)*exp(-e)/2 + x**2*(3*I*a*c**2*d*exp(2*e) - 3*I*a*c**2*d)*exp(-e)/4 + x
*(I*a*c**3*exp(2*e) - I*a*c**3)*exp(-e)/2, True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(88) = 176.

Time = 0.05 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.40

$$
\begin{aligned}
&\int (c+dx)^3(a+ia\sinh(e+fx))\,dx \\
&= \frac{1}{4}\,ad^3x^4 + acd^2x^3 + \frac{3}{2}\,ac^2dx^2 + ac^3x + \frac{3}{2}i\,ac^2d\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) \\
&\quad + \frac{3}{2}i\,acd^2\left(\frac{(f^2x^2e^e - 2\,fxe^e + 2\,e^e)e^{(fx)}}{f^3} + \frac{(f^2x^2 + 2\,fx + 2)e^{(-fx-e)}}{f^3}\right) \\
&\quad + \frac{1}{2}i\,ad^3\left(\frac{(f^3x^3e^e - 3\,f^2x^2e^e + 6\,fxe^e - 6\,e^e)e^{(fx)}}{f^4} + \frac{(f^3x^3 + 3\,f^2x^2 + 6\,fx + 6)e^{(-fx-e)}}{f^4}\right) \\
&\quad + \frac{i\,ac^3\cosh(fx+e)}{f}
\end{aligned}
$$

input
```
integrate((d*x+c)^3*(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + 3/2*I*a*c^2*d*((
f*x*e^e - e^e)*e^(f*x)/f^2 + (f*x + 1)*e^(-f*x - e)/f^2) + 3/2*I*a*c*d^2*(
(f^2*x^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f^3 + (f^2*x^2 + 2*f*x + 2)*e^(-
f*x - e)/f^3) + 1/2*I*a*d^3*((f^3*x^3*e^e - 3*f^2*x^2*e^e + 6*f*x*e^e - 6*
e^e)*e^(f*x)/f^4 + (f^3*x^3 + 3*f^2*x^2 + 6*f*x + 6)*e^(-f*x - e)/f^4) + I
*a*c^3*cosh(f*x + e)/f
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(88) = 176$.

Time = 0.14 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.67

$$\int (c+dx)^3(a+ia\sinh(e+fx))\,dx = \frac{1}{4}ad^3x^4 + acd^2x^3 + \frac{3}{2}ac^2dx^2 + ac^3x$$
$$-\frac{(-i\,ad^3f^3x^3 - 3i\,acd^2f^3x^2 - 3i\,ac^2df^3x + 3i\,ad^3f^2x^2 - i\,ac^3f^3 + 6i\,acd^2f^2x + 3i\,ac^2df^2 - 6i\,ad^3fx -}{2\,f^4}$$
$$-\frac{(-i\,ad^3f^3x^3 - 3i\,acd^2f^3x^2 - 3i\,ac^2df^3x - 3i\,ad^3f^2x^2 - i\,ac^3f^3 - 6i\,acd^2f^2x - 3i\,ac^2df^2 - 6i\,ad^3fx -}{2\,f^4}$$

input
```
integrate((d*x+c)^3*(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

output
```
1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x - 1/2*(-I*a*d^3*f^
3*x^3 - 3*I*a*c*d^2*f^3*x^2 - 3*I*a*c^2*d*f^3*x + 3*I*a*d^3*f^2*x^2 - I*a*
c^3*f^3 + 6*I*a*c*d^2*f^2*x + 3*I*a*c^2*d*f^2 - 6*I*a*d^3*f*x - 6*I*a*c*d^
2*f + 6*I*a*d^3)*e^(f*x + e)/f^4 - 1/2*(-I*a*d^3*f^3*x^3 - 3*I*a*c*d^2*f^3
*x^2 - 3*I*a*c^2*d*f^3*x - 3*I*a*d^3*f^2*x^2 - I*a*c^3*f^3 - 6*I*a*c*d^2*f
^2*x - 3*I*a*c^2*d*f^2 - 6*I*a*d^3*f*x - 6*I*a*c*d^2*f - 6*I*a*d^3)*e^(-f*
x - e)/f^4
```

Mupad [B] (verification not implemented)

Time = 1.24 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.00

$$\int (c+dx)^3(a+ia\sinh(e+fx))\,dx = \frac{\cosh(e+f\,x)\,\left(a\,c^3\,f^2+6\,a\,c\,d^2\right)\,1\mathrm{i}}{f^3} - \frac{\sinh(e+f\,x)\,\left(a\,c^2\,d\,f^2+2\,a\,d^3\right)\,3\mathrm{i}}{f^4} + \frac{a\,d^3\,x^4}{4} + a\,c^3\,x + \frac{x\cosh(e+f\,x)\,\left(a\,c^2\,d\,f^2+2\,a\,d^3\right)\,3\mathrm{i}}{f^3} + \frac{3\,a\,c^2\,d\,x^2}{2} + a\,c\,d^2\,x^3 + \frac{a\,d^3\,x^3\cosh(e+f\,x)\,1\mathrm{i}}{f} - \frac{a\,d^3\,x^2\sinh(e+f\,x)\,3\mathrm{i}}{f^2} - \frac{a\,c\,d^2\,x\sinh(e+f\,x)\,6\mathrm{i}}{f^2} + \frac{a\,c\,d^2\,x^2\cosh(e+f\,x)\,3\mathrm{i}}{f}$$

input
```
int((a + a*sinh(e + f*x)*1i)*(c + d*x)^3,x)
```

output
```
(cosh(e + f*x)*(a*c^3*f^2 + 6*a*c*d^2)*1i)/f^3 - (sinh(e + f*x)*(2*a*d^3 +
 a*c^2*d*f^2)*3i)/f^4 + (a*d^3*x^4)/4 + a*c^3*x + (x*cosh(e + f*x)*(2*a*d^
3 + a*c^2*d*f^2)*3i)/f^3 + (3*a*c^2*d*x^2)/2 + a*c*d^2*x^3 + (a*d^3*x^3*co
sh(e + f*x)*1i)/f - (a*d^3*x^2*sinh(e + f*x)*3i)/f^2 - (a*c*d^2*x*sinh(e +
 f*x)*6i)/f^2 + (a*c*d^2*x^2*cosh(e + f*x)*3i)/f
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.14

$$\int (c+dx)^3(a+ia\sinh(e+fx))\,dx = \frac{a(4\cosh(fx+e)\,c^3f^3i+12\cosh(fx+e)\,c^2d\,f^3ix+12\cosh(fx+e)\,c\,d^2f^3i\,x^2+24\cosh(fx+e)\,c\,d^2f}{}$$

input
```
int((d*x+c)^3*(a+I*a*sinh(f*x+e)),x)
```

output
```
(a*(4*cosh(e + f*x)*c**3*f**3*i + 12*cosh(e + f*x)*c**2*d*f**3*i*x + 12*co
sh(e + f*x)*c*d**2*f**3*i*x**2 + 24*cosh(e + f*x)*c*d**2*f*i + 4*cosh(e +
f*x)*d**3*f**3*i*x**3 + 24*cosh(e + f*x)*d**3*f*i*x - 12*sinh(e + f*x)*c**
2*d*f**2*i - 24*sinh(e + f*x)*c*d**2*f**2*i*x - 12*sinh(e + f*x)*d**3*f**2
*i*x**2 - 24*sinh(e + f*x)*d**3*i + 4*c**3*f**4*x + 6*c**2*d*f**4*x**2 + 4
*c*d**2*f**4*x**3 + d**3*f**4*x**4))/(4*f**4)
```

3.97 $\int (c+dx)^2(a+ia\sinh(e+fx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 74

$$\int (c+dx)^2(a+ia\sinh(e+fx))\,dx = \frac{a(c+dx)^3}{3d} + \frac{2iad^2\cosh(e+fx)}{f^3} + \frac{ia(c+dx)^2\cosh(e+fx)}{f} - \frac{2iad(c+dx)\sinh(e+fx)}{f^2}$$

output
```
1/3*a*(d*x+c)^3/d+2*I*a*d^2*cosh(f*x+e)/f^3+I*a*(d*x+c)^2*cosh(f*x+e)/f-2*
I*a*d*(d*x+c)*sinh(f*x+e)/f^2
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19

$$\int (c+dx)^2(a+ia\sinh(e+fx))\,dx = \frac{a(f^3x(3c^2+3cdx+d^2x^2)+3i(c^2f^2+2cdf^2x+d^2(2+f^2x^2))\cosh(e+fx)-6idf(c+dx)\sinh(e+fx}{3f^3}$$

input
```
Integrate[(c + d*x)^2*(a + I*a*Sinh[e + f*x]),x]
```

output
```
(a*(f^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) + (3*I)*(c^2*f^2 + 2*c*d*f^2*x + d^2
*(2 + f^2*x^2))*Cosh[e + f*x] - (6*I)*d*f*(c + d*x)*Sinh[e + f*x]))/(3*f^3
)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2(a+ia\sinh(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2(a+a\sin(ie+ifx))dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(a(c+dx)^2+ia(c+dx)^2\sinh(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2iad(c+dx)\sinh(e+fx)}{f^2}+\frac{ia(c+dx)^2\cosh(e+fx)}{f}+\frac{a(c+dx)^3}{3d}+\frac{2iad^2\cosh(e+fx)}{f^3}$$

input
```
Int[(c + d*x)^2*(a + I*a*Sinh[e + f*x]),x]
```

output
```
(a*(c + d*x)^3)/(3*d) + ((2*I)*a*d^2*Cosh[e + f*x])/f^3 + (I*a*(c + d*x)^2
*Cosh[e + f*x])/f - ((2*I)*a*d*(c + d*x)*Sinh[e + f*x])/f^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

method	result
parallelrisch	$\frac{\left(i\left((dx+c)^2 f^2+2d^2\right)\cosh(fx+e)-2i(dx+c)fd\sinh(fx+e)+x\left(\frac{1}{3}x^2d^2+cdx+c^2\right)f^3+ic^2f^2+2id^2\right)a}{f^3}$
risch	$\frac{a d^2 x^3}{3}+adcx^2+ac^2x+\frac{ac^3}{3d}+\frac{ia(d^2x^2f^2+2cd\,f^2x+c^2f^2-2d^2fx-2cdf+2d^2)e^{fx+e}}{2f^3}+\frac{ia(d^2x^2f^2+2cd$
parts	$\frac{a(dx+c)^3}{3d}+\frac{ia\left(\frac{d^2\left((fx+e)^2\cosh(fx+e)-2(fx+e)\sinh(fx+e)+2\cosh(fx+e)\right)}{f^2}-\frac{2d^2e((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f^2}+\frac{2dc}{}\right.}{f}$
derivativedivides	$\frac{d^2a(fx+e)^3}{3f^2}+\frac{id^2a\left((fx+e)^2\cosh(fx+e)-2(fx+e)\sinh(fx+e)+2\cosh(fx+e)\right)}{f^2}-\frac{d^2ea(fx+e)^2}{f^2}-\frac{2id^2ea((fx+e)\cosh(fx+e)-\sinh}{f^2}$
default	$\frac{d^2a(fx+e)^3}{3f^2}+\frac{id^2a\left((fx+e)^2\cosh(fx+e)-2(fx+e)\sinh(fx+e)+2\cosh(fx+e)\right)}{f^2}-\frac{d^2ea(fx+e)^2}{f^2}-\frac{2id^2ea((fx+e)\cosh(fx+e)-\sinh}{f^2}$
orering	$\frac{(d^4f^4x^5+5cd^3f^4x^4+10c^2d^2f^4x^3+9c^3d\,f^4x^2+3c^4f^4x-12d^4f^2x^3-42cd^3f^2x^2-48c^2d^2f^2x-12c^3d\,f^2-48d^4x-12d^3}{3f^4(dx+c)^2}$

input
```
int((d*x+c)^2*(a+I*a*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
(I*((d*x+c)^2*f^2+2*d^2)*cosh(f*x+e)-2*I*(d*x+c)*f*d*sinh(f*x+e)+x*(1/3*x^
2*d^2+c*d*x+c^2)*f^3+I*c^2*f^2+2*I*d^2)*a/f^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(66) = 132.

Time = 0.10 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.32

$$\int (c+dx)^2(a+ia\sinh(e+fx))\,dx = \frac{\left(3i\,ad^2f^2x^2+3i\,ac^2f^2+6i\,acdf+6i\,ad^2-6\left(-i\,acdf^2-i\,ad^2f\right)x-3\left(-i\,ad^2f^2x^2-i\,ac^2f^2+2i\,acdf\right.\right.}{6\,f^3}$$

input
```
integrate((d*x+c)^2*(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
1/6*(3*I*a*d^2*f^2*x^2 + 3*I*a*c^2*f^2 + 6*I*a*c*d*f + 6*I*a*d^2 - 6*(-I*a
*c*d*f^2 - I*a*d^2*f)*x - 3*(-I*a*d^2*f^2*x^2 - I*a*c^2*f^2 + 2*I*a*c*d*f
- 2*I*a*d^2 + 2*(-I*a*c*d*f^2 + I*a*d^2*f)*x)*e^(2*f*x + 2*e) + 2*(a*d^2*f
^3*x^3 + 3*a*c*d*f^3*x^2 + 3*a*c^2*f^3*x)*e^(f*x + e))*e^(-f*x - e)/f^3
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 314, normalized size of antiderivative = 4.24

$$\int (c+dx)^2(a+ia\sinh(e+fx))\,dx = ac^2x + acdx^2 + \frac{ad^2x^3}{3} + \begin{cases} \frac{\left(\left(2iac^2f^5+4iacdf^5x+4iacdf^4+2iad^2f^5x^2+4iad^2f^4x+4iad^2f^3\right)e^{-fx}+\left(2iac^2f^5e^{2e}+4iacdf^5xe^{2e}-4iacdf^4e^{2e}+2iad^2f^5x^2e^{2e}-4iad^2f^4\right.\right.}{4f^6} \\ \frac{x^3\left(iad^2e^{2e}-iad^2\right)e^{-e}}{6} + \frac{x^2\left(iacde^{2e}-iacd\right)e^{-e}}{2} + \frac{x\left(iac^2e^{2e}-iac^2\right)e^{-e}}{2} \end{cases}$$

input
```
integrate((d*x+c)**2*(a+I*a*sinh(f*x+e)),x)
```

output
```
a*c**2*x + a*c*d*x**2 + a*d**2*x**3/3 + Piecewise((((2*I*a*c**2*f**5 + 4*I
*a*c*d*f**5*x + 4*I*a*c*d*f**4 + 2*I*a*d**2*f**5*x**2 + 4*I*a*d**2*f**4*x
+ 4*I*a*d**2*f**3)*exp(-f*x) + (2*I*a*c**2*f**5*exp(2*e) + 4*I*a*c*d*f**5*
x*exp(2*e) - 4*I*a*c*d*f**4*exp(2*e) + 2*I*a*d**2*f**5*x**2*exp(2*e) - 4*I
*a*d**2*f**4*x*exp(2*e) + 4*I*a*d**2*f**3*exp(2*e))*exp(f*x))*exp(-e)/(4*f
**6), Ne(f**6*exp(e), 0)), (x**3*(I*a*d**2*exp(2*e) - I*a*d**2)*exp(-e)/6
+ x**2*(I*a*c*d*exp(2*e) - I*a*c*d)*exp(-e)/2 + x*(I*a*c**2*exp(2*e) - I*a
*c**2)*exp(-e)/2, True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(66) = 132$.

Time = 0.06 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.91

$$\begin{aligned}\int (c+dx)^2(a+ia\sinh(e+fx))\,dx &= \frac{1}{3}\,ad^2x^3 + acdx^2 + ac^2x + i\,acd\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) \\ &\quad + \frac{1}{2}i\,ad^2\left(\frac{(f^2x^2e^e - 2\,fxe^e + 2\,e^e)e^{(fx)}}{f^3} + \frac{(f^2x^2 + 2\,fx + 2)e^{(-fx-e)}}{f^3}\right) \\ &\quad + \frac{i\,ac^2\cosh\left(fx+e\right)}{f}\end{aligned}$$

input
```
integrate((d*x+c)^2*(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + I*a*c*d*((f*x*e^e - e^e)*e^(f*x)/f^2
 + (f*x + 1)*e^(-f*x - e)/f^2) + 1/2*I*a*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2
*e^e)*e^(f*x)/f^3 + (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + I*a*c^2*cosh
(f*x + e)/f
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(66) = 132$.

Time = 0.13 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.03

$$\int (c+dx)^2(a+ia\sinh(e+fx))\,dx$$
$$= \frac{1}{3}ad^2x^3 + acdx^2 + ac^2x - \frac{(-i\,ad^2f^2x^2 - 2i\,acdf^2x - i\,ac^2f^2 + 2i\,ad^2fx + 2i\,acdf - 2i\,ad^2)e^{(fx+e)}}{2\,f^3} + \frac{(i\,ad^2f^2x^2 + 2i\,acdf^2x + i\,ac^2f^2 + 2i\,ad^2fx + 2i\,acdf + 2i\,ad^2)e^{(-fx-e)}}{2\,f^3}$$

input
```
integrate((d*x+c)^2*(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

output
```
1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x - 1/2*(-I*a*d^2*f^2*x^2 - 2*I*a*c*d*f^
2*x - I*a*c^2*f^2 + 2*I*a*d^2*f*x + 2*I*a*c*d*f - 2*I*a*d^2)*e^(f*x + e)/f
^3 + 1/2*(I*a*d^2*f^2*x^2 + 2*I*a*c*d*f^2*x + I*a*c^2*f^2 + 2*I*a*d^2*f*x
+ 2*I*a*c*d*f + 2*I*a*d^2)*e^(-f*x - e)/f^3
```

Mupad [B] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.59

$$\int (c+dx)^2(a+ia\sinh(e+fx))\,dx$$
$$= \frac{-\frac{a\,f\,\left(6\mathrm{i}\,x\sinh(e+f\,x)\,d^2+6\mathrm{i}\,c\sinh(e+f\,x)\,d\right)}{3} + \frac{a\,f^2\,\left(c^2\cosh(e+f\,x)\,3\mathrm{i}+d^2\,x^2\cosh(e+f\,x)\,3\mathrm{i}+c\,d\,x\cosh(e+f\,x)\,6\mathrm{i}\right)}{3} + a\,d^2\cosh(e+f}{f^3} + \frac{a\,(3\,c^2\,x + 3\,c\,d\,x^2 + d^2\,x^3)}{3}$$

input
```
int((a + a*sinh(e + f*x)*1i)*(c + d*x)^2,x)
```

output
```
((a*f^2*(c^2*cosh(e + f*x)*3i + d^2*x^2*cosh(e + f*x)*3i + c*d*x*cosh(e +
f*x)*6i))/3 - (a*f*(d^2*x*sinh(e + f*x)*6i + c*d*sinh(e + f*x)*6i))/3 + a*
d^2*cosh(e + f*x)*2i)/f^3 + (a*(3*c^2*x + d^2*x^3 + 3*c*d*x^2))/3
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.65

$$\int (c+dx)^2(a+ia\sinh(e+fx))\,dx = \frac{a(3\cosh(fx+e)\,c^2f^2i+6\cosh(fx+e)\,cd\,f^2ix+3\cosh(fx+e)\,d^2f^2i\,x^2+6\cosh(fx+e)\,d^2i-6\sin}{3f^3}$$

input
```
int((d*x+c)^2*(a+I*a*sinh(f*x+e)),x)
```

output
```
(a*(3*cosh(e + f*x)*c**2*f**2*i + 6*cosh(e + f*x)*c*d*f**2*i*x + 3*cosh(e
+ f*x)*d**2*f**2*i*x**2 + 6*cosh(e + f*x)*d**2*i - 6*sinh(e + f*x)*c*d*f*i
 - 6*sinh(e + f*x)*d**2*f*i*x + 3*c**2*f**3*x + 3*c*d*f**3*x**2 + d**2*f**
3*x**3))/(3*f**3)
```

3.98 $\int (c+dx)(a+ia\sinh(e+fx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int (c+dx)(a+ia\sinh(e+fx))\,dx = \frac{a(c+dx)^2}{2d}+\frac{ia(c+dx)\cosh(e+fx)}{f} - \frac{iad\sinh(e+fx)}{f^2}$$

output
```
1/2*a*(d*x+c)^2/d+I*a*(d*x+c)*cosh(f*x+e)/f-I*a*d*sinh(f*x+e)/f^2
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (c+dx)(a+ia\sinh(e+fx))\,dx = \frac{a(f^2x(2c+dx)+2if(c+dx)\cosh(e+fx)-2id\sinh(e+fx))}{2f^2}$$

input
```
Integrate[(c + d*x)*(a + I*a*Sinh[e + f*x]),x]
```

output
```
(a*(f^2*x*(2*c + d*x) + (2*I)*f*(c + d*x)*Cosh[e + f*x] - (2*I)*d*Sinh[e +
 f*x]))/(2*f^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(a+ia\sinh(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)(a+a\sin(ie+ifx))dx$$

$$\Big\downarrow \text{3798}$$

$$\int (a(c+dx)+ia(c+dx)\sinh(e+fx))dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ia(c+dx)\cosh(e+fx)}{f}+\frac{a(c+dx)^2}{2d}-\frac{iad\sinh(e+fx)}{f^2}$$

input
```
Int[(c + d*x)*(a + I*a*Sinh[e + f*x]),x]
```

output
```
(a*(c + d*x)^2)/(2*d) + (I*a*(c + d*x)*Cosh[e + f*x])/f - (I*a*d*Sinh[e +
f*x])/f^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

method	result
parallelrisch	$\frac{a\left(i(dx+c)f\cosh(fx+e)-i\sinh(fx+e)d+\left(x\left(\frac{dx}{2}+c\right)f+ic\right)f\right)}{f^{2}}$
risch	$\frac{ad\,x^{2}}{2}+acx+\frac{ia(dxf+cf-d)\mathrm{e}^{fx+e}}{2f^{2}}+\frac{ia(dxf+cf+d)\mathrm{e}^{-fx-e}}{2f^{2}}$
parts	$a\left(\frac{1}{2}d\,x^{2}+cx\right)+\frac{ia\left(\frac{d((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}-\frac{de\cosh(fx+e)}{f}+c\cosh(fx+e)\right)}{f}$
derivativedivides	$\frac{\frac{da(fx+e)^{2}}{2f}+\frac{ida((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}-\frac{dea(fx+e)}{f}-\frac{idea\cosh(fx+e)}{f}+ca(fx+e)+ica\cosh(fx+e)}{f}$
default	$\frac{\frac{da(fx+e)^{2}}{2f}+\frac{ida((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}-\frac{dea(fx+e)}{f}-\frac{idea\cosh(fx+e)}{f}+ca(fx+e)+ica\cosh(fx+e)}{f}$
orering	$\frac{(d^{3}f^{2}x^{4}+4c\,d^{2}f^{2}x^{3}+5c^{2}d\,f^{2}x^{2}+2c^{3}f^{2}x-6d^{3}x^{2}-12c\,d^{2}x-4c^{2}d)(a+ia\sinh(fx+e))}{2f^{2}(dx+c)^{2}}+\frac{(2x^{2}d^{2}+4cdx+c^{2})(d(a+ia\ldots}{\ldots}$

input
```
int((d*x+c)*(a+I*a*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
a*(I*(d*x+c)*f*cosh(f*x+e)-I*sinh(f*x+e)*d+(x*(1/2*d*x+c)*f+I*c)*f)/f^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.62

$$\int (c+dx)(a+ia\sinh(e+fx))\,dx = \frac{\left(i\,adfx+i\,acf+i\,ad+(i\,adfx+i\,acf-i\,ad)e^{(2\,fx+2\,e)}+(adf^2x^2+2\,acf^2x)e^{(fx+e)}\right)e^{(-fx-e)}}{2\,f^2}$$

input
```
integrate((d*x+c)*(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*(I*a*d*f*x + I*a*c*f + I*a*d + (I*a*d*f*x + I*a*c*f - I*a*d)*e^(2*f*x
+ 2*e) + (a*d*f^2*x^2 + 2*a*c*f^2*x)*e^(f*x + e))*e^(-f*x - e)/f^2
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 162, normalized size of antiderivative = 3.24

$$\int (c+dx)(a+ia\sinh(e+fx))\,dx = acx+\frac{adx^2}{2} + \begin{cases} \frac{((2iacf^3+2iadf^3x+2iadf^2)e^{-fx}+(2iacf^3e^{2e}+2iadf^3xe^{2e}-2iadf^2e^{2e})e^{fx})e^{-e}}{4f^4} & \text{for } f^4e^e\neq 0 \\ \frac{x^2(iade^{2e}-iad)e^{-e}}{4}+\frac{x(iace^{2e}-iac)e^{-e}}{2} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*(a+I*a*sinh(f*x+e)),x)
```

output
```
a*c*x + a*d*x**2/2 + Piecewise((((2*I*a*c*f**3 + 2*I*a*d*f**3*x + 2*I*a*d*
f**2)*exp(-f*x) + (2*I*a*c*f**3*exp(2*e) + 2*I*a*d*f**3*x*exp(2*e) - 2*I*a
*d*f**2*exp(2*e))*exp(f*x))*exp(-e)/(4*f**4), Ne(f**4*exp(e), 0)), (x**2*(
I*a*d*exp(2*e) - I*a*d)*exp(-e)/4 + x*(I*a*c*exp(2*e) - I*a*c)*exp(-e)/2,
True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.32

$$\int (c+dx)(a+ia\sinh(e+fx))\,dx = \frac{1}{2}\,adx^2+acx+\frac{1}{2}i\,ad\left(\frac{(fxe^e-e^e)e^{(fx)}}{f^2}+\frac{(fx+1)e^{(-fx-e)}}{f^2}\right)+\frac{i\,ac\cosh\left(fx+e\right)}{f}$$

input
```
integrate((d*x+c)*(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
1/2*a*d*x^2 + a*c*x + 1/2*I*a*d*((f*x*e^e - e^e)*e^(f*x)/f^2 + (f*x + 1)*e
^(-f*x - e)/f^2) + I*a*c*cosh(f*x + e)/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.38

$$\int (c+dx)(a+ia\sinh(e+fx))\,dx = \frac{1}{2}\,adx^2+acx-\frac{(-i\,adfx-i\,acf+i\,ad)e^{(fx+e)}}{2\,f^2}-\frac{(-i\,adfx-i\,acf-i\,ad)e^{(-fx-e)}}{2\,f^2}$$

input
```
integrate((d*x+c)*(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

output
```
1/2*a*d*x^2 + a*c*x - 1/2*(-I*a*d*f*x - I*a*c*f + I*a*d)*e^(f*x + e)/f^2 -
 1/2*(-I*a*d*f*x - I*a*c*f - I*a*d)*e^(-f*x - e)/f^2
```

Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12

$$\int (c+dx)(a+ia\sinh(e+fx))\,dx = \frac{\frac{a\,f\,(c\cosh(e+f\,x)\,2\mathrm{i}+d\,x\cosh(e+f\,x)\,2\mathrm{i})}{2}-a\,d\sinh(e+f\,x)\,1\mathrm{i}}{f^2}+\frac{a\,(d\,x^2+2\,c\,x)}{2}$$

input
```
int((a + a*sinh(e + f*x)*1i)*(c + d*x),x)
```

output
```
((a*f*(c*cosh(e + f*x)*2i + d*x*cosh(e + f*x)*2i))/2 - a*d*sinh(e + f*x)*1
i)/f^2 + (a*(2*c*x + d*x^2))/2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.10

$$\int (c+dx)(a+ia\sinh(e+fx))\,dx = \frac{a(2\cosh(fx+e)\,cfi+2\cosh(fx+e)\,dfix-2\sinh(fx+e)\,di+2cf^2x+df^2x^2)}{2f^2}$$

input
```
int((d*x+c)*(a+I*a*sinh(f*x+e)),x)
```

output
```
(a*(2*cosh(e + f*x)*c*f*i + 2*cosh(e + f*x)*d*f*i*x - 2*sinh(e + f*x)*d*i
+ 2*c*f**2*x + d*f**2*x**2))/(2*f**2)
```

3.99 $\int \frac{a+ia\sinh(e+fx)}{c+dx}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \frac{a+ia\sinh(e+fx)}{c+dx}\,dx=\frac{a\log(c+dx)}{d}+\frac{ia\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d}+\frac{ia\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{d}$$

output
```
a*ln(d*x+c)/d-I*a*Chi(c*f/d+f*x)*sinh(-e+c*f/d)/d+I*a*cosh(-e+c*f/d)*Shi(c
*f/d+f*x)/d
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

$$\int \frac{a+ia\sinh(e+fx)}{c+dx}\,dx=\frac{a\left(\log(c+dx)+i\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)+i\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)\right)}{d}$$

input
```
Integrate[(a + I*a*Sinh[e + f*x])/(c + d*x),x]
```

output
```
(a*(Log[c + d*x] + I*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] + I*Cosh[
e - (c*f)/d]*SinhIntegral[f*(c/d + x)]))/d
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+ia\sinh(e+fx)}{c+dx}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+a\sin(ie+ifx)}{c+dx}dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(\frac{a}{c+dx}+\frac{ia\sinh(e+fx)}{c+dx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ia\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d}+\frac{ia\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d}+\frac{a\log(c+dx)}{d}$$

input
```
Int[(a + I*a*Sinh[e + f*x])/(c + d*x),x]
```

output
```
(a*Log[c + d*x])/d + (I*a*CoshIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d
 + (I*a*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.37

method	result	size
risch	$\frac{a\ln(dx+c)}{d}+\frac{ia\,\mathrm{e}^{\frac{cf-de}{d}}\operatorname{expIntegral}_1\left(fx+e+\frac{cf-de}{d}\right)}{2d}-\frac{ia\,\mathrm{e}^{-\frac{cf-de}{d}}\operatorname{expIntegral}_1\left(-fx-e-\frac{cf-de}{d}\right)}{2d}$	96

input
```
int((a+I*a*sinh(f*x+e))/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*ln(d*x+c)/d+1/2*I*a/d*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/2*I*a/d
*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.13

$$\int \frac{a+ia\sinh(e+fx)}{c+dx}\,dx = \frac{i\,a\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(\frac{de-cf}{d}\right)}-i\,a\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-\frac{de-cf}{d}\right)}+2\,a\log\left(\frac{dx+c}{d}\right)}{2\,d}$$

input
```
integrate((a+I*a*sinh(f*x+e))/(d*x+c),x, algorithm="fricas")
```

output
```
1/2*(I*a*Ei((d*f*x + c*f)/d)*e^((d*e - c*f)/d) - I*a*Ei(-(d*f*x + c*f)/d)*
e^(-(d*e - c*f)/d) + 2*a*log((d*x + c)/d))/d
```

Sympy [F]

$$\int \frac{a + ia\sinh(e + fx)}{c + dx}\,dx = ia\left(\int \left(-\frac{i}{c + dx}\right)dx + \int \frac{\sinh(e + fx)}{c + dx}\,dx\right)$$

input
```
integrate((a+I*a*sinh(f*x+e))/(d*x+c),x)
```

output
```
I*a*(Integral(-I/(c + d*x), x) + Integral(sinh(e + f*x)/(c + d*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int \frac{a + ia\sinh(e + fx)}{c + dx}\,dx = \frac{1}{2}ia\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_1\left(\frac{(dx+c)f}{d}\right)}{d} - \frac{e^{\left(e-\frac{cf}{d}\right)}E_1\left(-\frac{(dx+c)f}{d}\right)}{d}\right) + \frac{a\log(dx + c)}{d}$$

input
```
integrate((a+I*a*sinh(f*x+e))/(d*x+c),x, algorithm="maxima")
```

output
```
1/2*I*a*(e^(-e + c*f/d)*exp_integral_e(1, (d*x + c)*f/d)/d - e^(e - c*f/d)
*exp_integral_e(1, -(d*x + c)*f/d)/d) + a*log(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.99

$$\int \frac{a+ia\sinh(e+fx)}{c+dx}\,dx = -\frac{-i\,a\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}+i\,a\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)}-2\,a\log\left(dx+c\right)}{2\,d}$$

input
```
integrate((a+I*a*sinh(f*x+e))/(d*x+c),x, algorithm="giac")
```

output
```
-1/2*(-I*a*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + I*a*Ei(-(d*f*x + c*f)/d)*e^
(-e + c*f/d) - 2*a*log(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+ia\sinh(e+fx)}{c+dx}\,dx = \int \frac{a+a\sinh(e+f\,x)\,1\mathrm{i}}{c+d\,x}\,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)/(c + d*x),x)
```

output
```
int((a + a*sinh(e + f*x)*1i)/(c + d*x), x)
```

Reduce [F]

$$\int \frac{a+ia\sinh(e+fx)}{c+dx}\,dx = \frac{a\left(\left(\int \frac{\sinh(fx+e)}{dx+c}dx\right)di+\log(dx+c)\right)}{d}$$

input
```
int((a+I*a*sinh(f*x+e))/(d*x+c),x)
```

output
```
(a*(int(sinh(e + f*x)/(c + d*x),x)*d*i + log(c + d*x)))/d
```

3.100 $\int \frac{a+ia\sinh(e+fx)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \frac{a+ia\sinh(e+fx)}{(c+dx)^2}\,dx = -\frac{a}{d(c+dx)}+\frac{iaf\cosh\left(e-\frac{cf}{d}\right)\operatorname{Chi}\left(\frac{cf}{d}+fx\right)}{d^2} - \frac{ia\sinh(e+fx)}{d(c+dx)}+\frac{iaf\sinh\left(e-\frac{cf}{d}\right)\operatorname{Shi}\left(\frac{cf}{d}+fx\right)}{d^2}$$

output
```
-a/d/(d*x+c)+I*a*f*cosh(-e+c*f/d)*Chi(c*f/d+f*x)/d^2-I*a*sinh(f*x+e)/d/(d*
x+c)-I*a*f*sinh(-e+c*f/d)*Shi(c*f/d+f*x)/d^2
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.87

$$\int \frac{a+ia\sinh(e+fx)}{(c+dx)^2}\,dx = \frac{ia\left(f(c+dx)\cosh\left(e-\frac{cf}{d}\right)\operatorname{Chi}\left(f\left(\frac{c}{d}+x\right)\right)-d(-i+\sinh(e+fx))+f(c+dx)\sinh\left(e-\frac{cf}{d}\right)\operatorname{Shi}\left(f\left(\frac{c}{d}+\right.\right.\right.}{d^2(c+dx)}$$

input
```
Integrate[(a + I*a*Sinh[e + f*x])/(c + d*x)^2,x]
```

output
```
(I*a*(f*(c + d*x)*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] - d*(-I + Si
nh[e + f*x]) + f*(c + d*x)*Sinh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)]))/(
d^2*(c + d*x))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + ia\sinh(e+fx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a + a\sin(ie+ifx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(\frac{a}{(c+dx)^2} + \frac{ia\sinh(e+fx)}{(c+dx)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{iaf\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d^2} + \frac{iaf\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d^2} - \frac{ia\sinh(e+fx)}{d(c+dx)} - \frac{a}{d(c+dx)}$$

input
```
Int[(a + I*a*Sinh[e + f*x])/(c + d*x)^2,x]
```

output
```
-(a/(d*(c + d*x))) + (I*a*f*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])
/d^2 - (I*a*Sinh[e + f*x])/(d*(c + d*x)) + (I*a*f*Sinh[e - (c*f)/d]*SinhIn
tegral[(c*f)/d + f*x])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.61

method	result
risch	$-\frac{a}{d(dx+c)}+\frac{iaf\,\mathrm{e}^{-fx-e}}{2d(dxf+cf)}-\frac{iaf\,\mathrm{e}^{\frac{cf-de}{d}}\,\mathrm{expIntegral}_1\left(fx+e+\frac{cf-de}{d}\right)}{2d^2}-\frac{iaf\,\mathrm{e}^{fx+e}}{2d^2\left(\frac{cf}{d}+fx\right)}-\frac{iaf\,\mathrm{e}^{-\frac{cf-de}{d}}\,\mathrm{expIntegral}_1\left(-fx-\right.}{2d^2}$

input
```
int((a+I*a*sinh(f*x+e))/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-a/d/(d*x+c)+1/2*I*a*f*exp(-f*x-e)/d/(d*f*x+c*f)-1/2*I*a*f/d^2*exp((c*f-d*
e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/2*I*a*f/d^2*exp(f*x+e)/(c*f/d+f*x)-1/2*I*a
*f/d^2*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.41

$$\int \frac{a+ia\sinh(e+fx)}{(c+dx)^2}\,dx$$
$$=\frac{\left(-i\,ade^{(2\,fx+2\,e)}+i\,ad+\left((i\,adfx+i\,acf)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(\frac{de-cf}{d}\right)}+(i\,adfx+i\,acf)\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-\frac{de-cf}{d}\right)}\right.\right.}{2\left(d^3x+cd^2\right)}$$

input
```
integrate((a+I*a*sinh(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*(-I*a*d*e^(2*f*x + 2*e) + I*a*d + ((I*a*d*f*x + I*a*c*f)*Ei((d*f*x + c
*f)/d)*e^((d*e - c*f)/d) + (I*a*d*f*x + I*a*c*f)*Ei(-(d*f*x + c*f)/d)*e^(-
(d*e - c*f)/d) - 2*a*d)*e^(f*x + e))*e^(-f*x - e)/(d^3*x + c*d^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + ia\sinh(e + fx)}{(c + dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*sinh(f*x+e))/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \frac{a + ia\sinh(e + fx)}{(c + dx)^2}\,dx = \frac{1}{2}i\,a\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d} - \frac{e^{\left(e-\frac{cf}{d}\right)}E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d}\right) - \frac{a}{d^2x + cd}$$

input
```
integrate((a+I*a*sinh(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*I*a*(e^(-e + c*f/d)*exp_integral_e(2, (d*x + c)*f/d)/((d*x + c)*d) - e
^(e - c*f/d)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c)*d)) - a/(d^2*x +
 c*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(91) = 182$.

Time = 0.16 (sec) , antiderivative size = 630, normalized size of antiderivative = 6.63

$$\int \frac{a+ia\sinh(e+fx)}{(c+dx)^2}\,dx$$
$$= \frac{1}{2} i a \left(\frac{\left((dx+c)\left(\frac{de}{dx+c} - \frac{cf}{dx+c} + f\right) f^2 \mathrm{Ei}\left(\frac{(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf}{d}\right) e^{\left(\frac{de-cf}{d}\right)} - def^2\mathrm{Ei}\left(\frac{(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c} \cdots\right.}{d} \cdots \right.\right.}{\left((dx+c)d^4\left(\frac{de}{dx+c}-\frac{cf}{dx+c} \cdots\right.\right.} \cdots \right.$$
$$- \frac{a}{(dx+c)d}$$

input
```
integrate((a+I*a*sinh(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/2*I*a*(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(((d*x + c)*
(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e - c*f)/d) - d*
e*f^2*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^
((d*e - c*f)/d) + c*f^3*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)
- d*e + c*f)/d)*e^((d*e - c*f)/d) - d*f^2*e^((d*x + c)*(d*e/(d*x + c) - c*
f/(d*x + c) + f)/d))*d^2/((((d*x + c)*d^4*(d*e/(d*x + c) - c*f/(d*x + c) +
f) - d^5*e + c*d^4*f)*f) + ((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*
f^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(
-(d*e - c*f)/d) - d*e*f^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) +
f) - d*e + c*f)/d)*e^(-(d*e - c*f)/d) + c*f^3*Ei(-((d*x + c)*(d*e/(d*x + c
) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(d*e - c*f)/d) + d*f^2*e^(-(d*x
 + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d))*d^2/((((d*x + c)*d^4*(d*e/(d*
x + c) - c*f/(d*x + c) + f) - d^5*e + c*d^4*f)*f)) - a/((d*x + c)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+ia\sinh(e+fx)}{(c+dx)^2}\,dx=\int \frac{a+a\sinh(e+f\,x)\;1\mathrm{i}}{{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)/(c + d*x)^2,x)
```

output
```
int((a + a*sinh(e + f*x)*1i)/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{a+ia\sinh(e+fx)}{(c+dx)^2}\,dx = \frac{a\left(e^{2e}\left(\int \frac{e^{fx}}{d^2x^2+2cdx+c^2}dx\right)c^2i+e^{2e}\left(\int \frac{e^{fx}}{d^2x^2+2cdx+c^2}dx\right)cdix+2e^e x-\left(\int \frac{1}{e^{fx}c^2+2e^{fx}cdx+e^{fx}d^2x^2}dx\right)c^2i-\left(\int\right.\right.}{2e^e c\,(dx+c)}$$

input
```
int((a+I*a*sinh(f*x+e))/(d*x+c)^2,x)
```

output
```
(a*(e**(2*e)*int(e**(f*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*c**2*i + e**(2*e
)*int(e**(f*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*c*d*i*x + 2*e**e*x - int(1/
(e**(f*x)*c**2 + 2*e**(f*x)*c*d*x + e**(f*x)*d**2*x**2),x)*c**2*i - int(1/
(e**(f*x)*c**2 + 2*e**(f*x)*c*d*x + e**(f*x)*d**2*x**2),x)*c*d*i*x))/(2*e*
*e*c*(c + d*x))
```

3.101 $\int \frac{a+ia\sinh(e+fx)}{(c+dx)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 131

$$\int \frac{a+ia\sinh(e+fx)}{(c+dx)^3} dx = -\frac{a}{2d(c+dx)^2} - \frac{iaf\cosh(e+fx)}{2d^2(c+dx)} + \frac{iaf^2\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{2d^3} - \frac{ia\sinh(e+fx)}{2d(c+dx)^2} + \frac{iaf^2\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{2d^3}$$

output
```
-1/2*a/d/(d*x+c)^2-1/2*I*a*f*cosh(f*x+e)/d^2/(d*x+c)-1/2*I*a*f^2*Chi(c*f/d
+f*x)*sinh(-e+c*f/d)/d^3-1/2*I*a*sinh(f*x+e)/d/(d*x+c)^2+1/2*I*a*f^2*cosh(
-e+c*f/d)*Shi(c*f/d+f*x)/d^3
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.83

$$\int \frac{a+ia\sinh(e+fx)}{(c+dx)^3} dx = \frac{ia\left(f^2(c+dx)^2\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right) - d(f(c+dx)\cosh(e+fx) + d(-i+\sinh(e+fx))) + f^2(\right.}{2d^3(c+dx)^2}$$

input
```
Integrate[(a + I*a*Sinh[e + f*x])/(c + d*x)^3,x]
```

output
```
((I/2)*a*(f^2*(c + d*x)^2*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] - d*
(f*(c + d*x)*Cosh[e + f*x] + d*(-I + Sinh[e + f*x])) + f^2*(c + d*x)^2*Cos
h[e - (c*f)/d]*SinhIntegral[f*(c/d + x)]))/(d^3*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+ia\sinh(e+fx)}{(c+dx)^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{a+a\sin(ie+ifx)}{(c+dx)^3}dx$$
$$\Big\downarrow \text{3798}$$
$$\int \left(\frac{a}{(c+dx)^3}+\frac{ia\sinh(e+fx)}{(c+dx)^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{iaf^2\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{2d^3}+\frac{iaf^2\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{2d^3}-\frac{iaf\cosh(e+fx)}{2d^2(c+dx)}-\frac{ia\sinh(e+fx)}{2d(c+dx)^2}-\frac{a}{2d(c+dx)^2}$$

input
```
Int[(a + I*a*Sinh[e + f*x])/(c + d*x)^3,x]
```

output
```
-1/2*a/(d*(c + d*x)^2) - ((I/2)*a*f*Cosh[e + f*x])/(d^2*(c + d*x)) + ((I/2
)*a*f^2*CoshIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d^3 - ((I/2)*a*Sinh
[e + f*x])/(d*(c + d*x)^2) + ((I/2)*a*f^2*Cosh[e - (c*f)/d]*SinhIntegral[(
c*f)/d + f*x])/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(119) = 238.

Time = 0.48 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.31

method	result
risch	$-\frac{a}{2d(dx+c)^2}-\frac{ia\,f^3 \mathrm{e}^{-fx-e}x}{4d(d^2x^2f^2+2cd\,f^2x+c^2f^2)}-\frac{ia\,f^3\mathrm{e}^{-fx-e}c}{4d^2(d^2x^2f^2+2cd\,f^2x+c^2f^2)}+\frac{ia\,f^2\mathrm{e}^{-fx-e}}{4d(d^2x^2f^2+2cd\,f^2x+c^2f^2)}+\frac{ia\,f^2\mathrm{e}^{\frac{cf-de}{d}}\,\mathrm{e}}{}$

input
```
int((a+I*a*sinh(f*x+e))/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/d/(d*x+c)^2-1/4*I*a*f^3*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*
f^2)*x-1/4*I*a*f^3*exp(-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c+1/4
*I*a*f^2*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)+1/4*I*a*f^2/d^3*e
xp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/4*I*a*f^2/d^3*exp(f*x+e)/(c*f/d+
f*x)^2-1/4*I*a*f^2/d^3*exp(f*x+e)/(c*f/d+f*x)-1/4*I*a*f^2/d^3*exp(-(c*f-d*
e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.69

$$\int \frac{a + ia\sinh(e + fx)}{(c + dx)^3}\,dx$$
$$= \frac{\left(-i\,ad^2fx - i\,acdf + i\,ad^2 + (-i\,ad^2fx - i\,acdf - i\,ad^2)e^{(2\,fx+2\,e)} - \left(2\,ad^2 - (i\,ad^2f^2x^2 + 2i\,acdf^2x + \right.\right.}{4\,(d^5x^2 + 2}$$

input
```
integrate((a+I*a*sinh(f*x+e))/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/4*(-I*a*d^2*f*x - I*a*c*d*f + I*a*d^2 + (-I*a*d^2*f*x - I*a*c*d*f - I*a*
d^2)*e^(2*f*x + 2*e) - (2*a*d^2 - (I*a*d^2*f^2*x^2 + 2*I*a*c*d*f^2*x + I*a
*c^2*f^2)*Ei((d*f*x + c*f)/d)*e^((d*e - c*f)/d) - (-I*a*d^2*f^2*x^2 - 2*I*
a*c*d*f^2*x - I*a*c^2*f^2)*Ei(-(d*f*x + c*f)/d)*e^(-(d*e - c*f)/d))*e^(f*x
 + e))*e^(-f*x - e)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + ia\sinh(e + fx)}{(c + dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*sinh(f*x+e))/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.76

$$\int \frac{a+ia\sinh(e+fx)}{(c+dx)^3}\,dx = \frac{1}{2}i\,a\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2d} - \frac{e^{\left(e-\frac{cf}{d}\right)}E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}\right) - \frac{a}{2\left(d^3x^2+2\,cd^2x+c^2d\right)}$$

input
```
integrate((a+I*a*sinh(f*x+e))/(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/2*I*a*(e^(-e + c*f/d)*exp_integral_e(3, (d*x + c)*f/d)/((d*x + c)^2*d) -
 e^(e - c*f/d)*exp_integral_e(3, -(d*x + c)*f/d)/((d*x + c)^2*d)) - 1/2*a/
(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(115) = 230.

Time = 0.13 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.46

$$\int \frac{a+ia\sinh(e+fx)}{(c+dx)^3}\,dx$$
$$= \frac{i\,ad^2f^2x^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)} - i\,ad^2f^2x^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)} + 2i\,acdf^2x\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)} - 2i\,acdf^2x}{}$$

input
```
integrate((a+I*a*sinh(f*x+e))/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/4*(I*a*d^2*f^2*x^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) - I*a*d^2*f^2*x^2*E
i(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + 2*I*a*c*d*f^2*x*Ei((d*f*x + c*f)/d)*e
^(e - c*f/d) - 2*I*a*c*d*f^2*x*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + I*a*c
^2*f^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) - I*a*c^2*f^2*Ei(-(d*f*x + c*f)/d
)*e^(-e + c*f/d) - I*a*d^2*f*x*e^(f*x + e) - I*a*d^2*f*x*e^(-f*x - e) - I*
a*c*d*f*e^(f*x + e) - I*a*c*d*f*e^(-f*x - e) - I*a*d^2*e^(f*x + e) + I*a*d
^2*e^(-f*x - e) - 2*a*d^2)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + ia\sinh(e + fx)}{(c + dx)^3}\,dx = \int \frac{a + a\sinh(e + f\,x)\,1\mathrm{i}}{(c + d\,x)^3}\,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)/(c + d*x)^3,x)
```

output
```
int((a + a*sinh(e + f*x)*1i)/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{a + ia\sinh(e + fx)}{(c + dx)^3}\,dx = \text{too large to display}$$

input
```
int((a+I*a*sinh(f*x+e))/(d*x+c)^3,x)
```

output
```
(a*(2*e**(e + f*x)*cosh(e + f*x)*c**3*d*f**3*i + 2*e**(e + f*x)*cosh(e + f
*x)*c**2*d**2*f**3*i*x - 8*e**(e + f*x)*cosh(e + f*x)*c*d**3*f*i - 8*e**(e
 + f*x)*cosh(e + f*x)*d**4*f*i*x - e**(2*e + 2*f*x)*c**2*d**2*f**3*i*x + e
**(2*e + 2*f*x)*c**2*d**2*f**2*i + 2*e**(2*e + 2*f*x)*c*d**3*f*i + 4*e**(2
*e + 2*f*x)*d**4*f*i*x - e**(2*e + f*x)*int((e**(f*x)*x)/(c**5*f**2 + 3*c*
*4*d*f**2*x + 3*c**3*d**2*f**2*x**2 - 4*c**3*d**2 + c**2*d**3*f**2*x**3 -
12*c**2*d**3*x - 12*c*d**4*x**2 - 4*d**5*x**3),x)*c**7*d**2*f**6*i - 2*e**
(2*e + f*x)*int((e**(f*x)*x)/(c**5*f**2 + 3*c**4*d*f**2*x + 3*c**3*d**2*f*
*2*x**2 - 4*c**3*d**2 + c**2*d**3*f**2*x**3 - 12*c**2*d**3*x - 12*c*d**4*x
**2 - 4*d**5*x**3),x)*c**6*d**3*f**6*i*x - 2*e**(2*e + f*x)*int((e**(f*x)*
x)/(c**5*f**2 + 3*c**4*d*f**2*x + 3*c**3*d**2*f**2*x**2 - 4*c**3*d**2 + c*
*2*d**3*f**2*x**3 - 12*c**2*d**3*x - 12*c*d**4*x**2 - 4*d**5*x**3),x)*c**6
*d**3*f**5*i - e**(2*e + f*x)*int((e**(f*x)*x)/(c**5*f**2 + 3*c**4*d*f**2*
x + 3*c**3*d**2*f**2*x**2 - 4*c**3*d**2 + c**2*d**3*f**2*x**3 - 12*c**2*d*
*3*x - 12*c*d**4*x**2 - 4*d**5*x**3),x)*c**5*d**4*f**6*i*x**2 - 4*e**(2*e
+ f*x)*int((e**(f*x)*x)/(c**5*f**2 + 3*c**4*d*f**2*x + 3*c**3*d**2*f**2*x*
*2 - 4*c**3*d**2 + c**2*d**3*f**2*x**3 - 12*c**2*d**3*x - 12*c*d**4*x**2 -
 4*d**5*x**3),x)*c**5*d**4*f**5*i*x + 6*e**(2*e + f*x)*int((e**(f*x)*x)/(c
**5*f**2 + 3*c**4*d*f**2*x + 3*c**3*d**2*f**2*x**2 - 4*c**3*d**2 + c**2*d*
*3*f**2*x**3 - 12*c**2*d**3*x - 12*c*d**4*x**2 - 4*d**5*x**3),x)*c**5*d...
```

3.102 $\int (c+dx)^3(a+ia\sinh(e+fx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 232

$$\begin{aligned}\int (c+dx)^3(a+ia\sinh(e+fx))^2\,dx &= \frac{3a^2d(c+dx)^2}{8f^2}+\frac{3a^2(c+dx)^4}{8d}+\frac{12ia^2d^2(c+dx)\cosh(e+fx)}{f^3}\\&\quad+\frac{2ia^2(c+dx)^3\cosh(e+fx)}{f}-\frac{12ia^2d^3\sinh(e+fx)}{f^4}-\frac{6ia^2d(c+dx)^2\sinh(e+fx)}{f^2}\\&\quad-\frac{3a^2d^2(c+dx)\cosh(e+fx)\sinh(e+fx)}{4f^3}-\frac{a^2(c+dx)^3\cosh(e+fx)\sinh(e+fx)}{2f}\\&\quad+\frac{3a^2d^3\sinh^2(e+fx)}{8f^4}+\frac{3a^2d(c+dx)^2\sinh^2(e+fx)}{4f^2}\end{aligned}$$

output
```
3/8*a^2*d*(d*x+c)^2/f^2+3/8*a^2*(d*x+c)^4/d+12*I*a^2*d^2*(d*x+c)*cosh(f*x+
e)/f^3+2*I*a^2*(d*x+c)^3*cosh(f*x+e)/f-12*I*a^2*d^3*sinh(f*x+e)/f^4-6*I*a^
2*d*(d*x+c)^2*sinh(f*x+e)/f^2-3/4*a^2*d^2*(d*x+c)*cosh(f*x+e)*sinh(f*x+e)/
f^3-1/2*a^2*(d*x+c)^3*cosh(f*x+e)*sinh(f*x+e)/f+3/8*a^2*d^3*sinh(f*x+e)^2/
f^4+3/4*a^2*d*(d*x+c)^2*sinh(f*x+e)^2/f^2
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.95

$$\int (c+dx)^3(a+ia\sinh(e+fx))^2\,dx$$
$$= \frac{a^2(6f^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)+32if(c+dx)\left(c^2f^2+2cdf^2x+d^2(6+f^2x^2)\right)\cosh(e+fx)+3}{}$$

input
```
Integrate[(c + d*x)^3*(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
(a^2*(6*f^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + (32*I)*f*(c +
d*x)*(c^2*f^2 + 2*c*d*f^2*x + d^2*(6 + f^2*x^2))*Cosh[e + f*x] + 3*d*(2*c^
2*f^2 + 4*c*d*f^2*x + d^2*(1 + 2*f^2*x^2))*Cosh[2*(e + f*x)] - (96*I)*d*(c
^2*f^2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Sinh[e + f*x] - 2*f*(c + d*x)*(2
*c^2*f^2 + 4*c*d*f^2*x + d^2*(3 + 2*f^2*x^2))*Sinh[2*(e + f*x)]))/(16*f^4)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3(a+ia\sinh(e+fx))^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (c+dx)^3(a+a\sin(ie+ifx))^2dx$$

$$\Big\downarrow \; 3798$$

$$\int \left(-a^2(c+dx)^3\sinh^2(e+fx)+2ia^2(c+dx)^3\sinh(e+fx)+a^2(c+dx)^3\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{12ia^2d^2(c+dx)\cosh(e+fx)}{f^3} - \frac{3a^2d^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{4f^3} + \frac{3a^2d(c+dx)^2\sinh^2(e+fx)}{4f^2} - \frac{6ia^2d(c+dx)^2\sinh(e+fx)}{f^2} + \frac{2ia^2(c+dx)^3\cosh(e+fx)}{f} - \frac{a^2(c+dx)^3\sinh(e+fx)\cosh(e+fx)}{2f} + \frac{3a^2d(c+dx)^2}{8f^2} + \frac{3a^2(c+dx)^4}{8d} + \frac{3a^2d^3\sinh^2(e+fx)}{8f^4} - \frac{12ia^2d^3\sinh(e+fx)}{f^4}$$

input
```
Int[(c + d*x)^3*(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
(3*a^2*d*(c + d*x)^2)/(8*f^2) + (3*a^2*(c + d*x)^4)/(8*d) + ((12*I)*a^2*d^
2*(c + d*x)*Cosh[e + f*x])/f^3 + ((2*I)*a^2*(c + d*x)^3*Cosh[e + f*x])/f -
 ((12*I)*a^2*d^3*Sinh[e + f*x])/f^4 - ((6*I)*a^2*d*(c + d*x)^2*Sinh[e + f*
x])/f^2 - (3*a^2*d^2*(c + d*x)*Cosh[e + f*x]*Sinh[e + f*x])/(4*f^3) - (a^2
*(c + d*x)^3*Cosh[e + f*x]*Sinh[e + f*x])/(2*f) + (3*a^2*d^3*Sinh[e + f*x]
^2)/(8*f^4) + (3*a^2*d*(c + d*x)^2*Sinh[e + f*x]^2)/(4*f^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.82

method	result
parallelrisch	$\frac{2a^2\left(-\frac{\left((dx+c)^2f^2+\frac{3d^2}{2}\right)(dx+c)f\sinh(2fx+2e)}{8}+\frac{3d\left((dx+c)^2f^2+\frac{d^2}{2}\right)\cosh(2fx+2e)}{16}+i(dx+c)\left((dx+c)^2f^2+6d^2\right)f\cosh(\right.}{f^4}$
risch	$\frac{3a^2d^3x^4}{8}+\frac{3a^2d^2c\,x^3}{2}+\frac{9a^2d\,c^2x^2}{4}+\frac{3a^2c^3x}{2}+\frac{3a^2c^4}{8d}-\frac{a^2\left(4d^3x^3f^3+12c\,d^2f^3x^2+12c^2d\,f^3x-6d^3f^2x^2+4c^3\right.}{32}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display
orering	Expression too large to display

input
```
int((d*x+c)^3*(a+I*a*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
2*a^2*(-1/8*((d*x+c)^2*f^2+3/2*d^2)*(d*x+c)*f*sinh(2*f*x+2*e)+3/16*d*((d*x
+c)^2*f^2+1/2*d^2)*cosh(2*f*x+2*e)+I*(d*x+c)*((d*x+c)^2*f^2+6*d^2)*f*cosh(
f*x+e)-3*I*((d*x+c)^2*f^2+2*d^2)*d*sinh(f*x+e)+3/4*(1/2*d*x+c)*x*(1/2*x^2*
d^2+c*d*x+c^2)*f^4+I*c^3*f^3-3/16*c^2*d*f^2+6*I*c*d^2*f-3/32*d^3)/f^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 596 vs. 2(212) = 424.

Time = 0.10 (sec) , antiderivative size = 596, normalized size of antiderivative = 2.57

$$\int (c+dx)^3(a+ia\sinh(e+fx))^2\,dx$$
$$=\frac{\left(4\,a^2d^3f^3x^3+4\,a^2c^3f^3+6\,a^2c^2df^2+6\,a^2cd^2f+3\,a^2d^3+6\,(2\,a^2cd^2f^3+a^2d^3f^2)x^2+6\,(2\,a^2c^2df^3+2\,a^2\right.}{}$$

input
```
integrate((d*x+c)^3*(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/32*(4*a^2*d^3*f^3*x^3 + 4*a^2*c^3*f^3 + 6*a^2*c^2*d*f^2 + 6*a^2*c*d^2*f
+ 3*a^2*d^3 + 6*(2*a^2*c*d^2*f^3 + a^2*d^3*f^2)*x^2 + 6*(2*a^2*c^2*d*f^3 +
 2*a^2*c*d^2*f^2 + a^2*d^3*f)*x - (4*a^2*d^3*f^3*x^3 + 4*a^2*c^3*f^3 - 6*a
^2*c^2*d*f^2 + 6*a^2*c*d^2*f - 3*a^2*d^3 + 6*(2*a^2*c*d^2*f^3 - a^2*d^3*f^
2)*x^2 + 6*(2*a^2*c^2*d*f^3 - 2*a^2*c*d^2*f^2 + a^2*d^3*f)*x)*e^(4*f*x + 4
*e) - 32*(-I*a^2*d^3*f^3*x^3 - I*a^2*c^3*f^3 + 3*I*a^2*c^2*d*f^2 - 6*I*a^2
*c*d^2*f + 6*I*a^2*d^3 + 3*(-I*a^2*c*d^2*f^3 + I*a^2*d^3*f^2)*x^2 + 3*(-I*
a^2*c^2*d*f^3 + 2*I*a^2*c*d^2*f^2 - 2*I*a^2*d^3*f)*x)*e^(3*f*x + 3*e) + 12
*(a^2*d^3*f^4*x^4 + 4*a^2*c*d^2*f^4*x^3 + 6*a^2*c^2*d*f^4*x^2 + 4*a^2*c^3*
f^4*x)*e^(2*f*x + 2*e) - 32*(-I*a^2*d^3*f^3*x^3 - I*a^2*c^3*f^3 - 3*I*a^2*
c^2*d*f^2 - 6*I*a^2*c*d^2*f - 6*I*a^2*d^3 + 3*(-I*a^2*c*d^2*f^3 - I*a^2*d^
3*f^2)*x^2 + 3*(-I*a^2*c^2*d*f^3 - 2*I*a^2*c*d^2*f^2 - 2*I*a^2*d^3*f)*x)*e
^(f*x + e))*e^(-2*f*x - 2*e)/f^4
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 1134, normalized size of antiderivative = 4.89

$$\int (c+dx)^3(a+ia\sinh(e+fx))^2\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**3*(a+I*a*sinh(f*x+e))**2,x)
```

output
```
3*a**2*c**3*x/2 + 9*a**2*c**2*d*x**2/4 + 3*a**2*c*d**2*x**3/2 + 3*a**2*d**
3*x**4/8 + Piecewise((((128*a**2*c**3*f**15*exp(e) + 384*a**2*c**2*d*f**15
*x*exp(e) + 192*a**2*c**2*d*f**14*exp(e) + 384*a**2*c*d**2*f**15*x**2*exp(
e) + 384*a**2*c*d**2*f**14*x*exp(e) + 192*a**2*c*d**2*f**13*exp(e) + 128*a
**2*d**3*f**15*x**3*exp(e) + 192*a**2*d**3*f**14*x**2*exp(e) + 192*a**2*d*
*3*f**13*x*exp(e) + 96*a**2*d**3*f**12*exp(e))*exp(-2*f*x) + (-128*a**2*c*
*3*f**15*exp(5*e) - 384*a**2*c**2*d*f**15*x*exp(5*e) + 192*a**2*c**2*d*f**
14*exp(5*e) - 384*a**2*c*d**2*f**15*x**2*exp(5*e) + 384*a**2*c*d**2*f**14*
x*exp(5*e) - 192*a**2*c*d**2*f**13*exp(5*e) - 128*a**2*d**3*f**15*x**3*exp
(5*e) + 192*a**2*d**3*f**14*x**2*exp(5*e) - 192*a**2*d**3*f**13*x*exp(5*e)
 + 96*a**2*d**3*f**12*exp(5*e))*exp(2*f*x) + (1024*I*a**2*c**3*f**15*exp(2
*e) + 3072*I*a**2*c**2*d*f**15*x*exp(2*e) + 3072*I*a**2*c**2*d*f**14*exp(2
*e) + 3072*I*a**2*c*d**2*f**15*x**2*exp(2*e) + 6144*I*a**2*c*d**2*f**14*x*
exp(2*e) + 6144*I*a**2*c*d**2*f**13*exp(2*e) + 1024*I*a**2*d**3*f**15*x**3
*exp(2*e) + 3072*I*a**2*d**3*f**14*x**2*exp(2*e) + 6144*I*a**2*d**3*f**13*
x*exp(2*e) + 6144*I*a**2*d**3*f**12*exp(2*e))*exp(-f*x) + (1024*I*a**2*c**
3*f**15*exp(4*e) + 3072*I*a**2*c**2*d*f**15*x*exp(4*e) - 3072*I*a**2*c**2*
d*f**14*exp(4*e) + 3072*I*a**2*c*d**2*f**15*x**2*exp(4*e) - 6144*I*a**2*c*
d**2*f**14*x*exp(4*e) + 6144*I*a**2*c*d**2*f**13*exp(4*e) + 1024*I*a**2*d*
*3*f**15*x**3*exp(4*e) - 3072*I*a**2*d**3*f**14*x**2*exp(4*e) + 6144*I*...
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(212) = 424$.

Time = 0.07 (sec) , antiderivative size = 525, normalized size of antiderivative = 2.26

$$\int (c+dx)^3(a+ia\sinh(e+fx))^2\,dx = \frac{1}{4}a^2d^3x^4 + a^2cd^2x^3 + \frac{3}{2}a^2c^2dx^2 + \frac{3}{16}\left(4x^2 - \frac{\left(2fxe^{(2e)} - e^{(2e)}\right)e^{(2fx)}}{f^2} + \frac{(2fx+1)e^{(-2fx-2e)}}{f^2}\right)a^2c^2d + \frac{1}{16}\left(8x^3 - \frac{3\left(2f^2x^2e^{(2e)} - 2fxe^{(2e)} + e^{(2e)}\right)e^{(2fx)}}{f^3} + \frac{3(2f^2x^2+2fx+1)e^{(-2fx-2e)}}{f^3}\right)a^2cd^2 + \frac{1}{32}\left(4x^4 - \frac{\left(4f^3x^3e^{(2e)} - 6f^2x^2e^{(2e)} + 6fxe^{(2e)} - 3e^{(2e)}\right)e^{(2fx)}}{f^4} + \frac{(4f^3x^3+6f^2x^2+6fx+3)e^{(-2fx-2e)}}{f^4}\right)a^2d^3 + \frac{1}{8}a^2c^3\left(4x - \frac{e^{(2fx+2e)}}{f} + \frac{e^{(-2fx-2e)}}{f}\right) + a^2c^3x + 3ia^2c^2d\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + 3ia^2cd^2\left(\frac{(f^2x^2e^e - 2fxe^e + 2e^e)e^{(fx)}}{f^3} + \frac{(f^2x^2+2fx+2)e^{(-fx-e)}}{f^3}\right) + ia^2d^3\left(\frac{(f^3x^3e^e - 3f^2x^2e^e + 6fxe^e - 6e^e)e^{(fx)}}{f^4} + \frac{(f^3x^3+3f^2x^2+6fx+6)e^{(-fx-e)}}{f^4}\right) + \frac{2ia^2c^3\cosh(fx+e)}{f}$$

input
```
integrate((d*x+c)^3*(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/4*a^2*d^3*x^4 + a^2*c*d^2*x^3 + 3/2*a^2*c^2*d*x^2 + 3/16*(4*x^2 - (2*f*x
*e^(2*e) - e^(2*e))*e^(2*f*x)/f^2 + (2*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*a^2*
c^2*d + 1/16*(8*x^3 - 3*(2*f^2*x^2*e^(2*e) - 2*f*x*e^(2*e) + e^(2*e))*e^(2
*f*x)/f^3 + 3*(2*f^2*x^2 + 2*f*x + 1)*e^(-2*f*x - 2*e)/f^3)*a^2*c*d^2 + 1/
32*(4*x^4 - (4*f^3*x^3*e^(2*e) - 6*f^2*x^2*e^(2*e) + 6*f*x*e^(2*e) - 3*e^(
2*e))*e^(2*f*x)/f^4 + (4*f^3*x^3 + 6*f^2*x^2 + 6*f*x + 3)*e^(-2*f*x - 2*e)
/f^4)*a^2*d^3 + 1/8*a^2*c^3*(4*x - e^(2*f*x + 2*e)/f + e^(-2*f*x - 2*e)/f)
 + a^2*c^3*x + 3*I*a^2*c^2*d*((f*x*e^e - e^e)*e^(f*x)/f^2 + (f*x + 1)*e^(-
f*x - e)/f^2) + 3*I*a^2*c*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f
^3 + (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + I*a^2*d^3*((f^3*x^3*e^e - 3
*f^2*x^2*e^e + 6*f*x*e^e - 6*e^e)*e^(f*x)/f^4 + (f^3*x^3 + 3*f^2*x^2 + 6*f
*x + 6)*e^(-f*x - e)/f^4) + 2*I*a^2*c^3*cosh(f*x + e)/f
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(212) = 424$.

Time = 0.14 (sec) , antiderivative size = 580, normalized size of antiderivative = 2.50

$$\int (c+dx)^3(a+ia\sinh(e+fx))^2\,dx = \frac{3}{8}a^2d^3x^4 + \frac{3}{2}a^2cd^2x^3 + \frac{9}{4}a^2c^2dx^2 + \frac{3}{2}a^2c^3x$$
$$-\frac{(4a^2d^3f^3x^3 + 12a^2cd^2f^3x^2 + 12a^2c^2df^3x - 6a^2d^3f^2x^2 + 4a^2c^3f^3 - 12a^2cd^2f^2x - 6a^2c^2df^2 + 6a^2d^3}{32f^4}$$
$$+\frac{(ia^2d^3f^3x^3 + 3ia^2cd^2f^3x^2 + 3ia^2c^2df^3x - 3ia^2d^3f^2x^2 + ia^2c^3f^3 - 6ia^2cd^2f^2x - 3ia^2c^2df^2 + 6ia^2d}{f^4}$$
$$+\frac{(ia^2d^3f^3x^3 + 3ia^2cd^2f^3x^2 + 3ia^2c^2df^3x + 3ia^2d^3f^2x^2 + ia^2c^3f^3 + 6ia^2cd^2f^2x + 3ia^2c^2df^2 + 6ia^2d}{f^4}$$
$$+\frac{(4a^2d^3f^3x^3 + 12a^2cd^2f^3x^2 + 12a^2c^2df^3x + 6a^2d^3f^2x^2 + 4a^2c^3f^3 + 12a^2cd^2f^2x + 6a^2c^2df^2 + 6a^2d^3}{32f^4}$$

input
```
integrate((d*x+c)^3*(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
3/8*a^2*d^3*x^4 + 3/2*a^2*c*d^2*x^3 + 9/4*a^2*c^2*d*x^2 + 3/2*a^2*c^3*x -
1/32*(4*a^2*d^3*f^3*x^3 + 12*a^2*c*d^2*f^3*x^2 + 12*a^2*c^2*d*f^3*x - 6*a^
2*d^3*f^2*x^2 + 4*a^2*c^3*f^3 - 12*a^2*c*d^2*f^2*x - 6*a^2*c^2*d*f^2 + 6*a
^2*d^3*f*x + 6*a^2*c*d^2*f - 3*a^2*d^3)*e^(2*f*x + 2*e)/f^4 + (I*a^2*d^3*f
^3*x^3 + 3*I*a^2*c*d^2*f^3*x^2 + 3*I*a^2*c^2*d*f^3*x - 3*I*a^2*d^3*f^2*x^2
 + I*a^2*c^3*f^3 - 6*I*a^2*c*d^2*f^2*x - 3*I*a^2*c^2*d*f^2 + 6*I*a^2*d^3*f
*x + 6*I*a^2*c*d^2*f - 6*I*a^2*d^3)*e^(f*x + e)/f^4 + (I*a^2*d^3*f^3*x^3 +
 3*I*a^2*c*d^2*f^3*x^2 + 3*I*a^2*c^2*d*f^3*x + 3*I*a^2*d^3*f^2*x^2 + I*a^2
*c^3*f^3 + 6*I*a^2*c*d^2*f^2*x + 3*I*a^2*c^2*d*f^2 + 6*I*a^2*d^3*f*x + 6*I
*a^2*c*d^2*f + 6*I*a^2*d^3)*e^(-f*x - e)/f^4 + 1/32*(4*a^2*d^3*f^3*x^3 + 1
2*a^2*c*d^2*f^3*x^2 + 12*a^2*c^2*d*f^3*x + 6*a^2*d^3*f^2*x^2 + 4*a^2*c^3*f
^3 + 12*a^2*c*d^2*f^2*x + 6*a^2*c^2*d*f^2 + 6*a^2*d^3*f*x + 6*a^2*c*d^2*f
+ 3*a^2*d^3)*e^(-2*f*x - 2*e)/f^4
```

Mupad [B] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.69

$$\int (c+dx)^3(a+ia\sinh(e+fx))^2\,dx$$
$$= \frac{a^2\left(3\,d^3\cosh(2\,e+2\,f\,x)+24\,c^3\,f^4\,x-4\,c^3\,f^3\sinh(2\,e+2\,f\,x)+6\,d^3\,f^4\,x^4+6\,c^2\,d\,f^2\cosh(2\,e+2\,f\,x)\right.}{}$$

input
```
int((a + a*sinh(e + f*x)*1i)^2*(c + d*x)^3,x)
```

output
```
(a^2*(3*d^3*cosh(2*e + 2*f*x) - d^3*sinh(e + f*x)*192i + c^3*f^3*cosh(e +
f*x)*32i + 24*c^3*f^4*x - 4*c^3*f^3*sinh(2*e + 2*f*x) + 6*d^3*f^4*x^4 + 6*
c^2*d*f^2*cosh(2*e + 2*f*x) + 36*c^2*d*f^4*x^2 + 24*c*d^2*f^4*x^3 + d^3*f^
3*x^3*cosh(e + f*x)*32i - d^3*f^2*x^2*sinh(e + f*x)*96i + c*d^2*f*cosh(e +
 f*x)*192i + d^3*f*x*cosh(e + f*x)*192i + 6*d^3*f^2*x^2*cosh(2*e + 2*f*x)
- 4*d^3*f^3*x^3*sinh(2*e + 2*f*x) - 6*c*d^2*f*sinh(2*e + 2*f*x) - c^2*d*f^
2*sinh(e + f*x)*96i - 6*d^3*f*x*sinh(2*e + 2*f*x) + c^2*d*f^3*x*cosh(e + f
*x)*96i - c*d^2*f^2*x*sinh(e + f*x)*192i + 12*c*d^2*f^2*x*cosh(2*e + 2*f*x
) + c*d^2*f^3*x^2*cosh(e + f*x)*96i - 12*c^2*d*f^3*x*sinh(2*e + 2*f*x) - 1
2*c*d^2*f^3*x^2*sinh(2*e + 2*f*x)))/(16*f^4)
```

Reduce [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 755, normalized size of antiderivative = 3.25

$$\int (c+dx)^3(a+ia\sinh(e+fx))^2\,dx$$
$$= \frac{a^2\left(3d^3+6e^{4fx+4e}c^2d\,f^2-6e^{4fx+4e}c\,d^2f-4e^{4fx+4e}d^3f^3x^3+6e^{4fx+4e}d^3f^2x^2-6e^{4fx+4e}d^3fx+32e^{3fx+3e}\right.}{}$$

input
```
int((d*x+c)^3*(a+I*a*sinh(f*x+e))^2,x)
```

output

```
(a**2*( - 4*e**(4*e + 4*f*x)*c**3*f**3 - 12*e**(4*e + 4*f*x)*c**2*d*f**3*x
 + 6*e**(4*e + 4*f*x)*c**2*d*f**2 - 12*e**(4*e + 4*f*x)*c*d**2*f**3*x**2 +
 12*e**(4*e + 4*f*x)*c*d**2*f**2*x - 6*e**(4*e + 4*f*x)*c*d**2*f - 4*e**(4
*e + 4*f*x)*d**3*f**3*x**3 + 6*e**(4*e + 4*f*x)*d**3*f**2*x**2 - 6*e**(4*e
 + 4*f*x)*d**3*f*x + 3*e**(4*e + 4*f*x)*d**3 + 32*e**(3*e + 3*f*x)*c**3*f*
*3*i + 96*e**(3*e + 3*f*x)*c**2*d*f**3*i*x - 96*e**(3*e + 3*f*x)*c**2*d*f*
*2*i + 96*e**(3*e + 3*f*x)*c*d**2*f**3*i*x**2 - 192*e**(3*e + 3*f*x)*c*d**
2*f**2*i*x + 192*e**(3*e + 3*f*x)*c*d**2*f*i + 32*e**(3*e + 3*f*x)*d**3*f*
*3*i*x**3 - 96*e**(3*e + 3*f*x)*d**3*f**2*i*x**2 + 192*e**(3*e + 3*f*x)*d*
*3*f*i*x - 192*e**(3*e + 3*f*x)*d**3*i + 48*e**(2*e + 2*f*x)*c**3*f**4*x +
 72*e**(2*e + 2*f*x)*c**2*d*f**4*x**2 + 48*e**(2*e + 2*f*x)*c*d**2*f**4*x*
*3 + 12*e**(2*e + 2*f*x)*d**3*f**4*x**4 + 32*e**(e + f*x)*c**3*f**3*i + 96
*e**(e + f*x)*c**2*d*f**3*i*x + 96*e**(e + f*x)*c**2*d*f**2*i + 96*e**(e +
 f*x)*c*d**2*f**3*i*x**2 + 192*e**(e + f*x)*c*d**2*f**2*i*x + 192*e**(e +
f*x)*c*d**2*f*i + 32*e**(e + f*x)*d**3*f**3*i*x**3 + 96*e**(e + f*x)*d**3*
f**2*i*x**2 + 192*e**(e + f*x)*d**3*f*i*x + 192*e**(e + f*x)*d**3*i + 4*c*
*3*f**3 + 12*c**2*d*f**3*x + 6*c**2*d*f**2 + 12*c*d**2*f**3*x**2 + 12*c*d*
*2*f**2*x + 6*c*d**2*f + 4*d**3*f**3*x**3 + 6*d**3*f**2*x**2 + 6*d**3*f*x
+ 3*d**3))/(32*e**(2*e + 2*f*x)*f**4)
```

3.103 $\int (c+dx)^2(a+ia\sinh(e+fx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 174

$$\int (c+dx)^2(a+ia\sinh(e+fx))^2\,dx = \frac{a^2d^2x}{4f^2}+\frac{a^2(c+dx)^3}{2d}+\frac{4ia^2d^2\cosh(e+fx)}{f^3} + \frac{2ia^2(c+dx)^2\cosh(e+fx)}{f} - \frac{4ia^2d(c+dx)\sinh(e+fx)}{f^2} - \frac{a^2d^2\cosh(e+fx)\sinh(e+fx)}{4f^3} - \frac{a^2(c+dx)^2\cosh(e+fx)\sinh(e+fx)}{2f} + \frac{a^2d(c+dx)\sinh^2(e+fx)}{2f^2}$$

output

```
1/4*a^2*d^2*x/f^2+1/2*a^2*(d*x+c)^3/d+4*I*a^2*d^2*cosh(f*x+e)/f^3+2*I*a^2*
(d*x+c)^2*cosh(f*x+e)/f-4*I*a^2*d*(d*x+c)*sinh(f*x+e)/f^2-1/4*a^2*d^2*cosh
(f*x+e)*sinh(f*x+e)/f^3-1/2*a^2*(d*x+c)^2*cosh(f*x+e)*sinh(f*x+e)/f+1/2*a^
2*d*(d*x+c)*sinh(f*x+e)^2/f^2
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.09

$$\int (c+dx)^2(a+ia\sinh(e+fx))^2\,dx$$
$$= \frac{a^2(12c^2f^3x+12cdf^3x^2+4d^2f^3x^3+16i(c^2f^2+2cdf^2x+d^2(2+f^2x^2))\cosh(e+fx)+2df(c+dx)\cosh}{}$$

input
```
Integrate[(c + d*x)^2*(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
(a^2*(12*c^2*f^3*x + 12*c*d*f^3*x^2 + 4*d^2*f^3*x^3 + (16*I)*(c^2*f^2 + 2*
c*d*f^2*x + d^2*(2 + f^2*x^2))*Cosh[e + f*x] + 2*d*f*(c + d*x)*Cosh[2*(e +
 f*x)] - (32*I)*c*d*f*Sinh[e + f*x] - (32*I)*d^2*f*x*Sinh[e + f*x] - d^2*S
inh[2*(e + f*x)] - 2*c^2*f^2*Sinh[2*(e + f*x)] - 4*c*d*f^2*x*Sinh[2*(e + f
*x)] - 2*d^2*f^2*x^2*Sinh[2*(e + f*x)]))/(8*f^3)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2(a+ia\sinh(e+fx))^2\,dx$$
$$\Big\downarrow \; 3042$$
$$\int (c+dx)^2(a+a\sin(ie+ifx))^2dx$$
$$\Big\downarrow \; 3798$$
$$\int \left(-a^2(c+dx)^2\sinh^2(e+fx)+2ia^2(c+dx)^2\sinh(e+fx)+a^2(c+dx)^2\right)dx$$
$$\Big\downarrow \; 2009$$

$$\frac{a^2d(c+dx)\sinh^2(e+fx)}{2f^2}-\frac{4ia^2d(c+dx)\sinh(e+fx)}{f^2}+\frac{2ia^2(c+dx)^2\cosh(e+fx)}{f}-\frac{a^2(c+dx)^2\sinh(e+fx)\cosh(e+fx)}{2f}+\frac{a^2(c+dx)^3}{2d}+\frac{4ia^2d^2\cosh(e+fx)}{f^3}-\frac{a^2d^2\sinh(e+fx)\cosh(e+fx)}{4f^3}+\frac{a^2d^2x}{4f^2}$$

input
```
Int[(c + d*x)^2*(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
(a^2*d^2*x)/(4*f^2) + (a^2*(c + d*x)^3)/(2*d) + ((4*I)*a^2*d^2*Cosh[e + f*
x])/f^3 + ((2*I)*a^2*(c + d*x)^2*Cosh[e + f*x])/f - ((4*I)*a^2*d*(c + d*x)
*Sinh[e + f*x])/f^2 - (a^2*d^2*Cosh[e + f*x]*Sinh[e + f*x])/(4*f^3) - (a^2
*(c + d*x)^2*Cosh[e + f*x]*Sinh[e + f*x])/(2*f) + (a^2*d*(c + d*x)*Sinh[e
+ f*x]^2)/(2*f^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.81

method	result
parallelrisch	$\frac{2a^2\left(\frac{\left(-(dx+c)^2f^2-\frac{d^2}{2}\right)\sinh(2fx+2e)}{8}+\frac{df(dx+c)\cosh(2fx+2e)}{8}+i\left((dx+c)^2f^2+2d^2\right)\cosh(fx+e)-2i(dx+c)fd\sinh(fx\right.}{f^3}$
risch	$\frac{a^2d^2x^3}{2}+\frac{3a^2dc\,x^2}{2}+\frac{3a^2c^2x}{2}+\frac{a^2c^3}{2d}-\frac{a^2(2d^2x^2f^2+4cd\,f^2x+2c^2f^2-2d^2fx-2cdf+d^2)\,\mathrm{e}^{2fx+2e}}{16f^3}+\frac{ia^2(d^2x^2}{}$
parts	$\frac{a^2(dx+c)^3}{3d}+\frac{2ia^2\left(\frac{d^2\left((fx+e)^2\cosh(fx+e)-2(fx+e)\sinh(fx+e)+2\cosh(fx+e)\right)}{f^2}-\frac{2d^2e((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f^2}+\right.}{f}$
derivativedivides	$\frac{\frac{d^2a^2(fx+e)^3}{3f^2}+\frac{2id^2a^2\left((fx+e)^2\cosh(fx+e)-2(fx+e)\sinh(fx+e)+2\cosh(fx+e)\right)}{f^2}-\frac{d^2a^2\left(\frac{(fx+e)^2\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(f}{}\right.}{}}{}$
default	$\frac{\frac{d^2a^2(fx+e)^3}{3f^2}+\frac{2id^2a^2\left((fx+e)^2\cosh(fx+e)-2(fx+e)\sinh(fx+e)+2\cosh(fx+e)\right)}{f^2}-\frac{d^2a^2\left(\frac{(fx+e)^2\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(f}{}\right.}{}}{}$
orering	Expression too large to display

input
```
int((d*x+c)^2*(a+I*a*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
2*a^2/f^3*(1/8*(-(d*x+c)^2*f^2-1/2*d^2)*sinh(2*f*x+2*e)+1/8*d*f*(d*x+c)*co
sh(2*f*x+2*e)+I*((d*x+c)^2*f^2+2*d^2)*cosh(f*x+e)-2*I*(d*x+c)*f*d*sinh(f*x
+e)+3/4*x*(1/3*x^2*d^2+c*d*x+c^2)*f^3+I*c^2*f^2-1/8*c*d*f+2*I*d^2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(158) = 316.

Time = 0.10 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.02

$$\int (c+dx)^2(a+ia\sinh(e+fx))^2\,dx$$

$$=\frac{\left(2\,a^2d^2f^2x^2+2\,a^2c^2f^2+2\,a^2cdf+a^2d^2+2\,(2\,a^2cdf^2+a^2d^2f)x-(2\,a^2d^2f^2x^2+2\,a^2c^2f^2-2\,a^2cdf+a\right.}{}$$

input
```
integrate((d*x+c)^2*(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/16*(2*a^2*d^2*f^2*x^2 + 2*a^2*c^2*f^2 + 2*a^2*c*d*f + a^2*d^2 + 2*(2*a^2
*c*d*f^2 + a^2*d^2*f)*x - (2*a^2*d^2*f^2*x^2 + 2*a^2*c^2*f^2 - 2*a^2*c*d*f
 + a^2*d^2 + 2*(2*a^2*c*d*f^2 - a^2*d^2*f)*x)*e^(4*f*x + 4*e) - 16*(-I*a^2
*d^2*f^2*x^2 - I*a^2*c^2*f^2 + 2*I*a^2*c*d*f - 2*I*a^2*d^2 + 2*(-I*a^2*c*d
*f^2 + I*a^2*d^2*f)*x)*e^(3*f*x + 3*e) + 8*(a^2*d^2*f^3*x^3 + 3*a^2*c*d*f^
3*x^2 + 3*a^2*c^2*f^3*x)*e^(2*f*x + 2*e) - 16*(-I*a^2*d^2*f^2*x^2 - I*a^2*
c^2*f^2 - 2*I*a^2*c*d*f - 2*I*a^2*d^2 + 2*(-I*a^2*c*d*f^2 - I*a^2*d^2*f)*x
)*e^(f*x + e))*e^(-2*f*x - 2*e)/f^3
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 694, normalized size of antiderivative = 3.99

$$\int (c+dx)^2(a+ia\sinh(e+fx))^2\,dx=\frac{3a^2c^2x}{2}+\frac{3a^2cdx^2}{2}+\frac{a^2d^2x^3}{2}$$

$$+\begin{cases}\frac{((32a^2c^2f^{11}e^e+64a^2cdf^{11}xe^e+32a^2cdf^{10}e^e+32a^2d^2f^{11}x^2e^e+32a^2d^2f^{10}xe^e+16a^2d^2f^9e^e)e^{-2fx}+(-32a^2c^2f^{11}e^{5e}-64a^2cdf^{11}xe^{5e}+3}{} \\ \frac{x^3(-a^2d^2e^{4e}+4ia^2d^2e^{3e}-4ia^2d^2e^e-a^2d^2)e^{-2e}}{12}+\frac{x^2(-a^2cde^{4e}+4ia^2cde^{3e}-4ia^2cde^e-a^2cd)e^{-2e}}{4}+\frac{x(-a^2c^2e^{4e}+4ia^2c^2e^{3e}-4ia^2c}{4}\end{cases}$$

input
```
integrate((d*x+c)**2*(a+I*a*sinh(f*x+e))**2,x)
```

output
```
3*a**2*c**2*x/2 + 3*a**2*c*d*x**2/2 + a**2*d**2*x**3/2 + Piecewise((((32*a
**2*c**2*f**11*exp(e) + 64*a**2*c*d*f**11*x*exp(e) + 32*a**2*c*d*f**10*exp
(e) + 32*a**2*d**2*f**11*x**2*exp(e) + 32*a**2*d**2*f**10*x*exp(e) + 16*a*
*2*d**2*f**9*exp(e))*exp(-2*f*x) + (-32*a**2*c**2*f**11*exp(5*e) - 64*a**2
*c*d*f**11*x*exp(5*e) + 32*a**2*c*d*f**10*exp(5*e) - 32*a**2*d**2*f**11*x*
*2*exp(5*e) + 32*a**2*d**2*f**10*x*exp(5*e) - 16*a**2*d**2*f**9*exp(5*e))*
exp(2*f*x) + (256*I*a**2*c**2*f**11*exp(2*e) + 512*I*a**2*c*d*f**11*x*exp(
2*e) + 512*I*a**2*c*d*f**10*exp(2*e) + 256*I*a**2*d**2*f**11*x**2*exp(2*e)
 + 512*I*a**2*d**2*f**10*x*exp(2*e) + 512*I*a**2*d**2*f**9*exp(2*e))*exp(-
f*x) + (256*I*a**2*c**2*f**11*exp(4*e) + 512*I*a**2*c*d*f**11*x*exp(4*e) -
 512*I*a**2*c*d*f**10*exp(4*e) + 256*I*a**2*d**2*f**11*x**2*exp(4*e) - 512
*I*a**2*d**2*f**10*x*exp(4*e) + 512*I*a**2*d**2*f**9*exp(4*e))*exp(f*x))*e
xp(-3*e)/(256*f**12), Ne(f**12*exp(3*e), 0)), (x**3*(-a**2*d**2*exp(4*e) +
 4*I*a**2*d**2*exp(3*e) - 4*I*a**2*d**2*exp(e) - a**2*d**2)*exp(-2*e)/12 +
 x**2*(-a**2*c*d*exp(4*e) + 4*I*a**2*c*d*exp(3*e) - 4*I*a**2*c*d*exp(e) -
a**2*c*d)*exp(-2*e)/4 + x*(-a**2*c**2*exp(4*e) + 4*I*a**2*c**2*exp(3*e) -
4*I*a**2*c**2*exp(e) - a**2*c**2)*exp(-2*e)/4, True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(158) = 316$.

Time = 0.06 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.87

$$
\begin{aligned}
&\int (c+dx)^2(a+ia\sinh(e+fx))^2\,dx \\
&= \frac{1}{3}\,a^2d^2x^3 + a^2cdx^2 + \frac{1}{8}\left(4\,x^2 - \frac{\left(2\,fxe^{(2\,e)} - e^{(2\,e)}\right)e^{(2\,fx)}}{f^2} + \frac{(2\,fx+1)e^{(-2\,fx-2\,e)}}{f^2}\right)a^2cd \\
&\quad + \frac{1}{48}\left(8\,x^3 - \frac{3\left(2\,f^2x^2e^{(2\,e)} - 2\,fxe^{(2\,e)} + e^{(2\,e)}\right)e^{(2\,fx)}}{f^3} + \frac{3\,(2\,f^2x^2 + 2\,fx + 1)e^{(-2\,fx-2\,e)}}{f^3}\right)a^2d^2 \\
&\quad + \frac{1}{8}\,a^2c^2\left(4\,x - \frac{e^{(2\,fx+2\,e)}}{f} + \frac{e^{(-2\,fx-2\,e)}}{f}\right) + a^2c^2x \\
&\quad + 2i\,a^2cd\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) \\
&\quad + i\,a^2d^2\left(\frac{(f^2x^2e^e - 2\,fxe^e + 2\,e^e)e^{(fx)}}{f^3} + \frac{(f^2x^2 + 2\,fx + 2)e^{(-fx-e)}}{f^3}\right) \\
&\quad + \frac{2i\,a^2c^2\cosh\left(fx+e\right)}{f}
\end{aligned}
$$

input
```
integrate((d*x+c)^2*(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*d^2*x^3 + a^2*c*d*x^2 + 1/8*(4*x^2 - (2*f*x*e^(2*e) - e^(2*e))*e^(
2*f*x)/f^2 + (2*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*a^2*c*d + 1/48*(8*x^3 - 3*(
2*f^2*x^2*e^(2*e) - 2*f*x*e^(2*e) + e^(2*e))*e^(2*f*x)/f^3 + 3*(2*f^2*x^2
+ 2*f*x + 1)*e^(-2*f*x - 2*e)/f^3)*a^2*d^2 + 1/8*a^2*c^2*(4*x - e^(2*f*x +
 2*e)/f + e^(-2*f*x - 2*e)/f) + a^2*c^2*x + 2*I*a^2*c*d*((f*x*e^e - e^e)*e
^(f*x)/f^2 + (f*x + 1)*e^(-f*x - e)/f^2) + I*a^2*d^2*((f^2*x^2*e^e - 2*f*x
*e^e + 2*e^e)*e^(f*x)/f^3 + (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + 2*I*
a^2*c^2*cosh(f*x + e)/f
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(158) = 316$.

Time = 0.13 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.91

$$\begin{aligned}
&\int (c+dx)^2(a+ia\sinh(e+fx))^2\,dx \\
&= \frac{1}{2}\,a^2d^2x^3 + \frac{3}{2}\,a^2cdx^2 + \frac{3}{2}\,a^2c^2x \\
&\quad - \frac{(2\,a^2d^2f^2x^2 + 4\,a^2cdf^2x + 2\,a^2c^2f^2 - 2\,a^2d^2fx - 2\,a^2cdf + a^2d^2)e^{(2\,fx+2\,e)}}{16\,f^3} \\
&\quad + \frac{(i\,a^2d^2f^2x^2 + 2i\,a^2cdf^2x + i\,a^2c^2f^2 - 2i\,a^2d^2fx - 2i\,a^2cdf + 2i\,a^2d^2)e^{(fx+e)}}{f^3} \\
&\quad - \frac{(-i\,a^2d^2f^2x^2 - 2i\,a^2cdf^2x - i\,a^2c^2f^2 - 2i\,a^2d^2fx - 2i\,a^2cdf - 2i\,a^2d^2)e^{(-fx-e)}}{f^3} \\
&\quad + \frac{(2\,a^2d^2f^2x^2 + 4\,a^2cdf^2x + 2\,a^2c^2f^2 + 2\,a^2d^2fx + 2\,a^2cdf + a^2d^2)e^{(-2\,fx-2\,e)}}{16\,f^3}
\end{aligned}$$

input
```
integrate((d*x+c)^2*(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
1/2*a^2*d^2*x^3 + 3/2*a^2*c*d*x^2 + 3/2*a^2*c^2*x - 1/16*(2*a^2*d^2*f^2*x^
2 + 4*a^2*c*d*f^2*x + 2*a^2*c^2*f^2 - 2*a^2*d^2*f*x - 2*a^2*c*d*f + a^2*d^
2)*e^(2*f*x + 2*e)/f^3 + (I*a^2*d^2*f^2*x^2 + 2*I*a^2*c*d*f^2*x + I*a^2*c^
2*f^2 - 2*I*a^2*d^2*f*x - 2*I*a^2*c*d*f + 2*I*a^2*d^2)*e^(f*x + e)/f^3 - (
-I*a^2*d^2*f^2*x^2 - 2*I*a^2*c*d*f^2*x - I*a^2*c^2*f^2 - 2*I*a^2*d^2*f*x -
 2*I*a^2*c*d*f - 2*I*a^2*d^2)*e^(-f*x - e)/f^3 + 1/16*(2*a^2*d^2*f^2*x^2 +
 4*a^2*c*d*f^2*x + 2*a^2*c^2*f^2 + 2*a^2*d^2*f*x + 2*a^2*c*d*f + a^2*d^2)*
e^(-2*f*x - 2*e)/f^3
```

Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.25

$$\int (c+dx)^2(a+ia\sinh(e+fx))^2\,dx = \frac{a^2\,(12\,c^2\,x+12\,c\,d\,x^2+4\,d^2\,x^3)}{8} + \frac{a^2\,(-d^2\sinh(2\,e+2\,f\,x)+d^2\cosh(e+f\,x)\,32\mathrm{i})}{8} + \frac{a^2\,f^2\,(-2\,c^2\sinh(2\,e+2\,f\,x)-2\,d^2\,x^2\sinh(2\,e+2\,f\,x)-4\,c\,d\,x\sinh(2\,e+2\,f\,x)+c^2\cosh(e+f}{8}$$

input
```
int((a + a*sinh(e + f*x)*1i)^2*(c + d*x)^2,x)
```

output
```
(a^2*(12*c^2*x + 4*d^2*x^3 + 12*c*d*x^2))/8 + ((a^2*(d^2*cosh(e + f*x)*32i
 - d^2*sinh(2*e + 2*f*x)))/8 + (a^2*f^2*(c^2*cosh(e + f*x)*16i - 2*c^2*sin
h(2*e + 2*f*x) + d^2*x^2*cosh(e + f*x)*16i - 2*d^2*x^2*sinh(2*e + 2*f*x) +
 c*d*x*cosh(e + f*x)*32i - 4*c*d*x*sinh(2*e + 2*f*x)))/8 - (a^2*f*(d^2*x*s
inh(e + f*x)*32i - 2*d^2*x*cosh(2*e + 2*f*x) + c*d*sinh(e + f*x)*32i - 2*c
*d*cosh(2*e + 2*f*x)))/8)/f^3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.47

$$\int (c+dx)^2(a+ia\sinh(e+fx))^2\,dx = \frac{a^2\left(d^2+2c^2f^2+32e^{3fx+3e}cd\,f^2ix+32e^{fx+e}cd\,f^2ix+2e^{4fx+4e}cdf-2e^{4fx+4e}d^2f^2x^2+2e^{4fx+4e}d^2fx+1}{}$$

input

```
int((d*x+c)^2*(a+I*a*sinh(f*x+e))^2,x)
```

output

```
(a**2*( - 2*e**(4*e + 4*f*x)*c**2*f**2 - 4*e**(4*e + 4*f*x)*c*d*f**2*x + 2
*e**(4*e + 4*f*x)*c*d*f - 2*e**(4*e + 4*f*x)*d**2*f**2*x**2 + 2*e**(4*e +
4*f*x)*d**2*f*x - e**(4*e + 4*f*x)*d**2 + 16*e**(3*e + 3*f*x)*c**2*f**2*i
+ 32*e**(3*e + 3*f*x)*c*d*f**2*i*x - 32*e**(3*e + 3*f*x)*c*d*f*i + 16*e**(
3*e + 3*f*x)*d**2*f**2*i*x**2 - 32*e**(3*e + 3*f*x)*d**2*f*i*x + 32*e**(3*
e + 3*f*x)*d**2*i + 24*e**(2*e + 2*f*x)*c**2*f**3*x + 24*e**(2*e + 2*f*x)*
c*d*f**3*x**2 + 8*e**(2*e + 2*f*x)*d**2*f**3*x**3 + 16*e**(e + f*x)*c**2*f
**2*i + 32*e**(e + f*x)*c*d*f**2*i*x + 32*e**(e + f*x)*c*d*f*i + 16*e**(e
+ f*x)*d**2*f**2*i*x**2 + 32*e**(e + f*x)*d**2*f*i*x + 32*e**(e + f*x)*d**
2*i + 2*c**2*f**2 + 4*c*d*f**2*x + 2*c*d*f + 2*d**2*f**2*x**2 + 2*d**2*f*x
 + d**2))/(16*e**(2*e + 2*f*x)*f**3)
```

3.104 $\int (c+dx)(a+ia\sinh(e+fx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int (c+dx)(a+ia\sinh(e+fx))^2\,dx = \frac{3a^2(c+dx)^2}{4d} + \frac{2ia^2(c+dx)\cosh(e+fx)}{f} - \frac{2ia^2d\sinh(e+fx)}{f^2} - \frac{a^2(c+dx)\cosh(e+fx)\sinh(e+fx)}{2f} + \frac{a^2d\sinh^2(e+fx)}{4f^2}$$

output
```
3/4*a^2*(d*x+c)^2/d+2*I*a^2*(d*x+c)*cosh(f*x+e)/f-2*I*a^2*d*sinh(f*x+e)/f^
2-1/2*a^2*(d*x+c)*cosh(f*x+e)*sinh(f*x+e)/f+1/4*a^2*d*sinh(f*x+e)^2/f^2
```

Mathematica [A] (verified)

Time = 12.50 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.84

$$\int (c+dx)(a+ia\sinh(e+fx))^2\,dx$$
$$=\frac{a^2(16if(c+dx)\cosh(e+fx)+d\cosh(2(e+fx))-2(3(e+fx)(de-2cf-dfx)+8id\sinh(e+fx)+}{8f^2}$$

input
```
Integrate[(c + d*x)*(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
(a^2*((16*I)*f*(c + d*x)*Cosh[e + f*x] + d*Cosh[2*(e + f*x)] - 2*(3*(e + f
*x)*(d*e - 2*c*f - d*f*x) + (8*I)*d*Sinh[e + f*x] + f*(c + d*x)*Sinh[2*(e
+ f*x)])))/(8*f^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(a+ia\sinh(e+fx))^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (c+dx)(a+a\sin(ie+ifx))^2dx$$
$$\Big\downarrow \text{3798}$$
$$\int \left(-\left(a^2(c+dx)\sinh^2(e+fx)\right)+2ia^2(c+dx)\sinh(e+fx)+a^2(c+dx)\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{2ia^2(c+dx)\cosh(e+fx)}{f} - \frac{a^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{2f} + \frac{3a^2(c+dx)^2}{4d} + \frac{a^2d\sinh^2(e+fx)}{4f^2} - \frac{2ia^2d\sinh(e+fx)}{f^2}$$

input
```
Int[(c + d*x)*(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
(3*a^2*(c + d*x)^2)/(4*d) + ((2*I)*a^2*(c + d*x)*Cosh[e + f*x])/f - ((2*I)
*a^2*d*Sinh[e + f*x])/f^2 - (a^2*(c + d*x)*Cosh[e + f*x]*Sinh[e + f*x])/(2
*f) + (a^2*d*Sinh[e + f*x]^2)/(4*f^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.82

method	result
parallelrisch	$\frac{2\left(-\frac{f(dx+c)\sinh(2fx+2e)}{8}+\frac{d\cosh(2fx+2e)}{16}+i(dx+c)f\cosh(fx+e)-i\sinh(fx+e)d+\frac{3x\left(\frac{dx}{2}+c\right)f^{2}}{4}+icf-\frac{d}{16}\right)a^{2}}{f^{2}}$
risch	$\frac{3a^{2}d\,x^{2}}{4}+\frac{3a^{2}cx}{2}-\frac{a^{2}(2dxf+2cf-d)\mathrm{e}^{2fx+2e}}{16f^{2}}+\frac{ia^{2}(dxf+cf-d)\mathrm{e}^{fx+e}}{f^{2}}+\frac{ia^{2}(dxf+cf+d)\mathrm{e}^{-fx-e}}{f^{2}}+\frac{a^{2}(2dxf}{}$
parts	$a^{2}\left(\frac{1}{2}d\,x^{2}+cx\right)+\frac{2ia^{2}\left(\frac{d((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}-\frac{de\cosh(fx+e)}{f}+c\cosh(fx+e)\right)}{f}-\frac{a^{2}\left(\frac{d\left(\frac{(fx+e)\cosh}{}\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{d\,a^{2}(fx+e)^{2}}{2f}+\frac{2id\,a^{2}((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}-\frac{d\,a^{2}\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e)^{2}}{4}-\frac{\cosh(fx+e)^{2}}{4}\right)}{f}-\frac{de\,a}{}}{}$
default	$\frac{\frac{d\,a^{2}(fx+e)^{2}}{2f}+\frac{2id\,a^{2}((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}-\frac{d\,a^{2}\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e)^{2}}{4}-\frac{\cosh(fx+e)^{2}}{4}\right)}{f}-\frac{de\,a}{}}{}$
orering	$\frac{(2d^{5}f^{4}x^{6}+12c\,d^{4}f^{4}x^{5}+28c^{2}d^{3}f^{4}x^{4}+32c^{3}d^{2}f^{4}x^{3}+18c^{4}d\,f^{4}x^{2}-15d^{5}f^{2}x^{4}+4c^{5}f^{4}x-60c\,d^{4}f^{2}x^{3}-85c^{2}d^{3}f^{2}x^{2}-50c^{3}}{4f^{4}(dx+c)^{4}}$

input
```
int((d*x+c)*(a+I*a*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
2*(-1/8*f*(d*x+c)*sinh(2*f*x+2*e)+1/16*d*cosh(2*f*x+2*e)+I*(d*x+c)*f*cosh(
f*x+e)-I*sinh(f*x+e)*d+3/4*x*(1/2*d*x+c)*f^2+I*c*f-1/16*d)*a^2/f^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.61

$$\int(c+dx)(a+ia\sinh(e+fx))^{2}\,dx$$
$$=\frac{(2\,a^{2}dfx+2\,a^{2}cf+a^{2}d-(2\,a^{2}dfx+2\,a^{2}cf-a^{2}d)e^{(4\,fx+4\,e)}-16\,(-i\,a^{2}dfx-i\,a^{2}cf+i\,a^{2}d)e^{(3\,fx+3\,e)}+}{16\,f^{2}}$$

input
```
integrate((d*x+c)*(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/16*(2*a^2*d*f*x + 2*a^2*c*f + a^2*d - (2*a^2*d*f*x + 2*a^2*c*f - a^2*d)*
e^(4*f*x + 4*e) - 16*(-I*a^2*d*f*x - I*a^2*c*f + I*a^2*d)*e^(3*f*x + 3*e)
+ 12*(a^2*d*f^2*x^2 + 2*a^2*c*f^2*x)*e^(2*f*x + 2*e) - 16*(-I*a^2*d*f*x -
I*a^2*c*f - I*a^2*d)*e^(f*x + e))*e^(-2*f*x - 2*e)/f^2
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 357, normalized size of antiderivative = 3.50

$$\int (c+dx)(a+ia\sinh(e+fx))^2\,dx = \frac{3a^2cx}{2} + \frac{3a^2dx^2}{4} + \begin{cases} \frac{((32a^2cf^7e^e+32a^2df^7xe^e+16a^2df^6e^e)e^{-2fx}+(-32a^2cf^7e^{5e}-32a^2df^7xe^{5e}+16a^2df^6e^{5e})e^{2fx}+(256ia^2cf^7e^{2e}+256ia^2df^7xe^{2e}+256ia^2}{256f^8} \\ \frac{x^2(-a^2de^{4e}+4ia^2de^{3e}-4ia^2de^e-a^2d)e^{-2e}}{8} + \frac{x(-a^2ce^{4e}+4ia^2ce^{3e}-4ia^2ce^e-a^2c)e^{-2e}}{4} \end{cases}$$

input
```
integrate((d*x+c)*(a+I*a*sinh(f*x+e))**2,x)
```

output
```
3*a**2*c*x/2 + 3*a**2*d*x**2/4 + Piecewise((((32*a**2*c*f**7*exp(e) + 32*a
**2*d*f**7*x*exp(e) + 16*a**2*d*f**6*exp(e))*exp(-2*f*x) + (-32*a**2*c*f**
7*exp(5*e) - 32*a**2*d*f**7*x*exp(5*e) + 16*a**2*d*f**6*exp(5*e))*exp(2*f*
x) + (256*I*a**2*c*f**7*exp(2*e) + 256*I*a**2*d*f**7*x*exp(2*e) + 256*I*a*
*2*d*f**6*exp(2*e))*exp(-f*x) + (256*I*a**2*c*f**7*exp(4*e) + 256*I*a**2*d
*f**7*x*exp(4*e) - 256*I*a**2*d*f**6*exp(4*e))*exp(f*x))*exp(-3*e)/(256*f*
*8), Ne(f**8*exp(3*e), 0)), (x**2*(-a**2*d*exp(4*e) + 4*I*a**2*d*exp(3*e)
- 4*I*a**2*d*exp(e) - a**2*d)*exp(-2*e)/8 + x*(-a**2*c*exp(4*e) + 4*I*a**2
*c*exp(3*e) - 4*I*a**2*c*exp(e) - a**2*c)*exp(-2*e)/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.64

$$\int (c+dx)(a+ia\sinh(e+fx))^2\,dx = \frac{1}{2}\,a^2dx^2 + \frac{1}{16}\left(4\,x^2 - \frac{\left(2\,fxe^{(2\,e)} - e^{(2\,e)}\right)e^{(2\,fx)}}{f^2} + \frac{(2\,fx+1)e^{(-2\,fx-2\,e)}}{f^2}\right)a^2d + \frac{1}{8}\,a^2c\left(4\,x - \frac{e^{(2\,fx+2\,e)}}{f} + \frac{e^{(-2\,fx-2\,e)}}{f}\right) + a^2cx + i\,a^2d\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + \frac{2i\,a^2c\cosh(fx+e)}{f}$$

input
```
integrate((d*x+c)*(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/2*a^2*d*x^2 + 1/16*(4*x^2 - (2*f*x*e^(2*e) - e^(2*e))*e^(2*f*x)/f^2 + (2
*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*a^2*d + 1/8*a^2*c*(4*x - e^(2*f*x + 2*e)/f
 + e^(-2*f*x - 2*e)/f) + a^2*c*x + I*a^2*d*((f*x*e^e - e^e)*e^(f*x)/f^2 +
(f*x + 1)*e^(-f*x - e)/f^2) + 2*I*a^2*c*cosh(f*x + e)/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.52

$$\int (c+dx)(a+ia\sinh(e+fx))^2\,dx = \frac{3}{4}\,a^2dx^2 + \frac{3}{2}\,a^2cx - \frac{(2\,a^2dfx + 2\,a^2cf - a^2d)e^{(2\,fx+2\,e)}}{16\,f^2} + \frac{(i\,a^2dfx + i\,a^2cf - i\,a^2d)e^{(fx+e)}}{f^2} + \frac{(i\,a^2dfx + i\,a^2cf + i\,a^2d)e^{(-fx-e)}}{f^2} + \frac{(2\,a^2dfx + 2\,a^2cf + a^2d)e^{(-2\,fx-2\,e)}}{16\,f^2}$$

input
```
integrate((d*x+c)*(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
3/4*a^2*d*x^2 + 3/2*a^2*c*x - 1/16*(2*a^2*d*f*x + 2*a^2*c*f - a^2*d)*e^(2*
f*x + 2*e)/f^2 + (I*a^2*d*f*x + I*a^2*c*f - I*a^2*d)*e^(f*x + e)/f^2 + (I*
a^2*d*f*x + I*a^2*c*f + I*a^2*d)*e^(-f*x - e)/f^2 + 1/16*(2*a^2*d*f*x + 2*
a^2*c*f + a^2*d)*e^(-2*f*x - 2*e)/f^2
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.02

$$\int (c+dx)(a+ia\sinh(e+fx))^2\,dx = \frac{a^2\,(6\,d\,x^2+12\,c\,x)}{8} - \frac{\frac{a^2\,(-d\cosh(2\,e+2\,f\,x)+d\sinh(e+f\,x)\,16\mathrm{i})}{8} - \frac{a^2\,f\,(c\cosh(e+f\,x)\,16\mathrm{i}-2\,c\sinh(2\,e+2\,f\,x)-2\,d\,x\sinh(2\,e+2\,f\,x)+d\,x\cosh(e+f\,x)\,16\mathrm{i})}{8}}{f^2}$$

input
```
int((a + a*sinh(e + f*x)*1i)^2*(c + d*x),x)
```

output
```
(a^2*(12*c*x + 6*d*x^2))/8 - ((a^2*(d*sinh(e + f*x)*16i - d*cosh(2*e + 2*f
*x)))/8 - (a^2*f*(c*cosh(e + f*x)*16i - 2*c*sinh(2*e + 2*f*x) - 2*d*x*sinh
(2*e + 2*f*x) + d*x*cosh(e + f*x)*16i))/8)/f^2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.85

$$\int (c+dx)(a+ia\sinh(e+fx))^2\,dx = \frac{a^2\big(-2e^{4fx+4e}cf-2e^{4fx+4e}dfx+e^{4fx+4e}d+16e^{3fx+3e}cfi+16e^{3fx+3e}dfix-16e^{3fx+3e}di+24e^{2fx+2e}c\,f^2}{16e^{2fx+2e}f^2}$$

input
```
int((d*x+c)*(a+I*a*sinh(f*x+e))^2,x)
```

output
```
(a**2*( - 2*e**(4*e + 4*f*x)*c*f - 2*e**(4*e + 4*f*x)*d*f*x + e**(4*e + 4*
f*x)*d + 16*e**(3*e + 3*f*x)*c*f*i + 16*e**(3*e + 3*f*x)*d*f*i*x - 16*e**(
3*e + 3*f*x)*d*i + 24*e**(2*e + 2*f*x)*c*f**2*x + 12*e**(2*e + 2*f*x)*d*f*
*2*x**2 + 16*e**(e + f*x)*c*f*i + 16*e**(e + f*x)*d*f*i*x + 16*e**(e + f*x
)*d*i + 2*c*f + 2*d*f*x + d))/(16*e**(2*e + 2*f*x)*f**2)
```

3.105 $\int \frac{(a+ia\sinh(e+fx))^2}{c+dx}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 149

$$\int \frac{(a+ia\sinh(e+fx))^2}{c+dx}\,dx = -\frac{a^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{2d} + \frac{3a^2\log(c+dx)}{2d} + \frac{2ia^2\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d} + \frac{2ia^2\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{d} - \frac{a^2\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2cf}{d}+2fx\right)}{2d}$$

output
```
-1/2*a^2*cosh(-2*e+2*c*f/d)*Chi(2*c*f/d+2*f*x)/d+3/2*a^2*ln(d*x+c)/d-2*I*a
^2*Chi(c*f/d+f*x)*sinh(-e+c*f/d)/d+2*I*a^2*cosh(-e+c*f/d)*Shi(c*f/d+f*x)/d
+1/2*a^2*sinh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/d
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.79

$$\int \frac{(a+ia\sinh(e+fx))^2}{c+dx}\,dx = -\frac{a^2\left(\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Chi}\left(\frac{2f(c+dx)}{d}\right)-3\log(c+dx)-4i\operatorname{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)-4i\cosh\left(e-\frac{cf}{d}\right.\right.}{2d}$$

input
```
Integrate[(a + I*a*Sinh[e + f*x])^2/(c + d*x),x]
```

output
```
-1/2*(a^2*(Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*f*(c + d*x))/d] - 3*Log[c
 + d*x] - (4*I)*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] - (4*I)*Cosh[e
 - (c*f)/d]*SinhIntegral[f*(c/d + x)] + Sinh[2*e - (2*c*f)/d]*SinhIntegral
[(2*f*(c + d*x))/d]))/d
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3799, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\sinh(e+fx))^2}{c+dx}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+a\sin(ie+ifx))^2}{c+dx}dx$$

$$\Big\downarrow \text{ 3799}$$

$$4a^2\int \frac{\sinh^4\left(\frac{e}{2}+\frac{fx}{2}-\frac{i\pi}{4}\right)}{c+dx}dx$$

$$\Big\downarrow \text{ 3042}$$

$$4a^2 \int \frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^4}{c+dx}dx$$

$$\Big\downarrow \; 3793$$

$$4a^2 \int \left(-\frac{\cosh(2e+2fx)}{8(c+dx)}+\frac{i\sinh(e+fx)}{2(c+dx)}+\frac{3}{8(c+dx)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$4a^2 \left(\frac{i\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{2d}-\frac{\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{8d}-\frac{\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{8d}+\frac{i}{}\right.$$

input
```
Int[(a + I*a*Sinh[e + f*x])^2/(c + d*x),x]
```

output
```
4*a^2*(-1/8*(Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/d + (3
*Log[c + d*x])/(8*d) + ((I/2)*CoshIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d
])/d + ((I/2)*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d - (Sinh[2*e
 - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(8*d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.30

method	result
risch	$-\frac{i a^{2} \mathrm{e}^{-\frac{c f -d e}{d}} \mathrm{expIntegral}_{1}\left(-f x -e -\frac{c f -d e}{d}\right)}{d}+\frac{3 a^{2} \ln \left(d x +c \right)}{2 d}+\frac{a^{2} \mathrm{e}^{-\frac{2 \left(c f -d e \right)}{d}} \mathrm{expIntegral}_{1}\left(-2 f x -2 e -\frac{2 \left(c f -d e \right)}{d}\right)}{4 d}+\frac{a^{2} \mathrm{e}^{2\cdots}}{}$

input
```
int((a+I*a*sinh(f*x+e))^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-I*a^2/d*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)+3/2*a^2*ln(d*x+c)/d+1/
4*a^2/d*exp(-2*(c*f-d*e)/d)*Ei(1,-2*f*x-2*e-2*(c*f-d*e)/d)+1/4*a^2/d*exp(2
*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)+I*a^2/d*exp((c*f-d*e)/d)*Ei(1,
f*x+e+(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00

$$\int \frac{(a+ia\sinh(e+fx))^2}{c+dx}\,dx = -\frac{a^2\mathrm{Ei}\left(\frac{2\,(dfx+cf)}{d}\right)e^{\left(\frac{2\,(de-cf)}{d}\right)}-4i\,a^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(\frac{de-cf}{d}\right)}+4i\,a^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-\frac{de-cf}{d}\right)}+a^2\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\cdots\right.}{4\,d}$$

input
```
integrate((a+I*a*sinh(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

output
```
-1/4*(a^2*Ei(2*(d*f*x + c*f)/d)*e^(2*(d*e - c*f)/d) - 4*I*a^2*Ei((d*f*x +
c*f)/d)*e^((d*e - c*f)/d) + 4*I*a^2*Ei(-(d*f*x + c*f)/d)*e^(-(d*e - c*f)/d
) + a^2*Ei(-2*(d*f*x + c*f)/d)*e^(-2*(d*e - c*f)/d) - 6*a^2*log((d*x + c)/
d))/d
```

Sympy [F]

$$\int \frac{(a+ia\sinh(e+fx))^2}{c+dx}\,dx = -a^2\left(\int \frac{\sinh^2(e+fx)}{c+dx}\,dx + \int \left(-\frac{2i\sinh(e+fx)}{c+dx}\right)dx + \int \left(-\frac{1}{c+dx}\right)dx\right)$$

input
```
integrate((a+I*a*sinh(f*x+e))**2/(d*x+c),x)
```

output
```
-a**2*(Integral(sinh(e + f*x)**2/(c + d*x), x) + Integral(-2*I*sinh(e + f*
x)/(c + d*x), x) + Integral(-1/(c + d*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.01

$$\int \frac{(a+ia\sinh(e+fx))^2}{c+dx}\,dx = \frac{1}{4}a^2\left(\frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_1\left(\frac{2(dx+c)f}{d}\right)}{d} + \frac{e^{\left(2e-\frac{2cf}{d}\right)}E_1\left(-\frac{2(dx+c)f}{d}\right)}{d} + \frac{2\log(dx+c)}{d}\right) + i\,a^2\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_1\left(\frac{(dx+c)f}{d}\right)}{d} - \frac{e^{\left(e-\frac{cf}{d}\right)}E_1\left(-\frac{(dx+c)f}{d}\right)}{d}\right) + \frac{a^2\log(dx+c)}{d}$$

input
```
integrate((a+I*a*sinh(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

output
```
1/4*a^2*(e^(-2*e + 2*c*f/d)*exp_integral_e(1, 2*(d*x + c)*f/d)/d + e^(2*e
- 2*c*f/d)*exp_integral_e(1, -2*(d*x + c)*f/d)/d + 2*log(d*x + c)/d) + I*a
^2*(e^(-e + c*f/d)*exp_integral_e(1, (d*x + c)*f/d)/d - e^(e - c*f/d)*exp_
integral_e(1, -(d*x + c)*f/d)/d) + a^2*log(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.91

$$\int \frac{(a+ia\sinh(e+fx))^2}{c+dx}\,dx = -\frac{a^2\mathrm{Ei}\left(\frac{2\,(dfx+cf)}{d}\right)e^{\left(2\,e-\frac{2\,cf}{d}\right)}-4i\,a^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}+4i\,a^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)}+a^2\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)}{4\,d}$$

input
```
integrate((a+I*a*sinh(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

output
```
-1/4*(a^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) - 4*I*a^2*Ei((d*f*x + c*
f)/d)*e^(e - c*f/d) + 4*I*a^2*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + a^2*Ei
(-2*(d*f*x + c*f)/d)*e^(-2*e + 2*c*f/d) - 6*a^2*log(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\sinh(e+fx))^2}{c+dx}\,dx = \int \frac{\left(a+a\sinh(e+f\,x)\ 1\mathrm{i}\right)^2}{c+d\,x}\,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)^2/(c + d*x),x)
```

output
```
int((a + a*sinh(e + f*x)*1i)^2/(c + d*x), x)
```

Reduce [F]

$$\int \frac{(a+ia\sinh(e+fx))^2}{c+dx}\,dx = \frac{a^2\left(-\left(\int \frac{\sinh(fx+e)^2}{dx+c}dx\right)d+2\left(\int \frac{\sinh(fx+e)}{dx+c}dx\right)di+\log(dx+c)\right)}{d}$$

input
```
int((a+I*a*sinh(f*x+e))^2/(d*x+c),x)
```

output
```
(a**2*( - int(sinh(e + f*x)**2/(c + d*x),x)*d + 2*int(sinh(e + f*x)/(c + d
*x),x)*d*i + log(c + d*x)))/d
```

3.106 $\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 170

$$\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^2}\,dx = -\frac{4a^2\cosh^4\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)} + \frac{2ia^2f\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{d^2} - \frac{a^2f\text{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d^2} + \frac{2ia^2f\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{d^2} - \frac{a^2f\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2cf}{d}+2fx\right)}{d^2}$$

output

```
-4*a^2*cosh(1/2*e+1/4*I*Pi+1/2*f*x)^4/d/(d*x+c)+2*I*a^2*f*cosh(-e+c*f/d)*C
hi(c*f/d+f*x)/d^2+a^2*f*Chi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/d^2-2*I*a^2*
f*sinh(-e+c*f/d)*Shi(c*f/d+f*x)/d^2-a^2*f*cosh(-2*e+2*c*f/d)*Shi(2*c*f/d+2
*f*x)/d^2
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.26

$$\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^2}\,dx$$
$$= \frac{a^2\left(-3d+d\cosh(2(e+fx))+4if(c+dx)\cosh\left(e-\frac{cf}{d}\right)\mathrm{Chi}\left(f\left(\frac{c}{d}+x\right)\right)-2f(c+dx)\mathrm{Chi}\left(\frac{2f(c+dx)}{d}\right)\sin\right.}{}$$

input
```
Integrate[(a + I*a*Sinh[e + f*x])^2/(c + d*x)^2,x]
```

output
```
(a^2*(-3*d + d*Cosh[2*(e + f*x)] + (4*I)*f*(c + d*x)*Cosh[e - (c*f)/d]*Cos
hIntegral[f*(c/d + x)] - 2*f*(c + d*x)*CoshIntegral[(2*f*(c + d*x))/d]*Sin
h[2*e - (2*c*f)/d] - (4*I)*d*Sinh[e + f*x] + (4*I)*c*f*Sinh[e - (c*f)/d]*S
inhIntegral[f*(c/d + x)] + (4*I)*d*f*x*Sinh[e - (c*f)/d]*SinhIntegral[f*(c
/d + x)] - 2*c*f*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d] - 2
*d*f*x*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d]))/(2*d^2*(c +
 d*x))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3799, 3042, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+a\sin(ie+ifx))^2}{(c+dx)^2}dx$$
$$\Big\downarrow \text{3799}$$

$$4a^2\int \frac{\sinh^4\left(\frac{e}{2}+\frac{fx}{2}-\frac{i\pi}{4}\right)}{(c+dx)^2}dx$$

$\Big\downarrow$ 3042

$$4a^2\int \frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^4}{(c+dx)^2}dx$$

$\Big\downarrow$ 3794

$$4a^2\left(\frac{2if\int\left(\frac{\cosh(e+fx)}{4(c+dx)}+\frac{i\sinh(2e+2fx)}{8(c+dx)}\right)dx}{d}-\frac{\cosh^4\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{d(c+dx)}\right)$$

$\Big\downarrow$ 2009

$$4a^2\left(\frac{2if\left(\frac{i\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{8d}+\frac{\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{4d}+\frac{\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{4d}+\frac{i\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2}{\cdots}\right.}{8d}\right.}{d}\right.$$

input
```
Int[(a + I*a*Sinh[e + f*x])^2/(c + d*x)^2,x]
```

output
```
4*a^2*(-(Cosh[e/2 + (I/4)*Pi + (f*x)/2]^4/(d*(c + d*x))) + ((2*I)*f*((Cosh
[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/(4*d) + ((I/8)*CoshIntegral[(2*
c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/d + (Sinh[e - (c*f)/d]*SinhIntegral
[(c*f)/d + f*x])/(4*d) + ((I/8)*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)
/d + 2*f*x])/d))/d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.84

method	result
risch	$-\frac{i a^{2} f \,{\mathrm e}^{f x +e}}{d^{2} \left(\frac{c f}{d}+f x \right)}-\frac{i a^{2} f \,{\mathrm e}^{-\frac{c f -d e}{d}} \operatorname{expIntegral}_{1}\left(-f x -e -\frac{c f -d e}{d}\right)}{d^{2}}-\frac{3 a^{2}}{2 d \left(d x +c \right)}+\frac{f \,a^{2} {\mathrm e}^{-2 f x -2 e}}{4 d \left(d x f +c f \right)}-\frac{f \,a^{2} {\mathrm e}^{\frac{2 c f -2 d e}{d}} \operatorname{expIntegral}_{1}}{2 d^{2}}$

input
```
int((a+I*a*sinh(f*x+e))^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-I*a^2*f/d^2*exp(f*x+e)/(c*f/d+f*x)-I*a^2*f/d^2*exp(-(c*f-d*e)/d)*Ei(1,-f*
x-e-(c*f-d*e)/d)-3/2*a^2/d/(d*x+c)+1/4*f*a^2*exp(-2*f*x-2*e)/d/(d*f*x+c*f)
-1/2*f*a^2/d^2*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)+1/4*f*a^2/
d^2*exp(2*f*x+2*e)/(c*f/d+f*x)+1/2*f*a^2/d^2*exp(-2*(c*f-d*e)/d)*Ei(1,-2*f
*x-2*e-2*(c*f-d*e)/d)+I*a^2*f*exp(-f*x-e)/d/(d*f*x+c*f)-I*a^2*f/d^2*exp((c
*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.56

$$\int \frac{(a + ia\sinh(e + fx))^2}{(c + dx)^2} \, dx$$
$$= \frac{\left(a^2de^{(4\,fx+4\,e)} - 4i\,a^2de^{(3\,fx+3\,e)} + 4i\,a^2de^{(fx+e)} + a^2d - 2\left(3\,a^2d + (a^2dfx + a^2cf)\text{Ei}\left(\frac{2\,(dfx+cf)}{d}\right)e^{\left(\frac{2\,(de-}{d}\right.}\right.}{}$$

input
```
integrate((a+I*a*sinh(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/4*(a^2*d*e^(4*f*x + 4*e) - 4*I*a^2*d*e^(3*f*x + 3*e) + 4*I*a^2*d*e^(f*x
+ e) + a^2*d - 2*(3*a^2*d + (a^2*d*f*x + a^2*c*f)*Ei(2*(d*f*x + c*f)/d)*e^
(2*(d*e - c*f)/d) + 2*(-I*a^2*d*f*x - I*a^2*c*f)*Ei((d*f*x + c*f)/d)*e^((d
*e - c*f)/d) + 2*(-I*a^2*d*f*x - I*a^2*c*f)*Ei(-(d*f*x + c*f)/d)*e^(-(d*e
- c*f)/d) - (a^2*d*f*x + a^2*c*f)*Ei(-2*(d*f*x + c*f)/d)*e^(-2*(d*e - c*f)
/d))*e^(2*f*x + 2*e))*e^(-2*f*x - 2*e)/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{(a + ia\sinh(e + fx))^2}{(c + dx)^2} \, dx = -a^2 \left(\int \frac{\sinh^2(e + fx)}{c^2 + 2cdx + d^2x^2} \, dx + \int \left(-\frac{2i\sinh(e + fx)}{c^2 + 2cdx + d^2x^2} \right) dx + \int \left(-\frac{1}{c^2 + 2cdx + d^2x^2} \right) dx \right)$$

input
```
integrate((a+I*a*sinh(f*x+e))**2/(d*x+c)**2,x)
```

output
```
-a**2*(Integral(sinh(e + f*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integr
al(-2*I*sinh(e + f*x)/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(-1/(c**2
 + 2*c*d*x + d**2*x**2), x))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.08

$$\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^2}\,dx$$
$$=\frac{1}{4}a^2\left(\frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_2\left(\frac{2(dx+c)f}{d}\right)}{(dx+c)d}+\frac{e^{\left(2e-\frac{2cf}{d}\right)}E_2\left(-\frac{2(dx+c)f}{d}\right)}{(dx+c)d}-\frac{2}{d^2x+cd}\right)$$
$$+i\,a^2\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d}-\frac{e^{\left(e-\frac{cf}{d}\right)}E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d}\right)-\frac{a^2}{d^2x+cd}$$

input
```
integrate((a+I*a*sinh(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/4*a^2*(e^(-2*e + 2*c*f/d)*exp_integral_e(2, 2*(d*x + c)*f/d)/((d*x + c)*
d) + e^(2*e - 2*c*f/d)*exp_integral_e(2, -2*(d*x + c)*f/d)/((d*x + c)*d) -
 2/(d^2*x + c*d)) + I*a^2*(e^(-e + c*f/d)*exp_integral_e(2, (d*x + c)*f/d)
/((d*x + c)*d) - e^(e - c*f/d)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c
)*d)) - a^2/(d^2*x + c*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1134 vs. $2(159) = 318$.

Time = 0.18 (sec) , antiderivative size = 1134, normalized size of antiderivative = 6.67

$$\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*sinh(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/4*(2*(d*x + c)*a^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(2*((d*x +
 c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(2*(d*e - c*f)/d
) - 2*a^2*d*e*f^2*Ei(2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*
e + c*f)/d)*e^(2*(d*e - c*f)/d) + 2*a^2*c*f^3*Ei(2*((d*x + c)*(d*e/(d*x +
c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(2*(d*e - c*f)/d) - 4*I*(d*x + c
)*a^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(((d*x + c)*(d*e/(d*x + c)
 - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e - c*f)/d) + 4*I*a^2*d*e*f^2*
Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e
- c*f)/d) - 4*I*a^2*c*f^3*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f
) - d*e + c*f)/d)*e^((d*e - c*f)/d) - 4*I*(d*x + c)*a^2*(d*e/(d*x + c) - c
*f/(d*x + c) + f)*f^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) -
 d*e + c*f)/d)*e^(-(d*e - c*f)/d) + 4*I*a^2*d*e*f^2*Ei(-((d*x + c)*(d*e/(d
*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(d*e - c*f)/d) - 4*I*a^2*
c*f^3*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e
^(-(d*e - c*f)/d) - 2*(d*x + c)*a^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^
2*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(
-2*(d*e - c*f)/d) + 2*a^2*d*e*f^2*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*f/(d
*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d) - 2*a^2*c*f^3*Ei(-2*((d*
x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*
f)/d) - a^2*d*f^2*e^(2*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^2}\,dx=\int \frac{{\left(a+a\,\mathrm{sinh}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^2}{{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)^2/(c + d*x)^2,x)
```

output
```
int((a + a*sinh(e + f*x)*1i)^2/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^2}\,dx$$
$$= \frac{a^2\left(-e^{3e}\left(\int \frac{e^{2fx}}{d^2x^2+2cdx+c^2}dx\right)c^2 - e^{3e}\left(\int \frac{e^{2fx}}{d^2x^2+2cdx+c^2}dx\right)cdx + 4e^{2e}\left(\int \frac{e^{fx}}{d^2x^2+2cdx+c^2}dx\right)c^2i + 4e^{2e}\left(\int \frac{e}{d^2x^2+}\right.\right.}{}$$

input
```
int((a+I*a*sinh(f*x+e))^2/(d*x+c)^2,x)
```

output
```
(a**2*( - e**(3*e)*int(e**(2*f*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*c**2 - e
**(3*e)*int(e**(2*f*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*c*d*x + 4*e**(2*e)*
int(e**(f*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*c**2*i + 4*e**(2*e)*int(e**(f
*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*c*d*i*x - e**e*int(1/(e**(2*e + 2*f*x)
*c**2 + 2*e**(2*e + 2*f*x)*c*d*x + e**(2*e + 2*f*x)*d**2*x**2),x)*c**2 - e
**e*int(1/(e**(2*e + 2*f*x)*c**2 + 2*e**(2*e + 2*f*x)*c*d*x + e**(2*e + 2*
f*x)*d**2*x**2),x)*c*d*x + 6*e**e*x - 4*int(1/(e**(f*x)*c**2 + 2*e**(f*x)*
c*d*x + e**(f*x)*d**2*x**2),x)*c**2*i - 4*int(1/(e**(f*x)*c**2 + 2*e**(f*x
)*c*d*x + e**(f*x)*d**2*x**2),x)*c*d*i*x))/(4*e**e*c*(c + d*x))
```

3.107 $\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 236

$$\begin{aligned}\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^3}\,dx = &-\frac{2a^2\cosh^4\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)^2} \\ &-\frac{a^2f^2\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Chi}\left(\frac{2cf}{d}+2fx\right)}{d^3} \\ &+\frac{ia^2f^2\operatorname{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d^3} \\ &-\frac{4a^2f\cosh^3\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d^2(c+dx)} \\ &+\frac{ia^2f^2\cosh\left(e-\frac{cf}{d}\right)\operatorname{Shi}\left(\frac{cf}{d}+fx\right)}{d^3} \\ &-\frac{a^2f^2\sinh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(\frac{2cf}{d}+2fx\right)}{d^3}\end{aligned}$$

output
```
-2*a^2*cosh(1/2*e+1/4*I*Pi+1/2*f*x)^4/d/(d*x+c)^2-a^2*f^2*cosh(-2*e+2*c*f/
d)*Chi(2*c*f/d+2*f*x)/d^3-I*a^2*f^2*Chi(c*f/d+f*x)*sinh(-e+c*f/d)/d^3-4*a^
2*f*cosh(1/2*e+1/4*I*Pi+1/2*f*x)^3*sinh(1/2*e+1/4*I*Pi+1/2*f*x)/d^2/(d*x+c
)+I*a^2*f^2*cosh(-e+c*f/d)*Shi(c*f/d+f*x)/d^3+a^2*f^2*sinh(-2*e+2*c*f/d)*S
hi(2*c*f/d+2*f*x)/d^3
```

Mathematica [A] (verified)

Time = 1.93 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.84

$$\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^3}\,dx$$
$$= \frac{a^2\left(-4f^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2f(c+dx)}{d}\right)+4if^2\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)+\frac{d(-3d-4if(c+dx)\cosh(e+fx)+d}{}\right.}{}$$

input
```
Integrate[(a + I*a*Sinh[e + f*x])^2/(c + d*x)^3,x]
```

output
```
(a^2*(-4*f^2*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*f*(c + d*x))/d] + (4*I)
*f^2*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] + (d*(-3*d - (4*I)*f*(c +
 d*x)*Cosh[e + f*x] + d*Cosh[2*(e + f*x)] - (4*I)*d*Sinh[e + f*x] + 2*c*f*
Sinh[2*(e + f*x)] + 2*d*f*x*Sinh[2*(e + f*x)]))/(c + d*x)^2 + (4*I)*f^2*Co
sh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)] - 4*f^2*Sinh[2*e - (2*c*f)/d]*Si
nhIntegral[(2*f*(c + d*x))/d]))/(4*d^3)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.39, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 3799, 3042, 3795, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+a\sin(ie+ifx))^2}{(c+dx)^3}dx$$
$$\Big\downarrow \text{3799}$$
$$4a^2\int \frac{\sinh^4\left(\frac{e}{2}+\frac{fx}{2}-\frac{i\pi}{4}\right)}{(c+dx)^3}dx$$

$$\Big\downarrow 3042$$

$$4a^2\int \frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^4}{(c+dx)^3}dx$$

$$\Big\downarrow 3795$$

$$4a^2\left(\frac{2f^2\int \frac{\cosh^4\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{c+dx}dx}{d^2}-\frac{3f^2\int \frac{\cosh^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{c+dx}dx}{2d^2}-\frac{f\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{d^2(c+dx)}-\frac{\cosh^4}{2}\right.$$

$$\Big\downarrow 3042$$

$$4a^2\left(-\frac{3f^2\int \frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^2}{c+dx}dx}{2d^2}+\frac{2f^2\int \frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^4}{c+dx}dx}{d^2}-\frac{f\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{d^2(c+dx)}-\frac{\cosh^4}{2}\right.$$

$$\Big\downarrow 3793$$

$$4a^2\left(-\frac{3f^2\int \left(\frac{i\sinh(e+fx)}{2(c+dx)}+\frac{1}{2(c+dx)}\right)dx}{2d^2}+\frac{2f^2\int \left(-\frac{\cosh(2e+2fx)}{8(c+dx)}+\frac{i\sinh(e+fx)}{2(c+dx)}+\frac{3}{8(c+dx)}\right)dx}{d^2}-\frac{f\sinh\left(\frac{e}{2}+\frac{fx}{2}+\right.}{d}\right.$$

$$\Big\downarrow 2009$$

$$4a^2\left(-\frac{3f^2\left(\frac{i\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{2d}+\frac{i\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{2d}+\frac{\log(c+dx)}{2d}\right)}{2d^2}+\frac{2f^2\left(\frac{i\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{2d}-\frac{\text{Ch}}{}\right.}{}\right.$$

input
```
Int[(a + I*a*Sinh[e + f*x])^2/(c + d*x)^3,x]
```

output
```
4*a^2*(-1/2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^4/(d*(c + d*x)^2) - (f*Cosh[e/2
 + (I/4)*Pi + (f*x)/2]^3*Sinh[e/2 + (I/4)*Pi + (f*x)/2])/(d^2*(c + d*x)) -
 (3*f^2*(Log[c + d*x]/(2*d) + ((I/2)*CoshIntegral[(c*f)/d + f*x]*Sinh[e -
(c*f)/d])/d + ((I/2)*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d))/(2
*d^2) + (2*f^2*(-1/8*(Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x
])/d + (3*Log[c + d*x])/(8*d) + ((I/2)*CoshIntegral[(c*f)/d + f*x]*Sinh[e
- (c*f)/d])/d + ((I/2)*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d -
(Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(8*d)))/d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 624 vs. 2(214) = 428.

Time = 1.29 (sec) , antiderivative size = 625, normalized size of antiderivative = 2.65

method	result
risch	$-\frac{i a^2 f^2 \mathrm{e}^{fx+e}}{2 d^3 \left(\frac{cf}{d}+fx\right)^2}-\frac{i a^2 f^2 \mathrm{e}^{fx+e}}{2 d^3 \left(\frac{cf}{d}+fx\right)}-\frac{i a^2 f^2 \mathrm{e}^{-\frac{cf-de}{d}} \operatorname{expIntegral}_1\left(-fx-e-\frac{cf-de}{d}\right)}{2 d^3}-\frac{3 a^2}{4 d (dx+c)^2}-\frac{f^3 a^2 \mathrm{e}^{-2fx-2e} x}{4 d (d^2 x^2 f^2+2cd\, f^2 x+c$

input
```
int((a+I*a*sinh(f*x+e))^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*a^2*f^2/d^3*exp(f*x+e)/(c*f/d+f*x)^2-1/2*I*a^2*f^2/d^3*exp(f*x+e)/(
c*f/d+f*x)-1/2*I*a^2*f^2/d^3*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)-3/
4*a^2/d/(d*x+c)^2-1/4*f^3*a^2*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c
^2*f^2)*x-1/4*f^3*a^2*exp(-2*f*x-2*e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2
)*c+1/8*f^2*a^2*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)+1/2*f^
2*a^2/d^3*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)+1/8*f^2*a^2/d^3
*exp(2*f*x+2*e)/(c*f/d+f*x)^2+1/4*f^2*a^2/d^3*exp(2*f*x+2*e)/(c*f/d+f*x)+1
/2*f^2*a^2/d^3*exp(-2*(c*f-d*e)/d)*Ei(1,-2*f*x-2*e-2*(c*f-d*e)/d)-1/2*I*a^
2*f^3*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*x-1/2*I*a^2*f^3*exp(
-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c+1/2*I*a^2*f^2*exp(-f*x-e)/
d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)+1/2*I*a^2*f^2/d^3*exp((c*f-d*e)/d)*Ei(
1,f*x+e+(c*f-d*e)/d)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 454 vs. 2(209) = 418.

Time = 0.11 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.92

$$\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^3}\,dx =$$

$$-\frac{\left(2\,a^2 d^2 fx + 2\,a^2 cdf - a^2 d^2 - (2\,a^2 d^2 fx + 2\,a^2 cdf + a^2 d^2)e^{(4\,fx+4\,e)} + 4\,(i\,a^2 d^2 fx + i\,a^2 cdf + i\,a^2 d^2)e\right.}{}$$

input
```
integrate((a+I*a*sinh(f*x+e))^2/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/8*(2*a^2*d^2*f*x + 2*a^2*c*d*f - a^2*d^2 - (2*a^2*d^2*f*x + 2*a^2*c*d*f
 + a^2*d^2)*e^(4*f*x + 4*e) + 4*(I*a^2*d^2*f*x + I*a^2*c*d*f + I*a^2*d^2)*
e^(3*f*x + 3*e) + 2*(3*a^2*d^2 + 2*(a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^
2*c^2*f^2)*Ei(2*(d*f*x + c*f)/d)*e^(2*(d*e - c*f)/d) + 2*(-I*a^2*d^2*f^2*x
^2 - 2*I*a^2*c*d*f^2*x - I*a^2*c^2*f^2)*Ei((d*f*x + c*f)/d)*e^((d*e - c*f)
/d) + 2*(I*a^2*d^2*f^2*x^2 + 2*I*a^2*c*d*f^2*x + I*a^2*c^2*f^2)*Ei(-(d*f*x
 + c*f)/d)*e^(-(d*e - c*f)/d) + 2*(a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2
*c^2*f^2)*Ei(-2*(d*f*x + c*f)/d)*e^(-2*(d*e - c*f)/d))*e^(2*f*x + 2*e) + 4
*(I*a^2*d^2*f*x + I*a^2*c*d*f - I*a^2*d^2)*e^(f*x + e))*e^(-2*f*x - 2*e)/(
d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^3}\,dx = -a^2\left(\int \frac{\sinh^2(e+fx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \left(-\frac{2i\sinh(e+fx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\right)dx + \int \left(-\frac{1}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\right)dx\right)$$

input
```
integrate((a+I*a*sinh(f*x+e))**2/(d*x+c)**3,x)
```

output
```
-a**2*(Integral(sinh(e + f*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3
*x**3), x) + Integral(-2*I*sinh(e + f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**
2 + d**3*x**3), x) + Integral(-1/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3
*x**3), x))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.87

$$\int \frac{(a + ia\sinh(e + fx))^2}{(c + dx)^3}\, dx = -\frac{1}{4}\, a^2 \left(\frac{1}{d^3x^2 + 2\, cd^2x + c^2d} - \frac{e^{\left(-2\, e + \frac{2\, cf}{d}\right)} E_3\left(\frac{2\, (dx+c)f}{d}\right)}{(dx + c)^2 d} - \frac{e^{\left(2\, e - \frac{2\, cf}{d}\right)} E_3\left(-\frac{2\, (dx+c)f}{d}\right)}{(dx + c)^2 d}\right) + i\, a^2 \left(\frac{e^{\left(-e + \frac{cf}{d}\right)} E_3\left(\frac{(dx+c)f}{d}\right)}{(dx + c)^2 d} - \frac{e^{\left(e - \frac{cf}{d}\right)} E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx + c)^2 d}\right) - \frac{a^2}{2\, (d^3x^2 + 2\, cd^2x + c^2d)}$$

input
```
integrate((a+I*a*sinh(f*x+e))^2/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/4*a^2*(1/(d^3*x^2 + 2*c*d^2*x + c^2*d) - e^(-2*e + 2*c*f/d)*exp_integra
l_e(3, 2*(d*x + c)*f/d)/((d*x + c)^2*d) - e^(2*e - 2*c*f/d)*exp_integral_e
(3, -2*(d*x + c)*f/d)/((d*x + c)^2*d)) + I*a^2*(e^(-e + c*f/d)*exp_integra
l_e(3, (d*x + c)*f/d)/((d*x + c)^2*d) - e^(e - c*f/d)*exp_integral_e(3, -(
d*x + c)*f/d)/((d*x + c)^2*d)) - 1/2*a^2/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 682 vs. 2(209) = 418.

Time = 0.12 (sec) , antiderivative size = 682, normalized size of antiderivative = 2.89

$$\int \frac{(a + ia\sinh(e + fx))^2}{(c + dx)^3}\, dx = \text{Too large to display}$$

input
```
integrate((a+I*a*sinh(f*x+e))^2/(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/8*(4*a^2*d^2*f^2*x^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) - 4*I*a^2*
d^2*f^2*x^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + 4*I*a^2*d^2*f^2*x^2*Ei(-(d
*f*x + c*f)/d)*e^(-e + c*f/d) + 4*a^2*d^2*f^2*x^2*Ei(-2*(d*f*x + c*f)/d)*e
^(-2*e + 2*c*f/d) + 8*a^2*c*d*f^2*x*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d
) - 8*I*a^2*c*d*f^2*x*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + 8*I*a^2*c*d*f^2*
x*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + 8*a^2*c*d*f^2*x*Ei(-2*(d*f*x + c*f
)/d)*e^(-2*e + 2*c*f/d) + 4*a^2*c^2*f^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c
*f/d) - 4*I*a^2*c^2*f^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + 4*I*a^2*c^2*f^
2*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + 4*a^2*c^2*f^2*Ei(-2*(d*f*x + c*f)/
d)*e^(-2*e + 2*c*f/d) - 2*a^2*d^2*f*x*e^(2*f*x + 2*e) + 4*I*a^2*d^2*f*x*e^
(f*x + e) + 4*I*a^2*d^2*f*x*e^(-f*x - e) + 2*a^2*d^2*f*x*e^(-2*f*x - 2*e)
- 2*a^2*c*d*f*e^(2*f*x + 2*e) + 4*I*a^2*c*d*f*e^(f*x + e) + 4*I*a^2*c*d*f*
e^(-f*x - e) + 2*a^2*c*d*f*e^(-2*f*x - 2*e) - a^2*d^2*e^(2*f*x + 2*e) + 4*
I*a^2*d^2*e^(f*x + e) - 4*I*a^2*d^2*e^(-f*x - e) - a^2*d^2*e^(-2*f*x - 2*e
) + 6*a^2*d^2)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^3}\,dx = \int \frac{\left(a+a\sinh(e+f\,x)\ 1\mathrm{i}\right)^2}{\left(c+d\,x\right)^3}\,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)^2/(c + d*x)^3,x)
```

output
```
int((a + a*sinh(e + f*x)*1i)^2/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^3}\,dx = \frac{a^2\left(-e^{3e}\left(\int \frac{e^{2fx}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}\,dx\right)c^2d-2e^{3e}\left(\int \frac{e^{2fx}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}\,dx\right)c\,d^2x-e^{3e}\left(\int \frac{e^{2fx}}{d^3x^3+3c\,d^2x^2+3c^2dx+}\right.\right.}{}$$

input
```
int((a+I*a*sinh(f*x+e))^2/(d*x+c)^3,x)
```

output

```
(a**2*( - e**(3*e)*int(e**(2*f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**
3*x**3),x)*c**2*d - 2*e**(3*e)*int(e**(2*f*x)/(c**3 + 3*c**2*d*x + 3*c*d**
2*x**2 + d**3*x**3),x)*c*d**2*x - e**(3*e)*int(e**(2*f*x)/(c**3 + 3*c**2*d
*x + 3*c*d**2*x**2 + d**3*x**3),x)*d**3*x**2 + 4*e**(2*e)*int(e**(f*x)/(c*
*3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*c**2*d*i + 8*e**(2*e)*int(
e**(f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*c*d**2*i*x + 4
*e**(2*e)*int(e**(f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*
d**3*i*x**2 - e**e*int(1/(e**(2*e + 2*f*x)*c**3 + 3*e**(2*e + 2*f*x)*c**2*
d*x + 3*e**(2*e + 2*f*x)*c*d**2*x**2 + e**(2*e + 2*f*x)*d**3*x**3),x)*c**2
*d - 2*e**e*int(1/(e**(2*e + 2*f*x)*c**3 + 3*e**(2*e + 2*f*x)*c**2*d*x + 3
*e**(2*e + 2*f*x)*c*d**2*x**2 + e**(2*e + 2*f*x)*d**3*x**3),x)*c*d**2*x -
e**e*int(1/(e**(2*e + 2*f*x)*c**3 + 3*e**(2*e + 2*f*x)*c**2*d*x + 3*e**(2*
e + 2*f*x)*c*d**2*x**2 + e**(2*e + 2*f*x)*d**3*x**3),x)*d**3*x**2 - 3*e**e
 - 4*int(1/(e**(f*x)*c**3 + 3*e**(f*x)*c**2*d*x + 3*e**(f*x)*c*d**2*x**2 +
 e**(f*x)*d**3*x**3),x)*c**2*d*i - 8*int(1/(e**(f*x)*c**3 + 3*e**(f*x)*c**
2*d*x + 3*e**(f*x)*c*d**2*x**2 + e**(f*x)*d**3*x**3),x)*c*d**2*i*x - 4*int
(1/(e**(f*x)*c**3 + 3*e**(f*x)*c**2*d*x + 3*e**(f*x)*c*d**2*x**2 + e**(f*x
)*d**3*x**3),x)*d**3*i*x**2))/(4*e**e*d*(c**2 + 2*c*d*x + d**2*x**2))
```

3.108 $\int \frac{(c+dx)^3}{a+ia\sinh(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 132

$$\int \frac{(c+dx)^3}{a+ia\sinh(e+fx)}\,dx = \frac{(c+dx)^3}{af} - \frac{6d(c+dx)^2\log\left(1+ie^{e+fx}\right)}{af^2} - \frac{12d^2(c+dx)\operatorname{PolyLog}\left(2,-ie^{e+fx}\right)}{af^3} + \frac{12d^3\operatorname{PolyLog}\left(3,-ie^{e+fx}\right)}{af^4} + \frac{(c+dx)^3\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af}$$

output
```
(d*x+c)^3/a/f-6*d*(d*x+c)^2*ln(1+I*exp(f*x+e))/a/f^2-12*d^2*(d*x+c)*polylo
g(2,-I*exp(f*x+e))/a/f^3+12*d^3*polylog(3,-I*exp(f*x+e))/a/f^4+(d*x+c)^3*t
anh(1/2*e+1/4*I*Pi+1/2*f*x)/a/f
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.56

$$\int \frac{(c+dx)^3}{a+ia\sinh(e+fx)}\,dx$$
$$= \frac{2\left(\frac{3de^e\left(\frac{e^{-e}(c+dx)^3}{3d}+\frac{(i+e^{-e})(c+dx)^2\log(1-ie^{-e-fx})}{f}-\frac{2ide^{-e}(-i+e^e)\left(f(c+dx)\operatorname{PolyLog}(2,ie^{-e-fx})+d\operatorname{PolyLog}(3,ie^{-e-fx})\right)}{f^3}\right)}{-1-ie^e}+\frac{}{(\cosh(\frac{e}{2})}\right.}{af}$$

input
```
Integrate[(c + d*x)^3/(a + I*a*Sinh[e + f*x]),x]
```

output
```
(2*((3*d*E^e*((c + d*x)^3/(3*d*E^e) + ((I + E^(-e))*(c + d*x)^2*Log[1 - I*
E^(-e - f*x)])/f - ((2*I)*d*(-I + E^e)*(f*(c + d*x)*PolyLog[2, I*E^(-e - f
*x)] + d*PolyLog[3, I*E^(-e - f*x)]))/(E^e*f^3)))/(-1 - I*E^e) + ((c + d*x
)^3*Sinh[(f*x)/2])/((Cosh[e/2] + I*Sinh[e/2])*(Cosh[(e + f*x)/2] + I*Sinh[
(e + f*x)/2]))))/(a*f)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 3799, 25, 25, 3042, 4672, 26, 3042, 26, 4199, 26, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{a+ia\sinh(e+fx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c+dx)^3}{a+a\sin(ie+ifx)}dx$$
$$\Big\downarrow \text{3799}$$

$$\frac{\int -(c+dx)^3\text{csch}^2\left(\frac{e}{2}+\frac{fx}{2}-\frac{i\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -(c+dx)^3\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \; 25$$

$$\frac{\int (c+dx)^3\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int (c+dx)^3\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \; 4672$$

$$\frac{\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6id\int -i(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}}{2a}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6d\int (c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}}{2a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6d\int -i(c+dx)^2\tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}}{2a}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{6id\int (c+dx)^2\tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}}{2a}$$

$$\Big\downarrow \; 4199$$

$$\frac{\frac{6id\left(2i\int \frac{ie^{e+fx}(c+dx)^2}{1+ie^{e+fx}}dx-\frac{i(c+dx)^3}{3d}\right)}{f}+\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}}{2a}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{6id\left(-2\int \frac{e^{e+fx}(c+dx)^2}{1+ie^{e+fx}}dx-\frac{i(c+dx)^3}{3d}\right)}{f}+\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}}{2a}$$

$\Big\downarrow$ 2620

$$\frac{\frac{6id\left(-2\left(\frac{2id\int (c+dx)\log\left(1+ie^{e+fx}\right)dx}{f}-\frac{i(c+dx)^2\log\left(1+ie^{e+fx}\right)}{f}\right)-\frac{i(c+dx)^3}{3d}\right)}{f}+\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}}{2a}$$

$\Big\downarrow$ 3011

$$\frac{\frac{6id\left(-2\left(\frac{2id\left(\frac{d\int \text{PolyLog}\left(2,-ie^{e+fx}\right)dx}{f}-\frac{(c+dx)\text{PolyLog}\left(2,-ie^{e+fx}\right)}{f}\right)}{f}-\frac{i(c+dx)^2\log\left(1+ie^{e+fx}\right)}{f}\right)-\frac{i(c+dx)^3}{3d}\right)}{f}+\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}}{2a}$$

$\Big\downarrow$ 2720

$$\frac{\frac{6id\left(-2\left(\frac{2id\left(\frac{d\int e^{-e-fx}\text{PolyLog}\left(2,-ie^{e+fx}\right)de^{e+fx}}{f^2}-\frac{(c+dx)\text{PolyLog}\left(2,-ie^{e+fx}\right)}{f}\right)}{f}-\frac{i(c+dx)^2\log\left(1+ie^{e+fx}\right)}{f}\right)-\frac{i(c+dx)^3}{3d}\right)}{f}+\frac{2(c+dx)^3\tanh\left(\right.}{f}}{2a}$$

$\Big\downarrow$ 7143

$$\frac{\frac{6id\left(-2\left(\frac{2id\left(\frac{d\,\text{PolyLog}\left(3,-ie^{e+fx}\right)}{f^2}-\frac{(c+dx)\text{PolyLog}\left(2,-ie^{e+fx}\right)}{f}\right)}{f}-\frac{i(c+dx)^2\log\left(1+ie^{e+fx}\right)}{f}\right)-\frac{i(c+dx)^3}{3d}\right)}{f}+\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}}{2a}$$

input

```
Int[(c + d*x)^3/(a + I*a*Sinh[e + f*x]),x]
```

output

```
(((6*I)*d*(((-1/3*I)*(c + d*x)^3)/d - 2*(((-I)*(c + d*x)^2*Log[1 + I*E^(e
+ f*x)])/f + ((2*I)*d*(-(((c + d*x)*PolyLog[2, (-I)*E^(e + f*x)])/f) + (d*
PolyLog[3, (-I)*E^(e + f*x)])/f^2))/f)))/f + (2*(c + d*x)^3*Tanh[e/2 + (I/
4)*Pi + (f*x)/2])/f)/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(119) = 238$.

Time = 0.40 (sec) , antiderivative size = 435, normalized size of antiderivative = 3.30

method	result
risch	$\frac{2i(d^3x^3+3c\,d^2x^2+3c^2dx+c^3)}{fa(e^{fx+e}-i)}-\frac{4d^3e^3}{a\,f^4}+\frac{12d^2ce\ln(e^{fx+e}-i)}{a\,f^3}-\frac{12d^2c\,\mathrm{polylog}(2,-ie^{fx+e})}{a\,f^3}+\frac{6d^2c\,x^2}{af}+\frac{6d^3e^2\ln(e^{fx+e})}{a\,f^4}$

input
```
int((d*x+c)^3/(a+I*a*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/f/a/(exp(f*x+e)-I)-4/a/f^4*d^3*e^3
+12/a/f^3*d^2*c*e*ln(exp(f*x+e)-I)-12/a/f^3*d^2*c*polylog(2,-I*exp(f*x+e))
+6/a/f*d^2*c*x^2+6/a/f^4*d^3*e^2*ln(exp(f*x+e))-12/a/f^3*d^2*c*ln(1+I*exp(
f*x+e))*e-12/a/f^2*d^2*c*ln(1+I*exp(f*x+e))*x+12*d^3*polylog(3,-I*exp(f*x+
e))/a/f^4+6/a/f^3*d^2*c*e^2-6/a/f^2*d^3*ln(1+I*exp(f*x+e))*x^2+6/a/f^2*d*l
n(exp(f*x+e))*c^2-6/a/f^4*d^3*e^2*ln(exp(f*x+e)-I)-12/a/f^3*d^3*polylog(2,
-I*exp(f*x+e))*x+2/a/f*d^3*x^3-6/a/f^2*d*ln(exp(f*x+e)-I)*c^2-12/a/f^3*d^2
*c*e*ln(exp(f*x+e))+6/a/f^4*d^3*ln(1+I*exp(f*x+e))*e^2+12/a/f^2*d^2*c*e*x-
6/a/f^3*d^3*e^2*x
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(114) = 228.

Time = 0.11 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.76

$$\int \frac{(c+dx)^3}{a+ia\sinh(e+fx)}\,dx = -\frac{2\left(i\,d^3e^3-3i\,cd^2e^2f+3i\,c^2def^2-i\,c^3f^3+6\left(-i\,d^3fx-i\,cd^2f+(d^3fx+cd^2f)e^{(fx+e)}\right)\mathrm{Li}_2\left(-i\,e^{(fx+e}\right.\right.}{}$$

input
```
integrate((d*x+c)^3/(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
-2*(I*d^3*e^3 - 3*I*c*d^2*e^2*f + 3*I*c^2*d*e*f^2 - I*c^3*f^3 + 6*(-I*d^3*
f*x - I*c*d^2*f + (d^3*f*x + c*d^2*f)*e^(f*x + e))*dilog(-I*e^(f*x + e)) -
 (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + d^3*e^3 - 3*c*d^2*e^2*f
+ 3*c^2*d*e*f^2)*e^(f*x + e) + 3*(-I*d^3*e^2 + 2*I*c*d^2*e*f - I*c^2*d*f^2
 + (d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*e^(f*x + e))*log(e^(f*x + e) - I) +
 3*(-I*d^3*f^2*x^2 - 2*I*c*d^2*f^2*x + I*d^3*e^2 - 2*I*c*d^2*e*f + (d^3*f^
2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*e^(f*x + e))*log(I*e^(f*x +
 e) + 1) - 6*(d^3*e^(f*x + e) - I*d^3)*polylog(3, -I*e^(f*x + e)))/(a*f^4*
e^(f*x + e) - I*a*f^4)
```

Sympy [F]

$$\int \frac{(c+dx)^3}{a+ia\sinh(e+fx)}\,dx = \frac{2ic^3+6ic^2dx+6icd^2x^2+2id^3x^3}{afe^ee^{fx}-iaf} - \frac{6id\left(\int \frac{c^2}{e^ee^{fx}-i}\,dx+\int \frac{d^2x^2}{e^ee^{fx}-i}\,dx+\int \frac{2cdx}{e^ee^{fx}-i}\,dx\right)}{af}$$

input
```
integrate((d*x+c)**3/(a+I*a*sinh(f*x+e)),x)
```

output
```
(2*I*c**3 + 6*I*c**2*d*x + 6*I*c*d**2*x**2 + 2*I*d**3*x**3)/(a*f*exp(e)*ex
p(f*x) - I*a*f) - 6*I*d*(Integral(c**2/(exp(e)*exp(f*x) - I), x) + Integra
l(d**2*x**2/(exp(e)*exp(f*x) - I), x) + Integral(2*c*d*x/(exp(e)*exp(f*x)
- I), x))/(a*f)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(114) = 228$.

Time = 0.22 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.80

$$\int \frac{(c+dx)^3}{a+ia\sinh(e+fx)}\,dx = 6\,c^2d\left(\frac{xe^{(fx+e)}}{afe^{(fx+e)}-i\,af}-\frac{\log\left(\left(e^{(fx+e)}-i\right)e^{(-e)}\right)}{af^2}\right)-\frac{2\,c^3}{(i\,ae^{(-fx-e)}-a)f} - \frac{2\left(-i\,d^3x^3-3i\,cd^2x^2\right)}{afe^{(fx+e)}-i\,af}-\frac{12\left(fx\log\left(i\,e^{(fx+e)}+1\right)+\mathrm{Li}_2\left(-i\,e^{(fx+e)}\right)\right)cd^2}{af^3} - \frac{6\left(f^2x^2\log\left(i\,e^{(fx+e)}+1\right)+2\,fx\mathrm{Li}_2\left(-i\,e^{(fx+e)}\right)-2\,\mathrm{Li}_3\left(-i\,e^{(fx+e)}\right)\right)d^3}{af^4} + \frac{2\left(d^3f^3x^3+3\,cd^2f^3x^2\right)}{af^4}$$

input
```
integrate((d*x+c)^3/(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
6*c^2*d*(x*e^(f*x + e)/(a*f*e^(f*x + e) - I*a*f) - log((e^(f*x + e) - I)*e
^(-e))/(a*f^2)) - 2*c^3/((I*a*e^(-f*x - e) - a)*f) - 2*(-I*d^3*x^3 - 3*I*c
*d^2*x^2)/(a*f*e^(f*x + e) - I*a*f) - 12*(f*x*log(I*e^(f*x + e) + 1) + dil
og(-I*e^(f*x + e)))*c*d^2/(a*f^3) - 6*(f^2*x^2*log(I*e^(f*x + e) + 1) + 2*
f*x*dilog(-I*e^(f*x + e)) - 2*polylog(3, -I*e^(f*x + e)))*d^3/(a*f^4) + 2*
(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2)/(a*f^4)
```

Giac [F]

$$\int \frac{(c+dx)^3}{a+ia\sinh(e+fx)}\,dx = \int \frac{(dx+c)^3}{i\,a\sinh\left(fx+e\right)+a}\,dx$$

input
```
integrate((d*x+c)^3/(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/(I*a*sinh(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{a+ia\sinh(e+fx)}\,dx = \int \frac{(c+d\,x)^3}{a+a\sinh\left(e+f\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((c + d*x)^3/(a + a*sinh(e + f*x)*1i),x)
```

output
```
int((c + d*x)^3/(a + a*sinh(e + f*x)*1i), x)
```

Reduce [F]

$$\int \frac{(c+dx)^3}{a+ia\sinh(e+fx)}\,dx = \frac{\left(\int \frac{x^3}{\sinh(fx+e)i+1}dx\right)d^3+3\left(\int \frac{x^2}{\sinh(fx+e)i+1}dx\right)c\,d^2+3\left(\int \frac{x}{\sinh(fx+e)i+1}dx\right)c^2d+\left(\int \frac{1}{\sinh(fx+e)i+1}dx\right)c^3}{a}$$

input
```
int((d*x+c)^3/(a+I*a*sinh(f*x+e)),x)
```

output
```
(int(x**3/(sinh(e + f*x)*i + 1),x)*d**3 + 3*int(x**2/(sinh(e + f*x)*i + 1)
,x)*c*d**2 + 3*int(x/(sinh(e + f*x)*i + 1),x)*c**2*d + int(1/(sinh(e + f*x
)*i + 1),x)*c**3)/a
```

3.109 $\int \frac{(c+dx)^2}{a+ia\sinh(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 101

$$\int \frac{(c+dx)^2}{a+ia\sinh(e+fx)}\,dx = \frac{(c+dx)^2}{af} - \frac{4d(c+dx)\log\left(1+ie^{e+fx}\right)}{af^2} - \frac{4d^2\,\mathrm{PolyLog}\left(2,-ie^{e+fx}\right)}{af^3} + \frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af}$$

output
```
(d*x+c)^2/a/f-4*d*(d*x+c)*ln(1+I*exp(f*x+e))/a/f^2-4*d^2*polylog(2,-I*exp(
f*x+e))/a/f^3+(d*x+c)^2*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/a/f
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.50

$$\int \frac{(c+dx)^2}{a+ia\sinh(e+fx)}\,dx = \frac{2\left(\frac{\frac{if(c+dx)\left(f(c+dx)+2d(1+ie^e)\log\left(1-ie^{-e-fx}\right)\right)}{-i+e^e}+2d^2\,\mathrm{PolyLog}\left(2,ie^{-e-fx}\right)}{f^2} + \frac{(c+dx)^2\sinh\left(\frac{fx}{2}\right)}{\left(\cosh\left(\frac{e}{2}\right)+i\sinh\left(\frac{e}{2}\right)\right)\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}\right)}{af}$$

input
```
Integrate[(c + d*x)^2/(a + I*a*Sinh[e + f*x]),x]
```

output
```
(2*(((I*f*(c + d*x)*(f*(c + d*x) + 2*d*(1 + I*E^e)*Log[1 - I*E^(-e - f*x)]
))/(-I + E^e) + 2*d^2*PolyLog[2, I*E^(-e - f*x)])/f^2 + ((c + d*x)^2*Sinh[
(f*x)/2])/((Cosh[e/2] + I*Sinh[e/2])*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)
/2]))))/(a*f)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.14, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 3799, 25, 25, 3042, 4672, 26, 3042, 26, 4199, 26, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{a+ia\sinh(e+fx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(c+dx)^2}{a+a\sin(ie+ifx)}dx$$

$$\Big\downarrow \text{ 3799}$$

$$\frac{\int -(c+dx)^2\text{csch}^2\left(\frac{e}{2}+\frac{fx}{2}-\frac{i\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int -(c+dx)^2\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int (c+dx)^2\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (c+dx)^2\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \text{ 4672}$$

$$\frac{\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4id\int -i(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}}{2a}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4d\int (c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}}{2a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4d\int -i(c+dx)\tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}}{2a}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{4id\int (c+dx)\tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}}{2a}$$

$$\Big\downarrow \; 4199$$

$$\frac{\frac{4id\left(2i\int \frac{ie^{e+fx}(c+dx)}{1+ie^{e+fx}}dx-\frac{i(c+dx)^2}{2d}\right)}{f}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}}{2a}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{4id\left(-2\int \frac{e^{e+fx}(c+dx)}{1+ie^{e+fx}}dx-\frac{i(c+dx)^2}{2d}\right)}{f}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}}{2a}$$

$$\Big\downarrow \; 2620$$

$$\frac{\frac{4id\left(-2\left(\frac{id\int \log\left(1+ie^{e+fx}\right)dx}{f}-\frac{i(c+dx)\log\left(1+ie^{e+fx}\right)}{f}\right)-\frac{i(c+dx)^2}{2d}\right)}{f}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}}{2a}$$

$$\Big\downarrow \; 2715$$

$$\frac{\frac{4id\left(-2\left(\frac{id\int e^{-e-fx}\log\left(1+ie^{e+fx}\right)de^{e+fx}}{f^2}-\frac{i(c+dx)\log\left(1+ie^{e+fx}\right)}{f}\right)-\frac{i(c+dx)^2}{2d}\right)}{f}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}}{2a}$$

$$\Big\downarrow \; 2838$$

$$\frac{\frac{4id\left(-2\left(-\frac{i(c+dx)\log\left(1+ie^{e+fx}\right)}{f}-\frac{id\,\text{PolyLog}\left(2,-ie^{e+fx}\right)}{f^2}\right)-\frac{i(c+dx)^2}{2d}\right)}{f}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}}{2a}$$

input
```
Int[(c + d*x)^2/(a + I*a*Sinh[e + f*x]),x]
```

output
```
(((4*I)*d*(((-1/2*I)*(c + d*x)^2)/d - 2*(((-I)*(c + d*x)*Log[1 + I*E^(e +
f*x)])/f - (I*d*PolyLog[2, (-I)*E^(e + f*x)])/f^2)))/f + (2*(c + d*x)^2*Ta
nh[e/2 + (I/4)*Pi + (f*x)/2])/f)/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(90) = 180.

Time = 0.27 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.25

method	result
risch	$\frac{2i(x^2d^2+2cdx+c^2)}{fa(e^{fx+e}-i)} - \frac{4d\ln(e^{fx+e}-i)c}{a f^2} + \frac{4d\ln(e^{fx+e})c}{a f^2} + \frac{2d^2x^2}{af} + \frac{4d^2ex}{a f^2} + \frac{2d^2e^2}{a f^3} - \frac{4d^2\ln(1+ie^{fx+e})x}{a f^2} - \frac{4d^2\ln(1+}{a j}$

input
```
int((d*x+c)^2/(a+I*a*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2*I*(d^2*x^2+2*c*d*x+c^2)/f/a/(exp(f*x+e)-I)-4/a/f^2*d*ln(exp(f*x+e)-I)*c+
4/a/f^2*d*ln(exp(f*x+e))*c+2/a/f*d^2*x^2+4/a/f^2*d^2*e*x+2/a/f^3*d^2*e^2-4
/a/f^2*d^2*ln(1+I*exp(f*x+e))*x-4/a/f^3*d^2*ln(1+I*exp(f*x+e))*e-4*d^2*pol
ylog(2,-I*exp(f*x+e))/a/f^3+4/a/f^3*d^2*e*ln(exp(f*x+e)-I)-4/a/f^3*d^2*e*l
n(exp(f*x+e))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(86) = 172.

Time = 0.10 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.99

$$\int \frac{(c+dx)^2}{a+ia\sinh(e+fx)}\,dx = -\frac{2\left(-i\,d^2e^2+2i\,cdef-i\,c^2f^2+2\left(d^2e^{(fx+e)}-i\,d^2\right)\mathrm{Li}_2\left(-i\,e^{(fx+e)}\right)-\left(d^2f^2x^2+2\,cdf^2x-d^2e^2+2\,cde\right.}{}$$

input
```
integrate((d*x+c)^2/(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
-2*(-I*d^2*e^2 + 2*I*c*d*e*f - I*c^2*f^2 + 2*(d^2*e^(f*x + e) - I*d^2)*dil
og(-I*e^(f*x + e)) - (d^2*f^2*x^2 + 2*c*d*f^2*x - d^2*e^2 + 2*c*d*e*f)*e^(
f*x + e) + 2*(I*d^2*e - I*c*d*f - (d^2*e - c*d*f)*e^(f*x + e))*log(e^(f*x
+ e) - I) + 2*(-I*d^2*f*x - I*d^2*e + (d^2*f*x + d^2*e)*e^(f*x + e))*log(I
*e^(f*x + e) + 1))/(a*f^3*e^(f*x + e) - I*a*f^3)
```

Sympy [F]

$$\int \frac{(c+dx)^2}{a+ia\sinh(e+fx)}\,dx = \frac{2ic^2+4icdx+2id^2x^2}{afe^ee^{fx}-iaf} - \frac{4id\left(\int \frac{c}{e^ee^{fx}-i}\,dx+\int \frac{dx}{e^ee^{fx}-i}\,dx\right)}{af}$$

input
```
integrate((d*x+c)**2/(a+I*a*sinh(f*x+e)),x)
```

output
```
(2*I*c**2 + 4*I*c*d*x + 2*I*d**2*x**2)/(a*f*exp(e)*exp(f*x) - I*a*f) - 4*I
*d*(Integral(c/(exp(e)*exp(f*x) - I), x) + Integral(d*x/(exp(e)*exp(f*x) -
 I), x))/(a*f)
```

Maxima [F]

$$\int \frac{(c+dx)^2}{a+ia\sinh(e+fx)}\,dx = \int \frac{(dx+c)^2}{i\,a\sinh\left(fx+e\right)+a}\,dx$$

input
```
integrate((d*x+c)^2/(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
-2*d^2*(-I*x^2/(a*f*e^(f*x + e) - I*a*f) + 2*I*integrate(x/(a*f*e^(f*x + e
) - I*a*f), x)) + 4*c*d*(x*e^(f*x + e)/(a*f*e^(f*x + e) - I*a*f) - log((e^
(f*x + e) - I)*e^(-e))/(a*f^2)) - 2*c^2/((I*a*e^(-f*x - e) - a)*f)
```

Giac [F]

$$\int \frac{(c+dx)^2}{a+ia\sinh(e+fx)}\,dx = \int \frac{(dx+c)^2}{i\,a\sinh\left(fx+e\right)+a}\,dx$$

input
```
integrate((d*x+c)^2/(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(I*a*sinh(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{a+ia\sinh(e+fx)}\,dx = \int \frac{(c+d\,x)^2}{a+a\sinh\left(e+f\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((c + d*x)^2/(a + a*sinh(e + f*x)*1i),x)
```

output
```
int((c + d*x)^2/(a + a*sinh(e + f*x)*1i), x)
```

Reduce [F]

$$\int \frac{(c+dx)^2}{a+ia\sinh(e+fx)}\,dx$$
$$=\frac{\left(\int \frac{x^2}{\sinh(fx+e)i+1}dx\right)d^2+2\left(\int \frac{x}{\sinh(fx+e)i+1}dx\right)cd+\left(\int \frac{1}{\sinh(fx+e)i+1}dx\right)c^2}{a}$$

input
```
int((d*x+c)^2/(a+I*a*sinh(f*x+e)),x)
```

output
```
(int(x**2/(sinh(e + f*x)*i + 1),x)*d**2 + 2*int(x/(sinh(e + f*x)*i + 1),x)
*c*d + int(1/(sinh(e + f*x)*i + 1),x)*c**2)/a
```

3.110 $\int \frac{c+dx}{a+ia\sinh(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \frac{c+dx}{a+ia\sinh(e+fx)}\,dx = -\frac{2d\log\left(\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)}{af^2}+\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af}$$

output
```
-2*d*ln(cosh(1/2*e+1/4*I*Pi+1/2*f*x))/a/f^2+(d*x+c)*tanh(1/2*e+1/4*I*Pi+1/
2*f*x)/a/f
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 185 vs. 2(63) = 126.

Time = 0.35 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.94

$$\int \frac{c+dx}{a+ia\sinh(e+fx)}\,dx$$
$$= \frac{idfx\cosh\left(e+\frac{fx}{2}\right)+\cosh\left(\frac{fx}{2}\right)\left(-2id\arctan\left(\operatorname{sech}\left(e+\frac{fx}{2}\right)\sinh\left(\frac{fx}{2}\right)\right)-d\log(\cosh(e+fx))\right)+2cf\sinh}{af^2\left(\cosh\left(\frac{e}{2}\right)+i\sinh\left(\frac{e}{2}\right)\right)(\mathrm{co}}$$

input
```
Integrate[(c + d*x)/(a + I*a*Sinh[e + f*x]),x]
```

output
```
(I*d*f*x*Cosh[e + (f*x)/2] + Cosh[(f*x)/2]*((-2*I)*d*ArcTan[Sech[e + (f*x)
/2]*Sinh[(f*x)/2]] - d*Log[Cosh[e + f*x]]) + 2*c*f*Sinh[(f*x)/2] + d*f*x*S
inh[(f*x)/2] + 2*d*ArcTan[Sech[e + (f*x)/2]*Sinh[(f*x)/2]]*Sinh[e + (f*x)/
2] - I*d*Log[Cosh[e + f*x]]*Sinh[e + (f*x)/2])/(a*f^2*(Cosh[e/2] + I*Sinh[
e/2])*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3799, 25, 25, 3042, 4672, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{a+ia\sinh(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c+dx}{a+a\sin(ie+ifx)}dx$$

$$\Big\downarrow \text{3799}$$

$$\frac{\int -\left((c+dx)\text{csch}^2\left(\frac{e}{2}+\frac{fx}{2}-\frac{i\pi}{4}\right)\right)dx}{2a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\left((c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)dx}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int (c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (c+dx)\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \text{4672}$$

$$\frac{\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2id\int -i\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}}{2a}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2d\int \tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}}{2a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2d\int -i\tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}}{2a}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{2id\int \tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}}{2a}$$

$$\Big\downarrow \; 3956$$

$$\frac{\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)}{f^2}}{2a}$$

input
```
Int[(c + d*x)/(a + I*a*Sinh[e + f*x]),x]
```

output
```
((-4*d*Log[Cosh[e/2 + (I/4)*Pi + (f*x)/2]])/f^2 + (2*(c + d*x)*Tanh[e/2 +
(I/4)*Pi + (f*x)/2])/f)/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

method	result
risch	$\frac{2dx}{af}+\frac{2de}{a\,f^{2}}+\frac{2i(dx+c)}{fa\left(\mathrm{e}^{fx+e}-i\right)}-\frac{2d\ln\left(\mathrm{e}^{fx+e}-i\right)}{a\,f^{2}}$
parallelrisch	$\frac{2\left(-\tanh\left(\frac{fx}{2}+\frac{e}{2}\right)+i\right)d\ln\left(1-\tanh\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-2\left(-\tanh\left(\frac{fx}{2}+\frac{e}{2}\right)+i\right)d\ln\left(-i+\tanh\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+f\left((-1+i)xd\tanh\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{f^{2}a\left(-\tanh\left(\frac{fx}{2}+\frac{e}{2}\right)+i\right)}$

input
```
int((d*x+c)/(a+I*a*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2*d/a/f*x+2*d/a/f^2*e+2*I*(d*x+c)/f/a/(exp(f*x+e)-I)-2*d/a/f^2*ln(exp(f*x+
e)-I)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{c+dx}{a+ia\sinh(e+fx)}\,dx = \frac{2\left(dfxe^{(fx+e)}+i\,cf-\left(de^{(fx+e)}-i\,d\right)\log\left(e^{(fx+e)}-i\right)\right)}{af^2e^{(fx+e)}-i\,af^2}$$

input
```
integrate((d*x+c)/(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
2*(d*f*x*e^(f*x + e) + I*c*f - (d*e^(f*x + e) - I*d)*log(e^(f*x + e) - I))
/(a*f^2*e^(f*x + e) - I*a*f^2)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{c+dx}{a+ia\sinh(e+fx)}\,dx = \frac{2ic+2idx}{afe^ee^{fx}-iaf}+\frac{2dx}{af}-\frac{2d\log\left(e^{fx}-ie^{-e}\right)}{af^2}$$

input
```
integrate((d*x+c)/(a+I*a*sinh(f*x+e)),x)
```

output
```
(2*I*c + 2*I*d*x)/(a*f*exp(e)*exp(f*x) - I*a*f) + 2*d*x/(a*f) - 2*d*log(ex
p(f*x) - I*exp(-e))/(a*f**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19

$$\int \frac{c+dx}{a+ia\sinh(e+fx)}\,dx = 2\,d\left(\frac{xe^{(fx+e)}}{afe^{(fx+e)}-i\,af}-\frac{\log\left(\left(e^{(fx+e)}-i\right)e^{(-e)}\right)}{af^2}\right) - \frac{2\,c}{\left(i\,ae^{(-fx-e)}-a\right)f}$$

input
```
integrate((d*x+c)/(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
2*d*(x*e^(f*x + e)/(a*f*e^(f*x + e) - I*a*f) - log((e^(f*x + e) - I)*e^(-e
))/(a*f^2)) - 2*c/((I*a*e^(-f*x - e) - a)*f)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

$$\int \frac{c+dx}{a+ia\sinh(e+fx)}\,dx$$
$$=\frac{2\left(dfxe^{(fx+e)}-de^{(fx+e)}\log\left(e^{(fx+e)}-i\right)+i\,cf+i\,d\log\left(e^{(fx+e)}-i\right)\right)}{af^2e^{(fx+e)}-i\,af^2}$$

input
```
integrate((d*x+c)/(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

output
```
2*(d*f*x*e^(f*x + e) - d*e^(f*x + e)*log(e^(f*x + e) - I) + I*c*f + I*d*lo
g(e^(f*x + e) - I))/(a*f^2*e^(f*x + e) - I*a*f^2)
```

Mupad [B] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{c+dx}{a+ia\sinh(e+fx)}\,dx=\frac{(c+d\,x)\;2\mathrm{i}}{a\,f\,\left(\mathrm{e}^{e+f\,x}-\mathrm{i}\right)}+\frac{2\,d\,x}{a\,f}-\frac{2\,d\,\ln\left(\mathrm{e}^{f\,x}\,\mathrm{e}^{e}-\mathrm{i}\right)}{a\,f^2}$$

input
```
int((c + d*x)/(a + a*sinh(e + f*x)*1i),x)
```

output
```
((c + d*x)*2i)/(a*f*(exp(e + f*x) - 1i)) + (2*d*x)/(a*f) - (2*d*log(exp(f*
x)*exp(e) - 1i))/(a*f^2)
```

Reduce [F]

$$\int \frac{c+dx}{a+ia\sinh(e+fx)}\,dx = \frac{\left(\int \frac{x}{\sinh(fx+e)i+1}dx\right)d+\left(\int \frac{1}{\sinh(fx+e)i+1}dx\right)c}{a}$$

input
```
int((d*x+c)/(a+I*a*sinh(f*x+e)),x)
```

output
```
(int(x/(sinh(e + f*x)*i + 1),x)*d + int(1/(sinh(e + f*x)*i + 1),x)*c)/a
```

3.111 $\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx = \mathrm{Int}\left(\frac{1}{(c+dx)(a+ia\sinh(e+fx))},x\right)$$

output
```
Defer(Int)(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x)
```

Mathematica [N/A]

Not integrable

Time = 26.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx = \int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx$$

input
```
Integrate[1/((c + d*x)*(a + I*a*Sinh[e + f*x])),x]
```

output
```
Integrate[1/((c + d*x)*(a + I*a*Sinh[e + f*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c+dx)(a+a\sin(ie+ifx))}dx$$

$$\Big\downarrow \text{3807}$$

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}dx$$

input
```
Int[1/((c + d*x)*(a + I*a*Sinh[e + f*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(dx+c)\,(a+ia\sinh\,(fx+e))}dx$$

input
```
int(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x)
```

output
```
int(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 5.48

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx = \int \frac{1}{(dx+c)(i\,a\sinh\left(fx+e\right)+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
((-I*a*d*f*x - I*a*c*f + (a*d*f*x + a*c*f)*e^(f*x + e))*integral(2*I*d/(-I
*a*d^2*f*x^2 - 2*I*a*c*d*f*x - I*a*c^2*f + (a*d^2*f*x^2 + 2*a*c*d*f*x + a*
c^2*f)*e^(f*x + e)), x) + 2*I)/(-I*a*d*f*x - I*a*c*f + (a*d*f*x + a*c*f)*e
^(f*x + e))
```

Sympy [N/A]

Not integrable

Time = 3.58 (sec) , antiderivative size = 117, normalized size of antiderivative = 5.09

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx = \frac{2i}{-iacf-iadfx+(acfe^{e}+adfxe^{e})\,e^{fx}} + \frac{2id\int \frac{1}{c^{2}e^{e}e^{fx}-ic^{2}+2cdxe^{e}e^{fx}-2icdx+d^{2}x^{2}e^{e}e^{fx}-id^{2}x^{2}}\,dx}{af}$$

input
```
integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x)
```

output
```
2*I/(-I*a*c*f - I*a*d*f*x + (a*c*f*exp(e) + a*d*f*x*exp(e))*exp(f*x)) + 2*
I*d*Integral(1/(c**2*exp(e)*exp(f*x) - I*c**2 + 2*c*d*x*exp(e)*exp(f*x) -
2*I*c*d*x + d**2*x**2*exp(e)*exp(f*x) - I*d**2*x**2), x)/(a*f)
```

Maxima [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 102, normalized size of antiderivative = 4.43

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx = \int \frac{1}{(dx+c)(i\,a\sinh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
2*I*d*integrate(1/(-I*a*d^2*f*x^2 - 2*I*a*c*d*f*x - I*a*c^2*f + (a*d^2*f*x
^2*e^e + 2*a*c*d*f*x*e^e + a*c^2*f*e^e)*e^(f*x)), x) + 2*I/(-I*a*d*f*x - I
*a*c*f + (a*d*f*x*e^e + a*c*f*e^e)*e^(f*x))
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx = \int \frac{1}{(dx+c)(i\,a\sinh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)*(I*a*sinh(f*x + e) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx = \int \frac{1}{(a+a\sinh(e+f\,x)\ 1\mathrm{i})\ (c+d\,x)}\,dx$$

input
```
int(1/((a + a*sinh(e + f*x)*1i)*(c + d*x)),x)
```

output
```
int(1/((a + a*sinh(e + f*x)*1i)*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx = \frac{\int \frac{1}{\sinh(fx+e)ci+\sinh(fx+e)dix+c+dx}dx}{a}$$

input
```
int(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x)
```

output
```
int(1/(sinh(e + f*x)*c*i + sinh(e + f*x)*d*i*x + c + d*x),x)/a
```

3.112 $\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))} dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))} dx = \text{Int}\left(\frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}, x\right)$$

output
```
Defer(Int)(1/(d*x+c)^2/(a+I*a*sinh(f*x+e)),x)
```

Mathematica [N/A]

Not integrable

Time = 24.81 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))} dx = \int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))} dx$$

input
```
Integrate[1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])),x]
```

output
```
Integrate[1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{(c+dx)^2(a+a\sin(ie+ifx))}dx$$
$$\Big\downarrow \; 3807$$
$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}dx$$

input
```
Int[1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(dx+c)^2\,(a+ia\sinh\,(fx+e))}dx$$

input
```
int(1/(d*x+c)^2/(a+I*a*sinh(f*x+e)),x)
```

output
```
int(1/(d*x+c)^2/(a+I*a*sinh(f*x+e)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 204, normalized size of antiderivative = 8.87

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(i\,a\sinh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
((-I*a*d^2*f*x^2 - 2*I*a*c*d*f*x - I*a*c^2*f + (a*d^2*f*x^2 + 2*a*c*d*f*x
+ a*c^2*f)*e^(f*x + e))*integral(4*I*d/(-I*a*d^3*f*x^3 - 3*I*a*c*d^2*f*x^2
 - 3*I*a*c^2*d*f*x - I*a*c^3*f + (a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*
d*f*x + a*c^3*f)*e^(f*x + e)), x) + 2*I)/(-I*a*d^2*f*x^2 - 2*I*a*c*d*f*x -
 I*a*c^2*f + (a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*e^(f*x + e))
```

Sympy [N/A]

Not integrable

Time = 11.16 (sec) , antiderivative size = 187, normalized size of antiderivative = 8.13

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}\,dx = \frac{2i}{-iac^2f-2iacdfx-iad^2fx^2+(ac^2fe^e+2acdfxe^e+ad^2fx^2e^e)\,e^{fx}} + \frac{4id\int \frac{1}{c^3e^ee^{fx}-ic^3+3c^2dxe^ee^{fx}-3ic^2dx+3cd^2x^2e^ee^{fx}-3icd^2x^2+d^3x^3e^ee^{fx}-id^3x^3}\,dx}{af}$$

input
```
integrate(1/(d*x+c)**2/(a+I*a*sinh(f*x+e)),x)
```

output
```
2*I/(-I*a*c**2*f - 2*I*a*c*d*f*x - I*a*d**2*f*x**2 + (a*c**2*f*exp(e) + 2*
a*c*d*f*x*exp(e) + a*d**2*f*x**2*exp(e))*exp(f*x)) + 4*I*d*Integral(1/(c**
3*exp(e)*exp(f*x) - I*c**3 + 3*c**2*d*x*exp(e)*exp(f*x) - 3*I*c**2*d*x + 3
*c*d**2*x**2*exp(e)*exp(f*x) - 3*I*c*d**2*x**2 + d**3*x**3*exp(e)*exp(f*x)
 - I*d**3*x**3), x)/(a*f)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 158, normalized size of antiderivative = 6.87

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(i\,a\sinh{(fx+e)}+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
4*I*d*integrate(1/(-I*a*d^3*f*x^3 - 3*I*a*c*d^2*f*x^2 - 3*I*a*c^2*d*f*x -
I*a*c^3*f + (a*d^3*f*x^3*e^e + 3*a*c*d^2*f*x^2*e^e + 3*a*c^2*d*f*x*e^e + a
*c^3*f*e^e)*e^(f*x)), x) + 2*I/(-I*a*d^2*f*x^2 - 2*I*a*c*d*f*x - I*a*c^2*f
 + (a*d^2*f*x^2*e^e + 2*a*c*d*f*x*e^e + a*c^2*f*e^e)*e^(f*x))
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(i\,a\sinh{(fx+e)}+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)^2*(I*a*sinh(f*x + e) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}\,dx = \int \frac{1}{(a+a\sinh\left(e+f\,x\right)\,1\mathrm{i})\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/((a + a*sinh(e + f*x)*1i)*(c + d*x)^2),x)
```

output
```
int(1/((a + a*sinh(e + f*x)*1i)*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.91

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}\,dx = \frac{\left(\int -\frac{1}{\sinh(fx+e)c^2+2\sinh(fx+e)cdx+\sinh(fx+e)d^2x^2-c^2i-2cdix-d^2i\,x^2}dx\right)i}{a}$$

input
```
int(1/(d*x+c)^2/(a+I*a*sinh(f*x+e)),x)
```

output
```
(int(( - 1)/(sinh(e + f*x)*c**2 + 2*sinh(e + f*x)*c*d*x + sinh(e + f*x)*d*
*2*x**2 - c**2*i - 2*c*d*i*x - d**2*i*x**2),x)*i)/a
```

3.113 $\int \frac{(c+dx)^3}{(a+ia\sinh(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 305

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+ia\sinh(e+fx))^2}\,dx = {} & \frac{(c+dx)^3}{3a^2f}-\frac{2d(c+dx)^2\log\left(1+ie^{e+fx}\right)}{a^2f^2} \\ & +\frac{4d^3\log\left(\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)}{a^2f^4} \\ & -\frac{4d^2(c+dx)\,\text{PolyLog}\left(2,-ie^{e+fx}\right)}{a^2f^3} \\ & +\frac{4d^3\,\text{PolyLog}\left(3,-ie^{e+fx}\right)}{a^2f^4} \\ & +\frac{d(c+dx)^2\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{2a^2f^2} \\ & -\frac{2d^2(c+dx)\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{a^2f^3} \\ & +\frac{(c+dx)^3\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2f} \\ & +\frac{(c+dx)^3\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{6a^2f}\end{aligned}$$

output
```
1/3*(d*x+c)^3/a^2/f-2*d*(d*x+c)^2*ln(1+I*exp(f*x+e))/a^2/f^2+4*d^3*ln(cosh
(1/2*e+1/4*I*Pi+1/2*f*x))/a^2/f^4-4*d^2*(d*x+c)*polylog(2,-I*exp(f*x+e))/a
^2/f^3+4*d^3*polylog(3,-I*exp(f*x+e))/a^2/f^4+1/2*d*(d*x+c)^2*sech(1/2*e+1
/4*I*Pi+1/2*f*x)^2/a^2/f^2-2*d^2*(d*x+c)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/a^2/
f^3+1/3*(d*x+c)^3*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/a^2/f+1/6*(d*x+c)^3*sech(1/
2*e+1/4*I*Pi+1/2*f*x)^2*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/a^2/f
```

Mathematica [A] (verified)

Time = 2.66 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.55

$$\int \frac{(c+dx)^3}{(a+ia\sinh(e+fx))^2}\,dx = \frac{\frac{2d\left(-6d^2x+3c^2f^2x+3(1+ie^e)(2d^2-c^2f^2)x+3cdf^2x^2+d^2f^2x^3+6cd(1+ie^e)fx\log(1-ie^{-e-fx})+3d^2(1+ie^e)fx^2\log(1-ie^{-e-fx})+\frac{3(1+ie^e)(\cdots}{\cdots}\right.}{-1-ie^e}\cdots}{\cdots}$$

input
```
Integrate[(c + d*x)^3/(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
((2*d*(-6*d^2*x + 3*c^2*f^2*x + 3*(1 + I*E^e)*(2*d^2 - c^2*f^2)*x + 3*c*d*
f^2*x^2 + d^2*f^2*x^3 + 6*c*d*(1 + I*E^e)*f*x*Log[1 - I*E^(-e - f*x)] + 3*
d^2*(1 + I*E^e)*f*x^2*Log[1 - I*E^(-e - f*x)] + (3*(1 + I*E^e)*(-2*d^2 + c
^2*f^2)*Log[I - E^(e + f*x)])/f - 6*c*d*(1 + I*E^e)*PolyLog[2, I*E^(-e - f
*x)] - 6*d^2*(1 + I*E^e)*x*PolyLog[2, I*E^(-e - f*x)] - (6*d^2*(1 + I*E^e)
*PolyLog[3, I*E^(-e - f*x)])/f))/(-1 - I*E^e) + ((c + d*x)*(3*d*f*(c + d*x
)*Cosh[(f*x)/2] + (6*I)*d^2*Cosh[e + (f*x)/2] + I*(c^2*f^2 + 2*c*d*f^2*x +
 d^2*(-6 + f^2*x^2))*Cosh[e + (3*f*x)/2] + 3*(c^2*f^2 + 2*c*d*f^2*x + d^2*
(-4 + f^2*x^2))*Sinh[(f*x)/2] + (3*I)*d*f*(c + d*x)*Sinh[e + (f*x)/2]))/((
Cosh[e/2] + I*Sinh[e/2])*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])^3))/(3*
a^2*f^3)
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.01, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 3799, 3042, 4674, 3042, 4672, 26, 3042, 26, 3956, 4199, 26, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{(a+ia\sinh(e+fx))^2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(c+dx)^3}{(a+a\sin(ie+ifx))^2}dx$$

$$\Big\downarrow \text{ 3799}$$

$$\frac{\int (c+dx)^3\text{csch}^4\left(\frac{e}{2}+\frac{fx}{2}-\frac{i\pi}{4}\right)dx}{4a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (c+dx)^3\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^4dx}{4a^2}$$

$$\Big\downarrow \text{ 4674}$$

$$\frac{-\frac{4d^2\int (c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f^2}+\frac{2}{3}\int (c+dx)^3\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx+\frac{2d(c+dx)^2\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2}+\frac{2(c+dx)^3\tanh(\cdots}{\cdots}}{4a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{4d^2\int (c+dx)\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^2dx}{f^2}+\frac{2}{3}\int (c+dx)^3\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^2dx+\frac{2d(c+dx)^2\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2}+\frac{2(c+dx)^3\tanh(\cdots}{\cdots}}{4a^2}$$

$$\Big\downarrow \text{ 4672}$$

$$\frac{-\frac{4d^2\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2id\int -i\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)}{f^2}+\frac{2}{3}\left(\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6id\int -i(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}}{4a^2}$$

$\big\downarrow$ 26

$$\frac{-\frac{4d^2\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2d\int \tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)}{f^2}+\frac{2}{3}\left(\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6d\int (c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)+\frac{2}{}}{4a^2}$$

$\big\downarrow$ 3042

$$\frac{-\frac{4d^2\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2d\int -i\tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}\right)}{f^2}+\frac{2}{3}\left(\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6d\int -i(c+dx)^2\tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}\right)}{4a^2}$$

$\big\downarrow$ 26

$$\frac{-\frac{4d^2\left(\frac{2id\int \tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)}{f^2}+\frac{2}{3}\left(\frac{6id\int (c+dx)^2\tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)+\frac{2}{}}{4a^2}$$

$\big\downarrow$ 3956

$$\frac{\frac{2}{3}\left(\frac{6id\int (c+dx)^2\tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)-\frac{4d^2\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)}{f^2}\right)}{f^2}+\frac{2d}{}}{4a^2}$$

$\big\downarrow$ 4199

$$\frac{\frac{2}{3}\left(\frac{6id\left(2i\int \frac{ie^{e+fx}(c+dx)^2}{1+ie^{e+fx}}dx-\frac{i(c+dx)^3}{3d}\right)}{f}+\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)-\frac{4d^2\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)}{f^2}\right)}{f^2}}{4a^2}$$

$\big\downarrow$ 26

$$\frac{\frac{2}{3}\left(\frac{6id\left(-2\int \frac{e^{e+fx}(c+dx)^2}{1+ie^{e+fx}}dx-\frac{i(c+dx)^3}{3d}\right)}{f}+\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)-\frac{4d^2\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)}{f^2}\right)}{f^2}}{4a^2}$$

$\downarrow$ 2620

$$\frac{\frac{2}{3}\left(\frac{6id\left(-2\left(\frac{2id\int (c+dx)\log\left(1+ie^{e+fx}\right)dx}{f}-\frac{i(c+dx)^2\log\left(1+ie^{e+fx}\right)}{f}\right)-\frac{i(c+dx)^3}{3d}\right)}{f}+\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)-\frac{4d^2\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\right.}{f}\right.}{}}{4a^2}$$

$\downarrow$ 3011

$$\frac{2}{3}\left(\frac{6id\left(-2\left(\frac{2id\left(\frac{d\int \text{PolyLog}\left(2,-ie^{e+fx}\right)dx}{f}-\frac{(c+dx)\text{PolyLog}\left(2,-ie^{e+fx}\right)}{f}\right)}{f}-\frac{i(c+dx)^2\log\left(1+ie^{e+fx}\right)}{f}\right)-\frac{i(c+dx)^3}{3d}\right)}{f}+\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\right.}{f}\right.$$

$\downarrow$ 2720

$$\frac{2}{3}\left(\frac{6id\left(-2\left(\frac{2id\left(\frac{d\int e^{-e-fx}\text{PolyLog}\left(2,-ie^{e+fx}\right)de^{e+fx}}{f^2}-\frac{(c+dx)\text{PolyLog}\left(2,-ie^{e+fx}\right)}{f}\right)}{f}-\frac{i(c+dx)^2\log\left(1+ie^{e+fx}\right)}{f}\right)-\frac{i(c+dx)^3}{3d}\right)}{f}+2(c+dx)^3\text{t}\right.$$

$\downarrow$ 7143

$$-\frac{4d^2\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)}{f^2}\right)}{f^2}+\frac{2}{3}\left(\frac{6id\left(-2\left(\frac{2id\left(\frac{d\,\text{PolyLog}\left(3,-ie^{e+fx}\right)}{f^2}-\frac{(c+dx)\text{PolyLog}\left(2,-ie^{e+fx}\right)}{f}\right)}{f}-\frac{i(c+\right.}{}\right.}{f}\right.$$

input
```
Int[(c + d*x)^3/(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
((2*d*(c + d*x)^2*Sech[e/2 + (I/4)*Pi + (f*x)/2]^2)/f^2 + (2*(c + d*x)^3*S
ech[e/2 + (I/4)*Pi + (f*x)/2]^2*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(3*f) - (4
*d^2*((-4*d*Log[Cosh[e/2 + (I/4)*Pi + (f*x)/2]])/f^2 + (2*(c + d*x)*Tanh[e
/2 + (I/4)*Pi + (f*x)/2])/f))/f^2 + (2*(((6*I)*d*(((-1/3*I)*(c + d*x)^3)/d
 - 2*(((-I)*(c + d*x)^2*Log[1 + I*E^(e + f*x)])/f + ((2*I)*d*(-(((c + d*x)
*PolyLog[2, (-I)*E^(e + f*x)])/f) + (d*PolyLog[3, (-I)*E^(e + f*x)])/f^2))
/f)))/f + (2*(c + d*x)^3*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/f))/3)/(4*a^2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 722 vs. 2(249) = 498.

Time = 1.01 (sec) , antiderivative size = 723, normalized size of antiderivative = 2.37

method	result
risch	$\frac{-2if^2c^2dx+4icd^2-4id^3x\,\mathrm{e}^{2fx+2e}-\frac{2if^2d^3x^3}{3}+6f^2cd^2x^2\mathrm{e}^{fx+e}+6f^2c^2dx\,\mathrm{e}^{fx+e}-4fcd^2x\,\mathrm{e}^{fx+e}+4id^3x-2ifd^3x^2\mathrm{e}^{2fx+2e}-4icd^2}{}$

input
```
int((d*x+c)^3/(a+I*a*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*(-3*I*f^2*c^2*d*x+6*I*c*d^2-6*I*d^3*x*exp(2*f*x+2*e)-I*f^2*d^3*x^3+9*f
^2*c*d^2*x^2*exp(f*x+e)+9*f^2*c^2*d*x*exp(f*x+e)-6*f*c*d^2*x*exp(f*x+e)+6*
I*d^3*x-3*I*f*d^3*x^2*exp(2*f*x+2*e)-6*I*c*d^2*exp(2*f*x+2*e)+3*f^2*d^3*x^
3*exp(f*x+e)-3*f*d^3*x^2*exp(f*x+e)-3*f*c^2*d*exp(f*x+e)-3*I*f^2*c*d^2*x^2
+3*f^2*c^3*exp(f*x+e)-I*c^3*f^2-6*I*f*c*d^2*x*exp(2*f*x+2*e)-3*I*f*c^2*d*e
xp(2*f*x+2*e)-12*d^3*x*exp(f*x+e)-12*c*d^2*exp(f*x+e))/(exp(f*x+e)-I)^3/f^
3/a^2+2/3/a^2/f*d^3*x^3+2/a^2/f^4*d^3*ln(1+I*exp(f*x+e))*e^2-2/a^2/f^2*d*l
n(exp(f*x+e)-I)*c^2+2/a^2/f^2*d*ln(exp(f*x+e))*c^2-2/a^2/f^4*d^3*ln(exp(f*
x+e)-I)*e^2+2/a^2/f^4*d^3*ln(exp(f*x+e))*e^2-4/3/a^2/f^4*d^3*e^3+4*d^3*pol
ylog(3,-I*exp(f*x+e))/a^2/f^4+4/a^2/f^2*d^2*c*e*x-4/a^2/f^2*d^2*c*ln(1+I*e
xp(f*x+e))*x-4/a^2/f^3*d^2*c*ln(1+I*exp(f*x+e))*e+4/a^2/f^3*d^2*ln(exp(f*x
+e)-I)*c*e-4/a^2/f^3*d^2*ln(exp(f*x+e))*c*e-2/a^2/f^3*d^3*e^2*x-2/a^2/f^2*
d^3*ln(1+I*exp(f*x+e))*x^2-4/a^2/f^3*d^3*polylog(2,-I*exp(f*x+e))*x+2/a^2/
f*d^2*c*x^2+2/a^2/f^3*d^2*c*e^2-4/a^2/f^3*d^2*c*polylog(2,-I*exp(f*x+e))+4
/a^2/f^4*d^3*ln(exp(f*x+e)-I)-4/a^2/f^4*d^3*ln(exp(f*x+e))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 917 vs. 2(239) = 478.

Time = 0.10 (sec) , antiderivative size = 917, normalized size of antiderivative = 3.01

$$\int \frac{(c+dx)^3}{(a+ia\sinh(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
-2/3*(-I*d^3*e^3 - 3*I*c^2*d*e*f^2 + I*c^3*f^3 + 6*I*d^3*e + 3*(I*c*d^2*e^
2 - 2*I*c*d^2)*f + 6*(I*d^3*f*x + I*c*d^2*f + (d^3*f*x + c*d^2*f)*e^(3*f*x
 + 3*e) + 3*(-I*d^3*f*x - I*c*d^2*f)*e^(2*f*x + 2*e) - 3*(d^3*f*x + c*d^2*
f)*e^(f*x + e))*dilog(-I*e^(f*x + e)) - (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + d
^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - 6*d^3*e + 3*(c^2*d*f^3 - 2*d^3*f)
*x)*e^(3*f*x + 3*e) + 3*(I*d^3*f^3*x^3 + I*d^3*e^3 - 6*I*d^3*e + (3*I*c^2*
d*e + I*c^2*d)*f^2 + (3*I*c*d^2*f^3 + I*d^3*f^2)*x^2 + (-3*I*c*d^2*e^2 + 2
*I*c*d^2)*f + (3*I*c^2*d*f^3 + 2*I*c*d^2*f^2 - 4*I*d^3*f)*x)*e^(2*f*x + 2*
e) + 3*(d^3*f^2*x^2 + d^3*e^3 - c^3*f^3 - 6*d^3*e + (3*c^2*d*e + c^2*d)*f^
2 - (3*c*d^2*e^2 - 4*c*d^2)*f + 2*(c*d^2*f^2 - d^3*f)*x)*e^(f*x + e) + 3*(
I*d^3*e^2 - 2*I*c*d^2*e*f + I*c^2*d*f^2 - 2*I*d^3 + (d^3*e^2 - 2*c*d^2*e*f
 + c^2*d*f^2 - 2*d^3)*e^(3*f*x + 3*e) + 3*(-I*d^3*e^2 + 2*I*c*d^2*e*f - I*
c^2*d*f^2 + 2*I*d^3)*e^(2*f*x + 2*e) - 3*(d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^
2 - 2*d^3)*e^(f*x + e))*log(e^(f*x + e) - I) + 3*(I*d^3*f^2*x^2 + 2*I*c*d^
2*f^2*x - I*d^3*e^2 + 2*I*c*d^2*e*f + (d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e
^2 + 2*c*d^2*e*f)*e^(3*f*x + 3*e) + 3*(-I*d^3*f^2*x^2 - 2*I*c*d^2*f^2*x +
I*d^3*e^2 - 2*I*c*d^2*e*f)*e^(2*f*x + 2*e) - 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*
x - d^3*e^2 + 2*c*d^2*e*f)*e^(f*x + e))*log(I*e^(f*x + e) + 1) - 6*(d^3*e^
(3*f*x + 3*e) - 3*I*d^3*e^(2*f*x + 2*e) - 3*d^3*e^(f*x + e) + I*d^3)*polyl
og(3, -I*e^(f*x + e)))/(a^2*f^4*e^(3*f*x + 3*e) - 3*I*a^2*f^4*e^(2*f*x ...
```

Sympy [F]

$$\int \frac{(c+dx)^3}{(a+ia\sinh(e+fx))^2}\,dx$$
$$= \frac{-2ic^3f^2 - 6ic^2df^2x - 6icd^2f^2x^2 + 12icd^2 - 2id^3f^2x^3 + 12id^3x + (-6ic^2dfe^{2e} - 12icd^2fxe^{2e} - 12icd^2e^{\cdots}}{3a^2f^{\cdots}}$$
$$- \frac{2id\left(\int \left(-\frac{2d^2}{e^e e^{fx}-i}\right)dx + \int \frac{c^2f^2}{e^e e^{fx}-i}\,dx + \int \frac{d^2f^2x^2}{e^e e^{fx}-i}\,dx + \int \frac{2cdf^2x}{e^e e^{fx}-i}\,dx\right)}{a^2f^3}$$

input
```
integrate((d*x+c)**3/(a+I*a*sinh(f*x+e))**2,x)
```

output
```
(-2*I*c**3*f**2 - 6*I*c**2*d*f**2*x - 6*I*c*d**2*f**2*x**2 + 12*I*c*d**2 -
 2*I*d**3*f**2*x**3 + 12*I*d**3*x + (-6*I*c**2*d*f*exp(2*e) - 12*I*c*d**2*
f*x*exp(2*e) - 12*I*c*d**2*exp(2*e) - 6*I*d**3*f*x**2*exp(2*e) - 12*I*d**3
*x*exp(2*e))*exp(2*f*x) + (6*c**3*f**2*exp(e) + 18*c**2*d*f**2*x*exp(e) -
6*c**2*d*f*exp(e) + 18*c*d**2*f**2*x**2*exp(e) - 12*c*d**2*f*x*exp(e) - 24
*c*d**2*exp(e) + 6*d**3*f**2*x**3*exp(e) - 6*d**3*f*x**2*exp(e) - 24*d**3*
x*exp(e))*exp(f*x))/(3*a**2*f**3*exp(3*e)*exp(3*f*x) - 9*I*a**2*f**3*exp(2
*e)*exp(2*f*x) - 9*a**2*f**3*exp(e)*exp(f*x) + 3*I*a**2*f**3) - 2*I*d*(Int
egral(-2*d**2/(exp(e)*exp(f*x) - I), x) + Integral(c**2*f**2/(exp(e)*exp(f
*x) - I), x) + Integral(d**2*f**2*x**2/(exp(e)*exp(f*x) - I), x) + Integra
l(2*c*d*f**2*x/(exp(e)*exp(f*x) - I), x))/(a**2*f**3)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 635 vs. $2(239) = 478$.

Time = 0.26 (sec) , antiderivative size = 635, normalized size of antiderivative = 2.08

$$\int \frac{(c+dx)^3}{(a+ia\sinh(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
2*c^2*d*((f*x*e^(3*f*x + 3*e) - (3*I*f*x*e^(2*e) + I*e^(2*e))*e^(2*f*x) -
e^(f*x + e))/(a^2*f^2*e^(3*f*x + 3*e) - 3*I*a^2*f^2*e^(2*f*x + 2*e) - 3*a^
2*f^2*e^(f*x + e) + I*a^2*f^2) - log(-I*(I*e^(f*x + e) + 1)*e^(-e))/(a^2*f
^2)) + 2/3*c^3*(3*e^(-f*x - e)/((3*a^2*e^(-f*x - e) - 3*I*a^2*e^(-2*f*x -
2*e) - a^2*e^(-3*f*x - 3*e) + I*a^2)*f) + I/((3*a^2*e^(-f*x - e) - 3*I*a^2
*e^(-2*f*x - 2*e) - a^2*e^(-3*f*x - 3*e) + I*a^2)*f)) - 2/3*(I*d^3*f^2*x^3
 + 3*I*c*d^2*f^2*x^2 - 6*I*d^3*x - 6*I*c*d^2 - 3*(-I*d^3*f*x^2*e^(2*e) - 2
*I*c*d^2*e^(2*e) + 2*(-I*c*d^2*f*e^(2*e) - I*d^3*e^(2*e))*x)*e^(2*f*x) - 3
*(d^3*f^2*x^3*e^e - 4*c*d^2*e^e + (3*c*d^2*f^2*e^e - d^3*f*e^e)*x^2 - 2*(c
*d^2*f*e^e + 2*d^3*e^e)*x)*e^(f*x))/(a^2*f^3*e^(3*f*x + 3*e) - 3*I*a^2*f^3
*e^(2*f*x + 2*e) - 3*a^2*f^3*e^(f*x + e) + I*a^2*f^3) - 4*(f*x*log(I*e^(f*
x + e) + 1) + dilog(-I*e^(f*x + e)))*c*d^2/(a^2*f^3) - 4*d^3*x/(a^2*f^3) -
 2*(f^2*x^2*log(I*e^(f*x + e) + 1) + 2*f*x*dilog(-I*e^(f*x + e)) - 2*polyl
og(3, -I*e^(f*x + e)))*d^3/(a^2*f^4) + 4*d^3*log(e^(f*x + e) - I)/(a^2*f^4
) + 2/3*(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2)/(a^2*f^4)
```

Giac [F]

$$\int \frac{(c+dx)^3}{(a+ia\sinh(e+fx))^2}\,dx = \int \frac{(dx+c)^3}{(i\,a\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^3/(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/(I*a*sinh(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{(a+ia\sinh(e+fx))^2}\,dx = \int \frac{(c+d\,x)^3}{(a+a\sinh(e+f\,x)\,1\mathrm{i})^2}\,dx$$

input
```
int((c + d*x)^3/(a + a*sinh(e + f*x)*1i)^2,x)
```

output
```
int((c + d*x)^3/(a + a*sinh(e + f*x)*1i)^2, x)
```

Reduce [F]

$$\int \frac{(c+dx)^3}{(a+ia\sinh(e+fx))^2}\,dx$$
$$= \frac{-\left(\int \frac{x^3}{\sinh(fx+e)^2-2\sinh(fx+e)i-1}dx\right)d^3 - 3\left(\int \frac{x^2}{\sinh(fx+e)^2-2\sinh(fx+e)i-1}dx\right)c\,d^2 - 3\left(\int \frac{x}{\sinh(fx+e)^2-2\sinh(fx+e)i-}\right.}{a^2}$$

input
```
int((d*x+c)^3/(a+I*a*sinh(f*x+e))^2,x)
```

output
```
( - int(x**3/(sinh(e + f*x)**2 - 2*sinh(e + f*x)*i - 1),x)*d**3 - 3*int(x*
*2/(sinh(e + f*x)**2 - 2*sinh(e + f*x)*i - 1),x)*c*d**2 - 3*int(x/(sinh(e
+ f*x)**2 - 2*sinh(e + f*x)*i - 1),x)*c**2*d - int(1/(sinh(e + f*x)**2 - 2
*sinh(e + f*x)*i - 1),x)*c**3)/a**2
```

3.114 $\int \frac{(c+dx)^2}{(a+ia\sinh(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 241

$$\int \frac{(c+dx)^2}{(a+ia\sinh(e+fx))^2}\,dx = \frac{(c+dx)^2}{3a^2f} - \frac{4d(c+dx)\log\left(1+ie^{e+fx}\right)}{3a^2f^2} - \frac{4d^2\,\text{PolyLog}\left(2,-ie^{e+fx}\right)}{3a^2f^3} + \frac{d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2f^2} - \frac{2d^2\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2f^3} + \frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)^2\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{6a^2f}$$

output

```
1/3*(d*x+c)^2/a^2/f-4/3*d*(d*x+c)*ln(1+I*exp(f*x+e))/a^2/f^2-4/3*d^2*polyl
og(2,-I*exp(f*x+e))/a^2/f^3+1/3*d*(d*x+c)*sech(1/2*e+1/4*I*Pi+1/2*f*x)^2/a
^2/f^2-2/3*d^2*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/a^2/f^3+1/3*(d*x+c)^2*tanh(1/2
*e+1/4*I*Pi+1/2*f*x)/a^2/f+1/6*(d*x+c)^2*sech(1/2*e+1/4*I*Pi+1/2*f*x)^2*ta
nh(1/2*e+1/4*I*Pi+1/2*f*x)/a^2/f
```

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.12

$$\int \frac{(c+dx)^2}{(a+ia\sinh(e+fx))^2}\,dx = \frac{\frac{2if(c+dx)\left(f(c+dx)+2d(1+ie^e)\log\left(1-ie^{-e-fx}\right)\right)}{-i+e^e} + 4d^2\,\text{PolyLog}\left(2, ie^{-e-fx}\right) + \frac{2df(c+dx)\cosh\left(\frac{fx}{2}\right)+2id^2\cosh\left(e+\frac{fx}{2}\right)+i(c^2f^2}{}}{3a^2f^3}$$

input
```
Integrate[(c + d*x)^2/(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
(((2*I)*f*(c + d*x)*(f*(c + d*x) + 2*d*(1 + I*E^e)*Log[1 - I*E^(-e - f*x)]
))/(-I + E^e) + 4*d^2*PolyLog[2, I*E^(-e - f*x)] + (2*d*f*(c + d*x)*Cosh[(
f*x)/2] + (2*I)*d^2*Cosh[e + (f*x)/2] + I*(c^2*f^2 + 2*c*d*f^2*x + d^2*(-2
 + f^2*x^2))*Cosh[e + (3*f*x)/2] + (3*c^2*f^2 + 6*c*d*f^2*x + d^2*(-4 + 3*
f^2*x^2))*Sinh[(f*x)/2] + (2*I)*d*f*(c + d*x)*Sinh[e + (f*x)/2])/((Cosh[e/
2] + I*Sinh[e/2])*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])^3))/(3*a^2*f^3
)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 3799, 3042, 4674, 3042, 4254, 24, 4672, 26, 3042, 26, 4199, 26, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(a+ia\sinh(e+fx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(c+dx)^2}{(a+a\sin(ie+ifx))^2}dx$$

$$\Big\downarrow \; 3799$$

$$\frac{\int (c+dx)^2 \text{csch}^4\left(\frac{e}{2}+\frac{fx}{2}-\frac{i\pi}{4}\right)dx}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (c+dx)^2 \csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^4 dx}{4a^2}$$

$$\Big\downarrow \text{4674}$$

$$\frac{\frac{2}{3}\int (c+dx)^2 \text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx-\frac{4d^2\int \text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{3f^2}+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right.}{3f}}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2}{3}\int (c+dx)^2 \csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^2 dx-\frac{4d^2\int \csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^2 dx}{3f^2}+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f}}{4a^2}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\frac{2}{3}\int (c+dx)^2 \csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^2 dx-\frac{8id^2\int 1d\left(-i\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)}{3f^3}+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{f}{2}\right.}{}}{4a^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{2}{3}\int (c+dx)^2 \csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^2 dx+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f}-\frac{8d^2\,\text{ta}}{}}{4a^2}$$

$$\Big\downarrow \text{4672}$$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4id\int -i(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f}}{4a^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4d\int (c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{sec}}{3f}}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4d\int -i(c+dx)\tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}\right)+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{se}}{3f}}{4a^2}$$

$\downarrow$ 26

$$\frac{\frac{2}{3}\left(\frac{4id\int(c+dx)\tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{sec}}{3f}}{4a^2}$$

$\downarrow$ 4199

$$\frac{\frac{2}{3}\left(\frac{4id\left(2i\int\frac{ie^{e+fx}(c+dx)}{1+ie^{e+fx}}dx-\frac{i(c+dx)^2}{2d}\right)}{f}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right.}{3f}}{4a^2}$$

$\downarrow$ 26

$$\frac{\frac{2}{3}\left(\frac{4id\left(-2\int\frac{e^{e+fx}(c+dx)}{1+ie^{e+fx}}dx-\frac{i(c+dx)^2}{2d}\right)}{f}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right.}{3f}}{4a^2}$$

$\downarrow$ 2620

$$\frac{\frac{2}{3}\left(\frac{4id\left(-2\left(\frac{id\int\log\left(1+ie^{e+fx}\right)dx}{f}-\frac{i(c+dx)\log\left(1+ie^{e+fx}\right)}{f}\right)-\frac{i(c+dx)^2}{2d}\right)}{f}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}}{4a^2}$$

$\downarrow$ 2715

$$\frac{\frac{2}{3}\left(\frac{4id\left(-2\left(\frac{id\int e^{-e-fx}\log\left(1+ie^{e+fx}\right)de^{e+fx}}{f^2}-\frac{i(c+dx)\log\left(1+ie^{e+fx}\right)}{f}\right)-\frac{i(c+dx)^2}{2d}\right)}{f}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)+\frac{4d(c+dx)\text{sech}^2(}{3f^2}}{4a^2}$$

$\downarrow$ 2838

$$\frac{\frac{2}{3}\left(\frac{4id\left(-2\left(-\frac{i(c+dx)\log\left(1+ie^{e+fx}\right)}{f}-\frac{id\,\text{PolyLog}\left(2,-ie^{e+fx}\right)}{f^2}\right)-\frac{i(c+dx)^2}{2d}\right)}{f}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\right.}{3f^2}}{4a^2}$$

input
```
Int[(c + d*x)^2/(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
((4*d*(c + d*x)*Sech[e/2 + (I/4)*Pi + (f*x)/2]^2)/(3*f^2) - (8*d^2*Tanh[e/
2 + (I/4)*Pi + (f*x)/2])/(3*f^3) + (2*(c + d*x)^2*Sech[e/2 + (I/4)*Pi + (f
*x)/2]^2*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(3*f) + (2*(((4*I)*d*(((-1/2*I)*(
c + d*x)^2)/d - 2*(((-I)*(c + d*x)*Log[1 + I*E^(e + f*x)])/f - (I*d*PolyLo
g[2, (-I)*E^(e + f*x)])/f^2)))/f + (2*(c + d*x)^2*Tanh[e/2 + (I/4)*Pi + (f
*x)/2])/f))/3)/(4*a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.55

method	result
risch	$\frac{-\frac{4if^2cdx}{3}-\frac{4if\,d^2x\,\mathrm{e}^{2fx+2e}}{3}-\frac{4ifcd\,\mathrm{e}^{2fx+2e}}{3}-\frac{4f\,d^2x\,\mathrm{e}^{fx+e}}{3}-\frac{4fcd\,\mathrm{e}^{fx+e}}{3}-\frac{2if^2d^2x^2}{3}+\frac{4id^2}{3}-\frac{8\,\mathrm{e}^{fx+e}d^2}{3}-\frac{4id^2\mathrm{e}^{2fx+2e}}{3}-\frac{2ic^2f^2}{3}+2d^2f}{\left(\mathrm{e}^{fx+e}-i\right)^3f^3a^2}$

input
```
int((d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*(-2*I*f^2*c*d*x-2*I*f*d^2*x*exp(2*f*x+2*e)-2*I*f*c*d*exp(2*f*x+2*e)-2*
f*d^2*x*exp(f*x+e)-2*f*c*d*exp(f*x+e)-I*f^2*d^2*x^2+2*I*d^2-4*exp(f*x+e)*d
^2-2*I*d^2*exp(2*f*x+2*e)-I*c^2*f^2+3*d^2*f^2*x^2*exp(f*x+e)+6*c*d*f^2*x*e
xp(f*x+e)+3*c^2*f^2*exp(f*x+e))/(exp(f*x+e)-I)^3/f^3/a^2-4/3/a^2/f^2*d*ln(
exp(f*x+e)-I)*c+4/3/a^2/f^2*d*ln(exp(f*x+e))*c+2/3/a^2/f*d^2*x^2+4/3/a^2/f
^2*d^2*e*x+2/3/a^2/f^3*d^2*e^2-4/3/a^2/f^2*d^2*ln(1+I*exp(f*x+e))*x-4/3/a^
2/f^3*d^2*ln(1+I*exp(f*x+e))*e-4/3*d^2*polylog(2,-I*exp(f*x+e))/a^2/f^3+4/
3/a^2/f^3*d^2*e*ln(exp(f*x+e)-I)-4/3/a^2/f^3*d^2*e*ln(exp(f*x+e))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 482 vs. 2(180) = 360.

Time = 0.10 (sec) , antiderivative size = 482, normalized size of antiderivative = 2.00

$$\int \frac{(c+dx)^2}{(a+ia\sinh(e+fx))^2}\,dx = -\frac{2\left(i\,d^2e^2-2i\,cdef+i\,c^2f^2-2i\,d^2+2\left(d^2e^{(3\,fx+3\,e)}-3i\,d^2e^{(2\,fx+2\,e)}-3\,d^2e^{(fx+e)}+i\,d^2\right)\mathrm{Li}_2\left(-i\,e^{(fx+e}\right.\right.}{}$$

input
```
integrate((d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
-2/3*(I*d^2*e^2 - 2*I*c*d*e*f + I*c^2*f^2 - 2*I*d^2 + 2*(d^2*e^(3*f*x + 3*
e) - 3*I*d^2*e^(2*f*x + 2*e) - 3*d^2*e^(f*x + e) + I*d^2)*dilog(-I*e^(f*x
+ e)) - (d^2*f^2*x^2 + 2*c*d*f^2*x - d^2*e^2 + 2*c*d*e*f)*e^(3*f*x + 3*e)
+ (3*I*d^2*f^2*x^2 - 3*I*d^2*e^2 + 2*I*d^2 + 2*(3*I*c*d*e + I*c*d)*f + 2*(
3*I*c*d*f^2 + I*d^2*f)*x)*e^(2*f*x + 2*e) - (3*d^2*e^2 + 3*c^2*f^2 - 2*d^2
*f*x - 4*d^2 - 2*(3*c*d*e + c*d)*f)*e^(f*x + e) + 2*(-I*d^2*e + I*c*d*f -
(d^2*e - c*d*f)*e^(3*f*x + 3*e) + 3*(I*d^2*e - I*c*d*f)*e^(2*f*x + 2*e) +
3*(d^2*e - c*d*f)*e^(f*x + e))*log(e^(f*x + e) - I) + 2*(I*d^2*f*x + I*d^2
*e + (d^2*f*x + d^2*e)*e^(3*f*x + 3*e) + 3*(-I*d^2*f*x - I*d^2*e)*e^(2*f*x
 + 2*e) - 3*(d^2*f*x + d^2*e)*e^(f*x + e))*log(I*e^(f*x + e) + 1))/(a^2*f^
3*e^(3*f*x + 3*e) - 3*I*a^2*f^3*e^(2*f*x + 2*e) - 3*a^2*f^3*e^(f*x + e) +
I*a^2*f^3)
```

Sympy [F]

$$\int \frac{(c+dx)^2}{(a+ia\sinh(e+fx))^2}\,dx$$
$$= \frac{-2ic^2f^2 - 4icdf^2x - 2id^2f^2x^2 + 4id^2 + (-4icdfe^{2e} - 4id^2fxe^{2e} - 4id^2e^{2e})\,e^{2fx} + (6c^2f^2e^e + 12cdf^2xe^e}{3a^2f^3e^{3e}e^{3fx} - 9ia^2f^3e^{2e}e^{2fx} - 9a^2f^3e^ee^{fx} + 3ia^2f^3}$$
$$- \frac{4id\left(\int \frac{c}{e^e e^{fx} - i}\,dx + \int \frac{dx}{e^e e^{fx} - i}\,dx\right)}{3a^2f}$$

input
```
integrate((d*x+c)**2/(a+I*a*sinh(f*x+e))**2,x)
```

output
```
(-2*I*c**2*f**2 - 4*I*c*d*f**2*x - 2*I*d**2*f**2*x**2 + 4*I*d**2 + (-4*I*c
*d*f*exp(2*e) - 4*I*d**2*f*x*exp(2*e) - 4*I*d**2*exp(2*e))*exp(2*f*x) + (6
*c**2*f**2*exp(e) + 12*c*d*f**2*x*exp(e) - 4*c*d*f*exp(e) + 6*d**2*f**2*x*
*2*exp(e) - 4*d**2*f*x*exp(e) - 8*d**2*exp(e))*exp(f*x))/(3*a**2*f**3*exp(
3*e)*exp(3*f*x) - 9*I*a**2*f**3*exp(2*e)*exp(2*f*x) - 9*a**2*f**3*exp(e)*e
xp(f*x) + 3*I*a**2*f**3) - 4*I*d*(Integral(c/(exp(e)*exp(f*x) - I), x) + I
ntegral(d*x/(exp(e)*exp(f*x) - I), x))/(3*a**2*f)
```

Maxima [F]

$$\int \frac{(c+dx)^2}{(a+ia\sinh(e+fx))^2}\,dx = \int \frac{(dx+c)^2}{(i\,a\sinh{(fx+e)}+a)^2}\,dx$$

input
```
integrate((d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
-2/3*d^2*((I*f^2*x^2 - 2*(-I*f*x*e^(2*e) - I*e^(2*e))*e^(2*f*x) - (3*f^2*x
^2*e^e - 2*f*x*e^e - 4*e^e)*e^(f*x) - 2*I)/(a^2*f^3*e^(3*f*x + 3*e) - 3*I*
a^2*f^3*e^(2*f*x + 2*e) - 3*a^2*f^3*e^(f*x + e) + I*a^2*f^3) + 6*I*integra
te(1/3*x/(a^2*f*e^(f*x + e) - I*a^2*f), x)) + 4/3*c*d*((f*x*e^(3*f*x + 3*e
) - (3*I*f*x*e^(2*e) + I*e^(2*e))*e^(2*f*x) - e^(f*x + e))/(a^2*f^2*e^(3*f
*x + 3*e) - 3*I*a^2*f^2*e^(2*f*x + 2*e) - 3*a^2*f^2*e^(f*x + e) + I*a^2*f^
2) - log(-I*(I*e^(f*x + e) + 1)*e^(-e))/(a^2*f^2)) + 2/3*c^2*(3*e^(-f*x -
e)/((3*a^2*e^(-f*x - e) - 3*I*a^2*e^(-2*f*x - 2*e) - a^2*e^(-3*f*x - 3*e)
+ I*a^2)*f) + I/((3*a^2*e^(-f*x - e) - 3*I*a^2*e^(-2*f*x - 2*e) - a^2*e^(-
3*f*x - 3*e) + I*a^2)*f))
```

Giac [F]

$$\int \frac{(c+dx)^2}{(a+ia\sinh(e+fx))^2}\,dx = \int \frac{(dx+c)^2}{(i\,a\sinh{(fx+e)}+a)^2}\,dx$$

input
```
integrate((d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(I*a*sinh(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{(a+ia\sinh(e+fx))^2}\,dx = \int \frac{(c+d\,x)^2}{(a+a\sinh\left(e+f\,x\right)\,1\mathrm{i})^2}\,dx$$

input
```
int((c + d*x)^2/(a + a*sinh(e + f*x)*1i)^2,x)
```

output
```
int((c + d*x)^2/(a + a*sinh(e + f*x)*1i)^2, x)
```

Reduce [F]

$$\int \frac{(c+dx)^2}{(a+ia\sinh(e+fx))^2}\,dx = \frac{-\left(\int \frac{x^2}{\sinh(fx+e)^2-2\sinh(fx+e)i-1}dx\right)d^2-2\left(\int \frac{x}{\sinh(fx+e)^2-2\sinh(fx+e)i-1}dx\right)cd-\left(\int \frac{1}{\sinh(fx+e)^2-2\sinh(fx+e)i-1}\right.}{a^2}$$

input
```
int((d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x)
```

output
```
( - int(x**2/(sinh(e + f*x)**2 - 2*sinh(e + f*x)*i - 1),x)*d**2 - 2*int(x/
(sinh(e + f*x)**2 - 2*sinh(e + f*x)*i - 1),x)*c*d - int(1/(sinh(e + f*x)**
2 - 2*sinh(e + f*x)*i - 1),x)*c**2)/a**2
```

3.115 $\int \frac{c+dx}{(a+ia\sinh(e+fx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 158

$$\int \frac{c+dx}{(a+ia\sinh(e+fx))^2}\,dx = -\frac{2d\log\left(\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)}{3a^2f^2}+\frac{d\,\mathrm{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{6a^2f^2}+\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2f}+\frac{(c+dx)\mathrm{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{6a^2f}$$

output
```
-2/3*d*ln(cosh(1/2*e+1/4*I*Pi+1/2*f*x))/a^2/f^2+1/6*d*sech(1/2*e+1/4*I*Pi+
1/2*f*x)^2/a^2/f^2+1/3*(d*x+c)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/a^2/f+1/6*(d*x
+c)*sech(1/2*e+1/4*I*Pi+1/2*f*x)^2*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/a^2/f
```

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.53

$$\int \frac{c+dx}{(a+ia\sinh(e+fx))^2}\,dx = \frac{\left(-i\cosh\left(\frac{1}{2}(e+fx)\right)+\sinh\left(\frac{1}{2}(e+fx)\right)\right)\left(d\cosh\left(\frac{1}{2}(e+fx)\right)\left(-2i+3e+3fx-6\arctan\left(\tanh\left(\frac{1}{2}(e+\right.\right.\right.\right.}{}$$

input
```
Integrate[(c + d*x)/(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
(((-I)*Cosh[(e + f*x)/2] + Sinh[(e + f*x)/2])*(d*Cosh[(e + f*x)/2]*(-2*I +
 3*e + 3*f*x - 6*ArcTan[Tanh[(e + f*x)/2]] + (3*I)*Log[Cosh[e + f*x]]) + C
osh[(3*(e + f*x))/2]*(-(d*e) + 2*c*f + d*f*x + 2*d*ArcTan[Tanh[(e + f*x)/2
]] - I*d*Log[Cosh[e + f*x]]) + (2*I)*((-I)*d + 2*d*e - 3*c*f - d*f*x - 4*d
*ArcTan[Tanh[(e + f*x)/2]] + d*Cosh[e + f*x]*(e + f*x - 2*ArcTan[Tanh[(e +
 f*x)/2]] + I*Log[Cosh[e + f*x]]) + (2*I)*d*Log[Cosh[e + f*x]])*Sinh[(e +
f*x)/2]))/(6*a^2*f^2*(-I + Sinh[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3799, 3042, 4673, 3042, 4672, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a+ia\sinh(e+fx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{c+dx}{(a+a\sin(ie+ifx))^2}dx$$

$$\Big\downarrow \text{ 3799}$$

$$\frac{\int (c+dx)\text{csch}^4\left(\frac{e}{2}+\frac{fx}{2}-\frac{i\pi}{4}\right)dx}{4a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (c+dx)\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^4dx}{4a^2}$$

$$\Big\downarrow \text{ 4673}$$

$$\frac{\frac{2}{3}\int (c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx+\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f}+\frac{2d\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}}{4a^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2}{3}\int (c+dx)\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^2dx+\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f}+\frac{2d\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}}{4a^2}$$

$\Big\downarrow$ 4672

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2id\int -i\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)+\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f}+\frac{2d\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\right.}{3f^2}}{4a^2}$$

$\Big\downarrow$ 26

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2d\int \tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)+\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f}+\frac{2d\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}}{4a^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2d\int -i\tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}\right)+\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f}+\frac{2d\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right.}{3f^2}}{4a^2}$$

$\Big\downarrow$ 26

$$\frac{\frac{2}{3}\left(\frac{2id\int \tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)+\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f}+\frac{2d\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}}{4a^2}$$

$\Big\downarrow$ 3956

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)}{f^2}\right)+\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f}+\frac{2d\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}}{4a^2}$$

input
```
Int[(c + d*x)/(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
((2*d*Sech[e/2 + (I/4)*Pi + (f*x)/2]^2)/(3*f^2) + (2*(c + d*x)*Sech[e/2 +
(I/4)*Pi + (f*x)/2]^2*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(3*f) + (2*((-4*d*Lo
g[Cosh[e/2 + (I/4)*Pi + (f*x)/2]])/f^2 + (2*(c + d*x)*Tanh[e/2 + (I/4)*Pi
+ (f*x)/2])/f))/3)/(4*a^2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.72

method	result
risch	$\frac{2dx}{3f\,a^2}+\frac{2de}{3f^2a^2}-\frac{2i\left(3ifdx\,\mathrm{e}^{fx+e}+3ifc\,\mathrm{e}^{fx+e}-id\,\mathrm{e}^{fx+e}+dxf+\mathrm{e}^{2fx+2e}d+cf\right)}{3\left(\mathrm{e}^{fx+e}-i\right)^3f^2a^2}-\frac{2d\ln\left(\mathrm{e}^{fx+e}-i\right)}{3f^2a^2}$
parallelrisch	$18\left(i\cosh\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{i\cosh\left(\frac{3fx}{2}+\frac{3e}{2}\right)}{3}-\frac{\sinh\left(\frac{3fx}{2}+\frac{3e}{2}\right)}{3}-\sinh\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d\ln\left(1-\tanh\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-18\left(i\cosh\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{i\cosh(}{}\right.$

input
```
int((d*x+c)/(a+I*a*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*d/f/a^2*x+2/3*d/f^2/a^2*e-2/3*I*(3*I*f*d*x*exp(f*x+e)+3*I*f*c*exp(f*x+
e)-I*d*exp(f*x+e)+d*x*f+exp(2*f*x+2*e)*d+c*f)/(exp(f*x+e)-I)^3/f^2/a^2-2/3
*d/f^2/a^2*ln(exp(f*x+e)-I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.01

$$\int\frac{c+dx}{(a+ia\sinh(e+fx))^2}\,dx$$
$$=\frac{2\left(dfxe^{(3\,fx+3\,e)}-i\,cf-(3i\,dfx+i\,d)e^{(2\,fx+2\,e)}+(3\,cf-d)e^{(fx+e)}-\left(de^{(3\,fx+3\,e)}-3i\,de^{(2\,fx+2\,e)}-3\,de^{(}\right.\right.}{3\left(a^2f^2e^{(3\,fx+3\,e)}-3i\,a^2f^2e^{(2\,fx+2\,e)}-3\,a^2f^2e^{(fx+e)}+i\,a^2f^2\right)}$$

input
```
integrate((d*x+c)/(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
2/3*(d*f*x*e^(3*f*x + 3*e) - I*c*f - (3*I*d*f*x + I*d)*e^(2*f*x + 2*e) + (
3*c*f - d)*e^(f*x + e) - (d*e^(3*f*x + 3*e) - 3*I*d*e^(2*f*x + 2*e) - 3*d*
e^(f*x + e) + I*d)*log(e^(f*x + e) - I))/(a^2*f^2*e^(3*f*x + 3*e) - 3*I*a^
2*f^2*e^(2*f*x + 2*e) - 3*a^2*f^2*e^(f*x + e) + I*a^2*f^2)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.06

$$\int \frac{c+dx}{(a+ia\sinh(e+fx))^2}\,dx$$
$$= \frac{-2icf-2idfx-2ide^{2e}e^{2fx}+\left(6cfe^{e}+6dfxe^{e}-2de^{e}\right)e^{fx}}{3a^2f^2e^{3e}e^{3fx}-9ia^2f^2e^{2e}e^{2fx}-9a^2f^2e^{e}e^{fx}+3ia^2f^2} + \frac{2dx}{3a^2f} - \frac{2d\log\left(e^{fx}-ie^{-e}\right)}{3a^2f^2}$$

input
```
integrate((d*x+c)/(a+I*a*sinh(f*x+e))**2,x)
```

output
```
(-2*I*c*f - 2*I*d*f*x - 2*I*d*exp(2*e)*exp(2*f*x) + (6*c*f*exp(e) + 6*d*f*
x*exp(e) - 2*d*exp(e))*exp(f*x))/(3*a**2*f**2*exp(3*e)*exp(3*f*x) - 9*I*a*
*2*f**2*exp(2*e)*exp(2*f*x) - 9*a**2*f**2*exp(e)*exp(f*x) + 3*I*a**2*f**2)
 + 2*d*x/(3*a**2*f) - 2*d*log(exp(f*x) - I*exp(-e))/(3*a**2*f**2)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(110) = 220.

Time = 0.07 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.61

$$\int \frac{c+dx}{(a+ia\sinh(e+fx))^2}\,dx$$
$$= \frac{2}{3}\,d\left(\frac{fxe^{(3\,fx+3\,e)}-\left(3i\,fxe^{(2\,e)}+i\,e^{(2\,e)}\right)e^{(2\,fx)}-e^{(fx+e)}}{a^2f^2e^{(3\,fx+3\,e)}-3i\,a^2f^2e^{(2\,fx+2\,e)}-3\,a^2f^2e^{(fx+e)}+i\,a^2f^2}-\frac{\log\left(-i\left(i\,e^{(fx+e)}+1\right)e^{(-e)}\right)}{a^2f^2}\right) + \frac{2}{3}\,c\left(\frac{3\,e^{(-fx-e)}}{\left(3\,a^2e^{(-fx-e)}-3i\,a^2e^{(-2\,fx-2\,e)}-a^2e^{(-3\,fx-3\,e)}+i\,a^2\right)f}+\frac{i}{\left(3\,a^2e^{(-fx-e)}-3i\,a^2e^{(-2\,fx-2\,e)}-a^2e^{(-\right.}}\right.$$

input
```
integrate((d*x+c)/(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
2/3*d*((f*x*e^(3*f*x + 3*e) - (3*I*f*x*e^(2*e) + I*e^(2*e))*e^(2*f*x) - e^
(f*x + e))/(a^2*f^2*e^(3*f*x + 3*e) - 3*I*a^2*f^2*e^(2*f*x + 2*e) - 3*a^2*
f^2*e^(f*x + e) + I*a^2*f^2) - log(-I*(I*e^(f*x + e) + 1)*e^(-e))/(a^2*f^2
)) + 2/3*c*(3*e^(-f*x - e)/((3*a^2*e^(-f*x - e) - 3*I*a^2*e^(-2*f*x - 2*e)
 - a^2*e^(-3*f*x - 3*e) + I*a^2)*f) + I/((3*a^2*e^(-f*x - e) - 3*I*a^2*e^(
-2*f*x - 2*e) - a^2*e^(-3*f*x - 3*e) + I*a^2)*f))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.23

$$\int \frac{c+dx}{(a+ia\sinh(e+fx))^2}\,dx = \frac{2\left(dfxe^{(3\,fx+3\,e)}-3i\,dfxe^{(2\,fx+2\,e)}+3\,cfe^{(fx+e)}-de^{(3\,fx+3\,e)}\log\left(e^{(fx+e)}-i\right)+3i\,de^{(2\,fx+2\,e)}\log\left(e^{(fx+e)}\right.\right.}{3\left(a^2f^2e^{(3\,fx+3\,e)}-3i\,a^2f^2e^{(2\,fx+2\,e)}-\right.}$$

input
```
integrate((d*x+c)/(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
2/3*(d*f*x*e^(3*f*x + 3*e) - 3*I*d*f*x*e^(2*f*x + 2*e) + 3*c*f*e^(f*x + e)
 - d*e^(3*f*x + 3*e)*log(e^(f*x + e) - I) + 3*I*d*e^(2*f*x + 2*e)*log(e^(f
*x + e) - I) + 3*d*e^(f*x + e)*log(e^(f*x + e) - I) - I*c*f - I*d*e^(2*f*x
 + 2*e) - d*e^(f*x + e) - I*d*log(e^(f*x + e) - I))/(a^2*f^2*e^(3*f*x + 3*
e) - 3*I*a^2*f^2*e^(2*f*x + 2*e) - 3*a^2*f^2*e^(f*x + e) + I*a^2*f^2)
```

Mupad [B] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.01

$$\int \frac{c+dx}{(a+ia\sinh(e+fx))^2}\,dx = -\frac{\frac{2\,d\,\ln\left(e^{f\,x}\,e^{e}-i\right)}{3}+e^{e+f\,x}\left(-\frac{d\,2i}{3}+c\,f\,2i+d\,\ln\left(e^{f\,x}\,e^{e}-i\right)\,2i\right)+\frac{2\,d\,e^{2\,e+2\,f\,x}}{3}+f\left(\frac{2\,c}{3}+2\,d\,x\,e^{2\,e+2\,f\,x}+\frac{d\,x\,e^{3}}{}\right.}{a^2\,f^2\,\left(1+e^{e+f\,x}\,1i\right)^3}$$

input
```
int((c + d*x)/(a + a*sinh(e + f*x)*1i)^2,x)
```

output
```
-((2*d*log(exp(f*x)*exp(e) - 1i))/3 + exp(e + f*x)*(c*f*2i - (d*2i)/3 + d*
log(exp(f*x)*exp(e) - 1i)*2i) + (2*d*exp(2*e + 2*f*x))/3 + f*((2*c)/3 + 2*
d*x*exp(2*e + 2*f*x) + (d*x*exp(3*e + 3*f*x)*2i)/3) - 2*d*exp(2*e + 2*f*x)
*log(exp(f*x)*exp(e) - 1i) - (d*exp(3*e + 3*f*x)*log(exp(f*x)*exp(e) - 1i)
*2i)/3)/(a^2*f^2*(exp(e + f*x)*1i + 1)^3)
```

Reduce [F]

$$\int \frac{c+dx}{(a+ia\sinh(e+fx))^2}\,dx = \frac{-\left(\int \frac{x}{\sinh(fx+e)^2-2\sinh(fx+e)i-1}dx\right)d-\left(\int \frac{1}{\sinh(fx+e)^2-2\sinh(fx+e)i-1}dx\right)c}{a^2}$$

input
```
int((d*x+c)/(a+I*a*sinh(f*x+e))^2,x)
```

output
```
( - (int(x/(sinh(e + f*x)**2 - 2*sinh(e + f*x)*i - 1),x)*d + int(1/(sinh(e
 + f*x)**2 - 2*sinh(e + f*x)*i - 1),x)*c))/a**2
```

3.116 $\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}\,dx = \text{Int}\left(\frac{1}{(c+dx)(a+ia\sinh(e+fx))^2},x\right)$$

output
```
Defer(Int)(1/(d*x+c)/(a+I*a*sinh(f*x+e))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 24.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}\,dx = \int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}\,dx$$

input
```
Integrate[1/((c + d*x)*(a + I*a*Sinh[e + f*x])^2),x]
```

output
```
Integrate[1/((c + d*x)*(a + I*a*Sinh[e + f*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c+dx)(a+a\sin(ie+ifx))^2}dx$$

$$\Big\downarrow \text{3807}$$

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}dx$$

input
```
Int[1/((c + d*x)*(a + I*a*Sinh[e + f*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(dx+c)\left(a+ia\sinh\left(fx+e\right)\right)^2}dx$$

input
```
int(1/(d*x+c)/(a+I*a*sinh(f*x+e))^2,x)
```

output
```
int(1/(d*x+c)/(a+I*a*sinh(f*x+e))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 793, normalized size of antiderivative = 34.48

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(i\,a\sinh{(fx+e)}+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
(-2*I*d^2*f^2*x^2 - 4*I*c*d*f^2*x - 2*I*c^2*f^2 + 4*I*d^2 - 2*(-I*d^2*f*x
- I*c*d*f + 2*I*d^2)*e^(2*f*x + 2*e) + 2*(3*d^2*f^2*x^2 + 3*c^2*f^2 + c*d*
f - 4*d^2 + (6*c*d*f^2 + d^2*f)*x)*e^(f*x + e) - 3*(-I*a^2*d^3*f^3*x^3 - 3
*I*a^2*c*d^2*f^3*x^2 - 3*I*a^2*c^2*d*f^3*x - I*a^2*c^3*f^3 - (a^2*d^3*f^3*
x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*e^(3*f*x + 3*
e) + 3*(I*a^2*d^3*f^3*x^3 + 3*I*a^2*c*d^2*f^3*x^2 + 3*I*a^2*c^2*d*f^3*x +
I*a^2*c^3*f^3)*e^(2*f*x + 2*e) + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2
+ 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*e^(f*x + e))*integral(-2*(-I*d^3*f^2*x^
2 - 2*I*c*d^2*f^2*x - I*c^2*d*f^2 + 6*I*d^3)/(-3*I*a^2*d^4*f^3*x^4 - 12*I*
a^2*c*d^3*f^3*x^3 - 18*I*a^2*c^2*d^2*f^3*x^2 - 12*I*a^2*c^3*d*f^3*x - 3*I*
a^2*c^4*f^3 + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3
*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*e^(f*x + e)), x))/(3*I*a^2*d^3*f^3
*x^3 + 9*I*a^2*c*d^2*f^3*x^2 + 9*I*a^2*c^2*d*f^3*x + 3*I*a^2*c^3*f^3 + 3*(
a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*e
^(3*f*x + 3*e) - 9*(I*a^2*d^3*f^3*x^3 + 3*I*a^2*c*d^2*f^3*x^2 + 3*I*a^2*c^
2*d*f^3*x + I*a^2*c^3*f^3)*e^(2*f*x + 2*e) - 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*
d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*e^(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 606, normalized size of antiderivative = 26.35

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(i\,a\sinh\left(fx+e\right)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
-2*(I*d^2*f^2*x^2 + 2*I*c*d*f^2*x + I*c^2*f^2 - 2*I*d^2 + (-I*d^2*f*x*e^(2
*e) - I*c*d*f*e^(2*e) + 2*I*d^2*e^(2*e))*e^(2*f*x) - (3*d^2*f^2*x^2*e^e +
3*c^2*f^2*e^e + c*d*f*e^e - 4*d^2*e^e + (6*c*d*f^2*e^e + d^2*f*e^e)*x)*e^(
f*x))/(3*I*a^2*d^3*f^3*x^3 + 9*I*a^2*c*d^2*f^3*x^2 + 9*I*a^2*c^2*d*f^3*x +
 3*I*a^2*c^3*f^3 + 3*(a^2*d^3*f^3*x^3*e^(3*e) + 3*a^2*c*d^2*f^3*x^2*e^(3*e
) + 3*a^2*c^2*d*f^3*x*e^(3*e) + a^2*c^3*f^3*e^(3*e))*e^(3*f*x) - 9*(I*a^2*
d^3*f^3*x^3*e^(2*e) + 3*I*a^2*c*d^2*f^3*x^2*e^(2*e) + 3*I*a^2*c^2*d*f^3*x*
e^(2*e) + I*a^2*c^3*f^3*e^(2*e))*e^(2*f*x) - 9*(a^2*d^3*f^3*x^3*e^e + 3*a^
2*c*d^2*f^3*x^2*e^e + 3*a^2*c^2*d*f^3*x*e^e + a^2*c^3*f^3*e^e)*e^(f*x)) -
integrate(2/3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 6*d^3)/(a^2*d^4*f
^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x +
 a^2*c^4*f^3 - (-I*a^2*d^4*f^3*x^4*e^e - 4*I*a^2*c*d^3*f^3*x^3*e^e - 6*I*a
^2*c^2*d^2*f^3*x^2*e^e - 4*I*a^2*c^3*d*f^3*x*e^e - I*a^2*c^4*f^3*e^e)*e^(f
*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(i\,a\sinh\left(fx+e\right)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)*(I*a*sinh(f*x + e) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}\,dx = \int \frac{1}{\left(a+a\sinh\left(e+f\,x\right)\,1\mathrm{i}\right)^2\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/((a + a*sinh(e + f*x)*1i)^2*(c + d*x)),x)
```

output
```
int(1/((a + a*sinh(e + f*x)*1i)^2*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.57

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}\,dx = -\frac{\int \frac{1}{\sinh(fx+e)^2c+\sinh(fx+e)^2dx-2\sinh(fx+e)ci-2\sinh(fx+e)dix-c-dx}dx}{a^2}$$

input
```
int(1/(d*x+c)/(a+I*a*sinh(f*x+e))^2,x)
```

output
```
( - int(1/(sinh(e + f*x)**2*c + sinh(e + f*x)**2*d*x - 2*sinh(e + f*x)*c*i
 - 2*sinh(e + f*x)*d*i*x - c - d*x),x))/a**2
```

3.117 $\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2}\,dx = \operatorname{Int}\left(\frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2},x\right)$$

output
```
Defer(Int)(1/(d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 25.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2}\,dx = \int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2}\,dx$$

input
```
Integrate[1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])^2),x]
```

output
```
Integrate[1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(c+dx)^2(a+a\sin(ie+ifx))^2}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2}dx$$

input
```
Int[1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(dx+c)^2(a+ia\sinh(fx+e))^2}dx$$

input
```
int(1/(d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x)
```

output
```
int(1/(d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 965, normalized size of antiderivative = 41.96

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2}\,dx=\int \frac{1}{(dx+c)^2(i\,a\sinh{(fx+e)}+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
(-2*I*d^2*f^2*x^2 - 4*I*c*d*f^2*x - 2*I*c^2*f^2 + 12*I*d^2 - 4*(-I*d^2*f*x
 - I*c*d*f + 3*I*d^2)*e^(2*f*x + 2*e) + 2*(3*d^2*f^2*x^2 + 3*c^2*f^2 + 2*c
*d*f - 12*d^2 + 2*(3*c*d*f^2 + d^2*f)*x)*e^(f*x + e) - 3*(-I*a^2*d^4*f^3*x
^4 - 4*I*a^2*c*d^3*f^3*x^3 - 6*I*a^2*c^2*d^2*f^3*x^2 - 4*I*a^2*c^3*d*f^3*x
 - I*a^2*c^4*f^3 - (a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*
f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*e^(3*f*x + 3*e) + 3*(I*a^2*d^4*
f^3*x^4 + 4*I*a^2*c*d^3*f^3*x^3 + 6*I*a^2*c^2*d^2*f^3*x^2 + 4*I*a^2*c^3*d*
f^3*x + I*a^2*c^4*f^3)*e^(2*f*x + 2*e) + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*
f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*e^(f*x
+ e))*integral(-4*(-I*d^3*f^2*x^2 - 2*I*c*d^2*f^2*x - I*c^2*d*f^2 + 12*I*d
^3)/(-3*I*a^2*d^5*f^3*x^5 - 15*I*a^2*c*d^4*f^3*x^4 - 30*I*a^2*c^2*d^3*f^3*
x^3 - 30*I*a^2*c^3*d^2*f^3*x^2 - 15*I*a^2*c^4*d*f^3*x - 3*I*a^2*c^5*f^3 +
3*(a^2*d^5*f^3*x^5 + 5*a^2*c*d^4*f^3*x^4 + 10*a^2*c^2*d^3*f^3*x^3 + 10*a^2
*c^3*d^2*f^3*x^2 + 5*a^2*c^4*d*f^3*x + a^2*c^5*f^3)*e^(f*x + e)), x))/(3*I
*a^2*d^4*f^3*x^4 + 12*I*a^2*c*d^3*f^3*x^3 + 18*I*a^2*c^2*d^2*f^3*x^2 + 12*
I*a^2*c^3*d*f^3*x + 3*I*a^2*c^4*f^3 + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3
*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*e^(3*f*x +
 3*e) - 9*(I*a^2*d^4*f^3*x^4 + 4*I*a^2*c*d^3*f^3*x^3 + 6*I*a^2*c^2*d^2*f^3
*x^2 + 4*I*a^2*c^3*d*f^3*x + I*a^2*c^4*f^3)*e^(2*f*x + 2*e) - 9*(a^2*d^4*f
^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)**2/(a+I*a*sinh(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 723, normalized size of antiderivative = 31.43

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(i\,a\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
-2*(I*d^2*f^2*x^2 + 2*I*c*d*f^2*x + I*c^2*f^2 - 6*I*d^2 + 2*(-I*d^2*f*x*e^
(2*e) - I*c*d*f*e^(2*e) + 3*I*d^2*e^(2*e))*e^(2*f*x) - (3*d^2*f^2*x^2*e^e
+ 3*c^2*f^2*e^e + 2*c*d*f*e^e - 12*d^2*e^e + 2*(3*c*d*f^2*e^e + d^2*f*e^e)
*x)*e^(f*x))/(3*I*a^2*d^4*f^3*x^4 + 12*I*a^2*c*d^3*f^3*x^3 + 18*I*a^2*c^2*
d^2*f^3*x^2 + 12*I*a^2*c^3*d*f^3*x + 3*I*a^2*c^4*f^3 + 3*(a^2*d^4*f^3*x^4*
e^(3*e) + 4*a^2*c*d^3*f^3*x^3*e^(3*e) + 6*a^2*c^2*d^2*f^3*x^2*e^(3*e) + 4*
a^2*c^3*d*f^3*x*e^(3*e) + a^2*c^4*f^3*e^(3*e))*e^(3*f*x) - 9*(I*a^2*d^4*f^
3*x^4*e^(2*e) + 4*I*a^2*c*d^3*f^3*x^3*e^(2*e) + 6*I*a^2*c^2*d^2*f^3*x^2*e^
(2*e) + 4*I*a^2*c^3*d*f^3*x*e^(2*e) + I*a^2*c^4*f^3*e^(2*e))*e^(2*f*x) - 9
*(a^2*d^4*f^3*x^4*e^e + 4*a^2*c*d^3*f^3*x^3*e^e + 6*a^2*c^2*d^2*f^3*x^2*e^
e + 4*a^2*c^3*d*f^3*x*e^e + a^2*c^4*f^3*e^e)*e^(f*x)) - integrate(4/3*(d^3
*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 12*d^3)/(a^2*d^5*f^3*x^5 + 5*a^2*c*
d^4*f^3*x^4 + 10*a^2*c^2*d^3*f^3*x^3 + 10*a^2*c^3*d^2*f^3*x^2 + 5*a^2*c^4*
d*f^3*x + a^2*c^5*f^3 - (-I*a^2*d^5*f^3*x^5*e^e - 5*I*a^2*c*d^4*f^3*x^4*e^
e - 10*I*a^2*c^2*d^3*f^3*x^3*e^e - 10*I*a^2*c^3*d^2*f^3*x^2*e^e - 5*I*a^2*
c^4*d*f^3*x*e^e - I*a^2*c^5*f^3*e^e)*e^(f*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(i\,a\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)^2*(I*a*sinh(f*x + e) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2}\,dx = \int \frac{1}{(a+a\sinh(e+f\,x)\;1\mathrm{i})^2\,(c+d\,x)^2}\,dx$$

input
```
int(1/((a + a*sinh(e + f*x)*1i)^2*(c + d*x)^2),x)
```

output
```
int(1/((a + a*sinh(e + f*x)*1i)^2*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 4.65

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2}\,dx = -\frac{\int \frac{1}{\sinh(fx+e)^2c^2+2\sinh(fx+e)^2cdx+\sinh(fx+e)^2d^2x^2-2\sinh(fx+e)c^2i-4\sinh(fx+e)cdix-2\sinh(fx+e)d^2i\,x^2-c^2-2cdx-d^2x^2}dx}{a^2}$$

input
```
int(1/(d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x)
```

output
```
( - int(1/(sinh(e + f*x)**2*c**2 + 2*sinh(e + f*x)**2*c*d*x + sinh(e + f*x
)**2*d**2*x**2 - 2*sinh(e + f*x)*c**2*i - 4*sinh(e + f*x)*c*d*i*x - 2*sinh
(e + f*x)*d**2*i*x**2 - c**2 - 2*c*d*x - d**2*x**2),x))/a**2
```

3.118 $\int x^4\sqrt{a+ia\sinh(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 181

$$\int x^4\sqrt{a+ia\sinh(e+fx)}\,dx = -\frac{384x\sqrt{a+ia\sinh(e+fx)}}{f^4} - \frac{16x^3\sqrt{a+ia\sinh(e+fx)}}{f^2} + \frac{768\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f^5} + \frac{96x^2\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f^3} + \frac{2x^4\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f}$$

output
```
-384*x*(a+I*a*sinh(f*x+e))^(1/2)/f^4-16*x^3*(a+I*a*sinh(f*x+e))^(1/2)/f^2+
768*(a+I*a*sinh(f*x+e))^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/f^5+96*x^2*(a+I
*a*sinh(f*x+e))^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/f^3+2*x^4*(a+I*a*sinh(f
*x+e))^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/f
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.78

$$\int x^4\sqrt{a+ia\sinh(e+fx)}\,dx$$
$$=\frac{2\left(i(384+192ifx+48f^2x^2+8if^3x^3+f^4x^4)\cosh\left(\frac{1}{2}(e+fx)\right)+(384-192ifx+48f^2x^2-8if^3x^3+\right.}{f^5\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}$$

input
```
Integrate[x^4*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

output
```
(2*(I*(384 + (192*I)*f*x + 48*f^2*x^2 + (8*I)*f^3*x^3 + f^4*x^4)*Cosh[(e +
 f*x)/2] + (384 - (192*I)*f*x + 48*f^2*x^2 - (8*I)*f^3*x^3 + f^4*x^4)*Sinh
[(e + f*x)/2])*Sqrt[a + I*a*Sinh[e + f*x]])/(f^5*(Cosh[(e + f*x)/2] + I*Si
nh[(e + f*x)/2]))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.13, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {3042, 3800, 3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{a+ia\sinh(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^4\sqrt{a+a\sin(ie+ifx)}dx$$

$$\Big\downarrow \text{3800}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int x^4\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int x^4\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx$$

$\Big\downarrow$ 3777

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2x^4\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{8i\int -ix^3\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)$$

$\Big\downarrow$ 26

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2x^4\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{8\int x^3\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)$$

$\Big\downarrow$ 3042

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2x^4\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{8\int -ix^3\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}\right)$$

$\Big\downarrow$ 26

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8i\int x^3\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2x^4\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)$$

$\Big\downarrow$ 3777

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8i\left(\frac{2ix^3\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6i\int x^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)}{f}+\frac{2x^4\sinh\left(\frac{e}{2}+\right.}{f}\right.$$

$\Big\downarrow$ 3042

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8i\left(\frac{2ix^3\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6i\int x^2\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx}{f}\right)}{f}+\frac{2x^4\sinh\left(\frac{e}{2}+\right.}{f}\right.$$

$\Big\downarrow$ 3777

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8i\left(\frac{2ix^3\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6i\left(\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\int -ix\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right.}{f}}{f}\right.$$

$\downarrow$ 26

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8i\left(\frac{2ix^3\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6i\left(\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4\int x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)}{f}}{f}\right.$$

$\downarrow$ 3042

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8i\left(\frac{2ix^3\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6i\left(\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4\int -ix\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}\right.}{f}}{f}\right.$$

$\downarrow$ 26

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8i\left(\frac{2ix^3\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6i\left(\frac{4i\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)}{f}}{f}\right.$$

$\downarrow$ 3777

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8i\left(\frac{2ix^3\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6i\left(\frac{4i\left(\frac{2ix\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2i\int\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right.}{f}\right.}{f}\right.}{f}\right.$$

$\downarrow$ 3042

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8i\left(\frac{2ix^3\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6i\left(\frac{4i\left(\frac{2ix\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2i\int\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx}{f}\right.}{f}\right.}{f}\right.}{f}\right.$$

$\downarrow$ 3118

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8i\left(\frac{2ix^3\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6i\left(\frac{4i\left(\frac{2ix\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2}\right)}{f}+\right.}{f}\right.}{f}\right.$$

input
```
Int[x^4*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

output
```
Sech[e/2 + (I/4)*Pi + (f*x)/2]*((2*x^4*Sinh[e/2 + (I/4)*Pi + (f*x)/2])/f +
 ((8*I)*(((2*I)*x^3*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/f - ((6*I)*((2*x^2*Sin
h[e/2 + (I/4)*Pi + (f*x)/2])/f + ((4*I)*(((2*I)*x*Cosh[e/2 + (I/4)*Pi + (f
*x)/2])/f - ((4*I)*Sinh[e/2 + (I/4)*Pi + (f*x)/2])/f^2))/f))/f))/f)*Sqrt[a
 + I*a*Sinh[e + f*x]]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.96

method	result
risch	$\frac{i\sqrt{2}\,\sqrt{a\left(i\mathrm{e}^{2fx+2e}-i+2\,\mathrm{e}^{fx+e}\right)\mathrm{e}^{-fx-e}}\,\left(ix^4f^4+f^4x^4\mathrm{e}^{fx+e}+8ix^3f^3-8f^3x^3\mathrm{e}^{fx+e}+48ix^2f^2+48f^2x^2\mathrm{e}^{fx+e}+192ixf-192fx\,\mathrm{e}^{fx+e}\right.}{\left(i\mathrm{e}^{2fx+2e}-i+2\,\mathrm{e}^{fx+e}\right)f^5}$

input
```
int(x^4*(a+I*a*sinh(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I*2^(1/2)*(a*(I*exp(2*f*x+2*e)-I+2*exp(f*x+e))*exp(-f*x-e))^(1/2)/(I*exp(2
*f*x+2*e)-I+2*exp(f*x+e))*(I*x^4*f^4+f^4*x^4*exp(f*x+e)+8*I*x^3*f^3-8*f^3*
x^3*exp(f*x+e)+48*I*x^2*f^2+48*f^2*x^2*exp(f*x+e)+192*I*x*f-192*f*x*exp(f*
x+e)+384*I+384*exp(f*x+e))*(exp(f*x+e)-I)/f^5
```

Fricas [F(-2)]

Exception generated.

$$\int x^4\sqrt{a+ia\sinh(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^4\sqrt{a+ia\sinh(e+fx)}\,dx=\int x^4\sqrt{ia\left(\sinh\left(e+fx\right)-i\right)}\,dx$$

input
```
integrate(x**4*(a+I*a*sinh(f*x+e))**(1/2),x)
```

output
```
Integral(x**4*sqrt(I*a*(sinh(e + f*x) - I)), x)
```

Maxima [F]

$$\int x^4\sqrt{a+ia\sinh(e+fx)}\,dx=\int \sqrt{i\,a\sinh\left(fx+e\right)+a}x^4\,dx$$

input
```
integrate(x^4*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(I*a*sinh(f*x + e) + a)*x^4, x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.74

$$\int x^4\sqrt{a+ia\sinh(e+fx)}\,dx$$
$$=\frac{\left((i+1)\sqrt{a}f^4x^4e^{(fx+e)}+(i-1)\sqrt{a}f^4x^4-(8i+8)\sqrt{a}f^3x^3e^{(fx+e)}+(8i-8)\sqrt{a}f^3x^3+(48i+48)\sqrt{a}\cdots\right.}{\cdots}$$

input
```
integrate(x^4*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
((I + 1)*sqrt(a)*f^4*x^4*e^(f*x + e) + (I - 1)*sqrt(a)*f^4*x^4 - (8*I + 8)
*sqrt(a)*f^3*x^3*e^(f*x + e) + (8*I - 8)*sqrt(a)*f^3*x^3 + (48*I + 48)*sqr
t(a)*f^2*x^2*e^(f*x + e) + (48*I - 48)*sqrt(a)*f^2*x^2 - (192*I + 192)*sqr
t(a)*f*x*e^(f*x + e) + (192*I - 192)*sqrt(a)*f*x + (384*I + 384)*sqrt(a)*e
^(f*x + e) + (384*I - 384)*sqrt(a))*e^(-1/2*f*x - 1/2*e)/f^5
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.82

$$\int x^4\sqrt{a+ia\sinh(e+fx)}\,dx$$
$$=\frac{\sqrt{2}\left(\mathrm{e}^{e+f\,x}+1\mathrm{i}\right)\,\sqrt{a\,\mathrm{e}^{-e-f\,x}\,\left(\mathrm{e}^{e+f\,x}-\mathrm{i}\right)^2\,1\mathrm{i}}\,\left(384\,\mathrm{e}^{e+f\,x}+f\,x\,192\mathrm{i}+f^2\,x^2\,48\mathrm{i}+f^3\,x^3\,8\mathrm{i}+f^4\,x^4\,1\mathrm{i}+48\,f^2\right.}{f^5\,\left(\mathrm{e}^{2\,e+2\,f\,x}+1\right)}$$

input
```
int(x^4*(a + a*sinh(e + f*x)*1i)^(1/2),x)
```

output
```
(2^(1/2)*(exp(e + f*x) + 1i)*(a*exp(- e - f*x)*(exp(e + f*x) - 1i)^2*1i)^(
1/2)*(384*exp(e + f*x) + f*x*192i + f^2*x^2*48i + f^3*x^3*8i + f^4*x^4*1i
+ 48*f^2*x^2*exp(e + f*x) - 8*f^3*x^3*exp(e + f*x) + f^4*x^4*exp(e + f*x)
- 192*f*x*exp(e + f*x) + 384i))/(f^5*(exp(2*e + 2*f*x) + 1))
```

Reduce [F]

$$\int x^4\sqrt{a+ia\sinh(e+fx)}\,dx=\sqrt{a}\left(\int\sqrt{\sinh(fx+e)\,i+1}\,x^4dx\right)$$

input
```
int(x^4*(a+I*a*sinh(f*x+e))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sinh(e + f*x)*i + 1)*x**4,x)
```

3.119 $\int x^3 \sqrt{a+ia\sinh(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 136

$$\int x^3\sqrt{a+ia\sinh(e+fx)}\,dx = -\frac{96\sqrt{a+ia\sinh(e+fx)}}{f^4}-\frac{12x^2\sqrt{a+ia\sinh(e+fx)}}{f^2}+\frac{48x\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f^3}+\frac{2x^3\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f}$$

output
```
-96*(a+I*a*sinh(f*x+e))^(1/2)/f^4-12*x^2*(a+I*a*sinh(f*x+e))^(1/2)/f^2+48*
x*(a+I*a*sinh(f*x+e))^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/f^3+2*x^3*(a+I*a*
sinh(f*x+e))^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/f
```

Mathematica [A] (verified)

Time = 2.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.92

$$\int x^3\sqrt{a+ia\sinh(e+fx)}\,dx = \frac{2\left(i(48i+24fx+6if^2x^2+f^3x^3)\cosh\left(\frac{1}{2}(e+fx)\right)+(-48i+24fx-6if^2x^2+f^3x^3)\sinh\left(\frac{1}{2}(e+fx)\right)\right)}{f^4\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}$$

input
```
Integrate[x^3*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

output
```
(2*(I*(48*I + 24*f*x + (6*I)*f^2*x^2 + f^3*x^3)*Cosh[(e + f*x)/2] + (-48*I
 + 24*f*x - (6*I)*f^2*x^2 + f^3*x^3)*Sinh[(e + f*x)/2])*Sqrt[a + I*a*Sinh[
e + f*x]])/(f^4*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.21, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3800, 3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+ia\sinh(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^3\sqrt{a+a\sin(ie+ifx)}dx$$

$$\Big\downarrow \text{3800}$$

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int x^3\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int x^3\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2x^3\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6i\int -ix^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2x^3\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6\int x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)$$

$\downarrow$ 3042

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2x^3\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6\int -ix^2\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}\right)$$

$\downarrow$ 26

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{6i\int x^2\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2x^3\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)$$

$\downarrow$ 3777

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\int x\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)}{f}+\frac{2x^3\sinh\left(\frac{e}{2}+\right.}{f}\right.$$

$\downarrow$ 3042

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx}{f}\right)}{f}+\frac{2x^3\sinh\left(\frac{e}{2}+\right.}{f}\right.$$

$\downarrow$ 3777

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2i\int -i\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)}{f}\right.}{f}\right.$$

$\downarrow$ 26

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2\int\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)}{f}\right)}{f}\right.$$

$\Big\downarrow$ 3042

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2\int-i\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}\right)}{f}\right)}{f}\right.$$

$\Big\downarrow$ 26

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\left(\frac{2i\int\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)}{f}\right)}{f}\right.$$

$\Big\downarrow$ 3118

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2}\right)}{f}\right)}{f}+\right.$$

input
```
Int[x^3*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

output
```
Sech[e/2 + (I/4)*Pi + (f*x)/2]*((2*x^3*Sinh[e/2 + (I/4)*Pi + (f*x)/2])/f +
 ((6*I)*(((2*I)*x^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/f - ((4*I)*((-4*Cosh[e
/2 + (I/4)*Pi + (f*x)/2])/f^2 + (2*x*Sinh[e/2 + (I/4)*Pi + (f*x)/2])/f))/f
))/f)*Sqrt[a + I*a*Sinh[e + f*x]]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.11

method	result
risch	$\frac{i\sqrt{2}\,\sqrt{a\left(i\mathrm{e}^{2fx+2e}-i+2\,\mathrm{e}^{fx+e}\right)\mathrm{e}^{-fx-e}}\,\left(ix^3f^3+f^3x^3\mathrm{e}^{fx+e}+6ix^2f^2-6f^2x^2\mathrm{e}^{fx+e}+24ixf+24fx\,\mathrm{e}^{fx+e}+48i-48\,\mathrm{e}^{fx+e}\right)\left(\mathrm{e}^{fx+e}-i\right)}{\left(i\mathrm{e}^{2fx+2e}-i+2\,\mathrm{e}^{fx+e}\right)f^4}$

input
```
int(x^3*(a+I*a*sinh(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I*2^(1/2)*(a*(I*exp(2*f*x+2*e)-I+2*exp(f*x+e))*exp(-f*x-e))^(1/2)/(I*exp(2
*f*x+2*e)-I+2*exp(f*x+e))*(I*x^3*f^3+f^3*x^3*exp(f*x+e)+6*I*x^2*f^2-6*f^2*
x^2*exp(f*x+e)+24*I*x*f+24*f*x*exp(f*x+e)+48*I-48*exp(f*x+e))*(exp(f*x+e)-
I)/f^4
```

Fricas [F(-2)]

Exception generated.

$$\int x^3\sqrt{a+ia\sinh(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^3\sqrt{a+ia\sinh(e+fx)}\,dx = \int x^3\sqrt{ia\left(\sinh\left(e+fx\right)-i\right)}\,dx$$

input
```
integrate(x**3*(a+I*a*sinh(f*x+e))**(1/2),x)
```

output
```
Integral(x**3*sqrt(I*a*(sinh(e + f*x) - I)), x)
```

Maxima [F]

$$\int x^3\sqrt{a+ia\sinh(e+fx)}\,dx = \int \sqrt{i\,a\sinh\left(fx+e\right)+a}x^3\,dx$$

input
```
integrate(x^3*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(I*a*sinh(f*x + e) + a)*x^3, x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.78

$$\int x^3\sqrt{a+ia\sinh(e+fx)}\,dx$$
$$= \frac{\left((i+1)\,\sqrt{a}f^3x^3e^{(fx+e)} + (i-1)\,\sqrt{a}f^3x^3 - (6i+6)\,\sqrt{a}f^2x^2e^{(fx+e)} + (6i-6)\,\sqrt{a}f^2x^2 + (24i+24)\,\sqrt{\ldots}\right.}{f^4}$$

input
```
integrate(x^3*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
((I + 1)*sqrt(a)*f^3*x^3*e^(f*x + e) + (I - 1)*sqrt(a)*f^3*x^3 - (6*I + 6)
*sqrt(a)*f^2*x^2*e^(f*x + e) + (6*I - 6)*sqrt(a)*f^2*x^2 + (24*I + 24)*sqr
t(a)*f*x*e^(f*x + e) + (24*I - 24)*sqrt(a)*f*x - (48*I + 48)*sqrt(a)*e^(f*
x + e) + (48*I - 48)*sqrt(a))*e^(-1/2*f*x - 1/2*e)/f^4
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.93

$$\int x^3\sqrt{a+ia\sinh(e+fx)}\,dx$$
$$=\frac{\sqrt{2}\left(\mathrm{e}^{e+f\,x}+1\mathrm{i}\right)\,\sqrt{a\,\mathrm{e}^{-e-f\,x}\,{\left(\mathrm{e}^{e+f\,x}-\mathrm{i}\right)}^2\,1\mathrm{i}}\,\left(f^3\,x^3\,\mathrm{e}^{e+f\,x}+f\,x\,24\mathrm{i}+f^2\,x^2\,6\mathrm{i}+f^3\,x^3\,1\mathrm{i}-6\,f^2\,x^2\,\mathrm{e}^{e+f\,x}\right.}{f^4\,\left(\mathrm{e}^{2\,e+2\,f\,x}+1\right)}$$

input
```
int(x^3*(a + a*sinh(e + f*x)*1i)^(1/2),x)
```

output
```
(2^(1/2)*(exp(e + f*x) + 1i)*(a*exp(- e - f*x)*(exp(e + f*x) - 1i)^2*1i)^(
1/2)*(f*x*24i - 48*exp(e + f*x) + f^2*x^2*6i + f^3*x^3*1i - 6*f^2*x^2*exp(
e + f*x) + f^3*x^3*exp(e + f*x) + 24*f*x*exp(e + f*x) + 48i))/(f^4*(exp(2*
e + 2*f*x) + 1))
```

Reduce [F]

$$\int x^3\sqrt{a+ia\sinh(e+fx)}\,dx=\sqrt{a}\left(\int\sqrt{\sinh\left(fx+e\right)i+1}\,x^3dx\right)$$

input
```
int(x^3*(a+I*a*sinh(f*x+e))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sinh(e + f*x)*i + 1)*x**3,x)
```

3.120 $\int x^2\sqrt{a+ia\sinh(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 111

$$\int x^2\sqrt{a+ia\sinh(e+fx)}\,dx=-\frac{8x\sqrt{a+ia\sinh(e+fx)}}{f^2}+\frac{16\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f^3}+\frac{2x^2\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f}$$

output
```
-8*x*(a+I*a*sinh(f*x+e))^(1/2)/f^2+16*(a+I*a*sinh(f*x+e))^(1/2)*tanh(1/2*e
+1/4*I*Pi+1/2*f*x)/f^3+2*x^2*(a+I*a*sinh(f*x+e))^(1/2)*tanh(1/2*e+1/4*I*Pi
+1/2*f*x)/f
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int x^2\sqrt{a+ia\sinh(e+fx)}\,dx=\frac{2\left(i(8+4ifx+f^2x^2)\cosh\left(\frac{1}{2}(e+fx)\right)+(8-4ifx+f^2x^2)\sinh\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a+ia\sinh(e+fx)}}{f^3\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}$$

input
```
Integrate[x^2*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

output
```
(2*(I*(8 + (4*I)*f*x + f^2*x^2)*Cosh[(e + f*x)/2] + (8 - (4*I)*f*x + f^2*x
^2)*Sinh[(e + f*x)/2])*Sqrt[a + I*a*Sinh[e + f*x]])/(f^3*(Cosh[(e + f*x)/2
] + I*Sinh[(e + f*x)/2]))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3800, 3042, 3777, 26, 3042, 26, 3777, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+ia\sinh(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2\sqrt{a+a\sin(ie+ifx)}dx$$

$$\Big\downarrow \text{3800}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int x^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int x^2\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\int -ix\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)$$

$$\Big\downarrow \text{26}$$

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4\int x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)$$

$\downarrow$ 3042

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4\int -ix\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}\right)$$

$\downarrow$ 26

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{4i\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)$$

$\downarrow$ 3777

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{4i\left(\frac{2ix\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2i\int\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)}{f}+\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}\right.}{f}\right.$$

$\downarrow$ 3042

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{4i\left(\frac{2ix\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2i\int\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx}{f}\right)}{f}+\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}\right.}{f}\right.$$

$\downarrow$ 3118

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{4i\left(\frac{2ix\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2}\right)}{f}+\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\right.}{f}\right.$$

input
```
Int[x^2*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

output
```
Sech[e/2 + (I/4)*Pi + (f*x)/2]*((2*x^2*Sinh[e/2 + (I/4)*Pi + (f*x)/2])/f +
 ((4*I)*(((2*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/f - ((4*I)*Sinh[e/2 + (I
/4)*Pi + (f*x)/2])/f^2))/f)*Sqrt[a + I*a*Sinh[e + f*x]]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.15

method	result	size
risch	$\frac{i\sqrt{2}\,\sqrt{a\left(i\mathrm{e}^{2fx+2e}-i+2\,\mathrm{e}^{fx+e}\right)\mathrm{e}^{-fx-e}}\,\left(ix^{2}f^{2}+f^{2}x^{2}\mathrm{e}^{fx+e}+4ixf-4fx\,\mathrm{e}^{fx+e}+8i+8\,\mathrm{e}^{fx+e}\right)\left(\mathrm{e}^{fx+e}-i\right)}{\left(i\mathrm{e}^{2fx+2e}-i+2\,\mathrm{e}^{fx+e}\right)f^{3}}$	128

input
```
int(x^2*(a+I*a*sinh(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I*2^(1/2)*(a*(I*exp(2*f*x+2*e)-I+2*exp(f*x+e))*exp(-f*x-e))^(1/2)/(I*exp(2
*f*x+2*e)-I+2*exp(f*x+e))*(I*x^2*f^2+f^2*x^2*exp(f*x+e)+4*I*x*f-4*f*x*exp(
f*x+e)+8*I+8*exp(f*x+e))*(exp(f*x+e)-I)/f^3
```

Fricas [F(-2)]

Exception generated.

$$\int x^2\sqrt{a+ia\sinh(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^2\sqrt{a+ia\sinh(e+fx)}\,dx = \int x^2\sqrt{ia\left(\sinh\left(e+fx\right)-i\right)}\,dx$$

input
```
integrate(x**2*(a+I*a*sinh(f*x+e))**(1/2),x)
```

output
```
Integral(x**2*sqrt(I*a*(sinh(e + f*x) - I)), x)
```

Maxima [F]

$$\int x^2\sqrt{a+ia\sinh(e+fx)}\,dx=\int\sqrt{i\,a\sinh\left(fx+e\right)+a}x^2\,dx$$

input
```
integrate(x^2*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(I*a*sinh(f*x + e) + a)*x^2, x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.70

$$\int x^2\sqrt{a+ia\sinh(e+fx)}\,dx$$
$$=\frac{\left((i+1)\ \sqrt{a}f^2x^2e^{(fx+e)}+(i-1)\ \sqrt{a}f^2x^2-(4i+4)\ \sqrt{a}fxe^{(fx+e)}+(4i-4)\ \sqrt{a}fx+(8i+8)\ \sqrt{a}e^{(fx+}\right.}{f^3}$$

input
```
integrate(x^2*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
((I + 1)*sqrt(a)*f^2*x^2*e^(f*x + e) + (I - 1)*sqrt(a)*f^2*x^2 - (4*I + 4)
*sqrt(a)*f*x*e^(f*x + e) + (4*I - 4)*sqrt(a)*f*x + (8*I + 8)*sqrt(a)*e^(f*
x + e) + (8*I - 8)*sqrt(a))*e^(-1/2*f*x - 1/2*e)/f^3
```

Mupad [B] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.83

$$\int x^2\sqrt{a+ia\sinh(e+fx)}\,dx$$
$$=\frac{\sqrt{2}\,\sqrt{a\,{\mathrm{e}}^{-e-f\,x}\,{\left({\mathrm{e}}^{e+f\,x}-\mathrm{i}\right)}^2}\,1\mathrm{i}\,\left(8\,{\mathrm{e}}^{e+f\,x}+f\,x\,4\mathrm{i}+f^2\,x^2\,1\mathrm{i}+f^2\,x^2\,{\mathrm{e}}^{e+f\,x}-4\,f\,x\,{\mathrm{e}}^{e+f\,x}+8\mathrm{i}\right)}{f^3\,\left({\mathrm{e}}^{e+f\,x}-\mathrm{i}\right)}$$

input
```
int(x^2*(a + a*sinh(e + f*x)*1i)^(1/2),x)
```

output
```
(2^(1/2)*(a*exp(- e - f*x)*(exp(e + f*x) - 1i)^2*1i)^(1/2)*(8*exp(e + f*x)
 + f*x*4i + f^2*x^2*1i + f^2*x^2*exp(e + f*x) - 4*f*x*exp(e + f*x) + 8i))/
(f^3*(exp(e + f*x) - 1i))
```

Reduce [F]

$$\int x^2\sqrt{a+ia\sinh(e+fx)}\,dx=\sqrt{a}\left(\int\sqrt{\sinh\left(fx+e\right)i+1}\,x^2dx\right)$$

input
```
int(x^2*(a+I*a*sinh(f*x+e))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sinh(e + f*x)*i + 1)*x**2,x)
```

3.121 $\int x\sqrt{a+ia\sinh(e+fx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 66

$$\int x\sqrt{a+ia\sinh(e+fx)}\,dx = -\frac{4\sqrt{a+ia\sinh(e+fx)}}{f^2} + \frac{2x\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f}$$

output
```
-4*(a+I*a*sinh(f*x+e))^(1/2)/f^2+2*x*(a+I*a*sinh(f*x+e))^(1/2)*tanh(1/2*e+
1/4*I*Pi+1/2*f*x)/f
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.32

$$\int x\sqrt{a+ia\sinh(e+fx)}\,dx = \frac{2\left((-2+ifx)\cosh\left(\frac{1}{2}(e+fx)\right)+(-2i+fx)\sinh\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a+ia\sinh(e+fx)}}{f^2\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}$$

input
```
Integrate[x*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

output
```
(2*((-2 + I*f*x)*Cosh[(e + f*x)/2] + (-2*I + f*x)*Sinh[(e + f*x)/2])*Sqrt[
a + I*a*Sinh[e + f*x]])/(f^2*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.36, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3800, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+ia\sinh(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\sqrt{a+a\sin(ie+ifx)}dx$$

$$\Big\downarrow \text{3800}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int x\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2i\int -i\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2\int \sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2\int -i\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}\right)$$

$$\Big\downarrow \; 26$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2i\int \sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)$$

$$\Big\downarrow \; 3118$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2}\right)$$

input
```
Int[x*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

output
```
Sech[e/2 + (I/4)*Pi + (f*x)/2]*((-4*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/f^2 +
(2*x*Sinh[e/2 + (I/4)*Pi + (f*x)/2])/f)*Sqrt[a + I*a*Sinh[e + f*x]]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.59

method	result	size
risch	$\frac{i\sqrt{2}\,\sqrt{a\left(i\mathrm{e}^{2fx+2e}-i+2\,\mathrm{e}^{fx+e}\right)\mathrm{e}^{-fx-e}}\,\left(ixf+fx\,\mathrm{e}^{fx+e}+2i-2\,\mathrm{e}^{fx+e}\right)\left(\mathrm{e}^{fx+e}-i\right)}{\left(i\mathrm{e}^{2fx+2e}-i+2\,\mathrm{e}^{fx+e}\right)f^{2}}$	105

input
```
int(x*(a+I*a*sinh(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I*2^(1/2)*(a*(I*exp(2*f*x+2*e)-I+2*exp(f*x+e))*exp(-f*x-e))^(1/2)/(I*exp(2
*f*x+2*e)-I+2*exp(f*x+e))*(I*f*x+f*x*exp(f*x+e)+2*I-2*exp(f*x+e))*(exp(f*x
+e)-I)/f^2
```

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{a+ia\sinh(e+fx)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x\sqrt{a+ia\sinh(e+fx)}\,dx=\int x\sqrt{ia\left(\sinh\left(e+fx\right)-i\right)}\,dx$$

input
```
integrate(x*(a+I*a*sinh(f*x+e))**(1/2),x)
```

output
```
Integral(x*sqrt(I*a*(sinh(e + f*x) - I)), x)
```

Maxima [F]

$$\int x\sqrt{a+ia\sinh(e+fx)}\,dx=\int\sqrt{i\,a\sinh\left(fx+e\right)+a}x\,dx$$

input
```
integrate(x*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(I*a*sinh(f*x + e) + a)*x, x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int x\sqrt{a+ia\sinh(e+fx)}\,dx$$
$$=\frac{\left((i+1)\sqrt{a}fxe^{(fx+e)}+(i-1)\sqrt{a}fx-(2i+2)\sqrt{a}e^{(fx+e)}+(2i-2)\sqrt{a}\right)e^{\left(-\frac{1}{2}fx-\frac{1}{2}e\right)}}{f^2}$$

input
```
integrate(x*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
((I + 1)*sqrt(a)*f*x*e^(f*x + e) + (I - 1)*sqrt(a)*f*x - (2*I + 2)*sqrt(a)
*e^(f*x + e) + (2*I - 2)*sqrt(a))*e^(-1/2*f*x - 1/2*e)/f^2
```

Mupad [B] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.21

$$\int x\sqrt{a+ia\sinh(e+fx)}\,dx$$
$$=\frac{\sqrt{2}\left(e^{e+fx}+1i\right)\left(f\,x\,e^{e+fx}+f\,x\,1i-2\,e^{e+fx}+2i\right)\sqrt{a\,e^{-e-fx}\left(e^{e+fx}-i\right)^2 1i}}{f^2\left(e^{2e+2fx}+1\right)}$$

input
```
int(x*(a + a*sinh(e + f*x)*1i)^(1/2),x)
```

output
```
(2^(1/2)*(exp(e + f*x) + 1i)*(f*x*1i - 2*exp(e + f*x) + f*x*exp(e + f*x) +
 2i)*(a*exp(- e - f*x)*(exp(e + f*x) - 1i)^2*1i)^(1/2))/(f^2*(exp(2*e + 2*
f*x) + 1))
```

Reduce [F]

$$\int x\sqrt{a+ia\sinh(e+fx)}\,dx=\sqrt{a}\left(\int\sqrt{\sinh(fx+e)\,i+1}\,xdx\right)$$

input
```
int(x*(a+I*a*sinh(f*x+e))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sinh(e + f*x)*i + 1)*x,x)
```

3.122 $\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 125

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x}\,dx = i\text{Chi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{1}{4}(2e-i\pi)\right)\sqrt{a+ia\sinh(e+fx)}+i\cosh\left(\frac{1}{4}(2e-i\pi)\right)\text{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\text{Shi}\left(\frac{fx}{2}\right)$$

output
```
Chi(1/2*f*x)*sech(1/2*e+1/4*I*Pi+1/2*f*x)*cosh(1/2*e+1/4*I*Pi)*(a+I*a*sinh
(f*x+e))^(1/2)+sinh(1/2*e+1/4*I*Pi)*sech(1/2*e+1/4*I*Pi+1/2*f*x)*(a+I*a*si
nh(f*x+e))^(1/2)*Shi(1/2*f*x)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x}\,dx = \frac{\sqrt{a+ia\sinh(e+fx)}\left(\text{Chi}\left(\frac{fx}{2}\right)\left(\cosh\left(\frac{e}{2}\right)+i\sinh\left(\frac{e}{2}\right)\right)+\left(i\cosh\left(\frac{e}{2}\right)+\sinh\left(\frac{e}{2}\right)\right)\text{Shi}\left(\frac{fx}{2}\right)\right)}{\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)}$$

input
```
Integrate[Sqrt[a + I*a*Sinh[e + f*x]]/x,x]
```

output
```
(Sqrt[a + I*a*Sinh[e + f*x]]*(CoshIntegral[(f*x)/2]*(Cosh[e/2] + I*Sinh[e/
2]) + (I*Cosh[e/2] + Sinh[e/2])*SinhIntegral[(f*x)/2]))/(Cosh[(e + f*x)/2]
 + I*Sinh[(e + f*x)/2])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.71, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3800, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+a\sin(ie+ifx)}}{x}dx$$

$$\Big\downarrow \text{3800}$$

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int \frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int\frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)}{x}dx$$

$$\Big\downarrow \text{3784}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(i\sinh\left(\frac{1}{4}(2e-i\pi)\right)\int\frac{\cosh\left(\frac{fx}{2}\right)}{x}dx+\cosh\left(\frac{1}{4}(2e-i\pi)\right)\int\frac{i\sinh}{}\right.$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(i\sinh\left(\frac{1}{4}(2e-i\pi)\right)\int\frac{\cosh\left(\frac{fx}{2}\right)}{x}dx+i\cosh\left(\frac{1}{4}(2e-i\pi)\right)\int\frac{\sinh}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(i\sinh\left(\frac{1}{4}(2e-i\pi)\right)\int\frac{\sin\left(\frac{ifx}{2}+\frac{\pi}{2}\right)}{x}dx+i\cosh\left(\frac{1}{4}(2e-i\pi)\right)\int\right.$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(i\sinh\left(\frac{1}{4}(2e-i\pi)\right)\int\frac{\sin\left(\frac{ifx}{2}+\frac{\pi}{2}\right)}{x}dx+\cosh\left(\frac{1}{4}(2e-i\pi)\right)\int\frac{\mathrm{s}}{}\right.$$

$$\Big\downarrow \text{3779}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(i\sinh\left(\frac{1}{4}(2e-i\pi)\right)\int\frac{\sin\left(\frac{ifx}{2}+\frac{\pi}{2}\right)}{x}dx+i\cosh\left(\frac{1}{4}(2e-i\pi)\right)\mathrm{Sh}\right.$$

$$\Big\downarrow \text{3782}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(i\sinh\left(\frac{1}{4}(2e-i\pi)\right)\mathrm{Chi}\left(\frac{fx}{2}\right)+i\cosh\left(\frac{1}{4}(2e-i\pi)\right)\mathrm{Shi}\left(\frac{fx}{2}\right)\right)$$

input
```
Int[Sqrt[a + I*a*Sinh[e + f*x]]/x,x]
```

output
```
Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]]*(I*CoshIntegral
[(f*x)/2]*Sinh[(2*e - I*Pi)/4] + I*Cosh[(2*e - I*Pi)/4]*SinhIntegral[(f*x)
/2])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\sqrt{a + ia \sinh(fx + e)}}{x} dx$$

input
```
int((a+I*a*sinh(f*x+e))^(1/2)/x,x)
```

output
```
int((a+I*a*sinh(f*x+e))^(1/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a + ia \sinh(e + fx)}}{x} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*sinh(f*x+e))^(1/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{a + ia \sinh(e + fx)}}{x} dx = \int \frac{\sqrt{ia (\sinh(e + fx) - i)}}{x} dx$$

input
```
integrate((a+I*a*sinh(f*x+e))**(1/2)/x,x)
```

output
```
Integral(sqrt(I*a*(sinh(e + f*x) - I))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x}\,dx=\int \frac{\sqrt{i\,a\sinh\left(fx+e\right)+a}}{x}\,dx$$

input
```
integrate((a+I*a*sinh(f*x+e))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(I*a*sinh(f*x + e) + a)/x, x)
```

Giac [F]

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x}\,dx=\int \frac{\sqrt{i\,a\sinh\left(fx+e\right)+a}}{x}\,dx$$

input
```
integrate((a+I*a*sinh(f*x+e))^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(I*a*sinh(f*x + e) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x}\,dx=\int \frac{\sqrt{a+a\sinh\left(e+f\,x\right)\,1{}\mathrm{i}}}{x}\,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)^(1/2)/x,x)
```

output
```
int((a + a*sinh(e + f*x)*1i)^(1/2)/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{a + ia\sinh(e + fx)}}{x}\, dx = \sqrt{a} \left(\int \frac{\sqrt{\sinh (fx + e)\, i + 1}}{x} dx \right)$$

input
```
int((a+I*a*sinh(f*x+e))^(1/2)/x,x)
```

output
```
sqrt(a)*int(sqrt(sinh(e + f*x)*i + 1)/x,x)
```

3.123 $\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 149

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^2}\,dx = -\frac{\sqrt{a+ia\sinh(e+fx)}}{x}+\frac{1}{2}f\text{Chi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{1}{4}(2e+i\pi)\right)\sqrt{a+ia\sinh(e+fx)}+\frac{1}{2}f\cosh\left(\frac{1}{4}(2e+i\pi)\right)\text{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\text{Shi}\left(\frac{fx}{2}\right)$$

output
```
-(a+I*a*sinh(f*x+e))^(1/2)/x+1/2*f*Chi(1/2*f*x)*sech(1/2*e+1/4*I*Pi+1/2*f*
x)*sinh(1/2*e+1/4*I*Pi)*(a+I*a*sinh(f*x+e))^(1/2)+1/2*f*cosh(1/2*e+1/4*I*P
i)*sech(1/2*e+1/4*I*Pi+1/2*f*x)*(a+I*a*sinh(f*x+e))^(1/2)*Shi(1/2*f*x)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^2}\,dx$$
$$= \frac{\sqrt{a+ia\sinh(e+fx)}\left(fx\text{Chi}\left(\frac{fx}{2}\right)\left(i\cosh\left(\frac{e}{2}\right)+\sinh\left(\frac{e}{2}\right)\right)-2\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)+}{2x\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}$$

input
```
Integrate[Sqrt[a + I*a*Sinh[e + f*x]]/x^2,x]
```

output
```
(Sqrt[a + I*a*Sinh[e + f*x]]*(f*x*CoshIntegral[(f*x)/2]*(I*Cosh[e/2] + Sin
h[e/2]) - 2*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]) + f*x*(Cosh[e/2] + I
*Sinh[e/2])*SinhIntegral[(f*x)/2]))/(2*x*(Cosh[(e + f*x)/2] + I*Sinh[(e +
f*x)/2]))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.82, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3800, 3042, 3778, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{a+a\sin(ie+ifx)}}{x^2}dx$$
$$\Big\downarrow \text{3800}$$
$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int \frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x^2}dx$$
$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int\frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)}{x^2}dx$$

$$\Big\downarrow \text{3778}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{1}{2}if\int-\frac{i\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x}dx-\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x}\right)$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{1}{2}f\int\frac{\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x}dx-\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{1}{2}f\int-\frac{i\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)}{x}dx-\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x}\right)$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{2}if\int\frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)}{x}dx-\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x}\right)$$

$$\Big\downarrow \text{3784}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{2}if\left(i\sinh\left(\frac{1}{4}(2e+i\pi)\right)\int\frac{\cosh\left(\frac{fx}{2}\right)}{x}dx+\cosh\left(\frac{1}{4}(2e+i\pi)\right)\right.\right.$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{2}if\left(i\sinh\left(\frac{1}{4}(2e+i\pi)\right)\int\frac{\cosh\left(\frac{fx}{2}\right)}{x}dx+i\cosh\left(\frac{1}{4}(2e+i\pi)\right)\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{2}if\left(i\sinh\left(\frac{1}{4}(2e+i\pi)\right)\int\frac{\sin\left(\frac{ifx}{2}+\frac{\pi}{2}\right)}{x}dx+i\cosh\left(\frac{1}{4}(2e+\right.\right.\right.$$

$\downarrow$ 26

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{2}if\left(i\sinh\left(\frac{1}{4}(2e+i\pi)\right)\int\frac{\sin\left(\frac{ifx}{2}+\frac{\pi}{2}\right)}{x}dx+\cosh\left(\frac{1}{4}(2e+i\right.\right.\right.$$

$\downarrow$ 3779

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{2}if\left(i\sinh\left(\frac{1}{4}(2e+i\pi)\right)\int\frac{\sin\left(\frac{ifx}{2}+\frac{\pi}{2}\right)}{x}dx+i\cosh\left(\frac{1}{4}(2e+\right.\right.\right.$$

$\downarrow$ 3782

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{2}if\left(i\sinh\left(\frac{1}{4}(2e+i\pi)\right)\text{Chi}\left(\frac{fx}{2}\right)+i\cosh\left(\frac{1}{4}(2e+i\pi)\right)\text{Shi}\left(\right.\right.\right.$$

input
```
Int[Sqrt[a + I*a*Sinh[e + f*x]]/x^2,x]
```

output
```
Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]]*(-(Cosh[e/2 + (
I/4)*Pi + (f*x)/2]/x) - (I/2)*f*(I*CoshIntegral[(f*x)/2]*Sinh[(2*e + I*Pi)
/4] + I*Cosh[(2*e + I*Pi)/4]*SinhIntegral[(f*x)/2]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\sqrt{a + ia \sinh (fx + e)}}{x^2} dx$$

input
```
int((a+I*a*sinh(f*x+e))^(1/2)/x^2,x)
```

output
```
int((a+I*a*sinh(f*x+e))^(1/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*sinh(f*x+e))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^2}\,dx = \int \frac{\sqrt{ia\left(\sinh\left(e+fx\right)-i\right)}}{x^2}\,dx$$

input
```
integrate((a+I*a*sinh(f*x+e))**(1/2)/x**2,x)
```

output
```
Integral(sqrt(I*a*(sinh(e + f*x) - I))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^2}\,dx = \int \frac{\sqrt{i\,a\sinh\left(fx+e\right)+a}}{x^2}\,dx$$

input
```
integrate((a+I*a*sinh(f*x+e))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(I*a*sinh(f*x + e) + a)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^2}\,dx=\int \frac{\sqrt{i\,a\sinh\left(fx+e\right)+a}}{x^2}\,dx$$

input
```
integrate((a+I*a*sinh(f*x+e))^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(I*a*sinh(f*x + e) + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^2}\,dx=\int \frac{\sqrt{a+a\sinh\left(e+f\,x\right)\,1{}\mathrm{i}}}{x^2}\,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)^(1/2)/x^2,x)
```

output
```
int((a + a*sinh(e + f*x)*1i)^(1/2)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^2}\,dx=\sqrt{a}\left(\int \frac{\sqrt{\sinh\left(fx+e\right)i+1}}{x^2}dx\right)$$

input
```
int((a+I*a*sinh(f*x+e))^(1/2)/x^2,x)
```

output
```
sqrt(a)*int(sqrt(sinh(e + f*x)*i + 1)/x**2,x)
```

3.124 $\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 204

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^3}\,dx = -\frac{\sqrt{a+ia\sinh(e+fx)}}{2x^2} + \frac{1}{8}if^2\text{Chi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{1}{4}(2e-i\pi)\right)\sqrt{a+ia\sinh(e+fx)} + \frac{1}{8}if^2\cosh\left(\frac{1}{4}(2e-i\pi)\right)\text{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\text{Shi}\left(\frac{fx}{2}\right) - \frac{f\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{4x}$$

output
```
-1/2*(a+I*a*sinh(f*x+e))^(1/2)/x^2+1/8*f^2*Chi(1/2*f*x)*sech(1/2*e+1/4*I*P
i+1/2*f*x)*cosh(1/2*e+1/4*I*Pi)*(a+I*a*sinh(f*x+e))^(1/2)+1/8*f^2*sinh(1/2
*e+1/4*I*Pi)*sech(1/2*e+1/4*I*Pi+1/2*f*x)*(a+I*a*sinh(f*x+e))^(1/2)*Shi(1/
2*f*x)-1/4*f*(a+I*a*sinh(f*x+e))^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/x
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^3}\,dx$$
$$= \frac{\sqrt{a+ia\sinh(e+fx)}\left(-4\cosh\left(\frac{1}{2}(e+fx)\right)-2ifx\cosh\left(\frac{1}{2}(e+fx)\right)+f^2x^2\mathrm{Chi}\left(\frac{fx}{2}\right)\left(\cosh\left(\frac{e}{2}\right)+i\sinh\right.\right.}{8x^2\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sin\right.}$$

input
```
Integrate[Sqrt[a + I*a*Sinh[e + f*x]]/x^3,x]
```

output
```
(Sqrt[a + I*a*Sinh[e + f*x]]*(-4*Cosh[(e + f*x)/2] - (2*I)*f*x*Cosh[(e + f
*x)/2] + f^2*x^2*CoshIntegral[(f*x)/2]*(Cosh[e/2] + I*Sinh[e/2]) - (4*I)*S
inh[(e + f*x)/2] - 2*f*x*Sinh[(e + f*x)/2] + f^2*x^2*(I*Cosh[e/2] + Sinh[e
/2])*SinhIntegral[(f*x)/2]))/(8*x^2*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/
2]))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.78, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 3800, 3042, 3778, 26, 3042, 26, 3778, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sqrt{a+a\sin(ie+ifx)}}{x^3}dx$$
$$\Big\downarrow \; 3800$$
$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int \frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int \frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)}{x^3}dx$$

$$\Big\downarrow \text{3778}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{1}{4}if\int -\frac{i\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x^2}dx-\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{2x^2}\right)$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{1}{4}f\int \frac{\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x^2}dx-\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{2x^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{1}{4}f\int -\frac{i\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)}{x^2}dx-\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{2x^2}\right)$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{4}if\int \frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)}{x^2}dx-\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{2x^2}\right)$$

$$\Big\downarrow \text{3778}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{4}if\left(\frac{1}{2}if\int \frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x}dx-\frac{i\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x}\right)-\right.$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{4}if\left(\frac{1}{2}if\int \frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)}{x}dx-\frac{i\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x}\right)-\right.$$

$$\Big\downarrow \text{3784}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{4}if\left(\frac{1}{2}if\left(i\sinh\left(\frac{1}{4}(2e-i\pi)\right)\int\frac{\cosh\left(\frac{fx}{2}\right)}{x}dx+\cosh\left(\frac{1}{4}(2e-\right.\right.\right.\right.$$

$\downarrow$ 26

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{4}if\left(\frac{1}{2}if\left(i\sinh\left(\frac{1}{4}(2e-i\pi)\right)\int\frac{\cosh\left(\frac{fx}{2}\right)}{x}dx+i\cosh\left(\frac{1}{4}(2e\right.\right.\right.\right.$$

$\downarrow$ 3042

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{4}if\left(\frac{1}{2}if\left(i\sinh\left(\frac{1}{4}(2e-i\pi)\right)\int\frac{\sin\left(\frac{ifx}{2}+\frac{\pi}{2}\right)}{x}dx+i\cosh\left(\frac{1}{4}\right.\right.\right.\right.$$

$\downarrow$ 26

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{4}if\left(\frac{1}{2}if\left(i\sinh\left(\frac{1}{4}(2e-i\pi)\right)\int\frac{\sin\left(\frac{ifx}{2}+\frac{\pi}{2}\right)}{x}dx+\cosh\left(\frac{1}{4}(2\right.\right.\right.\right.$$

$\downarrow$ 3779

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{4}if\left(\frac{1}{2}if\left(i\sinh\left(\frac{1}{4}(2e-i\pi)\right)\int\frac{\sin\left(\frac{ifx}{2}+\frac{\pi}{2}\right)}{x}dx+i\cosh\left(\frac{1}{4}\right.\right.\right.\right.$$

$\downarrow$ 3782

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{4}if\left(\frac{1}{2}if\left(i\sinh\left(\frac{1}{4}(2e-i\pi)\right)\operatorname{Chi}\left(\frac{fx}{2}\right)+i\cosh\left(\frac{1}{4}(2e-i\pi)\right)\right.\right.\right.$$

input
```
Int[Sqrt[a + I*a*Sinh[e + f*x]]/x^3,x]
```

output
```
Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]]*(-1/2*Cosh[e/2
+ (I/4)*Pi + (f*x)/2]/x^2 - (I/4)*f*(((-I)*Sinh[e/2 + (I/4)*Pi + (f*x)/2])
/x + (I/2)*f*(I*CoshIntegral[(f*x)/2]*Sinh[(2*e - I*Pi)/4] + I*Cosh[(2*e -
 I*Pi)/4]*SinhIntegral[(f*x)/2])))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\sqrt{a + ia \sinh{(fx + e)}}}{x^3} dx$$

input
```
int((a+I*a*sinh(f*x+e))^(1/2)/x^3,x)
```

output
```
int((a+I*a*sinh(f*x+e))^(1/2)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a + ia \sinh(e + fx)}}{x^3} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*sinh(f*x+e))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{a + ia \sinh(e + fx)}}{x^3} dx = \int \frac{\sqrt{ia (\sinh{(e + fx)} - i)}}{x^3} dx$$

input
```
integrate((a+I*a*sinh(f*x+e))**(1/2)/x**3,x)
```

output
```
Integral(sqrt(I*a*(sinh(e + f*x) - I))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{a + ia\sinh(e + fx)}}{x^3}\,dx = \int \frac{\sqrt{i\,a\sinh\left(fx + e\right) + a}}{x^3}\,dx$$

input
```
integrate((a+I*a*sinh(f*x+e))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(I*a*sinh(f*x + e) + a)/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt{a + ia\sinh(e + fx)}}{x^3}\,dx = \int \frac{\sqrt{i\,a\sinh\left(fx + e\right) + a}}{x^3}\,dx$$

input
```
integrate((a+I*a*sinh(f*x+e))^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(I*a*sinh(f*x + e) + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + ia\sinh(e + fx)}}{x^3}\,dx = \int \frac{\sqrt{a + a\sinh\left(e + f\,x\right)\,1\mathrm{i}}}{x^3}\,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)^(1/2)/x^3,x)
```

output
```
int((a + a*sinh(e + f*x)*1i)^(1/2)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{a + ia\sinh(e + fx)}}{x^3}\,dx = \sqrt{a}\left(\int \frac{\sqrt{\sinh(fx + e)\,i + 1}}{x^3}dx\right)$$

input
```
int((a+I*a*sinh(f*x+e))^(1/2)/x^3,x)
```

output
```
sqrt(a)*int(sqrt(sinh(e + f*x)*i + 1)/x**3,x)
```

3.125 $\int x^3(a+ia\sinh(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 377

$$\begin{aligned}\int x^3(a+ia\sinh(e+fx))^{3/2}\,dx =&-\frac{1280a\sqrt{a+ia\sinh(e+fx)}}{9f^4}-\frac{16ax^2\sqrt{a+ia\sinh(e+fx)}}{f^2}\\&-\frac{64a\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{27f^4}\\&-\frac{8ax^2\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{3f^2}\\&+\frac{32ax\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{9f^3}\\&+\frac{4ax^3\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{3f}\\&+\frac{640ax\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{9f^3}\\&+\frac{8ax^3\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3f}\end{aligned}$$

output
```
-1280/9*a*(a+I*a*sinh(f*x+e))^(1/2)/f^4-16*a*x^2*(a+I*a*sinh(f*x+e))^(1/2)
/f^2-64/27*a*cosh(1/2*e+1/4*I*Pi+1/2*f*x)^2*(a+I*a*sinh(f*x+e))^(1/2)/f^4-
8/3*a*x^2*cosh(1/2*e+1/4*I*Pi+1/2*f*x)^2*(a+I*a*sinh(f*x+e))^(1/2)/f^2+32/
9*a*x*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*sinh(1/2*e+1/4*I*Pi+1/2*f*x)*(a+I*a*sin
h(f*x+e))^(1/2)/f^3+4/3*a*x^3*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*sinh(1/2*e+1/4*
I*Pi+1/2*f*x)*(a+I*a*sinh(f*x+e))^(1/2)/f+640/9*a*x*(a+I*a*sinh(f*x+e))^(1
/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/f^3+8/3*a*x^3*(a+I*a*sinh(f*x+e))^(1/2)*t
anh(1/2*e+1/4*I*Pi+1/2*f*x)/f
```

Mathematica [A] (verified)

Time = 7.56 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.71

$$\int x^3(a+ia\sinh(e+fx))^{3/2}\,dx =$$

$$-\frac{a(-i+\sinh(e+fx))\sqrt{a+ia\sinh(e+fx)}\big(81(48i+24fx+6if^2x^2+f^3x^3)\cosh\left(\frac{1}{2}(e+fx)\right)+(-16i$$

input
```
Integrate[x^3*(a + I*a*Sinh[e + f*x])^(3/2),x]
```

output
```
-1/27*(a*(-I + Sinh[e + f*x])*Sqrt[a + I*a*Sinh[e + f*x]]*(81*(48*I + 24*f
*x + (6*I)*f^2*x^2 + f^3*x^3)*Cosh[(e + f*x)/2] + (-16*I + 24*f*x - (18*I)
*f^2*x^2 + 9*f^3*x^3)*Cosh[(3*(e + f*x))/2] - 3888*Sinh[(e + f*x)/2] - (19
44*I)*f*x*Sinh[(e + f*x)/2] - 486*f^2*x^2*Sinh[(e + f*x)/2] - (81*I)*f^3*x
^3*Sinh[(e + f*x)/2] - 16*Sinh[(3*(e + f*x))/2] + (24*I)*f*x*Sinh[(3*(e +
f*x))/2] - 18*f^2*x^2*Sinh[(3*(e + f*x))/2] + (9*I)*f^3*x^3*Sinh[(3*(e + f
*x))/2]))/(f^4*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])^3)
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.06, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 1.095$, Rules used = {3042, 3800, 3042, 3792, 3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118, 3791, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+ia\sinh(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int x^3(a+a\sin(ie+ifx))^{3/2}dx$$

$$\Big\downarrow \text{ 3800}$$

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int x^3\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx$$

$$\Big\downarrow \text{ 3042}$$

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int x^3\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx$$

$$\Big\downarrow \text{ 3792}$$

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int x\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{3f^2}+\frac{2}{3}\int x^3\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx$$

$$\Big\downarrow \text{ 3042}$$

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\int x^3\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx-$$

$$\Big\downarrow \text{ 3777}$$

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\left(\frac{2x^3\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6i}{}\right.\right.$$

$$\Big\downarrow \text{ 26}$$

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\left(\frac{2x^3\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6\int}{}\right.\right.$$

$$\Big\downarrow \text{ 3042}$$

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\left(\frac{2x^3\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6\int\right.\right.$$

$\downarrow$ 26

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\left(\frac{6i\int x^2\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}\right.\right.$$

$\downarrow$ 3777

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\int}{}\right.}{f}\right.\right.$$

$\downarrow$ 3042

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\int}{}\right.}{f}\right.\right.$$

$\downarrow$ 3777

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\left(\right.}{}\right.}{}\right.\right.$$

$\downarrow$ 26

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\left(\right.}{}\right.}{}\right.\right.$$

$\downarrow$ 3042

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\left(\right.}{}\right.}{}\right.\right.$$

$\downarrow$ 26

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\left(\right.}{}\right.}{}\right.\right.$$

$\downarrow$ 3118

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}-\frac{4x^2\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}+\frac{2}{3}\left(\frac{6}{-}\right.\right.$$

$\downarrow$ 3791

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\left(\frac{2}{3}\int x\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx-\frac{4\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{9f^2}+\frac{2x\sinh\left(\frac{e}{2}+\right.}{}\right.}{3f^2}\right.$$

↓ 3042

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\left(\frac{2}{3}\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx-\frac{4\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{9f^2}+\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}\right.}{}\right.}{3f^2}\right.$$

↓ 3777

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\left(\frac{2}{3}\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2i\int-i\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)-\frac{4\cosh^3\left(\frac{e}{2}+\frac{fx}{2}\right.}{9f^2}\right.}{3f^2}\right.$$

↓ 26

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\left(\frac{2}{3}\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2\int\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)-\frac{4\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\right.}{9f^2}\right.}{3f^2}\right.$$

↓ 3042

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\left(\frac{2}{3}\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2\int -i\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}\right)-\frac{4\cosh^3\left(\frac{e}{2}+\frac{fx}{2}\right.}{9f^2}\right.}{3f^2}\right.$$

$\downarrow$ 26

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\left(\frac{2}{3}\left(\frac{2i\int\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)-\frac{4\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\right.}{9f^2}\right.}{3f^2}\right.$$

$\downarrow$ 3118

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{4x^2\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}+\frac{2}{3}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\left(\frac{2x}{}\right.}{}\right.}{}\right.\right.$$

input
```
Int[x^3*(a + I*a*Sinh[e + f*x])^(3/2),x]
```

output
```
2*a*Sech[e/2 + (I/4)*Pi + (f*x)/2]*((-4*x^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]
^3)/(3*f^2) + (2*x^3*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sinh[e/2 + (I/4)*Pi
+ (f*x)/2])/(3*f) + (8*((-4*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^3)/(9*f^2) + (2
*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sinh[e/2 + (I/4)*Pi + (f*x)/2])/(3*f)
+ (2*((-4*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/f^2 + (2*x*Sinh[e/2 + (I/4)*Pi +
 (f*x)/2])/f))/3))/(3*f^2) + (2*((2*x^3*Sinh[e/2 + (I/4)*Pi + (f*x)/2])/f
+ ((6*I)*(((2*I)*x^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/f - ((4*I)*((-4*Cosh[
e/2 + (I/4)*Pi + (f*x)/2])/f^2 + (2*x*Sinh[e/2 + (I/4)*Pi + (f*x)/2])/f))/
f))/f))/3)*Sqrt[a + I*a*Sinh[e + f*x]]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int x^3(a + ia\sinh(fx + e))^{\frac{3}{2}}\, dx$$

input
```
int(x^3*(a+I*a*sinh(f*x+e))^(3/2),x)
```

output
```
int(x^3*(a+I*a*sinh(f*x+e))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^3(a + ia\sinh(e + fx))^{3/2}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int x^3(a + ia\sinh(e + fx))^{3/2}\, dx = \text{Timed out}$$

input
```
integrate(x**3*(a+I*a*sinh(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^3(a+ia\sinh(e+fx))^{3/2}\,dx=\int (i\,a\sinh\left(fx+e\right)+a)^{\frac{3}{2}}x^3\,dx$$

input
```
integrate(x^3*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((I*a*sinh(f*x + e) + a)^(3/2)*x^3, x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.66

$$\int x^3(a+ia\sinh(e+fx))^{3/2}\,dx=$$
$$-\frac{\left(-(9i-9)\ a^{\frac{3}{2}}f^3x^3e^{(3\,fx+3\,e)}-(81i+81)\ a^{\frac{3}{2}}f^3x^3e^{(2\,fx+2\,e)}-(81i-81)\ a^{\frac{3}{2}}f^3x^3e^{(fx+e)}-(9i+9)\ a^{\frac{3}{2}}f^3x\right.}{}$$

input
```
integrate(x^3*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
-1/54*(-(9*I - 9)*a^(3/2)*f^3*x^3*e^(3*f*x + 3*e) - (81*I + 81)*a^(3/2)*f^
3*x^3*e^(2*f*x + 2*e) - (81*I - 81)*a^(3/2)*f^3*x^3*e^(f*x + e) - (9*I + 9
)*a^(3/2)*f^3*x^3 + (18*I - 18)*a^(3/2)*f^2*x^2*e^(3*f*x + 3*e) + (486*I +
 486)*a^(3/2)*f^2*x^2*e^(2*f*x + 2*e) - (486*I - 486)*a^(3/2)*f^2*x^2*e^(f
*x + e) - (18*I + 18)*a^(3/2)*f^2*x^2 - (24*I - 24)*a^(3/2)*f*x*e^(3*f*x +
 3*e) - (1944*I + 1944)*a^(3/2)*f*x*e^(2*f*x + 2*e) - (1944*I - 1944)*a^(3
/2)*f*x*e^(f*x + e) - (24*I + 24)*a^(3/2)*f*x + (16*I - 16)*a^(3/2)*e^(3*f
*x + 3*e) + (3888*I + 3888)*a^(3/2)*e^(2*f*x + 2*e) - (3888*I - 3888)*a^(3
/2)*e^(f*x + e) - (16*I + 16)*a^(3/2))*e^(-3/2*f*x - 3/2*e)/f^4
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a + ia\sinh(e + fx))^{3/2}\,dx = \int x^3\,(a + a\sinh(e + f\,x)\;1\mathrm{i})^{3/2}\,dx$$

input
```
int(x^3*(a + a*sinh(e + f*x)*1i)^(3/2),x)
```

output
```
int(x^3*(a + a*sinh(e + f*x)*1i)^(3/2), x)
```

Reduce [F]

$$\int x^3(a + ia\sinh(e + fx))^{3/2}\,dx = \sqrt{a}\,a\left(\int\sqrt{\sinh(fx + e)\,i + 1}\,x^3dx + \left(\int\sqrt{\sinh(fx + e)\,i + 1}\,\sinh(fx + e)\,x^3dx\right)i\right)$$

input
```
int(x^3*(a+I*a*sinh(f*x+e))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(sinh(e + f*x)*i + 1)*x**3,x) + int(sqrt(sinh(e + f*x)*
i + 1)*sinh(e + f*x)*x**3,x)*i)
```

3.126 $\int x^2(a+ia\sinh(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 303

$$\begin{aligned}\int x^2(a+ia\sinh(e+fx))^{3/2}\,dx &= -\frac{32ax\sqrt{a+ia\sinh(e+fx)}}{3f^2}\\
&-\frac{16ax\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{9f^2}\\
&+\frac{4ax^2\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{3f}\\
&+\frac{224a\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{9f^3}\\
&+\frac{8ax^2\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3f}\\
&+\frac{32a\sinh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{27f^3}\end{aligned}$$

output
```
-32/3*a*x*(a+I*a*sinh(f*x+e))^(1/2)/f^2-16/9*a*x*cosh(1/2*e+1/4*I*Pi+1/2*f
*x)^2*(a+I*a*sinh(f*x+e))^(1/2)/f^2+4/3*a*x^2*cosh(1/2*e+1/4*I*Pi+1/2*f*x)
*sinh(1/2*e+1/4*I*Pi+1/2*f*x)*(a+I*a*sinh(f*x+e))^(1/2)/f+224/9*a*(a+I*a*s
inh(f*x+e))^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/f^3+8/3*a*x^2*(a+I*a*sinh(f
*x+e))^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/f+32/27*a*sinh(1/2*e+1/4*I*Pi+1/
2*f*x)^2*(a+I*a*sinh(f*x+e))^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/f^3
```

Mathematica [A] (verified)

Time = 7.37 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.57

$$\int x^2(a+ia\sinh(e+fx))^{3/2}\,dx =$$
$$-\frac{a\left(81(8+4ifx+f^2x^2)\cosh\left(\frac{1}{2}(e+fx)\right)+(8-12ifx+9f^2x^2)\cosh\left(\frac{3}{2}(e+fx)\right)+2i(-4(80-42ifx+\ldots\right.}{27f^3\left(\cosh\left(\frac{1}{2}(e+fx\ldots\right.\right.}$$

input
```
Integrate[x^2*(a + I*a*Sinh[e + f*x])^(3/2),x]
```

output
```
-1/27*(a*(81*(8 + (4*I)*f*x + f^2*x^2)*Cosh[(e + f*x)/2] + (8 - (12*I)*f*x
 + 9*f^2*x^2)*Cosh[(3*(e + f*x))/2] + (2*I)*(-4*(80 - (42*I)*f*x + 9*f^2*x
^2) + (8 + (12*I)*f*x + 9*f^2*x^2)*Cosh[e + f*x])*Sinh[(e + f*x)/2])*(-I +
 Sinh[e + f*x])*Sqrt[a + I*a*Sinh[e + f*x]])/(f^3*(Cosh[(e + f*x)/2] + I*S
inh[(e + f*x)/2])^3)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.93, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3800, 3042, 3792, 3042, 3113, 2009, 3777, 26, 3042, 26, 3777, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+ia\sinh(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2(a+a\sin(ie+ifx))^{3/2}dx$$

$$\Big\downarrow \text{3800}$$

$$2a\,\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int x^2\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int x^2\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx$$

$\big\downarrow$ 3792

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{9f^2}+\frac{2}{3}\int x^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx-\right.$$

$\big\downarrow$ 3042

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{9f^2}+\frac{2}{3}\int x^2\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx-\right.$$

$\big\downarrow$ 3113

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{16i\int\left(\sinh^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)+1\right)d\left(-i\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)}{9f^3}+\frac{2}{3}\right.$$

$\big\downarrow$ 2009

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2}{3}\int x^2\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx+\frac{16i\left(-\frac{1}{3}i\sinh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right.}{9f^3}\right.$$

$\big\downarrow$ 3777

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2}{3}\left(\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\int-ix\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)\right.$$

$\big\downarrow$ 26

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2}{3}\left(\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4\int x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)+\right.$$

$\big\downarrow$ 3042

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2}{3}\left(\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4\int -ix\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}\right)+\right.$$

$\big\downarrow$ 26

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2}{3}\left(\frac{4i\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)+\right.$$

$\big\downarrow$ 3777

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2}{3}\left(\frac{4i\left(\frac{2ix\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2i\int\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)}{f}+\frac{2x^2\sinh\left(\frac{e}{2}\right.}{}\right.\right.$$

$\big\downarrow$ 3042

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2}{3}\left(\frac{4i\left(\frac{2ix\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2i\int\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx}{f}\right)}{f}+\frac{2x^2\sinh\left(\frac{e}{2}\right.}{}\right.\right.$$

$\big\downarrow$ 3118

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{16i\left(-\frac{1}{3}i\sinh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)-i\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)}{9f^3}+\frac{2}{3}\left(\frac{4i}{}\right.\right.$$

input
```
Int[x^2*(a + I*a*Sinh[e + f*x])^(3/2),x]
```

output
```
2*a*Sech[e/2 + (I/4)*Pi + (f*x)/2]*((-8*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^3
)/(9*f^2) + (2*x^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sinh[e/2 + (I/4)*Pi +
(f*x)/2])/(3*f) + (((16*I)/9)*((-I)*Sinh[e/2 + (I/4)*Pi + (f*x)/2] - (I/3)
*Sinh[e/2 + (I/4)*Pi + (f*x)/2]^3))/f^3 + (2*((2*x^2*Sinh[e/2 + (I/4)*Pi +
 (f*x)/2])/f + ((4*I)*(((2*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/f - ((4*I)
*Sinh[e/2 + (I/4)*Pi + (f*x)/2])/f^2))/f))/3)*Sqrt[a + I*a*Sinh[e + f*x]]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int x^2(a+ia\sinh(fx+e))^{\frac{3}{2}}\,dx$$

input
```
int(x^2*(a+I*a*sinh(f*x+e))^(3/2),x)
```

output
```
int(x^2*(a+I*a*sinh(f*x+e))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2(a+ia\sinh(e+fx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^2(a+ia\sinh(e+fx))^{3/2}\,dx=\int x^2(ia(\sinh{(e+fx)}-i))^{\frac{3}{2}}\,dx$$

input
```
integrate(x**2*(a+I*a*sinh(f*x+e))**(3/2),x)
```

output
```
Integral(x**2*(I*a*(sinh(e + f*x) - I))**(3/2), x)
```

Maxima [F]

$$\int x^2(a+ia\sinh(e+fx))^{3/2}\,dx=\int(i\,a\sinh{(fx+e)}+a)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((I*a*sinh(f*x + e) + a)^(3/2)*x^2, x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.59

$$\int x^2(a+ia\sinh(e+fx))^{3/2}\,dx=$$
$$-\frac{\left(-(9i-9)\,a^{\frac{3}{2}}f^2x^2e^{(3\,fx+3\,e)}-(81i+81)\,a^{\frac{3}{2}}f^2x^2e^{(2\,fx+2\,e)}-(81i-81)\,a^{\frac{3}{2}}f^2x^2e^{(fx+e)}-(9i+9)\,a^{\frac{3}{2}}f^2x\right.}{}$$

input
```
integrate(x^2*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
-1/54*(-(9*I - 9)*a^(3/2)*f^2*x^2*e^(3*f*x + 3*e) - (81*I + 81)*a^(3/2)*f^
2*x^2*e^(2*f*x + 2*e) - (81*I - 81)*a^(3/2)*f^2*x^2*e^(f*x + e) - (9*I + 9
)*a^(3/2)*f^2*x^2 + (12*I - 12)*a^(3/2)*f*x*e^(3*f*x + 3*e) + (324*I + 324
)*a^(3/2)*f*x*e^(2*f*x + 2*e) - (324*I - 324)*a^(3/2)*f*x*e^(f*x + e) - (1
2*I + 12)*a^(3/2)*f*x - (8*I - 8)*a^(3/2)*e^(3*f*x + 3*e) - (648*I + 648)*
a^(3/2)*e^(2*f*x + 2*e) - (648*I - 648)*a^(3/2)*e^(f*x + e) - (8*I + 8)*a^
(3/2))*e^(-3/2*f*x - 3/2*e)/f^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+ia\sinh(e+fx))^{3/2}\,dx=\int x^2\,(a+a\sinh(e+f\,x)\,1\mathrm{i})^{3/2}\,dx$$

input
```
int(x^2*(a + a*sinh(e + f*x)*1i)^(3/2),x)
```

output
```
int(x^2*(a + a*sinh(e + f*x)*1i)^(3/2), x)
```

Reduce [F]

$$\int x^2(a+ia\sinh(e+fx))^{3/2}\,dx=\sqrt{a}\,a\left(\int\sqrt{\sinh(fx+e)\,i+1}\,x^2dx+\left(\int\sqrt{\sinh(fx+e)\,i+1}\,\sinh(fx+e)\,x^2dx\right)i\right)$$

input
```
int(x^2*(a+I*a*sinh(f*x+e))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(sinh(e + f*x)*i + 1)*x**2,x) + int(sqrt(sinh(e + f*x)*
i + 1)*sinh(e + f*x)*x**2,x)*i)
```

3.127 $\int x(a+ia\sinh(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 185

$$\begin{aligned}\int x(a+ia\sinh(e+fx))^{3/2}\,dx &= -\frac{16a\sqrt{a+ia\sinh(e+fx)}}{3f^2} \\ &- \frac{8a\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{9f^2} \\ &+ \frac{4ax\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{3f} \\ &+ \frac{8ax\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3f}\end{aligned}$$

output

```
-16/3*a*(a+I*a*sinh(f*x+e))^(1/2)/f^2-8/9*a*cosh(1/2*e+1/4*I*Pi+1/2*f*x)^2
*(a+I*a*sinh(f*x+e))^(1/2)/f^2+4/3*a*x*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*sinh(1
/2*e+1/4*I*Pi+1/2*f*x)*(a+I*a*sinh(f*x+e))^(1/2)/f+8/3*a*x*(a+I*a*sinh(f*x
+e))^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/f
```

Mathematica [A] (verified)

Time = 3.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.75

$$\int x(a+ia\sinh(e+fx))^{3/2}\,dx =$$
$$-\frac{a\left(27(2i+fx)\cosh\left(\frac{1}{2}(e+fx)\right)+(-2i+3fx)\cosh\left(\frac{3}{2}(e+fx)\right)+2i(28i-12fx+(2i+3fx)\cosh(e+\right.}{9f^2\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx\right.\right.}$$

input
```
Integrate[x*(a + I*a*Sinh[e + f*x])^(3/2),x]
```

output
```
-1/9*(a*(27*(2*I + f*x)*Cosh[(e + f*x)/2] + (-2*I + 3*f*x)*Cosh[(3*(e + f*
x))/2] + (2*I)*(28*I - 12*f*x + (2*I + 3*f*x)*Cosh[e + f*x])*Sinh[(e + f*x
)/2])*(-I + Sinh[e + f*x])*Sqrt[a + I*a*Sinh[e + f*x]])/(f^2*(Cosh[(e + f*
x)/2] + I*Sinh[(e + f*x)/2])^3)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 3800, 3042, 3791, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+ia\sinh(e+fx))^{3/2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int x(a+a\sin(ie+ifx))^{3/2}dx$$
$$\Big\downarrow \text{3800}$$
$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int x\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx$$
$$\Big\downarrow \text{3042}$$

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx$$

$\big\downarrow$ 3791

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2}{3}\int x\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx-\frac{4\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{9f^2}+\frac{2x\text{ si}}{}\right.$$

$\big\downarrow$ 3042

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2}{3}\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx-\frac{4\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{9f^2}+\frac{2x\text{ sin}}{}\right.$$

$\big\downarrow$ 3777

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2}{3}\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2i\int-i\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)-\right.$$

$\big\downarrow$ 26

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2}{3}\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2\int\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)-\frac{4\text{ co}}{}\right.$$

$\big\downarrow$ 3042

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2}{3}\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2\int-i\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}\right)-\frac{4}{}\right.$$

$\big\downarrow$ 26

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2}{3}\left(\frac{2i\int\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)-\frac{4\text{ co}}{}\right.$$

$\big\downarrow$ 3118

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{4\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{9f^2}+\frac{2}{3}\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4\cosh\left(\right.}{}\right.\right.$$

input
```
Int[x*(a + I*a*Sinh[e + f*x])^(3/2),x]
```

output
```
2*a*Sech[e/2 + (I/4)*Pi + (f*x)/2]*((-4*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^3)/
(9*f^2) + (2*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sinh[e/2 + (I/4)*Pi + (f*x
)/2])/(3*f) + (2*((-4*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/f^2 + (2*x*Sinh[e/2
+ (I/4)*Pi + (f*x)/2])/f))/3)*Sqrt[a + I*a*Sinh[e + f*x]]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int x(a + ia\sinh{(fx + e)})^{\frac{3}{2}}\, dx$$

input
```
int(x*(a+I*a*sinh(f*x+e))^(3/2),x)
```

output
```
int(x*(a+I*a*sinh(f*x+e))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x(a + ia\sinh(e + fx))^{3/2}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x(a+ia\sinh(e+fx))^{3/2}\,dx=\int x(ia(\sinh{(e+fx)}-i))^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(a+I*a*sinh(f*x+e))**(3/2),x)
```

output
```
Integral(x*(I*a*(sinh(e + f*x) - I))**(3/2), x)
```

Maxima [F]

$$\int x(a+ia\sinh(e+fx))^{3/2}\,dx=\int(i\,a\sinh{(fx+e)}+a)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((I*a*sinh(f*x + e) + a)^(3/2)*x, x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.60

$$\int x(a+ia\sinh(e+fx))^{3/2}\,dx=$$
$$-\frac{\left(-(3i-3)\,a^{\frac{3}{2}}fxe^{(3\,fx+3\,e)}-(27i+27)\,a^{\frac{3}{2}}fxe^{(2\,fx+2\,e)}-(27i-27)\,a^{\frac{3}{2}}fxe^{(fx+e)}-(3i+3)\,a^{\frac{3}{2}}fx+(2i\right.}{18\,f^2}$$

input
```
integrate(x*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
-1/18*(-(3*I - 3)*a^(3/2)*f*x*e^(3*f*x + 3*e) - (27*I + 27)*a^(3/2)*f*x*e^
(2*f*x + 2*e) - (27*I - 27)*a^(3/2)*f*x*e^(f*x + e) - (3*I + 3)*a^(3/2)*f*
x + (2*I - 2)*a^(3/2)*e^(3*f*x + 3*e) + (54*I + 54)*a^(3/2)*e^(2*f*x + 2*e
) - (54*I - 54)*a^(3/2)*e^(f*x + e) - (2*I + 2)*a^(3/2))*e^(-3/2*f*x - 3/2
*e)/f^2
```

Mupad [F(-1)]

Timed out.

$$\int x(a+ia\sinh(e+fx))^{3/2}\,dx=\int x\,(a+a\sinh(e+f\,x)\,\,1\mathrm{i})^{3/2}\,dx$$

input
```
int(x*(a + a*sinh(e + f*x)*1i)^(3/2),x)
```

output
```
int(x*(a + a*sinh(e + f*x)*1i)^(3/2), x)
```

Reduce [F]

$$\int x(a+ia\sinh(e+fx))^{3/2}\,dx=\sqrt{a}\,a\left(\left(\int\sqrt{\sinh(fx+e)\,i+1}\,\sinh(fx+e)\,xdx\right)i+\int\sqrt{\sinh(fx+e)\,i+1}\,xdx\right)$$

input
```
int(x*(a+I*a*sinh(f*x+e))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(sinh(e + f*x)*i + 1)*sinh(e + f*x)*x,x)*i + int(sqrt(s
inh(e + f*x)*i + 1)*x,x))
```

3.128 $\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 261

$$\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x}\,dx = \frac{3}{2}ia\text{Chi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{1}{4}(2e-i\pi)\right)\sqrt{a+ia\sinh(e+fx)} + \frac{1}{2}ia\text{Chi}\left(\frac{3fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{1}{4}(6e+i\pi)\right)\sqrt{a+ia\sinh(e+fx)} + \frac{3}{2}ia\cosh\left(\frac{1}{4}(2e-i\pi)\right)\text{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\text{Shi}\left(\frac{fx}{2}\right) + \frac{1}{2}ia\cosh\left(\frac{1}{4}(6e+i\pi)\right)\text{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\text{Shi}\left(\frac{3fx}{2}\right)$$

output
```
3/2*a*Chi(1/2*f*x)*sech(1/2*e+1/4*I*Pi+1/2*f*x)*cosh(1/2*e+1/4*I*Pi)*(a+I*
a*sinh(f*x+e))^(1/2)+1/2*I*a*Chi(3/2*f*x)*sech(1/2*e+1/4*I*Pi+1/2*f*x)*sin
h(3/2*e+1/4*I*Pi)*(a+I*a*sinh(f*x+e))^(1/2)+3/2*a*sinh(1/2*e+1/4*I*Pi)*sec
h(1/2*e+1/4*I*Pi+1/2*f*x)*(a+I*a*sinh(f*x+e))^(1/2)*Shi(1/2*f*x)+1/2*I*a*c
osh(3/2*e+1/4*I*Pi)*sech(1/2*e+1/4*I*Pi+1/2*f*x)*(a+I*a*sinh(f*x+e))^(1/2)
*Shi(3/2*f*x)
```

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.56

$$\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x}\,dx = \frac{a\sqrt{a+ia\sinh(e+fx)}\left(3\text{Chi}\left(\frac{fx}{2}\right)\left(\cosh\left(\frac{e}{2}\right)+i\sinh\left(\frac{e}{2}\right)\right)-\text{Chi}\left(\frac{3fx}{2}\right)\left(\text{co}\right.\right.}{2\left(\cosh\left(\frac{1}{2}(e+\right.\right.}$$

input
```
Integrate[(a + I*a*Sinh[e + f*x])^(3/2)/x,x]
```

output
```
(a*Sqrt[a + I*a*Sinh[e + f*x]]*(3*CoshIntegral[(f*x)/2]*(Cosh[e/2] + I*Sin
h[e/2]) - CoshIntegral[(3*f*x)/2]*(Cosh[(3*e)/2] - I*Sinh[(3*e)/2]) + (I*C
osh[e/2] + Sinh[e/2])*(3*SinhIntegral[(f*x)/2] + (1 + (2*I)*Sinh[e])*SinhI
ntegral[(3*f*x)/2])))/(2*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.57, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3800, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+a\sin(ie+ifx))^{3/2}}{x}dx$$

$$\Big\downarrow \text{3800}$$

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int \frac{\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int \frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3}{x}dx$$

$$\Big\downarrow \text{3793}$$

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int\left(\frac{3i\sinh\left(\frac{1}{4}(2e-i\pi)+\frac{fx}{2}\right)}{4x}+\frac{i\sinh\left(\frac{1}{4}(6e+i\pi)+\frac{3fx}{2}\right)}{4x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{3}{4}i\sinh\left(\frac{1}{4}(2e-i\pi)\right)\text{Chi}\left(\frac{fx}{2}\right)+\frac{1}{4}i\sinh\left(\frac{1}{4}(6e+i\pi)\right)\text{Chi}\left(\frac{3}{-}\right.\right.$$

input
```
Int[(a + I*a*Sinh[e + f*x])^(3/2)/x,x]
```

output
```
2*a*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]]*(((3*I)/4)*
CoshIntegral[(f*x)/2]*Sinh[(2*e - I*Pi)/4] + (I/4)*CoshIntegral[(3*f*x)/2]
*Sinh[(6*e + I*Pi)/4] + ((3*I)/4)*Cosh[(2*e - I*Pi)/4]*SinhIntegral[(f*x)/
2] + (I/4)*Cosh[(6*e + I*Pi)/4]*SinhIntegral[(3*f*x)/2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int \frac{(a + ia\sinh(fx + e))^{\frac{3}{2}}}{x}dx$$

input
```
int((a+I*a*sinh(f*x+e))^(3/2)/x,x)
```

output
```
int((a+I*a*sinh(f*x+e))^(3/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a + ia\sinh(e + fx))^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*sinh(f*x+e))^(3/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{(a + ia\sinh(e + fx))^{3/2}}{x}\,dx = \int \frac{(ia(\sinh(e + fx) - i))^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+I*a*sinh(f*x+e))**(3/2)/x,x)
```

output
```
Integral((I*a*(sinh(e + f*x) - I))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x}\,dx = \int \frac{(i\,a\sinh{(fx+e)}+a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+I*a*sinh(f*x+e))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate((I*a*sinh(f*x + e) + a)^(3/2)/x, x)
```

Giac [F]

$$\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x}\,dx = \int \frac{(i\,a\sinh{(fx+e)}+a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+I*a*sinh(f*x+e))^(3/2)/x,x, algorithm="giac")
```

output
```
integrate((I*a*sinh(f*x + e) + a)^(3/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x}\,dx = \int \frac{(a+a\sinh(e+f\,x)\ 1\mathrm{i})^{3/2}}{x}\,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)^(3/2)/x,x)
```

output
```
int((a + a*sinh(e + f*x)*1i)^(3/2)/x, x)
```

Reduce [F]

$$\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x}\,dx = \sqrt{a}\,a\left(\int \frac{\sqrt{\sinh(fx+e)\,i+1}}{x}dx + \left(\int \frac{\sqrt{\sinh(fx+e)\,i+1}\,\sinh(fx+e)}{x}dx\right)i\right)$$

input
```
int((a+I*a*sinh(f*x+e))^(3/2)/x,x)
```

output
```
sqrt(a)*a*(int(sqrt(sinh(e + f*x)*i + 1)/x,x) + int((sqrt(sinh(e + f*x)*i
+ 1)*sinh(e + f*x))/x,x)*i)
```

3.129 $\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 302

$$\begin{aligned}
\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x^2}\,dx &= \\
&-\frac{2a\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{x}-\frac{3}{4}af\operatorname{Chi}\left(\frac{3fx}{2}\right)\operatorname{sech}\left(\frac{e}{2}\right. \\
&\left.+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{1}{4}(6e-i\pi)\right)\sqrt{a+ia\sinh(e+fx)} \\
&+\frac{3}{4}af\operatorname{Chi}\left(\frac{fx}{2}\right)\operatorname{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{1}{4}(2e+i\pi)\right)\sqrt{a+ia\sinh(e+fx)} \\
&+\frac{3}{4}af\cosh\left(\frac{1}{4}(2e+i\pi)\right)\operatorname{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\operatorname{Shi}\left(\frac{fx}{2}\right) \\
&-\frac{3}{4}af\cosh\left(\frac{1}{4}(6e-i\pi)\right)\operatorname{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\operatorname{Shi}\left(\frac{3fx}{2}\right)
\end{aligned}$$

output
```
-2*a*cosh(1/2*e+1/4*I*Pi+1/2*f*x)^2*(a+I*a*sinh(f*x+e))^(1/2)/x+3/4*I*a*f*
Chi(3/2*f*x)*sech(1/2*e+1/4*I*Pi+1/2*f*x)*cosh(3/2*e+1/4*I*Pi)*(a+I*a*sinh
(f*x+e))^(1/2)+3/4*a*f*Chi(1/2*f*x)*sech(1/2*e+1/4*I*Pi+1/2*f*x)*sinh(1/2*
e+1/4*I*Pi)*(a+I*a*sinh(f*x+e))^(1/2)+3/4*a*f*cosh(1/2*e+1/4*I*Pi)*sech(1/
2*e+1/4*I*Pi+1/2*f*x)*(a+I*a*sinh(f*x+e))^(1/2)*Shi(1/2*f*x)+3/4*I*a*f*sin
h(3/2*e+1/4*I*Pi)*sech(1/2*e+1/4*I*Pi+1/2*f*x)*(a+I*a*sinh(f*x+e))^(1/2)*S
hi(3/2*f*x)
```

Mathematica [A] (verified)

Time = 1.87 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.80

$$\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x^2}\,dx = \frac{a(-i+\sinh(e+fx))\sqrt{a+ia\sinh(e+fx)}\left(-6i\cosh\left(\frac{1}{2}(e+fx)\right)+2i\,\text{co}\right.}{}$$

input
```
Integrate[(a + I*a*Sinh[e + f*x])^(3/2)/x^2,x]
```

output
```
(a*(-I + Sinh[e + f*x])*Sqrt[a + I*a*Sinh[e + f*x]]*((-6*I)*Cosh[(e + f*x)
/2] + (2*I)*Cosh[(3*(e + f*x))/2] - 3*f*x*CoshIntegral[(f*x)/2]*(Cosh[e/2]
 - I*Sinh[e/2]) - 3*f*x*CoshIntegral[(3*f*x)/2]*(Cosh[(3*e)/2] + I*Sinh[(3
*e)/2]) + 6*Sinh[(e + f*x)/2] + 2*Sinh[(3*(e + f*x))/2] + (3*I)*f*x*Cosh[e
/2]*SinhIntegral[(f*x)/2] - 3*f*x*Sinh[e/2]*SinhIntegral[(f*x)/2] - (3*I)*
f*x*Cosh[(3*e)/2]*SinhIntegral[(3*f*x)/2] - 3*f*x*Sinh[(3*e)/2]*SinhIntegr
al[(3*f*x)/2]))/(4*x*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])^3)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.61, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3800, 3042, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+a\sin(ie+ifx))^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{3800}$$

$$2a\,\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int \frac{\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int\frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3}{x^2}dx$$

$$\Big\downarrow \text{3794}$$

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{3}{2}if\int\left(\frac{\cosh\left(\frac{3e}{2}+\frac{3fx}{2}+\frac{i\pi}{4}\right)}{4x}-\frac{i\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{4x}\right)dx-\text{c}\right.$$

$$\Big\downarrow \text{2009}$$

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{3}{2}if\left(\frac{1}{4}i\sinh\left(\frac{1}{4}(6e-i\pi)\right)\text{Chi}\left(\frac{3fx}{2}\right)-\frac{1}{4}i\sinh\left(\frac{1}{4}(2e+i\pi)\right)\right.\right.$$

input
```
Int[(a + I*a*Sinh[e + f*x])^(3/2)/x^2,x]
```

output
```
2*a*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]]*(-(Cosh[e/2
 + (I/4)*Pi + (f*x)/2]^3/x) + ((3*I)/2)*f*((I/4)*CoshIntegral[(3*f*x)/2]*S
inh[(6*e - I*Pi)/4] - (I/4)*CoshIntegral[(f*x)/2]*Sinh[(2*e + I*Pi)/4] - (
I/4)*Cosh[(2*e + I*Pi)/4]*SinhIntegral[(f*x)/2] + (I/4)*Cosh[(6*e - I*Pi)/
4]*SinhIntegral[(3*f*x)/2]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int \frac{(a+ia\sinh(fx+e))^{\frac{3}{2}}}{x^2}dx$$

input
```
int((a+I*a*sinh(f*x+e))^(3/2)/x^2,x)
```

output
```
int((a+I*a*sinh(f*x+e))^(3/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*sinh(f*x+e))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x^2}\,dx = \int \frac{(ia(\sinh(e+fx)-i))^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+I*a*sinh(f*x+e))**(3/2)/x**2,x)
```

output
```
Integral((I*a*(sinh(e + f*x) - I))**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x^2}\,dx = \int \frac{(i\,a\sinh(fx+e)+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+I*a*sinh(f*x+e))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((I*a*sinh(f*x + e) + a)^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x^2}\,dx = \int \frac{(i\,a\sinh(fx+e)+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+I*a*sinh(f*x+e))^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((I*a*sinh(f*x + e) + a)^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x^2}\,dx=\int \frac{(a+a\sinh(e+f\,x)\;1\mathrm{i})^{3/2}}{x^2}\,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)^(3/2)/x^2,x)
```

output
```
int((a + a*sinh(e + f*x)*1i)^(3/2)/x^2, x)
```

Reduce [F]

$$\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x^2}\,dx=\sqrt{a}\,a\left(\int \frac{\sqrt{\sinh\left(fx+e\right)i+1}}{x^2}dx+\left(\int \frac{\sqrt{\sinh\left(fx+e\right)i+1}\,\sinh\left(fx+e\right)}{x^2}dx\right)i\right)$$

input
```
int((a+I*a*sinh(f*x+e))^(3/2)/x^2,x)
```

output
```
sqrt(a)*a*(int(sqrt(sinh(e + f*x)*i + 1)/x**2,x) + int((sqrt(sinh(e + f*x)
*i + 1)*sinh(e + f*x))/x**2,x)*i)
```

3.130 $\int x^3(a+ia\sinh(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 638

$$\begin{aligned}\int x^3(a+ia\sinh(c+dx))^{5/2}\,dx = & -\frac{265216a^2\sqrt{a+ia\sinh(c+dx)}}{1125d^4}-\frac{128a^2x^2\sqrt{a+ia\sinh(c+dx)}}{5d^2}\\ & -\frac{17408a^2\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{3375d^4}\\ & -\frac{64a^2x^2\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{15d^2}\\ & -\frac{384a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{625d^4}\\ & -\frac{48a^2x^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{25d^2}\\ & +\frac{8704a^2x\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{1125d^3}\\ & +\frac{32a^2x^3\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{15d}\\ & +\frac{192a^2x\cosh^3\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{125d^3}\\ & +\frac{8a^2x^3\cosh^3\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{5d}\\ & +\frac{132608a^2x\sqrt{a+ia\sinh(c+dx)}\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{1125d^3}\\ & +\frac{64a^2x^3\sqrt{a+ia\sinh(c+dx)}\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{15d}\end{aligned}$$

output
```
-265216/1125*a^2*(a+I*a*sinh(d*x+c))^(1/2)/d^4-128/5*a^2*x^2*(a+I*a*sinh(d
*x+c))^(1/2)/d^2-17408/3375*a^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^2*(a+I*a*sinh
(d*x+c))^(1/2)/d^4-64/15*a^2*x^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^2*(a+I*a*sin
h(d*x+c))^(1/2)/d^2-384/625*a^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^4*(a+I*a*sinh
(d*x+c))^(1/2)/d^4-48/25*a^2*x^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^4*(a+I*a*sin
h(d*x+c))^(1/2)/d^2+8704/1125*a^2*x*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*sinh(1/2*
c+1/4*I*Pi+1/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)/d^3+32/15*a^2*x^3*cosh(1/2*c
+1/4*I*Pi+1/2*d*x)*sinh(1/2*c+1/4*I*Pi+1/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)/
d+192/125*a^2*x*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^3*sinh(1/2*c+1/4*I*Pi+1/2*d*x
)*(a+I*a*sinh(d*x+c))^(1/2)/d^3+8/5*a^2*x^3*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^3
*sinh(1/2*c+1/4*I*Pi+1/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)/d+132608/1125*a^2*
x*(a+I*a*sinh(d*x+c))^(1/2)*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/d^3+64/15*a^2*x^3
*(a+I*a*sinh(d*x+c))^(1/2)*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2918 vs. 2(638) = 1276.

Time = 15.39 (sec) , antiderivative size = 2918, normalized size of antiderivative = 4.57

$$\int x^3(a+ia\sinh(c+dx))^{5/2}\,dx = \text{Result too large to show}$$

input
```
Integrate[x^3*(a + I*a*Sinh[c + d*x])^(5/2),x]
```

output
```
(2*(((-1/135000 - I/135000)*Cosh[5*(c/2 + (d*x)/2)])/d^3 + ((1/135000 + I/
135000)*Sinh[5*(c/2 + (d*x)/2)])/d^3)*(1296*I - (3240*I)*c + (4050*I)*c^2
- (3375*I)*c^3 + (6480*I)*(c/2 + (d*x)/2) - (16200*I)*c*(c/2 + (d*x)/2) +
(20250*I)*c^2*(c/2 + (d*x)/2) + (16200*I)*(c/2 + (d*x)/2)^2 - (40500*I)*c*
(c/2 + (d*x)/2)^2 + (27000*I)*(c/2 + (d*x)/2)^3 - 50000*Cosh[2*(c/2 + (d*x
)/2)] + 75000*c*Cosh[2*(c/2 + (d*x)/2)] - 56250*c^2*Cosh[2*(c/2 + (d*x)/2)
] + 28125*c^3*Cosh[2*(c/2 + (d*x)/2)] - 150000*(c/2 + (d*x)/2)*Cosh[2*(c/2
 + (d*x)/2)] + 225000*c*(c/2 + (d*x)/2)*Cosh[2*(c/2 + (d*x)/2)] - 168750*c
^2*(c/2 + (d*x)/2)*Cosh[2*(c/2 + (d*x)/2)] - 225000*(c/2 + (d*x)/2)^2*Cosh
[2*(c/2 + (d*x)/2)] + 337500*c*(c/2 + (d*x)/2)^2*Cosh[2*(c/2 + (d*x)/2)] -
 225000*(c/2 + (d*x)/2)^3*Cosh[2*(c/2 + (d*x)/2)] - (8100000*I)*Cosh[4*(c/
2 + (d*x)/2)] + (4050000*I)*c*Cosh[4*(c/2 + (d*x)/2)] - (1012500*I)*c^2*Co
sh[4*(c/2 + (d*x)/2)] + (168750*I)*c^3*Cosh[4*(c/2 + (d*x)/2)] - (8100000*
I)*(c/2 + (d*x)/2)*Cosh[4*(c/2 + (d*x)/2)] + (4050000*I)*c*(c/2 + (d*x)/2)
*Cosh[4*(c/2 + (d*x)/2)] - (1012500*I)*c^2*(c/2 + (d*x)/2)*Cosh[4*(c/2 + (
d*x)/2)] - (4050000*I)*(c/2 + (d*x)/2)^2*Cosh[4*(c/2 + (d*x)/2)] + (202500
0*I)*c*(c/2 + (d*x)/2)^2*Cosh[4*(c/2 + (d*x)/2)] - (1350000*I)*(c/2 + (d*x
)/2)^3*Cosh[4*(c/2 + (d*x)/2)] + 8100000*Cosh[6*(c/2 + (d*x)/2)] + 4050000
*c*Cosh[6*(c/2 + (d*x)/2)] + 1012500*c^2*Cosh[6*(c/2 + (d*x)/2)] + 168750*
c^3*Cosh[6*(c/2 + (d*x)/2)] - 8100000*(c/2 + (d*x)/2)*Cosh[6*(c/2 + (d*...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+ia\sinh(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int x^3(a+a\sin(ic+idx))^{5/2}dx$$

$$\Big\downarrow \text{ 3800}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\int x^3\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx$$

$$\Big\downarrow \text{ 3042}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\int x^3\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^5dx$$

$\Big\downarrow$ 3792

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{24\int x\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{25d^2}+\frac{4}{5}\int x^3\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)d\right.$$

$\Big\downarrow$ 3042

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{24\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^5dx}{25d^2}+\frac{4}{5}\int x^3\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx\right.$$

$\Big\downarrow$ 3791

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{24\left(\frac{4}{5}\int x\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx-\frac{4\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{25d^2}+\frac{2x\sinh\left(\frac{c}{2}\right.}{}\right.}{25d^2}\right.$$

$\Big\downarrow$ 3042

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{24\left(\frac{4}{5}\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx-\frac{4\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{25d^2}+\frac{2x\sinh\left(\frac{c}{2}+\right.}{}\right.}{25d^2}\right.$$

$\Big\downarrow$ 3791

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{24\left(\frac{4}{5}\left(\frac{2}{3}\int x\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx-\frac{4\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{9d^2}+\frac{2x\sinh\left(\right.}{}\right.\right.}{}\right.$$

$\Big\downarrow$ 3042

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{24\left(\frac{4}{5}\left(\frac{2}{3}\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)dx-\frac{4\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{9d^2}+\frac{2x\sinh\left(\right.}{}\right.\right.}{}\right.$$

$\Big\downarrow$ 3777

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{24\left(\frac{4}{5}\left(\frac{2}{3}\left(\frac{2x\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2i\int -i\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)-\frac{4\cosh^3\left(\right.}{}\right.\right.}{}\right.$$

$\downarrow$ 26

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{24\left(\frac{4}{5}\left(\frac{2}{3}\left(\frac{2x\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2\int\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)-\frac{4\cosh^3\left(\frac{c}{2}+\right.}{9d^2}\right.\right.}{}\right.$$

$\downarrow$ 3042

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{24\left(\frac{4}{5}\left(\frac{2}{3}\left(\frac{2x\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2\int -i\sin\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}\right)-\frac{4\cosh^3\left(\right.}{9}\right.\right.}{}\right.$$

$\downarrow$ 26

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{24\left(\frac{4}{5}\left(\frac{2}{3}\left(\frac{2i\int\sin\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}+\frac{2x\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)-\frac{4\cosh^3\left(\frac{c}{2}+\right.}{9d^2}\right.\right.}{}\right.$$

$\downarrow$ 3118

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\int x^3\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx-\frac{12x^2\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{25d^2}+\right.$$

$\downarrow$ 3792

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int x\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{3d^2}+\frac{2}{3}\int x^3\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right.\right.$$

$\big\downarrow$ 3042

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{3d^2}+\frac{2}{3}\int x^3\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)d\right.\right.$$

$\big\downarrow$ 3777

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{2x^3\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\right.\right.\right.$$

$\big\downarrow$ 26

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{2x^3\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\right.\right.\right.$$

$\big\downarrow$ 3042

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{2x^3\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\right.\right.\right.$$

$\big\downarrow$ 26

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{6i\int x^2\sin\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)d}{d}\right.\right.\right.$$

$\downarrow$ 3777

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\right.}{d}\right.\right.\right.$$

$\downarrow$ 3042

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\right.}{d}\right.\right.\right.$$

$\downarrow$ 3777

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\right.}{}\right.\right.\right.$$

$\downarrow$ 26

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\right.}{}\right.\right.\right.$$

$\downarrow$ 3042

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\right.}{}\right.\right.\right.$$

$\downarrow$ 26

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\right.}{}\right.\right.\right.$$

$\downarrow$ 3118

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{3d^2}-\frac{4x^2\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{3d^2}+\frac{2}{3}\right.\right.$$

$\downarrow$ 3791

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\left(\frac{2}{3}\int x\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx-\frac{4\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{9d^2}+\frac{2x\sinh\left(\right.}{}\right.}{3d^2}\right.\right.$$

$\downarrow$ 3042

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\left(\frac{2}{3}\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)dx-\frac{4\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{9d^2}+\frac{2x\sinh\left(\frac{c}{2}\right.}{}\right.}{3d^2}\right.\right.$$

$$\Big\downarrow \; 3777$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\left(\frac{2}{3}\left(\frac{2x\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2i\int -i\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)-\frac{4\cosh^3\left(\right.}{}\right.}{3d^2}\right.\right.$$

input
```
Int[x^3*(a + I*a*Sinh[c + d*x])^(5/2),x]
```

output
```
$Aborted
```

Maple [F]

$$\int x^3(a+ia\sinh(dx+c))^{\frac{5}{2}}\,dx$$

input
```
int(x^3*(a+I*a*sinh(d*x+c))^(5/2),x)
```

output
```
int(x^3*(a+I*a*sinh(d*x+c))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^3(a+ia\sinh(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int x^3(a+ia\sinh(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+I*a*sinh(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^3(a+ia\sinh(c+dx))^{5/2}\,dx = \int (i\,a\sinh{(dx+c)}+a)^{\frac{5}{2}}x^3\,dx$$

input
```
integrate(x^3*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(5/2)*x^3, x)
```

Giac [F]

$$\int x^3(a+ia\sinh(c+dx))^{5/2}\,dx=\int(i\,a\sinh(dx+c)+a)^{\frac{5}{2}}x^3\,dx$$

input
```
integrate(x^3*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(5/2)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a+ia\sinh(c+dx))^{5/2}\,dx=\int x^3\,(a+a\sinh(c+d\,x)\,1\mathrm{i})^{5/2}\,dx$$

input
```
int(x^3*(a + a*sinh(c + d*x)*1i)^(5/2),x)
```

output
```
int(x^3*(a + a*sinh(c + d*x)*1i)^(5/2), x)
```

Reduce [F]

$$\int x^3(a+ia\sinh(c+dx))^{5/2}\,dx=\sqrt{a}\,a^2\left(-\left(\int\sqrt{\sinh(dx+c)\,i+1}\,\sinh(dx+c)^2\,x^3dx\right)+\int\sqrt{\sinh(dx+c)\,i+1}\,x^3dx+2\left(\int\sqrt{\sinh(dx+c)\,i+1}\,\sinh(dx+c)\,x^3dx\right)i\right)$$

input
```
int(x^3*(a+I*a*sinh(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*( - int(sqrt(sinh(c + d*x)*i + 1)*sinh(c + d*x)**2*x**3,x) +
int(sqrt(sinh(c + d*x)*i + 1)*x**3,x) + 2*int(sqrt(sinh(c + d*x)*i + 1)*si
nh(c + d*x)*x**3,x)*i)
```

3.131 $\int x^2(a+ia\sinh(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 506

$$\begin{aligned}
\int x^2(a+ia\sinh(c+dx))^{5/2}\,dx &= -\frac{256a^2x\sqrt{a+ia\sinh(c+dx)}}{15d^2} \\
&- \frac{128a^2x\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{45d^2} \\
&- \frac{32a^2x\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{25d^2} \\
&+ \frac{32a^2x^2\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{15d} \\
&+ \frac{8a^2x^2\cosh^3\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{5d} \\
&+ \frac{9536a^2\sqrt{a+ia\sinh(c+dx)}\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{225d^3} \\
&+ \frac{64a^2x^2\sqrt{a+ia\sinh(c+dx)}\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{15d} \\
&+ \frac{2432a^2\sinh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{675d^3} \\
&+ \frac{64a^2\sinh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{125d^3}
\end{aligned}$$

output
```
-256/15*a^2*x*(a+I*a*sinh(d*x+c))^(1/2)/d^2-128/45*a^2*x*cosh(1/2*c+1/4*I*
Pi+1/2*d*x)^2*(a+I*a*sinh(d*x+c))^(1/2)/d^2-32/25*a^2*x*cosh(1/2*c+1/4*I*P
i+1/2*d*x)^4*(a+I*a*sinh(d*x+c))^(1/2)/d^2+32/15*a^2*x^2*cosh(1/2*c+1/4*I*
Pi+1/2*d*x)*sinh(1/2*c+1/4*I*Pi+1/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)/d+8/5*a
^2*x^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^3*sinh(1/2*c+1/4*I*Pi+1/2*d*x)*(a+I*a*
sinh(d*x+c))^(1/2)/d+9536/225*a^2*(a+I*a*sinh(d*x+c))^(1/2)*tanh(1/2*c+1/4
*I*Pi+1/2*d*x)/d^3+64/15*a^2*x^2*(a+I*a*sinh(d*x+c))^(1/2)*tanh(1/2*c+1/4*
I*Pi+1/2*d*x)/d+2432/675*a^2*sinh(1/2*c+1/4*I*Pi+1/2*d*x)^2*(a+I*a*sinh(d*
x+c))^(1/2)*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/d^3+64/125*a^2*sinh(1/2*c+1/4*I*P
i+1/2*d*x)^4*(a+I*a*sinh(d*x+c))^(1/2)*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/d^3
```

Mathematica [A] (verified)

Time = 9.67 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.59

$$\int x^2(a+ia\sinh(c+dx))^{5/2}\,dx=\frac{a^2\sqrt{a+ia\sinh(c+dx)}\left(33750i(8+4idx+d^2x^2)\cosh\left(\frac{1}{2}(c+dx)\right)+625(8i+12dx+9id^2x\right.}{}$$

input
```
Integrate[x^2*(a + I*a*Sinh[c + d*x])^(5/2),x]
```

output
```
(a^2*Sqrt[a + I*a*Sinh[c + d*x]]*((33750*I)*(8 + (4*I)*d*x + d^2*x^2)*Cosh
[(c + d*x)/2] + 625*(8*I + 12*d*x + (9*I)*d^2*x^2)*Cosh[(3*(c + d*x))/2] -
 (216*I)*Cosh[(5*(c + d*x))/2] + 540*d*x*Cosh[(5*(c + d*x))/2] - (675*I)*d
^2*x^2*Cosh[(5*(c + d*x))/2] + 270000*Sinh[(c + d*x)/2] - (135000*I)*d*x*S
inh[(c + d*x)/2] + 33750*d^2*x^2*Sinh[(c + d*x)/2] - 5000*Sinh[(3*(c + d*x
))/2] - (7500*I)*d*x*Sinh[(3*(c + d*x))/2] - 5625*d^2*x^2*Sinh[(3*(c + d*x
))/2] - 216*Sinh[(5*(c + d*x))/2] + (540*I)*d*x*Sinh[(5*(c + d*x))/2] - 67
5*d^2*x^2*Sinh[(5*(c + d*x))/2]))/(6750*d^3*(Cosh[(c + d*x)/2] + I*Sinh[(c
 + d*x)/2]))
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.91, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {3042, 3800, 3042, 3792, 3042, 3113, 2009, 3792, 3042, 3113, 2009, 3777, 26, 3042, 26, 3777, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+ia\sinh(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2(a+a\sin(ic+idx))^{5/2}dx$$

$$\Big\downarrow \text{3800}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\int x^2\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\int x^2\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^5dx$$

$$\Big\downarrow \text{3792}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{8\int\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{25d^2}+\frac{4}{5}\int x^2\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx\right.$$

$$\Big\downarrow \text{3042}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{8\int\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^5dx}{25d^2}+\frac{4}{5}\int x^2\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx-\right.$$

$$\Big\downarrow \text{3113}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{16i\int\left(\sinh^4\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)+2\sinh^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)+1\right)d(-i\sin}{25d^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\int x^2\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx+\frac{16i\left(-\frac{1}{5}i\sinh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right.}{}\right.$$

$\downarrow$ 3792

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{9d^2}+\frac{2}{3}\int x^2\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)d\right.\right.$$

$\downarrow$ 3042

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{9d^2}+\frac{2}{3}\int x^2\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)dx\right.\right.$$

$\downarrow$ 3113

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{16i\int\left(\sinh^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)+1\right)d\left(-i\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{9d^3}+\right.\right.$$

$\downarrow$ 2009

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\int x^2\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)dx+\frac{16i\left(-\frac{1}{3}i\sinh^3\left(\frac{c}{2}+\frac{dx}{2}+\right.\right.}{9d}\right.\right.$$

$\downarrow$ 3777

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\left(\frac{2x^2\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4i\int-ix\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right.\right.\right.$$

$\downarrow$ 26

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\left(\frac{2x^2\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4\int x\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)+\right.\right.$$

$\downarrow$ 3042

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\left(\frac{2x^2\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4\int-ix\sin\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}\right)\right.\right.$$

$\downarrow$ 26

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\left(\frac{4i\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}+\frac{2x^2\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)+\right.\right.$$

$\downarrow$ 3777

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\left(\frac{4i\left(\frac{2ix\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2i\int\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{d}+\frac{2x^2\sin}{}\right.\right.\right.$$

$\downarrow$ 3042

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\left(\frac{4i\left(\frac{2ix\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2i\int\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)dx}{d}\right)}{d}+\frac{2x^2\sinh}{}\right.\right.\right.$$

$\downarrow$ 3118

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{16i\left(-\frac{1}{5}i\sinh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)-\frac{2}{3}i\sinh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)-i\sinh\left(\right.\right.}{25d^3}\right.$$

input
```
Int[x^2*(a + I*a*Sinh[c + d*x])^(5/2),x]
```

output
```
4*a^2*Sech[c/2 + (I/4)*Pi + (d*x)/2]*((-8*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]
^5)/(25*d^2) + (2*x^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^4*Sinh[c/2 + (I/4)*Pi
 + (d*x)/2])/(5*d) + (((16*I)/25)*((-I)*Sinh[c/2 + (I/4)*Pi + (d*x)/2] - (
(2*I)/3)*Sinh[c/2 + (I/4)*Pi + (d*x)/2]^3 - (I/5)*Sinh[c/2 + (I/4)*Pi + (d
*x)/2]^5))/d^3 + (4*((-8*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^3)/(9*d^2) + (2*
x^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^2*Sinh[c/2 + (I/4)*Pi + (d*x)/2])/(3*d)
 + (((16*I)/9)*((-I)*Sinh[c/2 + (I/4)*Pi + (d*x)/2] - (I/3)*Sinh[c/2 + (I/
4)*Pi + (d*x)/2]^3))/d^3 + (2*((2*x^2*Sinh[c/2 + (I/4)*Pi + (d*x)/2])/d +
((4*I)*(((2*I)*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2])/d - ((4*I)*Sinh[c/2 + (I/
4)*Pi + (d*x)/2])/d^2))/d))/3))/5)*Sqrt[a + I*a*Sinh[c + d*x]]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int x^2(a + ia \sinh (dx + c))^{\frac{5}{2}} dx$$

input
```
int(x^2*(a+I*a*sinh(d*x+c))^(5/2),x)
```

output
```
int(x^2*(a+I*a*sinh(d*x+c))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2(a + ia \sinh(c + dx))^{5/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int x^2(a + ia \sinh(c + dx))^{5/2} dx = \text{Timed out}$$

input
```
integrate(x**2*(a+I*a*sinh(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2(a+ia\sinh(c+dx))^{5/2}\,dx=\int (i\,a\sinh\left(dx+c\right)+a)^{\frac{5}{2}}x^2\,dx$$

input
```
integrate(x^2*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(5/2)*x^2, x)
```

Giac [F]

$$\int x^2(a+ia\sinh(c+dx))^{5/2}\,dx=\int (i\,a\sinh\left(dx+c\right)+a)^{\frac{5}{2}}x^2\,dx$$

input
```
integrate(x^2*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(5/2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+ia\sinh(c+dx))^{5/2}\,dx=\int x^2\,\left(a+a\sinh(c+d\,x)\;1\mathrm{i}\right)^{5/2}\,dx$$

input
```
int(x^2*(a + a*sinh(c + d*x)*1i)^(5/2),x)
```

output
```
int(x^2*(a + a*sinh(c + d*x)*1i)^(5/2), x)
```

Reduce [F]

$$\int x^2(a+ia\sinh(c+dx))^{5/2}\,dx=\sqrt{a}\,a^2\left(-\left(\int\sqrt{\sinh\left(dx+c\right)i+1}\,\sinh\left(dx+c\right)^2x^2dx\right)+\int\sqrt{\sinh\left(dx+c\right)i+1}\,x^2dx+2\left(\int\sqrt{\sinh\left(dx+c\right)i+1}\,\sinh\left(dx+c\right)x^2dx\right)i\right)$$

input
```
int(x^2*(a+I*a*sinh(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*( - int(sqrt(sinh(c + d*x)*i + 1)*sinh(c + d*x)**2*x**2,x) +
int(sqrt(sinh(c + d*x)*i + 1)*x**2,x) + 2*int(sqrt(sinh(c + d*x)*i + 1)*si
nh(c + d*x)*x**2,x)*i)
```

3.132 $\int x(a+ia\sinh(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 312

$$\begin{aligned}\int x(a+ia\sinh(c+dx))^{5/2}\,dx &= -\frac{128a^2\sqrt{a+ia\sinh(c+dx)}}{15d^2}\\ &\quad-\frac{64a^2\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{45d^2}\\ &\quad-\frac{16a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{25d^2}\\ &\quad+\frac{32a^2x\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{15d}\\ &\quad+\frac{8a^2x\cosh^3\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{5d}\\ &\quad+\frac{64a^2x\sqrt{a+ia\sinh(c+dx)}\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{15d}\end{aligned}$$

output
```
-128/15*a^2*(a+I*a*sinh(d*x+c))^(1/2)/d^2-64/45*a^2*cosh(1/2*c+1/4*I*Pi+1/
2*d*x)^2*(a+I*a*sinh(d*x+c))^(1/2)/d^2-16/25*a^2*cosh(1/2*c+1/4*I*Pi+1/2*d
*x)^4*(a+I*a*sinh(d*x+c))^(1/2)/d^2+32/15*a^2*x*cosh(1/2*c+1/4*I*Pi+1/2*d*
x)*sinh(1/2*c+1/4*I*Pi+1/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)/d+8/5*a^2*x*cosh
(1/2*c+1/4*I*Pi+1/2*d*x)^3*sinh(1/2*c+1/4*I*Pi+1/2*d*x)*(a+I*a*sinh(d*x+c)
)^(1/2)/d+64/15*a^2*x*(a+I*a*sinh(d*x+c))^(1/2)*tanh(1/2*c+1/4*I*Pi+1/2*d*
x)/d
```

Mathematica [A] (verified)

Time = 9.25 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.70

$$\int x(a+ia\sinh(c+dx))^{5/2}\,dx = \frac{a^2(-i+\sinh(c+dx))^2\sqrt{a+ia\sinh(c+dx)}\left(2250(2-idx)\cosh\left(\frac{1}{2}(c+dx)\right)+(-250-37\right.}{}$$

input
```
Integrate[x*(a + I*a*Sinh[c + d*x])^(5/2),x]
```

output
```
(a^2*(-I + Sinh[c + d*x])^2*Sqrt[a + I*a*Sinh[c + d*x]]*(2250*(2 - I*d*x)*
Cosh[(c + d*x)/2] + (-250 - (375*I)*d*x)*Cosh[(3*(c + d*x))/2] - 18*Cosh[(
5*(c + d*x))/2] + (45*I)*d*x*Cosh[(5*(c + d*x))/2] + (4500*I)*Sinh[(c + d*
x)/2] - 2250*d*x*Sinh[(c + d*x)/2] + (250*I)*Sinh[(3*(c + d*x))/2] + 375*d
*x*Sinh[(3*(c + d*x))/2] - (18*I)*Sinh[(5*(c + d*x))/2] + 45*d*x*Sinh[(5*(
c + d*x))/2]))/(450*d^2*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^5)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.84, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {3042, 3800, 3042, 3791, 3042, 3791, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+ia\sinh(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x(a+a\sin(ic+idx))^{5/2}dx$$

$$\Big\downarrow \text{3800}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\int x\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx$$

$$\Big\downarrow \ 3042$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^5dx$$

$$\Big\downarrow \ 3791$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\int x\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx-\frac{4\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{25d^2}+\frac{2x\text{ s}}{}\right.$$

$$\Big\downarrow \ 3042$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx-\frac{4\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{25d^2}+\frac{2x\text{ si}}{}\right.$$

$$\Big\downarrow \ 3791$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\int x\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx-\frac{4\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{9d^2}+\frac{2}{}\right.\right.$$

$$\Big\downarrow \ 3042$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)dx-\frac{4\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{9d^2}+\frac{2x}{}\right.\right.$$

$$\Big\downarrow \ 3777$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\left(\frac{2x\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2i\int-i\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)\right.\right.$$

$$\Big\downarrow \ 26$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\left(\frac{2x\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2\int\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)-\frac{4}{}\right.\right.$$

$$\Big\downarrow \ 3042$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\left(\frac{2x\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2\int-i\sin\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}\right)-\right.\right.$$

$$\Big\downarrow \ 26$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\left(\frac{2i\int\sin\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}+\frac{2x\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)-\frac{4}{}\right.\right.$$

$\downarrow$ 3118

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(-\frac{4\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{25d^2}+\frac{4}{5}\left(-\frac{4\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{9d^2}+\frac{2}{3}\left(\frac{2x}{}\right.\right.\right.$$

input
```
Int[x*(a + I*a*Sinh[c + d*x])^(5/2),x]
```

output
```
4*a^2*Sech[c/2 + (I/4)*Pi + (d*x)/2]*((-4*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^5
)/(25*d^2) + (2*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^4*Sinh[c/2 + (I/4)*Pi + (
d*x)/2])/(5*d) + (4*((-4*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^3)/(9*d^2) + (2*x*
Cosh[c/2 + (I/4)*Pi + (d*x)/2]^2*Sinh[c/2 + (I/4)*Pi + (d*x)/2])/(3*d) + (
2*((-4*Cosh[c/2 + (I/4)*Pi + (d*x)/2])/d^2 + (2*x*Sinh[c/2 + (I/4)*Pi + (d
*x)/2])/d))/3))/5)*Sqrt[a + I*a*Sinh[c + d*x]]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int x(a+ia\sinh{(dx+c)})^{\frac{5}{2}}\,dx$$

input
```
int(x*(a+I*a*sinh(d*x+c))^(5/2),x)
```

output
```
int(x*(a+I*a*sinh(d*x+c))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x(a+ia\sinh(c+dx))^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int x(a + ia\sinh(c + dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(x*(a+I*a*sinh(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x(a + ia\sinh(c + dx))^{5/2}\,dx = \int (i\,a\sinh\left(dx + c\right) + a)^{\frac{5}{2}}x\,dx$$

input
```
integrate(x*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(5/2)*x, x)
```

Giac [F]

$$\int x(a + ia\sinh(c + dx))^{5/2}\,dx = \int (i\,a\sinh\left(dx + c\right) + a)^{\frac{5}{2}}x\,dx$$

input
```
integrate(x*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(5/2)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+ia\sinh(c+dx))^{5/2}\,dx=\int x\,\left(a+a\sinh(c+d\,x)\ 1\mathrm{i}\right)^{5/2}\,dx$$

input
```
int(x*(a + a*sinh(c + d*x)*1i)^(5/2),x)
```

output
```
int(x*(a + a*sinh(c + d*x)*1i)^(5/2), x)
```

Reduce [F]

$$\int x(a+ia\sinh(c+dx))^{5/2}\,dx=\sqrt{a}\,a^2\left(-\left(\int\sqrt{\sinh(dx+c)\,i+1}\,\sinh(dx+c)^2\,xdx\right)+2\left(\int\sqrt{\sinh(dx+c)\,i+1}\,\sinh(dx+c)\,xdx\right)i+\int\sqrt{\sinh(dx+c)\,i+1}\,xdx\right)$$

input
```
int(x*(a+I*a*sinh(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*( - int(sqrt(sinh(c + d*x)*i + 1)*sinh(c + d*x)**2*x,x) + 2*i
nt(sqrt(sinh(c + d*x)*i + 1)*sinh(c + d*x)*x,x)*i + int(sqrt(sinh(c + d*x)
*i + 1)*x,x))
```

3.133 $\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 403

$$\begin{aligned}\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x}\,dx &= -\frac{1}{4}ia^2\text{Chi}\left(\frac{5dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{i\pi}{4}\right.\\
&\left.+\frac{dx}{2}\right)\sinh\left(\frac{5c}{2}-\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}+\frac{5}{2}ia^2\text{Chi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{i\pi}{4}\right.\\
&\left.+\frac{dx}{2}\right)\sinh\left(\frac{1}{4}(2c-i\pi)\right)\sqrt{a+ia\sinh(c+dx)}+\frac{5}{4}ia^2\text{Chi}\left(\frac{3dx}{2}\right)\text{sech}\left(\frac{c}{2}\right.\\
&\left.+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{1}{4}(6c+i\pi)\right)\sqrt{a+ia\sinh(c+dx)}\\
&+\frac{5}{2}ia^2\cosh\left(\frac{1}{4}(2c-i\pi)\right)\text{sech}\left(\frac{c}{2}+\frac{i\pi}{4}\right.\\
&\left.+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\text{Shi}\left(\frac{dx}{2}\right)+\frac{5}{4}ia^2\cosh\left(\frac{1}{4}(6c+i\pi)\right)\text{sech}\left(\frac{c}{2}\right.\\
&\left.+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\text{Shi}\left(\frac{3dx}{2}\right)-\frac{1}{4}ia^2\cosh\left(\frac{5c}{2}\right.\\
&\left.-\frac{i\pi}{4}\right)\text{sech}\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\text{Shi}\left(\frac{5dx}{2}\right)\end{aligned}$$

output
```
-1/4*a^2*Chi(5/2*d*x)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*cosh(5/2*c+1/4*I*Pi)*(a
+I*a*sinh(d*x+c))^(1/2)+5/2*a^2*Chi(1/2*d*x)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*
cosh(1/2*c+1/4*I*Pi)*(a+I*a*sinh(d*x+c))^(1/2)+5/4*I*a^2*Chi(3/2*d*x)*sech
(1/2*c+1/4*I*Pi+1/2*d*x)*sinh(3/2*c+1/4*I*Pi)*(a+I*a*sinh(d*x+c))^(1/2)+5/
2*a^2*sinh(1/2*c+1/4*I*Pi)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*(a+I*a*sinh(d*x+c)
)^(1/2)*Shi(1/2*d*x)+5/4*I*a^2*cosh(3/2*c+1/4*I*Pi)*sech(1/2*c+1/4*I*Pi+1/
2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)*Shi(3/2*d*x)-1/4*a^2*sinh(5/2*c+1/4*I*Pi)
*sech(1/2*c+1/4*I*Pi+1/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)*Shi(5/2*d*x)
```

Mathematica [A] (verified)

Time = 4.32 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.60

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x}\,dx = \frac{a^2(-i+\sinh(c+dx))^2\sqrt{a+ia\sinh(c+dx)}\left(\cosh\left(\frac{5c}{2}\right)\text{Chi}\left(\frac{5dx}{2}\right)-10\text{Chi}\right.}{}$$

input
```
Integrate[(a + I*a*Sinh[c + d*x])^(5/2)/x,x]
```

output
```
(a^2*(-I + Sinh[c + d*x])^2*Sqrt[a + I*a*Sinh[c + d*x]]*(Cosh[(5*c)/2]*Cos
hIntegral[(5*d*x)/2] - 10*CoshIntegral[(d*x)/2]*(Cosh[c/2] + I*Sinh[c/2])
+ 5*CoshIntegral[(3*d*x)/2]*(Cosh[(3*c)/2] - I*Sinh[(3*c)/2]) + I*CoshInte
gral[(5*d*x)/2]*Sinh[(5*c)/2] - (10*I)*Cosh[c/2]*SinhIntegral[(d*x)/2] - 1
0*Sinh[c/2]*SinhIntegral[(d*x)/2] - (5*I)*Cosh[(3*c)/2]*SinhIntegral[(3*d*
x)/2] + 5*Sinh[(3*c)/2]*SinhIntegral[(3*d*x)/2] + I*Cosh[(5*c)/2]*SinhInte
gral[(5*d*x)/2] + Sinh[(5*c)/2]*SinhIntegral[(5*d*x)/2]))/(4*(Cosh[(c + d*
x)/2] + I*Sinh[(c + d*x)/2])^5)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.51, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3800, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x}\,dx$$

$\Big\downarrow$ 3042

$$\int \frac{(a+a\sin(ic+idx))^{5/2}}{x}dx$$

$\Big\downarrow$ 3800

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\int \frac{\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{x}dx$$

$\Big\downarrow$ 3042

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\int \frac{\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^5}{x}dx$$

$\Big\downarrow$ 3793

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\int \left(\frac{5i\sinh\left(\frac{1}{4}(2c-i\pi)+\frac{dx}{2}\right)}{8x}+\frac{5i\sinh\left(\frac{1}{4}(6c+i\pi)+\frac{3dx}{2}\right)}{16x}-\frac{i\,\text{si}$$

$\Big\downarrow$ 2009

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(-\frac{1}{16}i\sinh\left(\frac{5c}{2}-\frac{i\pi}{4}\right)\text{Chi}\left(\frac{5dx}{2}\right)+\frac{5}{8}i\sinh\left(\frac{1}{4}(2c-i\pi)\right)\text{Chi}\left(\right.\right.$$

input
```
Int[(a + I*a*Sinh[c + d*x])^(5/2)/x,x]
```

output
```
4*a^2*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*((-1/16*I
)*CoshIntegral[(5*d*x)/2]*Sinh[(5*c)/2 - (I/4)*Pi] + ((5*I)/8)*CoshIntegra
l[(d*x)/2]*Sinh[(2*c - I*Pi)/4] + ((5*I)/16)*CoshIntegral[(3*d*x)/2]*Sinh[
(6*c + I*Pi)/4] + ((5*I)/8)*Cosh[(2*c - I*Pi)/4]*SinhIntegral[(d*x)/2] + (
(5*I)/16)*Cosh[(6*c + I*Pi)/4]*SinhIntegral[(3*d*x)/2] - (I/16)*Cosh[(5*c)
/2 - (I/4)*Pi]*SinhIntegral[(5*d*x)/2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int \frac{(a + ia \sinh(dx + c))^{\frac{5}{2}}}{x} dx$$

input
```
int((a+I*a*sinh(d*x+c))^(5/2)/x,x)
```

output
```
int((a+I*a*sinh(d*x+c))^(5/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*sinh(d*x+c))^(5/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*sinh(d*x+c))**(5/2)/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x}\,dx = \int \frac{(i\,a\sinh\left(dx+c\right)+a)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate((a+I*a*sinh(d*x+c))^(5/2)/x,x, algorithm="maxima")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(5/2)/x, x)
```

Giac [F]

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x}\,dx=\int \frac{(i\,a\sinh(dx+c)+a)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate((a+I*a*sinh(d*x+c))^(5/2)/x,x, algorithm="giac")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(5/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x}\,dx=\int \frac{(a+a\sinh(c+d\,x)\ 1\mathrm{i})^{5/2}}{x}\,dx$$

input
```
int((a + a*sinh(c + d*x)*1i)^(5/2)/x,x)
```

output
```
int((a + a*sinh(c + d*x)*1i)^(5/2)/x, x)
```

Reduce [F]

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x}\,dx=\sqrt{a}\,a^2\left(\int \frac{\sqrt{\sinh(dx+c)\,i+1}}{x}dx - \left(\int \frac{\sqrt{\sinh(dx+c)\,i+1}\ \sinh(dx+c)^2}{x}dx\right) + 2\left(\int \frac{\sqrt{\sinh(dx+c)\,i+1}\ \sinh(dx+c)}{x}dx\right)i\right)$$

input
```
int((a+I*a*sinh(d*x+c))^(5/2)/x,x)
```

output

```
sqrt(a)*a**2*(int(sqrt(sinh(c + d*x)*i + 1)/x,x) - int((sqrt(sinh(c + d*x)
*i + 1)*sinh(c + d*x)**2)/x,x) + 2*int((sqrt(sinh(c + d*x)*i + 1)*sinh(c +
 d*x))/x,x)*i)
```

3.134 $\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 444

$$\begin{aligned}
&\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^2}\,dx = \\
&-\frac{4a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{x} \\
&-\frac{5}{8}a^2d\text{Chi}\left(\frac{5dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{5c}{2}\right. \\
&\left.+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}-\frac{15}{8}a^2d\text{Chi}\left(\frac{3dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{i\pi}{4}\right. \\
&\left.+\frac{dx}{2}\right)\sinh\left(\frac{1}{4}(6c-i\pi)\right)\sqrt{a+ia\sinh(c+dx)}+\frac{5}{4}a^2d\text{Chi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{c}{2}\right. \\
&\left.+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{1}{4}(2c+i\pi)\right)\sqrt{a+ia\sinh(c+dx)} \\
&+\frac{5}{4}a^2d\cosh\left(\frac{1}{4}(2c+i\pi)\right)\text{sech}\left(\frac{c}{2}+\frac{i\pi}{4}\right. \\
&\left.+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\text{Shi}\left(\frac{dx}{2}\right)-\frac{15}{8}a^2d\cosh\left(\frac{1}{4}(6c-i\pi)\right)\text{sech}\left(\frac{c}{2}\right. \\
&\left.+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\text{Shi}\left(\frac{3dx}{2}\right)-\frac{5}{8}a^2d\cosh\left(\frac{5c}{2}\right. \\
&\left.+\frac{i\pi}{4}\right)\text{sech}\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\text{Shi}\left(\frac{5dx}{2}\right)
\end{aligned}$$

output
```
-4*a^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^4*(a+I*a*sinh(d*x+c))^(1/2)/x-5/8*a^2*
d*Chi(5/2*d*x)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*sinh(5/2*c+1/4*I*Pi)*(a+I*a*si
nh(d*x+c))^(1/2)+15/8*I*a^2*d*Chi(3/2*d*x)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*co
sh(3/2*c+1/4*I*Pi)*(a+I*a*sinh(d*x+c))^(1/2)+5/4*a^2*d*Chi(1/2*d*x)*sech(1
/2*c+1/4*I*Pi+1/2*d*x)*sinh(1/2*c+1/4*I*Pi)*(a+I*a*sinh(d*x+c))^(1/2)+5/4*
a^2*d*cosh(1/2*c+1/4*I*Pi)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*(a+I*a*sinh(d*x+c)
)^(1/2)*Shi(1/2*d*x)+15/8*I*a^2*d*sinh(3/2*c+1/4*I*Pi)*sech(1/2*c+1/4*I*Pi
+1/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)*Shi(3/2*d*x)-5/8*a^2*d*cosh(5/2*c+1/4*
I*Pi)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)*Shi(5/2*d*x)
```

Mathematica [A] (verified)

Time = 5.01 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.78

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^2}\,dx = \frac{a^2(-i+\sinh(c+dx))^2\sqrt{a+ia\sinh(c+dx)}\left(20\cosh\left(\frac{1}{2}(c+dx)\right)-10\,\mathrm{co}\right.}{}$$

input
```
Integrate[(a + I*a*Sinh[c + d*x])^(5/2)/x^2,x]
```

output
```
(a^2*(-I + Sinh[c + d*x])^2*Sqrt[a + I*a*Sinh[c + d*x]]*(20*Cosh[(c + d*x)
/2] - 10*Cosh[(3*(c + d*x))/2] - 2*Cosh[(5*(c + d*x))/2] + (5*I)*d*x*Cosh[
(5*c)/2]*CoshIntegral[(5*d*x)/2] - (10*I)*d*x*CoshIntegral[(d*x)/2]*(Cosh[
c/2] - I*Sinh[c/2]) + 15*d*x*CoshIntegral[(3*d*x)/2]*((-I)*Cosh[(3*c)/2] +
 Sinh[(3*c)/2]) + 5*d*x*CoshIntegral[(5*d*x)/2]*Sinh[(5*c)/2] + (20*I)*Sin
h[(c + d*x)/2] + (10*I)*Sinh[(3*(c + d*x))/2] - (2*I)*Sinh[(5*(c + d*x))/2
] - 10*d*x*Cosh[c/2]*SinhIntegral[(d*x)/2] - (10*I)*d*x*Sinh[c/2]*SinhInte
gral[(d*x)/2] + 15*d*x*Cosh[(3*c)/2]*SinhIntegral[(3*d*x)/2] - (15*I)*d*x*
Sinh[(3*c)/2]*SinhIntegral[(3*d*x)/2] + 5*d*x*Cosh[(5*c)/2]*SinhIntegral[(
5*d*x)/2] + (5*I)*d*x*Sinh[(5*c)/2]*SinhIntegral[(5*d*x)/2]))/(8*x*(Cosh[(
c + d*x)/2] + I*Sinh[(c + d*x)/2])^5)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.54, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3800, 3042, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+a\sin(ic+idx))^{5/2}}{x^2}dx$$

$$\Big\downarrow \text{3800}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\int \frac{\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\int \frac{\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^5}{x^2}dx$$

$$\Big\downarrow \text{3794}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{5}{2}id\int \left(\frac{3\cosh\left(\frac{3c}{2}+\frac{3dx}{2}+\frac{i\pi}{4}\right)}{16x}-\frac{i\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{8x}+\frac{i\sinh}{}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{5}{2}id\left(\frac{1}{16}i\sinh\left(\frac{5c}{2}+\frac{i\pi}{4}\right)\text{Chi}\left(\frac{5dx}{2}\right)+\frac{3}{16}i\sinh\left(\frac{1}{4}(6c-i\pi)\right)\right.\right.$$

input
```
Int[(a + I*a*Sinh[c + d*x])^(5/2)/x^2,x]
```

output
```
4*a^2*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*(-(Cosh[c
/2 + (I/4)*Pi + (d*x)/2]^5/x) + ((5*I)/2)*d*((I/16)*CoshIntegral[(5*d*x)/2
]*Sinh[(5*c)/2 + (I/4)*Pi] + ((3*I)/16)*CoshIntegral[(3*d*x)/2]*Sinh[(6*c
- I*Pi)/4] - (I/8)*CoshIntegral[(d*x)/2]*Sinh[(2*c + I*Pi)/4] - (I/8)*Cosh
[(2*c + I*Pi)/4]*SinhIntegral[(d*x)/2] + ((3*I)/16)*Cosh[(6*c - I*Pi)/4]*S
inhIntegral[(3*d*x)/2] + (I/16)*Cosh[(5*c)/2 + (I/4)*Pi]*SinhIntegral[(5*d
*x)/2]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int \frac{(a + ia \sinh(dx + c))^{\frac{5}{2}}}{x^2} dx$$

input
```
int((a+I*a*sinh(d*x+c))^(5/2)/x^2,x)
```

output
```
int((a+I*a*sinh(d*x+c))^(5/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a + ia \sinh(c + dx))^{5/2}}{x^2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*sinh(d*x+c))^(5/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + ia \sinh(c + dx))^{5/2}}{x^2} dx = \text{Timed out}$$

input
```
integrate((a+I*a*sinh(d*x+c))**(5/2)/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^2}\,dx = \int \frac{(i\,a\sinh\left(dx+c\right)+a)^{\frac{5}{2}}}{x^2}\,dx$$

input
```
integrate((a+I*a*sinh(d*x+c))^(5/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(5/2)/x^2, x)
```

Giac [F]

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^2}\,dx = \int \frac{(i\,a\sinh\left(dx+c\right)+a)^{\frac{5}{2}}}{x^2}\,dx$$

input
```
integrate((a+I*a*sinh(d*x+c))^(5/2)/x^2,x, algorithm="giac")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(5/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^2}\,dx = \int \frac{(a+a\sinh(c+d\,x)\ 1\mathrm{i})^{5/2}}{x^2}\,dx$$

input
```
int((a + a*sinh(c + d*x)*1i)^(5/2)/x^2,x)
```

output
```
int((a + a*sinh(c + d*x)*1i)^(5/2)/x^2, x)
```

Reduce [F]

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^2}\,dx = \sqrt{a}\,a^2\left(\int \frac{\sqrt{\sinh(dx+c)\,i+1}}{x^2}dx - \left(\int \frac{\sqrt{\sinh(dx+c)\,i+1}\,\sinh(dx+c)^2}{x^2}dx\right) + 2\left(\int \frac{\sqrt{\sinh(dx+c)\,i+1}\,\sinh(dx+c)}{x^2}dx\right)i\right)$$

input

```
int((a+I*a*sinh(d*x+c))^(5/2)/x^2,x)
```

output

```
sqrt(a)*a**2*(int(sqrt(sinh(c + d*x)*i + 1)/x**2,x) - int((sqrt(sinh(c + d
*x)*i + 1)*sinh(c + d*x)**2)/x**2,x) + 2*int((sqrt(sinh(c + d*x)*i + 1)*si
nh(c + d*x))/x**2,x)*i)
```

3.135 $\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 536

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^3}\,dx = -\frac{2a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{x^2} - \frac{25}{32}ia^2d^2\text{Chi}\left(\frac{5dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{5c}{2}-\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)} + \frac{5}{16}ia^2d^2\text{Chi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{1}{4}(2c-i\pi)\right)\sqrt{a+ia\sinh(c+dx)} + \frac{45}{32}ia^2d^2\text{Chi}\left(\frac{3dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{1}{4}(6c+i\pi)\right)\sqrt{a+ia\sinh(c+dx)} - \frac{5a^2d\cosh^3\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{x} + \frac{5}{16}ia^2d^2\cosh\left(\frac{1}{4}(2c-i\pi)\right)\text{sech}\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\text{Shi}\left(\frac{dx}{2}\right) + \frac{45}{32}ia^2d^2\cosh\left(\frac{1}{4}(6c+i\pi)\right)\text{sech}\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\text{Shi}\left(\frac{3dx}{2}\right) - \frac{25}{32}ia^2d^2\cosh\left(\frac{5c}{2}-\frac{i\pi}{4}\right)\text{sech}\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\text{Shi}\left(\frac{5dx}{2}\right)$$

output
```
-2*a^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^4*(a+I*a*sinh(d*x+c))^(1/2)/x^2-25/32*
a^2*d^2*Chi(5/2*d*x)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*cosh(5/2*c+1/4*I*Pi)*(a+
I*a*sinh(d*x+c))^(1/2)+5/16*a^2*d^2*Chi(1/2*d*x)*sech(1/2*c+1/4*I*Pi+1/2*d
*x)*cosh(1/2*c+1/4*I*Pi)*(a+I*a*sinh(d*x+c))^(1/2)+45/32*I*a^2*d^2*Chi(3/2
*d*x)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*sinh(3/2*c+1/4*I*Pi)*(a+I*a*sinh(d*x+c)
)^(1/2)-5*a^2*d*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^3*sinh(1/2*c+1/4*I*Pi+1/2*d*x
)*(a+I*a*sinh(d*x+c))^(1/2)/x+5/16*a^2*d^2*sinh(1/2*c+1/4*I*Pi)*sech(1/2*c
+1/4*I*Pi+1/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)*Shi(1/2*d*x)+45/32*I*a^2*d^2*
cosh(3/2*c+1/4*I*Pi)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2
)*Shi(3/2*d*x)-25/32*a^2*d^2*sinh(5/2*c+1/4*I*Pi)*sech(1/2*c+1/4*I*Pi+1/2*
d*x)*(a+I*a*sinh(d*x+c))^(1/2)*Shi(5/2*d*x)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4751 vs. 2(536) = 1072.

Time = 10.21 (sec) , antiderivative size = 4751, normalized size of antiderivative = 8.86

$$\int \frac{(a + ia\sinh(c + dx))^{5/2}}{x^3}\, dx = \text{Result too large to show}$$

input
```
Integrate[(a + I*a*Sinh[c + d*x])^(5/2)/x^3,x]
```

output
```
(2*((1/128 + I/128)*Cosh[5*(c/2 + (d*x)/2)] - (1/128 + I/128)*Sinh[5*(c/2
+ (d*x)/2)])*(a + I*a*Sinh[c + d*x])^(5/2)*((-4*I)*d^3 - (10*I)*c*d^3 + (2
0*I)*d^3*(c/2 + (d*x)/2) + 20*d^3*Cosh[2*(c/2 + (d*x)/2)] + 30*c*d^3*Cosh[
2*(c/2 + (d*x)/2)] - 60*d^3*(c/2 + (d*x)/2)*Cosh[2*(c/2 + (d*x)/2)] + (40*
I)*d^3*Cosh[4*(c/2 + (d*x)/2)] + (20*I)*c*d^3*Cosh[4*(c/2 + (d*x)/2)] - (4
0*I)*d^3*(c/2 + (d*x)/2)*Cosh[4*(c/2 + (d*x)/2)] - 40*d^3*Cosh[6*(c/2 + (d
*x)/2)] + 20*c*d^3*Cosh[6*(c/2 + (d*x)/2)] - 40*d^3*(c/2 + (d*x)/2)*Cosh[6
*(c/2 + (d*x)/2)] - (20*I)*d^3*Cosh[8*(c/2 + (d*x)/2)] + (30*I)*c*d^3*Cosh
[8*(c/2 + (d*x)/2)] - (60*I)*d^3*(c/2 + (d*x)/2)*Cosh[8*(c/2 + (d*x)/2)] +
 4*d^3*Cosh[10*(c/2 + (d*x)/2)] - 10*c*d^3*Cosh[10*(c/2 + (d*x)/2)] + 20*d
^3*(c/2 + (d*x)/2)*Cosh[10*(c/2 + (d*x)/2)] - (10*I)*c^2*d^3*Cosh[c/2 - 5*
(c/2 + (d*x)/2)]*CoshIntegral[(d*x)/2] + (40*I)*c*d^3*(c/2 + (d*x)/2)*Cosh
[c/2 - 5*(c/2 + (d*x)/2)]*CoshIntegral[(d*x)/2] - (40*I)*d^3*(c/2 + (d*x)/
2)^2*Cosh[c/2 - 5*(c/2 + (d*x)/2)]*CoshIntegral[(d*x)/2] + 10*c^2*d^3*Cosh
[c/2 + 5*(c/2 + (d*x)/2)]*CoshIntegral[(d*x)/2] - 40*c*d^3*(c/2 + (d*x)/2)
*Cosh[c/2 + 5*(c/2 + (d*x)/2)]*CoshIntegral[(d*x)/2] + 40*d^3*(c/2 + (d*x)
/2)^2*Cosh[c/2 + 5*(c/2 + (d*x)/2)]*CoshIntegral[(d*x)/2] - 45*c^2*d^3*Cos
h[(3*c)/2 - 5*(c/2 + (d*x)/2)]*CoshIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)]
+ 180*c*d^3*(c/2 + (d*x)/2)*Cosh[(3*c)/2 - 5*(c/2 + (d*x)/2)]*CoshIntegral
[(-3*c)/2 + 3*(c/2 + (d*x)/2)] - 180*d^3*(c/2 + (d*x)/2)^2*Cosh[(3*c)/2...
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 408, normalized size of antiderivative = 0.76, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3800, 3042, 3795, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+a\sin(ic+idx))^{5/2}}{x^3}dx$$

$$\Big\downarrow \text{ 3800}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\int\frac{\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{x^3}dx$$

$\Big\downarrow$ 3042

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\int\frac{\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^5}{x^3}dx$$

$\Big\downarrow$ 3795

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{25}{8}d^2\int\frac{\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{x}dx-\frac{5}{2}d^2\int\frac{\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{x}dx\right.$$

$\Big\downarrow$ 3042

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(-\frac{5}{2}d^2\int\frac{\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3}{x}dx+\frac{25}{8}d^2\int\frac{\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^5}{x}dx\right.$$

$\Big\downarrow$ 3793

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(-\frac{5}{2}d^2\int\left(\frac{3i\sinh\left(\frac{1}{4}(2c-i\pi)+\frac{dx}{2}\right)}{4x}+\frac{i\sinh\left(\frac{1}{4}(6c+i\pi)+\frac{3dx}{2}\right)}{4x}\right.\right.$$

$\Big\downarrow$ 2009

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(-\frac{5}{2}d^2\left(\frac{3}{4}i\sinh\left(\frac{1}{4}(2c-i\pi)\right)\text{Chi}\left(\frac{dx}{2}\right)+\frac{1}{4}i\sinh\left(\frac{1}{4}(6c+i\pi)\right)\right.\right.$$

input
```
Int[(a + I*a*Sinh[c + d*x])^(5/2)/x^3,x]
```

output
```
4*a^2*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*(-1/2*Cos
h[c/2 + (I/4)*Pi + (d*x)/2]^5/x^2 - (5*d*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^4*
Sinh[c/2 + (I/4)*Pi + (d*x)/2])/(4*x) - (5*d^2*(((3*I)/4)*CoshIntegral[(d*
x)/2]*Sinh[(2*c - I*Pi)/4] + (I/4)*CoshIntegral[(3*d*x)/2]*Sinh[(6*c + I*P
i)/4] + ((3*I)/4)*Cosh[(2*c - I*Pi)/4]*SinhIntegral[(d*x)/2] + (I/4)*Cosh[
(6*c + I*Pi)/4]*SinhIntegral[(3*d*x)/2]))/2 + (25*d^2*((-1/16*I)*CoshInteg
ral[(5*d*x)/2]*Sinh[(5*c)/2 - (I/4)*Pi] + ((5*I)/8)*CoshIntegral[(d*x)/2]*
Sinh[(2*c - I*Pi)/4] + ((5*I)/16)*CoshIntegral[(3*d*x)/2]*Sinh[(6*c + I*Pi
)/4] + ((5*I)/8)*Cosh[(2*c - I*Pi)/4]*SinhIntegral[(d*x)/2] + ((5*I)/16)*C
osh[(6*c + I*Pi)/4]*SinhIntegral[(3*d*x)/2] - (I/16)*Cosh[(5*c)/2 - (I/4)*
Pi]*SinhIntegral[(5*d*x)/2]))/8)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int \frac{(a + ia\sinh{(dx + c)})^{\frac{5}{2}}}{x^3} dx$$

input
```
int((a+I*a*sinh(d*x+c))^(5/2)/x^3,x)
```

output
```
int((a+I*a*sinh(d*x+c))^(5/2)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a + ia\sinh(c + dx))^{5/2}}{x^3} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*sinh(d*x+c))^(5/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*sinh(d*x+c))**(5/2)/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^3}\,dx = \int \frac{(i\,a\sinh\left(dx+c\right)+a)^{\frac{5}{2}}}{x^3}\,dx$$

input
```
integrate((a+I*a*sinh(d*x+c))^(5/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(5/2)/x^3, x)
```

Giac [F]

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^3}\,dx = \int \frac{(i\,a\sinh\left(dx+c\right)+a)^{\frac{5}{2}}}{x^3}\,dx$$

input
```
integrate((a+I*a*sinh(d*x+c))^(5/2)/x^3,x, algorithm="giac")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(5/2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^3}\,dx=\int \frac{(a+a\sinh(c+d\,x)\ 1\mathrm{i})^{5/2}}{x^3}\,dx$$

input
```
int((a + a*sinh(c + d*x)*1i)^(5/2)/x^3,x)
```

output
```
int((a + a*sinh(c + d*x)*1i)^(5/2)/x^3, x)
```

Reduce [F]

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^3}\,dx=\sqrt{a}\,a^2\left(\int \frac{\sqrt{\sinh(dx+c)\,i+1}}{x^3}dx -\left(\int \frac{\sqrt{\sinh(dx+c)\,i+1}\,\sinh(dx+c)^2}{x^3}dx\right) +2\left(\int \frac{\sqrt{\sinh(dx+c)\,i+1}\,\sinh(dx+c)}{x^3}dx\right)i\right)$$

input
```
int((a+I*a*sinh(d*x+c))^(5/2)/x^3,x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sinh(c + d*x)*i + 1)/x**3,x) - int((sqrt(sinh(c + d
*x)*i + 1)*sinh(c + d*x)**2)/x**3,x) + 2*int((sqrt(sinh(c + d*x)*i + 1)*si
nh(c + d*x))/x**3,x)*i)
```

3.136 $\int \frac{x^3}{\sqrt{a+ia\sinh(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 493

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+ia\sinh(e+fx)}}\,dx &= \frac{4ix^3\operatorname{arctanh}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}} \\
&+ \frac{12ix^2\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}} \\
&- \frac{12ix^2\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{PolyLog}\left(2,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}} \\
&- \frac{48ix\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^3\sqrt{a+ia\sinh(e+fx)}} \\
&+ \frac{48ix\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{PolyLog}\left(3,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^3\sqrt{a+ia\sinh(e+fx)}} \\
&+ \frac{96i\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{PolyLog}\left(4,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^4\sqrt{a+ia\sinh(e+fx)}} \\
&- \frac{96i\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{PolyLog}\left(4,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^4\sqrt{a+ia\sinh(e+fx)}}\end{aligned}$$

output
```
-4*I*x^3*arctanh(exp(1/2*e+3/4*I*Pi+1/2*f*x))*cosh(1/2*e+1/4*I*Pi+1/2*f*x)
/f/(a+I*a*sinh(f*x+e))^(1/2)+12*I*x^2*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog
(2,exp(1/2*e+3/4*I*Pi+1/2*f*x))/f^2/(a+I*a*sinh(f*x+e))^(1/2)-12*I*x^2*cos
h(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(2,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/f^2/(a+I
*a*sinh(f*x+e))^(1/2)-48*I*x*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(3,exp(1/
2*e+3/4*I*Pi+1/2*f*x))/f^3/(a+I*a*sinh(f*x+e))^(1/2)+48*I*x*cosh(1/2*e+1/4
*I*Pi+1/2*f*x)*polylog(3,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/f^3/(a+I*a*sinh(f*x
+e))^(1/2)+96*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(4,exp(1/2*e+3/4*I*Pi+
1/2*f*x))/f^4/(a+I*a*sinh(f*x+e))^(1/2)-96*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*
polylog(4,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/f^4/(a+I*a*sinh(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{\sqrt{a+ia\sinh(e+fx)}}\,dx = \frac{(1-i)(-1)^{3/4}\left(2ie^3\arctan\left(\sqrt[4]{-1}e^{\frac{1}{2}(e+fx)}\right)+e^3\log\left(1-(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)+f^3x^3\log\left(1-(-1)^{3/4}e^{\frac{1}{2}(e+\cdots}\right.\right.}{\cdots}$$

input
```
Integrate[x^3/Sqrt[a + I*a*Sinh[e + f*x]],x]
```

output
```
((1 - I)*(-1)^(3/4)*((2*I)*e^3*ArcTan[(-1)^(1/4)*E^((e + f*x)/2)] + e^3*Lo
g[1 - (-1)^(3/4)*E^((e + f*x)/2)] + f^3*x^3*Log[1 - (-1)^(3/4)*E^((e + f*x
)/2)] - e^3*Log[1 + (-1)^(3/4)*E^((e + f*x)/2)] - f^3*x^3*Log[1 + (-1)^(3/
4)*E^((e + f*x)/2)] - 6*f^2*x^2*PolyLog[2, -((-1)^(3/4)*E^((e + f*x)/2))]
+ 6*f^2*x^2*PolyLog[2, (-1)^(3/4)*E^((e + f*x)/2)] + 24*f*x*PolyLog[3, -((
-1)^(3/4)*E^((e + f*x)/2))] - 24*f*x*PolyLog[3, (-1)^(3/4)*E^((e + f*x)/2)
] - 48*PolyLog[4, -((-1)^(3/4)*E^((e + f*x)/2))] + 48*PolyLog[4, (-1)^(3/4
)*E^((e + f*x)/2)])*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))/(f^4*Sqrt[a
 + I*a*Sinh[e + f*x]])
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.58, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3800, 3042, 4670, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+ia\sinh(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{x^3}{\sqrt{a+a\sin(ie+ifx)}}dx$$

$$\Big\downarrow \text{3800}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\int x^3\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\int x^3\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx}{\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \text{4670}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{6i\int x^2\log\left(1-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}-\frac{6i\int x^2\log\left(1+e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}+\frac{4ix^3\text{arctanh}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f}\right)}{\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \text{3011}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(-\frac{6i\left(\frac{4\int x\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}-\frac{2x^2\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f}\right)}{f}+\frac{6i\left(\frac{4\int x\,\text{PolyLog}\left(2,e^{\frac{1}{4}(2e-i\pi)+\frac{f\cdots}{2}}\right.}{f}\right.}{\cdots}\right.}{\sqrt{a+ia\sinh(e+fx)}}$$

$\downarrow$ 7163

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(-\frac{6i\left(\frac{4\left(\frac{2x\,\text{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f}-\frac{2\int\text{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}\right)}{f}-\frac{2x^2\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f}\right)}{f}\right.}{\sqrt{a+ia}}$$

$\downarrow$ 2720

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(-\frac{6i\left(\frac{4\left(\frac{2x\,\text{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f}-\frac{4\int e^{\frac{1}{4}(i\pi-2e)-\frac{fx}{2}}\,\text{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)de^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}}{f^2}\right)}{f}-\frac{2x^2\,\text{Pol}}{}\right.}{f}\right.}{}$$

$\downarrow$ 7143

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{4ix^3\text{arctanh}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f}-\frac{6i\left(\frac{4\left(\frac{2x\,\text{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f}-\frac{4\,\text{PolyLog}\left(4,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2}\right)}{f}-\frac{2x^2}{}\right.}{f}\right.}{\sqrt{a+ia\sin}}$$

input
```
Int[x^3/Sqrt[a + I*a*Sinh[e + f*x]],x]
```

output
```
(Cosh[e/2 + (I/4)*Pi + (f*x)/2]*(((4*I)*x^3*ArcTanh[E^((2*e - I*Pi)/4 + (f
*x)/2)])/f - ((6*I)*((-2*x^2*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/f
+ (4*((2*x*PolyLog[3, -E^((2*e - I*Pi)/4 + (f*x)/2)])/f - (4*PolyLog[4, -E
^((2*e - I*Pi)/4 + (f*x)/2)])/f^2))/f))/f + ((6*I)*((-2*x^2*PolyLog[2, E^(
(2*e - I*Pi)/4 + (f*x)/2)])/f + (4*((2*x*PolyLog[3, E^((2*e - I*Pi)/4 + (f
*x)/2)])/f - (4*PolyLog[4, E^((2*e - I*Pi)/4 + (f*x)/2)])/f^2))/f))/f))/Sq
rt[a + I*a*Sinh[e + f*x]]
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{x^3}{\sqrt{a + ia\sinh(fx + e)}}dx$$

input
```
int(x^3/(a+I*a*sinh(f*x+e))^(1/2),x)
```

output
```
int(x^3/(a+I*a*sinh(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{x^3}{\sqrt{a + ia\sinh(e + fx)}}\,dx = \int \frac{x^3}{\sqrt{i\,a\sinh(fx + e) + a}}\,dx$$

input
```
integrate(x^3/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(-2*I*sqrt(1/2*I*a*e^(-f*x - e))*x^3*e^(f*x + e)/(a*e^(f*x + e) -
I*a), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{x^3}{\sqrt{ia\left(\sinh\left(e+fx\right)-i\right)}}\,dx$$

input
```
integrate(x**3/(a+I*a*sinh(f*x+e))**(1/2),x)
```

output
```
Integral(x**3/sqrt(I*a*(sinh(e + f*x) - I)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{x^3}{\sqrt{i\,a\sinh\left(fx+e\right)+a}}\,dx$$

input
```
integrate(x^3/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(I*a*sinh(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{x^3}{\sqrt{i\,a\sinh\left(fx+e\right)+a}}\,dx$$

input
```
integrate(x^3/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/sqrt(I*a*sinh(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{x^3}{\sqrt{a+a\sinh\left(e+f\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int(x^3/(a + a*sinh(e + f*x)*1i)^(1/2),x)
```

output
```
int(x^3/(a + a*sinh(e + f*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{a+ia\sinh(e+fx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(fx+e)i+1}\,x^3}{\sinh(fx+e)^2+1}dx-\left(\int \frac{\sqrt{\sinh(fx+e)i+1}\,\sinh(fx+e)x^3}{\sinh(fx+e)^2+1}dx\right)i\right)}{a}$$

input
```
int(x^3/(a+I*a*sinh(f*x+e))^(1/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sinh(e + f*x)*i + 1)*x**3)/(sinh(e + f*x)**2 + 1),x) -
 int((sqrt(sinh(e + f*x)*i + 1)*sinh(e + f*x)*x**3)/(sinh(e + f*x)**2 + 1)
,x)*i))/a
```

3.137 $\int \frac{x^2}{\sqrt{a+ia\sinh(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 349

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+ia\sinh(e+fx)}}\,dx =& \frac{4ix^2\text{arctanh}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}}\\&+\frac{8ix\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}}\\&-\frac{8ix\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\text{PolyLog}\left(2,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}}\\&-\frac{16i\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\text{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^3\sqrt{a+ia\sinh(e+fx)}}\\&+\frac{16i\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\text{PolyLog}\left(3,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^3\sqrt{a+ia\sinh(e+fx)}}\end{aligned}$$

output
```
-4*I*x^2*arctanh(exp(1/2*e+3/4*I*Pi+1/2*f*x))*cosh(1/2*e+1/4*I*Pi+1/2*f*x)
/f/(a+I*a*sinh(f*x+e))^(1/2)+8*I*x*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(2,
exp(1/2*e+3/4*I*Pi+1/2*f*x))/f^2/(a+I*a*sinh(f*x+e))^(1/2)-8*I*x*cosh(1/2*
e+1/4*I*Pi+1/2*f*x)*polylog(2,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/f^2/(a+I*a*sin
h(f*x+e))^(1/2)-16*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(3,exp(1/2*e+3/4*
I*Pi+1/2*f*x))/f^3/(a+I*a*sinh(f*x+e))^(1/2)+16*I*cosh(1/2*e+1/4*I*Pi+1/2*
f*x)*polylog(3,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/f^3/(a+I*a*sinh(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{\sqrt{a+ia\sinh(e+fx)}}\,dx = \frac{(1+i)(-1)^{3/4}\left(-2ie^2\arctan\left(\sqrt[4]{-1}e^{\frac{1}{2}(e+fx)}\right)-e^2\log\left(1-(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)+f^2x^2\log\left(1-(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right.\right.}{}$$

input
```
Integrate[x^2/Sqrt[a + I*a*Sinh[e + f*x]],x]
```

output
```
((1 + I)*(-1)^(3/4)*((-2*I)*e^2*ArcTan[(-1)^(1/4)*E^((e + f*x)/2)] - e^2*L
og[1 - (-1)^(3/4)*E^((e + f*x)/2)] + f^2*x^2*Log[1 - (-1)^(3/4)*E^((e + f*
x)/2)] + e^2*Log[1 + (-1)^(3/4)*E^((e + f*x)/2)] - f^2*x^2*Log[1 + (-1)^(3
/4)*E^((e + f*x)/2)] - 4*f*x*PolyLog[2, -((-1)^(3/4)*E^((e + f*x)/2))] + 4
*f*x*PolyLog[2, (-1)^(3/4)*E^((e + f*x)/2)] + 8*PolyLog[3, -((-1)^(3/4)*E^
((e + f*x)/2))] - 8*PolyLog[3, (-1)^(3/4)*E^((e + f*x)/2)])*((-I)*Cosh[(e
+ f*x)/2] + Sinh[(e + f*x)/2]))/(f^3*Sqrt[a + I*a*Sinh[e + f*x]])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.60, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3800, 3042, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+ia\sinh(e+fx)}}\,dx$$

$\Big\downarrow$ 3042

$$\int \frac{x^2}{\sqrt{a+a\sin(ie+ifx)}}dx$$

$\Big\downarrow$ 3800

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\int x^2\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{\sqrt{a+ia\sinh(e+fx)}}$$

$\Big\downarrow$ 3042

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\int x^2\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx}{\sqrt{a+ia\sinh(e+fx)}}$$

$\Big\downarrow$ 4670

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{4i\int x\log\left(1-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}-\frac{4i\int x\log\left(1+e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}+\frac{4ix^2\text{arctanh}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f}\right)}{\sqrt{a+ia\sinh(e+fx)}}$$

$\Big\downarrow$ 3011

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(-\frac{4i\left(\frac{2\int \text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}-\frac{2x\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f}\right)}{f}+\frac{4i\left(\frac{2\int \text{PolyLog}\left(2,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)d\right.}{f}}{f}\right.}{\sqrt{a+ia\sinh(e+fx)}}$$

$\Big\downarrow$ 2720

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(-\frac{4i\left(\frac{4\int e^{\frac{1}{4}(i\pi-2e)-\frac{fx}{2}}\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)de^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}}{f^2}-\frac{2x\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f}\right)}{f}+\frac{4i\left(\frac{4\int\right.}{}}{}\right.}{\sqrt{a+ia\sinh(e}}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{4ix^2\text{arctanh}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f}-\frac{4i\left(\frac{4\,\text{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2}-\frac{2x\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f}\right)}{f}+\frac{4i\left(\cdots\right.}{}\right.}{\sqrt{a+ia\sinh(e+fx)}}$$

input
```
Int[x^2/Sqrt[a + I*a*Sinh[e + f*x]],x]
```

output
```
(Cosh[e/2 + (I/4)*Pi + (f*x)/2]*(((4*I)*x^2*ArcTanh[E^((2*e - I*Pi)/4 + (f
*x)/2)])/f - ((4*I)*((-2*x*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/f +
(4*PolyLog[3, -E^((2*e - I*Pi)/4 + (f*x)/2)])/f^2))/f + ((4*I)*((-2*x*Poly
Log[2, E^((2*e - I*Pi)/4 + (f*x)/2)])/f + (4*PolyLog[3, E^((2*e - I*Pi)/4
+ (f*x)/2)])/f^2))/f))/Sqrt[a + I*a*Sinh[e + f*x]]
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{a + ia \sinh(fx + e)}} dx$$

input
```
int(x^2/(a+I*a*sinh(f*x+e))^(1/2),x)
```

output
```
int(x^2/(a+I*a*sinh(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{a + ia \sinh(e + fx)}}\, dx = \int \frac{x^2}{\sqrt{i\, a \sinh(fx + e) + a}}\, dx$$

input
```
integrate(x^2/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(-2*I*sqrt(1/2*I*a*e^(-f*x - e))*x^2*e^(f*x + e)/(a*e^(f*x + e) -
I*a), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a + ia\sinh(e + fx)}}\,dx = \int \frac{x^2}{\sqrt{ia\left(\sinh\left(e + fx\right) - i\right)}}\,dx$$

input
```
integrate(x**2/(a+I*a*sinh(f*x+e))**(1/2),x)
```

output
```
Integral(x**2/sqrt(I*a*(sinh(e + f*x) - I)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a + ia\sinh(e + fx)}}\,dx = \int \frac{x^2}{\sqrt{i\,a\sinh\left(fx + e\right) + a}}\,dx$$

input
```
integrate(x^2/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(I*a*sinh(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a + ia\sinh(e + fx)}}\,dx = \int \frac{x^2}{\sqrt{i\,a\sinh\left(fx + e\right) + a}}\,dx$$

input
```
integrate(x^2/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(I*a*sinh(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{x^2}{\sqrt{a+a\sinh\left(e+f\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int(x^2/(a + a*sinh(e + f*x)*1i)^(1/2),x)
```

output
```
int(x^2/(a + a*sinh(e + f*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+ia\sinh(e+fx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(fx+e)i+1}\,x^2}{\sinh(fx+e)^2+1}dx-\left(\int \frac{\sqrt{\sinh(fx+e)i+1}\,\sinh(fx+e)x^2}{\sinh(fx+e)^2+1}dx\right)i\right)}{a}$$

input
```
int(x^2/(a+I*a*sinh(f*x+e))^(1/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sinh(e + f*x)*i + 1)*x**2)/(sinh(e + f*x)**2 + 1),x) -
 int((sqrt(sinh(e + f*x)*i + 1)*sinh(e + f*x)*x**2)/(sinh(e + f*x)**2 + 1)
,x)*i))/a
```

3.138 $\int \frac{x}{\sqrt{a+ia\sinh(e+fx)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 207

$$\int \frac{x}{\sqrt{a+ia\sinh(e+fx)}}\,dx = \frac{4ix\operatorname{arctanh}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}} + \frac{4i\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}} - \frac{4i\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{PolyLog}\left(2,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}}$$

output

```
-4*I*x*arctanh(exp(1/2*e+3/4*I*Pi+1/2*f*x))*cosh(1/2*e+1/4*I*Pi+1/2*f*x)/f
/(a+I*a*sinh(f*x+e))^(1/2)+4*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(2,exp(
1/2*e+3/4*I*Pi+1/2*f*x))/f^2/(a+I*a*sinh(f*x+e))^(1/2)-4*I*cosh(1/2*e+1/4*
I*Pi+1/2*f*x)*polylog(2,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/f^2/(a+I*a*sinh(f*x+
e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.86

$$\int \frac{x}{\sqrt{a+ia\sinh(e+fx)}}\,dx$$
$$=\frac{(2-2i)(-1)^{3/4}\left(ie\arctan\left(\sqrt[4]{-1}e^{\frac{1}{2}(e+fx)}\right)+\frac{1}{2}(e+fx)\log\left(1-(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)-\frac{1}{2}(e+fx)\log\left(1+\right.\right.}{f}$$

input
```
Integrate[x/Sqrt[a + I*a*Sinh[e + f*x]],x]
```

output
```
((2 - 2*I)*(-1)^(3/4)*(I*e*ArcTan[(-1)^(1/4)*E^((e + f*x)/2)] + ((e + f*x)
*Log[1 - (-1)^(3/4)*E^((e + f*x)/2)])/2 - ((e + f*x)*Log[1 + (-1)^(3/4)*E^
((e + f*x)/2)])/2 - PolyLog[2, -((-1)^(3/4)*E^((e + f*x)/2))] + PolyLog[2,
 (-1)^(3/4)*E^((e + f*x)/2)])*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))/(
f^2*Sqrt[a + I*a*Sinh[e + f*x]])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.65, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 3800, 3042, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+ia\sinh(e+fx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{x}{\sqrt{a+a\sin(ie+ifx)}}dx$$
$$\Big\downarrow \text{3800}$$
$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\int x\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\int x\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx}{\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \text{4670}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{2i\int\log\left(1-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}-\frac{2i\int\log\left(1+e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}+\frac{4ix\text{arctanh}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f}\right)}{\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{4i\int e^{\frac{1}{4}(i\pi-2e)-\frac{fx}{2}}\log\left(1-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)de^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}}{f^2}-\frac{4i\int e^{\frac{1}{4}(i\pi-2e)-\frac{fx}{2}}\log\left(1+e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)de^{\frac{1}{4}(2e-}}{f^2}\right.}{\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{4ix\text{arctanh}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f}+\frac{4i\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2}-\frac{4i\,\text{PolyLog}\left(2,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2}\right)}{\sqrt{a+ia\sinh(e+fx)}}$$

input

```
Int[x/Sqrt[a + I*a*Sinh[e + f*x]],x]
```

output

```
(Cosh[e/2 + (I/4)*Pi + (f*x)/2]*(((4*I)*x*ArcTanh[E^((2*e - I*Pi)/4 + (f*x
)/2)])/f + ((4*I)*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/f^2 - ((4*I)*
PolyLog[2, E^((2*e - I*Pi)/4 + (f*x)/2)])/f^2))/Sqrt[a + I*a*Sinh[e + f*x]
]
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{x}{\sqrt{a+ia\sinh\left(fx+e\right)}}dx$$

input
```
int(x/(a+I*a*sinh(f*x+e))^(1/2),x)
```

output
```
int(x/(a+I*a*sinh(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{x}{\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{x}{\sqrt{i\,a\sinh\left(fx+e\right)+a}}\,dx$$

input
```
integrate(x/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(-2*I*sqrt(1/2*I*a*e^(-f*x - e))*x*e^(f*x + e)/(a*e^(f*x + e) - I*
a), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{x}{\sqrt{ia\left(\sinh\left(e+fx\right)-i\right)}}\,dx$$

input
```
integrate(x/(a+I*a*sinh(f*x+e))**(1/2),x)
```

output
```
Integral(x/sqrt(I*a*(sinh(e + f*x) - I)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{x}{\sqrt{i\,a\sinh\left(fx+e\right)+a}}\,dx$$

input
```
integrate(x/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(I*a*sinh(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{a+ia\sinh(e+fx)}}\,dx=\int \frac{x}{\sqrt{i\,a\sinh\left(fx+e\right)+a}}\,dx$$

input
```
integrate(x/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(I*a*sinh(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+ia\sinh(e+fx)}}\,dx=\int \frac{x}{\sqrt{a+a\sinh\left(e+f\,x\right)\,1{}\mathrm{i}}}\,dx$$

input
```
int(x/(a + a*sinh(e + f*x)*1i)^(1/2),x)
```

output
```
int(x/(a + a*sinh(e + f*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{a+ia\sinh(e+fx)}}\,dx = \frac{\sqrt{a}\left(-\left(\int \frac{\sqrt{\sinh(fx+e)i+1}\,\sinh(fx+e)x}{\sinh(fx+e)^2+1}dx\right)i+\int \frac{\sqrt{\sinh(fx+e)i+1}\,x}{\sinh(fx+e)^2+1}dx\right)}{a}$$

input
```
int(x/(a+I*a*sinh(f*x+e))^(1/2),x)
```

output
```
(sqrt(a)*( - int((sqrt(sinh(e + f*x)*i + 1)*sinh(e + f*x)*x)/(sinh(e + f*x
)**2 + 1),x)*i + int((sqrt(sinh(e + f*x)*i + 1)*x)/(sinh(e + f*x)**2 + 1),
x)))/a
```

3.139 $\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx = \text{Int}\left(\frac{1}{x\sqrt{a+ia\sinh(e+fx)}},x\right)$$

output
```
Defer(Int)(1/x/(a+I*a*sinh(f*x+e))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 3.51 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx$$

input
```
Integrate[1/(x*Sqrt[a + I*a*Sinh[e + f*x]]),x]
```

output
```
Integrate[1/(x*Sqrt[a + I*a*Sinh[e + f*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{x\sqrt{a+a\sin(ie+ifx)}}dx$$

$$\Big\downarrow \text{ 3807}$$

$$\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}dx$$

input
```
Int[1/(x*Sqrt[a + I*a*Sinh[e + f*x]]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{x\sqrt{a+ia\sinh\left(fx+e\right)}}dx$$

input
```
int(1/x/(a+I*a*sinh(f*x+e))^(1/2),x)
```

output
```
int(1/x/(a+I*a*sinh(f*x+e))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.90

$$\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{1}{\sqrt{i\,a\sinh\left(fx+e\right)+ax}}\,dx$$

input
```
integrate(1/x/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(-2*I*sqrt(1/2*I*a*e^(-f*x - e))*e^(f*x + e)/(a*x*e^(f*x + e) - I*
a*x), x)
```

Sympy [N/A]

Not integrable

Time = 2.82 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{1}{x\sqrt{ia\left(\sinh\left(e+fx\right)-i\right)}}\,dx$$

input
```
integrate(1/x/(a+I*a*sinh(f*x+e))**(1/2),x)
```

output
```
Integral(1/(x*sqrt(I*a*(sinh(e + f*x) - I))), x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{1}{\sqrt{i\,a\sinh\left(fx+e\right)+ax}}\,dx$$

input
```
integrate(1/x/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(I*a*sinh(f*x + e) + a)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{1}{\sqrt{i\,a\sinh\left(fx+e\right)+a}x}\,dx$$

input
```
integrate(1/x/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(I*a*sinh(f*x + e) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{1}{x\,\sqrt{a+a\sinh\left(e+f\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int(1/(x*(a + a*sinh(e + f*x)*1i)^(1/2)),x)
```

output
```
int(1/(x*(a + a*sinh(e + f*x)*1i)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.43

$$\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx$$
$$= \frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(fx+e)i+1}}{\sinh(fx+e)^2x+x}dx - \left(\int \frac{\sqrt{\sinh(fx+e)i+1}\,\sinh(fx+e)}{\sinh(fx+e)^2x+x}dx\right)i\right)}{a}$$

input
```
int(1/x/(a+I*a*sinh(f*x+e))^(1/2),x)
```

output
```
(sqrt(a)*(int(sqrt(sinh(e + f*x)*i + 1)/(sinh(e + f*x)**2*x + x),x) - int(
(sqrt(sinh(e + f*x)*i + 1)*sinh(e + f*x))/(sinh(e + f*x)**2*x + x),x)*i))/
a
```

3.140 $\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx = \operatorname{Int}\left(\frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}},x\right)$$

output
```
Defer(Int)(1/x^2/(a+I*a*sinh(f*x+e))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 3.57 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx$$

input
```
Integrate[1/(x^2*Sqrt[a + I*a*Sinh[e + f*x]]),x]
```

output
```
Integrate[1/(x^2*Sqrt[a + I*a*Sinh[e + f*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{x^2\sqrt{a+a\sin(ie+ifx)}}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}dx$$

input
```
Int[1/(x^2*Sqrt[a + I*a*Sinh[e + f*x]]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^2\sqrt{a+ia\sinh\left(fx+e\right)}}dx$$

input
```
int(1/x^2/(a+I*a*sinh(f*x+e))^(1/2),x)
```

output
```
int(1/x^2/(a+I*a*sinh(f*x+e))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.10

$$\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{1}{\sqrt{i\,a\sinh\left(fx+e\right)+ax^2}}\,dx$$

input
```
integrate(1/x^2/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(-2*I*sqrt(1/2*I*a*e^(-f*x - e))*e^(f*x + e)/(a*x^2*e^(f*x + e) -
I*a*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 5.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{1}{x^2\sqrt{ia\left(\sinh\left(e+fx\right)-i\right)}}\,dx$$

input
```
integrate(1/x**2/(a+I*a*sinh(f*x+e))**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(I*a*(sinh(e + f*x) - I))), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{1}{\sqrt{i\,a\sinh\left(fx+e\right)+ax^2}}\,dx$$

input
```
integrate(1/x^2/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(I*a*sinh(f*x + e) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{1}{\sqrt{i\,a\sinh\left(fx+e\right)+a}x^2}\,dx$$

input
```
integrate(1/x^2/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(I*a*sinh(f*x + e) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{1}{x^2\,\sqrt{a+a\sinh\left(e+f\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int(1/(x^2*(a + a*sinh(e + f*x)*1i)^(1/2)),x)
```

output
```
int(1/(x^2*(a + a*sinh(e + f*x)*1i)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.81

$$\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(fx+e)i+1}}{\sinh(fx+e)^2x^2+x^2}dx-\left(\int \frac{\sqrt{\sinh(fx+e)i+1}\,\sinh(fx+e)}{\sinh(fx+e)^2x^2+x^2}dx\right)i\right)}{a}$$

input
```
int(1/x^2/(a+I*a*sinh(f*x+e))^(1/2),x)
```

output
```
(sqrt(a)*(int(sqrt(sinh(e + f*x)*i + 1)/(sinh(e + f*x)**2*x**2 + x**2),x)
- int((sqrt(sinh(e + f*x)*i + 1)*sinh(e + f*x))/(sinh(e + f*x)**2*x**2 + x
**2),x)*i))/a
```

3.141 $\int \frac{x^3}{(a+ia\sinh(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 807

$$\int \frac{x^3}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

output
```
3*x^2/a/f^2/(a+I*a*sinh(f*x+e))^(1/2)+24*I*x*arctanh(exp(1/2*e+3/4*I*Pi+1/
2*f*x))*cosh(1/2*e+1/4*I*Pi+1/2*f*x)/a/f^3/(a+I*a*sinh(f*x+e))^(1/2)-I*x^3
*arctanh(exp(1/2*e+3/4*I*Pi+1/2*f*x))*cosh(1/2*e+1/4*I*Pi+1/2*f*x)/a/f/(a+
I*a*sinh(f*x+e))^(1/2)-24*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(2,exp(1/2
*e+3/4*I*Pi+1/2*f*x))/a/f^4/(a+I*a*sinh(f*x+e))^(1/2)+3*I*x^2*cosh(1/2*e+1
/4*I*Pi+1/2*f*x)*polylog(2,exp(1/2*e+3/4*I*Pi+1/2*f*x))/a/f^2/(a+I*a*sinh(
f*x+e))^(1/2)+24*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(2,-exp(1/2*e+3/4*I
*Pi+1/2*f*x))/a/f^4/(a+I*a*sinh(f*x+e))^(1/2)-3*I*x^2*cosh(1/2*e+1/4*I*Pi+
1/2*f*x)*polylog(2,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/a/f^2/(a+I*a*sinh(f*x+e))
^(1/2)-12*I*x*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(3,exp(1/2*e+3/4*I*Pi+1/
2*f*x))/a/f^3/(a+I*a*sinh(f*x+e))^(1/2)+12*I*x*cosh(1/2*e+1/4*I*Pi+1/2*f*x
)*polylog(3,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/a/f^3/(a+I*a*sinh(f*x+e))^(1/2)+
24*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(4,exp(1/2*e+3/4*I*Pi+1/2*f*x))/a
/f^4/(a+I*a*sinh(f*x+e))^(1/2)-24*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(4
,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/a/f^4/(a+I*a*sinh(f*x+e))^(1/2)+1/2*x^3*tan
h(1/2*e+1/4*I*Pi+1/2*f*x)/a/f/(a+I*a*sinh(f*x+e))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.02 (sec) , antiderivative size = 546, normalized size of antiderivative = 0.68

$$\int \frac{x^3}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \frac{\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)\left(f^2x^2(6+ifx)\left(\cosh\left(\frac{1}{2}(e+fx\right)\right.\right.}{}$$

input
```
Integrate[x^3/(a + I*a*Sinh[e + f*x])^(3/2),x]
```

output
```
((Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])*(f^2*x^2*(6 + I*f*x)*(Cosh[(e +
 f*x)/2] + I*Sinh[(e + f*x)/2]) + (1/2 - I/2)*(-1)^(3/4)*(-48*e*ArcTanh[(-
1)^(3/4)*E^((e + f*x)/2)] + 2*e^3*ArcTanh[(-1)^(3/4)*E^((e + f*x)/2)] - 24
*e*Log[1 - (-1)^(3/4)*E^((e + f*x)/2)] + e^3*Log[1 - (-1)^(3/4)*E^((e + f*
x)/2)] - 24*f*x*Log[1 - (-1)^(3/4)*E^((e + f*x)/2)] + f^3*x^3*Log[1 - (-1)
^(3/4)*E^((e + f*x)/2)] + 24*e*Log[1 + (-1)^(3/4)*E^((e + f*x)/2)] - e^3*L
og[1 + (-1)^(3/4)*E^((e + f*x)/2)] + 24*f*x*Log[1 + (-1)^(3/4)*E^((e + f*x
)/2)] - f^3*x^3*Log[1 + (-1)^(3/4)*E^((e + f*x)/2)] - 6*(-8 + f^2*x^2)*Pol
yLog[2, -((-1)^(3/4)*E^((e + f*x)/2))] + 6*(-8 + f^2*x^2)*PolyLog[2, (-1)^
(3/4)*E^((e + f*x)/2)] + 24*f*x*PolyLog[3, -((-1)^(3/4)*E^((e + f*x)/2))]
- 24*f*x*PolyLog[3, (-1)^(3/4)*E^((e + f*x)/2)] - 48*PolyLog[4, -((-1)^(3/
4)*E^((e + f*x)/2))] + 48*PolyLog[4, (-1)^(3/4)*E^((e + f*x)/2)])*(Cosh[(e
 + f*x)/2] + I*Sinh[(e + f*x)/2])^2 + 2*f^3*x^3*Sinh[(e + f*x)/2]))/(2*f^4
*(a + I*a*Sinh[e + f*x])^(3/2))
```

Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 475, normalized size of antiderivative = 0.59, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3800, 3042, 4674, 3042, 4670, 2715, 2838, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+ia\sinh(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{x^3}{(a+a\sin(ie+ifx))^{3/2}}dx$$

$\Big\downarrow$ 3800

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\int x^3\text{sech}^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$\Big\downarrow$ 3042

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\int x^3\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$\Big\downarrow$ 4674

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(-\frac{12\int x\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f^2}+\frac{1}{2}\int x^3\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx+\frac{6x^2\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2}+\frac{x^3\tanh\left(\frac{e}{2}+\right.}{}\right.}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$\Big\downarrow$ 3042

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(-\frac{12\int x\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx}{f^2}+\frac{1}{2}\int x^3\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx+\frac{6x^2\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2}+\frac{x^3\tanh\left(\frac{e}{2}+\frac{f}{2}\right.}{}\right.}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$\Big\downarrow$ 4670

$$\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(-\frac{12\left(\frac{2i\int\log\left(1-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}-\frac{2i\int\log\left(1+e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}+\frac{4ix\text{arctanh}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f}\right)}{f^2}+\frac{1}{2}\left(\frac{6i\int}{}\right.\right.$$

$\Big\downarrow$ 2715

$$\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(-\frac{12\left(\frac{4i\int e^{\frac{1}{4}(i\pi-2e)-\frac{fx}{2}}\log\left(1-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)de^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}}{f^2}-\frac{4i\int e^{\frac{1}{4}(i\pi-2e)-\frac{fx}{2}}\log\left(1+e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)de^{\frac{1}{4}(2e-}}{f^2}\right.}{f^2}\right.$$

$$\downarrow 2838$$

$$\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{1}{2}\left(\frac{6i\int x^2\log\left(1-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}-\frac{6i\int x^2\log\left(1+e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}+\frac{4ix^3\text{arctanh}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f}\right)\right.$$

$$\downarrow 3011$$

$$\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{1}{2}\left(-\frac{6i\left(\frac{4\int x\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}-\frac{2x^2\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f}\right)}{f}+\frac{6i\left(\frac{4\int x\,\text{PolyLog}\left(2,e^{\frac{1}{4}(2e-i\pi}\right.}{f}\right.}{}\right.\right.$$

$$\downarrow 7163$$

$$\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{1}{2}\left(-\frac{6i\left(\frac{4\left(\frac{2x\,\text{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f}-\frac{2\int\text{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}\right)}{f}-\frac{2x^2\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right.}{f}\right.}{f}\right.\right.$$

$$\downarrow 2720$$

$$\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{1}{2}\left(-\frac{6i\left(\frac{4\left(\frac{2x\,\text{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f}-\frac{4\int e^{\frac{1}{4}(i\pi-2e)-\frac{fx}{2}}\,\text{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)de^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}}{f^2}\right)}{f}-\frac{2x}{}\right.}{f}\right.\right.$$

$$\Big\downarrow \textbf{7143}$$

$$\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{1}{2}\left(\frac{4ix^3\text{arctanh}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f}-\frac{6i\left(\frac{4\left(\frac{2x\,\text{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f}-\frac{4\,\text{PolyLog}\left(4,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2}\right)}{f}-\right.}{f}\right.\right.$$

input
```
Int[x^3/(a + I*a*Sinh[e + f*x])^(3/2),x]
```

output
```
(Cosh[e/2 + (I/4)*Pi + (f*x)/2]*((-12*(((4*I)*x*ArcTanh[E^((2*e - I*Pi)/4
+ (f*x)/2)])/f + ((4*I)*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/f^2 - (
(4*I)*PolyLog[2, E^((2*e - I*Pi)/4 + (f*x)/2)])/f^2))/f^2 + (((4*I)*x^3*Ar
cTanh[E^((2*e - I*Pi)/4 + (f*x)/2)])/f - ((6*I)*((-2*x^2*PolyLog[2, -E^((2
*e - I*Pi)/4 + (f*x)/2)])/f + (4*((2*x*PolyLog[3, -E^((2*e - I*Pi)/4 + (f*
x)/2)])/f - (4*PolyLog[4, -E^((2*e - I*Pi)/4 + (f*x)/2)])/f^2))/f))/f + ((
6*I)*((-2*x^2*PolyLog[2, E^((2*e - I*Pi)/4 + (f*x)/2)])/f + (4*((2*x*PolyL
og[3, E^((2*e - I*Pi)/4 + (f*x)/2)])/f - (4*PolyLog[4, E^((2*e - I*Pi)/4 +
 (f*x)/2)])/f^2))/f))/f)/2 + (6*x^2*Sech[e/2 + (I/4)*Pi + (f*x)/2])/f^2 +
(x^3*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/f))/(2
*a*Sqrt[a + I*a*Sinh[e + f*x]])
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{x^3}{(a + ia\sinh(fx + e))^{\frac{3}{2}}} dx$$

input
```
int(x^3/(a+I*a*sinh(f*x+e))^(3/2),x)
```

output
```
int(x^3/(a+I*a*sinh(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \frac{x^3}{(a + ia\sinh(e + fx))^{3/2}} dx = \int \frac{x^3}{(i\,a\sinh(fx + e) + a)^{\frac{3}{2}}} dx$$

input
```
integrate(x^3/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
((a^2*f^2*e^(2*f*x + 2*e) - 2*I*a^2*f^2*e^(f*x + e) - a^2*f^2)*integral(1/
2*(-I*f^2*x^3 + 24*I*x)*sqrt(1/2*I*a*e^(-f*x - e))*e^(f*x + e)/(a^2*f^2*e^
(f*x + e) - I*a^2*f^2), x) + ((-I*f*x^3 - 6*I*x^2)*e^(2*f*x + 2*e) + (f*x^
3 - 6*x^2)*e^(f*x + e))*sqrt(1/2*I*a*e^(-f*x - e)))/(a^2*f^2*e^(2*f*x + 2*
e) - 2*I*a^2*f^2*e^(f*x + e) - a^2*f^2)
```

Sympy [F]

$$\int \frac{x^3}{(a + ia\sinh(e + fx))^{3/2}} dx = \int \frac{x^3}{(ia(\sinh(e + fx) - i))^{\frac{3}{2}}} dx$$

input
```
integrate(x**3/(a+I*a*sinh(f*x+e))**(3/2),x)
```

output
```
Integral(x**3/(I*a*(sinh(e + f*x) - I))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{x^3}{\left(i\,a\sinh\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(I*a*sinh(f*x + e) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^3}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{x^3}{\left(i\,a\sinh\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^3/(I*a*sinh(f*x + e) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{x^3}{{\left(a+a\,\mathrm{sinh}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}\,dx$$

input
```
int(x^3/(a + a*sinh(e + f*x)*1i)^(3/2),x)
```

output
```
int(x^3/(a + a*sinh(e + f*x)*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(fx+e)i+1}\,x^3}{\sinh(fx+e)^3i+\sinh(fx+e)^2+\sinh(fx+e)i+1}dx - \left(\int \frac{\sqrt{\sinh(fx+e)i+1}\,\sinh(f\cdots}{\sinh(fx+e)^3i+\sinh(fx+e)^2+\mathrm{si}\cdots}\right.\right.}{a^2}$$

input
```
int(x^3/(a+I*a*sinh(f*x+e))^(3/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sinh(e + f*x)*i + 1)*x**3)/(sinh(e + f*x)**3*i + sinh(
e + f*x)**2 + sinh(e + f*x)*i + 1),x) - int((sqrt(sinh(e + f*x)*i + 1)*sin
h(e + f*x)*x**3)/(sinh(e + f*x)**3*i + sinh(e + f*x)**2 + sinh(e + f*x)*i
+ 1),x)*i))/a**2
```

3.142 $\int \frac{x^2}{(a+ia\sinh(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 506

$$\begin{aligned}
\int \frac{x^2}{(a+ia\sinh(e+fx))^{3/2}}\,dx &= \frac{2x}{af^2\sqrt{a+ia\sinh(e+fx)}} \\
&- \frac{4\arctan\left(\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af^3\sqrt{a+ia\sinh(e+fx)}} \\
&+ \frac{ix^2\operatorname{arctanh}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af\sqrt{a+ia\sinh(e+fx)}} \\
&+ \frac{2ix\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^2\sqrt{a+ia\sinh(e+fx)}} \\
&- \frac{2ix\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{PolyLog}\left(2,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^2\sqrt{a+ia\sinh(e+fx)}} \\
&- \frac{4i\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^3\sqrt{a+ia\sinh(e+fx)}} \\
&+ \frac{4i\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{PolyLog}\left(3,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^3\sqrt{a+ia\sinh(e+fx)}} + \frac{x^2\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+ia\sinh(e+fx)}}
\end{aligned}$$

output
```
2*x/a/f^2/(a+I*a*sinh(f*x+e))^(1/2)-4*arctan(sinh(1/2*e+1/4*I*Pi+1/2*f*x))
*cosh(1/2*e+1/4*I*Pi+1/2*f*x)/a/f^3/(a+I*a*sinh(f*x+e))^(1/2)-I*x^2*arctan
h(exp(1/2*e+3/4*I*Pi+1/2*f*x))*cosh(1/2*e+1/4*I*Pi+1/2*f*x)/a/f/(a+I*a*sin
h(f*x+e))^(1/2)+2*I*x*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(2,exp(1/2*e+3/4
*I*Pi+1/2*f*x))/a/f^2/(a+I*a*sinh(f*x+e))^(1/2)-2*I*x*cosh(1/2*e+1/4*I*Pi+
1/2*f*x)*polylog(2,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/a/f^2/(a+I*a*sinh(f*x+e))
^(1/2)-4*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(3,exp(1/2*e+3/4*I*Pi+1/2*f
*x))/a/f^3/(a+I*a*sinh(f*x+e))^(1/2)+4*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*poly
log(3,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/a/f^3/(a+I*a*sinh(f*x+e))^(1/2)+1/2*x^
2*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/a/f/(a+I*a*sinh(f*x+e))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.08 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \frac{\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)\left(fx(4+ifx)\left(\cosh\left(\frac{1}{2}(e+fx)\right)\right.\right.}{}$$

input
```
Integrate[x^2/(a + I*a*Sinh[e + f*x])^(3/2),x]
```

output
```
((Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])*(f*x*(4 + I*f*x)*(Cosh[(e + f*x
)/2] + I*Sinh[(e + f*x)/2]) - (1/2 - I/2)*(-1)^(3/4)*(-16*ArcTanh[(-1)^(3/
4)*E^((e + f*x)/2)] + 2*e^2*ArcTanh[(-1)^(3/4)*E^((e + f*x)/2)] + e^2*Log[
1 - (-1)^(3/4)*E^((e + f*x)/2)] - f^2*x^2*Log[1 - (-1)^(3/4)*E^((e + f*x)/
2)] - e^2*Log[1 + (-1)^(3/4)*E^((e + f*x)/2)] + f^2*x^2*Log[1 + (-1)^(3/4)
*E^((e + f*x)/2)] + 4*f*x*PolyLog[2, -((-1)^(3/4)*E^((e + f*x)/2))] - 4*f*
x*PolyLog[2, (-1)^(3/4)*E^((e + f*x)/2)] - 8*PolyLog[3, -((-1)^(3/4)*E^((e
 + f*x)/2))] + 8*PolyLog[3, (-1)^(3/4)*E^((e + f*x)/2)])*(Cosh[(e + f*x)/2
] + I*Sinh[(e + f*x)/2])^2 + 2*f^2*x^2*Sinh[(e + f*x)/2]))/(2*f^3*(a + I*a
*Sinh[e + f*x])^(3/2))
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.63, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3800, 3042, 4674, 3042, 4257, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+ia\sinh(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{x^2}{(a+a\sin(ie+ifx))^{3/2}}dx$$

$$\Big\downarrow \ 3800$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\int x^2\text{sech}^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\int x^2\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \ 4674$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(-\frac{4\int \text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f^2}+\frac{1}{2}\int x^2\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx+\frac{4x\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2}+\frac{x^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\right.}{}\right.}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(-\frac{4\int \csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx}{f^2}+\frac{1}{2}\int x^2\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx+\frac{4x\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2}+\frac{x^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i}{4}\right.}{}\right.}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \ 4257$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\left(\frac{1}{2}\int x^2\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx-\frac{8\arctan\left(\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)}{f^3}+\frac{4x\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2}+\frac{x^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right.}{}\right.\right.}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \text{4670}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{1}{2}\left(\frac{4i\int x\log\left(1-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}-\frac{4i\int x\log\left(1+e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}+\frac{4ix^2\text{arctanh}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f}\right)\right.}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \text{3011}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{1}{2}\left(-\frac{4i\left(\frac{2\int \text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}-\frac{2x\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f}\right)}{f}+\frac{4i\left(\frac{2\int \text{PolyLog}\left(2,e^{\frac{1}{4}(2e-i\pi)+\frac{f}{2}}\right.}{f}\right.}{}\right.\right.}{}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{1}{2}\left(-\frac{4i\left(\frac{4\int e^{\frac{1}{4}(i\pi-2e)-\frac{fx}{2}}\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)de^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}}{f^2}-\frac{2x\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f}\right)}{f}+\frac{4i}{}\right.\right.}{}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(-\frac{8\arctan\left(\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)}{f^3}+\frac{1}{2}\left(\frac{4ix^2\text{arctanh}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f}-\frac{4i\left(\frac{4\,\text{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2}-\frac{2x}{}\right.}{f}\right.\right.}{}$$

input
```
Int[x^2/(a + I*a*Sinh[e + f*x])^(3/2),x]
```

output
```
(Cosh[e/2 + (I/4)*Pi + (f*x)/2]*((-8*ArcTan[Sinh[e/2 + (I/4)*Pi + (f*x)/2]
])/f^3 + (((4*I)*x^2*ArcTanh[E^((2*e - I*Pi)/4 + (f*x)/2)])/f - ((4*I)*((-
2*x*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/f + (4*PolyLog[3, -E^((2*e
- I*Pi)/4 + (f*x)/2)])/f^2))/f + ((4*I)*((-2*x*PolyLog[2, E^((2*e - I*Pi)/
4 + (f*x)/2)])/f + (4*PolyLog[3, E^((2*e - I*Pi)/4 + (f*x)/2)])/f^2))/f)/2
 + (4*x*Sech[e/2 + (I/4)*Pi + (f*x)/2])/f^2 + (x^2*Sech[e/2 + (I/4)*Pi + (
f*x)/2]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/f))/(2*a*Sqrt[a + I*a*Sinh[e + f*x
]])
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{x^2}{(a + ia\sinh(fx + e))^{\frac{3}{2}}}dx$$

input
```
int(x^2/(a+I*a*sinh(f*x+e))^(3/2),x)
```

output
```
int(x^2/(a+I*a*sinh(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \frac{x^2}{(a+ia\sinh(e+fx))^{3/2}}\,dx=\int \frac{x^2}{(i\,a\sinh\left(fx+e\right)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
((a^2*f^2*e^(2*f*x + 2*e) - 2*I*a^2*f^2*e^(f*x + e) - a^2*f^2)*integral(1/
2*(-I*f^2*x^2 + 8*I)*sqrt(1/2*I*a*e^(-f*x - e))*e^(f*x + e)/(a^2*f^2*e^(f*
x + e) - I*a^2*f^2), x) + ((-I*f*x^2 - 4*I*x)*e^(2*f*x + 2*e) + (f*x^2 - 4
*x)*e^(f*x + e))*sqrt(1/2*I*a*e^(-f*x - e)))/(a^2*f^2*e^(2*f*x + 2*e) - 2*
I*a^2*f^2*e^(f*x + e) - a^2*f^2)
```

Sympy [F]

$$\int \frac{x^2}{(a+ia\sinh(e+fx))^{3/2}}\,dx=\int \frac{x^2}{(ia\left(\sinh\left(e+fx\right)-i\right))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(a+I*a*sinh(f*x+e))**(3/2),x)
```

output
```
Integral(x**2/(I*a*(sinh(e + f*x) - I))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+ia\sinh(e+fx))^{3/2}}\,dx=\int \frac{x^2}{(i\,a\sinh\left(fx+e\right)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(I*a*sinh(f*x + e) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{x^2}{(i\,a\sinh\left(fx+e\right)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/(I*a*sinh(f*x + e) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{x^2}{\left(a+a\sinh\left(e+f\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

input
```
int(x^2/(a + a*sinh(e + f*x)*1i)^(3/2),x)
```

output
```
int(x^2/(a + a*sinh(e + f*x)*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(fx+e)i+1}\,x^2}{\sinh(fx+e)^3 i+\sinh(fx+e)^2+\sinh(fx+e)i+1}dx - \left(\int \frac{\sqrt{\sinh(fx+e)i+1}\,\sinh(f}{\sinh(fx+e)^3 i+\sinh(fx+e)^2+\sin}\right.\right.}{a^2}$$

input
```
int(x^2/(a+I*a*sinh(f*x+e))^(3/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sinh(e + f*x)*i + 1)*x**2)/(sinh(e + f*x)**3*i + sinh(
e + f*x)**2 + sinh(e + f*x)*i + 1),x) - int((sqrt(sinh(e + f*x)*i + 1)*sin
h(e + f*x)*x**2)/(sinh(e + f*x)**3*i + sinh(e + f*x)**2 + sinh(e + f*x)*i
+ 1),x)*i))/a**2
```

3.143 $\int \frac{x}{(a+ia\sinh(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 288

$$\int \frac{x}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \frac{1}{af^2\sqrt{a+ia\sinh(e+fx)}} + \frac{ix\,\text{arctanh}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af\sqrt{a+ia\sinh(e+fx)}} + \frac{i\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^2\sqrt{a+ia\sinh(e+fx)}} - \frac{i\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\text{PolyLog}\left(2,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^2\sqrt{a+ia\sinh(e+fx)}} + \frac{x\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+ia\sinh(e+fx)}}$$

output
```
1/a/f^2/(a+I*a*sinh(f*x+e))^(1/2)-I*x*arctanh(exp(1/2*e+3/4*I*Pi+1/2*f*x))
*cosh(1/2*e+1/4*I*Pi+1/2*f*x)/a/f/(a+I*a*sinh(f*x+e))^(1/2)+I*cosh(1/2*e+1
/4*I*Pi+1/2*f*x)*polylog(2,exp(1/2*e+3/4*I*Pi+1/2*f*x))/a/f^2/(a+I*a*sinh(
f*x+e))^(1/2)-I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(2,-exp(1/2*e+3/4*I*Pi
+1/2*f*x))/a/f^2/(a+I*a*sinh(f*x+e))^(1/2)+1/2*x*tanh(1/2*e+1/4*I*Pi+1/2*f
*x)/a/f/(a+I*a*sinh(f*x+e))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.73 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.90

$$\int \frac{x}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \frac{\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)\left((2+ifx)\left(\cosh\left(\frac{1}{2}(e+fx)\right)+\right.\right.}{}$$

input
```
Integrate[x/(a + I*a*Sinh[e + f*x])^(3/2),x]
```

output
```
((Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])*((2 + I*f*x)*(Cosh[(e + f*x)/2]
 + I*Sinh[(e + f*x)/2]) + (1 - I)*(-1)^(3/4)*(I*e*ArcTan[(-1)^(1/4)*E^((e
+ f*x)/2)] + ((e + f*x)*Log[1 - (-1)^(3/4)*E^((e + f*x)/2)])/2 - ((e + f*x
)*Log[1 + (-1)^(3/4)*E^((e + f*x)/2)])/2 - PolyLog[2, -((-1)^(3/4)*E^((e +
 f*x)/2))] + PolyLog[2, (-1)^(3/4)*E^((e + f*x)/2)])*(Cosh[(e + f*x)/2] +
I*Sinh[(e + f*x)/2])^2 + 2*f*x*Sinh[(e + f*x)/2]))/(2*f^2*(a + I*a*Sinh[e
+ f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.75, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3800, 3042, 4673, 3042, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+ia\sinh(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{x}{(a+a\sin(ie+ifx))^{3/2}}dx$$

$$\Big\downarrow \text{3800}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\int x\text{sech}^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\int x\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \text{4673}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{1}{2}\int x\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx+\frac{2\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2}+\frac{x\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{1}{2}\int x\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx+\frac{2\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2}+\frac{x\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \text{4670}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{1}{2}\left(\frac{2i\int\log\left(1-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}-\frac{2i\int\log\left(1+e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}+\frac{4ix\text{arctanh}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f}\right)+\frac{2}{\cdots}}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{1}{2}\left(\frac{4i\int e^{\frac{1}{4}(i\pi-2e)-\frac{fx}{2}}\log\left(1-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)de^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}}{f^2}-\frac{4i\int e^{\frac{1}{4}(i\pi-2e)-\frac{fx}{2}}\log\left(1+e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)de^{\frac{1}{4}\cdots}}{f^2}\cdots\right.\right.}{2a\sqrt{a+ia\sinh(e+\cdots}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{1}{2}\left(\frac{4ix\text{arctanh}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f}+\frac{4i\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2}-\frac{4i\,\text{PolyLog}\left(2,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2}\right)+\cdots\right.}{2a\sqrt{a+ia\sinh(e+fx)}}$$

input

```
Int[x/(a + I*a*Sinh[e + f*x])^(3/2),x]
```

output
```
(Cosh[e/2 + (I/4)*Pi + (f*x)/2]*((((4*I)*x*ArcTanh[E^((2*e - I*Pi)/4 + (f*
x)/2)])/f + ((4*I)*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/f^2 - ((4*I)
*PolyLog[2, E^((2*e - I*Pi)/4 + (f*x)/2)])/f^2)/2 + (2*Sech[e/2 + (I/4)*Pi
 + (f*x)/2])/f^2 + (x*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Tanh[e/2 + (I/4)*Pi +
 (f*x)/2])/f))/(2*a*Sqrt[a + I*a*Sinh[e + f*x]])
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Maple [F]

$$\int \frac{x}{(a+ia\sinh(fx+e))^{\frac{3}{2}}}dx$$

input
```
int(x/(a+I*a*sinh(f*x+e))^(3/2),x)
```

output
```
int(x/(a+I*a*sinh(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \frac{x}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{x}{(i\,a\sinh(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
((a^2*f^2*e^(2*f*x + 2*e) - 2*I*a^2*f^2*e^(f*x + e) - a^2*f^2)*integral(-1
/2*I*sqrt(1/2*I*a*e^(-f*x - e))*x*e^(f*x + e)/(a^2*e^(f*x + e) - I*a^2), x
) + ((-I*f*x - 2*I)*e^(2*f*x + 2*e) + (f*x - 2)*e^(f*x + e))*sqrt(1/2*I*a*
e^(-f*x - e)))/(a^2*f^2*e^(2*f*x + 2*e) - 2*I*a^2*f^2*e^(f*x + e) - a^2*f^
2)
```

Sympy [F]

$$\int \frac{x}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{x}{(ia\,(\sinh{(e+fx)}-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+I*a*sinh(f*x+e))**(3/2),x)
```

output
```
Integral(x/(I*a*(sinh(e + f*x) - I))**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{x}{(i\,a\sinh{(fx+e)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(I*a*sinh(f*x + e) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{x}{(i\,a\sinh{(fx+e)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(x/(I*a*sinh(f*x + e) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{x}{\left(a+a\sinh\left(e+f\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

input
```
int(x/(a + a*sinh(e + f*x)*1i)^(3/2),x)
```

output
```
int(x/(a + a*sinh(e + f*x)*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{x}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \frac{\sqrt{a}\left(-\left(\int \frac{\sqrt{\sinh(fx+e)i+1}\,\sinh(fx+e)x}{\sinh(fx+e)^3i+\sinh(fx+e)^2+\sinh(fx+e)i+1}dx\right)i+\int \frac{\sqrt{\sinh(fx+e}}{\sinh(fx+e)^3i+\sinh(fx+e}\right.}{a^2}$$

input
```
int(x/(a+I*a*sinh(f*x+e))^(3/2),x)
```

output
```
(sqrt(a)*( - int((sqrt(sinh(e + f*x)*i + 1)*sinh(e + f*x)*x)/(sinh(e + f*x
)**3*i + sinh(e + f*x)**2 + sinh(e + f*x)*i + 1),x)*i + int((sqrt(sinh(e +
 f*x)*i + 1)*x)/(sinh(e + f*x)**3*i + sinh(e + f*x)**2 + sinh(e + f*x)*i +
 1),x)))/a**2
```

3.144 $\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx = \text{Int}\left(\frac{1}{x(a+ia\sinh(e+fx))^{3/2}},x\right)$$

output
```
Defer(Int)(1/x/(a+I*a*sinh(f*x+e))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 20.46 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx$$

input
```
Integrate[1/(x*(a + I*a*Sinh[e + f*x])^(3/2)),x]
```

output
```
Integrate[1/(x*(a + I*a*Sinh[e + f*x])^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{x(a+a\sin(ie+ifx))^{3/2}}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}dx$$

input
```
Int[1/(x*(a + I*a*Sinh[e + f*x])^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{x\left(a+ia\sinh\left(fx+e\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x/(a+I*a*sinh(f*x+e))^(3/2),x)
```

output
```
int(1/x/(a+I*a*sinh(f*x+e))^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 209, normalized size of antiderivative = 9.95

$$\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{1}{(i\,a\sinh(fx+e)+a)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
((a^2*f^2*x^2*e^(2*f*x + 2*e) - 2*I*a^2*f^2*x^2*e^(f*x + e) - a^2*f^2*x^2)
*integral(1/2*(-I*f^2*x^2 + 8*I)*sqrt(1/2*I*a*e^(-f*x - e))*e^(f*x + e)/(a
^2*f^2*x^3*e^(f*x + e) - I*a^2*f^2*x^3), x) + ((-I*f*x + 2*I)*e^(2*f*x + 2
*e) + (f*x + 2)*e^(f*x + e))*sqrt(1/2*I*a*e^(-f*x - e)))/(a^2*f^2*x^2*e^(2
*f*x + 2*e) - 2*I*a^2*f^2*x^2*e^(f*x + e) - a^2*f^2*x^2)
```

Sympy [N/A]

Not integrable

Time = 29.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{1}{x\,(ia\,(\sinh(e+fx)-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(a+I*a*sinh(f*x+e))**(3/2),x)
```

output
```
Integral(1/(x*(I*a*(sinh(e + f*x) - I))**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{1}{(i\,a\sinh\left(fx+e\right)+a)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*sinh(f*x + e) + a)^(3/2)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{1}{(i\,a\sinh\left(fx+e\right)+a)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((I*a*sinh(f*x + e) + a)^(3/2)*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.74 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{1}{x\,{\left(a+a\,\mathrm{sinh}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}\,dx$$

input
```
int(1/(x*(a + a*sinh(e + f*x)*1i)^(3/2)),x)
```

output
```
int(1/(x*(a + a*sinh(e + f*x)*1i)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 112, normalized size of antiderivative = 5.33

$$\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(fx+e)i+1}}{\sinh(fx+e)^3ix+\sinh(fx+e)^2x+\sinh(fx+e)ix+x}dx-\left(\int \frac{\sqrt{\sinh(fx+e)i+}}{\sinh(fx+e)^3ix+\sinh(fx+}\right.\right.}{a^2}$$

input
```
int(1/x/(a+I*a*sinh(f*x+e))^(3/2),x)
```

output
```
(sqrt(a)*(int(sqrt(sinh(e + f*x)*i + 1)/(sinh(e + f*x)**3*i*x + sinh(e + f
*x)**2*x + sinh(e + f*x)*i*x + x),x) - int((sqrt(sinh(e + f*x)*i + 1)*sinh
(e + f*x))/(sinh(e + f*x)**3*i*x + sinh(e + f*x)**2*x + sinh(e + f*x)*i*x
+ x),x)*i))/a**2
```

3.145 $\int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}\,dx = \text{Int}\left(\frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}},x\right)$$

output
```
Defer(Int)(1/x^2/(a+I*a*sinh(f*x+e))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 22.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}\,dx$$

input
```
Integrate[1/(x^2*(a + I*a*Sinh[e + f*x])^(3/2)),x]
```

output
```
Integrate[1/(x^2*(a + I*a*Sinh[e + f*x])^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{x^2(a+a\sin(ie+ifx))^{3/2}}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}dx$$

input
```
Int[1/(x^2*(a + I*a*Sinh[e + f*x])^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^2\left(a+ia\sinh\left(fx+e\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x^2/(a+I*a*sinh(f*x+e))^(3/2),x)
```

output
```
int(1/x^2/(a+I*a*sinh(f*x+e))^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 209, normalized size of antiderivative = 9.95

$$\int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{1}{(i\,a\sinh\left(fx+e\right)+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
((a^2*f^2*x^3*e^(2*f*x + 2*e) - 2*I*a^2*f^2*x^3*e^(f*x + e) - a^2*f^2*x^3)
*integral(1/2*(-I*f^2*x^2 + 24*I)*sqrt(1/2*I*a*e^(-f*x - e))*e^(f*x + e)/(
a^2*f^2*x^4*e^(f*x + e) - I*a^2*f^2*x^4), x) + ((-I*f*x + 4*I)*e^(2*f*x +
2*e) + (f*x + 4)*e^(f*x + e))*sqrt(1/2*I*a*e^(-f*x - e)))/(a^2*f^2*x^3*e^(
2*f*x + 2*e) - 2*I*a^2*f^2*x^3*e^(f*x + e) - a^2*f^2*x^3)
```

Sympy [N/A]

Not integrable

Time = 53.90 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{1}{x^2\left(ia\left(\sinh\left(e+fx\right)-i\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(a+I*a*sinh(f*x+e))**(3/2),x)
```

output
```
Integral(1/(x**2*(I*a*(sinh(e + f*x) - I))**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{1}{(i\,a\sinh(fx+e)+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*sinh(f*x + e) + a)^(3/2)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{1}{(i\,a\sinh(fx+e)+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((I*a*sinh(f*x + e) + a)^(3/2)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.77 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{1}{x^2\,{\left(a+a\sinh\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}\,dx$$

input
```
int(1/(x^2*(a + a*sinh(e + f*x)*1i)^(3/2)),x)
```

output
```
int(1/(x^2*(a + a*sinh(e + f*x)*1i)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 128, normalized size of antiderivative = 6.10

$$\int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(fx+e)i+1}}{\sinh(fx+e)^3 i\,x^2+\sinh(fx+e)^2x^2+\sinh(fx+e)i\,x^2+x^2}dx-\left(\int \frac{\sqrt{\sinh(}}{\sinh(fx+e)^3 i\,x^2+\mathrm{s}}\right.\right.}{a^2}$$

input
```
int(1/x^2/(a+I*a*sinh(f*x+e))^(3/2),x)
```

output
```
(sqrt(a)*(int(sqrt(sinh(e + f*x)*i + 1)/(sinh(e + f*x)**3*i*x**2 + sinh(e
+ f*x)**2*x**2 + sinh(e + f*x)*i*x**2 + x**2),x) - int((sqrt(sinh(e + f*x)
*i + 1)*sinh(e + f*x))/(sinh(e + f*x)**3*i*x**2 + sinh(e + f*x)**2*x**2 +
sinh(e + f*x)*i*x**2 + x**2),x)*i))/a**2
```

3.146 $\int \frac{x^3}{(a+ia\sinh(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 1016

$$\int \frac{x^3}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

output
```
-1/a^2/d^4/(a+I*a*sinh(d*x+c))^(1/2)+9/8*x^2/a^2/d^2/(a+I*a*sinh(d*x+c))^(
1/2)-9/8*I*x^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog(2,-exp(1/2*c+3/4*I*Pi+
1/2*d*x))/a^2/d^2/(a+I*a*sinh(d*x+c))^(1/2)+10*I*cosh(1/2*c+1/4*I*Pi+1/2*d
*x)*polylog(2,-exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/d^4/(a+I*a*sinh(d*x+c))^(1
/2)-9*I*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog(4,-exp(1/2*c+3/4*I*Pi+1/2*d*x
))/a^2/d^4/(a+I*a*sinh(d*x+c))^(1/2)-9/2*I*x*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*
polylog(3,exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/d^3/(a+I*a*sinh(d*x+c))^(1/2)-3
/8*I*x^3*arctanh(exp(1/2*c+3/4*I*Pi+1/2*d*x))*cosh(1/2*c+1/4*I*Pi+1/2*d*x)
/a^2/d/(a+I*a*sinh(d*x+c))^(1/2)+9/8*I*x^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*po
lylog(2,exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/d^2/(a+I*a*sinh(d*x+c))^(1/2)+9/2
*I*x*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog(3,-exp(1/2*c+3/4*I*Pi+1/2*d*x))/
a^2/d^3/(a+I*a*sinh(d*x+c))^(1/2)+10*I*x*arctanh(exp(1/2*c+3/4*I*Pi+1/2*d*
x))*cosh(1/2*c+1/4*I*Pi+1/2*d*x)/a^2/d^3/(a+I*a*sinh(d*x+c))^(1/2)-10*I*co
sh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog(2,exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/d^4/
(a+I*a*sinh(d*x+c))^(1/2)+9*I*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog(4,exp(1
/2*c+3/4*I*Pi+1/2*d*x))/a^2/d^4/(a+I*a*sinh(d*x+c))^(1/2)+1/4*x^2*sech(1/2
*c+1/4*I*Pi+1/2*d*x)^2/a^2/d^2/(a+I*a*sinh(d*x+c))^(1/2)-1/2*x*tanh(1/2*c+
1/4*I*Pi+1/2*d*x)/a^2/d^3/(a+I*a*sinh(d*x+c))^(1/2)+3/16*x^3*tanh(1/2*c+1/
4*I*Pi+1/2*d*x)/a^2/d/(a+I*a*sinh(d*x+c))^(1/2)+1/8*x^3*sech(1/2*c+1/4*I*P
i+1/2*d*x)^2*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a^2/d/(a+I*a*sinh(d*x+c))^(1/...
```

Mathematica [A] (verified)

Time = 2.90 (sec) , antiderivative size = 1200, normalized size of antiderivative = 1.18

$$\int \frac{x^3}{(a + ia\sinh(c + dx))^{5/2}} \, dx = \text{Too large to display}$$

input
```
Integrate[x^3/(a + I*a*Sinh[c + d*x])^(5/2),x]
```

output
```
((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(-48*Cosh[(c + d*x)/2] + (8*I)*
c*Cosh[(c + d*x)/2] + 70*c^2*Cosh[(c + d*x)/2] - (11*I)*c^3*Cosh[(c + d*x)
/2] - (8*I)*(c + d*x)*Cosh[(c + d*x)/2] - 140*c*(c + d*x)*Cosh[(c + d*x)/2
] + (33*I)*c^2*(c + d*x)*Cosh[(c + d*x)/2] + 70*(c + d*x)^2*Cosh[(c + d*x)
/2] - (33*I)*c*(c + d*x)^2*Cosh[(c + d*x)/2] + (11*I)*(c + d*x)^3*Cosh[(c
+ d*x)/2] + 16*Cosh[(3*(c + d*x))/2] + (8*I)*c*Cosh[(3*(c + d*x))/2] - 18*
c^2*Cosh[(3*(c + d*x))/2] - (3*I)*c^3*Cosh[(3*(c + d*x))/2] - (8*I)*(c + d
*x)*Cosh[(3*(c + d*x))/2] + 36*c*(c + d*x)*Cosh[(3*(c + d*x))/2] + (9*I)*c
^2*(c + d*x)*Cosh[(3*(c + d*x))/2] - 18*(c + d*x)^2*Cosh[(3*(c + d*x))/2]
- (9*I)*c*(c + d*x)^2*Cosh[(3*(c + d*x))/2] + (3*I)*(c + d*x)^3*Cosh[(3*(c
 + d*x))/2] + (1 - I)*(-1)^(3/4)*(-160*c*ArcTanh[(-1)^(3/4)*E^((c + d*x)/2
)] + 6*c^3*ArcTanh[(-1)^(3/4)*E^((c + d*x)/2)] - 80*c*Log[1 - (-1)^(3/4)*E
^((c + d*x)/2)] + 3*c^3*Log[1 - (-1)^(3/4)*E^((c + d*x)/2)] - 80*d*x*Log[1
 - (-1)^(3/4)*E^((c + d*x)/2)] + 3*d^3*x^3*Log[1 - (-1)^(3/4)*E^((c + d*x)
/2)] + 80*c*Log[1 + (-1)^(3/4)*E^((c + d*x)/2)] - 3*c^3*Log[1 + (-1)^(3/4)
*E^((c + d*x)/2)] + 80*d*x*Log[1 + (-1)^(3/4)*E^((c + d*x)/2)] - 3*d^3*x^3
*Log[1 + (-1)^(3/4)*E^((c + d*x)/2)] - 2*(-80 + 9*d^2*x^2)*PolyLog[2, -((-
1)^(3/4)*E^((c + d*x)/2))] + 2*(-80 + 9*d^2*x^2)*PolyLog[2, (-1)^(3/4)*E^(
(c + d*x)/2)] + 72*d*x*PolyLog[3, -((-1)^(3/4)*E^((c + d*x)/2))] - 72*d*x*
PolyLog[3, (-1)^(3/4)*E^((c + d*x)/2)] - 144*PolyLog[4, -((-1)^(3/4)*E^...
```

Rubi [A] (verified)

Time = 2.81 (sec) , antiderivative size = 737, normalized size of antiderivative = 0.73, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.905$, Rules used = {3042, 3800, 3042, 4674, 3042, 4673, 3042, 4670, 2715, 2838, 4674, 3042, 4670, 2715, 2838, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+ia\sinh(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{x^3}{(a+a\sin(ic+idx))^{5/2}}dx$$

$$\Big\downarrow \text{3800}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\int x^3\text{sech}^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\int x^3\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^5dx}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$\downarrow$ 4674

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(-\frac{2\int x\text{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d^2}+\frac{3}{4}\int x^3\text{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx+\frac{x^2\text{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d^2}+\frac{x^3\tanh\left(\frac{c}{2}+\frac{d}{2}\right.}{}\right.}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(-\frac{2\int x\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{d^2}+\frac{3}{4}\int x^3\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx+\frac{x^2\text{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d^2}+\frac{x^3\tanh\left(\frac{c}{2}+\frac{dx}{2}\right.}{}\right.}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$\downarrow$ 4673

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(-\frac{2\left(\frac{1}{2}\int x\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx+\frac{2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d^2}+\frac{x\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{d^2}+\frac{3}{4}\int x^3\csc\left(\frac{ic}{2}\right.\right.}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(-\frac{2\left(\frac{1}{2}\int x\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)dx+\frac{2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d^2}+\frac{x\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{d^2}+\frac{3}{4}\int x^3\csc\left(\frac{ic}{2}+\right.\right.}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$\downarrow$ 4670

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(-\frac{2\left(\frac{1}{2}\left(\frac{2i\int\log\left(1-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)dx}{d}-\frac{2i\int\log\left(1+e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)dx}{d}+\frac{4ix\text{arctanh}\left(e^{\frac{dx}{2}+\frac{1}{4}(2c-i\pi)}\right)}{d}\right)+\frac{2\text{sech}\left(\frac{c}{2}\right.}{d}}{d^2}\right.}{4a^2}$$

$$\Big\downarrow \text{ 2715}$$

$$\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(-\frac{2\left(\frac{1}{2}\left(\frac{4i\int e^{\frac{1}{4}(i\pi-2c)-\frac{dx}{2}}\log\left(1-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)de^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}}{d^2}-\frac{4i\int e^{\frac{1}{4}(i\pi-2c)-\frac{dx}{2}}\log\left(1+e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)de^{\frac{1}{4}(2c}}{d^2}\right.\right.}{d^2}\right.$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\int x^3\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx-\frac{2\left(\frac{1}{2}\left(\frac{4ix\text{arctanh}\left(e^{\frac{dx}{2}+\frac{1}{4}(2c-i\pi)}\right)}{d}+\frac{4i\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{d^2}-\frac{4i\,\text{P}}{}\right.\right.}{}\right.}{4a^2}$$

$$\Big\downarrow \text{ 4674}$$

$$\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(-\frac{12\int x\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d^2}+\frac{1}{2}\int x^3\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx+\frac{6x^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d^2}+\frac{x^3\tanh\left(\frac{c}{2}\right.}{}\right.\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(-\frac{12\int x\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)dx}{d^2}+\frac{1}{2}\int x^3\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)dx+\frac{6x^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d^2}+\frac{x^3\tanh\left(\frac{c}{2}+\right.}{}\right.\right.$$

$\downarrow$ 4670

$$\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(-\frac{12\left(\frac{2i\int\log\left(1-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)dx}{d}-\frac{2i\int\log\left(1+e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)dx}{d}+\frac{4ix\,\text{arctanh}\left(e^{\frac{dx}{2}+\frac{1}{4}(2c-i\pi)}\right)}{d}\right)}{d^2}+\frac{1}{2}\left(\frac{6}{}\right.\right.\right.$$

$\downarrow$ 2715

$$\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(-\frac{12\left(\frac{4i\int e^{\frac{1}{4}(i\pi-2c)-\frac{dx}{2}}\log\left(1-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)de^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}}{d^2}-\frac{4i\int e^{\frac{1}{4}(i\pi-2c)-\frac{dx}{2}}\log\left(1+e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)de^{\frac{1}{4}(}}{d^2}\right.}{d^2}\right.\right.$$

$\downarrow$ 2838

$$\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{6i\int x^2\log\left(1-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)dx}{d}-\frac{6i\int x^2\log\left(1+e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)dx}{d}+\frac{4ix^3\text{arctanh}\left(e^{\frac{dx}{2}+\frac{1}{4}(2c-i\pi)}\right.}{d}\right.\right.\right.$$

$\downarrow$ 3011

$$\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(\frac{1}{2}\left(-\frac{6i\left(\frac{4\int x\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)dx}{d}-\frac{2x^2\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{d}\right)}{d}+\frac{6i\left(\frac{4\int x\,\text{PolyLog}\left(2,e^{\frac{1}{4}(2c\cdots}\right.\right.}{d}\right.\right.\right.\right.$$

$$\Big\downarrow \text{7163}$$

$$\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(\frac{1}{2}\left(-\frac{6i\left(\frac{4\left(\frac{2x\,\text{PolyLog}\left(3,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{d}-\frac{2\int \text{PolyLog}\left(3,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)dx}{d}\right)}{d}-\frac{2x^2\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2c-i\pi)+\cdots}\right.}{d}\right.}{d}\right.\right.\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{x^2\text{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d^2}+\frac{x^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\text{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{2d}-\frac{2\left(\frac{1}{2}\left(\frac{4ix\,\text{arctanh}\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{d}+\frac{4i\,\cdots}{}\right.\right.}{}\right.$$

$$\Big\downarrow \text{7143}$$

$$\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{4ix^3\operatorname{arctanh}\left(e^{\frac{dx}{2}+\frac{1}{4}(2c-i\pi)}\right)}{d}-\frac{6i\left(\frac{4\left(\frac{2x\,\text{PolyLog}\left(3,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{d}-\frac{4\,\text{PolyLog}\left(4,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{d^2}\right.}{d}\right.}{d}\right.\right.\right.$$

input
```
Int[x^3/(a + I*a*Sinh[c + d*x])^(5/2),x]
```

output
```
(Cosh[c/2 + (I/4)*Pi + (d*x)/2]*((x^2*Sech[c/2 + (I/4)*Pi + (d*x)/2]^3)/d^
2 + (x^3*Sech[c/2 + (I/4)*Pi + (d*x)/2]^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/
(2*d) - (2*((((4*I)*x*ArcTanh[E^((2*c - I*Pi)/4 + (d*x)/2)])/d + ((4*I)*Po
lyLog[2, -E^((2*c - I*Pi)/4 + (d*x)/2)])/d^2 - ((4*I)*PolyLog[2, E^((2*c -
 I*Pi)/4 + (d*x)/2)])/d^2)/2 + (2*Sech[c/2 + (I/4)*Pi + (d*x)/2])/d^2 + (x
*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/d))/d^2 +
(3*((-12*(((4*I)*x*ArcTanh[E^((2*c - I*Pi)/4 + (d*x)/2)])/d + ((4*I)*PolyL
og[2, -E^((2*c - I*Pi)/4 + (d*x)/2)])/d^2 - ((4*I)*PolyLog[2, E^((2*c - I*
Pi)/4 + (d*x)/2)])/d^2))/d^2 + (((4*I)*x^3*ArcTanh[E^((2*c - I*Pi)/4 + (d*
x)/2)])/d - ((6*I)*((-2*x^2*PolyLog[2, -E^((2*c - I*Pi)/4 + (d*x)/2)])/d +
 (4*((2*x*PolyLog[3, -E^((2*c - I*Pi)/4 + (d*x)/2)])/d - (4*PolyLog[4, -E^
((2*c - I*Pi)/4 + (d*x)/2)])/d^2))/d))/d + ((6*I)*((-2*x^2*PolyLog[2, E^((
2*c - I*Pi)/4 + (d*x)/2)])/d + (4*((2*x*PolyLog[3, E^((2*c - I*Pi)/4 + (d*
x)/2)])/d - (4*PolyLog[4, E^((2*c - I*Pi)/4 + (d*x)/2)])/d^2))/d))/d)/2 +
(6*x^2*Sech[c/2 + (I/4)*Pi + (d*x)/2])/d^2 + (x^3*Sech[c/2 + (I/4)*Pi + (d
*x)/2]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/d))/4))/(4*a^2*Sqrt[a + I*a*Sinh[c
+ d*x]])
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{x^3}{(a + ia \sinh(dx + c))^{\frac{5}{2}}} dx$$

input
```
int(x^3/(a+I*a*sinh(d*x+c))^(5/2),x)
```

output
```
int(x^3/(a+I*a*sinh(d*x+c))^(5/2),x)
```

Fricas [F]

$$\int \frac{x^3}{(a + ia\sinh(c + dx))^{5/2}}\, dx = \int \frac{x^3}{(i\, a \sinh(dx + c) + a)^{\frac{5}{2}}}\, dx$$

input
```
integrate(x^3/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/8*(8*(a^3*d^4*e^(4*d*x + 4*c) - 4*I*a^3*d^4*e^(3*d*x + 3*c) - 6*a^3*d^4*
e^(2*d*x + 2*c) + 4*I*a^3*d^4*e^(d*x + c) + a^3*d^4)*integral(1/16*(-3*I*d
^2*x^3 + 80*I*x)*sqrt(1/2*I*a*e^(-d*x - c))*e^(d*x + c)/(a^3*d^2*e^(d*x +
c) - I*a^3*d^2), x) + ((-3*I*d^3*x^3 - 18*I*d^2*x^2 + 8*I*d*x + 16*I)*e^(4
*d*x + 4*c) - (11*d^3*x^3 + 70*d^2*x^2 - 8*d*x - 48)*e^(3*d*x + 3*c) + (-1
1*I*d^3*x^3 + 70*I*d^2*x^2 + 8*I*d*x - 48*I)*e^(2*d*x + 2*c) - (3*d^3*x^3
- 18*d^2*x^2 - 8*d*x + 16)*e^(d*x + c))*sqrt(1/2*I*a*e^(-d*x - c)))/(a^3*d
^4*e^(4*d*x + 4*c) - 4*I*a^3*d^4*e^(3*d*x + 3*c) - 6*a^3*d^4*e^(2*d*x + 2*
c) + 4*I*a^3*d^4*e^(d*x + c) + a^3*d^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{(a + ia\sinh(c + dx))^{5/2}}\, dx = \text{Timed out}$$

input
```
integrate(x**3/(a+I*a*sinh(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^3}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{x^3}{(i\,a\sinh\left(dx+c\right)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(I*a*sinh(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^3}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{x^3}{(i\,a\sinh\left(dx+c\right)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(x^3/(I*a*sinh(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{x^3}{\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

input
```
int(x^3/(a + a*sinh(c + d*x)*1i)^(5/2),x)
```

output
```
int(x^3/(a + a*sinh(c + d*x)*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^3}{(a+ia\sinh(c+dx))^{5/2}}\,dx = -\frac{\int \frac{x^3}{\sqrt{\sinh(dx+c)i+1}\,\sinh(dx+c)^2-2\sqrt{\sinh(dx+c)i+1}\,\sinh(dx+c)i-\sqrt{\sinh(dx+c)i+1}}dx}{\sqrt{a}\,a^2}$$

input
```
int(x^3/(a+I*a*sinh(d*x+c))^(5/2),x)
```

output
```
( - int(x**3/(sqrt(sinh(c + d*x)*i + 1)*sinh(c + d*x)**2 - 2*sqrt(sinh(c +
 d*x)*i + 1)*sinh(c + d*x)*i - sqrt(sinh(c + d*x)*i + 1)),x))/(sqrt(a)*a**
2)
```

3.147 $\int \frac{x^2}{(a+ia\sinh(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 689

$$\int \frac{x^2}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \frac{3x}{4a^2d^2\sqrt{a+ia\sinh(c+dx)}} - \frac{5\arctan\left(\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{3a^2d^3\sqrt{a+ia\sinh(c+dx)}} + \frac{3ix^2\text{arctanh}\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{8a^2d\sqrt{a+ia\sinh(c+dx)}} + \frac{3ix\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\text{PolyLog}\left(2,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{4a^2d^2\sqrt{a+ia\sinh(c+dx)}} - \frac{3ix\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\text{PolyLog}\left(2,e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{4a^2d^2\sqrt{a+ia\sinh(c+dx)}} - \frac{3i\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\text{PolyLog}\left(3,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{2a^2d^3\sqrt{a+ia\sinh(c+dx)}} + \frac{3i\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\text{PolyLog}\left(3,e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{2a^2d^3\sqrt{a+ia\sinh(c+dx)}} + \frac{x\,\text{sech}^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{6a^2d^2\sqrt{a+ia\sinh(c+dx)}} - \frac{\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{6a^2d^3\sqrt{a+ia\sinh(c+dx)}} + \frac{3x^2\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{16a^2d\sqrt{a+ia\sinh(c+dx)}} + \frac{x^2\text{sech}^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{8a^2d\sqrt{a+ia\sinh(c+dx)}}$$

output
```
3/4*x/a^2/d^2/(a+I*a*sinh(d*x+c))^(1/2)-5/3*arctan(sinh(1/2*c+1/4*I*Pi+1/2
*d*x))*cosh(1/2*c+1/4*I*Pi+1/2*d*x)/a^2/d^3/(a+I*a*sinh(d*x+c))^(1/2)-3/8*
I*x^2*arctanh(exp(1/2*c+3/4*I*Pi+1/2*d*x))*cosh(1/2*c+1/4*I*Pi+1/2*d*x)/a^
2/d/(a+I*a*sinh(d*x+c))^(1/2)+3/4*I*x*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog
(2,exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/d^2/(a+I*a*sinh(d*x+c))^(1/2)-3/4*I*x*
cosh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog(2,-exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/d
^2/(a+I*a*sinh(d*x+c))^(1/2)-3/2*I*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog(3,
exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/d^3/(a+I*a*sinh(d*x+c))^(1/2)+3/2*I*cosh(
1/2*c+1/4*I*Pi+1/2*d*x)*polylog(3,-exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/d^3/(a
+I*a*sinh(d*x+c))^(1/2)+1/6*x*sech(1/2*c+1/4*I*Pi+1/2*d*x)^2/a^2/d^2/(a+I*
a*sinh(d*x+c))^(1/2)-1/6*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a^2/d^3/(a+I*a*sinh(
d*x+c))^(1/2)+3/16*x^2*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a^2/d/(a+I*a*sinh(d*x+
c))^(1/2)+1/8*x^2*sech(1/2*c+1/4*I*Pi+1/2*d*x)^2*tanh(1/2*c+1/4*I*Pi+1/2*d
*x)/a^2/d/(a+I*a*sinh(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.41 (sec) , antiderivative size = 482, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(4dx(4+3idx)\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right.\right.}{}$$

input
```
Integrate[x^2/(a + I*a*Sinh[c + d*x])^(5/2),x]
```

output
```
((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(4*d*x*(4 + (3*I)*d*x)*(Cosh[(c
 + d*x)/2] + I*Sinh[(c + d*x)/2]) + (-8*I + 36*d*x + (9*I)*d^2*x^2)*(Cosh[
(c + d*x)/2] + I*Sinh[(c + d*x)/2])^3 - (1/2 - I/2)*(-1)^(3/4)*(-160*ArcTa
nh[(-1)^(3/4)*E^((c + d*x)/2)] + 18*c^2*ArcTanh[(-1)^(3/4)*E^((c + d*x)/2)
] + 9*c^2*Log[1 - (-1)^(3/4)*E^((c + d*x)/2)] - 9*d^2*x^2*Log[1 - (-1)^(3/
4)*E^((c + d*x)/2)] - 9*c^2*Log[1 + (-1)^(3/4)*E^((c + d*x)/2)] + 9*d^2*x^
2*Log[1 + (-1)^(3/4)*E^((c + d*x)/2)] + 36*d*x*PolyLog[2, -((-1)^(3/4)*E^(
(c + d*x)/2))] - 36*d*x*PolyLog[2, (-1)^(3/4)*E^((c + d*x)/2)] - 72*PolyLo
g[3, -((-1)^(3/4)*E^((c + d*x)/2))] + 72*PolyLog[3, (-1)^(3/4)*E^((c + d*x
)/2)])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^4 + 24*d^2*x^2*Sinh[(c +
d*x)/2] + 2*(-8 + 9*d^2*x^2)*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2*S
inh[(c + d*x)/2]))/(48*d^3*(a + I*a*Sinh[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 484, normalized size of antiderivative = 0.70, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 3800, 3042, 4674, 3042, 4255, 3042, 4257, 4674, 3042, 4257, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+ia\sinh(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{x^2}{(a+a\sin(ic+idx))^{5/2}}dx$$

$$\Big\downarrow \text{3800}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\int x^2\text{sech}^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\int x^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^5dx}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$\Big\downarrow \text{4674}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(-\frac{2\int \text{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{3d^2}+\frac{3}{4}\int x^2\text{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx+\frac{2x\text{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{3d^2}+\frac{x^2\tanh\left(\frac{c}{2}+\frac{dx}{2}\right.}{}\right.}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(-\frac{2\int \csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{3d^2}+\frac{3}{4}\int x^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx+\frac{2x\text{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{3d^2}+\frac{x^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\right.}{}\right.}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(-\frac{2\left(\frac{1}{2}\int \text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx+\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{3d^2}+\frac{3}{4}\int x^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx+\frac{2}{\ldots}\right.}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(-\frac{2\left(\frac{1}{2}\int \csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)dx+\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{3d^2}+\frac{3}{4}\int x^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx+\frac{2x\ldots}{\ldots}\right.}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\int x^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx-\frac{2\left(\frac{\arctan\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{d}+\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{3d^2}+\frac{2x\text{se}\ldots}{\ldots}\right.}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$\Big\downarrow \text{4674}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(-\frac{4\int \text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d^2}+\frac{1}{2}\int x^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx+\frac{4x\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d^2}+\frac{x^2\tanh\left(\frac{c}{2}+\frac{dx}{2}\ldots\right.}{\ldots}\right.\right.}{\ldots}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(-\frac{4\int \csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)dx}{d^2}+\frac{1}{2}\int x^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)dx+\frac{4x\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d^2}+\frac{x^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\ldots\right.}{\ldots}\right.\right.}{4\ldots}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(\frac{1}{2}\int x^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)dx-\frac{8\arctan\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{d^3}+\frac{4x\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d^2}+\frac{x^2\tanh\left(\frac{c}{2}+\ldots\right.}{\ldots}\right.\right.}{\ldots}$$

$$\Big\downarrow \; 4670$$

$$\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{4i\int x\log\left(1-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)dx}{d}-\frac{4i\int x\log\left(1+e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)dx}{d}+\frac{4ix^2\text{arctanh}\left(e^{\frac{dx}{2}+\frac{1}{4}(2c-i\pi)}\right)}{d}\right.\right.\right.$$

$$\Big\downarrow \; 3011$$

$$\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(\frac{1}{2}\left(-\frac{4i\left(\frac{2\int \text{PolyLog}\left(2,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)dx}{d}-\frac{2x\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{d}\right)}{d}+\frac{4i\left(\frac{2\int \text{PolyLog}\left(2,e^{\frac{1}{4}(2c-i\pi)}\right.}{d}\right.}{}\right.\right.\right.$$

$$\Big\downarrow \; 2720$$

$$\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(\frac{1}{2}\left(-\frac{4i\left(\frac{4\int e^{\frac{1}{4}(i\pi-2c)-\frac{dx}{2}}\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)de^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}}{d^2}-\frac{2x\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{d}\right)}{d}+\right.\right.\right.$$

$$\Big\downarrow \; 7143$$

$$\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(-\frac{8\arctan\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{d^3}+\frac{1}{2}\left(\frac{4ix^2\text{arctanh}\left(e^{\frac{dx}{2}+\frac{1}{4}(2c-i\pi)}\right)}{d}-\frac{4i\left(\frac{4\,\text{PolyLog}\left(3,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{d^2}\right.}{d}\right.\right.\right.$$

input
```
Int[x^2/(a + I*a*Sinh[c + d*x])^(5/2),x]
```

output
```
(Cosh[c/2 + (I/4)*Pi + (d*x)/2]*((2*x*Sech[c/2 + (I/4)*Pi + (d*x)/2]^3)/(3
*d^2) + (x^2*Sech[c/2 + (I/4)*Pi + (d*x)/2]^3*Tanh[c/2 + (I/4)*Pi + (d*x)/
2])/(2*d) - (2*(ArcTan[Sinh[c/2 + (I/4)*Pi + (d*x)/2]]/d + (Sech[c/2 + (I/
4)*Pi + (d*x)/2]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/d))/(3*d^2) + (3*((-8*Arc
Tan[Sinh[c/2 + (I/4)*Pi + (d*x)/2]])/d^3 + (((4*I)*x^2*ArcTanh[E^((2*c - I
*Pi)/4 + (d*x)/2)])/d - ((4*I)*((-2*x*PolyLog[2, -E^((2*c - I*Pi)/4 + (d*x
)/2)])/d + (4*PolyLog[3, -E^((2*c - I*Pi)/4 + (d*x)/2)])/d^2))/d + ((4*I)*
((-2*x*PolyLog[2, E^((2*c - I*Pi)/4 + (d*x)/2)])/d + (4*PolyLog[3, E^((2*c
 - I*Pi)/4 + (d*x)/2)])/d^2))/d)/2 + (4*x*Sech[c/2 + (I/4)*Pi + (d*x)/2])/
d^2 + (x^2*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/
d))/4))/(4*a^2*Sqrt[a + I*a*Sinh[c + d*x]])
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{x^2}{(a + ia \sinh(dx + c))^{\frac{5}{2}}} dx$$

input
```
int(x^2/(a+I*a*sinh(d*x+c))^(5/2),x)
```

output
```
int(x^2/(a+I*a*sinh(d*x+c))^(5/2),x)
```

Fricas [F]

$$\int \frac{x^2}{(a+ia\sinh(c+dx))^{5/2}}\,dx=\int \frac{x^2}{(i\,a\sinh\left(dx+c\right)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/24*(24*(a^3*d^3*e^(4*d*x + 4*c) - 4*I*a^3*d^3*e^(3*d*x + 3*c) - 6*a^3*d^
3*e^(2*d*x + 2*c) + 4*I*a^3*d^3*e^(d*x + c) + a^3*d^3)*integral(1/48*(-9*I
*d^2*x^2 + 80*I)*sqrt(1/2*I*a*e^(-d*x - c))*e^(d*x + c)/(a^3*d^2*e^(d*x +
c) - I*a^3*d^2), x) + ((-9*I*d^2*x^2 - 36*I*d*x + 8*I)*e^(4*d*x + 4*c) - (
33*d^2*x^2 + 140*d*x - 8)*e^(3*d*x + 3*c) + (-33*I*d^2*x^2 + 140*I*d*x + 8
*I)*e^(2*d*x + 2*c) - (9*d^2*x^2 - 36*d*x - 8)*e^(d*x + c))*sqrt(1/2*I*a*e
^(-d*x - c)))/(a^3*d^3*e^(4*d*x + 4*c) - 4*I*a^3*d^3*e^(3*d*x + 3*c) - 6*a
^3*d^3*e^(2*d*x + 2*c) + 4*I*a^3*d^3*e^(d*x + c) + a^3*d^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+ia\sinh(c+dx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(x**2/(a+I*a*sinh(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{x^2}{(i\,a\sinh\left(dx+c\right)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(I*a*sinh(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^2}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{x^2}{(i\,a\sinh\left(dx+c\right)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(x^2/(I*a*sinh(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{x^2}{{\left(a+a\sinh\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}\,dx$$

input
```
int(x^2/(a + a*sinh(c + d*x)*1i)^(5/2),x)
```

output
```
int(x^2/(a + a*sinh(c + d*x)*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+ia\sinh(c+dx))^{5/2}}\,dx = -\frac{\int \frac{x^2}{\sqrt{\sinh(dx+c)i+1}\,\sinh(dx+c)^2-2\sqrt{\sinh(dx+c)i+1}\,\sinh(dx+c)i-\sqrt{\sinh(dx+c)i+1}}dx}{\sqrt{a}\,a^2}$$

input
```
int(x^2/(a+I*a*sinh(d*x+c))^(5/2),x)
```

output
```
( - int(x**2/(sqrt(sinh(c + d*x)*i + 1)*sinh(c + d*x)**2 - 2*sqrt(sinh(c +
 d*x)*i + 1)*sinh(c + d*x)*i - sqrt(sinh(c + d*x)*i + 1)),x))/(sqrt(a)*a**
2)
```

3.148 $\int \frac{x}{(a+ia\sinh(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 416

$$\begin{aligned}\int \frac{x}{(a+ia\sinh(c+dx))^{5/2}}\,dx &= \frac{3}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}} \\ &+ \frac{3ix\operatorname{arctanh}\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{8a^2d\sqrt{a+ia\sinh(c+dx)}} \\ &+ \frac{3i\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}} \\ &- \frac{3i\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}} \\ &+ \frac{\operatorname{sech}^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{12a^2d^2\sqrt{a+ia\sinh(c+dx)}} + \frac{3x\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{16a^2d\sqrt{a+ia\sinh(c+dx)}} \\ &+ \frac{x\operatorname{sech}^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{8a^2d\sqrt{a+ia\sinh(c+dx)}}\end{aligned}$$

output
```
3/8/a^2/d^2/(a+I*a*sinh(d*x+c))^(1/2)-3/8*I*x*arctanh(exp(1/2*c+3/4*I*Pi+1
/2*d*x))*cosh(1/2*c+1/4*I*Pi+1/2*d*x)/a^2/d/(a+I*a*sinh(d*x+c))^(1/2)+3/8*
I*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog(2,exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/
d^2/(a+I*a*sinh(d*x+c))^(1/2)-3/8*I*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog(2
,-exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/d^2/(a+I*a*sinh(d*x+c))^(1/2)+1/12*sech
(1/2*c+1/4*I*Pi+1/2*d*x)^2/a^2/d^2/(a+I*a*sinh(d*x+c))^(1/2)+3/16*x*tanh(1
/2*c+1/4*I*Pi+1/2*d*x)/a^2/d/(a+I*a*sinh(d*x+c))^(1/2)+1/8*x*sech(1/2*c+1/
4*I*Pi+1/2*d*x)^2*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a^2/d/(a+I*a*sinh(d*x+c))^(
1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.05 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.81

$$\int \frac{x}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(4(2+3idx)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+\right.\right.}{}$$

input
```
Integrate[x/(a + I*a*Sinh[c + d*x])^(5/2),x]
```

output
```
((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(4*(2 + (3*I)*d*x)*(Cosh[(c + d
*x)/2] + I*Sinh[(c + d*x)/2]) + 9*(2 + I*d*x)*(Cosh[(c + d*x)/2] + I*Sinh[
(c + d*x)/2])^3 + (9 - 9*I)*(-1)^(3/4)*(I*c*ArcTan[(-1)^(1/4)*E^((c + d*x)
/2)] + ((c + d*x)*Log[1 - (-1)^(3/4)*E^((c + d*x)/2)])/2 - ((c + d*x)*Log[
1 + (-1)^(3/4)*E^((c + d*x)/2)])/2 - PolyLog[2, -((-1)^(3/4)*E^((c + d*x)/
2))] + PolyLog[2, (-1)^(3/4)*E^((c + d*x)/2)])*(Cosh[(c + d*x)/2] + I*Sinh
[(c + d*x)/2])^4 + 24*d*x*Sinh[(c + d*x)/2] + 18*d*x*(Cosh[(c + d*x)/2] +
I*Sinh[(c + d*x)/2])^2*Sinh[(c + d*x)/2]))/(48*d^2*(a + I*a*Sinh[c + d*x])
^(5/2))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.72, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 3800, 3042, 4673, 3042, 4673, 3042, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+ia\sinh(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{x}{(a+a\sin(ic+idx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3800}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\int x\text{sech}^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\int x\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^5dx}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$\Big\downarrow \text{ 4673}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\int x\text{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx+\frac{\text{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{3d^2}+\frac{x\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\text{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{2d}\right)}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\int x\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx+\frac{\text{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{3d^2}+\frac{x\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\text{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{2d}\right)}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$\Big\downarrow \text{ 4673}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(\frac{1}{2}\int x\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx+\frac{2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d^2}+\frac{x\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)+\text{sec}}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(\frac{1}{2}\int x\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)dx+\frac{2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d^2}+\frac{x\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)+\text{sec}}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$\downarrow$ 4670

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{2i\int\log\left(1-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)dx}{d}-\frac{2i\int\log\left(1+e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)dx}{d}+\frac{4ix\text{arctanh}\left(e^{\frac{dx}{2}+\frac{1}{4}(2c-i\pi)}\right)}{d}\right)+\right.\right.}{4a^2\sqrt{a+ia\text{si}}}$$

$\downarrow$ 2715

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{4i\int e^{\frac{1}{4}(i\pi-2c)-\frac{dx}{2}}\log\left(1-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)de^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}}{d^2}-\frac{4i\int e^{\frac{1}{4}(i\pi-2c)-\frac{dx}{2}}\log\left(1+e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)\text{c}}{d^2}\right.\right.\right.}{}$$

$\downarrow$ 2838

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{4ix\text{arctanh}\left(e^{\frac{dx}{2}+\frac{1}{4}(2c-i\pi)}\right)}{d}+\frac{4i\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{d^2}-\frac{4i\,\text{PolyLog}\left(2,e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{d^2}\right)\right.\right.}{4a^2\sqrt{a+ia\text{s}}}$$

input
```
Int[x/(a + I*a*Sinh[c + d*x])^(5/2),x]
```

output
```
(Cosh[c/2 + (I/4)*Pi + (d*x)/2]*(Sech[c/2 + (I/4)*Pi + (d*x)/2]^3/(3*d^2)
+ (x*Sech[c/2 + (I/4)*Pi + (d*x)/2]^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(2*d
) + (3*((((4*I)*x*ArcTanh[E^((2*c - I*Pi)/4 + (d*x)/2)])/d + ((4*I)*PolyLo
g[2, -E^((2*c - I*Pi)/4 + (d*x)/2)])/d^2 - ((4*I)*PolyLog[2, E^((2*c - I*P
i)/4 + (d*x)/2)])/d^2)/2 + (2*Sech[c/2 + (I/4)*Pi + (d*x)/2])/d^2 + (x*Sec
h[c/2 + (I/4)*Pi + (d*x)/2]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/d))/4))/(4*a^2
*Sqrt[a + I*a*Sinh[c + d*x]])
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Maple [F]

$$\int \frac{x}{(a+ia\sinh(dx+c))^{\frac{5}{2}}}dx$$

input
```
int(x/(a+I*a*sinh(d*x+c))^(5/2),x)
```

output
```
int(x/(a+I*a*sinh(d*x+c))^(5/2),x)
```

Fricas [F]

$$\int \frac{x}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{x}{(i\,a\sinh(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/24*(24*(a^3*d^2*e^(4*d*x + 4*c) - 4*I*a^3*d^2*e^(3*d*x + 3*c) - 6*a^3*d^
2*e^(2*d*x + 2*c) + 4*I*a^3*d^2*e^(d*x + c) + a^3*d^2)*integral(-3/16*I*sq
rt(1/2*I*a*e^(-d*x - c))*x*e^(d*x + c)/(a^3*e^(d*x + c) - I*a^3), x) - (9*
(I*d*x + 2*I)*e^(4*d*x + 4*c) + (33*d*x + 70)*e^(3*d*x + 3*c) - (-33*I*d*x
 + 70*I)*e^(2*d*x + 2*c) + 9*(d*x - 2)*e^(d*x + c))*sqrt(1/2*I*a*e^(-d*x -
 c)))/(a^3*d^2*e^(4*d*x + 4*c) - 4*I*a^3*d^2*e^(3*d*x + 3*c) - 6*a^3*d^2*e
^(2*d*x + 2*c) + 4*I*a^3*d^2*e^(d*x + c) + a^3*d^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x/(a+I*a*sinh(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{x}{(i\,a\sinh{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(x/(I*a*sinh(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{x}{(i\,a\sinh{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(x/(I*a*sinh(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a + ia\sinh(c + dx))^{5/2}}\,dx = \int \frac{x}{\left(a + a\sinh\left(c + d\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

input
```
int(x/(a + a*sinh(c + d*x)*1i)^(5/2),x)
```

output
```
int(x/(a + a*sinh(c + d*x)*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{x}{(a + ia\sinh(c + dx))^{5/2}}\,dx = -\frac{\int \frac{x}{\sqrt{\sinh(dx+c)i+1}\,\sinh(dx+c)^2 - 2\sqrt{\sinh(dx+c)i+1}\,\sinh(dx+c)i - \sqrt{\sinh(dx+c)i+1}}dx}{\sqrt{a}\,a^2}$$

input
```
int(x/(a+I*a*sinh(d*x+c))^(5/2),x)
```

output
```
( - int(x/(sqrt(sinh(c + d*x)*i + 1)*sinh(c + d*x)**2 - 2*sqrt(sinh(c + d*
x)*i + 1)*sinh(c + d*x)*i - sqrt(sinh(c + d*x)*i + 1)),x))/(sqrt(a)*a**2)
```

3.149 $\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx = \text{Int}\left(\frac{1}{x(a+ia\sinh(c+dx))^{5/2}},x\right)$$

output
```
Defer(Int)(1/x/(a+I*a*sinh(d*x+c))^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 35.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx$$

input
```
Integrate[1/(x*(a + I*a*Sinh[c + d*x])^(5/2)),x]
```

output
```
Integrate[1/(x*(a + I*a*Sinh[c + d*x])^(5/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{x(a+a\sin(ic+idx))^{5/2}}dx$$

$$\Big\downarrow \; 3807$$

$$\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}dx$$

input
```
Int[1/(x*(a + I*a*Sinh[c + d*x])^(5/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{x\left(a+ia\sinh\left(dx+c\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(1/x/(a+I*a*sinh(d*x+c))^(5/2),x)
```

output
```
int(1/x/(a+I*a*sinh(d*x+c))^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 396, normalized size of antiderivative = 18.86

$$\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{(i\,a\sinh\left(dx+c\right)+a)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(1/x/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/24*(24*(a^3*d^4*x^4*e^(4*d*x + 4*c) - 4*I*a^3*d^4*x^4*e^(3*d*x + 3*c) -
6*a^3*d^4*x^4*e^(2*d*x + 2*c) + 4*I*a^3*d^4*x^4*e^(d*x + c) + a^3*d^4*x^4)
*integral(1/48*(-9*I*d^4*x^4 + 80*I*d^2*x^2 - 384*I)*sqrt(1/2*I*a*e^(-d*x
- c))*e^(d*x + c)/(a^3*d^4*x^5*e^(d*x + c) - I*a^3*d^4*x^5), x) + ((-9*I*d
^3*x^3 + 18*I*d^2*x^2 + 8*I*d*x - 48*I)*e^(4*d*x + 4*c) - (33*d^3*x^3 - 70
*d^2*x^2 - 8*d*x + 144)*e^(3*d*x + 3*c) + (-33*I*d^3*x^3 - 70*I*d^2*x^2 +
8*I*d*x + 144*I)*e^(2*d*x + 2*c) - (9*d^3*x^3 + 18*d^2*x^2 - 8*d*x - 48)*e
^(d*x + c))*sqrt(1/2*I*a*e^(-d*x - c)))/(a^3*d^4*x^4*e^(4*d*x + 4*c) - 4*I
*a^3*d^4*x^4*e^(3*d*x + 3*c) - 6*a^3*d^4*x^4*e^(2*d*x + 2*c) + 4*I*a^3*d^4
*x^4*e^(d*x + c) + a^3*d^4*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(a+I*a*sinh(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx=\int \frac{1}{(i\,a\sinh{(dx+c)}+a)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(1/x/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*sinh(d*x + c) + a)^(5/2)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx=\int \frac{1}{(i\,a\sinh{(dx+c)}+a)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(1/x/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((I*a*sinh(d*x + c) + a)^(5/2)*x), x)
```

Mupad [N/A]

Not integrable

Time = 2.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx=\int \frac{1}{x\,{\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}\,dx$$

input
```
int(1/(x*(a + a*sinh(c + d*x)*1i)^(5/2)),x)
```

output
```
int(1/(x*(a + a*sinh(c + d*x)*1i)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.33

$$\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx = -\frac{\int \frac{1}{\sqrt{\sinh(dx+c)i+1}\,\sinh(dx+c)^2x-2\sqrt{\sinh(dx+c)i+1}\,\sinh(dx+c)ix-\sqrt{\sinh(dx+c)i+1}\,x}dx}{\sqrt{a}\,a^2}$$

input
```
int(1/x/(a+I*a*sinh(d*x+c))^(5/2),x)
```

output
```
( - int(1/(sqrt(sinh(c + d*x)*i + 1)*sinh(c + d*x)**2*x - 2*sqrt(sinh(c +
d*x)*i + 1)*sinh(c + d*x)*i*x - sqrt(sinh(c + d*x)*i + 1)*x),x))/(sqrt(a)*
a**2)
```

3.150 $\int \frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x}\,dx = \text{Int}\left(\frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x},x\right)$$

output
```
Defer(Int)((a+I*a*sinh(f*x+e))^(1/3)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 3.96 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x}\,dx = \int \frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x}\,dx$$

input
```
Integrate[(a + I*a*Sinh[e + f*x])^(1/3)/x,x]
```

output
```
Integrate[(a + I*a*Sinh[e + f*x])^(1/3)/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{a+a\sin(ie+ifx)}}{x}dx$$

$$\Big\downarrow \text{3807}$$

$$\int \frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x}dx$$

input
```
Int[(a + I*a*Sinh[e + f*x])^(1/3)/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{(a+ia\sinh(fx+e))^{\frac{1}{3}}}{x}dx$$

input
```
int((a+I*a*sinh(f*x+e))^(1/3)/x,x)
```

output
```
int((a+I*a*sinh(f*x+e))^(1/3)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*sinh(f*x+e))^(1/3)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [N/A]

Not integrable

Time = 2.39 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x}\,dx = \int \frac{\sqrt[3]{ia\left(\sinh\left(e+fx\right)-i\right)}}{x}\,dx$$

input
```
integrate((a+I*a*sinh(f*x+e))**(1/3)/x,x)
```

output
```
Integral((I*a*(sinh(e + f*x) - I))**(1/3)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x}\,dx = \int \frac{\left(i\,a\sinh\left(fx+e\right)+a\right)^{\frac{1}{3}}}{x}\,dx$$

input
```
integrate((a+I*a*sinh(f*x+e))^(1/3)/x,x, algorithm="maxima")
```

output
```
integrate((I*a*sinh(f*x + e) + a)^(1/3)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x}\,dx = \int \frac{(i\,a\sinh\left(fx+e\right)+a)^{\frac{1}{3}}}{x}\,dx$$

input
```
integrate((a+I*a*sinh(f*x+e))^(1/3)/x,x, algorithm="giac")
```

output
```
integrate((I*a*sinh(f*x + e) + a)^(1/3)/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x}\,dx = \int \frac{{\left(a+a\sinh(e+f\,x)\,\,1\mathrm{i}\right)}^{1/3}}{x}\,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)^(1/3)/x,x)
```

output
```
int((a + a*sinh(e + f*x)*1i)^(1/3)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x}\,dx = a^{\frac{1}{3}}\left(\int \frac{(\sinh(fx+e)\,i+1)^{\frac{1}{3}}}{x}dx\right)$$

input
```
int((a+I*a*sinh(f*x+e))^(1/3)/x,x)
```

output
```
a**(1/3)*int((sinh(e + f*x)*i + 1)**(1/3)/x,x)
```

3.151 $\int (c+dx)^m (a+ia\sinh(e+fx))^n\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (c+dx)^m (a+ia\sinh(e+fx))^n\,dx = \text{Int}((c+dx)^m (a+ia\sinh(e+fx))^n, x)$$

output
```
Defer(Int)((d*x+c)^m*(a+I*a*sinh(f*x+e))^n,x)
```

Mathematica [N/A]

Not integrable

Time = 2.82 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (c+dx)^m (a+ia\sinh(e+fx))^n\,dx = \int (c+dx)^m (a+ia\sinh(e+fx))^n\,dx$$

input
```
Integrate[(c + d*x)^m*(a + I*a*Sinh[e + f*x])^n,x]
```

output
```
Integrate[(c + d*x)^m*(a + I*a*Sinh[e + f*x])^n, x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(a+ia\sinh(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^m(a+a\sin(ie+ifx))^n dx$$

$$\Big\downarrow \text{3807}$$

$$\int (c+dx)^m(a+ia\sinh(e+fx))^n dx$$

input
```
Int[(c + d*x)^m*(a + I*a*Sinh[e + f*x])^n,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int (dx+c)^m\,(a+ia\sinh(fx+e))^n\,dx$$

input
```
int((d*x+c)^m*(a+I*a*sinh(f*x+e))^n,x)
```

output
```
int((d*x+c)^m*(a+I*a*sinh(f*x+e))^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.09

$$\int (c+dx)^m(a+ia\sinh(e+fx))^n\,dx = \int (dx+c)^m(i\,a\sinh(fx+e)+a)^n\,dx$$

input
```
integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*(1/2*(I*a*e^(2*f*x + 2*e) + 2*a*e^(f*x + e) - I*a)*e^
(-f*x - e))^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^m(a+ia\sinh(e+fx))^n\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**m*(a+I*a*sinh(f*x+e))**n,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (c+dx)^m(a+ia\sinh(e+fx))^n\,dx = \int (dx+c)^m(i\,a\sinh(fx+e)+a)^n\,dx$$

input
```
integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*(I*a*sinh(f*x + e) + a)^n, x)
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (c+dx)^m(a+ia\sinh(e+fx))^n\,dx = \int (dx+c)^m(i\,a\sinh(fx+e)+a)^n\,dx$$

input
```
integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*(I*a*sinh(f*x + e) + a)^n, x)
```

Mupad [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int (c+dx)^m(a+ia\sinh(e+fx))^n\,dx = \int (a+a\sinh(e+f\,x)\;1\mathrm{i})^n\,(c+d\,x)^m\,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)^n*(c + d*x)^m,x)
```

output
```
int((a + a*sinh(e + f*x)*1i)^n*(c + d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (c+dx)^m(a+ia\sinh(e+fx))^n\,dx = \int (dx+c)^m(\sinh(fx+e)\,ai+a)^n\,dx$$

input
```
int((d*x+c)^m*(a+I*a*sinh(f*x+e))^n,x)
```

output
```
int((c + d*x)**m*(sinh(e + f*x)*a*i + a)**n,x)
```

3.152 $\int (c+dx)^m(a+ia\sinh(e+fx))^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 410

$$\int (c+dx)^m(a+ia\sinh(e+fx))^3\,dx = \frac{5a^3(c+dx)^{1+m}}{2d(1+m)} - \frac{i3^{-1-m}a^3e^{3e-\frac{3cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3f(c+dx)}{d}\right)}{8f} - \frac{3\,2^{-3-m}a^3e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2f(c+dx)}{d}\right)}{f} + \frac{15ia^3e^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{8f} + \frac{15ia^3e^{-e+\frac{cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{f(c+dx)}{d}\right)}{8f} + \frac{3\,2^{-3-m}a^3e^{-2e+\frac{2cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)}{f} - \frac{i3^{-1-m}a^3e^{-3e+\frac{3cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{3f(c+dx)}{d}\right)}{8f}$$

output
```
5/2*a^3*(d*x+c)^(1+m)/d/(1+m)-1/8*I*3^(-1-m)*a^3*exp(3*e-3*c*f/d)*(d*x+c)^
m*GAMMA(1+m,-3*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)-3*2^(-3-m)*a^3*exp(2*e-2*
c*f/d)*(d*x+c)^m*GAMMA(1+m,-2*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+15/8*I*a^3
*exp(e-c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+15/8*
I*a^3*exp(-e+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)+3
*2^(-3-m)*a^3*exp(-2*e+2*c*f/d)*(d*x+c)^m*GAMMA(1+m,2*f*(d*x+c)/d)/f/((f*(
d*x+c)/d)^m)-1/8*I*3^(-1-m)*a^3*exp(-3*e+3*c*f/d)*(d*x+c)^m*GAMMA(1+m,3*f*
(d*x+c)/d)/f/((f*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00

$$\int (c+dx)^m(a+ia\sinh(e+fx))^3\,dx = \frac{5a^3(c+dx)^{1+m}}{2d(1+m)} - \frac{i3^{-1-m}a^3e^{3e-\frac{3cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3f(c+dx)}{d}\right)}{8f} - \frac{3\,2^{-3-m}a^3e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2f(c+dx)}{d}\right)}{f} + \frac{15ia^3e^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{8f} + \frac{15ia^3e^{-e+\frac{cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{f(c+dx)}{d}\right)}{8f} + \frac{3\,2^{-3-m}a^3e^{-2e+\frac{2cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)}{f} - \frac{i3^{-1-m}a^3e^{-3e+\frac{3cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{3f(c+dx)}{d}\right)}{8f}$$

input
```
Integrate[(c + d*x)^m*(a + I*a*Sinh[e + f*x])^3,x]
```

output
```
(5*a^3*(c + d*x)^(1 + m))/(2*d*(1 + m)) - ((I/8)*3^(-1 - m)*a^3*E^(3*e - (
3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-3*f*(c + d*x))/d])/(f*(-((f*(c + d*x)
)/d))^m) - (3*2^(-3 - m)*a^3*E^(2*e - (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m,
(-2*f*(c + d*x))/d])/(f*(-((f*(c + d*x))/d))^m) + (((15*I)/8)*a^3*E^(e - (
c*f)/d)*(c + d*x)^m*Gamma[1 + m, -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/
d))^m) + (((15*I)/8)*a^3*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c +
 d*x))/d])/(f*((f*(c + d*x))/d)^m) + (3*2^(-3 - m)*a^3*E^(-2*e + (2*c*f)/d
)*(c + d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m) - (
(I/8)*3^(-1 - m)*a^3*E^(-3*e + (3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (3*f*(c
 + d*x))/d])/(f*((f*(c + d*x))/d)^m)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 394, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 3799, 25, 25, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(a+ia\sinh(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^m(a+a\sin(ie+ifx))^3dx$$

$$\Big\downarrow \text{3799}$$

$$8a^3\int -(c+dx)^m\sinh^6\left(\frac{e}{2}+\frac{fx}{2}-\frac{i\pi}{4}\right)dx$$

$$\Big\downarrow \text{25}$$

$$-8a^3\int -(c+dx)^m\cosh^6\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx$$

$$\Big\downarrow \text{25}$$

$$8a^3\int (c+dx)^m\cosh^6\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$8a^3 \int (c+dx)^m \sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^6 dx$$

$$\Big\downarrow \text{3793}$$

$$8a^3 \int \left(-\frac{3}{16}\cosh(2e+2fx)(c+dx)^m+\frac{15}{32}i\sinh(e+fx)(c+dx)^m-\frac{1}{32}i\sinh(3e+3fx)(c+dx)^m+\frac{5}{16}(c+d\right.$$

$$\Big\downarrow \text{2009}$$

$$8a^3\left(-\frac{i3^{-m-1}e^{3e-\frac{3cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3f(c+dx)}{d}\right)}{64f}-\frac{3\,2^{-m-6}e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-}}{f}\right.$$

input
```
Int[(c + d*x)^m*(a + I*a*Sinh[e + f*x])^3,x]
```

output
```
8*a^3*((5*(c + d*x)^(1 + m))/(16*d*(1 + m)) - ((I/64)*3^(-1 - m)*E^(3*e -
(3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-3*f*(c + d*x))/d])/(f*(-((f*(c + d*x
))/d))^m) - (3*2^(-6 - m)*E^(2*e - (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-2
*f*(c + d*x))/d])/(f*(-((f*(c + d*x))/d))^m) + (((15*I)/64)*E^(e - (c*f)/d
)*(c + d*x)^m*Gamma[1 + m, -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/d))^m)
 + (((15*I)/64)*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d]
)/(f*((f*(c + d*x))/d)^m) + (3*2^(-6 - m)*E^(-2*e + (2*c*f)/d)*(c + d*x)^m
*Gamma[1 + m, (2*f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m) - ((I/64)*3^(-1
- m)*E^(-3*e + (3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (3*f*(c + d*x))/d])/(f*
((f*(c + d*x))/d)^m))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int (dx+c)^{m} (a+ia\sinh(fx+e))^{3} dx$$

input
```
int((d*x+c)^m*(a+I*a*sinh(f*x+e))^3,x)
```

output
```
int((d*x+c)^m*(a+I*a*sinh(f*x+e))^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.91

$$\int (c+dx)^m(a+ia\sinh(e+fx))^3 dx = \frac{(-i\,a^3dm-i\,a^3d)e^{\left(-\frac{dm\log\left(\frac{3f}{d}\right)+3\,de-3\,cf}{d}\right)}\Gamma\left(m+1,\frac{3\,(dfx+cf)}{d}\right)+9\,(a^3dm+a^3d)e^{\left(-\frac{dm\log\left(\frac{2f}{d}\right)+2\,de-2\,cf}{d}\right)}\Gamma\left(m\right.}{}$$

input
```
integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/24*((-I*a^3*d*m - I*a^3*d)*e^(-(d*m*log(3*f/d) + 3*d*e - 3*c*f)/d)*gamma
(m + 1, 3*(d*f*x + c*f)/d) + 9*(a^3*d*m + a^3*d)*e^(-(d*m*log(2*f/d) + 2*d
*e - 2*c*f)/d)*gamma(m + 1, 2*(d*f*x + c*f)/d) - 45*(-I*a^3*d*m - I*a^3*d)
*e^(-(d*m*log(f/d) + d*e - c*f)/d)*gamma(m + 1, (d*f*x + c*f)/d) - 45*(-I*
a^3*d*m - I*a^3*d)*e^(-(d*m*log(-f/d) - d*e + c*f)/d)*gamma(m + 1, -(d*f*x
 + c*f)/d) - 9*(a^3*d*m + a^3*d)*e^(-(d*m*log(-2*f/d) - 2*d*e + 2*c*f)/d)*
gamma(m + 1, -2*(d*f*x + c*f)/d) + (-I*a^3*d*m - I*a^3*d)*e^(-(d*m*log(-3*
f/d) - 3*d*e + 3*c*f)/d)*gamma(m + 1, -3*(d*f*x + c*f)/d) + 60*(a^3*d*f*x
+ a^3*c*f)*(d*x + c)^m)/(d*f*m + d*f)
```

Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m(a+ia\sinh(e+fx))^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)**m*(a+I*a*sinh(f*x+e))**3,x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.91

$$\int (c+dx)^m (a+ia\sinh(e+fx))^3\,dx =$$

$$-\frac{1}{8}i\left(\frac{(dx+c)^{m+1}e^{\left(-3e+\frac{3cf}{d}\right)}E_{-m}\left(\frac{3(dx+c)f}{d}\right)}{d}-\frac{3(dx+c)^{m+1}e^{\left(-e+\frac{cf}{d}\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d}+\frac{3(dx+c)^{m+1}e\cdots}{\cdots}\right.$$

$$+\frac{3}{4}\left(\frac{(dx+c)^{m+1}e^{\left(-2e+\frac{2cf}{d}\right)}E_{-m}\left(\frac{2(dx+c)f}{d}\right)}{d}+\frac{(dx+c)^{m+1}e^{\left(2e-\frac{2cf}{d}\right)}E_{-m}\left(-\frac{2(dx+c)f}{d}\right)}{d}+\frac{2(dx+c)^{m+\cdots}}{d(m+1)}\right.$$

$$+\frac{3}{2}i\left(\frac{(dx+c)^{m+1}e^{\left(-e+\frac{cf}{d}\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d}-\frac{(dx+c)^{m+1}e^{\left(e-\frac{cf}{d}\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)a^3$$

$$+\frac{(dx+c)^{m+1}a^3}{d(m+1)}$$

input
```
integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^3,x, algorithm="maxima")
```

output
```
-1/8*I*((d*x + c)^(m + 1)*e^(-3*e + 3*c*f/d)*exp_integral_e(-m, 3*(d*x + c
)*f/d)/d - 3*(d*x + c)^(m + 1)*e^(-e + c*f/d)*exp_integral_e(-m, (d*x + c)
*f/d)/d + 3*(d*x + c)^(m + 1)*e^(e - c*f/d)*exp_integral_e(-m, -(d*x + c)*
f/d)/d - (d*x + c)^(m + 1)*e^(3*e - 3*c*f/d)*exp_integral_e(-m, -3*(d*x +
c)*f/d)/d)*a^3 + 3/4*((d*x + c)^(m + 1)*e^(-2*e + 2*c*f/d)*exp_integral_e(
-m, 2*(d*x + c)*f/d)/d + (d*x + c)^(m + 1)*e^(2*e - 2*c*f/d)*exp_integral_
e(-m, -2*(d*x + c)*f/d)/d + 2*(d*x + c)^(m + 1)/(d*(m + 1)))*a^3 + 3/2*I*(
(d*x + c)^(m + 1)*e^(-e + c*f/d)*exp_integral_e(-m, (d*x + c)*f/d)/d - (d*
x + c)^(m + 1)*e^(e - c*f/d)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*a^3 + (
d*x + c)^(m + 1)*a^3/(d*(m + 1))
```

Giac [F]

$$\int (c+dx)^m(a+ia\sinh(e+fx))^3\,dx=\int (i\,a\sinh{(fx+e)}+a)^3(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((I*a*sinh(f*x + e) + a)^3*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m(a+ia\sinh(e+fx))^3\,dx=\int {\left(a+a\,\mathrm{sinh}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^3\,{\left(c+d\,x\right)}^m\,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)^3*(c + d*x)^m,x)
```

output
```
int((a + a*sinh(e + f*x)*1i)^3*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m(a+ia\sinh(e+fx))^3\,dx$$
$$=\frac{a^3\left(-e^{6fx+6e}(dx+c)^m\,dim+9e^{3fx+5e}\left(\int \frac{e^{2fx}(dx+c)^m}{dx+c}dx\right)d^2m^2+9e^{3fx+5e}\left(\int \frac{e^{2fx}(dx+c)^m}{dx+c}dx\right)d^2m-9e^{3fx}\cdots\right)}{}$$

input
```
int((d*x+c)^m*(a+I*a*sinh(f*x+e))^3,x)
```

output

```
(a**3*( - e**(6*e + 6*f*x)*(c + d*x)**m*d*i*m - e**(6*e + 6*f*x)*(c + d*x)
**m*d*i - 9*e**(5*e + 5*f*x)*(c + d*x)**m*d*m - 9*e**(5*e + 5*f*x)*(c + d*
x)**m*d + 45*e**(4*e + 4*f*x)*(c + d*x)**m*d*i*m + 45*e**(4*e + 4*f*x)*(c
+ d*x)**m*d*i + 60*e**(3*e + 3*f*x)*(c + d*x)**m*c*f + 60*e**(3*e + 3*f*x)
*(c + d*x)**m*d*f*x + 45*e**(2*e + 2*f*x)*(c + d*x)**m*d*i*m + 45*e**(2*e
+ 2*f*x)*(c + d*x)**m*d*i + 9*e**(e + f*x)*(c + d*x)**m*d*m + 9*e**(e + f*
x)*(c + d*x)**m*d - (c + d*x)**m*d*i*m - (c + d*x)**m*d*i + e**(6*e + 3*f*
x)*int((e**(3*f*x)*(c + d*x)**m)/(c + d*x),x)*d**2*i*m**2 + e**(6*e + 3*f*
x)*int((e**(3*f*x)*(c + d*x)**m)/(c + d*x),x)*d**2*i*m + 9*e**(5*e + 3*f*x
)*int((e**(2*f*x)*(c + d*x)**m)/(c + d*x),x)*d**2*m**2 + 9*e**(5*e + 3*f*x
)*int((e**(2*f*x)*(c + d*x)**m)/(c + d*x),x)*d**2*m - 45*e**(4*e + 3*f*x)*
int((e**(f*x)*(c + d*x)**m)/(c + d*x),x)*d**2*i*m**2 - 45*e**(4*e + 3*f*x)
*int((e**(f*x)*(c + d*x)**m)/(c + d*x),x)*d**2*i*m + e**(3*e + 3*f*x)*int(
(c + d*x)**m/(e**(3*e + 3*f*x)*c + e**(3*e + 3*f*x)*d*x),x)*d**2*i*m**2 +
e**(3*e + 3*f*x)*int((c + d*x)**m/(e**(3*e + 3*f*x)*c + e**(3*e + 3*f*x)*d
*x),x)*d**2*i*m - 45*e**(2*e + 3*f*x)*int((c + d*x)**m/(e**(f*x)*c + e**(f
*x)*d*x),x)*d**2*i*m**2 - 45*e**(2*e + 3*f*x)*int((c + d*x)**m/(e**(f*x)*c
 + e**(f*x)*d*x),x)*d**2*i*m - 9*e**(e + 3*f*x)*int((c + d*x)**m/(e**(2*f*
x)*c + e**(2*f*x)*d*x),x)*d**2*m**2 - 9*e**(e + 3*f*x)*int((c + d*x)**m/(e
**(2*f*x)*c + e**(2*f*x)*d*x),x)*d**2*m))/(24*e**(3*e + 3*f*x)*d*f*(m +...
```

3.153 $\int (c+dx)^m(a+ia\sinh(e+fx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 268

$$\begin{aligned}
\int (c+dx)^m(a+ia\sinh(e+fx))^2\,dx &= \frac{3a^2(c+dx)^{1+m}}{2d(1+m)} - \frac{2^{-3-m}a^2e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2f(c+dx)}{d}\right)}{f} \\
&+ \frac{ia^2e^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{f} \\
&+ \frac{ia^2e^{-e+\frac{cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{f(c+dx)}{d}\right)}{f} \\
&+ \frac{2^{-3-m}a^2e^{-2e+\frac{2cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)}{f}
\end{aligned}$$

output
```
3/2*a^2*(d*x+c)^(1+m)/d/(1+m)-2^(-3-m)*a^2*exp(2*e-2*c*f/d)*(d*x+c)^m*GAMM
A(1+m,-2*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+I*a^2*exp(e-c*f/d)*(d*x+c)^m*GA
MMA(1+m,-f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+I*a^2*exp(-e+c*f/d)*(d*x+c)^m*G
AMMA(1+m,f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)+2^(-3-m)*a^2*exp(-2*e+2*c*f/d)*(
d*x+c)^m*GAMMA(1+m,2*f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.85

$$\int (c+dx)^m(a+ia\sinh(e+fx))^2\,dx$$
$$=\frac{1}{8}a^2(c+dx)^m\left(\frac{12(c+dx)}{d(1+m)}-\frac{2^{-m}e^{2e-\frac{2cf}{d}}\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2f(c+dx)}{d}\right)}{f}+\frac{8ie^{e-\frac{cf}{d}}\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{f}+\frac{8ie^{-e+\frac{cf}{d}}\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{f(c+dx)}{d}\right)}{f}+\frac{2^{-m}e^{-2e+\frac{2cf}{d}}\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)}{f}\right)$$

input
```
Integrate[(c + d*x)^m*(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
(a^2*(c + d*x)^m*((12*(c + d*x))/(d*(1 + m)) - (E^(2*e - (2*c*f)/d)*Gamma[
1 + m, (-2*f*(c + d*x))/d])/(2^m*f*(-((f*(c + d*x))/d))^m) + ((8*I)*E^(e -
 (c*f)/d)*Gamma[1 + m, -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/d))^m) + (
(8*I)*E^(-e + (c*f)/d)*Gamma[1 + m, (f*(c + d*x))/d])/(f*((f*(c + d*x))/d)
^m) + (E^(-2*e + (2*c*f)/d)*Gamma[1 + m, (2*f*(c + d*x))/d])/(2^m*f*((f*(c
 + d*x))/d)^m)))/8
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 3799, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m (a+ia\sinh(e+fx))^2\,dx$$

$\Big\downarrow$ 3042

$$\int (c+dx)^m (a+a\sin(ie+ifx))^2dx$$

$\Big\downarrow$ 3799

$$4a^2\int (c+dx)^m\sinh^4\left(\frac{e}{2}+\frac{fx}{2}-\frac{i\pi}{4}\right)dx$$

$\Big\downarrow$ 3042

$$4a^2\int (c+dx)^m\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^4dx$$

$\Big\downarrow$ 3793

$$4a^2\int \left(-\frac{1}{8}\cosh(2e+2fx)(c+dx)^m+\frac{1}{2}i\sinh(e+fx)(c+dx)^m+\frac{3}{8}(c+dx)^m\right)dx$$

$\Big\downarrow$ 2009

$$4a^2\left(-\frac{2^{-m-5}e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2f(c+dx)}{d}\right)}{f}+\frac{ie^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+\right.}{4f}\right.$$

input
```
Int[(c + d*x)^m*(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
4*a^2*((3*(c + d*x)^(1 + m))/(8*d*(1 + m)) - (2^(-5 - m)*E^(2*e - (2*c*f)/
d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(c + d*x))/d])/(f*(-((f*(c + d*x))/d))^m
) + ((I/4)*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1 + m, -((f*(c + d*x))/d)])/(
f*(-((f*(c + d*x))/d))^m) + ((I/4)*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 +
m, (f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m) + (2^(-5 - m)*E^(-2*e + (2*c*
f)/d)*(c + d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m)
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int (dx+c)^m (a+ia\sinh(fx+e))^2\,dx$$

input
```
int((d*x+c)^m*(a+I*a*sinh(f*x+e))^2,x)
```

output
```
int((d*x+c)^m*(a+I*a*sinh(f*x+e))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.97

$$\int (c+dx)^m(a+ia\sinh(e+fx))^2\,dx = \frac{(a^2dm+a^2d)e^{\left(-\frac{dm\log\left(\frac{2f}{d}\right)+2de-2cf}{d}\right)}\Gamma\left(m+1,\frac{2(dfx+cf)}{d}\right)-8(-i\,a^2dm-i\,a^2d)e^{\left(-\frac{dm\log\left(\frac{f}{d}\right)+de-cf}{d}\right)}\Gamma\left(m+\right.}{}$$

input
```
integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/8*((a^2*d*m + a^2*d)*e^(-(d*m*log(2*f/d) + 2*d*e - 2*c*f)/d)*gamma(m + 1
, 2*(d*f*x + c*f)/d) - 8*(-I*a^2*d*m - I*a^2*d)*e^(-(d*m*log(f/d) + d*e -
c*f)/d)*gamma(m + 1, (d*f*x + c*f)/d) - 8*(-I*a^2*d*m - I*a^2*d)*e^(-(d*m*
log(-f/d) - d*e + c*f)/d)*gamma(m + 1, -(d*f*x + c*f)/d) - (a^2*d*m + a^2*
d)*e^(-(d*m*log(-2*f/d) - 2*d*e + 2*c*f)/d)*gamma(m + 1, -2*(d*f*x + c*f)/
d) + 12*(a^2*d*f*x + a^2*c*f)*(d*x + c)^m)/(d*f*m + d*f)
```

Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m(a+ia\sinh(e+fx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)**m*(a+I*a*sinh(f*x+e))**2,x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.78

$$\begin{aligned}
&\int (c+dx)^m(a+ia\sinh(e+fx))^2\,dx \\
&= \frac{1}{4}\left(\frac{(dx+c)^{m+1}e^{\left(-2\,e+\frac{2\,cf}{d}\right)}E_{-m}\left(\frac{2\,(dx+c)f}{d}\right)}{d}+\frac{(dx+c)^{m+1}e^{\left(2\,e-\frac{2\,cf}{d}\right)}E_{-m}\left(-\frac{2\,(dx+c)f}{d}\right)}{d}+\frac{2\,(dx+c)^{m+1}}{d(m+1)}\right) \\
&\quad+i\left(\frac{(dx+c)^{m+1}e^{\left(-e+\frac{cf}{d}\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d}-\frac{(dx+c)^{m+1}e^{\left(e-\frac{cf}{d}\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)a^2 \\
&\quad+\frac{(dx+c)^{m+1}a^2}{d(m+1)}
\end{aligned}$$

input
```
integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/4*((d*x + c)^(m + 1)*e^(-2*e + 2*c*f/d)*exp_integral_e(-m, 2*(d*x + c)*f
/d)/d + (d*x + c)^(m + 1)*e^(2*e - 2*c*f/d)*exp_integral_e(-m, -2*(d*x + c
)*f/d)/d + 2*(d*x + c)^(m + 1)/(d*(m + 1)))*a^2 + I*((d*x + c)^(m + 1)*e^(
-e + c*f/d)*exp_integral_e(-m, (d*x + c)*f/d)/d - (d*x + c)^(m + 1)*e^(e -
 c*f/d)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*a^2 + (d*x + c)^(m + 1)*a^2/
(d*(m + 1))
```

Giac [F]

$$\int (c+dx)^m(a+ia\sinh(e+fx))^2\,dx=\int (i\,a\sinh\left(fx+e\right)+a)^2(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((I*a*sinh(f*x + e) + a)^2*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m(a+ia\sinh(e+fx))^2\,dx=\int (a+a\sinh(e+f\,x)\,\,1\mathrm{i})^2\,(c+d\,x)^m\,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)^2*(c + d*x)^m,x)
```

output
```
int((a + a*sinh(e + f*x)*1i)^2*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m(a+ia\sinh(e+fx))^2\,dx$$
$$= \frac{a^2\left(-e^{4fx+4e}(dx+c)^m\,dm-e^{4fx+4e}(dx+c)^m\,d+8e^{3fx+3e}(dx+c)^m\,dim+8e^{3fx+3e}(dx+c)^m\,di+12e^{2}\right.}{}$$

input
```
int((d*x+c)^m*(a+I*a*sinh(f*x+e))^2,x)
```

output
```
(a**2*( - e**(4*e + 4*f*x)*(c + d*x)**m*d*m - e**(4*e + 4*f*x)*(c + d*x)**
m*d + 8*e**(3*e + 3*f*x)*(c + d*x)**m*d*i*m + 8*e**(3*e + 3*f*x)*(c + d*x)
**m*d*i + 12*e**(2*e + 2*f*x)*(c + d*x)**m*c*f + 12*e**(2*e + 2*f*x)*(c +
d*x)**m*d*f*x + 8*e**(e + f*x)*(c + d*x)**m*d*i*m + 8*e**(e + f*x)*(c + d*
x)**m*d*i + (c + d*x)**m*d*m + (c + d*x)**m*d + e**(4*e + 2*f*x)*int((e**(
2*f*x)*(c + d*x)**m)/(c + d*x),x)*d**2*m**2 + e**(4*e + 2*f*x)*int((e**(2*
f*x)*(c + d*x)**m)/(c + d*x),x)*d**2*m - 8*e**(3*e + 2*f*x)*int((e**(f*x)*
(c + d*x)**m)/(c + d*x),x)*d**2*i*m**2 - 8*e**(3*e + 2*f*x)*int((e**(f*x)*
(c + d*x)**m)/(c + d*x),x)*d**2*i*m - e**(2*e + 2*f*x)*int((c + d*x)**m/(e
**(2*e + 2*f*x)*c + e**(2*e + 2*f*x)*d*x),x)*d**2*m**2 - e**(2*e + 2*f*x)*
int((c + d*x)**m/(e**(2*e + 2*f*x)*c + e**(2*e + 2*f*x)*d*x),x)*d**2*m - 8
*e**(e + 2*f*x)*int((c + d*x)**m/(e**(f*x)*c + e**(f*x)*d*x),x)*d**2*i*m**
2 - 8*e**(e + 2*f*x)*int((c + d*x)**m/(e**(f*x)*c + e**(f*x)*d*x),x)*d**2*
i*m))/(8*e**(2*e + 2*f*x)*d*f*(m + 1))
```

3.154 $\int (c+dx)^m(a+ia\sinh(e+fx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 135

$$\int (c+dx)^m(a+ia\sinh(e+fx))\,dx = \frac{a(c+dx)^{1+m}}{d(1+m)} + \frac{iae^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{2f} + \frac{iae^{-e+\frac{cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{f(c+dx)}{d}\right)}{2f}$$

output
```
a*(d*x+c)^(1+m)/d/(1+m)+1/2*I*a*exp(e-c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c
)/d)/f/((-f*(d*x+c)/d)^m)+1/2*I*a*exp(-e+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*x
+c)/d)/f/((f*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.53

$$\int (c+dx)^m(a+ia\sinh(e+fx))\,dx =$$

$$-\frac{ae^{-e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m}\left(-2ie^{e+\frac{cf}{d}}f(c+dx)\left(-\frac{f^2(c+dx)^2}{d^2}\right)^m+de^{2e}(1+m)\left(f\left(\frac{c}{d}+x\right)\right)^m\Gamma(\right.}{2df(1+m)\left(\cosh\left(\frac{1}{2}(e+fx)\right)+\right.}$$

input
```
Integrate[(c + d*x)^m*(a + I*a*Sinh[e + f*x]),x]
```

output
```
-1/2*(a*E^(-e - (c*f)/d)*(c + d*x)^m*((-2*I)*E^(e + (c*f)/d)*f*(c + d*x)*(
-((f^2*(c + d*x)^2)/d^2))^m + d*E^(2*e)*(1 + m)*(f*(c/d + x))^m*Gamma[1 +
m, -((f*(c + d*x))/d)] + d*E^((2*c*f)/d)*(1 + m)*(-((f*(c + d*x))/d))^m*Ga
mma[1 + m, (f*(c + d*x))/d])*(-I + Sinh[e + f*x]))/(d*f*(1 + m)*(-((f^2*(c
 + d*x)^2)/d^2))^m*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])^2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(a+ia\sinh(e+fx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)^m(a+a\sin(ie+ifx))dx$$

$$\Big\downarrow \text{ 3798}$$

$$\int \left(a(c+dx)^m+ia(c+dx)^m\sinh(e+fx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{iae^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f}+\frac{iae^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{f(c+dx)}{d}\right)}{2f}+\frac{a(c+dx)^{m+1}}{d(m+1)}$$

input
```
Int[(c + d*x)^m*(a + I*a*Sinh[e + f*x]),x]
```

output
```
(a*(c + d*x)^(1 + m))/(d*(1 + m)) + ((I/2)*a*E^(e - (c*f)/d)*(c + d*x)^m*G
amma[1 + m, -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/d))^m) + ((I/2)*a*E^(
-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(f*((f*(c + d*x))
/d)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [F]

$$\int (dx+c)^m\,(a+ia\sinh(fx+e))\,dx$$

input
```
int((d*x+c)^m*(a+I*a*sinh(f*x+e)),x)
```

output
```
int((d*x+c)^m*(a+I*a*sinh(f*x+e)),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.99

$$\int (c+dx)^m(a+ia\sinh(e+fx))\,dx$$
$$=\frac{(i\,adm+i\,ad)e^{\left(-\frac{dm\log\left(\frac{f}{d}\right)+de-cf}{d}\right)}\Gamma\left(m+1,\frac{dfx+cf}{d}\right)+(i\,adm+i\,ad)e^{\left(-\frac{dm\log\left(-\frac{f}{d}\right)-de+cf}{d}\right)}\Gamma\left(m+1,-\frac{dfx+cf}{d}\right.}{2\,(dfm+df)}$$

input
```
integrate((d*x+c)^m*(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*((I*a*d*m + I*a*d)*e^(-(d*m*log(f/d) + d*e - c*f)/d)*gamma(m + 1, (d*f
*x + c*f)/d) + (I*a*d*m + I*a*d)*e^(-(d*m*log(-f/d) - d*e + c*f)/d)*gamma(
m + 1, -(d*f*x + c*f)/d) + 2*(a*d*f*x + a*c*f)*(d*x + c)^m)/(d*f*m + d*f)
```

Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m(a+ia\sinh(e+fx))\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)**m*(a+I*a*sinh(f*x+e)),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.75

$$\int (c+dx)^m(a+ia\sinh(e+fx))\,dx$$
$$= \frac{1}{2}i\left(\frac{(dx+c)^{m+1}e^{\left(-e+\frac{cf}{d}\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d} - \frac{(dx+c)^{m+1}e^{\left(e-\frac{cf}{d}\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)a$$
$$+ \frac{(dx+c)^{m+1}a}{d(m+1)}$$

input
```
integrate((d*x+c)^m*(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
1/2*I*((d*x + c)^(m + 1)*e^(-e + c*f/d)*exp_integral_e(-m, (d*x + c)*f/d)/
d - (d*x + c)^(m + 1)*e^(e - c*f/d)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*
a + (d*x + c)^(m + 1)*a/(d*(m + 1))
```

Giac [F]

$$\int (c+dx)^m(a+ia\sinh(e+fx))\,dx = \int (i\,a\sinh(fx+e)+a)(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((I*a*sinh(f*x + e) + a)*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m(a+ia\sinh(e+fx))\,dx = \int (a+a\sinh(e+f\,x)\;1\mathrm{i})\;(c+d\,x)^m\,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)*(c + d*x)^m,x)
```

output
```
int((a + a*sinh(e + f*x)*1i)*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m(a+ia\sinh(e+fx))\,dx$$
$$= \frac{a\Big(e^{2fx+2e}(dx+c)^m\,dim + e^{2fx+2e}(dx+c)^m\,di + 2e^{fx+e}(dx+c)^m\,cf + 2e^{fx+e}(dx+c)^m\,dfx + (dx+c)^{m}}{}$$

input
```
int((d*x+c)^m*(a+I*a*sinh(f*x+e)),x)
```

output
```
(a*(e**(2*e + 2*f*x)*(c + d*x)**m*d*i*m + e**(2*e + 2*f*x)*(c + d*x)**m*d*
i + 2*e**(e + f*x)*(c + d*x)**m*c*f + 2*e**(e + f*x)*(c + d*x)**m*d*f*x +
(c + d*x)**m*d*i*m + (c + d*x)**m*d*i - e**(2*e + f*x)*int((e**(f*x)*(c +
d*x)**m)/(c + d*x),x)*d**2*i*m**2 - e**(2*e + f*x)*int((e**(f*x)*(c + d*x)
**m)/(c + d*x),x)*d**2*i*m - e**(f*x)*int((c + d*x)**m/(e**(f*x)*c + e**(f
*x)*d*x),x)*d**2*i*m**2 - e**(f*x)*int((c + d*x)**m/(e**(f*x)*c + e**(f*x)
*d*x),x)*d**2*i*m))/(2*e**(e + f*x)*d*f*(m + 1))
```

3.155 $\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx=\text{Int}\left(\frac{(c+dx)^m}{a+ia\sinh(e+fx)},x\right)$$

output
```
Defer(Int)((d*x+c)^m/(a+I*a*sinh(f*x+e)),x)
```

Mathematica [N/A]

Not integrable

Time = 3.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx=\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx$$

input
```
Integrate[(c + d*x)^m/(a + I*a*Sinh[e + f*x]),x]
```

output
```
Integrate[(c + d*x)^m/(a + I*a*Sinh[e + f*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{(c+dx)^m}{a+a\sin(ie+ifx)}dx$$
$$\Big\downarrow \text{ 3807}$$
$$\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}dx$$

input
```
Int[(c + d*x)^m/(a + I*a*Sinh[e + f*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(dx+c)^m}{a+ia\sinh(fx+e)}dx$$

input
```
int((d*x+c)^m/(a+I*a*sinh(f*x+e)),x)
```

output
```
int((d*x+c)^m/(a+I*a*sinh(f*x+e)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.74

$$\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx = \int \frac{(dx+c)^m}{i\,a\sinh\left(fx+e\right)+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
((a*f*e^(f*x + e) - I*a*f)*integral(-2*I*(d*x + c)^m*d*m/(-I*a*d*f*x - I*a
*c*f + (a*d*f*x + a*c*f)*e^(f*x + e)), x) + 2*I*(d*x + c)^m)/(a*f*e^(f*x +
 e) - I*a*f)
```

Sympy [N/A]

Not integrable

Time = 12.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx = -\frac{i\int \frac{(c+dx)^m}{\sinh\left(e+fx\right)-i}\,dx}{a}$$

input
```
integrate((d*x+c)**m/(a+I*a*sinh(f*x+e)),x)
```

output
```
-I*Integral((c + d*x)**m/(sinh(e + f*x) - I), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx = \int \frac{(dx+c)^m}{i\,a\sinh\left(fx+e\right)+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(I*a*sinh(f*x + e) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx = \int \frac{(dx+c)^m}{i\,a\sinh\left(fx+e\right)+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/(I*a*sinh(f*x + e) + a), x)
```

Mupad [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx = \int \frac{(c+d\,x)^m}{a+a\sinh\left(e+f\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((c + d*x)^m/(a + a*sinh(e + f*x)*1i),x)
```

output
```
int((c + d*x)^m/(a + a*sinh(e + f*x)*1i), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx = \frac{\int \frac{(dx+c)^m}{\sinh(fx+e)i+1}dx}{a}$$

input
```
int((d*x+c)^m/(a+I*a*sinh(f*x+e)),x)
```

output
```
int((c + d*x)**m/(sinh(e + f*x)*i + 1),x)/a
```

3.156 $\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx = \mathrm{Int}\left(\frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2},x\right)$$

output
```
Defer(Int)((d*x+c)^m/(a+I*a*sinh(f*x+e))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 12.87 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx = \int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx$$

input
```
Integrate[(c + d*x)^m/(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
Integrate[(c + d*x)^m/(a + I*a*Sinh[e + f*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c+dx)^m}{(a+a\sin(ie+ifx))^2}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}dx$$

input
```
Int[(c + d*x)^m/(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(dx+c)^m}{(a+ia\sinh(fx+e))^2}dx$$

input
```
int((d*x+c)^m/(a+I*a*sinh(f*x+e))^2,x)
```

output
```
int((d*x+c)^m/(a+I*a*sinh(f*x+e))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 696, normalized size of antiderivative = 30.26

$$\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx = \int \frac{(dx+c)^m}{(i\,a\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
-(2*(I*d^2*f^2*x^2 + 2*I*c*d*f^2*x + I*c^2*f^2 - I*d^2*m^2 + I*d^2*m + (I*
d^2*f*m*x + I*d^2*m^2 + (I*c*d*f - I*d^2)*m)*e^(2*f*x + 2*e) - (3*d^2*f^2*
x^2 + 3*c^2*f^2 - 2*d^2*m^2 - (c*d*f - 2*d^2)*m + (6*c*d*f^2 - d^2*f*m)*x)
*e^(f*x + e))*(d*x + c)^m + 3*(-I*a^2*d^2*f^3*x^2 - 2*I*a^2*c*d*f^3*x - I*
a^2*c^2*f^3 - (a^2*d^2*f^3*x^2 + 2*a^2*c*d*f^3*x + a^2*c^2*f^3)*e^(3*f*x +
 3*e) + 3*(I*a^2*d^2*f^3*x^2 + 2*I*a^2*c*d*f^3*x + I*a^2*c^2*f^3)*e^(2*f*x
 + 2*e) + 3*(a^2*d^2*f^3*x^2 + 2*a^2*c*d*f^3*x + a^2*c^2*f^3)*e^(f*x + e))
*integral(-2*(I*d^3*f^2*m*x^2 + 2*I*c*d^2*f^2*m*x - I*d^3*m^3 + 3*I*d^3*m^
2 + (I*c^2*d*f^2 - 2*I*d^3)*m)*(d*x + c)^m/(-3*I*a^2*d^3*f^3*x^3 - 9*I*a^2
*c*d^2*f^3*x^2 - 9*I*a^2*c^2*d*f^3*x - 3*I*a^2*c^3*f^3 + 3*(a^2*d^3*f^3*x^
3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*e^(f*x + e)), x
))/(3*I*a^2*d^2*f^3*x^2 + 6*I*a^2*c*d*f^3*x + 3*I*a^2*c^2*f^3 + 3*(a^2*d^2
*f^3*x^2 + 2*a^2*c*d*f^3*x + a^2*c^2*f^3)*e^(3*f*x + 3*e) - 9*(I*a^2*d^2*f
^3*x^2 + 2*I*a^2*c*d*f^3*x + I*a^2*c^2*f^3)*e^(2*f*x + 2*e) - 9*(a^2*d^2*f
^3*x^2 + 2*a^2*c*d*f^3*x + a^2*c^2*f^3)*e^(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**m/(a+I*a*sinh(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx = \int \frac{(dx+c)^m}{(i\,a\sinh{(fx+e)}+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(I*a*sinh(f*x + e) + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx = \int \frac{(dx+c)^m}{(i\,a\sinh{(fx+e)}+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/(I*a*sinh(f*x + e) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx = \int \frac{(c+d\,x)^m}{(a+a\sinh{(e+f\,x)}\,1\mathrm{i})^2}\,dx$$

input
```
int((c + d*x)^m/(a + a*sinh(e + f*x)*1i)^2,x)
```

output
```
int((c + d*x)^m/(a + a*sinh(e + f*x)*1i)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx = -\frac{\int \frac{(dx+c)^m}{\sinh(fx+e)^2-2\sinh(fx+e)i-1}dx}{a^2}$$

input
```
int((d*x+c)^m/(a+I*a*sinh(f*x+e))^2,x)
```

output
```
( - int((c + d*x)**m/(sinh(e + f*x)**2 - 2*sinh(e + f*x)*i - 1),x))/a**2
```

3.157 $\int (c+dx)^3(a+b\sinh(e+fx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 89

$$\int (c+dx)^3(a+b\sinh(e+fx))\,dx = \frac{a(c+dx)^4}{4d}+\frac{6bd^2(c+dx)\cosh(e+fx)}{f^3}+\frac{b(c+dx)^3\cosh(e+fx)}{f}-\frac{6bd^3\sinh(e+fx)}{f^4}-\frac{3bd(c+dx)^2\sinh(e+fx)}{f^2}$$

output
```
1/4*a*(d*x+c)^4/d+6*b*d^2*(d*x+c)*cosh(f*x+e)/f^3+b*(d*x+c)^3*cosh(f*x+e)/
f-6*b*d^3*sinh(f*x+e)/f^4-3*b*d*(d*x+c)^2*sinh(f*x+e)/f^2
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.38

$$\int (c+dx)^3(a+b\sinh(e+fx))\,dx = \frac{1}{4}ax\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)+\frac{b(c+dx)\left(c^2f^2+2cdf^2x+d^2(6+f^2x^2)\right)\cosh(e+fx)}{f^3}-\frac{3bd(c^2f^2+2cdf^2x+d^2(2+f^2x^2))\sinh(e+fx)}{f^4}$$

input
```
Integrate[(c + d*x)^3*(a + b*Sinh[e + f*x]),x]
```

output
```
(a*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))/4 + (b*(c + d*x)*(c^2*f^
2 + 2*c*d*f^2*x + d^2*(6 + f^2*x^2))*Cosh[e + f*x])/f^3 - (3*b*d*(c^2*f^2
+ 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Sinh[e + f*x])/f^4
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3(a+b\sinh(e+fx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)^3(a-ib\sin(ie+ifx))dx$$

$$\Big\downarrow \text{ 3798}$$

$$\int \left(a(c+dx)^3+b(c+dx)^3\sinh(e+fx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a(c+dx)^4}{4d}+\frac{6bd^2(c+dx)\cosh(e+fx)}{f^3}-\frac{3bd(c+dx)^2\sinh(e+fx)}{f^2}+\frac{b(c+dx)^3\cosh(e+fx)}{f}-\frac{6bd^3\sinh(e+fx)}{f^4}$$

input
```
Int[(c + d*x)^3*(a + b*Sinh[e + f*x]),x]
```

output
```
(a*(c + d*x)^4)/(4*d) + (6*b*d^2*(c + d*x)*Cosh[e + f*x])/f^3 + (b*(c + d*
x)^3*Cosh[e + f*x])/f - (6*b*d^3*Sinh[e + f*x])/f^4 - (3*b*d*(c + d*x)^2*S
inh[e + f*x])/f^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.24

method	result
parallelrisch	$\frac{b(dx+c)f\left((dx+c)^2f^2+6d^2\right)\cosh(fx+e)-3bd\left((dx+c)^2f^2+2d^2\right)\sinh(fx+e)+\left(a\left(\frac{1}{2}x^2d^2+cdx+c^2\right)\left(\frac{dx}{2}+c\right)x f^3+b c \cdots\right)}{f^4}$ …
risch	$\frac{a d^3x^4}{4}+a d^2c x^3+\frac{3ad c^2x^2}{2}+a c^3x+\frac{a c^4}{4d}+\frac{b(d^3x^3f^3+3c d^2f^3x^2+3c^2d f^3x-3d^3f^2x^2+c^3f^3-6c d^2f^2 \cdots)}{2f^4}$ …
parts	$\frac{a(dx+c)^4}{4d}+\frac{b\left(\frac{d^3\left((fx+e)^3\cosh(fx+e)-3(fx+e)^2\sinh(fx+e)+6(fx+e)\cosh(fx+e)-6\sinh(fx+e)\right)}{f^3}-\frac{3d^3e\left((fx+e)^2\cosh(fx \cdots\right)}{\cdots}\right)}{\cdots}$ …
orering	$\frac{(d^5f^4x^6+6c d^4f^4x^5+15c^2d^3f^4x^4+20c^3d^2f^4x^3+14c^4d f^4x^2-24d^5f^2x^4+4c^5f^4x-96c d^4f^2x^3-156c^2d^3f^2x^2-120c^3 \cdots}{4f^4(dx+c)^2}$ …
derivativedivides	$\frac{\frac{d^3a(fx+e)^4}{4f^3}+\frac{d^3b\left((fx+e)^3\cosh(fx+e)-3(fx+e)^2\sinh(fx+e)+6(fx+e)\cosh(fx+e)-6\sinh(fx+e)\right)}{f^3}-\frac{d^3ea(fx+e)^3}{f^3}-\frac{3d^3eb\left((f \cdots\right)}{\cdots}}{\cdots}$ …
default	$\frac{\frac{d^3a(fx+e)^4}{4f^3}+\frac{d^3b\left((fx+e)^3\cosh(fx+e)-3(fx+e)^2\sinh(fx+e)+6(fx+e)\cosh(fx+e)-6\sinh(fx+e)\right)}{f^3}-\frac{d^3ea(fx+e)^3}{f^3}-\frac{3d^3eb\left((f \cdots\right)}{\cdots}}{\cdots}$ …

input
```
int((d*x+c)^3*(a+b*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
(b*(d*x+c)*f*((d*x+c)^2*f^2+6*d^2)*cosh(f*x+e)-3*b*d*((d*x+c)^2*f^2+2*d^2)
*sinh(f*x+e)+(a*(1/2*x^2*d^2+c*d*x+c^2)*(1/2*d*x+c)*x*f^3+b*c^3*f^2+6*b*c*
d^2)*f)/f^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.89

$$\int (c+dx)^3(a+b\sinh(e+fx))\,dx = \frac{ad^3f^4x^4 + 4\,acd^2f^4x^3 + 6\,ac^2df^4x^2 + 4\,ac^3f^4x + 4\,(bd^3f^3x^3 + 3\,bcd^2f^3x^2 + bc^3f^3 + 6\,bcd^2f + 3\,(bc^2df^3}{4\,f^4}$$

input
```
integrate((d*x+c)^3*(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
1/4*(a*d^3*f^4*x^4 + 4*a*c*d^2*f^4*x^3 + 6*a*c^2*d*f^4*x^2 + 4*a*c^3*f^4*x
 + 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + b*c^3*f^3 + 6*b*c*d^2*f + 3*(b*c
^2*d*f^3 + 2*b*d^3*f)*x)*cosh(f*x + e) - 12*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2
*x + b*c^2*d*f^2 + 2*b*d^3)*sinh(f*x + e))/f^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(88) = 176.

Time = 0.28 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.97

$$\int (c+dx)^3(a+b\sinh(e+fx))\,dx = \begin{cases} ac^3x + \frac{3ac^2dx^2}{2} + acd^2x^3 + \frac{ad^3x^4}{4} + \frac{bc^3\cosh(e+fx)}{f} + \frac{3bc^2dx\cosh(e+fx)}{f} - \frac{3bc^2d\sinh(e+fx)}{f^2} + \frac{3bcd^2x^2\cosh(e+fx)}{f} - \\ (a+b\sinh(e))\left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right) \end{cases}$$

input
```
integrate((d*x+c)**3*(a+b*sinh(f*x+e)),x)
```

output
```
Piecewise((a*c**3*x + 3*a*c**2*d*x**2/2 + a*c*d**2*x**3 + a*d**3*x**4/4 +
b*c**3*cosh(e + f*x)/f + 3*b*c**2*d*x*cosh(e + f*x)/f - 3*b*c**2*d*sinh(e
+ f*x)/f**2 + 3*b*c*d**2*x**2*cosh(e + f*x)/f - 6*b*c*d**2*x*sinh(e + f*x)
/f**2 + 6*b*c*d**2*cosh(e + f*x)/f**3 + b*d**3*x**3*cosh(e + f*x)/f - 3*b*
d**3*x**2*sinh(e + f*x)/f**2 + 6*b*d**3*x*cosh(e + f*x)/f**3 - 6*b*d**3*si
nh(e + f*x)/f**4, Ne(f, 0)), ((a + b*sinh(e))*(c**3*x + 3*c**2*d*x**2/2 +
c*d**2*x**3 + d**3*x**4/4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(87) = 174$.

Time = 0.05 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.63

$$\int (c+dx)^3(a+b\sinh(e+fx))\,dx = \frac{1}{4}\,ad^3x^4 + acd^2x^3 + \frac{3}{2}\,ac^2dx^2 + ac^3x + \frac{3}{2}\,bc^2d\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + \frac{3}{2}\,bcd^2\left(\frac{(f^2x^2e^e - 2\,fxe^e + 2\,e^e)e^{(fx)}}{f^3} + \frac{(f^2x^2 + 2\,fx + 2)e^{(-fx-e)}}{f^3}\right) + \frac{1}{2}\,bd^3\left(\frac{(f^3x^3e^e - 3\,f^2x^2e^e + 6\,fxe^e - 6\,e^e)e^{(fx)}}{f^4} + \frac{(f^3x^3 + 3\,f^2x^2 + 6\,fx + 6)e^{(-fx-e)}}{f^4}\right) + \frac{bc^3\cosh(fx+e)}{f}$$

input
```
integrate((d*x+c)^3*(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + 3/2*b*c^2*d*((f*
x*e^e - e^e)*e^(f*x)/f^2 + (f*x + 1)*e^(-f*x - e)/f^2) + 3/2*b*c*d^2*((f^2
*x^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f^3 + (f^2*x^2 + 2*f*x + 2)*e^(-f*x
- e)/f^3) + 1/2*b*d^3*((f^3*x^3*e^e - 3*f^2*x^2*e^e + 6*f*x*e^e - 6*e^e)*e
^(f*x)/f^4 + (f^3*x^3 + 3*f^2*x^2 + 6*f*x + 6)*e^(-f*x - e)/f^4) + b*c^3*c
osh(f*x + e)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(87) = 174$.

Time = 0.12 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.90

$$\int (c+dx)^3(a+b\sinh(e+fx))\,dx = \frac{1}{4}\,ad^3x^4 + acd^2x^3 + \frac{3}{2}\,ac^2dx^2 + ac^3x + \frac{(bd^3f^3x^3 + 3\,bcd^2f^3x^2 + 3\,bc^2df^3x - 3\,bd^3f^2x^2 + bc^3f^3 - 6\,bcd^2f^2x - 3\,bc^2df^2 + 6\,bd^3fx + 6\,bcd^2f -}{2\,f^4} + \frac{(bd^3f^3x^3 + 3\,bcd^2f^3x^2 + 3\,bc^2df^3x + 3\,bd^3f^2x^2 + bc^3f^3 + 6\,bcd^2f^2x + 3\,bc^2df^2 + 6\,bd^3fx + 6\,bcd^2f +}{2\,f^4}$$

input
```
integrate((d*x+c)^3*(a+b*sinh(f*x+e)),x, algorithm="giac")
```

output
```
1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + 1/2*(b*d^3*f^3*x
^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x - 3*b*d^3*f^2*x^2 + b*c^3*f^3 - 6
*b*c*d^2*f^2*x - 3*b*c^2*d*f^2 + 6*b*d^3*f*x + 6*b*c*d^2*f - 6*b*d^3)*e^(f
*x + e)/f^4 + 1/2*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + 3
*b*d^3*f^2*x^2 + b*c^3*f^3 + 6*b*c*d^2*f^2*x + 3*b*c^2*d*f^2 + 6*b*d^3*f*x
 + 6*b*c*d^2*f + 6*b*d^3)*e^(-f*x - e)/f^4
```

Mupad [B] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.10

$$\int (c+dx)^3(a+b\sinh(e+fx))\,dx = \frac{\cosh(e+f\,x)\,\left(b\,c^3\,f^2+6\,b\,c\,d^2\right)}{f^3} - \frac{3\sinh(e+f\,x)\,\left(b\,c^2\,d\,f^2+2\,b\,d^3\right)}{f^4} + \frac{a\,d^3\,x^4}{4} + a\,c^3\,x + \frac{3\,x\cosh(e+f\,x)\,\left(b\,c^2\,d\,f^2+2\,b\,d^3\right)}{f^3} + \frac{3\,a\,c^2\,d\,x^2}{2} + a\,c\,d^2\,x^3 + \frac{b\,d^3\,x^3\cosh(e+f\,x)}{f} - \frac{3\,b\,d^3\,x^2\sinh(e+f\,x)}{f^2} - \frac{6\,b\,c\,d^2\,x\sinh(e+f\,x)}{f^2} + \frac{3\,b\,c\,d^2\,x^2\cosh(e+f\,x)}{f}$$

input
```
int((a + b*sinh(e + f*x))*(c + d*x)^3,x)
```

output
```
(cosh(e + f*x)*(b*c^3*f^2 + 6*b*c*d^2))/f^3 - (3*sinh(e + f*x)*(2*b*d^3 +
b*c^2*d*f^2))/f^4 + (a*d^3*x^4)/4 + a*c^3*x + (3*x*cosh(e + f*x)*(2*b*d^3
+ b*c^2*d*f^2))/f^3 + (3*a*c^2*d*x^2)/2 + a*c*d^2*x^3 + (b*d^3*x^3*cosh(e
+ f*x))/f - (3*b*d^3*x^2*sinh(e + f*x))/f^2 - (6*b*c*d^2*x*sinh(e + f*x))/
f^2 + (3*b*c*d^2*x^2*cosh(e + f*x))/f
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.39

$$\int (c+dx)^3(a+b\sinh(e+fx))\,dx$$
$$= \frac{4\cosh(fx+e)\,b\,c^3f^3+12\cosh(fx+e)\,b\,c^2d\,f^3x+12\cosh(fx+e)\,bc\,d^2f^3x^2+24\cosh(fx+e)\,bc\,d^2f}{}$$

input
```
int((d*x+c)^3*(a+b*sinh(f*x+e)),x)
```

output
```
(4*cosh(e + f*x)*b*c**3*f**3 + 12*cosh(e + f*x)*b*c**2*d*f**3*x + 12*cosh(
e + f*x)*b*c*d**2*f**3*x**2 + 24*cosh(e + f*x)*b*c*d**2*f + 4*cosh(e + f*x
)*b*d**3*f**3*x**3 + 24*cosh(e + f*x)*b*d**3*f*x - 12*sinh(e + f*x)*b*c**2
*d*f**2 - 24*sinh(e + f*x)*b*c*d**2*f**2*x - 12*sinh(e + f*x)*b*d**3*f**2*
x**2 - 24*sinh(e + f*x)*b*d**3 + 4*a*c**3*f**4*x + 6*a*c**2*d*f**4*x**2 +
4*a*c*d**2*f**4*x**3 + a*d**3*f**4*x**4)/(4*f**4)
```

3.158 $\int (c+dx)^2(a+b\sinh(e+fx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 67

$$\int (c+dx)^2(a+b\sinh(e+fx))\,dx = \frac{a(c+dx)^3}{3d}+\frac{2bd^2\cosh(e+fx)}{f^3}+\frac{b(c+dx)^2\cosh(e+fx)}{f}-\frac{2bd(c+dx)\sinh(e+fx)}{f^2}$$

output
```
1/3*a*(d*x+c)^3/d+2*b*d^2*cosh(f*x+e)/f^3+b*(d*x+c)^2*cosh(f*x+e)/f-2*b*d*
(d*x+c)*sinh(f*x+e)/f^2
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.24

$$\int (c+dx)^2(a+b\sinh(e+fx))\,dx = \frac{1}{3}ax\left(3c^2+3cdx+d^2x^2\right)+\frac{b(c^2f^2+2cdf^2x+d^2(2+f^2x^2))\cosh(e+fx)}{f^3}-\frac{2bd(c+dx)\sinh(e+fx)}{f^2}$$

input
```
Integrate[(c + d*x)^2*(a + b*Sinh[e + f*x]),x]
```

output
```
(a*x*(3*c^2 + 3*c*d*x + d^2*x^2))/3 + (b*(c^2*f^2 + 2*c*d*f^2*x + d^2*(2 +
 f^2*x^2))*Cosh[e + f*x])/f^3 - (2*b*d*(c + d*x)*Sinh[e + f*x])/f^2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2(a+b\sinh(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2(a-ib\sin(ie+ifx))dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(a(c+dx)^2+b(c+dx)^2\sinh(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(c+dx)^3}{3d}-\frac{2bd(c+dx)\sinh(e+fx)}{f^2}+\frac{b(c+dx)^2\cosh(e+fx)}{f}+\frac{2bd^2\cosh(e+fx)}{f^3}$$

input
```
Int[(c + d*x)^2*(a + b*Sinh[e + f*x]),x]
```

output
```
(a*(c + d*x)^3)/(3*d) + (2*b*d^2*Cosh[e + f*x])/f^3 + (b*(c + d*x)^2*Cosh[
e + f*x])/f - (2*b*d*(c + d*x)*Sinh[e + f*x])/f^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.24

method	result
parallelrisch	$\frac{b\left(\left(dx+c\right)^{2} f^{2}+2d^{2}\right) \cosh \left(fx+e\right)-2bdf\left(dx+c\right) \sinh \left(fx+e\right)+a\left(\frac{1}{3}x^{2} d^{2}+cdx+c^{2}\right) x\,f^{3}+b\,c^{2} f^{2}+2b\,d^{2}}{f^{3}}$
risch	$\frac{a\,d^{2} x^{3}}{3}+adc\,x^{2}+a\,c^{2} x+\frac{a\,c^{3}}{3d}+\frac{b\left(d^{2} x^{2} f^{2}+2cd\,f^{2} x+c^{2} f^{2}-2d^{2} fx-2cdf+2d^{2}\right) {\mathrm e}^{fx+e}}{2f^{3}}+\frac{b\left(d^{2} x^{2} f^{2}+2cd\,f^{2}\right.}{}$
parts	$\frac{a\left(dx+c\right)^{3}}{3d}+\frac{b\left(\frac{d^{2}\left(\left(fx+e\right)^{2} \cosh \left(fx+e\right)-2\left(fx+e\right) \sinh \left(fx+e\right)+2\cosh \left(fx+e\right)\right)}{f^{2}}-\frac{2d^{2} e\left(\left(fx+e\right) \cosh \left(fx+e\right)-\sinh \left(fx+e\right)\right)}{f^{2}}+\frac{2dc(}{}\right.}{f}$
derivativedivides	$\frac{\frac{d^{2} a\left(fx+e\right)^{3}}{3f^{2}}+\frac{d^{2} b\left(\left(fx+e\right)^{2} \cosh \left(fx+e\right)-2\left(fx+e\right) \sinh \left(fx+e\right)+2\cosh \left(fx+e\right)\right)}{f^{2}}-\frac{d^{2} ea\left(fx+e\right)^{2}}{f^{2}}-\frac{2d^{2} eb\left(\left(fx+e\right) \cosh \left(fx+e\right)-\sinh \left(\right.\right.}{f^{2}}}{}$
default	$\frac{\frac{d^{2} a\left(fx+e\right)^{3}}{3f^{2}}+\frac{d^{2} b\left(\left(fx+e\right)^{2} \cosh \left(fx+e\right)-2\left(fx+e\right) \sinh \left(fx+e\right)+2\cosh \left(fx+e\right)\right)}{f^{2}}-\frac{d^{2} ea\left(fx+e\right)^{2}}{f^{2}}-\frac{2d^{2} eb\left(\left(fx+e\right) \cosh \left(fx+e\right)-\sinh \left(\right.\right.}{f^{2}}}{}$
orering	$\frac{\left(d^{4} f^{4} x^{5}+5c\,d^{3} f^{4} x^{4}+10c^{2} d^{2} f^{4} x^{3}+9c^{3} d\,f^{4} x^{2}+3c^{4} f^{4} x-12d^{4} f^{2} x^{3}-42c\,d^{3} f^{2} x^{2}-48c^{2} d^{2} f^{2} x-12c^{3} d\,f^{2}-48d^{4} x-12d^{3}\right.}{3f^{4} \left(dx+c\right)^{2}}$

input
```
int((d*x+c)^2*(a+b*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
(b*((d*x+c)^2*f^2+2*d^2)*cosh(f*x+e)-2*b*d*f*(d*x+c)*sinh(f*x+e)+a*(1/3*x^
2*d^2+c*d*x+c^2)*x*f^3+b*c^2*f^2+2*b*d^2)/f^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.52

$$\int (c+dx)^2(a+b\sinh(e+fx))\,dx$$
$$= \frac{ad^2f^3x^3 + 3\,acdf^3x^2 + 3\,ac^2f^3x + 3\,(bd^2f^2x^2 + 2\,bcdf^2x + bc^2f^2 + 2\,bd^2)\cosh(fx+e) - 6\,(bd^2fx + bc}{3\,f^3}$$

input
```
integrate((d*x+c)^2*(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
1/3*(a*d^2*f^3*x^3 + 3*a*c*d*f^3*x^2 + 3*a*c^2*f^3*x + 3*(b*d^2*f^2*x^2 +
2*b*c*d*f^2*x + b*c^2*f^2 + 2*b*d^2)*cosh(f*x + e) - 6*(b*d^2*f*x + b*c*d*
f)*sinh(f*x + e))/f^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(65) = 130.

Time = 0.21 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.25

$$\int (c+dx)^2(a+b\sinh(e+fx))\,dx$$
$$= \begin{cases} ac^2x + acdx^2 + \frac{ad^2x^3}{3} + \frac{bc^2\cosh(e+fx)}{f} + \frac{2bcdx\cosh(e+fx)}{f} - \frac{2bcd\sinh(e+fx)}{f^2} + \frac{bd^2x^2\cosh(e+fx)}{f} - \frac{2bd^2x\sinh(e+fx)}{f^2} \\ (a+b\sinh(e))\left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) \end{cases}$$

input
```
integrate((d*x+c)**2*(a+b*sinh(f*x+e)),x)
```

output
```
Piecewise((a*c**2*x + a*c*d*x**2 + a*d**2*x**3/3 + b*c**2*cosh(e + f*x)/f
+ 2*b*c*d*x*cosh(e + f*x)/f - 2*b*c*d*sinh(e + f*x)/f**2 + b*d**2*x**2*cos
h(e + f*x)/f - 2*b*d**2*x*sinh(e + f*x)/f**2 + 2*b*d**2*cosh(e + f*x)/f**3
, Ne(f, 0)), ((a + b*sinh(e))*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(65) = 130$.

Time = 0.05 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.07

$$\int (c+dx)^2(a+b\sinh(e+fx))\,dx = \frac{1}{3}\,ad^2x^3 + acdx^2 + ac^2x + bcd\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + \frac{1}{2}\,bd^2\left(\frac{(f^2x^2e^e - 2\,fxe^e + 2\,e^e)e^{(fx)}}{f^3} + \frac{(f^2x^2 + 2\,fx + 2)e^{(-fx-e)}}{f^3}\right) + \frac{bc^2\cosh(fx+e)}{f}$$

input
```
integrate((d*x+c)^2*(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + b*c*d*((f*x*e^e - e^e)*e^(f*x)/f^2 +
 (f*x + 1)*e^(-f*x - e)/f^2) + 1/2*b*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e^e
)*e^(f*x)/f^3 + (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + b*c^2*cosh(f*x +
 e)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(65) = 130$.

Time = 0.12 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.18

$$\int (c+dx)^2(a+b\sinh(e+fx))\,dx = \frac{1}{3}\,ad^2x^3 + acdx^2 + ac^2x + \frac{(bd^2f^2x^2 + 2\,bcdf^2x + bc^2f^2 - 2\,bd^2fx - 2\,bcdf + 2\,bd^2)e^{(fx+e)}}{2\,f^3} + \frac{(bd^2f^2x^2 + 2\,bcdf^2x + bc^2f^2 + 2\,bd^2fx + 2\,bcdf + 2\,bd^2)e^{(-fx-e)}}{2\,f^3}$$

input
```
integrate((d*x+c)^2*(a+b*sinh(f*x+e)),x, algorithm="giac")
```

output
```
1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + 1/2*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x +
 b*c^2*f^2 - 2*b*d^2*f*x - 2*b*c*d*f + 2*b*d^2)*e^(f*x + e)/f^3 + 1/2*(b*d
^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2 + 2*b*d^2*f*x + 2*b*c*d*f + 2*b*d^2
)*e^(-f*x - e)/f^3
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.64

$$\int (c+dx)^2(a+b\sinh(e+fx))\,dx = \frac{a\,d^2\,x^3}{3} + \frac{\cosh(e+f\,x)\,\left(b\,c^2\,f^2+2\,b\,d^2\right)}{f^3} + a\,c^2\,x + a\,c\,d\,x^2 - \frac{2\,b\,d^2\,x\sinh(e+f\,x)}{f^2} + \frac{b\,d^2\,x^2\cosh(e+f\,x)}{f} - \frac{2\,b\,c\,d\sinh(e+f\,x)}{f^2} + \frac{2\,b\,c\,d\,x\cosh(e+f\,x)}{f}$$

input
```
int((a + b*sinh(e + f*x))*(c + d*x)^2,x)
```

output
```
(a*d^2*x^3)/3 + (cosh(e + f*x)*(2*b*d^2 + b*c^2*f^2))/f^3 + a*c^2*x + a*c*
d*x^2 - (2*b*d^2*x*sinh(e + f*x))/f^2 + (b*d^2*x^2*cosh(e + f*x))/f - (2*b
*c*d*sinh(e + f*x))/f^2 + (2*b*c*d*x*cosh(e + f*x))/f
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.85

$$\int (c+dx)^2(a+b\sinh(e+fx))\,dx = \frac{3\cosh(fx+e)\,b\,c^2f^2 + 6\cosh(fx+e)\,bcd\,f^2x + 3\cosh(fx+e)\,b\,d^2f^2x^2 + 6\cosh(fx+e)\,b\,d^2 - 6\sinh}{3f^3}$$

input
```
int((d*x+c)^2*(a+b*sinh(f*x+e)),x)
```

output
```
(3*cosh(e + f*x)*b*c**2*f**2 + 6*cosh(e + f*x)*b*c*d*f**2*x + 3*cosh(e + f
*x)*b*d**2*f**2*x**2 + 6*cosh(e + f*x)*b*d**2 - 6*sinh(e + f*x)*b*c*d*f -
6*sinh(e + f*x)*b*d**2*f*x + 3*a*c**2*f**3*x + 3*a*c*d*f**3*x**2 + a*d**2*
f**3*x**3)/(3*f**3)
```

3.159 $\int (c+dx)(a+b\sinh(e+fx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int (c+dx)(a+b\sinh(e+fx))\,dx=\frac{a(c+dx)^2}{2d}+\frac{b(c+dx)\cosh(e+fx)}{f}-\frac{bd\sinh(e+fx)}{f^2}$$

output
```
1/2*a*(d*x+c)^2/d+b*(d*x+c)*cosh(f*x+e)/f-b*d*sinh(f*x+e)/f^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int (c+dx)(a+b\sinh(e+fx))\,dx=\frac{1}{2}ax(2c+dx)+\frac{b(c+dx)\cosh(e+fx)}{f}-\frac{bd\sinh(e+fx)}{f^2}$$

input
```
Integrate[(c + d*x)*(a + b*Sinh[e + f*x]),x]
```

output
```
(a*x*(2*c + d*x))/2 + (b*(c + d*x)*Cosh[e + f*x])/f - (b*d*Sinh[e + f*x])/
f^2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(a+b\sinh(e+fx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)(a-ib\sin(ie+ifx))dx$$

$$\Big\downarrow \text{ 3798}$$

$$\int (a(c+dx)+b(c+dx)\sinh(e+fx))dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a(c+dx)^2}{2d}+\frac{b(c+dx)\cosh(e+fx)}{f}-\frac{bd\sinh(e+fx)}{f^2}$$

input
```
Int[(c + d*x)*(a + b*Sinh[e + f*x]),x]
```

output
```
(a*(c + d*x)^2)/(2*d) + (b*(c + d*x)*Cosh[e + f*x])/f - (b*d*Sinh[e + f*x]
)/f^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

method	result
parallelrisch	$\frac{(dx+c)bf\cosh(fx+e)-\sinh(fx+e)bd+f\left(ax\left(\frac{dx}{2}+c\right)f+bc\right)}{f^2}$
risch	$\frac{ad\,x^2}{2}+acx+\frac{b(dxf+cf-d)\mathrm{e}^{fx+e}}{2f^2}+\frac{b(dxf+cf+d)\mathrm{e}^{-fx-e}}{2f^2}$
parts	$a\left(\frac{1}{2}d\,x^2+cx\right)+\frac{b\left(\frac{d((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}-\frac{de\cosh(fx+e)}{f}+c\cosh(fx+e)\right)}{f}$
derivativedivides	$\frac{\frac{da(fx+e)^2}{2f}+\frac{db((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}-\frac{dea(fx+e)}{f}-\frac{deb\cosh(fx+e)}{f}+ca(fx+e)+bc\cosh(fx+e)}{f}$
default	$\frac{\frac{da(fx+e)^2}{2f}+\frac{db((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}-\frac{dea(fx+e)}{f}-\frac{deb\cosh(fx+e)}{f}+ca(fx+e)+bc\cosh(fx+e)}{f}$
orering	$\frac{(d^3f^2x^4+4c\,d^2f^2x^3+5c^2d\,f^2x^2+2c^3f^2x-6d^3x^2-12c\,d^2x-4c^2d)(a+b\sinh(fx+e))}{2f^2(dx+c)^2}+\frac{(2x^2d^2+4cdx+c^2)(d(a+b\sin\ldots}{(d\ldots}$

input
```
int((d*x+c)*(a+b*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*b*f*cosh(f*x+e)-sinh(f*x+e)*b*d+f*(a*x*(1/2*d*x+c)*f+b*c))/f^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int (c+dx)(a+b\sinh(e+fx))\,dx = \frac{adf^2x^2+2\,acf^2x-2\,bd\sinh\left(fx+e\right)+2\left(bdfx+bcf\right)\cosh\left(fx+e\right)}{2\,f^2}$$

input
```
integrate((d*x+c)*(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*(a*d*f^2*x^2 + 2*a*c*f^2*x - 2*b*d*sinh(f*x + e) + 2*(b*d*f*x + b*c*f)
*cosh(f*x + e))/f^2
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.51

$$\int (c+dx)(a+b\sinh(e+fx))\,dx = \begin{cases} acx+\frac{adx^2}{2}+\frac{bc\cosh\left(e+fx\right)}{f}+\frac{bdx\cosh\left(e+fx\right)}{f}-\frac{bd\sinh\left(e+fx\right)}{f^2} & \text{for } f\neq 0 \\ \left(a+b\sinh\left(e\right)\right)\left(cx+\frac{dx^2}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*(a+b*sinh(f*x+e)),x)
```

output
```
Piecewise((a*c*x + a*d*x**2/2 + b*c*cosh(e + f*x)/f + b*d*x*cosh(e + f*x)/
f - b*d*sinh(e + f*x)/f**2, Ne(f, 0)), ((a + b*sinh(e))*(c*x + d*x**2/2),
True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int (c+dx)(a+b\sinh(e+fx))\,dx = \frac{1}{2}\,adx^2 + acx + \frac{1}{2}\,bd\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + \frac{bc\cosh\left(fx+e\right)}{f}$$

input
```
integrate((d*x+c)*(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
1/2*a*d*x^2 + a*c*x + 1/2*b*d*((f*x*e^e - e^e)*e^(f*x)/f^2 + (f*x + 1)*e^(
-f*x - e)/f^2) + b*c*cosh(f*x + e)/f
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.42

$$\int (c+dx)(a+b\sinh(e+fx))\,dx = \frac{1}{2}\,adx^2 + acx + \frac{(bdfx + bcf - bd)e^{(fx+e)}}{2\,f^2} + \frac{(bdfx + bcf + bd)e^{(-fx-e)}}{2\,f^2}$$

input
```
integrate((d*x+c)*(a+b*sinh(f*x+e)),x, algorithm="giac")
```

output
```
1/2*a*d*x^2 + a*c*x + 1/2*(b*d*f*x + b*c*f - b*d)*e^(f*x + e)/f^2 + 1/2*(b
*d*f*x + b*c*f + b*d)*e^(-f*x - e)/f^2
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int (c+dx)(a+b\sinh(e+fx))\,dx$$
$$= \frac{f\,(b\,c\cosh(e+f\,x)+b\,d\,x\cosh(e+f\,x))-b\,d\sinh(e+f\,x)}{f^2}+a\,c\,x+\frac{a\,d\,x^2}{2}$$

input
```
int((a + b*sinh(e + f*x))*(c + d*x),x)
```

output
```
(f*(b*c*cosh(e + f*x) + b*d*x*cosh(e + f*x)) - b*d*sinh(e + f*x))/f^2 + a*
c*x + (a*d*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.24

$$\int (c+dx)(a+b\sinh(e+fx))\,dx$$
$$= \frac{2\cosh(fx+e)\,bcf+2\cosh(fx+e)\,bdfx-2\sinh(fx+e)\,bd+2ac\,f^2x+ad\,f^2x^2}{2f^2}$$

input
```
int((d*x+c)*(a+b*sinh(f*x+e)),x)
```

output
```
(2*cosh(e + f*x)*b*c*f + 2*cosh(e + f*x)*b*d*f*x - 2*sinh(e + f*x)*b*d + 2
*a*c*f**2*x + a*d*f**2*x**2)/(2*f**2)
```

3.160 $\int \frac{a+b\sinh(e+fx)}{c+dx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 64

$$\int \frac{a+b\sinh(e+fx)}{c+dx}\,dx = \frac{a\log(c+dx)}{d} + \frac{b\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d} + \frac{b\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{d}$$

output
```
a*ln(d*x+c)/d-b*Chi(c*f/d+f*x)*sinh(-e+c*f/d)/d+b*cosh(-e+c*f/d)*Shi(c*f/d
+f*x)/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int \frac{a+b\sinh(e+fx)}{c+dx}\,dx = \frac{a\log(c+dx)+b\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)+b\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)}{d}$$

input
```
Integrate[(a + b*Sinh[e + f*x])/(c + d*x),x]
```

output
```
(a*Log[c + d*x] + b*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] + b*Cosh[e
 - (c*f)/d]*SinhIntegral[f*(c/d + x)])/d
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sinh(e+fx)}{c+dx}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a-ib\sin(ie+ifx)}{c+dx}dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(\frac{a}{c+dx}+\frac{b\sinh(e+fx)}{c+dx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\log(c+dx)}{d}+\frac{b\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d}+\frac{b\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d}$$

input
```
Int[(a + b*Sinh[e + f*x])/(c + d*x),x]
```

output
```
(a*Log[c + d*x])/d + (b*CoshIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d +
 (b*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.47

method	result	size
risch	$\frac{a\ln(dx+c)}{d}+\frac{b\,\mathrm{e}^{\frac{cf-de}{d}}\,\mathrm{expIntegral}_1\left(fx+e+\frac{cf-de}{d}\right)}{2d}-\frac{b\,\mathrm{e}^{-\frac{cf-de}{d}}\,\mathrm{expIntegral}_1\left(-fx-e-\frac{cf-de}{d}\right)}{2d}$	94

input
```
int((a+b*sinh(f*x+e))/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*ln(d*x+c)/d+1/2*b/d*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/2*b/d*exp
(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.73

$$\int \frac{a+b\sinh(e+fx)}{c+dx}\,dx = \frac{\left(b\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)-b\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{de-cf}{d}\right)+2\,a\log\left(dx+c\right)-\left(b\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)+b\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\sinh\left(-\right.}{2\,d}$$

input
```
integrate((a+b*sinh(f*x+e))/(d*x+c),x, algorithm="fricas")
```

output
```
1/2*((b*Ei((d*f*x + c*f)/d) - b*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d)
 + 2*a*log(d*x + c) - (b*Ei((d*f*x + c*f)/d) + b*Ei(-(d*f*x + c*f)/d))*sin
h(-(d*e - c*f)/d))/d
```

Sympy [F]

$$\int \frac{a + b\sinh(e + fx)}{c + dx} dx = \int \frac{a + b\sinh(e + fx)}{c + dx} dx$$

input
```
integrate((a+b*sinh(f*x+e))/(d*x+c),x)
```

output
```
Integral((a + b*sinh(e + f*x))/(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.11

$$\int \frac{a + b\sinh(e + fx)}{c + dx} dx = \frac{1}{2}b\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_1\left(\frac{(dx+c)f}{d}\right)}{d} - \frac{e^{\left(e-\frac{cf}{d}\right)}E_1\left(-\frac{(dx+c)f}{d}\right)}{d}\right) + \frac{a\log(dx + c)}{d}$$

input
```
integrate((a+b*sinh(f*x+e))/(d*x+c),x, algorithm="maxima")
```

output
```
1/2*b*(e^(-e + c*f/d)*exp_integral_e(1, (d*x + c)*f/d)/d - e^(e - c*f/d)*e
xp_integral_e(1, -(d*x + c)*f/d)/d) + a*log(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06

$$\int \frac{a+b\sinh(e+fx)}{c+dx}\,dx = \frac{b\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)} - b\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)} + 2\,a\log\left(dx+c\right)}{2\,d}$$

input
```
integrate((a+b*sinh(f*x+e))/(d*x+c),x, algorithm="giac")
```

output
```
1/2*(b*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) - b*Ei(-(d*f*x + c*f)/d)*e^(-e +
c*f/d) + 2*a*log(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sinh(e+fx)}{c+dx}\,dx = \int \frac{a+b\sinh(e+f\,x)}{c+d\,x}\,dx$$

input
```
int((a + b*sinh(e + f*x))/(c + d*x),x)
```

output
```
int((a + b*sinh(e + f*x))/(c + d*x), x)
```

Reduce [F]

$$\int \frac{a+b\sinh(e+fx)}{c+dx}\,dx = \frac{\left(\int \frac{\sinh(fx+e)}{dx+c}dx\right)bd + \log(dx+c)\,a}{d}$$

input
```
int((a+b*sinh(f*x+e))/(d*x+c),x)
```

output
```
(int(sinh(e + f*x)/(c + d*x),x)*b*d + log(c + d*x)*a)/d
```

3.161 $\int \frac{a+b\sinh(e+fx)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 87

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^2}\,dx = -\frac{a}{d(c+dx)}+\frac{bf\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{d^2} - \frac{b\sinh(e+fx)}{d(c+dx)}+\frac{bf\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{d^2}$$

output
```
-a/d/(d*x+c)+b*f*cosh(-e+c*f/d)*Chi(c*f/d+f*x)/d^2-b*sinh(f*x+e)/d/(d*x+c)
-b*f*sinh(-e+c*f/d)*Shi(c*f/d+f*x)/d^2
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^2}\,dx = \frac{bf\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)-\frac{d(a+b\sinh(e+fx))}{c+dx}+bf\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)}{d^2}$$

input
```
Integrate[(a + b*Sinh[e + f*x])/(c + d*x)^2,x]
```

output
```
(b*f*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] - (d*(a + b*Sinh[e + f*x]
))/(c + d*x) + b*f*Sinh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)])/d^2
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a-ib\sin(ie+ifx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(\frac{a}{(c+dx)^2}+\frac{b\sinh(e+fx)}{(c+dx)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{d(c+dx)}+\frac{bf\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{bf\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{b\sinh(e+fx)}{d(c+dx)}$$

input
```
Int[(a + b*Sinh[e + f*x])/(c + d*x)^2,x]
```

output
```
-(a/(d*(c + d*x))) + (b*f*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/d
^2 - (b*Sinh[e + f*x])/(d*(c + d*x)) + (b*f*Sinh[e - (c*f)/d]*SinhIntegral
[(c*f)/d + f*x])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.71

method	result
risch	$-\frac{a}{d(dx+c)}+\frac{fb\,\mathrm{e}^{-fx-e}}{2d(dxf+cf)}-\frac{fb\,\mathrm{e}^{\frac{cf-de}{d}}\operatorname{expIntegral}_1\left(fx+e+\frac{cf-de}{d}\right)}{2d^2}-\frac{fb\,\mathrm{e}^{fx+e}}{2d^2\left(\frac{cf}{d}+fx\right)}-\frac{fb\,\mathrm{e}^{-\frac{cf-de}{d}}\operatorname{expIntegral}_1\left(-fx-e\right.}{2d^2}$

input
```
int((a+b*sinh(f*x+e))/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-a/d/(d*x+c)+1/2*f*b*exp(-f*x-e)/d/(d*f*x+c*f)-1/2*f*b/d^2*exp((c*f-d*e)/d
)*Ei(1,f*x+e+(c*f-d*e)/d)-1/2*f*b/d^2*exp(f*x+e)/(c*f/d+f*x)-1/2*f*b/d^2*e
xp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.86

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^2}\,dx = -\frac{2\,bd\sinh\left(fx+e\right)+2\,ad-\left((bdfx+bcf)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)+(bdfx+bcf)\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{de-cf}{d}\right)+\left((b\right.}{2\left(d^3x+cd^2\right)}$$

input
```
integrate((a+b*sinh(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*b*d*sinh(f*x + e) + 2*a*d - ((b*d*f*x + b*c*f)*Ei((d*f*x + c*f)/d)
 + (b*d*f*x + b*c*f)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) + ((b*d*f*
x + b*c*f)*Ei((d*f*x + c*f)/d) - (b*d*f*x + b*c*f)*Ei(-(d*f*x + c*f)/d))*s
inh(-(d*e - c*f)/d))/(d^3*x + c*d^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sinh(f*x+e))/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^2}\,dx = \frac{1}{2}b\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d} - \frac{e^{\left(e-\frac{cf}{d}\right)}E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d}\right) - \frac{a}{d^2x+cd}$$

input
```
integrate((a+b*sinh(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*b*(e^(-e + c*f/d)*exp_integral_e(2, (d*x + c)*f/d)/((d*x + c)*d) - e^(
e - c*f/d)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c)*d)) - a/(d^2*x + c
*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(90) = 180$.

Time = 0.14 (sec) , antiderivative size = 630, normalized size of antiderivative = 7.24

$$\int \frac{a + b\sinh(e + fx)}{(c + dx)^2} \, dx$$
$$= \frac{1}{2}b\left(\frac{\left((dx + c)\left(\frac{de}{dx+c} - \frac{cf}{dx+c} + f\right)f^2\mathrm{Ei}\left(\frac{(dx+c)\left(\frac{de}{dx+c} - \frac{cf}{dx+c} + f\right) - de + cf}{d}\right)e^{\left(\frac{de-cf}{d}\right)} - def^2\mathrm{Ei}\left(\frac{(dx+c)\left(\frac{de}{dx+c} - \frac{cf}{dx+c}\right.}{d}\right.\right.}{\left((dx + c)d^4\left(\frac{de}{dx+c} - \frac{cf}{dx+c}\right.\right.}\right.$$
$$- \frac{a}{(dx + c)d}$$

input
```
integrate((a+b*sinh(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/2*b*(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(((d*x + c)*(d
*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e - c*f)/d) - d*e*
f^2*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((
d*e - c*f)/d) + c*f^3*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) -
d*e + c*f)/d)*e^((d*e - c*f)/d) - d*f^2*e^((d*x + c)*(d*e/(d*x + c) - c*f/
(d*x + c) + f)/d))*d^2/(((d*x + c)*d^4*(d*e/(d*x + c) - c*f/(d*x + c) + f)
 - d^5*e + c*d^4*f)*f) + ((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^
2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(
d*e - c*f)/d) - d*e*f^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)
 - d*e + c*f)/d)*e^(-(d*e - c*f)/d) + c*f^3*Ei(-((d*x + c)*(d*e/(d*x + c)
- c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(d*e - c*f)/d) + d*f^2*e^(-(d*x +
 c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d))*d^2/(((d*x + c)*d^4*(d*e/(d*x
+ c) - c*f/(d*x + c) + f) - d^5*e + c*d^4*f)*f)) - a/((d*x + c)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^2}\,dx = \int \frac{a+b\sinh(e+f\,x)}{(c+d\,x)^2}\,dx$$

input
```
int((a + b*sinh(e + f*x))/(c + d*x)^2,x)
```

output
```
int((a + b*sinh(e + f*x))/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^2}\,dx = \frac{e^{2e}\left(\int \frac{e^{fx}}{d^2x^2+2cdx+c^2}dx\right)b\,c^2+e^{2e}\left(\int \frac{e^{fx}}{d^2x^2+2cdx+c^2}dx\right)bcdx+2e^{e}ax-\left(\int \frac{1}{e^{fx}c^2+2e^{fx}cdx+e^{fx}d^2x^2}dx\right)b\,c^2-\left(\int\right.}{2e^{e}c\,(dx+c)}$$

input
```
int((a+b*sinh(f*x+e))/(d*x+c)^2,x)
```

output
```
(e**(2*e)*int(e**(f*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*b*c**2 + e**(2*e)*i
nt(e**(f*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*b*c*d*x + 2*e**e*a*x - int(1/(
e**(f*x)*c**2 + 2*e**(f*x)*c*d*x + e**(f*x)*d**2*x**2),x)*b*c**2 - int(1/(
e**(f*x)*c**2 + 2*e**(f*x)*c*d*x + e**(f*x)*d**2*x**2),x)*b*c*d*x)/(2*e**e
*c*(c + d*x))
```

3.162 $\int \frac{a+b\sinh(e+fx)}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 123

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^3}\,dx = -\frac{a}{2d(c+dx)^2}-\frac{bf\cosh(e+fx)}{2d^2(c+dx)} + \frac{bf^2\mathrm{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{2d^3}-\frac{b\sinh(e+fx)}{2d(c+dx)^2} + \frac{bf^2\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(\frac{cf}{d}+fx\right)}{2d^3}$$

output
```
-1/2*a/d/(d*x+c)^2-1/2*b*f*cosh(f*x+e)/d^2/(d*x+c)-1/2*b*f^2*Chi(c*f/d+f*x
)*sinh(-e+c*f/d)/d^3-1/2*b*sinh(f*x+e)/d/(d*x+c)^2+1/2*b*f^2*cosh(-e+c*f/d
)*Shi(c*f/d+f*x)/d^3
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.77

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^3}\,dx = \frac{bf^2\mathrm{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)-\frac{d(bf(c+dx)\cosh(e+fx)+d(a+b\sinh(e+fx)))}{(c+dx)^2}+bf^2\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(f\left(\frac{c}{d}+x\right)\right)}{2d^3}$$

input
```
Integrate[(a + b*Sinh[e + f*x])/(c + d*x)^3,x]
```

output
```
(b*f^2*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] - (d*(b*f*(c + d*x)*Cos
h[e + f*x] + d*(a + b*Sinh[e + f*x])))/(c + d*x)^2 + b*f^2*Cosh[e - (c*f)/
d]*SinhIntegral[f*(c/d + x)])/(2*d^3)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a-ib\sin(ie+ifx)}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(\frac{a}{(c+dx)^3}+\frac{b\sinh(e+fx)}{(c+dx)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{2d(c+dx)^2}+\frac{bf^2\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{2d^3}+\frac{bf^2\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{2d^3}-\frac{bf\cosh(e+fx)}{2d^2(c+dx)}-\frac{b\sinh(e+fx)}{2d(c+dx)^2}$$

input
```
Int[(a + b*Sinh[e + f*x])/(c + d*x)^3,x]
```

output
```
-1/2*a/(d*(c + d*x)^2) - (b*f*Cosh[e + f*x])/(2*d^2*(c + d*x)) + (b*f^2*Co
shIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/(2*d^3) - (b*Sinh[e + f*x])/(
2*d*(c + d*x)^2) + (b*f^2*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/(
2*d^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(115) = 230$.

Time = 0.35 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.41

method	result
risch	$-\frac{a}{2d(dx+c)^2} - \frac{f^3 b\,\mathrm{e}^{-fx-e} x}{4d(d^2x^2f^2+2cd\,f^2x+c^2f^2)} - \frac{f^3 b\,\mathrm{e}^{-fx-e} c}{4d^2(d^2x^2f^2+2cd\,f^2x+c^2f^2)} + \frac{f^2 b\,\mathrm{e}^{-fx-e}}{4d(d^2x^2f^2+2cd\,f^2x+c^2f^2)} + \frac{f^2 b\,\mathrm{e}^{\frac{cf-de}{d}}\,\mathrm{ex}}{}$

input
```
int((a+b*sinh(f*x+e))/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/d/(d*x+c)^2-1/4*f^3*b*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^
2)*x-1/4*f^3*b*exp(-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c+1/4*f^2
*b*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)+1/4*f^2*b/d^3*exp((c*f-
d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/4*f^2*b/d^3*exp(f*x+e)/(c*f/d+f*x)^2-1/4
*f^2*b/d^3*exp(f*x+e)/(c*f/d+f*x)-1/4*f^2*b/d^3*exp(-(c*f-d*e)/d)*Ei(1,-f*
x-e-(c*f-d*e)/d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(115) = 230$.

Time = 0.09 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.23

$$\int \frac{a + b\sinh(e + fx)}{(c + dx)^3}\,dx = -\frac{2\,bd^2\sinh\left(fx + e\right) + 2\,ad^2 + 2\left(bd^2fx + bcdf\right)\cosh\left(fx + e\right) - \left(\left(bd^2f^2x^2 + 2\,bcdf^2x + bc^2f^2\right)\mathrm{Ei}\left(\frac{dfx + c}{d}\right.\right.}{}$$

input
```
integrate((a+b*sinh(f*x+e))/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*b*d^2*sinh(f*x + e) + 2*a*d^2 + 2*(b*d^2*f*x + b*c*d*f)*cosh(f*x +
 e) - ((b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2)*Ei((d*f*x + c*f)/d) - (
b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*
e - c*f)/d) + ((b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2)*Ei((d*f*x + c*f
)/d) + (b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2)*Ei(-(d*f*x + c*f)/d))*s
inh(-(d*e - c*f)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\sinh(e + fx)}{(c + dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sinh(f*x+e))/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.80

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^3}\,dx = \frac{1}{2}\,b\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2d} - \frac{e^{\left(e-\frac{cf}{d}\right)}E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}\right) - \frac{a}{2\,(d^3x^2+2\,cd^2x+c^2d)}$$

input
```
integrate((a+b*sinh(f*x+e))/(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/2*b*(e^(-e + c*f/d)*exp_integral_e(3, (d*x + c)*f/d)/((d*x + c)^2*d) - e
^(e - c*f/d)*exp_integral_e(3, -(d*x + c)*f/d)/((d*x + c)^2*d)) - 1/2*a/(d
^3*x^2 + 2*c*d^2*x + c^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(115) = 230.

Time = 0.11 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.59

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^3}\,dx = \frac{bd^2f^2x^2\text{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)} - bd^2f^2x^2\text{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)} + 2\,bcdf^2x\text{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)} - 2\,bcdf^2x\text{Ei}\left(-\right.}{}$$

input
```
integrate((a+b*sinh(f*x+e))/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/4*(b*d^2*f^2*x^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) - b*d^2*f^2*x^2*Ei(-(
d*f*x + c*f)/d)*e^(-e + c*f/d) + 2*b*c*d*f^2*x*Ei((d*f*x + c*f)/d)*e^(e -
c*f/d) - 2*b*c*d*f^2*x*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + b*c^2*f^2*Ei(
(d*f*x + c*f)/d)*e^(e - c*f/d) - b*c^2*f^2*Ei(-(d*f*x + c*f)/d)*e^(-e + c*
f/d) - b*d^2*f*x*e^(f*x + e) - b*d^2*f*x*e^(-f*x - e) - b*c*d*f*e^(f*x + e
) - b*c*d*f*e^(-f*x - e) - b*d^2*e^(f*x + e) + b*d^2*e^(-f*x - e) - 2*a*d^
2)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^3}\,dx = \int \frac{a+b\sinh(e+f\,x)}{(c+d\,x)^3}\,dx$$

input
```
int((a + b*sinh(e + f*x))/(c + d*x)^3,x)
```

output
```
int((a + b*sinh(e + f*x))/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^3}\,dx = \text{too large to display}$$

input
```
int((a+b*sinh(f*x+e))/(d*x+c)^3,x)
```

output
```
(2*e**(e + f*x)*cosh(e + f*x)*b*c**3*d*f**3 + 2*e**(e + f*x)*cosh(e + f*x)
*b*c**2*d**2*f**3*x - 8*e**(e + f*x)*cosh(e + f*x)*b*c*d**3*f - 8*e**(e +
f*x)*cosh(e + f*x)*b*d**4*f*x - e**(2*e + 2*f*x)*b*c**2*d**2*f**3*x + e**(
2*e + 2*f*x)*b*c**2*d**2*f**2 + 2*e**(2*e + 2*f*x)*b*c*d**3*f + 4*e**(2*e
+ 2*f*x)*b*d**4*f*x - e**(2*e + f*x)*int((e**(f*x)*x)/(c**5*f**2 + 3*c**4*
d*f**2*x + 3*c**3*d**2*f**2*x**2 - 4*c**3*d**2 + c**2*d**3*f**2*x**3 - 12*
c**2*d**3*x - 12*c*d**4*x**2 - 4*d**5*x**3),x)*b*c**7*d**2*f**6 - 2*e**(2*
e + f*x)*int((e**(f*x)*x)/(c**5*f**2 + 3*c**4*d*f**2*x + 3*c**3*d**2*f**2*
x**2 - 4*c**3*d**2 + c**2*d**3*f**2*x**3 - 12*c**2*d**3*x - 12*c*d**4*x**2
 - 4*d**5*x**3),x)*b*c**6*d**3*f**6*x - 2*e**(2*e + f*x)*int((e**(f*x)*x)/
(c**5*f**2 + 3*c**4*d*f**2*x + 3*c**3*d**2*f**2*x**2 - 4*c**3*d**2 + c**2*
d**3*f**2*x**3 - 12*c**2*d**3*x - 12*c*d**4*x**2 - 4*d**5*x**3),x)*b*c**6*
d**3*f**5 - e**(2*e + f*x)*int((e**(f*x)*x)/(c**5*f**2 + 3*c**4*d*f**2*x +
 3*c**3*d**2*f**2*x**2 - 4*c**3*d**2 + c**2*d**3*f**2*x**3 - 12*c**2*d**3*
x - 12*c*d**4*x**2 - 4*d**5*x**3),x)*b*c**5*d**4*f**6*x**2 - 4*e**(2*e + f
*x)*int((e**(f*x)*x)/(c**5*f**2 + 3*c**4*d*f**2*x + 3*c**3*d**2*f**2*x**2
- 4*c**3*d**2 + c**2*d**3*f**2*x**3 - 12*c**2*d**3*x - 12*c*d**4*x**2 - 4*
d**5*x**3),x)*b*c**5*d**4*f**5*x + 6*e**(2*e + f*x)*int((e**(f*x)*x)/(c**5
*f**2 + 3*c**4*d*f**2*x + 3*c**3*d**2*f**2*x**2 - 4*c**3*d**2 + c**2*d**3*
f**2*x**3 - 12*c**2*d**3*x - 12*c*d**4*x**2 - 4*d**5*x**3),x)*b*c**5*d*...
```

3.163 $\int (c+dx)^3(a+b\sinh(e+fx))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 237

$$\begin{aligned}\int (c+dx)^3(a+b\sinh(e+fx))^2\,dx = & -\frac{3b^2d(c+dx)^2}{8f^2}+\frac{a^2(c+dx)^4}{4d}-\frac{b^2(c+dx)^4}{8d} \\ & +\frac{12abd^2(c+dx)\cosh(e+fx)}{f^3} \\ & +\frac{2ab(c+dx)^3\cosh(e+fx)}{f} \\ & -\frac{12abd^3\sinh(e+fx)}{f^4} \\ & -\frac{6abd(c+dx)^2\sinh(e+fx)}{f^2} \\ & +\frac{3b^2d^2(c+dx)\cosh(e+fx)\sinh(e+fx)}{4f^3} \\ & +\frac{b^2(c+dx)^3\cosh(e+fx)\sinh(e+fx)}{2f} \\ & -\frac{3b^2d^3\sinh^2(e+fx)}{8f^4} \\ & -\frac{3b^2d(c+dx)^2\sinh^2(e+fx)}{4f^2}\end{aligned}$$

output
```
-3/8*b^2*d*(d*x+c)^2/f^2+1/4*a^2*(d*x+c)^4/d-1/8*b^2*(d*x+c)^4/d+12*a*b*d^
2*(d*x+c)*cosh(f*x+e)/f^3+2*a*b*(d*x+c)^3*cosh(f*x+e)/f-12*a*b*d^3*sinh(f*
x+e)/f^4-6*a*b*d*(d*x+c)^2*sinh(f*x+e)/f^2+3/4*b^2*d^2*(d*x+c)*cosh(f*x+e)
*sinh(f*x+e)/f^3+1/2*b^2*(d*x+c)^3*cosh(f*x+e)*sinh(f*x+e)/f-3/8*b^2*d^3*s
inh(f*x+e)^2/f^4-3/4*b^2*d*(d*x+c)^2*sinh(f*x+e)^2/f^2
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.99

$$\int (c+dx)^3(a+b\sinh(e+fx))^2\,dx$$
$$=\frac{32abf(c+dx)\left(c^2f^2+2cdf^2x+d^2(6+f^2x^2)\right)\cosh(e+fx)-3b^2d(2c^2f^2+4cdf^2x+d^2(1+2f^2x^2))\,\mathrm{co}}{}$$

input
```
Integrate[(c + d*x)^3*(a + b*Sinh[e + f*x])^2,x]
```

output
```
(32*a*b*f*(c + d*x)*(c^2*f^2 + 2*c*d*f^2*x + d^2*(6 + f^2*x^2))*Cosh[e + f
*x] - 3*b^2*d*(2*c^2*f^2 + 4*c*d*f^2*x + d^2*(1 + 2*f^2*x^2))*Cosh[2*(e +
f*x)] + 2*((2*a^2 - b^2)*f^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)
 - 48*a*b*d*(c^2*f^2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Sinh[e + f*x] + b^
2*f*(c + d*x)*(2*c^2*f^2 + 4*c*d*f^2*x + d^2*(3 + 2*f^2*x^2))*Sinh[2*(e +
f*x)]))/(16*f^4)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3(a+b\sinh(e+fx))^2\,dx$$
$$\Big\downarrow \; 3042$$

$$\int (c+dx)^3(a-ib\sin(ie+ifx))^2dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(a^2(c+dx)^3+2ab(c+dx)^3\sinh(e+fx)+b^2(c+dx)^3\sinh^2(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(c+dx)^4}{4d}+\frac{12abd^2(c+dx)\cosh(e+fx)}{f^3}-\frac{6abd(c+dx)^2\sinh(e+fx)}{f^2}+\frac{2ab(c+dx)^3\cosh(e+fx)}{f}-\frac{12abd^3\sinh(e+fx)}{f^4}+\frac{3b^2d^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{4f^3}-\frac{3b^2d(c+dx)^2\sinh^2(e+fx)}{4f^2}+\frac{b^2(c+dx)^3\sinh(e+fx)\cosh(e+fx)}{2f}-\frac{3b^2d(c+dx)^2}{8f^2}-\frac{b^2(c+dx)^4}{8d}-\frac{3b^2d^3\sinh^2(e+fx)}{8f^4}$$

input
```
Int[(c + d*x)^3*(a + b*Sinh[e + f*x])^2,x]
```

output
```
(-3*b^2*d*(c + d*x)^2)/(8*f^2) + (a^2*(c + d*x)^4)/(4*d) - (b^2*(c + d*x)^
4)/(8*d) + (12*a*b*d^2*(c + d*x)*Cosh[e + f*x])/f^3 + (2*a*b*(c + d*x)^3*C
osh[e + f*x])/f - (12*a*b*d^3*Sinh[e + f*x])/f^4 - (6*a*b*d*(c + d*x)^2*Si
nh[e + f*x])/f^2 + (3*b^2*d^2*(c + d*x)*Cosh[e + f*x]*Sinh[e + f*x])/(4*f^
3) + (b^2*(c + d*x)^3*Cosh[e + f*x]*Sinh[e + f*x])/(2*f) - (3*b^2*d^3*Sinh
[e + f*x]^2)/(8*f^4) - (3*b^2*d*(c + d*x)^2*Sinh[e + f*x]^2)/(4*f^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.90

method	result
parallelrisch	$\frac{4\left((dx+c)^2f^2+\frac{3d^2}{2}\right)b^2(dx+c)f\sinh(2fx+2e)-6b^2d\left((dx+c)^2f^2+\frac{d^2}{2}\right)\cosh(2fx+2e)+32ba(dx+c)f\left((dx+c)^2f^2+6d\right.}{}$
risch	$\frac{a^2d^3x^4}{4}-\frac{d^3b^2x^4}{8}+a^2d^2c\,x^3-\frac{d^2b^2c\,x^3}{2}+\frac{3a^2d\,c^2x^2}{2}-\frac{3d\,b^2c^2x^2}{4}+a^2c^3x-\frac{b^2c^3x}{2}+\frac{a^2c^4}{4d}-\frac{b^2c^4}{8d}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display
orering	Expression too large to display

input
```
int((d*x+c)^3*(a+b*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*(4*((d*x+c)^2*f^2+3/2*d^2)*b^2*(d*x+c)*f*sinh(2*f*x+2*e)-6*b^2*d*((d*
x+c)^2*f^2+1/2*d^2)*cosh(2*f*x+2*e)+32*b*a*(d*x+c)*f*((d*x+c)^2*f^2+6*d^2)
*cosh(f*x+e)-96*b*a*d*((d*x+c)^2*f^2+2*d^2)*sinh(f*x+e)+16*(a^2-1/2*b^2)*(
1/2*x^2*d^2+c*d*x+c^2)*(1/2*d*x+c)*x*f^4+32*a*b*c^3*f^3+6*b^2*c^2*d*f^2+19
2*a*b*c*d^2*f+3*d^3*b^2)/f^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.76

$$\int (c+dx)^3(a+b\sinh(e+fx))^2\,dx$$
$$=\frac{2\left(2a^2-b^2\right)d^3f^4x^4+8\left(2a^2-b^2\right)cd^2f^4x^3+12\left(2a^2-b^2\right)c^2df^4x^2+8\left(2a^2-b^2\right)c^3f^4x-3\left(2b^2d^3f^2x^2+\right.}{}$$

input
```
integrate((d*x+c)^3*(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/16*(2*(2*a^2 - b^2)*d^3*f^4*x^4 + 8*(2*a^2 - b^2)*c*d^2*f^4*x^3 + 12*(2*
a^2 - b^2)*c^2*d*f^4*x^2 + 8*(2*a^2 - b^2)*c^3*f^4*x - 3*(2*b^2*d^3*f^2*x^
2 + 4*b^2*c*d^2*f^2*x + 2*b^2*c^2*d*f^2 + b^2*d^3)*cosh(f*x + e)^2 - 3*(2*
b^2*d^3*f^2*x^2 + 4*b^2*c*d^2*f^2*x + 2*b^2*c^2*d*f^2 + b^2*d^3)*sinh(f*x
+ e)^2 + 32*(a*b*d^3*f^3*x^3 + 3*a*b*c*d^2*f^3*x^2 + a*b*c^3*f^3 + 6*a*b*c
*d^2*f + 3*(a*b*c^2*d*f^3 + 2*a*b*d^3*f)*x)*cosh(f*x + e) - 4*(24*a*b*d^3*
f^2*x^2 + 48*a*b*c*d^2*f^2*x + 24*a*b*c^2*d*f^2 + 48*a*b*d^3 - (2*b^2*d^3*
f^3*x^3 + 6*b^2*c*d^2*f^3*x^2 + 2*b^2*c^3*f^3 + 3*b^2*c*d^2*f + 3*(2*b^2*c
^2*d*f^3 + b^2*d^3*f)*x)*cosh(f*x + e))*sinh(f*x + e))/f^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 779 vs. $2(240) = 480$.

Time = 0.44 (sec) , antiderivative size = 779, normalized size of antiderivative = 3.29

$$\int (c+dx)^3(a+b\sinh(e+fx))^2\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**3*(a+b*sinh(f*x+e))**2,x)
```

output
```
Piecewise((a**2*c**3*x + 3*a**2*c**2*d*x**2/2 + a**2*c*d**2*x**3 + a**2*d*
*3*x**4/4 + 2*a*b*c**3*cosh(e + f*x)/f + 6*a*b*c**2*d*x*cosh(e + f*x)/f -
6*a*b*c**2*d*sinh(e + f*x)/f**2 + 6*a*b*c*d**2*x**2*cosh(e + f*x)/f - 12*a
*b*c*d**2*x*sinh(e + f*x)/f**2 + 12*a*b*c*d**2*cosh(e + f*x)/f**3 + 2*a*b*
d**3*x**3*cosh(e + f*x)/f - 6*a*b*d**3*x**2*sinh(e + f*x)/f**2 + 12*a*b*d*
*3*x*cosh(e + f*x)/f**3 - 12*a*b*d**3*sinh(e + f*x)/f**4 + b**2*c**3*x*sin
h(e + f*x)**2/2 - b**2*c**3*x*cosh(e + f*x)**2/2 + b**2*c**3*sinh(e + f*x)
*cosh(e + f*x)/(2*f) + 3*b**2*c**2*d*x**2*sinh(e + f*x)**2/4 - 3*b**2*c**2
*d*x**2*cosh(e + f*x)**2/4 + 3*b**2*c**2*d*x*sinh(e + f*x)*cosh(e + f*x)/(
2*f) - 3*b**2*c**2*d*sinh(e + f*x)**2/(4*f**2) + b**2*c*d**2*x**3*sinh(e +
 f*x)**2/2 - b**2*c*d**2*x**3*cosh(e + f*x)**2/2 + 3*b**2*c*d**2*x**2*sinh
(e + f*x)*cosh(e + f*x)/(2*f) - 3*b**2*c*d**2*x*sinh(e + f*x)**2/(4*f**2)
- 3*b**2*c*d**2*x*cosh(e + f*x)**2/(4*f**2) + 3*b**2*c*d**2*sinh(e + f*x)*
cosh(e + f*x)/(4*f**3) + b**2*d**3*x**4*sinh(e + f*x)**2/8 - b**2*d**3*x**
4*cosh(e + f*x)**2/8 + b**2*d**3*x**3*sinh(e + f*x)*cosh(e + f*x)/(2*f) -
3*b**2*d**3*x**2*sinh(e + f*x)**2/(8*f**2) - 3*b**2*d**3*x**2*cosh(e + f*x
)**2/(8*f**2) + 3*b**2*d**3*x*sinh(e + f*x)*cosh(e + f*x)/(4*f**3) - 3*b**
2*d**3*sinh(e + f*x)**2/(8*f**4), Ne(f, 0)), ((a + b*sinh(e))**2*(c**3*x +
 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(223) = 446$.

Time = 0.09 (sec) , antiderivative size = 520, normalized size of antiderivative = 2.19

$$\int (c+dx)^3(a+b\sinh(e+fx))^2\,dx = \frac{1}{4}\,a^2d^3x^4 + a^2cd^2x^3 + \frac{3}{2}\,a^2c^2dx^2$$
$$-\frac{3}{16}\left(4\,x^2 - \frac{\left(2\,fxe^{(2\,e)} - e^{(2\,e)}\right)e^{(2\,fx)}}{f^2} + \frac{(2\,fx+1)e^{(-2\,fx-2\,e)}}{f^2}\right)b^2c^2d$$
$$-\frac{1}{16}\left(8\,x^3 - \frac{3\left(2\,f^2x^2e^{(2\,e)} - 2\,fxe^{(2\,e)} + e^{(2\,e)}\right)e^{(2\,fx)}}{f^3} + \frac{3\,(2\,f^2x^2 + 2\,fx + 1)e^{(-2\,fx-2\,e)}}{f^3}\right)b^2cd^2$$
$$-\frac{1}{32}\left(4\,x^4 - \frac{\left(4\,f^3x^3e^{(2\,e)} - 6\,f^2x^2e^{(2\,e)} + 6\,fxe^{(2\,e)} - 3\,e^{(2\,e)}\right)e^{(2\,fx)}}{f^4} + \frac{(4\,f^3x^3 + 6\,f^2x^2 + 6\,fx + 3)e^{(-2}}{f^4}\right.$$
$$-\frac{1}{8}\,b^2c^3\left(4\,x - \frac{e^{(2\,fx+2\,e)}}{f} + \frac{e^{(-2\,fx-2\,e)}}{f}\right) + a^2c^3x$$
$$+3\,abc^2d\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right)$$
$$+3\,abcd^2\left(\frac{(f^2x^2e^e - 2\,fxe^e + 2\,e^e)e^{(fx)}}{f^3} + \frac{(f^2x^2 + 2\,fx + 2)e^{(-fx-e)}}{f^3}\right)$$
$$+abd^3\left(\frac{(f^3x^3e^e - 3\,f^2x^2e^e + 6\,fxe^e - 6\,e^e)e^{(fx)}}{f^4} + \frac{(f^3x^3 + 3\,f^2x^2 + 6\,fx + 6)e^{(-fx-e)}}{f^4}\right)$$
$$+\frac{2\,abc^3\cosh\,(fx+e)}{f}$$

input
```
integrate((d*x+c)^3*(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/4*a^2*d^3*x^4 + a^2*c*d^2*x^3 + 3/2*a^2*c^2*d*x^2 - 3/16*(4*x^2 - (2*f*x
*e^(2*e) - e^(2*e))*e^(2*f*x)/f^2 + (2*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*b^2*
c^2*d - 1/16*(8*x^3 - 3*(2*f^2*x^2*e^(2*e) - 2*f*x*e^(2*e) + e^(2*e))*e^(2
*f*x)/f^3 + 3*(2*f^2*x^2 + 2*f*x + 1)*e^(-2*f*x - 2*e)/f^3)*b^2*c*d^2 - 1/
32*(4*x^4 - (4*f^3*x^3*e^(2*e) - 6*f^2*x^2*e^(2*e) + 6*f*x*e^(2*e) - 3*e^(
2*e))*e^(2*f*x)/f^4 + (4*f^3*x^3 + 6*f^2*x^2 + 6*f*x + 3)*e^(-2*f*x - 2*e)
/f^4)*b^2*d^3 - 1/8*b^2*c^3*(4*x - e^(2*f*x + 2*e)/f + e^(-2*f*x - 2*e)/f)
 + a^2*c^3*x + 3*a*b*c^2*d*((f*x*e^e - e^e)*e^(f*x)/f^2 + (f*x + 1)*e^(-f*
x - e)/f^2) + 3*a*b*c*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f^3 +
 (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + a*b*d^3*((f^3*x^3*e^e - 3*f^2*x
^2*e^e + 6*f*x*e^e - 6*e^e)*e^(f*x)/f^4 + (f^3*x^3 + 3*f^2*x^2 + 6*f*x + 6
)*e^(-f*x - e)/f^4) + 2*a*b*c^3*cosh(f*x + e)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(223) = 446$.

Time = 0.12 (sec) , antiderivative size = 598, normalized size of antiderivative = 2.52

$$\int (c+dx)^3(a+b\sinh(e+fx))^2\,dx$$
$$= \frac{1}{4}a^2d^3x^4 - \frac{1}{8}b^2d^3x^4 + a^2cd^2x^3 - \frac{1}{2}b^2cd^2x^3 + \frac{3}{2}a^2c^2dx^2 - \frac{3}{4}b^2c^2dx^2 + a^2c^3x - \frac{1}{2}b^2c^3x$$
$$+ \frac{(4\,b^2d^3f^3x^3 + 12\,b^2cd^2f^3x^2 + 12\,b^2c^2df^3x - 6\,b^2d^3f^2x^2 + 4\,b^2c^3f^3 - 12\,b^2cd^2f^2x - 6\,b^2c^2df^2 + 6\,b^2d^3f}{32\,f^4}$$
$$+ \frac{(abd^3f^3x^3 + 3\,abcd^2f^3x^2 + 3\,abc^2df^3x - 3\,abd^3f^2x^2 + abc^3f^3 - 6\,abcd^2f^2x - 3\,abc^2df^2 + 6\,abd^3fx +}{f^4}$$
$$+ \frac{(abd^3f^3x^3 + 3\,abcd^2f^3x^2 + 3\,abc^2df^3x + 3\,abd^3f^2x^2 + abc^3f^3 + 6\,abcd^2f^2x + 3\,abc^2df^2 + 6\,abd^3fx +}{f^4}$$
$$- \frac{(4\,b^2d^3f^3x^3 + 12\,b^2cd^2f^3x^2 + 12\,b^2c^2df^3x + 6\,b^2d^3f^2x^2 + 4\,b^2c^3f^3 + 12\,b^2cd^2f^2x + 6\,b^2c^2df^2 + 6\,b^2d^3f}{32\,f^4}$$

input
```
integrate((d*x+c)^3*(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
1/4*a^2*d^3*x^4 - 1/8*b^2*d^3*x^4 + a^2*c*d^2*x^3 - 1/2*b^2*c*d^2*x^3 + 3/
2*a^2*c^2*d*x^2 - 3/4*b^2*c^2*d*x^2 + a^2*c^3*x - 1/2*b^2*c^3*x + 1/32*(4*
b^2*d^3*f^3*x^3 + 12*b^2*c*d^2*f^3*x^2 + 12*b^2*c^2*d*f^3*x - 6*b^2*d^3*f^
2*x^2 + 4*b^2*c^3*f^3 - 12*b^2*c*d^2*f^2*x - 6*b^2*c^2*d*f^2 + 6*b^2*d^3*f
*x + 6*b^2*c*d^2*f - 3*b^2*d^3)*e^(2*f*x + 2*e)/f^4 + (a*b*d^3*f^3*x^3 + 3
*a*b*c*d^2*f^3*x^2 + 3*a*b*c^2*d*f^3*x - 3*a*b*d^3*f^2*x^2 + a*b*c^3*f^3 -
 6*a*b*c*d^2*f^2*x - 3*a*b*c^2*d*f^2 + 6*a*b*d^3*f*x + 6*a*b*c*d^2*f - 6*a
*b*d^3)*e^(f*x + e)/f^4 + (a*b*d^3*f^3*x^3 + 3*a*b*c*d^2*f^3*x^2 + 3*a*b*c
^2*d*f^3*x + 3*a*b*d^3*f^2*x^2 + a*b*c^3*f^3 + 6*a*b*c*d^2*f^2*x + 3*a*b*c
^2*d*f^2 + 6*a*b*d^3*f*x + 6*a*b*c*d^2*f + 6*a*b*d^3)*e^(-f*x - e)/f^4 - 1
/32*(4*b^2*d^3*f^3*x^3 + 12*b^2*c*d^2*f^3*x^2 + 12*b^2*c^2*d*f^3*x + 6*b^2
*d^3*f^2*x^2 + 4*b^2*c^3*f^3 + 12*b^2*c*d^2*f^2*x + 6*b^2*c^2*d*f^2 + 6*b^
2*d^3*f*x + 6*b^2*c*d^2*f + 3*b^2*d^3)*e^(-2*f*x - 2*e)/f^4
```

Mupad [B] (verification not implemented)

Time = 2.76 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.03

$$\int (c+dx)^3(a+b\sinh(e+fx))^2\,dx = a^2\,c^3\,x-\frac{b^2\,c^3\,x}{2}+\frac{a^2\,d^3\,x^4}{4}-\frac{b^2\,d^3\,x^4}{8}+\frac{3\,a^2\,c^2\,d\,x^2}{2}+a^2\,c\,d^2\,x^3-\frac{3\,b^2\,c^2\,d\,x^2}{4}-\frac{b^2\,c\,d^2\,x^3}{2}-\frac{3\,b^2\,d^3\cosh(2\,e+2\,f\,x)}{16\,f^4}+\frac{b^2\,c^3\sinh(2\,e+2\,f\,x)}{4\,f}+\frac{2\,a\,b\,c^3\cosh(e+f\,x)}{f}-\frac{12\,a\,b\,d^3\sinh(e+f\,x)}{f^4}-\frac{3\,b^2\,d^3\,x^2\cosh(2\,e+2\,f\,x)}{8\,f^2}+\frac{b^2\,d^3\,x^3\sinh(2\,e+2\,f\,x)}{4\,f}-\frac{3\,b^2\,c^2\,d\cosh(2\,e+2\,f\,x)}{8\,f^2}+\frac{3\,b^2\,c\,d^2\sinh(2\,e+2\,f\,x)}{8\,f^3}+\frac{3\,b^2\,d^3\,x\sinh(2\,e+2\,f\,x)}{8\,f^3}-\frac{3\,b^2\,c\,d^2\,x\cosh(2\,e+2\,f\,x)}{4\,f^2}+\frac{3\,b^2\,c^2\,d\,x\sinh(2\,e+2\,f\,x)}{4\,f}+\frac{12\,a\,b\,c\,d^2\cosh(e+f\,x)}{f^3}-\frac{6\,a\,b\,c^2\,d\sinh(e+f\,x)}{f^2}+\frac{12\,a\,b\,d^3\,x\cosh(e+f\,x)}{f^3}+\frac{3\,b^2\,c\,d^2\,x^2\sinh(2\,e+2\,f\,x)}{4\,f}+\frac{2\,a\,b\,d^3\,x^3\cosh(e+f\,x)}{f}-\frac{6\,a\,b\,d^3\,x^2\sinh(e+f\,x)}{f^2}+\frac{6\,a\,b\,c\,d^2\,x^2\cosh(e+f\,x)}{f}+\frac{6\,a\,b\,c^2\,d\,x\cosh(e+f\,x)}{f}-\frac{12\,a\,b\,c\,d^2\,x\sinh(e+f\,x)}{f^2}$$

input
```
int((a + b*sinh(e + f*x))^2*(c + d*x)^3,x)
```

output
```
a^2*c^3*x - (b^2*c^3*x)/2 + (a^2*d^3*x^4)/4 - (b^2*d^3*x^4)/8 + (3*a^2*c^2
*d*x^2)/2 + a^2*c*d^2*x^3 - (3*b^2*c^2*d*x^2)/4 - (b^2*c*d^2*x^3)/2 - (3*b
^2*d^3*cosh(2*e + 2*f*x))/(16*f^4) + (b^2*c^3*sinh(2*e + 2*f*x))/(4*f) + (
2*a*b*c^3*cosh(e + f*x))/f - (12*a*b*d^3*sinh(e + f*x))/f^4 - (3*b^2*d^3*x
^2*cosh(2*e + 2*f*x))/(8*f^2) + (b^2*d^3*x^3*sinh(2*e + 2*f*x))/(4*f) - (3
*b^2*c^2*d*cosh(2*e + 2*f*x))/(8*f^2) + (3*b^2*c*d^2*sinh(2*e + 2*f*x))/(8
*f^3) + (3*b^2*d^3*x*sinh(2*e + 2*f*x))/(8*f^3) - (3*b^2*c*d^2*x*cosh(2*e
+ 2*f*x))/(4*f^2) + (3*b^2*c^2*d*x*sinh(2*e + 2*f*x))/(4*f) + (12*a*b*c*d^
2*cosh(e + f*x))/f^3 - (6*a*b*c^2*d*sinh(e + f*x))/f^2 + (12*a*b*d^3*x*cos
h(e + f*x))/f^3 + (3*b^2*c*d^2*x^2*sinh(2*e + 2*f*x))/(4*f) + (2*a*b*d^3*x
^3*cosh(e + f*x))/f - (6*a*b*d^3*x^2*sinh(e + f*x))/f^2 + (6*a*b*c*d^2*x^2
*cosh(e + f*x))/f + (6*a*b*c^2*d*x*cosh(e + f*x))/f - (12*a*b*c*d^2*x*sinh
(e + f*x))/f^2
```

Reduce [B] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 940, normalized size of antiderivative = 3.97

$$\int (c+dx)^3(a+b\sinh(e+fx))^2\,dx$$
$$= \frac{-3e^{4fx+4e}b^2d^3 - 4b^2c^3f^3 - 3b^2d^3 - 12b^2c^2d\,f^3x - 12b^2c\,d^2f^3x^2 - 12b^2c\,d^2f^2x - 6e^{4fx+4e}b^2c^2d\,f^2 + 6e^{4f}}{}$$

input
```
int((d*x+c)^3*(a+b*sinh(f*x+e))^2,x)
```

output

```
(4*e**(4*e + 4*f*x)*b**2*c**3*f**3 + 12*e**(4*e + 4*f*x)*b**2*c**2*d*f**3*
x - 6*e**(4*e + 4*f*x)*b**2*c**2*d*f**2 + 12*e**(4*e + 4*f*x)*b**2*c*d**2*
f**3*x**2 - 12*e**(4*e + 4*f*x)*b**2*c*d**2*f**2*x + 6*e**(4*e + 4*f*x)*b*
*2*c*d**2*f + 4*e**(4*e + 4*f*x)*b**2*d**3*f**3*x**3 - 6*e**(4*e + 4*f*x)*
b**2*d**3*f**2*x**2 + 6*e**(4*e + 4*f*x)*b**2*d**3*f*x - 3*e**(4*e + 4*f*x
)*b**2*d**3 + 32*e**(3*e + 3*f*x)*a*b*c**3*f**3 + 96*e**(3*e + 3*f*x)*a*b*
c**2*d*f**3*x - 96*e**(3*e + 3*f*x)*a*b*c**2*d*f**2 + 96*e**(3*e + 3*f*x)*
a*b*c*d**2*f**3*x**2 - 192*e**(3*e + 3*f*x)*a*b*c*d**2*f**2*x + 192*e**(3*
e + 3*f*x)*a*b*c*d**2*f + 32*e**(3*e + 3*f*x)*a*b*d**3*f**3*x**3 - 96*e**(
3*e + 3*f*x)*a*b*d**3*f**2*x**2 + 192*e**(3*e + 3*f*x)*a*b*d**3*f*x - 192*
e**(3*e + 3*f*x)*a*b*d**3 + 32*e**(2*e + 2*f*x)*a**2*c**3*f**4*x + 48*e**(
2*e + 2*f*x)*a**2*c**2*d*f**4*x**2 + 32*e**(2*e + 2*f*x)*a**2*c*d**2*f**4*
x**3 + 8*e**(2*e + 2*f*x)*a**2*d**3*f**4*x**4 - 16*e**(2*e + 2*f*x)*b**2*c
**3*f**4*x - 24*e**(2*e + 2*f*x)*b**2*c**2*d*f**4*x**2 - 16*e**(2*e + 2*f*
x)*b**2*c*d**2*f**4*x**3 - 4*e**(2*e + 2*f*x)*b**2*d**3*f**4*x**4 + 32*e**
(e + f*x)*a*b*c**3*f**3 + 96*e**(e + f*x)*a*b*c**2*d*f**3*x + 96*e**(e + f
*x)*a*b*c**2*d*f**2 + 96*e**(e + f*x)*a*b*c*d**2*f**3*x**2 + 192*e**(e + f
*x)*a*b*c*d**2*f**2*x + 192*e**(e + f*x)*a*b*c*d**2*f + 32*e**(e + f*x)*a*
b*d**3*f**3*x**3 + 96*e**(e + f*x)*a*b*d**3*f**2*x**2 + 192*e**(e + f*x)*a
*b*d**3*f*x + 192*e**(e + f*x)*a*b*d**3 - 4*b**2*c**3*f**3 - 12*b**2*c*...
```

3.164 $\int (c+dx)^2(a+b\sinh(e+fx))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 182

$$\int (c+dx)^2(a+b\sinh(e+fx))^2\,dx = -\frac{b^2d^2x}{4f^2}+\frac{a^2(c+dx)^3}{3d} - \frac{b^2(c+dx)^3}{6d}+\frac{4abd^2\cosh(e+fx)}{f^3} + \frac{2ab(c+dx)^2\cosh(e+fx)}{f} - \frac{4abd(c+dx)\sinh(e+fx)}{f^2} + \frac{b^2d^2\cosh(e+fx)\sinh(e+fx)}{4f^3} + \frac{b^2(c+dx)^2\cosh(e+fx)\sinh(e+fx)}{2f} - \frac{b^2d(c+dx)\sinh^2(e+fx)}{2f^2}$$

output
```
-1/4*b^2*d^2*x/f^2+1/3*a^2*(d*x+c)^3/d-1/6*b^2*(d*x+c)^3/d+4*a*b*d^2*cosh(
f*x+e)/f^3+2*a*b*(d*x+c)^2*cosh(f*x+e)/f-4*a*b*d*(d*x+c)*sinh(f*x+e)/f^2+1
/4*b^2*d^2*cosh(f*x+e)*sinh(f*x+e)/f^3+1/2*b^2*(d*x+c)^2*cosh(f*x+e)*sinh(
f*x+e)/f-1/2*b^2*d*(d*x+c)*sinh(f*x+e)^2/f^2
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.37

$$\int (c+dx)^2(a+b\sinh(e+fx))^2\,dx$$
$$= \frac{24a^2c^2f^3x - 12b^2c^2f^3x + 24a^2cdf^3x^2 - 12b^2cdf^3x^2 + 8a^2d^2f^3x^3 - 4b^2d^2f^3x^3 + 48ab(c^2f^2 + 2cdf^2x + d}{}$$

input
```
Integrate[(c + d*x)^2*(a + b*Sinh[e + f*x])^2,x]
```

output
```
(24*a^2*c^2*f^3*x - 12*b^2*c^2*f^3*x + 24*a^2*c*d*f^3*x^2 - 12*b^2*c*d*f^3
*x^2 + 8*a^2*d^2*f^3*x^3 - 4*b^2*d^2*f^3*x^3 + 48*a*b*(c^2*f^2 + 2*c*d*f^2
*x + d^2*(2 + f^2*x^2))*Cosh[e + f*x] - 6*b^2*d*f*(c + d*x)*Cosh[2*(e + f*
x)] - 96*a*b*c*d*f*Sinh[e + f*x] - 96*a*b*d^2*f*x*Sinh[e + f*x] + 3*b^2*d^
2*Sinh[2*(e + f*x)] + 6*b^2*c^2*f^2*Sinh[2*(e + f*x)] + 12*b^2*c*d*f^2*x*S
inh[2*(e + f*x)] + 6*b^2*d^2*f^2*x^2*Sinh[2*(e + f*x)])/(24*f^3)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2(a+b\sinh(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2(a-ib\sin(ie+ifx))^2dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(a^2(c+dx)^2+2ab(c+dx)^2\sinh(e+fx)+b^2(c+dx)^2\sinh^2(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(c+dx)^3}{3d} - \frac{4abd(c+dx)\sinh(e+fx)}{f^2} + \frac{2ab(c+dx)^2\cosh(e+fx)}{f} + \frac{4abd^2\cosh(e+fx)}{f^3} - \frac{b^2d(c+dx)\sinh^2(e+fx)}{2f^2} + \frac{b^2(c+dx)^2\sinh(e+fx)\cosh(e+fx)}{2f} - \frac{b^2(c+dx)^3}{6d} + \frac{b^2d^2\sinh(e+fx)\cosh(e+fx)}{4f^3} - \frac{b^2d^2x}{4f^2}$$

input
```
Int[(c + d*x)^2*(a + b*Sinh[e + f*x])^2,x]
```

output
```
-1/4*(b^2*d^2*x)/f^2 + (a^2*(c + d*x)^3)/(3*d) - (b^2*(c + d*x)^3)/(6*d) +
 (4*a*b*d^2*Cosh[e + f*x])/f^3 + (2*a*b*(c + d*x)^2*Cosh[e + f*x])/f - (4*
a*b*d*(c + d*x)*Sinh[e + f*x])/f^2 + (b^2*d^2*Cosh[e + f*x]*Sinh[e + f*x])
/(4*f^3) + (b^2*(c + d*x)^2*Cosh[e + f*x]*Sinh[e + f*x])/(2*f) - (b^2*d*(c
 + d*x)*Sinh[e + f*x]^2)/(2*f^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.86

method	result
parallelrisch	$\frac{\left((dx+c)^2 f^2+\frac{d^2}{2}\right)b^2 \sinh(2fx+2e)-b^2 df(dx+c)\cosh(2fx+2e)+8ba\left((dx+c)^2 f^2+2d^2\right)\cosh(fx+e)-16abdf(dx+c)\sinh}{4f^3}$
risch	$\frac{a^2d^2x^3}{3}-\frac{d^2b^2x^3}{6}+a^2dc\,x^2-\frac{d\,b^2c\,x^2}{2}+a^2c^2x-\frac{b^2c^2x}{2}+\frac{a^2c^3}{3d}-\frac{b^2c^3}{6d}+\frac{b^2(2d^2x^2f^2+4cd\,f^2x+2c^2}{1}$
parts	$\frac{a^2(dx+c)^3}{3d}+\frac{b^2\left(\frac{d^2\left(\frac{(fx+e)^2\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e)^3}{6}-\frac{(fx+e)\cosh(fx+e)^2}{2}+\frac{\cosh(fx+e)\sinh(fx+e)}{4}+\frac{fx}{4}+\frac{e}{4}\right)}{f^2}\right.}{}$
derivativedivides	$\frac{d^2a^2(fx+e)^3}{3f^2}+\frac{2d^2ab\left((fx+e)^2\cosh(fx+e)-2(fx+e)\sinh(fx+e)+2\cosh(fx+e)\right)}{f^2}+\frac{d^2b^2\left(\frac{(fx+e)^2\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx}{}\right.}{}$
default	$\frac{d^2a^2(fx+e)^3}{3f^2}+\frac{2d^2ab\left((fx+e)^2\cosh(fx+e)-2(fx+e)\sinh(fx+e)+2\cosh(fx+e)\right)}{f^2}+\frac{d^2b^2\left(\frac{(fx+e)^2\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx}{}\right.}{}$
orering	Expression too large to display

input
```
int((d*x+c)^2*(a+b*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(((d*x+c)^2*f^2+1/2*d^2)*b^2*sinh(2*f*x+2*e)-b^2*d*f*(d*x+c)*cosh(2*f*
x+2*e)+8*b*a*((d*x+c)^2*f^2+2*d^2)*cosh(f*x+e)-16*a*b*d*f*(d*x+c)*sinh(f*x
+e)+4*(a^2-1/2*b^2)*(1/3*x^2*d^2+c*d*x+c^2)*x*f^3+8*a*b*c^2*f^2+b^2*c*d*f+
16*a*d^2*b)/f^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.36

$$\int (c+dx)^2(a+b\sinh(e+fx))^2\,dx$$
$$=\frac{2\,(2\,a^2-b^2)d^2f^3x^3+6\,(2\,a^2-b^2)cdf^3x^2+6\,(2\,a^2-b^2)c^2f^3x-3\,(b^2d^2fx+b^2cdf)\cosh\,(fx+e)^2-3\,(b}{}$$

input
```
integrate((d*x+c)^2*(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/12*(2*(2*a^2 - b^2)*d^2*f^3*x^3 + 6*(2*a^2 - b^2)*c*d*f^3*x^2 + 6*(2*a^2
 - b^2)*c^2*f^3*x - 3*(b^2*d^2*f*x + b^2*c*d*f)*cosh(f*x + e)^2 - 3*(b^2*d
^2*f*x + b^2*c*d*f)*sinh(f*x + e)^2 + 24*(a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*
x + a*b*c^2*f^2 + 2*a*b*d^2)*cosh(f*x + e) - 3*(16*a*b*d^2*f*x + 16*a*b*c*
d*f - (2*b^2*d^2*f^2*x^2 + 4*b^2*c*d*f^2*x + 2*b^2*c^2*f^2 + b^2*d^2)*cosh
(f*x + e))*sinh(f*x + e))/f^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(177) = 354$.

Time = 0.32 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.51

$$\int (c+dx)^2(a+b\sinh(e+fx))^2\,dx$$
$$= \begin{cases} a^2c^2x + a^2cdx^2 + \frac{a^2d^2x^3}{3} + \frac{2abc^2\cosh(e+fx)}{f} + \frac{4abcdx\cosh(e+fx)}{f} - \frac{4abcd\sinh(e+fx)}{f^2} + \frac{2abd^2x^2\cosh(e+fx)}{f} - \frac{4abd^2x}{} \\ (a+b\sinh(e))^2\left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) \end{cases}$$

input
```
integrate((d*x+c)**2*(a+b*sinh(f*x+e))**2,x)
```

output
```
Piecewise((a**2*c**2*x + a**2*c*d*x**2 + a**2*d**2*x**3/3 + 2*a*b*c**2*cos
h(e + f*x)/f + 4*a*b*c*d*x*cosh(e + f*x)/f - 4*a*b*c*d*sinh(e + f*x)/f**2
+ 2*a*b*d**2*x**2*cosh(e + f*x)/f - 4*a*b*d**2*x*sinh(e + f*x)/f**2 + 4*a*
b*d**2*cosh(e + f*x)/f**3 + b**2*c**2*x*sinh(e + f*x)**2/2 - b**2*c**2*x*c
osh(e + f*x)**2/2 + b**2*c**2*sinh(e + f*x)*cosh(e + f*x)/(2*f) + b**2*c*d
*x**2*sinh(e + f*x)**2/2 - b**2*c*d*x**2*cosh(e + f*x)**2/2 + b**2*c*d*x*s
inh(e + f*x)*cosh(e + f*x)/f - b**2*c*d*sinh(e + f*x)**2/(2*f**2) + b**2*d
**2*x**3*sinh(e + f*x)**2/6 - b**2*d**2*x**3*cosh(e + f*x)**2/6 + b**2*d**
2*x**2*sinh(e + f*x)*cosh(e + f*x)/(2*f) - b**2*d**2*x*sinh(e + f*x)**2/(4
*f**2) - b**2*d**2*x*cosh(e + f*x)**2/(4*f**2) + b**2*d**2*sinh(e + f*x)*c
osh(e + f*x)/(4*f**3), Ne(f, 0)), ((a + b*sinh(e))**2*(c**2*x + c*d*x**2 +
 d**2*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.77

$$\int (c+dx)^2(a+b\sinh(e+fx))^2\,dx$$
$$= \frac{1}{3}a^2d^2x^3 + a^2cdx^2 - \frac{1}{8}\left(4x^2 - \frac{(2fxe^{(2e)} - e^{(2e)})e^{(2fx)}}{f^2} + \frac{(2fx+1)e^{(-2fx-2e)}}{f^2}\right)b^2cd$$
$$- \frac{1}{48}\left(8x^3 - \frac{3(2f^2x^2e^{(2e)} - 2fxe^{(2e)} + e^{(2e)})e^{(2fx)}}{f^3} + \frac{3(2f^2x^2 + 2fx + 1)e^{(-2fx-2e)}}{f^3}\right)b^2d^2$$
$$- \frac{1}{8}b^2c^2\left(4x - \frac{e^{(2fx+2e)}}{f} + \frac{e^{(-2fx-2e)}}{f}\right) + a^2c^2x$$
$$+ 2abcd\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right)$$
$$+ abd^2\left(\frac{(f^2x^2e^e - 2fxe^e + 2e^e)e^{(fx)}}{f^3} + \frac{(f^2x^2 + 2fx + 2)e^{(-fx-e)}}{f^3}\right)$$
$$+ \frac{2abc^2\cosh(fx+e)}{f}$$

input
```
integrate((d*x+c)^2*(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*d^2*x^3 + a^2*c*d*x^2 - 1/8*(4*x^2 - (2*f*x*e^(2*e) - e^(2*e))*e^(
2*f*x)/f^2 + (2*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*b^2*c*d - 1/48*(8*x^3 - 3*(
2*f^2*x^2*e^(2*e) - 2*f*x*e^(2*e) + e^(2*e))*e^(2*f*x)/f^3 + 3*(2*f^2*x^2
+ 2*f*x + 1)*e^(-2*f*x - 2*e)/f^3)*b^2*d^2 - 1/8*b^2*c^2*(4*x - e^(2*f*x +
 2*e)/f + e^(-2*f*x - 2*e)/f) + a^2*c^2*x + 2*a*b*c*d*((f*x*e^e - e^e)*e^(
f*x)/f^2 + (f*x + 1)*e^(-f*x - e)/f^2) + a*b*d^2*((f^2*x^2*e^e - 2*f*x*e^e
 + 2*e^e)*e^(f*x)/f^3 + (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + 2*a*b*c^
2*cosh(f*x + e)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(170) = 340$.

Time = 0.12 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.89

$$\int (c+dx)^2(a+b\sinh(e+fx))^2\,dx$$
$$= \frac{1}{3}a^2d^2x^3 - \frac{1}{6}b^2d^2x^3 + a^2cdx^2 - \frac{1}{2}b^2cdx^2 + a^2c^2x - \frac{1}{2}b^2c^2x$$
$$+ \frac{(2\,b^2d^2f^2x^2 + 4\,b^2cdf^2x + 2\,b^2c^2f^2 - 2\,b^2d^2fx - 2\,b^2cdf + b^2d^2)e^{(2\,fx+2\,e)}}{16\,f^3}$$
$$+ \frac{(abd^2f^2x^2 + 2\,abcdf^2x + abc^2f^2 - 2\,abd^2fx - 2\,abcdf + 2\,abd^2)e^{(fx+e)}}{f^3}$$
$$+ \frac{(abd^2f^2x^2 + 2\,abcdf^2x + abc^2f^2 + 2\,abd^2fx + 2\,abcdf + 2\,abd^2)e^{(-fx-e)}}{f^3}$$
$$- \frac{(2\,b^2d^2f^2x^2 + 4\,b^2cdf^2x + 2\,b^2c^2f^2 + 2\,b^2d^2fx + 2\,b^2cdf + b^2d^2)e^{(-2\,fx-2\,e)}}{16\,f^3}$$

input

```
integrate((d*x+c)^2*(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

output

```
1/3*a^2*d^2*x^3 - 1/6*b^2*d^2*x^3 + a^2*c*d*x^2 - 1/2*b^2*c*d*x^2 + a^2*c^
2*x - 1/2*b^2*c^2*x + 1/16*(2*b^2*d^2*f^2*x^2 + 4*b^2*c*d*f^2*x + 2*b^2*c^
2*f^2 - 2*b^2*d^2*f*x - 2*b^2*c*d*f + b^2*d^2)*e^(2*f*x + 2*e)/f^3 + (a*b*
d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2 - 2*a*b*d^2*f*x - 2*a*b*c*d*f
 + 2*a*b*d^2)*e^(f*x + e)/f^3 + (a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^
2*f^2 + 2*a*b*d^2*f*x + 2*a*b*c*d*f + 2*a*b*d^2)*e^(-f*x - e)/f^3 - 1/16*(
2*b^2*d^2*f^2*x^2 + 4*b^2*c*d*f^2*x + 2*b^2*c^2*f^2 + 2*b^2*d^2*f*x + 2*b^
2*c*d*f + b^2*d^2)*e^(-2*f*x - 2*e)/f^3
```

Mupad [B] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.54

$$\int (c+dx)^2(a+b\sinh(e+fx))^2\,dx = a^2\,c^2\,x - \frac{b^2\,c^2\,x}{2} + \frac{a^2\,d^2\,x^3}{3} - \frac{b^2\,d^2\,x^3}{6} + \frac{b^2\,c^2\sinh(2\,e+2\,f\,x)}{4\,f} + \frac{b^2\,d^2\sinh(2\,e+2\,f\,x)}{8\,f^3} + a^2\,c\,d\,x^2 - \frac{b^2\,c\,d\,x^2}{2} + \frac{2\,a\,b\,c^2\cosh(e+f\,x)}{f} + \frac{4\,a\,b\,d^2\cosh(e+f\,x)}{f^3} + \frac{b^2\,d^2\,x^2\sinh(2\,e+2\,f\,x)}{4\,f} - \frac{b^2\,c\,d\cosh(2\,e+2\,f\,x)}{4\,f^2} - \frac{b^2\,d^2\,x\cosh(2\,e+2\,f\,x)}{4\,f^2} - \frac{4\,a\,b\,c\,d\sinh(e+f\,x)}{f^2} - \frac{4\,a\,b\,d^2\,x\sinh(e+f\,x)}{f^2} + \frac{2\,a\,b\,d^2\,x^2\cosh(e+f\,x)}{f} + \frac{b^2\,c\,d\,x\sinh(2\,e+2\,f\,x)}{2\,f} + \frac{4\,a\,b\,c\,d\,x\cosh(e+f\,x)}{f}$$

input

```
int((a + b*sinh(e + f*x))^2*(c + d*x)^2,x)
```

output

```
a^2*c^2*x - (b^2*c^2*x)/2 + (a^2*d^2*x^3)/3 - (b^2*d^2*x^3)/6 + (b^2*c^2*s
inh(2*e + 2*f*x))/(4*f) + (b^2*d^2*sinh(2*e + 2*f*x))/(8*f^3) + a^2*c*d*x^
2 - (b^2*c*d*x^2)/2 + (2*a*b*c^2*cosh(e + f*x))/f + (4*a*b*d^2*cosh(e + f*
x))/f^3 + (b^2*d^2*x^2*sinh(2*e + 2*f*x))/(4*f) - (b^2*c*d*cosh(2*e + 2*f*
x))/(4*f^2) - (b^2*d^2*x*cosh(2*e + 2*f*x))/(4*f^2) - (4*a*b*c*d*sinh(e +
f*x))/f^2 - (4*a*b*d^2*x*sinh(e + f*x))/f^2 + (2*a*b*d^2*x^2*cosh(e + f*x)
)/f + (b^2*c*d*x*sinh(2*e + 2*f*x))/(2*f) + (4*a*b*c*d*x*cosh(e + f*x))/f
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 554, normalized size of antiderivative = 3.04

$$\int (c+dx)^2(a+b\sinh(e+fx))^2\,dx = \frac{16e^{2fx+2e}a^2d^2f^3x^3 - 24e^{2fx+2e}b^2c^2f^3x - 96e^{3fx+3e}ab\,d^2fx + 3e^{4fx+4e}b^2d^2 - 6b^2c^2f^2 + 48e^{2fx+2e}a^2cd\,f^3}{}$$

input
```
int((d*x+c)^2*(a+b*sinh(f*x+e))^2,x)
```

output
```
(6*e**(4*e + 4*f*x)*b**2*c**2*f**2 + 12*e**(4*e + 4*f*x)*b**2*c*d*f**2*x -
 6*e**(4*e + 4*f*x)*b**2*c*d*f + 6*e**(4*e + 4*f*x)*b**2*d**2*f**2*x**2 -
6*e**(4*e + 4*f*x)*b**2*d**2*f*x + 3*e**(4*e + 4*f*x)*b**2*d**2 + 48*e**(3
*e + 3*f*x)*a*b*c**2*f**2 + 96*e**(3*e + 3*f*x)*a*b*c*d*f**2*x - 96*e**(3*
e + 3*f*x)*a*b*c*d*f + 48*e**(3*e + 3*f*x)*a*b*d**2*f**2*x**2 - 96*e**(3*e
 + 3*f*x)*a*b*d**2*f*x + 96*e**(3*e + 3*f*x)*a*b*d**2 + 48*e**(2*e + 2*f*x
)*a**2*c**2*f**3*x + 48*e**(2*e + 2*f*x)*a**2*c*d*f**3*x**2 + 16*e**(2*e +
 2*f*x)*a**2*d**2*f**3*x**3 - 24*e**(2*e + 2*f*x)*b**2*c**2*f**3*x - 24*e*
*(2*e + 2*f*x)*b**2*c*d*f**3*x**2 - 8*e**(2*e + 2*f*x)*b**2*d**2*f**3*x**3
 + 48*e**(e + f*x)*a*b*c**2*f**2 + 96*e**(e + f*x)*a*b*c*d*f**2*x + 96*e**
(e + f*x)*a*b*c*d*f + 48*e**(e + f*x)*a*b*d**2*f**2*x**2 + 96*e**(e + f*x)
*a*b*d**2*f*x + 96*e**(e + f*x)*a*b*d**2 - 6*b**2*c**2*f**2 - 12*b**2*c*d*
f**2*x - 6*b**2*c*d*f - 6*b**2*d**2*f**2*x**2 - 6*b**2*d**2*f*x - 3*b**2*d
**2)/(48*e**(2*e + 2*f*x)*f**3)
```

3.165 $\int (c+dx)(a+b\sinh(e+fx))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 113

$$\int (c+dx)(a+b\sinh(e+fx))^2\,dx = \frac{a^2(c+dx)^2}{2d} - \frac{b^2(c+dx)^2}{4d} + \frac{2ab(c+dx)\cosh(e+fx)}{f} - \frac{2abd\sinh(e+fx)}{f^2} + \frac{b^2(c+dx)\cosh(e+fx)\sinh(e+fx)}{2f} - \frac{b^2d\sinh^2(e+fx)}{4f^2}$$

output
```
1/2*a^2*(d*x+c)^2/d-1/4*b^2*(d*x+c)^2/d+2*a*b*(d*x+c)*cosh(f*x+e)/f-2*a*b*
d*sinh(f*x+e)/f^2+1/2*b^2*(d*x+c)*cosh(f*x+e)*sinh(f*x+e)/f-1/4*b^2*d*sinh
(f*x+e)^2/f^2
```

Mathematica [A] (verified)

Time = 4.17 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.87

$$\int (c+dx)(a+b\sinh(e+fx))^2\,dx =$$
$$-\frac{2(2a^2-b^2)(e+fx)(-2cf+d(e-fx))-16abf(c+dx)\cosh(e+fx)+b^2d\cosh(2(e+fx))+16ab}{8f^2}$$

input
```
Integrate[(c + d*x)*(a + b*Sinh[e + f*x])^2,x]
```

output
```
-1/8*(2*(2*a^2 - b^2)*(e + f*x)*(-2*c*f + d*(e - f*x)) - 16*a*b*f*(c + d*x
)*Cosh[e + f*x] + b^2*d*Cosh[2*(e + f*x)] + 16*a*b*d*Sinh[e + f*x] - 2*b^2
*f*(c + d*x)*Sinh[2*(e + f*x)])/f^2
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(a+b\sinh(e+fx))^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (c+dx)(a-ib\sin(ie+ifx))^2dx$$
$$\Big\downarrow \text{3798}$$
$$\int \left(a^2(c+dx)+2ab(c+dx)\sinh(e+fx)+b^2(c+dx)\sinh^2(e+fx)\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{a^2(c+dx)^2}{2d}+\frac{2ab(c+dx)\cosh(e+fx)}{f}-\frac{2abd\sinh(e+fx)}{f^2}+\frac{b^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{2f}-\frac{b^2(c+dx)^2}{4d}-\frac{b^2d\sinh^2(e+fx)}{4f^2}$$

input
```
Int[(c + d*x)*(a + b*Sinh[e + f*x])^2,x]
```

output
```
(a^2*(c + d*x)^2)/(2*d) - (b^2*(c + d*x)^2)/(4*d) + (2*a*b*(c + d*x)*Cosh[
e + f*x])/f - (2*a*b*d*Sinh[e + f*x])/f^2 + (b^2*(c + d*x)*Cosh[e + f*x]*S
inh[e + f*x])/(2*f) - (b^2*d*Sinh[e + f*x]^2)/(4*f^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.98

method	result
parallelrisch	$\frac{2b^2 f(dx+c)\sinh(2fx+2e)-b^2 d\cosh(2fx+2e)+16abf(dx+c)\cosh(fx+e)-16dab\sinh(fx+e)+((-2d\,x^2-4cx)f^2+d)}{8f^2}$
risch	$\frac{a^2 d\,x^2}{2}+a^2 cx-\frac{b^2 d\,x^2}{4}-\frac{b^2 cx}{2}+\frac{b^2(2dxf+2cf-d)\mathrm{e}^{2fx+2e}}{16f^2}+\frac{ab(dxf+cf-d)\mathrm{e}^{fx+e}}{f^2}+\frac{ab(dxf+cf+d)\mathrm{e}^{-f}}{f^2}$
parts	$a^2\left(\frac{1}{2}d\,x^2+cx\right)+\frac{b^2\left(\frac{d\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e)^2}{4}-\frac{\cosh(fx+e)^2}{4}\right)}{f}-\frac{de\left(\frac{\cosh(fx+e)\sinh(fx+e)}{2}-\frac{fx}{2}\right.}{f}\right.}{f}$
derivativedivides	$\frac{\frac{d\,a^2(fx+e)^2}{2f}+\frac{2dab((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}+\frac{d\,b^2\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e)^2}{4}-\frac{\cosh(fx+e)^2}{4}\right)}{f}-\frac{de\,a^2}{}}{}$
default	$\frac{\frac{d\,a^2(fx+e)^2}{2f}+\frac{2dab((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}+\frac{d\,b^2\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e)^2}{4}-\frac{\cosh(fx+e)^2}{4}\right)}{f}-\frac{de\,a^2}{}}{}$
orering	$\frac{(2d^5f^4x^6+12c\,d^4f^4x^5+28c^2d^3f^4x^4+32c^3d^2f^4x^3+18c^4d\,f^4x^2-15d^5f^2x^4+4c^5f^4x-60c\,d^4f^2x^3-85c^2d^3f^2x^2-50c^3}{4f^4(dx+c)^4}$

input
```
int((d*x+c)*(a+b*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*b^2*f*(d*x+c)*sinh(2*f*x+2*e)-b^2*d*cosh(2*f*x+2*e)+16*a*b*f*(d*x+c
)*cosh(f*x+e)-16*d*a*b*sinh(f*x+e)+((-2*d*x^2-4*c*x)*f^2+d)*b^2+16*c*a*b*f
+8*a^2*(1/2*d*x+c)*f^2*x)/f^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.13

$$\int (c+dx)(a+b\sinh(e+fx))^2\,dx$$
$$=\frac{2\,(2\,a^2-b^2)df^2x^2+4\,(2\,a^2-b^2)cf^2x-b^2d\cosh\left(fx+e\right)^2-b^2d\sinh\left(fx+e\right)^2+16\left(abdfx+abcf\right)\cos}{8\,f^2}$$

input
```
integrate((d*x+c)*(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/8*(2*(2*a^2 - b^2)*d*f^2*x^2 + 4*(2*a^2 - b^2)*c*f^2*x - b^2*d*cosh(f*x
+ e)^2 - b^2*d*sinh(f*x + e)^2 + 16*(a*b*d*f*x + a*b*c*f)*cosh(f*x + e) -
4*(4*a*b*d - (b^2*d*f*x + b^2*c*f)*cosh(f*x + e))*sinh(f*x + e))/f^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(105) = 210$.

Time = 0.22 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.94

$$\int (c+dx)(a+b\sinh(e+fx))^2\,dx$$
$$= \begin{cases} a^2cx + \frac{a^2dx^2}{2} + \frac{2abc\cosh(e+fx)}{f} + \frac{2abdx\cosh(e+fx)}{f} - \frac{2abd\sinh(e+fx)}{f^2} + \frac{b^2cx\sinh^2(e+fx)}{2} - \frac{b^2cx\cosh^2(e+fx)}{2} + \frac{b^2c\sin \\ (a+b\sinh(e))^2\left(cx+\frac{dx^2}{2}\right) \end{cases}$$

input
```
integrate((d*x+c)*(a+b*sinh(f*x+e))**2,x)
```

output
```
Piecewise((a**2*c*x + a**2*d*x**2/2 + 2*a*b*c*cosh(e + f*x)/f + 2*a*b*d*x*
cosh(e + f*x)/f - 2*a*b*d*sinh(e + f*x)/f**2 + b**2*c*x*sinh(e + f*x)**2/2
 - b**2*c*x*cosh(e + f*x)**2/2 + b**2*c*sinh(e + f*x)*cosh(e + f*x)/(2*f)
+ b**2*d*x**2*sinh(e + f*x)**2/4 - b**2*d*x**2*cosh(e + f*x)**2/4 + b**2*d
*x*sinh(e + f*x)*cosh(e + f*x)/(2*f) - b**2*d*sinh(e + f*x)**2/(4*f**2), N
e(f, 0)), ((a + b*sinh(e))**2*(c*x + d*x**2/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.45

$$\int (c+dx)(a+b\sinh(e+fx))^2\,dx$$
$$= \frac{1}{2}a^2dx^2 - \frac{1}{16}\left(4x^2 - \frac{(2fxe^{(2e)} - e^{(2e)})e^{(2fx)}}{f^2} + \frac{(2fx+1)e^{(-2fx-2e)}}{f^2}\right)b^2d$$
$$- \frac{1}{8}b^2c\left(4x - \frac{e^{(2fx+2e)}}{f} + \frac{e^{(-2fx-2e)}}{f}\right) + a^2cx$$
$$+ abd\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + \frac{2abc\cosh(fx+e)}{f}$$

input
```
integrate((d*x+c)*(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/2*a^2*d*x^2 - 1/16*(4*x^2 - (2*f*x*e^(2*e) - e^(2*e))*e^(2*f*x)/f^2 + (2
*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*b^2*d - 1/8*b^2*c*(4*x - e^(2*f*x + 2*e)/f
 + e^(-2*f*x - 2*e)/f) + a^2*c*x + a*b*d*((f*x*e^e - e^e)*e^(f*x)/f^2 + (f
*x + 1)*e^(-f*x - e)/f^2) + 2*a*b*c*cosh(f*x + e)/f
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.41

$$\begin{aligned}\int (c+dx)(a+b\sinh(e+fx))^2\,dx &= \frac{1}{2}\,a^2dx^2 - \frac{1}{4}\,b^2dx^2 + a^2cx - \frac{1}{2}\,b^2cx \\ &\quad + \frac{(2\,b^2dfx + 2\,b^2cf - b^2d)e^{(2\,fx+2\,e)}}{16\,f^2} \\ &\quad + \frac{(abdfx + abcf - abd)e^{(fx+e)}}{f^2} \\ &\quad + \frac{(abdfx + abcf + abd)e^{(-fx-e)}}{f^2} \\ &\quad - \frac{(2\,b^2dfx + 2\,b^2cf + b^2d)e^{(-2\,fx-2\,e)}}{16\,f^2}\end{aligned}$$

input
```
integrate((d*x+c)*(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
1/2*a^2*d*x^2 - 1/4*b^2*d*x^2 + a^2*c*x - 1/2*b^2*c*x + 1/16*(2*b^2*d*f*x
+ 2*b^2*c*f - b^2*d)*e^(2*f*x + 2*e)/f^2 + (a*b*d*f*x + a*b*c*f - a*b*d)*e
^(f*x + e)/f^2 + (a*b*d*f*x + a*b*c*f + a*b*d)*e^(-f*x - e)/f^2 - 1/16*(2*
b^2*d*f*x + 2*b^2*c*f + b^2*d)*e^(-2*f*x - 2*e)/f^2
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.19

$$\int (c+dx)(a+b\sinh(e+fx))^2\,dx = \frac{a^2\,d\,x^2}{2}-\frac{b^2\,d\,x^2}{4}+a^2\,c\,x-\frac{b^2\,c\,x}{2}-\frac{b^2\,d\cosh(e+f\,x)^2}{4\,f^2}+\frac{b^2\,c\cosh(e+f\,x)\,\sinh(e+f\,x)}{2\,f}+\frac{2\,a\,b\,c\cosh(e+f\,x)}{f}-\frac{2\,a\,b\,d\sinh(e+f\,x)}{f^2}+\frac{2\,a\,b\,d\,x\cosh(e+f\,x)}{f}+\frac{b^2\,d\,x\cosh(e+f\,x)\,\sinh(e+f\,x)}{2\,f}$$

input
```
int((a + b*sinh(e + f*x))^2*(c + d*x),x)
```

output
```
(a^2*d*x^2)/2 - (b^2*d*x^2)/4 + a^2*c*x - (b^2*c*x)/2 - (b^2*d*cosh(e + f*
x)^2)/(4*f^2) + (b^2*c*cosh(e + f*x)*sinh(e + f*x))/(2*f) + (2*a*b*c*cosh(
e + f*x))/f - (2*a*b*d*sinh(e + f*x))/f^2 + (2*a*b*d*x*cosh(e + f*x))/f +
(b^2*d*x*cosh(e + f*x)*sinh(e + f*x))/(2*f)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.31

$$\int (c+dx)(a+b\sinh(e+fx))^2\,dx = \frac{2e^{4fx+4e}b^2cf+2e^{4fx+4e}b^2dfx-e^{4fx+4e}b^2d+16e^{3fx+3e}abcf+16e^{3fx+3e}abdfx-16e^{3fx+3e}abd+16e^{2fx+}}{}$$

input
```
int((d*x+c)*(a+b*sinh(f*x+e))^2,x)
```

output

```
(2*e**(4*e + 4*f*x)*b**2*c*f + 2*e**(4*e + 4*f*x)*b**2*d*f*x - e**(4*e + 4
*f*x)*b**2*d + 16*e**(3*e + 3*f*x)*a*b*c*f + 16*e**(3*e + 3*f*x)*a*b*d*f*x
 - 16*e**(3*e + 3*f*x)*a*b*d + 16*e**(2*e + 2*f*x)*a**2*c*f**2*x + 8*e**(2
*e + 2*f*x)*a**2*d*f**2*x**2 - 8*e**(2*e + 2*f*x)*b**2*c*f**2*x - 4*e**(2*
e + 2*f*x)*b**2*d*f**2*x**2 + 16*e**(e + f*x)*a*b*c*f + 16*e**(e + f*x)*a*
b*d*f*x + 16*e**(e + f*x)*a*b*d - 2*b**2*c*f - 2*b**2*d*f*x - b**2*d)/(16*
e**(2*e + 2*f*x)*f**2)
```

3.166 $\int \frac{(a+b\sinh(e+fx))^2}{c+dx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 156

$$\int \frac{(a+b\sinh(e+fx))^2}{c+dx}\,dx = \frac{b^2\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Chi}\left(\frac{2cf}{d}+2fx\right)}{2d}+\frac{a^2\log(c+dx)}{d} - \frac{b^2\log(c+dx)}{2d}+\frac{2ab\mathrm{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d} + \frac{2ab\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(\frac{cf}{d}+fx\right)}{d} + \frac{b^2\sinh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(\frac{2cf}{d}+2fx\right)}{2d}$$

output
```
1/2*b^2*cosh(-2*e+2*c*f/d)*Chi(2*c*f/d+2*f*x)/d+a^2*ln(d*x+c)/d-1/2*b^2*ln
(d*x+c)/d-2*a*b*Chi(c*f/d+f*x)*sinh(-e+c*f/d)/d+2*a*b*cosh(-e+c*f/d)*Shi(c
*f/d+f*x)/d-1/2*b^2*sinh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/d
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sinh(e+fx))^2}{c+dx}\,dx$$
$$=\frac{b^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2f(c+dx)}{d}\right)+2a^2\log(c+dx)-b^2\log(c+dx)+4ab\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)+}{2d}$$

input
```
Integrate[(a + b*Sinh[e + f*x])^2/(c + d*x),x]
```

output
```
(b^2*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*f*(c + d*x))/d] + 2*a^2*Log[c +
 d*x] - b^2*Log[c + d*x] + 4*a*b*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/
d] + 4*a*b*Cosh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)] + b^2*Sinh[2*e - (2
*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d])/(2*d)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sinh(e+fx))^2}{c+dx}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a-ib\sin(ie+ifx))^2}{c+dx}dx$$
$$\Big\downarrow \text{3798}$$
$$\int \left(\frac{a^2}{c+dx}+\frac{2ab\sinh(e+fx)}{c+dx}+\frac{b^2\sinh^2(e+fx)}{c+dx}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{a^2\log(c+dx)}{d}+\frac{2ab\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d}+\frac{2ab\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d}+\frac{b^2\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{2d}+\frac{b^2\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{2d}-\frac{b^2\log(c+dx)}{2d}$$

input
```
Int[(a + b*Sinh[e + f*x])^2/(c + d*x),x]
```

output
```
(b^2*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(2*d) + (a^2*L
og[c + d*x])/d - (b^2*Log[c + d*x])/(2*d) + (2*a*b*CoshIntegral[(c*f)/d +
f*x]*Sinh[e - (c*f)/d])/d + (2*a*b*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d
+ f*x])/d + (b^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(2
*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.29

method	result
risch	$-\frac{ab\,e^{-\frac{cf-de}{d}}\,\text{expIntegral}_1\left(-fx-e-\frac{cf-de}{d}\right)}{d}+\frac{a^2\ln(dx+c)}{d}-\frac{b^2\ln(dx+c)}{2d}-\frac{b^2 e^{\frac{2cf-2de}{d}}\,\text{expIntegral}_1\left(2fx+2e+\frac{2cf-2de}{d}\right)}{4d}$

input
```
int((a+b*sinh(f*x+e))^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-a*b/d*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)+a^2*ln(d*x+c)/d-1/2*b^2*
ln(d*x+c)/d-1/4*b^2/d*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)-1/4
*b^2/d*exp(-2*(c*f-d*e)/d)*Ei(1,-2*f*x-2*e-2*(c*f-d*e)/d)+a*b/d*exp((c*f-d
*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.49

$$\int \frac{(a+b\sinh(e+fx))^2}{c+dx}\,dx = \frac{4\left(ab\text{Ei}\left(\frac{dfx+cf}{d}\right)-ab\text{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{de-cf}{d}\right)+\left(b^2\text{Ei}\left(\frac{2\,(dfx+cf)}{d}\right)+b^2\text{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)\right)\cosh\left(-\frac{2\,(de-}{d}\right.}{}$$

input
```
integrate((a+b*sinh(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

output
```
1/4*(4*(a*b*Ei((d*f*x + c*f)/d) - a*b*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c
*f)/d) + (b^2*Ei(2*(d*f*x + c*f)/d) + b^2*Ei(-2*(d*f*x + c*f)/d))*cosh(-2*
(d*e - c*f)/d) + 2*(2*a^2 - b^2)*log(d*x + c) - 4*(a*b*Ei((d*f*x + c*f)/d)
 + a*b*Ei(-(d*f*x + c*f)/d))*sinh(-(d*e - c*f)/d) - (b^2*Ei(2*(d*f*x + c*f
)/d) - b^2*Ei(-2*(d*f*x + c*f)/d))*sinh(-2*(d*e - c*f)/d))/d
```

Sympy [F]

$$\int \frac{(a+b\sinh(e+fx))^2}{c+dx}\,dx = \int \frac{(a+b\sinh{(e+fx)})^2}{c+dx}\,dx$$

input
```
integrate((a+b*sinh(f*x+e))**2/(d*x+c),x)
```

output
```
Integral((a + b*sinh(e + f*x))**2/(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

$$\int \frac{(a+b\sinh(e+fx))^2}{c+dx}\,dx$$
$$= -\frac{1}{4}b^2\left(\frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_1\left(\frac{2(dx+c)f}{d}\right)}{d}+\frac{e^{\left(2e-\frac{2cf}{d}\right)}E_1\left(-\frac{2(dx+c)f}{d}\right)}{d}+\frac{2\log(dx+c)}{d}\right)$$
$$+ab\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_1\left(\frac{(dx+c)f}{d}\right)}{d}-\frac{e^{\left(e-\frac{cf}{d}\right)}E_1\left(-\frac{(dx+c)f}{d}\right)}{d}\right)+\frac{a^2\log(dx+c)}{d}$$

input
```
integrate((a+b*sinh(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

output
```
-1/4*b^2*(e^(-2*e + 2*c*f/d)*exp_integral_e(1, 2*(d*x + c)*f/d)/d + e^(2*e
 - 2*c*f/d)*exp_integral_e(1, -2*(d*x + c)*f/d)/d + 2*log(d*x + c)/d) + a*
b*(e^(-e + c*f/d)*exp_integral_e(1, (d*x + c)*f/d)/d - e^(e - c*f/d)*exp_i
ntegral_e(1, -(d*x + c)*f/d)/d) + a^2*log(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sinh(e+fx))^2}{c+dx}\,dx$$
$$=\frac{b^2\mathrm{Ei}\left(\frac{2(dfx+cf)}{d}\right)e^{\left(2e-\frac{2cf}{d}\right)}+4ab\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}-4ab\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)}+b^2\mathrm{Ei}\left(-\frac{2(dfx+cf)}{d}\right)e^{(-2}}{4d}$$

input
```
integrate((a+b*sinh(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

output
```
1/4*(b^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) + 4*a*b*Ei((d*f*x + c*f)/
d)*e^(e - c*f/d) - 4*a*b*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + b^2*Ei(-2*(
d*f*x + c*f)/d)*e^(-2*e + 2*c*f/d) + 4*a^2*log(d*x + c) - 2*b^2*log(d*x +
c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sinh(e+fx))^2}{c+dx}\,dx = \int \frac{\left(a+b\sinh(e+f\,x)\right)^2}{c+d\,x}\,dx$$

input
```
int((a + b*sinh(e + f*x))^2/(c + d*x),x)
```

output
```
int((a + b*sinh(e + f*x))^2/(c + d*x), x)
```

Reduce [F]

$$\int \frac{(a+b\sinh(e+fx))^2}{c+dx}\,dx = \frac{\left(\int \frac{\sinh(fx+e)^2}{dx+c}dx\right)b^2d+2\left(\int \frac{\sinh(fx+e)}{dx+c}dx\right)abd+\log(dx+c)\,a^2}{d}$$

input
```
int((a+b*sinh(f*x+e))^2/(d*x+c),x)
```

output
```
(int(sinh(e + f*x)**2/(c + d*x),x)*b**2*d + 2*int(sinh(e + f*x)/(c + d*x),
x)*a*b*d + log(c + d*x)*a**2)/d
```

3.167 $\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 183

$$\begin{aligned}\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^2} dx = & -\frac{a^2}{d(c+dx)}+\frac{2abf\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{d^2} \\ & +\frac{b^2f\text{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d^2}-\frac{2ab\sinh(e+fx)}{d(c+dx)} \\ & -\frac{b^2\sinh^2(e+fx)}{d(c+dx)}+\frac{2abf\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{d^2} \\ & +\frac{b^2f\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2cf}{d}+2fx\right)}{d^2}\end{aligned}$$

output
```
-a^2/d/(d*x+c)+2*a*b*f*cosh(-e+c*f/d)*Chi(c*f/d+f*x)/d^2-b^2*f*Chi(2*c*f/d
+2*f*x)*sinh(-2*e+2*c*f/d)/d^2-2*a*b*sinh(f*x+e)/d/(d*x+c)-b^2*sinh(f*x+e)
^2/d/(d*x+c)-2*a*b*f*sinh(-e+c*f/d)*Shi(c*f/d+f*x)/d^2+b^2*f*cosh(-2*e+2*c
*f/d)*Shi(2*c*f/d+2*f*x)/d^2
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.27

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^2}\,dx = \frac{-2a^2d+b^2d-b^2d\cosh(2(e+fx))+4abf(c+dx)\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)+2b^2f(c+dx)\text{Chi}\left(\frac{2f}{}\right.}{}$$

input
```
Integrate[(a + b*Sinh[e + f*x])^2/(c + d*x)^2,x]
```

output
```
(-2*a^2*d + b^2*d - b^2*d*Cosh[2*(e + f*x)] + 4*a*b*f*(c + d*x)*Cosh[e - (
c*f)/d]*CoshIntegral[f*(c/d + x)] + 2*b^2*f*(c + d*x)*CoshIntegral[(2*f*(c
 + d*x))/d]*Sinh[2*e - (2*c*f)/d] - 4*a*b*d*Sinh[e + f*x] + 4*a*b*c*f*Sinh
[e - (c*f)/d]*SinhIntegral[f*(c/d + x)] + 4*a*b*d*f*x*Sinh[e - (c*f)/d]*Si
nhIntegral[f*(c/d + x)] + 2*b^2*c*f*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*
f*(c + d*x))/d] + 2*b^2*d*f*x*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c +
 d*x))/d])/(2*d^2*(c + d*x))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a-ib\sin(ie+ifx))^2}{(c+dx)^2}dx$$

$$\Big\downarrow \; 3798$$

$$\int\left(\frac{a^2}{(c+dx)^2}+\frac{2ab\sinh(e+fx)}{(c+dx)^2}+\frac{b^2\sinh^2(e+fx)}{(c+dx)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{d(c+dx)}+\frac{2abf\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{2abf\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{2ab\sinh(e+fx)}{d(c+dx)}+\frac{b^2f\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d^2}+\frac{b^2f\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d^2}-\frac{b^2\sinh^2(e+fx)}{d(c+dx)}$$

input
```
Int[(a + b*Sinh[e + f*x])^2/(c + d*x)^2,x]
```

output
```
-(a^2/(d*(c + d*x))) + (2*a*b*f*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f
*x])/d^2 + (b^2*f*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/d
^2 - (2*a*b*Sinh[e + f*x])/(d*(c + d*x)) - (b^2*Sinh[e + f*x]^2)/(d*(c + d
*x)) + (2*a*b*f*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d^2 + (b^2*
f*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.74

method	result
risch	$-\frac{fab\,\mathrm{e}^{fx+e}}{d^2\left(\frac{cf}{d}+fx\right)}-\frac{fab\,\mathrm{e}^{-\frac{cf-de}{d}}\,\mathrm{expIntegral}_1\left(-fx-e-\frac{cf-de}{d}\right)}{d^2}-\frac{a^2}{d(dx+c)}+\frac{b^2}{2(dx+c)d}-\frac{f\,b^2\mathrm{e}^{-2fx-2e}}{4d(dxf+cf)}+\frac{f\,b^2\mathrm{e}^{\frac{2cf-2de}{d}}\,\mathrm{e}}{}$

input
```
int((a+b*sinh(f*x+e))^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/d^2*f*a*b*exp(f*x+e)/(c*f/d+f*x)-1/d^2*f*a*b*exp(-(c*f-d*e)/d)*Ei(1,-f*
x-e-(c*f-d*e)/d)-a^2/d/(d*x+c)+1/2*b^2/(d*x+c)/d-1/4*f*b^2*exp(-2*f*x-2*e)
/d/(d*f*x+c*f)+1/2*f*b^2/d^2*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)
/d)-1/4*b^2*f/d^2*exp(2*f*x+2*e)/(c*f/d+f*x)-1/2*b^2*f/d^2*exp(-2*(c*f-d*e
)/d)*Ei(1,-2*f*x-2*e-2*(c*f-d*e)/d)+f*a*b*exp(-f*x-e)/d/(d*f*x+c*f)-f*a*b/
d^2*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.95

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^2}\,dx =$$

$$-\frac{b^2d\cosh\left(fx+e\right)^2+b^2d\sinh\left(fx+e\right)^2+4\,abd\sinh\left(fx+e\right)+(2\,a^2-b^2)d-2\left((abdfx+abcf)\mathrm{Ei}\left(\frac{df}{}\right.\right.}{}$$

input
```
integrate((a+b*sinh(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(b^2*d*cosh(f*x + e)^2 + b^2*d*sinh(f*x + e)^2 + 4*a*b*d*sinh(f*x + e
) + (2*a^2 - b^2)*d - 2*((a*b*d*f*x + a*b*c*f)*Ei((d*f*x + c*f)/d) + (a*b*
d*f*x + a*b*c*f)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) - ((b^2*d*f*x
+ b^2*c*f)*Ei(2*(d*f*x + c*f)/d) - (b^2*d*f*x + b^2*c*f)*Ei(-2*(d*f*x + c*
f)/d))*cosh(-2*(d*e - c*f)/d) + 2*((a*b*d*f*x + a*b*c*f)*Ei((d*f*x + c*f)/
d) - (a*b*d*f*x + a*b*c*f)*Ei(-(d*f*x + c*f)/d))*sinh(-(d*e - c*f)/d) + ((
b^2*d*f*x + b^2*c*f)*Ei(2*(d*f*x + c*f)/d) + (b^2*d*f*x + b^2*c*f)*Ei(-2*(
d*f*x + c*f)/d))*sinh(-2*(d*e - c*f)/d))/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^2}\,dx=\int \frac{(a+b\sinh{(e+fx)})^2}{(c+dx)^2}\,dx$$

input
```
integrate((a+b*sinh(f*x+e))**2/(d*x+c)**2,x)
```

output
```
Integral((a + b*sinh(e + f*x))**2/(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.99

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^2}\,dx = -\frac{1}{4}b^2\left(\frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_2\left(\frac{2(dx+c)f}{d}\right)}{(dx+c)d}+\frac{e^{\left(2e-\frac{2cf}{d}\right)}E_2\left(-\frac{2(dx+c)f}{d}\right)}{(dx+c)d}-\frac{2}{d^2x+cd}\right) + ab\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d}-\frac{e^{\left(e-\frac{cf}{d}\right)}E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d}\right)-\frac{a^2}{d^2x+cd}$$

input
```
integrate((a+b*sinh(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/4*b^2*(e^(-2*e + 2*c*f/d)*exp_integral_e(2, 2*(d*x + c)*f/d)/((d*x + c)
*d) + e^(2*e - 2*c*f/d)*exp_integral_e(2, -2*(d*x + c)*f/d)/((d*x + c)*d)
- 2/(d^2*x + c*d)) + a*b*(e^(-e + c*f/d)*exp_integral_e(2, (d*x + c)*f/d)/
((d*x + c)*d) - e^(e - c*f/d)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c)
*d)) - a^2/(d^2*x + c*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1135 vs. $2(186) = 372$.

Time = 0.19 (sec) , antiderivative size = 1135, normalized size of antiderivative = 6.20

$$\int \frac{(a + b \sinh(e + fx))^2}{(c + dx)^2} \, dx = \text{Too large to display}$$

input
```
integrate((a+b*sinh(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/4*(2*(d*x + c)*b^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(2*((d*x +
c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(2*(d*e - c*f)/d)
 - 2*b^2*d*e*f^2*Ei(2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e
 + c*f)/d)*e^(2*(d*e - c*f)/d) + 2*b^2*c*f^3*Ei(2*((d*x + c)*(d*e/(d*x + c
) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(2*(d*e - c*f)/d) + 4*(d*x + c)*a
*b*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(((d*x + c)*(d*e/(d*x + c) -
c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e - c*f)/d) - 4*a*b*d*e*f^2*Ei(((
d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e - c*f
)/d) + 4*a*b*c*f^3*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e
 + c*f)/d)*e^((d*e - c*f)/d) + 4*(d*x + c)*a*b*(d*e/(d*x + c) - c*f/(d*x +
 c) + f)*f^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*
f)/d)*e^(-(d*e - c*f)/d) - 4*a*b*d*e*f^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c
*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(d*e - c*f)/d) + 4*a*b*c*f^3*Ei(-((d
*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(d*e - c*f
)/d) - 2*(d*x + c)*b^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(-2*((d*x
 + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f
)/d) + 2*b^2*d*e*f^2*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)
- d*e + c*f)/d)*e^(-2*(d*e - c*f)/d) - 2*b^2*c*f^3*Ei(-2*((d*x + c)*(d*e/(
d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d) - b^2*d
*f^2*e^(2*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d) - 4*a*b*d*f^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^2}\,dx=\int \frac{(a+b\sinh(e+f\,x))^2}{(c+d\,x)^2}\,dx$$

input
```
int((a + b*sinh(e + f*x))^2/(c + d*x)^2,x)
```

output
```
int((a + b*sinh(e + f*x))^2/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^2}\,dx = \frac{e^{3e}\left(\int \frac{e^{2fx}}{d^2x^2+2cdx+c^2}dx\right)b^2c^2+e^{3e}\left(\int \frac{e^{2fx}}{d^2x^2+2cdx+c^2}dx\right)b^2cdx+4e^{2e}\left(\int \frac{e^{fx}}{d^2x^2+2cdx+c^2}dx\right)ab\,c^2+4e^{2e}\left(\int \frac{e}{d^2x^2+}\right.}{}$$

input
```
int((a+b*sinh(f*x+e))^2/(d*x+c)^2,x)
```

output
```
(e**(3*e)*int(e**(2*f*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*b**2*c**2 + e**(3
*e)*int(e**(2*f*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*b**2*c*d*x + 4*e**(2*e)
*int(e**(f*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a*b*c**2 + 4*e**(2*e)*int(e*
*(f*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a*b*c*d*x + e**e*int(1/(e**(2*e + 2
*f*x)*c**2 + 2*e**(2*e + 2*f*x)*c*d*x + e**(2*e + 2*f*x)*d**2*x**2),x)*b**
2*c**2 + e**e*int(1/(e**(2*e + 2*f*x)*c**2 + 2*e**(2*e + 2*f*x)*c*d*x + e*
*(2*e + 2*f*x)*d**2*x**2),x)*b**2*c*d*x + 4*e**e*a**2*x - 2*e**e*b**2*x -
4*int(1/(e**(f*x)*c**2 + 2*e**(f*x)*c*d*x + e**(f*x)*d**2*x**2),x)*a*b*c**
2 - 4*int(1/(e**(f*x)*c**2 + 2*e**(f*x)*c*d*x + e**(f*x)*d**2*x**2),x)*a*b
*c*d*x)/(4*e**e*c*(c + d*x))
```

3.168 $\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 242

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^3}\,dx = -\frac{a^2}{2d(c+dx)^2}-\frac{abf\cosh(e+fx)}{d^2(c+dx)}+\frac{b^2f^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{d^3}+\frac{abf^2\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d^3}-\frac{ab\sinh(e+fx)}{d(c+dx)^2}-\frac{b^2f\cosh(e+fx)\sinh(e+fx)}{d^2(c+dx)}-\frac{b^2\sinh^2(e+fx)}{2d(c+dx)^2}+\frac{abf^2\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{d^3}+\frac{b^2f^2\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2cf}{d}+2fx\right)}{d^3}$$

output
```
-1/2*a^2/d/(d*x+c)^2-a*b*f*cosh(f*x+e)/d^2/(d*x+c)+b^2*f^2*cosh(-2*e+2*c*f
/d)*Chi(2*c*f/d+2*f*x)/d^3-a*b*f^2*Chi(c*f/d+f*x)*sinh(-e+c*f/d)/d^3-a*b*s
inh(f*x+e)/d/(d*x+c)^2-b^2*f*cosh(f*x+e)*sinh(f*x+e)/d^2/(d*x+c)-1/2*b^2*s
inh(f*x+e)^2/d/(d*x+c)^2+a*b*f^2*cosh(-e+c*f/d)*Shi(c*f/d+f*x)/d^3-b^2*f^2
*sinh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/d^3
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.63

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^3}\,dx$$
$$=\frac{-2a^2d^2+b^2d^2-4abcdf\cosh(e+fx)-4abd^2fx\cosh(e+fx)-b^2d^2\cosh(2(e+fx))+4b^2f^2(c+dx)^2}{}$$

input
```
Integrate[(a + b*Sinh[e + f*x])^2/(c + d*x)^3,x]
```

output
```
(-2*a^2*d^2 + b^2*d^2 - 4*a*b*c*d*f*Cosh[e + f*x] - 4*a*b*d^2*f*x*Cosh[e +
 f*x] - b^2*d^2*Cosh[2*(e + f*x)] + 4*b^2*f^2*(c + d*x)^2*Cosh[2*e - (2*c*
f)/d]*CoshIntegral[(2*f*(c + d*x))/d] + 4*a*b*f^2*(c + d*x)^2*CoshIntegral
[f*(c/d + x)]*Sinh[e - (c*f)/d] - 4*a*b*d^2*Sinh[e + f*x] - 2*b^2*c*d*f*Si
nh[2*(e + f*x)] - 2*b^2*d^2*f*x*Sinh[2*(e + f*x)] + 4*a*b*c^2*f^2*Cosh[e -
 (c*f)/d]*SinhIntegral[f*(c/d + x)] + 8*a*b*c*d*f^2*x*Cosh[e - (c*f)/d]*Si
nhIntegral[f*(c/d + x)] + 4*a*b*d^2*f^2*x^2*Cosh[e - (c*f)/d]*SinhIntegral
[f*(c/d + x)] + 4*b^2*c^2*f^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c +
 d*x))/d] + 8*b^2*c*d*f^2*x*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d
*x))/d] + 4*b^2*d^2*f^2*x^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d
*x))/d])/(4*d^3*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^3}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{(a - ib\sin(ie + ifx))^2}{(c + dx)^3} dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(\frac{a^2}{(c + dx)^3} + \frac{2ab\sinh(e + fx)}{(c + dx)^3} + \frac{b^2\sinh^2(e + fx)}{(c + dx)^3}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{2d(c + dx)^2} + \frac{abf^2\text{Chi}\left(xf + \frac{cf}{d}\right)\sinh\left(e - \frac{cf}{d}\right)}{d^3} + \frac{abf^2\cosh\left(e - \frac{cf}{d}\right)\text{Shi}\left(xf + \frac{cf}{d}\right)}{d^3} - \frac{abf\cosh(e + fx)}{d^2(c + dx)} - \frac{ab\sinh(e + fx)}{d(c + dx)^2} + \frac{b^2f^2\text{Chi}\left(2xf + \frac{2cf}{d}\right)\cosh\left(2e - \frac{2cf}{d}\right)}{d^3} + \frac{b^2f^2\sinh\left(2e - \frac{2cf}{d}\right)\text{Shi}\left(2xf + \frac{2cf}{d}\right)}{d^3} - \frac{b^2f\sinh(e + fx)\cosh(e + fx)}{d^2(c + dx)} - \frac{b^2\sinh^2(e + fx)}{2d(c + dx)^2}$$

input
```
Int[(a + b*Sinh[e + f*x])^2/(c + d*x)^3,x]
```

output
```
-1/2*a^2/(d*(c + d*x)^2) - (a*b*f*Cosh[e + f*x])/(d^2*(c + d*x)) + (b^2*f^
2*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/d^3 + (a*b*f^2*Co
shIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d^3 - (a*b*Sinh[e + f*x])/(d*
(c + d*x)^2) - (b^2*f*Cosh[e + f*x]*Sinh[e + f*x])/(d^2*(c + d*x)) - (b^2*
Sinh[e + f*x]^2)/(2*d*(c + d*x)^2) + (a*b*f^2*Cosh[e - (c*f)/d]*SinhIntegr
al[(c*f)/d + f*x])/d^3 + (b^2*f^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*
f)/d + 2*f*x])/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(242) = 484$.

Time = 1.18 (sec) , antiderivative size = 626, normalized size of antiderivative = 2.59

method	result
risch	$-\frac{f^{2} a b \,\mathrm{e}^{f x +e}}{2 d^{3} \left(\frac{c f}{d}+f x \right)^{2}}-\frac{f^{2} a b \,\mathrm{e}^{f x +e}}{2 d^{3} \left(\frac{c f}{d}+f x \right)}-\frac{f^{2} a b \,\mathrm{e}^{-\frac{c f -d e}{d}} \operatorname{expIntegral}_{1}\left(-f x -e -\frac{c f -d e}{d}\right)}{2 d^{3}}-\frac{a^{2}}{2 d \left(d x +c \right)^{2}}+\frac{b^{2}}{4 \left(d x +c \right)^{2} d}+\frac{f^{3} \cdots}{4 d \left(d^{2} x^{2} \cdots\right.}$

input
```
int((a+b*sinh(f*x+e))^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/d^3*f^2*a*b*exp(f*x+e)/(c*f/d+f*x)^2-1/2/d^3*f^2*a*b*exp(f*x+e)/(c*f/
d+f*x)-1/2/d^3*f^2*a*b*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)-1/2*a^2/
d/(d*x+c)^2+1/4*b^2/(d*x+c)^2/d+1/4*f^3*b^2*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2
+2*c*d*f^2*x+c^2*f^2)*x+1/4*f^3*b^2*exp(-2*f*x-2*e)/d^2/(d^2*f^2*x^2+2*c*d
*f^2*x+c^2*f^2)*c-1/8*f^2*b^2*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c
^2*f^2)-1/2*f^2*b^2/d^3*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)-1
/8*f^2*b^2/d^3*exp(2*f*x+2*e)/(c*f/d+f*x)^2-1/4*f^2*b^2/d^3*exp(2*f*x+2*e)
/(c*f/d+f*x)-1/2*f^2*b^2/d^3*exp(-2*(c*f-d*e)/d)*Ei(1,-2*f*x-2*e-2*(c*f-d*
e)/d)-1/2*f^3*a*b*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*x-1/2*f^
3*a*b*exp(-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c+1/2*f^2*a*b*exp(
-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)+1/2*f^2*a*b/d^3*exp((c*f-d*e)/
d)*Ei(1,f*x+e+(c*f-d*e)/d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(242) = 484$.

Time = 0.12 (sec) , antiderivative size = 590, normalized size of antiderivative = 2.44

$$\int \frac{(a + b\sinh(e + fx))^2}{(c + dx)^3}\, dx =$$
$$-\frac{b^2d^2\cosh\left(fx+e\right)^2 + b^2d^2\sinh\left(fx+e\right)^2 + \left(2\,a^2-b^2\right)d^2 + 4\left(abd^2fx + abcdf\right)\cosh\left(fx+e\right) - 2\left((ab\right.}{}$$

input
```
integrate((a+b*sinh(f*x+e))^2/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(b^2*d^2*cosh(f*x + e)^2 + b^2*d^2*sinh(f*x + e)^2 + (2*a^2 - b^2)*d^
2 + 4*(a*b*d^2*f*x + a*b*c*d*f)*cosh(f*x + e) - 2*((a*b*d^2*f^2*x^2 + 2*a*
b*c*d*f^2*x + a*b*c^2*f^2)*Ei((d*f*x + c*f)/d) - (a*b*d^2*f^2*x^2 + 2*a*b*
c*d*f^2*x + a*b*c^2*f^2)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) - 2*((
b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2*c^2*f^2)*Ei(2*(d*f*x + c*f)/d) + (
b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2*c^2*f^2)*Ei(-2*(d*f*x + c*f)/d))*c
osh(-2*(d*e - c*f)/d) + 4*(a*b*d^2 + (b^2*d^2*f*x + b^2*c*d*f)*cosh(f*x +
e))*sinh(f*x + e) + 2*((a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2)*E
i((d*f*x + c*f)/d) + (a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2)*Ei(
-(d*f*x + c*f)/d))*sinh(-(d*e - c*f)/d) + 2*((b^2*d^2*f^2*x^2 + 2*b^2*c*d*
f^2*x + b^2*c^2*f^2)*Ei(2*(d*f*x + c*f)/d) - (b^2*d^2*f^2*x^2 + 2*b^2*c*d*
f^2*x + b^2*c^2*f^2)*Ei(-2*(d*f*x + c*f)/d))*sinh(-2*(d*e - c*f)/d))/(d^5*
x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{(a + b\sinh(e + fx))^2}{(c + dx)^3}\, dx = \int \frac{\left(a + b\sinh\left(e + fx\right)\right)^2}{\left(c + dx\right)^3}\, dx$$

input
```
integrate((a+b*sinh(f*x+e))**2/(d*x+c)**3,x)
```

output
```
Integral((a + b*sinh(e + f*x))**2/(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^3}\,dx$$
$$=\frac{1}{4}b^2\left(\frac{1}{d^3x^2+2\,cd^2x+c^2d}-\frac{e^{\left(-2\,e+\frac{2\,cf}{d}\right)}E_3\left(\frac{2\,(dx+c)f}{d}\right)}{(dx+c)^2d}-\frac{e^{\left(2\,e-\frac{2\,cf}{d}\right)}E_3\left(-\frac{2\,(dx+c)f}{d}\right)}{(dx+c)^2d}\right)$$
$$+ab\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}-\frac{e^{\left(e-\frac{cf}{d}\right)}E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}\right)-\frac{a^2}{2\,(d^3x^2+2\,cd^2x+c^2d)}$$

input
```
integrate((a+b*sinh(f*x+e))^2/(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/4*b^2*(1/(d^3*x^2 + 2*c*d^2*x + c^2*d) - e^(-2*e + 2*c*f/d)*exp_integral
_e(3, 2*(d*x + c)*f/d)/((d*x + c)^2*d) - e^(2*e - 2*c*f/d)*exp_integral_e(
3, -2*(d*x + c)*f/d)/((d*x + c)^2*d)) + a*b*(e^(-e + c*f/d)*exp_integral_e
(3, (d*x + c)*f/d)/((d*x + c)^2*d) - e^(e - c*f/d)*exp_integral_e(3, -(d*x
 + c)*f/d)/((d*x + c)^2*d)) - 1/2*a^2/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 678 vs. 2(242) = 484.

Time = 0.13 (sec) , antiderivative size = 678, normalized size of antiderivative = 2.80

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sinh(f*x+e))^2/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/8*(4*b^2*d^2*f^2*x^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) + 4*a*b*d^2
*f^2*x^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) - 4*a*b*d^2*f^2*x^2*Ei(-(d*f*x
+ c*f)/d)*e^(-e + c*f/d) + 4*b^2*d^2*f^2*x^2*Ei(-2*(d*f*x + c*f)/d)*e^(-2*
e + 2*c*f/d) + 8*b^2*c*d*f^2*x*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) + 8
*a*b*c*d*f^2*x*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) - 8*a*b*c*d*f^2*x*Ei(-(d*
f*x + c*f)/d)*e^(-e + c*f/d) + 8*b^2*c*d*f^2*x*Ei(-2*(d*f*x + c*f)/d)*e^(-
2*e + 2*c*f/d) + 4*b^2*c^2*f^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) + 4
*a*b*c^2*f^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) - 4*a*b*c^2*f^2*Ei(-(d*f*x
+ c*f)/d)*e^(-e + c*f/d) + 4*b^2*c^2*f^2*Ei(-2*(d*f*x + c*f)/d)*e^(-2*e +
2*c*f/d) - 2*b^2*d^2*f*x*e^(2*f*x + 2*e) - 4*a*b*d^2*f*x*e^(f*x + e) - 4*a
*b*d^2*f*x*e^(-f*x - e) + 2*b^2*d^2*f*x*e^(-2*f*x - 2*e) - 2*b^2*c*d*f*e^(
2*f*x + 2*e) - 4*a*b*c*d*f*e^(f*x + e) - 4*a*b*c*d*f*e^(-f*x - e) + 2*b^2*
c*d*f*e^(-2*f*x - 2*e) - b^2*d^2*e^(2*f*x + 2*e) - 4*a*b*d^2*e^(f*x + e) +
 4*a*b*d^2*e^(-f*x - e) - b^2*d^2*e^(-2*f*x - 2*e) - 4*a^2*d^2 + 2*b^2*d^2
)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^3}\,dx = \int \frac{(a+b\sinh(e+f\,x))^2}{(c+d\,x)^3}\,dx$$

input
```
int((a + b*sinh(e + f*x))^2/(c + d*x)^3,x)
```

output
```
int((a + b*sinh(e + f*x))^2/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^3}\,dx = \frac{e^{3e}\left(\int \frac{e^{2fx}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)b^2c^2d+2e^{3e}\left(\int \frac{e^{2fx}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)b^2c\,d^2x+e^{3e}\left(\int \frac{e^{2fx}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}\right.}{}$$

input
```
int((a+b*sinh(f*x+e))^2/(d*x+c)^3,x)
```

output

```
(e**(3*e)*int(e**(2*f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x
)*b**2*c**2*d + 2*e**(3*e)*int(e**(2*f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x*
*2 + d**3*x**3),x)*b**2*c*d**2*x + e**(3*e)*int(e**(2*f*x)/(c**3 + 3*c**2*
d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b**2*d**3*x**2 + 4*e**(2*e)*int(e**(f*
x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a*b*c**2*d + 8*e**(2
*e)*int(e**(f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a*b*c*
d**2*x + 4*e**(2*e)*int(e**(f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3
*x**3),x)*a*b*d**3*x**2 + e**e*int(1/(e**(2*e + 2*f*x)*c**3 + 3*e**(2*e +
2*f*x)*c**2*d*x + 3*e**(2*e + 2*f*x)*c*d**2*x**2 + e**(2*e + 2*f*x)*d**3*x
**3),x)*b**2*c**2*d + 2*e**e*int(1/(e**(2*e + 2*f*x)*c**3 + 3*e**(2*e + 2*
f*x)*c**2*d*x + 3*e**(2*e + 2*f*x)*c*d**2*x**2 + e**(2*e + 2*f*x)*d**3*x**
3),x)*b**2*c*d**2*x + e**e*int(1/(e**(2*e + 2*f*x)*c**3 + 3*e**(2*e + 2*f*
x)*c**2*d*x + 3*e**(2*e + 2*f*x)*c*d**2*x**2 + e**(2*e + 2*f*x)*d**3*x**3)
,x)*b**2*d**3*x**2 - 2*e**e*a**2 + e**e*b**2 - 4*int(1/(e**(f*x)*c**3 + 3*
e**(f*x)*c**2*d*x + 3*e**(f*x)*c*d**2*x**2 + e**(f*x)*d**3*x**3),x)*a*b*c*
*2*d - 8*int(1/(e**(f*x)*c**3 + 3*e**(f*x)*c**2*d*x + 3*e**(f*x)*c*d**2*x*
*2 + e**(f*x)*d**3*x**3),x)*a*b*c*d**2*x - 4*int(1/(e**(f*x)*c**3 + 3*e**(
f*x)*c**2*d*x + 3*e**(f*x)*c*d**2*x**2 + e**(f*x)*d**3*x**3),x)*a*b*d**3*x
**2)/(4*e**e*d*(c**2 + 2*c*d*x + d**2*x**2))
```

3.169 $\int \frac{(c+dx)^3}{a+b\sinh(e+fx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 404

$$\begin{aligned}\int \frac{(c+dx)^3}{a+b\sinh(e+fx)}\,dx =& \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f}-\frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f}\\
&+\frac{3d(c+dx)^2\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f^2}\\
&-\frac{3d(c+dx)^2\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f^2}\\
&-\frac{6d^2(c+dx)\,\text{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f^3}\\
&+\frac{6d^2(c+dx)\,\text{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f^3}\\
&+\frac{6d^3\,\text{PolyLog}\left(4,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f^4}\\
&-\frac{6d^3\,\text{PolyLog}\left(4,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f^4}\end{aligned}$$

output
```
(d*x+c)^3*ln(1+b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(1/2)/f-(d*x+c)
^3*ln(1+b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(1/2)/f+3*d*(d*x+c)^2*
polylog(2,-b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(1/2)/f^2-3*d*(d*x+
c)^2*polylog(2,-b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(1/2)/f^2-6*d^
2*(d*x+c)*polylog(3,-b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(1/2)/f^3
+6*d^2*(d*x+c)*polylog(3,-b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(1/2
)/f^3+6*d^3*polylog(4,-b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(1/2)/f
^4-6*d^3*polylog(4,-b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(1/2)/f^4
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.79

$$\int \frac{(c+dx)^3}{a+b\sinh(e+fx)}\,dx$$
$$= \frac{(c+dx)^3 \log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)-(c+dx)^3 \log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)+\frac{3d\left(f^2(c+dx)^2 \operatorname{PolyLog}\left(2,\frac{be^{e+fx}}{-a+\sqrt{a^2+b^2}}\right)-2df(c+dx)\right.}{}}{}$$

input
```
Integrate[(c + d*x)^3/(a + b*Sinh[e + f*x]),x]
```

output
```
((c + d*x)^3*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])] - (c + d*x)^3*
Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])] + (3*d*(f^2*(c + d*x)^2*Pol
yLog[2, (b*E^(e + f*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*f*(c + d*x)*PolyLog[
3, (b*E^(e + f*x))/(-a + Sqrt[a^2 + b^2])] + 2*d^2*PolyLog[4, (b*E^(e + f*
x))/(-a + Sqrt[a^2 + b^2])]))/f^3 - (3*d*(f^2*(c + d*x)^2*PolyLog[2, -((b*
E^(e + f*x))/(a + Sqrt[a^2 + b^2]))] - 2*d*f*(c + d*x)*PolyLog[3, -((b*E^(
e + f*x))/(a + Sqrt[a^2 + b^2]))] + 2*d^2*PolyLog[4, -((b*E^(e + f*x))/(a
+ Sqrt[a^2 + b^2]))]))/f^3)/(Sqrt[a^2 + b^2]*f)
```

Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 377, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3803, 25, 2694, 27, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{a+b\sinh(e+fx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(c+dx)^3}{a-ib\sin(ie+ifx)}dx$$

$$\Big\downarrow \text{ 3803}$$

$$2\int -\frac{e^{e+fx}(c+dx)^3}{-2e^{e+fx}a-be^{2(e+fx)}+b}dx$$

$$\Big\downarrow \text{ 25}$$

$$-2\int \frac{e^{e+fx}(c+dx)^3}{-2e^{e+fx}a-be^{2(e+fx)}+b}dx$$

$$\Big\downarrow \text{ 2694}$$

$$-2\left(\frac{b\int -\frac{e^{e+fx}(c+dx)^3}{2\left(a+be^{e+fx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{e+fx}(c+dx)^3}{2\left(a+be^{e+fx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-2\left(\frac{b\int \frac{e^{e+fx}(c+dx)^3}{a+be^{e+fx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{e+fx}(c+dx)^3}{a+be^{e+fx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)$$

$$\Big\downarrow \text{ 2620}$$

$$-2\left(\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{3d\int(c+dx)^2\log\left(\frac{e^{e+fx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{bf}-\frac{3d\int(c+dx)^2\log\left(\right.}{b}\right.}{2\sqrt{a^2+b^2}}\right.$$

$\downarrow$ 3011

$$-2\left(\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{3d\left(\frac{2d\int(c+dx)\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)dx}{f}-\frac{(c+dx)^2\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)^3\,\mathrm{l}\cdots}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 7163

$$-2\left(\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{3d\left(\frac{2d\left(\frac{(c+dx)\,\text{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f}-\frac{d\int\text{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)dx}{f}\right)}{f}-\frac{(c+dx)^2\,\text{PolyLog}\left(2,-\frac{be\cdots}{a+\cdots}\right.}{f}\cdots\right.}{bf}\cdots\right.}{2\sqrt{a^2+b^2}}\cdots\right.$$

$\downarrow$ 2720

$$-2\left(\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{3d\left(\frac{2d\left(\frac{(c+dx)\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f}-\frac{d\int e^{-e-fx}\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)de^{e+fx}}{f^2}\right)}{f}-\frac{(c+dx)^2\operatorname{Pol}}{}\right.}{bf}\right.}{2\sqrt{a^2+b^2}}\right.$$

$\downarrow$ 7143

$$-2\left(\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{3d\left(\frac{2d\left(\frac{(c+dx)\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f}-\frac{d\operatorname{PolyLog}\left(4,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f^2}\right)}{f}-\frac{(c+dx)^2\operatorname{PolyLog}\left(2,-\frac{be^{e+f}}{a+\sqrt{a^2}}\right.}{f}\right.}{bf}\right.}{2\sqrt{a^2+b^2}}\right.$$

input
```
Int[(c + d*x)^3/(a + b*Sinh[e + f*x]),x]
```

output
```
-2*(-1/2*(b*(((c + d*x)^3*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])])/
(b*f) - (3*d*(-(((c + d*x)^2*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 +
b^2]))])/f) + (2*d*(((c + d*x)*PolyLog[3, -((b*E^(e + f*x))/(a - Sqrt[a^2
+ b^2]))])/f - (d*PolyLog[4, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/f^
2))/f))/(b*f)))/Sqrt[a^2 + b^2] + (b*(((c + d*x)^3*Log[1 + (b*E^(e + f*x))
/(a + Sqrt[a^2 + b^2])])/(b*f) - (3*d*(-(((c + d*x)^2*PolyLog[2, -((b*E^(e
 + f*x))/(a + Sqrt[a^2 + b^2]))])/f) + (2*d*(((c + d*x)*PolyLog[3, -((b*E^
(e + f*x))/(a + Sqrt[a^2 + b^2]))])/f - (d*PolyLog[4, -((b*E^(e + f*x))/(a
 + Sqrt[a^2 + b^2]))])/f^2))/f))/(b*f)))/(2*Sqrt[a^2 + b^2]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(dx+c)^3}{a+b\sinh(fx+e)}dx$$

input
```
int((d*x+c)^3/(a+b*sinh(f*x+e)),x)
```

output
```
int((d*x+c)^3/(a+b*sinh(f*x+e)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1004 vs. $2(362) = 724$.

Time = 0.14 (sec) , antiderivative size = 1004, normalized size of antiderivative = 2.49

$$\int \frac{(c+dx)^3}{a+b\sinh(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
(6*b*d^3*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(f*x + e) + a*sinh(f*x +
e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2))/b) - 6*b*d
^3*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(f*x + e) + a*sinh(f*x + e) - (
b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2))/b) + 3*(b*d^3*f^
2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh
(f*x + e) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^
2 + b^2)/b^2) - b)/b + 1) - 3*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f
^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(f*x + e) + a*sinh(f*x + e) - (b*co
sh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b*d^3*
e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sqrt((a^2 + b^2)/b^2)
*log(2*b*cosh(f*x + e) + 2*b*sinh(f*x + e) + 2*b*sqrt((a^2 + b^2)/b^2) + 2
*a) - (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sqrt((a^
2 + b^2)/b^2)*log(2*b*cosh(f*x + e) + 2*b*sinh(f*x + e) - 2*b*sqrt((a^2 +
b^2)/b^2) + 2*a) + (b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x +
b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*sqrt((a^2 + b^2)/b^2)*log(-
(a*cosh(f*x + e) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*s
qrt((a^2 + b^2)/b^2) - b)/b) - (b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^
2*d*f^3*x + b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*sqrt((a^2 + b^2
)/b^2)*log(-(a*cosh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x + e) + b*sinh
(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b) - 6*(b*d^3*f*x + b*c*d^2*f)*sq...
```

Sympy [F]

$$\int \frac{(c+dx)^3}{a+b\sinh(e+fx)}\,dx = \int \frac{(c+dx)^3}{a+b\sinh(e+fx)}\,dx$$

input
```
integrate((d*x+c)**3/(a+b*sinh(f*x+e)),x)
```

output
```
Integral((c + d*x)**3/(a + b*sinh(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(c+dx)^3}{a+b\sinh(e+fx)}\,dx = \int \frac{(dx+c)^3}{b\sinh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^3/(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
c^3*log((b*e^(-f*x - e) - a - sqrt(a^2 + b^2))/(b*e^(-f*x - e) - a + sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*f) + integrate(2*d^3*x^3/(b*(e^(f*x + e) - e
^(-f*x - e)) + 2*a) + 6*c*d^2*x^2/(b*(e^(f*x + e) - e^(-f*x - e)) + 2*a) +
 6*c^2*d*x/(b*(e^(f*x + e) - e^(-f*x - e)) + 2*a), x)
```

Giac [F]

$$\int \frac{(c+dx)^3}{a+b\sinh(e+fx)}\,dx = \int \frac{(dx+c)^3}{b\sinh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^3/(a+b*sinh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/(b*sinh(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{a+b\sinh(e+fx)}\,dx = \int \frac{(c+d\,x)^3}{a+b\sinh\left(e+f\,x\right)}\,dx$$

input
```
int((c + d*x)^3/(a + b*sinh(e + f*x)),x)
```

output
```
int((c + d*x)^3/(a + b*sinh(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(c+dx)^3}{a+b\sinh(e+fx)}\,dx = \frac{2\sqrt{a^2+b^2}\,atan\left(\frac{e^{fx+e}bi+ai}{\sqrt{a^2+b^2}}\right)c^3i+2e^e\left(\int \frac{e^{fx}x^3}{e^{2fx+2e}b+2e^{fx+e}a-b}dx\right)a^2d^3f+2e^e\left(\int \frac{e^{fx}x^3}{e^{2fx+2e}b+2e^{fx+e}a-b}dx\right)b^2d^3f}{}$$

input
```
int((d*x+c)^3/(a+b*sinh(f*x+e)),x)
```

output
```
(2*(sqrt(a**2 + b**2)*atan((e**(e + f*x)*b*i + a*i)/sqrt(a**2 + b**2))*c**
3*i + e**e*int((e**(f*x)*x**3)/(e**(2*e + 2*f*x)*b + 2*e**(e + f*x)*a - b)
,x)*a**2*d**3*f + e**e*int((e**(f*x)*x**3)/(e**(2*e + 2*f*x)*b + 2*e**(e +
 f*x)*a - b),x)*b**2*d**3*f + 3*e**e*int((e**(f*x)*x**2)/(e**(2*e + 2*f*x)
*b + 2*e**(e + f*x)*a - b),x)*a**2*c*d**2*f + 3*e**e*int((e**(f*x)*x**2)/(
e**(2*e + 2*f*x)*b + 2*e**(e + f*x)*a - b),x)*b**2*c*d**2*f + 3*e**e*int((
e**(f*x)*x)/(e**(2*e + 2*f*x)*b + 2*e**(e + f*x)*a - b),x)*a**2*c**2*d*f +
 3*e**e*int((e**(f*x)*x)/(e**(2*e + 2*f*x)*b + 2*e**(e + f*x)*a - b),x)*b*
*2*c**2*d*f))/(f*(a**2 + b**2))
```

3.170 $\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 296

$$\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}\,dx = \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f} - \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f} + \frac{2d(c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f^2} - \frac{2d(c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f^2} - \frac{2d^2\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f^3} + \frac{2d^2\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f^3}$$

output
```
(d*x+c)^2*ln(1+b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(1/2)/f-(d*x+c)
^2*ln(1+b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(1/2)/f+2*d*(d*x+c)*po
lylog(2,-b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(1/2)/f^2-2*d*(d*x+c)
*polylog(2,-b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(1/2)/f^2-2*d^2*po
lylog(3,-b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(1/2)/f^3+2*d^2*polyl
og(3,-b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(1/2)/f^3
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.79

$$\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}\,dx = \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)-(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)+\frac{2d\left(f(c+dx)\operatorname{PolyLog}\left(2,\frac{be^{e+fx}}{-a+\sqrt{a^2+b^2}}\right)-d\operatorname{PolyLog}\left(3,\right.\right.}{f^2}}{\sqrt{a^2+b^2}f}$$

input
```
Integrate[(c + d*x)^2/(a + b*Sinh[e + f*x]),x]
```

output
```
((c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])] - (c + d*x)^2*
Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])] + (2*d*(f*(c + d*x)*PolyLog
[2, (b*E^(e + f*x))/(-a + Sqrt[a^2 + b^2])] - d*PolyLog[3, (b*E^(e + f*x))
/(-a + Sqrt[a^2 + b^2])]))/f^2 - (2*d*(f*(c + d*x)*PolyLog[2, -((b*E^(e +
f*x))/(a + Sqrt[a^2 + b^2]))] - d*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a
^2 + b^2]))]))/f^2)/(Sqrt[a^2 + b^2]*f)
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3042, 3803, 25, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(c+dx)^2}{a-ib\sin(ie+ifx)}dx$$

$$\Big\downarrow \text{ 3803}$$

$$2\int -\frac{e^{e+fx}(c+dx)^2}{-2e^{e+fx}a-be^{2(e+fx)}+b}dx$$

$$\Big\downarrow \ 25$$

$$-2\int \frac{e^{e+fx}(c+dx)^2}{-2e^{e+fx}a-be^{2(e+fx)}+b}dx$$

$$\Big\downarrow \ 2694$$

$$-2\left(\frac{b\int -\frac{e^{e+fx}(c+dx)^2}{2\left(a+be^{e+fx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{e+fx}(c+dx)^2}{2\left(a+be^{e+fx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)$$

$$\Big\downarrow \ 27$$

$$-2\left(\frac{b\int \frac{e^{e+fx}(c+dx)^2}{a+be^{e+fx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{e+fx}(c+dx)^2}{a+be^{e+fx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)$$

$$\Big\downarrow \ 2620$$

$$-2\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{2d\int(c+dx)\log\left(\frac{e^{e+fx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{bf}-\frac{2d\int(c+dx)\log\left(\frac{}{a-}\right.}{bf}\right.}{2\sqrt{a^2+b^2}}\right.$$

$$\Big\downarrow \ 3011$$

$$-2\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{2d\left(\frac{d\int \text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)dx}{f}-\frac{(c+dx)\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be}{a-\sqrt{}}\right.}{bf}\right.}{}\right.$$

$$\Big\downarrow \ 2720$$

$$-2\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{2d\left(\frac{d\int e^{-e-fx}\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)de^{e+fx}}{f^2}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left((c+d\right.}{}\right.$$

$\downarrow$ 7143

$$-2\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{2d\left(\frac{d\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f^2}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+f}}{a-\sqrt{a^2-}}\right.}{bf}\right.}{}\right.$$

input
```
Int[(c + d*x)^2/(a + b*Sinh[e + f*x]),x]
```

output
```
-2*(-1/2*(b*(((c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])])/
(b*f) - (2*d*(-(((c + d*x)*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^
2]))])/f) + (d*PolyLog[3, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/f^2))
/(b*f)))/Sqrt[a^2 + b^2] + (b*(((c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a + S
qrt[a^2 + b^2])])/(b*f) - (2*d*(-(((c + d*x)*PolyLog[2, -((b*E^(e + f*x))/
(a + Sqrt[a^2 + b^2]))])/f) + (d*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a^
2 + b^2]))])/f^2))/(b*f)))/(2*Sqrt[a^2 + b^2]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(dx+c)^2}{a+b\sinh(fx+e)}dx$$

input
```
int((d*x+c)^2/(a+b*sinh(f*x+e)),x)
```

output
```
int((d*x+c)^2/(a+b*sinh(f*x+e)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 708 vs. $2(264) = 528$.

Time = 0.11 (sec) , antiderivative size = 708, normalized size of antiderivative = 2.39

$$\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2/(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
-(2*b*d^2*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(f*x + e) + a*sinh(f*x +
 e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2))/b) - 2*b*
d^2*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(f*x + e) + a*sinh(f*x + e) -
(b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2))/b) - 2*(b*d^2*f
*x + b*c*d*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(f*x + e) + a*sinh(f*x +
e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b + 1)
 + 2*(b*d^2*f*x + b*c*d*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(f*x + e) +
a*sinh(f*x + e) - (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2
) - b)/b + 1) + (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sqrt((a^2 + b^2)/b^2
)*log(2*b*cosh(f*x + e) + 2*b*sinh(f*x + e) + 2*b*sqrt((a^2 + b^2)/b^2) +
2*a) - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sqrt((a^2 + b^2)/b^2)*log(2*b
*cosh(f*x + e) + 2*b*sinh(f*x + e) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b
*d^2*f^2*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*c*d*e*f)*sqrt((a^2 + b^2)/b
^2)*log(-(a*cosh(f*x + e) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*
x + e))*sqrt((a^2 + b^2)/b^2) - b)/b) + (b*d^2*f^2*x^2 + 2*b*c*d*f^2*x - b
*d^2*e^2 + 2*b*c*d*e*f)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(f*x + e) + a*si
nh(f*x + e) - (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) -
b)/b))/((a^2 + b^2)*f^3)
```

Sympy [F]

$$\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}\,dx = \int \frac{(c+dx)^2}{a+b\sinh{(e+fx)}}\,dx$$

input
```
integrate((d*x+c)**2/(a+b*sinh(f*x+e)),x)
```

output
```
Integral((c + d*x)**2/(a + b*sinh(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}\,dx = \int \frac{(dx+c)^2}{b\sinh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^2/(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
c^2*log((b*e^(-f*x - e) - a - sqrt(a^2 + b^2))/(b*e^(-f*x - e) - a + sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*f) + integrate(2*d^2*x^2/(b*(e^(f*x + e) - e
^(-f*x - e)) + 2*a) + 4*c*d*x/(b*(e^(f*x + e) - e^(-f*x - e)) + 2*a), x)
```

Giac [F]

$$\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}\,dx = \int \frac{(dx+c)^2}{b\sinh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^2/(a+b*sinh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(b*sinh(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}\,dx = \int \frac{(c+d\,x)^2}{a+b\sinh(e+f\,x)}\,dx$$

input
```
int((c + d*x)^2/(a + b*sinh(e + f*x)),x)
```

output
```
int((c + d*x)^2/(a + b*sinh(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}\,dx$$
$$= \frac{2\sqrt{a^2+b^2}\,atan\left(\frac{e^{fx+e}bi+ai}{\sqrt{a^2+b^2}}\right)c^2i+2e^e\left(\int \frac{e^{fx}x^2}{e^{2fx+2e}b+2e^{fx+e}a-b}dx\right)a^2d^2f+2e^e\left(\int \frac{e^{fx}x^2}{e^{2fx+2e}b+2e^{fx+e}a-b}dx\right)b^2d^2f}{f\left(a^2+b^2\right)}$$

input
```
int((d*x+c)^2/(a+b*sinh(f*x+e)),x)
```

output
```
(2*(sqrt(a**2 + b**2)*atan((e**(e + f*x)*b*i + a*i)/sqrt(a**2 + b**2))*c**
2*i + e**e*int((e**(f*x)*x**2)/(e**(2*e + 2*f*x)*b + 2*e**(e + f*x)*a - b)
,x)*a**2*d**2*f + e**e*int((e**(f*x)*x**2)/(e**(2*e + 2*f*x)*b + 2*e**(e +
 f*x)*a - b),x)*b**2*d**2*f + 2*e**e*int((e**(f*x)*x)/(e**(2*e + 2*f*x)*b
+ 2*e**(e + f*x)*a - b),x)*a**2*c*d*f + 2*e**e*int((e**(f*x)*x)/(e**(2*e +
 2*f*x)*b + 2*e**(e + f*x)*a - b),x)*b**2*c*d*f))/(f*(a**2 + b**2))
```

3.171 $\int \frac{c+dx}{a+b\sinh(e+fx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 187

$$\int \frac{c+dx}{a+b\sinh(e+fx)}\,dx = \frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f} - \frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f} + \frac{d\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f^2} - \frac{d\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f^2}$$

output
```
(d*x+c)*ln(1+b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(1/2)/f-(d*x+c)*l
n(1+b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(1/2)/f+d*polylog(2,-b*exp
(f*x+e)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(1/2)/f^2-d*polylog(2,-b*exp(f*x+e)
/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(1/2)/f^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.76

$$\int \frac{c+dx}{a+b\sinh(e+fx)}\,dx = \frac{f(c+dx)\left(\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)-\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)\right)+d\,\text{PolyLog}\left(2,\frac{be^{e+fx}}{-a+\sqrt{a^2+b^2}}\right)-d\,\text{PolyLog}\left(2,-\frac{}{a}\right.}{\sqrt{a^2+b^2}\,f^2}$$

input
```
Integrate[(c + d*x)/(a + b*Sinh[e + f*x]),x]
```

output
```
(f*(c + d*x)*(Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])] - Log[1 + (b*
E^(e + f*x))/(a + Sqrt[a^2 + b^2])]) + d*PolyLog[2, (b*E^(e + f*x))/(-a +
Sqrt[a^2 + b^2])] - d*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))]
)/(Sqrt[a^2 + b^2]*f^2)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{a+b\sinh(e+fx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{c+dx}{a-ib\sin(ie+ifx)}dx$$

$$\Big\downarrow \text{ 3803}$$

$$2\int -\frac{e^{e+fx}(c+dx)}{-2e^{e+fx}a-be^{2(e+fx)}+b}dx$$

$$\Big\downarrow \text{ 25}$$

$$-2\int \frac{e^{e+fx}(c+dx)}{-2e^{e+fx}a-be^{2(e+fx)}+b}dx$$

$$\Big\downarrow \text{ 2694}$$

$$-2\left(\frac{b\int -\frac{e^{e+fx}(c+dx)}{2\left(a+be^{e+fx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{e+fx}(c+dx)}{2\left(a+be^{e+fx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-2\left(\frac{b\int \frac{e^{e+fx}(c+dx)}{a+be^{e+fx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{e+fx}(c+dx)}{a+be^{e+fx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)$$

$\Big\downarrow$ 2620

$$-2\left(\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{d\int \log\left(\frac{e^{e+fx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{bf}-\frac{d\int \log\left(\frac{e^{e+fx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bf}\right)}{2\sqrt{a^2+b^2}}\right.$$

$\Big\downarrow$ 2715

$$-2\left(\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{d\int e^{-e-fx}\log\left(\frac{e^{e+fx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{e+fx}}{bf^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{bf}-\frac{d\int e^{-e-fx}\log\left(\right.}{}\right.}{2\sqrt{a^2+b^2}}\right.$$

$\Big\downarrow$ 2838

$$-2\left(\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}+\frac{d\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{bf^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{bf}+\frac{d\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{bf^2}\right.}{2\sqrt{a^2+b^2}}\right.$$

input

```
Int[(c + d*x)/(a + b*Sinh[e + f*x]),x]
```

output

```
-2*(-1/2*(b*(((c + d*x)*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])])/(b
*f) + (d*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/(b*f^2)))/S
qrt[a^2 + b^2] + (b*(((c + d*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^
2])])/(b*f) + (d*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/(b*
f^2)))/(2*Sqrt[a^2 + b^2]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(167) = 334$.

Time = 0.19 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.10

method	result
risch	$-\frac{2c\,\mathrm{arctanh}\left(\frac{2b\,\mathrm{e}^{fx+e}+2a}{2\sqrt{a^2+b^2}}\right)}{f\sqrt{a^2+b^2}}+\frac{d\ln\left(\frac{-b\,\mathrm{e}^{fx+e}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)x}{f\sqrt{a^2+b^2}}-\frac{d\ln\left(\frac{b\,\mathrm{e}^{fx+e}+\sqrt{a^2+b^2}+a}{a+\sqrt{a^2+b^2}}\right)x}{f\sqrt{a^2+b^2}}+\frac{d\ln\left(\frac{-b\,\mathrm{e}^{fx+e}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)}{f^2\sqrt{a^2+b^2}}$

input
```
int((d*x+c)/(a+b*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-2/f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(f*x+e)+2*a)/(a^2+b^2)^(1/2))+1
/f*d/(a^2+b^2)^(1/2)*ln((-b*exp(f*x+e)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1
/2)))*x-1/f*d/(a^2+b^2)^(1/2)*ln((b*exp(f*x+e)+(a^2+b^2)^(1/2)+a)/(a+(a^2+
b^2)^(1/2)))*x+1/f^2*d/(a^2+b^2)^(1/2)*ln((-b*exp(f*x+e)+(a^2+b^2)^(1/2)-a
)/(-a+(a^2+b^2)^(1/2)))*e-1/f^2*d/(a^2+b^2)^(1/2)*ln((b*exp(f*x+e)+(a^2+b^
2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*e+1/f^2*d/(a^2+b^2)^(1/2)*dilog((-b*exp(f
*x+e)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/f^2*d/(a^2+b^2)^(1/2)*dil
og((b*exp(f*x+e)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/f^2*d*e/(a^2+b^
2)^(1/2)*arctanh(1/2*(2*b*exp(f*x+e)+2*a)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(165) = 330$.

Time = 0.09 (sec) , antiderivative size = 455, normalized size of antiderivative = 2.43

$$\int \frac{c+dx}{a+b\sinh(e+fx)}\,dx = \frac{bd\sqrt{\frac{a^2+b^2}{b^2}}\mathrm{Li}_2\left(\frac{a\cosh(fx+e)+a\sinh(fx+e)+(b\cosh(fx+e)+b\sinh(fx+e))\sqrt{\frac{a^2+b^2}{b^2}}-b}{b}+1\right)-bd\sqrt{\frac{a^2+b^2}{b^2}}\mathrm{Li}_2\left(\frac{a\cosh(fx+e)+a}{}\right.}{}$$

input
```
integrate((d*x+c)/(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
(b*d*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(f*x + e) + a*sinh(f*x + e) + (b*c
osh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - b*d*sq
rt((a^2 + b^2)/b^2)*dilog((a*cosh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x
 + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b*d*e - b*c*
f)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(f*x + e) + 2*b*sinh(f*x + e) + 2*b*s
qrt((a^2 + b^2)/b^2) + 2*a) - (b*d*e - b*c*f)*sqrt((a^2 + b^2)/b^2)*log(2*
b*cosh(f*x + e) + 2*b*sinh(f*x + e) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (
b*d*f*x + b*d*e)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(f*x + e) + a*sinh(f*x
+ e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b) -
 (b*d*f*x + b*d*e)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(f*x + e) + a*sinh(f*
x + e) - (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b)
)/((a^2 + b^2)*f^2)
```

Sympy [F]

$$\int \frac{c+dx}{a+b\sinh(e+fx)}\,dx=\int \frac{c+dx}{a+b\sinh{(e+fx)}}\,dx$$

input
```
integrate((d*x+c)/(a+b*sinh(f*x+e)),x)
```

output
```
Integral((c + d*x)/(a + b*sinh(e + f*x)), x)
```

Maxima [F]

$$\int \frac{c+dx}{a+b\sinh(e+fx)}\,dx=\int \frac{dx+c}{b\sinh{(fx+e)}+a}\,dx$$

input
```
integrate((d*x+c)/(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
d*integrate(2*x/(b*(e^(f*x + e) - e^(-f*x - e)) + 2*a), x) + c*log((b*e^(-
f*x - e) - a - sqrt(a^2 + b^2))/(b*e^(-f*x - e) - a + sqrt(a^2 + b^2)))/(s
qrt(a^2 + b^2)*f)
```

Giac [F]

$$\int \frac{c+dx}{a+b\sinh(e+fx)}\,dx = \int \frac{dx+c}{b\sinh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)/(a+b*sinh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)/(b*sinh(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{a+b\sinh(e+fx)}\,dx = \int \frac{c+d\,x}{a+b\sinh(e+f\,x)}\,dx$$

input
```
int((c + d*x)/(a + b*sinh(e + f*x)),x)
```

output
```
int((c + d*x)/(a + b*sinh(e + f*x)), x)
```

Reduce [F]

$$\int \frac{c+dx}{a+b\sinh(e+fx)}\,dx = \frac{2\sqrt{a^2+b^2}\,atan\left(\frac{e^{fx+e}bi+ai}{\sqrt{a^2+b^2}}\right)ci+2e^e\left(\int \frac{e^{fx}x}{e^{2fx+2e}b+2e^{fx+e}a-b}dx\right)a^2df+2e^e\left(\int \frac{e^{fx}x}{e^{2fx+2e}b+2e^{fx+e}a-b}dx\right)b^2df}{f\,(a^2+b^2)}$$

input
```
int((d*x+c)/(a+b*sinh(f*x+e)),x)
```

output
```
(2*(sqrt(a**2 + b**2)*atan((e**(e + f*x)*b*i + a*i)/sqrt(a**2 + b**2))*c*i
 + e**e*int((e**(f*x)*x)/(e**(2*e + 2*f*x)*b + 2*e**(e + f*x)*a - b),x)*a*
*2*d*f + e**e*int((e**(f*x)*x)/(e**(2*e + 2*f*x)*b + 2*e**(e + f*x)*a - b)
,x)*b**2*d*f))/(f*(a**2 + b**2))
```

3.172 $\int \frac{1}{(c+dx)(a+b\sinh(e+fx))}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))}\,dx = \mathrm{Int}\left(\frac{1}{(c+dx)(a+b\sinh(e+fx))},x\right)$$

output
```
Defer(Int)(1/(d*x+c)/(a+b*sinh(f*x+e)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))}\,dx = \int \frac{1}{(c+dx)(a+b\sinh(e+fx))}\,dx$$

input
```
Integrate[1/((c + d*x)*(a + b*Sinh[e + f*x])),x]
```

output
```
Integrate[1/((c + d*x)*(a + b*Sinh[e + f*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(c+dx)(a-ib\sin(ie+ifx))}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))}dx$$

input
```
Int[1/((c + d*x)*(a + b*Sinh[e + f*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)\left(a+b\sinh\left(fx+e\right)\right)}dx$$

input
```
int(1/(d*x+c)/(a+b*sinh(f*x+e)),x)
```

output
```
int(1/(d*x+c)/(a+b*sinh(f*x+e)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))}\,dx = \int \frac{1}{(dx+c)(b\sinh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
integral(1/(a*d*x + a*c + (b*d*x + b*c)*sinh(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 10.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))}\,dx = \int \frac{1}{(a+b\sinh(e+fx))(c+dx)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*sinh(f*x+e)),x)
```

output
```
Integral(1/((a + b*sinh(e + f*x))*(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))}\,dx = \int \frac{1}{(dx+c)(b\sinh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(1/((d*x + c)*(b*sinh(f*x + e) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))}\,dx = \int \frac{1}{(dx+c)(b\sinh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*sinh(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)*(b*sinh(f*x + e) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))}\,dx = \int \frac{1}{(a+b\sinh(e+f\,x))\,(c+d\,x)}\,dx$$

input
```
int(1/((a + b*sinh(e + f*x))*(c + d*x)),x)
```

output
```
int(1/((a + b*sinh(e + f*x))*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))}\,dx = \int \frac{1}{\sinh(fx+e)\,bc+\sinh(fx+e)\,bdx+ac+adx}dx$$

input
```
int(1/(d*x+c)/(a+b*sinh(f*x+e)),x)
```

output
```
int(1/(sinh(e + f*x)*b*c + sinh(e + f*x)*b*d*x + a*c + a*d*x),x)
```

3.173 $\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}\,dx = \text{Int}\left(\frac{1}{(c+dx)^2(a+b\sinh(e+fx))},x\right)$$

output
```
Defer(Int)(1/(d*x+c)^2/(a+b*sinh(f*x+e)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}\,dx = \int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}\,dx$$

input
```
Integrate[1/((c + d*x)^2*(a + b*Sinh[e + f*x])),x]
```

output
```
Integrate[1/((c + d*x)^2*(a + b*Sinh[e + f*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(c+dx)^2(a-ib\sin(ie+ifx))}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}dx$$

input
```
Int[1/((c + d*x)^2*(a + b*Sinh[e + f*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)^2 (a+b\sinh(fx+e))}dx$$

input
```
int(1/(d*x+c)^2/(a+b*sinh(f*x+e)),x)
```

output
```
int(1/(d*x+c)^2/(a+b*sinh(f*x+e)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.55

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(b\sinh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2
)*sinh(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 57.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}\,dx = \int \frac{1}{(a+b\sinh(e+fx))(c+dx)^2}\,dx$$

input
```
integrate(1/(d*x+c)**2/(a+b*sinh(f*x+e)),x)
```

output
```
Integral(1/((a + b*sinh(e + f*x))*(c + d*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(b\sinh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(1/((d*x + c)^2*(b*sinh(f*x + e) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(b\sinh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*sinh(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)^2*(b*sinh(f*x + e) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}\,dx = \int \frac{1}{(a+b\sinh(e+f\,x))\,(c+d\,x)^2}\,dx$$

input
```
int(1/((a + b*sinh(e + f*x))*(c + d*x)^2),x)
```

output
```
int(1/((a + b*sinh(e + f*x))*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 128, normalized size of antiderivative = 6.40

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}\,dx$$
$$=2e^e\left(\int \frac{e^{fx}}{e^{2fx+2e}b\,c^2+2e^{2fx+2e}bcdx+e^{2fx+2e}b\,d^2x^2+2e^{fx+e}a\,c^2+4e^{fx+e}acdx+2e^{fx+e}a\,d^2x^2-b\,c^2-2b\right.$$

input
```
int(1/(d*x+c)^2/(a+b*sinh(f*x+e)),x)
```

output
```
2*e**e*int(e**(f*x)/(e**(2*e + 2*f*x)*b*c**2 + 2*e**(2*e + 2*f*x)*b*c*d*x
+ e**(2*e + 2*f*x)*b*d**2*x**2 + 2*e**(e + f*x)*a*c**2 + 4*e**(e + f*x)*a*
c*d*x + 2*e**(e + f*x)*a*d**2*x**2 - b*c**2 - 2*b*c*d*x - b*d**2*x**2),x)
```

3.174 $\int \frac{(c+dx)^2}{(a+b\sinh(e+fx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 549

$$\int \frac{(c+dx)^2}{(a+b\sinh(e+fx))^2}\,dx = -\frac{(c+dx)^2}{(a^2+b^2)f} + \frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)f^2} + \frac{a(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}f} + \frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)f^2} - \frac{a(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}f} + \frac{2d^2\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)f^3} + \frac{2ad(c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}f^2} + \frac{2d^2\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)f^3} - \frac{2ad(c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}f^2} - \frac{2ad^2\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}f^3} + \frac{2ad^2\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}f^3} - \frac{b(c+dx)^2\cosh(e+fx)}{(a^2+b^2)f(a+b\sinh(e+fx))}$$

output
```
-(d*x+c)^2/(a^2+b^2)/f+2*d*(d*x+c)*ln(1+b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/
(a^2+b^2)/f^2+a*(d*x+c)^2*ln(1+b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)
^(3/2)/f+2*d*(d*x+c)*ln(1+b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/f^2-
a*(d*x+c)^2*ln(1+b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/f+2*d^2
*polylog(2,-b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/f^3+2*a*d*(d*x+c)*
polylog(2,-b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/f^2+2*d^2*pol
ylog(2,-b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/f^3-2*a*d*(d*x+c)*poly
log(2,-b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/f^2-2*a*d^2*polyl
og(3,-b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/f^3+2*a*d^2*polylo
g(3,-b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/f^3-b*(d*x+c)^2*cos
h(f*x+e)/(a^2+b^2)/f/(a+b*sinh(f*x+e))
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.78

$$\int \frac{(c+dx)^2}{(a+b\sinh(e+fx))^2}\,dx$$

$$= \frac{-f^2(c+dx)^2 + 2df(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right) + 2df(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right) + 2d^2\operatorname{PolyLog}\left(2, \frac{\quad}{-a \ldots}\right.}{\ldots}$$

input
```
Integrate[(c + d*x)^2/(a + b*Sinh[e + f*x])^2,x]
```

output
```
(-(f^2*(c + d*x)^2) + 2*d*f*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^
2 + b^2])] + 2*d*f*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])
] + 2*d^2*PolyLog[2, (b*E^(e + f*x))/(-a + Sqrt[a^2 + b^2])] + 2*d^2*PolyL
og[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))] - (a*(-(f^2*(c + d*x)^2*Lo
g[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])]) + f^2*(c + d*x)^2*Log[1 + (b
*E^(e + f*x))/(a + Sqrt[a^2 + b^2])] - 2*d*f*(c + d*x)*PolyLog[2, (b*E^(e
+ f*x))/(-a + Sqrt[a^2 + b^2])] + 2*d*f*(c + d*x)*PolyLog[2, -((b*E^(e + f
*x))/(a + Sqrt[a^2 + b^2]))] + 2*d^2*PolyLog[3, (b*E^(e + f*x))/(-a + Sqrt
[a^2 + b^2])] - 2*d^2*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))]
))/Sqrt[a^2 + b^2] - (b*f^2*(c + d*x)^2*Cosh[e + f*x])/(a + b*Sinh[e + f*x
]))/((a^2 + b^2)*f^3)
```

Rubi [A] (verified)

Time = 2.70 (sec) , antiderivative size = 520, normalized size of antiderivative = 0.95, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3805, 3042, 3803, 25, 2694, 27, 2620, 3011, 2720, 6095, 2620, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(a+b\sinh(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+dx)^2}{(a-ib\sin(ie+ifx))^2}dx$$

$$\Big\downarrow \text{3805}$$

$$\frac{a\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}dx}{a^2+b^2}+\frac{2bd\int \frac{(c+dx)\cosh(e+fx)}{a+b\sinh(e+fx)}dx}{f(a^2+b^2)}-\frac{b(c+dx)^2\cosh(e+fx)}{f(a^2+b^2)(a+b\sinh(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{(c+dx)^2}{a-ib\sin(ie+ifx)}dx}{a^2+b^2}+\frac{2bd\int \frac{(c+dx)\cosh(e+fx)}{a+b\sinh(e+fx)}dx}{f(a^2+b^2)}-\frac{b(c+dx)^2\cosh(e+fx)}{f(a^2+b^2)(a+b\sinh(e+fx))}$$

$$\Big\downarrow \text{3803}$$

$$\frac{2a\int -\frac{e^{e+fx}(c+dx)^2}{-2e^{e+fx}a-be^{2(e+fx)}+b}dx}{a^2+b^2}+\frac{2bd\int \frac{(c+dx)\cosh(e+fx)}{a+b\sinh(e+fx)}dx}{f(a^2+b^2)}-\frac{b(c+dx)^2\cosh(e+fx)}{f(a^2+b^2)(a+b\sinh(e+fx))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2a\int \frac{e^{e+fx}(c+dx)^2}{-2e^{e+fx}a-be^{2(e+fx)}+b}dx}{a^2+b^2}+\frac{2bd\int \frac{(c+dx)\cosh(e+fx)}{a+b\sinh(e+fx)}dx}{f(a^2+b^2)}-\frac{b(c+dx)^2\cosh(e+fx)}{f(a^2+b^2)(a+b\sinh(e+fx))}$$

$$\Big\downarrow \text{2694}$$

$$-\frac{2a\left(\frac{b\int -\frac{e^{e+fx}(c+dx)^2}{2\left(a+be^{e+fx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{e+fx}(c+dx)^2}{2\left(a+be^{e+fx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{2bd\int \frac{(c+dx)\cosh(e+fx)}{a+b\sinh(e+fx)}dx}{f(a^2+b^2)}-\frac{b(c+dx)^2\cosh(e+fx)}{f(a^2+b^2)(a+b\sinh(e+fx))}$$

$$\Big\downarrow \; 27$$

$$-\frac{2a\left(\frac{b\int \frac{e^{e+fx}(c+dx)^2}{a+be^{e+fx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{e+fx}(c+dx)^2}{a+be^{e+fx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{2bd\int \frac{(c+dx)\cosh(e+fx)}{a+b\sinh(e+fx)}dx}{f\left(a^2+b^2\right)}-\frac{b(c+dx)^2\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}$$

$$\Big\downarrow \; 2620$$

$$-\frac{2a\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{2d\int (c+dx)\log\left(\frac{e^{e+fx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{bf}-\frac{2d\int (c+dx)\log\left(\frac{e^{e+fx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bf}\right.}{2\sqrt{a^2+b^2}}\right.}{a^2+b^2}$$

$$\frac{2bd\int \frac{(c+dx)\cosh(e+fx)}{a+b\sinh(e+fx)}dx}{f\left(a^2+b^2\right)}-\frac{b(c+dx)^2\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}$$

$$\Big\downarrow \; 3011$$

$$-\frac{2a\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{2d\left(\frac{d\int \text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)dx}{f}-\frac{(c+dx)\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right.}{bf}\right.}{}\right.}{a^2+b^2}$$

$$\frac{2bd\int \frac{(c+dx)\cosh(e+fx)}{a+b\sinh(e+fx)}dx}{f\left(a^2+b^2\right)}-\frac{b(c+dx)^2\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}$$

$$\Big\downarrow \; 2720$$

$$-\frac{2a\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{2d\left(\frac{d\int e^{-e-fx}\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)de^{e+fx}}{f^2}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)^2\log}{}\right.}{}\right.}{a^2+b^2}$$

$$\frac{2bd\int\frac{(c+dx)\cosh(e+fx)}{a+b\sinh(e+fx)}dx}{f\left(a^2+b^2\right)}-\frac{b(c+dx)^2\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}$$

$\downarrow$ 6095

$$-\frac{2a\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{2d\left(\frac{d\int e^{-e-fx}\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)de^{e+fx}}{f^2}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)^2\log}{}\right.}{}\right.}{a^2+b^2}$$

$$\frac{2bd\left(\int\frac{e^{e+fx}(c+dx)}{a+be^{e+fx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{e+fx}(c+dx)}{a+be^{e+fx}+\sqrt{a^2+b^2}}dx-\frac{(c+dx)^2}{2bd}\right)}{f\left(a^2+b^2\right)}-$$
$$\frac{b(c+dx)^2\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}$$

$\downarrow$ 2620

$$-\frac{2a\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{2d\left(\frac{d\int e^{-e-fx}\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)de^{e+fx}}{f^2}-\frac{(c+dx)\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)^2\log \cdots}{\cdots}\right.\cdots}{\cdots}\cdots\right)}{a^2+b^2}\cdots$$

$$\frac{2bd\left(-\frac{d\int\log\left(\frac{e^{e+fx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bf}-\frac{d\int\log\left(\frac{e^{e+fx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bf}+\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{bf}+\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{(c+dx)\cdots}{2bd}\cdots\right)}{f\left(a^2+b^2\right)}\cdots$$

$$\frac{b(c+dx)^2\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}$$

$$\Big\downarrow \text{ 2715}$$

$$-\frac{2a\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{2d\left(\frac{d\int e^{-e-fx}\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)de^{e+fx}}{f^2}-\frac{(c+dx)\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)^2\log \cdots}{\cdots}\right.\cdots}{\cdots}\cdots\right)}{a^2+b^2}\cdots$$

$$\frac{2bd\left(-\frac{d\int e^{-e-fx}\log\left(\frac{e^{e+fx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{e+fx}}{bf^2}-\frac{d\int e^{-e-fx}\log\left(\frac{e^{e+fx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{e+fx}}{bf^2}+\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{bf}+\frac{(c+dx)\log\left(\cdots\right.}{\cdots}\cdots\right)}{f\left(a^2+b^2\right)}\cdots$$

$$\frac{b(c+dx)^2\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}$$

$$\Big\downarrow \text{ 2838}$$

$$-\frac{2a\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{2d\left(\frac{d\int e^{-e-fx}\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)de^{e+fx}}{f^2}-\frac{(c+dx)\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)^2\log\cdots}{\cdots}\right.}{\cdots}\right.}{a^2+b^2}$$

$$\frac{2bd\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{bf}+\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}+\frac{d\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{bf^2}+\frac{d\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{bf^2}-\frac{(c+dx\cdots}{2bd\cdots}\right.}{f\left(a^2+b^2\right)}$$

$$\frac{b(c+dx)^2\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}$$

$$\Big\downarrow \text{7143}$$

$$\frac{2bd\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{bf}+\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}+\frac{d\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{bf^2}+\frac{d\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{bf^2}-\frac{(c+dx\cdots}{2bd\cdots}\right.}{f\left(a^2+b^2\right)}$$

$$\frac{2a\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{2d\left(\frac{d\,\text{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f^2}-\frac{(c+dx)\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{bf}-\cdots\right.}{\cdots}\right.}{a^2+b^2}$$

$$\frac{b(c+dx)^2\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}$$

input
```
Int[(c + d*x)^2/(a + b*Sinh[e + f*x])^2,x]
```

output
```
(2*b*d*(-1/2*(c + d*x)^2/(b*d) + ((c + d*x)*Log[1 + (b*E^(e + f*x))/(a - S
qrt[a^2 + b^2])])/(b*f) + ((c + d*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2
 + b^2])])/(b*f) + (d*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))]
)/(b*f^2) + (d*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/(b*f^
2)))/((a^2 + b^2)*f) - (2*a*(-1/2*(b*(((c + d*x)^2*Log[1 + (b*E^(e + f*x))
/(a - Sqrt[a^2 + b^2])])/(b*f) - (2*d*(-(((c + d*x)*PolyLog[2, -((b*E^(e +
 f*x))/(a - Sqrt[a^2 + b^2]))])/f) + (d*PolyLog[3, -((b*E^(e + f*x))/(a -
Sqrt[a^2 + b^2]))])/f^2))/(b*f)))/Sqrt[a^2 + b^2] + (b*(((c + d*x)^2*Log[1
 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])])/(b*f) - (2*d*(-(((c + d*x)*Poly
Log[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/f) + (d*PolyLog[3, -((b*
E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/f^2))/(b*f)))/(2*Sqrt[a^2 + b^2])))/
(a^2 + b^2) - (b*(c + d*x)^2*Cosh[e + f*x])/((a^2 + b^2)*f*(a + b*Sinh[e +
 f*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(dx+c)^2}{(a+b\sinh(fx+e))^2}dx$$

input
```
int((d*x+c)^2/(a+b*sinh(f*x+e))^2,x)
```

output
```
int((d*x+c)^2/(a+b*sinh(f*x+e))^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3957 vs. $2(503) = 1006$.

Time = 0.17 (sec) , antiderivative size = 3957, normalized size of antiderivative = 7.21

$$\int \frac{(c+dx)^2}{(a+b\sinh(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2/(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{(a+b\sinh(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2/(a+b*sinh(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)^2}{(a+b\sinh(e+fx))^2}\,dx = \int \frac{(dx+c)^2}{(b\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^2/(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
2*a*d^2*f*integrate(x^2*e^(f*x + e)/(a^2*b*f*e^(2*f*x + 2*e) + b^3*f*e^(2*
f*x + 2*e) + 2*a^3*f*e^(f*x + e) + 2*a*b^2*f*e^(f*x + e) - a^2*b*f - b^3*f
), x) + 4*a*c*d*f*integrate(x*e^(f*x + e)/(a^2*b*f*e^(2*f*x + 2*e) + b^3*f
*e^(2*f*x + 2*e) + 2*a^3*f*e^(f*x + e) + 2*a*b^2*f*e^(f*x + e) - a^2*b*f -
 b^3*f), x) + 2*b*c*d*(a*log((b*e^(f*x + e) + a - sqrt(a^2 + b^2))/(b*e^(f
*x + e) + a + sqrt(a^2 + b^2)))/((a^2*b + b^3)*sqrt(a^2 + b^2)*f^2) - 2*(f
*x + e)/((a^2*b + b^3)*f^2) + log(b*e^(2*f*x + 2*e) + 2*a*e^(f*x + e) - b)
/((a^2*b + b^3)*f^2)) - 4*a*d^2*integrate(x*e^(f*x + e)/(a^2*b*f*e^(2*f*x
+ 2*e) + b^3*f*e^(2*f*x + 2*e) + 2*a^3*f*e^(f*x + e) + 2*a*b^2*f*e^(f*x +
e) - a^2*b*f - b^3*f), x) + 4*b*d^2*integrate(x/(a^2*b*f*e^(2*f*x + 2*e) +
 b^3*f*e^(2*f*x + 2*e) + 2*a^3*f*e^(f*x + e) + 2*a*b^2*f*e^(f*x + e) - a^2
*b*f - b^3*f), x) + c^2*(a*log((b*e^(-f*x - e) - a - sqrt(a^2 + b^2))/(b*e
^(-f*x - e) - a + sqrt(a^2 + b^2)))/((a^2 + b^2)^(3/2)*f) - 2*(a*e^(-f*x -
 e) + b)/((a^2*b + b^3 + 2*(a^3 + a*b^2)*e^(-f*x - e) - (a^2*b + b^3)*e^(-
2*f*x - 2*e))*f)) - 2*a*c*d*log((b*e^(f*x + e) + a - sqrt(a^2 + b^2))/(b*e
^(f*x + e) + a + sqrt(a^2 + b^2)))/((a^2 + b^2)^(3/2)*f^2) + 2*(b*d^2*x^2
+ 2*b*c*d*x - (a*d^2*x^2*e^e + 2*a*c*d*x*e^e)*e^(f*x))/(a^2*b*f + b^3*f -
(a^2*b*f*e^(2*e) + b^3*f*e^(2*e))*e^(2*f*x) - 2*(a^3*f*e^e + a*b^2*f*e^e)*
e^(f*x))
```

Giac [F]

$$\int \frac{(c+dx)^2}{(a+b\sinh(e+fx))^2}\,dx = \int \frac{(dx+c)^2}{(b\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^2/(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(b*sinh(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{(a+b\sinh(e+fx))^2}\,dx = \int \frac{(c+d\,x)^2}{(a+b\sinh(e+f\,x))^2}\,dx$$

input
```
int((c + d*x)^2/(a + b*sinh(e + f*x))^2,x)
```

output
```
int((c + d*x)^2/(a + b*sinh(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{(c+dx)^2}{(a+b\sinh(e+fx))^2}\,dx = \text{too large to display}$$

input
```
int((d*x+c)^2/(a+b*sinh(f*x+e))^2,x)
```

output

```
(4*e**(2*e + 2*f*x)*sqrt(a**2 + b**2)*atan((e**(e + f*x)*b*i + a*i)/sqrt(a
**2 + b**2))*a**3*b*c*d*f*i + 2*e**(2*e + 2*f*x)*sqrt(a**2 + b**2)*atan((e
**(e + f*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**3*b*d**2*i + 2*e**(2*e + 2*f*
x)*sqrt(a**2 + b**2)*atan((e**(e + f*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b*
*3*c**2*f**2*i + 4*e**(2*e + 2*f*x)*sqrt(a**2 + b**2)*atan((e**(e + f*x)*b
*i + a*i)/sqrt(a**2 + b**2))*a*b**3*c*d*f*i + 2*e**(2*e + 2*f*x)*sqrt(a**2
 + b**2)*atan((e**(e + f*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b**3*d**2*i +
8*e**(e + f*x)*sqrt(a**2 + b**2)*atan((e**(e + f*x)*b*i + a*i)/sqrt(a**2 +
 b**2))*a**4*c*d*f*i + 4*e**(e + f*x)*sqrt(a**2 + b**2)*atan((e**(e + f*x)
*b*i + a*i)/sqrt(a**2 + b**2))*a**4*d**2*i + 4*e**(e + f*x)*sqrt(a**2 + b*
*2)*atan((e**(e + f*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b**2*c**2*f**2*i
 + 8*e**(e + f*x)*sqrt(a**2 + b**2)*atan((e**(e + f*x)*b*i + a*i)/sqrt(a**
2 + b**2))*a**2*b**2*c*d*f*i + 4*e**(e + f*x)*sqrt(a**2 + b**2)*atan((e**(
e + f*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b**2*d**2*i - 4*sqrt(a**2 + b*
*2)*atan((e**(e + f*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**3*b*c*d*f*i - 2*sq
rt(a**2 + b**2)*atan((e**(e + f*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**3*b*d*
*2*i - 2*sqrt(a**2 + b**2)*atan((e**(e + f*x)*b*i + a*i)/sqrt(a**2 + b**2)
)*a*b**3*c**2*f**2*i - 4*sqrt(a**2 + b**2)*atan((e**(e + f*x)*b*i + a*i)/s
qrt(a**2 + b**2))*a*b**3*c*d*f*i - 2*sqrt(a**2 + b**2)*atan((e**(e + f*x)*
b*i + a*i)/sqrt(a**2 + b**2))*a*b**3*d**2*i - 4*e**(3*e + 2*f*x)*int((e...
```

3.175 $\int \frac{c+dx}{(a+b\sinh(e+fx))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 254

$$\int \frac{c+dx}{(a+b\sinh(e+fx))^2}\,dx = \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}f} - \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}f} + \frac{d\log(a+b\sinh(e+fx))}{(a^2+b^2)f^2} + \frac{ad\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}f^2} - \frac{ad\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}f^2} - \frac{b(c+dx)\cosh(e+fx)}{(a^2+b^2)f(a+b\sinh(e+fx))}$$

output
```
a*(d*x+c)*ln(1+b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/f-a*(d*x+
c)*ln(1+b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/f+d*ln(a+b*sinh(
f*x+e))/(a^2+b^2)/f^2+a*d*polylog(2,-b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/(a^
2+b^2)^(3/2)/f^2-a*d*polylog(2,-b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2
)^(3/2)/f^2-b*(d*x+c)*cosh(f*x+e)/(a^2+b^2)/f/(a+b*sinh(f*x+e))
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.76

$$\int \frac{c+dx}{(a+b\sinh(e+fx))^2}\,dx$$
$$= \frac{d\log(a+b\sinh(e+fx)) + \frac{a\left(f(c+dx)\left(\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)-\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)\right)+d\,\text{PolyLog}\left(2,\frac{be^{e+fx}}{-a+\sqrt{a^2+b^2}}\right)-d\,\text{PolyLog}\left(2,-\right.}{\sqrt{a^2+b^2}}}{(a^2+b^2)\,f^2}$$

input
```
Integrate[(c + d*x)/(a + b*Sinh[e + f*x])^2,x]
```

output
```
(d*Log[a + b*Sinh[e + f*x]] + (a*(f*(c + d*x)*(Log[1 + (b*E^(e + f*x))/(a
- Sqrt[a^2 + b^2])] - Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])]) + d*
PolyLog[2, (b*E^(e + f*x))/(-a + Sqrt[a^2 + b^2])] - d*PolyLog[2, -((b*E^(
e + f*x))/(a + Sqrt[a^2 + b^2]))]))/Sqrt[a^2 + b^2] - (b*f*(c + d*x)*Cosh[
e + f*x])/(a + b*Sinh[e + f*x]))/((a^2 + b^2)*f^2)
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3805, 3042, 3147, 16, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a+b\sinh(e+fx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{c+dx}{(a-ib\sin(ie+ifx))^2}dx$$
$$\Big\downarrow \text{3805}$$
$$\frac{a\int \frac{c+dx}{a+b\sinh(e+fx)}dx}{a^2+b^2}+\frac{bd\int \frac{\cosh(e+fx)}{a+b\sinh(e+fx)}dx}{f\,(a^2+b^2)}-\frac{b(c+dx)\cosh(e+fx)}{f\,(a^2+b^2)\,(a+b\sinh(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{c+dx}{a-ib\sin(ie+ifx)}dx}{a^2+b^2}+\frac{bd\int \frac{\cos(ie+ifx)}{a-ib\sin(ie+ifx)}dx}{f(a^2+b^2)}-\frac{b(c+dx)\cosh(e+fx)}{f(a^2+b^2)(a+b\sinh(e+fx))}$$

$$\Big\downarrow \text{3147}$$

$$\frac{a\int \frac{c+dx}{a-ib\sin(ie+ifx)}dx}{a^2+b^2}+\frac{d\int \frac{1}{a+b\sinh(e+fx)}d(b\sinh(e+fx))}{f^2(a^2+b^2)}-\frac{b(c+dx)\cosh(e+fx)}{f(a^2+b^2)(a+b\sinh(e+fx))}$$

$$\Big\downarrow \text{16}$$

$$\frac{a\int \frac{c+dx}{a-ib\sin(ie+ifx)}dx}{a^2+b^2}-\frac{b(c+dx)\cosh(e+fx)}{f(a^2+b^2)(a+b\sinh(e+fx))}+\frac{d\log(a+b\sinh(e+fx))}{f^2(a^2+b^2)}$$

$$\Big\downarrow \text{3803}$$

$$\frac{2a\int -\frac{e^{e+fx}(c+dx)}{-2e^{e+fx}a-be^{2(e+fx)}+b}dx}{a^2+b^2}-\frac{b(c+dx)\cosh(e+fx)}{f(a^2+b^2)(a+b\sinh(e+fx))}+\frac{d\log(a+b\sinh(e+fx))}{f^2(a^2+b^2)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2a\int \frac{e^{e+fx}(c+dx)}{-2e^{e+fx}a-be^{2(e+fx)}+b}dx}{a^2+b^2}-\frac{b(c+dx)\cosh(e+fx)}{f(a^2+b^2)(a+b\sinh(e+fx))}+\frac{d\log(a+b\sinh(e+fx))}{f^2(a^2+b^2)}$$

$$\Big\downarrow \text{2694}$$

$$-\frac{2a\left(\frac{b\int -\frac{e^{e+fx}(c+dx)}{2\left(a+be^{e+fx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{e+fx}(c+dx)}{2\left(a+be^{e+fx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}-\frac{b(c+dx)\cosh(e+fx)}{f(a^2+b^2)(a+b\sinh(e+fx))}+\frac{d\log(a+b\sinh(e+fx))}{f^2(a^2+b^2)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2a\left(\frac{b\int \frac{e^{e+fx}(c+dx)}{a+be^{e+fx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{e+fx}(c+dx)}{a+be^{e+fx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}-\frac{b(c+dx)\cosh(e+fx)}{f(a^2+b^2)(a+b\sinh(e+fx))}+\frac{d\log(a+b\sinh(e+fx))}{f^2(a^2+b^2)}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{2a\left(\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{d\int \log\left(\frac{e^{e+fx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{bf}-\frac{d\int \log\left(\frac{e^{e+fx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bf}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}-\frac{b(c+dx)\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}+\frac{d\log(a+b\sinh(e+fx))}{f^2\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{2a\left(\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{d\int e^{-e-fx}\log\left(\frac{e^{e+fx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{e+fx}}{bf^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{bf}-\frac{d\int e^{-e-fx}\log\left(\frac{e^{e+fx}b}{a-\sqrt{a^2+b^2}}+1\right.}{bf^2}}{2\sqrt{a^2+b^2}}}{a^2+b^2}$$

$$\frac{b(c+dx)\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}+\frac{d\log(a+b\sinh(e+fx))}{f^2\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{2a\left(\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}+\frac{d\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{bf^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{bf}+\frac{d\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{bf^2}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}-\frac{b(c+dx)\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}+\frac{d\log(a+b\sinh(e+fx))}{f^2\left(a^2+b^2\right)}$$

input
```
Int[(c + d*x)/(a + b*Sinh[e + f*x])^2,x]
```

output
```
(d*Log[a + b*Sinh[e + f*x]])/((a^2 + b^2)*f^2) - (2*a*(-1/2*(b*(((c + d*x)
*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])])/(b*f) + (d*PolyLog[2, -((
b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/(b*f^2)))/Sqrt[a^2 + b^2] + (b*(((
c + d*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])])/(b*f) + (d*PolyLo
g[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/(b*f^2)))/(2*Sqrt[a^2 + b^
2])))/(a^2 + b^2) - (b*(c + d*x)*Cosh[e + f*x])/((a^2 + b^2)*f*(a + b*Sinh
[e + f*x]))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(234) = 468$.

Time = 0.32 (sec) , antiderivative size = 519, normalized size of antiderivative = 2.04

method	result
risch	$\frac{2(dx+c)\left(a\,\mathrm{e}^{fx+e}-b\right)}{f\left(a^{2}+b^{2}\right)\left(\mathrm{e}^{2fx+2e}b+2a\,\mathrm{e}^{fx+e}-b\right)}-\frac{2d\ln\left(\mathrm{e}^{fx+e}\right)}{f^{2}\left(a^{2}+b^{2}\right)}+\frac{d\ln\left(\mathrm{e}^{2fx+2e}b+2a\,\mathrm{e}^{fx+e}-b\right)}{f^{2}\left(a^{2}+b^{2}\right)}-\frac{2ac\,\operatorname{arctanh}\left(\frac{2b\,\mathrm{e}^{fx+e}+2a}{2\sqrt{a^{2}+b^{2}}}\right)}{f\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}+\frac{da\ln\Big(}{}$

input
```
int((d*x+c)/(a+b*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
2*(d*x+c)*(a*exp(f*x+e)-b)/f/(a^2+b^2)/(exp(2*f*x+2*e)*b+2*a*exp(f*x+e)-b)
-2/f^2/(a^2+b^2)*d*ln(exp(f*x+e))+1/f^2/(a^2+b^2)*d*ln(exp(2*f*x+2*e)*b+2*
a*exp(f*x+e)-b)-2/f/(a^2+b^2)^(3/2)*a*c*arctanh(1/2*(2*b*exp(f*x+e)+2*a)/(
a^2+b^2)^(1/2))+1/f/(a^2+b^2)^(3/2)*d*a*ln((-b*exp(f*x+e)+(a^2+b^2)^(1/2)-
a)/(-a+(a^2+b^2)^(1/2)))*x-1/f/(a^2+b^2)^(3/2)*d*a*ln((b*exp(f*x+e)+(a^2+b
^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/f^2/(a^2+b^2)^(3/2)*d*a*ln((-b*exp(f
*x+e)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*e-1/f^2/(a^2+b^2)^(3/2)*d*a
*ln((b*exp(f*x+e)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*e+1/f^2/(a^2+b^2
)^(3/2)*d*a*dilog((-b*exp(f*x+e)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-
1/f^2/(a^2+b^2)^(3/2)*d*a*dilog((b*exp(f*x+e)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b
^2)^(1/2)))+2/f^2/(a^2+b^2)^(3/2)*a*d*e*arctanh(1/2*(2*b*exp(f*x+e)+2*a)/(
a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1717 vs. $2(232) = 464$.

Time = 0.12 (sec) , antiderivative size = 1717, normalized size of antiderivative = 6.76

$$\int \frac{c+dx}{(a+b\sinh(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
(2*(a^2*b + b^3)*d*e - 2*(a^2*b + b^3)*c*f - 2*((a^2*b + b^3)*d*f*x + (a^2
*b + b^3)*d*e)*cosh(f*x + e)^2 - 2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*
e)*sinh(f*x + e)^2 + (a*b^2*d*cosh(f*x + e)^2 + a*b^2*d*sinh(f*x + e)^2 +
2*a^2*b*d*cosh(f*x + e) - a*b^2*d + 2*(a*b^2*d*cosh(f*x + e) + a^2*b*d)*si
nh(f*x + e))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(f*x + e) + a*sinh(f*x + e
) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b + 1)
- (a*b^2*d*cosh(f*x + e)^2 + a*b^2*d*sinh(f*x + e)^2 + 2*a^2*b*d*cosh(f*x
+ e) - a*b^2*d + 2*(a*b^2*d*cosh(f*x + e) + a^2*b*d)*sinh(f*x + e))*sqrt((
a^2 + b^2)/b^2)*dilog((a*cosh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x + e
) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (a*b^2*d*f*x + a*
b^2*d*e - (a*b^2*d*f*x + a*b^2*d*e)*cosh(f*x + e)^2 - (a*b^2*d*f*x + a*b^2
*d*e)*sinh(f*x + e)^2 - 2*(a^2*b*d*f*x + a^2*b*d*e)*cosh(f*x + e) - 2*(a^2
*b*d*f*x + a^2*b*d*e + (a*b^2*d*f*x + a*b^2*d*e)*cosh(f*x + e))*sinh(f*x +
 e))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(f*x + e) + a*sinh(f*x + e) + (b*co
sh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b) + (a*b^2*d*f*
x + a*b^2*d*e - (a*b^2*d*f*x + a*b^2*d*e)*cosh(f*x + e)^2 - (a*b^2*d*f*x +
 a*b^2*d*e)*sinh(f*x + e)^2 - 2*(a^2*b*d*f*x + a^2*b*d*e)*cosh(f*x + e) -
2*(a^2*b*d*f*x + a^2*b*d*e + (a*b^2*d*f*x + a*b^2*d*e)*cosh(f*x + e))*sinh
(f*x + e))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(f*x + e) + a*sinh(f*x + e) -
 (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx}{(a+b\sinh(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)/(a+b*sinh(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{c+dx}{(a+b\sinh(e+fx))^2}\,dx=\int \frac{dx+c}{(b\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)/(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
(2*a*f*integrate(x*e^(f*x + e)/(a^2*b*f*e^(2*f*x + 2*e) + b^3*f*e^(2*f*x +
 2*e) + 2*a^3*f*e^(f*x + e) + 2*a*b^2*f*e^(f*x + e) - a^2*b*f - b^3*f), x)
 + b*(a*log((b*e^(f*x + e) + a - sqrt(a^2 + b^2))/(b*e^(f*x + e) + a + sqr
t(a^2 + b^2)))/((a^2*b + b^3)*sqrt(a^2 + b^2)*f^2) - 2*(f*x + e)/((a^2*b +
 b^3)*f^2) + log(b*e^(2*f*x + 2*e) + 2*a*e^(f*x + e) - b)/((a^2*b + b^3)*f
^2)) - 2*(a*x*e^(f*x + e) - b*x)/(a^2*b*f + b^3*f - (a^2*b*f*e^(2*e) + b^3
*f*e^(2*e))*e^(2*f*x) - 2*(a^3*f*e^e + a*b^2*f*e^e)*e^(f*x)) - a*log((b*e^
(f*x + e) + a - sqrt(a^2 + b^2))/(b*e^(f*x + e) + a + sqrt(a^2 + b^2)))/((
a^2 + b^2)^(3/2)*f^2))*d + c*(a*log((b*e^(-f*x - e) - a - sqrt(a^2 + b^2))
/(b*e^(-f*x - e) - a + sqrt(a^2 + b^2)))/((a^2 + b^2)^(3/2)*f) - 2*(a*e^(-
f*x - e) + b)/((a^2*b + b^3 + 2*(a^3 + a*b^2)*e^(-f*x - e) - (a^2*b + b^3)
*e^(-2*f*x - 2*e))*f))
```

Giac [F]

$$\int \frac{c+dx}{(a+b\sinh(e+fx))^2}\,dx=\int \frac{dx+c}{(b\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)/(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)/(b*sinh(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{(a+b\sinh(e+fx))^2}\,dx=\int \frac{c+d\,x}{\left(a+b\sinh\left(e+f\,x\right)\right)^2}\,dx$$

input
```
int((c + d*x)/(a + b*sinh(e + f*x))^2,x)
```

output
```
int((c + d*x)/(a + b*sinh(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{c+dx}{(a+b\sinh(e+fx))^2}\,dx=\text{too large to display}$$

input
```
int((d*x+c)/(a+b*sinh(f*x+e))^2,x)
```

output
```
(2*e**(2*e + 2*f*x)*sqrt(a**2 + b**2)*atan((e**(e + f*x)*b*i + a*i)/sqrt(a
**2 + b**2))*a**3*b*d*i + 2*e**(2*e + 2*f*x)*sqrt(a**2 + b**2)*atan((e**(e
 + f*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b**3*c*f*i + 2*e**(2*e + 2*f*x)*sq
rt(a**2 + b**2)*atan((e**(e + f*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b**3*d*
i + 4*e**(e + f*x)*sqrt(a**2 + b**2)*atan((e**(e + f*x)*b*i + a*i)/sqrt(a*
*2 + b**2))*a**4*d*i + 4*e**(e + f*x)*sqrt(a**2 + b**2)*atan((e**(e + f*x)
*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b**2*c*f*i + 4*e**(e + f*x)*sqrt(a**2
+ b**2)*atan((e**(e + f*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b**2*d*i - 2
*sqrt(a**2 + b**2)*atan((e**(e + f*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**3*b
*d*i - 2*sqrt(a**2 + b**2)*atan((e**(e + f*x)*b*i + a*i)/sqrt(a**2 + b**2)
)*a*b**3*c*f*i - 2*sqrt(a**2 + b**2)*atan((e**(e + f*x)*b*i + a*i)/sqrt(a*
*2 + b**2))*a*b**3*d*i - 4*e**(3*e + 2*f*x)*int((e**(f*x)*x)/(e**(4*e + 4*
f*x)*b**2 + 4*e**(3*e + 3*f*x)*a*b + 4*e**(2*e + 2*f*x)*a**2 - 2*e**(2*e +
 2*f*x)*b**2 - 4*e**(e + f*x)*a*b + b**2),x)*a**5*b**2*d*f**2 - 8*e**(3*e
+ 2*f*x)*int((e**(f*x)*x)/(e**(4*e + 4*f*x)*b**2 + 4*e**(3*e + 3*f*x)*a*b
+ 4*e**(2*e + 2*f*x)*a**2 - 2*e**(2*e + 2*f*x)*b**2 - 4*e**(e + f*x)*a*b +
 b**2),x)*a**3*b**4*d*f**2 - 4*e**(3*e + 2*f*x)*int((e**(f*x)*x)/(e**(4*e
+ 4*f*x)*b**2 + 4*e**(3*e + 3*f*x)*a*b + 4*e**(2*e + 2*f*x)*a**2 - 2*e**(2
*e + 2*f*x)*b**2 - 4*e**(e + f*x)*a*b + b**2),x)*a*b**6*d*f**2 + e**(2*e +
 2*f*x)*log(e**(2*e + 2*f*x)*b + 2*e**(e + f*x)*a - b)*a**4*b*d + 2*e**...
```

3.176 $\int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2} dx = \text{Int}\left(\frac{1}{(c+dx)(a+b\sinh(e+fx))^2},x\right)$$

output
```
Defer(Int)(1/(d*x+c)/(a+b*sinh(f*x+e))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 25.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2} dx = \int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2} dx$$

input
```
Integrate[1/((c + d*x)*(a + b*Sinh[e + f*x])^2),x]
```

output
```
Integrate[1/((c + d*x)*(a + b*Sinh[e + f*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(c+dx)(a-ib\sin(ie+ifx))^2}dx$$

$$\Big\downarrow \text{ 3807}$$

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2}dx$$

input
```
Int[1/((c + d*x)*(a + b*Sinh[e + f*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)\,(a+b\sinh\,(fx+e))^2}dx$$

input
```
int(1/(d*x+c)/(a+b*sinh(f*x+e))^2,x)
```

output
```
int(1/(d*x+c)/(a+b*sinh(f*x+e))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.75

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(b\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*d*x + a^2*c + (b^2*d*x + b^2*c)*sinh(f*x + e)^2 + 2*(a*b*d
*x + a*b*c)*sinh(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)/(a+b*sinh(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 406, normalized size of antiderivative = 20.30

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(b\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
-2*(a*e^(f*x + e) - b)/(a^2*b*c*f + b^3*c*f + (a^2*b*d*f + b^3*d*f)*x - (a
^2*b*c*f*e^(2*e) + b^3*c*f*e^(2*e) + (a^2*b*d*f*e^(2*e) + b^3*d*f*e^(2*e))
*x)*e^(2*f*x) - 2*(a^3*c*f*e^e + a*b^2*c*f*e^e + (a^3*d*f*e^e + a*b^2*d*f*
e^e)*x)*e^(f*x)) + integrate(2*(b*d - (a*d*f*x*e^e + (c*f*e^e + d*e^e)*a)*
e^(f*x))/(a^2*b*c^2*f + b^3*c^2*f + (a^2*b*d^2*f + b^3*d^2*f)*x^2 + 2*(a^2
*b*c*d*f + b^3*c*d*f)*x - (a^2*b*c^2*f*e^(2*e) + b^3*c^2*f*e^(2*e) + (a^2*
b*d^2*f*e^(2*e) + b^3*d^2*f*e^(2*e))*x^2 + 2*(a^2*b*c*d*f*e^(2*e) + b^3*c*
d*f*e^(2*e))*x)*e^(2*f*x) - 2*(a^3*c^2*f*e^e + a*b^2*c^2*f*e^e + (a^3*d^2*
f*e^e + a*b^2*d^2*f*e^e)*x^2 + 2*(a^3*c*d*f*e^e + a*b^2*c*d*f*e^e)*x)*e^(f
*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(b\sinh{(fx+e)}+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)*(b*sinh(f*x + e) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2}\,dx = \int \frac{1}{(a+b\sinh{(e+f\,x)})^2\,(c+d\,x)}\,dx$$

input
```
int(1/((a + b*sinh(e + f*x))^2*(c + d*x)),x)
```

output
```
int(1/((a + b*sinh(e + f*x))^2*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.30

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2}\,dx$$
$$=\int \frac{1}{\sinh(fx+e)^2 b^2c+\sinh(fx+e)^2 b^2dx+2\sinh(fx+e)abc+2\sinh(fx+e)abdx+a^2c+a^2dx}dx$$

input
```
int(1/(d*x+c)/(a+b*sinh(f*x+e))^2,x)
```

output
```
int(1/(sinh(e + f*x)**2*b**2*c + sinh(e + f*x)**2*b**2*d*x + 2*sinh(e + f*
x)*a*b*c + 2*sinh(e + f*x)*a*b*d*x + a**2*c + a**2*d*x),x)
```

3.177 $\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2} dx = \mathrm{Int}\left(\frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2}, x\right)$$

output
```
Defer(Int)(1/(d*x+c)^2/(a+b*sinh(f*x+e))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 25.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2} dx = \int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2} dx$$

input
```
Integrate[1/((c + d*x)^2*(a + b*Sinh[e + f*x])^2),x]
```

output
```
Integrate[1/((c + d*x)^2*(a + b*Sinh[e + f*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c+dx)^2(a-ib\sin(ie+ifx))^2}dx$$

$$\Big\downarrow \text{3807}$$

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2}dx$$

input
```
Int[1/((c + d*x)^2*(a + b*Sinh[e + f*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(dx+c\right)^2 \left(a+b\sinh\left(fx+e\right)\right)^2}dx$$

input
```
int(1/(d*x+c)^2/(a+b*sinh(f*x+e))^2,x)
```

output
```
int(1/(d*x+c)^2/(a+b*sinh(f*x+e))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 4.80

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(b\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d
*x + b^2*c^2)*sinh(f*x + e)^2 + 2*(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*si
nh(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)**2/(a+b*sinh(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 606, normalized size of antiderivative = 30.30

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(b\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
-2*(a*e^(f*x + e) - b)/(a^2*b*c^2*f + b^3*c^2*f + (a^2*b*d^2*f + b^3*d^2*f
)*x^2 + 2*(a^2*b*c*d*f + b^3*c*d*f)*x - (a^2*b*c^2*f*e^(2*e) + b^3*c^2*f*e
^(2*e) + (a^2*b*d^2*f*e^(2*e) + b^3*d^2*f*e^(2*e))*x^2 + 2*(a^2*b*c*d*f*e^
(2*e) + b^3*c*d*f*e^(2*e))*x)*e^(2*f*x) - 2*(a^3*c^2*f*e^e + a*b^2*c^2*f*e
^e + (a^3*d^2*f*e^e + a*b^2*d^2*f*e^e)*x^2 + 2*(a^3*c*d*f*e^e + a*b^2*c*d*
f*e^e)*x)*e^(f*x)) + integrate(2*(2*b*d - (a*d*f*x*e^e + (c*f*e^e + 2*d*e^
e)*a)*e^(f*x))/(a^2*b*c^3*f + b^3*c^3*f + (a^2*b*d^3*f + b^3*d^3*f)*x^3 +
3*(a^2*b*c*d^2*f + b^3*c*d^2*f)*x^2 + 3*(a^2*b*c^2*d*f + b^3*c^2*d*f)*x -
(a^2*b*c^3*f*e^(2*e) + b^3*c^3*f*e^(2*e) + (a^2*b*d^3*f*e^(2*e) + b^3*d^3*
f*e^(2*e))*x^3 + 3*(a^2*b*c*d^2*f*e^(2*e) + b^3*c*d^2*f*e^(2*e))*x^2 + 3*(
a^2*b*c^2*d*f*e^(2*e) + b^3*c^2*d*f*e^(2*e))*x)*e^(2*f*x) - 2*(a^3*c^3*f*e
^e + a*b^2*c^3*f*e^e + (a^3*d^3*f*e^e + a*b^2*d^3*f*e^e)*x^3 + 3*(a^3*c*d^
2*f*e^e + a*b^2*c*d^2*f*e^e)*x^2 + 3*(a^3*c^2*d*f*e^e + a*b^2*c^2*d*f*e^e)
*x)*e^(f*x)), x)
```

Giac [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(b\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)^2*(b*sinh(f*x + e) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2}\,dx = \int \frac{1}{\left(a+b\sinh\left(e+f\,x\right)\right)^2\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/((a + b*sinh(e + f*x))^2*(c + d*x)^2),x)
```

output
```
int(1/((a + b*sinh(e + f*x))^2*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 6.05

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2}\,dx$$
$$= \int \frac{1}{\sinh\left(fx+e\right)^2 b^2c^2+2\sinh\left(fx+e\right)^2 b^2cdx+\sinh\left(fx+e\right)^2 b^2d^2x^2+2\sinh\left(fx+e\right)ab\,c^2+4\sinh\left(f\right.}$$

input
```
int(1/(d*x+c)^2/(a+b*sinh(f*x+e))^2,x)
```

output
```
int(1/(sinh(e + f*x)**2*b**2*c**2 + 2*sinh(e + f*x)**2*b**2*c*d*x + sinh(e
 + f*x)**2*b**2*d**2*x**2 + 2*sinh(e + f*x)*a*b*c**2 + 4*sinh(e + f*x)*a*b
*c*d*x + 2*sinh(e + f*x)*a*b*d**2*x**2 + a**2*c**2 + 2*a**2*c*d*x + a**2*d
**2*x**2),x)
```

3.178 $\int \frac{e+fx}{(a+b\sinh(c+dx))^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 544

$$\int \frac{e+fx}{(a+b\sinh(c+dx))^3}\,dx = \frac{3a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{5/2}d} - \frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}d} - \frac{3a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{5/2}d} + \frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}d} + \frac{3af\log(a+b\sinh(c+dx))}{2(a^2+b^2)^2d^2} + \frac{3a^2f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{5/2}d^2} - \frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}d^2} - \frac{3a^2f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{5/2}d^2} + \frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}d^2} - \frac{b(e+fx)\cosh(c+dx)}{2(a^2+b^2)d(a+b\sinh(c+dx))^2} - \frac{f}{2(a^2+b^2)d^2(a+b\sinh(c+dx))} - \frac{3ab(e+fx)\cosh(c+dx)}{2(a^2+b^2)^2d(a+b\sinh(c+dx))}$$

output
```
3/2*a^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(5/2)/d-1
/2*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d-3/2*a^
2*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(5/2)/d+1/2*(f*
x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d+3/2*a*f*ln(a
+b*sinh(d*x+c))/(a^2+b^2)^2/d^2+3/2*a^2*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+
b^2)^(1/2)))/(a^2+b^2)^(5/2)/d^2-1/2*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2
)^(1/2)))/(a^2+b^2)^(3/2)/d^2-3/2*a^2*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/(a^2+b^2)^(5/2)/d^2+1/2*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^
(1/2)))/(a^2+b^2)^(3/2)/d^2-1/2*b*(f*x+e)*cosh(d*x+c)/(a^2+b^2)/d/(a+b*sin
h(d*x+c))^2-1/2*f/(a^2+b^2)/d^2/(a+b*sinh(d*x+c))-3/2*a*b*(f*x+e)*cosh(d*x
+c)/(a^2+b^2)^2/d/(a+b*sinh(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 4.73 (sec) , antiderivative size = 773, normalized size of antiderivative = 1.42

$$\int \frac{e+fx}{(a+b\sinh(c+dx))^3}\,dx =$$

$$-\frac{-3a\sqrt{-(a^2+b^2)^2}f(c+dx)+6a^2\sqrt{a^2+b^2}f\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)-4a^2\sqrt{-a^2-b^2}de\,\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+2b^2\sqrt{-a^2-b^2}de\,\text{arctanh}\left(\cdots\right.}{\cdots}$$

input
```
Integrate[(e + f*x)/(a + b*Sinh[c + d*x])^3,x]
```

output
```
-1/2*(-((-3*a*Sqrt[-(a^2 + b^2)^2]*f*(c + d*x) + 6*a^2*Sqrt[a^2 + b^2]*f*A
rcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]] - 4*a^2*Sqrt[-a^2 - b^2]*d*e*A
rcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*b^2*Sqrt[-a^2 - b^2]*d*e*A
rcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 6*a^2*Sqrt[-a^2 - b^2]*f*Arc
Tanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 4*a^2*Sqrt[-a^2 - b^2]*c*f*Arc
Tanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 2*b^2*Sqrt[-a^2 - b^2]*c*f*Arc
Tanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*a^2*Sqrt[-a^2 - b^2]*f*(c +
d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - b^2*Sqrt[-a^2 - b^2]
*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*a^2*Sqrt[-
a^2 - b^2]*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + b^
2*Sqrt[-a^2 - b^2]*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
])] + 3*a*Sqrt[-(a^2 + b^2)^2]*f*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d
*x)))] + Sqrt[-a^2 - b^2]*(2*a^2 - b^2)*f*PolyLog[2, (b*E^(c + d*x))/(-a +
 Sqrt[a^2 + b^2])] + Sqrt[-a^2 - b^2]*(-2*a^2 + b^2)*f*PolyLog[2, -((b*E^(
c + d*x))/(a + Sqrt[a^2 + b^2]))])/Sqrt[-(a^2 + b^2)^2]) + (b*(a^2 + b^2)*
d*(e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2 + ((a^2 + b^2)*f + 3*a*
b*d*(e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x]))/((a^2 + b^2)^2*d^2)
```

Rubi [A] (verified)

Time = 3.79 (sec) , antiderivative size = 834, normalized size of antiderivative = 1.53, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 1.056$, Rules used = {3042, 3806, 26, 3042, 3147, 17, 3805, 3042, 3147, 16, 3803, 25, 2694, 27, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{(a+b\sinh(c+dx))^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{e+fx}{(a-ib\sin(ic+idx))^3}dx$$
$$\Big\downarrow \text{3806}$$

$$\frac{a\int \frac{e+fx}{(a+b\sinh(c+dx))^2}dx}{a^2+b^2}+\frac{ib\int \frac{i(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2(a^2+b^2)}+\frac{bf\int \frac{\cosh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2d(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}$$

$\downarrow$ 26

$$\frac{a\int \frac{e+fx}{(a+b\sinh(c+dx))^2}dx}{a^2+b^2}-\frac{b\int \frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2(a^2+b^2)}+\frac{bf\int \frac{\cosh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2d(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{a\int \frac{e+fx}{(a-ib\sin(ic+idx))^2}dx}{a^2+b^2}-\frac{b\int \frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2(a^2+b^2)}+\frac{bf\int \frac{\cos(ic+idx)}{(a-ib\sin(ic+idx))^2}dx}{2d(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}$$

$\downarrow$ 3147

$$\frac{f\int \frac{1}{(a+b\sinh(c+dx))^2}d(b\sinh(c+dx))}{2d^2(a^2+b^2)}+\frac{a\int \frac{e+fx}{(a-ib\sin(ic+idx))^2}dx}{a^2+b^2}-\frac{b\int \frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}$$

$\downarrow$ 17

$$\frac{a\int \frac{e+fx}{(a-ib\sin(ic+idx))^2}dx}{a^2+b^2}-\frac{b\int \frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2(a^2+b^2)}-\frac{f}{2d^2(a^2+b^2)(a+b\sinh(c+dx))}-\frac{b(e+fx)\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}$$

$\downarrow$ 3805

$$-\frac{b\int \frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2(a^2+b^2)}+\frac{a\left(\frac{a\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{bf\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{a^2+b^2}-\frac{f}{2d^2(a^2+b^2)(a+b\sinh(c+dx))}-\frac{b(e+fx)\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}$$

$\downarrow$ 3042

$$-\frac{b\int \frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2\left(a^2+b^2\right)}+\frac{a\left(\frac{a\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a^2+b^2}+\frac{bf\int \frac{\cos(ic+idx)}{a-ib\sin(ic+idx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{a^2+b^2}-\frac{f}{2d^2\left(a^2+b^2\right)(a+b\sinh(c+dx))}-\frac{b(e+fx)\cosh(c+dx)}{2d\left(a^2+b^2\right)(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{ 3147}$$

$$\frac{a\left(\frac{f\int \frac{1}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{d^2(a^2+b^2)}+\frac{a\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a^2+b^2}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{a^2+b^2}-\frac{b\int \frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2\left(a^2+b^2\right)}-\frac{f}{2d^2\left(a^2+b^2\right)(a+b\sinh(c+dx))}-\frac{b(e+fx)\cosh(c+dx)}{2d\left(a^2+b^2\right)(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{a\left(\frac{a\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{a^2+b^2}-\frac{b\int \frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2\left(a^2+b^2\right)}-\frac{f}{2d^2\left(a^2+b^2\right)(a+b\sinh(c+dx))}-\frac{b(e+fx)\cosh(c+dx)}{2d\left(a^2+b^2\right)(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{ 3803}$$

$$\frac{a\left(\frac{2a\int -\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{a^2+b^2}-\frac{b\int \frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2\left(a^2+b^2\right)}-\frac{f}{2d^2\left(a^2+b^2\right)(a+b\sinh(c+dx))}-\frac{b(e+fx)\cosh(c+dx)}{2d\left(a^2+b^2\right)(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{a\left(-\frac{2a\int \frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{a^2+b^2}-\frac{b\int \frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2\left(a^2+b^2\right)}-\frac{f}{2d^2\left(a^2+b^2\right)(a+b\sinh(c+dx))}-\frac{b(e+fx)\cosh(c+dx)}{2d\left(a^2+b^2\right)(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{ 2694}$$

$$\frac{a\left(-\frac{2a\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{a^2+b^2}-\frac{b\int \frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2\left(a^2+b^2\right)}-\frac{f}{2d^2\left(a^2+b^2\right)(a+b\sinh(c+dx))}-\frac{b(e+fx)\cosh(c+dx)}{2d\left(a^2+b^2\right)(a+b\sinh(c+dx))^2}$$

$\big\downarrow$ 27

$$\frac{a\left(-\frac{2a\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{a^2+b^2}-\frac{b\int \frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2\left(a^2+b^2\right)}-\frac{f}{2d^2\left(a^2+b^2\right)(a+b\sinh(c+dx))}-\frac{b(e+fx)\cosh(c+dx)}{2d\left(a^2+b^2\right)(a+b\sinh(c+dx))^2}$$

$\big\downarrow$ 2620

$$\frac{a\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{f\log(a-}{d^2}\right.}{a^2+b^2}$$

$$\frac{b\int \frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2\left(a^2+b^2\right)}-\frac{f}{2d^2\left(a^2+b^2\right)(a+b\sinh(c+dx))}-\frac{b(e+fx)\cosh(c+dx)}{2d\left(a^2+b^2\right)(a+b\sinh(c+dx))^2}$$

$\big\downarrow$ 2715

$$\frac{a\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right.}{bd^2}}{2\sqrt{a^2+b^2}}\right.}{a^2+b^2}\right.}{a^2+b^2}$$

$$\frac{b\int\frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2(a^2+b^2)}-\frac{f}{2d^2(a^2+b^2)(a+b\sinh(c+dx))}-\frac{b(e+fx)\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}$$

$\downarrow$ 2838

$$-\frac{b\int\frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2(a^2+b^2)}+$$

$$\frac{a\left(-\frac{2a\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{f\log\ldots\right.}{a^2+b^2}$$

$$\frac{f}{2d^2(a^2+b^2)(a+b\sinh(c+dx))}-\frac{b(e+fx)\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}$$

$\downarrow$ 7293

$$-\frac{b\int \left(\frac{e+fx}{b(a+b\sinh(c+dx))}-\frac{a(e+fx)}{b(a+b\sinh(c+dx))^2}\right)dx}{2\left(a^2+b^2\right)}+$$

$$\frac{a\left(-\frac{2a\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{f\log}{\cdots}\right.}{a^2+b^2}$$

$$\frac{f}{2d^2\left(a^2+b^2\right)(a+b\sinh(c+dx))}-\frac{b(e+fx)\cosh(c+dx)}{2d\left(a^2+b^2\right)(a+b\sinh(c+dx))^2}$$

$\downarrow$ 2009

$$-\frac{f}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))}-$$

$$\frac{b\left(-\frac{(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)a^2}{b(a^2+b^2)^{3/2}d}+\frac{(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)a^2}{b(a^2+b^2)^{3/2}d}-\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)a^2}{b(a^2+b^2)^{3/2}d^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)a^2}{b(a^2+b^2)^{3/2}d^2}\right.}{\cdots}$$

$$\frac{a\left(-\frac{b(e+fx)\cosh(c+dx)}{(a^2+b^2)d(a+b\sinh(c+dx))}+\frac{f\log(a+b\sinh(c+dx))}{(a^2+b^2)d^2}-\frac{2a\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log}{\cdots}\right.}{\cdots}\right.}{a^2+b^2}\right.}{a^2+b^2}$$

$$\frac{b(e+fx)\cosh(c+dx)}{2\left(a^2+b^2\right)d(a+b\sinh(c+dx))^2}$$

input

```
Int[(e + f*x)/(a + b*Sinh[c + d*x])^3,x]
```

output
```
-1/2*(b*(e + f*x)*Cosh[c + d*x])/((a^2 + b^2)*d*(a + b*Sinh[c + d*x])^2) -
 f/(2*(a^2 + b^2)*d^2*(a + b*Sinh[c + d*x])) - (b*(-((a^2*(e + f*x)*Log[1
+ (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)^(3/2)*d)) + ((e +
 f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]*d
) + (a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*(a^2
 + b^2)^(3/2)*d) - ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
)])/(b*Sqrt[a^2 + b^2]*d) - (a*f*Log[a + b*Sinh[c + d*x]])/(b*(a^2 + b^2)*
d^2) - (a^2*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^
2 + b^2)^(3/2)*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]
))])/(b*Sqrt[a^2 + b^2]*d^2) + (a^2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^2) - (f*PolyLog[2, -((b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^2) + (a*(e + f*x)*Cosh[c
 + d*x])/((a^2 + b^2)*d*(a + b*Sinh[c + d*x]))))/(2*(a^2 + b^2)) + (a*((f*
Log[a + b*Sinh[c + d*x]])/((a^2 + b^2)*d^2) - (2*a*(-1/2*(b*(((e + f*x)*Lo
g[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2)))/Sqrt[a^2 + b^2] + (b*(((e +
 f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(2*Sqrt[a^2 + b^2])
))/(a^2 + b^2) - (b*(e + f*x)*Cosh[c + d*x])/((a^2 + b^2)*d*(a + b*Sinh[c
+ d*x]))))/(a^2 + b^2)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

rule 3806
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(-b)*(c + d*x)^m*Cos[e + f*x]*((a + b*Sin[e + f*x])^(n +
 1)/(f*(n + 1)*(a^2 - b^2))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m*(a
 + b*Sin[e + f*x])^(n + 1), x], x] - Simp[b*((n + 2)/((n + 1)*(a^2 - b^2)))
   Int[(c + d*x)^m*Sin[e + f*x]*(a + b*Sin[e + f*x])^(n + 1), x], x] + Simp
[b*d*(m/(f*(n + 1)*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*Cos[e + f*x]*(a +
b*Sin[e + f*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2
 - b^2, 0] && ILtQ[n, -2] && IGtQ[m, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1231 vs. 2(480) = 960.

Time = 0.59 (sec) , antiderivative size = 1232, normalized size of antiderivative = 2.26

method	result	size
risch	Expression too large to display	1232

input
```
int((f*x+e)/(a+b*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
(2*a^2*b*d*f*x*exp(3*d*x+3*c)-b^3*d*f*x*exp(3*d*x+3*c)+6*a^3*d*f*x*exp(2*d
*x+2*c)+2*a^2*b*d*e*exp(3*d*x+3*c)-3*a*b^2*d*f*x*exp(2*d*x+2*c)-b^3*d*e*ex
p(3*d*x+3*c)+6*a^3*d*e*exp(2*d*x+2*c)-10*a^2*b*d*f*x*exp(d*x+c)-a^2*b*f*ex
p(3*d*x+3*c)-3*a*b^2*d*e*exp(2*d*x+2*c)-b^3*d*f*x*exp(d*x+c)-b^3*f*exp(3*d
*x+3*c)-2*a^3*f*exp(2*d*x+2*c)-10*a^2*b*d*e*exp(d*x+c)+3*a*b^2*d*f*x-2*a*b
^2*f*exp(2*d*x+2*c)-b^3*d*e*exp(d*x+c)+a^2*b*f*exp(d*x+c)+3*a*b^2*d*e+b^3*
f*exp(d*x+c))/d^2/(a^2+b^2)^2/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)^2-1/(a^2
+b^2)^(5/2)/d^2*b^2*f*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-
3/(a^2+b^2)^2/d^2*a*f*ln(exp(d*x+c))+3/2/(a^2+b^2)^2/d^2*a*f*ln(b*exp(2*d*
x+2*c)+2*a*exp(d*x+c)-b)-2/(a^2+b^2)^(5/2)/d*a^2*e*arctanh(1/2*(2*b*exp(d*
x+c)+2*a)/(a^2+b^2)^(1/2))+2/(a^2+b^2)^(5/2)/d^2*a^2*f*c*arctanh(1/2*(2*b*
exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/(a^2+b^2)^(5/2)/d*a^2*f*ln((-b*exp(d*x+
c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/(a^2+b^2)^(5/2)/d*a^2*f*ln
((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/(a^2+b^2)^(5/2)
/d^2*a^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/
(a^2+b^2)^(5/2)/d^2*a^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)
^(1/2)))*c+1/(a^2+b^2)^(5/2)/d^2*a^2*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2
)-a)/(-a+(a^2+b^2)^(1/2)))-1/(a^2+b^2)^(5/2)/d^2*a^2*f*dilog((b*exp(d*x+c)
+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/(a^2+b^2)^(5/2)/d*b^2*e*arctanh
(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/2/(a^2+b^2)^(5/2)/d*b^2*f*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6396 vs. $2(476) = 952$.

Time = 0.21 (sec) , antiderivative size = 6396, normalized size of antiderivative = 11.76

$$\int \frac{e+fx}{(a+b\sinh(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e+fx}{(a+b\sinh(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)/(a+b*sinh(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e+fx}{(a+b\sinh(c+dx))^3}\,dx = \int \frac{fx+e}{(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate((f*x+e)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*(4*a^2*d*integrate(x*e^(d*x + c)/(a^4*b*d*e^(2*d*x + 2*c) + 2*a^2*b^3*
d*e^(2*d*x + 2*c) + b^5*d*e^(2*d*x + 2*c) + 2*a^5*d*e^(d*x + c) + 4*a^3*b^
2*d*e^(d*x + c) + 2*a*b^4*d*e^(d*x + c) - a^4*b*d - 2*a^2*b^3*d - b^5*d),
x) - 2*b^2*d*integrate(x*e^(d*x + c)/(a^4*b*d*e^(2*d*x + 2*c) + 2*a^2*b^3*
d*e^(2*d*x + 2*c) + b^5*d*e^(2*d*x + 2*c) + 2*a^5*d*e^(d*x + c) + 4*a^3*b^
2*d*e^(d*x + c) + 2*a*b^4*d*e^(d*x + c) - a^4*b*d - 2*a^2*b^3*d - b^5*d),
x) + 3*a*b*(a*log((b*e^(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(d*x + c) + a
 + sqrt(a^2 + b^2)))/((a^4*b + 2*a^2*b^3 + b^5)*sqrt(a^2 + b^2)*d^2) - 2*(
d*x + c)/((a^4*b + 2*a^2*b^3 + b^5)*d^2) + log(b*e^(2*d*x + 2*c) + 2*a*e^(
d*x + c) - b)/((a^4*b + 2*a^2*b^3 + b^5)*d^2)) + 2*(3*a*b^2*d*x - (a^2*b*e
^(3*c) + b^3*e^(3*c) - (2*a^2*b*d*e^(3*c) - b^3*d*e^(3*c))*x)*e^(3*d*x) -
(2*a^3*e^(2*c) + 2*a*b^2*e^(2*c) - 3*(2*a^3*d*e^(2*c) - a*b^2*d*e^(2*c))*x
)*e^(2*d*x) + (a^2*b*e^c + b^3*e^c - (10*a^2*b*d*e^c + b^3*d*e^c)*x)*e^(d*
x))/(a^4*b^2*d^2 + 2*a^2*b^4*d^2 + b^6*d^2 + (a^4*b^2*d^2*e^(4*c) + 2*a^2*
b^4*d^2*e^(4*c) + b^6*d^2*e^(4*c))*e^(4*d*x) + 4*(a^5*b*d^2*e^(3*c) + 2*a^
3*b^3*d^2*e^(3*c) + a*b^5*d^2*e^(3*c))*e^(3*d*x) + 2*(2*a^6*d^2*e^(2*c) +
3*a^4*b^2*d^2*e^(2*c) - b^6*d^2*e^(2*c))*e^(2*d*x) - 4*(a^5*b*d^2*e^c + 2*
a^3*b^3*d^2*e^c + a*b^5*d^2*e^c)*e^(d*x)) - 3*a^2*log((b*e^(d*x + c) + a -
 sqrt(a^2 + b^2))/(b*e^(d*x + c) + a + sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2
 + b^4)*sqrt(a^2 + b^2)*d^2))*f + 1/2*e*((2*a^2 - b^2)*log((b*e^(-d*x -...
```

Giac [F]

$$\int \frac{e+fx}{(a+b\sinh(c+dx))^3}\,dx=\int \frac{fx+e}{(b\sinh{(dx+c)}+a)^3}\,dx$$

input
```
integrate((f*x+e)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)/(b*sinh(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{(a+b\sinh(c+dx))^3}\,dx = \int \frac{e+f\,x}{\left(a+b\sinh\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int((e + f*x)/(a + b*sinh(c + d*x))^3,x)
```

output
```
int((e + f*x)/(a + b*sinh(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{e+fx}{(a+b\sinh(c+dx))^3}\,dx = \text{too large to display}$$

input
```
int((f*x+e)/(a+b*sinh(d*x+c))^3,x)
```

output
```
(16*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(
a**2 + b**2))*a**7*b**2*f*i + 40*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((
e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**5*b**4*f*i + 24*e**(4*c + 4*
d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*
*3*b**6*d*e*i + 8*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*
i + a*i)/sqrt(a**2 + b**2))*a**3*b**6*f*i - 12*e**(4*c + 4*d*x)*sqrt(a**2
+ b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b**8*d*e*i - 16
*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**
2 + b**2))*a*b**8*f*i + 64*e**(3*c + 3*d*x)*sqrt(a**2 + b**2)*atan((e**(c
+ d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**8*b*f*i + 160*e**(3*c + 3*d*x)*sqr
t(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**6*b**3*
f*i + 96*e**(3*c + 3*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/
sqrt(a**2 + b**2))*a**4*b**5*d*e*i + 32*e**(3*c + 3*d*x)*sqrt(a**2 + b**2)
*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**4*b**5*f*i - 48*e**(3
*c + 3*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b*
*2))*a**2*b**7*d*e*i - 64*e**(3*c + 3*d*x)*sqrt(a**2 + b**2)*atan((e**(c +
 d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b**7*f*i + 64*e**(2*c + 2*d*x)*sq
rt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**9*f*i
+ 128*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqr
t(a**2 + b**2))*a**7*b**2*f*i + 96*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*a...
```

3.179 $\int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3}\,dx = \text{Int}\left(\frac{1}{(e+fx)(a+b\sinh(c+dx))^3},x\right)$$

output
```
Defer(Int)(1/(f*x+e)/(a+b*sinh(d*x+c))^3,x)
```

Mathematica [N/A]

Not integrable

Time = 44.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3}\,dx = \int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3}\,dx$$

input
```
Integrate[1/((e + f*x)*(a + b*Sinh[c + d*x])^3),x]
```

output
```
Integrate[1/((e + f*x)*(a + b*Sinh[c + d*x])^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(e+fx)(a-ib\sin(ic+idx))^3}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3}dx$$

input
```
Int[1/((e + f*x)*(a + b*Sinh[c + d*x])^3),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(fx+e)\left(a+b\sinh\left(dx+c\right)\right)^{3}}dx$$

input
```
int(1/(f*x+e)/(a+b*sinh(d*x+c))^3,x)
```

output
```
int(1/(f*x+e)/(a+b*sinh(d*x+c))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 4.15

$$\int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3}\,dx=\int \frac{1}{(fx+e)(b\sinh{(dx+c)}+a)^3}\,dx$$

input
```
integrate(1/(f*x+e)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(1/(a^3*f*x + a^3*e + (b^3*f*x + b^3*e)*sinh(d*x + c)^3 + 3*(a*b^2
*f*x + a*b^2*e)*sinh(d*x + c)^2 + 3*(a^2*b*f*x + a^2*b*e)*sinh(d*x + c)),
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3}\,dx=\text{Timed out}$$

input
```
integrate(1/(f*x+e)/(a+b*sinh(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 2.47 (sec) , antiderivative size = 1651, normalized size of antiderivative = 82.55

$$\int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3}\,dx=\int \frac{1}{(fx+e)(b\sinh{(dx+c)}+a)^3}\,dx$$

input
```
integrate(1/(f*x+e)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
(3*a*b^2*d*f*x + 3*a*b^2*d*e + ((2*d*e + f)*a^2*b*e^(3*c) - (d*e - f)*b^3*
e^(3*c) + (2*a^2*b*d*f*e^(3*c) - b^3*d*f*e^(3*c))*x)*e^(3*d*x) + (2*(3*d*e
 + f)*a^3*e^(2*c) - (3*d*e - 2*f)*a*b^2*e^(2*c) + 3*(2*a^3*d*f*e^(2*c) - a
*b^2*d*f*e^(2*c))*x)*e^(2*d*x) - ((10*d*e + f)*a^2*b*e^c + (d*e + f)*b^3*e
^c + (10*a^2*b*d*f*e^c + b^3*d*f*e^c)*x)*e^(d*x))/(a^4*b^2*d^2*e^2 + 2*a^2
*b^4*d^2*e^2 + b^6*d^2*e^2 + (a^4*b^2*d^2*f^2 + 2*a^2*b^4*d^2*f^2 + b^6*d^
2*f^2)*x^2 + 2*(a^4*b^2*d^2*e*f + 2*a^2*b^4*d^2*e*f + b^6*d^2*e*f)*x + (a^
4*b^2*d^2*e^2*e^(4*c) + 2*a^2*b^4*d^2*e^2*e^(4*c) + b^6*d^2*e^2*e^(4*c) +
(a^4*b^2*d^2*f^2*e^(4*c) + 2*a^2*b^4*d^2*f^2*e^(4*c) + b^6*d^2*f^2*e^(4*c)
)*x^2 + 2*(a^4*b^2*d^2*e*f*e^(4*c) + 2*a^2*b^4*d^2*e*f*e^(4*c) + b^6*d^2*e
*f*e^(4*c))*x)*e^(4*d*x) + 4*(a^5*b*d^2*e^2*e^(3*c) + 2*a^3*b^3*d^2*e^2*e^
(3*c) + a*b^5*d^2*e^2*e^(3*c) + (a^5*b*d^2*f^2*e^(3*c) + 2*a^3*b^3*d^2*f^2
*e^(3*c) + a*b^5*d^2*f^2*e^(3*c))*x^2 + 2*(a^5*b*d^2*e*f*e^(3*c) + 2*a^3*b
^3*d^2*e*f*e^(3*c) + a*b^5*d^2*e*f*e^(3*c))*x)*e^(3*d*x) + 2*(2*a^6*d^2*e^
2*e^(2*c) + 3*a^4*b^2*d^2*e^2*e^(2*c) - b^6*d^2*e^2*e^(2*c) + (2*a^6*d^2*f
^2*e^(2*c) + 3*a^4*b^2*d^2*f^2*e^(2*c) - b^6*d^2*f^2*e^(2*c))*x^2 + 2*(2*a
^6*d^2*e*f*e^(2*c) + 3*a^4*b^2*d^2*e*f*e^(2*c) - b^6*d^2*e*f*e^(2*c))*x)*e
^(2*d*x) - 4*(a^5*b*d^2*e^2*e^c + 2*a^3*b^3*d^2*e^2*e^c + a*b^5*d^2*e^2*e^
c + (a^5*b*d^2*f^2*e^c + 2*a^3*b^3*d^2*f^2*e^c + a*b^5*d^2*f^2*e^c)*x^2 +
2*(a^5*b*d^2*e*f*e^c + 2*a^3*b^3*d^2*e*f*e^c + a*b^5*d^2*e*f*e^c)*x)*e^...
```

Giac [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3}\,dx = \int \frac{1}{(fx+e)(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate(1/(f*x+e)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((f*x + e)*(b*sinh(d*x + c) + a)^3), x)
```

Mupad [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3} \, dx = \int \frac{1}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)^3} \, dx$$

input
```
int(1/((e + f*x)*(a + b*sinh(c + d*x))^3),x)
```

output
```
int(1/((e + f*x)*(a + b*sinh(c + d*x))^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 5.05

$$\int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3} \, dx$$
$$= \int \frac{1}{\sinh\left(dx+c\right)^3 b^3 e+\sinh\left(dx+c\right)^3 b^3 f x+3\sinh\left(dx+c\right)^2 a\,b^2 e+3\sinh\left(dx+c\right)^2 a\,b^2 f x+3\sinh\left(dx\right.}$$

input
```
int(1/(f*x+e)/(a+b*sinh(d*x+c))^3,x)
```

output
```
int(1/(sinh(c + d*x)**3*b**3*e + sinh(c + d*x)**3*b**3*f*x + 3*sinh(c + d*
x)**2*a*b**2*e + 3*sinh(c + d*x)**2*a*b**2*f*x + 3*sinh(c + d*x)*a**2*b*e
+ 3*sinh(c + d*x)*a**2*b*f*x + a**3*e + a**3*f*x),x)
```

3.180 $\int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\,dx = \text{Int}\left(\frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3},x\right)$$

output
```
Defer(Int)(1/(f*x+e)^2/(a+b*sinh(d*x+c))^3,x)
```

Mathematica [N/A]

Not integrable

Time = 43.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\,dx = \int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\,dx$$

input
```
Integrate[1/((e + f*x)^2*(a + b*Sinh[c + d*x])^3),x]
```

output
```
Integrate[1/((e + f*x)^2*(a + b*Sinh[c + d*x])^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(e+fx)^2(a-ib\sin(ic+idx))^3}dx$$

$$\Big\downarrow \text{ 3807}$$

$$\int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}dx$$

input
```
Int[1/((e + f*x)^2*(a + b*Sinh[c + d*x])^3),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(fx+e)^2\,(a+b\sinh(dx+c))^3}dx$$

input
```
int(1/(f*x+e)^2/(a+b*sinh(d*x+c))^3,x)
```

output
```
int(1/(f*x+e)^2/(a+b*sinh(d*x+c))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 141, normalized size of antiderivative = 7.05

$$\int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\,dx = \int \frac{1}{(fx+e)^2(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate(1/(f*x+e)^2/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(1/(a^3*f^2*x^2 + 2*a^3*e*f*x + a^3*e^2 + (b^3*f^2*x^2 + 2*b^3*e*f
*x + b^3*e^2)*sinh(d*x + c)^3 + 3*(a*b^2*f^2*x^2 + 2*a*b^2*e*f*x + a*b^2*e
^2)*sinh(d*x + c)^2 + 3*(a^2*b*f^2*x^2 + 2*a^2*b*e*f*x + a^2*b*e^2)*sinh(d
*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(f*x+e)**2/(a+b*sinh(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 3.83 (sec) , antiderivative size = 2122, normalized size of antiderivative = 106.10

$$\int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\,dx = \int \frac{1}{(fx+e)^2(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate(1/(f*x+e)^2/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
(3*a*b^2*d*f*x + 3*a*b^2*d*e + (2*(d*e + f)*a^2*b*e^(3*c) - (d*e - 2*f)*b^
3*e^(3*c) + (2*a^2*b*d*f*e^(3*c) - b^3*d*f*e^(3*c))*x)*e^(3*d*x) + (2*(3*d
*e + 2*f)*a^3*e^(2*c) - (3*d*e - 4*f)*a*b^2*e^(2*c) + 3*(2*a^3*d*f*e^(2*c)
 - a*b^2*d*f*e^(2*c))*x)*e^(2*d*x) - (2*(5*d*e + f)*a^2*b*e^c + (d*e + 2*f
)*b^3*e^c + (10*a^2*b*d*f*e^c + b^3*d*f*e^c)*x)*e^(d*x))/(a^4*b^2*d^2*e^3
+ 2*a^2*b^4*d^2*e^3 + b^6*d^2*e^3 + (a^4*b^2*d^2*f^3 + 2*a^2*b^4*d^2*f^3 +
 b^6*d^2*f^3)*x^3 + 3*(a^4*b^2*d^2*e*f^2 + 2*a^2*b^4*d^2*e*f^2 + b^6*d^2*e
*f^2)*x^2 + 3*(a^4*b^2*d^2*e^2*f + 2*a^2*b^4*d^2*e^2*f + b^6*d^2*e^2*f)*x
+ (a^4*b^2*d^2*e^3*e^(4*c) + 2*a^2*b^4*d^2*e^3*e^(4*c) + b^6*d^2*e^3*e^(4*
c) + (a^4*b^2*d^2*f^3*e^(4*c) + 2*a^2*b^4*d^2*f^3*e^(4*c) + b^6*d^2*f^3*e^
(4*c))*x^3 + 3*(a^4*b^2*d^2*e*f^2*e^(4*c) + 2*a^2*b^4*d^2*e*f^2*e^(4*c) +
b^6*d^2*e*f^2*e^(4*c))*x^2 + 3*(a^4*b^2*d^2*e^2*f*e^(4*c) + 2*a^2*b^4*d^2*
e^2*f*e^(4*c) + b^6*d^2*e^2*f*e^(4*c))*x)*e^(4*d*x) + 4*(a^5*b*d^2*e^3*e^(
3*c) + 2*a^3*b^3*d^2*e^3*e^(3*c) + a*b^5*d^2*e^3*e^(3*c) + (a^5*b*d^2*f^3*
e^(3*c) + 2*a^3*b^3*d^2*f^3*e^(3*c) + a*b^5*d^2*f^3*e^(3*c))*x^3 + 3*(a^5*
b*d^2*e*f^2*e^(3*c) + 2*a^3*b^3*d^2*e*f^2*e^(3*c) + a*b^5*d^2*e*f^2*e^(3*c
))*x^2 + 3*(a^5*b*d^2*e^2*f*e^(3*c) + 2*a^3*b^3*d^2*e^2*f*e^(3*c) + a*b^5*
d^2*e^2*f*e^(3*c))*x)*e^(3*d*x) + 2*(2*a^6*d^2*e^3*e^(2*c) + 3*a^4*b^2*d^2
*e^3*e^(2*c) - b^6*d^2*e^3*e^(2*c) + (2*a^6*d^2*f^3*e^(2*c) + 3*a^4*b^2*d^
2*f^3*e^(2*c) - b^6*d^2*f^3*e^(2*c))*x^3 + 3*(2*a^6*d^2*e*f^2*e^(2*c) +...
```

Giac [N/A]

Not integrable

Time = 52.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\,dx=\int \frac{1}{(fx+e)^2(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate(1/(f*x+e)^2/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((f*x + e)^2*(b*sinh(d*x + c) + a)^3), x)
```

Mupad [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\,dx = \int \frac{1}{(e+f\,x)^2\,(a+b\sinh\left(c+d\,x\right))^3}\,dx$$

input
```
int(1/((e + f*x)^2*(a + b*sinh(c + d*x))^3),x)
```

output
```
int(1/((e + f*x)^2*(a + b*sinh(c + d*x))^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 9.05

$$\int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\,dx$$
$$= \int \frac{}{\sinh\left(dx+c\right)^3 b^3e^2+2\sinh\left(dx+c\right)^3 b^3efx+\sinh\left(dx+c\right)^3 b^3f^2x^2+3\sinh\left(dx+c\right)^2 a\,b^2e^2+6\sinh(}$$

input
```
int(1/(f*x+e)^2/(a+b*sinh(d*x+c))^3,x)
```

output
```
int(1/(sinh(c + d*x)**3*b**3*e**2 + 2*sinh(c + d*x)**3*b**3*e*f*x + sinh(c
 + d*x)**3*b**3*f**2*x**2 + 3*sinh(c + d*x)**2*a*b**2*e**2 + 6*sinh(c + d*
x)**2*a*b**2*e*f*x + 3*sinh(c + d*x)**2*a*b**2*f**2*x**2 + 3*sinh(c + d*x)
*a**2*b*e**2 + 6*sinh(c + d*x)*a**2*b*e*f*x + 3*sinh(c + d*x)*a**2*b*f**2*
x**2 + a**3*e**2 + 2*a**3*e*f*x + a**3*f**2*x**2),x)
```

3.181 $\int (c+dx)^m (a+b\sinh(e+fx))^n\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (c+dx)^m (a+b\sinh(e+fx))^n\,dx = \text{Int}((c+dx)^m (a+b\sinh(e+fx))^n, x)$$

output
```
Defer(Int)((d*x+c)^m*(a+b*sinh(f*x+e))^n,x)
```

Mathematica [N/A]

Not integrable

Time = 2.90 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m (a+b\sinh(e+fx))^n\,dx = \int (c+dx)^m (a+b\sinh(e+fx))^n\,dx$$

input
```
Integrate[(c + d*x)^m*(a + b*Sinh[e + f*x])^n,x]
```

output
```
Integrate[(c + d*x)^m*(a + b*Sinh[e + f*x])^n, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(a+b\sinh(e+fx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)^m(a-ib\sin(ie+ifx))^ndx$$

$$\Big\downarrow \text{ 3807}$$

$$\int (c+dx)^m(a+b\sinh(e+fx))^ndx$$

input
```
Int[(c + d*x)^m*(a + b*Sinh[e + f*x])^n,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (dx+c)^m\,(a+b\sinh\,(fx+e))^n\,dx$$

input
```
int((d*x+c)^m*(a+b*sinh(f*x+e))^n,x)
```

output
```
int((d*x+c)^m*(a+b*sinh(f*x+e))^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+b\sinh(e+fx))^n\,dx=\int (dx+c)^m(b\sinh(fx+e)+a)^n\,dx$$

input
```
integrate((d*x+c)^m*(a+b*sinh(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*(b*sinh(f*x + e) + a)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^m(a+b\sinh(e+fx))^n\,dx=\text{Timed out}$$

input
```
integrate((d*x+c)**m*(a+b*sinh(f*x+e))**n,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+b\sinh(e+fx))^n\,dx=\int (dx+c)^m(b\sinh(fx+e)+a)^n\,dx$$

input
```
integrate((d*x+c)^m*(a+b*sinh(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*(b*sinh(f*x + e) + a)^n, x)
```

Giac [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+b\sinh(e+fx))^n\,dx = \int (dx+c)^m(b\sinh(fx+e)+a)^n\,dx$$

input
```
integrate((d*x+c)^m*(a+b*sinh(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*(b*sinh(f*x + e) + a)^n, x)
```

Mupad [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+b\sinh(e+fx))^n\,dx = \int (a+b\sinh(e+f\,x))^n\,(c+d\,x)^m\,dx$$

input
```
int((a + b*sinh(e + f*x))^n*(c + d*x)^m,x)
```

output
```
int((a + b*sinh(e + f*x))^n*(c + d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+b\sinh(e+fx))^n\,dx = \int (dx+c)^m\,(\sinh(fx+e)\,b+a)^n\,dx$$

input
```
int((d*x+c)^m*(a+b*sinh(f*x+e))^n,x)
```

output
```
int((c + d*x)**m*(sinh(e + f*x)*b + a)**n,x)
```

3.182 $\int (c+dx)^m (a+b\sinh(e+fx))^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 543

$$\begin{aligned}
&\int (c+dx)^m(a+b\sinh(e+fx))^3\,dx\\
&=\frac{a^3(c+dx)^{1+m}}{d(1+m)}-\frac{3ab^2(c+dx)^{1+m}}{2d(1+m)}\\
&+\frac{3^{-1-m}b^3e^{3e-\frac{3cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3f(c+dx)}{d}\right)}{8f}\\
&+\frac{3\ 2^{-3-m}ab^2e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2f(c+dx)}{d}\right)}{f}\\
&+\frac{3a^2be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{2f}\\
&-\frac{3b^3e^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{8f}\\
&+\frac{3a^2be^{-e+\frac{cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{f(c+dx)}{d}\right)}{2f}\\
&-\frac{3b^3e^{-e+\frac{cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{f(c+dx)}{d}\right)}{8f}\\
&-\frac{3\ 2^{-3-m}ab^2e^{-2e+\frac{2cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)}{f}\\
&+\frac{3^{-1-m}b^3e^{-3e+\frac{3cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{3f(c+dx)}{d}\right)}{8f}
\end{aligned}$$

output
```
a^3*(d*x+c)^(1+m)/d/(1+m)-3/2*a*b^2*(d*x+c)^(1+m)/d/(1+m)+1/8*3^(-1-m)*b^3
*exp(3*e-3*c*f/d)*(d*x+c)^m*GAMMA(1+m,-3*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)
+3*2^(-3-m)*a*b^2*exp(2*e-2*c*f/d)*(d*x+c)^m*GAMMA(1+m,-2*f*(d*x+c)/d)/f/(
(-f*(d*x+c)/d)^m)+3/2*a^2*b*exp(e-c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c)/d)
/f/((-f*(d*x+c)/d)^m)-3/8*b^3*exp(e-c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c)/
d)/f/((-f*(d*x+c)/d)^m)+3/2*a^2*b*exp(-e+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*x
+c)/d)/f/((f*(d*x+c)/d)^m)-3/8*b^3*exp(-e+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*
x+c)/d)/f/((f*(d*x+c)/d)^m)-3*2^(-3-m)*a*b^2*exp(-2*e+2*c*f/d)*(d*x+c)^m*G
AMMA(1+m,2*f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)+1/8*3^(-1-m)*b^3*exp(-3*e+3*c*
f/d)*(d*x+c)^m*GAMMA(1+m,3*f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 448, normalized size of antiderivative = 0.83

$$\int (c+dx)^m(a+b\sinh(e+fx))^3\,dx = \frac{2^{-3-m}3^{-1-m}e^{-3\left(e+\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m}\left(2^mb^3de^{6e}(1+m)\left(\frac{f(c+dx)}{d}\right)^m\Gamma\left(1+m,-\frac{3f(c+dx)}{d}\right)+3\right.}{}$$

input
```
Integrate[(c + d*x)^m*(a + b*Sinh[e + f*x])^3,x]
```

output
```
(2^(-3 - m)*3^(-1 - m)*(c + d*x)^m*(2^m*b^3*d*E^(6*e)*(1 + m)*((f*(c + d*x
))/d)^m*Gamma[1 + m, (-3*f*(c + d*x))/d] + 3^(2 + m)*a*b^2*d*E^(5*e + (c*f
)/d)*(1 + m)*(f*(c/d + x))^m*Gamma[1 + m, (-2*f*(c + d*x))/d] - 2^m*3^(2 +
 m)*b*(-4*a^2 + b^2)*d*E^(4*e + (2*c*f)/d)*(1 + m)*((f*(c + d*x))/d)^m*Gam
ma[1 + m, -((f*(c + d*x))/d)] - 2^m*3^(2 + m)*b*(-4*a^2 + b^2)*d*E^(2*e +
(4*c*f)/d)*(1 + m)*(-((f*(c + d*x))/d))^m*Gamma[1 + m, (f*(c + d*x))/d] -
3^(2 + m)*a*b^2*d*E^(e + (5*c*f)/d)*(1 + m)*(-((f*(c + d*x))/d))^m*Gamma[1
 + m, (2*f*(c + d*x))/d] + 2^m*E^((3*c*f)/d)*(4*3^(1 + m)*a*(2*a^2 - 3*b^2
)*E^(3*e)*f*(c + d*x)*(-((f^2*(c + d*x)^2)/d^2))^m + b^3*d*E^((3*c*f)/d)*(
1 + m)*(-((f*(c + d*x))/d))^m*Gamma[1 + m, (3*f*(c + d*x))/d])))/(d*E^(3*(
e + (c*f)/d))*f*(1 + m)*(-((f^2*(c + d*x)^2)/d^2))^m)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m (a+b\sinh(e+fx))^3\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (c+dx)^m (a-ib\sin(ie+ifx))^3dx$$

$$\Big\downarrow \ 3798$$

$$\int \left(a^3(c+dx)^m+3a^2b(c+dx)^m\sinh(e+fx)+3ab^2(c+dx)^m\sinh^2(e+fx)+b^3(c+dx)^m\sinh^3(e+fx)\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a^3(c+dx)^{m+1}}{d(m+1)}+\frac{3a^2be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f}+\frac{3a^2be^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{f(c+dx)}{d}\right)}{2f}+\frac{3ab^22^{-m-3}e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2f(c+dx)}{d}\right)}{f}-\frac{3ab^22^{-m-3}e^{\frac{2cf}{d}-2e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2f(c+dx)}{d}\right)}{f}-\frac{3ab^2(c+dx)^{m+1}}{2d(m+1)}+\frac{b^33^{-m-1}e^{3e-\frac{3cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3f(c+dx)}{d}\right)}{8f}-\frac{3b^3e^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{f(c+dx)}{d}\right)}{8f}-\frac{3b^3e^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{f(c+dx)}{d}\right)}{8f}+\frac{b^33^{-m-1}e^{\frac{3cf}{d}-3e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{3f(c+dx)}{d}\right)}{8f}$$

input
```
Int[(c + d*x)^m*(a + b*Sinh[e + f*x])^3,x]
```

output
```
(a^3*(c + d*x)^(1 + m))/(d*(1 + m)) - (3*a*b^2*(c + d*x)^(1 + m))/(2*d*(1
+ m)) + (3^(-1 - m)*b^3*E^(3*e - (3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-3*f
*(c + d*x))/d])/(8*f*(-((f*(c + d*x))/d))^m) + (3*2^(-3 - m)*a*b^2*E^(2*e
- (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(c + d*x))/d])/(f*(-((f*(c + d
*x))/d))^m) + (3*a^2*b*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1 + m, -((f*(c +
d*x))/d)])/(2*f*(-((f*(c + d*x))/d))^m) - (3*b^3*E^(e - (c*f)/d)*(c + d*x)
^m*Gamma[1 + m, -((f*(c + d*x))/d)])/(8*f*(-((f*(c + d*x))/d))^m) + (3*a^2
*b*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(2*f*((f*(c
 + d*x))/d)^m) - (3*b^3*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c +
d*x))/d])/(8*f*((f*(c + d*x))/d)^m) - (3*2^(-3 - m)*a*b^2*E^(-2*e + (2*c*f
)/d)*(c + d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m)
+ (3^(-1 - m)*b^3*E^(-3*e + (3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (3*f*(c +
d*x))/d])/(8*f*((f*(c + d*x))/d)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [F]

$$\int (dx+c)^m (a+b\sinh(fx+e))^3 \, dx$$

input
```
int((d*x+c)^m*(a+b*sinh(f*x+e))^3,x)
```

output
```
int((d*x+c)^m*(a+b*sinh(f*x+e))^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 829, normalized size of antiderivative = 1.53

$$\int (c+dx)^m (a+b\sinh(e+fx))^3 \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^m*(a+b*sinh(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/24*((b^3*d*m + b^3*d)*cosh((d*m*log(3*f/d) + 3*d*e - 3*c*f)/d)*gamma(m +
 1, 3*(d*f*x + c*f)/d) - 9*(a*b^2*d*m + a*b^2*d)*cosh((d*m*log(2*f/d) + 2*
d*e - 2*c*f)/d)*gamma(m + 1, 2*(d*f*x + c*f)/d) + 9*((4*a^2*b - b^3)*d*m +
 (4*a^2*b - b^3)*d)*cosh((d*m*log(f/d) + d*e - c*f)/d)*gamma(m + 1, (d*f*x
 + c*f)/d) + 9*((4*a^2*b - b^3)*d*m + (4*a^2*b - b^3)*d)*cosh((d*m*log(-f/
d) - d*e + c*f)/d)*gamma(m + 1, -(d*f*x + c*f)/d) + 9*(a*b^2*d*m + a*b^2*d
)*cosh((d*m*log(-2*f/d) - 2*d*e + 2*c*f)/d)*gamma(m + 1, -2*(d*f*x + c*f)/
d) + (b^3*d*m + b^3*d)*cosh((d*m*log(-3*f/d) - 3*d*e + 3*c*f)/d)*gamma(m +
 1, -3*(d*f*x + c*f)/d) - (b^3*d*m + b^3*d)*gamma(m + 1, 3*(d*f*x + c*f)/d
)*sinh((d*m*log(3*f/d) + 3*d*e - 3*c*f)/d) + 9*(a*b^2*d*m + a*b^2*d)*gamma
(m + 1, 2*(d*f*x + c*f)/d)*sinh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d) - 9*((
4*a^2*b - b^3)*d*m + (4*a^2*b - b^3)*d)*gamma(m + 1, (d*f*x + c*f)/d)*sinh
((d*m*log(f/d) + d*e - c*f)/d) - 9*((4*a^2*b - b^3)*d*m + (4*a^2*b - b^3)*
d)*gamma(m + 1, -(d*f*x + c*f)/d)*sinh((d*m*log(-f/d) - d*e + c*f)/d) - 9*
(a*b^2*d*m + a*b^2*d)*gamma(m + 1, -2*(d*f*x + c*f)/d)*sinh((d*m*log(-2*f/
d) - 2*d*e + 2*c*f)/d) - (b^3*d*m + b^3*d)*gamma(m + 1, -3*(d*f*x + c*f)/d
)*sinh((d*m*log(-3*f/d) - 3*d*e + 3*c*f)/d) + 12*((2*a^3 - 3*a*b^2)*d*f*x
+ (2*a^3 - 3*a*b^2)*c*f)*cosh(m*log(d*x + c)) + 12*((2*a^3 - 3*a*b^2)*d*f*
x + (2*a^3 - 3*a*b^2)*c*f)*sinh(m*log(d*x + c)))/(d*f*m + d*f)
```

Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m(a+b\sinh(e+fx))^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)**m*(a+b*sinh(f*x+e))**3,x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 377, normalized size of antiderivative = 0.69

$$\int (c+dx)^m(a+b\sinh(e+fx))^3\,dx$$
$$= \frac{3}{2}\left(\frac{(dx+c)^{m+1}e^{\left(-e+\frac{cf}{d}\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d} - \frac{(dx+c)^{m+1}e^{\left(e-\frac{cf}{d}\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)a^2b$$
$$-\frac{3}{4}\left(\frac{(dx+c)^{m+1}e^{\left(-2\,e+\frac{2\,cf}{d}\right)}E_{-m}\left(\frac{2\,(dx+c)f}{d}\right)}{d} + \frac{(dx+c)^{m+1}e^{\left(2\,e-\frac{2\,cf}{d}\right)}E_{-m}\left(-\frac{2\,(dx+c)f}{d}\right)}{d} + \frac{2\,(dx+c)^{m+}}{d(m+1)}\right.$$
$$+\frac{1}{8}\left(\frac{(dx+c)^{m+1}e^{\left(-3\,e+\frac{3\,cf}{d}\right)}E_{-m}\left(\frac{3\,(dx+c)f}{d}\right)}{d} - \frac{3\,(dx+c)^{m+1}e^{\left(-e+\frac{cf}{d}\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d} + \frac{3\,(dx+c)^{m+1}e}{}\right.$$
$$+\frac{(dx+c)^{m+1}a^3}{d(m+1)}$$

input
```
integrate((d*x+c)^m*(a+b*sinh(f*x+e))^3,x, algorithm="maxima")
```

output
```
3/2*((d*x + c)^(m + 1)*e^(-e + c*f/d)*exp_integral_e(-m, (d*x + c)*f/d)/d
- (d*x + c)^(m + 1)*e^(e - c*f/d)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*a^
2*b - 3/4*((d*x + c)^(m + 1)*e^(-2*e + 2*c*f/d)*exp_integral_e(-m, 2*(d*x
+ c)*f/d)/d + (d*x + c)^(m + 1)*e^(2*e - 2*c*f/d)*exp_integral_e(-m, -2*(d
*x + c)*f/d)/d + 2*(d*x + c)^(m + 1)/(d*(m + 1)))*a*b^2 + 1/8*((d*x + c)^(
m + 1)*e^(-3*e + 3*c*f/d)*exp_integral_e(-m, 3*(d*x + c)*f/d)/d - 3*(d*x +
 c)^(m + 1)*e^(-e + c*f/d)*exp_integral_e(-m, (d*x + c)*f/d)/d + 3*(d*x +
c)^(m + 1)*e^(e - c*f/d)*exp_integral_e(-m, -(d*x + c)*f/d)/d - (d*x + c)^
(m + 1)*e^(3*e - 3*c*f/d)*exp_integral_e(-m, -3*(d*x + c)*f/d)/d)*b^3 + (d
*x + c)^(m + 1)*a^3/(d*(m + 1))
```

Giac [F]

$$\int (c+dx)^m(a+b\sinh(e+fx))^3\,dx=\int (b\sinh\left(fx+e\right)+a)^3(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+b*sinh(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e) + a)^3*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m(a+b\sinh(e+fx))^3\,dx=\int \left(a+b\sinh(e+f\,x)\right)^3\,\left(c+d\,x\right)^m\,dx$$

input
```
int((a + b*sinh(e + f*x))^3*(c + d*x)^m,x)
```

output
```
int((a + b*sinh(e + f*x))^3*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m (a+b\sinh(e+fx))^3\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^m*(a+b*sinh(f*x+e))^3,x)
```

output
```
(e**(6*e + 6*f*x)*(c + d*x)**m*b**3*d*m + e**(6*e + 6*f*x)*(c + d*x)**m*b*
*3*d + 9*e**(5*e + 5*f*x)*(c + d*x)**m*a*b**2*d*m + 9*e**(5*e + 5*f*x)*(c
+ d*x)**m*a*b**2*d + 36*e**(4*e + 4*f*x)*(c + d*x)**m*a**2*b*d*m + 36*e**(
4*e + 4*f*x)*(c + d*x)**m*a**2*b*d - 9*e**(4*e + 4*f*x)*(c + d*x)**m*b**3*
d*m - 9*e**(4*e + 4*f*x)*(c + d*x)**m*b**3*d + 24*e**(3*e + 3*f*x)*(c + d*
x)**m*a**3*c*f + 24*e**(3*e + 3*f*x)*(c + d*x)**m*a**3*d*f*x - 36*e**(3*e
+ 3*f*x)*(c + d*x)**m*a*b**2*c*f - 36*e**(3*e + 3*f*x)*(c + d*x)**m*a*b**2
*d*f*x + 36*e**(2*e + 2*f*x)*(c + d*x)**m*a**2*b*d*m + 36*e**(2*e + 2*f*x)
*(c + d*x)**m*a**2*b*d - 9*e**(2*e + 2*f*x)*(c + d*x)**m*b**3*d*m - 9*e**(
2*e + 2*f*x)*(c + d*x)**m*b**3*d - 9*e**(e + f*x)*(c + d*x)**m*a*b**2*d*m
- 9*e**(e + f*x)*(c + d*x)**m*a*b**2*d + (c + d*x)**m*b**3*d*m + (c + d*x)
**m*b**3*d - e**(6*e + 3*f*x)*int((e**(3*f*x)*(c + d*x)**m)/(c + d*x),x)*b
**3*d**2*m**2 - e**(6*e + 3*f*x)*int((e**(3*f*x)*(c + d*x)**m)/(c + d*x),x
)*b**3*d**2*m - 9*e**(5*e + 3*f*x)*int((e**(2*f*x)*(c + d*x)**m)/(c + d*x)
,x)*a*b**2*d**2*m**2 - 9*e**(5*e + 3*f*x)*int((e**(2*f*x)*(c + d*x)**m)/(c
 + d*x),x)*a*b**2*d**2*m - 36*e**(4*e + 3*f*x)*int((e**(f*x)*(c + d*x)**m)
/(c + d*x),x)*a**2*b*d**2*m**2 - 36*e**(4*e + 3*f*x)*int((e**(f*x)*(c + d*
x)**m)/(c + d*x),x)*a**2*b*d**2*m + 9*e**(4*e + 3*f*x)*int((e**(f*x)*(c +
d*x)**m)/(c + d*x),x)*b**3*d**2*m**2 + 9*e**(4*e + 3*f*x)*int((e**(f*x)*(c
 + d*x)**m)/(c + d*x),x)*b**3*d**2*m - e**(3*e + 3*f*x)*int((c + d*x)**...
```

3.183 $\int (c+dx)^m(a+b\sinh(e+fx))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 281

$$\begin{aligned}
\int (c+dx)^m(a+b\sinh(e+fx))^2\,dx &= \frac{a^2(c+dx)^{1+m}}{d(1+m)}-\frac{b^2(c+dx)^{1+m}}{2d(1+m)} \\
&+\frac{2^{-3-m}b^2e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2f(c+dx)}{d}\right)}{f} \\
&+\frac{abe^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{f} \\
&+\frac{abe^{-e+\frac{cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{f(c+dx)}{d}\right)}{f} \\
&-\frac{2^{-3-m}b^2e^{-2e+\frac{2cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)}{f}
\end{aligned}$$

output
```
a^2*(d*x+c)^(1+m)/d/(1+m)-1/2*b^2*(d*x+c)^(1+m)/d/(1+m)+2^(-3-m)*b^2*exp(2
*e-2*c*f/d)*(d*x+c)^m*GAMMA(1+m,-2*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+a*b*e
xp(e-c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+a*b*exp
(-e+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)-2^(-3-m)*b
^2*exp(-2*e+2*c*f/d)*(d*x+c)^m*GAMMA(1+m,2*f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m
)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.90

$$\int (c+dx)^m(a+b\sinh(e+fx))^2\,dx$$

$$= \frac{(c+dx)^m\left(8a^2f(c+dx)-4b^2f(c+dx)+2^{-m}b^2de^{2e-\frac{2cf}{d}}(1+m)\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2f(c+dx)}{d}\right)\right.}{}$$

input
```
Integrate[(c + d*x)^m*(a + b*Sinh[e + f*x])^2,x]
```

output
```
((c + d*x)^m*(8*a^2*f*(c + d*x) - 4*b^2*f*(c + d*x) + (b^2*d*E^(2*e - (2*c
*f)/d)*(1 + m)*Gamma[1 + m, (-2*f*(c + d*x))/d])/(2^m*(-((f*(c + d*x))/d))
^m) + (8*a*b*d*E^(e - (c*f)/d)*(1 + m)*Gamma[1 + m, -((f*(c + d*x))/d)])/(
-((f*(c + d*x))/d))^m + (8*a*b*d*E^(-e + (c*f)/d)*(1 + m)*Gamma[1 + m, (f*
(c + d*x))/d])/((f*(c + d*x))/d)^m - (b^2*d*E^(-2*e + (2*c*f)/d)*(1 + m)*G
amma[1 + m, (2*f*(c + d*x))/d])/(2^m*((f*(c + d*x))/d)^m)))/(8*d*f*(1 + m)
)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(a+b\sinh(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^m(a-ib\sin(ie+ifx))^2dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(a^2(c+dx)^m+2ab(c+dx)^m\sinh(e+fx)+b^2(c+dx)^m\sinh^2(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(c+dx)^{m+1}}{d(m+1)}+\frac{abe^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{f(c+dx)}{d}\right)}{f}+\frac{abe^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{f(c+dx)}{d}\right)}{f}+\frac{b^22^{-m-3}e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2f(c+dx)}{d}\right)}{f}-\frac{b^22^{-m-3}e^{\frac{2cf}{d}-2e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2f(c+dx)}{d}\right)}{f}-\frac{b^2(c+dx)^{m+1}}{2d(m+1)}$$

input
```
Int[(c + d*x)^m*(a + b*Sinh[e + f*x])^2,x]
```

output
```
(a^2*(c + d*x)^(1 + m))/(d*(1 + m)) - (b^2*(c + d*x)^(1 + m))/(2*d*(1 + m)
) + (2^(-3 - m)*b^2*E^(2*e - (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(c
+ d*x))/d])/(f*(-((f*(c + d*x))/d))^m) + (a*b*E^(e - (c*f)/d)*(c + d*x)^m*
Gamma[1 + m, -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/d))^m) + (a*b*E^(-e
+ (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(f*((f*(c + d*x))/d)
^m) - (2^(-3 - m)*b^2*E^(-2*e + (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (2*f*(
c + d*x))/d])/(f*((f*(c + d*x))/d)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [F]

$$\int (dx+c)^m (a+b\sinh(fx+e))^2\,dx$$

input
```
int((d*x+c)^m*(a+b*sinh(f*x+e))^2,x)
```

output
```
int((d*x+c)^m*(a+b*sinh(f*x+e))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.84

$$\int (c+dx)^m(a+b\sinh(e+fx))^2\,dx = -\frac{(b^2dm+b^2d)\cosh\left(\frac{dm\log\left(\frac{2f}{d}\right)+2de-2cf}{d}\right)\Gamma\left(m+1,\frac{2(dfx+cf)}{d}\right)-8(abdm+abd)\cosh\left(\frac{dm\log\left(\frac{f}{d}\right)+de-cf}{d}\right.}{}$$

input
```
integrate((d*x+c)^m*(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
-1/8*((b^2*d*m + b^2*d)*cosh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d)*gamma(m +
 1, 2*(d*f*x + c*f)/d) - 8*(a*b*d*m + a*b*d)*cosh((d*m*log(f/d) + d*e - c*
f)/d)*gamma(m + 1, (d*f*x + c*f)/d) - 8*(a*b*d*m + a*b*d)*cosh((d*m*log(-f
/d) - d*e + c*f)/d)*gamma(m + 1, -(d*f*x + c*f)/d) - (b^2*d*m + b^2*d)*cos
h((d*m*log(-2*f/d) - 2*d*e + 2*c*f)/d)*gamma(m + 1, -2*(d*f*x + c*f)/d) -
(b^2*d*m + b^2*d)*gamma(m + 1, 2*(d*f*x + c*f)/d)*sinh((d*m*log(2*f/d) + 2
*d*e - 2*c*f)/d) + 8*(a*b*d*m + a*b*d)*gamma(m + 1, (d*f*x + c*f)/d)*sinh(
(d*m*log(f/d) + d*e - c*f)/d) + 8*(a*b*d*m + a*b*d)*gamma(m + 1, -(d*f*x +
 c*f)/d)*sinh((d*m*log(-f/d) - d*e + c*f)/d) + (b^2*d*m + b^2*d)*gamma(m +
 1, -2*(d*f*x + c*f)/d)*sinh((d*m*log(-2*f/d) - 2*d*e + 2*c*f)/d) - 4*((2*
a^2 - b^2)*d*f*x + (2*a^2 - b^2)*c*f)*cosh(m*log(d*x + c)) - 4*((2*a^2 - b
^2)*d*f*x + (2*a^2 - b^2)*c*f)*sinh(m*log(d*x + c)))/(d*f*m + d*f)
```

Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m (a+b\sinh(e+fx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)**m*(a+b*sinh(f*x+e))**2,x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.74

$$\int (c+dx)^m(a+b\sinh(e+fx))^2\,dx$$
$$=\left(\frac{(dx+c)^{m+1}e^{\left(-e+\frac{cf}{d}\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d}-\frac{(dx+c)^{m+1}e^{\left(e-\frac{cf}{d}\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)ab$$
$$-\frac{1}{4}\left(\frac{(dx+c)^{m+1}e^{\left(-2\,e+\frac{2\,cf}{d}\right)}E_{-m}\left(\frac{2\,(dx+c)f}{d}\right)}{d}+\frac{(dx+c)^{m+1}e^{\left(2\,e-\frac{2\,cf}{d}\right)}E_{-m}\left(-\frac{2\,(dx+c)f}{d}\right)}{d}+\frac{2\,(dx+c)^{m+1}}{d(m+1)}\right.$$
$$+\frac{(dx+c)^{m+1}a^2}{d(m+1)}$$

input
```
integrate((d*x+c)^m*(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
((d*x + c)^(m + 1)*e^(-e + c*f/d)*exp_integral_e(-m, (d*x + c)*f/d)/d - (d
*x + c)^(m + 1)*e^(e - c*f/d)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*a*b -
1/4*((d*x + c)^(m + 1)*e^(-2*e + 2*c*f/d)*exp_integral_e(-m, 2*(d*x + c)*f
/d)/d + (d*x + c)^(m + 1)*e^(2*e - 2*c*f/d)*exp_integral_e(-m, -2*(d*x + c
)*f/d)/d + 2*(d*x + c)^(m + 1)/(d*(m + 1)))*b^2 + (d*x + c)^(m + 1)*a^2/(d
*(m + 1))
```

Giac [F]

$$\int (c+dx)^m(a+b\sinh(e+fx))^2\,dx=\int (b\sinh(fx+e)+a)^2(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e) + a)^2*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m(a+b\sinh(e+fx))^2\,dx=\int (a+b\sinh(e+f\,x))^2\,(c+d\,x)^m\,dx$$

input
```
int((a + b*sinh(e + f*x))^2*(c + d*x)^m,x)
```

output
```
int((a + b*sinh(e + f*x))^2*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m(a+b\sinh(e+fx))^2\,dx$$
$$=\frac{e^{4fx+4e}(dx+c)^m\,b^2dm+e^{4fx+4e}(dx+c)^m\,b^2d+8e^{3fx+3e}(dx+c)^m\,abdm+8e^{3fx+3e}(dx+c)^m\,abd+8e^{2}}{}$$

input
```
int((d*x+c)^m*(a+b*sinh(f*x+e))^2,x)
```

output
```
(e**(4*e + 4*f*x)*(c + d*x)**m*b**2*d*m + e**(4*e + 4*f*x)*(c + d*x)**m*b*
*2*d + 8*e**(3*e + 3*f*x)*(c + d*x)**m*a*b*d*m + 8*e**(3*e + 3*f*x)*(c + d
*x)**m*a*b*d + 8*e**(2*e + 2*f*x)*(c + d*x)**m*a**2*c*f + 8*e**(2*e + 2*f*
x)*(c + d*x)**m*a**2*d*f*x - 4*e**(2*e + 2*f*x)*(c + d*x)**m*b**2*c*f - 4*
e**(2*e + 2*f*x)*(c + d*x)**m*b**2*d*f*x + 8*e**(e + f*x)*(c + d*x)**m*a*b
*d*m + 8*e**(e + f*x)*(c + d*x)**m*a*b*d - (c + d*x)**m*b**2*d*m - (c + d*
x)**m*b**2*d - e**(4*e + 2*f*x)*int((e**(2*f*x)*(c + d*x)**m)/(c + d*x),x)
*b**2*d**2*m**2 - e**(4*e + 2*f*x)*int((e**(2*f*x)*(c + d*x)**m)/(c + d*x)
,x)*b**2*d**2*m - 8*e**(3*e + 2*f*x)*int((e**(f*x)*(c + d*x)**m)/(c + d*x)
,x)*a*b*d**2*m**2 - 8*e**(3*e + 2*f*x)*int((e**(f*x)*(c + d*x)**m)/(c + d*
x),x)*a*b*d**2*m + e**(2*e + 2*f*x)*int((c + d*x)**m/(e**(2*e + 2*f*x)*c +
 e**(2*e + 2*f*x)*d*x),x)*b**2*d**2*m**2 + e**(2*e + 2*f*x)*int((c + d*x)*
*m/(e**(2*e + 2*f*x)*c + e**(2*e + 2*f*x)*d*x),x)*b**2*d**2*m - 8*e**(e +
2*f*x)*int((c + d*x)**m/(e**(f*x)*c + e**(f*x)*d*x),x)*a*b*d**2*m**2 - 8*e
**(e + 2*f*x)*int((c + d*x)**m/(e**(f*x)*c + e**(f*x)*d*x),x)*a*b*d**2*m)/
(8*e**(2*e + 2*f*x)*d*f*(m + 1))
```

3.184 $\int (c+dx)^m (a+b\sinh(e+fx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 131

$$\int (c+dx)^m (a+b\sinh(e+fx))\,dx = \frac{a(c+dx)^{1+m}}{d(1+m)} + \frac{be^{e-\frac{cf}{d}}(c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{2f} + \frac{be^{-e+\frac{cf}{d}}(c+dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1+m,\frac{f(c+dx)}{d}\right)}{2f}$$

output
```
a*(d*x+c)^(1+m)/d/(1+m)+1/2*b*exp(e-c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c)/
d)/f/((-f*(d*x+c)/d)^m)+1/2*b*exp(-e+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*x+c)/
d)/f/((f*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.90

$$\int (c+dx)^m(a+b\sinh(e+fx))\,dx=\frac{1}{2}(c+dx)^m\left(\frac{2a(c+dx)}{d(1+m)}+\frac{be^{e-\frac{cf}{d}}\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{f}+\frac{be^{-e+\frac{cf}{d}}\left(f\left(\frac{c}{d}+x\right)\right)^{-m}\Gamma\left(1+m,\frac{f(c+dx)}{d}\right)}{f}\right)$$

input
```
Integrate[(c + d*x)^m*(a + b*Sinh[e + f*x]),x]
```

output
```
((c + d*x)^m*((2*a*(c + d*x))/(d*(1 + m)) + (b*E^(e - (c*f)/d)*Gamma[1 + m
, -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/d))^m) + (b*E^(-e + (c*f)/d)*Ga
mma[1 + m, (f*(c + d*x))/d])/(f*(f*(c/d + x))^m)))/2
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(a+b\sinh(e+fx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)^m(a-ib\sin(ie+ifx))dx$$

$$\Big\downarrow \text{ 3798}$$

$$\int \left(a(c+dx)^m + b(c+dx)^m \sinh(e+fx)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(c+dx)^{m+1}}{d(m+1)} + \frac{be^{e-\frac{cf}{d}}(c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f} + \frac{be^{\frac{cf}{d}-e}(c+dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,\frac{f(c+dx)}{d}\right)}{2f}$$

input
```
Int[(c + d*x)^m*(a + b*Sinh[e + f*x]),x]
```

output
```
(a*(c + d*x)^(1 + m))/(d*(1 + m)) + (b*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1
 + m, -((f*(c + d*x))/d)])/(2*f*(-((f*(c + d*x))/d))^m) + (b*E^(-e + (c*f)
/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(2*f*((f*(c + d*x))/d)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [F]

$$\int (dx+c)^m (a+b\sinh (fx+e))\, dx$$

input
```
int((d*x+c)^m*(a+b*sinh(f*x+e)),x)
```

output
```
int((d*x+c)^m*(a+b*sinh(f*x+e)),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.90

$$\int (c+dx)^m(a+b\sinh(e+fx))\, dx = \frac{(bdm+bd)\cosh\left(\frac{dm\log\left(\frac{f}{d}\right)+de-cf}{d}\right)\Gamma\left(m+1,\frac{dfx+cf}{d}\right)+(bdm+bd)\cosh\left(\frac{dm\log\left(-\frac{f}{d}\right)-de+cf}{d}\right)\Gamma\left(m+1,-\right.}{}$$

input
```
integrate((d*x+c)^m*(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*((b*d*m + b*d)*cosh((d*m*log(f/d) + d*e - c*f)/d)*gamma(m + 1, (d*f*x
+ c*f)/d) + (b*d*m + b*d)*cosh((d*m*log(-f/d) - d*e + c*f)/d)*gamma(m + 1,
 -(d*f*x + c*f)/d) - (b*d*m + b*d)*gamma(m + 1, (d*f*x + c*f)/d)*sinh((d*m
*log(f/d) + d*e - c*f)/d) - (b*d*m + b*d)*gamma(m + 1, -(d*f*x + c*f)/d)*s
inh((d*m*log(-f/d) - d*e + c*f)/d) + 2*(a*d*f*x + a*c*f)*cosh(m*log(d*x +
c)) + 2*(a*d*f*x + a*c*f)*sinh(m*log(d*x + c)))/(d*f*m + d*f)
```

Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m(a+b\sinh(e+fx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)**m*(a+b*sinh(f*x+e)),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.77

$$\int (c+dx)^m(a+b\sinh(e+fx))\,dx = \frac{1}{2}\left(\frac{(dx+c)^{m+1}e^{\left(-e+\frac{cf}{d}\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d} - \frac{(dx+c)^{m+1}e^{\left(e-\frac{cf}{d}\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)b + \frac{(dx+c)^{m+1}a}{d(m+1)}$$

input
```
integrate((d*x+c)^m*(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
1/2*((d*x + c)^(m + 1)*e^(-e + c*f/d)*exp_integral_e(-m, (d*x + c)*f/d)/d
- (d*x + c)^(m + 1)*e^(e - c*f/d)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*b
+ (d*x + c)^(m + 1)*a/(d*(m + 1))
```

Giac [F]

$$\int (c+dx)^m(a+b\sinh(e+fx))\,dx = \int (b\sinh(fx+e)+a)(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+b*sinh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e) + a)*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m(a+b\sinh(e+fx))\,dx = \int (a+b\sinh(e+f\,x))\,\left(c+d\,x\right)^m\,dx$$

input
```
int((a + b*sinh(e + f*x))*(c + d*x)^m,x)
```

output
```
int((a + b*sinh(e + f*x))*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m(a+b\sinh(e+fx))\,dx$$
$$= \frac{e^{2fx+2e}(dx+c)^m\,bdm + e^{2fx+2e}(dx+c)^m\,bd + 2e^{fx+e}(dx+c)^m\,acf + 2e^{fx+e}(dx+c)^m\,adfx + (dx+c)^{\cdots}}{\cdots}$$

input
```
int((d*x+c)^m*(a+b*sinh(f*x+e)),x)
```

output
```
(e**(2*e + 2*f*x)*(c + d*x)**m*b*d*m + e**(2*e + 2*f*x)*(c + d*x)**m*b*d +
 2*e**(e + f*x)*(c + d*x)**m*a*c*f + 2*e**(e + f*x)*(c + d*x)**m*a*d*f*x +
 (c + d*x)**m*b*d*m + (c + d*x)**m*b*d - e**(2*e + f*x)*int((e**(f*x)*(c +
 d*x)**m)/(c + d*x),x)*b*d**2*m**2 - e**(2*e + f*x)*int((e**(f*x)*(c + d*x
)**m)/(c + d*x),x)*b*d**2*m - e**(f*x)*int((c + d*x)**m/(e**(f*x)*c + e**(
f*x)*d*x),x)*b*d**2*m**2 - e**(f*x)*int((c + d*x)**m/(e**(f*x)*c + e**(f*x
)*d*x),x)*b*d**2*m)/(2*e**(e + f*x)*d*f*(m + 1))
```

3.185 $\int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx=\operatorname{Int}\left(\frac{(c+dx)^m}{a+b\sinh(e+fx)},x\right)$$

output
```
Defer(Int)((d*x+c)^m/(a+b*sinh(f*x+e)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx=\int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx$$

input
```
Integrate[(c + d*x)^m/(a + b*Sinh[e + f*x]),x]
```

output
```
Integrate[(c + d*x)^m/(a + b*Sinh[e + f*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{a+b\sinh(e+fx)} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(c+dx)^m}{a-ib\sin(ie+ifx)}dx$$

$$\Big\downarrow \text{ 3807}$$

$$\int \frac{(c+dx)^m}{a+b\sinh(e+fx)}dx$$

input
```
Int[(c + d*x)^m/(a + b*Sinh[e + f*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(dx+c)^m}{a+b\sinh(fx+e)}dx$$

input
```
int((d*x+c)^m/(a+b*sinh(f*x+e)),x)
```

output
```
int((d*x+c)^m/(a+b*sinh(f*x+e)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx=\int \frac{(dx+c)^m}{b\sinh{(fx+e)}+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m/(b*sinh(f*x + e) + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx=\int \frac{(c+dx)^m}{a+b\sinh{(e+fx)}}\,dx$$

input
```
integrate((d*x+c)**m/(a+b*sinh(f*x+e)),x)
```

output
```
Integral((c + d*x)**m/(a + b*sinh(e + f*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx=\int \frac{(dx+c)^m}{b\sinh{(fx+e)}+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(b*sinh(f*x + e) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx=\int \frac{(dx+c)^m}{b\sinh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*sinh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/(b*sinh(f*x + e) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx=\int \frac{(c+d\,x)^m}{a+b\sinh(e+f\,x)}\,dx$$

input
```
int((c + d*x)^m/(a + b*sinh(e + f*x)),x)
```

output
```
int((c + d*x)^m/(a + b*sinh(e + f*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.40

$$\int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx = 2e^e\left(\int \frac{e^{fx}(dx+c)^m}{e^{2fx+2e}b+2e^{fx+e}a-b}dx\right)$$

input
```
int((d*x+c)^m/(a+b*sinh(f*x+e)),x)
```

output
```
2*e**e*int((e**(f*x)*(c + d*x)**m)/(e**(2*e + 2*f*x)*b + 2*e**(e + f*x)*a
- b),x)
```

3.186 $\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx = \text{Int}\left(\frac{(c+dx)^m}{(a+b\sinh(e+fx))^2},x\right)$$

output
```
Defer(Int)((d*x+c)^m/(a+b*sinh(f*x+e))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 4.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx = \int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx$$

input
```
Integrate[(c + d*x)^m/(a + b*Sinh[e + f*x])^2,x]
```

output
```
Integrate[(c + d*x)^m/(a + b*Sinh[e + f*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c+dx)^m}{(a-ib\sin(ie+ifx))^2}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}dx$$

input
```
Int[(c + d*x)^m/(a + b*Sinh[e + f*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(dx+c)^m}{(a+b\sinh(fx+e))^2}dx$$

input
```
int((d*x+c)^m/(a+b*sinh(f*x+e))^2,x)
```

output
```
int((d*x+c)^m/(a+b*sinh(f*x+e))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx=\int \frac{(dx+c)^m}{(b\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m/(b^2*sinh(f*x + e)^2 + 2*a*b*sinh(f*x + e) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 30.75 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx=\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx$$

input
```
integrate((d*x+c)**m/(a+b*sinh(f*x+e))**2,x)
```

output
```
Integral((c + d*x)**m/(a + b*sinh(e + f*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx=\int \frac{(dx+c)^m}{(b\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(b*sinh(f*x + e) + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx = \int \frac{(dx+c)^m}{(b\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/(b*sinh(f*x + e) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx = \int \frac{(c+d\,x)^m}{(a+b\sinh(e+f\,x))^2}\,dx$$

input
```
int((c + d*x)^m/(a + b*sinh(e + f*x))^2,x)
```

output
```
int((c + d*x)^m/(a + b*sinh(e + f*x))^2, x)
```

Reduce [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 62433, normalized size of antiderivative = 3121.65

$$\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2} \, dx = \text{Too large to display}$$

input
```
int((d*x+c)^m/(a+b*sinh(f*x+e))^2,x)
```

output
```
(4*(4*e**(e + f*x)*(c + d*x)**m*a*c**2*f - (c + d*x)**m*b*c**2*f - (c + d*
x)**m*b*c*d*m + 4*e**(5*e + 2*f*x)*int((e**(3*f*x)*(c + d*x)**m*x)/(2*e**(
4*e + 4*f*x)*b**2*c**3*f**2 + 2*e**(4*e + 4*f*x)*b**2*c**2*d*f**2*x + e**(
4*e + 4*f*x)*b**2*c**2*d*f*m + e**(4*e + 4*f*x)*b**2*c*d**2*f*m*x - e**(4*
e + 4*f*x)*b**2*c*d**2*m**2 - e**(4*e + 4*f*x)*b**2*d**3*m**2*x + 8*e**(3*
e + 3*f*x)*a*b*c**3*f**2 + 8*e**(3*e + 3*f*x)*a*b*c**2*d*f**2*x + 4*e**(3*
e + 3*f*x)*a*b*c**2*d*f*m + 4*e**(3*e + 3*f*x)*a*b*c*d**2*f*m*x - 4*e**(3*
e + 3*f*x)*a*b*c*d**2*m**2 - 4*e**(3*e + 3*f*x)*a*b*d**3*m**2*x + 8*e**(2*
e + 2*f*x)*a**2*c**3*f**2 + 8*e**(2*e + 2*f*x)*a**2*c**2*d*f**2*x + 4*e**(
2*e + 2*f*x)*a**2*c**2*d*f*m + 4*e**(2*e + 2*f*x)*a**2*c*d**2*f*m*x - 4*e*
*(2*e + 2*f*x)*a**2*c*d**2*m**2 - 4*e**(2*e + 2*f*x)*a**2*d**3*m**2*x - 4*
e**(2*e + 2*f*x)*b**2*c**3*f**2 - 4*e**(2*e + 2*f*x)*b**2*c**2*d*f**2*x -
2*e**(2*e + 2*f*x)*b**2*c**2*d*f*m - 2*e**(2*e + 2*f*x)*b**2*c*d**2*f*m*x
+ 2*e**(2*e + 2*f*x)*b**2*c*d**2*m**2 + 2*e**(2*e + 2*f*x)*b**2*d**3*m**2*
x - 8*e**(e + f*x)*a*b*c**3*f**2 - 8*e**(e + f*x)*a*b*c**2*d*f**2*x - 4*e*
*(e + f*x)*a*b*c**2*d*f*m - 4*e**(e + f*x)*a*b*c*d**2*f*m*x + 4*e**(e + f*
x)*a*b*c*d**2*m**2 + 4*e**(e + f*x)*a*b*d**3*m**2*x + 2*b**2*c**3*f**2 + 2
*b**2*c**2*d*f**2*x + b**2*c**2*d*f*m + b**2*c*d**2*f*m*x - b**2*c*d**2*m*
*2 - b**2*d**3*m**2*x),x)*a*b**2*c**4*d*f**4 - 2*e**(5*e + 2*f*x)*int((e**
(3*f*x)*(c + d*x)**m*x)/(2*e**(4*e + 4*f*x)*b**2*c**3*f**2 + 2*e**(4*e ...
```

3.187 $\int \frac{(e+fx)^3\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 163

$$\int \frac{(e+fx)^3\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{i(e+fx)^3}{ad}-\frac{i(e+fx)^4}{4af}-\frac{6if(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2}-\frac{12if^2(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}+\frac{12if^3\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^4}+\frac{i(e+fx)^3\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}$$

output

```
I*(f*x+e)^3/a/d-1/4*I*(f*x+e)^4/a/f-6*I*f*(f*x+e)^2*ln(1+I*exp(d*x+c))/a/d
^2-12*I*f^2*(f*x+e)*polylog(2,-I*exp(d*x+c))/a/d^3+12*I*f^3*polylog(3,-I*e
xp(d*x+c))/a/d^4+I*(f*x+e)^3*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a/d
```

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.31

$$\int \frac{(e+fx)^3\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{-\frac{8(e+fx)^3}{d(-i+e^c)} - ix(4e^3+6e^2fx+4ef^2x^2+f^3x^3) - \frac{24if(e+fx)^2\log(1-ie^{-c-dx})}{d^2} + \frac{48if^2(d(e+fx)\,\text{PolyLog}(2,ie^{-c-dx})+f}{d^4}}{4a}$$

input
```
Integrate[((e + f*x)^3*Sinh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-8*(e + f*x)^3)/(d*(-I + E^c)) - I*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 +
f^3*x^3) - ((24*I)*f*(e + f*x)^2*Log[1 - I*E^(-c - d*x)])/d^2 + ((48*I)*f^
2*(d*(e + f*x)*PolyLog[2, I*E^(-c - d*x)] + f*PolyLog[3, I*E^(-c - d*x)]))
/d^4 + ((8*I)*(e + f*x)^3*Sinh[(d*x)/2])/(d*(Cosh[c/2] + I*Sinh[c/2])*(Cos
h[(c + d*x)/2] + I*Sinh[(c + d*x)/2])))/(4*a)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.03, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.586$, Rules used = {6091, 17, 3042, 3799, 25, 25, 3042, 4672, 26, 3042, 26, 4199, 26, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$\Big\downarrow \text{6091}$$
$$i\int \frac{(e+fx)^3}{i\sinh(c+dx)a+a}dx - \frac{i\int (e+fx)^3dx}{a}$$
$$\Big\downarrow \text{17}$$
$$i\int \frac{(e+fx)^3}{i\sinh(c+dx)a+a}dx - \frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \; 3042$$

$$i\int \frac{(e+fx)^3}{\sin(ic+idx)a+a}dx-\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \; 3799$$

$$\frac{i\int -(e+fx)^3\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \; 25$$

$$-\frac{i\int -(e+fx)^3\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \; 25$$

$$\frac{i\int (e+fx)^3\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \; 3042$$

$$\frac{i\int (e+fx)^3\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}-\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \; 4672$$

$$\frac{i\left(\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{6if\int -i(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \; 26$$

$$\frac{i\left(\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{6f\int (e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \; 3042$$

$$\frac{i\left(\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{6f\int -i(e+fx)^2\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \; 26$$

$$\frac{i\left(\frac{6if\int (e+fx)^2\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \text{4199}$$

$$\frac{i\left(\frac{6if\left(2i\int \frac{ie^{c+dx}(e+fx)^2}{1+ie^{c+dx}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\left(\frac{6if\left(-2\int \frac{e^{c+dx}(e+fx)^2}{1+ie^{c+dx}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \text{2620}$$

$$\frac{i\left(\frac{6if\left(-2\left(\frac{2if\int (e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \text{3011}$$

$$\frac{i\left(\frac{6if\left(-2\left(\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\right.}{d}}{2a}$$

$$\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \text{2720}$$

$$\frac{i\left(\frac{6if\left(-2\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\operatorname{ta}}{}\right.}{2a}$$

$$\frac{i(e+fx)^4}{4af}$$

$\big\downarrow$ 7143

$$\frac{i\left(\frac{6if\left(-2\left(\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right.}{2a}$$

$$\frac{i(e+fx)^4}{4af}$$

input
```
Int[((e + f*x)^3*Sinh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-1/4*I)*(e + f*x)^4)/(a*f) + ((I/2)*(((6*I)*f*(((-1/3*I)*(e + f*x)^3)/f
- 2*(((-I)*(e + f*x)^2*Log[1 + I*E^(c + d*x)])/d + ((2*I)*f*(-(((e + f*x)*
PolyLog[2, (-I)*E^(c + d*x)])/d) + (f*PolyLog[3, (-I)*E^(c + d*x)])/d^2))/
d)))/d + (2*(e + f*x)^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/d))/a
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 578 vs. 2(142) = 284.

Time = 0.42 (sec) , antiderivative size = 579, normalized size of antiderivative = 3.55

method	result
risch	$\frac{6if^2ec^2}{a\,d^3} - \frac{12c\,f^2e\arctan\left(e^{dx+c}\right)}{a\,d^3} - \frac{12ic\,f^2e\ln\left(e^{dx+c}\right)}{a\,d^3} + \frac{12if^2ecx}{a\,d^2} + \frac{6ic\,f^2e\ln\left(1+e^{2dx+2c}\right)}{a\,d^3} - \frac{12if^2e\ln\left(1+ie^{dx+c}\right)c}{a\,d^3} -$

input
```
int((f*x+e)^3*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-I/a*f^2*e*x^3-3/2*I/a*f*e^2*x^2-1/4*I/a*f^3*x^4-I/a*e^3*x-12/a/d^3*c*f^2*
e*arctan(exp(d*x+c))+6*I/a/d^3*f^2*e*c^2-6*I/a/d^2*f^3*ln(1+I*exp(d*x+c))*
x^2+6*I/a/d*f^2*e*x^2+6*I/a/d^2*e^2*f*ln(exp(d*x+c))+6*I/a/d^4*f^3*ln(1+I*
exp(d*x+c))*c^2-12*I/a/d^3*f^3*polylog(2,-I*exp(d*x+c))*x+6*I/a/d^3*c*f^2*
e*ln(1+exp(2*d*x+2*c))-12*I/a/d^3*f^2*e*ln(1+I*exp(d*x+c))*c-12*I/a/d^2*f^
2*e*ln(1+I*exp(d*x+c))*x-12*I/a/d^3*c*f^2*e*ln(exp(d*x+c))+12*I/a/d^2*f^2*
e*c*x+2*I/a/d*f^3*x^3-2*(f^3*x^3+3*e*f^2*x^2+3*e^2*f*x+e^3)/d/a/(exp(d*x+c
)-I)+6/a/d^4*c^2*f^3*arctan(exp(d*x+c))+6/a/d^2*e^2*f*arctan(exp(d*x+c))-4
*I/a/d^4*f^3*c^3-6*I/a/d^3*f^3*c^2*x-12*I/a/d^3*f^2*e*polylog(2,-I*exp(d*x
+c))-3*I/a/d^2*e^2*f*ln(1+exp(2*d*x+2*c))+6*I/a/d^4*c^2*f^3*ln(exp(d*x+c))
-3*I/a/d^4*c^2*f^3*ln(1+exp(2*d*x+2*c))+12*I*f^3*polylog(3,-I*exp(d*x+c))/
a/d^4-1/4*I/a/f*e^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(131) = 262$.

Time = 0.11 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.80

$$\int \frac{(e+fx)^3\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{d^4f^3x^4+4\,d^4ef^2x^3+6\,d^4e^2fx^2+4\,d^4e^3x+8\,d^3e^3-24\,cd^2e^2f+24\,c^2def^2-8\,c^3f^3+48\,(df^3x+de$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/4*(d^4*f^3*x^4 + 4*d^4*e*f^2*x^3 + 6*d^4*e^2*f*x^2 + 4*d^4*e^3*x + 8*d^
3*e^3 - 24*c*d^2*e^2*f + 24*c^2*d*e*f^2 - 8*c^3*f^3 + 48*(d*f^3*x + d*e*f^
2 + (I*d*f^3*x + I*d*e*f^2)*e^(d*x + c))*dilog(-I*e^(d*x + c)) - (-I*d^4*f
^3*x^4 + 24*I*c*d^2*e^2*f - 24*I*c^2*d*e*f^2 + 8*I*c^3*f^3 - 4*(I*d^4*e*f^
2 - 2*I*d^3*f^3)*x^3 - 6*(I*d^4*e^2*f - 4*I*d^3*e*f^2)*x^2 - 4*(I*d^4*e^3
- 6*I*d^3*e^2*f)*x)*e^(d*x + c) + 24*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3 +
(I*d^2*e^2*f - 2*I*c*d*e*f^2 + I*c^2*f^3)*e^(d*x + c))*log(e^(d*x + c) - I
) + 24*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3 + (I*d^2*f^3*x
^2 + 2*I*d^2*e*f^2*x + 2*I*c*d*e*f^2 - I*c^2*f^3)*e^(d*x + c))*log(I*e^(d*
x + c) + 1) + 48*(-I*f^3*e^(d*x + c) - f^3)*polylog(3, -I*e^(d*x + c)))/(a
*d^4*e^(d*x + c) - I*a*d^4)
```

Sympy [F]

$$\int \frac{(e+fx)^3\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{-2e^3-6e^2fx-6ef^2x^2-2f^3x^3}{ade^ce^{dx}-iad}$$
$$-\frac{i\left(\int\left(-\frac{ide^3}{e^ce^{dx}-i}\right)dx+\int\frac{6ie^2f}{e^ce^{dx}-i}\,dx+\int\frac{6if^3x^2}{e^ce^{dx}-i}\,dx+\int\left(-\frac{idf^3x^3}{e^ce^{dx}-i}\right)dx+\int\frac{12ief^2x}{e^ce^{dx}-i}\,dx+\int\frac{de^3e^ce^{dx}}{e^ce^{dx}-i}\,dx+\int\right.}{ad}$$

input
```
integrate((f*x+e)**3*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
(-2*e**3 - 6*e**2*f*x - 6*e*f**2*x**2 - 2*f**3*x**3)/(a*d*exp(c)*exp(d*x)
- I*a*d) - I*(Integral(-I*d*e**3/(exp(c)*exp(d*x) - I), x) + Integral(6*I*
e**2*f/(exp(c)*exp(d*x) - I), x) + Integral(6*I*f**3*x**2/(exp(c)*exp(d*x)
 - I), x) + Integral(-I*d*f**3*x**3/(exp(c)*exp(d*x) - I), x) + Integral(1
2*I*e*f**2*x/(exp(c)*exp(d*x) - I), x) + Integral(d*e**3*exp(c)*exp(d*x)/(
exp(c)*exp(d*x) - I), x) + Integral(-3*I*d*e*f**2*x**2/(exp(c)*exp(d*x) -
I), x) + Integral(-3*I*d*e**2*f*x/(exp(c)*exp(d*x) - I), x) + Integral(d*f
**3*x**3*exp(c)*exp(d*x)/(exp(c)*exp(d*x) - I), x) + Integral(3*d*e*f**2*x
**2*exp(c)*exp(d*x)/(exp(c)*exp(d*x) - I), x) + Integral(3*d*e**2*f*x*exp(
c)*exp(d*x)/(exp(c)*exp(d*x) - I), x))/(a*d)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(131) = 262$.

Time = 0.16 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.95

$$\int \frac{(e+fx)^3\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{3}{2}e^2f\left(\frac{-i\,dx^2+(dx^2e^c-4\,xe^c)e^{(dx)}}{i\,ade^{(dx+c)}+ad}-\frac{4i\,\log\left(\left(e^{(dx+c)}-i\right)e^{(-c)}\right)}{ad^2}\right)$$
$$-e^3\left(\frac{i\,(dx+c)}{ad}+\frac{2}{(ae^{(-dx-c)}+i\,a)d}\right)$$
$$-\frac{df^3x^4+24\,ef^2x^2+4\,(def^2+2\,f^3)x^3+(i\,df^3x^4e^c+4i\,def^2x^3e^c)e^{(dx)}}{4\,(ade^{(dx+c)}-i\,ad)}$$
$$-\frac{12i\left(dx\log\left(i\,e^{(dx+c)}+1\right)+\mathrm{Li}_2(-i\,e^{(dx+c)})\right)ef^2}{ad^3}$$
$$-\frac{6i\left(d^2x^2\log\left(i\,e^{(dx+c)}+1\right)+2\,dx\mathrm{Li}_2(-i\,e^{(dx+c)})-2\,\mathrm{Li}_3(-i\,e^{(dx+c)})\right)f^3}{ad^4}$$
$$-\frac{2\,(-i\,d^3f^3x^3-3i\,d^3ef^2x^2)}{ad^4}$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
3/2*e^2*f*((-I*d*x^2 + (d*x^2*e^c - 4*x*e^c)*e^(d*x))/(I*a*d*e^(d*x + c) +
 a*d) - 4*I*log((e^(d*x + c) - I)*e^(-c))/(a*d^2)) - e^3*(I*(d*x + c)/(a*d
) + 2/((a*e^(-d*x - c) + I*a)*d)) - 1/4*(d*f^3*x^4 + 24*e*f^2*x^2 + 4*(d*e
*f^2 + 2*f^3)*x^3 + (I*d*f^3*x^4*e^c + 4*I*d*e*f^2*x^3*e^c)*e^(d*x))/(a*d*
e^(d*x + c) - I*a*d) - 12*I*(d*x*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x
+ c)))*e*f^2/(a*d^3) - 6*I*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-
I*e^(d*x + c)) - 2*polylog(3, -I*e^(d*x + c)))*f^3/(a*d^4) - 2*(-I*d^3*f^3
*x^3 - 3*I*d^3*e*f^2*x^2)/(a*d^4)
```

Giac [F]

$$\int \frac{(e+fx)^3\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\sinh(dx+c)}{i\,a\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sinh(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)\,\left(e+f\,x\right)^3}{a+a\sinh\left(c+d\,x\right)\,1i}\,dx$$

input
```
int((sinh(c + d*x)*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i),x)
```

output
```
int((sinh(c + d*x)*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{4\left(\int \frac{x^3}{\sinh(dx+c)-i}dx\right)f^3+12\left(\int \frac{x^2}{\sinh(dx+c)-i}dx\right)e\,f^2+12\left(\int \frac{x}{\sinh(dx+c)-i}dx\right)e^2f+4\left(\int \frac{1}{\sinh(dx+c)-i}dx\right)e^3 -}{4a}$$

input
```
int((f*x+e)^3*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
(4*int(x**3/(sinh(c + d*x) - i),x)*f**3 + 12*int(x**2/(sinh(c + d*x) - i),
x)*e*f**2 + 12*int(x/(sinh(c + d*x) - i),x)*e**2*f + 4*int(1/(sinh(c + d*x
) - i),x)*e**3 - 4*e**3*i*x - 6*e**2*f*i*x**2 - 4*e*f**2*i*x**3 - f**3*i*x
**4)/(4*a)
```

3.188 $\int \frac{(e+fx)^2 \sinh(c+dx)}{a+ia \sinh(c+dx)} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 130

$$\int \frac{(e+fx)^2 \sinh(c+dx)}{a+ia \sinh(c+dx)} \, dx = \frac{i(e+fx)^2}{ad} - \frac{i(e+fx)^3}{3af} - \frac{4if(e+fx) \log\left(1+ie^{c+dx}\right)}{ad^2} - \frac{4if^2 \operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3} + \frac{i(e+fx)^2 \tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}$$

output

```
I*(f*x+e)^2/a/d-1/3*I*(f*x+e)^3/a/f-4*I*f*(f*x+e)*ln(1+I*exp(d*x+c))/a/d^2
-4*I*f^2*polylog(2,-I*exp(d*x+c))/a/d^3+I*(f*x+e)^2*tanh(1/2*c+1/4*I*Pi+1/
2*d*x)/a/d
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.44

$$\int \frac{(e+fx)^2 \sinh(c+dx)}{a+ia \sinh(c+dx)} \, dx$$
$$= \frac{-ix(3e^2+3efx+f^2x^2) + \frac{-6d(e+fx)\left(d(e+fx)+2(1+ie^c)f \log\left(1-ie^{-c-dx}\right)\right)+12(1+ie^c)f^2 \operatorname{PolyLog}\left(2,ie^{-c-dx}\right)}{d^3(-i+e^c)} + \frac{}{d\left(\cosh\left(\frac{c}{2}\right)+\right.}}{3a}$$

input
```
Integrate[((e + f*x)^2*Sinh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-I)*x*(3*e^2 + 3*e*f*x + f^2*x^2) + (-6*d*(e + f*x)*(d*(e + f*x) + 2*(1
+ I*E^c)*f*Log[1 - I*E^(-c - d*x)]) + 12*(1 + I*E^c)*f^2*PolyLog[2, I*E^(-
c - d*x)])/(d^3*(-I + E^c)) + ((6*I)*(e + f*x)^2*Sinh[(d*x)/2])/(d*(Cosh[c
/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])))/(3*a)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.552$, Rules used = {6091, 17, 3042, 3799, 25, 25, 3042, 4672, 26, 3042, 26, 4199, 26, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2 \sinh(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6091}$$

$$i\int \frac{(e+fx)^2}{i\sinh(c+dx)a+a}dx-\frac{i\int (e+fx)^2dx}{a}$$

$$\Big\downarrow \text{17}$$

$$i\int \frac{(e+fx)^2}{i\sinh(c+dx)a+a}dx-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{3042}$$

$$i\int \frac{(e+fx)^2}{\sin(ic+idx)a+a}dx-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{3799}$$

$$\frac{i\int -(e+fx)^2\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{25}$$

$$-\frac{i\int -(e+fx)^2\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{i\int (e+fx)^2\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{i\int (e+fx)^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{ 4672}$$

$$\frac{i\left(\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4if\int -i(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{i\left(\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4f\int (e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{i\left(\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4f\int -i(e+fx)\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{i\left(\frac{4if\int (e+fx)\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{ 4199}$$

$$\frac{i\left(\frac{4if\left(2i\int \frac{ie^{c+dx}(e+fx)}{1+ie^{c+dx}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{i\left(\frac{4if\left(-2\int \frac{e^{c+dx}(e+fx)}{1+ie^{c+dx}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{2620}$$

$$\frac{i\left(\frac{4if\left(-2\left(\frac{if\int \log\left(1+ie^{c+dx}\right)dx}{d}-\frac{i(e+fx)\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{2715}$$

$$\frac{i\left(\frac{4if\left(-2\left(\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{i(e+fx)\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{2838}$$

$$\frac{i\left(\frac{4if\left(-2\left(-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{i(e+fx)\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}$$

input

```
Int[((e + f*x)^2*Sinh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output

```
((-1/3*I)*(e + f*x)^3)/(a*f) + ((I/2)*(((4*I)*f*(((-1/2*I)*(e + f*x)^2)/f
 - 2*(((-I)*(e + f*x)*Log[1 + I*E^(c + d*x)])/d - (I*f*PolyLog[2, (-I)*E^(c
 + d*x)])/d^2)))/d + (2*(e + f*x)^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/d))/a
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(112) = 224.

Time = 0.31 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.47

method	result
risch	$-\frac{i f^2 x^3}{3a}-\frac{i f e\, x^2}{a}-\frac{2 i e f \ln\left(1+\mathrm{e}^{2 d x+2 c}\right)}{a\, d^2}-\frac{i e^2 x}{a}-\frac{2\left(x^2 f^2+2 e f x+e^2\right)}{d a\left(\mathrm{e}^{d x+c}-i\right)}+\frac{4 i f^2 c x}{a\, d^2}+\frac{4 e f \arctan\left(\mathrm{e}^{d x+c}\right)}{a\, d^2}+\frac{2 i f^2 x^2}{a d}+$

input
```
int((f*x+e)^2*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I/a*f^2*x^3-I/a*f*e*x^2-2*I/a/d^2*e*f*ln(1+exp(2*d*x+2*c))-I/a*e^2*x-
2*(f^2*x^2+2*e*f*x+e^2)/d/a/(exp(d*x+c)-I)+4*I/a/d^2*f^2*c*x+4/a/d^2*e*f*a
rctan(exp(d*x+c))+2*I/a/d*f^2*x^2+2*I/a/d^3*f^2*c^2+4*I/a/d^2*e*f*ln(exp(d
*x+c))-4*I*f^2*polylog(2,-I*exp(d*x+c))/a/d^3-1/3*I/a/f*e^3-4*I/a/d^3*c*f^
2*ln(exp(d*x+c))+2*I/a/d^3*c*f^2*ln(1+exp(2*d*x+2*c))-4*I/a/d^3*f^2*ln(1+I
*exp(d*x+c))*c-4/a/d^3*c*f^2*arctan(exp(d*x+c))-4*I/a/d^2*f^2*ln(1+I*exp(d
*x+c))*x
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(103) = 206.

Time = 0.09 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.01

$$\int \frac{(e+fx)^2\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{d^3f^2x^3+3\,d^3efx^2+3\,d^3e^2x+6\,d^2e^2-12\,cdef+6\,c^2f^2+12\,\left(i\,f^2e^{(dx+c)}+f^2\right)\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)-(-i$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/3*(d^3*f^2*x^3 + 3*d^3*e*f*x^2 + 3*d^3*e^2*x + 6*d^2*e^2 - 12*c*d*e*f +
 6*c^2*f^2 + 12*(I*f^2*e^(d*x + c) + f^2)*dilog(-I*e^(d*x + c)) - (-I*d^3*
f^2*x^3 + 12*I*c*d*e*f - 6*I*c^2*f^2 - 3*(I*d^3*e*f - 2*I*d^2*f^2)*x^2 - 3
*(I*d^3*e^2 - 4*I*d^2*e*f)*x)*e^(d*x + c) + 12*(d*e*f - c*f^2 + (I*d*e*f -
 I*c*f^2)*e^(d*x + c))*log(e^(d*x + c) - I) + 12*(d*f^2*x + c*f^2 + (I*d*f
^2*x + I*c*f^2)*e^(d*x + c))*log(I*e^(d*x + c) + 1))/(a*d^3*e^(d*x + c) -
I*a*d^3)
```

Sympy [F]

$$\int \frac{(e+fx)^2\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{-2e^2-4efx-2f^2x^2}{ade^ce^{dx}-iad} - \frac{i\left(\int\left(-\frac{ide^2}{e^ce^{dx}-i}\right)dx+\int\frac{4ief}{e^ce^{dx}-i}\,dx+\int\frac{4if^2x}{e^ce^{dx}-i}\,dx+\int\left(-\frac{idf^2x^2}{e^ce^{dx}-i}\right)dx+\int\frac{de^2e^ce^{dx}}{e^ce^{dx}-i}\,dx+\int\left(-\frac{2idefx}{e^ce^{dx}-i}\right)dx\right.}{ad}$$

input
```
integrate((f*x+e)**2*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
(-2*e**2 - 4*e*f*x - 2*f**2*x**2)/(a*d*exp(c)*exp(d*x) - I*a*d) - I*(Integ
ral(-I*d*e**2/(exp(c)*exp(d*x) - I), x) + Integral(4*I*e*f/(exp(c)*exp(d*x
) - I), x) + Integral(4*I*f**2*x/(exp(c)*exp(d*x) - I), x) + Integral(-I*d
*f**2*x**2/(exp(c)*exp(d*x) - I), x) + Integral(d*e**2*exp(c)*exp(d*x)/(ex
p(c)*exp(d*x) - I), x) + Integral(-2*I*d*e*f*x/(exp(c)*exp(d*x) - I), x) +
 Integral(d*f**2*x**2*exp(c)*exp(d*x)/(exp(c)*exp(d*x) - I), x) + Integral
(2*d*e*f*x*exp(c)*exp(d*x)/(exp(c)*exp(d*x) - I), x))/(a*d)
```

Maxima [F]

$$\int \frac{(e+fx)^2\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\sinh(dx+c)}{i\,a\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
1/3*f^2*((-I*d*x^3*e^(d*x + c) - d*x^3 - 6*x^2)/(a*d*e^(d*x + c) - I*a*d)
+ 12*integrate(x/(a*d*e^(d*x + c) - I*a*d), x)) + e*f*((-I*d*x^2 + (d*x^2*
e^c - 4*x*e^c)*e^(d*x))/(I*a*d*e^(d*x + c) + a*d) - 4*I*log((e^(d*x + c) -
 I)*e^(-c))/(a*d^2)) - e^2*(I*(d*x + c)/(a*d) + 2/((a*e^(-d*x - c) + I*a)*
d))
```

Giac [F]

$$\int \frac{(e+fx)^2\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\sinh(dx+c)}{i\,a\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sinh(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)\,\left(e+f\,x\right)^2}{a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((sinh(c + d*x)*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i),x)
```

output
```
int((sinh(c + d*x)*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{3\left(\int \frac{x^2}{\sinh(dx+c)-i}dx\right)f^2+6\left(\int \frac{x}{\sinh(dx+c)-i}dx\right)ef+3\left(\int \frac{1}{\sinh(dx+c)-i}dx\right)e^2-3e^2ix-3efi\,x^2-f^2i\,x^3}{3a}$$

input
```
int((f*x+e)^2*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
(3*int(x**2/(sinh(c + d*x) - i),x)*f**2 + 6*int(x/(sinh(c + d*x) - i),x)*e
*f + 3*int(1/(sinh(c + d*x) - i),x)*e**2 - 3*e**2*i*x - 3*e*f*i*x**2 - f**
2*i*x**3)/(3*a)
```

3.189 $\int \frac{(e+fx)\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 87

$$\int \frac{(e+fx)\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i(e+fx)^2}{2af} - \frac{2if\log\left(\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2} + \frac{i(e+fx)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}$$

output
```
-1/2*I*(f*x+e)^2/a/f-2*I*f*ln(cosh(1/2*c+1/4*I*Pi+1/2*d*x))/a/d^2+I*(f*x+e
)*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 239 vs. 2(87) = 174.

Time = 0.69 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.75

$$\int \frac{(e+fx)\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{-2dfx\cosh\left(c+\frac{dx}{2}\right) - i\cosh\left(\frac{dx}{2}\right)\left(d^2x(2e+fx)+4if\arctan\left(\operatorname{sech}\left(c+\frac{dx}{2}\right)\sinh\left(\frac{dx}{2}\right)\right)+2f\log(\cosh(c\right.}{}$$

input
```
Integrate[((e + f*x)*Sinh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(-2*d*f*x*Cosh[c + (d*x)/2] - I*Cosh[(d*x)/2]*(d^2*x*(2*e + f*x) + (4*I)*f
*ArcTan[Sech[c + (d*x)/2]*Sinh[(d*x)/2]] + 2*f*Log[Cosh[c + d*x]]) + (4*I)
*d*e*Sinh[(d*x)/2] + (2*I)*d*f*x*Sinh[(d*x)/2] + 2*d^2*e*x*Sinh[c + (d*x)/
2] + d^2*f*x^2*Sinh[c + (d*x)/2] + (4*I)*f*ArcTan[Sech[c + (d*x)/2]*Sinh[(
d*x)/2]]*Sinh[c + (d*x)/2] + 2*f*Log[Cosh[c + d*x]]*Sinh[c + (d*x)/2])/(2*
a*d^2*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6091, 17, 3042, 3799, 25, 25, 3042, 4672, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6091}$$

$$i\int \frac{e+fx}{i\sinh(c+dx)a+a}dx-\frac{i\int (e+fx)dx}{a}$$

$$\Big\downarrow \text{ 17}$$

$$i\int \frac{e+fx}{i\sinh(c+dx)a+a}dx-\frac{i(e+fx)^2}{2af}$$

$$\Big\downarrow \text{ 3042}$$

$$i\int \frac{e+fx}{\sin(ic+idx)a+a}dx-\frac{i(e+fx)^2}{2af}$$

$$\Big\downarrow \text{ 3799}$$

$$\frac{i\int -\left((e+fx)\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)\right)dx}{2a}-\frac{i(e+fx)^2}{2af}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{i\int -\left((e+fx)\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)dx}{2a}-\frac{i(e+fx)^2}{2af}$$

$\big\downarrow$ 25

$$\frac{i\int (e+fx)\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^2}{2af}$$

$\big\downarrow$ 3042

$$\frac{i\int (e+fx)\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}-\frac{i(e+fx)^2}{2af}$$

$\big\downarrow$ 4672

$$\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2if\int -i\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^2}{2af}$$

$\big\downarrow$ 26

$$\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2f\int \tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^2}{2af}$$

$\big\downarrow$ 3042

$$\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2f\int -i\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^2}{2af}$$

$\big\downarrow$ 26

$$\frac{i\left(\frac{2if\int \tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}+\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^2}{2af}$$

$\big\downarrow$ 3956

$$\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4f\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{d^2}\right)}{2a}-\frac{i(e+fx)^2}{2af}$$

input
```
Int[((e + f*x)*Sinh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-1/2*I)*(e + f*x)^2)/(a*f) + ((I/2)*((-4*f*Log[Cosh[c/2 + (I/4)*Pi + (d*
x)/2]])/d^2 + (2*(e + f*x)*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/d))/a
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

method	result
risch	$-\frac{if\,x^2}{2a}-\frac{iex}{a}+\frac{2ifx}{ad}+\frac{2ifc}{a\,d^2}-\frac{2(fx+e)}{da\left(e^{dx+c}-i\right)}-\frac{2if\ln\left(e^{dx+c}-i\right)}{a\,d^2}$
parallelrisch	$\frac{2f\left(-1-i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\ln\left(1-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2f\left(1+i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\ln\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\left(i\left(\frac{fx}{2}+e\right)d+(-1-i)f\right)\right.}{\left(i-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)d^2a}$

input
```
int((f*x+e)*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/a*f*x^2-I/a*e*x+2*I*f/a/d*x+2*I*f/a/d^2*c-2*(f*x+e)/d/a/(exp(d*x+c)
-I)-2*I*f/a/d^2*ln(exp(d*x+c)-I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.10

$$\int \frac{(e+fx)\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{d^2fx^2+2\,d^2ex+4\,de-(-i\,d^2fx^2-2\,(i\,d^2e-2i\,df)x)e^{(dx+c)}+4\,\left(i\,fe^{(dx+c)}+f\right)\log\left(e^{(dx+c)}-i\right)}{2\,\left(ad^2e^{(dx+c)}-i\,ad^2\right)}$$

input
```
integrate((f*x+e)*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(d^2*f*x^2 + 2*d^2*e*x + 4*d*e - (-I*d^2*f*x^2 - 2*(I*d^2*e - 2*I*d*f
)*x)*e^(d*x + c) + 4*(I*f*e^(d*x + c) + f)*log(e^(d*x + c) - I))/(a*d^2*e^
(d*x + c) - I*a*d^2)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

$$\int \frac{(e+fx)\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{-2e-2fx}{ade^{c}e^{dx}-iad} - \frac{ifx^2}{2a} + \frac{x(-ide+2if)}{ad} - \frac{2if\log\left(e^{dx}-ie^{-c}\right)}{ad^2}$$

input
```
integrate((f*x+e)*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
(-2*e - 2*f*x)/(a*d*exp(c)*exp(d*x) - I*a*d) - I*f*x**2/(2*a) + x*(-I*d*e
+ 2*I*f)/(a*d) - 2*I*f*log(exp(d*x) - I*exp(-c))/(a*d**2)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.24

$$\int \frac{(e+fx)\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{1}{2}f\left(\frac{-i\,dx^2+(dx^2e^c-4\,xe^c)e^{(dx)}}{i\,ade^{(dx+c)}+ad} - \frac{4i\,\log\left(\left(e^{(dx+c)}-i\right)e^{(-c)}\right)}{ad^2}\right) - e\left(\frac{i\,(dx+c)}{ad} + \frac{2}{(ae^{(-dx-c)}+i\,a)d}\right)$$

input
```
integrate((f*x+e)*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*f*((-I*d*x^2 + (d*x^2*e^c - 4*x*e^c)*e^(d*x))/(I*a*d*e^(d*x + c) + a*d
) - 4*I*log((e^(d*x + c) - I)*e^(-c))/(a*d^2)) - e*(I*(d*x + c)/(a*d) + 2/
((a*e^(-d*x - c) + I*a)*d))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.28

$$\int \frac{(e+fx)\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\,d^2fx^2e^{(dx+c)}+d^2fx^2+2i\,d^2exe^{(dx+c)}+2\,d^2ex-4i\,dfxe^{(dx+c)}+4i\,fe^{(dx+c)}\log\left(e^{(dx+c)}-i\right)+4\,de+}{2\left(ad^2e^{(dx+c)}-i\,ad^2\right)}$$

input
```
integrate((f*x+e)*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(I*d^2*f*x^2*e^(d*x + c) + d^2*f*x^2 + 2*I*d^2*e*x*e^(d*x + c) + 2*d^
2*e*x - 4*I*d*f*x*e^(d*x + c) + 4*I*f*e^(d*x + c)*log(e^(d*x + c) - I) + 4
*d*e + 4*f*log(e^(d*x + c) - I))/(a*d^2*e^(d*x + c) - I*a*d^2)
```

Mupad [B] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

$$\int \frac{(e+fx)\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{f\,x^2\,1\mathrm{i}}{2\,a} - \frac{2\,(e+f\,x)}{a\,d\,\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)} + \frac{x\,(2\,f-d\,e)\,1\mathrm{i}}{a\,d} - \frac{f\,\ln\left(\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-\mathrm{i}\right)\,2\mathrm{i}}{a\,d^2}$$

input
```
int((sinh(c + d*x)*(e + f*x))/(a + a*sinh(c + d*x)*1i),x)
```

output
```
(x*(2*f - d*e)*1i)/(a*d) - (2*(e + f*x))/(a*d*(exp(c + d*x) - 1i)) - (f*x^
2*1i)/(2*a) - (f*log(exp(d*x)*exp(c) - 1i)*2i)/(a*d^2)
```

Reduce [F]

$$\int \frac{(e+fx)\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$=\frac{2\left(\int \frac{x}{\sinh(dx+c)-i}dx\right)f+2\left(\int \frac{1}{\sinh(dx+c)-i}dx\right)e-2eix-fi\,x^2}{2a}$$

input
```
int((f*x+e)*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
(2*int(x/(sinh(c + d*x) - i),x)*f + 2*int(1/(sinh(c + d*x) - i),x)*e - 2*e
*i*x - f*i*x**2)/(2*a)
```

3.190 $\int \frac{\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 35

$$\int \frac{\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{ix}{a}-\frac{\cosh(c+dx)}{d(a+ia\sinh(c+dx))}$$

output
```
-I*x/a-cosh(d*x+c)/d/(a+I*a*sinh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.74

$$\int \frac{\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{i\cosh(c+dx)\left(1-\frac{\operatorname{arcsinh}(\sinh(c+dx))(-i+\sinh(c+dx))}{\sqrt{\cosh^2(c+dx)}}\right)}{ad(-i+\sinh(c+dx))}$$

input
```
Integrate[Sinh[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(I*Cosh[c + d*x]*(1 - (ArcSinh[Sinh[c + d*x]]*(-I + Sinh[c + d*x]))/Sqrt[C
osh[c + d*x]^2]))/(a*d*(-I + Sinh[c + d*x]))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3042, 26, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh (c+d x)}{a+i a \sinh (c+d x)} \, dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{i \sin (i c+i d x)}{a+a \sin (i c+i d x)}dx$$

$$\Big\downarrow \ 26$$

$$-i \int \frac{\sin (i c+i d x)}{\sin (i c+i d x) a+a}dx$$

$$\Big\downarrow \ 3214$$

$$-i \left(\frac{x}{a}-\int \frac{1}{i \sinh (c+d x) a+a}dx\right)$$

$$\Big\downarrow \ 3042$$

$$-i \left(\frac{x}{a}-\int \frac{1}{\sin (i c+i d x) a+a}dx\right)$$

$$\Big\downarrow \ 3127$$

$$-i \left(\frac{x}{a}-\frac{i \cosh (c+d x)}{d (a+i a \sinh (c+d x))}\right)$$

input
```
Int[Sinh[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(-I)*(x/a - (I*Cosh[c + d*x])/(d*(a + I*a*Sinh[c + d*x])))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{ix}{a}-\frac{2}{da\left(\mathrm{e}^{dx+c}-i\right)}$	28
parallelrisch	$-\frac{dx+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)(idx-2)}{da\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	44
derivativedivides	$\frac{-i\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4i}{-2i+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da}$	57
default	$\frac{-i\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4i}{-2i+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da}$	57

input
```
int(sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-I*x/a-2/d/a/(exp(d*x+c)-I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{-i\,dxe^{(dx+c)}-dx-2}{ade^{(dx+c)}-i\,ad}$$

input
```
integrate(sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(-I*d*x*e^(d*x + c) - d*x - 2)/(a*d*e^(d*x + c) - I*a*d)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int \frac{\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{2}{ade^{c}e^{dx}-iad}-\frac{ix}{a}$$

input
```
integrate(sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-2/(a*d*exp(c)*exp(d*x) - I*a*d) - I*x/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\,(dx+c)}{ad}-\frac{2}{(ae^{(-dx-c)}+i\,a)d}$$

input
```
integrate(sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-I*(d*x + c)/(a*d) - 2/((a*e^(-d*x - c) + I*a)*d)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{\frac{i\,(dx+c)}{a}+\frac{2i}{a\left(i\,e^{(dx+c)}+1\right)}}{d}$$

input
```
integrate(sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-(I*(d*x + c)/a + 2*I/(a*(I*e^(d*x + c) + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{x\,1\mathrm{i}}{a}-\frac{2}{a\,d\,\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)}$$

input
```
int(sinh(c + d*x)/(a + a*sinh(c + d*x)*1i),x)
```

output
```
- (x*1i)/a - 2/(a*d*(exp(c + d*x) - 1i))
```

Reduce [F]

$$\int \frac{\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{i\left(\int \frac{1}{\sinh(dx+c)i+1}dx-x\right)}{a}$$

input
```
int(sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
(i*(int(1/(sinh(c + d*x)*i + 1),x) - x))/a
```

3.191 $\int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(sinh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 40.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

input
```
Integrate[Sinh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Integrate[Sinh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}dx$$

$$\Big\downarrow \text{ 6111}$$

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}dx$$

input
```
Int[Sinh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sinh(dx+c)}{(fx+e)(a+ia\sinh(dx+c))}dx$$

input
```
int(sinh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 5.31

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
((-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))*integral(-(d*f*x +
 d*e - (-I*d*f*x - I*d*e)*e^(d*x + c) + 2*f)/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f
*x - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c)), x) -
2)/(-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))
```

Sympy [N/A]

Not integrable

Time = 12.59 (sec) , antiderivative size = 425, normalized size of antiderivative = 14.66

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = -\frac{2}{-iade-iadfx+(adee^c+adfxe^c)\,e^{dx}} - \frac{i\left(\int\left(-\frac{2if}{e^2e^ce^{dx}-ie^2+2efxe^ce^{dx}-2iefx+f^2x^2e^ce^{dx}-if^2x^2}\right)dx+\int\left(-\frac{ide}{e^2e^ce^{dx}-ie^2+2efxe^ce^{dx}-2iefx+f^2x^2e^ce^{dx}-if^2x^2}\right)d\right.}{}$$

input
```
integrate(sinh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-2/(-I*a*d*e - I*a*d*f*x + (a*d*e*exp(c) + a*d*f*x*exp(c))*exp(d*x)) - I*(
Integral(-2*I*f/(e**2*exp(c)*exp(d*x) - I*e**2 + 2*e*f*x*exp(c)*exp(d*x) -
 2*I*e*f*x + f**2*x**2*exp(c)*exp(d*x) - I*f**2*x**2), x) + Integral(-I*d*
e/(e**2*exp(c)*exp(d*x) - I*e**2 + 2*e*f*x*exp(c)*exp(d*x) - 2*I*e*f*x + f
**2*x**2*exp(c)*exp(d*x) - I*f**2*x**2), x) + Integral(-I*d*f*x/(e**2*exp(
c)*exp(d*x) - I*e**2 + 2*e*f*x*exp(c)*exp(d*x) - 2*I*e*f*x + f**2*x**2*exp
(c)*exp(d*x) - I*f**2*x**2), x) + Integral(d*e*exp(c)*exp(d*x)/(e**2*exp(c
)*exp(d*x) - I*e**2 + 2*e*f*x*exp(c)*exp(d*x) - 2*I*e*f*x + f**2*x**2*exp(
c)*exp(d*x) - I*f**2*x**2), x) + Integral(d*f*x*exp(c)*exp(d*x)/(e**2*exp(
c)*exp(d*x) - I*e**2 + 2*e*f*x*exp(c)*exp(d*x) - 2*I*e*f*x + f**2*x**2*exp
(c)*exp(d*x) - I*f**2*x**2), x))/(a*d)
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 4.00

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-2*f*integrate(1/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x^
2*e^c + 2*a*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)), x) - 2/(-I*a*d*f*x - I*a*
d*e + (a*d*f*x*e^c + a*d*e*e^c)*e^(d*x)) - I*log(f*x + e)/(a*f)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sinh(d*x + c)/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(c+d\,x)}{(e+f\,x)\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(sinh(c + d*x)/((e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(sinh(c + d*x)/((e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.62

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \frac{i\left(\left(\int \frac{1}{\sinh(dx+c)ei+\sinh(dx+c)fix+e+fx}dx\right)f-\log(fx+e)\right)}{af}$$

input
```
int(sinh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
(i*(int(1/(sinh(c + d*x)*e*i + sinh(c + d*x)*f*i*x + e + f*x),x)*f - log(e
 + f*x)))/(a*f)
```

3.192 $\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} \, dx = \text{Int}\left(\frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}, x\right)$$

output
```
Defer(Int)(sinh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 28.79 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} \, dx = \int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} \, dx$$

input
```
Integrate[Sinh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Integrate[Sinh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

$$\Big\downarrow \text{ 6111}$$

$$\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}dx$$

input
```
Int[Sinh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sinh(dx+c)}{(fx+e)^2\,(a+ia\sinh(dx+c))}dx$$

input
```
int(sinh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 232, normalized size of antiderivative = 8.00

$$\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx=\int \frac{\sinh\left(dx+c\right)}{\left(fx+e\right)^2\left(i\,a\sinh\left(dx+c\right)+a\right)}\,dx$$

input
```
integrate(sinh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
((-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x
+ a*d*e^2)*e^(d*x + c))*integral(-(d*f*x + d*e - (-I*d*f*x - I*d*e)*e^(d*x
 + c) + 4*f)/(-I*a*d*f^3*x^3 - 3*I*a*d*e*f^2*x^2 - 3*I*a*d*e^2*f*x - I*a*d
*e^3 + (a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x + a*d*e^3)*e^(d*x +
c)), x) - 2)/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x^2 +
2*a*d*e*f*x + a*d*e^2)*e^(d*x + c))
```

Sympy [N/A]

Not integrable

Time = 24.16 (sec) , antiderivative size = 631, normalized size of antiderivative = 21.76

$$\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$
$$=-\frac{2}{-iade^2-2iadefx-iadf^2x^2+\left(ade^2e^c+2adefxe^c+adf^2x^2e^c\right)e^{dx}}$$
$$-\frac{i\left(\int\left(-\frac{4if}{e^3e^ce^{dx}-ie^3+3e^2fxe^ce^{dx}-3ie^2fx+3ef^2x^2e^ce^{dx}-3ief^2x^2+f^3x^3e^ce^{dx}-if^3x^3}\right)dx+\int\left(-\frac{}{e^3e^ce^{dx}-ie^3+3e^2fxe^ce^{dx}-3i\cdots}\right.\right.}{}$$

input
```
integrate(sinh(d*x+c)/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
-2/(-I*a*d*e**2 - 2*I*a*d*e*f*x - I*a*d*f**2*x**2 + (a*d*e**2*exp(c) + 2*a
*d*e*f*x*exp(c) + a*d*f**2*x**2*exp(c))*exp(d*x)) - I*(Integral(-4*I*f/(e*
*3*exp(c)*exp(d*x) - I*e**3 + 3*e**2*f*x*exp(c)*exp(d*x) - 3*I*e**2*f*x +
3*e*f**2*x**2*exp(c)*exp(d*x) - 3*I*e*f**2*x**2 + f**3*x**3*exp(c)*exp(d*x
) - I*f**3*x**3), x) + Integral(-I*d*e/(e**3*exp(c)*exp(d*x) - I*e**3 + 3*
e**2*f*x*exp(c)*exp(d*x) - 3*I*e**2*f*x + 3*e*f**2*x**2*exp(c)*exp(d*x) -
3*I*e*f**2*x**2 + f**3*x**3*exp(c)*exp(d*x) - I*f**3*x**3), x) + Integral(
-I*d*f*x/(e**3*exp(c)*exp(d*x) - I*e**3 + 3*e**2*f*x*exp(c)*exp(d*x) - 3*I
*e**2*f*x + 3*e*f**2*x**2*exp(c)*exp(d*x) - 3*I*e*f**2*x**2 + f**3*x**3*ex
p(c)*exp(d*x) - I*f**3*x**3), x) + Integral(d*e*exp(c)*exp(d*x)/(e**3*exp(
c)*exp(d*x) - I*e**3 + 3*e**2*f*x*exp(c)*exp(d*x) - 3*I*e**2*f*x + 3*e*f**
2*x**2*exp(c)*exp(d*x) - 3*I*e*f**2*x**2 + f**3*x**3*exp(c)*exp(d*x) - I*f
**3*x**3), x) + Integral(d*f*x*exp(c)*exp(d*x)/(e**3*exp(c)*exp(d*x) - I*e
**3 + 3*e**2*f*x*exp(c)*exp(d*x) - 3*I*e**2*f*x + 3*e*f**2*x**2*exp(c)*exp
(d*x) - 3*I*e*f**2*x**2 + f**3*x**3*exp(c)*exp(d*x) - I*f**3*x**3), x))/(a
*d)
```

Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 194, normalized size of antiderivative = 6.69

$$\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)}{(fx+e)^2(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-4*f*integrate(1/(-I*a*d*f^3*x^3 - 3*I*a*d*e*f^2*x^2 - 3*I*a*d*e^2*f*x - I
*a*d*e^3 + (a*d*f^3*x^3*e^c + 3*a*d*e*f^2*x^2*e^c + 3*a*d*e^2*f*x*e^c + a*
d*e^3*e^c)*e^(d*x)), x) + (d*f*x + d*e - (-I*d*f*x*e^c - I*d*e*e^c)*e^(d*x
) - 2*f)/(-I*a*d*f^3*x^2 - 2*I*a*d*e*f^2*x - I*a*d*e^2*f + (a*d*f^3*x^2*e^
c + 2*a*d*e*f^2*x*e^c + a*d*e^2*f*e^c)*e^(d*x))
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)}{\left(fx+e\right)^2\left(i\,a\sinh\left(dx+c\right)+a\right)}\,dx$$

input
```
integrate(sinh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sinh(d*x + c)/((f*x + e)^2*(I*a*sinh(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(c+d\,x)}{\left(e+f\,x\right)^2\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(sinh(c + d*x)/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(sinh(c + d*x)/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.34

$$\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \frac{\int \frac{\sinh(dx+c)}{\sinh(dx+c)e^2i+2\sinh(dx+c)efix+\sinh(dx+c)f^2i\,x^2+e^2+2efx+f^2x^2}dx}{a}$$

input
```
int(sinh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sinh(c + d*x)/(sinh(c + d*x)*e**2*i + 2*sinh(c + d*x)*e*f*i*x + sinh(c
 + d*x)*f**2*i*x**2 + e**2 + 2*e*f*x + f**2*x**2),x)/a
```

3.193 $\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 241

$$\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{(e+fx)^3}{ad}+\frac{(e+fx)^4}{4af}-\frac{6if^2(e+fx)\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^3\cosh(c+dx)}{ad} + \frac{6f(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2} + \frac{12f^2(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3} - \frac{12f^3\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^4} + \frac{6if^3\sinh(c+dx)}{ad^4}+\frac{3if(e+fx)^2\sinh(c+dx)}{ad^2} - \frac{(e+fx)^3\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}$$

output
```
-(f*x+e)^3/a/d+1/4*(f*x+e)^4/a/f-6*I*f^2*(f*x+e)*cosh(d*x+c)/a/d^3-I*(f*x+
e)^3*cosh(d*x+c)/a/d+6*f*(f*x+e)^2*ln(1+I*exp(d*x+c))/a/d^2+12*f^2*(f*x+e)
*polylog(2,-I*exp(d*x+c))/a/d^3-12*f^3*polylog(3,-I*exp(d*x+c))/a/d^4+6*I*
f^3*sinh(d*x+c)/a/d^4+3*I*f*(f*x+e)^2*sinh(d*x+c)/a/d^2-(f*x+e)^3*tanh(1/2
*c+1/4*I*Pi+1/2*d*x)/a/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2619 vs. 2(241) = 482.

Time = 3.34 (sec) , antiderivative size = 2619, normalized size of antiderivative = 10.87

$$\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Sinh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-10*I)*d^3*e^3*E^c*Cosh[(d*x)/2] - 2*d^3*e^3*E^(2*c)*Cosh[(d*x)/2] - (6*
I)*d^2*e^2*E^c*f*Cosh[(d*x)/2] + 6*d^2*e^2*E^(2*c)*f*Cosh[(d*x)/2] - (12*I
)*d*e*E^c*f^2*Cosh[(d*x)/2] - 12*d*e*E^(2*c)*f^2*Cosh[(d*x)/2] - (12*I)*E^
c*f^3*Cosh[(d*x)/2] + 12*E^(2*c)*f^3*Cosh[(d*x)/2] - (4*I)*d^4*e^3*E^c*x*C
osh[(d*x)/2] + 4*d^4*e^3*E^(2*c)*x*Cosh[(d*x)/2] - (30*I)*d^3*e^2*E^c*f*x*
Cosh[(d*x)/2] - 6*d^3*e^2*E^(2*c)*f*x*Cosh[(d*x)/2] - (12*I)*d^2*e*E^c*f^2
*x*Cosh[(d*x)/2] + 12*d^2*e*E^(2*c)*f^2*x*Cosh[(d*x)/2] - (12*I)*d*E^c*f^3
*x*Cosh[(d*x)/2] - 12*d*E^(2*c)*f^3*x*Cosh[(d*x)/2] - (6*I)*d^4*e^2*E^c*f*
x^2*Cosh[(d*x)/2] + 6*d^4*e^2*E^(2*c)*f*x^2*Cosh[(d*x)/2] - (30*I)*d^3*e*E
^c*f^2*x^2*Cosh[(d*x)/2] - 6*d^3*e*E^(2*c)*f^2*x^2*Cosh[(d*x)/2] - (6*I)*d
^2*E^c*f^3*x^2*Cosh[(d*x)/2] + 6*d^2*E^(2*c)*f^3*x^2*Cosh[(d*x)/2] - (4*I)
*d^4*e*E^c*f^2*x^3*Cosh[(d*x)/2] + 4*d^4*e*E^(2*c)*f^2*x^3*Cosh[(d*x)/2] -
 (10*I)*d^3*E^c*f^3*x^3*Cosh[(d*x)/2] - 2*d^3*E^(2*c)*f^3*x^3*Cosh[(d*x)/2
] - I*d^4*E^c*f^3*x^4*Cosh[(d*x)/2] + d^4*E^(2*c)*f^3*x^4*Cosh[(d*x)/2] -
2*d^3*e^3*Cosh[(3*d*x)/2] - (2*I)*d^3*e^3*E^(3*c)*Cosh[(3*d*x)/2] - 6*d^2*
e^2*f*Cosh[(3*d*x)/2] + (6*I)*d^2*e^2*E^(3*c)*f*Cosh[(3*d*x)/2] - 12*d*e*f
^2*Cosh[(3*d*x)/2] - (12*I)*d*e*E^(3*c)*f^2*Cosh[(3*d*x)/2] - 12*f^3*Cosh[
(3*d*x)/2] + (12*I)*E^(3*c)*f^3*Cosh[(3*d*x)/2] - 6*d^3*e^2*f*x*Cosh[(3*d*
x)/2] - (6*I)*d^3*e^2*E^(3*c)*f*x*Cosh[(3*d*x)/2] - 12*d^2*e*f^2*x*Cosh[(3
*d*x)/2] + (12*I)*d^2*e*E^(3*c)*f^2*x*Cosh[(3*d*x)/2] - 12*d*f^3*x*Cosh...
```

Rubi [A] (verified)

Time = 2.21 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.10, number of steps used = 30, number of rules used = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.935$, Rules used = {6091, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117, 6091, 17, 3042, 3799, 25, 25, 3042, 4672, 26, 3042, 26, 4199, 26, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6091}$$

$$i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int (e+fx)^3\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int -i(e+fx)^3\sin(ic+idx)dx}{a}$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\int (e+fx)^3\sin(ic+idx)dx}{a}$$

$$\Big\downarrow \text{ 3777}$$

$$i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\cosh(c+dx)dx}{d}}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}$$

$$\Big\downarrow \text{ 3777}$$

$$i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int (e+fx)\sinh(c+dx)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int -i(e+fx)\sin(ic+idx)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int (e+fx)\sin(ic+idx)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{ 3777}$$

$$i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{ 3117}$$

$$i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{ 6091}$$

$$\frac{i\left(i\int \frac{(e+fx)^3}{i\sinh(c+dx)a+a}dx-\frac{i\int (e+fx)^3dx}{a}\right)-\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$\downarrow$ 17

$$\frac{i\left(i\int \frac{(e+fx)^3}{i\sinh(c+dx)a+a}dx-\frac{i(e+fx)^4}{4af}\right)-\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$\downarrow$ 3042

$$\frac{i\left(i\int \frac{(e+fx)^3}{\sin(ic+idx)a+a}dx-\frac{i(e+fx)^4}{4af}\right)-\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$\downarrow$ 3799

$$\frac{i\left(\frac{i\int -(e+fx)^3\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$\downarrow$ 25

$$\frac{i\left(-\frac{i\int -(e+fx)^3\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$\downarrow$ 25

$$\frac{i\left(\frac{i\int(e+fx)^3\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}}{}$$

$\downarrow$ 3042

$$i\left(\frac{i\int(e+fx)^3\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$\downarrow$ 4672

$$i\left(\frac{i\left(\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{6if\int-i(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$\downarrow$ 26

$$i\left(\frac{i\left(\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{6f\int(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$\downarrow$ 3042

$$i\left(\frac{i\left(\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{6f\int -i(e+fx)^2\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$\downarrow$ 26

$$i\left(\frac{i\left(\frac{6if\int (e+fx)^2\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$\downarrow$ 4199

$$i\left(\frac{i\left(\frac{6if\left(2i\int \frac{ie^{c+dx}(e+fx)^2}{1+ie^{c+dx}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$\downarrow$ 26

$$i\left(\frac{i\left(\frac{6if\left(-2\int \frac{e^{c+dx}(e+fx)^2}{1+ie^{c+dx}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$\downarrow$ 2620

$$i\left(\frac{i\left(\frac{6if\left(-2\left(\frac{2if\int(e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}\right)-$$

$$\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$\downarrow$ 3011

$$i\left(\frac{i\left(\frac{6if\left(-2\left(\frac{2if\left(\frac{f\int\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\right.}{d}\right.}{2a}\right.$$

$$\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$\downarrow$ 2720

$$i\left(\frac{i\left(\frac{6if\left(-2\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)}{}\right.}{2a}\right.$$

$$\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$\downarrow$ 7143

$$i\left(\frac{i\left(\frac{6if\left(-2\left(\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\right.}{d}\right.}{2a}\right.$$

$$\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

input
```
Int[((e + f*x)^3*Sinh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
-(((I*(e + f*x)^3*Cosh[c + d*x])/d - ((3*I)*f*(((e + f*x)^2*Sinh[c + d*x])
/d + ((2*I)*f*((I*(e + f*x)*Cosh[c + d*x])/d - (I*f*Sinh[c + d*x])/d^2))/d
))/d)/a) + I*(((-1/4*I)*(e + f*x)^4)/(a*f) + ((I/2)*(((6*I)*f*(((-1/3*I)*(
e + f*x)^3)/f - 2*(((-I)*(e + f*x)^2*Log[1 + I*E^(c + d*x)])/d + ((2*I)*f*
(-(((e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d) + (f*PolyLog[3, (-I)*E^(c +
 d*x)])/d^2))/d)))/d + (2*(e + f*x)^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/d))/
a)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 698 vs. 2(222) = 444.

Time = 0.84 (sec) , antiderivative size = 699, normalized size of antiderivative = 2.90

method	result
risch	$-\frac{12 f^{2} e c \ln\left(\mathrm{e}^{d x+c}-i\right)}{a\, d^{3}}+\frac{12 f^{2} e \ln\left(1+i \mathrm{e}^{d x+c}\right) c}{a\, d^{3}}+\frac{12 f^{2} e \ln\left(1+i \mathrm{e}^{d x+c}\right) x}{a\, d^{2}}-\frac{i\left(d^{3} x^{3} f^{3}+3 d^{3} e\, f^{2} x^{2}+3 d^{3} e^{2} f x+3 d^{2} f^{3} x^{2}+d^{3} e^{3}+\cdots\right.}{2 d^{4}\cdots}$

input
```
int((f*x+e)^3*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*(d^3*f^3*x^3+3*d^3*e*f^2*x^2+3*d^3*e^2*f*x+3*d^2*f^3*x^2+d^3*e^3+6*
d^2*e*f^2*x+3*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2+6*f^3)/d^4/a*exp(-d*x-c)+12/a/
d^3*f^2*e*c*ln(exp(d*x+c))-12/a/d^3*f^2*e*c*ln(exp(d*x+c)-I)-12/a/d^2*f^2*
e*c*x+12/a/d^3*f^2*e*ln(1+I*exp(d*x+c))*c+12/a/d^2*f^2*e*ln(1+I*exp(d*x+c)
)*x-1/2*I*(d^3*f^3*x^3+3*d^3*e*f^2*x^2+3*d^3*e^2*f*x-3*d^2*f^3*x^2+d^3*e^3
-6*d^2*e*f^2*x-3*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2-6*f^3)/d^4/a*exp(d*x+c)-2*I
*(f^3*x^3+3*e*f^2*x^2+3*e^2*f*x+e^3)/d/a/(exp(d*x+c)-I)-6/a/d*f^2*e*x^2-6/
a/d^2*f*ln(exp(d*x+c))*e^2+6/a/d^3*f^3*c^2*x-6/a/d^4*f^3*ln(1+I*exp(d*x+c)
)*c^2+12/a/d^3*f^3*polylog(2,-I*exp(d*x+c))*x+6/a/d^2*f*ln(exp(d*x+c)-I)*e
^2+6/a/d^2*f^3*ln(1+I*exp(d*x+c))*x^2-6/a/d^4*f^3*c^2*ln(exp(d*x+c))-6/a/d
^3*f^2*e*c^2+12/a/d^3*f^2*e*polylog(2,-I*exp(d*x+c))+6/a/d^4*f^3*c^2*ln(ex
p(d*x+c)-I)+1/a*f^2*e*x^3+3/2*a*f*e^2*x^2+1/a*e^3*x-12*f^3*polylog(3,-I*ex
p(d*x+c))/a/d^4+4/a/d^4*f^3*c^3-2/a/d*f^3*x^3+1/4/a*f^3*x^4+1/4/a/f*e^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 823 vs. $2(213) = 426$.

Time = 0.12 (sec) , antiderivative size = 823, normalized size of antiderivative = 3.41

$$\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
-1/4*(2*d^3*f^3*x^3 + 2*d^3*e^3 + 6*d^2*e^2*f + 12*d*e*f^2 + 12*f^3 + 6*(d
^3*e*f^2 + d^2*f^3)*x^2 + 6*(d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)*x - 48*((d
*f^3*x + d*e*f^2)*e^(2*d*x + 2*c) - (I*d*f^3*x + I*d*e*f^2)*e^(d*x + c))*d
ilog(-I*e^(d*x + c)) + 2*(I*d^3*f^3*x^3 + I*d^3*e^3 - 3*I*d^2*e^2*f + 6*I*
d*e*f^2 - 6*I*f^3 + 3*(I*d^3*e*f^2 - I*d^2*f^3)*x^2 + 3*(I*d^3*e^2*f - 2*I
*d^2*e*f^2 + 2*I*d*f^3)*x)*e^(3*d*x + 3*c) - (d^4*f^3*x^4 - 2*d^3*e^3 - 6*
(4*c - 1)*d^2*e^2*f + 12*(2*c^2 - 1)*d*e*f^2 - 4*(2*c^3 - 3)*f^3 + 2*(2*d^
4*e*f^2 - 5*d^3*f^3)*x^3 + 6*(d^4*e^2*f - 5*d^3*e*f^2 + d^2*f^3)*x^2 + 2*(
2*d^4*e^3 - 15*d^3*e^2*f + 6*d^2*e*f^2 - 6*d*f^3)*x)*e^(2*d*x + 2*c) - (-I
*d^4*f^3*x^4 - 10*I*d^3*e^3 - 6*(-4*I*c + I)*d^2*e^2*f - 12*(2*I*c^2 + I)*
d*e*f^2 - 4*(-2*I*c^3 + 3*I)*f^3 - 2*(2*I*d^4*e*f^2 + I*d^3*f^3)*x^3 - 6*(
I*d^4*e^2*f + I*d^3*e*f^2 + I*d^2*f^3)*x^2 - 2*(2*I*d^4*e^3 + 3*I*d^3*e^2*
f + 6*I*d^2*e*f^2 + 6*I*d*f^3)*x)*e^(d*x + c) - 24*((d^2*e^2*f - 2*c*d*e*f
^2 + c^2*f^3)*e^(2*d*x + 2*c) - (I*d^2*e^2*f - 2*I*c*d*e*f^2 + I*c^2*f^3)*
e^(d*x + c))*log(e^(d*x + c) - I) - 24*((d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c
*d*e*f^2 - c^2*f^3)*e^(2*d*x + 2*c) - (I*d^2*f^3*x^2 + 2*I*d^2*e*f^2*x + 2
*I*c*d*e*f^2 - I*c^2*f^3)*e^(d*x + c))*log(I*e^(d*x + c) + 1) + 48*(f^3*e^
(2*d*x + 2*c) - I*f^3*e^(d*x + c))*polylog(3, -I*e^(d*x + c)))/(a*d^4*e^(2
*d*x + 2*c) - I*a*d^4*e^(d*x + c))
```

Sympy [F]

$$\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{-2ie^3 - 6ie^2fx - 6ief^2x^2 - 2if^3x^3}{ade^ce^{dx} - iad}$$
$$- \frac{i\left(\int \frac{ide^3}{e^ce^{2dx}-ie^{dx}}\,dx + \int \frac{idf^3x^3}{e^ce^{2dx}-ie^{dx}}\,dx + \int \frac{de^3e^ce^{dx}}{e^ce^{2dx}-ie^{dx}}\,dx + \int \frac{de^3e^{3c}e^{3dx}}{e^ce^{2dx}-ie^{dx}}\,dx + \int \left(-\frac{12e^2fe^ce^{dx}}{e^ce^{2dx}-ie^{dx}}\right)dx + \int \left(-\right.\right.}{}$$

input
```
integrate((f*x+e)**3*sinh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
(-2*I*e**3 - 6*I*e**2*f*x - 6*I*e*f**2*x**2 - 2*I*f**3*x**3)/(a*d*exp(c)*e
xp(d*x) - I*a*d) - I*(Integral(I*d*e**3/(exp(c)*exp(2*d*x) - I*exp(d*x)),
x) + Integral(I*d*f**3*x**3/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integra
l(d*e**3*exp(c)*exp(d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(d
*e**3*exp(3*c)*exp(3*d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(
-12*e**2*f*exp(c)*exp(d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral
(-12*f**3*x**2*exp(c)*exp(d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Inte
gral(3*I*d*e*f**2*x**2/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(3*I
*d*e**2*f*x/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(I*d*e**3*exp(2
*c)*exp(2*d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(d*f**3*x**3
*exp(c)*exp(d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(d*f**3*x*
*3*exp(3*c)*exp(3*d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(-24
*e*f**2*x*exp(c)*exp(d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(
I*d*f**3*x**3*exp(2*c)*exp(2*d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + I
ntegral(3*d*e*f**2*x**2*exp(c)*exp(d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)),
x) + Integral(3*d*e*f**2*x**2*exp(3*c)*exp(3*d*x)/(exp(c)*exp(2*d*x) - I*e
xp(d*x)), x) + Integral(3*d*e**2*f*x*exp(c)*exp(d*x)/(exp(c)*exp(2*d*x) -
I*exp(d*x)), x) + Integral(3*d*e**2*f*x*exp(3*c)*exp(3*d*x)/(exp(c)*exp(2*
d*x) - I*exp(d*x)), x) + Integral(3*I*d*e*f**2*x**2*exp(2*c)*exp(2*d*x)/(e
xp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(3*I*d*e**2*f*x*exp(2*c)*e...
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(213) = 426$.

Time = 0.25 (sec) , antiderivative size = 671, normalized size of antiderivative = 2.78

$$\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-3/2*e^2*f*(2*x*e^(d*x + c)/(a*d*e^(d*x + c) - I*a*d) + (I*d^2*x^2*e^c + I
*d*x*e^c - (-I*d*x*e^(3*c) + I*e^(3*c))*e^(2*d*x) - (d^2*x^2*e^(2*c) - 3*d
*x*e^(2*c) + e^(2*c))*e^(d*x) + (d*x + 1)*e^(-d*x) + I*e^c)/(a*d^2*e^(d*x
+ 2*c) - I*a*d^2*e^c) - 4*log((e^(d*x + c) - I)*e^(-c))/(a*d^2)) + 1/2*e^3
*(2*(d*x + c)/(a*d) + (-5*I*e^(-d*x - c) + 1)/((I*a*e^(-d*x - c) + a*e^(-2
*d*x - 2*c))*d) - I*e^(-d*x - c)/(a*d)) + 1/4*(-I*d^4*f^3*x^4 - 12*I*d*e*f
^2 + 2*(-2*I*d^4*e*f^2 - 5*I*d^3*f^3)*x^3 - 12*I*f^3 + 6*(-5*I*d^3*e*f^2 -
 I*d^2*f^3)*x^2 + 12*(-I*d^2*e*f^2 - I*d*f^3)*x + 2*(-I*d^3*f^3*x^3*e^(2*c
) + 3*(-I*d^3*e*f^2 + I*d^2*f^3)*x^2*e^(2*c) + 6*(I*d^2*e*f^2 - I*d*f^3)*x
*e^(2*c) + 6*(-I*d*e*f^2 + I*f^3)*e^(2*c))*e^(2*d*x) + (d^4*f^3*x^4*e^c +
2*(2*d^4*e*f^2 - d^3*f^3)*x^3*e^c - 6*(d^3*e*f^2 - d^2*f^3)*x^2*e^c + 12*(
d^2*e*f^2 - d*f^3)*x*e^c - 12*(d*e*f^2 - f^3)*e^c)*e^(d*x))/(a*d^4*e^(d*x
+ c) - I*a*d^4) + 12*(d*x*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c)))*
e*f^2/(a*d^3) + 6*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-I*e^(d*x
+ c)) - 2*polylog(3, -I*e^(d*x + c)))*f^3/(a*d^4) - 2*(d^3*f^3*x^3 + 3*d^3
*e*f^2*x^2)/(a*d^4)
```

Giac [F]

$$\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3 \sinh(dx+c)^2}{i\,a\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sinh(d*x + c)^2/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{\sinh(c+d\,x)^2\,(e+f\,x)^3}{a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((sinh(c + d*x)^2*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i),x)
```

output
```
int((sinh(c + d*x)^2*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$=\frac{-4\cosh\left(dx+c\right)d^3e^3i-12\cosh\left(dx+c\right)d^3e^2fix-12\cosh\left(dx+c\right)d^3e\,f^2i\,x^2-4\cosh\left(dx+c\right)d^3f^3i\,x^3}{}$$

input
```
int((f*x+e)^3*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
( - 4*cosh(c + d*x)*d**3*e**3*i - 12*cosh(c + d*x)*d**3*e**2*f*i*x - 12*co
sh(c + d*x)*d**3*e*f**2*i*x**2 - 4*cosh(c + d*x)*d**3*f**3*i*x**3 - 24*cos
h(c + d*x)*d*e*f**2*i - 24*cosh(c + d*x)*d*f**3*i*x - 4*int(x**3/(sinh(c +
 d*x)*i + 1),x)*d**4*f**3 - 12*int(x**2/(sinh(c + d*x)*i + 1),x)*d**4*e*f*
*2 - 12*int(x/(sinh(c + d*x)*i + 1),x)*d**4*e**2*f - 4*int(1/(sinh(c + d*x
)*i + 1),x)*d**4*e**3 + 12*sinh(c + d*x)*d**2*e**2*f*i + 24*sinh(c + d*x)*
d**2*e*f**2*i*x + 12*sinh(c + d*x)*d**2*f**3*i*x**2 + 24*sinh(c + d*x)*f**
3*i + 4*d**4*e**3*x + 6*d**4*e**2*f*x**2 + 4*d**4*e*f**2*x**3 + d**4*f**3*
x**4)/(4*a*d**4)
```

3.194 $\int \frac{(e+fx)^2 \sinh^2(c+dx)}{a+ia \sinh(c+dx)} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 184

$$\int \frac{(e+fx)^2 \sinh^2(c+dx)}{a+ia \sinh(c+dx)} \, dx = -\frac{(e+fx)^2}{ad} + \frac{(e+fx)^3}{3af} - \frac{2if^2 \cosh(c+dx)}{ad^3} - \frac{i(e+fx)^2 \cosh(c+dx)}{ad} + \frac{4f(e+fx) \log\left(1+ie^{c+dx}\right)}{ad^2} + \frac{4f^2 \operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3} + \frac{2if(e+fx) \sinh(c+dx)}{ad^2} - \frac{(e+fx)^2 \tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}$$

output

```
-(f*x+e)^2/a/d+1/3*(f*x+e)^3/a/f-2*I*f^2*cosh(d*x+c)/a/d^3-I*(f*x+e)^2*cos
h(d*x+c)/a/d+4*f*(f*x+e)*ln(1+I*exp(d*x+c))/a/d^2+4*f^2*polylog(2,-I*exp(d
*x+c))/a/d^3+2*I*f*(f*x+e)*sinh(d*x+c)/a/d^2-(f*x+e)^2*tanh(1/2*c+1/4*I*Pi
+1/2*d*x)/a/d
```

Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.41

$$\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{x(3e^2+3efx+f^2x^2)+\frac{6\left(\frac{d(e+fx)\left(-id(e+fx)+2(-i+e^c)f\log\left(1-ie^{-c-dx}\right)\right)}{-i+e^c}-2f^2\operatorname{PolyLog}\left(2,ie^{-c-dx}\right)\right)}{d^3}-\frac{3i\cosh(dx)\left(\left(2f^2+d^2(e\cdots\right.\right.}{\cdots}}{\cdots}$$

input
```
Integrate[((e + f*x)^2*Sinh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(x*(3*e^2 + 3*e*f*x + f^2*x^2) + (6*((d*(e + f*x)*((-I)*d*(e + f*x) + 2*(-
I + E^c)*f*Log[1 - I*E^(-c - d*x)]))/(-I + E^c) - 2*f^2*PolyLog[2, I*E^(-c
 - d*x)]))/d^3 - ((3*I)*Cosh[d*x]*((2*f^2 + d^2*(e + f*x)^2)*Cosh[c] - 2*d
*f*(e + f*x)*Sinh[c]))/d^3 - ((3*I)*(-2*d*f*(e + f*x)*Cosh[c] + (2*f^2 + d
^2*(e + f*x)^2)*Sinh[c])*Sinh[d*x])/d^3 - (6*(e + f*x)^2*Sinh[(d*x)/2])/(d
*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])))/(3*
a)
```

Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.11, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.839, Rules used = {6091, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118, 6091, 17, 3042, 3799, 25, 25, 3042, 4672, 26, 3042, 26, 4199, 26, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6091}$$

$$i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int (e+fx)^2\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$i \int \frac{(e+fx)^2 \sinh(c+dx)}{i \sinh(c+dx) a + a} dx - \frac{i \int -i(e+fx)^2 \sin(ic+idx) dx}{a}$$

$$\Big\downarrow \text{26}$$

$$i \int \frac{(e+fx)^2 \sinh(c+dx)}{i \sinh(c+dx) a + a} dx - \frac{\int (e+fx)^2 \sin(ic+idx) dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$i \int \frac{(e+fx)^2 \sinh(c+dx)}{i \sinh(c+dx) a + a} dx - \frac{\frac{i(e+fx)^2 \cosh(c+dx)}{d} - \frac{2if \int (e+fx) \cosh(c+dx) dx}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$i \int \frac{(e+fx)^2 \sinh(c+dx)}{i \sinh(c+dx) a + a} dx - \frac{\frac{i(e+fx)^2 \cosh(c+dx)}{d} - \frac{2if \int (e+fx) \sin\left(ic+idx+\frac{\pi}{2}\right) dx}{d}}{a}$$

$$\Big\downarrow \text{3777}$$

$$i \int \frac{(e+fx)^2 \sinh(c+dx)}{i \sinh(c+dx) a + a} dx - \frac{\frac{i(e+fx)^2 \cosh(c+dx)}{d} - \frac{2if \left(\frac{(e+fx) \sinh(c+dx)}{d} - \frac{if \int -i \sinh(c+dx) dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$i \int \frac{(e+fx)^2 \sinh(c+dx)}{i \sinh(c+dx) a + a} dx - \frac{\frac{i(e+fx)^2 \cosh(c+dx)}{d} - \frac{2if \left(\frac{(e+fx) \sinh(c+dx)}{d} - \frac{f \int \sinh(c+dx) dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$i \int \frac{(e+fx)^2 \sinh(c+dx)}{i \sinh(c+dx) a + a} dx - \frac{\frac{i(e+fx)^2 \cosh(c+dx)}{d} - \frac{2if \left(\frac{(e+fx) \sinh(c+dx)}{d} - \frac{f \int -i \sin(ic+idx) dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$i \int \frac{(e+fx)^2 \sinh(c+dx)}{i \sinh(c+dx) a + a} dx - \frac{\frac{i(e+fx)^2 \cosh(c+dx)}{d} - \frac{2if \left(\frac{(e+fx) \sinh(c+dx)}{d} + \frac{if \int \sin(ic+idx) dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{3118}$$

$$i \int \frac{(e+fx)^2 \sinh(c+dx)}{i \sinh(c+dx) a + a} dx - \frac{\frac{i(e+fx)^2 \cosh(c+dx)}{d} - \frac{2if \left(\frac{(e+fx) \sinh(c+dx)}{d} - \frac{f \cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \text{6091}$$

$$i\left(i\int \frac{(e+fx)^2}{i\sinh(c+dx)a+a}dx-\frac{i\int (e+fx)^2dx}{a}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \text{ 17}$$

$$i\left(i\int \frac{(e+fx)^2}{i\sinh(c+dx)a+a}dx-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$i\left(i\int \frac{(e+fx)^2}{\sin(ic+idx)a+a}dx-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \text{ 3799}$$

$$i\left(\frac{i\int -(e+fx)^2\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \text{ 25}$$

$$i\left(-\frac{i\int -(e+fx)^2\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \text{ 25}$$

$$i\left(\frac{i\int (e+fx)^2\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$i\left(\frac{i\int(e+fx)^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \text{4672}$$

$$i\left(\frac{i\left(\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4if\int-i(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i\left(\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4f\int(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{i\left(\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4f\int-i(e+fx)\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i\left(\frac{4if\int(e+fx)\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$\downarrow$ 4199

$$i\left(\frac{i\left(\frac{4if\left(2i\int\frac{ie^{c+dx}(e+fx)}{1+ie^{c+dx}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$\downarrow$ 26

$$i\left(\frac{i\left(\frac{4if\left(-2\int\frac{e^{c+dx}(e+fx)}{1+ie^{c+dx}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$\downarrow$ 2620

$$i\left(\frac{i\left(\frac{4if\left(-2\left(\frac{if\int\log\left(1+ie^{c+dx}\right)dx}{d}-\frac{i(e+fx)\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$\downarrow$ 2715

$$i\left(\frac{i\left(\frac{4if\left(-2\left(\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{i(e+fx)\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}\right)$$

$$\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \text{2838}$$

$$i\left(\frac{i\left(\frac{4if\left(-2\left(-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{i(e+fx)\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}\right)-$$

$$\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

input
```
Int[((e + f*x)^2*Sinh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
-(((I*(e + f*x)^2*Cosh[c + d*x])/d - ((2*I)*f*(-((f*Cosh[c + d*x])/d^2) +
((e + f*x)*Sinh[c + d*x])/d))/d)/a) + I*(((-1/3*I)*(e + f*x)^3)/(a*f) + ((
I/2)*(((4*I)*f*(((-1/2*I)*(e + f*x)^2)/f - 2*(((-I)*(e + f*x)*Log[1 + I*E^
(c + d*x)])/d - (I*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2)))/d + (2*(e + f*x)
^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/d))/a)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(168) = 336$.

Time = 0.65 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.09

method	result
risch	$\frac{f^2x^3}{3a}+\frac{fe\,x^2}{a}+\frac{e^2x}{a}+\frac{e^3}{3af}-\frac{i(d^2x^2f^2+2d^2efx+d^2e^2-2d\,f^2x-2def+2f^2)\,\mathrm{e}^{dx+c}}{2d^3a}-\frac{i(d^2x^2f^2+2d^2efx+d^2e^2+2d\,f^2x+2}{2d^3a}$

input
```
int((f*x+e)^2*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/3/a*f^2*x^3+1/a*f*e*x^2+1/a*e^2*x+1/3/a/f*e^3-1/2*I*(d^2*f^2*x^2+2*d^2*e
*f*x+d^2*e^2-2*d*f^2*x-2*d*e*f+2*f^2)/d^3/a*exp(d*x+c)-1/2*I*(d^2*f^2*x^2+
2*d^2*e*f*x+d^2*e^2+2*d*f^2*x+2*d*e*f+2*f^2)/d^3/a*exp(-d*x-c)-2*I*(f^2*x^
2+2*e*f*x+e^2)/d/a/(exp(d*x+c)-I)+4/a/d^2*f*ln(exp(d*x+c)-I)*e-4/a/d^2*f*l
n(exp(d*x+c))*e-2/a/d*f^2*x^2-4/a/d^2*f^2*c*x-2/a/d^3*f^2*c^2+4/a/d^2*f^2*
ln(1+I*exp(d*x+c))*x+4/a/d^3*f^2*ln(1+I*exp(d*x+c))*c+4*f^2*polylog(2,-I*e
xp(d*x+c))/a/d^3-4/a/d^3*f^2*c*ln(exp(d*x+c)-I)+4/a/d^3*f^2*c*ln(exp(d*x+c
))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(161) = 322$.

Time = 0.12 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.58

$$\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{3\,d^2f^2x^2+3\,d^2e^2+6\,def+6\,f^2+6\,(d^2ef+df^2)x-24\left(f^2e^{(2\,dx+2\,c)}-i\,f^2e^{(dx+c)}\right)\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)+3$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
-1/6*(3*d^2*f^2*x^2 + 3*d^2*e^2 + 6*d*e*f + 6*f^2 + 6*(d^2*e*f + d*f^2)*x
- 24*(f^2*e^(2*d*x + 2*c) - I*f^2*e^(d*x + c))*dilog(-I*e^(d*x + c)) + 3*(
I*d^2*f^2*x^2 + I*d^2*e^2 - 2*I*d*e*f + 2*I*f^2 + 2*(I*d^2*e*f - I*d*f^2)*
x)*e^(3*d*x + 3*c) - (2*d^3*f^2*x^3 - 3*d^2*e^2 - 6*(4*c - 1)*d*e*f + 6*(2
*c^2 - 1)*f^2 + 3*(2*d^3*e*f - 5*d^2*f^2)*x^2 + 6*(d^3*e^2 - 5*d^2*e*f + d
*f^2)*x)*e^(2*d*x + 2*c) - (-2*I*d^3*f^2*x^3 - 15*I*d^2*e^2 - 6*(-4*I*c +
I)*d*e*f - 6*(2*I*c^2 + I)*f^2 - 3*(2*I*d^3*e*f + I*d^2*f^2)*x^2 - 6*(I*d^
3*e^2 + I*d^2*e*f + I*d*f^2)*x)*e^(d*x + c) - 24*((d*e*f - c*f^2)*e^(2*d*x
 + 2*c) - (I*d*e*f - I*c*f^2)*e^(d*x + c))*log(e^(d*x + c) - I) - 24*((d*f
^2*x + c*f^2)*e^(2*d*x + 2*c) - (I*d*f^2*x + I*c*f^2)*e^(d*x + c))*log(I*e
^(d*x + c) + 1))/(a*d^3*e^(2*d*x + 2*c) - I*a*d^3*e^(d*x + c))
```

Sympy [F]

$$\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{-2ie^2-4iefx-2if^2x^2}{ade^ce^{dx}-iad} - \frac{i\left(\int \frac{ide^2}{e^ce^{2dx}-ie^{dx}}\,dx+\int \frac{idf^2x^2}{e^ce^{2dx}-ie^{dx}}\,dx+\int \frac{de^2e^ce^{dx}}{e^ce^{2dx}-ie^{dx}}\,dx+\int \frac{de^2e^{3c}e^{3dx}}{e^ce^{2dx}-ie^{dx}}\,dx+\int \left(-\frac{8efe^ce^{dx}}{e^ce^{2dx}-ie^{dx}}\right)dx+\int \left(-\right.$$

input
```
integrate((f*x+e)**2*sinh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
(-2*I*e**2 - 4*I*e*f*x - 2*I*f**2*x**2)/(a*d*exp(c)*exp(d*x) - I*a*d) - I*
(Integral(I*d*e**2/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(I*d*f**
2*x**2/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(d*e**2*exp(c)*exp(d
*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(d*e**2*exp(3*c)*exp(3*
d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(-8*e*f*exp(c)*exp(d*x
)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(-8*f**2*x*exp(c)*exp(d*x
)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(2*I*d*e*f*x/(exp(c)*exp(
2*d*x) - I*exp(d*x)), x) + Integral(I*d*e**2*exp(2*c)*exp(2*d*x)/(exp(c)*e
xp(2*d*x) - I*exp(d*x)), x) + Integral(d*f**2*x**2*exp(c)*exp(d*x)/(exp(c)
*exp(2*d*x) - I*exp(d*x)), x) + Integral(d*f**2*x**2*exp(3*c)*exp(3*d*x)/(
exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(I*d*f**2*x**2*exp(2*c)*exp(
2*d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(2*d*e*f*x*exp(c)*ex
p(d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(2*d*e*f*x*exp(3*c)*
exp(3*d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(2*I*d*e*f*x*exp
(2*c)*exp(2*d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x))*exp(-c)/(2*a*d)
```

Maxima [F]

$$\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\sinh(dx+c)^2}{i\,a\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-e*f*(2*x*e^(d*x + c)/(a*d*e^(d*x + c) - I*a*d) + (I*d^2*x^2*e^c + I*d*x*e
^c - (-I*d*x*e^(3*c) + I*e^(3*c))*e^(2*d*x) - (d^2*x^2*e^(2*c) - 3*d*x*e^(
2*c) + e^(2*c))*e^(d*x) + (d*x + 1)*e^(-d*x) + I*e^c)/(a*d^2*e^(d*x + 2*c)
 - I*a*d^2*e^c) - 4*log((e^(d*x + c) - I)*e^(-c))/(a*d^2)) - 1/6*f^2*((2*I
*d^3*x^3 + 15*I*d^2*x^2 + 6*I*d*x - 3*(-I*d^2*x^2*e^(2*c) + 2*I*d*x*e^(2*c
) - 2*I*e^(2*c))*e^(2*d*x) - (2*d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c -
6*e^c)*e^(d*x) + 6*I)/(a*d^3*e^(d*x + c) - I*a*d^3) - 24*I*integrate(x/(a*
d*e^(d*x + c) - I*a*d), x)) + 1/2*e^2*(2*(d*x + c)/(a*d) + (-5*I*e^(-d*x -
 c) + 1)/((I*a*e^(-d*x - c) + a*e^(-2*d*x - 2*c))*d) - I*e^(-d*x - c)/(a*d
))
```

Giac [F]

$$\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\sinh(dx+c)^2}{i\,a\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sinh(d*x + c)^2/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{\sinh(c+d\,x)^2\,(e+f\,x)^2}{a+a\sinh(c+d\,x)\,1\mathrm{i}}\,dx$$

input
```
int((sinh(c + d*x)^2*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i),x)
```

output
```
int((sinh(c + d*x)^2*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2 \sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{-3\cosh(dx+c)\,d^2e^2i - 6\cosh(dx+c)\,d^2efix - 3\cosh(dx+c)\,d^2f^2i\,x^2 - 6\cosh(dx+c)\,f^2i - 3\Big(\int \frac{}{\mathrm{si}}}{}$$

input
```
int((f*x+e)^2*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
( - 3*cosh(c + d*x)*d**2*e**2*i - 6*cosh(c + d*x)*d**2*e*f*i*x - 3*cosh(c
+ d*x)*d**2*f**2*i*x**2 - 6*cosh(c + d*x)*f**2*i - 3*int(x**2/(sinh(c + d*
x)*i + 1),x)*d**3*f**2 - 6*int(x/(sinh(c + d*x)*i + 1),x)*d**3*e*f - 3*int
(1/(sinh(c + d*x)*i + 1),x)*d**3*e**2 + 6*sinh(c + d*x)*d*e*f*i + 6*sinh(c
 + d*x)*d*f**2*i*x + 3*d**3*e**2*x + 3*d**3*e*f*x**2 + d**3*f**2*x**3)/(3*
a*d**3)
```

3.195 $\int \frac{(e+fx)\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 119

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{(e+fx)^2}{2af} - \frac{i(e+fx)\cosh(c+dx)}{ad} + \frac{2f\log\left(\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2} + \frac{if\sinh(c+dx)}{ad^2} - \frac{(e+fx)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}$$

output
```
1/2*(f*x+e)^2/a/f-I*(f*x+e)*cosh(d*x+c)/a/d+2*f*ln(cosh(1/2*c+1/4*I*Pi+1/2
*d*x))/a/d^2+I*f*sinh(d*x+c)/a/d^2-(f*x+e)*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a/
d
```

Mathematica [A] (verified)

Time = 2.83 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.00

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{\left(-i\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(\sinh\left(\frac{1}{2}(c+dx)\right)\left(i(2i+c+dx)(2de-cf+dfx)-4f\arctan\right.\right.}{}$$

input
```
Integrate[((e + f*x)*Sinh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(((-I)*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])*(Sinh[(c + d*x)/2]*(I*(2*I +
 c + d*x)*(2*d*e - c*f + d*f*x) - 4*f*ArcTan[Tanh[(c + d*x)/2]] + 2*d*(e +
 f*x)*Cosh[c + d*x] + (2*I)*f*Log[Cosh[c + d*x]] - 2*f*Sinh[c + d*x]) + Co
sh[(c + d*x)/2]*(2*c*d*e - (2*I)*c*f - c^2*f + 2*d^2*e*x - (2*I)*d*f*x + d
^2*f*x^2 + (4*I)*f*ArcTan[Tanh[(c + d*x)/2]] - (2*I)*d*(e + f*x)*Cosh[c +
d*x] + 2*f*Log[Cosh[c + d*x]] + (2*I)*f*Sinh[c + d*x])))/(2*a*d^2*(-I + Si
nh[c + d*x]))
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.09, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.621, Rules used = {6091, 3042, 26, 3777, 3042, 3117, 6091, 17, 3042, 3799, 25, 25, 3042, 4672, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$\Big\downarrow \ 6091$$

$$i\int \frac{(e+fx)\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int (e+fx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \ 3042$$

$$i\int \frac{(e+fx)\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int -i(e+fx)\sin(ic+idx)dx}{a}$$

$$\Big\downarrow \ 26$$

$$i\int \frac{(e+fx)\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\int (e+fx)\sin(ic+idx)dx}{a}$$

$$\Big\downarrow \ 3777$$

$$i\int \frac{(e+fx)\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$i\int \frac{(e+fx)\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}$$

$$\Big\downarrow \text{3117}$$

$$i\int \frac{(e+fx)\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}$$

$$\Big\downarrow \text{6091}$$

$$i\left(i\int \frac{e+fx}{i\sinh(c+dx)a+a}dx-\frac{i\int (e+fx)dx}{a}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}$$

$$\Big\downarrow \text{17}$$

$$i\left(i\int \frac{e+fx}{i\sinh(c+dx)a+a}dx-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}$$

$$\Big\downarrow \text{3042}$$

$$i\left(i\int \frac{e+fx}{\sin(ic+idx)a+a}dx-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}$$

$$\Big\downarrow \text{3799}$$

$$i\left(\frac{i\int -\left((e+fx)\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)\right)dx}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}$$

$$\Big\downarrow \text{25}$$

$$i\left(-\frac{i\int -\left((e+fx)\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)dx}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}$$

$$\Big\downarrow \text{25}$$

$$i\left(\frac{i\int (e+fx)\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{i\int (e+fx)\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}$$

$$\Big\downarrow \text{4672}$$

$$i\left(\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2if\int -i\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}$$

$\downarrow$ 26

$$i\left(\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2f\int \tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}$$

$\downarrow$ 3042

$$i\left(\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2f\int -i\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}$$

$\downarrow$ 26

$$i\left(\frac{i\left(\frac{2if\int \tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}+\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}$$

$\downarrow$ 3956

$$i\left(\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4f\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{d^2}\right)}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}$$

input
```
Int[((e + f*x)*Sinh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
-(((I*(e + f*x)*Cosh[c + d*x])/d - (I*f*Sinh[c + d*x])/d^2)/a) + I*(((-1/2
*I)*(e + f*x)^2)/(a*f) + ((I/2)*((-4*f*Log[Cosh[c/2 + (I/4)*Pi + (d*x)/2]]
)/d^2 + (2*(e + f*x)*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/d))/a)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.13

method	result
risch	$\frac{f\,x^{2}}{2a}+\frac{ex}{a}-\frac{i(dxf+de-f)\mathrm{e}^{dx+c}}{2a\,d^{2}}-\frac{i(dxf+de+f)\mathrm{e}^{-dx-c}}{2a\,d^{2}}-\frac{2fx}{ad}-\frac{2fc}{a\,d^{2}}-\frac{2i(fx+e)}{da\left(\mathrm{e}^{dx+c}-i\right)}+\frac{2f\ln\left(\mathrm{e}^{dx+c}-i\right)}{a\,d^{2}}$
parallelrisch	$\frac{4f\left(i\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)-\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\ln\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4f\left(i\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)-\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+((i\cdots}{}$

input
```
int((f*x+e)*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*f*x^2+1/a*e*x-1/2*I*(d*f*x+d*e-f)/a/d^2*exp(d*x+c)-1/2*I*(d*f*x+d*e+
f)/a/d^2*exp(-d*x-c)-2*f*x/a/d-2*f/a/d^2*c-2*I*(f*x+e)/d/a/(exp(d*x+c)-I)+
2*f/a/d^2*ln(exp(d*x+c)-I)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.45

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{dfx+de-(-i\,dfx-i\,de+i\,f)e^{(3\,dx+3\,c)}-(d^2fx^2-de+(2\,d^2e-5\,df)x+f)e^{(2\,dx+2\,c)}-(-i\,d^2fx^2}{2\,(ad^2e^{(2\,dx+2\,c)}-i\,ac}$$

input

```
integrate((f*x+e)*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output

```
-1/2*(d*f*x + d*e - (-I*d*f*x - I*d*e + I*f)*e^(3*d*x + 3*c) - (d^2*f*x^2
- d*e + (2*d^2*e - 5*d*f)*x + f)*e^(2*d*x + 2*c) - (-I*d^2*f*x^2 - 5*I*d*e
 + (-2*I*d^2*e - I*d*f)*x - I*f)*e^(d*x + c) - 4*(f*e^(2*d*x + 2*c) - I*f*
e^(d*x + c))*log(e^(d*x + c) - I) + f)/(a*d^2*e^(2*d*x + 2*c) - I*a*d^2*e^
(d*x + c))
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.88

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{-2ie-2ifx}{ade^ce^{dx}-iad} + \begin{cases} \frac{((-2iad^3e-2iad^3fx-2iad^2f)e^{-dx}+(-2iad^3ee^{2c}-2iad^3fxe^{2c}+2iad^2fe^{2c})e^{dx})e^{-c}}{4a^2d^4} & \text{for } a^2d^4e^c\neq 0 \\ \frac{x^2(-ife^{2c}+if)e^{-c}}{4a}+\frac{x(-iee^{2c}+ie)e^{-c}}{2a} & \text{otherwise} \end{cases} + \frac{fx^2}{2a}+\frac{x(de-2f)}{ad}+\frac{2f\log\left(e^{dx}-ie^{-c}\right)}{ad^2}$$

input

```
integrate((f*x+e)*sinh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
(-2*I*e - 2*I*f*x)/(a*d*exp(c)*exp(d*x) - I*a*d) + Piecewise((((-2*I*a*d**
3*e - 2*I*a*d**3*f*x - 2*I*a*d**2*f)*exp(-d*x) + (-2*I*a*d**3*e*exp(2*c) -
 2*I*a*d**3*f*x*exp(2*c) + 2*I*a*d**2*f*exp(2*c))*exp(d*x))*exp(-c)/(4*a**
2*d**4), Ne(a**2*d**4*exp(c), 0)), (x**2*(-I*f*exp(2*c) + I*f)*exp(-c)/(4*
a) + x*(-I*e*exp(2*c) + I*e)*exp(-c)/(2*a), True)) + f*x**2/(2*a) + x*(d*e
 - 2*f)/(a*d) + 2*f*log(exp(d*x) - I*exp(-c))/(a*d**2)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(100) = 200$.

Time = 0.12 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.00

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{1}{2}f\left(\frac{2\,xe^{(dx+c)}}{ade^{(dx+c)}-i\,ad}+\frac{i\,d^2x^2e^c+i\,dxe^c-\left(-i\,dxe^{(3\,c)}+i\,e^{(3\,c)}\right)e^{(2\,dx)}-\left(d^2x^2e^{(2\,c)}-3\,dxe^{(2\,c)}+e^{(2\,c)}\right.}{ad^2e^{(dx+2\,c)}-i\,ad^2e^c}\right. + \frac{1}{2}e\left(\frac{2\,(dx+c)}{ad}+\frac{-5i\,e^{(-dx-c)}+1}{\left(i\,ae^{(-dx-c)}+ae^{(-2\,dx-2\,c)}\right)d}-\frac{i\,e^{(-dx-c)}}{ad}\right)$$

input
```
integrate((f*x+e)*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*f*(2*x*e^(d*x + c)/(a*d*e^(d*x + c) - I*a*d) + (I*d^2*x^2*e^c + I*d*x
*e^c - (-I*d*x*e^(3*c) + I*e^(3*c))*e^(2*d*x) - (d^2*x^2*e^(2*c) - 3*d*x*e
^(2*c) + e^(2*c))*e^(d*x) + (d*x + 1)*e^(-d*x) + I*e^c)/(a*d^2*e^(d*x + 2*
c) - I*a*d^2*e^c) - 4*log((e^(d*x + c) - I)*e^(-c))/(a*d^2)) + 1/2*e*(2*(d
*x + c)/(a*d) + (-5*I*e^(-d*x - c) + 1)/((I*a*e^(-d*x - c) + a*e^(-2*d*x -
 2*c))*d) - I*e^(-d*x - c)/(a*d))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(100) = 200$.

Time = 0.12 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.11

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{d^2fx^2e^{(2\,dx+2\,c)} - i\,d^2fx^2e^{(dx+c)} + 2\,d^2exe^{(2\,dx+2\,c)} - 2i\,d^2exe^{(dx+c)} - i\,dfxe^{(3\,dx+3\,c)} - 5\,dfxe^{(2\,dx+2\,c)} - i\,c}{}$$

input
```
integrate((f*x+e)*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(d^2*f*x^2*e^(2*d*x + 2*c) - I*d^2*f*x^2*e^(d*x + c) + 2*d^2*e*x*e^(2*
d*x + 2*c) - 2*I*d^2*e*x*e^(d*x + c) - I*d*f*x*e^(3*d*x + 3*c) - 5*d*f*x*e
^(2*d*x + 2*c) - I*d*f*x*e^(d*x + c) - d*f*x - I*d*e*e^(3*d*x + 3*c) - d*e
*e^(2*d*x + 2*c) - 5*I*d*e*e^(d*x + c) + 4*f*e^(2*d*x + 2*c)*log(e^(d*x +
c) - I) - 4*I*f*e^(d*x + c)*log(e^(d*x + c) - I) - d*e + I*f*e^(3*d*x + 3*
c) + f*e^(2*d*x + 2*c) - I*f*e^(d*x + c) - f)/(a*d^2*e^(2*d*x + 2*c) - I*a
*d^2*e^(d*x + c))
```

Mupad [B] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.20

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{f\,x^2}{2\,a} + \mathrm{e}^{c+d\,x}\left(\frac{(f-d\,e)\,1\mathrm{i}}{2\,a\,d^2} - \frac{f\,x\,1\mathrm{i}}{2\,a\,d}\right) - \mathrm{e}^{-c-d\,x}\left(\frac{(f+d\,e)\,1\mathrm{i}}{2\,a\,d^2} + \frac{f\,x\,1\mathrm{i}}{2\,a\,d}\right) - \frac{(e+f\,x)\,2\mathrm{i}}{a\,d\,(\mathrm{e}^{c+d\,x}-\mathrm{i})} - \frac{x\,(2\,f-d\,e)}{a\,d} + \frac{2\,f\,\ln\left(\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-\mathrm{i}\right)}{a\,d^2}$$

input
```
int((sinh(c + d*x)^2*(e + f*x))/(a + a*sinh(c + d*x)*1i),x)
```

output
```
exp(c + d*x)*(((f - d*e)*1i)/(2*a*d^2) - (f*x*1i)/(2*a*d)) - exp(- c - d*x
)*(((f + d*e)*1i)/(2*a*d^2) + (f*x*1i)/(2*a*d)) + (f*x^2)/(2*a) - ((e + f*
x)*2i)/(a*d*(exp(c + d*x) - 1i)) - (x*(2*f - d*e))/(a*d) + (2*f*log(exp(d*
x)*exp(c) - 1i))/(a*d^2)
```

Reduce [F]

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{-2\cosh(dx+c)\,dei-2\cosh(dx+c)\,dfix-2\left(\int \frac{x}{\sinh(dx+c)i+1}dx\right)d^2f-2\left(\int \frac{1}{\sinh(dx+c)i+1}dx\right)d^2e+2\sin}{2a\,d^2}$$

input
```
int((f*x+e)*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
( - 2*cosh(c + d*x)*d*e*i - 2*cosh(c + d*x)*d*f*i*x - 2*int(x/(sinh(c + d*
x)*i + 1),x)*d**2*f - 2*int(1/(sinh(c + d*x)*i + 1),x)*d**2*e + 2*sinh(c +
 d*x)*f*i + 2*d**2*e*x + d**2*f*x**2)/(2*a*d**2)
```

3.196 $\int \frac{\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 52

$$\int \frac{\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{x}{a} - \frac{i\cosh(c+dx)}{ad} - \frac{i\cosh(c+dx)}{ad(1+i\sinh(c+dx))}$$

output
```
x/a-I*cosh(d*x+c)/a/d-I*cosh(d*x+c)/a/d/(1+I*sinh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int \frac{\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\cosh(c+dx)\left(\frac{\operatorname{arcsinh}(\sinh(c+dx))}{\sqrt{\cosh^2(c+dx)}} + \frac{-2-i\sinh(c+dx)}{-i+\sinh(c+dx)}\right)}{ad}$$

input
```
Integrate[Sinh[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(Cosh[c + d*x]*(ArcSinh[Sinh[c + d*x]]/Sqrt[Cosh[c + d*x]^2] + (-2 - I*Sin
h[c + d*x])/(-I + Sinh[c + d*x])))/(a*d)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 25, 3225, 26, 3042, 26, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ic+idx)^2}{a+a\sin(ic+idx)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ic+idx)^2}{\sin(ic+idx)a+a}dx$$

$$\Big\downarrow \text{3225}$$

$$-\frac{\int -\frac{i\sinh(c+dx)}{i\sinh(c+dx)+1}dx}{a}-\frac{i\cosh(c+dx)}{ad}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int \frac{\sinh(c+dx)}{i\sinh(c+dx)+1}dx}{a}-\frac{i\cosh(c+dx)}{ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\int -\frac{i\sin(ic+idx)}{\sin(ic+idx)+1}dx}{a}-\frac{i\cosh(c+dx)}{ad}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\sin(ic+idx)}{\sin(ic+idx)+1}dx}{a}-\frac{i\cosh(c+dx)}{ad}$$

$$\Big\downarrow \text{3214}$$

$$\frac{x-\int \frac{1}{i\sinh(c+dx)+1}dx}{a}-\frac{i\cosh(c+dx)}{ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x-\int \frac{1}{\sin(ic+idx)+1}dx}{a}-\frac{i\cosh(c+dx)}{ad}$$

$$\Big\downarrow \text{3127}$$

$$\frac{x-\frac{i\cosh(c+dx)}{d(1+i\sinh(c+dx))}}{a}-\frac{i\cosh(c+dx)}{ad}$$

input
```
Int[Sinh[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-I)*Cosh[c + d*x])/(a*d) + (x - (I*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x
]))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3225
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Simp[1/d
Int[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15

method	result	size
risch	$\frac{x}{a}-\frac{i e^{dx+c}}{2ad}-\frac{i e^{-dx-c}}{2ad}-\frac{2i}{da\left(e^{dx+c}-i\right)}$	60
derivativedivides	$\frac{-\frac{i}{1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{8i}{8\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-8}-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da}$	86
default	$\frac{-\frac{i}{1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{8i}{8\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-8}-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da}$	86
parallelrisch	$\frac{(-2dx-3i)\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)+2idx\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\sinh\left(\frac{3dx}{2}+\frac{3c}{2}\right)+3\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)+\cosh\left(\frac{3dx}{2}+\frac{3c}{2}\right)}{2da\left(i\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)-\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	99

input
```
int(sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
x/a-1/2*I/a/d*exp(d*x+c)-1/2*I/a/d*exp(-d*x-c)-2*I/d/a/(exp(d*x+c)-I)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.33

$$\int \frac{\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{(2\,dx-1)e^{(2\,dx+2\,c)}+(-2i\,dx-5i)e^{(dx+c)}-i\,e^{(3\,dx+3\,c)}-1}{2\left(ade^{(2\,dx+2\,c)}-i\,ade^{(dx+c)}\right)}$$

input
```
integrate(sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((2*d*x - 1)*e^(2*d*x + 2*c) + (-2*I*d*x - 5*I)*e^(d*x + c) - I*e^(3*d
*x + 3*c) - 1)/(a*d*e^(2*d*x + 2*c) - I*a*d*e^(d*x + c))
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.90

$$\int \frac{\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \begin{cases} \frac{(-2iade^{2c}e^{dx}-2iade^{-dx})e^{-c}}{4a^2d^2} & \text{for } a^2d^2e^c \neq 0 \\ x\left(\frac{(-ie^{2c}+2e^c+i)e^{-c}}{2a}-\frac{1}{a}\right) & \text{otherwise} \end{cases} - \frac{2i}{ade^ce^{dx}-iad} + \frac{x}{a}$$

input
```
integrate(sinh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
Piecewise(((-2*I*a*d*exp(2*c)*exp(d*x) - 2*I*a*d*exp(-d*x))*exp(-c)/(4*a**
2*d**2), Ne(a**2*d**2*exp(c), 0)), (x*((-I*exp(2*c) + 2*exp(c) + I)*exp(-c
)/(2*a) - 1/a), True)) - 2*I/(a*d*exp(c)*exp(d*x) - I*a*d) + x/a
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.42

$$\int \frac{\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{dx+c}{ad} + \frac{-5i\,e^{(-dx-c)}+1}{2\left(i\,ae^{(-dx-c)}+ae^{(-2\,dx-2\,c)}\right)d} - \frac{i\,e^{(-dx-c)}}{2\,ad}$$

input
```
integrate(sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
(d*x + c)/(a*d) + 1/2*(-5*I*e^(-d*x - c) + 1)/((I*a*e^(-d*x - c) + a*e^(-2
*d*x - 2*c))*d) - 1/2*I*e^(-d*x - c)/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.17

$$\int \frac{\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\frac{2\,(dx+c)}{a}-\frac{i\,e^{(dx+c)}}{a}-\frac{\left(5i\,e^{(dx+c)}+1\right)e^{(-dx-c)}}{a\left(e^{(dx+c)}-i\right)}}{2\,d}$$

input
```
integrate(sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*(d*x + c)/a - I*e^(d*x + c)/a - (5*I*e^(d*x + c) + 1)*e^(-d*x - c)/
(a*(e^(d*x + c) - I)))/d
```

Mupad [B] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int \frac{\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{x}{a}-\frac{2\mathrm{i}}{a\,d\,\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)}-\frac{\mathrm{e}^{c+d\,x}\,1\mathrm{i}}{2\,a\,d}-\frac{\mathrm{e}^{-c-d\,x}\,1\mathrm{i}}{2\,a\,d}$$

input
```
int(sinh(c + d*x)^2/(a + a*sinh(c + d*x)*1i),x)
```

output
```
x/a - 2i/(a*d*(exp(c + d*x) - 1i)) - (exp(c + d*x)*1i)/(2*a*d) - (exp(- c
- d*x)*1i)/(2*a*d)
```

Reduce [F]

$$\int \frac{\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{-\cosh\left(dx+c\right)i-\left(\int \frac{1}{\sinh(dx+c)i+1}dx\right)d+dx}{ad}$$

input
```
int(sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
( - cosh(c + d*x)*i - int(1/(sinh(c + d*x)*i + 1),x)*d + d*x)/(a*d)
```

3.197 $\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=\text{Int}\left(\frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(sinh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 24.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

input
```
Integrate[Sinh[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Integrate[Sinh[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))} \, dx$$

$$\Big\downarrow \; 6111$$

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}dx$$

input
```
Int[Sinh[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\sinh(dx+c)^2}{(fx+e)(a+ia\sinh(dx+c))}dx$$

input
```
int(sinh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 203, normalized size of antiderivative = 6.55

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)^2}{(fx+e)(i\,a\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
((-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))*integral(1/2*(d*f*
x + d*e + (-I*d*f*x - I*d*e)*e^(3*d*x + 3*c) + (d*f*x + d*e)*e^(2*d*x + 2*
c) + (-I*d*f*x - I*d*e - 4*I*f)*e^(d*x + c))/((a*d*f^2*x^2 + 2*a*d*e*f*x +
 a*d*e^2)*e^(2*d*x + 2*c) - (I*a*d*f^2*x^2 + 2*I*a*d*e*f*x + I*a*d*e^2)*e^
(d*x + c)), x) - 2*I)/(-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c
))
```

Sympy [N/A]

Not integrable

Time = 110.62 (sec) , antiderivative size = 950, normalized size of antiderivative = 30.65

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = -\frac{2i}{-iade-iadfx+\left(adee^{c}+adfxe^{c}\right)e^{dx}} - \frac{i\left(\int \frac{ide}{e^{2}e^{c}e^{2dx}-ie^{2}e^{dx}+2efxe^{c}e^{2dx}-2iefxe^{dx}+f^{2}x^{2}e^{c}e^{2dx}-if^{2}x^{2}e^{dx}}\,dx + \int \frac{4fe^{c}e^{dx}}{e^{2}e^{c}e^{2dx}-ie^{2}e^{dx}+2efxe^{c}e^{2dx}-2iefxe^{dx}+f^{2}x^{2}e^{c}e^{2dx}}\right.}{}$$

input
```
integrate(sinh(d*x+c)**2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-2*I/(-I*a*d*e - I*a*d*f*x + (a*d*e*exp(c) + a*d*f*x*exp(c))*exp(d*x)) - I
*(Integral(I*d*e/(e**2*exp(c)*exp(2*d*x) - I*e**2*exp(d*x) + 2*e*f*x*exp(c
)*exp(2*d*x) - 2*I*e*f*x*exp(d*x) + f**2*x**2*exp(c)*exp(2*d*x) - I*f**2*x
**2*exp(d*x)), x) + Integral(4*f*exp(c)*exp(d*x)/(e**2*exp(c)*exp(2*d*x) -
 I*e**2*exp(d*x) + 2*e*f*x*exp(c)*exp(2*d*x) - 2*I*e*f*x*exp(d*x) + f**2*x
**2*exp(c)*exp(2*d*x) - I*f**2*x**2*exp(d*x)), x) + Integral(I*d*f*x/(e**2
*exp(c)*exp(2*d*x) - I*e**2*exp(d*x) + 2*e*f*x*exp(c)*exp(2*d*x) - 2*I*e*f
*x*exp(d*x) + f**2*x**2*exp(c)*exp(2*d*x) - I*f**2*x**2*exp(d*x)), x) + In
tegral(d*e*exp(c)*exp(d*x)/(e**2*exp(c)*exp(2*d*x) - I*e**2*exp(d*x) + 2*e
*f*x*exp(c)*exp(2*d*x) - 2*I*e*f*x*exp(d*x) + f**2*x**2*exp(c)*exp(2*d*x)
- I*f**2*x**2*exp(d*x)), x) + Integral(d*e*exp(3*c)*exp(3*d*x)/(e**2*exp(c
)*exp(2*d*x) - I*e**2*exp(d*x) + 2*e*f*x*exp(c)*exp(2*d*x) - 2*I*e*f*x*exp
(d*x) + f**2*x**2*exp(c)*exp(2*d*x) - I*f**2*x**2*exp(d*x)), x) + Integral
(I*d*e*exp(2*c)*exp(2*d*x)/(e**2*exp(c)*exp(2*d*x) - I*e**2*exp(d*x) + 2*e
*f*x*exp(c)*exp(2*d*x) - 2*I*e*f*x*exp(d*x) + f**2*x**2*exp(c)*exp(2*d*x)
- I*f**2*x**2*exp(d*x)), x) + Integral(d*f*x*exp(c)*exp(d*x)/(e**2*exp(c)*
exp(2*d*x) - I*e**2*exp(d*x) + 2*e*f*x*exp(c)*exp(2*d*x) - 2*I*e*f*x*exp(d
*x) + f**2*x**2*exp(c)*exp(2*d*x) - I*f**2*x**2*exp(d*x)), x) + Integral(d
*f*x*exp(3*c)*exp(3*d*x)/(e**2*exp(c)*exp(2*d*x) - I*e**2*exp(d*x) + 2*e*f
*x*exp(c)*exp(2*d*x) - 2*I*e*f*x*exp(d*x) + f**2*x**2*exp(c)*exp(2*d*x)...
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 177, normalized size of antiderivative = 5.71

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)^2}{(fx+e)(i\,a\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-2*I*f*integrate(1/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*
x^2*e^c + 2*a*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)), x) - 1/2*I*e^(-c + d*e/
f)*exp_integral_e(1, (f*x + e)*d/f)/(a*f) + 1/2*I*e^(c - d*e/f)*exp_integr
al_e(1, -(f*x + e)*d/f)/(a*f) - 2*I/(-I*a*d*f*x - I*a*d*e + (a*d*f*x*e^c +
 a*d*e*e^c)*e^(d*x)) + log(f*x + e)/(a*f)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)^2}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sinh(d*x + c)^2/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(c+d\,x)^2}{(e+f\,x)\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(sinh(c + d*x)^2/((e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(sinh(c + d*x)^2/((e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \frac{\int \frac{\sinh(dx+c)^2}{\sinh(dx+c)ei+\sinh(dx+c)fix+e+fx}dx}{a}$$

input
```
int(sinh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sinh(c + d*x)**2/(sinh(c + d*x)*e*i + sinh(c + d*x)*f*i*x + e + f*x),x
)/a
```

3.198 $\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} \, dx = \text{Int}\left(\frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(sinh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 14.33 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} \, dx = \int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} \, dx$$

input
```
Integrate[Sinh[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Integrate[Sinh[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$
$$\Big\downarrow \text{6111}$$
$$\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}dx$$

input
```
Int[Sinh[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\sinh(dx+c)^2}{(fx+e)^2(a+ia\sinh(dx+c))}dx$$

input
```
int(sinh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 281, normalized size of antiderivative = 9.06

$$\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)^2}{(fx+e)^2(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
((-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x
+ a*d*e^2)*e^(d*x + c))*integral(1/2*(d*f*x + d*e + (-I*d*f*x - I*d*e)*e^(
3*d*x + 3*c) + (d*f*x + d*e)*e^(2*d*x + 2*c) + (-I*d*f*x - I*d*e - 8*I*f)*
e^(d*x + c))/((a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x + a*d*e^3)*e^
(2*d*x + 2*c) - (I*a*d*f^3*x^3 + 3*I*a*d*e*f^2*x^2 + 3*I*a*d*e^2*f*x + I*a
*d*e^3)*e^(d*x + c)), x) - 2*I)/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^
2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**2/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 270, normalized size of antiderivative = 8.71

$$\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} dx = \int \frac{\sinh(dx+c)^2}{(fx+e)^2(i\,a\sinh(dx+c)+a)} dx$$

input
```
integrate(sinh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-4*I*f*integrate(1/(-I*a*d*f^3*x^3 - 3*I*a*d*e*f^2*x^2 - 3*I*a*d*e^2*f*x -
 I*a*d*e^3 + (a*d*f^3*x^3*e^c + 3*a*d*e*f^2*x^2*e^c + 3*a*d*e^2*f*x*e^c +
a*d*e^3*e^c)*e^(d*x)), x) - (-I*d*f*x - I*d*e + (d*f*x*e^c + d*e*e^c)*e^(d
*x) + 2*I*f)/(-I*a*d*f^3*x^2 - 2*I*a*d*e*f^2*x - I*a*d*e^2*f + (a*d*f^3*x^
2*e^c + 2*a*d*e*f^2*x*e^c + a*d*e^2*f*e^c)*e^(d*x)) - 1/2*I*e^(-c + d*e/f)
*exp_integral_e(2, (f*x + e)*d/f)/((f*x + e)*a*f) + 1/2*I*e^(c - d*e/f)*ex
p_integral_e(2, -(f*x + e)*d/f)/((f*x + e)*a*f)
```

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} dx = \int \frac{\sinh(dx+c)^2}{(fx+e)^2(i\,a\sinh(dx+c)+a)} dx$$

input
```
integrate(sinh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sinh(d*x + c)^2/((f*x + e)^2*(I*a*sinh(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(c+d\,x)^2}{(e+f\,x)^2\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(sinh(c + d*x)^2/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(sinh(c + d*x)^2/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.26

$$\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \frac{\int \frac{\sinh(dx+c)^2}{\sinh(dx+c)e^2i+2\sinh(dx+c)efix+\sinh(dx+c)f^2i\,x^2+e^2+2efx+f^2x^2}dx}{a}$$

input
```
int(sinh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sinh(c + d*x)**2/(sinh(c + d*x)*e**2*i + 2*sinh(c + d*x)*e*f*i*x + sin
h(c + d*x)*f**2*i*x**2 + e**2 + 2*e*f*x + f**2*x**2),x)/a
```

3.199 $\int \frac{(e+fx)^3 \sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 378

$$\begin{aligned}\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx &= \frac{3if(e+fx)^2}{8ad^2}-\frac{i(e+fx)^3}{ad}+\frac{3i(e+fx)^4}{8af}\\ &+\frac{6f^2(e+fx)\cosh(c+dx)}{ad^3}+\frac{(e+fx)^3\cosh(c+dx)}{ad}\\ &+\frac{6if(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2}\\ &+\frac{12if^2(e+fx)\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}\\ &-\frac{12if^3\,\mathrm{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^4}\\ &-\frac{6f^3\sinh(c+dx)}{ad^4}-\frac{3f(e+fx)^2\sinh(c+dx)}{ad^2}\\ &-\frac{3if^2(e+fx)\cosh(c+dx)\sinh(c+dx)}{4ad^3}\\ &-\frac{i(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{2ad}\\ &+\frac{3if^3\sinh^2(c+dx)}{8ad^4}+\frac{3if(e+fx)^2\sinh^2(c+dx)}{4ad^2}\\ &-\frac{i(e+fx)^3\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}\end{aligned}$$

output
```
3/8*I*f*(f*x+e)^2/a/d^2-I*(f*x+e)^3/a/d+3/8*I*(f*x+e)^4/a/f+6*f^2*(f*x+e)*
cosh(d*x+c)/a/d^3+(f*x+e)^3*cosh(d*x+c)/a/d+6*I*f*(f*x+e)^2*ln(1+I*exp(d*x
+c))/a/d^2+12*I*f^2*(f*x+e)*polylog(2,-I*exp(d*x+c))/a/d^3-12*I*f^3*polylo
g(3,-I*exp(d*x+c))/a/d^4-6*f^3*sinh(d*x+c)/a/d^4-3*f*(f*x+e)^2*sinh(d*x+c)
/a/d^2-3/4*I*f^2*(f*x+e)*cosh(d*x+c)*sinh(d*x+c)/a/d^3-1/2*I*(f*x+e)^3*cos
h(d*x+c)*sinh(d*x+c)/a/d+3/8*I*f^3*sinh(d*x+c)^2/a/d^4+3/4*I*f*(f*x+e)^2*s
inh(d*x+c)^2/a/d^2-I*(f*x+e)^3*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a/d
```

Mathematica [A] (verified)

Time = 4.42 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.99

$$\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$= \frac{24ie^3x+36ie^2fx^2+24ief^2x^3+6if^3x^4+\frac{32(e+fx)^3}{d(-i+e^c)}+\frac{96f^2(e+fx)\cosh(c+dx)}{d^3}+\frac{16(e+fx)^3\cosh(c+dx)}{d}+\frac{3if^3\cosh(2}{d^4}}{}$$

input
```
Integrate[((e + f*x)^3*Sinh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((24*I)*e^3*x + (36*I)*e^2*f*x^2 + (24*I)*e*f^2*x^3 + (6*I)*f^3*x^4 + (32*
(e + f*x)^3)/(d*(-I + E^c)) + (96*f^2*(e + f*x)*Cosh[c + d*x])/d^3 + (16*(
e + f*x)^3*Cosh[c + d*x])/d + ((3*I)*f^3*Cosh[2*(c + d*x)])/d^4 + ((6*I)*f
*(e + f*x)^2*Cosh[2*(c + d*x)])/d^2 + ((96*I)*f*(e + f*x)^2*Log[1 - I*E^(-
c - d*x)])/d^2 - ((192*I)*f^2*(d*(e + f*x)*PolyLog[2, I*E^(-c - d*x)] + f*
PolyLog[3, I*E^(-c - d*x)]))/d^4 - ((32*I)*(e + f*x)^3*Sinh[(d*x)/2])/(d*(
Cosh[c/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])) - (96*
f^3*Sinh[c + d*x])/d^4 - (48*f*(e + f*x)^2*Sinh[c + d*x])/d^2 - ((6*I)*f^2
*(e + f*x)*Sinh[2*(c + d*x)])/d^3 - ((4*I)*(e + f*x)^3*Sinh[2*(c + d*x)])/
d)/(16*a)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6091}$$

$$i\int \frac{(e+fx)^3\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int (e+fx)^3\sinh^2(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$i\int \frac{(e+fx)^3\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int -(e+fx)^3\sin(ic+idx)^2dx}{a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{i\int (e+fx)^3\sin(ic+idx)^2dx}{a}+i\int \frac{(e+fx)^3\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 3792}$$

$$\frac{i\left(\frac{3f^2\int -((e+fx)\sinh^2(c+dx))dx}{2d^2}+\frac{1}{2}\int (e+fx)^3dx+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{a}+$$
$$i\int \frac{(e+fx)^3\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 17}$$

$$\frac{i\left(\frac{3f^2\int -((e+fx)\sinh^2(c+dx))dx}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}+$$
$$i\int \frac{(e+fx)^3\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 25}$$

$$\frac{i\left(-\frac{3f^2\int (e+fx)\sinh^2(c+dx)dx}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}+$$
$$i\int \frac{(e+fx)^3\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{i\left(-\frac{3f^2\int -((e+fx)\sin(ic+idx)^2)dx}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}+ i\int \frac{(e+fx)^3\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$\big\downarrow$ 25

$$\frac{i\left(\frac{3f^2\int (e+fx)\sin(ic+idx)^2dx}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}+ i\int \frac{(e+fx)^3\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$\big\downarrow$ 3791

$$\frac{i\left(\frac{3f^2\left(\frac{1}{2}\int (e+fx)dx+\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+f}{8}\right.}{a}$$

$$i\int \frac{(e+fx)^3\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$\big\downarrow$ 17

$$i\int \frac{(e+fx)^3\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx+ \frac{i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$\big\downarrow$ 6091

$$i\left(i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int (e+fx)^3\sinh(c+dx)dx}{a}\right)+ \frac{i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$\big\downarrow$ 3042

$$i\left(i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int -i(e+fx)^3\sin(ic+idx)dx}{a}\right)+ \frac{i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$i\left(i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\int (e+fx)^3\sin(ic+idx)dx}{a}\right)+\frac{i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$$\Big\downarrow \text{3777}$$

$$i\left(i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\cosh(c+dx)dx}{d}}{a}\right)+\frac{i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$i\left(i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}\right)+\frac{i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$$\Big\downarrow \text{3777}$$

$$i\left(i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}\right)}{d}}{a}\right)+\frac{i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\left(i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int(e+fx)\sinh(c+dx)dx}{d}\right)}{d}}{a}\right)+i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$\downarrow$ 3042

$$\frac{i\left(i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int -i(e+fx)\sin(ic+idx)dx}{d}\right)}{d}}{a}\right)+i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$\downarrow$ 26

$$\frac{i\left(i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int(e+fx)\sin(ic+idx)dx}{d}\right)}{d}}{a}\right)+i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$\downarrow$ 3777

$$\frac{i\left(i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{d}\right)}{d}}{a}\right)+i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$\downarrow$ 3042

$$\frac{i\left(i\left(i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right.}{d}\right.}{d}}{a}\right.\right.}{}$$

$$\frac{i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right.}{a}$$

$\downarrow$ 3117

$$\frac{i\left(i\left(i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}\right)+\right.}{}$$

$$\frac{i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right.}{a}$$

$\downarrow$ 6091

$$\frac{i\left(i\left(i\left(i\int \frac{(e+fx)^3}{i\sinh(c+dx)a+a}dx-\frac{i\int(e+fx)^3dx}{a}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx}{d}\right.}{d}\right.}{d}}{a}\right.\right.}{}$$

$$\frac{i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right.}{a}$$

$\downarrow$ 17

$$i\left(i\left(i\int\frac{(e+fx)^3}{i\sinh(c+dx)a+a}dx-\frac{i(e+fx)^4}{4af}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\,s}{}\right.}{d}\right.}{d}}{a}\right.$$

$$\frac{i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$\downarrow$ 3042

$$i\left(i\left(i\int\frac{(e+fx)^3}{\sin(ic+idx)a+a}dx-\frac{i(e+fx)^4}{4af}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\,\sin}{}\right.}{d}\right.}{d}}{a}\right.$$

$$\frac{i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$\downarrow$ 3799

$$i\left(i\left(\frac{i\int-(e+fx)^3\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)}{}\right.}{}\right.}{d}}{a}\right.$$

$$\frac{i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$\downarrow$ 25

$$\frac{i\left(i\left(i\left(-\frac{i\int -(e+fx)^3\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+f}{}\right.}{}\right.}{d}}{a}\right)}{}}{}$$

$$\frac{i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$\downarrow$ 25

$$\frac{i\left(i\left(i\left(\frac{i\int (e+fx)^3\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh}{d}\right.}{}\right.}{d}}{a}\right)}{}}{}$$

$$\frac{i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$\downarrow$ 3042

$$\frac{i\left(i\left(i\left(\frac{i\int (e+fx)^3\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh}{d}\right.}{}\right.}{d}}{a}\right)}{}}{}$$

$$\frac{i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$\downarrow$ 4672

$$i\left(i\left(\frac{i\left(\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{6if\int -i(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e\cdots}{}\right.}{}}{}\right.$$

$$\frac{i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right.}{a}$$

input
```
Int[((e + f*x)^3*Sinh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1005 vs. $2(342) = 684$.

Time = 1.10 (sec) , antiderivative size = 1006, normalized size of antiderivative = 2.66

method	result
risch	$\frac{3ie^2f\ln(1+e^{2dx+2c})}{a\,d^2}+\frac{12ie\,f^2\operatorname{polylog}(2,-ie^{dx+c})}{a\,d^3}+\frac{12if^3\operatorname{polylog}(2,-ie^{dx+c})x}{a\,d^3}+\frac{12c\,f^2e\arctan(e^{dx+c})}{a\,d^3}+\frac{12ic\,f^2e\ln(e\cdots}{a\,d^3}$

input
```
int((f*x+e)^3*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
3/2*I/a*f^2*e*x^3+9/4*I/a*f*e^2*x^2-1/32*I*(4*d^3*f^3*x^3+12*d^3*e*f^2*x^2
+12*d^3*e^2*f*x-6*d^2*f^3*x^2+4*d^3*e^3-12*d^2*e*f^2*x-6*d^2*e^2*f+6*d*f^3
*x+6*d*e*f^2-3*f^3)/d^4/a*exp(2*d*x+2*c)+3/8*I/a*f^3*x^4+3/2*I/a*e^3*x+3/8
*I/a/f*e^4+12/a/d^3*c*f^2*e*arctan(exp(d*x+c))+6*I/a/d^3*f^3*c^2*x+3*I/a/d
^4*c^2*f^3*ln(1+exp(2*d*x+2*c))-6*I/a/d*e*f^2*x^2-6*I/a/d^3*e*f^2*c^2-6*I/
a/d^4*c^2*f^3*ln(exp(d*x+c))-6*I/a/d^2*e^2*f*ln(exp(d*x+c))-6*I/a/d^4*f^3*
ln(1+I*exp(d*x+c))*c^2+3*I/a/d^2*e^2*f*ln(1+exp(2*d*x+2*c))+6*I/a/d^2*f^3*
ln(1+I*exp(d*x+c))*x^2+12*I/a/d^3*e*f^2*polylog(2,-I*exp(d*x+c))+12*I/a/d^
3*f^3*polylog(2,-I*exp(d*x+c))*x-12*I/a/d^2*e*f^2*c*x+12*I/a/d^3*c*f^2*e*l
n(exp(d*x+c))+12*I/a/d^3*e*f^2*ln(1+I*exp(d*x+c))*c+2*(f^3*x^3+3*e*f^2*x^2
+3*e^2*f*x+e^3)/d/a/(exp(d*x+c)-I)+1/32*I*(4*d^3*f^3*x^3+12*d^3*e*f^2*x^2+
12*d^3*e^2*f*x+6*d^2*f^3*x^2+4*d^3*e^3+12*d^2*e*f^2*x+6*d^2*e^2*f+6*d*f^3*
x+6*d*e*f^2+3*f^3)/d^4/a*exp(-2*d*x-2*c)-12*I*f^3*polylog(3,-I*exp(d*x+c))
/a/d^4-6/a/d^2*e^2*f*arctan(exp(d*x+c))+4*I/a/d^4*f^3*c^3-2*I/a/d*f^3*x^3-
6*I/a/d^3*c*f^2*e*ln(1+exp(2*d*x+2*c))+12*I/a/d^2*e*f^2*ln(1+I*exp(d*x+c))
*x-6/a/d^4*c^2*f^3*arctan(exp(d*x+c))+1/2*(d^3*f^3*x^3+3*d^3*e*f^2*x^2+3*d
^3*e^2*f*x-3*d^2*f^3*x^2+d^3*e^3-6*d^2*e*f^2*x-3*d^2*e^2*f+6*d*f^3*x+6*d*e
*f^2-6*f^3)/d^4/a*exp(d*x+c)+1/2*(d^3*f^3*x^3+3*d^3*e*f^2*x^2+3*d^3*e^2*f*
x+3*d^2*f^3*x^2+d^3*e^3+6*d^2*e*f^2*x+3*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2+6*f^
3)/d^4/a*exp(-d*x-c)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1044 vs. $2(326) = 652$.

Time = 0.12 (sec) , antiderivative size = 1044, normalized size of antiderivative = 2.76

$$\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/32*(4*d^3*f^3*x^3 + 4*d^3*e^3 + 6*d^2*e^2*f + 6*d*e*f^2 + 3*f^3 + 6*(2*d
^3*e*f^2 + d^2*f^3)*x^2 + 6*(2*d^3*e^2*f + 2*d^2*e*f^2 + d*f^3)*x - 384*((
-I*d*f^3*x - I*d*e*f^2)*e^(3*d*x + 3*c) - (d*f^3*x + d*e*f^2)*e^(2*d*x + 2
*c))*dilog(-I*e^(d*x + c)) + (-4*I*d^3*f^3*x^3 - 4*I*d^3*e^3 + 6*I*d^2*e^2
*f - 6*I*d*e*f^2 + 3*I*f^3 - 6*(2*I*d^3*e*f^2 - I*d^2*f^3)*x^2 - 6*(2*I*d^
3*e^2*f - 2*I*d^2*e*f^2 + I*d*f^3)*x)*e^(5*d*x + 5*c) + 3*(4*d^3*f^3*x^3 +
 4*d^3*e^3 - 14*d^2*e^2*f + 30*d*e*f^2 - 31*f^3 + 2*(6*d^3*e*f^2 - 7*d^2*f
^3)*x^2 + 2*(6*d^3*e^2*f - 14*d^2*e*f^2 + 15*d*f^3)*x)*e^(4*d*x + 4*c) - 4
*(-3*I*d^4*f^3*x^4 + 4*I*d^3*e^3 + 12*(4*I*c - I)*d^2*e^2*f + 24*(-2*I*c^2
 + I)*d*e*f^2 + 8*(2*I*c^3 - 3*I)*f^3 + 4*(-3*I*d^4*e*f^2 + 5*I*d^3*f^3)*x
^3 + 6*(-3*I*d^4*e^2*f + 10*I*d^3*e*f^2 - 2*I*d^2*f^3)*x^2 + 12*(-I*d^4*e^
3 + 5*I*d^3*e^2*f - 2*I*d^2*e*f^2 + 2*I*d*f^3)*x)*e^(3*d*x + 3*c) + 4*(3*d
^4*f^3*x^4 + 20*d^3*e^3 - 12*(4*c - 1)*d^2*e^2*f + 24*(2*c^2 + 1)*d*e*f^2
- 8*(2*c^3 - 3)*f^3 + 4*(3*d^4*e*f^2 + d^3*f^3)*x^3 + 6*(3*d^4*e^2*f + 2*d
^3*e*f^2 + 2*d^2*f^3)*x^2 + 12*(d^4*e^3 + d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^
3)*x)*e^(2*d*x + 2*c) - 3*(4*I*d^3*f^3*x^3 + 4*I*d^3*e^3 + 14*I*d^2*e^2*f
+ 30*I*d*e*f^2 + 31*I*f^3 + 2*(6*I*d^3*e*f^2 + 7*I*d^2*f^3)*x^2 + 2*(6*I*d
^3*e^2*f + 14*I*d^2*e*f^2 + 15*I*d*f^3)*x)*e^(d*x + c) - 192*((-I*d^2*e^2*
f + 2*I*c*d*e*f^2 - I*c^2*f^3)*e^(3*d*x + 3*c) - (d^2*e^2*f - 2*c*d*e*f^2
+ c^2*f^3)*e^(2*d*x + 2*c))*log(e^(d*x + c) - I) - 192*((-I*d^2*f^3*x^2...
```

Sympy [F]

$$\int \frac{(e+fx)^3 \sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)**3*sinh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
(2*e**3 + 6*e**2*f*x + 6*e*f**2*x**2 + 2*f**3*x**3)/(a*d*exp(c)*exp(d*x) -
 I*a*d) - I*(Integral(-I*d*e**3/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + I
ntegral(-I*d*f**3*x**3/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-
d*e**3*exp(c)*exp(d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-
4*d*e**3*exp(3*c)*exp(3*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Inte
gral(d*e**3*exp(5*c)*exp(5*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + I
ntegral(-3*I*d*e*f**2*x**2/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integr
al(-3*I*d*e**2*f*x/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(4*I*d
*e**3*exp(2*c)*exp(2*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integra
l(I*d*e**3*exp(4*c)*exp(4*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + In
tegral(-24*I*e**2*f*exp(2*c)*exp(2*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x))
, x) + Integral(-24*I*f**3*x**2*exp(2*c)*exp(2*d*x)/(exp(c)*exp(3*d*x) - I
*exp(2*d*x)), x) + Integral(-d*f**3*x**3*exp(c)*exp(d*x)/(exp(c)*exp(3*d*x
) - I*exp(2*d*x)), x) + Integral(-4*d*f**3*x**3*exp(3*c)*exp(3*d*x)/(exp(c
)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(d*f**3*x**3*exp(5*c)*exp(5*d*x
)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(4*I*d*f**3*x**3*exp(2*
c)*exp(2*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(I*d*f**3*x
**3*exp(4*c)*exp(4*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(
-48*I*e*f**2*x*exp(2*c)*exp(2*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x)
+ Integral(-3*d*e*f**2*x**2*exp(c)*exp(d*x)/(exp(c)*exp(3*d*x) - I*exp(...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3 \sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\sinh\left(dx+c\right)^3}{i\,a\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sinh(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{\sinh(c+d\,x)^3\,(e+f\,x)^3}{a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((sinh(c + d*x)^3*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i),x)
```

output
```
int((sinh(c + d*x)^3*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$=\frac{6\cosh\left(dx+c\right)^2d^4e^2fi\,x^2+\cosh\left(dx+c\right)^2d^4f^3i\,x^4+6\cosh\left(dx+c\right)^2d^2e^2fi+3\cosh\left(dx+c\right)^2d^2f^3i\,x^2}{}$$

input
```
int((f*x+e)^3*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

output
```
(6*cosh(c + d*x)**2*d**4*e**2*f*i*x**2 + cosh(c + d*x)**2*d**4*f**3*i*x**4
 + 6*cosh(c + d*x)**2*d**2*e**2*f*i + 3*cosh(c + d*x)**2*d**2*f**3*i*x**2
+ 3*cosh(c + d*x)**2*f**3*i - 12*cosh(c + d*x)*sinh(c + d*x)*d**3*e**2*f*i
*x - 4*cosh(c + d*x)*sinh(c + d*x)*d**3*f**3*i*x**3 - 6*cosh(c + d*x)*sinh
(c + d*x)*d*f**3*i*x + 24*cosh(c + d*x)*d**3*e**2*f*x + 8*cosh(c + d*x)*d*
*3*f**3*x**3 + 48*cosh(c + d*x)*d*f**3*x + 8*int(sinh(c + d*x)**3/(sinh(c
+ d*x)*i + 1),x)*d**4*e**3 + 8*int(( - x**3)/(sinh(c + d*x) - i),x)*d**4*f
**3 + 24*int(( - x)/(sinh(c + d*x) - i),x)*d**4*e**2*f + 24*int((sinh(c +
d*x)**3*x**2)/(sinh(c + d*x)*i + 1),x)*d**4*e*f**2 - 6*sinh(c + d*x)**2*d*
*4*e**2*f*i*x**2 - sinh(c + d*x)**2*d**4*f**3*i*x**4 + 3*sinh(c + d*x)**2*
d**2*f**3*i*x**2 - 24*sinh(c + d*x)*d**2*e**2*f - 24*sinh(c + d*x)*d**2*f*
*3*x**2 - 48*sinh(c + d*x)*f**3 + 12*d**4*e**2*f*i*x**2 + 2*d**4*f**3*i*x*
*4)/(8*a*d**4)
```

3.200 $\int \frac{(e+fx)^2 \sinh^3(c+dx)}{a+ia\sinh(c+dx)} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 287

$$\begin{aligned}\int \frac{(e+fx)^2 \sinh^3(c+dx)}{a+ia\sinh(c+dx)} \, dx &= \frac{if^2x}{4ad^2} - \frac{i(e+fx)^2}{ad} + \frac{i(e+fx)^3}{2af} \\ &+ \frac{2f^2\cosh(c+dx)}{ad^3} + \frac{(e+fx)^2\cosh(c+dx)}{ad} \\ &+ \frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2} \\ &+ \frac{4if^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3} - \frac{2f(e+fx)\sinh(c+dx)}{ad^2} \\ &- \frac{if^2\cosh(c+dx)\sinh(c+dx)}{4ad^3} \\ &- \frac{i(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{2ad} \\ &+ \frac{if(e+fx)\sinh^2(c+dx)}{2ad^2} \\ &- \frac{i(e+fx)^2\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}\end{aligned}$$

output
```
1/4*I*f^2*x/a/d^2-I*(f*x+e)^2/a/d+1/2*I*(f*x+e)^3/a/f+2*f^2*cosh(d*x+c)/a/
d^3+(f*x+e)^2*cosh(d*x+c)/a/d+4*I*f*(f*x+e)*ln(1+I*exp(d*x+c))/a/d^2+4*I*f
^2*polylog(2,-I*exp(d*x+c))/a/d^3-2*f*(f*x+e)*sinh(d*x+c)/a/d^2-1/4*I*f^2*
cosh(d*x+c)*sinh(d*x+c)/a/d^3-1/2*I*(f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/a/d+
1/2*I*f*(f*x+e)*sinh(d*x+c)^2/a/d^2-I*(f*x+e)^2*tanh(1/2*c+1/4*I*Pi+1/2*d*
x)/a/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1661 vs. $2(287) = 574$.

Time = 3.35 (sec) , antiderivative size = 1661, normalized size of antiderivative = 5.79

$$\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Sinh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-6*I)*d^2*e^2*E^c*Cosh[(3*d*x)/2] + 6*d^2*e^2*E^(4*c)*Cosh[(3*d*x)/2] -
(14*I)*d*e*E^c*f*Cosh[(3*d*x)/2] - 14*d*e*E^(4*c)*f*Cosh[(3*d*x)/2] - (15*
I)*E^c*f^2*Cosh[(3*d*x)/2] + 15*E^(4*c)*f^2*Cosh[(3*d*x)/2] - (12*I)*d^2*e
*E^c*f*x*Cosh[(3*d*x)/2] + 12*d^2*e*E^(4*c)*f*x*Cosh[(3*d*x)/2] - (14*I)*d
*E^c*f^2*x*Cosh[(3*d*x)/2] - 14*d*E^(4*c)*f^2*x*Cosh[(3*d*x)/2] - (6*I)*d^
2*E^c*f^2*x^2*Cosh[(3*d*x)/2] + 6*d^2*E^(4*c)*f^2*x^2*Cosh[(3*d*x)/2] + 2*
d^2*e^2*Cosh[(5*d*x)/2] - (2*I)*d^2*e^2*E^(5*c)*Cosh[(5*d*x)/2] + 2*d*e*f*
Cosh[(5*d*x)/2] + (2*I)*d*e*E^(5*c)*f*Cosh[(5*d*x)/2] + f^2*Cosh[(5*d*x)/2
] - I*E^(5*c)*f^2*Cosh[(5*d*x)/2] + 4*d^2*e*f*x*Cosh[(5*d*x)/2] - (4*I)*d^
2*e*E^(5*c)*f*x*Cosh[(5*d*x)/2] + 2*d*f^2*x*Cosh[(5*d*x)/2] + (2*I)*d*E^(5
*c)*f^2*x*Cosh[(5*d*x)/2] + 2*d^2*f^2*x^2*Cosh[(5*d*x)/2] - (2*I)*d^2*E^(5
*c)*f^2*x^2*Cosh[(5*d*x)/2] + 8*E^(2*c)*Cosh[(d*x)/2]*(2*(1 - I*E^c)*f^2 +
 2*d*(1 + I*E^c)*f*(e + f*x) + d^2*(5 - I*E^c)*(e + f*x)^2 + d^3*(1 + I*E^
c)*x*(3*e^2 + 3*e*f*x + f^2*x^2) + 8*d*(1 + I*E^c)*f*(e + f*x)*Log[1 - I*E
^(-c - d*x)]) - 40*d^2*e^2*E^(2*c)*Sinh[(d*x)/2] - (8*I)*d^2*e^2*E^(3*c)*S
inh[(d*x)/2] - 16*d*e*E^(2*c)*f*Sinh[(d*x)/2] + (16*I)*d*e*E^(3*c)*f*Sinh[
(d*x)/2] - 16*E^(2*c)*f^2*Sinh[(d*x)/2] - (16*I)*E^(3*c)*f^2*Sinh[(d*x)/2]
 - 24*d^3*e^2*E^(2*c)*x*Sinh[(d*x)/2] + (24*I)*d^3*e^2*E^(3*c)*x*Sinh[(d*x
)/2] - 80*d^2*e*E^(2*c)*f*x*Sinh[(d*x)/2] - (16*I)*d^2*e*E^(3*c)*f*x*Sinh[
(d*x)/2] - 16*d*E^(2*c)*f^2*x*Sinh[(d*x)/2] + (16*I)*d*E^(3*c)*f^2*x*Si...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6091}$$

$$i\int \frac{(e+fx)^2\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int (e+fx)^2\sinh^2(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$i\int \frac{(e+fx)^2\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int -(e+fx)^2\sin(ic+idx)^2dx}{a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{i\int (e+fx)^2\sin(ic+idx)^2dx}{a}+i\int \frac{(e+fx)^2\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \textbf{3792}$$

$$\frac{i\left(\frac{f^2\int -\sinh^2(c+dx)dx}{2d^2}+\frac{1}{2}\int (e+fx)^2dx+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{a}+ i\int \frac{(e+fx)^2\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \textbf{17}$$

$$\frac{i\left(\frac{f^2\int -\sinh^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}+ i\int \frac{(e+fx)^2\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \textbf{25}$$

$$\frac{i\left(-\frac{f^2\int \sinh^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}+ i\int \frac{(e+fx)^2\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{i\left(-\frac{f^2\int -\sin(ic+idx)^2dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}+ i\int \frac{(e+fx)^2\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \textbf{25}$$

$$\frac{i\left(\frac{f^2\int \sin(ic+idx)^2dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}+ i\int \frac{(e+fx)^2\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \textbf{3115}$$

$$\frac{i\left(\frac{f^2\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}+ i\int \frac{(e+fx)^2\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \textbf{24}$$

$$\frac{i\int \frac{(e+fx)^2\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx+i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\Big\downarrow \text{ 6091}$$

$$\frac{i\left(i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int (e+fx)^2\sinh(c+dx)dx}{a}\right)+i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{i\left(i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int -i(e+fx)^2\sin(ic+idx)dx}{a}\right)+i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{i\left(i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\int (e+fx)^2\sin(ic+idx)dx}{a}\right)+i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{i\left(i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\cosh(c+dx)dx}{d}}{a}\right)+i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{i\left(i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}\right)+i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\Big\downarrow \text{3777}$$

$$i\left(i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{d}}{a}\right)+\frac{i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$i\left(i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)}{d}}{a}\right)+\frac{i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$i\left(i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}}{a}\right)+\frac{i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$i\left(i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}\right)}{d}}{a}\right)+\frac{i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\Big\downarrow \text{3118}$$

$$i\left(i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}\right)+\frac{i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\Big\downarrow \text{6091}$$

$$\frac{i\left(i\left(i\int \frac{(e+fx)^2}{i\sinh(c+dx)a+a}dx-\frac{i\int (e+fx)^2dx}{a}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}\right)+i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\Big\downarrow \text{17}$$

$$\frac{i\left(i\left(i\int \frac{(e+fx)^2}{i\sinh(c+dx)a+a}dx-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}\right)+i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\left(i\left(i\int \frac{(e+fx)^2}{\sin(ic+idx)a+a}dx-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}\right)+i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\Big\downarrow \text{3799}$$

$$\frac{i\left(i\left(\frac{i\int -(e+fx)^2\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}\right)+i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\Big\downarrow \text{25}$$

$$i\left(i\left(-\frac{i\int -(e+fx)^2\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right.}{d}}{a}\right.$$

$$\frac{i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$\downarrow$ 25

$$i\left(i\left(\frac{i\int(e+fx)^2\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}\right)$$

$$\frac{i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$\downarrow$ 3042

$$i\left(i\left(\frac{i\int(e+fx)^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}\right)$$

$$\frac{i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$\downarrow$ 4672

$$i\left(i\left(\frac{i\left(\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4if\int -i(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+f}{}\right.}{}}{a}\right.$$

$$\frac{i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$\downarrow$ 26

$$i\left(i\left(\frac{i\left(\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4f\int(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh}{}\right.}{}}{a}\right.$$

$$\frac{i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$\downarrow$ 3042

$$i\left(i\left(\frac{i\left(\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4f\int-i(e+fx)\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left((e+fx\right.}{}}{a}\right.$$

$$\frac{i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

input
```
Int[((e + f*x)^2*Sinh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 559 vs. 2(257) = 514.

Time = 0.80 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.95

method	result
risch	$\frac{ie^3}{2af}+\frac{3ie^2x}{2a}+\frac{if^2x^3}{2a}+\frac{3ife\,x^2}{2a}-\frac{4ief\ln\left(e^{dx+c}\right)}{a\,d^2}+\frac{\left(d^2x^2f^2+2d^2efx+d^2e^2-2d\,f^2x-2def+2f^2\right)e^{dx+c}}{2d^3a}+\frac{(d^2x^2f^2+2d}{}$

input
```
int((f*x+e)^2*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/a/f*e^3+3/2*I/a*e^2*x+1/2*I/a*f^2*x^3+3/2*I/a*f*e*x^2-4*I/a/d^2*e*f*
ln(exp(d*x+c))+1/2*(d^2*f^2*x^2+2*d^2*e*f*x+d^2*e^2-2*d*f^2*x-2*d*e*f+2*f^
2)/d^3/a*exp(d*x+c)+1/2*(d^2*f^2*x^2+2*d^2*e*f*x+d^2*e^2+2*d*f^2*x+2*d*e*f
+2*f^2)/d^3/a*exp(-d*x-c)-2*I/a/d^3*f^2*c^2+2*(f^2*x^2+2*e*f*x+e^2)/d/a/(e
xp(d*x+c)-I)+4*I/a/d^3*f^2*ln(1+I*exp(d*x+c))*c-4/a/d^2*e*f*arctan(exp(d*x
+c))-1/16*I*(2*d^2*f^2*x^2+4*d^2*e*f*x+2*d^2*e^2-2*d*f^2*x-2*d*e*f+f^2)/d^
3/a*exp(2*d*x+2*c)+2*I/a/d^2*e*f*ln(1+exp(2*d*x+2*c))+1/16*I*(2*d^2*f^2*x^
2+4*d^2*e*f*x+2*d^2*e^2+2*d*f^2*x+2*d*e*f+f^2)/d^3/a*exp(-2*d*x-2*c)-2*I/a
/d*f^2*x^2+4*I/a/d^3*c*f^2*ln(exp(d*x+c))+4*I*f^2*polylog(2,-I*exp(d*x+c))
/a/d^3-4*I/a/d^2*f^2*c*x+4*I/a/d^2*f^2*ln(1+I*exp(d*x+c))*x+4/a/d^3*c*f^2*
arctan(exp(d*x+c))-2*I/a/d^3*c*f^2*ln(1+exp(2*d*x+2*c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 595 vs. 2(244) = 488.

Time = 0.09 (sec) , antiderivative size = 595, normalized size of antiderivative = 2.07

$$\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{2\,d^2f^2x^2+2\,d^2e^2+2\,def+f^2+2\,(2\,d^2ef+df^2)x-64\left(-i\,f^2e^{(3\,dx+3\,c)}-f^2e^{(2\,dx+2\,c)}\right)\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)+}{}$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/16*(2*d^2*f^2*x^2 + 2*d^2*e^2 + 2*d*e*f + f^2 + 2*(2*d^2*e*f + d*f^2)*x
- 64*(-I*f^2*e^(3*d*x + 3*c) - f^2*e^(2*d*x + 2*c))*dilog(-I*e^(d*x + c))
+ (-2*I*d^2*f^2*x^2 - 2*I*d^2*e^2 + 2*I*d*e*f - I*f^2 - 2*(2*I*d^2*e*f - I
*d*f^2)*x)*e^(5*d*x + 5*c) + (6*d^2*f^2*x^2 + 6*d^2*e^2 - 14*d*e*f + 15*f^
2 + 2*(6*d^2*e*f - 7*d*f^2)*x)*e^(4*d*x + 4*c) - 8*(-I*d^3*f^2*x^3 + I*d^2
*e^2 + 2*(4*I*c - I)*d*e*f + 2*(-2*I*c^2 + I)*f^2 + (-3*I*d^3*e*f + 5*I*d^
2*f^2)*x^2 + (-3*I*d^3*e^2 + 10*I*d^2*e*f - 2*I*d*f^2)*x)*e^(3*d*x + 3*c)
+ 8*(d^3*f^2*x^3 + 5*d^2*e^2 - 2*(4*c - 1)*d*e*f + 2*(2*c^2 + 1)*f^2 + (3*
d^3*e*f + d^2*f^2)*x^2 + (3*d^3*e^2 + 2*d^2*e*f + 2*d*f^2)*x)*e^(2*d*x + 2
*c) + (-6*I*d^2*f^2*x^2 - 6*I*d^2*e^2 - 14*I*d*e*f - 15*I*f^2 - 2*(6*I*d^2
*e*f + 7*I*d*f^2)*x)*e^(d*x + c) - 64*((-I*d*e*f + I*c*f^2)*e^(3*d*x + 3*c
) - (d*e*f - c*f^2)*e^(2*d*x + 2*c))*log(e^(d*x + c) - I) - 64*((-I*d*f^2*
x - I*c*f^2)*e^(3*d*x + 3*c) - (d*f^2*x + c*f^2)*e^(2*d*x + 2*c))*log(I*e^
(d*x + c) + 1))/(a*d^3*e^(3*d*x + 3*c) - I*a*d^3*e^(2*d*x + 2*c))
```

Sympy [F]

$$\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{2e^2+4efx+2f^2x^2}{ade^ce^{dx}-iad} - \frac{i\left(\int\left(-\frac{ide^2}{e^ce^{3dx}-ie^{2dx}}\right)dx + \int\left(-\frac{idf^2x^2}{e^ce^{3dx}-ie^{2dx}}\right)dx + \int\left(-\frac{de^2e^ce^{dx}}{e^ce^{3dx}-ie^{2dx}}\right)dx + \int\left(-\frac{4de^2e^{3c}e^{3dx}}{e^ce^{3dx}-ie^{2dx}}\right)dx + \int \frac{de}{e^ce^{3}}\cdots\right.}{}$$

input
```
integrate((f*x+e)**2*sinh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
(2*e**2 + 4*e*f*x + 2*f**2*x**2)/(a*d*exp(c)*exp(d*x) - I*a*d) - I*(Integr
al(-I*d*e**2/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-I*d*f**2*x
**2/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-d*e**2*exp(c)*exp(d
*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-4*d*e**2*exp(3*c)*e
xp(3*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(d*e**2*exp(5*c
)*exp(5*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-2*I*d*e*f*
x/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(4*I*d*e**2*exp(2*c)*ex
p(2*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(I*d*e**2*exp(4*
c)*exp(4*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-16*I*e*f*
exp(2*c)*exp(2*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-16*
I*f**2*x*exp(2*c)*exp(2*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Inte
gral(-d*f**2*x**2*exp(c)*exp(d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) +
 Integral(-4*d*f**2*x**2*exp(3*c)*exp(3*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*
d*x)), x) + Integral(d*f**2*x**2*exp(5*c)*exp(5*d*x)/(exp(c)*exp(3*d*x) -
I*exp(2*d*x)), x) + Integral(4*I*d*f**2*x**2*exp(2*c)*exp(2*d*x)/(exp(c)*e
xp(3*d*x) - I*exp(2*d*x)), x) + Integral(I*d*f**2*x**2*exp(4*c)*exp(4*d*x)
/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-2*d*e*f*x*exp(c)*exp(d
*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-8*d*e*f*x*exp(3*c)*
exp(3*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(2*d*e*f*x*exp
(5*c)*exp(5*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(8*I*...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\sinh{(dx+c)}^3}{i\,a\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sinh(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{\sinh(c+d\,x)^3\,(e+f\,x)^2}{a+a\sinh{(c+d\,x)}\;1\mathrm{i}}\,dx$$

input
```
int((sinh(c + d*x)^3*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i),x)
```

output
```
int((sinh(c + d*x)^3*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$=\frac{\cosh{(dx+c)}^2\,d^2efi\,x^2+\cosh{(dx+c)}^2\,efi-2\cosh{(dx+c)}\sinh{(dx+c)}\,defix+4\cosh{(dx+c)}\,defx}{}$$

input
```
int((f*x+e)^2*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

output
```
(cosh(c + d*x)**2*d**2*e*f*i*x**2 + cosh(c + d*x)**2*e*f*i - 2*cosh(c + d*
x)*sinh(c + d*x)*d*e*f*i*x + 4*cosh(c + d*x)*d*e*f*x + 2*int(sinh(c + d*x)
**3/(sinh(c + d*x)*i + 1),x)*d**2*e**2 + 4*int(( - x)/(sinh(c + d*x) - i),
x)*d**2*e*f + 2*int((sinh(c + d*x)**3*x**2)/(sinh(c + d*x)*i + 1),x)*d**2*
f**2 - sinh(c + d*x)**2*d**2*e*f*i*x**2 - 4*sinh(c + d*x)*e*f + 2*d**2*e*f
*i*x**2)/(2*a*d**2)
```

3.201 $\int \frac{(e+fx)\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 170

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{3i(e+fx)^2}{4af}+\frac{(e+fx)\cosh(c+dx)}{ad}+\frac{2if\log\left(\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2}-\frac{f\sinh(c+dx)}{ad^2}-\frac{i(e+fx)\cosh(c+dx)\sinh(c+dx)}{2ad}+\frac{if\sinh^2(c+dx)}{4ad^2}-\frac{i(e+fx)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}$$

output
```
3/4*I*(f*x+e)^2/a/f+(f*x+e)*cosh(d*x+c)/a/d+2*I*f*ln(cosh(1/2*c+1/4*I*Pi+1
/2*d*x))/a/d^2-f*sinh(d*x+c)/a/d^2-1/2*I*(f*x+e)*cosh(d*x+c)*sinh(d*x+c)/a
/d+1/4*I*f*sinh(d*x+c)^2/a/d^2-I*(f*x+e)*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a/d
```

Mathematica [A] (verified)

Time = 3.42 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.91

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)\left(-8id(e+fx)\cosh(c+dx)+f\cosh(2(c+d\right.\right.}{}$$

input
```
Integrate[((e + f*x)*Sinh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(Cosh[(c + d*x)/2]*((-8*I)*d*(e
 + f*x)*Cosh[c + d*x] + f*Cosh[2*(c + d*x)] + 2*(6*c*d*e - (4*I)*c*f - 3*c
^2*f + 6*d^2*e*x - (4*I)*d*f*x + 3*d^2*f*x^2 + (8*I)*f*ArcTan[Tanh[(c + d*
x)/2]] + 4*f*Log[Cosh[c + d*x]] + (4*I)*f*Sinh[c + d*x] - d*(e + f*x)*Sinh
[2*(c + d*x)])) + Sinh[(c + d*x)/2]*(8*d*(e + f*x)*Cosh[c + d*x] + I*(f*Co
sh[2*(c + d*x)] + 2*((8*I)*d*e + 6*c*d*e - (4*I)*c*f - 3*c^2*f + 6*d^2*e*x
 + (4*I)*d*f*x + 3*d^2*f*x^2 + (8*I)*f*ArcTan[Tanh[(c + d*x)/2]] + 4*f*Log
[Cosh[c + d*x]] + (4*I)*f*Sinh[c + d*x] - d*(e + f*x)*Sinh[2*(c + d*x)])))
))/(8*a*d^2*(-I + Sinh[c + d*x]))
```

Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.16, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.793$, Rules used = {6091, 3042, 25, 3791, 17, 6091, 3042, 26, 3777, 3042, 3117, 6091, 17, 3042, 3799, 25, 25, 3042, 4672, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$\Big\downarrow \; 6091$$
$$i\int \frac{(e+fx)\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int (e+fx)\sinh^2(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$i\int \frac{(e+fx)\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int -\left((e+fx)\sin(ic+idx)^2\right)dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{i\int (e+fx)\sin(ic+idx)^2dx}{a}+i\int \frac{(e+fx)\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{3791}$$

$$\frac{i\left(\frac{1}{2}\int (e+fx)dx+\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{a}+i\int \frac{(e+fx)\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{17}$$

$$i\int \frac{(e+fx)\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \text{6091}$$

$$i\left(i\int \frac{(e+fx)\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int (e+fx)\sinh(c+dx)dx}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$i\left(i\int \frac{(e+fx)\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int -i(e+fx)\sin(ic+idx)dx}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$i\left(i\int \frac{(e+fx)\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\int (e+fx)\sin(ic+idx)dx}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \text{3777}$$

$$i\left(i\int \frac{(e+fx)\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$i\left(i\int \frac{(e+fx)\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \; 3117$$

$$i\left(i\int \frac{(e+fx)\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \; 6091$$

$$i\left(i\left(i\int \frac{e+fx}{i\sinh(c+dx)a+a}dx-\frac{i\int(e+fx)dx}{a}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \; 17$$

$$i\left(i\left(i\int \frac{e+fx}{i\sinh(c+dx)a+a}dx-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$i\left(i\left(i\int \frac{e+fx}{\sin(ic+idx)a+a}dx-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \; 3799$$

$$i\left(i\left(\frac{i\int -\left((e+fx)\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)\right)dx}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\downarrow 25$$

$$i\left(i\left(-\frac{i\int -\left((e+fx)\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)dx}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\downarrow 25$$

$$i\left(i\left(\frac{i\int (e+fx)\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\downarrow 3042$$

$$i\left(i\left(\frac{i\int (e+fx)\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\downarrow 4672$$

$$i\left(i\left(\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2if\int -i\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\downarrow 26$$

$$i\left(i\left(\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2f\int \tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\downarrow 3042$$

$$i\left(i\left(\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2f\int -i\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \; 26$$

$$i\left(i\left(\frac{i\left(\frac{2if\int\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}+\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \; 3956$$

$$\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}+i\left(i\left(\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4f\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{d^2}\right)}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}\right)$$

input
```
Int[((e + f*x)*Sinh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(I*((e + f*x)^2/(4*f) - ((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (f
*Sinh[c + d*x]^2)/(4*d^2)))/a + I*(-(((I*(e + f*x)*Cosh[c + d*x])/d - (I*f
*Sinh[c + d*x])/d^2)/a) + I*(((-1/2*I)*(e + f*x)^2)/(a*f) + ((I/2)*((-4*f*
Log[Cosh[c/2 + (I/4)*Pi + (d*x)/2]])/d^2 + (2*(e + f*x)*Tanh[c/2 + (I/4)*P
i + (d*x)/2])/d))/a))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.16

method	result
risch	$\frac{3 i f x^{2}}{4 a}+\frac{3 i e x}{2 a}-\frac{i \left(2 d x f+2 d e-f\right) {\mathrm e}^{2 d x+2 c}}{16 a \,d^{2}}+\frac{\left(d x f+d e-f\right) {\mathrm e}^{d x+c}}{2 a \,d^{2}}+\frac{\left(d x f+d e+f\right) {\mathrm e}^{-d x-c}}{2 a \,d^{2}}+\frac{i \left(2 d x f+2 d e+f\right) {\mathrm e}^{-2 d x-2 c}}{16 a \,d^{2}}$
parallelrisch	$32 f \left(i \cosh \left(\frac{d x}{2}+\frac{c}{2}\right)-\sinh \left(\frac{d x}{2}+\frac{c}{2}\right)\right) \ln \left(-i+\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)\right)-32 f \left(i \cosh \left(\frac{d x}{2}+\frac{c}{2}\right)-\sinh \left(\frac{d x}{2}+\frac{c}{2}\right)\right) \ln \left(\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)+($

input
```
int((f*x+e)*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
3/4*I/a*f*x^2+3/2*I/a*e*x-1/16*I*(2*d*f*x+2*d*e-f)/a/d^2*exp(2*d*x+2*c)+1/
2*(d*f*x+d*e-f)/a/d^2*exp(d*x+c)+1/2*(d*f*x+d*e+f)/a/d^2*exp(-d*x-c)+1/16*
I*(2*d*f*x+2*d*e+f)/a/d^2*exp(-2*d*x-2*c)-2*I*f/a/d*x-2*I*f/a/d^2*c+2*(f*x
+e)/d/a/(exp(d*x+c)-I)+2*I*f/a/d^2*ln(exp(d*x+c)-I)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.35

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{2\,dfx + 2\,de + (-2i\,dfx - 2i\,de + i\,f)e^{(5\,dx+5\,c)} + (6\,dfx + 6\,de - 7\,f)e^{(4\,dx+4\,c)} - 4\,(-3i\,d^2fx^2 + 2i\,de +}{}$$

input
```
integrate((f*x+e)*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/16*(2*d*f*x + 2*d*e + (-2*I*d*f*x - 2*I*d*e + I*f)*e^(5*d*x + 5*c) + (6*
d*f*x + 6*d*e - 7*f)*e^(4*d*x + 4*c) - 4*(-3*I*d^2*f*x^2 + 2*I*d*e + 2*(-3
*I*d^2*e + 5*I*d*f)*x - 2*I*f)*e^(3*d*x + 3*c) + 4*(3*d^2*f*x^2 + 10*d*e +
 2*(3*d^2*e + d*f)*x + 2*f)*e^(2*d*x + 2*c) + (-6*I*d*f*x - 6*I*d*e - 7*I*
f)*e^(d*x + c) - 32*(-I*f*e^(3*d*x + 3*c) - f*e^(2*d*x + 2*c))*log(e^(d*x
+ c) - I) + f)/(a*d^2*e^(3*d*x + 3*c) - I*a*d^2*e^(2*d*x + 2*c))
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.33

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{2e+2fx}{ade^ce^{dx}-iad}$$
$$+\begin{cases} \frac{\left(\left(512a^3d^7ee^{2c}+512a^3d^7fxe^{2c}+512a^3d^6fe^{2c}\right)e^{-dx}+\left(512a^3d^7ee^{4c}+512a^3d^7fxe^{4c}-512a^3d^6fe^{4c}\right)e^{dx}+\left(128ia^3d^7ee^c+128ia^3d^7fxe^c+6\right.}{1024a^4d^8} \\ \frac{x^2\left(-ife^{4c}+2fe^{3c}-2fe^c-if\right)e^{-2c}}{8a}+\frac{x\left(-iee^{4c}+2ee^{3c}-2ee^c-ie\right)e^{-2c}}{4a} \end{cases}$$
$$+\frac{3ifx^2}{4a}+\frac{x(3ide-4if)}{2ad}+\frac{2if\log\left(e^{dx}-ie^{-c}\right)}{ad^2}$$

input
```
integrate((f*x+e)*sinh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
(2*e + 2*f*x)/(a*d*exp(c)*exp(d*x) - I*a*d) + Piecewise((((512*a**3*d**7*e
*exp(2*c) + 512*a**3*d**7*f*x*exp(2*c) + 512*a**3*d**6*f*exp(2*c))*exp(-d*
x) + (512*a**3*d**7*e*exp(4*c) + 512*a**3*d**7*f*x*exp(4*c) - 512*a**3*d**
6*f*exp(4*c))*exp(d*x) + (128*I*a**3*d**7*e*exp(c) + 128*I*a**3*d**7*f*x*e
xp(c) + 64*I*a**3*d**6*f*exp(c))*exp(-2*d*x) + (-128*I*a**3*d**7*e*exp(5*c
) - 128*I*a**3*d**7*f*x*exp(5*c) + 64*I*a**3*d**6*f*exp(5*c))*exp(2*d*x))*
exp(-3*c)/(1024*a**4*d**8), Ne(a**4*d**8*exp(3*c), 0)), (x**2*(-I*f*exp(4*
c) + 2*f*exp(3*c) - 2*f*exp(c) - I*f)*exp(-2*c)/(8*a) + x*(-I*e*exp(4*c) +
 2*e*exp(3*c) - 2*e*exp(c) - I*e)*exp(-2*c)/(4*a), True)) + 3*I*f*x**2/(4*
a) + x*(3*I*d*e - 4*I*f)/(2*a*d) + 2*I*f*log(exp(d*x) - I*exp(-c))/(a*d**2
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(144) = 288.

Time = 0.12 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.01

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{12i\,d^2fx^2e^{(3\,dx+3\,c)}+12\,d^2fx^2e^{(2\,dx+2\,c)}+24i\,d^2exe^{(3\,dx+3\,c)}+24\,d^2exe^{(2\,dx+2\,c)}-2i\,dfxe^{(5\,dx+5\,c)}+6\,dfxe}{}$$

input
```
integrate((f*x+e)*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/16*(12*I*d^2*f*x^2*e^(3*d*x + 3*c) + 12*d^2*f*x^2*e^(2*d*x + 2*c) + 24*I
*d^2*e*x*e^(3*d*x + 3*c) + 24*d^2*e*x*e^(2*d*x + 2*c) - 2*I*d*f*x*e^(5*d*x
 + 5*c) + 6*d*f*x*e^(4*d*x + 4*c) - 40*I*d*f*x*e^(3*d*x + 3*c) + 8*d*f*x*e
^(2*d*x + 2*c) - 6*I*d*f*x*e^(d*x + c) + 2*d*f*x - 2*I*d*e*e^(5*d*x + 5*c)
 + 6*d*e*e^(4*d*x + 4*c) - 8*I*d*e*e^(3*d*x + 3*c) + 40*d*e*e^(2*d*x + 2*c
) - 6*I*d*e*e^(d*x + c) + 32*I*f*e^(3*d*x + 3*c)*log(e^(d*x + c) - I) + 32
*f*e^(2*d*x + 2*c)*log(e^(d*x + c) - I) + 2*d*e + I*f*e^(5*d*x + 5*c) - 7*
f*e^(4*d*x + 4*c) + 8*I*f*e^(3*d*x + 3*c) + 8*f*e^(2*d*x + 2*c) - 7*I*f*e^
(d*x + c) + f)/(a*d^2*e^(3*d*x + 3*c) - I*a*d^2*e^(2*d*x + 2*c))
```

Mupad [B] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.26

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \mathrm{e}^{-c-d\,x}\left(\frac{f+d\,e}{2\,a\,d^2}+\frac{f\,x}{2\,a\,d}\right) + \mathrm{e}^{-2\,c-2\,d\,x}\left(\frac{(f+2\,d\,e)\ 1\mathrm{i}}{16\,a\,d^2}+\frac{f\,x\,1\mathrm{i}}{8\,a\,d}\right) + \mathrm{e}^{2\,c+2\,d\,x}\left(\frac{(f-2\,d\,e)\ 1\mathrm{i}}{16\,a\,d^2}-\frac{f\,x\,1\mathrm{i}}{8\,a\,d}\right) - \mathrm{e}^{c+d\,x}\left(\frac{f-d\,e}{2\,a\,d^2}-\frac{f\,x}{2\,a\,d}\right)+\frac{f\,x^2\,3\mathrm{i}}{4\,a}+\frac{2\,(e+f\,x)}{a\,d\,\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)} - \frac{x\,(4\,f-3\,d\,e)\ 1\mathrm{i}}{2\,a\,d}+\frac{f\,\ln\left(\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-\mathrm{i}\right)\ 2\mathrm{i}}{a\,d^2}$$

input
```
int((sinh(c + d*x)^3*(e + f*x))/(a + a*sinh(c + d*x)*1i),x)
```

output
```
exp(- c - d*x)*((f + d*e)/(2*a*d^2) + (f*x)/(2*a*d)) + exp(- 2*c - 2*d*x)*
(((f + 2*d*e)*1i)/(16*a*d^2) + (f*x*1i)/(8*a*d)) + exp(2*c + 2*d*x)*(((f -
 2*d*e)*1i)/(16*a*d^2) - (f*x*1i)/(8*a*d)) - exp(c + d*x)*((f - d*e)/(2*a*
d^2) - (f*x)/(2*a*d)) + (f*x^2*3i)/(4*a) + (2*(e + f*x))/(a*d*(exp(c + d*x
) - 1i)) - (x*(4*f - 3*d*e)*1i)/(2*a*d) + (f*log(exp(d*x)*exp(c) - 1i)*2i)
/(a*d^2)
```

Reduce [F]

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{\cosh(dx+c)^2 d^2 fi\,x^2 + \cosh(dx+c)^2 fi - 2\cosh(dx+c)\sinh(dx+c)\,dfix + 4\cosh(dx+c)\,dfx + 4(}{4a}$$

input
```
int((f*x+e)*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

output
```
(cosh(c + d*x)**2*d**2*f*i*x**2 + cosh(c + d*x)**2*f*i - 2*cosh(c + d*x)*s
inh(c + d*x)*d*f*i*x + 4*cosh(c + d*x)*d*f*x + 4*int(sinh(c + d*x)**3/(sin
h(c + d*x)*i + 1),x)*d**2*e + 4*int(( - x)/(sinh(c + d*x) - i),x)*d**2*f -
 sinh(c + d*x)**2*d**2*f*i*x**2 - 4*sinh(c + d*x)*f + 2*d**2*f*i*x**2)/(4*
a*d**2)
```

3.202 $\int \frac{\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 83

$$\int \frac{\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{3ix}{2a} + \frac{2\cosh(c+dx)}{ad} - \frac{3i\cosh(c+dx)\sinh(c+dx)}{2ad} - \frac{\cosh(c+dx)\sinh^2(c+dx)}{d(a+ia\sinh(c+dx))}$$

output
```
3/2*I*x/a+2*cosh(d*x+c)/a/d-3/2*I*cosh(d*x+c)*sinh(d*x+c)/a/d-cosh(d*x+c)*
sinh(d*x+c)^2/d/(a+I*a*sinh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.31

$$\int \frac{\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\cosh(c+dx)\left(3\text{arcsinh}(\sinh(c+dx))\sqrt{1+i\sinh(c+dx)}+\sqrt{1-i\sinh(c+dx)}(-4i+\sinh(c+dx)-\right.}{2ad\sqrt{1-i\sinh(c+dx)}(-i+\sinh(c+dx))}$$

input
```
Integrate[Sinh[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(Cosh[c + d*x]*(3*ArcSinh[Sinh[c + d*x]]*Sqrt[1 + I*Sinh[c + d*x]] + Sqrt[
1 - I*Sinh[c + d*x]]*(-4*I + Sinh[c + d*x] - I*Sinh[c + d*x]^2)))/(2*a*d*S
qrt[1 - I*Sinh[c + d*x]]*(-I + Sinh[c + d*x]))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3042, 26, 3246, 26, 3042, 26, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{a+ia\sinh(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i\sin(ic+idx)^3}{a+a\sin(ic+idx)}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{\sin(ic+idx)^3}{\sin(ic+idx)a+a}dx$$
$$\Big\downarrow \text{3246}$$
$$i\left(\frac{i\sinh^2(c+dx)\cosh(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{\int i\sinh(c+dx)(2a-3ia\sinh(c+dx))dx}{a^2}\right)$$
$$\Big\downarrow \text{26}$$
$$i\left(\frac{i\sinh^2(c+dx)\cosh(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{i\int \sinh(c+dx)(2a-3ia\sinh(c+dx))dx}{a^2}\right)$$
$$\Big\downarrow \text{3042}$$
$$i\left(\frac{i\sinh^2(c+dx)\cosh(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{i\int -i\sin(ic+idx)(2a-3a\sin(ic+idx))dx}{a^2}\right)$$
$$\Big\downarrow \text{26}$$
$$i\left(\frac{i\sinh^2(c+dx)\cosh(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{\int \sin(ic+idx)(2a-3a\sin(ic+idx))dx}{a^2}\right)$$

$$\Big\downarrow \text{3213}$$

$$i\left(\frac{i\sinh^2(c+dx)\cosh(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{\frac{2ia\cosh(c+dx)}{d}+\frac{3a\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{3ax}{2}}{a^2}\right)$$

input
```
Int[Sinh[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

output
```
I*((I*Cosh[c + d*x]*Sinh[c + d*x]^2)/(d*(a + I*a*Sinh[c + d*x])) - ((-3*a*
x)/2 + ((2*I)*a*Cosh[c + d*x])/d + (3*a*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
)/a^2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3246
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e +
 f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Simp[d/(a*b)   Int[(c + d*
Sin[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[
e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] ||
EqQ[c, 0])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.14

method	result
risch	$\frac{3ix}{2a}-\frac{i\,\mathrm{e}^{2dx+2c}}{8ad}+\frac{\mathrm{e}^{dx+c}}{2ad}+\frac{\mathrm{e}^{-dx-c}}{2ad}+\frac{i\,\mathrm{e}^{-2dx-2c}}{8ad}+\frac{2}{da\left(\mathrm{e}^{dx+c}-i\right)}$
parallelrisch	$\frac{(12idx-4)\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)+(-12dx-28i)\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\sinh\left(\frac{5dx}{2}+\frac{5c}{2}\right)+3i\sinh\left(\frac{3dx}{2}+\frac{3c}{2}\right)+3\cosh\left(\frac{3dx}{2}+\frac{3c}{2}\right)+\cosh}{8ad\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$
derivativedivides	$\frac{-\frac{3i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{i}{2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{16\left(-\frac{1}{16}-\frac{i}{32}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{2i}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{i}{2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{3i\ln\left(1+\tan\right.}{}}{da}$
default	$\frac{-\frac{3i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{i}{2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{16\left(-\frac{1}{16}-\frac{i}{32}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{2i}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{i}{2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{3i\ln\left(1+\tan\right.}{}}{da}$

input
```
int(sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
3/2*I*x/a-1/8*I/a/d*exp(2*d*x+2*c)+1/2/a/d*exp(d*x+c)+1/2/a/d*exp(-d*x-c)+
1/8*I/a/d*exp(-2*d*x-2*c)+2/d/a/(exp(d*x+c)-I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.16

$$\int \frac{\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{4\,(-3i\,dx+i)e^{(3\,dx+3\,c)}-4\,(3\,dx+5)e^{(2\,dx+2\,c)}+i\,e^{(5\,dx+5\,c)}-3\,e^{(4\,dx+4\,c)}+3i\,e^{(dx+c)}-1}{8\left(ade^{(3\,dx+3\,c)}-i\,ade^{(2\,dx+2\,c)}\right)}$$

input
```
integrate(sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/8*(4*(-3*I*d*x + I)*e^(3*d*x + 3*c) - 4*(3*d*x + 5)*e^(2*d*x + 2*c) + I
*e^(5*d*x + 5*c) - 3*e^(4*d*x + 4*c) + 3*I*e^(d*x + c) - 1)/(a*d*e^(3*d*x
+ 3*c) - I*a*d*e^(2*d*x + 2*c))
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.11

$$\int \frac{\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \begin{cases} \frac{(-32ia^3d^3e^{5c}e^{2dx}+128a^3d^3e^{4c}e^{dx}+128a^3d^3e^{2c}e^{-dx}+32ia^3d^3e^{c}e^{-2dx})e^{-3c}}{256a^4d^4} & \text{for } a^4d^4e^{3c} \neq 0 \\ x\left(\frac{(-ie^{4c}+2e^{3c}+6ie^{2c}-2e^{c}-i)e^{-2c}}{4a} - \frac{3i}{2a}\right) & \text{otherwise} \end{cases} + \frac{2}{ade^ce^{dx}-iad} + \frac{3ix}{2a}$$

input
```
integrate(sinh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
Piecewise(((-32*I*a**3*d**3*exp(5*c)*exp(2*d*x) + 128*a**3*d**3*exp(4*c)*e
xp(d*x) + 128*a**3*d**3*exp(2*c)*exp(-d*x) + 32*I*a**3*d**3*exp(c)*exp(-2*
d*x))*exp(-3*c)/(256*a**4*d**4), Ne(a**4*d**4*exp(3*c), 0)), (x*((-I*exp(4
*c) + 2*exp(3*c) + 6*I*exp(2*c) - 2*exp(c) - I)*exp(-2*c)/(4*a) - 3*I/(2*a
)), True)) + 2/(a*d*exp(c)*exp(d*x) - I*a*d) + 3*I*x/(2*a)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.18

$$\int \frac{\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{3i\,(dx+c)}{2\,ad} + \frac{3i\,e^{(-dx-c)}+20\,e^{(-2\,dx-2\,c)}+1}{8\,(i\,ae^{(-2\,dx-2\,c)}+ae^{(-3\,dx-3\,c)})d} + \frac{i\,(-4i\,e^{(-dx-c)}+e^{(-2\,dx-2\,c)})}{8\,ad}$$

input
```
integrate(sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
3/2*I*(d*x + c)/(a*d) + 1/8*(3*I*e^(-d*x - c) + 20*e^(-2*d*x - 2*c) + 1)/(
(I*a*e^(-2*d*x - 2*c) + a*e^(-3*d*x - 3*c))*d) + 1/8*I*(-4*I*e^(-d*x - c)
+ e^(-2*d*x - 2*c))/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{-\frac{12i\,(dx+c)}{a} - \frac{\left(20\,e^{(2\,dx+2\,c)}-3i\,e^{(dx+c)}+1\right)e^{(-2\,dx-2\,c)}}{a\left(e^{(dx+c)}-i\right)} + \frac{i\,ae^{(2\,dx+2\,c)}-4\,ae^{(dx+c)}}{a^2}}{8\,d}$$

input
```
integrate(sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/8*(-12*I*(d*x + c)/a - (20*e^(2*d*x + 2*c) - 3*I*e^(d*x + c) + 1)*e^(-2
*d*x - 2*c)/(a*(e^(d*x + c) - I)) + (I*a*e^(2*d*x + 2*c) - 4*a*e^(d*x + c)
)/a^2)/d
```

Mupad [B] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.13

$$\int \frac{\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{x\,3\mathrm{i}}{2\,a} + \frac{2}{a\,d\,\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)} + \frac{\mathrm{e}^{c+d\,x}}{2\,a\,d} + \frac{\mathrm{e}^{-c-d\,x}}{2\,a\,d} + \frac{\mathrm{e}^{-2\,c-2\,d\,x}\,1\mathrm{i}}{8\,a\,d} - \frac{\mathrm{e}^{2\,c+2\,d\,x}\,1\mathrm{i}}{8\,a\,d}$$

input
```
int(sinh(c + d*x)^3/(a + a*sinh(c + d*x)*1i),x)
```

output
```
(x*3i)/(2*a) + 2/(a*d*(exp(c + d*x) - 1i)) + exp(c + d*x)/(2*a*d) + exp(-
c - d*x)/(2*a*d) + (exp(- 2*c - 2*d*x)*1i)/(8*a*d) - (exp(2*c + 2*d*x)*1i)
/(8*a*d)
```

Reduce [F]

$$\int \frac{\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\int \frac{\sinh(dx+c)^3}{\sinh(dx+c)i+1}dx}{a}$$

input
```
int(sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sinh(c + d*x)**3/(sinh(c + d*x)*i + 1),x)/a
```

3.203 $\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(sinh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 34.42 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

input
```
Integrate[Sinh[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Integrate[Sinh[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))} \, dx$$

$$\Big\downarrow \text{ 6111}$$

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}dx$$

input
```
Int[Sinh[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\sinh(dx+c)^3}{(fx+e)(a+ia\sinh(dx+c))}dx$$

input
```
int(sinh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 249, normalized size of antiderivative = 8.03

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)^3}{(fx+e)(i\,a\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
((-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))*integral(-1/4*(d*f
*x + d*e - (-I*d*f*x - I*d*e)*e^(5*d*x + 5*c) - (d*f*x + d*e)*e^(4*d*x + 4
*c) + 4*(-I*d*f*x - I*d*e)*e^(3*d*x + 3*c) - 4*(d*f*x + d*e + 2*f)*e^(2*d*
x + 2*c) - (I*d*f*x + I*d*e)*e^(d*x + c))/((a*d*f^2*x^2 + 2*a*d*e*f*x + a*
d*e^2)*e^(3*d*x + 3*c) - (I*a*d*f^2*x^2 + 2*I*a*d*e*f*x + I*a*d*e^2)*e^(2*
d*x + 2*c)), x) + 2)/(-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c)
)
```

Sympy [N/A]

Not integrable

Time = 111.01 (sec) , antiderivative size = 1452, normalized size of antiderivative = 46.84

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)**3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
2/(-I*a*d*e - I*a*d*f*x + (a*d*e*exp(c) + a*d*f*x*exp(c))*exp(d*x)) - I*(I
ntegral(-I*d*e/(e**2*exp(c)*exp(3*d*x) - I*e**2*exp(2*d*x) + 2*e*f*x*exp(c
)*exp(3*d*x) - 2*I*e*f*x*exp(2*d*x) + f**2*x**2*exp(c)*exp(3*d*x) - I*f**2
*x**2*exp(2*d*x)), x) + Integral(-I*d*f*x/(e**2*exp(c)*exp(3*d*x) - I*e**2
*exp(2*d*x) + 2*e*f*x*exp(c)*exp(3*d*x) - 2*I*e*f*x*exp(2*d*x) + f**2*x**2
*exp(c)*exp(3*d*x) - I*f**2*x**2*exp(2*d*x)), x) + Integral(8*I*f*exp(2*c)
*exp(2*d*x)/(e**2*exp(c)*exp(3*d*x) - I*e**2*exp(2*d*x) + 2*e*f*x*exp(c)*e
xp(3*d*x) - 2*I*e*f*x*exp(2*d*x) + f**2*x**2*exp(c)*exp(3*d*x) - I*f**2*x*
*2*exp(2*d*x)), x) + Integral(-d*e*exp(c)*exp(d*x)/(e**2*exp(c)*exp(3*d*x)
 - I*e**2*exp(2*d*x) + 2*e*f*x*exp(c)*exp(3*d*x) - 2*I*e*f*x*exp(2*d*x) +
f**2*x**2*exp(c)*exp(3*d*x) - I*f**2*x**2*exp(2*d*x)), x) + Integral(-4*d*
e*exp(3*c)*exp(3*d*x)/(e**2*exp(c)*exp(3*d*x) - I*e**2*exp(2*d*x) + 2*e*f*
x*exp(c)*exp(3*d*x) - 2*I*e*f*x*exp(2*d*x) + f**2*x**2*exp(c)*exp(3*d*x) -
 I*f**2*x**2*exp(2*d*x)), x) + Integral(d*e*exp(5*c)*exp(5*d*x)/(e**2*exp(
c)*exp(3*d*x) - I*e**2*exp(2*d*x) + 2*e*f*x*exp(c)*exp(3*d*x) - 2*I*e*f*x*
exp(2*d*x) + f**2*x**2*exp(c)*exp(3*d*x) - I*f**2*x**2*exp(2*d*x)), x) + I
ntegral(4*I*d*e*exp(2*c)*exp(2*d*x)/(e**2*exp(c)*exp(3*d*x) - I*e**2*exp(2
*d*x) + 2*e*f*x*exp(c)*exp(3*d*x) - 2*I*e*f*x*exp(2*d*x) + f**2*x**2*exp(c
)*exp(3*d*x) - I*f**2*x**2*exp(2*d*x)), x) + Integral(I*d*e*exp(4*c)*exp(4
*d*x)/(e**2*exp(c)*exp(3*d*x) - I*e**2*exp(2*d*x) + 2*e*f*x*exp(c)*exp(...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sinh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)^3}{(fx+e)(i\,a\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sinh(d*x + c)^3/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(c+d\,x)^3}{(e+f\,x)\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(sinh(c + d*x)^3/((e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(sinh(c + d*x)^3/((e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \frac{\int \frac{\sinh(dx+c)^3}{\sinh(dx+c)ei+\sinh(dx+c)fix+e+fx}dx}{a}$$

input
```
int(sinh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sinh(c + d*x)**3/(sinh(c + d*x)*e*i + sinh(c + d*x)*f*i*x + e + f*x),x
)/a
```

3.204 $\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(sinh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 15.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

input
```
Integrate[Sinh[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Integrate[Sinh[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$
$$\Big\downarrow \text{6111}$$
$$\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}dx$$

input
```
Int[Sinh[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\sinh\left(dx+c\right)^{3}}{\left(fx+e\right)^{2}\left(a+ia\sinh\left(dx+c\right)\right)}dx$$

input
```
int(sinh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 327, normalized size of antiderivative = 10.55

$$\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh{(dx+c)}^3}{(fx+e)^2(i\,a\sinh{(dx+c)}+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
((-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x
+ a*d*e^2)*e^(d*x + c))*integral(-1/4*(d*f*x + d*e - (-I*d*f*x - I*d*e)*e^
(5*d*x + 5*c) - (d*f*x + d*e)*e^(4*d*x + 4*c) + 4*(-I*d*f*x - I*d*e)*e^(3*
d*x + 3*c) - 4*(d*f*x + d*e + 4*f)*e^(2*d*x + 2*c) - (I*d*f*x + I*d*e)*e^(
d*x + c))/((a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x + a*d*e^3)*e^(3*
d*x + 3*c) - (I*a*d*f^3*x^3 + 3*I*a*d*e*f^2*x^2 + 3*I*a*d*e^2*f*x + I*a*d*
e^3)*e^(2*d*x + 2*c)), x) + 2)/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2
 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**3/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sinh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)^3}{(fx+e)^2(i\,a\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sinh(d*x + c)^3/((f*x + e)^2*(I*a*sinh(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(c+d\,x)^3}{(e+f\,x)^2\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(sinh(c + d*x)^3/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(sinh(c + d*x)^3/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.26

$$\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \frac{\int \frac{\sinh(dx+c)^3}{\sinh(dx+c)e^2i+2\sinh(dx+c)efix+\sinh(dx+c)f^2i\,x^2+e^2+2efx+f^2x^2}dx}{a}$$

input
```
int(sinh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sinh(c + d*x)**3/(sinh(c + d*x)*e**2*i + 2*sinh(c + d*x)*e*f*i*x + sin
h(c + d*x)*f**2*i*x**2 + e**2 + 2*e*f*x + f**2*x**2),x)/a
```

3.205 $\int \frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 313

$$\begin{aligned}\int \frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx =& -\frac{i(e+fx)^3}{ad}-\frac{2(e+fx)^3\operatorname{arctanh}\left(e^{c+dx}\right)}{ad} \\ &+\frac{6if(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2} \\ &-\frac{3f(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2} \\ &+\frac{12if^2(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3} \\ &+\frac{3f(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{ad^2} \\ &+\frac{6f^2(e+fx)\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3} \\ &-\frac{12if^3\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^4} \\ &-\frac{6f^2(e+fx)\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{ad^3} \\ &-\frac{6f^3\operatorname{PolyLog}\left(4,-e^{c+dx}\right)}{ad^4}+\frac{6f^3\operatorname{PolyLog}\left(4,e^{c+dx}\right)}{ad^4} \\ &-\frac{i(e+fx)^3\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}\end{aligned}$$

output
```
-I*(f*x+e)^3/a/d-2*(f*x+e)^3*arctanh(exp(d*x+c))/a/d+6*I*f*(f*x+e)^2*ln(1+
I*exp(d*x+c))/a/d^2-3*f*(f*x+e)^2*polylog(2,-exp(d*x+c))/a/d^2+12*I*f^2*(f
*x+e)*polylog(2,-I*exp(d*x+c))/a/d^3+3*f*(f*x+e)^2*polylog(2,exp(d*x+c))/a
/d^2+6*f^2*(f*x+e)*polylog(3,-exp(d*x+c))/a/d^3-12*I*f^3*polylog(3,-I*exp(
d*x+c))/a/d^4-6*f^2*(f*x+e)*polylog(3,exp(d*x+c))/a/d^3-6*f^3*polylog(4,-e
xp(d*x+c))/a/d^4+6*f^3*polylog(4,exp(d*x+c))/a/d^4-I*(f*x+e)^3*tanh(1/2*c+
1/4*I*Pi+1/2*d*x)/a/d
```

Mathematica [A] (verified)

Time = 2.38 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$= \frac{\frac{2(e+fx)^3}{-i+e^c} + \frac{6if(e+fx)^2\log(1-ie^{-c-dx})}{d} + (e+fx)^3\log\left(1-e^{c+dx}\right) - (e+fx)^3\log\left(1+e^{c+dx}\right) - \frac{3f(e+fx)^2\,\text{PolyL}}{d}}{}$$

input
```
Integrate[((e + f*x)^3*Csch[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((2*(e + f*x)^3)/(-I + E^c) + ((6*I)*f*(e + f*x)^2*Log[1 - I*E^(-c - d*x)]
)/d + (e + f*x)^3*Log[1 - E^(c + d*x)] - (e + f*x)^3*Log[1 + E^(c + d*x)]
 - (3*f*(e + f*x)^2*PolyLog[2, -E^(c + d*x)])/d + (3*f*(e + f*x)^2*PolyLog[
2, E^(c + d*x)])/d - ((12*I)*f^2*(d*(e + f*x)*PolyLog[2, I*E^(-c - d*x)] +
 f*PolyLog[3, I*E^(-c - d*x)]))/d^3 + (6*f^2*(e + f*x)*PolyLog[3, -E^(c +
d*x)])/d^2 - (6*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/d^2 - (6*f^3*PolyLo
g[4, -E^(c + d*x)])/d^3 + (6*f^3*PolyLog[4, E^(c + d*x)])/d^3 - ((2*I)*(e
+ f*x)^3*Sinh[(d*x)/2])/((Cosh[c/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] + I*
Sinh[(c + d*x)/2])))/(a*d)
```

Rubi [A] (verified)

Time = 2.01 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.04, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.759$, Rules used = {6109, 3042, 26, 3799, 25, 25, 3042, 4670, 3011, 4672, 26, 3042, 26, 4199, 26, 2620, 3011, 2720, 7143, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6109}$$

$$\frac{\int (e+fx)^3\text{csch}(c+dx)dx}{a}-i\int \frac{(e+fx)^3}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int i(e+fx)^3\csc(ic+idx)dx}{a}-i\int \frac{(e+fx)^3}{\sin(ic+idx)a+a}dx$$

$$\Big\downarrow \text{ 26}$$

$$\frac{i\int (e+fx)^3\csc(ic+idx)dx}{a}-i\int \frac{(e+fx)^3}{\sin(ic+idx)a+a}dx$$

$$\Big\downarrow \text{ 3799}$$

$$\frac{i\int (e+fx)^3\csc(ic+idx)dx}{a}-\frac{i\int -(e+fx)^3\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{i\int (e+fx)^3\csc(ic+idx)dx}{a}+\frac{i\int -(e+fx)^3\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{i\int (e+fx)^3\csc(ic+idx)dx}{a}-\frac{i\int (e+fx)^3\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{i\int (e+fx)^3\csc(ic+idx)dx}{a}-\frac{i\int (e+fx)^3\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \text{ 4670}$$

$$\frac{i\left(\frac{3if\int (e+fx)^2\log\left(1-e^{c+dx}\right)dx}{d}-\frac{3if\int (e+fx)^2\log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)^3\text{arctanh}\left(e^{c+dx}\right)}{d}\right)}{a}-\frac{i\int (e+fx)^3\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \text{3011}$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$
$$\frac{i\int (e+fx)^3\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \text{4672}$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$
$$\frac{i\left(\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{6if\int -i(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$
$$\frac{i\left(\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{6f\int (e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{i\left(\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{6f\int-i(e+fx)^2\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}\right)}{2a}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{i\left(\frac{6if\int(e+fx)^2\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}$$

$$\Big\downarrow \text{4199}$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{i\left(\frac{6if\left(2i\int\frac{ie^{c+dx}(e+fx)^2}{1+ie^{c+dx}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{i\left(\frac{6if\left(-2\int\frac{e^{c+dx}(e+fx)^2}{1+ie^{c+dx}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}$$

$$\Big\downarrow \text{2620}$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{i\left(\frac{6if\left(-2\left(\frac{2if\int(e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}$$

$$\Big\downarrow \text{3011}$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{i\left(\frac{6if\left(-2\left(\frac{2if\left(\frac{f\int\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\right.}{d}\right.}{2a}$$

$$\Big\downarrow \text{2720}$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{i\left(\frac{6if\left(-2\left(\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\,\text{ta}}{}\right.}{2a}$$

$$\Big\downarrow \text{7143}$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{i\left(\frac{6if\left(-2\left(\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right.}{2a}$$

$$\Big\downarrow \text{7163}$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{d}-\frac{f\int\operatorname{PolyLog}\left(3,-e^{c+dx}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{d}-\frac{f\int}{}\right.}{d}\right.}{}\right.}{a}$$

$$\frac{i\left(\frac{6if\left(-2\left(\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right.}{2a}$$

$$\Big\downarrow \text{2720}$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\text{PolyLog}(3,-e^{c+dx})}{d}-\frac{f\int e^{-c-dx}\text{PolyLog}(3,-e^{c+dx})de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\text{PolyLog}(2,-e^{c+dx})}{d}\right)}{d}+\frac{3if\left(\frac{2f\left(\frac{(e+fx)\text{PolyLog}(3}{d}\right.}{}\right.}{}\right.}{a}$$

$$\frac{i\left(\frac{6if\left(-2\left(\frac{2if\left(\frac{f\text{PolyLog}(3,-ie^{c+dx})}{d^2}-\frac{(e+fx)\text{PolyLog}(2,-ie^{c+dx})}{d}\right)}{d}-\frac{i(e+fx)^2\log(1+ie^{c+dx})}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right.}{2a}$$

$\downarrow$ 7143

$$\frac{i\left(\frac{2i(e+fx)^3\text{arctanh}(e^{c+dx})}{d}-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\text{PolyLog}(3,-e^{c+dx})}{d}-\frac{f\text{PolyLog}(4,-e^{c+dx})}{d^2}\right)}{d}-\frac{(e+fx)^2\text{PolyLog}(2,-e^{c+dx})}{d}\right)}{d}+\frac{3if\left(\frac{2f\left(\frac{(e}{}\right.}{}\right.}{}\right.}{a}$$

$$\frac{i\left(\frac{6if\left(-2\left(\frac{2if\left(\frac{f\text{PolyLog}(3,-ie^{c+dx})}{d^2}-\frac{(e+fx)\text{PolyLog}(2,-ie^{c+dx})}{d}\right)}{d}-\frac{i(e+fx)^2\log(1+ie^{c+dx})}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right.}{2a}$$

input

```
Int[((e + f*x)^3*Csch[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(I*(((2*I)*(e + f*x)^3*ArcTanh[E^(c + d*x)])/d - ((3*I)*f*(-(((e + f*x)^2*
PolyLog[2, -E^(c + d*x)])/d) + (2*f*(((e + f*x)*PolyLog[3, -E^(c + d*x)])/
d - (f*PolyLog[4, -E^(c + d*x)])/d^2))/d))/d + ((3*I)*f*(-(((e + f*x)^2*Po
lyLog[2, E^(c + d*x)])/d) + (2*f*(((e + f*x)*PolyLog[3, E^(c + d*x)])/d -
(f*PolyLog[4, E^(c + d*x)])/d^2))/d))/d))/a - ((I/2)*(((6*I)*f*(((-1/3*I)*
(e + f*x)^3)/f - 2*(((-I)*(e + f*x)^2*Log[1 + I*E^(c + d*x)])/d + ((2*I)*f
*(-(((e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d) + (f*PolyLog[3, (-I)*E^(c
+ d*x)])/d^2))/d)))/d + (2*(e + f*x)^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/d))
/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6109
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csch[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a +
 b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1033 vs. $2(288) = 576$.

Time = 0.70 (sec) , antiderivative size = 1034, normalized size of antiderivative = 3.30

method	result	size
risch	Expression too large to display	1034

input
```
int((f*x+e)^3*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/a/d*e^3*ln(exp(d*x+c)+1)+1/a/d*e^3*ln(exp(d*x+c)-1)+6*I/a/d^3*f^3*c^2*x
-6*I/a/d*e*f^2*x^2-6*I/a/d^3*e*f^2*c^2-6*I/a/d^4*c^2*f^3*ln(exp(d*x+c))-6*
I/a/d^2*e^2*f*ln(exp(d*x+c))-6*I/a/d^4*f^3*ln(1+I*exp(d*x+c))*c^2+6*I/a/d^
2*f^3*ln(1+I*exp(d*x+c))*x^2+12*I/a/d^3*e*f^2*polylog(2,-I*exp(d*x+c))+12*
I/a/d^3*f^3*polylog(2,-I*exp(d*x+c))*x-12*I/a/d^2*e*f^2*c*x+12*I/a/d^3*c*f
^2*e*ln(exp(d*x+c))+12*I/a/d^3*e*f^2*ln(1+I*exp(d*x+c))*c-6/a/d^3*f^3*poly
log(3,exp(d*x+c))*x-1/a/d*f^3*ln(exp(d*x+c)+1)*x^3-3/a/d^2*f^3*polylog(2,-
exp(d*x+c))*x^2+6/a/d^3*f^3*polylog(3,-exp(d*x+c))*x+6/a/d^3*e*f^2*polylog
(3,-exp(d*x+c))-6/a/d^3*e*f^2*polylog(3,exp(d*x+c))-1/a/d^4*c^3*f^3*ln(exp
(d*x+c)-1)-3/a/d^2*e^2*f*polylog(2,-exp(d*x+c))+3/a/d^2*e^2*f*polylog(2,ex
p(d*x+c))+1/a/d^4*c^3*f^3*ln(1-exp(d*x+c))+1/a/d*f^3*ln(1-exp(d*x+c))*x^3+
3/a/d^2*f^3*polylog(2,exp(d*x+c))*x^2+2*(f^3*x^3+3*e*f^2*x^2+3*e^2*f*x+e^3
)/d/a/(exp(d*x+c)-I)-6*f^3*polylog(4,-exp(d*x+c))/a/d^4+6*f^3*polylog(4,ex
p(d*x+c))/a/d^4-12*I*f^3*polylog(3,-I*exp(d*x+c))/a/d^4-12*I/a/d^3*c*e*f^2
*ln(exp(d*x+c)-I)+4*I/a/d^4*f^3*c^3-2*I/a/d*f^3*x^3+12*I/a/d^2*e*f^2*ln(1+
I*exp(d*x+c))*x+3/a/d*e^2*f*ln(1-exp(d*x+c))*x-3/a/d*e^2*f*ln(exp(d*x+c)+1
)*x-3/a/d^2*e^2*c*f*ln(exp(d*x+c)-1)+3/a/d^3*c^2*e*f^2*ln(exp(d*x+c)-1)+3/
a/d^2*e^2*f*ln(1-exp(d*x+c))*c+6*I/a/d^4*c^2*f^3*ln(exp(d*x+c)-I)+6*I/a/d^
2*e^2*f*ln(exp(d*x+c)-I)+3/a/d*e*f^2*ln(1-exp(d*x+c))*x^2+6/a/d^2*e*f^2*po
lylog(2,exp(d*x+c))*x-3/a/d^3*e*f^2*ln(1-exp(d*x+c))*c^2-3/a/d*e*f^2*ln...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1000 vs. $2(276) = 552$.

Time = 0.14 (sec) , antiderivative size = 1000, normalized size of antiderivative = 3.19

$$\int \frac{(e+fx)^3 \text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(2*d^3*e^3 - 6*c*d^2*e^2*f + 6*c^2*d*e*f^2 - 2*c^3*f^3 + 12*(d*f^3*x + d*e
*f^2 - (-I*d*f^3*x - I*d*e*f^2)*e^(d*x + c))*dilog(-I*e^(d*x + c)) - 3*(-I
*d^2*f^3*x^2 - 2*I*d^2*e*f^2*x - I*d^2*e^2*f + (d^2*f^3*x^2 + 2*d^2*e*f^2*
x + d^2*e^2*f)*e^(d*x + c))*dilog(-e^(d*x + c)) - 3*(I*d^2*f^3*x^2 + 2*I*d
^2*e*f^2*x + I*d^2*e^2*f - (d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f)*e^(d*
x + c))*dilog(e^(d*x + c)) - 2*(I*d^3*f^3*x^3 + 3*I*d^3*e*f^2*x^2 + 3*I*d^
3*e^2*f*x + 3*I*c*d^2*e^2*f - 3*I*c^2*d*e*f^2 + I*c^3*f^3)*e^(d*x + c) + (
I*d^3*f^3*x^3 + 3*I*d^3*e*f^2*x^2 + 3*I*d^3*e^2*f*x + I*d^3*e^3 - (d^3*f^3
*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + d^3*e^3)*e^(d*x + c))*log(e^(d*x
+ c) + 1) + 6*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3 - (-I*d^2*e^2*f + 2*I*c*d
*e*f^2 - I*c^2*f^3)*e^(d*x + c))*log(e^(d*x + c) - I) + (-I*d^3*e^3 + 3*I*
c*d^2*e^2*f - 3*I*c^2*d*e*f^2 + I*c^3*f^3 + (d^3*e^3 - 3*c*d^2*e^2*f + 3*c
^2*d*e*f^2 - c^3*f^3)*e^(d*x + c))*log(e^(d*x + c) - 1) + 6*(d^2*f^3*x^2 +
 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3 - (-I*d^2*f^3*x^2 - 2*I*d^2*e*f^2*x
 - 2*I*c*d*e*f^2 + I*c^2*f^3)*e^(d*x + c))*log(I*e^(d*x + c) + 1) + (-I*d^
3*f^3*x^3 - 3*I*d^3*e*f^2*x^2 - 3*I*d^3*e^2*f*x - 3*I*c*d^2*e^2*f + 3*I*c^
2*d*e*f^2 - I*c^3*f^3 + (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + 3
*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f^3)*e^(d*x + c))*log(-e^(d*x + c) + 1)
 - 6*(f^3*e^(d*x + c) - I*f^3)*polylog(4, -e^(d*x + c)) + 6*(f^3*e^(d*x +
c) - I*f^3)*polylog(4, e^(d*x + c)) - 12*(I*f^3*e^(d*x + c) + f^3)*poly...
```

Sympy [F]

$$\int \frac{(e+fx)^3 \text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e^3 \operatorname{csch}(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{f^3x^3 \operatorname{csch}(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{3ef^2x^2 \operatorname{csch}(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{3e^2fx \operatorname{csch}(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)**3*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e**3*csch(c + d*x)/(sinh(c + d*x) - I), x) + Integral(f**3*x*
*3*csch(c + d*x)/(sinh(c + d*x) - I), x) + Integral(3*e*f**2*x**2*csch(c +
 d*x)/(sinh(c + d*x) - I), x) + Integral(3*e**2*f*x*csch(c + d*x)/(sinh(c
+ d*x) - I), x))/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(276) = 552.

Time = 0.25 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.85

$$\int \frac{(e+fx)^3 \text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e^3*(log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d) - 2/((a*e^
(-d*x - c) + I*a)*d)) - 6*I*e^2*f*x/(a*d) - 3*(d*x*log(e^(d*x + c) + 1) +
dilog(-e^(d*x + c)))*e^2*f/(a*d^2) + 3*(d*x*log(-e^(d*x + c) + 1) + dilog(
e^(d*x + c)))*e^2*f/(a*d^2) + 6*I*e^2*f*log(I*e^(d*x + c) + 1)/(a*d^2) + 2
*(f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x)/(a*d*e^(d*x + c) - I*a*d) - 3*(d^2*x^
2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x
+ c)))*e*f^2/(a*d^3) + 3*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d
*x + c)) - 2*polylog(3, e^(d*x + c)))*e*f^2/(a*d^3) + 12*I*(d*x*log(I*e^(d
*x + c) + 1) + dilog(-I*e^(d*x + c)))*e*f^2/(a*d^3) - (d^3*x^3*log(e^(d*x
+ c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c))
 + 6*polylog(4, -e^(d*x + c)))*f^3/(a*d^4) + (d^3*x^3*log(-e^(d*x + c) + 1
) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polyl
og(4, e^(d*x + c)))*f^3/(a*d^4) + 6*I*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*
d*x*dilog(-I*e^(d*x + c)) - 2*polylog(3, -I*e^(d*x + c)))*f^3/(a*d^4) + 2*
(-I*d^3*f^3*x^3 - 3*I*d^3*e*f^2*x^2)/(a*d^4)
```

Giac [F]

$$\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\,\text{csch}\,(dx+c)}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*csch(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(e+f\,x)^3}{\sinh\,(c+d\,x)\,\left(a+a\sinh\,(c+d\,x)\,1\mathrm{i}\right)}\,dx$$

input
```
int((e + f*x)^3/(sinh(c + d*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)^3/(sinh(c + d*x)*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3 \operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{\left(\int \frac{\operatorname{csch}(dx+c)}{\sinh(dx+c)i+1}dx\right)e^3+\left(\int \frac{\operatorname{csch}(dx+c)x^3}{\sinh(dx+c)i+1}dx\right)f^3+3\left(\int \frac{\operatorname{csch}(dx+c)x^2}{\sinh(dx+c)i+1}dx\right)e\,f^2+3\left(\int \frac{\operatorname{csch}(dx+c)x}{\sinh(dx+c)i+1}dx\right)e^2f}{a}$$

input
```
int((f*x+e)^3*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
(int(csch(c + d*x)/(sinh(c + d*x)*i + 1),x)*e**3 + int((csch(c + d*x)*x**3
)/(sinh(c + d*x)*i + 1),x)*f**3 + 3*int((csch(c + d*x)*x**2)/(sinh(c + d*x
)*i + 1),x)*e*f**2 + 3*int((csch(c + d*x)*x)/(sinh(c + d*x)*i + 1),x)*e**2
*f)/a
```

3.206 $\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 224

$$\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i(e+fx)^2}{ad}-\frac{2(e+fx)^2\text{arctanh}\left(e^{c+dx}\right)}{ad}+\frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2}-\frac{2f(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}+\frac{4if^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}+\frac{2f(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2}+\frac{2f^2\,\text{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3}-\frac{2f^2\,\text{PolyLog}\left(3,e^{c+dx}\right)}{ad^3}-\frac{i(e+fx)^2\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}$$

output
```
-I*(f*x+e)^2/a/d-2*(f*x+e)^2*arctanh(exp(d*x+c))/a/d+4*I*f*(f*x+e)*ln(1+I*
exp(d*x+c))/a/d^2-2*f*(f*x+e)*polylog(2,-exp(d*x+c))/a/d^2+4*I*f^2*polylog
(2,-I*exp(d*x+c))/a/d^3+2*f*(f*x+e)*polylog(2,exp(d*x+c))/a/d^2+2*f^2*poly
log(3,-exp(d*x+c))/a/d^3-2*f^2*polylog(3,exp(d*x+c))/a/d^3-I*(f*x+e)^2*tan
h(1/2*c+1/4*I*Pi+1/2*d*x)/a/d
```

Mathematica [A] (verified)

Time = 1.76 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.23

$$\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$= \frac{(e+fx)^2\log\left(1-e^{c+dx}\right)-(e+fx)^2\log\left(1+e^{c+dx}\right)+\frac{2d(e+fx)\left(-id(e+fx)+2(-i+e^c)f\log\left(1-ie^{-c-dx}\right)\right)-4(-i+e^c)f}{d^2(-1-ie^c)}}{}$$

input
```
Integrate[((e + f*x)^2*Csch[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((e + f*x)^2*Log[1 - E^(c + d*x)] - (e + f*x)^2*Log[1 + E^(c + d*x)] + (2*
d*(e + f*x)*((-I)*d*(e + f*x) + 2*(-I + E^c)*f*Log[1 - I*E^(-c - d*x)]) -
4*(-I + E^c)*f^2*PolyLog[2, I*E^(-c - d*x)])/(d^2*(-1 - I*E^c)) - (2*f*(d*
(e + f*x)*PolyLog[2, -E^(c + d*x)] - f*PolyLog[3, -E^(c + d*x)]))/d^2 + (2
*f*(d*(e + f*x)*PolyLog[2, E^(c + d*x)] - f*PolyLog[3, E^(c + d*x)]))/d^2
- ((2*I)*(e + f*x)^2*Sinh[(d*x)/2])/((Cosh[c/2] + I*Sinh[c/2])*(Cosh[(c +
d*x)/2] + I*Sinh[(c + d*x)/2])))/(a*d)
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.05, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.690$, Rules used = {6109, 3042, 26, 3799, 25, 25, 3042, 4670, 3011, 2720, 4672, 26, 3042, 26, 4199, 26, 2620, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$\Big\downarrow \; 6109$$

$$\frac{\int (e+fx)^2\text{csch}(c+dx)dx}{a}-i\int \frac{(e+fx)^2}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \; 3042$$

$$\frac{\int i(e+fx)^2\csc(ic+idx)dx}{a}-i\int\frac{(e+fx)^2}{\sin(ic+idx)a+a}dx$$

$$\Big\downarrow \; 26$$

$$\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}-i\int\frac{(e+fx)^2}{\sin(ic+idx)a+a}dx$$

$$\Big\downarrow \; 3799$$

$$\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}-\frac{i\int-(e+fx)^2\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \; 25$$

$$\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}+\frac{i\int-(e+fx)^2\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \; 25$$

$$\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}-\frac{i\int(e+fx)^2\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \; 3042$$

$$\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}-\frac{i\int(e+fx)^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \; 4670$$

$$\frac{i\left(\frac{2if\int(e+fx)\log(1-e^{c+dx})dx}{d}-\frac{2if\int(e+fx)\log(1+e^{c+dx})dx}{d}+\frac{2i(e+fx)^2\text{arctanh}(e^{c+dx})}{d}\right)}{a}-\frac{i\int(e+fx)^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \; 3011$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int\text{PolyLog}(2,-e^{c+dx})dx}{d}-\frac{(e+fx)\text{PolyLog}(2,-e^{c+dx})}{d}\right)}{d}+\frac{2if\left(\frac{f\int\text{PolyLog}(2,e^{c+dx})dx}{d}-\frac{(e+fx)\text{PolyLog}(2,e^{c+dx})}{d}\right)}{d}+\frac{2i(e+fx)^2\text{ar}}{}\right.}{a}$$

$$\frac{i\int(e+fx)^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \; 2720$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right.}{d}}{a}$$

$$\frac{i\int(e+fx)^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \text{4672}$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right.}{d}}{a}$$

$$\frac{i\left(\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4if\int -i(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right.}{d}}{a}$$

$$\frac{i\left(\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4f\int(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right.}{d}}{a}$$

$$\frac{i\left(\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4f\int -i(e+fx)\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}\right)}{2a}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,e^{\cdots}\right.}{d}\right.}{d}\cdots\right.}{a}$$

$$\frac{i\left(\frac{4if\int(e+fx)\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}$$

$\big\downarrow$ 4199

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,e^{\cdots}\right.}{d}\right.}{d}\cdots\right.}{a}$$

$$\frac{i\left(\frac{4if\left(2i\int\frac{ie^{c+dx}(e+fx)}{1+ie^{c+dx}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}$$

$\big\downarrow$ 26

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,e^{\cdots}\right.}{d}\right.}{d}\cdots\right.}{a}$$

$$\frac{i\left(\frac{4if\left(-2\int\frac{e^{c+dx}(e+fx)}{1+ie^{c+dx}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}$$

$\big\downarrow$ 2620

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,e^{\cdots}\right.}{d}\right.}{d}\cdots\right.}{a}$$

$$\frac{i\left(\frac{4if\left(-2\left(\frac{if\int\log\left(1+ie^{c+dx}\right)dx}{d}-\frac{i(e+fx)\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}$$

$\big\downarrow$ 2715

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right.}{d}}{a}$$

$$\frac{i\left(\frac{4if\left(-2\left(\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{i(e+fx)\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}$$

$\downarrow$ 2838

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right.}{d}}{a}$$

$$\frac{i\left(\frac{4if\left(-2\left(-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{i(e+fx)\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}$$

$\downarrow$ 7143

$$\frac{i\left(\frac{2i(e+fx)^2\operatorname{arctanh}(e^{c+dx})}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right.}{d}}{a}$$

$$\frac{i\left(\frac{4if\left(-2\left(-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{i(e+fx)\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}$$

input
```
Int[((e + f*x)^2*Csch[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(I*(((2*I)*(e + f*x)^2*ArcTanh[E^(c + d*x)])/d - ((2*I)*f*(-(((e + f*x)*Po
lyLog[2, -E^(c + d*x)])/d) + (f*PolyLog[3, -E^(c + d*x)])/d^2))/d + ((2*I)
*f*(-(((e + f*x)*PolyLog[2, E^(c + d*x)])/d) + (f*PolyLog[3, E^(c + d*x)])
/d^2))/d))/a - ((I/2)*(((4*I)*f*(((-1/2*I)*(e + f*x)^2)/f - 2*(((-I)*(e +
f*x)*Log[1 + I*E^(c + d*x)])/d - (I*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2)))
/d + (2*(e + f*x)^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/d))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6109
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csch[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a +
 b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(204) = 408$.

Time = 0.55 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.56

method	result
risch	$-\frac{f^2 \ln(1-e^{dx+c})c^2}{a\,d^3} - \frac{2ef\,\mathrm{polylog}(2,-e^{dx+c})}{a\,d^2} + \frac{2ef\,\mathrm{polylog}(2,e^{dx+c})}{a\,d^2} + \frac{c^2 f^2 \ln(e^{dx+c}-1)}{a\,d^3} - \frac{f^2 \ln(e^{dx+c}+1)x^2}{ad} - 2f^2$ …

input
```
int((f*x+e)^2*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-4*I/a/d^2*e*f*ln(exp(d*x+c))-4*I/a/d^2*f^2*c*x+4*I/a/d^2*f^2*ln(1+I*exp(d
*x+c))*x+4*I/a/d^3*f^2*ln(1+I*exp(d*x+c))*c+4*I/a/d^3*c*f^2*ln(exp(d*x+c))
-1/a/d^3*f^2*ln(1-exp(d*x+c))*c^2-2/a/d^2*e*f*polylog(2,-exp(d*x+c))+2/a/d
^2*e*f*polylog(2,exp(d*x+c))+1/a/d^3*c^2*f^2*ln(exp(d*x+c)-1)-1/a/d*f^2*ln
(exp(d*x+c)+1)*x^2-2/a/d^2*f^2*polylog(2,-exp(d*x+c))*x+1/a/d*f^2*ln(1-exp
(d*x+c))*x^2+2/a/d^2*f^2*polylog(2,exp(d*x+c))*x+4*I/a/d^2*e*f*ln(exp(d*x+
c)-I)-4*I/a/d^3*c*f^2*ln(exp(d*x+c)-I)+2*f^2*polylog(3,-exp(d*x+c))/a/d^3-
2*f^2*polylog(3,exp(d*x+c))/a/d^3-2*I/a/d*f^2*x^2-2*I/a/d^3*f^2*c^2+4*I*f^
2*polylog(2,-I*exp(d*x+c))/a/d^3-1/a/d*e^2*ln(exp(d*x+c)+1)+1/a/d*e^2*ln(e
xp(d*x+c)-1)+2*(f^2*x^2+2*e*f*x+e^2)/d/a/(exp(d*x+c)-I)-2/a/d*e*f*ln(exp(d
*x+c)+1)*x+2/a/d*e*f*ln(1-exp(d*x+c))*x-2/a/d^2*e*c*f*ln(exp(d*x+c)-1)+2/a
/d^2*e*f*ln(1-exp(d*x+c))*c
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 559 vs. 2(194) = 388.

Time = 0.11 (sec) , antiderivative size = 559, normalized size of antiderivative = 2.50

$$\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{2\,d^2e^2-4\,cdef+2\,c^2f^2-4\left(-i\,f^2e^{(dx+c)}-f^2\right)\text{Li}_2\left(-i\,e^{(dx+c)}\right)-2\left(-i\,df^2x-i\,def+\left(df^2x+def\right)e^{(dx\cdots}\right.}{}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(2*d^2*e^2 - 4*c*d*e*f + 2*c^2*f^2 - 4*(-I*f^2*e^(d*x + c) - f^2)*dilog(-I
*e^(d*x + c)) - 2*(-I*d*f^2*x - I*d*e*f + (d*f^2*x + d*e*f)*e^(d*x + c))*d
ilog(-e^(d*x + c)) - 2*(I*d*f^2*x + I*d*e*f - (d*f^2*x + d*e*f)*e^(d*x + c
))*dilog(e^(d*x + c)) - 2*(I*d^2*f^2*x^2 + 2*I*d^2*e*f*x + 2*I*c*d*e*f - I
*c^2*f^2)*e^(d*x + c) + (I*d^2*f^2*x^2 + 2*I*d^2*e*f*x + I*d^2*e^2 - (d^2*
f^2*x^2 + 2*d^2*e*f*x + d^2*e^2)*e^(d*x + c))*log(e^(d*x + c) + 1) + 4*(d*
e*f - c*f^2 - (-I*d*e*f + I*c*f^2)*e^(d*x + c))*log(e^(d*x + c) - I) + (-I
*d^2*e^2 + 2*I*c*d*e*f - I*c^2*f^2 + (d^2*e^2 - 2*c*d*e*f + c^2*f^2)*e^(d*
x + c))*log(e^(d*x + c) - 1) + 4*(d*f^2*x + c*f^2 - (-I*d*f^2*x - I*c*f^2)
*e^(d*x + c))*log(I*e^(d*x + c) + 1) + (-I*d^2*f^2*x^2 - 2*I*d^2*e*f*x - 2
*I*c*d*e*f + I*c^2*f^2 + (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)
*e^(d*x + c))*log(-e^(d*x + c) + 1) + 2*(f^2*e^(d*x + c) - I*f^2)*polylog(
3, -e^(d*x + c)) - 2*(f^2*e^(d*x + c) - I*f^2)*polylog(3, e^(d*x + c)))/(a
*d^3*e^(d*x + c) - I*a*d^3)
```

Sympy [F]

$$\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= -\frac{i\left(\int \frac{e^2\operatorname{csch}(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{f^2x^2\operatorname{csch}(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{2efx\operatorname{csch}(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)**2*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e**2*csch(c + d*x)/(sinh(c + d*x) - I), x) + Integral(f**2*x*
*2*csch(c + d*x)/(sinh(c + d*x) - I), x) + Integral(2*e*f*x*csch(c + d*x)/
(sinh(c + d*x) - I), x))/a
```

Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.55

$$\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= -e^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{ad} - \frac{\log\left(e^{(-dx-c)}-1\right)}{ad} - \frac{2}{(ae^{(-dx-c)}+i\,a)d}\right) - \frac{2i\,f^2x^2}{ad}$$
$$- \frac{4i\,efx}{ad} + \frac{2\,(f^2x^2+2\,efx)}{ade^{(dx+c)}-i\,ad} - \frac{2\left(dx\log\left(e^{(dx+c)}+1\right)+\text{Li}_2\left(-e^{(dx+c)}\right)\right)ef}{ad^2}$$
$$+ \frac{2\left(dx\log\left(-e^{(dx+c)}+1\right)+\text{Li}_2\left(e^{(dx+c)}\right)\right)ef}{ad^2} + \frac{4i\,ef\log\left(i\,e^{(dx+c)}+1\right)}{ad^2}$$
$$- \frac{\left(d^2x^2\log\left(e^{(dx+c)}+1\right)+2\,dx\text{Li}_2\left(-e^{(dx+c)}\right)-2\,\text{Li}_3(-e^{(dx+c)})\right)f^2}{ad^3}$$
$$+ \frac{\left(d^2x^2\log\left(-e^{(dx+c)}+1\right)+2\,dx\text{Li}_2\left(e^{(dx+c)}\right)-2\,\text{Li}_3(e^{(dx+c)})\right)f^2}{ad^3}$$
$$+ \frac{4i\left(dx\log\left(i\,e^{(dx+c)}+1\right)+\text{Li}_2\left(-i\,e^{(dx+c)}\right)\right)f^2}{ad^3}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e^2*(log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d) - 2/((a*e^
(-d*x - c) + I*a)*d)) - 2*I*f^2*x^2/(a*d) - 4*I*e*f*x/(a*d) + 2*(f^2*x^2 +
 2*e*f*x)/(a*d*e^(d*x + c) - I*a*d) - 2*(d*x*log(e^(d*x + c) + 1) + dilog(
-e^(d*x + c)))*e*f/(a*d^2) + 2*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x +
 c)))*e*f/(a*d^2) + 4*I*e*f*log(I*e^(d*x + c) + 1)/(a*d^2) - (d^2*x^2*log(
e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))
*f^2/(a*d^3) + (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) -
 2*polylog(3, e^(d*x + c)))*f^2/(a*d^3) + 4*I*(d*x*log(I*e^(d*x + c) + 1)
+ dilog(-I*e^(d*x + c)))*f^2/(a*d^3)
```

Giac [F]

$$\int \frac{(e+fx)^2 \text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2 \text{csch}\,(dx+c)}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*csch(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^2}{\sinh\,(c+d\,x)\,\left(a+a\sinh\,(c+d\,x)\,1\text{i}\right)}\,dx$$

input
```
int((e + f*x)^2/(sinh(c + d*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)^2/(sinh(c + d*x)*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2 \text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(\int \frac{\text{csch}(dx+c)}{\sinh(dx+c)i+1}dx\right)e^2 + \left(\int \frac{\text{csch}(dx+c)x^2}{\sinh(dx+c)i+1}dx\right)f^2 + 2\left(\int \frac{\text{csch}(dx+c)x}{\sinh(dx+c)i+1}dx\right)ef}{a}$$

input
```
int((f*x+e)^2*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
(int(csch(c + d*x)/(sinh(c + d*x)*i + 1),x)*e**2 + int((csch(c + d*x)*x**2
)/(sinh(c + d*x)*i + 1),x)*f**2 + 2*int((csch(c + d*x)*x)/(sinh(c + d*x)*i
 + 1),x)*e*f)/a
```

3.207 $\int \frac{(e+fx)\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 126

$$\int \frac{(e+fx)\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{2(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{ad} + \frac{2if\log\left(\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2} - \frac{f\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2} + \frac{f\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)}{ad^2} - \frac{i(e+fx)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}$$

output
```
-2*(f*x+e)*arctanh(exp(d*x+c))/a/d+2*I*f*ln(cosh(1/2*c+1/4*I*Pi+1/2*d*x))/
a/d^2-f*polylog(2,-exp(d*x+c))/a/d^2+f*polylog(2,exp(d*x+c))/a/d^2-I*(f*x+
e)*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 257 vs. 2(126) = 252.

Time = 2.37 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.04

$$\int \frac{(e+fx)\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(f(c+dx)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)-2f\arctan\right.}{}$$

input
```
Integrate[((e + f*x)*Csch[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(f*(c + d*x)*(Cosh[(c + d*x)/2]
 + I*Sinh[(c + d*x)/2]) - 2*f*ArcTan[Tanh[(c + d*x)/2]]*(Cosh[(c + d*x)/2]
 + I*Sinh[(c + d*x)/2]) + I*f*Log[Cosh[c + d*x]]*(Cosh[(c + d*x)/2] + I*Si
nh[(c + d*x)/2]) + (d*(e + f*x)*(Log[1 - E^(c + d*x)] - Log[1 + E^(c + d*x
)]) - f*PolyLog[2, -E^(c + d*x)] + f*PolyLog[2, E^(c + d*x)])*(Cosh[(c + d
*x)/2] + I*Sinh[(c + d*x)/2]) - (2*I)*d*(e + f*x)*Sinh[(c + d*x)/2]))/(d^2
*(a + I*a*Sinh[c + d*x]))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.05, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6109, 3042, 26, 3799, 25, 25, 3042, 4670, 2715, 2838, 4672, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6109}$$

$$\frac{\int (e+fx)\text{csch}(c+dx)dx}{a}-i\int \frac{e+fx}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int i(e+fx)\csc(ic+idx)dx}{a}-i\int \frac{e+fx}{\sin(ic+idx)a+a}dx$$

$$\Big\downarrow \text{ 26}$$

$$\frac{i\int (e+fx)\csc(ic+idx)dx}{a}-i\int \frac{e+fx}{\sin(ic+idx)a+a}dx$$

$$\Big\downarrow \text{ 3799}$$

$$\frac{i\int (e+fx)\csc(ic+idx)dx}{a}-\frac{i\int -\left((e+fx)\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)\right)dx}{2a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{i\int (e+fx)\csc(ic+idx)dx}{a}+\frac{i\int -\left((e+fx)\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)dx}{2a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{i\int (e+fx)\csc(ic+idx)dx}{a}-\frac{i\int (e+fx)\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{i\int (e+fx)\csc(ic+idx)dx}{a}-\frac{i\int (e+fx)\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \text{ 4670}$$

$$\frac{i\left(\frac{if\int \log(1-e^{c+dx})dx}{d}-\frac{if\int \log(1+e^{c+dx})dx}{d}+\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}\right)}{a}-\frac{i\int (e+fx)\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{i\left(\frac{if\int e^{-c-dx}\log(1-e^{c+dx})de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log(1+e^{c+dx})de^{c+dx}}{d^2}+\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}\right)}{a}-\frac{i\int (e+fx)\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)}{a}-\frac{i\int (e+fx)\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \text{ 4672}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)}{a}-\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2if\int-i\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)}{a}-\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2f\int\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)}{a}-\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2f\int-i\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}\right)}{2a}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)}{a}-\frac{i\left(\frac{2if\int\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}+\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}$$

$$\Big\downarrow \text{3956}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)}{a}-\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4f\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{d^2}\right)}{2a}$$

input
```
Int[((e + f*x)*Csch[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(I*(((2*I)*(e + f*x)*ArcTanh[E^(c + d*x)])/d + (I*f*PolyLog[2, -E^(c + d*x
)])/d^2 - (I*f*PolyLog[2, E^(c + d*x)])/d^2))/a - ((I/2)*((-4*f*Log[Cosh[c
/2 + (I/4)*Pi + (d*x)/2]])/d^2 + (2*(e + f*x)*Tanh[c/2 + (I/4)*Pi + (d*x)/
2])/d))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6109
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csch[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a +
 b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.67

method	result
risch	$\frac{2fx+2e}{da\left(\mathrm{e}^{dx+c}-i\right)}+\frac{f\ln\left(1-\mathrm{e}^{dx+c}\right)c}{a\,d^{2}}+\frac{f\ln\left(1-\mathrm{e}^{dx+c}\right)x}{ad}-\frac{f\,\mathrm{polylog}\left(2,-\mathrm{e}^{dx+c}\right)}{a\,d^{2}}+\frac{f\,\mathrm{polylog}\left(2,\mathrm{e}^{dx+c}\right)}{a\,d^{2}}-\frac{e\ln\left(\mathrm{e}^{dx+c}+1\right)}{ad}+$ e l

input
```
int((f*x+e)*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*(f*x+e)/d/a/(exp(d*x+c)-I)+1/a/d^2*f*ln(1-exp(d*x+c))*c+1/a/d*f*ln(1-exp
(d*x+c))*x-f*polylog(2,-exp(d*x+c))/a/d^2+f*polylog(2,exp(d*x+c))/a/d^2-1/
a/d*e*ln(exp(d*x+c)+1)+1/a/d*e*ln(exp(d*x+c)-1)-1/a/d^2*c*f*ln(exp(d*x+c)-
1)-1/a/d*f*ln(exp(d*x+c)+1)*x+2*I*f/a/d^2*ln(exp(d*x+c)-I)-2*I/a/d^2*f*ln(
exp(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(104) = 208$.

Time = 0.11 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.67

$$\int \frac{(e+fx)\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{-2i\,dfxe^{(dx+c)} + 2\,de - \left(fe^{(dx+c)} - i\,f\right)\text{Li}_2\left(-e^{(dx+c)}\right) + \left(fe^{(dx+c)} - i\,f\right)\text{Li}_2\left(e^{(dx+c)}\right) + \left(i\,dfx + i\,de - (d\right.}{}$$

input
```
integrate((f*x+e)*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(-2*I*d*f*x*e^(d*x + c) + 2*d*e - (f*e^(d*x + c) - I*f)*dilog(-e^(d*x + c)
) + (f*e^(d*x + c) - I*f)*dilog(e^(d*x + c)) + (I*d*f*x + I*d*e - (d*f*x +
 d*e)*e^(d*x + c))*log(e^(d*x + c) + 1) - 2*(-I*f*e^(d*x + c) - f)*log(e^(
d*x + c) - I) + (-I*d*e + I*c*f + (d*e - c*f)*e^(d*x + c))*log(e^(d*x + c)
 - 1) + (-I*d*f*x - I*c*f + (d*f*x + c*f)*e^(d*x + c))*log(-e^(d*x + c) +
1))/(a*d^2*e^(d*x + c) - I*a*d^2)
```

Sympy [F]

$$\int \frac{(e+fx)\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e\,\text{csch}\,(c+dx)}{\sinh\,(c+dx)-i}\,dx + \int \frac{fx\,\text{csch}\,(c+dx)}{\sinh\,(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e*csch(c + d*x)/(sinh(c + d*x) - I), x) + Integral(f*x*csch(c
 + d*x)/(sinh(c + d*x) - I), x))/a
```

Maxima [F]

$$\int \frac{(e+fx)\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(fx+e)\,\text{csch}\,(dx+c)}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
2*f*(x*e^(d*x + c)/(I*a*d*e^(d*x + c) + a*d) + I*log((e^(d*x + c) - I)*e^(
-c))/(a*d^2) + integrate(1/2*x/(a*e^(d*x + c) + a), x) + integrate(1/2*x/(
a*e^(d*x + c) - a), x)) - e*(log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c
) - 1)/(a*d) - 2/((a*e^(-d*x - c) + I*a)*d))
```

Giac [F]

$$\int \frac{(e+fx)\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(fx+e)\,\text{csch}\,(dx+c)}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*csch(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{e+f\,x}{\sinh\,(c+d\,x)\,\left(a+a\sinh\,(c+d\,x)\,1\mathrm{i}\right)}\,dx$$

input
```
int((e + f*x)/(sinh(c + d*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)/(sinh(c + d*x)*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(\int \frac{\text{csch}(dx+c)}{\sinh(dx+c)i+1}dx\right)e+\left(\int \frac{\text{csch}(dx+c)x}{\sinh(dx+c)i+1}dx\right)f}{a}$$

input
```
int((f*x+e)*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
(int(csch(c + d*x)/(sinh(c + d*x)*i + 1),x)*e + int((csch(c + d*x)*x)/(sin
h(c + d*x)*i + 1),x)*f)/a
```

3.208 $\int \frac{\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 41

$$\int \frac{\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx=-\frac{\operatorname{arctanh}(\cosh(c+dx))}{ad}+\frac{\cosh(c+dx)}{d(a+ia\sinh(c+dx))}$$

output
```
-arctanh(cosh(d*x+c))/a/d+cosh(d*x+c)/d/(a+I*a*sinh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.27

$$\int \frac{\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{\operatorname{sech}(c+dx)\left(-1+\operatorname{arctanh}\left(\sqrt{\cosh^2(c+dx)}\right)\sqrt{\cosh^2(c+dx)}+i\sinh(c+dx)\right)}{ad}$$

input
```
Integrate[Csch[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

output
```
-((Sech[c + d*x]*(-1 + ArcTanh[Sqrt[Cosh[c + d*x]^2]]*Sqrt[Cosh[c + d*x]^2
] + I*Sinh[c + d*x]))/(a*d))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 26, 3226, 26, 3042, 26, 3127, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\sin(ic+idx)(a+a\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\sin(ic+idx)(\sin(ic+idx)a+a)}dx$$

$$\Big\downarrow \text{3226}$$

$$i\left(\frac{\int -i\operatorname{csch}(c+dx)dx}{a}-\int \frac{1}{i\sinh(c+dx)a+a}dx\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\int \frac{1}{i\sinh(c+dx)a+a}dx-\frac{i\int \operatorname{csch}(c+dx)dx}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(-\int \frac{1}{\sin(ic+idx)a+a}dx-\frac{i\int i\csc(ic+idx)dx}{a}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{\int \csc(ic+idx)dx}{a}-\int \frac{1}{\sin(ic+idx)a+a}dx\right)$$

$$\Big\downarrow \text{3127}$$

$$i\left(\frac{\int \csc(ic+idx)dx}{a}-\frac{i\cosh(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow \text{4257}$$

$$i\left(\frac{i\text{arctanh}(\cosh(c+dx))}{ad}-\frac{i\cosh(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

input
```
Int[Csch[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

output
```
I*((I*ArcTanh[Cosh[c + d*x]])/(a*d) - (I*Cosh[c + d*x])/(d*(a + I*a*Sinh[c
 + d*x])))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3226
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]),
 x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2i}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}}{da}$	36
default	$\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2i}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}}{da}$	36
risch	$\frac{2}{da\left(\mathrm{e}^{dx+c}-i\right)}-\frac{\ln\left(\mathrm{e}^{dx+c}+1\right)}{da}+\frac{\ln\left(\mathrm{e}^{dx+c}-1\right)}{da}$	54
parallelrisch	$\frac{\left(i-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{da\left(i-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	61

input
```
int(csch(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(ln(tanh(1/2*d*x+1/2*c))-2*I/(-I+tanh(1/2*d*x+1/2*c)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.39

$$\int \frac{\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{\left(e^{(dx+c)}-i\right)\log\left(e^{(dx+c)}+1\right)-\left(e^{(dx+c)}-i\right)\log\left(e^{(dx+c)}-1\right)-2}{ade^{(dx+c)}-i\,ad}$$

input
```
integrate(csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-((e^(d*x + c) - I)*log(e^(d*x + c) + 1) - (e^(d*x + c) - I)*log(e^(d*x +
c) - 1) - 2)/(a*d*e^(d*x + c) - I*a*d)
```

Sympy [F]

$$\int \frac{\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int \frac{\operatorname{csch}(c+dx)}{\sinh(c+dx)-i}\,dx}{a}$$

input
```
integrate(csch(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(csch(c + d*x)/(sinh(c + d*x) - I), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.51

$$\int \frac{\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{\log\left(e^{(-dx-c)}+1\right)}{ad} + \frac{\log\left(e^{(-dx-c)}-1\right)}{ad} + \frac{2}{(ae^{(-dx-c)}+i\,a)d}$$

input
```
integrate(csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-log(e^(-d*x - c) + 1)/(a*d) + log(e^(-d*x - c) - 1)/(a*d) + 2/((a*e^(-d*x
 - c) + I*a)*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.17

$$\int \frac{\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{\frac{\log\left(e^{(dx+c)}+1\right)}{a} - \frac{\log\left(e^{(dx+c)}-1\right)}{a} - \frac{2}{a\left(e^{(dx+c)}-i\right)}}{d}$$

input
```
integrate(csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-(log(e^(d*x + c) + 1)/a - log(e^(d*x + c) - 1)/a - 2/(a*(e^(d*x + c) - I)
))/d
```

Mupad [B] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.37

$$\int \frac{\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-a^2\,d^2}}{a\,d}\right)}{\sqrt{-a^2\,d^2}}+\frac{2}{a\,d\,\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)}$$

input
```
int(1/(sinh(c + d*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
2/(a*d*(exp(c + d*x) - 1i)) - (2*atan((exp(d*x)*exp(c)*(-a^2*d^2)^(1/2))/(
a*d)))/(-a^2*d^2)^(1/2)
```

Reduce [F]

$$\int \frac{\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\int \frac{\operatorname{csch}(dx+c)}{\sinh(dx+c)i+1}dx}{a}$$

input
```
int(csch(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(csch(c + d*x)/(sinh(c + d*x)*i + 1),x)/a
```

3.209 $\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=\operatorname{Int}\left(\frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 34.68 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

input
```
Integrate[Csch[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Integrate[Csch[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$
$$\Big\downarrow \ 6111$$
$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}dx$$

input
```
Int[Csch[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{csch}(dx+c)}{(fx+e)(a+ia\sinh(dx+c))}dx$$

input
```
int(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 235, normalized size of antiderivative = 8.10

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)}{(fx+e)(i\,a\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
((-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))*integral(2*((d*f*x
 + d*e + f)*e^(2*d*x + 2*c) - (I*d*f*x + I*d*e)*e^(d*x + c) - f)/(I*a*d*f^
2*x^2 + 2*I*a*d*e*f*x + I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*
e^(3*d*x + 3*c) + (-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2)*e^(2*d*x +
2*c) - (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c)), x) + 2)/(-I*a*d
*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))
```

Sympy [N/A]

Not integrable

Time = 19.33 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=-\frac{i\int \frac{\operatorname{csch}\left(c+dx\right)}{e\sinh\left(c+dx\right)-ie+fx\sinh\left(c+dx\right)-ifx}\,dx}{a}$$

input
```
integrate(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(csch(c + d*x)/(e*sinh(c + d*x) - I*e + f*x*sinh(c + d*x) - I*f
*x), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 169, normalized size of antiderivative = 5.83

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
2*f*integrate(1/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x^2
*e^c + 2*a*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)), x) + 2/(-I*a*d*f*x - I*a*d
*e + (a*d*f*x*e^c + a*d*e*e^c)*e^(d*x)) + 2*integrate(1/2/(a*f*x + a*e + (
a*f*x*e^c + a*e*e^c)*e^(d*x)), x) + 2*integrate(-1/2/(a*f*x + a*e - (a*f*x
*e^c + a*e*e^c)*e^(d*x)), x)
```

Giac [N/A]

Not integrable

Time = 20.48 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(csch(d*x + c)/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.57 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{1}{\sinh(c+d\,x)\,(e+f\,x)\,(a+a\sinh(c+d\,x)\,1\mathrm{i})}\,dx$$

input
```
int(1/(sinh(c + d*x)*(e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(1/(sinh(c + d*x)*(e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \frac{\int \frac{\operatorname{csch}(dx+c)}{\sinh(dx+c)ei+\sinh(dx+c)fix+e+fx}dx}{a}$$

input
```
int(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(csch(c + d*x)/(sinh(c + d*x)*e*i + sinh(c + d*x)*f*i*x + e + f*x),x)/a
```

3.210 $\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(csch(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 41.98 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

input
```
Integrate[Csch[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Integrate[Csch[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$
$$\Big\downarrow \; 6111$$
$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}dx$$

input
```
Int[Csch[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{csch}(dx+c)}{(fx+e)^2\,(a+ia\sinh(dx+c))}dx$$

input
```
int(csch(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 341, normalized size of antiderivative = 11.76

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)}{(fx+e)^2\left(i\,a\sinh\left(dx+c\right)+a\right)}\,dx$$

input
```
integrate(csch(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
((-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x
+ a*d*e^2)*e^(d*x + c))*integral(2*((d*f*x + d*e + 2*f)*e^(2*d*x + 2*c) -
(I*d*f*x + I*d*e)*e^(d*x + c) - 2*f)/(I*a*d*f^3*x^3 + 3*I*a*d*e*f^2*x^2 +
3*I*a*d*e^2*f*x + I*a*d*e^3 + (a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f
*x + a*d*e^3)*e^(3*d*x + 3*c) + (-I*a*d*f^3*x^3 - 3*I*a*d*e*f^2*x^2 - 3*I*
a*d*e^2*f*x - I*a*d*e^3)*e^(2*d*x + 2*c) - (a*d*f^3*x^3 + 3*a*d*e*f^2*x^2
+ 3*a*d*e^2*f*x + a*d*e^3)*e^(d*x + c)), x) + 2)/(-I*a*d*f^2*x^2 - 2*I*a*d
*e*f*x - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 277, normalized size of antiderivative = 9.55

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)}{(fx+e)^2(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
4*f*integrate(1/(-I*a*d*f^3*x^3 - 3*I*a*d*e*f^2*x^2 - 3*I*a*d*e^2*f*x - I*
a*d*e^3 + (a*d*f^3*x^3*e^c + 3*a*d*e*f^2*x^2*e^c + 3*a*d*e^2*f*x*e^c + a*d
*e^3*e^c)*e^(d*x)), x) + 2/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (
a*d*f^2*x^2*e^c + 2*a*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)) + 2*integrate(1/
2/(a*f^2*x^2 + 2*a*e*f*x + a*e^2 + (a*f^2*x^2*e^c + 2*a*e*f*x*e^c + a*e^2*
e^c)*e^(d*x)), x) + 2*integrate(-1/2/(a*f^2*x^2 + 2*a*e*f*x + a*e^2 - (a*f
^2*x^2*e^c + 2*a*e*f*x*e^c + a*e^2*e^c)*e^(d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 1.86 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{1}{\sinh\left(c+d\,x\right)\,{\left(e+f\,x\right)}^2\,\left(a+a\,\sinh\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

input
```
int(1/(sinh(c + d*x)*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(1/(sinh(c + d*x)*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.34

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \frac{\int \frac{\operatorname{csch}(dx+c)}{\sinh(dx+c)e^2i+2\sinh(dx+c)efix+\sinh(dx+c)f^2i\,x^2+e^2+2efx+f^2x^2}dx}{a}$$

input
```
int(csch(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(csch(c + d*x)/(sinh(c + d*x)*e**2*i + 2*sinh(c + d*x)*e*f*i*x + sinh(c
 + d*x)*f**2*i*x**2 + e**2 + 2*e*f*x + f**2*x**2),x)/a
```

3.211 $\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 419

$$\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{2(e+fx)^3}{ad}+\frac{2i(e+fx)^3\text{arctanh}\left(e^{c+dx}\right)}{ad} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{6f(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2} + \frac{3f(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{ad^2} + \frac{3if(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2} + \frac{12f^2(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3} - \frac{3if(e+fx)^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2} + \frac{3f^2(e+fx)\,\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{ad^3} - \frac{6if^2(e+fx)\,\text{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3} - \frac{12f^3\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^4} + \frac{6if^2(e+fx)\,\text{PolyLog}\left(3,e^{c+dx}\right)}{ad^3} - \frac{3f^3\,\text{PolyLog}\left(3,e^{2(c+dx)}\right)}{2ad^4} + \frac{6if^3\,\text{PolyLog}\left(4,-e^{c+dx}\right)}{ad^4} - \frac{6if^3\,\text{PolyLog}\left(4,e^{c+dx}\right)}{ad^4} - \frac{(e+fx)^3\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}$$

output
```
-2*(f*x+e)^3/a/d+6*I*f^2*(f*x+e)*polylog(3,exp(d*x+c))/a/d^3-(f*x+e)^3*cot
h(d*x+c)/a/d+6*f*(f*x+e)^2*ln(1+I*exp(d*x+c))/a/d^2+3*f*(f*x+e)^2*ln(1-exp
(2*d*x+2*c))/a/d^2-6*I*f^2*(f*x+e)*polylog(3,-exp(d*x+c))/a/d^3+12*f^2*(f*
x+e)*polylog(2,-I*exp(d*x+c))/a/d^3-6*I*f^3*polylog(4,exp(d*x+c))/a/d^4+3*
f^2*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a/d^3+6*I*f^3*polylog(4,-exp(d*x+c))
/a/d^4-12*f^3*polylog(3,-I*exp(d*x+c))/a/d^4+2*I*(f*x+e)^3*arctanh(exp(d*x
+c))/a/d-3/2*f^3*polylog(3,exp(2*d*x+2*c))/a/d^4-3*I*f*(f*x+e)^2*polylog(2
,exp(d*x+c))/a/d^2+3*I*f*(f*x+e)^2*polylog(2,-exp(d*x+c))/a/d^2-(f*x+e)^3*
tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a/d
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1205 vs. 2(419) = 838.

Time = 8.47 (sec) , antiderivative size = 1205, normalized size of antiderivative = 2.88

$$\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Csch[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-6*I)*E^c*f*((e + f*x)^3/(3*E^c*f) + ((I + E^(-c))*(e + f*x)^2*Log[1 - I
*E^(-c - d*x)])/d - ((2*I)*(-I + E^c)*f*(d*(e + f*x)*PolyLog[2, I*E^(-c -
d*x)] + f*PolyLog[3, I*E^(-c - d*x)]))/(d^3*E^c)))/(a*d*(-I + E^c)) + (I*d
^3*e^2*(-1 + E^(2*c))*(d*e + (3*I)*f)*x + d^3*e^2*(1 - E^(2*c))*(I*d*e + 3
*f)*x - 2*d^3*(e + f*x)^3 + 3*d^2*e*(-1 + E^(2*c))*f*((-I)*d*e + 2*f)*x*Lo
g[1 - E^(-c - d*x)] + 3*d^2*(-1 + E^(2*c))*f^2*((-I)*d*e + f)*x^2*Log[1 -
E^(-c - d*x)] - I*d^3*(-1 + E^(2*c))*f^3*x^3*Log[1 - E^(-c - d*x)] + 3*d^2
*e*(-1 + E^(2*c))*f*(I*d*e + 2*f)*x*Log[1 + E^(-c - d*x)] + 3*d^2*(-1 + E^
(2*c))*f^2*(I*d*e + f)*x^2*Log[1 + E^(-c - d*x)] + I*d^3*(-1 + E^(2*c))*f^
3*x^3*Log[1 + E^(-c - d*x)] + d^2*e^2*(-1 + E^(2*c))*((-I)*d*e + 3*f)*Log[
1 - E^(c + d*x)] + d^2*e^2*(-1 + E^(2*c))*(I*d*e + 3*f)*Log[1 + E^(c + d*x
)] + 3*d*e*(1 - E^(2*c))*f*(I*d*e + 2*f)*PolyLog[2, -E^(-c - d*x)] + 6*d*(
1 - E^(2*c))*f^2*(I*d*e + f)*x*PolyLog[2, -E^(-c - d*x)] - (3*I)*d^2*(-1 +
 E^(2*c))*f^3*x^2*PolyLog[2, -E^(-c - d*x)] + (3*I)*d*e*(-1 + E^(2*c))*(d*
e + (2*I)*f)*f*PolyLog[2, E^(-c - d*x)] + (6*I)*d*(-1 + E^(2*c))*(d*e + I*
f)*f^2*x*PolyLog[2, E^(-c - d*x)] + (3*I)*d^2*(-1 + E^(2*c))*f^3*x^2*PolyL
og[2, E^(-c - d*x)] - 6*(-1 + E^(2*c))*f^2*(I*d*e + f)*PolyLog[3, -E^(-c -
 d*x)] - (6*I)*d*(-1 + E^(2*c))*f^3*x*PolyLog[3, -E^(-c - d*x)] + (6*I)*(-
1 + E^(2*c))*(d*e + I*f)*f^2*PolyLog[3, E^(-c - d*x)] + (6*I)*d*(-1 + E^(2
*c))*f^3*x*PolyLog[3, E^(-c - d*x)] - (6*I)*(-1 + E^(2*c))*f^3*PolyLog[...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6109}$$

$$\frac{\int (e+fx)^3\text{csch}^2(c+dx)dx}{a}-i\int \frac{(e+fx)^3\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int -(e+fx)^3\csc(ic+idx)^2dx}{a}-i\int \frac{(e+fx)^3\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int (e+fx)^3 \csc(ic+idx)^2dx}{a}-i\int \frac{(e+fx)^3\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$\big\downarrow$ 4672

$$-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{3if\int -i(e+fx)^2\coth(c+dx)dx}{d}}{a}-i\int \frac{(e+fx)^3\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$\big\downarrow$ 26

$$-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{3f\int (e+fx)^2\coth(c+dx)dx}{d}}{a}-i\int \frac{(e+fx)^3\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$\big\downarrow$ 3042

$$-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{3f\int -i(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}-i\int \frac{(e+fx)^3\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$\big\downarrow$ 26

$$-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\int (e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}}{a}-i\int \frac{(e+fx)^3\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$\big\downarrow$ 4201

$$-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\int \frac{e^{2c+2dx-i\pi}(e+fx)^2}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}-i\int \frac{(e+fx)^3\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$\big\downarrow$ 2620

$$-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int (e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}-i\int \frac{(e+fx)^3\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$\big\downarrow$ 3011

$$-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int \text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{2d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+\cdots}{3\cdots}}{d}}{a} \cdots$$
$$i\int \frac{(e+fx)^3\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$\big\downarrow$ 2720

$$-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}(2,-}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$$i\int\frac{(e+fx)^3\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$\Big\downarrow$ 6109

$$-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}(2,-}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$$i\left(\frac{\int(e+fx)^3\text{csch}(c+dx)dx}{a}-i\int\frac{(e+fx)^3}{i\sinh(c+dx)a+a}dx\right)$$

$\Big\downarrow$ 3042

$$-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}(2,-}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$$i\left(\frac{\int i(e+fx)^3\csc(ic+idx)dx}{a}-i\int\frac{(e+fx)^3}{\sin(ic+idx)a+a}dx\right)$$

$\Big\downarrow$ 26

$$-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}(2,-}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$$i\left(\frac{i\int(e+fx)^3\csc(ic+idx)dx}{a}-i\int\frac{(e+fx)^3}{\sin(ic+idx)a+a}dx\right)$$

$\Big\downarrow$ 3799

$$-\frac{-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$$i\left(\frac{i\int(e+fx)^3\csc(ic+idx)dx}{a}-\frac{i\int-(e+fx)^3\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)dx}{2a}\right)$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$$i\left(\frac{i\int(e+fx)^3\csc(ic+idx)dx}{a}+\frac{i\int-(e+fx)^3\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}\right)$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$$i\left(\frac{i\int(e+fx)^3\csc(ic+idx)dx}{a}-\frac{i\int(e+fx)^3\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}\right)$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$$i\left(\frac{i\int(e+fx)^3\csc(ic+idx)dx}{a}-\frac{i\int(e+fx)^3\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}\right)$$

$$\Big\downarrow \; 4670$$

$$-i\left(\frac{i\left(\frac{3if\int(e+fx)^2\log\left(1-e^{c+dx}\right)dx}{d}-\frac{3if\int(e+fx)^2\log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)^3\text{arctanh}\left(e^{c+dx}\right)}{d}\right)}{a}-\frac{i\int(e+fx)^3\csc\left(\frac{ic}{2}+\cdots\right.}{2a}\cdots\right.$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2\cdots}\right)}{2d}\cdots\right)}{d}\cdots\right)\cdots\right)}{d}\cdots}{a}$$

$\downarrow$ 3011

$$-i\left(\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,e^{c+\cdots}\right)}{d}\cdots\right)}{d}\cdots\right)}{a}\cdots\right.$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2\cdots}\right)}{2d}\cdots\right)}{d}\cdots\right)\cdots\right)}{d}\cdots}{a}$$

$\downarrow$ 4672

$$-i\left(\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,e^{c+\cdots}\right)}{d}\cdots\right)}{d}\cdots\right)}{a}\cdots\right.$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2\cdots}\right)}{2d}\cdots\right)}{d}\cdots\right)\cdots\right)}{d}\cdots}{a}$$

$\downarrow$ 26

$$-i\left(\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+\ldots}\right.}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2\ldots}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$\downarrow$ 3042

$$-i\left(\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+\ldots}\right.}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2\ldots}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$\downarrow$ 26

$$-i\left(\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+\ldots}\right.}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2\ldots}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$\downarrow$ 4199

$$-i\left(\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+\cdots}\right.}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2\cdots}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$\downarrow$ 26

$$-i\left(\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+\cdots}\right.}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2\cdots}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$\downarrow$ 2620

$$-i\left(\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+\cdots}\right.}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2\cdots}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$\downarrow$ 3011

$$-i\left(\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+\right.}}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2\right.}}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$\downarrow$ 2720

$$-i\left(\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+\right.}}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2\right.}}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$\downarrow$ 7143

$$-i\left(\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+}\right.}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\operatorname{PolyLog}\left(3,-e^{2c+2dx-i\pi}\right)}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}$$

input
```
Int[((e + f*x)^3*Csch[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1603 vs. $2(390) = 780$.

Time = 1.10 (sec) , antiderivative size = 1604, normalized size of antiderivative = 3.83

method	result	size
risch	Expression too large to display	1604

input
```
int((f*x+e)^3*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-4/a/d*f^3*x^3+8/a/d^4*c^3*f^3+6*I*f^3*polylog(4,-exp(d*x+c))/a/d^4+I/a/d*
f^3*ln(exp(d*x+c)+1)*x^3+12/a/d^2*e*f^2*ln(1+I*exp(d*x+c))*x-24/a/d^2*e*f^
2*c*x+6/a/d^2*e*f^2*ln(exp(d*x+c)+1)*x+6/a/d^2*e*f^2*ln(1-exp(d*x+c))*x+6/
a/d^3*e*f^2*ln(1-exp(d*x+c))*c+12/a/d^3*e*f^2*ln(1+I*exp(d*x+c))*c+24/a/d^
3*c*e*f^2*ln(exp(d*x+c))-6/a/d^3*c*e*f^2*ln(1+exp(2*d*x+2*c))-6/a/d^3*c*e*
f^2*ln(exp(d*x+c)-1)+I/a/d^4*c^3*f^3*ln(exp(d*x+c)-1)-2*I*(f^3*x^3*exp(2*d
*x+2*c)+3*e*f^2*x^2*exp(2*d*x+2*c)+3*e^2*f*x*exp(2*d*x+2*c)-2*x^3*f^3-I*ex
p(d*x+c)*f^3*x^3+e^3*exp(2*d*x+2*c)-6*e*f^2*x^2-3*I*exp(d*x+c)*e*f^2*x^2-6
*e^2*f*x-3*I*exp(d*x+c)*e^2*f*x-2*e^3-I*exp(d*x+c)*e^3)/(exp(2*d*x+2*c)-1)
/(exp(d*x+c)-I)/d/a-12/a/d*e*f^2*x^2+I/a/d*e^3*ln(exp(d*x+c)+1)+3/a/d^2*e^
2*f*ln(1+exp(2*d*x+2*c))+3/a/d^2*e^2*f*ln(exp(d*x+c)+1)-6*I*f^3*polylog(4,
exp(d*x+c))/a/d^4-I/a/d*f^3*ln(1-exp(d*x+c))*x^3-3*I/a/d^2*f^3*polylog(2,e
xp(d*x+c))*x^2+6*I/a/d^3*f^3*polylog(3,exp(d*x+c))*x+3*I/a/d^2*f^3*polylog
(2,-exp(d*x+c))*x^2-6*I/a/d^3*f^3*polylog(3,-exp(d*x+c))*x+6*I/a/d^4*c^2*f
^3*arctan(exp(d*x+c))-I/a/d^4*c^3*f^3*ln(1-exp(d*x+c))+3*I/a/d^2*e^2*f*pol
ylog(2,-exp(d*x+c))-3*I/a/d^2*e^2*f*polylog(2,exp(d*x+c))+6*I/a/d^2*e^2*f*
arctan(exp(d*x+c))-6*I/a/d^3*e*f^2*polylog(3,-exp(d*x+c))+6*I/a/d^3*e*f^2*
polylog(3,exp(d*x+c))-6*f^3*polylog(3,-exp(d*x+c))/a/d^4-6*f^3*polylog(3,e
xp(d*x+c))/a/d^4-12*f^3*polylog(3,-I*exp(d*x+c))/a/d^4+3/a/d^2*e^2*f*ln(ex
p(d*x+c)-1)+12/a/d^3*c^2*f^3*x-3/a/d^4*c^2*f^3*ln(1-exp(d*x+c))-6/a/d^4...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2562 vs. $2(375) = 750$.

Time = 0.14 (sec) , antiderivative size = 2562, normalized size of antiderivative = 6.11

$$\int \frac{(e+fx)^3 \operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
(4*I*d^3*e^3 - 12*I*c*d^2*e^2*f + 12*I*c^2*d*e*f^2 - 4*I*c^3*f^3 - 12*(-I*
d*f^3*x - I*d*e*f^2 - (d*f^3*x + d*e*f^2)*e^(3*d*x + 3*c) + (I*d*f^3*x + I
*d*e*f^2)*e^(2*d*x + 2*c) + (d*f^3*x + d*e*f^2)*e^(d*x + c))*dilog(-I*e^(d
*x + c)) - 3*(d^2*f^3*x^2 + d^2*e^2*f - 2*I*d*e*f^2 + 2*(d^2*e*f^2 - I*d*f
^3)*x + (-I*d^2*f^3*x^2 - I*d^2*e^2*f - 2*d*e*f^2 + 2*(-I*d^2*e*f^2 - d*f^
3)*x)*e^(3*d*x + 3*c) - (d^2*f^3*x^2 + d^2*e^2*f - 2*I*d*e*f^2 + 2*(d^2*e*
f^2 - I*d*f^3)*x)*e^(2*d*x + 2*c) + (I*d^2*f^3*x^2 + I*d^2*e^2*f + 2*d*e*f
^2 + 2*(I*d^2*e*f^2 + d*f^3)*x)*e^(d*x + c))*dilog(-e^(d*x + c)) + 3*(d^2*
f^3*x^2 + d^2*e^2*f + 2*I*d*e*f^2 + 2*(d^2*e*f^2 + I*d*f^3)*x - (I*d^2*f^3
*x^2 + I*d^2*e^2*f - 2*d*e*f^2 + 2*(I*d^2*e*f^2 - d*f^3)*x)*e^(3*d*x + 3*c
) - (d^2*f^3*x^2 + d^2*e^2*f + 2*I*d*e*f^2 + 2*(d^2*e*f^2 + I*d*f^3)*x)*e^
(2*d*x + 2*c) - (-I*d^2*f^3*x^2 - I*d^2*e^2*f + 2*d*e*f^2 + 2*(-I*d^2*e*f^
2 + d*f^3)*x)*e^(d*x + c))*dilog(e^(d*x + c)) - 4*(d^3*f^3*x^3 + 3*d^3*e*f
^2*x^2 + 3*d^3*e^2*f*x + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f^3)*e^(3*d*x
 + 3*c) - 2*(-I*d^3*f^3*x^3 - 3*I*d^3*e*f^2*x^2 - 3*I*d^3*e^2*f*x + I*d^3*
e^3 - 6*I*c*d^2*e^2*f + 6*I*c^2*d*e*f^2 - 2*I*c^3*f^3)*e^(2*d*x + 2*c) + 2
*(d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x - d^3*e^3 + 6*c*d^2*e^2*f
- 6*c^2*d*e*f^2 + 2*c^3*f^3)*e^(d*x + c) - (d^3*f^3*x^3 + d^3*e^3 - 3*I*d^
2*e^2*f + 3*(d^3*e*f^2 - I*d^2*f^3)*x^2 + 3*(d^3*e^2*f - 2*I*d^2*e*f^2)*x
- (I*d^3*f^3*x^3 + I*d^3*e^3 + 3*d^2*e^2*f - 3*(-I*d^3*e*f^2 - d^2*f^3)...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*csch(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 939 vs. 2(375) = 750.

Time = 0.29 (sec) , antiderivative size = 939, normalized size of antiderivative = 2.24

$$\int \frac{(e+fx)^3 \text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-e^3*(2*(e^(-d*x - c) - I*e^(-2*d*x - 2*c) + 2*I)/((a*e^(-d*x - c) - I*a*e
^(-2*d*x - 2*c) - a*e^(-3*d*x - 3*c) + I*a)*d) - I*log(e^(-d*x - c) + 1)/(
a*d) + I*log(e^(-d*x - c) - 1)/(a*d)) - 12*e^2*f*x/(a*d) + 3*e^2*f*log(e^(
d*x + c) + 1)/(a*d^2) + 6*e^2*f*log(e^(d*x + c) - I)/(a*d^2) + 3*e^2*f*log
(e^(d*x + c) - 1)/(a*d^2) - 2*(-2*I*f^3*x^3 - 6*I*e*f^2*x^2 - 6*I*e^2*f*x
- (-I*f^3*x^3*e^(2*c) - 3*I*e*f^2*x^2*e^(2*c) - 3*I*e^2*f*x*e^(2*c))*e^(2*
d*x) + (f^3*x^3*e^c + 3*e*f^2*x^2*e^c + 3*e^2*f*x*e^c)*e^(d*x))/(a*d*e^(3*
d*x + 3*c) - I*a*d*e^(2*d*x + 2*c) - a*d*e^(d*x + c) + I*a*d) + 12*(d*x*lo
g(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c)))*e*f^2/(a*d^3) + I*(d^3*x^3*l
og(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^
(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*f^3/(a*d^4) - I*(d^3*x^3*log(-e^(
d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c
)) + 6*polylog(4, e^(d*x + c)))*f^3/(a*d^4) + 6*(d^2*x^2*log(I*e^(d*x + c)
 + 1) + 2*d*x*dilog(-I*e^(d*x + c)) - 2*polylog(3, -I*e^(d*x + c)))*f^3/(a
*d^4) - 3*(-I*d*e^2*f - 2*e*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x
 + c)))/(a*d^3) + 3*(-I*d*e^2*f + 2*e*f^2)*(d*x*log(-e^(d*x + c) + 1) + di
log(e^(d*x + c)))/(a*d^3) + 3*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog
(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*(-I*d*e*f^2 + f^3)/(a*d^4) - 3*
(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -
e^(d*x + c)))*(-I*d*e*f^2 - f^3)/(a*d^4) + 1/4*(I*d^4*f^3*x^4 - 4*(-I*d...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^3}{\sinh(c+d\,x)^2\,(a+a\sinh(c+d\,x)\,1\mathrm{i})}\,dx$$

input
```
int((e + f*x)^3/(sinh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)^3/(sinh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(\int \frac{\operatorname{csch}(dx+c)^2}{\sinh(dx+c)i+1}dx\right)e^3+\left(\int \frac{\operatorname{csch}(dx+c)^2x^3}{\sinh(dx+c)i+1}dx\right)f^3+3\left(\int \frac{\operatorname{csch}(dx+c)^2x^2}{\sinh(dx+c)i+1}dx\right)e\,f^2+3\left(\int \frac{\operatorname{csch}(dx+c)^2x}{\sinh(dx+c)i+1}dx\right)e^2f}{a}$$

input
```
int((f*x+e)^3*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
(int(csch(c + d*x)**2/(sinh(c + d*x)*i + 1),x)*e**3 + int((csch(c + d*x)**
2*x**3)/(sinh(c + d*x)*i + 1),x)*f**3 + 3*int((csch(c + d*x)**2*x**2)/(sin
h(c + d*x)*i + 1),x)*e*f**2 + 3*int((csch(c + d*x)**2*x)/(sinh(c + d*x)*i
+ 1),x)*e**2*f)/a
```

3.212 $\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 296

$$\begin{aligned}\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx &= -\frac{2(e+fx)^2}{ad}+\frac{2i(e+fx)^2\operatorname{arctanh}\left(e^{c+dx}\right)}{ad}\\ &\quad-\frac{(e+fx)^2\coth(c+dx)}{ad}+\frac{4f(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2}\\ &\quad+\frac{2f(e+fx)\log\left(1-e^{2(c+dx)}\right)}{ad^2}\\ &\quad+\frac{2if(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}\\ &\quad+\frac{4f^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}\\ &\quad-\frac{2if(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{ad^2}\\ &\quad+\frac{f^2\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{ad^3}\\ &\quad-\frac{2if^2\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3}+\frac{2if^2\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{ad^3}\\ &\quad-\frac{(e+fx)^2\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}\end{aligned}$$

output
```
-2*(f*x+e)^2/a/d+2*I*(f*x+e)^2*arctanh(exp(d*x+c))/a/d-(f*x+e)^2*coth(d*x+
c)/a/d+4*f*(f*x+e)*ln(1+I*exp(d*x+c))/a/d^2+2*f*(f*x+e)*ln(1-exp(2*d*x+2*c
))/a/d^2+2*I*f*(f*x+e)*polylog(2,-exp(d*x+c))/a/d^2+4*f^2*polylog(2,-I*exp
(d*x+c))/a/d^3-2*I*f*(f*x+e)*polylog(2,exp(d*x+c))/a/d^2+f^2*polylog(2,exp
(2*d*x+2*c))/a/d^3-2*I*f^2*polylog(3,-exp(d*x+c))/a/d^3+2*I*f^2*polylog(3,
exp(d*x+c))/a/d^3-(f*x+e)^2*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a/d
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 803 vs. 2(296) = 592.

Time = 7.96 (sec) , antiderivative size = 803, normalized size of antiderivative = 2.71

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Csch[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-4*I)*E^c*f*((e + f*x)^2/(2*E^c*f) + ((I + E^(-c))*(e + f*x)*Log[1 - I*E
^(-c - d*x)])/d - ((1 + I*E^c)*f*PolyLog[2, I*E^(-c - d*x)])/(d^2*E^c)))/(
a*d*(-I + E^c)) + (I*d^2*e*(-1 + E^(2*c))*(d*e + (2*I)*f)*x + d^2*e*(1 - E
^(2*c))*(I*d*e + 2*f)*x - 2*d^2*(e + f*x)^2 + 2*d*(-1 + E^(2*c))*f*((-I)*d
*e + f)*x*Log[1 - E^(-c - d*x)] - I*d^2*(-1 + E^(2*c))*f^2*x^2*Log[1 - E^(
-c - d*x)] + 2*d*(-1 + E^(2*c))*f*(I*d*e + f)*x*Log[1 + E^(-c - d*x)] + I*
d^2*(-1 + E^(2*c))*f^2*x^2*Log[1 + E^(-c - d*x)] + d*e*(-1 + E^(2*c))*((-I
)*d*e + 2*f)*Log[1 - E^(c + d*x)] + d*e*(-1 + E^(2*c))*(I*d*e + 2*f)*Log[1
 + E^(c + d*x)] - 2*(-1 + E^(2*c))*f*(I*d*e + f)*PolyLog[2, -E^(-c - d*x)]
 - (2*I)*d*(-1 + E^(2*c))*f^2*x*PolyLog[2, -E^(-c - d*x)] + (2*I)*(-1 + E^
(2*c))*(d*e + I*f)*f*PolyLog[2, E^(-c - d*x)] + (2*I)*d*(-1 + E^(2*c))*f^2
*x*PolyLog[2, E^(-c - d*x)] - (2*I)*(-1 + E^(2*c))*f^2*PolyLog[3, -E^(-c -
 d*x)] + (2*I)*(-1 + E^(2*c))*f^2*PolyLog[3, E^(-c - d*x)])/(a*d^3*(-1 + E
^(2*c))) + (Sech[c/2]*Sech[c/2 + (d*x)/2]*(-(e^2*Sinh[(d*x)/2]) - 2*e*f*x*
Sinh[(d*x)/2] - f^2*x^2*Sinh[(d*x)/2]))/(2*a*d) + (Csch[c/2]*Csch[c/2 + (d
*x)/2]*(e^2*Sinh[(d*x)/2] + 2*e*f*x*Sinh[(d*x)/2] + f^2*x^2*Sinh[(d*x)/2])
)/(2*a*d) - (2*(e^2*Sinh[(d*x)/2] + 2*e*f*x*Sinh[(d*x)/2] + f^2*x^2*Sinh[(
d*x)/2]))/(a*d*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2
 + (d*x)/2]))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6109}$$

$$\frac{\int (e+fx)^2\text{csch}^2(c+dx)dx}{a}-i\int \frac{(e+fx)^2\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int -(e+fx)^2\csc(ic+idx)^2dx}{a}-i\int \frac{(e+fx)^2\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int (e+fx)^2\csc(ic+idx)^2dx}{a}-i\int \frac{(e+fx)^2\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 4672}$$

$$-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2if\int -i(e+fx)\coth(c+dx)dx}{d}}{a}-i\int \frac{(e+fx)^2\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2f\int (e+fx)\coth(c+dx)dx}{d}}{a}-i\int \frac{(e+fx)^2\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2f\int -i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}-i\int \frac{(e+fx)^2\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}}{a}-i\int \frac{(e+fx)^2\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 4201}$$

$$-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\int \frac{e^{2c+2dx-i\pi}(e+fx)}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}-i\int \frac{(e+fx)^2\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \; 2620$$

$$-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int\log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}-i\int\frac{(e+fx)^2\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \; 2715$$

$$-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\log\left(1+e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}-i\int\frac{(e+fx)^2\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \; 2838$$

$$-i\int\frac{(e+fx)^2\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \; 6109$$

$$-i\left(\frac{\int(e+fx)^2\text{csch}(c+dx)dx}{a}-i\int\frac{(e+fx)^2}{i\sinh(c+dx)a+a}dx\right)-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \; 3042$$

$$-i\left(\frac{\int i(e+fx)^2\csc(ic+idx)dx}{a}-i\int\frac{(e+fx)^2}{\sin(ic+idx)a+a}dx\right)-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \; 26$$

$$-i\left(\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}-i\int\frac{(e+fx)^2}{\sin(ic+idx)a+a}dx\right)-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \text{3799}$$

$$\frac{-i\left(\frac{i\int (e+fx)^2\csc(ic+idx)dx}{a}-\frac{i\int -(e+fx)^2\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)dx}{2a}\right)-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{-i\left(\frac{i\int (e+fx)^2\csc(ic+idx)dx}{a}+\frac{i\int -(e+fx)^2\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}\right)-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{-i\left(\frac{i\int (e+fx)^2\csc(ic+idx)dx}{a}-\frac{i\int (e+fx)^2\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}\right)-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-i\left(\frac{i\int (e+fx)^2\csc(ic+idx)dx}{a}-\frac{i\int (e+fx)^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}\right)-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \text{4670}$$

$$\frac{-i\left(\frac{i\left(\frac{2if\int (e+fx)\log\left(1-e^{c+dx}\right)dx}{d}-\frac{2if\int (e+fx)\log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)^2\text{arctanh}\left(e^{c+dx}\right)}{d}\right)}{a}-\frac{i\int (e+fx)^2\csc\left(\frac{ic}{2}+\frac{i
…}{2}\right.}{2a}\right.-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \text{3011}$$

$$-i\left(\frac{i\left(-\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}+2i(e+f\cdots\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \; 2720$$

$$-i\left(\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyL}\cdots}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \; 4672$$

$$-i\left(\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyL}\cdots}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \; 26$$

$$-i\left(\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyL}}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$\downarrow$ 3042

$$-i\left(\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyL}}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$\downarrow$ 26

$$-i\left(\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyL}}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$\downarrow$ 4199

$$-i\left(\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyL}\cdots}{\cdots}\right.}{d}\cdots\right.}{a}\cdots\right.$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \; 26$$

$$-i\left(\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyL}\cdots}{\cdots}\right.}{d}\cdots\right.}{a}\cdots\right.$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \; 2620$$

$$-i\left(\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyL}\cdots}{\cdots}\right.}{d}\cdots\right.}{a}\cdots\right.$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \; 2715$$

$$-i\left(\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyL}}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \text{2838}$$

$$-i\left(\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyL}}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

input
```
Int[((e + f*x)^2*Csch[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 888 vs. 2(275) = 550.

Time = 0.78 (sec) , antiderivative size = 889, normalized size of antiderivative = 3.00

method	result
risch	$-\frac{4f^2c^2}{a\,d^3}-\frac{4f^2x^2}{ad}+\frac{if^2\ln(\mathrm{e}^{dx+c}+1)x^2}{ad}+\frac{if^2\ln(1-\mathrm{e}^{dx+c})c^2}{a\,d^3}-\frac{4ic\,f^2\arctan(\mathrm{e}^{dx+c})}{a\,d^3}+\frac{2ief\operatorname{polylog}(2,-\mathrm{e}^{dx+c})}{a\,d^2}-\frac{2ief}{}$

input
```
int((f*x+e)^2*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*I*f^2*polylog(3,exp(d*x+c))/a/d^3-4/a/d^3*f^2*c^2-4/a/d*f^2*x^2+I/a/d*f^
2*ln(exp(d*x+c)+1)*x^2+I/a/d^3*f^2*ln(1-exp(d*x+c))*c^2-2*I/a/d^2*e*f*ln(1
-exp(d*x+c))*c+2*I/a/d*e*f*ln(exp(d*x+c)+1)*x-2*I/a/d*e*f*ln(1-exp(d*x+c))
*x+2*I/a/d^2*e*c*f*ln(exp(d*x+c)-1)-2*I*f^2*polylog(3,-exp(d*x+c))/a/d^3-I
/a/d*e^2*ln(exp(d*x+c)-1)+2*f^2*polylog(2,-exp(d*x+c))/a/d^3+2*f^2*polylog
(2,exp(d*x+c))/a/d^3+2/a/d^2*f^2*ln(1-exp(d*x+c))*x+4/a/d^2*f^2*ln(1+I*exp
(d*x+c))*x+2/a/d^3*f^2*ln(1-exp(d*x+c))*c+4/a/d^3*f^2*ln(1+I*exp(d*x+c))*c
-2/a/d^3*f^2*c*ln(1+exp(2*d*x+2*c))-2/a/d^3*f^2*c*ln(exp(d*x+c)-1)-8/a/d^2
*e*f*ln(exp(d*x+c))+2/a/d^2*e*f*ln(1+exp(2*d*x+2*c))+2/a/d^2*e*f*ln(exp(d*
x+c)+1)+2/a/d^2*e*f*ln(exp(d*x+c)-1)+8/a/d^3*c*f^2*ln(exp(d*x+c))-8/a/d^2*
f^2*c*x+2/a/d^2*f^2*ln(exp(d*x+c)+1)*x+I/a/d*e^2*ln(exp(d*x+c)+1)-4*I/a/d^
3*c*f^2*arctan(exp(d*x+c))+2*I/a/d^2*e*f*polylog(2,-exp(d*x+c))-2*I/a/d^2*
e*f*polylog(2,exp(d*x+c))+4*I/a/d^2*e*f*arctan(exp(d*x+c))+2*I/a/d^2*f^2*p
olylog(2,-exp(d*x+c))*x-I/a/d*f^2*ln(1-exp(d*x+c))*x^2-2*I/a/d^2*f^2*polyl
og(2,exp(d*x+c))*x-I/a/d^3*c^2*f^2*ln(exp(d*x+c)-1)+4*f^2*polylog(2,-I*exp
(d*x+c))/a/d^3-2*I*(f^2*x^2*exp(2*d*x+2*c)+2*e*f*x*exp(2*d*x+2*c)+e^2*exp(
2*d*x+2*c)-2*x^2*f^2-I*exp(d*x+c)*f^2*x^2-4*e*f*x-2*I*exp(d*x+c)*e*f*x-2*e
^2-I*exp(d*x+c)*e^2)/(exp(2*d*x+2*c)-1)/(exp(d*x+c)-I)/d/a
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1355 vs. $2(263) = 526$.

Time = 0.12 (sec) , antiderivative size = 1355, normalized size of antiderivative = 4.58

$$\int \frac{(e+fx)^2 \text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
(4*I*d^2*e^2 - 8*I*c*d*e*f + 4*I*c^2*f^2 + 4*(f^2*e^(3*d*x + 3*c) - I*f^2*
e^(2*d*x + 2*c) - f^2*e^(d*x + c) + I*f^2)*dilog(-I*e^(d*x + c)) - 2*(d*f^
2*x + d*e*f - I*f^2 + (-I*d*f^2*x - I*d*e*f - f^2)*e^(3*d*x + 3*c) - (d*f^
2*x + d*e*f - I*f^2)*e^(2*d*x + 2*c) + (I*d*f^2*x + I*d*e*f + f^2)*e^(d*x
+ c))*dilog(-e^(d*x + c)) + 2*(d*f^2*x + d*e*f + I*f^2 - (I*d*f^2*x + I*d*
e*f - f^2)*e^(3*d*x + 3*c) - (d*f^2*x + d*e*f + I*f^2)*e^(2*d*x + 2*c) - (
-I*d*f^2*x - I*d*e*f + f^2)*e^(d*x + c))*dilog(e^(d*x + c)) - 4*(d^2*f^2*x
^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*e^(3*d*x + 3*c) - 2*(-I*d^2*f^2*x^
2 - 2*I*d^2*e*f*x + I*d^2*e^2 - 4*I*c*d*e*f + 2*I*c^2*f^2)*e^(2*d*x + 2*c)
 + 2*(d^2*f^2*x^2 + 2*d^2*e*f*x - d^2*e^2 + 4*c*d*e*f - 2*c^2*f^2)*e^(d*x
+ c) - (d^2*f^2*x^2 + d^2*e^2 - 2*I*d*e*f + 2*(d^2*e*f - I*d*f^2)*x - (I*d
^2*f^2*x^2 + I*d^2*e^2 + 2*d*e*f - 2*(-I*d^2*e*f - d*f^2)*x)*e^(3*d*x + 3*
c) - (d^2*f^2*x^2 + d^2*e^2 - 2*I*d*e*f + 2*(d^2*e*f - I*d*f^2)*x)*e^(2*d*
x + 2*c) - (-I*d^2*f^2*x^2 - I*d^2*e^2 - 2*d*e*f - 2*(I*d^2*e*f + d*f^2)*x
)*e^(d*x + c))*log(e^(d*x + c) + 1) - 4*(-I*d*e*f + I*c*f^2 - (d*e*f - c*f
^2)*e^(3*d*x + 3*c) + (I*d*e*f - I*c*f^2)*e^(2*d*x + 2*c) + (d*e*f - c*f^2
)*e^(d*x + c))*log(e^(d*x + c) - I) + (d^2*e^2 - 2*(c - I)*d*e*f + (c^2 -
2*I*c)*f^2 + (-I*d^2*e^2 - 2*(-I*c - 1)*d*e*f + (-I*c^2 - 2*c)*f^2)*e^(3*d
*x + 3*c) - (d^2*e^2 - 2*(c - I)*d*e*f + (c^2 - 2*I*c)*f^2)*e^(2*d*x + 2*c
) + (I*d^2*e^2 - 2*(I*c + 1)*d*e*f + (I*c^2 + 2*c)*f^2)*e^(d*x + c))*lo...
```

Sympy [F]

$$\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e^2\operatorname{csch}^2(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{f^2x^2\operatorname{csch}^2(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{2efx\operatorname{csch}^2(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)**2*csch(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e**2*csch(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(f**2
*x**2*csch(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(2*e*f*x*csch(c +
 d*x)**2/(sinh(c + d*x) - I), x))/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 601 vs. 2(263) = 526.

Time = 0.29 (sec) , antiderivative size = 601, normalized size of antiderivative = 2.03

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-e^2*(2*(e^(-d*x - c) - I*e^(-2*d*x - 2*c) + 2*I)/((a*e^(-d*x - c) - I*a*e
^(-2*d*x - 2*c) - a*e^(-3*d*x - 3*c) + I*a)*d) - I*log(e^(-d*x - c) + 1)/(
a*d) + I*log(e^(-d*x - c) - 1)/(a*d)) - 2*f^2*x^2/(a*d) - 8*e*f*x/(a*d) -
2*(-2*I*f^2*x^2 - 4*I*e*f*x - (-I*f^2*x^2*e^(2*c) - 2*I*e*f*x*e^(2*c))*e^(
2*d*x) + (f^2*x^2*e^c + 2*e*f*x*e^c)*e^(d*x))/(a*d*e^(3*d*x + 3*c) - I*a*d
*e^(2*d*x + 2*c) - a*d*e^(d*x + c) + I*a*d) + 2*e*f*log(e^(d*x + c) + 1)/(
a*d^2) + 4*e*f*log(e^(d*x + c) - I)/(a*d^2) + 2*e*f*log(e^(d*x + c) - 1)/(
a*d^2) + I*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*p
olylog(3, -e^(d*x + c)))*f^2/(a*d^3) - I*(d^2*x^2*log(-e^(d*x + c) + 1) +
2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*f^2/(a*d^3) + 4*(d*x
*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c)))*f^2/(a*d^3) - 2*(-I*d*e*f
 - f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a*d^3) + 2*(-I*d
*e*f + f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a*d^3) + 1/3
*(I*d^3*f^2*x^3 - 3*(-I*d*e*f + f^2)*d^2*x^2)/(a*d^3) - 1/3*(I*d^3*f^2*x^3
 - 3*(-I*d*e*f - f^2)*d^2*x^2)/(a*d^3)
```

Giac [F]

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\,\text{csch}\,(dx+c)^2}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*csch(d*x + c)^2/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{{\left(e+f\,x\right)}^2}{{\mathrm{sinh}\left(c+d\,x\right)}^2\,\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

input
```
int((e + f*x)^2/(sinh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)^2/(sinh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(\int \frac{\mathrm{csch}(dx+c)^2}{\sinh(dx+c)i+1}dx\right)e^2+\left(\int \frac{\mathrm{csch}(dx+c)^2x^2}{\sinh(dx+c)i+1}dx\right)f^2+2\left(\int \frac{\mathrm{csch}(dx+c)^2x}{\sinh(dx+c)i+1}dx\right)ef}{a}$$

input
```
int((f*x+e)^2*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
(int(csch(c + d*x)**2/(sinh(c + d*x)*i + 1),x)*e**2 + int((csch(c + d*x)**
2*x**2)/(sinh(c + d*x)*i + 1),x)*f**2 + 2*int((csch(c + d*x)**2*x)/(sinh(c
 + d*x)*i + 1),x)*e*f)/a
```

3.213 $\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 163

$$\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{ad} - \frac{(e+fx)\coth(c+dx)}{ad} + \frac{2f\log\left(\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2} + \frac{f\log(\sinh(c+dx))}{ad^2} + \frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2} - \frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2} - \frac{(e+fx)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}$$

output
```
2*I*(f*x+e)*arctanh(exp(d*x+c))/a/d-(f*x+e)*coth(d*x+c)/a/d+2*f*ln(cosh(1/
2*c+1/4*I*Pi+1/2*d*x))/a/d^2+f*ln(sinh(d*x+c))/a/d^2+I*f*polylog(2,-exp(d*
x+c))/a/d^2-I*f*polylog(2,exp(d*x+c))/a/d^2-(f*x+e)*tanh(1/2*c+1/4*I*Pi+1/
2*d*x)/a/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 366 vs. $2(163) = 326$.

Time = 3.10 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.25

$$\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(-d(e+fx)\cosh\left(\frac{1}{2}(c+dx)\right)\left(i+\coth\left(\frac{1}{2}(c+dx)\right)\right)+4if\,\text{arct}\right.}{}$$

input
```
Integrate[((e + f*x)*Csch[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(-(d*(e + f*x)*Cosh[(c + d*x)/2
]*(I + Coth[(c + d*x)/2])) + (4*I)*f*ArcTan[Tanh[(c + d*x)/2]]*(Cosh[(c +
d*x)/2] + I*Sinh[(c + d*x)/2]) + 2*f*Log[Cosh[c + d*x]]*(Cosh[(c + d*x)/2]
 + I*Sinh[(c + d*x)/2]) + 2*(f*(c + d*x) + (f - I*d*(e + f*x))*Log[1 - E^(
-c - d*x)] + (f + I*d*(e + f*x))*Log[1 + E^(-c - d*x)] - I*f*PolyLog[2, -E
^(-c - d*x)] + I*f*PolyLog[2, E^(-c - d*x)])*(Cosh[(c + d*x)/2] + I*Sinh[(
c + d*x)/2]) - 4*d*(e + f*x)*Sinh[(c + d*x)/2] + 2*f*(c + d*x)*((-I)*Cosh[
(c + d*x)/2] + Sinh[(c + d*x)/2]) - I*d*(e + f*x)*Sinh[(c + d*x)/2]*(-I +
Tanh[(c + d*x)/2])))/(2*d^2*(a + I*a*Sinh[c + d*x]))
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.07, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.793$, Rules used = {6109, 3042, 25, 4672, 26, 3042, 26, 3956, 6109, 3042, 26, 3799, 25, 25, 3042, 4670, 2715, 2838, 4672, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6109}$$

$$\frac{\int (e+fx)\text{csch}^2(c+dx)dx}{a}-i\int \frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -\left((e+fx)\csc(ic+idx)^2\right)dx}{a}-i\int \frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int (e+fx)\csc(ic+idx)^2dx}{a}-i\int \frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{4672}$$

$$-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{if\int -i\coth(c+dx)dx}{d}}{a}-i\int \frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{26}$$

$$-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\int \coth(c+dx)dx}{d}}{a}-i\int \frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\int -i\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}-i\int \frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{26}$$

$$-\frac{\frac{(e+fx)\coth(c+dx)}{d}+\frac{if\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}}{a}-i\int \frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{3956}$$

$$-i\int \frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \text{6109}$$

$$-i\left(\frac{\int (e+fx)\text{csch}(c+dx)dx}{a}-i\int \frac{e+fx}{i\sinh(c+dx)a+a}dx\right)-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{\int i(e+fx)\csc(ic+idx)dx}{a}-i\int\frac{e+fx}{\sin(ic+idx)a+a}dx\right)-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow 26$$

$$-i\left(\frac{i\int(e+fx)\csc(ic+idx)dx}{a}-i\int\frac{e+fx}{\sin(ic+idx)a+a}dx\right)-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow 3799$$

$$-i\left(\frac{i\int(e+fx)\csc(ic+idx)dx}{a}-\frac{i\int-\left((e+fx)\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)\right)dx}{2a}\right)-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow 25$$

$$-i\left(\frac{i\int(e+fx)\csc(ic+idx)dx}{a}+\frac{i\int-\left((e+fx)\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)dx}{2a}\right)-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow 25$$

$$-i\left(\frac{i\int(e+fx)\csc(ic+idx)dx}{a}-\frac{i\int(e+fx)\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}\right)-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow 3042$$

$$-i\left(\frac{i\int(e+fx)\csc(ic+idx)dx}{a}-\frac{i\int(e+fx)\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}\right)-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow 4670$$

$$-i\left(\frac{i\left(\frac{if\int\log(1-e^{c+dx})dx}{d}-\frac{if\int\log(1+e^{c+dx})dx}{d}+\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}\right)}{a}-\frac{i\int(e+fx)\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}\right)-$$

$$\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \text{2715}$$

$$-i\left(\frac{i\left(\frac{if\int e^{-c-dx}\log(1-e^{c+dx})de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log(1+e^{c+dx})de^{c+dx}}{d^2}+\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}\right)}{a}-\frac{i\int(e+fx)\csc\left(\frac{ic}{2}\right.}{2a}\right.$$

$$\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \text{2838}$$

$$-i\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)}{a}-\frac{i\int(e+fx)\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}\right)-$$

$$\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \text{4672}$$

$$-i\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)}{a}-\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2if\int -i\tan\right.}{2a}\right.$$

$$\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{a}-\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2f\int\tanh\left(\frac{c}{2}\right.}{}\right.}{2a}\right.$$

$$\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$\big\downarrow$ 3042

$$-i\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{a}-\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2f\int-i\tan}{}\right.}{2a}\right.$$

$$\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$\big\downarrow$ 26

$$-i\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{a}-\frac{i\left(\frac{2if\int\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}+\frac{2(e+fx)\tanh}{}\right.}{2a}\right.$$

$$\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$\big\downarrow$ 3956

$$-i\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{a}-\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4f\log\left(\cosh\right.}{}\right.}{2a}\right.$$

$$\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

input
```
Int[((e + f*x)*Csch[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
-((((e + f*x)*Coth[c + d*x])/d - (f*Log[(-I)*Sinh[c + d*x]])/d^2)/a) - I*(
(I*(((2*I)*(e + f*x)*ArcTanh[E^(c + d*x)])/d + (I*f*PolyLog[2, -E^(c + d*x
)])/d^2 - (I*f*PolyLog[2, E^(c + d*x)])/d^2))/a - ((I/2)*((-4*f*Log[Cosh[c
/2 + (I/4)*Pi + (d*x)/2]])/d^2 + (2*(e + f*x)*Tanh[c/2 + (I/4)*Pi + (d*x)/
2])/d))/a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6109
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csch[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a +
 b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 IGtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(143) = 286$.

Time = 0.72 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.05

method	result
risch	$-\frac{2i\left(fx\,\mathrm{e}^{2dx+2c}+e\,\mathrm{e}^{2dx+2c}-2fx-i\mathrm{e}^{dx+c}fx-2e-i\mathrm{e}^{dx+c}e\right)}{\left(\mathrm{e}^{2dx+2c}-1\right)\left(\mathrm{e}^{dx+c}-i\right)da}+\frac{if\ln\left(\mathrm{e}^{dx+c}+1\right)x}{ad}-\frac{if\ln\left(1-\mathrm{e}^{dx+c}\right)x}{ad}+\frac{ie\ln\left(\mathrm{e}^{dx+c}+1\right)}{ad}-\frac{ie}{}$

input
```
int((f*x+e)*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2*I*(f*x*exp(2*d*x+2*c)+e*exp(2*d*x+2*c)-2*f*x-I*exp(d*x+c)*f*x-2*e-I*exp
(d*x+c)*e)/(exp(2*d*x+2*c)-1)/(exp(d*x+c)-I)/d/a+I/a/d*f*ln(exp(d*x+c)+1)*
x-I/a/d*f*ln(1-exp(d*x+c))*x+I/a/d*e*ln(exp(d*x+c)+1)-I/a/d*e*ln(exp(d*x+c
)-1)-I/a/d^2*f*ln(1-exp(d*x+c))*c+I/a/d^2*c*f*ln(exp(d*x+c)-1)+1/a/d^2*f*l
n(1+exp(2*d*x+2*c))+1/a/d^2*f*ln(exp(d*x+c)+1)+1/a/d^2*f*ln(exp(d*x+c)-1)-
4/a/d^2*f*ln(exp(d*x+c))+2*I/a/d^2*f*arctan(exp(d*x+c))+I*f*polylog(2,-exp
(d*x+c))/a/d^2-I*f*polylog(2,exp(d*x+c))/a/d^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(139) = 278$.

Time = 0.13 (sec) , antiderivative size = 506, normalized size of antiderivative = 3.10

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{4i\,de-2i\,cf+\left(i\,fe^{(3\,dx+3\,c)}+fe^{(2\,dx+2\,c)}-i\,fe^{(dx+c)}-f\right)\operatorname{Li}_2\left(-e^{(dx+c)}\right)+\left(-i\,fe^{(3\,dx+3\,c)}-fe^{(2\,dx+2\,c)}\right.}{}$$

input
```
integrate((f*x+e)*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(4*I*d*e - 2*I*c*f + (I*f*e^(3*d*x + 3*c) + f*e^(2*d*x + 2*c) - I*f*e^(d*x
 + c) - f)*dilog(-e^(d*x + c)) + (-I*f*e^(3*d*x + 3*c) - f*e^(2*d*x + 2*c)
 + I*f*e^(d*x + c) + f)*dilog(e^(d*x + c)) - 2*(2*d*f*x + c*f)*e^(3*d*x +
3*c) - 2*(-I*d*f*x + I*d*e - I*c*f)*e^(2*d*x + 2*c) + 2*(d*f*x - d*e + c*f
)*e^(d*x + c) - (d*f*x + d*e - (I*d*f*x + I*d*e + f)*e^(3*d*x + 3*c) - (d*
f*x + d*e - I*f)*e^(2*d*x + 2*c) - (-I*d*f*x - I*d*e - f)*e^(d*x + c) - I*
f)*log(e^(d*x + c) + 1) + 2*(f*e^(3*d*x + 3*c) - I*f*e^(2*d*x + 2*c) - f*e
^(d*x + c) + I*f)*log(e^(d*x + c) - I) + (d*e - (c - I)*f + (-I*d*e + (I*c
 + 1)*f)*e^(3*d*x + 3*c) - (d*e - (c - I)*f)*e^(2*d*x + 2*c) + (I*d*e + (-
I*c - 1)*f)*e^(d*x + c))*log(e^(d*x + c) - 1) + (d*f*x + c*f + (-I*d*f*x -
 I*c*f)*e^(3*d*x + 3*c) - (d*f*x + c*f)*e^(2*d*x + 2*c) + (I*d*f*x + I*c*f
)*e^(d*x + c))*log(-e^(d*x + c) + 1))/(a*d^2*e^(3*d*x + 3*c) - I*a*d^2*e^(
2*d*x + 2*c) - a*d^2*e^(d*x + c) + I*a*d^2)
```

Sympy [F]

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e\operatorname{csch}^2(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{fx\operatorname{csch}^2(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)*csch(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e*csch(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(f*x*csc
h(c + d*x)**2/(sinh(c + d*x) - I), x))/a
```

Maxima [F]

$$\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(fx+e)\,\text{csch}\,(dx+c)^2}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-(4*I*d*integrate(1/4*x/(a*d*e^(d*x + c) + a*d), x) + 4*I*d*integrate(1/4*
x/(a*d*e^(d*x + c) - a*d), x) + 2*(x*e^(3*d*x + 3*c) - I*x)/(a*d*e^(3*d*x
+ 3*c) - I*a*d*e^(2*d*x + 2*c) - a*d*e^(d*x + c) + I*a*d) + 2*(d*x + c)/(a
*d^2) - 2*log((e^(d*x + c) - I)*e^(-c))/(a*d^2) - log(e^(d*x + c) + 1)/(a*
d^2) - log(e^(d*x + c) - 1)/(a*d^2))*f - e*(2*(e^(-d*x - c) - I*e^(-2*d*x
- 2*c) + 2*I)/((a*e^(-d*x - c) - I*a*e^(-2*d*x - 2*c) - a*e^(-3*d*x - 3*c)
 + I*a)*d) - I*log(e^(-d*x - c) + 1)/(a*d) + I*log(e^(-d*x - c) - 1)/(a*d)
)
```

Giac [F]

$$\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(fx+e)\,\text{csch}\,(dx+c)^2}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*csch(d*x + c)^2/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{e+f\,x}{{\mathrm{sinh}\left(c+d\,x\right)}^2\,\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)} \,d x$$

input
```
int((e + f*x)/(sinh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)/(sinh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx=\frac{\left(\int \frac{\mathrm{csch}\left(d x +c \right)^{2}}{\sinh \left(d x +c \right) i +1}d x \right) e +\left(\int \frac{\mathrm{csch}\left(d x +c \right)^{2} x}{\sinh \left(d x +c \right) i +1}d x \right) f}{a}$$

input
```
int((f*x+e)*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
(int(csch(c + d*x)**2/(sinh(c + d*x)*i + 1),x)*e + int((csch(c + d*x)**2*x
)/(sinh(c + d*x)*i + 1),x)*f)/a
```

3.214 $\int \frac{\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 57

$$\int \frac{\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{i\text{arctanh}(\cosh(c+dx))}{ad} - \frac{2\coth(c+dx)}{ad} + \frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}$$

output
```
I*arctanh(cosh(d*x+c))/a/d-2*coth(d*x+c)/a/d+coth(d*x+c)/d/(a+I*a*sinh(d*x
+c))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \frac{\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{\text{sech}(c+dx)\left(i - i\text{arctanh}\left(\sqrt{\cosh^2(c+dx)}\right)\sqrt{\cosh^2(c+dx)} + \text{csch}(c+dx) + 2\sinh(c+dx)\right)}{ad}$$

input
```
Integrate[Csch[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
```

output
```
-((Sech[c + d*x]*(I - I*ArcTanh[Sqrt[Cosh[c + d*x]^2]]*Sqrt[Cosh[c + d*x]^
2] + Csch[c + d*x] + 2*Sinh[c + d*x]))/(a*d))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3042, 25, 3247, 3042, 25, 3227, 25, 26, 3042, 25, 26, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{1}{\sin(ic+idx)^2(a+a\sin(ic+idx))}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{1}{\sin(ic+idx)^2(\sin(ic+idx)a+a)}dx$$

$$\Big\downarrow \ 3247$$

$$\frac{\int \operatorname{csch}^2(c+dx)(2a-ia\sinh(c+dx))dx}{a^2}+\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int -\frac{2a-a\sin(ic+idx)}{\sin(ic+idx)^2}dx}{a^2}+\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}$$

$$\Big\downarrow \ 25$$

$$\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{\int \frac{2a-a\sin(ic+idx)}{\sin(ic+idx)^2}dx}{a^2}$$

$$\Big\downarrow \ 3227$$

$$\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{2a\int -\operatorname{csch}^2(c+dx)dx-a\int -i\operatorname{csch}(c+dx)dx}{a^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{-2a\int \operatorname{csch}^2(c+dx)dx-a\int -i\operatorname{csch}(c+dx)dx}{a^2}$$

$$\Big\downarrow \; 26$$

$$\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{-2a\int \operatorname{csch}^2(c+dx)dx+ia\int \operatorname{csch}(c+dx)dx}{a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{ia\int i\csc(ic+idx)dx-2a\int -\csc(ic+idx)^2dx}{a^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{ia\int i\csc(ic+idx)dx+2a\int \csc(ic+idx)^2dx}{a^2}$$

$$\Big\downarrow \; 26$$

$$\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{2a\int \csc(ic+idx)^2dx-a\int \csc(ic+idx)dx}{a^2}$$

$$\Big\downarrow \; 4254$$

$$\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{\frac{2ia\int 1d(-i\coth(c+dx))}{d}-a\int \csc(ic+idx)dx}{a^2}$$

$$\Big\downarrow \; 24$$

$$\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{\frac{2a\coth(c+dx)}{d}-a\int \csc(ic+idx)dx}{a^2}$$

$$\Big\downarrow \; 4257$$

$$\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{\frac{2a\coth(c+dx)}{d}-\frac{ia\text{arctanh}(\cosh(c+dx))}{d}}{a^2}$$

input

```
Int[Csch[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
```

output

```
-((((-I)*a*ArcTanh[Cosh[c + d*x]])/d + (2*a*Coth[c + d*x])/d)/a^2) + Coth[
c + d*x]/(d*(a + I*a*Sinh[c + d*x]))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3247
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Simp[d/(a*(b*c - a*d))
   Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[
c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-2i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da}$	63
default	$\frac{-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-2i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da}$	63
parallelrisch	$\frac{2\left(i-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\coth\left(\frac{dx}{2}+\frac{c}{2}\right)+6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{2\left(-1-i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)da}$	86
risch	$-\frac{2i\left(-i\,\mathrm{e}^{dx+c}+\mathrm{e}^{2dx+2c}-2\right)}{\left(\mathrm{e}^{2dx+2c}-1\right)\left(\mathrm{e}^{dx+c}-i\right)da}+\frac{i\ln\left(\mathrm{e}^{dx+c}+1\right)}{da}-\frac{i\ln\left(\mathrm{e}^{dx+c}-1\right)}{da}$	91

input
```
int(csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2/d/a*(-tanh(1/2*d*x+1/2*c)-4/(-I+tanh(1/2*d*x+1/2*c))-1/tanh(1/2*d*x+1/
2*c)-2*I*ln(tanh(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(53) = 106.

Time = 0.10 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.56

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{\left(i\,e^{(3\,dx+3\,c)}+e^{(2\,dx+2\,c)}-i\,e^{(dx+c)}-1\right)\log\left(e^{(dx+c)}+1\right)+\left(-i\,e^{(3\,dx+3\,c)}-e^{(2\,dx+2\,c)}+i\,e^{(dx+c)}+1\right)\log\left(e\right.}{ade^{(3\,dx+3\,c)}-i\,ade^{(2\,dx+2\,c)}-ade^{(dx+c)}+i\,ad}$$

input
```
integrate(csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
((I*e^(3*d*x + 3*c) + e^(2*d*x + 2*c) - I*e^(d*x + c) - 1)*log(e^(d*x + c)
 + 1) + (-I*e^(3*d*x + 3*c) - e^(2*d*x + 2*c) + I*e^(d*x + c) + 1)*log(e^(
d*x + c) - 1) - 2*I*e^(2*d*x + 2*c) - 2*e^(d*x + c) + 4*I)/(a*d*e^(3*d*x +
 3*c) - I*a*d*e^(2*d*x + 2*c) - a*d*e^(d*x + c) + I*a*d)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int \frac{\operatorname{csch}^2(c+dx)}{\sinh(c+dx)-i}\,dx}{a}$$

input
```
integrate(csch(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(csch(c + d*x)**2/(sinh(c + d*x) - I), x)/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(53) = 106$.

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.91

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{2\left(e^{(-dx-c)} - i\,e^{(-2\,dx-2\,c)} + 2i\right)}{(ae^{(-dx-c)} - i\,ae^{(-2\,dx-2\,c)} - ae^{(-3\,dx-3\,c)} + i\,a)d} + \frac{i\,\log\left(e^{(-dx-c)}+1\right)}{ad} - \frac{i\,\log\left(e^{(-dx-c)}-1\right)}{ad}$$

input
```
integrate(csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-2*(e^(-d*x - c) - I*e^(-2*d*x - 2*c) + 2*I)/((a*e^(-d*x - c) - I*a*e^(-2*
d*x - 2*c) - a*e^(-3*d*x - 3*c) + I*a)*d) + I*log(e^(-d*x - c) + 1)/(a*d)
- I*log(e^(-d*x - c) - 1)/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.58

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{-\frac{i\log\left(e^{(dx+c)}+1\right)}{a}+\frac{i\log\left(e^{(dx+c)}-1\right)}{a}-\frac{2\left(e^{(2\,dx+2\,c)}-i\,e^{(dx+c)}-2\right)}{a\left(i\,e^{(3\,dx+3\,c)}+e^{(2\,dx+2\,c)}-i\,e^{(dx+c)}-1\right)}}{d}$$

input
```
integrate(csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-(-I*log(e^(d*x + c) + 1)/a + I*log(e^(d*x + c) - 1)/a - 2*(e^(2*d*x + 2*c
) - I*e^(d*x + c) - 2)/(a*(I*e^(3*d*x + 3*c) + e^(2*d*x + 2*c) - I*e^(d*x
+ c) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.14

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\frac{2\,\mathrm{e}^{c+d\,x}}{a\,d}-\frac{4\mathrm{i}}{a\,d}+\frac{\mathrm{e}^{2\,c+2\,d\,x}\,2\mathrm{i}}{a\,d}}{\mathrm{e}^{c+d\,x}+\mathrm{e}^{2\,c+2\,d\,x}\,1\mathrm{i}-\mathrm{e}^{3\,c+3\,d\,x}-\mathrm{i}} - \frac{\ln\left(\mathrm{e}^{c+d\,x}\,2\mathrm{i}-2\mathrm{i}\right)\,1\mathrm{i}}{a\,d}+\frac{\ln\left(\mathrm{e}^{c+d\,x}\,2\mathrm{i}+2\mathrm{i}\right)\,1\mathrm{i}}{a\,d}$$

input
```
int(1/(sinh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
((2*exp(c + d*x))/(a*d) - 4i/(a*d) + (exp(2*c + 2*d*x)*2i)/(a*d))/(exp(c +
 d*x) + exp(2*c + 2*d*x)*1i - exp(3*c + 3*d*x) - 1i) - (log(exp(c + d*x)*2
i - 2i)*1i)/(a*d) + (log(exp(c + d*x)*2i + 2i)*1i)/(a*d)
```

Reduce [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\int \frac{\operatorname{csch}(dx+c)^2}{\sinh(dx+c)i+1}dx}{a}$$

input
```
int(csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(csch(c + d*x)**2/(sinh(c + d*x)*i + 1),x)/a
```

3.215 $\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(csch(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 67.50 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

input
```
Integrate[Csch[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Integrate[Csch[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$
$$\Big\downarrow \text{ 6111}$$
$$\int \frac{\text{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}dx$$

input
```
Int[Csch[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\text{csch}\,(dx+c)^2}{(fx+e)\,(a+ia\sinh\,(dx+c))}dx$$

input
```
int(csch(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 342, normalized size of antiderivative = 11.03

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)^2}{(fx+e)(i\,a\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
((I*a*d*f*x + I*a*d*e + (a*d*f*x + a*d*e)*e^(3*d*x + 3*c) + (-I*a*d*f*x -
I*a*d*e)*e^(2*d*x + 2*c) - (a*d*f*x + a*d*e)*e^(d*x + c))*integral(-2*((I*
d*f*x + I*d*e + I*f)*e^(2*d*x + 2*c) + (d*f*x + d*e + f)*e^(d*x + c) - 2*I
*f)/(I*a*d*f^2*x^2 + 2*I*a*d*e*f*x + I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*
x + a*d*e^2)*e^(3*d*x + 3*c) + (-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2
)*e^(2*d*x + 2*c) - (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c)), x)
 - 2*I*e^(2*d*x + 2*c) - 2*e^(d*x + c) + 4*I)/(I*a*d*f*x + I*a*d*e + (a*d*
f*x + a*d*e)*e^(3*d*x + 3*c) + (-I*a*d*f*x - I*a*d*e)*e^(2*d*x + 2*c) - (a
*d*f*x + a*d*e)*e^(d*x + c))
```

Sympy [N/A]

Not integrable

Time = 20.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=-\frac{i\int \frac{\operatorname{csch}^2\left(c+dx\right)}{e\sinh\left(c+dx\right)-ie+fx\sinh\left(c+dx\right)-ifx}\,dx}{a}$$

input
```
integrate(csch(d*x+c)**2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(csch(c + d*x)**2/(e*sinh(c + d*x) - I*e + f*x*sinh(c + d*x) -
I*f*x), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 330, normalized size of antiderivative = 10.65

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)^2}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-4*I*f*integrate(1/(-2*I*a*d*f^2*x^2 - 4*I*a*d*e*f*x - 2*I*a*d*e^2 + 2*(a*
d*f^2*x^2*e^c + 2*a*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)), x) - 4*(I*e^(2*d*
x + 2*c) + e^(d*x + c) - 2*I)/(2*I*a*d*f*x + 2*I*a*d*e + 2*(a*d*f*x*e^(3*c
) + a*d*e*e^(3*c))*e^(3*d*x) - 2*(I*a*d*f*x*e^(2*c) + I*a*d*e*e^(2*c))*e^(
2*d*x) - 2*(a*d*f*x*e^c + a*d*e*e^c)*e^(d*x)) - 4*integrate(-1/4*(I*d*f*x
+ I*d*e + f)/(a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2 - (a*d*f^2*x^2*e^c + 2*a
*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)), x) - 4*integrate(1/4*(I*d*f*x + I*d*
e - f)/(a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2 + (a*d*f^2*x^2*e^c + 2*a*d*e*f
*x*e^c + a*d*e^2*e^c)*e^(d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{1}{\sinh\left(c+d\,x\right)^2\,\left(e+f\,x\right)\,\left(a+a\,\sinh\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

input
```
int(1/(sinh(c + d*x)^2*(e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(1/(sinh(c + d*x)^2*(e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \frac{\int \frac{\operatorname{csch}(dx+c)^2}{\sinh(dx+c)ei+\sinh(dx+c)fix+e+fx}dx}{a}$$

input
```
int(csch(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(csch(c + d*x)**2/(sinh(c + d*x)*e*i + sinh(c + d*x)*f*i*x + e + f*x),x
)/a
```

3.216 $\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(csch(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 78.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

input
```
Integrate[Csch[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Integrate[Csch[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$
$$\Big\downarrow \text{6111}$$
$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}dx$$

input
```
Int[Csch[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{csch}(dx+c)^2}{(fx+e)^2\,(a+ia\sinh(dx+c))}dx$$

input
```
int(csch(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 500, normalized size of antiderivative = 16.13

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)^2}{(fx+e)^2(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
((I*a*d*f^2*x^2 + 2*I*a*d*e*f*x + I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x +
 a*d*e^2)*e^(3*d*x + 3*c) + (-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2)*e
^(2*d*x + 2*c) - (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c))*integr
al(-2*((I*d*f*x + I*d*e + 2*I*f)*e^(2*d*x + 2*c) + (d*f*x + d*e + 2*f)*e^(
d*x + c) - 4*I*f)/(I*a*d*f^3*x^3 + 3*I*a*d*e*f^2*x^2 + 3*I*a*d*e^2*f*x + I
*a*d*e^3 + (a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x + a*d*e^3)*e^(3*
d*x + 3*c) + (-I*a*d*f^3*x^3 - 3*I*a*d*e*f^2*x^2 - 3*I*a*d*e^2*f*x - I*a*d
*e^3)*e^(2*d*x + 2*c) - (a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x + a
*d*e^3)*e^(d*x + c)), x) - 2*I*e^(2*d*x + 2*c) - 2*e^(d*x + c) + 4*I)/(I*a
*d*f^2*x^2 + 2*I*a*d*e*f*x + I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*
e^2)*e^(3*d*x + 3*c) + (-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2)*e^(2*d
*x + 2*c) - (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**2/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 477, normalized size of antiderivative = 15.39

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)^2}{\left(fx+e\right)^2(i\,a\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-4*I*f*integrate(1/(-I*a*d*f^3*x^3 - 3*I*a*d*e*f^2*x^2 - 3*I*a*d*e^2*f*x -
 I*a*d*e^3 + (a*d*f^3*x^3*e^c + 3*a*d*e*f^2*x^2*e^c + 3*a*d*e^2*f*x*e^c +
a*d*e^3*e^c)*e^(d*x)), x) - 4*(I*e^(2*d*x + 2*c) + e^(d*x + c) - 2*I)/(2*I
*a*d*f^2*x^2 + 4*I*a*d*e*f*x + 2*I*a*d*e^2 + 2*(a*d*f^2*x^2*e^(3*c) + 2*a*
d*e*f*x*e^(3*c) + a*d*e^2*e^(3*c))*e^(3*d*x) - 2*(I*a*d*f^2*x^2*e^(2*c) +
2*I*a*d*e*f*x*e^(2*c) + I*a*d*e^2*e^(2*c))*e^(2*d*x) - 2*(a*d*f^2*x^2*e^c
+ 2*a*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)) - 4*integrate(-1/4*(I*d*f*x + I*
d*e + 2*f)/(a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x + a*d*e^3 - (a*d
*f^3*x^3*e^c + 3*a*d*e*f^2*x^2*e^c + 3*a*d*e^2*f*x*e^c + a*d*e^3*e^c)*e^(d
*x)), x) - 4*integrate(1/4*(I*d*f*x + I*d*e - 2*f)/(a*d*f^3*x^3 + 3*a*d*e*
f^2*x^2 + 3*a*d*e^2*f*x + a*d*e^3 + (a*d*f^3*x^3*e^c + 3*a*d*e*f^2*x^2*e^c
 + 3*a*d*e^2*f*x*e^c + a*d*e^3*e^c)*e^(d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 1.64 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{1}{\sinh\left(c+d\,x\right)^2\,\left(e+f\,x\right)^2\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(1/(sinh(c + d*x)^2*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(1/(sinh(c + d*x)^2*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.26

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \frac{\int \frac{\operatorname{csch}(dx+c)^2}{\sinh(dx+c)e^2i+2\sinh(dx+c)efix+\sinh(dx+c)f^2i\,x^2+e^2+2efx+f^2x^2}dx}{a}$$

input
```
int(csch(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(csch(c + d*x)**2/(sinh(c + d*x)*e**2*i + 2*sinh(c + d*x)*e*f*i*x + sin
h(c + d*x)*f**2*i*x**2 + e**2 + 2*e*f*x + f**2*x**2),x)/a
```

3.217 $\int \frac{(e+fx)^3 \operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 546

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{2i(e+fx)^3}{ad} - \frac{6f^2(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{ad^3} + \frac{3(e+fx)^3\text{arctanh}\left(e^{c+dx}\right)}{ad} + \frac{i(e+fx)^3\coth(c+dx)}{ad} - \frac{3f(e+fx)^2\text{csch}(c+dx)}{2ad^2} - \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{2ad} - \frac{6if(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2} - \frac{3if(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{ad^2} - \frac{3f^3\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^4} + \frac{9f(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{2ad^2} - \frac{12if^2(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3} + \frac{3f^3\,\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^4} - \frac{9f(e+fx)^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{2ad^2} - \frac{3if^2(e+fx)\,\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{ad^3} - \frac{9f^2(e+fx)\,\text{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3} + \frac{12if^3\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^4} + \frac{9f^2(e+fx)\,\text{PolyLog}\left(3,e^{c+dx}\right)}{ad^3} + \frac{3if^3\,\text{PolyLog}\left(3,e^{2(c+dx)}\right)}{2ad^4} + \frac{9f^3\,\text{PolyLog}\left(4,-e^{c+dx}\right)}{ad^4} - \frac{9f^3\,\text{PolyLog}\left(4,e^{c+dx}\right)}{ad^4} + \frac{i(e+fx)^3\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}$$

output
```
12*I*f^3*polylog(3,-I*exp(d*x+c))/a/d^4-6*f^2*(f*x+e)*arctanh(exp(d*x+c))/
a/d^3+3*(f*x+e)^3*arctanh(exp(d*x+c))/a/d+I*(f*x+e)^3*tanh(1/2*c+1/4*I*Pi+
1/2*d*x)/a/d-3/2*f*(f*x+e)^2*csch(d*x+c)/a/d^2-1/2*(f*x+e)^3*coth(d*x+c)*c
sch(d*x+c)/a/d-6*I*f*(f*x+e)^2*ln(1+I*exp(d*x+c))/a/d^2+2*I*(f*x+e)^3/a/d-
3*f^3*polylog(2,-exp(d*x+c))/a/d^4+9/2*f*(f*x+e)^2*polylog(2,-exp(d*x+c))/
a/d^2+3/2*I*f^3*polylog(3,exp(2*d*x+2*c))/a/d^4+3*f^3*polylog(2,exp(d*x+c)
)/a/d^4-9/2*f*(f*x+e)^2*polylog(2,exp(d*x+c))/a/d^2-3*I*f*(f*x+e)^2*ln(1-e
xp(2*d*x+2*c))/a/d^2-9*f^2*(f*x+e)*polylog(3,-exp(d*x+c))/a/d^3-12*I*f^2*(
f*x+e)*polylog(2,-I*exp(d*x+c))/a/d^3+9*f^2*(f*x+e)*polylog(3,exp(d*x+c))/
a/d^3+I*(f*x+e)^3*coth(d*x+c)/a/d+9*f^3*polylog(4,-exp(d*x+c))/a/d^4-9*f^3
*polylog(4,exp(d*x+c))/a/d^4-3*I*f^2*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a/d
^3
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2585 vs. 2(546) = 1092.

Time = 108.77 (sec) , antiderivative size = 2585, normalized size of antiderivative = 4.73

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Csch[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(-6*E^c*f*((e + f*x)^3/(3*E^c*f) + ((I + E^(-c))*(e + f*x)^2*Log[1 - I*E^(
-c - d*x)])/d - ((2*I)*(-I + E^c)*f*(d*(e + f*x)*PolyLog[2, I*E^(-c - d*x)
] + f*PolyLog[3, I*E^(-c - d*x)]))/(d^3*E^c)))/(a*d*(-I + E^c)) + ((12*I)*
d^3*e^2*E^(2*c)*f*x + (12*I)*d^3*e*E^(2*c)*f^2*x^2 + (4*I)*d^3*E^(2*c)*f^3
*x^3 - 6*d^3*e^3*ArcTanh[E^(c + d*x)] + 6*d^3*e^3*E^(2*c)*ArcTanh[E^(c + d
*x)] + 12*d*e*f^2*ArcTanh[E^(c + d*x)] - 12*d*e*E^(2*c)*f^2*ArcTanh[E^(c +
 d*x)] + 9*d^3*e^2*f*x*Log[1 - E^(c + d*x)] - 9*d^3*e^2*E^(2*c)*f*x*Log[1
- E^(c + d*x)] - 6*d*f^3*x*Log[1 - E^(c + d*x)] + 6*d*E^(2*c)*f^3*x*Log[1
- E^(c + d*x)] + 9*d^3*e*f^2*x^2*Log[1 - E^(c + d*x)] - 9*d^3*e*E^(2*c)*f^
2*x^2*Log[1 - E^(c + d*x)] + 3*d^3*f^3*x^3*Log[1 - E^(c + d*x)] - 3*d^3*E^
(2*c)*f^3*x^3*Log[1 - E^(c + d*x)] - 9*d^3*e^2*f*x*Log[1 + E^(c + d*x)] +
9*d^3*e^2*E^(2*c)*f*x*Log[1 + E^(c + d*x)] + 6*d*f^3*x*Log[1 + E^(c + d*x)
] - 6*d*E^(2*c)*f^3*x*Log[1 + E^(c + d*x)] - 9*d^3*e*f^2*x^2*Log[1 + E^(c
+ d*x)] + 9*d^3*e*E^(2*c)*f^2*x^2*Log[1 + E^(c + d*x)] - 3*d^3*f^3*x^3*Log
[1 + E^(c + d*x)] + 3*d^3*E^(2*c)*f^3*x^3*Log[1 + E^(c + d*x)] + (6*I)*d^2
*e^2*f*Log[1 - E^(2*(c + d*x))] - (6*I)*d^2*e^2*E^(2*c)*f*Log[1 - E^(2*(c
+ d*x))] + (12*I)*d^2*e*f^2*x*Log[1 - E^(2*(c + d*x))] - (12*I)*d^2*e*E^(2
*c)*f^2*x*Log[1 - E^(2*(c + d*x))] + (6*I)*d^2*f^3*x^2*Log[1 - E^(2*(c + d
*x))] - (6*I)*d^2*E^(2*c)*f^3*x^2*Log[1 - E^(2*(c + d*x))] + 3*(-1 + E^(2*
c))*f*(-2*f^2 + 3*d^2*(e + f*x)^2)*PolyLog[2, -E^(c + d*x)] - 3*(-1 + E...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6109}$$

$$\frac{\int (e+fx)^3\text{csch}^3(c+dx)dx}{a}-i\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int -i(e+fx)^3\csc(ic+idx)^3dx}{a}-i\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{i\int (e+fx)^3\csc(ic+idx)^3dx}{a}-i\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$\Big\downarrow$ 4674

$$-\frac{i\left(-\frac{3f^2\int -i(e+fx)\text{csch}(c+dx)dx}{d^2}+\frac{1}{2}\int -i(e+fx)^3\text{csch}(c+dx)dx-\frac{3if(e+fx)^2\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)^3\coth(c+dx)\text{csch}(}{2d}\right.}{a}$$
$$i\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$\Big\downarrow$ 26

$$-\frac{i\left(\frac{3if^2\int (e+fx)\text{csch}(c+dx)dx}{d^2}-\frac{1}{2}i\int (e+fx)^3\text{csch}(c+dx)dx-\frac{3if(e+fx)^2\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$
$$i\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$\Big\downarrow$ 3042

$$-\frac{i\left(\frac{3if^2\int i(e+fx)\csc(ic+idx)dx}{d^2}-\frac{1}{2}i\int i(e+fx)^3\csc(ic+idx)dx-\frac{3if(e+fx)^2\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)^3\coth(c+dx)\text{csch}(c+d}{2d}\right.}{a}$$
$$i\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$\Big\downarrow$ 26

$$-\frac{i\left(-\frac{3f^2\int (e+fx)\csc(ic+idx)dx}{d^2}+\frac{1}{2}\int (e+fx)^3\csc(ic+idx)dx-\frac{3if(e+fx)^2\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$
$$i\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$\Big\downarrow$ 4670

$$-\frac{i\left(-\frac{3f^2\left(\frac{if\int \log\left(1-e^{c+dx}\right)dx}{d}-\frac{if\int \log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)}{d^2}+\frac{1}{2}\left(\frac{3if\int (e+fx)^2\log(1-e^{c+dx})dx}{d}-\frac{3if\int (e+fx)^2}{}\right.\right.}{a}$$
$$i\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$\Big\downarrow$ 2715

$$-\frac{i\left(-\frac{3f^2\left(\frac{if\int e^{-c-dx}\log\left(1-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log\left(1+e^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)}{d^2}+\frac{1}{2}\left(\frac{3if\int(e+fx)^2\log(1-e^{c}}{d}\right.\right.}{}$$

$$i\int\frac{(e+fx)^3\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{2838}$$

$$-\frac{i\left(\frac{1}{2}\left(\frac{3if\int(e+fx)^2\log(1-e^{c+dx})dx}{d}-\frac{3if\int(e+fx)^2\log(1+e^{c+dx})dx}{d}+\frac{2i(e+fx)^3\text{arctanh}(e^{c+dx})}{d}\right)-\frac{3f^2\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c}\right.}{d}\right.}{}\right.}{a}$$

$$i\int\frac{(e+fx)^3\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{3011}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c}\right.}{d}\right.}{d}\right.\right.}{}$$

$$i\int\frac{(e+fx)^3\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{6109}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c}\right.}{d}\right.}{d}\right.\right.}{}$$

$$i\left(\frac{\int(e+fx)^3\text{csch}^2(c+dx)dx}{a}-i\int\frac{(e+fx)^3\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c}\right.}{d}\right.}{d}\right.\right.}{i\left(\frac{\int-(e+fx)^3\csc(ic+idx)^2dx}{a}-i\int\frac{(e+fx)^3\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)}$$

$\downarrow$ 25

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c}\right.}{d}\right.}{d}\right.\right.}{i\left(-\frac{\int(e+fx)^3\csc(ic+idx)^2dx}{a}-i\int\frac{(e+fx)^3\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)}$$

$\downarrow$ 4672

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c}\right.}{d}\right.}{d}\right.\right.}{i\left(-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{3if\int-i(e+fx)^2\coth(c+dx)dx}{d}}{a}-i\int\frac{(e+fx)^3\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)}$$

$\downarrow$ 26

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c}\right.}{d}\right.}{d}\right.\right.}{i\left(-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{3f\int(e+fx)^2\coth(c+dx)dx}{d}}{a}-i\int\frac{(e+fx)^3\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)}$$

$\downarrow$ 3042

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c}\cdots\right)}{d}\cdots\right)}{d}\cdots\right)\cdots\right)}{i\left(-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{3f\int -i(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}-i\int\frac{(e+fx)^3\operatorname{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)}$$

$$\Big\downarrow \; 26$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c}\cdots\right)}{d}\cdots\right)}{d}\cdots\right)\cdots\right)}{i\left(-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\int (e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}}{a}-i\int\frac{(e+fx)^3\operatorname{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)}$$

$$\Big\downarrow \; 4201$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c}\cdots\right)}{d}\cdots\right)}{d}\cdots\right)\cdots\right)}{i\left(-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\int\frac{e^{2c+2dx-i\pi}(e+fx)^2}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}-i\int\frac{(e+fx)^3\operatorname{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)}$$

$$\Big\downarrow \; 2620$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c}\cdots\right)}{d}\cdots\right)}{d}\cdots\right)\cdots\right)}{i\left(-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int (e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}-i\int\frac{(e+fx)^3\operatorname{csch}(\cdots}{i\sinh(c+dx\cdots}\cdots\right)}$$

$$\Big\downarrow \; 3011$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c}\right.}{d}\right.}{d}\right.\right.}{i\left(-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{2d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i}{}\right.}{d}}{a}\right.}$$

$\downarrow$ 2720

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c}\right.}{d}\right.}{d}\right.\right.}{i\left(-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(\right.}{}\right.}{d}\right.\right.}{d}}{a}\right.}$$

$\downarrow$ 6109

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c}\right.}{d}\right.}{d}\right.\right.}{i\left(-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(\right.}{}\right.}{d}\right.\right.}{d}}{a}\right.}$$

$\downarrow$ 3042

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c\cdots}\right.}{d}\cdots\right.}{d}\cdots\right.\right.}{i\left(-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(\cdots\right.}{\cdots}\right.}{d}\cdots\right.\right.}{d}\cdots\right.}{a}\cdots}$$

$\downarrow$ 26

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c\cdots}\right.}{d}\cdots\right.}{d}\cdots\right.\right.}{i\left(-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(\cdots\right.}{\cdots}\right.}{d}\cdots\right.\right.}{d}\cdots\right.}{a}\cdots}$$

$\downarrow$ 3799

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c\cdots}\right.}{d}\cdots\right.}{d}\cdots\right.\right.}{i\left(-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(\cdots\right.}{\cdots}\right.}{d}\cdots\right.\right.}{d}\cdots\right.}{a}\cdots}$$

$\downarrow$ 25

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c\cdots}\right)}{d}\cdots\right)}{d}\cdots\right.\right.}{i\left(-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(\cdots\right)}{\cdots}\right)}{d}\cdots\right.\right.}{d}\cdots}{a}\cdots\right.}$$

$\Big\downarrow$ 25

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c\cdots}\right)}{d}\cdots\right)}{d}\cdots\right.\right.}{i\left(-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(\cdots\right)}{\cdots}\right)}{d}\cdots\right.\right.}{d}\cdots}{a}\cdots\right.}$$

$\Big\downarrow$ 3042

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c\cdots}\right)}{d}\cdots\right)}{d}\cdots\right.\right.}{i\left(-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(\cdots\right)}{\cdots}\right)}{d}\cdots\right.\right.}{d}\cdots}{a}\cdots\right.}$$

$\Big\downarrow$ 4670

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c}\right.}{d}\right.}{d}\right.\right.}{i\left(-i\left(\frac{i\left(\frac{3if\int(e+fx)^2\log\left(1-e^{c+dx}\right)dx}{d}-\frac{3if\int(e+fx)^2\log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)^3\operatorname{arctanh}\left(e^{c+dx}\right)}{d}\right)}{a}-\frac{i\int(e+fx)^3\csc\left(\frac{ic}{2}\right.}{2a}\right.\right.}$$

input
```
Int[((e + f*x)^3*Csch[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2123 vs. $2(504) = 1008$.

Time = 1.40 (sec) , antiderivative size = 2124, normalized size of antiderivative = 3.89

method	result	size
risch	Expression too large to display	2124

input
```
int((f*x+e)^3*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
3*I/a/d^4*c^2*f^3*ln(1-exp(d*x+c))+6*I/a/d^4*c^2*f^3*ln(1+I*exp(d*x+c))+12
*I/a/d^2*e^2*f*ln(exp(d*x+c))-3*I/a/d^2*e^2*f*ln(1+exp(2*d*x+2*c))-3*I/a/d
^2*e^2*f*ln(exp(d*x+c)+1)-3*I/a/d^2*e^2*f*ln(exp(d*x+c)-1)+12*I/a/d^3*e*f^
2*c^2-12*I/a/d^3*c^2*f^3*x-6*I/a/d^3*e*f^2*polylog(2,-exp(d*x+c))-12*I/a/d
^3*e*f^2*polylog(2,-I*exp(d*x+c))-6*I/a/d^3*e*f^2*polylog(2,exp(d*x+c))+12
*I/a/d*e*f^2*x^2+12*I/a/d^4*c^2*f^3*ln(exp(d*x+c))-3*I/a/d^2*f^3*ln(exp(d*
x+c)+1)*x^2-6*I/a/d^3*f^3*polylog(2,-exp(d*x+c))*x-3*I/a/d^2*f^3*ln(1-exp(
d*x+c))*x^2-6*I/a/d^3*f^3*polylog(2,exp(d*x+c))*x-6*I/a/d^2*f^3*ln(1+I*exp
(d*x+c))*x^2-12*I/a/d^3*f^3*polylog(2,-I*exp(d*x+c))*x-3*I/a/d^4*c^2*f^3*l
n(1+exp(2*d*x+2*c))-3*I/a/d^4*c^2*f^3*ln(exp(d*x+c)-1)-12/a/d^3*c*f^2*e*ar
ctan(exp(d*x+c))-3*f^3*polylog(2,-exp(d*x+c))/a/d^4+3*f^3*polylog(2,exp(d*
x+c))/a/d^4+3/2/a/d*e^3*ln(exp(d*x+c)+1)-3/2/a/d*e^3*ln(exp(d*x+c)-1)-(I*d
*e^3*exp(d*x+c)+3*I*f^3*x^2*exp(d*x+c)+3*I*e^2*f*exp(d*x+c)+3*d*e^3*exp(4*
d*x+4*c)+3*e^2*f*exp(4*d*x+4*c)-6*I*e*f^2*x*exp(3*d*x+3*c)-3*I*d*f^3*x^3*e
xp(3*d*x+3*c)+9*d*e*f^2*x^2*exp(4*d*x+4*c)+9*d*e^2*f*x*exp(4*d*x+4*c)-9*I*
d*e*f^2*x^2*exp(3*d*x+3*c)+12*d*e*f^2*x^2+12*d*e^2*f*x+3*I*d*e*f^2*x^2*exp
(d*x+c)+3*I*d*e^2*f*x*exp(d*x+c)-9*I*d*e^2*f*x*exp(3*d*x+3*c)+6*e*f^2*x*ex
p(4*d*x+4*c)-3*I*f^3*x^2*exp(3*d*x+3*c)+3*d*f^3*x^3*exp(4*d*x+4*c)-3*I*d*e
^3*exp(3*d*x+3*c)-3*I*e^2*f*exp(3*d*x+3*c)+6*I*e*f^2*x*exp(d*x+c)+I*d*f^3*
x^3*exp(d*x+c)+4*d*e^3-3*f^3*x^2*exp(2*d*x+2*c)-5*f^3*x^3*d*exp(2*d*x+2...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4252 vs. $2(485) = 970$.

Time = 0.19 (sec) , antiderivative size = 4252, normalized size of antiderivative = 7.79

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*csch(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1320 vs. $2(485) = 970$.

Time = 0.39 (sec) , antiderivative size = 1320, normalized size of antiderivative = 2.42

$$\int \frac{(e+fx)^3 \operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-1/2*e^3*(2*(-I*e^(-d*x - c) - 5*e^(-2*d*x - 2*c) + 3*I*e^(-3*d*x - 3*c) +
 3*e^(-4*d*x - 4*c) + 4)/((a*e^(-d*x - c) - 2*I*a*e^(-2*d*x - 2*c) - 2*a*e
^(-3*d*x - 3*c) + I*a*e^(-4*d*x - 4*c) + a*e^(-5*d*x - 5*c) + I*a)*d) - 3*
log(e^(-d*x - c) + 1)/(a*d) + 3*log(e^(-d*x - c) - 1)/(a*d)) + 6*I*e^2*f*x
/(a*d) - 6*I*e^2*f*log(I*e^(d*x + c) + 1)/(a*d^2) - (4*d*f^3*x^3 + 12*d*e*
f^2*x^2 + 12*d*e^2*f*x + 3*(d*f^3*x^3*e^(4*c) + e^2*f*e^(4*c) + (3*d*e*f^2
 + f^3)*x^2*e^(4*c) + (3*d*e^2*f + 2*e*f^2)*x*e^(4*c))*e^(4*d*x) - 3*(I*d*
f^3*x^3*e^(3*c) + I*e^2*f*e^(3*c) + (3*I*d*e*f^2 + I*f^3)*x^2*e^(3*c) + (3
*I*d*e^2*f + 2*I*e*f^2)*x*e^(3*c))*e^(3*d*x) - (5*d*f^3*x^3*e^(2*c) + 3*e^
2*f*e^(2*c) + 3*(5*d*e*f^2 + f^3)*x^2*e^(2*c) + 3*(5*d*e^2*f + 2*e*f^2)*x*
e^(2*c))*e^(2*d*x) + (I*d*f^3*x^3*e^c + 3*I*e^2*f*e^c - 3*(-I*d*e*f^2 - I*
f^3)*x^2*e^c - 3*(-I*d*e^2*f - 2*I*e*f^2)*x*e^c)*e^(d*x))/(a*d^2*e^(5*d*x
+ 5*c) - I*a*d^2*e^(4*d*x + 4*c) - 2*a*d^2*e^(3*d*x + 3*c) + 2*I*a*d^2*e^(
2*d*x + 2*c) + a*d^2*e^(d*x + c) - I*a*d^2) - 12*I*(d*x*log(I*e^(d*x + c)
+ 1) + dilog(-I*e^(d*x + c)))*e*f^2/(a*d^3) + 3/2*(d^3*x^3*log(e^(d*x + c)
 + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) + 6
*polylog(4, -e^(d*x + c)))*f^3/(a*d^4) - 3/2*(d^3*x^3*log(-e^(d*x + c) + 1
) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polyl
og(4, e^(d*x + c)))*f^3/(a*d^4) - 6*I*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*
d*x*dilog(-I*e^(d*x + c)) - 2*polylog(3, -I*e^(d*x + c)))*f^3/(a*d^4) -...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{{\left(e+f\,x\right)}^3}{{\mathrm{sinh}\left(c+d\,x\right)}^3\,\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

input
```
int((e + f*x)^3/(sinh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)^3/(sinh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$=\frac{\left(\int \frac{\mathrm{csch}(dx+c)^{3}}{\sinh(dx+c)i+1}dx\right)e^{3}+\left(\int \frac{\mathrm{csch}(dx+c)^{3}x^{3}}{\sinh(dx+c)i+1}dx\right)f^{3}+3\left(\int \frac{\mathrm{csch}(dx+c)^{3}x^{2}}{\sinh(dx+c)i+1}dx\right)e\,f^{2}+3\left(\int \frac{\mathrm{csch}(dx+c)^{3}x}{\sinh(dx+c)i+1}dx\right)e^{2}f}{a}$$

input
```
int((f*x+e)^3*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

output
```
(int(csch(c + d*x)**3/(sinh(c + d*x)*i + 1),x)*e**3 + int((csch(c + d*x)**
3*x**3)/(sinh(c + d*x)*i + 1),x)*f**3 + 3*int((csch(c + d*x)**3*x**2)/(sin
h(c + d*x)*i + 1),x)*e*f**2 + 3*int((csch(c + d*x)**3*x)/(sinh(c + d*x)*i
+ 1),x)*e**2*f)/a
```

3.218 $\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 368

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{2i(e+fx)^2}{ad}+\frac{3(e+fx)^2\text{arctanh}\left(e^{c+dx}\right)}{ad} - \frac{f^2\text{arctanh}(\cosh(c+dx))}{ad^3} + \frac{i(e+fx)^2\coth(c+dx)}{ad} - \frac{f(e+fx)\text{csch}(c+dx)}{ad^2} - \frac{(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{2ad} - \frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2} - \frac{2if(e+fx)\log\left(1-e^{2(c+dx)}\right)}{ad^2} + \frac{3f(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2} - \frac{4if^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3} - \frac{3f(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2} - \frac{if^2\,\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{ad^3} - \frac{3f^2\,\text{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3} + \frac{3f^2\,\text{PolyLog}\left(3,e^{c+dx}\right)}{ad^3} + \frac{i(e+fx)^2\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}$$

output

```
2*I*(f*x+e)^2/a/d+3*(f*x+e)^2*arctanh(exp(d*x+c))/a/d-f^2*arctanh(cosh(d*x
+c))/a/d^3+I*(f*x+e)^2*coth(d*x+c)/a/d-f*(f*x+e)*csch(d*x+c)/a/d^2-1/2*(f*
x+e)^2*coth(d*x+c)*csch(d*x+c)/a/d-4*I*f*(f*x+e)*ln(1+I*exp(d*x+c))/a/d^2-
2*I*f*(f*x+e)*ln(1-exp(2*d*x+2*c))/a/d^2+3*f*(f*x+e)*polylog(2,-exp(d*x+c)
)/a/d^2-4*I*f^2*polylog(2,-I*exp(d*x+c))/a/d^3-3*f*(f*x+e)*polylog(2,exp(d
*x+c))/a/d^2-I*f^2*polylog(2,exp(2*d*x+2*c))/a/d^3-3*f^2*polylog(3,-exp(d*
x+c))/a/d^3+3*f^2*polylog(3,exp(d*x+c))/a/d^3+I*(f*x+e)^2*tanh(1/2*c+1/4*I
*Pi+1/2*d*x)/a/d
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1496 vs. $2(368) = 736$.

Time = 8.57 (sec) , antiderivative size = 1496, normalized size of antiderivative = 4.07

$$\int \frac{(e + fx)^2 \text{csch}^3(c + dx)}{a + ia \sinh(c + dx)} \, dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Csch[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(-4*E^c*f*((e + f*x)^2/(2*E^c*f) + ((I + E^(-c))*(e + f*x)*Log[1 - I*E^(-c
 - d*x)])/d - ((1 + I*E^c)*f*PolyLog[2, I*E^(-c - d*x)])/(d^2*E^c)))/(a*d*
(-I + E^c)) + (-(d*(-1 + E^(2*c))*(3*d^2*e^2 - (4*I)*d*e*f - 2*f^2)*x) + d
*(-1 + E^(2*c))*(3*d^2*e^2 + (4*I)*d*e*f - 2*f^2)*x + (4*I)*d^2*(e + f*x)^
2 - 2*d*(-1 + E^(2*c))*(3*d*e + (2*I)*f)*f*x*Log[1 - E^(-c - d*x)] - 3*d^2
*(-1 + E^(2*c))*f^2*x^2*Log[1 - E^(-c - d*x)] + 2*d*(-1 + E^(2*c))*(3*d*e
- (2*I)*f)*f*x*Log[1 + E^(-c - d*x)] + 3*d^2*(-1 + E^(2*c))*f^2*x^2*Log[1
+ E^(-c - d*x)] - (-1 + E^(2*c))*(3*d^2*e^2 + (4*I)*d*e*f - 2*f^2)*Log[1 -
 E^(c + d*x)] + (-1 + E^(2*c))*(3*d^2*e^2 - (4*I)*d*e*f - 2*f^2)*Log[1 + E
^(c + d*x)] - 2*(-1 + E^(2*c))*(3*d*e - (2*I)*f)*f*PolyLog[2, -E^(-c - d*x
)] - 6*d*(-1 + E^(2*c))*f^2*x*PolyLog[2, -E^(-c - d*x)] + 2*(-1 + E^(2*c))
*(3*d*e + (2*I)*f)*f*PolyLog[2, E^(-c - d*x)] + 6*d*(-1 + E^(2*c))*f^2*x*P
olyLog[2, E^(-c - d*x)] - 6*(-1 + E^(2*c))*f^2*PolyLog[3, -E^(-c - d*x)] +
 6*(-1 + E^(2*c))*f^2*PolyLog[3, E^(-c - d*x)])/(2*a*d^3*(-1 + E^(2*c))) +
 (Csch[c]*Csch[c + d*x]^2*(2*e*f*Cosh[(d*x)/2] + 2*f^2*x*Cosh[(d*x)/2] + 2
*e*f*Cosh[(3*d*x)/2] + 2*f^2*x*Cosh[(3*d*x)/2] + (5*I)*d*e^2*Cosh[c - (d*x
)/2] + (10*I)*d*e*f*x*Cosh[c - (d*x)/2] + (5*I)*d*f^2*x^2*Cosh[c - (d*x)/2
] - I*d*e^2*Cosh[c + (d*x)/2] - (2*I)*d*e*f*x*Cosh[c + (d*x)/2] - I*d*f^2*
x^2*Cosh[c + (d*x)/2] - 2*e*f*Cosh[2*c + (d*x)/2] - 2*f^2*x*Cosh[2*c + (d*
x)/2] + I*d*e^2*Cosh[c + (3*d*x)/2] + (2*I)*d*e*f*x*Cosh[c + (3*d*x)/2]...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6109}$$

$$\frac{\int (e+fx)^2\text{csch}^3(c+dx)dx}{a}-i\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -i(e+fx)^2\csc(ic+idx)^3dx}{a}-i\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int (e+fx)^2\csc(ic+idx)^3dx}{a}-i\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{4674}$$

$$-\frac{i\left(-\frac{f^2\int -i\text{csch}(c+dx)dx}{d^2}+\frac{1}{2}\int -i(e+fx)^2\text{csch}(c+dx)dx-\frac{if(e+fx)\text{csch}(c+dx)}{d^2}-\frac{i(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}-i\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\left(\frac{if^2\int \text{csch}(c+dx)dx}{d^2}-\frac{1}{2}i\int (e+fx)^2\text{csch}(c+dx)dx-\frac{if(e+fx)\text{csch}(c+dx)}{d^2}-\frac{i(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}-i\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$-\frac{i\left(\frac{if^2\int i\csc(ic+idx)dx}{d^2}-\frac{1}{2}i\int i(e+fx)^2\csc(ic+idx)dx-\frac{if(e+fx)\text{csch}(c+dx)}{d^2}-\frac{i(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}-i\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\left(-\frac{f^2\int \csc(ic+idx)dx}{d^2}+\frac{1}{2}\int (e+fx)^2\csc(ic+idx)dx-\frac{if(e+fx)\text{csch}(c+dx)}{d^2}-\frac{i(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}-i\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{4257}$$

$$-\frac{i\left(\frac{1}{2}\int (e+fx)^2\csc(ic+idx)dx-\frac{if^2\text{arctanh}(\cosh(c+dx))}{d^3}-\frac{if(e+fx)\text{csch}(c+dx)}{d^2}-\frac{i(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}-i\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{4670}$$

$$-\frac{i\left(\frac{1}{2}\left(\frac{2if\int (e+fx)\log(1-e^{c+dx})dx}{d}-\frac{2if\int (e+fx)\log(1+e^{c+dx})dx}{d}+\frac{2i(e+fx)^2\text{arctanh}(e^{c+dx})}{d}\right)-\frac{if^2\text{arctanh}(\cosh(c+dx))}{d^3}-\right.}{a} i\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{3011}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int \text{PolyLog}(2,-e^{c+dx})dx}{d}-\frac{(e+fx)\text{PolyLog}(2,-e^{c+dx})}{d}\right)}{d}+\frac{2if\left(\frac{f\int \text{PolyLog}(2,e^{c+dx})dx}{d}-\frac{(e+fx)\text{PolyLog}(2,e^{c+dx})}{d}\right)}{d}+\frac{2i(e+\right.\right.}{a} i\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{2720}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}(2,-e^{c+dx})de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}(2,-e^{c+dx})}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}(2,e^{c+dx})de^{c+dx}}{d^2}-\frac{(e+fx)\text{Poly}\right.}{d}\right.\right.}{} i\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{6109}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}}{}\right.}{d}\right.\right.}{i\left(\frac{\int(e+fx)^2\operatorname{csch}^2(c+dx)dx}{a}-i\int\frac{(e+fx)^2\operatorname{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}}{}\right.}{d}\right.\right.}{i\left(\frac{\int-(e+fx)^2\csc(ic+idx)^2dx}{a}-i\int\frac{(e+fx)^2\operatorname{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}}{}\right.}{d}\right.\right.}{i\left(-\frac{\int(e+fx)^2\csc(ic+idx)^2dx}{a}-i\int\frac{(e+fx)^2\operatorname{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)}$$

$$\Big\downarrow \text{4672}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}}{}\right.}{d}\right.\right.}{i\left(-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2if\int-i(e+fx)\coth(c+dx)dx}{d}}{a}-i\int\frac{(e+fx)^2\operatorname{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyL}}{}\right.}{d}\right.\right.}{i\left(-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2f\int(e+fx)\coth(c+dx)dx}{d}}{a}-i\int\frac{(e+fx)^2\operatorname{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyL}}{}\right.}{d}\right.\right.}{i\left(-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2f\int-i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}-i\int\frac{(e+fx)^2\operatorname{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyL}}{}\right.}{d}\right.\right.}{i\left(-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\int(e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}}{a}-i\int\frac{(e+fx)^2\operatorname{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyL}}{}\right.}{d}\right.\right.}{i\left(-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\int\frac{e^{2c+2dx-i\pi}(e+fx)}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}-i\int\frac{(e+fx)^2\operatorname{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyL}}{}\right.}{d}\right.\right.}{i\left(-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int\log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}-i\int\frac{(e+fx)^2\text{csch}(c+d}{i\sinh(c+dx)a+a}\right.}$$

$$\Big\downarrow \textbf{2715}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyL}}{}\right.}{d}\right.\right.}{i\left(-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\log\left(1+e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}-i\int\frac{(e}{}\right.}$$

$$\Big\downarrow \textbf{2838}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyL}}{}\right.}{d}\right.\right.}{i\left(-i\int\frac{(e+fx)^2\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+}{2}\right.}{d}}{a}\right.}$$

$$\Big\downarrow \textbf{6109}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyL}}{}\right.}{d}\right.\right.}{i\left(-i\left(\frac{\int(e+fx)^2\operatorname{csch}(c+dx)dx}{a}-i\int\frac{(e+fx)^2}{i\sinh(c+dx)a+a}dx\right)-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-}\right)}{4d^2}\right.\right.}{}}{a}\right.}$$

$$\Big\downarrow \; 3042$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyL}}{}\right.}{d}\right.\right.}{i\left(-i\left(\frac{\int i(e+fx)^2\csc(ic+idx)dx}{a}-i\int\frac{(e+fx)^2}{\sin(ic+idx)a+a}dx\right)-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}\right.\right.}{}}{a}\right.}$$

$$\Big\downarrow \; 26$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyL}}{}\right.}{d}\right.\right.}{i\left(-i\left(\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}-i\int\frac{(e+fx)^2}{\sin(ic+idx)a+a}dx\right)-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}\right.\right.}{}}{a}\right.}$$

$$\Big\downarrow \; 3799$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyL}}{}\right.}{d}\right.\right.}{i\left(-i\left(\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}-\frac{i\int-(e+fx)^2\operatorname{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)dx}{2a}\right)-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{Pol}}{}\right.\right.}{}\right.}$$

$\downarrow$ 25

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyL}}{}\right.}{d}\right.\right.}{i\left(-i\left(\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}+\frac{i\int-(e+fx)^2\operatorname{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}\right)-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{Pol}}{}\right.\right.}{}\right.}$$

$\downarrow$ 25

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyL}}{}\right.}{d}\right.\right.}{i\left(-i\left(\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}-\frac{i\int(e+fx)^2\operatorname{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}\right)-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLo}}{}\right.\right.}{}\right.}$$

$\downarrow$ 3042

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyI}}{}\right.}{d}\right.\right.}{i\left(-i\left(\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}-\frac{i\int(e+fx)^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}\right)-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLo}}{}\right.\right.}{}\right.}$$

$$\Big\downarrow \text{4670}$$

$$-i\left(-i\left(\frac{i\left(\frac{2if\int(e+fx)\log(1-e^{c+dx})dx}{d}-\frac{2if\int(e+fx)\log(1+e^{c+dx})dx}{d}+\frac{2i(e+fx)^2\operatorname{arctanh}(e^{c+dx})}{d}\right)}{a}-\frac{i\int(e+fx)^2\csc\left(\frac{ic}{2}\right.}{2a}\right.\right.$$

$$\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}}{d}\right.}{d}\right.\right.}{}$$

input
```
Int[((e + f*x)^2*Csch[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1146 vs. 2(343) = 686.

Time = 1.00 (sec) , antiderivative size = 1147, normalized size of antiderivative = 3.12

method	result	size
risch	Expression too large to display	1147

input
```
int((f*x+e)^2*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
3/a/d*e*f*ln(exp(d*x+c)+1)*x-(6*d*e*f*x*exp(4*d*x+4*c)-2*I*f^2*x*exp(3*d*x
+3*c)+I*d*f^2*x^2*exp(d*x+c)-3*I*d*f^2*x^2*exp(3*d*x+3*c)+I*d*e^2*exp(d*x+
c)+4*d*f^2*x^2+4*d*e^2+8*d*e*f*x-5*e^2*d*exp(2*d*x+2*c)-10*e*f*x*d*exp(2*d
*x+2*c)-5*f^2*x^2*d*exp(2*d*x+2*c)+3*d*f^2*x^2*exp(4*d*x+4*c)+2*I*f^2*x*ex
p(d*x+c)+2*I*exp(d*x+c)*e*f+2*I*d*e*f*x*exp(d*x+c)-6*I*d*e*f*x*exp(3*d*x+3
*c)-2*I*e*f*exp(3*d*x+3*c)-3*I*d*e^2*exp(3*d*x+3*c)+3*d*e^2*exp(4*d*x+4*c)
+2*f^2*x*exp(4*d*x+4*c)+2*e*f*exp(4*d*x+4*c)-2*f^2*x*exp(2*d*x+2*c)-2*e*f*
exp(2*d*x+2*c))/(exp(2*d*x+2*c)-1)^2/d^2/(exp(d*x+c)-I)/a+3/2/a/d^3*f^2*ln
(1-exp(d*x+c))*c^2+3/2/a/d*f^2*ln(exp(d*x+c)+1)*x^2-1/a/d^3*f^2*ln(exp(d*x
+c)+1)+1/a/d^3*f^2*ln(exp(d*x+c)-1)+3/2/a/d*e^2*ln(exp(d*x+c)+1)-3/2/a/d*e
^2*ln(exp(d*x+c)-1)-3*f^2*polylog(3,-exp(d*x+c))/a/d^3+3*f^2*polylog(3,exp
(d*x+c))/a/d^3+3/a/d^2*f^2*polylog(2,-exp(d*x+c))*x-3/2/a/d*f^2*ln(1-exp(d
*x+c))*x^2-3/a/d^2*f^2*polylog(2,exp(d*x+c))*x-3/a/d^2*e*f*polylog(2,exp(d
*x+c))+3/a/d^2*e*f*polylog(2,-exp(d*x+c))-3/2/a/d^3*c^2*f^2*ln(exp(d*x+c)-
1)+4*I/a/d^3*c^2*f^2+4*I/a/d*f^2*x^2-2*I/a/d^3*f^2*polylog(2,exp(d*x+c))-2
*I/a/d^3*f^2*polylog(2,-exp(d*x+c))-3/a/d*e*f*ln(1-exp(d*x+c))*x+8*I/a/d^2
*e*f*ln(exp(d*x+c))-2*I/a/d^3*f^2*ln(1-exp(d*x+c))*c-4*I/a/d^3*f^2*ln(1+I*
exp(d*x+c))*c-8*I/a/d^3*f^2*c*ln(exp(d*x+c))-2*I/a/d^2*e*f*ln(1+exp(2*d*x+
2*c))-2*I/a/d^2*e*f*ln(exp(d*x+c)+1)-2*I/a/d^2*e*f*ln(exp(d*x+c)-1)-2*I/a/
d^2*f^2*ln(exp(d*x+c)+1)*x-2*I/a/d^2*f^2*ln(1-exp(d*x+c))*x-4*I/a/d^2*f...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2215 vs. 2(329) = 658.

Time = 0.13 (sec) , antiderivative size = 2215, normalized size of antiderivative = 6.02

$$\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
-1/2*(8*d^2*e^2 - 16*c*d*e*f + 8*c^2*f^2 + 8*(I*f^2*e^(5*d*x + 5*c) + f^2*
e^(4*d*x + 4*c) - 2*I*f^2*e^(3*d*x + 3*c) - 2*f^2*e^(2*d*x + 2*c) + I*f^2*
e^(d*x + c) + f^2)*dilog(-I*e^(d*x + c)) + 2*(3*I*d*f^2*x + 3*I*d*e*f + 2*
f^2 - (3*d*f^2*x + 3*d*e*f - 2*I*f^2)*e^(5*d*x + 5*c) + (3*I*d*f^2*x + 3*I
*d*e*f + 2*f^2)*e^(4*d*x + 4*c) + 2*(3*d*f^2*x + 3*d*e*f - 2*I*f^2)*e^(3*d
*x + 3*c) + 2*(-3*I*d*f^2*x - 3*I*d*e*f - 2*f^2)*e^(2*d*x + 2*c) - (3*d*f^
2*x + 3*d*e*f - 2*I*f^2)*e^(d*x + c))*dilog(-e^(d*x + c)) + 2*(-3*I*d*f^2*
x - 3*I*d*e*f + 2*f^2 + (3*d*f^2*x + 3*d*e*f + 2*I*f^2)*e^(5*d*x + 5*c) +
(-3*I*d*f^2*x - 3*I*d*e*f + 2*f^2)*e^(4*d*x + 4*c) - 2*(3*d*f^2*x + 3*d*e*
f + 2*I*f^2)*e^(3*d*x + 3*c) + 2*(3*I*d*f^2*x + 3*I*d*e*f - 2*f^2)*e^(2*d*
x + 2*c) + (3*d*f^2*x + 3*d*e*f + 2*I*f^2)*e^(d*x + c))*dilog(e^(d*x + c))
 + 8*(-I*d^2*f^2*x^2 - 2*I*d^2*e*f*x - 2*I*c*d*e*f + I*c^2*f^2)*e^(5*d*x +
 5*c) - 2*(d^2*f^2*x^2 - 3*d^2*e^2 + 2*(4*c - 1)*d*e*f - 4*c^2*f^2 + 2*(d^
2*e*f - d*f^2)*x)*e^(4*d*x + 4*c) + 2*(5*I*d^2*f^2*x^2 - 3*I*d^2*e^2 + 2*(
8*I*c - I)*d*e*f - 8*I*c^2*f^2 + 2*(5*I*d^2*e*f - I*d*f^2)*x)*e^(3*d*x + 3
*c) + 2*(3*d^2*f^2*x^2 - 5*d^2*e^2 + 2*(8*c - 1)*d*e*f - 8*c^2*f^2 + 2*(3*
d^2*e*f - d*f^2)*x)*e^(2*d*x + 2*c) + 2*(-3*I*d^2*f^2*x^2 + I*d^2*e^2 + 2*
(-4*I*c + I)*d*e*f + 4*I*c^2*f^2 + 2*(-3*I*d^2*e*f + I*d*f^2)*x)*e^(d*x +
c) - (-3*I*d^2*f^2*x^2 - 3*I*d^2*e^2 - 4*d*e*f + 2*I*f^2 - 2*(3*I*d^2*e*f
+ 2*d*f^2)*x + (3*d^2*f^2*x^2 + 3*d^2*e^2 - 4*I*d*e*f - 2*f^2 + 2*(3*d^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)**2*csch(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 863 vs. $2(329) = 658$.

Time = 0.35 (sec) , antiderivative size = 863, normalized size of antiderivative = 2.35

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-1/2*e^2*(2*(-I*e^(-d*x - c) - 5*e^(-2*d*x - 2*c) + 3*I*e^(-3*d*x - 3*c) +
 3*e^(-4*d*x - 4*c) + 4)/((a*e^(-d*x - c) - 2*I*a*e^(-2*d*x - 2*c) - 2*a*e
^(-3*d*x - 3*c) + I*a*e^(-4*d*x - 4*c) + a*e^(-5*d*x - 5*c) + I*a)*d) - 3*
log(e^(-d*x - c) + 1)/(a*d) + 3*log(e^(-d*x - c) - 1)/(a*d)) + 2*I*f^2*x^2
/(a*d) + 4*I*e*f*x/(a*d) - (4*d*f^2*x^2 + 8*d*e*f*x + (3*d*f^2*x^2*e^(4*c)
 + 2*e*f*e^(4*c) + 2*(3*d*e*f + f^2)*x*e^(4*c))*e^(4*d*x) + (-3*I*d*f^2*x^
2*e^(3*c) - 2*I*e*f*e^(3*c) - 2*(3*I*d*e*f + I*f^2)*x*e^(3*c))*e^(3*d*x) -
 (5*d*f^2*x^2*e^(2*c) + 2*e*f*e^(2*c) + 2*(5*d*e*f + f^2)*x*e^(2*c))*e^(2*
d*x) + (I*d*f^2*x^2*e^c + 2*I*e*f*e^c - 2*(-I*d*e*f - I*f^2)*x*e^c)*e^(d*x
))/(a*d^2*e^(5*d*x + 5*c) - I*a*d^2*e^(4*d*x + 4*c) - 2*a*d^2*e^(3*d*x + 3
*c) + 2*I*a*d^2*e^(2*d*x + 2*c) + a*d^2*e^(d*x + c) - I*a*d^2) - 4*I*e*f*l
og(I*e^(d*x + c) + 1)/(a*d^2) + 3/2*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*
dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*f^2/(a*d^3) - 3/2*(d^2*x
^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x
+ c)))*f^2/(a*d^3) - 4*I*(d*x*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c
)))*f^2/(a*d^3) + (2*I*d*e*f + f^2)*x/(a*d^2) + (2*I*d*e*f - f^2)*x/(a*d^2
) + (3*d*e*f - 2*I*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(
a*d^3) - (3*d*e*f + 2*I*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c
)))/(a*d^3) - (2*I*d*e*f + f^2)*log(e^(d*x + c) + 1)/(a*d^3) - (2*I*d*e*f
- f^2)*log(e^(d*x + c) - 1)/(a*d^3) + 1/2*(d^3*f^2*x^3 + (3*d*e*f + 2*I...
```

Giac [F]

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\operatorname{csch}\left(dx+c\right)^3}{i\,a\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*csch(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(e+f\,x)^2}{\sinh\left(c+d\,x\right)^3\,\left(a+a\sinh\left(c+d\,x\right)\,1\text{i}\right)}\,dx$$

input
```
int((e + f*x)^2/(sinh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)^2/(sinh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(\int \frac{\text{csch}(dx+c)^3}{\sinh(dx+c)i+1}dx\right)e^2+\left(\int \frac{\text{csch}(dx+c)^3x^2}{\sinh(dx+c)i+1}dx\right)f^2+2\left(\int \frac{\text{csch}(dx+c)^3x}{\sinh(dx+c)i+1}dx\right)ef}{a}$$

input
```
int((f*x+e)^2*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

output
```
(int(csch(c + d*x)**3/(sinh(c + d*x)*i + 1),x)*e**2 + int((csch(c + d*x)**
3*x**2)/(sinh(c + d*x)*i + 1),x)*f**2 + 2*int((csch(c + d*x)**3*x)/(sinh(c
 + d*x)*i + 1),x)*e*f)/a
```

3.219 $\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 214

$$\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{3(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{ad}+\frac{i(e+fx)\coth(c+dx)}{ad}-\frac{f\text{csch}(c+dx)}{2ad^2}-\frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2ad}-\frac{2if\log\left(\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2}-\frac{if\log(\sinh(c+dx))}{ad^2}+\frac{3f\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{2ad^2}-\frac{3f\,\text{PolyLog}\left(2,e^{c+dx}\right)}{2ad^2}+\frac{i(e+fx)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}$$

output
```
3*(f*x+e)*arctanh(exp(d*x+c))/a/d+I*(f*x+e)*coth(d*x+c)/a/d-1/2*f*csch(d*x
+c)/a/d^2-1/2*(f*x+e)*coth(d*x+c)*csch(d*x+c)/a/d-2*I*f*ln(cosh(1/2*c+1/4*
I*Pi+1/2*d*x))/a/d^2-I*f*ln(sinh(d*x+c))/a/d^2+3/2*f*polylog(2,-exp(d*x+c)
)/a/d^2-3/2*f*polylog(2,exp(d*x+c))/a/d^2+I*(f*x+e)*tanh(1/2*c+1/4*I*Pi+1/
2*d*x)/a/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 461 vs. $2(214) = 428$.

Time = 3.43 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.15

$$\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(2i(if+2d(e+fx))\cosh\left(\frac{1}{2}(c+dx)\right)\left(i+\coth\left(\frac{1}{2}(c+dx)\right)\right)-\right.}{}$$

input
```
Integrate[((e + f*x)*Csch[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*((2*I)*(I*f + 2*d*(e + f*x))*Co
sh[(c + d*x)/2]*(I + Coth[(c + d*x)/2]) - d*(e + f*x)*(I + Coth[(c + d*x)/
2])*Csch[(c + d*x)/2] - 8*f*(c + d*x)*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x
)/2]) + 16*f*ArcTan[Tanh[(c + d*x)/2]]*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*
x)/2]) + 4*((-2*I)*f*(c + d*x) - ((2*I)*f + 3*d*(e + f*x))*Log[1 - E^(-c -
 d*x)] + ((-2*I)*f + 3*d*(e + f*x))*Log[1 + E^(-c - d*x)] - 3*f*PolyLog[2,
 -E^(-c - d*x)] + 3*f*PolyLog[2, E^(-c - d*x)])*(Cosh[(c + d*x)/2] + I*Sin
h[(c + d*x)/2]) + (16*I)*d*(e + f*x)*Sinh[(c + d*x)/2] + 8*f*Log[Cosh[c +
d*x]]*((-I)*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2]) + 2*(f + (2*I)*d*(e + f
*x))*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*Tanh[(c + d*x)/2] - I*d*(e
+ f*x)*Sech[(c + d*x)/2]*(-I + Tanh[(c + d*x)/2])))/(8*d^2*(a + I*a*Sinh[c
 + d*x]))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$\Big\downarrow \ 6109$$

$$\frac{\int (e+fx)\text{csch}^3(c+dx)dx}{a}-i\int \frac{(e+fx)\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -i(e+fx)\csc(ic+idx)^3dx}{a}-i\int \frac{(e+fx)\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int (e+fx)\csc(ic+idx)^3dx}{a}-i\int \frac{(e+fx)\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{4673}$$

$$-\frac{i\left(\frac{1}{2}\int -i(e+fx)\text{csch}(c+dx)dx-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}-i\int \frac{(e+fx)\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\left(-\frac{1}{2}i\int (e+fx)\text{csch}(c+dx)dx-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}-i\int \frac{(e+fx)\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$-\frac{i\left(-\frac{1}{2}i\int i(e+fx)\csc(ic+idx)dx-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}-i\int \frac{(e+fx)\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\left(\frac{1}{2}\int (e+fx)\csc(ic+idx)dx-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}-i\int \frac{(e+fx)\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{4670}$$

$$-\frac{i\left(\frac{1}{2}\left(\frac{if\int \log\left(1-e^{c+dx}\right)dx}{d}-\frac{if\int \log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+}{2d}\right.}{a}$$
$$i\int \frac{(e+fx)\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{2715}$$

$$-\frac{i\left(\frac{1}{2}\left(\frac{if\int e^{-c-dx}\log(1-e^{c+dx})de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log(1+e^{c+dx})de^{c+dx}}{d^2}+\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+}{}\right.}{a}$$

$$i\int \frac{(e+fx)\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$\Big\downarrow$ 2838

$$-i\int \frac{(e+fx)\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\text{ PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\text{ PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+d}{2d}\right.}{a}$$

$\Big\downarrow$ 6109

$$-i\left(\frac{\int(e+fx)\text{csch}^2(c+dx)dx}{a}-i\int\frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\text{ PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\text{ PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+d}{2d}\right.}{a}$$

$\Big\downarrow$ 3042

$$-i\left(\frac{\int-\left((e+fx)\csc(ic+idx)^2\right)dx}{a}-i\int\frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\text{ PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\text{ PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+d}{2d}\right.}{a}$$

$\Big\downarrow$ 25

$$-i\left(-\frac{\int(e+fx)\csc(ic+idx)^2dx}{a}-i\int\frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\text{ PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\text{ PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+d}{2d}\right.}{a}$$

$\Big\downarrow$ 4672

$$-i\left(-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{if\int-i\coth(c+dx)dx}{d}}{a}-i\int\frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\text{ PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\text{ PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+d}{2d}\right.}{a}$$

$\Big\downarrow$ 26

$$-i\left(-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\int\coth(c+dx)dx}{d}}{a}-i\int\frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)-$$

$$\frac{i\Bigg(\frac{1}{2}\Bigg(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\Bigg)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+a}{2d}}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-i\left(-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\int-i\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}-i\int\frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)-$$

$$\frac{i\Bigg(\frac{1}{2}\Bigg(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\Bigg)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+a}{2d}}{a}$$

$$\Big\downarrow \text{ 26}$$

$$-i\left(-\frac{\frac{(e+fx)\coth(c+dx)}{d}+\frac{if\int\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}}{a}-i\int\frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)-$$

$$\frac{i\Bigg(\frac{1}{2}\Bigg(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\Bigg)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+a}{2d}}{a}$$

$$\Big\downarrow \text{ 3956}$$

$$-i\left(-i\int\frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}\right)-$$

$$\frac{i\Bigg(\frac{1}{2}\Bigg(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\Bigg)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+a}{2d}}{a}$$

$$\Big\downarrow \text{ 6109}$$

$$-i\left(-i\left(\frac{\int(e+fx)\text{csch}(c+dx)dx}{a}-i\int\frac{e+fx}{i\sinh(c+dx)a+a}dx\right)-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}\right)-$$

$$\frac{i\Bigg(\frac{1}{2}\Bigg(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\Bigg)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+a}{2d}}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-i\left(-i\left(\frac{\int i(e+fx)\csc(ic+idx)dx}{a}-i\int\frac{e+fx}{\sin(ic+idx)a+a}dx\right)-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}\right)-$$
$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+d}{2d}\right.}{a}$$

$$\Big\downarrow \text{26}$$

$$-i\left(-i\left(\frac{i\int(e+fx)\csc(ic+idx)dx}{a}-i\int\frac{e+fx}{\sin(ic+idx)a+a}dx\right)-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}\right)-$$
$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+d}{2d}\right.}{a}$$

$$\Big\downarrow \text{3799}$$

$$-i\left(-i\left(\frac{i\int(e+fx)\csc(ic+idx)dx}{a}-\frac{i\int-\left((e+fx)\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)\right)dx}{2a}\right)-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh}{d^2}}{a}\right.$$
$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+d}{2d}\right.}{a}$$

$$\Big\downarrow \text{25}$$

$$-i\left(-i\left(\frac{i\int(e+fx)\csc(ic+idx)dx}{a}+\frac{i\int-\left((e+fx)\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)dx}{2a}\right)-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh}{d^2}}{a}\right.$$
$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+d}{2d}\right.}{a}$$

$$\Big\downarrow \text{25}$$

$$-i\left(-i\left(\frac{i\int(e+fx)\csc(ic+idx)dx}{a}-\frac{i\int(e+fx)\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}\right)-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx}{d^2}}{a}\right.$$
$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+d}{2d}\right.}{a}$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-i\left(\frac{i\int(e+fx)\csc(ic+idx)dx}{a}-\frac{i\int(e+fx)\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}\right)-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx}{d^2}}{a}\right.$$

$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+d}{2d}\right.}{a}$$

$$\Big\downarrow \; 4670$$

$$-i\left(-i\left(\frac{i\left(\frac{if\int\log(1-e^{c+dx})dx}{d}-\frac{if\int\log(1+e^{c+dx})dx}{d}+\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}\right)}{a}-\frac{i\int(e+fx)\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}\right.\right.$$

$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+d}{2d}\right.}{a}$$

$$\Big\downarrow \; 2715$$

$$-i\left(-i\left(\frac{i\left(\frac{if\int e^{-c-dx}\log(1-e^{c+dx})de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log(1+e^{c+dx})de^{c+dx}}{d^2}+\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}\right)}{a}-\frac{i\int(e+fx)\text{cs}}{}\right.\right.$$

$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+d}{2d}\right.}{a}$$

$$\Big\downarrow \; 2838$$

$$-i\left(-i\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)}{a}-\frac{i\int(e+fx)\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2d}{2a}\right.\right.$$

$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+d}{2d}\right.}{a}$$

$$\Big\downarrow \; 4672$$

$$-i\left(-i\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)}{a}-\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2if\int\right.}{2a}\right.\right.$$

$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+}{2d}\right.}{a}$$

$$\downarrow 26$$

$$-i\left(-i\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)}{a}-\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2f\int t\right.}{2a}\right.\right.$$

$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+d}{2d}\right.}{a}$$

input
```
Int[((e + f*x)*Csch[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 440 vs. 2(185) = 370.

Time = 0.93 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.06

method	result
risch	$-\frac{-3idfx\,e^{3dx+3c}-5\,e^{2dx+2c}dfx+3dfx\,e^{4dx+4c}-3ide\,e^{3dx+3c}-5\,e^{2dx+2c}de+3de\,e^{4dx+4c}+ide\,e^{dx+c}+4dxf+f\,e^{4dx+4c}-ie^{3dx+3c}}{\left(e^{2dx+2c}-1\right)^2d^2\left(e^{dx+c}-i\right)a}$

input
```
int((f*x+e)*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-(-3*I*d*f*x*exp(3*d*x+3*c)-5*exp(2*d*x+2*c)*d*f*x+3*d*f*x*exp(4*d*x+4*c)-
3*I*d*e*exp(3*d*x+3*c)-5*exp(2*d*x+2*c)*d*e+3*d*e*exp(4*d*x+4*c)+I*d*e*exp
(d*x+c)+4*d*x*f+f*exp(4*d*x+4*c)-I*exp(3*d*x+3*c)*f+I*d*f*x*exp(d*x+c)+4*d
*e-f*exp(2*d*x+2*c)+I*exp(d*x+c)*f)/(exp(2*d*x+2*c)-1)^2/d^2/(exp(d*x+c)-I
)/a+3/2*f*polylog(2,-exp(d*x+c))/a/d^2-3/2*f*polylog(2,exp(d*x+c))/a/d^2+2
/a/d^2*f*arctan(exp(d*x+c))+3/2/a/d*e*ln(exp(d*x+c)+1)-3/2/a/d*e*ln(exp(d*
x+c)-1)+3/2/a/d^2*c*f*ln(exp(d*x+c)-1)-3/2/a/d^2*f*ln(1-exp(d*x+c))*c-I/a/
d^2*f*ln(1+exp(2*d*x+2*c))-I/a/d^2*f*ln(exp(d*x+c)+1)-I/a/d^2*f*ln(exp(d*x
+c)-1)+4*I/a/d^2*f*ln(exp(d*x+c))+3/2/a/d*f*ln(exp(d*x+c)+1)*x-3/2/a/d*f*l
n(1-exp(d*x+c))*x
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 817 vs. $2(181) = 362$.

Time = 0.12 (sec) , antiderivative size = 817, normalized size of antiderivative = 3.82

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(8*d*e - 4*c*f - 3*(f*e^(5*d*x + 5*c) - I*f*e^(4*d*x + 4*c) - 2*f*e^(
3*d*x + 3*c) + 2*I*f*e^(2*d*x + 2*c) + f*e^(d*x + c) - I*f)*dilog(-e^(d*x
+ c)) + 3*(f*e^(5*d*x + 5*c) - I*f*e^(4*d*x + 4*c) - 2*f*e^(3*d*x + 3*c) +
 2*I*f*e^(2*d*x + 2*c) + f*e^(d*x + c) - I*f)*dilog(e^(d*x + c)) + 4*(-2*I
*d*f*x - I*c*f)*e^(5*d*x + 5*c) - 2*(d*f*x - 3*d*e + (2*c - 1)*f)*e^(4*d*x
 + 4*c) + 2*(5*I*d*f*x - 3*I*d*e + (4*I*c - I)*f)*e^(3*d*x + 3*c) + 2*(3*d
*f*x - 5*d*e + (4*c - 1)*f)*e^(2*d*x + 2*c) + 2*(-3*I*d*f*x + I*d*e + (-2*
I*c + I)*f)*e^(d*x + c) - (-3*I*d*f*x - 3*I*d*e + (3*d*f*x + 3*d*e - 2*I*f
)*e^(5*d*x + 5*c) + (-3*I*d*f*x - 3*I*d*e - 2*f)*e^(4*d*x + 4*c) - 2*(3*d*
f*x + 3*d*e - 2*I*f)*e^(3*d*x + 3*c) - 2*(-3*I*d*f*x - 3*I*d*e - 2*f)*e^(2
*d*x + 2*c) + (3*d*f*x + 3*d*e - 2*I*f)*e^(d*x + c) - 2*f)*log(e^(d*x + c)
 + 1) + 4*(I*f*e^(5*d*x + 5*c) + f*e^(4*d*x + 4*c) - 2*I*f*e^(3*d*x + 3*c)
 - 2*f*e^(2*d*x + 2*c) + I*f*e^(d*x + c) + f)*log(e^(d*x + c) - I) - (3*I*
d*e + (-3*I*c - 2)*f - (3*d*e - (3*c - 2*I)*f)*e^(5*d*x + 5*c) + (3*I*d*e
+ (-3*I*c - 2)*f)*e^(4*d*x + 4*c) + 2*(3*d*e - (3*c - 2*I)*f)*e^(3*d*x + 3
*c) - 2*(3*I*d*e + (-3*I*c - 2)*f)*e^(2*d*x + 2*c) - (3*d*e - (3*c - 2*I)*
f)*e^(d*x + c))*log(e^(d*x + c) - 1) + 3*(-I*d*f*x - I*c*f + (d*f*x + c*f)
*e^(5*d*x + 5*c) + (-I*d*f*x - I*c*f)*e^(4*d*x + 4*c) - 2*(d*f*x + c*f)*e^
(3*d*x + 3*c) + 2*(I*d*f*x + I*c*f)*e^(2*d*x + 2*c) + (d*f*x + c*f)*e^(d*x
 + c))*log(-e^(d*x + c) + 1))/(a*d^2*e^(5*d*x + 5*c) - I*a*d^2*e^(4*d*x...
```

Sympy [F]

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e\operatorname{csch}^3(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{fx\operatorname{csch}^3(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)*csch(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e*csch(c + d*x)**3/(sinh(c + d*x) - I), x) + Integral(f*x*csc
h(c + d*x)**3/(sinh(c + d*x) - I), x))/a
```

Maxima [F]

$$\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(fx+e)\,\text{csch}\,(dx+c)^3}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-(24*d*integrate(1/16*x/(a*d*e^(d*x + c) + a*d), x) + 24*d*integrate(1/16*
x/(a*d*e^(d*x + c) - a*d), x) + 8*(2*d*x*e^(5*d*x + 5*c) + 2*I*d*x + (I*d*
x*e^(4*c) + I*e^(4*c))*e^(4*d*x) - (d*x*e^(3*c) - e^(3*c))*e^(3*d*x) + (-I
*d*x*e^(2*c) - I*e^(2*c))*e^(2*d*x) + (d*x*e^c - e^c)*e^(d*x))/(8*I*a*d^2*
e^(5*d*x + 5*c) + 8*a*d^2*e^(4*d*x + 4*c) - 16*I*a*d^2*e^(3*d*x + 3*c) - 1
6*a*d^2*e^(2*d*x + 2*c) + 8*I*a*d^2*e^(d*x + c) + 8*a*d^2) - 2*I*(d*x + c)
/(a*d^2) + 2*I*log((e^(d*x + c) - I)*e^(-c))/(a*d^2) + I*log(e^(d*x + c) +
 1)/(a*d^2) + I*log(e^(d*x + c) - 1)/(a*d^2))*f - 1/2*e*(2*(-I*e^(-d*x - c
) - 5*e^(-2*d*x - 2*c) + 3*I*e^(-3*d*x - 3*c) + 3*e^(-4*d*x - 4*c) + 4)/((
a*e^(-d*x - c) - 2*I*a*e^(-2*d*x - 2*c) - 2*a*e^(-3*d*x - 3*c) + I*a*e^(-4
*d*x - 4*c) + a*e^(-5*d*x - 5*c) + I*a)*d) - 3*log(e^(-d*x - c) + 1)/(a*d)
 + 3*log(e^(-d*x - c) - 1)/(a*d))
```

Giac [F]

$$\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(fx+e)\,\text{csch}\,(dx+c)^3}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*csch(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{e+f\,x}{\sinh\left(c+d\,x\right)^3\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int((e + f*x)/(sinh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)/(sinh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(\int \frac{\operatorname{csch}(dx+c)^3}{\sinh(dx+c)i+1}dx\right)e+\left(\int \frac{\operatorname{csch}(dx+c)^3x}{\sinh(dx+c)i+1}dx\right)f}{a}$$

input
```
int((f*x+e)*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

output
```
(int(csch(c + d*x)**3/(sinh(c + d*x)*i + 1),x)*e + int((csch(c + d*x)**3*x
)/(sinh(c + d*x)*i + 1),x)*f)/a
```

3.220 $\int \frac{\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{3\operatorname{arctanh}(\cosh(c+dx))}{2ad} + \frac{2i\coth(c+dx)}{ad} - \frac{3\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} + \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{d(a+ia\sinh(c+dx))}$$

output
```
3/2*arctanh(cosh(d*x+c))/a/d+2*I*coth(d*x+c)/a/d-3/2*coth(d*x+c)*csch(d*x+
c)/a/d+coth(d*x+c)*csch(d*x+c)/d/(a+I*a*sinh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{4i\operatorname{csch}(2(c+dx)) - 3\operatorname{sech}(c+dx) + 3\operatorname{arctanh}\left(\sqrt{\cosh^2(c+dx)}\right)\sqrt{\cosh^2(c+dx)}\operatorname{sech}(c+dx) - \operatorname{csch}^2}{2ad}$$

input
```
Integrate[Csch[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((4*I)*Csch[2*(c + d*x)] - 3*Sech[c + d*x] + 3*ArcTanh[Sqrt[Cosh[c + d*x]^
2]]*Sqrt[Cosh[c + d*x]^2]*Sech[c + d*x] - Csch[c + d*x]^2*Sech[c + d*x] +
(4*I)*Tanh[c + d*x])/(2*a*d)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.11, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.792$, Rules used = {3042, 26, 3247, 26, 3042, 26, 3227, 25, 26, 3042, 25, 26, 4254, 24, 4255, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\sin(ic+idx)^3(a+a\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\sin(ic+idx)^3(\sin(ic+idx)a+a)}dx$$

$$\Big\downarrow \text{3247}$$

$$-i\left(\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{\int -i\text{csch}^3(c+dx)(3a-2ia\sinh(c+dx))dx}{a^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i\int \text{csch}^3(c+dx)(3a-2ia\sinh(c+dx))dx}{a^2}+\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i\int -\frac{i(3a-2a\sin(ic+idx))}{\sin(ic+idx)^3}dx}{a^2}+\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{\int \frac{3a-2a\sin(ic+idx)}{\sin(ic+idx)^3}dx}{a^2}+\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow \text{3227}$$

$$-i\left(\frac{-2a\int -\text{csch}^2(c+dx)dx+3a\int i\text{csch}^3(c+dx)dx}{a^2}+\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow \text{25}$$

$$-i\left(\frac{2a\int \text{csch}^2(c+dx)dx+3a\int i\text{csch}^3(c+dx)dx}{a^2}+\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{2a\int \text{csch}^2(c+dx)dx+3ia\int \text{csch}^3(c+dx)dx}{a^2}+\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{2a\int -\csc(ic+idx)^2dx+3ia\int -i\csc(ic+idx)^3dx}{a^2}+\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow \text{25}$$

$$-i\left(\frac{3ia\int -i\csc(ic+idx)^3dx-2a\int \csc(ic+idx)^2dx}{a^2}+\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{3a\int \csc(ic+idx)^3dx-2a\int \csc(ic+idx)^2dx}{a^2}+\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow \text{4254}$$

$$-i\left(\frac{3a\int \csc(ic+idx)^3dx-\frac{2ia\int 1d(-i\coth(c+dx))}{d}}{a^2}+\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow \text{24}$$

$$-i\left(\frac{-\frac{2a\coth(c+dx)}{d}+3a\int \csc(ic+idx)^3dx}{a^2}+\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow \text{4255}$$

$$-i\left(\frac{-\frac{2a\coth(c+dx)}{d}+3a\left(\frac{1}{2}\int -i\text{csch}(c+dx)dx-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a^2}+\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow 26$$

$$-i\left(\frac{-\frac{2a\coth(c+dx)}{d}+3a\left(-\frac{1}{2}i\int \operatorname{csch}(c+dx)dx-\frac{i\coth(c+dx)\operatorname{csch}(c+dx)}{2d}\right)}{a^2}+\frac{i\coth(c+dx)\operatorname{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow 3042$$

$$-i\left(\frac{-\frac{2a\coth(c+dx)}{d}+3a\left(-\frac{1}{2}i\int i\csc(ic+idx)dx-\frac{i\coth(c+dx)\operatorname{csch}(c+dx)}{2d}\right)}{a^2}+\frac{i\coth(c+dx)\operatorname{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow 26$$

$$-i\left(\frac{-\frac{2a\coth(c+dx)}{d}+3a\left(\frac{1}{2}\int \csc(ic+idx)dx-\frac{i\coth(c+dx)\operatorname{csch}(c+dx)}{2d}\right)}{a^2}+\frac{i\coth(c+dx)\operatorname{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow 4257$$

$$-i\left(\frac{-\frac{2a\coth(c+dx)}{d}+3a\left(\frac{i\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{i\coth(c+dx)\operatorname{csch}(c+dx)}{2d}\right)}{a^2}+\frac{i\coth(c+dx)\operatorname{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

input
```
Int[Csch[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(-I)*(((-2*a*Coth[c + d*x])/d + 3*a*(((I/2)*ArcTanh[Cosh[c + d*x]])/d - ((
I/2)*Coth[c + d*x]*Csch[c + d*x])/d))/a^2 + (I*Coth[c + d*x]*Csch[c + d*x]
)/(d*(a + I*a*Sinh[c + d*x])))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3247
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Simp[d/(a*(b*c - a*d))
   Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[
c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{2i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2}-\frac{1}{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{2i}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-6\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{8i}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}}{4da}$
default	$\frac{2i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2}-\frac{1}{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{2i}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-6\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{8i}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}}{4da}$
parallelrisch	$\frac{12\left(i-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+i\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^2+3i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+3\coth\left(\frac{dx}{2}+\frac{c}{2}\right)+24}{8da\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$
risch	$-\frac{-3i e^{3dx+3c}+i e^{dx+c}+3\,e^{4dx+4c}-5\,e^{2dx+2c}+4}{\left(e^{2dx+2c}-1\right)^2\left(e^{dx+c}-i\right)da}+\frac{3\ln\left(e^{dx+c}+1\right)}{2da}-\frac{3\ln\left(e^{dx+c}-1\right)}{2da}$

input
```
int(csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a*(2*I*tanh(1/2*d*x+1/2*c)+1/2*tanh(1/2*d*x+1/2*c)^2-1/2/tanh(1/2*d*
x+1/2*c)^2+2*I/tanh(1/2*d*x+1/2*c)-6*ln(tanh(1/2*d*x+1/2*c))+8*I/(-I+tanh(
1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(79) = 158.

Time = 0.11 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.69

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$=\frac{3\left(e^{(5\,dx+5\,c)}-i\,e^{(4\,dx+4\,c)}-2\,e^{(3\,dx+3\,c)}+2i\,e^{(2\,dx+2\,c)}+e^{(dx+c)}-i\right)\log\left(e^{(dx+c)}+1\right)-3\left(e^{(5\,dx+5\,c)}-i\,e^{(4}\right.}{2\left(ade^{(5\,dx+5\,c)}-i\,ade^{(4\,dx+4\,c)}-\right.}$$

input
```
integrate(csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(3*(e^(5*d*x + 5*c) - I*e^(4*d*x + 4*c) - 2*e^(3*d*x + 3*c) + 2*I*e^(2
*d*x + 2*c) + e^(d*x + c) - I)*log(e^(d*x + c) + 1) - 3*(e^(5*d*x + 5*c) -
 I*e^(4*d*x + 4*c) - 2*e^(3*d*x + 3*c) + 2*I*e^(2*d*x + 2*c) + e^(d*x + c)
 - I)*log(e^(d*x + c) - 1) - 6*e^(4*d*x + 4*c) + 6*I*e^(3*d*x + 3*c) + 10*
e^(2*d*x + 2*c) - 2*I*e^(d*x + c) - 8)/(a*d*e^(5*d*x + 5*c) - I*a*d*e^(4*d
*x + 4*c) - 2*a*d*e^(3*d*x + 3*c) + 2*I*a*d*e^(2*d*x + 2*c) + a*d*e^(d*x +
 c) - I*a*d)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int \frac{\operatorname{csch}^3(c+dx)}{\sinh(c+dx)-i}\,dx}{a}$$

input
```
integrate(csch(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(csch(c + d*x)**3/(sinh(c + d*x) - I), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.79

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{-i\,e^{(-dx-c)} - 5\,e^{(-2\,dx-2\,c)} + 3i\,e^{(-3\,dx-3\,c)} + 3\,e^{(-4\,dx-4\,c)} + 4}{\left(ae^{(-dx-c)} - 2i\,ae^{(-2\,dx-2\,c)} - 2\,ae^{(-3\,dx-3\,c)} + i\,ae^{(-4\,dx-4\,c)} + ae^{(-5\,dx-5\,c)} + i\,a\right)d} + \frac{3\,\log\left(e^{(-dx-c)}+1\right)}{2\,ad} - \frac{3\,\log\left(e^{(-dx-c)}-1\right)}{2\,ad}$$

input
```
integrate(csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-(-I*e^(-d*x - c) - 5*e^(-2*d*x - 2*c) + 3*I*e^(-3*d*x - 3*c) + 3*e^(-4*d*
x - 4*c) + 4)/((a*e^(-d*x - c) - 2*I*a*e^(-2*d*x - 2*c) - 2*a*e^(-3*d*x -
3*c) + I*a*e^(-4*d*x - 4*c) + a*e^(-5*d*x - 5*c) + I*a)*d) + 3/2*log(e^(-d
*x - c) + 1)/(a*d) - 3/2*log(e^(-d*x - c) - 1)/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\frac{3\log\left(e^{(dx+c)}+1\right)}{a} - \frac{3\log\left(e^{(dx+c)}-1\right)}{a} - \frac{2\left(e^{(3\,dx+3\,c)}-2i\,e^{(2\,dx+2\,c)}+e^{(dx+c)}+2i\right)}{a\left(e^{(2\,dx+2\,c)}-1\right)^2} - \frac{4i}{a\left(i\,e^{(dx+c)}+1\right)}}{2\,d}$$

input
```
integrate(csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(3*log(e^(d*x + c) + 1)/a - 3*log(e^(d*x + c) - 1)/a - 2*(e^(3*d*x + 3
*c) - 2*I*e^(2*d*x + 2*c) + e^(d*x + c) + 2*I)/(a*(e^(2*d*x + 2*c) - 1)^2)
 - 4*I/(a*(I*e^(d*x + c) + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.52

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{3\operatorname{atan}\left(\frac{e^{d\,x}\,e^{c}\,\sqrt{-a^2\,d^2}}{a\,d}\right)}{\sqrt{-a^2\,d^2}} - \frac{2}{a\,d\,\left(e^{c+d\,x}-\mathrm{i}\right)} - \frac{e^{c+d\,x}}{a\,d\,\left(e^{2\,c+2\,d\,x}-1\right)} - \frac{2\,e^{c+d\,x}}{a\,d\,\left(e^{2\,c+2\,d\,x}-1\right)^2} + \frac{2\mathrm{i}}{a\,d\,\left(e^{2\,c+2\,d\,x}-1\right)}$$

input
```
int(1/(sinh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
(3*atan((exp(d*x)*exp(c)*(-a^2*d^2)^(1/2))/(a*d)))/(-a^2*d^2)^(1/2) - 2/(a
*d*(exp(c + d*x) - 1i)) + 2i/(a*d*(exp(2*c + 2*d*x) - 1)) - exp(c + d*x)/(
a*d*(exp(2*c + 2*d*x) - 1)) - (2*exp(c + d*x))/(a*d*(exp(2*c + 2*d*x) - 1)
^2)
```

Reduce [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\int \frac{\operatorname{csch}(dx+c)^3}{\sinh(dx+c)i+1}dx}{a}$$

input
```
int(csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(csch(c + d*x)**3/(sinh(c + d*x)*i + 1),x)/a
```

3.221 $\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=\operatorname{Int}\left(\frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(csch(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Mathematica [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=\$\text{Aborted}$$

input
```
Integrate[Csch[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$
$$\Big\downarrow \text{ 6111}$$
$$\int \frac{\text{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}dx$$

input
```
Int[Csch[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\text{csch}\,(dx+c)^3}{(fx+e)\,(a+ia\sinh\,(dx+c))}dx$$

input
```
int(csch(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 903, normalized size of antiderivative = 29.13

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)^3}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(4*d*f*x + 4*d*e + (3*d*f*x + 3*d*e - f)*e^(4*d*x + 4*c) - (3*I*d*f*x + 3
*I*d*e - I*f)*e^(3*d*x + 3*c) - (5*d*f*x + 5*d*e - f)*e^(2*d*x + 2*c) - (-
I*d*f*x - I*d*e + I*f)*e^(d*x + c) - (-I*a*d^2*f^2*x^2 - 2*I*a*d^2*e*f*x -
 I*a*d^2*e^2 + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*e^(5*d*x + 5*c)
 + (-I*a*d^2*f^2*x^2 - 2*I*a*d^2*e*f*x - I*a*d^2*e^2)*e^(4*d*x + 4*c) - 2*
(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*e^(3*d*x + 3*c) - 2*(-I*a*d^2*
f^2*x^2 - 2*I*a*d^2*e*f*x - I*a*d^2*e^2)*e^(2*d*x + 2*c) + (a*d^2*f^2*x^2
+ 2*a*d^2*e*f*x + a*d^2*e^2)*e^(d*x + c))*integral((4*d*f^2*x + 4*d*e*f -
(3*d^2*f^2*x^2 + 3*d^2*e^2 + 2*d*e*f - 2*f^2 + 2*(3*d^2*e*f + d*f^2)*x)*e^
(2*d*x + 2*c) + (3*I*d^2*f^2*x^2 + 3*I*d^2*e^2 + 2*I*d*e*f - 2*I*f^2 - 2*(
-3*I*d^2*e*f - I*d*f^2)*x)*e^(d*x + c))/(I*a*d^2*f^3*x^3 + 3*I*a*d^2*e*f^2
*x^2 + 3*I*a*d^2*e^2*f*x + I*a*d^2*e^3 + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^
2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*e^(3*d*x + 3*c) + (-I*a*d^2*f^3*x^3 - 3*I
*a*d^2*e*f^2*x^2 - 3*I*a*d^2*e^2*f*x - I*a*d^2*e^3)*e^(2*d*x + 2*c) - (a*d
^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*e^(d*x + c))
, x))/(-I*a*d^2*f^2*x^2 - 2*I*a*d^2*e*f*x - I*a*d^2*e^2 + (a*d^2*f^2*x^2 +
 2*a*d^2*e*f*x + a*d^2*e^2)*e^(5*d*x + 5*c) + (-I*a*d^2*f^2*x^2 - 2*I*a*d^
2*e*f*x - I*a*d^2*e^2)*e^(4*d*x + 4*c) - 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x
+ a*d^2*e^2)*e^(3*d*x + 3*c) - 2*(-I*a*d^2*f^2*x^2 - 2*I*a*d^2*e*f*x - I*a
*d^2*e^2)*e^(2*d*x + 2*c) + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)...
```

Sympy [N/A]

Not integrable

Time = 57.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = -\frac{i\int \frac{\operatorname{csch}^3(c+dx)}{e\sinh(c+dx)-ie+fx\sinh(c+dx)-ifx}\,dx}{a}$$

input
```
integrate(csch(d*x+c)**3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(csch(c + d*x)**3/(e*sinh(c + d*x) - I*e + f*x*sinh(c + d*x) -
I*f*x), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 775, normalized size of antiderivative = 25.00

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)^3}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-8*f*integrate(1/(-4*I*a*d*f^2*x^2 - 8*I*a*d*e*f*x - 4*I*a*d*e^2 + 4*(a*d*
f^2*x^2*e^c + 2*a*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)), x) - 8*(4*d*f*x + 4
*d*e + (3*d*f*x*e^(4*c) + (3*d*e - f)*e^(4*c))*e^(4*d*x) + (-3*I*d*f*x*e^(
3*c) + (-3*I*d*e + I*f)*e^(3*c))*e^(3*d*x) - (5*d*f*x*e^(2*c) + (5*d*e - f
)*e^(2*c))*e^(2*d*x) + (I*d*f*x*e^c + (I*d*e - I*f)*e^c)*e^(d*x))/(-8*I*a*
d^2*f^2*x^2 - 16*I*a*d^2*e*f*x - 8*I*a*d^2*e^2 + 8*(a*d^2*f^2*x^2*e^(5*c)
+ 2*a*d^2*e*f*x*e^(5*c) + a*d^2*e^2*e^(5*c))*e^(5*d*x) - 8*(I*a*d^2*f^2*x^
2*e^(4*c) + 2*I*a*d^2*e*f*x*e^(4*c) + I*a*d^2*e^2*e^(4*c))*e^(4*d*x) - 16*
(a*d^2*f^2*x^2*e^(3*c) + 2*a*d^2*e*f*x*e^(3*c) + a*d^2*e^2*e^(3*c))*e^(3*d
*x) - 16*(-I*a*d^2*f^2*x^2*e^(2*c) - 2*I*a*d^2*e*f*x*e^(2*c) - I*a*d^2*e^2
*e^(2*c))*e^(2*d*x) + 8*(a*d^2*f^2*x^2*e^c + 2*a*d^2*e*f*x*e^c + a*d^2*e^2
*e^c)*e^(d*x)) - 8*integrate(1/16*(3*d^2*f^2*x^2 + 3*d^2*e^2 + 2*I*d*e*f -
 2*f^2 + 2*(3*d^2*e*f + I*d*f^2)*x)/(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3
*a*d^2*e^2*f*x + a*d^2*e^3 + (a*d^2*f^3*x^3*e^c + 3*a*d^2*e*f^2*x^2*e^c +
3*a*d^2*e^2*f*x*e^c + a*d^2*e^3*e^c)*e^(d*x)), x) - 8*integrate(-1/16*(3*d
^2*f^2*x^2 + 3*d^2*e^2 - 2*I*d*e*f - 2*f^2 + 2*(3*d^2*e*f - I*d*f^2)*x)/(a
*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 - (a*d^2*f^
3*x^3*e^c + 3*a*d^2*e*f^2*x^2*e^c + 3*a*d^2*e^2*f*x*e^c + a*d^2*e^3*e^c)*e
^(d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 4.75 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{1}{\sinh\left(c+d\,x\right)^3\,\left(e+f\,x\right)\,\left(a+a\,\sinh\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

input
```
int(1/(sinh(c + d*x)^3*(e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(1/(sinh(c + d*x)^3*(e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \frac{\int \frac{\operatorname{csch}(dx+c)^3}{\sinh(dx+c)ei+\sinh(dx+c)fix+e+fx}dx}{a}$$

input
```
int(csch(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(csch(c + d*x)**3/(sinh(c + d*x)*e*i + sinh(c + d*x)*f*i*x + e + f*x),x
)/a
```

3.222 $\int \frac{\text{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{\text{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx=\text{Int}\left(\frac{\text{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(csch(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Mathematica [F(-1)]

Timed out.

$$\int \frac{\text{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx=\$\text{Aborted}$$

input
```
Integrate[Csch[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$
$$\Big\downarrow \text{ 6111}$$
$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}dx$$

input
```
Int[Csch[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{csch}\left(dx+c\right)^{3}}{\left(fx+e\right)^{2}\left(a+ia\sinh\left(dx+c\right)\right)}dx$$

input
```
int(csch(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 1144, normalized size of antiderivative = 36.90

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)^3}{\left(fx+e\right)^2\left(i\,a\sinh\left(dx+c\right)+a\right)}\,dx$$

input
```
integrate(csch(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
-(4*d*f*x + 4*d*e + (3*d*f*x + 3*d*e - 2*f)*e^(4*d*x + 4*c) - (3*I*d*f*x +
 3*I*d*e - 2*I*f)*e^(3*d*x + 3*c) - (5*d*f*x + 5*d*e - 2*f)*e^(2*d*x + 2*c
) - (-I*d*f*x - I*d*e + 2*I*f)*e^(d*x + c) - (-I*a*d^2*f^3*x^3 - 3*I*a*d^2
*e*f^2*x^2 - 3*I*a*d^2*e^2*f*x - I*a*d^2*e^3 + (a*d^2*f^3*x^3 + 3*a*d^2*e*
f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*e^(5*d*x + 5*c) + (-I*a*d^2*f^3*x^3
 - 3*I*a*d^2*e*f^2*x^2 - 3*I*a*d^2*e^2*f*x - I*a*d^2*e^3)*e^(4*d*x + 4*c)
- 2*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*e^(3
*d*x + 3*c) - 2*(-I*a*d^2*f^3*x^3 - 3*I*a*d^2*e*f^2*x^2 - 3*I*a*d^2*e^2*f*
x - I*a*d^2*e^3)*e^(2*d*x + 2*c) + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*
a*d^2*e^2*f*x + a*d^2*e^3)*e^(d*x + c))*integral((8*d*f^2*x + 8*d*e*f - (3
*d^2*f^2*x^2 + 3*d^2*e^2 + 4*d*e*f - 6*f^2 + 2*(3*d^2*e*f + 2*d*f^2)*x)*e^
(2*d*x + 2*c) + (3*I*d^2*f^2*x^2 + 3*I*d^2*e^2 + 4*I*d*e*f - 6*I*f^2 - 2*(
-3*I*d^2*e*f - 2*I*d*f^2)*x)*e^(d*x + c))/(I*a*d^2*f^4*x^4 + 4*I*a*d^2*e*f
^3*x^3 + 6*I*a*d^2*e^2*f^2*x^2 + 4*I*a*d^2*e^3*f*x + I*a*d^2*e^4 + (a*d^2*
f^4*x^4 + 4*a*d^2*e*f^3*x^3 + 6*a*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*f*x + a*d^
2*e^4)*e^(3*d*x + 3*c) + (-I*a*d^2*f^4*x^4 - 4*I*a*d^2*e*f^3*x^3 - 6*I*a*d
^2*e^2*f^2*x^2 - 4*I*a*d^2*e^3*f*x - I*a*d^2*e^4)*e^(2*d*x + 2*c) - (a*d^2
*f^4*x^4 + 4*a*d^2*e*f^3*x^3 + 6*a*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*f*x + a*d
^2*e^4)*e^(d*x + c)), x))/(-I*a*d^2*f^3*x^3 - 3*I*a*d^2*e*f^2*x^2 - 3*I*a*
d^2*e^2*f*x - I*a*d^2*e^3 + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**3/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 975, normalized size of antiderivative = 31.45

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^3}{\left(fx+e\right)^2\left(i\,a\sinh\left(dx+c\right)+a\right)}\,dx$$

input
```
integrate(csch(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-8*f*integrate(1/(-2*I*a*d*f^3*x^3 - 6*I*a*d*e*f^2*x^2 - 6*I*a*d*e^2*f*x -
 2*I*a*d*e^3 + 2*(a*d*f^3*x^3*e^c + 3*a*d*e*f^2*x^2*e^c + 3*a*d*e^2*f*x*e^
c + a*d*e^3*e^c)*e^(d*x)), x) - 8*(4*d*f*x + 4*d*e + (3*d*f*x*e^(4*c) + (3
*d*e - 2*f)*e^(4*c))*e^(4*d*x) + (-3*I*d*f*x*e^(3*c) + (-3*I*d*e + 2*I*f)*
e^(3*c))*e^(3*d*x) - (5*d*f*x*e^(2*c) + (5*d*e - 2*f)*e^(2*c))*e^(2*d*x) +
 (I*d*f*x*e^c + (I*d*e - 2*I*f)*e^c)*e^(d*x))/(-8*I*a*d^2*f^3*x^3 - 24*I*a
*d^2*e*f^2*x^2 - 24*I*a*d^2*e^2*f*x - 8*I*a*d^2*e^3 + 8*(a*d^2*f^3*x^3*e^(
5*c) + 3*a*d^2*e*f^2*x^2*e^(5*c) + 3*a*d^2*e^2*f*x*e^(5*c) + a*d^2*e^3*e^(
5*c))*e^(5*d*x) - 8*(I*a*d^2*f^3*x^3*e^(4*c) + 3*I*a*d^2*e*f^2*x^2*e^(4*c)
 + 3*I*a*d^2*e^2*f*x*e^(4*c) + I*a*d^2*e^3*e^(4*c))*e^(4*d*x) - 16*(a*d^2*
f^3*x^3*e^(3*c) + 3*a*d^2*e*f^2*x^2*e^(3*c) + 3*a*d^2*e^2*f*x*e^(3*c) + a*
d^2*e^3*e^(3*c))*e^(3*d*x) - 16*(-I*a*d^2*f^3*x^3*e^(2*c) - 3*I*a*d^2*e*f^
2*x^2*e^(2*c) - 3*I*a*d^2*e^2*f*x*e^(2*c) - I*a*d^2*e^3*e^(2*c))*e^(2*d*x)
 + 8*(a*d^2*f^3*x^3*e^c + 3*a*d^2*e*f^2*x^2*e^c + 3*a*d^2*e^2*f*x*e^c + a*
d^2*e^3*e^c)*e^(d*x)) - 8*integrate(1/16*(3*d^2*f^2*x^2 + 3*d^2*e^2 + 4*I*
d*e*f - 6*f^2 + 2*(3*d^2*e*f + 2*I*d*f^2)*x)/(a*d^2*f^4*x^4 + 4*a*d^2*e*f^
3*x^3 + 6*a*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*f*x + a*d^2*e^4 + (a*d^2*f^4*x^4
*e^c + 4*a*d^2*e*f^3*x^3*e^c + 6*a*d^2*e^2*f^2*x^2*e^c + 4*a*d^2*e^3*f*x*e
^c + a*d^2*e^4*e^c)*e^(d*x)), x) - 8*integrate(-1/16*(3*d^2*f^2*x^2 + 3*d^
2*e^2 - 4*I*d*e*f - 6*f^2 + 2*(3*d^2*e*f - 2*I*d*f^2)*x)/(a*d^2*f^4*x^4...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 7.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{1}{\sinh\left(c+d\,x\right)^3\,\left(e+f\,x\right)^2\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(1/(sinh(c + d*x)^3*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(1/(sinh(c + d*x)^3*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.26

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \frac{\int \frac{\operatorname{csch}(dx+c)^3}{\sinh(dx+c)e^2i+2\sinh(dx+c)efix+\sinh(dx+c)f^2i\,x^2+e^2+2efx+f^2x^2}dx}{a}$$

input
```
int(csch(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(csch(c + d*x)**3/(sinh(c + d*x)*e**2*i + 2*sinh(c + d*x)*e*f*i*x + sin
h(c + d*x)*f**2*i*x**2 + e**2 + 2*e*f*x + f**2*x**2),x)/a
```

3.223 $\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 453

$$\begin{aligned}\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx =\ & \frac{(e+fx)^4}{4bf} - \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} \\ & + \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} \\ & - \frac{3af(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} \\ & + \frac{3af(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} \\ & + \frac{6af^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3} \\ & - \frac{6af^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3} \\ & - \frac{6af^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^4} \\ & + \frac{6af^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^4}\end{aligned}$$

output
```
1/4*(f*x+e)^4/b/f-a*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^
2+b^2)^(1/2)/d+a*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b
^2)^(1/2)/d-3*a*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b
/(a^2+b^2)^(1/2)/d^2+3*a*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^
(1/2)))/b/(a^2+b^2)^(1/2)/d^2+6*a*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(
a^2+b^2)^(1/2)))/b/(a^2+b^2)^(1/2)/d^3-6*a*f^2*(f*x+e)*polylog(3,-b*exp(d*
x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(1/2)/d^3-6*a*f^3*polylog(4,-b*exp(d
*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(1/2)/d^4+6*a*f^3*polylog(4,-b*exp(
d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(1/2)/d^4
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 608, normalized size of antiderivative = 1.34

$$\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{x(4e^3+6e^2fx+4ef^2x^2+f^3x^3)}{4b} - \frac{a\left(-2d^3e^3\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+3d^3e^2fx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+3d^3ef^2x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+d^3f^3x^3\ldots\right)}{\ldots}$$

input
```
Integrate[((e + f*x)^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))/(4*b) - (a*(-2*d^3*e^3*Arc
Tanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 3*d^3*e^2*f*x*Log[1 + (b*E^(c
+ d*x))/(a - Sqrt[a^2 + b^2])] + 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2])] + d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2])] - 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] -
3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - d^3*f^3*x
^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 3*d^2*f*(e + f*x)^2*Po
lyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 3*d^2*f*(e + f*x)^2*Pol
yLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*d*e*f^2*PolyLog[3, (
b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 6*d*f^3*x*PolyLog[3, (b*E^(c + d*
x))/(-a + Sqrt[a^2 + b^2])] + 6*d*e*f^2*PolyLog[3, -((b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2]))] + 6*d*f^3*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2]))] + 6*f^3*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 6*f
^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(b*Sqrt[a^2 + b^
2]*d^4)
```

Rubi [A] (verified)

Time = 1.88 (sec) , antiderivative size = 399, normalized size of antiderivative = 0.88, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {6091, 17, 3042, 3803, 25, 2694, 27, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6091}$$

$$\frac{\int (e+fx)^3dx}{b}-\frac{a\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{(e+fx)^4}{4bf}-\frac{a\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(e+fx)^4}{4bf}-\frac{a\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b}$$

$$\Big\downarrow \text{ 3803}$$

$$\frac{(e+fx)^4}{4bf}-\frac{2a\int -\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2a\int \frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}+\frac{(e+fx)^4}{4bf}$$

$$\Big\downarrow \text{ 2694}$$

$$\frac{2a\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^4}{4bf}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2a\left(\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^4}{4bf}$$

$\downarrow$ 2620

$$\frac{2a\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{b}$$

$$\frac{(e+fx)^4}{4bf}$$

$\downarrow$ 3011

$$\frac{2a\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c}}{a-\sqrt{}}\right.}{bd}\right.}{}\right.}{b}$$

$$\frac{(e+fx)^4}{4bf}$$

$\downarrow$ 7163

$$2a\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b}}\right.}{d}}{bd}}{2\sqrt{a^2+b^2}}\right.$$

$$\frac{(e+fx)^4}{4bf}$$

$\big\downarrow$ 2720

$$2a\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(\right.}{d}}{bd}}{2\sqrt{a^2+b^2}}\right.$$

$$\frac{(e+fx)^4}{4bf}$$

$\downarrow$ 7143

$$2a\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}}{bd}}{2\sqrt{a^2+b^2}}\right.$$

$$\frac{(e+fx)^4}{4bf}$$

input
```
Int[((e + f*x)^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(e + f*x)^4/(4*b*f) + (2*a*(-1/2*(b*(((e + f*x)^3*Log[1 + (b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c
+ d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^3
*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x
)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (2*f*(((e +
 f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d - (f*PolyLog
[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/(2*Sqrt[a
^2 + b^2])))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(fx+e)^3 \sinh(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1112 vs. $2(409) = 818$.

Time = 0.13 (sec) , antiderivative size = 1112, normalized size of antiderivative = 2.45

$$\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*((a^2 + b^2)*d^4*f^3*x^4 + 4*(a^2 + b^2)*d^4*e*f^2*x^3 + 6*(a^2 + b^2)
*d^4*e^2*f*x^2 + 4*(a^2 + b^2)*d^4*e^3*x - 24*a*b*f^3*sqrt((a^2 + b^2)/b^2
)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sin
h(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 24*a*b*f^3*sqrt((a^2 + b^2)/b^2)*p
olylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 12*(a*b*d^2*f^3*x^2 + 2*a*b*d^2*e*f^2
*x + a*b*d^2*e^2*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(
d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/
b + 1) + 12*(a*b*d^2*f^3*x^2 + 2*a*b*d^2*e*f^2*x + a*b*d^2*e^2*f)*sqrt((a^
2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 4*(a*b*d^3*e^3 - 3*
a*b*c*d^2*e^2*f + 3*a*b*c^2*d*e*f^2 - a*b*c^3*f^3)*sqrt((a^2 + b^2)/b^2)*l
og(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a
) - 4*(a*b*d^3*e^3 - 3*a*b*c*d^2*e^2*f + 3*a*b*c^2*d*e*f^2 - a*b*c^3*f^3)*
sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt
((a^2 + b^2)/b^2) + 2*a) - 4*(a*b*d^3*f^3*x^3 + 3*a*b*d^3*e*f^2*x^2 + 3*a*
b*d^3*e^2*f*x + 3*a*b*c*d^2*e^2*f - 3*a*b*c^2*d*e*f^2 + a*b*c^3*f^3)*sqrt(
(a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c
) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 4*(a*b*d^3*f^3*x^3 +
3*a*b*d^3*e*f^2*x^2 + 3*a*b*d^3*e^2*f*x + 3*a*b*c*d^2*e^2*f - 3*a*b*c^2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e^3*(a*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + s
qrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b*d) - (d*x + c)/(b*d)) + 1/4*(f^3*x^4 +
 4*e*f^2*x^3 + 6*e^2*f*x^2)/b - integrate(2*(a*f^3*x^3*e^c + 3*a*e*f^2*x^2
*e^c + 3*a*e^2*f*x*e^c)*e^(d*x)/(b^2*e^(2*d*x + 2*c) + 2*a*b*e^(d*x + c) -
 b^2), x)
```

Giac [F]

$$\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)\,\left(e+f\,x\right)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((sinh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{-8\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)a\,e^3i-8e^c\left(\int\frac{e^{dx}x^3}{e^{2dx+2c}b+2e^{dx+c}a-b}dx\right)a^3d\,f^3-8e^c\left(\int\frac{e^{dx}x^3}{e^{2dx+2c}b+2e^{dx+c}a-b}dx\right)a\,b}{}$$

input
```
int((f*x+e)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 8*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a
*e**3*i - 8*e**c*int((e**(d*x)*x**3)/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*
a - b),x)*a**3*d*f**3 - 8*e**c*int((e**(d*x)*x**3)/(e**(2*c + 2*d*x)*b + 2
*e**(c + d*x)*a - b),x)*a*b**2*d*f**3 - 24*e**c*int((e**(d*x)*x**2)/(e**(2
*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**3*d*e*f**2 - 24*e**c*int((e**(
d*x)*x**2)/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a*b**2*d*e*f**2
- 24*e**c*int((e**(d*x)*x)/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*
a**3*d*e**2*f - 24*e**c*int((e**(d*x)*x)/(e**(2*c + 2*d*x)*b + 2*e**(c + d
*x)*a - b),x)*a*b**2*d*e**2*f + 4*a**2*d*e**3*x + 6*a**2*d*e**2*f*x**2 + 4
*a**2*d*e*f**2*x**3 + a**2*d*f**3*x**4 + 4*b**2*d*e**3*x + 6*b**2*d*e**2*f
*x**2 + 4*b**2*d*e*f**2*x**3 + b**2*d*f**3*x**4)/(4*b*d*(a**2 + b**2))
```

3.224 $\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 337

$$\begin{aligned}\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx =\ & \frac{(e+fx)^3}{3bf}-\frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d}\\ &+\frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d}\\ &-\frac{2af(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}\\ &+\frac{2af(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}\\ &+\frac{2af^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3}\\ &-\frac{2af^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3}\end{aligned}$$

output
```
1/3*(f*x+e)^3/b/f-a*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^
2+b^2)^(1/2)/d+a*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b
^2)^(1/2)/d-2*a*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(
a^2+b^2)^(1/2)/d^2+2*a*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2
)))/b/(a^2+b^2)^(1/2)/d^2+2*a*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/
2)))/b/(a^2+b^2)^(1/2)/d^3-2*a*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1
/2)))/b/(a^2+b^2)^(1/2)/d^3
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{x(3e^2+3efx+f^2x^2)}{3b} - \frac{a\left(-2d^2e^2\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+2d^2efx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+d^2f^2x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-2d^2efx\log\right.}{}$$

input
```
Integrate[((e + f*x)^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(x*(3*e^2 + 3*e*f*x + f^2*x^2))/(3*b) - (a*(-2*d^2*e^2*ArcTanh[(a + b*E^(c
 + d*x))/Sqrt[a^2 + b^2]] + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[
a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])]
- 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - d^2*f^2*x^2
*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*f*(e + f*x)*PolyLog[
2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*f*(e + f*x)*PolyLog[2, -(
(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2*f^2*PolyLog[3, (b*E^(c + d*x))
/(-a + Sqrt[a^2 + b^2])] + 2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))]))/(b*Sqrt[a^2 + b^2]*d^3)
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.91, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {6091, 17, 3042, 3803, 25, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6091}$$

$$\frac{\int (e+fx)^2dx}{b}-\frac{a\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{17}$$

$$\frac{(e+fx)^3}{3bf}-\frac{a\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^3}{3bf}-\frac{a\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b}$$

$$\Big\downarrow \text{3803}$$

$$\frac{(e+fx)^3}{3bf}-\frac{2a\int -\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{2a\int \frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}+\frac{(e+fx)^3}{3bf}$$

$$\Big\downarrow \text{2694}$$

$$\frac{2a\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^3}{3bf}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a\left(\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^3}{3bf}$$

$\downarrow$ 2620

$$\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right.}{2\sqrt{a^2+b^2}}}{b}$$

$$\frac{(e+fx)^3}{3bf}$$

$\downarrow$ 3011

$$\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right.}{bd}\right.}{}}{b}$$

$$\frac{(e+fx)^3}{3bf}$$

$\downarrow$ 2720

$$\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(-\right.}{}\right.}{}}{b}$$

$$\frac{(e+fx)^3}{3bf}$$

$$\Big\downarrow \; 7143$$

$$\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\right.}{}\right.}{b}$$

$$\frac{(e+fx)^3}{3bf}$$

input
```
Int[((e + f*x)^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(e + f*x)^3/(3*b*f) + (2*a*(-1/2*(b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - S
qrt[a^2 + b^2]))])/d^2))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^2*Log[1
+ (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyL
og[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/(2*Sqrt[a^2 + b^2])))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(fx+e)^2 \sinh(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 782 vs. 2(303) = 606.

Time = 0.12 (sec) , antiderivative size = 782, normalized size of antiderivative = 2.32

$$\int \frac{(e+fx)^2 \sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/3*((a^2 + b^2)*d^3*f^2*x^3 + 3*(a^2 + b^2)*d^3*e*f*x^2 + 3*(a^2 + b^2)*d
^3*e^2*x + 6*a*b*f^2*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a
*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
)/b) - 6*a*b*f^2*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sin
h(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b)
 - 6*(a*b*d*f^2*x + a*b*d*e*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c
) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2) - b)/b + 1) + 6*(a*b*d*f^2*x + a*b*d*e*f)*sqrt((a^2 + b^2)/b^2)*dilo
g((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))
*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 3*(a*b*d^2*e^2 - 2*a*b*c*d*e*f + a*b*
c^2*f^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) +
 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 3*(a*b*d^2*e^2 - 2*a*b*c*d*e*f + a*b*c
^2*f^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) -
2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 3*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x +
2*a*b*c*d*e*f - a*b*c^2*f^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) +
 a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^
2) - b)/b) + 3*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + 2*a*b*c*d*e*f - a*b*c^
2*f^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b))/((a^2*b +
b^3)*d^3)
```

Sympy [F]

$$\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)^2\sinh{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*sinh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e^2*(a*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + s
qrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b*d) - (d*x + c)/(b*d)) + 1/3*(f^2*x^3 +
 3*e*f*x^2)/b - integrate(2*(a*f^2*x^2*e^c + 2*a*e*f*x*e^c)*e^(d*x)/(b^2*e
^(2*d*x + 2*c) + 2*a*b*e^(d*x + c) - b^2), x)
```

Giac [F]

$$\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+dx)\,(e+fx)^2}{a+b\sinh(c+dx)}\,dx$$

input
```
int((sinh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-6\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)a\,e^2i - 6e^c\left(\int \frac{e^{dx}x^2}{e^{2dx+2c}b+2e^{dx+c}a-b}dx\right)a^3d\,f^2 - 6e^c\left(\int \frac{e^{dx}x^2}{e^{2dx+2c}b+2e^{dx+c}a-b}dx\right)a\,b$$

input
```
int((f*x+e)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 6*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a
*e**2*i - 6*e**c*int((e**(d*x)*x**2)/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*
a - b),x)*a**3*d*f**2 - 6*e**c*int((e**(d*x)*x**2)/(e**(2*c + 2*d*x)*b + 2
*e**(c + d*x)*a - b),x)*a*b**2*d*f**2 - 12*e**c*int((e**(d*x)*x)/(e**(2*c
+ 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**3*d*e*f - 12*e**c*int((e**(d*x)*x
)/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a*b**2*d*e*f + 3*a**2*d*e
**2*x + 3*a**2*d*e*f*x**2 + a**2*d*f**2*x**3 + 3*b**2*d*e**2*x + 3*b**2*d*
e*f*x**2 + b**2*d*f**2*x**3)/(3*b*d*(a**2 + b**2))
```

3.225 $\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 220

$$\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{(e+fx)^2}{2bf} - \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} - \frac{af\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} + \frac{af\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}$$

output
```
1/2*(f*x+e)^2/b/f-a*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+
b^2)^(1/2)/d+a*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)^
(1/2)/d-a*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(1/2)
/d^2+a*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(1/2)/d^
2
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.86

$$\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{x(2e+fx)}{2b} + \frac{a\left(d\left(2\text{earctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) - fx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right) + fx\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right) - f\,\text{PolyLog}\left(2, \frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right) \cdots}{b\sqrt{a^2+b^2}d^2}$$

input
```
Integrate[((e + f*x)*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(x*(2*e + f*x))/(2*b) + (a*(d*(2*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 +
b^2]] - f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*x*Log[1 + (
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]) - f*PolyLog[2, (b*E^(c + d*x))/(-a
+ Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
)]))/(b*Sqrt[a^2 + b^2]*d^2)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6091, 17, 3042, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6091}$$

$$\frac{\int (e+fx)dx}{b} - \frac{a\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{(e+fx)^2}{2bf} - \frac{a\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^2}{2bf}-\frac{a\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{b}$$

$$\Big\downarrow \text{3803}$$

$$\frac{(e+fx)^2}{2bf}-\frac{2a\int -\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{2a\int \frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}+\frac{(e+fx)^2}{2bf}$$

$$\Big\downarrow \text{2694}$$

$$\frac{2a\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^2}{2bf}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^2}{2bf}$$

$$\Big\downarrow \text{2620}$$

$$\frac{2a\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^2}{2bf}$$

$$\Big\downarrow \text{2715}$$

$$\frac{2a\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)\ldots}{bd^2}\right.}{2\sqrt{a^2+b^2}}\right.}{b}$$

$$\frac{(e+fx)^2}{2bf}$$

$\Big\downarrow$ 2838

$$\frac{2a\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^2}{2bf}$$

input
```
Int[((e + f*x)*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(e + f*x)^2/(2*b*f) + (2*a*(-1/2*(b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2]))])/(b*d^2)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[1 + (b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2]))])/(b*d^2)))/(2*Sqrt[a^2 + b^2])))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(198) = 396$.

Time = 0.22 (sec) , antiderivative size = 440, normalized size of antiderivative = 2.00

method	result
risch	$\frac{f\,x^2}{2b} + \frac{ex}{b} + \frac{2ae\,\operatorname{arctanh}\left(\frac{2b\,\mathrm{e}^{dx+c}+2a}{2\sqrt{a^2+b^2}}\right)}{db\sqrt{a^2+b^2}} - \frac{af\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)x}{db\sqrt{a^2+b^2}} + \frac{af\ln\left(\frac{b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}+a}{a+\sqrt{a^2+b^2}}\right)x}{db\sqrt{a^2+b^2}} - \frac{af\ln\left(\frac{-b\,\mathrm{e}^{\cdots}}{\cdots}\right.}{d^{\cdots}}$

input
```
int((f*x+e)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*f*x^2+1/b*e*x+2/d/b*a*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+
2*a)/(a^2+b^2)^(1/2))-1/d/b*a*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2
)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d/b*a*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x
+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2/b*a*f/(a^2+b^2)^(1/2)*
ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d^2/b*a*f/(
a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-
1/d^2/b*a*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a
^2+b^2)^(1/2)))+1/d^2/b*a*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^
(1/2)+a)/(a+(a^2+b^2)^(1/2)))-2/d^2/b*a*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2
*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(196) = 392$.

Time = 0.10 (sec) , antiderivative size = 500, normalized size of antiderivative = 2.27

$$\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{(a^2+b^2)d^2fx^2 + 2\,(a^2+b^2)d^2ex - 2\,abf\sqrt{\frac{a^2+b^2}{b^2}}\operatorname{Li}_2\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2+b^2}{b^2}}}{b}\right.}{}$$

input
```
integrate((f*x+e)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((a^2 + b^2)*d^2*f*x^2 + 2*(a^2 + b^2)*d^2*e*x - 2*a*b*f*sqrt((a^2 + b
^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*s
inh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*a*b*f*sqrt((a^2 + b^2)
/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(a*b*d*e - a*b*c*f)*sqrt(
(a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2
 + b^2)/b^2) + 2*a) - 2*(a*b*d*e - a*b*c*f)*sqrt((a^2 + b^2)/b^2)*log(2*b*
cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(
a*b*d*f*x + a*b*c*f)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(
d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/
b) + 2*(a*b*d*f*x + a*b*c*f)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) +
 a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^
2) - b)/b))/((a^2*b + b^3)*d^2)
```

Sympy [F]

$$\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\sinh{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*sinh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*(4*a*integrate(x*e^(d*x + c)/(b^2*e^(2*d*x + 2*c) + 2*a*b*e^(d*x + c)
 - b^2), x) - x^2/b)*f - e*(a*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(
b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b*d) - (d*x + c)/(
b*d))
```

Giac [F]

$$\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((sinh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-4\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)aei-4e^c\left(\int\frac{e^{dx}x}{e^{2dx+2c}b+2e^{dx+c}a-b}dx\right)a^3df-4e^c\left(\int\frac{e^{dx}x}{e^{2dx+2c}b+2e^{dx+c}a-b}dx\right)a\,b^2df}{2bd\,(a^2+b^2)}$$

input
```
int((f*x+e)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 4*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a
*e*i - 4*e**c*int((e**(d*x)*x)/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)
,x)*a**3*d*f - 4*e**c*int((e**(d*x)*x)/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x
)*a - b),x)*a*b**2*d*f + 2*a**2*d*e*x + a**2*d*f*x**2 + 2*b**2*d*e*x + b**
2*d*f*x**2)/(2*b*d*(a**2 + b**2))
```

3.226 $\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{x}{b} + \frac{2a\text{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d}$$

output
```
x/b+2*a*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/b/(a^2+b^2)^(1/
2)/d
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19

$$\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{c}{d} + x - \frac{2a\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}d}}{b}$$

input
```
Integrate[Sinh[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

output
```
(c/d + x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(Sqrt[
-a^2 - b^2]*d))/b
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3042, 26, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ic+idx)}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ic+idx)}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3214}$$

$$-i\left(\frac{ix}{b}-\frac{ia\int \frac{1}{a+b\sinh(c+dx)}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{ix}{b}-\frac{ia\int \frac{1}{a-ib\sin(ic+idx)}dx}{b}\right)$$

$$\Big\downarrow \text{3139}$$

$$-i\left(\frac{ix}{b}-\frac{2a\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{bd}\right)$$

$$\Big\downarrow \text{1083}$$

$$-i\left(\frac{4a\int \frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4(a^2+b^2)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{bd}+\frac{ix}{b}\right)$$

$$\Big\downarrow \text{217}$$

$$-i\left(\frac{ix}{b}-\frac{2ia\operatorname{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{bd\sqrt{a^2+b^2}}\right)$$

input
```
Int[Sinh[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

output
```
(-I)*((I*x)/b - ((2*I)*a*ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b^2])])/(
b*Sqrt[a^2 + b^2]*d))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.52

method	result	size
derivativedivides	$\frac{-\frac{2a\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}+\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}}{d}$	82
default	$\frac{-\frac{2a\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}+\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}}{d}$	82
risch	$\frac{x}{b}+\frac{a\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,db}-\frac{a\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,db}$	124

input
```
int(sinh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/b*a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2
+b^2)^(1/2))-1/b*ln(tanh(1/2*d*x+1/2*c)-1)+1/b*ln(1+tanh(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(51) = 102.

Time = 0.10 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.44

$$\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{(a^2+b^2)dx+\sqrt{a^2+b^2}a\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+2\,a^2+b^2+2\,(b^2\cosh(dx+c)+ab)\sinh(dx+c)+2\,\sqrt{}}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2\,a\cosh(dx+c)+2\,(b\cosh(dx+c)+a)\sinh}\right.}{(a^2b+b^3)d}$$

input
```
integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
((a^2 + b^2)*d*x + sqrt(a^2 + b^2)*a*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d
*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b
)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a
))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(
d*x + c) + a)*sinh(d*x + c) - b)))/((a^2*b + b^3)*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 30.72 (sec) , antiderivative size = 269, normalized size of antiderivative = 4.98

$$\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \begin{cases} \tilde{\infty}x & \text{for}\: a=0 \wedge b=0 \wedge d=0 \\ \frac{x}{b} & \text{for}\: a=0 \\ \frac{\cosh{(c+dx)}}{ad} & \text{for}\: b=0 \\ \frac{x\sinh{(c)}}{a+b\sinh{(c)}} & \text{for}\: d=0 \\ \frac{dx\tanh{\left(\frac{c}{2}+\frac{dx}{2}\right)}}{bd\tanh{\left(\frac{c}{2}+\frac{dx}{2}\right)}-ibd} - \frac{idx}{bd\tanh{\left(\frac{c}{2}+\frac{dx}{2}\right)}-ibd} - \frac{2}{bd\tanh{\left(\frac{c}{2}+\frac{dx}{2}\right)}-ibd} & \text{for}\: a=-ib \\ \frac{dx\tanh{\left(\frac{c}{2}+\frac{dx}{2}\right)}}{bd\tanh{\left(\frac{c}{2}+\frac{dx}{2}\right)}+ibd} + \frac{idx}{bd\tanh{\left(\frac{c}{2}+\frac{dx}{2}\right)}+ibd} - \frac{2}{bd\tanh{\left(\frac{c}{2}+\frac{dx}{2}\right)}+ibd} & \text{for}\: a=ib \\ \frac{a\log{\left(\tanh{\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}}{bd\sqrt{a^2+b^2}} - \frac{a\log{\left(\tanh{\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}}{bd\sqrt{a^2+b^2}} + \frac{x}{b} & \text{otherwise} \end{cases}$$

input
```
integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/b, Eq(a, 0)), (cosh(
c + d*x)/(a*d), Eq(b, 0)), (x*sinh(c)/(a + b*sinh(c)), Eq(d, 0)), (d*x*tan
h(c/2 + d*x/2)/(b*d*tanh(c/2 + d*x/2) - I*b*d) - I*d*x/(b*d*tanh(c/2 + d*x
/2) - I*b*d) - 2/(b*d*tanh(c/2 + d*x/2) - I*b*d), Eq(a, -I*b)), (d*x*tanh(
c/2 + d*x/2)/(b*d*tanh(c/2 + d*x/2) + I*b*d) + I*d*x/(b*d*tanh(c/2 + d*x/2
) + I*b*d) - 2/(b*d*tanh(c/2 + d*x/2) + I*b*d), Eq(a, I*b)), (a*log(tanh(c
/2 + d*x/2) - b/a - sqrt(a**2 + b**2)/a)/(b*d*sqrt(a**2 + b**2)) - a*log(t
anh(c/2 + d*x/2) - b/a + sqrt(a**2 + b**2)/a)/(b*d*sqrt(a**2 + b**2)) + x/
b, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.57

$$\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}bd} + \frac{dx+c}{bd}$$

input
```
integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-a*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*b*d) + (d*x + c)/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.56

$$\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{a\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b} - \frac{dx+c}{b}}{d}$$

input
```
integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-(a*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c)
 + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b) - (d*x + c)/b)/d
```

Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.24

$$\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{x}{b} - \frac{a\,\ln\left(\frac{2\,a\,e^{c+d\,x}}{b^2} - \frac{2\,a\,\left(b-a\,e^{c+d\,x}\right)}{b^2\,\sqrt{a^2+b^2}}\right)}{b\,d\,\sqrt{a^2+b^2}} + \frac{a\,\ln\left(\frac{2\,a\,e^{c+d\,x}}{b^2} + \frac{2\,a\,\left(b-a\,e^{c+d\,x}\right)}{b^2\,\sqrt{a^2+b^2}}\right)}{b\,d\,\sqrt{a^2+b^2}}$$

input
```
int(sinh(c + d*x)/(a + b*sinh(c + d*x)),x)
```

output
```
x/b - (a*log((2*a*exp(c + d*x))/b^2 - (2*a*(b - a*exp(c + d*x)))/(b^2*(a^2
 + b^2)^(1/2))))/(b*d*(a^2 + b^2)^(1/2)) + (a*log((2*a*exp(c + d*x))/b^2 +
 (2*a*(b - a*exp(c + d*x)))/(b^2*(a^2 + b^2)^(1/2))))/(b*d*(a^2 + b^2)^(1/
2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.24

$$\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{-2\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)ai + a^2dx + b^2dx}{bd\,(a^2+b^2)}$$

input
```
int(sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a
*i + a**2*d*x + b**2*d*x)/(b*d*(a**2 + b**2))
```

3.227 $\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \text{Int}\left(\frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 2.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

input
```
Integrate[Sinh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Sinh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

$$\Big\downarrow \; 6111$$

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Sinh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(dx+c)}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sinh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.38

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-2*a*integrate(-e^(d*x + c)/(b^2*f*x + b^2*e - (b^2*f*x*e^(2*c) + b^2*e*e^
(2*c))*e^(2*d*x) - 2*(a*b*f*x*e^c + a*b*e*e^c)*e^(d*x)), x) + log(f*x + e)
/(b*f)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sinh(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(c+d\,x)}{(e+f\,x)\,(a+b\sinh(c+d\,x))}\,dx$$

input
```
int(sinh(c + d*x)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(sinh(c + d*x)/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$= \frac{-\left(\int \frac{1}{\sinh(dx+c)be+\sinh(dx+c)bfx+ae+afx}dx\right)af+\log(fx+e)}{bf}$$

input
```
int(sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
( - int(1/(sinh(c + d*x)*b*e + sinh(c + d*x)*b*f*x + a*e + a*f*x),x)*a*f +
 log(e + f*x))/(b*f)
```

3.228 $\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+b\sinh(c+dx)} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 551

$$\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a(e+fx)^4}{4b^2f}+\frac{6f^2(e+fx)\cosh(c+dx)}{bd^3}+\frac{(e+fx)^3\cosh(c+dx)}{bd}+\frac{a^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d}-\frac{a^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d}+\frac{3a^2f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d^2}-\frac{3a^2f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d^2}-\frac{6a^2f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d^3}+\frac{6a^2f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d^3}+\frac{6a^2f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d^4}-\frac{6a^2f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d^4}-\frac{6f^3\sinh(c+dx)}{bd^4}-\frac{3f(e+fx)^2\sinh(c+dx)}{bd^2}$$

output
```
-1/4*a*(f*x+e)^4/b^2/f+6*f^2*(f*x+e)*cosh(d*x+c)/b/d^3+(f*x+e)^3*cosh(d*x+
c)/b/d+a^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)^
(1/2)/d-a^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)
^(1/2)/d+3*a^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^
2/(a^2+b^2)^(1/2)/d^2-3*a^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/b^2/(a^2+b^2)^(1/2)/d^2-6*a^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+
c)/(a-(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)^(1/2)/d^3+6*a^2*f^2*(f*x+e)*polylog(
3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)^(1/2)/d^3+6*a^2*f^3*pol
ylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)^(1/2)/d^4-6*a^2*f^
3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)^(1/2)/d^4-6*f
^3*sinh(d*x+c)/b/d^4-3*f*(f*x+e)^2*sinh(d*x+c)/b/d^2
```

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 979, normalized size of antiderivative = 1.78

$$\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-4a\sqrt{a^2+b^2}d^4e^3x-6a\sqrt{a^2+b^2}d^4e^2fx^2-4a\sqrt{a^2+b^2}d^4ef^2x^3-a\sqrt{a^2+b^2}d^4f^3x^4-8a^2d^3e^3\operatorname{arctanh}\Big(}{}$$

input
```
Integrate[((e + f*x)^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-4*a*Sqrt[a^2 + b^2]*d^4*e^3*x - 6*a*Sqrt[a^2 + b^2]*d^4*e^2*f*x^2 - 4*a*
Sqrt[a^2 + b^2]*d^4*e*f^2*x^3 - a*Sqrt[a^2 + b^2]*d^4*f^3*x^4 - 8*a^2*d^3*
e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 4*b*Sqrt[a^2 + b^2]*d^3
*e^3*Cosh[c + d*x] + 24*b*Sqrt[a^2 + b^2]*d*e*f^2*Cosh[c + d*x] + 12*b*Sqr
t[a^2 + b^2]*d^3*e^2*f*x*Cosh[c + d*x] + 24*b*Sqrt[a^2 + b^2]*d*f^3*x*Cosh
[c + d*x] + 12*b*Sqrt[a^2 + b^2]*d^3*e*f^2*x^2*Cosh[c + d*x] + 4*b*Sqrt[a^
2 + b^2]*d^3*f^3*x^3*Cosh[c + d*x] + 12*a^2*d^3*e^2*f*x*Log[1 + (b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2])] + 12*a^2*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x)
)/(a - Sqrt[a^2 + b^2])] + 4*a^2*d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2])] - 12*a^2*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])] - 12*a^2*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2])] - 4*a^2*d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]
 + 12*a^2*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^
2])] - 12*a^2*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2]))] - 24*a^2*d*e*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^
2])] - 24*a^2*d*f^3*x*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] +
 24*a^2*d*e*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 24*
a^2*d*f^3*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 24*a^2*
f^3*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 24*a^2*f^3*PolyLo
g[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 12*b*Sqrt[a^2 + b^2]*d...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.97 (sec) , antiderivative size = 501, normalized size of antiderivative = 0.91, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6091, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117, 6091, 17, 3042, 3803, 25, 2694, 27, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6091}$$

$$\frac{\int (e+fx)^3\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -i(e+fx)^3\sin(ic+idx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\int (e+fx)^3\sin(ic+idx)dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\cosh(c+dx)dx}{d}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int (e+fx)\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int -i(e+fx)\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int (e+fx)\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{3117}$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{6091}$$

$$-\frac{a\left(\frac{\int(e+fx)^3dx}{b}-\frac{a\int\frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{a\left(\frac{(e+fx)^4}{4bf}-\frac{a\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$\Big\downarrow$ 3042

$$-\frac{a\left(\frac{(e+fx)^4}{4bf}-\frac{a\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$\Big\downarrow$ 3803

$$-\frac{a\left(\frac{(e+fx)^4}{4bf}-\frac{2a\int -\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$\Big\downarrow$ 25

$$-\frac{a\left(\frac{2a\int \frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}+\frac{(e+fx)^4}{4bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$\Big\downarrow$ 2694

$$-\frac{a\left(\frac{2a\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^4}{4bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\left(\frac{2a\left(\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^4}{4bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \; 2620$$

$$-\frac{a\Bigg(\frac{2a\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\Big(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)\cdots}{bd}}{2\sqrt{a^2+b^2}}\cdots}{b}\cdots}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \; 3011$$

$$-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{\cdots}{a-\cdots}\cdots\right)}{bd}\cdots\right)}{\cdots}\cdots\right)}{b}\cdots\right)}{b}\cdots$$

$$\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{7163}$$

$$
-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int \text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{\cdots}}\right)}{d}\cdots}{bd}\cdots}{2\sqrt{a^2+b^2}}\cdots}{\cdots}\cdots}{\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}}
$$

$$\Big\downarrow \; 2720$$

$$a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{Poly}}{}}{bd}}{2\sqrt{a^2+b^2}}}{}\right.$$

$$-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$\downarrow$ 7143

$$-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+}}\right.}{d}}{bd}}{2\sqrt{a^2+b^2}}}{}}{}$$

$$\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

input
```
Int[((e + f*x)^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-((a*((e + f*x)^4/(4*b*f) + (2*a*(-1/2*(b*(((e + f*x)^3*Log[1 + (b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b
*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -(
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x
))/(a - Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e +
f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e
 + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (2*f*
(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d - (f*P
olyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/(2*
Sqrt[a^2 + b^2])))/b))/b) - (I*((I*(e + f*x)^3*Cosh[c + d*x])/d - ((3*I)*f
*(((e + f*x)^2*Sinh[c + d*x])/d + ((2*I)*f*((I*(e + f*x)*Cosh[c + d*x])/d
- (I*f*Sinh[c + d*x])/d^2))/d))/d))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(fx+e)^3 \sinh(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2612 vs. $2(507) = 1014$.

Time = 0.17 (sec) , antiderivative size = 2612, normalized size of antiderivative = 4.74

$$\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(2*(a^2*b + b^3)*d^3*f^3*x^3 + 2*(a^2*b + b^3)*d^3*e^3 + 6*(a^2*b + b^
3)*d^2*e^2*f + 12*(a^2*b + b^3)*d*e*f^2 + 12*(a^2*b + b^3)*f^3 + 6*((a^2*b
 + b^3)*d^3*e*f^2 + (a^2*b + b^3)*d^2*f^3)*x^2 + 2*((a^2*b + b^3)*d^3*f^3*
x^3 + (a^2*b + b^3)*d^3*e^3 - 3*(a^2*b + b^3)*d^2*e^2*f + 6*(a^2*b + b^3)*
d*e*f^2 - 6*(a^2*b + b^3)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - (a^2*b + b^3)
*d^2*f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^2*b + b^3)*d^2*e*f^2 + 2
*(a^2*b + b^3)*d*f^3)*x)*cosh(d*x + c)^2 + 2*((a^2*b + b^3)*d^3*f^3*x^3 +
(a^2*b + b^3)*d^3*e^3 - 3*(a^2*b + b^3)*d^2*e^2*f + 6*(a^2*b + b^3)*d*e*f^
2 - 6*(a^2*b + b^3)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - (a^2*b + b^3)*d^2*f
^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^2*b + b^3)*d^2*e*f^2 + 2*(a^2*
b + b^3)*d*f^3)*x)*sinh(d*x + c)^2 + 12*((a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*
e*f^2*x + a^2*b*d^2*e^2*f)*cosh(d*x + c) + (a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^
2*e*f^2*x + a^2*b*d^2*e^2*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a
*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqr
t((a^2 + b^2)/b^2) - b)/b + 1) - 12*((a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*e*f^
2*x + a^2*b*d^2*e^2*f)*cosh(d*x + c) + (a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*e*
f^2*x + a^2*b*d^2*e^2*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cos
h(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2) - b)/b + 1) - 4*((a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a
^2*b*c^2*d*e*f^2 - a^2*b*c^3*f^3)*cosh(d*x + c) + (a^2*b*d^3*e^3 - 3*a^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*e^3*(2*a^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c)
- a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2*d) - 2*(d*x + c)*a/(b^2*d) +
e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d)) - 1/4*(a*d^4*f^3*x^4*e^c + 4*a*d^4
*e*f^2*x^3*e^c + 6*a*d^4*e^2*f*x^2*e^c - 2*(b*d^3*f^3*x^3*e^(2*c) + 3*(d^3
*e*f^2 - d^2*f^3)*b*x^2*e^(2*c) + 3*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*b*
x*e^(2*c) - 3*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*b*e^(2*c))*e^(d*x) - 2*(b*d^
3*f^3*x^3 + 3*(d^3*e*f^2 + d^2*f^3)*b*x^2 + 3*(d^3*e^2*f + 2*d^2*e*f^2 + 2
*d*f^3)*b*x + 3*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*b)*e^(-d*x))*e^(-c)/(b^2*d
^4) + integrate(2*(a^2*f^3*x^3*e^c + 3*a^2*e*f^2*x^2*e^c + 3*a^2*e^2*f*x*e
^c)*e^(d*x)/(b^3*e^(2*d*x + 2*c) + 2*a*b^2*e^(d*x + c) - b^3), x)
```

Giac [F]

$$\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sinh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\sinh(c+d\,x)^2\,(e+f\,x)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((sinh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Too large to display}$$

input
```
int((f*x+e)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output

```
(8*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**2
*d**3*e**3*i + 4*cosh(c + d*x)*a**2*b*d**3*e**3 + 12*cosh(c + d*x)*a**2*b*
d**3*e**2*f*x + 12*cosh(c + d*x)*a**2*b*d**3*e*f**2*x**2 + 4*cosh(c + d*x)
*a**2*b*d**3*f**3*x**3 + 24*cosh(c + d*x)*a**2*b*d*e*f**2 + 24*cosh(c + d*
x)*a**2*b*d*f**3*x + 4*cosh(c + d*x)*b**3*d**3*e**3 + 12*cosh(c + d*x)*b**
3*d**3*e**2*f*x + 12*cosh(c + d*x)*b**3*d**3*e*f**2*x**2 + 4*cosh(c + d*x)
*b**3*d**3*f**3*x**3 + 24*cosh(c + d*x)*b**3*d*e*f**2 + 24*cosh(c + d*x)*b
**3*d*f**3*x + 8*e**c*int((e**(d*x)*x**3)/(e**(2*c + 2*d*x)*b + 2*e**(c +
d*x)*a - b),x)*a**4*d**4*f**3 + 8*e**c*int((e**(d*x)*x**3)/(e**(2*c + 2*d*
x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b**2*d**4*f**3 + 24*e**c*int((e**(d*x
)*x**2)/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*d**4*e*f**2 +
24*e**c*int((e**(d*x)*x**2)/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)
*a**2*b**2*d**4*e*f**2 + 24*e**c*int((e**(d*x)*x)/(e**(2*c + 2*d*x)*b + 2*
e**(c + d*x)*a - b),x)*a**4*d**4*e**2*f + 24*e**c*int((e**(d*x)*x)/(e**(2*
c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b**2*d**4*e**2*f - 12*sinh(c
+ d*x)*a**2*b*d**2*e**2*f - 24*sinh(c + d*x)*a**2*b*d**2*e*f**2*x - 12*sin
h(c + d*x)*a**2*b*d**2*f**3*x**2 - 24*sinh(c + d*x)*a**2*b*f**3 - 12*sinh(
c + d*x)*b**3*d**2*e**2*f - 24*sinh(c + d*x)*b**3*d**2*e*f**2*x - 12*sinh(
c + d*x)*b**3*d**2*f**3*x**2 - 24*sinh(c + d*x)*b**3*f**3 - 4*a**3*d**4*e*
*3*x - 6*a**3*d**4*e**2*f*x**2 - 4*a**3*d**4*e*f**2*x**3 - a**3*d**4*f*...
```

3.229 $\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 407

$$\begin{aligned}\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = &-\frac{a(e+fx)^3}{3b^2f}+\frac{2f^2\cosh(c+dx)}{bd^3}\\ &+\frac{(e+fx)^2\cosh(c+dx)}{bd}\\ &+\frac{a^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d}\\ &-\frac{a^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d}\\ &+\frac{2a^2f(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d^2}\\ &-\frac{2a^2f(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d^2}\\ &-\frac{2a^2f^2\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d^3}\\ &+\frac{2a^2f^2\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d^3}\\ &-\frac{2f(e+fx)\sinh(c+dx)}{bd^2}\end{aligned}$$

output
```
-1/3*a*(f*x+e)^3/b^2/f+2*f^2*cosh(d*x+c)/b/d^3+(f*x+e)^2*cosh(d*x+c)/b/d+a
^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)^(1/2)/d-
a^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)^(1/2)/d
+2*a^2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/(a^2+b^2
)^(1/2)/d^2-2*a^2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b
^2/(a^2+b^2)^(1/2)/d^2-2*a^2*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2
)))/b^2/(a^2+b^2)^(1/2)/d^3+2*a^2*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)
^(1/2)))/b^2/(a^2+b^2)^(1/2)/d^3-2*f*(f*x+e)*sinh(d*x+c)/b/d^2
```

Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.11

$$\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{-ax(3e^2+3efx+f^2x^2)+\frac{3a^2\left(-2d^2e^2\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+2d^2efx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+d^2f^2x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-2d^2e\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[((e + f*x)^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-(a*x*(3*e^2 + 3*e*f*x + f^2*x^2)) + (3*a^2*(-2*d^2*e^2*ArcTanh[(a + b*E^
(c + d*x))/Sqrt[a^2 + b^2]] + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - d^2*f^2*x
^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*f*(e + f*x)*PolyLo
g[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*f*(e + f*x)*PolyLog[2,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2*f^2*PolyLog[3, (b*E^(c + d*x
))/(-a + Sqrt[a^2 + b^2])] + 2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[
a^2 + b^2]))]))/(Sqrt[a^2 + b^2]*d^3) + (3*b*Cosh[d*x]*((2*f^2 + d^2*(e +
f*x)^2)*Cosh[c] - 2*d*f*(e + f*x)*Sinh[c]))/d^3 + (3*b*(-2*d*f*(e + f*x)*C
osh[c] + (2*f^2 + d^2*(e + f*x)^2)*Sinh[c])*Sinh[d*x])/d^3)/(3*b^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.21 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.92, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6091, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118, 6091, 17, 3042, 3803, 25, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6091}$$

$$\frac{\int (e+fx)^2\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -i(e+fx)^2\sin(ic+idx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\int (e+fx)^2\sin(ic+idx)dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\cosh(c+dx)dx}{d}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)}{d}\right)}{b}$$

$\big\downarrow$ 3042

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}$$

$\big\downarrow$ 26

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}\right)}{d}\right)}{b}$$

$\big\downarrow$ 3118

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$\big\downarrow$ 6091

$$-\frac{a\left(\frac{\int (e+fx)^2dx}{b}-\frac{a\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$\big\downarrow$ 17

$$-\frac{a\left(\frac{(e+fx)^3}{3bf}-\frac{a\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$\big\downarrow$ 3042

$$-\frac{a\left(\frac{(e+fx)^3}{3bf}-\frac{a\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$\big\downarrow$ 3803

$$-\frac{a\left(\frac{(e+fx)^3}{3bf}-\frac{2a\int -\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$$\Big\downarrow \textbf{25}$$

$$-\frac{a\left(\frac{2a\int \frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}+\frac{(e+fx)^3}{3bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$$\Big\downarrow \textbf{2694}$$

$$-\frac{a\left(\frac{2a\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^3}{3bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{a\left(\frac{2a\left(\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^3}{3bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$$\Big\downarrow \textbf{2620}$$

$$-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{b}\right)}{b}$$

$$\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$\big\downarrow$ 3011

$$-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right.}{bd}\right.}{}\right)}{b}\right)}{b}$$

$$\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$\big\downarrow$ 2720

$$-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\cdots}{\cdots}\right.}{\cdots}\right)}{b}\cdots}{b}$$

$$\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$$\Big\downarrow \; 7143$$

$$-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right.}{}\right)}{b}\right.}{b}$$

$$\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

input
```
Int[((e + f*x)^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-((a*((e + f*x)^3/(3*b*f) + (2*a*(-1/2*(b*(((e + f*x)^2*Log[1 + (b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^2*
Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)
*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/(2*Sqrt[a^2 + b^2
])))/b))/b) - (I*((I*(e + f*x)^2*Cosh[c + d*x])/d - ((2*I)*f*(-((f*Cosh[c
+ d*x])/d^2) + ((e + f*x)*Sinh[c + d*x])/d))/d))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(fx+e)^2 \sinh(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1689 vs. $2(373) = 746$.

Time = 0.11 (sec) , antiderivative size = 1689, normalized size of antiderivative = 4.15

$$\int \frac{(e+fx)^2 \sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(3*(a^2*b + b^3)*d^2*f^2*x^2 + 3*(a^2*b + b^3)*d^2*e^2 + 6*(a^2*b + b^
3)*d*e*f + 6*(a^2*b + b^3)*f^2 + 3*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b
^3)*d^2*e^2 - 2*(a^2*b + b^3)*d*e*f + 2*(a^2*b + b^3)*f^2 + 2*((a^2*b + b^
3)*d^2*e*f - (a^2*b + b^3)*d*f^2)*x)*cosh(d*x + c)^2 + 3*((a^2*b + b^3)*d^
2*f^2*x^2 + (a^2*b + b^3)*d^2*e^2 - 2*(a^2*b + b^3)*d*e*f + 2*(a^2*b + b^3
)*f^2 + 2*((a^2*b + b^3)*d^2*e*f - (a^2*b + b^3)*d*f^2)*x)*sinh(d*x + c)^2
 + 12*((a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c) + (a^2*b*d*f^2*x + a^2*
b*d*e*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*s
inh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) -
 b)/b + 1) - 12*((a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c) + (a^2*b*d*f^
2*x + a^2*b*d*e*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x
+ c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b
^2)/b^2) - b)/b + 1) - 6*((a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2
)*cosh(d*x + c) + (a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) +
 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 6*((a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f +
a^2*b*c^2*f^2)*cosh(d*x + c) + (a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^
2*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*si
nh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 6*((a^2*b*d^2*f^2*x^2 + 2
*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2)*cosh(d*x + c) + (a^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*e^2*(2*a^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c)
- a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2*d) - 2*(d*x + c)*a/(b^2*d) +
e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d)) - 1/6*(2*a*d^3*f^2*x^3*e^c + 6*a*d
^3*e*f*x^2*e^c - 3*(b*d^2*f^2*x^2*e^(2*c) + 2*(d^2*e*f - d*f^2)*b*x*e^(2*c
) - 2*(d*e*f - f^2)*b*e^(2*c))*e^(d*x) - 3*(b*d^2*f^2*x^2 + 2*(d^2*e*f + d
*f^2)*b*x + 2*(d*e*f + f^2)*b)*e^(-d*x))*e^(-c)/(b^2*d^3) + integrate(2*(a
^2*f^2*x^2*e^c + 2*a^2*e*f*x*e^c)*e^(d*x)/(b^3*e^(2*d*x + 2*c) + 2*a*b^2*e
^(d*x + c) - b^3), x)
```

Giac [F]

$$\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sinh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)^2\,(e+f\,x)^2}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((sinh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2 \sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{6\sqrt{a^2+b^2}\, atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right) a^2d^2e^2i + 3\cosh(dx+c)\, a^2b\, d^2e^2 + 6\cosh(dx+c)\, a^2b\, d^2efx + 3\cosh(dx+}{}$$

input
```
int((f*x+e)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(6*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**2
*d**2*e**2*i + 3*cosh(c + d*x)*a**2*b*d**2*e**2 + 6*cosh(c + d*x)*a**2*b*d
**2*e*f*x + 3*cosh(c + d*x)*a**2*b*d**2*f**2*x**2 + 6*cosh(c + d*x)*a**2*b
*f**2 + 3*cosh(c + d*x)*b**3*d**2*e**2 + 6*cosh(c + d*x)*b**3*d**2*e*f*x +
 3*cosh(c + d*x)*b**3*d**2*f**2*x**2 + 6*cosh(c + d*x)*b**3*f**2 + 6*e**c*
int((e**(d*x)*x**2)/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*d*
*3*f**2 + 6*e**c*int((e**(d*x)*x**2)/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*
a - b),x)*a**2*b**2*d**3*f**2 + 12*e**c*int((e**(d*x)*x)/(e**(2*c + 2*d*x)
*b + 2*e**(c + d*x)*a - b),x)*a**4*d**3*e*f + 12*e**c*int((e**(d*x)*x)/(e*
*(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b**2*d**3*e*f - 6*sinh(c
+ d*x)*a**2*b*d*e*f - 6*sinh(c + d*x)*a**2*b*d*f**2*x - 6*sinh(c + d*x)*b*
*3*d*e*f - 6*sinh(c + d*x)*b**3*d*f**2*x - 3*a**3*d**3*e**2*x - 3*a**3*d**
3*e*f*x**2 - a**3*d**3*f**2*x**3 - 3*a*b**2*d**3*e**2*x - 3*a*b**2*d**3*e*
f*x**2 - a*b**2*d**3*f**2*x**3)/(3*b**2*d**3*(a**2 + b**2))
```

3.230 $\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 262

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a(e+fx)^2}{2b^2f}+\frac{(e+fx)\cosh(c+dx)}{bd} + \frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d} - \frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d} + \frac{a^2f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d^2} - \frac{a^2f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d^2} - \frac{f\sinh(c+dx)}{bd^2}$$

output
```
-1/2*a*(f*x+e)^2/b^2/f+(f*x+e)*cosh(d*x+c)/b/d+a^2*(f*x+e)*ln(1+b*exp(d*x+
c)/(a-(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)^(1/2)/d-a^2*(f*x+e)*ln(1+b*exp(d*x+c
)/(a+(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)^(1/2)/d+a^2*f*polylog(2,-b*exp(d*x+c)
/(a-(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)^(1/2)/d^2-a^2*f*polylog(2,-b*exp(d*x+c
)/(a+(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)^(1/2)/d^2-f*sinh(d*x+c)/b/d^2
```

Mathematica [A] (verified)

Time = 2.45 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{a(c+dx)(cf-d(2e+fx))+2bd(e+fx)\cosh(c+dx)+\frac{2a^2\left(-2de\,\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+2cf\,\text{arctanh}\left(\frac{a+be^{c+d}}{\sqrt{a^2+b^2}}\right.\right.}{}}{}$$

input
```
Integrate[((e + f*x)*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(a*(c + d*x)*(c*f - d*(2*e + f*x)) + 2*b*d*(e + f*x)*Cosh[c + d*x] + (2*a^
2*(-2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*c*f*ArcTanh[(a
+ b*E^(c + d*x))/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2])] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - f*PolyLog
[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/Sqrt[a^2 + b^2] - 2*b*f*Si
nh[c + d*x])/(2*b^2*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.30 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.98, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {6091, 3042, 26, 3777, 3042, 3117, 6091, 17, 3042, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \ 6091$$

$$\frac{\int (e+fx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -i(e+fx)\sin(ic+idx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\int (e+fx)\sin(ic+idx)dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{a\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}$$

$$\Big\downarrow \text{3117}$$

$$-\frac{a\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \text{6091}$$

$$-\frac{a\left(\frac{\int (e+fx)dx}{b}-\frac{a\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{a\left(\frac{(e+fx)^2}{2bf}-\frac{a\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\left(\frac{(e+fx)^2}{2bf}-\frac{a\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \text{3803}$$

$$-\frac{a\left(\frac{(e+fx)^2}{2bf}-\frac{2a\int -\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a\left(\frac{2a\int \frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}+\frac{(e+fx)^2}{2bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \text{ 2694}$$

$$-\frac{a\left(\frac{2a\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^2}{2bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a\left(\frac{2a\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^2}{2bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \text{ 2620}$$

$$-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^2}{2bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \text{ 2715}$$

$$-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right.}{bd^2}\right.}{2\sqrt{a^2+b^2}}\right.}{b}\right.}{b}$$

$$\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$\Big\downarrow$ 2838

$$-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+f}{2b}\right.}{b}$$

$$\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

input
```
Int[((e + f*x)*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-((a*((e + f*x)^2/(2*b*f) + (2*a*(-1/2*(b*(((e + f*x)*Log[1 + (b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - S
qrt[a^2 + b^2]))])/(b*d^2)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[1 + (b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(2*Sqrt[a^2 + b^2])))/b))/b) - (I*((I
*(e + f*x)*Cosh[c + d*x])/d - (I*f*Sinh[c + d*x])/d^2))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(240) = 480$.

Time = 0.36 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.95

method	result
risch	$-\frac{af\,x^2}{2b^2}-\frac{aex}{b^2}+\frac{(dxf+de-f)\mathrm{e}^{dx+c}}{2b\,d^2}+\frac{(dxf+de+f)\mathrm{e}^{-dx-c}}{2b\,d^2}-\frac{2a^2e\,\operatorname{arctanh}\left(\frac{2b\,\mathrm{e}^{dx+c}+2a}{2\sqrt{a^2+b^2}}\right)}{d\,b^2\sqrt{a^2+b^2}}+\frac{a^2f\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}}{-a+\sqrt{a^2+b^2}}\right.}{d\,b^2\sqrt{a^2+b^2}}$

input
```
int((f*x+e)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/b^2*f*x^2-a/b^2*e*x+1/2*(d*f*x+d*e-f)/b/d^2*exp(d*x+c)+1/2*(d*f*x+d
*e+f)/b/d^2*exp(-d*x-c)-2/d*a^2/b^2*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp
(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/d*a^2/b^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x
+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d*a^2/b^2*f/(a^2+b^2)^(1/
2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2*a^2/b^
2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/
2)))*c-1/d^2*a^2/b^2*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)
/(a+(a^2+b^2)^(1/2)))*c+1/d^2*a^2/b^2*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+
c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2*a^2/b^2*f/(a^2+b^2)^(1/2
)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/d^2*a^2/b^
2*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 946 vs. $2(238) = 476$.

Time = 0.11 (sec) , antiderivative size = 946, normalized size of antiderivative = 3.61

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + ((a^2*b + b^3)*d*f*x + (a^2
*b + b^3)*d*e - (a^2*b + b^3)*f)*cosh(d*x + c)^2 + ((a^2*b + b^3)*d*f*x +
(a^2*b + b^3)*d*e - (a^2*b + b^3)*f)*sinh(d*x + c)^2 + 2*(a^2*b*f*cosh(d*x
 + c) + a^2*b*f*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c
) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2) - b)/b + 1) - 2*(a^2*b*f*cosh(d*x + c) + a^2*b*f*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x +
 c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*((a^2*b*d*e -
 a^2*b*c*f)*cosh(d*x + c) + (a^2*b*d*e - a^2*b*c*f)*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 +
 b^2)/b^2) + 2*a) + 2*((a^2*b*d*e - a^2*b*c*f)*cosh(d*x + c) + (a^2*b*d*e
- a^2*b*c*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) +
2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*((a^2*b*d*f*x + a
^2*b*c*f)*cosh(d*x + c) + (a^2*b*d*f*x + a^2*b*c*f)*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*((a^2*b*d*f*x + a^2*b
*c*f)*cosh(d*x + c) + (a^2*b*d*f*x + a^2*b*c*f)*sinh(d*x + c))*sqrt((a^2 +
 b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*
sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (a^2*b + b^3)*f - ((a^3 + a
*b^2)*d^2*f*x^2 + 2*(a^3 + a*b^2)*d^2*e*x)*cosh(d*x + c) - ((a^3 + a*b^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(4*a^2*integrate(x*e^(d*x + c)/(b^3*e^(2*d*x + 2*c) + 2*a*b^2*e^(d*x +
 c) - b^3), x) - (a*d^2*x^2*e^c - (b*d*x*e^(2*c) - b*e^(2*c))*e^(d*x) - (b
*d*x + b)*e^(-d*x))*e^(-c)/(b^2*d^2))*f + 1/2*e*(2*a^2*log((b*e^(-d*x - c)
 - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(sqrt(a^2
+ b^2)*b^2*d) - 2*(d*x + c)*a/(b^2*d) + e^(d*x + c)/(b*d) + e^(-d*x - c)/(
b*d))
```

Giac [F]

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*sinh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)^2\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((sinh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{4\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)a^2dei+2\cosh(dx+c)\,a^2bde+2\cosh(dx+c)\,a^2bdfx+2\cosh(dx+c)\,b^3de}{}$$

input
```
int((f*x+e)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(4*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**2
*d*e*i + 2*cosh(c + d*x)*a**2*b*d*e + 2*cosh(c + d*x)*a**2*b*d*f*x + 2*cos
h(c + d*x)*b**3*d*e + 2*cosh(c + d*x)*b**3*d*f*x + 4*e**c*int((e**(d*x)*x)
/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*d**2*f + 4*e**c*int((
e**(d*x)*x)/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b**2*d**2*
f - 2*sinh(c + d*x)*a**2*b*f - 2*sinh(c + d*x)*b**3*f - 2*a**3*d**2*e*x -
a**3*d**2*f*x**2 - 2*a*b**2*d**2*e*x - a*b**2*d**2*f*x**2)/(2*b**2*d**2*(a
**2 + b**2))
```

3.231 $\int \frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 71

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=-\frac{ax}{b^2}-\frac{2a^2\text{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d}+\frac{\cosh(c+dx)}{bd}$$

output
```
-a*x/b^2-2*a^2*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/b^2/(a^2
+b^2)^(1/2)/d+cosh(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\frac{-a\left(c+dx-\frac{2a\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}\right)+b\cosh(c+dx)}{b^2d}$$

input
```
Integrate[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

output
```
(-(a*(c + d*x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/S
qrt[-a^2 - b^2])) + b*Cosh[c + d*x])/(b^2*d)
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 25, 3225, 26, 27, 3042, 26, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ic+idx)^2}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ic+idx)^2}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3225}$$

$$\frac{\cosh(c+dx)}{bd}-\frac{i\int -\frac{ia\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\cosh(c+dx)}{bd}-\frac{\int \frac{a\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\cosh(c+dx)}{bd}-\frac{a\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh(c+dx)}{bd}-\frac{a\int -\frac{i\sin(ic+idx)}{a-ib\sin(ic+idx)}dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\cosh(c+dx)}{bd}+\frac{ia\int \frac{\sin(ic+idx)}{a-ib\sin(ic+idx)}dx}{b}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\cosh(c+dx)}{bd}+\frac{ia\left(\frac{ix}{b}-\frac{ia\int \frac{1}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh(c+dx)}{bd}+\frac{ia\left(\frac{ix}{b}-\frac{ia\int \frac{1}{a-ib\sin(ic+idx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3139}$$

$$\frac{\cosh(c+dx)}{bd}+\frac{ia\left(\frac{ix}{b}-\frac{2a\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{bd}\right)}{b}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\cosh(c+dx)}{bd}+\frac{ia\left(\frac{4a\int \frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4\left(a^2+b^2\right)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{bd}+\frac{ix}{b}\right)}{b}$$

$$\Big\downarrow \text{217}$$

$$\frac{\cosh(c+dx)}{bd}+\frac{ia\left(\frac{ix}{b}-\frac{2ia\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{bd\sqrt{a^2+b^2}}\right)}{b}$$

input

```
Int[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

output

```
(I*a*((I*x)/b - ((2*I)*a*ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b^2])])/(
b*Sqrt[a^2 + b^2]*d)))/b + Cosh[c + d*x]/(b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3225
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Simp[1/d
Int[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.70

method	result
derivativedivides	$\frac{-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{1}{b\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{a\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}+\frac{2a^2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\sqrt{a^2+b^2}}\right.}{b^2\sqrt{a^2+b^2}}}{d}$
default	$\frac{-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{1}{b\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{a\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}+\frac{2a^2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\sqrt{a^2+b^2}}\right.}{b^2\sqrt{a^2+b^2}}}{d}$
risch	$-\frac{ax}{b^2}+\frac{\mathrm{e}^{dx+c}}{2bd}+\frac{\mathrm{e}^{-dx-c}}{2bd}+\frac{a^2\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d\,b^2}-\frac{a^2\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d\,b^2}$

input
```
int(sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b/(tanh(1/2*d*x+1/2*c)-1)+a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)+1/b/(1+t
anh(1/2*d*x+1/2*c))-a/b^2*ln(1+tanh(1/2*d*x+1/2*c))+2*a^2/b^2/(a^2+b^2)^(1
/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(68) = 136.

Time = 0.11 (sec) , antiderivative size = 331, normalized size of antiderivative = 4.66

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{2\left(a^3+ab^2\right)dx\cosh\left(dx+c\right)-a^2b-b^3-\left(a^2b+b^3\right)\cosh\left(dx+c\right)^2-\left(a^2b+b^3\right)\sinh\left(dx+c\right)^2-2\left(a^2\right.}{}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(2*(a^3 + a*b^2)*d*x*cosh(d*x + c) - a^2*b - b^3 - (a^2*b + b^3)*cosh
(d*x + c)^2 - (a^2*b + b^3)*sinh(d*x + c)^2 - 2*(a^2*cosh(d*x + c) + a^2*s
inh(d*x + c))*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)
^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(
d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*c
osh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c
) + a)*sinh(d*x + c) - b)) + 2*((a^3 + a*b^2)*d*x - (a^2*b + b^3)*cosh(d*x
 + c))*sinh(d*x + c))/((a^2*b^2 + b^4)*d*cosh(d*x + c) + (a^2*b^2 + b^4)*d
*sinh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.68

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{a^2\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b^2d} - \frac{(dx+c)a}{b^2d} + \frac{e^{(dx+c)}}{2\,bd} + \frac{e^{(-dx-c)}}{2\,bd}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
a^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2*d) - (d*x + c)*a/(b^2*d) + 1/2*e^(d*x +
c)/(b*d) + 1/2*e^(-d*x - c)/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.56

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{2\,a^2\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^2} - \frac{2\,(dx+c)a}{b^2} + \frac{e^{(dx+c)}}{b} + \frac{e^{(-dx-c)}}{b}}{2\,d}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*a^2*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d
*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) - 2*(d*x + c)*a/
b^2 + e^(d*x + c)/b + e^(-d*x - c)/b)/d
```

Mupad [B] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.34

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{e^{c+d\,x}}{2\,b\,d} + \frac{e^{-c-d\,x}}{2\,b\,d} - \frac{a\,x}{b^2} - \frac{a^2\,\ln\left(-\frac{2\,a^2\,e^{c+d\,x}}{b^3} - \frac{2\,a^2\,\left(b-a\,e^{c+d\,x}\right)}{b^3\,\sqrt{a^2+b^2}}\right)}{b^2\,d\,\sqrt{a^2+b^2}} + \frac{a^2\,\ln\left(\frac{2\,a^2\,\left(b-a\,e^{c+d\,x}\right)}{b^3\,\sqrt{a^2+b^2}} - \frac{2\,a^2\,e^{c+d\,x}}{b^3}\right)}{b^2\,d\,\sqrt{a^2+b^2}}$$

input
```
int(sinh(c + d*x)^2/(a + b*sinh(c + d*x)),x)
```

output
```
exp(c + d*x)/(2*b*d) + exp(- c - d*x)/(2*b*d) - (a*x)/b^2 - (a^2*log(- (2*
a^2*exp(c + d*x))/b^3 - (2*a^2*(b - a*exp(c + d*x)))/(b^3*(a^2 + b^2)^(1/2
))))/(b^2*d*(a^2 + b^2)^(1/2)) + (a^2*log((2*a^2*(b - a*exp(c + d*x)))/(b^
3*(a^2 + b^2)^(1/2)) - (2*a^2*exp(c + d*x))/b^3))/(b^2*d*(a^2 + b^2)^(1/2)
)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.31

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\frac{2\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)a^2i+\cosh\left(dx+c\right)a^2b+\cosh\left(dx+c\right)b^3-a^3dx-a\,b^2dx}{b^2d\left(a^2+b^2\right)}$$

input
```
int(sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(2*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**2
*i + cosh(c + d*x)*a**2*b + cosh(c + d*x)*b**3 - a**3*d*x - a*b**2*d*x)/(b
**2*d*(a**2 + b**2))
```

3.232 $\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 8.65 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[Sinh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Sinh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx$$

$$\Big\downarrow \text{ 6111}$$

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Sinh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(dx+c)^2}{(fx+e)\,(a+b\sinh(dx+c))}dx$$

input
```
int(sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sinh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 158, normalized size of antiderivative = 5.64

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
2*a^2*integrate(-e^(d*x + c)/(b^3*f*x + b^3*e - (b^3*f*x*e^(2*c) + b^3*e*e
^(2*c))*e^(2*d*x) - 2*(a*b^2*f*x*e^c + a*b^2*e*e^c)*e^(d*x)), x) + 1/2*e^(
-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b*f) - 1/2*e^(c - d*e/f)*exp
_integral_e(1, -(f*x + e)*d/f)/(b*f) - a*log(f*x + e)/(b^2*f)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\sinh\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sinh(d*x + c)^2/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\sinh(c+d\,x)^2}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(sinh(c + d*x)^2/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(sinh(c + d*x)^2/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 781, normalized size of antiderivative = 27.89

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$= \frac{2e^{dx+c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)a^2i+e^{dx+c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)b^2i+2e^{dx+c}\cosh\left(dx+c\right)a^2b+2e}{}$$

input
```
int(sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
(2*e**(c + d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2
+ b**2))*a**2*i + e**(c + d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i +
a*i)/sqrt(a**2 + b**2))*b**2*i + 2*e**(c + d*x)*cosh(c + d*x)*a**2*b + 2*e
**(c + d*x)*cosh(c + d*x)*b**3 - e**(4*c + d*x)*int((e**(3*d*x)*x)/(e**(2*
c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x + 2*e**(c + d*x)*a*e + 2*e**(c + d
*x)*a*f*x - b*e - b*f*x),x)*a**2*b**2*d*f - e**(4*c + d*x)*int((e**(3*d*x)
*x)/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x + 2*e**(c + d*x)*a*e +
2*e**(c + d*x)*a*f*x - b*e - b*f*x),x)*b**4*d*f - 2*e**(2*c + d*x)*int(e**
(d*x)/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x + 2*e**(c + d*x)*a*e
+ 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x)*a**2*b**2*d*e - 2*e**(2*c + d*x)*
int(e**(d*x)/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x + 2*e**(c + d*
x)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x)*b**4*d*e - e**(c + d*x)*lo
g(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*a**3 - e**(c + d*x)*log(e**(2
*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*a*b**2 + e**(d*x)*int(1/(e**(2*c + 3
*d*x)*b*e + e**(2*c + 3*d*x)*b*f*x + 2*e**(c + 2*d*x)*a*e + 2*e**(c + 2*d*
x)*a*f*x - e**(d*x)*b*e - e**(d*x)*b*f*x),x)*a**2*b**2*d*e + e**(d*x)*int(
1/(e**(2*c + 3*d*x)*b*e + e**(2*c + 3*d*x)*b*f*x + 2*e**(c + 2*d*x)*a*e +
2*e**(c + 2*d*x)*a*f*x - e**(d*x)*b*e - e**(d*x)*b*f*x),x)*b**4*d*e - a**2
*b - b**3)/(2*e**(c + d*x)*b**2*d*e*(a**2 + b**2))
```

3.233 $\int \frac{(e+fx)^3 \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 699

$$\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{3f(e+fx)^2}{8bd^2}+\frac{a^2(e+fx)^4}{4b^3f}-\frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cosh(c+dx)}{b^2d^3} - \frac{a(e+fx)^3\cosh(c+dx)}{b^2d} - \frac{a^3(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d} + \frac{a^3(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d} - \frac{3a^3f(e+fx)^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d^2} + \frac{3a^3f(e+fx)^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d^2} + \frac{6a^3f^2(e+fx)\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d^3} - \frac{6a^3f^2(e+fx)\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d^3} - \frac{6a^3f^3\,\mathrm{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d^4} + \frac{6a^3f^3\,\mathrm{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d^4} + \frac{6af^3\sinh(c+dx)}{b^2d^4}+\frac{3af(e+fx)^2\sinh(c+dx)}{b^2d^2} + \frac{3f^2(e+fx)\cosh(c+dx)\sinh(c+dx)}{4bd^3} + \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{3f^3\sinh^2(c+dx)}{8bd^4}-\frac{3f(e+fx)^2\sinh^2(c+dx)}{4bd^2}$$

output
```
-3/8*f*(f*x+e)^2/b/d^2+1/4*a^2*(f*x+e)^4/b^3/f-1/8*(f*x+e)^4/b/f-6*a*f^2*(
f*x+e)*cosh(d*x+c)/b^2/d^3-a*(f*x+e)^3*cosh(d*x+c)/b^2/d-a^3*(f*x+e)^3*ln(
1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/(a^2+b^2)^(1/2)/d+a^3*(f*x+e)^3*ln
(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/(a^2+b^2)^(1/2)/d-3*a^3*f*(f*x+e)
^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/(a^2+b^2)^(1/2)/d^2+3*
a^3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/(a^2+b^2)
^(1/2)/d^2+6*a^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/
b^3/(a^2+b^2)^(1/2)/d^3-6*a^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+
b^2)^(1/2)))/b^3/(a^2+b^2)^(1/2)/d^3-6*a^3*f^3*polylog(4,-b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))/b^3/(a^2+b^2)^(1/2)/d^4+6*a^3*f^3*polylog(4,-b*exp(d*x+c
)/(a+(a^2+b^2)^(1/2)))/b^3/(a^2+b^2)^(1/2)/d^4+6*a*f^3*sinh(d*x+c)/b^2/d^4
+3*a*f*(f*x+e)^2*sinh(d*x+c)/b^2/d^2+3/4*f^2*(f*x+e)*cosh(d*x+c)*sinh(d*x+
c)/b/d^3+1/2*(f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/b/d-3/8*f^3*sinh(d*x+c)^2/b
/d^4-3/4*f*(f*x+e)^2*sinh(d*x+c)^2/b/d^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1407 vs. $2(699) = 1398$.

Time = 3.38 (sec) , antiderivative size = 1407, normalized size of antiderivative = 2.01

$$\int \frac{(e+fx)^3 \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(16*a^2*Sqrt[a^2 + b^2]*d^4*e^3*x - 8*b^2*Sqrt[a^2 + b^2]*d^4*e^3*x + 24*a
^2*Sqrt[a^2 + b^2]*d^4*e^2*f*x^2 - 12*b^2*Sqrt[a^2 + b^2]*d^4*e^2*f*x^2 +
16*a^2*Sqrt[a^2 + b^2]*d^4*e*f^2*x^3 - 8*b^2*Sqrt[a^2 + b^2]*d^4*e*f^2*x^3
 + 4*a^2*Sqrt[a^2 + b^2]*d^4*f^3*x^4 - 2*b^2*Sqrt[a^2 + b^2]*d^4*f^3*x^4 +
 32*a^3*d^3*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 16*a*b*Sqrt
[a^2 + b^2]*d^3*e^3*Cosh[c + d*x] - 96*a*b*Sqrt[a^2 + b^2]*d*e*f^2*Cosh[c
+ d*x] - 48*a*b*Sqrt[a^2 + b^2]*d^3*e^2*f*x*Cosh[c + d*x] - 96*a*b*Sqrt[a^
2 + b^2]*d*f^3*x*Cosh[c + d*x] - 48*a*b*Sqrt[a^2 + b^2]*d^3*e*f^2*x^2*Cosh
[c + d*x] - 16*a*b*Sqrt[a^2 + b^2]*d^3*f^3*x^3*Cosh[c + d*x] - 6*b^2*Sqrt[
a^2 + b^2]*d^2*e^2*f*Cosh[2*(c + d*x)] - 3*b^2*Sqrt[a^2 + b^2]*f^3*Cosh[2*
(c + d*x)] - 12*b^2*Sqrt[a^2 + b^2]*d^2*e*f^2*x*Cosh[2*(c + d*x)] - 6*b^2*
Sqrt[a^2 + b^2]*d^2*f^3*x^2*Cosh[2*(c + d*x)] - 48*a^3*d^3*e^2*f*x*Log[1 +
 (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 48*a^3*d^3*e*f^2*x^2*Log[1 + (b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 16*a^3*d^3*f^3*x^3*Log[1 + (b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2])] + 48*a^3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2])] + 48*a^3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])] + 16*a^3*d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2])] - 48*a^3*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a +
Sqrt[a^2 + b^2])] + 48*a^3*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))] + 96*a^3*d*e*f^2*PolyLog[3, (b*E^(c + d*x))/(-a...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6091}$$

$$\frac{\int (e+fx)^3\sinh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -(e+fx)^3\sin(ic+idx)^2dx}{b}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{a\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\int (e+fx)^3\sin(ic+idx)^2dx}{b}$$

$\big\downarrow$ 3792

$$-\frac{a\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{3f^2\int -((e+fx)\sinh^2(c+dx))dx}{2d^2}+\frac{1}{2}\int (e+fx)^3dx+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}}{b}$$

$\big\downarrow$ 17

$$-\frac{a\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{3f^2\int -((e+fx)\sinh^2(c+dx))dx}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}$$

$\big\downarrow$ 25

$$-\frac{a\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{-\frac{3f^2\int (e+fx)\sinh^2(c+dx)dx}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}$$

$\big\downarrow$ 3042

$$-\frac{a\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{-\frac{3f^2\int -((e+fx)\sin(ic+idx)^2)dx}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}$$

$\big\downarrow$ 25

$$-\frac{a\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{3f^2\int (e+fx)\sin(ic+idx)^2dx}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}$$

$\big\downarrow$ 3791

$$-\frac{a\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{3f^2\left(\frac{1}{2}\int (e+fx)dx+\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}$$

$\big\downarrow$ 17

$$-\frac{a\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}$$

$$\Big\downarrow \text{ 6091}$$

$$-\frac{a\left(\frac{\int (e+fx)^3\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -i(e+fx)^3\sin(ic+idx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\int (e+fx)^3\sin(ic+idx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3777}$$

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\cosh(c+dx)dx}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \ 3777$$

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int-i(e+fx)\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \ 26$$

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int(e+fx)\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int-i(e+fx)\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 26

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int(e+fx)\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 3777

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{d}\right)}{d}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 3042

$$-\frac{-\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 3117$$

$$-\frac{-\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 6091$$

$$-\frac{-\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)^3dx}{b}-\frac{a\int\frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}\right)}{b}$$

$\downarrow$ 17

$$-\frac{-\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\left(\frac{(e+fx)^4}{4bf}-\frac{a\int\frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}\right)}{b}$$

$\downarrow$ 3042

$$-\frac{-\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\left(\frac{(e+fx)^4}{4bf}-\frac{a\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 3803

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-$$

$$\frac{a\left(-\frac{a\left(\frac{(e+fx)^4}{4bf}-\frac{2a\int-\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right.}{d}\right.}{d}\right.}{b}\right.}{b}$$

$\Big\downarrow$ 25

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-$$

$$\frac{a\left(-\frac{a\left(\frac{2a\int\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}+\frac{(e+fx)^4}{4bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right.}{d}\right.}{d}\right.}{b}\right.}{b}$$

$\Big\downarrow$ 2694

$$-\frac{-\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-$$

$$\frac{a\left(-\frac{a\left(\frac{2a\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^4}{4bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\right.}{}\right.}{b}\right.}{b}$$

$\downarrow$ 27

$$-\frac{-\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-$$

$$\frac{a\left(-\frac{a\left(\frac{2a\left(\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^4}{4bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+f}{}\right.}{}\right.}{d}\right.}{b}\right.}{b}$$

$\downarrow$ 2620

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}-}{b}$$

$$\frac{a\left(-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{b}\right.}{b}\right.}{b}$$

input
```
Int[((e + f*x)^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^3\sinh(dx+c)^3}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5191 vs. 2(643) = 1286.

Time = 0.22 (sec) , antiderivative size = 5191, normalized size of antiderivative = 7.43

$$\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/8*e^3*(8*a^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c)
 - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^3*d) + (4*a*e^(-d*x - c) - b)*
e^(2*d*x + 2*c)/(b^2*d) - 4*(2*a^2 - b^2)*(d*x + c)/(b^3*d) + (4*a*e^(-d*x
 - c) + b*e^(-2*d*x - 2*c))/(b^2*d)) + 1/32*(4*(2*a^2*d^4*f^3*e^(2*c) - b^
2*d^4*f^3*e^(2*c))*x^4 + 16*(2*a^2*d^4*e*f^2*e^(2*c) - b^2*d^4*e*f^2*e^(2*
c))*x^3 + 24*(2*a^2*d^4*e^2*f*e^(2*c) - b^2*d^4*e^2*f*e^(2*c))*x^2 + (4*b^
2*d^3*f^3*x^3*e^(4*c) + 6*(2*d^3*e*f^2 - d^2*f^3)*b^2*x^2*e^(4*c) + 6*(2*d
^3*e^2*f - 2*d^2*e*f^2 + d*f^3)*b^2*x*e^(4*c) - 3*(2*d^2*e^2*f - 2*d*e*f^2
 + f^3)*b^2*e^(4*c))*e^(2*d*x) - 16*(a*b*d^3*f^3*x^3*e^(3*c) + 3*(d^3*e*f^
2 - d^2*f^3)*a*b*x^2*e^(3*c) + 3*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*a*b*x
*e^(3*c) - 3*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*a*b*e^(3*c))*e^(d*x) - 16*(a*
b*d^3*f^3*x^3*e^c + 3*(d^3*e*f^2 + d^2*f^3)*a*b*x^2*e^c + 3*(d^3*e^2*f + 2
*d^2*e*f^2 + 2*d*f^3)*a*b*x*e^c + 3*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*a*b*e^
c)*e^(-d*x) - (4*b^2*d^3*f^3*x^3 + 6*(2*d^3*e*f^2 + d^2*f^3)*b^2*x^2 + 6*(
2*d^3*e^2*f + 2*d^2*e*f^2 + d*f^3)*b^2*x + 3*(2*d^2*e^2*f + 2*d*e*f^2 + f^
3)*b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^4) - integrate(2*(a^3*f^3*x^3*e^c + 3*
a^3*e*f^2*x^2*e^c + 3*a^3*e^2*f*x*e^c)*e^(d*x)/(b^4*e^(2*d*x + 2*c) + 2*a*
b^3*e^(d*x + c) - b^4), x)
```

Giac [F]

$$\int \frac{(e+fx)^3 \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\sinh\left(dx+c\right)^3}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sinh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\sinh(c+d\,x)^3\,(e+f\,x)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((sinh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\sinh\left(dx+c\right)^3}{a+b\sinh\left(dx+c\right)}dx$$

input
```
int((f*x+e)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

3.234 $\int \frac{(e+fx)^2 \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 522

$$\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{f^2x}{4bd^2}+\frac{a^2(e+fx)^3}{3b^3f}-\frac{(e+fx)^3}{6bf} -\frac{2af^2\cosh(c+dx)}{b^2d^3}-\frac{a(e+fx)^2\cosh(c+dx)}{b^2d} -\frac{a^3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d} +\frac{a^3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d} -\frac{2a^3f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d^2} +\frac{2a^3f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d^2} +\frac{2a^3f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d^3} -\frac{2a^3f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d^3} +\frac{2af(e+fx)\sinh(c+dx)}{b^2d^2} +\frac{f^2\cosh(c+dx)\sinh(c+dx)}{4bd^3} +\frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{2bd} -\frac{f(e+fx)\sinh^2(c+dx)}{2bd^2}$$

output
```
-1/4*f^2*x/b/d^2+1/3*a^2*(f*x+e)^3/b^3/f-1/6*(f*x+e)^3/b/f-2*a*f^2*cosh(d*
x+c)/b^2/d^3-a*(f*x+e)^2*cosh(d*x+c)/b^2/d-a^3*(f*x+e)^2*ln(1+b*exp(d*x+c)
/(a-(a^2+b^2)^(1/2)))/b^3/(a^2+b^2)^(1/2)/d+a^3*(f*x+e)^2*ln(1+b*exp(d*x+c
)/(a+(a^2+b^2)^(1/2)))/b^3/(a^2+b^2)^(1/2)/d-2*a^3*f*(f*x+e)*polylog(2,-b*
exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/(a^2+b^2)^(1/2)/d^2+2*a^3*f*(f*x+e)*po
lylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/(a^2+b^2)^(1/2)/d^2+2*a^3*f
^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/(a^2+b^2)^(1/2)/d^3-2*
a^3*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/(a^2+b^2)^(1/2)/d
^3+2*a*f*(f*x+e)*sinh(d*x+c)/b^2/d^2+1/4*f^2*cosh(d*x+c)*sinh(d*x+c)/b/d^3
+1/2*(f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/b/d-1/2*f*(f*x+e)*sinh(d*x+c)^2/b/d
^2
```

Mathematica [A] (verified)

Time = 3.54 (sec) , antiderivative size = 740, normalized size of antiderivative = 1.42

$$\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{24a^2e^2x-12b^2e^2x+24a^2efx^2-12b^2efx^2+8a^2f^2x^3-4b^2f^2x^3+\frac{48a^3e^2\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}d}-\frac{24abe^2\cosh(c}{d}}{}$$

input
```
Integrate[((e + f*x)^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(24*a^2*e^2*x - 12*b^2*e^2*x + 24*a^2*e*f*x^2 - 12*b^2*e*f*x^2 + 8*a^2*f^2
*x^3 - 4*b^2*f^2*x^3 + (48*a^3*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 +
b^2]])/(Sqrt[a^2 + b^2]*d) - (24*a*b*e^2*Cosh[c + d*x])/d - (48*a*b*f^2*Co
sh[c + d*x])/d^3 - (48*a*b*e*f*x*Cosh[c + d*x])/d - (24*a*b*f^2*x^2*Cosh[c
 + d*x])/d - (6*b^2*e*f*Cosh[2*(c + d*x)])/d^2 - (6*b^2*f^2*x*Cosh[2*(c +
d*x)])/d^2 - (48*a^3*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])
/(Sqrt[a^2 + b^2]*d) - (24*a^3*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2])])/(Sqrt[a^2 + b^2]*d) + (48*a^3*e*f*x*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])])/(Sqrt[a^2 + b^2]*d) + (24*a^3*f^2*x^2*Log[1 + (b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2])])/(Sqrt[a^2 + b^2]*d) - (48*a^3*f*(e + f*
x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])])/(Sqrt[a^2 + b^2]*d^
2) + (48*a^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
))])/(Sqrt[a^2 + b^2]*d^2) + (48*a^3*f^2*PolyLog[3, (b*E^(c + d*x))/(-a +
Sqrt[a^2 + b^2])])/(Sqrt[a^2 + b^2]*d^3) - (48*a^3*f^2*PolyLog[3, -((b*E^(
c + d*x))/(a + Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*d^3) + (48*a*b*e*f*Sin
h[c + d*x])/d^2 + (48*a*b*f^2*x*Sinh[c + d*x])/d^2 + (6*b^2*e^2*Sinh[2*(c
+ d*x)])/d + (3*b^2*f^2*Sinh[2*(c + d*x)])/d^3 + (12*b^2*e*f*x*Sinh[2*(c +
 d*x)])/d + (6*b^2*f^2*x^2*Sinh[2*(c + d*x)])/d)/(24*b^3)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \ 6091$$

$$\frac{\int (e+fx)^2\sinh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -(e+fx)^2\sin(ic+idx)^2dx}{b}$$

$$\Big\downarrow \ 25$$

$$-\frac{a\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\int (e+fx)^2\sin(ic+idx)^2dx}{b}$$

$$\Big\downarrow \text{3792}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{f^2\int -\sinh^2(c+dx)dx}{2d^2}+\frac{1}{2}\int (e+fx)^2dx+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}}{b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{f^2\int -\sinh^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{-\frac{f^2\int \sinh^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{-\frac{f^2\int -\sin(ic+idx)^2dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{f^2\int \sin(ic+idx)^2dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{f^2\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}$$

$$\Big\downarrow \text{24}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}$$

$$\Big\downarrow \text{6091}$$

$$-\frac{a\left(\frac{\int (e+fx)^2\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -i(e+fx)^2\sin(ic+idx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\int (e+fx)^2\sin(ic+idx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\cosh(c+dx)dx}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3777}$$

$$-\frac{-\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int-i\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{-\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{-\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int-i\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{-\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3118}$$

$$-\frac{-\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 6091}$$

$$-\frac{-\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)^2dx}{b}-\frac{a\int\frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{-\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\left(\frac{(e+fx)^3}{3bf}-\frac{a\int\frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{-\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\left(\frac{(e+fx)^3}{3bf}-\frac{a\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3803}$$

$$-\frac{-\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\left(\frac{(e+fx)^3}{3bf}-\frac{2a\int-\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{-\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\left(\frac{2a\int\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}+\frac{(e+fx)^3}{3bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 2694}$$

$$
\frac{-\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\left(\frac{2a\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^3}{3bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f}{}\right.}{d}\right.}{b}}{b}
$$

$\downarrow$ 27

$$
\frac{-\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\left(\frac{2a\left(\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^3}{3bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx}{d^2}\right.}{d}\right.}{b}}{b}
$$

$\downarrow$ 2620

$$-\frac{\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-$$

$$\frac{a\left(-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\cdots\right.}{bd}\cdots\right.}{2\sqrt{a^2+b^2}}\cdots\right.}{b}\cdots\right.}{b}\cdots\right.}{b}$$

$\downarrow$ 3011

$$-\frac{-\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-$$

$$a\left(-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+\cdots}}\right.}{bd}\right.}{\cdots}\right)}{b}}{b}\right.$$

↓ 2720

$$-\frac{-\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-$$

$$a\left(-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2}{}\right.}{}}{b}\right.}{b}\right.$$

input
```
Int[((e + f*x)^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^2 \sinh(dx+c)^3}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3247 vs. $2(478) = 956$.

Time = 0.16 (sec) , antiderivative size = 3247, normalized size of antiderivative = 6.22

$$\int \frac{(e+fx)^2 \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^2 \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2 \sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/8*e^2*(8*a^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c)
 - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^3*d) + (4*a*e^(-d*x - c) - b)*
e^(2*d*x + 2*c)/(b^2*d) - 4*(2*a^2 - b^2)*(d*x + c)/(b^3*d) + (4*a*e^(-d*x
 - c) + b*e^(-2*d*x - 2*c))/(b^2*d)) + 1/48*(8*(2*a^2*d^3*f^2*e^(2*c) - b^
2*d^3*f^2*e^(2*c))*x^3 + 24*(2*a^2*d^3*e*f*e^(2*c) - b^2*d^3*e*f*e^(2*c))*
x^2 + 3*(2*b^2*d^2*f^2*x^2*e^(4*c) + 2*(2*d^2*e*f - d*f^2)*b^2*x*e^(4*c) -
 (2*d*e*f - f^2)*b^2*e^(4*c))*e^(2*d*x) - 24*(a*b*d^2*f^2*x^2*e^(3*c) + 2*
(d^2*e*f - d*f^2)*a*b*x*e^(3*c) - 2*(d*e*f - f^2)*a*b*e^(3*c))*e^(d*x) - 2
4*(a*b*d^2*f^2*x^2*e^c + 2*(d^2*e*f + d*f^2)*a*b*x*e^c + 2*(d*e*f + f^2)*a
*b*e^c)*e^(-d*x) - 3*(2*b^2*d^2*f^2*x^2 + 2*(2*d^2*e*f + d*f^2)*b^2*x + (2
*d*e*f + f^2)*b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^3) - integrate(2*(a^3*f^2*x
^2*e^c + 2*a^3*e*f*x*e^c)*e^(d*x)/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x +
c) - b^4), x)
```

Giac [F]

$$\int \frac{(e+fx)^2 \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2 \sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sinh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\sinh(c+d\,x)^3\,(e+f\,x)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((sinh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{too large to display}$$

input
```
int((f*x+e)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output

```
( - 96*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sq
rt(a**2 + b**2))*a**3*b**2*d**2*e**2*i + 6*e**(4*c + 4*d*x)*a**2*b**4*d**2
*e**2 + 12*e**(4*c + 4*d*x)*a**2*b**4*d**2*e*f*x + 6*e**(4*c + 4*d*x)*a**2
*b**4*d**2*f**2*x**2 - 6*e**(4*c + 4*d*x)*a**2*b**4*d*e*f - 6*e**(4*c + 4*
d*x)*a**2*b**4*d*f**2*x + 3*e**(4*c + 4*d*x)*a**2*b**4*f**2 + 6*e**(4*c +
4*d*x)*b**6*d**2*e**2 + 12*e**(4*c + 4*d*x)*b**6*d**2*e*f*x + 6*e**(4*c +
4*d*x)*b**6*d**2*f**2*x**2 - 6*e**(4*c + 4*d*x)*b**6*d*e*f - 6*e**(4*c + 4
*d*x)*b**6*d*f**2*x + 3*e**(4*c + 4*d*x)*b**6*f**2 - 24*e**(3*c + 3*d*x)*a
**3*b**3*d**2*e**2 - 48*e**(3*c + 3*d*x)*a**3*b**3*d**2*e*f*x - 24*e**(3*c
 + 3*d*x)*a**3*b**3*d**2*f**2*x**2 + 48*e**(3*c + 3*d*x)*a**3*b**3*d*e*f +
 48*e**(3*c + 3*d*x)*a**3*b**3*d*f**2*x - 48*e**(3*c + 3*d*x)*a**3*b**3*f*
*2 - 24*e**(3*c + 3*d*x)*a*b**5*d**2*e**2 - 48*e**(3*c + 3*d*x)*a*b**5*d**
2*e*f*x - 24*e**(3*c + 3*d*x)*a*b**5*d**2*f**2*x**2 + 48*e**(3*c + 3*d*x)*
a*b**5*d*e*f + 48*e**(3*c + 3*d*x)*a*b**5*d*f**2*x - 48*e**(3*c + 3*d*x)*a
*b**5*f**2 + 192*e**(2*c + 2*d*x)*int(x**2/(e**(4*c + 4*d*x)*b + 2*e**(3*c
 + 3*d*x)*a - e**(2*c + 2*d*x)*b),x)*a**6*b*d**3*f**2 + 192*e**(2*c + 2*d*
x)*int(x**2/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*
b),x)*a**4*b**3*d**3*f**2 + 384*e**(2*c + 2*d*x)*int(x/(e**(4*c + 4*d*x)*b
 + 2*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b),x)*a**6*b*d**3*e*f + 384*e**
(2*c + 2*d*x)*int(x/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a - e**(2*...
```

3.235 $\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 329

$$\begin{aligned}\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{a^2(e+fx)^2}{2b^3f}-\frac{(e+fx)^2}{4bf}-\frac{a(e+fx)\cosh(c+dx)}{b^2d}\\
&\quad-\frac{a^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d}\\
&\quad+\frac{a^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d}\\
&\quad-\frac{a^3f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d^2}\\
&\quad+\frac{a^3f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d^2}+\frac{af\sinh(c+dx)}{b^2d^2}\\
&\quad+\frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{2bd}-\frac{f\sinh^2(c+dx)}{4bd^2}\end{aligned}$$

output
```
1/2*a^2*(f*x+e)^2/b^3/f-1/4*(f*x+e)^2/b/f-a*(f*x+e)*cosh(d*x+c)/b^2/d-a^3*
(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/(a^2+b^2)^(1/2)/d+a^3*(
f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/(a^2+b^2)^(1/2)/d-a^3*f*
polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/(a^2+b^2)^(1/2)/d^2+a^3*f
*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/(a^2+b^2)^(1/2)/d^2+a*f*
sinh(d*x+c)/b^2/d^2+1/2*(f*x+e)*cosh(d*x+c)*sinh(d*x+c)/b/d-1/4*f*sinh(d*x
+c)^2/b/d^2
```

Mathematica [A] (verified)

Time = 4.71 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.93

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{2(2a^2-b^2)(c+dx)(cf-d(2e+fx))+8abd(e+fx)\cosh(c+dx)+b^2f\cosh(2(c+dx))+\frac{8a^3\left(-2de\right.}{}}{}$$

input
```
Integrate[((e + f*x)*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/8*(2*(2*a^2 - b^2)*(c + d*x)*(c*f - d*(2*e + f*x)) + 8*a*b*d*(e + f*x)*
Cosh[c + d*x] + b^2*f*Cosh[2*(c + d*x)] + (8*a^3*(-2*d*e*ArcTanh[(a + b*E^
(c + d*x))/Sqrt[a^2 + b^2]] + 2*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 +
 b^2]] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - f*(c
 + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + f*PolyLog[2, (b*E
^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - f*PolyLog[2, -((b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2]))]))/Sqrt[a^2 + b^2] - 8*a*b*f*Sinh[c + d*x] - 2*b^2*d*(e
+ f*x)*Sinh[2*(c + d*x)])/(b^3*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.82 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.99, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.808$, Rules used = {6091, 3042, 25, 3791, 17, 6091, 3042, 26, 3777, 3042, 3117, 6091, 17, 3042, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6091}$$

$$\frac{\int (e+fx)\sinh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -\left((e+fx)\sin(ic+idx)^2\right)dx}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\int (e+fx)\sin(ic+idx)^2dx}{b}$$

$$\Big\downarrow \text{3791}$$

$$-\frac{a\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{1}{2}\int (e+fx)dx+\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}}{b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{a\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}$$

$$\Big\downarrow \text{6091}$$

$$-\frac{a\left(\frac{\int (e+fx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -i(e+fx)\sin(ic+idx)dx}{b}\right)}{b}$$

$$\Big\downarrow \ 26$$

$$-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\int(e+fx)\sin(ic+idx)dx}{b}\right)}{b}$$

$$\Big\downarrow \ 3777$$

$$-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \ 3117$$

$$-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{b}$$

$$\Big\downarrow \ 6091$$

$$-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)dx}{b}-\frac{a\int\frac{e+fx}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{b}$$

$\downarrow$ 17

$$-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{a\left(\frac{(e+fx)^2}{2bf}-\frac{a\int\frac{e+fx}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{b}$$

$\downarrow$ 3042

$$-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{a\left(\frac{(e+fx)^2}{2bf}-\frac{a\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{b}$$

$\downarrow$ 3803

$$-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{a\left(\frac{(e+fx)^2}{2bf}-\frac{2a\int-\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{b}$$

$\downarrow$ 25

$$-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{a\left(\frac{2a\int\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}+\frac{(e+fx)^2}{2bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{2694}$$

$$-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{a\left(\frac{2a\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^2}{2bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{a\left(\frac{2a\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^2}{2bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(a-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^2}{2bf}}{b}\right)}{b}$$

$$\Big\downarrow \ 2715$$

$$-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(a-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}\right.}{bd^2}}{2\sqrt{a^2+b^2}}}{b}}{b}\right)}{b}$$

$$\Big\downarrow \ 2838$$

$$-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-$$

$$\frac{a\left(-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+f}{2b}\right.}{b}\right.}{b}$$

input
```
Int[((e + f*x)*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
-(((e + f*x)^2/(4*f) - ((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (f*
Sinh[c + d*x]^2)/(4*d^2))/b) - (a*(-((a*((e + f*x)^2/(2*b*f) + (2*a*(-1/2*
(b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (f*
PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2)))/Sqrt[a^2 +
 b^2] + (b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*
d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(2
*Sqrt[a^2 + b^2])))/b))/b) - (I*((I*(e + f*x)*Cosh[c + d*x])/d - (I*f*Sinh
[c + d*x])/d^2))/b))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.79

method	result
risch	$\frac{a^2 f x^2}{2b^3} - \frac{f x^2}{4b} + \frac{a^2 e x}{b^3} - \frac{e x}{2b} + \frac{(2dxf+2de-f)\mathrm{e}^{2dx+2c}}{16b\,d^2} - \frac{a(dxf+de-f)\mathrm{e}^{dx+c}}{2b^2d^2} - \frac{a(dxf+de+f)\mathrm{e}^{-dx-c}}{2b^2d^2} - \frac{(2dxf+2de+}{16b}$

input
```
int((f*x+e)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^3*a^2*f*x^2-1/4/b*f*x^2+1/b^3*a^2*e*x-1/2/b*e*x+1/16*(2*d*f*x+2*d*e-
f)/b/d^2*exp(2*d*x+2*c)-1/2*a*(d*f*x+d*e-f)/b^2/d^2*exp(d*x+c)-1/2*a*(d*f*
x+d*e+f)/b^2/d^2*exp(-d*x-c)-1/16*(2*d*f*x+2*d*e+f)/b/d^2*exp(-2*d*x-2*c)+
2/d*a^3/b^3*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(
1/2))-1/d*a^3/b^3*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(
-a+(a^2+b^2)^(1/2)))*x+1/d*a^3/b^3*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2
+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2*a^3/b^3*f/(a^2+b^2)^(1/2)*ln((
-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d^2*a^3/b^3*f/(
a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-
1/d^2*a^3/b^3*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-
a+(a^2+b^2)^(1/2)))+1/d^2*a^3/b^3*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a
^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-2/d^2*a^3/b^3*f*c/(a^2+b^2)^(1/2)*ar
ctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1727 vs. 2(299) = 598.

Time = 0.14 (sec) , antiderivative size = 1727, normalized size of antiderivative = 5.25

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/16*((2*(a^2*b^2 + b^4)*d*f*x + 2*(a^2*b^2 + b^4)*d*e - (a^2*b^2 + b^4)*f
)*cosh(d*x + c)^4 + (2*(a^2*b^2 + b^4)*d*f*x + 2*(a^2*b^2 + b^4)*d*e - (a^
2*b^2 + b^4)*f)*sinh(d*x + c)^4 - 2*(a^2*b^2 + b^4)*d*f*x - 8*((a^3*b + a*
b^3)*d*f*x + (a^3*b + a*b^3)*d*e - (a^3*b + a*b^3)*f)*cosh(d*x + c)^3 - 4*
(2*(a^3*b + a*b^3)*d*f*x + 2*(a^3*b + a*b^3)*d*e - 2*(a^3*b + a*b^3)*f - (
2*(a^2*b^2 + b^4)*d*f*x + 2*(a^2*b^2 + b^4)*d*e - (a^2*b^2 + b^4)*f)*cosh(
d*x + c))*sinh(d*x + c)^3 - 2*(a^2*b^2 + b^4)*d*e + 4*((2*a^4 + a^2*b^2 -
b^4)*d^2*f*x^2 + 2*(2*a^4 + a^2*b^2 - b^4)*d^2*e*x)*cosh(d*x + c)^2 + 2*(2
*(2*a^4 + a^2*b^2 - b^4)*d^2*f*x^2 + 4*(2*a^4 + a^2*b^2 - b^4)*d^2*e*x + 3
*(2*(a^2*b^2 + b^4)*d*f*x + 2*(a^2*b^2 + b^4)*d*e - (a^2*b^2 + b^4)*f)*cos
h(d*x + c)^2 - 12*((a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*d*e - (a^3*b +
a*b^3)*f)*cosh(d*x + c))*sinh(d*x + c)^2 - 16*(a^3*b*f*cosh(d*x + c)^2 + 2
*a^3*b*f*cosh(d*x + c)*sinh(d*x + c) + a^3*b*f*sinh(d*x + c)^2)*sqrt((a^2
+ b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 16*(a^3*b*f*cosh(d*x
+ c)^2 + 2*a^3*b*f*cosh(d*x + c)*sinh(d*x + c) + a^3*b*f*sinh(d*x + c)^2)*
sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 16*((a^3*b*
d*e - a^3*b*c*f)*cosh(d*x + c)^2 + 2*(a^3*b*d*e - a^3*b*c*f)*cosh(d*x + c)
*sinh(d*x + c) + (a^3*b*d*e - a^3*b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/16*(32*a^3*integrate(x*e^(d*x + c)/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*
x + c) - b^4), x) - (4*(2*a^2*d^2*e^(2*c) - b^2*d^2*e^(2*c))*x^2 + (2*b^2*
d*x*e^(4*c) - b^2*e^(4*c))*e^(2*d*x) - 8*(a*b*d*x*e^(3*c) - a*b*e^(3*c))*e
^(d*x) - 8*(a*b*d*x*e^c + a*b*e^c)*e^(-d*x) - (2*b^2*d*x + b^2)*e^(-2*d*x)
)*e^(-2*c)/(b^3*d^2))*f - 1/8*e*(8*a^3*log((b*e^(-d*x - c) - a - sqrt(a^2
+ b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^3*d) +
(4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) - 4*(2*a^2 - b^2)*(d*x + c)
/(b^3*d) + (4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d))
```

Giac [F]

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*sinh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)^3\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((sinh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{64e^{2dx+2c}\left(\int \frac{x}{e^{4dx+4c}b+2e^{3dx+3c}a-e^{2dx+2c}b}dx\right)a^6b\,d^2f+64e^{2dx+2c}\left(\int \frac{x}{e^{4dx+4c}b+2e^{3dx+3c}a-e^{2dx+2c}b}dx\right)a^4b^3d^2f-1}{}$$

input
```
int((f*x+e)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
( - 32*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sq
rt(a**2 + b**2))*a**3*b**2*d*e*i + 2*e**(4*c + 4*d*x)*a**2*b**4*d*e + 2*e*
*(4*c + 4*d*x)*a**2*b**4*d*f*x - e**(4*c + 4*d*x)*a**2*b**4*f + 2*e**(4*c
+ 4*d*x)*b**6*d*e + 2*e**(4*c + 4*d*x)*b**6*d*f*x - e**(4*c + 4*d*x)*b**6*
f - 8*e**(3*c + 3*d*x)*a**3*b**3*d*e - 8*e**(3*c + 3*d*x)*a**3*b**3*d*f*x
+ 8*e**(3*c + 3*d*x)*a**3*b**3*f - 8*e**(3*c + 3*d*x)*a*b**5*d*e - 8*e**(3
*c + 3*d*x)*a*b**5*d*f*x + 8*e**(3*c + 3*d*x)*a*b**5*f + 64*e**(2*c + 2*d*
x)*int(x/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b),
x)*a**6*b*d**2*f + 64*e**(2*c + 2*d*x)*int(x/(e**(4*c + 4*d*x)*b + 2*e**(3
*c + 3*d*x)*a - e**(2*c + 2*d*x)*b),x)*a**4*b**3*d**2*f + 16*e**(2*c + 2*d
*x)*a**4*b**2*d**2*e*x + 8*e**(2*c + 2*d*x)*a**4*b**2*d**2*f*x**2 + 8*e**(
2*c + 2*d*x)*a**2*b**4*d**2*e*x + 4*e**(2*c + 2*d*x)*a**2*b**4*d**2*f*x**2
 - 8*e**(2*c + 2*d*x)*b**6*d**2*e*x - 4*e**(2*c + 2*d*x)*b**6*d**2*f*x**2
- 128*e**(c + 2*d*x)*int(x/(e**(2*c + 3*d*x)*b + 2*e**(c + 2*d*x)*a - e**(
d*x)*b),x)*a**7*d**2*f - 160*e**(c + 2*d*x)*int(x/(e**(2*c + 3*d*x)*b + 2*
e**(c + 2*d*x)*a - e**(d*x)*b),x)*a**5*b**2*d**2*f - 32*e**(c + 2*d*x)*int
(x/(e**(2*c + 3*d*x)*b + 2*e**(c + 2*d*x)*a - e**(d*x)*b),x)*a**3*b**4*d**
2*f + 32*e**(c + d*x)*a**5*b*d*f*x + 32*e**(c + d*x)*a**5*b*f - 8*e**(c +
d*x)*a**3*b**3*d*e + 24*e**(c + d*x)*a**3*b**3*d*f*x + 24*e**(c + d*x)*a**
3*b**3*f - 8*e**(c + d*x)*a*b**5*d*e - 8*e**(c + d*x)*a*b**5*d*f*x - 8*...
```

3.236 $\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{(2a^2-b^2)\,x}{2b^3} + \frac{2a^3\text{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}d} - \frac{a\cosh(c+dx)}{b^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2bd}$$

output
```
1/2*(2*a^2-b^2)*x/b^3+2*a^3*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1
/2))/b^3/(a^2+b^2)^(1/2)/d-a*cosh(d*x+c)/b^2/d+1/2*cosh(d*x+c)*sinh(d*x+c)
/b/d
```

Mathematica [A] (verified)

Time = 2.85 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{-2(-2a^2+b^2)(c+dx) - \frac{8a^3\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - 4ab\cosh(c+dx) + b^2\sinh(2(c+dx))}{4b^3d}$$

input
```
Integrate[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

output
```
(-2*(-2*a^2 + b^2)*(c + d*x) - (8*a^3*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqr
t[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - 4*a*b*Cosh[c + d*x] + b^2*Sinh[2*(c + d
*x)])/(4*b^3*d)
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.16, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 26, 3272, 3042, 3502, 26, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ic+idx)^3}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ic+idx)^3}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3272}$$

$$i\left(\frac{i\int \frac{2a\sinh^2(c+dx)+b\sinh(c+dx)+a}{a+b\sinh(c+dx)}dx}{2b}-\frac{i\sinh(c+dx)\cosh(c+dx)}{2bd}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{i\int \frac{-2a\sin(ic+idx)^2-ib\sin(ic+idx)+a}{a-ib\sin(ic+idx)}dx}{2b}-\frac{i\sinh(c+dx)\cosh(c+dx)}{2bd}\right)$$

$$\Big\downarrow \text{3502}$$

$$i\left(\frac{i\left(\frac{2a\cosh(c+dx)}{bd}+\frac{i\int -\frac{i\left(ab-\left(2a^2-b^2\right)\sinh(c+dx)\right)}{a+b\sinh(c+dx)}dx}{b}\right)}{2b}-\frac{i\sinh(c+dx)\cosh(c+dx)}{2bd}\right)$$

$$\Big\downarrow \text{ 26}$$

$$i\left(\frac{i\left(\frac{\int \frac{ab-\left(2a^2-b^2\right)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{2a\cosh(c+dx)}{bd}\right)}{2b}-\frac{i\sinh(c+dx)\cosh(c+dx)}{2bd}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$i\left(\frac{i\left(\frac{2a\cosh(c+dx)}{bd}+\frac{\int \frac{ab+i\left(2a^2-b^2\right)\sin(ic+idx)}{a-ib\sin(ic+idx)}dx}{b}\right)}{2b}-\frac{i\sinh(c+dx)\cosh(c+dx)}{2bd}\right)$$

$$\Big\downarrow \text{ 3214}$$

$$i\left(\frac{i\left(\frac{\frac{2a^3\int \frac{1}{a+b\sinh(c+dx)}dx}{b}-\frac{x\left(2a^2-b^2\right)}{b}}{b}+\frac{2a\cosh(c+dx)}{bd}\right)}{2b}-\frac{i\sinh(c+dx)\cosh(c+dx)}{2bd}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$i\left(\frac{i\left(\frac{2a\cosh(c+dx)}{bd}+\frac{-\frac{x\left(2a^2-b^2\right)}{b}+\frac{2a^3\int \frac{1}{a-ib\sin(ic+idx)}dx}{b}}{b}\right)}{2b}-\frac{i\sinh(c+dx)\cosh(c+dx)}{2bd}\right)$$

$$\Big\downarrow \text{ 3139}$$

$$i\left(\frac{i\left(\frac{2a\cosh(c+dx)}{bd}+\frac{-\frac{x(2a^2-b^2)}{b}-\frac{4ia^3\int\frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{bd}}{b}\right)}{2b}-\frac{i\sinh(c+dx)\cosh(c+\right.$$

$$\Big\downarrow \text{1083}$$

$$i\left(\frac{i\left(\frac{2a\cosh(c+dx)}{bd}+\frac{-\frac{x(2a^2-b^2)}{b}+\frac{8ia^3\int\frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4(a^2+b^2)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{bd}}{b}\right)}{2b}-\frac{i\sinh(c+dx)\cosh(c+dx)}{2bd}\right)$$

$$\Big\downarrow \text{217}$$

$$i\left(\frac{i\left(\frac{\frac{4a^3\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{bd\sqrt{a^2+b^2}}-\frac{x(2a^2-b^2)}{b}}{b}+\frac{2a\cosh(c+dx)}{bd}\right)}{2b}-\frac{i\sinh(c+dx)\cosh(c+dx)}{2bd}\right)$$

input
```
Int[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

output
```
I*(((I/2)*((-(((2*a^2 - b^2)*x)/b) + (4*a^3*ArcTanh[Tanh[(c + d*x)/2]/(2*S
qrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]*d))/b + (2*a*Cosh[c + d*x])/(b*d)))/b
 - ((I/2)*Cosh[c + d*x]*Sinh[c + d*x])/(b*d))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.79

method	result
derivativedivides	$\frac{\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-b-2a}{2b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-2a^2+b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^3}-\frac{1}{2b\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{-b+2a}{2b^2\left(1+\tanh\left(\frac{dx}{2}\right.\right.}}{d}$
default	$\frac{\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-b-2a}{2b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-2a^2+b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^3}-\frac{1}{2b\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{-b+2a}{2b^2\left(1+\tanh\left(\frac{dx}{2}\right.\right.}}{d}$
risch	$\frac{x\,a^2}{b^3}-\frac{x}{2b}+\frac{e^{2dx+2c}}{8bd}-\frac{a\,e^{dx+c}}{2b^2d}-\frac{a\,e^{-dx-c}}{2b^2d}-\frac{e^{-2dx-2c}}{8bd}+\frac{a^3\ln\left(e^{dx+c}+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d\,b^3}-\frac{a^3\ln\left(e^{dx-}\right.}{}$

input
```
int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/b/(tanh(1/2*d*x+1/2*c)-1)^2-1/2*(-b-2*a)/b^2/(tanh(1/2*d*x+1/2*c)
-1)+1/2/b^3*(-2*a^2+b^2)*ln(tanh(1/2*d*x+1/2*c)-1)-1/2/b/(1+tanh(1/2*d*x+1
/2*c))^2-1/2*(-b+2*a)/b^2/(1+tanh(1/2*d*x+1/2*c))+1/2*(2*a^2-b^2)/b^3*ln(1
+tanh(1/2*d*x+1/2*c))-2*a^3/b^3/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*
d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(100) = 200$.

Time = 0.10 (sec) , antiderivative size = 601, normalized size of antiderivative = 5.62

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/8*(4*(2*a^4 + a^2*b^2 - b^4)*d*x*cosh(d*x + c)^2 + (a^2*b^2 + b^4)*cosh(
d*x + c)^4 + (a^2*b^2 + b^4)*sinh(d*x + c)^4 - a^2*b^2 - b^4 - 4*(a^3*b +
a*b^3)*cosh(d*x + c)^3 - 4*(a^3*b + a*b^3 - (a^2*b^2 + b^4)*cosh(d*x + c))
*sinh(d*x + c)^3 + 2*(2*(2*a^4 + a^2*b^2 - b^4)*d*x + 3*(a^2*b^2 + b^4)*co
sh(d*x + c)^2 - 6*(a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*(a^3*
cosh(d*x + c)^2 + 2*a^3*cosh(d*x + c)*sinh(d*x + c) + a^3*sinh(d*x + c)^2)
*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*co
sh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*
sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^
2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d
*x + c) - b)) - 4*(a^3*b + a*b^3)*cosh(d*x + c) - 4*(a^3*b + a*b^3 - 2*(2*
a^4 + a^2*b^2 - b^4)*d*x*cosh(d*x + c) - (a^2*b^2 + b^4)*cosh(d*x + c)^3 +
 3*(a^3*b + a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^2*b^3 + b^5)*d*cosh
(d*x + c)^2 + 2*(a^2*b^3 + b^5)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2*b^3 +
 b^5)*d*sinh(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.53

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a^3\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b^3d} - \frac{\left(4\,ae^{(-dx-c)}-b\right)e^{(2\,dx+2\,c)}}{8\,b^2d} + \frac{(2\,a^2-b^2)(dx+c)}{2\,b^3d} - \frac{4\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}}{8\,b^2d}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-a^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt
(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^3*d) - 1/8*(4*a*e^(-d*x - c) - b)*e^(2*d*
x + 2*c)/(b^2*d) + 1/2*(2*a^2 - b^2)*(d*x + c)/(b^3*d) - 1/8*(4*a*e^(-d*x
- c) + b*e^(-2*d*x - 2*c))/(b^2*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.41

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{8\,a^3\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^3} - \frac{4\,(2\,a^2-b^2)(dx+c)}{b^3} - \frac{be^{(2\,dx+2\,c)}-4\,ae^{(dx+c)}}{b^2} + \frac{\left(4\,abe^{(dx+c)}+b^2\right)e^{(-2\,dx-2\,c)}}{b^3}}{8\,d}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/8*(8*a^3*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(
d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^3) - 4*(2*a^2 - b^
2)*(d*x + c)/b^3 - (b*e^(2*d*x + 2*c) - 4*a*e^(d*x + c))/b^2 + (4*a*b*e^(d
*x + c) + b^2)*e^(-2*d*x - 2*c)/b^3)/d
```

Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.98

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{x\,(2\,a^2-b^2)}{2\,b^3}-\frac{e^{-2\,c-2\,d\,x}}{8\,b\,d}+\frac{e^{2\,c+2\,d\,x}}{8\,b\,d}-\frac{a\,e^{-c-d\,x}}{2\,b^2\,d}-\frac{a\,e^{c+d\,x}}{2\,b^2\,d}-\frac{a^3\,\ln\left(\frac{2\,a^3\,e^{c+d\,x}}{b^4}-\frac{2\,a^3\,\left(b-a\,e^{c+d\,x}\right)}{b^4\,\sqrt{a^2+b^2}}\right)}{b^3\,d\,\sqrt{a^2+b^2}}+\frac{a^3\,\ln\left(\frac{2\,a^3\,e^{c+d\,x}}{b^4}+\frac{2\,a^3\,\left(b-a\,e^{c+d\,x}\right)}{b^4\,\sqrt{a^2+b^2}}\right)}{b^3\,d\,\sqrt{a^2+b^2}}$$

input
```
int(sinh(c + d*x)^3/(a + b*sinh(c + d*x)),x)
```

output
```
(x*(2*a^2 - b^2))/(2*b^3) - exp(- 2*c - 2*d*x)/(8*b*d) + exp(2*c + 2*d*x)/
(8*b*d) - (a*exp(- c - d*x))/(2*b^2*d) - (a*exp(c + d*x))/(2*b^2*d) - (a^3
*log((2*a^3*exp(c + d*x))/b^4 - (2*a^3*(b - a*exp(c + d*x)))/(b^4*(a^2 + b
^2)^(1/2))))/(b^3*d*(a^2 + b^2)^(1/2)) + (a^3*log((2*a^3*exp(c + d*x))/b^4
 + (2*a^3*(b - a*exp(c + d*x)))/(b^4*(a^2 + b^2)^(1/2))))/(b^3*d*(a^2 + b^
2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.21

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-16e^{2dx+2c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)a^3i+e^{4dx+4c}a^2b^2+e^{4dx+4c}b^4-4e^{3dx+3c}a^3b-4e^{3dx+3c}a\,b^3+8e^{2dx+}}{8e^{2dx+2c}b^3d\left(a^2+b^2\right)}$$

input
```
int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
( - 16*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sq
rt(a**2 + b**2))*a**3*i + e**(4*c + 4*d*x)*a**2*b**2 + e**(4*c + 4*d*x)*b*
*4 - 4*e**(3*c + 3*d*x)*a**3*b - 4*e**(3*c + 3*d*x)*a*b**3 + 8*e**(2*c + 2
*d*x)*a**4*d*x + 4*e**(2*c + 2*d*x)*a**2*b**2*d*x - 4*e**(2*c + 2*d*x)*b**
4*d*x - 4*e**(c + d*x)*a**3*b - 4*e**(c + d*x)*a*b**3 - a**2*b**2 - b**4)/
(8*e**(2*c + 2*d*x)*b**3*d*(a**2 + b**2))
```

3.237 $\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx = \text{Int}\left(\frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 10.83 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx = \int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx$$

input
```
Integrate[Sinh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Sinh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx$$

$$\Big\downarrow \; 6111$$

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Sinh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sinh (dx+c)^3}{(fx+e)\,(a+b\sinh (dx+c))}dx$$

input
```
int(sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sinh(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 236, normalized size of antiderivative = 8.43

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-2*a^3*integrate(-e^(d*x + c)/(b^4*f*x + b^4*e - (b^4*f*x*e^(2*c) + b^4*e*
e^(2*c))*e^(2*d*x) - 2*(a*b^3*f*x*e^c + a*b^3*e*e^c)*e^(d*x)), x) - 1/4*e^
(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b*f) - 1/2*a*e^(-c +
d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^2*f) + 1/2*a*e^(c - d*e/f)*exp_
integral_e(1, -(f*x + e)*d/f)/(b^2*f) - 1/4*e^(2*c - 2*d*e/f)*exp_integral
_e(1, -2*(f*x + e)*d/f)/(b*f) + 1/2*(2*a^2 - b^2)*log(f*x + e)/(b^3*f)
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh{(dx+c)}^3}{(fx+e)(b\sinh{(dx+c)}+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sinh(d*x + c)^3/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(c+d\,x)^3}{(e+f\,x)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(sinh(c + d*x)^3/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(sinh(c + d*x)^3/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 267, normalized size of antiderivative = 9.54

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$= \frac{e^{4c}\left(\int \frac{e^{4dx}}{e^{2dx+2c}be+e^{2dx+2c}bfx+2e^{dx+c}ae+2e^{dx+c}afx-be-bfx}dx\right)bf+6e^{c}\left(\int \frac{e^{dx}}{e^{2dx+2c}be+e^{2dx+2c}bfx+2e^{dx+c}ae+2e^{dx+c}afx-be-}\right.}{4bf}$$

input
```
int(sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
(e**(4*c)*int(e**(4*d*x)/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x +
2*e**(c + d*x)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x)*b*f + 6*e**c*i
nt(e**(d*x)/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x + 2*e**(c + d*x
)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x)*a*f - int(1/(e**(4*c + 4*d*
x)*b*e + e**(4*c + 4*d*x)*b*f*x + 2*e**(3*c + 3*d*x)*a*e + 2*e**(3*c + 3*d
*x)*a*f*x - e**(2*c + 2*d*x)*b*e - e**(2*c + 2*d*x)*b*f*x),x)*b*f - 3*log(
e + f*x))/(4*b*f)
```

3.238 $\int \frac{(e+fx)^3 \operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 605

$$\int \frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{2(e+fx)^3\operatorname{arctanh}\left(e^{c+dx}\right)}{ad} - \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} - \frac{3f(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2} + \frac{3f(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{ad^2} - \frac{3bf(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{3bf(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{6f^2(e+fx)\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3} - \frac{6f^2(e+fx)\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{ad^3} + \frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{6f^3\operatorname{PolyLog}\left(4,-e^{c+dx}\right)}{ad^4} + \frac{6f^3\operatorname{PolyLog}\left(4,e^{c+dx}\right)}{ad^4} - \frac{6bf^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4} + \frac{6bf^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4}$$

output
```
-2*(f*x+e)^3*arctanh(exp(d*x+c))/a/d-b*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2
+b^2)^(1/2)))/a/(a^2+b^2)^(1/2)/d+b*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/a/(a^2+b^2)^(1/2)/d-3*f*(f*x+e)^2*polylog(2,-exp(d*x+c))/a/d^2+
3*f*(f*x+e)^2*polylog(2,exp(d*x+c))/a/d^2-3*b*f*(f*x+e)^2*polylog(2,-b*exp
(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(1/2)/d^2+3*b*f*(f*x+e)^2*polylog
(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(1/2)/d^2+6*f^2*(f*x+e)*
polylog(3,-exp(d*x+c))/a/d^3-6*f^2*(f*x+e)*polylog(3,exp(d*x+c))/a/d^3+6*b
*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(1/2
)/d^3-6*b*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+
b^2)^(1/2)/d^3-6*f^3*polylog(4,-exp(d*x+c))/a/d^4+6*f^3*polylog(4,exp(d*x+
c))/a/d^4-6*b*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)
^(1/2)/d^4+6*b*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2
)^(1/2)/d^4
```

Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 734, normalized size of antiderivative = 1.21

$$\int \frac{(e+fx)^3 \operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{d^3(e+fx)^3 \log\left(1-e^{c+dx}\right) - d^3(e+fx)^3 \log\left(1+e^{c+dx}\right) - 3f\left(d^2(e+fx)^2 \operatorname{PolyLog}\left(2,-e^{c+dx}\right) - 2df(\right.}{}$$

input
```
Integrate[((e + f*x)^3*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(d^3*(e + f*x)^3*Log[1 - E^(c + d*x)] - d^3*(e + f*x)^3*Log[1 + E^(c + d*x
)] - 3*f*(d^2*(e + f*x)^2*PolyLog[2, -E^(c + d*x)] - 2*d*f*(e + f*x)*PolyL
og[3, -E^(c + d*x)] + 2*f^2*PolyLog[4, -E^(c + d*x)]) + 3*f*(d^2*(e + f*x)
^2*PolyLog[2, E^(c + d*x)] - 2*d*f*(e + f*x)*PolyLog[3, E^(c + d*x)] + 2*f
^2*PolyLog[4, E^(c + d*x)]) + (b*(2*d^3*e^3*ArcTanh[(a + b*E^(c + d*x))/Sq
rt[a^2 + b^2]] - 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
])] - 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^3
*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 3*d^3*e^2*f*x*Lo
g[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 3*d^3*e*f^2*x^2*Log[1 + (b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2])] - 3*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-
a + Sqrt[a^2 + b^2])] + 3*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))] + 6*d*e*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a
^2 + b^2])] + 6*d*f^3*x*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
 - 6*d*e*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*d*f^
3*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*f^3*PolyLog[4
, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 6*f^3*PolyLog[4, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))]))/Sqrt[a^2 + b^2])/(a*d^4)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.51 (sec) , antiderivative size = 558, normalized size of antiderivative = 0.92, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {6109, 3042, 26, 3803, 25, 2694, 27, 2620, 3011, 4670, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6109}$$

$$\frac{\int (e+fx)^3\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int i(e+fx)^3\csc(ic+idx)dx}{a}-\frac{b\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int (e+fx)^3\csc(ic+idx)dx}{a}-\frac{b\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{a}$$

$$\Big\downarrow \text{3803}$$

$$-\frac{2b\int -\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{2b\int \frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{2694}$$

$$\frac{2b\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\left(\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{2620}$$

$$\frac{2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d\right.\right.}{2\sqrt{a^2+b^2}}\right.}{a}$$

$$\frac{i\int (e+fx)^3\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{3011}$$

$$2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c}}{a-\sqrt{}}\right.}{bd}\right.}{}\right.$$

$$a$$

$$\frac{i\int(e+fx)^3\operatorname{csc}(ic+idx)dx}{a}$$

$\downarrow$ 4670

$$2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c}}{a-\sqrt{}}\right.}{bd}\right.}{}\right.$$

$$a$$

$$\frac{i\left(\frac{3if\int(e+fx)^2\log(1-e^{c+dx})dx}{d}-\frac{3if\int(e+fx)^2\log(1+e^{c+dx})dx}{d}+\frac{2i(e+fx)^3\operatorname{arctanh}(e^{c+dx})}{d}\right)}{a}$$

$\downarrow$ 3011

$$2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c}}{a-\sqrt{}}\right.}{bd}\right.}{}\right.$$

$$a$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}(2,-e^{c+dx})dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,-e^{c+dx})}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}(2,e^{c+dx})dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,e^{c+dx})}{d}\right)}{d}\right.}{a}$$

$\downarrow$ 7163

$$2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b}}\right.}{d}}{bd}\right.}{2\sqrt{a^2+b^2}}\right.$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-e^{c+dx}\right)}{d}-\frac{f\int\text{PolyLog}\left(3,-e^{c+dx}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,e^{c+dx}\right)}{d}-\frac{f\int}{}\right.}{d}\right.}{}\right.}{a}$$

$\downarrow$ 2720

$$2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\,\text{PolyLog}(\cdots}{d}}{bd}}{2\sqrt{a^2+b^2}}\right.$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-e^{c+dx}\right)}{d}-\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(3,-e^{c+dx}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(2f\left(\frac{(e+fx)\,\text{PolyLog}(3\cdots}{d}\right.\right.}{}\right.}{a}$$

$\downarrow$ 7143

$$2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)$$

$$\frac{i\left(\frac{2i(e+fx)^3\operatorname{arctanh}(e^{c+dx})}{d}-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}(3,-e^{c+dx})}{d}-\frac{f\operatorname{PolyLog}(4,-e^{c+dx})}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,-e^{c+dx})}{d}\right)}{d}+\frac{3if\left(\frac{2f\left(\frac{(e}{}\right.}{}\right.}{}\right.}{a}$$

input
```
Int[((e + f*x)^3*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(I*(((2*I)*(e + f*x)^3*ArcTanh[E^(c + d*x)])/d - ((3*I)*f*(-(((e + f*x)^2*
PolyLog[2, -E^(c + d*x)])/d) + (2*f*(((e + f*x)*PolyLog[3, -E^(c + d*x)])/
d - (f*PolyLog[4, -E^(c + d*x)])/d^2))/d))/d + ((3*I)*f*(-(((e + f*x)^2*Po
lyLog[2, E^(c + d*x)])/d) + (2*f*(((e + f*x)*PolyLog[3, E^(c + d*x)])/d -
(f*PolyLog[4, E^(c + d*x)])/d^2))/d))/d))/a + (2*b*(-1/2*(b*(((e + f*x)^3*
Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)
^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (2*f*(((e +
f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d - (f*PolyLog[
4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/Sqrt[a^2 +
 b^2] + (b*(((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(
b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2
))/d))/(b*d)))/(2*Sqrt[a^2 + b^2])))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6109
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csch[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a +
 b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(fx+e)^3 \operatorname{csch}(dx+c)}{a+b\sinh(dx+c)}\,dx$$

input
```
int((f*x+e)^3*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1645 vs. $2(554) = 1108$.

Time = 0.13 (sec) , antiderivative size = 1645, normalized size of antiderivative = 2.72

$$\int \frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(6*b^2*f^3*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x
 + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*
b^2*f^3*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c
) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*(a^2
 + b^2)*f^3*polylog(4, cosh(d*x + c) + sinh(d*x + c)) + 6*(a^2 + b^2)*f^3*
polylog(4, -cosh(d*x + c) - sinh(d*x + c)) + 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^
2*e*f^2*x + b^2*d^2*e^2*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) +
a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2
) - b)/b + 1) - 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*sq
rt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
 + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^2*d^3*e^3
- 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*sqrt((a^2 + b^2)/b^
2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) +
 2*a) + (b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3
)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sq
rt((a^2 + b^2)/b^2) + 2*a) + (b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^
2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*sqrt(
(a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c
) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b^2*d^3*f^3*x^3 + 3*
b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d...
```

Sympy [F]

$$\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**3*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*csch(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\,\text{csch}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e^3*(b*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + s
qrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a*d) + log(e^(-d*x - c) + 1)/(a*d) - log
(e^(-d*x - c) - 1)/(a*d)) - 3*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x +
c)))*e^2*f/(a*d^2) + 3*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))*e^
2*f/(a*d^2) - 3*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c))
- 2*polylog(3, -e^(d*x + c)))*e*f^2/(a*d^3) + 3*(d^2*x^2*log(-e^(d*x + c)
+ 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*e*f^2/(a*d^3)
 - (d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*p
olylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*f^3/(a*d^4) + (d^3*x
^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3,
 e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))*f^3/(a*d^4) - integrate(2*(b*f^
3*x^3*e^c + 3*b*e*f^2*x^2*e^c + 3*b*e^2*f*x*e^c)*e^(d*x)/(a*b*e^(2*d*x + 2
*c) + 2*a^2*e^(d*x + c) - a*b), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+f\,x)^3}{\sinh(c+d\,x)\,\left(a+b\sinh(c+d\,x)\right)}\,dx$$

input
```
int((e + f*x)^3/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^3/(sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\frac{-2\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)b\,e^3i+4e^{2c}\left(\int\frac{e^{2dx}x^3}{e^{4dx+4c}b+2e^{3dx+3c}a-2e^{2dx+2c}b-2e^{dx+c}a+b}dx\right)a^3d\,f^3+4e^{2c}\left(\int\frac{}{e^{4dx+}}\right.}{}$$

input
```
int((f*x+e)^3*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*b
*e**3*i + 4*e**(2*c)*int((e**(2*d*x)*x**3)/(e**(4*c + 4*d*x)*b + 2*e**(3*c
 + 3*d*x)*a - 2*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**3*d*f**3
+ 4*e**(2*c)*int((e**(2*d*x)*x**3)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x
)*a - 2*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a*b**2*d*f**3 + 12*e
**(2*c)*int((e**(2*d*x)*x**2)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a -
 2*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**3*d*e*f**2 + 12*e**(2*
c)*int((e**(2*d*x)*x**2)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a - 2*e*
*(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a*b**2*d*e*f**2 + 12*e**(2*c)*
int((e**(2*d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a - 2*e**(2*c
+ 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**3*d*e**2*f + 12*e**(2*c)*int((e**
(2*d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a - 2*e**(2*c + 2*d*x)
*b - 2*e**(c + d*x)*a + b),x)*a*b**2*d*e**2*f + log(e**(c + d*x) - 1)*a**2
*e**3 + log(e**(c + d*x) - 1)*b**2*e**3 - log(e**(c + d*x) + 1)*a**2*e**3
- log(e**(c + d*x) + 1)*b**2*e**3)/(a*d*(a**2 + b**2))
```

3.239 $\int \frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 433

$$\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{2(e+fx)^2\text{arctanh}\left(e^{c+dx}\right)}{ad} - \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} - \frac{2f(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2} + \frac{2f(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2} - \frac{2bf(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{2bf(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{2f^2\,\text{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3} - \frac{2f^2\,\text{PolyLog}\left(3,e^{c+dx}\right)}{ad^3} + \frac{2bf^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{2bf^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3}$$

output
```
-2*(f*x+e)^2*arctanh(exp(d*x+c))/a/d-b*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2
+b^2)^(1/2)))/a/(a^2+b^2)^(1/2)/d+b*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/a/(a^2+b^2)^(1/2)/d-2*f*(f*x+e)*polylog(2,-exp(d*x+c))/a/d^2+2*
f*(f*x+e)*polylog(2,exp(d*x+c))/a/d^2-2*b*f*(f*x+e)*polylog(2,-b*exp(d*x+c
)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(1/2)/d^2+2*b*f*(f*x+e)*polylog(2,-b*ex
p(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(1/2)/d^2+2*f^2*polylog(3,-exp(d
*x+c))/a/d^3-2*f^2*polylog(3,exp(d*x+c))/a/d^3+2*b*f^2*polylog(3,-b*exp(d*
x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(1/2)/d^3-2*b*f^2*polylog(3,-b*exp(d
*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(1/2)/d^3
```

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.05

$$\int \frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{d^2(e+fx)^2\log\left(1-e^{c+dx}\right)-d^2(e+fx)^2\log\left(1+e^{c+dx}\right)-2df(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)+2df(e+$$

input
```
Integrate[((e + f*x)^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(d^2*(e + f*x)^2*Log[1 - E^(c + d*x)] - d^2*(e + f*x)^2*Log[1 + E^(c + d*x
)] - 2*d*f*(e + f*x)*PolyLog[2, -E^(c + d*x)] + 2*d*f*(e + f*x)*PolyLog[2,
 E^(c + d*x)] + 2*f^2*PolyLog[3, -E^(c + d*x)] - 2*f^2*PolyLog[3, E^(c + d
*x)] + (b*(2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 2*d^2*
e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^2*f^2*x^2*Log[1 +
 (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2])] - 2*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2
+ b^2])] + 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2]))] + 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*f^2*P
olyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/Sqrt[a^2 + b^2])/(a*
d^3)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.01 (sec) , antiderivative size = 406, normalized size of antiderivative = 0.94, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {6109, 3042, 26, 3803, 25, 2694, 27, 2620, 3011, 2720, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6109}$$

$$\frac{\int (e+fx)^2\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int i(e+fx)^2\csc(ic+idx)dx}{a}-\frac{b\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int (e+fx)^2\csc(ic+idx)dx}{a}-\frac{b\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{a}$$

$$\Big\downarrow \text{3803}$$

$$-\frac{2b\int -\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{2b\int \frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{2694}$$

$$\frac{2b\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\left(\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{2620}$$

$$\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right.}{a}$$

$$\frac{i\int (e+fx)^2\csc(ic+idx)dx}{a}$$

$\Big\downarrow$ 3011

$$\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right.}{bd}\right.}{}}{a}$$

$$\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}$$

$\Big\downarrow$ 2720

$$\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\right.}{}\right.}{}}{a}$$

$$\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}$$

$\Big\downarrow$ 4670

$$\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\right.}{}\right.}{}}{a}$$

$$\frac{i\left(\frac{2if\int(e+fx)\log\left(1-e^{c+dx}\right)dx}{d}-\frac{2if\int(e+fx)\log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)^2\text{arctanh}\left(e^{c+dx}\right)}{d}\right)}{a}$$

$\downarrow$ 3011

$$\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\cdots\right)}{b\cdots}\cdots\right)}{\cdots}\cdots\right)}{a}$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}+\frac{2i(e+fx)^2\operatorname{ar}\cdots}{\cdots}\cdots\right)}{a}$$

$\downarrow$ 2720

$$\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\cdots\right)}{b\cdots}\cdots\right)}{\cdots}\cdots\right)}{a}$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{\cdots}\right)}{d}\cdots\right)}{d}\cdots\right)}{a}$$

$\downarrow$ 7143

$$\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\right.}{}\right.}{a}$$

$$\frac{i\left(\frac{2i(e+fx)^2\text{arctanh}(e^{c+dx})}{d}-\frac{2if\left(\frac{f\,\text{PolyLog}\left(3,-e^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\,\text{PolyLog}\left(3,e^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{c+\right.}}{d}\right.}{d}\right.}{a}$$

input
```
Int[((e + f*x)^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(I*(((2*I)*(e + f*x)^2*ArcTanh[E^(c + d*x)])/d - ((2*I)*f*(-(((e + f*x)*Po
lyLog[2, -E^(c + d*x)])/d) + (f*PolyLog[3, -E^(c + d*x)])/d^2))/d + ((2*I)
*f*(-(((e + f*x)*PolyLog[2, E^(c + d*x)])/d) + (f*PolyLog[3, E^(c + d*x)])
/d^2))/d))/a + (2*b*(-1/2*(b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2
 + b^2]))])/d^2))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^2*Log[1 + (b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -
((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/(2*Sqrt[a^2 + b^2])))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6109
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csch[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a +
 b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1096 vs. 2(394) = 788.

Time = 0.13 (sec) , antiderivative size = 1096, normalized size of antiderivative = 2.53

$$\int \frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(2*b^2*f^2*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x
+ c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 2*b
^2*f^2*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c)
 - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 2*(a^2
+ b^2)*f^2*polylog(3, cosh(d*x + c) + sinh(d*x + c)) + 2*(a^2 + b^2)*f^2*p
olylog(3, -cosh(d*x + c) - sinh(d*x + c)) - 2*(b^2*d*f^2*x + b^2*d*e*f)*sq
rt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
 + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(b^2*d*f^2*
x + b^2*d*e*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x +
 c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1
) + (b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*sqrt((a^2 + b^2)/b^2)*log(
2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) -
 (b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*sqrt((a^2 + b^2)/b^2)*log(2*b
*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b
^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*sqrt((a^2
+ b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (b^2*d^2*f^2*x^2 + 2*b^2*d
^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh
(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2) - b)/b) + 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*dil...
```

Sympy [F]

$$\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2\,\text{csch}\,(c+dx)}{a+b\sinh\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*csch(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\,\text{csch}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e^2*(b*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + s
qrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a*d) + log(e^(-d*x - c) + 1)/(a*d) - log
(e^(-d*x - c) - 1)/(a*d)) - 2*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x +
c)))*e*f/(a*d^2) + 2*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))*e*f/
(a*d^2) - (d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*po
lylog(3, -e^(d*x + c)))*f^2/(a*d^3) + (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d
*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*f^2/(a*d^3) - integrate
(2*(b*f^2*x^2*e^c + 2*b*e*f*x*e^c)*e^(d*x)/(a*b*e^(2*d*x + 2*c) + 2*a^2*e^
(d*x + c) - a*b), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+f\,x)^2}{\sinh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^2/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^2/(sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\frac{-2\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)b\,e^2i+4e^{2c}\left(\int \frac{e^{2dx}x^2}{e^{4dx+4c}b+2e^{3dx+3c}a-2e^{2dx+2c}b-2e^{dx+c}a+b}dx\right)a^3d\,f^2+4e^{2c}\left(\int \frac{}{e^{4dx+}}\right.}{}$$

input
```
int((f*x+e)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*b
*e**2*i + 4*e**(2*c)*int((e**(2*d*x)*x**2)/(e**(4*c + 4*d*x)*b + 2*e**(3*c
 + 3*d*x)*a - 2*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**3*d*f**2
+ 4*e**(2*c)*int((e**(2*d*x)*x**2)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x
)*a - 2*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a*b**2*d*f**2 + 8*e*
*(2*c)*int((e**(2*d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a - 2*e
**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**3*d*e*f + 8*e**(2*c)*int((
e**(2*d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a - 2*e**(2*c + 2*d
*x)*b - 2*e**(c + d*x)*a + b),x)*a*b**2*d*e*f + log(e**(c + d*x) - 1)*a**2
*e**2 + log(e**(c + d*x) - 1)*b**2*e**2 - log(e**(c + d*x) + 1)*a**2*e**2
- log(e**(c + d*x) + 1)*b**2*e**2)/(a*d*(a**2 + b**2))
```

3.240 $\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 261

$$\begin{aligned}\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx =& -\frac{2(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{ad} \\ &- \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} \\ &+ \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} - \frac{f\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2} \\ &+ \frac{f\,\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2} - \frac{bf\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} \\ &+ \frac{bf\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2}\end{aligned}$$

output
```
-2*(f*x+e)*arctanh(exp(d*x+c))/a/d-b*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2
)^(1/2)))/a/(a^2+b^2)^(1/2)/d+b*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/
2)))/a/(a^2+b^2)^(1/2)/d-f*polylog(2,-exp(d*x+c))/a/d^2+f*polylog(2,exp(d*
x+c))/a/d^2-b*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(
1/2)/d^2+b*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(1/2
)/d^2
```

Mathematica [A] (verified)

Time = 2.01 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.02

$$\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{d(e+fx)\left(\log\left(1-e^{c+dx}\right)-\log\left(1+e^{c+dx}\right)\right)-f\,\text{PolyLog}\left(2,-e^{c+dx}\right)+f\,\text{PolyLog}\left(2,e^{c+dx}\right)-\frac{b\left(-2de\text{a}\right.}{}}{}$$

input
```
Integrate[((e + f*x)*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(d*(e + f*x)*(Log[1 - E^(c + d*x)] - Log[1 + E^(c + d*x)]) - f*PolyLog[2,
-E^(c + d*x)] + f*PolyLog[2, E^(c + d*x)] - (b*(-2*d*e*ArcTanh[(a + b*E^(c
 + d*x))/Sqrt[a^2 + b^2]] + 2*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b
^2]] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - f*(c +
 d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + f*PolyLog[2, (b*E^(
c + d*x))/(-a + Sqrt[a^2 + b^2])] - f*PolyLog[2, -((b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2]))]))/Sqrt[a^2 + b^2])/(a*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.25 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {6109, 3042, 26, 3803, 25, 2694, 27, 2620, 2715, 2838, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6109}$$

$$\frac{\int (e+fx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int i(e+fx)\csc(ic+idx)dx}{a}-\frac{b\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a}$$

$\Big\downarrow$ 26

$$\frac{i\int (e+fx)\csc(ic+idx)dx}{a}-\frac{b\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a}$$

$\Big\downarrow$ 3803

$$-\frac{2b\int -\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx}{a}$$

$\Big\downarrow$ 25

$$\frac{2b\int \frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx}{a}$$

$\Big\downarrow$ 2694

$$\frac{2b\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx}{a}$$

$\Big\downarrow$ 27

$$\frac{2b\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx}{a}$$

$\Big\downarrow$ 2620

$$\frac{2b\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}+$$
$$\frac{i\int (e+fx)\csc(ic+idx)dx}{a}$$

$\Big\downarrow$ 2715

$$\frac{2b\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d}{bd^2}\right.}{2\sqrt{a^2+b^2}}\right.}{a}+\frac{i\int(e+fx)\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{2b\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int(e+fx)\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{ 4670}$$

$$\frac{2b\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\left(\frac{if\int\log\left(1-e^{c+dx}\right)dx}{d}-\frac{if\int\log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{2b\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\left(\frac{if\int e^{-c-dx}\log\left(1-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log\left(1+e^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{2b\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)}{a}$$

input
```
Int[((e + f*x)*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(I*(((2*I)*(e + f*x)*ArcTanh[E^(c + d*x)])/d + (I*f*PolyLog[2, -E^(c + d*x
)])/d^2 - (I*f*PolyLog[2, E^(c + d*x)])/d^2))/a + (2*b*(-1/2*(b*(((e + f*x
)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -(
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2)))/Sqrt[a^2 + b^2] + (b*((
(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyL
og[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(2*Sqrt[a^2 + b
^2])))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6109
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csch[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a +
 b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(238) = 476$.

Time = 0.33 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.04

method	result
risch	$\frac{2eb\,\operatorname{arctanh}\left(\frac{2b\,\mathrm{e}^{dx+c}+2a}{2\sqrt{a^2+b^2}}\right)}{da\sqrt{a^2+b^2}}-\frac{e\ln\left(\mathrm{e}^{dx+c}+1\right)}{ad}+\frac{e\ln\left(\mathrm{e}^{dx+c}-1\right)}{ad}-\frac{fb\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)x}{da\sqrt{a^2+b^2}}+\frac{fb\ln\left(\frac{b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}\dots}{a+\sqrt{a^2+b^2}}\right.}{da\sqrt{a^2+b^2}}$

input
```
int((f*x+e)*csch(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/d*e*b/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2)
)-1/a/d*e*ln(exp(d*x+c)+1)+1/a/d*e*ln(exp(d*x+c)-1)-1/d*f*b/a/(a^2+b^2)^(1
/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d*f*b/a
/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*
x-1/d^2*f*b/a/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^
2+b^2)^(1/2)))*c+1/d^2*f*b/a/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1
/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d^2*f*b/a/(a^2+b^2)^(1/2)*dilog((-b*exp(d*
x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/d^2*f*b/a/(a^2+b^2)^(1/2)*
dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/a/d*f*ln(exp
(d*x+c)+1)*x-1/d^2*f/a*dilog(exp(d*x+c)+1)-1/d^2*f*dilog(exp(d*x+c))/a-2/d
^2*c*f*b/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2
))-1/a/d^2*c*f*ln(exp(d*x+c)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 649 vs. 2(234) = 468.

Time = 0.11 (sec) , antiderivative size = 649, normalized size of antiderivative = 2.49

$$\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(b^2*f*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (
b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - b^2
*f*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cos
h(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (a^2 + b
^2)*f*dilog(cosh(d*x + c) + sinh(d*x + c)) + (a^2 + b^2)*f*dilog(-cosh(d*x
 + c) - sinh(d*x + c)) - (b^2*d*e - b^2*c*f)*sqrt((a^2 + b^2)/b^2)*log(2*b
*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b
^2*d*e - b^2*c*f)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d
*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^2*d*f*x + b^2*c*f)*sqrt((a
^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b^2*d*f*x + b^2*c*f)*s
qrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
 + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + ((a^2 + b^2)*d*f*
x + (a^2 + b^2)*d*e)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - ((a^2 + b^2)
*d*e - (a^2 + b^2)*c*f)*log(cosh(d*x + c) + sinh(d*x + c) - 1) - ((a^2 + b
^2)*d*f*x + (a^2 + b^2)*c*f)*log(-cosh(d*x + c) - sinh(d*x + c) + 1))/((a^
3 + a*b^2)*d^2)
```

Sympy [F]

$$\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\,\text{csch}\,(c+dx)}{a+b\sinh\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*csch(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{csch}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e*(b*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqr
t(a^2 + b^2)))/(sqrt(a^2 + b^2)*a*d) + log(e^(-d*x - c) + 1)/(a*d) - log(e
^(-d*x - c) - 1)/(a*d)) + 2*f*integrate(2*x/((b*(e^(d*x + c) - e^(-d*x - c
)) + 2*a)*(e^(d*x + c) - e^(-d*x - c))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{e+f\,x}{\sinh\left(c+d\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-2\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)bei+4e^{2c}\left(\int\frac{e^{2dx}x}{e^{4dx+4c}b+2e^{3dx+3c}a-2e^{2dx+2c}b-2e^{dx+c}a+b}dx\right)a^3df+4e^{2c}\left(\int\frac{}{e^{4dx+4c}b+}\right.}{}$$

input
```
int((f*x+e)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*b
*e*i + 4*e**(2*c)*int((e**(2*d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d
*x)*a - 2*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**3*d*f + 4*e**(2
*c)*int((e**(2*d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a - 2*e**(
2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a*b**2*d*f + log(e**(c + d*x) -
1)*a**2*e + log(e**(c + d*x) - 1)*b**2*e - log(e**(c + d*x) + 1)*a**2*e -
log(e**(c + d*x) + 1)*b**2*e)/(a*d*(a**2 + b**2))
```

3.241 $\int \frac{\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 64

$$\int \frac{\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\text{arctanh}(\cosh(c+dx))}{ad} + \frac{2b\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}$$

output
```
-arctanh(cosh(d*x+c))/a/d+2*b*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^
(1/2))/a/(a^2+b^2)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.28

$$\int \frac{\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{-\frac{2b\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - \log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right) + \log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

input
```
Integrate[Csch[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

output
```
((-2*b*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2
] - Log[Cosh[(c + d*x)/2]] + Log[Sinh[(c + d*x)/2]])/(a*d)
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 26, 3226, 26, 3042, 26, 3139, 1083, 217, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3226}$$

$$i\left(\frac{ib\int \frac{1}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i\text{csch}(c+dx)dx}{a}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{ib\int \frac{1}{a+b\sinh(c+dx)}dx}{a}-\frac{i\int \text{csch}(c+dx)dx}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{ib\int \frac{1}{a-ib\sin(ic+idx)}dx}{a}-\frac{i\int i\csc(ic+idx)dx}{a}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{ib\int \frac{1}{a-ib\sin(ic+idx)}dx}{a}+\frac{\int \csc(ic+idx)dx}{a}\right)$$

$\Big\downarrow$ 3139

$$i\left(\frac{2b\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{ad}+\frac{\int \csc(ic+idx)dx}{a}\right)$$

$\Big\downarrow$ 1083

$$i\left(\frac{\int \csc(ic+idx)dx}{a}-\frac{4b\int \frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4(a^2+b^2)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{ad}\right)$$

$\Big\downarrow$ 217

$$i\left(\frac{\int \csc(ic+idx)dx}{a}+\frac{2ib\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}\right)$$

$\Big\downarrow$ 4257

$$i\left(\frac{2ib\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}+\frac{i\text{arctanh}(\cosh(c+dx))}{ad}\right)$$

input
```
Int[Csch[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

output
```
I*((I*ArcTanh[Cosh[c + d*x]])/(a*d) + ((2*I)*b*ArcTanh[Tanh[(c + d*x)/2]/(
2*Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3226
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]),
 x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2b\,\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{a\sqrt{a^{2}+b^{2}}}}{d}$	63
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2b\,\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{a\sqrt{a^{2}+b^{2}}}}{d}$	63
risch	$-\frac{\ln\left(\mathrm{e}^{dx+c}+1\right)}{da}+\frac{b\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^{2}+b^{2}}+a^{2}+b^{2}}{\sqrt{a^{2}+b^{2}}\,b}\right)}{\sqrt{a^{2}+b^{2}}\,da}-\frac{b\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^{2}+b^{2}}-a^{2}-b^{2}}{\sqrt{a^{2}+b^{2}}\,b}\right)}{\sqrt{a^{2}+b^{2}}\,da}+\frac{\ln\left(\mathrm{e}^{dx+c}-1\right)}{da}$	152

input
```
int(csch(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a*ln(tanh(1/2*d*x+1/2*c))-2*b/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*ta
nh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(61) = 122$.

Time = 0.12 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.48

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\sqrt{a^2+b^2}\,b\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+2\,a^2+b^2+2\,(b^2\cosh(dx+c)+ab)\sinh(dx+c)+2\,\sqrt{a^2+b^2}(b\cosh(dx+c)}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2\,a\cosh(dx+c)+2\,(b\cosh(dx+c)+a)\sinh(dx+c)-b}\right.}{}$$

input
```
integrate(csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(sqrt(a^2 + b^2)*b*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*
cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) +
2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c
)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh
(d*x + c) - b)) - (a^2 + b^2)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + (a^
2 + b^2)*log(cosh(d*x + c) + sinh(d*x + c) - 1))/((a^3 + a*b^2)*d)
```

Sympy [F]

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(csch(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.75

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}ad} - \frac{\log\left(e^{(-dx-c)}+1\right)}{ad} + \frac{\log\left(e^{(-dx-c)}-1\right)}{ad}$$

input
```
integrate(csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-b*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*a*d) - log(e^(-d*x - c) + 1)/(a*d) + log(e^(-
d*x - c) - 1)/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.59

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{b\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a} + \frac{\log\left(e^{(dx+c)}+1\right)}{a} - \frac{\log\left(\left|e^{(dx+c)}-1\right|\right)}{a}}{d}$$

input
```
integrate(csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-(b*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c)
 + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a) + log(e^(d*x + c) + 1)/a
- log(abs(e^(d*x + c) - 1))/a)/d
```

Mupad [B] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 347, normalized size of antiderivative = 5.42

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\ln\left(32\,a-32\,a\,e^{d\,x}\,e^{c}\right)}{a\,d}-\frac{\ln\left(32\,a+32\,a\,e^{d\,x}\,e^{c}\right)}{a\,d}$$
$$+\frac{b\,\ln\left(128\,a^5\,e^{d\,x}\,e^{c}-64\,a^2\,b^3-64\,a^4\,b+32\,a\,b^3\,\sqrt{a^2+b^2}+64\,a^3\,b\,\sqrt{a^2+b^2}+160\,a^3\,b^2\,e^{d\,x}\,e^{c}-128\,a^4\right.}{d\,a^3+d\,a\,b^2}$$
$$-\frac{b\,\ln\left(64\,a^4\,b+64\,a^2\,b^3-128\,a^5\,e^{d\,x}\,e^{c}+32\,a\,b^3\,\sqrt{a^2+b^2}+64\,a^3\,b\,\sqrt{a^2+b^2}-160\,a^3\,b^2\,e^{d\,x}\,e^{c}-128\,a^4\right.}{d\,a^3+d\,a\,b^2}$$

input
```
int(1/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
log(32*a - 32*a*exp(d*x)*exp(c))/(a*d) - log(32*a + 32*a*exp(d*x)*exp(c))/
(a*d) + (b*log(128*a^5*exp(d*x)*exp(c) - 64*a^2*b^3 - 64*a^4*b + 32*a*b^3*
(a^2 + b^2)^(1/2) + 64*a^3*b*(a^2 + b^2)^(1/2) + 160*a^3*b^2*exp(d*x)*exp(
c) - 128*a^4*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2) + 32*a*b^4*exp(d*x)*exp(c)
- 96*a^2*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a^3*d
+ a*b^2*d) - (b*log(64*a^4*b + 64*a^2*b^3 - 128*a^5*exp(d*x)*exp(c) + 32*a
*b^3*(a^2 + b^2)^(1/2) + 64*a^3*b*(a^2 + b^2)^(1/2) - 160*a^3*b^2*exp(d*x)
*exp(c) - 128*a^4*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2) - 32*a*b^4*exp(d*x)*ex
p(c) - 96*a^2*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a
^3*d + a*b^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.77

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\frac{-2\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)bi+\log\left(e^{dx+c}-1\right)a^2+\log\left(e^{dx+c}-1\right)b^2-\log\left(e^{dx+c}+1\right)a^2-\log\left(e^{dx+c}\right.}{ad\left(a^2+b^2\right)}$$

input
```
int(csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*b
*i + log(e**(c + d*x) - 1)*a**2 + log(e**(c + d*x) - 1)*b**2 - log(e**(c +
 d*x) + 1)*a**2 - log(e**(c + d*x) + 1)*b**2)/(a*d*(a**2 + b**2))
```

3.242 $\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 4.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[Csch[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Csch[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$\Big\downarrow \text{ 6111}$$
$$\int \frac{\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Csch[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\text{csch}\,(dx+c)}{(fx+e)\,(a+b\sinh\,(dx+c))}dx$$

input
```
int(csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(csch(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 6.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

input
```
integrate(csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(csch(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(csch(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{1}{\sinh\left(c+d\,x\right)\,\left(e+f\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/(sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)}{\sinh\left(dx+c\right)be+\sinh\left(dx+c\right)bfx+ae+afx}dx$$

input
```
int(csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(csch(c + d*x)/(sinh(c + d*x)*b*e + sinh(c + d*x)*b*f*x + a*e + a*f*x),
x)
```

3.243 $\int \frac{(e+fx)^3\mathrm{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 745

$$\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{(e+fx)^3}{ad}+\frac{2b(e+fx)^3\text{arctanh}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d} - \frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d} + \frac{3f(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{ad^2} + \frac{3bf(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{a^2d^2} - \frac{3bf(e+fx)^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{a^2d^2} + \frac{3b^2f(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d^2} - \frac{3b^2f(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d^2} + \frac{3f^2(e+fx)\,\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{ad^3} - \frac{6bf^2(e+fx)\,\text{PolyLog}\left(3,-e^{c+dx}\right)}{a^2d^3} + \frac{6bf^2(e+fx)\,\text{PolyLog}\left(3,e^{c+dx}\right)}{a^2d^3} - \frac{6b^2f^2(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d^3} + \frac{6b^2f^2(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d^3} - \frac{3f^3\,\text{PolyLog}\left(3,e^{2(c+dx)}\right)}{2ad^4} + \frac{6bf^3\,\text{PolyLog}\left(4,-e^{c+dx}\right)}{a^2d^4} - \frac{6bf^3\,\text{PolyLog}\left(4,e^{c+dx}\right)}{a^2d^4} + \frac{6b^2f^3\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d^4} - \frac{6b^2f^3\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d^4}$$

output
```
-(f*x+e)^3/a/d+2*b*(f*x+e)^3*arctanh(exp(d*x+c))/a^2/d-(f*x+e)^3*coth(d*x+
c)/a/d+b^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^
(1/2)/d-b^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)
^(1/2)/d+3*f*(f*x+e)^2*ln(1-exp(2*d*x+2*c))/a/d^2+3*b*f*(f*x+e)^2*polylog(
2,-exp(d*x+c))/a^2/d^2-3*b*f*(f*x+e)^2*polylog(2,exp(d*x+c))/a^2/d^2+3*b^2
*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(1
/2)/d^2-3*b^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2
/(a^2+b^2)^(1/2)/d^2+3*f^2*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a/d^3-6*b*f^2
*(f*x+e)*polylog(3,-exp(d*x+c))/a^2/d^3+6*b*f^2*(f*x+e)*polylog(3,exp(d*x+
c))/a^2/d^3-6*b^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))
/a^2/(a^2+b^2)^(1/2)/d^3+6*b^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2
+b^2)^(1/2)))/a^2/(a^2+b^2)^(1/2)/d^3-3/2*f^3*polylog(3,exp(2*d*x+2*c))/a/
d^4+6*b*f^3*polylog(4,-exp(d*x+c))/a^2/d^4-6*b*f^3*polylog(4,exp(d*x+c))/a
^2/d^4+6*b^2*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2
)^(1/2)/d^4-6*b^2*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^
2+b^2)^(1/2)/d^4
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1493 vs. $2(745) = 1490$.

Time = 8.20 (sec) , antiderivative size = 1493, normalized size of antiderivative = 2.00

$$\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-((-(d^3*e^2*(-1 + E^(2*c))*(b*d*e - 3*a*f)*x) + d^3*e^2*(-1 + E^(2*c))*(b
*d*e + 3*a*f)*x + 2*a*d^3*(e + f*x)^3 + 3*d^2*e*(-1 + E^(2*c))*f*(b*d*e -
2*a*f)*x*Log[1 - E^(-c - d*x)] + 3*d^2*(-1 + E^(2*c))*f^2*(b*d*e - a*f)*x^
2*Log[1 - E^(-c - d*x)] + b*d^3*(-1 + E^(2*c))*f^3*x^3*Log[1 - E^(-c - d*x
)] - 3*d^2*e*(-1 + E^(2*c))*f*(b*d*e + 2*a*f)*x*Log[1 + E^(-c - d*x)] - 3*
d^2*(-1 + E^(2*c))*f^2*(b*d*e + a*f)*x^2*Log[1 + E^(-c - d*x)] - b*d^3*(-1
 + E^(2*c))*f^3*x^3*Log[1 + E^(-c - d*x)] + d^2*e^2*(-1 + E^(2*c))*(b*d*e
- 3*a*f)*Log[1 - E^(c + d*x)] - d^2*e^2*(-1 + E^(2*c))*(b*d*e + 3*a*f)*Log
[1 + E^(c + d*x)] + 3*d*e*(-1 + E^(2*c))*f*(b*d*e + 2*a*f)*PolyLog[2, -E^(
-c - d*x)] + 6*d*(-1 + E^(2*c))*f^2*(b*d*e + a*f)*x*PolyLog[2, -E^(-c - d*
x)] + 3*b*d^2*(-1 + E^(2*c))*f^3*x^2*PolyLog[2, -E^(-c - d*x)] - 3*d*e*(-1
 + E^(2*c))*f*(b*d*e - 2*a*f)*PolyLog[2, E^(-c - d*x)] - 6*d*(-1 + E^(2*c)
)*f^2*(b*d*e - a*f)*x*PolyLog[2, E^(-c - d*x)] - 3*b*d^2*(-1 + E^(2*c))*f^
3*x^2*PolyLog[2, E^(-c - d*x)] + 6*(-1 + E^(2*c))*f^2*(b*d*e + a*f)*PolyLo
g[3, -E^(-c - d*x)] + 6*b*d*(-1 + E^(2*c))*f^3*x*PolyLog[3, -E^(-c - d*x)]
 + 6*(-1 + E^(2*c))*f^2*(-(b*d*e) + a*f)*PolyLog[3, E^(-c - d*x)] - 6*b*d*
(-1 + E^(2*c))*f^3*x*PolyLog[3, E^(-c - d*x)] + 6*b*(-1 + E^(2*c))*f^3*Pol
yLog[4, -E^(-c - d*x)] - 6*b*(-1 + E^(2*c))*f^3*PolyLog[4, E^(-c - d*x)])/
(a^2*d^4*(-1 + E^(2*c)))) + (b^2*(-2*d^3*e^3*ArcTanh[(a + b*E^(c + d*x))/S
qrt[a^2 + b^2]] + 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 4.54 (sec) , antiderivative size = 715, normalized size of antiderivative = 0.96, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {6109, 3042, 25, 4672, 26, 3042, 26, 4201, 2620, 3011, 2720, 6109, 3042, 26, 3803, 25, 2694, 27, 2620, 3011, 4670, 3011, 7143, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6109}$$

$$\frac{\int (e+fx)^3\text{csch}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \ 3042$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -(e+fx)^3\csc(ic+idx)^2dx}{a}$$

$$\Big\downarrow \ 25$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\int (e+fx)^3\csc(ic+idx)^2dx}{a}$$

$$\Big\downarrow \ 4672$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{3if\int -i(e+fx)^2\coth(c+dx)dx}{d}}{a}$$

$$\Big\downarrow \ 26$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{3f\int (e+fx)^2\coth(c+dx)dx}{d}}{a}$$

$$\Big\downarrow \ 3042$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{3f\int -i(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}$$

$$\Big\downarrow \ 26$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\int (e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}}{a}$$

$$\Big\downarrow \ 4201$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\int \frac{e^{2c+2dx-i\pi}(e+fx)^2}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}$$

$$\Big\downarrow \ 2620$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int (e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}$$

$$\Big\downarrow \ 3011$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int \text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{2d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)}{3f}\right.}{d}}{a}-\frac{b\int\frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-$$

$\downarrow$ 2720

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}-\frac{b\int\frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-$$

$\downarrow$ 6109

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^3\text{csch}(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-$$

$\downarrow$ 3042

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}-\frac{b\left(\frac{\int i(e+fx)^3\csc(ic+idx)dx}{a}-\frac{b\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{a}\right)}{a}-$$

$\downarrow$ 26

$$-\frac{b\left(\frac{i\int(e+fx)^3\csc(ic+idx)dx}{a}-\frac{b\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{a}\right)}{a}-\cdots$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{a}$$

$\downarrow$ 3803

$$-\frac{b\left(-\frac{2b\int-\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int(e+fx)^3\csc(ic+idx)dx}{a}\right)}{a}-\cdots$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{a}$$

$\downarrow$ 25

$$-\frac{b\left(\frac{2b\int\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int(e+fx)^3\csc(ic+idx)dx}{a}\right)}{a}-\cdots$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{a}$$

$\downarrow$ 2694

$$-\frac{b\left(\frac{2b\left(\frac{b\int-\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int-\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int(e+fx)^3\csc(ic+idx)dx}{a}\right)}{a}-\cdots$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{a}$$

$\downarrow$ 27

$$-\frac{b\left(\frac{2b\left(\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx}{a}\right)}{a}-$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c\ldots}\right)}{2d}\ldots\right)}{d}\ldots\right.\right.}{d}\ldots}{a}$$

$\downarrow$ 2620

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)\ldots}{bd}\right.}{2\sqrt{a^2+b^2}}\ldots\right.}{a}\ldots\right.}{a}$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c\ldots}\right)}{2d}\ldots\right)}{d}\ldots\right.\right.}{d}\ldots}{a}$$

$\downarrow$ 3011

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{b}{a-\cdots}\right.}{bd}\cdots\right.}{\cdots}\right)}{a}\cdots\right.}{a}$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c\cdots}\right.}{2d}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{a}$$

$\downarrow$ 4670

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{b}{a-\cdots}\right.}{bd}\right.}{\cdots}\right)}{a}}{\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2\cdots}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}}$$

$\downarrow$ 3011

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{b}{a-\cdots}\right.}{bd}\cdots\right.}{\cdots}\right.}{a}\cdots}{\cdots}$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c\cdots}\right)}{2d}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{a}$$

$\downarrow$ 7143

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{b}{a-}\right.}{bd}\right.}{}\right)}{a}}{}}{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\,\text{PolyLog}\left(3,-e^{2c+2dx-i\pi}\right)}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}$$

$\downarrow$ 7163

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c}}{a+\sqrt{}}\right.}{d}}{bd}}{2\sqrt{a^2+b^2}}}{}\right)}{}\right)}{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\operatorname{PolyLog}\left(3,-e^{2c+2dx-i\pi}\right)}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}$$

$\downarrow$ 2720

$$-\frac{\frac{\coth(c+dx)(e+fx)^3}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\,\mathrm{PolyLog}\left(3,-e^{2c+2dx-i\pi}\right)}{4d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}}{d}}{a}$$

$$b\left(\frac{i\left(\frac{2i\,\mathrm{arctanh}\left(e^{c+dx}\right)(e+fx)^3}{d}-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\,\mathrm{PolyLog}\left(3,-e^{c+dx}\right)}{d}-\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(3,-e^{c+dx}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}\right)}{a}\right.$$

$\downarrow$ 7143

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+\cdots}}\right)}{d}\cdots}{bd}\cdots\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\operatorname{PolyLog}\left(3,-e^{2c+2dx-i\pi}\right)}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}\cdots}{a}$$

input

```
Int[((e + f*x)^3*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-((((e + f*x)^3*Coth[c + d*x])/d + ((3*I)*f*(((-1/3*I)*(e + f*x)^3)/f + (2
*I)*(((e + f*x)^2*Log[1 + E^(2*c - I*Pi + 2*d*x)])/(2*d) - (f*(-1/2*((e +
f*x)*PolyLog[2, -E^(2*c - I*Pi + 2*d*x)])/d + (f*PolyLog[3, -E^(2*c - I*Pi
 + 2*d*x)])/(4*d^2)))/d)))/d)/a) - (b*((I*(((2*I)*(e + f*x)^3*ArcTanh[E^(c
 + d*x)])/d - ((3*I)*f*(-(((e + f*x)^2*PolyLog[2, -E^(c + d*x)])/d) + (2*f
*(((e + f*x)*PolyLog[3, -E^(c + d*x)])/d - (f*PolyLog[4, -E^(c + d*x)])/d^
2))/d))/d + ((3*I)*f*(-(((e + f*x)^2*PolyLog[2, E^(c + d*x)])/d) + (2*f*((
(e + f*x)*PolyLog[3, E^(c + d*x)])/d - (f*PolyLog[4, E^(c + d*x)])/d^2))/d
))/d))/a + (2*b*(-1/2*(b*(((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2]))])/d^2))/d))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^3*Log[1 + (b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog
[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyL
og[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/(2*Sqrt[a^2 + b^2]))
)/a))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6109
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csch[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a +
 b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(fx+e)^3 \operatorname{csch}(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6416 vs. 2(691) = 1382.

Time = 0.22 (sec) , antiderivative size = 6416, normalized size of antiderivative = 8.61

$$\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\,\text{csch}\,(dx+c)^2}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
e^3*(b^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a +
sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^2*d) + b*log(e^(-d*x - c) + 1)/(a^2*d
) - b*log(e^(-d*x - c) - 1)/(a^2*d) + 2/((a*e^(-2*d*x - 2*c) - a)*d)) - 6*
e^2*f*x/(a*d) + 3*e^2*f*log(e^(d*x + c) + 1)/(a*d^2) + 3*e^2*f*log(e^(d*x
+ c) - 1)/(a*d^2) - 2*(f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x)/(a*d*e^(2*d*x +
2*c) - a*d) + (d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)
) - 6*d*x*polylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*b*f^3/(a^
2*d^4) - (d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6
*d*x*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))*b*f^3/(a^2*d^4)
+ 3*(b*d*e^2*f + 2*a*e*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)
))/(a^2*d^3) - 3*(b*d*e^2*f - 2*a*e*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilo
g(e^(d*x + c)))/(a^2*d^3) + 3*(b*d*e*f^2 + a*f^3)*(d^2*x^2*log(e^(d*x + c)
 + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))/(a^2*d^4)
- 3*(b*d*e*f^2 - a*f^3)*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*
x + c)) - 2*polylog(3, e^(d*x + c)))/(a^2*d^4) - 1/4*(b*d^4*f^3*x^4 + 4*(b
*d*e*f^2 + a*f^3)*d^3*x^3 + 6*(b*d^2*e^2*f + 2*a*d*e*f^2)*d^2*x^2)/(a^2*d^
4) + 1/4*(b*d^4*f^3*x^4 + 4*(b*d*e*f^2 - a*f^3)*d^3*x^3 + 6*(b*d^2*e^2*f -
 2*a*d*e*f^2)*d^2*x^2)/(a^2*d^4) + integrate(2*(b^2*f^3*x^3*e^c + 3*b^2*e*
f^2*x^2*e^c + 3*b^2*e^2*f*x*e^c)*e^(d*x)/(a^2*b*e^(2*d*x + 2*c) + 2*a^3*e^
(d*x + c) - a^2*b), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^3}{\sinh\left(c+d\,x\right)^2\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^3/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^3/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^3*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(2*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a
**2 + b**2))*b**2*e**3*i - 2*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*
i)/sqrt(a**2 + b**2))*b**2*e**3*i + 8*e**(5*c + 2*d*x)*int((e**(3*d*x)*x**
3)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e
**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*d
*f**3 + 8*e**(5*c + 2*d*x)*int((e**(3*d*x)*x**3)/(e**(6*c + 6*d*x)*b + 2*e
**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*
c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b**2*d*f**3 + 24*e**(5*c + 2*
d*x)*int((e**(3*d*x)*x**2)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*
e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b + 2*e**(c
 + d*x)*a - b),x)*a**4*d*e*f**2 + 24*e**(5*c + 2*d*x)*int((e**(3*d*x)*x**2
)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e*
*(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b*
*2*d*e*f**2 + 24*e**(5*c + 2*d*x)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b +
 2*e**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a + 3*e*
*(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*d*e**2*f + 24*e**(5*c + 2
*d*x)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e*
*(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b + 2*e**(c +
 d*x)*a - b),x)*a**2*b**2*d*e**2*f - e**(2*c + 2*d*x)*log(e**(c + d*x) - 1
)*a**2*b*e**3 - e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*b**3*e**3 + e**(...
```

3.244 $\int \frac{(e+fx)^2 \operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 535

$$\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{(e+fx)^2}{ad}+\frac{2b(e+fx)^2\operatorname{arctanh}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^2\coth(c+dx)}{ad} + \frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d} - \frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d} + \frac{2f(e+fx)\log\left(1-e^{2(c+dx)}\right)}{ad^2} + \frac{2bf(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{a^2d^2} - \frac{2bf(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{a^2d^2} + \frac{2b^2f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d^2} - \frac{2b^2f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d^2} + \frac{f^2\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{ad^3} - \frac{2bf^2\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{a^2d^3} + \frac{2bf^2\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{a^2d^3} - \frac{2b^2f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d^3} + \frac{2b^2f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d^3}$$

output
```
-(f*x+e)^2/a/d+2*b*(f*x+e)^2*arctanh(exp(d*x+c))/a^2/d-(f*x+e)^2*coth(d*x+
c)/a/d+b^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^
(1/2)/d-b^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)
^(1/2)/d+2*f*(f*x+e)*ln(1-exp(2*d*x+2*c))/a/d^2+2*b*f*(f*x+e)*polylog(2,-e
xp(d*x+c))/a^2/d^2-2*b*f*(f*x+e)*polylog(2,exp(d*x+c))/a^2/d^2+2*b^2*f*(f*
x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(1/2)/d^2-
2*b^2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)
^(1/2)/d^2+f^2*polylog(2,exp(2*d*x+2*c))/a/d^3-2*b*f^2*polylog(3,-exp(d*x+
c))/a^2/d^3+2*b*f^2*polylog(3,exp(d*x+c))/a^2/d^3-2*b^2*f^2*polylog(3,-b*e
xp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(1/2)/d^3+2*b^2*f^2*polylog(3
,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(1/2)/d^3
```

Mathematica [A] (warning: unable to verify)

Time = 7.70 (sec) , antiderivative size = 920, normalized size of antiderivative = 1.72

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-((-(d^2*e*(-1 + E^(2*c))*(b*d*e - 2*a*f)*x) + d^2*e*(-1 + E^(2*c))*(b*d*e
 + 2*a*f)*x + 2*a*d^2*(e + f*x)^2 + 2*d*(-1 + E^(2*c))*f*(b*d*e - a*f)*x*L
og[1 - E^(-c - d*x)] + b*d^2*(-1 + E^(2*c))*f^2*x^2*Log[1 - E^(-c - d*x)]
- 2*d*(-1 + E^(2*c))*f*(b*d*e + a*f)*x*Log[1 + E^(-c - d*x)] - b*d^2*(-1 +
 E^(2*c))*f^2*x^2*Log[1 + E^(-c - d*x)] + d*e*(-1 + E^(2*c))*(b*d*e - 2*a*
f)*Log[1 - E^(c + d*x)] - d*e*(-1 + E^(2*c))*(b*d*e + 2*a*f)*Log[1 + E^(c
+ d*x)] + 2*(-1 + E^(2*c))*f*(b*d*e + a*f)*PolyLog[2, -E^(-c - d*x)] + 2*b
*d*(-1 + E^(2*c))*f^2*x*PolyLog[2, -E^(-c - d*x)] + 2*(-1 + E^(2*c))*f*(-(
b*d*e) + a*f)*PolyLog[2, E^(-c - d*x)] - 2*b*d*(-1 + E^(2*c))*f^2*x*PolyLo
g[2, E^(-c - d*x)] + 2*b*(-1 + E^(2*c))*f^2*PolyLog[3, -E^(-c - d*x)] - 2*
b*(-1 + E^(2*c))*f^2*PolyLog[3, E^(-c - d*x)])/(a^2*d^3*(-1 + E^(2*c)))) +
 (b^2*(-2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*d^2*e*f
*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b
*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2])] + 2*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b
^2])] - 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
)] - 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f^2*Poly
Log[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^2*Sqrt[a^2 + b^2]*d^
3) + (Sech[c/2]*Sech[c/2 + (d*x)/2]*(-(e^2*Sinh[(d*x)/2]) - 2*e*f*x*Sin...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 3.10 (sec) , antiderivative size = 523, normalized size of antiderivative = 0.98, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.893$, Rules used = {6109, 3042, 25, 4672, 26, 3042, 26, 4201, 2620, 2715, 2838, 6109, 3042, 26, 3803, 25, 2694, 27, 2620, 3011, 2720, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6109}$$

$$\frac{\int (e+fx)^2\text{csch}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -(e+fx)^2\csc(ic+idx)^2dx}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\int (e+fx)^2\csc(ic+idx)^2dx}{a}$$

$$\Big\downarrow \text{4672}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2if\int -i(e+fx)\coth(c+dx)dx}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2f\int (e+fx)\coth(c+dx)dx}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2f\int -i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}}{a}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\int \frac{e^{2c+2dx-i\pi}(e+fx)}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int \log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \text{2715}$$

$$\frac{-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\log\left(1+e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \text{2838}$$

$$\frac{-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \text{6109}$$

$$\frac{-\frac{b\left(\frac{\int (e+fx)^2\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{b\left(\frac{\int i(e+fx)^2\csc(ic+idx)dx}{a}-\frac{b\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{a}\right)}{a}-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{b\left(\frac{i\int (e+fx)^2\csc(ic+idx)dx}{a}-\frac{b\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{a}\right)}{a}-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \text{3803}$$

$$-\frac{b\left(-\frac{2b\int -\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx}{a}\right)}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \; 25$$

$$-\frac{b\left(\frac{2b\int \frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx}{a}\right)}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \; 2694$$

$$-\frac{b\left(\frac{2b\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx}{a}\right)}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \; 27$$

$$-\frac{b\left(\frac{2b\left(\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx}{a}\right)}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \; 2620$$

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}\right)}{a}$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$\downarrow$ 3011

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+\cdots\right)}{bd}\cdots\right)}{\cdots}\right)}{a}\right)}{a}$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$\downarrow$ 2720

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\cdots\right.}{\cdots}\right.}{\cdots}\right)}{a}}{a}$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \; 4670$$

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log(\cdots}{\cdots}\right.}{\cdots}\right)}{a}\cdots}{\cdots}$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$\downarrow$ 3011

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\cdots\right.}{\cdots}\right.}{\cdots}\right)}{a}}{\cdots}$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$\Big\downarrow$ 2720

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\cdots\right.}{\cdots}\right.}{\cdots}\right)}{a}\cdots}{\cdots}$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \; 7143$$

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right.}{}\right)}{a}}{}$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

input
```
Int[((e + f*x)^2*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-((((e + f*x)^2*Coth[c + d*x])/d + ((2*I)*f*(((-1/2*I)*(e + f*x)^2)/f + (2
*I)*(((e + f*x)*Log[1 + E^(2*c - I*Pi + 2*d*x)])/(2*d) + (f*PolyLog[2, -E^
(2*c - I*Pi + 2*d*x)])/(4*d^2))))/d)/a) - (b*((I*(((2*I)*(e + f*x)^2*ArcTa
nh[E^(c + d*x)])/d - ((2*I)*f*(-(((e + f*x)*PolyLog[2, -E^(c + d*x)])/d) +
 (f*PolyLog[3, -E^(c + d*x)])/d^2))/d + ((2*I)*f*(-(((e + f*x)*PolyLog[2,
E^(c + d*x)])/d) + (f*PolyLog[3, E^(c + d*x)])/d^2))/d))/a + (2*b*(-1/2*(b
*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) - (2*
f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) +
 (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/Sq
rt[a^2 + b^2] + (b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
)/d^2))/(b*d)))/(2*Sqrt[a^2 + b^2])))/a))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6109
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csch[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a +
 b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3805 vs. 2(495) = 990.

Time = 0.19 (sec) , antiderivative size = 3805, normalized size of antiderivative = 7.11

$$\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2\,\text{csch}^2\,(c+dx)}{a+b\sinh\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*csch(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\,\text{csch}\,(dx+c)^2}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
e^2*(b^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a +
sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^2*d) + b*log(e^(-d*x - c) + 1)/(a^2*d
) - b*log(e^(-d*x - c) - 1)/(a^2*d) + 2/((a*e^(-2*d*x - 2*c) - a)*d)) - 4*
e*f*x/(a*d) - 2*(f^2*x^2 + 2*e*f*x)/(a*d*e^(2*d*x + 2*c) - a*d) + 2*e*f*lo
g(e^(d*x + c) + 1)/(a*d^2) + 2*e*f*log(e^(d*x + c) - 1)/(a*d^2) + (d^2*x^2
*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x +
 c)))*b*f^2/(a^2*d^3) - (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*
x + c)) - 2*polylog(3, e^(d*x + c)))*b*f^2/(a^2*d^3) + 2*(b*d*e*f + a*f^2)
*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^2*d^3) - 2*(b*d*e*f -
 a*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^2*d^3) - 1/3*(
b*d^3*f^2*x^3 + 3*(b*d*e*f + a*f^2)*d^2*x^2)/(a^2*d^3) + 1/3*(b*d^3*f^2*x^
3 + 3*(b*d*e*f - a*f^2)*d^2*x^2)/(a^2*d^3) + integrate(2*(b^2*f^2*x^2*e^c
+ 2*b^2*e*f*x*e^c)*e^(d*x)/(a^2*b*e^(2*d*x + 2*c) + 2*a^3*e^(d*x + c) - a^
2*b), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^2}{\sinh\left(c+d\,x\right)^2\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^2/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^2/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^2*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(2*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a
**2 + b**2))*b**2*e**2*i - 2*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*
i)/sqrt(a**2 + b**2))*b**2*e**2*i + 8*e**(5*c + 2*d*x)*int((e**(3*d*x)*x**
2)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e
**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*d
*f**2 + 8*e**(5*c + 2*d*x)*int((e**(3*d*x)*x**2)/(e**(6*c + 6*d*x)*b + 2*e
**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*
c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b**2*d*f**2 + 16*e**(5*c + 2*
d*x)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e**
(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b + 2*e**(c +
d*x)*a - b),x)*a**4*d*e*f + 16*e**(5*c + 2*d*x)*int((e**(3*d*x)*x)/(e**(6*
c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e**(3*c + 3
*d*x)*a + 3*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b**2*d*e*f
- e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a**2*b*e**2 - e**(2*c + 2*d*x)*lo
g(e**(c + d*x) - 1)*b**3*e**2 + e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*a**
2*b*e**2 + e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*b**3*e**2 - 2*e**(2*c +
2*d*x)*a**3*e**2 - 2*e**(2*c + 2*d*x)*a*b**2*e**2 - 8*e**(3*c)*int((e**(3*
d*x)*x**2)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d*x)
*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x
)*a**4*d*f**2 - 8*e**(3*c)*int((e**(3*d*x)*x**2)/(e**(6*c + 6*d*x)*b + ...
```

3.245 $\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 306

$$\begin{aligned}\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{2b(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{a^2d}-\frac{(e+fx)\coth(c+dx)}{ad}\\ &\quad+\frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d}\\ &\quad-\frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d}\\ &\quad+\frac{f\log(\sinh(c+dx))}{ad^2}+\frac{bf\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{a^2d^2}\\ &\quad-\frac{bf\,\text{PolyLog}\left(2,e^{c+dx}\right)}{a^2d^2}+\frac{b^2f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2}\\ &\quad-\frac{b^2f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2}\end{aligned}$$

output
```
2*b*(f*x+e)*arctanh(exp(d*x+c))/a^2/d-(f*x+e)*coth(d*x+c)/a/d+b^2*(f*x+e)*
ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(1/2)/d-b^2*(f*x+e)*l
n(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(1/2)/d+f*ln(sinh(d*x+
c))/a/d^2+b*f*polylog(2,-exp(d*x+c))/a^2/d^2-b*f*polylog(2,exp(d*x+c))/a^2
/d^2+b^2*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(1/2
)/d^2-b^2*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(1/
2)/d^2
```

Mathematica [A] (verified)

Time = 5.24 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.14

$$\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{ad(e+fx)\coth\left(\frac{1}{2}(c+dx)\right)-2\Big(af(c+dx)+(af-bd(e+fx))\log\left(1-e^{-c-dx}\right)+(af+bd(e+fx}{}$$

input
```
Integrate[((e + f*x)*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*(a*d*(e + f*x)*Coth[(c + d*x)/2] - 2*(a*f*(c + d*x) + (a*f - b*d*(e +
 f*x))*Log[1 - E^(-c - d*x)] + (a*f + b*d*(e + f*x))*Log[1 + E^(-c - d*x)]
 - b*f*PolyLog[2, -E^(-c - d*x)] + b*f*PolyLog[2, E^(-c - d*x)]) - (2*b^2*
(-2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*c*f*ArcTanh[(a +
b*E^(c + d*x))/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2])] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - f*PolyLog[2
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/Sqrt[a^2 + b^2] + a*d*(e + f
*x)*Tanh[(c + d*x)/2])/(a^2*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.66 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.99, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.808, Rules used = {6109, 3042, 25, 4672, 26, 3042, 26, 3956, 6109, 3042, 26, 3803, 25, 2694, 27, 2620, 2715, 2838, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6109}$$

$$\frac{\int (e+fx)\text{csch}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -\left((e+fx)\csc(ic+idx)^2\right)dx}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\int (e+fx)\csc(ic+idx)^2dx}{a}$$

$$\Big\downarrow \text{4672}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{if\int -i\coth(c+dx)dx}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\int \coth(c+dx)dx}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\int -i\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}+\frac{if\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}}{a}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \text{6109}$$

$$-\frac{b\left(\frac{\int (e+fx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\left(\frac{\int i(e+fx)\csc(ic+idx)dx}{a}-\frac{b\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a}\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\left(\frac{i\int (e+fx)\csc(ic+idx)dx}{a}-\frac{b\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a}\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \text{3803}$$

$$-\frac{b\left(-\frac{2b\int -\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx}{a}\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\left(\frac{2b\int \frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx}{a}\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \text{2694}$$

$$-\frac{b\left(\frac{2b\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx}{a}\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \; 27$$

$$-\frac{b\left(\frac{2b\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx}{a}\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \; 2620$$

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int(e+f\cdots}{\cdots}\cdots\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \; 2715$$

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\cdots\right.}{bd^2}\cdots\right.}{2\sqrt{a^2+b^2}}\cdots\right.}{a}\cdots\right.}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$\downarrow$ 2838

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int(e\cdots}{\cdots}\cdots\right.}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$\downarrow$ 4670

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\left(\frac{if\cdots}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$\downarrow$ 2715

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\left(\frac{if\ldots}{\ldots}\right.}{\ldots}\right)}{a}$$

$$\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$\Big\downarrow$ 2838

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\left(\frac{2i(\ldots}{\ldots}\right.}{\ldots}\right)}{a}$$

$$\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

input
```
Int[((e + f*x)*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-((((e + f*x)*Coth[c + d*x])/d - (f*Log[(-I)*Sinh[c + d*x]])/d^2)/a) - (b*
((I*(((2*I)*(e + f*x)*ArcTanh[E^(c + d*x)])/d + (I*f*PolyLog[2, -E^(c + d*
x)])/d^2 - (I*f*PolyLog[2, E^(c + d*x)])/d^2))/a + (2*b*(-1/2*(b*(((e + f*
x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -
((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2)))/Sqrt[a^2 + b^2] + (b*(
((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*Poly
Log[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(2*Sqrt[a^2 +
b^2])))/a))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6109
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csch[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a +
 b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(283) = 566$.

Time = 0.44 (sec) , antiderivative size = 626, normalized size of antiderivative = 2.05

method	result
risch	$-\frac{2(fx+e)}{da\left(e^{2dx+2c}-1\right)}+\frac{bf\ln\left(e^{dx+c}+1\right)x}{a^2d}+\frac{b^2f\ln\left(\frac{-b\,e^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)c}{a^2d^2\sqrt{a^2+b^2}}-\frac{b^2f\ln\left(\frac{b\,e^{dx+c}+\sqrt{a^2+b^2}+a}{a+\sqrt{a^2+b^2}}\right)c}{a^2d^2\sqrt{a^2+b^2}}+\frac{cbf\ln\left(e^{dx+c}-\right.}{a^2d^2}$

input
```
int((f*x+e)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(f*x+e)/a/(exp(2*d*x+2*c)-1)+1/a^2/d*b*f*ln(exp(d*x+c)+1)*x+1/a^2/d^2
*b^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^
(1/2)))*c-1/a^2/d^2*b^2*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)
+a)/(a+(a^2+b^2)^(1/2)))*c+1/a^2/d^2*c*b*f*ln(exp(d*x+c)-1)+1/a/d^2*f*ln(e
xp(d*x+c)-1)+1/a/d^2*f*ln(exp(d*x+c)+1)+1/a^2/d^2*b^2*f/(a^2+b^2)^(1/2)*di
log((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/a^2/d^2*b^2*
f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2
)))-2/a^2/d*b^2*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^
2)^(1/2))+2/a^2/d^2*c*b^2*f/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*
a)/(a^2+b^2)^(1/2))+1/a^2/d*b*e*ln(exp(d*x+c)+1)-1/a^2/d*b*e*ln(exp(d*x+c)
-1)-2/a/d^2*f*ln(exp(d*x+c))+1/a^2/d^2*b*f*dilog(exp(d*x+c)+1)+1/a^2/d^2*b
*f*dilog(exp(d*x+c))-1/a^2/d*b^2*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b
^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/a^2/d*b^2*f/(a^2+b^2)^(1/2)*ln((-b*e
xp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1830 vs. $2(279) = 558$.

Time = 0.15 (sec) , antiderivative size = 1830, normalized size of antiderivative = 5.98

$$\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(2*(a^3 + a*b^2)*d*e - 2*(a^3 + a*b^2)*c*f + 2*((a^3 + a*b^2)*d*f*x + (a^
3 + a*b^2)*c*f)*cosh(d*x + c)^2 + 4*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c
*f)*cosh(d*x + c)*sinh(d*x + c) + 2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c
*f)*sinh(d*x + c)^2 - (b^3*f*cosh(d*x + c)^2 + 2*b^3*f*cosh(d*x + c)*sinh(
d*x + c) + b^3*f*sinh(d*x + c)^2 - b^3*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*c
osh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2) - b)/b + 1) + (b^3*f*cosh(d*x + c)^2 + 2*b^3*f*cosh(d*x +
 c)*sinh(d*x + c) + b^3*f*sinh(d*x + c)^2 - b^3*f)*sqrt((a^2 + b^2)/b^2)*d
ilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^3*d*e - b^3*c*f - (b^3*d*e - b^
3*c*f)*cosh(d*x + c)^2 - 2*(b^3*d*e - b^3*c*f)*cosh(d*x + c)*sinh(d*x + c)
 - (b^3*d*e - b^3*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh
(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^3*d*
e - b^3*c*f - (b^3*d*e - b^3*c*f)*cosh(d*x + c)^2 - 2*(b^3*d*e - b^3*c*f)*
cosh(d*x + c)*sinh(d*x + c) - (b^3*d*e - b^3*c*f)*sinh(d*x + c)^2)*sqrt((a
^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 +
 b^2)/b^2) + 2*a) + (b^3*d*f*x + b^3*c*f - (b^3*d*f*x + b^3*c*f)*cosh(d*x
+ c)^2 - 2*(b^3*d*f*x + b^3*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d*f*x
+ b^3*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) +
a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/...
```

Sympy [F]

$$\int \frac{(e+fx)\mathrm{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)\,\mathrm{csch}^2\,(c+dx)}{a+b\sinh\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*csch(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{csch}\,(dx+c)^2}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
(4*b^2*integrate(1/2*x*e^(d*x + c)/(a^2*b*e^(2*d*x + 2*c) + 2*a^3*e^(d*x +
 c) - a^2*b), x) - 4*b*d*integrate(1/4*x/(a^2*d*e^(d*x + c) + a^2*d), x) -
 4*b*d*integrate(1/4*x/(a^2*d*e^(d*x + c) - a^2*d), x) - a*((d*x + c)/(a^2
*d^2) - log(e^(d*x + c) + 1)/(a^2*d^2)) - a*((d*x + c)/(a^2*d^2) - log(e^(
d*x + c) - 1)/(a^2*d^2)) - 2*x/(a*d*e^(2*d*x + 2*c) - a*d))*f + e*(b^2*log
((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b
^2)))/(sqrt(a^2 + b^2)*a^2*d) + b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^
(-d*x - c) - 1)/(a^2*d) + 2/((a*e^(-2*d*x - 2*c) - a)*d))
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{e+f\,x}{\sinh\left(c+d\,x\right)^2\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\frac{2e^{2dx+2c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)b^2ei-2\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)b^2ei+8e^{2dx+5c}\left(\int\frac{}{e^{6dx+6c}b+2e^{5dx+5c}a-}\right.}{}$$

input
```
int((f*x+e)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(2*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a
**2 + b**2))*b**2*e*i - 2*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/
sqrt(a**2 + b**2))*b**2*e*i + 8*e**(5*c + 2*d*x)*int((e**(3*d*x)*x)/(e**(6
*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e**(3*c +
3*d*x)*a + 3*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*d*f + 8*e*
*(5*c + 2*d*x)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)
*a - 3*e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b +
2*e**(c + d*x)*a - b),x)*a**2*b**2*d*f - e**(2*c + 2*d*x)*log(e**(c + d*x)
 - 1)*a**2*b*e - e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*b**3*e + e**(2*c +
 2*d*x)*log(e**(c + d*x) + 1)*a**2*b*e + e**(2*c + 2*d*x)*log(e**(c + d*x)
 + 1)*b**3*e - 2*e**(2*c + 2*d*x)*a**3*e - 2*e**(2*c + 2*d*x)*a*b**2*e - 8
*e**(3*c)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a -
3*e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b + 2*e**
(c + d*x)*a - b),x)*a**4*d*f - 8*e**(3*c)*int((e**(3*d*x)*x)/(e**(6*c + 6*
d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*
a + 3*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b**2*d*f + log(e*
*(c + d*x) - 1)*a**2*b*e + log(e**(c + d*x) - 1)*b**3*e - log(e**(c + d*x)
 + 1)*a**2*b*e - log(e**(c + d*x) + 1)*b**3*e)/(a**2*d*(e**(2*c + 2*d*x)*a
**2 + e**(2*c + 2*d*x)*b**2 - a**2 - b**2))
```

3.246 $\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b\operatorname{arctanh}(\cosh(c+dx))}{a^2d} - \frac{2b^2\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d} - \frac{\coth(c+dx)}{ad}$$

output
```
b*arctanh(cosh(d*x+c))/a^2/d-2*b^2*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+
b^2)^(1/2))/a^2/(a^2+b^2)^(1/2)/d-coth(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 1.71 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.41

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a\coth\left(\frac{1}{2}(c+dx)\right) + 2b\left(-\frac{2b\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - \log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right) + \log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)\right.}{2a^2d}$$

input
```
Integrate[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*(a*Coth[(c + d*x)/2] + 2*b*((-2*b*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sq
rt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - Log[Cosh[(c + d*x)/2]] + Log[Sinh[(c +
 d*x)/2]]) + a*Tanh[(c + d*x)/2])/(a^2*d)
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.11, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 25, 3281, 27, 3042, 26, 3226, 26, 3042, 26, 3139, 1083, 217, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3281}$$

$$-\frac{\int \frac{b\,\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\coth(c+dx)}{ad}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\coth(c+dx)}{ad}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{b\int \frac{i}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{\coth(c+dx)}{ad}-\frac{ib\int \frac{1}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx}{a}$$

$$\Big\downarrow \; 3226$$

$$-\frac{\coth(c+dx)}{ad}-\frac{ib\left(\frac{ib\int \frac{1}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i\text{csch}(c+dx)dx}{a}\right)}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{\coth(c+dx)}{ad}-\frac{ib\left(\frac{ib\int \frac{1}{a+b\sinh(c+dx)}dx}{a}-\frac{i\int \text{csch}(c+dx)dx}{a}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\coth(c+dx)}{ad}-\frac{ib\left(\frac{ib\int \frac{1}{a-ib\sin(ic+idx)}dx}{a}-\frac{i\int i\csc(ic+idx)dx}{a}\right)}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{\coth(c+dx)}{ad}-\frac{ib\left(\frac{ib\int \frac{1}{a-ib\sin(ic+idx)}dx}{a}+\frac{\int \csc(ic+idx)dx}{a}\right)}{a}$$

$$\Big\downarrow \; 3139$$

$$-\frac{\coth(c+dx)}{ad}-\frac{ib\left(\frac{2b\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{ad}+\frac{\int \csc(ic+idx)dx}{a}\right)}{a}$$

$$\Big\downarrow \; 1083$$

$$-\frac{\coth(c+dx)}{ad}-\frac{ib\left(\frac{\int \csc(ic+idx)dx}{a}-\frac{4b\int \frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4\left(a^2+b^2\right)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{ad}\right)}{a}$$

$$\Big\downarrow \; 217$$

$$-\frac{\coth(c+dx)}{ad}-\frac{ib\left(\frac{\int \csc(ic+idx)dx}{a}+\frac{2ib\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}\right)}{a}$$

$$\Big\downarrow \; 4257$$

$$-\frac{\coth(c+dx)}{ad}-\frac{ib\left(\frac{2ib\operatorname{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}+\frac{i\operatorname{arctanh}(\cosh(c+dx))}{ad}\right)}{a}$$

input
```
Int[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

output
```
((-I)*b*((I*ArcTanh[Cosh[c + d*x]])/(a*d) + ((2*I)*b*ArcTanh[Tanh[(c + d*x
)/2]/(2*Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d)))/a - Coth[c + d*x]/(a*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3226
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]),
 x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{2b^2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}}{d}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{2b^2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}}{d}$
risch	$-\frac{2}{da\left(\mathrm{e}^{2dx+2c}-1\right)}+\frac{b^2\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d\,a^2}-\frac{b^2\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d\,a^2}-\frac{b\ln\left(\mathrm{e}^{dx+c}-1\right)}{d\,a^2}+$

input
```
int(csch(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/a*tanh(1/2*d*x+1/2*c)-1/2/a/tanh(1/2*d*x+1/2*c)-1/a^2*b*ln(tanh(
1/2*d*x+1/2*c))+2*b^2/a^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/
2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(77) = 154$.

Time = 0.13 (sec) , antiderivative size = 479, normalized size of antiderivative = 5.99

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{2\,a^3+2\,ab^2-\left(b^2\cosh\left(dx+c\right)^2+2\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b^2\sinh\left(dx+c\right)^2-b^2\right)\sqrt{a^2+b^2}\,\mathrm{l}}{}$$

input
```
integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(2*a^3 + 2*a*b^2 - (b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x +
c) + b^2*sinh(d*x + c)^2 - b^2)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 +
 b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x
 + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d
*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) +
 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + (a^2*b + b^3 - (a^2*b + b^3
)*cosh(d*x + c)^2 - 2*(a^2*b + b^3)*cosh(d*x + c)*sinh(d*x + c) - (a^2*b +
 b^3)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (a^2*b + b
^3 - (a^2*b + b^3)*cosh(d*x + c)^2 - 2*(a^2*b + b^3)*cosh(d*x + c)*sinh(d*
x + c) - (a^2*b + b^3)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c)
- 1))/((a^4 + a^2*b^2)*d*cosh(d*x + c)^2 + 2*(a^4 + a^2*b^2)*d*cosh(d*x +
c)*sinh(d*x + c) + (a^4 + a^2*b^2)*d*sinh(d*x + c)^2 - (a^4 + a^2*b^2)*d)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\operatorname{csch}^2{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate(csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(csch(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.71

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\frac{b^2\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a^2d}+\frac{b\log\left(e^{(-dx-c)}+1\right)}{a^2d}-\frac{b\log\left(e^{(-dx-c)}-1\right)}{a^2d}+\frac{2}{(ae^{(-2\,dx-2\,c)}-a)d}$$

input
```
integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
b^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*a^2*d) + b*log(e^(-d*x - c) + 1)/(a^2*d) - b
*log(e^(-d*x - c) - 1)/(a^2*d) + 2/((a*e^(-2*d*x - 2*c) - a)*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.54

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{b^2\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^2}+\frac{b\log\left(e^{(dx+c)}+1\right)}{a^2}-\frac{b\log\left(\left|e^{(dx+c)}-1\right|\right)}{a^2}-\frac{2}{a\left(e^{(2\,dx+2\,c)}-1\right)}}{d}$$

input
```
integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
(b^2*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c
) + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^2) + b*log(e^(d*x + c) +
1)/a^2 - b*log(abs(e^(d*x + c) - 1))/a^2 - 2/(a*(e^(2*d*x + 2*c) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 360, normalized size of antiderivative = 4.50

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2}{a\,d-a\,d\,\mathrm{e}^{2\,c+2\,d\,x}} + \frac{b^2\,\ln\left(128\,a^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-64\,a\,b^3-64\,a^3\,b-32\,b^3\,\sqrt{a^2+b^2}+32\,b^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-64\,a^2\,b\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,\mathrm{e}^{d\,x}}{d\,a^4+d\,a^2\,b^2} - \frac{b^2\,\ln\left(32\,b^3\,\sqrt{a^2+b^2}-64\,a\,b^3-64\,a^3\,b+128\,a^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+32\,b^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+64\,a^2\,b\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,\mathrm{e}^{d\,x}}{d\,a^4+d\,a^2\,b^2} - \frac{b\,\ln\left(32\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-32\right)}{a^2\,d} + \frac{b\,\ln\left(32\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+32\right)}{a^2\,d}$$

input
```
int(1/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
2/(a*d - a*d*exp(2*c + 2*d*x)) + (b^2*log(128*a^4*exp(d*x)*exp(c) - 64*a*b
^3 - 64*a^3*b - 32*b^3*(a^2 + b^2)^(1/2) + 32*b^4*exp(d*x)*exp(c) - 64*a^2
*b*(a^2 + b^2)^(1/2) + 160*a^2*b^2*exp(d*x)*exp(c) + 128*a^3*exp(d*x)*exp(
c)*(a^2 + b^2)^(1/2) + 96*a*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2))*(a^2 +
b^2)^(1/2))/(a^4*d + a^2*b^2*d) - (b^2*log(32*b^3*(a^2 + b^2)^(1/2) - 64*a
*b^3 - 64*a^3*b + 128*a^4*exp(d*x)*exp(c) + 32*b^4*exp(d*x)*exp(c) + 64*a^
2*b*(a^2 + b^2)^(1/2) + 160*a^2*b^2*exp(d*x)*exp(c) - 128*a^3*exp(d*x)*exp
(c)*(a^2 + b^2)^(1/2) - 96*a*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2))*(a^2 +
 b^2)^(1/2))/(a^4*d + a^2*b^2*d) - (b*log(32*exp(d*x)*exp(c) - 32))/(a^2*d
) + (b*log(32*exp(d*x)*exp(c) + 32))/(a^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 330, normalized size of antiderivative = 4.12

$$\int \frac{\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2e^{2dx+2c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)b^2i - 2\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)b^2i - e^{2dx+2c}\log\left(e^{dx+c}-1\right)a^2b - e^{2c}}{}$$

input
```
int(csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(2*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a
**2 + b**2))*b**2*i - 2*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sq
rt(a**2 + b**2))*b**2*i - e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a**2*b -
e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*b**3 + e**(2*c + 2*d*x)*log(e**(c +
 d*x) + 1)*a**2*b + e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*b**3 - 2*e**(2*
c + 2*d*x)*a**3 - 2*e**(2*c + 2*d*x)*a*b**2 + log(e**(c + d*x) - 1)*a**2*b
 + log(e**(c + d*x) - 1)*b**3 - log(e**(c + d*x) + 1)*a**2*b - log(e**(c +
 d*x) + 1)*b**3)/(a**2*d*(e**(2*c + 2*d*x)*a**2 + e**(2*c + 2*d*x)*b**2 -
a**2 - b**2))
```

3.247 $\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\operatorname{Int}\left(\frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 84.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[Csch[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Csch[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx$$

$$\Big\downarrow \text{6111}$$

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Csch[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(dx+c)^2}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(csch(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 6.90 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}^2\left(c+dx\right)}{(a+b\sinh\left(c+dx\right))\left(e+fx\right)}\,dx$$

input
```
integrate(csch(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(csch(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 304, normalized size of antiderivative = 10.86

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
4*b^2*integrate(-1/2*e^(d*x + c)/(a^2*b*f*x + a^2*b*e - (a^2*b*f*x*e^(2*c)
 + a^2*b*e*e^(2*c))*e^(2*d*x) - 2*(a^3*f*x*e^c + a^3*e*e^c)*e^(d*x)), x) +
 2/(a*d*f*x + a*d*e - (a*d*f*x*e^(2*c) + a*d*e*e^(2*c))*e^(2*d*x)) - 4*int
egrate(-1/4*(b*d*f*x + b*d*e + a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d
*e^2 - (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)), x
) - 4*integrate(1/4*(b*d*f*x + b*d*e - a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x
 + a^2*d*e^2 + (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(
d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{1}{\sinh(c+d\,x)^2\,(e+f\,x)\,(a+b\sinh(c+d\,x))}\,dx$$

input
```
int(1/(sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.43

$$\int \frac{\text{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\text{csch}(dx+c)^2}{\sinh(dx+c)\,be+\sinh(dx+c)\,bfx+ae+afx}dx$$

input
```
int(csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(csch(c + d*x)**2/(sinh(c + d*x)*b*e + sinh(c + d*x)*b*f*x + a*e + a*f*
x),x)
```

3.248 $\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 1053

$$\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output
```
-6*b^2*f^3*polylog(4,-exp(d*x+c))/a^3/d^4+6*b^2*f^3*polylog(4,exp(d*x+c))/
a^3/d^4+3/2*b*f^3*polylog(3,exp(2*d*x+2*c))/a^2/d^4-2*b^2*(f*x+e)^3*arctan
h(exp(d*x+c))/a^3/d+(f*x+e)^3*arctanh(exp(d*x+c))/a/d+3*f^3*polylog(4,-exp
(d*x+c))/a/d^4-3*f^3*polylog(4,exp(d*x+c))/a/d^4-6*b^3*f^2*(f*x+e)*polylog
(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^(1/2)/d^3+6*b^3*f^2*(f
*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^(1/2)/d^3
+3*b^3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/(a^2+b
^2)^(1/2)/d^2-3*b^3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)
))/a^3/(a^2+b^2)^(1/2)/d^2+6*b^2*f^2*(f*x+e)*polylog(3,-exp(d*x+c))/a^3/d^
3-3*b^2*f*(f*x+e)^2*polylog(2,-exp(d*x+c))/a^3/d^2-3*b*f*(f*x+e)^2*ln(1-ex
p(2*d*x+2*c))/a^2/d^2-6*b^2*f^2*(f*x+e)*polylog(3,exp(d*x+c))/a^3/d^3-3*b*
f^2*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a^2/d^3+3*b^2*f*(f*x+e)^2*polylog(2,
exp(d*x+c))/a^3/d^2+b*(f*x+e)^3/a^2/d-6*f^2*(f*x+e)*arctanh(exp(d*x+c))/a/
d^3-3/2*f*(f*x+e)^2*csch(d*x+c)/a/d^2-1/2*(f*x+e)^3*coth(d*x+c)*csch(d*x+c
)/a/d+3/2*f*(f*x+e)^2*polylog(2,-exp(d*x+c))/a/d^2-3/2*f*(f*x+e)^2*polylog
(2,exp(d*x+c))/a/d^2-3*f^2*(f*x+e)*polylog(3,-exp(d*x+c))/a/d^3+3*f^2*(f*x
+e)*polylog(3,exp(d*x+c))/a/d^3+b*(f*x+e)^3*coth(d*x+c)/a^2/d-3*f^3*polylo
g(2,-exp(d*x+c))/a/d^4+3*f^3*polylog(2,exp(d*x+c))/a/d^4+6*b^3*f^3*polylog
(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^(1/2)/d^4-6*b^3*f^3*po
lylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^(1/2)/d^4+b^3*...
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2801 vs. $2(1053) = 2106$.

Time = 9.00 (sec) , antiderivative size = 2801, normalized size of antiderivative = 2.66

$$\int \frac{(e+fx)^3 \operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Csch[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(12*a*b*d^3*e^2*E^(2*c)*f*x + 12*a*b*d^3*e*E^(2*c)*f^2*x^2 + 4*a*b*d^3*E^(
2*c)*f^3*x^3 - 2*a^2*d^3*e^3*ArcTanh[E^(c + d*x)] + 4*b^2*d^3*e^3*ArcTanh[
E^(c + d*x)] + 2*a^2*d^3*e^3*E^(2*c)*ArcTanh[E^(c + d*x)] - 4*b^2*d^3*e^3*
E^(2*c)*ArcTanh[E^(c + d*x)] + 12*a^2*d*e*f^2*ArcTanh[E^(c + d*x)] - 12*a^
2*d*e*E^(2*c)*f^2*ArcTanh[E^(c + d*x)] + 3*a^2*d^3*e^2*f*x*Log[1 - E^(c +
d*x)] - 6*b^2*d^3*e^2*f*x*Log[1 - E^(c + d*x)] - 3*a^2*d^3*e^2*E^(2*c)*f*x
*Log[1 - E^(c + d*x)] + 6*b^2*d^3*e^2*E^(2*c)*f*x*Log[1 - E^(c + d*x)] - 6
*a^2*d*f^3*x*Log[1 - E^(c + d*x)] + 6*a^2*d*E^(2*c)*f^3*x*Log[1 - E^(c + d
*x)] + 3*a^2*d^3*e*f^2*x^2*Log[1 - E^(c + d*x)] - 6*b^2*d^3*e*f^2*x^2*Log[
1 - E^(c + d*x)] - 3*a^2*d^3*e*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] + 6*b^
2*d^3*e*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] + a^2*d^3*f^3*x^3*Log[1 - E^(
c + d*x)] - 2*b^2*d^3*f^3*x^3*Log[1 - E^(c + d*x)] - a^2*d^3*E^(2*c)*f^3*x
^3*Log[1 - E^(c + d*x)] + 2*b^2*d^3*E^(2*c)*f^3*x^3*Log[1 - E^(c + d*x)] -
 3*a^2*d^3*e^2*f*x*Log[1 + E^(c + d*x)] + 6*b^2*d^3*e^2*f*x*Log[1 + E^(c +
 d*x)] + 3*a^2*d^3*e^2*E^(2*c)*f*x*Log[1 + E^(c + d*x)] - 6*b^2*d^3*e^2*E^
(2*c)*f*x*Log[1 + E^(c + d*x)] + 6*a^2*d*f^3*x*Log[1 + E^(c + d*x)] - 6*a^
2*d*E^(2*c)*f^3*x*Log[1 + E^(c + d*x)] - 3*a^2*d^3*e*f^2*x^2*Log[1 + E^(c
+ d*x)] + 6*b^2*d^3*e*f^2*x^2*Log[1 + E^(c + d*x)] + 3*a^2*d^3*e*E^(2*c)*f
^2*x^2*Log[1 + E^(c + d*x)] - 6*b^2*d^3*e*E^(2*c)*f^2*x^2*Log[1 + E^(c + d
*x)] - a^2*d^3*f^3*x^3*Log[1 + E^(c + d*x)] + 2*b^2*d^3*f^3*x^3*Log[1 +...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6109}$$

$$\frac{\int (e+fx)^3\text{csch}^3(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i(e+fx)^3\csc(ic+idx)^3dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\int (e+fx)^3\csc(ic+idx)^3dx}{a}$$

$$\Big\downarrow \text{4674}$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(-\frac{3f^2\int -i(e+fx)\text{csch}(c+dx)dx}{d^2}+\frac{1}{2}\int -i(e+fx)^3\text{csch}(c+dx)dx-\frac{3if(e+fx)^2\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(\frac{3if^2\int (e+fx)\text{csch}(c+dx)dx}{d^2}-\frac{1}{2}i\int (e+fx)^3\text{csch}(c+dx)dx-\frac{3if(e+fx)^2\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(\frac{3if^2\int i(e+fx)\csc(ic+idx)dx}{d^2}-\frac{1}{2}i\int i(e+fx)^3\csc(ic+idx)dx-\frac{3if(e+fx)^2\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(-\frac{3f^2\int (e+fx)\csc(ic+idx)dx}{d^2}+\frac{1}{2}\int (e+fx)^3\csc(ic+idx)dx-\frac{3if(e+fx)^2\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(-\frac{3f^2\left(\frac{if\int \log\left(1-e^{c+dx}\right)dx}{d}-\frac{if\int \log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)}{d^2}+\frac{1}{2}\left(\frac{3if\int (e+fx)^2\log(1-e^{c+dx})dx}{d}-\frac{3if\int (e+fx)^2\log\ldots}{d}\ldots\right.\ldots\right)}{a}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-$$
$$i\left(-\frac{3f^2\left(\frac{if\int e^{-c-dx}\log\left(1-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log\left(1+e^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)}{d^2}+\frac{1}{2}\left(\frac{3if\int(e+fx)^2\log(1-e^{c+a}}{d}\right.\right.$$

$\downarrow$ 2838

$$-\frac{b\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-$$
$$\frac{i\left(\frac{1}{2}\left(\frac{3if\int(e+fx)^2\log(1-e^{c+dx})dx}{d}-\frac{3if\int(e+fx)^2\log(1+e^{c+dx})dx}{d}+\frac{2i(e+fx)^3\text{arctanh}(e^{c+dx})}{d}\right)-\frac{3f^2\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+d}\right.}{d}\right.}{}\right.}{a}$$

$\downarrow$ 3011

$$-\frac{b\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-$$
$$i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\text{ PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{ PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\text{ PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{ PolyLog}\left(2,e^{c+d}\right.}{d}\right.}{d}\right.\right.$$

$\downarrow$ 6109

$$-\frac{b\left(\frac{\int(e+fx)^3\text{csch}^2(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-$$
$$i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\text{ PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{ PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\text{ PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{ PolyLog}\left(2,e^{c+d}\right.}{d}\right.}{d}\right.\right.$$

$\downarrow$ 3042

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c}\right.}{d}\right.}{d}\right.\right.}{\frac{b\left(-\frac{b\int\frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int-(e+fx)^3\csc(ic+idx)^2dx}{a}\right)}{a}}$$

$$\Big\downarrow \; 25$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c}\right.}{d}\right.}{d}\right.\right.}{\frac{b\left(-\frac{b\int\frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\int(e+fx)^3\csc(ic+idx)^2dx}{a}\right)}{a}}$$

$$\Big\downarrow \; 4672$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c}\right.}{d}\right.}{d}\right.\right.}{\frac{b\left(-\frac{b\int\frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{3if\int-i(e+fx)^2\coth(c+dx)dx}{d}}{a}\right)}{a}}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{3f\int(e+fx)^2\coth(c+dx)dx}{d}}{a}\right)}{a}-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+d}\right.}{d}\right.}{d}\right.\right.}{}$$

$$\Big\downarrow \; 3042$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c}\right.}{d}\cdots\right.}{d}\cdots\right.\right.}{\cdots}$$

$$\frac{b\left(-\frac{b\int \frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{3f\int -i(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}\right)}{a}$$

$\downarrow$ 26

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c}\right.}{d}\cdots\right.}{d}\cdots\right.\right.}{\cdots}$$

$$\frac{b\left(-\frac{b\int \frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\int (e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}}{a}\right)}{a}$$

$\downarrow$ 4201

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c}\right.}{d}\cdots\right.}{d}\cdots\right.\right.}{\cdots}$$

$$\frac{b\left(-\frac{b\int \frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\int \frac{e^{2c+2dx-i\pi}(e+fx)^2}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}\right)}{a}$$

$\downarrow$ 2620

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c}\right.}{d}\cdots\right.}{d}\cdots\right.\right.}{\cdots}$$

$$\frac{b\left(-\frac{b\int \frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int (e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}\right)}{a}$$

$\downarrow$ 3011

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c}\right.}{d}\right.}{d}\right.\right.}{b\left(-\frac{b\int\frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{2d}-\frac{(e+fx)\operatorname{PolyLog}}{}\right.}{d}\right.\right.}{d}}{a}\right.}a$$

$\downarrow$ 2720

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c}\right.}{d}\right.}{d}\right.\right.}{b\left(-\frac{b\int\frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c-}}{4d^2}\right.}{d}\right.\right.}{d}}{a}\right.}a$$

$\downarrow$ 6109

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c}\right.}{d}\right.}{d}\right.\right.}{b\left(-\frac{b\left(\frac{\int(e+fx)^3\operatorname{csch}(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{Poly}}{}\right.}{}\right.\right.}{}\right.}{a}$$

$$\downarrow \text{3042}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c}\right.}{d}\right.}{d}\right.\right.}{b\left(-\frac{b\left(\frac{\int i(e+fx)^3\csc(ic+idx)dx}{a}-\frac{b\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{a}\right)}{a}-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{Pol}}{}\right.}{}\right.\right.}{}\right.}{a}$$

$$\downarrow \text{26}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c}\right.}{d}\right.}{d}\right.\right.}{b\left(-\frac{b\left(\frac{i\int(e+fx)^3\csc(ic+idx)dx}{a}-\frac{b\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{a}\right)}{a}-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{Pol}}{}\right.}{}\right.\right.}{}\right.}{a}$$

$\downarrow$ 3803

$$-\frac{i\Bigg(\frac{1}{2}\Bigg(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\Big(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\big(2,e^{c}}{d}}{d}}{b\Bigg(-\frac{b\left(-\frac{2b\int-\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int(e+fx)^3\csc(ic+idx)dx}{a}\right)}{a}-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\Big(2i\Big(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\Big(\frac{f\int e^{-2}}{}}{}}{}}{a}$$

$\downarrow$ 25

$$-\frac{i\Bigg(\frac{1}{2}\Bigg(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\Big(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\big(2,e^{c}}{d}}{d}}{b\Bigg(-\frac{b\left(\frac{2b\int\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int(e+fx)^3\csc(ic+idx)dx}{a}\right)}{a}-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\Big(2i\Big(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\Big(\frac{f\int e^{-2c-2}}{}}{}}{}}{a}$$

$\downarrow$ 2694

$$-\frac{b\left(-\frac{b\left(\frac{2b\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx}{a}\right)}{a}-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+\cdots}{}\right.\right.}{}\right.}{a}$$

$$\frac{}{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+d\cdots}\right.}{d}\right.}{d}\right.\right.}$$

$\downarrow$ 27

$$-\frac{b\left(-\frac{b\left(\frac{2b\left(\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx}{a}\right)}{a}-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1\cdots\right.}{2\cdots}\right.\right.}{}\right.}{a}$$

$$\frac{}{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+d\cdots}\right.}{d}\right.}{d}\right.\right.}$$

$\downarrow$ 2620

$$-\frac{b\left(-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}\cdots\right.}{bd}\cdots\right.}{2\sqrt{a^2+b^2}}\cdots\right.}{a}\cdots\right.}{a}\cdots\right.}{i\left(\frac{1}{2}\left(\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,e^{c+d\cdots}\right.}{d}\cdots\right.}{d}\cdots\right.\right.\right.}$$

input
```
Int[((e + f*x)^3*Csch[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^3\,\mathrm{csch}\,(dx+c)^3}{a+b\sinh\,(dx+c)}dx$$

input
```
int((f*x+e)^3*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18159 vs. 2(977) = 1954.

Time = 0.44 (sec) , antiderivative size = 18159, normalized size of antiderivative = 17.25

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*csch(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\,\text{csch}\,(dx+c)^3}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*e^3*(2*b^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c)
 - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3*d) - 2*(a*e^(-d*x - c) + 2*b
*e^(-2*d*x - 2*c) + a*e^(-3*d*x - 3*c) - 2*b)/((2*a^2*e^(-2*d*x - 2*c) - a
^2*e^(-4*d*x - 4*c) - a^2)*d) - (a^2 - 2*b^2)*log(e^(-d*x - c) + 1)/(a^3*d
) + (a^2 - 2*b^2)*log(e^(-d*x - c) - 1)/(a^3*d)) - (2*b*d*f^3*x^3 + 6*b*d*
e*f^2*x^2 + 6*b*d*e^2*f*x + (a*d*f^3*x^3*e^(3*c) + 3*a*e^2*f*e^(3*c) + 3*(
d*e*f^2 + f^3)*a*x^2*e^(3*c) + 3*(d*e^2*f + 2*e*f^2)*a*x*e^(3*c))*e^(3*d*x
) - 2*(b*d*f^3*x^3*e^(2*c) + 3*b*d*e*f^2*x^2*e^(2*c) + 3*b*d*e^2*f*x*e^(2*
c))*e^(2*d*x) + (a*d*f^3*x^3*e^c - 3*a*e^2*f*e^c + 3*(d*e*f^2 - f^3)*a*x^2
*e^c + 3*(d*e^2*f - 2*e*f^2)*a*x*e^c)*e^(d*x))/(a^2*d^2*e^(4*d*x + 4*c) -
2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2) + 3*(b*d*e^2*f + a*e*f^2)*x/(a^2*d^2)
 + 3*(b*d*e^2*f - a*e*f^2)*x/(a^2*d^2) - 3*(b*d*e^2*f + a*e*f^2)*log(e^(d*
x + c) + 1)/(a^2*d^3) - 3*(b*d*e^2*f - a*e*f^2)*log(e^(d*x + c) - 1)/(a^2*
d^3) + 1/2*(d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) -
 6*d*x*polylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*(a^2*f^3 - 2
*b^2*f^3)/(a^3*d^4) - 1/2*(d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog
(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))
*(a^2*f^3 - 2*b^2*f^3)/(a^3*d^4) + 3/2*(a^2*d*e*f^2 - 2*b^2*d*e*f^2 - 2*a*
b*f^3)*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polyl
og(3, -e^(d*x + c)))/(a^3*d^4) - 3/2*(a^2*d*e*f^2 - 2*b^2*d*e*f^2 + 2*a...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+f\,x)^3}{\sinh\left(c+d\,x\right)^3\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^3/(sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^3/(sinh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{too large to display}$$

input
```
int((f*x+e)^3*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output

```
( - 16*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sq
rt(a**2 + b**2))*b**4*d**3*e**3*i + 32*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*
atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*b**4*d**3*e**3*i - 16*sqr
t(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*b**4*d**3*
e**3*i - 256*e**(7*c + 4*d*x)*int((e**(3*d*x)*x**3)/(e**(8*c + 8*d*x)*b +
2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**
(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c +
d*x)*a + b),x)*a**6*d**4*f**3 - 256*e**(7*c + 4*d*x)*int((e**(3*d*x)*x**3)
/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**
(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c
+ 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**4*b**2*d**4*f**3 - 768*e**(7*c +
4*d*x)*int((e**(3*d*x)*x**2)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a -
4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**
(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**6*d**
4*e*f**2 - 768*e**(7*c + 4*d*x)*int((e**(3*d*x)*x**2)/(e**(8*c + 8*d*x)*b
+ 2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e
**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c
+ d*x)*a + b),x)*a**4*b**2*d**4*e*f**2 - 768*e**(7*c + 4*d*x)*int((e**(3*d
*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b -
 6*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4...
```

3.249 $\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 725

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b(e+fx)^2}{a^2d}+\frac{(e+fx)^2\text{arctanh}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^2\text{arctanh}\left(e^{c+dx}\right)}{a^3d} - \frac{f^2\text{arctanh}(\cosh(c+dx))}{ad^3} + \frac{b(e+fx)^2\coth(c+dx)}{a^2d} - \frac{f(e+fx)\text{csch}(c+dx)}{ad^2} - \frac{(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{2ad} - \frac{b^3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}\,d} + \frac{b^3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}\,d} - \frac{2bf(e+fx)\log\left(1-e^{2(c+dx)}\right)}{a^2d^2} + \frac{f(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2} - \frac{2b^2f(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{a^3d^2} - \frac{f(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2} + \frac{2b^2f(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)}{a^3d^2} - \frac{2b^3f(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}\,d^2} + \frac{2b^3f(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}\,d^2} - \frac{bf^2\,\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{a^2d^3} - \frac{f^2\,\text{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3} + \frac{2b^2f^2\,\text{PolyLog}\left(3,-e^{c+dx}\right)}{a^3d^3} + \frac{f^2\,\text{PolyLog}\left(3,e^{c+dx}\right)}{ad^3} - \frac{2b^2f^2\,\text{PolyLog}\left(3,e^{c+dx}\right)}{a^3d^3} + \frac{2b^3f^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}\,d^3} - \frac{2b^3f^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}\,d^3}$$

output
```
b*(f*x+e)^2/a^2/d+(f*x+e)^2*arctanh(exp(d*x+c))/a/d-2*b^2*(f*x+e)^2*arctan
h(exp(d*x+c))/a^3/d-f^2*arctanh(cosh(d*x+c))/a/d^3+b*(f*x+e)^2*coth(d*x+c)
/a^2/d-f*(f*x+e)*csch(d*x+c)/a/d^2-1/2*(f*x+e)^2*coth(d*x+c)*csch(d*x+c)/a
/d-b^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^(1/2
)/d+b^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^(1/
2)/d-2*b*f*(f*x+e)*ln(1-exp(2*d*x+2*c))/a^2/d^2+f*(f*x+e)*polylog(2,-exp(d
*x+c))/a/d^2-2*b^2*f*(f*x+e)*polylog(2,-exp(d*x+c))/a^3/d^2-f*(f*x+e)*poly
log(2,exp(d*x+c))/a/d^2+2*b^2*f*(f*x+e)*polylog(2,exp(d*x+c))/a^3/d^2-2*b^
3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^(1/
2)/d^2+2*b^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/(a
^2+b^2)^(1/2)/d^2-b*f^2*polylog(2,exp(2*d*x+2*c))/a^2/d^3-f^2*polylog(3,-e
xp(d*x+c))/a/d^3+2*b^2*f^2*polylog(3,-exp(d*x+c))/a^3/d^3+f^2*polylog(3,ex
p(d*x+c))/a/d^3-2*b^2*f^2*polylog(3,exp(d*x+c))/a^3/d^3+2*b^3*f^2*polylog(
3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^(1/2)/d^3-2*b^3*f^2*pol
ylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^(1/2)/d^3
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1532 vs. $2(725) = 1450$.

Time = 8.07 (sec) , antiderivative size = 1532, normalized size of antiderivative = 2.11

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Csch[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(8*a*b*d^2*e*E^(2*c)*f*x + 4*a*b*d^2*E^(2*c)*f^2*x^2 - 2*a^2*d^2*e^2*ArcTa
nh[E^(c + d*x)] + 4*b^2*d^2*e^2*ArcTanh[E^(c + d*x)] + 2*a^2*d^2*e^2*E^(2*
c)*ArcTanh[E^(c + d*x)] - 4*b^2*d^2*e^2*E^(2*c)*ArcTanh[E^(c + d*x)] + 4*a
^2*f^2*ArcTanh[E^(c + d*x)] - 4*a^2*E^(2*c)*f^2*ArcTanh[E^(c + d*x)] + 2*a
^2*d^2*e*f*x*Log[1 - E^(c + d*x)] - 4*b^2*d^2*e*f*x*Log[1 - E^(c + d*x)] -
 2*a^2*d^2*e*E^(2*c)*f*x*Log[1 - E^(c + d*x)] + 4*b^2*d^2*e*E^(2*c)*f*x*Lo
g[1 - E^(c + d*x)] + a^2*d^2*f^2*x^2*Log[1 - E^(c + d*x)] - 2*b^2*d^2*f^2*
x^2*Log[1 - E^(c + d*x)] - a^2*d^2*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] +
2*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] - 2*a^2*d^2*e*f*x*Log[1 + E
^(c + d*x)] + 4*b^2*d^2*e*f*x*Log[1 + E^(c + d*x)] + 2*a^2*d^2*e*E^(2*c)*f
*x*Log[1 + E^(c + d*x)] - 4*b^2*d^2*e*E^(2*c)*f*x*Log[1 + E^(c + d*x)] - a
^2*d^2*f^2*x^2*Log[1 + E^(c + d*x)] + 2*b^2*d^2*f^2*x^2*Log[1 + E^(c + d*x
)] + a^2*d^2*E^(2*c)*f^2*x^2*Log[1 + E^(c + d*x)] - 2*b^2*d^2*E^(2*c)*f^2*
x^2*Log[1 + E^(c + d*x)] + 4*a*b*d*e*f*Log[1 - E^(2*(c + d*x))] - 4*a*b*d*
e*E^(2*c)*f*Log[1 - E^(2*(c + d*x))] + 4*a*b*d*f^2*x*Log[1 - E^(2*(c + d*x
))] - 4*a*b*d*E^(2*c)*f^2*x*Log[1 - E^(2*(c + d*x))] + 2*(a^2 - 2*b^2)*d*(
-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -E^(c + d*x)] - 2*(a^2 - 2*b^2)*d*(-1
 + E^(2*c))*f*(e + f*x)*PolyLog[2, E^(c + d*x)] + 2*a*b*f^2*PolyLog[2, E^(
2*(c + d*x))] - 2*a*b*E^(2*c)*f^2*PolyLog[2, E^(2*(c + d*x))] + 2*a^2*f^2*
PolyLog[3, -E^(c + d*x)] - 4*b^2*f^2*PolyLog[3, -E^(c + d*x)] - 2*a^2*E...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6109}$$

$$\frac{\int (e+fx)^2\text{csch}^3(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i(e+fx)^2\csc(ic+idx)^3dx}{a}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\int (e+fx)^2\csc(ic+idx)^3dx}{a}$$

$$\Big\downarrow \text{4674}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(-\frac{f^2\int -i\text{csch}(c+dx)dx}{d^2}+\frac{1}{2}\int -i(e+fx)^2\text{csch}(c+dx)dx-\frac{if(e+fx)\text{csch}(c+dx)}{d^2}-\frac{i(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(\frac{if^2\int \text{csch}(c+dx)dx}{d^2}-\frac{1}{2}i\int (e+fx)^2\text{csch}(c+dx)dx-\frac{if(e+fx)\text{csch}(c+dx)}{d^2}-\frac{i(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(\frac{if^2\int i\csc(ic+idx)dx}{d^2}-\frac{1}{2}i\int i(e+fx)^2\csc(ic+idx)dx-\frac{if(e+fx)\text{csch}(c+dx)}{d^2}-\frac{i(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(-\frac{f^2\int \csc(ic+idx)dx}{d^2}+\frac{1}{2}\int (e+fx)^2\csc(ic+idx)dx-\frac{if(e+fx)\text{csch}(c+dx)}{d^2}-\frac{i(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(\frac{1}{2}\int (e+fx)^2\csc(ic+idx)dx-\frac{if^2\text{arctanh}(\cosh(c+dx))}{d^3}-\frac{if(e+fx)\text{csch}(c+dx)}{d^2}-\frac{i(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(\frac{1}{2}\left(\frac{2if\int (e+fx)\log(1-e^{c+dx})dx}{d}-\frac{2if\int (e+fx)\log(1+e^{c+dx})dx}{d}+\frac{2i(e+fx)^2\text{arctanh}(e^{c+dx})}{d}\right)-\frac{if^2\text{arctanh}(\cosh(c+dx))}{d^3}-\frac{i}{}\right.}{a}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}+\frac{2i(e+fx}{}\right.\right.}{a}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}}{d}\right.}{d}\right.\right.}{}$$

$$\Big\downarrow \text{6109}$$

$$-\frac{b\left(\frac{\int (e+fx)^2\text{csch}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}}{d}\right.}{d}\right.\right.}{}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyL}}{}\right.}{d}\right.\right.}{}$$

$$\frac{b\left(-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -(e+fx)^2\csc(ic+idx)^2dx}{a}\right)}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyI}}{}\right.}{d}\right.\right.}{}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\int(e+fx)^2\csc(ic+idx)^2dx}{a}\right)}{a}$$

$\downarrow$ 4672

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyI}}{}\right.}{d}\right.\right.}{}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2if\int -i(e+fx)\coth(c+dx)dx}{d}}{a}\right)}{a}$$

$\downarrow$ 26

$$-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2f\int(e+fx)\coth(c+dx)dx}{d}}{a}\right)}{a}-$$

$$\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}}{d}\right.}{d}\right.\right.}{}$$

$\downarrow$ 3042

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyI}}{}\right.}{d}\right.\right.}{}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2f\int -i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}\right)}{a}$$

$\downarrow$ 26

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyL}\cdots}{}\right.}{d}\cdots\right.\right.}{\cdots}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\int(e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}}{a}\right)}{a}$$

$\downarrow$ 4201

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyL}\cdots}{}\right.}{d}\cdots\right.\right.}{\cdots}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\int\frac{e^{2c+2dx-i\pi}(e+fx)}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}\right)}{a}$$

$\downarrow$ 2620

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyL}\cdots}{}\right.}{d}\cdots\right.\right.}{\cdots}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int\log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}\right)}{a}$$

$\downarrow$ 2715

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyL}\cdots}{}\right.}{d}\cdots\right.\right.}{\cdots}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\log\left(1+e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}\right)\cdots\right.}{d}}{a}\cdots\right.}{a}$$

$$\Big\downarrow \; 2838$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyL}\cdots}{\cdots}\right.}{d}\cdots\right.\right.}{\frac{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}\right)}{a}}$$

$$\Big\downarrow \; 6109$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyL}\cdots}{\cdots}\right.}{d}\cdots\right.\right.}{\frac{b\left(-\frac{b\left(\frac{\int(e+fx)^2\operatorname{csch}(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right.\cdots}{2d}\cdots\right.\right.}{d}}{a}\cdots\right.}{a}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyL}\cdots}{\cdots}\right.}{d}\cdots\right.\right.}{\frac{b\left(-\frac{b\left(\frac{\int i(e+fx)^2\csc(ic+idx)dx}{a}-\frac{b\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{a}\right)}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\cdots}\right.}{2d}\cdots\right.\right.}{d}}{a}\cdots\right.}{a}}$$

$$\Big\downarrow \; 26$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyL}\cdots}{\cdots}\right)}{d}\cdots\right.\right.}{\frac{b\left(-\frac{b\left(\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}-\frac{b\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{a}\right)}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\cdots}\right)}{2d}\cdots\right.\right.}{d}}{a}\cdots\right.}{a}}$$

$$\downarrow \text{3803}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyL}\cdots}{\cdots}\right)}{d}\cdots\right.\right.}{\frac{b\left(-\frac{b\left(-\frac{2b\int-\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}\right)}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1\cdots\right.}{\cdots}\right.\right.}{d}}{a}\cdots\right.}{a}}$$

$$\downarrow \text{25}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyL}\cdots}{\cdots}\right)}{d}\cdots\right.\right.}{\frac{b\left(-\frac{b\left(\frac{2b\int\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}\right)}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2\cdots}\right.}{2d}\right.\right.}{d}}{a}\cdots\right.}{a}}$$

$$\downarrow \text{2694}$$

$$-\frac{b\left(-\frac{b\left(\frac{2b\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx}{a}\right)}{a}-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\mathrm{Po}\cdots}{\cdots}\right.\right.}{\cdots}\right.}{a}$$

$$i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\cdots}{d}\right.}{d}\right.\right.$$

$$\Big\downarrow \; 27$$

$$-\frac{b\left(-\frac{b\left(\frac{2b\left(\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx}{a}\right)}{a}-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-\cdots\right.}{4d\cdots}\right.\right.}{\cdots}\right.}{a}$$

$$i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\cdots}{d}\right.}{d}\right.\right.$$

$$\Big\downarrow \; 2620$$

$$-\frac{b\left(-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+\cdots\right.}{bd}\cdots}{2\sqrt{a^2+b^2}}\right)}{a}\cdots\right)}{a}\cdots\right)}{\cdots}$$

$$i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\cdots}{d}\right.}{d}\cdots\right.\right.$$

input
```
Int[((e + f*x)^2*Csch[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int\frac{(fx+e)^2\operatorname{csch}(dx+c)^3}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10341 vs. 2(676) = 1352.

Time = 0.35 (sec) , antiderivative size = 10341, normalized size of antiderivative = 14.26

$$\int \frac{(e+fx)^2 \mathrm{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \mathrm{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*csch(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^2 \mathrm{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2 \operatorname{csch}(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*e^2*(2*b^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c)
 - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3*d) - 2*(a*e^(-d*x - c) + 2*b
*e^(-2*d*x - 2*c) + a*e^(-3*d*x - 3*c) - 2*b)/((2*a^2*e^(-2*d*x - 2*c) - a
^2*e^(-4*d*x - 4*c) - a^2)*d) - (a^2 - 2*b^2)*log(e^(-d*x - c) + 1)/(a^3*d
) + (a^2 - 2*b^2)*log(e^(-d*x - c) - 1)/(a^3*d)) - (2*b*d*f^2*x^2 + 4*b*d*
e*f*x + (a*d*f^2*x^2*e^(3*c) + 2*a*e*f*e^(3*c) + 2*(d*e*f + f^2)*a*x*e^(3*
c))*e^(3*d*x) - 2*(b*d*f^2*x^2*e^(2*c) + 2*b*d*e*f*x*e^(2*c))*e^(2*d*x) +
(a*d*f^2*x^2*e^c - 2*a*e*f*e^c + 2*(d*e*f - f^2)*a*x*e^c)*e^(d*x))/(a^2*d^
2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2) + (2*b*d*e*f + a*
f^2)*x/(a^2*d^2) + (2*b*d*e*f - a*f^2)*x/(a^2*d^2) - (2*b*d*e*f + a*f^2)*l
og(e^(d*x + c) + 1)/(a^2*d^3) - (2*b*d*e*f - a*f^2)*log(e^(d*x + c) - 1)/(
a^2*d^3) + 1/2*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) -
 2*polylog(3, -e^(d*x + c)))*(a^2*f^2 - 2*b^2*f^2)/(a^3*d^3) - 1/2*(d^2*x^
2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x +
 c)))*(a^2*f^2 - 2*b^2*f^2)/(a^3*d^3) + (a^2*d*e*f - 2*b^2*d*e*f - 2*a*b*f
^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^3*d^3) - (a^2*d*e*
f - 2*b^2*d*e*f + 2*a*b*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c
)))/(a^3*d^3) + 1/6*((a^2*f^2 - 2*b^2*f^2)*d^3*x^3 + 3*(a^2*d*e*f - 2*b^2*
d*e*f + 2*a*b*f^2)*d^2*x^2)/(a^3*d^3) - 1/6*((a^2*f^2 - 2*b^2*f^2)*d^3*x^3
 + 3*(a^2*d*e*f - 2*b^2*d*e*f - 2*a*b*f^2)*d^2*x^2)/(a^3*d^3) - integra...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{{\left(e+f\,x\right)}^2}{{\mathrm{sinh}\left(c+d\,x\right)}^3\,\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)} \,d x$$

input
```
int((e + f*x)^2/(sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^2/(sinh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{too large to display}$$

input
```
int((f*x+e)^2*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
( - 4*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqr
t(a**2 + b**2))*b**4*d**2*e**2*i + 8*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*at
an((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*b**4*d**2*e**2*i - 4*sqrt(a
**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*b**4*d**2*e**
2*i - 64*e**(7*c + 4*d*x)*int((e**(3*d*x)*x**2)/(e**(8*c + 8*d*x)*b + 2*e*
*(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c
 + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)
*a + b),x)*a**6*d**3*f**2 - 64*e**(7*c + 4*d*x)*int((e**(3*d*x)*x**2)/(e**
(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c
+ 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d
*x)*b - 2*e**(c + d*x)*a + b),x)*a**4*b**2*d**3*f**2 - 128*e**(7*c + 4*d*x
)*int((e**(3*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*
c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3
*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**6*d**3*e*f -
128*e**(7*c + 4*d*x)*int((e**(3*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c +
7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x
)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),
x)*a**4*b**2*d**3*e*f + 32*e**(6*c + 4*d*x)*int((e**(2*d*x)*x**2)/(e**(8*c
 + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*
d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*...
```

3.250 $\int \frac{(e+fx)\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 420

$$\begin{aligned}\int \frac{(e+fx)\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{ad}-\frac{2b^2(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{a^3d} \\ &\quad+\frac{b(e+fx)\coth(c+dx)}{a^2d}-\frac{f\operatorname{csch}(c+dx)}{2ad^2} \\ &\quad-\frac{(e+fx)\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} \\ &\quad-\frac{b^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}d} \\ &\quad+\frac{b^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}d} \\ &\quad-\frac{bf\log(\sinh(c+dx))}{a^2d^2}+\frac{f\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{2ad^2} \\ &\quad-\frac{b^2f\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{a^3d^2}-\frac{f\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{2ad^2} \\ &\quad+\frac{b^2f\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{a^3d^2}-\frac{b^3f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}d^2} \\ &\quad+\frac{b^3f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}d^2}\end{aligned}$$

output
```
(f*x+e)*arctanh(exp(d*x+c))/a/d-2*b^2*(f*x+e)*arctanh(exp(d*x+c))/a^3/d+b*
(f*x+e)*coth(d*x+c)/a^2/d-1/2*f*csch(d*x+c)/a/d^2-1/2*(f*x+e)*coth(d*x+c)*
csch(d*x+c)/a/d-b^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/(a^
2+b^2)^(1/2)/d+b^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/(a^2
+b^2)^(1/2)/d-b*f*ln(sinh(d*x+c))/a^2/d^2+1/2*f*polylog(2,-exp(d*x+c))/a/d
^2-b^2*f*polylog(2,-exp(d*x+c))/a^3/d^2-1/2*f*polylog(2,exp(d*x+c))/a/d^2+
b^2*f*polylog(2,exp(d*x+c))/a^3/d^2-b^3*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+
b^2)^(1/2)))/a^3/(a^2+b^2)^(1/2)/d^2+b^3*f*polylog(2,-b*exp(d*x+c)/(a+(a^2
+b^2)^(1/2)))/a^3/(a^2+b^2)^(1/2)/d^2
```

Mathematica [A] (warning: unable to verify)

Time = 8.22 (sec) , antiderivative size = 617, normalized size of antiderivative = 1.47

$$\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{\left(2bde\cosh\left(\frac{1}{2}(c+dx)\right)-af\cosh\left(\frac{1}{2}(c+dx)\right)-2bcf\cosh\left(\frac{1}{2}(c+dx)\right)+2bf(c+dx)\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{4a^2d^2}$$
$$+\frac{(-de+cf-f(c+dx))\text{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8ad^2}$$
$$-\frac{2abf(c+dx)+(2abf+a^2(de+dfx)-2b^2(de+dfx))\log\left(1-e^{-c-dx}\right)+(2abf-a^2(de+dfx)+2b}{2a}$$
$$-\frac{b^3\left(-2de\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+2cf\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+f(c+dx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-f(c+dx)\log\right.}{a^3\sqrt{a^2+b^2}d^2}$$
$$+\frac{(-de+cf-f(c+dx))\text{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8ad^2}$$
$$+\frac{\text{sech}\left(\frac{1}{2}(c+dx)\right)\left(2bde\sinh\left(\frac{1}{2}(c+dx)\right)+af\sinh\left(\frac{1}{2}(c+dx)\right)-2bcf\sinh\left(\frac{1}{2}(c+dx)\right)+2bf(c+dx)\right.}{4a^2d^2}$$

input
```
Integrate[((e + f*x)*Csch[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
((2*b*d*e*Cosh[(c + d*x)/2] - a*f*Cosh[(c + d*x)/2] - 2*b*c*f*Cosh[(c + d*
x)/2] + 2*b*f*(c + d*x)*Cosh[(c + d*x)/2])*Csch[(c + d*x)/2])/(4*a^2*d^2)
+ ((-(d*e) + c*f - f*(c + d*x))*Csch[(c + d*x)/2]^2)/(8*a*d^2) - (2*a*b*f*
(c + d*x) + (2*a*b*f + a^2*(d*e + d*f*x) - 2*b^2*(d*e + d*f*x))*Log[1 - E^
(-c - d*x)] + (2*a*b*f - a^2*(d*e + d*f*x) + 2*b^2*(d*e + d*f*x))*Log[1 +
E^(-c - d*x)] + (a^2 - 2*b^2)*f*PolyLog[2, -E^(-c - d*x)] - (a^2 - 2*b^2)*
f*PolyLog[2, E^(-c - d*x)])/(2*a^3*d^2) - (b^3*(-2*d*e*ArcTanh[(a + b*E^(c
 + d*x))/Sqrt[a^2 + b^2]] + 2*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b
^2]] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - f*(c +
 d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + f*PolyLog[2, (b*E^(
c + d*x))/(-a + Sqrt[a^2 + b^2])] - f*PolyLog[2, -((b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2]))]))/(a^3*Sqrt[a^2 + b^2]*d^2) + ((-(d*e) + c*f - f*(c + d*x
))*Sech[(c + d*x)/2]^2)/(8*a*d^2) + (Sech[(c + d*x)/2]*(2*b*d*e*Sinh[(c +
d*x)/2] + a*f*Sinh[(c + d*x)/2] - 2*b*c*f*Sinh[(c + d*x)/2] + 2*b*f*(c + d
*x)*Sinh[(c + d*x)/2]))/(4*a^2*d^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6109}$$

$$\frac{\int (e+fx)\operatorname{csch}^3(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{b\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i(e+fx)\csc(ic+idx)^3dx}{a}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{b\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\int (e+fx)\csc(ic+idx)^3dx}{a}$$

$$\Big\downarrow \text{ 4673}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(\frac{1}{2}\int -i(e+fx)\text{csch}(c+dx)dx-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(-\frac{1}{2}i\int (e+fx)\text{csch}(c+dx)dx-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(-\frac{1}{2}i\int i(e+fx)\csc(ic+idx)dx-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(\frac{1}{2}\int (e+fx)\csc(ic+idx)dx-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{ 4670}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(\frac{1}{2}\left(\frac{if\int \log(1-e^{c+dx})dx}{d}-\frac{if\int \log(1+e^{c+dx})dx}{d}+\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{ 2715}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(\frac{1}{2}\left(\frac{if\int e^{-c-dx}\log(1-e^{c+dx})de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log(1+e^{c+dx})de^{c+dx}}{d^2}+\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{ 2838}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+a}{2d}}{a}$$

$$\Big\downarrow \textbf{6109}$$

$$-\frac{b\left(\frac{\int(e+fx)\text{csch}^2(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+a}{2d}}{a}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{b\left(-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int-\left((e+fx)\csc(ic+idx)^2\right)dx}{a}\right)}{a}-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+a}{2d}}{a}$$

$$\Big\downarrow \textbf{25}$$

$$-\frac{b\left(-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\int(e+fx)\csc(ic+idx)^2dx}{a}\right)}{a}-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+a}{2d}}{a}$$

$$\Big\downarrow \textbf{4672}$$

$$-\frac{b\left(-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{if\int-i\coth(c+dx)dx}{d}}{a}\right)}{a}-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+a}{2d}}{a}$$

$$\Big\downarrow \textbf{26}$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\int \coth(c+dx)dx}{d}}{a}\right)}{a}-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+d}{2d}\right.}{a}$$

$\big\downarrow$ 3042

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\int -i\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}\right)}{a}-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+d}{2d}\right.}{a}$$

$\big\downarrow$ 26

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}+\frac{if\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}}{a}\right)}{a}-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+d}{2d}\right.}{a}$$

$\big\downarrow$ 3956

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}\right)}{a}-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+d}{2d}\right.}{a}$$

$\big\downarrow$ 6109

$$-\frac{b\left(-\frac{b\left(\frac{\int (e+fx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}\right)}{a}-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+d}{2d}\right.}{a}$$

$\big\downarrow$ 3042

$$-\frac{b\left(-\frac{b\left(\frac{\int i(e+fx)\csc(ic+idx)dx}{a}-\frac{b\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{a}\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}\right)}{a}-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+a}{2d}\right.}{a}$$

$\Big\downarrow$ 26

$$-\frac{b\left(-\frac{b\left(\frac{i\int(e+fx)\csc(ic+idx)dx}{a}-\frac{b\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{a}\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}\right)}{a}-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+a}{2d}\right.}{a}$$

$\Big\downarrow$ 3803

$$-\frac{b\left(-\frac{b\left(-\frac{2b\int-\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int(e+fx)\csc(ic+idx)dx}{a}\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}\right)}{a}-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+a}{2d}\right.}{a}$$

$\Big\downarrow$ 25

$$-\frac{b\left(-\frac{b\left(\frac{2b\int\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int(e+fx)\csc(ic+idx)dx}{a}\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}\right)}{a}-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+a}{2d}\right.}{a}$$

$\Big\downarrow$ 2694

$$-\frac{b\left(-\frac{b\left(\frac{2b\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx}{a}\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c-}{d^2}}{a}\right.}{a}$$

$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+d}{2d}\right.}{a}$$

$\downarrow$ 27

$$-\frac{b\left(-\frac{b\left(\frac{2b\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx}{a}\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}\right)}{a}-$$

$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+d}{2d}\right.}{a}$$

$\downarrow$ 2620

$$-\frac{b\left(-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int(e+}{}\right)}{a}}{a}$$

$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+d}{2d}\right.}{a}$$

$\downarrow$ 2715

$$-\frac{b\left(-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}\cdots\right.}{bd^2}\cdots\right.}{2\sqrt{a^2+b^2}}\cdots\right.}{a}\cdots\right.}{a}\cdots\right.}{a}$$

$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\operatorname{arctanh}(e^{c+dx})}{d}+\frac{if\operatorname{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\operatorname{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\operatorname{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\operatorname{csch}(c+\cdots}{2d}\cdots\right.}{a}$$

$\downarrow$ 2838

$$-\frac{b\left(-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int(\cdots}{}\right)}{a}\cdots\right)}{a}$$

$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\operatorname{arctanh}(e^{c+dx})}{d}+\frac{if\operatorname{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\operatorname{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\operatorname{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\operatorname{csch}(c+a\cdots}{2d}\right.}{a}$$

$\downarrow$ 4670

$$-\frac{b\left(-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\left(\cdots\right.}{\cdots}\right.}{a}\right.}{a}$$

$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\operatorname{arctanh}(e^{c+dx})}{d}+\frac{if\,\mathrm{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\mathrm{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\operatorname{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\operatorname{csch}(c+\cdots}{2d}\right.}{a}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{b\left(b-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i(\cdots}{\cdots}\right)}{a}\cdots}{\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+\cdots}{2d}\cdots\right)}{a}}$$

input
```
Int[((e + f*x)*Csch[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 860 vs. $2(386) = 772$.

Time = 0.59 (sec) , antiderivative size = 861, normalized size of antiderivative = 2.05

method	result
risch	$-\frac{adfx\,e^{3dx+3c}+ade\,e^{3dx+3c}-2bdfx\,e^{2dx+2c}+adfx\,e^{dx+c}+af\,e^{3dx+3c}-2bde\,e^{2dx+2c}+ade\,e^{dx+c}+2bdfx-af\,e^{dx+c}+2bde}{d^2a^2\left(e^{2dx+2c}-1\right)^2}+\frac{2b^3\cdots}{\cdots}$

input
```
int((f*x+e)*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-(a*d*f*x*exp(3*d*x+3*c)+a*d*e*exp(3*d*x+3*c)-2*b*d*f*x*exp(2*d*x+2*c)+a*d
*f*x*exp(d*x+c)+a*f*exp(3*d*x+3*c)-2*b*d*e*exp(2*d*x+2*c)+a*d*e*exp(d*x+c)
+2*b*d*f*x-a*f*exp(d*x+c)+2*b*d*e)/d^2/a^2/(exp(2*d*x+2*c)-1)^2+2/d/a^3*b^
3*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d^
2/a^3*c*b^3*f/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(
1/2))-1/d^2/a^3*b^3*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)
/(-a+(a^2+b^2)^(1/2)))*c+1/2/a/d*e*ln(exp(d*x+c)+1)-1/2/a/d*e*ln(exp(d*x+c
)-1)+1/d^2/a^3*b^3*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(
a+(a^2+b^2)^(1/2)))*c-1/d/a^3*b^2*f*ln(exp(d*x+c)+1)*x-1/d^2/a^2*b*f*ln(ex
p(d*x+c)-1)-1/d^2/a^2*b*f*ln(exp(d*x+c)+1)-1/d^2/a^3*b^2*f*dilog(exp(d*x+c
)+1)-1/d^2/a^3*b^2*f*dilog(exp(d*x+c))+2/d^2/a^2*b*f*ln(exp(d*x+c))+1/2/a/
d^2*c*f*ln(exp(d*x+c)-1)+1/2/a/d*f*ln(exp(d*x+c)+1)*x-1/d/a^3*b^2*e*ln(exp
(d*x+c)+1)+1/d/a^3*b^2*e*ln(exp(d*x+c)-1)-1/d^2/a^3*c*b^2*f*ln(exp(d*x+c)-
1)-1/d^2/a^3*b^3*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)
/(-a+(a^2+b^2)^(1/2)))+1/d^2/a^3*b^3*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)
+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/2/d^2*f/a*dilog(exp(d*x+c)+1)+1
/2/d^2*f*dilog(exp(d*x+c))/a-1/d/a^3*b^3*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+
c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d/a^3*b^3*f/(a^2+b^2)^(1/2
)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4720 vs. $2(380) = 760$.

Time = 0.20 (sec) , antiderivative size = 4720, normalized size of antiderivative = 11.24

$$\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\,\text{csch}^3\,(c+dx)}{a+b\sinh\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*csch(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{csch}\,(dx+c)^3}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-(8*b^3*integrate(1/4*x*e^(d*x + c)/(a^3*b*e^(2*d*x + 2*c) + 2*a^4*e^(d*x
+ c) - a^3*b), x) + 8*a^2*d*integrate(1/16*x/(a^3*d*e^(d*x + c) + a^3*d),
x) - 16*b^2*d*integrate(1/16*x/(a^3*d*e^(d*x + c) + a^3*d), x) + 8*a^2*d*i
ntegrate(1/16*x/(a^3*d*e^(d*x + c) - a^3*d), x) - 16*b^2*d*integrate(1/16*
x/(a^3*d*e^(d*x + c) - a^3*d), x) - a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x
+ c) + 1)/(a^3*d^2)) - a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) - 1)/(a^
3*d^2)) - (2*b*d*x*e^(2*d*x + 2*c) - 2*b*d*x - (a*d*x*e^(3*c) + a*e^(3*c))
*e^(3*d*x) - (a*d*x*e^c - a*e^c)*e^(d*x))/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2
*d^2*e^(2*d*x + 2*c) + a^2*d^2))*f - 1/2*e*(2*b^3*log((b*e^(-d*x - c) - a
- sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2
)*a^3*d) - 2*(a*e^(-d*x - c) + 2*b*e^(-2*d*x - 2*c) + a*e^(-3*d*x - 3*c) -
 2*b)/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) - (a^2 - 2
*b^2)*log(e^(-d*x - c) + 1)/(a^3*d) + (a^2 - 2*b^2)*log(e^(-d*x - c) - 1)/
(a^3*d))
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{e+f\,x}{\sinh\left(c+d\,x\right)^3\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)/(sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(sinh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output

```
( - 4*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqr
t(a**2 + b**2))*b**4*d*e*i + 8*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e*
*(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*b**4*d*e*i - 4*sqrt(a**2 + b**2)*
atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*b**4*d*e*i - 64*e**(7*c +
 4*d*x)*int((e**(3*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*
e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3
*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**6*d**2*
f - 64*e**(7*c + 4*d*x)*int((e**(3*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c
 + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*
d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a +
b),x)*a**4*b**2*d**2*f + 32*e**(6*c + 4*d*x)*int((e**(2*d*x)*x)/(e**(8*c +
 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*
x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b
- 2*e**(c + d*x)*a + b),x)*a**5*b*d**2*f + 32*e**(6*c + 4*d*x)*int((e**(2*
d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b
- 6*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e
**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**3*b**3*d**2*f - 4*e**(4*c
+ 4*d*x)*log(e**(c + d*x) - 1)*a**5*f - e**(4*c + 4*d*x)*log(e**(c + d*x)
- 1)*a**4*b*d*e - 4*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**3*b**2*f + e
**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**2*b**3*d*e + 2*e**(4*c + 4*d*x...
```

3.251 $\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 113

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{(a^2-2b^2)\operatorname{arctanh}(\cosh(c+dx))}{2a^3d} + \frac{2b^3\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}d} + \frac{b\coth(c+dx)}{a^2d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}$$

output
```
1/2*(a^2-2*b^2)*arctanh(cosh(d*x+c))/a^3/d+2*b^3*arctanh((b-a*tanh(1/2*d*x
+1/2*c))/(a^2+b^2)^(1/2))/a^3/(a^2+b^2)^(1/2)/d+b*coth(d*x+c)/a^2/d-1/2*co
th(d*x+c)*csch(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 2.14 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.48

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{\frac{16b^3\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-4ab\coth\left(\frac{1}{2}(c+dx)\right)+a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)-4(a^2-2b^2)\log\left(\cosh\left(\frac{1}{2}(c+\right.\right.}{8a^3d}$$

input
```
Integrate[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/8*((16*b^3*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^
2 - b^2] - 4*a*b*Coth[(c + d*x)/2] + a^2*Csch[(c + d*x)/2]^2 - 4*(a^2 - 2*
b^2)*Log[Cosh[(c + d*x)/2]] + 4*(a^2 - 2*b^2)*Log[Sinh[(c + d*x)/2]] + a^2
*Sech[(c + d*x)/2]^2 - 4*a*b*Tanh[(c + d*x)/2])/(a^3*d)
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.22, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {3042, 26, 3281, 26, 3042, 25, 3534, 25, 3042, 26, 3480, 26, 3042, 26, 3139, 1083, 217, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int -\frac{i}{\sin(ic+idx)^3(a-ib\sin(ic+idx))}dx$$
$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{1}{\sin(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3281}$$

$$-i\left(\frac{\int -\frac{i\operatorname{csch}^2(c+dx)\left(b\sinh^2(c+dx)+a\sinh(c+dx)+2b\right)}{a+b\sinh(c+dx)}dx}{2a}-\frac{i\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{i\int \frac{\operatorname{csch}^2(c+dx)\left(b\sinh^2(c+dx)+a\sinh(c+dx)+2b\right)}{a+b\sinh(c+dx)}dx}{2a}-\frac{i\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{i\int -\frac{-b\sin(ic+idx)^2-ia\sin(ic+idx)+2b}{\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx}{2a}-\frac{i\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{25}$$

$$-i\left(\frac{i\int \frac{-b\sin(ic+idx)^2-ia\sin(ic+idx)+2b}{\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx}{2a}-\frac{i\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{3534}$$

$$-i\left(\frac{i\left(\frac{\int -\frac{\operatorname{csch}(c+dx)\left(a^2+b\sinh(c+dx)a-2b^2\right)}{a+b\sinh(c+dx)}dx}{a}+\frac{2b\coth(c+dx)}{ad}\right)}{2a}-\frac{i\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{25}$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}-\frac{\int \frac{\operatorname{csch}(c+dx)\left(a^2+b\sinh(c+dx)a-2b^2\right)}{a+b\sinh(c+dx)}dx}{a}\right)}{2a}-\frac{i\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}-\frac{\int\frac{i\left(a^2-ib\sin(ic+idx)a-2b^2\right)}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$\downarrow$ 26

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}-\frac{i\int\frac{a^2-ib\sin(ic+idx)a-2b^2}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$\downarrow$ 3480

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}-\frac{i\left(\frac{\left(a^2-2b^2\right)\int-i\text{csch}(c+dx)dx}{a}-\frac{2ib^3\int\frac{1}{a+b\sinh(c+dx)}dx}{a}\right)}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$\downarrow$ 26

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}-\frac{i\left(-\frac{i\left(a^2-2b^2\right)\int\text{csch}(c+dx)dx}{a}-\frac{2ib^3\int\frac{1}{a+b\sinh(c+dx)}dx}{a}\right)}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$\downarrow$ 3042

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}-\frac{i\left(-\frac{i\left(a^2-2b^2\right)\int i\csc(ic+idx)dx}{a}-\frac{2ib^3\int\frac{1}{a-ib\sin(ic+idx)}dx}{a}\right)}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}-\frac{i\left(\frac{(a^2-2b^2)\int\csc(ic+idx)dx}{a}-\frac{2ib^3\int\frac{1}{a-ib\sin(ic+idx)}dx}{a}\right)}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{3139}$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}-\frac{i\left(\frac{(a^2-2b^2)\int\csc(ic+idx)dx}{a}-\frac{4b^3\int\frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{ad}\right)}{a}\right)}{2a}-\frac{i\coth(c+\right.$$

$$\Big\downarrow \text{1083}$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}-\frac{i\left(\frac{(a^2-2b^2)\int\csc(ic+idx)dx}{a}+\frac{8b^3\int\frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4(a^2+b^2)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{ad}\right)}{a}\right)}{2a}-\frac{i\coth(c+dx)c}{2ac}\right.$$

$$\Big\downarrow \text{217}$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}-\frac{i\left(\frac{(a^2-2b^2)\int\csc(ic+idx)dx}{a}-\frac{4ib^3\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}\right)}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$\downarrow$ 4257

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}-\frac{i\left(\frac{i(a^2-2b^2)\text{arctanh}(\cosh(c+dx))}{ad}-\frac{4ib^3\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}\right)}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

input
```
Int[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

output
```
(-I)*(((I/2)*(((-I)*((I*(a^2 - 2*b^2)*ArcTanh[Cosh[c + d*x]])/(a*d) - ((4*
I)*b^3*ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*
d)))/a + (2*b*Coth[c + d*x])/(a*d)))/a - ((I/2)*Coth[c + d*x]*Csch[c + d*x
])/(a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{2b^3\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(-2a^2+4b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}}{d}$
default	$\frac{\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{2b^3\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(-2a^2+4b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}}{d}$
risch	$-\frac{e^{3dx+3c}a-2b\,e^{2dx+2c}+a\,e^{dx+c}+2b}{d\,a^2\left(e^{2dx+2c}-1\right)^2}+\frac{b^3\ln\left(e^{dx+c}+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d\,a^3}-\frac{b^3\ln\left(e^{dx+c}+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d\,a^3}+$

input
```
int(csch(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4/a^2*(1/2*tanh(1/2*d*x+1/2*c)^2*a+2*b*tanh(1/2*d*x+1/2*c))-2/a^3*b
^3/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/
2))-1/8/a/tanh(1/2*d*x+1/2*c)^2+1/4/a^3*(-2*a^2+4*b^2)*ln(tanh(1/2*d*x+1/2
*c))+1/2/a^2*b/tanh(1/2*d*x+1/2*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1203 vs. $2(106) = 212$.

Time = 0.16 (sec) , antiderivative size = 1203, normalized size of antiderivative = 10.65

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(4*a^3*b + 4*a*b^3 + 2*(a^4 + a^2*b^2)*cosh(d*x + c)^3 + 2*(a^4 + a^2
*b^2)*sinh(d*x + c)^3 - 4*(a^3*b + a*b^3)*cosh(d*x + c)^2 - 2*(2*a^3*b + 2
*a*b^3 - 3*(a^4 + a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(b^3*cosh(d*
x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*sinh(d*x + c)^4 - 2*b
^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x + c)^2
 + 4*(b^3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b
^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) +
 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^
2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*
x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b))
 + 2*(a^4 + a^2*b^2)*cosh(d*x + c) - ((a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c
)^4 + 4*(a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 - a^2
*b^2 - 2*b^4)*sinh(d*x + c)^4 + a^4 - a^2*b^2 - 2*b^4 - 2*(a^4 - a^2*b^2 -
 2*b^4)*cosh(d*x + c)^2 - 2*(a^4 - a^2*b^2 - 2*b^4 - 3*(a^4 - a^2*b^2 - 2*
b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^4 - a^2*b^2 - 2*b^4)*cosh(d*
x + c)^3 - (a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(
d*x + c) + sinh(d*x + c) + 1) + ((a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)^4 +
 4*(a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 - a^2*b^2
- 2*b^4)*sinh(d*x + c)^4 + a^4 - a^2*b^2 - 2*b^4 - 2*(a^4 - a^2*b^2 - 2*b^
4)*cosh(d*x + c)^2 - 2*(a^4 - a^2*b^2 - 2*b^4 - 3*(a^4 - a^2*b^2 - 2*b^...
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\operatorname{csch}^3{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate(csch(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(csch(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.87

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b^3\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a^3d} + \frac{ae^{(-dx-c)}+2\,be^{(-2\,dx-2\,c)}+ae^{(-3\,dx-3\,c)}-2\,b}{(2\,a^2e^{(-2\,dx-2\,c)}-a^2e^{(-4\,dx-4\,c)}-a^2)d} + \frac{(a^2-2\,b^2)\log\left(e^{(-dx-c)}+1\right)}{2\,a^3d} - \frac{(a^2-2\,b^2)\log\left(e^{(-dx-c)}-1\right)}{2\,a^3d}$$

input
```
integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-b^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt
(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3*d) + (a*e^(-d*x - c) + 2*b*e^(-2*d*x -
2*c) + a*e^(-3*d*x - 3*c) - 2*b)/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x
- 4*c) - a^2)*d) + 1/2*(a^2 - 2*b^2)*log(e^(-d*x - c) + 1)/(a^3*d) - 1/2*(
a^2 - 2*b^2)*log(e^(-d*x - c) - 1)/(a^3*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.56

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{2\,b^3\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^3} - \frac{(a^2-2\,b^2)\log\left(e^{(dx+c)}+1\right)}{a^3} + \frac{(a^2-2\,b^2)\log\left(\left|e^{(dx+c)}-1\right|\right)}{a^3} + \frac{2\,(ae^{(3\,dx+3\,c)}-2\,be^{(2\,dx+2\,c)}+ae^{(}}{a^2\left(e^{(2\,dx+2\,c)}-1\right)^2}}{2\,d}$$

input
```
integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(2*b^3*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(
d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3) - (a^2 - 2*b^2)
*log(e^(d*x + c) + 1)/a^3 + (a^2 - 2*b^2)*log(abs(e^(d*x + c) - 1))/a^3 +
2*(a*e^(3*d*x + 3*c) - 2*b*e^(2*d*x + 2*c) + a*e^(d*x + c) + 2*b)/(a^2*(e^
(2*d*x + 2*c) - 1)^2))/d
```

Mupad [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 776, normalized size of antiderivative = 6.87

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
exp(c + d*x)/(a*d - a*d*exp(2*c + 2*d*x)) - (2*exp(c + d*x))/(a*d - 2*a*d*
exp(2*c + 2*d*x) + a*d*exp(4*c + 4*d*x)) - (2*b)/(a^2*d - a^2*d*exp(2*c +
2*d*x)) - log(4*a^4 + 24*b^4 - 20*a^2*b^2 - 4*a^4*exp(d*x)*exp(c) - 24*b^4
*exp(d*x)*exp(c) + 20*a^2*b^2*exp(d*x)*exp(c))/(2*a*d) + log(4*a^4 + 24*b^
4 - 20*a^2*b^2 + 4*a^4*exp(d*x)*exp(c) + 24*b^4*exp(d*x)*exp(c) - 20*a^2*b
^2*exp(d*x)*exp(c))/(2*a*d) - (b^3*log(16*a^5*b - 48*a*b^5 - 24*b^5*(a^2 +
 b^2)^(1/2) - 32*a^3*b^3 - 40*a^2*b^3*(a^2 + b^2)^(1/2) - 32*a^6*exp(d*x)*
exp(c) + 24*b^6*exp(d*x)*exp(c) + 16*a^4*b*(a^2 + b^2)^(1/2) + 112*a^2*b^4
*exp(d*x)*exp(c) + 56*a^4*b^2*exp(d*x)*exp(c) - 32*a^5*exp(d*x)*exp(c)*(a^
2 + b^2)^(1/2) + 72*a*b^4*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2) + 72*a^3*b^2*e
xp(d*x)*exp(c)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a^5*d + a^3*b^2*d) +
 (b^3*log(24*b^5*(a^2 + b^2)^(1/2) - 48*a*b^5 + 16*a^5*b - 32*a^3*b^3 + 40
*a^2*b^3*(a^2 + b^2)^(1/2) - 32*a^6*exp(d*x)*exp(c) + 24*b^6*exp(d*x)*exp(
c) - 16*a^4*b*(a^2 + b^2)^(1/2) + 112*a^2*b^4*exp(d*x)*exp(c) + 56*a^4*b^2
*exp(d*x)*exp(c) + 32*a^5*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2) - 72*a*b^4*exp
(d*x)*exp(c)*(a^2 + b^2)^(1/2) - 72*a^3*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^(1
/2))*(a^2 + b^2)^(1/2))/(a^5*d + a^3*b^2*d) + (b^2*log(4*a^4 + 24*b^4 - 20
*a^2*b^2 - 4*a^4*exp(d*x)*exp(c) - 24*b^4*exp(d*x)*exp(c) + 20*a^2*b^2*exp
(d*x)*exp(c)))/(a^3*d) - (b^2*log(4*a^4 + 24*b^4 - 20*a^2*b^2 + 4*a^4*exp(
d*x)*exp(c) + 24*b^4*exp(d*x)*exp(c) - 20*a^2*b^2*exp(d*x)*exp(c)))/(a^...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 724, normalized size of antiderivative = 6.41

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-e^{4dx+4c}\log\left(e^{dx+c}-1\right)a^4+e^{4dx+4c}\log\left(e^{dx+c}-1\right)a^2b^2+e^{4dx+4c}\log\left(e^{dx+c}+1\right)a^4-e^{4dx+4c}\log\left(e^{dx+c}+\right.}{}$$

input
```
int(csch(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
( - 4*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqr
t(a**2 + b**2))*b**3*i + 8*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c
+ d*x)*b*i + a*i)/sqrt(a**2 + b**2))*b**3*i - 4*sqrt(a**2 + b**2)*atan((e*
*(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*b**3*i - e**(4*c + 4*d*x)*log(e**
(c + d*x) - 1)*a**4 + e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**2*b**2 + 2
*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*b**4 + e**(4*c + 4*d*x)*log(e**(c
+ d*x) + 1)*a**4 - e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a**2*b**2 - 2*e*
*(4*c + 4*d*x)*log(e**(c + d*x) + 1)*b**4 + 2*e**(4*c + 4*d*x)*a**3*b + 2*
e**(4*c + 4*d*x)*a*b**3 - 2*e**(3*c + 3*d*x)*a**4 - 2*e**(3*c + 3*d*x)*a**
2*b**2 + 2*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a**4 - 2*e**(2*c + 2*d*x
)*log(e**(c + d*x) - 1)*a**2*b**2 - 4*e**(2*c + 2*d*x)*log(e**(c + d*x) -
1)*b**4 - 2*e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*a**4 + 2*e**(2*c + 2*d*
x)*log(e**(c + d*x) + 1)*a**2*b**2 + 4*e**(2*c + 2*d*x)*log(e**(c + d*x) +
 1)*b**4 - 2*e**(c + d*x)*a**4 - 2*e**(c + d*x)*a**2*b**2 - log(e**(c + d*
x) - 1)*a**4 + log(e**(c + d*x) - 1)*a**2*b**2 + 2*log(e**(c + d*x) - 1)*b
**4 + log(e**(c + d*x) + 1)*a**4 - log(e**(c + d*x) + 1)*a**2*b**2 - 2*log
(e**(c + d*x) + 1)*b**4 - 2*a**3*b - 2*a*b**3)/(2*a**3*d*(e**(4*c + 4*d*x)
*a**2 + e**(4*c + 4*d*x)*b**2 - 2*e**(2*c + 2*d*x)*a**2 - 2*e**(2*c + 2*d*
x)*b**2 + a**2 + b**2))
```

3.252 $\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx = \operatorname{Int}\left(\frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(csch(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 72.76 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx = \int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx$$

input
```
Integrate[Csch[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Csch[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx$$
$$\Big\downarrow \text{ 6111}$$
$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Csch[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(dx+c)^3}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(csch(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(csch(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 19.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}^3\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

input
```
integrate(csch(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(csch(c + d*x)**3/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Maxima [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 757, normalized size of antiderivative = 27.04

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-8*b^3*integrate(-1/4*e^(d*x + c)/(a^3*b*f*x + a^3*b*e - (a^3*b*f*x*e^(2*c
) + a^3*b*e*e^(2*c))*e^(2*d*x) - 2*(a^4*f*x*e^c + a^4*e*e^c)*e^(d*x)), x)
- (2*b*d*f*x + 2*b*d*e + (a*d*f*x*e^(3*c) + (d*e - f)*a*e^(3*c))*e^(3*d*x)
 - 2*(b*d*f*x*e^(2*c) + b*d*e*e^(2*c))*e^(2*d*x) + (a*d*f*x*e^c + (d*e + f
)*a*e^c)*e^(d*x))/(a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + a^2*d^2*e^2 + (a^2*
d^2*f^2*x^2*e^(4*c) + 2*a^2*d^2*e*f*x*e^(4*c) + a^2*d^2*e^2*e^(4*c))*e^(4*
d*x) - 2*(a^2*d^2*f^2*x^2*e^(2*c) + 2*a^2*d^2*e*f*x*e^(2*c) + a^2*d^2*e^2*
e^(2*c))*e^(2*d*x)) - 8*integrate(1/16*(2*b^2*d^2*e^2 + 2*a*b*d*e*f - (d^2
*e^2 - 2*f^2)*a^2 - (a^2*d^2*f^2 - 2*b^2*d^2*f^2)*x^2 - 2*(a^2*d^2*e*f - 2
*b^2*d^2*e*f - a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^
3*d^2*e^2*f*x + a^3*d^2*e^3 - (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e
^c + 3*a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)*e^(d*x)), x) - 8*integrate(-
1/16*(2*b^2*d^2*e^2 - 2*a*b*d*e*f - (d^2*e^2 - 2*f^2)*a^2 - (a^2*d^2*f^2 -
 2*b^2*d^2*f^2)*x^2 - 2*(a^2*d^2*e*f - 2*b^2*d^2*e*f + a*b*d*f^2)*x)/(a^3*
d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*e^3 + (a^3
*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c + a^3*d
^2*e^3*e^c)*e^(d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{1}{\sinh\left(c+d\,x\right)^3\,\left(e+f\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/(sinh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(sinh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.43

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$=\int \frac{\operatorname{csch}(dx+c)^3}{\sinh(dx+c)\,be+\sinh(dx+c)\,bfx+ae+afx}dx$$

input
```
int(csch(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(csch(c + d*x)**3/(sinh(c + d*x)*b*e + sinh(c + d*x)*b*f*x + a*e + a*f*
x),x)
```

3.253 $\int \frac{(e+fx)^3\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 139

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{i(e+fx)^4}{4af}-\frac{2i(e+fx)^3\log\left(1+ie^{c+dx}\right)}{ad} - \frac{6if(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^2} + \frac{12if^2(e+fx)\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^3} - \frac{12if^3\operatorname{PolyLog}\left(4,-ie^{c+dx}\right)}{ad^4}$$

output
```
1/4*I*(f*x+e)^4/a/f-2*I*(f*x+e)^3*ln(1+I*exp(d*x+c))/a/d-6*I*f*(f*x+e)^2*p
olylog(2,-I*exp(d*x+c))/a/d^2+12*I*f^2*(f*x+e)*polylog(3,-I*exp(d*x+c))/a/
d^3-12*I*f^3*polylog(4,-I*exp(d*x+c))/a/d^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.85

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$=\frac{i\left(\frac{(e+fx)^4}{f}-\frac{8(e+fx)^3\log(1+ie^{c+dx})}{d}-\frac{24f\left(d^2(e+fx)^2\operatorname{PolyLog}(2,-ie^{c+dx})-2df(e+fx)\operatorname{PolyLog}(3,-ie^{c+dx})+2f^2\operatorname{PolyLog}(4,-ie^{c+dx})\right)}{d^4}\right)}{4a}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((I/4)*((e + f*x)^4/f - (8*(e + f*x)^3*Log[1 + I*E^(c + d*x)])/d - (24*f*(
d^2*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)] - 2*d*f*(e + f*x)*PolyLog[3,
(-I)*E^(c + d*x)] + 2*f^2*PolyLog[4, (-I)*E^(c + d*x)]))/d^4))/a
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {6093, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6093}$$

$$2\int \frac{e^{c+dx}(e+fx)^3}{ie^{c+dx}a+a}dx+\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \text{2620}$$

$$2\left(\frac{3if\int (e+fx)^2\log\left(1+ie^{c+dx}\right)dx}{ad}-\frac{i(e+fx)^3\log\left(1+ie^{c+dx}\right)}{ad}\right)+\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \text{3011}$$

$$2\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}(2,-ie^{c+dx})dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,-ie^{c+dx})}{d}\right)}{ad}-\frac{i(e+fx)^3\log\left(1+ie^{c+dx}\right)}{ad}\right)+\frac{i(e+fx)^4}{4af}$$

$\big\downarrow$ 7163

$$2\left(\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d}-\frac{f\int\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,-ie^{c+dx})}{d}\right)}{ad}-\frac{i(e+fx)^3\log\left(1+ie^{c+dx}\right)}{ad}\right)+\frac{i(e+fx)^4}{4af}$$

$\big\downarrow$ 2720

$$2\left(\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,-ie^{c+dx})}{d}\right)}{ad}-\frac{i(e+fx)^3\log\left(1+ie^{c+dx}\right)}{ad}\right)+\frac{i(e+fx)^4}{4af}$$

$\big\downarrow$ 7143

$$2\left(\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-ie^{c+dx}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,-ie^{c+dx})}{d}\right)}{ad}-\frac{i(e+fx)^3\log\left(1+ie^{c+dx}\right)}{ad}\right)+\frac{i(e+fx)^4}{4af}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((I/4)*(e + f*x)^4)/(a*f) + 2*(((-I)*(e + f*x)^3*Log[1 + I*E^(c + d*x)])/(
a*d) + ((3*I)*f*(-(((e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/d) + (2*f*((
(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/d - (f*PolyLog[4, (-I)*E^(c + d*x)
])/d^2))/d))/(a*d))
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6093
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + Simp[2   Int[(e + f*x)^m*(E^(c + d*x)/(a + b*E^(c + d*x))), x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[a^2 + b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 646 vs. $2(124) = 248$.

Time = 4.70 (sec) , antiderivative size = 647, normalized size of antiderivative = 4.65

method	result
risch	$-\frac{2i\ln\left(e^{dx+c}-i\right)e^{3}}{da}+\frac{2i\ln\left(e^{dx+c}\right)e^{3}}{da}+\frac{3if^{3}c^{4}}{2d^{4}a}+\frac{if^{2}e\,x^{3}}{a}+\frac{3if\,e^{2}x^{2}}{2a}+\frac{if^{3}x^{4}}{4a}-\frac{ie^{3}x}{a}-\frac{ie^{4}}{4af}-\frac{12if^{3}\,\mathrm{polylog}(4,-ie^{d}}{a\,d^{4}}$

input
```
int((f*x+e)^3*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
I/a*f^2*e*x^3-12*I*f^3*polylog(4,-I*exp(d*x+c))/a/d^4+3/2*I/a*f*e^2*x^2-2*
I/d/a*ln(exp(d*x+c)-I)*e^3+2*I/d/a*ln(exp(d*x+c))*e^3+3/2*I/d^4/a*f^3*c^4+
1/4*I/a*f^3*x^4+2*I/d^3/a*f^3*c^3*x-2*I/d/a*f^3*ln(1+I*exp(d*x+c))*x^3-2*I
/d^4/a*f^3*ln(1+I*exp(d*x+c))*c^3-6*I/d^2/a*f^3*polylog(2,-I*exp(d*x+c))*x
^2+12*I/d^3/a*f^3*polylog(3,-I*exp(d*x+c))*x-4*I/d^3/a*f^2*e*c^3+12*I/d^3/
a*f^2*e*polylog(3,-I*exp(d*x+c))+3*I/d^2/a*f*e^2*c^2-6*I/d^2/a*f*e^2*polyl
og(2,-I*exp(d*x+c))+2*I/d^4/a*c^3*f^3*ln(exp(d*x+c)-I)-2*I/d^4/a*c^3*f^3*l
n(exp(d*x+c))-6*I/d^2/a*f^2*e*c^2*x-6*I/d/a*f^2*e*ln(1+I*exp(d*x+c))*x^2+6
*I/d^3/a*f^2*e*ln(1+I*exp(d*x+c))*c^2-12*I/d^2/a*f^2*e*polylog(2,-I*exp(d*
x+c))*x+6*I/d/a*f*e^2*c*x-6*I/d/a*f*e^2*ln(1+I*exp(d*x+c))*x-6*I/d^2/a*f*e
^2*ln(1+I*exp(d*x+c))*c-6*I/d^3/a*c^2*f^2*e*ln(exp(d*x+c)-I)+6*I/d^3/a*c^2
*f^2*e*ln(exp(d*x+c))+6*I/d^2/a*c*f*e^2*ln(exp(d*x+c)-I)-6*I/d^2/a*c*f*e^2
*ln(exp(d*x+c))-I/a*e^3*x-1/4*I/a/f*e^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(114) = 228.

Time = 0.08 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.15

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{i\,d^4f^3x^4+4i\,d^4ef^2x^3+6i\,d^4e^2fx^2+4i\,d^4e^3x+8i\,cd^3e^3-12i\,c^2d^2e^2f+8i\,c^3def^2-2i\,c^4f^3-48i\,f^3\mathrm{pol}}{}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(I*d^4*f^3*x^4 + 4*I*d^4*e*f^2*x^3 + 6*I*d^4*e^2*f*x^2 + 4*I*d^4*e^3*x
 + 8*I*c*d^3*e^3 - 12*I*c^2*d^2*e^2*f + 8*I*c^3*d*e*f^2 - 2*I*c^4*f^3 - 48
*I*f^3*polylog(4, -I*e^(d*x + c)) - 24*(I*d^2*f^3*x^2 + 2*I*d^2*e*f^2*x +
I*d^2*e^2*f)*dilog(-I*e^(d*x + c)) - 8*(I*d^3*e^3 - 3*I*c*d^2*e^2*f + 3*I*
c^2*d*e*f^2 - I*c^3*f^3)*log(e^(d*x + c) - I) - 8*(I*d^3*f^3*x^3 + 3*I*d^3
*e*f^2*x^2 + 3*I*d^3*e^2*f*x + 3*I*c*d^2*e^2*f - 3*I*c^2*d*e*f^2 + I*c^3*f
^3)*log(I*e^(d*x + c) + 1) - 48*(-I*d*f^3*x - I*d*e*f^2)*polylog(3, -I*e^(
d*x + c)))/(a*d^4)
```

Sympy [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx =$$
$$-\frac{i\left(\int \frac{e^3\cosh(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{f^3x^3\cosh(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{3ef^2x^2\cosh(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{3e^2fx\cosh(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e**3*cosh(c + d*x)/(sinh(c + d*x) - I), x) + Integral(f**3*x**
*3*cosh(c + d*x)/(sinh(c + d*x) - I), x) + Integral(3*e*f**2*x**2*cosh(c +
 d*x)/(sinh(c + d*x) - I), x) + Integral(3*e**2*f*x*cosh(c + d*x)/(sinh(c
+ d*x) - I), x))/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(114) = 228$.

Time = 0.16 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.90

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\,e^3\log\left(i\,a\sinh\left(dx+c\right)+a\right)}{ad} - \frac{6i\left(dx\log\left(i\,e^{(dx+c)}+1\right)+\mathrm{Li}_2(-i\,e^{(dx+c)})\right)e^2f}{ad^2} - \frac{i\left(f^3x^4+4\,ef^2x^3+6\,e^2fx^2\right)}{4\,a} - \frac{6i\left(d^2x^2\log\left(i\,e^{(dx+c)}+1\right)+2\,dx\mathrm{Li}_2(-i\,e^{(dx+c)})-2\,\mathrm{Li}_3(-i\,e^{(dx+c)})\right)ef^2}{ad^3} - \frac{2i\left(d^3x^3\log\left(i\,e^{(dx+c)}+1\right)+3\,d^2x^2\mathrm{Li}_2(-i\,e^{(dx+c)})-6\,dx\mathrm{Li}_3(-i\,e^{(dx+c)})+6\,\mathrm{Li}_4(-i\,e^{(dx+c)})\right)f^3}{ad^4} + \frac{i\,d^4f^3x^4+4i\,d^4ef^2x^3+6i\,d^4e^2fx^2}{2\,ad^4}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-I*e^3*log(I*a*sinh(d*x + c) + a)/(a*d) - 6*I*(d*x*log(I*e^(d*x + c) + 1)
+ dilog(-I*e^(d*x + c)))*e^2*f/(a*d^2) - 1/4*I*(f^3*x^4 + 4*e*f^2*x^3 + 6*
e^2*f*x^2)/a - 6*I*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-I*e^(d*x
 + c)) - 2*polylog(3, -I*e^(d*x + c)))*e*f^2/(a*d^3) - 2*I*(d^3*x^3*log(I*
e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-I*e^(d*x + c)) - 6*d*x*polylog(3, -I*e
^(d*x + c)) + 6*polylog(4, -I*e^(d*x + c)))*f^3/(a*d^4) + 1/2*(I*d^4*f^3*x
^4 + 4*I*d^4*e*f^2*x^3 + 6*I*d^4*e^2*f*x^2)/(a*d^4)
```

Giac [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh\left(dx+c\right)}{i\,a\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,\left(e+f\,x\right)^3}{a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i),x)
```

output
```
int((cosh(c + d*x)*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(\int \frac{\cosh(dx+c)x^3}{\sinh(dx+c)i+1}dx\right)d\,f^3+3\left(\int \frac{\cosh(dx+c)x^2}{\sinh(dx+c)i+1}dx\right)de\,f^2+3\left(\int \frac{\cosh(dx+c)x}{\sinh(dx+c)i+1}dx\right)d\,e^2 f-\log(\sinh\left(dx+c\right)i+}{ad}$$

input
```
int((f*x+e)^3*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
(int((cosh(c + d*x)*x**3)/(sinh(c + d*x)*i + 1),x)*d*f**3 + 3*int((cosh(c
+ d*x)*x**2)/(sinh(c + d*x)*i + 1),x)*d*e*f**2 + 3*int((cosh(c + d*x)*x)/(
sinh(c + d*x)*i + 1),x)*d*e**2*f - log(sinh(c + d*x)*i + 1)*e**3*i)/(a*d)
```

3.254 $\int \frac{(e+fx)^2\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 106

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{i(e+fx)^3}{3af} - \frac{2i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad} - \frac{4if(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^2} + \frac{4if^2\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^3}$$

output
```
1/3*I*(f*x+e)^3/a/f-2*I*(f*x+e)^2*ln(1+I*exp(d*x+c))/a/d-4*I*f*(f*x+e)*pol
ylog(2,-I*exp(d*x+c))/a/d^2+4*I*f^2*polylog(3,-I*exp(d*x+c))/a/d^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{i\left(d^2(e+fx)^2\left(d(e+fx)-6f\log\left(1+ie^{c+dx}\right)\right)-12df^2(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)+12f^3\operatorname{PolyLog}\right.}{3ad^3f}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((I/3)*(d^2*(e + f*x)^2*(d*(e + f*x) - 6*f*Log[1 + I*E^(c + d*x)]) - 12*d*
f^2*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)] + 12*f^3*PolyLog[3, (-I)*E^(c +
 d*x)]))/(a*d^3*f)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {6093, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6093}$$

$$2\int \frac{e^{c+dx}(e+fx)^2}{ie^{c+dx}a+a}dx+\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{2620}$$

$$2\left(\frac{2if\int (e+fx)\log\left(1+ie^{c+dx}\right)dx}{ad}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad}\right)+\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{3011}$$

$$2\left(\frac{2if\left(\frac{f\int \text{PolyLog}(2,-ie^{c+dx})dx}{d}-\frac{(e+fx)\text{PolyLog}(2,-ie^{c+dx})}{d}\right)}{ad}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad}\right)+\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{2720}$$

$$2\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}(2,-ie^{c+dx})de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}(2,-ie^{c+dx})}{d}\right)}{ad}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad}\right)+\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{7143}$$

$$2\left(\frac{2if\left(\frac{f\,\mathrm{PolyLog}(3,-ie^{c+dx})}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}(2,-ie^{c+dx})}{d}\right)}{ad}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad}\right)+\frac{i(e+fx)^3}{3af}$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((I/3)*(e + f*x)^3)/(a*f) + 2*(((-I)*(e + f*x)^2*Log[1 + I*E^(c + d*x)])/(
a*d) + ((2*I)*f*(-(((e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d) + (f*PolyLo
g[3, (-I)*E^(c + d*x)])/d^2))/(a*d))
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6093
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + Simp[2   Int[(e + f*x)^m*(E^(c + d*x)/(a + b*E^(c + d*x))), x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[a^2 + b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(94) = 188$.

Time = 1.74 (sec) , antiderivative size = 405, normalized size of antiderivative = 3.82

method	result
risch	$\frac{2 i f^{2} c^{2} \ln \left({\mathrm e}^{d x +c}\right)}{d^{3} a}-\frac{4 i e f \ln \left(1+i {\mathrm e}^{d x +c}\right) c}{d^{2} a}+\frac{i f^{2} x^{3}}{3 a}+\frac{2 i f^{2} \ln \left(1+i {\mathrm e}^{d x +c}\right) c^{2}}{d^{3} a}-\frac{i e^{2} x}{a}-\frac{2 i f^{2} c^{2} x}{d^{2} a}-\frac{4 i e f c \ln \left({\mathrm e}^{d x +c}\right)}{d^{2} a}-\frac{2 i f^{2}}{}$

input
```
int((f*x+e)^2*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*I/d^3/a*f^2*c^2*ln(exp(d*x+c))-4*I/d^2/a*e*f*ln(1+I*exp(d*x+c))*c+1/3*I/
a*f^2*x^3+2*I/d^3/a*f^2*ln(1+I*exp(d*x+c))*c^2-I/a*e^2*x-2*I/d^2/a*f^2*c^2
*x-4*I/d^2/a*e*f*c*ln(exp(d*x+c))-2*I/d/a*f^2*ln(1+I*exp(d*x+c))*x^2+4*I/d
/a*e*f*c*x+4*I*f^2*polylog(3,-I*exp(d*x+c))/a/d^3+2*I/d^2/a*e*f*c^2-4/3*I/
d^3/a*f^2*c^3+2*I/d/a*ln(exp(d*x+c))*e^2+I/a*f*e*x^2-1/3*I/a/f*e^3+4*I/d^2
/a*e*f*c*ln(exp(d*x+c)-I)-4*I/d^2/a*f^2*polylog(2,-I*exp(d*x+c))*x-4*I/d/a
*e*f*ln(1+I*exp(d*x+c))*x-2*I/d/a*ln(exp(d*x+c)-I)*e^2-2*I/d^3/a*f^2*c^2*l
n(exp(d*x+c)-I)-4*I/d^2/a*e*f*polylog(2,-I*exp(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(86) = 172$.

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.74

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{i\,d^3f^2x^3 + 3i\,d^3efx^2 + 3i\,d^3e^2x + 6i\,cd^2e^2 - 6i\,c^2def + 2i\,c^3f^2 + 12i\,f^2\mathrm{polylog}\left(3, -i\,e^{(dx+c)}\right) - 12\,(i\,df^2$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/3*(I*d^3*f^2*x^3 + 3*I*d^3*e*f*x^2 + 3*I*d^3*e^2*x + 6*I*c*d^2*e^2 - 6*I
*c^2*d*e*f + 2*I*c^3*f^2 + 12*I*f^2*polylog(3, -I*e^(d*x + c)) - 12*(I*d*f
^2*x + I*d*e*f)*dilog(-I*e^(d*x + c)) - 6*(I*d^2*e^2 - 2*I*c*d*e*f + I*c^2
*f^2)*log(e^(d*x + c) - I) - 6*(I*d^2*f^2*x^2 + 2*I*d^2*e*f*x + 2*I*c*d*e*
f - I*c^2*f^2)*log(I*e^(d*x + c) + 1))/(a*d^3)
```

Sympy [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= -\frac{i\left(\int \frac{e^2\cosh(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{f^2x^2\cosh(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{2efx\cosh(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e**2*cosh(c + d*x)/(sinh(c + d*x) - I), x) + Integral(f**2*x**
*2*cosh(c + d*x)/(sinh(c + d*x) - I), x) + Integral(2*e*f*x*cosh(c + d*x)/
(sinh(c + d*x) - I), x))/a
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.55

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\,e^2\log\left(i\,a\sinh\left(dx+c\right)+a\right)}{ad} - \frac{i\,f^2x^3+3i\,efx^2}{3\,a} - \frac{4i\,\left(dx\log\left(i\,e^{(dx+c)}+1\right)+\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)\right)ef}{ad^2} - \frac{2i\,\left(d^2x^2\log\left(i\,e^{(dx+c)}+1\right)+2\,dx\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)-2\,\mathrm{Li}_3\left(-i\,e^{(dx+c)}\right)\right)f^2}{ad^3} - \frac{2\left(-i\,d^3f^2x^3-3i\,d^3efx^2\right)}{3\,ad^3}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-I*e^2*log(I*a*sinh(d*x + c) + a)/(a*d) - 1/3*(I*f^2*x^3 + 3*I*e*f*x^2)/a
- 4*I*(d*x*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c)))*e*f/(a*d^2) - 2
*I*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-I*e^(d*x + c)) - 2*polyl
og(3, -I*e^(d*x + c)))*f^2/(a*d^3) - 2/3*(-I*d^3*f^2*x^3 - 3*I*d^3*e*f*x^2
)/(a*d^3)
```

Giac [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh\left(dx+c\right)}{i\,a\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{\cosh(c+d\,x)\,\left(e+f\,x\right)^2}{a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i),x)
```

output
```
int((cosh(c + d*x)*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(\int \frac{\cosh(dx+c)x^2}{\sinh(dx+c)i+1}dx\right)d\,f^2+2\left(\int \frac{\cosh(dx+c)x}{\sinh(dx+c)i+1}dx\right)def-\log(\sinh\left(dx+c\right)i+1)\,e^2i}{ad}$$

input
```
int((f*x+e)^2*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
(int((cosh(c + d*x)*x**2)/(sinh(c + d*x)*i + 1),x)*d*f**2 + 2*int((cosh(c
+ d*x)*x)/(sinh(c + d*x)*i + 1),x)*d*e*f - log(sinh(c + d*x)*i + 1)*e**2*i
)/(a*d)
```

3.255 $\int \frac{(e+fx)\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 73

$$\int \frac{(e+fx)\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{i(e+fx)^2}{2af} - \frac{2i(e+fx)\log\left(1+ie^{c+dx}\right)}{ad} - \frac{2if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^2}$$

output
```
1/2*I*(f*x+e)^2/a/f-2*I*(f*x+e)*ln(1+I*exp(d*x+c))/a/d-2*I*f*polylog(2,-I*
exp(d*x+c))/a/d^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int \frac{(e+fx)\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{i\left(d(e+fx)\left(d(e+fx)-4f\log\left(1+ie^{c+dx}\right)\right)-4f^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)\right)}{2ad^2f}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((I/2)*(d*(e + f*x)*(d*(e + f*x) - 4*f*Log[1 + I*E^(c + d*x)]) - 4*f^2*Pol
yLog[2, (-I)*E^(c + d*x)]))/(a*d^2*f)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6093, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cosh(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6093}$$

$$2\int \frac{e^{c+dx}(e+fx)}{ie^{c+dx}a+a}dx+\frac{i(e+fx)^2}{2af}$$

$$\Big\downarrow \text{2620}$$

$$2\left(\frac{if\int \log\left(1+ie^{c+dx}\right)dx}{ad}-\frac{i(e+fx)\log\left(1+ie^{c+dx}\right)}{ad}\right)+\frac{i(e+fx)^2}{2af}$$

$$\Big\downarrow \text{2715}$$

$$2\left(\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{ad^2}-\frac{i(e+fx)\log\left(1+ie^{c+dx}\right)}{ad}\right)+\frac{i(e+fx)^2}{2af}$$

$$\Big\downarrow \text{2838}$$

$$2\left(-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^2}-\frac{i(e+fx)\log\left(1+ie^{c+dx}\right)}{ad}\right)+\frac{i(e+fx)^2}{2af}$$

input
```
Int[((e + f*x)*Cosh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((I/2)*(e + f*x)^2)/(a*f) + 2*(((-I)*(e + f*x)*Log[1 + I*E^(c + d*x)])/(a*
d) - (I*f*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2))
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6093
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + Simp[2   Int[(e + f*x)^m*(E^(c + d*x)/(a + b*E^(c + d*x))), x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(64) = 128$.

Time = 0.82 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.58

method	result
risch	$\frac{if\,x^2}{2a} - \frac{iex}{a} - \frac{2i\ln\left(e^{dx+c}-i\right)e}{da} + \frac{2i\ln\left(e^{dx+c}\right)e}{da} + \frac{2ifcx}{da} + \frac{if\,c^2}{d^2a} - \frac{2if\ln\left(1+ie^{dx+c}\right)x}{da} - \frac{2if\ln\left(1+ie^{dx+c}\right)c}{d^2a} - \frac{2if\,\mathrm{po}}{}$

input
```
int((f*x+e)*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/a*f*x^2-I/a*e*x-2*I/d/a*ln(exp(d*x+c)-I)*e+2*I/d/a*ln(exp(d*x+c))*e+
2*I/d/a*f*c*x+I/d^2/a*f*c^2-2*I/d/a*f*ln(1+I*exp(d*x+c))*x-2*I/d^2/a*f*ln(
1+I*exp(d*x+c))*c-2*I*f*polylog(2,-I*exp(d*x+c))/a/d^2+2*I/d^2/a*c*f*ln(ex
p(d*x+c)-I)-2*I/d^2/a*c*f*ln(exp(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \frac{(e+fx)\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{i\,d^2fx^2 + 2i\,d^2ex + 4i\,cde - 2i\,c^2f - 4i\,f\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right) - 4\,(i\,de - i\,cf)\log\left(e^{(dx+c)} - i\right) - 4\,(i\,dfx + i\,c}{2\,ad^2}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(I*d^2*f*x^2 + 2*I*d^2*e*x + 4*I*c*d*e - 2*I*c^2*f - 4*I*f*dilog(-I*e^
(d*x + c)) - 4*(I*d*e - I*c*f)*log(e^(d*x + c) - I) - 4*(I*d*f*x + I*c*f)*
log(I*e^(d*x + c) + 1))/(a*d^2)
```

Sympy [F]

$$\int \frac{(e+fx)\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e\cosh(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{fx\cosh(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e*cosh(c + d*x)/(sinh(c + d*x) - I), x) + Integral(f*x*cosh(c
 + d*x)/(sinh(c + d*x) - I), x))/a
```

Maxima [F]

$$\int \frac{(e+fx)\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)}{i\,a\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*f*(-I*x^2/a + 4*integrate(x/(a*e^(d*x + c) - I*a), x)) - I*e*log(I*a*s
inh(d*x + c) + a)/(a*d)
```

Giac [F]

$$\int \frac{(e+fx)\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)}{i\,a\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,(e+f\,x)}{a+a\sinh(c+d\,x)\,1\mathrm{i}}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x))/(a + a*sinh(c + d*x)*1i),x)
```

output
```
int((cosh(c + d*x)*(e + f*x))/(a + a*sinh(c + d*x)*1i), x)
```

Reduce [F]

$$\int \frac{(e+fx)\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(\int \frac{\cosh(dx+c)x}{\sinh(dx+c)i+1}dx\right)df - \log(\sinh(dx+c)\,i+1)\,ei}{ad}$$

input
```
int((f*x+e)*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
(int((cosh(c + d*x)*x)/(sinh(c + d*x)*i + 1),x)*d*f - log(sinh(c + d*x)*i
+ 1)*e*i)/(a*d)
```

3.256 $\int \frac{\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 23

$$\int \frac{\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\log(i-\sinh(c+dx))}{ad}$$

output
```
-I*ln(I-sinh(d*x+c))/a/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\log(i-\sinh(c+dx))}{ad}$$

input
```
Integrate[Cosh[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-I)*Log[I - Sinh[c + d*x]])/(a*d)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{a+ia\sinh(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(ic+idx)}{a+a\sin(ic+idx)}dx$$
$$\Big\downarrow \text{3146}$$
$$-\frac{i\int \frac{1}{i\sinh(c+dx)a+a}d(ia\sinh(c+dx))}{ad}$$
$$\Big\downarrow \text{16}$$
$$-\frac{i\log(a+ia\sinh(c+dx))}{ad}$$

input
```
Int[Cosh[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-I)*Log[a + I*a*Sinh[c + d*x]])/(a*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

method	result	size
risch	$\frac{ix}{a}+\frac{2ic}{da}-\frac{2i\ln\left(\mathrm{e}^{dx+c}-i\right)}{da}$	38
derivativedivides	$-\frac{i\ln\left(a^{2}\sinh\left(dx+c\right)^{2}+a^{2}\right)}{2da}+\frac{\arctan\left(\sinh\left(dx+c\right)\right)}{ad}$	42
default	$-\frac{i\ln\left(a^{2}\sinh\left(dx+c\right)^{2}+a^{2}\right)}{2da}+\frac{\arctan\left(\sinh\left(dx+c\right)\right)}{ad}$	42

input
```
int(cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
I*x/a+2*I/d/a*c-2*I/d/a*ln(exp(d*x+c)-I)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int\frac{\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx=\frac{i\,dx-2i\,\log\left(e^{(dx+c)}-i\right)}{ad}$$

input
```
integrate(cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(I*d*x - 2*I*log(e^(d*x + c) - I))/(a*d)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{ix}{a} - \frac{2i\log\left(e^{dx}-ie^{-c}\right)}{ad}$$

input
```
integrate(cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
I*x/a - 2*I*log(exp(d*x) - I*exp(-c))/(a*d)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\,\log\left(i\,a\sinh\left(dx+c\right)+a\right)}{ad}$$

input
```
integrate(cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-I*log(I*a*sinh(d*x + c) + a)/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \frac{\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{-\frac{i\,(dx+c)}{a}+\frac{2i\,\log\left(e^{(dx+c)}-i\right)}{a}}{d}$$

input
```
integrate(cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-(-I*(d*x + c)/a + 2*I*log(e^(d*x + c) - I)/a)/d
```

Mupad [B] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{\ln\left(\sinh(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{a\,d}$$

input
```
int(cosh(c + d*x)/(a + a*sinh(c + d*x)*1i),x)
```

output
```
-(log(sinh(c + d*x) - 1i)*1i)/(a*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{\log(\sinh\left(dx+c\right)i+1)\,i}{ad}$$

input
```
int(cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
( - log(sinh(c + d*x)*i + 1)*i)/(a*d)
```

3.257 $\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(cosh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 5.75 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

input
```
Integrate[Cosh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Integrate[Cosh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

$$\Big\downarrow \; 6111$$

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}dx$$

input
```
Int[Cosh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\cosh\left(dx+c\right)}{\left(fx+e\right)\left(a+ia\sinh\left(dx+c\right)\right)}dx$$

input
```
int(cosh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral((-I*e^(d*x + c) + 1)/(-I*a*f*x - I*a*e + (a*f*x + a*e)*e^(d*x + c
)), x)
```

Sympy [N/A]

Not integrable

Time = 5.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = -\frac{i\int \frac{\cosh(c+dx)}{e\sinh(c+dx)-ie+fx\sinh(c+dx)-ifx}\,dx}{a}$$

input
```
integrate(cosh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(cosh(c + d*x)/(e*sinh(c + d*x) - I*e + f*x*sinh(c + d*x) - I*f
*x), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.66

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-I*log(f*x + e)/(a*f) + 2*integrate(1/(-I*a*f*x - I*a*e + (a*f*x*e^c + a*e
*e^c)*e^(d*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)}{(e+f\,x)\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(cosh(c + d*x)/((e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(cosh(c + d*x)/((e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \frac{\int \frac{\cosh(dx+c)}{\sinh(dx+c)ei+\sinh(dx+c)fix+e+fx}dx}{a}$$

input
```
int(cosh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(cosh(c + d*x)/(sinh(c + d*x)*e*i + sinh(c + d*x)*f*i*x + e + f*x),x)/a
```

3.258 $\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} \, dx = \text{Int}\left(\frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(cosh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 25.79 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} \, dx = \int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} \, dx$$

input
```
Integrate[Cosh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Integrate[Cosh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}dx$$
$$\Big\downarrow \text{ 6111}$$
$$\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}dx$$

input
```
Int[Cosh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\cosh(dx+c)}{(fx+e)^2(a+ia\sinh(dx+c))}dx$$

input
```
int(cosh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.21

$$\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)}{(fx+e)^2(i\,a\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral((-I*e^(d*x + c) + 1)/(-I*a*f^2*x^2 - 2*I*a*e*f*x - I*a*e^2 + (a*f
^2*x^2 + 2*a*e*f*x + a*e^2)*e^(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 24.95 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.45

$$\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = -\frac{i\int \frac{\cosh\left(c+dx\right)}{e^2\sinh\left(c+dx\right)-ie^2+2efx\sinh\left(c+dx\right)-2iefx+f^2x^2\sinh\left(c+dx\right)-if^2x^2}\,dx}{a}$$

input
```
integrate(cosh(d*x+c)/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(cosh(c + d*x)/(e**2*sinh(c + d*x) - I*e**2 + 2*e*f*x*sinh(c +
d*x) - 2*I*e*f*x + f**2*x**2*sinh(c + d*x) - I*f**2*x**2), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.59

$$\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)}{(fx+e)^2(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
I/(a*f^2*x + a*e*f) + 2*integrate(1/(-I*a*f^2*x^2 - 2*I*a*e*f*x - I*a*e^2
+ (a*f^2*x^2*e^c + 2*a*e*f*x*e^c + a*e^2*e^c)*e^(d*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)}{(fx+e)^2(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)/((f*x + e)^2*(I*a*sinh(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)}{(e+f\,x)^2\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(cosh(c + d*x)/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(cosh(c + d*x)/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.34

$$\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \frac{\int \frac{\cosh(dx+c)}{\sinh(dx+c)e^2i+2\sinh(dx+c)efix+\sinh(dx+c)f^2i\,x^2+e^2+2efx+f^2x^2}dx}{a}$$

input
```
int(cosh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(cosh(c + d*x)/(sinh(c + d*x)*e**2*i + 2*sinh(c + d*x)*e*f*i*x + sinh(c
 + d*x)*f**2*i*x**2 + e**2 + 2*e*f*x + f**2*x**2),x)/a
```

3.259 $\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 108

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{(e+fx)^4}{4af} - \frac{6if^2(e+fx)\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^3\cosh(c+dx)}{ad} + \frac{6if^3\sinh(c+dx)}{ad^4} + \frac{3if(e+fx)^2\sinh(c+dx)}{ad^2}$$

output
```
1/4*(f*x+e)^4/a/f-6*I*f^2*(f*x+e)*cosh(d*x+c)/a/d^3-I*(f*x+e)^3*cosh(d*x+c
)/a/d+6*I*f^3*sinh(d*x+c)/a/d^4+3*I*f*(f*x+e)^2*sinh(d*x+c)/a/d^2
```

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.98

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{d^4x(4e^3+6e^2fx+4ef^2x^2+f^3x^3) - 4id(e+fx)\left(6f^2+d^2(e+fx)^2\right)\cosh(c+dx) + 12if\left(2f^2+d^2(e\right.}{4ad^4}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(d^4*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3) - (4*I)*d*(e + f*x)*(6*
f^2 + d^2*(e + f*x)^2)*Cosh[c + d*x] + (12*I)*f*(2*f^2 + d^2*(e + f*x)^2)*
Sinh[c + d*x])/(4*a*d^4)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {6097, 17, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6097}$$

$$\frac{\int (e+fx)^3dx}{a}-\frac{i\int (e+fx)^3\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{17}$$

$$\frac{(e+fx)^4}{4af}-\frac{i\int (e+fx)^3\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^4}{4af}-\frac{i\int -i(e+fx)^3\sin(ic+idx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{(e+fx)^4}{4af}-\frac{\int (e+fx)^3\sin(ic+idx)dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(e+fx)^4}{4af}-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\cosh(c+dx)dx}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^4}{4af}-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}$$

$$\Big\downarrow \ 3777$$

$$\frac{(e+fx)^4}{4af}-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \ 26$$

$$\frac{(e+fx)^4}{4af}-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int (e+fx)\sinh(c+dx)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \ 3042$$

$$\frac{(e+fx)^4}{4af}-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int -i(e+fx)\sin(ic+idx)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \ 26$$

$$\frac{(e+fx)^4}{4af}-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int (e+fx)\sin(ic+idx)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \ 3777$$

$$\frac{(e+fx)^4}{4af}-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \ 3042$$

$$\frac{(e+fx)^4}{4af}-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \ 3117$$

$$\frac{(e+fx)^4}{4af}-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(e + f*x)^4/(4*a*f) - ((I*(e + f*x)^3*Cosh[c + d*x])/d - ((3*I)*f*(((e + f
*x)^2*Sinh[c + d*x])/d + ((2*I)*f*((I*(e + f*x)*Cosh[c + d*x])/d - (I*f*Si
nh[c + d*x])/d^2))/d))/d)/a
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6097
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cosh[c
 + d*x]^(n - 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*
Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && E
qQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(102) = 204.

Time = 13.60 (sec) , antiderivative size = 266, normalized size of antiderivative = 2.46

method	result
risch	$\frac{f^3x^4}{4a}+\frac{f^2e\,x^3}{a}+\frac{3f\,e^2x^2}{2a}+\frac{e^3x}{a}+\frac{e^4}{4af}-\frac{i(d^3x^3f^3+3d^3e\,f^2x^2+3d^3e^2fx-3d^2f^3x^2+d^3e^3-6d^2e\,f^2x-3d^2e^2}{2d^4a}$
derivativedivides	$-\frac{-6icde\,f^2((dx+c)\cosh(dx+c)-\sinh(dx+c))-3ic\,d^2e^2f\cosh(dx+c)+id^3e^3\cosh(dx+c)+3ide\,f^2\left((dx+c)^2\cosh(dx\right.}{}$
default	$-\frac{-6icde\,f^2((dx+c)\cosh(dx+c)-\sinh(dx+c))-3ic\,d^2e^2f\cosh(dx+c)+id^3e^3\cosh(dx+c)+3ide\,f^2\left((dx+c)^2\cosh(dx\right.}{}$

input
```
int((f*x+e)^3*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*f^3*x^4+1/a*f^2*e*x^3+3/2/a*f*e^2*x^2+1/a*e^3*x+1/4/a/f*e^4-1/2*I*(d
^3*f^3*x^3+3*d^3*e*f^2*x^2+3*d^3*e^2*f*x-3*d^2*f^3*x^2+d^3*e^3-6*d^2*e*f^2
*x-3*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2-6*f^3)/d^4/a*exp(d*x+c)-1/2*I*(d^3*f^3*
x^3+3*d^3*e*f^2*x^2+3*d^3*e^2*f*x+3*d^2*f^3*x^2+d^3*e^3+6*d^2*e*f^2*x+3*d^
2*e^2*f+6*d*f^3*x+6*d*e*f^2+6*f^3)/d^4/a*exp(-d*x-c)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(98) = 196.

Time = 0.09 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.44

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$=\frac{\left(-2i\,d^3f^3x^3-2i\,d^3e^3-6i\,d^2e^2f-12i\,def^2-12i\,f^3-6\left(i\,d^3ef^2+i\,d^2f^3\right)x^2-6\left(i\,d^3e^2f+2i\,d^2ef^2+\right.\right.}{}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/4*(-2*I*d^3*f^3*x^3 - 2*I*d^3*e^3 - 6*I*d^2*e^2*f - 12*I*d*e*f^2 - 12*I*
f^3 - 6*(I*d^3*e*f^2 + I*d^2*f^3)*x^2 - 6*(I*d^3*e^2*f + 2*I*d^2*e*f^2 + 2
*I*d*f^3)*x - 2*(I*d^3*f^3*x^3 + I*d^3*e^3 - 3*I*d^2*e^2*f + 6*I*d*e*f^2 -
 6*I*f^3 + 3*(I*d^3*e*f^2 - I*d^2*f^3)*x^2 + 3*(I*d^3*e^2*f - 2*I*d^2*e*f^
2 + 2*I*d*f^3)*x)*e^(2*d*x + 2*c) + (d^4*f^3*x^4 + 4*d^4*e*f^2*x^3 + 6*d^4
*e^2*f*x^2 + 4*d^4*e^3*x)*e^(d*x + c))*e^(-d*x - c)/(a*d^4)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 518, normalized size of antiderivative = 4.80

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$= \begin{cases} \frac{((-2iad^7e^3-6iad^7e^2fx-6iad^7ef^2x^2-2iad^7f^3x^3-6iad^6e^2f-12iad^6ef^2x-6iad^6f^3x^2-12iad^5ef^2-12iad^5f^3x-12iad^4f^3)e^{-dx}+(-2iad^7 \cdots}{} \\ \frac{x^4(-if^3e^{2c}+if^3)e^{-c}}{8a}+\frac{x^3(-ief^2e^{2c}+ief^2)e^{-c}}{2a}+\frac{x^2(-3ie^2fe^{2c}+3ie^2f)e^{-c}}{4a}+\frac{x(-ie^3e^{2c}+ie^3)e^{-c}}{2a} \end{cases}$$
$$+\frac{e^3x}{a}+\frac{3e^2fx^2}{2a}+\frac{ef^2x^3}{a}+\frac{f^3x^4}{4a}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
Piecewise((((-2*I*a*d**7*e**3 - 6*I*a*d**7*e**2*f*x - 6*I*a*d**7*e*f**2*x*
*2 - 2*I*a*d**7*f**3*x**3 - 6*I*a*d**6*e**2*f - 12*I*a*d**6*e*f**2*x - 6*I
*a*d**6*f**3*x**2 - 12*I*a*d**5*e*f**2 - 12*I*a*d**5*f**3*x - 12*I*a*d**4*
f**3)*exp(-d*x) + (-2*I*a*d**7*e**3*exp(2*c) - 6*I*a*d**7*e**2*f*x*exp(2*c
) - 6*I*a*d**7*e*f**2*x**2*exp(2*c) - 2*I*a*d**7*f**3*x**3*exp(2*c) + 6*I*
a*d**6*e**2*f*exp(2*c) + 12*I*a*d**6*e*f**2*x*exp(2*c) + 6*I*a*d**6*f**3*x
**2*exp(2*c) - 12*I*a*d**5*e*f**2*exp(2*c) - 12*I*a*d**5*f**3*x*exp(2*c) +
 12*I*a*d**4*f**3*exp(2*c))*exp(d*x))*exp(-c)/(4*a**2*d**8), Ne(a**2*d**8*
exp(c), 0)), (x**4*(-I*f**3*exp(2*c) + I*f**3)*exp(-c)/(8*a) + x**3*(-I*e*
f**2*exp(2*c) + I*e*f**2)*exp(-c)/(2*a) + x**2*(-3*I*e**2*f*exp(2*c) + 3*I
*e**2*f)*exp(-c)/(4*a) + x*(-I*e**3*exp(2*c) + I*e**3)*exp(-c)/(2*a), True
)) + e**3*x/a + 3*e**2*f*x**2/(2*a) + e*f**2*x**3/a + f**3*x**4/(4*a)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(98) = 196$.

Time = 0.23 (sec) , antiderivative size = 373, normalized size of antiderivative = 3.45

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$= \frac{3}{2}e^2f\left(\frac{2\,xe^{(dx+c)}}{ade^{(dx+c)}-i\,ad} - \frac{i\,d^2x^2e^c + i\,dxe^c - \left(-i\,dxe^{(3\,c)} + i\,e^{(3\,c)}\right)e^{(2\,dx)} - \left(d^2x^2e^{(2\,c)} - 3\,dxe^{(2\,c)} + e^{(2\,c)}\right.}{ad^2e^{(dx+2\,c)} - i\,ad^2e^c}\right.$$

$$+ \frac{1}{2}e^3\left(\frac{2\,(dx+c)}{ad} - \frac{i\,e^{(dx+c)}}{ad} - \frac{i\,e^{(-dx-c)}}{ad}\right)$$

$$+ \frac{\left(2\,d^3x^3e^c + 3\left(-i\,d^2x^2e^{(2\,c)} + 2i\,dxe^{(2\,c)} - 2i\,e^{(2\,c)}\right)e^{(dx)} + 3\left(-i\,d^2x^2 - 2i\,dx - 2i\right)e^{(-dx)}\right)ef^2e^{(-c)}}{2\,ad^3}$$

$$+ \frac{\left(d^4x^4e^c + 2\left(-i\,d^3x^3e^{(2\,c)} + 3i\,d^2x^2e^{(2\,c)} - 6i\,dxe^{(2\,c)} + 6i\,e^{(2\,c)}\right)e^{(dx)} + 2\left(-i\,d^3x^3 - 3i\,d^2x^2 - 6i\,dx - 6\right.}{4\,ad^4}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
3/2*e^2*f*(2*x*e^(d*x + c)/(a*d*e^(d*x + c) - I*a*d) - (I*d^2*x^2*e^c + I*
d*x*e^c - (-I*d*x*e^(3*c) + I*e^(3*c))*e^(2*d*x) - (d^2*x^2*e^(2*c) - 3*d*
x*e^(2*c) + e^(2*c))*e^(d*x) + (d*x + 1)*e^(-d*x) + I*e^c)/(a*d^2*e^(d*x +
 2*c) - I*a*d^2*e^c)) + 1/2*e^3*(2*(d*x + c)/(a*d) - I*e^(d*x + c)/(a*d) -
 I*e^(-d*x - c)/(a*d)) + 1/2*(2*d^3*x^3*e^c + 3*(-I*d^2*x^2*e^(2*c) + 2*I*
d*x*e^(2*c) - 2*I*e^(2*c))*e^(d*x) + 3*(-I*d^2*x^2 - 2*I*d*x - 2*I)*e^(-d*
x))*e*f^2*e^(-c)/(a*d^3) + 1/4*(d^4*x^4*e^c + 2*(-I*d^3*x^3*e^(2*c) + 3*I*
d^2*x^2*e^(2*c) - 6*I*d*x*e^(2*c) + 6*I*e^(2*c))*e^(d*x) + 2*(-I*d^3*x^3 -
 3*I*d^2*x^2 - 6*I*d*x - 6*I)*e^(-d*x))*f^3*e^(-c)/(a*d^4)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(98) = 196$.

Time = 0.12 (sec) , antiderivative size = 355, normalized size of antiderivative = 3.29

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{\left(d^4f^3x^4e^{(dx+c)} + 4\,d^4ef^2x^3e^{(dx+c)} - 2i\,d^3f^3x^3e^{(2\,dx+2\,c)} + 6\,d^4e^2fx^2e^{(dx+c)} - 2i\,d^3f^3x^3 - 6i\,d^3ef^2x^2e^{(2\,dx}\right.}{}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/4*(d^4*f^3*x^4*e^(d*x + c) + 4*d^4*e*f^2*x^3*e^(d*x + c) - 2*I*d^3*f^3*x
^3*e^(2*d*x + 2*c) + 6*d^4*e^2*f*x^2*e^(d*x + c) - 2*I*d^3*f^3*x^3 - 6*I*d
^3*e*f^2*x^2*e^(2*d*x + 2*c) + 4*d^4*e^3*x*e^(d*x + c) - 6*I*d^3*e*f^2*x^2
 - 6*I*d^3*e^2*f*x*e^(2*d*x + 2*c) + 6*I*d^2*f^3*x^2*e^(2*d*x + 2*c) - 6*I
*d^3*e^2*f*x - 6*I*d^2*f^3*x^2 - 2*I*d^3*e^3*e^(2*d*x + 2*c) + 12*I*d^2*e*
f^2*x*e^(2*d*x + 2*c) - 2*I*d^3*e^3 - 12*I*d^2*e*f^2*x + 6*I*d^2*e^2*f*e^(
2*d*x + 2*c) - 12*I*d*f^3*x*e^(2*d*x + 2*c) - 6*I*d^2*e^2*f - 12*I*d*f^3*x
 - 12*I*d*e*f^2*e^(2*d*x + 2*c) - 12*I*d*e*f^2 + 12*I*f^3*e^(2*d*x + 2*c)
- 12*I*f^3)*e^(-d*x - c)/(a*d^4)
```

Mupad [B] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.49

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = e^{c+dx}\left(\frac{(-d^3e^3+3d^2e^2f-6def^2+6f^3)\,1i}{2ad^4} - \frac{f^3x^3\,1i}{2ad} + \frac{f^2x^2(f-de)\,3i}{2ad^2} - \frac{fx(d^2e^2-2def+2f^2)\,3i}{2ad^3}\right) - e^{-c-dx}\left(\frac{(d^3e^3+3d^2e^2f+6def^2+6f^3)\,1i}{2ad^4} + \frac{f^3x^3\,1i}{2ad} + \frac{f^2x^2(f+de)\,3i}{2ad^2} + \frac{fx(d^2e^2+2def+2f^2)\,3i}{2ad^3}\right) + \frac{e^3x}{a} + \frac{f^3x^4}{4a} + \frac{3e^2fx^2}{2a} + \frac{ef^2x^3}{a}$$

input
```
int((cosh(c + d*x)^2*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i),x)
```

output
```
exp(c + d*x)*(((6*f^3 - d^3*e^3 + 3*d^2*e^2*f - 6*d*e*f^2)*1i)/(2*a*d^4) -
 (f^3*x^3*1i)/(2*a*d) + (f^2*x^2*(f - d*e)*3i)/(2*a*d^2) - (f*x*(2*f^2 + d
^2*e^2 - 2*d*e*f)*3i)/(2*a*d^3)) - exp(- c - d*x)*(((6*f^3 + d^3*e^3 + 3*d
^2*e^2*f + 6*d*e*f^2)*1i)/(2*a*d^4) + (f^3*x^3*1i)/(2*a*d) + (f^2*x^2*(f +
 d*e)*3i)/(2*a*d^2) + (f*x*(2*f^2 + d^2*e^2 + 2*d*e*f)*3i)/(2*a*d^3)) + (e
^3*x)/a + (f^3*x^4)/(4*a) + (3*e^2*f*x^2)/(2*a) + (e*f^2*x^3)/a
```

Reduce [F]

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(\int \frac{\cosh(dx+c)^2}{\sinh(dx+c)i+1}dx\right)e^3 + \left(\int \frac{\cosh(dx+c)^2x^3}{\sinh(dx+c)i+1}dx\right)f^3 + 3\left(\int \frac{\cosh(dx+c)^2x^2}{\sinh(dx+c)i+1}dx\right)ef^2 + 3\left(\int \frac{\cosh(dx+c)^2x}{\sinh(dx+c)i+1}dx\right)e^2f}{a}$$

input
```
int((f*x+e)^3*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
(int(cosh(c + d*x)**2/(sinh(c + d*x)*i + 1),x)*e**3 + int((cosh(c + d*x)**
2*x**3)/(sinh(c + d*x)*i + 1),x)*f**3 + 3*int((cosh(c + d*x)**2*x**2)/(sin
h(c + d*x)*i + 1),x)*e*f**2 + 3*int((cosh(c + d*x)**2*x)/(sinh(c + d*x)*i
+ 1),x)*e**2*f)/a
```

3.260 $\int \frac{(e+fx)^2 \cosh^2(c+dx)}{a+ia \sinh(c+dx)} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 82

$$\int \frac{(e+fx)^2 \cosh^2(c+dx)}{a+ia\sinh(c+dx)} \, dx = \frac{(e+fx)^3}{3af} - \frac{2if^2\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^2\cosh(c+dx)}{ad} + \frac{2if(e+fx)\sinh(c+dx)}{ad^2}$$

output
```
1/3*(f*x+e)^3/a/f-2*I*f^2*cosh(d*x+c)/a/d^3-I*(f*x+e)^2*cosh(d*x+c)/a/d+2*
I*f*(f*x+e)*sinh(d*x+c)/a/d^2
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \frac{(e+fx)^2 \cosh^2(c+dx)}{a+ia\sinh(c+dx)} \, dx = \frac{d^3x(3e^2+3efx+f^2x^2) - 3i(2f^2+d^2(e+fx)^2)\cosh(c+dx) + 6idf(e+fx)\sinh(c+dx)}{3ad^3}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(d^3*x*(3*e^2 + 3*e*f*x + f^2*x^2) - (3*I)*(2*f^2 + d^2*(e + f*x)^2)*Cosh[
c + d*x] + (6*I)*d*f*(e + f*x)*Sinh[c + d*x])/(3*a*d^3)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {6097, 17, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6097}$$

$$\frac{\int (e+fx)^2dx}{a}-\frac{i\int (e+fx)^2\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{17}$$

$$\frac{(e+fx)^3}{3af}-\frac{i\int (e+fx)^2\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^3}{3af}-\frac{i\int -i(e+fx)^2\sin(ic+idx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{(e+fx)^3}{3af}-\frac{\int (e+fx)^2\sin(ic+idx)dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(e+fx)^3}{3af}-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\cosh(c+dx)dx}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^3}{3af}-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(e+fx)^3}{3af}-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{d}}{a}$$

$\Big\downarrow$ 26

$$\frac{(e+fx)^3}{3af}-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)}{d}}{a}$$

$\Big\downarrow$ 3042

$$\frac{(e+fx)^3}{3af}-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}}{a}$$

$\Big\downarrow$ 26

$$\frac{(e+fx)^3}{3af}-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}\right)}{d}}{a}$$

$\Big\downarrow$ 3118

$$\frac{(e+fx)^3}{3af}-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(e + f*x)^3/(3*a*f) - ((I*(e + f*x)^2*Cosh[c + d*x])/d - ((2*I)*f*(-((f*Co
sh[c + d*x])/d^2) + ((e + f*x)*Sinh[c + d*x])/d))/d)/a
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6097
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cosh[c
 + d*x]^(n - 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*
Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && E
qQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(77) = 154$.

Time = 5.50 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.95

method	result
risch	$\frac{f^2x^3}{3a}+\frac{fe\,x^2}{a}+\frac{e^2x}{a}+\frac{e^3}{3af}-\frac{i(d^2x^2f^2+2d^2efx+d^2e^2-2d\,f^2x-2def+2f^2)\,\mathrm{e}^{dx+c}}{2d^3a}-\frac{i(d^2x^2f^2+2d^2efx+d^2e}{}$
derivativedivides	$-\frac{ic^2f^2\cosh(dx+c)-2icdef\cosh(dx+c)-2icf^2((dx+c)\cosh(dx+c)-\sinh(dx+c))+id^2e^2\cosh(dx+c)+2idef((dx+c}{}$
default	$-\frac{ic^2f^2\cosh(dx+c)-2icdef\cosh(dx+c)-2icf^2((dx+c)\cosh(dx+c)-\sinh(dx+c))+id^2e^2\cosh(dx+c)+2idef((dx+c}{}$

input
```
int((f*x+e)^2*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/3/a*f^2*x^3+1/a*f*e*x^2+1/a*e^2*x+1/3/a/f*e^3-1/2*I*(d^2*f^2*x^2+2*d^2*e
*f*x+d^2*e^2-2*d*f^2*x-2*d*e*f+2*f^2)/d^3/a*exp(d*x+c)-1/2*I*(d^2*f^2*x^2+
2*d^2*e*f*x+d^2*e^2+2*d*f^2*x+2*d*e*f+2*f^2)/d^3/a*exp(-d*x-c)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(74) = 148.

Time = 0.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.95

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(-3i\,d^2f^2x^2-3i\,d^2e^2-6i\,def-6i\,f^2-6\left(i\,d^2ef+i\,df^2\right)x-3\left(i\,d^2f^2x^2+i\,d^2e^2-2i\,def+2i\,f^2+2\left(\right.\right.}{6\,ad^3}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/6*(-3*I*d^2*f^2*x^2 - 3*I*d^2*e^2 - 6*I*d*e*f - 6*I*f^2 - 6*(I*d^2*e*f +
 I*d*f^2)*x - 3*(I*d^2*f^2*x^2 + I*d^2*e^2 - 2*I*d*e*f + 2*I*f^2 + 2*(I*d^
2*e*f - I*d*f^2)*x)*e^(2*d*x + 2*c) + 2*(d^3*f^2*x^3 + 3*d^3*e*f*x^2 + 3*d
^3*e^2*x)*e^(d*x + c))*e^(-d*x - c)/(a*d^3)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.88

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \begin{cases} \frac{\left(\left(-2iad^5e^2-4iad^5efx-2iad^5f^2x^2-4iad^4ef-4iad^4f^2x-4iad^3f^2\right)e^{-dx}+\left(-2iad^5e^2e^{2c}-4iad^5efxe^{2c}-2iad^5f^2x^2e^{2c}+4iad^4efe^{2c}+4iad^4f\right.\right.}{4a^2d^6} \\ \frac{x^3\left(-if^2e^{2c}+if^2\right)e^{-c}}{6a}+\frac{x^2\left(-iefe^{2c}+ief\right)e^{-c}}{2a}+\frac{x\left(-ie^2e^{2c}+ie^2\right)e^{-c}}{2a} \end{cases} + \frac{e^2x}{a}+\frac{efx^2}{a}+\frac{f^2x^3}{3a}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
Piecewise((((-2*I*a*d**5*e**2 - 4*I*a*d**5*e*f*x - 2*I*a*d**5*f**2*x**2 -
4*I*a*d**4*e*f - 4*I*a*d**4*f**2*x - 4*I*a*d**3*f**2)*exp(-d*x) + (-2*I*a*
d**5*e**2*exp(2*c) - 4*I*a*d**5*e*f*x*exp(2*c) - 2*I*a*d**5*f**2*x**2*exp(
2*c) + 4*I*a*d**4*e*f*exp(2*c) + 4*I*a*d**4*f**2*x*exp(2*c) - 4*I*a*d**3*f
**2*exp(2*c))*exp(d*x))*exp(-c)/(4*a**2*d**6), Ne(a**2*d**6*exp(c), 0)), (
x**3*(-I*f**2*exp(2*c) + I*f**2)*exp(-c)/(6*a) + x**2*(-I*e*f*exp(2*c) + I
*e*f)*exp(-c)/(2*a) + x*(-I*e**2*exp(2*c) + I*e**2)*exp(-c)/(2*a), True))
+ e**2*x/a + e*f*x**2/a + f**2*x**3/(3*a)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(74) = 148$.

Time = 0.16 (sec) , antiderivative size = 270, normalized size of antiderivative = 3.29

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= ef\left(\frac{2\,xe^{(dx+c)}}{ade^{(dx+c)}-i\,ad} - \frac{i\,d^2x^2e^c + i\,dxe^c - \left(-i\,dxe^{(3\,c)} + i\,e^{(3\,c)}\right)e^{(2\,dx)} - \left(d^2x^2e^{(2\,c)} - 3\,dxe^{(2\,c)} + e^{(2\,c)}\right)e^{(}}{ad^2e^{(dx+2\,c)} - i\,ad^2e^c}\right.$$
$$\left. + \frac{1}{2}\,e^2\left(\frac{2\,(dx+c)}{ad} - \frac{i\,e^{(dx+c)}}{ad} - \frac{i\,e^{(-dx-c)}}{ad}\right)\right.$$
$$+ \frac{\left(2\,d^3x^3e^c + 3\left(-i\,d^2x^2e^{(2\,c)} + 2i\,dxe^{(2\,c)} - 2i\,e^{(2\,c)}\right)e^{(dx)} + 3\left(-i\,d^2x^2 - 2i\,dx - 2i\right)e^{(-dx)}\right)f^2e^{(-c)}}{6\,ad^3}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
e*f*(2*x*e^(d*x + c)/(a*d*e^(d*x + c) - I*a*d) - (I*d^2*x^2*e^c + I*d*x*e^
c - (-I*d*x*e^(3*c) + I*e^(3*c))*e^(2*d*x) - (d^2*x^2*e^(2*c) - 3*d*x*e^(2
*c) + e^(2*c))*e^(d*x) + (d*x + 1)*e^(-d*x) + I*e^c)/(a*d^2*e^(d*x + 2*c)
- I*a*d^2*e^c)) + 1/2*e^2*(2*(d*x + c)/(a*d) - I*e^(d*x + c)/(a*d) - I*e^(
-d*x - c)/(a*d)) + 1/6*(2*d^3*x^3*e^c + 3*(-I*d^2*x^2*e^(2*c) + 2*I*d*x*e^
(2*c) - 2*I*e^(2*c))*e^(d*x) + 3*(-I*d^2*x^2 - 2*I*d*x - 2*I)*e^(-d*x))*f^
2*e^(-c)/(a*d^3)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(74) = 148.

Time = 0.12 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.54

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{\left(2\,d^3f^2x^3e^{(dx+c)}+6\,d^3efx^2e^{(dx+c)}-3i\,d^2f^2x^2e^{(2\,dx+2\,c)}+6\,d^3e^2xe^{(dx+c)}-3i\,d^2f^2x^2-6i\,d^2efxe^{(2\,dx+2\,c)}\right.}{}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(2*d^3*f^2*x^3*e^(d*x + c) + 6*d^3*e*f*x^2*e^(d*x + c) - 3*I*d^2*f^2*x
^2*e^(2*d*x + 2*c) + 6*d^3*e^2*x*e^(d*x + c) - 3*I*d^2*f^2*x^2 - 6*I*d^2*e
*f*x*e^(2*d*x + 2*c) - 6*I*d^2*e*f*x - 3*I*d^2*e^2*e^(2*d*x + 2*c) + 6*I*d
*f^2*x*e^(2*d*x + 2*c) - 3*I*d^2*e^2 - 6*I*d*f^2*x + 6*I*d*e*f*e^(2*d*x +
2*c) - 6*I*d*e*f - 6*I*f^2*e^(2*d*x + 2*c) - 6*I*f^2)*e^(-d*x - c)/(a*d^3)
```

Mupad [B] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.04

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{e^2\,x}{a}-\mathrm{e}^{-c-d\,x}\left(\frac{\left(d^2\,e^2+2\,d\,e\,f+2\,f^2\right)\,1\mathrm{i}}{2\,a\,d^3}+\frac{f^2\,x^2\,1\mathrm{i}}{2\,a\,d}+\frac{f\,x\,\left(f+d\,e\right)\,1\mathrm{i}}{a\,d^2}\right)-\mathrm{e}^{c+d\,x}\left(\frac{\left(d^2\,e^2-2\,d\,e\,f+2\,f^2\right)\,1\mathrm{i}}{2\,a\,d^3}+\frac{f^2\,x^2\,1\mathrm{i}}{2\,a\,d}-\frac{f\,x\,\left(f-d\,e\right)\,1\mathrm{i}}{a\,d^2}\right)+\frac{f^2\,x^3}{3\,a}+\frac{e\,f\,x^2}{a}$$

input
```
int((cosh(c + d*x)^2*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i),x)
```

output
```
(e^2*x)/a - exp(- c - d*x)*(((2*f^2 + d^2*e^2 + 2*d*e*f)*1i)/(2*a*d^3) + (
f^2*x^2*1i)/(2*a*d) + (f*x*(f + d*e)*1i)/(a*d^2)) - exp(c + d*x)*(((2*f^2
+ d^2*e^2 - 2*d*e*f)*1i)/(2*a*d^3) + (f^2*x^2*1i)/(2*a*d) - (f*x*(f - d*e)
*1i)/(a*d^2)) + (f^2*x^3)/(3*a) + (e*f*x^2)/a
```

Reduce [F]

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$=\frac{\left(\int \frac{\cosh(dx+c)^2}{\sinh(dx+c)i+1}dx\right)e^2+\left(\int \frac{\cosh(dx+c)^2x^2}{\sinh(dx+c)i+1}dx\right)f^2+2\left(\int \frac{\cosh(dx+c)^2x}{\sinh(dx+c)i+1}dx\right)ef}{a}$$

input
```
int((f*x+e)^2*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
(int(cosh(c + d*x)**2/(sinh(c + d*x)*i + 1),x)*e**2 + int((cosh(c + d*x)**
2*x**2)/(sinh(c + d*x)*i + 1),x)*f**2 + 2*int((cosh(c + d*x)**2*x)/(sinh(c
 + d*x)*i + 1),x)*e*f)/a
```

3.261 $\int \frac{(e+fx)\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 56

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx=\frac{(e+fx)^2}{2af}-\frac{i(e+fx)\cosh(c+dx)}{ad}+\frac{if\sinh(c+dx)}{ad^2}$$

output
```
1/2*(f*x+e)^2/a/f-I*(f*x+e)*cosh(d*x+c)/a/d+I*f*sinh(d*x+c)/a/d^2
```

Mathematica [A] (verified)

Time = 2.56 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{(c+dx)(-2de+cf-dfx)+2id(e+fx)\cosh(c+dx)-2if\sinh(c+dx)}{2ad^2}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
-1/2*((c + d*x)*(-2*d*e + c*f - d*f*x) + (2*I)*d*(e + f*x)*Cosh[c + d*x] -
 (2*I)*f*Sinh[c + d*x])/(a*d^2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {6097, 17, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6097}$$

$$\frac{\int (e+fx)dx}{a}-\frac{i\int (e+fx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{17}$$

$$\frac{(e+fx)^2}{2af}-\frac{i\int (e+fx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^2}{2af}-\frac{i\int -i(e+fx)\sin(ic+idx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{(e+fx)^2}{2af}-\frac{\int (e+fx)\sin(ic+idx)dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(e+fx)^2}{2af}-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^2}{2af}-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}$$

$$\Big\downarrow \text{3117}$$

$$\frac{(e+fx)^2}{2af}-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}$$

input
```
Int[((e + f*x)*Cosh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(e + f*x)^2/(2*a*f) - ((I*(e + f*x)*Cosh[c + d*x])/d - (I*f*Sinh[c + d*x])
/d^2)/a
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6097
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cosh[c
 + d*x]^(n - 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*
Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && E
qQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.25

method	result	size
risch	$\frac{f\,x^{2}}{2a}+\frac{ex}{a}-\frac{i(dxf+de-f)\,{\mathrm e}^{dx+c}}{2a\,d^{2}}-\frac{i(dxf+de+f)\,{\mathrm e}^{-dx-c}}{2a\,d^{2}}$	70
derivativedivides	$-\frac{-ifc\cosh(dx+c)+i\cosh(dx+c)de+if((dx+c)\cosh(dx+c)-\sinh(dx+c))+fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^{2}}{2}}{d^{2}a}$	84
default	$-\frac{-ifc\cosh(dx+c)+i\cosh(dx+c)de+if((dx+c)\cosh(dx+c)-\sinh(dx+c))+fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^{2}}{2}}{d^{2}a}$	84

input
```
int((f*x+e)*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*f*x^2+1/a*e*x-1/2*I*(d*f*x+d*e-f)/a/d^2*exp(d*x+c)-1/2*I*(d*f*x+d*e+
f)/a/d^2*exp(-d*x-c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.36

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$=\frac{\left(-i\,dfx-i\,de+\left(-i\,dfx-i\,de+i\,f\right)e^{(2\,dx+2\,c)}+\left(d^2fx^2+2\,d^2ex\right)e^{(dx+c)}-i\,f\right)e^{(-dx-c)}}{2\,ad^2}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(-I*d*f*x - I*d*e + (-I*d*f*x - I*d*e + I*f)*e^(2*d*x + 2*c) + (d^2*f*
x^2 + 2*d^2*e*x)*e^(d*x + c) - I*f)*e^(-d*x - c)/(a*d^2)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.98

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \begin{cases} \frac{\left(\left(-2iad^3e-2iad^3fx-2iad^2f\right)e^{-dx}+\left(-2iad^3ee^{2c}-2iad^3fxe^{2c}+2iad^2fe^{2c}\right)e^{dx}\right)e^{-c}}{4a^2d^4} & \text{for } a^2d^4e^c\neq 0 \\ \frac{x^2\left(-ife^{2c}+if\right)e^{-c}}{4a}+\frac{x\left(-iee^{2c}+ie\right)e^{-c}}{2a} & \text{otherwise} \end{cases} + \frac{ex}{a} + \frac{fx^2}{2a}$$

input
```
integrate((f*x+e)*cosh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
Piecewise((((-2*I*a*d**3*e - 2*I*a*d**3*f*x - 2*I*a*d**2*f)*exp(-d*x) + (-
2*I*a*d**3*e*exp(2*c) - 2*I*a*d**3*f*x*exp(2*c) + 2*I*a*d**2*f*exp(2*c))*e
xp(d*x))*exp(-c)/(4*a**2*d**4), Ne(a**2*d**4*exp(c), 0)), (x**2*(-I*f*exp(
2*c) + I*f)*exp(-c)/(4*a) + x*(-I*e*exp(2*c) + I*e)*exp(-c)/(2*a), True))
+ e*x/a + f*x**2/(2*a)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(53) = 106.

Time = 0.12 (sec) , antiderivative size = 188, normalized size of antiderivative = 3.36

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{1}{2}f\left(\frac{2\,xe^{(dx+c)}}{ade^{(dx+c)}-i\,ad} - \frac{i\,d^2x^2e^c+i\,dxe^c-\left(-i\,dxe^{(3\,c)}+i\,e^{(3\,c)}\right)e^{(2\,dx)}-\left(d^2x^2e^{(2\,c)}-3\,dxe^{(2\,c)}+e^{(2\,c)}\right)e}{ad^2e^{(dx+2\,c)}-i\,ad^2e^c}\right) + \frac{1}{2}e\left(\frac{2\,(dx+c)}{ad}-\frac{i\,e^{(dx+c)}}{ad}-\frac{i\,e^{(-dx-c)}}{ad}\right)$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*f*(2*x*e^(d*x + c)/(a*d*e^(d*x + c) - I*a*d) - (I*d^2*x^2*e^c + I*d*x*
e^c - (-I*d*x*e^(3*c) + I*e^(3*c))*e^(2*d*x) - (d^2*x^2*e^(2*c) - 3*d*x*e^
(2*c) + e^(2*c))*e^(d*x) + (d*x + 1)*e^(-d*x) + I*e^c)/(a*d^2*e^(d*x + 2*c
) - I*a*d^2*e^c)) + 1/2*e*(2*(d*x + c)/(a*d) - I*e^(d*x + c)/(a*d) - I*e^(
-d*x - c)/(a*d))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.71

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(d^2fx^2e^{(dx+c)} + 2\,d^2exe^{(dx+c)} - i\,dfxe^{(2\,dx+2\,c)} - i\,dfx - i\,dee^{(2\,dx+2\,c)} - i\,de + i\,fe^{(2\,dx+2\,c)} - i\,f\right)e^{(-dx-c)}}{2\,ad^2}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(d^2*f*x^2*e^(d*x + c) + 2*d^2*e*x*e^(d*x + c) - I*d*f*x*e^(2*d*x + 2*
c) - I*d*f*x - I*d*e*e^(2*d*x + 2*c) - I*d*e + I*f*e^(2*d*x + 2*c) - I*f)*
e^(-d*x - c)/(a*d^2)
```

Mupad [B] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.55

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{f\,x^2}{2\,a} + \mathrm{e}^{c+d\,x}\left(\frac{(f-d\,e)\ 1\mathrm{i}}{2\,a\,d^2} - \frac{f\,x\,1\mathrm{i}}{2\,a\,d}\right) - \mathrm{e}^{-c-d\,x}\left(\frac{(f+d\,e)\ 1\mathrm{i}}{2\,a\,d^2} + \frac{f\,x\,1\mathrm{i}}{2\,a\,d}\right) + \frac{e\,x}{a}$$

input
```
int((cosh(c + d*x)^2*(e + f*x))/(a + a*sinh(c + d*x)*1i),x)
```

output
```
exp(c + d*x)*(((f - d*e)*1i)/(2*a*d^2) - (f*x*1i)/(2*a*d)) - exp(- c - d*x
)*(((f + d*e)*1i)/(2*a*d^2) + (f*x*1i)/(2*a*d)) + (f*x^2)/(2*a) + (e*x)/a
```

Reduce [F]

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(\int \frac{\cosh(dx+c)^2}{\sinh(dx+c)i+1}dx\right)e+\left(\int \frac{\cosh(dx+c)^2x}{\sinh(dx+c)i+1}dx\right)f}{a}$$

input
```
int((f*x+e)*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
(int(cosh(c + d*x)**2/(sinh(c + d*x)*i + 1),x)*e + int((cosh(c + d*x)**2*x
)/(sinh(c + d*x)*i + 1),x)*f)/a
```

3.262 $\int \frac{\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 22

$$\int \frac{\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx=\frac{x}{a}-\frac{i\cosh(c+dx)}{ad}$$

output
```
x/a-I*cosh(d*x+c)/a/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 139 vs. 2(22) = 44.

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 6.32

$$\int \frac{\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx \\ =\frac{\cosh^3(c+dx)\left(-2\arcsin\left(\frac{\sqrt{1-i\sinh(c+dx)}}{\sqrt{2}}\right)\sqrt{1-i\sinh(c+dx)}+\sqrt{1+i\sinh(c+dx)}-i\sqrt{1+i\sinh(c}{ad\sqrt{1+i\sinh(c+dx)}(-i+\sinh(c+dx))(i+\sinh(c+dx))^2}$$

input
```
Integrate[Cosh[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(Cosh[c + d*x]^3*(-2*ArcSin[Sqrt[1 - I*Sinh[c + d*x]]/Sqrt[2]]*Sqrt[1 - I*
Sinh[c + d*x]] + Sqrt[1 + I*Sinh[c + d*x]] - I*Sqrt[1 + I*Sinh[c + d*x]]*S
inh[c + d*x]))/(a*d*Sqrt[1 + I*Sinh[c + d*x]]*(-I + Sinh[c + d*x])*(I + Si
nh[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(ic+idx)^2}{a+a\sin(ic+idx)}dx$$

$$\Big\downarrow \text{ 3161}$$

$$\frac{\int 1dx}{a}-\frac{i\cosh(c+dx)}{ad}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{x}{a}-\frac{i\cosh(c+dx)}{ad}$$

input
```
Int[Cosh[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
```

output
```
x/a - (I*Cosh[c + d*x])/(a*d)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

method	result	size
risch	$\frac{x}{a}-\frac{i e^{dx+c}}{2ad}-\frac{i e^{-dx-c}}{2ad}$	40
derivativedivides	$\frac{\frac{2i}{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2}-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{i}{1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	70
default	$\frac{\frac{2i}{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2}-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{i}{1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	70

input
```
int(cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
x/a-1/2*I/a/d*exp(d*x+c)-1/2*I/a/d*exp(-d*x-c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(2\,dxe^{(dx+c)} - i\,e^{(2\,dx+2\,c)} - i\right)e^{(-dx-c)}}{2\,ad}$$

input
```
integrate(cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*d*x*e^(d*x + c) - I*e^(2*d*x + 2*c) - I)*e^(-d*x - c)/(a*d)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.55

$$\int \frac{\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \begin{cases} \frac{(-2iade^{2c}e^{dx}-2iade^{-dx})e^{-c}}{4a^2d^2} & \text{for } a^2d^2e^c \neq 0 \\ x\left(\frac{(-ie^{2c}+2e^c+i)e^{-c}}{2a} - \frac{1}{a}\right) & \text{otherwise} \end{cases} + \frac{x}{a}$$

input
```
integrate(cosh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
Piecewise(((-2*I*a*d*exp(2*c)*exp(d*x) - 2*I*a*d*exp(-d*x))*exp(-c)/(4*a**
2*d**2), Ne(a**2*d**2*exp(c), 0)), (x*((-I*exp(2*c) + 2*exp(c) + I)*exp(-c
)/(2*a) - 1/a), True)) + x/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(20) = 40.

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{dx+c}{ad} - \frac{i\,e^{(dx+c)}}{2\,ad} - \frac{i\,e^{(-dx-c)}}{2\,ad}$$

input
```
integrate(cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
(d*x + c)/(a*d) - 1/2*I*e^(d*x + c)/(a*d) - 1/2*I*e^(-d*x - c)/(a*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(20) = 40.

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.86

$$\int \frac{\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\frac{2\,(dx+c)}{a} - \frac{i\,e^{(dx+c)}}{a} - \frac{i\,e^{(-dx-c)}}{a}}{2\,d}$$

input
```
integrate(cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*(d*x + c)/a - I*e^(d*x + c)/a - I*e^(-d*x - c)/a)/d
```

Mupad [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{x}{a} - \frac{\frac{\mathrm{e}^{c+d\,x}\,1\mathrm{i}}{2} + \frac{\mathrm{e}^{-c-d\,x}\,1\mathrm{i}}{2}}{a\,d}$$

input
```
int(cosh(c + d*x)^2/(a + a*sinh(c + d*x)*1i),x)
```

output
```
x/a - ((exp(c + d*x)*1i)/2 + (exp(- c - d*x)*1i)/2)/(a*d)
```

Reduce [F]

$$\int \frac{\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx=\frac{\int \frac{\cosh(dx+c)^2}{\sinh(dx+c)i+1}dx}{a}$$

input
```
int(cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(cosh(c + d*x)**2/(sinh(c + d*x)*i + 1),x)/a
```

3.263 $\int \frac{\cosh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 76

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \frac{\log(e+fx)}{af} - \frac{i\text{Chi}\left(\frac{de}{f}+dx\right)\sinh\left(c-\frac{de}{f}\right)}{af} - \frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{af}$$

output
```
ln(f*x+e)/a/f-I*Chi(d*e/f+d*x)*sinh(c-d*e/f)/a/f-I*cosh(c-d*e/f)*Shi(d*e/f
+d*x)/a/f
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \frac{\log(e+fx) - i\text{Chi}\left(d\left(\frac{e}{f}+x\right)\right)\sinh\left(c-\frac{de}{f}\right) - i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(d\left(\frac{e}{f}+x\right)\right)}{af}$$

input
```
Integrate[Cosh[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
(Log[e + f*x] - I*CoshIntegral[d*(e/f + x)]*Sinh[c - (d*e)/f] - I*Cosh[c -
 (d*e)/f]*SinhIntegral[d*(e/f + x)])/(a*f)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {6097, 16, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}dx$$

$$\Big\downarrow \text{6097}$$

$$\frac{\int \frac{1}{e+fx}dx}{a}-\frac{i\int \frac{\sinh(c+dx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{16}$$

$$\frac{\log(e+fx)}{af}-\frac{i\int \frac{\sinh(c+dx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\log(e+fx)}{af}-\frac{i\int -\frac{i\sin(ic+idx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\log(e+fx)}{af}-\frac{\int \frac{\sin(ic+idx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{3784}$$

$$\frac{\log(e+fx)}{af}-\frac{i\sinh\left(c-\frac{de}{f}\right)\int \frac{\cosh\left(\frac{de}{f}+dx\right)}{e+fx}dx+\cosh\left(c-\frac{de}{f}\right)\int \frac{i\sinh\left(\frac{de}{f}+dx\right)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\log(e+fx)}{af}-\frac{i\sinh\left(c-\frac{de}{f}\right)\int\frac{\cosh\left(\frac{de}{f}+dx\right)}{e+fx}dx+i\cosh\left(c-\frac{de}{f}\right)\int\frac{\sinh\left(\frac{de}{f}+dx\right)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\log(e+fx)}{af}-\frac{i\sinh\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{ide}{f}+idx+\frac{\pi}{2}\right)}{e+fx}dx+i\cosh\left(c-\frac{de}{f}\right)\int-\frac{i\sin\left(\frac{ide}{f}+idx\right)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{\log(e+fx)}{af}-\frac{i\sinh\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{ide}{f}+idx+\frac{\pi}{2}\right)}{e+fx}dx+\cosh\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{ide}{f}+idx\right)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{ 3779}$$

$$\frac{\log(e+fx)}{af}-\frac{i\sinh\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{ide}{f}+idx+\frac{\pi}{2}\right)}{e+fx}dx+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}}{a}$$

$$\Big\downarrow \text{ 3782}$$

$$\frac{\log(e+fx)}{af}-\frac{\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}}{a}$$

input
```
Int[Cosh[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Log[e + f*x]/(a*f) - ((I*CoshIntegral[(d*e)/f + d*x]*Sinh[c - (d*e)/f])/f
+ (I*Cosh[c - (d*e)/f]*SinhIntegral[(d*e)/f + d*x])/f)/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6097
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cosh[c
 + d*x]^(n - 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*
Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && E
qQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 10.61 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.36

method	result	size
risch	$\frac{\ln(fx+e)}{af}+\frac{ie^{\frac{cf-de}{f}}\operatorname{expIntegral}_1\left(-dx-c-\frac{-cf+de}{f}\right)}{2af}-\frac{ie^{-\frac{cf-de}{f}}\operatorname{expIntegral}_1\left(dx+c-\frac{cf-de}{f}\right)}{2af}$	103

input
```
int(cosh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
ln(f*x+e)/a/f+1/2*I/a/f*exp((c*f-d*e)/f)*Ei(1,-d*x-c-(-c*f+d*e)/f)-1/2*I/a
/f*exp(-(c*f-d*e)/f)*Ei(1,d*x+c-(c*f-d*e)/f)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \frac{i\,\mathrm{Ei}\left(-\frac{dfx+de}{f}\right)e^{\left(\frac{de-cf}{f}\right)} - i\,\mathrm{Ei}\left(\frac{dfx+de}{f}\right)e^{\left(-\frac{de-cf}{f}\right)} + 2\,\log\left(\frac{fx+e}{f}\right)}{2\,af}$$

input
```
integrate(cosh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(I*Ei(-(d*f*x + d*e)/f)*e^((d*e - c*f)/f) - I*Ei((d*f*x + d*e)/f)*e^(-
(d*e - c*f)/f) + 2*log((f*x + e)/f))/(a*f)
```

Sympy [F]

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = -\frac{i\int \frac{\cosh^2(c+dx)}{e\sinh(c+dx)-ie+fx\sinh(c+dx)-ifx}\,dx}{a}$$

input
```
integrate(cosh(d*x+c)**2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(cosh(c + d*x)**2/(e*sinh(c + d*x) - I*e + f*x*sinh(c + d*x) -
I*f*x), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = -\frac{i\,e^{\left(-c+\frac{de}{f}\right)}E_1\left(\frac{(fx+e)d}{f}\right)}{2\,af} + \frac{i\,e^{\left(c-\frac{de}{f}\right)}E_1\left(-\frac{(fx+e)d}{f}\right)}{2\,af} + \frac{\log\left(fx+e\right)}{af}$$

input
```
integrate(cosh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*I*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(a*f) + 1/2*I*e^(c
- d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(a*f) + log(f*x + e)/(a*f)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = -\frac{\left(i\,\mathrm{Ei}\left(\frac{dfx+de}{f}\right)e^{\left(2\,c-\frac{de}{f}\right)} - i\,\mathrm{Ei}\left(-\frac{dfx+de}{f}\right)e^{\left(\frac{de}{f}\right)} - 2\,e^{c}\log\left(i\,fx+i\,e\right)\right)e^{(-c)}}{2\,af}$$

input
```
integrate(cosh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(I*Ei((d*f*x + d*e)/f)*e^(2*c - d*e/f) - I*Ei(-(d*f*x + d*e)/f)*e^(d*
e/f) - 2*e^c*log(I*f*x + I*e))*e^(-c)/(a*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)^2}{(e+f\,x)\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(cosh(c + d*x)^2/((e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(cosh(c + d*x)^2/((e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [F]

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \frac{\int \frac{\cosh(dx+c)^2}{\sinh(dx+c)ei+\sinh(dx+c)fix+e+fx}dx}{a}$$

input
```
int(cosh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(cosh(c + d*x)**2/(sinh(c + d*x)*e*i + sinh(c + d*x)*f*i*x + e + f*x),x
)/a
```

3.264 $\int \frac{\cosh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 103

$$\int \frac{\cosh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = -\frac{1}{af(e+fx)} - \frac{id\cosh\left(c-\frac{de}{f}\right)\mathrm{Chi}\left(\frac{de}{f}+dx\right)}{af^2} + \frac{i\sinh(c+dx)}{af(e+fx)} - \frac{id\sinh\left(c-\frac{de}{f}\right)\mathrm{Shi}\left(\frac{de}{f}+dx\right)}{af^2}$$

output
```
-1/a/f/(f*x+e)-I*d*cosh(c-d*e/f)*Chi(d*e/f+d*x)/a/f^2+I*sinh(d*x+c)/a/f/(f
*x+e)-I*d*sinh(c-d*e/f)*Shi(d*e/f+d*x)/a/f^2
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.83

$$\int \frac{\cosh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = -\frac{i\left(d(e+fx)\cosh\left(c-\frac{de}{f}\right)\mathrm{Chi}\left(d\left(\frac{e}{f}+x\right)\right)-f(i+\sinh(c+dx))+d(e+fx)\sinh\left(c-\frac{de}{f}\right)\mathrm{Shi}\left(d\left(\frac{e}{f}\right.\right.\right.}{af^2(e+fx)}$$

input
```
Integrate[Cosh[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
((-I)*(d*(e + f*x)*Cosh[c - (d*e)/f]*CoshIntegral[d*(e/f + x)] - f*(I + Si
nh[c + d*x]) + d*(e + f*x)*Sinh[c - (d*e)/f]*SinhIntegral[d*(e/f + x)]))/(
a*f^2*(e + f*x))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {6097, 17, 3042, 26, 3778, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}dx$$

$$\Big\downarrow \text{6097}$$

$$\frac{\int \frac{1}{(e+fx)^2}dx}{a}-\frac{i\int \frac{\sinh(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{17}$$

$$-\frac{1}{af(e+fx)}-\frac{i\int \frac{\sinh(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{af(e+fx)}-\frac{i\int -\frac{i\sin(ic+idx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{1}{af(e+fx)}-\frac{\int \frac{\sin(ic+idx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{1}{af(e+fx)}-\frac{\frac{id\int \frac{\cosh(c+dx)}{e+fx}dx}{f}-\frac{i\sinh(c+dx)}{f(e+fx)}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{af(e+fx)}-\frac{\frac{id\int\frac{\sin\left(ic+idx+\frac{\pi}{2}\right)}{e+fx}dx}{f}-\frac{i\sinh(c+dx)}{f(e+fx)}}{a}$$

$\Big\downarrow$ 3784

$$-\frac{1}{af(e+fx)}-\frac{\frac{id\left(\cosh\left(c-\frac{de}{f}\right)\int\frac{\cosh\left(\frac{de}{f}+dx\right)}{e+fx}dx-i\sinh\left(c-\frac{de}{f}\right)\int\frac{i\sinh\left(\frac{de}{f}+dx\right)}{e+fx}dx\right)}{f}-\frac{i\sinh(c+dx)}{f(e+fx)}}{a}$$

$\Big\downarrow$ 26

$$-\frac{1}{af(e+fx)}-\frac{\frac{id\left(\sinh\left(c-\frac{de}{f}\right)\int\frac{\sinh\left(\frac{de}{f}+dx\right)}{e+fx}dx+\cosh\left(c-\frac{de}{f}\right)\int\frac{\cosh\left(\frac{de}{f}+dx\right)}{e+fx}dx\right)}{f}-\frac{i\sinh(c+dx)}{f(e+fx)}}{a}$$

$\Big\downarrow$ 3042

$$-\frac{1}{af(e+fx)}-\frac{\frac{id\left(\sinh\left(c-\frac{de}{f}\right)\int-\frac{i\sin\left(\frac{ide}{f}+idx\right)}{e+fx}dx+\cosh\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{ide}{f}+idx+\frac{\pi}{2}\right)}{e+fx}dx\right)}{f}-\frac{i\sinh(c+dx)}{f(e+fx)}}{a}$$

$\Big\downarrow$ 26

$$-\frac{1}{af(e+fx)}-\frac{\frac{id\left(\cosh\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{ide}{f}+idx+\frac{\pi}{2}\right)}{e+fx}dx-i\sinh\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{ide}{f}+idx\right)}{e+fx}dx\right)}{f}-\frac{i\sinh(c+dx)}{f(e+fx)}}{a}$$

$\Big\downarrow$ 3779

$$-\frac{1}{af(e+fx)}-\frac{\frac{id\left(\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}+\cosh\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{ide}{f}+idx+\frac{\pi}{2}\right)}{e+fx}dx\right)}{f}-\frac{i\sinh(c+dx)}{f(e+fx)}}{a}$$

$\Big\downarrow$ 3782

$$-\frac{1}{af(e+fx)}-\frac{\frac{id\left(\frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}-\frac{i\sinh(c+dx)}{f(e+fx)}}{a}$$

input
```
Int[Cosh[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
-(1/(a*f*(e + f*x))) - (((-I)*Sinh[c + d*x])/(f*(e + f*x)) + (I*d*((Cosh[c
 - (d*e)/f]*CoshIntegral[(d*e)/f + d*x])/f + (Sinh[c - (d*e)/f]*SinhIntegr
al[(d*e)/f + d*x])/f))/f)/a
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6097
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cosh[c
 + d*x]^(n - 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*
Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && E
qQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 25.81 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.59

method	result
risch	$-\frac{1}{af(fx+e)}+\frac{id\,\mathrm{e}^{dx+c}}{2a\,f^2\left(\frac{de}{f}+dx\right)}+\frac{id\,\mathrm{e}^{\frac{cf-de}{f}}\,\mathrm{expIntegral}_1\left(-dx-c-\frac{-cf+de}{f}\right)}{2a\,f^2}-\frac{id\,\mathrm{e}^{-dx-c}}{2af(dxf+de)}+\frac{id\,\mathrm{e}^{-\frac{cf-de}{f}}\,\mathrm{expIntegral}_1\left(\right.}{2a\,f^2}$

input
```
int(cosh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/a/f/(f*x+e)+1/2*I*d/a/f^2*exp(d*x+c)/(d*e/f+d*x)+1/2*I*d/a/f^2*exp((c*f
-d*e)/f)*Ei(1,-d*x-c-(-c*f+d*e)/f)-1/2*I/a*d*exp(-d*x-c)/f/(d*f*x+d*e)+1/2
*I/a*d/f^2*exp(-(c*f-d*e)/f)*Ei(1,d*x+c-(c*f-d*e)/f)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.25

$$\int \frac{\cosh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \frac{\left(i\,fe^{(2\,dx+2\,c)}+\left((-i\,dfx-i\,de)\mathrm{Ei}\left(-\frac{dfx+de}{f}\right)e^{\left(\frac{de-cf}{f}\right)}+(-i\,dfx-i\,de)\mathrm{Ei}\left(\frac{dfx+de}{f}\right)e^{\left(-\frac{de-cf}{f}\right)}-2\,f\right)e^{(dx}\right.}{2\,(af^3x+aef^2)}$$

input
```
integrate(cosh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/2*(I*f*e^(2*d*x + 2*c) + ((-I*d*f*x - I*d*e)*Ei(-(d*f*x + d*e)/f)*e^((d*
e - c*f)/f) + (-I*d*f*x - I*d*e)*Ei((d*f*x + d*e)/f)*e^(-(d*e - c*f)/f) -
2*f)*e^(d*x + c) - I*f)*e^(-d*x - c)/(a*f^3*x + a*e*f^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**2/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.89

$$\int \frac{\cosh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = -\frac{1}{af^2x+aef} - \frac{i\,e^{\left(-c+\frac{de}{f}\right)}E_2\left(\frac{(fx+e)d}{f}\right)}{2\,(fx+e)af} + \frac{i\,e^{\left(c-\frac{de}{f}\right)}E_2\left(-\frac{(fx+e)d}{f}\right)}{2\,(fx+e)af}$$

input
```
integrate(cosh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-1/(a*f^2*x + a*e*f) - 1/2*I*e^(-c + d*e/f)*exp_integral_e(2, (f*x + e)*d/
f)/((f*x + e)*a*f) + 1/2*I*e^(c - d*e/f)*exp_integral_e(2, -(f*x + e)*d/f)
/((f*x + e)*a*f)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(97) = 194$.

Time = 0.18 (sec) , antiderivative size = 572, normalized size of antiderivative = 5.55

$$\int \frac{\cosh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx =$$

$$-\frac{\left(i\,(fx+e)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)d^2\mathrm{Ei}\left(-\frac{(fx+e)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)+de-cf}{f}\right)e^{\left(\frac{de-cf}{f}\right)}+i\,d^3e\mathrm{Ei}\left(-\frac{(fx+e)\left(d-\frac{de}{fx+e}\right.}{f}\right.\right.}{}$$

input
```
integrate(cosh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(I*(f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e))*d^2*Ei(-((f*x + e)*(
d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e - c*f)/f)*e^((d*e - c*f)/f) + I*d
^3*e*Ei(-((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e - c*f)/f)*e^
((d*e - c*f)/f) - I*c*d^2*f*Ei(-((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x +
 e)) + d*e - c*f)/f)*e^((d*e - c*f)/f) + I*(f*x + e)*(d - d*e/(f*x + e) +
c*f/(f*x + e))*d^2*Ei(((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e
 - c*f)/f)*e^(-(d*e - c*f)/f) + I*d^3*e*Ei(((f*x + e)*(d - d*e/(f*x + e) +
 c*f/(f*x + e)) + d*e - c*f)/f)*e^(-(d*e - c*f)/f) - I*c*d^2*f*Ei(((f*x +
e)*(d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e - c*f)/f)*e^(-(d*e - c*f)/f)
- I*d^2*f*e^((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e))/f) + I*d^2*f*e^
(-(f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e))/f) + 2*d^2*f)*f^2/(((f*x +
 e)*a*(d - d*e/(f*x + e) + c*f/(f*x + e))*f^4 + a*d*e*f^4 - a*c*f^5)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{{\cosh\left(c+d\,x\right)}^2}{{\left(e+f\,x\right)}^2\,\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

input
```
int(cosh(c + d*x)^2/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(cosh(c + d*x)^2/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [F]

$$\int \frac{\cosh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \frac{\int \frac{\cosh(dx+c)^2}{\sinh(dx+c)e^2i+2\sinh(dx+c)efix+\sinh(dx+c)f^2i\,x^2+e^2+2efx+f^2x^2}dx}{a}$$

input
```
int(cosh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(cosh(c + d*x)**2/(sinh(c + d*x)*e**2*i + 2*sinh(c + d*x)*e*f*i*x + sin
h(c + d*x)*f**2*i*x**2 + e**2 + 2*e*f*x + f**2*x**2),x)/a
```

3.265 $\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 231

$$\begin{aligned}\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx =& -\frac{3if^3x}{8ad^3}-\frac{i(e+fx)^3}{4ad}-\frac{6f^3\cosh(c+dx)}{ad^4}\\ &-\frac{3f(e+fx)^2\cosh(c+dx)}{ad^2}\\ &+\frac{6f^2(e+fx)\sinh(c+dx)}{ad^3}+\frac{(e+fx)^3\sinh(c+dx)}{ad}\\ &+\frac{3if^3\cosh(c+dx)\sinh(c+dx)}{8ad^4}\\ &+\frac{3if(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{4ad^2}\\ &-\frac{3if^2(e+fx)\sinh^2(c+dx)}{4ad^3}\\ &-\frac{i(e+fx)^3\sinh^2(c+dx)}{2ad}\end{aligned}$$

output
```
-3/8*I*f^3*x/a/d^3-1/4*I*(f*x+e)^3/a/d-6*f^3*cosh(d*x+c)/a/d^4-3*f*(f*x+e)
^2*cosh(d*x+c)/a/d^2+6*f^2*(f*x+e)*sinh(d*x+c)/a/d^3+(f*x+e)^3*sinh(d*x+c)
/a/d+3/8*I*f^3*cosh(d*x+c)*sinh(d*x+c)/a/d^4+3/4*I*f*(f*x+e)^2*cosh(d*x+c)
*sinh(d*x+c)/a/d^2-3/4*I*f^2*(f*x+e)*sinh(d*x+c)^2/a/d^3-1/2*I*(f*x+e)^3*s
inh(d*x+c)^2/a/d
```

Mathematica [A] (verified)

Time = 1.67 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.58

$$\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{-96f(2f^2+d^2(e+fx)^2)\cosh(c+dx)-4id(e+fx)\left(3f^2+2d^2(e+fx)^2\right)\cosh(2(c+dx))+4(8d(e+}{32ad^4}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(-96*f*(2*f^2 + d^2*(e + f*x)^2)*Cosh[c + d*x] - (4*I)*d*(e + f*x)*(3*f^2
+ 2*d^2*(e + f*x)^2)*Cosh[2*(c + d*x)] + 4*(8*d*(e + f*x)*(6*f^2 + d^2*(e
+ f*x)^2) + (3*I)*f*(f^2 + 2*d^2*(e + f*x)^2)*Cosh[c + d*x])*Sinh[c + d*x]
)/(32*a*d^4)
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.97, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.742$, Rules used = {6097, 3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118, 5969, 3042, 25, 3792, 17, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6097}$$

$$\frac{\int (e+fx)^3\cosh(c+dx)dx}{a}-\frac{i\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (e+fx)^3\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{a}-\frac{i\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3if\int -i(e+fx)^2\sinh(c+dx)dx}{d}}{a}-\frac{i\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int(e+fx)^2\sinh(c+dx)dx}{d}}{a}-\frac{i\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int -i(e+fx)^2\sin(ic+idx)dx}{d}}{a}-\frac{i\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\int(e+fx)^2\sin(ic+idx)dx}{d}}{a}-\frac{i\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\cosh(c+dx)dx}{d}\right)}{d}}{a}-\frac{i\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}}{a}-\frac{i\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{d}\right)}{d}}{a}-\frac{i\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}\right)}{d}\right)}{d}}{a}-\frac{i\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}\right)}{d}}{a}-\frac{i\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int\sin(ic+idx)dx}{d}\right)}{d}\right)}{d}}{a}-\frac{i\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3118}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}-\frac{i\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{5969}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}-\frac{i\left(\frac{(e+fx)^3\sinh^2(c+dx)}{2d}-\frac{3f\int(e+fx)^2\sinh^2(c+dx)dx}{2d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}-\frac{i\left(\frac{(e+fx)^3\sinh^2(c+dx)}{2d}-\frac{3f\int-(e+fx)^2\sin(ic+idx)^2dx}{2d}\right)}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}-\frac{i\left(\frac{(e+fx)^3\sinh^2(c+dx)}{2d}+\frac{3f\int(e+fx)^2\sin(ic+idx)^2dx}{2d}\right)}{a}$$

$$\Big\downarrow \; 3792$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}-\frac{i\left(\frac{3f\left(\frac{f^2\int-\sinh^2(c+dx)dx}{2d^2}+\frac{1}{2}\int(e+fx)^2dx+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \; 17$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}-\frac{i\left(\frac{3f\left(\frac{f^2\int-\sinh^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}-\frac{i\left(\frac{3f\left(-\frac{f^2\int\sinh^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}-\frac{i\left(\frac{(e+fx)^3\sinh^2(c+dx)}{2d}+\frac{3f\left(-\frac{f^2\int-\sin(ic+idx)^2dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}\right)}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}-\frac{i\left(\frac{(e+fx)^3\sinh^2(c+dx)}{2d}+\frac{3f\left(\frac{f^2\int\sin(ic+idx)^2dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}\right)}{a}$$

$$\Big\downarrow \; 3115$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}-\frac{i\left(\frac{3f\left(\frac{f^2\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \; 24$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}-\frac{i\left(\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}\right)}{a}$$

input

```
Int[((e + f*x)^3*Cosh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output

```
(((e + f*x)^3*Sinh[c + d*x])/d + ((3*I)*f*((I*(e + f*x)^2*Cosh[c + d*x])/d
 - ((2*I)*f*(-((f*Cosh[c + d*x])/d^2) + ((e + f*x)*Sinh[c + d*x])/d))/d))/
d)/a - (I*(((e + f*x)^3*Sinh[c + d*x]^2)/(2*d) + (3*f*((e + f*x)^3/(6*f) -
 ((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (f*(e + f*x)*Sinh[c + d
*x]^2)/(2*d^2) + (f^2*(x/2 - (Cosh[c + d*x]*Sinh[c + d*x])/(2*d)))/(2*d^2)
))/(2*d)))/a
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6097
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cosh[c
 + d*x]^(n - 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*
Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && E
qQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 428 vs. 2(213) = 426.

Time = 31.39 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.86

method	result
risch	$-\frac{i\left(4d^3x^3f^3+12d^3e\,f^2x^2+12d^3e^2fx-6d^2f^3x^2+4d^3e^3-12d^2e\,f^2x-6d^2e^2f+6d\,f^3x+6de\,f^2-3f^3\right)\mathrm{e}^{2dx+2c}}{32d^4a}+\frac{(d^3x}{}$
derivativedivides	$-\frac{3id^2e^2f\left(\frac{(dx+c)\cosh(dx+c)^2}{2}-\frac{\sinh(dx+c)\cosh(dx+c)}{4}-\frac{dx}{4}-\frac{c}{4}\right)-6icde\,f^2\left(\frac{(dx+c)\cosh(dx+c)^2}{2}-\frac{\sinh(dx+c)\cosh(dx+c)}{4}\right.}{}$
default	$-\frac{3id^2e^2f\left(\frac{(dx+c)\cosh(dx+c)^2}{2}-\frac{\sinh(dx+c)\cosh(dx+c)}{4}-\frac{dx}{4}-\frac{c}{4}\right)-6icde\,f^2\left(\frac{(dx+c)\cosh(dx+c)^2}{2}-\frac{\sinh(dx+c)\cosh(dx+c)}{4}\right.}{}$

input
```
int((f*x+e)^3*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/32*I*(4*d^3*f^3*x^3+12*d^3*e*f^2*x^2+12*d^3*e^2*f*x-6*d^2*f^3*x^2+4*d^3
*e^3-12*d^2*e*f^2*x-6*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2-3*f^3)/d^4/a*exp(2*d*x
+2*c)+1/2*(d^3*f^3*x^3+3*d^3*e*f^2*x^2+3*d^3*e^2*f*x-3*d^2*f^3*x^2+d^3*e^3
-6*d^2*e*f^2*x-3*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2-6*f^3)/d^4/a*exp(d*x+c)-1/2
*(d^3*f^3*x^3+3*d^3*e*f^2*x^2+3*d^3*e^2*f*x+3*d^2*f^3*x^2+d^3*e^3+6*d^2*e*
f^2*x+3*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2+6*f^3)/d^4/a*exp(-d*x-c)-1/32*I*(4*d
^3*f^3*x^3+12*d^3*e*f^2*x^2+12*d^3*e^2*f*x+6*d^2*f^3*x^2+4*d^3*e^3+12*d^2*
e*f^2*x+6*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2+3*f^3)/a/d^4*exp(-2*d*x-2*c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.75

$$\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(-4i\,d^3f^3x^3-4i\,d^3e^3-6i\,d^2e^2f-6i\,def^2-3i\,f^3-6\left(2i\,d^3ef^2+i\,d^2f^3\right)x^2-6\left(2i\,d^3e^2f+2i\,d^2ef^2+\right.\right.}{}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/32*(-4*I*d^3*f^3*x^3 - 4*I*d^3*e^3 - 6*I*d^2*e^2*f - 6*I*d*e*f^2 - 3*I*f
^3 - 6*(2*I*d^3*e*f^2 + I*d^2*f^3)*x^2 - 6*(2*I*d^3*e^2*f + 2*I*d^2*e*f^2
+ I*d*f^3)*x + (-4*I*d^3*f^3*x^3 - 4*I*d^3*e^3 + 6*I*d^2*e^2*f - 6*I*d*e*f
^2 + 3*I*f^3 - 6*(2*I*d^3*e*f^2 - I*d^2*f^3)*x^2 - 6*(2*I*d^3*e^2*f - 2*I*
d^2*e*f^2 + I*d*f^3)*x)*e^(4*d*x + 4*c) + 16*(d^3*f^3*x^3 + d^3*e^3 - 3*d^
2*e^2*f + 6*d*e*f^2 - 6*f^3 + 3*(d^3*e*f^2 - d^2*f^3)*x^2 + 3*(d^3*e^2*f -
 2*d^2*e*f^2 + 2*d*f^3)*x)*e^(3*d*x + 3*c) - 16*(d^3*f^3*x^3 + d^3*e^3 + 3
*d^2*e^2*f + 6*d*e*f^2 + 6*f^3 + 3*(d^3*e*f^2 + d^2*f^3)*x^2 + 3*(d^3*e^2*
f + 2*d^2*e*f^2 + 2*d*f^3)*x)*e^(d*x + c))*e^(-2*d*x - 2*c)/(a*d^4)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1040 vs. $2(214) = 428$.

Time = 0.56 (sec) , antiderivative size = 1040, normalized size of antiderivative = 4.50

$$\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
Piecewise((((-2048*a**3*d**15*e**3*exp(2*c) - 6144*a**3*d**15*e**2*f*x*exp
(2*c) - 6144*a**3*d**15*e*f**2*x**2*exp(2*c) - 2048*a**3*d**15*f**3*x**3*e
xp(2*c) - 6144*a**3*d**14*e**2*f*exp(2*c) - 12288*a**3*d**14*e*f**2*x*exp(
2*c) - 6144*a**3*d**14*f**3*x**2*exp(2*c) - 12288*a**3*d**13*e*f**2*exp(2*
c) - 12288*a**3*d**13*f**3*x*exp(2*c) - 12288*a**3*d**12*f**3*exp(2*c))*ex
p(-d*x) + (2048*a**3*d**15*e**3*exp(4*c) + 6144*a**3*d**15*e**2*f*x*exp(4*
c) + 6144*a**3*d**15*e*f**2*x**2*exp(4*c) + 2048*a**3*d**15*f**3*x**3*exp(
4*c) - 6144*a**3*d**14*e**2*f*exp(4*c) - 12288*a**3*d**14*e*f**2*x*exp(4*c
) - 6144*a**3*d**14*f**3*x**2*exp(4*c) + 12288*a**3*d**13*e*f**2*exp(4*c)
+ 12288*a**3*d**13*f**3*x*exp(4*c) - 12288*a**3*d**12*f**3*exp(4*c))*exp(d
*x) + (-512*I*a**3*d**15*e**3*exp(c) - 1536*I*a**3*d**15*e**2*f*x*exp(c) -
 1536*I*a**3*d**15*e*f**2*x**2*exp(c) - 512*I*a**3*d**15*f**3*x**3*exp(c)
- 768*I*a**3*d**14*e**2*f*exp(c) - 1536*I*a**3*d**14*e*f**2*x*exp(c) - 768
*I*a**3*d**14*f**3*x**2*exp(c) - 768*I*a**3*d**13*e*f**2*exp(c) - 768*I*a*
*3*d**13*f**3*x*exp(c) - 384*I*a**3*d**12*f**3*exp(c))*exp(-2*d*x) + (-512
*I*a**3*d**15*e**3*exp(5*c) - 1536*I*a**3*d**15*e**2*f*x*exp(5*c) - 1536*I
*a**3*d**15*e*f**2*x**2*exp(5*c) - 512*I*a**3*d**15*f**3*x**3*exp(5*c) + 7
68*I*a**3*d**14*e**2*f*exp(5*c) + 1536*I*a**3*d**14*e*f**2*x*exp(5*c) + 76
8*I*a**3*d**14*f**3*x**2*exp(5*c) - 768*I*a**3*d**13*e*f**2*exp(5*c) - 768
*I*a**3*d**13*f**3*x*exp(5*c) + 384*I*a**3*d**12*f**3*exp(5*c))*exp(2*d...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 618 vs. 2(207) = 414.

Time = 0.14 (sec) , antiderivative size = 618, normalized size of antiderivative = 2.68

$$\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{\left(4i\,d^3f^3x^3e^{(4\,dx+4\,c)}-16\,d^3f^3x^3e^{(3\,dx+3\,c)}+16\,d^3f^3x^3e^{(dx+c)}+4i\,d^3f^3x^3+12i\,d^3ef^2x^2e^{(4\,dx+4\,c)}-48\,d\ldots\right.}{}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/32*(4*I*d^3*f^3*x^3*e^(4*d*x + 4*c) - 16*d^3*f^3*x^3*e^(3*d*x + 3*c) +
16*d^3*f^3*x^3*e^(d*x + c) + 4*I*d^3*f^3*x^3 + 12*I*d^3*e*f^2*x^2*e^(4*d*x
 + 4*c) - 48*d^3*e*f^2*x^2*e^(3*d*x + 3*c) + 48*d^3*e*f^2*x^2*e^(d*x + c)
+ 12*I*d^3*e*f^2*x^2 + 12*I*d^3*e^2*f*x*e^(4*d*x + 4*c) - 6*I*d^2*f^3*x^2*
e^(4*d*x + 4*c) - 48*d^3*e^2*f*x*e^(3*d*x + 3*c) + 48*d^2*f^3*x^2*e^(3*d*x
 + 3*c) + 48*d^3*e^2*f*x*e^(d*x + c) + 48*d^2*f^3*x^2*e^(d*x + c) + 12*I*d
^3*e^2*f*x + 6*I*d^2*f^3*x^2 + 4*I*d^3*e^3*e^(4*d*x + 4*c) - 12*I*d^2*e*f^
2*x*e^(4*d*x + 4*c) - 16*d^3*e^3*e^(3*d*x + 3*c) + 96*d^2*e*f^2*x*e^(3*d*x
 + 3*c) + 16*d^3*e^3*e^(d*x + c) + 96*d^2*e*f^2*x*e^(d*x + c) + 4*I*d^3*e^
3 + 12*I*d^2*e*f^2*x - 6*I*d^2*e^2*f*e^(4*d*x + 4*c) + 6*I*d*f^3*x*e^(4*d*
x + 4*c) + 48*d^2*e^2*f*e^(3*d*x + 3*c) - 96*d*f^3*x*e^(3*d*x + 3*c) + 48*
d^2*e^2*f*e^(d*x + c) + 96*d*f^3*x*e^(d*x + c) + 6*I*d^2*e^2*f + 6*I*d*f^3
*x + 6*I*d*e*f^2*e^(4*d*x + 4*c) - 96*d*e*f^2*e^(3*d*x + 3*c) + 96*d*e*f^2
*e^(d*x + c) + 6*I*d*e*f^2 - 3*I*f^3*e^(4*d*x + 4*c) + 96*f^3*e^(3*d*x + 3
*c) + 96*f^3*e^(d*x + c) + 3*I*f^3)*e^(-2*d*x - 2*c)/(a*d^4)
```

Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.94

$$\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -e^{c+dx}\left(\frac{-d^3e^3+3d^2e^2f-6def^2+6f^3}{2ad^4}-\frac{f^3x^3}{2ad}+\frac{3f^2x^2(f-de)}{2ad^2}-\frac{3fx(d^2e^2-2def+2f^2)}{2ad^3}\right) - e^{-2c-2dx}\left(\frac{(4d^3e^3+6d^2e^2f+6def^2+3f^3)\,1i}{32ad^4}+\frac{f^3x^3\,1i}{8ad}+\frac{fx(2d^2e^2+2def+f^2)\,3i}{16ad^3}+\frac{f^2x^2(f+2de)\,3i}{16ad^2}\right) + e^{2c+2dx}\left(\frac{(-4d^3e^3+6d^2e^2f-6def^2+3f^3)\,1i}{32ad^4}-\frac{f^3x^3\,1i}{8ad}-\frac{fx(2d^2e^2-2def+f^2)\,3i}{16ad^3}+\frac{f^2x^2(f-2de)\,3i}{16ad^2}\right) - e^{-c-dx}\left(\frac{d^3e^3+3d^2e^2f+6def^2+6f^3}{2ad^4}+\frac{f^3x^3}{2ad}+\frac{3f^2x^2(f+de)}{2ad^2}+\frac{3fx(d^2e^2+2def+2f^2)}{2ad^3}\right)$$

input
```
int((cosh(c + d*x)^3*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i),x)
```

output
```
exp(2*c + 2*d*x)*(((3*f^3 - 4*d^3*e^3 + 6*d^2*e^2*f - 6*d*e*f^2)*1i)/(32*a
*d^4) - (f^3*x^3*1i)/(8*a*d) - (f*x*(f^2 + 2*d^2*e^2 - 2*d*e*f)*3i)/(16*a*
d^3) + (f^2*x^2*(f - 2*d*e)*3i)/(16*a*d^2)) - exp(- 2*c - 2*d*x)*(((3*f^3
+ 4*d^3*e^3 + 6*d^2*e^2*f + 6*d*e*f^2)*1i)/(32*a*d^4) + (f^3*x^3*1i)/(8*a*
d) + (f*x*(f^2 + 2*d^2*e^2 + 2*d*e*f)*3i)/(16*a*d^3) + (f^2*x^2*(f + 2*d*e
)*3i)/(16*a*d^2)) - exp(c + d*x)*((6*f^3 - d^3*e^3 + 3*d^2*e^2*f - 6*d*e*f
^2)/(2*a*d^4) - (f^3*x^3)/(2*a*d) + (3*f^2*x^2*(f - d*e))/(2*a*d^2) - (3*f
*x*(2*f^2 + d^2*e^2 - 2*d*e*f))/(2*a*d^3)) - exp(- c - d*x)*((6*f^3 + d^3*
e^3 + 3*d^2*e^2*f + 6*d*e*f^2)/(2*a*d^4) + (f^3*x^3)/(2*a*d) + (3*f^2*x^2*
(f + d*e))/(2*a*d^2) + (3*f*x*(2*f^2 + d^2*e^2 + 2*d*e*f))/(2*a*d^3))
```

Reduce [F]

$$\int \frac{(e+fx)^3 \cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{\left(\int \frac{\cosh(dx+c)^3}{\sinh(dx+c)i+1}dx\right)e^3 + \left(\int \frac{\cosh(dx+c)^3 x^3}{\sinh(dx+c)i+1}dx\right)f^3 + 3\left(\int \frac{\cosh(dx+c)^3 x^2}{\sinh(dx+c)i+1}dx\right)e\,f^2 + 3\left(\int \frac{\cosh(dx+c)^3 x}{\sinh(dx+c)i+1}dx\right)e^2 f}{a}$$

input
```
int((f*x+e)^3*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

output
```
(int(cosh(c + d*x)**3/(sinh(c + d*x)*i + 1),x)*e**3 + int((cosh(c + d*x)**
3*x**3)/(sinh(c + d*x)*i + 1),x)*f**3 + 3*int((cosh(c + d*x)**3*x**2)/(sin
h(c + d*x)*i + 1),x)*e*f**2 + 3*int((cosh(c + d*x)**3*x)/(sinh(c + d*x)*i
+ 1),x)*e**2*f)/a
```

3.266 $\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 157

$$\begin{aligned}\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = &-\frac{i(e+fx)^2}{4ad}-\frac{2f(e+fx)\cosh(c+dx)}{ad^2}\\ &+\frac{2f^2\sinh(c+dx)}{ad^3}+\frac{(e+fx)^2\sinh(c+dx)}{ad}\\ &+\frac{if(e+fx)\cosh(c+dx)\sinh(c+dx)}{2ad^2}\\ &-\frac{if^2\sinh^2(c+dx)}{4ad^3}-\frac{i(e+fx)^2\sinh^2(c+dx)}{2ad}\end{aligned}$$

output

```
-1/4*I*(f*x+e)^2/a/d-2*f*(f*x+e)*cosh(d*x+c)/a/d^2+2*f^2*sinh(d*x+c)/a/d^3
+(f*x+e)^2*sinh(d*x+c)/a/d+1/2*I*f*(f*x+e)*cosh(d*x+c)*sinh(d*x+c)/a/d^2-1
/4*I*f^2*sinh(d*x+c)^2/a/d^3-1/2*I*(f*x+e)^2*sinh(d*x+c)^2/a/d
```

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.63

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{-32df(e+fx)\cosh(c+dx)-2i(f^2+2d^2(e+fx)^2)\cosh(2(c+dx))+8(2(2f^2+d^2(e+fx)^2)+idf(e}{16ad^3}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(-32*d*f*(e + f*x)*Cosh[c + d*x] - (2*I)*(f^2 + 2*d^2*(e + f*x)^2)*Cosh[2*
(c + d*x)] + 8*(2*(2*f^2 + d^2*(e + f*x)^2) + I*d*f*(e + f*x)*Cosh[c + d*x
])*Sinh[c + d*x])/(16*a*d^3)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {6097, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117, 5969, 3042, 25, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$\Big\downarrow \ 6097$$

$$\frac{\int (e+fx)^2\cosh(c+dx)dx}{a}-\frac{i\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{a}-\frac{i\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \ 3777$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}}{a}-\frac{i\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int(e+fx)\sinh(c+dx)dx}{d}}{a}-\frac{i\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int-i(e+fx)\sin(ic+idx)dx}{d}}{a}-\frac{i\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int(e+fx)\sin(ic+idx)dx}{d}}{a}-\frac{i\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{d}}{a}-\frac{i\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}}{a}-\frac{i\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3117}$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}}{a}-\frac{i\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{5969}$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}}{a}-\frac{i\left(\frac{(e+fx)^2\sinh^2(c+dx)}{2d}-\frac{f\int(e+fx)\sinh^2(c+dx)dx}{d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}}{a}-\frac{i\left(\frac{(e+fx)^2\sinh^2(c+dx)}{2d}-\frac{f\int-((e+fx)\sin(ic+idx)^2)dx}{d}\right)}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}}{a}-\frac{i\left(\frac{(e+fx)^2\sinh^2(c+dx)}{2d}+\frac{f\int(e+fx)\sin(ic+idx)^2dx}{d}\right)}{a}$$

$$\Big\downarrow \; 3791$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}}{a}-\frac{i\left(\frac{f\left(\frac{1}{2}\int(e+fx)dx+\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \; 17$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}}{a}-\frac{i\left(\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}\right)}{a}$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(((e + f*x)^2*Sinh[c + d*x])/d + ((2*I)*f*((I*(e + f*x)*Cosh[c + d*x])/d -
 (I*f*Sinh[c + d*x])/d^2))/d)/a - (I*(((e + f*x)^2*Sinh[c + d*x]^2)/(2*d)
+ (f*((e + f*x)^2/(4*f) - ((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) +
(f*Sinh[c + d*x]^2)/(4*d^2)))/d))/a
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6097
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cosh[c
 + d*x]^(n - 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*
Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && E
qQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 14.28 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.54

method	result
risch	$-\frac{i(2d^2x^2f^2+4d^2efx+2d^2e^2-2d\,f^2x-2def+f^2)\mathrm{e}^{2dx+2c}}{16d^3a}+\frac{(d^2x^2f^2+2d^2efx+d^2e^2-2d\,f^2x-2def+2f^2)\mathrm{e}^{dx+c}}{2d^3a}-\frac{(d^2x^2f^2+$

input
```
int((f*x+e)^2*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/16*I*(2*d^2*f^2*x^2+4*d^2*e*f*x+2*d^2*e^2-2*d*f^2*x-2*d*e*f+f^2)/d^3/a*
exp(2*d*x+2*c)+1/2*(d^2*f^2*x^2+2*d^2*e*f*x+d^2*e^2-2*d*f^2*x-2*d*e*f+2*f^
2)/d^3/a*exp(d*x+c)-1/2*(d^2*f^2*x^2+2*d^2*e*f*x+d^2*e^2+2*d*f^2*x+2*d*e*f
+2*f^2)/d^3/a*exp(-d*x-c)-1/16*I*(2*d^2*f^2*x^2+4*d^2*e*f*x+2*d^2*e^2+2*d*
f^2*x+2*d*e*f+f^2)/a/d^3*exp(-2*d*x-2*c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.45

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$=\frac{\left(-2i\,d^2f^2x^2-2i\,d^2e^2-2i\,def-i\,f^2-2\,(2i\,d^2ef+i\,df^2)x+(-2i\,d^2f^2x^2-2i\,d^2e^2+2i\,def-i\,f^2-2\right.}{}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/16*(-2*I*d^2*f^2*x^2 - 2*I*d^2*e^2 - 2*I*d*e*f - I*f^2 - 2*(2*I*d^2*e*f
+ I*d*f^2)*x + (-2*I*d^2*f^2*x^2 - 2*I*d^2*e^2 + 2*I*d*e*f - I*f^2 - 2*(2*
I*d^2*e*f - I*d*f^2)*x)*e^(4*d*x + 4*c) + 8*(d^2*f^2*x^2 + d^2*e^2 - 2*d*e
*f + 2*f^2 + 2*(d^2*e*f - d*f^2)*x)*e^(3*d*x + 3*c) - 8*(d^2*f^2*x^2 + d^2
*e^2 + 2*d*e*f + 2*f^2 + 2*(d^2*e*f + d*f^2)*x)*e^(d*x + c))*e^(-2*d*x - 2
*c)/(a*d^3)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(138) = 276$.

Time = 0.41 (sec) , antiderivative size = 631, normalized size of antiderivative = 4.02

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$= \begin{cases} \frac{((-512a^3d^{11}e^2e^{2c}-1024a^3d^{11}efxe^{2c}-512a^3d^{11}f^2x^2e^{2c}-1024a^3d^{10}efe^{2c}-1024a^3d^{10}f^2xe^{2c}-1024a^3d^9f^2e^{2c})e^{-dx}+(512a^3d^{11}e^2e^{4c}+10}{} \\ \frac{x^3(-if^2e^{4c}+2f^2e^{3c}+2f^2e^c+if^2)e^{-2c}}{12a}+\frac{x^2(-iefe^{4c}+2efe^{3c}+2efe^c+ief)e^{-2c}}{4a}+\frac{x(-ie^2e^{4c}+2e^2e^{3c}+2e^2e^c+ie^2)e^{-2c}}{4a} \end{cases}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
Piecewise((((-512*a**3*d**11*e**2*exp(2*c) - 1024*a**3*d**11*e*f*x*exp(2*c
) - 512*a**3*d**11*f**2*x**2*exp(2*c) - 1024*a**3*d**10*e*f*exp(2*c) - 102
4*a**3*d**10*f**2*x*exp(2*c) - 1024*a**3*d**9*f**2*exp(2*c))*exp(-d*x) + (
512*a**3*d**11*e**2*exp(4*c) + 1024*a**3*d**11*e*f*x*exp(4*c) + 512*a**3*d
**11*f**2*x**2*exp(4*c) - 1024*a**3*d**10*e*f*exp(4*c) - 1024*a**3*d**10*f
**2*x*exp(4*c) + 1024*a**3*d**9*f**2*exp(4*c))*exp(d*x) + (-128*I*a**3*d**
11*e**2*exp(c) - 256*I*a**3*d**11*e*f*x*exp(c) - 128*I*a**3*d**11*f**2*x**
2*exp(c) - 128*I*a**3*d**10*e*f*exp(c) - 128*I*a**3*d**10*f**2*x*exp(c) -
64*I*a**3*d**9*f**2*exp(c))*exp(-2*d*x) + (-128*I*a**3*d**11*e**2*exp(5*c)
 - 256*I*a**3*d**11*e*f*x*exp(5*c) - 128*I*a**3*d**11*f**2*x**2*exp(5*c) +
 128*I*a**3*d**10*e*f*exp(5*c) + 128*I*a**3*d**10*f**2*x*exp(5*c) - 64*I*a
**3*d**9*f**2*exp(5*c))*exp(2*d*x))*exp(-3*c)/(1024*a**4*d**12), Ne(a**4*d
**12*exp(3*c), 0)), (x**3*(-I*f**2*exp(4*c) + 2*f**2*exp(3*c) + 2*f**2*exp
(c) + I*f**2)*exp(-2*c)/(12*a) + x**2*(-I*e*f*exp(4*c) + 2*e*f*exp(3*c) +
2*e*f*exp(c) + I*e*f)*exp(-2*c)/(4*a) + x*(-I*e**2*exp(4*c) + 2*e**2*exp(3
*c) + 2*e**2*exp(c) + I*e**2)*exp(-2*c)/(4*a), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(141) = 282.

Time = 0.13 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.15

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{\left(2i\,d^2f^2x^2e^{(4\,dx+4\,c)} - 8\,d^2f^2x^2e^{(3\,dx+3\,c)} + 8\,d^2f^2x^2e^{(dx+c)} + 2i\,d^2f^2x^2 + 4i\,d^2efxe^{(4\,dx+4\,c)} - 16\,d^2efx\right.}{}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/16*(2*I*d^2*f^2*x^2*e^(4*d*x + 4*c) - 8*d^2*f^2*x^2*e^(3*d*x + 3*c) + 8
*d^2*f^2*x^2*e^(d*x + c) + 2*I*d^2*f^2*x^2 + 4*I*d^2*e*f*x*e^(4*d*x + 4*c)
 - 16*d^2*e*f*x*e^(3*d*x + 3*c) + 16*d^2*e*f*x*e^(d*x + c) + 4*I*d^2*e*f*x
 + 2*I*d^2*e^2*e^(4*d*x + 4*c) - 2*I*d*f^2*x*e^(4*d*x + 4*c) - 8*d^2*e^2*e
^(3*d*x + 3*c) + 16*d*f^2*x*e^(3*d*x + 3*c) + 8*d^2*e^2*e^(d*x + c) + 16*d
*f^2*x*e^(d*x + c) + 2*I*d^2*e^2 + 2*I*d*f^2*x - 2*I*d*e*f*e^(4*d*x + 4*c)
 + 16*d*e*f*e^(3*d*x + 3*c) + 16*d*e*f*e^(d*x + c) + 2*I*d*e*f + I*f^2*e^(
4*d*x + 4*c) - 16*f^2*e^(3*d*x + 3*c) + 16*f^2*e^(d*x + c) + I*f^2)*e^(-2*
d*x - 2*c)/(a*d^3)
```

Mupad [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.73

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = e^{c+d\,x}\left(\frac{d^2\,e^2-2\,d\,e\,f+2\,f^2}{2\,a\,d^3}+\frac{f^2\,x^2}{2\,a\,d}-\frac{f\,x\,(f-d\,e)}{a\,d^2}\right) - e^{-2\,c-2\,d\,x}\left(\frac{(2\,d^2\,e^2+2\,d\,e\,f+f^2)\ 1\mathrm{i}}{16\,a\,d^3}+\frac{f^2\,x^2\ 1\mathrm{i}}{8\,a\,d}+\frac{f\,x\,(f+2\,d\,e)\ 1\mathrm{i}}{8\,a\,d^2}\right) - e^{2\,c+2\,d\,x}\left(\frac{(2\,d^2\,e^2-2\,d\,e\,f+f^2)\ 1\mathrm{i}}{16\,a\,d^3}+\frac{f^2\,x^2\ 1\mathrm{i}}{8\,a\,d}-\frac{f\,x\,(f-2\,d\,e)\ 1\mathrm{i}}{8\,a\,d^2}\right) - e^{-c-d\,x}\left(\frac{d^2\,e^2+2\,d\,e\,f+2\,f^2}{2\,a\,d^3}+\frac{f^2\,x^2}{2\,a\,d}+\frac{f\,x\,(f+d\,e)}{a\,d^2}\right)$$

input

```
int((cosh(c + d*x)^3*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i),x)
```

output

```
exp(c + d*x)*((2*f^2 + d^2*e^2 - 2*d*e*f)/(2*a*d^3) + (f^2*x^2)/(2*a*d) -
(f*x*(f - d*e))/(a*d^2)) - exp(- 2*c - 2*d*x)*(((f^2 + 2*d^2*e^2 + 2*d*e*f
)*1i)/(16*a*d^3) + (f^2*x^2*1i)/(8*a*d) + (f*x*(f + 2*d*e)*1i)/(8*a*d^2))
- exp(2*c + 2*d*x)*(((f^2 + 2*d^2*e^2 - 2*d*e*f)*1i)/(16*a*d^3) + (f^2*x^2
*1i)/(8*a*d) - (f*x*(f - 2*d*e)*1i)/(8*a*d^2)) - exp(- c - d*x)*((2*f^2 +
d^2*e^2 + 2*d*e*f)/(2*a*d^3) + (f^2*x^2)/(2*a*d) + (f*x*(f + d*e))/(a*d^2)
)
```

Reduce [F]

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(\int \frac{\cosh(dx+c)^3}{\sinh(dx+c)i+1}dx\right)e^2+\left(\int \frac{\cosh(dx+c)^3x^2}{\sinh(dx+c)i+1}dx\right)f^2+2\left(\int \frac{\cosh(dx+c)^3x}{\sinh(dx+c)i+1}dx\right)ef}{a}$$

input

```
int((f*x+e)^2*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

output
```
(int(cosh(c + d*x)**3/(sinh(c + d*x)*i + 1),x)*e**2 + int((cosh(c + d*x)**
3*x**2)/(sinh(c + d*x)*i + 1),x)*f**2 + 2*int((cosh(c + d*x)**3*x)/(sinh(c
 + d*x)*i + 1),x)*e*f)/a
```

3.267 $\int \frac{(e+fx)\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 98

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{ifx}{4ad}-\frac{f\cosh(c+dx)}{ad^2}+\frac{(e+fx)\sinh(c+dx)}{ad}+\frac{if\cosh(c+dx)\sinh(c+dx)}{4ad^2}-\frac{i(e+fx)\sinh^2(c+dx)}{2ad}$$

output
```
-1/4*I*f*x/a/d-f*cosh(d*x+c)/a/d^2+(f*x+e)*sinh(d*x+c)/a/d+1/4*I*f*cosh(d*
x+c)*sinh(d*x+c)/a/d^2-1/2*I*(f*x+e)*sinh(d*x+c)^2/a/d
```

Mathematica [A] (verified)

Time = 2.95 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.61

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{if\cosh(c+dx)(4i+\sinh(c+dx))+d(e+fx)(-i\cosh(2(c+dx))+4\sinh(c+dx))}{4ad^2}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(I*f*Cosh[c + d*x]*(4*I + Sinh[c + d*x]) + d*(e + f*x)*((-I)*Cosh[2*(c + d
*x)] + 4*Sinh[c + d*x]))/(4*a*d^2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.96, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {6097, 3042, 3777, 26, 3042, 26, 3118, 5969, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6097}$$

$$\frac{\int (e+fx)\cosh(c+dx)dx}{a}-\frac{i\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{a}-\frac{i\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}}{a}-\frac{i\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}}{a}-\frac{i\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}}{a}-\frac{i\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}}{a}-\frac{i\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3118}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{a}-\frac{i\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{5969}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{a}-\frac{i\left(\frac{(e+fx)\sinh^2(c+dx)}{2d}-\frac{f\int\sinh^2(c+dx)dx}{2d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{a}-\frac{i\left(\frac{(e+fx)\sinh^2(c+dx)}{2d}-\frac{f\int-\sin(ic+idx)^2dx}{2d}\right)}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{a}-\frac{i\left(\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\int\sin(ic+idx)^2dx}{2d}\right)}{a}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{a}-\frac{i\left(\frac{f\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}+\frac{(e+fx)\sinh^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{a}-\frac{i\left(\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}\right)}{a}$$

input
```
Int[((e + f*x)*Cosh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(-((f*Cosh[c + d*x])/d^2) + ((e + f*x)*Sinh[c + d*x])/d)/a - (I*(((e + f*x
)*Sinh[c + d*x]^2)/(2*d) + (f*(x/2 - (Cosh[c + d*x]*Sinh[c + d*x])/(2*d)))
/(2*d)))/a
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6097
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cosh[c
 + d*x]^(n - 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*
Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && E
qQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 5.76 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.15

method	result
risch	$-\frac{i(2dxf+2de-f)\mathrm{e}^{2dx+2c}}{16a\,d^2}+\frac{(dxf+de-f)\mathrm{e}^{dx+c}}{2a\,d^2}-\frac{(dxf+de+f)\mathrm{e}^{-dx-c}}{2a\,d^2}-\frac{i(2dxf+2de+f)\mathrm{e}^{-2dx-2c}}{16a\,d^2}$
derivativedivides	$\frac{\frac{icf\cosh(dx+c)^2}{2}-\frac{ide\cosh(dx+c)^2}{2}-if\left(\frac{(dx+c)\cosh(dx+c)^2}{2}-\frac{\sinh(dx+c)\cosh(dx+c)}{4}-\frac{dx}{4}-\frac{c}{4}\right)-\sinh(dx+c)cf+\sinh(dx+c}{d^2a}$
default	$\frac{\frac{icf\cosh(dx+c)^2}{2}-\frac{ide\cosh(dx+c)^2}{2}-if\left(\frac{(dx+c)\cosh(dx+c)^2}{2}-\frac{\sinh(dx+c)\cosh(dx+c)}{4}-\frac{dx}{4}-\frac{c}{4}\right)-\sinh(dx+c)cf+\sinh(dx+c}{d^2a}$

input
```
int((f*x+e)*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/16*I*(2*d*f*x+2*d*e-f)/a/d^2*exp(2*d*x+2*c)+1/2*(d*f*x+d*e-f)/a/d^2*exp
(d*x+c)-1/2*(d*f*x+d*e+f)/a/d^2*exp(-d*x-c)-1/16*I*(2*d*f*x+2*d*e+f)/a/d^2
*exp(-2*d*x-2*c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.94

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$=\frac{\left(-2i\,dfx-2i\,de+(-2i\,dfx-2i\,de+i\,f)e^{(4\,dx+4\,c)}+8\,(dfx+de-f)e^{(3\,dx+3\,c)}-8\,(dfx+de+f)e^{(dx+}\right.}{16\,ad^2}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/16*(-2*I*d*f*x - 2*I*d*e + (-2*I*d*f*x - 2*I*d*e + I*f)*e^(4*d*x + 4*c)
+ 8*(d*f*x + d*e - f)*e^(3*d*x + 3*c) - 8*(d*f*x + d*e + f)*e^(d*x + c) -
I*f)*e^(-2*d*x - 2*c)/(a*d^2)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(82) = 164$.

Time = 0.28 (sec) , antiderivative size = 321, normalized size of antiderivative = 3.28

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \begin{cases} \frac{((-512a^3d^7ee^{2c}-512a^3d^7fxe^{2c}-512a^3d^6fe^{2c})e^{-dx}+(512a^3d^7ee^{4c}+512a^3d^7fxe^{4c}-512a^3d^6fe^{4c})e^{dx}+(-128ia^3d^7ee^c-128ia^3d^7fxe^c-6}{1024a^4d^8} \\ \frac{x^2(-ife^{4c}+2fe^{3c}+2fe^c+if)e^{-2c}}{8a}+\frac{x(-iee^{4c}+2ee^{3c}+2ee^c+ie)e^{-2c}}{4a} \end{cases}$$

input
```
integrate((f*x+e)*cosh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
Piecewise((((-512*a**3*d**7*e*exp(2*c) - 512*a**3*d**7*f*x*exp(2*c) - 512*
a**3*d**6*f*exp(2*c))*exp(-d*x) + (512*a**3*d**7*e*exp(4*c) + 512*a**3*d**
7*f*x*exp(4*c) - 512*a**3*d**6*f*exp(4*c))*exp(d*x) + (-128*I*a**3*d**7*e*
exp(c) - 128*I*a**3*d**7*f*x*exp(c) - 64*I*a**3*d**6*f*exp(c))*exp(-2*d*x)
 + (-128*I*a**3*d**7*e*exp(5*c) - 128*I*a**3*d**7*f*x*exp(5*c) + 64*I*a**3
*d**6*f*exp(5*c))*exp(2*d*x))*exp(-3*c)/(1024*a**4*d**8), Ne(a**4*d**8*exp
(3*c), 0)), (x**2*(-I*f*exp(4*c) + 2*f*exp(3*c) + 2*f*exp(c) + I*f)*exp(-2
*c)/(8*a) + x*(-I*e*exp(4*c) + 2*e*exp(3*c) + 2*e*exp(c) + I*e)*exp(-2*c)/
(4*a), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.41

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{\left(2i\,dfxe^{(4\,dx+4\,c)} - 8\,dfxe^{(3\,dx+3\,c)} + 8\,dfxe^{(dx+c)} + 2i\,dfx + 2i\,dee^{(4\,dx+4\,c)} - 8\,dee^{(3\,dx+3\,c)} + 8\,dee^{(dx+c)}}{16\,ad^2}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/16*(2*I*d*f*x*e^(4*d*x + 4*c) - 8*d*f*x*e^(3*d*x + 3*c) + 8*d*f*x*e^(d*
x + c) + 2*I*d*f*x + 2*I*d*e*e^(4*d*x + 4*c) - 8*d*e*e^(3*d*x + 3*c) + 8*d
*e*e^(d*x + c) + 2*I*d*e - I*f*e^(4*d*x + 4*c) + 8*f*e^(3*d*x + 3*c) + 8*f
*e^(d*x + c) + I*f)*e^(-2*d*x - 2*c)/(a*d^2)
```

Mupad [B] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.47

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\mathrm{e}^{-c-d\,x}\left(\frac{f+d\,e}{2\,a\,d^2}+\frac{f\,x}{2\,a\,d}\right) - \mathrm{e}^{-2\,c-2\,d\,x}\left(\frac{(f+2\,d\,e)\ 1\mathrm{i}}{16\,a\,d^2}+\frac{f\,x\,1\mathrm{i}}{8\,a\,d}\right) + \mathrm{e}^{2\,c+2\,d\,x}\left(\frac{(f-2\,d\,e)\ 1\mathrm{i}}{16\,a\,d^2}-\frac{f\,x\,1\mathrm{i}}{8\,a\,d}\right) - \mathrm{e}^{c+d\,x}\left(\frac{f-d\,e}{2\,a\,d^2}-\frac{f\,x}{2\,a\,d}\right)$$

input
```
int((cosh(c + d*x)^3*(e + f*x))/(a + a*sinh(c + d*x)*1i),x)
```

output
```
exp(2*c + 2*d*x)*(((f - 2*d*e)*1i)/(16*a*d^2) - (f*x*1i)/(8*a*d)) - exp(-
2*c - 2*d*x)*(((f + 2*d*e)*1i)/(16*a*d^2) + (f*x*1i)/(8*a*d)) - exp(- c -
d*x)*((f + d*e)/(2*a*d^2) + (f*x)/(2*a*d)) - exp(c + d*x)*((f - d*e)/(2*a*
d^2) - (f*x)/(2*a*d))
```

Reduce [F]

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(\int \frac{\cosh(dx+c)^3}{\sinh(dx+c)i+1}dx\right)e+\left(\int \frac{\cosh(dx+c)^3x}{\sinh(dx+c)i+1}dx\right)f}{a}$$

input
```
int((f*x+e)*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

output
```
(int(cosh(c + d*x)**3/(sinh(c + d*x)*i + 1),x)*e + int((cosh(c + d*x)**3*x
)/(sinh(c + d*x)*i + 1),x)*f)/a
```

3.268 $\int \frac{\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx=\frac{i(a-ia\sinh(c+dx))^2}{2a^3d}$$

output
```
1/2*I*(a-I*a*sinh(d*x+c))^2/a^3/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx=\frac{(2-i\sinh(c+dx))\sinh(c+dx)}{2ad}$$

input
```
Integrate[Cosh[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((2 - I*Sinh[c + d*x])*Sinh[c + d*x])/(2*a*d)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3146, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(ic+idx)^3}{a+a\sin(ic+idx)}dx$$

$$\Big\downarrow \text{ 3146}$$

$$-\frac{i\int(a-ia\sinh(c+dx))d(ia\sinh(c+dx))}{a^3d}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{i(a-ia\sinh(c+dx))^2}{2a^3d}$$

input
```
Int[Cosh[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((I/2)*(a - I*a*Sinh[c + d*x])^2)/(a^3*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 5.70 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$-\frac{i\left(\frac{\sinh(dx+c)^2}{2}+i\sinh(dx+c)\right)}{da}$	30
default	$-\frac{i\left(\frac{\sinh(dx+c)^2}{2}+i\sinh(dx+c)\right)}{da}$	30
risch	$-\frac{i e^{2dx+2c}}{8ad}+\frac{e^{dx+c}}{2ad}-\frac{e^{-dx-c}}{2ad}-\frac{i e^{-2dx-2c}}{8da}$	69

input
```
int(cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-I/d/a*(1/2*sinh(d*x+c)^2+I*sinh(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(21) = 42.

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int \frac{\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(-i\,e^{(4\,dx+4\,c)}+4\,e^{(3\,dx+3\,c)}-4\,e^{(dx+c)}-i\right)e^{(-2\,dx-2\,c)}}{8\,ad}$$

input
```
integrate(cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/8*(-I*e^(4*d*x + 4*c) + 4*e^(3*d*x + 3*c) - 4*e^(d*x + c) - I)*e^(-2*d*x
 - 2*c)/(a*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(20) = 40$.

Time = 0.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 4.93

$$\int \frac{\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \begin{cases} \frac{(-32ia^3d^3e^{5c}e^{2dx}+128a^3d^3e^{4c}e^{dx}-128a^3d^3e^{2c}e^{-dx}-32ia^3d^3e^{c}e^{-2dx})e^{-3c}}{256a^4d^4} & \text{for } a^4d^4e^{3c}\neq 0 \\ \frac{x(-ie^{4c}+2e^{3c}+2e^{c}+i)e^{-2c}}{4a} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
Piecewise(((-32*I*a**3*d**3*exp(5*c)*exp(2*d*x) + 128*a**3*d**3*exp(4*c)*e
xp(d*x) - 128*a**3*d**3*exp(2*c)*exp(-d*x) - 32*I*a**3*d**3*exp(c)*exp(-2*
d*x))*exp(-3*c)/(256*a**4*d**4), Ne(a**4*d**4*exp(3*c), 0)), (x*(-I*exp(4*
c) + 2*exp(3*c) + 2*exp(c) + I)*exp(-2*c)/(4*a), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(21) = 42$.

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.22

$$\int \frac{\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(4i\,e^{(-dx-c)}+1\right)e^{(2\,dx+2\,c)}}{8\,ad} - \frac{i\left(-4i\,e^{(-dx-c)}+e^{(-2\,dx-2\,c)}\right)}{8\,ad}$$

input
```
integrate(cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/8*I*(4*I*e^(-d*x - c) + 1)*e^(2*d*x + 2*c)/(a*d) - 1/8*I*(-4*I*e^(-d*x
- c) + e^(-2*d*x - 2*c))/(a*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(21) = 42.

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.04

$$\int \frac{\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{\frac{(4\,e^{(dx+c)}+i)e^{(-2\,dx-2\,c)}}{a} + \frac{i\,ae^{(2\,dx+2\,c)}-4\,ae^{(dx+c)}}{a^2}}{8\,d}$$

input
```
integrate(cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/8*((4*e^(d*x + c) + I)*e^(-2*d*x - 2*c)/a + (I*a*e^(2*d*x + 2*c) - 4*a*
e^(d*x + c))/a^2)/d
```

Mupad [B] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{4\sinh(c+d\,x)-\cosh(2\,c+2\,d\,x)\ 1\mathrm{i}}{4\,a\,d}$$

input
```
int(cosh(c + d*x)^3/(a + a*sinh(c + d*x)*1i),x)
```

output
```
(4*sinh(c + d*x) - cosh(2*c + 2*d*x)*1i)/(4*a*d)
```

Reduce [F]

$$\int \frac{\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\int \frac{\cosh(dx+c)^3}{\sinh(dx+c)i+1}dx}{a}$$

input
```
int(cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(cosh(c + d*x)**3/(sinh(c + d*x)*i + 1),x)/a
```

3.269 $\int \frac{\cosh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 131

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{af} - \frac{i\text{Chi}\left(\frac{2de}{f}+2dx\right)\sinh\left(2c-\frac{2de}{f}\right)}{2af} + \frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{af} - \frac{i\cosh\left(2c-\frac{2de}{f}\right)\text{Shi}\left(\frac{2de}{f}+2dx\right)}{2af}$$

output
```
cosh(c-d*e/f)*Chi(d*e/f+d*x)/a/f-1/2*I*Chi(2*d*e/f+2*d*x)*sinh(2*c-2*d*e/f
)/a/f+sinh(c-d*e/f)*Shi(d*e/f+d*x)/a/f-1/2*I*cosh(2*c-2*d*e/f)*Shi(2*d*e/f
+2*d*x)/a/f
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \frac{2\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(d\left(\frac{e}{f}+x\right)\right)-i\left(\text{Chi}\left(\frac{2d(e+fx)}{f}\right)\sinh\left(2c-\frac{2de}{f}\right)+2i\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(d\left(\frac{e}{f}+x\right)\right)\right.}{2af}$$

input
```
Integrate[Cosh[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
(2*Cosh[c - (d*e)/f]*CoshIntegral[d*(e/f + x)] - I*(CoshIntegral[(2*d*(e +
 f*x))/f]*Sinh[2*c - (2*d*e)/f] + (2*I)*Sinh[c - (d*e)/f]*SinhIntegral[d*(
e/f + x)] + Cosh[2*c - (2*d*e)/f]*SinhIntegral[(2*d*(e + f*x))/f]))/(2*a*f
)
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.581$, Rules used = {6097, 3042, 3784, 26, 3042, 26, 3779, 3782, 5971, 27, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

$$\Big\downarrow \text{ 6097}$$

$$\frac{\int \frac{\cosh(c+dx)}{e+fx}dx}{a}-\frac{i\int \frac{\cosh(c+dx)\sinh(c+dx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\sin\left(ic+idx+\frac{\pi}{2}\right)}{e+fx}dx}{a}-\frac{i\int \frac{\cosh(c+dx)\sinh(c+dx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{3784}$$

$$\frac{\cosh\left(c-\frac{de}{f}\right)\int\frac{\cosh\left(\frac{de}{f}+dx\right)}{e+fx}dx-i\sinh\left(c-\frac{de}{f}\right)\int\frac{i\sinh\left(\frac{de}{f}+dx\right)}{e+fx}dx}{a}-\frac{i\int\frac{\cosh(c+dx)\sinh(c+dx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\sinh\left(c-\frac{de}{f}\right)\int\frac{\sinh\left(\frac{de}{f}+dx\right)}{e+fx}dx+\cosh\left(c-\frac{de}{f}\right)\int\frac{\cosh\left(\frac{de}{f}+dx\right)}{e+fx}dx}{a}-\frac{i\int\frac{\cosh(c+dx)\sinh(c+dx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh\left(c-\frac{de}{f}\right)\int-\frac{i\sin\left(\frac{ide}{f}+idx\right)}{e+fx}dx+\cosh\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{ide}{f}+idx+\frac{\pi}{2}\right)}{e+fx}dx}{a}-\frac{i\int\frac{\cosh(c+dx)\sinh(c+dx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\cosh\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{ide}{f}+idx+\frac{\pi}{2}\right)}{e+fx}dx-i\sinh\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{ide}{f}+idx\right)}{e+fx}dx}{a}-\frac{i\int\frac{\cosh(c+dx)\sinh(c+dx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{3779}$$

$$\frac{\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}+\cosh\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{ide}{f}+idx+\frac{\pi}{2}\right)}{e+fx}dx}{a}-\frac{i\int\frac{\cosh(c+dx)\sinh(c+dx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{3782}$$

$$\frac{\frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{i\int\frac{\cosh(c+dx)\sinh(c+dx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{5971}$$

$$\frac{\frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{i\int\frac{\sinh(2c+2dx)}{2(e+fx)}dx}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{i\int\frac{\sinh(2c+2dx)}{e+fx}dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{i\int -\frac{i\sin(2ic+2idx)}{e+fx}dx}{2a}$$

$\downarrow$ 26

$$\frac{\frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{\int \frac{\sin(2ic+2idx)}{e+fx}dx}{2a}$$

$\downarrow$ 3784

$$\frac{\frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{i\sinh\left(2c-\frac{2de}{f}\right)\int \frac{\cosh\left(\frac{2de}{f}+2dx\right)}{e+fx}dx+\cosh\left(2c-\frac{2de}{f}\right)\int \frac{i\sinh\left(\frac{2de}{f}+2dx\right)}{e+fx}dx}{2a}$$

$\downarrow$ 26

$$\frac{\frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{i\sinh\left(2c-\frac{2de}{f}\right)\int \frac{\cosh\left(\frac{2de}{f}+2dx\right)}{e+fx}dx+i\cosh\left(2c-\frac{2de}{f}\right)\int \frac{\sinh\left(\frac{2de}{f}+2dx\right)}{e+fx}dx}{2a}$$

$\downarrow$ 3042

$$\frac{\frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{i\sinh\left(2c-\frac{2de}{f}\right)\int \frac{\sin\left(\frac{2ide}{f}+2idx+\frac{\pi}{2}\right)}{e+fx}dx+i\cosh\left(2c-\frac{2de}{f}\right)\int -\frac{i\sin\left(\frac{2ide}{f}+2idx\right)}{e+fx}dx}{2a}$$

$\downarrow$ 26

$$\frac{\frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{i\sinh\left(2c-\frac{2de}{f}\right)\int \frac{\sin\left(\frac{2ide}{f}+2idx+\frac{\pi}{2}\right)}{e+fx}dx+\cosh\left(2c-\frac{2de}{f}\right)\int \frac{\sin\left(\frac{2ide}{f}+2idx\right)}{e+fx}dx}{2a}$$

$\downarrow$ 3779

$$\frac{\frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{i\sinh\left(2c-\frac{2de}{f}\right)\int\frac{\sin\left(\frac{2ide}{f}+2idx+\frac{\pi}{2}\right)}{e+fx}dx+\frac{i\cosh\left(2c-\frac{2de}{f}\right)\text{Shi}\left(\frac{2de}{f}+2dx\right)}{f}}{2a}$$

$$\Big\downarrow \text{ 3782}$$

$$\frac{\frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{\frac{i\sinh\left(2c-\frac{2de}{f}\right)\text{Chi}\left(\frac{2de}{f}+2dx\right)}{f}+\frac{i\cosh\left(2c-\frac{2de}{f}\right)\text{Shi}\left(\frac{2de}{f}+2dx\right)}{f}}{2a}$$

input
```
Int[Cosh[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
((Cosh[c - (d*e)/f]*CoshIntegral[(d*e)/f + d*x])/f + (Sinh[c - (d*e)/f]*Si
nhIntegral[(d*e)/f + d*x])/f)/a - ((I*CoshIntegral[(2*d*e)/f + 2*d*x]*Sinh
[2*c - (2*d*e)/f])/f + (I*Cosh[2*c - (2*d*e)/f]*SinhIntegral[(2*d*e)/f + 2
*d*x])/f)/(2*a)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6097
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cosh[c
 + d*x]^(n - 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*
Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && E
qQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 27.98 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.37

method	result
risch	$-\frac{e^{-\frac{cf-de}{f}} \operatorname{expIntegral}_1\left(dx+c-\frac{cf-de}{f}\right)}{2af}+\frac{i e^{\frac{2cf-2de}{f}} \operatorname{expIntegral}_1\left(-2dx-2c-\frac{2(-cf+de)}{f}\right)}{4af}-\frac{e^{\frac{cf-de}{f}} \operatorname{expIntegral}_1\left(-dx-c-\right.}{2af}$

input
```
int(cosh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/f*exp(-(c*f-d*e)/f)*Ei(1,d*x+c-(c*f-d*e)/f)+1/4*I/a/f*exp(2*(c*f-d*
e)/f)*Ei(1,-2*d*x-2*c-2*(-c*f+d*e)/f)-1/2/a/f*exp((c*f-d*e)/f)*Ei(1,-d*x-c
-(-c*f+d*e)/f)-1/4*I/a/f*exp(-2*(c*f-d*e)/f)*Ei(1,2*d*x+2*c-2*(c*f-d*e)/f)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.97

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$
$$= \frac{i\operatorname{Ei}\left(-\frac{2\,(dfx+de)}{f}\right)e^{\left(\frac{2\,(de-cf)}{f}\right)} + 2\operatorname{Ei}\left(-\frac{dfx+de}{f}\right)e^{\left(\frac{de-cf}{f}\right)} + 2\operatorname{Ei}\left(\frac{dfx+de}{f}\right)e^{\left(-\frac{de-cf}{f}\right)} - i\operatorname{Ei}\left(\frac{2\,(dfx+de)}{f}\right)e^{\left(-\frac{2\,(de-cf)}{f}\right)}}{4\,af}$$

input
```
integrate(cosh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(I*Ei(-2*(d*f*x + d*e)/f)*e^(2*(d*e - c*f)/f) + 2*Ei(-(d*f*x + d*e)/f)
*e^((d*e - c*f)/f) + 2*Ei((d*f*x + d*e)/f)*e^(-(d*e - c*f)/f) - I*Ei(2*(d*
f*x + d*e)/f)*e^(-2*(d*e - c*f)/f))/(a*f)
```

Sympy [F]

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = -\frac{i\int \frac{\cosh^3(c+dx)}{e\sinh(c+dx)-ie+fx\sinh(c+dx)-ifx}\,dx}{a}$$

input
```
integrate(cosh(d*x+c)**3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(cosh(c + d*x)**3/(e*sinh(c + d*x) - I*e + f*x*sinh(c + d*x) -
I*f*x), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cosh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.10

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = -\frac{\left(i\,\mathrm{Ei}\left(\frac{2\,(dfx+de)}{f}\right)e^{\left(4\,c-\frac{2\,de}{f}\right)}-2\,\mathrm{Ei}\left(\frac{dfx+de}{f}\right)e^{\left(3\,c-\frac{de}{f}\right)}-2\,\mathrm{Ei}\left(-\frac{dfx+de}{f}\right)e^{\left(c+\frac{de}{f}\right)}-i\,\mathrm{Ei}\left(-\frac{2\,(dfx+de)}{f}\right)e^{\left(\frac{2\,de}{f}\right.}}{4\,af}$$

input
```
integrate(cosh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*(I*Ei(2*(d*f*x + d*e)/f)*e^(4*c - 2*d*e/f) - 2*Ei((d*f*x + d*e)/f)*e^
(3*c - d*e/f) - 2*Ei(-(d*f*x + d*e)/f)*e^(c + d*e/f) - I*Ei(-2*(d*f*x + d*
e)/f)*e^(2*d*e/f) + 3*I*e^(2*c)*log(f*x + e) - 3*I*e^(2*c)*log(I*f*x + I*e
))*e^(-2*c)/(a*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)^3}{(e+f\,x)\,\left(a+a\sinh\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

input
```
int(cosh(c + d*x)^3/((e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(cosh(c + d*x)^3/((e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [F]

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \frac{\int \frac{\cosh(dx+c)^3}{\sinh(dx+c)ei+\sinh(dx+c)fix+e+fx}dx}{a}$$

input
```
int(cosh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(cosh(c + d*x)**3/(sinh(c + d*x)*e*i + sinh(c + d*x)*f*i*x + e + f*x),x
)/a
```

3.270 $\int \frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 180

$$\int \frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = -\frac{\cosh(c+dx)}{af(e+fx)} - \frac{id\cosh\left(2c-\frac{2de}{f}\right)\text{Chi}\left(\frac{2de}{f}+2dx\right)}{af^2} + \frac{d\text{Chi}\left(\frac{de}{f}+dx\right)\sinh\left(c-\frac{de}{f}\right)}{af^2} + \frac{i\sinh(2c+2dx)}{2af(e+fx)} + \frac{d\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{af^2} - \frac{id\sinh\left(2c-\frac{2de}{f}\right)\text{Shi}\left(\frac{2de}{f}+2dx\right)}{af^2}$$

output
```
-cosh(d*x+c)/a/f/(f*x+e)-I*d*cosh(2*c-2*d*e/f)*Chi(2*d*e/f+2*d*x)/a/f^2+d*
Chi(d*e/f+d*x)*sinh(c-d*e/f)/a/f^2+1/2*I*sinh(2*d*x+2*c)/a/f/(f*x+e)+d*cos
h(c-d*e/f)*Shi(d*e/f+d*x)/a/f^2-I*d*sinh(2*c-2*d*e/f)*Shi(2*d*e/f+2*d*x)/a
/f^2
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.18

$$\int \frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$
$$= \frac{-2f\cosh(c+dx)-2id(e+fx)\cosh\left(2c-\frac{2de}{f}\right)\text{Chi}\left(\frac{2d(e+fx)}{f}\right)+2d(e+fx)\text{Chi}\left(d\left(\frac{e}{f}+x\right)\right)\sinh\left(c-\right.}{}$$

input
```
Integrate[Cosh[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
(-2*f*Cosh[c + d*x] - (2*I)*d*(e + f*x)*Cosh[2*c - (2*d*e)/f]*CoshIntegral
[(2*d*(e + f*x))/f] + 2*d*(e + f*x)*CoshIntegral[d*(e/f + x)]*Sinh[c - (d*
e)/f] + I*f*Sinh[2*(c + d*x)] + 2*d*e*Cosh[c - (d*e)/f]*SinhIntegral[d*(e/
f + x)] + 2*d*f*x*Cosh[c - (d*e)/f]*SinhIntegral[d*(e/f + x)] - (2*I)*d*e*
Sinh[2*c - (2*d*e)/f]*SinhIntegral[(2*d*(e + f*x))/f] - (2*I)*d*f*x*Sinh[2
*c - (2*d*e)/f]*SinhIntegral[(2*d*(e + f*x))/f])/(2*a*f^2*(e + f*x))
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.04, number of steps used = 24, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.774$, Rules used = {6097, 3042, 3778, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782, 5971, 27, 3042, 26, 3778, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$
$$\Big\downarrow \; 6097$$
$$\frac{\int \frac{\cosh(c+dx)}{(e+fx)^2}dx}{a}-\frac{i\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)^2}dx}{a}$$
$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\sin\left(ic+idx+\frac{\pi}{2}\right)}{(e+fx)^2}dx}{a}-\frac{i\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{3778}$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}+\frac{id\int -\frac{i\sinh(c+dx)}{e+fx}dx}{f}}{a}-\frac{i\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{d\int \frac{\sinh(c+dx)}{e+fx}dx}{f}-\frac{\cosh(c+dx)}{f(e+fx)}}{a}-\frac{i\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}+\frac{d\int -\frac{i\sin(ic+idx)}{e+fx}dx}{f}}{a}-\frac{i\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\int \frac{\sin(ic+idx)}{e+fx}dx}{f}}{a}-\frac{i\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{3784}$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(i\sinh\left(c-\frac{de}{f}\right)\int \frac{\cosh\left(\frac{de}{f}+dx\right)}{e+fx}dx+\cosh\left(c-\frac{de}{f}\right)\int \frac{i\sinh\left(\frac{de}{f}+dx\right)}{e+fx}dx\right)}{f}}{a}-\frac{i\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(i\sinh\left(c-\frac{de}{f}\right)\int \frac{\cosh\left(\frac{de}{f}+dx\right)}{e+fx}dx+i\cosh\left(c-\frac{de}{f}\right)\int \frac{\sinh\left(\frac{de}{f}+dx\right)}{e+fx}dx\right)}{f}}{a}-\frac{i\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(i\sinh\left(c-\frac{de}{f}\right)\int \frac{\sin\left(\frac{ide}{f}+idx+\frac{\pi}{2}\right)}{e+fx}dx+i\cosh\left(c-\frac{de}{f}\right)\int -\frac{i\sin\left(\frac{ide}{f}+idx\right)}{e+fx}dx\right)}{f}}{a}-\frac{i\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(i\sinh\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{ide}{f}+idx+\frac{\pi}{2}\right)}{e+fx}dx+\cosh\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{ide}{f}+idx\right)}{e+fx}dx\right)}{f}}{a}-\frac{i\int\frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{3779}$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(i\sinh\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{ide}{f}+idx+\frac{\pi}{2}\right)}{e+fx}dx+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}}{a}-\frac{i\int\frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{3782}$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}}{a}-\frac{i\int\frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{5971}$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}}{a}-\frac{i\int\frac{\sinh(2c+2dx)}{2(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}}{a}-\frac{i\int\frac{\sinh(2c+2dx)}{(e+fx)^2}dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}}{a}-\frac{i\int-\frac{i\sin(2ic+2idx)}{(e+fx)^2}dx}{2a}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}}{a}-\frac{\int\frac{\sin(2ic+2idx)}{(e+fx)^2}dx}{2a}$$

$$\Big\downarrow \text{3778}$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}}{a}-\frac{\frac{2id\int\frac{\cosh(2c+2dx)}{e+fx}dx}{f}-\frac{i\sinh(2c+2dx)}{f(e+fx)}}{2a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}}{a}-\frac{\frac{2id\int\frac{\sin\left(2ic+2idx+\frac{\pi}{2}\right)}{e+fx}dx}{f}-\frac{i\sinh(2c+2dx)}{f(e+fx)}}{2a}$$

$$\Big\downarrow \text{ 3784}$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}}{a}-\frac{\frac{2id\left(\cosh\left(2c-\frac{2de}{f}\right)\int\frac{\cosh\left(\frac{2de}{f}+2dx\right)}{e+fx}dx-i\sinh\left(2c-\frac{2de}{f}\right)\int\frac{i\sinh\left(\frac{2de}{f}+2dx\right)}{e+fx}dx\right)}{f}-\frac{i\sinh(2c+2dx)}{f(e+fx)}}{2a}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}}{a}-\frac{\frac{2id\left(\sinh\left(2c-\frac{2de}{f}\right)\int\frac{\sinh\left(\frac{2de}{f}+2dx\right)}{e+fx}dx+\cosh\left(2c-\frac{2de}{f}\right)\int\frac{\cosh\left(\frac{2de}{f}+2dx\right)}{e+fx}dx\right)}{f}-\frac{i\sinh(2c+2dx)}{f(e+fx)}}{2a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}}{a}-\frac{\frac{2id\left(\sinh\left(2c-\frac{2de}{f}\right)\int-\frac{i\sin\left(\frac{2ide}{f}+2idx\right)}{e+fx}dx+\cosh\left(2c-\frac{2de}{f}\right)\int\frac{\sin\left(\frac{2ide}{f}+2idx+\frac{\pi}{2}\right)}{e+fx}dx\right)}{f}-\frac{i\sinh(2c+2dx)}{f(e+fx)}}{2a}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}}{a}-\frac{\frac{2id\left(\cosh\left(2c-\frac{2de}{f}\right)\int\frac{\sin\left(\frac{2ide}{f}+2idx+\frac{\pi}{2}\right)}{e+fx}dx-i\sinh\left(2c-\frac{2de}{f}\right)\int\frac{\sin\left(\frac{2ide}{f}+2idx\right)}{e+fx}dx\right)}{f}-\frac{i\sinh(2c+2dx)}{f(e+fx)}}{2a}$$

$$\Big\downarrow \text{3779}$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}}{a}-\frac{\frac{2id\left(\frac{\sinh\left(2c-\frac{2de}{f}\right)\text{Shi}\left(\frac{2de}{f}+2dx\right)}{f}+\cosh\left(2c-\frac{2de}{f}\right)\int\frac{\sin\left(\frac{2ide}{f}+2idx+\frac{\pi}{2}\right)}{e+fx}dx\right)}{f}-\frac{i\sinh(2c+2dx)}{f(e+fx)}}{2a}$$

$$\Big\downarrow \text{3782}$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}}{a}-\frac{\frac{2id\left(\frac{\cosh\left(2c-\frac{2de}{f}\right)\text{Chi}\left(\frac{2de}{f}+2dx\right)}{f}+\frac{\sinh\left(2c-\frac{2de}{f}\right)\text{Shi}\left(\frac{2de}{f}+2dx\right)}{f}\right)}{f}-\frac{i\sinh(2c+2dx)}{f(e+fx)}}{2a}$$

input

```
Int[Cosh[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output

```
(-(Cosh[c + d*x]/(f*(e + f*x))) - (I*d*((I*CoshIntegral[(d*e)/f + d*x]*Sin
h[c - (d*e)/f])/f + (I*Cosh[c - (d*e)/f]*SinhIntegral[(d*e)/f + d*x])/f))/
f)/a - (((-I)*Sinh[2*c + 2*d*x])/(f*(e + f*x)) + ((2*I)*d*((Cosh[2*c - (2*
d*e)/f]*CoshIntegral[(2*d*e)/f + 2*d*x])/f + (Sinh[2*c - (2*d*e)/f]*SinhIn
tegral[(2*d*e)/f + 2*d*x])/f))/f)/(2*a)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6097
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cosh[c
 + d*x]^(n - 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*
Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && E
qQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 113.99 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.66

method	result
risch	$-\frac{d\,{\mathrm e}^{-d x -c}}{2 a f \left(d x f +d e \right)}+\frac{d\,{\mathrm e}^{-\frac{c f -d e}{f}} \operatorname{expIntegral}_{1}\left(d x +c -\frac{c f -d e}{f}\right)}{2 a \,f^{2}}-\frac{d\,{\mathrm e}^{d x +c}}{2 f^{2} a \left(\frac{d e}{f}+d x \right)}-\frac{d\,{\mathrm e}^{\frac{c f -d e}{f}} \operatorname{expIntegral}_{1}\left(-d x -c -\frac{-c f +d e}{f}\right)}{2 f^{2} a}-$

input
```
int(cosh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*d/a*exp(-d*x-c)/f/(d*f*x+d*e)+1/2*d/a/f^2*exp(-(c*f-d*e)/f)*Ei(1,d*x+
c-(c*f-d*e)/f)-1/2/f^2*d/a*exp(d*x+c)/(d*e/f+d*x)-1/2/f^2*d/a*exp((c*f-d*e
)/f)*Ei(1,-d*x-c-(-c*f+d*e)/f)+1/4*I*d/a/f^2*exp(2*d*x+2*c)/(d*e/f+d*x)+1/
2*I*d/a/f^2*exp(2*(c*f-d*e)/f)*Ei(1,-2*d*x-2*c-2*(-c*f+d*e)/f)-1/4*I/a*d*e
xp(-2*d*x-2*c)/f/(d*f*x+d*e)+1/2*I/a*d/f^2*exp(-2*(c*f-d*e)/f)*Ei(1,2*d*x+
2*c-2*(c*f-d*e)/f)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.26

$$\int \frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \frac{\left(i\,fe^{(4\,dx+4\,c)}-2\,fe^{(3\,dx+3\,c)}-2\left((i\,dfx+i\,de)\mathrm{Ei}\left(-\frac{2\,(dfx+de)}{f}\right)e^{\left(\frac{2\,(de-cf)}{f}\right)}+(dfx+de)\mathrm{Ei}\left(-\frac{dfx+de}{f}\right)e^{\left(\frac{de}{}\right.}\right.\right.}{}$$

input
```
integrate(cosh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/4*(I*f*e^(4*d*x + 4*c) - 2*f*e^(3*d*x + 3*c) - 2*((I*d*f*x + I*d*e)*Ei(-
2*(d*f*x + d*e)/f)*e^(2*(d*e - c*f)/f) + (d*f*x + d*e)*Ei(-(d*f*x + d*e)/f
)*e^((d*e - c*f)/f) - (d*f*x + d*e)*Ei((d*f*x + d*e)/f)*e^(-(d*e - c*f)/f)
 + (I*d*f*x + I*d*e)*Ei(2*(d*f*x + d*e)/f)*e^(-2*(d*e - c*f)/f))*e^(2*d*x
+ 2*c) - 2*f*e^(d*x + c) - I*f)*e^(-2*d*x - 2*c)/(a*f^3*x + a*e*f^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**3/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cosh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1080 vs. $2(172) = 344$.

Time = 0.20 (sec) , antiderivative size = 1080, normalized size of antiderivative = 6.00

$$\int \frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*(2*I*(f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e))*d^2*Ei(-2*((f*x +
e)*(d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e - c*f)/f)*e^(2*(d*e - c*f)/f)
 + 2*I*d^3*e*Ei(-2*((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e -
c*f)/f)*e^(2*(d*e - c*f)/f) - 2*I*c*d^2*f*Ei(-2*((f*x + e)*(d - d*e/(f*x +
 e) + c*f/(f*x + e)) + d*e - c*f)/f)*e^(2*(d*e - c*f)/f) + 2*(f*x + e)*(d
- d*e/(f*x + e) + c*f/(f*x + e))*d^2*Ei(-((f*x + e)*(d - d*e/(f*x + e) + c
*f/(f*x + e)) + d*e - c*f)/f)*e^((d*e - c*f)/f) + 2*d^3*e*Ei(-((f*x + e)*(
d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e - c*f)/f)*e^((d*e - c*f)/f) - 2*c
*d^2*f*Ei(-((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e - c*f)/f)*
e^((d*e - c*f)/f) - 2*(f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e))*d^2*Ei
(((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e - c*f)/f)*e^(-(d*e -
 c*f)/f) - 2*d^3*e*Ei(((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e
 - c*f)/f)*e^(-(d*e - c*f)/f) + 2*c*d^2*f*Ei(((f*x + e)*(d - d*e/(f*x + e)
 + c*f/(f*x + e)) + d*e - c*f)/f)*e^(-(d*e - c*f)/f) + 2*I*(f*x + e)*(d -
d*e/(f*x + e) + c*f/(f*x + e))*d^2*Ei(2*((f*x + e)*(d - d*e/(f*x + e) + c*
f/(f*x + e)) + d*e - c*f)/f)*e^(-2*(d*e - c*f)/f) + 2*I*d^3*e*Ei(2*((f*x +
 e)*(d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e - c*f)/f)*e^(-2*(d*e - c*f)/
f) - 2*I*c*d^2*f*Ei(2*((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e
 - c*f)/f)*e^(-2*(d*e - c*f)/f) - I*d^2*f*e^(2*(f*x + e)*(d - d*e/(f*x + e
) + c*f/(f*x + e))/f) + 2*d^2*f*e^((f*x + e)*(d - d*e/(f*x + e) + c*f/(...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)^3}{(e+f\,x)^2\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(cosh(c + d*x)^3/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(cosh(c + d*x)^3/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [F]

$$\int \frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)^3}{\left(fx+e\right)^2\left(a+ia\sinh\left(dx+c\right)\right)}dx$$

input
```
int(cosh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

3.271 $\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 463

$$\int \frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{3if(e+fx)^2}{2ad^2}-\frac{6f^2(e+fx)\arctan\left(e^{c+dx}\right)}{ad^3}+\frac{(e+fx)^3\arctan\left(e^{c+dx}\right)}{ad}+\frac{3if^2(e+fx)\log\left(1+e^{2(c+dx)}\right)}{ad^3}+\frac{3if^3\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^4}-\frac{3if(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{2ad^2}-\frac{3if^3\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{ad^4}+\frac{3if(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{2ad^2}+\frac{3if^3\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2ad^4}+\frac{3if^2(e+fx)\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^3}-\frac{3if^2(e+fx)\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{ad^3}-\frac{3if^3\operatorname{PolyLog}\left(4,-ie^{c+dx}\right)}{ad^4}+\frac{3if^3\operatorname{PolyLog}\left(4,ie^{c+dx}\right)}{ad^4}+\frac{3f(e+fx)^2\operatorname{sech}(c+dx)}{2ad^2}+\frac{i(e+fx)^3\operatorname{sech}^2(c+dx)}{2ad}-\frac{3if(e+fx)^2\tanh(c+dx)}{2ad^2}+\frac{(e+fx)^3\operatorname{sech}(c+dx)\tanh(c+dx)}{2ad}$$

output
```
3*I*f^3*polylog(4,I*exp(d*x+c))/a/d^4-6*f^2*(f*x+e)*arctan(exp(d*x+c))/a/d
^3+(f*x+e)^3*arctan(exp(d*x+c))/a/d-3*I*f^3*polylog(2,I*exp(d*x+c))/a/d^4-
3/2*I*f*(f*x+e)^2*tanh(d*x+c)/a/d^2+3*I*f^2*(f*x+e)*polylog(3,-I*exp(d*x+c
))/a/d^3+3/2*I*f^3*polylog(2,-exp(2*d*x+2*c))/a/d^4-3*I*f^2*(f*x+e)*polylo
g(3,I*exp(d*x+c))/a/d^3+3/2*I*f*(f*x+e)^2*polylog(2,I*exp(d*x+c))/a/d^2-3*
I*f^3*polylog(4,-I*exp(d*x+c))/a/d^4+3*I*f^3*polylog(2,-I*exp(d*x+c))/a/d^
4+1/2*I*(f*x+e)^3*sech(d*x+c)^2/a/d-3/2*I*f*(f*x+e)^2*polylog(2,-I*exp(d*x
+c))/a/d^2+3/2*f*(f*x+e)^2*sech(d*x+c)/a/d^2-3/2*I*f*(f*x+e)^2/a/d^2+3*I*f
^2*(f*x+e)*ln(1+exp(2*d*x+2*c))/a/d^3+1/2*(f*x+e)^3*sech(d*x+c)*tanh(d*x+c
)/a/d
```

Mathematica [A] (verified)

Time = 7.82 (sec) , antiderivative size = 828, normalized size of antiderivative = 1.79

$$\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Sech[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
-1/8*((e + f*x)^4/f + (4*(1 - I*E^c)*(e + f*x)^3*Log[1 + I*E^(-c - d*x)])/
d + ((12*I)*(I + E^c)*f*(d^2*(e + f*x)^2*PolyLog[2, (-I)*E^(-c - d*x)] + 2
*f*(d*(e + f*x)*PolyLog[3, (-I)*E^(-c - d*x)] + f*PolyLog[4, (-I)*E^(-c -
d*x)])))/d^4)/(a*(I + E^c)) - (-4*d^2*e*(1 + I*E^c)*f*(d^2*e^2 - 12*f^2)*x
 + (-12*f^2 + d^2*(e + f*x)^2)^2 + 12*d*(1 + I*E^c)*f^2*(d^2*e^2 - 4*f^2)*
x*Log[1 - I*E^(-c - d*x)] + 12*d^3*e*(1 + I*E^c)*f^3*x^2*Log[1 - I*E^(-c -
 d*x)] + 4*d^3*(1 + I*E^c)*f^4*x^3*Log[1 - I*E^(-c - d*x)] + 4*d*e*(1 + I*
E^c)*f*(d^2*e^2 - 12*f^2)*Log[I - E^(c + d*x)] + 12*(1 + I*E^c)*f^2*(-(d^2
*e^2) + 4*f^2)*PolyLog[2, I*E^(-c - d*x)] - 24*d^2*e*(1 + I*E^c)*f^3*x*Pol
yLog[2, I*E^(-c - d*x)] - 12*d^2*(1 + I*E^c)*f^4*x^2*PolyLog[2, I*E^(-c -
d*x)] - 24*d*e*(1 + I*E^c)*f^3*PolyLog[3, I*E^(-c - d*x)] - 24*d*(1 + I*E^
c)*f^4*x*PolyLog[3, I*E^(-c - d*x)] - 24*(1 + I*E^c)*f^4*PolyLog[4, I*E^(-
c - d*x)])/(8*a*d^4*(-I + E^c)*f) + (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 +
f^3*x^3))/(8*a*(Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2] + I*Sinh[c/2])) + ((I/
2)*(e + f*x)^3)/(a*d*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 + (d*x)/2])^2) - ((
3*I)*(e^2*f*Sinh[(d*x)/2] + 2*e*f^2*x*Sinh[(d*x)/2] + f^3*x^2*Sinh[(d*x)/2
]))/(a*d^2*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 + (
d*x)/2]))
```

Rubi [A] (verified)

Time = 3.07 (sec) , antiderivative size = 442, normalized size of antiderivative = 0.95, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.724, Rules used = {6105, 3042, 4674, 3042, 4668, 2715, 2838, 3011, 5974, 3042, 4672, 26, 3042, 26, 4201, 2620, 2715, 2838, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6105}$$

$$\frac{\int (e+fx)^3\text{sech}^3(c+dx)dx}{a}-\frac{i\int (e+fx)^3\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{a}-\frac{i\int (e+fx)^3\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4674}$$

$$\frac{-\frac{3f^2\int (e+fx)\text{sech}(c+dx)dx}{d^2}+\frac{1}{2}\int (e+fx)^3\text{sech}(c+dx)dx+\frac{3f(e+fx)^2\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)^3\tanh(c+dx)\text{sech}(c+dx)}{2d}}{a}-\frac{i\int (e+fx)^3\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{3f^2\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{d^2}+\frac{1}{2}\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{3f(e+fx)^2\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)^3\tanh(c+dx)\text{sech}(c+}{2d}}{a}\frac{i\int (e+fx)^3\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4668}$$

$$\frac{-\frac{3f^2\left(-\frac{if\int \log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int \log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)}{d^2}+\frac{1}{2}\left(-\frac{3if\int (e+fx)^2\log(1-ie^{c+dx})dx}{d}+\frac{3if\int (e+fx)^2\log(}{d}\right.}{a}\frac{i\int (e+fx)^3\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$\Big\downarrow$ 2715

$$\frac{-\frac{3f^2\left(-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)}{d^2}+\frac{1}{2}\left(-\frac{3if\int(e+fx)^2\log(1-ie^{c+dx})}{d}\right.}{a}$$

$$\frac{i\int(e+fx)^3\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$\Big\downarrow$ 2838

$$\frac{\frac{1}{2}\left(-\frac{3if\int(e+fx)^2\log(1-ie^{c+dx})dx}{d}+\frac{3if\int(e+fx)^2\log(1+ie^{c+dx})dx}{d}+\frac{2(e+fx)^3\arctan(e^{c+dx})}{d}\right)-\frac{3f^2\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\text{ Po}}{}\right.}{}}{a}$$

$$\frac{i\int(e+fx)^3\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$\Big\downarrow$ 3011

$$\frac{\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\text{ PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{ PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\text{ PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{ PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}\right.}{}$$

$$\frac{i\int(e+fx)^3\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$\Big\downarrow$ 5974

$$\frac{\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\text{ PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{ PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\text{ PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{ PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}\right.}{}$$

$$\frac{i\left(\frac{3f\int(e+fx)^2\text{sech}^2(c+dx)dx}{2d}-\frac{(e+fx)^3\text{sech}^2(c+dx)}{2d}\right)}{a}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}\right.}{\frac{i\left(-\frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{2d}+\frac{3f\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{2d}\right)}{a}}$$

$$\Big\downarrow \text{ 4672}$$

$$\frac{\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}\right.}{\frac{i\left(-\frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{2d}+\frac{3f\left(\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2if\int-i(e+fx)\tanh(c+dx)dx}{d}\right)}{2d}\right)}{a}}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}\right.}{\frac{i\left(\frac{3f\left(\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2f\int(e+fx)\tanh(c+dx)dx}{d}\right)}{2d}-\frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{2d}\right)}{a}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}\right.}{\frac{i\left(-\frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{2d}+\frac{3f\left(\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2f\int-i(e+fx)\tan(ic+idx)dx}{d}\right)}{2d}\right)}{a}}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}\right.}{}$$

$$\frac{i\left(-\frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{2d}+\frac{3f\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\int(e+fx)\tan(ic+idx)dx}{d}\right)}{2d}\right)}{a}$$

$$\Big\downarrow \text{4201}$$

$$\frac{\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}\right.}{}$$

$$\frac{i\left(-\frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{2d}+\frac{3f\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\int\frac{e^{2(c+dx)}(e+fx)}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}\right)}{a}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}\right.}{}$$

$$\frac{i\left(-\frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{2d}+\frac{3f\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int\log\left(1+e^{2(c+dx)}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}\right)}{a}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}\right.}{a}$$

$$i\left(-\frac{(e+fx)^3\text{sech}^2(c+dx)}{2d}+\frac{3f\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int e^{-2(c+dx)}\log\left(1+e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right.}{d}\right.}{2d}\right.$$

$$\Big\downarrow \text{2838}$$

$$\frac{\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}\right.}{a}$$

$$\frac{i\left(-\frac{(e+fx)^3\text{sech}^2(c+dx)}{2d}+\frac{3f\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}\right)}{a}$$

$$\Big\downarrow \text{7163}$$

$$\frac{\frac{1}{2}\left(\frac{3if\left(\frac{2f\left(\frac{(e+fx)\text{PolyLog}\left(3,-ie^{c+dx}\right)}{d}-\frac{f\int\text{PolyLog}\left(3,-ie^{c+dx}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\text{PolyLog}\left(3,ie^{c+dx}\right)}{d}-\right.}{d}\right.}{}\right.}{a}$$

$$\frac{i\left(-\frac{(e+fx)^3\text{sech}^2(c+dx)}{2d}+\frac{3f\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}\right)}{a}$$

$\Big\downarrow$ 2720

$$\frac{\frac{1}{2}\left(\frac{3if\left(\frac{2f\left(\frac{(e+fx)\text{PolyLog}\left(3,-ie^{c+dx}\right)}{d}-\frac{f\int e^{-c-dx}\text{PolyLog}\left(3,-ie^{c+dx}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\text{PolyLog}(}{d}\right.}{}\right.}{}\right.}{a}$$

$$i\left(-\frac{(e+fx)^3\text{sech}^2(c+dx)}{2d}+\frac{3f\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}\right)$$

$\Big\downarrow$ 7143

$$-\frac{3f^2\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d^2}+\frac{1}{2}\left(\frac{2(e+fx)^3\arctan\left(e^{c+dx}\right)}{d}+\frac{3if\left(\frac{2f\left(\frac{(e+fx)\text{PolyLog}(3,}{d}\right.}{}\right.}{}\right.$$

$$\frac{i\left(-\frac{(e+fx)^3\text{sech}^2(c+dx)}{2d}+\frac{3f\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}\right)}{a}$$

input
```
Int[((e + f*x)^3*Sech[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-3*f^2*((2*(e + f*x)*ArcTan[E^(c + d*x)])/d - (I*f*PolyLog[2, (-I)*E^(c
+ d*x)])/d^2 + (I*f*PolyLog[2, I*E^(c + d*x)])/d^2))/d^2 + ((2*(e + f*x)^3
*ArcTan[E^(c + d*x)])/d + ((3*I)*f*(-(((e + f*x)^2*PolyLog[2, (-I)*E^(c +
d*x)])/d) + (2*f*(((e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/d - (f*PolyLog[
4, (-I)*E^(c + d*x)])/d^2))/d))/d - ((3*I)*f*(-(((e + f*x)^2*PolyLog[2, I*
E^(c + d*x)])/d) + (2*f*(((e + f*x)*PolyLog[3, I*E^(c + d*x)])/d - (f*Poly
Log[4, I*E^(c + d*x)])/d^2))/d))/d)/2 + (3*f*(e + f*x)^2*Sech[c + d*x])/(2
*d^2) + ((e + f*x)^3*Sech[c + d*x]*Tanh[c + d*x])/(2*d))/a - (I*(-1/2*((e
+ f*x)^3*Sech[c + d*x]^2)/d + (3*f*(((2*I)*f*(((-1/2*I)*(e + f*x)^2)/f + (
2*I)*(((e + f*x)*Log[1 + E^(2*(c + d*x))])/(2*d) + (f*PolyLog[2, -E^(2*(c
+ d*x))])/(4*d^2))))/d + ((e + f*x)^2*Tanh[c + d*x])/d))/(2*d)))/a
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6105
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Sech[
c + d*x]^(n + 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)
*Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] &&
EqQ[a^2 + b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1079 vs. $2(416) = 832$.

Time = 33.68 (sec) , antiderivative size = 1080, normalized size of antiderivative = 2.33

method	result	size
risch	Expression too large to display	1080

input
```
int((f*x+e)^3*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-3*I*f^3*polylog(4,-I*exp(d*x+c))/a/d^4+6/a/d^4*f^3*c*arctan(exp(d*x+c))-1
/a/d^4*f^3*c^3*arctan(exp(d*x+c))-6/a/d^3*e*f^2*arctan(exp(d*x+c))+6*I/a/d
^4*f^3*polylog(2,-I*exp(d*x+c))-3*I/a/d^2*f^3*x^2-3*I/a/d^4*f^3*c^2+1/a/d*
e^3*arctan(exp(d*x+c))+3*I*f^3*polylog(4,I*exp(d*x+c))/a/d^4+6*I/a/d^4*f^3
*ln(1+I*exp(d*x+c))*c+3/2*I/a/d^2*e^2*f*polylog(2,I*exp(d*x+c))-3/2*I/a/d^
2*e^2*f*polylog(2,-I*exp(d*x+c))-3*I/a/d^4*f^3*c*ln(1+exp(2*d*x+2*c))+6*I/
a/d^4*f^3*c*ln(exp(d*x+c))+1/2*I/a/d*f^3*ln(1-I*exp(d*x+c))*x^3+3/2*I/a/d^
2*f^3*polylog(2,I*exp(d*x+c))*x^2-3*I/a/d^3*f^3*polylog(3,I*exp(d*x+c))*x-
1/2*I/a/d*f^3*ln(1+I*exp(d*x+c))*x^3-3/2*I/a/d^2*f^3*polylog(2,-I*exp(d*x+
c))*x^2+3*I/a/d^3*f^3*polylog(3,-I*exp(d*x+c))*x+1/2*I/a/d^4*f^3*ln(1-I*ex
p(d*x+c))*c^3-1/2*I/a/d^4*f^3*ln(1+I*exp(d*x+c))*c^3-3*I/a/d^3*e*f^2*polyl
og(3,I*exp(d*x+c))+3*I/a/d^3*e*f^2*polylog(3,-I*exp(d*x+c))+3*I/a/d^3*e*f^
2*ln(1+exp(2*d*x+2*c))-6*I/a/d^3*e*f^2*ln(exp(d*x+c))+3/a/d^3*f^2*c^2*e*ar
ctan(exp(d*x+c))-3/a/d^2*e^2*f*c*arctan(exp(d*x+c))-6*I/a/d^3*f^3*c*x+6*I/
a/d^3*f^3*ln(1+I*exp(d*x+c))*x+(d*f^3*x^3*exp(d*x+c)+3*d*e*f^2*x^2*exp(d*x
+c)+3*d*e^2*f*x*exp(d*x+c)+d*e^3*exp(d*x+c)+3*f^3*x^2*exp(d*x+c)-3*I*f^3*x
^2+6*e*f^2*x*exp(d*x+c)-6*I*e*f^2*x+3*e^2*f*exp(d*x+c)-3*I*e^2*f)/(exp(d*x
+c)-I)^2/d^2/a+3/2*I/a/d*e^2*f*ln(1-I*exp(d*x+c))*x+3/2*I/a/d^2*e^2*f*ln(1
-I*exp(d*x+c))*c-3/2*I/a/d*e^2*f*ln(1+I*exp(d*x+c))*x-3/2*I/a/d^2*e^2*f*ln
(1+I*exp(d*x+c))*c+3/2*I/a/d*e*f^2*ln(1-I*exp(d*x+c))*x^2+3*I/a/d^2*e*f...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1461 vs. $2(390) = 780$.

Time = 0.11 (sec) , antiderivative size = 1461, normalized size of antiderivative = 3.16

$$\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(-6*I*d^2*e^2*f + 12*I*c*d*e*f^2 - 6*I*c^2*f^3 - 3*(I*d^2*f^3*x^2 + 2*
I*d^2*e*f^2*x + I*d^2*e^2*f + (-I*d^2*f^3*x^2 - 2*I*d^2*e*f^2*x - I*d^2*e^
2*f)*e^(2*d*x + 2*c) - 2*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f)*e^(d*x
+ c))*dilog(I*e^(d*x + c)) - 3*(-I*d^2*f^3*x^2 - 2*I*d^2*e*f^2*x - I*d^2*e
^2*f + 4*I*f^3 + (I*d^2*f^3*x^2 + 2*I*d^2*e*f^2*x + I*d^2*e^2*f - 4*I*f^3)
*e^(2*d*x + 2*c) + 2*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f - 4*f^3)*e^(
d*x + c))*dilog(-I*e^(d*x + c)) - 6*(I*d^2*f^3*x^2 + 2*I*d^2*e*f^2*x + 2*I
*c*d*e*f^2 - I*c^2*f^3)*e^(2*d*x + 2*c) + 2*(d^3*f^3*x^3 + d^3*e^3 + 3*d^2
*e^2*f - 12*c*d*e*f^2 + 6*c^2*f^3 + 3*(d^3*e*f^2 - d^2*f^3)*x^2 + 3*(d^3*e
^2*f - 2*d^2*e*f^2)*x)*e^(d*x + c) + (-I*d^3*e^3 + 3*I*c*d^2*e^2*f - 3*I*c
^2*d*e*f^2 + I*c^3*f^3 + (I*d^3*e^3 - 3*I*c*d^2*e^2*f + 3*I*c^2*d*e*f^2 -
I*c^3*f^3)*e^(2*d*x + 2*c) + 2*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 -
c^3*f^3)*e^(d*x + c))*log(e^(d*x + c) + I) + (I*d^3*e^3 - 3*I*c*d^2*e^2*f
- 3*(-I*c^2 + 4*I)*d*e*f^2 + (-I*c^3 + 12*I*c)*f^3 + (-I*d^3*e^3 + 3*I*c*d
^2*e^2*f - 3*(I*c^2 - 4*I)*d*e*f^2 + (I*c^3 - 12*I*c)*f^3)*e^(2*d*x + 2*c)
 - 2*(d^3*e^3 - 3*c*d^2*e^2*f + 3*(c^2 - 4)*d*e*f^2 - (c^3 - 12*c)*f^3)*e^
(d*x + c))*log(e^(d*x + c) - I) + (I*d^3*f^3*x^3 + 3*I*d^3*e*f^2*x^2 + 3*I
*c*d^2*e^2*f - 3*I*c^2*d*e*f^2 + (I*c^3 - 12*I*c)*f^3 - 3*(-I*d^3*e^2*f +
4*I*d*f^3)*x + (-I*d^3*f^3*x^3 - 3*I*d^3*e*f^2*x^2 - 3*I*c*d^2*e^2*f + 3*I
*c^2*d*e*f^2 + (-I*c^3 + 12*I*c)*f^3 - 3*(I*d^3*e^2*f - 4*I*d*f^3)*x)*e...
```

Sympy [F]

$$\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e^3\,\text{sech}\,(c+dx)}{\sinh\,(c+dx)-i}\,dx+\int \frac{f^3x^3\,\text{sech}\,(c+dx)}{\sinh\,(c+dx)-i}\,dx+\int \frac{3ef^2x^2\,\text{sech}\,(c+dx)}{\sinh\,(c+dx)-i}\,dx+\int \frac{3e^2fx\,\text{sech}\,(c+dx)}{\sinh\,(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)**3*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e**3*sech(c + d*x)/(sinh(c + d*x) - I), x) + Integral(f**3*x*
*3*sech(c + d*x)/(sinh(c + d*x) - I), x) + Integral(3*e*f**2*x**2*sech(c +
 d*x)/(sinh(c + d*x) - I), x) + Integral(3*e**2*f*x*sech(c + d*x)/(sinh(c
+ d*x) - I), x))/a
```

Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 685, normalized size of antiderivative = 1.48

$$\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*e^3*(4*e^(-d*x - c)/((4*I*a*e^(-d*x - c) + 2*a*e^(-2*d*x - 2*c) - 2*a
)*d) + I*log(e^(-d*x - c) + I)/(a*d) - I*log(I*e^(-d*x - c) + 1)/(a*d)) +
3/2*I*(d*x*log(-I*e^(d*x + c) + 1) + dilog(I*e^(d*x + c)))*e^2*f/(a*d^2) -
 6*I*e*f^2*x/(a*d^2) + (-3*I*f^3*x^2 - 6*I*e*f^2*x - 3*I*e^2*f + (d*f^3*x^
3*e^c + 3*e^2*f*e^c + 3*(d*e*f^2 + f^3)*x^2*e^c + 3*(d*e^2*f + 2*e*f^2)*x*
e^c)*e^(d*x))/(a*d^2*e^(2*d*x + 2*c) - 2*I*a*d^2*e^(d*x + c) - a*d^2) - 3/
2*I*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-I*e^(d*x + c)) - 2*poly
log(3, -I*e^(d*x + c)))*e*f^2/(a*d^3) + 3/2*I*(d^2*x^2*log(-I*e^(d*x + c)
+ 1) + 2*d*x*dilog(I*e^(d*x + c)) - 2*polylog(3, I*e^(d*x + c)))*e*f^2/(a*
d^3) + 6*I*e*f^2*log(I*e^(d*x + c) + 1)/(a*d^3) - 1/2*I*(d^3*x^3*log(I*e^(
d*x + c) + 1) + 3*d^2*x^2*dilog(-I*e^(d*x + c)) - 6*d*x*polylog(3, -I*e^(d
*x + c)) + 6*polylog(4, -I*e^(d*x + c)))*f^3/(a*d^4) + 1/2*I*(d^3*x^3*log(
-I*e^(d*x + c) + 1) + 3*d^2*x^2*dilog(I*e^(d*x + c)) - 6*d*x*polylog(3, I*
e^(d*x + c)) + 6*polylog(4, I*e^(d*x + c)))*f^3/(a*d^4) - 3/2*I*(d^2*e^2*f
 - 4*f^3)*(d*x*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c)))/(a*d^4) - 1
/8*(I*d^4*f^3*x^4 + 4*I*d^4*e*f^2*x^3 + 6*I*d^4*e^2*f*x^2)/(a*d^4) + 1/8*(
I*d^4*f^3*x^4 + 4*I*d^4*e*f^2*x^3 - 6*(-I*d^2*e^2*f + 4*I*f^3)*d^2*x^2)/(a
*d^4)
```

Giac [F]

$$\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\,\text{sech}\,(dx+c)}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sech(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^3}{\cosh(c+d\,x)\,(a+a\sinh(c+d\,x)\,1\mathrm{i})}\,dx$$

input
```
int((e + f*x)^3/(cosh(c + d*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)^3/(cosh(c + d*x)*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(\int \frac{\operatorname{sech}(dx+c)}{\sinh(dx+c)i+1}dx\right)e^3+\left(\int \frac{\operatorname{sech}(dx+c)x^3}{\sinh(dx+c)i+1}dx\right)f^3+3\left(\int \frac{\operatorname{sech}(dx+c)x^2}{\sinh(dx+c)i+1}dx\right)e\,f^2+3\left(\int \frac{\operatorname{sech}(dx+c)x}{\sinh(dx+c)i+1}dx\right)e^2f}{a}$$

input
```
int((f*x+e)^3*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
(int(sech(c + d*x)/(sinh(c + d*x)*i + 1),x)*e**3 + int((sech(c + d*x)*x**3
)/(sinh(c + d*x)*i + 1),x)*f**3 + 3*int((sech(c + d*x)*x**2)/(sinh(c + d*x
)*i + 1),x)*e*f**2 + 3*int((sech(c + d*x)*x)/(sinh(c + d*x)*i + 1),x)*e**2
*f)/a
```

3.272 $\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 268

$$\begin{aligned}\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx &= \frac{(e+fx)^2\arctan\left(e^{c+dx}\right)}{ad} \\ &\quad - \frac{f^2\arctan(\sinh(c+dx))}{ad^3} + \frac{if^2\log(\cosh(c+dx))}{ad^3} \\ &\quad - \frac{if(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^2} \\ &\quad + \frac{if(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{ad^2} \\ &\quad + \frac{if^2\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^3} \\ &\quad - \frac{if^2\,\text{PolyLog}\left(3,ie^{c+dx}\right)}{ad^3} + \frac{f(e+fx)\text{sech}(c+dx)}{ad^2} \\ &\quad + \frac{i(e+fx)^2\text{sech}^2(c+dx)}{2ad} - \frac{if(e+fx)\tanh(c+dx)}{ad^2} \\ &\quad + \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{2ad}\end{aligned}$$

output
```
(f*x+e)^2*arctan(exp(d*x+c))/a/d-f^2*arctan(sinh(d*x+c))/a/d^3+I*f^2*ln(co
sh(d*x+c))/a/d^3-I*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/a/d^2+I*f*(f*x+e)*po
lylog(2,I*exp(d*x+c))/a/d^2+I*f^2*polylog(3,-I*exp(d*x+c))/a/d^3-I*f^2*pol
ylog(3,I*exp(d*x+c))/a/d^3+f*(f*x+e)*sech(d*x+c)/a/d^2+1/2*I*(f*x+e)^2*sec
h(d*x+c)^2/a/d-I*f*(f*x+e)*tanh(d*x+c)/a/d^2+1/2*(f*x+e)^2*sech(d*x+c)*tan
h(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 5.28 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.98

$$\int \frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{\frac{\frac{(e+fx)^3}{f}+\frac{3(1-ie^c)(e+fx)^2\log\left(1+ie^{-c-dx}\right)}{d}+\frac{6i(i+e^c)f\left(d(e+fx)\operatorname{PolyLog}\left(2,-ie^{-c-dx}\right)+f\operatorname{PolyLog}\left(3,-ie^{-c-dx}\right)\right)}{d^3}}{i+e^c}+\frac{3d^2e^2x-12f^2x-3(1+ie^c}{}}{}$$

input
```
Integrate[((e + f*x)^2*Sech[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
-1/6*(((e + f*x)^3/f + (3*(1 - I*E^c)*(e + f*x)^2*Log[1 + I*E^(-c - d*x)])
/d + ((6*I)*(I + E^c)*f*(d*(e + f*x)*PolyLog[2, (-I)*E^(-c - d*x)] + f*Pol
yLog[3, (-I)*E^(-c - d*x)]))/d^3)/(I + E^c) + (3*d^2*e^2*x - 12*f^2*x - 3*
(1 + I*E^c)*(d^2*e^2 - 4*f^2)*x + 3*d^2*e*f*x^2 + d^2*f^2*x^3 + 6*d*e*(1 +
 I*E^c)*f*x*Log[1 - I*E^(-c - d*x)] + 3*d*(1 + I*E^c)*f^2*x^2*Log[1 - I*E^
(-c - d*x)] + (3*(1 + I*E^c)*(d^2*e^2 - 4*f^2)*Log[I - E^(c + d*x)])/d - 6
*e*(1 + I*E^c)*f*PolyLog[2, I*E^(-c - d*x)] - 6*(1 + I*E^c)*f^2*x*PolyLog[
2, I*E^(-c - d*x)] - (6*(1 + I*E^c)*f^2*PolyLog[3, I*E^(-c - d*x)])/d)/(d^
2*(-I + E^c)) - x*(3*e^2 + 3*e*f*x + f^2*x^2)*Sech[c] - ((3*I)*(e + f*x)^2
)/(d*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2) + ((12*I)*f*(e + f*x)*Si
nh[(d*x)/2])/(d^2*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] + I*Sinh[(c
 + d*x)/2])))/a
```

Rubi [A] (verified)

Time = 1.74 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.94, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.552, Rules used = {6105, 3042, 4674, 3042, 4257, 4668, 3011, 2720, 5974, 3042, 4672, 26, 3042, 26, 3956, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6105}$$

$$\frac{\int (e+fx)^2\text{sech}^3(c+dx)dx}{a}-\frac{i\int (e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{a}-\frac{i\int (e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4674}$$

$$\frac{-\frac{f^2\int \text{sech}(c+dx)dx}{d^2}+\frac{1}{2}\int (e+fx)^2\text{sech}(c+dx)dx+\frac{f(e+fx)\text{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}(c+dx)}{2d}}{a}-\frac{i\int (e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{f^2\int \csc\left(ic+idx+\frac{\pi}{2}\right)dx}{d^2}+\frac{1}{2}\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{f(e+fx)\text{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}(c+dx)}{2d}}{a}-\frac{i\int (e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{1}{2}\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f^2\arctan(\sinh(c+dx))}{d^3}+\frac{f(e+fx)\text{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}(c+dx)}{2d}}{a}-\frac{i\int (e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4668}$$

$$\frac{\frac{1}{2}\left(-\frac{2if\int(e+fx)\log(1-ie^{c+dx})dx}{d}+\frac{2if\int(e+fx)\log(1+ie^{c+dx})dx}{d}+\frac{2(e+fx)^2\arctan(e^{c+dx})}{d}\right)-\frac{f^2\arctan(\sinh(c+dx))}{d^3}+\frac{f(e+fx)\mathrm{S}}{}}{a}$$

$$\frac{i\int(e+fx)^2\mathrm{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$\downarrow$ 3011

$$\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int \mathrm{PolyLog}(2,-ie^{c+dx})dx}{d}-\frac{(e+fx)\mathrm{PolyLog}(2,-ie^{c+dx})}{d}\right)}{d}-\frac{2if\left(\frac{f\int \mathrm{PolyLog}(2,ie^{c+dx})dx}{d}-\frac{(e+fx)\mathrm{PolyLog}(2,ie^{c+dx})}{d}\right)}{d}+\frac{2(e+fx)^2\mathrm{a}}{}\right.}{a}$$

$$\frac{i\int(e+fx)^2\mathrm{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$\downarrow$ 2720

$$\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\mathrm{PolyLog}(2,-ie^{c+dx})de^{c+dx}}{d^2}-\frac{(e+fx)\mathrm{PolyLog}(2,-ie^{c+dx})}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\mathrm{PolyLog}(2,ie^{c+dx})de^{c+dx}}{d^2}-\frac{(e+fx)\mathrm{PolyLog}(2,i}{d}\right.}{d}\right.}{a}$$

$$\frac{i\int(e+fx)^2\mathrm{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$\downarrow$ 5974

$$\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\mathrm{PolyLog}(2,-ie^{c+dx})de^{c+dx}}{d^2}-\frac{(e+fx)\mathrm{PolyLog}(2,-ie^{c+dx})}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\mathrm{PolyLog}(2,ie^{c+dx})de^{c+dx}}{d^2}-\frac{(e+fx)\mathrm{PolyLog}(2,i}{d}\right.}{d}\right.}{a}$$

$$\frac{i\left(\frac{f\int(e+fx)\mathrm{sech}^2(c+dx)dx}{d}-\frac{(e+fx)^2\mathrm{sech}^2(c+dx)}{2d}\right)}{a}$$

$\downarrow$ 3042

$$\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\mathrm{PolyLog}(2,-ie^{c+dx})de^{c+dx}}{d^2}-\frac{(e+fx)\mathrm{PolyLog}(2,-ie^{c+dx})}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\mathrm{PolyLog}(2,ie^{c+dx})de^{c+dx}}{d^2}-\frac{(e+fx)\mathrm{PolyLog}(2,i}{d}\right.}{d}\right.}{a}$$

$$\frac{i\left(-\frac{(e+fx)^2\mathrm{sech}^2(c+dx)}{2d}+\frac{f\int(e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{d}\right)}{a}$$

$$\Big\downarrow \; 4672$$

$$\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,i\right.}{d}\right.}{d}\right.}{a}$$

$$\frac{i\left(-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}+\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{if\int -i\tanh(c+dx)dx}{d}\right)}{d}\right)}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,i\right.}{d}\right.}{d}\right.}{a}$$

$$\frac{i\left(\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int\tanh(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,i\right.}{d}\right.}{d}\right.}{a}$$

$$\frac{i\left(-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}+\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int -i\tan(ic+idx)dx}{d}\right)}{d}\right)}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,i\right.}{d}\right.}{d}\right.}{a}$$

$$\frac{i\left(-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}+\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}+\frac{if\int\tan(ic+idx)dx}{d}\right)}{d}\right)}{a}$$

$$\Big\downarrow \; 3956$$

$$\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,i\right.}{d}\right.}{d}\right.}{a}$$

$$\frac{i\left(\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \; 7143$$

$$\frac{-\frac{f^2\arctan(\sinh(c+dx))}{d^3}+\frac{1}{2}\left(\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}+\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,ie\right.}{d^2}\right.}{}\right.}{a}$$

$$\frac{i\left(\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{2d}\right)}{a}$$

input
```
Int[((e + f*x)^2*Sech[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(-((f^2*ArcTan[Sinh[c + d*x]])/d^3) + ((2*(e + f*x)^2*ArcTan[E^(c + d*x)])
/d + ((2*I)*f*(-(((e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d) + (f*PolyLog[
3, (-I)*E^(c + d*x)])/d^2))/d - ((2*I)*f*(-(((e + f*x)*PolyLog[2, I*E^(c +
 d*x)])/d) + (f*PolyLog[3, I*E^(c + d*x)])/d^2))/d)/2 + (f*(e + f*x)*Sech[
c + d*x])/d^2 + ((e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*d))/a - (I*(-
1/2*((e + f*x)^2*Sech[c + d*x]^2)/d + (f*(-((f*Log[Cosh[c + d*x]])/d^2) +
((e + f*x)*Tanh[c + d*x])/d))/d))/a
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6105
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Sech[
c + d*x]^(n + 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)
*Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] &&
EqQ[a^2 + b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(248) = 496$.

Time = 13.68 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.05

method	result
risch	$\frac{d\,x^2 f^2 \mathrm{e}^{dx+c}+2 d e f x\,\mathrm{e}^{dx+c}+d\,e^2 \mathrm{e}^{dx+c}-2 i f^2 x+2 f^2 x\,\mathrm{e}^{dx+c}-2 i e f+2 e f\,\mathrm{e}^{dx+c}}{\left(\mathrm{e}^{dx+c}-i\right)^2 d^2 a}+\frac{i f^2\,\mathrm{polylog}\left(3,-i\mathrm{e}^{dx+c}\right)}{a\,d^3}-\frac{i\,\mathrm{polylog}\left(2,-i\mathrm{e}^{dx+c}\right)}{a\,d^2}$

input
```
int((f*x+e)^2*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(d*x^2*f^2*exp(d*x+c)+2*d*e*f*x*exp(d*x+c)+d*e^2*exp(d*x+c)-2*I*f^2*x+2*f^
2*x*exp(d*x+c)-2*I*e*f+2*e*f*exp(d*x+c))/(exp(d*x+c)-I)^2/d^2/a+I*f^2*poly
log(3,-I*exp(d*x+c))/a/d^3-I/a/d^2*polylog(2,-I*exp(d*x+c))*f^2*x+I/a/d^2*
e*f*polylog(2,I*exp(d*x+c))+I/a/d^3*f^2*ln(1+exp(2*d*x+2*c))+1/a/d*e^2*arc
tan(exp(d*x+c))+1/a/d^3*c^2*f^2*arctan(exp(d*x+c))+1/2*I/a/d^3*ln(1+I*exp(
d*x+c))*c^2*f^2-2*I/a/d^3*f^2*ln(exp(d*x+c))+1/2*I/a/d*ln(1-I*exp(d*x+c))*
f^2*x^2+I/a/d*ln(1-I*exp(d*x+c))*e*f*x-2/a/d^2*c*e*f*arctan(exp(d*x+c))-I/
a/d^2*ln(1+I*exp(d*x+c))*c*e*f-I/a/d^2*e*f*polylog(2,-I*exp(d*x+c))-I*f^2*
polylog(3,I*exp(d*x+c))/a/d^3-1/2*I/a/d*ln(1+I*exp(d*x+c))*f^2*x^2+I/a/d^2
*polylog(2,I*exp(d*x+c))*f^2*x-1/2*I/a/d^3*ln(1-I*exp(d*x+c))*c^2*f^2-I/a/
d*ln(1+I*exp(d*x+c))*e*f*x+I/a/d^2*ln(1-I*exp(d*x+c))*c*e*f-2/a/d^3*f^2*ar
ctan(exp(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 805 vs. $2(235) = 470$.

Time = 0.13 (sec) , antiderivative size = 805, normalized size of antiderivative = 3.00

$$\int \frac{(e+fx)^2\mathrm{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(-4*I*d*e*f + 4*I*c*f^2 - 2*(I*d*f^2*x + I*d*e*f + (-I*d*f^2*x - I*d*e
*f)*e^(2*d*x + 2*c) - 2*(d*f^2*x + d*e*f)*e^(d*x + c))*dilog(I*e^(d*x + c)
) - 2*(-I*d*f^2*x - I*d*e*f + (I*d*f^2*x + I*d*e*f)*e^(2*d*x + 2*c) + 2*(d
*f^2*x + d*e*f)*e^(d*x + c))*dilog(-I*e^(d*x + c)) - 4*(I*d*f^2*x + I*c*f^
2)*e^(2*d*x + 2*c) + 2*(d^2*f^2*x^2 + d^2*e^2 + 2*d*e*f - 4*c*f^2 + 2*(d^2
*e*f - d*f^2)*x)*e^(d*x + c) + (-I*d^2*e^2 + 2*I*c*d*e*f - I*c^2*f^2 + (I*
d^2*e^2 - 2*I*c*d*e*f + I*c^2*f^2)*e^(2*d*x + 2*c) + 2*(d^2*e^2 - 2*c*d*e*
f + c^2*f^2)*e^(d*x + c))*log(e^(d*x + c) + I) + (I*d^2*e^2 - 2*I*c*d*e*f
+ (I*c^2 - 4*I)*f^2 + (-I*d^2*e^2 + 2*I*c*d*e*f + (-I*c^2 + 4*I)*f^2)*e^(2
*d*x + 2*c) - 2*(d^2*e^2 - 2*c*d*e*f + (c^2 - 4)*f^2)*e^(d*x + c))*log(e^(
d*x + c) - I) + (I*d^2*f^2*x^2 + 2*I*d^2*e*f*x + 2*I*c*d*e*f - I*c^2*f^2 +
 (-I*d^2*f^2*x^2 - 2*I*d^2*e*f*x - 2*I*c*d*e*f + I*c^2*f^2)*e^(2*d*x + 2*c
) - 2*(d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*e^(d*x + c))*log(I
*e^(d*x + c) + 1) + (-I*d^2*f^2*x^2 - 2*I*d^2*e*f*x - 2*I*c*d*e*f + I*c^2*
f^2 + (I*d^2*f^2*x^2 + 2*I*d^2*e*f*x + 2*I*c*d*e*f - I*c^2*f^2)*e^(2*d*x +
 2*c) + 2*(d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*e^(d*x + c))*l
og(-I*e^(d*x + c) + 1) - 2*(I*f^2*e^(2*d*x + 2*c) + 2*f^2*e^(d*x + c) - I*
f^2)*polylog(3, I*e^(d*x + c)) - 2*(-I*f^2*e^(2*d*x + 2*c) - 2*f^2*e^(d*x
+ c) + I*f^2)*polylog(3, -I*e^(d*x + c)))/(a*d^3*e^(2*d*x + 2*c) - 2*I*a*d
^3*e^(d*x + c) - a*d^3)
```

Sympy [F]

$$\int \frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e^2\operatorname{sech}(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{f^2x^2\operatorname{sech}(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{2efx\operatorname{sech}(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)**2*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e**2*sech(c + d*x)/(sinh(c + d*x) - I), x) + Integral(f**2*x**
*2*sech(c + d*x)/(sinh(c + d*x) - I), x) + Integral(2*e*f*x*sech(c + d*x)/
(sinh(c + d*x) - I), x))/a
```

Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.44

$$\int \frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx =$$

$$-\frac{1}{2}e^2\left(\frac{4\,e^{(-dx-c)}}{-2\left(-2i\,ae^{(-dx-c)}-ae^{(-2\,dx-2\,c)}+a\right)d}+\frac{i\,\log\left(e^{(-dx-c)}+i\right)}{ad}-\frac{i\,\log\left(i\,e^{(-dx-c)}+1\right)}{ad}\right)$$
$$+\frac{-2i\,f^2x-2i\,ef+\left(df^2x^2e^c+2\,efe^c+2\,(def+f^2)xe^c\right)e^{(dx)}}{ad^2e^{(2\,dx+2\,c)}-2i\,ad^2e^{(dx+c)}-ad^2}$$
$$-\frac{i\left(dx\log\left(i\,e^{(dx+c)}+1\right)+\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)\right)ef}{ad^2}$$
$$+\frac{i\left(dx\log\left(-i\,e^{(dx+c)}+1\right)+\mathrm{Li}_2\left(i\,e^{(dx+c)}\right)\right)ef}{ad^2}-\frac{2i\,f^2x}{ad^2}$$
$$-\frac{i\left(d^2x^2\log\left(i\,e^{(dx+c)}+1\right)+2\,dx\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)-2\,\mathrm{Li}_3\left(-i\,e^{(dx+c)}\right)\right)f^2}{2\,ad^3}$$
$$+\frac{i\left(d^2x^2\log\left(-i\,e^{(dx+c)}+1\right)+2\,dx\mathrm{Li}_2\left(i\,e^{(dx+c)}\right)-2\,\mathrm{Li}_3\left(i\,e^{(dx+c)}\right)\right)f^2}{2\,ad^3}$$
$$+\frac{2i\,f^2\log\left(i\,e^{(dx+c)}+1\right)}{ad^3}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*e^2*(4*e^(-d*x - c)/((4*I*a*e^(-d*x - c) + 2*a*e^(-2*d*x - 2*c) - 2*a
)*d) + I*log(e^(-d*x - c) + I)/(a*d) - I*log(I*e^(-d*x - c) + 1)/(a*d)) +
(-2*I*f^2*x - 2*I*e*f + (d*f^2*x^2*e^c + 2*e*f*e^c + 2*(d*e*f + f^2)*x*e^c
)*e^(d*x))/(a*d^2*e^(2*d*x + 2*c) - 2*I*a*d^2*e^(d*x + c) - a*d^2) - I*(d*
x*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c)))*e*f/(a*d^2) + I*(d*x*log
(-I*e^(d*x + c) + 1) + dilog(I*e^(d*x + c)))*e*f/(a*d^2) - 2*I*f^2*x/(a*d^
2) - 1/2*I*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-I*e^(d*x + c)) -
 2*polylog(3, -I*e^(d*x + c)))*f^2/(a*d^3) + 1/2*I*(d^2*x^2*log(-I*e^(d*x
+ c) + 1) + 2*d*x*dilog(I*e^(d*x + c)) - 2*polylog(3, I*e^(d*x + c)))*f^2/
(a*d^3) + 2*I*f^2*log(I*e^(d*x + c) + 1)/(a*d^3)
```

Giac [F]

$$\int \frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\operatorname{sech}(dx+c)}{i\,a\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sech(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(e+f\,x)^2}{\cosh(c+d\,x)\,(a+a\sinh(c+d\,x)\,1\mathrm{i})}\,dx$$

input
```
int((e + f*x)^2/(cosh(c + d*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)^2/(cosh(c + d*x)*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx=\frac{\left(\int \frac{\operatorname{sech}(dx+c)}{\sinh(dx+c)i+1}dx\right)e^2+\left(\int \frac{\operatorname{sech}(dx+c)x^2}{\sinh(dx+c)i+1}dx\right)f^2+2\left(\int \frac{\operatorname{sech}(dx+c)x}{\sinh(dx+c)i+1}dx\right)ef}{a}$$

input
```
int((f*x+e)^2*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
(int(sech(c + d*x)/(sinh(c + d*x)*i + 1),x)*e**2 + int((sech(c + d*x)*x**2
)/(sinh(c + d*x)*i + 1),x)*f**2 + 2*int((sech(c + d*x)*x)/(sinh(c + d*x)*i
 + 1),x)*e*f)/a
```

3.273 $\int \frac{(e+fx)\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 161

$$\begin{aligned}\int \frac{(e+fx)\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx &= \frac{(e+fx)\arctan\left(e^{c+dx}\right)}{ad} - \frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{2ad^2} \\ &\quad + \frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{2ad^2} + \frac{f\operatorname{sech}(c+dx)}{2ad^2} \\ &\quad + \frac{i(e+fx)\operatorname{sech}^2(c+dx)}{2ad} - \frac{if\tanh(c+dx)}{2ad^2} \\ &\quad + \frac{(e+fx)\operatorname{sech}(c+dx)\tanh(c+dx)}{2ad}\end{aligned}$$

output
```
(f*x+e)*arctan(exp(d*x+c))/a/d-1/2*I*f*polylog(2,-I*exp(d*x+c))/a/d^2+1/2*
I*f*polylog(2,I*exp(d*x+c))/a/d^2+1/2*f*sech(d*x+c)/a/d^2+1/2*I*(f*x+e)*se
ch(d*x+c)^2/a/d-1/2*I*f*tanh(d*x+c)/a/d^2+1/2*(f*x+e)*sech(d*x+c)*tanh(d*x
+c)/a/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 400 vs. 2(161) = 322.

Time = 2.33 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.48

$$\int \frac{(e+fx)\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{-2id(e+fx)+(c+dx)(cf-d(2e+fx))\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^2+(1-i)\left(\frac{1}{2}d^2fx\right.}{}$$

input
```
Integrate[((e + f*x)*Sech[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
-1/4*((-2*I)*d*(e + f*x) + (c + d*x)*(c*f - d*(2*e + f*x))*(Cosh[(c + d*x)
/2] + I*Sinh[(c + d*x)/2])^2 + (1 - I)*((d^2*f*x^2)/2 + d*e*(c + d*x) - (1
 - I)*(d*e - c*f)*(c + d*x) + (1 - I)*f*(c + d*x)*Log[1 + I*E^(-c - d*x)]
+ (1 - I)*(d*e - c*f)*Log[I + E^(c + d*x)] - (1 - I)*f*PolyLog[2, (-I)*E^(
-c - d*x)])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2 + (1 + I)*((d^2*f*
x^2)/2 + d*e*(c + d*x) - (1 + I)*(d*e - c*f)*(c + d*x) + (1 + I)*f*(c + d*
x)*Log[1 - I*E^(-c - d*x)] + (1 + I)*(d*e - c*f)*Log[I - E^(c + d*x)] - (1
 + I)*f*PolyLog[2, I*E^(-c - d*x)])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/
2])^2 - 4*f*Sinh[(c + d*x)/2]*((-I)*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])
)/(d^2*(a + I*a*Sinh[c + d*x]))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {6105, 3042, 4673, 3042, 4668, 2715, 2838, 5974, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{sech}(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6105}$$

$$\frac{\int (e+fx)\text{sech}^3(c+dx)dx}{a}-\frac{i\int (e+fx)\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{a}-\frac{i\int (e+fx)\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4673}$$

$$\frac{\frac{1}{2}\int (e+fx)\text{sech}(c+dx)dx+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{a}-\frac{i\int (e+fx)\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{2}\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{a}-\frac{i\int (e+fx)\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4668}$$

$$\frac{\frac{1}{2}\left(-\frac{if\int \log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int \log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{a}-\frac{i\int (e+fx)\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\frac{1}{2}\left(-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh}{}}{a}-\frac{i\int (e+fx)\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{a}-\frac{i\int (e+fx)\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{5974}$$

$$\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan(e^{c+dx})}{d}-\frac{if\operatorname{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{if\operatorname{PolyLog}(2,ie^{c+dx})}{d^2}\right)+\frac{f\operatorname{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}}{a}-\frac{i\left(\frac{f\int\operatorname{sech}^2(c+dx)dx}{2d}-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan(e^{c+dx})}{d}-\frac{if\operatorname{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{if\operatorname{PolyLog}(2,ie^{c+dx})}{d^2}\right)+\frac{f\operatorname{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}}{a}-\frac{i\left(-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}+\frac{f\int\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{2d}\right)}{a}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan(e^{c+dx})}{d}-\frac{if\operatorname{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{if\operatorname{PolyLog}(2,ie^{c+dx})}{d^2}\right)+\frac{f\operatorname{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}}{a}-\frac{i\left(-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}+\frac{if\int 1d(-i\tanh(c+dx))}{2d^2}\right)}{a}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan(e^{c+dx})}{d}-\frac{if\operatorname{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{if\operatorname{PolyLog}(2,ie^{c+dx})}{d^2}\right)+\frac{f\operatorname{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}}{a}-\frac{i\left(\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}\right)}{a}$$

input
```
Int[((e + f*x)*Sech[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-I)*(-1/2*((e + f*x)*Sech[c + d*x]^2)/d + (f*Tanh[c + d*x])/(2*d^2)))/a
+ (((2*(e + f*x)*ArcTan[E^(c + d*x)])/d - (I*f*PolyLog[2, (-I)*E^(c + d*x)
])/d^2 + (I*f*PolyLog[2, I*E^(c + d*x)])/d^2)/2 + (f*Sech[c + d*x])/(2*d^2
) + ((e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*d))/a
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6105
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Sech[
c + d*x]^(n + 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)
*Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] &&
EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 5.38 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.35

method	result
risch	$\frac{dfx\,\mathrm{e}^{dx+c}+de\,\mathrm{e}^{dx+c}+\mathrm{e}^{dx+c}f-if}{\left(\mathrm{e}^{dx+c}-i\right)^{2}d^{2}a}+\frac{e\arctan\left(\mathrm{e}^{dx+c}\right)}{da}+\frac{if\ln\left(1-i\mathrm{e}^{dx+c}\right)x}{2da}+\frac{if\ln\left(1-i\mathrm{e}^{dx+c}\right)c}{2d^{2}a}+\frac{if\,\mathrm{polylog}\left(2,i\mathrm{e}^{dx+c}\right)}{2a\,d^{2}}-$

input
```
int((f*x+e)*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(d*f*x*exp(d*x+c)+d*e*exp(d*x+c)+exp(d*x+c)*f-I*f)/(exp(d*x+c)-I)^2/d^2/a+
1/d/a*e*arctan(exp(d*x+c))+1/2*I/d/a*f*ln(1-I*exp(d*x+c))*x+1/2*I/d^2/a*f*
ln(1-I*exp(d*x+c))*c+1/2*I*f*polylog(2,I*exp(d*x+c))/a/d^2-1/2*I/d/a*f*ln(
1+I*exp(d*x+c))*x-1/2*I/d^2/a*f*ln(1+I*exp(d*x+c))*c-1/2*I*f*polylog(2,-I*
exp(d*x+c))/a/d^2-1/d^2/a*f*c*arctan(exp(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(135) = 270.

Time = 0.11 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.19

$$\int \frac{(e+fx)\mathrm{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{\left(i\,fe^{(2\,dx+2\,c)}+2\,fe^{(dx+c)}-i\,f\right)\mathrm{Li}_2\left(i\,e^{(dx+c)}\right)+\left(-i\,fe^{(2\,dx+2\,c)}-2\,fe^{(dx+c)}+i\,f\right)\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)+2\,(df\cdots}{\cdots}$$

input
```
integrate((f*x+e)*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((I*f*e^(2*d*x + 2*c) + 2*f*e^(d*x + c) - I*f)*dilog(I*e^(d*x + c)) +
(-I*f*e^(2*d*x + 2*c) - 2*f*e^(d*x + c) + I*f)*dilog(-I*e^(d*x + c)) + 2*(
d*f*x + d*e + f)*e^(d*x + c) + (-I*d*e + I*c*f + (I*d*e - I*c*f)*e^(2*d*x
+ 2*c) + 2*(d*e - c*f)*e^(d*x + c))*log(e^(d*x + c) + I) + (I*d*e - I*c*f
+ (-I*d*e + I*c*f)*e^(2*d*x + 2*c) - 2*(d*e - c*f)*e^(d*x + c))*log(e^(d*x
 + c) - I) + (I*d*f*x + I*c*f + (-I*d*f*x - I*c*f)*e^(2*d*x + 2*c) - 2*(d*
f*x + c*f)*e^(d*x + c))*log(I*e^(d*x + c) + 1) + (-I*d*f*x - I*c*f + (I*d*
f*x + I*c*f)*e^(2*d*x + 2*c) + 2*(d*f*x + c*f)*e^(d*x + c))*log(-I*e^(d*x
+ c) + 1) - 2*I*f)/(a*d^2*e^(2*d*x + 2*c) - 2*I*a*d^2*e^(d*x + c) - a*d^2)
```

Sympy [F]

$$\int \frac{(e+fx)\mathrm{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e\,\mathrm{sech}\,(c+dx)}{\sinh\,(c+dx)-i}\,dx+\int \frac{fx\,\mathrm{sech}\,(c+dx)}{\sinh\,(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e*sech(c + d*x)/(sinh(c + d*x) - I), x) + Integral(f*x*sech(c
 + d*x)/(sinh(c + d*x) - I), x))/a
```

Maxima [F]

$$\int \frac{(e+fx)\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(fx+e)\,\text{sech}\,(dx+c)}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
f*(((d*x*e^c + e^c)*e^(d*x) - I)/(a*d^2*e^(2*d*x + 2*c) - 2*I*a*d^2*e^(d*x
 + c) - a*d^2) + 2*integrate(1/4*x/(a*e^(d*x + c) + I*a), x) + 2*integrate
(1/4*x/(a*e^(d*x + c) - I*a), x)) - 1/2*e*(4*e^(-d*x - c)/((4*I*a*e^(-d*x
- c) + 2*a*e^(-2*d*x - 2*c) - 2*a)*d) + I*log(e^(-d*x - c) + I)/(a*d) - I*
log(I*e^(-d*x - c) + 1)/(a*d))
```

Giac [F]

$$\int \frac{(e+fx)\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(fx+e)\,\text{sech}\,(dx+c)}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*sech(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{e+f\,x}{\cosh\,(c+d\,x)\,\,(a+a\sinh\,(c+d\,x)\,\,1\text{i})}\,dx$$

input
```
int((e + f*x)/(cosh(c + d*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)/(cosh(c + d*x)*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx=\frac{\left(\int \frac{\text{sech}(dx+c)}{\sinh(dx+c)i+1}dx\right)e+\left(\int \frac{\text{sech}(dx+c)x}{\sinh(dx+c)i+1}dx\right)f}{a}$$

input
```
int((f*x+e)*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
(int(sech(c + d*x)/(sinh(c + d*x)*i + 1),x)*e + int((sech(c + d*x)*x)/(sin
h(c + d*x)*i + 1),x)*f)/a
```

3.274 $\int \frac{\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 42

$$\int \frac{\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx=\frac{\arctan(\sinh(c+dx))}{2ad}+\frac{i}{2d(a+ia\sinh(c+dx))}$$

output
```
1/2*arctan(sinh(d*x+c))/a/d+1/2*I/d/(a+I*a*sinh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \frac{\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx=\frac{\arctan(\sinh(c+dx))+\frac{1}{-i+\sinh(c+dx)}}{2ad}$$

input
```
Integrate[Sech[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(ArcTan[Sinh[c + d*x]] + (-I + Sinh[c + d*x])^(-1))/(2*a*d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 3146, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ic+idx)(a+a\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3146}$$

$$-\frac{ia\int \frac{1}{(a-ia\sinh(c+dx))(i\sinh(c+dx)a+a)^2}d(ia\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$-\frac{ia\int \left(\frac{1}{2(\sinh^2(c+dx)a^2+a^2)a}+\frac{1}{2(i\sinh(c+dx)a+a)^2a}\right)d(ia\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia\left(\frac{i\arctan(\sinh(c+dx))}{2a^2}-\frac{1}{2a(a+ia\sinh(c+dx))}\right)}{d}$$

input
```
Int[Sech[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-I)*a*(((I/2)*ArcTan[Sinh[c + d*x]])/a^2 - 1/(2*a*(a + I*a*Sinh[c + d*x]
))))/d
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 4.76 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.52

method	result	size
risch	$\frac{e^{dx+c}}{\left(e^{dx+c}-i\right)^{2} d a}-\frac{i \ln\left(e^{dx+c}-i\right)}{2 d a}+\frac{i \ln\left(e^{dx+c}+i\right)}{2 d a}$	64
derivativedivides	$\frac{\frac{i \ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+i\right)}{2}-\frac{i}{\left(-i+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{i \ln\left(-i+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{2}-\frac{1}{-i+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}}{d a}$	75
default	$\frac{\frac{i \ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+i\right)}{2}-\frac{i}{\left(-i+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{i \ln\left(-i+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{2}-\frac{1}{-i+\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}}{d a}$	75

input
```
int(sech(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
exp(d*x+c)/(exp(d*x+c)-I)^2/d/a-1/2*I/d/a*ln(exp(d*x+c)-I)+1/2*I/d/a*ln(ex
p(d*x+c)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(34) = 68$.

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.43

$$\int \frac{\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(i\,e^{(2\,dx+2\,c)}+2\,e^{(dx+c)}-i\right)\log\left(e^{(dx+c)}+i\right)+\left(-i\,e^{(2\,dx+2\,c)}-2\,e^{(dx+c)}+i\right)\log\left(e^{(dx+c)}-i\right)+2\,e^{(dx+c)}}{2\left(ade^{(2\,dx+2\,c)}-2i\,ade^{(dx+c)}-ad\right)}$$

input
```
integrate(sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((I*e^(2*d*x + 2*c) + 2*e^(d*x + c) - I)*log(e^(d*x + c) + I) + (-I*e^
(2*d*x + 2*c) - 2*e^(d*x + c) + I)*log(e^(d*x + c) - I) + 2*e^(d*x + c))/(
a*d*e^(2*d*x + 2*c) - 2*I*a*d*e^(d*x + c) - a*d)
```

Sympy [F]

$$\int \frac{\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int \frac{\operatorname{sech}(c+dx)}{\sinh(c+dx)-i}\,dx}{a}$$

input
```
integrate(sech(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(sech(c + d*x)/(sinh(c + d*x) - I), x)/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(34) = 68.

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.07

$$\int \frac{\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{2\,e^{(-dx-c)}}{-2\left(-2i\,ae^{(-dx-c)}-ae^{(-2\,dx-2\,c)}+a\right)d} - \frac{i\,\log\left(e^{(-dx-c)}+i\right)}{2\,ad} + \frac{i\,\log\left(i\,e^{(-dx-c)}+1\right)}{2\,ad}$$

input
```
integrate(sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-2*e^(-d*x - c)/((4*I*a*e^(-d*x - c) + 2*a*e^(-2*d*x - 2*c) - 2*a)*d) - 1/
2*I*log(e^(-d*x - c) + I)/(a*d) + 1/2*I*log(I*e^(-d*x - c) + 1)/(a*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(34) = 68.

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.43

$$\int \frac{\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{-\frac{i\,\log\left(e^{(dx+c)}-e^{(-dx-c)}+2i\right)}{a} + \frac{i\,\log\left(e^{(dx+c)}-e^{(-dx-c)}-2i\right)}{a} + \frac{-i\,e^{(dx+c)}+i\,e^{(-dx-c)}-6}{a\left(e^{(dx+c)}-e^{(-dx-c)}-2i\right)}}{4\,d}$$

input
```
integrate(sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*(-I*log(e^(d*x + c) - e^(-d*x - c) + 2*I)/a + I*log(e^(d*x + c) - e^(
-d*x - c) - 2*I)/a + (-I*e^(d*x + c) + I*e^(-d*x - c) - 6)/(a*(e^(d*x + c)
 - e^(-d*x - c) - 2*I)))/d
```

Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.76

$$\int \frac{\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\,\sqrt{a^2\,d^2}}{a\,d}\right)}{\sqrt{a^2\,d^2}} + \frac{1}{a\,d\,\left(e^{c+d\,x}-\mathrm{i}\right)} - \frac{1\mathrm{i}}{a\,d\,\left(1+e^{c+d\,x}\,1\mathrm{i}\right)^2}$$

input
```
int(1/(cosh(c + d*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
atan((exp(d*x)*exp(c)*(a^2*d^2)^(1/2))/(a*d))/(a^2*d^2)^(1/2) + 1/(a*d*(ex
p(c + d*x) - 1i)) - 1i/(a*d*(exp(c + d*x)*1i + 1)^2)
```

Reduce [F]

$$\int \frac{\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\int \frac{\operatorname{sech}(dx+c)}{\sinh(dx+c)i+1}dx}{a}$$

input
```
int(sech(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sech(c + d*x)/(sinh(c + d*x)*i + 1),x)/a
```

3.275 $\int \frac{\text{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\text{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\text{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 85.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\text{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\text{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

input
```
Integrate[Sech[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Integrate[Sech[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

$$\Big\downarrow \text{ 6111}$$

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}dx$$

input
```
Int[Sech[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{sech}(dx+c)}{(fx+e)\,(a+ia\sinh(dx+c))}dx$$

input
```
int(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 391, normalized size of antiderivative = 13.48

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}(dx+c)}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-((d*f*x + d*e - f)*e^(d*x + c) - (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e
^2 - (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*e^(2*d*x + 2*c) + 2*(I*a*
d^2*f^2*x^2 + 2*I*a*d^2*e*f*x + I*a*d^2*e^2)*e^(d*x + c))*integral((-2*I*f
^2 + (d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2 - 2*f^2)*e^(d*x + c))/(a*d^2*f^3
*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 + (a*d^2*f^3*x^3 +
3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*e^(2*d*x + 2*c)), x) + I*
f)/(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2 - (a*d^2*f^2*x^2 + 2*a*d^2*e
*f*x + a*d^2*e^2)*e^(2*d*x + 2*c) + 2*(I*a*d^2*f^2*x^2 + 2*I*a*d^2*e*f*x +
 I*a*d^2*e^2)*e^(d*x + c))
```

Sympy [N/A]

Not integrable

Time = 4.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=-\frac{i\int \frac{\operatorname{sech}(c+dx)}{e\sinh(c+dx)-ie+fx\sinh(c+dx)-ifx}\,dx}{a}$$

input
```
integrate(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(sech(c + d*x)/(e*sinh(c + d*x) - I*e + f*x*sinh(c + d*x) - I*f
*x), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 327, normalized size of antiderivative = 11.28

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(dx+c)}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-((d*f*x*e^c + (d*e - f)*e^c)*e^(d*x) + I*f)/(a*d^2*f^2*x^2 + 2*a*d^2*e*f*
x + a*d^2*e^2 - (a*d^2*f^2*x^2*e^(2*c) + 2*a*d^2*e*f*x*e^(2*c) + a*d^2*e^2
*e^(2*c))*e^(2*d*x) + 2*(I*a*d^2*f^2*x^2*e^c + 2*I*a*d^2*e*f*x*e^c + I*a*d
^2*e^2*e^c)*e^(d*x)) + 2*integrate((d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2 -
4*f^2)/(-4*I*a*d^2*f^3*x^3 - 12*I*a*d^2*e*f^2*x^2 - 12*I*a*d^2*e^2*f*x - 4
*I*a*d^2*e^3 + 4*(a*d^2*f^3*x^3*e^c + 3*a*d^2*e*f^2*x^2*e^c + 3*a*d^2*e^2*
f*x*e^c + a*d^2*e^3*e^c)*e^(d*x)), x) + 2*integrate(1/(4*I*a*f*x + 4*I*a*e
 + 4*(a*f*x*e^c + a*e*e^c)*e^(d*x)), x)
```

Giac [N/A]

Not integrable

Time = 2.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(dx+c)}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sech(d*x + c)/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)\,\left(e+f\,x\right)\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)*(e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(1/(cosh(c + d*x)*(e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \frac{\int \frac{\operatorname{sech}(dx+c)}{\sinh(dx+c)ei+\sinh(dx+c)fix+e+fx}dx}{a}$$

input
```
int(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sech(c + d*x)/(sinh(c + d*x)*e*i + sinh(c + d*x)*f*i*x + e + f*x),x)/a
```

3.276 $\int \frac{\text{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\text{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\text{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(sech(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Mathematica [F(-1)]

Timed out.

$$\int \frac{\text{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \$\text{Aborted}$$

input
```
Integrate[Sech[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

$$\Big\downarrow \text{ 6111}$$

$$\int \frac{\text{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}dx$$

input
```
Int[Sech[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\text{sech}\,(dx+c)}{(fx+e)^2\,(a+ia\sinh\,(dx+c))}dx$$

input
```
int(sech(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sech(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 511, normalized size of antiderivative = 17.62

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)}{(fx+e)^2(i\,a\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sech(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-((d*f*x + d*e - 2*f)*e^(d*x + c) - (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3
*a*d^2*e^2*f*x + a*d^2*e^3 - (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*
e^2*f*x + a*d^2*e^3)*e^(2*d*x + 2*c) + 2*(I*a*d^2*f^3*x^3 + 3*I*a*d^2*e*f^
2*x^2 + 3*I*a*d^2*e^2*f*x + I*a*d^2*e^3)*e^(d*x + c))*integral((-6*I*f^2 +
 (d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2 - 6*f^2)*e^(d*x + c))/(a*d^2*f^4*x^4
 + 4*a*d^2*e*f^3*x^3 + 6*a*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*f*x + a*d^2*e^4 +
 (a*d^2*f^4*x^4 + 4*a*d^2*e*f^3*x^3 + 6*a*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*f*
x + a*d^2*e^4)*e^(2*d*x + 2*c)), x) + 2*I*f)/(a*d^2*f^3*x^3 + 3*a*d^2*e*f^
2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 - (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 +
 3*a*d^2*e^2*f*x + a*d^2*e^3)*e^(2*d*x + 2*c) + 2*(I*a*d^2*f^3*x^3 + 3*I*a
*d^2*e*f^2*x^2 + 3*I*a*d^2*e^2*f*x + I*a*d^2*e^3)*e^(d*x + c))
```

Sympy [N/A]

Not integrable

Time = 39.41 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.45

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = -\frac{i\int \frac{\operatorname{sech}\left(c+dx\right)}{e^2\sinh\left(c+dx\right)-ie^2+2efx\sinh\left(c+dx\right)-2iefx+f^2x^2\sinh\left(c+dx\right)-if^2x^2}\,dx}{a}$$

input
```
integrate(sech(d*x+c)/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(sech(c + d*x)/(e**2*sinh(c + d*x) - I*e**2 + 2*e*f*x*sinh(c +
d*x) - 2*I*e*f*x + f**2*x**2*sinh(c + d*x) - I*f**2*x**2), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 436, normalized size of antiderivative = 15.03

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}(dx+c)}{(fx+e)^2(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sech(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-((d*f*x*e^c + (d*e - 2*f)*e^c)*e^(d*x) + 2*I*f)/(a*d^2*f^3*x^3 + 3*a*d^2*
e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 - (a*d^2*f^3*x^3*e^(2*c) + 3*a*d^2
*e*f^2*x^2*e^(2*c) + 3*a*d^2*e^2*f*x*e^(2*c) + a*d^2*e^3*e^(2*c))*e^(2*d*x
) + 2*(I*a*d^2*f^3*x^3*e^c + 3*I*a*d^2*e*f^2*x^2*e^c + 3*I*a*d^2*e^2*f*x*e
^c + I*a*d^2*e^3*e^c)*e^(d*x)) + 2*integrate((d^2*f^2*x^2 + 2*d^2*e*f*x +
d^2*e^2 - 12*f^2)/(-4*I*a*d^2*f^4*x^4 - 16*I*a*d^2*e*f^3*x^3 - 24*I*a*d^2*
e^2*f^2*x^2 - 16*I*a*d^2*e^3*f*x - 4*I*a*d^2*e^4 + 4*(a*d^2*f^4*x^4*e^c +
4*a*d^2*e*f^3*x^3*e^c + 6*a*d^2*e^2*f^2*x^2*e^c + 4*a*d^2*e^3*f*x*e^c + a*
d^2*e^4*e^c)*e^(d*x)), x) + 2*integrate(1/(4*I*a*f^2*x^2 + 8*I*a*e*f*x + 4
*I*a*e^2 + 4*(a*f^2*x^2*e^c + 2*a*e*f*x*e^c + a*e^2*e^c)*e^(d*x)), x)
```

Giac [N/A]

Not integrable

Time = 42.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}(dx+c)}{(fx+e)^2(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sech(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sech(d*x + c)/((f*x + e)^2*(I*a*sinh(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$
$$=\int \frac{1}{\cosh(c+d\,x)\,\left(e+f\,x\right)^2\,\left(a+a\sinh(c+d\,x)\,1\mathrm{i}\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(1/(cosh(c + d*x)*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.34

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$
$$=\frac{\int \frac{\operatorname{sech}(dx+c)}{\sinh(dx+c)e^2i+2\sinh(dx+c)efix+\sinh(dx+c)f^2i\,x^2+e^2+2efx+f^2x^2}dx}{a}$$

input
```
int(sech(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sech(c + d*x)/(sinh(c + d*x)*e**2*i + 2*sinh(c + d*x)*e*f*i*x + sinh(c
 + d*x)*f**2*i*x**2 + e**2 + 2*e*f*x + f**2*x**2),x)/a
```

3.277 $\int \frac{(e+fx)^3 \operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 450

$$\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{2(e+fx)^3}{3ad}-\frac{if(e+fx)^2\arctan\left(e^{c+dx}\right)}{ad^2}+\frac{if^3\arctan(\sinh(c+dx))}{ad^4}-\frac{2f(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{ad^2}+\frac{f^3\log(\cosh(c+dx))}{ad^4}-\frac{f^2(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}+\frac{f^2(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{ad^3}-\frac{2f^2(e+fx)\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{ad^3}+\frac{f^3\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^4}-\frac{f^3\,\text{PolyLog}\left(3,ie^{c+dx}\right)}{ad^4}+\frac{f^3\,\text{PolyLog}\left(3,-e^{2(c+dx)}\right)}{ad^4}-\frac{if^2(e+fx)\text{sech}(c+dx)}{ad^3}+\frac{f(e+fx)^2\text{sech}^2(c+dx)}{2ad^2}+\frac{i(e+fx)^3\text{sech}^3(c+dx)}{3ad}-\frac{f^2(e+fx)\tanh(c+dx)}{ad^3}+\frac{2(e+fx)^3\tanh(c+dx)}{3ad}-\frac{if(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{2ad^2}+\frac{(e+fx)^3\text{sech}^2(c+dx)\tanh(c+dx)}{3ad}$$

output

```
2/3*(f*x+e)^3/a/d-I*f*(f*x+e)^2*arctan(exp(d*x+c))/a/d^2-I*f^2*(f*x+e)*sec
h(d*x+c)/a/d^3-2*f*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/a/d^2+f^3*ln(cosh(d*x+c)
)/a/d^4-f^2*(f*x+e)*polylog(2,-I*exp(d*x+c))/a/d^3+f^2*(f*x+e)*polylog(2,I
*exp(d*x+c))/a/d^3-2*f^2*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/a/d^3+f^3*poly
log(3,-I*exp(d*x+c))/a/d^4-f^3*polylog(3,I*exp(d*x+c))/a/d^4+f^3*polylog(3
,-exp(2*d*x+2*c))/a/d^4-1/2*I*f*(f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/a/d^2+1/
2*f*(f*x+e)^2*sech(d*x+c)^2/a/d^2+1/3*I*(f*x+e)^3*sech(d*x+c)^3/a/d-f^2*(f
*x+e)*tanh(d*x+c)/a/d^3+2/3*(f*x+e)^3*tanh(d*x+c)/a/d+I*f^3*arctan(sinh(d*
x+c))/a/d^4+1/3*(f*x+e)^3*sech(d*x+c)^2*tanh(d*x+c)/a/d
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1078 vs. $2(450) = 900$.

Time = 8.14 (sec) , antiderivative size = 1078, normalized size of antiderivative = 2.40

$$\int \frac{(e+fx)^3 \operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Sech[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-1/2*I)*f*((e + f*x)^3/f + (3*(1 - I*E^c)*(e + f*x)^2*Log[1 + I*E^(-c -
d*x)])/d + ((6*I)*(I + E^c)*f*(d*(e + f*x)*PolyLog[2, (-I)*E^(-c - d*x)] +
 f*PolyLog[3, (-I)*E^(-c - d*x)]))/d^3))/(a*d*(I + E^c)) + ((I/6)*f*(15*d^
2*e^2*x - 12*f^2*x - 3*(1 + I*E^c)*(5*d^2*e^2 - 4*f^2)*x + 15*d^2*e*f*x^2
+ 5*d^2*f^2*x^3 + 30*d*e*(1 + I*E^c)*f*x*Log[1 - I*E^(-c - d*x)] + 15*d*(1
 + I*E^c)*f^2*x^2*Log[1 - I*E^(-c - d*x)] + (3*(1 + I*E^c)*(5*d^2*e^2 - 4*
f^2)*Log[I - E^(c + d*x)])/d - 30*e*(1 + I*E^c)*f*PolyLog[2, I*E^(-c - d*x
)] - 30*(1 + I*E^c)*f^2*x*PolyLog[2, I*E^(-c - d*x)] - (30*(1 + I*E^c)*f^2
*PolyLog[3, I*E^(-c - d*x)])/d))/(a*d^3*(-I + E^c)) + (e^3*Sinh[(d*x)/2] +
 3*e^2*f*x*Sinh[(d*x)/2] + 3*e*f^2*x^2*Sinh[(d*x)/2] + f^3*x^3*Sinh[(d*x)/
2])/(2*a*d*(Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2 + (d*x)/2] - I*Sinh[c/2 + (
d*x)/2])) + (e^3*Sinh[(d*x)/2] + 3*e^2*f*x*Sinh[(d*x)/2] + 3*e*f^2*x^2*Sin
h[(d*x)/2] + f^3*x^3*Sinh[(d*x)/2])/(3*a*d*(Cosh[c/2] + I*Sinh[c/2])*(Cosh
[c/2 + (d*x)/2] + I*Sinh[c/2 + (d*x)/2])^3) + (I*d*e^3*Cosh[c/2] + 3*e^2*f
*Cosh[c/2] + (3*I)*d*e^2*f*x*Cosh[c/2] + 6*e*f^2*x*Cosh[c/2] + (3*I)*d*e*f
^2*x^2*Cosh[c/2] + 3*f^3*x^2*Cosh[c/2] + I*d*f^3*x^3*Cosh[c/2] + d*e^3*Sin
h[c/2] + (3*I)*e^2*f*Sinh[c/2] + 3*d*e^2*f*x*Sinh[c/2] + (6*I)*e*f^2*x*Sin
h[c/2] + 3*d*e*f^2*x^2*Sinh[c/2] + (3*I)*f^3*x^2*Sinh[c/2] + d*f^3*x^3*Sin
h[c/2])/(6*a*d^2*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[c/2 + (d*x)/2] + I*Sinh[c
/2 + (d*x)/2])^2) + (5*d^2*e^3*Sinh[(d*x)/2] - 12*e*f^2*Sinh[(d*x)/2] +...
```

Rubi [A] (verified)

Time = 2.98 (sec) , antiderivative size = 443, normalized size of antiderivative = 0.98, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.710, Rules used = {6105, 3042, 4674, 3042, 4672, 26, 3042, 26, 3956, 4201, 2620, 3011, 2720, 5974, 3042, 4674, 3042, 4257, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6105}$$

$$\frac{\int (e+fx)^3\text{sech}^4(c+dx)dx}{a}-\frac{i\int (e+fx)^3\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)^4dx}{a}-\frac{i\int (e+fx)^3\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 4674}$$

$$\frac{-\frac{f^2\int (e+fx)\text{sech}^2(c+dx)dx}{d^2}+\frac{2}{3}\int (e+fx)^3\text{sech}^2(c+dx)dx+\frac{f(e+fx)^2\text{sech}^2(c+dx)}{2d^2}+\frac{(e+fx)^3\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int (e+fx)^3\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{f^2\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{d^2}+\frac{2}{3}\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx+\frac{f(e+fx)^2\text{sech}^2(c+dx)}{2d^2}+\frac{(e+fx)^3\tanh(c+dx)\text{sech}^2}{3d}}{a}$$
$$\frac{i\int (e+fx)^3\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 4672}$$

$$\frac{-\frac{f^2\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{if\int -i\tanh(c+dx)dx}{d}\right)}{d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{3if\int -i(e+fx)^2\tanh(c+dx)dx}{d}\right)+\frac{f(e+fx)^2\text{sech}^2(c+dx)}{2d^2}}{a}$$
$$\frac{i\int (e+fx)^3\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$\downarrow$ 26

$$\frac{-\frac{f^2\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int\tanh(c+dx)dx}{d}\right)}{d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{3f\int(e+fx)^2\tanh(c+dx)dx}{d}\right)+\frac{f(e+fx)^2\text{sech}^2(c+dx)}{2d^2}+\frac{(e+\ldots}{}}{a}$$

$$\frac{i\int(e+fx)^3\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$\downarrow$ 3042

$$\frac{-\frac{f^2\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int-i\tan(ic+idx)dx}{d}\right)}{d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{3f\int-i(e+fx)^2\tan(ic+idx)dx}{d}\right)+\frac{f(e+fx)^2\text{sech}^2(c+dx)}{2d^2}+\ldots}{a}$$

$$\frac{i\int(e+fx)^3\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$\downarrow$ 26

$$\frac{-\frac{f^2\left(\frac{(e+fx)\tanh(c+dx)}{d}+\frac{if\int\tan(ic+idx)dx}{d}\right)}{d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\int(e+fx)^2\tan(ic+idx)dx}{d}\right)+\frac{f(e+fx)^2\text{sech}^2(c+dx)}{2d^2}+\frac{(e\ldots}{}}{a}$$

$$\frac{i\int(e+fx)^3\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$\downarrow$ 3956

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\int(e+fx)^2\tan(ic+idx)dx}{d}\right)-\frac{f^2\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d^2}+\frac{f(e+fx)^2\text{sech}^2(c+dx)}{2d^2}+\frac{(e+f\ldots}{}}{a}$$

$$\frac{i\int(e+fx)^3\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$\downarrow$ 4201

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\int\frac{e^{2(c+dx)}(e+fx)^2}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}\right)-\frac{f^2\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d^2}+\frac{f(e+fx)^2\text{sech}^2(c+\ldots}{2d^2}}{a}$$

$$\frac{i\int(e+fx)^3\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$\downarrow$ 2620

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int(e+fx)\log\left(1+e^{2(c+dx)}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}\right)-\frac{f^2\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cdots}{}\right.}{d^2}\cdots}{a}$$

$$\frac{i\int(e+fx)^3\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3011}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int\text{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{2d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}\cdots\right.}{a}$$

$$\frac{i\int(e+fx)^3\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots\right.}{a}$$

$$\frac{i\int(e+fx)^3\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{5974}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots\right.}{a}$$

$$\frac{i\left(\frac{f\int(e+fx)^2\text{sech}^3(c+dx)dx}{d}-\frac{(e+fx)^3\text{sech}^3(c+dx)}{3d}\right)}{a}$$

$\downarrow$ 3042

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right.}{d}\right.\right.}{d}\right.}{a}$$

$$\frac{i\left(-\frac{(e+fx)^3\operatorname{sech}^3(c+dx)}{3d}+\frac{f\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{d}\right)}{a}$$

$\downarrow$ 4674

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right.}{d}\right.\right.}{d}\right.}{a}$$

$$\frac{i\left(\frac{f\left(-\frac{f^2\int\operatorname{sech}(c+dx)dx}{d^2}+\frac{1}{2}\int(e+fx)^2\operatorname{sech}(c+dx)dx+\frac{f(e+fx)\operatorname{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}\right)}{d}-\frac{(e+fx)^3\operatorname{sech}^3(c+dx)}{3d}\right)}{a}$$

$\downarrow$ 3042

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right.}{d}\right.\right.}{d}\right.}{a}$$

$$\frac{i\left(-\frac{(e+fx)^3\operatorname{sech}^3(c+dx)}{3d}+\frac{f\left(-\frac{f^2\int\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{d^2}+\frac{1}{2}\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{f(e+fx)\operatorname{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\operatorname{sech}(c+\right.}{d}\right.}{a}$$

$\downarrow$ 4257

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right.}{d}\right.\right.}{d}\right.}{a}$$

$$\frac{i\left(-\frac{(e+fx)^3\text{sech}^3(c+dx)}{3d}+\frac{f\left(\frac{1}{2}\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f^2\arctan(\sinh(c+dx))}{d^3}+\frac{f(e+fx)\text{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}(c+dx)}{2d}\right.}{d}\right.}{a}$$

$$\Big\downarrow \ 4668$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right.}{d}\right.\right.}{d}\right.}{a}$$

$$\frac{i\left(-\frac{(e+fx)^3\text{sech}^3(c+dx)}{3d}+\frac{f\left(\frac{1}{2}\left(-\frac{2if\int(e+fx)\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{2if\int(e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}\right)-\frac{f^2\arctan(\sinh(c\ldots}{d^3}\right.}{d}\right.}{a}$$

$$\Big\downarrow \ 3011$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right.}{d}\right.\right.}{d}\right.}{a}$$

$$\frac{i\left(-\frac{(e+fx)^3\text{sech}^3(c+dx)}{3d}+\frac{f\left(\frac{1}{2}\left(\frac{2if\left(\frac{f\int\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{Poly}\ldots}{}\right.}{d}\right.\right.}{}\right.}{a}$$

$\downarrow$ 2720

$$\frac{\frac{2}{3}\Bigg(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\Bigg(2i\Bigg(\frac{(e+fx)^2\log(e^{2(c+dx)}+1)}{2d}-\frac{f\Big(\frac{f\int e^{-2(c+dx)}\text{PolyLog}(2,-e^{2(c+dx)})de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}(2,-e^{2(c+dx)})}{2d}\cdots}{d}\cdots}{d}\cdots}{a}$$

$$i\Bigg(-\frac{(e+fx)^3\text{sech}^3(c+dx)}{3d}+\frac{f\Bigg(\frac{1}{2}\Bigg(\frac{2if\Big(\frac{f\int e^{-c-dx}\text{PolyLog}(2,-ie^{c+dx})de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}(2,-ie^{c+dx})}{d}\Big)}{d}-\frac{2if\Big(\frac{f\int e^{-c-dx}\text{PolyLog}(2,ie^{\cdots}}{d^2}\cdots}{\cdots}\cdots}{\cdots}\cdots$$

$\downarrow$ 7143

$$\frac{-\frac{f^2\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d^2}+\frac{2}{3}\Bigg(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\Bigg(2i\Bigg(\frac{(e+fx)^2\log(e^{2(c+dx)}+1)}{2d}-\frac{f\Big(\frac{f\text{PolyLog}(3,-e^{2(c+dx)})}{4d^2}-\frac{(e\cdots}{\cdots}}{d\cdots}\cdots}{d}\cdots}{a}$$

$$\frac{i\Bigg(-\frac{(e+fx)^3\text{sech}^3(c+dx)}{3d}+\frac{f\Bigg(-\frac{f^2\arctan(\sinh(c+dx))}{d^3}+\frac{1}{2}\Bigg(\frac{2(e+fx)^2\arctan(e^{c+dx})}{d}+\frac{2if\Big(\frac{f\text{PolyLog}(3,-ie^{c+dx})}{d^2}-\frac{(e+fx)\text{PolyLog}(2,-ie^{c+dx})}{d}\cdots}{d}\cdots}{\cdots}\cdots}{a}$$

input
```
Int[((e + f*x)^3*Sech[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((f*(e + f*x)^2*Sech[c + d*x]^2)/(2*d^2) + ((e + f*x)^3*Sech[c + d*x]^2*Ta
nh[c + d*x])/(3*d) - (f^2*(-((f*Log[Cosh[c + d*x]])/d^2) + ((e + f*x)*Tanh
[c + d*x])/d))/d^2 + (2*(((3*I)*f*(((-1/3*I)*(e + f*x)^3)/f + (2*I)*(((e +
 f*x)^2*Log[1 + E^(2*(c + d*x))])/(2*d) - (f*(-1/2*((e + f*x)*PolyLog[2, -
E^(2*(c + d*x))])/d + (f*PolyLog[3, -E^(2*(c + d*x))])/(4*d^2)))/d)))/d +
((e + f*x)^3*Tanh[c + d*x])/d))/3)/a - (I*(-1/3*((e + f*x)^3*Sech[c + d*x]
^3)/d + (f*(-((f^2*ArcTan[Sinh[c + d*x]])/d^3) + ((2*(e + f*x)^2*ArcTan[E^
(c + d*x)])/d + ((2*I)*f*(-(((e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d) +
(f*PolyLog[3, (-I)*E^(c + d*x)])/d^2))/d - ((2*I)*f*(-(((e + f*x)*PolyLog[
2, I*E^(c + d*x)])/d) + (f*PolyLog[3, I*E^(c + d*x)])/d^2))/d)/2 + (f*(e +
 f*x)*Sech[c + d*x])/d^2 + ((e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
))/d))/a
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6105
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Sech[
c + d*x]^(n + 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)
*Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] &&
EqQ[a^2 + b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1020 vs. 2(424) = 848.

Time = 142.72 (sec) , antiderivative size = 1021, normalized size of antiderivative = 2.27

method	result
risch	$\frac{4f^3x^3}{3ad}+\frac{4f^2e\,x^2}{ad}+\frac{4f^2e\,c^2}{a\,d^3}-\frac{4f^3c^2x}{a\,d^3}-\frac{2f^3c^2\ln\left(1+e^{2dx+2c}\right)}{a\,d^4}-\frac{3f^2e\operatorname{polylog}\left(2,ie^{dx+c}\right)}{a\,d^3}-\frac{8f^3c^3}{3a\,d^4}+\frac{f^3\ln\left(1+e^{2dx+2c}\right)}{a\,d^4}$

input
```
int((f*x+e)^3*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
3*f^3*polylog(3,I*exp(d*x+c))/a/d^4+4/3/a/d*f^3*x^3-3/a/d^3*f^2*e*polylog(
2,I*exp(d*x+c))-5/a/d^3*f^2*e*polylog(2,-I*exp(d*x+c))+4/a/d*f^2*e*x^2+4/a
/d^3*f^2*e*c^2-4/a/d^3*f^3*c^2*x-3/2/a/d^2*f^3*ln(1-I*exp(d*x+c))*x^2-3/a/
d^3*f^3*polylog(2,I*exp(d*x+c))*x-5/2/a/d^2*f^3*ln(1+I*exp(d*x+c))*x^2-5/a
/d^3*f^3*polylog(2,-I*exp(d*x+c))*x+3/2/a/d^4*f^3*c^2*ln(1-I*exp(d*x+c))+5
/2/a/d^4*f^3*c^2*ln(1+I*exp(d*x+c))-2/a/d^4*f^3*c^2*ln(1+exp(2*d*x+2*c))-8
/3/a/d^4*f^3*c^3+1/a/d^4*f^3*ln(1+exp(2*d*x+2*c))-2/a/d^4*f^3*ln(exp(d*x+c
))+1/3*I*(6*I*f^2*e+8*d^2*f^3*x^3*exp(d*x+c)-3*d*f^3*x^2*exp(3*d*x+3*c)+6*
I*e*f^2*exp(2*d*x+2*c)+24*d^2*e*f^2*x^2*exp(d*x+c)-6*d*e*f^2*x*exp(3*d*x+3
*c)-12*I*d^2*e*f^2*x^2-12*I*d^2*e^2*f*x+24*d^2*e^2*f*x*exp(d*x+c)-3*d*e^2*
f*exp(3*d*x+3*c)-3*d*f^3*x^2*exp(d*x+c)-6*f^3*x*exp(3*d*x+3*c)-4*I*d^2*x^3
*f^3+6*I*f^3*x+8*d^2*e^3*exp(d*x+c)-6*d*e*f^2*x*exp(d*x+c)-6*e*f^2*exp(3*d
*x+3*c)-4*I*d^2*e^3-3*d*e^2*f*exp(d*x+c)-6*f^3*x*exp(d*x+c)+6*I*f^3*x*exp(
2*d*x+2*c)-6*e*f^2*exp(d*x+c))/(exp(d*x+c)+I)/(exp(d*x+c)-I)^3/d^3/a+4/a/d
^3*f^2*e*c*ln(1+exp(2*d*x+2*c))-8/a/d^3*f^2*e*c*ln(exp(d*x+c))+8/a/d^2*f^2
*e*c*x-3/a/d^2*f^2*e*ln(1-I*exp(d*x+c))*x-3/a/d^3*f^2*e*ln(1-I*exp(d*x+c))
*c-5/a/d^2*f^2*e*ln(1+I*exp(d*x+c))*x-I/a/d^4*f^3*c^2*arctan(exp(d*x+c))-I
/a/d^2*f*e^2*arctan(exp(d*x+c))+2*I/a/d^3*f^2*c*e*arctan(exp(d*x+c))+4/a/d
^4*f^3*c^2*ln(exp(d*x+c))-2/a/d^2*f*e^2*ln(1+exp(2*d*x+2*c))+4/a/d^2*f*e^2
*ln(exp(d*x+c))+2*I/a/d^4*f^3*arctan(exp(d*x+c))+5*f^3*polylog(3,-I*exp...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1405 vs. $2(412) = 824$.

Time = 0.11 (sec) , antiderivative size = 1405, normalized size of antiderivative = 3.12

$$\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/6*(8*d^3*e^3 - 24*c*d^2*e^2*f + 12*(2*c^2 - 1)*d*e*f^2 - 4*(2*c^3 - 3*c)
*f^3 + 18*(d*f^3*x + d*e*f^2 - (d*f^3*x + d*e*f^2)*e^(4*d*x + 4*c) - 2*(-I
*d*f^3*x - I*d*e*f^2)*e^(3*d*x + 3*c) - 2*(-I*d*f^3*x - I*d*e*f^2)*e^(d*x
+ c))*dilog(I*e^(d*x + c)) + 30*(d*f^3*x + d*e*f^2 - (d*f^3*x + d*e*f^2)*e
^(4*d*x + 4*c) - 2*(-I*d*f^3*x - I*d*e*f^2)*e^(3*d*x + 3*c) - 2*(-I*d*f^3*
x - I*d*e*f^2)*e^(d*x + c))*dilog(-I*e^(d*x + c)) + 4*(2*d^3*f^3*x^3 + 6*d
^3*e*f^2*x^2 + 6*c*d^2*e^2*f - 6*c^2*d*e*f^2 + (2*c^3 - 3*c)*f^3 + 3*(2*d^
3*e^2*f - d*f^3)*x)*e^(4*d*x + 4*c) - 2*(8*I*d^3*f^3*x^3 + 3*(8*I*c + I)*d
^2*e^2*f + 6*(-4*I*c^2 + I)*d*e*f^2 + 4*(2*I*c^3 - 3*I*c)*f^3 + 3*(8*I*d^3
*e*f^2 + I*d^2*f^3)*x^2 + 6*(4*I*d^3*e^2*f + I*d^2*e*f^2 - I*d*f^3)*x)*e^(
3*d*x + 3*c) - 12*(d*f^3*x + d*e*f^2)*e^(2*d*x + 2*c) - 2*(3*I*d^2*f^3*x^2
 - 8*I*d^3*e^3 + 3*(8*I*c + I)*d^2*e^2*f + 6*(-4*I*c^2 + I)*d*e*f^2 + 4*(2
*I*c^3 - 3*I*c)*f^3 + 6*(I*d^2*e*f^2 - I*d*f^3)*x)*e^(d*x + c) + 9*(d^2*e^
2*f - 2*c*d*e*f^2 + c^2*f^3 - (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*e^(4*d*x
 + 4*c) - 2*(-I*d^2*e^2*f + 2*I*c*d*e*f^2 - I*c^2*f^3)*e^(3*d*x + 3*c) - 2
*(-I*d^2*e^2*f + 2*I*c*d*e*f^2 - I*c^2*f^3)*e^(d*x + c))*log(e^(d*x + c) +
 I) + 3*(5*d^2*e^2*f - 10*c*d*e*f^2 + (5*c^2 - 4)*f^3 - (5*d^2*e^2*f - 10*
c*d*e*f^2 + (5*c^2 - 4)*f^3)*e^(4*d*x + 4*c) - 2*(-5*I*d^2*e^2*f + 10*I*c*
d*e*f^2 + (-5*I*c^2 + 4*I)*f^3)*e^(3*d*x + 3*c) - 2*(-5*I*d^2*e^2*f + 10*I
*c*d*e*f^2 + (-5*I*c^2 + 4*I)*f^3)*e^(d*x + c))*log(e^(d*x + c) - I) + ...
```

Sympy [F]

$$\int \frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e^3\operatorname{sech}^2(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{f^3x^3\operatorname{sech}^2(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{3ef^2x^2\operatorname{sech}^2(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{3e^2fx\operatorname{sech}^2(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)**3*sech(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e**3*sech(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(f**3
*x**3*sech(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(3*e*f**2*x**2*se
ch(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(3*e**2*f*x*sech(c + d*x)
**2/(sinh(c + d*x) - I), x))/a
```

Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 730, normalized size of antiderivative = 1.62

$$\int \frac{(e+fx)^3 \operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
1/2*e^2*f*(24*(4*I*d*x*e^(4*d*x + 4*c) + (8*d*x*e^(3*c) + e^(3*c))*e^(3*d*
x) + e^(d*x + c))/(12*I*a*d^2*e^(4*d*x + 4*c) + 24*a*d^2*e^(3*d*x + 3*c) +
 24*a*d^2*e^(d*x + c) - 12*I*a*d^2) - 3*log((e^(d*x + c) + I)*e^(-c))/(a*d
^2) - 5*log(-I*(I*e^(d*x + c) + 1)*e^(-c))/(a*d^2)) + 4/3*e^3*(2*e^(-d*x -
 c)/((2*a*e^(-d*x - c) + 2*a*e^(-3*d*x - 3*c) - I*a*e^(-4*d*x - 4*c) + I*a
)*d) + I/((2*a*e^(-d*x - c) + 2*a*e^(-3*d*x - 3*c) - I*a*e^(-4*d*x - 4*c)
+ I*a)*d)) + (4*I*d^2*f^3*x^3 + 12*I*d^2*e*f^2*x^2 - 6*I*f^3*x - 6*I*e*f^2
 + 3*(d*f^3*x^2*e^(3*c) + 2*e*f^2*e^(3*c) + 2*(d*e*f^2 + f^3)*x*e^(3*c))*e
^(3*d*x) - 6*(I*f^3*x*e^(2*c) + I*e*f^2*e^(2*c))*e^(2*d*x) - (8*d^2*f^3*x^
3*e^c - 6*e*f^2*e^c + 3*(8*d^2*e*f^2 - d*f^3)*x^2*e^c - 6*(d*e*f^2 + f^3)*
x*e^c)*e^(d*x))/(3*I*a*d^3*e^(4*d*x + 4*c) + 6*a*d^3*e^(3*d*x + 3*c) + 6*a
*d^3*e^(d*x + c) - 3*I*a*d^3) - 5*(d*x*log(I*e^(d*x + c) + 1) + dilog(-I*e
^(d*x + c)))*e*f^2/(a*d^3) - 3*(d*x*log(-I*e^(d*x + c) + 1) + dilog(I*e^(d
*x + c)))*e*f^2/(a*d^3) - 2*f^3*x/(a*d^3) - 5/2*(d^2*x^2*log(I*e^(d*x + c)
 + 1) + 2*d*x*dilog(-I*e^(d*x + c)) - 2*polylog(3, -I*e^(d*x + c)))*f^3/(a
*d^4) - 3/2*(d^2*x^2*log(-I*e^(d*x + c) + 1) + 2*d*x*dilog(I*e^(d*x + c))
- 2*polylog(3, I*e^(d*x + c)))*f^3/(a*d^4) + 2*f^3*log(e^(d*x + c) - I)/(a
*d^4) + 4/3*(d^3*f^3*x^3 + 3*d^3*e*f^2*x^2)/(a*d^4)
```

Giac [F]

$$\int \frac{(e+fx)^3 \operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3 \operatorname{sech}(dx+c)^2}{i\,a\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sech(d*x + c)^2/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(e+f\,x)^3}{\cosh\left(c+d\,x\right)^2\,\left(a+a\sinh\left(c+d\,x\right)\,1\text{i}\right)}\,dx$$

input
```
int((e + f*x)^3/(cosh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)^3/(cosh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$=\frac{\left(\int \frac{\text{sech}(dx+c)^2}{\sinh(dx+c)i+1}dx\right)e^3+\left(\int \frac{\text{sech}(dx+c)^2x^3}{\sinh(dx+c)i+1}dx\right)f^3+3\left(\int \frac{\text{sech}(dx+c)^2x^2}{\sinh(dx+c)i+1}dx\right)e\,f^2+3\left(\int \frac{\text{sech}(dx+c)^2x}{\sinh(dx+c)i+1}dx\right)e^2f}{a}$$

input
```
int((f*x+e)^3*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
(int(sech(c + d*x)**2/(sinh(c + d*x)*i + 1),x)*e**3 + int((sech(c + d*x)**
2*x**3)/(sinh(c + d*x)*i + 1),x)*f**3 + 3*int((sech(c + d*x)**2*x**2)/(sin
h(c + d*x)*i + 1),x)*e*f**2 + 3*int((sech(c + d*x)**2*x)/(sinh(c + d*x)*i
+ 1),x)*e**2*f)/a
```

3.278 $\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 325

$$\begin{aligned}\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx &= \frac{2(e+fx)^2}{3ad}-\frac{2if(e+fx)\arctan\left(e^{c+dx}\right)}{3ad^2}\\ &-\frac{4f(e+fx)\log\left(1+e^{2(c+dx)}\right)}{3ad^2}\\ &-\frac{f^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{3ad^3}+\frac{f^2\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{3ad^3}\\ &-\frac{2f^2\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{3ad^3}-\frac{if^2\text{sech}(c+dx)}{3ad^3}\\ &+\frac{f(e+fx)\text{sech}^2(c+dx)}{3ad^2}+\frac{i(e+fx)^2\text{sech}^3(c+dx)}{3ad}\\ &-\frac{f^2\tanh(c+dx)}{3ad^3}+\frac{2(e+fx)^2\tanh(c+dx)}{3ad}\\ &-\frac{if(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{3ad^2}\\ &+\frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{3ad}\end{aligned}$$

output
```
2/3*(f*x+e)^2/a/d-2/3*I*f*(f*x+e)*arctan(exp(d*x+c))/a/d^2-4/3*f*(f*x+e)*l
n(1+exp(2*d*x+2*c))/a/d^2-1/3*f^2*polylog(2,-I*exp(d*x+c))/a/d^3+1/3*f^2*p
olylog(2,I*exp(d*x+c))/a/d^3-2/3*f^2*polylog(2,-exp(2*d*x+2*c))/a/d^3-1/3*
I*f^2*sech(d*x+c)/a/d^3+1/3*f*(f*x+e)*sech(d*x+c)^2/a/d^2+1/3*I*(f*x+e)^2*
sech(d*x+c)^3/a/d-1/3*f^2*tanh(d*x+c)/a/d^3+2/3*(f*x+e)^2*tanh(d*x+c)/a/d-
1/3*I*f*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/a/d^2+1/3*(f*x+e)^2*sech(d*x+c)^2*
tanh(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 3.92 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.77

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\frac{10id(e+fx)\left(d(e+fx)+2(1+ie^c)f\log\left(1-ie^{-c-dx}\right)\right)}{-i+e^c}+\frac{6\left(d(e+fx)\left(d(e+fx)+2(1-ie^c)f\log\left(1+ie^{-c-dx}\right)\right)+2i(i+e^c)f^2\,\text{PolyLog}\left(2,-ie^{-c-dx}\right.\right.}{-1+ie^c}}{}$$

input
```
Integrate[((e + f*x)^2*Sech[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(((10*I)*d*(e + f*x)*(d*(e + f*x) + 2*(1 + I*E^c)*f*Log[1 - I*E^(-c - d*x)
]))/(-I + E^c) + (6*(d*(e + f*x)*(d*(e + f*x) + 2*(1 - I*E^c)*f*Log[1 + I*
E^(-c - d*x)]) + (2*I)*(I + E^c)*f^2*PolyLog[2, (-I)*E^(-c - d*x)]))/(-1 +
 I*E^c) + 20*f^2*PolyLog[2, I*E^(-c - d*x)] + ((-2*I)*f^2*Cosh[c] + 2*d*f*
(e + f*x)*Cosh[d*x] - (2*I)*d^2*e^2*Cosh[c + d*x] + (4*I)*f^2*Cosh[c + d*x
] - (4*I)*d^2*e*f*x*Cosh[c + d*x] - (2*I)*d^2*f^2*x^2*Cosh[c + d*x] + 2*d*
e*f*Cosh[2*c + d*x] + 2*d*f^2*x*Cosh[2*c + d*x] + (4*I)*d^2*e^2*Cosh[c + 2
*d*x] - (2*I)*f^2*Cosh[c + 2*d*x] + (8*I)*d^2*e*f*x*Cosh[c + 2*d*x] + (4*I
)*d^2*f^2*x^2*Cosh[c + 2*d*x] + 8*d^2*e^2*Sinh[d*x] - 2*f^2*Sinh[d*x] + 16
*d^2*e*f*x*Sinh[d*x] + 8*d^2*f^2*x^2*Sinh[d*x] + d^2*e^2*Sinh[2*(c + d*x)]
 - 2*f^2*Sinh[2*(c + d*x)] + 2*d^2*e*f*x*Sinh[2*(c + d*x)] + d^2*f^2*x^2*S
inh[2*(c + d*x)] + 2*f^2*Sinh[2*c + d*x])/((Cosh[c/2] - I*Sinh[c/2])*(Cosh
[c/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] - I*Sinh[(c + d*x)/2])*(Cosh[(c +
d*x)/2] + I*Sinh[(c + d*x)/2])^3))/(12*a*d^3)
```

Rubi [A] (verified)

Time = 2.08 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.95, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.677$, Rules used = {6105, 3042, 4674, 3042, 4254, 24, 4672, 26, 3042, 26, 4201, 2620, 2715, 2838, 5974, 3042, 4673, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6105}$$

$$\frac{\int (e+fx)^2\text{sech}^4(c+dx)dx}{a}-\frac{i\int (e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^4dx}{a}-\frac{i\int (e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 4674}$$

$$\frac{-\frac{f^2\int \text{sech}^2(c+dx)dx}{3d^2}+\frac{2}{3}\int (e+fx)^2\text{sech}^2(c+dx)dx+\frac{f(e+fx)\text{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int (e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{f^2\int \csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{3d^2}+\frac{2}{3}\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx+\frac{f(e+fx)\text{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int (e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 4254}$$

$$\frac{-\frac{if^2\int 1d(-i\tanh(c+dx))}{3d^3}+\frac{2}{3}\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx+\frac{f(e+fx)\text{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int (e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \; 24$$

$$\frac{\frac{2}{3}\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{f(e+fx)\text{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int (e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \; 4672$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2if\int -i(e+fx)\tanh(c+dx)dx}{d}\right)-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{f(e+fx)\text{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int (e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2f\int (e+fx)\tanh(c+dx)dx}{d}\right)-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{f(e+fx)\text{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int (e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2f\int -i(e+fx)\tan(ic+idx)dx}{d}\right)-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{f(e+fx)\text{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int (e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\int (e+fx)\tan(ic+idx)dx}{d}\right)-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{f(e+fx)\text{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int (e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \; 4201$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\int \frac{e^{2(c+dx)}(e+fx)}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}\right)-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{f(e+fx)\text{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+dx)}{3d}}{a}-\frac{i\int (e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int\log\left(1+e^{2(c+dx)}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{f(e+fx)\text{sech}^2(c}{3d^2}}{a}$$

$$\frac{i\int(e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int e^{-2(c+dx)}\log\left(1+e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)-\frac{f^2\tanh(c+dx)}{3d^3}+}{a}$$

$$\frac{i\int(e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{2838}$$

$$\frac{-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)+\frac{f(e+fx)\text{sec}}{3d^2}}{a}$$

$$\frac{i\int(e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{5974}$$

$$\frac{-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)+\frac{f(e+fx)\text{sec}}{3d^2}}{a}$$

$$\frac{i\left(\frac{2f\int(e+fx)\text{sech}^3(c+dx)dx}{3d}-\frac{(e+fx)^2\text{sech}^3(c+dx)}{3d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)+\frac{f(e+fx)\text{sec}}{3d^2}}{a}$$

$$\frac{i\left(-\frac{(e+fx)^2\text{sech}^3(c+dx)}{3d}+\frac{2f\int(e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d}\right)}{a}$$

$$\Big\downarrow \text{4673}$$

$$\frac{-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)+\frac{f(e+fx)\text{sec}}{3d^2}}{a}$$

$$\frac{i\left(\frac{2f\left(\frac{1}{2}\int(e+fx)\text{sech}(c+dx)dx+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)}{3d}-\frac{(e+fx)^2\text{sech}^3(c+dx)}{3d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)+\frac{f(e+fx)\text{sec}}{3d^2}}{a}$$

$$\frac{i\left(-\frac{(e+fx)^2\text{sech}^3(c+dx)}{3d}+\frac{2f\left(\frac{1}{2}\int(e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)}{3d}\right)}{a}$$

$$\Big\downarrow \text{4668}$$

$$\frac{-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)+\frac{f(e+fx)\text{sec}}{3d^2}}{a}$$

$$\frac{i\left(-\frac{(e+fx)^2\text{sech}^3(c+dx)}{3d}+\frac{2f\left(\frac{1}{2}\left(-\frac{if\int\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+}{}\right.}{3d}\right.}{a}$$

$$\Big\downarrow \text{2715}$$

$$\frac{-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)+\frac{f(e+fx)\text{sech}}{3d^2}}{a}$$

$$\frac{i\left(-\frac{(e+fx)^2\text{sech}^3(c+dx)}{3d}+\frac{2f\left(\frac{1}{2}\left(-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{f\text{sec}}{}\right.}{3d}\right.}{a}$$

$\Big\downarrow$ 2838

$$\frac{-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)+\frac{f(e+fx)\text{sech}}{3d^2}}{a}$$

$$\frac{i\left(-\frac{(e+fx)^2\text{sech}^3(c+dx)}{3d}+\frac{2f\left(\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+}{2}\right.}{3d}\right.}{a}$$

input
```
Int[((e + f*x)^2*Sech[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((f*(e + f*x)*Sech[c + d*x]^2)/(3*d^2) - (f^2*Tanh[c + d*x])/(3*d^3) + ((e
 + f*x)^2*Sech[c + d*x]^2*Tanh[c + d*x])/(3*d) + (2*(((2*I)*f*(((-1/2*I)*(
e + f*x)^2)/f + (2*I)*(((e + f*x)*Log[1 + E^(2*(c + d*x))])/(2*d) + (f*Pol
yLog[2, -E^(2*(c + d*x))])/(4*d^2))))/d + ((e + f*x)^2*Tanh[c + d*x])/d))/
3)/a - (I*(-1/3*((e + f*x)^2*Sech[c + d*x]^3)/d + (2*f*(((2*(e + f*x)*ArcT
an[E^(c + d*x)])/d - (I*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + (I*f*PolyLog
[2, I*E^(c + d*x)])/d^2)/2 + (f*Sech[c + d*x])/(2*d^2) + ((e + f*x)*Sech[c
 + d*x]*Tanh[c + d*x])/(2*d)))/(3*d)))/a
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6105
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Sech[
c + d*x]^(n + 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)
*Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] &&
EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 44.99 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.57

method	result
risch	$-\frac{2ife\arctan(e^{dx+c})}{3a\,d^2}+\frac{4f^2c\ln(1+e^{2dx+2c})}{3a\,d^3}+\frac{4f^2c^2}{3a\,d^3}+\frac{2icf^2\arctan(e^{dx+c})}{3a\,d^3}+\frac{8f^2cx}{3a\,d^2}-\frac{4ef\ln(1+e^{2dx+2c})}{3a\,d^2}+\frac{8ef\ln(e}{3a\,}$

input
```
int((f*x+e)^2*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I/a/d^2*f*e*arctan(exp(d*x+c))+4/3/a/d^3*f^2*c*ln(1+exp(2*d*x+2*c))+4
/3/a/d^3*f^2*c^2+2/3*I/a/d^3*f^2*c*arctan(exp(d*x+c))+8/3/a/d^2*f^2*c*x-4/
3/a/d^2*e*f*ln(1+exp(2*d*x+2*c))+8/3/a/d^2*e*f*ln(exp(d*x+c))-8/3/a/d^3*c*
f^2*ln(exp(d*x+c))-f^2*polylog(2,I*exp(d*x+c))/a/d^3-1/a/d^2*f^2*ln(1-I*ex
p(d*x+c))*x+4/3/a/d*f^2*x^2-5/3/a/d^2*f^2*ln(1+I*exp(d*x+c))*x-5/3/a/d^3*f
^2*ln(1+I*exp(d*x+c))*c-1/a/d^3*f^2*ln(1-I*exp(d*x+c))*c-5/3*f^2*polylog(2
,-I*exp(d*x+c))/a/d^3+2/3*I*(-2*I*d^2*x^2*f^2+4*d^2*x^2*f^2*exp(d*x+c)-d*f
^2*x*exp(3*d*x+3*c)-4*I*d^2*e*f*x+8*d^2*e*f*x*exp(d*x+c)-d*e*f*exp(3*d*x+3
*c)-2*I*d^2*e^2+I*f^2*exp(2*d*x+2*c)+4*d^2*e^2*exp(d*x+c)-exp(d*x+c)*d*f^2
*x-f^2*exp(3*d*x+3*c)-exp(d*x+c)*d*e*f+I*f^2-f^2*exp(d*x+c))/(exp(d*x+c)+I
)/(exp(d*x+c)-I)^3/d^3/a
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 714 vs. 2(281) = 562.

Time = 0.11 (sec) , antiderivative size = 714, normalized size of antiderivative = 2.20

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/3*(4*d^2*e^2 - 8*c*d*e*f + 2*(2*c^2 - 1)*f^2 - 2*f^2*e^(2*d*x + 2*c) - 3
*(f^2*e^(4*d*x + 4*c) - 2*I*f^2*e^(3*d*x + 3*c) - 2*I*f^2*e^(d*x + c) - f^
2)*dilog(I*e^(d*x + c)) - 5*(f^2*e^(4*d*x + 4*c) - 2*I*f^2*e^(3*d*x + 3*c)
 - 2*I*f^2*e^(d*x + c) - f^2)*dilog(-I*e^(d*x + c)) + 4*(d^2*f^2*x^2 + 2*d
^2*e*f*x + 2*c*d*e*f - c^2*f^2)*e^(4*d*x + 4*c) - 2*(4*I*d^2*f^2*x^2 + (8*
I*c + I)*d*e*f + (-4*I*c^2 + I)*f^2 + (8*I*d^2*e*f + I*d*f^2)*x)*e^(3*d*x
+ 3*c) - 2*(-4*I*d^2*e^2 + (8*I*c + I)*d*e*f + I*d*f^2*x + (-4*I*c^2 + I)*
f^2)*e^(d*x + c) + 3*(d*e*f - c*f^2 - (d*e*f - c*f^2)*e^(4*d*x + 4*c) - 2*
(-I*d*e*f + I*c*f^2)*e^(3*d*x + 3*c) - 2*(-I*d*e*f + I*c*f^2)*e^(d*x + c))
*log(e^(d*x + c) + I) + 5*(d*e*f - c*f^2 - (d*e*f - c*f^2)*e^(4*d*x + 4*c)
 - 2*(-I*d*e*f + I*c*f^2)*e^(3*d*x + 3*c) - 2*(-I*d*e*f + I*c*f^2)*e^(d*x
+ c))*log(e^(d*x + c) - I) + 5*(d*f^2*x + c*f^2 - (d*f^2*x + c*f^2)*e^(4*d
*x + 4*c) - 2*(-I*d*f^2*x - I*c*f^2)*e^(3*d*x + 3*c) - 2*(-I*d*f^2*x - I*c
*f^2)*e^(d*x + c))*log(I*e^(d*x + c) + 1) + 3*(d*f^2*x + c*f^2 - (d*f^2*x
+ c*f^2)*e^(4*d*x + 4*c) - 2*(-I*d*f^2*x - I*c*f^2)*e^(3*d*x + 3*c) - 2*(-
I*d*f^2*x - I*c*f^2)*e^(d*x + c))*log(-I*e^(d*x + c) + 1))/(a*d^3*e^(4*d*x
 + 4*c) - 2*I*a*d^3*e^(3*d*x + 3*c) - 2*I*a*d^3*e^(d*x + c) - a*d^3)
```

Sympy [F]

$$\int \frac{(e+fx)^2\mathrm{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e^2\operatorname{sech}^2(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{f^2x^2\operatorname{sech}^2(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{2efx\operatorname{sech}^2(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)**2*sech(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e**2*sech(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(f**2
*x**2*sech(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(2*e*f*x*sech(c +
 d*x)**2/(sinh(c + d*x) - I), x))/a
```

Maxima [F]

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\,\text{sech}\,(dx+c)^2}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
4*f^2*((2*I*d^2*x^2 + (d*x*e^(3*c) + e^(3*c))*e^(3*d*x) - (4*d^2*x^2*e^c -
 d*x*e^c - e^c)*e^(d*x) - I*e^(2*d*x + 2*c) - I)/(6*I*a*d^3*e^(4*d*x + 4*c
) + 12*a*d^3*e^(3*d*x + 3*c) + 12*a*d^3*e^(d*x + c) - 6*I*a*d^3) + I*integ
rate(1/4*x/(a*d*e^(d*x + c) + I*a*d), x) - 5*I*integrate(1/12*x/(a*d*e^(d*
x + c) - I*a*d), x)) + 1/3*e*f*(24*(4*I*d*x*e^(4*d*x + 4*c) + (8*d*x*e^(3*
c) + e^(3*c))*e^(3*d*x) + e^(d*x + c))/(12*I*a*d^2*e^(4*d*x + 4*c) + 24*a*
d^2*e^(3*d*x + 3*c) + 24*a*d^2*e^(d*x + c) - 12*I*a*d^2) - 3*log((e^(d*x +
 c) + I)*e^(-c))/(a*d^2) - 5*log(-I*(I*e^(d*x + c) + 1)*e^(-c))/(a*d^2)) +
 4/3*e^2*(2*e^(-d*x - c)/((2*a*e^(-d*x - c) + 2*a*e^(-3*d*x - 3*c) - I*a*e
^(-4*d*x - 4*c) + I*a)*d) + I/((2*a*e^(-d*x - c) + 2*a*e^(-3*d*x - 3*c) -
I*a*e^(-4*d*x - 4*c) + I*a)*d))
```

Giac [F]

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\,\text{sech}\,(dx+c)^2}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sech(d*x + c)^2/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^2}{\cosh\left(c+d\,x\right)^2\,\left(a+a\sinh\left(c+d\,x\right)\,1\text{i}\right)}\,dx$$

input
```
int((e + f*x)^2/(cosh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)^2/(cosh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(\int \frac{\text{sech}(dx+c)^2}{\sinh(dx+c)i+1}dx\right)e^2+\left(\int \frac{\text{sech}(dx+c)^2x^2}{\sinh(dx+c)i+1}dx\right)f^2+2\left(\int \frac{\text{sech}(dx+c)^2x}{\sinh(dx+c)i+1}dx\right)ef}{a}$$

input
```
int((f*x+e)^2*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
(int(sech(c + d*x)**2/(sinh(c + d*x)*i + 1),x)*e**2 + int((sech(c + d*x)**
2*x**2)/(sinh(c + d*x)*i + 1),x)*f**2 + 2*int((sech(c + d*x)**2*x)/(sinh(c
 + d*x)*i + 1),x)*e*f)/a
```

3.279 $\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 158

$$\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{if\arctan(\sinh(c+dx))}{6ad^2} - \frac{2f\log(\cosh(c+dx))}{3ad^2} + \frac{f\text{sech}^2(c+dx)}{6ad^2} + \frac{i(e+fx)\text{sech}^3(c+dx)}{3ad} + \frac{2(e+fx)\tanh(c+dx)}{3ad} - \frac{if\text{sech}(c+dx)\tanh(c+dx)}{6ad^2} + \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{3ad}$$

output
```
-1/6*I*f*arctan(sinh(d*x+c))/a/d^2-2/3*f*ln(cosh(d*x+c))/a/d^2+1/6*f*sech(
d*x+c)^2/a/d^2+1/3*I*(f*x+e)*sech(d*x+c)^3/a/d+2/3*(f*x+e)*tanh(d*x+c)/a/d
-1/6*I*f*sech(d*x+c)*tanh(d*x+c)/a/d^2+1/3*(f*x+e)*sech(d*x+c)^2*tanh(d*x+
c)/a/d
```

Mathematica [A] (verified)

Time = 2.87 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.23

$$\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{2d(e+fx)(\cosh(2(c+dx))-2i\sinh(c+dx))+\cosh(c+dx)\left(-de-if+cf-2f\arctan\left(\tanh\left(\frac{1}{2}(c+\right.\right.\right.}{6ad^2\left(\cosh\left(\frac{1}{2}(c+dx)\right)-i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(\right.}$$

input
```
Integrate[((e + f*x)*Sech[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(2*d*(e + f*x)*(Cosh[2*(c + d*x)] - (2*I)*Sinh[c + d*x]) + Cosh[c + d*x]*(
-(d*e) - I*f + c*f - 2*f*ArcTan[Tanh[(c + d*x)/2]] + (4*I)*f*Log[Cosh[c +
d*x]] - I*(d*e - c*f + 2*f*ArcTan[Tanh[(c + d*x)/2]] - (4*I)*f*Log[Cosh[c
+ d*x]])*Sinh[c + d*x]))/(6*a*d^2*(Cosh[(c + d*x)/2] - I*Sinh[(c + d*x)/2]
)*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(-I + Sinh[c + d*x]))
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.483$, Rules used = {6105, 3042, 4673, 3042, 4672, 26, 3042, 26, 3956, 5974, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$\Big\downarrow \text{6105}$$
$$\frac{\int (e+fx)\text{sech}^4(c+dx)dx}{a}-\frac{i\int (e+fx)\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^4dx}{a}-\frac{i\int (e+fx)\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$
$$\Big\downarrow \text{4673}$$

$$\frac{\frac{2}{3}\int(e+fx)\text{sech}^2(c+dx)dx+\frac{f\text{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int(e+fx)\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{2}{3}\int(e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx+\frac{f\text{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int(e+fx)\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \textbf{4672}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{if\int-i\tanh(c+dx)dx}{d}\right)+\frac{f\text{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int(e+fx)\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \textbf{26}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int\tanh(c+dx)dx}{d}\right)+\frac{f\text{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int(e+fx)\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int-i\tan(ic+idx)dx}{d}\right)+\frac{f\text{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int(e+fx)\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \textbf{26}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}+\frac{if\int\tan(ic+idx)dx}{d}\right)+\frac{f\text{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int(e+fx)\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \textbf{3956}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)+\frac{f\text{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int(e+fx)\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 5974}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)+\frac{f\text{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\left(\frac{f\int \text{sech}^3(c+dx)dx}{3d}-\frac{(e+fx)\text{sech}^3(c+dx)}{3d}\right)}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)+\frac{f\text{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\left(-\frac{(e+fx)\text{sech}^3(c+dx)}{3d}+\frac{f\int \csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d}\right)}{a}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)+\frac{f\text{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\left(\frac{f\left(\frac{1}{2}\int \text{sech}(c+dx)dx+\frac{\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)}{3d}-\frac{(e+fx)\text{sech}^3(c+dx)}{3d}\right)}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)+\frac{f\text{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\left(-\frac{(e+fx)\text{sech}^3(c+dx)}{3d}+\frac{f\left(\frac{\tanh(c+dx)\text{sech}(c+dx)}{2d}+\frac{1}{2}\int \csc\left(ic+idx+\frac{\pi}{2}\right)dx\right)}{3d}\right)}{a}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)+\frac{f\text{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\left(\frac{f\left(\frac{\arctan(\sinh(c+dx))}{2d}+\frac{\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)}{3d}-\frac{(e+fx)\text{sech}^3(c+dx)}{3d}\right)}{a}$$

input
```
Int[((e + f*x)*Sech[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((f*Sech[c + d*x]^2)/(6*d^2) + ((e + f*x)*Sech[c + d*x]^2*Tanh[c + d*x])/(
3*d) + (2*(-((f*Log[Cosh[c + d*x]])/d^2) + ((e + f*x)*Tanh[c + d*x])/d))/3
)/a - (I*(-1/3*((e + f*x)*Sech[c + d*x]^3)/d + (f*(ArcTan[Sinh[c + d*x]]/(
2*d) + (Sech[c + d*x]*Tanh[c + d*x])/(2*d)))/(3*d)))/a
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6105
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Sech[
c + d*x]^(n + 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)
*Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] &&
EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 16.60 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.91

method	result	size
risch	$\frac{4fx}{3ad}+\frac{4fc}{3a\,d^{2}}-\frac{i\left(-8dfx\,{\mathrm e}^{dx+c}+f\,{\mathrm e}^{3dx+3c}-8de\,{\mathrm e}^{dx+c}+{\mathrm e}^{dx+c}f+4idfx+4ied\right)}{3\left({\mathrm e}^{dx+c}+i\right)\left({\mathrm e}^{dx+c}-i\right)^{3}d^{2}a}-\frac{f\ln\left({\mathrm e}^{dx+c}+i\right)}{2a\,d^{2}}-\frac{5f\ln\left({\mathrm e}^{dx+c}-i\right)}{6a\,d^{2}}$	143

input
```
int((f*x+e)*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
4/3*f*x/a/d+4/3*f/a/d^2*c-1/3*I*(-8*d*f*x*exp(d*x+c)+f*exp(3*d*x+3*c)-8*d*
e*exp(d*x+c)+exp(d*x+c)*f+4*I*d*f*x+4*I*e*d)/(exp(d*x+c)+I)/(exp(d*x+c)-I)
^3/d^2/a-1/2*f/a/d^2*ln(exp(d*x+c)+I)-5/6*f/a/d^2*ln(exp(d*x+c)-I)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.27

$$\int \frac{(e+fx)\mathrm{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{8\,dfxe^{(4\,dx+4\,c)} + 8\,de - 2\,(8i\,dfx + i\,f)e^{(3\,dx+3\,c)} - 2\,(-8i\,de + i\,f)e^{(dx+c)} - 3\,\left(fe^{(4\,dx+4\,c)} - 2i\,fe^{(3\,dx+3\,c)}\right.}{6\,\left(ad^2e^{(4\,dx+4\,c)} - 2i\,ad^2e^{(3\,dx-}\right.}$$

input
```
integrate((f*x+e)*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(8*d*f*x*e^(4*d*x + 4*c) + 8*d*e - 2*(8*I*d*f*x + I*f)*e^(3*d*x + 3*c)
 - 2*(-8*I*d*e + I*f)*e^(d*x + c) - 3*(f*e^(4*d*x + 4*c) - 2*I*f*e^(3*d*x
+ 3*c) - 2*I*f*e^(d*x + c) - f)*log(e^(d*x + c) + I) - 5*(f*e^(4*d*x + 4*c
) - 2*I*f*e^(3*d*x + 3*c) - 2*I*f*e^(d*x + c) - f)*log(e^(d*x + c) - I))/(
a*d^2*e^(4*d*x + 4*c) - 2*I*a*d^2*e^(3*d*x + 3*c) - 2*I*a*d^2*e^(d*x + c)
- a*d^2)
```

Sympy [F]

$$\int \frac{(e+fx)\mathrm{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e\,\mathrm{sech}^2\,(c+dx)}{\sinh\,(c+dx)-i}\,dx + \int \frac{fx\,\mathrm{sech}^2\,(c+dx)}{\sinh\,(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)*sech(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e*sech(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(f*x*sec
h(c + d*x)**2/(sinh(c + d*x) - I), x))/a
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.59

$$\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$=\frac{1}{6}f\left(\frac{24\left(4i\,dxe^{(4\,dx+4\,c)}+\left(8\,dxe^{(3\,c)}+e^{(3\,c)}\right)e^{(3\,dx)}+e^{(dx+c)}\right)}{12i\,ad^2e^{(4\,dx+4\,c)}+24\,ad^2e^{(3\,dx+3\,c)}+24\,ad^2e^{(dx+c)}-12i\,ad^2}-\frac{3\,\log\left(\left(e^{(dx+c)}+i\right)e^{(-c)}\right)}{ad^2}-\frac{5\,\log\left(-\right.}{}\right.$$
$$+\frac{4}{3}e\left(\frac{2\,e^{(-dx-c)}}{\left(2\,ae^{(-dx-c)}+2\,ae^{(-3\,dx-3\,c)}-i\,ae^{(-4\,dx-4\,c)}+i\,a\right)d}+\frac{i}{\left(2\,ae^{(-dx-c)}+2\,ae^{(-3\,dx-3\,c)}-i\,ae^{(-4\,dx-4\,c)}\right.}\right.$$

input
```
integrate((f*x+e)*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
1/6*f*(24*(4*I*d*x*e^(4*d*x + 4*c) + (8*d*x*e^(3*c) + e^(3*c))*e^(3*d*x) +
 e^(d*x + c))/(12*I*a*d^2*e^(4*d*x + 4*c) + 24*a*d^2*e^(3*d*x + 3*c) + 24*
a*d^2*e^(d*x + c) - 12*I*a*d^2) - 3*log((e^(d*x + c) + I)*e^(-c))/(a*d^2)
- 5*log(-I*(I*e^(d*x + c) + 1)*e^(-c))/(a*d^2)) + 4/3*e*(2*e^(-d*x - c)/((
2*a*e^(-d*x - c) + 2*a*e^(-3*d*x - 3*c) - I*a*e^(-4*d*x - 4*c) + I*a)*d) +
 I/((2*a*e^(-d*x - c) + 2*a*e^(-3*d*x - 3*c) - I*a*e^(-4*d*x - 4*c) + I*a)
*d))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.65

$$\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$=\frac{8\,dfxe^{(4\,dx+4\,c)}-16i\,dfxe^{(3\,dx+3\,c)}+16i\,dee^{(dx+c)}-3\,fe^{(4\,dx+4\,c)}\log\left(e^{(dx+c)}+i\right)+6i\,fe^{(3\,dx+3\,c)}\log\left(e^{(dx+\right.}}{}$$

input
```
integrate((f*x+e)*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(8*d*f*x*e^(4*d*x + 4*c) - 16*I*d*f*x*e^(3*d*x + 3*c) + 16*I*d*e*e^(d*
x + c) - 3*f*e^(4*d*x + 4*c)*log(e^(d*x + c) + I) + 6*I*f*e^(3*d*x + 3*c)*
log(e^(d*x + c) + I) + 6*I*f*e^(d*x + c)*log(e^(d*x + c) + I) - 5*f*e^(4*d
*x + 4*c)*log(e^(d*x + c) - I) + 10*I*f*e^(3*d*x + 3*c)*log(e^(d*x + c) -
I) + 10*I*f*e^(d*x + c)*log(e^(d*x + c) - I) + 8*d*e - 2*I*f*e^(3*d*x + 3*
c) - 2*I*f*e^(d*x + c) + 3*f*log(e^(d*x + c) + I) + 5*f*log(e^(d*x + c) -
I))/(a*d^2*e^(4*d*x + 4*c) - 2*I*a*d^2*e^(3*d*x + 3*c) - 2*I*a*d^2*e^(d*x
+ c) - a*d^2)
```

Mupad [B] (verification not implemented)

Time = 3.51 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.30

$$\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{4\,f\,x}{3\,a\,d} - \frac{f+3\,d\,e+3\,d\,f\,x}{3\,a\,d^2\,\left(1-{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{c+d\,x}\,2{}\mathrm{i}\right)} - \frac{5\,f\,\ln\left(f+f\,{\mathrm{e}}^{c+d\,x}\,1{}\mathrm{i}\right)}{6\,a\,d^2} - \frac{\left(e+f\,x\right)\,2{}\mathrm{i}}{3\,a\,d\,\left(3\,{\mathrm{e}}^{c+d\,x}+{\mathrm{e}}^{2\,c+2\,d\,x}\,3{}\mathrm{i}-{\mathrm{e}}^{3\,c+3\,d\,x}-\mathrm{i}\right)} - \frac{\left(e+f\,x\right)\,1{}\mathrm{i}}{2\,a\,d\,\left({\mathrm{e}}^{c+d\,x}+1{}\mathrm{i}\right)} - \frac{f\,\ln\left(-1+{\mathrm{e}}^{c+d\,x}\,1{}\mathrm{i}\right)}{2\,a\,d^2} + \frac{\left(3\,d\,e-2\,f+3\,d\,f\,x\right)\,1{}\mathrm{i}}{6\,a\,d^2\,\left({\mathrm{e}}^{c+d\,x}-\mathrm{i}\right)}$$

input
```
int((e + f*x)/(cosh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
(4*f*x)/(3*a*d) - (f + 3*d*e + 3*d*f*x)/(3*a*d^2*(exp(c + d*x)*2i - exp(2*
c + 2*d*x) + 1)) - (5*f*log(f + f*exp(c + d*x)*1i))/(6*a*d^2) - ((e + f*x)
*2i)/(3*a*d*(3*exp(c + d*x) + exp(2*c + 2*d*x)*3i - exp(3*c + 3*d*x) - 1i)
) - ((e + f*x)*1i)/(2*a*d*(exp(c + d*x) + 1i)) - (f*log(exp(c + d*x)*1i -
1))/(2*a*d^2) + ((3*d*e - 2*f + 3*d*f*x)*1i)/(6*a*d^2*(exp(c + d*x) - 1i))
```

Reduce [F]

$$\int \frac{(e+fx)\mathrm{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(\int \frac{\mathrm{sech}(dx+c)^2}{\sinh(dx+c)i+1}dx\right)e+\left(\int \frac{\mathrm{sech}(dx+c)^2x}{\sinh(dx+c)i+1}dx\right)f}{a}$$

input
```
int((f*x+e)*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
(int(sech(c + d*x)**2/(sinh(c + d*x)*i + 1),x)*e + int((sech(c + d*x)**2*x
)/(sinh(c + d*x)*i + 1),x)*f)/a
```

3.280 $\int \frac{\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 47

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{i\operatorname{sech}(c+dx)}{3d(a+ia\sinh(c+dx))}+\frac{2\tanh(c+dx)}{3ad}$$

output
```
1/3*I*sech(d*x+c)/d/(a+I*a*sinh(d*x+c))+2/3*tanh(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\operatorname{sech}(c+dx)(\cosh(2(c+dx))-2i\sinh(c+dx))}{3ad(-i+\sinh(c+dx))}$$

input
```
Integrate[Sech[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(Sech[c + d*x]*(Cosh[2*(c + d*x)] - (2*I)*Sinh[c + d*x]))/(3*a*d*(-I + Sin
h[c + d*x]))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3042, 3151, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\cos(ic+idx)^2(a+a\sin(ic+idx))}dx$$

$$\Big\downarrow \text{ 3151}$$

$$\frac{2\int \text{sech}^2(c+dx)dx}{3a}+\frac{i\text{sech}(c+dx)}{3d(a+ia\sinh(c+dx))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\int \csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{3a}+\frac{i\text{sech}(c+dx)}{3d(a+ia\sinh(c+dx))}$$

$$\Big\downarrow \text{ 4254}$$

$$\frac{2i\int 1d(-i\tanh(c+dx))}{3ad}+\frac{i\text{sech}(c+dx)}{3d(a+ia\sinh(c+dx))}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{2\tanh(c+dx)}{3ad}+\frac{i\text{sech}(c+dx)}{3d(a+ia\sinh(c+dx))}$$

input
```
Int[Sech[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((I/3)*Sech[c + d*x])/(d*(a + I*a*Sinh[c + d*x])) + (2*Tanh[c + d*x])/(3*a
*d)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 15.97 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

method	result	size
risch	$\frac{4 i \left(2 \,{\mathrm e}^{d x +c}-i\right)}{3 \left({\mathrm e}^{d x +c}-i\right)^{3} \left({\mathrm e}^{d x +c}+i\right) d a}$	43
derivativedivides	$\frac{-\frac{2}{3 \left(-i+\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{i}{\left(-i+\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{3}{2 \left(-i+\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)\right)}+\frac{2}{4 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)+4 i}}{d a}$	75
default	$\frac{-\frac{2}{3 \left(-i+\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{i}{\left(-i+\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{3}{2 \left(-i+\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)\right)}+\frac{2}{4 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)+4 i}}{d a}$	75

input
```
int(sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
4/3*I*(2*exp(d*x+c)-I)/(exp(d*x+c)-I)^3/(exp(d*x+c)+I)/d/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

$$\int \frac{\mathrm{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{4\left(-2i\,e^{(dx+c)}-1\right)}{3\left(ade^{(4\,dx+4\,c)}-2i\,ade^{(3\,dx+3\,c)}-2i\,ade^{(dx+c)}-ad\right)}$$

input
```
integrate(sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-4/3*(-2*I*e^(d*x + c) - 1)/(a*d*e^(4*d*x + 4*c) - 2*I*a*d*e^(3*d*x + 3*c)
 - 2*I*a*d*e^(d*x + c) - a*d)
```

Sympy [F]

$$\int \frac{\mathrm{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int \frac{\mathrm{sech}^2(c+dx)}{\sinh(c+dx)-i}\,dx}{a}$$

input
```
integrate(sech(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(sech(c + d*x)**2/(sinh(c + d*x) - I), x)/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(39) = 78.

Time = 0.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.21

$$\int \frac{\mathrm{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{8\,e^{(-dx-c)}}{3\left(2\,ae^{(-dx-c)}+2\,ae^{(-3\,dx-3\,c)}-i\,ae^{(-4\,dx-4\,c)}+i\,a\right)d} + \frac{4i}{3\left(2\,ae^{(-dx-c)}+2\,ae^{(-3\,dx-3\,c)}-i\,ae^{(-4\,dx-4\,c)}+i\,a\right)d}$$

input
```
integrate(sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
8/3*e^(-d*x - c)/((2*a*e^(-d*x - c) + 2*a*e^(-3*d*x - 3*c) - I*a*e^(-4*d*x
 - 4*c) + I*a)*d) + 4/3*I/((2*a*e^(-d*x - c) + 2*a*e^(-3*d*x - 3*c) - I*a*
e^(-4*d*x - 4*c) + I*a)*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\frac{3}{a\left(i\,e^{(dx+c)}-1\right)} - \frac{-3i\,e^{(2\,dx+2\,c)}-12\,e^{(dx+c)}+5i}{a\left(e^{(dx+c)}-i\right)^3}}{6\,d}$$

input
```
integrate(sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(3/(a*(I*e^(d*x + c) - 1)) - (-3*I*e^(2*d*x + 2*c) - 12*e^(d*x + c) +
5*I)/(a*(e^(d*x + c) - I)^3))/d
```

Mupad [B] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{4\left(1+e^{c+d\,x}\,2i\right)\,\left(e^{c+d\,x}+1i\right)^2}{3\,a\,d\,\left(e^{2\,c+2\,d\,x}+1\right)^3}$$

input
```
int(1/(cosh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
(4*(exp(c + d*x)*2i + 1)*(exp(c + d*x) + 1i)^2)/(3*a*d*(exp(2*c + 2*d*x) +
 1)^3)
```

Reduce [F]

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\int \frac{\operatorname{sech}(dx+c)^2}{\sinh(dx+c)i+1}dx}{a}$$

input
```
int(sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sech(c + d*x)**2/(sinh(c + d*x)*i + 1),x)/a
```

3.281 $\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=\operatorname{Int}\left(\frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(sech(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Mathematica [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=\text{\$Aborted}$$

input
```
Integrate[Sech[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

$$\Big\downarrow \; 6111$$

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}dx$$

input
```
Int[Sech[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{sech}\left(dx+c\right)^2}{\left(fx+e\right)\left(a+ia\sinh\left(dx+c\right)\right)}dx$$

input
```
int(sech(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sech(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 768, normalized size of antiderivative = 24.77

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}\left(dx+c\right)^2}{(fx+e)(i\,a\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sech(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/3*(4*d^2*f^2*x^2 + 8*d^2*e*f*x + 4*d^2*e^2 - 2*f^2*e^(2*d*x + 2*c) - 2*
f^2 + (I*d*f^2*x + I*d*e*f - 2*I*f^2)*e^(3*d*x + 3*c) + (8*I*d^2*f^2*x^2 +
 8*I*d^2*e^2 + I*d*e*f - 2*I*f^2 + (16*I*d^2*e*f + I*d*f^2)*x)*e^(d*x + c)
 - 3*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3 - (a
*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*e^(4*d*x +
 4*c) + 2*(I*a*d^3*f^3*x^3 + 3*I*a*d^3*e*f^2*x^2 + 3*I*a*d^3*e^2*f*x + I*a
*d^3*e^3)*e^(3*d*x + 3*c) + 2*(I*a*d^3*f^3*x^3 + 3*I*a*d^3*e*f^2*x^2 + 3*I
*a*d^3*e^2*f*x + I*a*d^3*e^3)*e^(d*x + c))*integral(-1/3*(4*d^2*f^3*x^2 +
8*d^2*e*f^2*x + 4*d^2*e^2*f - 6*f^3 - (I*d^2*f^3*x^2 + 2*I*d^2*e*f^2*x + I
*d^2*e^2*f - 6*I*f^3)*e^(d*x + c))/(a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*
a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4 + (a*d^3*f^4*x^4 + 4*a*d^3
*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4)*e^(2*d*x +
 2*c)), x))/(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e
^3 - (a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*e^(
4*d*x + 4*c) + 2*(I*a*d^3*f^3*x^3 + 3*I*a*d^3*e*f^2*x^2 + 3*I*a*d^3*e^2*f*
x + I*a*d^3*e^3)*e^(3*d*x + 3*c) + 2*(I*a*d^3*f^3*x^3 + 3*I*a*d^3*e*f^2*x^
2 + 3*I*a*d^3*e^2*f*x + I*a*d^3*e^3)*e^(d*x + c))
```

Sympy [N/A]

Not integrable

Time = 4.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=-\frac{i\int \frac{\operatorname{sech}^2\left(c+dx\right)}{e\sinh\left(c+dx\right)-ie+fx\sinh\left(c+dx\right)-ifx}\,dx}{a}$$

input
```
integrate(sech(d*x+c)**2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(sech(c + d*x)**2/(e*sinh(c + d*x) - I*e + f*x*sinh(c + d*x) -
I*f*x), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 632, normalized size of antiderivative = 20.39

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(dx+c)^2}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sech(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-4*I*f*integrate(1/(8*I*a*d*f^2*x^2 + 16*I*a*d*e*f*x + 8*I*a*d*e^2 + 8*(a*
d*f^2*x^2*e^c + 2*a*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)), x) - 1/3*(4*d^2*f
^2*x^2 + 8*d^2*e*f*x + 4*d^2*e^2 - 2*f^2*e^(2*d*x + 2*c) - 2*f^2 + (I*d*f^
2*x*e^(3*c) + (I*d*e*f - 2*I*f^2)*e^(3*c))*e^(3*d*x) + (8*I*d^2*f^2*x^2*e^
c + (16*I*d^2*e*f + I*d*f^2)*x*e^c + (8*I*d^2*e^2 + I*d*e*f - 2*I*f^2)*e^c
)*e^(d*x))/(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^
3 - (a*d^3*f^3*x^3*e^(4*c) + 3*a*d^3*e*f^2*x^2*e^(4*c) + 3*a*d^3*e^2*f*x*e
^(4*c) + a*d^3*e^3*e^(4*c))*e^(4*d*x) + 2*(I*a*d^3*f^3*x^3*e^(3*c) + 3*I*a
*d^3*e*f^2*x^2*e^(3*c) + 3*I*a*d^3*e^2*f*x*e^(3*c) + I*a*d^3*e^3*e^(3*c))*
e^(3*d*x) + 2*(I*a*d^3*f^3*x^3*e^c + 3*I*a*d^3*e*f^2*x^2*e^c + 3*I*a*d^3*e
^2*f*x*e^c + I*a*d^3*e^3*e^c)*e^(d*x)) - 4*integrate(1/24*(5*d^2*f^3*x^2 +
 10*d^2*e*f^2*x + 5*d^2*e^2*f - 12*f^3)/(a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3
 + 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4 - (-I*a*d^3*f^4*x^4*e
^c - 4*I*a*d^3*e*f^3*x^3*e^c - 6*I*a*d^3*e^2*f^2*x^2*e^c - 4*I*a*d^3*e^3*f
*x*e^c - I*a*d^3*e^4*e^c)*e^(d*x)), x)
```

Giac [N/A]

Not integrable

Time = 59.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^2}{(fx+e)(i\,a\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sech(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sech(d*x + c)^2/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.52 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^2\,\left(e+f\,x\right)\,\left(a+a\sinh\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)^2*(e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(1/(cosh(c + d*x)^2*(e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \frac{\int \frac{\operatorname{sech}(dx+c)^2}{\sinh(dx+c)ei+\sinh(dx+c)fix+e+fx}dx}{a}$$

input
```
int(sech(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sech(c + d*x)**2/(sinh(c + d*x)*e*i + sinh(c + d*x)*f*i*x + e + f*x),x
)/a
```

3.282 $\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx=\text{Int}\left(\frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(sech(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Mathematica [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx=\text{\$Aborted}$$

input
```
Integrate[Sech[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}dx$$

$$\Big\downarrow \text{6111}$$

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}dx$$

input
```
Int[Sech[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{sech}\left(dx+c\right)^2}{\left(fx+e\right)^2\left(a+ia\sinh\left(dx+c\right)\right)}dx$$

input
```
int(sech(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sech(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 919, normalized size of antiderivative = 29.65

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^2}{\left(fx+e\right)^2\left(i\,a\sinh\left(dx+c\right)+a\right)}\,dx$$

input
```
integrate(sech(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
-1/3*(4*d^2*f^2*x^2 + 8*d^2*e*f*x + 4*d^2*e^2 - 6*f^2*e^(2*d*x + 2*c) - 6*
f^2 - 2*(-I*d*f^2*x - I*d*e*f + 3*I*f^2)*e^(3*d*x + 3*c) - 2*(-4*I*d^2*f^2
*x^2 - 4*I*d^2*e^2 - I*d*e*f + 3*I*f^2 + (-8*I*d^2*e*f - I*d*f^2)*x)*e^(d*
x + c) - 3*(a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*
d^3*e^3*f*x + a*d^3*e^4 - (a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2
*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4)*e^(4*d*x + 4*c) + 2*(I*a*d^3*f^4*x
^4 + 4*I*a*d^3*e*f^3*x^3 + 6*I*a*d^3*e^2*f^2*x^2 + 4*I*a*d^3*e^3*f*x + I*a
*d^3*e^4)*e^(3*d*x + 3*c) + 2*(I*a*d^3*f^4*x^4 + 4*I*a*d^3*e*f^3*x^3 + 6*I
*a*d^3*e^2*f^2*x^2 + 4*I*a*d^3*e^3*f*x + I*a*d^3*e^4)*e^(d*x + c))*integra
l(-2/3*(4*d^2*f^3*x^2 + 8*d^2*e*f^2*x + 4*d^2*e^2*f - 12*f^3 + (-I*d^2*f^3
*x^2 - 2*I*d^2*e*f^2*x - I*d^2*e^2*f + 12*I*f^3)*e^(d*x + c))/(a*d^3*f^5*x
^5 + 5*a*d^3*e*f^4*x^4 + 10*a*d^3*e^2*f^3*x^3 + 10*a*d^3*e^3*f^2*x^2 + 5*a
*d^3*e^4*f*x + a*d^3*e^5 + (a*d^3*f^5*x^5 + 5*a*d^3*e*f^4*x^4 + 10*a*d^3*e
^2*f^3*x^3 + 10*a*d^3*e^3*f^2*x^2 + 5*a*d^3*e^4*f*x + a*d^3*e^5)*e^(2*d*x
+ 2*c)), x))/(a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*
a*d^3*e^3*f*x + a*d^3*e^4 - (a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e
^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4)*e^(4*d*x + 4*c) + 2*(I*a*d^3*f^4
*x^4 + 4*I*a*d^3*e*f^3*x^3 + 6*I*a*d^3*e^2*f^2*x^2 + 4*I*a*d^3*e^3*f*x + I
*a*d^3*e^4)*e^(3*d*x + 3*c) + 2*(I*a*d^3*f^4*x^4 + 4*I*a*d^3*e*f^3*x^3 + 6
*I*a*d^3*e^2*f^2*x^2 + 4*I*a*d^3*e^3*f*x + I*a*d^3*e^4)*e^(d*x + c))
```

Sympy [N/A]

Not integrable

Time = 40.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.35

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = -\frac{i\int \frac{\operatorname{sech}^2(c+dx)}{e^2\sinh(c+dx)-ie^2+2efx\sinh(c+dx)-2iefx+f^2x^2\sinh(c+dx)-if^2x^2}\,dx}{a}$$

input
```
integrate(sech(d*x+c)**2/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(sech(c + d*x)**2/(e**2*sinh(c + d*x) - I*e**2 + 2*e*f*x*sinh(c
 + d*x) - 2*I*e*f*x + f**2*x**2*sinh(c + d*x) - I*f**2*x**2), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 762, normalized size of antiderivative = 24.58

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(dx+c)^2}{(fx+e)^2(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sech(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-4*I*f*integrate(1/(4*I*a*d*f^3*x^3 + 12*I*a*d*e*f^2*x^2 + 12*I*a*d*e^2*f*
x + 4*I*a*d*e^3 + 4*(a*d*f^3*x^3*e^c + 3*a*d*e*f^2*x^2*e^c + 3*a*d*e^2*f*x
*e^c + a*d*e^3*e^c)*e^(d*x)), x) - 2/3*(2*d^2*f^2*x^2 + 4*d^2*e*f*x + 2*d^
2*e^2 - 3*f^2*e^(2*d*x + 2*c) - 3*f^2 + (I*d*f^2*x*e^(3*c) + (I*d*e*f - 3*
I*f^2)*e^(3*c))*e^(3*d*x) + (4*I*d^2*f^2*x^2*e^c + (8*I*d^2*e*f + I*d*f^2)
*x*e^c + (4*I*d^2*e^2 + I*d*e*f - 3*I*f^2)*e^c)*e^(d*x))/(a*d^3*f^4*x^4 +
4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4 - (a
*d^3*f^4*x^4*e^(4*c) + 4*a*d^3*e*f^3*x^3*e^(4*c) + 6*a*d^3*e^2*f^2*x^2*e^(
4*c) + 4*a*d^3*e^3*f*x*e^(4*c) + a*d^3*e^4*e^(4*c))*e^(4*d*x) + 2*(I*a*d^3
*f^4*x^4*e^(3*c) + 4*I*a*d^3*e*f^3*x^3*e^(3*c) + 6*I*a*d^3*e^2*f^2*x^2*e^(
3*c) + 4*I*a*d^3*e^3*f*x*e^(3*c) + I*a*d^3*e^4*e^(3*c))*e^(3*d*x) + 2*(I*a
*d^3*f^4*x^4*e^c + 4*I*a*d^3*e*f^3*x^3*e^c + 6*I*a*d^3*e^2*f^2*x^2*e^c + 4
*I*a*d^3*e^3*f*x*e^c + I*a*d^3*e^4*e^c)*e^(d*x)) - 4*integrate(1/12*(5*d^2
*f^3*x^2 + 10*d^2*e*f^2*x + 5*d^2*e^2*f - 24*f^3)/(a*d^3*f^5*x^5 + 5*a*d^3
*e*f^4*x^4 + 10*a*d^3*e^2*f^3*x^3 + 10*a*d^3*e^3*f^2*x^2 + 5*a*d^3*e^4*f*x
 + a*d^3*e^5 - (-I*a*d^3*f^5*x^5*e^c - 5*I*a*d^3*e*f^4*x^4*e^c - 10*I*a*d^
3*e^2*f^3*x^3*e^c - 10*I*a*d^3*e^3*f^2*x^2*e^c - 5*I*a*d^3*e^4*f*x*e^c - I
*a*d^3*e^5*e^c)*e^(d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sech(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^2\,{\left(e+f\,x\right)}^2\,\left(a+a\,\sinh\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)^2*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(1/(cosh(c + d*x)^2*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.26

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \frac{\int \frac{\operatorname{sech}(dx+c)^2}{\sinh(dx+c)e^2i+2\sinh(dx+c)efix+\sinh(dx+c)f^2i\,x^2+e^2+2efx+f^2x^2}dx}{a}$$

input
```
int(sech(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sech(c + d*x)**2/(sinh(c + d*x)*e**2*i + 2*sinh(c + d*x)*e*f*i*x + sin
h(c + d*x)*f**2*i*x**2 + e**2 + 2*e*f*x + f**2*x**2),x)/a
```

3.283 $\int \frac{(e+fx)^3 \operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 667

$$\int \frac{(e+fx)^3\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{if(e+fx)^2}{2ad^2}-\frac{5f^2(e+fx)\arctan\left(e^{c+dx}\right)}{ad^3}+\frac{3(e+fx)^3\arctan\left(e^{c+dx}\right)}{4ad}+\frac{if^2(e+fx)\log\left(1+e^{2(c+dx)}\right)}{ad^3}+\frac{5if^3\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{2ad^4}-\frac{9if(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{8ad^2}-\frac{5if^3\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{2ad^4}+\frac{9if(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{8ad^2}+\frac{if^3\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2ad^4}+\frac{9if^2(e+fx)\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{4ad^3}-\frac{9if^2(e+fx)\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{4ad^3}-\frac{9if^3\operatorname{PolyLog}\left(4,-ie^{c+dx}\right)}{4ad^4}+\frac{9if^3\operatorname{PolyLog}\left(4,ie^{c+dx}\right)}{4ad^4}-\frac{f^3\operatorname{sech}(c+dx)}{4ad^4}+\frac{9f(e+fx)^2\operatorname{sech}(c+dx)}{8ad^2}-\frac{if^2(e+fx)\operatorname{sech}^2(c+dx)}{4ad^3}+\frac{f(e+fx)^2\operatorname{sech}^3(c+dx)}{4ad^2}+\frac{i(e+fx)^3\operatorname{sech}^4(c+dx)}{4ad}+\frac{if^3\tanh(c+dx)}{4ad^4}-\frac{if(e+fx)^2\tanh(c+dx)}{2ad^2}-\frac{f^2(e+fx)\operatorname{sech}(c+dx)\tanh(c+dx)}{4ad^3}+\frac{3(e+fx)^3\operatorname{sech}(c+dx)\tanh(c+dx)}{8ad}-\frac{if(e+fx)^2\operatorname{sech}^2(c+dx)\tanh(c+dx)}{4ad^2}+\frac{(e+fx)^3\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4ad}$$

output
```
9/4*I*f^3*polylog(4,I*exp(d*x+c))/a/d^4-5*f^2*(f*x+e)*arctan(exp(d*x+c))/a
/d^3+3/4*(f*x+e)^3*arctan(exp(d*x+c))/a/d-5/2*I*f^3*polylog(2,I*exp(d*x+c)
)/a/d^4-1/2*I*f*(f*x+e)^2*tanh(d*x+c)/a/d^2+9/4*I*f^2*(f*x+e)*polylog(3,-I
*exp(d*x+c))/a/d^3+1/2*I*f^3*polylog(2,-exp(2*d*x+2*c))/a/d^4-9/4*I*f^2*(f
*x+e)*polylog(3,I*exp(d*x+c))/a/d^3+9/8*I*f*(f*x+e)^2*polylog(2,I*exp(d*x+
c))/a/d^2+1/4*I*f^3*tanh(d*x+c)/a/d^4+1/4*I*(f*x+e)^3*sech(d*x+c)^4/a/d-1/
4*I*f*(f*x+e)^2*sech(d*x+c)^2*tanh(d*x+c)/a/d^2-9/4*I*f^3*polylog(4,-I*exp
(d*x+c))/a/d^4-1/4*f^3*sech(d*x+c)/a/d^4+9/8*f*(f*x+e)^2*sech(d*x+c)/a/d^2
-1/4*I*f^2*(f*x+e)*sech(d*x+c)^2/a/d^3+1/4*f*(f*x+e)^2*sech(d*x+c)^3/a/d^2
+5/2*I*f^3*polylog(2,-I*exp(d*x+c))/a/d^4-9/8*I*f*(f*x+e)^2*polylog(2,-I*e
xp(d*x+c))/a/d^2-1/2*I*f*(f*x+e)^2/a/d^2-1/4*f^2*(f*x+e)*sech(d*x+c)*tanh(
d*x+c)/a/d^3+3/8*(f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/a/d+I*f^2*(f*x+e)*ln(1+
exp(2*d*x+2*c))/a/d^3+1/4*(f*x+e)^3*sech(d*x+c)^3*tanh(d*x+c)/a/d
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2008 vs. $2(667) = 1334$.

Time = 8.83 (sec) , antiderivative size = 2008, normalized size of antiderivative = 3.01

$$\int \frac{(e+fx)^3 \operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Sech[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(-3*E^c*((d^2*e^3*x)/E^c - (4*e*f^2*x)/E^c - (e*(1 - I*E^c)*(d^2*e^2 - 4*f
^2)*x)/E^c + (3*d^2*e^2*f*x^2)/(2*E^c) - (2*f^3*x^2)/E^c + (d^2*e*f^2*x^3)
/E^c + (d^2*f^3*x^4)/(4*E^c) + ((1 - I*E^c)*f*(3*d^2*e^2 - 4*f^2)*x*Log[1
+ I*E^(-c - d*x)])/(d*E^c) + (3*d*e*(1 - I*E^c)*f^2*x^2*Log[1 + I*E^(-c -
d*x)])/E^c + (d*(1 - I*E^c)*f^3*x^3*Log[1 + I*E^(-c - d*x)])/E^c + (e*(1 -
 I*E^c)*(d^2*e^2 - 4*f^2)*Log[I + E^(c + d*x)])/(d*E^c) - ((1 - I*E^c)*f*(
3*d^2*e^2 - 4*f^2)*PolyLog[2, (-I)*E^(-c - d*x)])/(d^2*E^c) - (6*e*(1 - I*
E^c)*f^2*x*PolyLog[2, (-I)*E^(-c - d*x)])/E^c - (3*(1 - I*E^c)*f^3*x^2*Pol
yLog[2, (-I)*E^(-c - d*x)])/E^c - (6*e*(1 - I*E^c)*f^2*PolyLog[3, (-I)*E^(
-c - d*x)])/(d*E^c) - (6*(1 - I*E^c)*f^3*x*PolyLog[3, (-I)*E^(-c - d*x)])/
(d*E^c) - (6*(1 - I*E^c)*f^3*PolyLog[4, (-I)*E^(-c - d*x)])/(d^2*E^c)))/(8
*a*d^2*(I + E^c)) - (-12*d^2*e*(1 + I*E^c)*f*(3*d^2*e^2 - 28*f^2)*x + (28*
f^2 - 3*d^2*(e + f*x)^2)^2 + 12*d*(1 + I*E^c)*f^2*(9*d^2*e^2 - 28*f^2)*x*L
og[1 - I*E^(-c - d*x)] + 108*d^3*e*(1 + I*E^c)*f^3*x^2*Log[1 - I*E^(-c - d
*x)] + 36*d^3*(1 + I*E^c)*f^4*x^3*Log[1 - I*E^(-c - d*x)] + 12*d*e*(1 + I*
E^c)*f*(3*d^2*e^2 - 28*f^2)*Log[I - E^(c + d*x)] + 12*(1 + I*E^c)*f^2*(-9*
d^2*e^2 + 28*f^2)*PolyLog[2, I*E^(-c - d*x)] - 216*d^2*e*(1 + I*E^c)*f^3*x
*PolyLog[2, I*E^(-c - d*x)] - 108*d^2*(1 + I*E^c)*f^4*x^2*PolyLog[2, I*E^(
-c - d*x)] - 216*d*e*(1 + I*E^c)*f^3*PolyLog[3, I*E^(-c - d*x)] - 216*d*(1
 + I*E^c)*f^4*x*PolyLog[3, I*E^(-c - d*x)] - 216*(1 + I*E^c)*f^4*PolyLo...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6105}$$

$$\frac{\int (e+fx)^3\text{sech}^5(c+dx)dx}{a}-\frac{i\int (e+fx)^3\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)^5dx}{a}-\frac{i\int (e+fx)^3\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 4674}$$

$$\frac{-\frac{f^2\int(e+fx)\text{sech}^3(c+dx)dx}{2d^2}+\frac{3}{4}\int(e+fx)^3\text{sech}^3(c+dx)dx+\frac{f(e+fx)^2\text{sech}^3(c+dx)}{4d^2}+\frac{(e+fx)^3\tanh(c+dx)\text{sech}^3(c+dx)}{4d}}{a}-\frac{i\int(e+fx)^3\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{f^2\int(e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{2d^2}+\frac{3}{4}\int(e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx+\frac{f(e+fx)^2\text{sech}^3(c+dx)}{4d^2}+\frac{(e+fx)^3\tanh(c+dx)\text{sech}^3}{4d}}{a}\frac{i\int(e+fx)^3\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$\Big\downarrow$ 4673

$$\frac{-\frac{f^2\left(\frac{1}{2}\int(e+fx)\text{sech}(c+dx)dx+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)}{2d^2}+\frac{3}{4}\int(e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx+\frac{f(e+fx)}{}}{a}\frac{i\int(e+fx)^3\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{f^2\left(\frac{1}{2}\int(e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)}{2d^2}+\frac{3}{4}\int(e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx+\frac{f(e+}{}}{a}\frac{i\int(e+fx)^3\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$\Big\downarrow$ 4668

$$\frac{-\frac{f^2\left(\frac{1}{2}\left(-\frac{if\int\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)}{2d^2}+\frac{3}{4}\int(e+}{a}\frac{i\int(e+fx)^3\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$\Big\downarrow$ 2715

$$\frac{-\frac{f^2\left(\frac{1}{2}\left(-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{se}}{2d}\right.}{2d^2}}{a}\frac{i\int(e+fx)^3\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$\Big\downarrow$ 2838

$$\frac{\frac{3}{4}\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx-\frac{f^2\left(\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\mathrm{sech}(c+dx)}{2d^2}}{2d^2}}{a}$$

$$\frac{i\int (e+fx)^3\mathrm{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$\Big\downarrow$ 4674

$$\frac{\frac{3}{4}\left(-\frac{3f^2\int (e+fx)\mathrm{sech}(c+dx)dx}{d^2}+\frac{1}{2}\int (e+fx)^3\mathrm{sech}(c+dx)dx+\frac{3f(e+fx)^2\mathrm{sech}(c+dx)}{2d^2}+\frac{(e+fx)^3\tanh(c+dx)\mathrm{sech}(c+dx)}{2d}\right)}{}$$

$$\frac{i\int (e+fx)^3\mathrm{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3}{4}\left(-\frac{3f^2\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{d^2}+\frac{1}{2}\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{3f(e+fx)^2\mathrm{sech}(c+dx)}{2d^2}+\frac{(e+fx)^3\tanh(c+dx)\mathrm{sec}}{2d}\right.}{}$$

$$\frac{i\int (e+fx)^3\mathrm{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$\Big\downarrow$ 4668

$$\frac{\frac{3}{4}\left(-\frac{3f^2\left(-\frac{if\int \log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int \log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)}{d^2}+\frac{1}{2}\left(-\frac{3if\int (e+fx)^2\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{3if\int (e+fx)^2}{}\right.\right.}{}$$

$$\frac{i\int (e+fx)^3\mathrm{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$\Big\downarrow$ 2715

$$\frac{\frac{3}{4}\left(-\frac{3f^2\left(-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)}{d^2}+\frac{1}{2}\left(-\frac{3if\int (e+fx)^2\log\left(1-ie^{c\cdots}\right)}{d}\right.\right.}{}$$

$$\frac{i\int (e+fx)^3\mathrm{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$\Big\downarrow$ 2838

$$\frac{3}{4}\left(\frac{1}{2}\left(-\frac{3if\int(e+fx)^2\log(1-ie^{c+dx})dx}{d}+\frac{3if\int(e+fx)^2\log(1+ie^{c+dx})dx}{d}+\frac{2(e+fx)^3\arctan(e^{c+dx})}{d}\right)-\frac{3f^2\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\right.}{}\right.$$

$$\frac{i\int(e+fx)^3\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3011}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,ie^{c+}\right.}{d}\right.}{d}\right.\right.$$

$$\frac{i\int(e+fx)^3\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{5974}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,ie^{c+}\right.}{d}\right.}{d}\right.\right.$$

$$\frac{i\left(\frac{3f\int(e+fx)^2\text{sech}^4(c+dx)dx}{4d}-\frac{(e+fx)^3\text{sech}^4(c+dx)}{4d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,ie^{c+}\right.}{d}\right.}{d}\right.\right.$$

$$\frac{i\left(-\frac{(e+fx)^3\text{sech}^4(c+dx)}{4d}+\frac{3f\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^4dx}{4d}\right)}{a}$$

$$\Big\downarrow \text{4674}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+}\right.}{d}\right.}{d}\right.\right.}{i\left(\frac{3f\left(-\frac{f^2\int\operatorname{sech}^2(c+dx)dx}{3d^2}+\frac{2}{3}\int(e+fx)^2\operatorname{sech}^2(c+dx)dx+\frac{f(e+fx)\operatorname{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+dx)\operatorname{sech}^2(c+dx)}{3d}\right)}{4d}-\frac{(e+fx)^3\operatorname{sech}^4(c+d}{4d}\right.}$$

$$a$$

$\Big\downarrow$ 3042

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+}\right.}{d}\right.}{d}\right.\right.}{i\left(-\frac{(e+fx)^3\operatorname{sech}^4(c+dx)}{4d}+\frac{3f\left(-\frac{f^2\int\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{3d^2}+\frac{2}{3}\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx+\frac{f(e+fx)\operatorname{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+dx)\operatorname{sec}}{3d}\right.}{4d}\right.}$$

$$a$$

$\Big\downarrow$ 4254

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+}\right.}{d}\right.}{d}\right.\right.}{i\left(-\frac{(e+fx)^3\operatorname{sech}^4(c+dx)}{4d}+\frac{3f\left(-\frac{if^2\int 1d(-i\tanh(c+dx))}{3d^3}+\frac{2}{3}\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx+\frac{f(e+fx)\operatorname{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+dx)\operatorname{sec}}{3d}\right.}{4d}\right.}$$

$$a$$

$\Big\downarrow$ 24

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+}\right.}{d}\right.}{d}\right.\right.}{i\left(-\frac{(e+fx)^3\operatorname{sech}^4(c+dx)}{4d}+\frac{3f\left(\frac{2}{3}\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{f(e+fx)\operatorname{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+dx)\operatorname{sech}^2(c+dx)}{3d}\right)}{4d}\right.}$$

$$a$$

$$\Big\downarrow \ 4672$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+\cdots}\right.}{d}\right.}{d}\right.\right.}{i\left(-\frac{(e+fx)^3\operatorname{sech}^4(c+dx)}{4d}+\frac{3f\left(\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2if\int-i(e+fx)\tanh(c+dx)dx}{d}\right)-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{f(e+fx)\operatorname{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh\cdots}{}\right.}{4d}\right.}$$

$$a$$

$$\Big\downarrow \ 26$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+\cdots}\right.}{d}\right.}{d}\right.\right.}{i\left(\frac{3f\left(\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2f\int(e+fx)\tanh(c+dx)dx}{d}\right)-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{f(e+fx)\operatorname{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+dx)\operatorname{sech}^2(c+dx)}{3d}\right)}{4d}-\frac{(e+fx)\cdots}{}\right.}$$

$$a$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+\cdots}\right.}{d}\right.}{d}\right.\right.}{i\left(-\frac{(e+fx)^3\operatorname{sech}^4(c+dx)}{4d}+\frac{3f\left(\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2f\int-i(e+fx)\tan(ic+idx)dx}{d}\right)-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{f(e+fx)\operatorname{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(\cdots}{}\right.}{4d}\right.}$$

$$a$$

$$\Big\downarrow \ 26$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+\right.}{d}}{d}\right.\right.}{i\left(-\frac{(e+fx)^3\operatorname{sech}^4(c+dx)}{4d}+\frac{3f\left(\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\int(e+fx)\tan(ic+idx)dx}{d}\right)-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{f(e+fx)\operatorname{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+\right.}{4d}\right.}{a}$$

$\downarrow$ 4201

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+\right.}{d}}{d}\right.\right.}{i\left(-\frac{(e+fx)^3\operatorname{sech}^4(c+dx)}{4d}+\frac{3f\left(\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\int\frac{e^{2(c+dx)}(e+fx)}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}\right)-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{f(e+fx)\operatorname{sech}^2(c+dx)}{3d^2}+\right.}{4d}\right.}{a}$$

$\downarrow$ 2620

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+\right.}{d}}{d}\right.\right.}{i\left(-\frac{(e+fx)^3\operatorname{sech}^4(c+dx)}{4d}+\frac{3f\left(\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int\log\left(1+e^{2(c+dx)}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)-\frac{f^2\tanh(\right.}{3d^3}}{4d}\right.}{a}$$

$\downarrow$ 2715

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,ie^{c+}\right.}{d}\right.}{d}\right.\right.}{i\left(-\frac{(e+fx)^3\mathrm{sech}^4(c+dx)}{4d}+\frac{3f\left(\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int e^{-2(c+dx)}\log\left(1+e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}\right)-\frac{i(e+f}{2f}\right.}{d}\right.\right.}{4d}\right.}$$

$$a$$

$\downarrow$ 2838

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,ie^{c+}\right.}{d}\right.}{d}\right.\right.}{i\left(-\frac{(e+fx)^3\mathrm{sech}^4(c+dx)}{4d}+\frac{3f\left(-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+}{2}\right.}{d}\right.\right.}{4d}\right.}$$

$$a$$

$\downarrow$ 7163

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\left(\frac{(e+fx)\,\mathrm{PolyLog}\left(3,-ie^{c+dx}\right)}{d}-\frac{f\int\mathrm{PolyLog}\left(3,-ie^{c+dx}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\,\mathrm{PolyLog}\left(3,ie^{c+dx}\right.}{d}\right.}{}\right.}{}\right.\right.}{i\left(-\frac{(e+fx)^3\mathrm{sech}^4(c+dx)}{4d}+\frac{3f\left(-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+}{2}\right.}{d}\right.\right.}{4d}\right.}$$

$$a$$

input
```
Int[((e + f*x)^3*Sech[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1908 vs. $2(589) = 1178$.

Time = 1.41 (sec) , antiderivative size = 1909, normalized size of antiderivative = 2.86

Expression too large to display

input
```
int((f*x+e)^3*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

output
```
1/4*(18*I*d^3*e^2*f*x*exp(2*d*x+2*c)-36*I*d^2*e*f^2*x*exp(4*d*x+4*c)-44*I*
d^2*e*f^2*x*exp(2*d*x+2*c)-18*I*d^3*e^2*f*x*exp(4*d*x+4*c)-18*I*d^3*e*f^2*
x^2*exp(4*d*x+4*c)+18*I*d^3*e*f^2*x^2*exp(2*d*x+2*c)+9*d^3*e*f^2*x^2*exp(5
*d*x+5*c)-6*I*d^3*e^3*exp(4*d*x+4*c)+3*d^3*f^3*x^3*exp(5*d*x+5*c)+2*d^3*f^
3*x^3*exp(3*d*x+3*c)+6*I*d^3*e^3*exp(2*d*x+2*c)-2*d*e*f^2*exp(5*d*x+5*c)-2
*d*f^3*x*exp(5*d*x+5*c)+9*d^2*f^3*x^2*exp(5*d*x+5*c)+9*d^2*e^2*f*exp(5*d*x
+5*c)+18*d^2*e*f^2*x*exp(5*d*x+5*c)+3*d^3*e^3*exp(5*d*x+5*c)+4*I*f^3*exp(2
*d*x+2*c)+2*d^3*e^3*exp(3*d*x+3*c)+2*I*f^3*exp(4*d*x+4*c)-8*I*d^2*e*f^2*x+
6*d^3*e^2*f*x*exp(3*d*x+3*c)+6*d^3*e*f^2*x^2*exp(3*d*x+3*c)+9*d^3*e^2*f*x*
exp(5*d*x+5*c)-22*I*d^2*f^3*x^2*exp(2*d*x+2*c)-22*I*d^2*e^2*f*exp(2*d*x+2*
c)+6*I*d^3*f^3*x^3*exp(2*d*x+2*c)-6*I*d^3*f^3*x^3*exp(4*d*x+4*c)-18*I*d^2*
f^3*x^2*exp(4*d*x+4*c)-18*I*d^2*e^2*f*exp(4*d*x+4*c)-2*f^3*exp(5*d*x+5*c)-
4*I*d^2*f^3*x^2-4*I*d^2*e^2*f+2*I*f^3-4*d*e*f^2*exp(3*d*x+3*c)+16*d^2*e*f^
2*x*exp(3*d*x+3*c)-d^2*e^2*f*exp(d*x+c)-2*d*f^3*x*exp(d*x+c)-2*d*e*f^2*exp
(d*x+c)-2*d^2*e*f^2*x*exp(d*x+c)+9*d^3*e*f^2*x^2*exp(d*x+c)+8*d^2*f^3*x^2*
exp(3*d*x+3*c)+8*d^2*e^2*f*exp(3*d*x+3*c)+3*d^3*e^3*exp(d*x+c)-4*f^3*exp(3
*d*x+3*c)-2*f^3*exp(d*x+c)-d^2*f^3*x^2*exp(d*x+c)+3*d^3*f^3*x^3*exp(d*x+c)
-4*d*f^3*x*exp(3*d*x+3*c)+9*d^3*e^2*f*x*exp(d*x+c))/(exp(d*x+c)+I)^2/(exp(
d*x+c)-I)^4/d^4/a+5/a/d^4*f^3*c*arctan(exp(d*x+c))-3/4/a/d^4*f^3*c^3*arcta
n(exp(d*x+c))-5/a/d^3*e*f^2*arctan(exp(d*x+c))+3/4/a/d*e^3*arctan(exp(d...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3854 vs. 2(560) = 1120.

Time = 0.14 (sec) , antiderivative size = 3854, normalized size of antiderivative = 5.78

$$\int \frac{(e+fx)^3 \operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^3 \operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e^3\operatorname{sech}^3(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{f^3x^3\operatorname{sech}^3(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{3ef^2x^2\operatorname{sech}^3(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{3e^2fx\operatorname{sech}^3(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)**3*sech(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e**3*sech(c + d*x)**3/(sinh(c + d*x) - I), x) + Integral(f**3
*x**3*sech(c + d*x)**3/(sinh(c + d*x) - I), x) + Integral(3*e*f**2*x**2*se
ch(c + d*x)**3/(sinh(c + d*x) - I), x) + Integral(3*e**2*f*x*sech(c + d*x)
**3/(sinh(c + d*x) - I), x))/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)^3*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e+fx)^3\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\,\text{sech}\,(dx+c)^3}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sech(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^3}{\cosh\,(c+d\,x)^3\,\left(a+a\sinh\,(c+d\,x)\,1\mathrm{i}\right)}\,dx$$

input
```
int((e + f*x)^3/(cosh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)^3/(cosh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$=\frac{\left(\int\frac{\text{sech}(dx+c)^3}{\sinh(dx+c)i+1}dx\right)e^3+\left(\int\frac{\text{sech}(dx+c)^3x^3}{\sinh(dx+c)i+1}dx\right)f^3+3\left(\int\frac{\text{sech}(dx+c)^3x^2}{\sinh(dx+c)i+1}dx\right)e\,f^2+3\left(\int\frac{\text{sech}(dx+c)^3x}{\sinh(dx+c)i+1}dx\right)e^2f}{a}$$

input
```
int((f*x+e)^3*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

output
```
(int(sech(c + d*x)**3/(sinh(c + d*x)*i + 1),x)*e**3 + int((sech(c + d*x)**
3*x**3)/(sinh(c + d*x)*i + 1),x)*f**3 + 3*int((sech(c + d*x)**3*x**2)/(sin
h(c + d*x)*i + 1),x)*e*f**2 + 3*int((sech(c + d*x)**3*x)/(sinh(c + d*x)*i
+ 1),x)*e**2*f)/a
```

3.284 $\int \frac{(e+fx)^2\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 423

$$\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{3(e+fx)^2\arctan\left(e^{c+dx}\right)}{4ad} - \frac{5f^2\arctan(\sinh(c+dx))}{6ad^3} + \frac{if^2\log(\cosh(c+dx))}{3ad^3} - \frac{3if(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{4ad^2} + \frac{3if(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{4ad^2} + \frac{3if^2\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{4ad^3} - \frac{3if^2\,\text{PolyLog}\left(3,ie^{c+dx}\right)}{4ad^3} + \frac{3f(e+fx)\text{sech}(c+dx)}{4ad^2} - \frac{if^2\text{sech}^2(c+dx)}{12ad^3} + \frac{f(e+fx)\text{sech}^3(c+dx)}{6ad^2} + \frac{i(e+fx)^2\text{sech}^4(c+dx)}{4ad} - \frac{if(e+fx)\tanh(c+dx)}{3ad^2} - \frac{f^2\text{sech}(c+dx)\tanh(c+dx)}{12ad^3} + \frac{3(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{8ad} - \frac{if(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{6ad^2} + \frac{(e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)}{4ad}$$

output
```
3/4*(f*x+e)^2*arctan(exp(d*x+c))/a/d-5/6*f^2*arctan(sinh(d*x+c))/a/d^3+3/4
*I*f^2*polylog(3,-I*exp(d*x+c))/a/d^3+1/4*I*(f*x+e)^2*sech(d*x+c)^4/a/d-3/
4*I*f^2*polylog(3,I*exp(d*x+c))/a/d^3-3/4*I*f*(f*x+e)*polylog(2,-I*exp(d*x
+c))/a/d^2-1/3*I*f*(f*x+e)*tanh(d*x+c)/a/d^2+3/4*f*(f*x+e)*sech(d*x+c)/a/d
^2-1/6*I*f*(f*x+e)*sech(d*x+c)^2*tanh(d*x+c)/a/d^2+1/6*f*(f*x+e)*sech(d*x+
c)^3/a/d^2-1/12*I*f^2*sech(d*x+c)^2/a/d^3+1/3*I*f^2*ln(cosh(d*x+c))/a/d^3-
1/12*f^2*sech(d*x+c)*tanh(d*x+c)/a/d^3+3/8*(f*x+e)^2*sech(d*x+c)*tanh(d*x+
c)/a/d+3/4*I*f*(f*x+e)*polylog(2,I*exp(d*x+c))/a/d^2+1/4*(f*x+e)^2*sech(d*
x+c)^3*tanh(d*x+c)/a/d
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1363 vs. 2(423) = 846.

Time = 7.81 (sec) , antiderivative size = 1363, normalized size of antiderivative = 3.22

$$\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Sech[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
-1/8*(E^c*((3*d^2*e^2*x)/E^c - (4*f^2*x)/E^c - ((1 - I*E^c)*(3*d^2*e^2 - 4
*f^2)*x)/E^c + (3*d^2*e*f*x^2)/E^c + (d^2*f^2*x^3)/E^c + (6*d*e*(1 - I*E^c
)*f*x*Log[1 + I*E^(-c - d*x)])/E^c + (3*d*(1 - I*E^c)*f^2*x^2*Log[1 + I*E^
(-c - d*x)])/E^c + ((1 - I*E^c)*(3*d^2*e^2 - 4*f^2)*Log[I + E^(c + d*x)])/
(d*E^c) - (6*e*(1 - I*E^c)*f*PolyLog[2, (-I)*E^(-c - d*x)])/E^c - (6*(1 -
I*E^c)*f^2*x*PolyLog[2, (-I)*E^(-c - d*x)])/E^c - (6*(1 - I*E^c)*f^2*PolyL
og[3, (-I)*E^(-c - d*x)])/(d*E^c)))/(a*d^2*(I + E^c)) - (9*d^2*e^2*x - 28*
f^2*x - (1 + I*E^c)*(9*d^2*e^2 - 28*f^2)*x + 9*d^2*e*f*x^2 + 3*d^2*f^2*x^3
 + 18*d*e*(1 + I*E^c)*f*x*Log[1 - I*E^(-c - d*x)] + 9*d*(1 + I*E^c)*f^2*x^
2*Log[1 - I*E^(-c - d*x)] + ((1 + I*E^c)*(9*d^2*e^2 - 28*f^2)*Log[I - E^(c
 + d*x)])/d - 18*e*(1 + I*E^c)*f*PolyLog[2, I*E^(-c - d*x)] - 18*(1 + I*E^
c)*f^2*x*PolyLog[2, I*E^(-c - d*x)] - (18*(1 + I*E^c)*f^2*PolyLog[3, I*E^(
-c - d*x)])/d)/(24*a*d^2*(-I + E^c)) + ((3*e^2*x*Cosh[c])/(4*a) + (3*e^2*x
*Sinh[c])/(4*a))/(1 + Cosh[2*c] + Sinh[2*c]) + ((3*e*f*x^2*Cosh[c])/(4*a)
+ (3*e*f*x^2*Sinh[c])/(4*a))/(1 + Cosh[2*c] + Sinh[2*c]) + ((f^2*x^3*Cosh[
c])/(4*a) + (f^2*x^3*Sinh[c])/(4*a))/(1 + Cosh[2*c] + Sinh[2*c]) - ((I/8)*
(e^2 + 2*e*f*x + f^2*x^2))/(a*d*(Cosh[c/2 + (d*x)/2] - I*Sinh[c/2 + (d*x)/
2])^2) + ((I/2)*(e*f*Sinh[(d*x)/2] + f^2*x*Sinh[(d*x)/2]))/(a*d^2*(Cosh[c/
2] - I*Sinh[c/2])*(Cosh[c/2 + (d*x)/2] - I*Sinh[c/2 + (d*x)/2])) + ((I/8)*
(e^2 + 2*e*f*x + f^2*x^2))/(a*d*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 + (d*...
```

Rubi [A] (verified)

Time = 3.03 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.95, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.742, Rules used = {6105, 3042, 4674, 3042, 4255, 3042, 4257, 4674, 3042, 4257, 4668, 3011, 2720, 5974, 3042, 4673, 3042, 4672, 26, 3042, 26, 3956, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6105}$$

$$\frac{\int (e+fx)^2\text{sech}^5(c+dx)dx}{a}-\frac{i\int (e+fx)^2\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^5dx}{a}-\frac{i\int (e+fx)^2\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 4674}$$

$$\frac{-\frac{f^2\int \text{sech}^3(c+dx)dx}{6d^2}+\frac{3}{4}\int (e+fx)^2\text{sech}^3(c+dx)dx+\frac{f(e+fx)\text{sech}^3(c+dx)}{6d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}^3(c+dx)}{4d}}{a}-\frac{i\int (e+fx)^2\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{f^2\int \csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{6d^2}+\frac{3}{4}\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx+\frac{f(e+fx)\text{sech}^3(c+dx)}{6d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}^3(c+dx)}{4d}}{a} \cdots \frac{i\int (e+fx)^2\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{-\frac{f^2\left(\frac{1}{2}\int \text{sech}(c+dx)dx+\frac{\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)}{6d^2}+\frac{3}{4}\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx+\frac{f(e+fx)\text{sech}^3(c+dx)}{6d^2}+\frac{(e+fx)^2\text{ta}\cdots}{}}{a} \cdots \frac{i\int (e+fx)^2\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{f^2\left(\frac{\tanh(c+dx)\text{sech}(c+dx)}{2d}+\frac{1}{2}\int \csc\left(ic+idx+\frac{\pi}{2}\right)dx\right)}{6d^2}+\frac{3}{4}\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx+\frac{f(e+fx)\text{sech}^3(c+dx)}{6d^2}+\frac{(e+fx)}{}}{a}$$

$$\frac{i\int (e+fx)^2\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{3}{4}\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx-\frac{f^2\left(\frac{\arctan(\sinh(c+dx))}{2d}+\frac{\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)}{6d^2}+\frac{f(e+fx)\text{sech}^3(c+dx)}{6d^2}+\frac{(e+fx)^2\tanh(}{}}{a}$$

$$\frac{i\int (e+fx)^2\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4674}$$

$$\frac{\frac{3}{4}\left(-\frac{f^2\int \text{sech}(c+dx)dx}{d^2}+\frac{1}{2}\int (e+fx)^2\text{sech}(c+dx)dx+\frac{f(e+fx)\text{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)-\frac{f^2\left(\text{arct}\right.}{}}{a}$$

$$\frac{i\int (e+fx)^2\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{4}\left(-\frac{f^2\int \csc\left(ic+idx+\frac{\pi}{2}\right)dx}{d^2}+\frac{1}{2}\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{f(e+fx)\text{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)}{a}$$

$$\frac{i\int (e+fx)^2\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f^2\arctan(\sinh(c+dx))}{d^3}+\frac{f(e+fx)\text{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)-}{a}$$

$$\frac{i\int (e+fx)^2\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4668}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(-\frac{2if\int(e+fx)\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{2if\int(e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}\right)-\frac{f^2\arctan(\sinh(c+dx))}{d^3}+\frac{f(e+f}{}\right.}{}$$

$$\frac{i\int(e+fx)^2\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3011}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}+\frac{2(e+f}{}\right.\right.}{}$$

$$\frac{i\int(e+fx)^2\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLo}}{d}\right.}{d}\right.\right.}{}$$

$$\frac{i\int(e+fx)^2\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{5974}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLo}}{d}\right.}{d}\right.\right.}{}$$

$$\frac{i\left(\frac{f\int(e+fx)\text{sech}^4(c+dx)dx}{2d}-\frac{(e+fx)^2\text{sech}^4(c+dx)}{4d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLo}}{d}\right.}{d}\right.\right.}{}$$

$$\frac{i\left(-\frac{(e+fx)^2\text{sech}^4(c+dx)}{4d}+\frac{f\int(e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^4dx}{2d}\right)}{a}$$

$$\Big\downarrow \; 4673$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLo}}{d}\right.}{d}\right.\right.}{a}$$

$$\frac{i\left(\frac{f\left(\frac{2}{3}\int(e+fx)\operatorname{sech}^2(c+dx)dx+\frac{f\operatorname{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\operatorname{sech}^2(c+dx)}{3d}\right)}{2d}-\frac{(e+fx)^2\operatorname{sech}^4(c+dx)}{4d}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLo}}{d}\right.}{d}\right.\right.}{a}$$

$$\frac{i\left(-\frac{(e+fx)^2\operatorname{sech}^4(c+dx)}{4d}+\frac{f\left(\frac{2}{3}\int(e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx+\frac{f\operatorname{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\operatorname{sech}^2(c+dx)}{3d}\right)}{2d}\right)}{a}$$

$$\Big\downarrow \; 4672$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLo}}{d}\right.}{d}\right.\right.}{a}$$

$$\frac{i\left(-\frac{(e+fx)^2\operatorname{sech}^4(c+dx)}{4d}+\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{if\int -i\tanh(c+dx)dx}{d}\right)+\frac{f\operatorname{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\operatorname{sech}^2(c+dx)}{3d}\right)}{2d}\right)}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLo}}{d}\right.}{d}\right.\right.}{i\left(\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int\tanh(c+dx)dx}{d}\right)+\frac{f\operatorname{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\operatorname{sech}^2(c+dx)}{3d}\right)}{2d}-\frac{(e+fx)^2\operatorname{sech}^4(c+dx)}{4d}\right)}$$

$$a$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLo}}{d}\right.}{d}\right.\right.}{i\left(-\frac{(e+fx)^2\operatorname{sech}^4(c+dx)}{4d}+\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int -i\tan(ic+idx)dx}{d}\right)+\frac{f\operatorname{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\operatorname{sech}^2(c+dx)}{3d}\right)}{2d}\right)}$$

$$a$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLo}}{d}\right.}{d}\right.\right.}{i\left(-\frac{(e+fx)^2\operatorname{sech}^4(c+dx)}{4d}+\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}+\frac{if\int\tan(ic+idx)dx}{d}\right)+\frac{f\operatorname{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\operatorname{sech}^2(c+dx)}{3d}\right)}{2d}\right)}$$

$$a$$

$$\Big\downarrow \text{3956}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLo}}{d}\right.}{d}\right.\right.}{i\left(\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)+\frac{f\operatorname{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\operatorname{sech}^2(c+dx)}{3d}\right)}{2d}-\frac{(e+fx)^2\operatorname{sech}^4(c+dx)}{4d}\right)}$$

$$a$$

$$\Big\downarrow \; 7143$$

$$\frac{-\frac{f^2\left(\frac{\arctan(\sinh(c+dx))}{2d}+\frac{\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)}{6d^2}+\frac{3}{4}\left(-\frac{f^2\arctan(\sinh(c+dx))}{d^3}+\frac{1}{2}\left(\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}+\frac{2if\left(\frac{f\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}\cdots\right.}{\cdots}\right.\right.}{}$$

$$\frac{i\left(\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)+\frac{f\text{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^2(c+dx)}{3d}\right)}{2d}-\frac{(e+fx)^2\text{sech}^4(c+dx)}{4d}\right)}{a}$$

input
```
Int[((e + f*x)^2*Sech[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((f*(e + f*x)*Sech[c + d*x]^3)/(6*d^2) + ((e + f*x)^2*Sech[c + d*x]^3*Tanh
[c + d*x])/(4*d) - (f^2*(ArcTan[Sinh[c + d*x]]/(2*d) + (Sech[c + d*x]*Tanh
[c + d*x])/(2*d)))/(6*d^2) + (3*(-((f^2*ArcTan[Sinh[c + d*x]])/d^3) + ((2*
(e + f*x)^2*ArcTan[E^(c + d*x)])/d + ((2*I)*f*(-(((e + f*x)*PolyLog[2, (-I
)*E^(c + d*x)])/d) + (f*PolyLog[3, (-I)*E^(c + d*x)])/d^2))/d - ((2*I)*f*(
-(((e + f*x)*PolyLog[2, I*E^(c + d*x)])/d) + (f*PolyLog[3, I*E^(c + d*x)])
/d^2))/d)/2 + (f*(e + f*x)*Sech[c + d*x])/d^2 + ((e + f*x)^2*Sech[c + d*x]
*Tanh[c + d*x])/(2*d)))/4)/a - (I*(-1/4*((e + f*x)^2*Sech[c + d*x]^4)/d +
(f*((f*Sech[c + d*x]^2)/(6*d^2) + ((e + f*x)*Sech[c + d*x]^2*Tanh[c + d*x]
)/(3*d) + (2*(-((f*Log[Cosh[c + d*x]])/d^2) + ((e + f*x)*Tanh[c + d*x])/d)
)/3))/(2*d)))/a
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6105
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Sech[
c + d*x]^(n + 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)
*Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] &&
EqQ[a^2 + b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 960 vs. $2(373) = 746$.

Time = 140.19 (sec) , antiderivative size = 961, normalized size of antiderivative = 2.27

method	result
risch	$\frac{18d^2efx\,e^{dx+c}-2f^2e^{dx+c}+9d^2f^2x^2e^{5dx+5c}+16d\,f^2x\,e^{3dx+3c}+16def\,e^{3dx+3c}+18d^2efx\,e^{5dx+5c}+12d^2efx\,e^{3dx+3c}-4f^2e^{3dx+3c}\ldots}{}$

input
```
int((f*x+e)^2*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/12*(18*d^2*e*f*x*exp(d*x+c)-2*f^2*exp(d*x+c)+9*d^2*f^2*x^2*exp(5*d*x+5*c
)-36*I*d^2*e*f*x*exp(4*d*x+4*c)+36*I*d^2*e*f*x*exp(2*d*x+2*c)+16*d*f^2*x*e
xp(3*d*x+3*c)+16*d*e*f*exp(3*d*x+3*c)-36*I*d*f^2*x*exp(4*d*x+4*c)-36*I*d*e
*f*exp(4*d*x+4*c)+18*d^2*e*f*x*exp(5*d*x+5*c)-18*I*d^2*f^2*x^2*exp(4*d*x+4
*c)+12*d^2*e*f*x*exp(3*d*x+3*c)-44*I*d*f^2*x*exp(2*d*x+2*c)-44*I*d*e*f*exp
(2*d*x+2*c)+18*I*d^2*f^2*x^2*exp(2*d*x+2*c)-4*f^2*exp(3*d*x+3*c)-8*I*d*e*f
-8*I*d*f^2*x+9*d^2*x^2*f^2*exp(d*x+c)+18*d*e*f*exp(5*d*x+5*c)+18*d*f^2*x*e
xp(5*d*x+5*c)+6*d^2*f^2*x^2*exp(3*d*x+3*c)-18*I*d^2*e^2*exp(4*d*x+4*c)+18*
I*d^2*e^2*exp(2*d*x+2*c)-2*exp(d*x+c)*d*f^2*x-2*exp(d*x+c)*d*e*f+9*d^2*e^2
*exp(d*x+c)+6*d^2*e^2*exp(3*d*x+3*c)-2*f^2*exp(5*d*x+5*c)+9*d^2*e^2*exp(5*
d*x+5*c))/(exp(d*x+c)+I)^2/(exp(d*x+c)-I)^4/d^3/a-5/3/a/d^3*f^2*arctan(exp
(d*x+c))-3/8*I/a/d*ln(1+I*exp(d*x+c))*f^2*x^2+3/8*I/a/d^3*ln(1+I*exp(d*x+c
))*c^2*f^2-3/8*I/a/d^3*ln(1-I*exp(d*x+c))*c^2*f^2+3/4*I*f^2*polylog(3,-I*e
xp(d*x+c))/a/d^3+1/3*I/a/d^3*f^2*ln(1+exp(2*d*x+2*c))-3/4*I*f^2*polylog(3,
I*exp(d*x+c))/a/d^3-3/4*I/a/d^2*e*f*polylog(2,-I*exp(d*x+c))+3/4*I/a/d*ln(
1-I*exp(d*x+c))*e*f*x-2/3*I/a/d^3*f^2*ln(exp(d*x+c))-3/2/a/d^2*c*e*f*arcta
n(exp(d*x+c))+3/4*I/a/d^2*polylog(2,I*exp(d*x+c))*f^2*x-3/4*I/a/d*ln(1+I*e
xp(d*x+c))*e*f*x+3/4*I/a/d^2*e*f*polylog(2,I*exp(d*x+c))+3/4/a/d*e^2*arcta
n(exp(d*x+c))+3/4/a/d^3*c^2*f^2*arctan(exp(d*x+c))+3/8*I/a/d*ln(1-I*exp(d*
x+c))*f^2*x^2-3/4*I/a/d^2*ln(1+I*exp(d*x+c))*c*e*f+3/4*I/a/d^2*ln(1-I*e...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2066 vs. $2(358) = 716$.

Time = 0.13 (sec) , antiderivative size = 2066, normalized size of antiderivative = 4.88

$$\int \frac{(e+fx)^2 \text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/24*(-16*I*d*e*f + 16*I*c*f^2 - 18*(I*d*f^2*x + I*d*e*f + (-I*d*f^2*x - I
*d*e*f)*e^(6*d*x + 6*c) - 2*(d*f^2*x + d*e*f)*e^(5*d*x + 5*c) + (-I*d*f^2*
x - I*d*e*f)*e^(4*d*x + 4*c) - 4*(d*f^2*x + d*e*f)*e^(3*d*x + 3*c) + (I*d*
f^2*x + I*d*e*f)*e^(2*d*x + 2*c) - 2*(d*f^2*x + d*e*f)*e^(d*x + c))*dilog(
I*e^(d*x + c)) - 18*(-I*d*f^2*x - I*d*e*f + (I*d*f^2*x + I*d*e*f)*e^(6*d*x
 + 6*c) + 2*(d*f^2*x + d*e*f)*e^(5*d*x + 5*c) + (I*d*f^2*x + I*d*e*f)*e^(4
*d*x + 4*c) + 4*(d*f^2*x + d*e*f)*e^(3*d*x + 3*c) + (-I*d*f^2*x - I*d*e*f)
*e^(2*d*x + 2*c) + 2*(d*f^2*x + d*e*f)*e^(d*x + c))*dilog(-I*e^(d*x + c))
- 16*(I*d*f^2*x + I*c*f^2)*e^(6*d*x + 6*c) + 2*(9*d^2*f^2*x^2 + 9*d^2*e^2
+ 18*d*e*f - 2*(8*c + 1)*f^2 + 2*(9*d^2*e*f + d*f^2)*x)*e^(5*d*x + 5*c) -
4*(9*I*d^2*f^2*x^2 + 9*I*d^2*e^2 + 18*I*d*e*f + 4*I*c*f^2 + 2*(9*I*d^2*e*f
 + 11*I*d*f^2)*x)*e^(4*d*x + 4*c) + 4*(3*d^2*f^2*x^2 + 3*d^2*e^2 + 8*d*e*f
 - 2*(8*c + 1)*f^2 + 2*(3*d^2*e*f - 4*d*f^2)*x)*e^(3*d*x + 3*c) - 4*(-9*I*
d^2*f^2*x^2 - 9*I*d^2*e^2 + 22*I*d*e*f - 4*I*c*f^2 + 18*(-I*d^2*e*f + I*d*
f^2)*x)*e^(2*d*x + 2*c) + 2*(9*d^2*f^2*x^2 + 9*d^2*e^2 - 2*d*e*f - 2*(8*c
+ 1)*f^2 + 18*(d^2*e*f - d*f^2)*x)*e^(d*x + c) - 3*(3*I*d^2*e^2 - 6*I*c*d*
e*f + (3*I*c^2 - 4*I)*f^2 + (-3*I*d^2*e^2 + 6*I*c*d*e*f + (-3*I*c^2 + 4*I)
*f^2)*e^(6*d*x + 6*c) - 2*(3*d^2*e^2 - 6*c*d*e*f + (3*c^2 - 4)*f^2)*e^(5*d
*x + 5*c) + (-3*I*d^2*e^2 + 6*I*c*d*e*f + (-3*I*c^2 + 4*I)*f^2)*e^(4*d*x +
 4*c) - 4*(3*d^2*e^2 - 6*c*d*e*f + (3*c^2 - 4)*f^2)*e^(3*d*x + 3*c) + (...
```

Sympy [F]

$$\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e^2\,\text{sech}^3\,(c+dx)}{\sinh\,(c+dx)-i}\,dx+\int \frac{f^2x^2\,\text{sech}^3\,(c+dx)}{\sinh\,(c+dx)-i}\,dx+\int \frac{2efx\,\text{sech}^3\,(c+dx)}{\sinh\,(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)**2*sech(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e**2*sech(c + d*x)**3/(sinh(c + d*x) - I), x) + Integral(f**2
*x**2*sech(c + d*x)**3/(sinh(c + d*x) - I), x) + Integral(2*e*f*x*sech(c +
 d*x)**3/(sinh(c + d*x) - I), x))/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\,\text{sech}\,(dx+c)^3}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sech(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^2}{\cosh\,(c+d\,x)^3\,\left(a+a\sinh\,(c+d\,x)\,1\text{i}\right)}\,dx$$

input
```
int((e + f*x)^2/(cosh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)^2/(cosh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(\int \frac{\text{sech}(dx+c)^3}{\sinh(dx+c)i+1}dx\right)e^2+\left(\int \frac{\text{sech}(dx+c)^3x^2}{\sinh(dx+c)i+1}dx\right)f^2+2\left(\int \frac{\text{sech}(dx+c)^3x}{\sinh(dx+c)i+1}dx\right)ef}{a}$$

input
```
int((f*x+e)^2*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

output
```
(int(sech(c + d*x)**3/(sinh(c + d*x)*i + 1),x)*e**2 + int((sech(c + d*x)**
3*x**2)/(sinh(c + d*x)*i + 1),x)*f**2 + 2*int((sech(c + d*x)**3*x)/(sinh(c
 + d*x)*i + 1),x)*e*f)/a
```

3.285 $\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 233

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{3(e+fx)\arctan\left(e^{c+dx}\right)}{4ad} - \frac{3if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{8ad^2} + \frac{3if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{8ad^2} + \frac{3f\text{sech}(c+dx)}{8ad^2} + \frac{f\text{sech}^3(c+dx)}{12ad^2} + \frac{i(e+fx)\text{sech}^4(c+dx)}{4ad} - \frac{if\tanh(c+dx)}{4ad^2} + \frac{3(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{8ad} + \frac{(e+fx)\text{sech}^3(c+dx)\tanh(c+dx)}{4ad} + \frac{if\tanh^3(c+dx)}{12ad^2}$$

output
```
3/4*(f*x+e)*arctan(exp(d*x+c))/a/d-3/8*I*f*polylog(2,-I*exp(d*x+c))/a/d^2+
3/8*I*f*polylog(2,I*exp(d*x+c))/a/d^2+3/8*f*sech(d*x+c)/a/d^2+1/12*f*sech(
d*x+c)^3/a/d^2+1/4*I*(f*x+e)*sech(d*x+c)^4/a/d-1/4*I*f*tanh(d*x+c)/a/d^2+3
/8*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/a/d+1/4*(f*x+e)*sech(d*x+c)^3*tanh(d*x+
c)/a/d+1/12*I*f*tanh(d*x+c)^3/a/d^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 617 vs. 2(233) = 466.

Time = 4.20 (sec) , antiderivative size = 617, normalized size of antiderivative = 2.65

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{2(f+6id(e+fx)) + \frac{6id(e+fx)}{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^2} - 9(c+dx)(cf-d(2e+fx))\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\,\text{s}\right.}{}$$

input
```
Integrate[((e + f*x)*Sech[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(2*(f + (6*I)*d*(e + f*x)) + ((6*I)*d*(e + f*x))/(Cosh[(c + d*x)/2] + I*Si
nh[(c + d*x)/2])^2 - 9*(c + d*x)*(c*f - d*(2*e + f*x))*(Cosh[(c + d*x)/2]
+ I*Sinh[(c + d*x)/2])^2 - (9 - 9*I)*((d^2*f*x^2)/2 + d*e*(c + d*x) - (1 -
 I)*(d*e - c*f)*(c + d*x) + (1 - I)*f*(c + d*x)*Log[1 + I*E^(-c - d*x)] +
(1 - I)*(d*e - c*f)*Log[I + E^(c + d*x)] - (1 - I)*f*PolyLog[2, (-I)*E^(-c
 - d*x)])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2 - (9 + 9*I)*((d^2*f*
x^2)/2 + d*e*(c + d*x) - (1 + I)*(d*e - c*f)*(c + d*x) + (1 + I)*f*(c + d*
x)*Log[1 - I*E^(-c - d*x)] + (1 + I)*(d*e - c*f)*Log[I - E^(c + d*x)] - (1
 + I)*f*PolyLog[2, I*E^(-c - d*x)])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/
2])^2 - ((6*I)*d*(e + f*x)*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2)/(C
osh[(c + d*x)/2] - I*Sinh[(c + d*x)/2])^2 - ((4*I)*f*Sinh[(c + d*x)/2])/(C
osh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]) + ((12*I)*f*(Cosh[(c + d*x)/2] + I
*Sinh[(c + d*x)/2])^2*Sinh[(c + d*x)/2])/(Cosh[(c + d*x)/2] - I*Sinh[(c +
d*x)/2]) + 28*f*Sinh[(c + d*x)/2]*((-I)*Cosh[(c + d*x)/2] + Sinh[(c + d*x)
/2]))/(48*d^2*(a + I*a*Sinh[c + d*x]))
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.94, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {6105, 3042, 4673, 3042, 4673, 3042, 4668, 2715, 2838, 5974, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6105}$$

$$\frac{\int (e+fx)\text{sech}^5(c+dx)dx}{a}-\frac{i\int (e+fx)\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^5dx}{a}-\frac{i\int (e+fx)\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4673}$$

$$\frac{\frac{3}{4}\int (e+fx)\text{sech}^3(c+dx)dx+\frac{f\text{sech}^3(c+dx)}{12d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^3(c+dx)}{4d}}{a}-\frac{i\int (e+fx)\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{4}\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx+\frac{f\text{sech}^3(c+dx)}{12d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^3(c+dx)}{4d}}{a}-\frac{i\int (e+fx)\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4673}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\int (e+fx)\text{sech}(c+dx)dx+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)+\frac{f\text{sech}^3(c+dx)}{12d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}}{4d}}{a}$$
$$\frac{i\int (e+fx)\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)+\frac{f\text{sech}^3(c+dx)}{12d^2}+\frac{(e+fx)\tanh(c+}{4}}{a}$$
$$\frac{i\int (e+fx)\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4668}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(-\frac{if\int \log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int \log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)}{a}$$

$$\frac{i\int(e+fx)\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)}{}\right)}{a}$$

$$\frac{i\int(e+fx)\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)}{a}$$

$$\frac{i\int(e+fx)\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 5974}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)}{a}$$

$$\frac{i\left(\frac{f\int \text{sech}^4(c+dx)dx}{4d}-\frac{(e+fx)\text{sech}^4(c+dx)}{4d}\right)}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)}{a}$$

$$\frac{i\left(-\frac{(e+fx)\text{sech}^4(c+dx)}{4d}+\frac{f\int \csc\left(ic+idx+\frac{\pi}{2}\right)^4dx}{4d}\right)}{a}$$

$$\Big\downarrow \text{ 4254}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)}{a}$$

$$\frac{i\left(-\frac{(e+fx)\text{sech}^4(c+dx)}{4d}+\frac{if\int\left(1-\tanh^2(c+dx)\right)d(-i\tanh(c+dx))}{4d^2}\right)}{a}$$

$\Big\downarrow$ 2009

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\operatorname{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}\right)}{a}$$
$$\frac{i\left(-\frac{(e+fx)\operatorname{sech}^4(c+dx)}{4d}+\frac{if\left(\frac{1}{3}i\tanh^3(c+dx)-i\tanh(c+dx)\right)}{4d^2}\right)}{a}$$

input
```
Int[((e + f*x)*Sech[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((f*Sech[c + d*x]^3)/(12*d^2) + ((e + f*x)*Sech[c + d*x]^3*Tanh[c + d*x])/
(4*d) + (3*(((2*(e + f*x)*ArcTan[E^(c + d*x)])/d - (I*f*PolyLog[2, (-I)*E^
(c + d*x)])/d^2 + (I*f*PolyLog[2, I*E^(c + d*x)])/d^2)/2 + (f*Sech[c + d*x
])/(2*d^2) + ((e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*d)))/4)/a - (I*(-1
/4*((e + f*x)*Sech[c + d*x]^4)/d + ((I/4)*f*((-I)*Tanh[c + d*x] + (I/3)*Ta
nh[c + d*x]^3))/d^2))/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6105
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Sech[
c + d*x]^(n + 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)
*Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] &&
EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 40.15 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.70

method	result
risch	$\frac{9de\,e^{dx+c}+9f\,e^{5dx+5c}+8f\,e^{3dx+3c}-e^{dx+c}f-18ide\,e^{4dx+4c}-18idfx\,e^{4dx+4c}+9de\,e^{5dx+5c}+6dfx\,e^{3dx+3c}-22if\,e^{2dx+2c}+18ide\,e^{2}}{12\left(e^{dx+c}+i\right)^{2}\left(e^{dx+c}-i\right)^{4}d^{2}a}$

input
```
int((f*x+e)*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/12*(9*d*e*exp(d*x+c)+9*f*exp(5*d*x+5*c)+8*f*exp(3*d*x+3*c)-exp(d*x+c)*f-
18*I*d*e*exp(4*d*x+4*c)-18*I*d*f*x*exp(4*d*x+4*c)+9*d*e*exp(5*d*x+5*c)+6*d
*f*x*exp(3*d*x+3*c)-22*I*f*exp(2*d*x+2*c)+18*I*d*e*exp(2*d*x+2*c)+9*d*f*x*
exp(d*x+c)+9*d*f*x*exp(5*d*x+5*c)-18*I*f*exp(4*d*x+4*c)+18*I*d*f*x*exp(2*d
*x+2*c)+6*d*e*exp(3*d*x+3*c)-4*I*f)/(exp(d*x+c)+I)^2/(exp(d*x+c)-I)^4/d^2/
a+3/4/d/a*e*arctan(exp(d*x+c))+3/8*I/d/a*f*ln(1-I*exp(d*x+c))*x+3/8*I/d^2/
a*f*ln(1-I*exp(d*x+c))*c+3/8*I*f*polylog(2,I*exp(d*x+c))/a/d^2-3/8*I/d/a*f
*ln(1+I*exp(d*x+c))*x-3/8*I/d^2/a*f*ln(1+I*exp(d*x+c))*c-3/8*I*f*polylog(2
,-I*exp(d*x+c))/a/d^2-3/4/d^2/a*f*c*arctan(exp(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 917 vs. 2(197) = 394.

Time = 0.11 (sec) , antiderivative size = 917, normalized size of antiderivative = 3.94

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/24*(9*(-I*f*e^(6*d*x + 6*c) - 2*f*e^(5*d*x + 5*c) - I*f*e^(4*d*x + 4*c)
 - 4*f*e^(3*d*x + 3*c) + I*f*e^(2*d*x + 2*c) - 2*f*e^(d*x + c) + I*f)*dilo
g(I*e^(d*x + c)) + 9*(I*f*e^(6*d*x + 6*c) + 2*f*e^(5*d*x + 5*c) + I*f*e^(4
*d*x + 4*c) + 4*f*e^(3*d*x + 3*c) - I*f*e^(2*d*x + 2*c) + 2*f*e^(d*x + c)
- I*f)*dilog(-I*e^(d*x + c)) - 18*(d*f*x + d*e + f)*e^(5*d*x + 5*c) + 36*(
I*d*f*x + I*d*e + I*f)*e^(4*d*x + 4*c) - 4*(3*d*f*x + 3*d*e + 4*f)*e^(3*d*
x + 3*c) + 4*(-9*I*d*f*x - 9*I*d*e + 11*I*f)*e^(2*d*x + 2*c) - 2*(9*d*f*x
+ 9*d*e - f)*e^(d*x + c) + 9*(I*d*e - I*c*f + (-I*d*e + I*c*f)*e^(6*d*x +
6*c) - 2*(d*e - c*f)*e^(5*d*x + 5*c) + (-I*d*e + I*c*f)*e^(4*d*x + 4*c) -
4*(d*e - c*f)*e^(3*d*x + 3*c) + (I*d*e - I*c*f)*e^(2*d*x + 2*c) - 2*(d*e -
 c*f)*e^(d*x + c))*log(e^(d*x + c) + I) + 9*(-I*d*e + I*c*f + (I*d*e - I*c
*f)*e^(6*d*x + 6*c) + 2*(d*e - c*f)*e^(5*d*x + 5*c) + (I*d*e - I*c*f)*e^(4
*d*x + 4*c) + 4*(d*e - c*f)*e^(3*d*x + 3*c) + (-I*d*e + I*c*f)*e^(2*d*x +
2*c) + 2*(d*e - c*f)*e^(d*x + c))*log(e^(d*x + c) - I) + 9*(-I*d*f*x - I*c
*f + (I*d*f*x + I*c*f)*e^(6*d*x + 6*c) + 2*(d*f*x + c*f)*e^(5*d*x + 5*c) +
 (I*d*f*x + I*c*f)*e^(4*d*x + 4*c) + 4*(d*f*x + c*f)*e^(3*d*x + 3*c) + (-I
*d*f*x - I*c*f)*e^(2*d*x + 2*c) + 2*(d*f*x + c*f)*e^(d*x + c))*log(I*e^(d*
x + c) + 1) + 9*(I*d*f*x + I*c*f + (-I*d*f*x - I*c*f)*e^(6*d*x + 6*c) - 2*
(d*f*x + c*f)*e^(5*d*x + 5*c) + (-I*d*f*x - I*c*f)*e^(4*d*x + 4*c) - 4*(d*
f*x + c*f)*e^(3*d*x + 3*c) + (I*d*f*x + I*c*f)*e^(2*d*x + 2*c) - 2*(d*f...
```

Sympy [F]

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e\,\text{sech}^3(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{fx\,\text{sech}^3(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)*sech(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e*sech(c + d*x)**3/(sinh(c + d*x) - I), x) + Integral(f*x*sec
h(c + d*x)**3/(sinh(c + d*x) - I), x))/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{sech}\,(dx+c)^3}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*sech(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{e+f\,x}{\cosh\,(c+d\,x)^3\,\left(a+a\sinh\,(c+d\,x)\,1\text{i}\right)}\,dx$$

input
```
int((e + f*x)/(cosh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)/(cosh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(\int \frac{\text{sech}(dx+c)^3}{\sinh(dx+c)i+1}dx\right)e+\left(\int \frac{\text{sech}(dx+c)^3x}{\sinh(dx+c)i+1}dx\right)f}{a}$$

input
```
int((f*x+e)*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

output
```
(int(sech(c + d*x)**3/(sinh(c + d*x)*i + 1),x)*e + int((sech(c + d*x)**3*x
)/(sinh(c + d*x)*i + 1),x)*f)/a
```

3.286 $\int \frac{\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 97

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{3\arctan(\sinh(c+dx))}{8ad} - \frac{i}{8d(a-ia\sinh(c+dx))} + \frac{i}{4d(a+ia\sinh(c+dx))} + \frac{ia^3}{8d\left(a^2+ia^2\sinh(c+dx)\right)^2}$$

output
```
3/8*arctan(sinh(d*x+c))/a/d-1/8*I/d/(a-I*a*sinh(d*x+c))+1/4*I/d/(a+I*a*sin
h(d*x+c))+1/8*I*a^3/d/(a^2+I*a^2*sinh(d*x+c))^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\operatorname{sech}^2(c+dx)\left(2-3i\arctan(\sinh(c+dx))+3(-i+\arctan(\sinh(c+dx)))\sinh(c+dx)+(3-3i\operatorname{arcta}\right.}{8ad(-i+\sinh(c+dx))}$$

input
```
Integrate[Sech[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(Sech[c + d*x]^2*(2 - (3*I)*ArcTan[Sinh[c + d*x]] + 3*(-I + ArcTan[Sinh[c
+ d*x]])*Sinh[c + d*x] + (3 - (3*I)*ArcTan[Sinh[c + d*x]])*Sinh[c + d*x]^2
 + 3*ArcTan[Sinh[c + d*x]]*Sinh[c + d*x]^3))/(8*a*d*(-I + Sinh[c + d*x]))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{3042, 3146, 54, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ic+idx)^3(a+a\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3146}$$

$$-\frac{ia^3\int \frac{1}{(a-ia\sinh(c+dx))^2(i\sinh(c+dx)a+a)^3}d(ia\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$-\frac{ia^3\int \left(\frac{1}{8a^3(a-ia\sinh(c+dx))^2}+\frac{1}{4a^3(i\sinh(c+dx)a+a)^2}+\frac{1}{4a^2(i\sinh(c+dx)a+a)^3}+\frac{3}{8a^3\left(\sinh^2(c+dx)a^2+a^2\right)}\right)d(ia\sinh(c+dx}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^3\left(\frac{3i\arctan(\sinh(c+dx))}{8a^4}+\frac{1}{8a^3(a-ia\sinh(c+dx))}-\frac{1}{4a^3(a+ia\sinh(c+dx))}-\frac{1}{8a^2(a+ia\sinh(c+dx))^2}\right)}{d}$$

input
```
Int[Sech[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-I)*a^3*((((3*I)/8)*ArcTan[Sinh[c + d*x]])/a^4 + 1/(8*a^3*(a - I*a*Sinh[
c + d*x])) - 1/(8*a^2*(a + I*a*Sinh[c + d*x])^2) - 1/(4*a^3*(a + I*a*Sinh[
c + d*x]))))/d
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 39.33 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.29

method	result
risch	$\frac{6i\,e^{2dx+2c}-6i\,e^{4dx+4c}+3\,e^{5dx+5c}+2\,e^{3dx+3c}+3\,e^{dx+c}}{4\left(e^{dx+c}+i\right)^{2}\left(e^{dx+c}-i\right)^{4}da}-\frac{3i\ln\left(e^{dx+c}-i\right)}{8da}+\frac{3i\ln\left(e^{dx+c}+i\right)}{8da}$
derivativedivides	$\frac{\frac{i}{2\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}-\frac{3i\ln\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-\frac{3i}{2\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{1}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}-\frac{1}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{}{4(\mathrm{ta}}}{da}$
default	$\frac{\frac{i}{2\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}-\frac{3i\ln\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-\frac{3i}{2\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{1}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}-\frac{1}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{}{4(\mathrm{ta}}}{da}$

input
```
int(sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/4*(6*I*exp(2*d*x+2*c)-6*I*exp(4*d*x+4*c)+3*exp(5*d*x+5*c)+2*exp(3*d*x+3*
c)+3*exp(d*x+c))/(exp(d*x+c)+I)^2/(exp(d*x+c)-I)^4/d/a-3/8*I/d/a*ln(exp(d*
x+c)-I)+3/8*I/d/a*ln(exp(d*x+c)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(77) = 154$.

Time = 0.11 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.96

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{3\left(-i\,e^{(6\,dx+6\,c)}-2\,e^{(5\,dx+5\,c)}-i\,e^{(4\,dx+4\,c)}-4\,e^{(3\,dx+3\,c)}+i\,e^{(2\,dx+2\,c)}-2\,e^{(dx+c)}+i\right)\log\left(e^{(dx+c)}+i\right)+}{8\left(ade^{(6\,dx+6\,c)}-\right.}$$

input
```
integrate(sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/8*(3*(-I*e^(6*d*x + 6*c) - 2*e^(5*d*x + 5*c) - I*e^(4*d*x + 4*c) - 4*e^
(3*d*x + 3*c) + I*e^(2*d*x + 2*c) - 2*e^(d*x + c) + I)*log(e^(d*x + c) + I
) + 3*(I*e^(6*d*x + 6*c) + 2*e^(5*d*x + 5*c) + I*e^(4*d*x + 4*c) + 4*e^(3*
d*x + 3*c) - I*e^(2*d*x + 2*c) + 2*e^(d*x + c) - I)*log(e^(d*x + c) - I) -
 6*e^(5*d*x + 5*c) + 12*I*e^(4*d*x + 4*c) - 4*e^(3*d*x + 3*c) - 12*I*e^(2*
d*x + 2*c) - 6*e^(d*x + c))/(a*d*e^(6*d*x + 6*c) - 2*I*a*d*e^(5*d*x + 5*c)
 + a*d*e^(4*d*x + 4*c) - 4*I*a*d*e^(3*d*x + 3*c) - a*d*e^(2*d*x + 2*c) - 2
*I*a*d*e^(d*x + c) - a*d)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int \frac{\operatorname{sech}^3(c+dx)}{\sinh(c+dx)-i}\,dx}{a}$$

input
```
integrate(sech(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(sech(c + d*x)**3/(sinh(c + d*x) - I), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(77) = 154$.

Time = 0.12 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.78

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{-\frac{6i\log\left(e^{(dx+c)}-e^{(-dx-c)}+2i\right)}{a}+\frac{6i\log\left(e^{(dx+c)}-e^{(-dx-c)}-2i\right)}{a}-\frac{2\left(3e^{(dx+c)}-3e^{(-dx-c)}+10i\right)}{a\left(ie^{(dx+c)}-ie^{(-dx-c)}-2\right)}+\frac{-9i\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2-52e\cdots}{a\left(e^{(dx+c)}-e^{(-d\cdots}\right.}}{32\,d}$$

input
```
integrate(sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/32*(-6*I*log(e^(d*x + c) - e^(-d*x - c) + 2*I)/a + 6*I*log(e^(d*x + c)
- e^(-d*x - c) - 2*I)/a - 2*(3*e^(d*x + c) - 3*e^(-d*x - c) + 10*I)/(a*(I*
e^(d*x + c) - I*e^(-d*x - c) - 2)) + (-9*I*(e^(d*x + c) - e^(-d*x - c))^2
- 52*e^(d*x + c) + 52*e^(-d*x - c) + 84*I)/(a*(e^(d*x + c) - e^(-d*x - c)
- 2*I)^2))/d
```

Mupad [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.41

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{a^2\,d^2}}{a\,d}\right)}{4\,\sqrt{a^2\,d^2}} + \frac{1}{2\,a\,d\,\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)} + \frac{1}{4\,a\,d\,\left(\mathrm{e}^{c+d\,x}+1\mathrm{i}\right)} - \frac{1\mathrm{i}}{4\,a\,d\,\left(\mathrm{e}^{c+d\,x}+1\mathrm{i}\right)^2} - \frac{1\mathrm{i}}{a\,d\,\left(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)^3} + \frac{1\mathrm{i}}{2\,a\,d\,\left(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)^4}$$

input
```
int(1/(cosh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
(3*atan((exp(d*x)*exp(c)*(a^2*d^2)^(1/2))/(a*d)))/(4*(a^2*d^2)^(1/2)) + 1/
(2*a*d*(exp(c + d*x) - 1i)) + 1/(4*a*d*(exp(c + d*x) + 1i)) - 1i/(4*a*d*(e
xp(c + d*x) + 1i)^2) - 1i/(a*d*(exp(c + d*x)*1i + 1)^3) + 1i/(2*a*d*(exp(c
 + d*x)*1i + 1)^4)
```

Reduce [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\int \frac{\operatorname{sech}(dx+c)^3}{\sinh(dx+c)i+1}\,dx}{a}$$

input
```
int(sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sech(c + d*x)**3/(sinh(c + d*x)*i + 1),x)/a
```

3.287 $\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=\operatorname{Int}\left(\frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(sech(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 49.47 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

input
```
Integrate[Sech[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Integrate[Sech[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

$$\Big\downarrow \text{ 6111}$$

$$\int \frac{\text{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}dx$$

input
```
Int[Sech[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\text{sech}\left(dx+c\right)^3}{\left(fx+e\right)\left(a+ia\sinh\left(dx+c\right)\right)}dx$$

input
```
int(sech(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sech(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 1783, normalized size of antiderivative = 57.52

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(dx+c)^3}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sech(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/12*(4*I*d^2*f^3*x^2 + 8*I*d^2*e*f^2*x + 4*I*d^2*e^2*f - 6*I*f^3 + (9*d^
3*f^3*x^3 + 9*d^3*e^3 - 9*d^2*e^2*f - 2*d*e*f^2 + 6*f^3 + 9*(3*d^3*e*f^2 -
 d^2*f^3)*x^2 + (27*d^3*e^2*f - 18*d^2*e*f^2 - 2*d*f^3)*x)*e^(5*d*x + 5*c)
 - 6*(3*I*d^3*f^3*x^3 + 3*I*d^3*e^3 - 3*I*d^2*e^2*f + I*f^3 + 3*(3*I*d^3*e
*f^2 - I*d^2*f^3)*x^2 + 3*(3*I*d^3*e^2*f - 2*I*d^2*e*f^2)*x)*e^(4*d*x + 4*
c) + 2*(3*d^3*f^3*x^3 + 3*d^3*e^3 - 4*d^2*e^2*f - 2*d*e*f^2 + 6*f^3 + (9*d
^3*e*f^2 - 4*d^2*f^3)*x^2 + (9*d^3*e^2*f - 8*d^2*e*f^2 - 2*d*f^3)*x)*e^(3*
d*x + 3*c) - 2*(-9*I*d^3*f^3*x^3 - 9*I*d^3*e^3 - 11*I*d^2*e^2*f + 6*I*f^3
+ (-27*I*d^3*e*f^2 - 11*I*d^2*f^3)*x^2 + (-27*I*d^3*e^2*f - 22*I*d^2*e*f^2
)*x)*e^(2*d*x + 2*c) + (9*d^3*f^3*x^3 + 9*d^3*e^3 + d^2*e^2*f - 2*d*e*f^2
+ 6*f^3 + (27*d^3*e*f^2 + d^2*f^3)*x^2 + (27*d^3*e^2*f + 2*d^2*e*f^2 - 2*d
*f^3)*x)*e^(d*x + c) - 12*(a*d^4*f^4*x^4 + 4*a*d^4*e*f^3*x^3 + 6*a*d^4*e^2
*f^2*x^2 + 4*a*d^4*e^3*f*x + a*d^4*e^4 - (a*d^4*f^4*x^4 + 4*a*d^4*e*f^3*x^
3 + 6*a*d^4*e^2*f^2*x^2 + 4*a*d^4*e^3*f*x + a*d^4*e^4)*e^(6*d*x + 6*c) + 2
*(I*a*d^4*f^4*x^4 + 4*I*a*d^4*e*f^3*x^3 + 6*I*a*d^4*e^2*f^2*x^2 + 4*I*a*d^
4*e^3*f*x + I*a*d^4*e^4)*e^(5*d*x + 5*c) - (a*d^4*f^4*x^4 + 4*a*d^4*e*f^3*
x^3 + 6*a*d^4*e^2*f^2*x^2 + 4*a*d^4*e^3*f*x + a*d^4*e^4)*e^(4*d*x + 4*c) +
 4*(I*a*d^4*f^4*x^4 + 4*I*a*d^4*e*f^3*x^3 + 6*I*a*d^4*e^2*f^2*x^2 + 4*I*a*
d^4*e^3*f*x + I*a*d^4*e^4)*e^(3*d*x + 3*c) + (a*d^4*f^4*x^4 + 4*a*d^4*e*f^
3*x^3 + 6*a*d^4*e^2*f^2*x^2 + 4*a*d^4*e^3*f*x + a*d^4*e^4)*e^(2*d*x + 2...
```

Sympy [N/A]

Not integrable

Time = 9.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = -\frac{i\int \frac{\operatorname{sech}^3(c+dx)}{e\sinh(c+dx)-ie+fx\sinh(c+dx)-ifx}\,dx}{a}$$

input
```
integrate(sech(d*x+c)**3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(sech(c + d*x)**3/(e*sinh(c + d*x) - I*e + f*x*sinh(c + d*x) -
I*f*x), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sech(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [N/A]

Not integrable

Time = 153.67 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(dx+c)^3}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sech(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sech(d*x + c)^3/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.98 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^3\,\left(e+f\,x\right)\,\left(a+a\,\sinh\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)^3*(e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(1/(cosh(c + d*x)^3*(e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \frac{\int \frac{\operatorname{sech}(dx+c)^3}{\sinh(dx+c)ei+\sinh(dx+c)fix+e+fx}dx}{a}$$

input
```
int(sech(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sech(c + d*x)**3/(sinh(c + d*x)*e*i + sinh(c + d*x)*f*i*x + e + f*x),x
)/a
```

3.288 $\int \frac{\text{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{\text{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\text{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(sech(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Mathematica [F(-1)]

Timed out.

$$\int \frac{\text{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \$\text{Aborted}$$

input
```
Integrate[Sech[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}dx$$
$$\Big\downarrow \text{6111}$$
$$\int \frac{\text{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}dx$$

input
```
Int[Sech[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\text{sech}\,(dx+c)^3}{(fx+e)^2\,(a+ia\sinh\,(dx+c))}dx$$

input
```
int(sech(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sech(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 2026, normalized size of antiderivative = 65.35

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}\left(dx+c\right)^3}{\left(fx+e\right)^2\left(i\,a\sinh\left(dx+c\right)+a\right)}\,dx$$

input
```
integrate(sech(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
-1/12*(8*I*d^2*f^3*x^2 + 16*I*d^2*e*f^2*x + 8*I*d^2*e^2*f - 24*I*f^3 + 3*(
3*d^3*f^3*x^3 + 3*d^3*e^3 - 6*d^2*e^2*f - 2*d*e*f^2 + 8*f^3 + 3*(3*d^3*e*f
^2 - 2*d^2*f^3)*x^2 + (9*d^3*e^2*f - 12*d^2*e*f^2 - 2*d*f^3)*x)*e^(5*d*x +
 5*c) - 6*(3*I*d^3*f^3*x^3 + 3*I*d^3*e^3 - 6*I*d^2*e^2*f + 4*I*f^3 + 3*(3*
I*d^3*e*f^2 - 2*I*d^2*f^3)*x^2 + 3*(3*I*d^3*e^2*f - 4*I*d^2*e*f^2)*x)*e^(4
*d*x + 4*c) + 2*(3*d^3*f^3*x^3 + 3*d^3*e^3 - 8*d^2*e^2*f - 6*d*e*f^2 + 24*
f^3 + (9*d^3*e*f^2 - 8*d^2*f^3)*x^2 + (9*d^3*e^2*f - 16*d^2*e*f^2 - 6*d*f^
3)*x)*e^(3*d*x + 3*c) - 2*(-9*I*d^3*f^3*x^3 - 9*I*d^3*e^3 - 22*I*d^2*e^2*f
 + 24*I*f^3 + (-27*I*d^3*e*f^2 - 22*I*d^2*f^3)*x^2 + (-27*I*d^3*e^2*f - 44
*I*d^2*e*f^2)*x)*e^(2*d*x + 2*c) + (9*d^3*f^3*x^3 + 9*d^3*e^3 + 2*d^2*e^2*
f - 6*d*e*f^2 + 24*f^3 + (27*d^3*e*f^2 + 2*d^2*f^3)*x^2 + (27*d^3*e^2*f +
4*d^2*e*f^2 - 6*d*f^3)*x)*e^(d*x + c) - 12*(a*d^4*f^5*x^5 + 5*a*d^4*e*f^4*
x^4 + 10*a*d^4*e^2*f^3*x^3 + 10*a*d^4*e^3*f^2*x^2 + 5*a*d^4*e^4*f*x + a*d^
4*e^5 - (a*d^4*f^5*x^5 + 5*a*d^4*e*f^4*x^4 + 10*a*d^4*e^2*f^3*x^3 + 10*a*d
^4*e^3*f^2*x^2 + 5*a*d^4*e^4*f*x + a*d^4*e^5)*e^(6*d*x + 6*c) + 2*(I*a*d^4
*f^5*x^5 + 5*I*a*d^4*e*f^4*x^4 + 10*I*a*d^4*e^2*f^3*x^3 + 10*I*a*d^4*e^3*f
^2*x^2 + 5*I*a*d^4*e^4*f*x + I*a*d^4*e^5)*e^(5*d*x + 5*c) - (a*d^4*f^5*x^5
 + 5*a*d^4*e*f^4*x^4 + 10*a*d^4*e^2*f^3*x^3 + 10*a*d^4*e^3*f^2*x^2 + 5*a*d
^4*e^4*f*x + a*d^4*e^5)*e^(4*d*x + 4*c) + 4*(I*a*d^4*f^5*x^5 + 5*I*a*d^4*e
*f^4*x^4 + 10*I*a*d^4*e^2*f^3*x^3 + 10*I*a*d^4*e^3*f^2*x^2 + 5*I*a*d^4*...
```

Sympy [N/A]

Not integrable

Time = 40.80 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.35

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = -\frac{i\int \frac{\operatorname{sech}^3(c+dx)}{e^2\sinh(c+dx)-ie^2+2efx\sinh(c+dx)-2iefx+f^2x^2\sinh(c+dx)-if^2x^2}\,dx}{a}$$

input
```
integrate(sech(d*x+c)**3/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(sech(c + d*x)**3/(e**2*sinh(c + d*x) - I*e**2 + 2*e*f*x*sinh(c
 + d*x) - 2*I*e*f*x + f**2*x**2*sinh(c + d*x) - I*f**2*x**2), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sech(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sech(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 2.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{1}{\cosh(c+d\,x)^3\,(e+f\,x)^2\,\left(a+a\sinh(c+d\,x)\,1\mathrm{i}\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)^3*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(1/(cosh(c + d*x)^3*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.26

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \frac{\int \frac{\operatorname{sech}(dx+c)^3}{\sinh(dx+c)e^2i+2\sinh(dx+c)efix+\sinh(dx+c)f^2i\,x^2+e^2+2efx+f^2x^2}dx}{a}$$

input
```
int(sech(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sech(c + d*x)**3/(sinh(c + d*x)*e**2*i + 2*sinh(c + d*x)*e*f*i*x + sin
h(c + d*x)*f**2*i*x**2 + e**2 + 2*e*f*x + f**2*x**2),x)/a
```

3.289 $\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 356

$$\begin{aligned}\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx &= -\frac{(e+fx)^4}{4bf}+\frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd}\\
&\quad+\frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd}\\
&\quad+\frac{3f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}\\
&\quad+\frac{3f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}\\
&\quad-\frac{6f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3}\\
&\quad-\frac{6f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^3}\\
&\quad+\frac{6f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^4}\\
&\quad+\frac{6f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^4}\end{aligned}$$

output
```
-1/4*(f*x+e)^4/b/f+(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/d+(f
*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/d+3*f*(f*x+e)^2*polylog(2
,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/d^2+3*f*(f*x+e)^2*polylog(2,-b*exp(d
*x+c)/(a+(a^2+b^2)^(1/2)))/b/d^2-6*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))/b/d^3-6*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)
^(1/2)))/b/d^3+6*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/d^4+6*
f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/d^4
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.92

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-\frac{(e+fx)^4}{f} + \frac{4(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d} + \frac{4(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d} + \frac{12f\left(d^2(e+fx)^2\operatorname{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right)-2df(e+fx)\right.}{}}{}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-((e + f*x)^4/f) + (4*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])])/d + (4*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]
)/d + (12*f*(d^2*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b
^2])] - 2*d*f*(e + f*x)*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
 + 2*f^2*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]))/d^4 + (12*f*
(d^2*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2*
d*f*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 2*f^2
*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/d^4)/(4*b)
```

Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6095, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6095}$$

$$\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^4}{4bf}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^4}{4bf}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{3f\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^4}{4bf}$$

$$\Big\downarrow \text{7163}$$

$$-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int \operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int \operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^4}{4bf}$$

$\Big\downarrow$ 2720

$$-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^4}{4bf}$$

$\Big\downarrow$ 7143

$$-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^4}{4bf}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/4*(e + f*x)^4/(b*f) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2])])/(b*d) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt
[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]
))])/d^2))/d))/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))])/d^2))/d))/(b*d)
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(fx+e)^3 \cosh (dx+c)}{a+b\sinh (dx+c)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(328) = 656$.

Time = 0.13 (sec) , antiderivative size = 882, normalized size of antiderivative = 2.48

$$\int \frac{(e+fx)^3 \cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/4*(d^4*f^3*x^4 + 4*d^4*e*f^2*x^3 + 6*d^4*e^2*f*x^2 + 4*d^4*e^3*x - 24*f
^3*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 24*f^3*polylog(4, (a*cosh(d*x + c
) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2))/b) - 12*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f)*dilog((a*cosh(d*x
 + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2) - b)/b + 1) - 12*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f)*dilog
((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*
sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 4*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e
*f^2 - c^3*f^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2
+ b^2)/b^2) + 2*a) - 4*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)
*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2
*a) - 4*(d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + 3*c*d^2*e^2*f - 3
*c^2*d*e*f^2 + c^3*f^3)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(
d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 4*(d^3*f^3*x^3
 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f
^3)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 24*(d*f^3*x + d*e*f^2)*polylog(3,
(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*s
qrt((a^2 + b^2)/b^2))/b) + 24*(d*f^3*x + d*e*f^2)*polylog(3, (a*cosh(d*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
e^3*log(b*sinh(d*x + c) + a)/(b*d) + 1/4*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*
x^2)/b - integrate(-2*(b*f^3*x^3 + 3*b*e*f^2*x^2 + 3*b*e^2*f*x - (a*f^3*x^
3*e^c + 3*a*e*f^2*x^2*e^c + 3*a*e^2*f*x*e^c)*e^(d*x))/(b^2*e^(2*d*x + 2*c)
 + 2*a*b*e^(d*x + c) - b^2), x)
```

Giac [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,(e+f\,x)^3}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{e^{2c}\left(\int \frac{e^{2dx}x^3}{e^{2dx+2c}b+2e^{dx+c}a-b}dx\right)bd\,f^3+3e^{2c}\left(\int \frac{e^{2dx}x^2}{e^{2dx+2c}b+2e^{dx+c}a-b}dx\right)bde\,f^2+3e^{2c}\left(\int \frac{e^{2dx}x}{e^{2dx+2c}b+2e^{dx+c}a-b}dx\right)bd\,e}{}$$

input
```
int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
(e**(2*c)*int((e**(2*d*x)*x**3)/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b
),x)*b*d*f**3 + 3*e**(2*c)*int((e**(2*d*x)*x**2)/(e**(2*c + 2*d*x)*b + 2*e
**(c + d*x)*a - b),x)*b*d*e*f**2 + 3*e**(2*c)*int((e**(2*d*x)*x)/(e**(2*c
+ 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*b*d*e**2*f + int(x**3/(e**(2*c + 2*d
*x)*b + 2*e**(c + d*x)*a - b),x)*b*d*f**3 + 3*int(x**2/(e**(2*c + 2*d*x)*b
 + 2*e**(c + d*x)*a - b),x)*b*d*e*f**2 + 3*int(x/(e**(2*c + 2*d*x)*b + 2*e
**(c + d*x)*a - b),x)*b*d*e**2*f + log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)
*a - b)*e**3 - d*e**3*x)/(b*d)
```

3.290 $\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 264

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{(e+fx)^3}{3bf}+\frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd}+\frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd}+\frac{2f(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{2f(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}-\frac{2f^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3}-\frac{2f^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^3}$$

output
```
-1/3*(f*x+e)^3/b/f+(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/d+(f
*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/d+2*f*(f*x+e)*polylog(2,-
b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/d^2+2*f*(f*x+e)*polylog(2,-b*exp(d*x+c
)/(a+(a^2+b^2)^(1/2)))/b/d^2-2*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1
/2)))/b/d^3-2*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/d^3
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.92

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-\frac{(e+fx)^3}{f}+\frac{3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}+\frac{3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}+\frac{6f\left(d(e+fx)\operatorname{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right)-f\operatorname{PolyLog}\left(3,\right.\right.}{d^3}}{3b}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-((e + f*x)^3/f) + (3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])])/d + (3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]
)/d + (6*f*(d*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
 - f*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]))/d^3 + (6*f*(d*(e
 + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - f*PolyLog[3
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/d^3)/(3*b)
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {6095, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\Big\downarrow \text{ 6095}$$
$$\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}$$
$$\Big\downarrow \text{ 2620}$$

$$-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^3}{3bf}$$

$$\Big\downarrow \text{ 3011}$$

$$-\frac{2f\left(\frac{f\int \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^3}{3bf}$$

$$\Big\downarrow \text{ 2720}$$

$$-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^3}{3bf}$$

$$\Big\downarrow \text{ 7143}$$

$$-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^3}{3bf}$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]
)/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[
a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
])/d^2))/(b*d)
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(fx+e)^2 \cosh(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 609 vs. 2(242) = 484.

Time = 0.13 (sec) , antiderivative size = 609, normalized size of antiderivative = 2.31

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{d^3f^2x^3+3\,d^3efx^2+3\,d^3e^2x+6\,f^2\mathrm{polylog}\left(3,\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2+b^2}{b^2}}}{b}\right)+}{}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/3*(d^3*f^2*x^3 + 3*d^3*e*f*x^2 + 3*d^3*e^2*x + 6*f^2*polylog(3, (a*cosh
(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2))/b) + 6*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) -
(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*(d*f^2*x
 + d*e*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*
sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 6*(d*f^2*x + d*e*f)*dil
og((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c)
)*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*(d^2*e^2 - 2*c*d*e*f + c^2*f^2)*lo
g(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
 - 3*(d^2*e^2 - 2*c*d*e*f + c^2*f^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x
+ c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 3*(d^2*f^2*x^2 + 2*d^2*e*f*x + 2
*c*d*e*f - c^2*f^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 3*(d^2*f^2*x^2 + 2
*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c)
- (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b))/(b*d^
3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
e^2*log(b*sinh(d*x + c) + a)/(b*d) + 1/3*(f^2*x^3 + 3*e*f*x^2)/b - integra
te(-2*(b*f^2*x^2 + 2*b*e*f*x - (a*f^2*x^2*e^c + 2*a*e*f*x*e^c)*e^(d*x))/(b
^2*e^(2*d*x + 2*c) + 2*a*b*e^(d*x + c) - b^2), x)
```

Giac [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\cosh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\cosh(c+d\,x)\,(e+f\,x)^2}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\frac{e^{2c}\left(\int \frac{e^{2dx}x^2}{e^{2dx+2c}b+2e^{dx+c}a-b}dx\right)bd\,f^2+2e^{2c}\left(\int \frac{e^{2dx}x}{e^{2dx+2c}b+2e^{dx+c}a-b}dx\right)bdef+\left(\int \frac{x^2}{e^{2dx+2c}b+2e^{dx+c}a-b}dx\right)bd\,f^2+2}{bd}$$

input
```
int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
(e**(2*c)*int((e**(2*d*x)*x**2)/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b
),x)*b*d*f**2 + 2*e**(2*c)*int((e**(2*d*x)*x)/(e**(2*c + 2*d*x)*b + 2*e**(
c + d*x)*a - b),x)*b*d*e*f + int(x**2/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)
*a - b),x)*b*d*f**2 + 2*int(x/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),
x)*b*d*e*f + log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*e**2 - d*e**2*
x)/(b*d)
```

3.291 $\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 170

$$\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{(e+fx)^2}{2bf}+\frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd}+\frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}$$

output
```
-1/2*(f*x+e)^2/b/f+(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/d+(f*x
+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/d+f*polylog(2,-b*exp(d*x+c)/(
a-(a^2+b^2)^(1/2)))/b/d^2+f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b
/d^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.92

$$\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{-d(e+fx)\left(de+dfx-2f\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-2f\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)+2f^2\operatorname{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right.}{2bd^2f}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-(d*(e + f*x)*(d*e + d*f*x - 2*f*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2])] - 2*f*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])) + 2*f^2*Poly
Log[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f^2*PolyLog[2, -((b*E^(
c + d*x))/(a + Sqrt[a^2 + b^2]))])/(2*b*d^2*f)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6095}$$

$$\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}$$

$$\Big\downarrow \text{2838}$$

$$\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}$$

input
```
Int[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2
) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. 2(156) = 312.

Time = 0.75 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.42

method	result
risch	$-\frac{f\,x^2}{2b}+\frac{ex}{b}-\frac{f\,c^2}{d^2 b}+\frac{f\ln\left(\frac{-b\,e^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)c}{d^2 b}+\frac{f\ln\left(\frac{b\,e^{dx+c}+\sqrt{a^2+b^2}+a}{a+\sqrt{a^2+b^2}}\right)c}{d^2 b}+\frac{2cf\ln\left(e^{dx+c}\right)}{d^2 b}-\frac{cf\ln\left(b\,e^{2dx+2c}+\right.}{d^2 b}$

input
```
int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*f*x^2+1/b*e*x-1/d^2/b*f*c^2+1/d^2/b*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(
1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d^2/b*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)
+a)/(a+(a^2+b^2)^(1/2)))*c+2/d^2/b*c*f*ln(exp(d*x+c))-1/d^2/b*c*f*ln(b*exp
(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/d/b*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b
)-2/d/b*e*ln(exp(d*x+c))-2/d/b*f*c*x+1/d^2/b*f*dilog((b*exp(d*x+c)+(a^2+b^
2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/d/b*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)
-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d/b*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a
+(a^2+b^2)^(1/2)))*x+1/d^2/b*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a
+(a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(154) = 308$.

Time = 0.13 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.24

$$\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{d^2fx^2 + 2\,d^2ex - 2\,f\mathrm{Li}_2\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2+b^2}{b^2}}-b}{b}+1\right) - 2\,f\mathrm{Li}_2\left(\frac{a\cosh(d}{}\right.}{}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(d^2*f*x^2 + 2*d^2*e*x - 2*f*dilog((a*cosh(d*x + c) + a*sinh(d*x + c)
 + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) -
 2*f*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(d*e - c*f)*log(2*b*cosh(d
*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(d*e -
c*f)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2)
 + 2*a) - 2*(d*f*x + c*f)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cos
h(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*(d*f*x + c
*f)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 + b^2)/b^2) - b)/b))/(b*d^2)
```

Sympy [F]

$$\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\cosh{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*cosh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*f*(x^2/b - integrate(4*(a*x*e^(d*x + c) - b*x)/(b^2*e^(2*d*x + 2*c) +
2*a*b*e^(d*x + c) - b^2), x)) + e*log(b*sinh(d*x + c) + a)/(b*d)
```

Giac [F]

$$\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{e^{2c}\left(\int \frac{e^{2dx}x}{e^{2dx+2c}b+2e^{dx+c}a-b}dx\right)bdf+\left(\int \frac{x}{e^{2dx+2c}b+2e^{dx+c}a-b}dx\right)bdf+\log\left(e^{2dx+2c}b+2e^{dx+c}a-b\right)e-dex}{bd}$$

input
```
int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
(e**(2*c)*int((e**(2*d*x)*x)/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x
)*b*d*f + int(x/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*b*d*f + log
(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*e - d*e*x)/(b*d)
```

3.292 $\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 18

$$\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\log(a+b\sinh(c+dx))}{bd}$$

output
```
ln(a+b*sinh(d*x+c))/b/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\log(a+b\sinh(c+dx))}{bd}$$

input
```
Integrate[Cosh[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

output
```
Log[a + b*Sinh[c + d*x]]/(b*d)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3147, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(ic+idx)}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \; 3147$$

$$\frac{\int \frac{1}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{bd}$$

$$\Big\downarrow \; 16$$

$$\frac{\log(a+b\sinh(c+dx))}{bd}$$

input
```
Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

output
```
Log[a + b*Sinh[c + d*x]]/(b*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\ln(a+b\sinh(dx+c))}{bd}$	19
default	$\frac{\ln(a+b\sinh(dx+c))}{bd}$	19
risch	$-\frac{x}{b}-\frac{2c}{bd}+\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{b}-1\right)}{bd}$	48

input
```
int(cosh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
ln(a+b*sinh(d*x+c))/b/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(18) = 36.

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.44

$$\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx=-\frac{dx-\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{bd}$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(d*x - log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))))/(b*d
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(14) = 28$.

Time = 0.77 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.28

$$\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \begin{cases} \frac{x\cosh(c)}{a} & \text{for } b=0 \wedge d=0 \\ \frac{\sinh(c+dx)}{ad} & \text{for } b=0 \\ \frac{x\cosh(c)}{a+b\sinh(c)} & \text{for } d=0 \\ \frac{\log\left(\frac{a}{b}+\sinh(c+dx)\right)}{bd} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Piecewise((x*cosh(c)/a, Eq(b, 0) & Eq(d, 0)), (sinh(c + d*x)/(a*d), Eq(b,
0)), (x*cosh(c)/(a + b*sinh(c)), Eq(d, 0)), (log(a/b + sinh(c + d*x))/(b*d
), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\log(b\sinh(dx+c)+a)}{bd}$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
log(b*sinh(d*x + c) + a)/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.83

$$\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx=\frac{\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{bd}$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(b*d)
```

Mupad [B] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx=\frac{\ln\left(a+b\sinh(c+d\,x)\right)}{b\,d}$$

input
```
int(cosh(c + d*x)/(a + b*sinh(c + d*x)),x)
```

output
```
log(a + b*sinh(c + d*x))/(b*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx=\frac{\log(a+b\sinh\left(dx+c\right))}{bd}$$

input
```
int(cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
log(sinh(c + d*x)*b + a)/(b*d)
```

3.293 $\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 4.91 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[Cosh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Cosh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$\Big\downarrow \text{ 6111}$$
$$\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Cosh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(dx+c\right)}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(cosh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.58

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
log(f*x + e)/(b*f) - 1/2*integrate(-4*(a*e^(d*x + c) - b)/(b^2*f*x + b^2*e
 - (b^2*f*x*e^(2*c) + b^2*e*e^(2*c))*e^(2*d*x) - 2*(a*b*f*x*e^c + a*b*e*e^
c)*e^(d*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(cosh(c + d*x)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(cosh(c + d*x)/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 159, normalized size of antiderivative = 6.12

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$= \frac{-2e^c\left(\int \frac{e^{dx}}{e^{2dx+2c}be+e^{2dx+2c}bfx+2e^{dx+c}ae+2e^{dx+c}afx-be-bfx}dx\right)af+2\left(\int \frac{1}{e^{2dx+2c}be+e^{2dx+2c}bfx+2e^{dx+c}ae+2e^{dx+c}afx-be-}\right.}{bf}$$

input
```
int(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*e**c*int(e**(d*x)/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x + 2
*e**(c + d*x)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x)*a*f + 2*int(1/(
e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x + 2*e**(c + d*x)*a*e + 2*e**
(c + d*x)*a*f*x - b*e - b*f*x),x)*b*f + log(e + f*x))/(b*f)
```

3.294 $\int \frac{(e+fx)^3 \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 527

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a(e+fx)^4}{4b^2f}+\frac{6f^2(e+fx)\cosh(c+dx)}{bd^3}+\frac{(e+fx)^3\cosh(c+dx)}{bd}+\frac{\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{3\sqrt{a^2+b^2}f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2}-\frac{3\sqrt{a^2+b^2}f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^2}-\frac{6\sqrt{a^2+b^2}f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3}+\frac{6\sqrt{a^2+b^2}f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^3}+\frac{6\sqrt{a^2+b^2}f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^4}-\frac{6\sqrt{a^2+b^2}f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^4}-\frac{6f^3\sinh(c+dx)}{bd^4}-\frac{3f(e+fx)^2\sinh(c+dx)}{bd^2}$$

output
```
-1/4*a*(f*x+e)^4/b^2/f+6*f^2*(f*x+e)*cosh(d*x+c)/b/d^3+(f*x+e)^3*cosh(d*x+
c)/b/d+(a^2+b^2)^(1/2)*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^
2/d-(a^2+b^2)^(1/2)*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d
+3*(a^2+b^2)^(1/2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2))
)/b^2/d^2-3*(a^2+b^2)^(1/2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/b^2/d^2-6*(a^2+b^2)^(1/2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(
a-(a^2+b^2)^(1/2)))/b^2/d^3+6*(a^2+b^2)^(1/2)*f^2*(f*x+e)*polylog(3,-b*exp
(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d^3+6*(a^2+b^2)^(1/2)*f^3*polylog(4,-b*ex
p(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d^4-6*(a^2+b^2)^(1/2)*f^3*polylog(4,-b*e
xp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d^4-6*f^3*sinh(d*x+c)/b/d^4-3*f*(f*x+e)
^2*sinh(d*x+c)/b/d^2
```

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 933, normalized size of antiderivative = 1.77

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{4ad^4e^3x+6ad^4e^2fx^2+4ad^4ef^2x^3+ad^4f^3x^4+8\sqrt{a^2+b^2}d^3e^3\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)-4bd^3e^3\cosh(c+d}{}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/4*(4*a*d^4*e^3*x + 6*a*d^4*e^2*f*x^2 + 4*a*d^4*e*f^2*x^3 + a*d^4*f^3*x^
4 + 8*Sqrt[a^2 + b^2]*d^3*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]]
 - 4*b*d^3*e^3*Cosh[c + d*x] - 24*b*d*e*f^2*Cosh[c + d*x] - 12*b*d^3*e^2*f
*x*Cosh[c + d*x] - 24*b*d*f^3*x*Cosh[c + d*x] - 12*b*d^3*e*f^2*x^2*Cosh[c
+ d*x] - 4*b*d^3*f^3*x^3*Cosh[c + d*x] - 12*Sqrt[a^2 + b^2]*d^3*e^2*f*x*Lo
g[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 12*Sqrt[a^2 + b^2]*d^3*e*f^
2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 4*Sqrt[a^2 + b^2]*d
^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 12*Sqrt[a^2 +
b^2]*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 12*Sqrt[
a^2 + b^2]*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] +
4*Sqrt[a^2 + b^2]*d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
)] - 12*Sqrt[a^2 + b^2]*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a +
 Sqrt[a^2 + b^2])] + 12*Sqrt[a^2 + b^2]*d^2*f*(e + f*x)^2*PolyLog[2, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 24*Sqrt[a^2 + b^2]*d*e*f^2*PolyLog[
3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 24*Sqrt[a^2 + b^2]*d*f^3*x*Po
lyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 24*Sqrt[a^2 + b^2]*d*e*
f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 24*Sqrt[a^2 + b
^2]*d*f^3*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 24*Sqrt
[a^2 + b^2]*f^3*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 24*Sq
rt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] ...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.60 (sec) , antiderivative size = 501, normalized size of antiderivative = 0.95, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {6099, 17, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117, 3803, 25, 2694, 27, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \; 6099$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)^3dx}{b^2}+\frac{\int (e+fx)^3\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{17}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int (e+fx)^3\sinh(c+dx)dx}{b}-\frac{a(e+fx)^4}{4b^2f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}+\frac{\int -i(e+fx)^3\sin(ic+idx)dx}{b}-\frac{a(e+fx)^4}{4b^2f}$$

$$\Big\downarrow \text{26}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\int (e+fx)^3\sin(ic+idx)dx}{b}-\frac{a(e+fx)^4}{4b^2f}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\cosh(c+dx)dx}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}$$

$$\Big\downarrow \text{26}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int (e+fx)\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int -i(e+fx)\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}$$

$\downarrow$ 26

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int (e+fx)\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}$$

$\downarrow$ 3777

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}\right)}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}$$

$\downarrow$ 3042

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}$$

$\downarrow$ 3117

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{3803}$$

$$\frac{2\left(a^2+b^2\right)\int -\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\left(a^2+b^2\right)\int \frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{2694}$$

$$-\frac{2\left(a^2+b^2\right)\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2(a^2+b^2)\left(\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$\downarrow$ 2620

$$-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e\cdots}{a-\cdots}\right.}{bd}\cdots\right.}{2\sqrt{a^2+b^2}}\cdots\right)}{b^2}$$

$$\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$\downarrow$ 3011

$$-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+\cdots}{\cdots}\right.\cdots}{\cdots}\cdots\right)}{b^2}$$

$$\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$\downarrow$ 7163

$$-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(}{}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}}{\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}}$$

↓ 2720

$$-\frac{2\left(a^2+b^2\right)\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+\cdots}{\cdots}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{\cdots}$$

$$\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\downarrow \ 7143$$

$$-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\right.}{d}}{bd}\right)}{2\sqrt{a^2+b^2}}\right.}{}$$

$$\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/4*(a*(e + f*x)^4)/(b^2*f) - (2*(a^2 + b^2)*(-1/2*(b*(((e + f*x)^3*Log[1
 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*Po
lyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*
PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -(
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/Sqrt[a^2 + b^2]
 + (b*(((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d)
- (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/d)
)/(b*d)))/(2*Sqrt[a^2 + b^2])))/b^2 - (I*((I*(e + f*x)^3*Cosh[c + d*x])/d
- ((3*I)*f*(((e + f*x)^2*Sinh[c + d*x])/d + ((2*I)*f*((I*(e + f*x)*Cosh[c
+ d*x])/d - (I*f*Sinh[c + d*x])/d^2))/d))/d))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(fx+e)^3 \cosh(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2020 vs. $2(483) = 966$.

Time = 0.21 (sec) , antiderivative size = 2020, normalized size of antiderivative = 3.83

$$\int \frac{(e+fx)^3 \cosh^2(c+dx)}{a+b\sinh(c+dx)} dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(2*b*d^3*f^3*x^3 + 2*b*d^3*e^3 + 6*b*d^2*e^2*f + 12*b*d*e*f^2 + 12*b*f
^3 + 6*(b*d^3*e*f^2 + b*d^2*f^3)*x^2 + 2*(b*d^3*f^3*x^3 + b*d^3*e^3 - 3*b*
d^2*e^2*f + 6*b*d*e*f^2 - 6*b*f^3 + 3*(b*d^3*e*f^2 - b*d^2*f^3)*x^2 + 3*(b
*d^3*e^2*f - 2*b*d^2*e*f^2 + 2*b*d*f^3)*x)*cosh(d*x + c)^2 + 2*(b*d^3*f^3*
x^3 + b*d^3*e^3 - 3*b*d^2*e^2*f + 6*b*d*e*f^2 - 6*b*f^3 + 3*(b*d^3*e*f^2 -
 b*d^2*f^3)*x^2 + 3*(b*d^3*e^2*f - 2*b*d^2*e*f^2 + 2*b*d*f^3)*x)*sinh(d*x
+ c)^2 + 12*((b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*cosh(d*x + c)
 + (b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 12*((b*d^2*f^3*x^2
+ 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*cosh(d*x + c) + (b*d^2*f^3*x^2 + 2*b*d^2*
e*f^2*x + b*d^2*e^2*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(
d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2) - b)/b + 1) - 4*((b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f
^2 - b*c^3*f^3)*cosh(d*x + c) + (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e
*f^2 - b*c^3*f^3)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x +
c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 4*((b*d^3*e^3
- 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*cosh(d*x + c) + (b*d^3*e^
3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*e^3*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) - e^(-d*x - c)/(b*d) -
 2*sqrt(a^2 + b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x -
 c) - a + sqrt(a^2 + b^2)))/(b^2*d)) - 1/4*(a*d^4*f^3*x^4*e^c + 4*a*d^4*e*
f^2*x^3*e^c + 6*a*d^4*e^2*f*x^2*e^c - 2*(b*d^3*f^3*x^3*e^(2*c) + 3*(d^3*e*
f^2 - d^2*f^3)*b*x^2*e^(2*c) + 3*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*b*x*e
^(2*c) - 3*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*b*e^(2*c))*e^(d*x) - 2*(b*d^3*f
^3*x^3 + 3*(d^3*e*f^2 + d^2*f^3)*b*x^2 + 3*(d^3*e^2*f + 2*d^2*e*f^2 + 2*d*
f^3)*b*x + 3*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*b)*e^(-d*x))*e^(-c)/(b^2*d^4)
 + integrate(2*((a^2*f^3*e^c + b^2*f^3*e^c)*x^3 + 3*(a^2*e*f^2*e^c + b^2*e
*f^2*e^c)*x^2 + 3*(a^2*e^2*f*e^c + b^2*e^2*f*e^c)*x)*e^(d*x)/(b^3*e^(2*d*x
 + 2*c) + 2*a*b^2*e^(d*x + c) - b^3), x)
```

Giac [F]

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\cosh(c+d\,x)^2\,(e+f\,x)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Too large to display}$$

input
```
int((f*x+e)^3*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(8*e**(c + d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2
+ b**2))*b*d**3*e**3*i + 2*e**(2*c + 2*d*x)*b**2*d**3*e**3 + 6*e**(2*c + 2
*d*x)*b**2*d**3*e**2*f*x + 6*e**(2*c + 2*d*x)*b**2*d**3*e*f**2*x**2 + 2*e*
*(2*c + 2*d*x)*b**2*d**3*f**3*x**3 - 6*e**(2*c + 2*d*x)*b**2*d**2*e**2*f -
 12*e**(2*c + 2*d*x)*b**2*d**2*e*f**2*x - 6*e**(2*c + 2*d*x)*b**2*d**2*f**
3*x**2 + 12*e**(2*c + 2*d*x)*b**2*d*e*f**2 + 12*e**(2*c + 2*d*x)*b**2*d*f*
*3*x - 12*e**(2*c + 2*d*x)*b**2*f**3 - 16*e**(c + d*x)*int(x**3/(e**(2*c +
 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**3*d**4*f**3 - 16*e**(c + d*x)*int(
x**3/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a*b**2*d**4*f**3 - 48*
e**(c + d*x)*int(x**2/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**3*
d**4*e*f**2 - 48*e**(c + d*x)*int(x**2/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x
)*a - b),x)*a*b**2*d**4*e*f**2 - 48*e**(c + d*x)*int(x/(e**(2*c + 2*d*x)*b
 + 2*e**(c + d*x)*a - b),x)*a**3*d**4*e**2*f - 48*e**(c + d*x)*int(x/(e**(
2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a*b**2*d**4*e**2*f - 4*e**(c + d
*x)*a*b*d**4*e**3*x - 6*e**(c + d*x)*a*b*d**4*e**2*f*x**2 - 4*e**(c + d*x)
*a*b*d**4*e*f**2*x**3 - e**(c + d*x)*a*b*d**4*f**3*x**4 + 8*e**(d*x)*int(x
**3/(e**(2*c + 3*d*x)*b + 2*e**(c + 2*d*x)*a - e**(d*x)*b),x)*a**2*b*d**4*
f**3 + 8*e**(d*x)*int(x**3/(e**(2*c + 3*d*x)*b + 2*e**(c + 2*d*x)*a - e**(
d*x)*b),x)*b**3*d**4*f**3 + 24*e**(d*x)*int(x**2/(e**(2*c + 3*d*x)*b + 2*e
**(c + 2*d*x)*a - e**(d*x)*b),x)*a**2*b*d**4*e*f**2 + 24*e**(d*x)*int(x...
```

3.295 $\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 389

$$\begin{aligned}\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = &-\frac{a(e+fx)^3}{3b^2f}+\frac{2f^2\cosh(c+dx)}{bd^3}\\ &+\frac{(e+fx)^2\cosh(c+dx)}{bd}\\ &+\frac{\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d}\\ &-\frac{\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d}\\ &+\frac{2\sqrt{a^2+b^2}f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2}\\ &-\frac{2\sqrt{a^2+b^2}f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^2}\\ &-\frac{2\sqrt{a^2+b^2}f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3}\\ &+\frac{2\sqrt{a^2+b^2}f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^3}\\ &-\frac{2f(e+fx)\sinh(c+dx)}{bd^2}\end{aligned}$$

output
```
-1/3*a*(f*x+e)^3/b^2/f+2*f^2*cosh(d*x+c)/b/d^3+(f*x+e)^2*cosh(d*x+c)/b/d+(
a^2+b^2)^(1/2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d-(a^2
+b^2)^(1/2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d+2*(a^2+
b^2)^(1/2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d^2-
2*(a^2+b^2)^(1/2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b
^2/d^2-2*(a^2+b^2)^(1/2)*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/
b^2/d^3+2*(a^2+b^2)^(1/2)*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))
/b^2/d^3-2*f*(f*x+e)*sinh(d*x+c)/b/d^2
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.15

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{ad^3x(3e^2+3efx+f^2x^2)+3\sqrt{a^2+b^2}\left(2d^2e^2\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)-2d^2efx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-d^2f^2\right.}{}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/3*(a*d^3*x*(3*e^2 + 3*e*f*x + f^2*x^2) + 3*Sqrt[a^2 + b^2]*(2*d^2*e^2*A
rcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 2*d^2*e*f*x*Log[1 + (b*E^(c
+ d*x))/(a - Sqrt[a^2 + b^2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2])] + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])] + d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*d*f*
(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*d*f*(e +
f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 2*f^2*PolyLog[
3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*f^2*PolyLog[3, -((b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2]))]) - 3*b*Cosh[d*x]*((2*f^2 + d^2*(e + f*x)^2)*
Cosh[c] - 2*d*f*(e + f*x)*Sinh[c]) + 3*b*(2*d*f*(e + f*x)*Cosh[c] - (2*f^2
 + d^2*(e + f*x)^2)*Sinh[c])*Sinh[d*x])/(b^2*d^3)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.94 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.97, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.679$, Rules used = {6099, 17, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118, 3803, 25, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6099}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)^2dx}{b^2}+\frac{\int (e+fx)^2\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{17}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int (e+fx)^2\sinh(c+dx)dx}{b}-\frac{a(e+fx)^3}{3b^2f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}+\frac{\int -i(e+fx)^2\sin(ic+idx)dx}{b}-\frac{a(e+fx)^3}{3b^2f}$$

$$\Big\downarrow \text{26}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\int (e+fx)^2\sin(ic+idx)dx}{b}-\frac{a(e+fx)^3}{3b^2f}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\cosh(c+dx)dx}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}$$

$$\Big\downarrow \; 26$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}$$

$$\Big\downarrow \; 3042$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}$$

$$\Big\downarrow \; 26$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}$$

$$\Big\downarrow \; 3118$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$$\Big\downarrow \; 3803$$

$$\frac{2(a^2+b^2)\int -\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$\Big\downarrow$ 25

$$-\frac{2\left(a^2+b^2\right)\int \frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$\Big\downarrow$ 2694

$$-\frac{2\left(a^2+b^2\right)\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$\Big\downarrow$ 27

$$-\frac{2\left(a^2+b^2\right)\left(\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$\Big\downarrow$ 2620

$$-\frac{2\left(a^2+b^2\right)\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+\cdots}}{a-\sqrt{\cdots}}\cdots\right)}{bd}\cdots\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{b^2}$$
$$\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$\Big\downarrow$ 3011

$$-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{b^2}$$

$$\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$\downarrow$ 2720

$$-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\cdots\right.}{\cdots}\right.}{b^2}$$

$$\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$\downarrow$ 7143

$$-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be}{a-\sqrt{}}\right.}{bd}\right.}{}\right.}{b^2}$$

$$\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

input

```
Int[((e + f*x)^2*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output

```
-1/3*(a*(e + f*x)^3)/(b^2*f) - (2*(a^2 + b^2)*(-1/2*(b*(((e + f*x)^2*Log[1
 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*Poly
Log[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/Sqrt[a^2 + b^2] + (b*(
((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (2*f*
(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (
f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/(2*S
qrt[a^2 + b^2])))/b^2 - (I*((I*(e + f*x)^2*Cosh[c + d*x])/d - ((2*I)*f*(-(
(f*Cosh[c + d*x])/d^2) + ((e + f*x)*Sinh[c + d*x])/d))/d))/b
```

Defintions of rubi rules used

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(fx+e)^2 \cosh(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1313 vs. $2(355) = 710$.

Time = 0.12 (sec) , antiderivative size = 1313, normalized size of antiderivative = 3.38

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(3*b*d^2*f^2*x^2 + 3*b*d^2*e^2 + 6*b*d*e*f + 6*b*f^2 + 3*(b*d^2*f^2*x^
2 + b*d^2*e^2 - 2*b*d*e*f + 2*b*f^2 + 2*(b*d^2*e*f - b*d*f^2)*x)*cosh(d*x
+ c)^2 + 3*(b*d^2*f^2*x^2 + b*d^2*e^2 - 2*b*d*e*f + 2*b*f^2 + 2*(b*d^2*e*f
 - b*d*f^2)*x)*sinh(d*x + c)^2 + 12*((b*d*f^2*x + b*d*e*f)*cosh(d*x + c) +
 (b*d*f^2*x + b*d*e*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(
d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2) - b)/b + 1) - 12*((b*d*f^2*x + b*d*e*f)*cosh(d*x + c) + (b*d*
f^2*x + b*d*e*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x +
c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2
)/b^2) - b)/b + 1) - 6*((b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*cosh(d*x + c
) + (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)/
b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2)
 + 2*a) + 6*((b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*cosh(d*x + c) + (b*d^2*
e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*
b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 6
*((b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*cosh(d*x + c)
+ (b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*sinh(d*x + c))
*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 6*((b*d^2*f^2*x
^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*cosh(d*x + c) + (b*d^2*f^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\cosh{(dx+c)}^2}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*e^2*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) - e^(-d*x - c)/(b*d) -
 2*sqrt(a^2 + b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x -
 c) - a + sqrt(a^2 + b^2)))/(b^2*d)) - 1/6*(2*a*d^3*f^2*x^3*e^c + 6*a*d^3*
e*f*x^2*e^c - 3*(b*d^2*f^2*x^2*e^(2*c) + 2*(d^2*e*f - d*f^2)*b*x*e^(2*c) -
 2*(d*e*f - f^2)*b*e^(2*c))*e^(d*x) - 3*(b*d^2*f^2*x^2 + 2*(d^2*e*f + d*f^
2)*b*x + 2*(d*e*f + f^2)*b)*e^(-d*x))*e^(-c)/(b^2*d^3) + integrate(2*((a^2
*f^2*e^c + b^2*f^2*e^c)*x^2 + 2*(a^2*e*f*e^c + b^2*e*f*e^c)*x)*e^(d*x)/(b^
3*e^(2*d*x + 2*c) + 2*a*b^2*e^(d*x + c) - b^3), x)
```

Giac [F]

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\cosh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\cosh(c+d\,x)^2\,(e+f\,x)^2}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\frac{-24a^2f^2-18b^2f^2+6e^{2dx+2c}b^2f^2+3b^2d^2e^2+12e^{dx}\left(\int \frac{x^2}{e^{3dx+2c}b+2e^{2dx+c}a-e^{dx}b}dx\right)b^3d^3f^2-9b^2d^2f^2x^2+3\cdots}{}$$

input
```
int((f*x+e)^2*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(12*e**(c + d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2
 + b**2))*b*d**2*e**2*i + 3*e**(2*c + 2*d*x)*b**2*d**2*e**2 + 6*e**(2*c +
2*d*x)*b**2*d**2*e*f*x + 3*e**(2*c + 2*d*x)*b**2*d**2*f**2*x**2 - 6*e**(2*
c + 2*d*x)*b**2*d*e*f - 6*e**(2*c + 2*d*x)*b**2*d*f**2*x + 6*e**(2*c + 2*d
*x)*b**2*f**2 - 24*e**(c + d*x)*int(x**2/(e**(2*c + 2*d*x)*b + 2*e**(c + d
*x)*a - b),x)*a**3*d**3*f**2 - 24*e**(c + d*x)*int(x**2/(e**(2*c + 2*d*x)*
b + 2*e**(c + d*x)*a - b),x)*a*b**2*d**3*f**2 - 48*e**(c + d*x)*int(x/(e**
(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**3*d**3*e*f - 48*e**(c + d*x)
*int(x/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a*b**2*d**3*e*f - 6*
e**(c + d*x)*a*b*d**3*e**2*x - 6*e**(c + d*x)*a*b*d**3*e*f*x**2 - 2*e**(c
+ d*x)*a*b*d**3*f**2*x**3 + 12*e**(d*x)*int(x**2/(e**(2*c + 3*d*x)*b + 2*e
**(c + 2*d*x)*a - e**(d*x)*b),x)*a**2*b*d**3*f**2 + 12*e**(d*x)*int(x**2/(
e**(2*c + 3*d*x)*b + 2*e**(c + 2*d*x)*a - e**(d*x)*b),x)*b**3*d**3*f**2 +
24*e**(d*x)*int(x/(e**(2*c + 3*d*x)*b + 2*e**(c + 2*d*x)*a - e**(d*x)*b),x
)*a**2*b*d**3*e*f + 24*e**(d*x)*int(x/(e**(2*c + 3*d*x)*b + 2*e**(c + 2*d*
x)*a - e**(d*x)*b),x)*b**3*d**3*e*f - 24*a**2*d**2*e*f*x - 12*a**2*d**2*f*
*2*x**2 - 24*a**2*d*e*f - 24*a**2*d*f**2*x - 24*a**2*f**2 + 3*b**2*d**2*e*
*2 - 18*b**2*d**2*e*f*x - 9*b**2*d**2*f**2*x**2 - 18*b**2*d*e*f - 18*b**2*
d*f**2*x - 18*b**2*f**2)/(6*e**(c + d*x)*b**3*d**3)
```

3.296 $\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 250

$$\begin{aligned}\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx &= -\frac{a(e+fx)^2}{2b^2f}+\frac{(e+fx)\cosh(c+dx)}{bd} \\ &+\frac{\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d} \\ &-\frac{\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d} \\ &+\frac{\sqrt{a^2+b^2}f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2} \\ &-\frac{\sqrt{a^2+b^2}f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^2}-\frac{f\sinh(c+dx)}{bd^2}\end{aligned}$$

output
```
-1/2*a*(f*x+e)^2/b^2/f+(f*x+e)*cosh(d*x+c)/b/d+(a^2+b^2)^(1/2)*(f*x+e)*ln(
1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d-(a^2+b^2)^(1/2)*(f*x+e)*ln(1+b*e
xp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d+(a^2+b^2)^(1/2)*f*polylog(2,-b*exp(d*
x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d^2-(a^2+b^2)^(1/2)*f*polylog(2,-b*exp(d*x+c
)/(a+(a^2+b^2)^(1/2)))/b^2/d^2-f*sinh(d*x+c)/b/d^2
```

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.03

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{a(c+dx)(cf-d(2e+fx))+2bd(e+fx)\cosh(c+dx)+2\sqrt{a^2+b^2}\left(-2de\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+2cf\operatorname{arc}\right.}{}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(a*(c + d*x)*(c*f - d*(2*e + f*x)) + 2*b*d*(e + f*x)*Cosh[c + d*x] + 2*Sqr
t[a^2 + b^2]*(-2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*c*f*
ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log[1 + (b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2])] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
- f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) - 2*b*f*Sinh[c +
 d*x])/(2*b^2*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {6099, 17, 3042, 26, 3777, 3042, 3117, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\Big\downarrow \text{6099}$$
$$\frac{(a^2+b^2)\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)dx}{b^2}+\frac{\int (e+fx)\sinh(c+dx)dx}{b}$$
$$\Big\downarrow \text{17}$$

$$\frac{(a^2+b^2)\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int (e+fx)\sinh(c+dx)dx}{b}-\frac{a(e+fx)^2}{2b^2f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}+\frac{\int -i(e+fx)\sin(ic+idx)dx}{b}-\frac{a(e+fx)^2}{2b^2f}$$

$$\Big\downarrow \text{26}$$

$$\frac{(a^2+b^2)\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\int (e+fx)\sin(ic+idx)dx}{b}-\frac{a(e+fx)^2}{2b^2f}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(a^2+b^2)\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}\right)}{b}-\frac{a(e+fx)^2}{2b^2f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}-\frac{a(e+fx)^2}{2b^2f}$$

$$\Big\downarrow \text{3117}$$

$$\frac{(a^2+b^2)\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \text{3803}$$

$$\frac{2(a^2+b^2)\int -\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2(a^2+b^2)\int \frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \text{2694}$$

$$-\frac{2(a^2+b^2)\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{2\left(a^2+b^2\right)\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \; 2620$$

$$-\frac{2\left(a^2+b^2\right)\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{b^2}$$
$$\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \; 2715$$

$$-\frac{2\left(a^2+b^2\right)\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{\cdots}{a-\cdots}\right.}{\cdots}\right.}{2\sqrt{a^2+b^2}}\right.}{b^2}$$
$$\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \; 2838$$

$$-\frac{2\left(a^2+b^2\right)\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right.}{b^2}$$
$$\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

input
```
Int[((e + f*x)*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*(a*(e + f*x)^2)/(b^2*f) - (2*(a^2 + b^2)*(-1/2*(b*(((e + f*x)*Log[1 +
 (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)
*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(2*Sqrt[a^2 + b^2])))/b^
2 - (I*((I*(e + f*x)*Cosh[c + d*x])/d - (I*f*Sinh[c + d*x])/d^2))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 900 vs. $2(228) = 456$.

Time = 2.13 (sec) , antiderivative size = 901, normalized size of antiderivative = 3.60

method	result
risch	$-\frac{af\,x^2}{2b^2}-\frac{aex}{b^2}+\frac{(dxf+de-f)\mathrm{e}^{dx+c}}{2b\,d^2}+\frac{(dxf+de+f)\mathrm{e}^{-dx-c}}{2b\,d^2}-\frac{2a^2e\,\operatorname{arctanh}\left(\frac{2b\,\mathrm{e}^{dx+c}+2a}{2\sqrt{a^2+b^2}}\right)}{d\,b^2\sqrt{a^2+b^2}}-\frac{2e\,\operatorname{arctanh}\left(\frac{2b\,\mathrm{e}^{dx+c}+2a}{2\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$

input
```
int((f*x+e)*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/b^2*f*x^2-a/b^2*e*x+1/2*(d*f*x+d*e-f)/b/d^2*exp(d*x+c)+1/2*(d*f*x+d
*e+f)/b/d^2*exp(-d*x-c)-2/d/b^2*a^2*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp
(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d
*x+c)+2*a)/(a^2+b^2)^(1/2))+1/d*a^2/b^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c
)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d/b^2*a^2*f/(a^2+b^2)^(1/2)
*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2/b^2*a^2*
f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)
))*c-1/d^2/b^2*a^2*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(
a+(a^2+b^2)^(1/2)))*c+1/d^2/b^2*a^2*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)
+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2/b^2*a^2*f/(a^2+b^2)^(1/2)*
dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/d*f/(a^2+b^2
)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d*f
/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*
x+1/d^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^
2)^(1/2)))*c-1/d^2*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(
a+(a^2+b^2)^(1/2)))*c+1/d^2*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^
2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x
+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/d^2/b^2*a^2*f*c/(a^2+b^2)^(1
/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/d^2*f*c/(a^2+b^2)^
(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 742 vs. $2(226) = 452$.

Time = 0.14 (sec) , antiderivative size = 742, normalized size of antiderivative = 2.97

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(b*d*f*x + b*d*e + (b*d*f*x + b*d*e - b*f)*cosh(d*x + c)^2 + (b*d*f*x
+ b*d*e - b*f)*sinh(d*x + c)^2 + 2*(b*f*cosh(d*x + c) + b*f*sinh(d*x + c))
*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(
d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(b*f*cos
h(d*x + c) + b*f*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x +
c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2
)/b^2) - b)/b + 1) - 2*((b*d*e - b*c*f)*cosh(d*x + c) + (b*d*e - b*c*f)*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x +
c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*((b*d*e - b*c*f)*cosh(d*x + c) +
 (b*d*e - b*c*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c
) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*((b*d*f*x + b
*c*f)*cosh(d*x + c) + (b*d*f*x + b*c*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^
2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
 + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*((b*d*f*x + b*c*f)*cosh(d*x + c)
+ (b*d*f*x + b*c*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x
+ c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b
^2)/b^2) - b)/b) + b*f - (a*d^2*f*x^2 + 2*a*d^2*e*x)*cosh(d*x + c) - (a*d^
2*f*x^2 + 2*a*d^2*e*x - 2*(b*d*f*x + b*d*e - b*f)*cosh(d*x + c))*sinh(d*x
+ c))/(b^2*d^2*cosh(d*x + c) + b^2*d^2*sinh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(4*(a^2*e^c + b^2*e^c)*integrate(x*e^(d*x)/(b^3*e^(2*d*x + 2*c) + 2*a*
b^2*e^(d*x + c) - b^3), x) - (a*d^2*x^2*e^c - (b*d*x*e^(2*c) - b*e^(2*c))*
e^(d*x) - (b*d*x + b)*e^(-d*x))*e^(-c)/(b^2*d^2))*f - 1/2*e*(2*(d*x + c)*a
/(b^2*d) - e^(d*x + c)/(b*d) - e^(-d*x - c)/(b*d) - 2*sqrt(a^2 + b^2)*log(
(b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^
2)))/(b^2*d))
```

Giac [F]

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^2\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{4e^{dx+c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)bdei + e^{2dx+2c}b^2de + e^{2dx+2c}b^2dfx - e^{2dx+2c}b^2f - 8e^{dx+c}\left(\int \frac{x}{e^{2dx+2c}b+2e^{dx}}\right.}{}$$

input
```
int((f*x+e)*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(4*e**(c + d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2
+ b**2))*b*d*e*i + e**(2*c + 2*d*x)*b**2*d*e + e**(2*c + 2*d*x)*b**2*d*f*x
 - e**(2*c + 2*d*x)*b**2*f - 8*e**(c + d*x)*int(x/(e**(2*c + 2*d*x)*b + 2*
e**(c + d*x)*a - b),x)*a**3*d**2*f - 8*e**(c + d*x)*int(x/(e**(2*c + 2*d*x
)*b + 2*e**(c + d*x)*a - b),x)*a*b**2*d**2*f - 2*e**(c + d*x)*a*b*d**2*e*x
 - e**(c + d*x)*a*b*d**2*f*x**2 + 4*e**(d*x)*int(x/(e**(2*c + 3*d*x)*b + 2
*e**(c + 2*d*x)*a - e**(d*x)*b),x)*a**2*b*d**2*f + 4*e**(d*x)*int(x/(e**(2
*c + 3*d*x)*b + 2*e**(c + 2*d*x)*a - e**(d*x)*b),x)*b**3*d**2*f - 4*a**2*d
*f*x - 4*a**2*f + b**2*d*e - 3*b**2*d*f*x - 3*b**2*f)/(2*e**(c + d*x)*b**3
*d**2)
```

3.297 $\int \frac{\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 68

$$\int \frac{\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=-\frac{ax}{b^2}-\frac{2\sqrt{a^2+b^2}\,\mathrm{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\cosh(c+dx)}{bd}$$

output
```
-a*x/b^2-2*(a^2+b^2)^(1/2)*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/
2))/b^2/d+cosh(d*x+c)/b/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 458, normalized size of antiderivative = 6.74

$$\int \frac{\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\frac{\cosh(c+dx)\left(-2\sqrt{a-ib}\sqrt{a+ib}\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{b(i+\sinh(c+dx))}{a-ib}}}{\sqrt{-\frac{b(-i+\sinh(c+dx))}{a+ib}}}\right)\sqrt{1+i\sinh(c+dx)}+2(a-ib)\mathrm{arctanh}\left(\right.\right.}{}$$

input
```
Integrate[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

output
```
(Cosh[c + d*x]*(-2*Sqrt[a - I*b]*Sqrt[a + I*b]*ArcTanh[Sqrt[-((b*(I + Sinh
[c + d*x]))/(a - I*b))]/Sqrt[-((b*(-I + Sinh[c + d*x]))/(a + I*b))]]*Sqrt[
1 + I*Sinh[c + d*x]] + 2*(a - I*b)*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I + S
inh[c + d*x]))/(a - I*b))])/(Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[c + d*x]))
/(a + I*b))])]*Sqrt[1 + I*Sinh[c + d*x]] + Sqrt[a + I*b]*Sqrt[-((b*(-I + S
inh[c + d*x]))/(a + I*b))]*(-2*(-1)^(3/4)*Sqrt[b]*ArcSin[((-1)^(1/4)*Sqrt[
a - I*b]*Sqrt[-((b*(I + Sinh[c + d*x]))/(a - I*b))])/(Sqrt[2]*Sqrt[b])] +
Sqrt[a - I*b]*Sqrt[1 + I*Sinh[c + d*x]]*Sqrt[-((b*(I + Sinh[c + d*x]))/(a
- I*b))])))/(Sqrt[a - I*b]*Sqrt[a + I*b]*b*d*Sqrt[1 + I*Sinh[c + d*x]]*Sqr
t[-((b*(-I + Sinh[c + d*x]))/(a + I*b))]*Sqrt[-((b*(I + Sinh[c + d*x]))/(a
 - I*b))])
```

Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 3174, 26, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ic+idx)^2}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3174}$$

$$\frac{\cosh(c+dx)}{bd}+\frac{i\int -\frac{i(b-a\sinh(c+dx))}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{b-a\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\cosh(c+dx)}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh(c+dx)}{bd}+\frac{\int \frac{b+ia\sin(ic+idx)}{a-ib\sin(ic+idx)}dx}{b}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\frac{(a^2+b^2)\int \frac{1}{a+b\sinh(c+dx)}dx}{b}-\frac{ax}{b}}{b}+\frac{\cosh(c+dx)}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh(c+dx)}{bd}+\frac{-\frac{ax}{b}+\frac{(a^2+b^2)\int \frac{1}{a-ib\sin(ic+idx)}dx}{b}}{b}$$

$$\Big\downarrow \text{3139}$$

$$\frac{\cosh(c+dx)}{bd}+\frac{-\frac{ax}{b}-\frac{2i(a^2+b^2)\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{bd}}{b}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\cosh(c+dx)}{bd}+\frac{-\frac{ax}{b}+\frac{4i(a^2+b^2)\int \frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4\left(a^2+b^2\right)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{bd}}{b}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{2\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{bd}-\frac{ax}{b}}{b}+\frac{\cosh(c+dx)}{bd}$$

input
```
Int[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

output
```
(-((a*x)/b) + (2*Sqrt[a^2 + b^2]*ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b
^2])])/(b*d))/b + Cosh[c + d*x]/(b*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3174
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(b*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
 0] && IntegersQ[2*m, 2*p]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.79

method	result
risch	$-\frac{ax}{b^2}+\frac{e^{dx+c}}{2bd}+\frac{e^{-dx-c}}{2bd}+\frac{\sqrt{a^2+b^2}\,\ln\left(e^{dx+c}-\frac{-a+\sqrt{a^2+b^2}}{b}\right)}{d\,b^2}-\frac{\sqrt{a^2+b^2}\,\ln\left(e^{dx+c}+\frac{a+\sqrt{a^2+b^2}}{b}\right)}{d\,b^2}$
derivativedivides	$\frac{\frac{1}{b\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{a\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}-\frac{2\left(-a^2-b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\right.}{2\sqrt{a}}\right.}{b^2\sqrt{a^2+b^2}}}{d}$
default	$\frac{\frac{1}{b\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{a\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}-\frac{2\left(-a^2-b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\right.}{2\sqrt{a}}\right.}{b^2\sqrt{a^2+b^2}}}{d}$

input
```
int(cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-a*x/b^2+1/2/b/d*exp(d*x+c)+1/2/b/d*exp(-d*x-c)+(a^2+b^2)^(1/2)/d/b^2*ln(e
xp(d*x+c)-(-a+(a^2+b^2)^(1/2))/b)-(a^2+b^2)^(1/2)/d/b^2*ln(exp(d*x+c)+(a+(
a^2+b^2)^(1/2))/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(65) = 130.

Time = 0.12 (sec) , antiderivative size = 259, normalized size of antiderivative = 3.81

$$\int \frac{\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{2\,adx\cosh\left(dx+c\right)-b\cosh\left(dx+c\right)^2-b\sinh\left(dx+c\right)^2-2\,\sqrt{a^2+b^2}(\cosh\left(dx+c\right)+\sinh\left(dx+c\right))\,\mathrm{l}}{}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(2*a*d*x*cosh(d*x + c) - b*cosh(d*x + c)^2 - b*sinh(d*x + c)^2 - 2*sq
rt(a^2 + b^2)*(cosh(d*x + c) + sinh(d*x + c))*log((b^2*cosh(d*x + c)^2 + b
^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x +
 c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x
 + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2
*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 2*(a*d*x - b*cosh(d*x + c))*s
inh(d*x + c) - b)/(b^2*d*cosh(d*x + c) + b^2*d*sinh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.71

$$\int \frac{\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{(dx+c)a}{b^2d} + \frac{e^{(dx+c)}}{2\,bd} + \frac{e^{(-dx-c)}}{2\,bd} + \frac{\sqrt{a^2+b^2}\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{b^2d}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-(d*x + c)*a/(b^2*d) + 1/2*e^(d*x + c)/(b*d) + 1/2*e^(-d*x - c)/(b*d) + sq
rt(a^2 + b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) -
 a + sqrt(a^2 + b^2)))/(b^2*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.62

$$\int \frac{\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= -\frac{\frac{2\,(dx+c)a}{b^2} - \frac{e^{(dx+c)}}{b} - \frac{e^{(-dx-c)}}{b} - \frac{2\,\sqrt{a^2+b^2}\,\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{b^2}}{2\,d}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(2*(d*x + c)*a/b^2 - e^(d*x + c)/b - e^(-d*x - c)/b - 2*sqrt(a^2 + b^
2)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c)
+ 2*a + 2*sqrt(a^2 + b^2)))/b^2)/d
```

Mupad [B] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.78

$$\int \frac{\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\mathrm{e}^{c+d\,x}}{2\,b\,d} - \frac{2\,\mathrm{atan}\left(\frac{a\,\sqrt{-b^4\,d^2}}{b^2\,d\,\sqrt{a^2+b^2}} + \frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-b^4\,d^2}}{b\,d\,\sqrt{a^2+b^2}}\right)\,\sqrt{a^2+b^2}}{\sqrt{-b^4\,d^2}} + \frac{\mathrm{e}^{-c-d\,x}}{2\,b\,d} - \frac{a\,x}{b^2}$$

input
```
int(cosh(c + d*x)^2/(a + b*sinh(c + d*x)),x)
```

output
```
exp(c + d*x)/(2*b*d) - (2*atan((a*(-b^4*d^2)^(1/2))/(b^2*d*(a^2 + b^2)^(1/
2)) + (exp(d*x)*exp(c)*(-b^4*d^2)^(1/2))/(b*d*(a^2 + b^2)^(1/2)))*(a^2 + b
^2)^(1/2))/(-b^4*d^2)^(1/2) + exp(- c - d*x)/(2*b*d) - (a*x)/b^2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.28

$$\int \frac{\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{4e^{dx+c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)i+e^{2dx+2c}b-2e^{dx+c}adx+b}{2e^{dx+c}b^2d}$$

input

```
int(cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output

```
(4*e**(c + d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2
+ b**2))*i + e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a*d*x + b)/(2*e**(c + d*x
)*b**2*d)
```

3.298 $\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \text{Int}\left(\frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}, x\right)$$

output
```
Defer(Int)(cosh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 9.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

input
```
Integrate[Cosh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Cosh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

$$\Big\downarrow \; 6111$$

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Cosh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(dx+c\right)^{2}}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(cosh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(cosh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 166, normalized size of antiderivative = 5.93

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
2*(a^2*e^c + b^2*e^c)*integrate(-e^(d*x)/(b^3*f*x + b^3*e - (b^3*f*x*e^(2*
c) + b^3*e*e^(2*c))*e^(2*d*x) - 2*(a*b^2*f*x*e^c + a*b^2*e*e^c)*e^(d*x)),
x) + 1/2*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b*f) - 1/2*e^(c
- d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b*f) - a*log(f*x + e)/(b^2*f)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)^2/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)^2}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(cosh(c + d*x)^2/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(cosh(c + d*x)^2/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 238, normalized size of antiderivative = 8.50

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$= \frac{e^{4c}\left(\int \frac{e^{3dx}}{e^{2dx+2c}be+e^{2dx+2c}bfx+2e^{dx+c}ae+2e^{dx+c}afx-be-bfx}dx\right)+2e^{2c}\left(\int \frac{e^{dx}}{e^{2dx+2c}be+e^{2dx+2c}bfx+2e^{dx+c}ae+2e^{dx+c}afx-be-bf}\right.}{2e^{c}}$$

input
```
int(cosh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
(e**(4*c)*int(e**(3*d*x)/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x +
2*e**(c + d*x)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x) + 2*e**(2*c)*i
nt(e**(d*x)/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x + 2*e**(c + d*x
)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x) + int(1/(e**(2*c + 3*d*x)*b
*e + e**(2*c + 3*d*x)*b*f*x + 2*e**(c + 2*d*x)*a*e + 2*e**(c + 2*d*x)*a*f*
x - e**(d*x)*b*e - e**(d*x)*b*f*x),x))/(2*e**c)
```

3.299 $\int \frac{(e+fx)^3 \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 642

$$\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{3f^3x}{8bd^3}+\frac{(e+fx)^3}{4bd}-\frac{(a^2+b^2)(e+fx)^4}{4b^3f} + \frac{6af^3\cosh(c+dx)}{b^2d^4}+\frac{3af(e+fx)^2\cosh(c+dx)}{b^2d^2} + \frac{(a^2+b^2)(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d} + \frac{(a^2+b^2)(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d} + \frac{3(a^2+b^2)f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2} + \frac{3(a^2+b^2)f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^2} - \frac{6(a^2+b^2)f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3} - \frac{6(a^2+b^2)f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^3} + \frac{6(a^2+b^2)f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^4} + \frac{6(a^2+b^2)f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^4} - \frac{6af^2(e+fx)\sinh(c+dx)}{b^2d^3}-\frac{a(e+fx)^3\sinh(c+dx)}{b^2d} - \frac{3f^3\cosh(c+dx)\sinh(c+dx)}{8bd^4} - \frac{3f(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{4bd^2} + \frac{3f^2(e+fx)\sinh^2(c+dx)}{4bd^3}+\frac{(e+fx)^3\sinh^2(c+dx)}{2bd}$$

output
```
3/8*f^3*x/b/d^3+1/4*(f*x+e)^3/b/d-1/4*(a^2+b^2)*(f*x+e)^4/b^3/f+6*a*f^3*co
sh(d*x+c)/b^2/d^4+3*a*f*(f*x+e)^2*cosh(d*x+c)/b^2/d^2+(a^2+b^2)*(f*x+e)^3*
ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d+(a^2+b^2)*(f*x+e)^3*ln(1+b*ex
p(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d+3*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-b*e
xp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^2+3*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-
b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^2-6*(a^2+b^2)*f^2*(f*x+e)*polylog(
3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^3-6*(a^2+b^2)*f^2*(f*x+e)*polyl
og(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^3+6*(a^2+b^2)*f^3*polylog(4,
-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^4+6*(a^2+b^2)*f^3*polylog(4,-b*ex
p(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^4-6*a*f^2*(f*x+e)*sinh(d*x+c)/b^2/d^3-
a*(f*x+e)^3*sinh(d*x+c)/b^2/d-3/8*f^3*cosh(d*x+c)*sinh(d*x+c)/b/d^4-3/4*f*
(f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/b/d^2+3/4*f^2*(f*x+e)*sinh(d*x+c)^2/b/d^
3+1/2*(f*x+e)^3*sinh(d*x+c)^2/b/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1977 vs. $2(642) = 1284$.

Time = 8.34 (sec) , antiderivative size = 1977, normalized size of antiderivative = 3.08

$$\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(32*(a^2 + b^2)*e^3*x*Coth[c] + 48*(a^2 + b^2)*e^2*f*x^2*Coth[c] + 32*(a^2
 + b^2)*e*f^2*x^3*Coth[c] + 8*(a^2 + b^2)*f^3*x^4*Coth[c] - (16*(a^2 + b^2
)*(4*e^3*E^(2*c)*x + 6*e^2*E^(2*c)*f*x^2 + 4*e*E^(2*c)*f^2*x^3 + E^(2*c)*f
^3*x^4 + (4*a*Sqrt[a^2 + b^2]*e^3*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b
^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (4*a*Sqrt[-a^2 - b^2]*e^3*ArcTanh[(a + b*
E^(c + d*x))/Sqrt[a^2 + b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) - (2*e^3*E^(2*c)*L
og[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))])/d + (2*e^3*Log[2*a*E^(c + d*x
) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a
*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e^2*E^(2*c)*f*x*Log[1 + (b*E^(2
*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f^2*x^2*Log[1 +
(b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f
^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d +
 (2*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])]
)/d - (2*E^(2*c)*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^
2)*E^(2*c)])])/d + (6*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2
 + b^2)*E^(2*c)])])/d - (6*e^2*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^
c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f^2*x^2*Log[1 + (b*E^(2*c + d*x)
)/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f^2*x^2*Log[1 + (
b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (2*f^3*x^3*Log[
1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (2*E^(2...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 3.67 (sec) , antiderivative size = 591, normalized size of antiderivative = 0.92, number of steps used = 30, number of rules used = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 1.036$, Rules used = {6099, 3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118, 5969, 3042, 25, 3792, 17, 25, 3042, 25, 3115, 24, 6095, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6099}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)^3\cosh(c+dx)dx}{b^2}+\frac{\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$\Big\downarrow$ 3042

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)^3\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b^2}+\frac{\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$\Big\downarrow$ 3777

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3if\int -i(e+fx)^2\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$\Big\downarrow$ 26

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int (e+fx)^2\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$\Big\downarrow$ 3042

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int -i(e+fx)^2\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$\Big\downarrow$ 26

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\int (e+fx)^2\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$\Big\downarrow$ 3777

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\cosh(c+dx)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{d}\right)}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)}{d}\right)}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}\right)}{d}\right)}{b^2}+\frac{\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$\Big\downarrow$ 26

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}\right)}{d}\right)}{d}\right)}{b^2}+\frac{\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$\Big\downarrow$ 3118

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$\Big\downarrow$ 5969

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}-\frac{3f\int (e+fx)^2\sinh^2(c+dx)dx}{2d}}{b}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$\Big\downarrow$ 3042

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}-\frac{3f\int -(e+fx)^2\sin(ic+idx)^2dx}{2d}}{b}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$\downarrow$ 25

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}+\frac{3f\int (e+fx)^2\sin(ic+idx)^2dx}{2d}}{b}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$\downarrow$ 3792

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{3f\left(\frac{f^2\int -\sinh^2(c+dx)dx}{2d^2}+\frac{1}{2}\int (e+fx)^2dx+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$\downarrow$ 17

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{3f\left(\frac{f^2\int -\sinh^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$\downarrow$ 25

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{3f\left(-\frac{f^2\int \sinh^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}+\frac{3f\left(-\frac{f^2\int -\sin(ic+idx)^2dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}}{b}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \; 25$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}+\frac{3f\left(\frac{f^2\int \sin(ic+idx)^2dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}}{b}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \; 3115$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{3f\left(\frac{f^2\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$\downarrow$ 24

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}+\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}$$

$\downarrow$ 6095

$$\frac{\left(a^2+b^2\right)\left(\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^4}{4bf}\right)}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}+\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}$$

$\downarrow$ 2620

$$\frac{\left(a^2+b^2\right)\left(-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log}{}\right.}{b^2}$$

$$\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}+$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}$$

$$\Big\downarrow \; 3011$$

$$\frac{\left(a^2+b^2\right)\left(-\frac{3f\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}\right.}{bd}\right.}{b^2}$$

$$\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}+$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}$$

$$\Big\downarrow \; 7163$$

$$\frac{(a^2+b^2)\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int \operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\cdots\right.}{}\cdots\right.}{}\cdots\right)}{\cdots}$$

$$\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}+$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}$$

$$\Big\downarrow \; 2720$$

$$\frac{(a^2+b^2)\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\cdots\right)}{\cdots}$$

$$\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}+$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}$$

$$\Big\downarrow \; 7143$$

$$\frac{(a^2+b^2)\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+f\cdots}{}\right.}{}\right.}{}\right.}{}$$

$$\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}+$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
((a^2 + b^2)*(-1/4*(e + f*x)^4/(b*f) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x)
)/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - S
qrt[a^2 + b^2]))])/d^2))/d))/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/b^2 - (a*(((e + f*x)^3*Sinh[c
 + d*x])/d + ((3*I)*f*((I*(e + f*x)^2*Cosh[c + d*x])/d - ((2*I)*f*(-((f*Co
sh[c + d*x])/d^2) + ((e + f*x)*Sinh[c + d*x])/d))/d))/d))/b^2 + (((e + f*x
)^3*Sinh[c + d*x]^2)/(2*d) + (3*f*((e + f*x)^3/(6*f) - ((e + f*x)^2*Cosh[c
 + d*x]*Sinh[c + d*x])/(2*d) + (f*(e + f*x)*Sinh[c + d*x]^2)/(2*d^2) + (f^
2*(x/2 - (Cosh[c + d*x]*Sinh[c + d*x])/(2*d)))/(2*d^2)))/(2*d))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(fx+e)^3 \cosh(dx+c)^3}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4371 vs. $2(602) = 1204$.

Time = 0.17 (sec) , antiderivative size = 4371, normalized size of antiderivative = 6.81

$$\int \frac{(e+fx)^3 \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh\left(dx+c\right)^3}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/8*e^3*((4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) - 8*(a^2 + b^2)*(
d*x + c)/(b^3*d) - (4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d) - 8*(a^
2 + b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d)) + 1/32*(
8*(a^2*d^4*f^3*e^(2*c) + b^2*d^4*f^3*e^(2*c))*x^4 + 32*(a^2*d^4*e*f^2*e^(2
*c) + b^2*d^4*e*f^2*e^(2*c))*x^3 + 48*(a^2*d^4*e^2*f*e^(2*c) + b^2*d^4*e^2
*f*e^(2*c))*x^2 + (4*b^2*d^3*f^3*x^3*e^(4*c) + 6*(2*d^3*e*f^2 - d^2*f^3)*b
^2*x^2*e^(4*c) + 6*(2*d^3*e^2*f - 2*d^2*e*f^2 + d*f^3)*b^2*x*e^(4*c) - 3*(
2*d^2*e^2*f - 2*d*e*f^2 + f^3)*b^2*e^(4*c))*e^(2*d*x) - 16*(a*b*d^3*f^3*x^
3*e^(3*c) + 3*(d^3*e*f^2 - d^2*f^3)*a*b*x^2*e^(3*c) + 3*(d^3*e^2*f - 2*d^2
*e*f^2 + 2*d*f^3)*a*b*x*e^(3*c) - 3*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*a*b*e^
(3*c))*e^(d*x) + 16*(a*b*d^3*f^3*x^3*e^c + 3*(d^3*e*f^2 + d^2*f^3)*a*b*x^2
*e^c + 3*(d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)*a*b*x*e^c + 3*(d^2*e^2*f + 2*
d*e*f^2 + 2*f^3)*a*b*e^c)*e^(-d*x) + (4*b^2*d^3*f^3*x^3 + 6*(2*d^3*e*f^2 +
 d^2*f^3)*b^2*x^2 + 6*(2*d^3*e^2*f + 2*d^2*e*f^2 + d*f^3)*b^2*x + 3*(2*d^2
*e^2*f + 2*d*e*f^2 + f^3)*b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^4) - integrate(
-2*((a^2*b*f^3 + b^3*f^3)*x^3 + 3*(a^2*b*e*f^2 + b^3*e*f^2)*x^2 + 3*(a^2*b
*e^2*f + b^3*e^2*f)*x - ((a^3*f^3*e^c + a*b^2*f^3*e^c)*x^3 + 3*(a^3*e*f^2*
e^c + a*b^2*e*f^2*e^c)*x^2 + 3*(a^3*e^2*f*e^c + a*b^2*e^2*f*e^c)*x)*e^(d*x
))/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) - b^4), x)
```

Giac [F]

$$\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\cosh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^3\,(e+f\,x)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh\left(dx+c\right)^3}{a+b\sinh\left(dx+c\right)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

3.300 $\int \frac{(e+fx)^2 \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 465

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{(e+fx)^2}{4bd} - \frac{(a^2+b^2)(e+fx)^3}{3b^3f} + \frac{2af(e+fx)\cosh(c+dx)}{b^2d^2} + \frac{(a^2+b^2)(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d} + \frac{(a^2+b^2)(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d} + \frac{2(a^2+b^2)f(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2} + \frac{2(a^2+b^2)f(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^2} - \frac{2(a^2+b^2)f^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3} - \frac{2(a^2+b^2)f^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^3} - \frac{2af^2\sinh(c+dx)}{b^2d^3} - \frac{a(e+fx)^2\sinh(c+dx)}{b^2d} - \frac{f(e+fx)\cosh(c+dx)\sinh(c+dx)}{2bd^2} + \frac{f^2\sinh^2(c+dx)}{4bd^3} + \frac{(e+fx)^2\sinh^2(c+dx)}{2bd}$$

output

```
1/4*(f*x+e)^2/b/d-1/3*(a^2+b^2)*(f*x+e)^3/b^3/f+2*a*f*(f*x+e)*cosh(d*x+c)/
b^2/d^2+(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d+(
a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d+2*(a^2+b^2
)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^2+2*(a^2+b^
2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^2-2*(a^2+b
^2)*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^3-2*(a^2+b^2)*f
^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^3-2*a*f^2*sinh(d*x+c
)/b^2/d^3-a*(f*x+e)^2*sinh(d*x+c)/b^2/d-1/2*f*(f*x+e)*cosh(d*x+c)*sinh(d*x
+c)/b/d^2+1/4*f^2*sinh(d*x+c)^2/b/d^3+1/2*(f*x+e)^2*sinh(d*x+c)^2/b/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1253 vs. 2(465) = 930.

Time = 8.69 (sec) , antiderivative size = 1253, normalized size of antiderivative = 2.69

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(8*(a^2 + b^2)*x*(3*e^2 + 3*e*f*x + f^2*x^2)*Coth[c] - (8*(a^2 + b^2)*(6*e
^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2
]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*
d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt
[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTa
nh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqr
t[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]]
)/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c +
d*x)))])/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))
])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c
)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 +
b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a
^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*
E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/
(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2
*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b
*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x
^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*
(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a
^2 + b^2)*E^(2*c)]))])/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.41 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.92, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.679$, Rules used = {6099, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117, 5969, 3042, 25, 3791, 17, 6095, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6099}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)^2\cosh(c+dx)dx}{b^2}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b^2}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int (e+fx)\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int -i(e+fx)\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int (e+fx)\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{ 3117}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \text{ 5969}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}-\frac{f\int (e+fx)\sinh^2(c+dx)dx}{d}}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b^2}$$

$\downarrow$ 3042

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}-\frac{f\int -((e+fx)\sin(ic+idx)^2)dx}{d}}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b^2}$$

$\downarrow$ 25

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}+\frac{f\int (e+fx)\sin(ic+idx)^2dx}{d}}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b^2}$$

$\downarrow$ 3791

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{f\left(\frac{1}{2}\int (e+fx)dx+\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b^2}$$

$\downarrow$ 17

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b^2}+\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}$$

$\downarrow$ 6095

$$\frac{(a^2+b^2)\left(\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b^2}+\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}$$

$\downarrow$ 2620

$$\frac{(a^2+b^2)\left(-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log}{}\right.}{b^2}$$

$$\frac{\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b^2}+\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}$$

$\downarrow$ 3011

$$\frac{(a^2+b^2)\left(-\frac{2f\left(\frac{f\int\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}}{}\right.}{bd}\right.}{b^2}$$

$$\frac{\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b^2}+\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}$$

$\downarrow$ 2720

$$\frac{(a^2+b^2)\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right.}{d^2}\right.}{}\right.}{b^2}$$

$$\frac{\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b^2}+\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}$$

$\downarrow$ 7143

$$\frac{(a^2+b^2)\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{b}{a+}\right.}{d}\right.}{bd}\right.}{b^2}$$

$$\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b^2}+\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
((a^2 + b^2)*(-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x)
)/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x
))/(a - Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt
[a^2 + b^2]))])/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2]))])/d^2))/(b*d)))/b^2 - (a*(((e + f*x)^2*Sinh[c + d*x])/d + (
(2*I)*f*((I*(e + f*x)*Cosh[c + d*x])/d - (I*f*Sinh[c + d*x])/d^2))/d))/b^2
 + (((e + f*x)^2*Sinh[c + d*x]^2)/(2*d) + (f*((e + f*x)^2/(4*f) - ((e + f*
x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (f*Sinh[c + d*x]^2)/(4*d^2)))/d)/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(fx+e)^2 \cosh (dx+c)^3}{a+b\sinh (dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2726 vs. $2(435) = 870$.

Time = 0.15 (sec) , antiderivative size = 2726, normalized size of antiderivative = 5.86

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/48*(6*b^2*d^2*f^2*x^2 + 6*b^2*d^2*e^2 + 6*b^2*d*e*f + 3*(2*b^2*d^2*f^2*x
^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)
*x)*cosh(d*x + c)^4 + 3*(2*b^2*d^2*f^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f +
 b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)*sinh(d*x + c)^4 + 3*b^2*f^2 -
24*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e
*f - a*b*d*f^2)*x)*cosh(d*x + c)^3 - 12*(2*a*b*d^2*f^2*x^2 + 2*a*b*d^2*e^2
 - 4*a*b*d*e*f + 4*a*b*f^2 + 4*(a*b*d^2*e*f - a*b*d*f^2)*x - (2*b^2*d^2*f^
2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f
^2)*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 16*((a^2 + b^2)*d^3*f^2*x^3 + 3*(a
^2 + b^2)*d^3*e*f*x^2 + 3*(a^2 + b^2)*d^3*e^2*x + 6*(a^2 + b^2)*c*d^2*e^2
- 6*(a^2 + b^2)*c^2*d*e*f + 2*(a^2 + b^2)*c^3*f^2)*cosh(d*x + c)^2 - 2*(8*
(a^2 + b^2)*d^3*f^2*x^3 + 24*(a^2 + b^2)*d^3*e*f*x^2 + 24*(a^2 + b^2)*d^3*
e^2*x + 48*(a^2 + b^2)*c*d^2*e^2 - 48*(a^2 + b^2)*c^2*d*e*f + 16*(a^2 + b^
2)*c^3*f^2 - 9*(2*b^2*d^2*f^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2
+ 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)*cosh(d*x + c)^2 + 36*(a*b*d^2*f^2*x^2 +
 a*b*d^2*e^2 - 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f - a*b*d*f^2)*x)*co
sh(d*x + c))*sinh(d*x + c)^2 + 6*(2*b^2*d^2*e*f + b^2*d*f^2)*x + 24*(a*b*d
^2*f^2*x^2 + a*b*d^2*e^2 + 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f + a*b*
d*f^2)*x)*cosh(d*x + c) + 96*(((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*co
sh(d*x + c)^2 + 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*cosh(d*x + ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/8*e^2*((4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) - 8*(a^2 + b^2)*(
d*x + c)/(b^3*d) - (4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d) - 8*(a^
2 + b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d)) + 1/48*(
16*(a^2*d^3*f^2*e^(2*c) + b^2*d^3*f^2*e^(2*c))*x^3 + 48*(a^2*d^3*e*f*e^(2*
c) + b^2*d^3*e*f*e^(2*c))*x^2 + 3*(2*b^2*d^2*f^2*x^2*e^(4*c) + 2*(2*d^2*e*
f - d*f^2)*b^2*x*e^(4*c) - (2*d*e*f - f^2)*b^2*e^(4*c))*e^(2*d*x) - 24*(a*
b*d^2*f^2*x^2*e^(3*c) + 2*(d^2*e*f - d*f^2)*a*b*x*e^(3*c) - 2*(d*e*f - f^2
)*a*b*e^(3*c))*e^(d*x) + 24*(a*b*d^2*f^2*x^2*e^c + 2*(d^2*e*f + d*f^2)*a*b
*x*e^c + 2*(d*e*f + f^2)*a*b*e^c)*e^(-d*x) + 3*(2*b^2*d^2*f^2*x^2 + 2*(2*d
^2*e*f + d*f^2)*b^2*x + (2*d*e*f + f^2)*b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^3
) - integrate(-2*((a^2*b*f^2 + b^3*f^2)*x^2 + 2*(a^2*b*e*f + b^3*e*f)*x -
((a^3*f^2*e^c + a*b^2*f^2*e^c)*x^2 + 2*(a^3*e*f*e^c + a*b^2*e*f*e^c)*x)*e^
(d*x))/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) - b^4), x)
```

Giac [F]

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\cosh{(dx+c)}^3}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\cosh(c+d\,x)^3\,(e+f\,x)^2}{a+b\sinh{(c+d\,x)}}\,dx$$

input
```
int((cosh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Too large to display}$$

input
```
int((f*x+e)^2*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
(6*e**(4*c + 4*d*x)*b**4*d**2*e**2 + 12*e**(4*c + 4*d*x)*b**4*d**2*e*f*x +
 6*e**(4*c + 4*d*x)*b**4*d**2*f**2*x**2 - 6*e**(4*c + 4*d*x)*b**4*d*e*f -
6*e**(4*c + 4*d*x)*b**4*d*f**2*x + 3*e**(4*c + 4*d*x)*b**4*f**2 - 24*e**(3
*c + 3*d*x)*a*b**3*d**2*e**2 - 48*e**(3*c + 3*d*x)*a*b**3*d**2*e*f*x - 24*
e**(3*c + 3*d*x)*a*b**3*d**2*f**2*x**2 + 48*e**(3*c + 3*d*x)*a*b**3*d*e*f
+ 48*e**(3*c + 3*d*x)*a*b**3*d*f**2*x - 48*e**(3*c + 3*d*x)*a*b**3*f**2 +
192*e**(2*c + 2*d*x)*int(x**2/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a -
 e**(2*c + 2*d*x)*b),x)*a**4*b*d**3*f**2 + 288*e**(2*c + 2*d*x)*int(x**2/(
e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b),x)*a**2*b*
*3*d**3*f**2 + 96*e**(2*c + 2*d*x)*int(x**2/(e**(4*c + 4*d*x)*b + 2*e**(3*
c + 3*d*x)*a - e**(2*c + 2*d*x)*b),x)*b**5*d**3*f**2 + 384*e**(2*c + 2*d*x
)*int(x/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b),x
)*a**4*b*d**3*e*f + 576*e**(2*c + 2*d*x)*int(x/(e**(4*c + 4*d*x)*b + 2*e**
(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b),x)*a**2*b**3*d**3*e*f + 192*e**(2*c
+ 2*d*x)*int(x/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*
x)*b),x)*b**5*d**3*e*f + 48*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x)*b + 2*e*
*(c + d*x)*a - b)*a**2*b**2*d**2*e**2 + 48*e**(2*c + 2*d*x)*log(e**(2*c +
2*d*x)*b + 2*e**(c + d*x)*a - b)*b**4*d**2*e**2 - 48*e**(2*c + 2*d*x)*a**2
*b**2*d**3*e**2*x + 48*e**(2*c + 2*d*x)*a**2*b**2*d**3*e*f*x**2 + 16*e**(2
*c + 2*d*x)*a**2*b**2*d**3*f**2*x**3 - 48*e**(2*c + 2*d*x)*b**4*d**3*e*...
```

3.301 $\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 298

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{fx}{4bd}-\frac{(a^2+b^2)(e+fx)^2}{2b^3f}+\frac{af\cosh(c+dx)}{b^2d}+\frac{(a^2+b^2)(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d}+\frac{(a^2+b^2)(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d}+\frac{(a^2+b^2)f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{(a^2+b^2)f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^2}-\frac{a(e+fx)\sinh(c+dx)}{b^2d}-\frac{f\cosh(c+dx)\sinh(c+dx)}{4bd^2}+\frac{(e+fx)\sinh^2(c+dx)}{2bd}$$

output
```
1/4*f*x/b/d-1/2*(a^2+b^2)*(f*x+e)^2/b^3/f+a*f*cosh(d*x+c)/b^2/d^2+(a^2+b^2
)*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d+(a^2+b^2)*(f*x+e)*l
n(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d+(a^2+b^2)*f*polylog(2,-b*exp(d
*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^2+(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/(a+
(a^2+b^2)^(1/2)))/b^3/d^2-a*(f*x+e)*sinh(d*x+c)/b^2/d-1/4*f*cosh(d*x+c)*si
nh(d*x+c)/b/d^2+1/2*(f*x+e)*sinh(d*x+c)^2/b/d
```

Mathematica [A] (warning: unable to verify)

Time = 3.29 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.40

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{8abf\cosh(c+dx)+2b^2d(e+fx)\cosh(2(c+dx))+4(a^2+b^2)\Bigg(-2de(c+dx)+2cf(c+dx)-f(c+}{}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(8*a*b*f*Cosh[c + d*x] + 2*b^2*d*(e + f*x)*Cosh[2*(c + d*x)] + 4*(a^2 + b^
2)*(-2*d*e*(c + d*x) + 2*c*f*(c + d*x) - f*(c + d*x)^2 + (4*a*Sqrt[a^2 + b
^2]*d*e*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b^2)^2]
 - (4*a*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^
2]])/(-a^2 - b^2)^(3/2) + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[
a^2 + b^2])] + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
] - 2*c*f*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*d*e*Log[2*a*E^(
c + d*x) + b*(-1 + E^(2*(c + d*x)))] + 2*f*PolyLog[2, (b*E^(c + d*x))/(-a
+ Sqrt[a^2 + b^2])] + 2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
]))]) - 8*a*b*d*(e + f*x)*Sinh[c + d*x] - b^2*f*Sinh[2*(c + d*x)])/(8*b^3*
d^2)
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.92, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {6099, 3042, 3777, 26, 3042, 26, 3118, 5969, 3042, 25, 3115, 24, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6099}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)\cosh(c+dx)dx}{b^2}+\frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b^2}+\frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3118}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}$$

$$\Big\downarrow \text{5969}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}-\frac{f\int \sinh^2(c+dx)dx}{2d}}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}-\frac{f\int -\sin(ic+idx)^2dx}{2d}}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\int \sin(ic+idx)^2dx}{2d}}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}$$

$$\Big\downarrow \text{3115}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{f\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}+\frac{(e+fx)\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}$$

$\downarrow$ 6095

$$\frac{(a^2+b^2)\left(\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}$$

$\downarrow$ 2620

$$\frac{(a^2+b^2)\left(-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\right.}{b^2}$$
$$\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}$$

$\downarrow$ 2715

$$\frac{(a^2+b^2)\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+f\right.}{b^2}$$
$$\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}$$

$\downarrow$ 2838

$$\frac{(a^2+b^2)\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\right.}{b^2}$$
$$\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}$$

input
```
Int[((e + f*x)*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
((a^2 + b^2)*(-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2]))])/(b*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])
/(b*d^2)))/b^2 - (a*(-((f*Cosh[c + d*x])/d^2) + ((e + f*x)*Sinh[c + d*x])/
d))/b^2 + (((e + f*x)*Sinh[c + d*x]^2)/(2*d) + (f*(x/2 - (Cosh[c + d*x]*Si
nh[c + d*x])/(2*d)))/(2*d))/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 974 vs. $2(278) = 556$.

Time = 5.82 (sec) , antiderivative size = 975, normalized size of antiderivative = 3.27

method	result
risch	$\frac{a^2 f \ln\left(\frac{b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}+a}{a+\sqrt{a^2+b^2}}\right) c}{d^2 b^3} + \frac{2c\,a^2 f \ln(\mathrm{e}^{dx+c})}{d^2 b^3} - \frac{a^2 f\,c^2}{d^2 b^3} - \frac{2a^2 e \ln(\mathrm{e}^{dx+c})}{d\,b^3} + \frac{a^2 e \ln\left(b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}^{dx+c}-b\right)}{d\,b^3} + \frac{a^2 f\,\mathrm{dil}}{}$

input
```
int((f*x+e)*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d^2/b^3*a^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c
+2/d^2/b^3*c*a^2*f*ln(exp(d*x+c))-1/d^2/b^3*a^2*f*c^2-2/d/b^3*a^2*e*ln(exp
(d*x+c))+1/d/b^3*a^2*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/d^2/b^3*a^2
*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/d^2/b^3
*a^2*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d^2/b
*f*c^2+1/d/b*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d/b*e*ln(exp(d*x+c)
)+1/d^2/b*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/
d^2/b*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-2/d/
b^3*a^2*f*c*x+1/d/b^3*a^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+
b^2)^(1/2)))*x+1/d/b^3*a^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b
^2)^(1/2)))*x+1/d^2/b^3*a^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^
2+b^2)^(1/2)))*c-1/2*a*(d*f*x+d*e-f)/b^2/d^2*exp(d*x+c)-1/d^2/b^3*c*a^2*f*
ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d/b*f*c*x+1/d^2/b*f*ln((-b*exp(d*x
+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d^2/b*f*ln((b*exp(d*x+c)+
(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2/d^2/b*c*f*ln(exp(d*x+c))-1/d^2
/b*c*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/d/b*f*ln((-b*exp(d*x+c)+(a^
2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d/b*f*ln((b*exp(d*x+c)+(a^2+b^2)
^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/2/b^3*a^2*f*x^2-1/2/b*f*x^2+1/b^3*a^2*e
*x+1/16*(2*d*f*x+2*d*e-f)/b/d^2*exp(2*d*x+2*c)+1/16*(2*d*f*x+2*d*e+f)/b/d^
2*exp(-2*d*x-2*c)+1/b*e*x+1/2*a*(d*f*x+d*e+f)/b^2/d^2*exp(-d*x-c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1416 vs. $2(276) = 552$.

Time = 0.13 (sec) , antiderivative size = 1416, normalized size of antiderivative = 4.75

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/16*(2*b^2*d*f*x + (2*b^2*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c)^4 + (2
*b^2*d*f*x + 2*b^2*d*e - b^2*f)*sinh(d*x + c)^4 + 2*b^2*d*e - 8*(a*b*d*f*x
 + a*b*d*e - a*b*f)*cosh(d*x + c)^3 - 4*(2*a*b*d*f*x + 2*a*b*d*e - 2*a*b*f
 - (2*b^2*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c))*sinh(d*x + c)^3 + b^2*
f - 8*((a^2 + b^2)*d^2*f*x^2 + 2*(a^2 + b^2)*d^2*e*x + 4*(a^2 + b^2)*c*d*e
 - 2*(a^2 + b^2)*c^2*f)*cosh(d*x + c)^2 - 2*(4*(a^2 + b^2)*d^2*f*x^2 + 8*(
a^2 + b^2)*d^2*e*x + 16*(a^2 + b^2)*c*d*e - 8*(a^2 + b^2)*c^2*f - 3*(2*b^2
*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c)^2 + 12*(a*b*d*f*x + a*b*d*e - a*
b*f)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*(a*b*d*f*x + a*b*d*e + a*b*f)*cosh
(d*x + c) + 16*((a^2 + b^2)*f*cosh(d*x + c)^2 + 2*(a^2 + b^2)*f*cosh(d*x +
 c)*sinh(d*x + c) + (a^2 + b^2)*f*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c)
+ a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2) - b)/b + 1) + 16*((a^2 + b^2)*f*cosh(d*x + c)^2 + 2*(a^2 + b^2)*f*cosh
(d*x + c)*sinh(d*x + c) + (a^2 + b^2)*f*sinh(d*x + c)^2)*dilog((a*cosh(d*x
 + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2) - b)/b + 1) + 16*(((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x +
 c)^2 + 2*((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c)
+ ((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*sinh(d*x + c)^2)*log(2*b*cosh(d*x +
c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 16*(((a^2 + b^
2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c)^2 + 2*((a^2 + b^2)*d*e - (a^2 +...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/8*e*((4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) - 8*(a^2 + b^2)*(d*
x + c)/(b^3*d) - (4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d) - 8*(a^2
+ b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d)) + 1/16*f*(
(8*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*x^2 + (2*b^2*d*x*e^(4*c) - b^2*e^(4
*c))*e^(2*d*x) - 8*(a*b*d*x*e^(3*c) - a*b*e^(3*c))*e^(d*x) + 8*(a*b*d*x*e^
c + a*b*e^c)*e^(-d*x) + (2*b^2*d*x + b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^2) -
 2*integrate(16*((a^3*e^c + a*b^2*e^c)*x*e^(d*x) - (a^2*b + b^3)*x)/(b^4*e
^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) - b^4), x))
```

Giac [F]

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh\left(dx+c\right)^3}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^3\,(e+f\,x)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{2e^{4dx+4c}b^4de - 24a^2b^2f + 2b^4de - e^{4dx+4c}b^4f - 48a^2b^2dfx - 8e^{3dx+3c}ab^3dfx + 16e^{2dx+2c}\log\left(e^{2dx+2c}b + 2\right.}{}$$

input
```
int((f*x+e)*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
(2*e**(4*c + 4*d*x)*b**4*d*e + 2*e**(4*c + 4*d*x)*b**4*d*f*x - e**(4*c + 4
*d*x)*b**4*f - 8*e**(3*c + 3*d*x)*a*b**3*d*e - 8*e**(3*c + 3*d*x)*a*b**3*d
*f*x + 8*e**(3*c + 3*d*x)*a*b**3*f + 64*e**(2*c + 2*d*x)*int(x/(e**(4*c +
4*d*x)*b + 2*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b),x)*a**4*b*d**2*f + 9
6*e**(2*c + 2*d*x)*int(x/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a - e**(
2*c + 2*d*x)*b),x)*a**2*b**3*d**2*f + 32*e**(2*c + 2*d*x)*int(x/(e**(4*c +
 4*d*x)*b + 2*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b),x)*b**5*d**2*f + 16
*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*a**2*b**2
*d*e + 16*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*
b**4*d*e - 16*e**(2*c + 2*d*x)*a**2*b**2*d**2*e*x + 8*e**(2*c + 2*d*x)*a**
2*b**2*d**2*f*x**2 - 16*e**(2*c + 2*d*x)*b**4*d**2*e*x + 8*e**(2*c + 2*d*x
)*b**4*d**2*f*x**2 - 128*e**(c + 2*d*x)*int(x/(e**(2*c + 3*d*x)*b + 2*e**(
c + 2*d*x)*a - e**(d*x)*b),x)*a**5*d**2*f - 224*e**(c + 2*d*x)*int(x/(e**(
2*c + 3*d*x)*b + 2*e**(c + 2*d*x)*a - e**(d*x)*b),x)*a**3*b**2*d**2*f - 96
*e**(c + 2*d*x)*int(x/(e**(2*c + 3*d*x)*b + 2*e**(c + 2*d*x)*a - e**(d*x)*
b),x)*a*b**4*d**2*f + 32*e**(c + d*x)*a**3*b*d*f*x + 32*e**(c + d*x)*a**3*
b*f + 8*e**(c + d*x)*a*b**3*d*e + 40*e**(c + d*x)*a*b**3*d*f*x + 40*e**(c
+ d*x)*a*b**3*f - 32*a**4*d*f*x - 16*a**4*f - 48*a**2*b**2*d*f*x - 24*a**2
*b**2*f + 2*b**4*d*e - 14*b**4*d*f*x - 7*b**4*f)/(16*e**(2*c + 2*d*x)*b**5
*d**2)
```

3.302 $\int \frac{\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{b^3d} - \frac{a\sinh(c+dx)}{b^2d} + \frac{\sinh^2(c+dx)}{2bd}$$

output
```
(a^2+b^2)*ln(a+b*sinh(d*x+c))/b^3/d-a*sinh(d*x+c)/b^2/d+1/2*sinh(d*x+c)^2/
b/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{-((a^2+b^2)\log(a+b\sinh(c+dx))) + ab\sinh(c+dx) - \frac{1}{2}b^2\sinh^2(c+dx)}{b^3d}$$

input
```
Integrate[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

output
```
-((-((a^2 + b^2)*Log[a + b*Sinh[c + d*x]]) + a*b*Sinh[c + d*x] - (b^2*Sinh
[c + d*x]^2)/2)/(b^3*d))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3147, 25, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ic+idx)^3}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3147}$$

$$-\frac{\int -\frac{\sinh^2(c+dx)b^2+b^2}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^3d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sinh^2(c+dx)b^2+b^2}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^3d}$$

$$\Big\downarrow \text{476}$$

$$\frac{\int \left(-a+b\sinh(c+dx)+\frac{a^2+b^2}{a+b\sinh(c+dx)}\right)d(b\sinh(c+dx))}{b^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\left(a^2+b^2\right)\log(a+b\sinh(c+dx))+ab\sinh(c+dx)-\frac{1}{2}b^2\sinh^2(c+dx)}{b^3d}$$

input
```
Int[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

output
```
-((-((a^2 + b^2)*Log[a + b*Sinh[c + d*x]]) + a*b*Sinh[c + d*x] - (b^2*Sinh
[c + d*x]^2)/2)/(b^3*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 5.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-\frac{-\frac{\sinh(dx+c)^2 b}{2}+a\sinh(dx+c)}{b^2}+\frac{\left(a^2+b^2\right)\ln(a+b\sinh(dx+c))}{b^3}}{d}$
default	$\frac{-\frac{-\frac{\sinh(dx+c)^2 b}{2}+a\sinh(dx+c)}{b^2}+\frac{\left(a^2+b^2\right)\ln(a+b\sinh(dx+c))}{b^3}}{d}$
risch	$-\frac{x a^2}{b^3}-\frac{x}{b}+\frac{e^{2dx+2c}}{8bd}-\frac{a\,e^{dx+c}}{2b^2 d}+\frac{a\,e^{-dx-c}}{2b^2 d}+\frac{e^{-2dx-2c}}{8bd}-\frac{2a^2 c}{b^3 d}-\frac{2c}{bd}+\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{b}-1\right)a^2}{b^3 d}$

input
```
int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^2*(-1/2*sinh(d*x+c)^2*b+a*sinh(d*x+c))+(a^2+b^2)/b^3*ln(a+b*sinh
(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(57) = 114$.

Time = 0.11 (sec) , antiderivative size = 327, normalized size of antiderivative = 5.54

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b^2\cosh{(dx+c)}^4 + b^2\sinh{(dx+c)}^4 - 8\,(a^2+b^2)dx\cosh{(dx+c)}^2 - 4\,ab\cosh{(dx+c)}^3 + 4\,(b^2\cosh{(dx}}{}$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/8*(b^2*cosh(d*x + c)^4 + b^2*sinh(d*x + c)^4 - 8*(a^2 + b^2)*d*x*cosh(d*
x + c)^2 - 4*a*b*cosh(d*x + c)^3 + 4*(b^2*cosh(d*x + c) - a*b)*sinh(d*x +
c)^3 + 4*a*b*cosh(d*x + c) + 2*(3*b^2*cosh(d*x + c)^2 - 4*(a^2 + b^2)*d*x
- 6*a*b*cosh(d*x + c))*sinh(d*x + c)^2 + b^2 + 8*((a^2 + b^2)*cosh(d*x + c
)^2 + 2*(a^2 + b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + b^2)*sinh(d*x + c
)^2)*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(b^2
*cosh(d*x + c)^3 - 4*(a^2 + b^2)*d*x*cosh(d*x + c) - 3*a*b*cosh(d*x + c)^2
 + a*b)*sinh(d*x + c))/(b^3*d*cosh(d*x + c)^2 + 2*b^3*d*cosh(d*x + c)*sinh
(d*x + c) + b^3*d*sinh(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(57) = 114$.

Time = 0.05 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.15

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\left(4\,ae^{(-dx-c)}-b\right)e^{(2\,dx+2\,c)}}{8\,b^2d} + \frac{(a^2+b^2)(dx+c)}{b^3d} + \frac{4\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}}{8\,b^2d} + \frac{(a^2+b^2)\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{b^3d}$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/8*(4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) + (a^2 + b^2)*(d*x + c
)/(b^3*d) + 1/8*(4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d) + (a^2 + b
^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.56

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2-4\,a\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{b^2}+\frac{8\,(a^2+b^2)\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{b^3}}{8\,d}$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/8*((b*(e^(d*x + c) - e^(-d*x - c))^2 - 4*a*(e^(d*x + c) - e^(-d*x - c)))
/b^2 + 8*(a^2 + b^2)*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/b^3)/d
```

Mupad [B] (verification not implemented)

Time = 1.24 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.03

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{e^{-2\,c-2\,d\,x}}{8\,b\,d} - \frac{x\,(a^2+b^2)}{b^3} + \frac{e^{2\,c+2\,d\,x}}{8\,b\,d} + \frac{\ln\left(2\,a\,e^{d\,x}\,e^{c}-b+b\,e^{2\,c}\,e^{2\,d\,x}\right)\,(a^2+b^2)}{b^3\,d} + \frac{a\,e^{-c-d\,x}}{2\,b^2\,d} - \frac{a\,e^{c+d\,x}}{2\,b^2\,d}$$

input
```
int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)),x)
```

output
```
exp(- 2*c - 2*d*x)/(8*b*d) - (x*(a^2 + b^2))/b^3 + exp(2*c + 2*d*x)/(8*b*d
) + (log(2*a*exp(d*x)*exp(c) - b + b*exp(2*c)*exp(2*d*x))*(a^2 + b^2))/(b^
3*d) + (a*exp(- c - d*x))/(2*b^2*d) - (a*exp(c + d*x))/(2*b^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 181, normalized size of antiderivative = 3.07

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{e^{4dx+4c}b^2-4e^{3dx+3c}ab+8e^{2dx+2c}\log\left(e^{2dx+2c}b+2e^{dx+c}a-b\right)a^2+8e^{2dx+2c}\log\left(e^{2dx+2c}b+2e^{dx+c}a-b\right)b^2}{8e^{2dx+2c}b^3d}$$

input
```
int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
(e**(4*c + 4*d*x)*b**2 - 4*e**(3*c + 3*d*x)*a*b + 8*e**(2*c + 2*d*x)*log(e
**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*a**2 + 8*e**(2*c + 2*d*x)*log(e*
*(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*b**2 - 8*e**(2*c + 2*d*x)*a**2*d*
x - 8*e**(2*c + 2*d*x)*b**2*d*x + 4*e**(c + d*x)*a*b + b**2)/(8*e**(2*c +
2*d*x)*b**3*d)
```

3.303 $\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(cosh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 28.96 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[Cosh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Cosh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx$$

$$\Big\downarrow \text{6111}$$

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Cosh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(dx+c)^3}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(cosh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)^3}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(cosh(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 251, normalized size of antiderivative = 8.96

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)^3}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*e^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b*f) + 1/2*a*e^
(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^2*f) + 1/2*a*e^(c - d*e/f
)*exp_integral_e(1, -(f*x + e)*d/f)/(b^2*f) - 1/4*e^(2*c - 2*d*e/f)*exp_in
tegral_e(1, -2*(f*x + e)*d/f)/(b*f) + (a^2 + b^2)*log(f*x + e)/(b^3*f) - 1
/8*integrate(16*(a^2*b + b^3 - (a^3*e^c + a*b^2*e^c)*e^(d*x))/(b^4*f*x + b
^4*e - (b^4*f*x*e^(2*c) + b^4*e*e^(2*c))*e^(2*d*x) - 2*(a*b^3*f*x*e^c + a*
b^3*e*e^c)*e^(d*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)^3/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)^3}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(cosh(c + d*x)^3/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(cosh(c + d*x)^3/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 334, normalized size of antiderivative = 11.93

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$= \frac{e^{4c}\left(\int \frac{e^{4dx}}{e^{2dx+2c}be+e^{2dx+2c}bfx+2e^{dx+c}ae+2e^{dx+c}afx-be-bfx}dx\right)bf-6e^{c}\left(\int \frac{e^{dx}}{e^{2dx+2c}be+e^{2dx+2c}bfx+2e^{dx+c}ae+2e^{dx+c}afx-be-}\right.}{}$$

input
```
int(cosh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
(e**(4*c)*int(e**(4*d*x)/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x +
2*e**(c + d*x)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x)*b*f - 6*e**c*i
nt(e**(d*x)/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x + 2*e**(c + d*x
)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x)*a*f + int(1/(e**(4*c + 4*d*
x)*b*e + e**(4*c + 4*d*x)*b*f*x + 2*e**(3*c + 3*d*x)*a*e + 2*e**(3*c + 3*d
*x)*a*f*x - e**(2*c + 2*d*x)*b*e - e**(2*c + 2*d*x)*b*f*x),x)*b*f + 6*int(
1/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x + 2*e**(c + d*x)*a*e + 2*
e**(c + d*x)*a*f*x - b*e - b*f*x),x)*b*f + 3*log(e + f*x))/(4*b*f)
```

3.304 $\int \frac{(e+fx)^3 \operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 786

$$\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2a(e+fx)^3\arctan\left(e^{c+dx}\right)}{(a^2+b^2)\,d} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d} - \frac{b(e+fx)^3\log\left(1+e^{2(c+dx)}\right)}{(a^2+b^2)\,d} - \frac{3iaf(e+fx)^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{(a^2+b^2)\,d^2} + \frac{3iaf(e+fx)^2\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{(a^2+b^2)\,d^2} + \frac{3bf(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d^2} + \frac{3bf(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d^2} - \frac{3bf(e+fx)^2\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2\,(a^2+b^2)\,d^2} + \frac{6iaf^2(e+fx)\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{(a^2+b^2)\,d^3} - \frac{6iaf^2(e+fx)\,\text{PolyLog}\left(3,ie^{c+dx}\right)}{(a^2+b^2)\,d^3} - \frac{6bf^2(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d^3} - \frac{6bf^2(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d^3} + \frac{3bf^2(e+fx)\,\text{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2\,(a^2+b^2)\,d^3} - \frac{6iaf^3\,\text{PolyLog}\left(4,-ie^{c+dx}\right)}{(a^2+b^2)\,d^4} + \frac{6iaf^3\,\text{PolyLog}\left(4,ie^{c+dx}\right)}{(a^2+b^2)\,d^4} + \frac{6bf^3\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d^4} + \frac{6bf^3\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d^4}$$

output
```
2*a*(f*x+e)^3*arctan(exp(d*x+c))/(a^2+b^2)/d+b*(f*x+e)^3*ln(1+b*exp(d*x+c)
/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d+b*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/(a^2+b^2)/d-b*(f*x+e)^3*ln(1+exp(2*d*x+2*c))/(a^2+b^2)/d+3*I*a*
f*(f*x+e)^2*polylog(2,I*exp(d*x+c))/(a^2+b^2)/d^2-6*I*a*f^3*polylog(4,-I*e
xp(d*x+c))/(a^2+b^2)/d^4+3*b*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b
^2)^(1/2)))/(a^2+b^2)/d^2+3*b*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+
b^2)^(1/2)))/(a^2+b^2)/d^2-3/2*b*f*(f*x+e)^2*polylog(2,-exp(2*d*x+2*c))/(a
^2+b^2)/d^2-6*I*a*f^2*(f*x+e)*polylog(3,I*exp(d*x+c))/(a^2+b^2)/d^3+6*I*a*
f^2*(f*x+e)*polylog(3,-I*exp(d*x+c))/(a^2+b^2)/d^3-6*b*f^2*(f*x+e)*polylog
(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d^3-6*b*f^2*(f*x+e)*polylo
g(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d^3+3/2*b*f^2*(f*x+e)*pol
ylog(3,-exp(2*d*x+2*c))/(a^2+b^2)/d^3-3*I*a*f*(f*x+e)^2*polylog(2,-I*exp(d
*x+c))/(a^2+b^2)/d^2+6*I*a*f^3*polylog(4,I*exp(d*x+c))/(a^2+b^2)/d^4+6*b*f
^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d^4+6*b*f^3*poly
log(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d^4-3/4*b*f^3*polylog(4
,-exp(2*d*x+2*c))/(a^2+b^2)/d^4
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3078 vs. 2(786) = 1572.

Time = 10.97 (sec) , antiderivative size = 3078, normalized size of antiderivative = 3.92

$$\int \frac{(e+fx)^3 \operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(8*b*d^4*e^3*E^(2*c)*x + 12*b*d^4*e^2*E^(2*c)*f*x^2 + 8*b*d^4*e*E^(2*c)*f^
2*x^3 + 2*b*d^4*E^(2*c)*f^3*x^4 + 8*a*d^3*e^3*ArcTan[E^(c + d*x)] + 8*a*d^
3*e^3*E^(2*c)*ArcTan[E^(c + d*x)] + (12*I)*a*d^3*e^2*f*x*Log[1 - I*E^(c +
d*x)] + (12*I)*a*d^3*e^2*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] + (12*I)*a*d^3
*e*f^2*x^2*Log[1 - I*E^(c + d*x)] + (12*I)*a*d^3*e*E^(2*c)*f^2*x^2*Log[1 -
 I*E^(c + d*x)] + (4*I)*a*d^3*f^3*x^3*Log[1 - I*E^(c + d*x)] + (4*I)*a*d^3
*E^(2*c)*f^3*x^3*Log[1 - I*E^(c + d*x)] - (12*I)*a*d^3*e^2*f*x*Log[1 + I*E
^(c + d*x)] - (12*I)*a*d^3*e^2*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] - (12*I)
*a*d^3*e*f^2*x^2*Log[1 + I*E^(c + d*x)] - (12*I)*a*d^3*e*E^(2*c)*f^2*x^2*L
og[1 + I*E^(c + d*x)] - (4*I)*a*d^3*f^3*x^3*Log[1 + I*E^(c + d*x)] - (4*I)
*a*d^3*E^(2*c)*f^3*x^3*Log[1 + I*E^(c + d*x)] - 4*b*d^3*e^3*Log[1 + E^(2*(
c + d*x))] - 4*b*d^3*e^3*E^(2*c)*Log[1 + E^(2*(c + d*x))] - 12*b*d^3*e^2*f
*x*Log[1 + E^(2*(c + d*x))] - 12*b*d^3*e^2*E^(2*c)*f*x*Log[1 + E^(2*(c + d
*x))] - 12*b*d^3*e*f^2*x^2*Log[1 + E^(2*(c + d*x))] - 12*b*d^3*e*E^(2*c)*f
^2*x^2*Log[1 + E^(2*(c + d*x))] - 4*b*d^3*f^3*x^3*Log[1 + E^(2*(c + d*x))]
 - 4*b*d^3*E^(2*c)*f^3*x^3*Log[1 + E^(2*(c + d*x))] - (12*I)*a*d^2*(1 + E^
(2*c))*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)] + (12*I)*a*d^2*(1 + E^(2
*c))*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)] - 6*b*d^2*e^2*f*PolyLog[2, -E
^(2*(c + d*x))] - 6*b*d^2*e^2*E^(2*c)*f*PolyLog[2, -E^(2*(c + d*x))] - 12*
b*d^2*e*f^2*x*PolyLog[2, -E^(2*(c + d*x))] - 12*b*d^2*e*E^(2*c)*f^2*x*P...
```

Rubi [A] (verified)

Time = 3.01 (sec) , antiderivative size = 690, normalized size of antiderivative = 0.88, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {6107, 6095, 2620, 3011, 7163, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6107}$$

$$\frac{b^2\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{6095}$$

$$\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^4}{4bf}\right)}{a^2+b^2}+\frac{\int (e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$\downarrow$ 2620

$$\frac{b^2\left(-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+}}{\sqrt{a^2+}}\right.}{bd}\right.}{a^2+b^2}$$
$$\frac{\int (e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$\downarrow$ 3011

$$\frac{b^2\left(-\frac{3f\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2}{}\right.}{bd}\right.}{a^2+b^2}$$
$$\frac{\int (e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$\downarrow$ 7163

$$b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int \text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\text{Poly}}{}\right.}{}\right.}{}\right.$$

$$\frac{\int (e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$\downarrow$ 2720

$$b^2 \left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd} - \frac{3f\left(\frac{2f}{\cdots}\right.}{\cdots}\right.$$

$$\frac{\int (e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\int (e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+$$

$$b^2 \left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd} - \frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}(\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$$\Big\downarrow \text{7293}$$

$$\frac{\int \left(a(e+fx)^3\text{sech}(c+dx)-b(e+fx)^3\tanh(c+dx)\right)dx}{a^2+b^2}+$$

$$b^2 \left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd} - \frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}(\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$$\Big\downarrow \text{2009}$$

$$b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}(}{}\right.}{}\right.}{}\right.$$

$$\frac{2a(e+fx)^3\arctan\left(e^{c+dx}\right)}{d}-\frac{6iaf^3\,\text{PolyLog}\left(4,-ie^{c+dx}\right)}{d^4}+\frac{6iaf^3\,\text{PolyLog}\left(4,ie^{c+dx}\right)}{d^4}+\frac{6iaf^2(e+fx)\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{d^3}-\frac{6iaf^2(e+fx)}{}$$

input
```
Int[((e + f*x)^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(b^2*(-1/4*(e + f*x)^4/(b*f) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 + b^2])])/(b*d) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2]))])/d^2))/d))/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/(a^2 + b^2) + ((b*(e + f*x)^4)/(4*f)
+ (2*a*(e + f*x)^3*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)^3*Log[1 + E^(2*(c
 + d*x))])/d - ((3*I)*a*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/d^2 +
((3*I)*a*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/d^2 - (3*b*f*(e + f*x)^2
*PolyLog[2, -E^(2*(c + d*x))])/(2*d^2) + ((6*I)*a*f^2*(e + f*x)*PolyLog[3,
 (-I)*E^(c + d*x)])/d^3 - ((6*I)*a*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)]
)/d^3 + (3*b*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*d^3) - ((6*I)*
a*f^3*PolyLog[4, (-I)*E^(c + d*x)])/d^4 + ((6*I)*a*f^3*PolyLog[4, I*E^(c +
 d*x)])/d^4 - (3*b*f^3*PolyLog[4, -E^(2*(c + d*x))])/(4*d^4))/(a^2 + b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^3 \operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1718 vs. $2(720) = 1440$.

Time = 0.17 (sec) , antiderivative size = 1718, normalized size of antiderivative = 2.19

$$\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(6*b*f^3*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*b*f^3*polylog(4, (a*cosh(
d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2))/b) + 3*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*dilog
((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*
sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 3*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b
*d^2*e^2*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) +
b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*(-I*a*d^2*f^3*x^2 +
 b*d^2*f^3*x^2 - 2*I*a*d^2*e*f^2*x + 2*b*d^2*e*f^2*x - I*a*d^2*e^2*f + b*d
^2*e^2*f)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) - 3*(I*a*d^2*f^3*x^2 +
b*d^2*f^3*x^2 + 2*I*a*d^2*e*f^2*x + 2*b*d^2*e*f^2*x + I*a*d^2*e^2*f + b*d^
2*e^2*f)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) + (b*d^3*e^3 - 3*b*c*d^
2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*
x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b*d^3*e^3 - 3*b*c*d^2*e^2*f +
 3*b*c^2*d*e*f^2 - b*c^3*f^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) -
2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*
b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3)*log(-(a*cos
h(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2) - b)/b) + (b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*
f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3)*log(-(a*cosh(d*x +...
```

Sympy [F]

$$\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)^3 \text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\,\text{sech}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e^3*(2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - b*log(-2*a*e^(-d*x - c) +
 b*e^(-2*d*x - 2*c) - b)/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a
^2 + b^2)*d)) + integrate(4*f^3*x^3/((b*(e^(d*x + c) - e^(-d*x - c)) + 2*a
)*(e^(d*x + c) + e^(-d*x - c))) + 12*e*f^2*x^2/((b*(e^(d*x + c) - e^(-d*x
- c)) + 2*a)*(e^(d*x + c) + e^(-d*x - c))) + 12*e^2*f*x/((b*(e^(d*x + c) -
 e^(-d*x - c)) + 2*a)*(e^(d*x + c) + e^(-d*x - c))), x)
```

Giac [F]

$$\int \frac{(e+fx)^3 \text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\,\text{sech}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sech(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^3}{\cosh\,(c+d\,x)\,\,(a+b\sinh\,(c+d\,x))}\,dx$$

input
```
int((e + f*x)^3/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^3/(cosh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3 \text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2atan\left(e^{dx+c}\right) a\,e^3+4e^{2c}\left(\int \frac{e^{2dx}x^3}{e^{4dx+4c}b+2e^{3dx+3c}a+2e^{dx+c}a-b}dx\right) a^2d\,f^3+4e^{2c}\left(\int \frac{e^{2dx}x^3}{e^{4dx+4c}b+2e^{3dx+3c}a+2e^{dx+c}a-b}dx\right)}{}$$

input
```
int((f*x+e)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
(2*atan(e**(c + d*x))*a*e**3 + 4*e**(2*c)*int((e**(2*d*x)*x**3)/(e**(4*c +
 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a**2*d*f**3 +
4*e**(2*c)*int((e**(2*d*x)*x**3)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*
a + 2*e**(c + d*x)*a - b),x)*b**2*d*f**3 + 12*e**(2*c)*int((e**(2*d*x)*x**
2)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a
**2*d*e*f**2 + 12*e**(2*c)*int((e**(2*d*x)*x**2)/(e**(4*c + 4*d*x)*b + 2*e
**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*b**2*d*e*f**2 + 12*e**(2*c)*i
nt((e**(2*d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d
*x)*a - b),x)*a**2*d*e**2*f + 12*e**(2*c)*int((e**(2*d*x)*x)/(e**(4*c + 4*
d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*b**2*d*e**2*f - l
og(e**(2*c + 2*d*x) + 1)*b*e**3 + log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*
a - b)*b*e**3)/(d*(a**2 + b**2))
```

3.305 $\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 558

$$\int \frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2a(e+fx)^2\arctan\left(e^{c+dx}\right)}{(a^2+b^2)\,d} + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d} + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d} - \frac{b(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{(a^2+b^2)\,d} - \frac{2iaf(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{(a^2+b^2)\,d^2} + \frac{2iaf(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{(a^2+b^2)\,d^2} + \frac{2bf(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d^2} + \frac{2bf(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d^2} - \frac{bf(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{(a^2+b^2)\,d^2} + \frac{2iaf^2\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{(a^2+b^2)\,d^3} - \frac{2iaf^2\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{(a^2+b^2)\,d^3} - \frac{2bf^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d^3} - \frac{2bf^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d^3} + \frac{bf^2\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2\,(a^2+b^2)\,d^3}$$

output
```
2*a*(f*x+e)^2*arctan(exp(d*x+c))/(a^2+b^2)/d+b*(f*x+e)^2*ln(1+b*exp(d*x+c)
/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d+b*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/(a^2+b^2)/d-b*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/(a^2+b^2)/d-2*I*a*
f*(f*x+e)*polylog(2,-I*exp(d*x+c))/(a^2+b^2)/d^2+2*I*a*f*(f*x+e)*polylog(2
,I*exp(d*x+c))/(a^2+b^2)/d^2+2*b*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2
+b^2)^(1/2)))/(a^2+b^2)/d^2+2*b*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+
b^2)^(1/2)))/(a^2+b^2)/d^2-b*f*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2
)/d^2+2*I*a*f^2*polylog(3,-I*exp(d*x+c))/(a^2+b^2)/d^3-2*I*a*f^2*polylog(3
,I*exp(d*x+c))/(a^2+b^2)/d^3-2*b*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^
(1/2)))/(a^2+b^2)/d^3-2*b*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))
/(a^2+b^2)/d^3+1/2*b*f^2*polylog(3,-exp(2*d*x+2*c))/(a^2+b^2)/d^3
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1639 vs. $2(558) = 1116$.

Time = 10.35 (sec) , antiderivative size = 1639, normalized size of antiderivative = 2.94

$$\int \frac{(e+fx)^2 \text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(12*b*d^3*e^2*E^(2*c)*x - 12*b*d^3*e^2*(1 + E^(2*c))*x - 12*b*d^3*e*f*x^2
- 4*b*d^3*f^2*x^3 + 12*a*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] + 6*b*d
^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*a*d*e*(1
+ E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - Poly
Log[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) + 6*b*d*e*(1 + E^(2*
c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x)
)]) + (6*I)*a*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*x^2*
Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*PolyLo
g[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I*E^(c
 + d*x)]) + b*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c + d*
x))]) - 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d*x))
]))/(6*(a^2 + b^2)*d^3*(1 + E^(2*c))) - (b*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*
f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c +
d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)
^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)
^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt
[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*
c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) +
(3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d - (3*e^2*E^(2*c)
*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e*f*x*Log[1 + ...
```

Rubi [A] (verified)

Time = 2.19 (sec) , antiderivative size = 501, normalized size of antiderivative = 0.90, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6107, 6095, 2620, 3011, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6107}$$

$$\frac{b^2\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{6095}$$

$$\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{2620}$$

$$\frac{b^2\left(-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}-\cdots}\right.}{bd}\right.}{a^2+b^2}\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{3011}$$

$$\frac{b^2\left(-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c}}{a+\sqrt{}}\right.}{d}\right.}{bd}\right.}{a^2+b^2}$$

$$\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{2720}$$

$$\frac{b^2\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\cdots\right.}{bd}\right.}{a^2+b^2}$$

$$\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+$$

$$\frac{b^2\left(-\frac{2f\left(\frac{f\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right.}{bd}\right.}{a^2+b^2}$$

$$\Big\downarrow \text{7293}$$

$$\frac{\int \left(a(e+fx)^2\text{sech}(c+dx)-b(e+fx)^2\tanh(c+dx)\right)dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right.}{bd}}{a^2+b^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right.}{bd}}{a^2+b^2}$$

$$\frac{\frac{2a(e+fx)^2\arctan(e^{c+dx})}{d}+\frac{2iaf^2\,\text{PolyLog}(3,-ie^{c+dx})}{d^3}-\frac{2iaf^2\,\text{PolyLog}(3,ie^{c+dx})}{d^3}-\frac{2iaf(e+fx)\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{2iaf(e+fx)\,\text{P}}{}}{a^2+b^2}$$

input
```
Int[((e + f*x)^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(b^2*(-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2]))])/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2]))])/d^2))/(b*d)))/(a^2 + b^2) + ((b*(e + f*x)^3)/(3*f) + (2*a*(e + f*
x)^2*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/d -
 ((2*I)*a*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + ((2*I)*a*f*(e +
f*x)*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*(e + f*x)*PolyLog[2, -E^(2*(c +
 d*x))])/d^2 + ((2*I)*a*f^2*PolyLog[3, (-I)*E^(c + d*x)])/d^3 - ((2*I)*a*f
^2*PolyLog[3, I*E^(c + d*x)])/d^3 + (b*f^2*PolyLog[3, -E^(2*(c + d*x))])/(
2*d^3))/(a^2 + b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1094 vs. $2(512) = 1024$.

Time = 0.13 (sec) , antiderivative size = 1094, normalized size of antiderivative = 1.96

$$\int \frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(2*b*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
 + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 2*b*f^2*polylog(3, (a*cosh
(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2))/b) - 2*(b*d*f^2*x + b*d*e*f)*dilog((a*cosh(d*x + c) + a*sin
h(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b
)/b + 1) - 2*(b*d*f^2*x + b*d*e*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c
) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1)
+ 2*(-I*a*d*f^2*x + b*d*f^2*x - I*a*d*e*f + b*d*e*f)*dilog(I*cosh(d*x + c)
 + I*sinh(d*x + c)) + 2*(I*a*d*f^2*x + b*d*f^2*x + I*a*d*e*f + b*d*e*f)*di
log(-I*cosh(d*x + c) - I*sinh(d*x + c)) - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2
*f^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2
) + 2*a) - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*log(2*b*cosh(d*x + c) + 2
*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b*d^2*f^2*x^2 + 2*b
*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x +
 c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) -
(b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*log(-(a*cosh(d*x
 + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2) - b)/b) - (I*a*d^2*e^2 - b*d^2*e^2 - 2*I*a*c*d*e*f + 2*b*c*d*e*f
 + I*a*c^2*f^2 - b*c^2*f^2)*log(cosh(d*x + c) + sinh(d*x + c) + I) - (-I*a
*d^2*e^2 - b*d^2*e^2 + 2*I*a*c*d*e*f + 2*b*c*d*e*f - I*a*c^2*f^2 - b*c^...
```

Sympy [F]

$$\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)^2 \operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\operatorname{sech}(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e^2*(2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - b*log(-2*a*e^(-d*x - c) +
 b*e^(-2*d*x - 2*c) - b)/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a
^2 + b^2)*d)) + integrate(4*f^2*x^2/((b*(e^(d*x + c) - e^(-d*x - c)) + 2*a
)*(e^(d*x + c) + e^(-d*x - c))) + 8*e*f*x/((b*(e^(d*x + c) - e^(-d*x - c))
 + 2*a)*(e^(d*x + c) + e^(-d*x - c))), x)
```

Giac [F]

$$\int \frac{(e+fx)^2 \operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\operatorname{sech}(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sech(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^2}{\cosh(c+d\,x)\,(a+b\sinh(c+d\,x))}\,dx$$

input
```
int((e + f*x)^2/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^2/(cosh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{2atan\left(e^{dx+c}\right)a\,e^2+4e^{2c}\left(\int \frac{e^{2dx}x^2}{e^{4dx+4c}b+2e^{3dx+3c}a+2e^{dx+c}a-b}dx\right)a^2d\,f^2+4e^{2c}\left(\int \frac{e^{2dx}x^2}{e^{4dx+4c}b+2e^{3dx+3c}a+2e^{dx+c}a-b}dx\right)}{}$$

input
```
int((f*x+e)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
(2*atan(e**(c + d*x))*a*e**2 + 4*e**(2*c)*int((e**(2*d*x)*x**2)/(e**(4*c +
 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a**2*d*f**2 +
4*e**(2*c)*int((e**(2*d*x)*x**2)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*
a + 2*e**(c + d*x)*a - b),x)*b**2*d*f**2 + 8*e**(2*c)*int((e**(2*d*x)*x)/(
e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a**2*
d*e*f + 8*e**(2*c)*int((e**(2*d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*
d*x)*a + 2*e**(c + d*x)*a - b),x)*b**2*d*e*f - log(e**(2*c + 2*d*x) + 1)*b
*e**2 + log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*b*e**2)/(d*(a**2 +
b**2))
```

3.306 $\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 334

$$
\begin{aligned}
\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{2a(e+fx)\arctan\left(e^{c+dx}\right)}{(a^2+b^2)\,d} + \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d} \\
&+ \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d} \\
&- \frac{b(e+fx)\log\left(1+e^{2(c+dx)}\right)}{(a^2+b^2)\,d} - \frac{iaf\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{(a^2+b^2)\,d^2} \\
&+ \frac{iaf\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{(a^2+b^2)\,d^2} + \frac{bf\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d^2} \\
&+ \frac{bf\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d^2} \\
&- \frac{bf\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2\,(a^2+b^2)\,d^2}
\end{aligned}
$$

output
```
2*a*(f*x+e)*arctan(exp(d*x+c))/(a^2+b^2)/d+b*(f*x+e)*ln(1+b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))/(a^2+b^2)/d+b*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/
2)))/(a^2+b^2)/d-b*(f*x+e)*ln(1+exp(2*d*x+2*c))/(a^2+b^2)/d-I*a*f*polylog(
2,-I*exp(d*x+c))/(a^2+b^2)/d^2+I*a*f*polylog(2,I*exp(d*x+c))/(a^2+b^2)/d^2
+b*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d^2+b*f*polylo
g(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d^2-1/2*b*f*polylog(2,-ex
p(2*d*x+2*c))/(a^2+b^2)/d^2
```

Mathematica [A] (warning: unable to verify)

Time = 2.55 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.56

$$\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{4ade\arctan\left(e^{c+dx}\right)-4acf\arctan\left(e^{c+dx}\right)+\frac{4ab\sqrt{a^2+b^2}de\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-(a^2+b^2)^2}}-\frac{4ab\sqrt{-(a^2+b^2)^2}de\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right.}{(-a^2-b^2)^{3/2}}}{}$$

input
```
Integrate[((e + f*x)*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(4*a*d*e*ArcTan[E^(c + d*x)] - 4*a*c*f*ArcTan[E^(c + d*x)] + (4*a*b*Sqrt[a
^2 + b^2]*d*e*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b
^2)^2] - (4*a*b*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[
a^2 + b^2]])/(-a^2 - b^2)^(3/2) + (2*I)*a*f*(c + d*x)*Log[1 - I*E^(c + d*x
)] - (2*I)*a*f*(c + d*x)*Log[1 + I*E^(c + d*x)] + 2*b*f*(c + d*x)*Log[1 +
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*b*f*(c + d*x)*Log[1 + (b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2])] - 2*b*d*e*Log[1 + E^(2*(c + d*x))] + 2*b*c*f
*Log[1 + E^(2*(c + d*x))] - 2*b*f*(c + d*x)*Log[1 + E^(2*(c + d*x))] - 2*b
*c*f*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*b*d*e*Log[2*a*E^(c +
 d*x) + b*(-1 + E^(2*(c + d*x)))] - (2*I)*a*f*PolyLog[2, (-I)*E^(c + d*x)]
 + (2*I)*a*f*PolyLog[2, I*E^(c + d*x)] + 2*b*f*PolyLog[2, (b*E^(c + d*x))/
(-a + Sqrt[a^2 + b^2])] + 2*b*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2]))] - b*f*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)*d^2)
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6107, 6095, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6107}$$

$$\frac{b^2\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{6095}$$

$$\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{2620}$$

$$\frac{b^2\left(-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right.}{a^2+b^2}$$

$$\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{2715}$$

$$\frac{b^2\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{b}{\sqrt{a}}\right.}{bd}\right.}{a^2+b^2}$$

$$\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+$$
$$\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)}{2bf}\right.}{a^2+b^2}$$

$\downarrow$ 7293

$$\frac{\int (a(e+fx)\text{sech}(c+dx)-b(e+fx)\tanh(c+dx))dx}{a^2+b^2}+$$
$$\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)}{2bf}\right.}{a^2+b^2}$$

$\downarrow$ 2009

$$\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)}{2bf}\right.}{a^2+b^2}$$
$$\frac{\frac{2a(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{iaf\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{iaf\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}-\frac{bf\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d^2}-\frac{b(e+fx)\log\left(e^{2(c+dx)}+1\right)}{d}+}{a^2+b^2}$$

input
```
Int[((e + f*x)*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(b^2*(-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/
(b*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)
))/(a^2 + b^2) + ((b*(e + f*x)^2)/(2*f) + (2*a*(e + f*x)*ArcTan[E^(c + d*x
)])/d - (b*(e + f*x)*Log[1 + E^(2*(c + d*x))])/d - (I*a*f*PolyLog[2, (-I)*
E^(c + d*x)])/d^2 + (I*a*f*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*PolyLog[2
, -E^(2*(c + d*x))])/(2*d^2))/(a^2 + b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 953 vs. 2(313) = 626.

Time = 3.72 (sec) , antiderivative size = 954, normalized size of antiderivative = 2.86

method	result
risch	$-\frac{2eb\ln\left(1+\mathrm{e}^{2dx+2c}\right)}{d(2a^2+2b^2)}+\frac{4ea\arctan\left(\mathrm{e}^{dx+c}\right)}{d(2a^2+2b^2)}+\frac{2eb\ln\left(b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}^{dx+c}-b\right)}{d(2a^2+2b^2)}+\frac{2fb\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)x}{d(2a^2+2b^2)}+\frac{2fb\ln\left(\right.}{}$

input
```
int((f*x+e)*sech(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/d*e/(2*a^2+2*b^2)*b*ln(1+exp(2*d*x+2*c))+4/d*e/(2*a^2+2*b^2)*a*arctan(e
xp(d*x+c))+2/d*e*b/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d
*f*b/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2
)))*x+2/d^2*f*b/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^
2+b^2)^(1/2)))*c+2/d*f*b/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)
/(a+(a^2+b^2)^(1/2)))*x+2/d^2*f*b/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)
^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2/d^2*f*b/(2*a^2+2*b^2)*dilog((-b*exp(d*x
+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+2/d^2*f*b/(2*a^2+2*b^2)*dilog
((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-2/d*f/(2*a^2+2*b^2)
*ln(1+I*exp(d*x+c))*b*x-2/d^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*b*c+2*I/d
*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*a*x-2*I/d^2*f/(2*a^2+2*b^2)*ln(1+I*exp
(d*x+c))*a*c-2/d*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*b*x-2/d^2*f/(2*a^2+2*b
^2)*ln(1-I*exp(d*x+c))*b*c+2*I/d^2*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))*a
+2*I/d^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*a*c-2/d^2*f/(2*a^2+2*b^2)*dilo
g(1+I*exp(d*x+c))*b-2*I/d^2*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))*a-2/d^2*
f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))*b-2*I/d*f/(2*a^2+2*b^2)*ln(1+I*exp(d
*x+c))*a*x+2/d^2*c*f/(2*a^2+2*b^2)*b*ln(1+exp(2*d*x+2*c))-4/d^2*c*f/(2*a^2
+2*b^2)*a*arctan(exp(d*x+c))-2/d^2*c*f*b/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)
+2*a*exp(d*x+c)-b)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 588, normalized size of antiderivative = 1.76

$$\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(b*f*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + b*f*dilog((a*cosh(d*x + c) +
 a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^
2) - b)/b + 1) + (I*a*f - b*f)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) +
(-I*a*f - b*f)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) + (b*d*e - b*c*f)
*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2
*a) + (b*d*e - b*c*f)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt
((a^2 + b^2)/b^2) + 2*a) + (b*d*f*x + b*c*f)*log(-(a*cosh(d*x + c) + a*sin
h(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b
)/b) + (b*d*f*x + b*c*f)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (I*a*d*e - b*
d*e - I*a*c*f + b*c*f)*log(cosh(d*x + c) + sinh(d*x + c) + I) + (-I*a*d*e
- b*d*e + I*a*c*f + b*c*f)*log(cosh(d*x + c) + sinh(d*x + c) - I) + (-I*a*
d*f*x - b*d*f*x - I*a*c*f - b*c*f)*log(I*cosh(d*x + c) + I*sinh(d*x + c) +
 1) + (I*a*d*f*x - b*d*f*x + I*a*c*f - b*c*f)*log(-I*cosh(d*x + c) - I*sin
h(d*x + c) + 1))/((a^2 + b^2)*d^2)
```

Sympy [F]

$$\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{sech}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e*(2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - b*log(-2*a*e^(-d*x - c) + b
*e^(-2*d*x - 2*c) - b)/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a^2
 + b^2)*d)) + 2*f*integrate(2*x/((b*(e^(d*x + c) - e^(-d*x - c)) + 2*a)*(e
^(d*x + c) + e^(-d*x - c))), x)
```

Giac [F]

$$\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{sech}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*sech(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{e+f\,x}{\cosh\,(c+d\,x)\,\,(a+b\sinh\,(c+d\,x))}\,dx$$

input
```
int((e + f*x)/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(cosh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{2atan\left(e^{dx+c}\right)ae+4e^{2c}\left(\int \frac{e^{2dx}x}{e^{4dx+4c}b+2e^{3dx+3c}a+2e^{dx+c}a-b}dx\right)a^2df+4e^{2c}\left(\int \frac{e^{2dx}x}{e^{4dx+4c}b+2e^{3dx+3c}a+2e^{dx+c}a-b}dx\right)b^2d}{d\left(a^2+b^2\right)}$$

input
```
int((f*x+e)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
(2*atan(e**(c + d*x))*a*e + 4*e**(2*c)*int((e**(2*d*x)*x)/(e**(4*c + 4*d*x
)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a**2*d*f + 4*e**(2*c
)*int((e**(2*d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c
+ d*x)*a - b),x)*b**2*d*f - log(e**(2*c + 2*d*x) + 1)*b*e + log(e**(2*c +
2*d*x)*b + 2*e**(c + d*x)*a - b)*b*e)/(d*(a**2 + b**2))
```

3.307 $\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{a\arctan(\sinh(c+dx))}{(a^2+b^2)\,d} - \frac{b\log(\cosh(c+dx))}{(a^2+b^2)\,d} + \frac{b\log(a+b\sinh(c+dx))}{(a^2+b^2)\,d}$$

output
```
a*arctan(sinh(d*x+c))/(a^2+b^2)/d-b*ln(cosh(d*x+c))/(a^2+b^2)/d+b*ln(a+b*s
inh(d*x+c))/(a^2+b^2)/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.65

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b\left(\left(-a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}-b\sinh(c+dx)\right)-2\sqrt{-b^2}\log(a+b\sinh(c+dx))+\left(a+\sqrt{-b^2}\right)\log\left(\sqrt{-\right.}\right.}{2\sqrt{-b^2}\,(a^2+b^2)\,d}$$

input
```
Integrate[Sech[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*(b*((-a + Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[c + d*x]] - 2*Sqrt[-b^2
]*Log[a + b*Sinh[c + d*x]] + (a + Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Sinh[c +
d*x]]))/(Sqrt[-b^2]*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 3147, 25, 479, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3147}$$

$$-\frac{b\int -\frac{1}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\int \frac{1}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{479}$$

$$-\frac{b\left(-\frac{\int \frac{a-b\sinh(c+dx)}{\sinh^2(c+dx)b^2+b^2}d(b\sinh(c+dx))}{a^2+b^2}-\frac{\log(a+b\sinh(c+dx))}{a^2+b^2}\right)}{d}$$

$$\Big\downarrow \text{452}$$

$$-\frac{b\left(-\frac{a\int \frac{1}{\sinh^2(c+dx)b^2+b^2}d(b\sinh(c+dx))-\int \frac{b\sinh(c+dx)}{\sinh^2(c+dx)b^2+b^2}d(b\sinh(c+dx))}{a^2+b^2}-\frac{\log(a+b\sinh(c+dx))}{a^2+b^2}\right)}{d}$$

$$\Big\downarrow \text{216}$$

$$-\frac{b\left(-\frac{\frac{a\arctan(\sinh(c+dx))}{b}-\int\frac{b\sinh(c+dx)}{\sinh^2(c+dx)b^2+b^2}d(b\sinh(c+dx))}{a^2+b^2}-\frac{\log(a+b\sinh(c+dx))}{a^2+b^2}\right)}{d}$$

$$\Big\downarrow \text{ 240}$$

$$-\frac{b\left(-\frac{\frac{a\arctan(\sinh(c+dx))}{b}-\frac{1}{2}\log\left(b^2\sinh^2(c+dx)+b^2\right)}{a^2+b^2}-\frac{\log(a+b\sinh(c+dx))}{a^2+b^2}\right)}{d}$$

input
```
Int[Sech[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

output
```
-((b*(-(Log[a + b*Sinh[c + d*x]]/(a^2 + b^2)) - ((a*ArcTan[Sinh[c + d*x]])
/b - Log[b^2 + b^2*Sinh[c + d*x]^2]/2)/(a^2 + b^2)))/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 479
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[d*(Log
[RemoveContent[c + d*x, x]]/(b*c^2 + a*d^2)), x] + Simp[b/(b*c^2 + a*d^2)
 Int[(c - d*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 3.60 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{a^{2}+b^{2}}+\frac{-b\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)+2a\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^{2}+b^{2}}}{d}$
default	$\frac{\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{a^{2}+b^{2}}+\frac{-b\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)+2a\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^{2}+b^{2}}}{d}$
risch	$\frac{2b\,d^{2}x}{a^{2}d^{2}+b^{2}d^{2}}+\frac{2bdc}{a^{2}d^{2}+b^{2}d^{2}}-\frac{2bx}{a^{2}+b^{2}}-\frac{2bc}{d(a^{2}+b^{2})}+\frac{i\ln\left(\mathrm{e}^{dx+c}+i\right)a}{(a^{2}+b^{2})d}-\frac{\ln\left(\mathrm{e}^{dx+c}+i\right)b}{(a^{2}+b^{2})d}-\frac{i\ln\left(\mathrm{e}^{dx+c}-i\right)a}{(a^{2}+b^{2})d}-\ln$

input
```
int(sech(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b/(a^2+b^2)*ln(tanh(1/2*d*x+1/2*c)^2*a-2*b*tanh(1/2*d*x+1/2*c)-a)+2/(
a^2+b^2)*(-1/2*b*ln(1+tanh(1/2*d*x+1/2*c)^2)+a*arctan(tanh(1/2*d*x+1/2*c))
))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.33

$$\int\frac{\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\frac{2\,a\arctan\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)+b\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)-b\log\left(\frac{2\,\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{(a^{2}+b^{2})d}$$

input
```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(2*a*arctan(cosh(d*x + c) + sinh(d*x + c)) + b*log(2*(b*sinh(d*x + c) + a)
/(cosh(d*x + c) - sinh(d*x + c))) - b*log(2*cosh(d*x + c)/(cosh(d*x + c) -
 sinh(d*x + c))))/((a^2 + b^2)*d)
```

Sympy [F]

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.38

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{2\,a\arctan\left(e^{(-dx-c)}\right)}{(a^2+b^2)d} + \frac{b\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^2+b^2)d} - \frac{b\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^2+b^2)d}$$

input
```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(-2*a*e^(-d*x - c) + b*e^
(-2*d*x - 2*c) - b)/((a^2 + b^2)*d) - b*log(e^(-2*d*x - 2*c) + 1)/((a^2 +
b^2)*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.75

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{2\,b^2\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^2b+b^3}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)a}{a^2+b^2}-\frac{b\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^2+b^2}}{2\,d}$$

input
```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*b^2*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^2*b + b^3) +
(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*a/(a^2 + b^2) - b*
log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^2 + b^2))/d
```

Mupad [B] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.87

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b\,\ln\left(2\,a^3\,e^{d\,x}\,e^{c}-4\,b^3-a^2\,b+4\,b^3\,e^{2\,c}\,e^{2\,d\,x}+a^2\,b\,e^{2\,c}\,e^{2\,d\,x}+8\,a\,b^2\,e^{d\,x}\,e^{c}\right)}{d\,a^2+d\,b^2} - \frac{\ln\left(e^{c+d\,x}+1\mathrm{i}\right)}{b\,d+a\,d\,1\mathrm{i}} - \frac{\ln\left(1+e^{c+d\,x}\,1\mathrm{i}\right)\,1\mathrm{i}}{a\,d+b\,d\,1\mathrm{i}}$$

input
```
int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
(b*log(2*a^3*exp(d*x)*exp(c) - 4*b^3 - a^2*b + 4*b^3*exp(2*c)*exp(2*d*x) +
 a^2*b*exp(2*c)*exp(2*d*x) + 8*a*b^2*exp(d*x)*exp(c)))/(a^2*d + b^2*d) - (
log(exp(c + d*x)*1i + 1)*1i)/(a*d + b*d*1i) - log(exp(c + d*x) + 1i)/(a*d*
1i + b*d)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2atan\left(e^{dx+c}\right)a-\log\left(e^{2dx+2c}+1\right)b+\log\left(e^{2dx+2c}b+2e^{dx+c}a-b\right)b}{d\left(a^2+b^2\right)}$$

input

```
int(sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output

```
(2*atan(e**(c + d*x))*a - log(e**(2*c + 2*d*x) + 1)*b + log(e**(2*c + 2*d*
x)*b + 2*e**(c + d*x)*a - b)*b)/(d*(a**2 + b**2))
```

3.308 $\int \frac{\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\text{Int}\left(\frac{\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 11.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[Sech[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Sech[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx$$

$$\Big\downarrow \text{ 6111}$$

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Sech[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(dx+c)}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sech(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

input
```
integrate(sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(sech(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sech(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Giac [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sech(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{1}{\cosh(c+d\,x)\,\left(e+f\,x\right)\,\left(a+b\sinh(c+d\,x)\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(cosh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(dx+c)}{\sinh(dx+c)\,be+\sinh(dx+c)\,bfx+ae+afx}dx$$

input
```
int(sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(sech(c + d*x)/(sinh(c + d*x)*b*e + sinh(c + d*x)*b*f*x + a*e + a*f*x),
x)
```

3.309 $\int \frac{(e+fx)^3 \operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 780

$$\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{a(e+fx)^3}{(a^2+b^2)\,d} - \frac{6bf(e+fx)^2\arctan\left(e^{c+dx}\right)}{(a^2+b^2)\,d^2} + \frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} - \frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} - \frac{3af(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{(a^2+b^2)\,d^2} + \frac{6ibf^2(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{(a^2+b^2)\,d^3} - \frac{6ibf^2(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{(a^2+b^2)\,d^3} + \frac{3b^2f(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^2} - \frac{3b^2f(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^2} - \frac{3af^2(e+fx)\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{(a^2+b^2)\,d^3} - \frac{6ibf^3\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{(a^2+b^2)\,d^4} + \frac{6ibf^3\,\text{PolyLog}\left(3,ie^{c+dx}\right)}{(a^2+b^2)\,d^4} - \frac{6b^2f^2(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^3} + \frac{6b^2f^2(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^3} + \frac{3af^3\,\text{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2\,(a^2+b^2)\,d^4} + \frac{6b^2f^3\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^4} - \frac{6b^2f^3\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^4} + \frac{b(e+fx)^3\text{sech}(c+dx)}{(a^2+b^2)\,d} + \frac{a(e+fx)^3\tanh(c+dx)}{(a^2+b^2)\,d}$$

output
```
a*(f*x+e)^3/(a^2+b^2)/d-6*b*f*(f*x+e)^2*arctan(exp(d*x+c))/(a^2+b^2)/d^2+b
^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d-b^2*
(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d-3*a*f*(
f*x+e)^2*ln(1+exp(2*d*x+2*c))/(a^2+b^2)/d^2-6*I*b*f^3*polylog(3,-I*exp(d*x
+c))/(a^2+b^2)/d^4+6*I*b*f^3*polylog(3,I*exp(d*x+c))/(a^2+b^2)/d^4+3*b^2*f
*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^
2-3*b^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)
^(3/2)/d^2-3*a*f^2*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)/d^3+6*I*b*
f^2*(f*x+e)*polylog(2,-I*exp(d*x+c))/(a^2+b^2)/d^3-6*I*b*f^2*(f*x+e)*polyl
og(2,I*exp(d*x+c))/(a^2+b^2)/d^3-6*b^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)
/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^3+6*b^2*f^2*(f*x+e)*polylog(3,-b*e
xp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^3+3/2*a*f^3*polylog(3,-ex
p(2*d*x+2*c))/(a^2+b^2)/d^4+6*b^2*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)
^(1/2)))/(a^2+b^2)^(3/2)/d^4-6*b^2*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2
)^(1/2)))/(a^2+b^2)^(3/2)/d^4+b*(f*x+e)^3*sech(d*x+c)/(a^2+b^2)/d+a*(f*x+e
)^3*tanh(d*x+c)/(a^2+b^2)/d
```

Mathematica [A] (warning: unable to verify)

Time = 7.12 (sec) , antiderivative size = 1072, normalized size of antiderivative = 1.37

$$\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-((f*(-12*a*d^3*e^2*E^(2*c)*x + 12*a*d^3*e^2*(1 + E^(2*c))*x + 12*a*d^3*e
*f*x^2 + 4*a*d^3*f^2*x^3 + 12*b*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)]
- 6*a*d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*b*
d*e*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)])
 - PolyLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) - 6*a*d*e*(1
+ E^(2*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c
 + d*x))]) + (6*I)*b*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d
^2*x^2*Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x
*PolyLog[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3,
 I*E^(c + d*x)]) - a*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*
(c + d*x))]) - 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c
+ d*x))]))))/((a^2 + b^2)*(1 + E^(2*c)))) + (2*b^2*(-2*d^3*e^3*ArcTanh[(a +
 b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2])] + 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[
a^2 + b^2])] + d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])]
- 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 3*d^3*e*f
^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - d^3*f^3*x^3*Log[1
+ (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 3*d^2*f*(e + f*x)^2*PolyLog[2,
(b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 3*d^2*f*(e + f*x)^2*PolyLog[2, -
((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*d*e*f^2*PolyLog[3, (b*E^(c...
```

Rubi [A] (verified)

Time = 3.35 (sec) , antiderivative size = 654, normalized size of antiderivative = 0.84, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {6107, 3042, 3803, 25, 2694, 27, 2620, 3011, 7163, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6107}$$

$$\frac{b^2\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{a^2+b^2}$$

$\downarrow$ 3803

$$\frac{2b^2\int -\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$\downarrow$ 25

$$\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\int \frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}$$

$\downarrow$ 2694

$$\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}$$

$\downarrow$ 27

$$\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}$$

$\downarrow$ 2620

$$\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{a^2+b^2}$$

$\downarrow$ 3011

$$\frac{\int (e+fx)^3 \text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-$$
$$\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be}{a-}\right.}{bd}\right.}{}\right.}{a^2+b^2}$$

$\downarrow$ 7163

$$\frac{\int (e+fx)^3 \text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-$$
$$\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int \text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+}}\right.}{d}}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{}$$

$\downarrow$ 2720

$$\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-$$

$$2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\text{PolyLog}}{}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.$$

$\downarrow$ 7143

$$\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-$$

$$2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.$$

$\downarrow$ 7293

$$\frac{\int \left(a(e+fx)^3\text{sech}^2(c+dx)-b(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)\right)dx}{a^2+b^2}-$$

$$2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}}{bd}\right)}{2\sqrt{a^2+b^2}}\right.$$

$$\downarrow \text{2009}$$

$$-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b}}\right)}{d}}{bd}\right)}{2\sqrt{a^2+b^2}}\right.}{\frac{3af^3\text{PolyLog}(3,-e^{2(c+dx)})}{2d^4}-\frac{3af^2(e+fx)\text{PolyLog}(2,-e^{2(c+dx)})}{d^3}-\frac{3af(e+fx)^2\log(e^{2(c+dx)}+1)}{d^2}+\frac{a(e+fx)^3\tanh(c+dx)}{d}+\frac{a(e+fx)^3}{d}}$$

input
```
Int[((e + f*x)^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-2*b^2*(-1/2*(b*(((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt
[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))
])/d^2))/d))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^3*Log[1 + (b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -(
(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/(2*Sqrt[a^2 + b^2])))/(a^2 +
 b^2) + ((a*(e + f*x)^3)/d - (6*b*f*(e + f*x)^2*ArcTan[E^(c + d*x)])/d^2 -
 (3*a*f*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/d^2 + ((6*I)*b*f^2*(e + f*x)
*PolyLog[2, (-I)*E^(c + d*x)])/d^3 - ((6*I)*b*f^2*(e + f*x)*PolyLog[2, I*E
^(c + d*x)])/d^3 - (3*a*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/d^3 -
((6*I)*b*f^3*PolyLog[3, (-I)*E^(c + d*x)])/d^4 + ((6*I)*b*f^3*PolyLog[3, I
*E^(c + d*x)])/d^4 + (3*a*f^3*PolyLog[3, -E^(2*(c + d*x))])/(2*d^4) + (b*(
e + f*x)^3*Sech[c + d*x])/d + (a*(e + f*x)^3*Tanh[c + d*x])/d)/(a^2 + b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^3 \operatorname{sech}(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 6367 vs. 2(712) = 1424.

Time = 0.26 (sec) , antiderivative size = 6367, normalized size of antiderivative = 8.16

$$\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^3\,\text{sech}^2\,(c+dx)}{a+b\sinh\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)**3*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*sech(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\,\text{sech}\,(dx+c)^2}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
3*a*e^2*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2
+ b^2)*d^2)) - 6*b*f^3*integrate(x^2*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) +
b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + 6*a*f^3*integrate(x^2/(a^2*d*
e^(2*d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 12*b*e*f^2*
integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a
^2*d + b^2*d), x) + 12*a*e*f^2*integrate(x/(a^2*d*e^(2*d*x + 2*c) + b^2*d*
e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + e^3*(b^2*log((b*e^(-d*x - c) - a -
sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^2 + b^2)^(3/2
)*d) + 2*(b*e^(-d*x - c) + a)/((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c))*
d)) - 6*b*e^2*f*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) - 2*(a*f^3*x^3 + 3*a
*e*f^2*x^2 + 3*a*e^2*f*x - (b*f^3*x^3*e^c + 3*b*e*f^2*x^2*e^c + 3*b*e^2*f*
x*e^c)*e^(d*x))/(a^2*d + b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)
) + integrate(-2*(b^2*f^3*x^3*e^c + 3*b^2*e*f^2*x^2*e^c + 3*b^2*e^2*f*x*e^
c)*e^(d*x)/(a^2*b + b^3 - (a^2*b*e^(2*c) + b^3*e^(2*c))*e^(2*d*x) - 2*(a^3
*e^c + a*b^2*e^c)*e^(d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^3}{\cosh\left(c+d\,x\right)^2\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^3/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^3/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output

```
(2*(e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(
a**2 + b**2))*b**2*e**3*i + sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i
)/sqrt(a**2 + b**2))*b**2*e**3*i + 4*e**(5*c + 2*d*x)*int((e**(3*d*x)*x**3
)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(
3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*d*f**3
 + 8*e**(5*c + 2*d*x)*int((e**(3*d*x)*x**3)/(e**(6*c + 6*d*x)*b + 2*e**(5*
c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x
)*b + 2*e**(c + d*x)*a - b),x)*a**2*b**2*d*f**3 + 4*e**(5*c + 2*d*x)*int((
e**(3*d*x)*x**3)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*
d*x)*b + 4*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)
,x)*b**4*d*f**3 + 12*e**(5*c + 2*d*x)*int((e**(3*d*x)*x**2)/(e**(6*c + 6*d
*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a -
 e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*d*e*f**2 + 24*e**(5*c
+ 2*d*x)*int((e**(3*d*x)*x**2)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a
+ e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b + 2*e**(c
 + d*x)*a - b),x)*a**2*b**2*d*e*f**2 + 12*e**(5*c + 2*d*x)*int((e**(3*d*x)
*x**2)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4
*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*b**4*d
*e*f**2 + 12*e**(5*c + 2*d*x)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e
**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - e**(2*c...
```

3.310 $\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 548

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{a(e+fx)^2}{(a^2+b^2)\,d} - \frac{4bf(e+fx)\arctan\left(e^{c+dx}\right)}{(a^2+b^2)\,d^2} + \frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} - \frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} - \frac{2af(e+fx)\log\left(1+e^{2(c+dx)}\right)}{(a^2+b^2)\,d^2} + \frac{2ibf^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{(a^2+b^2)\,d^3} - \frac{2ibf^2\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{(a^2+b^2)\,d^3} + \frac{2b^2f(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^2} - \frac{2b^2f(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^2} - \frac{af^2\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{(a^2+b^2)\,d^3} - \frac{2b^2f^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^3} + \frac{2b^2f^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^3} + \frac{b(e+fx)^2\text{sech}(c+dx)}{(a^2+b^2)\,d} + \frac{a(e+fx)^2\tanh(c+dx)}{(a^2+b^2)\,d}$$

output
```
a*(f*x+e)^2/(a^2+b^2)/d-4*b*f*(f*x+e)*arctan(exp(d*x+c))/(a^2+b^2)/d^2+b^2
*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d-b^2*(f
*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d-2*a*f*(f*
x+e)*ln(1+exp(2*d*x+2*c))/(a^2+b^2)/d^2+2*I*b*f^2*polylog(2,-I*exp(d*x+c))
/(a^2+b^2)/d^3-2*I*b*f^2*polylog(2,I*exp(d*x+c))/(a^2+b^2)/d^3+2*b^2*f*(f*
x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2-2*b^
2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d
^2-a*f^2*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)/d^3-2*b^2*f^2*polylog(3,-b*e
xp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^3+2*b^2*f^2*polylog(3,-b*
exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^3+b*(f*x+e)^2*sech(d*x+c
)/(a^2+b^2)/d+a*(f*x+e)^2*tanh(d*x+c)/(a^2+b^2)/d
```

Mathematica [A] (warning: unable to verify)

Time = 3.88 (sec) , antiderivative size = 633, normalized size of antiderivative = 1.16

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$=\frac{f\left(4ad^2ee^{2c}x-4ad^2e\left(1+e^{2c}\right)x+2ad^2e^{2c}fx^2-2ad^2\left(1+e^{2c}\right)fx^2-4bde\left(1+e^{2c}\right)\arctan\left(e^{c+dx}\right)+2ade\left(1+e^{2c}\right)\left(2dx-\log\left(1+e^{2(c+dx)}\right)\right)+2ib\left(1+e^{2\cdots}\right.}{(\cdots}$$

input
```
Integrate[((e + f*x)^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
((f*(4*a*d^2*e*E^(2*c)*x - 4*a*d^2*e*(1 + E^(2*c))*x + 2*a*d^2*E^(2*c)*f*x
^2 - 2*a*d^2*(1 + E^(2*c))*f*x^2 - 4*b*d*e*(1 + E^(2*c))*ArcTan[E^(c + d*x
)] + 2*a*d*e*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (2*I)*b*(1
 + E^(2*c))*f*(d*x*(-Log[1 - I*E^(c + d*x)] + Log[1 + I*E^(c + d*x)]) + Po
lyLog[2, (-I)*E^(c + d*x)] - PolyLog[2, I*E^(c + d*x)]) + a*(1 + E^(2*c))*
f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x))]))
)/((a^2 + b^2)*(1 + E^(2*c))) - (b^2*(2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x)
)/Sqrt[a^2 + b^2]] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*d^2
*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1
+ (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*d*f*(e + f*x)*PolyLog[2, (b*E
^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2]))] + 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a +
Sqrt[a^2 + b^2])] - 2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
]))]))/(a^2 + b^2)^(3/2) + (d^2*(e + f*x)^2*Sech[c + d*x]*(b + a*Sech[c]*S
inh[d*x]))/(a^2 + b^2))/d^3
```

Rubi [A] (verified)

Time = 2.53 (sec) , antiderivative size = 469, normalized size of antiderivative = 0.86, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6107, 3042, 3803, 25, 2694, 27, 2620, 3011, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6107}$$

$$\frac{b^2\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3803}$$

$$\frac{2b^2\int -\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\int \frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}$$

$$\Big\downarrow \; 2694$$

$$\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 2620$$

$$\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}}{2\sqrt{a^2+b^2}}}{a^2+b^2}$$

$$\Big\downarrow \; 3011$$

$$\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+\cdots\right)}{bd}\cdots\right)}{\cdots}\right)}{a^2+b^2}$$

$\downarrow$ 2720

$$\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\cdots\right)}{\cdots}\cdots\right)}{\cdots}\right)}{a^2+b^2}$$

$\downarrow$ 7143

$$\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\cdots\right)}{\cdots}\right)}{a^2+b^2}$$

$\downarrow$ 7293

$$\frac{\int \left(a(e+fx)^2\text{sech}^2(c+dx)-b(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)\right)dx}{a^2+b^2}-$$

$$\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\cdots\right.}{\cdots}\right.}{a^2+b^2}$$

$\downarrow$ 2009

$$-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right.}{bd}\cdots\right.}{\cdots}\right.}{a^2+b^2}$$

$$\frac{-\frac{af^2\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{d^3}-\frac{2af(e+fx)\log\left(e^{2(c+dx)}+1\right)}{d^2}+\frac{a(e+fx)^2\tanh(c+dx)}{d}+\frac{a(e+fx)^2}{d}-\frac{4bf(e+fx)\arctan\left(e^{c+dx}\right)}{d^2}+\frac{2ibf^2\cdots}{\cdots}}{a^2+b^2}$$

input
```
Int[((e + f*x)^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-2*b^2*(-1/2*(b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2
 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/
d^2))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2]))])/d^2))/(b*d)))/(2*Sqrt[a^2 + b^2])))/(a^2 + b^2) + ((a*(e
 + f*x)^2)/d - (4*b*f*(e + f*x)*ArcTan[E^(c + d*x)])/d^2 - (2*a*f*(e + f*x
)*Log[1 + E^(2*(c + d*x))])/d^2 + ((2*I)*b*f^2*PolyLog[2, (-I)*E^(c + d*x)
])/d^3 - ((2*I)*b*f^2*PolyLog[2, I*E^(c + d*x)])/d^3 - (a*f^2*PolyLog[2, -
E^(2*(c + d*x))])/d^3 + (b*(e + f*x)^2*Sech[c + d*x])/d + (a*(e + f*x)^2*T
anh[c + d*x])/d)/(a^2 + b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{sech}(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3582 vs. $2(502) = 1004$.

Time = 0.21 (sec) , antiderivative size = 3582, normalized size of antiderivative = 6.54

$$\int \frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2 \operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*sech(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\,\text{sech}\,(dx+c)^2}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
2*a*e*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2 +
b^2)*d^2)) - 4*b*f^2*integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2*
d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + 4*a*f^2*integrate(x/(a^2*d*e^(2*d
*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + e^2*(b^2*log((b*e
^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))
/((a^2 + b^2)^(3/2)*d) + 2*(b*e^(-d*x - c) + a)/((a^2 + b^2 + (a^2 + b^2)*
e^(-2*d*x - 2*c))*d)) - 4*b*e*f*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) - 2*
(a*f^2*x^2 + 2*a*e*f*x - (b*f^2*x^2*e^c + 2*b*e*f*x*e^c)*e^(d*x))/(a^2*d +
 b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)) + integrate(-2*(b^2*f^
2*x^2*e^c + 2*b^2*e*f*x*e^c)*e^(d*x)/(a^2*b + b^3 - (a^2*b*e^(2*c) + b^3*e
^(2*c))*e^(2*d*x) - 2*(a^3*e^c + a*b^2*e^c)*e^(d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{{\left(e+f\,x\right)}^2}{{\mathrm{cosh}\left(c+d\,x\right)}^2\,\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)} \,d x$$

input
```
int((e + f*x)^2/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^2/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Too large to display}$$

input
```
int((f*x+e)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(2*(e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(
a**2 + b**2))*b**2*e**2*i + sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i
)/sqrt(a**2 + b**2))*b**2*e**2*i + 4*e**(5*c + 2*d*x)*int((e**(3*d*x)*x**2
)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(
3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*d*f**2
 + 8*e**(5*c + 2*d*x)*int((e**(3*d*x)*x**2)/(e**(6*c + 6*d*x)*b + 2*e**(5*
c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x
)*b + 2*e**(c + d*x)*a - b),x)*a**2*b**2*d*f**2 + 4*e**(5*c + 2*d*x)*int((
e**(3*d*x)*x**2)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*
d*x)*b + 4*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)
,x)*b**4*d*f**2 + 8*e**(5*c + 2*d*x)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*
b + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - e**
(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*d*e*f + 16*e**(5*c + 2*d*x
)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c
+ 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a
- b),x)*a**2*b**2*d*e*f + 8*e**(5*c + 2*d*x)*int((e**(3*d*x)*x)/(e**(6*c +
 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)
*a - e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*b**4*d*e*f + e**(2*c +
2*d*x)*a**3*e**2 + e**(2*c + 2*d*x)*a*b**2*e**2 + e**(c + d*x)*a**2*b*e**2
 + e**(c + d*x)*b**3*e**2 + 4*e**(3*c)*int((e**(3*d*x)*x**2)/(e**(6*c +...
```

3.311 $\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 295

$$\begin{aligned}\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx =& -\frac{bf\arctan(\sinh(c+dx))}{(a^2+b^2)\,d^2} \\ &+\frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} \\ &-\frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} \\ &-\frac{af\log(\cosh(c+dx))}{(a^2+b^2)\,d^2}+\frac{b^2f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^2} \\ &-\frac{b^2f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^2} \\ &+\frac{b(e+fx)\text{sech}(c+dx)}{(a^2+b^2)\,d}+\frac{a(e+fx)\tanh(c+dx)}{(a^2+b^2)\,d}\end{aligned}$$

output
```
-b*f*arctan(sinh(d*x+c))/(a^2+b^2)/d^2+b^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a
^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d-b^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/(a^2+b^2)^(3/2)/d-a*f*ln(cosh(d*x+c))/(a^2+b^2)/d^2+b^2*f*polyl
og(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2-b^2*f*polylog(
2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2+b*(f*x+e)*sech(d*
x+c)/(a^2+b^2)/d+a*(f*x+e)*tanh(d*x+c)/(a^2+b^2)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.28 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.13

$$\int \frac{(e+fx)\mathrm{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{\frac{2f\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{ia-b} + \frac{2if\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a-ib} - \frac{f\log(\cosh(c+dx))}{a-ib} - \frac{f\log(\cosh(c+dx))}{a+ib} + \frac{2b^2\left(-2de\,\mathrm{arctanh}\left(\frac{a+be^{c+d}}{\sqrt{a^2+b^2}}\right.\right.}{}}{}$$

input
```
Integrate[((e + f*x)*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
((2*f*ArcTan[Tanh[(c + d*x)/2]])/(I*a - b) + ((2*I)*f*ArcTan[Tanh[(c + d*x
)/2]])/(a - I*b) - (f*Log[Cosh[c + d*x]])/(a - I*b) - (f*Log[Cosh[c + d*x]
])/(a + I*b) + (2*b^2*(-2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]]
 + 2*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log[1
+ (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1 + (b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2])] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2
+ b^2])] - f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^2 +
 b^2)^(3/2) + (2*d*(e + f*x)*Sech[c + d*x]*(b + a*Sinh[c + d*x]))/(a^2 + b
^2))/(2*d^2)
```

Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.93, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {6107, 3042, 3803, 25, 2694, 27, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6107}$$

$$\frac{b^2\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3803}$$

$$\frac{2b^2\int -\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\int \frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}$$

$$\Big\downarrow \text{2694}$$

$$\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}$$

$\Big\downarrow$ 2620

$$\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}$$

$\Big\downarrow$ 2715

$$\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}$$

$\Big\downarrow$ 2838

$$\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}$$

$\Big\downarrow$ 7293

$$\frac{\int \left(a(e+fx)\text{sech}^2(c+dx) - b(e+fx)\text{sech}(c+dx)\tanh(c+dx)\right) dx}{a^2+b^2} - \frac{2b^2\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}} - \frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{af\log(\cosh(c+dx))}{d^2}+\frac{a(e+fx)\tanh(c+dx)}{d}-\frac{bf\arctan(\sinh(c+dx))}{d^2}+\frac{b(e+fx)\text{sech}(c+dx)}{d}}{a^2+b^2} - \frac{2b^2\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}} - \frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}$$

input
```
Int[((e + f*x)*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-2*b^2*(-1/2*(b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2
)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
)/(b*d^2)))/(2*Sqrt[a^2 + b^2])))/(a^2 + b^2) + (-((b*f*ArcTan[Sinh[c + d*
x]])/d^2) - (a*f*Log[Cosh[c + d*x]])/d^2 + (b*(e + f*x)*Sech[c + d*x])/d +
 (a*(e + f*x)*Tanh[c + d*x])/d)/(a^2 + b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1927 vs. $2(275) = 550$.

Time = 12.43 (sec) , antiderivative size = 1928, normalized size of antiderivative = 6.54

method	result	size
risch	Expression too large to display	1928

input
```
int((f*x+e)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/(a^2+b^2)^2/d^2*a^3*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/(a^2+b^2)
/d^2*a*f*ln(exp(d*x+c))+2/(a^2+b^2)^(5/2)/d^2*a^4*f*arctanh(1/2*(2*b*exp(d
*x+c)+2*a)/(a^2+b^2)^(1/2))-4/(a^2+b^2)/d^2*b^3*f/(2*a^2+2*b^2)*arctan(exp
(d*x+c))-2*(f*x+e)*(-b*exp(d*x+c)+a)/d/(a^2+b^2)/(1+exp(2*d*x+2*c))-2/(a^2
+b^2)/d^2*a^3*f/(2*a^2+2*b^2)*ln(1+exp(2*d*x+2*c))+2/(a^2+b^2)/d^2*a^3*f/(
2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/2/(a^2+b^2)^2/d^2*a*b
^2*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/(a^2+b^2)^(3/2)/d^2*b^2*f/(2*
a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a^2*
c-2/(a^2+b^2)^(3/2)/d^2*b^2*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/
2)+a)/(a+(a^2+b^2)^(1/2)))*a^2*c+2/(a^2+b^2)^(3/2)/d*b^2*f/(2*a^2+2*b^2)*l
n((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a^2*x-2/(a^2+b^2
)^(3/2)/d*b^2*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+
b^2)^(1/2)))*a^2*x+2/(a^2+b^2)^(3/2)/d^2*c*b^2*f/(2*a^2+2*b^2)*arctanh(1/2
*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^2+2/(a^2+b^2)^(3/2)/d^2*c*b^4*f/(
2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/(a^2+b^2)
^(3/2)/d^2*b^4*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a
^2+b^2)^(1/2)))*c-2/(a^2+b^2)^(3/2)/d^2*b^4*f/(2*a^2+2*b^2)*ln((b*exp(d*x+
c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2/(a^2+b^2)^(3/2)/d^2*b^2*f/(
2*a^2+2*b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))
*a^2-2/(a^2+b^2)^(3/2)/d^2*b^2*f/(2*a^2+2*b^2)*dilog((b*exp(d*x+c)+(a^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1296 vs. $2(273) = 546$.

Time = 0.13 (sec) , antiderivative size = 1296, normalized size of antiderivative = 4.39

$$\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(2*(a^3 + a*b^2)*d*f*x*cosh(d*x + c)^2 + 2*(a^3 + a*b^2)*d*f*x*sinh(d*x +
c)^2 - 2*(a^3 + a*b^2)*d*e + (b^3*f*cosh(d*x + c)^2 + 2*b^3*f*cosh(d*x + c
)*sinh(d*x + c) + b^3*f*sinh(d*x + c)^2 + b^3*f)*sqrt((a^2 + b^2)/b^2)*dil
og((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c)
)*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^3*f*cosh(d*x + c)^2 + 2*b^3*f*cos
h(d*x + c)*sinh(d*x + c) + b^3*f*sinh(d*x + c)^2 + b^3*f)*sqrt((a^2 + b^2)
/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^3*d*e - b^3*c*f + (b^3*d
*e - b^3*c*f)*cosh(d*x + c)^2 + 2*(b^3*d*e - b^3*c*f)*cosh(d*x + c)*sinh(d
*x + c) + (b^3*d*e - b^3*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2
*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) +
(b^3*d*e - b^3*c*f + (b^3*d*e - b^3*c*f)*cosh(d*x + c)^2 + 2*(b^3*d*e - b^
3*c*f)*cosh(d*x + c)*sinh(d*x + c) + (b^3*d*e - b^3*c*f)*sinh(d*x + c)^2)*
sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt
((a^2 + b^2)/b^2) + 2*a) + (b^3*d*f*x + b^3*c*f + (b^3*d*f*x + b^3*c*f)*co
sh(d*x + c)^2 + 2*(b^3*d*f*x + b^3*c*f)*cosh(d*x + c)*sinh(d*x + c) + (b^3
*d*f*x + b^3*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x
+ c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b
^2)/b^2) - b)/b) - (b^3*d*f*x + b^3*c*f + (b^3*d*f*x + b^3*c*f)*cosh(d*x +
 c)^2 + 2*(b^3*d*f*x + b^3*c*f)*cosh(d*x + c)*sinh(d*x + c) + (b^3*d*f*...
```

Sympy [F]

$$\int \frac{(e+fx)\mathrm{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\,\mathrm{sech}^2\,(c+dx)}{a+b\sinh\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*sech(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)\,\text{sech}\,(dx+c)^2}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
(4*b^2*integrate(-1/2*x*e^(d*x + c)/(a^2*b + b^3 - (a^2*b*e^(2*c) + b^3*e^
(2*c))*e^(2*d*x) - 2*(a^3*e^c + a*b^2*e^c)*e^(d*x)), x) + 2*(b*x*e^(d*x +
c) - a*x)/(a^2*d + b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)) + 2*
a*x/((a^2 + b^2)*d) - 2*b*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) - a*log(e^
(2*d*x + 2*c) + 1)/((a^2 + b^2)*d^2))*f + e*(b^2*log((b*e^(-d*x - c) - a -
 sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^2 + b^2)^(3/
2)*d) + 2*(b*e^(-d*x - c) + a)/((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c))
*d))
```

Giac [F]

$$\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)\,\text{sech}\,(dx+c)^2}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*sech(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{e+f\,x}{\cosh\,(c+d\,x)^2\,\left(a+b\sinh\,(c+d\,x)\right)}\,dx$$

input
```
int((e + f*x)/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{2e^{2dx+2c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)b^2ei+2\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)b^2ei+8e^{2dx+5c}\left(\int\frac{}{e^{6dx+6c}b+2e^{5dx+5c}a+}\right.}{}$$

input
```
int((f*x+e)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(2*(e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(
a**2 + b**2))*b**2*e*i + sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/s
qrt(a**2 + b**2))*b**2*e*i + 4*e**(5*c + 2*d*x)*int((e**(3*d*x)*x)/(e**(6*
c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d
*x)*a - e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*d*f + 8*e**(5*c
 + 2*d*x)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a +
e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b + 2*e**(c +
 d*x)*a - b),x)*a**2*b**2*d*f + 4*e**(5*c + 2*d*x)*int((e**(3*d*x)*x)/(e**
(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*c +
3*d*x)*a - e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*b**4*d*f + e**(2*
c + 2*d*x)*a**3*e + e**(2*c + 2*d*x)*a*b**2*e + e**(c + d*x)*a**2*b*e + e*
*(c + d*x)*b**3*e + 4*e**(3*c)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*
e**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - e**(2*c +
 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*d*f + 8*e**(3*c)*int((e**(3*d*x)
*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e*
*(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b**2
*d*f + 4*e**(3*c)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d
*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b + 2
*e**(c + d*x)*a - b),x)*b**4*d*f))/(d*(e**(2*c + 2*d*x)*a**4 + 2*e**(2*c +
 2*d*x)*a**2*b**2 + e**(2*c + 2*d*x)*b**4 + a**4 + 2*a**2*b**2 + b**4))
```

3.312 $\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{2b^2\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} + \frac{\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{(a^2+b^2)\,d}$$

output
```
-2*b^2*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/
d+sech(d*x+c)*(b+a*sinh(d*x+c))/(a^2+b^2)/d
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.35

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{2b^2\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right) + b\sqrt{-a^2-b^2}\operatorname{sech}(c+dx) + a\sqrt{-a^2-b^2}\tanh(c+dx)}{(-a^2-b^2)^{3/2}\,d}$$

input
```
Integrate[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

output
```
-((2*b^2*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]] + b*Sqrt[-a^2
- b^2]*Sech[c + d*x] + a*Sqrt[-a^2 - b^2]*Tanh[c + d*x])/((-a^2 - b^2)^(3/
2)*d))
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3175, 25, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3175}$$

$$\frac{\operatorname{sech}(c+dx)(a\sinh(c+dx)+b)}{d(a^2+b^2)}-\frac{\int -\frac{b^2}{a+b\sinh(c+dx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b^2}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\operatorname{sech}(c+dx)(a\sinh(c+dx)+b)}{d(a^2+b^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^2\int \frac{1}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\operatorname{sech}(c+dx)(a\sinh(c+dx)+b)}{d(a^2+b^2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\operatorname{sech}(c+dx)(a\sinh(c+dx)+b)}{d(a^2+b^2)}+\frac{b^2\int \frac{1}{a-ib\sin(ic+idx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3139}$$

$$\frac{\text{sech}(c+dx)(a\sinh(c+dx)+b)}{d(a^2+b^2)}-\frac{2ib^2\int\frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d(a^2+b^2)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\text{sech}(c+dx)(a\sinh(c+dx)+b)}{d(a^2+b^2)}+\frac{4ib^2\int\frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4(a^2+b^2)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{d(a^2+b^2)}$$

$$\Big\downarrow \text{217}$$

$$\frac{2b^2\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{3/2}}+\frac{\text{sech}(c+dx)(a\sinh(c+dx)+b)}{d(a^2+b^2)}$$

input
```
Int[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

output
```
(2*b^2*ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*
d) + (Sech[c + d*x]*(b + a*Sinh[c + d*x]))/((a^2 + b^2)*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3175
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*
(a^2 - b^2)*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*
(a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ
[2*m, 2*p]
```

Maple [A] (verified)

Time = 12.64 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{\frac{2b^2 \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(-a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-b\right)}{\left(a^2+b^2\right)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}}{d}$
default	$\frac{\frac{2b^2 \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(-a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-b\right)}{\left(a^2+b^2\right)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}}{d}$
risch	$-\frac{2\left(-b\,\mathrm{e}^{dx+c}+a\right)}{d(a^2+b^2)\left(1+\mathrm{e}^{2dx+2c}\right)}+\frac{b^2\ln\left(\mathrm{e}^{dx+c}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}-a^4-2a^2b^2-b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}\,d}-\frac{b^2\ln\left(\mathrm{e}^{dx+c}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}\,d}$

input
```
int(sech(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*b^2/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+
b^2)^(1/2))-2/(a^2+b^2)*(-a*tanh(1/2*d*x+1/2*c)-b)/(1+tanh(1/2*d*x+1/2*c)^
2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(74) = 148$.

Time = 0.08 (sec) , antiderivative size = 353, normalized size of antiderivative = 4.58

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{2a^3+2ab^2-\left(b^2\cosh\left(dx+c\right)^2+2b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b^2\sinh\left(dx+c\right)^2+b^2\right)\sqrt{a^2+b^2}\,\mathrm{lo}}{(a^4+2a^2b^2+b^4)d\cosh\left(dx+c\right)^2+2\,(a}$$

input
```
integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(2*a^3 + 2*a*b^2 - (b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x +
c) + b^2*sinh(d*x + c)^2 + b^2)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 +
 b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x
 + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d
*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) +
 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - 2*(a^2*b + b^3)*cosh(d*x +
c) - 2*(a^2*b + b^3)*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x +
c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*
a^2*b^2 + b^4)*d*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\operatorname{sech}^2\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate(sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(sech(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.49

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b^2\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}d} + \frac{2\left(be^{(-dx-c)}+a\right)}{(a^2+b^2+(a^2+b^2)e^{(-2\,dx-2\,c)})d}$$

input
```
integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
b^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(
a^2 + b^2)))/((a^2 + b^2)^(3/2)*d) + 2*(b*e^(-d*x - c) + a)/((a^2 + b^2 +
(a^2 + b^2)*e^(-2*d*x - 2*c))*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.40

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{b^2\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}} + \frac{2\left(be^{(dx+c)}-a\right)}{(a^2+b^2)\left(e^{(2\,dx+2\,c)}+1\right)}}{d}$$

input
```
integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
(b^2*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c
) + 2*a + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(b*e^(d*x + c) - a)/((
a^2 + b^2)*(e^(2*d*x + 2*c) + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 413, normalized size of antiderivative = 5.36

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{2a}{d(a^2+b^2)}-\frac{2b\,e^{c+dx}}{d(a^2+b^2)}}{e^{2c+2dx}+1}$$
$$-\frac{2\,\mathrm{atan}\left(\left(\frac{b^3\sqrt{-a^6d^2-3a^4b^2d^2-3a^2b^4d^2-b^6d^2}}{2}+\frac{a^2b\sqrt{-a^6d^2-3a^4b^2d^2-3a^2b^4d^2-b^6d^2}}{2}\right)\left(e^{dx}\,e^{c}\left(\frac{2}{d\sqrt{b^4}(a^2+b^2)^2}+\frac{}{b^4}\right.\right.\right.}{\sqrt{-a^6d^2-3a^4b}}$$

input
```
int(1/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
- ((2*a)/(d*(a^2 + b^2)) - (2*b*exp(c + d*x))/(d*(a^2 + b^2)))/(exp(2*c +
2*d*x) + 1) - (2*atan(((b^3*(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3*a^4*b
^2*d^2)^(1/2))/2 + (a^2*b*(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3*a^4*b^2
*d^2)^(1/2))/2)*(exp(d*x)*exp(c)*(2/(d*(b^4)^(1/2)*(a^2 + b^2)^2) + (2*a*(
a^3*d*(b^4)^(1/2) + a*b^2*d*(b^4)^(1/2)))/(b^4*(-d^2*(a^2 + b^2)^3)^(1/2)*
(a^2 + b^2)*(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3*a^4*b^2*d^2)^(1/2)))
- (2*a*(b^3*d*(b^4)^(1/2) + a^2*b*d*(b^4)^(1/2)))/(b^4*(-d^2*(a^2 + b^2)^3
)^(1/2)*(a^2 + b^2)*(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3*a^4*b^2*d^2)^
(1/2))))*(b^4)^(1/2))/(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3*a^4*b^2*d^2
)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.71

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\frac{2e^{2dx+2c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)b^2i+2\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)b^2i+2e^{2dx+2c}a^3+2e^{2dx+2c}a\,b^2+2e^{c}}{d\left(e^{2dx+2c}a^4+2e^{2dx+2c}a^2b^2+e^{2dx+2c}b^4+a^4+2a^2b^2+b^4\right)}$$

input
```
int(sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(2*(e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(
a**2 + b**2))*b**2*i + sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqr
t(a**2 + b**2))*b**2*i + e**(2*c + 2*d*x)*a**3 + e**(2*c + 2*d*x)*a*b**2 +
 e**(c + d*x)*a**2*b + e**(c + d*x)*b**3))/(d*(e**(2*c + 2*d*x)*a**4 + 2*e
**(2*c + 2*d*x)*a**2*b**2 + e**(2*c + 2*d*x)*b**4 + a**4 + 2*a**2*b**2 + b
**4))
```

3.313 $\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \operatorname{Int}\left(\frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}, x\right)$$

output
```
Defer(Int)(sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 61.96 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

input
```
Integrate[Sech[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Sech[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx$$
$$\Big\downarrow \text{ 6111}$$
$$\int \frac{\text{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Sech[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\text{sech}\left(dx+c\right)^{2}}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sech(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}^2\left(c+dx\right)}{(a+b\sinh\left(c+dx\right))\left(e+fx\right)}\,dx$$

input
```
integrate(sech(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(sech(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 391, normalized size of antiderivative = 13.96

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
4*b^2*integrate(-1/2*e^(d*x + c)/(a^2*b*e + b^3*e + (a^2*b*f + b^3*f)*x -
(a^2*b*e*e^(2*c) + b^3*e*e^(2*c) + (a^2*b*f*e^(2*c) + b^3*f*e^(2*c))*x)*e^
(2*d*x) - 2*(a^3*e*e^c + a*b^2*e*e^c + (a^3*f*e^c + a*b^2*f*e^c)*x)*e^(d*x
)), x) + 2*(b*e^(d*x + c) - a)/(a^2*d*e + b^2*d*e + (a^2*d*f + b^2*d*f)*x
+ (a^2*d*e*e^(2*c) + b^2*d*e*e^(2*c) + (a^2*d*f*e^(2*c) + b^2*d*f*e^(2*c))
*x)*e^(2*d*x)) + 4*integrate(1/2*(b*f*e^(d*x + c) - a*f)/(a^2*d*e^2 + b^2*
d*e^2 + (a^2*d*f^2 + b^2*d*f^2)*x^2 + 2*(a^2*d*e*f + b^2*d*e*f)*x + (a^2*d
*e^2*e^(2*c) + b^2*d*e^2*e^(2*c) + (a^2*d*f^2*e^(2*c) + b^2*d*f^2*e^(2*c))
*x^2 + 2*(a^2*d*e*f*e^(2*c) + b^2*d*e*f*e^(2*c))*x)*e^(2*d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{1}{\cosh(c+d\,x)^2\,(e+f\,x)\,(a+b\sinh(c+d\,x))}\,dx$$

input
```
int(1/(cosh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(cosh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.43

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$=\int \frac{\operatorname{sech}(dx+c)^2}{\sinh(dx+c)\,be+\sinh(dx+c)\,bfx+ae+afx}dx$$

input
```
int(sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(sech(c + d*x)**2/(sinh(c + d*x)*b*e + sinh(c + d*x)*b*f*x + a*e + a*f*
x),x)
```

3.314 $\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 928

$$\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output
```
-b*f*(f*x+e)*tanh(d*x+c)/(a^2+b^2)/d^2+a*f*(f*x+e)*sech(d*x+c)/(a^2+b^2)/d
^2+I*a*f^2*polylog(3,-I*exp(d*x+c))/(a^2+b^2)/d^3-b^3*f*(f*x+e)*polylog(2,
-exp(2*d*x+2*c))/(a^2+b^2)^2/d^2-b^3*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/(a^2+b
^2)^2/d-a*f^2*arctan(sinh(d*x+c))/(a^2+b^2)/d^3+b*f^2*ln(cosh(d*x+c))/(a^2
+b^2)/d^3+2*I*a*b^2*f*(f*x+e)*polylog(2,I*exp(d*x+c))/(a^2+b^2)^2/d^2+2*I*
a*b^2*f^2*polylog(3,-I*exp(d*x+c))/(a^2+b^2)^2/d^3+I*a*f*(f*x+e)*polylog(2
,I*exp(d*x+c))/(a^2+b^2)/d^2-2*I*a*b^2*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/
(a^2+b^2)^2/d^2+a*(f*x+e)^2*arctan(exp(d*x+c))/(a^2+b^2)/d+1/2*b^3*f^2*pol
ylog(3,-exp(2*d*x+2*c))/(a^2+b^2)^2/d^3+1/2*b*(f*x+e)^2*sech(d*x+c)^2/(a^2
+b^2)/d+2*b^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+
b^2)^2/d^2+2*b^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a
^2+b^2)^2/d^2-2*I*a*b^2*f^2*polylog(3,I*exp(d*x+c))/(a^2+b^2)^2/d^3-I*a*f*
(f*x+e)*polylog(2,-I*exp(d*x+c))/(a^2+b^2)/d^2+b^3*(f*x+e)^2*ln(1+b*exp(d*
x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d+b^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a
+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d-2*b^3*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2
+b^2)^(1/2)))/(a^2+b^2)^2/d^3-2*b^3*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^
2)^(1/2)))/(a^2+b^2)^2/d^3+2*a*b^2*(f*x+e)^2*arctan(exp(d*x+c))/(a^2+b^2)^
2/d+1/2*a*(f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/(a^2+b^2)/d-I*a*f^2*polylog(3,
I*exp(d*x+c))/(a^2+b^2)/d^3
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3368 vs. 2(928) = 1856.

Time = 11.30 (sec) , antiderivative size = 3368, normalized size of antiderivative = 3.63

$$\int \frac{(e+fx)^2 \text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(12*b^3*d^3*e^2*E^(2*c)*x - 12*a^2*b*d*E^(2*c)*f^2*x - 12*b^3*d*E^(2*c)*f^
2*x + 12*b^3*d^3*e*E^(2*c)*f*x^2 + 4*b^3*d^3*E^(2*c)*f^2*x^3 + 6*a^3*d^2*e
^2*ArcTan[E^(c + d*x)] + 18*a*b^2*d^2*e^2*ArcTan[E^(c + d*x)] + 6*a^3*d^2*
e^2*E^(2*c)*ArcTan[E^(c + d*x)] + 18*a*b^2*d^2*e^2*E^(2*c)*ArcTan[E^(c + d
*x)] - 12*a^3*f^2*ArcTan[E^(c + d*x)] - 12*a*b^2*f^2*ArcTan[E^(c + d*x)] -
 12*a^3*E^(2*c)*f^2*ArcTan[E^(c + d*x)] - 12*a*b^2*E^(2*c)*f^2*ArcTan[E^(c
 + d*x)] + (6*I)*a^3*d^2*e*f*x*Log[1 - I*E^(c + d*x)] + (18*I)*a*b^2*d^2*e
*f*x*Log[1 - I*E^(c + d*x)] + (6*I)*a^3*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c +
 d*x)] + (18*I)*a*b^2*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] + (3*I)*a^3
*d^2*f^2*x^2*Log[1 - I*E^(c + d*x)] + (9*I)*a*b^2*d^2*f^2*x^2*Log[1 - I*E^
(c + d*x)] + (3*I)*a^3*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] + (9*I)*
a*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] - (6*I)*a^3*d^2*e*f*x*Log
[1 + I*E^(c + d*x)] - (18*I)*a*b^2*d^2*e*f*x*Log[1 + I*E^(c + d*x)] - (6*I
)*a^3*d^2*e*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] - (18*I)*a*b^2*d^2*e*E^(2*c
)*f*x*Log[1 + I*E^(c + d*x)] - (3*I)*a^3*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)
] - (9*I)*a*b^2*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] - (3*I)*a^3*d^2*E^(2*c)
*f^2*x^2*Log[1 + I*E^(c + d*x)] - (9*I)*a*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 +
I*E^(c + d*x)] - 6*b^3*d^2*e^2*Log[1 + E^(2*(c + d*x))] - 6*b^3*d^2*e^2*E^
(2*c)*Log[1 + E^(2*(c + d*x))] + 6*a^2*b*f^2*Log[1 + E^(2*(c + d*x))] + 6*
b^3*f^2*Log[1 + E^(2*(c + d*x))] + 6*a^2*b*E^(2*c)*f^2*Log[1 + E^(2*(c ...
```

Rubi [A] (verified)

Time = 3.24 (sec) , antiderivative size = 764, normalized size of antiderivative = 0.82, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {6107, 6107, 6095, 2620, 3011, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6107}$$

$$\frac{\int (e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{6107}$$

$$\frac{\int (e+fx)^2 \text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a^2+b^2}$$

$\Big\downarrow$ 6095

$$\frac{\int (e+fx)^2 \text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a^2+b^2}$$

$\Big\downarrow$ 2620

$$\frac{\int (e+fx)^2 \text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+$$
$$\frac{b^2\left(\frac{b^2\left(-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+\ldots}{3b\ldots}\right.}{a^2+b^2}\right.}{a^2+b^2}$$

$\Big\downarrow$ 3011

$$\frac{b^2\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c\ldots}}{a+\sqrt{\ldots}}\right)}{d}\ldots}{bd}\ldots\right.}{a^2+b^2}\right.}{a^2+b^2}$$

$$\frac{\int (e+fx)^2 \text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$\Big\downarrow$ 2720

$$b^2\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\cdots\right.}{bd}\cdots\right.}{a^2+b^2\cdots}\cdots\right.$$

$$\frac{\int(e+fx)^2\operatorname{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \ 7143$$

$$b^2\left(\frac{\int(e+fx)^2\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,\cdots\right.}{\cdots}\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.$$

$$a^2+b^2$$

$$\frac{\int(e+fx)^2\operatorname{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \ 7293$$

$$b^2\left(\frac{\int\left(a(e+fx)^2\operatorname{sech}(c+dx)-b(e+fx)^2\tanh(c+dx)\right)dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\cdots}{\cdots}\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.$$

$$a^2+b^2$$

$$\frac{\int\left(a(e+fx)^2\operatorname{sech}^3(c+dx)-b(e+fx)^2\operatorname{sech}^2(c+dx)\tanh(c+dx)\right)dx}{a^2+b^2}$$

$$\Big\downarrow \ 2009$$

$$b^2\left(\frac{b^2\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right.}{bd}}{a^2+b^2}\right.$$

$$-\frac{af^2\arctan(\sinh(c+dx))}{d^3}+\frac{a(e+fx)^2\arctan(e^{c+dx})}{d}+\frac{iaf^2\,\text{PolyLog}(3,-ie^{c+dx})}{d^3}-\frac{iaf^2\,\text{PolyLog}(3,ie^{c+dx})}{d^3}-\frac{iaf(e+fx)\,\text{PolyLog}(2,-i}{d^2}$$

input
```
Int[((e + f*x)^2*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(b^2*((b^2*(-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[
a^2 + b^2]))])/d^2))/(b*d)))/(a^2 + b^2) + ((b*(e + f*x)^3)/(3*f) + (2*a*(
e + f*x)^2*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)^2*Log[1 + E^(2*(c + d*x))
])/d - ((2*I)*a*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + ((2*I)*a*f
*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*(e + f*x)*PolyLog[2, -E^(
2*(c + d*x))])/d^2 + ((2*I)*a*f^2*PolyLog[3, (-I)*E^(c + d*x)])/d^3 - ((2*
I)*a*f^2*PolyLog[3, I*E^(c + d*x)])/d^3 + (b*f^2*PolyLog[3, -E^(2*(c + d*x
))])/(2*d^3))/(a^2 + b^2)))/(a^2 + b^2) + ((a*(e + f*x)^2*ArcTan[E^(c + d*
x)])/d - (a*f^2*ArcTan[Sinh[c + d*x]])/d^3 + (b*f^2*Log[Cosh[c + d*x]])/d^
3 - (I*a*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + (I*a*f*(e + f*x)*
PolyLog[2, I*E^(c + d*x)])/d^2 + (I*a*f^2*PolyLog[3, (-I)*E^(c + d*x)])/d^
3 - (I*a*f^2*PolyLog[3, I*E^(c + d*x)])/d^3 + (a*f*(e + f*x)*Sech[c + d*x]
)/d^2 + (b*(e + f*x)^2*Sech[c + d*x]^2)/(2*d) - (b*f*(e + f*x)*Tanh[c + d*
x])/d^2 + (a*(e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*d))/(a^2 + b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{sech}(dx+c)^3}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 10642 vs. $2(855) = 1710$.

Time = 0.30 (sec) , antiderivative size = 10642, normalized size of antiderivative = 11.47

$$\int \frac{(e+fx)^2 \operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)^2\,\text{sech}^3\,(c+dx)}{a+b\sinh\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*sech(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\,\text{sech}\,(dx+c)^3}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
a^3*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2
*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 +
 b^4*d^2), x) + 3*a*b^2*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*
x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d
^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 2*b^3*d^2*f^2*integrate(x^2/(a^4*d^2*e
^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) +
 a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 2*a^3*d^2*e*f*integrate(x*e^(d*x
 + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e
^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 6*a*b^2*d^2*e*f*
integrate(x*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x
+ 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x)
+ 4*b^3*d^2*e*f*integrate(x/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*
d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2),
 x) - a^2*b*f^2*(2*(d*x + c)/((a^4 + 2*a^2*b^2 + b^4)*d^3) - log(e^(2*d*x
+ 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d^3)) - b^3*f^2*(2*(d*x + c)/((a^4 +
2*a^2*b^2 + b^4)*d^3) - log(e^(2*d*x + 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*
d^3)) + (b^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2
*b^2 + b^4)*d) - b^3*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d)
 - (a^3 + 3*a*b^2)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) + (a*e
^(-d*x - c) + 2*b*e^(-2*d*x - 2*c) - a*e^(-3*d*x - 3*c))/((a^2 + b^2 + ...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\mathrm{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{{\left(e+f\,x\right)}^2}{{\mathrm{cosh}\left(c+d\,x\right)}^3\,\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)} \,dx$$

input
```
int((e + f*x)^2/(cosh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^2/(cosh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\mathrm{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{too large to display}$$

input
```
int((f*x+e)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
(2*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**3*b*d**2*e**2 + 6*e**(4*c + 4*d*
x)*atan(e**(c + d*x))*a*b**3*d**2*e**2 + 4*e**(2*c + 2*d*x)*atan(e**(c + d
*x))*a**3*b*d**2*e**2 + 12*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a*b**3*d**2
*e**2 + 2*atan(e**(c + d*x))*a**3*b*d**2*e**2 + 6*atan(e**(c + d*x))*a*b**
3*d**2*e**2 - 64*e**(7*c + 4*d*x)*int((e**(3*d*x)*x**2)/(e**(8*c + 8*d*x)*
b + 2*e**(7*c + 7*d*x)*a + 2*e**(6*c + 6*d*x)*b + 6*e**(5*c + 5*d*x)*a + 6
*e**(3*c + 3*d*x)*a - 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**5
*d**3*f**2 - 128*e**(7*c + 4*d*x)*int((e**(3*d*x)*x**2)/(e**(8*c + 8*d*x)*
b + 2*e**(7*c + 7*d*x)*a + 2*e**(6*c + 6*d*x)*b + 6*e**(5*c + 5*d*x)*a + 6
*e**(3*c + 3*d*x)*a - 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**3
*b**2*d**3*f**2 - 64*e**(7*c + 4*d*x)*int((e**(3*d*x)*x**2)/(e**(8*c + 8*d
*x)*b + 2*e**(7*c + 7*d*x)*a + 2*e**(6*c + 6*d*x)*b + 6*e**(5*c + 5*d*x)*a
 + 6*e**(3*c + 3*d*x)*a - 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*
a*b**4*d**3*f**2 - 128*e**(7*c + 4*d*x)*int((e**(3*d*x)*x)/(e**(8*c + 8*d*
x)*b + 2*e**(7*c + 7*d*x)*a + 2*e**(6*c + 6*d*x)*b + 6*e**(5*c + 5*d*x)*a
+ 6*e**(3*c + 3*d*x)*a - 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a
**5*d**3*e*f - 256*e**(7*c + 4*d*x)*int((e**(3*d*x)*x)/(e**(8*c + 8*d*x)*b
 + 2*e**(7*c + 7*d*x)*a + 2*e**(6*c + 6*d*x)*b + 6*e**(5*c + 5*d*x)*a + 6*
e**(3*c + 3*d*x)*a - 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**3*
b**2*d**3*e*f - 128*e**(7*c + 4*d*x)*int((e**(3*d*x)*x)/(e**(8*c + 8*d*...
```

3.315 $\int \frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 560

$$\int \frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2ab^2(e+fx)\arctan\left(e^{c+dx}\right)}{(a^2+b^2)^2 d} + \frac{a(e+fx)\arctan\left(e^{c+dx}\right)}{(a^2+b^2)\,d} + \frac{b^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d} + \frac{b^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d} - \frac{b^3(e+fx)\log\left(1+e^{2(c+dx)}\right)}{(a^2+b^2)^2 d} - \frac{iab^2f\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{(a^2+b^2)^2 d^2} - \frac{iaf\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{2(a^2+b^2)\,d^2} + \frac{iab^2f\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{(a^2+b^2)^2 d^2} + \frac{iaf\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{2(a^2+b^2)\,d^2} + \frac{b^3f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d^2} + \frac{b^3f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d^2} - \frac{b^3f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2(a^2+b^2)^2 d^2} + \frac{af\operatorname{sech}(c+dx)}{2(a^2+b^2)\,d^2} + \frac{b(e+fx)\operatorname{sech}^2(c+dx)}{2(a^2+b^2)\,d} - \frac{bf\tanh(c+dx)}{2(a^2+b^2)\,d^2} + \frac{a(e+fx)\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a^2+b^2)\,d}$$

output
```
2*a*b^2*(f*x+e)*arctan(exp(d*x+c))/(a^2+b^2)^2/d+a*(f*x+e)*arctan(exp(d*x+
c))/(a^2+b^2)/d+b^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^
2)^2/d+b^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d-b^
3*(f*x+e)*ln(1+exp(2*d*x+2*c))/(a^2+b^2)^2/d+I*a*b^2*f*polylog(2,I*exp(d*x
+c))/(a^2+b^2)^2/d^2-1/2*I*a*f*polylog(2,-I*exp(d*x+c))/(a^2+b^2)/d^2+1/2*
I*a*f*polylog(2,I*exp(d*x+c))/(a^2+b^2)/d^2-I*a*b^2*f*polylog(2,-I*exp(d*x
+c))/(a^2+b^2)^2/d^2+b^3*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a
^2+b^2)^2/d^2+b^3*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)
^2/d^2-1/2*b^3*f*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)^2/d^2+1/2*a*f*sech(d
*x+c)/(a^2+b^2)/d^2+1/2*b*(f*x+e)*sech(d*x+c)^2/(a^2+b^2)/d-1/2*b*f*tanh(d
*x+c)/(a^2+b^2)/d^2+1/2*a*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a^2+b^2)/d
```

Mathematica [A] (warning: unable to verify)

Time = 8.48 (sec) , antiderivative size = 832, normalized size of antiderivative = 1.49

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{b^3\left(-2de(c+dx)+2cf(c+dx)-f(c+dx)^2+\frac{4a\sqrt{a^2+b^2}de\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-(a^2+b^2)^2}}-\frac{4a\sqrt{-(a^2+b^2)^2}de\,\text{arctanh}\left(\frac{a+be^{c}}{\sqrt{a^2}}\right.}{(-a^2-b^2)^{3/2}}\right.}{}$$
$$-\frac{-2b^3de(c+dx)+2b^3cf(c+dx)-b^3f(c+dx)^2-2a^3de\arctan\left(e^{c+dx}\right)-6ab^2de\arctan\left(e^{c+dx}\right)+2a}{}$$
$$+\frac{\text{sech}(c+dx)(af-bf\sinh(c+dx))}{2(a^2+b^2)d^2}$$
$$+\frac{\text{sech}^2(c+dx)(bde-bcf+bf(c+dx)+ade\sinh(c+dx)-acf\sinh(c+dx)+af(c+dx)\sinh(c+d}{2(a^2+b^2)d^2}$$

input
```
Integrate[((e + f*x)*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(b^3*(-2*d*e*(c + d*x) + 2*c*f*(c + d*x) - f*(c + d*x)^2 + (4*a*Sqrt[a^2 +
 b^2]*d*e*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b^2)^
2] - (4*a*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 +
b^2]])/(-a^2 - b^2)^(3/2) + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2])] + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
])] - 2*c*f*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*d*e*Log[2*a*E
^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] + 2*f*PolyLog[2, (b*E^(c + d*x))/(-
a + Sqrt[a^2 + b^2])] + 2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2]))]))/(2*(a^2 + b^2)^2*d^2) - (-2*b^3*d*e*(c + d*x) + 2*b^3*c*f*(c + d*
x) - b^3*f*(c + d*x)^2 - 2*a^3*d*e*ArcTan[E^(c + d*x)] - 6*a*b^2*d*e*ArcTa
n[E^(c + d*x)] + 2*a^3*c*f*ArcTan[E^(c + d*x)] + 6*a*b^2*c*f*ArcTan[E^(c +
 d*x)] - I*a^3*f*(c + d*x)*Log[1 - I*E^(c + d*x)] - (3*I)*a*b^2*f*(c + d*x
)*Log[1 - I*E^(c + d*x)] + I*a^3*f*(c + d*x)*Log[1 + I*E^(c + d*x)] + (3*I
)*a*b^2*f*(c + d*x)*Log[1 + I*E^(c + d*x)] + 2*b^3*d*e*Log[1 + E^(2*(c + d
*x))] - 2*b^3*c*f*Log[1 + E^(2*(c + d*x))] + 2*b^3*f*(c + d*x)*Log[1 + E^(
2*(c + d*x))] + I*a*(a^2 + 3*b^2)*f*PolyLog[2, (-I)*E^(c + d*x)] - I*a*(a^
2 + 3*b^2)*f*PolyLog[2, I*E^(c + d*x)] + b^3*f*PolyLog[2, -E^(2*(c + d*x))
])/(2*(a^2 + b^2)^2*d^2) + (Sech[c + d*x]*(a*f - b*f*Sinh[c + d*x]))/(2*(a
^2 + b^2)*d^2) + (Sech[c + d*x]^2*(b*d*e - b*c*f + b*f*(c + d*x) + a*d*e*S
inh[c + d*x] - a*c*f*Sinh[c + d*x] + a*f*(c + d*x)*Sinh[c + d*x]))/(2*(...
```

Rubi [A] (verified)

Time = 1.99 (sec) , antiderivative size = 485, normalized size of antiderivative = 0.87, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6107, 6107, 6095, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6107}$$

$$\frac{\int (e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 6107}$$

$$\frac{\int (e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{6095}$$

$$\frac{\int (e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\int (e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int (e+f}{}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{2715}$$

$$\frac{b^2\left(\frac{b^2\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right.}{a^2+b^2}\right.}{a^2+b^2}$$

$$\frac{\int (e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{2838}$$

$$\frac{b^2\left(\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e}{}\right.}{a^2+b^2}\right.}{a^2+b^2}$$

$$\frac{\int (e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{7293}$$

$$b^2\left(\frac{\int (a(e+fx)\text{sech}(c+dx)-b(e+fx)\tanh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right.}{bd}\right.}{a^2+b^2}\right.$$
$$\frac{}{a^2+b^2}$$

$$\frac{\int \left(a(e+fx)\text{sech}^3(c+dx)-b(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)\right)dx}{a^2+b^2}$$

$$\Big\downarrow \text{2009}$$

$$b^2\left(\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{2a(e+\right.$$
$$\frac{}{a^2+b^2}$$

$$\frac{\frac{a(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{iaf\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{2d^2}+\frac{iaf\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{2d^2}+\frac{af\,\text{sech}(c+dx)}{2d^2}+\frac{a(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}-\frac{b}{}}{a^2+b^2}$$

input
```
Int[((e + f*x)*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(b^2*((b^2*(-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
]))])/(b*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(
b*d^2)))/(a^2 + b^2) + ((b*(e + f*x)^2)/(2*f) + (2*a*(e + f*x)*ArcTan[E^(c
 + d*x)])/d - (b*(e + f*x)*Log[1 + E^(2*(c + d*x))])/d - (I*a*f*PolyLog[2,
 (-I)*E^(c + d*x)])/d^2 + (I*a*f*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*Pol
yLog[2, -E^(2*(c + d*x))])/(2*d^2))/(a^2 + b^2)))/(a^2 + b^2) + ((a*(e + f
*x)*ArcTan[E^(c + d*x)])/d - ((I/2)*a*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2
+ ((I/2)*a*f*PolyLog[2, I*E^(c + d*x)])/d^2 + (a*f*Sech[c + d*x])/(2*d^2)
+ (b*(e + f*x)*Sech[c + d*x]^2)/(2*d) - (b*f*Tanh[c + d*x])/(2*d^2) + (a*(
e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*d))/(a^2 + b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2050 vs. 2(520) = 1040.

Time = 31.58 (sec) , antiderivative size = 2051, normalized size of antiderivative = 3.66

method	result	size
risch	Expression too large to display	2051

input
```
int((f*x+e)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/d^2/(a^2+b^2)^(1/2)*c*a*b*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2
*a)/(a^2+b^2)^(1/2))+2/d/(a^2+b^2)*a^3*e/(2*a^2+2*b^2)*arctan(exp(d*x+c))+
(a*d*f*x*exp(3*d*x+3*c)+a*d*e*exp(3*d*x+3*c)+2*b*d*f*x*exp(2*d*x+2*c)-a*d*
f*x*exp(d*x+c)+a*f*exp(3*d*x+3*c)+2*b*d*e*exp(2*d*x+2*c)-a*d*e*exp(d*x+c)+
b*f*exp(2*d*x+2*c)+a*f*exp(d*x+c)+b*f)/d^2/(a^2+b^2)/(1+exp(2*d*x+2*c))^2+
2/d/(a^2+b^2)*b^3*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a
+(a^2+b^2)^(1/2)))*x+2/d^2/(a^2+b^2)*b^3*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)
+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+2/d/(a^2+b^2)*b^3*f/(2*a^2+2*b
^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+2/d^2/(a^2+
b^2)*b^3*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^
(1/2)))*c-2/d/(a^2+b^2)*b^3*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*x-2/d^2/(a^
2+b^2)*b^3*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*c-2/d/(a^2+b^2)*b^3*f/(2*a^2
+2*b^2)*ln(1-I*exp(d*x+c))*x-2/d^2/(a^2+b^2)*b^3*f/(2*a^2+2*b^2)*ln(1-I*ex
p(d*x+c))*c+6/d/(a^2+b^2)*a*b^2*e/(2*a^2+2*b^2)*arctan(exp(d*x+c))-2/d^2/(
a^2+b^2)*c*a^3*f/(2*a^2+2*b^2)*arctan(exp(d*x+c))+I/d^2/(a^2+b^2)*a^3*f/(2
*a^2+2*b^2)*dilog(1-I*exp(d*x+c))+2/d^2/(a^2+b^2)*c*b^3*f/(2*a^2+2*b^2)*ln
(1+exp(2*d*x+2*c))-2/d^2/(a^2+b^2)*c*b^3*f/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*
c)+2*a*exp(d*x+c)-b)-I/d^2/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x
+c))+I/d/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*x+I/d^2/(a^2+b^2
)*a^3*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*c+1/d/(a^2+b^2)^(1/2)*a*b*e/(2...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4729 vs. 2(505) = 1010.

Time = 0.20 (sec) , antiderivative size = 4729, normalized size of antiderivative = 8.44

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\,\text{sech}^3\,(c+dx)}{a+b\sinh\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*sech(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{sech}\,(dx+c)^3}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
(b^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 + b
^4)*d) - b^3*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (a^3
+ 3*a*b^2)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) + (a*e^(-d*x -
 c) + 2*b*e^(-2*d*x - 2*c) - a*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^
2)*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d))*e + f*(((a*d*x*e^(
3*c) + a*e^(3*c))*e^(3*d*x) + (2*b*d*x*e^(2*c) + b*e^(2*c))*e^(2*d*x) - (a
*d*x*e^c - a*e^c)*e^(d*x) + b)/(a^2*d^2 + b^2*d^2 + (a^2*d^2*e^(4*c) + b^2
*d^2*e^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*e^(2*d*x))
 - 8*integrate(-1/4*(a*b^3*x*e^(d*x + c) - b^4*x)/(a^4*b + 2*a^2*b^3 + b^5
 - (a^4*b*e^(2*c) + 2*a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^5*e^
c + 2*a^3*b^2*e^c + a*b^4*e^c)*e^(d*x)), x) + 8*integrate(1/8*(2*b^3*x + (
a^3*e^c + 3*a*b^2*e^c)*x*e^(d*x))/(a^4 + 2*a^2*b^2 + b^4 + (a^4*e^(2*c) +
2*a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x))
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{e+f\,x}{{\cosh\left(c+d\,x\right)}^3\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)/(cosh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(cosh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
(e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**3*b*d*e + 3*e**(4*c + 4*d*x)*atan(
e**(c + d*x))*a*b**3*d*e + 2*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**3*b*d*
e + 6*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a*b**3*d*e + atan(e**(c + d*x))*
a**3*b*d*e + 3*atan(e**(c + d*x))*a*b**3*d*e - 32*e**(7*c + 4*d*x)*int((e*
*(3*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a + 2*e**(6*c + 6*d*x
)*b + 6*e**(5*c + 5*d*x)*a + 6*e**(3*c + 3*d*x)*a - 2*e**(2*c + 2*d*x)*b +
 2*e**(c + d*x)*a - b),x)*a**5*d**2*f - 64*e**(7*c + 4*d*x)*int((e**(3*d*x
)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a + 2*e**(6*c + 6*d*x)*b + 6
*e**(5*c + 5*d*x)*a + 6*e**(3*c + 3*d*x)*a - 2*e**(2*c + 2*d*x)*b + 2*e**(
c + d*x)*a - b),x)*a**3*b**2*d**2*f - 32*e**(7*c + 4*d*x)*int((e**(3*d*x)*
x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a + 2*e**(6*c + 6*d*x)*b + 6*e
**(5*c + 5*d*x)*a + 6*e**(3*c + 3*d*x)*a - 2*e**(2*c + 2*d*x)*b + 2*e**(c
+ d*x)*a - b),x)*a*b**4*d**2*f + 16*e**(6*c + 4*d*x)*int((e**(2*d*x)*x)/(e
**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a + 2*e**(6*c + 6*d*x)*b + 6*e**(5*
c + 5*d*x)*a + 6*e**(3*c + 3*d*x)*a - 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x
)*a - b),x)*a**4*b*d**2*f + 32*e**(6*c + 4*d*x)*int((e**(2*d*x)*x)/(e**(8*
c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a + 2*e**(6*c + 6*d*x)*b + 6*e**(5*c + 5
*d*x)*a + 6*e**(3*c + 3*d*x)*a - 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a -
 b),x)*a**2*b**3*d**2*f + 16*e**(6*c + 4*d*x)*int((e**(2*d*x)*x)/(e**(8*c
+ 8*d*x)*b + 2*e**(7*c + 7*d*x)*a + 2*e**(6*c + 6*d*x)*b + 6*e**(5*c + ...
```

3.316 $\int \frac{\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 119

$$\int \frac{\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{a(a^2+3b^2)\arctan(\sinh(c+dx))}{2(a^2+b^2)^2 d} - \frac{b^3\log(\cosh(c+dx))}{(a^2+b^2)^2 d} + \frac{b^3\log(a+b\sinh(c+dx))}{(a^2+b^2)^2 d} + \frac{\text{sech}^2(c+dx)(b+a\sinh(c+dx))}{2(a^2+b^2)d}$$

output
```
1/2*a*(a^2+3*b^2)*arctan(sinh(d*x+c))/(a^2+b^2)^2/d-b^3*ln(cosh(d*x+c))/(a
^2+b^2)^2/d+b^3*ln(a+b*sinh(d*x+c))/(a^2+b^2)^2/d+1/2*sech(d*x+c)^2*(b+a*s
inh(d*x+c))/(a^2+b^2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.10

$$\int \frac{\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{a(a^2+b^2)\arctan(\sinh(c+dx)) - b^2((ia+b)\log(i-\sinh(c+dx)) + (-ia+b)\log(i+\sinh(c+dx)) -}{2(a^2+b^2)^2}$$

input
```
Integrate[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

output
```
(a*(a^2 + b^2)*ArcTan[Sinh[c + d*x]] - b^2*((I*a + b)*Log[I - Sinh[c + d*x
]] + ((-I)*a + b)*Log[I + Sinh[c + d*x]] - 2*b*Log[a + b*Sinh[c + d*x]]) +
 b*(a^2 + b^2)*Sech[c + d*x]^2 + a*(a^2 + b^2)*Sech[c + d*x]*Tanh[c + d*x]
)/(2*(a^2 + b^2)^2*d)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.33, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3147, 496, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3147}$$

$$\frac{b^3\int \frac{1}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{496}$$

$$\frac{b^3\left(\frac{ab\sinh(c+dx)+b^2}{2b^2(a^2+b^2)(b^2\sinh^2(c+dx)+b^2)}-\frac{\int -\frac{a^2+b\sinh(c+dx)a+2b^2}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)}d(b\sinh(c+dx))}{2b^2(a^2+b^2)}\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b^3\left(\frac{\int \frac{a^2+b\sinh(c+dx)a+2b^2}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)}d(b\sinh(c+dx))}{2b^2(a^2+b^2)}+\frac{ab\sinh(c+dx)+b^2}{2b^2(a^2+b^2)(b^2\sinh^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \text{657}$$

$$\frac{b^3\left(\frac{\int\left(\frac{2b^2}{(a^2+b^2)(a+b\sinh(c+dx))}+\frac{a^3+3b^2a-2b^3\sinh(c+dx)}{(a^2+b^2)(\sinh^2(c+dx)b^2+b^2)}\right)d(b\sinh(c+dx))}{2b^2(a^2+b^2)}+\frac{ab\sinh(c+dx)+b^2}{2b^2(a^2+b^2)(b^2\sinh^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^3\left(\frac{\frac{a(a^2+3b^2)\arctan(\sinh(c+dx))}{b(a^2+b^2)}-\frac{b^2\log(b^2\sinh^2(c+dx)+b^2)}{a^2+b^2}+\frac{2b^2\log(a+b\sinh(c+dx))}{a^2+b^2}}{2b^2(a^2+b^2)}+\frac{ab\sinh(c+dx)+b^2}{2b^2(a^2+b^2)(b^2\sinh^2(c+dx)+b^2)}\right)}{d}$$

input
```
Int[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

output
```
(b^3*(((a*(a^2 + 3*b^2)*ArcTan[Sinh[c + d*x]])/(b*(a^2 + b^2)) + (2*b^2*Lo
g[a + b*Sinh[c + d*x]])/(a^2 + b^2) - (b^2*Log[b^2 + b^2*Sinh[c + d*x]^2])
/(a^2 + b^2))/(2*b^2*(a^2 + b^2)) + (b^2 + a*b*Sinh[c + d*x])/(2*b^2*(a^2
+ b^2)*(b^2 + b^2*Sinh[c + d*x]^2))))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 31.32 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.72

method	result
derivativedivides	$\frac{\frac{b^3 \ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{a^4+2a^2b^2+b^4}+\frac{2\left(\frac{\left(-\frac{1}{2}a^3-\frac{1}{2}ab^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-a^2b-b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(\frac{1}{2}a^3+\frac{1}{2}ab^2\right) \mathrm{t}}{\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}\right.}{a^4+2a^2}}{d}$
default	$\frac{\frac{b^3 \ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{a^4+2a^2b^2+b^4}+\frac{2\left(\frac{\left(-\frac{1}{2}a^3-\frac{1}{2}ab^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-a^2b-b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(\frac{1}{2}a^3+\frac{1}{2}ab^2\right) \mathrm{t}}{\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}\right.}{a^4+2a^2}}{d}$
risch	$\frac{2b^3d^2x}{a^4d^2+2a^2b^2d^2+b^4d^2}+\frac{2b^3dc}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{2b^3x}{a^4+2a^2b^2+b^4}-\frac{2b^3c}{d(a^4+2a^2b^2+b^4)}+\frac{e^{dx+c}\left(e^{2dx+2c}a+2b\,e^{dx+}\right.}{d(a^2+b^2)\left(1+e^{2dx+2c}\right.}$

input
```
int(sech(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^3/(a^4+2*a^2*b^2+b^4)*ln(tanh(1/2*d*x+1/2*c)^2*a-2*b*tanh(1/2*d*x+1
/2*c)-a)+2/(a^4+2*a^2*b^2+b^4)*(((-1/2*a^3-1/2*a*b^2)*tanh(1/2*d*x+1/2*c)^
3+(-a^2*b-b^3)*tanh(1/2*d*x+1/2*c)^2+(1/2*a^3+1/2*a*b^2)*tanh(1/2*d*x+1/2*
c))/(1+tanh(1/2*d*x+1/2*c)^2)^2-1/2*b^3*ln(1+tanh(1/2*d*x+1/2*c)^2)+1/2*(a
^3+3*a*b^2)*arctan(tanh(1/2*d*x+1/2*c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 893 vs. $2(115) = 230$.

Time = 0.13 (sec) , antiderivative size = 893, normalized size of antiderivative = 7.50

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
((a^3 + a*b^2)*cosh(d*x + c)^3 + (a^3 + a*b^2)*sinh(d*x + c)^3 + 2*(a^2*b
+ b^3)*cosh(d*x + c)^2 + (2*a^2*b + 2*b^3 + 3*(a^3 + a*b^2)*cosh(d*x + c))
*sinh(d*x + c)^2 + ((a^3 + 3*a*b^2)*cosh(d*x + c)^4 + 4*(a^3 + 3*a*b^2)*co
sh(d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a*b^2)*sinh(d*x + c)^4 + a^3 + 3*a*
b^2 + 2*(a^3 + 3*a*b^2)*cosh(d*x + c)^2 + 2*(a^3 + 3*a*b^2 + 3*(a^3 + 3*a*
b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3 + 3*a*b^2)*cosh(d*x + c)^3
 + (a^3 + 3*a*b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + si
nh(d*x + c)) - (a^3 + a*b^2)*cosh(d*x + c) + (b^3*cosh(d*x + c)^4 + 4*b^3*
cosh(d*x + c)*sinh(d*x + c)^3 + b^3*sinh(d*x + c)^4 + 2*b^3*cosh(d*x + c)^
2 + b^3 + 2*(3*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*
x + c)^3 + b^3*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(
cosh(d*x + c) - sinh(d*x + c))) - (b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x +
c)*sinh(d*x + c)^3 + b^3*sinh(d*x + c)^4 + 2*b^3*cosh(d*x + c)^2 + b^3 + 2
*(3*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 +
b^3*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sin
h(d*x + c))) - (a^3 + a*b^2 - 3*(a^3 + a*b^2)*cosh(d*x + c)^2 - 4*(a^2*b +
 b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x +
c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 +
2*a^2*b^2 + b^4)*d*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x
+ c)^2 + 2*(3*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 + (a^4 + 2*a^2*...
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate(sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(sech(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.82

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\frac{b^3\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^4+2\,a^2b^2+b^4)d}-\frac{b^3\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^4+2\,a^2b^2+b^4)d}-\frac{(a^3+3\,ab^2)\arctan\left(e^{(-dx-c)}\right)}{(a^4+2\,a^2b^2+b^4)d}+\frac{ae^{(-dx-c)}+2\,be^{(-2\,dx-2\,c)}-ae^{(-3\,dx-3\,c)}}{(a^2+b^2+2\,(a^2+b^2)e^{(-2\,dx-2\,c)}+(a^2+b^2)e^{(-4\,dx-4\,c)})d}$$

input
```
integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
b^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 + b^
4)*d) - b^3*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (a^3 +
 3*a*b^2)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) + (a*e^(-d*x -
c) + 2*b*e^(-2*d*x - 2*c) - a*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^2
)*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(115) = 230$.

Time = 0.14 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.37

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{4\,b^4\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^4b+2\,a^2b^3+b^5} - \frac{2\,b^3\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^4+2\,a^2b^2+b^4} + \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(a^3+3\,ab^2\right)}{a^4+2\,a^2b^2+b^4} + \frac{2\,(\cdots}{\cdots}}{4\,d}$$

input
```
integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/4*(4*b^4*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^4*b + 2*a^2*b
^3 + b^5) - 2*b^3*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^4 + 2*a^2*b^2
 + b^4) + (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(a^3 + 3
*a*b^2)/(a^4 + 2*a^2*b^2 + b^4) + 2*(b^3*(e^(d*x + c) - e^(-d*x - c))^2 +
2*a^3*(e^(d*x + c) - e^(-d*x - c)) + 2*a*b^2*(e^(d*x + c) - e^(-d*x - c))
 + 4*a^2*b + 8*b^3)/((a^4 + 2*a^2*b^2 + b^4)*((e^(d*x + c) - e^(-d*x - c))^
2 + 4)))/d
```

Mupad [B] (verification not implemented)

Time = 3.21 (sec) , antiderivative size = 381, normalized size of antiderivative = 3.20

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{2\left(a^2\,b+b^3\right)}{d\left(a^2+b^2\right)^2} + \frac{e^{c+d\,x}\left(a^3+a\,b^2\right)}{d\left(a^2+b^2\right)^2}}{e^{2\,c+2\,d\,x}+1} - \frac{\frac{2\,b}{d\left(a^2+b^2\right)} + \frac{2\,a\,e^{c+d\,x}}{d\left(a^2+b^2\right)}}{2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1}$$
$$- \frac{\ln\left(e^{c+d\,x}+1\mathrm{i}\right)\,\left(2\,b+a\,1\mathrm{i}\right)}{2\,\left(-d\,a^2+2\mathrm{i}\,d\,a\,b+d\,b^2\right)} - \frac{\ln\left(1+e^{c+d\,x}\,1\mathrm{i}\right)\,\left(a+b\,2\mathrm{i}\right)}{2\,\left(-1\mathrm{i}\,d\,a^2+2\,d\,a\,b+1\mathrm{i}\,d\,b^2\right)}$$
$$+ \frac{b^3\,\ln\left(2\,a^7\,e^{d\,x}\,e^{c}-16\,b^7-9\,a^2\,b^5-6\,a^4\,b^3-a^6\,b+16\,b^7\,e^{2\,c}\,e^{2\,d\,x}+a^6\,b\,e^{2\,c}\,e^{2\,d\,x}+18\,a^3\,b^4\,e^{d\,x}\,e^{c}+12\,\cdots\right)}{d\,a^4+2\,d\,a^2\,b^2+d\,b^4}$$

input
```
int(1/(cosh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
((2*(a^2*b + b^3))/(d*(a^2 + b^2)^2) + (exp(c + d*x)*(a*b^2 + a^3))/(d*(a^
2 + b^2)^2))/(exp(2*c + 2*d*x) + 1) - ((2*b)/(d*(a^2 + b^2)) + (2*a*exp(c
+ d*x))/(d*(a^2 + b^2)))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - (lo
g(exp(c + d*x) + 1i)*(a*1i + 2*b))/(2*(b^2*d - a^2*d + a*b*d*2i)) - (log(e
xp(c + d*x)*1i + 1)*(a + b*2i))/(2*(b^2*d*1i - a^2*d*1i + 2*a*b*d)) + (b^3
*log(2*a^7*exp(d*x)*exp(c) - 16*b^7 - 9*a^2*b^5 - 6*a^4*b^3 - a^6*b + 16*b
^7*exp(2*c)*exp(2*d*x) + a^6*b*exp(2*c)*exp(2*d*x) + 18*a^3*b^4*exp(d*x)*e
xp(c) + 12*a^5*b^2*exp(d*x)*exp(c) + 9*a^2*b^5*exp(2*c)*exp(2*d*x) + 6*a^4
*b^3*exp(2*c)*exp(2*d*x) + 32*a*b^6*exp(d*x)*exp(c)))/(a^4*d + b^4*d + 2*a
^2*b^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 519, normalized size of antiderivative = 4.36

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{e^{4dx+4c}atan\left(e^{dx+c}\right)a^3+3e^{4dx+4c}atan\left(e^{dx+c}\right)ab^2+2e^{2dx+2c}atan\left(e^{dx+c}\right)a^3+6e^{2dx+2c}atan\left(e^{dx+c}\right)ab^2+}{}$$

input
```
int(sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
(e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**3 + 3*e**(4*c + 4*d*x)*atan(e**(c
+ d*x))*a*b**2 + 2*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**3 + 6*e**(2*c +
2*d*x)*atan(e**(c + d*x))*a*b**2 + atan(e**(c + d*x))*a**3 + 3*atan(e**(c
+ d*x))*a*b**2 - e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x) + 1)*b**3 + e**(4*c
 + 4*d*x)*log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*b**3 - e**(4*c +
4*d*x)*a**2*b - e**(4*c + 4*d*x)*b**3 + e**(3*c + 3*d*x)*a**3 + e**(3*c +
3*d*x)*a*b**2 - 2*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x) + 1)*b**3 + 2*e**(
2*c + 2*d*x)*log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*b**3 - e**(c +
 d*x)*a**3 - e**(c + d*x)*a*b**2 - log(e**(2*c + 2*d*x) + 1)*b**3 + log(e*
*(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*b**3 - a**2*b - b**3)/(d*(e**(4*c
 + 4*d*x)*a**4 + 2*e**(4*c + 4*d*x)*a**2*b**2 + e**(4*c + 4*d*x)*b**4 + 2*
e**(2*c + 2*d*x)*a**4 + 4*e**(2*c + 2*d*x)*a**2*b**2 + 2*e**(2*c + 2*d*x)*
b**4 + a**4 + 2*a**2*b**2 + b**4))
```

3.317 $\int \frac{\text{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\text{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \text{Int}\left(\frac{\text{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}, x\right)$$

output
```
Defer(Int)(sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 62.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\text{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \int \frac{\text{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

input
```
Integrate[Sech[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Sech[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

$$\Big\downarrow \text{ 6111}$$

$$\int \frac{\text{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Sech[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}\left(dx+c\right)^{3}}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 4.68 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sech(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 2.94 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}^3\left(c+dx\right)}{(a+b\sinh\left(c+dx\right))\left(e+fx\right)}\,dx$$

input
```
integrate(sech(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(sech(c + d*x)**3/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Maxima [N/A]

Not integrable

Time = 1.75 (sec) , antiderivative size = 1100, normalized size of antiderivative = 39.29

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-(b*f - (a*d*f*x*e^(3*c) + (d*e - f)*a*e^(3*c))*e^(3*d*x) - (2*b*d*f*x*e^(
2*c) + (2*d*e - f)*b*e^(2*c))*e^(2*d*x) + (a*d*f*x*e^c + (d*e + f)*a*e^c)*
e^(d*x))/(a^2*d^2*e^2 + b^2*d^2*e^2 + (a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + 2*
(a^2*d^2*e*f + b^2*d^2*e*f)*x + (a^2*d^2*e^2*e^(4*c) + b^2*d^2*e^2*e^(4*c)
 + (a^2*d^2*f^2*e^(4*c) + b^2*d^2*f^2*e^(4*c))*x^2 + 2*(a^2*d^2*e*f*e^(4*c
) + b^2*d^2*e*f*e^(4*c))*x)*e^(4*d*x) + 2*(a^2*d^2*e^2*e^(2*c) + b^2*d^2*e
^2*e^(2*c) + (a^2*d^2*f^2*e^(2*c) + b^2*d^2*f^2*e^(2*c))*x^2 + 2*(a^2*d^2*
e*f*e^(2*c) + b^2*d^2*e*f*e^(2*c))*x)*e^(2*d*x)) + 8*integrate(1/8*(2*b^3*
d^2*f^2*x^2 + 4*b^3*d^2*e*f*x - 2*a^2*b*f^2 + 2*(d^2*e^2 - f^2)*b^3 + ((d^
2*e^2 - 2*f^2)*a^3*e^c + (3*d^2*e^2 - 2*f^2)*a*b^2*e^c + (a^3*d^2*f^2*e^c
+ 3*a*b^2*d^2*f^2*e^c)*x^2 + 2*(a^3*d^2*e*f*e^c + 3*a*b^2*d^2*e*f*e^c)*x)*
e^(d*x))/(a^4*d^2*e^3 + 2*a^2*b^2*d^2*e^3 + b^4*d^2*e^3 + (a^4*d^2*f^3 + 2
*a^2*b^2*d^2*f^3 + b^4*d^2*f^3)*x^3 + 3*(a^4*d^2*e*f^2 + 2*a^2*b^2*d^2*e*f
^2 + b^4*d^2*e*f^2)*x^2 + 3*(a^4*d^2*e^2*f + 2*a^2*b^2*d^2*e^2*f + b^4*d^2
*e^2*f)*x + (a^4*d^2*e^3*e^(2*c) + 2*a^2*b^2*d^2*e^3*e^(2*c) + b^4*d^2*e^3
*e^(2*c) + (a^4*d^2*f^3*e^(2*c) + 2*a^2*b^2*d^2*f^3*e^(2*c) + b^4*d^2*f^3*
e^(2*c))*x^3 + 3*(a^4*d^2*e*f^2*e^(2*c) + 2*a^2*b^2*d^2*e*f^2*e^(2*c) + b^
4*d^2*e*f^2*e^(2*c))*x^2 + 3*(a^4*d^2*e^2*f*e^(2*c) + 2*a^2*b^2*d^2*e^2*f*
e^(2*c) + b^4*d^2*e^2*f*e^(2*c))*x)*e^(2*d*x)), x) - 8*integrate(-1/4*(a*b
^3*e^(d*x + c) - b^4)/(a^4*b*e + 2*a^2*b^3*e + b^5*e + (a^4*b*f + 2*a^2...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 3.81 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^3\,\left(e+f\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(cosh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.43

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(dx+c)^3}{\sinh(dx+c)\,be+\sinh(dx+c)\,bfx+ae+afx}dx$$

input
```
int(sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(sech(c + d*x)**3/(sinh(c + d*x)*b*e + sinh(c + d*x)*b*f*x + a*e + a*f*
x),x)
```

3.318 $\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \operatorname{Int}\left(\frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)},x\right)$$

output
```
Defer(Int)(x^m*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 9.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
Integrate[(x^m*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
Integrate[(x^m*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)} \, dx$$
$$\Big\downarrow \text{ 6111}$$
$$\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}dx$$

input
```
Int[(x^m*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \cosh\left(dx+c\right)^3}{a+b\sinh\left(dx+c\right)}dx$$

input
```
int(x^m*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int(x^m*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh\left(dx+c\right)^3}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x^m*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(x^m*cosh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 2.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh^3\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate(x**m*cosh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(x**m*cosh(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh\left(dx+c\right)^3}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x^m*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(x^m*cosh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh\left(dx+c\right)^3}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x^m*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^m*cosh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh(c+d\,x)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((x^m*cosh(c + d*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((x^m*cosh(c + d*x)^3)/(a + b*sinh(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 2652, normalized size of antiderivative = 110.50

$$\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(x^m*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
(x**m*e**(4*c + 4*d*x)*b**4*m + x**m*e**(4*c + 4*d*x)*b**4 - 4*x**m*e**(3*
c + 3*d*x)*a*b**3*m - 4*x**m*e**(3*c + 3*d*x)*a*b**3 - e**(6*c + 2*d*x)*in
t((x**m*e**(4*d*x))/(e**(2*c + 2*d*x)*b*x + 2*e**(c + d*x)*a*x - b*x),x)*b
**5*m**2 - e**(6*c + 2*d*x)*int((x**m*e**(4*d*x))/(e**(2*c + 2*d*x)*b*x +
2*e**(c + d*x)*a*x - b*x),x)*b**5*m + 2*e**(5*c + 2*d*x)*int((x**m*e**(3*d
*x))/(e**(2*c + 2*d*x)*b*x + 2*e**(c + d*x)*a*x - b*x),x)*a*b**4*m**2 + 2*
e**(5*c + 2*d*x)*int((x**m*e**(3*d*x))/(e**(2*c + 2*d*x)*b*x + 2*e**(c + d
*x)*a*x - b*x),x)*a*b**4*m - 32*e**(3*c + 2*d*x)*int((x**m*e**(d*x))/(e**(
2*c + 2*d*x)*b*x + 2*e**(c + d*x)*a*x - b*x),x)*a**3*b**2*m**2 - 32*e**(3*
c + 2*d*x)*int((x**m*e**(d*x))/(e**(2*c + 2*d*x)*b*x + 2*e**(c + d*x)*a*x
- b*x),x)*a**3*b**2*m - 26*e**(3*c + 2*d*x)*int((x**m*e**(d*x))/(e**(2*c +
 2*d*x)*b*x + 2*e**(c + d*x)*a*x - b*x),x)*a*b**4*m**2 - 26*e**(3*c + 2*d*
x)*int((x**m*e**(d*x))/(e**(2*c + 2*d*x)*b*x + 2*e**(c + d*x)*a*x - b*x),x
)*a*b**4*m + 8*x**m*e**(2*c + 2*d*x)*a**2*b**2*d*x + 8*x**m*e**(2*c + 2*d*
x)*a**2*b**2*m + 8*x**m*e**(2*c + 2*d*x)*a**2*b**2 + 8*x**m*e**(2*c + 2*d*
x)*b**4*d*x + x**m*e**(2*c + 2*d*x)*b**4*m + x**m*e**(2*c + 2*d*x)*b**4 -
16*e**(2*c + 2*d*x)*int(x**m/(e**(4*c + 4*d*x)*b*x + 2*e**(3*c + 3*d*x)*a*
x - e**(2*c + 2*d*x)*b*x),x)*a**4*b*m**2 - 16*e**(2*c + 2*d*x)*int(x**m/(e
**(4*c + 4*d*x)*b*x + 2*e**(3*c + 3*d*x)*a*x - e**(2*c + 2*d*x)*b*x),x)*a*
*4*b*m - 24*e**(2*c + 2*d*x)*int(x**m/(e**(4*c + 4*d*x)*b*x + 2*e**(3*c...
```

3.319 $\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Int}\left(\frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)},x\right)$$

output
```
Defer(Int)(x^m*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 8.57 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
Integrate[(x^m*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
Integrate[(x^m*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)} \, dx$$

$$\Big\downarrow \text{ 6111}$$

$$\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

input
```
Int[(x^m*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \cosh\left(dx+c\right)^2}{a+b\sinh\left(dx+c\right)}dx$$

input
```
int(x^m*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int(x^m*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh\left(dx+c\right)^2}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x^m*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(x^m*cosh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh^2\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate(x**m*cosh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(x**m*cosh(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh\left(dx+c\right)^2}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x^m*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(x^m*cosh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh{(dx+c)}^2}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate(x^m*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^m*cosh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m\,{\cosh(c+d\,x)}^2}{a+b\,\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((x^m*cosh(c + d*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((x^m*cosh(c + d*x)^2)/(a + b*sinh(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 1317, normalized size of antiderivative = 54.88

$$\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(x^m*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(x**m*e**(2*c + 2*d*x)*b**2*m + x**m*e**(2*c + 2*d*x)*b**2 - e**(4*c + d*x
)*int((x**m*e**(3*d*x))/(e**(2*c + 2*d*x)*b*x + 2*e**(c + d*x)*a*x - b*x),
x)*b**3*m**2 - e**(4*c + d*x)*int((x**m*e**(3*d*x))/(e**(2*c + 2*d*x)*b*x
+ 2*e**(c + d*x)*a*x - b*x),x)*b**3*m + 8*e**(2*c + d*x)*int((x**m*e**(d*x
))/(e**(2*c + 2*d*x)*b*x + 2*e**(c + d*x)*a*x - b*x),x)*a**2*b*m**2 + 8*e*
*(2*c + d*x)*int((x**m*e**(d*x))/(e**(2*c + 2*d*x)*b*x + 2*e**(c + d*x)*a*
x - b*x),x)*a**2*b*m + 4*e**(2*c + d*x)*int((x**m*e**(d*x))/(e**(2*c + 2*d
*x)*b*x + 2*e**(c + d*x)*a*x - b*x),x)*b**3*m**2 + 4*e**(2*c + d*x)*int((x
**m*e**(d*x))/(e**(2*c + 2*d*x)*b*x + 2*e**(c + d*x)*a*x - b*x),x)*b**3*m
- 2*x**m*e**(c + d*x)*a*b*d*x - 2*x**m*e**(c + d*x)*a*b*m - 2*x**m*e**(c +
 d*x)*a*b + 8*e**(c + d*x)*int(x**m/(e**(2*c + 2*d*x)*b*x + 2*e**(c + d*x)
*a*x - b*x),x)*a**3*m**2 + 8*e**(c + d*x)*int(x**m/(e**(2*c + 2*d*x)*b*x +
 2*e**(c + d*x)*a*x - b*x),x)*a**3*m + 4*e**(c + d*x)*int(x**m/(e**(2*c +
2*d*x)*b*x + 2*e**(c + d*x)*a*x - b*x),x)*a*b**2*m**2 + 4*e**(c + d*x)*int
(x**m/(e**(2*c + 2*d*x)*b*x + 2*e**(c + d*x)*a*x - b*x),x)*a*b**2*m - 8*e*
*(c + d*x)*int(x**m/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**3*d*
m - 8*e**(c + d*x)*int(x**m/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)
*a**3*d - 8*e**(c + d*x)*int(x**m/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a -
 b),x)*a*b**2*d*m - 8*e**(c + d*x)*int(x**m/(e**(2*c + 2*d*x)*b + 2*e**(c
+ d*x)*a - b),x)*a*b**2*d - 4*e**(d*x)*int(x**m/(e**(2*c + 3*d*x)*b*x +...
```

3.320 $\int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Int}\left(\frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)},x\right)$$

output
```
Defer(Int)(x^m*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
Integrate[(x^m*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
Integrate[(x^m*Cosh[c + d*x])/(a + b*Sinh[c + d*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\Big\downarrow \text{ 6111}$$
$$\int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)}dx$$

input
```
Int[(x^m*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \cosh\left(dx+c\right)}{a+b\sinh\left(dx+c\right)}dx$$

input
```
int(x^m*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int(x^m*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate(x^m*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(x^m*cosh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate(x**m*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(x**m*cosh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 205, normalized size of antiderivative = 9.32

$$\int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate(x^m*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
x*e^(2*d*x + m*log(x) + 2*c)/(b*(m + 1)*e^(2*d*x + 2*c) + 2*a*(m + 1)*e^(d
*x + c) - b*(m + 1)) - 1/2*integrate(2*(2*a*d*x*e^(3*d*x + 3*c) - 2*a*(m +
 1)*e^(d*x + c) + b*(m + 1) - (2*b*d*x*e^(2*c) + b*(m + 1)*e^(2*c))*e^(2*d
*x))*x^m/(b^2*(m + 1)*e^(4*d*x + 4*c) + 4*a*b*(m + 1)*e^(3*d*x + 3*c) - 4*
a*b*(m + 1)*e^(d*x + c) + b^2*(m + 1) + 2*(2*a^2*(m + 1)*e^(2*c) - b^2*(m
+ 1)*e^(2*c))*e^(2*d*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh\left(dx+c\right)}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x^m*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^m*cosh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh(c+d\,x)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((x^m*cosh(c + d*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((x^m*cosh(c + d*x))/(a + b*sinh(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 181, normalized size of antiderivative = 8.23

$$\int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-2e^c\left(\int \frac{x^m e^{dx}}{e^{2dx+2c}b+2e^{dx+c}a-b}dx\right)am-2e^c\left(\int \frac{x^m e^{dx}}{e^{2dx+2c}b+2e^{dx+c}a-b}dx\right)a+x^m x+2\left(\int \frac{x^m}{e^{2dx+2c}b+2e^{dx+c}a-b}dx\right)bm}{b\,(m+1)}$$

input
```
int(x^m*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*e**c*int((x**m*e**(d*x))/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)
,x)*a*m - 2*e**c*int((x**m*e**(d*x))/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*
a - b),x)*a + x**m*x + 2*int(x**m/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a -
 b),x)*b*m + 2*int(x**m/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*b)/
(b*(m + 1))
```

3.321 $\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = -\frac{2f\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}-\frac{e+fx}{bd(a+b\sinh(c+dx))}$$

output
```
-2*f*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/b/(a^2+b^2)^(1/2)/
d^2-(f*x+e)/b/d/(a+b*sinh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \frac{\frac{2f\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-\frac{d(e+fx)}{a+b\sinh(c+dx)}}{bd^2}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

output
```
((2*f*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2]
 - (d*(e + f*x))/(a + b*Sinh[c + d*x]))/(b*d^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5987, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}dx$$

$$\Big\downarrow \text{5987}$$

$$\frac{f\int \frac{1}{a+b\sinh(c+dx)}dx}{bd}-\frac{e+fx}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{e+fx}{bd(a+b\sinh(c+dx))}+\frac{f\int \frac{1}{a-ib\sin(ic+idx)}dx}{bd}$$

$$\Big\downarrow \text{3139}$$

$$-\frac{e+fx}{bd(a+b\sinh(c+dx))}-\frac{2if\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{bd^2}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{e+fx}{bd(a+b\sinh(c+dx))}+\frac{4if\int \frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4(a^2+b^2)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{bd^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{2f\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}}-\frac{e+fx}{bd(a+b\sinh(c+dx))}$$

input
```
Int[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

output
```
(2*f*ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]*d^
2) - (e + f*x)/(b*d*(a + b*Sinh[c + d*x]))
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 5987
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(69) = 138$.

Time = 4.59 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.22

method	result	size
risch	$-\frac{2(fx+e)\mathrm{e}^{dx+c}}{bd\left(b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}^{dx+c}-b\right)}+\frac{f\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d^2b}-\frac{f\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d^2b}$	164

input
```
int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(f*x+e)/b/d*exp(d*x+c)/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/(a^2+b^2)^
(1/2)*f/d^2/b*ln(exp(d*x+c)+(a*(a^2+b^2)^(1/2)-a^2-b^2)/(a^2+b^2)^(1/2)/b)
-1/(a^2+b^2)^(1/2)*f/d^2/b*ln(exp(d*x+c)+(a*(a^2+b^2)^(1/2)+a^2+b^2)/(a^2+
b^2)^(1/2)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(71) = 142$.

Time = 0.08 (sec) , antiderivative size = 411, normalized size of antiderivative = 5.55

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$$
$$= \frac{\left(bf\cosh\left(dx+c\right)^2+bf\sinh\left(dx+c\right)^2+2\,af\cosh\left(dx+c\right)-bf+2\left(bf\cosh\left(dx+c\right)+af\right)\sinh\left(dx+c\right)\right.}{(a^2b^2+b^4)d^2\cosh\left(dx+c\right)^2}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
((b*f*cosh(d*x + c)^2 + b*f*sinh(d*x + c)^2 + 2*a*f*cosh(d*x + c) - b*f +
2*(b*f*cosh(d*x + c) + a*f)*sinh(d*x + c))*sqrt(a^2 + b^2)*log((b^2*cosh(d
*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b
^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c)
 + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh
(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - 2*((a^2 + b^2)*d
*f*x + (a^2 + b^2)*d*e)*cosh(d*x + c) - 2*((a^2 + b^2)*d*f*x + (a^2 + b^2)
*d*e)*sinh(d*x + c))/((a^2*b^2 + b^4)*d^2*cosh(d*x + c)^2 + (a^2*b^2 + b^4
)*d^2*sinh(d*x + c)^2 + 2*(a^3*b + a*b^3)*d^2*cosh(d*x + c) - (a^2*b^2 + b
^4)*d^2 + 2*((a^2*b^2 + b^4)*d^2*cosh(d*x + c) + (a^3*b + a*b^3)*d^2)*sinh
(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(71) = 142.

Time = 0.21 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.12

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = -f\left(\frac{2\,xe^{(dx+c)}}{b^2de^{(2\,dx+2\,c)}+2\,abde^{(dx+c)}-b^2d} - \frac{\log\left(\frac{be^{(dx+c)}+a-\sqrt{a^2+b^2}}{be^{(dx+c)}+a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}bd^2}\right) - \frac{2\,ee^{(-dx-c)}}{(2\,abe^{(-dx-c)}-b^2e^{(-2\,dx-2\,c)}+b^2)d}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-f*(2*x*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) - b^2*d)
- log((b*e^(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(d*x + c) + a + sqrt(a^2
+ b^2)))/(sqrt(a^2 + b^2)*b*d^2)) - 2*e*e^(-d*x - c)/((2*a*b*e^(-d*x - c)
- b^2*e^(-2*d*x - 2*c) + b^2)*d)
```

Giac [F]

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \int \frac{(fx+e)\cosh(dx+c)}{(b\sinh(dx+c)+a)^2}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)/(b*sinh(d*x + c) + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.69

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \frac{f\ln\left(\frac{2f\,(b-a\,e^{c+d\,x})}{b^2\,d\,\sqrt{a^2+b^2}}-\frac{2f\,e^{c+d\,x}}{b^2\,d}\right)}{b\,d^2\,\sqrt{a^2+b^2}} - \frac{f\ln\left(-\frac{2f\,e^{c+d\,x}}{b^2\,d}-\frac{2f\,(b-a\,e^{c+d\,x})}{b^2\,d\,\sqrt{a^2+b^2}}\right)}{b\,d^2\,\sqrt{a^2+b^2}} - \frac{2\,e^{c+d\,x}\,(a^2\,e+b^2\,e+a^2\,f\,x+b^2\,f\,x)}{d\,(a^2\,b+b^3)\,(2\,a\,e^{c+d\,x}-b+b\,e^{2\,c+2\,d\,x})}$$

input
```
int((cosh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x))^2,x)
```

output
```
(f*log((2*f*(b - a*exp(c + d*x)))/(b^2*d*(a^2 + b^2)^(1/2)) - (2*f*exp(c +
 d*x))/(b^2*d)))/(b*d^2*(a^2 + b^2)^(1/2)) - (f*log(- (2*f*exp(c + d*x))/(
b^2*d) - (2*f*(b - a*exp(c + d*x)))/(b^2*d*(a^2 + b^2)^(1/2))))/(b*d^2*(a^
2 + b^2)^(1/2)) - (2*exp(c + d*x)*(a^2*e + b^2*e + a^2*f*x + b^2*f*x))/(d*
(a^2*b + b^3)*(2*a*exp(c + d*x) - b + b*exp(2*c + 2*d*x)))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 296, normalized size of antiderivative = 4.00

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$$
$$= \frac{2e^{2dx+2c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)abfi+4e^{dx+c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)a^2fi-2\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}b}{\sqrt{a^2-}}\right.}{ab\,d^2\left(e^{2dx+2c}a^2b+e^{2dx+2c}b^3+2e^{dx+c}a^3-\right.}$$

input
```
int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x)
```

output
```
(2*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a
**2 + b**2))*a*b*f*i + 4*e**(c + d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)
*b*i + a*i)/sqrt(a**2 + b**2))*a**2*f*i - 2*sqrt(a**2 + b**2)*atan((e**(c
+ d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b*f*i + e**(2*c + 2*d*x)*a**2*b*d*e
 + e**(2*c + 2*d*x)*b**3*d*e - 2*e**(c + d*x)*a**3*d*f*x - 2*e**(c + d*x)*
a*b**2*d*f*x - a**2*b*d*e - b**3*d*e)/(a*b*d**2*(e**(2*c + 2*d*x)*a**2*b +
 e**(2*c + 2*d*x)*b**3 + 2*e**(c + d*x)*a**3 + 2*e**(c + d*x)*a*b**2 - a**
2*b - b**3))
```

3.322 $\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 234

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} - \frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} + \frac{2f^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3} - \frac{2f^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3} - \frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

output
```
2*f*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(1/2)/d^2-2
*f*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(1/2)/d^2+2*
f^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(1/2)/d^3-2*f
^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(1/2)/d^3-(f*x
+e)^2/b/d/(a+b*sinh(d*x+c))
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.86

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx =$$
$$-\frac{2f\left(d\left(2e\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)-fx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+fx\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)-f\operatorname{PolyLog}\left(2,\frac{be^{\cdots}}{-a+\cdots}\right.\cdots}{b\sqrt{a^2+b^2}d^3}$$
$$-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

output
```
(-2*f*(d*(2*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - f*x*Log[1 + (
b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*x*Log[1 + (b*E^(c + d*x))/(a + S
qrt[a^2 + b^2])]) - f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] +
 f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(b*Sqrt[a^2 + b^
2]*d^3) - (e + f*x)^2/(b*d*(a + b*Sinh[c + d*x]))
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5987, 3042, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}dx$$
$$\Big\downarrow \text{ 5987}$$
$$\frac{2f\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{bd}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$
$$\Big\downarrow \text{ 3042}$$

$$-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}+\frac{2f\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{bd}$$

$\Big\downarrow$ 3803

$$\frac{4f\int-\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{bd}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

$\Big\downarrow$ 25

$$-\frac{4f\int\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{bd}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

$\Big\downarrow$ 2694

$$-\frac{4f\left(\frac{b\int-\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int-\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{bd}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

$\Big\downarrow$ 27

$$-\frac{4f\left(\frac{b\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{bd}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

$\Big\downarrow$ 2620

$$-\frac{4f\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{bd}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

$\Big\downarrow$ 2715

$$-\frac{4f\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+\right.}{bd^2}}{2\sqrt{a^2+b^2}}}{bd}$$

$$\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \; 2838$$

$$-\frac{4f\left(\frac{b\left(\frac{f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{bd}-$$

$$\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

output
```
(-4*f*(-1/2*(b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])
/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2))
)/Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/
(b*d^2)))/(2*Sqrt[a^2 + b^2])))/(b*d) - (e + f*x)^2/(b*d*(a + b*Sinh[c + d
*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5987
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(214) = 428$.

Time = 4.38 (sec) , antiderivative size = 491, normalized size of antiderivative = 2.10

method	result
risch	$-\frac{2(x^2f^2+2efx+e^2)\mathrm{e}^{dx+c}}{bd(b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}^{dx+c}-b)}-\frac{4fe\ \operatorname{arctanh}\left(\frac{2b\,\mathrm{e}^{dx+c}+2a}{2\sqrt{a^2+b^2}}\right)}{d^2b\sqrt{a^2+b^2}}+\frac{2f^2\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)x}{d^2b\sqrt{a^2+b^2}}-\frac{2f^2\ln\left(\frac{b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}+}{a+\sqrt{a^2+b^2}}\right.}{d^2b\sqrt{a^2+b^2}}$

input
```
int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(f^2*x^2+2*e*f*x+e^2)/b/d*exp(d*x+c)/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b
)-4/d^2/b*f*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(
1/2))+2/d^2/b*f^2/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a
+(a^2+b^2)^(1/2)))*x-2/d^2/b*f^2/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2
)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+2/d^3/b*f^2/(a^2+b^2)^(1/2)*ln((-b*exp(d
*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2/d^3/b*f^2/(a^2+b^2)^(1/
2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2/d^3/b*f^2/
(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2
)))-2/d^3/b*f^2/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+
(a^2+b^2)^(1/2)))+4/d^3/b*f^2*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c
)+2*a)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1378 vs. $2(212) = 424$.

Time = 0.12 (sec) , antiderivative size = 1378, normalized size of antiderivative = 5.89

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
2*((b^2*f^2*cosh(d*x + c)^2 + b^2*f^2*sinh(d*x + c)^2 + 2*a*b*f^2*cosh(d*x
 + c) - b^2*f^2 + 2*(b^2*f^2*cosh(d*x + c) + a*b*f^2)*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^2*f^2*cosh(d*
x + c)^2 + b^2*f^2*sinh(d*x + c)^2 + 2*a*b*f^2*cosh(d*x + c) - b^2*f^2 + 2
*(b^2*f^2*cosh(d*x + c) + a*b*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*di
log((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b^2*d*e*f - b^2*c*f^2 - (b^2*d*e*f
 - b^2*c*f^2)*cosh(d*x + c)^2 - (b^2*d*e*f - b^2*c*f^2)*sinh(d*x + c)^2 -
2*(a*b*d*e*f - a*b*c*f^2)*cosh(d*x + c) - 2*(a*b*d*e*f - a*b*c*f^2 + (b^2*
d*e*f - b^2*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log
(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
- (b^2*d*e*f - b^2*c*f^2 - (b^2*d*e*f - b^2*c*f^2)*cosh(d*x + c)^2 - (b^2*
d*e*f - b^2*c*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*e*f - a*b*c*f^2)*cosh(d*x +
c) - 2*(a*b*d*e*f - a*b*c*f^2 + (b^2*d*e*f - b^2*c*f^2)*cosh(d*x + c))*sin
h(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c
) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^2*d*f^2*x + b^2*c*f^2 - (b^2*d*f
^2*x + b^2*c*f^2)*cosh(d*x + c)^2 - (b^2*d*f^2*x + b^2*c*f^2)*sinh(d*x + c
)^2 - 2*(a*b*d*f^2*x + a*b*c*f^2)*cosh(d*x + c) - 2*(a*b*d*f^2*x + a*b*c*f
^2 + (b^2*d*f^2*x + b^2*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)/(a+b*sinh(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)}{(b\sinh(dx+c)+a)^2}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-2*(x^2*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) - b^2*d)
- 2*integrate(x*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) -
 b^2*d), x))*f^2 - 2*e*f*(2*x*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d
*e^(d*x + c) - b^2*d) - log((b*e^(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(d*
x + c) + a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b*d^2)) - 2*e^2*e^(-d*x -
c)/((2*a*b*e^(-d*x - c) - b^2*e^(-2*d*x - 2*c) + b^2)*d)
```

Giac [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)}{(b\sinh(dx+c)+a)^2}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)/(b*sinh(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \int \frac{\cosh(c+d\,x)\,(e+f\,x)^2}{(a+b\sinh(c+d\,x))^2}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x))^2,x)
```

output
```
int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x)
```

output
```
(4*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a
**2 + b**2))*a**3*b*f**2*i + 4*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e*
*(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b**3*d*e*f*i + 4*e**(2*c + 2*d*
x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b*
*3*f**2*i + 8*e**(c + d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)
/sqrt(a**2 + b**2))*a**4*f**2*i + 8*e**(c + d*x)*sqrt(a**2 + b**2)*atan((e
**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b**2*d*e*f*i + 8*e**(c + d*
x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**2
*b**2*f**2*i - 4*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2
 + b**2))*a**3*b*f**2*i - 4*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i
)/sqrt(a**2 + b**2))*a*b**3*d*e*f*i - 4*sqrt(a**2 + b**2)*atan((e**(c + d*
x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b**3*f**2*i - 8*e**(3*c + 2*d*x)*int((e
**(d*x)*x)/(e**(4*c + 4*d*x)*b**2 + 4*e**(3*c + 3*d*x)*a*b + 4*e**(2*c + 2
*d*x)*a**2 - 2*e**(2*c + 2*d*x)*b**2 - 4*e**(c + d*x)*a*b + b**2),x)*a**5*
b**2*d**2*f**2 - 16*e**(3*c + 2*d*x)*int((e**(d*x)*x)/(e**(4*c + 4*d*x)*b*
*2 + 4*e**(3*c + 3*d*x)*a*b + 4*e**(2*c + 2*d*x)*a**2 - 2*e**(2*c + 2*d*x)
*b**2 - 4*e**(c + d*x)*a*b + b**2),x)*a**3*b**4*d**2*f**2 - 8*e**(3*c + 2*
d*x)*int((e**(d*x)*x)/(e**(4*c + 4*d*x)*b**2 + 4*e**(3*c + 3*d*x)*a*b + 4*
e**(2*c + 2*d*x)*a**2 - 2*e**(2*c + 2*d*x)*b**2 - 4*e**(c + d*x)*a*b + b**
2),x)*a*b**6*d**2*f**2 + 2*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x)*b + 2*...
```

3.323 $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 348

$$\begin{aligned}\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx &= \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} \\ &\quad - \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} \\ &\quad + \frac{6f^2(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3} \\ &\quad - \frac{6f^2(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3} \\ &\quad - \frac{6f^3\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^4} \\ &\quad + \frac{6f^3\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^4} - \frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}\end{aligned}$$

output
```
3*f*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(1/2)/d^2
-3*f*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(1/2)/d^
2+6*f^2*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(
1/2)/d^3-6*f^2*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2
+b^2)^(1/2)/d^3-6*f^3*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+
b^2)^(1/2)/d^4+6*f^3*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b
^2)^(1/2)/d^4-(f*x+e)^3/b/d/(a+b*sinh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.06

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \frac{3f\left(-2d^2e^2\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+2d^2efx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+d^2f^2x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-2d^2efx\log\left(\right.\right.}{} - \frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

output
```
(3*f*(-2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*d^2*e*f*
x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2])] + 2*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^
2])] - 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
] - 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f^2*PolyL
og[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(b*Sqrt[a^2 + b^2]*d^4)
- (e + f*x)^3/(b*d*(a + b*Sinh[c + d*x]))
```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5987, 3042, 3803, 25, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}dx$$

$$\Big\downarrow \text{ 5987}$$

$$\frac{3f\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{bd}-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}+\frac{3f\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{bd}$$

$$\Big\downarrow \text{ 3803}$$

$$\frac{6f\int -\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{bd}-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{6f\int \frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{bd}-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{ 2694}$$

$$-\frac{6f\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{bd}-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{6f\left(\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{bd}-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{ 2620}$$

$$-\frac{6f\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d}{bd}\right.}{2\sqrt{a^2+b^2}}}{bd}$$

$$\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$\downarrow$ 3011

$$-\frac{6f\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right.}{bd}\right.}{}}{bd}$$

$$\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$\downarrow$ 2720

$$-\frac{6f\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log}{}\right.}{}}{bd}$$

$$\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$\downarrow$ 7143

$$-\frac{6f\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right.}{}\right.}{bd}$$

$$\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

output
```
(-6*f*(-1/2*(b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d^
2))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2]))])/d^2))/(b*d)))/(2*Sqrt[a^2 + b^2])))/(b*d) - (e + f*x)^3/(b
*d*(a + b*Sinh[c + d*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5987
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(fx+e)^3 \cosh(dx+c)}{(a+b\sinh(dx+c))^2}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2420 vs. $2(316) = 632$.

Time = 0.17 (sec) , antiderivative size = 2420, normalized size of antiderivative = 6.95

$$\int \frac{(e+fx)^3 \cosh(c+dx)}{(a+b\sinh(c+dx))^2} dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
-(6*(b^2*d*f^3*x + b^2*d*e*f^2 - (b^2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c)
^2 - (b^2*d*f^3*x + b^2*d*e*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*f^3*x + a*b*d*
e*f^2)*cosh(d*x + c) - 2*(a*b*d*f^3*x + a*b*d*e*f^2 + (b^2*d*f^3*x + b^2*d
*e*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(
d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2) - b)/b + 1) - 6*(b^2*d*f^3*x + b^2*d*e*f^2 - (b^2*d*f^3*x + b
^2*d*e*f^2)*cosh(d*x + c)^2 - (b^2*d*f^3*x + b^2*d*e*f^2)*sinh(d*x + c)^2
- 2*(a*b*d*f^3*x + a*b*d*e*f^2)*cosh(d*x + c) - 2*(a*b*d*f^3*x + a*b*d*e*f
^2 + (b^2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 +
 b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*(b^2*d^2*e^2*f - 2*b
^2*c*d*e*f^2 + b^2*c^2*f^3 - (b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^
3)*cosh(d*x + c)^2 - (b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3)*sinh(
d*x + c)^2 - 2*(a*b*d^2*e^2*f - 2*a*b*c*d*e*f^2 + a*b*c^2*f^3)*cosh(d*x +
c) - 2*(a*b*d^2*e^2*f - 2*a*b*c*d*e*f^2 + a*b*c^2*f^3 + (b^2*d^2*e^2*f - 2
*b^2*c*d*e*f^2 + b^2*c^2*f^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^
2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b
^2) + 2*a) + 3*(b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3 - (b^2*d^2*e
^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3)*cosh(d*x + c)^2 - (b^2*d^2*e^2*f - 2
*b^2*c*d*e*f^2 + b^2*c^2*f^3)*sinh(d*x + c)^2 - 2*(a*b*d^2*e^2*f - 2*a*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)/(a+b*sinh(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)}{(b\sinh(dx+c)+a)^2}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-3*e^2*f*(2*x*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) - b
^2*d) - log((b*e^(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(d*x + c) + a + sqr
t(a^2 + b^2)))/(sqrt(a^2 + b^2)*b*d^2)) - 2*e^3*e^(-d*x - c)/((2*a*b*e^(-d
*x - c) - b^2*e^(-2*d*x - 2*c) + b^2)*d) - 2*(f^3*x^3*e^c + 3*e*f^2*x^2*e^
c)*e^(d*x)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) - b^2*d) + integra
te(6*(f^3*x^2*e^c + 2*e*f^2*x*e^c)*e^(d*x)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*
d*e^(d*x + c) - b^2*d), x)
```

Giac [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)}{(b\sinh(dx+c)+a)^2}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)/(b*sinh(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \int \frac{\cosh(c+d\,x)\,(e+f\,x)^3}{(a+b\sinh(c+d\,x))^2}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x))^2,x)
```

output
```
int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x)
```

output
```
(12*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(
a**2 + b**2))*a**3*b*d*e*f**2*i + 18*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*at
an((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**3*b*f**3*i + 6*e**(2*c +
 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))
*a*b**3*d**2*e**2*f*i + 12*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c
+ d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b**3*d*e*f**2*i + 12*e**(2*c + 2*d*
x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b*
*3*f**3*i + 24*e**(c + d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i
)/sqrt(a**2 + b**2))*a**4*d*e*f**2*i + 36*e**(c + d*x)*sqrt(a**2 + b**2)*a
tan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**4*f**3*i + 12*e**(c + d
*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**
2*b**2*d**2*e**2*f*i + 24*e**(c + d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x
)*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b**2*d*e*f**2*i + 24*e**(c + d*x)*sqr
t(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b**2*
f**3*i - 12*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b*
*2))*a**3*b*d*e*f**2*i - 18*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i
)/sqrt(a**2 + b**2))*a**3*b*f**3*i - 6*sqrt(a**2 + b**2)*atan((e**(c + d*x
)*b*i + a*i)/sqrt(a**2 + b**2))*a*b**3*d**2*e**2*f*i - 12*sqrt(a**2 + b**2
)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b**3*d*e*f**2*i - 12*
sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b*...
```

3.324 $\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = -\frac{2f\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} - \frac{e+fx}{bd(a+b\sinh(c+dx))}$$

output
```
-2*f*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/b/(a^2+b^2)^(1/2)/
d^2-(f*x+e)/b/d/(a+b*sinh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \frac{\frac{2f\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - \frac{d(e+fx)}{a+b\sinh(c+dx)}}{bd^2}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

output
```
((2*f*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2]
 - (d*(e + f*x))/(a + b*Sinh[c + d*x]))/(b*d^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5987, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}dx$$

$$\Big\downarrow \text{5987}$$

$$\frac{f\int \frac{1}{a+b\sinh(c+dx)}dx}{bd}-\frac{e+fx}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{e+fx}{bd(a+b\sinh(c+dx))}+\frac{f\int \frac{1}{a-ib\sin(ic+idx)}dx}{bd}$$

$$\Big\downarrow \text{3139}$$

$$-\frac{e+fx}{bd(a+b\sinh(c+dx))}-\frac{2if\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{bd^2}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{e+fx}{bd(a+b\sinh(c+dx))}+\frac{4if\int \frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4(a^2+b^2)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{bd^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{2f\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}}-\frac{e+fx}{bd(a+b\sinh(c+dx))}$$

input
```
Int[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

output
```
(2*f*ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]*d^
2) - (e + f*x)/(b*d*(a + b*Sinh[c + d*x]))
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 5987
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(69) = 138$.

Time = 4.19 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.22

method	result	size
risch	$-\frac{2(fx+e)\,\mathrm{e}^{dx+c}}{bd\left(b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}^{dx+c}-b\right)}+\frac{f\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d^2b}-\frac{f\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d^2b}$	164

input
```
int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(f*x+e)/b/d*exp(d*x+c)/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/(a^2+b^2)^
(1/2)*f/d^2/b*ln(exp(d*x+c)+(a*(a^2+b^2)^(1/2)-a^2-b^2)/(a^2+b^2)^(1/2)/b)
-1/(a^2+b^2)^(1/2)*f/d^2/b*ln(exp(d*x+c)+(a*(a^2+b^2)^(1/2)+a^2+b^2)/(a^2+
b^2)^(1/2)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(71) = 142$.

Time = 0.09 (sec) , antiderivative size = 411, normalized size of antiderivative = 5.55

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \frac{\left(bf\cosh\left(dx+c\right)^2 + bf\sinh\left(dx+c\right)^2 + 2\,af\cosh\left(dx+c\right) - bf + 2\left(bf\cosh\left(dx+c\right) + af\right)\sinh\left(dx+c\right.}{(a^2b^2+b^4)d^2\cosh\left(dx+c\right)^2 -}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
((b*f*cosh(d*x + c)^2 + b*f*sinh(d*x + c)^2 + 2*a*f*cosh(d*x + c) - b*f +
2*(b*f*cosh(d*x + c) + a*f)*sinh(d*x + c))*sqrt(a^2 + b^2)*log((b^2*cosh(d
*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b
^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c)
 + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh
(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - 2*((a^2 + b^2)*d
*f*x + (a^2 + b^2)*d*e)*cosh(d*x + c) - 2*((a^2 + b^2)*d*f*x + (a^2 + b^2)
*d*e)*sinh(d*x + c))/((a^2*b^2 + b^4)*d^2*cosh(d*x + c)^2 + (a^2*b^2 + b^4
)*d^2*sinh(d*x + c)^2 + 2*(a^3*b + a*b^3)*d^2*cosh(d*x + c) - (a^2*b^2 + b
^4)*d^2 + 2*((a^2*b^2 + b^4)*d^2*cosh(d*x + c) + (a^3*b + a*b^3)*d^2)*sinh
(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(71) = 142.

Time = 0.22 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.12

$$\begin{aligned}&\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx\\&= -f\left(\frac{2\,xe^{(dx+c)}}{b^2de^{(2\,dx+2\,c)}+2\,abde^{(dx+c)}-b^2d}-\frac{\log\left(\frac{be^{(dx+c)}+a-\sqrt{a^2+b^2}}{be^{(dx+c)}+a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}bd^2}\right)\\&\quad-\frac{2\,ee^{(-dx-c)}}{(2\,abe^{(-dx-c)}-b^2e^{(-2\,dx-2\,c)}+b^2)d}\end{aligned}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-f*(2*x*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) - b^2*d)
- log((b*e^(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(d*x + c) + a + sqrt(a^2
+ b^2)))/(sqrt(a^2 + b^2)*b*d^2)) - 2*e*e^(-d*x - c)/((2*a*b*e^(-d*x - c)
- b^2*e^(-2*d*x - 2*c) + b^2)*d)
```

Giac [F]

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \int \frac{(fx+e)\cosh(dx+c)}{(b\sinh(dx+c)+a)^2}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)/(b*sinh(d*x + c) + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.69

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \frac{f\ln\left(\frac{2f(b-a\,e^{c+dx})}{b^2d\sqrt{a^2+b^2}}-\frac{2f\,e^{c+dx}}{b^2d}\right)}{b\,d^2\sqrt{a^2+b^2}} - \frac{f\ln\left(-\frac{2f\,e^{c+dx}}{b^2d}-\frac{2f(b-a\,e^{c+dx})}{b^2d\sqrt{a^2+b^2}}\right)}{b\,d^2\sqrt{a^2+b^2}} - \frac{2\,e^{c+dx}\,(a^2e+b^2e+a^2fx+b^2fx)}{d\,(a^2b+b^3)\,(2a\,e^{c+dx}-b+b\,e^{2c+2dx})}$$

input
```
int((cosh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x))^2,x)
```

output
```
(f*log((2*f*(b - a*exp(c + d*x)))/(b^2*d*(a^2 + b^2)^(1/2)) - (2*f*exp(c +
 d*x))/(b^2*d)))/(b*d^2*(a^2 + b^2)^(1/2)) - (f*log(- (2*f*exp(c + d*x))/(
b^2*d) - (2*f*(b - a*exp(c + d*x)))/(b^2*d*(a^2 + b^2)^(1/2))))/(b*d^2*(a^
2 + b^2)^(1/2)) - (2*exp(c + d*x)*(a^2*e + b^2*e + a^2*f*x + b^2*f*x))/(d*
(a^2*b + b^3)*(2*a*exp(c + d*x) - b + b*exp(2*c + 2*d*x)))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 296, normalized size of antiderivative = 4.00

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$$
$$= \frac{2e^{2dx+2c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)abfi+4e^{dx+c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)a^2fi-2\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}b}{\sqrt{a^2-}}\right.}{ab\,d^2\left(e^{2dx+2c}a^2b+e^{2dx+2c}b^3+2e^{dx+c}a^3-\right.}$$

input
```
int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x)
```

output
```
(2*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a
**2 + b**2))*a*b*f*i + 4*e**(c + d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)
*b*i + a*i)/sqrt(a**2 + b**2))*a**2*f*i - 2*sqrt(a**2 + b**2)*atan((e**(c
+ d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b*f*i + e**(2*c + 2*d*x)*a**2*b*d*e
 + e**(2*c + 2*d*x)*b**3*d*e - 2*e**(c + d*x)*a**3*d*f*x - 2*e**(c + d*x)*
a*b**2*d*f*x - a**2*b*d*e - b**3*d*e)/(a*b*d**2*(e**(2*c + 2*d*x)*a**2*b +
 e**(2*c + 2*d*x)*b**3 + 2*e**(c + d*x)*a**3 + 2*e**(c + d*x)*a*b**2 - a**
2*b - b**3))
```

3.325 $\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 234

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} - \frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} + \frac{2f^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3} - \frac{2f^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3} - \frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

output
```
2*f*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(1/2)/d^2-2
*f*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(1/2)/d^2+2*
f^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(1/2)/d^3-2*f
^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(1/2)/d^3-(f*x
+e)^2/b/d/(a+b*sinh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.86

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx =$$
$$-\frac{2f\left(d\left(2e\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)-fx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+fx\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)-f\operatorname{PolyLog}\left(2,\frac{be^{\cdots}}{-a+\cdots}\right.\cdots}{b\sqrt{a^2+b^2}d^3}$$
$$-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

output
```
(-2*f*(d*(2*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - f*x*Log[1 + (
b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*x*Log[1 + (b*E^(c + d*x))/(a + S
qrt[a^2 + b^2])]) - f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] +
 f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(b*Sqrt[a^2 + b^
2]*d^3) - (e + f*x)^2/(b*d*(a + b*Sinh[c + d*x]))
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5987, 3042, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$$
$$\Big\downarrow \text{5987}$$
$$\frac{2f\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{bd}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$
$$\Big\downarrow \text{3042}$$

$$-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}+\frac{2f\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{bd}$$

$$\Big\downarrow \text{ 3803}$$

$$\frac{4f\int -\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{bd}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{4f\int \frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{bd}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{ 2694}$$

$$-\frac{4f\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{bd}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{4f\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{bd}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{ 2620}$$

$$-\frac{4f\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{bd}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{ 2715}$$

$$-\frac{4f\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+\right.}{bd^2}}{2\sqrt{a^2+b^2}}\right.}{bd}$$

$$\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{4f\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{bd}-$$

$$\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

output
```
(-4*f*(-1/2*(b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])
/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2))
)/Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/
(b*d^2)))/(2*Sqrt[a^2 + b^2])))/(b*d) - (e + f*x)^2/(b*d*(a + b*Sinh[c + d
*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5987
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. 2(214) = 428.

Time = 4.09 (sec) , antiderivative size = 491, normalized size of antiderivative = 2.10

method	result
risch	$-\frac{2(x^2f^2+2efx+e^2)\,\mathrm{e}^{dx+c}}{bd\left(b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}^{dx+c}-b\right)}-\frac{4fe\ \operatorname{arctanh}\left(\frac{2b\,\mathrm{e}^{dx+c}+2a}{2\sqrt{a^2+b^2}}\right)}{d^2b\sqrt{a^2+b^2}}+\frac{2f^2\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)x}{d^2b\sqrt{a^2+b^2}}-\frac{2f^2\ln\left(\frac{b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}+}{a+\sqrt{a^2+b^2}}\right.}{d^2b\sqrt{a^2+b^2}}$

input
```
int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(f^2*x^2+2*e*f*x+e^2)/b/d*exp(d*x+c)/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b
)-4/d^2/b*f*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(
1/2))+2/d^2/b*f^2/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a
+(a^2+b^2)^(1/2)))*x-2/d^2/b*f^2/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2
)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+2/d^3/b*f^2/(a^2+b^2)^(1/2)*ln((-b*exp(d
*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2/d^3/b*f^2/(a^2+b^2)^(1/
2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2/d^3/b*f^2/
(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2
)))-2/d^3/b*f^2/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+
(a^2+b^2)^(1/2)))+4/d^3/b*f^2*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c
)+2*a)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1378 vs. 2(212) = 424.

Time = 0.13 (sec) , antiderivative size = 1378, normalized size of antiderivative = 5.89

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
2*((b^2*f^2*cosh(d*x + c)^2 + b^2*f^2*sinh(d*x + c)^2 + 2*a*b*f^2*cosh(d*x
 + c) - b^2*f^2 + 2*(b^2*f^2*cosh(d*x + c) + a*b*f^2)*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^2*f^2*cosh(d*
x + c)^2 + b^2*f^2*sinh(d*x + c)^2 + 2*a*b*f^2*cosh(d*x + c) - b^2*f^2 + 2
*(b^2*f^2*cosh(d*x + c) + a*b*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*di
log((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b^2*d*e*f - b^2*c*f^2 - (b^2*d*e*f
 - b^2*c*f^2)*cosh(d*x + c)^2 - (b^2*d*e*f - b^2*c*f^2)*sinh(d*x + c)^2 -
2*(a*b*d*e*f - a*b*c*f^2)*cosh(d*x + c) - 2*(a*b*d*e*f - a*b*c*f^2 + (b^2*
d*e*f - b^2*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log
(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
- (b^2*d*e*f - b^2*c*f^2 - (b^2*d*e*f - b^2*c*f^2)*cosh(d*x + c)^2 - (b^2*
d*e*f - b^2*c*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*e*f - a*b*c*f^2)*cosh(d*x +
c) - 2*(a*b*d*e*f - a*b*c*f^2 + (b^2*d*e*f - b^2*c*f^2)*cosh(d*x + c))*sin
h(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c
) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^2*d*f^2*x + b^2*c*f^2 - (b^2*d*f
^2*x + b^2*c*f^2)*cosh(d*x + c)^2 - (b^2*d*f^2*x + b^2*c*f^2)*sinh(d*x + c
)^2 - 2*(a*b*d*f^2*x + a*b*c*f^2)*cosh(d*x + c) - 2*(a*b*d*f^2*x + a*b*c*f
^2 + (b^2*d*f^2*x + b^2*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)/(a+b*sinh(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)}{(b\sinh(dx+c)+a)^2}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-2*(x^2*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) - b^2*d)
- 2*integrate(x*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) -
 b^2*d), x))*f^2 - 2*e*f*(2*x*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d
*e^(d*x + c) - b^2*d) - log((b*e^(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(d*
x + c) + a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b*d^2)) - 2*e^2*e^(-d*x -
c)/((2*a*b*e^(-d*x - c) - b^2*e^(-2*d*x - 2*c) + b^2)*d)
```

Giac [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)}{(b\sinh(dx+c)+a)^2}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)/(b*sinh(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \int \frac{\cosh(c+d\,x)\,(e+f\,x)^2}{(a+b\sinh(c+d\,x))^2}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x))^2,x)
```

output
```
int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x)
```

output
```
(4*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a
**2 + b**2))*a**3*b*f**2*i + 4*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e*
*(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b**3*d*e*f*i + 4*e**(2*c + 2*d*
x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b*
*3*f**2*i + 8*e**(c + d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)
/sqrt(a**2 + b**2))*a**4*f**2*i + 8*e**(c + d*x)*sqrt(a**2 + b**2)*atan((e
**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b**2*d*e*f*i + 8*e**(c + d*
x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**2
*b**2*f**2*i - 4*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2
 + b**2))*a**3*b*f**2*i - 4*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i
)/sqrt(a**2 + b**2))*a*b**3*d*e*f*i - 4*sqrt(a**2 + b**2)*atan((e**(c + d*
x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b**3*f**2*i - 8*e**(3*c + 2*d*x)*int((e
**(d*x)*x)/(e**(4*c + 4*d*x)*b**2 + 4*e**(3*c + 3*d*x)*a*b + 4*e**(2*c + 2
*d*x)*a**2 - 2*e**(2*c + 2*d*x)*b**2 - 4*e**(c + d*x)*a*b + b**2),x)*a**5*
b**2*d**2*f**2 - 16*e**(3*c + 2*d*x)*int((e**(d*x)*x)/(e**(4*c + 4*d*x)*b*
*2 + 4*e**(3*c + 3*d*x)*a*b + 4*e**(2*c + 2*d*x)*a**2 - 2*e**(2*c + 2*d*x)
*b**2 - 4*e**(c + d*x)*a*b + b**2),x)*a**3*b**4*d**2*f**2 - 8*e**(3*c + 2*
d*x)*int((e**(d*x)*x)/(e**(4*c + 4*d*x)*b**2 + 4*e**(3*c + 3*d*x)*a*b + 4*
e**(2*c + 2*d*x)*a**2 - 2*e**(2*c + 2*d*x)*b**2 - 4*e**(c + d*x)*a*b + b**
2),x)*a*b**6*d**2*f**2 + 2*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x)*b + 2*...
```

3.326 $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 348

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} - \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} + \frac{6f^2(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3} - \frac{6f^2(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3} - \frac{6f^3\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^4} + \frac{6f^3\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^4} - \frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

output
```
3*f*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(1/2)/d^2
-3*f*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(1/2)/d^
2+6*f^2*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(
1/2)/d^3-6*f^2*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2
+b^2)^(1/2)/d^3-6*f^3*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+
b^2)^(1/2)/d^4+6*f^3*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b
^2)^(1/2)/d^4-(f*x+e)^3/b/d/(a+b*sinh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.06

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$$
$$= \frac{3f\left(-2d^2e^2\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+2d^2efx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+d^2f^2x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-2d^2efx\log\left(\right.\right.}{}$$
$$-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

output
```
(3*f*(-2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*d^2*e*f*
x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2])] + 2*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^
2])] - 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
] - 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f^2*PolyL
og[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(b*Sqrt[a^2 + b^2]*d^4)
- (e + f*x)^3/(b*d*(a + b*Sinh[c + d*x]))
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5987, 3042, 3803, 25, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}dx$$

$$\Big\downarrow \text{5987}$$

$$\frac{3f\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{bd}-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}+\frac{3f\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{bd}$$

$$\Big\downarrow \text{3803}$$

$$\frac{6f\int -\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{bd}-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{6f\int \frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{bd}-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{2694}$$

$$-\frac{6f\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{bd}-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$-\frac{6f\left(\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{bd}-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{6f\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d\cdots}{bd}\cdots\right.}{2\sqrt{a^2+b^2}}\cdots\right.}{bd}$$

$$\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$\downarrow$ 3011

$$-\frac{6f\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\cdots\right.}{bd}\cdots\right.}{\cdots}\cdots\right.}{bd}$$

$$\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$\downarrow$ 2720

$$-\frac{6f\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\cdots}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{bd}$$

$$\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$\downarrow$ 7143

$$-\frac{6f\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right.}{}\right.}{bd}$$

$$\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

output
```
(-6*f*(-1/2*(b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d^
2))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2]))])/d^2))/(b*d)))/(2*Sqrt[a^2 + b^2])))/(b*d) - (e + f*x)^3/(b
*d*(a + b*Sinh[c + d*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5987
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(fx+e)^3 \cosh(dx+c)}{(a+b\sinh(dx+c))^2}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2420 vs. $2(316) = 632$.

Time = 0.12 (sec) , antiderivative size = 2420, normalized size of antiderivative = 6.95

$$\int \frac{(e+fx)^3 \cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
-(6*(b^2*d*f^3*x + b^2*d*e*f^2 - (b^2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c)
^2 - (b^2*d*f^3*x + b^2*d*e*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*f^3*x + a*b*d*
e*f^2)*cosh(d*x + c) - 2*(a*b*d*f^3*x + a*b*d*e*f^2 + (b^2*d*f^3*x + b^2*d
*e*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(
d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2) - b)/b + 1) - 6*(b^2*d*f^3*x + b^2*d*e*f^2 - (b^2*d*f^3*x + b
^2*d*e*f^2)*cosh(d*x + c)^2 - (b^2*d*f^3*x + b^2*d*e*f^2)*sinh(d*x + c)^2
- 2*(a*b*d*f^3*x + a*b*d*e*f^2)*cosh(d*x + c) - 2*(a*b*d*f^3*x + a*b*d*e*f
^2 + (b^2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 +
 b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*(b^2*d^2*e^2*f - 2*b
^2*c*d*e*f^2 + b^2*c^2*f^3 - (b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^
3)*cosh(d*x + c)^2 - (b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3)*sinh(
d*x + c)^2 - 2*(a*b*d^2*e^2*f - 2*a*b*c*d*e*f^2 + a*b*c^2*f^3)*cosh(d*x +
c) - 2*(a*b*d^2*e^2*f - 2*a*b*c*d*e*f^2 + a*b*c^2*f^3 + (b^2*d^2*e^2*f - 2
*b^2*c*d*e*f^2 + b^2*c^2*f^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^
2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b
^2) + 2*a) + 3*(b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3 - (b^2*d^2*e
^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3)*cosh(d*x + c)^2 - (b^2*d^2*e^2*f - 2
*b^2*c*d*e*f^2 + b^2*c^2*f^3)*sinh(d*x + c)^2 - 2*(a*b*d^2*e^2*f - 2*a*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)/(a+b*sinh(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)}{(b\sinh(dx+c)+a)^2}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-3*e^2*f*(2*x*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) - b
^2*d) - log((b*e^(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(d*x + c) + a + sqr
t(a^2 + b^2)))/(sqrt(a^2 + b^2)*b*d^2)) - 2*e^3*e^(-d*x - c)/((2*a*b*e^(-d
*x - c) - b^2*e^(-2*d*x - 2*c) + b^2)*d) - 2*(f^3*x^3*e^c + 3*e*f^2*x^2*e^
c)*e^(d*x)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) - b^2*d) + integra
te(6*(f^3*x^2*e^c + 2*e*f^2*x*e^c)*e^(d*x)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*
d*e^(d*x + c) - b^2*d), x)
```

Giac [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)}{(b\sinh(dx+c)+a)^2}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)/(b*sinh(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \int \frac{\cosh(c+d\,x)\,(e+f\,x)^3}{(a+b\sinh(c+d\,x))^2}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x))^2,x)
```

output
```
int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x)
```

output
```
(12*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(
a**2 + b**2))*a**3*b*d*e*f**2*i + 18*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*at
an((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**3*b*f**3*i + 6*e**(2*c +
 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))
*a*b**3*d**2*e**2*f*i + 12*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c
+ d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b**3*d*e*f**2*i + 12*e**(2*c + 2*d*
x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b*
*3*f**3*i + 24*e**(c + d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i
)/sqrt(a**2 + b**2))*a**4*d*e*f**2*i + 36*e**(c + d*x)*sqrt(a**2 + b**2)*a
tan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**4*f**3*i + 12*e**(c + d
*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**
2*b**2*d**2*e**2*f*i + 24*e**(c + d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x
)*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b**2*d*e*f**2*i + 24*e**(c + d*x)*sqr
t(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b**2*
f**3*i - 12*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b*
*2))*a**3*b*d*e*f**2*i - 18*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i
)/sqrt(a**2 + b**2))*a**3*b*f**3*i - 6*sqrt(a**2 + b**2)*atan((e**(c + d*x
)*b*i + a*i)/sqrt(a**2 + b**2))*a*b**3*d**2*e**2*f*i - 12*sqrt(a**2 + b**2
)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b**3*d*e*f**2*i - 12*
sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b*...
```

3.327 $\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 112

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = -\frac{af\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d^2} - \frac{e+fx}{2bd(a+b\sinh(c+dx))^2} - \frac{f\cosh(c+dx)}{2\,(a^2+b^2)\,d^2(a+b\sinh(c+dx))}$$

output
```
-a*f*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/b/(a^2+b^2)^(3/2)/
d^2-1/2*(f*x+e)/b/d/(a+b*sinh(d*x+c))^2-1/2*f*cosh(d*x+c)/(a^2+b^2)/d^2/(a
+b*sinh(d*x+c))
```

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = -\frac{\frac{f\cosh(c+dx)}{(a^2+b^2)(a+b\sinh(c+dx))} + \frac{\frac{2af\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}} + \frac{d(e+fx)}{(a+b\sinh(c+dx))^2}}{b}}{2d^2}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

output
```
-1/2*((f*Cosh[c + d*x])/((a^2 + b^2)*(a + b*Sinh[c + d*x])) + ((2*a*f*ArcT
an[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(3/2) + (d*(e
 + f*x))/(a + b*Sinh[c + d*x])^2)/b)/d^2
```

Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5987, 3042, 3143, 25, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}dx$$

$$\Big\downarrow \text{5987}$$

$$\frac{f\int \frac{1}{(a+b\sinh(c+dx))^2}dx}{2bd}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}+\frac{f\int \frac{1}{(a-ib\sin(ic+idx))^2}dx}{2bd}$$

$$\Big\downarrow \text{3143}$$

$$\frac{f\left(-\frac{\int -\frac{a}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{f\left(\frac{\int \frac{a}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{f\left(\frac{a\int \frac{1}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}+\frac{f\left(-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}+\frac{a\int \frac{1}{a-ib\sin(ic+idx)}dx}{a^2+b^2}\right)}{2bd}$$

$$\Big\downarrow \textbf{3139}$$

$$-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}+\frac{f\left(-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}-\frac{2ia\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d(a^2+b^2)}\right)}{2bd}$$

$$\Big\downarrow \textbf{1083}$$

$$-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}+\frac{f\left(-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}+\frac{4ia\int \frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4\left(a^2+b^2\right)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{d(a^2+b^2)}\right)}{2bd}$$

$$\Big\downarrow \textbf{217}$$

$$\frac{f\left(\frac{2a\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{3/2}}-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}$$

input
```
Int[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

output
```
-1/2*(e + f*x)/(b*d*(a + b*Sinh[c + d*x])^2) + (f*((2*a*ArcTanh[Tanh[(c +
d*x)/2]/(2*Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (b*Cosh[c + d*x])/((
a^2 + b^2)*d*(a + b*Sinh[c + d*x]))))/(2*b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 5987
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(103) = 206$.

Time = 25.45 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.75

method	result
risch	$-\frac{2\,\mathrm{e}^{2dx+2c} a^2 d f x+2 b^2 d f x\,\mathrm{e}^{2dx+2c}+2\,\mathrm{e}^{2dx+2c} a^2 d e-a b f\,\mathrm{e}^{3dx+3c}+2 b^2 d e\,\mathrm{e}^{2dx+2c}-2 a^2 f\,\mathrm{e}^{2dx+2c}+b^2 f\,\mathrm{e}^{2dx+2c}+3 a f\,\mathrm{e}^{dx+c} b-b^2 f}{b\,d^2 \left(b\,\mathrm{e}^{2dx+2c}+2 a\,\mathrm{e}^{dx+c}-b\right)^2 (a^2+b^2)}$

input
```
int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/b*(2*exp(2*d*x+2*c)*a^2*d*f*x+2*b^2*d*f*x*exp(2*d*x+2*c)+2*exp(2*d*x+2*
c)*a^2*d*e-a*b*f*exp(3*d*x+3*c)+2*b^2*d*e*exp(2*d*x+2*c)-2*a^2*f*exp(2*d*x
+2*c)+b^2*f*exp(2*d*x+2*c)+3*a*f*exp(d*x+c)*b-b^2*f)/d^2/(b*exp(2*d*x+2*c)
+2*a*exp(d*x+c)-b)^2/(a^2+b^2)+1/2/(a^2+b^2)^(3/2)*f*a/d^2/b*ln(exp(d*x+c)
+(a*(a^2+b^2)^(3/2)-a^4-2*a^2*b^2-b^4)/b/(a^2+b^2)^(3/2))-1/2/(a^2+b^2)^(3
/2)*f*a/d^2/b*ln(exp(d*x+c)+(a*(a^2+b^2)^(3/2)+a^4+2*a^2*b^2+b^4)/b/(a^2+b
^2)^(3/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1230 vs. $2(105) = 210$.

Time = 0.12 (sec) , antiderivative size = 1230, normalized size of antiderivative = 10.98

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(2*(a^3*b + a*b^3)*f*cosh(d*x + c)^3 + 2*(a^3*b + a*b^3)*f*sinh(d*x +
c)^3 - 6*(a^3*b + a*b^3)*f*cosh(d*x + c) - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d*
f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*d*e - (2*a^4 + a^2*b^2 - b^4)*f)*cosh(d*x
+ c)^2 - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*d*
e - 3*(a^3*b + a*b^3)*f*cosh(d*x + c) - (2*a^4 + a^2*b^2 - b^4)*f)*sinh(d*
x + c)^2 + (a*b^2*f*cosh(d*x + c)^4 + a*b^2*f*sinh(d*x + c)^4 + 4*a^2*b*f*
cosh(d*x + c)^3 - 4*a^2*b*f*cosh(d*x + c) + a*b^2*f + 2*(2*a^3 - a*b^2)*f*
cosh(d*x + c)^2 + 4*(a*b^2*f*cosh(d*x + c) + a^2*b*f)*sinh(d*x + c)^3 + 2*
(3*a*b^2*f*cosh(d*x + c)^2 + 6*a^2*b*f*cosh(d*x + c) + (2*a^3 - a*b^2)*f)*
sinh(d*x + c)^2 + 4*(a*b^2*f*cosh(d*x + c)^3 + 3*a^2*b*f*cosh(d*x + c)^2 -
 a^2*b*f + (2*a^3 - a*b^2)*f*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b^2)
*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*
a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*
(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x +
 c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) +
2*(a^2*b^2 + b^4)*f + 2*(3*(a^3*b + a*b^3)*f*cosh(d*x + c)^2 - 3*(a^3*b +
a*b^3)*f - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*
d*e - (2*a^4 + a^2*b^2 - b^4)*f)*cosh(d*x + c))*sinh(d*x + c))/((a^4*b^3 +
 2*a^2*b^5 + b^7)*d^2*cosh(d*x + c)^4 + (a^4*b^3 + 2*a^2*b^5 + b^7)*d^2*si
nh(d*x + c)^4 + 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^2*cosh(d*x + c)^3 + 2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. 2(105) = 210.

Time = 0.25 (sec) , antiderivative size = 413, normalized size of antiderivative = 3.69

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$$
$$=\frac{1}{2}f\left(\frac{2\left(abe^{(3\,dx+3\,c)}-3\,abe^{(dx+c)}+b^2+\left(2\,a^2e^{(2\,c)}-b^2e^{(2\,c)}-2\,(a^2d\right.\right.}{a^2b^3d^2+b^5d^2+(a^2b^3d^2e^{(4\,c)}+b^5d^2e^{(4\,c)})e^{(4\,dx)}+4\,(a^3b^2d^2e^{(3\,c)}+ab^4d^2e^{(3\,c)})e^{(3\,dx)}+2\,(2\,a^4bd^2e^{(2\,c)}}\right.$$
$$-\frac{2\,ee^{(-2\,dx-2\,c)}}{(4\,ab^2e^{(-dx-c)}-4\,ab^2e^{(-3\,dx-3\,c)}+b^3e^{(-4\,dx-4\,c)}+b^3+2\,(2\,a^2b-b^3)e^{(-2\,dx-2\,c)})d}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*f*(2*(a*b*e^(3*d*x + 3*c) - 3*a*b*e^(d*x + c) + b^2 + (2*a^2*e^(2*c) -
 b^2*e^(2*c) - 2*(a^2*d*e^(2*c) + b^2*d*e^(2*c))*x)*e^(2*d*x))/(a^2*b^3*d^
2 + b^5*d^2 + (a^2*b^3*d^2*e^(4*c) + b^5*d^2*e^(4*c))*e^(4*d*x) + 4*(a^3*b
^2*d^2*e^(3*c) + a*b^4*d^2*e^(3*c))*e^(3*d*x) + 2*(2*a^4*b*d^2*e^(2*c) + a
^2*b^3*d^2*e^(2*c) - b^5*d^2*e^(2*c))*e^(2*d*x) - 4*(a^3*b^2*d^2*e^c + a*b
^4*d^2*e^c)*e^(d*x)) + a*log((b*e^(d*x + 2*c) + a*e^c - sqrt(a^2 + b^2)*e^
c)/(b*e^(d*x + 2*c) + a*e^c + sqrt(a^2 + b^2)*e^c))/((a^2*b + b^3)*sqrt(a^
2 + b^2)*d^2)) - 2*e*e^(-2*d*x - 2*c)/((4*a*b^2*e^(-d*x - c) - 4*a*b^2*e^(
-3*d*x - 3*c) + b^3*e^(-4*d*x - 4*c) + b^3 + 2*(2*a^2*b - b^3)*e^(-2*d*x -
 2*c))*d)
```

Giac [F]

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx=\int \frac{(fx+e)\cosh\left(dx+c\right)}{\left(b\sinh\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)/(b*sinh(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \int \frac{\cosh(c+d\,x)\,(e+f\,x)}{(a+b\sinh(c+d\,x))^3}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x))^3,x)
```

output
```
int((cosh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x))^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 756, normalized size of antiderivative = 6.75

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$$
$$= \frac{4e^{4dx+4c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)a\,b^2fi+16e^{3dx+3c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)a^2bfi+16e^{2dx+2c}\sqrt{a^2+b}}{}$$

input
```
int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x)
```

output

```
(4*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a
**2 + b**2))*a*b**2*f*i + 16*e**(3*c + 3*d*x)*sqrt(a**2 + b**2)*atan((e**(
c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b*f*i + 16*e**(2*c + 2*d*x)*sq
rt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**3*f*i
- 8*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(
a**2 + b**2))*a*b**2*f*i - 16*e**(c + d*x)*sqrt(a**2 + b**2)*atan((e**(c +
 d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b*f*i + 4*sqrt(a**2 + b**2)*atan(
(e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b**2*f*i - e**(4*c + 4*d*x)*
a**2*b**2*f - e**(4*c + 4*d*x)*b**4*f - 8*e**(2*c + 2*d*x)*a**4*d*e - 8*e*
*(2*c + 2*d*x)*a**4*d*f*x + 4*e**(2*c + 2*d*x)*a**4*f - 16*e**(2*c + 2*d*x
)*a**2*b**2*d*e - 16*e**(2*c + 2*d*x)*a**2*b**2*d*f*x + 2*e**(2*c + 2*d*x)
*a**2*b**2*f - 8*e**(2*c + 2*d*x)*b**4*d*e - 8*e**(2*c + 2*d*x)*b**4*d*f*x
 - 2*e**(2*c + 2*d*x)*b**4*f - 8*e**(c + d*x)*a**3*b*f - 8*e**(c + d*x)*a*
b**3*f + 3*a**2*b**2*f + 3*b**4*f)/(4*b*d**2*(e**(4*c + 4*d*x)*a**4*b**2 +
 2*e**(4*c + 4*d*x)*a**2*b**4 + e**(4*c + 4*d*x)*b**6 + 4*e**(3*c + 3*d*x)
*a**5*b + 8*e**(3*c + 3*d*x)*a**3*b**3 + 4*e**(3*c + 3*d*x)*a*b**5 + 4*e**
(2*c + 2*d*x)*a**6 + 6*e**(2*c + 2*d*x)*a**4*b**2 - 2*e**(2*c + 2*d*x)*b**
6 - 4*e**(c + d*x)*a**5*b - 8*e**(c + d*x)*a**3*b**3 - 4*e**(c + d*x)*a*b*
*5 + a**4*b**2 + 2*a**2*b**4 + b**6))
```

3.328 $\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 306

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{3/2}d^2} - \frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{3/2}d^2} + \frac{f^2\log(a+b\sinh(c+dx))}{b\left(a^2+b^2\right)d^3} + \frac{af^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{3/2}d^3} - \frac{af^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{3/2}d^3} - \frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2} - \frac{f(e+fx)\cosh(c+dx)}{\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))}$$

output
```
a*f*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^2-a
*f*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^2+f^
2*ln(a+b*sinh(d*x+c))/b/(a^2+b^2)/d^3+a*f^2*polylog(2,-b*exp(d*x+c)/(a-(a^
2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^3-a*f^2*polylog(2,-b*exp(d*x+c)/(a+(a^2
+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^3-1/2*(f*x+e)^2/b/d/(a+b*sinh(d*x+c))^2-
f*(f*x+e)*cosh(d*x+c)/(a^2+b^2)/d^2/(a+b*sinh(d*x+c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 623 vs. $2(306) = 612$.

Time = 9.94 (sec) , antiderivative size = 623, normalized size of antiderivative = 2.04

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \frac{f^2x\coth(c)}{b(a^2+b^2)d^2}$$
$$+\frac{2e^cf\left(-e^cfx+e^{-c}(-1+e^{2c})fx-\frac{aee^{-c}(-1+e^{2c})\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{ae^{-c}(-1+e^{2c})f\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}d}+\right.}{}$$
$$-\frac{f^2x\cosh(c)\text{csch}\left(\frac{c}{2}\right)\text{sech}\left(\frac{c}{2}\right)}{2b(a^2+b^2)d^2}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$
$$+\frac{\text{csch}\left(\frac{c}{2}\right)\text{sech}\left(\frac{c}{2}\right)(aef\cosh(c)+af^2x\cosh(c)+bef\sinh(dx)+bf^2x\sinh(dx))}{2b(a^2+b^2)d^2(a+b\sinh(c+dx))}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

output
```
(f^2*x*Coth[c])/(b*(a^2 + b^2)*d^2) + (2*E^c*f*(-(E^c*f*x) + ((-1 + E^(2*c
))*f*x)/E^c - (a*e*(-1 + E^(2*c))*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b
^2]])/(Sqrt[a^2 + b^2]*E^c) + (a*(-1 + E^(2*c))*f*ArcTanh[(a + b*E^(c + d*
x))/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d*E^c) + ((-1 + E^(2*c))*f*(-2*x +
(2*a*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-a^2 - b^2]*d) +
Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))]/d))/(2*E^c) + (a*(-1 + E^(
2*c))*f*(d*x*(Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]
)] - Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])]) + Pol
yLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - PolyLo
g[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/(2*d*Sqrt
[(a^2 + b^2)*E^(2*c)])))/(b*(a^2 + b^2)*d^2*(-1 + E^(2*c))) - (f^2*x*Cosh[
c]*Csch[c/2]*Sech[c/2])/(2*b*(a^2 + b^2)*d^2) - (e + f*x)^2/(2*b*d*(a + b*
Sinh[c + d*x])^2) + (Csch[c/2]*Sech[c/2]*(a*e*f*Cosh[c] + a*f^2*x*Cosh[c]
+ b*e*f*Sinh[d*x] + b*f^2*x*Sinh[d*x]))/(2*b*(a^2 + b^2)*d^2*(a + b*Sinh[c
 + d*x]))
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5987, 3042, 3805, 3042, 3147, 16, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}dx$$

$$\Big\downarrow \text{5987}$$

$$\frac{f\int \frac{e+fx}{(a+b\sinh(c+dx))^2}dx}{bd}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}+\frac{f\int \frac{e+fx}{(a-ib\sin(ic+idx))^2}dx}{bd}$$

$$\Big\downarrow \text{3805}$$

$$\frac{f\left(\frac{a\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{bf\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}+\frac{f\left(\frac{a\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a^2+b^2}+\frac{bf\int \frac{\cos(ic+idx)}{a-ib\sin(ic+idx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}$$

$$\Big\downarrow \text{3147}$$

$$-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}+\frac{f\left(\frac{f\int \frac{1}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{d^2(a^2+b^2)}+\frac{a\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a^2+b^2}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}$$

$$\Big\downarrow \text{16}$$

$$-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}+\frac{f\left(\frac{a\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}$$

$$\Big\downarrow \text{3803}$$

$$\frac{f\left(\frac{2a\int -\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{f\left(-\frac{2a\int \frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{2694}$$

$$\frac{f\left(-\frac{2a\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \ 27$$

$$\frac{f\left(-\frac{2a\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \ 2620$$

$$\frac{f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{f\log(a-}{d^2}\right.}{bd}$$

$$\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \ 2715$$

$$\frac{f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right.}{bd^2}}{2\sqrt{a^2+b^2}}\right.}{a^2+b^2}\right.}{bd}$$

$$\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \; 2838$$

$$\frac{f\left(-\frac{2a\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{f\log}{}\right.}{bd}$$

$$\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

output
```
-1/2*(e + f*x)^2/(b*d*(a + b*Sinh[c + d*x])^2) + (f*((f*Log[a + b*Sinh[c +
 d*x]])/((a^2 + b^2)*d^2) - (2*a*(-1/2*(b*(((e + f*x)*Log[1 + (b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - S
qrt[a^2 + b^2]))])/(b*d^2)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[1 + (b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(2*Sqrt[a^2 + b^2])))/(a^2 + b^2) - (
b*(e + f*x)*Cosh[c + d*x])/((a^2 + b^2)*d*(a + b*Sinh[c + d*x]))))/(b*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

rule 5987
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 804 vs. $2(284) = 568$.

Time = 26.98 (sec) , antiderivative size = 805, normalized size of antiderivative = 2.63

method	result
risch	$-\frac{2\left(a^{2} d\, f^{2} x^{2} \mathrm{e}^{2 d x+2 c}+b^{2} d\, f^{2} x^{2} \mathrm{e}^{2 d x+2 c}+2 a^{2} d e f x\, \mathrm{e}^{2 d x+2 c}-a b\, f^{2} x\, \mathrm{e}^{3 d x+3 c}+2 b^{2} d e f x\, \mathrm{e}^{2 d x+2 c}+a^{2} d\, e^{2} \mathrm{e}^{2 d x+2 c}-2 a^{2} f^{2} x\, \mathrm{e}^{2 d x+2 c}-\cdots\right.}{b\, d^{2}\left(b\, \mathrm{e}^{2 d x+2 c}+2 a\, \mathrm{e}^{d x} \cdots\right.}$

input
```
int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/b*(a^2*d*f^2*x^2*exp(2*d*x+2*c)+b^2*d*f^2*x^2*exp(2*d*x+2*c)+2*a^2*d*e*
f*x*exp(2*d*x+2*c)-a*b*f^2*x*exp(3*d*x+3*c)+2*b^2*d*e*f*x*exp(2*d*x+2*c)+a
^2*d*e^2*exp(2*d*x+2*c)-2*a^2*f^2*x*exp(2*d*x+2*c)-a*b*e*f*exp(3*d*x+3*c)+
b^2*d*e^2*exp(2*d*x+2*c)+b^2*f^2*x*exp(2*d*x+2*c)-2*a^2*e*f*exp(2*d*x+2*c)
+3*a*b*f^2*x*exp(d*x+c)+b^2*e*f*exp(2*d*x+2*c)+3*a*b*e*f*exp(d*x+c)-b^2*f^
2*x-b^2*e*f)/d^2/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)^2/(a^2+b^2)-2/(a^2+b^
2)/d^3*f^2/b*ln(exp(d*x+c))+1/(a^2+b^2)/d^3*f^2/b*ln(b*exp(2*d*x+2*c)+2*a*
exp(d*x+c)-b)-2/(a^2+b^2)^(3/2)/d^2*f/b*a*e*arctanh(1/2*(2*b*exp(d*x+c)+2*
a)/(a^2+b^2)^(1/2))+1/(a^2+b^2)^(3/2)/d^2*f^2/b*a*ln((-b*exp(d*x+c)+(a^2+b
^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/(a^2+b^2)^(3/2)/d^2*f^2/b*a*ln((b*e
xp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/(a^2+b^2)^(3/2)/d^3*
f^2/b*a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/(a^
2+b^2)^(3/2)/d^3*f^2/b*a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^
(1/2)))*c+1/(a^2+b^2)^(3/2)/d^3*f^2/b*a*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/
2)-a)/(-a+(a^2+b^2)^(1/2)))-1/(a^2+b^2)^(3/2)/d^3*f^2/b*a*dilog((b*exp(d*x
+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/(a^2+b^2)^(3/2)/d^3*f^2/b*a*
c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5233 vs. $2(282) = 564$.

Time = 0.21 (sec) , antiderivative size = 5233, normalized size of antiderivative = 17.10

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)/(a+b*sinh(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)}{(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
(2*a*d*integrate(x*e^(d*x + c)/(a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2
*d*x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + 2*a*b^3*d^2*e^(d*x + c) - a^2*b^2*
d^2 - b^4*d^2), x) + b*(a*log((b*e^(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(
d*x + c) + a + sqrt(a^2 + b^2)))/((a^2*b^2 + b^4)*sqrt(a^2 + b^2)*d^3) - 2
*(d*x + c)/((a^2*b^2 + b^4)*d^3) + log(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c)
 - b)/((a^2*b^2 + b^4)*d^3)) + 2*(a*b*x*e^(3*d*x + 3*c) - 3*a*b*x*e^(d*x +
 c) + b^2*x - ((a^2*d*e^(2*c) + b^2*d*e^(2*c))*x^2 - (2*a^2*e^(2*c) - b^2*
e^(2*c))*x)*e^(2*d*x))/(a^2*b^3*d^2 + b^5*d^2 + (a^2*b^3*d^2*e^(4*c) + b^5
*d^2*e^(4*c))*e^(4*d*x) + 4*(a^3*b^2*d^2*e^(3*c) + a*b^4*d^2*e^(3*c))*e^(3
*d*x) + 2*(2*a^4*b*d^2*e^(2*c) + a^2*b^3*d^2*e^(2*c) - b^5*d^2*e^(2*c))*e^
(2*d*x) - 4*(a^3*b^2*d^2*e^c + a*b^4*d^2*e^c)*e^(d*x)) - a*log((b*e^(d*x +
 c) + a - sqrt(a^2 + b^2))/(b*e^(d*x + c) + a + sqrt(a^2 + b^2)))/((a^2*b
+ b^3)*sqrt(a^2 + b^2)*d^3))*f^2 + e*f*(2*(a*b*e^(3*d*x + 3*c) - 3*a*b*e^(
d*x + c) + b^2 + (2*a^2*e^(2*c) - b^2*e^(2*c) - 2*(a^2*d*e^(2*c) + b^2*d*e
^(2*c))*x)*e^(2*d*x))/(a^2*b^3*d^2 + b^5*d^2 + (a^2*b^3*d^2*e^(4*c) + b^5*
d^2*e^(4*c))*e^(4*d*x) + 4*(a^3*b^2*d^2*e^(3*c) + a*b^4*d^2*e^(3*c))*e^(3*
d*x) + 2*(2*a^4*b*d^2*e^(2*c) + a^2*b^3*d^2*e^(2*c) - b^5*d^2*e^(2*c))*e^(
2*d*x) - 4*(a^3*b^2*d^2*e^c + a*b^4*d^2*e^c)*e^(d*x)) + a*log((b*e^(d*x +
2*c) + a*e^c - sqrt(a^2 + b^2)*e^c)/(b*e^(d*x + 2*c) + a*e^c + sqrt(a^2 +
b^2)*e^c))/((a^2*b + b^3)*sqrt(a^2 + b^2)*d^2)) - 2*e^2*e^(-2*d*x - 2*c...
```

Giac [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx=\int \frac{(fx+e)^2\cosh(dx+c)}{(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)/(b*sinh(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \int \frac{\cosh(c+d\,x)\,\left(e+f\,x\right)^2}{\left(a+b\sinh\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x))^3,x)
```

output
```
int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x)
```

output
```
(8*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a
**2 + b**2))*a**5*b**2*f**2*i + 24*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan
((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**3*b**4*f**2*i + 12*e**(4*c
 + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2
))*a*b**6*d*e*f*i + 16*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*
x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b**6*f**2*i + 32*e**(3*c + 3*d*x)*sqrt(
a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**6*b*f**2*
i + 96*e**(3*c + 3*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sq
rt(a**2 + b**2))*a**4*b**3*f**2*i + 48*e**(3*c + 3*d*x)*sqrt(a**2 + b**2)*
atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b**5*d*e*f*i + 64*e*
*(3*c + 3*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 +
 b**2))*a**2*b**5*f**2*i + 32*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**
(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**7*f**2*i + 80*e**(2*c + 2*d*x)*
sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**5*b*
*2*f**2*i + 48*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i +
 a*i)/sqrt(a**2 + b**2))*a**3*b**4*d*e*f*i + 16*e**(2*c + 2*d*x)*sqrt(a**2
 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**3*b**4*f**2*i
 - 24*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqr
t(a**2 + b**2))*a*b**6*d*e*f*i - 32*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*ata
n((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b**6*f**2*i - 32*e**(c ...
```

3.329 $\int \frac{(e+fx)^3 \cosh(c+dx)}{(a+b\sinh(c+dx))^3} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 631

$$\begin{aligned}
\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = &-\frac{3f(e+fx)^2}{2b(a^2+b^2)d^2}+\frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)d^3}\\
&+\frac{3af(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2b(a^2+b^2)^{3/2}d^2}\\
&+\frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)d^3}\\
&-\frac{3af(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{2b(a^2+b^2)^{3/2}d^2}\\
&+\frac{3f^3\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)d^4}\\
&+\frac{3af^2(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d^3}\\
&+\frac{3f^3\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)d^4}\\
&-\frac{3af^2(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d^3}\\
&-\frac{3af^3\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d^4}\\
&+\frac{3af^3\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d^4}\\
&-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}\\
&-\frac{3f(e+fx)^2\cosh(c+dx)}{2(a^2+b^2)d^2(a+b\sinh(c+dx))}
\end{aligned}$$

output
```
-3/2*f*(f*x+e)^2/b/(a^2+b^2)/d^2+3*f^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b
^2)^(1/2)))/b/(a^2+b^2)/d^3+3/2*a*f*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^
2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^2+3*f^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b
^2)^(1/2)))/b/(a^2+b^2)/d^3-3/2*a*f*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^2+3*f^3*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2
)^(1/2)))/b/(a^2+b^2)/d^4+3*a*f^2*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+
b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^3+3*f^3*polylog(2,-b*exp(d*x+c)/(a+(a^2+b
^2)^(1/2)))/b/(a^2+b^2)/d^4-3*a*f^2*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^
2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^3-3*a*f^3*polylog(3,-b*exp(d*x+c)/(a-(a
^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^4+3*a*f^3*polylog(3,-b*exp(d*x+c)/(a+(
a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^4-1/2*(f*x+e)^3/b/d/(a+b*sinh(d*x+c))
^2-3/2*f*(f*x+e)^2*cosh(d*x+c)/(a^2+b^2)/d^2/(a+b*sinh(d*x+c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5753 vs. $2(631) = 1262$.

Time = 13.58 (sec) , antiderivative size = 5753, normalized size of antiderivative = 9.12

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 3.14 (sec) , antiderivative size = 561, normalized size of antiderivative = 0.89, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {5987, 3042, 3805, 3042, 3803, 25, 2694, 27, 2620, 3011, 2720, 6095, 2620, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}dx$$

$$\Big\downarrow \text{5987}$$

$$\frac{3f\int \frac{(e+fx)^2}{(a+b\sinh(c+dx))^2}dx}{2bd}-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}+\frac{3f\int \frac{(e+fx)^2}{(a-ib\sin(ic+idx))^2}dx}{2bd}$$

$$\Big\downarrow \text{3805}$$

$$\frac{3f\left(\frac{a\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{2bf\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)^2\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}+\frac{3f\left(\frac{a\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{a^2+b^2}+\frac{2bf\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)^2\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}$$

$$\Big\downarrow \text{3803}$$

$$\frac{3f\left(\frac{2a\int -\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{2bf\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)^2\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{3f\left(-\frac{2a\int \frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{2bf\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)^2\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{2694}$$

$$\frac{3f\left(-\frac{2a\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{2bf\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)^2\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$\downarrow$ 27

$$\frac{3f\left(-\frac{2a\left(\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{2bf\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)^2\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$\downarrow$ 2620

$$\frac{3f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{a^2+b^2}\right.}{2bd}-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$\downarrow$ 3011

$$3f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\cdots\right)}{bd}\cdots\right)}{\cdots}\right)}{a^2+b^2}\cdots\right)$$

$$\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$\downarrow$ 2720

$$3f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\cdots}{\cdots}\right)}{\cdots}\right)}{a^2+b^2}\cdots\right)$$

$$\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$\downarrow$ 6095

$$3f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\text{lo}\cdots}{\cdots}\right.}{\cdots}\right)}{a^2+b^2}\cdots\right.$$

$$\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$\downarrow$ 2620

$$3f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log}{}\right.}{}\right)}{a^2+b^2}\right.$$

$$\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$\downarrow$ 2715

$$3f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log}{}\right.}{}\right)}{a^2+b^2}\right.$$

$$\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$\Big\downarrow$ 2838

$$3f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log \cdots\right.}{\cdots}\right)}{a^2+b^2}\cdots\right.}{\cdots}$$

$$\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$\Big\downarrow$ 7143

$$3f\left(\frac{2bf\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{d(a^2+b^2)}-\frac{2a\,\cdots}{\cdots}\right.$$

$$\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

output
```
-1/2*(e + f*x)^3/(b*d*(a + b*Sinh[c + d*x])^2) + (3*f*((2*b*f*(-1/2*(e + f
*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(
b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) +
(f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (f*Poly
Log[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/((a^2 + b^2)*d
) - (2*a*(-1/2*(b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])
/d^2))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))])/d^2))/(b*d)))/(2*Sqrt[a^2 + b^2])))/(a^2 + b^2) - (b*(e
 + f*x)^2*Cosh[c + d*x])/((a^2 + b^2)*d*(a + b*Sinh[c + d*x]))))/(2*b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

rule 5987
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(fx+e)^3 \cosh (dx+c)}{(a+b\sinh (dx+c))^3}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11757 vs. 2(575) = 1150.

Time = 0.32 (sec) , antiderivative size = 11757, normalized size of antiderivative = 18.63

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)/(a+b*sinh(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)}{(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
3*a*d*f^3*integrate(x^2*e^(d*x + c)/(a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2
*e^(2*d*x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + 2*a*b^3*d^2*e^(d*x + c) - a^2
*b^2*d^2 - b^4*d^2), x) + 6*a*d*e*f^2*integrate(x*e^(d*x + c)/(a^2*b^2*d^2
*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + 2*a
*b^3*d^2*e^(d*x + c) - a^2*b^2*d^2 - b^4*d^2), x) + 3*b*e*f^2*(a*log((b*e^
(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(d*x + c) + a + sqrt(a^2 + b^2)))/((
a^2*b^2 + b^4)*sqrt(a^2 + b^2)*d^3) - 2*(d*x + c)/((a^2*b^2 + b^4)*d^3) +
log(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - b)/((a^2*b^2 + b^4)*d^3)) - 6*a*
f^3*integrate(x*e^(d*x + c)/(a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*
x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + 2*a*b^3*d^2*e^(d*x + c) - a^2*b^2*d^2
 - b^4*d^2), x) + 6*b*f^3*integrate(x/(a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d
^2*e^(2*d*x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + 2*a*b^3*d^2*e^(d*x + c) - a
^2*b^2*d^2 - b^4*d^2), x) + 3/2*e^2*f*(2*(a*b*e^(3*d*x + 3*c) - 3*a*b*e^(d
*x + c) + b^2 + (2*a^2*e^(2*c) - b^2*e^(2*c) - 2*(a^2*d*e^(2*c) + b^2*d*e^
(2*c))*x)*e^(2*d*x))/(a^2*b^3*d^2 + b^5*d^2 + (a^2*b^3*d^2*e^(4*c) + b^5*d
^2*e^(4*c))*e^(4*d*x) + 4*(a^3*b^2*d^2*e^(3*c) + a*b^4*d^2*e^(3*c))*e^(3*d
*x) + 2*(2*a^4*b*d^2*e^(2*c) + a^2*b^3*d^2*e^(2*c) - b^5*d^2*e^(2*c))*e^(2
*d*x) - 4*(a^3*b^2*d^2*e^c + a*b^4*d^2*e^c)*e^(d*x)) + a*log((b*e^(d*x + 2
*c) + a*e^c - sqrt(a^2 + b^2)*e^c)/(b*e^(d*x + 2*c) + a*e^c + sqrt(a^2 + b
^2)*e^c))/((a^2*b + b^3)*sqrt(a^2 + b^2)*d^2)) - 2*e^3*e^(-2*d*x - 2*c)...
```

Giac [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx=\int \frac{(fx+e)^3\cosh(dx+c)}{(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)/(b*sinh(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \int \frac{\cosh(c+d\,x)\,\left(e+f\,x\right)^3}{\left(a+b\sinh\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x))^3,x)
```

output
```
int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x)
```

output
```
(192*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt
(a**2 + b**2))*a**5*b**2*d*e*f**2*i + 16*e**(4*c + 4*d*x)*sqrt(a**2 + b**2
)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**5*b**2*f**3*i + 576*
e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2
 + b**2))*a**3*b**4*d*e*f**2*i + 240*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*at
an((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**3*b**4*f**3*i + 144*e**(
4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b
**2))*a*b**6*d**2*e**2*f*i + 384*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((
e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b**6*d*e*f**2*i + 236*e**(4*c
 + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2
))*a*b**6*f**3*i + 768*e**(3*c + 3*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*
x)*b*i + a*i)/sqrt(a**2 + b**2))*a**6*b*d*e*f**2*i + 64*e**(3*c + 3*d*x)*s
qrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**6*b*f
**3*i + 2304*e**(3*c + 3*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a
*i)/sqrt(a**2 + b**2))*a**4*b**3*d*e*f**2*i + 960*e**(3*c + 3*d*x)*sqrt(a*
*2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**4*b**3*f**3
*i + 576*e**(3*c + 3*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/
sqrt(a**2 + b**2))*a**2*b**5*d**2*e**2*f*i + 1536*e**(3*c + 3*d*x)*sqrt(a*
*2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b**5*d*e*
f**2*i + 944*e**(3*c + 3*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i ...
```

3.330 $\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 112

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = -\frac{af\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d^2} - \frac{e+fx}{2bd(a+b\sinh(c+dx))^2} - \frac{f\cosh(c+dx)}{2\,(a^2+b^2)\,d^2(a+b\sinh(c+dx))}$$

output
```
-a*f*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/b/(a^2+b^2)^(3/2)/
d^2-1/2*(f*x+e)/b/d/(a+b*sinh(d*x+c))^2-1/2*f*cosh(d*x+c)/(a^2+b^2)/d^2/(a
+b*sinh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = -\frac{\frac{f\cosh(c+dx)}{(a^2+b^2)(a+b\sinh(c+dx))} + \frac{\frac{2af\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}} + \frac{d(e+fx)}{(a+b\sinh(c+dx))^2}}{b}}{2d^2}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

output
```
-1/2*((f*Cosh[c + d*x])/((a^2 + b^2)*(a + b*Sinh[c + d*x])) + ((2*a*f*ArcT
an[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(3/2) + (d*(e
 + f*x))/(a + b*Sinh[c + d*x])^2)/b)/d^2
```

Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5987, 3042, 3143, 25, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}dx$$

$$\Big\downarrow \text{ 5987}$$

$$\frac{f\int \frac{1}{(a+b\sinh(c+dx))^2}dx}{2bd}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}+\frac{f\int \frac{1}{(a-ib\sin(ic+idx))^2}dx}{2bd}$$

$$\Big\downarrow \text{ 3143}$$

$$\frac{f\left(-\frac{\int -\frac{a}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{f\left(\frac{\int \frac{a}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{f\left(\frac{a\int \frac{1}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}+\frac{f\left(-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}+\frac{a\int \frac{1}{a-ib\sin(ic+idx)}dx}{a^2+b^2}\right)}{2bd}$$

$$\Big\downarrow \text{3139}$$

$$-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}+\frac{f\left(-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}-\frac{2ia\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d(a^2+b^2)}\right)}{2bd}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}+\frac{f\left(-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}+\frac{4ia\int \frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4\left(a^2+b^2\right)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{d(a^2+b^2)}\right)}{2bd}$$

$$\Big\downarrow \text{217}$$

$$\frac{f\left(\frac{2a\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{3/2}}-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}$$

input
```
Int[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

output
```
-1/2*(e + f*x)/(b*d*(a + b*Sinh[c + d*x])^2) + (f*((2*a*ArcTanh[Tanh[(c +
d*x)/2]/(2*Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (b*Cosh[c + d*x])/((
a^2 + b^2)*d*(a + b*Sinh[c + d*x]))))/(2*b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 5987
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(103) = 206$.

Time = 18.87 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.75

method	result
risch	$-\frac{2\,e^{2dx+2c}a^{2}dfx+2b^{2}dfx\,e^{2dx+2c}+2\,e^{2dx+2c}a^{2}de-abf\,e^{3dx+3c}+2b^{2}de\,e^{2dx+2c}-2a^{2}f\,e^{2dx+2c}+b^{2}f\,e^{2dx+2c}+3af\,e^{dx+c}b-b^{2}f}{b\,d^{2}\left(b\,e^{2dx+2c}+2a\,e^{dx+c}-b\right)^{2}\left(a^{2}+b^{2}\right)}$

input
```
int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/b*(2*exp(2*d*x+2*c)*a^2*d*f*x+2*b^2*d*f*x*exp(2*d*x+2*c)+2*exp(2*d*x+2*
c)*a^2*d*e-a*b*f*exp(3*d*x+3*c)+2*b^2*d*e*exp(2*d*x+2*c)-2*a^2*f*exp(2*d*x
+2*c)+b^2*f*exp(2*d*x+2*c)+3*a*f*exp(d*x+c)*b-b^2*f)/d^2/(b*exp(2*d*x+2*c)
+2*a*exp(d*x+c)-b)^2/(a^2+b^2)+1/2/(a^2+b^2)^(3/2)*f*a/d^2/b*ln(exp(d*x+c)
+(a*(a^2+b^2)^(3/2)-a^4-2*a^2*b^2-b^4)/b/(a^2+b^2)^(3/2))-1/2/(a^2+b^2)^(3
/2)*f*a/d^2/b*ln(exp(d*x+c)+(a*(a^2+b^2)^(3/2)+a^4+2*a^2*b^2+b^4)/b/(a^2+b
^2)^(3/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1230 vs. $2(105) = 210$.

Time = 0.13 (sec) , antiderivative size = 1230, normalized size of antiderivative = 10.98

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(2*(a^3*b + a*b^3)*f*cosh(d*x + c)^3 + 2*(a^3*b + a*b^3)*f*sinh(d*x +
c)^3 - 6*(a^3*b + a*b^3)*f*cosh(d*x + c) - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d*
f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*d*e - (2*a^4 + a^2*b^2 - b^4)*f)*cosh(d*x
+ c)^2 - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*d*
e - 3*(a^3*b + a*b^3)*f*cosh(d*x + c) - (2*a^4 + a^2*b^2 - b^4)*f)*sinh(d*
x + c)^2 + (a*b^2*f*cosh(d*x + c)^4 + a*b^2*f*sinh(d*x + c)^4 + 4*a^2*b*f*
cosh(d*x + c)^3 - 4*a^2*b*f*cosh(d*x + c) + a*b^2*f + 2*(2*a^3 - a*b^2)*f*
cosh(d*x + c)^2 + 4*(a*b^2*f*cosh(d*x + c) + a^2*b*f)*sinh(d*x + c)^3 + 2*
(3*a*b^2*f*cosh(d*x + c)^2 + 6*a^2*b*f*cosh(d*x + c) + (2*a^3 - a*b^2)*f)*
sinh(d*x + c)^2 + 4*(a*b^2*f*cosh(d*x + c)^3 + 3*a^2*b*f*cosh(d*x + c)^2 -
 a^2*b*f + (2*a^3 - a*b^2)*f*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b^2)
*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*
a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*
(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x +
 c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) +
2*(a^2*b^2 + b^4)*f + 2*(3*(a^3*b + a*b^3)*f*cosh(d*x + c)^2 - 3*(a^3*b +
a*b^3)*f - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*
d*e - (2*a^4 + a^2*b^2 - b^4)*f)*cosh(d*x + c))*sinh(d*x + c))/((a^4*b^3 +
 2*a^2*b^5 + b^7)*d^2*cosh(d*x + c)^4 + (a^4*b^3 + 2*a^2*b^5 + b^7)*d^2*si
nh(d*x + c)^4 + 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^2*cosh(d*x + c)^3 + 2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(105) = 210$.

Time = 0.25 (sec) , antiderivative size = 413, normalized size of antiderivative = 3.69

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$$
$$= \frac{1}{2}f\left(\frac{2\left(abe^{(3\,dx+3\,c)} - 3\,abe^{(dx+c)} + b^2 + \left(2\,a^2e^{(2\,c)} - b^2e^{(2\,c)} - 2\,(a^2d\right.\right.}{a^2b^3d^2 + b^5d^2 + (a^2b^3d^2e^{(4\,c)} + b^5d^2e^{(4\,c)})e^{(4\,dx)} + 4\,(a^3b^2d^2e^{(3\,c)} + ab^4d^2e^{(3\,c)})e^{(3\,dx)} + 2\,(2\,a^4bd^2e^{(2\,c)}}\right.$$
$$- \frac{2\,ee^{(-2\,dx-2\,c)}}{(4\,ab^2e^{(-dx-c)} - 4\,ab^2e^{(-3\,dx-3\,c)} + b^3e^{(-4\,dx-4\,c)} + b^3 + 2\,(2\,a^2b - b^3)e^{(-2\,dx-2\,c)})d}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*f*(2*(a*b*e^(3*d*x + 3*c) - 3*a*b*e^(d*x + c) + b^2 + (2*a^2*e^(2*c) -
 b^2*e^(2*c) - 2*(a^2*d*e^(2*c) + b^2*d*e^(2*c))*x)*e^(2*d*x))/(a^2*b^3*d^
2 + b^5*d^2 + (a^2*b^3*d^2*e^(4*c) + b^5*d^2*e^(4*c))*e^(4*d*x) + 4*(a^3*b
^2*d^2*e^(3*c) + a*b^4*d^2*e^(3*c))*e^(3*d*x) + 2*(2*a^4*b*d^2*e^(2*c) + a
^2*b^3*d^2*e^(2*c) - b^5*d^2*e^(2*c))*e^(2*d*x) - 4*(a^3*b^2*d^2*e^c + a*b
^4*d^2*e^c)*e^(d*x)) + a*log((b*e^(d*x + 2*c) + a*e^c - sqrt(a^2 + b^2)*e^
c)/(b*e^(d*x + 2*c) + a*e^c + sqrt(a^2 + b^2)*e^c))/((a^2*b + b^3)*sqrt(a^
2 + b^2)*d^2)) - 2*e*e^(-2*d*x - 2*c)/((4*a*b^2*e^(-d*x - c) - 4*a*b^2*e^(
-3*d*x - 3*c) + b^3*e^(-4*d*x - 4*c) + b^3 + 2*(2*a^2*b - b^3)*e^(-2*d*x -
 2*c))*d)
```

Giac [F]

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \int \frac{(fx+e)\cosh(dx+c)}{(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)/(b*sinh(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx=\int \frac{\cosh(c+d\,x)\,\left(e+f\,x\right)}{\left(a+b\sinh\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x))^3,x)
```

output
```
int((cosh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x))^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 756, normalized size of antiderivative = 6.75

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$$
$$=\frac{4e^{4dx+4c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)a\,b^2fi+16e^{3dx+3c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)a^2bfi+16e^{2dx+2c}\sqrt{a^2+b^2}}{}$$

input
```
int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x)
```

output
```
(4*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a
**2 + b**2))*a*b**2*f*i + 16*e**(3*c + 3*d*x)*sqrt(a**2 + b**2)*atan((e**(
c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b*f*i + 16*e**(2*c + 2*d*x)*sq
rt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**3*f*i
- 8*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(
a**2 + b**2))*a*b**2*f*i - 16*e**(c + d*x)*sqrt(a**2 + b**2)*atan((e**(c +
 d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b*f*i + 4*sqrt(a**2 + b**2)*atan(
(e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b**2*f*i - e**(4*c + 4*d*x)*
a**2*b**2*f - e**(4*c + 4*d*x)*b**4*f - 8*e**(2*c + 2*d*x)*a**4*d*e - 8*e*
*(2*c + 2*d*x)*a**4*d*f*x + 4*e**(2*c + 2*d*x)*a**4*f - 16*e**(2*c + 2*d*x
)*a**2*b**2*d*e - 16*e**(2*c + 2*d*x)*a**2*b**2*d*f*x + 2*e**(2*c + 2*d*x)
*a**2*b**2*f - 8*e**(2*c + 2*d*x)*b**4*d*e - 8*e**(2*c + 2*d*x)*b**4*d*f*x
 - 2*e**(2*c + 2*d*x)*b**4*f - 8*e**(c + d*x)*a**3*b*f - 8*e**(c + d*x)*a*
b**3*f + 3*a**2*b**2*f + 3*b**4*f)/(4*b*d**2*(e**(4*c + 4*d*x)*a**4*b**2 +
 2*e**(4*c + 4*d*x)*a**2*b**4 + e**(4*c + 4*d*x)*b**6 + 4*e**(3*c + 3*d*x)
*a**5*b + 8*e**(3*c + 3*d*x)*a**3*b**3 + 4*e**(3*c + 3*d*x)*a*b**5 + 4*e**
(2*c + 2*d*x)*a**6 + 6*e**(2*c + 2*d*x)*a**4*b**2 - 2*e**(2*c + 2*d*x)*b**
6 - 4*e**(c + d*x)*a**5*b - 8*e**(c + d*x)*a**3*b**3 - 4*e**(c + d*x)*a*b*
*5 + a**4*b**2 + 2*a**2*b**4 + b**6))
```

3.331 $\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 306

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{3/2}d^2} - \frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{3/2}d^2} + \frac{f^2\log(a+b\sinh(c+dx))}{b\left(a^2+b^2\right)d^3} + \frac{af^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{3/2}d^3} - \frac{af^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{3/2}d^3} - \frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2} - \frac{f(e+fx)\cosh(c+dx)}{\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))}$$

output
```
a*f*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^2-a
*f*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^2+f^
2*ln(a+b*sinh(d*x+c))/b/(a^2+b^2)/d^3+a*f^2*polylog(2,-b*exp(d*x+c)/(a-(a^
2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^3-a*f^2*polylog(2,-b*exp(d*x+c)/(a+(a^2
+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^3-1/2*(f*x+e)^2/b/d/(a+b*sinh(d*x+c))^2-
f*(f*x+e)*cosh(d*x+c)/(a^2+b^2)/d^2/(a+b*sinh(d*x+c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 623 vs. $2(306) = 612$.

Time = 6.65 (sec) , antiderivative size = 623, normalized size of antiderivative = 2.04

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \frac{f^2x\coth(c)}{b(a^2+b^2)\,d^2} + \frac{2e^cf\left(-e^cfx+e^{-c}(-1+e^{2c})\,fx-\frac{aee^{-c}(-1+e^{2c})\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{ae^{-c}(-1+e^{2c})f\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}d}+\cdots\right.}{\cdots} - \frac{f^2x\cosh(c)\text{csch}\left(\frac{c}{2}\right)\text{sech}\left(\frac{c}{2}\right)}{2b(a^2+b^2)\,d^2} - \frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2} + \frac{\text{csch}\left(\frac{c}{2}\right)\text{sech}\left(\frac{c}{2}\right)(aef\cosh(c)+af^2x\cosh(c)+bef\sinh(dx)+bf^2x\sinh(dx))}{2b(a^2+b^2)\,d^2(a+b\sinh(c+dx))}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

output
```
(f^2*x*Coth[c])/(b*(a^2 + b^2)*d^2) + (2*E^c*f*(-(E^c*f*x) + ((-1 + E^(2*c
))*f*x)/E^c - (a*e*(-1 + E^(2*c))*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b
^2]])/(Sqrt[a^2 + b^2]*E^c) + (a*(-1 + E^(2*c))*f*ArcTanh[(a + b*E^(c + d*
x))/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d*E^c) + ((-1 + E^(2*c))*f*(-2*x +
(2*a*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-a^2 - b^2]*d) +
Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))]/d))/(2*E^c) + (a*(-1 + E^(
2*c))*f*(d*x*(Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]
)] - Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])]) + Pol
yLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - PolyLo
g[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/(2*d*Sqrt
[(a^2 + b^2)*E^(2*c)])))/(b*(a^2 + b^2)*d^2*(-1 + E^(2*c))) - (f^2*x*Cosh[
c]*Csch[c/2]*Sech[c/2])/(2*b*(a^2 + b^2)*d^2) - (e + f*x)^2/(2*b*d*(a + b*
Sinh[c + d*x])^2) + (Csch[c/2]*Sech[c/2]*(a*e*f*Cosh[c] + a*f^2*x*Cosh[c]
+ b*e*f*Sinh[d*x] + b*f^2*x*Sinh[d*x]))/(2*b*(a^2 + b^2)*d^2*(a + b*Sinh[c
 + d*x]))
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5987, 3042, 3805, 3042, 3147, 16, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}dx$$

$$\Big\downarrow \text{5987}$$

$$\frac{f\int \frac{e+fx}{(a+b\sinh(c+dx))^2}dx}{bd}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}+\frac{f\int \frac{e+fx}{(a-ib\sin(ic+idx))^2}dx}{bd}$$

$$\Big\downarrow \text{3805}$$

$$\frac{f\left(\frac{a\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{bf\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}+\frac{f\left(\frac{a\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a^2+b^2}+\frac{bf\int \frac{\cos(ic+idx)}{a-ib\sin(ic+idx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}$$

$$\Big\downarrow \text{3147}$$

$$-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}+\frac{f\left(\frac{f\int \frac{1}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{d^2(a^2+b^2)}+\frac{a\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a^2+b^2}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}$$

$$\Big\downarrow \text{16}$$

$$-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}+\frac{f\left(\frac{a\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}$$

$$\Big\downarrow \text{3803}$$

$$\frac{f\left(\frac{2a\int -\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{f\left(-\frac{2a\int \frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{2694}$$

$$\frac{f\left(-\frac{2a\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$\Big\downarrow$ 27

$$\frac{f\left(-\frac{2a\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$\Big\downarrow$ 2620

$$\frac{f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{f\log(a-}{d^2}\right.}{bd}$$

$$\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$\Big\downarrow$ 2715

$$\frac{f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right.}{bd^2}\right.}{2\sqrt{a^2+b^2}}\right.}{a^2+b^2}\right.}{bd}$$

$$\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{f\left(-\frac{2a\left(\frac{b\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{f\log\cdots}{\cdots}\right.}{bd}$$

$$\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

output
```
-1/2*(e + f*x)^2/(b*d*(a + b*Sinh[c + d*x])^2) + (f*((f*Log[a + b*Sinh[c +
 d*x]])/((a^2 + b^2)*d^2) - (2*a*(-1/2*(b*(((e + f*x)*Log[1 + (b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - S
qrt[a^2 + b^2]))])/(b*d^2)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[1 + (b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(2*Sqrt[a^2 + b^2])))/(a^2 + b^2) - (
b*(e + f*x)*Cosh[c + d*x])/((a^2 + b^2)*d*(a + b*Sinh[c + d*x]))))/(b*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

rule 5987
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 804 vs. $2(284) = 568$.

Time = 15.52 (sec) , antiderivative size = 805, normalized size of antiderivative = 2.63

method	result
risch	$-\frac{2\left(a^2 d f^2 x^2 \mathrm{e}^{2dx+2c}+b^2 d f^2 x^2 \mathrm{e}^{2dx+2c}+2a^2 def x\, \mathrm{e}^{2dx+2c}-ab f^2 x\, \mathrm{e}^{3dx+3c}+2b^2 def x\, \mathrm{e}^{2dx+2c}+a^2 d e^2 \mathrm{e}^{2dx+2c}-2a^2 f^2 x\, \mathrm{e}^{2dx+2c}-\cdots\right.}{b d^2 \left(b \mathrm{e}^{2dx+2c}+2a \mathrm{e}^{dx}\cdots\right.}$

input
```
int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/b*(a^2*d*f^2*x^2*exp(2*d*x+2*c)+b^2*d*f^2*x^2*exp(2*d*x+2*c)+2*a^2*d*e*
f*x*exp(2*d*x+2*c)-a*b*f^2*x*exp(3*d*x+3*c)+2*b^2*d*e*f*x*exp(2*d*x+2*c)+a
^2*d*e^2*exp(2*d*x+2*c)-2*a^2*f^2*x*exp(2*d*x+2*c)-a*b*e*f*exp(3*d*x+3*c)+
b^2*d*e^2*exp(2*d*x+2*c)+b^2*f^2*x*exp(2*d*x+2*c)-2*a^2*e*f*exp(2*d*x+2*c)
+3*a*b*f^2*x*exp(d*x+c)+b^2*e*f*exp(2*d*x+2*c)+3*a*b*e*f*exp(d*x+c)-b^2*f^
2*x-b^2*e*f)/d^2/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)^2/(a^2+b^2)-2/(a^2+b^
2)/d^3*f^2/b*ln(exp(d*x+c))+1/(a^2+b^2)/d^3*f^2/b*ln(b*exp(2*d*x+2*c)+2*a*
exp(d*x+c)-b)-2/(a^2+b^2)^(3/2)/d^2*f/b*a*e*arctanh(1/2*(2*b*exp(d*x+c)+2*
a)/(a^2+b^2)^(1/2))+1/(a^2+b^2)^(3/2)/d^2*f^2/b*a*ln((-b*exp(d*x+c)+(a^2+b
^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/(a^2+b^2)^(3/2)/d^2*f^2/b*a*ln((b*e
xp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/(a^2+b^2)^(3/2)/d^3*
f^2/b*a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/(a^
2+b^2)^(3/2)/d^3*f^2/b*a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^
(1/2)))*c+1/(a^2+b^2)^(3/2)/d^3*f^2/b*a*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/
2)-a)/(-a+(a^2+b^2)^(1/2)))-1/(a^2+b^2)^(3/2)/d^3*f^2/b*a*dilog((b*exp(d*x
+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/(a^2+b^2)^(3/2)/d^3*f^2/b*a*
c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5233 vs. $2(282) = 564$.

Time = 0.23 (sec) , antiderivative size = 5233, normalized size of antiderivative = 17.10

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)/(a+b*sinh(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)}{(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
(2*a*d*integrate(x*e^(d*x + c)/(a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2
*d*x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + 2*a*b^3*d^2*e^(d*x + c) - a^2*b^2*
d^2 - b^4*d^2), x) + b*(a*log((b*e^(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(
d*x + c) + a + sqrt(a^2 + b^2)))/((a^2*b^2 + b^4)*sqrt(a^2 + b^2)*d^3) - 2
*(d*x + c)/((a^2*b^2 + b^4)*d^3) + log(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c)
 - b)/((a^2*b^2 + b^4)*d^3)) + 2*(a*b*x*e^(3*d*x + 3*c) - 3*a*b*x*e^(d*x +
 c) + b^2*x - ((a^2*d*e^(2*c) + b^2*d*e^(2*c))*x^2 - (2*a^2*e^(2*c) - b^2*
e^(2*c))*x)*e^(2*d*x))/(a^2*b^3*d^2 + b^5*d^2 + (a^2*b^3*d^2*e^(4*c) + b^5
*d^2*e^(4*c))*e^(4*d*x) + 4*(a^3*b^2*d^2*e^(3*c) + a*b^4*d^2*e^(3*c))*e^(3
*d*x) + 2*(2*a^4*b*d^2*e^(2*c) + a^2*b^3*d^2*e^(2*c) - b^5*d^2*e^(2*c))*e^
(2*d*x) - 4*(a^3*b^2*d^2*e^c + a*b^4*d^2*e^c)*e^(d*x)) - a*log((b*e^(d*x +
 c) + a - sqrt(a^2 + b^2))/(b*e^(d*x + c) + a + sqrt(a^2 + b^2)))/((a^2*b
+ b^3)*sqrt(a^2 + b^2)*d^3))*f^2 + e*f*(2*(a*b*e^(3*d*x + 3*c) - 3*a*b*e^(
d*x + c) + b^2 + (2*a^2*e^(2*c) - b^2*e^(2*c) - 2*(a^2*d*e^(2*c) + b^2*d*e
^(2*c))*x)*e^(2*d*x))/(a^2*b^3*d^2 + b^5*d^2 + (a^2*b^3*d^2*e^(4*c) + b^5*
d^2*e^(4*c))*e^(4*d*x) + 4*(a^3*b^2*d^2*e^(3*c) + a*b^4*d^2*e^(3*c))*e^(3*
d*x) + 2*(2*a^4*b*d^2*e^(2*c) + a^2*b^3*d^2*e^(2*c) - b^5*d^2*e^(2*c))*e^(
2*d*x) - 4*(a^3*b^2*d^2*e^c + a*b^4*d^2*e^c)*e^(d*x)) + a*log((b*e^(d*x +
2*c) + a*e^c - sqrt(a^2 + b^2)*e^c)/(b*e^(d*x + 2*c) + a*e^c + sqrt(a^2 +
b^2)*e^c))/((a^2*b + b^3)*sqrt(a^2 + b^2)*d^2)) - 2*e^2*e^(-2*d*x - 2*c...
```

Giac [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx=\int \frac{(fx+e)^2\cosh(dx+c)}{(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)/(b*sinh(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \int \frac{\cosh(c+d\,x)\,\left(e+f\,x\right)^2}{\left(a+b\sinh\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x))^3,x)
```

output
```
int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x)
```

output

```
(8*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a
**2 + b**2))*a**5*b**2*f**2*i + 24*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan
((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**3*b**4*f**2*i + 12*e**(4*c
 + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2
))*a*b**6*d*e*f*i + 16*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*
x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b**6*f**2*i + 32*e**(3*c + 3*d*x)*sqrt(
a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**6*b*f**2*
i + 96*e**(3*c + 3*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sq
rt(a**2 + b**2))*a**4*b**3*f**2*i + 48*e**(3*c + 3*d*x)*sqrt(a**2 + b**2)*
atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b**5*d*e*f*i + 64*e*
*(3*c + 3*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 +
 b**2))*a**2*b**5*f**2*i + 32*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**
(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**7*f**2*i + 80*e**(2*c + 2*d*x)*
sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**5*b*
*2*f**2*i + 48*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i +
 a*i)/sqrt(a**2 + b**2))*a**3*b**4*d*e*f*i + 16*e**(2*c + 2*d*x)*sqrt(a**2
 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**3*b**4*f**2*i
 - 24*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqr
t(a**2 + b**2))*a*b**6*d*e*f*i - 32*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*ata
n((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b**6*f**2*i - 32*e**(c ...
```

3.332 $\int \frac{(e+fx)^3 \cosh(c+dx)}{(a+b\sinh(c+dx))^3} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 631

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = -\frac{3f(e+fx)^2}{2b(a^2+b^2)d^2} + \frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)d^3} + \frac{3af(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2b(a^2+b^2)^{3/2}d^2} + \frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)d^3} - \frac{3af(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{2b(a^2+b^2)^{3/2}d^2} + \frac{3f^3\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)d^4} + \frac{3af^2(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d^3} + \frac{3f^3\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)d^4} - \frac{3af^2(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d^3} - \frac{3af^3\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d^4} + \frac{3af^3\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d^4} - \frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2} - \frac{3f(e+fx)^2\cosh(c+dx)}{2(a^2+b^2)d^2(a+b\sinh(c+dx))}$$

output
```
-3/2*f*(f*x+e)^2/b/(a^2+b^2)/d^2+3*f^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b
^2)^(1/2)))/b/(a^2+b^2)/d^3+3/2*a*f*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^
2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^2+3*f^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b
^2)^(1/2)))/b/(a^2+b^2)/d^3-3/2*a*f*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^2+3*f^3*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2
)^(1/2)))/b/(a^2+b^2)/d^4+3*a*f^2*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+
b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^3+3*f^3*polylog(2,-b*exp(d*x+c)/(a+(a^2+b
^2)^(1/2)))/b/(a^2+b^2)/d^4-3*a*f^2*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^
2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^3-3*a*f^3*polylog(3,-b*exp(d*x+c)/(a-(a
^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^4+3*a*f^3*polylog(3,-b*exp(d*x+c)/(a+(
a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^4-1/2*(f*x+e)^3/b/d/(a+b*sinh(d*x+c))
^2-3/2*f*(f*x+e)^2*cosh(d*x+c)/(a^2+b^2)/d^2/(a+b*sinh(d*x+c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5753 vs. $2(631) = 1262$.

Time = 6.83 (sec) , antiderivative size = 5753, normalized size of antiderivative = 9.12

$$\int \frac{(e+fx)^3 \cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 3.00 (sec) , antiderivative size = 561, normalized size of antiderivative = 0.89, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {5987, 3042, 3805, 3042, 3803, 25, 2694, 27, 2620, 3011, 2720, 6095, 2620, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$$

$\Big\downarrow$ 5987

$$\frac{3f\int \frac{(e+fx)^2}{(a+b\sinh(c+dx))^2}dx}{2bd}-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$\Big\downarrow$ 3042

$$-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}+\frac{3f\int \frac{(e+fx)^2}{(a-ib\sin(ic+idx))^2}dx}{2bd}$$

$\Big\downarrow$ 3805

$$\frac{3f\left(\frac{a\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{2bf\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)^2\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$\Big\downarrow$ 3042

$$-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}+\frac{3f\left(\frac{a\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{a^2+b^2}+\frac{2bf\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)^2\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}$$

$\Big\downarrow$ 3803

$$\frac{3f\left(\frac{2a\int -\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{2bf\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)^2\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$\Big\downarrow$ 25

$$\frac{3f\left(-\frac{2a\int \frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{2bf\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)^2\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$\Big\downarrow$ 2694

$$\frac{3f\left(-\frac{2a\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{2bf\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)^2\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$\Big\downarrow$ 27

$$\frac{3f\left(-\frac{2a\left(\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{2bf\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)^2\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$\Big\downarrow$ 2620

$$\frac{3f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{a^2+b^2}\right.}{2bd}$$

$$\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$\Big\downarrow$ 3011

$$3f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right.}{bd}\right.}{}}{a^2+b^2}\right.$$

$$\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$\downarrow$ 2720

$$3f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\ldots}{}\right.}{}}{a^2+b^2}\right.$$

$$\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$\big\downarrow$ 6095

$$3f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\,\text{lo}\ldots}{\ldots}\right.}{\ldots}\right)}{a^2+b^2}\ldots\right.}{\ldots}$$

$$\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$\big\downarrow$ 2620

$$3f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\cdots}{\cdots}\right.}{\cdots}\right)}{a^2+b^2}\cdots\right.$$

$$\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$\downarrow$ 2715

$$3f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\cdots}{\cdots}\right.}{\cdots}\right)}{a^2+b^2}\cdots\right.$$

$$\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$\Big\downarrow$ 2838

$$3f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\,\text{lo}\cdots}{\cdots}\right.}{\cdots}\right)}{a^2+b^2}\cdots\right)$$

$$\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$\Big\downarrow$ 7143

$$3f\left(\frac{2bf\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{d(a^2+b^2)}-\frac{2a}{}\right.$$

$$\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

output
```
-1/2*(e + f*x)^3/(b*d*(a + b*Sinh[c + d*x])^2) + (3*f*((2*b*f*(-1/2*(e + f
*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(
b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) +
(f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (f*Poly
Log[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/((a^2 + b^2)*d
) - (2*a*(-1/2*(b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])
/d^2))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))])/d^2))/(b*d)))/(2*Sqrt[a^2 + b^2])))/(a^2 + b^2) - (b*(e
 + f*x)^2*Cosh[c + d*x])/((a^2 + b^2)*d*(a + b*Sinh[c + d*x]))))/(2*b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

rule 5987
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(fx+e)^3 \cosh(dx+c)}{(a+b\sinh(dx+c))^3}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11757 vs. 2(575) = 1150.

Time = 0.31 (sec) , antiderivative size = 11757, normalized size of antiderivative = 18.63

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)/(a+b*sinh(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)}{(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
3*a*d*f^3*integrate(x^2*e^(d*x + c)/(a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2
*e^(2*d*x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + 2*a*b^3*d^2*e^(d*x + c) - a^2
*b^2*d^2 - b^4*d^2), x) + 6*a*d*e*f^2*integrate(x*e^(d*x + c)/(a^2*b^2*d^2
*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + 2*a
*b^3*d^2*e^(d*x + c) - a^2*b^2*d^2 - b^4*d^2), x) + 3*b*e*f^2*(a*log((b*e^
(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(d*x + c) + a + sqrt(a^2 + b^2)))/((
a^2*b^2 + b^4)*sqrt(a^2 + b^2)*d^3) - 2*(d*x + c)/((a^2*b^2 + b^4)*d^3) +
log(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - b)/((a^2*b^2 + b^4)*d^3)) - 6*a*
f^3*integrate(x*e^(d*x + c)/(a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*
x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + 2*a*b^3*d^2*e^(d*x + c) - a^2*b^2*d^2
 - b^4*d^2), x) + 6*b*f^3*integrate(x/(a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d
^2*e^(2*d*x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + 2*a*b^3*d^2*e^(d*x + c) - a
^2*b^2*d^2 - b^4*d^2), x) + 3/2*e^2*f*(2*(a*b*e^(3*d*x + 3*c) - 3*a*b*e^(d
*x + c) + b^2 + (2*a^2*e^(2*c) - b^2*e^(2*c) - 2*(a^2*d*e^(2*c) + b^2*d*e^
(2*c))*x)*e^(2*d*x))/(a^2*b^3*d^2 + b^5*d^2 + (a^2*b^3*d^2*e^(4*c) + b^5*d
^2*e^(4*c))*e^(4*d*x) + 4*(a^3*b^2*d^2*e^(3*c) + a*b^4*d^2*e^(3*c))*e^(3*d
*x) + 2*(2*a^4*b*d^2*e^(2*c) + a^2*b^3*d^2*e^(2*c) - b^5*d^2*e^(2*c))*e^(2
*d*x) - 4*(a^3*b^2*d^2*e^c + a*b^4*d^2*e^c)*e^(d*x)) + a*log((b*e^(d*x + 2
*c) + a*e^c - sqrt(a^2 + b^2)*e^c)/(b*e^(d*x + 2*c) + a*e^c + sqrt(a^2 + b
^2)*e^c))/((a^2*b + b^3)*sqrt(a^2 + b^2)*d^2)) - 2*e^3*e^(-2*d*x - 2*c)...
```

Giac [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx=\int \frac{(fx+e)^3\cosh(dx+c)}{(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)/(b*sinh(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx=\int \frac{\cosh(c+d\,x)\,\left(e+f\,x\right)^3}{\left(a+b\sinh\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x))^3,x)
```

output
```
int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx=\text{too large to display}$$

input
```
int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x)
```

output
```
(192*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt
(a**2 + b**2))*a**5*b**2*d*e*f**2*i + 16*e**(4*c + 4*d*x)*sqrt(a**2 + b**2
)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**5*b**2*f**3*i + 576*
e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2
 + b**2))*a**3*b**4*d*e*f**2*i + 240*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*at
an((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**3*b**4*f**3*i + 144*e**(
4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b
**2))*a*b**6*d**2*e**2*f*i + 384*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((
e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b**6*d*e*f**2*i + 236*e**(4*c
 + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2
))*a*b**6*f**3*i + 768*e**(3*c + 3*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*
x)*b*i + a*i)/sqrt(a**2 + b**2))*a**6*b*d*e*f**2*i + 64*e**(3*c + 3*d*x)*s
qrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**6*b*f
**3*i + 2304*e**(3*c + 3*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a
*i)/sqrt(a**2 + b**2))*a**4*b**3*d*e*f**2*i + 960*e**(3*c + 3*d*x)*sqrt(a*
*2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**4*b**3*f**3
*i + 576*e**(3*c + 3*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/
sqrt(a**2 + b**2))*a**2*b**5*d**2*e**2*f*i + 1536*e**(3*c + 3*d*x)*sqrt(a*
*2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b**5*d*e*
f**2*i + 944*e**(3*c + 3*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i ...
```

3.333 $\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 448

$$\begin{aligned}
&\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= \frac{a(e+fx)^4}{4b^2f}-\frac{6f^3\cosh(c+dx)}{bd^4}-\frac{3f(e+fx)^2\cosh(c+dx)}{bd^2} \\
&\quad-\frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d} \\
&\quad-\frac{3af(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2}-\frac{3af(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^2} \\
&\quad+\frac{6af^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3}+\frac{6af^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^3} \\
&\quad-\frac{6af^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^4}-\frac{6af^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^4} \\
&\quad+\frac{6f^2(e+fx)\sinh(c+dx)}{bd^3}+\frac{(e+fx)^3\sinh(c+dx)}{bd}
\end{aligned}$$

output
```
1/4*a*(f*x+e)^4/b^2/f-6*f^3*cosh(d*x+c)/b/d^4-3*f*(f*x+e)^2*cosh(d*x+c)/b/
d^2-a*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d-a*(f*x+e)^3*l
n(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d-3*a*f*(f*x+e)^2*polylog(2,-b*e
xp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d^2-3*a*f*(f*x+e)^2*polylog(2,-b*exp(d*
x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d^2+6*a*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/
(a-(a^2+b^2)^(1/2)))/b^2/d^3+6*a*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a
^2+b^2)^(1/2)))/b^2/d^3-6*a*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)
))/b^2/d^4-6*a*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d^4+6*
f^2*(f*x+e)*sinh(d*x+c)/b/d^3+(f*x+e)^3*sinh(d*x+c)/b/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2809 vs. 2(448) = 896.

Time = 9.37 (sec) , antiderivative size = 2809, normalized size of antiderivative = 6.27

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
((a*(4*e^3*E^(2*c)*x + 6*e^2*E^(2*c)*f*x^2 + 4*e*E^(2*c)*f^2*x^3 + E^(2*c)
*f^3*x^4 + (4*a*Sqrt[a^2 + b^2]*e^3*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 -
 b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (4*a*Sqrt[-a^2 - b^2]*e^3*ArcTanh[(a +
b*E^(c + d*x))/Sqrt[a^2 + b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) - (2*e^3*E^(2*c)
*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))])/d + (2*e^3*Log[2*a*E^(c + d
*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/
(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e^2*E^(2*c)*f*x*Log[1 + (b*E^
(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f^2*x^2*Log[1
+ (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)
*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d
 + (2*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]
)])/d - (2*E^(2*c)*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 +
b^2)*E^(2*c)])])/d + (6*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a
^2 + b^2)*E^(2*c)])])/d - (6*e^2*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*
E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f^2*x^2*Log[1 + (b*E^(2*c + d*
x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f^2*x^2*Log[1 +
 (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (2*f^3*x^3*Lo
g[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (2*E^(2*
c)*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])
/d - (6*(-1 + E^(2*c))*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.57 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.01, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.594$, Rules used = {6113, 3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118, 6095, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3 \sinh(c+dx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6113}$$

$$\frac{\int (e+fx)^3\cosh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^3\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b}$$

$$\Big\downarrow \ 3777$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3if\int -i(e+fx)^2\sinh(c+dx)dx}{d}}{b}$$

$$\Big\downarrow \ 26$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int (e+fx)^2\sinh(c+dx)dx}{d}}{b}-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int -i(e+fx)^2\sin(ic+idx)dx}{d}}{b}$$

$$\Big\downarrow \ 26$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\int (e+fx)^2\sin(ic+idx)dx}{d}}{b}$$

$$\Big\downarrow \ 3777$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\cosh(c+dx)dx}{d}\right)}{d}}{b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}}{b}$$

$$\Big\downarrow \ 3777$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{d}\right)}{d}}{b}$$

$$\Big\downarrow \ 26$$

$$\frac{-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)}{d}\right)}{d}}{b}}{}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}\right)}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{3118}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{6095}$$

$$-\frac{a\left(\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^4}{4bf}\right)}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{a\left(-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c}\cdots}{\sqrt{a^2\cdots}}\right.}{bd}\cdots\right.}{b}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

$\Big\downarrow$ 3011

$$-\frac{a\left(-\frac{3f\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\cdots}{\cdots}\right.}{bd}\cdots\right.}{b}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

$\Big\downarrow$ 7163

$$-\frac{a\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int \operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{Pol}\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\cdots\right.}{\cdots}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

$\Big\downarrow$ 2720

$$-\frac{a\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(2\ldots\right.}{\ldots}\right.}{\ldots}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

$\Big\downarrow$ 7143

$$-\frac{a\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLo}\ldots}{}\right.}{}\right.}{\ldots}\right.}{\ldots}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-((a*(-1/4*(e + f*x)^4/(b*f) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 + b^2])])/(b*d) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2]))])/d^2))/d))/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/b) + (((e + f*x)^3*Sinh[c + d*x])/d +
 ((3*I)*f*((I*(e + f*x)^2*Cosh[c + d*x])/d - ((2*I)*f*(-((f*Cosh[c + d*x])
/d^2) + ((e + f*x)*Sinh[c + d*x])/d))/d))/d)/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(fx+e)^3 \cosh(dx+c) \sinh(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1976 vs. $2(420) = 840$.

Time = 0.14 (sec) , antiderivative size = 1976, normalized size of antiderivative = 4.41

$$\int \frac{(e+fx)^3 \cosh(c+dx) \sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
-1/4*(2*b*d^3*f^3*x^3 + 2*b*d^3*e^3 + 6*b*d^2*e^2*f + 12*b*d*e*f^2 + 12*b*
f^3 + 6*(b*d^3*e*f^2 + b*d^2*f^3)*x^2 - 2*(b*d^3*f^3*x^3 + b*d^3*e^3 - 3*b
*d^2*e^2*f + 6*b*d*e*f^2 - 6*b*f^3 + 3*(b*d^3*e*f^2 - b*d^2*f^3)*x^2 + 3*(
b*d^3*e^2*f - 2*b*d^2*e*f^2 + 2*b*d*f^3)*x)*cosh(d*x + c)^2 - 2*(b*d^3*f^3
*x^3 + b*d^3*e^3 - 3*b*d^2*e^2*f + 6*b*d*e*f^2 - 6*b*f^3 + 3*(b*d^3*e*f^2
- b*d^2*f^3)*x^2 + 3*(b*d^3*e^2*f - 2*b*d^2*e*f^2 + 2*b*d*f^3)*x)*sinh(d*x
 + c)^2 + 6*(b*d^3*e^2*f + 2*b*d^2*e*f^2 + 2*b*d*f^3)*x - (a*d^4*f^3*x^4 +
 4*a*d^4*e*f^2*x^3 + 6*a*d^4*e^2*f*x^2 + 4*a*d^4*e^3*x + 8*a*c*d^3*e^3 - 1
2*a*c^2*d^2*e^2*f + 8*a*c^3*d*e*f^2 - 2*a*c^4*f^3)*cosh(d*x + c) + 12*((a*
d^2*f^3*x^2 + 2*a*d^2*e*f^2*x + a*d^2*e^2*f)*cosh(d*x + c) + (a*d^2*f^3*x^
2 + 2*a*d^2*e*f^2*x + a*d^2*e^2*f)*sinh(d*x + c))*dilog((a*cosh(d*x + c) +
 a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^
2) - b)/b + 1) + 12*((a*d^2*f^3*x^2 + 2*a*d^2*e*f^2*x + a*d^2*e^2*f)*cosh(
d*x + c) + (a*d^2*f^3*x^2 + 2*a*d^2*e*f^2*x + a*d^2*e^2*f)*sinh(d*x + c))*
dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x +
 c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 4*((a*d^3*e^3 - 3*a*c*d^2*e^2*f +
 3*a*c^2*d*e*f^2 - a*c^3*f^3)*cosh(d*x + c) + (a*d^3*e^3 - 3*a*c*d^2*e^2*f
 + 3*a*c^2*d*e*f^2 - a*c^3*f^3)*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b
*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 4*((a*d^3*e^3 - 3*a*c*
d^2*e^2*f + 3*a*c^2*d*e*f^2 - a*c^3*f^3)*cosh(d*x + c) + (a*d^3*e^3 - 3...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-1/2*e^3*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d) +
 2*a*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^2*d)) - 1/4*(a*d^4
*f^3*x^4*e^c + 4*a*d^4*e*f^2*x^3*e^c + 6*a*d^4*e^2*f*x^2*e^c - 2*(b*d^3*f^
3*x^3*e^(2*c) + 3*(d^3*e*f^2 - d^2*f^3)*b*x^2*e^(2*c) + 3*(d^3*e^2*f - 2*d
^2*e*f^2 + 2*d*f^3)*b*x*e^(2*c) - 3*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*b*e^(2
*c))*e^(d*x) + 2*(b*d^3*f^3*x^3 + 3*(d^3*e*f^2 + d^2*f^3)*b*x^2 + 3*(d^3*e
^2*f + 2*d^2*e*f^2 + 2*d*f^3)*b*x + 3*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*b)*e
^(-d*x))*e^(-c)/(b^2*d^4) + integrate(-2*(a*b*f^3*x^3 + 3*a*b*e*f^2*x^2 +
3*a*b*e^2*f*x - (a^2*f^3*x^3*e^c + 3*a^2*e*f^2*x^2*e^c + 3*a^2*e^2*f*x*e^c
)*e^(d*x))/(b^3*e^(2*d*x + 2*c) + 2*a*b^2*e^(d*x + c) - b^3), x)
```

Giac [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)*sinh(d*x + c)/(b*sinh(d*x + c) + a), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)\,\left(e+f\,x\right)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)*sinh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*sinh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-3\cosh\left(dx+c\right)b\,d^2e^2f-6\cosh\left(dx+c\right)b\,d^2e\,f^2x-3\cosh\left(dx+c\right)b\,d^2f^3x^2-6\cosh\left(dx+c\right)b\,f^3-e}{}$$

input
```
int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 3*cosh(c + d*x)*b*d**2*e**2*f - 6*cosh(c + d*x)*b*d**2*e*f**2*x - 3*co
sh(c + d*x)*b*d**2*f**3*x**2 - 6*cosh(c + d*x)*b*f**3 - e**(2*c)*int((e**(
2*d*x)*x**3)/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a*b*d**4*f**3
- 3*e**(2*c)*int((e**(2*d*x)*x**2)/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a
- b),x)*a*b*d**4*e*f**2 - 3*e**(2*c)*int((e**(2*d*x)*x)/(e**(2*c + 2*d*x)*
b + 2*e**(c + d*x)*a - b),x)*a*b*d**4*e**2*f - int(x**3/(e**(2*c + 2*d*x)*
b + 2*e**(c + d*x)*a - b),x)*a*b*d**4*f**3 - 3*int(x**2/(e**(2*c + 2*d*x)*
b + 2*e**(c + d*x)*a - b),x)*a*b*d**4*e*f**2 - 3*int(x/(e**(2*c + 2*d*x)*b
 + 2*e**(c + d*x)*a - b),x)*a*b*d**4*e**2*f - log(sinh(c + d*x)*b + a)*a*d
**3*e**3 + sinh(c + d*x)*b*d**3*e**3 + 3*sinh(c + d*x)*b*d**3*e**2*f*x + 3
*sinh(c + d*x)*b*d**3*e*f**2*x**2 + sinh(c + d*x)*b*d**3*f**3*x**3 + 6*sin
h(c + d*x)*b*d*e*f**2 + 6*sinh(c + d*x)*b*d*f**3*x)/(b**2*d**4)
```

3.334 $\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 330

$$\begin{aligned}\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{a(e+fx)^3}{3b^2f}-\frac{2f(e+fx)\cosh(c+dx)}{bd^2} \\ &- \frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d} \\ &- \frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d} \\ &- \frac{2af(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2} \\ &- \frac{2af(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^2} \\ &+ \frac{2af^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3} \\ &+ \frac{2af^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^3} \\ &+ \frac{2f^2\sinh(c+dx)}{bd^3} \\ &+ \frac{(e+fx)^2\sinh(c+dx)}{bd}\end{aligned}$$

output
```
1/3*a*(f*x+e)^3/b^2/f-2*f*(f*x+e)*cosh(d*x+c)/b/d^2-a*(f*x+e)^2*ln(1+b*exp
(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d-a*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b
^2)^(1/2)))/b^2/d-2*a*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)
))/b^2/d^2-2*a*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/
d^2+2*a*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d^3+2*a*f^2*p
olylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d^3+2*f^2*sinh(d*x+c)/b/d^
3+(f*x+e)^2*sinh(d*x+c)/b/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1301 vs. $2(330) = 660$.

Time = 9.15 (sec) , antiderivative size = 1301, normalized size of antiderivative = 3.94

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
((2*a*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt
[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2
+ b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c +
d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]
*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d)
+ (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a
^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 +
E^(2*(c + d*x)))])/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(
c + d*x)))])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b
^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sq
rt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c
 - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c +
 d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f*x*Log[1 + (b*E^(2*
c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1
 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*
Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(
2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])
])/d - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c
 - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyL
og[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - ...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.67 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6113, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117, 6095, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6113}$$

$$\frac{\int (e+fx)^2\cosh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b}$$

$\Big\downarrow$ 3777

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}}{b}$$

$\Big\downarrow$ 26

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int (e+fx)\sinh(c+dx)dx}{d}}{b}-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$\Big\downarrow$ 3042

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int -i(e+fx)\sin(ic+idx)dx}{d}}{b}$$

$\Big\downarrow$ 26

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int (e+fx)\sin(ic+idx)dx}{d}}{b}$$

$\Big\downarrow$ 3777

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}\right)}{d}}{b}$$

$\Big\downarrow$ 3042

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}}{b}$$

$\Big\downarrow$ 3117

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}}{b}$$

$\Big\downarrow$ 6095

$$-\frac{a\left(\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}}{b}$$

$\Big\downarrow$ 2620

$$-\frac{a\left(-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+\cdots}}{\sqrt{a^2+b^{\cdots}}}\cdots\right.}{bd}\cdots\right.}{b}$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}}{b}$$

$\downarrow$ 3011

$$-\frac{a\left(-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{\cdots}}{a+\cdots}\right.}{d}\cdots\right.}{bd}\cdots\right.}{b}$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}}{b}$$

$\downarrow$ 2720

$$-\frac{a\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\cdots\right.}{bd}\cdots\right.}{b}$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}}{b}$$

$\downarrow$ 7143

$$-\frac{a\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right.}{d}\cdots\right.}{bd}\cdots\right.}{b}$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}}{b}$$

input

```
Int[((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-((a*(-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2]))])/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2]))])/d^2))/(b*d)))/b) + (((e + f*x)^2*Sinh[c + d*x])/d + ((2*I)*f*((I*
(e + f*x)*Cosh[c + d*x])/d - (I*f*Sinh[c + d*x])/d^2))/d)/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(fx+e)^2 \cosh (dx+c) \sinh (dx+c)}{a+b\sinh (dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1265 vs. $2(308) = 616$.

Time = 0.11 (sec) , antiderivative size = 1265, normalized size of antiderivative = 3.83

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
-1/6*(3*b*d^2*f^2*x^2 + 3*b*d^2*e^2 + 6*b*d*e*f + 6*b*f^2 - 3*(b*d^2*f^2*x
^2 + b*d^2*e^2 - 2*b*d*e*f + 2*b*f^2 + 2*(b*d^2*e*f - b*d*f^2)*x)*cosh(d*x
 + c)^2 - 3*(b*d^2*f^2*x^2 + b*d^2*e^2 - 2*b*d*e*f + 2*b*f^2 + 2*(b*d^2*e*
f - b*d*f^2)*x)*sinh(d*x + c)^2 + 6*(b*d^2*e*f + b*d*f^2)*x - 2*(a*d^3*f^2
*x^3 + 3*a*d^3*e*f*x^2 + 3*a*d^3*e^2*x + 6*a*c*d^2*e^2 - 6*a*c^2*d*e*f + 2
*a*c^3*f^2)*cosh(d*x + c) + 12*((a*d*f^2*x + a*d*e*f)*cosh(d*x + c) + (a*d
*f^2*x + a*d*e*f)*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c)
+ (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) +
12*((a*d*f^2*x + a*d*e*f)*cosh(d*x + c) + (a*d*f^2*x + a*d*e*f)*sinh(d*x +
 c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 6*((a*d^2*e^2 - 2*a*c*d*e*f
+ a*c^2*f^2)*cosh(d*x + c) + (a*d^2*e^2 - 2*a*c*d*e*f + a*c^2*f^2)*sinh(d*
x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b
^2) + 2*a) + 6*((a*d^2*e^2 - 2*a*c*d*e*f + a*c^2*f^2)*cosh(d*x + c) + (a*d
^2*e^2 - 2*a*c*d*e*f + a*c^2*f^2)*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2
*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 6*((a*d^2*f^2*x^2 +
2*a*d^2*e*f*x + 2*a*c*d*e*f - a*c^2*f^2)*cosh(d*x + c) + (a*d^2*f^2*x^2 +
2*a*d^2*e*f*x + 2*a*c*d*e*f - a*c^2*f^2)*sinh(d*x + c))*log(-(a*cosh(d*x +
 c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^
2)/b^2) - b)/b) + 6*((a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + 2*a*c*d*e*f - a*c...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\cosh(dx+c)\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-1/2*e^2*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d) +
 2*a*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^2*d)) - 1/6*(2*a*d
^3*f^2*x^3*e^c + 6*a*d^3*e*f*x^2*e^c - 3*(b*d^2*f^2*x^2*e^(2*c) + 2*(d^2*e
*f - d*f^2)*b*x*e^(2*c) - 2*(d*e*f - f^2)*b*e^(2*c))*e^(d*x) + 3*(b*d^2*f^
2*x^2 + 2*(d^2*e*f + d*f^2)*b*x + 2*(d*e*f + f^2)*b)*e^(-d*x))*e^(-c)/(b^2
*d^3) + integrate(-2*(a*b*f^2*x^2 + 2*a*b*e*f*x - (a^2*f^2*x^2*e^c + 2*a^2
*e*f*x*e^c)*e^(d*x))/(b^3*e^(2*d*x + 2*c) + 2*a*b^2*e^(d*x + c) - b^3), x)
```

Giac [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)*sinh(d*x + c)/(b*sinh(d*x + c) + a), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)\,(e+f\,x)^2}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)*sinh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*sinh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-2\cosh(dx+c)\,bdef - 2\cosh(dx+c)\,bd\,f^2x - e^{2c}\left(\int \frac{e^{2dx}x^2}{e^{2dx+2c}b+2e^{dx+c}a-b}dx\right)ab\,d^3f^2 - 2e^{2c}\left(\int \frac{e^{2dx}}{e^{2dx+2c}b+2e}\right.}{}$$

input
```
int((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*cosh(c + d*x)*b*d*e*f - 2*cosh(c + d*x)*b*d*f**2*x - e**(2*c)*int((e
**(2*d*x)*x**2)/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a*b*d**3*f*
*2 - 2*e**(2*c)*int((e**(2*d*x)*x)/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a
- b),x)*a*b*d**3*e*f - int(x**2/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b
),x)*a*b*d**3*f**2 - 2*int(x/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x
)*a*b*d**3*e*f - log(sinh(c + d*x)*b + a)*a*d**2*e**2 + sinh(c + d*x)*b*d*
*2*e**2 + 2*sinh(c + d*x)*b*d**2*e*f*x + sinh(c + d*x)*b*d**2*f**2*x**2 +
2*sinh(c + d*x)*b*f**2)/(b**2*d**3)
```

3.335 $\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 212

$$\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{a(e+fx)^2}{2b^2f} - \frac{f\cosh(c+dx)}{bd^2} - \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d} - \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d} - \frac{af\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2} - \frac{af\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^2} + \frac{(e+fx)\sinh(c+dx)}{bd}$$

output
```
1/2*a*(f*x+e)^2/b^2/f-f*cosh(d*x+c)/b/d^2-a*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(
a^2+b^2)^(1/2)))/b^2/d-a*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^
2/d-a*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d^2-a*f*polylog(2
,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d^2+(f*x+e)*sinh(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 2.40 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.78

$$\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-2bf\cosh(c+dx) - a\left(-2de(c+dx) + 2cf(c+dx) - f(c+dx)^2 + \frac{4a\sqrt{a^2+b^2}de\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-(a^2+b^2)^2}} - \frac{4a\sqrt{-(\cdots}}{\cdots}\right.}{\cdots}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-2*b*f*Cosh[c + d*x] - a*(-2*d*e*(c + d*x) + 2*c*f*(c + d*x) - f*(c + d*x
)^2 + (4*a*Sqrt[a^2 + b^2]*d*e*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]
])/Sqrt[-(a^2 + b^2)^2] - (4*a*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(
c + d*x))/Sqrt[a^2 + b^2]])/(-a^2 - b^2)^(3/2) + 2*f*(c + d*x)*Log[1 + (b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2])] - 2*c*f*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x
))] + 2*d*e*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] + 2*f*PolyLog[
2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f*PolyLog[2, -((b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2]))]) + 2*b*d*(e + f*x)*Sinh[c + d*x])/(2*b^2*d^2)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {6113, 3042, 3777, 26, 3042, 26, 3118, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\Big\downarrow \; 6113$$
$$\frac{\int (e+fx)\cosh(c+dx)dx}{b} - \frac{a\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{a\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}}{b}-\frac{a\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}}{b}$$

$$\Big\downarrow \text{3118}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b}-\frac{a\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{6095}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b}-\frac{a\left(\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{b}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b}-\frac{a\left(-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right.}{b}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b}-a\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be}{\sqrt{a^2}}\right.}{bd}}{b}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b}-a\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right.}{b}$$

input
```
Int[((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-((a*(-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/
(b*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)
))/b) + (-((f*Cosh[c + d*x])/d^2) + ((e + f*x)*Sinh[c + d*x])/d)/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(198) = 396$.

Time = 1.76 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.28

method	result
risch	$\frac{af\,x^2}{2b^2} - \frac{aex}{b^2} + \frac{(dxf+de-f)\mathrm{e}^{dx+c}}{2b\,d^2} - \frac{(dxf+de+f)\mathrm{e}^{-dx-c}}{2b\,d^2} - \frac{af\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)x}{d\,b^2} - \frac{af\ln\left(\frac{b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}+a}{a+\sqrt{a^2+b^2}}\right)x}{d\,b^2}$

input
```
int((f*x+e)*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERB
OSE)
```

output
```
1/2*a/b^2*f*x^2-a/b^2*e*x+1/2*(d*f*x+d*e-f)/b/d^2*exp(d*x+c)-1/2*(d*f*x+d*
e+f)/b/d^2*exp(-d*x-c)-1/d*a/b^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-
a+(a^2+b^2)^(1/2)))*x-1/d*a/b^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(
a^2+b^2)^(1/2)))*x-1/d^2*a/b^2*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(
-a+(a^2+b^2)^(1/2)))-1/d^2*a/b^2*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/
(a+(a^2+b^2)^(1/2)))+1/d^2*a/b^2*f*c^2-1/d^2*a/b^2*f*ln((-b*exp(d*x+c)+(a^
2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2/d^2*a/b^2*c*f*ln(exp(d*x+c))+1/d
^2*a/b^2*c*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/d^2*a/b^2*f*ln((b*exp
(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2/d*a/b^2*e*ln(exp(d*x+c
))+2/d*a/b^2*f*c*x-1/d*a/b^2*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 692 vs. $2(196) = 392$.

Time = 0.10 (sec) , antiderivative size = 692, normalized size of antiderivative = 3.26

$$\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
fricas")
```

output
```
-1/2*(b*d*f*x + b*d*e - (b*d*f*x + b*d*e - b*f)*cosh(d*x + c)^2 - (b*d*f*x
 + b*d*e - b*f)*sinh(d*x + c)^2 + b*f - (a*d^2*f*x^2 + 2*a*d^2*e*x + 4*a*c
*d*e - 2*a*c^2*f)*cosh(d*x + c) + 2*(a*f*cosh(d*x + c) + a*f*sinh(d*x + c)
)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
 + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(a*f*cosh(d*x + c) + a*f*sinh
(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*((a*d*e - a*c*f)*cos
h(d*x + c) + (a*d*e - a*c*f)*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*si
nh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*((a*d*e - a*c*f)*cosh(d
*x + c) + (a*d*e - a*c*f)*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(
d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*((a*d*f*x + a*c*f)*cosh(d*
x + c) + (a*d*f*x + a*c*f)*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d
*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b
) + 2*((a*d*f*x + a*c*f)*cosh(d*x + c) + (a*d*f*x + a*c*f)*sinh(d*x + c))*
log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (a*d^2*f*x^2 + 2*a*d^2*e*x + 4*a*c*d*e
 - 2*a*c^2*f + 2*(b*d*f*x + b*d*e - b*f)*cosh(d*x + c))*sinh(d*x + c))/(b^
2*d^2*cosh(d*x + c) + b^2*d^2*sinh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
maxima")
```

output
```
-1/2*e*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d) + 2
*a*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^2*d)) - 1/4*f*(2*(a*
d^2*x^2*e^c - (b*d*x*e^(2*c) - b*e^(2*c))*e^(d*x) + (b*d*x + b)*e^(-d*x))*
e^(-c)/(b^2*d^2) - integrate(8*(a^2*x*e^(d*x + c) - a*b*x)/(b^3*e^(2*d*x +
 2*c) + 2*a*b^2*e^(d*x + c) - b^3), x))
```

Giac [F]

$$\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)*sinh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*sinh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{-\cosh(dx+c)\,bf - e^{2c}\left(\int \frac{e^{2dx}x}{e^{2dx+2c}b+2e^{dx+c}a-b}dx\right)ab\,d^2f - \left(\int \frac{x}{e^{2dx+2c}b+2e^{dx+c}a-b}dx\right)ab\,d^2f - \log(a+b\sin}{b^2d^2}$$

input
```
int((f*x+e)*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - cosh(c + d*x)*b*f - e**(2*c)*int((e**(2*d*x)*x)/(e**(2*c + 2*d*x)*b +
2*e**(c + d*x)*a - b),x)*a*b*d**2*f - int(x/(e**(2*c + 2*d*x)*b + 2*e**(c
+ d*x)*a - b),x)*a*b*d**2*f - log(sinh(c + d*x)*b + a)*a*d*e + sinh(c + d*
x)*b*d*e + sinh(c + d*x)*b*d*f*x)/(b**2*d**2)
```

3.336 $\int \frac{\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 34

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a\log(a+b\sinh(c+dx))}{b^2d}+\frac{\sinh(c+dx)}{bd}$$

output
```
-a*ln(a+b*sinh(d*x+c))/b^2/d+sinh(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{a\log(a+b\sinh(c+dx))}{b^2}-\frac{\sinh(c+dx)}{b}}{d}$$

input
```
Integrate[(Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-(((a*Log[a + b*Sinh[c + d*x]])/b^2 - Sinh[c + d*x]/b)/d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 26, 3312, 26, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ic+idx)\cos(ic+idx)}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos(ic+idx)\sin(ic+idx)}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3312}$$

$$-\frac{i\int \frac{i\sinh(c+dx)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{bd}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b\sinh(c+dx)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^2d}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(1-\frac{a}{a+b\sinh(c+dx)}\right)d(b\sinh(c+dx))}{b^2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\sinh(c+dx)-a\log(a+b\sinh(c+dx))}{b^2d}$$

input
```
Int[(Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-(a*Log[a + b*Sinh[c + d*x]]) + b*Sinh[c + d*x])/(b^2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$\frac{\frac{\sinh(dx+c)}{b}-\frac{a\ln(a+b\sinh(dx+c))}{b^2}}{d}$	33
default	$\frac{\frac{\sinh(dx+c)}{b}-\frac{a\ln(a+b\sinh(dx+c))}{b^2}}{d}$	33
risch	$\frac{ax}{b^2}+\frac{e^{dx+c}}{2bd}-\frac{e^{-dx-c}}{2bd}+\frac{2ac}{b^2d}-\frac{a\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{b}-1\right)}{b^2d}$	82

input
```
int(cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b*sinh(d*x+c)-a/b^2*ln(a+b*sinh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(34) = 68.

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 3.88

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\frac{2\,adx\cosh\left(dx+c\right)+b\cosh\left(dx+c\right)^2+b\sinh\left(dx+c\right)^2-2\left(a\cosh\left(dx+c\right)+a\sinh\left(dx+c\right)\right)\log\left(\frac{2(}{\cosh(}\right.}{2\left(b^2d\cosh\left(dx+c\right)+b^2d\sinh\left(dx+c\right)\right)}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*a*d*x*cosh(d*x + c) + b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 - 2*(a*
cosh(d*x + c) + a*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c
) - sinh(d*x + c))) + 2*(a*d*x + b*cosh(d*x + c))*sinh(d*x + c) - b)/(b^2*
d*cosh(d*x + c) + b^2*d*sinh(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(27) = 54$.

Time = 1.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.91

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \begin{cases} \frac{x\sinh(c)\cosh(c)}{a} & \text{for } b=0 \wedge d=0 \\ \frac{\sinh^2(c+dx)}{2ad} & \text{for } b=0 \\ \frac{x\sinh(c)\cosh(c)}{a+b\sinh(c)} & \text{for } d=0 \\ -\frac{a\log\left(\frac{a}{b}+\sinh(c+dx)\right)}{b^2d} + \frac{\sinh(c+dx)}{bd} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Piecewise((x*sinh(c)*cosh(c)/a, Eq(b, 0) & Eq(d, 0)), (sinh(c + d*x)**2/(2
*a*d), Eq(b, 0)), (x*sinh(c)*cosh(c)/(a + b*sinh(c)), Eq(d, 0)), (-a*log(a
/b + sinh(c + d*x))/(b**2*d) + sinh(c + d*x)/(b*d), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(34) = 68$.

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.44

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{(dx+c)a}{b^2d} + \frac{e^{(dx+c)}}{2\,bd} - \frac{e^{(-dx-c)}}{2\,bd} - \frac{a\log\left(-2\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)} - b\right)}{b^2d}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-(d*x + c)*a/(b^2*d) + 1/2*e^(d*x + c)/(b*d) - 1/2*e^(-d*x - c)/(b*d) - a*
log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^2*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.76

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{e^{(dx+c)}-e^{(-dx-c)}}{b} - \frac{2\,a\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{b^2}}{2\,d}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*((e^(d*x + c) - e^(-d*x - c))/b - 2*a*log(abs(b*(e^(d*x + c) - e^(-d*x
 - c)) + 2*a))/b^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a\,\ln\left(a+b\sinh(c+d\,x)\right)-b\sinh(c+d\,x)}{b^2\,d}$$

input
```
int((cosh(c + d*x)*sinh(c + d*x))/(a + b*sinh(c + d*x)),x)
```

output
```
-(a*log(a + b*sinh(c + d*x)) - b*sinh(c + d*x))/(b^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{-\log(a+b\sinh\left(dx+c\right))\,a+b\sinh\left(dx+c\right)}{b^2d}$$

input
```
int(cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - log(sinh(c + d*x)*b + a)*a + sinh(c + d*x)*b)/(b**2*d)
```

3.337 $\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 20.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Cosh[c + d*x]*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
x]
```

output
```
Integrate[(Cosh[c + d*x]*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
 x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{6125}$$
$$\int \frac{\sinh(c+dx)\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Cosh[c + d*x]*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(dx+c)\sinh(dx+c)}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)\sinh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
fricas")
```

output
```
integral(cosh(d*x + c)*sinh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x
 + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 164, normalized size of antiderivative = 5.12

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)\sinh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
maxima")
```

output
```
-1/2*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b*f) - 1/2*e^(c - d*
e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b*f) - a*log(f*x + e)/(b^2*f) + 1/
4*integrate(-8*(a^2*e^(d*x + c) - a*b)/(b^3*f*x + b^3*e - (b^3*f*x*e^(2*c)
 + b^3*e*e^(2*c))*e^(2*d*x) - 2*(a*b^2*f*x*e^c + a*b^2*e*e^c)*e^(d*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)\sinh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
giac")
```

output
```
integrate(cosh(d*x + c)*sinh(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x
)
```

Mupad [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)}{(e+f\,x)\,\left(a+b\sinh(c+d\,x)\right)}\,dx$$

input
```
int((cosh(c + d*x)*sinh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((cosh(c + d*x)*sinh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 163, normalized size of antiderivative = 5.09

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$= \frac{e^{4c}\left(\int \frac{e^{3dx}}{e^{2dx+2c}be+e^{2dx+2c}bfx+2e^{dx+c}ae+2e^{dx+c}afx-be-bfx}dx\right)-\left(\int \frac{1}{e^{3dx+2c}be+e^{3dx+2c}bfx+2e^{2dx+c}ae+2e^{2dx+c}afx-e^{dx}be-e^{d\cdots}}\right.}{2e^{c}}$$

input
```
int(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
(e**(4*c)*int(e**(3*d*x)/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x +
2*e**(c + d*x)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x) - int(1/(e**(2
*c + 3*d*x)*b*e + e**(2*c + 3*d*x)*b*f*x + 2*e**(c + 2*d*x)*a*e + 2*e**(c
+ 2*d*x)*a*f*x - e**(d*x)*b*e - e**(d*x)*b*f*x),x))/(2*e**c)
```

3.338 $\int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx$

Optimal result

Integrand size = 34, antiderivative size = 683

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{3f(e+fx)^2}{8bd^2}+\frac{a^2(e+fx)^4}{4b^3f}+\frac{(e+fx)^4}{8bf}-\frac{6af^2(e+fx)\cosh(c+dx)}{b^2d^3}$$
$$-\frac{a(e+fx)^3\cosh(c+dx)}{b^2d}-\frac{3f^3\cosh^2(c+dx)}{8bd^4}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4bd^2}$$
$$-\frac{a\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d}+\frac{a\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d}$$
$$-\frac{3a\sqrt{a^2+b^2}f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}$$
$$+\frac{3a\sqrt{a^2+b^2}f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^2}$$
$$+\frac{6a\sqrt{a^2+b^2}f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3}$$
$$-\frac{6a\sqrt{a^2+b^2}f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^3}$$
$$-\frac{6a\sqrt{a^2+b^2}f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^4}+\frac{6a\sqrt{a^2+b^2}f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^4}$$
$$+\frac{6af^3\sinh(c+dx)}{b^2d^4}+\frac{3af(e+fx)^2\sinh(c+dx)}{b^2d^2}$$
$$+\frac{3f^2(e+fx)\cosh(c+dx)\sinh(c+dx)}{4bd^3}+\frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{2bd}$$

output
```
3/8*f*(f*x+e)^2/b/d^2+1/4*a^2*(f*x+e)^4/b^3/f+1/8*(f*x+e)^4/b/f-6*a*f^2*(f
*x+e)*cosh(d*x+c)/b^2/d^3-a*(f*x+e)^3*cosh(d*x+c)/b^2/d-3/8*f^3*cosh(d*x+c
)^2/b/d^4-3/4*f*(f*x+e)^2*cosh(d*x+c)^2/b/d^2-a*(a^2+b^2)^(1/2)*(f*x+e)^3*
ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d+a*(a^2+b^2)^(1/2)*(f*x+e)^3*l
n(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d-3*a*(a^2+b^2)^(1/2)*f*(f*x+e)^
2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^2+3*a*(a^2+b^2)^(1/2)
*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^2+6*a*(a^2
+b^2)^(1/2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d
^3-6*a*(a^2+b^2)^(1/2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1
/2)))/b^3/d^3-6*a*(a^2+b^2)^(1/2)*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)
^(1/2)))/b^3/d^4+6*a*(a^2+b^2)^(1/2)*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b
^2)^(1/2)))/b^3/d^4+6*a*f^3*sinh(d*x+c)/b^2/d^4+3*a*f*(f*x+e)^2*sinh(d*x+c
)/b^2/d^2+3/4*f^2*(f*x+e)*cosh(d*x+c)*sinh(d*x+c)/b/d^3+1/2*(f*x+e)^3*cosh
(d*x+c)*sinh(d*x+c)/b/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1971 vs. 2(683) = 1366.

Time = 8.90 (sec) , antiderivative size = 1971, normalized size of antiderivative = 2.89

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
(4*b^2*e^3*(c/d + x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^
2]])/(Sqrt[-a^2 - b^2]*d)) + 6*b^2*e^2*f*(x^2 - (2*a*(d*x*(Log[1 + (b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2])] - Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2])]) + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - PolyLog[2,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(Sqrt[a^2 + b^2]*d^2)) + 4*b^2
*e*f^2*(x^3 - (3*a*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])]
 - d^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[
2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*x*PolyLog[2, -((b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2]))] - 2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^
2 + b^2])] + 2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(Sqr
t[a^2 + b^2]*d^3)) + b^2*f^3*(x^4 - (4*a*(d^3*x^3*Log[1 + (b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2])] - d^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2])] + 3*d^2*x^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 3*d
^2*x^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*d*x*PolyLo
g[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 6*d*x*PolyLog[3, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))] + 6*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[
a^2 + b^2])] - 6*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(S
qrt[a^2 + b^2]*d^4)) + 2*e*f^2*(2*(4*a^2 + b^2)*x^3 - (6*a*(4*a^2 + 3*b^2)
*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^2*x^2*Log[1 +
 (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2, (b*E^(c + d*...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 3.88 (sec) , antiderivative size = 641, normalized size of antiderivative = 0.94, number of steps used = 30, number of rules used = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.853$, Rules used = {6113, 3042, 3792, 17, 3042, 3791, 17, 6099, 17, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117, 3803, 25, 2694, 27, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3 \sinh(c+dx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \; 6113$$

$$\frac{\int (e+fx)^3\cosh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^3\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx}{b}$$

$$\Big\downarrow \; 3792$$

$$\frac{\frac{3f^2\int (e+fx)\cosh^2(c+dx)dx}{2d^2}+\frac{1}{2}\int (e+fx)^3dx-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 17$$

$$\frac{\frac{3f^2\int (e+fx)\cosh^2(c+dx)dx}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{3f^2\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}$$

$$\Big\downarrow \; 3791$$

$$\frac{\frac{3f^2\left(\frac{1}{2}\int (e+fx)dx-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 17$$

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 6099$$

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)^3dx}{b^2}+\frac{\int(e+fx)^3\sinh(c+dx)dx}{b}\right)}{b}$$

$\Big\downarrow$ 17

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int(e+fx)^3\sinh(c+dx)dx}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}+\frac{\int -i(e+fx)^3\sin(ic+idx)dx}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{b}$$

$\Big\downarrow$ 26

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\int(e+fx)^3\sin(ic+idx)dx}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{b}$$

$\Big\downarrow$ 3777

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int(e+fx)^2\cosh(c+dx)dx}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{b}$$

$\Big\downarrow$ 3777

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{b}$$

$\Big\downarrow$ 26

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int(e+fx)\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int -i(e+fx)\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{b}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int(e+fx)\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{b}$$

$$\Big\downarrow \; 3777$$

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{d}\right)}{d}\right)}{b}-\frac{a(e+}{4b}}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{d}\right)}{b}-\right)}{b}$$

$\downarrow$ 3117

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}\right)}{b}$$

$\downarrow$ 3803

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}-}{b}$$

$$\frac{a\left(\frac{2(a^2+b^2)\int-\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(}{d^2}\right.}{d}\right.}{d}\right.}{b}\right.}{b}$$

$\downarrow$ 25

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}-}{b}$$

$$\frac{a\left(-\frac{2(a^2+b^2)\int\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh}{d}\right.}{d}\right.}{d}\right.}{b}\right.}{b}$$

$\downarrow$ 2694

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}-}{b}$$

$$\frac{a\left(-\frac{2(a^2+b^2)\left(\frac{b\int-\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int-\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c}{d}\right.}{}\right.}{}\right.}{b}$$

$\downarrow$ 27

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}-}{b}$$

$$\frac{a\left(-\frac{2(a^2+b^2)\left(\frac{b\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if}{}\right.}{}\right.}{b}\right.}{b}$$

$\downarrow$ 2620

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}-}{b}$$

$$\frac{a\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}}{a-\sqrt{a^2}}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{b^2}\right.}{b}$$

$\downarrow$ 3011

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}-}{b}$$

$$a\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3}{}\right.}{}}{b^2}\right.$$

$\downarrow$ 7163

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}-}{b}$$

$$a\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(}{d}}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{}\right.$$

↓ 2720

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}-}{b}$$

$$a\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+f}{}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{}\right.$$

↓ 7143

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}-}{b}$$

$$a\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\right.}{d}}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{}\right.$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
((e + f*x)^4/(8*f) - (3*f*(e + f*x)^2*Cosh[c + d*x]^2)/(4*d^2) + ((e + f*x
)^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (3*f^2*((e + f*x)^2/(4*f) - (f*Co
sh[c + d*x]^2)/(4*d^2) + ((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)))/(
2*d^2))/b - (a*(-1/4*(a*(e + f*x)^4)/(b^2*f) - (2*(a^2 + b^2)*(-1/2*(b*(((
e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-
(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (
2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d -
(f*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))
/Sqrt[a^2 + b^2] + (b*(((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2]))])/d^2))/d))/(b*d)))/(2*Sqrt[a^2 + b^2])))/b^2 - (I*((I*(e + f*x)^3*C
osh[c + d*x])/d - ((3*I)*f*(((e + f*x)^2*Sinh[c + d*x])/d + ((2*I)*f*((I*(
e + f*x)*Cosh[c + d*x])/d - (I*f*Sinh[c + d*x])/d^2))/d))/d))/b))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(fx+e)^3 \cosh (dx+c)^2 \sinh (dx+c)}{a+b\sinh (dx+c)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3847 vs. $2(627) = 1254$.

Time = 0.19 (sec) , antiderivative size = 3847, normalized size of antiderivative = 5.63

$$\int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh(c+dx)}{a+b\sinh(c+dx)} \, dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh(c+dx)}{a+b\sinh(c+dx)} \, dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)**2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/8*e^3*((4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) + 8*sqrt(a^2 + b^
2)*a*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt
(a^2 + b^2)))/(b^3*d) - 4*(2*a^2 + b^2)*(d*x + c)/(b^3*d) + (4*a*e^(-d*x -
 c) + b*e^(-2*d*x - 2*c))/(b^2*d)) + 1/32*(4*(2*a^2*d^4*f^3*e^(2*c) + b^2*
d^4*f^3*e^(2*c))*x^4 + 16*(2*a^2*d^4*e*f^2*e^(2*c) + b^2*d^4*e*f^2*e^(2*c)
)*x^3 + 24*(2*a^2*d^4*e^2*f*e^(2*c) + b^2*d^4*e^2*f*e^(2*c))*x^2 + (4*b^2*
d^3*f^3*x^3*e^(4*c) + 6*(2*d^3*e*f^2 - d^2*f^3)*b^2*x^2*e^(4*c) + 6*(2*d^3
*e^2*f - 2*d^2*e*f^2 + d*f^3)*b^2*x*e^(4*c) - 3*(2*d^2*e^2*f - 2*d*e*f^2 +
 f^3)*b^2*e^(4*c))*e^(2*d*x) - 16*(a*b*d^3*f^3*x^3*e^(3*c) + 3*(d^3*e*f^2
- d^2*f^3)*a*b*x^2*e^(3*c) + 3*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*a*b*x*e
^(3*c) - 3*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*a*b*e^(3*c))*e^(d*x) - 16*(a*b*
d^3*f^3*x^3*e^c + 3*(d^3*e*f^2 + d^2*f^3)*a*b*x^2*e^c + 3*(d^3*e^2*f + 2*d
^2*e*f^2 + 2*d*f^3)*a*b*x*e^c + 3*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*a*b*e^c)
*e^(-d*x) - (4*b^2*d^3*f^3*x^3 + 6*(2*d^3*e*f^2 + d^2*f^3)*b^2*x^2 + 6*(2*
d^3*e^2*f + 2*d^2*e*f^2 + d*f^3)*b^2*x + 3*(2*d^2*e^2*f + 2*d*e*f^2 + f^3)
*b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^4) - integrate(2*((a^3*f^3*e^c + a*b^2*f
^3*e^c)*x^3 + 3*(a^3*e*f^2*e^c + a*b^2*e*f^2*e^c)*x^2 + 3*(a^3*e^2*f*e^c +
 a*b^2*e^2*f*e^c)*x)*e^(d*x)/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) -
b^4), x)
```

Giac [F]

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)^2*sinh(d*x + c)/(b*sinh(d*x + c) + a),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^2\sinh(c+d\,x)\,\left(e+f\,x\right)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*sinh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)}{a+b\sinh\left(dx+c\right)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.339 $\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 510

$$
\begin{aligned}
&\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= \frac{f^2x}{4bd^2}+\frac{a^2(e+fx)^3}{3b^3f}+\frac{(e+fx)^3}{6bf}-\frac{2af^2\cosh(c+dx)}{b^2d^3}-\frac{a(e+fx)^2\cosh(c+dx)}{b^2d} \\
&\quad-\frac{f(e+fx)\cosh^2(c+dx)}{2bd^2}-\frac{a\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d} \\
&\quad+\frac{a\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d} \\
&\quad-\frac{2a\sqrt{a^2+b^2}f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2} \\
&\quad+\frac{2a\sqrt{a^2+b^2}f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^2} \\
&\quad+\frac{2a\sqrt{a^2+b^2}f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3} \\
&\quad-\frac{2a\sqrt{a^2+b^2}f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^3}+\frac{2af(e+fx)\sinh(c+dx)}{b^2d^2} \\
&\quad+\frac{f^2\cosh(c+dx)\sinh(c+dx)}{4bd^3}+\frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{2bd}
\end{aligned}
$$

output
```
1/4*f^2*x/b/d^2+1/3*a^2*(f*x+e)^3/b^3/f+1/6*(f*x+e)^3/b/f-2*a*f^2*cosh(d*x
+c)/b^2/d^3-a*(f*x+e)^2*cosh(d*x+c)/b^2/d-1/2*f*(f*x+e)*cosh(d*x+c)^2/b/d^
2-a*(a^2+b^2)^(1/2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d
+a*(a^2+b^2)^(1/2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d-
2*a*(a^2+b^2)^(1/2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))
/b^3/d^2+2*a*(a^2+b^2)^(1/2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2
)^(1/2)))/b^3/d^2+2*a*(a^2+b^2)^(1/2)*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+
b^2)^(1/2)))/b^3/d^3-2*a*(a^2+b^2)^(1/2)*f^2*polylog(3,-b*exp(d*x+c)/(a+(a
^2+b^2)^(1/2)))/b^3/d^3+2*a*f*(f*x+e)*sinh(d*x+c)/b^2/d^2+1/4*f^2*cosh(d*x
+c)*sinh(d*x+c)/b/d^3+1/2*(f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/b/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1188 vs. $2(510) = 1020$.

Time = 4.27 (sec) , antiderivative size = 1188, normalized size of antiderivative = 2.33

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
(6*b^2*e^2*(c/d + x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^
2]])/(Sqrt[-a^2 - b^2]*d)) + 6*b^2*e*f*(x^2 - (2*a*(d*x*(Log[1 + (b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2])] - Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])]) + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - PolyLog[2, -(
(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(Sqrt[a^2 + b^2]*d^2)) + 2*b^2*f
^2*(x^3 - (3*a*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d
^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2, (
b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*x*PolyLog[2, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))] - 2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 +
b^2])] + 2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(Sqrt[a^
2 + b^2]*d^3)) + f^2*(2*(4*a^2 + b^2)*x^3 - (6*a*(4*a^2 + 3*b^2)*(d^2*x^2*
Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^2*x^2*Log[1 + (b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2, (b*E^(c + d*x))/(-a + Sqr
t[a^2 + b^2])] - 2*d*x*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
] - 2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*PolyLog[3, -(
(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(Sqrt[a^2 + b^2]*d^3) - (24*a*b*
Cosh[d*x]*((2 + d^2*x^2)*Cosh[c] - 2*d*x*Sinh[c]))/d^3 + (3*b^2*Cosh[2*d*x
]*(-2*d*x*Cosh[2*c] + (1 + 2*d^2*x^2)*Sinh[2*c]))/d^3 - (24*a*b*(-2*d*x*Co
sh[c] + (2 + d^2*x^2)*Sinh[c])*Sinh[d*x])/d^3 + (3*b^2*((1 + 2*d^2*x^2)*Co
sh[2*c] - 2*d*x*Sinh[2*c])*Sinh[2*d*x])/d^3) + (6*e^2*((4*a^2 + b^2)*(c...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 3.07 (sec) , antiderivative size = 484, normalized size of antiderivative = 0.95, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.765$, Rules used = {6113, 3042, 3792, 17, 3042, 3115, 24, 6099, 17, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118, 3803, 25, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2 \sinh(c+dx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6113}$$

$$\frac{\int (e+fx)^2\cosh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx}{b}$$

$$\Big\downarrow \text{3792}$$

$$\frac{\frac{f^2\int \cosh^2(c+dx)dx}{2d^2}+\frac{1}{2}\int (e+fx)^2dx-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{17}$$

$$\frac{\frac{f^2\int \cosh^2(c+dx)dx}{2d^2}-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{f^2\int \sin\left(ic+idx+\frac{\pi}{2}\right)^2dx}{2d^2}-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{f^2\left(\frac{\int 1dx}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{24}$$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{6099}$$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)^2dx}{b^2}+\frac{\int(e+fx)^2\sinh(c+dx)dx}{b}\right)}{b}$$

$\Big\downarrow$ 17

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int(e+fx)^2\sinh(c+dx)dx}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{b}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}+\frac{\int-i(e+fx)^2\sin(ic+idx)dx}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{b}$$

$\Big\downarrow$ 26

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\int(e+fx)^2\sin(ic+idx)dx}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{b}$$

$\Big\downarrow$ 3777

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\cosh(c+dx)dx}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{b}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int-i\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int-i\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{b}$$

$\downarrow$ 3118

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}\right)}{b}$$

$\downarrow$ 3803

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{2(a^2+b^2)\int-\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}\right)}{b}$$

$\downarrow$ 25

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{2(a^2+b^2)\int\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}\right)}{b}$$

$\downarrow$ 2694

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{2(a^2+b^2)\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+\cdots}{d}\right.}{\cdots}\right.}{b}\right.}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{2(a^2+b^2)\left(\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cos\cdots}{\cdots}\right.}{d}\right.}{b}\right.}{b}$$

$$\Big\downarrow \; 2620$$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+\cdots}}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{b^2}\right.}{b}$$

$$\Big\downarrow \; 3011$$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-$$

$$a\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be}{a-\sqrt{}}\right.}{bd}\right.}{}}{b^2}\right.$$

$\downarrow$ 2720

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-$$

$$a\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left((e+\right.}{}}{b^2}\right.$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+d}}{a-\sqrt{a^2-}}\cdots\right.}{bd}\cdots\right.}{\cdots}\right)}{b^2}\cdots\right)}{\cdots}$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
((e + f*x)^3/(6*f) - (f*(e + f*x)*Cosh[c + d*x]^2)/(2*d^2) + ((e + f*x)^2*
Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (f^2*(x/2 + (Cosh[c + d*x]*Sinh[c + d
*x])/(2*d)))/(2*d^2))/b - (a*(-1/3*(a*(e + f*x)^3)/(b^2*f) - (2*(a^2 + b^2
)*(-1/2*(b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(
b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d^2))/
(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2]))])/d^2))/(b*d))/(2*Sqrt[a^2 + b^2])))/b^2 - (I*((I*(e + f*x)^2*C
osh[c + d*x])/d - ((2*I)*f*(-((f*Cosh[c + d*x])/d^2) + ((e + f*x)*Sinh[c +
 d*x])/d))/d))/b))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(fx+e)^2 \cosh (dx+c)^2 \sinh (dx+c)}{a+b\sinh (dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2410 vs. 2(466) = 932.

Time = 0.17 (sec) , antiderivative size = 2410, normalized size of antiderivative = 4.73

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
-1/48*(6*b^2*d^2*f^2*x^2 + 6*b^2*d^2*e^2 + 6*b^2*d*e*f - 3*(2*b^2*d^2*f^2*
x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2
)*x)*cosh(d*x + c)^4 - 3*(2*b^2*d^2*f^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f
+ b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)*sinh(d*x + c)^4 + 3*b^2*f^2 +
 24*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*
e*f - a*b*d*f^2)*x)*cosh(d*x + c)^3 + 12*(2*a*b*d^2*f^2*x^2 + 2*a*b*d^2*e^
2 - 4*a*b*d*e*f + 4*a*b*f^2 + 4*(a*b*d^2*e*f - a*b*d*f^2)*x - (2*b^2*d^2*f
^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*
f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 8*((2*a^2 + b^2)*d^3*f^2*x^3 + 3*
(2*a^2 + b^2)*d^3*e*f*x^2 + 3*(2*a^2 + b^2)*d^3*e^2*x)*cosh(d*x + c)^2 - 2
*(4*(2*a^2 + b^2)*d^3*f^2*x^3 + 12*(2*a^2 + b^2)*d^3*e*f*x^2 + 12*(2*a^2 +
 b^2)*d^3*e^2*x + 9*(2*b^2*d^2*f^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2
*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)*cosh(d*x + c)^2 - 36*(a*b*d^2*f^2*
x^2 + a*b*d^2*e^2 - 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f - a*b*d*f^2)*
x)*cosh(d*x + c))*sinh(d*x + c)^2 + 96*((a*b*d*f^2*x + a*b*d*e*f)*cosh(d*x
 + c)^2 + 2*(a*b*d*f^2*x + a*b*d*e*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b*d
*f^2*x + a*b*d*e*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d
*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
+ b^2)/b^2) - b)/b + 1) - 96*((a*b*d*f^2*x + a*b*d*e*f)*cosh(d*x + c)^2 +
2*(a*b*d*f^2*x + a*b*d*e*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b*d*f^2*x ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)**2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)^2\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/8*e^2*((4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) + 8*sqrt(a^2 + b^
2)*a*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt
(a^2 + b^2)))/(b^3*d) - 4*(2*a^2 + b^2)*(d*x + c)/(b^3*d) + (4*a*e^(-d*x -
 c) + b*e^(-2*d*x - 2*c))/(b^2*d)) + 1/48*(8*(2*a^2*d^3*f^2*e^(2*c) + b^2*
d^3*f^2*e^(2*c))*x^3 + 24*(2*a^2*d^3*e*f*e^(2*c) + b^2*d^3*e*f*e^(2*c))*x^
2 + 3*(2*b^2*d^2*f^2*x^2*e^(4*c) + 2*(2*d^2*e*f - d*f^2)*b^2*x*e^(4*c) - (
2*d*e*f - f^2)*b^2*e^(4*c))*e^(2*d*x) - 24*(a*b*d^2*f^2*x^2*e^(3*c) + 2*(d
^2*e*f - d*f^2)*a*b*x*e^(3*c) - 2*(d*e*f - f^2)*a*b*e^(3*c))*e^(d*x) - 24*
(a*b*d^2*f^2*x^2*e^c + 2*(d^2*e*f + d*f^2)*a*b*x*e^c + 2*(d*e*f + f^2)*a*b
*e^c)*e^(-d*x) - 3*(2*b^2*d^2*f^2*x^2 + 2*(2*d^2*e*f + d*f^2)*b^2*x + (2*d
*e*f + f^2)*b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^3) - integrate(2*((a^3*f^2*e^
c + a*b^2*f^2*e^c)*x^2 + 2*(a^3*e*f*e^c + a*b^2*e*f*e^c)*x)*e^(d*x)/(b^4*e
^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) - b^4), x)
```

Giac [F]

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)^2\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)^2*sinh(d*x + c)/(b*sinh(d*x + c) + a),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^2\sinh(c+d\,x)\,\left(e+f\,x\right)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*sinh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 96*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sq
rt(a**2 + b**2))*a*b**2*d**2*e**2*i + 6*e**(4*c + 4*d*x)*b**4*d**2*e**2 +
12*e**(4*c + 4*d*x)*b**4*d**2*e*f*x + 6*e**(4*c + 4*d*x)*b**4*d**2*f**2*x*
*2 - 6*e**(4*c + 4*d*x)*b**4*d*e*f - 6*e**(4*c + 4*d*x)*b**4*d*f**2*x + 3*
e**(4*c + 4*d*x)*b**4*f**2 - 24*e**(3*c + 3*d*x)*a*b**3*d**2*e**2 - 48*e**
(3*c + 3*d*x)*a*b**3*d**2*e*f*x - 24*e**(3*c + 3*d*x)*a*b**3*d**2*f**2*x**
2 + 48*e**(3*c + 3*d*x)*a*b**3*d*e*f + 48*e**(3*c + 3*d*x)*a*b**3*d*f**2*x
 - 48*e**(3*c + 3*d*x)*a*b**3*f**2 + 192*e**(2*c + 2*d*x)*int(x**2/(e**(4*
c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b),x)*a**4*b*d**3*f
**2 + 192*e**(2*c + 2*d*x)*int(x**2/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*
x)*a - e**(2*c + 2*d*x)*b),x)*a**2*b**3*d**3*f**2 + 384*e**(2*c + 2*d*x)*i
nt(x/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b),x)*a
**4*b*d**3*e*f + 384*e**(2*c + 2*d*x)*int(x/(e**(4*c + 4*d*x)*b + 2*e**(3*
c + 3*d*x)*a - e**(2*c + 2*d*x)*b),x)*a**2*b**3*d**3*e*f + 48*e**(2*c + 2*
d*x)*a**2*b**2*d**3*e**2*x + 48*e**(2*c + 2*d*x)*a**2*b**2*d**3*e*f*x**2 +
 16*e**(2*c + 2*d*x)*a**2*b**2*d**3*f**2*x**3 + 24*e**(2*c + 2*d*x)*b**4*d
**3*e**2*x + 24*e**(2*c + 2*d*x)*b**4*d**3*e*f*x**2 + 8*e**(2*c + 2*d*x)*b
**4*d**3*f**2*x**3 - 384*e**(c + 2*d*x)*int(x**2/(e**(2*c + 3*d*x)*b + 2*e
**(c + 2*d*x)*a - e**(d*x)*b),x)*a**5*d**3*f**2 - 480*e**(c + 2*d*x)*int(x
**2/(e**(2*c + 3*d*x)*b + 2*e**(c + 2*d*x)*a - e**(d*x)*b),x)*a**3*b**2...
```

3.340 $\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 321

$$\begin{aligned}\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{a^2(e+fx)^2}{2b^3f}+\frac{(e+fx)^2}{4bf}\\
&-\frac{a(e+fx)\cosh(c+dx)}{b^2d}\\
&-\frac{f\cosh^2(c+dx)}{4bd^2}\\
&-\frac{a\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d}\\
&+\frac{a\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d}\\
&-\frac{a\sqrt{a^2+b^2}f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}\\
&+\frac{a\sqrt{a^2+b^2}f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^2}\\
&+\frac{af\sinh(c+dx)}{b^2d^2}\\
&+\frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{2bd}\end{aligned}$$

output
```
1/2*a^2*(f*x+e)^2/b^3/f+1/4*(f*x+e)^2/b/f-a*(f*x+e)*cosh(d*x+c)/b^2/d-1/4*
f*cosh(d*x+c)^2/b/d^2-a*(a^2+b^2)^(1/2)*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+
b^2)^(1/2)))/b^3/d+a*(a^2+b^2)^(1/2)*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2
)^(1/2)))/b^3/d-a*(a^2+b^2)^(1/2)*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(
1/2)))/b^3/d^2+a*(a^2+b^2)^(1/2)*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1
/2)))/b^3/d^2+a*f*sinh(d*x+c)/b^2/d^2+1/2*(f*x+e)*cosh(d*x+c)*sinh(d*x+c)/
b/d
```

Mathematica [A] (warning: unable to verify)

Time = 1.79 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.81

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{2b^2e\left(\frac{c}{d}+x-\frac{2a\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}d}\right)+b^2f\left(x^2-\frac{2a\left(dx\left(\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)+\mathrm{PolyLog}\left(2,\right.\right.}{\sqrt{a^2+b^2}d^2}\right.}{}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
(2*b^2*e*(c/d + x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]
])/(Sqrt[-a^2 - b^2]*d)) + b^2*f*(x^2 - (2*a*(d*x*(Log[1 + (b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2])] - Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])
+ PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - PolyLog[2, -((b*E^(
c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(Sqrt[a^2 + b^2]*d^2)) + (2*e*((4*a^2
+ b^2)*(c + d*x) - (2*a*(4*a^2 + 3*b^2)*ArcTan[(b - a*Tanh[(c + d*x)/2])/S
qrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - 4*a*b*Cosh[c + d*x] + b^2*Sinh[2*(c +
 d*x)]))/d + (f*((4*a^2 + b^2)*(-c + d*x)*(c + d*x) - 8*a*b*d*x*Cosh[c + d
*x] - b^2*Cosh[2*(c + d*x)] - (2*a*(4*a^2 + 3*b^2)*(2*c*ArcTanh[(a + b*E^(
c + d*x))/Sqrt[a^2 + b^2]] + (c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2])] - (c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + P
olyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - PolyLog[2, -((b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2]))]))/Sqrt[a^2 + b^2] + 8*a*b*Sinh[c + d*x] + 2
*b^2*d*x*Sinh[2*(c + d*x)]))/d^2)/(8*b^3)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.61 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.01, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {6113, 3042, 3791, 17, 6099, 17, 3042, 26, 3777, 3042, 3117, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6113}$$

$$\frac{\int (e+fx)\cosh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx}{b}$$

$$\Big\downarrow \text{3791}$$

$$\frac{\frac{1}{2}\int (e+fx)dx-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{17}$$

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{6099}$$

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)dx}{b^2}+\frac{\int (e+fx)\sinh(c+dx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{17}$$

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{e+fx}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int(e+fx)\sinh(c+dx)dx}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}+\frac{\int-i(e+fx)\sin(ic+idx)dx}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\int(e+fx)\sin(ic+idx)dx}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{b}$$

$$\Big\downarrow \text{3117}$$

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{b}$$

$$\Big\downarrow \ 3803$$

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{2(a^2+b^2)\int -\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{b}$$

$$\Big\downarrow \ 25$$

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2(a^2+b^2)\int \frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{b}$$

$$\Big\downarrow \ 2694$$

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2(a^2+b^2)\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{b}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2(a^2+b^2)\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{b}$$

$$\Big\downarrow \ 2620$$

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{b^2}-\cdots\right)}{b}$$

$\Big\downarrow$ 2715

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}}{a-\sqrt{a^2+b^2}}\cdots\right)}{bd^2}\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{b^2}\cdots\right)}{b}$$

$\Big\downarrow$ 2838

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{b^2}-\cdots\right)}{b}$$

input
```
Int[((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
((e + f*x)^2/(4*f) - (f*Cosh[c + d*x]^2)/(4*d^2) + ((e + f*x)*Cosh[c + d*x
]*Sinh[c + d*x])/(2*d))/b - (a*(-1/2*(a*(e + f*x)^2)/(b^2*f) - (2*(a^2 + b
^2)*(-1/2*(b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(
b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2)))/
Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b
*d^2)))/(2*Sqrt[a^2 + b^2])))/b^2 - (I*((I*(e + f*x)*Cosh[c + d*x])/d - (I
*f*Sinh[c + d*x])/d^2))/b))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1011 vs. $2(293) = 586$.

Time = 5.04 (sec) , antiderivative size = 1012, normalized size of antiderivative = 3.15

method	result	size
risch	Expression too large to display	1012

input
```
int((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
1/2/b^3*a^2*f*x^2+1/4/b*f*x^2+1/b^3*a^2*e*x+1/2/b*e*x+1/16*(2*d*f*x+2*d*e-
f)/b/d^2*exp(2*d*x+2*c)-1/2*a*(d*f*x+d*e-f)/b^2/d^2*exp(d*x+c)-1/2*a*(d*f*
x+d*e+f)/b^2/d^2*exp(-d*x-c)-1/16*(2*d*f*x+2*d*e+f)/b/d^2*exp(-2*d*x-2*c)+
2/d*a^3/b^3*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(
1/2))+2/d*a/b*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)
^(1/2))-1/d*a^3/b^3*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)
/(-a+(a^2+b^2)^(1/2)))*x+1/d*a^3/b^3*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a
^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2*a^3/b^3*f/(a^2+b^2)^(1/2)*ln
((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d^2*a^3/b^3*f
/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*
c-1/d^2*a^3/b^3*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/
(-a+(a^2+b^2)^(1/2)))+1/d^2*a^3/b^3*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+
(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d*a/b*f/(a^2+b^2)^(1/2)*ln((-b*e
xp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d*a/b*f/(a^2+b^2)^(
1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2*a/b*
f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)
))*c+1/d^2*a/b*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a
^2+b^2)^(1/2)))*c-1/d^2*a/b*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^
2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/d^2*a/b*f/(a^2+b^2)^(1/2)*dilog((b*exp
(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-2/d^2*a^3/b^3*f*c/(a^2+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1284 vs. $2(291) = 582$.

Time = 0.13 (sec) , antiderivative size = 1284, normalized size of antiderivative = 4.00

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
-1/16*(2*b^2*d*f*x - (2*b^2*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c)^4 - (
2*b^2*d*f*x + 2*b^2*d*e - b^2*f)*sinh(d*x + c)^4 + 2*b^2*d*e + 8*(a*b*d*f*
x + a*b*d*e - a*b*f)*cosh(d*x + c)^3 + 4*(2*a*b*d*f*x + 2*a*b*d*e - 2*a*b*
f - (2*b^2*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c))*sinh(d*x + c)^3 + b^2
*f - 4*((2*a^2 + b^2)*d^2*f*x^2 + 2*(2*a^2 + b^2)*d^2*e*x)*cosh(d*x + c)^2
 - 2*(2*(2*a^2 + b^2)*d^2*f*x^2 + 4*(2*a^2 + b^2)*d^2*e*x + 3*(2*b^2*d*f*x
 + 2*b^2*d*e - b^2*f)*cosh(d*x + c)^2 - 12*(a*b*d*f*x + a*b*d*e - a*b*f)*c
osh(d*x + c))*sinh(d*x + c)^2 + 16*(a*b*f*cosh(d*x + c)^2 + 2*a*b*f*cosh(d
*x + c)*sinh(d*x + c) + a*b*f*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog
((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*
sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 16*(a*b*f*cosh(d*x + c)^2 + 2*a*b*f*co
sh(d*x + c)*sinh(d*x + c) + a*b*f*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*d
ilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 16*((a*b*d*e - a*b*c*f)*cosh(d*x +
 c)^2 + 2*(a*b*d*e - a*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b*d*e - a*b
*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*s
inh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 16*((a*b*d*e - a*b*c*f)*
cosh(d*x + c)^2 + 2*(a*b*d*e - a*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b
*d*e - a*b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x +
c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 16*((a*b*d*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)**2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)^2\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-1/16*(32*(a^3*e^c + a*b^2*e^c)*integrate(x*e^(d*x)/(b^4*e^(2*d*x + 2*c) +
 2*a*b^3*e^(d*x + c) - b^4), x) - (4*(2*a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))
*x^2 + (2*b^2*d*x*e^(4*c) - b^2*e^(4*c))*e^(2*d*x) - 8*(a*b*d*x*e^(3*c) -
a*b*e^(3*c))*e^(d*x) - 8*(a*b*d*x*e^c + a*b*e^c)*e^(-d*x) - (2*b^2*d*x + b
^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^2))*f - 1/8*e*((4*a*e^(-d*x - c) - b)*e^(2
*d*x + 2*c)/(b^2*d) + 8*sqrt(a^2 + b^2)*a*log((b*e^(-d*x - c) - a - sqrt(a
^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(b^3*d) - 4*(2*a^2 + b^
2)*(d*x + c)/(b^3*d) + (4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d))
```

Giac [F]

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)^2\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)^2*sinh(d*x + c)/(b*sinh(d*x + c) + a), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^2\,\sinh(c+d\,x)\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*sinh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-32e^{2dx+2c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)a\,b^2dei+2e^{4dx+4c}b^4de-16a^2b^2f-2b^4de-e^{4dx+4c}b^4f-32a^2b^2df}{}$$

input
```
int((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 32*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sq
rt(a**2 + b**2))*a*b**2*d*e*i + 2*e**(4*c + 4*d*x)*b**4*d*e + 2*e**(4*c +
4*d*x)*b**4*d*f*x - e**(4*c + 4*d*x)*b**4*f - 8*e**(3*c + 3*d*x)*a*b**3*d*
e - 8*e**(3*c + 3*d*x)*a*b**3*d*f*x + 8*e**(3*c + 3*d*x)*a*b**3*f + 64*e**
(2*c + 2*d*x)*int(x/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a - e**(2*c +
 2*d*x)*b),x)*a**4*b*d**2*f + 64*e**(2*c + 2*d*x)*int(x/(e**(4*c + 4*d*x)*
b + 2*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b),x)*a**2*b**3*d**2*f + 16*e*
*(2*c + 2*d*x)*a**2*b**2*d**2*e*x + 8*e**(2*c + 2*d*x)*a**2*b**2*d**2*f*x*
*2 + 8*e**(2*c + 2*d*x)*b**4*d**2*e*x + 4*e**(2*c + 2*d*x)*b**4*d**2*f*x**
2 - 128*e**(c + 2*d*x)*int(x/(e**(2*c + 3*d*x)*b + 2*e**(c + 2*d*x)*a - e*
*(d*x)*b),x)*a**5*d**2*f - 160*e**(c + 2*d*x)*int(x/(e**(2*c + 3*d*x)*b +
2*e**(c + 2*d*x)*a - e**(d*x)*b),x)*a**3*b**2*d**2*f - 32*e**(c + 2*d*x)*i
nt(x/(e**(2*c + 3*d*x)*b + 2*e**(c + 2*d*x)*a - e**(d*x)*b),x)*a*b**4*d**2
*f + 32*e**(c + d*x)*a**3*b*d*f*x + 32*e**(c + d*x)*a**3*b*f - 8*e**(c + d
*x)*a*b**3*d*e + 24*e**(c + d*x)*a*b**3*d*f*x + 24*e**(c + d*x)*a*b**3*f -
 32*a**4*d*f*x - 16*a**4*f - 32*a**2*b**2*d*f*x - 16*a**2*b**2*f - 2*b**4*
d*e - 2*b**4*d*f*x - b**4*f)/(16*e**(2*c + 2*d*x)*b**5*d**2)
```

3.341 $\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 95

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx=\frac{(2a^2+b^2)\,x}{2b^3}+\frac{2a\sqrt{a^2+b^2}\,\mathrm{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^3d}-\frac{\cosh(c+dx)(2a-b\sinh(c+dx))}{2b^2d}$$

output
```
1/2*(2*a^2+b^2)*x/b^3+2*a*(a^2+b^2)^(1/2)*arctanh((b-a*tanh(1/2*d*x+1/2*c)
)/(a^2+b^2)^(1/2))/b^3/d-1/2*cosh(d*x+c)*(2*a-b*sinh(d*x+c))/b^2/d
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.15

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx=\frac{4a^2c+2b^2c+4a^2dx+2b^2dx+8a\sqrt{-a^2-b^2}\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)-4ab\cosh(c+dx)+b^2\sinh(2(c}{4b^3d}$$

input
```
Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(4*a^2*c + 2*b^2*c + 4*a^2*d*x + 2*b^2*d*x + 8*a*Sqrt[-a^2 - b^2]*ArcTan[(
b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]] - 4*a*b*Cosh[c + d*x] + b^2*Sin
h[2*(c + d*x)])/(4*b^3*d)
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {3042, 26, 3344, 26, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ic+idx)\cos(ic+idx)^2}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos(ic+idx)^2\sin(ic+idx)}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3344}$$

$$-i\left(-\frac{\int \frac{i(ab-(2a^2+b^2)\sinh(c+dx))}{a+b\sinh(c+dx)}dx}{2b^2}-\frac{i\cosh(c+dx)(2a-b\sinh(c+dx))}{2b^2d}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{i\int \frac{ab-(2a^2+b^2)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{2b^2}-\frac{i\cosh(c+dx)(2a-b\sinh(c+dx))}{2b^2d}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{i\int \frac{ab+i(2a^2+b^2)\sin(ic+idx)}{a-ib\sin(ic+idx)}dx}{2b^2}-\frac{i\cosh(c+dx)(2a-b\sinh(c+dx))}{2b^2d}\right)$$

$\Big\downarrow$ 3214

$$-i\left(-\frac{i\left(\frac{2a(a^2+b^2)\int\frac{1}{a+b\sinh(c+dx)}dx}{b}-\frac{x(2a^2+b^2)}{b}\right)}{2b^2}-\frac{i\cosh(c+dx)(2a-b\sinh(c+dx))}{2b^2d}\right)$$

$\Big\downarrow$ 3042

$$-i\left(-\frac{i\left(-\frac{x(2a^2+b^2)}{b}+\frac{2a(a^2+b^2)\int\frac{1}{a-ib\sin(ic+idx)}dx}{b}\right)}{2b^2}-\frac{i\cosh(c+dx)(2a-b\sinh(c+dx))}{2b^2d}\right)$$

$\Big\downarrow$ 3139

$$-i\left(-\frac{i\left(-\frac{x(2a^2+b^2)}{b}-\frac{4ia(a^2+b^2)\int\frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{bd}\right)}{2b^2}-\frac{i\cosh(c+dx)(2a-b\sinh}{2b^2d}\right.$$

$\Big\downarrow$ 1083

$$-i\left(-\frac{i\left(-\frac{x(2a^2+b^2)}{b}+\frac{8ia(a^2+b^2)\int\frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4(a^2+b^2)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{bd}\right)}{2b^2}-\frac{i\cosh(c+dx)(2a-b\sinh(c+}{2b^2d}\right.$$

$\Big\downarrow$ 217

$$-i\left(-\frac{i\left(\frac{4a\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{bd}-\frac{x(2a^2+b^2)}{b}\right)}{2b^2}-\frac{i\cosh(c+dx)(2a-b\sinh(c+dx))}{2b^2d}\right)$$

input
```
Int[(Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-I)*(((-1/2*I)*(-(((2*a^2 + b^2)*x)/b) + (4*a*Sqrt[a^2 + b^2]*ArcTanh[Tan
h[(c + d*x)/2]/(2*Sqrt[a^2 + b^2])])/(b*d)))/b^2 - ((I/2)*Cosh[c + d*x]*(2
*a - b*Sinh[c + d*x]))/(b^2*d))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.76

method	result
risch	$\frac{x a^2}{b^3}+\frac{x}{2b}+\frac{e^{2dx+2c}}{8bd}-\frac{a e^{dx+c}}{2b^2 d}-\frac{a e^{-dx-c}}{2b^2 d}-\frac{e^{-2dx-2c}}{8bd}+\frac{\sqrt{a^2+b^2}\,a\ln\left(e^{dx+c}+\frac{a+\sqrt{a^2+b^2}}{b}\right)}{d\,b^3}-\frac{\sqrt{a^2+b^2}\cdots}{}$
derivativedivides	$\frac{-\frac{1}{2b\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{-b+2a}{2b^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{\left(2a^2+b^2\right)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2b^3}+\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-b-2a}{2b^2\left(\tanh\left(\frac{dx}{2}+\cdots\right.\right.}\cdots}{d}$
default	$\frac{-\frac{1}{2b\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{-b+2a}{2b^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{\left(2a^2+b^2\right)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2b^3}+\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-b-2a}{2b^2\left(\tanh\left(\frac{dx}{2}+\cdots\right.\right.}\cdots}{d}$

input
```
int(cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
x/b^3*a^2+1/2*x/b+1/8/b/d*exp(2*d*x+2*c)-1/2*a/b^2/d*exp(d*x+c)-1/2*a/b^2/
d*exp(-d*x-c)-1/8/b/d*exp(-2*d*x-2*c)+(a^2+b^2)^(1/2)*a/d/b^3*ln(exp(d*x+c
)+(a+(a^2+b^2)^(1/2))/b)-(a^2+b^2)^(1/2)*a/d/b^3*ln(exp(d*x+c)-(-a+(a^2+b^
2)^(1/2))/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(87) = 174$.

Time = 0.10 (sec) , antiderivative size = 446, normalized size of antiderivative = 4.69

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b^2\cosh(dx+c)^4 + b^2\sinh(dx+c)^4 + 4\,(2\,a^2+b^2)dx\cosh(dx+c)^2 - 4\,ab\cosh(dx+c)^3 + 4\,(b^2\cosh(a}{}$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/8*(b^2*cosh(d*x + c)^4 + b^2*sinh(d*x + c)^4 + 4*(2*a^2 + b^2)*d*x*cosh(
d*x + c)^2 - 4*a*b*cosh(d*x + c)^3 + 4*(b^2*cosh(d*x + c) - a*b)*sinh(d*x
+ c)^3 - 4*a*b*cosh(d*x + c) + 2*(3*b^2*cosh(d*x + c)^2 + 2*(2*a^2 + b^2)*
d*x - 6*a*b*cosh(d*x + c))*sinh(d*x + c)^2 + 8*(a*cosh(d*x + c)^2 + 2*a*co
sh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2)*sqrt(a^2 + b^2)*log((b^2*co
sh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 +
2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x
+ c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*
cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - b^2 + 4*(b^2
*cosh(d*x + c)^3 + 2*(2*a^2 + b^2)*d*x*cosh(d*x + c) - 3*a*b*cosh(d*x + c)
^2 - a*b)*sinh(d*x + c))/(b^3*d*cosh(d*x + c)^2 + 2*b^3*d*cosh(d*x + c)*si
nh(d*x + c) + b^3*d*sinh(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.68

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\left(4\,ae^{(-dx-c)}-b\right)e^{(2\,dx+2\,c)}}{8\,b^2d} - \frac{\sqrt{a^2+b^2}a\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{b^3d} + \frac{(2\,a^2+b^2)(dx+c)}{2\,b^3d} - \frac{4\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}}{8\,b^2d}$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-1/8*(4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) - sqrt(a^2 + b^2)*a*lo
g((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 +
b^2)))/(b^3*d) + 1/2*(2*a^2 + b^2)*(d*x + c)/(b^3*d) - 1/8*(4*a*e^(-d*x -
c) + b*e^(-2*d*x - 2*c))/(b^2*d)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.63

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{4\,(2\,a^2+b^2)(dx+c)}{b^3} + \frac{be^{(2\,dx+2\,c)}-4\,ae^{(dx+c)}}{b^2} - \frac{\left(4\,abe^{(dx+c)}+b^2\right)e^{(-2\,dx-2\,c)}}{b^3} - \frac{8\,(a^3+ab^2)\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^3}}{8\,d}$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/8*(4*(2*a^2 + b^2)*(d*x + c)/b^3 + (b*e^(2*d*x + 2*c) - 4*a*e^(d*x + c))
/b^2 - (4*a*b*e^(d*x + c) + b^2)*e^(-2*d*x - 2*c)/b^3 - 8*(a^3 + a*b^2)*lo
g(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c) + 2*a
 + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^3))/d
```

Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.23

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{e^{2c+2dx}}{8bd}-\frac{e^{-2c-2dx}}{8bd}+\frac{x(2a^2+b^2)}{2b^3}-\frac{a\,e^{-c-dx}}{2b^2d}-\frac{a\,e^{c+dx}}{2b^2d} - \frac{a\ln\left(\frac{2a\,e^{c+dx}\,(a^2+b^2)}{b^4}-\frac{2a\sqrt{a^2+b^2}\,(b-a\,e^{c+dx})}{b^4}\right)\sqrt{a^2+b^2}}{b^3d} + \frac{a\ln\left(\frac{2a\sqrt{a^2+b^2}\,(b-a\,e^{c+dx})}{b^4}+\frac{2a\,e^{c+dx}\,(a^2+b^2)}{b^4}\right)\sqrt{a^2+b^2}}{b^3d}$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x))/(a + b*sinh(c + d*x)),x)
```

output
```
exp(2*c + 2*d*x)/(8*b*d) - exp(- 2*c - 2*d*x)/(8*b*d) + (x*(2*a^2 + b^2))/
(2*b^3) - (a*exp(- c - d*x))/(2*b^2*d) - (a*exp(c + d*x))/(2*b^2*d) - (a*l
og((2*a*exp(c + d*x)*(a^2 + b^2))/b^4 - (2*a*(a^2 + b^2)^(1/2)*(b - a*exp(
c + d*x)))/b^4)*(a^2 + b^2)^(1/2))/(b^3*d) + (a*log((2*a*(a^2 + b^2)^(1/2)
*(b - a*exp(c + d*x)))/b^4 + (2*a*exp(c + d*x)*(a^2 + b^2))/b^4)*(a^2 + b^
2)^(1/2))/(b^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.55

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{-16e^{2dx+2c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)ai+e^{4dx+4c}b^2-4e^{3dx+3c}ab+8e^{2dx+2c}a^2dx+4e^{2dx+2c}b^2dx-4e^{dx+}}{8e^{2dx+2c}b^3d}$$

input
```
int(cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 16*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sq
rt(a**2 + b**2))*a*i + e**(4*c + 4*d*x)*b**2 - 4*e**(3*c + 3*d*x)*a*b + 8*
e**(2*c + 2*d*x)*a**2*d*x + 4*e**(2*c + 2*d*x)*b**2*d*x - 4*e**(c + d*x)*a
*b - b**2)/(8*e**(2*c + 2*d*x)*b**3*d)
```

3.342 $\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(cosh(d*x+c)^2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 11.70 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{6125}$$
$$\int \frac{\sinh(c+dx)\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Cosh[c + d*x]^2*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(dx+c)^2\sinh(dx+c)}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(cosh(d*x+c)^2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)^2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(cosh(d*x + c)^2*sinh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\text{Timed out}$$

input
```
integrate(cosh(d*x+c)**2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 243, normalized size of antiderivative = 7.15

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-2*(a^3*e^c + a*b^2*e^c)*integrate(-e^(d*x)/(b^4*f*x + b^4*e - (b^4*f*x*e^
(2*c) + b^4*e*e^(2*c))*e^(2*d*x) - 2*(a*b^3*f*x*e^c + a*b^3*e*e^c)*e^(d*x)
), x) - 1/4*e^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b*f) -
1/2*a*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^2*f) + 1/2*a*e^(c
 - d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b^2*f) - 1/4*e^(2*c - 2*d*e/f
)*exp_integral_e(1, -2*(f*x + e)*d/f)/(b*f) + 1/2*(2*a^2 + b^2)*log(f*x +
e)/(b^3*f)
```

Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
integrate(cosh(d*x + c)^2*sinh(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)),
 x)
```

Mupad [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)^2\sinh(c+d\,x)}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((cosh(c + d*x)^2*sinh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 265, normalized size of antiderivative = 7.79

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$= \frac{e^{4c}\left(\int \frac{e^{4dx}}{e^{2dx+2c}be+e^{2dx+2c}bfx+2e^{dx+c}ae+2e^{dx+c}afx-be-bfx}dx\right)bf-2e^{c}\left(\int \frac{e^{dx}}{e^{2dx+2c}be+e^{2dx+2c}bfx+2e^{dx+c}ae+2e^{dx+c}afx-be-}\right.}{4bf}$$

input
```
int(cosh(d*x+c)^2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
(e**(4*c)*int(e**(4*d*x)/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x +
2*e**(c + d*x)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x)*b*f - 2*e**c*i
nt(e**(d*x)/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x + 2*e**(c + d*x
)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x)*a*f - int(1/(e**(4*c + 4*d*
x)*b*e + e**(4*c + 4*d*x)*b*f*x + 2*e**(3*c + 3*d*x)*a*e + 2*e**(3*c + 3*d
*x)*a*f*x - e**(2*c + 2*d*x)*b*e - e**(2*c + 2*d*x)*b*f*x),x)*b*f + log(e
+ f*x))/(4*b*f)
```

3.343 $\int \frac{(e+fx)^3 \cosh^3(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} \, dx$

Optimal result

Integrand size = 34, antiderivative size = 864

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= -\frac{3af^3x}{8b^2d^3} - \frac{a(e+fx)^3}{4b^2d} + \frac{a(a^2+b^2)(e+fx)^4}{4b^4f} - \frac{6a^2f^3\cosh(c+dx)}{b^3d^4}$$
$$- \frac{40f^3\cosh(c+dx)}{9bd^4} - \frac{3a^2f(e+fx)^2\cosh(c+dx)}{b^3d^2}$$
$$- \frac{2f(e+fx)^2\cosh(c+dx)}{bd^2} - \frac{2f^3\cosh^3(c+dx)}{27bd^4}$$
$$- \frac{f(e+fx)^2\cosh^3(c+dx)}{3bd^2} - \frac{a(a^2+b^2)(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d}$$
$$- \frac{a(a^2+b^2)(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d}$$
$$- \frac{3a(a^2+b^2)f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}$$
$$- \frac{3a(a^2+b^2)f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^2}$$
$$+ \frac{6a(a^2+b^2)f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^3}$$
$$+ \frac{6a(a^2+b^2)f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^3}$$
$$- \frac{6a(a^2+b^2)f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^4} - \frac{6a(a^2+b^2)f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^4}$$
$$+ \frac{6a^2f^2(e+fx)\sinh(c+dx)}{b^3d^3} + \frac{40f^2(e+fx)\sinh(c+dx)}{9bd^3}$$
$$+ \frac{a^2(e+fx)^3\sinh(c+dx)}{b^3d} + \frac{2(e+fx)^3\sinh(c+dx)}{3bd}$$
$$+ \frac{3af^3\cosh(c+dx)\sinh(c+dx)}{8b^2d^4} + \frac{3af(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{4b^2d^2}$$
$$+ \frac{2f^2(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{9bd^3} + \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{3bd}$$
$$- \frac{3af^2(e+fx)\sinh^2(c+dx)}{4b^2d^3} - \frac{a(e+fx)^3\sinh^2(c+dx)}{2b^2d}$$

output
```
-3/8*a*f^3*x/b^2/d^3+1/4*a*(a^2+b^2)*(f*x+e)^4/b^4/f-1/2*a*(f*x+e)^3*sinh(
d*x+c)^2/b^2/d-6*a^2*f^3*cosh(d*x+c)/b^3/d^4-1/3*f*(f*x+e)^2*cosh(d*x+c)^3
/b/d^2+1/3*(f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)/b/d+6*a^2*f^2*(f*x+e)*sinh(
d*x+c)/b^3/d^3-3/4*a*f^2*(f*x+e)*sinh(d*x+c)^2/b^2/d^3+3/8*a*f^3*cosh(d*x+
c)*sinh(d*x+c)/b^2/d^4+2/9*f^2*(f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/b/d^3-3*a
^2*f*(f*x+e)^2*cosh(d*x+c)/b^3/d^2-40/9*f^3*cosh(d*x+c)/b/d^4-2/27*f^3*cos
h(d*x+c)^3/b/d^4-1/4*a*(f*x+e)^3/b^2/d+a^2*(f*x+e)^3*sinh(d*x+c)/b^3/d+6*a
*(a^2+b^2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d^
3+6*a*(a^2+b^2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b
^4/d^3-3*a*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2
)))/b^4/d^2-3*a*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)
^(1/2)))/b^4/d^2-a*(a^2+b^2)*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2
)))/b^4/d-a*(a^2+b^2)*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4
/d-6*a*(a^2+b^2)*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d^4-
6*a*(a^2+b^2)*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d^4+2/3
*(f*x+e)^3*sinh(d*x+c)/b/d-2*f*(f*x+e)^2*cosh(d*x+c)/b/d^2+40/9*f^2*(f*x+e
)*sinh(d*x+c)/b/d^3+3/4*a*f*(f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/b^2/d^2
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 5656 vs. $2(864) = 1728$.

Time = 31.07 (sec) , antiderivative size = 5656, normalized size of antiderivative = 6.55

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sinh(c+dx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6113}$$

$$\frac{\int (e+fx)^3\cosh^3(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^3\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{b}$$

$$\Big\downarrow \text{ 3792}$$

$$\frac{\frac{2f^2\int (e+fx)\cosh^3(c+dx)dx}{3d^2}+\frac{2}{3}\int (e+fx)^3\cosh(c+dx)dx-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\int (e+fx)^3\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh^2(c+\text{…}}{3d}}{b}$$

$$\Big\downarrow \text{ 3777}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3if\int -i(e+fx)^2\sinh(c+dx)dx}{d}\right)-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{(e+fx)^3\sinh\text{…}}{\text{…}}}{b}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int (e+fx)^2\sinh(c+dx)dx}{d}\right)-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{(e+fx)^3\sinh(c+\text{…}}{3\text{…}}}{b}$$

$\downarrow$ 3042

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$$\frac{\frac{2f^2\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int -i(e+fx)^2\sin(ic+idx)dx}{d}\right)-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{(e+fx)^3\sinh ...}{}}{b}$$

$\downarrow$ 26

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$$\frac{\frac{2f^2\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\int (e+fx)^2\sin(ic+idx)dx}{d}\right)-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{(e+fx)^3\sinh(c ...}{}}{b}$$

$\downarrow$ 3777

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$$\frac{\frac{2f^2\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\cosh(c+dx)dx}{d}\right)}{d}\right)-\frac{f(e+fx)^2\cosh^3(c ...}{3d^2}}{b}$$

$\downarrow$ 3042

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$$\frac{\frac{2f^2\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)-\frac{f(e+fx)^2\cos ...}{3d^2 ...}}{b}$$

$\downarrow$ 3777

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$$\frac{\frac{2f^2\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{d}\right)}{d}\right) ...}{b}$$

$\downarrow$ 26

$$\frac{-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)}{d}\right)}{d}\right)-}{b}}{}$$

$\downarrow$ 3042

$$\frac{-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}\right)}{d}\right)}{b}}{}$$

$\downarrow$ 26

$$\frac{-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}\right)}{d}\right)}{d}\right)-}{b}}{}$$

$\downarrow$ 3118

$$\frac{-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}\right.}{d}\right.}{d}\right.}{b}}{}$$

$\downarrow$ 3791

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\left(\frac{2}{3}\int (e+fx)\cosh(c+dx)dx-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\cdots}{\cdots}\cdots\right)}{b}$$

$\downarrow$ 3042

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\left(\frac{2}{3}\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\cdots\right)}{b}$$

$\downarrow$ 3777

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh\cdots}{d}\cdots\right)}{b}$$

$\downarrow$ 26

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(\cdots}{d}\cdots\right)}{b}$$

$\downarrow$ 3042

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sin}{d}\right.}{b}$$

$\downarrow$ 26

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh}{d}\right.}{b}$$

$\downarrow$ 3118

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+c}{d}\right.}{b}$$

$\downarrow$ 6099

$$-\frac{a\left(\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)^3\cosh(c+dx)dx}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}+$$

$$\frac{\frac{2f^2\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+c}{d}\right.}{b}$$

$\downarrow$ 3042

$$\frac{\frac{2f^2\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+c}{d}\right.}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)^3\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{2f^2\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+c}{d}\right.}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3if\int-i(e+fx)^2\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int(e+fx)^2\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}+$$

$$\frac{\frac{2f^2\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+c}{d}\right.}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2f^2\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+d}{d}\right.}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int-i(e+fx)^2\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}$$

$\downarrow$ 26

$$\frac{\frac{2f^2\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+d}{d}\right.}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\int(e+fx)^2\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}$$

$\downarrow$ 3777

$$\frac{\frac{2f^2\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+d}{d}\right.}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\cosh(c+dx)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c+d}{b}\right.}{b}$$

$\downarrow$ 3042

$$\frac{\frac{2f^2\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+}{d}\right.}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(}{}\right.}{b}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int\frac{(fx+e)^3\cosh(dx+c)^3\sinh(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7980 vs. $2(810) = 1620$.

Time = 0.20 (sec) , antiderivative size = 7980, normalized size of antiderivative = 9.24

$$\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)**3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)^3\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/24*e^3*((3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 + 3*b^2)*e^(-2*d*x - 2*c))
*e^(3*d*x + 3*c)/(b^3*d) + 24*(a^3 + a*b^2)*(d*x + c)/(b^4*d) + (3*a*b*e^(
-2*d*x - 2*c) + b^2*e^(-3*d*x - 3*c) + 3*(4*a^2 + 3*b^2)*e^(-d*x - c))/(b^
3*d) + 24*(a^3 + a*b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b
^4*d)) - 1/864*(216*(a^3*d^4*f^3*e^(3*c) + a*b^2*d^4*f^3*e^(3*c))*x^4 + 86
4*(a^3*d^4*e*f^2*e^(3*c) + a*b^2*d^4*e*f^2*e^(3*c))*x^3 + 1296*(a^3*d^4*e^
2*f*e^(3*c) + a*b^2*d^4*e^2*f*e^(3*c))*x^2 - 4*(9*b^3*d^3*f^3*x^3*e^(6*c)
+ 9*(3*d^3*e*f^2 - d^2*f^3)*b^3*x^2*e^(6*c) + 3*(9*d^3*e^2*f - 6*d^2*e*f^2
 + 2*d*f^3)*b^3*x*e^(6*c) - (9*d^2*e^2*f - 6*d*e*f^2 + 2*f^3)*b^3*e^(6*c))
*e^(3*d*x) + 27*(4*a*b^2*d^3*f^3*x^3*e^(5*c) + 6*(2*d^3*e*f^2 - d^2*f^3)*a
*b^2*x^2*e^(5*c) + 6*(2*d^3*e^2*f - 2*d^2*e*f^2 + d*f^3)*a*b^2*x*e^(5*c) -
 3*(2*d^2*e^2*f - 2*d*e*f^2 + f^3)*a*b^2*e^(5*c))*e^(2*d*x) + 108*(12*(d^2
*e^2*f - 2*d*e*f^2 + 2*f^3)*a^2*b*e^(4*c) + 9*(d^2*e^2*f - 2*d*e*f^2 + 2*f
^3)*b^3*e^(4*c) - (4*a^2*b*d^3*f^3*e^(4*c) + 3*b^3*d^3*f^3*e^(4*c))*x^3 -
3*(4*(d^3*e*f^2 - d^2*f^3)*a^2*b*e^(4*c) + 3*(d^3*e*f^2 - d^2*f^3)*b^3*e^(
4*c))*x^2 - 3*(4*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*a^2*b*e^(4*c) + 3*(d^
3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*b^3*e^(4*c))*x)*e^(d*x) + 108*(12*(d^2*e^
2*f + 2*d*e*f^2 + 2*f^3)*a^2*b*e^(2*c) + 9*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)
*b^3*e^(2*c) + (4*a^2*b*d^3*f^3*e^(2*c) + 3*b^3*d^3*f^3*e^(2*c))*x^3 + 3*(
4*(d^3*e*f^2 + d^2*f^3)*a^2*b*e^(2*c) + 3*(d^3*e*f^2 + d^2*f^3)*b^3*e^(...
```

Giac [F]

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\cosh(dx+c)^3\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)^3*sinh(d*x + c)/(b*sinh(d*x + c) + a),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^3\sinh(c+d\,x)\,\left(e+f\,x\right)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^3*sinh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh\left(dx+c\right)^3\sinh\left(dx+c\right)}{a+b\sinh\left(dx+c\right)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.344 $\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 623

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\begin{aligned}
&=-\frac{a(e+fx)^2}{4b^2d}+\frac{a(a^2+b^2)\,(e+fx)^3}{3b^4f}-\frac{2a^2f(e+fx)\cosh(c+dx)}{b^3d^2}\\
&-\frac{4f(e+fx)\cosh(c+dx)}{3bd^2}-\frac{2f(e+fx)\cosh^3(c+dx)}{9bd^2}\\
&-\frac{a(a^2+b^2)\,(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d}\\
&-\frac{a(a^2+b^2)\,(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d}\\
&-\frac{2a(a^2+b^2)\,f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}\\
&-\frac{2a(a^2+b^2)\,f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^2}\\
&+\frac{2a(a^2+b^2)\,f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^3}+\frac{2a(a^2+b^2)\,f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^3}\\
&+\frac{2a^2f^2\sinh(c+dx)}{b^3d^3}+\frac{14f^2\sinh(c+dx)}{9bd^3}+\frac{a^2(e+fx)^2\sinh(c+dx)}{b^3d}\\
&+\frac{2(e+fx)^2\sinh(c+dx)}{3bd}+\frac{af(e+fx)\cosh(c+dx)\sinh(c+dx)}{2b^2d^2}\\
&+\frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{3bd}-\frac{af^2\sinh^2(c+dx)}{4b^2d^3}\\
&-\frac{a(e+fx)^2\sinh^2(c+dx)}{2b^2d}+\frac{2f^2\sinh^3(c+dx)}{27bd^3}
\end{aligned}$$

output
```
-1/4*a*(f*x+e)^2/b^2/d+1/3*a*(a^2+b^2)*(f*x+e)^3/b^4/f-2*a^2*f*(f*x+e)*cos
h(d*x+c)/b^3/d^2-4/3*f*(f*x+e)*cosh(d*x+c)/b/d^2-2/9*f*(f*x+e)*cosh(d*x+c)
^3/b/d^2-a*(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/
d-a*(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d-2*a*(
a^2+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d^2-2*
a*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d^2
+2*a*(a^2+b^2)*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d^3+2*
a*(a^2+b^2)*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d^3+2*a^2
*f^2*sinh(d*x+c)/b^3/d^3+14/9*f^2*sinh(d*x+c)/b/d^3+a^2*(f*x+e)^2*sinh(d*x
+c)/b^3/d+2/3*(f*x+e)^2*sinh(d*x+c)/b/d+1/2*a*f*(f*x+e)*cosh(d*x+c)*sinh(d
*x+c)/b^2/d^2+1/3*(f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/b/d-1/4*a*f^2*sinh(d
*x+c)^2/b^2/d^3-1/2*a*(f*x+e)^2*sinh(d*x+c)^2/b^2/d+2/27*f^2*sinh(d*x+c)^3
/b/d^3
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1961 vs. $2(623) = 1246$.

Time = 7.72 (sec) , antiderivative size = 1961, normalized size of antiderivative = 3.15

$$\int \frac{(e+fx)^2 \cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
(f^2*((4*a*x^3)/(-1 + E^(2*c)) - 2*a*x^3*Coth[c] - (6*a*b^2*(d^2*x^2*Log[1
 + ((a - Sqrt[a^2 + b^2])*E^(-c - d*x))/b] - 2*d*x*PolyLog[2, ((-a + Sqrt[
a^2 + b^2])*E^(-c - d*x))/b] - 2*PolyLog[3, ((-a + Sqrt[a^2 + b^2])*E^(-c
- d*x))/b]))/(Sqrt[a^2 + b^2]*(-a + Sqrt[a^2 + b^2])*d^3) - (6*a*b^2*(d^2*
x^2*Log[1 + ((a + Sqrt[a^2 + b^2])*E^(-c - d*x))/b] - 2*d*x*PolyLog[2, -((
(a + Sqrt[a^2 + b^2])*E^(-c - d*x))/b)] - 2*PolyLog[3, -(((a + Sqrt[a^2 +
b^2])*E^(-c - d*x))/b)]))/(Sqrt[a^2 + b^2]*(a + Sqrt[a^2 + b^2])*d^3) + (6
*a^2*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*d*x*PolyL
og[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*PolyLog[3, (b*E^(c + d*x
))/(-a + Sqrt[a^2 + b^2])]))/(Sqrt[a^2 + b^2]*d^3) - (6*a^2*(d^2*x^2*Log[1
 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))] - 2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))]))/(Sqrt[a^2 + b^2]*d^3) + (6*b*Cosh[d*x]*(-2*d*x*Cosh[c] + (2
+ d^2*x^2)*Sinh[c]))/d^3 + (6*b*((2 + d^2*x^2)*Cosh[c] - 2*d*x*Sinh[c])*Si
nh[d*x])/d^3))/(12*b^2) - (e^2*((a*Log[a + b*Sinh[c + d*x]])/b^2 - Sinh[c
+ d*x]/b))/(2*d) + (e*f*(-2*b*Cosh[c + d*x] - a*(2*c*(c + d*x) - (c + d*x)
^2 + 2*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*(c + d
*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*Log[b - 2*a*E^(c
+ d*x) - b*E^(2*(c + d*x))] + 2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2
+ b^2])] + 2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) + 2*...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sinh(c+dx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6113}$$

$$\frac{\int (e+fx)^2\cosh^3(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{b}$$

$$\Big\downarrow \text{3792}$$

$$\frac{\frac{2f^2\int \cosh^3(c+dx)dx}{9d^2}+\frac{2}{3}\int (e+fx)^2\cosh(c+dx)dx-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int \sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{9d^2}+\frac{2}{3}\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}$$

$$\Big\downarrow \; 3113$$

$$-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2if^2\int \left(\sinh^2(c+dx)+1\right)d(-i\sinh(c+dx))}{9d^3}+\frac{2}{3}\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}$$

$$\Big\downarrow \; 3777$$

$$-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int (e+fx)\sinh(c+dx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int -i(e+fx)\sin(ic+idx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2 s}{}}{b}$$

$\Big\downarrow$ 26

$$-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int(e+fx)\sin(ic+idx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2 \sin}{}}{b}$$

$\Big\downarrow$ 3777

$$-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx}{9d^2}}{b}$$

$\Big\downarrow$ 3042

$$-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3}{9d^2}}{b}$$

$\Big\downarrow$ 3117

$$-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$

$\Big\downarrow$ 6099

$$-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)^2\cosh(c+dx)dx}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}+\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$

$\downarrow$ 3042

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$
$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}$$

$\downarrow$ 3777

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$
$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int-i(e+fx)\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}$$

$\downarrow$ 26

$$-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int(e+fx)\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}+$$
$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$

$\downarrow$ 3042

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$
$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int-i(e+fx)\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}$$

$\downarrow$ 26

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b}+\cdots$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int(e+fx)\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}$$

$\downarrow$ 3777

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b}+\cdots$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c\cdots}{b}\cdots\right)}{b}$$

$\downarrow$ 3042

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b}+\cdots$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sin\cdots}{b}\cdots\right)}{b}$$

$\downarrow$ 3117

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b}+\cdots$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\cdots\right)}{d}\cdots\right)}{b^2}\cdots\right)}{b}$$

$\downarrow$ 5969

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}-\frac{f\int(e+fx)\sinh^2(c+dx)dx}{d}}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sin}{}\right.}{d}\right.}{b^2}\right.}{b}$$

$\downarrow$ 3042

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}-\frac{f\int-\left((e+fx)\sin(ic+idx)^2\right)dx}{d}}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\right.}{d}\right.}{b^2}\right.}{b}$$

$\downarrow$ 25

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}+\frac{f\int(e+fx)\sin(ic+idx)^2dx}{d}}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sin}{}\right.}{d}\right.}{b^2}\right.}{b}$$

$\downarrow$ 3791

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{f\left(\frac{1}{2}\int(e+fx)dx+\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(\right.}{d}}{}\right.}{b}$$

$\downarrow$ 17

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b^2}+\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+d}{2d}\right.}{d}\right.}{b}$$

$\downarrow$ 6095

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\left(\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right.}{b^2}\right.}{b}$$

$\downarrow$ 2620

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$

$$a\left(\frac{(a^2+b^2)\left(-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right.}{b^2}\right.$$

$$\Big\downarrow \text{3011}$$

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$

$$a\left(\frac{(a^2+b^2)\left(-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\right.}{d}\right.}{bd}\right.}{b^2}\right.$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^2\cosh(dx+c)^3\sinh(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4887 vs. 2(581) = 1162.

Time = 0.17 (sec) , antiderivative size = 4887, normalized size of antiderivative = 7.84

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)**3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)^3\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/24*e^2*((3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 + 3*b^2)*e^(-2*d*x - 2*c))
*e^(3*d*x + 3*c)/(b^3*d) + 24*(a^3 + a*b^2)*(d*x + c)/(b^4*d) + (3*a*b*e^(
-2*d*x - 2*c) + b^2*e^(-3*d*x - 3*c) + 3*(4*a^2 + 3*b^2)*e^(-d*x - c))/(b^
3*d) + 24*(a^3 + a*b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b
^4*d)) - 1/432*(144*(a^3*d^3*f^2*e^(3*c) + a*b^2*d^3*f^2*e^(3*c))*x^3 + 43
2*(a^3*d^3*e*f*e^(3*c) + a*b^2*d^3*e*f*e^(3*c))*x^2 - 2*(9*b^3*d^2*f^2*x^2
*e^(6*c) + 6*(3*d^2*e*f - d*f^2)*b^3*x*e^(6*c) - 2*(3*d*e*f - f^2)*b^3*e^(
6*c))*e^(3*d*x) + 27*(2*a*b^2*d^2*f^2*x^2*e^(5*c) + 2*(2*d^2*e*f - d*f^2)*
a*b^2*x*e^(5*c) - (2*d*e*f - f^2)*a*b^2*e^(5*c))*e^(2*d*x) + 54*(8*(d*e*f
- f^2)*a^2*b*e^(4*c) + 6*(d*e*f - f^2)*b^3*e^(4*c) - (4*a^2*b*d^2*f^2*e^(4
*c) + 3*b^3*d^2*f^2*e^(4*c))*x^2 - 2*(4*(d^2*e*f - d*f^2)*a^2*b*e^(4*c) +
3*(d^2*e*f - d*f^2)*b^3*e^(4*c))*x)*e^(d*x) + 54*(8*(d*e*f + f^2)*a^2*b*e^
(2*c) + 6*(d*e*f + f^2)*b^3*e^(2*c) + (4*a^2*b*d^2*f^2*e^(2*c) + 3*b^3*d^2
*f^2*e^(2*c))*x^2 + 2*(4*(d^2*e*f + d*f^2)*a^2*b*e^(2*c) + 3*(d^2*e*f + d*
f^2)*b^3*e^(2*c))*x)*e^(-d*x) + 27*(2*a*b^2*d^2*f^2*x^2*e^c + 2*(2*d^2*e*f
 + d*f^2)*a*b^2*x*e^c + (2*d*e*f + f^2)*a*b^2*e^c)*e^(-2*d*x) + 2*(9*b^3*d
^2*f^2*x^2 + 6*(3*d^2*e*f + d*f^2)*b^3*x + 2*(3*d*e*f + f^2)*b^3)*e^(-3*d*
x))*e^(-3*c)/(b^4*d^3) + integrate(-2*((a^3*b*f^2 + a*b^3*f^2)*x^2 + 2*(a^
3*b*e*f + a*b^3*e*f)*x - ((a^4*f^2*e^c + a^2*b^2*f^2*e^c)*x^2 + 2*(a^4*e*f
*e^c + a^2*b^2*e*f*e^c)*x)*e^(d*x))/(b^5*e^(2*d*x + 2*c) + 2*a*b^4*e^(d...
```

Giac [F]

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\cosh{(dx+c)}^3\sinh{(dx+c)}}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)^3*sinh(d*x + c)/(b*sinh(d*x + c) + a),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^3\sinh(c+d\,x)\,\left(e+f\,x\right)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^3*sinh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
(18*e**(6*c + 6*d*x)*b**6*d**2*e**2 + 36*e**(6*c + 6*d*x)*b**6*d**2*e*f*x
+ 18*e**(6*c + 6*d*x)*b**6*d**2*f**2*x**2 - 12*e**(6*c + 6*d*x)*b**6*d*e*f
 - 12*e**(6*c + 6*d*x)*b**6*d*f**2*x + 4*e**(6*c + 6*d*x)*b**6*f**2 - 54*e
**(5*c + 5*d*x)*a*b**5*d**2*e**2 - 108*e**(5*c + 5*d*x)*a*b**5*d**2*e*f*x
- 54*e**(5*c + 5*d*x)*a*b**5*d**2*f**2*x**2 + 54*e**(5*c + 5*d*x)*a*b**5*d
*e*f + 54*e**(5*c + 5*d*x)*a*b**5*d*f**2*x - 27*e**(5*c + 5*d*x)*a*b**5*f*
*2 + 216*e**(4*c + 4*d*x)*a**2*b**4*d**2*e**2 + 432*e**(4*c + 4*d*x)*a**2*
b**4*d**2*e*f*x + 216*e**(4*c + 4*d*x)*a**2*b**4*d**2*f**2*x**2 - 432*e**(
4*c + 4*d*x)*a**2*b**4*d*e*f - 432*e**(4*c + 4*d*x)*a**2*b**4*d*f**2*x + 4
32*e**(4*c + 4*d*x)*a**2*b**4*f**2 + 162*e**(4*c + 4*d*x)*b**6*d**2*e**2 +
 324*e**(4*c + 4*d*x)*b**6*d**2*e*f*x + 162*e**(4*c + 4*d*x)*b**6*d**2*f**
2*x**2 - 324*e**(4*c + 4*d*x)*b**6*d*e*f - 324*e**(4*c + 4*d*x)*b**6*d*f**
2*x + 324*e**(4*c + 4*d*x)*b**6*f**2 + 3456*e**(3*c + 3*d*x)*int(x**2/(e**
(5*c + 5*d*x)*b + 2*e**(4*c + 4*d*x)*a - e**(3*c + 3*d*x)*b),x)*a**6*b*d**
3*f**2 + 6048*e**(3*c + 3*d*x)*int(x**2/(e**(5*c + 5*d*x)*b + 2*e**(4*c +
4*d*x)*a - e**(3*c + 3*d*x)*b),x)*a**4*b**3*d**3*f**2 + 2592*e**(3*c + 3*d
*x)*int(x**2/(e**(5*c + 5*d*x)*b + 2*e**(4*c + 4*d*x)*a - e**(3*c + 3*d*x)
*b),x)*a**2*b**5*d**3*f**2 + 6912*e**(3*c + 3*d*x)*int(x/(e**(5*c + 5*d*x)
*b + 2*e**(4*c + 4*d*x)*a - e**(3*c + 3*d*x)*b),x)*a**6*b*d**3*e*f + 12096
*e**(3*c + 3*d*x)*int(x/(e**(5*c + 5*d*x)*b + 2*e**(4*c + 4*d*x)*a - e*...
```

3.345 $\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 400

$$\begin{aligned}
&\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= -\frac{afx}{4b^2d}+\frac{a(a^2+b^2)\,(e+fx)^2}{2b^4f}-\frac{a^2f\cosh(c+dx)}{b^3d^2}-\frac{2f\cosh(c+dx)}{3bd^2} \\
&\quad-\frac{f\cosh^3(c+dx)}{9bd^2}-\frac{a(a^2+b^2)\,(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d} \\
&\quad-\frac{a(a^2+b^2)\,(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d}-\frac{a(a^2+b^2)\,f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2} \\
&\quad-\frac{a(a^2+b^2)\,f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^2}+\frac{a^2(e+fx)\sinh(c+dx)}{b^3d} \\
&\quad+\frac{2(e+fx)\sinh(c+dx)}{3bd}+\frac{af\cosh(c+dx)\sinh(c+dx)}{4b^2d^2} \\
&\quad+\frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{3bd}-\frac{a(e+fx)\sinh^2(c+dx)}{2b^2d}
\end{aligned}$$

output
```
-1/4*a*f*x/b^2/d+1/2*a*(a^2+b^2)*(f*x+e)^2/b^4/f-a^2*f*cosh(d*x+c)/b^3/d^2
-2/3*f*cosh(d*x+c)/b/d^2-1/9*f*cosh(d*x+c)^3/b/d^2-a*(a^2+b^2)*(f*x+e)*ln(
1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d-a*(a^2+b^2)*(f*x+e)*ln(1+b*exp(d
*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d-a*(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))/b^4/d^2-a*(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/b^4/d^2+a^2*(f*x+e)*sinh(d*x+c)/b^3/d+2/3*(f*x+e)*sinh(d*x+c)/b
/d+1/4*a*f*cosh(d*x+c)*sinh(d*x+c)/b^2/d^2+1/3*(f*x+e)*cosh(d*x+c)^2*sinh(
d*x+c)/b/d-1/2*a*(f*x+e)*sinh(d*x+c)^2/b^2/d
```

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 604, normalized size of antiderivative = 1.51

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{-36b^2de(-a\log(a+b\sinh(c+dx))+b\sinh(c+dx))+18b^2f\Big(2b\cosh(c+dx)+a\Big(2c(c+dx)-(c+$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
-1/72*(-36*b^2*d*e*(-(a*Log[a + b*Sinh[c + d*x]]) + b*Sinh[c + d*x]) + 18*
b^2*f*(2*b*Cosh[c + d*x] + a*(2*c*(c + d*x) - (c + d*x)^2 + 2*(c + d*x)*Lo
g[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*(c + d*x)*Log[1 + (b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c
+ d*x))] + 2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*PolyLo
g[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) - 2*b*d*x*Sinh[c + d*x]) +
 12*d*e*(3*a*(2*a^2 + b^2)*Log[a + b*Sinh[c + d*x]] - 3*b*(2*a^2 + b^2)*Si
nh[c + d*x] + 3*a*b^2*Sinh[c + d*x]^2 - 2*b^3*Sinh[c + d*x]^3) + f*(18*b*(
4*a^2 + b^2)*Cosh[c + d*x] + 18*a*b^2*d*x*Cosh[2*(c + d*x)] + 2*b^3*Cosh[3
*(c + d*x)] + 18*a*(2*a^2 + b^2)*(2*c*(c + d*x) - (c + d*x)^2 + 2*(c + d*x
)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*(c + d*x)*Log[1 + (b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*Log[b - 2*a*E^(c + d*x) - b*E^(2
*(c + d*x))] + 2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*Po
lyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) - 18*b*(4*a^2 + b^2)*d
*x*Sinh[c + d*x] - 9*a*b^2*Sinh[2*(c + d*x)] - 6*b^3*d*x*Sinh[3*(c + d*x)]
))/(b^4*d^2)
```

Rubi [A] (verified)

Time = 2.49 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.90, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.781$, Rules used = {6113, 3042, 3791, 3042, 3777, 26, 3042, 26, 3118, 6099, 3042, 3777, 26, 3042, 26, 3118, 5969, 3042, 25, 3115, 24, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \; 6113$$

$$\frac{\int (e+fx)\cosh^3(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{b}$$

$$\Big\downarrow \; 3791$$

$$\frac{\frac{2}{3}\int (e+fx)\cosh(c+dx)dx-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}$$

$$\Big\downarrow \; 3777$$

$$-\frac{a\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$\big\downarrow$ 3042

$$-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int-i\sin(ic+idx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}$$

$\big\downarrow$ 26

$$-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int\sin(ic+idx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}$$

$\big\downarrow$ 3118

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$\big\downarrow$ 6099

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)\cosh(c+dx)dx}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}$$

$\big\downarrow$ 3042

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}$$

$\big\downarrow$ 3777

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int-i\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}$$

$\downarrow$ 26

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}$$

$\downarrow$ 3042

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int-i\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}$$

$\downarrow$ 26

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}$$

$\downarrow$ 3118

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}\right)}{b}$$

$\downarrow$ 5969

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}-\frac{f\int\sinh^2(c+dx)dx}{2d}}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}-\frac{f\int-\sin(ic+idx)^2dx}{2d}}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}\right)}{b}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\int\sin(ic+idx)^2dx}{2d}}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}\right)}{b}$$

$$\Big\downarrow \; 3115$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{f\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}+\frac{(e+fx)\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}\right)}{b}$$

$$\Big\downarrow \; 24$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}\right)}{b}$$

$$\Big\downarrow \; 6095$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\left(\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}+\frac{(e+fx)\sinh^2(c+dx)}{2d}+f\left(\cdots\right.\right.}{b}$$

$$\Big\downarrow \; 2620$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\left(-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{b^2}-\frac{a}{}\right.}{b}$$

$$\Big\downarrow \textbf{2715}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}}\right.}{bd}\right.}{b^2}\right.}{b}$$

$$\Big\downarrow \textbf{2838}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{b^2}-\right.}{b}$$

input
```
Int[((e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-1/9*(f*Cosh[c + d*x]^3)/d^2 + ((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/
(3*d) + (2*(-((f*Cosh[c + d*x])/d^2) + ((e + f*x)*Sinh[c + d*x])/d))/3)/b
- (a*(((a^2 + b^2)*(-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/(b*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2]))])/(b*d^2)))/b^2 - (a*(-((f*Cosh[c + d*x])/d^2) + ((e + f*x)*Sinh[c +
d*x])/d))/b^2 + (((e + f*x)*Sinh[c + d*x]^2)/(2*d) + (f*(x/2 - (Cosh[c + d
*x]*Sinh[c + d*x])/(2*d)))/(2*d))/b))/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1101 vs. $2(372) = 744$.

Time = 13.59 (sec) , antiderivative size = 1102, normalized size of antiderivative = 2.76

method	result	size
risch	Expression too large to display	1102

input
```
int((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
2/d*a/b^2*e*ln(exp(d*x+c))+2/d*a^3/b^4*e*ln(exp(d*x+c))-1/d*a^3/b^4*e*ln(b
*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/d^2*a^3/b^4*f*dilog((-b*exp(d*x+c)+(a^
2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2*a^3/b^4*f*dilog((b*exp(d*x+c)+
(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/d^2*a^3/b^4*f*c^2-a/b^2*e*x+2/d*
a/b^2*f*c*x-1/d*a/b^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)
^(1/2)))*x-1/d*a/b^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1
/2)))*x-1/d^2*a/b^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(
1/2)))*c-2/d^2*a/b^2*c*f*ln(exp(d*x+c))+1/d^2*a/b^2*c*f*ln(b*exp(2*d*x+2*c
)+2*a*exp(d*x+c)-b)-1/d*a^3/b^4*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(
a^2+b^2)^(1/2)))*x-1/d^2*a^3/b^4*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-
a+(a^2+b^2)^(1/2)))*c-1/d^2*a^3/b^4*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/
(a+(a^2+b^2)^(1/2)))*c-2/d^2*a^3/b^4*c*f*ln(exp(d*x+c))+1/d^2*a^3/b^4*c*f*
ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/d^2*a/b^2*f*ln((b*exp(d*x+c)+(a^2+
b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d*a^3/b^4*f*ln((-b*exp(d*x+c)+(a^2+
b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d^2*a/b^2*f*c^2-1/d^2*a/b^2*f*dilo
g((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2*a/b^2*f*di
log((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d*a/b^2*e*ln(b
*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/16*a*(2*d*f*x+2*d*e-f)/b^2/d^2*exp(2*d
*x+2*c)-1/8*(4*a^2+3*b^2)*(d*f*x+d*e+f)/b^3/d^2*exp(-d*x-c)-1/16*a*(2*d*f*
x+2*d*e+f)/b^2/d^2*exp(-2*d*x-2*c)+1/2*a^3/b^4*f*x^2+1/2*a/b^2*f*x^2-a^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2465 vs. $2(370) = 740$.

Time $= 0.14$ (sec) , antiderivative size $= 2465$, normalized size of antiderivative $= 6.16$

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
1/144*(2*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^6 + 2*(3*b^3*d*f*
x + 3*b^3*d*e - b^3*f)*sinh(d*x + c)^6 - 6*b^3*d*f*x - 9*(2*a*b^2*d*f*x +
2*a*b^2*d*e - a*b^2*f)*cosh(d*x + c)^5 - 3*(6*a*b^2*d*f*x + 6*a*b^2*d*e -
3*a*b^2*f - 4*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c))*sinh(d*x +
c)^5 - 6*b^3*d*e + 18*((4*a^2*b + 3*b^3)*d*f*x + (4*a^2*b + 3*b^3)*d*e - (
4*a^2*b + 3*b^3)*f)*cosh(d*x + c)^4 + 3*(6*(4*a^2*b + 3*b^3)*d*f*x + 6*(4*
a^2*b + 3*b^3)*d*e + 10*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^2
- 6*(4*a^2*b + 3*b^3)*f - 15*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*cosh(
d*x + c))*sinh(d*x + c)^4 - 2*b^3*f + 72*((a^3 + a*b^2)*d^2*f*x^2 + 2*(a^3
 + a*b^2)*d^2*e*x + 4*(a^3 + a*b^2)*c*d*e - 2*(a^3 + a*b^2)*c^2*f)*cosh(d*
x + c)^3 + 2*(36*(a^3 + a*b^2)*d^2*f*x^2 + 72*(a^3 + a*b^2)*d^2*e*x + 144*
(a^3 + a*b^2)*c*d*e - 72*(a^3 + a*b^2)*c^2*f + 20*(3*b^3*d*f*x + 3*b^3*d*e
 - b^3*f)*cosh(d*x + c)^3 - 45*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*cos
h(d*x + c)^2 + 36*((4*a^2*b + 3*b^3)*d*f*x + (4*a^2*b + 3*b^3)*d*e - (4*a^
2*b + 3*b^3)*f)*cosh(d*x + c))*sinh(d*x + c)^3 - 18*((4*a^2*b + 3*b^3)*d*f
*x + (4*a^2*b + 3*b^3)*d*e + (4*a^2*b + 3*b^3)*f)*cosh(d*x + c)^2 + 6*(5*(
3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^4 - 3*(4*a^2*b + 3*b^3)*d*f
*x - 15*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*cosh(d*x + c)^3 - 3*(4*a^2
*b + 3*b^3)*d*e + 18*((4*a^2*b + 3*b^3)*d*f*x + (4*a^2*b + 3*b^3)*d*e - (4
*a^2*b + 3*b^3)*f)*cosh(d*x + c)^2 - 3*(4*a^2*b + 3*b^3)*f + 36*((a^3 +...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)**3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)^3\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-1/24*e*((3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 + 3*b^2)*e^(-2*d*x - 2*c))*e
^(3*d*x + 3*c)/(b^3*d) + 24*(a^3 + a*b^2)*(d*x + c)/(b^4*d) + (3*a*b*e^(-2
*d*x - 2*c) + b^2*e^(-3*d*x - 3*c) + 3*(4*a^2 + 3*b^2)*e^(-d*x - c))/(b^3*
d) + 24*(a^3 + a*b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^4
*d)) - 1/144*f*((72*(a^3*d^2*e^(3*c) + a*b^2*d^2*e^(3*c))*x^2 - 2*(3*b^3*d
*x*e^(6*c) - b^3*e^(6*c))*e^(3*d*x) + 9*(2*a*b^2*d*x*e^(5*c) - a*b^2*e^(5*
c))*e^(2*d*x) + 18*(4*a^2*b*e^(4*c) + 3*b^3*e^(4*c) - (4*a^2*b*d*e^(4*c) +
 3*b^3*d*e^(4*c))*x)*e^(d*x) + 18*(4*a^2*b*e^(2*c) + 3*b^3*e^(2*c) + (4*a^
2*b*d*e^(2*c) + 3*b^3*d*e^(2*c))*x)*e^(-d*x) + 9*(2*a*b^2*d*x*e^c + a*b^2*
e^c)*e^(-2*d*x) + 2*(3*b^3*d*x + b^3)*e^(-3*d*x))*e^(-3*c)/(b^4*d^2) - 9*i
ntegrate(32*((a^4*e^c + a^2*b^2*e^c)*x*e^(d*x) - (a^3*b + a*b^3)*x)/(b^5*e
^(2*d*x + 2*c) + 2*a*b^4*e^(d*x + c) - b^5), x))
```

Giac [F]

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)\cosh\left(dx+c\right)^3\sinh\left(dx+c\right)}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)^3*sinh(d*x + c)/(b*sinh(d*x + c) + a), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\cosh(c+d\,x)^3\sinh(c+d\,x)\,\left(e+f\,x\right)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^3*sinh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Too large to display}$$

input
```
int((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
(6*e**(6*c + 6*d*x)*b**6*d*e + 6*e**(6*c + 6*d*x)*b**6*d*f*x - 2*e**(6*c +
 6*d*x)*b**6*f - 18*e**(5*c + 5*d*x)*a*b**5*d*e - 18*e**(5*c + 5*d*x)*a*b*
*5*d*f*x + 9*e**(5*c + 5*d*x)*a*b**5*f + 72*e**(4*c + 4*d*x)*a**2*b**4*d*e
 + 72*e**(4*c + 4*d*x)*a**2*b**4*d*f*x - 72*e**(4*c + 4*d*x)*a**2*b**4*f +
 54*e**(4*c + 4*d*x)*b**6*d*e + 54*e**(4*c + 4*d*x)*b**6*d*f*x - 54*e**(4*
c + 4*d*x)*b**6*f + 1152*e**(3*c + 3*d*x)*int(x/(e**(5*c + 5*d*x)*b + 2*e*
*(4*c + 4*d*x)*a - e**(3*c + 3*d*x)*b),x)*a**6*b*d**2*f + 2016*e**(3*c + 3
*d*x)*int(x/(e**(5*c + 5*d*x)*b + 2*e**(4*c + 4*d*x)*a - e**(3*c + 3*d*x)*
b),x)*a**4*b**3*d**2*f + 864*e**(3*c + 3*d*x)*int(x/(e**(5*c + 5*d*x)*b +
2*e**(4*c + 4*d*x)*a - e**(3*c + 3*d*x)*b),x)*a**2*b**5*d**2*f - 144*e**(3
*c + 3*d*x)*log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*a**3*b**3*d*e -
 144*e**(3*c + 3*d*x)*log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*a*b**
5*d*e + 144*e**(3*c + 3*d*x)*a**3*b**3*d**2*e*x - 72*e**(3*c + 3*d*x)*a**3
*b**3*d**2*f*x**2 + 144*e**(3*c + 3*d*x)*a*b**5*d**2*e*x - 72*e**(3*c + 3*
d*x)*a*b**5*d**2*f*x**2 - 2304*e**(c + 3*d*x)*int(x/(e**(2*c + 4*d*x)*b +
2*e**(c + 3*d*x)*a - e**(2*d*x)*b),x)*a**7*d**2*f - 4608*e**(c + 3*d*x)*in
t(x/(e**(2*c + 4*d*x)*b + 2*e**(c + 3*d*x)*a - e**(2*d*x)*b),x)*a**5*b**2*
d**2*f - 2592*e**(c + 3*d*x)*int(x/(e**(2*c + 4*d*x)*b + 2*e**(c + 3*d*x)*
a - e**(2*d*x)*b),x)*a**3*b**4*d**2*f - 288*e**(c + 3*d*x)*int(x/(e**(2*c
+ 4*d*x)*b + 2*e**(c + 3*d*x)*a - e**(2*d*x)*b),x)*a*b**6*d**2*f - 288*...
```

3.346 $\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 85

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a(a^2+b^2)\log(a+b\sinh(c+dx))}{b^4d} + \frac{(a^2+b^2)\sinh(c+dx)}{b^3d} - \frac{a\sinh^2(c+dx)}{2b^2d} + \frac{\sinh^3(c+dx)}{3bd}$$

output
```
-a*(a^2+b^2)*ln(a+b*sinh(d*x+c))/b^4/d+(a^2+b^2)*sinh(d*x+c)/b^3/d-1/2*a*s
inh(d*x+c)^2/b^2/d+1/3*sinh(d*x+c)^3/b/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{-6a(a^2+b^2)\log(a+b\sinh(c+dx))+6b(a^2+b^2)\sinh(c+dx)-3ab^2\sinh^2(c+dx)+2b^3\sinh^3(c+dx}{6b^4d}$$

input
```
Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-6*a*(a^2 + b^2)*Log[a + b*Sinh[c + d*x]] + 6*b*(a^2 + b^2)*Sinh[c + d*x]
 - 3*a*b^2*Sinh[c + d*x]^2 + 2*b^3*Sinh[c + d*x]^3)/(6*b^4*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3042, 26, 3316, 26, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ic+idx)\cos(ic+idx)^3}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos(ic+idx)^3\sin(ic+idx)}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3316}$$

$$\frac{i\int -\frac{i\sinh(c+dx)\left(\sinh^2(c+dx)b^2+b^2\right)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^3d}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\sinh(c+dx)\left(\sinh^2(c+dx)b^2+b^2\right)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b\sinh(c+dx)\left(\sinh^2(c+dx)b^2+b^2\right)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^4d}$$

$$\Big\downarrow \text{522}$$

$$\frac{\int \left(\left(\frac{b^2}{a^2}+1\right)a^2-b\sinh(c+dx)a-\frac{(a^2+b^2)a}{a+b\sinh(c+dx)}+b^2\sinh^2(c+dx)\right)d(b\sinh(c+dx))}{b^4d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b(a^2+b^2)\sinh(c+dx)-a(a^2+b^2)\log(a+b\sinh(c+dx))-\frac{1}{2}ab^2\sinh^2(c+dx)+\frac{1}{3}b^3\sinh^3(c+dx)}{b^4d}$$

input
```
Int[(Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-(a*(a^2 + b^2)*Log[a + b*Sinh[c + d*x]]) + b*(a^2 + b^2)*Sinh[c + d*x] -
 (a*b^2*Sinh[c + d*x]^2)/2 + (b^3*Sinh[c + d*x]^3)/3)/(b^4*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 8.60 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\frac{\frac{\sinh(dx+c)^3 b^2}{3}-\frac{a\sinh(dx+c)^2 b}{2}+\sinh(dx+c)a^2+b^2\sinh(dx+c)}{b^3}-\frac{a\left(a^2+b^2\right)\ln(a+b\sinh(dx+c))}{b^4}}{d}$
default	$\frac{\frac{\frac{\sinh(dx+c)^3 b^2}{3}-\frac{a\sinh(dx+c)^2 b}{2}+\sinh(dx+c)a^2+b^2\sinh(dx+c)}{b^3}-\frac{a\left(a^2+b^2\right)\ln(a+b\sinh(dx+c))}{b^4}}{d}$
risch	$\frac{a^3x}{b^4}+\frac{ax}{b^2}+\frac{e^{3dx+3c}}{24bd}-\frac{a\,e^{2dx+2c}}{8b^2d}+\frac{e^{dx+c}a^2}{2b^3d}+\frac{3\,e^{dx+c}}{8bd}-\frac{e^{-dx-c}a^2}{2b^3d}-\frac{3\,e^{-dx-c}}{8bd}-\frac{a\,e^{-2dx-2c}}{8b^2d}-\frac{e^{-3d}}{24}$

input
```
int(cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^3*(1/3*sinh(d*x+c)^3*b^2-1/2*a*sinh(d*x+c)^2*b+sinh(d*x+c)*a^2+b^
2*sinh(d*x+c))-a*(a^2+b^2)/b^4*ln(a+b*sinh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 652 vs. 2(81) = 162.

Time = 0.10 (sec) , antiderivative size = 652, normalized size of antiderivative = 7.67

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/24*(b^3*cosh(d*x + c)^6 + b^3*sinh(d*x + c)^6 - 3*a*b^2*cosh(d*x + c)^5
+ 24*(a^3 + a*b^2)*d*x*cosh(d*x + c)^3 + 3*(2*b^3*cosh(d*x + c) - a*b^2)*s
inh(d*x + c)^5 + 3*(4*a^2*b + 3*b^3)*cosh(d*x + c)^4 + 3*(5*b^3*cosh(d*x +
 c)^2 - 5*a*b^2*cosh(d*x + c) + 4*a^2*b + 3*b^3)*sinh(d*x + c)^4 - 3*a*b^2
*cosh(d*x + c) + 2*(10*b^3*cosh(d*x + c)^3 - 15*a*b^2*cosh(d*x + c)^2 + 12
*(a^3 + a*b^2)*d*x + 6*(4*a^2*b + 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 -
b^3 - 3*(4*a^2*b + 3*b^3)*cosh(d*x + c)^2 + 3*(5*b^3*cosh(d*x + c)^4 - 10*
a*b^2*cosh(d*x + c)^3 + 24*(a^3 + a*b^2)*d*x*cosh(d*x + c) - 4*a^2*b - 3*b
^3 + 6*(4*a^2*b + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 24*((a^3 + a*b
^2)*cosh(d*x + c)^3 + 3*(a^3 + a*b^2)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a
^3 + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3 + a*b^2)*sinh(d*x + c)^3)
*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + 3*(2*b^3*c
osh(d*x + c)^5 - 5*a*b^2*cosh(d*x + c)^4 + 24*(a^3 + a*b^2)*d*x*cosh(d*x +
 c)^2 + 4*(4*a^2*b + 3*b^3)*cosh(d*x + c)^3 - a*b^2 - 2*(4*a^2*b + 3*b^3)*
cosh(d*x + c))*sinh(d*x + c))/(b^4*d*cosh(d*x + c)^3 + 3*b^4*d*cosh(d*x +
c)^2*sinh(d*x + c) + 3*b^4*d*cosh(d*x + c)*sinh(d*x + c)^2 + b^4*d*sinh(d*
x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(81) = 162$.

Time = 0.04 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.15

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= -\frac{\left(3\,abe^{(-dx-c)} - b^2 - 3\,(4\,a^2+3\,b^2)e^{(-2\,dx-2\,c)}\right)e^{(3\,dx+3\,c)}}{24\,b^3d} - \frac{(a^3+ab^2)(dx+c)}{b^4d}$$
$$- \frac{3\,abe^{(-2\,dx-2\,c)} + b^2e^{(-3\,dx-3\,c)} + 3\,(4\,a^2+3\,b^2)e^{(-dx-c)}}{24\,b^3d}$$
$$- \frac{(a^3+ab^2)\log\left(-2\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)} - b\right)}{b^4d}$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-1/24*(3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 + 3*b^2)*e^(-2*d*x - 2*c))*e^(3
*d*x + 3*c)/(b^3*d) - (a^3 + a*b^2)*(d*x + c)/(b^4*d) - 1/24*(3*a*b*e^(-2*
d*x - 2*c) + b^2*e^(-3*d*x - 3*c) + 3*(4*a^2 + 3*b^2)*e^(-d*x - c))/(b^3*d
) - (a^3 + a*b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^4*d)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.71

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{\frac{b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3 - 3\,ab\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2 + 12\,a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right) + 12\,b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{b^3} - \frac{24\,(a^3+ab^2)\log\left(\left|b\left(e^{(dx+c)}-e^{\cdots}\right.\right.\right.}{b^4}}{24\,d}$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/24*((b^2*(e^(d*x + c) - e^(-d*x - c))^3 - 3*a*b*(e^(d*x + c) - e^(-d*x -
 c))^2 + 12*a^2*(e^(d*x + c) - e^(-d*x - c)) + 12*b^2*(e^(d*x + c) - e^(-d
*x - c)))/b^3 - 24*(a^3 + a*b^2)*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) +
2*a))/b^4)/d
```

Mupad [B] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.12

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{x\,(a^3+a\,b^2)}{b^4} - \frac{\mathrm{e}^{-3\,c-3\,d\,x}}{24\,b\,d} + \frac{\mathrm{e}^{3\,c+3\,d\,x}}{24\,b\,d} - \frac{a\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,b^2\,d} - \frac{a\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,b^2\,d} - \frac{\mathrm{e}^{-c-d\,x}\,(4\,a^2+3\,b^2)}{8\,b^3\,d} - \frac{\ln\left(2\,a\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-b+b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\right)\,(a^3+a\,b^2)}{b^4\,d} + \frac{\mathrm{e}^{c+d\,x}\,(4\,a^2+3\,b^2)}{8\,b^3\,d}$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x))/(a + b*sinh(c + d*x)),x)
```

output
```
(x*(a*b^2 + a^3))/b^4 - exp(- 3*c - 3*d*x)/(24*b*d) + exp(3*c + 3*d*x)/(24
*b*d) - (a*exp(- 2*c - 2*d*x))/(8*b^2*d) - (a*exp(2*c + 2*d*x))/(8*b^2*d)
- (exp(- c - d*x)*(4*a^2 + 3*b^2))/(8*b^3*d) - (log(2*a*exp(d*x)*exp(c) -
b + b*exp(2*c)*exp(2*d*x))*(a*b^2 + a^3))/(b^4*d) + (exp(c + d*x)*(4*a^2 +
 3*b^2))/(8*b^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.95

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{e^{6dx+6c}b^3 - 3e^{5dx+5c}a\,b^2 + 12e^{4dx+4c}a^2b + 9e^{4dx+4c}b^3 - 24e^{3dx+3c}\log\left(e^{2dx+2c}b + 2e^{dx+c}a - b\right)a^3 - 24e^{3dx+}}{2}$$

input
```
int(cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
(e**(6*c + 6*d*x)*b**3 - 3*e**(5*c + 5*d*x)*a*b**2 + 12*e**(4*c + 4*d*x)*a
**2*b + 9*e**(4*c + 4*d*x)*b**3 - 24*e**(3*c + 3*d*x)*log(e**(2*c + 2*d*x)
*b + 2*e**(c + d*x)*a - b)*a**3 - 24*e**(3*c + 3*d*x)*log(e**(2*c + 2*d*x)
*b + 2*e**(c + d*x)*a - b)*a*b**2 + 24*e**(3*c + 3*d*x)*a**3*d*x + 24*e**(
3*c + 3*d*x)*a*b**2*d*x - 12*e**(2*c + 2*d*x)*a**2*b - 9*e**(2*c + 2*d*x)*
b**3 - 3*e**(c + d*x)*a*b**2 - b**3)/(24*e**(3*c + 3*d*x)*b**4*d)
```

3.347 $\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\operatorname{Int}\left(\frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(cosh(d*x+c)^3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 36.46 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{6125}$$
$$\int \frac{\sinh(c+dx)\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Cosh[c + d*x]^3*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(dx+c)^3 \sinh(dx+c)}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(cosh(d*x+c)^3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)^3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)^3\sinh\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(cosh(d*x + c)^3*sinh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 348, normalized size of antiderivative = 10.24

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)^3\sinh\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-1/8*e^(-3*c + 3*d*e/f)*exp_integral_e(1, 3*(f*x + e)*d/f)/(b*f) - 1/4*a*e
^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b^2*f) + 1/4*a*e^(2*
c - 2*d*e/f)*exp_integral_e(1, -2*(f*x + e)*d/f)/(b^2*f) - 1/8*e^(3*c - 3*
d*e/f)*exp_integral_e(1, -3*(f*x + e)*d/f)/(b*f) - 1/8*(4*a^2 + 3*b^2)*e^(
-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^3*f) - 1/8*(4*a^2*e^c + 3*
b^2*e^c)*e^(-d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b^3*f) - (a^3 + a*b
^2)*log(f*x + e)/(b^4*f) + 1/16*integrate(32*(a^3*b + a*b^3 - (a^4*e^c + a
^2*b^2*e^c)*e^(d*x))/(b^5*f*x + b^5*e - (b^5*f*x*e^(2*c) + b^5*e*e^(2*c))*
e^(2*d*x) - 2*(a*b^4*f*x*e^c + a*b^4*e*e^c)*e^(d*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)^3\sinh\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
integrate(cosh(d*x + c)^3*sinh(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)),
 x)
```

Mupad [N/A]

Not integrable

Time = 2.77 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{{\mathrm{cosh}\left(c+d\,x\right)}^3\,\mathrm{sinh}\left(c+d\,x\right)}{\left(e+f\,x\right)\,\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)}\,dx$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((cosh(c + d*x)^3*sinh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 336, normalized size of antiderivative = 9.88

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \frac{e^{6c}\left(\int \frac{e^{5dx}}{e^{2dx+2c}be+e^{2dx+2c}bfx+2e^{dx+c}ae+2e^{dx+c}afx-be-bfx}dx\right)+2e^{4c}\left(\int \frac{e^{3dx}}{e^{2dx+2c}be+e^{2dx+2c}bfx+2e^{dx+c}ae+2e^{dx+c}afx-be-bf}\right.}{}$$

input
```
int(cosh(d*x+c)^3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
(e**(6*c)*int(e**(5*d*x)/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x +
2*e**(c + d*x)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x) + 2*e**(4*c)*i
nt(e**(3*d*x)/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x + 2*e**(c + d
*x)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x) - e**c*int(1/(e**(5*c + 5
*d*x)*b*e + e**(5*c + 5*d*x)*b*f*x + 2*e**(4*c + 4*d*x)*a*e + 2*e**(4*c +
4*d*x)*a*f*x - e**(3*c + 3*d*x)*b*e - e**(3*c + 3*d*x)*b*f*x),x) - 2*int(1
/(e**(2*c + 3*d*x)*b*e + e**(2*c + 3*d*x)*b*f*x + 2*e**(c + 2*d*x)*a*e + 2
*e**(c + 2*d*x)*a*f*x - e**(d*x)*b*e - e**(d*x)*b*f*x),x))/(8*e**c)
```

3.348 $\int \frac{(e+fx)^3 \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 1021

$$\int \frac{(e+fx)^3 \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output
```
3/4*a*f^3*polylog(4,-exp(2*d*x+2*c))/(a^2+b^2)/d^4-6*I*f^3*polylog(4,-I*ex
p(d*x+c))/b/d^4+a*(f*x+e)^3*ln(1+exp(2*d*x+2*c))/(a^2+b^2)/d+6*I*f^3*polyl
og(4,I*exp(d*x+c))/b/d^4-6*I*a^2*f^2*(f*x+e)*polylog(3,-I*exp(d*x+c))/b/(a
^2+b^2)/d^3-3*I*a^2*f*(f*x+e)^2*polylog(2,I*exp(d*x+c))/b/(a^2+b^2)/d^2+6*
I*f^2*(f*x+e)*polylog(3,-I*exp(d*x+c))/b/d^3+3*I*f*(f*x+e)^2*polylog(2,I*e
xp(d*x+c))/b/d^2+6*I*a^2*f^2*(f*x+e)*polylog(3,I*exp(d*x+c))/b/(a^2+b^2)/d
^3-3/2*a*f^2*(f*x+e)*polylog(3,-exp(2*d*x+2*c))/(a^2+b^2)/d^3+3/2*a*f*(f*x
+e)^2*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)/d^2-2*a^2*(f*x+e)^3*arctan(exp(
d*x+c))/b/(a^2+b^2)/d-6*I*f^2*(f*x+e)*polylog(3,I*exp(d*x+c))/b/d^3-3*I*f*
(f*x+e)^2*polylog(2,-I*exp(d*x+c))/b/d^2+6*a*f^2*(f*x+e)*polylog(3,-b*exp(
d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d^3+6*a*f^2*(f*x+e)*polylog(3,-b*exp
(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d^3-3*a*f*(f*x+e)^2*polylog(2,-b*ex
p(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d^2-3*a*f*(f*x+e)^2*polylog(2,-b*e
xp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d^2+6*I*a^2*f^3*polylog(4,-I*exp(
d*x+c))/b/(a^2+b^2)/d^4+3*I*a^2*f*(f*x+e)^2*polylog(2,-I*exp(d*x+c))/b/(a^
2+b^2)/d^2+2*(f*x+e)^3*arctan(exp(d*x+c))/b/d-6*a*f^3*polylog(4,-b*exp(d*x
+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d^4-6*a*f^3*polylog(4,-b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))/(a^2+b^2)/d^4-a*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)
^(1/2)))/(a^2+b^2)/d-a*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a
^2+b^2)/d-6*I*a^2*f^3*polylog(4,I*exp(d*x+c))/b/(a^2+b^2)/d^4
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3078 vs. $2(1021) = 2042$.

Time = 10.74 (sec) , antiderivative size = 3078, normalized size of antiderivative = 3.01

$$\int \frac{(e+fx)^3 \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-8*a*d^4*e^3*E^(2*c)*x - 12*a*d^4*e^2*E^(2*c)*f*x^2 - 8*a*d^4*e*E^(2*c)*f
^2*x^3 - 2*a*d^4*E^(2*c)*f^3*x^4 + 8*b*d^3*e^3*ArcTan[E^(c + d*x)] + 8*b*d
^3*e^3*E^(2*c)*ArcTan[E^(c + d*x)] + (12*I)*b*d^3*e^2*f*x*Log[1 - I*E^(c +
 d*x)] + (12*I)*b*d^3*e^2*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] + (12*I)*b*d^
3*e*f^2*x^2*Log[1 - I*E^(c + d*x)] + (12*I)*b*d^3*e*E^(2*c)*f^2*x^2*Log[1
- I*E^(c + d*x)] + (4*I)*b*d^3*f^3*x^3*Log[1 - I*E^(c + d*x)] + (4*I)*b*d^
3*E^(2*c)*f^3*x^3*Log[1 - I*E^(c + d*x)] - (12*I)*b*d^3*e^2*f*x*Log[1 + I*
E^(c + d*x)] - (12*I)*b*d^3*e^2*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] - (12*I
)*b*d^3*e*f^2*x^2*Log[1 + I*E^(c + d*x)] - (12*I)*b*d^3*e*E^(2*c)*f^2*x^2*
Log[1 + I*E^(c + d*x)] - (4*I)*b*d^3*f^3*x^3*Log[1 + I*E^(c + d*x)] - (4*I
)*b*d^3*E^(2*c)*f^3*x^3*Log[1 + I*E^(c + d*x)] + 4*a*d^3*e^3*Log[1 + E^(2*
(c + d*x))] + 4*a*d^3*e^3*E^(2*c)*Log[1 + E^(2*(c + d*x))] + 12*a*d^3*e^2*
f*x*Log[1 + E^(2*(c + d*x))] + 12*a*d^3*e^2*E^(2*c)*f*x*Log[1 + E^(2*(c +
d*x))] + 12*a*d^3*e*f^2*x^2*Log[1 + E^(2*(c + d*x))] + 12*a*d^3*e*E^(2*c)*
f^2*x^2*Log[1 + E^(2*(c + d*x))] + 4*a*d^3*f^3*x^3*Log[1 + E^(2*(c + d*x))
] + 4*a*d^3*E^(2*c)*f^3*x^3*Log[1 + E^(2*(c + d*x))] - (12*I)*b*d^2*(1 + E
^(2*c))*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)] + (12*I)*b*d^2*(1 + E^(
2*c))*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)] + 6*a*d^2*e^2*f*PolyLog[2, -
E^(2*(c + d*x))] + 6*a*d^2*e^2*E^(2*c)*f*PolyLog[2, -E^(2*(c + d*x))] + 12
*a*d^2*e*f^2*x*PolyLog[2, -E^(2*(c + d*x))] + 12*a*d^2*e*E^(2*c)*f^2*x*...
```

Rubi [A] (verified)

Time = 4.18 (sec) , antiderivative size = 886, normalized size of antiderivative = 0.87, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6101, 3042, 4668, 3011, 6107, 6095, 2620, 3011, 7163, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3 \tanh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6101}$$

$$\frac{\int (e+fx)^3\text{sech}(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{b}$$

$$\Big\downarrow \text{4668}$$

$$-\frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{3if\int (e+fx)^2\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{3if\int (e+fx)^2\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)^3\arctan\left(e^{c+dx}\right)}{d}}{b}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}+}{b}$$

$$\Big\downarrow \text{6107}$$

$$-\frac{a\left(\frac{b^2\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}+}{b}$$

$$\Big\downarrow \text{6095}$$

$$-\frac{a\left(\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^4}{4bf}\right)}{a^2+b^2}+\frac{\int (e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}+}{b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{a\left(\frac{b^2\left(-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\cdots\right.}{a^2+b^2}\right.}{b}$$

$$\frac{\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}+\cdots}{b}$$

$\downarrow$ 3011

$$-\frac{a\left(\frac{b^2\left(-\frac{3f\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\cdots}{\cdots}\right.}{bd}\right.}{a^2+b^2}\right.}{\cdots}$$

$$\frac{\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}+\cdots}{b}$$

$\downarrow$ 7163

$$-\frac{a\left(\frac{b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int \operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{Po}}{}\right.}{}\right.}{}\right.}{}\right.}{\frac{\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d}-\frac{f\int \operatorname{PolyLog}\left(3,-ie^{c+dx}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{d}-\frac{f\int \operatorname{Po}}{}\right.}{d}\right.}{}}{b}}$$

$\downarrow$ 2720

$$-\frac{a\left(\frac{b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(2\ldots\right.}{\ldots}\ldots\right)}{\ldots}\ldots\right.}{\ldots}$$

$$\frac{\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,ie^{c\ldots}\right.}{d}\ldots\right.}{\ldots}\right.}{\ldots}\ldots}{b}$$

↓ 7143

$$-\frac{a\left(\frac{\int (e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\text{Pol}}{}}{bd}\right.}{}\right.}{}$$

$$\frac{\frac{2(e+fx)^3\arctan\left(e^{c+dx}\right)}{d}+\frac{3if\left(\frac{2f\left(\frac{(e+fx)\text{PolyLog}\left(3,-ie^{c+dx}\right)}{d}-\frac{f\text{PolyLog}\left(4,-ie^{c+dx}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\left(\frac{(e+fx)}{}\right.}{}\right.}{}}{b}$$

$\downarrow$ 7293

$$-\frac{a\left(\frac{\int\left(a(e+fx)^3\text{sech}(c+dx)-b(e+fx)^3\tanh(c+dx)\right)dx}{a^2+b^2}+\frac{b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\cdots\right)}{bd}\cdots\right)}{\cdots}\right)}{\cdots}$$

$$\frac{\frac{2(e+fx)^3\arctan\left(e^{c+dx}\right)}{d}+\frac{3if\left(\frac{2f\left(\frac{(e+fx)\text{PolyLog}\left(3,-ie^{c+dx}\right)}{d}-\frac{f\text{PolyLog}\left(4,-ie^{c+dx}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}}{b}$$

$\downarrow$ 2009

$$\frac{\frac{2\arctan\left(e^{c+dx}\right)(e+fx)^3}{d}+\frac{3if\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{d}-\frac{f\,\text{PolyLog}\left(4,-ie^{c+dx}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\left(\frac{(e+fx)}{}\right.}{}\right.}{}}{b}$$

$$a\left(\frac{\left(-\frac{(e+fx)^4}{4bf}+\frac{\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)(e+fx)^3}{bd}+\frac{\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)(e+fx)^3}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right.}{d^2}\right.}{d}\right.}{bd}\right.}{}\right.$$

input
```
Int[((e + f*x)^3*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
((2*(e + f*x)^3*ArcTan[E^(c + d*x)])/d + ((3*I)*f*(-(((e + f*x)^2*PolyLog[
2, (-I)*E^(c + d*x)])/d) + (2*f*(((e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/
d - (f*PolyLog[4, (-I)*E^(c + d*x)])/d^2))/d))/d - ((3*I)*f*(-(((e + f*x)^
2*PolyLog[2, I*E^(c + d*x)])/d) + (2*f*(((e + f*x)*PolyLog[3, I*E^(c + d*x
)])/d - (f*PolyLog[4, I*E^(c + d*x)])/d^2))/d))/d)/b - (a*((b^2*(-1/4*(e +
 f*x)^4/(b*f) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
])/(b*d) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b
*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d^2)
)/d))/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[
a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
)])/d^2))/d))/(b*d)))/(a^2 + b^2) + ((b*(e + f*x)^4)/(4*f) + (2*a*(e + f*x
)^3*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)^3*Log[1 + E^(2*(c + d*x))])/d -
((3*I)*a*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + ((3*I)*a*f*(e +
 f*x)^2*PolyLog[2, I*E^(c + d*x)])/d^2 - (3*b*f*(e + f*x)^2*PolyLog[2, -E^
(2*(c + d*x))])/(2*d^2) + ((6*I)*a*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*
x)])/d^3 - ((6*I)*a*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/d^3 + (3*b*f^
2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*d^3) - ((6*I)*a*f^3*PolyLo...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6101
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sech[
c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*Sech[c
+ d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^3 \tanh (dx+c)}{a+b\sinh (dx+c)}dx$$

input
```
int((f*x+e)^3*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1715, normalized size of antiderivative = 1.68

$$\int \frac{(e+fx)^3 \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(6*a*f^3*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
 + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*a*f^3*polylog(4, (a*cosh
(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2))/b) + 3*(a*d^2*f^3*x^2 + 2*a*d^2*e*f^2*x + a*d^2*e^2*f)*dilo
g((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))
*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 3*(a*d^2*f^3*x^2 + 2*a*d^2*e*f^2*x +
a*d^2*e^2*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*(a*d^2*f^3*x^2 + I
*b*d^2*f^3*x^2 + 2*a*d^2*e*f^2*x + 2*I*b*d^2*e*f^2*x + a*d^2*e^2*f + I*b*d
^2*e^2*f)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) - 3*(a*d^2*f^3*x^2 - I*
b*d^2*f^3*x^2 + 2*a*d^2*e*f^2*x - 2*I*b*d^2*e*f^2*x + a*d^2*e^2*f - I*b*d^
2*e^2*f)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) + (a*d^3*e^3 - 3*a*c*d^
2*e^2*f + 3*a*c^2*d*e*f^2 - a*c^3*f^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*
x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (a*d^3*e^3 - 3*a*c*d^2*e^2*f +
 3*a*c^2*d*e*f^2 - a*c^3*f^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) -
2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*
a*d^3*e^2*f*x + 3*a*c*d^2*e^2*f - 3*a*c^2*d*e*f^2 + a*c^3*f^3)*log(-(a*cos
h(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2) - b)/b) + (a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*
f*x + 3*a*c*d^2*e^2*f - 3*a*c^2*d*e*f^2 + a*c^3*f^3)*log(-(a*cosh(d*x +...
```

Sympy [F]

$$\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^3\tanh\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate((f*x+e)**3*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\tanh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e^3*(2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + a*log(-2*a*e^(-d*x - c) +
 b*e^(-2*d*x - 2*c) - b)/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/((a
^2 + b^2)*d)) + integrate(2*f^3*x^3*(e^(d*x + c) - e^(-d*x - c))/((b*(e^(d
*x + c) - e^(-d*x - c)) + 2*a)*(e^(d*x + c) + e^(-d*x - c))) + 6*e*f^2*x^2
*(e^(d*x + c) - e^(-d*x - c))/((b*(e^(d*x + c) - e^(-d*x - c)) + 2*a)*(e^(
d*x + c) + e^(-d*x - c))) + 6*e^2*f*x*(e^(d*x + c) - e^(-d*x - c))/((b*(e^
(d*x + c) - e^(-d*x - c)) + 2*a)*(e^(d*x + c) + e^(-d*x - c))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)^3*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\tanh(c+d\,x)\,\left(e+f\,x\right)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((tanh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((tanh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2atan\left(e^{dx+c}\right)b\,e^3+2e^{3c}\left(\int \frac{e^{3dx}x^3}{e^{4dx+4c}b+2e^{3dx+3c}a+2e^{dx+c}a-b}dx\right)a^2d\,f^3+2e^{3c}\left(\int \frac{e^{3dx}x^3}{e^{4dx+4c}b+2e^{3dx+3c}a+2e^{dx+c}a-b}dx\right)\cdots}{\cdots}$$

input
```
int((f*x+e)^3*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
(2*atan(e**(c + d*x))*b*e**3 + 2*e**(3*c)*int((e**(3*d*x)*x**3)/(e**(4*c +
 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a**2*d*f**3 +
2*e**(3*c)*int((e**(3*d*x)*x**3)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*
a + 2*e**(c + d*x)*a - b),x)*b**2*d*f**3 + 6*e**(3*c)*int((e**(3*d*x)*x**2
)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a*
*2*d*e*f**2 + 6*e**(3*c)*int((e**(3*d*x)*x**2)/(e**(4*c + 4*d*x)*b + 2*e**
(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*b**2*d*e*f**2 + 6*e**(3*c)*int(
(e**(3*d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)
*a - b),x)*a**2*d*e**2*f + 6*e**(3*c)*int((e**(3*d*x)*x)/(e**(4*c + 4*d*x)
*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*b**2*d*e**2*f - 2*e**
c*int((e**(d*x)*x**3)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c
 + d*x)*a - b),x)*a**2*d*f**3 - 2*e**c*int((e**(d*x)*x**3)/(e**(4*c + 4*d*
x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*b**2*d*f**3 - 6*e**
c*int((e**(d*x)*x**2)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c
 + d*x)*a - b),x)*a**2*d*e*f**2 - 6*e**c*int((e**(d*x)*x**2)/(e**(4*c + 4*
d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*b**2*d*e*f**2 - 6
*e**c*int((e**(d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(
c + d*x)*a - b),x)*a**2*d*e**2*f - 6*e**c*int((e**(d*x)*x)/(e**(4*c + 4*d*
x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*b**2*d*e**2*f + log
(e**(2*c + 2*d*x) + 1)*a*e**3 - log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)...
```

3.349 $\int \frac{(e+fx)^2 \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 716

$$\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)^2\arctan\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} - \frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} + \frac{a(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{\left(a^2+b^2\right)d} - \frac{2if(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{bd^2} + \frac{2ia^2f(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{b\left(a^2+b^2\right)d^2} + \frac{2if(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{bd^2} - \frac{2ia^2f(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{b\left(a^2+b^2\right)d^2} - \frac{2af(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d^2} - \frac{2af(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d^2} + \frac{af(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{\left(a^2+b^2\right)d^2} + \frac{2if^2\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{bd^3} - \frac{2ia^2f^2\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{b\left(a^2+b^2\right)d^3} - \frac{2if^2\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{bd^3} + \frac{2ia^2f^2\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{b\left(a^2+b^2\right)d^3} + \frac{2af^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d^3} + \frac{2af^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d^3} - \frac{af^2\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2\left(a^2+b^2\right)d^3}$$

output
```
2*(f*x+e)^2*arctan(exp(d*x+c))/b/d-2*a^2*(f*x+e)^2*arctan(exp(d*x+c))/b/(a
^2+b^2)/d-a*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d-a
*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d+a*(f*x+e)^2*
ln(1+exp(2*d*x+2*c))/(a^2+b^2)/d+2*I*f^2*polylog(3,-I*exp(d*x+c))/b/d^3-2*
I*f^2*polylog(3,I*exp(d*x+c))/b/d^3+2*I*a^2*f*(f*x+e)*polylog(2,-I*exp(d*x
+c))/b/(a^2+b^2)/d^2-2*I*a^2*f^2*polylog(3,-I*exp(d*x+c))/b/(a^2+b^2)/d^3-
2*a*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d^2-2
*a*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d^2+a*
f*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)/d^2-2*I*f*(f*x+e)*polylog(2
,-I*exp(d*x+c))/b/d^2+2*I*a^2*f^2*polylog(3,I*exp(d*x+c))/b/(a^2+b^2)/d^3+
2*I*f*(f*x+e)*polylog(2,I*exp(d*x+c))/b/d^2-2*I*a^2*f*(f*x+e)*polylog(2,I*
exp(d*x+c))/b/(a^2+b^2)/d^2+2*a*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(
1/2)))/(a^2+b^2)/d^3+2*a*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/
(a^2+b^2)/d^3-1/2*a*f^2*polylog(3,-exp(2*d*x+2*c))/(a^2+b^2)/d^3
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1640 vs. $2(716) = 1432$.

Time = 9.99 (sec) , antiderivative size = 1640, normalized size of antiderivative = 2.29

$$\int \frac{(e+fx)^2 \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-12*a*d^3*e^2*E^(2*c)*x + 12*a*d^3*e^2*(1 + E^(2*c))*x + 12*a*d^3*e*f*x^2
 + 4*a*d^3*f^2*x^3 + 12*b*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] - 6*a*
d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*b*d*e*(1
 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - Pol
yLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) - 6*a*d*e*(1 + E^(2
*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x
))]) + (6*I)*b*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*x^2
*Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*PolyL
og[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I*E^(
c + d*x)]) - a*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c + d
*x))]) - 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d*x)
)]))/(6*(a^2 + b^2)*d^3*(1 + E^(2*c))) + (a*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)
*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c +
 d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2
)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2
)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqr
t[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2
*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) +
 (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d - (3*e^2*E^(2*c
)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e*f*x*Log[1 +...
```

Rubi [A] (verified)

Time = 3.10 (sec) , antiderivative size = 633, normalized size of antiderivative = 0.88, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6101, 3042, 4668, 3011, 2720, 6107, 6095, 2620, 3011, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6101}$$

$$\frac{\int (e+fx)^2\text{sech}(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{b}$$

$\downarrow$ 4668

$$-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{2if\int (e+fx)\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{2if\int (e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}}{b}$$

$\downarrow$ 3011

$$-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}+\frac{2(e+fx)^2\text{arctan}}{d}}{b}$$

$\downarrow$ 2720

$$-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c+d}\right.}{d}\right.}{d}}{b}$$

$\downarrow$ 6107

$$-\frac{a\left(\frac{b^2\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c+d}\right.}{d}\right.}{d}}{b}$$

$\downarrow$ 6095

$$-\frac{a\left(\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c+d}\right.}{d}\right.}{d}}{b}$$

$\downarrow$ 2620

$$-\frac{a\left(\frac{b^2\left(-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+\cdots}{3\cdots}\right.}{a^2+b^2}\right.}{b}$$

$$\frac{\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c+d\cdots}\right.}{d}\right.}{d}}{b}$$

$\downarrow$ 3011

$$-\frac{a\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be\cdots}{a+\sqrt{\cdots}}\right.}{d}\right.}{bd}\right.}{a^2+b^2}\right.}{b}$$

$$\frac{\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c+d\cdots}\right.}{d}\right.}{d}}{b}$$

$\downarrow$ 2720

$$-\frac{a\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\cdots\right.}{bd}\right.}{a^2+b^2}\right.}{b}$$

$$\frac{\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c+d\cdots}\right.}{d}\right.}{d}}{b}$$

$\downarrow$ 7143

$$-\frac{a\left(\frac{\int (e+fx)^2\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\operatorname{PolyLog}(3\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}\right)}{b}$$

$$\frac{\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}+\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}\cdots}{b}$$

$\downarrow$ 7293

$$-\frac{a\left(\frac{\int \left(a(e+fx)^2\operatorname{sech}(c+dx)-b(e+fx)^2\tanh(c+dx)\right)dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\cdots\right)}{\cdots}\right)}{\cdots}\right)}{b}$$

$$\frac{\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}+\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}\cdots}{b}$$

$\downarrow$ 2009

$$\frac{\frac{2(e+fx)^2\arctan(e^{c+dx})}{d}+\frac{2if\left(\frac{f\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\,\text{PolyLog}\left(3,ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}}{b}$$

$$a\left(\frac{b^2\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right.}{bd}\right.}{a^2+b^2}\right.$$

input
```
Int[((e + f*x)^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
((2*(e + f*x)^2*ArcTan[E^(c + d*x)])/d + ((2*I)*f*(-(((e + f*x)*PolyLog[2,
 (-I)*E^(c + d*x)])/d) + (f*PolyLog[3, (-I)*E^(c + d*x)])/d^2))/d - ((2*I)
*f*(-(((e + f*x)*PolyLog[2, I*E^(c + d*x)])/d) + (f*PolyLog[3, I*E^(c + d*
x)])/d^2))/d)/b - (a*((b^2*(-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 +
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -
((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/(a^2 + b^2) + ((b*(e + f*x)^3
)/(3*f) + (2*a*(e + f*x)^2*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)^2*Log[1 +
 E^(2*(c + d*x))])/d - ((2*I)*a*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/
d^2 + ((2*I)*a*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*(e + f*x)
*PolyLog[2, -E^(2*(c + d*x))])/d^2 + ((2*I)*a*f^2*PolyLog[3, (-I)*E^(c + d
*x)])/d^3 - ((2*I)*a*f^2*PolyLog[3, I*E^(c + d*x)])/d^3 + (b*f^2*PolyLog[3
, -E^(2*(c + d*x))])/(2*d^3))/(a^2 + b^2)))/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6101
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sech[
c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*Sech[c
+ d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^2 \tanh(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 1083, normalized size of antiderivative = 1.51

$$\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(2*a*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 2*a*f^2*polylog(3, (a*cosh(
d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2))/b) - 2*(a*d*f^2*x + a*d*e*f)*dilog((a*cosh(d*x + c) + a*sinh
(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)
/b + 1) - 2*(a*d*f^2*x + a*d*e*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c)
 - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) +
 2*(a*d*f^2*x + I*b*d*f^2*x + a*d*e*f + I*b*d*e*f)*dilog(I*cosh(d*x + c) +
 I*sinh(d*x + c)) + 2*(a*d*f^2*x - I*b*d*f^2*x + a*d*e*f - I*b*d*e*f)*dilo
g(-I*cosh(d*x + c) - I*sinh(d*x + c)) - (a*d^2*e^2 - 2*a*c*d*e*f + a*c^2*f
^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2)
+ 2*a) - (a*d^2*e^2 - 2*a*c*d*e*f + a*c^2*f^2)*log(2*b*cosh(d*x + c) + 2*b
*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (a*d^2*f^2*x^2 + 2*a*d
^2*e*f*x + 2*a*c*d*e*f - a*c^2*f^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c
) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (a
*d^2*f^2*x^2 + 2*a*d^2*e*f*x + 2*a*c*d*e*f - a*c^2*f^2)*log(-(a*cosh(d*x +
 c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^
2)/b^2) - b)/b) + (a*d^2*e^2 + I*b*d^2*e^2 - 2*a*c*d*e*f - 2*I*b*c*d*e*f +
 a*c^2*f^2 + I*b*c^2*f^2)*log(cosh(d*x + c) + sinh(d*x + c) + I) + (a*d^2*
e^2 - I*b*d^2*e^2 - 2*a*c*d*e*f + 2*I*b*c*d*e*f + a*c^2*f^2 - I*b*c^2*f...
```

Sympy [F]

$$\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2\tanh\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate((f*x+e)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\tanh\left(dx+c\right)}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e^2*(2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + a*log(-2*a*e^(-d*x - c) +
 b*e^(-2*d*x - 2*c) - b)/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/((a
^2 + b^2)*d)) + integrate(2*f^2*x^2*(e^(d*x + c) - e^(-d*x - c))/((b*(e^(d
*x + c) - e^(-d*x - c)) + 2*a)*(e^(d*x + c) + e^(-d*x - c))) + 4*e*f*x*(e^
(d*x + c) - e^(-d*x - c))/((b*(e^(d*x + c) - e^(-d*x - c)) + 2*a)*(e^(d*x
+ c) + e^(-d*x - c))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\tanh(c+d\,x)\,\left(e+f\,x\right)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((tanh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((tanh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2atan\left(e^{dx+c}\right)b\,e^2+2e^{3c}\left(\int \frac{e^{3dx}x^2}{e^{4dx+4c}b+2e^{3dx+3c}a+2e^{dx+c}a-b}dx\right)a^2d\,f^2+2e^{3c}\left(\int \frac{e^{3dx}x^2}{e^{4dx+4c}b+2e^{3dx+3c}a+2e^{dx+c}a-b}dx\right)\ldots}{}$$

input
```
int((f*x+e)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
(2*atan(e**(c + d*x))*b*e**2 + 2*e**(3*c)*int((e**(3*d*x)*x**2)/(e**(4*c +
 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a**2*d*f**2 +
2*e**(3*c)*int((e**(3*d*x)*x**2)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*
a + 2*e**(c + d*x)*a - b),x)*b**2*d*f**2 + 4*e**(3*c)*int((e**(3*d*x)*x)/(
e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a**2*
d*e*f + 4*e**(3*c)*int((e**(3*d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*
d*x)*a + 2*e**(c + d*x)*a - b),x)*b**2*d*e*f - 2*e**c*int((e**(d*x)*x**2)/
(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a**2
*d*f**2 - 2*e**c*int((e**(d*x)*x**2)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d
*x)*a + 2*e**(c + d*x)*a - b),x)*b**2*d*f**2 - 4*e**c*int((e**(d*x)*x)/(e*
*(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a**2*d*
e*f - 4*e**c*int((e**(d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a +
 2*e**(c + d*x)*a - b),x)*b**2*d*e*f + log(e**(2*c + 2*d*x) + 1)*a*e**2 -
log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*a*e**2)/(d*(a**2 + b**2))
```

3.350 $\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 421

$$\begin{aligned}\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)\arctan\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} \\ &\quad - \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} \\ &\quad - \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} \\ &\quad + \frac{a(e+fx)\log\left(1+e^{2(c+dx)}\right)}{\left(a^2+b^2\right)d} - \frac{if\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{bd^2} \\ &\quad + \frac{ia^2f\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{b\left(a^2+b^2\right)d^2} + \frac{if\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{bd^2} \\ &\quad - \frac{ia^2f\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{b\left(a^2+b^2\right)d^2} - \frac{af\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d^2} \\ &\quad - \frac{af\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d^2} \\ &\quad + \frac{af\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2\left(a^2+b^2\right)d^2}\end{aligned}$$

output
```
2*(f*x+e)*arctan(exp(d*x+c))/b/d-2*a^2*(f*x+e)*arctan(exp(d*x+c))/b/(a^2+b
^2)/d-a*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d-a*(f*x+
e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d+a*(f*x+e)*ln(1+exp(2
*d*x+2*c))/(a^2+b^2)/d-I*f*polylog(2,-I*exp(d*x+c))/b/d^2+I*a^2*f*polylog(
2,-I*exp(d*x+c))/b/(a^2+b^2)/d^2+I*f*polylog(2,I*exp(d*x+c))/b/d^2-I*a^2*f
*polylog(2,I*exp(d*x+c))/b/(a^2+b^2)/d^2-a*f*polylog(2,-b*exp(d*x+c)/(a-(a
^2+b^2)^(1/2)))/(a^2+b^2)/d^2-a*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/
2)))/(a^2+b^2)/d^2+1/2*a*f*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)/d^2
```

Mathematica [A] (warning: unable to verify)

Time = 2.68 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.24

$$\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{4bde\arctan\left(e^{c+dx}\right) - 4bcf\arctan\left(e^{c+dx}\right) + \frac{4a^2(a^2+b^2)^{5/2}de\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\left(-(a^2+b^2)^2\right)^{3/2}} + \frac{4a^2\sqrt{-(a^2+b^2)^2}de\,\mathrm{arctanh}\left(\frac{a+be^{c-}}{\sqrt{a^2+}}\right.}{(-a^2-b^2)^{3/2}}}{}$$

input
```
Integrate[((e + f*x)*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(4*b*d*e*ArcTan[E^(c + d*x)] - 4*b*c*f*ArcTan[E^(c + d*x)] + (4*a^2*(a^2 +
 b^2)^(5/2)*d*e*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(-(a^2 + b^2
)^2)^(3/2) + (4*a^2*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/S
qrt[a^2 + b^2]])/(-a^2 - b^2)^(3/2) + (2*I)*b*f*(c + d*x)*Log[1 - I*E^(c +
 d*x)] - (2*I)*b*f*(c + d*x)*Log[1 + I*E^(c + d*x)] - 2*a*f*(c + d*x)*Log[
1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*a*f*(c + d*x)*Log[1 + (b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*a*d*e*Log[1 + E^(2*(c + d*x))] - 2*a
*c*f*Log[1 + E^(2*(c + d*x))] + 2*a*f*(c + d*x)*Log[1 + E^(2*(c + d*x))] +
 2*a*c*f*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] - 2*a*d*e*Log[2*a*E^
(c + d*x) + b*(-1 + E^(2*(c + d*x)))] - (2*I)*b*f*PolyLog[2, (-I)*E^(c + d
*x)] + (2*I)*b*f*PolyLog[2, I*E^(c + d*x)] - 2*a*f*PolyLog[2, (b*E^(c + d*
x))/(-a + Sqrt[a^2 + b^2])] - 2*a*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2]))] + a*f*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)*d^2)
```

Rubi [A] (verified)

Time = 2.03 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.93, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6101, 3042, 4668, 2715, 2838, 6107, 6095, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6101}$$

$$\frac{\int (e+fx)\text{sech}(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{b}$$

$$\Big\downarrow \text{4668}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{if\int \log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int \log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}}{b}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}}{b}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}}{b}$$

$$\Big\downarrow \text{6107}$$

$$-\frac{a\left(\frac{b^2\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}}{b}$$

$$\Big\downarrow \text{6095}$$

$$-\frac{a\left(\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}}{b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{a\left(\frac{b^2\left(-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int (e+\ldots}{\ldots}\right)}{b}$$

$$\frac{\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}}{b}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{a\left(\frac{b^2\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+\ldots\right)}{bd}\ldots\right)}{a^2+b^2}\ldots\right)}{b}$$

$$\frac{\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}}{b}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{a\left(\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\ldots\right)}{a^2+b^2}\ldots\right)}{b}$$

$$\frac{\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}}{b}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{a\left(\frac{\int (a(e+fx)\text{sech}(c+dx)-b(e+fx)\tanh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right.}{bd}\right.}{a^2+b^2}\right.}{b}$$

$$\frac{\frac{2(e+fx)\arctan(e^{c+dx})}{d}-\frac{if\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{if\,\text{PolyLog}(2,ie^{c+dx})}{d^2}}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2(e+fx)\arctan(e^{c+dx})}{d}-\frac{if\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{if\,\text{PolyLog}(2,ie^{c+dx})}{d^2}}{b}-$$

$$\frac{a\left(\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{2a(e+f}{}\right.}{b}$$

input
```
Int[((e + f*x)*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
((2*(e + f*x)*ArcTan[E^(c + d*x)])/d - (I*f*PolyLog[2, (-I)*E^(c + d*x)])/
d^2 + (I*f*PolyLog[2, I*E^(c + d*x)])/d^2)/b - (a*((b^2*(-1/2*(e + f*x)^2/
(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) +
 ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*Pol
yLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (f*PolyLog[2,
 -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(a^2 + b^2) + ((b*(e
 + f*x)^2)/(2*f) + (2*a*(e + f*x)*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)*Lo
g[1 + E^(2*(c + d*x))])/d - (I*a*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + (I*
a*f*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*PolyLog[2, -E^(2*(c + d*x))])/(2
*d^2))/(a^2 + b^2)))/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6101
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sech[
c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*Sech[c
+ d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1286 vs. $2(393) = 786$.

Time = 0.68 (sec) , antiderivative size = 1287, normalized size of antiderivative = 3.06

method	result	size
risch	Expression too large to display	1287

input
```
int((f*x+e)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/d^2*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))*a+2/d^2*f/(2*a^2+2*b^2)*dilog(
1-I*exp(d*x+c))*a-2/d^2*f/(2*a^2+2*b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/
2)-a)/(-a+(a^2+b^2)^(1/2)))*a-2/d^2*f/(2*a^2+2*b^2)*dilog((b*exp(d*x+c)+(a
^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a+2/d*e/(2*a^2+2*b^2)*a*ln(1+exp(2*d
*x+2*c))+4/d*e/(2*a^2+2*b^2)*b*arctan(exp(d*x+c))-2/d*e/(2*a^2+2*b^2)*a*ln
(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d*e/(2*a^2+2*b^2)*(a^2+b^2)^(1/2)*ar
ctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d^2*c*f/(2*a^2+2*b^2)*(a
^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d*e*b^2/
(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(
1/2))-2/d*e/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)
/(a^2+b^2)^(1/2))*a^2-2/d*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2
)-a)/(-a+(a^2+b^2)^(1/2)))*a*x-2/d*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b
^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a*x+2/d^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x
+c))*a*c+2/d^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*a*c-2/d^2*f/(2*a^2+2*b^2
)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a*c+2/d*f/(2*a^
2+2*b^2)*ln(1+I*exp(d*x+c))*a*x+2/d*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*a*x
-2/d^2*c*f/(2*a^2+2*b^2)*a*ln(1+exp(2*d*x+2*c))-4/d^2*c*f/(2*a^2+2*b^2)*b*
arctan(exp(d*x+c))+2/d^2*c*f/(2*a^2+2*b^2)*a*ln(b*exp(2*d*x+2*c)+2*a*exp(d
*x+c)-b)-2/d^2*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a
^2+b^2)^(1/2)))*a*c-2*I/d^2*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))*b+2*I...
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.40

$$\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(a*f*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + a*f*dilog((a*cosh(d*x + c)
+ a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2) - b)/b + 1) - (a*f + I*b*f)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) -
 (a*f - I*b*f)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) + (a*d*e - a*c*f)
*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2
*a) + (a*d*e - a*c*f)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt
((a^2 + b^2)/b^2) + 2*a) + (a*d*f*x + a*c*f)*log(-(a*cosh(d*x + c) + a*sin
h(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b
)/b) + (a*d*f*x + a*c*f)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (a*d*e + I*b*
d*e - a*c*f - I*b*c*f)*log(cosh(d*x + c) + sinh(d*x + c) + I) - (a*d*e - I
*b*d*e - a*c*f + I*b*c*f)*log(cosh(d*x + c) + sinh(d*x + c) - I) - (a*d*f*
x - I*b*d*f*x + a*c*f - I*b*c*f)*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1
) - (a*d*f*x + I*b*d*f*x + a*c*f + I*b*c*f)*log(-I*cosh(d*x + c) - I*sinh(
d*x + c) + 1))/((a^2 + b^2)*d^2)
```

Sympy [F]

$$\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\tanh{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\tanh{(dx+c)}}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e*(2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + a*log(-2*a*e^(-d*x - c) + b
*e^(-2*d*x - 2*c) - b)/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/((a^2
 + b^2)*d)) + f*integrate(2*x*(e^(d*x + c) - e^(-d*x - c))/((b*(e^(d*x + c
) - e^(-d*x - c)) + 2*a)*(e^(d*x + c) + e^(-d*x - c))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh(c+d\,x)\,\left(e+f\,x\right)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((tanh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((tanh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{2atan\left(e^{dx+c}\right)be+2e^{3c}\left(\int \frac{e^{3dx}x}{e^{4dx+4c}b+2e^{3dx+3c}a+2e^{dx+c}a-b}dx\right)a^2df+2e^{3c}\left(\int \frac{e^{3dx}x}{e^{4dx+4c}b+2e^{3dx+3c}a+2e^{dx+c}a-b}dx\right)b^2df}{}$$

input
```
int((f*x+e)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
(2*atan(e**(c + d*x))*b*e + 2*e**(3*c)*int((e**(3*d*x)*x)/(e**(4*c + 4*d*x
)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a**2*d*f + 2*e**(3*c
)*int((e**(3*d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c
+ d*x)*a - b),x)*b**2*d*f - 2*e**c*int((e**(d*x)*x)/(e**(4*c + 4*d*x)*b +
2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a**2*d*f - 2*e**c*int((e**
(d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b
),x)*b**2*d*f + log(e**(2*c + 2*d*x) + 1)*a*e - log(e**(2*c + 2*d*x)*b + 2
*e**(c + d*x)*a - b)*a*e)/(d*(a**2 + b**2))
```

3.351 $\int \frac{\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \frac{\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b\arctan(\sinh(c+dx))}{(a^2+b^2)\,d} + \frac{a\log(\cosh(c+dx))}{(a^2+b^2)\,d} - \frac{a\log(a+b\sinh(c+dx))}{(a^2+b^2)\,d}$$

output
```
b*arctan(sinh(d*x+c))/(a^2+b^2)/d+a*ln(cosh(d*x+c))/(a^2+b^2)/d-a*ln(a+b*s
inh(d*x+c))/(a^2+b^2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \frac{\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{(a-ib)\log(i-\sinh(c+dx)) + (a+ib)\log(i+\sinh(c+dx)) - 2a\log(a+b\sinh(c+dx))}{2(a^2+b^2)\,d}$$

input
```
Integrate[Tanh[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

output
```
((a - I*b)*Log[I - Sinh[c + d*x]] + (a + I*b)*Log[I + Sinh[c + d*x]] - 2*a
*Log[a + b*Sinh[c + d*x]])/(2*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 26, 3200, 25, 587, 16, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ic+idx)}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ic+idx)}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3200}$$

$$-\frac{\int -\frac{b\sinh(c+dx)}{(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b\sinh(c+dx)}{(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{587}$$

$$-\frac{\frac{a\int \frac{1}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{a^2+b^2}-\frac{\int \frac{b^2+a\sinh(c+dx)b}{\sinh^2(c+dx)b^2+b^2}d(b\sinh(c+dx))}{a^2+b^2}}{d}$$

$$\Big\downarrow \text{16}$$

$$-\frac{\frac{a\log(a+b\sinh(c+dx))}{a^2+b^2}-\frac{\int \frac{b^2+a\sinh(c+dx)b}{\sinh^2(c+dx)b^2+b^2}d(b\sinh(c+dx))}{a^2+b^2}}{d}$$

$$\Big\downarrow \ 452$$

$$-\frac{\frac{a\log(a+b\sinh(c+dx))}{a^2+b^2}-\frac{a\int \frac{b\sinh(c+dx)}{\sinh^2(c+dx)b^2+b^2}d(b\sinh(c+dx))+b^2\int \frac{1}{\sinh^2(c+dx)b^2+b^2}d(b\sinh(c+dx))}{a^2+b^2}}{d}$$

$$\Big\downarrow \ 216$$

$$-\frac{\frac{a\log(a+b\sinh(c+dx))}{a^2+b^2}-\frac{a\int \frac{b\sinh(c+dx)}{\sinh^2(c+dx)b^2+b^2}d(b\sinh(c+dx))+b\arctan(\sinh(c+dx))}{a^2+b^2}}{d}$$

$$\Big\downarrow \ 240$$

$$-\frac{\frac{a\log(a+b\sinh(c+dx))}{a^2+b^2}-\frac{\frac{1}{2}a\log\left(b^2\sinh^2(c+dx)+b^2\right)+b\arctan(\sinh(c+dx))}{a^2+b^2}}{d}$$

input
```
Int[Tanh[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

output
```
-(((a*Log[a + b*Sinh[c + d*x]])/(a^2 + b^2) - (b*ArcTan[Sinh[c + d*x]] + (
a*Log[b^2 + b^2*Sinh[c + d*x]^2])/2)/(a^2 + b^2))/d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 587
```
Int[(x_.)/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[(-
c)*(d/(b*c^2 + a*d^2))   Int[1/(c + d*x), x], x] + Simp[1/(b*c^2 + a*d^2)
 Int[(a*d + b*c*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
^2 + a*d^2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.41

method	result
derivativedivides	$\frac{\frac{2a\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)+4b\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^{2}+2b^{2}}-\frac{2a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{2a^{2}+2b^{2}}}{d}$
default	$\frac{\frac{2a\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)+4b\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^{2}+2b^{2}}-\frac{2a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{2a^{2}+2b^{2}}}{d}$
risch	$-\frac{2d^{2}ax}{a^{2}d^{2}+b^{2}d^{2}}-\frac{2dac}{a^{2}d^{2}+b^{2}d^{2}}+\frac{2ax}{a^{2}+b^{2}}+\frac{2ac}{d(a^{2}+b^{2})}+\frac{i\ln(\mathrm{e}^{dx+c}+i)b}{(a^{2}+b^{2})d}+\frac{\ln(\mathrm{e}^{dx+c}+i)a}{(a^{2}+b^{2})d}-\frac{i\ln(\mathrm{e}^{dx+c}-i)b}{(a^{2}+b^{2})d}+$

input
```
int(tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(4/(2*a^2+2*b^2)*(1/2*a*ln(1+tanh(1/2*d*x+1/2*c)^2)+b*arctan(tanh(1/2*
d*x+1/2*c)))-2*a/(2*a^2+2*b^2)*ln(tanh(1/2*d*x+1/2*c)^2*a-2*b*tanh(1/2*d*x
+1/2*c)-a))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.33

$$\int \frac{\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2\,b\arctan\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)-a\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)+a\log\left(\frac{2\,\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{(a^2+b^2)d}$$

input
```
integrate(tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(2*b*arctan(cosh(d*x + c) + sinh(d*x + c)) - a*log(2*(b*sinh(d*x + c) + a)
/(cosh(d*x + c) - sinh(d*x + c))) + a*log(2*cosh(d*x + c)/(cosh(d*x + c) -
 sinh(d*x + c))))/((a^2 + b^2)*d)
```

Sympy [F]

$$\int \frac{\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate(tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.38

$$\int \frac{\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{2\,b\arctan\left(e^{(-dx-c)}\right)}{(a^2+b^2)d} - \frac{a\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^2+b^2)d} + \frac{a\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^2+b^2)d}$$

input
```
integrate(tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - a*log(-2*a*e^(-d*x - c) + b*e^
(-2*d*x - 2*c) - b)/((a^2 + b^2)*d) + a*log(e^(-2*d*x - 2*c) + 1)/((a^2 +
b^2)*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.75

$$\int \frac{\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{2\,ab\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^2b+b^3} - \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)b}{a^2+b^2} - \frac{a\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^2+b^2}}{2\,d}$$

input
```
integrate(tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(2*a*b*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^2*b + b^3) -
 (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*b/(a^2 + b^2) - a
*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^2 + b^2))/d
```

Mupad [B] (verification not implemented)

Time = 2.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.88

$$\int \frac{\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\ln\left(e^{c+dx}+1i\right)}{a\,d-b\,d\,1i} - \frac{a\,\ln\left(8\,a^3\,e^{d\,x}\,e^{c}-b^3-4\,a^2\,b+b^3\,e^{2\,c}\,e^{2\,d\,x}+4\,a^2\,b\,e^{2\,c}\,e^{2\,d\,x}+2\,a\,b^2\,e^{d\,x}\,e^{c}\right)}{d\,a^2+d\,b^2} + \frac{\ln\left(1+e^{c+d\,x}\,1i\right)\,1i}{-b\,d+a\,d\,1i}$$

input
```
int(tanh(c + d*x)/(a + b*sinh(c + d*x)),x)
```

output
```
log(exp(c + d*x) + 1i)/(a*d - b*d*1i) + (log(exp(c + d*x)*1i + 1)*1i)/(a*d
*1i - b*d) - (a*log(8*a^3*exp(d*x)*exp(c) - b^3 - 4*a^2*b + b^3*exp(2*c)*e
xp(2*d*x) + 4*a^2*b*exp(2*c)*exp(2*d*x) + 2*a*b^2*exp(d*x)*exp(c)))/(a^2*d
 + b^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2atan\left(e^{dx+c}\right)b+\log\left(e^{2dx+2c}+1\right)a-\log\left(e^{2dx+2c}b+2e^{dx+c}a-b\right)a}{d\left(a^2+b^2\right)}$$

input
```
int(tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
(2*atan(e**(c + d*x))*b + log(e**(2*c + 2*d*x) + 1)*a - log(e**(2*c + 2*d*
x)*b + 2*e**(c + d*x)*a - b)*a)/(d*(a**2 + b**2))
```

3.352 $\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 10.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[Tanh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Tanh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$\Big\downarrow \text{ 6111}$$
$$\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Tanh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\tanh\left(dx+c\right)}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(tanh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh\left(c+dx\right)}{(a+b\sinh\left(c+dx\right))\left(e+fx\right)}\,dx$$

input
```
integrate(tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(tanh(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(tanh(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh(c+d\,x)}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(tanh(c + d*x)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(tanh(c + d*x)/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh\left(dx+c\right)}{\sinh\left(dx+c\right)be+\sinh\left(dx+c\right)bfx+ae+afx}dx$$

input
```
int(tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(tanh(c + d*x)/(sinh(c + d*x)*b*e + sinh(c + d*x)*b*f*x + a*e + a*f*x),
x)
```

3.353 $\int \frac{(e+fx)^3\mathrm{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 917

$$\int \frac{(e+fx)^3\mathrm{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output
```
(f*x+e)^3/b/d-a^2*(f*x+e)^3/b/(a^2+b^2)/d+6*a*f*(f*x+e)^2*arctan(exp(d*x+c
))/(a^2+b^2)/d^2-a*b*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2
+b^2)^(3/2)/d+a*b*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^
2)^(3/2)/d-3*f*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/b/d^2+3*a^2*f*(f*x+e)^2*ln(1
+exp(2*d*x+2*c))/b/(a^2+b^2)/d^2-6*I*a*f^2*(f*x+e)*polylog(2,-I*exp(d*x+c)
)/(a^2+b^2)/d^3-6*I*a*f^3*polylog(3,I*exp(d*x+c))/(a^2+b^2)/d^4-3*a*b*f*(f
*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2+3
*a*b*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3
/2)/d^2-3*f^2*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/b/d^3+3*a^2*f^2*(f*x+e)*p
olylog(2,-exp(2*d*x+2*c))/b/(a^2+b^2)/d^3+6*I*a*f^3*polylog(3,-I*exp(d*x+c
))/(a^2+b^2)/d^4+6*I*a*f^2*(f*x+e)*polylog(2,I*exp(d*x+c))/(a^2+b^2)/d^3+6
*a*b*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3
/2)/d^3-6*a*b*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^
2+b^2)^(3/2)/d^3+3/2*f^3*polylog(3,-exp(2*d*x+2*c))/b/d^4-3/2*a^2*f^3*poly
log(3,-exp(2*d*x+2*c))/b/(a^2+b^2)/d^4-6*a*b*f^3*polylog(4,-b*exp(d*x+c)/(
a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^4+6*a*b*f^3*polylog(4,-b*exp(d*x+c)/
(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^4-a*(f*x+e)^3*sech(d*x+c)/(a^2+b^2)
/d+(f*x+e)^3*tanh(d*x+c)/b/d-a^2*(f*x+e)^3*tanh(d*x+c)/b/(a^2+b^2)/d
```

Mathematica [A] (warning: unable to verify)

Time = 7.35 (sec) , antiderivative size = 1070, normalized size of antiderivative = 1.17

$$\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
((f*(12*b*d^3*e^2*E^(2*c)*x - 12*b*d^3*e^2*(1 + E^(2*c))*x - 12*b*d^3*e*f*
x^2 - 4*b*d^3*f^2*x^3 + 12*a*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] + 6
*b*d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*a*d*e
*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) -
PolyLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) + 6*b*d*e*(1 + E
^(2*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c +
d*x))]) + (6*I)*a*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*
x^2*Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*Po
lyLog[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I*
E^(c + d*x)]) + b*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c
+ d*x))]) - 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d
*x))])))/((a^2 + b^2)*(1 + E^(2*c))) + (2*a*b*(2*d^3*e^3*ArcTanh[(a + b*E^
(c + d*x))/Sqrt[a^2 + b^2]] - 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 + b^2])] - 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])] - d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 3*d
^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 3*d^3*e*f^2*x^
2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d^3*f^3*x^3*Log[1 + (b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 3*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^
(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 3*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 6*d*e*f^2*PolyLog[3, (b*E^(c + d*...
```

Rubi [A] (verified)

Time = 5.44 (sec) , antiderivative size = 792, normalized size of antiderivative = 0.86, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6117, 3042, 4672, 26, 3042, 26, 4201, 2620, 3011, 2720, 6107, 3042, 3803, 25, 2694, 27, 2620, 3011, 7143, 7163, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\tanh(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6117}$$

$$\frac{\int (e+fx)^3\text{sech}^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{b}$$

$$\Big\downarrow \text{4672}$$

$$-\frac{a\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{3if\int -i(e+fx)^2\tanh(c+dx)dx}{d}}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{3f\int (e+fx)^2\tanh(c+dx)dx}{d}}{b}-\frac{a\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{3f\int -i(e+fx)^2\tan(ic+idx)dx}{d}}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\int (e+fx)^2\tan(ic+idx)dx}{d}}{b}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{a\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\int \frac{e^{2(c+dx)}(e+fx)^2}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}}{b}$$

$\downarrow$ 2620

$$-\frac{a\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int (e+fx)\log\left(1+e^{2(c+dx)}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{b}$$

$\downarrow$ 3011

$$-\frac{a\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int \text{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{2d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{b}$$

$\downarrow$ 2720

$$-\frac{a\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\bigg(2i\bigg(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\bigg)}{d}}{b}$$

$\downarrow$ 6107

$$-\frac{a\left(\frac{b^2\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\bigg(2i\bigg(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\bigg)}{d}}{b}$$

$\downarrow$ 3042

$$\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)\right)}{d}}{b}$$

$$\frac{a\left(\frac{\int(e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{a^2+b^2}\right)}{b}$$

$\Big\downarrow$ 3803

$$-\frac{a\left(\frac{2b^2\int-\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{\int(e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)\right)}{d}}{b}$$

$\Big\downarrow$ 25

$$-\frac{a\left(\frac{\int(e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\int\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}\right)}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)\right)}{d}}{b}$$

$\Big\downarrow$ 2694

$$-\frac{a\left(\frac{\int(e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int-\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int-\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)\right)}{d}}{b}$$

$\Big\downarrow$ 27

$$-\frac{a\left(\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)\cdots\right)}{d}}{b}$$

$\Big\downarrow$ 2620

$$-\frac{a\left(\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\cdots}{\cdots}\right)}{\cdots}\right)}{a^2+b^2}\cdots\right)}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)\cdots\right)}{d}}{b}$$

$\Big\downarrow$ 3011

$$-\frac{a\left(\frac{\int (e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\cdots}{\cdots}\right)}{bd}\cdots\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{\cdots}\cdots\right)}{\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)\cdots\right)}{d}\cdots}{b}}$$

$$\Big\downarrow \; 7143$$

$$-\frac{a\left(\frac{\int (e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\cdots}{\cdots}\right)}{bd}\cdots\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

$$\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{b}$$

$\downarrow$ 7163

$$-\frac{a\left(\frac{\int (e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int \operatorname{PolyLo}\cdots}{\cdots}\right)}{d}\cdots\right)}{\cdots}\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{\cdots}\right)}{\cdots}$$

$$\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{b}$$

$$\Big\downarrow \; 2720$$

$$-\frac{a\left(\frac{\int (e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-\cdots}}{\cdots}\right.}{d}\cdots}{\cdots}\right)}{2\sqrt{a^2+b^2}\cdots}\cdots}{\cdots}\right)\cdots}{\cdots}$$

$$\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{b}$$

$\downarrow$ 7143

$$-\frac{a\left(\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\text{PolyLog}}{}\right)}{d}\right.}{}\right.}{2\sqrt{a^2+b^2}}\right.}{}\right)}{}$$

$$\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\text{PolyLog}\left(3,-e^{2(c+dx)}\right)}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{b}$$

$\downarrow$ 7293

$$-\frac{a\left(\frac{\int \left(a(e+fx)^3\text{sech}^2(c+dx)-b(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)\right)dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-\ldots\right.}{d}\ldots\right.}{\ldots}\ldots\right)}{\ldots}\ldots\right)}{\ldots}\ldots\right)}{\ldots}\ldots\right)}{\ldots}$$

$$\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\,\text{PolyLog}\left(3,-e^{2(c+dx)}\right)}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{b}$$

$\downarrow$ 2009

$$\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\,\text{PolyLog}\left(3,-e^{2(c+dx)}\right)}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{b}-$$

$$a\left(-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+c}}{a+\sqrt{a^2}}\right.}{d}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{}\right.$$

input
```
Int[((e + f*x)^3*Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(((3*I)*f*(((-1/3*I)*(e + f*x)^3)/f + (2*I)*(((e + f*x)^2*Log[1 + E^(2*(c
+ d*x))])/(2*d) - (f*(-1/2*((e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/d + (f
*PolyLog[3, -E^(2*(c + d*x))])/(4*d^2)))/d)))/d + ((e + f*x)^3*Tanh[c + d*
x])/d)/b - (a*((-2*b^2*(-1/2*(b*(((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2]))])/d^2))/d))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^3*Log
[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*
PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x
)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/(2*Sqrt[a^2 +
 b^2])))/(a^2 + b^2) + ((a*(e + f*x)^3)/d - (6*b*f*(e + f*x)^2*ArcTan[E^(c
 + d*x)])/d^2 - (3*a*f*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/d^2 + ((6*I)*
b*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d^3 - ((6*I)*b*f^2*(e + f*x)
*PolyLog[2, I*E^(c + d*x)])/d^3 - (3*a*f^2*(e + f*x)*PolyLog[2, -E^(2*(c +
 d*x))])/d^3 - ((6*I)*b*f^3*PolyLog[3, (-I)*E^(c + d*x)])/d^4 + ((6*I)*b*f
^3*PolyLog[3, I*E^(c + d*x)])/d^4 + (3*a*f^3*PolyLog[3, -E^(2*(c + d*x))])
/(2*d^4) + (b*(e + f*x)^3*Sech[c + d*x])/d + (a*(e + f*x)^3*Tanh[c + d*x])
/d)/(a^2 + b^2)))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6117
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x],
x] - Simp[a/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^3 \operatorname{sech}(dx+c)\tanh(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 6503 vs. $2(846) = 1692$.

Time = 0.24 (sec) , antiderivative size = 6503, normalized size of antiderivative = 7.09

$$\int \frac{(e+fx)^3\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^3 \text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^3\tanh(c+dx)\,\text{sech}\,(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*tanh(c + d*x)*sech(c + d*x)/(a + b*sinh(c + d*x)), x
)
```

Maxima [F]

$$\int \frac{(e+fx)^3 \text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\,\text{sech}\,(dx+c)\tanh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
3*b*e^2*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2
+ b^2)*d^2)) + 6*a*f^3*integrate(x^2*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) +
b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + 6*b*f^3*integrate(x^2/(a^2*d*
e^(2*d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + 12*a*e*f^2*
integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a
^2*d + b^2*d), x) + 12*b*e*f^2*integrate(x/(a^2*d*e^(2*d*x + 2*c) + b^2*d*
e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - e^3*(a*b*log((b*e^(-d*x - c) - a -
sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^2 + b^2)^(3/2
)*d) + 2*(a*e^(-d*x - c) - b)/((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c))*
d)) + 6*a*e^2*f*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) - 2*(b*f^3*x^3 + 3*b
*e*f^2*x^2 + 3*b*e^2*f*x + (a*f^3*x^3*e^c + 3*a*e*f^2*x^2*e^c + 3*a*e^2*f*
x*e^c)*e^(d*x))/(a^2*d + b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)
) - integrate(-2*(a*b*f^3*x^3*e^c + 3*a*b*e*f^2*x^2*e^c + 3*a*b*e^2*f*x*e^
c)*e^(d*x)/(a^2*b + b^3 - (a^2*b*e^(2*c) + b^3*e^(2*c))*e^(2*d*x) - 2*(a^3
*e^c + a*b^2*e^c)*e^(d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\tanh(c+d\,x)\,\left(e+f\,x\right)^3}{\cosh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((tanh(c + d*x)*(e + f*x)^3)/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((tanh(c + d*x)*(e + f*x)^3)/(cosh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{too large to display}$$

input
```
int((f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 4*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqr
t(a**2 + b**2))*a*b**2*d**3*e**3*i - 4*sqrt(a**2 + b**2)*atan((e**(c + d*x
)*b*i + a*i)/sqrt(a**2 + b**2))*a*b**2*d**3*e**3*i - 16*e**(5*c + 2*d*x)*i
nt((e**(3*d*x)*x**3)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c
+ 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a
- b),x)*a**5*d**4*f**3 - 32*e**(5*c + 2*d*x)*int((e**(3*d*x)*x**3)/(e**(6*
c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d
*x)*a - e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**3*b**2*d**4*f**3
- 16*e**(5*c + 2*d*x)*int((e**(3*d*x)*x**3)/(e**(6*c + 6*d*x)*b + 2*e**(5*
c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x
)*b + 2*e**(c + d*x)*a - b),x)*a*b**4*d**4*f**3 - 48*e**(5*c + 2*d*x)*int(
(e**(3*d*x)*x**2)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4
*d*x)*b + 4*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b
),x)*a**5*d**4*e*f**2 - 96*e**(5*c + 2*d*x)*int((e**(3*d*x)*x**2)/(e**(6*c
 + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*
x)*a - e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**3*b**2*d**4*e*f**2
 - 48*e**(5*c + 2*d*x)*int((e**(3*d*x)*x**2)/(e**(6*c + 6*d*x)*b + 2*e**(5
*c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*
x)*b + 2*e**(c + d*x)*a - b),x)*a*b**4*d**4*e*f**2 - 48*e**(5*c + 2*d*x)*i
nt((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c ...
```

3.354 $\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 648

$$
\begin{aligned}
\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx
&= \frac{(e+fx)^2}{bd} - \frac{a^2(e+fx)^2}{b(a^2+b^2)\,d} + \frac{4af(e+fx)\arctan\left(e^{c+dx}\right)}{(a^2+b^2)\,d^2} \\
&\quad - \frac{ab(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} + \frac{ab(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} \\
&\quad - \frac{2f(e+fx)\log\left(1+e^{2(c+dx)}\right)}{bd^2} + \frac{2a^2f(e+fx)\log\left(1+e^{2(c+dx)}\right)}{b(a^2+b^2)\,d^2} \\
&\quad - \frac{2iaf^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{(a^2+b^2)\,d^3} + \frac{2iaf^2\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{(a^2+b^2)\,d^3} \\
&\quad - \frac{2abf(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^2} + \frac{2abf(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^2} \\
&\quad - \frac{f^2\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{bd^3} + \frac{a^2f^2\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{b(a^2+b^2)\,d^3} \\
&\quad + \frac{2abf^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^3} - \frac{2abf^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^3} \\
&\quad - \frac{a(e+fx)^2\text{sech}(c+dx)}{(a^2+b^2)\,d} + \frac{(e+fx)^2\tanh(c+dx)}{bd} - \frac{a^2(e+fx)^2\tanh(c+dx)}{b(a^2+b^2)\,d}
\end{aligned}
$$

output
```
(f*x+e)^2/b/d-a^2*(f*x+e)^2/b/(a^2+b^2)/d+4*a*f*(f*x+e)*arctan(exp(d*x+c))
/(a^2+b^2)/d^2-a*b*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b
^2)^(3/2)/d+a*b*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)
^(3/2)/d-2*f*(f*x+e)*ln(1+exp(2*d*x+2*c))/b/d^2+2*a^2*f*(f*x+e)*ln(1+exp(2
*d*x+2*c))/b/(a^2+b^2)/d^2+2*I*a*f^2*polylog(2,I*exp(d*x+c))/(a^2+b^2)/d^3
-2*I*a*f^2*polylog(2,-I*exp(d*x+c))/(a^2+b^2)/d^3-2*a*b*f*(f*x+e)*polylog(
2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2+2*a*b*f*(f*x+e)*p
olylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2-f^2*polylo
g(2,-exp(2*d*x+2*c))/b/d^3+a^2*f^2*polylog(2,-exp(2*d*x+2*c))/b/(a^2+b^2)/
d^3+2*a*b*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)
/d^3-2*a*b*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2
)/d^3-a*(f*x+e)^2*sech(d*x+c)/(a^2+b^2)/d+(f*x+e)^2*tanh(d*x+c)/b/d-a^2*(f
*x+e)^2*tanh(d*x+c)/b/(a^2+b^2)/d
```

Mathematica [A] (warning: unable to verify)

Time = 3.88 (sec) , antiderivative size = 633, normalized size of antiderivative = 0.98

$$\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{\frac{f\left(4bd^2ee^{2c}x-4bd^2e\left(1+e^{2c}\right)x+2bd^2e^{2c}fx^2-2bd^2\left(1+e^{2c}\right)fx^2+4ade\left(1+e^{2c}\right)\arctan\left(e^{c+dx}\right)+2bde\left(1+e^{2c}\right)\left(2dx-\log\left(1+e^{2(c+dx)}\right)\right)+2ia\left(1+e^{2c}\right.}{(a^2}}{}$$

input
```
Integrate[((e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
((f*(4*b*d^2*e*E^(2*c)*x - 4*b*d^2*e*(1 + E^(2*c))*x + 2*b*d^2*E^(2*c)*f*x
^2 - 2*b*d^2*(1 + E^(2*c))*f*x^2 + 4*a*d*e*(1 + E^(2*c))*ArcTan[E^(c + d*x
)] + 2*b*d*e*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (2*I)*a*(1
 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - Pol
yLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) + b*(1 + E^(2*c))*f
*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x))])))
/((a^2 + b^2)*(1 + E^(2*c))) + (a*b*(2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))
/Sqrt[a^2 + b^2]] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*d^2*
e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1 +
 (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*d*f*(e + f*x)*PolyLog[2, (b*E^
(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))] + 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + S
qrt[a^2 + b^2])] - 2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
))]))/(a^2 + b^2)^(3/2) + (d^2*(e + f*x)^2*Sech[c + d*x]*(-a + b*Sech[c]*S
inh[d*x]))/(a^2 + b^2))/d^3
```

Rubi [A] (verified)

Time = 3.85 (sec) , antiderivative size = 573, normalized size of antiderivative = 0.88, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {6117, 3042, 4672, 26, 3042, 26, 4201, 2620, 2715, 2838, 6107, 3042, 3803, 25, 2694, 27, 2620, 3011, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\tanh(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6117}$$

$$\frac{\int (e+fx)^2\text{sech}^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{b}$$

$$\Big\downarrow \text{ 4672}$$

$$-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2if\int -i(e+fx)\tanh(c+dx)dx}{d}}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2f\int (e+fx)\tanh(c+dx)dx}{d}}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2f\int -i(e+fx)\tan(ic+idx)dx}{d}}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\int (e+fx)\tan(ic+idx)dx}{d}}{b}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\int \frac{e^{2(c+dx)}(e+fx)}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int \log\left(1+e^{2(c+dx)}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int e^{-2(c+dx)}\log\left(1+e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}$$

$$\Big\downarrow \text{ 6107}$$

$$-\frac{a\left(\frac{b^2\int\frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}-\frac{a\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{a^2+b^2}\right)}{b}$$

$$\Big\downarrow \text{ 3803}$$

$$-\frac{a\left(\frac{2b^2\int-\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{\int(e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{a\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\int\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}\right)}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}$$

$$\Big\downarrow \text{ 2694}$$

$$-\frac{a\left(\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}$$

$$\Big\downarrow \ 27$$

$$-\frac{a\left(\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}$$

$$\Big\downarrow \ 2620$$

$$-\frac{a\left(\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\cdots\right.}{\cdots}\right.}{\cdots}\right)}{a^2+b^2}\right)}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}$$

$$\Big\downarrow \ 3011$$

$$-\frac{a\left(\frac{\int (e+fx)^2\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{ PolyLog}}{}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}}{}\right.}{b}$$

$$\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\text{ PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}$$

$$\Big\downarrow \text{ 2720}$$

$$
-\frac{a\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e\cdots}{\cdots}\right)}{bd}\cdots\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{\cdots}\right)}{\cdots}
$$

$$
\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}
$$

$$\Big\downarrow \; 7143$$

$$-\frac{a\left(\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\cdots\right.}{d}\cdots\right.}{bd}\cdots\right.}{2\sqrt{a^2+b^2}}\cdots\right.}{\cdots}\cdots\right.}{b\cdots}$$

$$\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}$$

$\downarrow$ 7293

$$-\frac{a\left(\frac{\int\left(a(e+fx)^2\operatorname{sech}^2(c+dx)-b(e+fx)^2\operatorname{sech}(c+dx)\tanh(c+dx)\right)dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right.}{d^2}\right.}{\cdots}\right.}{2\sqrt{a^2+b^2}\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}$$

$\downarrow$ 2009

$$\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}-$$

$$a\left(-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right.}{}\right)}{a^2+b^2}\right.$$

input
```
Int[((e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(((2*I)*f*(((-1/2*I)*(e + f*x)^2)/f + (2*I)*(((e + f*x)*Log[1 + E^(2*(c +
d*x))])/(2*d) + (f*PolyLog[2, -E^(2*(c + d*x))])/(4*d^2))))/d + ((e + f*x)
^2*Tanh[c + d*x])/d)/b - (a*((-2*b^2*(-1/2*(b*(((e + f*x)^2*Log[1 + (b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -(
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x
)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e +
f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog
[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/(2*Sqrt[a^2 +
 b^2])))/(a^2 + b^2) + ((a*(e + f*x)^2)/d - (4*b*f*(e + f*x)*ArcTan[E^(c +
 d*x)])/d^2 - (2*a*f*(e + f*x)*Log[1 + E^(2*(c + d*x))])/d^2 + ((2*I)*b*f^
2*PolyLog[2, (-I)*E^(c + d*x)])/d^3 - ((2*I)*b*f^2*PolyLog[2, I*E^(c + d*x
)])/d^3 - (a*f^2*PolyLog[2, -E^(2*(c + d*x))])/d^3 + (b*(e + f*x)^2*Sech[c
 + d*x])/d + (a*(e + f*x)^2*Tanh[c + d*x])/d)/(a^2 + b^2)))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6117
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x],
x] - Simp[a/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{sech}(dx+c)\tanh(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3662 vs. $2(601) = 1202$.

Time = 0.21 (sec) , antiderivative size = 3662, normalized size of antiderivative = 5.65

$$\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2\tanh(c+dx)\,\text{sech}\,(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*tanh(c + d*x)*sech(c + d*x)/(a + b*sinh(c + d*x)), x
)
```

Maxima [F]

$$\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\,\text{sech}\,(dx+c)\tanh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
2*b*e*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2 +
b^2)*d^2)) + 4*a*f^2*integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2*
d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + 4*b*f^2*integrate(x/(a^2*d*e^(2*d
*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - e^2*(a*b*log((b*e
^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))
/((a^2 + b^2)^(3/2)*d) + 2*(a*e^(-d*x - c) - b)/((a^2 + b^2 + (a^2 + b^2)*
e^(-2*d*x - 2*c))*d)) + 4*a*e*f*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) - 2*
(b*f^2*x^2 + 2*b*e*f*x + (a*f^2*x^2*e^c + 2*a*e*f*x*e^c)*e^(d*x))/(a^2*d +
 b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)) - integrate(-2*(a*b*f^
2*x^2*e^c + 2*a*b*e*f*x*e^c)*e^(d*x)/(a^2*b + b^3 - (a^2*b*e^(2*c) + b^3*e
^(2*c))*e^(2*d*x) - 2*(a^3*e^c + a*b^2*e^c)*e^(d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh(c+d\,x)\,\left(e+f\,x\right)^2}{\cosh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((tanh(c + d*x)*(e + f*x)^2)/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((tanh(c + d*x)*(e + f*x)^2)/(cosh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output

```
( - 2*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqr
t(a**2 + b**2))*a*b**2*d**2*e**2*i - 2*sqrt(a**2 + b**2)*atan((e**(c + d*x
)*b*i + a*i)/sqrt(a**2 + b**2))*a*b**2*d**2*e**2*i - 8*e**(5*c + 2*d*x)*in
t((e**(3*d*x)*x**2)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c +
 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a -
 b),x)*a**5*d**3*f**2 - 16*e**(5*c + 2*d*x)*int((e**(3*d*x)*x**2)/(e**(6*c
 + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*
x)*a - e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**3*b**2*d**3*f**2 -
 8*e**(5*c + 2*d*x)*int((e**(3*d*x)*x**2)/(e**(6*c + 6*d*x)*b + 2*e**(5*c
+ 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*
b + 2*e**(c + d*x)*a - b),x)*a*b**4*d**3*f**2 - 16*e**(5*c + 2*d*x)*int((e
**(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)
*b + 4*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*
a**5*d**3*e*f - 32*e**(5*c + 2*d*x)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b
 + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - e**(
2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**3*b**2*d**3*e*f - 16*e**(5*c
+ 2*d*x)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e
**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b + 2*e**(c +
d*x)*a - b),x)*a*b**4*d**3*e*f + 4*e**(4*c + 2*d*x)*int((e**(2*d*x)*x)/(e*
*(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*...
```

3.355 $\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 335

$$\begin{aligned}\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{af\arctan(\sinh(c+dx))}{(a^2+b^2)\,d^2} - \frac{ab(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} \\ &\quad + \frac{ab(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} - \frac{f\log(\cosh(c+dx))}{bd^2} + \frac{a^2f\log(\cosh(c+dx))}{b\,(a^2+b^2)\,d^2} \\ &\quad - \frac{abf\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^2} + \frac{abf\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^2} \\ &\quad - \frac{a(e+fx)\text{sech}(c+dx)}{(a^2+b^2)\,d} + \frac{(e+fx)\tanh(c+dx)}{bd} - \frac{a^2(e+fx)\tanh(c+dx)}{b\,(a^2+b^2)\,d}\end{aligned}$$

output
```
a*f*arctan(sinh(d*x+c))/(a^2+b^2)/d^2-a*b*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^
2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d+a*b*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2
)^(1/2)))/(a^2+b^2)^(3/2)/d-f*ln(cosh(d*x+c))/b/d^2+a^2*f*ln(cosh(d*x+c))/
b/(a^2+b^2)/d^2-a*b*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^
2)^(3/2)/d^2+a*b*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^
(3/2)/d^2-a*(f*x+e)*sech(d*x+c)/(a^2+b^2)/d+(f*x+e)*tanh(d*x+c)/b/d-a^2*(f
*x+e)*tanh(d*x+c)/b/(a^2+b^2)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.47 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.99

$$\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{\frac{2f\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a-ib} + \frac{2f\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a+ib} + \frac{f\log(\cosh(c+dx))}{ia-b} - \frac{f\log(\cosh(c+dx))}{ia+b} - \frac{2ab\left(-2de\,\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right.\right.}{}}{}$$

input
```
Integrate[((e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
((2*f*ArcTan[Tanh[(c + d*x)/2]])/(a - I*b) + (2*f*ArcTan[Tanh[(c + d*x)/2]
])/(a + I*b) + (f*Log[Cosh[c + d*x]])/(I*a - b) - (f*Log[Cosh[c + d*x]])/(
I*a + b) - (2*a*b*(-2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2
*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log[1 + (b
*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1 + (b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2])] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^
2])] - f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^2 + b^2
)^(3/2) + (2*d*(e + f*x)*Sech[c + d*x]*(-a + b*Sinh[c + d*x]))/(a^2 + b^2)
)/(2*d^2)
```

Rubi [A] (verified)

Time = 2.16 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.93, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6117, 3042, 4672, 26, 3042, 26, 3956, 6107, 3042, 3803, 25, 2694, 27, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \; 6117$$

$$\frac{\int (e+fx)\text{sech}^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$\Big\downarrow$ 3042

$$-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{b}$$

$\Big\downarrow$ 4672

$$-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{if\int -i\tanh(c+dx)dx}{d}}{b}$$

$\Big\downarrow$ 26

$$\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int \tanh(c+dx)dx}{d}}{b}-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$\Big\downarrow$ 3042

$$-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int -i\tan(ic+idx)dx}{d}}{b}$$

$\Big\downarrow$ 26

$$-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\tanh(c+dx)}{d}+\frac{if\int \tan(ic+idx)dx}{d}}{b}$$

$\Big\downarrow$ 3956

$$\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$\Big\downarrow$ 6107

$$\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{b^2\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a^2+b^2}\right)}{b}$$

$$\Big\downarrow \text{3803}$$

$$\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{2b^2\int -\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\int \frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}\right)}{b}$$

$$\Big\downarrow \text{2694}$$

$$\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{b}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+\cdots\right.}{bd}\cdots\right)}{2\sqrt{\cdots}}\right)}{a^2+b^2}\cdots\right)}{b}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\cdots\right.}{\cdots}\right.}{\cdots}\right)}{a^2+b^2}\cdots\right)}{b}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+\cdots}}\right.}{bd^2}\cdots\right.}{\cdots}\right)}{a^2+b^2}\cdots\right)}{b}$$

$$\Big\downarrow \text{7293}$$

$$\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-$$

$$\frac{a\left(\frac{\int\left(a(e+fx)\text{sech}^2(c+dx)-b(e+fx)\text{sech}(c+dx)\tanh(c+dx)\right)dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\cdots\right.}{\cdots}\right.}{a^2+b^2}\right.}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-$$

$$\frac{a\left(\frac{-\frac{af\log(\cosh(c+dx))}{d^2}+\frac{a(e+fx)\tanh(c+dx)}{d}-\frac{bf\arctan(\sinh(c+dx))}{d^2}+\frac{b(e+fx)\text{sech}(c+dx)}{d}}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{b\cdots}{\sqrt{a\cdots}}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{\cdots}\right.}{b}$$

input
```
Int[((e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-((f*Log[Cosh[c + d*x]])/d^2) + ((e + f*x)*Tanh[c + d*x])/d)/b - (a*((-2*
b^2*(-1/2*(b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])]])/(
b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2)))/
Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2])]])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b
*d^2)))/(2*Sqrt[a^2 + b^2])))/(a^2 + b^2) + (-((b*f*ArcTan[Sinh[c + d*x]])
/d^2) - (a*f*Log[Cosh[c + d*x]])/d^2 + (b*(e + f*x)*Sech[c + d*x])/d + (a*
(e + f*x)*Tanh[c + d*x])/d)/(a^2 + b^2)))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6117
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x],
x] - Simp[a/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1857 vs. $2(315) = 630$.

Time = 3.77 (sec) , antiderivative size = 1858, normalized size of antiderivative = 5.55

method	result	size
risch	Expression too large to display	1858

input
```
int((f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERB
OSE)
```

output
```
-2/d^2/(a^2+b^2)^(1/2)*c*a*b*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2
*a)/(a^2+b^2)^(1/2))+2/(a^2+b^2)/d^2*b*f*ln(exp(d*x+c))-1/2/(a^2+b^2)^2/d^
2*b^3*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/(a^2+b^2)/d^2*b^3*f/(2*a^2
+2*b^2)*ln(1+exp(2*d*x+2*c))+1/(a^2+b^2)/d^2*b^3*f/(2*a^2+2*b^2)*ln(b*exp(
2*d*x+2*c)+2*a*exp(d*x+c)-b)+4/(a^2+b^2)/d^2*a^3*f/(2*a^2+2*b^2)*arctan(ex
p(d*x+c))-1/(a^2+b^2)^2/d^2*b*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)*a^2-
2/(a^2+b^2)^(3/2)/d^2*a*b^3*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*
a)/(a^2+b^2)^(1/2))+2/(a^2+b^2)^(5/2)/d^2*b*f*arctanh(1/2*(2*b*exp(d*x+c)+
2*a)/(a^2+b^2)^(1/2))*a^3+2/(a^2+b^2)^(5/2)/d^2*b^3*f*arctanh(1/2*(2*b*exp
(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a+4/(a^2+b^2)/d^2*b^2*f/(2*a^2+2*b^2)*a*arct
an(exp(d*x+c))-2/(a^2+b^2)/d^2*a^2*b*f/(2*a^2+2*b^2)*ln(1+exp(2*d*x+2*c))+
2/(a^2+b^2)/d^2*a^2*b*f/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b
)+2/(a^2+b^2)^(3/2)/d*a*b^3*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/
2)+a)/(a+(a^2+b^2)^(1/2)))*x-2/(a^2+b^2)^(3/2)/d*a^3*b*f/(2*a^2+2*b^2)*ln(
(-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+2/(a^2+b^2)^(3/2
)/d*a^3*b*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)
^(1/2)))*x-2/(a^2+b^2)^(3/2)/d*a*b^3*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^
2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-2/(a^2+b^2)^(3/2)/d^2*a^3*b*f/(2*a
^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/(a^2+b^2)^(1
/2)/d^2*a*b*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1338 vs. $2(313) = 626$.

Time = 0.18 (sec) , antiderivative size = 1338, normalized size of antiderivative = 3.99

$$\int \frac{(e+fx)\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
fricas")
```

output
```
(2*(a^2*b + b^3)*d*f*x*cosh(d*x + c)^2 + 2*(a^2*b + b^3)*d*f*x*sinh(d*x +
c)^2 - 2*(a^2*b + b^3)*d*e - (a*b^2*f*cosh(d*x + c)^2 + 2*a*b^2*f*cosh(d*x
 + c)*sinh(d*x + c) + a*b^2*f*sinh(d*x + c)^2 + a*b^2*f)*sqrt((a^2 + b^2)/
b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (a*b^2*f*cosh(d*x + c)^2 + 2
*a*b^2*f*cosh(d*x + c)*sinh(d*x + c) + a*b^2*f*sinh(d*x + c)^2 + a*b^2*f)*
sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (a*b^2*d*e
- a*b^2*c*f + (a*b^2*d*e - a*b^2*c*f)*cosh(d*x + c)^2 + 2*(a*b^2*d*e - a*b
^2*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d*e - a*b^2*c*f)*sinh(d*x + c
)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b
*sqrt((a^2 + b^2)/b^2) + 2*a) - (a*b^2*d*e - a*b^2*c*f + (a*b^2*d*e - a*b^
2*c*f)*cosh(d*x + c)^2 + 2*(a*b^2*d*e - a*b^2*c*f)*cosh(d*x + c)*sinh(d*x
+ c) + (a*b^2*d*e - a*b^2*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(
2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) -
 (a*b^2*d*f*x + a*b^2*c*f + (a*b^2*d*f*x + a*b^2*c*f)*cosh(d*x + c)^2 + 2*
(a*b^2*d*f*x + a*b^2*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d*f*x + a*b
^2*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*s
inh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) -
 b)/b) + (a*b^2*d*f*x + a*b^2*c*f + (a*b^2*d*f*x + a*b^2*c*f)*cosh(d*x ...
```

Sympy [F]

$$\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\tanh{(c+dx)}\operatorname{sech}{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*tanh(c + d*x)*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\operatorname{sech}(dx+c)\tanh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
maxima")
```

output
```
-(2*a*b*integrate(-x*e^(d*x + c)/(a^2*b + b^3 - (a^2*b*e^(2*c) + b^3*e^(2*
c))*e^(2*d*x) - 2*(a^3*e^c + a*b^2*e^c)*e^(d*x)), x) + 2*(a*x*e^(d*x + c)
+ b*x)/(a^2*d + b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)) - 2*b*x
/((a^2 + b^2)*d) - 2*a*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) + b*log(e^(2*
d*x + 2*c) + 1)/((a^2 + b^2)*d^2))*f - e*(a*b*log((b*e^(-d*x - c) - a - sq
rt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^2 + b^2)^(3/2)*
d) + 2*(a*e^(-d*x - c) - b)/((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c))*d)
)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh(c+d\,x)\,\,(e+f\,x)}{\cosh\left(c+d\,x\right)\,\,(a+b\sinh\left(c+d\,x\right))}\,dx$$

input
```
int((tanh(c + d*x)*(e + f*x))/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((tanh(c + d*x)*(e + f*x))/(cosh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqr
t(a**2 + b**2))*a*b**2*d*e*i - 2*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i
+ a*i)/sqrt(a**2 + b**2))*a*b**2*d*e*i - 8*e**(5*c + 2*d*x)*int((e**(3*d*x
)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e
**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**5*d**
2*f - 16*e**(5*c + 2*d*x)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5
*c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*
x)*b + 2*e**(c + d*x)*a - b),x)*a**3*b**2*d**2*f - 8*e**(5*c + 2*d*x)*int(
(e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*d*
x)*b + 4*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x
)*a*b**4*d**2*f - e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x) + 1)*a**4*f - 2*e*
*(2*c + 2*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2*b**2*f - e**(2*c + 2*d*x)*lo
g(e**(2*c + 2*d*x) + 1)*b**4*f + 2*e**(2*c + 2*d*x)*a**4*d*f*x + 2*e**(2*c
 + 2*d*x)*a**2*b**2*d*e + 4*e**(2*c + 2*d*x)*a**2*b**2*d*f*x + 2*e**(2*c +
 2*d*x)*b**4*d*e + 2*e**(2*c + 2*d*x)*b**4*d*f*x - 2*e**(c + d*x)*a**3*b*d
*e - 2*e**(c + d*x)*a*b**3*d*e - 8*e**(3*c)*int((e**(3*d*x)*x)/(e**(6*c +
6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*
a - e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**5*d**2*f - 16*e**(3*c
)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c
+ 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b + 2*e**(c + d*x)...
```

3.356 $\int \frac{\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 78

$$\int \frac{\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2ab\text{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} - \frac{\text{sech}(c+dx)(a-b\sinh(c+dx))}{(a^2+b^2)\,d}$$

output
```
2*a*b*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d
-sech(d*x+c)*(a-b*sinh(d*x+c))/(a^2+b^2)/d
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.33

$$\int \frac{\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{-2ab\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right) - a\sqrt{-a^2-b^2}\text{sech}(c+dx) + b\sqrt{-a^2-b^2}\tanh(c+dx)}{(-a^2-b^2)^{3/2}\,d}$$

input
```
Integrate[(Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-((-2*a*b*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]] - a*Sqrt[-a^2
 - b^2]*Sech[c + d*x] + b*Sqrt[-a^2 - b^2]*Tanh[c + d*x])/((-a^2 - b^2)^(3
/2)*d))
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 26, 3345, 26, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ic+idx)}{\cos(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ic+idx)}{\cos(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3345}$$

$$-i\left(-\frac{\int \frac{iab}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{i\text{sech}(c+dx)(a-b\sinh(c+dx))}{d(a^2+b^2)}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{i\int \frac{ab}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{i\text{sech}(c+dx)(a-b\sinh(c+dx))}{d(a^2+b^2)}\right)$$

$$\Big\downarrow \text{27}$$

$$-i\left(-\frac{iab\int \frac{1}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{i\text{sech}(c+dx)(a-b\sinh(c+dx))}{d(a^2+b^2)}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{iab\int \frac{1}{a-ib\sin(ic+idx)}dx}{a^2+b^2}-\frac{i\text{sech}(c+dx)(a-b\sinh(c+dx))}{d(a^2+b^2)}\right)$$

$$\Big\downarrow \text{3139}$$

$$-i\left(-\frac{2ab\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d(a^2+b^2)}-\frac{i\text{sech}(c+dx)(a-b\sinh(c+dx))}{d(a^2+b^2)}\right)$$

$$\Big\downarrow \text{1083}$$

$$-i\left(\frac{4ab\int \frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4(a^2+b^2)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{d(a^2+b^2)}-\frac{i\text{sech}(c+dx)(a-b\sinh(c+dx))}{d(a^2+b^2)}\right)$$

$$\Big\downarrow \text{217}$$

$$-i\left(-\frac{2iab\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{3/2}}-\frac{i\text{sech}(c+dx)(a-b\sinh(c+dx))}{d(a^2+b^2)}\right)$$

input
```
Int[(Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-I)*(((-2*I)*a*b*ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b^2])])/((a^2 +
b^2)^(3/2)*d) - (I*Sech[c + d*x]*(a - b*Sinh[c + d*x]))/((a^2 + b^2)*d))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{\frac{2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2a}{\left(a^{2}+b^{2}\right)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}-\frac{4ab\,\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\left(2a^{2}+2b^{2}\right)\sqrt{a^{2}+b^{2}}}}{d}$
default	$\frac{\frac{2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2a}{\left(a^{2}+b^{2}\right)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}-\frac{4ab\,\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\left(2a^{2}+2b^{2}\right)\sqrt{a^{2}+b^{2}}}}{d}$
risch	$-\frac{2\left(a\,{e}^{dx+c}+b\right)}{d\left(a^{2}+b^{2}\right)\left(1+{e}^{2dx+2c}\right)}+\frac{ba\ln\left({e}^{dx+c}+\frac{a\left(a^{2}+b^{2}\right)^{\frac{3}{2}}+a^{4}+2a^{2}b^{2}+b^{4}}{b\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}d}-\frac{ba\ln\left({e}^{dx+c}+\frac{a\left(a^{2}+b^{2}\right)^{\frac{3}{2}}-a^{4}-2a^{2}b^{2}-b^{4}}{b\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}d}$

input
```
int(sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/(a^2+b^2)*(b*tanh(1/2*d*x+1/2*c)-a)/(1+tanh(1/2*d*x+1/2*c)^2)-4*a*b
/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(
a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(75) = 150$.

Time = 0.08 (sec) , antiderivative size = 350, normalized size of antiderivative = 4.49

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{2\,a^2b+2\,b^3-\left(ab\cosh\left(dx+c\right)^2+2\,ab\cosh\left(dx+c\right)\sinh\left(dx+c\right)+ab\sinh\left(dx+c\right)^2+ab\right)\sqrt{a^2+b^2}}{\left(a^4+2\,a^2b^2+b^4\right)d\cosh\left(dx+c\right)^2+2\,(}$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(2*a^2*b + 2*b^3 - (a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x +
c) + a*b*sinh(d*x + c)^2 + a*b)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 +
 b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x
 + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d
*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) +
 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 2*(a^3 + a*b^2)*cosh(d*x +
c) + 2*(a^3 + a*b^2)*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x +
c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*
a^2*b^2 + b^4)*d*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d)
```

Sympy [F]

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh{(c+dx)}\operatorname{sech}{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(tanh(c + d*x)*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.50

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{ab\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}d} - \frac{2\left(ae^{(-dx-c)}-b\right)}{(a^2+b^2+(a^2+b^2)e^{(-2\,dx-2\,c)})d}$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-a*b*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt
(a^2 + b^2)))/((a^2 + b^2)^(3/2)*d) - 2*(a*e^(-d*x - c) - b)/((a^2 + b^2 +
 (a^2 + b^2)*e^(-2*d*x - 2*c))*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.36

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{ab\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}} + \frac{2\left(ae^{(dx+c)}+b\right)}{(a^2+b^2)(e^{(2\,dx+2\,c)}+1)}}{d}$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-(a*b*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x +
c) + 2*a + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(a*e^(d*x + c) + b)/(
(a^2 + b^2)*(e^(2*d*x + 2*c) + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.18

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{a\,b\,\ln\left(\frac{2\,a\,e^{c+d\,x}}{a^2+b^2}+\frac{2\,a\,\left(b-a\,e^{c+d\,x}\right)}{{\left(a^2+b^2\right)}^{3/2}}\right)}{d\,{\left(a^2+b^2\right)}^{3/2}} - \frac{a\,b\,\ln\left(\frac{2\,a\,e^{c+d\,x}}{a^2+b^2}-\frac{2\,a\,\left(b-a\,e^{c+d\,x}\right)}{{\left(a^2+b^2\right)}^{3/2}}\right)}{d\,{\left(a^2+b^2\right)}^{3/2}} - \frac{\frac{2\,b}{d\,\left(a^2+b^2\right)}+\frac{2\,a\,e^{c+d\,x}}{d\,\left(a^2+b^2\right)}}{e^{2\,c+2\,d\,x}+1}$$

input
```
int(tanh(c + d*x)/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
(a*b*log((2*a*exp(c + d*x))/(a^2 + b^2) + (2*a*(b - a*exp(c + d*x)))/(a^2
+ b^2)^(3/2)))/(d*(a^2 + b^2)^(3/2)) - (a*b*log((2*a*exp(c + d*x))/(a^2 +
b^2) - (2*a*(b - a*exp(c + d*x)))/(a^2 + b^2)^(3/2)))/(d*(a^2 + b^2)^(3/2)
) - ((2*b)/(d*(a^2 + b^2)) + (2*a*exp(c + d*x))/(d*(a^2 + b^2)))/(exp(2*c
+ 2*d*x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.71

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{-2e^{2dx+2c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)abi-2\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)abi+2e^{2dx+2c}a^2b+2e^{2dx+2c}b^3-2}{d\left(e^{2dx+2c}a^4+2e^{2dx+2c}a^2b^2+e^{2dx+2c}b^4+a^4+2a^2b^2+b^4\right)}$$

input
```
int(sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
(2*( - e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sq
rt(a**2 + b**2))*a*b*i - sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/s
qrt(a**2 + b**2))*a*b*i + e**(2*c + 2*d*x)*a**2*b + e**(2*c + 2*d*x)*b**3
- e**(c + d*x)*a**3 - e**(c + d*x)*a*b**2))/(d*(e**(2*c + 2*d*x)*a**4 + 2*
e**(2*c + 2*d*x)*a**2*b**2 + e**(2*c + 2*d*x)*b**4 + a**4 + 2*a**2*b**2 +
b**4))
```

3.357 $\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 68.72 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Sech[c + d*x]*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
x]
```

output
```
Integrate[(Sech[c + d*x]*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
 x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(c+dx)\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{6125}$$
$$\int \frac{\tanh(c+dx)\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Sech[c + d*x]*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\text{sech}\,(dx+c)\tanh\,(dx+c)}{(fx+e)\,(a+b\sinh\,(dx+c))}dx$$

input
```
int(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
fricas")
```

output
```
integral(sech(d*x + c)*tanh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x
 + c)), x)
```

Sympy [N/A]

Not integrable

Time = 2.78 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh(c+dx)\operatorname{sech}(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(tanh(c + d*x)*sech(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 386, normalized size of antiderivative = 12.06

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
maxima")
```

output
```
-2*a*b*integrate(-e^(d*x + c)/(a^2*b*e + b^3*e + (a^2*b*f + b^3*f)*x - (a^
2*b*e*e^(2*c) + b^3*e*e^(2*c) + (a^2*b*f*e^(2*c) + b^3*f*e^(2*c))*x)*e^(2*
d*x) - 2*(a^3*e*e^c + a*b^2*e*e^c + (a^3*f*e^c + a*b^2*f*e^c)*x)*e^(d*x)),
 x) - 2*(a*e^(d*x + c) + b)/(a^2*d*e + b^2*d*e + (a^2*d*f + b^2*d*f)*x + (
a^2*d*e*e^(2*c) + b^2*d*e*e^(2*c) + (a^2*d*f*e^(2*c) + b^2*d*f*e^(2*c))*x)
*e^(2*d*x)) - 2*integrate((a*f*e^(d*x + c) + b*f)/(a^2*d*e^2 + b^2*d*e^2 +
 (a^2*d*f^2 + b^2*d*f^2)*x^2 + 2*(a^2*d*e*f + b^2*d*e*f)*x + (a^2*d*e^2*e^
(2*c) + b^2*d*e^2*e^(2*c) + (a^2*d*f^2*e^(2*c) + b^2*d*f^2*e^(2*c))*x^2 +
2*(a^2*d*e*f*e^(2*c) + b^2*d*e*f*e^(2*c))*x)*e^(2*d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 1.73 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh(c+d\,x)}{\cosh\left(c+d\,x\right)\,\left(e+f\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(tanh(c + d*x)/(cosh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(tanh(c + d*x)/(cosh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.38

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{\sinh(dx+c)\,be+\sinh(dx+c)\,bfx+ae+afx}dx$$

input
```
int(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int((sech(c + d*x)*tanh(c + d*x))/(sinh(c + d*x)*b*e + sinh(c + d*x)*b*f*x
 + a*e + a*f*x),x)
```

3.358 $\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 1176

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output
```
I*f*(f*x+e)*polylog(2,I*exp(d*x+c))/b/d^2+a*b^2*(f*x+e)^2*ln(1+exp(2*d*x+2
*c))/(a^2+b^2)^2/d+a^2*f^2*arctan(sinh(d*x+c))/b/(a^2+b^2)/d^3+a*f*(f*x+e)
*tanh(d*x+c)/(a^2+b^2)/d^2+I*f^2*polylog(3,-I*exp(d*x+c))/b/d^3-a*f^2*ln(c
osh(d*x+c))/(a^2+b^2)/d^3+f*(f*x+e)*sech(d*x+c)/b/d^2-f^2*arctan(sinh(d*x+
c))/b/d^3-2*I*a^2*b*f*(f*x+e)*polylog(2,I*exp(d*x+c))/(a^2+b^2)^2/d^2-I*a^
2*f*(f*x+e)*polylog(2,I*exp(d*x+c))/b/(a^2+b^2)/d^2-a^2*(f*x+e)^2*arctan(e
xp(d*x+c))/b/(a^2+b^2)/d+(f*x+e)^2*arctan(exp(d*x+c))/b/d-2*a*b^2*f*(f*x+e
)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2-2*a*b^2*f*(
f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2-1/2*a*
(f*x+e)^2*sech(d*x+c)^2/(a^2+b^2)/d+1/2*(f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/
b/d-I*f^2*polylog(3,I*exp(d*x+c))/b/d^3+I*a^2*f*(f*x+e)*polylog(2,-I*exp(d
*x+c))/b/(a^2+b^2)/d^2+2*I*a^2*b*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/(a^2+b
^2)^2/d^2-1/2*a^2*(f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/b/(a^2+b^2)/d-2*I*a^2*
b*f^2*polylog(3,-I*exp(d*x+c))/(a^2+b^2)^2/d^3-I*a^2*f^2*polylog(3,-I*exp(
d*x+c))/b/(a^2+b^2)/d^3+2*I*a^2*b*f^2*polylog(3,I*exp(d*x+c))/(a^2+b^2)^2/
d^3+I*a^2*f^2*polylog(3,I*exp(d*x+c))/b/(a^2+b^2)/d^3+a*b^2*f*(f*x+e)*poly
log(2,-exp(2*d*x+2*c))/(a^2+b^2)^2/d^2-a^2*f*(f*x+e)*sech(d*x+c)/b/(a^2+b^
2)/d^2-1/2*a*b^2*f^2*polylog(3,-exp(2*d*x+2*c))/(a^2+b^2)^2/d^3-2*a^2*b*(f
*x+e)^2*arctan(exp(d*x+c))/(a^2+b^2)^2/d-I*f*(f*x+e)*polylog(2,-I*exp(d*x+
c))/b/d^2-a*b^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b...
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3390 vs. 2(1176) = 2352.

Time = 11.50 (sec) , antiderivative size = 3390, normalized size of antiderivative = 2.88

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Sech[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
(-12*a*b^2*d^3*e^2*E^(2*c)*x + 12*a^3*d*E^(2*c)*f^2*x + 12*a*b^2*d*E^(2*c)
*f^2*x - 12*a*b^2*d^3*e*E^(2*c)*f*x^2 - 4*a*b^2*d^3*E^(2*c)*f^2*x^3 - 6*a^
2*b*d^2*e^2*ArcTan[E^(c + d*x)] + 6*b^3*d^2*e^2*ArcTan[E^(c + d*x)] - 6*a^
2*b*d^2*e^2*E^(2*c)*ArcTan[E^(c + d*x)] + 6*b^3*d^2*e^2*E^(2*c)*ArcTan[E^(
c + d*x)] - 12*a^2*b*f^2*ArcTan[E^(c + d*x)] - 12*b^3*f^2*ArcTan[E^(c + d*
x)] - 12*a^2*b*E^(2*c)*f^2*ArcTan[E^(c + d*x)] - 12*b^3*E^(2*c)*f^2*ArcTan
[E^(c + d*x)] - (6*I)*a^2*b*d^2*e*f*x*Log[1 - I*E^(c + d*x)] + (6*I)*b^3*d
^2*e*f*x*Log[1 - I*E^(c + d*x)] - (6*I)*a^2*b*d^2*e*E^(2*c)*f*x*Log[1 - I*
E^(c + d*x)] + (6*I)*b^3*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] - (3*I)*
a^2*b*d^2*f^2*x^2*Log[1 - I*E^(c + d*x)] + (3*I)*b^3*d^2*f^2*x^2*Log[1 - I
*E^(c + d*x)] - (3*I)*a^2*b*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] + (
3*I)*b^3*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] + (6*I)*a^2*b*d^2*e*f*
x*Log[1 + I*E^(c + d*x)] - (6*I)*b^3*d^2*e*f*x*Log[1 + I*E^(c + d*x)] + (6
*I)*a^2*b*d^2*e*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] - (6*I)*b^3*d^2*e*E^(2*
c)*f*x*Log[1 + I*E^(c + d*x)] + (3*I)*a^2*b*d^2*f^2*x^2*Log[1 + I*E^(c + d
*x)] - (3*I)*b^3*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] + (3*I)*a^2*b*d^2*E^(2
*c)*f^2*x^2*Log[1 + I*E^(c + d*x)] - (3*I)*b^3*d^2*E^(2*c)*f^2*x^2*Log[1 +
 I*E^(c + d*x)] + 6*a*b^2*d^2*e^2*Log[1 + E^(2*(c + d*x))] + 6*a*b^2*d^2*e
^2*E^(2*c)*Log[1 + E^(2*(c + d*x))] - 6*a^3*f^2*Log[1 + E^(2*(c + d*x))] -
 6*a*b^2*f^2*Log[1 + E^(2*(c + d*x))] - 6*a^3*E^(2*c)*f^2*Log[1 + E^(2*...
```

Rubi [A] (verified)

Time = 5.43 (sec) , antiderivative size = 958, normalized size of antiderivative = 0.81, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6117, 3042, 4674, 3042, 4257, 4668, 3011, 2720, 6107, 6107, 6095, 2620, 3011, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\tanh(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6117}$$

$$\frac{\int (e+fx)^2\text{sech}^3(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{b}$$

$\Big\downarrow$ 4674

$$\frac{-\frac{f^2\int \text{sech}(c+dx)dx}{d^2}+\frac{1}{2}\int (e+fx)^2\text{sech}(c+dx)dx+\frac{f(e+fx)\text{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$\Big\downarrow$ 3042

$$-\frac{a\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{f^2\int \csc\left(ic+idx+\frac{\pi}{2}\right)dx}{d^2}+\frac{1}{2}\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{f(e+fx)\text{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}$$

$\Big\downarrow$ 4257

$$-\frac{a\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f^2\arctan(\sinh(c+dx))}{d^3}+\frac{f(e+fx)\text{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}$$

$\Big\downarrow$ 4668

$$-\frac{a\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(-\frac{2if\int (e+fx)\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{2if\int (e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}\right)-\frac{f^2\arctan(\sinh(c+dx))}{d^3}+\frac{f(e+fx)\text{S}}{}}{b}$$

$\Big\downarrow$ 3011

$$-\frac{a\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}+\frac{2(e+fx)^2\text{a}}{}\right.}{b}$$

$\Big\downarrow$ 2720

$$-\frac{a\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,i\right.}{d}\right.}{d}\right.}{b}$$

$\downarrow$ 6107

$$-\frac{a\left(\frac{\int (e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}\right)}{b}+\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,i\right.}{d}\right.}{d}\right.}{b}$$

$\downarrow$ 6107

$$-\frac{a\left(\frac{\int (e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a^2+b^2}\right)}{b}+\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,i\right.}{d}\right.}{d}\right.}{b}$$

$\downarrow$ 6095

$$-\frac{a\left(\frac{\int (e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}(c+\ldots}{a^2\ldots}\right.}{a^2+b^2}\right.}{b}+\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,i\right.}{d}\right.}{d}\right.}{b}$$

$\downarrow$ 2620

$$-\frac{a\left(\frac{\int (e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\cdots}{\cdots}\right.}{a^2+b^2}\cdots}{\cdots}\right)}{b}\cdots$$

$$\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,i\cdots\right)}{d}\cdots\right.}{d}\cdots\right.}{b}\cdots$$

$\big\downarrow$ 3011

$$-\frac{a\left(\frac{b^2\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\cdots\right)}{d}\cdots\right.}{bd}\cdots\right.}{a^2+b^2}\cdots\right.}{a^2+b^2}\cdots\right.}{\cdots}\cdots$$

$$\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,i\cdots\right)}{d}\cdots\right.}{d}\cdots\right.}{b}\cdots$$

$\big\downarrow$ 2720

$$-\frac{a\left(\frac{b^2\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c}}{d^2}\right.}{bd}\right.}{a^2+b^2}\right.}{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,i\right.}{d}\right.}{d}\right.}{b}}$$

$\downarrow$ 7143

$$-\frac{a\left(\frac{b^2\left(\frac{\int (e+fx)^2 \operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\right.}{d^2}\right.}{}\right.}{}\right)}{a^2+b^2}\right.}{-\frac{f^2\arctan(\sinh(c+dx))}{d^3}+\frac{1}{2}\left(\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}+\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,ie\right.}{d^2}\right.}{}\right.}{b}$$

$\downarrow$ 7293

$$-\frac{a\left(\frac{b^2\left(\frac{\int\left(a(e+fx)^2\operatorname{sech}(c+dx)-b(e+fx)^2\tanh(c+dx)\right)dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\operatorname{Pol}}{}\right.}{}\right.}{}\right)}{a^2+b^2}\right)}{}$$

$$\frac{-\frac{f^2\arctan(\sinh(c+dx))}{d^3}+\frac{1}{2}\left(\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}+\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,ie\right.}{d^2}\right.}{}\right.}{b}$$

$\downarrow$ 2009

$$-\frac{\arctan(\sinh(c+dx))f^2}{d^3}+\frac{(e+fx)\operatorname{sech}(c+dx)f}{d^2}+\frac{1}{2}\left(\frac{2\arctan(e^{c+dx})(e+fx)^2}{d}+\frac{2if\left(\frac{f\operatorname{PolyLog}(3,-ie^{c+dx})}{d^2}-\frac{(e+fx)\operatorname{PolyLog}(2,-ie^{c+dx})}{d}\right.}{d}\right.$$

$$b$$

$$a\left(\frac{\left(\frac{\left(-\frac{(e+fx)^3}{3bf}+\frac{\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)(e+fx)^2}{bd}+\frac{\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)(e+fx)^2}{bd}-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right.}{a^2+b^2}\right.}{}\right.$$

input
```
Int[((e + f*x)^2*Sech[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-((f^2*ArcTan[Sinh[c + d*x]])/d^3) + ((2*(e + f*x)^2*ArcTan[E^(c + d*x)])
/d + ((2*I)*f*(-(((e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d) + (f*PolyLog[
3, (-I)*E^(c + d*x)])/d^2))/d - ((2*I)*f*(-(((e + f*x)*PolyLog[2, I*E^(c +
 d*x)])/d) + (f*PolyLog[3, I*E^(c + d*x)])/d^2))/d)/2 + (f*(e + f*x)*Sech[
c + d*x])/d^2 + ((e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*d))/b - (a*((
b^2*((b^2*(-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + S
qrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2]))])/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2]))])/d^2))/(b*d)))/(a^2 + b^2) + ((b*(e + f*x)^3)/(3*f) + (2*a*(e
 + f*x)^2*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)^2*Log[1 + E^(2*(c + d*x))]
)/d - ((2*I)*a*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + ((2*I)*a*f*
(e + f*x)*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*(e + f*x)*PolyLog[2, -E^(2
*(c + d*x))])/d^2 + ((2*I)*a*f^2*PolyLog[3, (-I)*E^(c + d*x)])/d^3 - ((2*I
)*a*f^2*PolyLog[3, I*E^(c + d*x)])/d^3 + (b*f^2*PolyLog[3, -E^(2*(c + d*x)
)])/(2*d^3))/(a^2 + b^2)))/(a^2 + b^2) + ((a*(e + f*x)^2*ArcTan[E^(c + d*x
)])/d - (a*f^2*ArcTan[Sinh[c + d*x]])/d^3 + (b*f^2*Log[Cosh[c + d*x]])/d^3
 - (I*a*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + (I*a*f*(e + f*x...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6117
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x],
x] - Simp[a/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{sech}(dx+c)^2 \tanh(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 11164 vs. 2(1078) = 2156.

Time = 0.31 (sec) , antiderivative size = 11164, normalized size of antiderivative = 9.49

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2\tanh(c+dx)\,\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*sech(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*tanh(c + d*x)*sech(c + d*x)**2/(a + b*sinh(c + d*x))
, x)
```

Maxima [F]

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\,\text{sech}(dx+c)^2\tanh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-a^2*b*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*
b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^
2 + b^4*d^2), x) + b^3*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x
 + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^
2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - 2*a*b^2*d^2*f^2*integrate(x^2/(a^4*d^2*
e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c)
+ a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - 2*a^2*b*d^2*e*f*integrate(x*e^(
d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^
2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 2*b^3*d^2*e*f
*integrate(x*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x
 + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x)
 - 4*a*b^2*d^2*e*f*integrate(x/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^
(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^
2), x) + a^3*f^2*(2*(d*x + c)/((a^4 + 2*a^2*b^2 + b^4)*d^3) - log(e^(2*d*x
 + 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d^3)) + a*b^2*f^2*(2*(d*x + c)/((a^4
 + 2*a^2*b^2 + b^4)*d^3) - log(e^(2*d*x + 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^
4)*d^3)) - (a*b^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 +
2*a^2*b^2 + b^4)*d) - a*b^2*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 +
b^4)*d) - (a^2*b - b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) -
 (b*e^(-d*x - c) - 2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh(c+d\,x)\,\left(e+f\,x\right)^2}{{\cosh\left(c+d\,x\right)}^2\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((tanh(c + d*x)*(e + f*x)^2)/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((tanh(c + d*x)*(e + f*x)^2)/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))), x
)
```

Reduce [F]

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^2*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
(48*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**4*b*d*e*f + 16*e**(4*c + 4*d*x)
*atan(e**(c + d*x))*a**4*b*f**2 - 18*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a
**2*b**3*d**2*e**2 + 96*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**2*b**3*d*e*
f + 32*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**2*b**3*f**2 + 18*e**(4*c + 4
*d*x)*atan(e**(c + d*x))*b**5*d**2*e**2 + 48*e**(4*c + 4*d*x)*atan(e**(c +
 d*x))*b**5*d*e*f + 16*e**(4*c + 4*d*x)*atan(e**(c + d*x))*b**5*f**2 + 96*
e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**4*b*d*e*f + 32*e**(2*c + 2*d*x)*ata
n(e**(c + d*x))*a**4*b*f**2 - 36*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**2*
b**3*d**2*e**2 + 192*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**2*b**3*d*e*f +
 64*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**2*b**3*f**2 + 36*e**(2*c + 2*d*
x)*atan(e**(c + d*x))*b**5*d**2*e**2 + 96*e**(2*c + 2*d*x)*atan(e**(c + d*
x))*b**5*d*e*f + 32*e**(2*c + 2*d*x)*atan(e**(c + d*x))*b**5*f**2 + 48*ata
n(e**(c + d*x))*a**4*b*d*e*f + 16*atan(e**(c + d*x))*a**4*b*f**2 - 18*atan
(e**(c + d*x))*a**2*b**3*d**2*e**2 + 96*atan(e**(c + d*x))*a**2*b**3*d*e*f
 + 32*atan(e**(c + d*x))*a**2*b**3*f**2 + 18*atan(e**(c + d*x))*b**5*d**2*
e**2 + 48*atan(e**(c + d*x))*b**5*d*e*f + 16*atan(e**(c + d*x))*b**5*f**2
+ 576*e**(7*c + 4*d*x)*int((e**(3*d*x)*x**2)/(e**(8*c + 8*d*x)*b + 2*e**(7
*c + 7*d*x)*a + 2*e**(6*c + 6*d*x)*b + 6*e**(5*c + 5*d*x)*a + 6*e**(3*c +
3*d*x)*a - 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**6*d**3*f**2
+ 1200*e**(7*c + 4*d*x)*int((e**(3*d*x)*x**2)/(e**(8*c + 8*d*x)*b + 2*e...
```

3.359 $\int \frac{(e+fx)\mathbf{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 711

$$\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{(e+fx)\arctan\left(e^{c+dx}\right)}{bd} - \frac{2a^2b(e+fx)\arctan\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{a^2(e+fx)\arctan\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{ab^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2 d} - \frac{ab^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2 d} + \frac{ab^2(e+fx)\log\left(1+e^{2(c+dx)}\right)}{\left(a^2+b^2\right)^2 d} - \frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{2bd^2} + \frac{ia^2bf\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{\left(a^2+b^2\right)^2 d^2} + \frac{ia^2f\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{2b\left(a^2+b^2\right)d^2} + \frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{2bd^2} - \frac{ia^2bf\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{\left(a^2+b^2\right)^2 d^2} - \frac{ia^2f\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{2b\left(a^2+b^2\right)d^2} - \frac{ab^2f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2 d^2} - \frac{ab^2f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2 d^2} + \frac{ab^2f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2\left(a^2+b^2\right)^2 d^2} + \frac{f\text{sech}(c+dx)}{2bd^2} - \frac{a^2f\text{sech}(c+dx)}{2b\left(a^2+b^2\right)d^2} - \frac{a(e+fx)\text{sech}^2(c+dx)}{2\left(a^2+b^2\right)d} + \frac{af\tanh(c+dx)}{2\left(a^2+b^2\right)d^2} + \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{2bd} - \frac{a^2(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{2b\left(a^2+b^2\right)d}$$

output
```
(f*x+e)*arctan(exp(d*x+c))/b/d-2*a^2*b*(f*x+e)*arctan(exp(d*x+c))/(a^2+b^2
)^2/d-a^2*(f*x+e)*arctan(exp(d*x+c))/b/(a^2+b^2)/d-a*b^2*(f*x+e)*ln(1+b*ex
p(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d-a*b^2*(f*x+e)*ln(1+b*exp(d*x+c
)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d+a*b^2*(f*x+e)*ln(1+exp(2*d*x+2*c))/(a
^2+b^2)^2/d+I*a^2*b*f*polylog(2,-I*exp(d*x+c))/(a^2+b^2)^2/d^2-I*a^2*b*f*p
olylog(2,I*exp(d*x+c))/(a^2+b^2)^2/d^2-1/2*I*f*polylog(2,-I*exp(d*x+c))/b/
d^2+1/2*I*f*polylog(2,I*exp(d*x+c))/b/d^2-1/2*I*a^2*f*polylog(2,I*exp(d*x+
c))/b/(a^2+b^2)/d^2+1/2*I*a^2*f*polylog(2,-I*exp(d*x+c))/b/(a^2+b^2)/d^2-a
*b^2*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2-a*b^2*
f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2+1/2*a*b^2*f
*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)^2/d^2+1/2*f*sech(d*x+c)/b/d^2-1/2*a^
2*f*sech(d*x+c)/b/(a^2+b^2)/d^2-1/2*a*(f*x+e)*sech(d*x+c)^2/(a^2+b^2)/d+1/
2*a*f*tanh(d*x+c)/(a^2+b^2)/d^2+1/2*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/b/d-1/
2*a^2*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/b/(a^2+b^2)/d
```

Mathematica [A] (warning: unable to verify)

Time = 8.65 (sec) , antiderivative size = 821, normalized size of antiderivative = 1.15

$$\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{ab^2\left(-2de(c+dx)+2cf(c+dx)-f(c+dx)^2+\frac{4a\sqrt{a^2+b^2}de\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-(a^2+b^2)^2}}-\frac{4a\sqrt{-(a^2+b^2)^2}de\,\text{arctanh}\left(\cdots\right.}{(-a^2-b^2)^{3/2}}\cdots\right.}{\cdots}$$

$$+\frac{b\left(-2abde(c+dx)+2abcf(c+dx)-abf(c+dx)^2-2a^2de\arctan\left(e^{c+dx}\right)+2b^2de\arctan\left(e^{c+dx}\right)+2\cdots\right.}{\cdots}$$

$$+\frac{\text{sech}(c+dx)(bf+af\sinh(c+dx))}{2(a^2+b^2)d^2}$$

$$+\frac{\text{sech}^2(c+dx)(-ade+acf-af(c+dx)+bde\sinh(c+dx)-bcf\sinh(c+dx)+bf(c+dx)\sinh(c+\cdots}{2(a^2+b^2)d^2}$$

input
```
Integrate[((e + f*x)*Sech[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
-1/2*(a*b^2*(-2*d*e*(c + d*x) + 2*c*f*(c + d*x) - f*(c + d*x)^2 + (4*a*Sqr
t[a^2 + b^2]*d*e*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2
+ b^2)^2] - (4*a*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt
[a^2 + b^2]])/(-a^2 - b^2)^(3/2) + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2])] + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2])] - 2*c*f*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*d*e*Lo
g[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] + 2*f*PolyLog[2, (b*E^(c + d
*x))/(-a + Sqrt[a^2 + b^2])] + 2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[
a^2 + b^2]))]))/((a^2 + b^2)^2*d^2) + (b*(-2*a*b*d*e*(c + d*x) + 2*a*b*c*f
*(c + d*x) - a*b*f*(c + d*x)^2 - 2*a^2*d*e*ArcTan[E^(c + d*x)] + 2*b^2*d*e
*ArcTan[E^(c + d*x)] + 2*a^2*c*f*ArcTan[E^(c + d*x)] - 2*b^2*c*f*ArcTan[E^
(c + d*x)] - I*a^2*f*(c + d*x)*Log[1 - I*E^(c + d*x)] + I*b^2*f*(c + d*x)*
Log[1 - I*E^(c + d*x)] + I*a^2*f*(c + d*x)*Log[1 + I*E^(c + d*x)] - I*b^2*
f*(c + d*x)*Log[1 + I*E^(c + d*x)] + 2*a*b*d*e*Log[1 + E^(2*(c + d*x))] -
2*a*b*c*f*Log[1 + E^(2*(c + d*x))] + 2*a*b*f*(c + d*x)*Log[1 + E^(2*(c + d
*x))] + I*(a^2 - b^2)*f*PolyLog[2, (-I)*E^(c + d*x)] - I*(a^2 - b^2)*f*Pol
yLog[2, I*E^(c + d*x)] + a*b*f*PolyLog[2, -E^(2*(c + d*x))]))/(2*(a^2 + b^
2)^2*d^2) + (Sech[c + d*x]*(b*f + a*f*Sinh[c + d*x]))/(2*(a^2 + b^2)*d^2)
+ (Sech[c + d*x]^2*(-(a*d*e) + a*c*f - a*f*(c + d*x) + b*d*e*Sinh[c + d*x]
 - b*c*f*Sinh[c + d*x] + b*f*(c + d*x)*Sinh[c + d*x]))/(2*(a^2 + b^2)*d...
```

Rubi [A] (verified)

Time = 3.14 (sec) , antiderivative size = 600, normalized size of antiderivative = 0.84, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.469$, Rules used = {6117, 3042, 4673, 3042, 4668, 2715, 2838, 6107, 6107, 6095, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\tanh(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6117}$$

$$\frac{\int (e+fx)\text{sech}^3(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{b}$$

$$\Big\downarrow \text{4673}$$

$$\frac{\frac{1}{2}\int (e+fx)\text{sech}(c+dx)dx+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}$$

$$\Big\downarrow \text{4668}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(-\frac{if\int \log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int \log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\,\text{tanl}}{}}{b}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}$$

$$\Big\downarrow \text{6107}$$

$$-\frac{a\left(\frac{\int (e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}\right)}{b}+\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}$$

$\downarrow$ 6107

$$-\frac{a\left(\frac{\int (e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a^2+b^2}\right)}{b}+\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}$$

$\downarrow$ 6095

$$-\frac{a\left(\frac{\int (e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int (e+fx)\text{sech}(c+dx)}{a^2+}\right.}{a^2+b^2}\right.}{b}$$

$$\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}$$

$\downarrow$ 2620

$$-\frac{a\left(\frac{\int (e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right.}{a^2+b^2}\right.}{a^2+b^2}\right.}{b}$$

$$\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}$$

$\downarrow$ 2715

$$-\frac{a\left(\frac{b^2\left(\frac{b^2\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+\cdots}\right)}{bd}\cdots}{a^2+b^2}\right)}{a^2+b^2}\cdots\right)}{b}$$

$$\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}$$

$\downarrow$ 2838

$$-\frac{a\left(\frac{b^2\left(\frac{\int(e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\cdots\right)}{a^2+b^2}\right)}{a^2+b^2}\cdots\right)}{b}$$

$$\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}$$

$\downarrow$ 7293

$$-\frac{a\left(\frac{b^2\left(\frac{\int(a(e+fx)\text{sech}(c+dx)-b(e+fx)\tanh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\cdots\right)}{a^2+b^2}\right)}{a^2+b^2}\cdots\right)}{b}$$

$$\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}$$

$\downarrow$ 2009

$$\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan(e^{c+dx})}{d}-\frac{if\,\mathrm{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{if\,\mathrm{PolyLog}(2,ie^{c+dx})}{d^2}\right)+\frac{f\mathrm{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\mathrm{sech}(c+dx)}{2d}}{b}-$$

$$a\left(\frac{b^2\left(\frac{b^2\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{2a(e+\cdots}{}}{a^2+b^2}\right.$$

input
```
Int[((e + f*x)*Sech[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(((2*(e + f*x)*ArcTan[E^(c + d*x)])/d - (I*f*PolyLog[2, (-I)*E^(c + d*x)])
/d^2 + (I*f*PolyLog[2, I*E^(c + d*x)])/d^2)/2 + (f*Sech[c + d*x])/(2*d^2)
+ ((e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*d))/b - (a*((b^2*((b^2*(-1/2*
(e + f*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b
*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (
f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(a^2 + b
^2) + ((b*(e + f*x)^2)/(2*f) + (2*a*(e + f*x)*ArcTan[E^(c + d*x)])/d - (b*
(e + f*x)*Log[1 + E^(2*(c + d*x))])/d - (I*a*f*PolyLog[2, (-I)*E^(c + d*x)
])/d^2 + (I*a*f*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*PolyLog[2, -E^(2*(c
+ d*x))])/(2*d^2))/(a^2 + b^2)))/(a^2 + b^2) + ((a*(e + f*x)*ArcTan[E^(c +
 d*x)])/d - ((I/2)*a*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + ((I/2)*a*f*Poly
Log[2, I*E^(c + d*x)])/d^2 + (a*f*Sech[c + d*x])/(2*d^2) + (b*(e + f*x)*Se
ch[c + d*x]^2)/(2*d) - (b*f*Tanh[c + d*x])/(2*d^2) + (a*(e + f*x)*Sech[c +
 d*x]*Tanh[c + d*x])/(2*d))/(a^2 + b^2)))/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6117
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x],
x] - Simp[a/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2073 vs. $2(656) = 1312$.

Time = 11.63 (sec) , antiderivative size = 2074, normalized size of antiderivative = 2.92

method	result	size
risch	Expression too large to display	2074

input
```
int((f*x+e)*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
-(-b*d*f*x*exp(3*d*x+3*c)+2*a*d*f*x*exp(2*d*x+2*c)-b*d*e*exp(3*d*x+3*c)+2*
a*d*e*exp(2*d*x+2*c)+b*d*f*x*exp(d*x+c)-b*f*exp(3*d*x+3*c)+a*f*exp(2*d*x+2
*c)+b*d*e*exp(d*x+c)-f*b*exp(d*x+c)+a*f)/d^2/(a^2+b^2)/(1+exp(2*d*x+2*c))^
2+b^4/d^2/(a^2+b^2)^(3/2)*c*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*
a)/(a^2+b^2)^(1/2))-b^2/d^2/(a^2+b^2)^(1/2)*c*f/(2*a^2+2*b^2)*arctanh(1/2*
(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-b^2/d/(a^2+b^2)^(3/2)*e/(2*a^2+2*b^2
)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^2+2*b/d^2/(a^2+b^2)*
c*a^2*f/(2*a^2+2*b^2)*arctan(exp(d*x+c))+2*b^2/d/(a^2+b^2)*f/(2*a^2+2*b^2)
*ln(1+I*exp(d*x+c))*a*x+2*b^2/d/(a^2+b^2)*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c
))*a*x+I*b^3/d^2/(a^2+b^2)*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*c-2*b^2/d/(a
^2+b^2)*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)
^(1/2)))*a*x-2*b^2/d/(a^2+b^2)*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^
(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a*x-2*b^2/d^2/(a^2+b^2)*f/(2*a^2+2*b^2)*ln((
-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a*c-2*b^2/d^2/(a^2+
b^2)*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2
)))*a*c+2*b^2/d^2/(a^2+b^2)*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*a*c+2*b^2/d
^2/(a^2+b^2)*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*a*c-I*b/d^2/(a^2+b^2)*a^2*
f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))-I*b^3/d/(a^2+b^2)*f/(2*a^2+2*b^2)*ln
(1+I*exp(d*x+c))*x-I*b^3/d^2/(a^2+b^2)*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*
c+I*b/d^2/(a^2+b^2)*a^2*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))+I*b^3/d/(...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4993 vs. $2(635) = 1270$.

Time = 0.20 (sec) , antiderivative size = 4993, normalized size of antiderivative = 7.02

$$\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\tanh(c+dx)\,\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*tanh(c + d*x)*sech(c + d*x)**2/(a + b*sinh(c + d*x)), x
)
```

Maxima [F]

$$\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{sech}(dx+c)^2\tanh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-(a*b^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2
+ b^4)*d) - a*b^2*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) -
(a^2*b - b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) - (b*e^(-d*
x - c) - 2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 +
 b^2)*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d))*e + f*(((b*d*x*
e^(3*c) + b*e^(3*c))*e^(3*d*x) - (2*a*d*x*e^(2*c) + a*e^(2*c))*e^(2*d*x) -
 (b*d*x*e^c - b*e^c)*e^(d*x) - a)/(a^2*d^2 + b^2*d^2 + (a^2*d^2*e^(4*c) +
b^2*d^2*e^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*e^(2*d*
x)) + 4*integrate(-1/2*(a^2*b^2*x*e^(d*x + c) - a*b^3*x)/(a^4*b + 2*a^2*b^
3 + b^5 - (a^4*b*e^(2*c) + 2*a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*
(a^5*e^c + 2*a^3*b^2*e^c + a*b^4*e^c)*e^(d*x)), x) - 4*integrate(1/4*(2*a*
b^2*x + (a^2*b*e^c - b^3*e^c)*x*e^(d*x))/(a^4 + 2*a^2*b^2 + b^4 + (a^4*e^(
2*c) + 2*a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x))
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\tanh(c+d\,x)\,\left(e+f\,x\right)}{\cosh\left(c+d\,x\right)^2\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((tanh(c + d*x)*(e + f*x))/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((tanh(c + d*x)*(e + f*x))/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{too large to display}$$

input
```
int((f*x+e)*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output

```
(4*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**4*b*f - 3*e**(4*c + 4*d*x)*atan(
e**(c + d*x))*a**2*b**3*d*e + 8*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**2*b
**3*f + 3*e**(4*c + 4*d*x)*atan(e**(c + d*x))*b**5*d*e + 4*e**(4*c + 4*d*x
)*atan(e**(c + d*x))*b**5*f + 8*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**4*b
*f - 6*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**2*b**3*d*e + 16*e**(2*c + 2*
d*x)*atan(e**(c + d*x))*a**2*b**3*f + 6*e**(2*c + 2*d*x)*atan(e**(c + d*x)
)*b**5*d*e + 8*e**(2*c + 2*d*x)*atan(e**(c + d*x))*b**5*f + 4*atan(e**(c +
 d*x))*a**4*b*f - 3*atan(e**(c + d*x))*a**2*b**3*d*e + 8*atan(e**(c + d*x)
)*a**2*b**3*f + 3*atan(e**(c + d*x))*b**5*d*e + 4*atan(e**(c + d*x))*b**5*
f + 96*e**(7*c + 4*d*x)*int((e**(3*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c
 + 7*d*x)*a + 2*e**(6*c + 6*d*x)*b + 6*e**(5*c + 5*d*x)*a + 6*e**(3*c + 3*
d*x)*a - 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**6*d**2*f + 200
*e**(7*c + 4*d*x)*int((e**(3*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d
*x)*a + 2*e**(6*c + 6*d*x)*b + 6*e**(5*c + 5*d*x)*a + 6*e**(3*c + 3*d*x)*a
 - 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*b**2*d**2*f + 112*
e**(7*c + 4*d*x)*int((e**(3*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*
x)*a + 2*e**(6*c + 6*d*x)*b + 6*e**(5*c + 5*d*x)*a + 6*e**(3*c + 3*d*x)*a
- 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b**4*d**2*f + 8*e**
(7*c + 4*d*x)*int((e**(3*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*
a + 2*e**(6*c + 6*d*x)*b + 6*e**(5*c + 5*d*x)*a + 6*e**(3*c + 3*d*x)*a ...
```

3.360 $\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 122

$$\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b(a^2-b^2)\arctan(\sinh(c+dx))}{2(a^2+b^2)^2 d} + \frac{ab^2\log(\cosh(c+dx))}{(a^2+b^2)^2 d} - \frac{ab^2\log(a+b\sinh(c+dx))}{(a^2+b^2)^2 d} - \frac{\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))}{2(a^2+b^2) d}$$

output
```
-1/2*b*(a^2-b^2)*arctan(sinh(d*x+c))/(a^2+b^2)^2/d+a*b^2*ln(cosh(d*x+c))/(
a^2+b^2)^2/d-a*b^2*ln(a+b*sinh(d*x+c))/(a^2+b^2)^2/d-1/2*sech(d*x+c)^2*(a-
b*sinh(d*x+c))/(a^2+b^2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{i\left(-\frac{b\log(i-\sinh(c+dx))}{(a+ib)^2}+\frac{b\log(i+\sinh(c+dx))}{(a-ib)^2}-\frac{4iab^2\log(a+b\sinh(c+dx))}{(a^2+b^2)^2}-\frac{1}{(a+ib)(-i+\sinh(c+dx))}+\frac{1}{(a-ib)(i+\sinh(c+dx))}\right)}{4d}$$

input
```
Integrate[(Sech[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
((-1/4*I)*(-((b*Log[I - Sinh[c + d*x]])/(a + I*b)^2) + (b*Log[I + Sinh[c +
 d*x]])/(a - I*b)^2 - ((4*I)*a*b^2*Log[a + b*Sinh[c + d*x]])/(a^2 + b^2)^2
 - 1/((a + I*b)*(-I + Sinh[c + d*x])) + 1/((a - I*b)*(I + Sinh[c + d*x])))
)/d
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 3316, 26, 27, 593, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ic+idx)}{\cos(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ic+idx)}{\cos(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \; 3316$$

$$-\frac{i b^3 \int \frac{i \sinh(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \; 26$$

$$\frac{b^3 \int \frac{\sinh(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{b^2 \int \frac{b\sinh(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \; 593$$

$$\frac{b^2\left(\frac{\int -\frac{a-b\sinh(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)}d(b\sinh(c+dx))}{2(a^2+b^2)}-\frac{a-b\sinh(c+dx)}{2(a^2+b^2)(b^2\sinh^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{b^2\left(-\frac{\int \frac{a-b\sinh(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)}d(b\sinh(c+dx))}{2(a^2+b^2)}-\frac{a-b\sinh(c+dx)}{2(a^2+b^2)(b^2\sinh^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \; 657$$

$$\frac{b^2\left(-\frac{\int \left(\frac{2a}{\left(a^2+b^2\right)(a+b\sinh(c+dx))}+\frac{a^2-2b\sinh(c+dx)a-b^2}{\left(a^2+b^2\right)\left(\sinh^2(c+dx)b^2+b^2\right)}\right)d(b\sinh(c+dx))}{2(a^2+b^2)}-\frac{a-b\sinh(c+dx)}{2(a^2+b^2)(b^2\sinh^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{b^2\left(-\frac{\frac{\left(a^2-b^2\right)\arctan(\sinh(c+dx))}{b\left(a^2+b^2\right)}-\frac{a\log\left(b^2\sinh^2(c+dx)+b^2\right)}{a^2+b^2}+\frac{2a\log(a+b\sinh(c+dx))}{a^2+b^2}}{2(a^2+b^2)}-\frac{a-b\sinh(c+dx)}{2(a^2+b^2)(b^2\sinh^2(c+dx)+b^2)}\right)}{d}$$

input
```
Int[(Sech[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(b^2*(-1/2*(((a^2 - b^2)*ArcTan[Sinh[c + d*x]])/(b*(a^2 + b^2)) + (2*a*Log
[a + b*Sinh[c + d*x]])/(a^2 + b^2) - (a*Log[b^2 + b^2*Sinh[c + d*x]^2])/(a
^2 + b^2))/(a^2 + b^2) - (a - b*Sinh[c + d*x])/(2*(a^2 + b^2)*(b^2 + b^2*S
inh[c + d*x]^2))))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 593
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[(c + d*x)^(n + 1)*(c - d*x)*((a + b*x^2)^(p + 1)/(2*(p + 1)*(b*c^2 +
a*d^2))), x] - Simp[d/(2*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*
x^2)^(p + 1)*(c*n - d*(n + 2*p + 4)*x), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 6.88 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.74

method	result
derivativedivides	$-\frac{2\left(\frac{\left(\frac{1}{2}a^2b+\frac{1}{2}b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-a^3-ab^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(-\frac{1}{2}a^2b-\frac{1}{2}b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{b\left(-ab\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\cdots\right.}{\cdots}\right)}{a^4+2a^2b^2+b^4}\cdots}{d}$
default	$-\frac{2\left(\frac{\left(\frac{1}{2}a^2b+\frac{1}{2}b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-a^3-ab^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(-\frac{1}{2}a^2b-\frac{1}{2}b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{b\left(-ab\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\cdots\right.}{\cdots}\right)}{a^4+2a^2b^2+b^4}\cdots}{d}$
risch	$-\frac{2a\,b^2d^2x}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{2a\,b^2dc}{a^4d^2+2a^2b^2d^2+b^4d^2}+\frac{2a\,b^2x}{a^4+2a^2b^2+b^4}+\frac{2a\,b^2c}{d(a^4+2a^2b^2+b^4)}-\frac{e^{dx+c}\left(-b\,e^{2dx+2c}+2a\cdots\right.}{d(a^2+b^2)\left(1+e^{2dx}\cdots\right.}$

input
```
int(sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/(a^4+2*a^2*b^2+b^4)*(((1/2*a^2*b+1/2*b^3)*tanh(1/2*d*x+1/2*c)^3+(-
a^3-a*b^2)*tanh(1/2*d*x+1/2*c)^2+(-1/2*a^2*b-1/2*b^3)*tanh(1/2*d*x+1/2*c))
/(1+tanh(1/2*d*x+1/2*c)^2)^2+1/2*b*(-a*b*ln(1+tanh(1/2*d*x+1/2*c)^2)+(a^2-
b^2)*arctan(tanh(1/2*d*x+1/2*c))))-2*a*b^2/(2*a^4+4*a^2*b^2+2*b^4)*ln(tanh
(1/2*d*x+1/2*c)^2*a-2*b*tanh(1/2*d*x+1/2*c)-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 926 vs. $2(119) = 238$.

Time = 0.12 (sec) , antiderivative size = 926, normalized size of antiderivative = 7.59

$$\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
((a^2*b + b^3)*cosh(d*x + c)^3 + (a^2*b + b^3)*sinh(d*x + c)^3 - 2*(a^3 +
a*b^2)*cosh(d*x + c)^2 - (2*a^3 + 2*a*b^2 - 3*(a^2*b + b^3)*cosh(d*x + c))
*sinh(d*x + c)^2 - ((a^2*b - b^3)*cosh(d*x + c)^4 + 4*(a^2*b - b^3)*cosh(d
*x + c)*sinh(d*x + c)^3 + (a^2*b - b^3)*sinh(d*x + c)^4 + a^2*b - b^3 + 2*
(a^2*b - b^3)*cosh(d*x + c)^2 + 2*(a^2*b - b^3 + 3*(a^2*b - b^3)*cosh(d*x
+ c)^2)*sinh(d*x + c)^2 + 4*((a^2*b - b^3)*cosh(d*x + c)^3 + (a^2*b - b^3)
*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - (a^
2*b + b^3)*cosh(d*x + c) - (a*b^2*cosh(d*x + c)^4 + 4*a*b^2*cosh(d*x + c)*
sinh(d*x + c)^3 + a*b^2*sinh(d*x + c)^4 + 2*a*b^2*cosh(d*x + c)^2 + a*b^2
+ 2*(3*a*b^2*cosh(d*x + c)^2 + a*b^2)*sinh(d*x + c)^2 + 4*(a*b^2*cosh(d*x
+ c)^3 + a*b^2*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(
cosh(d*x + c) - sinh(d*x + c))) + (a*b^2*cosh(d*x + c)^4 + 4*a*b^2*cosh(d*
x + c)*sinh(d*x + c)^3 + a*b^2*sinh(d*x + c)^4 + 2*a*b^2*cosh(d*x + c)^2 +
 a*b^2 + 2*(3*a*b^2*cosh(d*x + c)^2 + a*b^2)*sinh(d*x + c)^2 + 4*(a*b^2*co
sh(d*x + c)^3 + a*b^2*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(c
osh(d*x + c) - sinh(d*x + c))) - (a^2*b + b^3 - 3*(a^2*b + b^3)*cosh(d*x +
 c)^2 + 4*(a^3 + a*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 +
b^4)*d*cosh(d*x + c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*
x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2
+ b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)...
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate(sech(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(tanh(c + d*x)*sech(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.79

$$\begin{aligned}\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx &= -\frac{ab^2\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^4+2\,a^2b^2+b^4)d} \\ &\quad+\frac{ab^2\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^4+2\,a^2b^2+b^4)d}+\frac{(a^2b-b^3)\arctan\left(e^{(-dx-c)}\right)}{(a^4+2\,a^2b^2+b^4)d} \\ &\quad+\frac{be^{(-dx-c)}-2\,ae^{(-2\,dx-2\,c)}-be^{(-3\,dx-3\,c)}}{(a^2+b^2+2\,(a^2+b^2)e^{(-2\,dx-2\,c)}+(a^2+b^2)e^{(-4\,dx-4\,c)})d}\end{aligned}$$

input
```
integrate(sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-a*b^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 +
 b^4)*d) + a*b^2*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) + (
a^2*b - b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) + (b*e^(-d*x
 - c) - 2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 +
b^2)*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(119) = 238$.

Time = 0.15 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.34

$$\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{4ab^3\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2a\right|\right)}{a^4b+2a^2b^3+b^5} - \frac{2ab^2\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^4+2a^2b^2+b^4} + \frac{\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)}-1\right)e^{(-dx-c)}\right)\right)(a^2b-b^3)}{a^4+2a^2b^2+b^4} + \cdots}{4d}$$

input
```
integrate(sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*(4*a*b^3*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^4*b + 2*a^
2*b^3 + b^5) - 2*a*b^2*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^4 + 2*a^
2*b^2 + b^4) + (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(a^
2*b - b^3)/(a^4 + 2*a^2*b^2 + b^4) + 2*(a*b^2*(e^(d*x + c) - e^(-d*x - c))
^2 - 2*a^2*b*(e^(d*x + c) - e^(-d*x - c)) - 2*b^3*(e^(d*x + c) - e^(-d*x -
 c)) + 4*a^3 + 8*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*((e^(d*x + c) - e^(-d*x -
 c))^2 + 4)))/d
```

Mupad [B] (verification not implemented)

Time = 2.88 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.76

$$\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{\frac{2a}{d(a^2+b^2)} - \frac{2b\,e^{c+dx}}{d(a^2+b^2)}}{2e^{2c+2dx}+e^{4c+4dx}+1} - \frac{\frac{2(a^3+ab^2)}{d(a^2+b^2)^2} - \frac{e^{c+dx}(a^2b+b^3)}{d(a^2+b^2)^2}}{e^{2c+2dx}+1} + \frac{b\ln\left(1+e^{c+dx}\,1i\right)}{2(-1i\,d\,a^2+2\,d\,a\,b+1i\,d\,b^2)}$$
$$-\frac{a\,b^2\ln\left(b^6e^{2c}e^{2dx}-14a^2b^4-a^4b^2-b^6+28a^3b^3e^{dx}e^c+14a^2b^4e^{2c}e^{2dx}+a^4b^2e^{2c}e^{2dx}+2ab^5e^{c}\cdots\right)}{d\,a^4+2\,d\,a^2b^2+d\,b^4}$$
$$+\frac{b\ln\left(e^{c+dx}+1i\right)1i}{2(-d\,a^2+2i\,d\,a\,b+d\,b^2)}$$

input
```
int(tanh(c + d*x)/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
((2*a)/(d*(a^2 + b^2)) - (2*b*exp(c + d*x))/(d*(a^2 + b^2)))/(2*exp(2*c +
2*d*x) + exp(4*c + 4*d*x) + 1) - ((2*(a*b^2 + a^3))/(d*(a^2 + b^2)^2) - (e
xp(c + d*x)*(a^2*b + b^3))/(d*(a^2 + b^2)^2))/(exp(2*c + 2*d*x) + 1) + (b*
log(exp(c + d*x) + 1i)*1i)/(2*(b^2*d - a^2*d + a*b*d*2i)) + (b*log(exp(c +
 d*x)*1i + 1))/(2*(b^2*d*1i - a^2*d*1i + 2*a*b*d)) - (a*b^2*log(b^6*exp(2*
c)*exp(2*d*x) - 14*a^2*b^4 - a^4*b^2 - b^6 + 28*a^3*b^3*exp(d*x)*exp(c) +
14*a^2*b^4*exp(2*c)*exp(2*d*x) + a^4*b^2*exp(2*c)*exp(2*d*x) + 2*a*b^5*exp
(d*x)*exp(c) + 2*a^5*b*exp(d*x)*exp(c)))/(a^4*d + b^4*d + 2*a^2*b^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 520, normalized size of antiderivative = 4.26

$$\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{-e^{4dx+4c}atan\left(e^{dx+c}\right)a^2b+e^{4dx+4c}atan\left(e^{dx+c}\right)b^3-2e^{2dx+2c}atan\left(e^{dx+c}\right)a^2b+2e^{2dx+2c}atan\left(e^{dx+c}\right)b^3-a}{}$$

input
```
int(sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**2*b + e**(4*c + 4*d*x)*atan(e**
(c + d*x))*b**3 - 2*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**2*b + 2*e**(2*c
 + 2*d*x)*atan(e**(c + d*x))*b**3 - atan(e**(c + d*x))*a**2*b + atan(e**(c
 + d*x))*b**3 + e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b**2 - e**(4*
c + 4*d*x)*log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*a*b**2 + e**(4*c
 + 4*d*x)*a**3 + e**(4*c + 4*d*x)*a*b**2 + e**(3*c + 3*d*x)*a**2*b + e**(3
*c + 3*d*x)*b**3 + 2*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b**2 - 2
*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*a*b**2 -
e**(c + d*x)*a**2*b - e**(c + d*x)*b**3 + log(e**(2*c + 2*d*x) + 1)*a*b**2
 - log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*a*b**2 + a**3 + a*b**2)/
(d*(e**(4*c + 4*d*x)*a**4 + 2*e**(4*c + 4*d*x)*a**2*b**2 + e**(4*c + 4*d*x
)*b**4 + 2*e**(2*c + 2*d*x)*a**4 + 4*e**(2*c + 2*d*x)*a**2*b**2 + 2*e**(2*
c + 2*d*x)*b**4 + a**4 + 2*a**2*b**2 + b**4))
```

3.361 $\int \frac{\text{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\text{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\text{Int}\left(\frac{\text{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(sech(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 69.93 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\text{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\text{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Sech[c + d*x]^2*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Sech[c + d*x]^2*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(c+dx)\text{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{6125}$$
$$\int \frac{\tanh(c+dx)\text{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Sech[c + d*x]^2*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\text{sech}\left(dx+c\right)^{2}\tanh\left(dx+c\right)}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(sech(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(sech(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 4.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}\left(dx+c\right)^2\tanh\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sech(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(sech(d*x + c)^2*tanh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 4.80 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\tanh\left(c+dx\right)\operatorname{sech}^2\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

input
```
integrate(sech(d*x+c)**2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(tanh(c + d*x)*sech(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)),
 x)
```

Maxima [N/A]

Not integrable

Time = 1.81 (sec) , antiderivative size = 1104, normalized size of antiderivative = 32.47

$$\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(dx+c)^2\tanh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sech(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
(a*f + (b*d*f*x*e^(3*c) + (d*e - f)*b*e^(3*c))*e^(3*d*x) - (2*a*d*f*x*e^(2
*c) + (2*d*e - f)*a*e^(2*c))*e^(2*d*x) - (b*d*f*x*e^c + (d*e + f)*b*e^c)*e
^(d*x))/(a^2*d^2*e^2 + b^2*d^2*e^2 + (a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + 2*(
a^2*d^2*e*f + b^2*d^2*e*f)*x + (a^2*d^2*e^2*e^(4*c) + b^2*d^2*e^2*e^(4*c)
+ (a^2*d^2*f^2*e^(4*c) + b^2*d^2*f^2*e^(4*c))*x^2 + 2*(a^2*d^2*e*f*e^(4*c)
 + b^2*d^2*e*f*e^(4*c))*x)*e^(4*d*x) + 2*(a^2*d^2*e^2*e^(2*c) + b^2*d^2*e^
2*e^(2*c) + (a^2*d^2*f^2*e^(2*c) + b^2*d^2*f^2*e^(2*c))*x^2 + 2*(a^2*d^2*e
*f*e^(2*c) + b^2*d^2*e*f*e^(2*c))*x)*e^(2*d*x)) - 4*integrate(1/4*(2*a*b^2
*d^2*f^2*x^2 + 4*a*b^2*d^2*e*f*x - 2*a^3*f^2 + 2*(d^2*e^2 - f^2)*a*b^2 + (
(d^2*e^2 + 2*f^2)*a^2*b*e^c - (d^2*e^2 - 2*f^2)*b^3*e^c + (a^2*b*d^2*f^2*e
^c - b^3*d^2*f^2*e^c)*x^2 + 2*(a^2*b*d^2*e*f*e^c - b^3*d^2*e*f*e^c)*x)*e^(
d*x))/(a^4*d^2*e^3 + 2*a^2*b^2*d^2*e^3 + b^4*d^2*e^3 + (a^4*d^2*f^3 + 2*a^
2*b^2*d^2*f^3 + b^4*d^2*f^3)*x^3 + 3*(a^4*d^2*e*f^2 + 2*a^2*b^2*d^2*e*f^2
+ b^4*d^2*e*f^2)*x^2 + 3*(a^4*d^2*e^2*f + 2*a^2*b^2*d^2*e^2*f + b^4*d^2*e^
2*f)*x + (a^4*d^2*e^3*e^(2*c) + 2*a^2*b^2*d^2*e^3*e^(2*c) + b^4*d^2*e^3*e^
(2*c) + (a^4*d^2*f^3*e^(2*c) + 2*a^2*b^2*d^2*f^3*e^(2*c) + b^4*d^2*f^3*e^(
2*c))*x^3 + 3*(a^4*d^2*e*f^2*e^(2*c) + 2*a^2*b^2*d^2*e*f^2*e^(2*c) + b^4*d
^2*e*f^2*e^(2*c))*x^2 + 3*(a^4*d^2*e^2*f*e^(2*c) + 2*a^2*b^2*d^2*e^2*f*e^(
2*c) + b^4*d^2*e^2*f*e^(2*c))*x)*e^(2*d*x)), x) + 4*integrate(-1/2*(a^2*b^
2*e^(d*x + c) - a*b^3)/(a^4*b*e + 2*a^2*b^3*e + b^5*e + (a^4*b*f + 2*a^...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\text{Timed out}$$

input
```
integrate(sech(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 4.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\tanh(c+d\,x)}{\cosh\left(c+d\,x\right)^2\,\left(e+f\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(tanh(c + d*x)/(cosh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(tanh(c + d*x)/(cosh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}(dx+c)^2\tanh\left(dx+c\right)}{\sinh\left(dx+c\right)be+\sinh\left(dx+c\right)bfx+ae+afx}dx$$

input
```
int(sech(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int((sech(c + d*x)**2*tanh(c + d*x))/(sinh(c + d*x)*b*e + sinh(c + d*x)*b*
f*x + a*e + a*f*x),x)
```

3.362 $\int \frac{(e+fx)^3 \cosh(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$

Optimal result

Integrand size = 34, antiderivative size = 606

$$\begin{aligned}\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{3f^3x}{8bd^3}+\frac{(e+fx)^3}{4bd}-\frac{a^2(e+fx)^4}{4b^3f}+\frac{6af^3\cosh(c+dx)}{b^2d^4}+\frac{3af(e+fx)^2\cosh(c+dx)}{b^2d^2}\\ &+\frac{a^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d}+\frac{a^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d}\\ &+\frac{3a^2f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{3a^2f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^2}\\ &-\frac{6a^2f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3}-\frac{6a^2f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^3}\\ &+\frac{6a^2f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^4}+\frac{6a^2f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^4}\\ &-\frac{6af^2(e+fx)\sinh(c+dx)}{b^2d^3}-\frac{a(e+fx)^3\sinh(c+dx)}{b^2d}\\ &-\frac{3f^3\cosh(c+dx)\sinh(c+dx)}{8bd^4}-\frac{3f(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{4bd^2}\\ &+\frac{3f^2(e+fx)\sinh^2(c+dx)}{4bd^3}+\frac{(e+fx)^3\sinh^2(c+dx)}{2bd}\end{aligned}$$

output
```
3/8*f^3*x/b/d^3+1/4*(f*x+e)^3/b/d-1/4*a^2*(f*x+e)^4/b^3/f+6*a*f^3*cosh(d*x
+c)/b^2/d^4+3*a*f*(f*x+e)^2*cosh(d*x+c)/b^2/d^2+a^2*(f*x+e)^3*ln(1+b*exp(d
*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d+a^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b
^2)^(1/2)))/b^3/d+3*a^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(
1/2)))/b^3/d^2+3*a^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2
)))/b^3/d^2-6*a^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))
/b^3/d^3-6*a^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^
3/d^3+6*a^2*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^4+6*a^2
*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^4-6*a*f^2*(f*x+e)*
sinh(d*x+c)/b^2/d^3-a*(f*x+e)^3*sinh(d*x+c)/b^2/d-3/8*f^3*cosh(d*x+c)*sinh
(d*x+c)/b/d^4-3/4*f*(f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/b/d^2+3/4*f^2*(f*x+e
)*sinh(d*x+c)^2/b/d^3+1/2*(f*x+e)^3*sinh(d*x+c)^2/b/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2684 vs. 2(606) = 1212.

Time = 14.44 (sec) , antiderivative size = 2684, normalized size of antiderivative = 4.43

$$\int \frac{(e+fx)^3 \cosh(c+dx) \sinh^2(c+dx)}{a+b\sinh(c+dx)} \, dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
-1/4*(e^3*Log[a + b*Sinh[c + d*x]])/(b*d) - (3*e^2*f*(-1/2*x^2/b + (x*Log[
1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (x*Log[1 + (b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt
[a^2 + b^2])]/(b*d^2) + PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
)]/(b*d^2)))/4 - (3*e*f^2*(-1/3*x^3/b + (x^2*Log[1 + (b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2])])/(b*d) + (x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])])/(b*d) + (2*x*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(
b*d^2) + (2*x*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2
) - (2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])])/(b*d^3) - (2*Po
lyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^3)))/4 - (f^3*(-1
/4*x^4/b + (x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (x
^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (3*x^2*PolyLog[
2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (3*x^2*PolyLog[2,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2) - (6*x*PolyLog[3, -((b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^3) - (6*x*PolyLog[3, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^3) + (6*PolyLog[4, (b*E^(c + d*x))/(
-a + Sqrt[a^2 + b^2])])/(b*d^4) + (6*PolyLog[4, -((b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2]))])/(b*d^4)))/4 + (e*f^2*(2*(4*a^2 + b^2)*x^3*Coth[c] - (2*(4
*a^2 + b^2)*(2*x^3 - (3*b^2*(-1 + E^(2*c))*(d^2*x^2*Log[1 + ((a - Sqrt[a^2
 + b^2])*E^(-c - d*x))/b] - 2*d*x*PolyLog[2, ((-a + Sqrt[a^2 + b^2])*E^...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 3.38 (sec) , antiderivative size = 591, normalized size of antiderivative = 0.98, number of steps used = 31, number of rules used = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.882$, Rules used = {6113, 5969, 3042, 25, 3792, 17, 25, 3042, 25, 3115, 24, 6113, 3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118, 6095, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3 \sinh^2(c+dx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6113}$$

$$\frac{\int (e+fx)^3 \cosh(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{5969}$$

$$\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}-\frac{3f\int(e+fx)^2\sinh^2(c+dx)dx}{2d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}-\frac{3f\int-(e+fx)^2\sin(ic+idx)^2dx}{2d}}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}+\frac{3f\int(e+fx)^2\sin(ic+idx)^2dx}{2d}}{b}$$

$$\Big\downarrow \text{3792}$$

$$\frac{\frac{3f\left(\frac{f^2\int-\sinh^2(c+dx)dx}{2d^2}+\frac{1}{2}\int(e+fx)^2dx+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{17}$$

$$\frac{\frac{3f\left(\frac{f^2\int-\sinh^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{3f\left(-\frac{f^2\int\sinh^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}+\frac{3f\left(-\frac{f^2\int-\sin(ic+idx)^2dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}+\frac{3f\left(\frac{f^2\int\sin(ic+idx)^2dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{3f\left(\frac{f^2\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{6113}$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{\int(e+fx)^3\cosh(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)^3\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3if\int-i(e+fx)^2\sinh(c+dx)dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \ 26$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int(e+fx)^2\sinh(c+dx)dx}{d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int-i(e+fx)^2\sin(ic+idx)dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \ 26$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\int(e+fx)^2\sin(ic+idx)dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \ 3777$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\cosh(c+dx)dx}{d}\right)}{d}}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}}{b}\right)}{b}$$

$$\Big\downarrow \; 3777$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int-i\sinh(c+dx)dx}{d}\right)}{d}\right)}{d}}{b}\right)}{b}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}\right)}{d}\right)}{d}}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int-i\sin(ic+idx)dx}{d}\right)}{d}\right)}{d}}{b}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int\sin(ic+idx)dx}{d}\right)}{d}\right)}{d}}{b}\right)}{b}$$

$$\Big\downarrow \text{3118}$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}\right)}{b}$$

$\downarrow$ 6095

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\left(\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^4}{4bf}\right)}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}\cdots\right.}{d}\right.}{d}}{b}\right.}{b}$$

$\downarrow$ 2620

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\left(-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\cdots\right.}{b}\right.}{b}$$

$\downarrow$ 3011

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-$$

$$a\left(-\frac{a\left(-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2}{}\right.}{bd}\right.}{b}\right.$$

$\downarrow$ 7163

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-$$

$$a\left(-\frac{a\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{Poly}}{}\right.}{}\right.}{}\right.}{}\right.$$

$\downarrow$ 2720

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-$$

$$a\left(-\frac{a\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 7143

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-$$

$$a\left(-\frac{a\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(2f\left(\frac{(e+fx)\operatorname{PolyLog}}{}\right.\right.}{}\right.}{}\right.$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(((e + f*x)^3*Sinh[c + d*x]^2)/(2*d) + (3*f*((e + f*x)^3/(6*f) - ((e + f*x
)^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (f*(e + f*x)*Sinh[c + d*x]^2)/(2*
d^2) + (f^2*(x/2 - (Cosh[c + d*x]*Sinh[c + d*x])/(2*d)))/(2*d^2)))/(2*d))/
b - (a*(-((a*(-1/4*(e + f*x)^4/(b*f) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x)
)/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - S
qrt[a^2 + b^2]))])/d^2))/d))/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/b) + (((e + f*x)^3*Sinh[c + d
*x])/d + ((3*I)*f*((I*(e + f*x)^2*Cosh[c + d*x])/d - ((2*I)*f*(-((f*Cosh[c
 + d*x])/d^2) + ((e + f*x)*Sinh[c + d*x])/d))/d))/d)/b))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(fx+e)^3 \cosh(dx+c) \sinh(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3891 vs. 2(566) = 1132.

Time = 0.16 (sec) , antiderivative size = 3891, normalized size of antiderivative = 6.42

$$\int \frac{(e+fx)^3 \cosh(c+dx) \sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
1/8*e^3*(8*(d*x + c)*a^2/(b^3*d) - (4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/
(b^2*d) + 8*a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d) +
(4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d)) + 1/32*(8*a^2*d^4*f^3*x^4
*e^(2*c) + 32*a^2*d^4*e*f^2*x^3*e^(2*c) + 48*a^2*d^4*e^2*f*x^2*e^(2*c) + (
4*b^2*d^3*f^3*x^3*e^(4*c) + 6*(2*d^3*e*f^2 - d^2*f^3)*b^2*x^2*e^(4*c) + 6*
(2*d^3*e^2*f - 2*d^2*e*f^2 + d*f^3)*b^2*x*e^(4*c) - 3*(2*d^2*e^2*f - 2*d*e
*f^2 + f^3)*b^2*e^(4*c))*e^(2*d*x) - 16*(a*b*d^3*f^3*x^3*e^(3*c) + 3*(d^3*
e*f^2 - d^2*f^3)*a*b*x^2*e^(3*c) + 3*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*a
*b*x*e^(3*c) - 3*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*a*b*e^(3*c))*e^(d*x) + 16
*(a*b*d^3*f^3*x^3*e^c + 3*(d^3*e*f^2 + d^2*f^3)*a*b*x^2*e^c + 3*(d^3*e^2*f
 + 2*d^2*e*f^2 + 2*d*f^3)*a*b*x*e^c + 3*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*a*
b*e^c)*e^(-d*x) + (4*b^2*d^3*f^3*x^3 + 6*(2*d^3*e*f^2 + d^2*f^3)*b^2*x^2 +
 6*(2*d^3*e^2*f + 2*d^2*e*f^2 + d*f^3)*b^2*x + 3*(2*d^2*e^2*f + 2*d*e*f^2
+ f^3)*b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^4) - integrate(-2*(a^2*b*f^3*x^3 +
 3*a^2*b*e*f^2*x^2 + 3*a^2*b*e^2*f*x - (a^3*f^3*x^3*e^c + 3*a^3*e*f^2*x^2*
e^c + 3*a^3*e^2*f*x*e^c)*e^(d*x))/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x +
c) - b^4), x)
```

Giac [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\cosh(dx+c)\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)*sinh(d*x + c)^2/(b*sinh(d*x + c) + a),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)^2\,(e+f\,x)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)*sinh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*sinh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(4*cosh(c + d*x)**2*b**2*d**3*e**3 + 6*cosh(c + d*x)**2*b**2*d**3*e**2*f*x
 + 6*cosh(c + d*x)**2*b**2*d**3*e*f**2*x**2 + 2*cosh(c + d*x)**2*b**2*d**3
*f**3*x**3 + 6*cosh(c + d*x)**2*b**2*d*e*f**2 + 3*cosh(c + d*x)**2*b**2*d*
f**3*x - 6*cosh(c + d*x)*sinh(c + d*x)*b**2*d**2*e**2*f - 12*cosh(c + d*x)
*sinh(c + d*x)*b**2*d**2*e*f**2*x - 6*cosh(c + d*x)*sinh(c + d*x)*b**2*d**
2*f**3*x**2 - 3*cosh(c + d*x)*sinh(c + d*x)*b**2*f**3 + 24*cosh(c + d*x)*a
*b*d**2*e**2*f + 48*cosh(c + d*x)*a*b*d**2*e*f**2*x + 24*cosh(c + d*x)*a*b
*d**2*f**3*x**2 + 48*cosh(c + d*x)*a*b*f**3 + 8*e**(2*c)*int((e**(2*d*x)*x
**3)/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b*d**4*f**3 + 24*
e**(2*c)*int((e**(2*d*x)*x**2)/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)
,x)*a**2*b*d**4*e*f**2 + 24*e**(2*c)*int((e**(2*d*x)*x)/(e**(2*c + 2*d*x)*
b + 2*e**(c + d*x)*a - b),x)*a**2*b*d**4*e**2*f + 8*int(x**3/(e**(2*c + 2*
d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b*d**4*f**3 + 24*int(x**2/(e**(2*c
+ 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b*d**4*e*f**2 + 24*int(x/(e**(2
*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b*d**4*e**2*f + 8*log(sinh(c
 + d*x)*b + a)*a**2*d**3*e**3 + 6*sinh(c + d*x)**2*b**2*d**3*e**2*f*x + 6*
sinh(c + d*x)**2*b**2*d**3*e*f**2*x**2 + 2*sinh(c + d*x)**2*b**2*d**3*f**3
*x**3 + 3*sinh(c + d*x)**2*b**2*d*f**3*x - 8*sinh(c + d*x)*a*b*d**3*e**3 -
 24*sinh(c + d*x)*a*b*d**3*e**2*f*x - 24*sinh(c + d*x)*a*b*d**3*e*f**2*x**
2 - 8*sinh(c + d*x)*a*b*d**3*f**3*x**3 - 48*sinh(c + d*x)*a*b*d*e*f**2 ...
```

3.363 $\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 437

$$\begin{aligned}
&\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= \frac{(e+fx)^2}{4bd} - \frac{a^2(e+fx)^3}{3b^3f} + \frac{2af(e+fx)\cosh(c+dx)}{b^2d^2} \\
&\quad + \frac{a^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d} + \frac{a^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d} \\
&\quad + \frac{2a^2f(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2} + \frac{2a^2f(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^2} \\
&\quad - \frac{2a^2f^2\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3} - \frac{2a^2f^2\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^3} \\
&\quad - \frac{2af^2\sinh(c+dx)}{b^2d^3} - \frac{a(e+fx)^2\sinh(c+dx)}{b^2d} \\
&\quad - \frac{f(e+fx)\cosh(c+dx)\sinh(c+dx)}{2bd^2} + \frac{f^2\sinh^2(c+dx)}{4bd^3} + \frac{(e+fx)^2\sinh^2(c+dx)}{2bd}
\end{aligned}$$

output
```
1/4*(f*x+e)^2/b/d-1/3*a^2*(f*x+e)^3/b^3/f+2*a*f*(f*x+e)*cosh(d*x+c)/b^2/d^
2+a^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d+a^2*(f*x+e)^2
*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d+2*a^2*f*(f*x+e)*polylog(2,-b
*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^2+2*a^2*f*(f*x+e)*polylog(2,-b*exp(
d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^2-2*a^2*f^2*polylog(3,-b*exp(d*x+c)/(a-(
a^2+b^2)^(1/2)))/b^3/d^3-2*a^2*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1
/2)))/b^3/d^3-2*a*f^2*sinh(d*x+c)/b^2/d^3-a*(f*x+e)^2*sinh(d*x+c)/b^2/d-1/
2*f*(f*x+e)*cosh(d*x+c)*sinh(d*x+c)/b/d^2+1/4*f^2*sinh(d*x+c)^2/b/d^3+1/2*
(f*x+e)^2*sinh(d*x+c)^2/b/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1453 vs. $2(437) = 874$.

Time = 9.02 (sec) , antiderivative size = 1453, normalized size of antiderivative = 3.32

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
-1/4*(e^2*Log[a + b*Sinh[c + d*x]])/(b*d) - (e*f*(-1/2*x^2/b + (x*Log[1 +
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (x*Log[1 + (b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2])])/(b*d) + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2
 + b^2])]/(b*d^2) + PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(
b*d^2)))/2 - (f^2*(-1/3*x^3/b + (x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
 + b^2])])/(b*d) + (x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b
*d) + (2*x*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) +
 (2*x*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2) - (2*P
olyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])])/(b*d^3) - (2*PolyLog[3,
 -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^3)))/4 + (f^2*(2*(4*a^2 +
 b^2)*x^3*Coth[c] - (2*(4*a^2 + b^2)*(2*x^3 - (3*b^2*(-1 + E^(2*c))*(d^2*x
^2*Log[1 + ((a - Sqrt[a^2 + b^2])*E^(-c - d*x))/b] - 2*d*x*PolyLog[2, ((-a
 + Sqrt[a^2 + b^2])*E^(-c - d*x))/b] - 2*PolyLog[3, ((-a + Sqrt[a^2 + b^2]
)*E^(-c - d*x))/b]))/(Sqrt[a^2 + b^2]*(-a + Sqrt[a^2 + b^2])*d^3) - (3*b^2
*(-1 + E^(2*c))*(d^2*x^2*Log[1 + ((a + Sqrt[a^2 + b^2])*E^(-c - d*x))/b] -
 2*d*x*PolyLog[2, -(((a + Sqrt[a^2 + b^2])*E^(-c - d*x))/b)] - 2*PolyLog[3
, -(((a + Sqrt[a^2 + b^2])*E^(-c - d*x))/b)]))/(Sqrt[a^2 + b^2]*(a + Sqrt[
a^2 + b^2])*d^3) + (3*a*(-1 + E^(2*c))*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b
^2])] - 2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]))/(Sqrt[a^...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.77 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.98, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {6113, 5969, 3042, 25, 3791, 17, 6113, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117, 6095, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2 \sinh^2(c+dx) \cosh(c+dx)}{a+b\sinh(c+dx)} \, dx$$

$$\Big\downarrow \text{6113}$$

$$\frac{\int (e+fx)^2 \cosh(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{5969}$$

$$\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}-\frac{f\int (e+fx)\sinh^2(c+dx)dx}{d}}{b}-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}-\frac{f\int -\left((e+fx)\sin(ic+idx)^2\right)dx}{d}}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}+\frac{f\int (e+fx)\sin(ic+idx)^2dx}{d}}{b}$$

$$\Big\downarrow \text{3791}$$

$$\frac{\frac{f\left(\frac{1}{2}\int (e+fx)dx+\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{17}$$

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{6113}$$

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{\int (e+fx)^2\cosh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int-i(e+fx)\sinh(c+dx)dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int(e+fx)\sinh(c+dx)dx}{d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int-i(e+fx)\sin(ic+idx)dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int(e+fx)\sin(ic+idx)dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{d}}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}}{b}\right)}{b}$$

$$\Big\downarrow \ 3117$$

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}}{b}\right)}{b}$$

$$\Big\downarrow \ 6095$$

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\left(\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}}{b}\right)}{b}$$

$$\Big\downarrow \ 2620$$

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\left(-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+\cdots}{3\cdots}\right.}{b}\cdots\right.}{b}$$

$$\Big\downarrow \ 3011$$

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-$$

$$a\left(-\frac{a\left(-\frac{2f\left(\frac{f\int \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{\cdots}}\right)}{d}\cdots\right)}{bd}\cdots\right)}{b}\cdots\right)$$

$\downarrow$ 2720

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-$$

$$a\left(-\frac{a\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\cdots\right)}{bd}\cdots\right)}{b}\cdots\right)$$

$\downarrow$ 7143

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-$$

$$\frac{a\left(-\frac{a\left(-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\cdots\right)}{bd}\cdots\right)}{b}\cdots\right)}{b}$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(((e + f*x)^2*Sinh[c + d*x]^2)/(2*d) + (f*((e + f*x)^2/(4*f) - ((e + f*x)*
Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (f*Sinh[c + d*x]^2)/(4*d^2)))/d)/b -
(a*(-((a*(-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2
 + b^2]))])/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))])/d^2))/(b*d)))/b) + (((e + f*x)^2*Sinh[c + d*x])/d + ((2*I)*f*
((I*(e + f*x)*Cosh[c + d*x])/d - (I*f*Sinh[c + d*x])/d^2))/d)/b))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(fx+e)^2 \cosh(dx+c) \sinh(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2414 vs. $2(407) = 814$.

Time = 0.14 (sec) , antiderivative size = 2414, normalized size of antiderivative = 5.52

$$\int \frac{(e+fx)^2 \cosh(c+dx) \sinh^2(c+dx)}{a+b\sinh(c+dx)} dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
1/48*(6*b^2*d^2*f^2*x^2 + 6*b^2*d^2*e^2 + 6*b^2*d*e*f + 3*(2*b^2*d^2*f^2*x
^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)
*x)*cosh(d*x + c)^4 + 3*(2*b^2*d^2*f^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f +
 b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)*sinh(d*x + c)^4 + 3*b^2*f^2 -
24*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e
*f - a*b*d*f^2)*x)*cosh(d*x + c)^3 - 12*(2*a*b*d^2*f^2*x^2 + 2*a*b*d^2*e^2
 - 4*a*b*d*e*f + 4*a*b*f^2 + 4*(a*b*d^2*e*f - a*b*d*f^2)*x - (2*b^2*d^2*f^
2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f
^2)*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 16*(a^2*d^3*f^2*x^3 + 3*a^2*d^3*e*
f*x^2 + 3*a^2*d^3*e^2*x + 6*a^2*c*d^2*e^2 - 6*a^2*c^2*d*e*f + 2*a^2*c^3*f^
2)*cosh(d*x + c)^2 - 2*(8*a^2*d^3*f^2*x^3 + 24*a^2*d^3*e*f*x^2 + 24*a^2*d^
3*e^2*x + 48*a^2*c*d^2*e^2 - 48*a^2*c^2*d*e*f + 16*a^2*c^3*f^2 - 9*(2*b^2*
d^2*f^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b
^2*d*f^2)*x)*cosh(d*x + c)^2 + 36*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d
*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f - a*b*d*f^2)*x)*cosh(d*x + c))*sinh(d*x
+ c)^2 + 6*(2*b^2*d^2*e*f + b^2*d*f^2)*x + 24*(a*b*d^2*f^2*x^2 + a*b*d^2*e
^2 + 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f + a*b*d*f^2)*x)*cosh(d*x + c
) + 96*((a^2*d*f^2*x + a^2*d*e*f)*cosh(d*x + c)^2 + 2*(a^2*d*f^2*x + a^2*d
*e*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*d*f^2*x + a^2*d*e*f)*sinh(d*x + c
)^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*si...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\cosh(dx+c)\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
1/8*e^2*(8*(d*x + c)*a^2/(b^3*d) - (4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/
(b^2*d) + 8*a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d) +
(4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d)) + 1/48*(16*a^2*d^3*f^2*x^
3*e^(2*c) + 48*a^2*d^3*e*f*x^2*e^(2*c) + 3*(2*b^2*d^2*f^2*x^2*e^(4*c) + 2*
(2*d^2*e*f - d*f^2)*b^2*x*e^(4*c) - (2*d*e*f - f^2)*b^2*e^(4*c))*e^(2*d*x)
 - 24*(a*b*d^2*f^2*x^2*e^(3*c) + 2*(d^2*e*f - d*f^2)*a*b*x*e^(3*c) - 2*(d*
e*f - f^2)*a*b*e^(3*c))*e^(d*x) + 24*(a*b*d^2*f^2*x^2*e^c + 2*(d^2*e*f + d
*f^2)*a*b*x*e^c + 2*(d*e*f + f^2)*a*b*e^c)*e^(-d*x) + 3*(2*b^2*d^2*f^2*x^2
 + 2*(2*d^2*e*f + d*f^2)*b^2*x + (2*d*e*f + f^2)*b^2)*e^(-2*d*x))*e^(-2*c)
/(b^3*d^3) - integrate(-2*(a^2*b*f^2*x^2 + 2*a^2*b*e*f*x - (a^3*f^2*x^2*e^
c + 2*a^3*e*f*x*e^c)*e^(d*x))/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) -
 b^4), x)
```

Giac [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\cosh(dx+c)\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)*sinh(d*x + c)^2/(b*sinh(d*x + c) + a),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)^2\,(e+f\,x)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)*sinh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*sinh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2\cosh\left(dx+c\right)^2 b^2d^2e^2+2\cosh\left(dx+c\right)^2 b^2d^2efx+\cosh\left(dx+c\right)^2 b^2d^2f^2x^2+\cosh\left(dx+c\right)^2 b^2f^2-2\,\mathrm{co}}{}$$

input
```
int((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(2*cosh(c + d*x)**2*b**2*d**2*e**2 + 2*cosh(c + d*x)**2*b**2*d**2*e*f*x +
cosh(c + d*x)**2*b**2*d**2*f**2*x**2 + cosh(c + d*x)**2*b**2*f**2 - 2*cosh
(c + d*x)*sinh(c + d*x)*b**2*d*e*f - 2*cosh(c + d*x)*sinh(c + d*x)*b**2*d*
f**2*x + 8*cosh(c + d*x)*a*b*d*e*f + 8*cosh(c + d*x)*a*b*d*f**2*x + 4*e**(
2*c)*int((e**(2*d*x)*x**2)/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*
a**2*b*d**3*f**2 + 8*e**(2*c)*int((e**(2*d*x)*x)/(e**(2*c + 2*d*x)*b + 2*e
**(c + d*x)*a - b),x)*a**2*b*d**3*e*f + 4*int(x**2/(e**(2*c + 2*d*x)*b + 2
*e**(c + d*x)*a - b),x)*a**2*b*d**3*f**2 + 8*int(x/(e**(2*c + 2*d*x)*b + 2
*e**(c + d*x)*a - b),x)*a**2*b*d**3*e*f + 4*log(sinh(c + d*x)*b + a)*a**2*
d**2*e**2 + 2*sinh(c + d*x)**2*b**2*d**2*e*f*x + sinh(c + d*x)**2*b**2*d**
2*f**2*x**2 - 4*sinh(c + d*x)*a*b*d**2*e**2 - 8*sinh(c + d*x)*a*b*d**2*e*f
*x - 4*sinh(c + d*x)*a*b*d**2*f**2*x**2 - 8*sinh(c + d*x)*a*b*f**2)/(4*b**
3*d**3)
```

3.364 $\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 278

$$\begin{aligned}\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{fx}{4bd}-\frac{a^2(e+fx)^2}{2b^3f}+\frac{af\cosh(c+dx)}{b^2d^2}+\frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d}\\ &+\frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d}+\frac{a^2f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}\\ &+\frac{a^2f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^2}-\frac{a(e+fx)\sinh(c+dx)}{b^2d}\\ &-\frac{f\cosh(c+dx)\sinh(c+dx)}{4bd^2}+\frac{(e+fx)\sinh^2(c+dx)}{2bd}\end{aligned}$$

output

```
1/4*f*x/b/d-1/2*a^2*(f*x+e)^2/b^3/f+a*f*cosh(d*x+c)/b^2/d^2+a^2*(f*x+e)*ln
(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d+a^2*(f*x+e)*ln(1+b*exp(d*x+c)/(
a+(a^2+b^2)^(1/2)))/b^3/d+a^2*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)
))/b^3/d^2+a^2*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^2-a*(f
*x+e)*sinh(d*x+c)/b^2/d-1/4*f*cosh(d*x+c)*sinh(d*x+c)/b/d^2+1/2*(f*x+e)*si
nh(d*x+c)^2/b/d
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.74

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= -\frac{e\log(a+b\sinh(c+dx))}{4bd} - \frac{1}{4}f\left(-\frac{x^2}{2b} + \frac{x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd} + \frac{x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd} + \frac{\operatorname{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right)}{bd^2} + \frac{\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}\right)$$
$$+ \frac{e((4a^2+b^2)\log(a+b\sinh(c+dx)) - 4ab\sinh(c+dx) + 2b^2\sinh^2(c+dx))}{4b^3d}$$
$$+ \frac{f\left(8ab\cosh(c+dx) + 2b^2dx\cosh(2(c+dx)) + (4a^2+b^2)\left(2c(c+dx) - (c+dx)^2 + 2(c+dx)\log\left(1\right.\right.\right.}{}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),
x]
```

output
```
-1/4*(e*Log[a + b*Sinh[c + d*x]])/(b*d) - (f*(-1/2*x^2/b + (x*Log[1 + (b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (x*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])])/(b*d) + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b
^2])]/(b*d^2) + PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(b*d^
2)))/4 + (e*((4*a^2 + b^2)*Log[a + b*Sinh[c + d*x]] - 4*a*b*Sinh[c + d*x]
+ 2*b^2*Sinh[c + d*x]^2))/(4*b^3*d) + (f*(8*a*b*Cosh[c + d*x] + 2*b^2*d*x*
Cosh[2*(c + d*x)] + (4*a^2 + b^2)*(2*c*(c + d*x) - (c + d*x)^2 + 2*(c + d*
x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*(c + d*x)*Log[1 + (b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*Log[b - 2*a*E^(c + d*x) - b*E^(
2*(c + d*x))] + 2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*P
olyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) - 8*a*b*d*x*Sinh[c +
d*x] - b^2*Sinh[2*(c + d*x)]))/(8*b^3*d^2)
```

Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.99, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.531, Rules used = {6113, 5969, 3042, 25, 3115, 24, 6113, 3042, 3777, 26, 3042, 26, 3118, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6113}$$

$$\frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{5969}$$

$$\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}-\frac{f\int \sinh^2(c+dx)dx}{2d}}{b}-\frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}-\frac{f\int -\sin(ic+idx)^2dx}{2d}}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\int \sin(ic+idx)^2dx}{2d}}{b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{f\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}+\frac{(e+fx)\sinh^2(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{6113}$$

$$\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(\frac{\int(e+fx)\cosh(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b}\right)}{b}$$

$\Big\downarrow$ 3777

$$\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int-i\sinh(c+dx)dx}{d}}{b}\right)}{b}$$

$\Big\downarrow$ 26

$$\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}}{b}-\frac{a\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int-i\sin(ic+idx)dx}{d}}{b}\right)}{b}$$

$\Big\downarrow$ 26

$$\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int\sin(ic+idx)dx}{d}}{b}\right)}{b}$$

$\Big\downarrow$ 3118

$$\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b}-\frac{a\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{6095}$$

$$\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b}-\frac{a\left(\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b}-\frac{a\left(-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2}\cdots}\right.}{bd}\right.}{b}\right.}{b}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b}-\frac{a\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+\cdots}}\right.}{bd}\right.}{b}\right.}{b}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b}-\frac{a\left(\frac{f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be}{\sqrt{a}}\right.}{bd}\right.}{b}\right.}{b}$$

input
```
Int[((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-((a*(-((a*(-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
]))])/(b*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(
b*d^2)))/b) + (-((f*Cosh[c + d*x])/d^2) + ((e + f*x)*Sinh[c + d*x])/d)/b))
/b) + (((e + f*x)*Sinh[c + d*x]^2)/(2*d) + (f*(x/2 - (Cosh[c + d*x]*Sinh[c
 + d*x])/(2*d)))/(2*d))/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 564 vs. 2(258) = 516.

Time = 7.20 (sec) , antiderivative size = 565, normalized size of antiderivative = 2.03

method	result
risch	$-\frac{a^2 f x^2}{2b^3}+\frac{a^2 e x}{b^3}+\frac{(2dxf+2de-f)\mathrm{e}^{2dx+2c}}{16b\,d^2}-\frac{a(dxf+de-f)\mathrm{e}^{dx+c}}{2b^2d^2}+\frac{a(dxf+de+f)\mathrm{e}^{-dx-c}}{2b^2d^2}+\frac{(2dxf+2de+f)\mathrm{e}^{-2dx-2c}}{16b\,d^2}$

input
```
int((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
-1/2/b^3*a^2*f*x^2+1/b^3*a^2*e*x+1/16*(2*d*f*x+2*d*e-f)/b/d^2*exp(2*d*x+2*
c)-1/2*a*(d*f*x+d*e-f)/b^2/d^2*exp(d*x+c)+1/2*a*(d*f*x+d*e+f)/b^2/d^2*exp(
-d*x-c)+1/16*(2*d*f*x+2*d*e+f)/b/d^2*exp(-2*d*x-2*c)-2/d/b^3*a^2*f*c*x-2/d
/b^3*a^2*e*ln(exp(d*x+c))+2/d^2/b^3*c*a^2*f*ln(exp(d*x+c))-1/d^2/b^3*c*a^2
*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/d^2/b^3*a^2*f*c^2+1/d^2/b^3*a^2
*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d^2/b^3*
a^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/d/b^3*a
^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d/b^3*
a^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2/b^3
*a^2*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/d^2
/b^3*a^2*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/d
/b^3*a^2*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1248 vs. $2(256) = 512$.

Time = 0.13 (sec) , antiderivative size = 1248, normalized size of antiderivative = 4.49

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
1/16*(2*b^2*d*f*x + (2*b^2*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c)^4 + (2
*b^2*d*f*x + 2*b^2*d*e - b^2*f)*sinh(d*x + c)^4 + 2*b^2*d*e - 8*(a*b*d*f*x
 + a*b*d*e - a*b*f)*cosh(d*x + c)^3 - 4*(2*a*b*d*f*x + 2*a*b*d*e - 2*a*b*f
 - (2*b^2*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c))*sinh(d*x + c)^3 + b^2*
f - 8*(a^2*d^2*f*x^2 + 2*a^2*d^2*e*x + 4*a^2*c*d*e - 2*a^2*c^2*f)*cosh(d*x
 + c)^2 - 2*(4*a^2*d^2*f*x^2 + 8*a^2*d^2*e*x + 16*a^2*c*d*e - 8*a^2*c^2*f
- 3*(2*b^2*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c)^2 + 12*(a*b*d*f*x + a*
b*d*e - a*b*f)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*(a*b*d*f*x + a*b*d*e + a
*b*f)*cosh(d*x + c) + 16*(a^2*f*cosh(d*x + c)^2 + 2*a^2*f*cosh(d*x + c)*si
nh(d*x + c) + a^2*f*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x +
 c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1
) + 16*(a^2*f*cosh(d*x + c)^2 + 2*a^2*f*cosh(d*x + c)*sinh(d*x + c) + a^2*
f*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 16*((a^2*d*e -
 a^2*c*f)*cosh(d*x + c)^2 + 2*(a^2*d*e - a^2*c*f)*cosh(d*x + c)*sinh(d*x +
 c) + (a^2*d*e - a^2*c*f)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sin
h(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 16*((a^2*d*e - a^2*c*f)*co
sh(d*x + c)^2 + 2*(a^2*d*e - a^2*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*d
*e - a^2*c*f)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) -
 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 16*((a^2*d*f*x + a^2*c*f)*cosh(d*x ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
1/8*e*(8*(d*x + c)*a^2/(b^3*d) - (4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b
^2*d) + 8*a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d) + (4
*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d)) + 1/16*f*((8*a^2*d^2*x^2*e^
(2*c) + (2*b^2*d*x*e^(4*c) - b^2*e^(4*c))*e^(2*d*x) - 8*(a*b*d*x*e^(3*c) -
 a*b*e^(3*c))*e^(d*x) + 8*(a*b*d*x*e^c + a*b*e^c)*e^(-d*x) + (2*b^2*d*x +
b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^2) - 2*integrate(16*(a^3*x*e^(d*x + c) -
a^2*b*x)/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) - b^4), x))
```

Giac [F]

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)\cosh(dx+c)\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)*sinh(d*x + c)^2/(b*sinh(d*x + c) + a), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)^2\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)*sinh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*sinh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\frac{2\cosh(dx+c)^2b^2de+\cosh(dx+c)^2b^2dfx-\cosh(dx+c)\sinh(dx+c)b^2f+4\cosh(dx+c)abf+4e}{}$$

input
```
int((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(2*cosh(c + d*x)**2*b**2*d*e + cosh(c + d*x)**2*b**2*d*f*x - cosh(c + d*x)
*sinh(c + d*x)*b**2*f + 4*cosh(c + d*x)*a*b*f + 4*e**(2*c)*int((e**(2*d*x)
*x)/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b*d**2*f + 4*int(x
/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b*d**2*f + 4*log(sinh
(c + d*x)*b + a)*a**2*d*e + sinh(c + d*x)**2*b**2*d*f*x - 4*sinh(c + d*x)*
a*b*d*e - 4*sinh(c + d*x)*a*b*d*f*x)/(4*b**3*d**2)
```

3.365 $\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 55

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{a^2\log(a+b\sinh(c+dx))}{b^3d} - \frac{a\sinh(c+dx)}{b^2d} + \frac{\sinh^2(c+dx)}{2bd}$$

output
```
a^2*ln(a+b*sinh(d*x+c))/b^3/d-a*sinh(d*x+c)/b^2/d+1/2*sinh(d*x+c)^2/b/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2a^2\log(a+b\sinh(c+dx)) - 2ab\sinh(c+dx) + b^2\sinh^2(c+dx)}{2b^3d}$$

input
```
Integrate[(Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(2*a^2*Log[a + b*Sinh[c + d*x]] - 2*a*b*Sinh[c + d*x] + b^2*Sinh[c + d*x]^
2)/(2*b^3*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3042, 25, 3312, 25, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ic+idx)^2\cos(ic+idx)}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos(ic+idx)\sin(ic+idx)^2}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3312}$$

$$-\frac{\int -\frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{bd}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^3d}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{a^2}{a+b\sinh(c+dx)}-a+b\sinh(c+dx)\right)d(b\sinh(c+dx))}{b^3d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\log(a+b\sinh(c+dx))-ab\sinh(c+dx)+\frac{1}{2}b^2\sinh^2(c+dx)}{b^3d}$$

input
```
Int[(Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(a^2*Log[a + b*Sinh[c + d*x]] - a*b*Sinh[c + d*x] + (b^2*Sinh[c + d*x]^2)/
2)/(b^3*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 2.85 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{a^2 \ln(a+b\sinh(dx+c))}{b^3 d}-\frac{a\sinh(dx+c)}{b^2 d}+\frac{\sinh(dx+c)^2}{2bd}$	54
default	$\frac{a^2 \ln(a+b\sinh(dx+c))}{b^3 d}-\frac{a\sinh(dx+c)}{b^2 d}+\frac{\sinh(dx+c)^2}{2bd}$	54
risch	$-\frac{x a^2}{b^3}+\frac{e^{2dx+2c}}{8bd}-\frac{a e^{dx+c}}{2b^2 d}+\frac{a e^{-dx-c}}{2b^2 d}+\frac{e^{-2dx-2c}}{8bd}-\frac{2a^2 c}{b^3 d}+\frac{a^2 \ln\left(e^{2dx+2c}+\frac{2a e^{dx+c}}{b}-1\right)}{b^3 d}$	124

input
```
int(cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
a^2*ln(a+b*sinh(d*x+c))/b^3/d-a*sinh(d*x+c)/b^2/d+1/2*sinh(d*x+c)^2/b/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. 2(53) = 106.

Time = 0.10 (sec) , antiderivative size = 309, normalized size of antiderivative = 5.62

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{8\,a^2dx\cosh\left(dx+c\right)^2-b^2\cosh\left(dx+c\right)^4-b^2\sinh\left(dx+c\right)^4+4\,ab\cosh\left(dx+c\right)^3-4\left(b^2\cosh\left(dx+c\right.\right.}{}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
-1/8*(8*a^2*d*x*cosh(d*x + c)^2 - b^2*cosh(d*x + c)^4 - b^2*sinh(d*x + c)^
4 + 4*a*b*cosh(d*x + c)^3 - 4*(b^2*cosh(d*x + c) - a*b)*sinh(d*x + c)^3 -
4*a*b*cosh(d*x + c) + 2*(4*a^2*d*x - 3*b^2*cosh(d*x + c)^2 + 6*a*b*cosh(d*
x + c))*sinh(d*x + c)^2 - b^2 - 8*(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x +
c)*sinh(d*x + c) + a^2*sinh(d*x + c)^2)*log(2*(b*sinh(d*x + c) + a)/(cosh(
d*x + c) - sinh(d*x + c))) + 4*(4*a^2*d*x*cosh(d*x + c) - b^2*cosh(d*x + c
)^3 + 3*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c))/(b^3*d*cosh(d*x + c)^2 +
 2*b^3*d*cosh(d*x + c)*sinh(d*x + c) + b^3*d*sinh(d*x + c)^2)
```

Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.58

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \begin{cases} \frac{x\sinh^2(c)\cosh(c)}{a} & \text{for } b=0 \wedge d=0 \\ \frac{\sinh^3(c+dx)}{3ad} & \text{for } b=0 \\ \frac{x\sinh^2(c)\cosh(c)}{a+b\sinh(c)} & \text{for } d=0 \\ \frac{a^2\log\left(\frac{a}{b}+\sinh(c+dx)\right)}{b^3d} - \frac{a\sinh(c+dx)}{b^2d} + \frac{\sinh^2(c+dx)}{2bd} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Piecewise((x*sinh(c)**2*cosh(c)/a, Eq(b, 0) & Eq(d, 0)), (sinh(c + d*x)**3
/(3*a*d), Eq(b, 0)), (x*sinh(c)**2*cosh(c)/(a + b*sinh(c)), Eq(d, 0)), (a*
*2*log(a/b + sinh(c + d*x))/(b**3*d) - a*sinh(c + d*x)/(b**2*d) + sinh(c +
 d*x)**2/(2*b*d), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(53) = 106.

Time = 0.05 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.16

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{(dx+c)a^2}{b^3d} - \frac{\left(4\,ae^{(-dx-c)}-b\right)e^{(2\,dx+2\,c)}}{8\,b^2d} + \frac{a^2\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{b^3d} + \frac{4\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}}{8\,b^2d}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
(d*x + c)*a^2/(b^3*d) - 1/8*(4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d)
 + a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d) + 1/8*(4*a*
e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.60

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{8\,a^2\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{b^3}+\frac{b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2-4\,a\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{b^2}}{8\,d}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/8*(8*a^2*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/b^3 + (b*(e^(d*x
 + c) - e^(-d*x - c))^2 - 4*a*(e^(d*x + c) - e^(-d*x - c)))/b^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{a^2\,\ln\left(a+b\sinh(c+d\,x)\right)+\frac{b^2\sinh(c+d\,x)^2}{2}-a\,b\sinh(c+d\,x)}{b^3\,d}$$

input
```
int((cosh(c + d*x)*sinh(c + d*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
(a^2*log(a + b*sinh(c + d*x)) + (b^2*sinh(c + d*x)^2)/2 - a*b*sinh(c + d*x
))/(b^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\cosh{(dx+c)}^2 b^2 + 2\log(a+b\sinh{(dx+c)})\,a^2 - 2\sinh{(dx+c)}\,ab}{2b^3d}$$

input
```
int(cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(cosh(c + d*x)**2*b**2 + 2*log(sinh(c + d*x)*b + a)*a**2 - 2*sinh(c + d*x)
*a*b)/(2*b**3*d)
```

3.366 $\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\text{Int}\left(\frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(cosh(d*x+c)*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 25.91 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Cosh[c + d*x]*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Cosh[c + d*x]*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

$$\Big\downarrow \text{ 6125}$$

$$\int \frac{\sinh^2(c+dx)\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Cosh[c + d*x]*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(dx+c)\sinh(dx+c)^2}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(cosh(d*x+c)*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\cosh(dx+c)\sinh(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(cosh(d*x + c)*sinh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\text{Timed out}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 235, normalized size of antiderivative = 6.91

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\cosh(dx+c)\sinh(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
1/4*e^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b*f) + 1/2*a*e^
(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^2*f) + 1/2*a*e^(c - d*e/f
)*exp_integral_e(1, -(f*x + e)*d/f)/(b^2*f) - 1/4*e^(2*c - 2*d*e/f)*exp_in
tegral_e(1, -2*(f*x + e)*d/f)/(b*f) + a^2*log(f*x + e)/(b^3*f) - 1/8*integ
rate(-16*(a^3*e^(d*x + c) - a^2*b)/(b^4*f*x + b^4*e - (b^4*f*x*e^(2*c) + b
^4*e*e^(2*c))*e^(2*d*x) - 2*(a*b^3*f*x*e^c + a*b^3*e*e^c)*e^(d*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
integrate(cosh(d*x + c)*sinh(d*x + c)^2/((f*x + e)*(b*sinh(d*x + c) + a)),
 x)
```

Mupad [N/A]

Not integrable

Time = 1.52 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)^2}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((cosh(c + d*x)*sinh(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((cosh(c + d*x)*sinh(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 334, normalized size of antiderivative = 9.82

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$= \frac{e^{4c}\left(\int \frac{e^{4dx}}{e^{2dx+2c}be+e^{2dx+2c}bfx+2e^{dx+c}ae+2e^{dx+c}afx-be-bfx}dx\right)bf+2e^{c}\left(\int \frac{e^{dx}}{e^{2dx+2c}be+e^{2dx+2c}bfx+2e^{dx+c}ae+2e^{dx+c}afx-be-}\right.}{}$$

input
```
int(cosh(d*x+c)*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
(e**(4*c)*int(e**(4*d*x)/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x +
2*e**(c + d*x)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x)*b*f + 2*e**c*i
nt(e**(d*x)/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x + 2*e**(c + d*x
)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x)*a*f + int(1/(e**(4*c + 4*d*
x)*b*e + e**(4*c + 4*d*x)*b*f*x + 2*e**(3*c + 3*d*x)*a*e + 2*e**(3*c + 3*d
*x)*a*f*x - e**(2*c + 2*d*x)*b*e - e**(2*c + 2*d*x)*b*f*x),x)*b*f - 2*int(
1/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x + 2*e**(c + d*x)*a*e + 2*
e**(c + d*x)*a*f*x - b*e - b*f*x),x)*b*f - log(e + f*x))/(4*b*f)
```

3.367 $\int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} \, dx$

Optimal result

Integrand size = 36, antiderivative size = 883

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= -\frac{3af(e+fx)^2}{8b^2d^2} - \frac{a^3(e+fx)^4}{4b^4f} - \frac{a(e+fx)^4}{8b^2f} + \frac{6a^2f^2(e+fx)\cosh(c+dx)}{b^3d^3}$$
$$+ \frac{4f^2(e+fx)\cosh(c+dx)}{3bd^3} + \frac{a^2(e+fx)^3\cosh(c+dx)}{b^3d} + \frac{3af^3\cosh^2(c+dx)}{8b^2d^4}$$
$$+ \frac{3af(e+fx)^2\cosh^2(c+dx)}{4b^2d^2} + \frac{2f^2(e+fx)\cosh^3(c+dx)}{9bd^3}$$
$$+ \frac{(e+fx)^3\cosh^3(c+dx)}{3bd} + \frac{a^2\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d}$$
$$- \frac{a^2\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d}$$
$$+ \frac{3a^2\sqrt{a^2+b^2}f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}$$
$$- \frac{3a^2\sqrt{a^2+b^2}f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^2}$$
$$- \frac{6a^2\sqrt{a^2+b^2}f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^3}$$
$$+ \frac{6a^2\sqrt{a^2+b^2}f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^3}$$
$$+ \frac{6a^2\sqrt{a^2+b^2}f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^4}$$
$$- \frac{6a^2\sqrt{a^2+b^2}f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^4} - \frac{6a^2f^3\sinh(c+dx)}{b^3d^4}$$
$$- \frac{14f^3\sinh(c+dx)}{9bd^4} - \frac{3a^2f(e+fx)^2\sinh(c+dx)}{b^3d^2} - \frac{2f(e+fx)^2\sinh(c+dx)}{3bd^2}$$
$$- \frac{3af^2(e+fx)\cosh(c+dx)\sinh(c+dx)}{4b^2d^3} - \frac{a(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{2b^2d}$$
$$- \frac{f(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{3bd^2} - \frac{2f^3\sinh^3(c+dx)}{27bd^4}$$

output
```
-3*a^2*(a^2+b^2)^(1/2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1
/2)))/b^4/d^2+3*a^2*(a^2+b^2)^(1/2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a
-(a^2+b^2)^(1/2)))/b^4/d^2+4/3*f^2*(f*x+e)*cosh(d*x+c)/b/d^3-2/3*f*(f*x+e)
^2*sinh(d*x+c)/b/d^2-1/8*a*(f*x+e)^4/b^2/f-14/9*f^3*sinh(d*x+c)/b/d^4+6*a^
2*(a^2+b^2)^(1/2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))
/b^4/d^3-6*a^2*(a^2+b^2)^(1/2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2
+b^2)^(1/2)))/b^4/d^3-6*a^2*f^3*sinh(d*x+c)/b^3/d^4+3/8*a*f^3*cosh(d*x+c)^
2/b^2/d^4+2/9*f^2*(f*x+e)*cosh(d*x+c)^3/b/d^3-3/8*a*f*(f*x+e)^2/b^2/d^2+a^
2*(f*x+e)^3*cosh(d*x+c)/b^3/d-1/4*a^3*(f*x+e)^4/b^4/f-2/27*f^3*sinh(d*x+c)
^3/b/d^4+1/3*(f*x+e)^3*cosh(d*x+c)^3/b/d-3/4*a*f^2*(f*x+e)*cosh(d*x+c)*sin
h(d*x+c)/b^2/d^3-3*a^2*f*(f*x+e)^2*sinh(d*x+c)/b^3/d^2+6*a^2*f^2*(f*x+e)*c
osh(d*x+c)/b^3/d^3+3/4*a*f*(f*x+e)^2*cosh(d*x+c)^2/b^2/d^2-1/2*a*(f*x+e)^3
*cosh(d*x+c)*sinh(d*x+c)/b^2/d-1/3*f*(f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/b
/d^2-a^2*(a^2+b^2)^(1/2)*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/
b^4/d+a^2*(a^2+b^2)^(1/2)*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))
/b^4/d-6*a^2*(a^2+b^2)^(1/2)*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2
)))/b^4/d^4+6*a^2*(a^2+b^2)^(1/2)*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)
^(1/2)))/b^4/d^4
```

Mathematica [A] (verified)

Time = 4.14 (sec) , antiderivative size = 1667, normalized size of antiderivative = 1.89

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*
x]),x]
```

output

```
-1/432*(432*a^3*d^4*e^3*x + 216*a*b^2*d^4*e^3*x + 648*a^3*d^4*e^2*f*x^2 +
324*a*b^2*d^4*e^2*f*x^2 + 432*a^3*d^4*e*f^2*x^3 + 216*a*b^2*d^4*e*f^2*x^3
+ 108*a^3*d^4*f^3*x^4 + 54*a*b^2*d^4*f^3*x^4 + 864*a^2*Sqrt[a^2 + b^2]*d^3
*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 432*a^2*b*d^3*e^3*Cosh
[c + d*x] - 108*b^3*d^3*e^3*Cosh[c + d*x] - 2592*a^2*b*d*e*f^2*Cosh[c + d*
x] - 648*b^3*d*e*f^2*Cosh[c + d*x] - 1296*a^2*b*d^3*e^2*f*x*Cosh[c + d*x]
- 324*b^3*d^3*e^2*f*x*Cosh[c + d*x] - 2592*a^2*b*d*f^3*x*Cosh[c + d*x] - 6
48*b^3*d*f^3*x*Cosh[c + d*x] - 1296*a^2*b*d^3*e*f^2*x^2*Cosh[c + d*x] - 32
4*b^3*d^3*e*f^2*x^2*Cosh[c + d*x] - 432*a^2*b*d^3*f^3*x^3*Cosh[c + d*x] -
108*b^3*d^3*f^3*x^3*Cosh[c + d*x] - 162*a*b^2*d^2*e^2*f*Cosh[2*(c + d*x)]
- 81*a*b^2*f^3*Cosh[2*(c + d*x)] - 324*a*b^2*d^2*e*f^2*x*Cosh[2*(c + d*x)]
 - 162*a*b^2*d^2*f^3*x^2*Cosh[2*(c + d*x)] - 36*b^3*d^3*e^3*Cosh[3*(c + d*
x)] - 24*b^3*d*e*f^2*Cosh[3*(c + d*x)] - 108*b^3*d^3*e^2*f*x*Cosh[3*(c + d
*x)] - 24*b^3*d*f^3*x*Cosh[3*(c + d*x)] - 108*b^3*d^3*e*f^2*x^2*Cosh[3*(c
+ d*x)] - 36*b^3*d^3*f^3*x^3*Cosh[3*(c + d*x)] - 1296*a^2*Sqrt[a^2 + b^2]*
d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 1296*a^2*Sqrt
[a^2 + b^2]*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] -
 432*a^2*Sqrt[a^2 + b^2]*d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
 + b^2])] + 1296*a^2*Sqrt[a^2 + b^2]*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])] + 1296*a^2*Sqrt[a^2 + b^2]*d^3*e*f^2*x^2*Log[1 + ...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3 \sinh^2(c+dx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6113}$$

$$\frac{\int (e+fx)^3\cosh^2(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 5970}$$

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\int (e+fx)^2\cosh^3(c+dx)dx}{d}}{b}-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{d}}{b}$$

$\downarrow$ 3792

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2f^2\int \cosh^3(c+dx)dx}{9d^2}+\frac{2}{3}\int (e+fx)^2\cosh(c+dx)dx-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{d}}{b}-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$\downarrow$ 3042

$$-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2f^2\int \sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{9d^2}+\frac{2}{3}\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{d}}{b}$$

$\downarrow$ 3113

$$-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\int \left(\sinh^2(c+dx)+1\right)d(-i\sinh(c+dx))}{9d^3}+\frac{2}{3}\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{d}}{b}$$

$\downarrow$ 2009

$$-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2}{3}\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{d}}{b}$$

$\downarrow$ 3777

$$-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\dots\right)}{d}}{b}$$

$\downarrow$ 26

$$\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int(e+fx)\sinh(c+dx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+\cdots}{}\right)}{d}}{b}$$

$\Big\downarrow$ 3042

$$\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int-i(e+fx)\sin(ic+idx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\cdots\right)}{d}}{b}$$

$\Big\downarrow$ 26

$$\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int(e+fx)\sin(ic+idx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e\cdots}{}\right)}{d}}{b}$$

$\Big\downarrow$ 3777

$$\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+\cdots}{}\right)}{d}}{b}$$

$\Big\downarrow$ 3042

$$\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\cdots\right)}{d}}{b}$$

$\Big\downarrow$ 3117

$$-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh}{}\right.}{d}\right.\right.}{d}}{b}$$

$\downarrow$ 6113

$$-\frac{a\left(\frac{\int(e+fx)^3\cosh^2(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh}{}\right.}{d}\right.\right.}{d}}{b}$$

$\downarrow$ 3042

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh}{}\right.}{d}\right.\right.}{d}}{b}$$
$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)^3\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx}{b}\right)}{b}$$

$\downarrow$ 3792

$$-\frac{a\left(\frac{\frac{3f^2\int(e+fx)\cosh^2(c+dx)dx}{2d^2}+\frac{1}{2}\int(e+fx)^3dx-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh}{}\right.}{d}\right.\right.}{d}}{b}$$

$\downarrow$ 17

$$-\frac{a\left(\frac{\frac{3f^2\int(e+fx)\cosh^2(c+dx)dx}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh}{}\right.}{d}\right.\right.}{d}}{b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh\cdots}{}\right.}{d}\right.\right.}{d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{3f^2\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}\right)}{b}$$

$\Big\downarrow$ 3791

$$-\frac{a\left(\frac{\frac{3f^2\left(\frac{1}{2}\int(e+fx)dx-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\int\cdots}{}\right.}{b}$$

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh\cdots}{}\right.}{d}\right.\right.}{d}}{b}$$

$\Big\downarrow$ 17

$$-\frac{a\left(\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\int\frac{(e\cdots}{}}{}\right.}{b}$$

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh\cdots}{}\right.}{d}\right.\right.}{d}}{b}$$

$\Big\downarrow$ 6099

$$-\frac{a\left(\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a\cdots}{}\right.}{}\right.}{b}$$

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh\cdots}{}\right.}{d}\right.\right.}{d}}{b}$$

$$\downarrow \text{17}$$

$$-\frac{a\left(\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{\left(a^{\cdots}\right.}{}\right.}{}\right.}{b}$$

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh\cdots}{}\right.}{d}\right.}{d}}{b}$$

$$\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh\cdots}{}\right.}{d}\right.}{d}}{b}$$

$$\frac{a\left(\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{\left(a^2+\cdots\right.}{}\right.}{}\right.}{b}$$

$$\downarrow \text{26}$$

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh\cdots}{}\right.}{d}\right.}{d}}{b}$$

$$\frac{a\left(\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{\left(a^2+\cdots\right.}{}\right.}{}\right.}{b}$$

$$\downarrow \text{3777}$$

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh\cdots}{}\right.}{d}\right.\right.}{d}}{b}$$

$$\frac{a\left(\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{\left(a^2+\cdots\right.}{}\right.}{}\right.}{b}$$

$\downarrow$ 3042

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh\cdots}{}\right.}{d}\right.\right.}{d}}{b}$$

$$\frac{a\left(\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{\left(a^2+\cdots\right.}{}\right.}{}\right.}{b}$$

$\downarrow$ 3777

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh\cdots}{}\right.}{d}\right.\right.}{d}}{b}$$

$$\frac{a\left(\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{\left(a^2+\cdots\right.}{}\right.}{}\right.}{b}$$

$\downarrow$ 26

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh}{}\right.}{d}\right.\right.}{d}}{b}$$

$$\frac{a\left(\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a^2+}{}\right.}{}\right.}{b}$$

$\downarrow$ 3042

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh}{}\right.}{d}\right.\right.}{d}}{b}$$

$$\frac{a\left(\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a^2+}{}\right.}{}\right.}{b}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^3 \cosh(dx+c)^2 \sinh(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7042 vs. $2(813) = 1626$.

Time = 0.23 (sec) , antiderivative size = 7042, normalized size of antiderivative = 7.98

$$\int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh^2(c+dx)}{a+b\sinh(c+dx)} dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh^2(c+dx)}{a+b\sinh(c+dx)} dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)**2*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)^2\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="maxima")
```

output
```
1/24*e^3*(24*sqrt(a^2 + b^2)*a^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2)
)/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(b^4*d) - (3*a*b*e^(-d*x - c) -
b^2 - 3*(4*a^2 + b^2)*e^(-2*d*x - 2*c))*e^(3*d*x + 3*c)/(b^3*d) - 12*(2*a^
3 + a*b^2)*(d*x + c)/(b^4*d) + (3*a*b*e^(-2*d*x - 2*c) + b^2*e^(-3*d*x - 3
*c) + 3*(4*a^2 + b^2)*e^(-d*x - c))/(b^3*d)) - 1/864*(108*(2*a^3*d^4*f^3*e
^(3*c) + a*b^2*d^4*f^3*e^(3*c))*x^4 + 432*(2*a^3*d^4*e*f^2*e^(3*c) + a*b^2
*d^4*e*f^2*e^(3*c))*x^3 + 648*(2*a^3*d^4*e^2*f*e^(3*c) + a*b^2*d^4*e^2*f*e
^(3*c))*x^2 - 4*(9*b^3*d^3*f^3*x^3*e^(6*c) + 9*(3*d^3*e*f^2 - d^2*f^3)*b^3
*x^2*e^(6*c) + 3*(9*d^3*e^2*f - 6*d^2*e*f^2 + 2*d*f^3)*b^3*x*e^(6*c) - (9*
d^2*e^2*f - 6*d*e*f^2 + 2*f^3)*b^3*e^(6*c))*e^(3*d*x) + 27*(4*a*b^2*d^3*f^
3*x^3*e^(5*c) + 6*(2*d^3*e*f^2 - d^2*f^3)*a*b^2*x^2*e^(5*c) + 6*(2*d^3*e^2
*f - 2*d^2*e*f^2 + d*f^3)*a*b^2*x*e^(5*c) - 3*(2*d^2*e^2*f - 2*d*e*f^2 + f
^3)*a*b^2*e^(5*c))*e^(2*d*x) + 108*(12*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*a^2
*b*e^(4*c) + 3*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*b^3*e^(4*c) - (4*a^2*b*d^3*
f^3*e^(4*c) + b^3*d^3*f^3*e^(4*c))*x^3 - 3*(4*(d^3*e*f^2 - d^2*f^3)*a^2*b*
e^(4*c) + (d^3*e*f^2 - d^2*f^3)*b^3*e^(4*c))*x^2 - 3*(4*(d^3*e^2*f - 2*d^2
*e*f^2 + 2*d*f^3)*a^2*b*e^(4*c) + (d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*b^3*
e^(4*c))*x)*e^(d*x) - 108*(12*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*a^2*b*e^(2*c
) + 3*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*b^3*e^(2*c) + (4*a^2*b*d^3*f^3*e^(2*
c) + b^3*d^3*f^3*e^(2*c))*x^3 + 3*(4*(d^3*e*f^2 + d^2*f^3)*a^2*b*e^(2*c...
```

Giac [F]

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh{(dx+c)}^2\sinh{(dx+c)}^2}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)^2*sinh(d*x + c)^2/(b*sinh(d*x + c) + a
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^2\sinh(c+d\,x)^2\,(e+f\,x)^3}{a+b\sinh{(c+d\,x)}}\,dx$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*sinh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)), x
)
```

Reduce [F]

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)^2}{a+b\sinh\left(dx+c\right)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.368 $\int \frac{(e+fx)^2 \cosh^2(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} \, dx$

Optimal result

Integrand size = 36, antiderivative size = 649

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= -\frac{af^2x}{4b^2d^2} - \frac{a^3(e+fx)^3}{3b^4f} - \frac{a(e+fx)^3}{6b^2f} + \frac{2a^2f^2\cosh(c+dx)}{b^3d^3} + \frac{4f^2\cosh(c+dx)}{9bd^3} + \frac{a^2(e+fx)^2\cosh(c+dx)}{b^3d} + \frac{af(e+fx)\cosh^2(c+dx)}{2b^2d^2} + \frac{2f^2\cosh^3(c+dx)}{27bd^3} + \frac{(e+fx)^2\cosh^3(c+dx)}{3bd} + \frac{a^2\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d} - \frac{a^2\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d} + \frac{2a^2\sqrt{a^2+b^2}f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2} - \frac{2a^2\sqrt{a^2+b^2}f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^2} - \frac{2a^2\sqrt{a^2+b^2}f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^3} + \frac{2a^2\sqrt{a^2+b^2}f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^3} - \frac{2a^2f(e+fx)\sinh(c+dx)}{b^3d^2} - \frac{4f(e+fx)\sinh(c+dx)}{9bd^2} - \frac{af^2\cosh(c+dx)\sinh(c+dx)}{4b^2d^3} - \frac{a(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{2b^2d} - \frac{2f(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{9bd^2}$$

output
```
-1/4*a*f^2*x/b^2/d^2-1/3*a^3*(f*x+e)^3/b^4/f-1/6*a*(f*x+e)^3/b^2/f+2*a^2*f
^2*cosh(d*x+c)/b^3/d^3+4/9*f^2*cosh(d*x+c)/b/d^3+a^2*(f*x+e)^2*cosh(d*x+c)
/b^3/d+1/2*a*f*(f*x+e)*cosh(d*x+c)^2/b^2/d^2+2/27*f^2*cosh(d*x+c)^3/b/d^3+
1/3*(f*x+e)^2*cosh(d*x+c)^3/b/d+a^2*(a^2+b^2)^(1/2)*(f*x+e)^2*ln(1+b*exp(d
*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d-a^2*(a^2+b^2)^(1/2)*(f*x+e)^2*ln(1+b*exp(
d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d+2*a^2*(a^2+b^2)^(1/2)*f*(f*x+e)*polylog(
2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d^2-2*a^2*(a^2+b^2)^(1/2)*f*(f*x+
e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d^2-2*a^2*(a^2+b^2)^(1
/2)*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d^3+2*a^2*(a^2+b^
2)^(1/2)*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d^3-2*a^2*f*
(f*x+e)*sinh(d*x+c)/b^3/d^2-4/9*f*(f*x+e)*sinh(d*x+c)/b/d^2-1/4*a*f^2*cosh
(d*x+c)*sinh(d*x+c)/b^2/d^3-1/2*a*(f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/b^2/d-
2/9*f*(f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/b/d^2
```

Mathematica [A] (verified)

Time = 2.68 (sec) , antiderivative size = 966, normalized size of antiderivative = 1.49

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{216a^3d^3e^2x+108ab^2d^3e^2x+216a^3d^3efx^2+108ab^2d^3efx^2+72a^3d^3f^2x^3+36ab^2d^3f^2x^3+432a^2\sqrt{a^2}}{}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*
x]),x]
```

output
```
-1/216*(216*a^3*d^3*e^2*x + 108*a*b^2*d^3*e^2*x + 216*a^3*d^3*e*f*x^2 + 10
8*a*b^2*d^3*e*f*x^2 + 72*a^3*d^3*f^2*x^3 + 36*a*b^2*d^3*f^2*x^3 + 432*a^2*
Sqrt[a^2 + b^2]*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 216
*a^2*b*d^2*e^2*Cosh[c + d*x] - 54*b^3*d^2*e^2*Cosh[c + d*x] - 432*a^2*b*f^
2*Cosh[c + d*x] - 108*b^3*f^2*Cosh[c + d*x] - 432*a^2*b*d^2*e*f*x*Cosh[c +
 d*x] - 108*b^3*d^2*e*f*x*Cosh[c + d*x] - 216*a^2*b*d^2*f^2*x^2*Cosh[c + d
*x] - 54*b^3*d^2*f^2*x^2*Cosh[c + d*x] - 54*a*b^2*d*e*f*Cosh[2*(c + d*x)]
- 54*a*b^2*d*f^2*x*Cosh[2*(c + d*x)] - 18*b^3*d^2*e^2*Cosh[3*(c + d*x)] -
4*b^3*f^2*Cosh[3*(c + d*x)] - 36*b^3*d^2*e*f*x*Cosh[3*(c + d*x)] - 18*b^3*
d^2*f^2*x^2*Cosh[3*(c + d*x)] - 432*a^2*Sqrt[a^2 + b^2]*d^2*e*f*x*Log[1 +
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 216*a^2*Sqrt[a^2 + b^2]*d^2*f^2*x
^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 432*a^2*Sqrt[a^2 + b^2
]*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 216*a^2*Sqrt[
a^2 + b^2]*d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 43
2*a^2*Sqrt[a^2 + b^2]*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[
a^2 + b^2])] + 432*a^2*Sqrt[a^2 + b^2]*d*f*(e + f*x)*PolyLog[2, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))] + 432*a^2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, (
b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 432*a^2*Sqrt[a^2 + b^2]*f^2*PolyL
og[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 432*a^2*b*d*e*f*Sinh[c +
 d*x] + 108*b^3*d*e*f*Sinh[c + d*x] + 432*a^2*b*d*f^2*x*Sinh[c + d*x] +...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sinh^2(c+dx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6113}$$

$$\frac{\int (e+fx)^2\cosh^2(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 5970}$$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\int (e+fx)\cosh^3(c+dx)dx}{3d}}{b}-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d}}{b}$$

$\Big\downarrow$ 3791

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\int (e+fx)\cosh(c+dx)dx-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$\Big\downarrow$ 3042

$$-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}$$

$\Big\downarrow$ 3777

$$-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}$$

$\Big\downarrow$ 26

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$\Big\downarrow$ 3042

$$-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}$$

$\Big\downarrow$ 26

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int\sin(ic+idx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$$\Big\downarrow \text{ 3118}$$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 6113}$$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{\int(e+fx)^2\cosh^2(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3792}$$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{\frac{f^2\int\cosh^2(c+dx)dx}{2d^2}+\frac{1}{2}\int(e+fx)^2dx-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{\frac{f^2\int\cosh^2(c+dx)dx}{2d^2}-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$\downarrow$ 3042

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{f^2\int\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx}{2d^2}-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}\right)}{b}$$

$\downarrow$ 3115

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{\frac{f^2\left(\frac{\int 1dx}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$\downarrow$ 24

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$\downarrow$ 6099

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+f}{b^2}\right.}{b}\right.}{b}$$

$\downarrow$ 17

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int(e+fx}{}\right.}{b}\right.}{b}$$

$\downarrow$ 3042

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}+\frac{\int-i(e}{}\right.}{b}\right.}{b}$$

$\downarrow$ 26

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\int(e+}{}\right.}{b}\right.}{b}$$

$\downarrow$ 3777

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e-}{}\right.}{}\right.}{}\right.}{b}$$

$\downarrow$ 3042

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-$$

$$\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e-\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right)}{b}$$

$\downarrow$ 3777

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-$$

$$\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e-\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right)}{b}$$

$\downarrow$ 26

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-$$

$$\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e-\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right)}{b}$$

$\downarrow$ 3042

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-$$

$$\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e\cdots}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{b}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-$$

$$\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e\cdots}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{b}$$

$$\Big\downarrow \; 3118$$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-$$

$$\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{a(e+f\cdots}{3b^2\cdots}\cdots\right)}{\cdots}\cdots\right)}{b}$$

$$\Big\downarrow \; 3803$$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-$$

$$\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{2(a^2+b^2)\int -\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}}{b^2}\right.}{}\right.}{b}$$

$\Big\downarrow$ 25

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-$$

$$\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{2(a^2+b^2)\int \frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}}{b^2}\right.}{}\right.}{b}$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^2\cosh(dx+c)^2\sinh(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4311 vs. 2(595) = 1190.

Time = 0.18 (sec) , antiderivative size = 4311, normalized size of antiderivative = 6.64

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)**2*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)^2\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="maxima")
```

output
```
1/24*e^2*(24*sqrt(a^2 + b^2)*a^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2)
)/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(b^4*d) - (3*a*b*e^(-d*x - c) -
b^2 - 3*(4*a^2 + b^2)*e^(-2*d*x - 2*c))*e^(3*d*x + 3*c)/(b^3*d) - 12*(2*a^
3 + a*b^2)*(d*x + c)/(b^4*d) + (3*a*b*e^(-2*d*x - 2*c) + b^2*e^(-3*d*x - 3
*c) + 3*(4*a^2 + b^2)*e^(-d*x - c))/(b^3*d)) - 1/432*(72*(2*a^3*d^3*f^2*e^
(3*c) + a*b^2*d^3*f^2*e^(3*c))*x^3 + 216*(2*a^3*d^3*e*f*e^(3*c) + a*b^2*d^
3*e*f*e^(3*c))*x^2 - 2*(9*b^3*d^2*f^2*x^2*e^(6*c) + 6*(3*d^2*e*f - d*f^2)*
b^3*x*e^(6*c) - 2*(3*d*e*f - f^2)*b^3*e^(6*c))*e^(3*d*x) + 27*(2*a*b^2*d^2
*f^2*x^2*e^(5*c) + 2*(2*d^2*e*f - d*f^2)*a*b^2*x*e^(5*c) - (2*d*e*f - f^2)
*a*b^2*e^(5*c))*e^(2*d*x) + 54*(8*(d*e*f - f^2)*a^2*b*e^(4*c) + 2*(d*e*f -
 f^2)*b^3*e^(4*c) - (4*a^2*b*d^2*f^2*e^(4*c) + b^3*d^2*f^2*e^(4*c))*x^2 -
2*(4*(d^2*e*f - d*f^2)*a^2*b*e^(4*c) + (d^2*e*f - d*f^2)*b^3*e^(4*c))*x)*e
^(d*x) - 54*(8*(d*e*f + f^2)*a^2*b*e^(2*c) + 2*(d*e*f + f^2)*b^3*e^(2*c) +
 (4*a^2*b*d^2*f^2*e^(2*c) + b^3*d^2*f^2*e^(2*c))*x^2 + 2*(4*(d^2*e*f + d*f
^2)*a^2*b*e^(2*c) + (d^2*e*f + d*f^2)*b^3*e^(2*c))*x)*e^(-d*x) - 27*(2*a*b
^2*d^2*f^2*x^2*e^c + 2*(2*d^2*e*f + d*f^2)*a*b^2*x*e^c + (2*d*e*f + f^2)*a
*b^2*e^c)*e^(-2*d*x) - 2*(9*b^3*d^2*f^2*x^2 + 6*(3*d^2*e*f + d*f^2)*b^3*x
+ 2*(3*d*e*f + f^2)*b^3)*e^(-3*d*x))*e^(-3*c)/(b^4*d^3) + integrate(2*((a^
4*f^2*e^c + a^2*b^2*f^2*e^c)*x^2 + 2*(a^4*e*f*e^c + a^2*b^2*e*f*e^c)*x)*e^
(d*x)/(b^5*e^(2*d*x + 2*c) + 2*a*b^4*e^(d*x + c) - b^5), x)
```

Giac [F]

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)^2}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)^2*sinh(d*x + c)^2/(b*sinh(d*x + c) + a
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^2\sinh(c+d\,x)^2\,(e+f\,x)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*sinh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)), x
)
```

Reduce [F]

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(864*e**(3*c + 3*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt
(a**2 + b**2))*a**2*b**3*d**2*e**2*i + 18*e**(6*c + 6*d*x)*b**6*d**2*e**2
+ 36*e**(6*c + 6*d*x)*b**6*d**2*e*f*x + 18*e**(6*c + 6*d*x)*b**6*d**2*f**2
*x**2 - 12*e**(6*c + 6*d*x)*b**6*d*e*f - 12*e**(6*c + 6*d*x)*b**6*d*f**2*x
 + 4*e**(6*c + 6*d*x)*b**6*f**2 - 54*e**(5*c + 5*d*x)*a*b**5*d**2*e**2 - 1
08*e**(5*c + 5*d*x)*a*b**5*d**2*e*f*x - 54*e**(5*c + 5*d*x)*a*b**5*d**2*f*
*2*x**2 + 54*e**(5*c + 5*d*x)*a*b**5*d*e*f + 54*e**(5*c + 5*d*x)*a*b**5*d*
f**2*x - 27*e**(5*c + 5*d*x)*a*b**5*f**2 + 216*e**(4*c + 4*d*x)*a**2*b**4*
d**2*e**2 + 432*e**(4*c + 4*d*x)*a**2*b**4*d**2*e*f*x + 216*e**(4*c + 4*d*
x)*a**2*b**4*d**2*f**2*x**2 - 432*e**(4*c + 4*d*x)*a**2*b**4*d*e*f - 432*e
**(4*c + 4*d*x)*a**2*b**4*d*f**2*x + 432*e**(4*c + 4*d*x)*a**2*b**4*f**2 +
 54*e**(4*c + 4*d*x)*b**6*d**2*e**2 + 108*e**(4*c + 4*d*x)*b**6*d**2*e*f*x
 + 54*e**(4*c + 4*d*x)*b**6*d**2*f**2*x**2 - 108*e**(4*c + 4*d*x)*b**6*d*e
*f - 108*e**(4*c + 4*d*x)*b**6*d*f**2*x + 108*e**(4*c + 4*d*x)*b**6*f**2 +
 3456*e**(3*c + 3*d*x)*int(x**2/(e**(5*c + 5*d*x)*b + 2*e**(4*c + 4*d*x)*a
 - e**(3*c + 3*d*x)*b),x)*a**6*b*d**3*f**2 + 4320*e**(3*c + 3*d*x)*int(x**
2/(e**(5*c + 5*d*x)*b + 2*e**(4*c + 4*d*x)*a - e**(3*c + 3*d*x)*b),x)*a**4
*b**3*d**3*f**2 + 864*e**(3*c + 3*d*x)*int(x**2/(e**(5*c + 5*d*x)*b + 2*e*
*(4*c + 4*d*x)*a - e**(3*c + 3*d*x)*b),x)*a**2*b**5*d**3*f**2 + 6912*e**(3
*c + 3*d*x)*int(x/(e**(5*c + 5*d*x)*b + 2*e**(4*c + 4*d*x)*a - e**(3*c ...
```

3.369 $\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 395

$$\begin{aligned}
\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx
&= -\frac{a^3(e+fx)^2}{2b^4f}-\frac{a(e+fx)^2}{4b^2f}+\frac{a^2(e+fx)\cosh(c+dx)}{b^3d}+\frac{af\cosh^2(c+dx)}{4b^2d^2}\\
&\quad+\frac{(e+fx)\cosh^3(c+dx)}{3bd}+\frac{a^2\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d}\\
&\quad-\frac{a^2\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d}+\frac{a^2\sqrt{a^2+b^2}f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}\\
&\quad-\frac{a^2\sqrt{a^2+b^2}f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^2}-\frac{a^2f\sinh(c+dx)}{b^3d^2}\\
&\quad-\frac{f\sinh(c+dx)}{3bd^2}-\frac{a(e+fx)\cosh(c+dx)\sinh(c+dx)}{2b^2d}-\frac{f\sinh^3(c+dx)}{9bd^2}
\end{aligned}$$

output
```
-1/2*a^3*(f*x+e)^2/b^4/f-1/4*a*(f*x+e)^2/b^2/f+a^2*(f*x+e)*cosh(d*x+c)/b^3
/d+1/4*a*f*cosh(d*x+c)^2/b^2/d^2+1/3*(f*x+e)*cosh(d*x+c)^3/b/d+a^2*(a^2+b^
2)^(1/2)*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d-a^2*(a^2+b^2
)^(1/2)*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d+a^2*(a^2+b^2)
^(1/2)*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d^2-a^2*(a^2+b^2
)^(1/2)*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d^2-a^2*f*sinh(
d*x+c)/b^3/d^2-1/3*f*sinh(d*x+c)/b/d^2-1/2*a*(f*x+e)*cosh(d*x+c)*sinh(d*x+
c)/b^2/d-1/9*f*sinh(d*x+c)^3/b/d^2
```

Mathematica [A] (verified)

Time = 2.27 (sec) , antiderivative size = 626, normalized size of antiderivative = 1.58

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{72a^3cde+36ab^2cde-36a^3c^2f-18ab^2c^2f+72a^3d^2ex+36ab^2d^2ex+36a^3d^2fx^2+18ab^2d^2fx^2+144}{}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
-1/72*(72*a^3*c*d*e + 36*a*b^2*c*d*e - 36*a^3*c^2*f - 18*a*b^2*c^2*f + 72*
a^3*d^2*e*x + 36*a*b^2*d^2*e*x + 36*a^3*d^2*f*x^2 + 18*a*b^2*d^2*f*x^2 + 1
44*a^2*Sqrt[a^2 + b^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] -
144*a^2*Sqrt[a^2 + b^2]*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] -
 72*a^2*b*d*e*Cosh[c + d*x] - 18*b^3*d*e*Cosh[c + d*x] - 72*a^2*b*d*f*x*Co
sh[c + d*x] - 18*b^3*d*f*x*Cosh[c + d*x] - 9*a*b^2*f*Cosh[2*(c + d*x)] - 6
*b^3*d*e*Cosh[3*(c + d*x)] - 6*b^3*d*f*x*Cosh[3*(c + d*x)] - 72*a^2*Sqrt[a
^2 + b^2]*c*f*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 72*a^2*Sqrt
[a^2 + b^2]*d*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 72*a^2*
Sqrt[a^2 + b^2]*c*f*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 72*a^
2*Sqrt[a^2 + b^2]*d*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 7
2*a^2*Sqrt[a^2 + b^2]*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
 + 72*a^2*Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2]))] + 72*a^2*b*f*Sinh[c + d*x] + 18*b^3*f*Sinh[c + d*x] + 18*a*b^2*d*e*
Sinh[2*(c + d*x)] + 18*a*b^2*d*f*x*Sinh[2*(c + d*x)] + 2*b^3*f*Sinh[3*(c +
 d*x)])/(b^4*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.49 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.99, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.676, Rules used = {6113, 5970, 3042, 3113, 2009, 6113, 3042, 3791, 17, 6099, 17, 3042, 26, 3777, 3042, 3117, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh^2(c+dx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6113}$$

$$\frac{\int (e+fx)\cosh^2(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{5970}$$

$$\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{f\int \cosh^3(c+dx)dx}{3d}}{b}-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{f\int \sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d}}{b}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\int \left(\sinh^2(c+dx)+1\right)d(-i\sinh(c+dx))}{3d^2}}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}$$

$$\Big\downarrow \text{6113}$$

$$-\frac{a\left(\frac{\int (e+fx)\cosh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx}{b}\right)}{b}$$

$\Big\downarrow$ 3791

$$-\frac{a\left(\frac{\frac{1}{2}\int (e+fx)dx-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}$$

$\Big\downarrow$ 17

$$-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}$$

$\Big\downarrow$ 6099

$$-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)dx}{b^2}+\frac{\int (e+fx)\sinh(c+dx)dx}{b}\right)}{b}\right)}{b}+\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}$$

$\Big\downarrow$ 17

$$-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{e+fx}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int(e+fx)\sinh(c+dx)dx}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{b}\right)}{b}+\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}+\frac{\int -i(e+fx)\sin(ic+idx)dx}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\int(e+fx)\sin(ic+idx)dx}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 3777$$

$$\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{b}-\frac{a(e\ldots}{2\ldots}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}\right.}{b}\right.}{b}$$

$\downarrow$ 3117

$$\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right.}{b}\right.}{b}\right.}{b}$$

$\downarrow$ 3803

$$\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{2\left(a^2+b^2\right)\int-\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\ldots\right.}{b}\right.}{b}\right.}{b}$$

$\downarrow$ 25

$$\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2\left(a^2+b^2\right)\int\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\ldots\right.}{b}\right.}{b}\right.}{b}$$

$\downarrow$ 2694

$$\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2\left(a^2+b^2\right)\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b^2}\cdots\right)}{b}\cdots\right)}{b}$$

$\big\downarrow$ 27

$$\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2\left(a^2+b^2\right)\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)\cdots}{2b^2f}\cdots\right)}{b}\cdots\right)}{b}$$

$\big\downarrow$ 2620

$$\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2\left(a^2+b^2\right)\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\cdots\right.}{\cdots}\right.}{b^2}\cdots\right.}{\cdots}\right.}{b}$$

$\downarrow$ 2715

$$\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2\left(a^2+b^2\right)\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de\cdots}{bd^2}\right.}{2\sqrt{a^2+b^2}}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{b}$$

$\downarrow$ 2838

$$\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2\left(a^2+b^2\right)\left(\frac{b\left(\frac{f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\cdots\right.}{\cdots}\right)}{b^2}\cdots\right.}{\cdots}\right)}{b}$$

input
```
Int[((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(((e + f*x)*Cosh[c + d*x]^3)/(3*d) - ((I/3)*f*((-I)*Sinh[c + d*x] - (I/3)*
Sinh[c + d*x]^3))/d^2)/b - (a*(((e + f*x)^2/(4*f) - (f*Cosh[c + d*x]^2)/(4
*d^2) + ((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*d))/b - (a*(-1/2*(a*(e
+ f*x)^2)/(b^2*f) - (2*(a^2 + b^2)*(-1/2*(b*(((e + f*x)*Log[1 + (b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2]))])/(b*d^2)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[1 + (b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(2*Sqrt[a^2 + b^2])))/b^2 - (I*((I*
(e + f*x)*Cosh[c + d*x])/d - (I*f*Sinh[c + d*x])/d^2))/b))/b))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5970
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1127 vs. $2(361) = 722$.

Time = 19.55 (sec) , antiderivative size = 1128, normalized size of antiderivative = 2.86

method	result	size
risch	Expression too large to display	1128

input
```
int((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURN
VERBOSE)
```

output
```
-1/16*a*(2*d*f*x+2*d*e-f)/b^2/d^2*exp(2*d*x+2*c)+1/8*(4*a^2+b^2)*(d*f*x+d*
e+f)/b^3/d^2*exp(-d*x-c)+1/16*a*(2*d*f*x+2*d*e+f)/b^2/d^2*exp(-2*d*x-2*c)+
1/8*(4*a^2*d*f*x+b^2*d*f*x+4*a^2*d*e+b^2*d*e-4*a^2*f-b^2*f)/b^3/d^2*exp(d*
x+c)+1/72*(3*d*f*x+3*d*e-f)/b/d^2*exp(3*d*x+3*c)+1/72*(3*d*f*x+3*d*e+f)/b/
d^2*exp(-3*d*x-3*c)-1/2*a^3/b^4*f*x^2-1/4*a/b^2*f*x^2-a^3/b^4*e*x-1/2*a/b^
2*e*x+1/d^2/b^2*a^2*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)
-a)/(-a+(a^2+b^2)^(1/2)))+2/d^2*a^4/b^4*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2
*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/d*a^4/b^4*f/(a^2+b^2)^(1/2)*ln((-b*e
xp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d*a^4/b^4*f/(a^2+b^
2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2*
a^4/b^4*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^
2)^(1/2)))*c-1/d^2*a^4/b^4*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1
/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/d^2*a^2/b^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d
*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d/b^2*a^2*f/(a^2+b^2)^(
1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2/b^2*
a^2*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/
2)))*c+2/d^2/b^2*a^2*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/
(a^2+b^2)^(1/2))+1/d*a^2/b^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)
^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-2/d*a^4/b^4*e/(a^2+b^2)^(1/2)*arctanh(1/
2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d*a^2/b^2*e/(a^2+b^2)^(1/2)*a...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2195 vs. $2(359) = 718$.

Time = 0.16 (sec) , antiderivative size = 2195, normalized size of antiderivative = 5.56

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
1/144*(2*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^6 + 2*(3*b^3*d*f*
x + 3*b^3*d*e - b^3*f)*sinh(d*x + c)^6 + 6*b^3*d*f*x - 9*(2*a*b^2*d*f*x +
2*a*b^2*d*e - a*b^2*f)*cosh(d*x + c)^5 - 3*(6*a*b^2*d*f*x + 6*a*b^2*d*e -
3*a*b^2*f - 4*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c))*sinh(d*x +
c)^5 + 6*b^3*d*e + 18*((4*a^2*b + b^3)*d*f*x + (4*a^2*b + b^3)*d*e - (4*a^
2*b + b^3)*f)*cosh(d*x + c)^4 + 3*(6*(4*a^2*b + b^3)*d*f*x + 6*(4*a^2*b +
b^3)*d*e + 10*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^2 - 6*(4*a^2
*b + b^3)*f - 15*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*cosh(d*x + c))*si
nh(d*x + c)^4 + 2*b^3*f - 36*((2*a^3 + a*b^2)*d^2*f*x^2 + 2*(2*a^3 + a*b^2
)*d^2*e*x)*cosh(d*x + c)^3 - 2*(18*(2*a^3 + a*b^2)*d^2*f*x^2 + 36*(2*a^3 +
 a*b^2)*d^2*e*x - 20*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^3 + 4
5*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*cosh(d*x + c)^2 - 36*((4*a^2*b +
 b^3)*d*f*x + (4*a^2*b + b^3)*d*e - (4*a^2*b + b^3)*f)*cosh(d*x + c))*sinh
(d*x + c)^3 + 18*((4*a^2*b + b^3)*d*f*x + (4*a^2*b + b^3)*d*e + (4*a^2*b +
 b^3)*f)*cosh(d*x + c)^2 + 6*(5*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x
 + c)^4 + 3*(4*a^2*b + b^3)*d*f*x - 15*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^
2*f)*cosh(d*x + c)^3 + 3*(4*a^2*b + b^3)*d*e + 18*((4*a^2*b + b^3)*d*f*x +
 (4*a^2*b + b^3)*d*e - (4*a^2*b + b^3)*f)*cosh(d*x + c)^2 + 3*(4*a^2*b + b
^3)*f - 18*((2*a^3 + a*b^2)*d^2*f*x^2 + 2*(2*a^3 + a*b^2)*d^2*e*x)*cosh(d*
x + c))*sinh(d*x + c)^2 + 144*(a^2*b*f*cosh(d*x + c)^3 + 3*a^2*b*f*cosh...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)**2*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)^2\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
1/144*(288*(a^4*e^c + a^2*b^2*e^c)*integrate(x*e^(d*x)/(b^5*e^(2*d*x + 2*c
) + 2*a*b^4*e^(d*x + c) - b^5), x) - (36*(2*a^3*d^2*e^(3*c) + a*b^2*d^2*e^
(3*c))*x^2 - 2*(3*b^3*d*x*e^(6*c) - b^3*e^(6*c))*e^(3*d*x) + 9*(2*a*b^2*d*
x*e^(5*c) - a*b^2*e^(5*c))*e^(2*d*x) + 18*(4*a^2*b*e^(4*c) + b^3*e^(4*c) -
 (4*a^2*b*d*e^(4*c) + b^3*d*e^(4*c))*x)*e^(d*x) - 18*(4*a^2*b*e^(2*c) + b^
3*e^(2*c) + (4*a^2*b*d*e^(2*c) + b^3*d*e^(2*c))*x)*e^(-d*x) - 9*(2*a*b^2*d
*x*e^c + a*b^2*e^c)*e^(-2*d*x) - 2*(3*b^3*d*x + b^3)*e^(-3*d*x))*e^(-3*c)/
(b^4*d^2))*f + 1/24*e*(24*sqrt(a^2 + b^2)*a^2*log((b*e^(-d*x - c) - a - sq
rt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(b^4*d) - (3*a*b*e^
(-d*x - c) - b^2 - 3*(4*a^2 + b^2)*e^(-2*d*x - 2*c))*e^(3*d*x + 3*c)/(b^3*
d) - 12*(2*a^3 + a*b^2)*(d*x + c)/(b^4*d) + (3*a*b*e^(-2*d*x - 2*c) + b^2*
e^(-3*d*x - 3*c) + 3*(4*a^2 + b^2)*e^(-d*x - c))/(b^3*d))
```

Giac [F]

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)^2\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)^2*sinh(d*x + c)^2/(b*sinh(d*x + c) + a),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\int \frac{\cosh(c+d\,x)^2\,\sinh(c+d\,x)^2\,(e+f\,x)}{a+b\sinh\,(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*sinh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\frac{72e^{dx+c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)a^2bdei-18e^{dx+c}\cosh\,(dx+c)\sinh\,(dx+c)\,a\,b^3dfx+8e^{dx+c}\sinh\,(dx}{}$$

input
```
int((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output

```
(72*e**(c + d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2
 + b**2))*a**2*b*d*e*i + 12*e**(c + d*x)*cosh(c + d*x)**3*b**4*d*e + 12*e*
*(c + d*x)*cosh(c + d*x)**3*b**4*d*f*x - 12*e**(c + d*x)*cosh(c + d*x)**2*
sinh(c + d*x)*b**4*f - 18*e**(c + d*x)*cosh(c + d*x)**2*a*b**3*d**2*e*x -
9*e**(c + d*x)*cosh(c + d*x)**2*a*b**3*d**2*f*x**2 + 9*e**(c + d*x)*cosh(c
 + d*x)**2*a*b**3*f - 18*e**(c + d*x)*cosh(c + d*x)*sinh(c + d*x)*a*b**3*d
*e - 18*e**(c + d*x)*cosh(c + d*x)*sinh(c + d*x)*a*b**3*d*f*x + 18*e**(2*c
 + 2*d*x)*a**2*b**2*d*e + 18*e**(2*c + 2*d*x)*a**2*b**2*d*f*x - 18*e**(2*c
 + 2*d*x)*a**2*b**2*f - 144*e**(c + d*x)*int(x/(e**(2*c + 2*d*x)*b + 2*e**
(c + d*x)*a - b),x)*a**5*d**2*f - 144*e**(c + d*x)*int(x/(e**(2*c + 2*d*x)
*b + 2*e**(c + d*x)*a - b),x)*a**3*b**2*d**2*f + 8*e**(c + d*x)*sinh(c + d
*x)**3*b**4*f + 18*e**(c + d*x)*sinh(c + d*x)**2*a*b**3*d**2*e*x + 9*e**(c
 + d*x)*sinh(c + d*x)**2*a*b**3*d**2*f*x**2 - 36*e**(c + d*x)*a**3*b*d**2*
e*x - 18*e**(c + d*x)*a**3*b*d**2*f*x**2 + 72*e**(d*x)*int(x/(e**(2*c + 3*
d*x)*b + 2*e**(c + 2*d*x)*a - e**(d*x)*b),x)*a**4*b*d**2*f + 72*e**(d*x)*i
nt(x/(e**(2*c + 3*d*x)*b + 2*e**(c + 2*d*x)*a - e**(d*x)*b),x)*a**2*b**3*d
**2*f - 72*a**4*d*f*x - 72*a**4*f + 18*a**2*b**2*d*e - 54*a**2*b**2*d*f*x
- 54*a**2*b**2*f)/(36*e**(c + d*x)*b**5*d**2)
```

3.370 $\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 141

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a(2a^2+b^2)\,x}{2b^4} - \frac{2a^2\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^4d} + \frac{(3a^2+b^2)\cosh(c+dx)}{3b^3d} - \frac{a\cosh(c+dx)\sinh(c+dx)}{2b^2d} + \frac{\cosh(c+dx)\sinh^2(c+dx)}{3bd}$$

output
```
-1/2*a*(2*a^2+b^2)*x/b^4-2*a^2*(a^2+b^2)^(1/2)*arctanh((b-a*tanh(1/2*d*x+1
/2*c))/(a^2+b^2)^(1/2))/b^4/d+1/3*(3*a^2+b^2)*cosh(d*x+c)/b^3/d-1/2*a*cosh
(d*x+c)*sinh(d*x+c)/b^2/d+1/3*cosh(d*x+c)*sinh(d*x+c)^2/b/d
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.87

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{3b(4a^2+b^2)\cosh(c+dx)+b^3\cosh(3(c+dx))-3a\left(2(2a^2+b^2)(c+dx)+8a\sqrt{-a^2-b^2}\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)+b^2\sinh(2(c+dx))\right)}{12b^4d}$$

input
```
Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(3*b*(4*a^2 + b^2)*Cosh[c + d*x] + b^3*Cosh[3*(c + d*x)] - 3*a*(2*(2*a^2 +
 b^2)*(c + d*x) + 8*a*Sqrt[-a^2 - b^2]*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sq
rt[-a^2 - b^2]] + b^2*Sinh[2*(c + d*x)]))/(12*b^4*d)
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.16, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.724$, Rules used = {3042, 25, 3368, 25, 3042, 25, 3529, 26, 3042, 26, 3528, 25, 3042, 3502, 27, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{\sin(ic+idx)^2\cos(ic+idx)^2}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{\cos(ic+idx)^2\sin(ic+idx)^2}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3368}$$

$$-\int -\frac{\sinh^2(c+dx)\left(\sinh^2(c+dx)+1\right)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\sinh^2(c+dx)\left(\sinh^2(c+dx)+1\right)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ic+idx)^2\left(1-\sin(ic+idx)^2\right)}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ic+idx)^2\left(1-\sin(ic+idx)^2\right)}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3529}$$

$$\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{i\int -\frac{i\sinh(c+dx)\left(3a\sinh^2(c+dx)-b\sinh(c+dx)+2a\right)}{a+b\sinh(c+dx)}dx}{3b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{\int \frac{\sinh(c+dx)\left(3a\sinh^2(c+dx)-b\sinh(c+dx)+2a\right)}{a+b\sinh(c+dx)}dx}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{\int -\frac{i\sin(ic+idx)\left(-3a\sin(ic+idx)^2+ib\sin(ic+idx)+2a\right)}{a-ib\sin(ic+idx)}dx}{3b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}+\frac{i\int \frac{\sin(ic+idx)\left(-3a\sin(ic+idx)^2+ib\sin(ic+idx)+2a\right)}{a-ib\sin(ic+idx)}dx}{3b}$$

$$\Big\downarrow \text{3528}$$

$$\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}+\frac{i\left(\frac{i\int -\frac{3a^2-b\sinh(c+dx)a+2\left(3a^2+b^2\right)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{2b}+\frac{3ia\sinh(c+dx)\cosh(c+dx)}{2bd}\right)}{3b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}+\frac{i\left(\frac{3ia\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\int\frac{3a^2-b\sinh(c+dx)a+2\left(3a^2+b^2\right)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{2b}\right)}{3b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}+\frac{i\left(\frac{3ia\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\int\frac{3a^2+ib\sin(ic+idx)a-2\left(3a^2+b^2\right)\sin(ic+idx)^2}{a-ib\sin(ic+idx)}dx}{2b}\right)}{3b}$$

$$\Big\downarrow \; 3502$$

$$\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}+\frac{i\left(\frac{3ia\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{2\left(3a^2+b^2\right)\cosh(c+dx)}{bd}+\frac{i\int-\frac{3i\left(a^2b-a\left(2a^2+b^2\right)\sinh(c+dx)\right)}{a+b\sinh(c+dx)}dx}{b}\right)}{2b}\right)}{3b}$$

$$\Big\downarrow \; 27$$

$$\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}+\frac{i\left(\frac{3ia\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{3\int\frac{a^2b-a\left(2a^2+b^2\right)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{2\left(3a^2+b^2\right)\cosh(c+dx)}{bd}\right)}{2b}\right)}{3b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}+\frac{i\left(\frac{3ia\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{2\left(3a^2+b^2\right)\cosh(c+dx)}{bd}+\frac{3\int\frac{ba^2+i\left(2a^2+b^2\right)\sin(ic+idx)a}{a-ib\sin(ic+idx)}dx}{b}\right)}{2b}\right)}{3b}$$

$\downarrow$ 3214

$$\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}+\frac{i\left(\frac{3ia\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{3\left(\frac{2a^2\left(a^2+b^2\right)\int\frac{1}{a+b\sinh(c+dx)}dx}{b}-\frac{ax\left(2a^2+b^2\right)}{b}\right)}{b}+\frac{2\left(3a^2+b^2\right)\cosh(c+dx)}{bd}\right)}{2b}\right)}{3b}$$

$\downarrow$ 3042

$$\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}+\frac{i\left(\frac{3ia\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{2\left(3a^2+b^2\right)\cosh(c+dx)}{bd}+\frac{3\left(-\frac{ax\left(2a^2+b^2\right)}{b}+\frac{2a^2\left(a^2+b^2\right)\int\frac{1}{a-ib\sin(ic+idx)}dx}{b}\right)}{b}\right)}{2b}\right)}{3b}$$

$\downarrow$ 3139

$$\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}+\frac{i\left(\frac{3ia\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{2\left(3a^2+b^2\right)\cosh(c+dx)}{bd}+\frac{3\left(-\frac{ax\left(2a^2+b^2\right)}{b}-\frac{4ia^2\left(a^2+b^2\right)\int\frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}\right)\right)}{bd}\right)}{b}\right)}{2b}\right)}{3b}$$

$\downarrow$ 1083

$$\frac{\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}+ i\left(\frac{3ia\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{2(3a^2+b^2)\cosh(c+dx)}{bd}+\frac{3\left(-\frac{ax(2a^2+b^2)}{b}+\frac{8ia^2(a^2+b^2)\int\frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4(a^2+b^2)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2\cdots\right)}{bd}\right)}{b}\right)}{2b}\right)}{3b}$$

$\downarrow$ 217

$$\frac{\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}+ i\left(\frac{3ia\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{3\left(\frac{4a^2\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{bd}-\frac{ax(2a^2+b^2)}{b}\right)}{b}+\frac{2(3a^2+b^2)\cosh(c+dx)}{bd}\right)}{2b}\right)}{3b}$$

input
```
Int[(Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(Cosh[c + d*x]*Sinh[c + d*x]^2)/(3*b*d) + ((I/3)*(((-1/2*I)*((3*(-((a*(2*a
^2 + b^2)*x)/b) + (4*a^2*Sqrt[a^2 + b^2]*ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt
[a^2 + b^2])])/(b*d)))/b + (2*(3*a^2 + b^2)*Cosh[c + d*x])/(b*d)))/b + (((
3*I)/2)*a*Cosh[c + d*x]*Sinh[c + d*x])/(b*d)))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3368
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3529
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n +
1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m + n + 2))   Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(
n + 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*
(a*d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c,
0])))
```

Maple [A] (verified)

Time = 8.36 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.73

method	result
risch	$-\frac{a^3 x}{b^4}-\frac{a x}{2 b^2}+\frac{e^{3 d x+3 c}}{24 b d}-\frac{a\,e^{2 d x+2 c}}{8 b^2 d}+\frac{e^{d x+c} a^2}{2 b^3 d}+\frac{e^{d x+c}}{8 b d}+\frac{e^{-d x-c} a^2}{2 b^3 d}+\frac{e^{-d x-c}}{8 b d}+\frac{a\,e^{-2 d x-2 c}}{8 b^2 d}+\frac{e^{-3d}}{24}$
derivativedivides	$\frac{-\frac{1}{3 b\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{a+b}{2 b^2\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{2 a^2+a b+b^2}{2 b^3\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\left(2 a^2+b^2\right)\ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}{2 b^4}+\frac{1}{3 b\left(1+\tanh\left(\frac{d}{2}\right.\right.}}{}$
default	$\frac{-\frac{1}{3 b\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{a+b}{2 b^2\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{2 a^2+a b+b^2}{2 b^3\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\left(2 a^2+b^2\right)\ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}{2 b^4}+\frac{1}{3 b\left(1+\tanh\left(\frac{d}{2}\right.\right.}}{}$

input
```
int(cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-a^3*x/b^4-1/2*a*x/b^2+1/24/b/d*exp(3*d*x+3*c)-1/8*a/b^2/d*exp(2*d*x+2*c)+
1/2/b^3/d*exp(d*x+c)*a^2+1/8/b/d*exp(d*x+c)+1/2/b^3/d*exp(-d*x-c)*a^2+1/8/
b/d*exp(-d*x-c)+1/8*a/b^2/d*exp(-2*d*x-2*c)+1/24/b/d*exp(-3*d*x-3*c)+(a^2+
b^2)^(1/2)*a^2/d/b^4*ln(exp(d*x+c)-(-a+(a^2+b^2)^(1/2))/b)-(a^2+b^2)^(1/2)
*a^2/d/b^4*ln(exp(d*x+c)+(a+(a^2+b^2)^(1/2))/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. 2(130) = 260.

Time = 0.11 (sec) , antiderivative size = 745, normalized size of antiderivative = 5.28

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fric
as")
```

output
```
1/24*(b^3*cosh(d*x + c)^6 + b^3*sinh(d*x + c)^6 - 3*a*b^2*cosh(d*x + c)^5
- 12*(2*a^3 + a*b^2)*d*x*cosh(d*x + c)^3 + 3*(2*b^3*cosh(d*x + c) - a*b^2)
*sinh(d*x + c)^5 + 3*(4*a^2*b + b^3)*cosh(d*x + c)^4 + 3*(5*b^3*cosh(d*x +
 c)^2 - 5*a*b^2*cosh(d*x + c) + 4*a^2*b + b^3)*sinh(d*x + c)^4 + 3*a*b^2*c
osh(d*x + c) + 2*(10*b^3*cosh(d*x + c)^3 - 15*a*b^2*cosh(d*x + c)^2 - 6*(2
*a^3 + a*b^2)*d*x + 6*(4*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + b^3
 + 3*(4*a^2*b + b^3)*cosh(d*x + c)^2 + 3*(5*b^3*cosh(d*x + c)^4 - 10*a*b^2
*cosh(d*x + c)^3 - 12*(2*a^3 + a*b^2)*d*x*cosh(d*x + c) + 4*a^2*b + b^3 +
6*(4*a^2*b + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 24*(a^2*cosh(d*x + c)
^3 + 3*a^2*cosh(d*x + c)^2*sinh(d*x + c) + 3*a^2*cosh(d*x + c)*sinh(d*x +
c)^2 + a^2*sinh(d*x + c)^3)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2
*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c
) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x +
 c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(
b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 3*(2*b^3*cosh(d*x + c)^5 - 5*a*
b^2*cosh(d*x + c)^4 - 12*(2*a^3 + a*b^2)*d*x*cosh(d*x + c)^2 + 4*(4*a^2*b
+ b^3)*cosh(d*x + c)^3 + a*b^2 + 2*(4*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x
 + c))/(b^4*d*cosh(d*x + c)^3 + 3*b^4*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*
b^4*d*cosh(d*x + c)*sinh(d*x + c)^2 + b^4*d*sinh(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**2*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.48

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\sqrt{a^2+b^2}a^2\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{b^4d} - \frac{\left(3\,abe^{(-dx-c)}-b^2-3\,(4\,a^2+b^2)e^{(-2\,dx-2\,c)}\right)e^{(3\,dx+3\,c)}}{24\,b^3d} - \frac{(2\,a^3+ab^2)(dx+c)}{2\,b^4d} + \frac{3\,abe^{(-2\,dx-2\,c)}+b^2e^{(-3\,dx-3\,c)}+3\,(4\,a^2+b^2)e^{(-dx-c)}}{24\,b^3d}$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxi
ma")
```

output
```
sqrt(a^2 + b^2)*a^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x
- c) - a + sqrt(a^2 + b^2)))/(b^4*d) - 1/24*(3*a*b*e^(-d*x - c) - b^2 - 3*
(4*a^2 + b^2)*e^(-2*d*x - 2*c))*e^(3*d*x + 3*c)/(b^3*d) - 1/2*(2*a^3 + a*b
^2)*(d*x + c)/(b^4*d) + 1/24*(3*a*b*e^(-2*d*x - 2*c) + b^2*e^(-3*d*x - 3*c
) + 3*(4*a^2 + b^2)*e^(-d*x - c))/(b^3*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.50

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{12\,(2\,a^3+ab^2)(dx+c)}{b^4} - \frac{b^2e^{(3\,dx+3\,c)}-3\,abe^{(2\,dx+2\,c)}+12\,a^2e^{(dx+c)}+3\,b^2e^{(dx+c)}}{b^3} - \frac{\left(3\,ab^2e^{(dx+c)}+b^3+3\,(4\,a^2b+b^3)e^{(2\,dx+2\,c)}\right)e^{(-3\,dx-}}{b^4}}{24\,d}$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac
")
```

output
```
-1/24*(12*(2*a^3 + a*b^2)*(d*x + c)/b^4 - (b^2*e^(3*d*x + 3*c) - 3*a*b*e^(
2*d*x + 2*c) + 12*a^2*e^(d*x + c) + 3*b^2*e^(d*x + c))/b^3 - (3*a*b^2*e^(d
*x + c) + b^3 + 3*(4*a^2*b + b^3)*e^(2*d*x + 2*c))*e^(-3*d*x - 3*c)/b^4 -
24*(a^4 + a^2*b^2)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(
2*b*e^(d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4))/d
```

Mupad [B] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.97

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{e^{-3c-3dx}}{24bd} - \frac{x(2a^3+ab^2)}{2b^4} + \frac{e^{3c+3dx}}{24bd} + \frac{a\,e^{-2c-2dx}}{8b^2d} - \frac{a\,e^{2c+2dx}}{8b^2d} + \frac{e^{c+dx}(4a^2+b^2)}{8b^3d} + \frac{e^{-c-dx}(4a^2+b^2)}{8b^3d} - \frac{a^2\ln\left(-\frac{2a^2\sqrt{a^2+b^2}\,(b-a\,e^{c+dx})}{b^5} - \frac{2a^2e^{c+dx}(a^2+b^2)}{b^5}\right)\sqrt{a^2+b^2}}{b^4d} + \frac{a^2\ln\left(\frac{2a^2\sqrt{a^2+b^2}\,(b-a\,e^{c+dx})}{b^5} - \frac{2a^2e^{c+dx}(a^2+b^2)}{b^5}\right)\sqrt{a^2+b^2}}{b^4d}$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
exp(- 3*c - 3*d*x)/(24*b*d) - (x*(a*b^2 + 2*a^3))/(2*b^4) + exp(3*c + 3*d*
x)/(24*b*d) + (a*exp(- 2*c - 2*d*x))/(8*b^2*d) - (a*exp(2*c + 2*d*x))/(8*b
^2*d) + (exp(c + d*x)*(4*a^2 + b^2))/(8*b^3*d) + (exp(- c - d*x)*(4*a^2 +
b^2))/(8*b^3*d) - (a^2*log(- (2*a^2*(a^2 + b^2)^(1/2)*(b - a*exp(c + d*x))
)/b^5 - (2*a^2*exp(c + d*x)*(a^2 + b^2))/b^5)*(a^2 + b^2)^(1/2))/(b^4*d) +
 (a^2*log((2*a^2*(a^2 + b^2)^(1/2)*(b - a*exp(c + d*x)))/b^5 - (2*a^2*exp(
c + d*x)*(a^2 + b^2))/b^5)*(a^2 + b^2)^(1/2))/(b^4*d)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.52

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{48e^{3dx+3c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)a^2i+e^{6dx+6c}b^3-3e^{5dx+5c}a\,b^2+12e^{4dx+4c}a^2b+3e^{4dx+4c}b^3-24e^{3dx+3}}{24e^{3dx+3c}b^4d}$$

input
```
int(cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(48*e**(3*c + 3*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(
a**2 + b**2))*a**2*i + e**(6*c + 6*d*x)*b**3 - 3*e**(5*c + 5*d*x)*a*b**2 +
 12*e**(4*c + 4*d*x)*a**2*b + 3*e**(4*c + 4*d*x)*b**3 - 24*e**(3*c + 3*d*x
)*a**3*d*x - 12*e**(3*c + 3*d*x)*a*b**2*d*x + 12*e**(2*c + 2*d*x)*a**2*b +
 3*e**(2*c + 2*d*x)*b**3 + 3*e**(c + d*x)*a*b**2 + b**3)/(24*e**(3*c + 3*d
*x)*b**4*d)
```

3.371 $\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 36

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(cosh(d*x+c)^2*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 10.36 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x
])),x]
```

output
```
Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

$$\Big\downarrow \text{ 6125}$$

$$\int \frac{\sinh^2(c+dx)\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Cosh[c + d*x]^2*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(dx+c)^2\sinh(dx+c)^2}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(cosh(d*x+c)^2*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)^2*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)^2\sinh(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
integral(cosh(d*x + c)^2*sinh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh
(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**2*sinh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 334, normalized size of antiderivative = 9.28

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)^2\sinh(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
2*(a^4*e^c + a^2*b^2*e^c)*integrate(-e^(d*x)/(b^5*f*x + b^5*e - (b^5*f*x*e
^(2*c) + b^5*e*e^(2*c))*e^(2*d*x) - 2*(a*b^4*f*x*e^c + a*b^4*e*e^c)*e^(d*x
)), x) + 1/8*e^(-3*c + 3*d*e/f)*exp_integral_e(1, 3*(f*x + e)*d/f)/(b*f) +
 1/4*a*e^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b^2*f) + 1/4
*a*e^(2*c - 2*d*e/f)*exp_integral_e(1, -2*(f*x + e)*d/f)/(b^2*f) - 1/8*e^(
3*c - 3*d*e/f)*exp_integral_e(1, -3*(f*x + e)*d/f)/(b*f) + 1/8*(4*a^2 + b^
2)*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^3*f) - 1/8*(4*a^2*e^
c + b^2*e^c)*e^(-d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b^3*f) - 1/2*(2
*a^3 + a*b^2)*log(f*x + e)/(b^4*f)
```

Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate(cosh(d*x + c)^2*sinh(d*x + c)^2/((f*x + e)*(b*sinh(d*x + c) + a)
), x)
```

Mupad [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{{\mathrm{cosh}\left(c+d\,x\right)}^2\,{\mathrm{sinh}\left(c+d\,x\right)}^2}{\left(e+f\,x\right)\,\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)}\,dx$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((cosh(c + d*x)^2*sinh(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))), x
)
```

Reduce [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 246, normalized size of antiderivative = 6.83

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \frac{e^{5c}\left(\int \frac{e^{5dx}}{e^{2dx+2c}be+e^{2dx+2c}bfx+2e^{dx+c}ae+2e^{dx+c}afx-be-bfx}dx\right)}{8} - \frac{e^{c}\left(\int \frac{e^{dx}}{e^{2dx+2c}be+e^{2dx+2c}bfx+2e^{dx+c}ae+2e^{dx+c}afx-be-bfx}dx\right)}{4} + \frac{\left(\int \frac{1}{e^{5dx+5c}be+e^{5dx+5c}bfx+2e^{4dx+4c}ae+2e^{4dx+4c}afx-e^{3dx+3c}be-e^{3dx+3c}bfx}dx\right)}{8}$$

input
```
int(cosh(d*x+c)^2*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
(e**(5*c)*int(e**(5*d*x)/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x +
2*e**(c + d*x)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x) - 2*e**c*int(e
**(d*x)/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x + 2*e**(c + d*x)*a*
e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x) + int(1/(e**(5*c + 5*d*x)*b*e +
 e**(5*c + 5*d*x)*b*f*x + 2*e**(4*c + 4*d*x)*a*e + 2*e**(4*c + 4*d*x)*a*f*
x - e**(3*c + 3*d*x)*b*e - e**(3*c + 3*d*x)*b*f*x),x))/8
```

3.372 $\int \frac{(e+fx)^3 \cosh^3(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$

Optimal result

Integrand size = 36, antiderivative size = 1123

$$\int \frac{(e+fx)^3 \cosh^3(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} \, dx = \text{Too large to display}$$

output
```
-6*a^2*(a^2+b^2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/
b^5/d^3-6*a^2*(a^2+b^2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(
1/2)))/b^5/d^3+3*a^2*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2
+b^2)^(1/2)))/b^5/d^2+3*a^2*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/
(a-(a^2+b^2)^(1/2)))/b^5/d^2+40/9*a*f^3*cosh(d*x+c)/b^2/d^4-45/256*f^3*cos
h(d*x+c)*sinh(d*x+c)/b/d^4-45/256*f^3*x/b/d^3+a^2*(a^2+b^2)*(f*x+e)^3*ln(1
+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^5/d+a^2*(a^2+b^2)*(f*x+e)^3*ln(1+b*ex
p(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^5/d+6*a^2*(a^2+b^2)*f^3*polylog(4,-b*exp(d
*x+c)/(a+(a^2+b^2)^(1/2)))/b^5/d^4+6*a^2*(a^2+b^2)*f^3*polylog(4,-b*exp(d*
x+c)/(a-(a^2+b^2)^(1/2)))/b^5/d^4+1/4*a^2*(f*x+e)^3/b^3/d+1/4*(f*x+e)^3*co
sh(d*x+c)^4/b/d+1/2*a^2*(f*x+e)^3*sinh(d*x+c)^2/b^3/d-3/128*f^3*cosh(d*x+c
)^3*sinh(d*x+c)/b/d^4+6*a^3*f^3*cosh(d*x+c)/b^4/d^4+9/32*f^2*(f*x+e)*cosh(
d*x+c)^2/b/d^3+2/27*a*f^3*cosh(d*x+c)^3/b^2/d^4+3/32*f^2*(f*x+e)*cosh(d*x+
c)^4/b/d^3+3/8*a^2*f^3*x/b^3/d^3-1/4*a^2*(a^2+b^2)*(f*x+e)^4/b^5/f+2*a*f*(
f*x+e)^2*cosh(d*x+c)/b^2/d^2-40/9*a*f^2*(f*x+e)*sinh(d*x+c)/b^2/d^3-9/32*f
*(f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/b/d^2-a^3*(f*x+e)^3*sinh(d*x+c)/b^4/d-2
/3*a*(f*x+e)^3*sinh(d*x+c)/b^2/d-3/32*(f*x+e)^3/b/d-3/4*a^2*f*(f*x+e)^2*co
sh(d*x+c)*sinh(d*x+c)/b^3/d^2-2/9*a*f^2*(f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/
b^2/d^3-6*a^3*f^2*(f*x+e)*sinh(d*x+c)/b^4/d^3+3/4*a^2*f^2*(f*x+e)*sinh(d*x
+c)^2/b^3/d^3-3/8*a^2*f^3*cosh(d*x+c)*sinh(d*x+c)/b^3/d^4-1/3*a*(f*x+e)...
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7510 vs. $2(1123) = 2246$.

Time = 27.20 (sec) , antiderivative size = 7510, normalized size of antiderivative = 6.69

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*
x]),x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sinh^2(c+dx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6113}$$

$$\frac{\int (e+fx)^3\cosh^3(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 5970}$$

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\int (e+fx)^2\cosh^4(c+dx)dx}{4d}}{b}-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^4dx}{4d}}{b}$$

$$\Big\downarrow \text{ 3792}$$

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{f^2\int \cosh^4(c+dx)dx}{8d^2}+\frac{3}{4}\int (e+fx)^2\cosh^2(c+dx)dx-\frac{f(e+fx)\cosh^4(c+dx)}{8d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{4d}}{b}-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{f^2\int \sin\left(ic+idx+\frac{\pi}{2}\right)^4dx}{8d^2}+\frac{3}{4}\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx-\frac{f(e+fx)\cosh^4(c+dx)}{8d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{4d}}{b}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{f^2\left(\frac{3}{4}\int\cosh^2(c+dx)dx+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{8d^2}+\frac{3}{4}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx-\frac{f(e+fx)\cosh^4(c+dx)}{8d^2}+\frac{(e+fx)^2\ldots}{\ldots}\right.}{4d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$\downarrow$ 3042

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{f^2\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\int\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx\right)}{8d^2}+\frac{3}{4}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx-\frac{f(e+fx)\cosh^4(c+dx)}{8d^2}+\frac{(e+\ldots}{\ldots}\right.}{4d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$\downarrow$ 3115

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{f^2\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{8d^2}+\frac{3}{4}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx-\frac{f(e+fx)\cosh^4(\ldots}{8d^2}\right.}{4d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$\downarrow$ 24

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx-\frac{f(e+fx)\cosh^4(c+dx)}{8d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\ldots\right.}{8d^2}\right.}{4d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$\downarrow$ 3792

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(\frac{f^2\int\cosh^2(c+dx)dx}{2d^2}+\frac{1}{2}\int(e+fx)^2dx-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}\right)-\frac{f(e+fx)\cosh^4(c+d\ldots}{8d^2}\right.}{4d}}{b}$$

$$\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$\downarrow$ 17

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(\frac{f^2\int\cosh^2(c+dx)dx}{2d^2}-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4(c+dx)}{8d^2}+\frac{f^2}{}\right.}{4d}}{b}$$

$$\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$\downarrow$ 3042

$$-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(\frac{f^2\int\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx}{2d^2}-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4(c+dx)}{8d^2}\right.}{4d}}{b}$$

$\downarrow$ 3115

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(\frac{f^2\left(\frac{\int 1dx}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)}{}\right.}{4d}}{b}$$

$$\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$\downarrow$ 24

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\,\mathrm{co}}{8d}\right.}{4d}}{b}$$

$$\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$\downarrow$ 6113

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\,\mathrm{co}}{8d}\right.}{4d}}{b}$$

$$\frac{a\left(\frac{\int(e+fx)^3\cosh^3(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$\downarrow$ 3042

$$\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\mathrm{co}}{8d}}{4d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)^3\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{b}\right)}{b}$$

$\downarrow$ 3792

$$\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\mathrm{co}}{8d}}{4d}}{b}$$

$$\frac{a\left(\frac{\frac{2f^2\int(e+fx)\cosh^3(c+dx)dx}{3d^2}+\frac{2}{3}\int(e+fx)^3\cosh(c+dx)dx-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right.}{b}$$

$\downarrow$ 3042

$$\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\mathrm{co}}{8d}}{4d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\int(e+fx)^3\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{(e+fx)^3\sinh(c+dx)}{3d}}{b}\right.}{b}$$

$\downarrow$ 3777

$$\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\mathrm{co}}{8d}}{4d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3if\int-i(e+fx)^2\sinh(c+dx)dx}{d}\right)-\frac{f(e+fx)^2\cosh^3(c+d}{3d^2}}{b}\right.}{b}$$

$\downarrow$ 26

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\mathrm{co}}{8d}}{4d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int(e+fx)^2\sinh(c+dx)dx}{d}\right)-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+}{b}\right.}{b}$$

$\downarrow$ 3042

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\mathrm{co}}{8d}}{4d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int-i(e+fx)^2\sin(ic+idx)dx}{d}\right)-\frac{f(e+fx)^2\cosh^3(c+dx}{3d^2}}{b}\right.}{b}$$

$\downarrow$ 26

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\mathrm{co}}{8d}}{4d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\int(e+fx)^2\sin(ic+idx)dx}{d}\right)-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}}{b}\right.}{b}$$

$\downarrow$ 3777

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\mathrm{co}}{8d}}{4d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\cosh(c+dx)dx}{d}\right.}{d}\right.}{b}\right.}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\mathrm{co}\cdots}{8d\cdots}}{4d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\cdots\right.}{d}\cdots\right.}{d}\cdots\right.}{b}\cdots\right.}{b}$$

$$\Big\downarrow \; 3777$$

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\mathrm{co}\cdots}{8d\cdots}}{4d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\cdots\right.}{d}\cdots\right.}{d}\cdots\right.}{b}\cdots\right.}{b}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\,\mathrm{co}\cdots}{8d\cdots}\cdots}{4d}}{b}\cdots$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\cdots\right.}{d}\cdots\right.}{d}\cdots\right.}{b}\cdots\right.}{b}\cdots$$

$\Big\downarrow$ 3042

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\,\mathrm{co}\cdots}{8d\cdots}\cdots}{4d}}{b}\cdots$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\cdots\right.}{d}\cdots\right.}{d}\cdots\right.}{b}\cdots\right.}{b}\cdots$$

$\Big\downarrow$ 26

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\mathrm{co}}{8d}\right.}{4d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\right.}{d}\right.}{d}\right.}{b}\right.}{b}$$

$\downarrow$ 3118

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\mathrm{co}}{8d}\right.}{4d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\right.}{}\right.}{b}\right.}{b}$$

$\downarrow$ 3791

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\mathrm{co}}{8d}}{4d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\left(\frac{2}{3}\int(e+fx)\cosh(c+dx)dx-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left((e\cdots\right.}{}\right.}{b}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^3\cosh(dx+c)^3\sinh(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12603 vs. 2(1051) = 2102.

Time = 0.30 (sec) , antiderivative size = 12603, normalized size of antiderivative = 11.22

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)**3*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)^3\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="maxima")
```

output
```
-1/192*e^3*((8*a*b^2*e^(-d*x - c) - 3*b^3 - 12*(2*a^2*b + b^3)*e^(-2*d*x -
 2*c) + 24*(4*a^3 + 3*a*b^2)*e^(-3*d*x - 3*c))*e^(4*d*x + 4*c)/(b^4*d) - 1
92*(a^4 + a^2*b^2)*(d*x + c)/(b^5*d) - (8*a*b^2*e^(-3*d*x - 3*c) + 3*b^3*e
^(-4*d*x - 4*c) + 24*(4*a^3 + 3*a*b^2)*e^(-d*x - c) + 12*(2*a^2*b + b^3)*e
^(-2*d*x - 2*c))/(b^4*d) - 192*(a^4 + a^2*b^2)*log(-2*a*e^(-d*x - c) + b*e
^(-2*d*x - 2*c) - b)/(b^5*d)) + 1/55296*(13824*(a^4*d^4*f^3*e^(4*c) + a^2*
b^2*d^4*f^3*e^(4*c))*x^4 + 55296*(a^4*d^4*e*f^2*e^(4*c) + a^2*b^2*d^4*e*f^
2*e^(4*c))*x^3 + 82944*(a^4*d^4*e^2*f*e^(4*c) + a^2*b^2*d^4*e^2*f*e^(4*c))
*x^2 + 27*(32*b^4*d^3*f^3*x^3*e^(8*c) + 24*(4*d^3*e*f^2 - d^2*f^3)*b^4*x^2
*e^(8*c) + 12*(8*d^3*e^2*f - 4*d^2*e*f^2 + d*f^3)*b^4*x*e^(8*c) - 3*(8*d^2
*e^2*f - 4*d*e*f^2 + f^3)*b^4*e^(8*c))*e^(4*d*x) - 256*(9*a*b^3*d^3*f^3*x^
3*e^(7*c) + 9*(3*d^3*e*f^2 - d^2*f^3)*a*b^3*x^2*e^(7*c) + 3*(9*d^3*e^2*f -
 6*d^2*e*f^2 + 2*d*f^3)*a*b^3*x*e^(7*c) - (9*d^2*e^2*f - 6*d*e*f^2 + 2*f^3
)*a*b^3*e^(7*c))*e^(3*d*x) - 864*(6*(2*d^2*e^2*f - 2*d*e*f^2 + f^3)*a^2*b^
2*e^(6*c) + 3*(2*d^2*e^2*f - 2*d*e*f^2 + f^3)*b^4*e^(6*c) - 4*(2*a^2*b^2*d
^3*f^3*e^(6*c) + b^4*d^3*f^3*e^(6*c))*x^3 - 6*(2*(2*d^3*e*f^2 - d^2*f^3)*a
^2*b^2*e^(6*c) + (2*d^3*e*f^2 - d^2*f^3)*b^4*e^(6*c))*x^2 - 6*(2*(2*d^3*e^
2*f - 2*d^2*e*f^2 + d*f^3)*a^2*b^2*e^(6*c) + (2*d^3*e^2*f - 2*d^2*e*f^2 +
d*f^3)*b^4*e^(6*c))*x)*e^(2*d*x) + 6912*(12*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3
)*a^3*b*e^(5*c) + 9*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*a*b^3*e^(5*c) - (4*...
```

Giac [F]

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)^3\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)^3*sinh(d*x + c)^2/(b*sinh(d*x + c) + a
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^3\sinh(c+d\,x)^2\,(e+f\,x)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^3*sinh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)), x
)
```

Reduce [F]

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh\left(dx+c\right)^3\sinh\left(dx+c\right)^2}{a+b\sinh\left(dx+c\right)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.373 $\int \frac{(e+fx)^2 \cosh^3(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} \, dx$

Optimal result

Integrand size = 36, antiderivative size = 792

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{a^2(e+fx)^2}{4b^3d} - \frac{3(e+fx)^2}{32bd} - \frac{a^2(a^2+b^2)(e+fx)^3}{3b^5f} + \frac{2a^3f(e+fx)\cosh(c+dx)}{b^4d^2} + \frac{4af(e+fx)\cosh(c+dx)}{3b^2d^2} + \frac{3f^2\cosh^2(c+dx)}{32bd^3} + \frac{2af(e+fx)\cosh^3(c+dx)}{9b^2d^2} + \frac{f^2\cosh^4(c+dx)}{32bd^3} + \frac{(e+fx)^2\cosh^4(c+dx)}{4bd} + \frac{a^2(a^2+b^2)(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d} + \frac{a^2(a^2+b^2)(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d} + \frac{2a^2(a^2+b^2)f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d^2} + \frac{2a^2(a^2+b^2)f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d^2} - \frac{2a^2(a^2+b^2)f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d^3} - \frac{2a^2(a^2+b^2)f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d^3} - \frac{2a^3f^2\sinh(c+dx)}{b^4d^3} - \frac{14af^2\sinh(c+dx)}{9b^2d^3} - \frac{a^3(e+fx)^2\sinh(c+dx)}{b^4d} - \frac{2a(e+fx)^2\sinh(c+dx)}{3b^2d} - \frac{a^2f(e+fx)\cosh(c+dx)\sinh(c+dx)}{2b^3d^2} - \frac{3f(e+fx)\cosh(c+dx)\sinh(c+dx)}{16bd^2} - \frac{a(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{3b^2d} - \frac{f(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{8bd^2} + \frac{a^2f^2\sinh^2(c+dx)}{4b^3d^3} + \frac{a^2(e+fx)^2\sinh^2(c+dx)}{2b^3d} - \frac{2af^2\sinh^3(c+dx)}{27b^2d^3}$$

output
```
2*a^2*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^5
/d^2+2*a^2*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2))
)/b^5/d^2-14/9*a*f^2*sinh(d*x+c)/b^2/d^3+a^2*(a^2+b^2)*(f*x+e)^2*ln(1+b*ex
p(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^5/d+a^2*(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x
+c)/(a-(a^2+b^2)^(1/2)))/b^5/d-2*a^2*(a^2+b^2)*f^2*polylog(3,-b*exp(d*x+c)
/(a+(a^2+b^2)^(1/2)))/b^5/d^3-2*a^2*(a^2+b^2)*f^2*polylog(3,-b*exp(d*x+c)/
(a-(a^2+b^2)^(1/2)))/b^5/d^3+1/4*a^2*(f*x+e)^2/b^3/d+3/32*f^2*cosh(d*x+c)^
2/b/d^3+1/32*f^2*cosh(d*x+c)^4/b/d^3+1/4*(f*x+e)^2*cosh(d*x+c)^4/b/d-2*a^3
*f^2*sinh(d*x+c)/b^4/d^3+1/4*a^2*f^2*sinh(d*x+c)^2/b^3/d^3+1/2*a^2*(f*x+e)
^2*sinh(d*x+c)^2/b^3/d-2/27*a*f^2*sinh(d*x+c)^3/b^2/d^3-1/3*a^2*(a^2+b^2)*
(f*x+e)^3/b^5/f+4/3*a*f*(f*x+e)*cosh(d*x+c)/b^2/d^2-3/16*f*(f*x+e)*cosh(d*
x+c)*sinh(d*x+c)/b/d^2-a^3*(f*x+e)^2*sinh(d*x+c)/b^4/d-2/3*a*(f*x+e)^2*sin
h(d*x+c)/b^2/d-3/32*(f*x+e)^2/b/d+2*a^3*f*(f*x+e)*cosh(d*x+c)/b^4/d^2+2/9*
a*f*(f*x+e)*cosh(d*x+c)^3/b^2/d^2-1/3*a*(f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c
)/b^2/d-1/8*f*(f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)/b/d^2-1/2*a^2*f*(f*x+e)*co
sh(d*x+c)*sinh(d*x+c)/b^3/d^2
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 5113 vs. $2(792) = 1584$.

Time = 15.35 (sec) , antiderivative size = 5113, normalized size of antiderivative = 6.46

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*
x]),x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sinh^2(c+dx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6113}$$

$$\frac{\int (e+fx)^2\cosh^3(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 5970}$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\int (e+fx)\cosh^4(c+dx)dx}{2d}}{b}-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^4dx}{2d}}{b}$$

$$\Big\downarrow \text{ 3791}$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\int (e+fx)\cosh^2(c+dx)dx-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}$$

$$\Big\downarrow \text{ 3791}$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(\frac{1}{2}\int (e+fx)dx-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \ 17$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \ 6113$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(\frac{\int(e+fx)^2\cosh^3(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{b}\right)}{b}$$

$$\Big\downarrow \ 3792$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(\frac{\frac{2f^2\int\cosh^3(c+dx)dx}{9d^2}+\frac{2}{3}\int(e+fx)^2\cosh(c+dx)dx-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{9d^2}+\frac{2}{3}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^2(c}{3d}}{b}\right.}{b}$$

$\Big\downarrow$ 3113

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2if^2\int\left(\sinh^2(c+dx)+1\right)d(-i\sinh(c+dx))}{9d^3}+\frac{2}{3}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh}{}}{b}\right.}{b}$$

$\Big\downarrow$ 2009

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c-}{}}{b}\right.}{b}$$

$\Big\downarrow$ 3777

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int-i(e+fx)\sinh(c+dx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(}{9d^2}}{b}\right.}{b}$$

$\Big\downarrow$ 26

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int(e+fx)\sinh(c+dx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+d}{9d^2}}{b}\right.}{b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int-i(e+fx)\sin(ic+idx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c}{9d^2}}{b}\right.}{b}$$

$\Big\downarrow$ 26

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int(e+fx)\sin(ic+idx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+}{9d^2}}{b}\right.}{b}$$

$\Big\downarrow$ 3777

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}}{b}\right.}{b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+}{9d^3}}{b}\right.}{b}$$

$\Big\downarrow$ 3117

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}\right.}{d}\right.}{b}\right.}{b}$$

$$\Big\downarrow \ 6099$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-$$

$$\frac{a\left(-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)^2\cosh(c+dx)dx}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}+\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}}{}\right.}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-$$

$$\frac{a\left(\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+\frac{(e+fx)^2\sinh(}{}}{b}\right.}{b}$$

$$\Big\downarrow \ 3777$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-$$

$$\frac{a\left(\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+\frac{(e+fx)^2\sinh(}{}}{b}\right.}{}$$

$\big\downarrow$ 26

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-$$

$$a\left(-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int(e+fx)\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}+\frac{2if^2\left(-\frac{1}{3}i\,\mathrm{s}\right.}{}\right.$$

$\big\downarrow$ 3042

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-$$

$$a\left(\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+\frac{(e+fx)^2\sinh(}{}}{b}\right.$$

$\big\downarrow$ 26

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-$$

$$a\left(\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+\frac{(e+fx)^2\sinh(}{}}{b}\right.$$

$\big\downarrow$ 3777

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-$$

$$a\left(\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+\frac{(e+fx)^2\sinh}{}}{b}\right.$$

$\downarrow$ 3042

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-$$

$$a\left(\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+\frac{(e+fx)^2\sinh}{}}{b}\right.$$

$\downarrow$ 3117

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-$$

$$a\left(\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+\frac{(e+fx)^2\sinh(}{}}{b}\right.$$

$\downarrow$ 5969

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-$$

$$a\left(\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+\frac{(e+fx)^2\sinh(}{}}{b}\right.$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^2 \cosh (dx+c)^3 \sinh (dx+c)^2}{a+b\sinh (dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7645 vs. $2(738) = 1476$.

Time = 0.22 (sec) , antiderivative size = 7645, normalized size of antiderivative = 9.65

$$\int \frac{(e+fx)^2 \cosh^3(c+dx) \sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \cosh^3(c+dx) \sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)**3*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)^3\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="maxima")
```

output
```
-1/192*e^2*((8*a*b^2*e^(-d*x - c) - 3*b^3 - 12*(2*a^2*b + b^3)*e^(-2*d*x -
 2*c) + 24*(4*a^3 + 3*a*b^2)*e^(-3*d*x - 3*c))*e^(4*d*x + 4*c)/(b^4*d) - 1
92*(a^4 + a^2*b^2)*(d*x + c)/(b^5*d) - (8*a*b^2*e^(-3*d*x - 3*c) + 3*b^3*e
^(-4*d*x - 4*c) + 24*(4*a^3 + 3*a*b^2)*e^(-d*x - c) + 12*(2*a^2*b + b^3)*e
^(-2*d*x - 2*c))/(b^4*d) - 192*(a^4 + a^2*b^2)*log(-2*a*e^(-d*x - c) + b*e
^(-2*d*x - 2*c) - b)/(b^5*d)) + 1/13824*(4608*(a^4*d^3*f^2*e^(4*c) + a^2*b
^2*d^3*f^2*e^(4*c))*x^3 + 13824*(a^4*d^3*e*f*e^(4*c) + a^2*b^2*d^3*e*f*e^(
4*c))*x^2 + 27*(8*b^4*d^2*f^2*x^2*e^(8*c) + 4*(4*d^2*e*f - d*f^2)*b^4*x*e^
(8*c) - (4*d*e*f - f^2)*b^4*e^(8*c))*e^(4*d*x) - 64*(9*a*b^3*d^2*f^2*x^2*e
^(7*c) + 6*(3*d^2*e*f - d*f^2)*a*b^3*x*e^(7*c) - 2*(3*d*e*f - f^2)*a*b^3*e
^(7*c))*e^(3*d*x) - 432*(2*(2*d*e*f - f^2)*a^2*b^2*e^(6*c) + (2*d*e*f - f^
2)*b^4*e^(6*c) - 2*(2*a^2*b^2*d^2*f^2*e^(6*c) + b^4*d^2*f^2*e^(6*c))*x^2 -
 2*(2*(2*d^2*e*f - d*f^2)*a^2*b^2*e^(6*c) + (2*d^2*e*f - d*f^2)*b^4*e^(6*c
))*x)*e^(2*d*x) + 1728*(8*(d*e*f - f^2)*a^3*b*e^(5*c) + 6*(d*e*f - f^2)*a*
b^3*e^(5*c) - (4*a^3*b*d^2*f^2*e^(5*c) + 3*a*b^3*d^2*f^2*e^(5*c))*x^2 - 2*
(4*(d^2*e*f - d*f^2)*a^3*b*e^(5*c) + 3*(d^2*e*f - d*f^2)*a*b^3*e^(5*c))*x)
*e^(d*x) + 1728*(8*(d*e*f + f^2)*a^3*b*e^(3*c) + 6*(d*e*f + f^2)*a*b^3*e^(
3*c) + (4*a^3*b*d^2*f^2*e^(3*c) + 3*a*b^3*d^2*f^2*e^(3*c))*x^2 + 2*(4*(d^2
*e*f + d*f^2)*a^3*b*e^(3*c) + 3*(d^2*e*f + d*f^2)*a*b^3*e^(3*c))*x)*e^(-d*
x) + 432*(2*(2*d*e*f + f^2)*a^2*b^2*e^(2*c) + (2*d*e*f + f^2)*b^4*e^(2*...
```

Giac [F]

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)^3\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)^3*sinh(d*x + c)^2/(b*sinh(d*x + c) + a
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^3\sinh(c+d\,x)^2\,(e+f\,x)^2}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^3*sinh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)), x
)
```

Reduce [F]

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\int \frac{(fx+e)^2\cosh{(dx+c)}^3\sinh{(dx+c)}^2}{a+b\sinh{(dx+c)}}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.374 $\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 499

$$
\begin{aligned}
\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx
&= \frac{a^2fx}{4b^3d}-\frac{3fx}{32bd}-\frac{a^2(a^2+b^2)(e+fx)^2}{2b^5f}+\frac{a^3f\cosh(c+dx)}{b^4d^2} \\
&+\frac{2af\cosh(c+dx)}{3b^2d^2}+\frac{af\cosh^3(c+dx)}{9b^2d^2}+\frac{(e+fx)\cosh^4(c+dx)}{4bd} \\
&+\frac{a^2(a^2+b^2)(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d} \\
&+\frac{a^2(a^2+b^2)(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d}+\frac{a^2(a^2+b^2)f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d^2} \\
&+\frac{a^2(a^2+b^2)f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d^2}-\frac{a^3(e+fx)\sinh(c+dx)}{b^4d} \\
&-\frac{2a(e+fx)\sinh(c+dx)}{3b^2d}-\frac{a^2f\cosh(c+dx)\sinh(c+dx)}{4b^3d^2} \\
&-\frac{3f\cosh(c+dx)\sinh(c+dx)}{32bd^2}-\frac{a(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{3b^2d} \\
&-\frac{f\cosh^3(c+dx)\sinh(c+dx)}{16bd^2}+\frac{a^2(e+fx)\sinh^2(c+dx)}{2b^3d}
\end{aligned}
$$

output
```
1/4*a^2*f*x/b^3/d-3/32*f*x/b/d-1/2*a^2*(a^2+b^2)*(f*x+e)^2/b^5/f+a^3*f*cos
h(d*x+c)/b^4/d^2+2/3*a*f*cosh(d*x+c)/b^2/d^2+1/9*a*f*cosh(d*x+c)^3/b^2/d^2
+1/4*(f*x+e)*cosh(d*x+c)^4/b/d+a^2*(a^2+b^2)*(f*x+e)*ln(1+b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))/b^5/d+a^2*(a^2+b^2)*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/b^5/d+a^2*(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2
)))/b^5/d^2+a^2*(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b
^5/d^2-a^3*(f*x+e)*sinh(d*x+c)/b^4/d-2/3*a*(f*x+e)*sinh(d*x+c)/b^2/d-1/4*a
^2*f*cosh(d*x+c)*sinh(d*x+c)/b^3/d^2-3/32*f*cosh(d*x+c)*sinh(d*x+c)/b/d^2-
1/3*a*(f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/b^2/d-1/16*f*cosh(d*x+c)^3*sinh(d*
x+c)/b/d^2+1/2*a^2*(f*x+e)*sinh(d*x+c)^2/b^3/d
```

Mathematica [A] (warning: unable to verify)

Time = 1.60 (sec) , antiderivative size = 904, normalized size of antiderivative = 1.81

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-144b^4de\log(a+b\sinh(c+dx))+72b^4f\left(dx\left(dx-2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)-2\,\mathrm{I}\right.}{}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
(-144*b^4*d*e*Log[a + b*Sinh[c + d*x]] + 72*b^4*f*(d*x*(d*x - 2*Log[1 + (b
*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2])]) - 2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*
PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) + 72*b^2*d*e*((4*a^2
 + b^2)*Log[a + b*Sinh[c + d*x]] - 4*a*b*Sinh[c + d*x] + 2*b^2*Sinh[c + d*
x]^2) + 24*d*e*(3*(16*a^4 + 12*a^2*b^2 + b^4)*Log[a + b*Sinh[c + d*x]] - 1
2*a*b*(4*a^2 + 3*b^2)*Sinh[c + d*x] + 6*b^2*(4*a^2 + 3*b^2)*Sinh[c + d*x]^
2 - 16*a*b^3*Sinh[c + d*x]^3 + 12*b^4*Sinh[c + d*x]^4) + 36*b^2*f*(8*a*b*C
osh[c + d*x] + 2*b^2*d*x*Cosh[2*(c + d*x)] + (4*a^2 + b^2)*(2*c*(c + d*x)
- (c + d*x)^2 + 2*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])]
 + 2*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*Log[b
- 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*PolyLog[2, (b*E^(c + d*x))/(-a
+ Sqrt[a^2 + b^2])] + 2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
)]) - 8*a*b*d*x*Sinh[c + d*x] - b^2*Sinh[2*(c + d*x)]) + f*(576*a*b*(2*a^2
 + b^2)*Cosh[c + d*x] + 72*b^2*(4*a^2 + b^2)*d*x*Cosh[2*(c + d*x)] + 32*a*
b^3*Cosh[3*(c + d*x)] + 36*b^4*d*x*Cosh[4*(c + d*x)] + 36*(16*a^4 + 12*a^2
*b^2 + b^4)*(2*c*(c + d*x) - (c + d*x)^2 + 2*(c + d*x)*Log[1 + (b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2])] + 2*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2])] - 2*c*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*Poly
Log[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*PolyLog[2, -((b*E^(c...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh^2(c+dx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6113}$$

$$\frac{\int (e+fx)\cosh^3(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 5970}$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\int \cosh^4(c+dx)dx}{4d}}{b}-\frac{a\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\int \sin\left(ic+idx+\frac{\pi}{2}\right)^4dx}{4d}}{b}$$

$$\Big\downarrow \ 3115$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\int \cosh^2(c+dx)dx+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{4d}}{b}-\frac{a\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\int \sin\left(ic+idx+\frac{\pi}{2}\right)^2dx\right)}{4d}}{b}$$

$$\Big\downarrow \ 3115$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{4d}}{b}-\frac{a\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \ 24$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \ 6113$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\int (e+fx)\cosh^3(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{b}\right)}{b}$$

$\Big\downarrow$ 3791

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\int(e+fx)\cosh(c+dx)dx-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}\right)}{b}$$

$\Big\downarrow$ 3777

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int-i\sinh(c+dx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}\right)}{b}$$

$\Big\downarrow$ 26

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int-i\sin(ic+idx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}\right)}{b}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int\sin(ic+idx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}\right)}{b}$$

$$\Big\downarrow \; 3118$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 6099$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)\cosh(c+ \cdots}{b^2} \cdots\right.}{b} \cdots\right.}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)\sin\left(ic+ \cdots\right.}{b^2} \cdots\right.}{b} \cdots\right.}{b}$$

$$\Big\downarrow \ 3777$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+\cdots}{d}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{b}\cdots$$

$$\Big\downarrow \ 26$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+\cdots}{d}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{b}\cdots$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+\cdots}{d}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{b}\cdots$$

$$\Big\downarrow \ 26$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+\cdots}{d}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{b}\cdots$$

$$\Big\downarrow \ 3118$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-$$

$$\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int(e+fx)\cosh(c+d}{b}\right.}{}\right.}{b}$$

$\downarrow$ 5969

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-$$

$$\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{(e+fx)\sinh^2(c+d}{2d}\right.}{}\right.}{b}$$

$\downarrow$ 3042

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-$$

$$\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{(e+fx)\sinh^2(c+d}{2d}\right.}{}\right.}{b}$$

$\downarrow$ 25

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-$$

$$\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{(e+fx)\sinh^2(c+d}{2d}\right.}{}\right.}{b}$$

$\downarrow$ 3115

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-$$

$$\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{f\left(\frac{\int 1dx}{2}-\sinh(c-\right.}{2\ldots}\right.}{\ldots}\right.}{b\ldots}$$

$$\Big\downarrow 24$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-$$

$$\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+\ldots}{d}\right.}{\ldots}\right.}{\ldots}\right.}{b\ldots}$$

$$\Big\downarrow 6095$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-$$

$$\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\left(\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+\ldots}{a+be^{c+dx}+\sqrt{\ldots}}\right.}{b^2}\right.}{\ldots}\right.}{b\ldots}$$

$$\Big\downarrow 2620$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-$$

$$a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\left(-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int\log\left(\frac{e}{a+}\right.}{}\right.}{}\right.}{}\right.$$

input
```
Int[((e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1216 vs. $2(463) = 926$.

Time = 56.19 (sec) , antiderivative size = 1217, normalized size of antiderivative = 2.44

method	result	size
risch	Expression too large to display	1217

input
```
int((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURN
VERBOSE)
```

output
```
1/d^2/b^3*a^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c
+2/d^2/b^3*c*a^2*f*ln(exp(d*x+c))+2/d^2*a^4/b^5*c*f*ln(exp(d*x+c))-1/d^2*a
^4/b^5*c*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/d*a^4/b^5*f*ln((-b*exp(
d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d*a^4/b^5*f*ln((b*exp(
d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2*a^4/b^5*f*ln((-b*ex
p(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d^2*a^4/b^5*f*ln((b*
exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d^2/b^3*a^2*f*c^2-2
/d/b^3*a^2*e*ln(exp(d*x+c))+1/d/b^3*a^2*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+
c)-b)+1/d^2/b^3*a^2*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2
)^(1/2)))+1/d^2/b^3*a^2*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b
^2)^(1/2)))-2/d/b^3*a^2*f*c*x+1/d/b^3*a^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1
/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d/b^3*a^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1
/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2/b^3*a^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^
(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/8*a*(4*a^2*d*f*x+3*b^2*d*f*x+4*a^2*d*e+
3*b^2*d*e-4*a^2*f-3*b^2*f)/b^4/d^2*exp(d*x+c)-1/d^2/b^3*c*a^2*f*ln(b*exp(2
*d*x+2*c)+2*a*exp(d*x+c)-b)+1/8*a*(4*a^2+3*b^2)*(d*f*x+d*e+f)/b^4/d^2*exp(
-d*x-c)+1/256*(4*d*f*x+4*d*e-f)/b/d^2*exp(4*d*x+4*c)+1/32*(4*a^2*d*f*x+2*b
^2*d*f*x+4*a^2*d*e+2*b^2*d*e-2*a^2*f-b^2*f)/b^3/d^2*exp(2*d*x+2*c)+1/256*(
4*d*f*x+4*d*e+f)/b/d^2*exp(-4*d*x-4*c)-1/72*a*(3*d*f*x+3*d*e-f)/b^2/d^2*ex
p(3*d*x+3*c)+1/32*(2*a^2+b^2)*(2*d*f*x+2*d*e+f)/b^3/d^2*exp(-2*d*x-2*c)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3795 vs. 2(461) = 922.

Time = 0.15 (sec) , antiderivative size = 3795, normalized size of antiderivative = 7.61

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)**3*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)^3\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/192*e*((8*a*b^2*e^(-d*x - c) - 3*b^3 - 12*(2*a^2*b + b^3)*e^(-2*d*x - 2
*c) + 24*(4*a^3 + 3*a*b^2)*e^(-3*d*x - 3*c))*e^(4*d*x + 4*c)/(b^4*d) - 192
*(a^4 + a^2*b^2)*(d*x + c)/(b^5*d) - (8*a*b^2*e^(-3*d*x - 3*c) + 3*b^3*e^(
-4*d*x - 4*c) + 24*(4*a^3 + 3*a*b^2)*e^(-d*x - c) + 12*(2*a^2*b + b^3)*e^(
-2*d*x - 2*c))/(b^4*d) - 192*(a^4 + a^2*b^2)*log(-2*a*e^(-d*x - c) + b*e^(
-2*d*x - 2*c) - b)/(b^5*d)) + 1/2304*f*((1152*(a^4*d^2*e^(4*c) + a^2*b^2*d
^2*e^(4*c))*x^2 + 9*(4*b^4*d*x*e^(8*c) - b^4*e^(8*c))*e^(4*d*x) - 32*(3*a*
b^3*d*x*e^(7*c) - a*b^3*e^(7*c))*e^(3*d*x) - 72*(2*a^2*b^2*e^(6*c) + b^4*e
^(6*c) - 2*(2*a^2*b^2*d*e^(6*c) + b^4*d*e^(6*c))*x)*e^(2*d*x) + 288*(4*a^3
*b*e^(5*c) + 3*a*b^3*e^(5*c) - (4*a^3*b*d*e^(5*c) + 3*a*b^3*d*e^(5*c))*x)*
e^(d*x) + 288*(4*a^3*b*e^(3*c) + 3*a*b^3*e^(3*c) + (4*a^3*b*d*e^(3*c) + 3*
a*b^3*d*e^(3*c))*x)*e^(-d*x) + 72*(2*a^2*b^2*e^(2*c) + b^4*e^(2*c) + 2*(2*
a^2*b^2*d*e^(2*c) + b^4*d*e^(2*c))*x)*e^(-2*d*x) + 32*(3*a*b^3*d*x*e^c + a
*b^3*e^c)*e^(-3*d*x) + 9*(4*b^4*d*x + b^4)*e^(-4*d*x))*e^(-4*c)/(b^5*d^2)
- 72*integrate(64*((a^5*e^c + a^3*b^2*e^c)*x*e^(d*x) - (a^4*b + a^2*b^3)*x
)/(b^6*e^(2*d*x + 2*c) + 2*a*b^5*e^(d*x + c) - b^6), x))
```

Giac [F]

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)\cosh(dx+c)^3\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)^3*sinh(d*x + c)^2/(b*sinh(d*x + c) + a),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^3\sinh(c+d\,x)^2\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^3*sinh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output

```
(36*e**(8*c + 8*d*x)*b**8*d*e + 36*e**(8*c + 8*d*x)*b**8*d*f*x - 9*e**(8*c
 + 8*d*x)*b**8*f - 96*e**(7*c + 7*d*x)*a*b**7*d*e - 96*e**(7*c + 7*d*x)*a*
b**7*d*f*x + 32*e**(7*c + 7*d*x)*a*b**7*f + 288*e**(6*c + 6*d*x)*a**2*b**6
*d*e + 288*e**(6*c + 6*d*x)*a**2*b**6*d*f*x - 144*e**(6*c + 6*d*x)*a**2*b*
*6*f + 144*e**(6*c + 6*d*x)*b**8*d*e + 144*e**(6*c + 6*d*x)*b**8*d*f*x - 7
2*e**(6*c + 6*d*x)*b**8*f - 1152*e**(5*c + 5*d*x)*a**3*b**5*d*e - 1152*e**
(5*c + 5*d*x)*a**3*b**5*d*f*x + 1152*e**(5*c + 5*d*x)*a**3*b**5*f - 864*e*
*(5*c + 5*d*x)*a*b**7*d*e - 864*e**(5*c + 5*d*x)*a*b**7*d*f*x + 864*e**(5*
c + 5*d*x)*a*b**7*f + 36864*e**(4*c + 4*d*x)*int(x/(e**(6*c + 6*d*x)*b + 2
*e**(5*c + 5*d*x)*a - e**(4*c + 4*d*x)*b),x)*a**8*b*d**2*f + 73728*e**(4*c
 + 4*d*x)*int(x/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - e**(4*c + 4*d
*x)*b),x)*a**6*b**3*d**2*f + 41472*e**(4*c + 4*d*x)*int(x/(e**(6*c + 6*d*x
)*b + 2*e**(5*c + 5*d*x)*a - e**(4*c + 4*d*x)*b),x)*a**4*b**5*d**2*f + 460
8*e**(4*c + 4*d*x)*int(x/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - e**(
4*c + 4*d*x)*b),x)*a**2*b**7*d**2*f + 2304*e**(4*c + 4*d*x)*log(e**(2*c +
2*d*x)*b + 2*e**(c + d*x)*a - b)*a**4*b**4*d*e + 2304*e**(4*c + 4*d*x)*log
(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*a**2*b**6*d*e - 2304*e**(4*c +
 4*d*x)*a**4*b**4*d**2*e*x + 1152*e**(4*c + 4*d*x)*a**4*b**4*d**2*f*x**2 -
 2304*e**(4*c + 4*d*x)*a**2*b**6*d**2*e*x + 1152*e**(4*c + 4*d*x)*a**2*b**
6*d**2*f*x**2 - 73728*e**(c + 4*d*x)*int(x/(e**(2*c + 5*d*x)*b + 2*e**(...
```

3.375 $\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 113

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{a^2(a^2+b^2)\log(a+b\sinh(c+dx))}{b^5d} - \frac{a(a^2+b^2)\sinh(c+dx)}{b^4d} + \frac{(a^2+b^2)\sinh^2(c+dx)}{2b^3d} - \frac{a\sinh^3(c+dx)}{3b^2d} + \frac{\sinh^4(c+dx)}{4bd}$$

output
```
a^2*(a^2+b^2)*ln(a+b*sinh(d*x+c))/b^5/d-a*(a^2+b^2)*sinh(d*x+c)/b^4/d+1/2*
(a^2+b^2)*sinh(d*x+c)^2/b^3/d-1/3*a*sinh(d*x+c)^3/b^2/d+1/4*sinh(d*x+c)^4/
b/d
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.87

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{12a^2(a^2+b^2)\log(a+b\sinh(c+dx)) - 12ab(a^2+b^2)\sinh(c+dx) + 6b^2(a^2+b^2)\sinh^2(c+dx) - 4ab^3\sinh\ldots}{12b^5d}$$

input
```
Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(12*a^2*(a^2 + b^2)*Log[a + b*Sinh[c + d*x]] - 12*a*b*(a^2 + b^2)*Sinh[c +
 d*x] + 6*b^2*(a^2 + b^2)*Sinh[c + d*x]^2 - 4*a*b^3*Sinh[c + d*x]^3 + 3*b^
4*Sinh[c + d*x]^4)/(12*b^5*d)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3042, 25, 3316, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ic+idx)^2\cos(ic+idx)^3}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos(ic+idx)^3\sin(ic+idx)^2}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3316}$$

$$\frac{\int \frac{\sinh^2(c+dx)\left(\sinh^2(c+dx)b^2+b^2\right)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b^2\sinh^2(c+dx)\left(\sinh^2(c+dx)b^2+b^2\right)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^5d}$$

$$\Big\downarrow \text{522}$$

$$\frac{\int \left(b^3\sinh^3(c+dx)-ab^2\sinh^2(c+dx)+b\left(a^2+b^2\right)\sinh(c+dx)-a\left(a^2+b^2\right)+\frac{a^2\left(a^2+b^2\right)}{a+b\sinh(c+dx)}\right)d(b\sinh(c+dx))}{b^5d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}b^2(a^2+b^2)\sinh^2(c+dx)-ab(a^2+b^2)\sinh(c+dx)+a^2(a^2+b^2)\log(a+b\sinh(c+dx))-\frac{1}{3}ab^3\sinh^3(c+dx}{b^5d}$$

input
```
Int[(Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(a^2*(a^2 + b^2)*Log[a + b*Sinh[c + d*x]] - a*b*(a^2 + b^2)*Sinh[c + d*x]
+ (b^2*(a^2 + b^2)*Sinh[c + d*x]^2)/2 - (a*b^3*Sinh[c + d*x]^3)/3 + (b^4*S
inh[c + d*x]^4)/4)/(b^5*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 24.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{-\frac{-\frac{\sinh(dx+c)^4 b^3}{4}+\frac{a\sinh(dx+c)^3 b^2}{3}-\frac{(a^2+b^2)\sinh(dx+c)^2 b}{2}+a(a^2+b^2)\sinh(dx+c)}{b^4}+\frac{a^2(a^2+b^2)\ln(a+b\sinh(dx+c))}{b^5}}{d}$
default	$\frac{-\frac{-\frac{\sinh(dx+c)^4 b^3}{4}+\frac{a\sinh(dx+c)^3 b^2}{3}-\frac{(a^2+b^2)\sinh(dx+c)^2 b}{2}+a(a^2+b^2)\sinh(dx+c)}{b^4}+\frac{a^2(a^2+b^2)\ln(a+b\sinh(dx+c))}{b^5}}{d}$
risch	$-\frac{a^4x}{b^5}-\frac{x\,a^2}{b^3}+\frac{e^{4dx+4c}}{64bd}-\frac{a\,e^{3dx+3c}}{24b^2d}+\frac{e^{2dx+2c}a^2}{8b^3d}+\frac{e^{2dx+2c}}{16bd}-\frac{a^3e^{dx+c}}{2b^4d}-\frac{3a\,e^{dx+c}}{8b^2d}+\frac{a^3e^{-dx-c}}{2b^4d}+3$

input
```
int(cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^4*(-1/4*sinh(d*x+c)^4*b^3+1/3*a*sinh(d*x+c)^3*b^2-1/2*(a^2+b^2)*
sinh(d*x+c)^2*b+a*(a^2+b^2)*sinh(d*x+c))+a^2*(a^2+b^2)/b^5*ln(a+b*sinh(d*x
+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1069 vs. $2(107) = 214$.

Time = 0.11 (sec) , antiderivative size = 1069, normalized size of antiderivative = 9.46

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fric
as")
```

output
```
1/192*(3*b^4*cosh(d*x + c)^8 + 3*b^4*sinh(d*x + c)^8 - 8*a*b^3*cosh(d*x +
c)^7 + 8*(3*b^4*cosh(d*x + c) - a*b^3)*sinh(d*x + c)^7 - 192*(a^4 + a^2*b^
2)*d*x*cosh(d*x + c)^4 + 12*(2*a^2*b^2 + b^4)*cosh(d*x + c)^6 + 4*(21*b^4*
cosh(d*x + c)^2 - 14*a*b^3*cosh(d*x + c) + 6*a^2*b^2 + 3*b^4)*sinh(d*x + c
)^6 - 24*(4*a^3*b + 3*a*b^3)*cosh(d*x + c)^5 + 24*(7*b^4*cosh(d*x + c)^3 -
 7*a*b^3*cosh(d*x + c)^2 - 4*a^3*b - 3*a*b^3 + 3*(2*a^2*b^2 + b^4)*cosh(d*
x + c))*sinh(d*x + c)^5 + 8*a*b^3*cosh(d*x + c) + 2*(105*b^4*cosh(d*x + c)
^4 - 140*a*b^3*cosh(d*x + c)^3 - 96*(a^4 + a^2*b^2)*d*x + 90*(2*a^2*b^2 +
b^4)*cosh(d*x + c)^2 - 60*(4*a^3*b + 3*a*b^3)*cosh(d*x + c))*sinh(d*x + c)
^4 + 3*b^4 + 24*(4*a^3*b + 3*a*b^3)*cosh(d*x + c)^3 + 8*(21*b^4*cosh(d*x +
 c)^5 - 35*a*b^3*cosh(d*x + c)^4 + 12*a^3*b + 9*a*b^3 - 96*(a^4 + a^2*b^2)
*d*x*cosh(d*x + c) + 30*(2*a^2*b^2 + b^4)*cosh(d*x + c)^3 - 30*(4*a^3*b +
3*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 12*(2*a^2*b^2 + b^4)*cosh(d*x
+ c)^2 + 12*(7*b^4*cosh(d*x + c)^6 - 14*a*b^3*cosh(d*x + c)^5 - 96*(a^4 +
a^2*b^2)*d*x*cosh(d*x + c)^2 + 15*(2*a^2*b^2 + b^4)*cosh(d*x + c)^4 + 2*a^
2*b^2 + b^4 - 20*(4*a^3*b + 3*a*b^3)*cosh(d*x + c)^3 + 6*(4*a^3*b + 3*a*b^
3)*cosh(d*x + c))*sinh(d*x + c)^2 + 192*((a^4 + a^2*b^2)*cosh(d*x + c)^4 +
 4*(a^4 + a^2*b^2)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^4 + a^2*b^2)*cosh(
d*x + c)^2*sinh(d*x + c)^2 + 4*(a^4 + a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)
^3 + (a^4 + a^2*b^2)*sinh(d*x + c)^4)*log(2*(b*sinh(d*x + c) + a)/(cosh...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**3*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(107) = 214$.

Time = 0.05 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.07

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\left(8\,ab^2e^{(-dx-c)} - 3\,b^3 - 12\,(2\,a^2b+b^3)e^{(-2\,dx-2\,c)} + 24\,(4\,a^3+3\,ab^2)e^{(-3\,dx-3\,c)}\right)e^{(4\,dx+4\,c)}}{192\,b^4d} + \frac{(a^4+a^2b^2)(dx+c)}{b^5d} + \frac{8\,ab^2e^{(-3\,dx-3\,c)} + 3\,b^3e^{(-4\,dx-4\,c)} + 24\,(4\,a^3+3\,ab^2)e^{(-dx-c)} + 12\,(2\,a^2b+b^3)e^{(-2\,dx-2\,c)}}{192\,b^4d} + \frac{(a^4+a^2b^2)\log\left(-2\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)} - b\right)}{b^5d}$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxi
ma")
```

output
```
-1/192*(8*a*b^2*e^(-d*x - c) - 3*b^3 - 12*(2*a^2*b + b^3)*e^(-2*d*x - 2*c)
 + 24*(4*a^3 + 3*a*b^2)*e^(-3*d*x - 3*c))*e^(4*d*x + 4*c)/(b^4*d) + (a^4 +
 a^2*b^2)*(d*x + c)/(b^5*d) + 1/192*(8*a*b^2*e^(-3*d*x - 3*c) + 3*b^3*e^(-
4*d*x - 4*c) + 24*(4*a^3 + 3*a*b^2)*e^(-d*x - c) + 12*(2*a^2*b + b^3)*e^(-
2*d*x - 2*c))/(b^4*d) + (a^4 + a^2*b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*
x - 2*c) - b)/(b^5*d)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.79

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{3\,b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^4 - 8\,ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3 + 24\,a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2 + 24\,b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2 - 96\,a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right) - \cdots}{b^4}}{192\,d}$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac
")
```

output
```
1/192*((3*b^3*(e^(d*x + c) - e^(-d*x - c))^4 - 8*a*b^2*(e^(d*x + c) - e^(-
d*x - c))^3 + 24*a^2*b*(e^(d*x + c) - e^(-d*x - c))^2 + 24*b^3*(e^(d*x + c
) - e^(-d*x - c))^2 - 96*a^3*(e^(d*x + c) - e^(-d*x - c)) - 96*a*b^2*(e^(d
*x + c) - e^(-d*x - c)))/b^4 + 192*(a^4 + a^2*b^2)*log(abs(b*(e^(d*x + c)
- e^(-d*x - c)) + 2*a))/b^5)/d
```

Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.11

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{e^{-4c-4dx}}{64\,b\,d} - \frac{x\,(a^4+a^2\,b^2)}{b^5} + \frac{e^{4c+4dx}}{64\,b\,d} + \frac{a\,e^{-3c-3dx}}{24\,b^2\,d} - \frac{a\,e^{3c+3dx}}{24\,b^2\,d} + \frac{\ln\left(2\,a\,e^{dx}\,e^{c} - b + b\,e^{2c}\,e^{2dx}\right)\,(a^4+a^2\,b^2)}{b^5\,d} - \frac{e^{c+dx}\,(4\,a^3+3\,a\,b^2)}{8\,b^4\,d} + \frac{e^{-c-dx}\,(4\,a^3+3\,a\,b^2)}{8\,b^4\,d} + \frac{e^{-2c-2dx}\,(2\,a^2+b^2)}{16\,b^3\,d} + \frac{e^{2c+2dx}\,(2\,a^2+b^2)}{16\,b^3\,d}$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
exp(- 4*c - 4*d*x)/(64*b*d) - (x*(a^4 + a^2*b^2))/b^5 + exp(4*c + 4*d*x)/(
64*b*d) + (a*exp(- 3*c - 3*d*x))/(24*b^2*d) - (a*exp(3*c + 3*d*x))/(24*b^2
*d) + (log(2*a*exp(d*x)*exp(c) - b + b*exp(2*c)*exp(2*d*x))*(a^4 + a^2*b^2
))/(b^5*d) - (exp(c + d*x)*(3*a*b^2 + 4*a^3))/(8*b^4*d) + (exp(- c - d*x)*
(3*a*b^2 + 4*a^3))/(8*b^4*d) + (exp(- 2*c - 2*d*x)*(2*a^2 + b^2))/(16*b^3*
d) + (exp(2*c + 2*d*x)*(2*a^2 + b^2))/(16*b^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.87

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{3e^{8dx+8c}b^4 - 8e^{7dx+7c}a\,b^3 + 24e^{6dx+6c}a^2b^2 + 12e^{6dx+6c}b^4 - 96e^{5dx+5c}a^3b - 72e^{5dx+5c}a\,b^3 + 192e^{4dx+4c}\log(\cdots}{}$$

input
```
int(cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(3*e**(8*c + 8*d*x)*b**4 - 8*e**(7*c + 7*d*x)*a*b**3 + 24*e**(6*c + 6*d*x)
*a**2*b**2 + 12*e**(6*c + 6*d*x)*b**4 - 96*e**(5*c + 5*d*x)*a**3*b - 72*e*
*(5*c + 5*d*x)*a*b**3 + 192*e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*b + 2*e*
*(c + d*x)*a - b)*a**4 + 192*e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*b + 2*e
**(c + d*x)*a - b)*a**2*b**2 - 192*e**(4*c + 4*d*x)*a**4*d*x - 192*e**(4*c
 + 4*d*x)*a**2*b**2*d*x + 96*e**(3*c + 3*d*x)*a**3*b + 72*e**(3*c + 3*d*x)
*a*b**3 + 24*e**(2*c + 2*d*x)*a**2*b**2 + 12*e**(2*c + 2*d*x)*b**4 + 8*e**
(c + d*x)*a*b**3 + 3*b**4)/(192*e**(4*c + 4*d*x)*b**5*d)
```

3.376 $\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 36

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(cosh(d*x+c)^3*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 29.56 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x
])),x]
```

output
```
Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

$$\Big\downarrow \text{ 6125}$$

$$\int \frac{\sinh^2(c+dx)\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Cosh[c + d*x]^3*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(dx+c\right)^{3}\sinh\left(dx+c\right)^{2}}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(cosh(d*x+c)^3*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)^3*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)^3\sinh\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
integral(cosh(d*x + c)^3*sinh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh
(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**3*sinh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 442, normalized size of antiderivative = 12.28

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)^3\sinh\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
1/16*e^(-4*c + 4*d*e/f)*exp_integral_e(1, 4*(f*x + e)*d/f)/(b*f) + 1/8*a*e
^(-3*c + 3*d*e/f)*exp_integral_e(1, 3*(f*x + e)*d/f)/(b^2*f) + 1/8*a*e^(3*
c - 3*d*e/f)*exp_integral_e(1, -3*(f*x + e)*d/f)/(b^2*f) - 1/16*e^(4*c - 4
*d*e/f)*exp_integral_e(1, -4*(f*x + e)*d/f)/(b*f) + 1/8*(2*a^2 + b^2)*e^(-
2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b^3*f) - 1/8*(2*a^2*e^(
2*c) + b^2*e^(2*c))*e^(-2*d*e/f)*exp_integral_e(1, -2*(f*x + e)*d/f)/(b^3*
f) + 1/8*(4*a^3 + 3*a*b^2)*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)
/(b^4*f) + 1/8*(4*a^3*e^c + 3*a*b^2*e^c)*e^(-d*e/f)*exp_integral_e(1, -(f*
x + e)*d/f)/(b^4*f) + (a^4 + a^2*b^2)*log(f*x + e)/(b^5*f) - 1/32*integrat
e(64*(a^4*b + a^2*b^3 - (a^5*e^c + a^3*b^2*e^c)*e^(d*x))/(b^6*f*x + b^6*e
- (b^6*f*x*e^(2*c) + b^6*e*e^(2*c))*e^(2*d*x) - 2*(a*b^5*f*x*e^c + a*b^5*e
*e^c)*e^(d*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)^3\sinh(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate(cosh(d*x + c)^3*sinh(d*x + c)^2/((f*x + e)*(b*sinh(d*x + c) + a)
), x)
```

Mupad [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)^3\sinh(c+d\,x)^2}{(e+f\,x)\,(a+b\sinh(c+d\,x))}\,dx$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((cosh(c + d*x)^3*sinh(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))), x
)
```

Reduce [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 511, normalized size of antiderivative = 14.19

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \frac{e^{6c}\left(\int \frac{e^{6dx}}{e^{2dx+2c}be+e^{2dx+2c}bfx+2e^{dx+c}ae+2e^{dx+c}afx-be-bfx}dx\right)bf+e^{4c}\left(\int \frac{e^{4dx}}{e^{2dx+2c}be+e^{2dx+2c}bfx+2e^{dx+c}ae+2e^{dx+c}afx-be-b}\right.}{}$$

input
```
int(cosh(d*x+c)^3*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
(e**(6*c)*int(e**(6*d*x)/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x +
2*e**(c + d*x)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x)*b*f + e**(4*c)
*int(e**(4*d*x)/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x + 2*e**(c +
 d*x)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x)*b*f + e**(2*c)*int(1/(e
**(6*c + 4*d*x)*b*e + e**(6*c + 4*d*x)*b*f*x + 2*e**(5*c + 3*d*x)*a*e + 2*
e**(5*c + 3*d*x)*a*f*x - e**(4*c + 2*d*x)*b*e - e**(4*c + 2*d*x)*b*f*x),x)
*b*f + 4*e**c*int(e**(d*x)/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x
+ 2*e**(c + d*x)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x)*a*f + int(1/
(e**(6*c + 6*d*x)*b*e + e**(6*c + 6*d*x)*b*f*x + 2*e**(5*c + 5*d*x)*a*e +
2*e**(5*c + 5*d*x)*a*f*x - e**(4*c + 4*d*x)*b*e - e**(4*c + 4*d*x)*b*f*x),
x)*b*f - 4*int(1/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x + 2*e**(c
+ d*x)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x)*b*f - 2*log(e + f*x))/
(16*b*f)
```

3.377 $\int \frac{(e+fx)^3 \sinh(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 1218

$$\int \frac{(e+fx)^3 \sinh(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)} \, dx = \text{Too large to display}$$

output
```
3*a^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)
/d^2+3*a^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2
+b^2)/d^2-6*a^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b
/(a^2+b^2)/d^3-6*a^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2
)))/b/(a^2+b^2)/d^3-6*I*a^3*f^2*(f*x+e)*polylog(3,I*exp(d*x+c))/b^2/(a^2+b
^2)/d^3-3*I*a^3*f*(f*x+e)^2*polylog(2,-I*exp(d*x+c))/b^2/(a^2+b^2)/d^2+(f*
x+e)^3*ln(1+exp(2*d*x+2*c))/b/d+a^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/b/(a^2+b^2)/d+a^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2
)))/b/(a^2+b^2)/d+6*a^2*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b
/(a^2+b^2)/d^4+6*a^2*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a
^2+b^2)/d^4-a^2*(f*x+e)^3*ln(1+exp(2*d*x+2*c))/b/(a^2+b^2)/d+3/4*f^3*polyl
og(4,-exp(2*d*x+2*c))/b/d^4+6*I*a*f^3*polylog(4,-I*exp(d*x+c))/b^2/d^4-2*a
*(f*x+e)^3*arctan(exp(d*x+c))/b^2/d-3/2*f^2*(f*x+e)*polylog(3,-exp(2*d*x+2
*c))/b/d^3+3/2*f*(f*x+e)^2*polylog(2,-exp(2*d*x+2*c))/b/d^2-1/4*(f*x+e)^4/
b/f+2*a^3*(f*x+e)^3*arctan(exp(d*x+c))/b^2/(a^2+b^2)/d-3/4*a^2*f^3*polylog
(4,-exp(2*d*x+2*c))/b/(a^2+b^2)/d^4-6*I*a*f^3*polylog(4,I*exp(d*x+c))/b^2/
d^4+3/2*a^2*f^2*(f*x+e)*polylog(3,-exp(2*d*x+2*c))/b/(a^2+b^2)/d^3-3/2*a^2
*f*(f*x+e)^2*polylog(2,-exp(2*d*x+2*c))/b/(a^2+b^2)/d^2-6*I*a*f^2*(f*x+e)*
polylog(3,-I*exp(d*x+c))/b^2/d^3-3*I*a*f*(f*x+e)^2*polylog(2,I*exp(d*x+c))
/b^2/d^2-6*I*a^3*f^3*polylog(4,-I*exp(d*x+c))/b^2/(a^2+b^2)/d^4+6*I*a^3...
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3251 vs. 2(1218) = 2436.

Time = 10.61 (sec) , antiderivative size = 3251, normalized size of antiderivative = 2.67

$$\int \frac{(e+fx)^3 \sinh(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
-1/4*(8*b*d^4*e^3*E^(2*c)*x + 12*b*d^4*e^2*E^(2*c)*f*x^2 + 8*b*d^4*e*E^(2*
c)*f^2*x^3 + 2*b*d^4*E^(2*c)*f^3*x^4 + 8*a*d^3*e^3*ArcTan[E^(c + d*x)] + 8
*a*d^3*e^3*E^(2*c)*ArcTan[E^(c + d*x)] + (12*I)*a*d^3*e^2*f*x*Log[1 - I*E^
(c + d*x)] + (12*I)*a*d^3*e^2*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] + (12*I)*
a*d^3*e*f^2*x^2*Log[1 - I*E^(c + d*x)] + (12*I)*a*d^3*e*E^(2*c)*f^2*x^2*Lo
g[1 - I*E^(c + d*x)] + (4*I)*a*d^3*f^3*x^3*Log[1 - I*E^(c + d*x)] + (4*I)*
a*d^3*E^(2*c)*f^3*x^3*Log[1 - I*E^(c + d*x)] - (12*I)*a*d^3*e^2*f*x*Log[1
+ I*E^(c + d*x)] - (12*I)*a*d^3*e^2*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] - (
12*I)*a*d^3*e*f^2*x^2*Log[1 + I*E^(c + d*x)] - (12*I)*a*d^3*e*E^(2*c)*f^2*
x^2*Log[1 + I*E^(c + d*x)] - (4*I)*a*d^3*f^3*x^3*Log[1 + I*E^(c + d*x)] -
(4*I)*a*d^3*E^(2*c)*f^3*x^3*Log[1 + I*E^(c + d*x)] - 4*b*d^3*e^3*Log[1 + E
^(2*(c + d*x))] - 4*b*d^3*e^3*E^(2*c)*Log[1 + E^(2*(c + d*x))] - 12*b*d^3*
e^2*f*x*Log[1 + E^(2*(c + d*x))] - 12*b*d^3*e^2*E^(2*c)*f*x*Log[1 + E^(2*(
c + d*x))] - 12*b*d^3*e*f^2*x^2*Log[1 + E^(2*(c + d*x))] - 12*b*d^3*e*E^(2
*c)*f^2*x^2*Log[1 + E^(2*(c + d*x))] - 4*b*d^3*f^3*x^3*Log[1 + E^(2*(c + d
*x))] - 4*b*d^3*E^(2*c)*f^3*x^3*Log[1 + E^(2*(c + d*x))] - (12*I)*a*d^2*(1
 + E^(2*c))*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)] + (12*I)*a*d^2*(1 +
 E^(2*c))*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)] - 6*b*d^2*e^2*f*PolyLog[
2, -E^(2*(c + d*x))] - 6*b*d^2*e^2*E^(2*c)*f*PolyLog[2, -E^(2*(c + d*x))]
- 12*b*d^2*e*f^2*x*PolyLog[2, -E^(2*(c + d*x))] - 12*b*d^2*e*E^(2*c)*f^...
```

Rubi [A] (verified)

Time = 5.67 (sec) , antiderivative size = 1036, normalized size of antiderivative = 0.85, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.594, Rules used = {6115, 3042, 26, 4201, 2620, 3011, 6101, 3042, 4668, 3011, 6107, 6095, 2620, 3011, 7163, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3 \sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6115}$$

$$\frac{\int (e+fx)^3\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -i(e+fx)^3\tan(ic+idx)dx}{b}$$

$\Big\downarrow$ 26

$$-\frac{a\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\int (e+fx)^3\tan(ic+idx)dx}{b}$$

$\Big\downarrow$ 4201

$$-\frac{a\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\int \frac{e^{2(c+dx)}(e+fx)^3}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^4}{4f}\right)}{b}$$

$\Big\downarrow$ 2620

$$-\frac{a\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\int (e+fx)^2\log\left(1+e^{2(c+dx)}\right)dx}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{b}$$

$\Big\downarrow$ 3011

$$-\frac{a\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{b}$$

$\Big\downarrow$ 6101

$$-\frac{a\left(\frac{\int (e+fx)^3\operatorname{sech}(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{b}$$

$\Big\downarrow$ 3042

$$-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{b}\right)}{b}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{b}$$

$\downarrow$ 4668

$$-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{3if\int(e+fx)^2\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{3if\int(e+fx)^2\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)^3\arctan\left(e^{c+dx}\right)}{d}}{b}\right)}{b}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{b}$$

$\downarrow$ 3011

$$-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{}}{b}\right.}{b}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{b}$$

$\downarrow$ 6107

$$-\frac{a\left(-\frac{a\left(\frac{b^2\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)^3\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,\ldots}{d}\right.}{d}}{b}\right.}{b}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{b}$$

$\big\downarrow$ 6095

$$-\frac{a\left(-\frac{a\left(\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^4}{4bf}\right)}{a^2+b^2}+\frac{\int (e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyL}}{}\right.}{}\right.}{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\int (e+fx)\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}$$

$$b$$

$\big\downarrow$ 2620

$$-\frac{a\left(-\frac{a\left(\frac{b^2\left(-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right.}{bd}\right.}{a^2+b^2}\right.}{b}\right.}{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\int (e+fx)\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}$$

$$b$$

$\big\downarrow$ 3011

$$-\frac{a\left(-\frac{a\left(\frac{b^2\left(-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\cdots\right)}{bd}\cdots\right)}{a^2+b^2}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{b}$$

$$\Big\downarrow \text{ 7163}$$

$$a\left(-\frac{a\left(\frac{b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int \operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+f}{}\right.}{}\right.}{}\right.}{}\right.}{}\right.}{}\right.$$

$$-\frac{}{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2d}-\frac{f\int \operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)dx}{2d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)\right)}$$

$$b$$

↓ 2720

$$-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2(c+dx)}\right)}{2d}-\frac{3f\left(\frac{f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2d}-\frac{f\int e^{-2(c+dx)}\,\text{PolyLog}\left(3,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}\right)}{d}-\frac{(e+fx)^2\,\text{PolyLog}}{2d}\cdots\right.}{2d}\cdots\right.\right.}{b}$$

$$a\left(\frac{\frac{2\arctan\left(e^{c+dx}\right)(e+fx)^3}{d}+\frac{3if\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{d}-\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(3,-ie^{c+dx}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\cdots}{\cdots}}{b}\cdots\right.$$

↓ 7143

$$-\frac{a\left(-\frac{a\left(\frac{\int (e+fx)^3\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyL}}{}\right)}{bd}}{}\right)}{}\right)}{}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2d}-\frac{f\operatorname{PolyLog}\left(4,-e^{2(c+dx)}\right)}{4d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)-\right.}{b}$$

↓ 7293

$$a\left(-\frac{a\left(\frac{\int\left(a(e+fx)^3\operatorname{sech}(c+dx)-b(e+fx)^3\tanh(c+dx)\right)dx}{a^2+b^2}+\frac{b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+}{}\right)}{bd}}{}\right)}{}\right)$$

$$-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2d}-\frac{f\operatorname{PolyLog}\left(4,-e^{2(c+dx)}\right)}{4d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)-\frac{3}{}\right)}{b}$$

↓ 2009

$$-\frac{a\left(\frac{\frac{2\arctan\left(e^{c+dx}\right)(e+fx)^3}{d}+\frac{3if\left(\frac{2f\left(\frac{(e+fx)\,\mathrm{PolyLog}\left(3,-ie^{c+dx}\right)}{d}-\frac{f\,\mathrm{PolyLog}\left(4,-ie^{c+dx}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\,\mathrm{P}}{}\right.}{}\right.}{}}{b}\right.}{}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2(c+dx)}\right)}{2d}-\frac{3f\left(\frac{f\left(\frac{(e+fx)\,\mathrm{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2d}-\frac{f\,\mathrm{PolyLog}\left(4,-e^{2(c+dx)}\right)}{4d^2}\right)}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)-\right.}{b}$$

input
```
Int[((e + f*x)^3*Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-((a*(((2*(e + f*x)^3*ArcTan[E^(c + d*x)])/d + ((3*I)*f*(-(((e + f*x)^2*Po
lyLog[2, (-I)*E^(c + d*x)])/d) + (2*f*(((e + f*x)*PolyLog[3, (-I)*E^(c + d
*x)])/d - (f*PolyLog[4, (-I)*E^(c + d*x)])/d^2))/d))/d - ((3*I)*f*(-(((e +
 f*x)^2*PolyLog[2, I*E^(c + d*x)])/d) + (2*f*(((e + f*x)*PolyLog[3, I*E^(c
 + d*x)])/d - (f*PolyLog[4, I*E^(c + d*x)])/d^2))/d))/d)/b - (a*((b^2*(-1/
4*(e + f*x)^4/(b*f) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])])/(b*d) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
)])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[
a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]
)/d^2))/d))/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))])/d^2))/d))/(b*d)))/(a^2 + b^2) + ((b*(e + f*x)^4)/(4*f) + (2*a*(e
 + f*x)^3*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)^3*Log[1 + E^(2*(c + d*x))]
)/d - ((3*I)*a*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + ((3*I)*a*
f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/d^2 - (3*b*f*(e + f*x)^2*PolyLog[
2, -E^(2*(c + d*x))])/(2*d^2) + ((6*I)*a*f^2*(e + f*x)*PolyLog[3, (-I)*E^(
c + d*x)])/d^3 - ((6*I)*a*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/d^3 + (
3*b*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*d^3) - ((6*I)*a*f^3*...
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6101
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sech[
c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*Sech[c
+ d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6115
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - S
imp[a/b   Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*(Tanh[c + d*x]^n/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^3 \sinh(dx+c)\tanh(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1962, normalized size of antiderivative = 1.61

$$\int \frac{(e+fx)^3 \sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
-1/4*((a^2 + b^2)*d^4*f^3*x^4 + 4*(a^2 + b^2)*d^4*e*f^2*x^3 + 6*(a^2 + b^2
)*d^4*e^2*f*x^2 + 4*(a^2 + b^2)*d^4*e^3*x - 24*a^2*f^3*polylog(4, (a*cosh(
d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2))/b) - 24*a^2*f^3*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c
) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 12*(a^
2*d^2*f^3*x^2 + 2*a^2*d^2*e*f^2*x + a^2*d^2*e^2*f)*dilog((a*cosh(d*x + c)
+ a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2) - b)/b + 1) - 12*(a^2*d^2*f^3*x^2 + 2*a^2*d^2*e*f^2*x + a^2*d^2*e^2*f)
*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 12*(I*a*b*d^2*f^3*x^2 - b^2*d^2*
f^3*x^2 + 2*I*a*b*d^2*e*f^2*x - 2*b^2*d^2*e*f^2*x + I*a*b*d^2*e^2*f - b^2*
d^2*e^2*f)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) + 12*(-I*a*b*d^2*f^3*x
^2 - b^2*d^2*f^3*x^2 - 2*I*a*b*d^2*e*f^2*x - 2*b^2*d^2*e*f^2*x - I*a*b*d^2
*e^2*f - b^2*d^2*e^2*f)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) - 4*(a^2
*d^3*e^3 - 3*a^2*c*d^2*e^2*f + 3*a^2*c^2*d*e*f^2 - a^2*c^3*f^3)*log(2*b*co
sh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 4*(a^
2*d^3*e^3 - 3*a^2*c*d^2*e^2*f + 3*a^2*c^2*d*e*f^2 - a^2*c^3*f^3)*log(2*b*c
osh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 4*(a
^2*d^3*f^3*x^3 + 3*a^2*d^3*e*f^2*x^2 + 3*a^2*d^3*e^2*f*x + 3*a^2*c*d^2*e^2
*f - 3*a^2*c^2*d*e*f^2 + a^2*c^3*f^3)*log(-(a*cosh(d*x + c) + a*sinh(d*...
```

Sympy [F]

$$\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)^3\sinh{(c+dx)}\tanh{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*sinh(c + d*x)*tanh(c + d*x)/(a + b*sinh(c + d*x)), x
)
```

Maxima [F]

$$\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\sinh(dx+c)\tanh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
e^3*(a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b + b^3)*d)
 + 2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/
((a^2 + b^2)*d) + (d*x + c)/(b*d)) + 1/4*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*
x^2)/b - integrate(2*(a^2*b*f^3*x^3 + 3*a^2*b*e*f^2*x^2 + 3*a^2*b*e^2*f*x
- (a^3*f^3*x^3*e^c + 3*a^3*e*f^2*x^2*e^c + 3*a^3*e^2*f*x*e^c)*e^(d*x))/(a^
2*b^2 + b^4 - (a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x) - 2*(a^3*b*e^c + a
*b^3*e^c)*e^(d*x)), x) - integrate(2*(b*f^3*x^3 + 3*b*e*f^2*x^2 + 3*b*e^2*
f*x + (a*f^3*x^3*e^c + 3*a*e*f^2*x^2*e^c + 3*a*e^2*f*x*e^c)*e^(d*x))/(a^2
+ b^2 + (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\sinh(c+d\,x)\,\tanh(c+d\,x)\,\left(e+f\,x\right)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((sinh(c + d*x)*tanh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)*tanh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Too large to display}$$

input
```
int((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*atan(e**(c + d*x))*a*b*e**3 + e**(4*c)*int((e**(4*d*x)*x**3)/(e**(4*
c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a**2*b*d*f*
*3 + e**(4*c)*int((e**(4*d*x)*x**3)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*
x)*a + 2*e**(c + d*x)*a - b),x)*b**3*d*f**3 + 3*e**(4*c)*int((e**(4*d*x)*x
**2)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)
*a**2*b*d*e*f**2 + 3*e**(4*c)*int((e**(4*d*x)*x**2)/(e**(4*c + 4*d*x)*b +
2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*b**3*d*e*f**2 + 3*e**(4*c)
*int((e**(4*d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c +
 d*x)*a - b),x)*a**2*b*d*e**2*f + 3*e**(4*c)*int((e**(4*d*x)*x)/(e**(4*c +
 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*b**3*d*e**2*f
- 2*e**(2*c)*int((e**(2*d*x)*x**3)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x
)*a + 2*e**(c + d*x)*a - b),x)*a**2*b*d*f**3 - 2*e**(2*c)*int((e**(2*d*x)*
x**3)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x
)*b**3*d*f**3 - 6*e**(2*c)*int((e**(2*d*x)*x**2)/(e**(4*c + 4*d*x)*b + 2*e
**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a**2*b*d*e*f**2 - 6*e**(2*c)*
int((e**(2*d*x)*x**2)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c
 + d*x)*a - b),x)*b**3*d*e*f**2 - 6*e**(2*c)*int((e**(2*d*x)*x)/(e**(4*c +
 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a**2*b*d*e**2*
f - 6*e**(2*c)*int((e**(2*d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)
*a + 2*e**(c + d*x)*a - b),x)*b**3*d*e**2*f + int(x**3/(e**(4*c + 4*d*x...
```

3.378 $\int \frac{(e+fx)^2 \sinh(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 861

$$\int \frac{(e+fx)^2 \sinh(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} \, dx = \text{Too large to display}$$

output

```
-1/3*(f*x+e)^3/b/f-2*a*(f*x+e)^2*arctan(exp(d*x+c))/b^2/d+2*a^3*(f*x+e)^2*
arctan(exp(d*x+c))/b^2/(a^2+b^2)/d+a^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2
+b^2)^(1/2)))/b/(a^2+b^2)/d+a^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(
1/2)))/b/(a^2+b^2)/d+(f*x+e)^2*ln(1+exp(2*d*x+2*c))/b/d-a^2*(f*x+e)^2*ln(1
+exp(2*d*x+2*c))/b/(a^2+b^2)/d-2*I*a^3*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/
b^2/(a^2+b^2)/d^2+2*I*a*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/b^2/d^2-2*I*a*f
^2*polylog(3,-I*exp(d*x+c))/b^2/d^3-2*I*a*f*(f*x+e)*polylog(2,I*exp(d*x+c)
)/b^2/d^2+2*a^2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(
a^2+b^2)/d^2+2*a^2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/
b/(a^2+b^2)/d^2+f*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/b/d^2-a^2*f*(f*x+e)*p
olylog(2,-exp(2*d*x+2*c))/b/(a^2+b^2)/d^2+2*I*a^3*f*(f*x+e)*polylog(2,I*ex
p(d*x+c))/b^2/(a^2+b^2)/d^2+2*I*a^3*f^2*polylog(3,-I*exp(d*x+c))/b^2/(a^2+
b^2)/d^3+2*I*a*f^2*polylog(3,I*exp(d*x+c))/b^2/d^3-2*I*a^3*f^2*polylog(3,I
*exp(d*x+c))/b^2/(a^2+b^2)/d^3-2*a^2*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b
^2)^(1/2)))/b/(a^2+b^2)/d^3-2*a^2*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)
^(1/2)))/b/(a^2+b^2)/d^3-1/2*f^2*polylog(3,-exp(2*d*x+2*c))/b/d^3+1/2*a^2*
f^2*polylog(3,-exp(2*d*x+2*c))/b/(a^2+b^2)/d^3
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1759 vs. 2(861) = 1722.

Time = 10.06 (sec) , antiderivative size = 1759, normalized size of antiderivative = 2.04

$$\int \frac{(e+fx)^2 \sinh(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)} \, dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
-1/6*(12*b*d^3*e^2*E^(2*c)*x - 12*b*d^3*e^2*(1 + E^(2*c))*x - 12*b*d^3*e*f
*x^2 - 4*b*d^3*f^2*x^3 + 12*a*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] +
6*b*d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*a*d*
e*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) -
 PolyLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) + 6*b*d*e*(1 +
E^(2*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c +
 d*x))]) + (6*I)*a*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2
*x^2*Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*P
olyLog[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I
*E^(c + d*x)]) + b*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c
 + d*x))]) - 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c +
d*x))]))/((a^2 + b^2)*d^3*(1 + E^(2*c))) - (a^2*(6*e^2*E^(2*c)*x + 6*e*E^(
2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^
(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 +
 b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 +
 b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))
/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*
E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*
d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d - (3*e^2*E^
(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e*f*x*Log...
```

Rubi [A] (verified)

Time = 4.27 (sec) , antiderivative size = 748, normalized size of antiderivative = 0.87, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6115, 3042, 26, 4201, 2620, 3011, 2720, 6101, 3042, 4668, 3011, 2720, 6107, 6095, 2620, 3011, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2 \sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6115}$$

$$\frac{\int (e+fx)^2\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -i(e+fx)^2\tan(ic+idx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\int (e+fx)^2\tan(ic+idx)dx}{b}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{a\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\int \frac{e^{2(c+dx)}(e+fx)^2}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{a\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int (e+fx)\log\left(1+e^{2(c+dx)}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{a\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int \text{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{2d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{a\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$$\Big\downarrow \text{6101}$$

$$-\frac{a\left(\frac{\int (e+fx)^2\operatorname{sech}(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\left(-\frac{a\int \frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{b}\right)}{b}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$$\Big\downarrow \text{4668}$$

$$-\frac{a\left(-\frac{a\int \frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{2if\int (e+fx)\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{2if\int (e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}}{b}\right)}{b}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2if\left(\frac{f\int\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{Poly}}{}\right.}{d}}{b}\right.}{b}\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$\Big\downarrow$ 2720

$$-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c}\right.}{d^2}\right.}{}}{b}\right.}{b}\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$\Big\downarrow$ 6107

$$-\frac{a\left(-\frac{a\left(\frac{b^2\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}}{d}\right.}{d}\right.}{b}\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$\Big\downarrow$ 6095

$$-\frac{a\left(-\frac{a\left(\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{Poly}\right.}{}}{}\right.}{}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$\downarrow$ 2620

$$-\frac{a\left(-\frac{a\left(\frac{b^2\left(-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right.}{a^2+b^2}\right.}{b}\right.}{}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$\downarrow$ 3011

$$-\frac{a\left(-\frac{a\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,\right.}{d}\right.}{bd}\right.}{a^2+b^2}\right.}{b}\right.}{\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}}$$

$$\Big\downarrow \; 2720$$

$$a\left(-\frac{a\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de\cdots}{d^2}\cdots\right.}{b\cdots}\cdots\right.}{a^2+b^2}\cdots\right.}{\cdots}\cdots\right.$$

$$-\frac{\cdots}{\cdots}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$\downarrow$ 7143

$$-\frac{a\left(-\frac{a\left(\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,\right.}{}\right.}{}\right.}{}\right.}{b}}{}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\,\text{PolyLog}\left(3,-e^{2(c+dx)}\right)}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$$\Big\downarrow \text{ 7293}$$

$$-\frac{a\left(-\frac{a\left(\frac{\int\left(a(e+fx)^2\operatorname{sech}(c+dx)-b(e+fx)^2\tanh(c+dx)\right)dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\operatorname{Po}\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{b\cdots}\right.}{\cdots}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{a\left(\frac{\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}+\frac{2if\left(\frac{f\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}}{b}-\frac{2if\left(\frac{f\,\text{PolyLog}\left(3,ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}\right.}{\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\,\text{PolyLog}\left(3,-e^{2(c+dx)}\right)}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}}$$

input
```
Int[((e + f*x)^2*Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
((-I)*(((-1/3*I)*(e + f*x)^3)/f + (2*I)*(((e + f*x)^2*Log[1 + E^(2*(c + d*
x))])/(2*d) - (f*(-1/2*((e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/d + (f*Pol
yLog[3, -E^(2*(c + d*x))])/(4*d^2)))/d)))/b - (a*(((2*(e + f*x)^2*ArcTan[E
^(c + d*x)])/d + ((2*I)*f*(-(((e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d) +
 (f*PolyLog[3, (-I)*E^(c + d*x)])/d^2))/d - ((2*I)*f*(-(((e + f*x)*PolyLog
[2, I*E^(c + d*x)])/d) + (f*PolyLog[3, I*E^(c + d*x)])/d^2))/d)/b - (a*((b
^2*(-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
]))])/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2]))])/d^2))/(b*d)))/(a^2 + b^2) + ((b*(e + f*x)^3)/(3*f) + (2*a*(e + f*x)
^2*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/d - (
(2*I)*a*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + ((2*I)*a*f*(e + f*
x)*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*(e + f*x)*PolyLog[2, -E^(2*(c + d
*x))])/d^2 + ((2*I)*a*f^2*PolyLog[3, (-I)*E^(c + d*x)])/d^3 - ((2*I)*a*f^2
*PolyLog[3, I*E^(c + d*x)])/d^3 + (b*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*
d^3))/(a^2 + b^2)))/b))/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6101
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sech[
c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*Sech[c
+ d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6115
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - S
imp[a/b   Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*(Tanh[c + d*x]^n/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^2 \sinh(dx+c)\tanh(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1248, normalized size of antiderivative = 1.45

$$\int \frac{(e+fx)^2 \sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
-1/3*((a^2 + b^2)*d^3*f^2*x^3 + 3*(a^2 + b^2)*d^3*e*f*x^2 + 3*(a^2 + b^2)*
d^3*e^2*x + 6*a^2*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*a^2*f^2*poly
log(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2))/b) - 6*(a^2*d*f^2*x + a^2*d*e*f)*dilog((a*cos
h(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2) - b)/b + 1) - 6*(a^2*d*f^2*x + a^2*d*e*f)*dilog((a*cosh(d*x
 + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2) - b)/b + 1) + 6*(I*a*b*d*f^2*x - b^2*d*f^2*x + I*a*b*d*e*f - b^2
*d*e*f)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) + 6*(-I*a*b*d*f^2*x - b^2
*d*f^2*x - I*a*b*d*e*f - b^2*d*e*f)*dilog(-I*cosh(d*x + c) - I*sinh(d*x +
c)) - 3*(a^2*d^2*e^2 - 2*a^2*c*d*e*f + a^2*c^2*f^2)*log(2*b*cosh(d*x + c)
+ 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 3*(a^2*d^2*e^2 -
2*a^2*c*d*e*f + a^2*c^2*f^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2
*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 3*(a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + 2
*a^2*c*d*e*f - a^2*c^2*f^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 3*(a^2*d^2
*f^2*x^2 + 2*a^2*d^2*e*f*x + 2*a^2*c*d*e*f - a^2*c^2*f^2)*log(-(a*cosh(d*x
 + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2) - b)/b) + 3*(I*a*b*d^2*e^2 - b^2*d^2*e^2 - 2*I*a*b*c*d*e*f + ...
```

Sympy [F]

$$\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2\sinh{(c+dx)}\tanh{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**2*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*sinh(c + d*x)*tanh(c + d*x)/(a + b*sinh(c + d*x)), x
)
```

Maxima [F]

$$\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\sinh{(dx+c)}\tanh{(dx+c)}}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
e^2*(a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b + b^3)*d)
 + 2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/
((a^2 + b^2)*d) + (d*x + c)/(b*d)) + 1/3*(f^2*x^3 + 3*e*f*x^2)/b - integra
te(2*(a^2*b*f^2*x^2 + 2*a^2*b*e*f*x - (a^3*f^2*x^2*e^c + 2*a^3*e*f*x*e^c)*
e^(d*x))/(a^2*b^2 + b^4 - (a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x) - 2*(a
^3*b*e^c + a*b^3*e^c)*e^(d*x)), x) - integrate(2*(b*f^2*x^2 + 2*b*e*f*x +
(a*f^2*x^2*e^c + 2*a*e*f*x*e^c)*e^(d*x))/(a^2 + b^2 + (a^2*e^(2*c) + b^2*e
^(2*c))*e^(2*d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)\,\tanh(c+d\,x)\,\left(e+f\,x\right)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((sinh(c + d*x)*tanh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)*tanh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{-2atan\left(e^{dx+c}\right)ab\,e^2+e^{4c}\left(\int \frac{e^{4dx}x^2}{e^{4dx+4c}b+2e^{3dx+3c}a+2e^{dx+c}a-b}dx\right)a^2bd\,f^2+e^{4c}\left(\int \frac{e^{4dx}x^2}{e^{4dx+4c}b+2e^{3dx+3c}a+2e^{dx+c}a-b}dx\right.}{}$$

input
```
int((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*atan(e**(c + d*x))*a*b*e**2 + e**(4*c)*int((e**(4*d*x)*x**2)/(e**(4*
c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a**2*b*d*f*
*2 + e**(4*c)*int((e**(4*d*x)*x**2)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*
x)*a + 2*e**(c + d*x)*a - b),x)*b**3*d*f**2 + 2*e**(4*c)*int((e**(4*d*x)*x
)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a*
*2*b*d*e*f + 2*e**(4*c)*int((e**(4*d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c
 + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*b**3*d*e*f - 2*e**(2*c)*int((e**(2*
d*x)*x**2)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a -
 b),x)*a**2*b*d*f**2 - 2*e**(2*c)*int((e**(2*d*x)*x**2)/(e**(4*c + 4*d*x)*
b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*b**3*d*f**2 - 4*e**(2*
c)*int((e**(2*d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c
 + d*x)*a - b),x)*a**2*b*d*e*f - 4*e**(2*c)*int((e**(2*d*x)*x)/(e**(4*c +
4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*b**3*d*e*f + in
t(x**2/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),
x)*a**2*b*d*f**2 + int(x**2/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2
*e**(c + d*x)*a - b),x)*b**3*d*f**2 + 2*int(x/(e**(4*c + 4*d*x)*b + 2*e**(
3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a**2*b*d*e*f + 2*int(x/(e**(4*c
+ 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*b**3*d*e*f +
log(e**(2*c + 2*d*x) + 1)*b**2*e**2 + log(e**(2*c + 2*d*x)*b + 2*e**(c + d
*x)*a - b)*a**2*e**2 - a**2*d*e**2*x - b**2*d*e**2*x)/(b*d*(a**2 + b**2...
```

3.379 $\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 516

$$\begin{aligned}
&\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx\\
&=-\frac{(e+fx)^2}{2bf}-\frac{2a(e+fx)\arctan\left(e^{c+dx}\right)}{b^2d}+\frac{2a^3(e+fx)\arctan\left(e^{c+dx}\right)}{b^2(a^2+b^2)\,d}\\
&+\frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)\,d}+\frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)\,d}\\
&+\frac{(e+fx)\log\left(1+e^{2(c+dx)}\right)}{bd}-\frac{a^2(e+fx)\log\left(1+e^{2(c+dx)}\right)}{b(a^2+b^2)\,d}\\
&+\frac{iaf\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{b^2d^2}-\frac{ia^3f\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{b^2(a^2+b^2)\,d^2}\\
&-\frac{iaf\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{b^2d^2}+\frac{ia^3f\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{b^2(a^2+b^2)\,d^2}\\
&+\frac{a^2f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)\,d^2}+\frac{a^2f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)\,d^2}\\
&+\frac{f\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2bd^2}-\frac{a^2f\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2b(a^2+b^2)\,d^2}
\end{aligned}$$

output
```
-1/2*(f*x+e)^2/b/f-2*a*(f*x+e)*arctan(exp(d*x+c))/b^2/d+2*a^3*(f*x+e)*arct
an(exp(d*x+c))/b^2/(a^2+b^2)/d+a^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^
(1/2)))/b/(a^2+b^2)/d+a^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b
/(a^2+b^2)/d+(f*x+e)*ln(1+exp(2*d*x+2*c))/b/d-a^2*(f*x+e)*ln(1+exp(2*d*x+2
*c))/b/(a^2+b^2)/d+I*a*f*polylog(2,-I*exp(d*x+c))/b^2/d^2-I*a^3*f*polylog(
2,-I*exp(d*x+c))/b^2/(a^2+b^2)/d^2-I*a*f*polylog(2,I*exp(d*x+c))/b^2/d^2+I
*a^3*f*polylog(2,I*exp(d*x+c))/b^2/(a^2+b^2)/d^2+a^2*f*polylog(2,-b*exp(d*
x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)/d^2+a^2*f*polylog(2,-b*exp(d*x+c)/(a
+(a^2+b^2)^(1/2)))/b/(a^2+b^2)/d^2+1/2*f*polylog(2,-exp(2*d*x+2*c))/b/d^2-
1/2*a^2*f*polylog(2,-exp(2*d*x+2*c))/b/(a^2+b^2)/d^2
```

Mathematica [A] (warning: unable to verify)

Time = 3.29 (sec) , antiderivative size = 578, normalized size of antiderivative = 1.12

$$\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{-2bde(c+dx)+2bcf(c+dx)-bf(c+dx)^2-4ade\arctan\left(e^{c+dx}\right)+4acf\arctan\left(e^{c+dx}\right)-2iaf(c+d}{}$$

input
```
Integrate[((e + f*x)*Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-2*b*d*e*(c + d*x) + 2*b*c*f*(c + d*x) - b*f*(c + d*x)^2 - 4*a*d*e*ArcTan
[E^(c + d*x)] + 4*a*c*f*ArcTan[E^(c + d*x)] - (2*I)*a*f*(c + d*x)*Log[1 -
I*E^(c + d*x)] + (2*I)*a*f*(c + d*x)*Log[1 + I*E^(c + d*x)] + 2*b*d*e*Log[
1 + E^(2*(c + d*x))] - 2*b*c*f*Log[1 + E^(2*(c + d*x))] + 2*b*f*(c + d*x)*
Log[1 + E^(2*(c + d*x))] + (2*I)*a*f*PolyLog[2, (-I)*E^(c + d*x)] - (2*I)*
a*f*PolyLog[2, I*E^(c + d*x)] + (a^2*(-2*d*e*(c + d*x) + 2*c*f*(c + d*x) -
 f*(c + d*x)^2 + (4*a*Sqrt[a^2 + b^2]*d*e*ArcTan[(a + b*E^(c + d*x))/Sqrt[
-a^2 - b^2]])/Sqrt[-(a^2 + b^2)^2] - (4*a*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh
[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/(-a^2 - b^2)^(3/2) + 2*f*(c + d*x)*
Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*f*(c + d*x)*Log[1 + (b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*f*Log[b - 2*a*E^(c + d*x) - b*E^
(2*(c + d*x))] + 2*d*e*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] + 2
*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f*PolyLog[2, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/b + b*f*PolyLog[2, -E^(2*(c + d*x
))])/(2*(a^2 + b^2)*d^2)
```

Rubi [A] (verified)

Time = 2.78 (sec) , antiderivative size = 471, normalized size of antiderivative = 0.91, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.633$, Rules used = {6115, 3042, 26, 4201, 2620, 2715, 2838, 6101, 3042, 4668, 2715, 2838, 6107, 6095, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6115}$$

$$\frac{\int (e+fx)\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -i(e+fx)\tan(ic+idx)dx}{b}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{a\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\int (e+fx)\tan(ic+idx)dx}{b}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{a\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\int \frac{e^{2(c+dx)}(e+fx)}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{a\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int \log\left(1+e^{2(c+dx)}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{a\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int e^{-2(c+dx)}\log\left(1+e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{a\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$$\Big\downarrow \text{6101}$$

$$-\frac{a\left(\frac{\int (e+fx)\text{sech}(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\left(-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{b}\right)}{b}-\frac{i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$$\Big\downarrow \text{4668}$$

$$-\frac{a\left(-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{if\int \log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int \log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}}{b}\right)}{b}-\frac{i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{a\left(-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}}{b}\right)}{b}-\frac{i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{a\left(-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}}{b}\right)}{b}-\frac{i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$$\Big\downarrow \text{6107}$$

$$-\frac{a\left(-\frac{a\left(\frac{b^2\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,\ldots\right.}{d^2}}{b}\ldots\right.}{b}$$

$$\frac{i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$$\Big\downarrow \text{6095}$$

$$-\frac{a\left(-\frac{a\left(\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\ldots}{\ldots}\right.}{b}$$

$$\frac{i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$\Big\downarrow$ 2620

$$-\frac{a\left(-\frac{a\left(\frac{b^2\left(-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int(e\cdots}{\cdots}\right)}{b}\cdots\right)}{b}\cdots$$

$$\frac{i\left(2i\left(\frac{f\,\text{PolyLog}(2,-e^{2(c+dx)})}{4d^2}+\frac{(e+fx)\log(e^{2(c+dx)}+1)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$\Big\downarrow$ 2715

$$-\frac{a\left(-\frac{a\left(\frac{b^2\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}\cdots}\cdots\right)}{bd}\cdots\right)}{a^2+b^2}\cdots\right)}{b}\cdots\right)}{b}\cdots$$

$$\frac{i\left(2i\left(\frac{f\,\text{PolyLog}(2,-e^{2(c+dx)})}{4d^2}+\frac{(e+fx)\log(e^{2(c+dx)}+1)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$\Big\downarrow$ 2838

$$-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e\cdots}{\cdots}\cdots\right)}{a^2+b^2}\cdots\right)}{b}\cdots\right)}{b}\cdots$$

$$\frac{i\left(2i\left(\frac{f\,\text{PolyLog}(2,-e^{2(c+dx)})}{4d^2}+\frac{(e+fx)\log(e^{2(c+dx)}+1)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$$\downarrow \text{7293}$$

$$-\frac{a\left(-\frac{a\left(\frac{\int (a(e+fx)\text{sech}(c+dx)-b(e+fx)\tanh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\cdots\right)}{a^2+b^2}\right)}{b}\cdots\right)}{b}\cdots$$

$$\frac{i\left(2i\left(\frac{f\,\text{PolyLog}(2,-e^{2(c+dx)})}{4d^2}+\frac{(e+fx)\log(e^{2(c+dx)}+1)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$$\downarrow \text{2009}$$

$$-\frac{a\left(\frac{\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}}{b}-\frac{a\left(\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots$$

$$\frac{i\left(2i\left(\frac{f\,\text{PolyLog}(2,-e^{2(c+dx)})}{4d^2}+\frac{(e+fx)\log(e^{2(c+dx)}+1)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

input

```
Int[((e + f*x)*Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
((-I)*(((-1/2*I)*(e + f*x)^2)/f + (2*I)*(((e + f*x)*Log[1 + E^(2*(c + d*x)
)])/(2*d) + (f*PolyLog[2, -E^(2*(c + d*x))])/(4*d^2))))/b - (a*(((2*(e + f
*x)*ArcTan[E^(c + d*x)])/d - (I*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + (I*f
*PolyLog[2, I*E^(c + d*x)])/d^2)/b - (a*((b^2*(-1/2*(e + f*x)^2/(b*f) + ((
e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x
)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -(
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (f*PolyLog[2, -((b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(a^2 + b^2) + ((b*(e + f*x)^2)
/(2*f) + (2*a*(e + f*x)*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)*Log[1 + E^(2
*(c + d*x))])/d - (I*a*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + (I*a*f*PolyLo
g[2, I*E^(c + d*x)])/d^2 - (b*f*PolyLog[2, -E^(2*(c + d*x))])/(2*d^2))/(a^
2 + b^2)))/b))/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6101
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sech[
c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*Sech[c
+ d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6115
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - S
imp[a/b   Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*(Tanh[c + d*x]^n/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3881 vs. $2(484) = 968$.

Time = 0.73 (sec) , antiderivative size = 3882, normalized size of antiderivative = 7.52

method	result	size
risch	Expression too large to display	3882

input
```
int((f*x+e)*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERB
OSE)
```

output
```
-1/d*f*b/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^
(1/2)))*x-1/d^2*f*b/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a
+(a^2+b^2)^(1/2)))*c-1/d*f*b/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2
)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2*f*b/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+
b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2/d*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c)
)*b*x+2/d^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*b*c+2/d*f/(2*a^2+2*b^2)*ln(
1-I*exp(d*x+c))*b*x+2/d^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*b*c-2/d^2*c*f
/(2*a^2+2*b^2)*b*ln(1+exp(2*d*x+2*c))+1/b/d^2*f/(a^2+b^2)*dilog((-b*exp(d*
x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a^2+1/b/d^2*f/(a^2+b^2)*dilo
g((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a^2+1/b/d^2*f/(a^2
+b^2)^(3/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*
a^3-1/b/d^2*f/(a^2+b^2)^(3/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a
^2+b^2)^(1/2)))*a^3+2*I/d^2*a*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))-2*I/d^
2*a*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))-1/d^2/b*f*c^2-2/d/b*e*ln(exp(d*x
+c))-b/d^2*f/(a^2+b^2)^(3/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^
2+b^2)^(1/2)))*a-1/2*b/d^2*c*f/(a^2+b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c
)-b)+1/2*b/d*f/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2
)^(1/2)))*x+1/2*b/d^2*f/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a
+(a^2+b^2)^(1/2)))*c+1/2*b/d*f/(a^2+b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+
a)/(a+(a^2+b^2)^(1/2)))*x+1/2*b/d^2*f/(a^2+b^2)*ln((b*exp(d*x+c)+(a^2+b...
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 682, normalized size of antiderivative = 1.32

$$\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
fricas")
```

output
```
-1/2*((a^2 + b^2)*d^2*f*x^2 + 2*(a^2 + b^2)*d^2*e*x - 2*a^2*f*dilog((a*cos
h(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2) - b)/b + 1) - 2*a^2*f*dilog((a*cosh(d*x + c) + a*sinh(d*x +
 c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1
) + 2*(I*a*b*f - b^2*f)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) + 2*(-I*a
*b*f - b^2*f)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) - 2*(a^2*d*e - a^2
*c*f)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2
) + 2*a) - 2*(a^2*d*e - a^2*c*f)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c)
 - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(a^2*d*f*x + a^2*c*f)*log(-(a*cosh
(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2) - b)/b) - 2*(a^2*d*f*x + a^2*c*f)*log(-(a*cosh(d*x + c) + a*
sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
- b)/b) + 2*(I*a*b*d*e - b^2*d*e - I*a*b*c*f + b^2*c*f)*log(cosh(d*x + c)
+ sinh(d*x + c) + I) + 2*(-I*a*b*d*e - b^2*d*e + I*a*b*c*f + b^2*c*f)*log(
cosh(d*x + c) + sinh(d*x + c) - I) + 2*(-I*a*b*d*f*x - b^2*d*f*x - I*a*b*c
*f - b^2*c*f)*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) + 2*(I*a*b*d*f*x
- b^2*d*f*x + I*a*b*c*f - b^2*c*f)*log(-I*cosh(d*x + c) - I*sinh(d*x + c)
+ 1))/((a^2*b + b^3)*d^2)
```

Sympy [F]

$$\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*sinh(c + d*x)*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\sinh(dx+c)\tanh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
maxima")
```

output
```
1/2*f*(x^2/b - integrate(-4*(a^3*x*e^(d*x + c) - a^2*b*x)/(a^2*b^2 + b^4 -
 (a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x) - 2*(a^3*b*e^c + a*b^3*e^c)*e^(
d*x)), x) - integrate(4*(a*x*e^(d*x + c) + b*x)/(a^2 + b^2 + (a^2*e^(2*c)
+ b^2*e^(2*c))*e^(2*d*x)), x)) + e*(a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*
x - 2*c) - b)/((a^2*b + b^3)*d) + 2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d)
 + b*log(e^(-2*d*x - 2*c) + 1)/((a^2 + b^2)*d) + (d*x + c)/(b*d))
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)\,\tanh(c+d\,x)\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((sinh(c + d*x)*tanh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)*tanh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-2atan\left(e^{dx+c}\right)abe + e^{4c}\left(\int \frac{e^{4dx}x}{e^{4dx+4c}b+2e^{3dx+3c}a+2e^{dx+c}a-b}dx\right)a^2bdf + e^{4c}\left(\int \frac{e^{4dx}x}{e^{4dx+4c}b+2e^{3dx+3c}a+2e^{dx+c}a-b}dx\right)b^3}{}$$

input
```
int((f*x+e)*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*atan(e**(c + d*x))*a*b*e + e**(4*c)*int((e**(4*d*x)*x)/(e**(4*c + 4*
d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a**2*b*d*f + e**(
4*c)*int((e**(4*d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**
(c + d*x)*a - b),x)*b**3*d*f - 2*e**(2*c)*int((e**(2*d*x)*x)/(e**(4*c + 4*
d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a**2*b*d*f - 2*e*
*(2*c)*int((e**(2*d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e
**(c + d*x)*a - b),x)*b**3*d*f + int(x/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3
*d*x)*a + 2*e**(c + d*x)*a - b),x)*a**2*b*d*f + int(x/(e**(4*c + 4*d*x)*b
+ 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*b**3*d*f + log(e**(2*c +
 2*d*x) + 1)*b**2*e + log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*a**2*
e - a**2*d*e*x - b**2*d*e*x)/(b*d*(a**2 + b**2))
```

3.380 $\int \frac{\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 74

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a\arctan(\sinh(c+dx))}{(a^2+b^2)\,d}+\frac{b\log(\cosh(c+dx))}{(a^2+b^2)\,d}+\frac{a^2\log(a+b\sinh(c+dx))}{b\,(a^2+b^2)\,d}$$

output
```
-a*arctan(sinh(d*x+c))/(a^2+b^2)/d+b*ln(cosh(d*x+c))/(a^2+b^2)/d+a^2*ln(a+
b*sinh(d*x+c))/b/(a^2+b^2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b(ia+b)\log(i-\sinh(c+dx))+b(-ia+b)\log(i+\sinh(c+dx))+2a^2\log(a+b\sinh(c+dx))}{2b\,(a^2+b^2)\,d}$$

input
```
Integrate[(Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(b*(I*a + b)*Log[I - Sinh[c + d*x]] + b*((-I)*a + b)*Log[I + Sinh[c + d*x]
] + 2*a^2*Log[a + b*Sinh[c + d*x]])/(2*b*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 25, 3316, 27, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ic+idx)^2}{\cos(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ic+idx)^2}{\cos(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3316}$$

$$\frac{b\int \frac{\sinh^2(c+dx)}{(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b^2\sinh^2(c+dx)}{(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))}{bd}$$

$$\Big\downarrow \text{615}$$

$$\frac{\int \left(\frac{a^2}{(a^2+b^2)(a+b\sinh(c+dx))}-\frac{b^2(a-b\sinh(c+dx))}{(a^2+b^2)(\sinh^2(c+dx)b^2+b^2)}\right)d(b\sinh(c+dx))}{bd}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{ab\arctan(\sinh(c+dx))}{a^2+b^2}+\frac{b^2\log(b^2\sinh^2(c+dx)+b^2)}{2(a^2+b^2)}+\frac{a^2\log(a+b\sinh(c+dx))}{a^2+b^2}}{bd}$$

input
```
Int[(Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-((a*b*ArcTan[Sinh[c + d*x]])/(a^2 + b^2)) + (a^2*Log[a + b*Sinh[c + d*x]
])/(a^2 + b^2) + (b^2*Log[b^2 + b^2*Sinh[c + d*x]^2])/(2*(a^2 + b^2)))/(b*
d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.78

method	result
derivativedivides	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}+\frac{4b\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-8a\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2+4b^2}-\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}+\frac{a^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\right.}{b\left(a^2+\right.}}{d}$
default	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}+\frac{4b\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-8a\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2+4b^2}-\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}+\frac{a^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\right.}{b\left(a^2+\right.}}{d}$
risch	$\frac{x}{b}-\frac{2b\,d^2x}{a^2d^2+b^2d^2}-\frac{2bdc}{a^2d^2+b^2d^2}-\frac{2a^2x}{b(a^2+b^2)}-\frac{2a^2c}{bd(a^2+b^2)}+\frac{i\ln(\mathrm{e}^{dx+c}-i)a}{(a^2+b^2)d}+\frac{\ln(\mathrm{e}^{dx+c}-i)b}{(a^2+b^2)d}-\frac{i\ln(\mathrm{e}^{dx+c}+}{(a^2+b^2)a}$

input
```
int(sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b*ln(tanh(1/2*d*x+1/2*c)-1)+8/(4*a^2+4*b^2)*(1/2*b*ln(1+tanh(1/2*d
*x+1/2*c)^2)-a*arctan(tanh(1/2*d*x+1/2*c)))-1/b*ln(1+tanh(1/2*d*x+1/2*c))+
a^2/b/(a^2+b^2)*ln(tanh(1/2*d*x+1/2*c)^2*a-2*b*tanh(1/2*d*x+1/2*c)-a))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.50

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{(a^2+b^2)dx+2\,ab\arctan\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)-a^2\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)-b^2\log\left(\frac{}{\cosh(}\right.}{(a^2b+b^3)d}$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-((a^2 + b^2)*d*x + 2*a*b*arctan(cosh(d*x + c) + sinh(d*x + c)) - a^2*log(
2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) - b^2*log(2*cosh(
d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/((a^2*b + b^3)*d)
```

Sympy [F]

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(sinh(c + d*x)*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.49

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{a^2\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^2b+b^3)d} + \frac{2\,a\arctan\left(e^{(-dx-c)}\right)}{(a^2+b^2)d} + \frac{b\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^2+b^2)d} + \frac{dx+c}{bd}$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b + b^3)*d) + 2*
a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a^2
 + b^2)*d) + (d*x + c)/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.64

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{2\,a^2\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^2b+b^3}-\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)a}{a^2+b^2}+\frac{b\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^2+b^2}}{2\,d}$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*a^2*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^2*b + b^3) -
(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*a/(a^2 + b^2) + b*
log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^2 + b^2))/d
```

Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.35

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\ln\left(\mathrm{e}^{c+d\,x}+1\mathrm{i}\right)}{b\,d+a\,d\,1\mathrm{i}}-\frac{x}{b}$$
$$+\frac{a^2\,\ln\left(a^2\,b^3-b^5-a^4\,b+2\,a^5\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+b^5\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+a^4\,b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-2\,a^3\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-a^2\,b^3\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+2\,a\,b^4}{d\,a^2\,b+d\,b^3}$$
$$+\frac{\ln\left(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)\,1\mathrm{i}}{a\,d+b\,d\,1\mathrm{i}}$$

input
```
int((sinh(c + d*x)*tanh(c + d*x))/(a + b*sinh(c + d*x)),x)
```

output
```
log(exp(c + d*x) + 1i)/(a*d*1i + b*d) - x/b + (log(exp(c + d*x)*1i + 1)*1i
)/(a*d + b*d*1i) + (a^2*log(a^2*b^3 - b^5 - a^4*b + 2*a^5*exp(d*x)*exp(c)
+ b^5*exp(2*c)*exp(2*d*x) + a^4*b*exp(2*c)*exp(2*d*x) - 2*a^3*b^2*exp(d*x)
*exp(c) - a^2*b^3*exp(2*c)*exp(2*d*x) + 2*a*b^4*exp(d*x)*exp(c)))/(b^3*d +
 a^2*b*d)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.23

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-2atan\left(e^{dx+c}\right)ab+\log\left(e^{2dx+2c}+1\right)b^2+\log\left(e^{2dx+2c}b+2e^{dx+c}a-b\right)a^2-a^2dx-b^2dx}{bd\left(a^2+b^2\right)}$$

input
```
int(sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*atan(e**(c + d*x))*a*b + log(e**(2*c + 2*d*x) + 1)*b**2 + log(e**(2*
c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*a**2 - a**2*d*x - b**2*d*x)/(b*d*(a**
2 + b**2))
```

3.381 $\int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 13.73 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Sinh[c + d*x]*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
x]
```

output
```
Integrate[(Sinh[c + d*x]*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
 x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

$$\Big\downarrow \ 6125$$

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Sinh[c + d*x]*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(dx+c)\tanh(dx+c)}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)\tanh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
fricas")
```

output
```
integral(sinh(d*x + c)*tanh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x
 + c)), x)
```

Sympy [N/A]

Not integrable

Time = 1.67 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(c+dx)\tanh(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(sinh(c + d*x)*tanh(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 252, normalized size of antiderivative = 7.88

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)\tanh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
maxima")
```

output
```
log(f*x + e)/(b*f) - 1/2*integrate(-4*(a^3*e^(d*x + c) - a^2*b)/(a^2*b^2*e
 + b^4*e + (a^2*b^2*f + b^4*f)*x - (a^2*b^2*e*e^(2*c) + b^4*e*e^(2*c) + (a
^2*b^2*f*e^(2*c) + b^4*f*e^(2*c))*x)*e^(2*d*x) - 2*(a^3*b*e*e^c + a*b^3*e*
e^c + (a^3*b*f*e^c + a*b^3*f*e^c)*x)*e^(d*x)), x) - 1/2*integrate(4*(a*e^(
d*x + c) + b)/(a^2*e + b^2*e + (a^2*f + b^2*f)*x + (a^2*e*e^(2*c) + b^2*e*
e^(2*c) + (a^2*f*e^(2*c) + b^2*f*e^(2*c))*x)*e^(2*d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(c+d\,x)\,\tanh(c+d\,x)}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((sinh(c + d*x)*tanh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((sinh(c + d*x)*tanh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 435, normalized size of antiderivative = 13.59

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$= \frac{-2e^{3c}\left(\int \frac{e^{3dx}}{e^{4dx+4c}be+e^{4dx+4c}bfx+2e^{3dx+3c}ae+2e^{3dx+3c}afx+2e^{dx+c}ae+2e^{dx+c}afx-be-bfx}dx\right)af - 2e^{2c}\left(\int \frac{}{e^{4dx+4c}be+e^{4dx+4c}}\right.}{}$$

input
```
int(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*e**(3*c)*int(e**(3*d*x)/(e**(4*c + 4*d*x)*b*e + e**(4*c + 4*d*x)*b*f
*x + 2*e**(3*c + 3*d*x)*a*e + 2*e**(3*c + 3*d*x)*a*f*x + 2*e**(c + d*x)*a*
e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x)*a*f - 2*e**(2*c)*int(e**(2*d*x)
/(e**(4*c + 4*d*x)*b*e + e**(4*c + 4*d*x)*b*f*x + 2*e**(3*c + 3*d*x)*a*e +
 2*e**(3*c + 3*d*x)*a*f*x + 2*e**(c + d*x)*a*e + 2*e**(c + d*x)*a*f*x - b*
e - b*f*x),x)*b*f - 2*e**c*int(e**(d*x)/(e**(4*c + 4*d*x)*b*e + e**(4*c +
4*d*x)*b*f*x + 2*e**(3*c + 3*d*x)*a*e + 2*e**(3*c + 3*d*x)*a*f*x + 2*e**(c
 + d*x)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x)*a*f + 2*int(1/(e**(4*
c + 4*d*x)*b*e + e**(4*c + 4*d*x)*b*f*x + 2*e**(3*c + 3*d*x)*a*e + 2*e**(3
*c + 3*d*x)*a*f*x + 2*e**(c + d*x)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*
x),x)*b*f + log(e + f*x))/(b*f)
```

3.382 $\int \frac{(e+fx)^3 \tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 1118

$$\int \frac{(e+fx)^3 \tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output
```
-(f*x+e)^3*sech(d*x+c)/b/d-3*a^3*f^2*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/b^
2/(a^2+b^2)/d^3-3*a^3*f*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/b^2/(a^2+b^2)/d^2-6
*a^2*f*(f*x+e)^2*arctan(exp(d*x+c))/b/(a^2+b^2)/d^2-6*I*a^2*f^3*polylog(3,
-I*exp(d*x+c))/b/(a^2+b^2)/d^4-6*I*a^2*f^2*(f*x+e)*polylog(2,I*exp(d*x+c))
/b/(a^2+b^2)/d^3+6*a^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1
/2)))/(a^2+b^2)^(3/2)/d^3-6*a^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^
2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^3-3*a^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+
c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2+3*a^2*f*(f*x+e)^2*polylog(2,-b
*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2+3/2*a^3*f^3*polylog(3
,-exp(2*d*x+2*c))/b^2/(a^2+b^2)/d^4+3*a*f^2*(f*x+e)*polylog(2,-exp(2*d*x+2
*c))/b^2/d^3+3*a*f*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/b^2/d^2-6*I*f^2*(f*x+e)*
polylog(2,-I*exp(d*x+c))/b/d^3-a^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2
)^(1/2)))/(a^2+b^2)^(3/2)/d+a^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(
1/2)))/(a^2+b^2)^(3/2)/d-6*a^2*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1
/2)))/(a^2+b^2)^(3/2)/d^4+6*a^2*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(
1/2)))/(a^2+b^2)^(3/2)/d^4-a*(f*x+e)^3/b^2/d+a^3*(f*x+e)^3*tanh(d*x+c)/b^2
/(a^2+b^2)/d+a^2*(f*x+e)^3*sech(d*x+c)/b/(a^2+b^2)/d+6*I*f^3*polylog(3,-I*
exp(d*x+c))/b/d^4-a*(f*x+e)^3*tanh(d*x+c)/b^2/d+a^3*(f*x+e)^3/b^2/(a^2+b^2
)/d-3/2*a*f^3*polylog(3,-exp(2*d*x+2*c))/b^2/d^4+6*f*(f*x+e)^2*arctan(exp(
d*x+c))/b/d^2-6*I*f^3*polylog(3,I*exp(d*x+c))/b/d^4+6*I*f^2*(f*x+e)*pol...
```

Mathematica [A] (warning: unable to verify)

Time = 7.53 (sec) , antiderivative size = 1071, normalized size of antiderivative = 0.96

$$\int \frac{(e+fx)^3 \tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
((f*(-12*a*d^3*e^2*E^(2*c)*x + 12*a*d^3*e^2*(1 + E^(2*c))*x + 12*a*d^3*e*f
*x^2 + 4*a*d^3*f^2*x^3 + 12*b*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] -
6*a*d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*b*d*
e*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) -
 PolyLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) - 6*a*d*e*(1 +
E^(2*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c +
 d*x))]) + (6*I)*b*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2
*x^2*Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*P
olyLog[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I
*E^(c + d*x)]) - a*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c
 + d*x))]) - 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c +
d*x))])))/((a^2 + b^2)*(1 + E^(2*c))) + (2*a^2*(-2*d^3*e^3*ArcTanh[(a + b*
E^(c + d*x))/Sqrt[a^2 + b^2]] + 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2])] + 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
 + b^2])] + d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 3
*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 3*d^3*e*f^2*
x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - d^3*f^3*x^3*Log[1 + (
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 3*d^2*f*(e + f*x)^2*PolyLog[2, (b*
E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 3*d^2*f*(e + f*x)^2*PolyLog[2, -((b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*d*e*f^2*PolyLog[3, (b*E^(c + ...
```

Rubi [A] (verified)

Time = 6.91 (sec) , antiderivative size = 949, normalized size of antiderivative = 0.85, number of steps used = 31, number of rules used = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 1.071$, Rules used = {6101, 5974, 3042, 4668, 3011, 2720, 6117, 3042, 4672, 26, 3042, 26, 4201, 2620, 3011, 2720, 6107, 3042, 3803, 25, 2694, 27, 2620, 3011, 7143, 7163, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6101}$$

$$\frac{\int (e+fx)^3\text{sech}(c+dx)\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{5974}$$

$$\frac{\frac{3f\int (e+fx)^2\text{sech}(c+dx)dx}{d}-\frac{(e+fx)^3\text{sech}(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{b}$$

$$\Big\downarrow \text{4668}$$

$$-\frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(-\frac{2if\int (e+fx)\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{2if\int (e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}(2,i}{d}\right.}{d}\right.}{d}}{b}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de}{d^2}\right.}{d}\right.}{d}}{b}$$

$$\Big\downarrow \text{6117}$$

$$-\frac{a\left(\frac{\int (e+fx)^3\text{sech}^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de}{d^2}\right.}{d}}{d}}{b}$$

$\downarrow$ 3042

$$\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de}{d^2}\right.}{d}}{d}}{b}$$
$$\frac{a\left(-\frac{a\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{b}\right)}{b}$$

$\downarrow$ 4672

$$\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de}{d^2}\right.}{d}}{d}}{b}$$
$$\frac{a\left(-\frac{a\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{3if\int -i(e+fx)^2\tanh(c+dx)dx}{d}}{b}\right)}{b}$$

$\downarrow$ 26

$$-\frac{a\left(\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{3f\int (e+fx)^2\tanh(c+dx)dx}{d}}{b}-\frac{a\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de}{d^2}\right.}{d}}{d}}{b}$$

$\downarrow$ 3042

$$\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de}{d^2}\right.}{d}}{d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{3f\int -i(e+fx)^2\tan(ic+idx)dx}{d}}{b}\right)}{b}$$

$\downarrow$ 26

$$\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de}{d^2}\right.}{d}}{d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\int(e+fx)^2\tan(ic+idx)dx}{d}}{b}\right)}{b}$$

$\downarrow$ 4201

$$\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de}{d^2}\right.}{d}}{d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\int\frac{e^{2(c+dx)}(e+fx)^2}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}}{b}\right)}{b}$$

$\downarrow$ 2620

$$\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de\cdots}{d^2}\cdots\right.}{d}\cdots\right.}{d}}{b}\cdots$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int(e+fx)\log\left(1+e^{2(c+dx)}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{b}\right)}{b}$$

$\downarrow$ 3011

$$\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de\cdots}{d^2}\cdots\right.}{d}\cdots\right.}{d}}{b}\cdots$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{2d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\cdots\right)}{2d}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{b}\cdots\right.}{b}$$

$\downarrow$ 2720

$$\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de\cdots}{d^2}\cdots\right.}{d}\cdots\right.}{d}}{b}\cdots$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{b}\cdots\right.}{b}$$

$\downarrow$ 6107

$$\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de\ldots}{d^2}\ldots\right.}{d}\ldots}{d}}{b}$$

$$\frac{a\left(-\frac{a\left(\frac{b^2\int\frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e\ldots}{}\right.}{}\ldots\right.\right.}{}\ldots\right.}{b}$$

$\downarrow$ 3042

$$\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de\ldots}{d^2}\ldots\right.}{d}\ldots}{d}}{b}$$

$$\frac{a\left(\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\ldots\right.}{d}}{b}\ldots\right.}{b}$$

$\downarrow$ 3803

$$\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de\cdots}{d^2}\cdots\right.}{d}\cdots\right.}{d}\cdots}{b}$$

$$\frac{a\left(-\frac{a\left(\frac{2b^2\int -\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\cdots\right.}{2d}\cdots\right.\right.}{\cdots}\cdots\right.}{b}\cdots$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de\cdots}{d^2}\cdots\right.}{d}\cdots\right.}{d}\cdots}{b}$$

$$\frac{a\left(-\frac{a\left(\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\int \frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}\right)}{b}+\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}\cdots\right.\right.}{\cdots}\cdots\right.}{b}\cdots$$

$$\Big\downarrow \; 2694$$

$$\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de\cdots}{d^2}\cdots\right.}{d}\cdots\right.}{d}\cdots}{b}$$

$$\frac{a\left(-\frac{a\left(\frac{\int(e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int-\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int-\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{b}+\frac{(e+fx)^3\tanh(c+\cdots}{d}\cdots\right.}{b}$$

$\downarrow$ 27

$$\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de\cdots}{d^2}\cdots\right.}{d}\cdots\right.}{d}\cdots}{b}$$

$$\frac{a\left(-\frac{a\left(\frac{\int(e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{b}+\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2\cdots\right.}{\cdots}\cdots\right.}{b}$$

$\downarrow$ 2620

$$\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}\cdots\right.}{d}\right.}{d}}{b}$$

$$a\left(-\frac{a\left(\frac{\int (e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\cdots\right.}{bd}\cdots\right.}{\cdots}\right.}{a^2+b^2}\right.}{b}\right.$$

$\downarrow$ 3011

$$\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de}{d^2}\right.}{d}\right.}{d}}{b}$$

$$a\left(-\frac{a\left(\frac{\int(e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{Po}}{}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{}\right.}{}\right.$$

↓ 7143

$$-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}+\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{d^2}-\cdots\right.}{d}\right.}{d}$$

$$b$$

$$a\left(-\frac{a\left(\frac{\int(e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{Po}\cdots}{\cdots}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{\cdots}\right.}{\cdots}\right.$$

↓ 7163

$$-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}+\frac{2if\left(\frac{f\,\mathrm{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\,\mathrm{PolyLog}\left(3,ie^{c+dx}\right)}{d^2}-\cdots\right.}{d}\right.}{d}$$

$$b$$

$$a\left(-\frac{a\left(\frac{\int(e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))\,dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int\mathrm{PolyLog}\left(\cdots\right.}{\cdots}\right.}{d}\right.}{\cdots}\right.}{2\sqrt{a^2+b^2}}\right.}{\cdots}\right.}{\cdots}\right.$$

↓ 2720

$$\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}+\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{d^2}-\cdots\right)}{d}\cdots\right)}{d}}{b}$$

$$a\left(-\frac{a\left(\frac{\int(e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))\,dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\,\mathrm{P}\cdots}{\cdots}\right)}{d}\cdots\right)}{\cdots}\cdots\right)}{2\sqrt{a^2+b^2}\cdots}\cdots}{\cdots}\cdots\right)}{\cdots}\cdots\right)$$

↓ 7143

$$-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}+\frac{2if\left(\frac{f\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\,\text{PolyLog}\left(3,ie^{c+dx}\right)}{d^2}-\cdots\right.}{d}\cdots\right.}{d}$$

$$b$$

$$a\left(-\frac{a\left(\frac{\int(e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))\,dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\,\text{PolyLog}\left(4,\cdots\right.}{\cdots}\right)}{d}\cdots\right.}{bd}\cdots\right)}{2\sqrt{a^2+b^2}}\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

↓ 7293

$$-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}+\frac{2if\left(\frac{f\,\mathrm{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\,\mathrm{PolyLog}\left(3,ie^{c+dx}\right)}{d^2}-\cdots\right.}{\cdots}\right.}{d}$$

$$b$$

$$a\left(a\left(\frac{\int\left(a(e+fx)^3\operatorname{sech}^2(c+dx)-b(e+fx)^3\operatorname{sech}(c+dx)\tanh(c+dx)\right)dx}{a^2+b^2}-\frac{2b^2\left(b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\mathrm{PolyLog}\left(3,-\frac{be^{c+\cdots}}{a+\sqrt{a\cdots}}\right)}{d}\cdots\right.}{\cdots}\right.}{\cdots}\right.\right.}{\cdots}\right.\right.$$

$$a-\frac{\cdots}{\cdots}$$

$\Big\downarrow$ 2009

$$\frac{\frac{3f\left(\frac{2\arctan\left(e^{c+dx}\right)(e+fx)^2}{d}+\frac{2if\left(\frac{f\,\mathrm{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\,\mathrm{PolyLog}\left(3,ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}\right)}{d}}{b}$$

$$a\left(\frac{\frac{\tanh(c+dx)(e+fx)^3}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{2d}-\frac{f\left(\frac{f\,\mathrm{PolyLog}\left(3,-e^{2(c+dx)}\right)}{4d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{b}-\frac{a\left(-\frac{6\cdots}{\cdots}\right.}{\cdots}\right.$$

input
```
Int[((e + f*x)^3*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
((3*f*((2*(e + f*x)^2*ArcTan[E^(c + d*x)])/d + ((2*I)*f*(-(((e + f*x)*Poly
Log[2, (-I)*E^(c + d*x)])/d) + (f*PolyLog[3, (-I)*E^(c + d*x)])/d^2))/d -
((2*I)*f*(-(((e + f*x)*PolyLog[2, I*E^(c + d*x)])/d) + (f*PolyLog[3, I*E^(
c + d*x)])/d^2))/d))/d - ((e + f*x)^3*Sech[c + d*x])/d)/b - (a*((((3*I)*f*
(((-1/3*I)*(e + f*x)^3)/f + (2*I)*(((e + f*x)^2*Log[1 + E^(2*(c + d*x))])/
(2*d) - (f*(-1/2*((e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/d + (f*PolyLog[3
, -E^(2*(c + d*x))])/(4*d^2)))/d)))/d + ((e + f*x)^3*Tanh[c + d*x])/d)/b -
 (a*((-2*b^2*(-1/2*(b*(((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - S
qrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2]))])/d^2))/d))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^3*Log[1 + (b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2,
 -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[
3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/(2*Sqrt[a^2 + b^2])))/(
a^2 + b^2) + ((a*(e + f*x)^3)/d - (6*b*f*(e + f*x)^2*ArcTan[E^(c + d*x)])/
d^2 - (3*a*f*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/d^2 + ((6*I)*b*f^2*(e +
 f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d^3 - ((6*I)*b*f^2*(e + f*x)*PolyLog[2
, I*E^(c + d*x)])/d^3 - (3*a*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))]...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6101
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sech[
c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*Sech[c
+ d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6117
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x],
x] - Simp[a/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^3 \tanh (dx+c)^2}{a+b\sinh (dx+c)}dx$$

input
```
int((f*x+e)^3*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 6494 vs. 2(1024) = 2048.

Time = 0.25 (sec) , antiderivative size = 6494, normalized size of antiderivative = 5.81

$$\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^3\tanh^2\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate((f*x+e)**3*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*tanh(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\tanh\left(dx+c\right)^2}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^3*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-3*a*e^2*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2
 + b^2)*d^2)) + 6*b*f^3*integrate(x^2*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) +
 b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 6*a*f^3*integrate(x^2/(a^2*d
*e^(2*d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + 12*b*e*f^2
*integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) +
a^2*d + b^2*d), x) - 12*a*e*f^2*integrate(x/(a^2*d*e^(2*d*x + 2*c) + b^2*d
*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + e^3*(a^2*log((b*e^(-d*x - c) - a -
 sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^2 + b^2)^(3/
2)*d) - 2*(b*e^(-d*x - c) + a)/((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c))
*d)) + 6*b*e^2*f*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) + 2*(a*f^3*x^3 + 3*
a*e*f^2*x^2 + 3*a*e^2*f*x - (b*f^3*x^3*e^c + 3*b*e*f^2*x^2*e^c + 3*b*e^2*f
*x*e^c)*e^(d*x))/(a^2*d + b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x
)) + integrate(-2*(a^2*f^3*x^3*e^c + 3*a^2*e*f^2*x^2*e^c + 3*a^2*e^2*f*x*e
^c)*e^(d*x)/(a^2*b + b^3 - (a^2*b*e^(2*c) + b^3*e^(2*c))*e^(2*d*x) - 2*(a^
3*e^c + a*b^2*e^c)*e^(d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)^3*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\tanh(c+d\,x)^2\,(e+f\,x)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((tanh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((tanh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3 \tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^3*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(12*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**4*b*d**2*e**2*f + 24*e**(2*c +
2*d*x)*atan(e**(c + d*x))*a**4*b*d*e*f**2 + 24*e**(2*c + 2*d*x)*atan(e**(c
 + d*x))*a**4*b*f**3 + 24*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**2*b**3*d*
*2*e**2*f + 48*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**2*b**3*d*e*f**2 + 48
*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**2*b**3*f**3 + 12*e**(2*c + 2*d*x)*
atan(e**(c + d*x))*b**5*d**2*e**2*f + 24*e**(2*c + 2*d*x)*atan(e**(c + d*x
))*b**5*d*e*f**2 + 24*e**(2*c + 2*d*x)*atan(e**(c + d*x))*b**5*f**3 + 12*a
tan(e**(c + d*x))*a**4*b*d**2*e**2*f + 24*atan(e**(c + d*x))*a**4*b*d*e*f*
*2 + 24*atan(e**(c + d*x))*a**4*b*f**3 + 24*atan(e**(c + d*x))*a**2*b**3*d
**2*e**2*f + 48*atan(e**(c + d*x))*a**2*b**3*d*e*f**2 + 48*atan(e**(c + d*
x))*a**2*b**3*f**3 + 12*atan(e**(c + d*x))*b**5*d**2*e**2*f + 24*atan(e**(
c + d*x))*b**5*d*e*f**2 + 24*atan(e**(c + d*x))*b**5*f**3 + 4*e**(2*c + 2*
d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*
*2*b**2*d**3*e**3*i + 4*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sq
rt(a**2 + b**2))*a**2*b**2*d**3*e**3*i + 16*e**(5*c + 2*d*x)*int((e**(3*d*
x)*x**3)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)*b +
 4*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**6
*d**4*f**3 + 32*e**(5*c + 2*d*x)*int((e**(3*d*x)*x**3)/(e**(6*c + 6*d*x)*b
 + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - e**(
2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*b**2*d**4*f**3 + 16*e**(...
```

3.383 $\int \frac{(e+fx)^2 \tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 772

$$\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a(e+fx)^2}{b^2d}+\frac{a^3(e+fx)^2}{b^2(a^2+b^2)\,d}+\frac{4f(e+fx)\arctan\left(e^{c+dx}\right)}{bd^2}-\frac{4a^2f(e+fx)\arctan\left(e^{c+dx}\right)}{b(a^2+b^2)\,d^2}+\frac{a^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}-\frac{a^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}+\frac{2af(e+fx)\log\left(1+e^{2(c+dx)}\right)}{b^2d^2}-\frac{2a^3f(e+fx)\log\left(1+e^{2(c+dx)}\right)}{b^2(a^2+b^2)\,d^2}-\frac{2if^2\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{bd^3}+\frac{2ia^2f^2\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{b(a^2+b^2)\,d^3}+\frac{2if^2\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{bd^3}-\frac{2ia^2f^2\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{b(a^2+b^2)\,d^3}+\frac{2a^2f(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^2}-\frac{2a^2f(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^2}+\frac{af^2\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{b^2d^3}-\frac{a^3f^2\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{b^2(a^2+b^2)\,d^3}-\frac{2a^2f^2\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^3}+\frac{2a^2f^2\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^3}-\frac{(e+fx)^2\mathrm{sech}(c+dx)}{bd}+\frac{a^2(e+fx)^2\mathrm{sech}(c+dx)}{b(a^2+b^2)\,d}$$

$$\frac{a(e+fx)^2\tanh(c+dx)}{} + \frac{a^3(e+fx)^2\tanh(c+dx)}{}$$

output
```
-a*(f*x+e)^2/b^2/d+a^3*(f*x+e)^2/b^2/(a^2+b^2)/d+4*f*(f*x+e)*arctan(exp(d*
x+c))/b/d^2-4*a^2*f*(f*x+e)*arctan(exp(d*x+c))/b/(a^2+b^2)/d^2+a^2*(f*x+e)
^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d-a^2*(f*x+e)^2*
ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d+2*a*f*(f*x+e)*ln(
1+exp(2*d*x+2*c))/b^2/d^2-2*a^3*f*(f*x+e)*ln(1+exp(2*d*x+2*c))/b^2/(a^2+b^
2)/d^2-2*I*a^2*f^2*polylog(2,I*exp(d*x+c))/b/(a^2+b^2)/d^3+2*I*a^2*f^2*pol
ylog(2,-I*exp(d*x+c))/b/(a^2+b^2)/d^3+2*I*f^2*polylog(2,I*exp(d*x+c))/b/d^
3-2*I*f^2*polylog(2,-I*exp(d*x+c))/b/d^3+2*a^2*f*(f*x+e)*polylog(2,-b*exp(
d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2-2*a^2*f*(f*x+e)*polylog(2,
-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2+a*f^2*polylog(2,-ex
p(2*d*x+2*c))/b^2/d^3-a^3*f^2*polylog(2,-exp(2*d*x+2*c))/b^2/(a^2+b^2)/d^3
-2*a^2*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^
3+2*a^2*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d
^3-(f*x+e)^2*sech(d*x+c)/b/d+a^2*(f*x+e)^2*sech(d*x+c)/b/(a^2+b^2)/d-a*(f*
x+e)^2*tanh(d*x+c)/b^2/d+a^3*(f*x+e)^2*tanh(d*x+c)/b^2/(a^2+b^2)/d
```

Mathematica [A] (warning: unable to verify)

Time = 4.83 (sec) , antiderivative size = 633, normalized size of antiderivative = 0.82

$$\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{f\left(4ad^2ee^{2c}x-4ad^2e\left(1+e^{2c}\right)x+2ad^2e^{2c}fx^2-2ad^2\left(1+e^{2c}\right)fx^2-4bde\left(1+e^{2c}\right)\arctan\left(e^{c+dx}\right)+2ade\left(1+e^{2c}\right)\left(2dx-\log\left(1+e^{2(c+dx)}\right)\right)+2ib(1\cdots}{\cdots}$$

input
```
Integrate[((e + f*x)^2*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-(((f*(4*a*d^2*e*E^(2*c)*x - 4*a*d^2*e*(1 + E^(2*c))*x + 2*a*d^2*E^(2*c)*f
*x^2 - 2*a*d^2*(1 + E^(2*c))*f*x^2 - 4*b*d*e*(1 + E^(2*c))*ArcTan[E^(c + d
*x)] + 2*a*d*e*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (2*I)*b*
(1 + E^(2*c))*f*(d*x*(-Log[1 - I*E^(c + d*x)] + Log[1 + I*E^(c + d*x)]) +
PolyLog[2, (-I)*E^(c + d*x)] - PolyLog[2, I*E^(c + d*x)]) + a*(1 + E^(2*c)
)*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x))]
)))/((a^2 + b^2)*(1 + E^(2*c))) + (a^2*(2*d^2*e^2*ArcTanh[(a + b*E^(c + d*
x))/Sqrt[a^2 + b^2]] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*d
^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[
1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*d*f*(e + f*x)*PolyLog[2, (b
*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*d*f*(e + f*x)*PolyLog[2, -((b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a
+ Sqrt[a^2 + b^2])] - 2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2]))]))/(a^2 + b^2)^(3/2) + (d^2*(e + f*x)^2*Sech[c + d*x]*(b + a*Sech[c]
*Sinh[d*x]))/(a^2 + b^2))/d^3)
```

Rubi [A] (verified)

Time = 5.41 (sec) , antiderivative size = 670, normalized size of antiderivative = 0.87, number of steps used = 29, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6101, 5974, 3042, 4668, 2715, 2838, 6117, 3042, 4672, 26, 3042, 26, 4201, 2620, 2715, 2838, 6107, 3042, 3803, 25, 2694, 27, 2620, 3011, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6101}$$

$$\frac{\int (e+fx)^2\text{sech}(c+dx)\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{5974}$$

$$\frac{\frac{2f\int (e+fx)\text{sech}(c+dx)dx}{d}-\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{b}$$

$\Big\downarrow$ 4668

$$-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(-\frac{if\int \log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int \log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)}{d}}{b}$$

$\Big\downarrow$ 2715

$$-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)}{d}}{b}$$

$\Big\downarrow$ 2838

$$-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}$$

$\Big\downarrow$ 6117

$$-\frac{a\left(\frac{\int (e+fx)^2\text{sech}^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{b}\right)}{b}$$

$\downarrow$ 4672

$$\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2if\int-i(e+fx)\tanh(c+dx)dx}{d}}{b}\right)}{b}$$

$\downarrow$ 26

$$-\frac{a\left(\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2f\int(e+fx)\tanh(c+dx)dx}{d}}{b}-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}$$

$\downarrow$ 3042

$$\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2f\int-i(e+fx)\tan(ic+idx)dx}{d}}{b}\right)}{b}$$

$\downarrow$ 26

$$\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\int(e+fx)\tan(ic+idx)dx}{d}}{b}\right)}{b}$$

$\downarrow$ 4201

$$\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\int\frac{e^{2(c+dx)}(e+fx)}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}\right)}{b}$$

$$\Big\downarrow \; 2620$$

$$\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int\log\left(1+e^{2(c+dx)}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}\right)}{b}$$

$$\Big\downarrow \; 2715$$

$$\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int e^{-2(c+dx)}\log\left(1+e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}\right)}{b}$$

$$\Big\downarrow \; 2838$$

$$\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}\right)}{b}$$

$$\Big\downarrow \; 6107$$

$$\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-$$

$$\frac{a\left(-\frac{a\left(\frac{b^2\int\frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\cdots}{}\right.\right.}{d}}{b}\cdots\right.}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}-\frac{a\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\cdots\right.}{b}\cdots\right.}{b}$$

$$\Big\downarrow \; 3803$$

$$\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-$$

$$\frac{a\left(-\frac{a\left(\frac{2b^2\int-\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{\int(e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}\cdots\right.\right.}{}}{b}\cdots\right.}{b}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-$$

$$\frac{a\left(-\frac{a\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\int\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}\right)}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\cdots\right.\right.}{}}{b}\cdots\right.}{b}$$

$$\Big\downarrow \; 2694$$

$$\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int-\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int-\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{b}+\frac{(e+fx)^2\tanh(c+d}{d}\cdots\right.}{b}$$

$\downarrow$ 27

$$\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{b}+\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2\cdots\right.}{\cdots}\right.}{b}$$

$\downarrow$ 2620

$$\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-$$

$$a\left(-\frac{a\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{b}{a-}\right.}{bd}\right.}{}\right.}{a^2+b^2}\right.}{b}\right.$$

$\downarrow$ 3011

$$\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-$$

$$a\left(-\frac{a\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\cdots\right.}{d}\cdots\right.}{bd}\cdots\right.}{2\sqrt{a^2+b^2}}\cdots\right.}{\cdots}\right.}{b}\cdots\right.$$

↓ 2720

$$\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-$$

$$a\left(-\frac{a\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx}{}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{}\right.}{}\right.$$

↓ 7143

$$\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-$$

$$a\left(-\frac{a\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{b}{a+}\right.}{d}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{}\right.}{b}\right.$$

↓ 7293

$$\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-$$

$$a\left(-\frac{a\left(\frac{\int\left(a(e+fx)^2\operatorname{sech}^2(c+dx)-b(e+fx)^2\operatorname{sech}(c+dx)\tanh(c+dx)\right)dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\cdots\right.}{b\cdots}\right.}{2\sqrt{a^2+b^2}}\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 2009

$$\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-$$

$$a\left(\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}-\frac{a-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\right.}{}\right.}{}}{}\right.$$

input `Int[((e + f*x)^2*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]`

output
```
((2*f*((2*(e + f*x)*ArcTan[E^(c + d*x)])/d - (I*f*PolyLog[2, (-I)*E^(c + d
*x)])/d^2 + (I*f*PolyLog[2, I*E^(c + d*x)])/d^2))/d - ((e + f*x)^2*Sech[c
+ d*x])/d)/b - (a*((((2*I)*f*(((-1/2*I)*(e + f*x)^2)/f + (2*I)*(((e + f*x)
*Log[1 + E^(2*(c + d*x))])/(2*d) + (f*PolyLog[2, -E^(2*(c + d*x))])/(4*d^2
))))/d + ((e + f*x)^2*Tanh[c + d*x])/d)/b - (a*((-2*b^2*(-1/2*(b*(((e + f*
x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e +
 f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (f*PolyLo
g[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/Sqrt[a^2 + b
^2] + (b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*
d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
)])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/(b
*d)))/(2*Sqrt[a^2 + b^2])))/(a^2 + b^2) + ((a*(e + f*x)^2)/d - (4*b*f*(e +
 f*x)*ArcTan[E^(c + d*x)])/d^2 - (2*a*f*(e + f*x)*Log[1 + E^(2*(c + d*x))]
)/d^2 + ((2*I)*b*f^2*PolyLog[2, (-I)*E^(c + d*x)])/d^3 - ((2*I)*b*f^2*Poly
Log[2, I*E^(c + d*x)])/d^3 - (a*f^2*PolyLog[2, -E^(2*(c + d*x))])/d^3 + (b
*(e + f*x)^2*Sech[c + d*x])/d + (a*(e + f*x)^2*Tanh[c + d*x])/d)/(a^2 + b^
2)))/b))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6101
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sech[
c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*Sech[c
+ d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6117
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x],
x] - Simp[a/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^2 \tanh(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3661 vs. 2(712) = 1424.

Time = 0.18 (sec) , antiderivative size = 3661, normalized size of antiderivative = 4.74

$$\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2\tanh^2{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**2*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*tanh(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\tanh{(dx+c)}^2}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^2*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-2*a*e*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2 +
 b^2)*d^2)) + 4*b*f^2*integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2
*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 4*a*f^2*integrate(x/(a^2*d*e^(2*
d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + e^2*(a^2*log((b*
e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2))
)/((a^2 + b^2)^(3/2)*d) - 2*(b*e^(-d*x - c) + a)/((a^2 + b^2 + (a^2 + b^2)
*e^(-2*d*x - 2*c))*d)) + 4*b*e*f*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) + 2
*(a*f^2*x^2 + 2*a*e*f*x - (b*f^2*x^2*e^c + 2*b*e*f*x*e^c)*e^(d*x))/(a^2*d
+ b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)) + integrate(-2*(a^2*f
^2*x^2*e^c + 2*a^2*e*f*x*e^c)*e^(d*x)/(a^2*b + b^3 - (a^2*b*e^(2*c) + b^3*
e^(2*c))*e^(2*d*x) - 2*(a^3*e^c + a*b^2*e^c)*e^(d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)^2*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\tanh(c+d\,x)^2\,(e+f\,x)^2}{a+b\,\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((tanh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((tanh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^2*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(4*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**4*b*d*e*f + 4*e**(2*c + 2*d*x)*a
tan(e**(c + d*x))*a**4*b*f**2 + 8*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**2
*b**3*d*e*f + 8*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**2*b**3*f**2 + 4*e**
(2*c + 2*d*x)*atan(e**(c + d*x))*b**5*d*e*f + 4*e**(2*c + 2*d*x)*atan(e**(
c + d*x))*b**5*f**2 + 4*atan(e**(c + d*x))*a**4*b*d*e*f + 4*atan(e**(c + d
*x))*a**4*b*f**2 + 8*atan(e**(c + d*x))*a**2*b**3*d*e*f + 8*atan(e**(c + d
*x))*a**2*b**3*f**2 + 4*atan(e**(c + d*x))*b**5*d*e*f + 4*atan(e**(c + d*x
))*b**5*f**2 + 2*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i
 + a*i)/sqrt(a**2 + b**2))*a**2*b**2*d**2*e**2*i + 2*sqrt(a**2 + b**2)*ata
n((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b**2*d**2*e**2*i + 8*e*
*(5*c + 2*d*x)*int((e**(3*d*x)*x**2)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d
*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b + 2
*e**(c + d*x)*a - b),x)*a**6*d**3*f**2 + 16*e**(5*c + 2*d*x)*int((e**(3*d*
x)*x**2)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)*b +
 4*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4
*b**2*d**3*f**2 + 8*e**(5*c + 2*d*x)*int((e**(3*d*x)*x**2)/(e**(6*c + 6*d*
x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a -
e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b**4*d**3*f**2 + 16*e**
(5*c + 2*d*x)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*
a + e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b + 2*...
```

3.384 $\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 385

$$\begin{aligned}\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{f\arctan(\sinh(c+dx))}{bd^2}-\frac{a^2f\arctan(\sinh(c+dx))}{b(a^2+b^2)d^2} \\ &+\frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d} \\ &-\frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d} \\ &+\frac{af\log(\cosh(c+dx))}{b^2d^2}-\frac{a^3f\log(\cosh(c+dx))}{b^2(a^2+b^2)d^2} \\ &+\frac{a^2f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d^2} \\ &-\frac{a^2f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d^2} \\ &-\frac{(e+fx)\text{sech}(c+dx)}{bd}+\frac{a^2(e+fx)\text{sech}(c+dx)}{b(a^2+b^2)d} \\ &-\frac{a(e+fx)\tanh(c+dx)}{b^2d}+\frac{a^3(e+fx)\tanh(c+dx)}{b^2(a^2+b^2)d}\end{aligned}$$

output
```
f*arctan(sinh(d*x+c))/b/d^2-a^2*f*arctan(sinh(d*x+c))/b/(a^2+b^2)/d^2+a^2*
(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d-a^2*(f*x+
e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d+a*f*ln(cosh(d*
x+c))/b^2/d^2-a^3*f*ln(cosh(d*x+c))/b^2/(a^2+b^2)/d^2+a^2*f*polylog(2,-b*e
xp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2-a^2*f*polylog(2,-b*exp(
d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2-(f*x+e)*sech(d*x+c)/b/d+a^
2*(f*x+e)*sech(d*x+c)/b/(a^2+b^2)/d-a*(f*x+e)*tanh(d*x+c)/b^2/d+a^3*(f*x+e
)*tanh(d*x+c)/b^2/(a^2+b^2)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.76 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.86

$$\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{-\frac{2if\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a-ib}+\frac{2if\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a+ib}+\frac{f\log(\cosh(c+dx))}{a-ib}+\frac{f\log(\cosh(c+dx))}{a+ib}+\frac{2a^2\left(-2de\,\text{arctanh}\left(\frac{a+be}{\sqrt{a^2}}\right.\right.}{}}{}$$

input
```
Integrate[((e + f*x)*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(((-2*I)*f*ArcTan[Tanh[(c + d*x)/2]])/(a - I*b) + ((2*I)*f*ArcTan[Tanh[(c
+ d*x)/2]])/(a + I*b) + (f*Log[Cosh[c + d*x]])/(a - I*b) + (f*Log[Cosh[c +
 d*x]])/(a + I*b) + (2*a^2*(-2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 +
b^2]] + 2*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + f*(c + d*x)*L
og[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1 + (b*E^(
c + d*x))/(a + Sqrt[a^2 + b^2])] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt
[a^2 + b^2])] - f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(
a^2 + b^2)^(3/2) - (2*d*(e + f*x)*Sech[c + d*x]*(b + a*Sinh[c + d*x]))/(a^
2 + b^2))/(2*d^2)
```

Rubi [A] (verified)

Time = 2.68 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.92, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {6101, 5974, 3042, 4257, 6117, 3042, 4672, 26, 3042, 26, 3956, 6107, 3042, 3803, 25, 2694, 27, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6101}$$

$$\frac{\int (e+fx)\text{sech}(c+dx)\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{5974}$$

$$\frac{\frac{f\int \text{sech}(c+dx)dx}{d}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)\text{sech}(c+dx)}{d}+\frac{f\int \csc\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{6117}$$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\int (e+fx)\text{sech}^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{b}\right)}{b}$$

$$\Big\downarrow \ 4672$$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{if\int-i\tanh(c+dx)dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \ 26$$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int\tanh(c+dx)dx}{d}}{b}-\frac{a\int\frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int-i\tan(ic+idx)dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \ 26$$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\tanh(c+dx)}{d}+\frac{if\int\tan(ic+idx)dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \ 3956$$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\int\frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \ 6107$$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{b^2\int\frac{e+fx}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int(e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{a^2+b^2}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 3803

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{2b^2\int-\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{\int(e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 25

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int(e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\int\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 2694

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e \cdots}{2\left(a+be^{c+dx}+ \cdots\right)}}{\sqrt{a^2+b \cdots}}\right)}{a^2+b^2}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}} \cdots}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 2620$$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-$$

$$a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int(e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{}\right.}{b}\right.$$

$$b$$

$\downarrow$ 2715

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-$$

$$a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int(e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e}{a+}\right.}{b}\right.}{2\sqrt{a^2+b^2}}\right.}{}\right.}{b}\right.$$

$$b$$

$\downarrow$ 2838

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int(e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+d}}{\sqrt{a^2+b^2}}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{}\right.}{b}\right.}{b}$$

$\downarrow$ 7293

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int\left(a(e+fx)\text{sech}^2(c+dx)-b(e+fx)\text{sech}(c+dx)\tanh(c+dx)\right)dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}\right.}{2\sqrt{a}}\right.}{}\right.}{b}\right.}{b}$$

$\downarrow$ 2009

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\operatorname{sech}(c+dx)}{d}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{-\frac{af\log(\cosh(c+dx))}{d^2}+\frac{a(e+fx)\tanh(c+dx)}{d}-\frac{bf\arctan(\sinh(c+dx))}{d^2}+\frac{b(e+fx)\operatorname{sech}(c+dx)}{d}}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{f}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.}{b}$$

input
```
Int[((e + f*x)*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
((f*ArcTan[Sinh[c + d*x]])/d^2 - ((e + f*x)*Sech[c + d*x])/d)/b - (a*((-((
f*Log[Cosh[c + d*x]])/d^2) + ((e + f*x)*Tanh[c + d*x])/d)/b - (a*((-2*b^2*
(-1/2*(b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d)
 + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2)))/Sqrt
[a^2 + b^2] + (b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2
)))/(2*Sqrt[a^2 + b^2])))/(a^2 + b^2) + (-((b*f*ArcTan[Sinh[c + d*x]])/d^2
) - (a*f*Log[Cosh[c + d*x]])/d^2 + (b*(e + f*x)*Sech[c + d*x])/d + (a*(e +
 f*x)*Tanh[c + d*x])/d)/(a^2 + b^2)))/b))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6101
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sech[
c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*Sech[c
+ d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6117
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x],
x] - Simp[a/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1927 vs. $2(365) = 730$.

Time = 1.04 (sec) , antiderivative size = 1928, normalized size of antiderivative = 5.01

method	result	size
risch	Expression too large to display	1928

input
```
int((f*x+e)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*(f*x+e)*(-b*exp(d*x+c)+a)/d/(a^2+b^2)/(1+exp(2*d*x+2*c))-2/d^2/(a^2+b^2)
^(3/2)*a^4*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2
)^(1/2)))*c+2/d^2/(a^2+b^2)^(3/2)*a^2*f/(2*a^2+2*b^2)*dilog((-b*exp(d*x+c)
+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*b^2-2/d^2/(a^2+b^2)^(3/2)*a^2*f/
(2*a^2+2*b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*
b^2+2/d^2/(a^2+b^2)^(1/2)*c*a^2*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c
)+2*a)/(a^2+b^2)^(1/2))+2/d^2/(a^2+b^2)^(3/2)*c*a^4*f/(2*a^2+2*b^2)*arctan
h(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d^2/(a^2+b^2)^(3/2)*b^2*f/(2
*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^2+2/d/(a^2
+b^2)^(3/2)*a^4*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(
a^2+b^2)^(1/2)))*x-2/d/(a^2+b^2)^(3/2)*a^4*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c
)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+2/d^2/(a^2+b^2)^(3/2)*a^4*f/(2
*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2
*b^2/d/(a^2+b^2)^(3/2)*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a
^2+b^2)^(1/2))*a^2+2/d^2/(a^2+b^2)^(1/2)*b^2*f/(2*a^2+2*b^2)*arctanh(1/2*(
2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/d^2/(a^2+b^2)^(3/2)*a^4*f/(2*a^2+2*
b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-2/d^2/(
a^2+b^2)^(3/2)*a^4*f/(2*a^2+2*b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/
(a+(a^2+b^2)^(1/2)))+4/d^2/(a^2+b^2)^(1/2)*a^2*f/(2*a^2+2*b^2)*arctanh(1/2
*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d^2/(a^2+b^2)*b^2*f/(2*a^2+2*b...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1337 vs. $2(363) = 726$.

Time = 0.14 (sec) , antiderivative size = 1337, normalized size of antiderivative = 3.47

$$\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(2*(a^3 + a*b^2)*d*f*x*cosh(d*x + c)^2 + 2*(a^3 + a*b^2)*d*f*x*sinh(d*x +
 c)^2 - 2*(a^3 + a*b^2)*d*e - (a^2*b*f*cosh(d*x + c)^2 + 2*a^2*b*f*cosh(d*
x + c)*sinh(d*x + c) + a^2*b*f*sinh(d*x + c)^2 + a^2*b*f)*sqrt((a^2 + b^2)
/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (a^2*b*f*cosh(d*x + c)^2 +
2*a^2*b*f*cosh(d*x + c)*sinh(d*x + c) + a^2*b*f*sinh(d*x + c)^2 + a^2*b*f)
*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(
d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (a^2*b*d*e
 - a^2*b*c*f + (a^2*b*d*e - a^2*b*c*f)*cosh(d*x + c)^2 + 2*(a^2*b*d*e - a^
2*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*b*d*e - a^2*b*c*f)*sinh(d*x +
c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*
b*sqrt((a^2 + b^2)/b^2) + 2*a) - (a^2*b*d*e - a^2*b*c*f + (a^2*b*d*e - a^2
*b*c*f)*cosh(d*x + c)^2 + 2*(a^2*b*d*e - a^2*b*c*f)*cosh(d*x + c)*sinh(d*x
 + c) + (a^2*b*d*e - a^2*b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log
(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
- (a^2*b*d*f*x + a^2*b*c*f + (a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)^2 + 2
*(a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*b*d*f*x + a^
2*b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*
sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
- b)/b) + (a^2*b*d*f*x + a^2*b*c*f + (a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x...
```

Sympy [F]

$$\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\tanh^2{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*tanh(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\tanh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
(2*a^2*integrate(-x*e^(d*x + c)/(a^2*b + b^3 - (a^2*b*e^(2*c) + b^3*e^(2*c
))*e^(2*d*x) - 2*(a^3*e^c + a*b^2*e^c)*e^(d*x)), x) - 2*(b*x*e^(d*x + c) -
 a*x)/(a^2*d + b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)) - 2*a*x/
((a^2 + b^2)*d) + 2*b*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) + a*log(e^(2*d
*x + 2*c) + 1)/((a^2 + b^2)*d^2))*f + e*(a^2*log((b*e^(-d*x - c) - a - sqr
t(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^2 + b^2)^(3/2)*d
) - 2*(b*e^(-d*x - c) + a)/((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c))*d))
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh(c+d\,x)^2\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((tanh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((tanh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(2*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**4*b*f + 4*e**(2*c + 2*d*x)*atan(
e**(c + d*x))*a**2*b**3*f + 2*e**(2*c + 2*d*x)*atan(e**(c + d*x))*b**5*f +
 2*atan(e**(c + d*x))*a**4*b*f + 4*atan(e**(c + d*x))*a**2*b**3*f + 2*atan
(e**(c + d*x))*b**5*f + 2*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c +
 d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b**2*d*e*i + 2*sqrt(a**2 + b**2)*
atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b**2*d*e*i + 8*e**(5
*c + 2*d*x)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a
+ e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b + 2*e**(c
 + d*x)*a - b),x)*a**6*d**2*f + 16*e**(5*c + 2*d*x)*int((e**(3*d*x)*x)/(e*
*(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*c +
 3*d*x)*a - e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*b**2*d**2*f
 + 8*e**(5*c + 2*d*x)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c +
 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b
 + 2*e**(c + d*x)*a - b),x)*a**2*b**4*d**2*f + e**(2*c + 2*d*x)*log(e**(2*
c + 2*d*x) + 1)*a**5*f + 2*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x) + 1)*a**3
*b**2*f + e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b**4*f - 2*e**(2*c
+ 2*d*x)*a**5*d*f*x - 2*e**(2*c + 2*d*x)*a**3*b**2*d*e - 4*e**(2*c + 2*d*x
)*a**3*b**2*d*f*x - 2*e**(2*c + 2*d*x)*a*b**4*d*e - 2*e**(2*c + 2*d*x)*a*b
**4*d*f*x - 2*e**(c + d*x)*a**4*b*d*f*x - 2*e**(c + d*x)*a**2*b**3*d*e - 4
*e**(c + d*x)*a**2*b**3*d*f*x - 2*e**(c + d*x)*b**5*d*e - 2*e**(c + d*x...
```

3.385 $\int \frac{\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 90

$$\int \frac{\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=-\frac{2a^2\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}-\frac{b\operatorname{sech}(c+dx)}{(a^2+b^2)\,d}-\frac{a\tanh(c+dx)}{(a^2+b^2)\,d}$$

output
```
-2*a^2*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/
d-b*sech(d*x+c)/(a^2+b^2)/d-a*tanh(d*x+c)/(a^2+b^2)/d
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.18

$$\int \frac{\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=-\frac{-b\sqrt{-a^2-b^2}\operatorname{sech}(c+dx)+a\left(2a\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)-\sqrt{-a^2-b^2}\tanh(c+dx)\right)}{(-a^2-b^2)^{3/2}\,d}$$

input
```
Integrate[Tanh[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

output
```
-((-(b*Sqrt[-a^2 - b^2]*Sech[c + d*x]) + a*(2*a*ArcTan[(b - a*Tanh[(c + d*
x)/2])/Sqrt[-a^2 - b^2]] - Sqrt[-a^2 - b^2]*Tanh[c + d*x]))/((-a^2 - b^2)^
(3/2)*d))
```

Rubi [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 25, 3206, 26, 3042, 26, 3086, 24, 3139, 1083, 217, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan(ic+idx)^2}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan(ic+idx)^2}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3206}$$

$$\frac{a^2\int \frac{1}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{a\int \text{sech}^2(c+dx)dx}{a^2+b^2}-\frac{ib\int i\text{sech}(c+dx)\tanh(c+dx)dx}{a^2+b^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{a^2\int \frac{1}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{a\int \text{sech}^2(c+dx)dx}{a^2+b^2}+\frac{b\int \text{sech}(c+dx)\tanh(c+dx)dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int \frac{1}{a-ib\sin(ic+idx)}dx}{a^2+b^2}-\frac{a\int \csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{a^2+b^2}+\frac{b\int -i\sec(ic+idx)\tan(ic+idx)dx}{a^2+b^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{a^2\int \frac{1}{a-ib\sin(ic+idx)}dx}{a^2+b^2}-\frac{a\int \csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{a^2+b^2}-\frac{ib\int \sec(ic+idx)\tan(ic+idx)dx}{a^2+b^2}$$

$$\Big\downarrow \ 3086$$

$$\frac{a^2\int \frac{1}{a-ib\sin(ic+idx)}dx}{a^2+b^2}-\frac{a\int \csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{a^2+b^2}-\frac{b\int 1d\text{sech}(c+dx)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \ 24$$

$$\frac{a^2\int \frac{1}{a-ib\sin(ic+idx)}dx}{a^2+b^2}-\frac{a\int \csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{a^2+b^2}-\frac{b\text{sech}(c+dx)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \ 3139$$

$$-\frac{a\int \csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{a^2+b^2}-\frac{2ia^2\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d\left(a^2+b^2\right)}-\frac{b\text{sech}(c+dx)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \ 1083$$

$$-\frac{a\int \csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{a^2+b^2}+\frac{4ia^2\int \frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4\left(a^2+b^2\right)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{d\left(a^2+b^2\right)}-\frac{b\text{sech}(c+dx)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \ 217$$

$$-\frac{a\int \csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{a^2+b^2}+\frac{2a^2\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}-\frac{b\text{sech}(c+dx)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \ 4254$$

$$-\frac{ia\int 1d(-i\tanh(c+dx))}{d\left(a^2+b^2\right)}+\frac{2a^2\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}-\frac{b\text{sech}(c+dx)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \ 24$$

$$\frac{2a^2\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}-\frac{a\tanh(c+dx)}{d\left(a^2+b^2\right)}-\frac{b\text{sech}(c+dx)}{d\left(a^2+b^2\right)}$$

input

```
Int[Tanh[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

output
```
(2*a^2*ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*
d) - (b*Sech[c + d*x])/((a^2 + b^2)*d) - (a*Tanh[c + d*x])/((a^2 + b^2)*d)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3206
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[a/(a^2 - b^2)   Int[(g*Tan[e + f*x])^p/Sin[e + f*x]
^2, x], x] + (-Simp[b*(g/(a^2 - b^2))   Int[(g*Tan[e + f*x])^(p - 1)/Cos[e
+ f*x], x], x] - Simp[a^2*(g^2/(a^2 - b^2))   Int[(g*Tan[e + f*x])^(p - 2)/
(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2
, 0] && IntegersQ[2*p] && GtQ[p, 1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{\frac{8a^2\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(4a^2+4b^2\right)\sqrt{a^2+b^2}}+\frac{-2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{\left(a^2+b^2\right)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}}{d}$
default	$\frac{\frac{8a^2\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(4a^2+4b^2\right)\sqrt{a^2+b^2}}+\frac{-2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{\left(a^2+b^2\right)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}}{d}$
risch	$\frac{-2b\,\mathrm{e}^{dx+c}+2a}{d(a^2+b^2)\left(1+\mathrm{e}^{2dx+2c}\right)}+\frac{a^2\ln\left(\mathrm{e}^{dx+c}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}-a^4-2a^2b^2-b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}d}-\frac{a^2\ln\left(\mathrm{e}^{dx+c}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}d}$

input
```
int(tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(8*a^2/(4*a^2+4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2
*c)-2*b)/(a^2+b^2)^(1/2))+2/(a^2+b^2)*(-a*tanh(1/2*d*x+1/2*c)-b)/(1+tanh(1
/2*d*x+1/2*c)^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(87) = 174$.

Time = 0.10 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.90

$$\int \frac{\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2\,a^3+2\,ab^2+\left(a^2\cosh\left(dx+c\right)^2+2\,a^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a^2\sinh\left(dx+c\right)^2+a^2\right)\sqrt{a^2+b^2}\log}{(a^4+2\,a^2b^2+b^4)d\cosh\left(dx+c\right)^2+2\,(a^4}$$

input
```
integrate(tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(2*a^3 + 2*a*b^2 + (a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x + c
) + a^2*sinh(d*x + c)^2 + a^2)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 +
b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x
+ c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*
x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) +
2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - 2*(a^2*b + b^3)*cosh(d*x + c
) - 2*(a^2*b + b^3)*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c
)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a
^2*b^2 + b^4)*d*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d)
```

Sympy [F]

$$\int \frac{\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh^2\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate(tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(tanh(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.28

$$\int \frac{\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{a^2\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}d} - \frac{2\left(be^{(-dx-c)}+a\right)}{(a^2+b^2+(a^2+b^2)e^{(-2\,dx-2\,c)})d}$$

input
```
integrate(tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
a^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(
a^2 + b^2)))/((a^2 + b^2)^(3/2)*d) - 2*(b*e^(-d*x - c) + a)/((a^2 + b^2 +
(a^2 + b^2)*e^(-2*d*x - 2*c))*d)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.20

$$\int \frac{\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{a^2\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}} - \frac{2\left(be^{(dx+c)}-a\right)}{(a^2+b^2)\left(e^{(2\,dx+2\,c)}+1\right)}}{d}$$

input
```
integrate(tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
(a^2*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c
) + 2*a + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b*e^(d*x + c) - a)/((
a^2 + b^2)*(e^(2*d*x + 2*c) + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 422, normalized size of antiderivative = 4.69

$$\int \frac{\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{2\,a}{d\,(a^2+b^2)} - \frac{2\,b\,e^{c+d\,x}}{d\,(a^2+b^2)}}{e^{2\,c+2\,d\,x}+1} - \frac{2\,\mathrm{atan}\left(\left(e^{d\,x}\,e^{c}\left(\frac{2\,a^2}{b^2\,d\,\sqrt{a^4}\,(a^2+b^2)^2} + \frac{2\left(a^3\,d\,\sqrt{a^4}+a\,b^2\,d\,\sqrt{a^4}\right)}{a\,b^2\,\sqrt{-d^2\,(a^2+b^2)^3}\,(a^2+b^2)\,\sqrt{-a^6\,d^2-3\,a^4\,b^2\,d^2-3\,a^2\,b^4\,d^2-b^6\,d^2}}\right) - \frac{}{a\,b^2\,\sqrt{-d^2\,(a^2+b}}\right.\right.}{\sqrt{-a^6\,d^2-3\,a}}$$

input
```
int(tanh(c + d*x)^2/(a + b*sinh(c + d*x)),x)
```

output
```
((2*a)/(d*(a^2 + b^2)) - (2*b*exp(c + d*x))/(d*(a^2 + b^2)))/(exp(2*c + 2*
d*x) + 1) - (2*atan((exp(d*x)*exp(c)*((2*a^2)/(b^2*d*(a^4)^(1/2)*(a^2 + b^
2)^2) + (2*(a^3*d*(a^4)^(1/2) + a*b^2*d*(a^4)^(1/2)))/(a*b^2*(-d^2*(a^2 +
b^2)^3)^(1/2)*(a^2 + b^2)*(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3*a^4*b^2
*d^2)^(1/2))) - (2*(b^3*d*(a^4)^(1/2) + a^2*b*d*(a^4)^(1/2)))/(a*b^2*(-d^2
*(a^2 + b^2)^3)^(1/2)*(a^2 + b^2)*(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3
*a^4*b^2*d^2)^(1/2)))*((b^3*(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3*a^4*b
^2*d^2)^(1/2))/2 + (a^2*b*(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3*a^4*b^2
*d^2)^(1/2))/2))*(a^4)^(1/2))/(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3*a^4
*b^2*d^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.37

$$\int \frac{\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2e^{2dx+2c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)a^2 i + 2\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)a^2 i - 2e^{2dx+2c}a^3 - 2e^{2dx+2c}a\,b^2 - 2e}{d\left(e^{2dx+2c}a^4+2e^{2dx+2c}a^2b^2+e^{2dx+2c}b^4+a^4+2a^2b^2+b^4\right)}$$

input
```
int(tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(2*(e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(
a**2 + b**2))*a**2*i + sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqr
t(a**2 + b**2))*a**2*i - e**(2*c + 2*d*x)*a**3 - e**(2*c + 2*d*x)*a*b**2 -
 e**(c + d*x)*a**2*b - e**(c + d*x)*b**3))/(d*(e**(2*c + 2*d*x)*a**4 + 2*e
**(2*c + 2*d*x)*a**2*b**2 + e**(2*c + 2*d*x)*b**4 + a**4 + 2*a**2*b**2 + b
**4))
```

3.386 $\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 50.77 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[Tanh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Tanh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx$$

$$\Big\downarrow \text{ 6111}$$

$$\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Tanh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(dx+c)^2}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(tanh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh^2\left(c+dx\right)}{(a+b\sinh\left(c+dx\right))\left(e+fx\right)}\,dx$$

input
```
integrate(tanh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(tanh(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 391, normalized size of antiderivative = 13.96

$$\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
2*a^2*integrate(-e^(d*x + c)/(a^2*b*e + b^3*e + (a^2*b*f + b^3*f)*x - (a^2
*b*e*e^(2*c) + b^3*e*e^(2*c) + (a^2*b*f*e^(2*c) + b^3*f*e^(2*c))*x)*e^(2*d
*x) - 2*(a^3*e*e^c + a*b^2*e*e^c + (a^3*f*e^c + a*b^2*f*e^c)*x)*e^(d*x)),
x) - 2*(b*e^(d*x + c) - a)/(a^2*d*e + b^2*d*e + (a^2*d*f + b^2*d*f)*x + (a
^2*d*e*e^(2*c) + b^2*d*e*e^(2*c) + (a^2*d*f*e^(2*c) + b^2*d*f*e^(2*c))*x)*
e^(2*d*x)) - integrate(2*(b*f*e^(d*x + c) - a*f)/(a^2*d*e^2 + b^2*d*e^2 +
(a^2*d*f^2 + b^2*d*f^2)*x^2 + 2*(a^2*d*e*f + b^2*d*e*f)*x + (a^2*d*e^2*e^(
2*c) + b^2*d*e^2*e^(2*c) + (a^2*d*f^2*e^(2*c) + b^2*d*f^2*e^(2*c))*x^2 + 2
*(a^2*d*e*f*e^(2*c) + b^2*d*e*f*e^(2*c))*x)*e^(2*d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh(c+d\,x)^2}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(tanh(c + d*x)^2/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(tanh(c + d*x)^2/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.43

$$\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$= \int \frac{\tanh (dx+c)^2}{\sinh (dx+c)\,be+\sinh (dx+c)\,bfx+ae+afx}dx$$

input
```
int(tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(tanh(c + d*x)**2/(sinh(c + d*x)*b*e + sinh(c + d*x)*b*f*x + a*e + a*f*
x),x)
```

3.387 $\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 1256

$$\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output
```
-f^2*ln(cosh(d*x+c))/b/d^3+2*a^2*b*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a
^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2+2*a^2*b*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/
(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2+1/2*a^3*(f*x+e)^2*sech(d*x+c)*tanh(d*
x+c)/b^2/(a^2+b^2)/d-I*a^3*f^2*polylog(3,I*exp(d*x+c))/b^2/(a^2+b^2)/d^3-I
*a*f*(f*x+e)*polylog(2,I*exp(d*x+c))/b^2/d^2-2*I*a^3*f*(f*x+e)*polylog(2,-
I*exp(d*x+c))/(a^2+b^2)^2/d^2-I*a^3*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/b^2
/(a^2+b^2)/d^2+a^2*b*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2
+b^2)^2/d-1/2*(f*x+e)^2*sech(d*x+c)^2/b/d+a^2*b*(f*x+e)^2*ln(1+b*exp(d*x+c
)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d-2*a^2*b*f^2*polylog(3,-b*exp(d*x+c)/(
a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^3-2*a^2*b*f^2*polylog(3,-b*exp(d*x+c)/(a
-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^3+f*(f*x+e)*tanh(d*x+c)/b/d^2+a*f^2*arcta
n(sinh(d*x+c))/b^2/d^3-a^2*b*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/(a^2+b^2)^2/d-
a^3*f^2*arctan(sinh(d*x+c))/b^2/(a^2+b^2)/d^3+a^2*f^2*ln(cosh(d*x+c))/b/(a
^2+b^2)/d^3-a*f*(f*x+e)*sech(d*x+c)/b^2/d^2+I*a*f^2*polylog(3,I*exp(d*x+c)
)/b^2/d^3+I*a^3*f*(f*x+e)*polylog(2,I*exp(d*x+c))/b^2/(a^2+b^2)/d^2+a^3*f*
(f*x+e)*sech(d*x+c)/b^2/(a^2+b^2)/d^2+2*I*a^3*f*(f*x+e)*polylog(2,I*exp(d*
x+c))/(a^2+b^2)^2/d^2+I*a^3*f^2*polylog(3,-I*exp(d*x+c))/b^2/(a^2+b^2)/d^3
+I*a*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/b^2/d^2-a^2*b*f*(f*x+e)*polylog(2,
-exp(2*d*x+2*c))/(a^2+b^2)^2/d^2-a^2*f*(f*x+e)*tanh(d*x+c)/b/(a^2+b^2)/d^2
+2*a^3*(f*x+e)^2*arctan(exp(d*x+c))/(a^2+b^2)^2/d-1/2*a*(f*x+e)^2*sech(...
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3390 vs. $2(1256) = 2512$.

Time = 11.56 (sec) , antiderivative size = 3390, normalized size of antiderivative = 2.70

$$\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
(12*a^2*b*d^3*e^2*E^(2*c)*x + 12*a^2*b*d*E^(2*c)*f^2*x + 12*b^3*d*E^(2*c)*
f^2*x + 12*a^2*b*d^3*e*E^(2*c)*f*x^2 + 4*a^2*b*d^3*E^(2*c)*f^2*x^3 + 6*a^3
*d^2*e^2*ArcTan[E^(c + d*x)] - 6*a*b^2*d^2*e^2*ArcTan[E^(c + d*x)] + 6*a^3
*d^2*e^2*E^(2*c)*ArcTan[E^(c + d*x)] - 6*a*b^2*d^2*e^2*E^(2*c)*ArcTan[E^(c
 + d*x)] + 12*a^3*f^2*ArcTan[E^(c + d*x)] + 12*a*b^2*f^2*ArcTan[E^(c + d*x
)] + 12*a^3*E^(2*c)*f^2*ArcTan[E^(c + d*x)] + 12*a*b^2*E^(2*c)*f^2*ArcTan[
E^(c + d*x)] + (6*I)*a^3*d^2*e*f*x*Log[1 - I*E^(c + d*x)] - (6*I)*a*b^2*d^
2*e*f*x*Log[1 - I*E^(c + d*x)] + (6*I)*a^3*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(
c + d*x)] - (6*I)*a*b^2*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] + (3*I)*a
^3*d^2*f^2*x^2*Log[1 - I*E^(c + d*x)] - (3*I)*a*b^2*d^2*f^2*x^2*Log[1 - I*
E^(c + d*x)] + (3*I)*a^3*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] - (3*I
)*a*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] - (6*I)*a^3*d^2*e*f*x*L
og[1 + I*E^(c + d*x)] + (6*I)*a*b^2*d^2*e*f*x*Log[1 + I*E^(c + d*x)] - (6*
I)*a^3*d^2*e*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] + (6*I)*a*b^2*d^2*e*E^(2*c
)*f*x*Log[1 + I*E^(c + d*x)] - (3*I)*a^3*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)
] + (3*I)*a*b^2*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] - (3*I)*a^3*d^2*E^(2*c)
*f^2*x^2*Log[1 + I*E^(c + d*x)] + (3*I)*a*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 +
I*E^(c + d*x)] - 6*a^2*b*d^2*e^2*Log[1 + E^(2*(c + d*x))] - 6*a^2*b*d^2*e^
2*E^(2*c)*Log[1 + E^(2*(c + d*x))] - 6*a^2*b*f^2*Log[1 + E^(2*(c + d*x))]
- 6*b^3*f^2*Log[1 + E^(2*(c + d*x))] - 6*a^2*b*E^(2*c)*f^2*Log[1 + E^(2...
```

Rubi [A] (verified)

Time = 7.06 (sec) , antiderivative size = 1026, normalized size of antiderivative = 0.82, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.735$, Rules used = {6117, 5974, 3042, 4672, 26, 3042, 26, 3956, 6117, 3042, 4674, 3042, 4257, 4668, 3011, 2720, 6107, 6107, 6095, 2620, 3011, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\tanh^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6117}$$

$$\frac{\int (e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{5974}$$

$$\frac{\frac{f\int(e+fx)\text{sech}^2(c+dx)dx}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}+\frac{f\int(e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{d}}{b}$$

$$\Big\downarrow \text{4672}$$

$$-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}+\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{if\int-i\tanh(c+dx)dx}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int\tanh(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}+\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int-i\tan(ic+idx)dx}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}+\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}+\frac{if\int\tan(ic+idx)dx}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{6117}$$

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{\int(e+fx)^2\text{sech}^3(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{b}\right)}{b}$$

$\Big\downarrow$ 4674

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{-\frac{f^2\int\text{sech}(c+dx)dx}{d^2}+\frac{1}{2}\int(e+fx)^2\text{sech}(c+dx)dx+\frac{f(e+fx)\text{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{f^2\int\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{d^2}+\frac{1}{2}\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{f(e+fx)\text{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}(}{2d}}{b}\right)}{b}$$

$\Big\downarrow$ 4257

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f^2\arctan(\sinh(c+dx))}{d^3}+\frac{f(e+fx)\text{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}(c+d}{2d}}{b}\right)}{b}$$

$\Big\downarrow$ 4668

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(-\frac{2if\int(e+fx)\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{2if\int(e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}\right)-\frac{f^2\arctan(\sinh}{d^3}}{b}\right)}{b}$$

$\Big\downarrow$ 3011

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{P}}{}\right.}{d}\right.}{}\right.}{b}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,\right.}{d^2}\right.}{}\right.}{}\right.}{}$$

$$\Big\downarrow \text{6107}$$

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int\frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}\right)}{b}+\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{Pol}}{}\right.}{d}\right.}{}\right.}{}$$

$$\Big\downarrow \text{6107}$$

$$a\left(-\frac{a\left(\frac{\int (e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a^2+b^2}\right)}{b}+\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e}{}\right.}{}\right.}{}\right.$$

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-$$

$\downarrow$ 6095

$$a\left(-\frac{a\left(\frac{\int (e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}(c+dx)(a}{a^2+b^2}\right.}{a^2+b^2}\right.}{b}\right.$$

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-$$

$\downarrow$ 2620

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\cdots\right.}{b\cdots}\cdots\right)}{a^2+b^2}\cdots\right)}{\cdots}\right)}{b}\cdots\right)}{\cdots}$$

$\Big\downarrow$ 3011

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{2d}}{b}-$$

$$a\left(-\frac{a\left(\frac{b^2\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(\right.}{d}\right.}{bd}\right.}{a^2+b^2}\right.}{a^2+b^2}\right.}{}\right.$$

↓ 2720

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{2d}}{b}-$$

$$a\left(-\frac{a\left(\frac{b^2\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}\right.}{}\right)}{a^2+b^2}}{}\right)}{}}{}\right.$$

↓ 7143

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{2d}}{b}-$$

$$a\left(-\frac{a\left(\frac{b^2\left(\frac{\int(e+fx)^2\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3\right.}{}\right.}{}\right.}{}\right)}{a^2+b^2}\right.}{}\right.$$

↓ 7293

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{2d}}{b}-$$

$$a\left(-\frac{a\left(\frac{b^2\left(\frac{\int\left(a(e+fx)^2\operatorname{sech}(c+dx)-b(e+fx)^2\tanh(c+dx)\right)dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{a^2+\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 2009

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{2d}}{b}-$$

$$a\left(\frac{-\frac{\arctan(\sinh(c+dx))f^2}{d^3}+\frac{(e+fx)\operatorname{sech}(c+dx)f}{d^2}+\frac{1}{2}\left(\frac{2\arctan\left(e^{c+dx}\right)(e+fx)^2}{d}+\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\right.}{}\right.}{b}\right.$$

input
```
Int[((e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-1/2*((e + f*x)^2*Sech[c + d*x]^2)/d + (f*(-((f*Log[Cosh[c + d*x]])/d^2)
+ ((e + f*x)*Tanh[c + d*x])/d))/d)/b - (a*((-((f^2*ArcTan[Sinh[c + d*x]])/
d^3) + ((2*(e + f*x)^2*ArcTan[E^(c + d*x)])/d + ((2*I)*f*(-(((e + f*x)*Pol
yLog[2, (-I)*E^(c + d*x)])/d) + (f*PolyLog[3, (-I)*E^(c + d*x)])/d^2))/d -
 ((2*I)*f*(-(((e + f*x)*PolyLog[2, I*E^(c + d*x)])/d) + (f*PolyLog[3, I*E^
(c + d*x)])/d^2))/d)/2 + (f*(e + f*x)*Sech[c + d*x])/d^2 + ((e + f*x)^2*Se
ch[c + d*x]*Tanh[c + d*x])/(2*d))/b - (a*((b^2*((b^2*(-1/3*(e + f*x)^3/(b*
f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) +
((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (2*f*
(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (
f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d^2))/(b*d) - (2*f
*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) +
(f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/(a^
2 + b^2) + ((b*(e + f*x)^3)/(3*f) + (2*a*(e + f*x)^2*ArcTan[E^(c + d*x)])/
d - (b*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/d - ((2*I)*a*f*(e + f*x)*Poly
Log[2, (-I)*E^(c + d*x)])/d^2 + ((2*I)*a*f*(e + f*x)*PolyLog[2, I*E^(c + d
*x)])/d^2 - (b*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/d^2 + ((2*I)*a*f^
2*PolyLog[3, (-I)*E^(c + d*x)])/d^3 - ((2*I)*a*f^2*PolyLog[3, I*E^(c + d*x
)])/d^3 + (b*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*d^3))/(a^2 + b^2)))/(a^2
 + b^2) + ((a*(e + f*x)^2*ArcTan[E^(c + d*x)])/d - (a*f^2*ArcTan[Sinh[c...
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6117
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x],
x] - Simp[a/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{sech}(dx+c) \tanh(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 10934 vs. $2(1156) = 2312$.

Time = 0.32 (sec) , antiderivative size = 10934, normalized size of antiderivative = 8.71

$$\int \frac{(e+fx)^2 \operatorname{sech}(c+dx) \tanh^2(c+dx)}{a+b\sinh(c+dx)} dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2\tanh^2{(c+dx)}\,\text{sech}\,(c+dx)}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**2*sech(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*tanh(c + d*x)**2*sech(c + d*x)/(a + b*sinh(c + d*x))
, x)
```

Maxima [F]

$$\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\,\text{sech}\,(dx+c)\tanh{(dx+c)}^2}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
a^3*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2
*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 +
 b^4*d^2), x) - a*b^2*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x
+ 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2
 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 2*a^2*b*d^2*f^2*integrate(x^2/(a^4*d^2*e
^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) +
 a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 2*a^3*d^2*e*f*integrate(x*e^(d*x
 + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e
^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - 2*a*b^2*d^2*e*f*
integrate(x*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x
+ 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x)
+ 4*a^2*b*d^2*e*f*integrate(x/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(
2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2
), x) + a^2*b*f^2*(2*(d*x + c)/((a^4 + 2*a^2*b^2 + b^4)*d^3) - log(e^(2*d*
x + 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d^3)) + b^3*f^2*(2*(d*x + c)/((a^4
+ 2*a^2*b^2 + b^4)*d^3) - log(e^(2*d*x + 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4
)*d^3)) + (a^2*b*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2
*a^2*b^2 + b^4)*d) - a^2*b*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b
^4)*d) - (a^3 - a*b^2)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) -
(a*e^(-d*x - c) + 2*b*e^(-2*d*x - 2*c) - a*e^(-3*d*x - 3*c))/((a^2 + b^...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \mathrm{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh(c+d\,x)^2\,(e+f\,x)^2}{\cosh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((tanh(c + d*x)^2*(e + f*x)^2)/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((tanh(c + d*x)^2*(e + f*x)^2)/(cosh(c + d*x)*(a + b*sinh(c + d*x))), x
)
```

Reduce [F]

$$\int \frac{(e+fx)^2 \mathrm{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
( - 48*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**5*b*d*e*f - 16*e**(4*c + 4*d
*x)*atan(e**(c + d*x))*a**5*b*f**2 + 18*e**(4*c + 4*d*x)*atan(e**(c + d*x)
)*a**3*b**3*d**2*e**2 - 96*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**3*b**3*d
*e*f - 32*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**3*b**3*f**2 - 18*e**(4*c
+ 4*d*x)*atan(e**(c + d*x))*a*b**5*d**2*e**2 - 48*e**(4*c + 4*d*x)*atan(e*
*(c + d*x))*a*b**5*d*e*f - 16*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a*b**5*f
**2 - 96*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**5*b*d*e*f - 32*e**(2*c + 2
*d*x)*atan(e**(c + d*x))*a**5*b*f**2 + 36*e**(2*c + 2*d*x)*atan(e**(c + d*
x))*a**3*b**3*d**2*e**2 - 192*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**3*b**
3*d*e*f - 64*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**3*b**3*f**2 - 36*e**(2
*c + 2*d*x)*atan(e**(c + d*x))*a*b**5*d**2*e**2 - 96*e**(2*c + 2*d*x)*atan
(e**(c + d*x))*a*b**5*d*e*f - 32*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a*b**
5*f**2 - 48*atan(e**(c + d*x))*a**5*b*d*e*f - 16*atan(e**(c + d*x))*a**5*b
*f**2 + 18*atan(e**(c + d*x))*a**3*b**3*d**2*e**2 - 96*atan(e**(c + d*x))*
a**3*b**3*d*e*f - 32*atan(e**(c + d*x))*a**3*b**3*f**2 - 18*atan(e**(c + d
*x))*a*b**5*d**2*e**2 - 48*atan(e**(c + d*x))*a*b**5*d*e*f - 16*atan(e**(c
 + d*x))*a*b**5*f**2 - 576*e**(7*c + 4*d*x)*int((e**(3*d*x)*x**2)/(e**(8*c
 + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a + 2*e**(6*c + 6*d*x)*b + 6*e**(5*c + 5*
d*x)*a + 6*e**(3*c + 3*d*x)*a - 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a -
b),x)*a**7*d**3*f**2 - 1200*e**(7*c + 4*d*x)*int((e**(3*d*x)*x**2)/(e**...
```

3.388 $\int \frac{(e+fx)\mathbf{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 760

$$\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= -\frac{a(e+fx)\arctan\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)\arctan\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} + \frac{a^3(e+fx)\arctan\left(e^{c+dx}\right)}{b^2\left(a^2+b^2\right)d} + \frac{a^2b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2 d} + \frac{a^2b(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2 d} - \frac{a^2b(e+fx)\log\left(1+e^{2(c+dx)}\right)}{\left(a^2+b^2\right)^2 d} + \frac{iaf\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{2b^2d^2} - \frac{ia^3f\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{\left(a^2+b^2\right)^2 d^2} - \frac{ia^3f\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{2b^2\left(a^2+b^2\right)d^2} - \frac{iaf\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{2b^2d^2} + \frac{ia^3f\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{\left(a^2+b^2\right)^2 d^2} + \frac{ia^3f\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{2b^2\left(a^2+b^2\right)d^2} + \frac{a^2bf\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2 d^2} + \frac{a^2bf\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2 d^2} - \frac{a^2bf\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2\left(a^2+b^2\right)^2 d^2} - \frac{af\text{sech}(c+dx)}{2b^2d^2} + \frac{a^3f\text{sech}(c+dx)}{2b^2\left(a^2+b^2\right)d^2} - \frac{(e+fx)\text{sech}^2(c+dx)}{2bd} + \frac{a^2(e+fx)\text{sech}^2(c+dx)}{2b\left(a^2+b^2\right)d} + \frac{f\tanh(c+dx)}{2bd^2} - \frac{a^2f\tanh(c+dx)}{2b\left(a^2+b^2\right)d^2} - \frac{a(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{2b^2d} + \frac{a^3(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{2b^2\left(a^2+b^2\right)d}$$

output
```
-a*(f*x+e)*arctan(exp(d*x+c))/b^2/d+2*a^3*(f*x+e)*arctan(exp(d*x+c))/(a^2+
b^2)^2/d+a^3*(f*x+e)*arctan(exp(d*x+c))/b^2/(a^2+b^2)/d+a^2*b*(f*x+e)*ln(1
+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d+a^2*b*(f*x+e)*ln(1+b*exp(
d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d-a^2*b*(f*x+e)*ln(1+exp(2*d*x+2*c
))/(a^2+b^2)^2/d-1/2*I*a^3*f*polylog(2,-I*exp(d*x+c))/b^2/(a^2+b^2)/d^2-1/
2*I*a*f*polylog(2,I*exp(d*x+c))/b^2/d^2+1/2*I*a*f*polylog(2,-I*exp(d*x+c))
/b^2/d^2+I*a^3*f*polylog(2,I*exp(d*x+c))/(a^2+b^2)^2/d^2+1/2*I*a^3*f*polyl
og(2,I*exp(d*x+c))/b^2/(a^2+b^2)/d^2-I*a^3*f*polylog(2,-I*exp(d*x+c))/(a^2
+b^2)^2/d^2+a^2*b*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)
^2/d^2+a^2*b*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^
2-1/2*a^2*b*f*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)^2/d^2-1/2*a*f*sech(d*x+
c)/b^2/d^2+1/2*a^3*f*sech(d*x+c)/b^2/(a^2+b^2)/d^2-1/2*(f*x+e)*sech(d*x+c)
^2/b/d+1/2*a^2*(f*x+e)*sech(d*x+c)^2/b/(a^2+b^2)/d+1/2*f*tanh(d*x+c)/b/d^2
-1/2*a^2*f*tanh(d*x+c)/b/(a^2+b^2)/d^2-1/2*a*(f*x+e)*sech(d*x+c)*tanh(d*x+
c)/b^2/d+1/2*a^3*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/b^2/(a^2+b^2)/d
```

Mathematica [A] (warning: unable to verify)

Time = 8.95 (sec) , antiderivative size = 823, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{a^2b\left(-2de(c+dx)+2cf(c+dx)-f(c+dx)^2+\frac{4a\sqrt{a^2+b^2}de\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-(a^2+b^2)^2}}-\frac{4a\sqrt{-(a^2+b^2)^2}de\text{arctanh}\left(\frac{a+b}{\sqrt{a}}\right.}{(-a^2-b^2)^{3/2}}\right.}{}$$

$$+\frac{a\left(2abde(c+dx)-2abcf(c+dx)+abf(c+dx)^2+2a^2de\arctan\left(e^{c+dx}\right)-2b^2de\arctan\left(e^{c+dx}\right)-2a\right.}{}$$

$$+\frac{\text{sech}(c+dx)(-af+bf\sinh(c+dx))}{2\left(a^2+b^2\right)d^2}$$

$$+\frac{\text{sech}^2(c+dx)(-bde+bcf-bf(c+dx)-ade\sinh(c+dx)+acf\sinh(c+dx)-af(c+dx)\sinh(c+}{2\left(a^2+b^2\right)d^2}$$

input
```
Integrate[((e + f*x)*Sech[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),
x]
```

output
```
(a^2*b*(-2*d*e*(c + d*x) + 2*c*f*(c + d*x) - f*(c + d*x)^2 + (4*a*Sqrt[a^2
 + b^2]*d*e*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b^2
)^2] - (4*a*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2
+ b^2]])/(-a^2 - b^2)^(3/2) + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 + b^2])] + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2])] - 2*c*f*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*d*e*Log[2*a
*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] + 2*f*PolyLog[2, (b*E^(c + d*x))/
(-a + Sqrt[a^2 + b^2])] + 2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))]))/(2*(a^2 + b^2)^2*d^2) + (a*(2*a*b*d*e*(c + d*x) - 2*a*b*c*f*(c
+ d*x) + a*b*f*(c + d*x)^2 + 2*a^2*d*e*ArcTan[E^(c + d*x)] - 2*b^2*d*e*Arc
Tan[E^(c + d*x)] - 2*a^2*c*f*ArcTan[E^(c + d*x)] + 2*b^2*c*f*ArcTan[E^(c +
 d*x)] + I*a^2*f*(c + d*x)*Log[1 - I*E^(c + d*x)] - I*b^2*f*(c + d*x)*Log[
1 - I*E^(c + d*x)] - I*a^2*f*(c + d*x)*Log[1 + I*E^(c + d*x)] + I*b^2*f*(c
 + d*x)*Log[1 + I*E^(c + d*x)] - 2*a*b*d*e*Log[1 + E^(2*(c + d*x))] + 2*a*
b*c*f*Log[1 + E^(2*(c + d*x))] - 2*a*b*f*(c + d*x)*Log[1 + E^(2*(c + d*x))
] - I*(a^2 - b^2)*f*PolyLog[2, (-I)*E^(c + d*x)] + I*(a^2 - b^2)*f*PolyLog
[2, I*E^(c + d*x)] - a*b*f*PolyLog[2, -E^(2*(c + d*x))]))/(2*(a^2 + b^2)^2
*d^2) + (Sech[c + d*x]*(-(a*f) + b*f*Sinh[c + d*x]))/(2*(a^2 + b^2)*d^2) +
 (Sech[c + d*x]^2*(-(b*d*e) + b*c*f - b*f*(c + d*x) - a*d*e*Sinh[c + d*x]
+ a*c*f*Sinh[c + d*x] - a*f*(c + d*x)*Sinh[c + d*x]))/(2*(a^2 + b^2)*d^...
```

Rubi [A] (verified)

Time = 4.29 (sec) , antiderivative size = 646, normalized size of antiderivative = 0.85, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6117, 5974, 3042, 4254, 24, 6117, 3042, 4673, 3042, 4668, 2715, 2838, 6107, 6107, 6095, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\tanh^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6117}$$

$$\frac{\int (e+fx)\text{sech}^2(c+dx)\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{5974}$$

$$\frac{\frac{f\int \text{sech}^2(c+dx)dx}{2d}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}+\frac{f\int \csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{2d}}{b}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}+\frac{if\int 1d(-i\tanh(c+dx))}{2d^2}}{b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{6117}$$

$$\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{\int (e+fx)\text{sech}^3(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{b}\right)}{b}$$

$$\Big\downarrow \text{4673}$$

$$\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{\frac{1}{2}\int (e+fx)\text{sech}(c+dx)dx+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}\right)}{b}$$

$\Big\downarrow$ 4668

$$\frac{a\left(-\frac{a\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(-\frac{if\int \log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int \log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+\cdots}{2\cdots}}{b}\right.}{b}\ \frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-$$

$\Big\downarrow$ 2715

$$\frac{a\left(-\frac{a\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{f\text{sec}\cdots}{\cdots}}{b}\right.}{b}\ \frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-$$

$\Big\downarrow$ 2838

$$\frac{a\left(-\frac{a\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+\cdots}{2\cdots}}{b}\right.}{b}\ \frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-$$

$\Big\downarrow$ 6107

$$\frac{a\left(-\frac{a\left(\frac{\int (e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}\right)}{b}+\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLo}\cdots}{\cdots}\right.}{b}\right.}{b}\ \frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-$$

$\Big\downarrow$ 6107

$$\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)\operatorname{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a^2+b^2}\right)}{b}+\frac{\frac{1}{2}\left(\frac{2(e+fx)\operatorname{arcta}}{d}\right.}{}\right.}{b}$$

$\downarrow$ 6095

$$\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)\operatorname{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int(e+fx)\operatorname{sech}(c+dx)(a-}{a^2+b^2}\right.}{a^2+b^2}\right.}{b}\right.}{}$$

$\downarrow$ 2620

$$\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}}{b}-$$

$$a\left(-\frac{a\left(\frac{\int(e+fx)\operatorname{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\cdots\right.}{a^2+b^2}\cdots}{a^2+b^2}\cdots}{b}\cdots\right.$$

$\downarrow$ 2715

$$\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}}{b}-$$

$$a\left(-\frac{a\left(\frac{b^2\left(\frac{b^2\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+\cdots}}\cdots\right)}{bd}\cdots\right.}{a^2+b^2}\cdots}{a^2+b^2}\cdots}{b}\cdots\right.$$

$\downarrow$ 2838

$$\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-$$

$$a\left(-\frac{a\left(\frac{b^2\left(\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\ldots\right)}{a^2+b^2}\right)}{a^2+b^2}\ldots\right)}{b}\ldots\right)$$

$\downarrow$ 7293

$$\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-$$

$$a\left(-\frac{a\left(\frac{b^2\left(\frac{\int (a(e+fx)\text{sech}(c+dx)-b(e+fx)\tanh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\ldots\right)}{bd}\ldots\right)}{a^2+b^2}\right)}{a^2+b^2}\ldots\right)}{b}\ldots\right)$$

$\downarrow$ 2009

$$\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}}{b}-$$

$$a\left(\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\operatorname{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}}{b}-\frac{a\left(\frac{b^2\left(\frac{b^2\left(\frac{f\ldots}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.$$

input
```
Int[((e + f*x)*Sech[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-1/2*((e + f*x)*Sech[c + d*x]^2)/d + (f*Tanh[c + d*x])/(2*d^2))/b - (a*((
((2*(e + f*x)*ArcTan[E^(c + d*x)])/d - (I*f*PolyLog[2, (-I)*E^(c + d*x)])/
d^2 + (I*f*PolyLog[2, I*E^(c + d*x)])/d^2)/2 + (f*Sech[c + d*x])/(2*d^2) +
 ((e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*d))/b - (a*((b^2*((b^2*(-1/2*(
e + f*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]
)])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*
d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (f
*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(a^2 + b^
2) + ((b*(e + f*x)^2)/(2*f) + (2*a*(e + f*x)*ArcTan[E^(c + d*x)])/d - (b*(
e + f*x)*Log[1 + E^(2*(c + d*x))])/d - (I*a*f*PolyLog[2, (-I)*E^(c + d*x)]
)/d^2 + (I*a*f*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*PolyLog[2, -E^(2*(c +
 d*x))])/(2*d^2))/(a^2 + b^2)))/(a^2 + b^2) + ((a*(e + f*x)*ArcTan[E^(c +
d*x)])/d - ((I/2)*a*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + ((I/2)*a*f*PolyL
og[2, I*E^(c + d*x)])/d^2 + (a*f*Sech[c + d*x])/(2*d^2) + (b*(e + f*x)*Sec
h[c + d*x]^2)/(2*d) - (b*f*Tanh[c + d*x])/(2*d^2) + (a*(e + f*x)*Sech[c +
d*x]*Tanh[c + d*x])/(2*d))/(a^2 + b^2)))/b))/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6117
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x],
x] - Simp[a/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2067 vs. 2(701) = 1402.

Time = 6.06 (sec) , antiderivative size = 2068, normalized size of antiderivative = 2.72

method	result	size
risch	Expression too large to display	2068

input
```
int((f*x+e)*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
I/d*a/(a^2+b^2)*b^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*x-1/d^2*a^3/(a^2+b^
2)^(3/2)*c*b*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1
/2))-1/d^2*a/(a^2+b^2)^(3/2)*c*b^3*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*
x+c)+2*a)/(a^2+b^2)^(1/2))+I/d^2*a/(a^2+b^2)*b^2*f/(2*a^2+2*b^2)*ln(1+I*ex
p(d*x+c))*c-I/d^2*a/(a^2+b^2)*b^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*c-I/d
*a/(a^2+b^2)*b^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*x+2/d*a^2/(a^2+b^2)*b*
f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x
-2/d^2*a^2/(a^2+b^2)*b*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*c-2/d^2*a^2/(a^2
+b^2)*b*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*c+2/d^2*a^2/(a^2+b^2)*b*f/(2*a^
2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+I/d^
2*a^3/(a^2+b^2)*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*c+I/d*a^3/(a^2+b^2)*f/(
2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*x-I/d^2*a/(a^2+b^2)*b^2*f/(2*a^2+2*b^2)*di
log(1-I*exp(d*x+c))-I/d^2*a^3/(a^2+b^2)*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))
*c-I/d*a^3/(a^2+b^2)*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*x-b/d*a*e/(2*a^2+2
*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/
d*a/(a^2+b^2)*b^2*e/(2*a^2+2*b^2)*arctan(exp(d*x+c))-2/d^2*a^2/(a^2+b^2)*b
*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))-2/d^2*a^2/(a^2+b^2)*b*f/(2*a^2+2*b^
2)*dilog(1-I*exp(d*x+c))+2/d^2*a^2/(a^2+b^2)*b*f/(2*a^2+2*b^2)*dilog((-b*e
xp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-2/d*a^2/(a^2+b^2)*b*e/(
2*a^2+2*b^2)*ln(1+exp(2*d*x+2*c))+2/d*a^2/(a^2+b^2)*b*e/(2*a^2+2*b^2)*l...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4903 vs. $2(680) = 1360$.

Time = 0.24 (sec) , antiderivative size = 4903, normalized size of antiderivative = 6.45

$$\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\tanh^2{(c+dx)}\,\text{sech}\,(c+dx)}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*tanh(c + d*x)**2*sech(c + d*x)/(a + b*sinh(c + d*x)), x
)
```

Maxima [F]

$$\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{sech}\,(dx+c)\tanh\,(dx+c)^2}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
(a^2*b*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 +
 b^4)*d) - a^2*b*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (
a^3 - a*b^2)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) - (a*e^(-d*x
 - c) + 2*b*e^(-2*d*x - 2*c) - a*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 +
b^2)*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d))*e - f*(((a*d*x*e
^(3*c) + a*e^(3*c))*e^(3*d*x) + (2*b*d*x*e^(2*c) + b*e^(2*c))*e^(2*d*x) -
(a*d*x*e^c - a*e^c)*e^(d*x) + b)/(a^2*d^2 + b^2*d^2 + (a^2*d^2*e^(4*c) + b
^2*d^2*e^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*e^(2*d*x
)) + 2*integrate(-(a^3*b*x*e^(d*x + c) - a^2*b^2*x)/(a^4*b + 2*a^2*b^3 + b
^5 - (a^4*b*e^(2*c) + 2*a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^5*
e^c + 2*a^3*b^2*e^c + a*b^4*e^c)*e^(d*x)), x) - 2*integrate(1/2*(2*a^2*b*x
 + (a^3*e^c - a*b^2*e^c)*x*e^(d*x))/(a^4 + 2*a^2*b^2 + b^4 + (a^4*e^(2*c)
+ 2*a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x))
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh(c+d\,x)^2\,(e+f\,x)}{\cosh(c+d\,x)\,(a+b\sinh(c+d\,x))}\,dx$$

input
```
int((tanh(c + d*x)^2*(e + f*x))/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((tanh(c + d*x)^2*(e + f*x))/(cosh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
( - 8*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**5*b*f + 6*e**(4*c + 4*d*x)*at
an(e**(c + d*x))*a**3*b**3*d*e - 16*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a*
*3*b**3*f - 6*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a*b**5*d*e - 8*e**(4*c +
 4*d*x)*atan(e**(c + d*x))*a*b**5*f - 16*e**(2*c + 2*d*x)*atan(e**(c + d*x
))*a**5*b*f + 12*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**3*b**3*d*e - 32*e*
*(2*c + 2*d*x)*atan(e**(c + d*x))*a**3*b**3*f - 12*e**(2*c + 2*d*x)*atan(e
**(c + d*x))*a*b**5*d*e - 16*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a*b**5*f
- 8*atan(e**(c + d*x))*a**5*b*f + 6*atan(e**(c + d*x))*a**3*b**3*d*e - 16*
atan(e**(c + d*x))*a**3*b**3*f - 6*atan(e**(c + d*x))*a*b**5*d*e - 8*atan(
e**(c + d*x))*a*b**5*f - 192*e**(7*c + 4*d*x)*int((e**(3*d*x)*x)/(e**(8*c
+ 8*d*x)*b + 2*e**(7*c + 7*d*x)*a + 2*e**(6*c + 6*d*x)*b + 6*e**(5*c + 5*d
*x)*a + 6*e**(3*c + 3*d*x)*a - 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b
),x)*a**7*d**2*f - 400*e**(7*c + 4*d*x)*int((e**(3*d*x)*x)/(e**(8*c + 8*d*
x)*b + 2*e**(7*c + 7*d*x)*a + 2*e**(6*c + 6*d*x)*b + 6*e**(5*c + 5*d*x)*a
+ 6*e**(3*c + 3*d*x)*a - 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a
**5*b**2*d**2*f - 224*e**(7*c + 4*d*x)*int((e**(3*d*x)*x)/(e**(8*c + 8*d*x
)*b + 2*e**(7*c + 7*d*x)*a + 2*e**(6*c + 6*d*x)*b + 6*e**(5*c + 5*d*x)*a +
 6*e**(3*c + 3*d*x)*a - 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a*
*3*b**4*d**2*f - 16*e**(7*c + 4*d*x)*int((e**(3*d*x)*x)/(e**(8*c + 8*d*x)*
b + 2*e**(7*c + 7*d*x)*a + 2*e**(6*c + 6*d*x)*b + 6*e**(5*c + 5*d*x)*a ...
```

3.389 $\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 121

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{a(a^2-b^2)\arctan(\sinh(c+dx))}{2(a^2+b^2)^2 d} - \frac{a^2 b\log(\cosh(c+dx))}{(a^2+b^2)^2 d} + \frac{a^2 b\log(a+b\sinh(c+dx))}{(a^2+b^2)^2 d} - \frac{\operatorname{sech}^2(c+dx)(b+a\sinh(c+dx))}{2(a^2+b^2)d}$$

output
```
1/2*a*(a^2-b^2)*arctan(sinh(d*x+c))/(a^2+b^2)^2/d-a^2*b*ln(cosh(d*x+c))/(a
^2+b^2)^2/d+a^2*b*ln(a+b*sinh(d*x+c))/(a^2+b^2)^2/d-1/2*sech(d*x+c)^2*(b+a
*sinh(d*x+c))/(a^2+b^2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{a((a^2+b^2)\arctan(\sinh(c+dx))+a((ia+b)\log(i-\sinh(c+dx))+(-ia+b)\log(i+\sinh(c+dx))}{2\,(a^2+b}$$

input
```
Integrate[(Sech[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*(a*((a^2 + b^2)*ArcTan[Sinh[c + d*x]] + a*((I*a + b)*Log[I - Sinh[c +
 d*x]] + ((-I)*a + b)*Log[I + Sinh[c + d*x]] - 2*b*Log[a + b*Sinh[c + d*x]
])) + b*(a^2 + b^2)*Sech[c + d*x]^2 + a*(a^2 + b^2)*Sech[c + d*x]*Tanh[c +
 d*x])/((a^2 + b^2)^2*d)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.21, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {3042, 25, 3316, 25, 27, 601, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{\sin(ic+idx)^2}{\cos(ic+idx)^3(a-ib\sin(ic+idx))}dx$$
$$\Big\downarrow \text{25}$$
$$-\int \frac{\sin(ic+idx)^2}{\cos(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3316}$$

$$-\frac{b^3\int -\frac{\sinh^2(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b^3\int \frac{\sinh^2(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{b^2\sinh^2(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{601}$$

$$\frac{b\left(-\frac{\int -\frac{ab^2(a-b\sinh(c+dx))}{\left(a^2+b^2\right)(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)}d(b\sinh(c+dx))}{2b^2}-\frac{ab\sinh(c+dx)+b^2}{2(a^2+b^2)\left(b^2\sinh^2(c+dx)+b^2\right)}\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{\int \frac{ab^2(a-b\sinh(c+dx))}{\left(a^2+b^2\right)(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)}d(b\sinh(c+dx))}{2b^2}-\frac{ab\sinh(c+dx)+b^2}{2(a^2+b^2)\left(b^2\sinh^2(c+dx)+b^2\right)}\right)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\left(\frac{a\int \frac{a-b\sinh(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)}d(b\sinh(c+dx))}{2(a^2+b^2)}-\frac{ab\sinh(c+dx)+b^2}{2(a^2+b^2)\left(b^2\sinh^2(c+dx)+b^2\right)}\right)}{d}$$

$$\Big\downarrow \text{657}$$

$$\frac{b\left(\frac{a\int \left(\frac{2a}{\left(a^2+b^2\right)(a+b\sinh(c+dx))}+\frac{a^2-2b\sinh(c+dx)a-b^2}{\left(a^2+b^2\right)\left(\sinh^2(c+dx)b^2+b^2\right)}\right)d(b\sinh(c+dx))}{2(a^2+b^2)}-\frac{ab\sinh(c+dx)+b^2}{2(a^2+b^2)\left(b^2\sinh^2(c+dx)+b^2\right)}\right)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(\frac{a\left(\frac{(a^2-b^2)\arctan(\sinh(c+dx))}{b(a^2+b^2)}-\frac{a\log\left(b^2\sinh^2(c+dx)+b^2\right)}{a^2+b^2}+\frac{2a\log(a+b\sinh(c+dx))}{a^2+b^2}\right)}{2(a^2+b^2)}-\frac{ab\sinh(c+dx)+b^2}{2(a^2+b^2)\left(b^2\sinh^2(c+dx)+b^2\right)}\right)}{d}$$

input
```
Int[(Sech[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(b*((a*(((a^2 - b^2)*ArcTan[Sinh[c + d*x]])/(b*(a^2 + b^2)) + (2*a*Log[a +
 b*Sinh[c + d*x]])/(a^2 + b^2) - (a*Log[b^2 + b^2*Sinh[c + d*x]^2])/(a^2 +
 b^2)))/(2*(a^2 + b^2)) - (b^2 + a*b*Sinh[c + d*x])/(2*(a^2 + b^2)*(b^2 +
b^2*Sinh[c + d*x]^2))))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.73

method	result
derivativedivides	$\frac{\frac{4a^2b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{4a^4+8a^2b^2+4b^4}+\frac{\frac{2\left(\left(\frac{1}{2}a^3+\frac{1}{2}ab^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(a^2b+b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(-\frac{1}{2}a^3-\frac{1}{2}ab^2\right)}{\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}}{a^4+2}}{d}$
default	$\frac{\frac{4a^2b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{4a^4+8a^2b^2+4b^4}+\frac{\frac{2\left(\left(\frac{1}{2}a^3+\frac{1}{2}ab^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(a^2b+b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(-\frac{1}{2}a^3-\frac{1}{2}ab^2\right)}{\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}}{a^4+2}}{d}$
risch	$\frac{2a^2b\,d^2x}{a^4d^2+2a^2b^2d^2+b^4d^2}+\frac{2a^2bdc}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{2a^2bx}{a^4+2a^2b^2+b^4}-\frac{2a^2bc}{d(a^4+2a^2b^2+b^4)}-\frac{e^{dx+c}\left(e^{2dx+2c}a+2b\,e^{dx+}\right.}{d(a^2+b^2)\left(1+e^{2dx+2c}\right.}$

input
```
int(sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(4*a^2*b/(4*a^4+8*a^2*b^2+4*b^4)*ln(tanh(1/2*d*x+1/2*c)^2*a-2*b*tanh(1
/2*d*x+1/2*c)-a)+2/(a^4+2*a^2*b^2+b^4)*(((1/2*a^3+1/2*a*b^2)*tanh(1/2*d*x+
1/2*c)^3+(a^2*b+b^3)*tanh(1/2*d*x+1/2*c)^2+(-1/2*a^3-1/2*a*b^2)*tanh(1/2*d
*x+1/2*c))/(1+tanh(1/2*d*x+1/2*c)^2)^2+1/2*a*(-a*b*ln(1+tanh(1/2*d*x+1/2*c
)^2)+(a^2-b^2)*arctan(tanh(1/2*d*x+1/2*c)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 917 vs. $2(117) = 234$.

Time = 0.11 (sec) , antiderivative size = 917, normalized size of antiderivative = 7.58

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
-((a^3 + a*b^2)*cosh(d*x + c)^3 + (a^3 + a*b^2)*sinh(d*x + c)^3 + 2*(a^2*b
 + b^3)*cosh(d*x + c)^2 + (2*a^2*b + 2*b^3 + 3*(a^3 + a*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^2 - ((a^3 - a*b^2)*cosh(d*x + c)^4 + 4*(a^3 - a*b^2)*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^3 - a*b^2)*sinh(d*x + c)^4 + a^3 - a*b^2 + 2
*(a^3 - a*b^2)*cosh(d*x + c)^2 + 2*(a^3 - a*b^2 + 3*(a^3 - a*b^2)*cosh(d*x
 + c)^2)*sinh(d*x + c)^2 + 4*((a^3 - a*b^2)*cosh(d*x + c)^3 + (a^3 - a*b^2
)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - (a
^3 + a*b^2)*cosh(d*x + c) - (a^2*b*cosh(d*x + c)^4 + 4*a^2*b*cosh(d*x + c)
*sinh(d*x + c)^3 + a^2*b*sinh(d*x + c)^4 + 2*a^2*b*cosh(d*x + c)^2 + a^2*b
 + 2*(3*a^2*b*cosh(d*x + c)^2 + a^2*b)*sinh(d*x + c)^2 + 4*(a^2*b*cosh(d*x
 + c)^3 + a^2*b*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/
(cosh(d*x + c) - sinh(d*x + c))) + (a^2*b*cosh(d*x + c)^4 + 4*a^2*b*cosh(d
*x + c)*sinh(d*x + c)^3 + a^2*b*sinh(d*x + c)^4 + 2*a^2*b*cosh(d*x + c)^2
+ a^2*b + 2*(3*a^2*b*cosh(d*x + c)^2 + a^2*b)*sinh(d*x + c)^2 + 4*(a^2*b*c
osh(d*x + c)^3 + a^2*b*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(
cosh(d*x + c) - sinh(d*x + c))) - (a^3 + a*b^2 - 3*(a^3 + a*b^2)*cosh(d*x
+ c)^2 - 4*(a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 +
 b^4)*d*cosh(d*x + c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d
*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2
 + b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c...
```

Sympy [F]

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh^2{(c+dx)}\operatorname{sech}{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(tanh(c + d*x)**2*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.81

$$\begin{aligned}\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{a^2b\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^4+2\,a^2b^2+b^4)d} \\ &\quad - \frac{a^2b\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^4+2\,a^2b^2+b^4)d} - \frac{(a^3-ab^2)\arctan\left(e^{(-dx-c)}\right)}{(a^4+2\,a^2b^2+b^4)d} \\ &\quad - \frac{ae^{(-dx-c)}+2\,be^{(-2\,dx-2\,c)}-ae^{(-3\,dx-3\,c)}}{(a^2+b^2+2\,(a^2+b^2)e^{(-2\,dx-2\,c)}+(a^2+b^2)e^{(-4\,dx-4\,c)})d}\end{aligned}$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
a^2*b*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 +
b^4)*d) - a^2*b*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (a
^3 - a*b^2)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) - (a*e^(-d*x
- c) + 2*b*e^(-2*d*x - 2*c) - a*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b
^2)*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(117) = 234$.

Time = 0.16 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.32

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{4\,a^2b^2\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^4b+2\,a^2b^3+b^5} - \frac{2\,a^2b\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^4+2\,a^2b^2+b^4} + \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(a^3-ab^2\right)}{a^4+2\,a^2b^2+b^4} +}{4\,d}$$

input

```
integrate(sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output

```
1/4*(4*a^2*b^2*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^4*b + 2*a
^2*b^3 + b^5) - 2*a^2*b*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^4 + 2*a
^2*b^2 + b^4) + (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(a
^3 - a*b^2)/(a^4 + 2*a^2*b^2 + b^4) + 2*(a^2*b*(e^(d*x + c) - e^(-d*x - c)
)^2 - 2*a^3*(e^(d*x + c) - e^(-d*x - c)) - 2*a*b^2*(e^(d*x + c) - e^(-d*x
- c)) - 4*b^3)/((a^4 + 2*a^2*b^2 + b^4)*((e^(d*x + c) - e^(-d*x - c))^2 +
4)))/d
```

Mupad [B] (verification not implemented)

Time = 2.93 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.80

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{2\,b}{d\,(a^2+b^2)}+\frac{2\,a\,e^{c+d\,x}}{d\,(a^2+b^2)}}{2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1} - \frac{\frac{2\,(a^2\,b+b^3)}{d\,(a^2+b^2)^2}+\frac{e^{c+d\,x}\,(a^3+a\,b^2)}{d\,(a^2+b^2)^2}}{e^{2\,c+2\,d\,x}+1} - \frac{a\,\ln\left(1+e^{c+d\,x}\,1\mathrm{i}\right)}{2\,(-1\mathrm{i}\,d\,a^2+2\,d\,a\,b+1\mathrm{i}\,d\,b^2)} + \frac{a^2\,b\,\ln\left(2\,a^7\,e^{d\,x}\,e^{c}-a^2\,b^5-14\,a^4\,b^3-a^6\,b+a^6\,b\,e^{2\,c}\,e^{2\,d\,x}+2\,a^3\,b^4\,e^{d\,x}\,e^{c}+28\,a^5\,b^2\,e^{d\,x}\,e^{c}+a^2\,b^5\,e^{2\,c}\,e^{2}\right.}{d\,a^4+2\,d\,a^2\,b^2+d\,b^4} - \frac{a\,\ln\left(e^{c+d\,x}+1\mathrm{i}\right)\,1\mathrm{i}}{2\,(-d\,a^2+2\mathrm{i}\,d\,a\,b+d\,b^2)}$$

input

```
int(tanh(c + d*x)^2/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
((2*b)/(d*(a^2 + b^2)) + (2*a*exp(c + d*x))/(d*(a^2 + b^2)))/(2*exp(2*c +
2*d*x) + exp(4*c + 4*d*x) + 1) - ((2*(a^2*b + b^3))/(d*(a^2 + b^2)^2) + (e
xp(c + d*x)*(a*b^2 + a^3))/(d*(a^2 + b^2)^2))/(exp(2*c + 2*d*x) + 1) - (a*
log(exp(c + d*x) + 1i)*1i)/(2*(b^2*d - a^2*d + a*b*d*2i)) - (a*log(exp(c +
 d*x)*1i + 1))/(2*(b^2*d*1i - a^2*d*1i + 2*a*b*d)) + (a^2*b*log(2*a^7*exp(
d*x)*exp(c) - a^2*b^5 - 14*a^4*b^3 - a^6*b + a^6*b*exp(2*c)*exp(2*d*x) + 2
*a^3*b^4*exp(d*x)*exp(c) + 28*a^5*b^2*exp(d*x)*exp(c) + a^2*b^5*exp(2*c)*e
xp(2*d*x) + 14*a^4*b^3*exp(2*c)*exp(2*d*x)))/(a^4*d + b^4*d + 2*a^2*b^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 520, normalized size of antiderivative = 4.30

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{e^{4dx+4c}atan\left(e^{dx+c}\right)a^3 - e^{4dx+4c}atan\left(e^{dx+c}\right)a\,b^2 + 2e^{2dx+2c}atan\left(e^{dx+c}\right)a^3 - 2e^{2dx+2c}atan\left(e^{dx+c}\right)a\,b^2 + a$$

input
```
int(sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**3 - e**(4*c + 4*d*x)*atan(e**(c +
d*x))*a*b**2 + 2*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**3 - 2*e**(2*c + 2*
d*x)*atan(e**(c + d*x))*a*b**2 + atan(e**(c + d*x))*a**3 - atan(e**(c + d*
x))*a*b**2 - e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2*b + e**(4*c +
 4*d*x)*log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*a**2*b + e**(4*c +
4*d*x)*a**2*b + e**(4*c + 4*d*x)*b**3 - e**(3*c + 3*d*x)*a**3 - e**(3*c +
3*d*x)*a*b**2 - 2*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2*b + 2*e*
*(2*c + 2*d*x)*log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*a**2*b + e**
(c + d*x)*a**3 + e**(c + d*x)*a*b**2 - log(e**(2*c + 2*d*x) + 1)*a**2*b +
log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*a**2*b + a**2*b + b**3)/(d*
(e**(4*c + 4*d*x)*a**4 + 2*e**(4*c + 4*d*x)*a**2*b**2 + e**(4*c + 4*d*x)*b
**4 + 2*e**(2*c + 2*d*x)*a**4 + 4*e**(2*c + 2*d*x)*a**2*b**2 + 2*e**(2*c +
 2*d*x)*b**4 + a**4 + 2*a**2*b**2 + b**4))
```

3.390 $\int \frac{\text{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\text{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\text{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(sech(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 64.64 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\text{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\text{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Sech[c + d*x]*Tanh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Sech[c + d*x]*Tanh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(c+dx)\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

$$\Big\downarrow \text{6125}$$

$$\int \frac{\tanh^2(c+dx)\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Sech[c + d*x]*Tanh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\text{sech}\,(dx+c)\tanh\,(dx+c)^2}{(fx+e)\,(a+b\sinh\,(dx+c))}dx$$

input
```
int(sech(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(sech(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 4.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}\left(dx+c\right)\tanh\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(sech(d*x + c)*tanh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 3.86 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\tanh^2\left(c+dx\right)\operatorname{sech}\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(tanh(c + d*x)**2*sech(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)),
 x)
```

Maxima [N/A]

Not integrable

Time = 1.97 (sec) , antiderivative size = 1102, normalized size of antiderivative = 32.41

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}(dx+c)\tanh(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input

```
integrate(sech(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output

```
(b*f - (a*d*f*x*e^(3*c) + (d*e - f)*a*e^(3*c))*e^(3*d*x) - (2*b*d*f*x*e^(2
*c) + (2*d*e - f)*b*e^(2*c))*e^(2*d*x) + (a*d*f*x*e^c + (d*e + f)*a*e^c)*e
^(d*x))/(a^2*d^2*e^2 + b^2*d^2*e^2 + (a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + 2*(
a^2*d^2*e*f + b^2*d^2*e*f)*x + (a^2*d^2*e^2*e^(4*c) + b^2*d^2*e^2*e^(4*c)
+ (a^2*d^2*f^2*e^(4*c) + b^2*d^2*f^2*e^(4*c))*x^2 + 2*(a^2*d^2*e*f*e^(4*c)
 + b^2*d^2*e*f*e^(4*c))*x)*e^(4*d*x) + 2*(a^2*d^2*e^2*e^(2*c) + b^2*d^2*e^
2*e^(2*c) + (a^2*d^2*f^2*e^(2*c) + b^2*d^2*f^2*e^(2*c))*x^2 + 2*(a^2*d^2*e
*f*e^(2*c) + b^2*d^2*e*f*e^(2*c))*x)*e^(2*d*x)) + 2*integrate(1/2*(2*a^2*b
*d^2*f^2*x^2 + 4*a^2*b*d^2*e*f*x + 2*b^3*f^2 + 2*(d^2*e^2 + f^2)*a^2*b + (
(d^2*e^2 + 2*f^2)*a^3*e^c - (d^2*e^2 - 2*f^2)*a*b^2*e^c + (a^3*d^2*f^2*e^c
 - a*b^2*d^2*f^2*e^c)*x^2 + 2*(a^3*d^2*e*f*e^c - a*b^2*d^2*e*f*e^c)*x)*e^(
d*x))/(a^4*d^2*e^3 + 2*a^2*b^2*d^2*e^3 + b^4*d^2*e^3 + (a^4*d^2*f^3 + 2*a^
2*b^2*d^2*f^3 + b^4*d^2*f^3)*x^3 + 3*(a^4*d^2*e*f^2 + 2*a^2*b^2*d^2*e*f^2
+ b^4*d^2*e*f^2)*x^2 + 3*(a^4*d^2*e^2*f + 2*a^2*b^2*d^2*e^2*f + b^4*d^2*e^
2*f)*x + (a^4*d^2*e^3*e^(2*c) + 2*a^2*b^2*d^2*e^3*e^(2*c) + b^4*d^2*e^3*e^
(2*c) + (a^4*d^2*f^3*e^(2*c) + 2*a^2*b^2*d^2*f^3*e^(2*c) + b^4*d^2*f^3*e^(
2*c))*x^3 + 3*(a^4*d^2*e*f^2*e^(2*c) + 2*a^2*b^2*d^2*e*f^2*e^(2*c) + b^4*d
^2*e*f^2*e^(2*c))*x^2 + 3*(a^4*d^2*e^2*f*e^(2*c) + 2*a^2*b^2*d^2*e^2*f*e^(
2*c) + b^4*d^2*e^2*f*e^(2*c))*x)*e^(2*d*x)), x) - 2*integrate(-(a^3*b*e^(d
*x + c) - a^2*b^2)/(a^4*b*e + 2*a^2*b^3*e + b^5*e + (a^4*b*f + 2*a^2*b^...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 3.50 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh(c+d\,x)^2}{\cosh\left(c+d\,x\right)\,\left(e+f\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(tanh(c + d*x)^2/(cosh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(tanh(c + d*x)^2/(cosh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(dx+c)\tanh\left(dx+c\right)^2}{\sinh\left(dx+c\right)be+\sinh\left(dx+c\right)bfx+ae+afx}dx$$

input
```
int(sech(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int((sech(c + d*x)*tanh(c + d*x)**2)/(sinh(c + d*x)*b*e + sinh(c + d*x)*b*
f*x + a*e + a*f*x),x)
```

3.391 $\int \frac{(e+fx)^3 \cosh(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} \, dx$

Optimal result

Integrand size = 34, antiderivative size = 792

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= -\frac{3af^3x}{8b^2d^3} - \frac{a(e+fx)^3}{4b^2d} + \frac{a^3(e+fx)^4}{4b^4f} - \frac{6a^2f^3\cosh(c+dx)}{b^3d^4} + \frac{14f^3\cosh(c+dx)}{9bd^4}$$
$$- \frac{3a^2f(e+fx)^2\cosh(c+dx)}{b^3d^2} + \frac{2f(e+fx)^2\cosh(c+dx)}{3bd^2} - \frac{2f^3\cosh^3(c+dx)}{27bd^4}$$
$$- \frac{a^3(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d} - \frac{a^3(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d}$$
$$- \frac{3a^3f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2} - \frac{3a^3f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^2}$$
$$+ \frac{6a^3f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^3} + \frac{6a^3f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^3}$$
$$- \frac{6a^3f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^4} - \frac{6a^3f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^4}$$
$$+ \frac{6a^2f^2(e+fx)\sinh(c+dx)}{b^3d^3} - \frac{4f^2(e+fx)\sinh(c+dx)}{3bd^3}$$
$$+ \frac{a^2(e+fx)^3\sinh(c+dx)}{b^3d} + \frac{3af^3\cosh(c+dx)\sinh(c+dx)}{8b^2d^4}$$
$$+ \frac{3af(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{4b^2d^2} - \frac{3af^2(e+fx)\sinh^2(c+dx)}{4b^2d^3}$$
$$- \frac{a(e+fx)^3\sinh^2(c+dx)}{2b^2d} - \frac{f(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{3bd^2}$$
$$+ \frac{2f^2(e+fx)\sinh^3(c+dx)}{9bd^3} + \frac{(e+fx)^3\sinh^3(c+dx)}{3bd}$$

output
```
-3/8*a*f^3*x/b^2/d^3-1/2*a*(f*x+e)^3*sinh(d*x+c)^2/b^2/d-6*a^2*f^3*cosh(d*
x+c)/b^3/d^4-1/3*f*(f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^2/b/d^2+6*a^2*f^2*(f*
x+e)*sinh(d*x+c)/b^3/d^3-3/4*a*f^2*(f*x+e)*sinh(d*x+c)^2/b^2/d^3+3/8*a*f^3
*cosh(d*x+c)*sinh(d*x+c)/b^2/d^4-3*a^2*f*(f*x+e)^2*cosh(d*x+c)/b^3/d^2+1/3
*(f*x+e)^3*sinh(d*x+c)^3/b/d+6*a^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+
(a^2+b^2)^(1/2)))/b^4/d^3+6*a^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^
2+b^2)^(1/2)))/b^4/d^3-3*a^3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b
^2)^(1/2)))/b^4/d^2-3*a^3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)
^(1/2)))/b^4/d^2-a^3*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/
d-a^3*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d-6*a^3*f^3*pol
ylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d^4-6*a^3*f^3*polylog(4,-b*e
xp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d^4+14/9*f^3*cosh(d*x+c)/b/d^4-2/27*f^3
*cosh(d*x+c)^3/b/d^4-1/4*a*(f*x+e)^3/b^2/d+a^2*(f*x+e)^3*sinh(d*x+c)/b^3/d
+2/9*f^2*(f*x+e)*sinh(d*x+c)^3/b/d^3+1/4*a^3*(f*x+e)^4/b^4/f+2/3*f*(f*x+e)
^2*cosh(d*x+c)/b/d^2-4/3*f^2*(f*x+e)*sinh(d*x+c)/b/d^3+3/4*a*f*(f*x+e)^2*c
osh(d*x+c)*sinh(d*x+c)/b^2/d^2
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 5656 vs. 2(792) = 1584.

Time = 27.26 (sec) , antiderivative size = 5656, normalized size of antiderivative = 7.14

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]
),x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sinh^3(c+dx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \ 6113$$

$$\frac{\int (e+fx)^3\cosh(c+dx)\sinh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \ 5969$$

$$\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{f\int (e+fx)^2\sinh^3(c+dx)dx}{d}}{b}-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{f\int i(e+fx)^2\sin(ic+idx)^3dx}{d}}{b}$$

$$\Big\downarrow \ 26$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\int (e+fx)^2\sin(ic+idx)^3dx}{d}}{b}$$

$$\Big\downarrow \ 3792$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2f^2\int -i\sinh^3(c+dx)dx}{9d^2}+\frac{2}{3}\int i(e+fx)^2\sinh(c+dx)dx+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)^2\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{d}}{b}$$

$$\Big\downarrow \ 26$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(-\frac{2if^2\int \sinh^3(c+dx)dx}{9d^2}+\frac{2}{3}i\int (e+fx)^2\sinh(c+dx)dx+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)^2\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{d}}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(-\frac{2if^2\int i\sin(ic+idx)^3dx}{9d^2}+\frac{2}{3}i\int -i(e+fx)^2\sin(ic+idx)dx+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)^2\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{d}}{b}$$

$\downarrow$ 26

$$\frac{-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2f^2\int \sin(ic+idx)^3dx}{9d^2}+\frac{2}{3}\int (e+fx)^2\sin(ic+idx)dx+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)^2\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{d}}{b}$$

$\downarrow$ 3113

$$\frac{-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2if^2\int \left(1-\cosh^2(c+dx)\right)d\cosh(c+dx)}{9d^3}+\frac{2}{3}\int (e+fx)^2\sin(ic+idx)dx+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)^2\sinh^2(c+dx)\cosh(c}{3d}\right.}{d}}{b}$$

$\downarrow$ 2009

$$\frac{-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2}{3}\int (e+fx)^2\sin(ic+idx)dx+\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)^2\sinh^2(c+dx)\cosh(c+}{3d}\right.}{d}}{b}$$

$\downarrow$ 3777

$$\frac{-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\cosh(c+dx)dx}{d}\right)+\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}-i(e-\right.}{d}}{b}$$

$\downarrow$ 3042

$$\frac{-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)+\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}-\right.}{d}}{b}$$

$\downarrow$ 3777

$$\frac{-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{d}\right)+\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e\cdots}{}}{d}}{b}}{}$$

$$\downarrow 26$$

$$\frac{-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)}{d}\right)+\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx\cdots}{}}{d}}{b}}{}$$

$$\downarrow 3042$$

$$\frac{-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}\right)+\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+\cdots}{}}{d}}{b}}{}$$

$$\downarrow 26$$

$$\frac{-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}\right)}{d}\right)+\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+f\cdots}{}}{d}}{b}}{}$$

$$\downarrow 3118$$

$$\frac{-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c\cdots}{d^2}\right)}{d}\cdots\right.}{d}}{b}}{}$$

$$\downarrow 6113$$

$$-\frac{a\left(\frac{\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c}{d^2}\right.}{d}\right.\right.}{d}}{b}$$

$\downarrow$ 5969

$$-\frac{a\left(\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}-\frac{3f\int (e+fx)^2\sinh^2(c+dx)dx}{2d}}{b}-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c}{d^2}\right.}{d}\right.\right.}{d}}{b}$$

$\downarrow$ 3042

$$\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c}{d^2}\right.}{d}\right.\right.}{d}}{b}$$
$$\frac{a\left(-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}-\frac{3f\int -(e+fx)^2\sin(ic+idx)^2dx}{2d}}{b}\right)}{b}$$

$\downarrow$ 25

$$\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c}{d^2}\right.}{d}\right.\right.}{d}}{b}$$
$$\frac{a\left(-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}+\frac{3f\int (e+fx)^2\sin(ic+idx)^2dx}{2d}}{b}\right)}{b}$$

$\downarrow$ 3792

$$-\frac{a\left(\frac{\frac{3f\left(\frac{f^2\int -\sinh^2(c+dx)dx}{2d^2}+\frac{1}{2}\int (e+fx)^2dx+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)^3\cosh(c+\cdots}{a+b\sinh\cdots}}{b}\cdots}{b}$$

$$\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c\cdots}{d^2}\cdots\right)}{d}\cdots\right)\cdots\right)}{d}\cdots}{b}$$

$\downarrow$ 17

$$-\frac{a\left(\frac{\frac{3f\left(\frac{f^2\int -\sinh^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\,s\cdots}{a+b\sinh(c+d\cdots}}{b}\cdots}{b}$$

$$\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c\cdots}{d^2}\cdots\right)}{d}\cdots\right)\cdots\right)}{d}\cdots}{b}$$

$\downarrow$ 25

$$-\frac{a\left(\frac{\frac{3f\left(-\frac{f^2\int \sinh^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\,s\cdots}{a+b\sinh(c+d\cdots}}{b}\cdots}{b}$$

$$\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c\cdots}{d^2}\cdots\right)}{d}\cdots\right)\cdots\right)}{d}\cdots}{b}$$

$\downarrow$ 3042

$$\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c\cdots}{d^2}\cdots\right)}{d}\cdots\right)\cdots\right)}{d}\cdots}{b}$$

$$\frac{a\left(-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}+\frac{3f\left(-\frac{f^2\int -\sin(ic+idx)^2dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+d\cdots}{2d}\cdots\right)}{2d}}{b}\cdots\right)}{b}$$

$\downarrow$ 25

$$\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c}{d^2}\right.}{d}\right.\right.}{d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}+\frac{3f\left(\frac{f^2\int\sin(ic+idx)^2dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\right.}{2d}}{b}\right.}{b}$$

$\Big\downarrow$ 3115

$$-\frac{a\left(\frac{\frac{3f\left(\frac{f^2\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx}{}\right.}{b}$$

$$\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c}{d^2}\right.}{d}\right.\right.}{d}}{b}$$

$\Big\downarrow$ 24

$$-\frac{a\left(\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^3c}{a}\right.}{b}$$

$$\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c}{d^2}\right.}{d}\right.\right.}{d}}{b}$$

$\Big\downarrow$ 6113

$$-\frac{a\left(\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{\int(e+fx)^3\ldots}{\ldots}\right)}{\ldots}\right)}{b}$$

$$\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c\ldots}{d^2}\right)}{d}\ldots\right)\right)}{d}}{b}$$

$\downarrow$ 3042

$$\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c\ldots}{d^2}\right)}{d}\ldots\right)\right)}{d}}{b}$$

$$\frac{a\left(\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\ldots}{a+b\ldots}}{\ldots}\right)}{\ldots}\right)}{b}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^3\cosh(dx+c)\sinh(dx+c)^3}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7020 vs. $2(738) = 1476$.

Time = 0.20 (sec) , antiderivative size = 7020, normalized size of antiderivative = 8.86

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/24*e^3*(24*(d*x + c)*a^3/(b^4*d) + 24*a^3*log(-2*a*e^(-d*x - c) + b*e^(
-2*d*x - 2*c) - b)/(b^4*d) + (3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 - b^2)*e
^(-2*d*x - 2*c))*e^(3*d*x + 3*c)/(b^3*d) + (3*a*b*e^(-2*d*x - 2*c) + b^2*e
^(-3*d*x - 3*c) + 3*(4*a^2 - b^2)*e^(-d*x - c))/(b^3*d)) - 1/864*(216*a^3*
d^4*f^3*x^4*e^(3*c) + 864*a^3*d^4*e*f^2*x^3*e^(3*c) + 1296*a^3*d^4*e^2*f*x
^2*e^(3*c) - 4*(9*b^3*d^3*f^3*x^3*e^(6*c) + 9*(3*d^3*e*f^2 - d^2*f^3)*b^3*
x^2*e^(6*c) + 3*(9*d^3*e^2*f - 6*d^2*e*f^2 + 2*d*f^3)*b^3*x*e^(6*c) - (9*d
^2*e^2*f - 6*d*e*f^2 + 2*f^3)*b^3*e^(6*c))*e^(3*d*x) + 27*(4*a*b^2*d^3*f^3
*x^3*e^(5*c) + 6*(2*d^3*e*f^2 - d^2*f^3)*a*b^2*x^2*e^(5*c) + 6*(2*d^3*e^2*
f - 2*d^2*e*f^2 + d*f^3)*a*b^2*x*e^(5*c) - 3*(2*d^2*e^2*f - 2*d*e*f^2 + f^
3)*a*b^2*e^(5*c))*e^(2*d*x) + 108*(12*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*a^2*
b*e^(4*c) - 3*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*b^3*e^(4*c) - (4*a^2*b*d^3*f
^3*e^(4*c) - b^3*d^3*f^3*e^(4*c))*x^3 - 3*(4*(d^3*e*f^2 - d^2*f^3)*a^2*b*e
^(4*c) - (d^3*e*f^2 - d^2*f^3)*b^3*e^(4*c))*x^2 - 3*(4*(d^3*e^2*f - 2*d^2*
e*f^2 + 2*d*f^3)*a^2*b*e^(4*c) - (d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*b^3*e
^(4*c))*x)*e^(d*x) + 108*(12*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*a^2*b*e^(2*c)
 - 3*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*b^3*e^(2*c) + (4*a^2*b*d^3*f^3*e^(2*c
) - b^3*d^3*f^3*e^(2*c))*x^3 + 3*(4*(d^3*e*f^2 + d^2*f^3)*a^2*b*e^(2*c) -
(d^3*e*f^2 + d^2*f^3)*b^3*e^(2*c))*x^2 + 3*(4*(d^3*e^2*f + 2*d^2*e*f^2 + 2
*d*f^3)*a^2*b*e^(2*c) - (d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)*b^3*e^(2*c)...
```

Giac [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\cosh(dx+c)\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)*sinh(d*x + c)^3/(b*sinh(d*x + c) + a),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)^3\,{(e+f\,x)}^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)*sinh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*sinh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3}{a+b\sinh\left(dx+c\right)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

3.392 $\int \frac{(e+fx)^2\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 565

$$\begin{aligned}
&\int \frac{(e+fx)^2\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= -\frac{a(e+fx)^2}{4b^2d}+\frac{a^3(e+fx)^3}{3b^4f}-\frac{2a^2f(e+fx)\cosh(c+dx)}{b^3d^2}+\frac{4f(e+fx)\cosh(c+dx)}{9bd^2} \\
&\quad-\frac{a^3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d}-\frac{a^3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d} \\
&\quad-\frac{2a^3f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}-\frac{2a^3f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^2} \\
&\quad+\frac{2a^3f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^3}+\frac{2a^3f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^3} \\
&\quad+\frac{2a^2f^2\sinh(c+dx)}{b^3d^3}-\frac{4f^2\sinh(c+dx)}{9bd^3}+\frac{a^2(e+fx)^2\sinh(c+dx)}{b^3d} \\
&\quad+\frac{af(e+fx)\cosh(c+dx)\sinh(c+dx)}{2b^2d^2}-\frac{af^2\sinh^2(c+dx)}{4b^2d^3} \\
&\quad-\frac{a(e+fx)^2\sinh^2(c+dx)}{2b^2d}-\frac{2f(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{9bd^2} \\
&\quad+\frac{2f^2\sinh^3(c+dx)}{27bd^3}+\frac{(e+fx)^2\sinh^3(c+dx)}{3bd}
\end{aligned}$$

output
```
-1/4*a*(f*x+e)^2/b^2/d+1/3*a^3*(f*x+e)^3/b^4/f-2*a^2*f*(f*x+e)*cosh(d*x+c)
/b^3/d^2+4/9*f*(f*x+e)*cosh(d*x+c)/b/d^2-a^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(
a-(a^2+b^2)^(1/2)))/b^4/d-a^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/
2)))/b^4/d-2*a^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^
4/d^2-2*a^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d^2
+2*a^3*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d^3+2*a^3*f^2*
polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d^3+2*a^2*f^2*sinh(d*x+c)
/b^3/d^3-4/9*f^2*sinh(d*x+c)/b/d^3+a^2*(f*x+e)^2*sinh(d*x+c)/b^3/d+1/2*a*f
*(f*x+e)*cosh(d*x+c)*sinh(d*x+c)/b^2/d^2-1/4*a*f^2*sinh(d*x+c)^2/b^2/d^3-1
/2*a*(f*x+e)^2*sinh(d*x+c)^2/b^2/d-2/9*f*(f*x+e)*cosh(d*x+c)*sinh(d*x+c)^2
/b/d^2+2/27*f^2*sinh(d*x+c)^3/b/d^3+1/3*(f*x+e)^2*sinh(d*x+c)^3/b/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1961 vs. $2(565) = 1130$.

Time = 7.88 (sec) , antiderivative size = 1961, normalized size of antiderivative = 3.47

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]
),x]
```

output
```
-1/12*(f^2*((4*a*x^3)/(-1 + E^(2*c)) - 2*a*x^3*Coth[c] - (6*a*b^2*(d^2*x^2
*Log[1 + ((a - Sqrt[a^2 + b^2])*E^(-c - d*x))/b] - 2*d*x*PolyLog[2, ((-a +
 Sqrt[a^2 + b^2])*E^(-c - d*x))/b] - 2*PolyLog[3, ((-a + Sqrt[a^2 + b^2])*
E^(-c - d*x))/b]))/(Sqrt[a^2 + b^2]*(-a + Sqrt[a^2 + b^2])*d^3) - (6*a*b^2
*(d^2*x^2*Log[1 + ((a + Sqrt[a^2 + b^2])*E^(-c - d*x))/b] - 2*d*x*PolyLog[
2, -(((a + Sqrt[a^2 + b^2])*E^(-c - d*x))/b)] - 2*PolyLog[3, -(((a + Sqrt[
a^2 + b^2])*E^(-c - d*x))/b)]))/(Sqrt[a^2 + b^2]*(a + Sqrt[a^2 + b^2])*d^3
) + (6*a^2*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*d*x
*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*PolyLog[3, (b*E^(c
 + d*x))/(-a + Sqrt[a^2 + b^2])]))/(Sqrt[a^2 + b^2]*d^3) - (6*a^2*(d^2*x^2
*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2, -((b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2*PolyLog[3, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))]))/(Sqrt[a^2 + b^2]*d^3) + (6*b*Cosh[d*x]*(-2*d*x*Cosh[c]
 + (2 + d^2*x^2)*Sinh[c]))/d^3 + (6*b*((2 + d^2*x^2)*Cosh[c] - 2*d*x*Sinh[
c])*Sinh[d*x])/d^3))/b^2 + (e^2*((a*Log[a + b*Sinh[c + d*x]])/b^2 - Sinh[c
 + d*x]/b))/(2*d) - (e*f*(-2*b*Cosh[c + d*x] - a*(2*c*(c + d*x) - (c + d*x
)^2 + 2*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*(c +
d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*Log[b - 2*a*E^(c
 + d*x) - b*E^(2*(c + d*x))] + 2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2
 + b^2])] + 2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) + 2...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sinh^3(c+dx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6113}$$

$$\frac{\int (e+fx)^2\cosh(c+dx)\sinh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 5969}$$

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2f\int (e+fx)\sinh^3(c+dx)dx}{3d}}{b}-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2f\int i(e+fx)\sin(ic+idx)^3dx}{3d}}{b}$$

$\Big\downarrow$ 26

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\int (e+fx)\sin(ic+idx)^3dx}{3d}}{b}$$

$\Big\downarrow$ 3791

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\int i(e+fx)\sinh(c+dx)dx+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}$$

$\Big\downarrow$ 26

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}i\int (e+fx)\sinh(c+dx)dx+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}$$

$\Big\downarrow$ 3042

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}i\int -i(e+fx)\sin(ic+idx)dx+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}$$

$\Big\downarrow$ 26

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\int (e+fx)\sin(ic+idx)dx+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}$$

$\Big\downarrow$ 3777

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$$\Big\downarrow \text{3117}$$

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$$\Big\downarrow \text{6113}$$

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+$$

$$\Big\downarrow \text{5969}$$

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}-\frac{f\int(e+fx)\sinh^2(c+dx)dx}{d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}-\frac{f\int-\left((e+fx)\sin(ic+idx)^2\right)dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}+\frac{f\int(e+fx)\sin(ic+idx)^2dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \; 3791$$

$$-\frac{a\left(\frac{\frac{f\left(\frac{1}{2}\int(e+fx)dx+\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}$$

$$\Big\downarrow \; 17$$

$$-\frac{a\left(\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}$$

$$\Big\downarrow \; 6113$$

$$-\frac{a\left(\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{\int(e+fx)^2\cosh(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}\right)}{b}+\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)}{b}\right.}{b}}{b}$$

$\downarrow$ 3777

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if}{}}{b}\right.}{b}}{b}$$

$\downarrow$ 26

$$-\frac{a\left(\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int(e+fx)\sinh(c+dx)dx}{d}}{b}-\frac{a\int}{}\right.}{b}}{b}+\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}$$

$\downarrow$ 3042

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int}{}}{b}\right.}{b}}{b}$$

$\downarrow$ 26

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if}{}}{b}\right.}{b}\right.}{b}$$

$\downarrow$ 3777

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if}{}}{}\right.}{b}\right.}{b}$$

$\downarrow$ 3042

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if}{}\right.}{b}\right.}{b}$$

$\downarrow$ 3117

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-$$

$$\frac{a\left(\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if}{\cdots}\right.}{b}\right.}{b}$$

$\downarrow$ 6095

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-$$

$$\frac{a\left(\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\left(\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx\cdots\right.}{b}\right.}{\cdots}\right.}{b}$$

$\downarrow$ 2620

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-$$

$$\frac{a\left(\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\left(-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int(e+fx)\log\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$\downarrow$ 3011

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-$$

$$a\left(\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}{b}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}-\frac{a\left(a-\frac{a\left(-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{Pol}}{}\right.}{bd}\right.}{}\right.}{}\right.$$

$\downarrow$ 2720

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-$$

$$a\left(\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}{b}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}-\frac{a\left(a-\frac{a\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right.}{bd}\right.}{}\right.}{}\right.$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^2 \cosh (dx+c) \sinh (dx+c)^3}{a+b\sinh (dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4263 vs. $2(525) = 1050$.

Time = 0.15 (sec) , antiderivative size = 4263, normalized size of antiderivative = 7.55

$$\int \frac{(e+fx)^2 \cosh(c+dx) \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \cosh(c+dx) \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^2 \cosh(c+dx) \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2 \cosh(dx+c) \sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/24*e^2*(24*(d*x + c)*a^3/(b^4*d) + 24*a^3*log(-2*a*e^(-d*x - c) + b*e^(
-2*d*x - 2*c) - b)/(b^4*d) + (3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 - b^2)*e
^(-2*d*x - 2*c))*e^(3*d*x + 3*c)/(b^3*d) + (3*a*b*e^(-2*d*x - 2*c) + b^2*e
^(-3*d*x - 3*c) + 3*(4*a^2 - b^2)*e^(-d*x - c))/(b^3*d)) - 1/432*(144*a^3*
d^3*f^2*x^3*e^(3*c) + 432*a^3*d^3*e*f*x^2*e^(3*c) - 2*(9*b^3*d^2*f^2*x^2*e
^(6*c) + 6*(3*d^2*e*f - d*f^2)*b^3*x*e^(6*c) - 2*(3*d*e*f - f^2)*b^3*e^(6*
c))*e^(3*d*x) + 27*(2*a*b^2*d^2*f^2*x^2*e^(5*c) + 2*(2*d^2*e*f - d*f^2)*a*
b^2*x*e^(5*c) - (2*d*e*f - f^2)*a*b^2*e^(5*c))*e^(2*d*x) + 54*(8*(d*e*f -
f^2)*a^2*b*e^(4*c) - 2*(d*e*f - f^2)*b^3*e^(4*c) - (4*a^2*b*d^2*f^2*e^(4*c
) - b^3*d^2*f^2*e^(4*c))*x^2 - 2*(4*(d^2*e*f - d*f^2)*a^2*b*e^(4*c) - (d^2
*e*f - d*f^2)*b^3*e^(4*c))*x)*e^(d*x) + 54*(8*(d*e*f + f^2)*a^2*b*e^(2*c)
- 2*(d*e*f + f^2)*b^3*e^(2*c) + (4*a^2*b*d^2*f^2*e^(2*c) - b^3*d^2*f^2*e^(
2*c))*x^2 + 2*(4*(d^2*e*f + d*f^2)*a^2*b*e^(2*c) - (d^2*e*f + d*f^2)*b^3*e
^(2*c))*x)*e^(-d*x) + 27*(2*a*b^2*d^2*f^2*x^2*e^c + 2*(2*d^2*e*f + d*f^2)*
a*b^2*x*e^c + (2*d*e*f + f^2)*a*b^2*e^c)*e^(-2*d*x) + 2*(9*b^3*d^2*f^2*x^2
 + 6*(3*d^2*e*f + d*f^2)*b^3*x + 2*(3*d*e*f + f^2)*b^3)*e^(-3*d*x))*e^(-3*
c)/(b^4*d^3) + integrate(-2*(a^3*b*f^2*x^2 + 2*a^3*b*e*f*x - (a^4*f^2*x^2*
e^c + 2*a^4*e*f*x*e^c)*e^(d*x))/(b^5*e^(2*d*x + 2*c) + 2*a*b^4*e^(d*x + c)
 - b^5), x)
```

Giac [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\cosh(dx+c)\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)*sinh(d*x + c)^3/(b*sinh(d*x + c) + a),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)^3\,(e+f\,x)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)*sinh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*sinh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output

```
(18*e**(6*c + 6*d*x)*b**6*d**2*e**2 + 36*e**(6*c + 6*d*x)*b**6*d**2*e*f*x
+ 18*e**(6*c + 6*d*x)*b**6*d**2*f**2*x**2 - 12*e**(6*c + 6*d*x)*b**6*d*e*f
 - 12*e**(6*c + 6*d*x)*b**6*d*f**2*x + 4*e**(6*c + 6*d*x)*b**6*f**2 - 54*e
**(5*c + 5*d*x)*a*b**5*d**2*e**2 - 108*e**(5*c + 5*d*x)*a*b**5*d**2*e*f*x
- 54*e**(5*c + 5*d*x)*a*b**5*d**2*f**2*x**2 + 54*e**(5*c + 5*d*x)*a*b**5*d
*e*f + 54*e**(5*c + 5*d*x)*a*b**5*d*f**2*x - 27*e**(5*c + 5*d*x)*a*b**5*f*
*2 + 216*e**(4*c + 4*d*x)*a**2*b**4*d**2*e**2 + 432*e**(4*c + 4*d*x)*a**2*
b**4*d**2*e*f*x + 216*e**(4*c + 4*d*x)*a**2*b**4*d**2*f**2*x**2 - 432*e**(
4*c + 4*d*x)*a**2*b**4*d*e*f - 432*e**(4*c + 4*d*x)*a**2*b**4*d*f**2*x + 4
32*e**(4*c + 4*d*x)*a**2*b**4*f**2 - 54*e**(4*c + 4*d*x)*b**6*d**2*e**2 -
108*e**(4*c + 4*d*x)*b**6*d**2*e*f*x - 54*e**(4*c + 4*d*x)*b**6*d**2*f**2*
x**2 + 108*e**(4*c + 4*d*x)*b**6*d*e*f + 108*e**(4*c + 4*d*x)*b**6*d*f**2*
x - 108*e**(4*c + 4*d*x)*b**6*f**2 + 3456*e**(3*c + 3*d*x)*int(x**2/(e**(5
*c + 5*d*x)*b + 2*e**(4*c + 4*d*x)*a - e**(3*c + 3*d*x)*b),x)*a**6*b*d**3*
f**2 + 2592*e**(3*c + 3*d*x)*int(x**2/(e**(5*c + 5*d*x)*b + 2*e**(4*c + 4*
d*x)*a - e**(3*c + 3*d*x)*b),x)*a**4*b**3*d**3*f**2 + 6912*e**(3*c + 3*d*x
)*int(x/(e**(5*c + 5*d*x)*b + 2*e**(4*c + 4*d*x)*a - e**(3*c + 3*d*x)*b),x
)*a**6*b*d**3*e*f + 5184*e**(3*c + 3*d*x)*int(x/(e**(5*c + 5*d*x)*b + 2*e*
*(4*c + 4*d*x)*a - e**(3*c + 3*d*x)*b),x)*a**4*b**3*d**3*e*f - 432*e**(3*c
 + 3*d*x)*log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*a**3*b**3*d**2...
```

3.393 $\int \frac{(e+fx)\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 348

$$\begin{aligned}\int \frac{(e+fx)\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx &= -\frac{afx}{4b^2d}+\frac{a^3(e+fx)^2}{2b^4f}-\frac{a^2f\cosh(c+dx)}{b^3d^2}+\frac{f\cosh(c+dx)}{3bd^2}-\frac{f\cosh^3(c+dx)}{9bd^2}\\ &-\frac{a^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d}-\frac{a^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d}\\ &-\frac{a^3f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}-\frac{a^3f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^2}\\ &+\frac{a^2(e+fx)\sinh(c+dx)}{b^3d}+\frac{af\cosh(c+dx)\sinh(c+dx)}{4b^2d^2}\\ &-\frac{a(e+fx)\sinh^2(c+dx)}{2b^2d}+\frac{(e+fx)\sinh^3(c+dx)}{3bd}\end{aligned}$$

output
```
-1/4*a*f*x/b^2/d+1/2*a^3*(f*x+e)^2/b^4/f-a^2*f*cosh(d*x+c)/b^3/d^2+1/3*f*c
osh(d*x+c)/b/d^2-1/9*f*cosh(d*x+c)^3/b/d^2-a^3*(f*x+e)*ln(1+b*exp(d*x+c)/(
a-(a^2+b^2)^(1/2)))/b^4/d-a^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)
))/b^4/d-a^3*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d^2-a^3*f*
polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d^2+a^2*(f*x+e)*sinh(d*x+
c)/b^3/d+1/4*a*f*cosh(d*x+c)*sinh(d*x+c)/b^2/d^2-1/2*a*(f*x+e)*sinh(d*x+c)
^2/b^2/d+1/3*(f*x+e)*sinh(d*x+c)^3/b/d
```

Mathematica [A] (warning: unable to verify)

Time = 0.81 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.30

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{36a^3c^2f - 36a^3d^2fx^2 + 72a^2bf\cosh(c+dx) - 18b^3f\cosh(c+dx) + 18ab^2dfx\cosh(2(c+dx)) + 2b^3 \ldots}{}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),
x]
```

output
```
-1/72*(36*a^3*c^2*f - 36*a^3*d^2*f*x^2 + 72*a^2*b*f*Cosh[c + d*x] - 18*b^3
*f*Cosh[c + d*x] + 18*a*b^2*d*f*x*Cosh[2*(c + d*x)] + 2*b^3*f*Cosh[3*(c +
d*x)] + 72*a^3*c*f*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 72*a^3
*d*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 72*a^3*c*f*Log[1 +
 (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 72*a^3*d*f*x*Log[1 + (b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2])] - 72*a^3*c*f*Log[b - 2*a*E^(c + d*x) - b*E^(2*
(c + d*x))] + 72*a^3*d*e*Log[a + b*Sinh[c + d*x]] + 72*a^3*f*PolyLog[2, (b
*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 72*a^3*f*PolyLog[2, -((b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2]))] - 72*a^2*b*d*e*Sinh[c + d*x] - 72*a^2*b*d*f*x*
Sinh[c + d*x] + 18*b^3*d*f*x*Sinh[c + d*x] + 36*a*b^2*d*e*Sinh[c + d*x]^2
- 24*b^3*d*e*Sinh[c + d*x]^3 - 9*a*b^2*f*Sinh[2*(c + d*x)] - 6*b^3*d*f*x*S
inh[3*(c + d*x)])/(b^4*d^2)
```

Rubi [A] (verified)

Time = 2.29 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.96, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.719$, Rules used = {6113, 5969, 3042, 26, 3113, 2009, 6113, 5969, 3042, 25, 3115, 24, 6113, 3042, 3777, 26, 3042, 26, 3118, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh^3(c+dx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$\Big\downarrow$ 6113

$$\frac{\int (e+fx)\cosh(c+dx)\sinh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$\Big\downarrow$ 5969

$$\frac{\frac{(e+fx)\sinh^3(c+dx)}{3d}-\frac{f\int \sinh^3(c+dx)dx}{3d}}{b}-\frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$\Big\downarrow$ 3042

$$-\frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\sinh^3(c+dx)}{3d}-\frac{f\int i\sin(ic+idx)^3dx}{3d}}{b}$$

$\Big\downarrow$ 26

$$-\frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\sinh^3(c+dx)}{3d}-\frac{if\int \sin(ic+idx)^3dx}{3d}}{b}$$

$\Big\downarrow$ 3113

$$\frac{\frac{f\int (1-\cosh^2(c+dx))d\cosh(c+dx)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b}-\frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$\Big\downarrow$ 2009

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b}-\frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$\Big\downarrow$ 6113

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b}-\frac{a\left(\frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$\Big\downarrow$ 5969

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}-\frac{f\int \sinh^2(c+dx)dx}{2d}}{b}-\frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}-\frac{f\int-\sin(ic+idx)^2dx}{2d}}{b}\right)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\int\sin(ic+idx)^2dx}{2d}}{b}\right)}{b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b}-\frac{a\left(\frac{\frac{f\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}+\frac{(e+fx)\sinh^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\int\frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{6113}$$

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(\frac{\int(e+fx)\cosh(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b}\right)}{b}\right)}{b}}{b}$$

$\Big\downarrow$ 3777

$$\frac{\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int-i\sinh(c+dx)dx}{d}}{b}\right)}{b}\right)}{b}}{b}$$

$\Big\downarrow$ 26

$$\frac{\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}}{b}-\frac{a\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}\right)}{b}}{b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int-i\sin(ic+idx)dx}{d}}{b}\right)}{b}\right)}{b}}{b}$$

$\Big\downarrow$ 26

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int\sin(ic+idx)dx}{d}}{b}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 3118

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b}-\frac{a\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 6095

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b}-\frac{a\left(\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}\right)}{b}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 2620

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b}-\frac{a\left(-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}\right)}{bd}\right)}{}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 2715

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b}-\frac{a\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e\ldots}{\ldots}\right.}{\ldots}\right)}{\ldots}\right)}{b}$$

$$\Big\downarrow \; 2838$$

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b}-\frac{a\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{b\ldots}{a+\ldots}\right)}{bd^2}\ldots\right.}{\ldots}\right)}{b}\right)}{b}$$

input
```
Int[((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
((f*(Cosh[c + d*x] - Cosh[c + d*x]^3/3))/(3*d^2) + ((e + f*x)*Sinh[c + d*x
]^3)/(3*d))/b - (a*(-((a*(-((a*(-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Log[1
+ (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))])/(b*d^2)))/b) + (-((f*Cosh[c + d*x])/d^2) + ((e + f*x)*S
inh[c + d*x])/d)/b))/b) + (((e + f*x)*Sinh[c + d*x]^2)/(2*d) + (f*(x/2 - (
Cosh[c + d*x]*Sinh[c + d*x])/(2*d)))/(2*d))/b))/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 670 vs. $2(322) = 644$.

Time = 28.71 (sec) , antiderivative size = 671, normalized size of antiderivative = 1.93

method	result
risch	$\frac{a^3 f x^2}{2b^4} - \frac{a^3 e x}{b^4} + \frac{(3dxf+3de-f)e^{3dx+3c}}{72b\,d^2} - \frac{a(2dxf+2de-f)e^{2dx+2c}}{16b^2d^2} + \frac{(4a^2dfx-b^2dfx+4a^2de-b^2de-4a^2f+b^2f)e^{dx+c}}{8b^3d^2} -$

input
```
int((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
1/2*a^3/b^4*f*x^2-a^3/b^4*e*x+1/72*(3*d*f*x+3*d*e-f)/b/d^2*exp(3*d*x+3*c)-
1/16*a*(2*d*f*x+2*d*e-f)/b^2/d^2*exp(2*d*x+2*c)+1/8*(4*a^2*d*f*x-b^2*d*f*x
+4*a^2*d*e-b^2*d*e-4*a^2*f+b^2*f)/b^3/d^2*exp(d*x+c)-1/8*(4*a^2-b^2)*(d*f*
x+d*e+f)/b^3/d^2*exp(-d*x-c)-1/16*a*(2*d*f*x+2*d*e+f)/b^2/d^2*exp(-2*d*x-2
*c)-1/72*(3*d*f*x+3*d*e+f)/b/d^2*exp(-3*d*x-3*c)+2/d*a^3/b^4*e*ln(exp(d*x+
c))+2/d*a^3/b^4*f*c*x-1/d*a^3/b^4*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(
-a+(a^2+b^2)^(1/2)))*x-1/d*a^3/b^4*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(
a+(a^2+b^2)^(1/2)))*x-1/d^2*a^3/b^4*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)
-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2*a^3/b^4*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1
/2)+a)/(a+(a^2+b^2)^(1/2)))+1/d^2*a^3/b^4*f*c^2-1/d^2*a^3/b^4*f*ln((-b*exp
(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d^2*a^3/b^4*f*ln((b*e
xp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/d^2*a^3/b^4*c*f*ln(b
*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d^2*a^3/b^4*c*f*ln(exp(d*x+c))-1/d*a^3
/b^4*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2129 vs. $2(320) = 640$.

Time = 0.14 (sec) , antiderivative size = 2129, normalized size of antiderivative = 6.12

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
1/144*(2*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^6 + 2*(3*b^3*d*f*
x + 3*b^3*d*e - b^3*f)*sinh(d*x + c)^6 - 6*b^3*d*f*x - 9*(2*a*b^2*d*f*x +
2*a*b^2*d*e - a*b^2*f)*cosh(d*x + c)^5 - 3*(6*a*b^2*d*f*x + 6*a*b^2*d*e -
3*a*b^2*f - 4*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c))*sinh(d*x +
c)^5 - 6*b^3*d*e + 18*((4*a^2*b - b^3)*d*f*x + (4*a^2*b - b^3)*d*e - (4*a^
2*b - b^3)*f)*cosh(d*x + c)^4 + 3*(6*(4*a^2*b - b^3)*d*f*x + 6*(4*a^2*b -
b^3)*d*e + 10*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^2 - 6*(4*a^2
*b - b^3)*f - 15*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*cosh(d*x + c))*si
nh(d*x + c)^4 - 2*b^3*f + 72*(a^3*d^2*f*x^2 + 2*a^3*d^2*e*x + 4*a^3*c*d*e
- 2*a^3*c^2*f)*cosh(d*x + c)^3 + 2*(36*a^3*d^2*f*x^2 + 72*a^3*d^2*e*x + 14
4*a^3*c*d*e - 72*a^3*c^2*f + 20*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x
 + c)^3 - 45*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*cosh(d*x + c)^2 + 36*
((4*a^2*b - b^3)*d*f*x + (4*a^2*b - b^3)*d*e - (4*a^2*b - b^3)*f)*cosh(d*x
 + c))*sinh(d*x + c)^3 - 18*((4*a^2*b - b^3)*d*f*x + (4*a^2*b - b^3)*d*e +
 (4*a^2*b - b^3)*f)*cosh(d*x + c)^2 + 6*(5*(3*b^3*d*f*x + 3*b^3*d*e - b^3*
f)*cosh(d*x + c)^4 - 3*(4*a^2*b - b^3)*d*f*x - 15*(2*a*b^2*d*f*x + 2*a*b^2
*d*e - a*b^2*f)*cosh(d*x + c)^3 - 3*(4*a^2*b - b^3)*d*e + 18*((4*a^2*b - b
^3)*d*f*x + (4*a^2*b - b^3)*d*e - (4*a^2*b - b^3)*f)*cosh(d*x + c)^2 - 3*(
4*a^2*b - b^3)*f + 36*(a^3*d^2*f*x^2 + 2*a^3*d^2*e*x + 4*a^3*c*d*e - 2*a^3
*c^2*f)*cosh(d*x + c))*sinh(d*x + c)^2 - 9*(2*a*b^2*d*f*x + 2*a*b^2*d*e...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh{(dx+c)}\sinh{(dx+c)}^3}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-1/24*e*(24*(d*x + c)*a^3/(b^4*d) + 24*a^3*log(-2*a*e^(-d*x - c) + b*e^(-2
*d*x - 2*c) - b)/(b^4*d) + (3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 - b^2)*e^(
-2*d*x - 2*c))*e^(3*d*x + 3*c)/(b^3*d) + (3*a*b*e^(-2*d*x - 2*c) + b^2*e^(
-3*d*x - 3*c) + 3*(4*a^2 - b^2)*e^(-d*x - c))/(b^3*d)) - 1/144*f*((72*a^3*
d^2*x^2*e^(3*c) - 2*(3*b^3*d*x*e^(6*c) - b^3*e^(6*c))*e^(3*d*x) + 9*(2*a*b
^2*d*x*e^(5*c) - a*b^2*e^(5*c))*e^(2*d*x) + 18*(4*a^2*b*e^(4*c) - b^3*e^(4
*c) - (4*a^2*b*d*e^(4*c) - b^3*d*e^(4*c))*x)*e^(d*x) + 18*(4*a^2*b*e^(2*c)
 - b^3*e^(2*c) + (4*a^2*b*d*e^(2*c) - b^3*d*e^(2*c))*x)*e^(-d*x) + 9*(2*a*
b^2*d*x*e^c + a*b^2*e^c)*e^(-2*d*x) + 2*(3*b^3*d*x + b^3)*e^(-3*d*x))*e^(-
3*c)/(b^4*d^2) - 9*integrate(32*(a^4*x*e^(d*x + c) - a^3*b*x)/(b^5*e^(2*d*
x + 2*c) + 2*a*b^4*e^(d*x + c) - b^5), x))
```

Giac [F]

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh{(dx+c)}\sinh{(dx+c)}^3}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)*sinh(d*x + c)^3/(b*sinh(d*x + c) + a), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)^3\,(e+f\,x)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)*sinh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*sinh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\frac{-2e^{6dx+6c}b^6f+18e^{2dx+2c}b^6f-96a^4b^2f-6b^6de+6e^{6dx+6c}b^6dfx-18e^{5dx+5c}ab^5de-72e^{2dx+2c}a^2b^4de+}{}$$

input
```
int((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output

```
(6*e**(6*c + 6*d*x)*b**6*d*e + 6*e**(6*c + 6*d*x)*b**6*d*f*x - 2*e**(6*c +
 6*d*x)*b**6*f - 18*e**(5*c + 5*d*x)*a*b**5*d*e - 18*e**(5*c + 5*d*x)*a*b*
*5*d*f*x + 9*e**(5*c + 5*d*x)*a*b**5*f + 72*e**(4*c + 4*d*x)*a**2*b**4*d*e
 + 72*e**(4*c + 4*d*x)*a**2*b**4*d*f*x - 72*e**(4*c + 4*d*x)*a**2*b**4*f -
 18*e**(4*c + 4*d*x)*b**6*d*e - 18*e**(4*c + 4*d*x)*b**6*d*f*x + 18*e**(4*
c + 4*d*x)*b**6*f + 1152*e**(3*c + 3*d*x)*int(x/(e**(5*c + 5*d*x)*b + 2*e*
*(4*c + 4*d*x)*a - e**(3*c + 3*d*x)*b),x)*a**6*b*d**2*f + 864*e**(3*c + 3*
d*x)*int(x/(e**(5*c + 5*d*x)*b + 2*e**(4*c + 4*d*x)*a - e**(3*c + 3*d*x)*b
),x)*a**4*b**3*d**2*f - 144*e**(3*c + 3*d*x)*log(e**(2*c + 2*d*x)*b + 2*e*
*(c + d*x)*a - b)*a**3*b**3*d*e + 144*e**(3*c + 3*d*x)*a**3*b**3*d**2*e*x
- 72*e**(3*c + 3*d*x)*a**3*b**3*d**2*f*x**2 - 2304*e**(c + 3*d*x)*int(x/(e
**(2*c + 4*d*x)*b + 2*e**(c + 3*d*x)*a - e**(2*d*x)*b),x)*a**7*d**2*f - 23
04*e**(c + 3*d*x)*int(x/(e**(2*c + 4*d*x)*b + 2*e**(c + 3*d*x)*a - e**(2*d
*x)*b),x)*a**5*b**2*d**2*f - 288*e**(c + 3*d*x)*int(x/(e**(2*c + 4*d*x)*b
+ 2*e**(c + 3*d*x)*a - e**(2*d*x)*b),x)*a**3*b**4*d**2*f - 288*e**(2*c + 2
*d*x)*a**4*b**2*d*f*x - 288*e**(2*c + 2*d*x)*a**4*b**2*f - 72*e**(2*c + 2*
d*x)*a**2*b**4*d*e - 72*e**(2*c + 2*d*x)*a**2*b**4*d*f*x - 72*e**(2*c + 2*
d*x)*a**2*b**4*f + 18*e**(2*c + 2*d*x)*b**6*d*e + 18*e**(2*c + 2*d*x)*b**6
*d*f*x + 18*e**(2*c + 2*d*x)*b**6*f + 288*e**(c + d*x)*a**5*b*d*f*x + 144*
e**(c + d*x)*a**5*b*f + 144*e**(c + d*x)*a**3*b**3*d*f*x + 72*e**(c + d...
```

3.394 $\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 76

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=-\frac{a^3\log(a+b\sinh(c+dx))}{b^4d}+\frac{a^2\sinh(c+dx)}{b^3d}-\frac{a\sinh^2(c+dx)}{2b^2d}+\frac{\sinh^3(c+dx)}{3bd}$$

output
```
-a^3*ln(a+b*sinh(d*x+c))/b^4/d+a^2*sinh(d*x+c)/b^3/d-1/2*a*sinh(d*x+c)^2/b
^2/d+1/3*sinh(d*x+c)^3/b/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx =\frac{-6a^3\log(a+b\sinh(c+dx))+6a^2b\sinh(c+dx)-3ab^2\sinh^2(c+dx)+2b^3\sinh^3(c+dx)}{6b^4d}$$

input
```
Integrate[(Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-6*a^3*Log[a + b*Sinh[c + d*x]] + 6*a^2*b*Sinh[c + d*x] - 3*a*b^2*Sinh[c
+ d*x]^2 + 2*b^3*Sinh[c + d*x]^3)/(6*b^4*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3042, 26, 3312, 26, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ic+idx)^3\cos(ic+idx)}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cos(ic+idx)\sin(ic+idx)^3}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3312}$$

$$\frac{i\int -\frac{i\sinh^3(c+dx)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{bd}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b^3\sinh^3(c+dx)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^4d}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(-\frac{a^3}{a+b\sinh(c+dx)}+a^2-b\sinh(c+dx)a+b^2\sinh^2(c+dx)\right)d(b\sinh(c+dx))}{b^4d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3(-\log(a+b\sinh(c+dx)))+a^2b\sinh(c+dx)-\frac{1}{2}ab^2\sinh^2(c+dx)+\frac{1}{3}b^3\sinh^3(c+dx)}{b^4d}$$

input
```
Int[(Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-(a^3*Log[a + b*Sinh[c + d*x]]) + a^2*b*Sinh[c + d*x] - (a*b^2*Sinh[c + d
*x]^2)/2 + (b^3*Sinh[c + d*x]^3)/3)/(b^4*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 12.64 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{\frac{\frac{\sinh(dx+c)^3 b^2}{3}-\frac{a\sinh(dx+c)^2 b}{2}+\sinh(dx+c)a^2}{b^3}-\frac{a^3\ln(a+b\sinh(dx+c))}{b^4}}{d}$
default	$\frac{\frac{\frac{\sinh(dx+c)^3 b^2}{3}-\frac{a\sinh(dx+c)^2 b}{2}+\sinh(dx+c)a^2}{b^3}-\frac{a^3\ln(a+b\sinh(dx+c))}{b^4}}{d}$
risch	$\frac{a^3x}{b^4}+\frac{e^{3dx+3c}}{24bd}-\frac{a\,e^{2dx+2c}}{8b^2d}+\frac{e^{dx+c}a^2}{2b^3d}-\frac{e^{dx+c}}{8bd}-\frac{e^{-dx-c}a^2}{2b^3d}+\frac{e^{-dx-c}}{8bd}-\frac{a\,e^{-2dx-2c}}{8b^2d}-\frac{e^{-3dx-3c}}{24bd}+$

input
```
int(cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^3*(1/3*sinh(d*x+c)^3*b^2-1/2*a*sinh(d*x+c)^2*b+sinh(d*x+c)*a^2)-a
^3/b^4*ln(a+b*sinh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 602 vs. 2(72) = 144.

Time = 0.13 (sec) , antiderivative size = 602, normalized size of antiderivative = 7.92

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/24*(b^3*cosh(d*x + c)^6 + b^3*sinh(d*x + c)^6 + 24*a^3*d*x*cosh(d*x + c)
^3 - 3*a*b^2*cosh(d*x + c)^5 + 3*(2*b^3*cosh(d*x + c) - a*b^2)*sinh(d*x +
c)^5 + 3*(4*a^2*b - b^3)*cosh(d*x + c)^4 + 3*(5*b^3*cosh(d*x + c)^2 - 5*a*
b^2*cosh(d*x + c) + 4*a^2*b - b^3)*sinh(d*x + c)^4 - 3*a*b^2*cosh(d*x + c)
 + 2*(10*b^3*cosh(d*x + c)^3 + 12*a^3*d*x - 15*a*b^2*cosh(d*x + c)^2 + 6*(
4*a^2*b - b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - b^3 - 3*(4*a^2*b - b^3)*co
sh(d*x + c)^2 + 3*(5*b^3*cosh(d*x + c)^4 + 24*a^3*d*x*cosh(d*x + c) - 10*a
*b^2*cosh(d*x + c)^3 - 4*a^2*b + b^3 + 6*(4*a^2*b - b^3)*cosh(d*x + c)^2)*
sinh(d*x + c)^2 - 24*(a^3*cosh(d*x + c)^3 + 3*a^3*cosh(d*x + c)^2*sinh(d*x
 + c) + 3*a^3*cosh(d*x + c)*sinh(d*x + c)^2 + a^3*sinh(d*x + c)^3)*log(2*(
b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + 3*(2*b^3*cosh(d*x
+ c)^5 + 24*a^3*d*x*cosh(d*x + c)^2 - 5*a*b^2*cosh(d*x + c)^4 + 4*(4*a^2*b
 - b^3)*cosh(d*x + c)^3 - a*b^2 - 2*(4*a^2*b - b^3)*cosh(d*x + c))*sinh(d*
x + c))/(b^4*d*cosh(d*x + c)^3 + 3*b^4*d*cosh(d*x + c)^2*sinh(d*x + c) + 3
*b^4*d*cosh(d*x + c)*sinh(d*x + c)^2 + b^4*d*sinh(d*x + c)^3)
```

Sympy [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.38

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \begin{cases} \frac{x\sinh^3(c)\cosh(c)}{a} & \text{for } b=0 \wedge d=0 \\ \frac{\sinh^4(c+dx)}{4ad} & \text{for } b=0 \\ \frac{x\sinh^3(c)\cosh(c)}{a+b\sinh(c)} & \text{for } d=0 \\ -\frac{a^3\log\left(\frac{a}{b}+\sinh(c+dx)\right)}{b^4d}+\frac{a^2\sinh(c+dx)}{b^3d}-\frac{a\sinh^2(c+dx)}{2b^2d}+\frac{\sinh^3(c+dx)}{3bd} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Piecewise((x*sinh(c)**3*cosh(c)/a, Eq(b, 0) & Eq(d, 0)), (sinh(c + d*x)**4
/(4*a*d), Eq(b, 0)), (x*sinh(c)**3*cosh(c)/(a + b*sinh(c)), Eq(d, 0)), (-a
**3*log(a/b + sinh(c + d*x))/(b**4*d) + a**2*sinh(c + d*x)/(b**3*d) - a*si
nh(c + d*x)**2/(2*b**2*d) + sinh(c + d*x)**3/(3*b*d), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(72) = 144$.

Time = 0.04 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.25

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= -\frac{(dx+c)a^3}{b^4d} - \frac{a^3\log\left(-2\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)} - b\right)}{b^4d} - \frac{\left(3\,abe^{(-dx-c)} - b^2 - 3\,(4\,a^2-b^2)e^{(-2\,dx-2\,c)}\right)e^{(3\,dx+3\,c)}}{24\,b^3d} - \frac{3\,abe^{(-2\,dx-2\,c)} + b^2e^{(-3\,dx-3\,c)} + 3\,(4\,a^2-b^2)e^{(-dx-c)}}{24\,b^3d}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-(d*x + c)*a^3/(b^4*d) - a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) -
b)/(b^4*d) - 1/24*(3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 - b^2)*e^(-2*d*x -
2*c))*e^(3*d*x + 3*c)/(b^3*d) - 1/24*(3*a*b*e^(-2*d*x - 2*c) + b^2*e^(-3*d
*x - 3*c) + 3*(4*a^2 - b^2)*e^(-d*x - c))/(b^3*d)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.54

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{24\,a^3\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{b^4} - \frac{b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3 - 3\,ab\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2 + 12\,a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{b^3}}{24\,d}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/24*(24*a^3*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/b^4 - (b^2*(e
^(d*x + c) - e^(-d*x - c))^3 - 3*a*b*(e^(d*x + c) - e^(-d*x - c))^2 + 12*a
^2*(e^(d*x + c) - e^(-d*x - c)))/b^3)/d
```

Mupad [B] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.83

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= -\frac{a^3\,\ln\left(a+b\sinh(c+d\,x)\right)-\frac{b^3\sinh(c+d\,x)^3}{3}+\frac{a\,b^2\sinh(c+d\,x)^2}{2}-a^2\,b\sinh(c+d\,x)}{b^4\,d}$$

input
```
int((cosh(c + d*x)*sinh(c + d*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
-(a^3*log(a + b*sinh(c + d*x)) - (b^3*sinh(c + d*x)^3)/3 + (a*b^2*sinh(c +
 d*x)^2)/2 - a^2*b*sinh(c + d*x))/(b^4*d)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-3\cosh\left(dx+c\right)^2 a\,b^2-6\log(a+b\sinh\left(dx+c\right))\,a^3+2\sinh\left(dx+c\right)^3 b^3+6\sinh\left(dx+c\right)a^2b}{6b^4d}$$

input
```
int(cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
( - 3*cosh(c + d*x)**2*a*b**2 - 6*log(sinh(c + d*x)*b + a)*a**3 + 2*sinh(c
 + d*x)**3*b**3 + 6*sinh(c + d*x)*a**2*b)/(6*b**4*d)
```

3.395 $\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(cosh(d*x+c)*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 34.89 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Cosh[c + d*x]*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Cosh[c + d*x]*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

$$\Big\downarrow \; 6125$$

$$\int \frac{\sinh^3(c+dx)\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Cosh[c + d*x]*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(dx+c)\sinh(dx+c)^3}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(cosh(d*x+c)*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(cosh(d*x + c)*sinh(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\text{Timed out}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 326, normalized size of antiderivative = 9.59

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-1/8*e^(-3*c + 3*d*e/f)*exp_integral_e(1, 3*(f*x + e)*d/f)/(b*f) - 1/4*a*e
^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b^2*f) + 1/4*a*e^(2*
c - 2*d*e/f)*exp_integral_e(1, -2*(f*x + e)*d/f)/(b^2*f) - 1/8*e^(3*c - 3*
d*e/f)*exp_integral_e(1, -3*(f*x + e)*d/f)/(b*f) - 1/8*(4*a^2 - b^2)*e^(-c
 + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^3*f) - 1/8*(4*a^2*e^c - b^2*
e^c)*e^(-d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b^3*f) - a^3*log(f*x +
e)/(b^4*f) + 1/16*integrate(-32*(a^4*e^(d*x + c) - a^3*b)/(b^5*f*x + b^5*e
 - (b^5*f*x*e^(2*c) + b^5*e*e^(2*c))*e^(2*d*x) - 2*(a*b^4*f*x*e^c + a*b^4*
e*e^c)*e^(d*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
integrate(cosh(d*x + c)*sinh(d*x + c)^3/((f*x + e)*(b*sinh(d*x + c) + a)),
 x)
```

Mupad [N/A]

Not integrable

Time = 2.96 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)^3}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((cosh(c + d*x)*sinh(c + d*x)^3)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((cosh(c + d*x)*sinh(c + d*x)^3)/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 336, normalized size of antiderivative = 9.88

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \frac{e^{6c}\left(\int \frac{e^{5dx}}{e^{2dx+2c}be+e^{2dx+2c}bfx+2e^{dx+c}ae+2e^{dx+c}afx-be-bfx}dx\right)-2e^{4c}\left(\int \frac{e^{3dx}}{e^{2dx+2c}be+e^{2dx+2c}bfx+2e^{dx+c}ae+2e^{dx+c}afx-be-bf}\right.}{}$$

input
```
int(cosh(d*x+c)*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
(e**(6*c)*int(e**(5*d*x)/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x +
2*e**(c + d*x)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x) - 2*e**(4*c)*i
nt(e**(3*d*x)/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x + 2*e**(c + d
*x)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x) - e**c*int(1/(e**(5*c + 5
*d*x)*b*e + e**(5*c + 5*d*x)*b*f*x + 2*e**(4*c + 4*d*x)*a*e + 2*e**(4*c +
4*d*x)*a*f*x - e**(3*c + 3*d*x)*b*e - e**(3*c + 3*d*x)*b*f*x),x) + 2*int(1
/(e**(2*c + 3*d*x)*b*e + e**(2*c + 3*d*x)*b*f*x + 2*e**(c + 2*d*x)*a*e + 2
*e**(c + 2*d*x)*a*f*x - e**(d*x)*b*e - e**(d*x)*b*f*x),x))/(8*e**c)
```

3.396 $\int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} \, dx$

Optimal result

Integrand size = 36, antiderivative size = 1022

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{3a^2f(e+fx)^2}{8b^3d^2}+\frac{a^4(e+fx)^4}{4b^5f}+\frac{a^2(e+fx)^4}{8b^3f}-\frac{(e+fx)^4}{32bf}$$
$$-\frac{6a^3f^2(e+fx)\cosh(c+dx)}{b^4d^3}-\frac{4af^2(e+fx)\cosh(c+dx)}{3b^2d^3}$$
$$-\frac{a^3(e+fx)^3\cosh(c+dx)}{b^4d}-\frac{3a^2f^3\cosh^2(c+dx)}{8b^3d^4}-\frac{3a^2f(e+fx)^2\cosh^2(c+dx)}{4b^3d^2}$$
$$-\frac{2af^2(e+fx)\cosh^3(c+dx)}{9b^2d^3}-\frac{a(e+fx)^3\cosh^3(c+dx)}{3b^2d}-\frac{3f^3\cosh(4c+4dx)}{1024bd^4}$$
$$-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128bd^2}-\frac{a^3\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d}$$
$$+\frac{a^3\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d}$$
$$-\frac{3a^3\sqrt{a^2+b^2}f(e+fx)^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d^2}$$
$$+\frac{3a^3\sqrt{a^2+b^2}f(e+fx)^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d^2}$$
$$+\frac{6a^3\sqrt{a^2+b^2}f^2(e+fx)\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d^3}$$
$$-\frac{6a^3\sqrt{a^2+b^2}f^2(e+fx)\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d^3}$$
$$-\frac{6a^3\sqrt{a^2+b^2}f^3\,\mathrm{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d^4}$$
$$+\frac{6a^3\sqrt{a^2+b^2}f^3\,\mathrm{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d^4}+\frac{6a^3f^3\sinh(c+dx)}{b^4d^4}$$
$$+\frac{14af^3\sinh(c+dx)}{9b^2d^4}+\frac{3a^3f(e+fx)^2\sinh(c+dx)}{b^4d^2}+\frac{2af(e+fx)^2\sinh(c+dx)}{3b^2d^2}$$
$$+\frac{3a^2f^2(e+fx)\cosh(c+dx)\sinh(c+dx)}{4b^3d^3}+\frac{a^2(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{2b^3d}$$
$$+\frac{af(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{3b^2d^2}+\frac{2af^3\sinh^3(c+dx)}{27b^2d^4}$$
$$+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256bd^3}+\frac{(e+fx)^3\sinh(4c+4dx)}{32bd}$$

output
```
-6*a^3*(a^2+b^2)^(1/2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1
/2)))/b^5/d^3+6*a^3*(a^2+b^2)^(1/2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a
-(a^2+b^2)^(1/2)))/b^5/d^3+3*a^3*(a^2+b^2)^(1/2)*f*(f*x+e)^2*polylog(2,-b*
exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^5/d^2-3*a^3*(a^2+b^2)^(1/2)*f*(f*x+e)^2*
polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^5/d^2+3*a^3*f*(f*x+e)^2*sin
h(d*x+c)/b^4/d^2-3/4*a^2*f*(f*x+e)^2*cosh(d*x+c)^2/b^3/d^2-2/9*a*f^2*(f*x+
e)*cosh(d*x+c)^3/b^2/d^3+1/2*a^2*(f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/b^3/d-6
*a^3*f^2*(f*x+e)*cosh(d*x+c)/b^4/d^3+3/4*a^2*f^2*(f*x+e)*cosh(d*x+c)*sinh(
d*x+c)/b^3/d^3+14/9*a*f^3*sinh(d*x+c)/b^2/d^4+3/256*f^2*(f*x+e)*sinh(4*d*x
+4*c)/b/d^3+6*a^3*f^3*sinh(d*x+c)/b^4/d^4+2/27*a*f^3*sinh(d*x+c)^3/b^2/d^4
-3/128*f*(f*x+e)^2*cosh(4*d*x+4*c)/b/d^2-3/8*a^2*f^3*cosh(d*x+c)^2/b^3/d^4
-1/3*a*(f*x+e)^3*cosh(d*x+c)^3/b^2/d+3/8*a^2*f*(f*x+e)^2/b^3/d^2-a^3*(f*x+
e)^3*cosh(d*x+c)/b^4/d+1/3*a*f*(f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/b^2/d^2
+1/32*(f*x+e)^3*sinh(4*d*x+4*c)/b/d-3/1024*f^3*cosh(4*d*x+4*c)/b/d^4+1/4*a
^4*(f*x+e)^4/b^5/f+6*a^3*(a^2+b^2)^(1/2)*f^3*polylog(4,-b*exp(d*x+c)/(a+(a
^2+b^2)^(1/2)))/b^5/d^4-6*a^3*(a^2+b^2)^(1/2)*f^3*polylog(4,-b*exp(d*x+c)/
(a-(a^2+b^2)^(1/2)))/b^5/d^4+a^3*(a^2+b^2)^(1/2)*(f*x+e)^3*ln(1+b*exp(d*x+
c)/(a+(a^2+b^2)^(1/2)))/b^5/d-a^3*(a^2+b^2)^(1/2)*(f*x+e)^3*ln(1+b*exp(d*x
+c)/(a-(a^2+b^2)^(1/2)))/b^5/d-4/3*a*f^2*(f*x+e)*cosh(d*x+c)/b^2/d^3+2/3*a
*f*(f*x+e)^2*sinh(d*x+c)/b^2/d^2-1/32*(f*x+e)^4/b/f+1/8*a^2*(f*x+e)^4/b...
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 5984 vs. $2(1022) = 2044$.

Time = 20.26 (sec) , antiderivative size = 5984, normalized size of antiderivative = 5.86

$$\int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh^3(c+dx)}{a+b\sinh(c+dx)} \, dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*
x]),x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3 \sinh^3(c+dx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6113}$$

$$\frac{\int (e+fx)^3 \cosh^2(c+dx)\sinh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{\int \left(\frac{1}{8}(e+fx)^3\cosh(4c+4dx)-\frac{1}{8}(e+fx)^3\right)dx}{b}-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 6113}$$

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-\frac{a\left(\frac{\int (e+fx)^3\cosh^2(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 5970}$$

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-\frac{a\left(\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\int (e+fx)^2\cosh^3(c+dx)dx}{d}}{b}-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{d}}{b}\right)}{b}$$

$\Big\downarrow$ 3792

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-\frac{a\left(\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2f^2\int\cosh^3(c+dx)dx}{9d^2}+\frac{2}{3}\int(e+fx)^2\cosh(c+dx)dx-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{d}}{b}-\frac{a\int\frac{(e+fx}{}\cdots\right)}{b}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2f^2\int\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{9d^2}+\frac{2}{3}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{2f(e+fx)\cosh^3(c\cdots}{9d^2}\cdots\right.}{d}}{b}\cdots\right)}{b}$$

$\Big\downarrow$ 3113

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\int\left(\sinh^2(c+dx)+1\right)d(-i\sinh(c+dx))}{9d^3}+\frac{2}{3}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{2f}{}\cdots\right.}{d}}{b}\cdots\right)}{b}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{\frac{f\left(\frac{2}{3}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e-\cdots}{\cdots}\right)}{d}}{b}\cdots\right.}{b}$$

$\downarrow$ 3777

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int-i(e+fx)\sinh(c+dx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+\cdots\right)}{9d\cdots}\cdots\right)}{d}}{b}\cdots\right.}{b}$$

$\downarrow$ 26

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int(e+fx)\sinh(c+dx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)\cdots\right)}{9d^3}\cdots\right)}{d}}{b}\cdots\right.}{b}$$

$\downarrow$ 3042

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int-i(e+fx)\sin(ic+idx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+\cdots\right)}{9d\cdots}\cdots\right)}{d}}{b}\cdots\right.}{b}$$

$\downarrow$ 26

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int(e+fx)\sin(ic+idx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx\cdots\right)}{9d^3}\cdots\right)}{d}}{b}\cdots\right)}{b}$$

$\downarrow$ 3777

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{d}\right)+\cdots\right)}{b}\cdots\right)}{b}$$

$\downarrow$ 3042

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\cdots\right)\cdots\right)}{b}\cdots\right)}{b}$$

$\downarrow$ 3117

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\cdots}{\cdots}\right)\cdots\right)}{b}\cdots\right)}{b}$$

$\downarrow$ 6113

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)^3\cosh^2(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3}{9d^2}\right.}{}\right.}{b}$$

$\downarrow$ 3042

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-\frac{a\left(\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(}{d^2}\right.}{d}\right.\right.}{d}}{b}\right.}{b}$$

$\downarrow$ 3792

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-\frac{a\left(-\frac{a\left(\frac{\frac{3f^2\int(e+fx)\cosh^2(c+dx)dx}{2d^2}+\frac{1}{2}\int(e+fx)^3dx-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{(e+}{}\right.}{}$$

$\downarrow$ 17

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$a\left(-\frac{a\left(\frac{\frac{3f^2\int(e+fx)\cosh^2(c+dx)dx}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{(e+fx)^3\cdots}{\cdots}\cdots\right.$$

$\downarrow$ 3042

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(\cdots}{d^2}\cdots\right)}{d}\cdots\right)\right)}{d}}{b}\cdots\right.$$

$\downarrow$ 3791

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$a\left(-\frac{a\left(\frac{\frac{3f^2\left(\frac{1}{2}\int(e+fx)dx-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\int\cdots}{\cdots}\cdots\right)}{b}\cdots\right.$$

$\downarrow$ 17

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$a\left(-\frac{a\left(\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\int\frac{(e}{}}{}\right.}{b}\right.$$

$\downarrow$ 6099

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$a\left(-\frac{a\left(\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a}{}\right.}{}\right.}{b}\right.$$

$\downarrow$ 17

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$a\left(-\frac{a\left(\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a}{}\right.}{}\right.}{b}\right.$$

$\downarrow$ 3042

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(}{d^2}\right.}{d}}{d}\right.\right.}{d}}{b}\right.$$

$\downarrow$ 26

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(}{d^2}\right.}{d}}{d}\right.\right.}{d}}{b}\right.$$

$\downarrow$ 3777

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(}{d^2}\right.}{d}}{d}\right.\right.}{d}}{b}\right.$$

$\downarrow$ 3042

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$a\left(\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(\ldots}{d^2}\right.}{d}\right.\right.}{d}}{b}\right.$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^3\cosh(dx+c)^2\sinh(dx+c)^3}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10658 vs. 2(942) = 1884.

Time = 0.29 (sec) , antiderivative size = 10658, normalized size of antiderivative = 10.43

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algor
ithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)**2*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)^2\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algor
ithm="maxima")
```

output
```
-1/192*e^3*(192*sqrt(a^2 + b^2)*a^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b
^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(b^5*d) + (8*a*b^2*e^(-d*x -
c) - 24*a^2*b*e^(-2*d*x - 2*c) - 3*b^3 + 24*(4*a^3 + a*b^2)*e^(-3*d*x - 3*
c))*e^(4*d*x + 4*c)/(b^4*d) - 24*(8*a^4 + 4*a^2*b^2 - b^4)*(d*x + c)/(b^5*
d) + (24*a^2*b*e^(-2*d*x - 2*c) + 8*a*b^2*e^(-3*d*x - 3*c) + 3*b^3*e^(-4*d
*x - 4*c) + 24*(4*a^3 + a*b^2)*e^(-d*x - c))/(b^4*d)) + 1/55296*(1728*(8*a
^4*d^4*f^3*e^(4*c) + 4*a^2*b^2*d^4*f^3*e^(4*c) - b^4*d^4*f^3*e^(4*c))*x^4
+ 6912*(8*a^4*d^4*e*f^2*e^(4*c) + 4*a^2*b^2*d^4*e*f^2*e^(4*c) - b^4*d^4*e*
f^2*e^(4*c))*x^3 + 10368*(8*a^4*d^4*e^2*f*e^(4*c) + 4*a^2*b^2*d^4*e^2*f*e^
(4*c) - b^4*d^4*e^2*f*e^(4*c))*x^2 + 27*(32*b^4*d^3*f^3*x^3*e^(8*c) + 24*(
4*d^3*e*f^2 - d^2*f^3)*b^4*x^2*e^(8*c) + 12*(8*d^3*e^2*f - 4*d^2*e*f^2 + d
*f^3)*b^4*x*e^(8*c) - 3*(8*d^2*e^2*f - 4*d*e*f^2 + f^3)*b^4*e^(8*c))*e^(4*
d*x) - 256*(9*a*b^3*d^3*f^3*x^3*e^(7*c) + 9*(3*d^3*e*f^2 - d^2*f^3)*a*b^3*
x^2*e^(7*c) + 3*(9*d^3*e^2*f - 6*d^2*e*f^2 + 2*d*f^3)*a*b^3*x*e^(7*c) - (9
*d^2*e^2*f - 6*d*e*f^2 + 2*f^3)*a*b^3*e^(7*c))*e^(3*d*x) + 1728*(4*a^2*b^2
*d^3*f^3*x^3*e^(6*c) + 6*(2*d^3*e*f^2 - d^2*f^3)*a^2*b^2*x^2*e^(6*c) + 6*(
2*d^3*e^2*f - 2*d^2*e*f^2 + d*f^3)*a^2*b^2*x*e^(6*c) - 3*(2*d^2*e^2*f - 2*
d*e*f^2 + f^3)*a^2*b^2*e^(6*c))*e^(2*d*x) + 6912*(12*(d^2*e^2*f - 2*d*e*f^
2 + 2*f^3)*a^3*b*e^(5*c) + 3*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*a*b^3*e^(5*c)
 - (4*a^3*b*d^3*f^3*e^(5*c) + a*b^3*d^3*f^3*e^(5*c))*x^3 - 3*(4*(d^3*e*...
```

Giac [F]

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)^3}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algor
ithm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)^2*sinh(d*x + c)^3/(b*sinh(d*x + c) + a
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^2\,\sinh(c+d\,x)^3\,{(e+f\,x)}^3}{a+b\,\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*sinh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)), x
)
```

Reduce [F]

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)^3}{a+b\sinh\left(dx+c\right)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

3.397 $\int \frac{(e+fx)^2 \cosh^2(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} \, dx$

Optimal result

Integrand size = 36, antiderivative size = 755

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{a^2f^2x}{4b^3d^2}+\frac{a^4(e+fx)^3}{3b^5f}+\frac{a^2(e+fx)^3}{6b^3f}-\frac{(e+fx)^3}{24bf}-\frac{2a^3f^2\cosh(c+dx)}{b^4d^3}$$
$$-\frac{4af^2\cosh(c+dx)}{9b^2d^3}-\frac{a^3(e+fx)^2\cosh(c+dx)}{b^4d}-\frac{a^2f(e+fx)\cosh^2(c+dx)}{2b^3d^2}$$
$$-\frac{2af^2\cosh^3(c+dx)}{27b^2d^3}-\frac{a(e+fx)^2\cosh^3(c+dx)}{3b^2d}$$
$$-\frac{f(e+fx)\cosh(4c+4dx)}{64bd^2}-\frac{a^3\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d}$$
$$+\frac{a^3\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d}$$
$$-\frac{2a^3\sqrt{a^2+b^2}f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d^2}$$
$$+\frac{2a^3\sqrt{a^2+b^2}f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d^2}$$
$$+\frac{2a^3\sqrt{a^2+b^2}f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d^3}$$
$$-\frac{2a^3\sqrt{a^2+b^2}f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d^3}+\frac{2a^3f(e+fx)\sinh(c+dx)}{b^4d^2}$$
$$+\frac{4af(e+fx)\sinh(c+dx)}{9b^2d^2}+\frac{a^2f^2\cosh(c+dx)\sinh(c+dx)}{4b^3d^3}$$
$$+\frac{a^2(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{2b^3d}+\frac{2af(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{9b^2d^2}$$
$$+\frac{f^2\sinh(4c+4dx)}{256bd^3}+\frac{(e+fx)^2\sinh(4c+4dx)}{32bd}$$

output
```
1/4*a^2*f^2*x/b^3/d^2+1/3*a^4*(f*x+e)^3/b^5/f+1/6*a^2*(f*x+e)^3/b^3/f-1/24
*(f*x+e)^3/b/f-2*a^3*f^2*cosh(d*x+c)/b^4/d^3-4/9*a*f^2*cosh(d*x+c)/b^2/d^3
-a^3*(f*x+e)^2*cosh(d*x+c)/b^4/d-1/2*a^2*f*(f*x+e)*cosh(d*x+c)^2/b^3/d^2-2
/27*a*f^2*cosh(d*x+c)^3/b^2/d^3-1/3*a*(f*x+e)^2*cosh(d*x+c)^3/b^2/d-1/64*f
*(f*x+e)*cosh(4*d*x+4*c)/b/d^2-a^3*(a^2+b^2)^(1/2)*(f*x+e)^2*ln(1+b*exp(d*
x+c)/(a-(a^2+b^2)^(1/2)))/b^5/d+a^3*(a^2+b^2)^(1/2)*(f*x+e)^2*ln(1+b*exp(d
*x+c)/(a+(a^2+b^2)^(1/2)))/b^5/d-2*a^3*(a^2+b^2)^(1/2)*f*(f*x+e)*polylog(2
,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^5/d^2+2*a^3*(a^2+b^2)^(1/2)*f*(f*x+e
)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^5/d^2+2*a^3*(a^2+b^2)^(1/
2)*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^5/d^3-2*a^3*(a^2+b^2
)^(1/2)*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^5/d^3+2*a^3*f*(
f*x+e)*sinh(d*x+c)/b^4/d^2+4/9*a*f*(f*x+e)*sinh(d*x+c)/b^2/d^2+1/4*a^2*f^2
*cosh(d*x+c)*sinh(d*x+c)/b^3/d^3+1/2*a^2*(f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)
/b^3/d+2/9*a*f*(f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/b^2/d^2+1/256*f^2*sinh(4*
d*x+4*c)/b/d^3+1/32*(f*x+e)^2*sinh(4*d*x+4*c)/b/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3579 vs. $2(755) = 1510$.

Time = 12.47 (sec) , antiderivative size = 3579, normalized size of antiderivative = 4.74

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*
x]),x]
```

output
```
-1/8*(e^2*(c/d + x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2
]])/(Sqrt[-a^2 - b^2]*d)))/b - (e*f*(x^2 - (2*a*(d*x*(Log[1 + (b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2])] - Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
]) + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - PolyLog[2, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(Sqrt[a^2 + b^2]*d^2)))/(8*b) - (f^
2*(x^3 - (3*a*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^
2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2, (b
*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*x*PolyLog[2, -((b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2]))] - 2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b
^2])] + 2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(Sqrt[a^2
 + b^2]*d^3)))/(24*b) - (f^2*(2*(4*a^2 + b^2)*x^3 - (6*a*(4*a^2 + 3*b^2)*(
d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^2*x^2*Log[1 + (
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2, (b*E^(c + d*x))/(
-a + Sqrt[a^2 + b^2])] - 2*d*x*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2]))] - 2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*PolyL
og[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(Sqrt[a^2 + b^2]*d^3) -
(24*a*b*Cosh[d*x]*((2 + d^2*x^2)*Cosh[c] - 2*d*x*Sinh[c]))/d^3 + (3*b^2*Co
sh[2*d*x]*(-2*d*x*Cosh[2*c] + (1 + 2*d^2*x^2)*Sinh[2*c]))/d^3 - (24*a*b*(-
2*d*x*Cosh[c] + (2 + d^2*x^2)*Sinh[c])*Sinh[d*x])/d^3 + (3*b^2*((1 + 2*d^2
*x^2)*Cosh[2*c] - 2*d*x*Sinh[2*c])*Sinh[2*d*x])/d^3))/(96*b^3) - (e^2*(...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sinh^3(c+dx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \ 6113$$

$$\frac{\int (e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \ 5971$$

$$\frac{\int \left(\frac{1}{8}(e+fx)^2\cosh(4c+4dx)-\frac{1}{8}(e+fx)^2\right)dx}{b}-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{6113}$$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(\frac{\int(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{5970}$$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\int(e+fx)\cosh^3(c+dx)dx}{3d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d}}{b}\right)}{b}$$

$$\Big\downarrow \text{3791}$$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\int(e+fx)\cosh(c+dx)dx-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+d\cdots}{3d}\right.\cdots}{3d}}{b}\cdots\right.}{b}$$

$\Big\downarrow$ 3777

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int-i\sinh(c+dx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh\cdots}{}\right.\cdots}{3d}}{b}\cdots\right.}{b}$$

$\Big\downarrow$ 26

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx\cdots}{a+b\sinh(c+\cdots}}{b}\cdots\right.}{b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int-i\sin(ic+idx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(\cdots}{}\right.\cdots}{3d}}{b}\cdots\right.}{b}$$

$\Big\downarrow$ 26

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int\sin(ic+idx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c\cdots}{}\right.\cdots}{3d}}{b}\cdots\right.}{b}$$

$\Big\downarrow$ 3118

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)\sin}{a+b\sinh(c+dx)}}{b}\right.}{b}$$

$\Big\downarrow$ 6113

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{\int(e+fx)^2\cosh^2(c+dx)dx}{b}\right.}{}\right.}{b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^2(c+d}{a+b\sinh(c+dx)}}{b}\right.}{}\right.}{b}$$

$\Big\downarrow$ 3792

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{\frac{f^2\int\cosh^2(c+dx)dx}{2d^2}+\frac{1}{2}}{}\right.}{}\right.}{b}$$

$\Big\downarrow$ 17

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{\frac{f^2\int\cosh^2(c+dx)dx}{2d^2}-\frac{f(\cdots}{\cdots}}{\cdots}\cdots\right.}{\cdots}\right.}{b}$$

$\downarrow$ 3042

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^2(c+\cdots}{a+b\sinh(c+dx)}\cdots}{b}\cdots\right.}{\cdots}\right.}{b}$$

$\downarrow$ 3115

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{\frac{f^2\left(\frac{\int 1dx}{2}+\frac{\sinh(c+dx)\cdots}{2\cdots}\right.}{2d^2}\cdots}{\cdots}\right.}{\cdots}\right.}{b}$$

$\downarrow$ 24

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}\cdots}{\cdots}\right.}{\cdots}\right.}{b}$$

$\downarrow$ 6099

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}\right.}{}\right.$$

$\downarrow$ 17

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}\right.}{}\right.$$

$\downarrow$ 3042

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}\right.}{}\right.$$

$\downarrow$ 26

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}\right.}{}\right.$$

$\downarrow$ 3777

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}\right.}{}\right.$$

$\downarrow$ 3042

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}\right.}{}\right.$$

$\downarrow$ 3777

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}\right.}{}\right.$$

$\downarrow$ 26

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}\right.}{}\right.$$

$\downarrow$ 3042

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}\right.}{}\right.$$

$\downarrow$ 26

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}\right.}{}\right.$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^2\cosh(dx+c)^2\sinh(dx+c)^3}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6459 vs. $2(693) = 1386$.

Time = 0.20 (sec) , antiderivative size = 6459, normalized size of antiderivative = 8.55

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algor
ithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)**2*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)^3}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algor
ithm="maxima")
```

output
```
-1/192*e^2*(192*sqrt(a^2 + b^2)*a^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b
^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(b^5*d) + (8*a*b^2*e^(-d*x -
c) - 24*a^2*b*e^(-2*d*x - 2*c) - 3*b^3 + 24*(4*a^3 + a*b^2)*e^(-3*d*x - 3*
c))*e^(4*d*x + 4*c)/(b^4*d) - 24*(8*a^4 + 4*a^2*b^2 - b^4)*(d*x + c)/(b^5*
d) + (24*a^2*b*e^(-2*d*x - 2*c) + 8*a*b^2*e^(-3*d*x - 3*c) + 3*b^3*e^(-4*d
*x - 4*c) + 24*(4*a^3 + a*b^2)*e^(-d*x - c))/(b^4*d)) + 1/13824*(576*(8*a^
4*d^3*f^2*e^(4*c) + 4*a^2*b^2*d^3*f^2*e^(4*c) - b^4*d^3*f^2*e^(4*c))*x^3 +
 1728*(8*a^4*d^3*e*f*e^(4*c) + 4*a^2*b^2*d^3*e*f*e^(4*c) - b^4*d^3*e*f*e^(
4*c))*x^2 + 27*(8*b^4*d^2*f^2*x^2*e^(8*c) + 4*(4*d^2*e*f - d*f^2)*b^4*x*e^
(8*c) - (4*d*e*f - f^2)*b^4*e^(8*c))*e^(4*d*x) - 64*(9*a*b^3*d^2*f^2*x^2*e
^(7*c) + 6*(3*d^2*e*f - d*f^2)*a*b^3*x*e^(7*c) - 2*(3*d*e*f - f^2)*a*b^3*e
^(7*c))*e^(3*d*x) + 864*(2*a^2*b^2*d^2*f^2*x^2*e^(6*c) + 2*(2*d^2*e*f - d*
f^2)*a^2*b^2*x*e^(6*c) - (2*d*e*f - f^2)*a^2*b^2*e^(6*c))*e^(2*d*x) + 1728
*(8*(d*e*f - f^2)*a^3*b*e^(5*c) + 2*(d*e*f - f^2)*a*b^3*e^(5*c) - (4*a^3*b
*d^2*f^2*e^(5*c) + a*b^3*d^2*f^2*e^(5*c))*x^2 - 2*(4*(d^2*e*f - d*f^2)*a^3
*b*e^(5*c) + (d^2*e*f - d*f^2)*a*b^3*e^(5*c))*x)*e^(d*x) - 1728*(8*(d*e*f
+ f^2)*a^3*b*e^(3*c) + 2*(d*e*f + f^2)*a*b^3*e^(3*c) + (4*a^3*b*d^2*f^2*e^
(3*c) + a*b^3*d^2*f^2*e^(3*c))*x^2 + 2*(4*(d^2*e*f + d*f^2)*a^3*b*e^(3*c)
+ (d^2*e*f + d*f^2)*a*b^3*e^(3*c))*x)*e^(-d*x) - 864*(2*a^2*b^2*d^2*f^2*x^
2*e^(2*c) + 2*(2*d^2*e*f + d*f^2)*a^2*b^2*x*e^(2*c) + (2*d*e*f + f^2)*a...
```

Giac [F]

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)^2\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algor
ithm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)^2*sinh(d*x + c)^3/(b*sinh(d*x + c) + a
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{{\cosh\left(c+d\,x\right)}^2\,{\sinh\left(c+d\,x\right)}^3\,{\left(e+f\,x\right)}^2}{a+b\,\sinh\left(c+d\,x\right)} \,d x$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*sinh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)), x
)
```

Reduce [F]

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2 \cosh(dx+c)^2 \sinh(dx+c)^3}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

3.398 $\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 465

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{a^4(e+fx)^2}{2b^5f}+\frac{a^2(e+fx)^2}{4b^3f}-\frac{(e+fx)^2}{16bf}-\frac{a^3(e+fx)\cosh(c+dx)}{b^4d} -\frac{a^2f\cosh^2(c+dx)}{4b^3d^2}-\frac{a(e+fx)\cosh^3(c+dx)}{3b^2d}-\frac{f\cosh(4c+4dx)}{128bd^2} -\frac{a^3\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d}+\frac{a^3\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d} -\frac{a^3\sqrt{a^2+b^2}f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d^2}+\frac{a^3\sqrt{a^2+b^2}f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d^2} +\frac{a^3f\sinh(c+dx)}{b^4d^2}+\frac{af\sinh(c+dx)}{3b^2d^2}+\frac{a^2(e+fx)\cosh(c+dx)\sinh(c+dx)}{2b^3d} +\frac{af\sinh^3(c+dx)}{9b^2d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32bd}$$

output
```
1/2*a^4*(f*x+e)^2/b^5/f+1/4*a^2*(f*x+e)^2/b^3/f-1/16*(f*x+e)^2/b/f-a^3*(f*
x+e)*cosh(d*x+c)/b^4/d-1/4*a^2*f*cosh(d*x+c)^2/b^3/d^2-1/3*a*(f*x+e)*cosh(
d*x+c)^3/b^2/d-1/128*f*cosh(4*d*x+4*c)/b/d^2-a^3*(a^2+b^2)^(1/2)*(f*x+e)*l
n(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^5/d+a^3*(a^2+b^2)^(1/2)*(f*x+e)*ln
(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^5/d-a^3*(a^2+b^2)^(1/2)*f*polylog(2
,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^5/d^2+a^3*(a^2+b^2)^(1/2)*f*polylog(
2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^5/d^2+a^3*f*sinh(d*x+c)/b^4/d^2+1/3
*a*f*sinh(d*x+c)/b^2/d^2+1/2*a^2*(f*x+e)*cosh(d*x+c)*sinh(d*x+c)/b^3/d+1/9
*a*f*sinh(d*x+c)^3/b^2/d^2+1/32*(f*x+e)*sinh(4*d*x+4*c)/b/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1627 vs. $2(465) = 930$.

Time = 3.74 (sec) , antiderivative size = 1627, normalized size of antiderivative = 3.50

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]
),x]
```

output
```
(1152*a^4*Sqrt[-(a^2 + b^2)^2]*c*d*e + 576*a^2*b^2*Sqrt[-(a^2 + b^2)^2]*c*
d*e - 144*b^4*Sqrt[-(a^2 + b^2)^2]*c*d*e - 576*a^4*Sqrt[-(a^2 + b^2)^2]*c^
2*f - 288*a^2*b^2*Sqrt[-(a^2 + b^2)^2]*c^2*f + 1152*a^4*Sqrt[-(a^2 + b^2)^
2]*d^2*e*x + 576*a^2*b^2*Sqrt[-(a^2 + b^2)^2]*d^2*e*x - 144*b^4*Sqrt[-(a^2
 + b^2)^2]*d^2*e*x + 576*a^4*Sqrt[-(a^2 + b^2)^2]*d^2*f*x^2 + 288*a^2*b^2*
Sqrt[-(a^2 + b^2)^2]*d^2*f*x^2 - 72*b^4*Sqrt[-(a^2 + b^2)^2]*d^2*f*x^2 - 2
304*a^5*Sqrt[a^2 + b^2]*d*e*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b
^2]] - 2304*a^3*b^2*Sqrt[a^2 + b^2]*d*e*ArcTan[(b - a*Tanh[(c + d*x)/2])/S
qrt[-a^2 - b^2]] - 2304*a^5*Sqrt[-a^2 - b^2]*c*f*ArcTanh[(a + b*E^(c + d*x
))/Sqrt[a^2 + b^2]] - 2304*a^3*b^2*Sqrt[-a^2 - b^2]*c*f*ArcTanh[(a + b*E^(
c + d*x))/Sqrt[a^2 + b^2]] - 288*a*b^4*Sqrt[-a^2 - b^2]*c*f*ArcTanh[(a + b
*E^(c + d*x))/Sqrt[a^2 + b^2]] - 1152*a^3*b*Sqrt[-(a^2 + b^2)^2]*d*e*Cosh[
c + d*x] - 288*a*b^3*Sqrt[-(a^2 + b^2)^2]*d*e*Cosh[c + d*x] - 1152*a^3*b*S
qrt[-(a^2 + b^2)^2]*d*f*x*Cosh[c + d*x] - 288*a*b^3*Sqrt[-(a^2 + b^2)^2]*d
*f*x*Cosh[c + d*x] - 144*a^2*b^2*Sqrt[-(a^2 + b^2)^2]*f*Cosh[2*(c + d*x)]
- 96*a*b^3*Sqrt[-(a^2 + b^2)^2]*d*e*Cosh[3*(c + d*x)] - 96*a*b^3*Sqrt[-(a^
2 + b^2)^2]*d*f*x*Cosh[3*(c + d*x)] - 9*b^4*Sqrt[-(a^2 + b^2)^2]*f*Cosh[4*
(c + d*x)] - 1152*a^5*Sqrt[-a^2 - b^2]*c*f*Log[1 + (b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2])] - 1152*a^3*b^2*Sqrt[-a^2 - b^2]*c*f*Log[1 + (b*E^(c + d*x)
)/(a - Sqrt[a^2 + b^2])] - 144*a*b^4*Sqrt[-a^2 - b^2]*c*f*Log[1 + (b*E^...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 3.09 (sec) , antiderivative size = 455, normalized size of antiderivative = 0.98, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.765$, Rules used = {6113, 5971, 2009, 6113, 5970, 3042, 3113, 2009, 6113, 3042, 3791, 17, 6099, 17, 3042, 26, 3777, 3042, 3117, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh^3(c+dx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \ 6113$$

$$\frac{\int (e+fx)\cosh^2(c+dx)\sinh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{5971}$$

$$\frac{\int \left(\frac{1}{8}(-e-fx)+\frac{1}{8}(e+fx)\cosh(4c+4dx)\right)dx}{b}-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{6113}$$

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-\frac{a\left(\frac{\int (e+fx)\cosh^2(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{5970}$$

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-\frac{a\left(\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{f\int \cosh^3(c+dx)dx}{3d}}{b}-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{f\int \sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d}}{b}\right)}{b}$$

$$\Big\downarrow \text{3113}$$

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\int \left(\sinh^2(c+dx)+1\right)d(-i\sinh(c+dx))}{3d^2}}{b}\right)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}\right)}{b}$$

$\Big\downarrow$ 6113

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)\cosh^2(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}\right)}{b}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-\frac{a\left(\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx}{b}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 3791

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-\frac{a\left(-\frac{a\left(\frac{\frac{1}{2}\int(e+fx)dx-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}\right)}{b}$$

$\Big\downarrow$ 17

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-\frac{a\left(-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\int\frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}\right)}{b}$$

$\downarrow$ 6099

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-$$

$$\frac{a\left(-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{e+fx}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)dx}{b^2}+\frac{\int(e+fx)\sinh(c+dx)dx}{b}\right)}{b}\right)}{b}+\ldots\right.}{b}$$

$\downarrow$ 17

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-$$

$$\frac{a\left(-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{e+fx}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int(e+fx)\sinh(c+dx)dx}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{b}\right)}{b}+\frac{(e+\ldots}{}\right.}{b}$$

$\downarrow$ 3042

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{e+\ldots}{a-ib\sin\ldots}}{b^2}\ldots\right.}{}\right.}{b}\right.}{b}$$

$\downarrow$ 26

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{e+\cdots}{a-ib\sin\cdots}}{b^2}\cdots\right.}{\cdots}\right.}{b}\cdots\right.}{b}$$

$\downarrow$ 3777

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{e+\cdots}{a-ib\sin\cdots}}{b^2}\cdots\right.}{\cdots}\right.}{b}\cdots\right.}{b}$$

$\downarrow$ 3042

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{e+\cdots}{a-ib\sin\cdots}}{b^2}\cdots\right.}{\cdots}\right.}{b}\cdots\right.}{b}$$

$\downarrow$ 3117

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-\frac{a\left(\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{e+\cdots}{a-ib\sin\cdots}}{b^2}\cdots\right.}{\cdots}\right.}{b}\cdots\right.}{b}$$

$\Big\downarrow$ 3803

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-\frac{a\left(\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(2(a^2+b^2)\int-\frac{\cdots}{-2e^{c}\cdots}\cdots\right.}{\cdots}\right.}{b}\cdots\right.}{b}$$

$\Big\downarrow$ 25

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-\frac{a\left(\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-2(a^2+b^2)\int\frac{\cdots}{-2e^{c}\cdots}\cdots\right.}{\cdots}\right.}{b}\cdots\right.}{b}$$

$\Big\downarrow$ 2694

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2\left(a^2+b^2\right)\left(\frac{b\int -\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}\right)}{b}$$

$\downarrow$ 27

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2\left(a^2+b^2\right)\left(\frac{b\int \frac{}{a\ldots}}{}\right.}{}\right.}{}\right)}{}\right.$$

$$b$$

$\downarrow$ 2620

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2\left(a^2+b^2\right)\left(\frac{b\left(\frac{(e\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

↓ 2715

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2\left(a^2+b^2\right)\left(\frac{b\left(\frac{(e\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

↓ 2838

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2\left(a^2+b^2\right)\left(\frac{b\left(\frac{f\,1}{\ }\right.}{\ }\right.}{\ }\right.}{\ }\right.}{\ }\right.$$

input
```
Int[((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
-((a*((((e + f*x)*Cosh[c + d*x]^3)/(3*d) - ((I/3)*f*((-I)*Sinh[c + d*x] -
(I/3)*Sinh[c + d*x]^3))/d^2)/b - (a*(((e + f*x)^2/(4*f) - (f*Cosh[c + d*x]
^2)/(4*d^2) + ((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*d))/b - (a*(-1/2*
(a*(e + f*x)^2)/(b^2*f) - (2*(a^2 + b^2)*(-1/2*(b*(((e + f*x)*Log[1 + (b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x)
)/(a - Sqrt[a^2 + b^2]))])/(b*d^2)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[
1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(
c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(2*Sqrt[a^2 + b^2])))/b^2 - (
I*((I*(e + f*x)*Cosh[c + d*x])/d - (I*f*Sinh[c + d*x])/d^2))/b))/b))/b))/b
) + (-1/16*(e + f*x)^2/f - (f*Cosh[4*c + 4*d*x])/(128*d^2) + ((e + f*x)*Si
nh[4*c + 4*d*x])/(32*d))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5970
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1212 vs. $2(425) = 850$.

Time = 103.05 (sec) , antiderivative size = 1213, normalized size of antiderivative = 2.61

method	result	size
risch	Expression too large to display	1213

input
```
int((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURN
VERBOSE)
```

output
```
-1/72*a*(3*d*f*x+3*d*e-f)/b^2/d^2*exp(3*d*x+3*c)+1/16*a^2*(2*d*f*x+2*d*e-f
)/b^3/d^2*exp(2*d*x+2*c)-1/16*a^2*(2*d*f*x+2*d*e+f)/b^3/d^2*exp(-2*d*x-2*c
)-1/d*a^5/b^5*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(
a^2+b^2)^(1/2)))*x+1/d*a^5/b^5*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2
)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2*a^5/b^5*f/(a^2+b^2)^(1/2)*ln((-b*e
xp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d^2*a^5/b^5*f/(a^2+
b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-2/d^
2*a^5/b^5*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(
1/2))+2/d*a^5/b^5*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+
b^2)^(1/2))-1/d^2*a^5/b^5*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)
^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/d^2*a^5/b^5*f/(a^2+b^2)^(1/2)*dilog((b*e
xp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/8*a*(4*a^2*d*f*x+b^2*d
*f*x+4*a^2*d*e+b^2*d*e-4*a^2*f-b^2*f)/b^4/d^2*exp(d*x+c)-1/72*a*(3*d*f*x+3
*d*e+f)/b^2/d^2*exp(-3*d*x-3*c)+2/d*a^3/b^3*e/(a^2+b^2)^(1/2)*arctanh(1/2*
(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d^2*a^3/b^3*f/(a^2+b^2)^(1/2)*dilo
g((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/d^2*a^3/b^3*f/
(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))
)+1/4/b^3*a^2*f*x^2+1/256*(4*d*f*x+4*d*e-f)/b/d^2*exp(4*d*x+4*c)-1/256*(4*
d*f*x+4*d*e+f)/b/d^2*exp(-4*d*x-4*c)-1/16/b*f*x^2+1/2/b^3*a^2*e*x-1/8*a*(4
*a^2+b^2)*(d*f*x+d*e+f)/b^4/d^2*exp(-d*x-c)-1/8/b*e*x+1/2/b^5*a^4*f*x^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3228 vs. $2(423) = 846$.

Time = 0.21 (sec) , antiderivative size = 3228, normalized size of antiderivative = 6.94

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)**2*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)^2\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/2304*(4608*(a^5*e^c + a^3*b^2*e^c)*integrate(x*e^(d*x)/(b^6*e^(2*d*x +
2*c) + 2*a*b^5*e^(d*x + c) - b^6), x) - (144*(8*a^4*d^2*e^(4*c) + 4*a^2*b^
2*d^2*e^(4*c) - b^4*d^2*e^(4*c))*x^2 + 9*(4*b^4*d*x*e^(8*c) - b^4*e^(8*c))
*e^(4*d*x) - 32*(3*a*b^3*d*x*e^(7*c) - a*b^3*e^(7*c))*e^(3*d*x) + 144*(2*a
^2*b^2*d*x*e^(6*c) - a^2*b^2*e^(6*c))*e^(2*d*x) + 288*(4*a^3*b*e^(5*c) + a
*b^3*e^(5*c) - (4*a^3*b*d*e^(5*c) + a*b^3*d*e^(5*c))*x)*e^(d*x) - 288*(4*a
^3*b*e^(3*c) + a*b^3*e^(3*c) + (4*a^3*b*d*e^(3*c) + a*b^3*d*e^(3*c))*x)*e^
(-d*x) - 144*(2*a^2*b^2*d*x*e^(2*c) + a^2*b^2*e^(2*c))*e^(-2*d*x) - 32*(3*
a*b^3*d*x*e^c + a*b^3*e^c)*e^(-3*d*x) - 9*(4*b^4*d*x + b^4)*e^(-4*d*x))*e^
(-4*c)/(b^5*d^2))*f - 1/192*e*(192*sqrt(a^2 + b^2)*a^3*log((b*e^(-d*x - c)
 - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(b^5*d) +
(8*a*b^2*e^(-d*x - c) - 24*a^2*b*e^(-2*d*x - 2*c) - 3*b^3 + 24*(4*a^3 + a*
b^2)*e^(-3*d*x - 3*c))*e^(4*d*x + 4*c)/(b^4*d) - 24*(8*a^4 + 4*a^2*b^2 - b
^4)*(d*x + c)/(b^5*d) + (24*a^2*b*e^(-2*d*x - 2*c) + 8*a*b^2*e^(-3*d*x - 3
*c) + 3*b^3*e^(-4*d*x - 4*c) + 24*(4*a^3 + a*b^2)*e^(-d*x - c))/(b^4*d))
```

Giac [F]

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)\cosh(dx+c)^2\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)^2*sinh(d*x + c)^3/(b*sinh(d*x + c) + a),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\int \frac{\cosh(c+d\,x)^2\sinh(c+d\,x)^3\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*sinh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e + fx)\cosh^2(c + dx)\sinh^3(c + dx)}{a + b\sinh(c + dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
( - 4608*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/
sqrt(a**2 + b**2))*a**3*b**4*d*e*i + 36*e**(8*c + 8*d*x)*b**8*d*e + 36*e**
(8*c + 8*d*x)*b**8*d*f*x - 9*e**(8*c + 8*d*x)*b**8*f - 96*e**(7*c + 7*d*x)
*a*b**7*d*e - 96*e**(7*c + 7*d*x)*a*b**7*d*f*x + 32*e**(7*c + 7*d*x)*a*b**
7*f + 288*e**(6*c + 6*d*x)*a**2*b**6*d*e + 288*e**(6*c + 6*d*x)*a**2*b**6*
d*f*x - 144*e**(6*c + 6*d*x)*a**2*b**6*f - 1152*e**(5*c + 5*d*x)*a**3*b**5
*d*e - 1152*e**(5*c + 5*d*x)*a**3*b**5*d*f*x + 1152*e**(5*c + 5*d*x)*a**3*
b**5*f - 288*e**(5*c + 5*d*x)*a*b**7*d*e - 288*e**(5*c + 5*d*x)*a*b**7*d*f
*x + 288*e**(5*c + 5*d*x)*a*b**7*f + 36864*e**(4*c + 4*d*x)*int(x/(e**(6*c
 + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - e**(4*c + 4*d*x)*b),x)*a**8*b*d**2*f
+ 55296*e**(4*c + 4*d*x)*int(x/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a
- e**(4*c + 4*d*x)*b),x)*a**6*b**3*d**2*f + 18432*e**(4*c + 4*d*x)*int(x/(
e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - e**(4*c + 4*d*x)*b),x)*a**4*b*
*5*d**2*f + 2304*e**(4*c + 4*d*x)*a**4*b**4*d**2*e*x + 1152*e**(4*c + 4*d*
x)*a**4*b**4*d**2*f*x**2 + 1152*e**(4*c + 4*d*x)*a**2*b**6*d**2*e*x + 576*
e**(4*c + 4*d*x)*a**2*b**6*d**2*f*x**2 - 288*e**(4*c + 4*d*x)*b**8*d**2*e*
x - 144*e**(4*c + 4*d*x)*b**8*d**2*f*x**2 - 73728*e**(c + 4*d*x)*int(x/(e*
*(2*c + 5*d*x)*b + 2*e**(c + 4*d*x)*a - e**(3*d*x)*b),x)*a**9*d**2*f - 129
024*e**(c + 4*d*x)*int(x/(e**(2*c + 5*d*x)*b + 2*e**(c + 4*d*x)*a - e**(3*
d*x)*b),x)*a**7*b**2*d**2*f - 59904*e**(c + 4*d*x)*int(x/(e**(2*c + 5*d...
```

3.399 $\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 184

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{(8a^4+4a^2b^2-b^4)\,x}{8b^5} + \frac{2a^3\sqrt{a^2+b^2}\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^5d} - \frac{a(3a^2+b^2)\cosh(c+dx)}{3b^4d} + \frac{(4a^2+b^2)\cosh(c+dx)\sinh(c+dx)}{8b^3d} - \frac{a\cosh(c+dx)\sinh^2(c+dx)}{3b^2d} + \frac{\cosh(c+dx)\sinh^3(c+dx)}{4bd}$$

output
```
1/8*(8*a^4+4*a^2*b^2-b^4)*x/b^5+2*a^3*(a^2+b^2)^(1/2)*arctanh((b-a*tanh(1/
2*d*x+1/2*c))/(a^2+b^2)^(1/2))/b^5/d-1/3*a*(3*a^2+b^2)*cosh(d*x+c)/b^4/d+1
/8*(4*a^2+b^2)*cosh(d*x+c)*sinh(d*x+c)/b^3/d-1/3*a*cosh(d*x+c)*sinh(d*x+c)
^2/b^2/d+1/4*cosh(d*x+c)*sinh(d*x+c)^3/b/d
```

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.83

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-24ab(4a^2+b^2)\cosh(c+dx)-8ab^3\cosh(3(c+dx))+3\Big(4(8a^4+4a^2b^2-b^4)(c+dx)+64a^3\sqrt{-a^2-}}{96b^5d}$$

input
```
Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-24*a*b*(4*a^2 + b^2)*Cosh[c + d*x] - 8*a*b^3*Cosh[3*(c + d*x)] + 3*(4*(8
*a^4 + 4*a^2*b^2 - b^4)*(c + d*x) + 64*a^3*Sqrt[-a^2 - b^2]*ArcTan[(b - a*
Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]] + 8*a^2*b^2*Sinh[2*(c + d*x)] + b^4*S
inh[4*(c + d*x)]))/(96*b^5*d)
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.21, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.828$, Rules used = {3042, 26, 3368, 26, 3042, 26, 3529, 3042, 25, 3528, 26, 3042, 26, 3528, 25, 3042, 3502, 27, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i\sin(ic+idx)^3\cos(ic+idx)^2}{a-ib\sin(ic+idx)}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{\cos(ic+idx)^2\sin(ic+idx)^3}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3368}$$

$$i\int -\frac{i\sinh^3(c+dx)\left(\sinh^2(c+dx)+1\right)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\sinh^3(c+dx)\left(\sinh^2(c+dx)+1\right)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ic+idx)^3\left(1-\sin(ic+idx)^2\right)}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ic+idx)^3\left(1-\sin(ic+idx)^2\right)}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3529}$$

$$i\left(\frac{i\int \frac{\sinh^2(c+dx)(4a\sinh^2(c+dx)-b\sinh(c+dx)+3a)}{a+b\sinh(c+dx)}dx}{4b}-\frac{i\sinh^3(c+dx)\cosh(c+dx)}{4bd}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{i\int -\frac{\sin(ic+idx)^2(-4a\sin(ic+idx)^2+ib\sin(ic+idx)+3a)}{a-ib\sin(ic+idx)}dx}{4b}-\frac{i\sinh^3(c+dx)\cosh(c+dx)}{4bd}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(-\frac{i\int \frac{\sin(ic+idx)^2(-4a\sin(ic+idx)^2+ib\sin(ic+idx)+3a)}{a-ib\sin(ic+idx)}dx}{4b}-\frac{i\sinh^3(c+dx)\cosh(c+dx)}{4bd}\right)$$

$$\Big\downarrow \text{3528}$$

$$i\left(-\frac{i\left(-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}+\frac{i\int -\frac{i\sinh(c+dx)\left(8a^2-b\sinh(c+dx)a+3\left(4a^2+b^2\right)\sinh^2(c+dx)\right)}{a+b\sinh(c+dx)}dx}{3b}\right)}{4b}-\frac{i\sinh^3(c+dx)\cosh(c+$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{i\left(\frac{\int \frac{\sinh(c+dx)\left(8a^2-b\sinh(c+dx)a+3\left(4a^2+b^2\right)\sinh^2(c+dx)\right)}{a+b\sinh(c+dx)}dx}{3b}-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}\right)}{4b}-\frac{i\sinh^3(c+dx)\cosh(c+dx)}{4bd}\right.$$

$\downarrow$ 3042

$$i\left(-\frac{i\left(-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}+\frac{\int -\frac{i\sin(ic+idx)\left(8a^2+ib\sin(ic+idx)a-3\left(4a^2+b^2\right)\sin(ic+idx)^2\right)}{a-ib\sin(ic+idx)}dx}{3b}\right)}{4b}-\frac{i\sinh^3(c+dx)\cosh(c+\right.$$

$\downarrow$ 26

$$i\left(-\frac{i\left(-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{i\int \frac{\sin(ic+idx)\left(8a^2+ib\sin(ic+idx)a-3\left(4a^2+b^2\right)\sin(ic+idx)^2\right)}{a-ib\sin(ic+idx)}dx}{3b}\right)}{4b}-\frac{i\sinh^3(c+dx)\cosh(c+\right.$$

$\downarrow$ 3528

$$i\left(-\frac{i\left(-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{i\left(\frac{i\int -\frac{8a\left(3a^2+b^2\right)\sinh^2(c+dx)-b\left(4a^2-3b^2\right)\sinh(c+dx)+3a\left(4a^2+b^2\right)}{a+b\sinh(c+dx)}dx}{2b}+\frac{3i\left(4a^2+b^2\right)\sinh(c+dx)\cosh(c+\right.}{2bd}}{3b}\right.}{4b}\right.$$

$\downarrow$ 25

$$i\left(-\frac{i\left(-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{i\left(\frac{3i\left(4a^2+b^2\right)\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\int\frac{8a\left(3a^2+b^2\right)\sinh^2(c+dx)-b\left(4a^2-3b^2\right)\sinh(c+dx)+3a\left(4a^2+b^2\right)}{a+b\sinh(c+dx)}}{2b}\right.}{3b}\right.}{4b}\right.$$

$\downarrow$ 3042

$$i\left(-\frac{i\left(-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{i\left(\frac{3i\left(4a^2+b^2\right)\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\int\frac{-8a\left(3a^2+b^2\right)\sin(ic+idx)^2+ib\left(4a^2-3b^2\right)\sin(ic+idx)+3a\left(4a^2+b\right.}{a-ib\sin(ic+idx)}}{2b}\right.}{3b}\right.}{4b}\right.$$

$\downarrow$ 3502

$$i\left(i-\frac{i\left(-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{i\left(\frac{3i\left(4a^2+b^2\right)\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{8a\left(3a^2+b^2\right)\cosh(c+dx)}{bd}+\frac{i\int -\frac{3i\left(ab\left(4a^2+b^2\right)-\left(8a^4+4b^2a^2-b^4\right.\right.}{a+b\sinh(c+dx)}}{b}\right.}{2b}\right.}{3b}\right.}{4b}\right.$$

$$\Big\downarrow 27$$

$$i\left(-\frac{i\left(-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{i\left(\frac{3i\left(4a^2+b^2\right)\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{3\int\frac{ab\left(4a^2+b^2\right)-\left(8a^4+4b^2a^2-b^4\right)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{8a\left(3a^2+b^2\right)c}{bd}\right.}{2b}\right.}{3b}\right.}{4b}\right.$$

$\Big\downarrow$ 3042

$$i\left(-\frac{i\left(-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{i\left(\frac{3i\left(4a^2+b^2\right)\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{8a\left(3a^2+b^2\right)\cosh(c+dx)}{bd}+\frac{3\int\frac{ab\left(4a^2+b^2\right)+i\left(8a^4+4b^2a^2-b^4\right)\sin}{a-ib\sin(ic+idx)}}{b}\right.}{2b}\right.}{3b}\right.}{4b}\right.$$

$\downarrow$ 3214

$$i\left(-\frac{i\left(-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{i\left(\frac{3i\left(4a^2+b^2\right)\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{3\left(\frac{8a^3\left(a^2+b^2\right)\int\frac{1}{a+b\sinh(c+dx)}dx}{b}-\frac{x\left(8a^4+4a^2b^2-b^4\right)}{b}\right)}{b}+\frac{8a(\cdots}{\cdots}\right)}{2b}\cdots\right)}{3b}\cdots\right)}{4b}\cdots\right)$$

$\big\downarrow$ 3042

$$i\left(-\frac{i\left(-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{i\left(\frac{3i\left(4a^2+b^2\right)\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{8a\left(3a^2+b^2\right)\cosh(c+dx)}{bd}+\frac{3\left(-\frac{x\left(8a^4+4a^2b^2-b^4\right)}{b}+\frac{8a^3\left(a^2+b^2\right)\cdots}{\cdots}\right.}{b}\cdots\right.}{2b}\cdots\right.}{3b}\cdots\right.}{4b}\cdots\right.$$

$\downarrow$ 3139

$$i\left(-\frac{i\left(-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{i\left(\frac{3i\left(4a^2+b^2\right)\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{8a\left(3a^2+b^2\right)\cosh(c+dx)}{bd}+\frac{3\left(-\frac{x\left(8a^4+4a^2b^2-b^4\right)}{b}-\frac{16ia^3\left(a^2+b^2\right.}{}\right.}{}\right)}{}\right)}{3b}\right)}{4b}\right.$$

↓ 1083

$$i\left(-\frac{i\left(-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{i\left(\frac{3i\left(4a^2+b^2\right)\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{8a\left(3a^2+b^2\right)\cosh(c+dx)}{bd}+\frac{3\left(-\frac{x\left(8a^4+4a^2b^2-b^4\right)}{b}+\frac{32ia^3\left(a^2+b^2\right.}{}\right.}{}\right.}{2b}\right.}{3b}\right.}{4b}\right.$$

↓ 217

$$i\left(-\frac{i\left(-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{i\left(\frac{3i\left(4a^2+b^2\right)\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{8a\left(3a^2+b^2\right)\cosh(c+dx)}{bd}+\frac{3\left(\frac{16a^3\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c\right.}{2\sqrt{a^2\cdots}}\right.}{bd}\cdots\right.}{b}\cdots\right)}{2b}\cdots\right)}{3b}\cdots\right)}{4b}\cdots\right.$$

input
```
Int[(Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
I*(((-1/4*I)*Cosh[c + d*x]*Sinh[c + d*x]^3)/(b*d) - ((I/4)*((-4*a*Cosh[c +
 d*x]*Sinh[c + d*x]^2)/(3*b*d) - ((I/3)*(((-1/2*I)*((3*(-(((8*a^4 + 4*a^2*
b^2 - b^4)*x)/b) + (16*a^3*Sqrt[a^2 + b^2]*ArcTanh[Tanh[(c + d*x)/2]/(2*Sq
rt[a^2 + b^2])])/(b*d)))/b + (8*a*(3*a^2 + b^2)*Cosh[c + d*x])/(b*d)))/b +
 (((3*I)/2)*(4*a^2 + b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(b*d)))/b))/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3368
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3529
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n +
1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m + n + 2))   Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(
n + 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*
(a*d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c,
0])))
```

Maple [A] (verified)

Time = 33.22 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.59

method	result
risch	$\frac{a^4 x}{b^5}+\frac{x\,a^2}{2b^3}-\frac{x}{8b}+\frac{\mathrm{e}^{4dx+4c}}{64bd}-\frac{a\,\mathrm{e}^{3dx+3c}}{24b^2d}+\frac{\mathrm{e}^{2dx+2c}a^2}{8b^3d}-\frac{a^3\mathrm{e}^{dx+c}}{2b^4d}-\frac{a\,\mathrm{e}^{dx+c}}{8b^2d}-\frac{a^3\mathrm{e}^{-dx-c}}{2b^4d}-\frac{a\,\mathrm{e}^{-dx-c}}{8b^2d}$ …
derivativedivides	$-\frac{1}{4b\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{-3b+2a}{6b^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{4a^2-4ab+3b^2}{8b^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{\left(8a^4+4a^2b^2-b^4\right)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8b^5}-\frac{8a^3-}{8b^4(1}$ …
default	$-\frac{1}{4b\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{-3b+2a}{6b^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{4a^2-4ab+3b^2}{8b^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{\left(8a^4+4a^2b^2-b^4\right)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8b^5}-\frac{8a^3-}{8b^4(1}$ …

input
```
int(cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
a^4*x/b^5+1/2*x/b^3*a^2-1/8*x/b+1/64/b/d*exp(4*d*x+4*c)-1/24*a/b^2/d*exp(3
*d*x+3*c)+1/8/b^3/d*exp(2*d*x+2*c)*a^2-1/2*a^3/b^4/d*exp(d*x+c)-1/8*a/b^2/
d*exp(d*x+c)-1/2*a^3/b^4/d*exp(-d*x-c)-1/8*a/b^2/d*exp(-d*x-c)-1/8/b^3/d*e
xp(-2*d*x-2*c)*a^2-1/24*a/b^2/d*exp(-3*d*x-3*c)-1/64/b/d*exp(-4*d*x-4*c)+(
a^2+b^2)^(1/2)*a^3/d/b^5*ln(exp(d*x+c)+(a+(a^2+b^2)^(1/2))/b)-(a^2+b^2)^(1
/2)*a^3/d/b^5*ln(exp(d*x+c)-(-a+(a^2+b^2)^(1/2))/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1134 vs. $2(171) = 342$.

Time = 0.12 (sec) , antiderivative size = 1134, normalized size of antiderivative = 6.16

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fric
as")
```

output
```
1/192*(3*b^4*cosh(d*x + c)^8 + 3*b^4*sinh(d*x + c)^8 - 8*a*b^3*cosh(d*x +
c)^7 + 24*a^2*b^2*cosh(d*x + c)^6 + 8*(3*b^4*cosh(d*x + c) - a*b^3)*sinh(d
*x + c)^7 + 24*(8*a^4 + 4*a^2*b^2 - b^4)*d*x*cosh(d*x + c)^4 + 4*(21*b^4*c
osh(d*x + c)^2 - 14*a*b^3*cosh(d*x + c) + 6*a^2*b^2)*sinh(d*x + c)^6 - 24*
a^2*b^2*cosh(d*x + c)^2 - 24*(4*a^3*b + a*b^3)*cosh(d*x + c)^5 + 24*(7*b^4
*cosh(d*x + c)^3 - 7*a*b^3*cosh(d*x + c)^2 + 6*a^2*b^2*cosh(d*x + c) - 4*a
^3*b - a*b^3)*sinh(d*x + c)^5 - 8*a*b^3*cosh(d*x + c) + 2*(105*b^4*cosh(d*
x + c)^4 - 140*a*b^3*cosh(d*x + c)^3 + 180*a^2*b^2*cosh(d*x + c)^2 + 12*(8
*a^4 + 4*a^2*b^2 - b^4)*d*x - 60*(4*a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x
 + c)^4 - 3*b^4 - 24*(4*a^3*b + a*b^3)*cosh(d*x + c)^3 + 8*(21*b^4*cosh(d*
x + c)^5 - 35*a*b^3*cosh(d*x + c)^4 + 60*a^2*b^2*cosh(d*x + c)^3 - 12*a^3*
b - 3*a*b^3 + 12*(8*a^4 + 4*a^2*b^2 - b^4)*d*x*cosh(d*x + c) - 30*(4*a^3*b
 + a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 12*(7*b^4*cosh(d*x + c)^6 - 1
4*a*b^3*cosh(d*x + c)^5 + 30*a^2*b^2*cosh(d*x + c)^4 + 12*(8*a^4 + 4*a^2*b
^2 - b^4)*d*x*cosh(d*x + c)^2 - 2*a^2*b^2 - 20*(4*a^3*b + a*b^3)*cosh(d*x
+ c)^3 - 6*(4*a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 192*(a^3*cos
h(d*x + c)^4 + 4*a^3*cosh(d*x + c)^3*sinh(d*x + c) + 6*a^3*cosh(d*x + c)^2
*sinh(d*x + c)^2 + 4*a^3*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*sinh(d*x + c)
^4)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b
*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**2*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.40

$$\begin{aligned}
&\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= -\frac{\sqrt{a^2+b^2}a^3\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{b^5d} \\
&\quad - \frac{\left(8\,ab^2e^{(-dx-c)}-24\,a^2be^{(-2\,dx-2\,c)}-3\,b^3+24\,(4\,a^3+ab^2)e^{(-3\,dx-3\,c)}\right)e^{(4\,dx+4\,c)}}{192\,b^4d} \\
&\quad + \frac{(8\,a^4+4\,a^2b^2-b^4)(dx+c)}{8\,b^5d} \\
&\quad - \frac{24\,a^2be^{(-2\,dx-2\,c)}+8\,ab^2e^{(-3\,dx-3\,c)}+3\,b^3e^{(-4\,dx-4\,c)}+24\,(4\,a^3+ab^2)e^{(-dx-c)}}{192\,b^4d}
\end{aligned}$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxi
ma")
```

output
```
-sqrt(a^2 + b^2)*a^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x
 - c) - a + sqrt(a^2 + b^2)))/(b^5*d) - 1/192*(8*a*b^2*e^(-d*x - c) - 24*a
^2*b*e^(-2*d*x - 2*c) - 3*b^3 + 24*(4*a^3 + a*b^2)*e^(-3*d*x - 3*c))*e^(4*
d*x + 4*c)/(b^4*d) + 1/8*(8*a^4 + 4*a^2*b^2 - b^4)*(d*x + c)/(b^5*d) - 1/1
92*(24*a^2*b*e^(-2*d*x - 2*c) + 8*a*b^2*e^(-3*d*x - 3*c) + 3*b^3*e^(-4*d*x
 - 4*c) + 24*(4*a^3 + a*b^2)*e^(-d*x - c))/(b^4*d)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.40

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{\frac{24\,(8\,a^4+4\,a^2b^2-b^4)(dx+c)}{b^5} + \frac{3\,b^3e^{(4\,dx+4\,c)}-8\,ab^2e^{(3\,dx+3\,c)}+24\,a^2be^{(2\,dx+2\,c)}-96\,a^3e^{(dx+c)}-24\,ab^2e^{(dx+c)}}{b^4} - \frac{(24\,a^2b^2e^{(2\,dx+2\,c)}+8\,ab^3}{}}{192\,d}$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac
")
```

output
```
1/192*(24*(8*a^4 + 4*a^2*b^2 - b^4)*(d*x + c)/b^5 + (3*b^3*e^(4*d*x + 4*c)
 - 8*a*b^2*e^(3*d*x + 3*c) + 24*a^2*b*e^(2*d*x + 2*c) - 96*a^3*e^(d*x + c)
 - 24*a*b^2*e^(d*x + c))/b^4 - (24*a^2*b^2*e^(2*d*x + 2*c) + 8*a*b^3*e^(d*
x + c) + 3*b^4 + 24*(4*a^3*b + a*b^3)*e^(3*d*x + 3*c))*e^(-4*d*x - 4*c)/b^
5 - 192*(a^5 + a^3*b^2)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))
/abs(2*b*e^(d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^5))/d
```

Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.79

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{x\,(8\,a^4+4\,a^2\,b^2-b^4)}{8\,b^5} - \frac{\mathrm{e}^{-4\,c-4\,d\,x}}{64\,b\,d} + \frac{\mathrm{e}^{4\,c+4\,d\,x}}{64\,b\,d} - \frac{a\,\mathrm{e}^{-3\,c-3\,d\,x}}{24\,b^2\,d} - \frac{a\,\mathrm{e}^{3\,c+3\,d\,x}}{24\,b^2\,d}$$
$$- \frac{\mathrm{e}^{c+d\,x}\,(4\,a^3+a\,b^2)}{8\,b^4\,d} - \frac{a^2\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,b^3\,d} + \frac{a^2\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,b^3\,d} - \frac{\mathrm{e}^{-c-d\,x}\,(4\,a^3+a\,b^2)}{8\,b^4\,d}$$
$$- \frac{a^3\,\ln\left(\frac{2\,a^3\,\mathrm{e}^{c+d\,x}\,(a^2+b^2)}{b^6} - \frac{2\,a^3\,\sqrt{a^2+b^2}\,(b-a\,\mathrm{e}^{c+d\,x})}{b^6}\right)\,\sqrt{a^2+b^2}}{b^5\,d}$$
$$+ \frac{a^3\,\ln\left(\frac{2\,a^3\,\sqrt{a^2+b^2}\,(b-a\,\mathrm{e}^{c+d\,x})}{b^6} + \frac{2\,a^3\,\mathrm{e}^{c+d\,x}\,(a^2+b^2)}{b^6}\right)\,\sqrt{a^2+b^2}}{b^5\,d}$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
(x*(8*a^4 - b^4 + 4*a^2*b^2))/(8*b^5) - exp(- 4*c - 4*d*x)/(64*b*d) + exp(
4*c + 4*d*x)/(64*b*d) - (a*exp(- 3*c - 3*d*x))/(24*b^2*d) - (a*exp(3*c + 3
*d*x))/(24*b^2*d) - (exp(c + d*x)*(a*b^2 + 4*a^3))/(8*b^4*d) - (a^2*exp(-
2*c - 2*d*x))/(8*b^3*d) + (a^2*exp(2*c + 2*d*x))/(8*b^3*d) - (exp(- c - d*
x)*(a*b^2 + 4*a^3))/(8*b^4*d) - (a^3*log((2*a^3*exp(c + d*x)*(a^2 + b^2))/
b^6 - (2*a^3*(a^2 + b^2)^(1/2)*(b - a*exp(c + d*x)))/b^6)*(a^2 + b^2)^(1/2
))/(b^5*d) + (a^3*log((2*a^3*(a^2 + b^2)^(1/2)*(b - a*exp(c + d*x)))/b^6 +
 (2*a^3*exp(c + d*x)*(a^2 + b^2))/b^6)*(a^2 + b^2)^(1/2))/(b^5*d)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.49

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-384e^{4dx+4c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)a^3i+3e^{8dx+8c}b^4-8e^{7dx+7c}a\,b^3+24e^{6dx+6c}a^2b^2-96e^{5dx+5c}a^3b-}{}$$

input
```
int(cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
( - 384*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/s
qrt(a**2 + b**2))*a**3*i + 3*e**(8*c + 8*d*x)*b**4 - 8*e**(7*c + 7*d*x)*a*
b**3 + 24*e**(6*c + 6*d*x)*a**2*b**2 - 96*e**(5*c + 5*d*x)*a**3*b - 24*e**
(5*c + 5*d*x)*a*b**3 + 192*e**(4*c + 4*d*x)*a**4*d*x + 96*e**(4*c + 4*d*x)
*a**2*b**2*d*x - 24*e**(4*c + 4*d*x)*b**4*d*x - 96*e**(3*c + 3*d*x)*a**3*b
 - 24*e**(3*c + 3*d*x)*a*b**3 - 24*e**(2*c + 2*d*x)*a**2*b**2 - 8*e**(c +
d*x)*a*b**3 - 3*b**4)/(192*e**(4*c + 4*d*x)*b**5*d)
```

3.400 $\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 36

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\text{Int}\left(\frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(cosh(d*x+c)^2*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 15.47 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x
])),x]
```

output
```
Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

$$\Big\downarrow \text{ 6125}$$

$$\int \frac{\sinh^3(c+dx)\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Cosh[c + d*x]^2*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(dx+c\right)^{2}\sinh\left(dx+c\right)^{3}}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(cosh(d*x+c)^2*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)^2*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
integral(cosh(d*x + c)^2*sinh(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh
(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**2*sinh(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 417, normalized size of antiderivative = 11.58

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-2*(a^5*e^c + a^3*b^2*e^c)*integrate(-e^(d*x)/(b^6*f*x + b^6*e - (b^6*f*x*
e^(2*c) + b^6*e*e^(2*c))*e^(2*d*x) - 2*(a*b^5*f*x*e^c + a*b^5*e*e^c)*e^(d*
x)), x) - 1/16*e^(-4*c + 4*d*e/f)*exp_integral_e(1, 4*(f*x + e)*d/f)/(b*f)
 - 1/8*a*e^(-3*c + 3*d*e/f)*exp_integral_e(1, 3*(f*x + e)*d/f)/(b^2*f) - 1
/4*a^2*e^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b^3*f) - 1/4
*a^2*e^(2*c - 2*d*e/f)*exp_integral_e(1, -2*(f*x + e)*d/f)/(b^3*f) + 1/8*a
*e^(3*c - 3*d*e/f)*exp_integral_e(1, -3*(f*x + e)*d/f)/(b^2*f) - 1/16*e^(4
*c - 4*d*e/f)*exp_integral_e(1, -4*(f*x + e)*d/f)/(b*f) - 1/8*(4*a^3 + a*b
^2)*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^4*f) + 1/8*(4*a^3*e
^c + a*b^2*e^c)*e^(-d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b^4*f) + 1/8
*(8*a^4 + 4*a^2*b^2 - b^4)*log(f*x + e)/(b^5*f)
```

Giac [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate(cosh(d*x + c)^2*sinh(d*x + c)^3/((f*x + e)*(b*sinh(d*x + c) + a)
), x)
```

Mupad [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{{\cosh\left(c+d\,x\right)}^2\,{\sinh\left(c+d\,x\right)}^3}{\left(e+f\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)} \,d x$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x)^3)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((cosh(c + d*x)^2*sinh(c + d*x)^3)/((e + f*x)*(a + b*sinh(c + d*x))), x
)
```

Reduce [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 445, normalized size of antiderivative = 12.36

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$= \frac{e^{6c}\left(\int \frac{e^{6dx}}{e^{2dx+2c}be+e^{2dx+2c}bfx+2e^{dx+c}ae+2e^{dx+c}afx-be-bfx}dx\right)bf - e^{4c}\left(\int \frac{e^{4dx}}{e^{2dx+2c}be+e^{2dx+2c}bfx+2e^{dx+c}ae+2e^{dx+c}afx-be-}\right.}{}$$

input
```
int(cosh(d*x+c)^2*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
(e**(6*c)*int(e**(6*d*x)/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x +
2*e**(c + d*x)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x)*b*f - e**(4*c)
*int(e**(4*d*x)/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x + 2*e**(c +
 d*x)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x)*b*f + e**(2*c)*int(1/(e
**(6*c + 4*d*x)*b*e + e**(6*c + 4*d*x)*b*f*x + 2*e**(5*c + 3*d*x)*a*e + 2*
e**(5*c + 3*d*x)*a*f*x - e**(4*c + 2*d*x)*b*e - e**(4*c + 2*d*x)*b*f*x),x)
*b*f + 4*e**c*int(e**(d*x)/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x
+ 2*e**(c + d*x)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x)*a*f - int(1/
(e**(6*c + 6*d*x)*b*e + e**(6*c + 6*d*x)*b*f*x + 2*e**(5*c + 5*d*x)*a*e +
2*e**(5*c + 5*d*x)*a*f*x - e**(4*c + 4*d*x)*b*e - e**(4*c + 4*d*x)*b*f*x),
x)*b*f - 2*log(e + f*x))/(16*b*f)
```

3.401 $\int \frac{(e+fx)^3 \cosh^3(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$

Optimal result

Integrand size = 36, antiderivative size = 1443

$$\int \frac{(e+fx)^3 \cosh^3(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx = \text{Too large to display}$$

output
```
45/256*a*f^3*x/b^2/d^3-40/9*a^2*f^3*cosh(d*x+c)/b^3/d^4-6*a^3*(a^2+b^2)*f^
3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^6/d^4-6*a^3*(a^2+b^2)*f^3
*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^6/d^4-a^3*(a^2+b^2)*(f*x+e
)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^6/d-a^3*(a^2+b^2)*(f*x+e)^3*l
n(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^6/d+6*a^3*(a^2+b^2)*f^2*(f*x+e)*po
lylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^6/d^3+6*a^3*(a^2+b^2)*f^2*(f*
x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^6/d^3-3*a^3*(a^2+b^2)*
f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^6/d^2-3*a^3*(a^
2+b^2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^6/d^2+3/
128*a*f^3*cosh(d*x+c)^3*sinh(d*x+c)/b^2/d^4-3*a^4*f*(f*x+e)^2*cosh(d*x+c)/
b^5/d^2-9/32*a*f^2*(f*x+e)*cosh(d*x+c)^2/b^2/d^3-1/3*a^2*f*(f*x+e)^2*cosh(
d*x+c)^3/b^3/d^2-3/32*a*f^2*(f*x+e)*cosh(d*x+c)^4/b^2/d^3+6*a^4*f^2*(f*x+e
)*sinh(d*x+c)/b^5/d^3-3/4*a^3*f^2*(f*x+e)*sinh(d*x+c)^2/b^4/d^3-1/2*a^3*(f
*x+e)^3*sinh(d*x+c)^2/b^4/d-3/400*f*(f*x+e)^2*cosh(5*d*x+5*c)/b/d^2-1/48*f
*(f*x+e)^2*cosh(3*d*x+3*c)/b/d^2-6*a^4*f^3*cosh(d*x+c)/b^5/d^4-2/27*a^2*f^
3*cosh(d*x+c)^3/b^3/d^4-1/4*a*(f*x+e)^3*cosh(d*x+c)^4/b^2/d-3/8*a^3*f^3*x/
b^4/d^3+1/4*a^3*(a^2+b^2)*(f*x+e)^4/b^6/f+3/8*a^3*f^3*cosh(d*x+c)*sinh(d*x
+c)/b^4/d^4+1/3*a^2*(f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)/b^3/d+3/1000*f^2*(
f*x+e)*sinh(5*d*x+5*c)/b/d^3+1/72*f^2*(f*x+e)*sinh(3*d*x+3*c)/b/d^3+40/9*a
^2*f^2*(f*x+e)*sinh(d*x+c)/b^3/d^3+45/256*a*f^3*cosh(d*x+c)*sinh(d*x+c)...
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 5147 vs. $2(1443) = 2886$.

Time = 10.65 (sec) , antiderivative size = 5147, normalized size of antiderivative = 3.57

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*
x]),x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sinh^3(c+dx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6113}$$

$$\frac{\int (e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{5971}$$

$$\frac{\int \left(-\frac{1}{8}\cosh(c+dx)(e+fx)^3+\frac{1}{16}\cosh(3c+3dx)(e+fx)^3+\frac{1}{16}\cosh(5c+5dx)(e+fx)^3\right)dx}{b}-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5da}{1000d^3}}{}$$
$$\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{6113}$$

$$\frac{\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5da}{1000d^3}}{}$$
$$\frac{a\left(\frac{\int (e+fx)^3\cosh^3(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{5970}$$

$$\frac{\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5da}{1000d^3}}{}$$
$$\frac{a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\int (e+fx)^2\cosh^4(c+dx)dx}{4d}}{b}-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$\downarrow$ 3042

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^4dx}{4d}}{b}\right)}{b}$$

$\downarrow$ 3792

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}$$

$$\frac{a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{f^2\int\cosh^4(c+dx)dx}{8d^2}+\frac{3}{4}\int(e+fx)^2\cosh^2(c+dx)dx-\frac{f(e+fx)\cosh^4(c+dx)}{8d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{4d}}{b}-\frac{a\int\frac{(e+fx}{}\cdots\right.}{b}$$

$\downarrow$ 3042

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{f^2\int\sin\left(ic+idx+\frac{\pi}{2}\right)^4dx}{8d^2}+\frac{3}{4}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx-\frac{f(e+fx)\cosh^4(c}{8d^2}\cdots\right.}{4d}}{b}\right.}{b}$$

$\downarrow$ 3115

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{f^2\left(\frac{3}{4}\int\cosh^2(c+dx)dx+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{8d^2}+\frac{3}{4}\int(e+fx)^2\sin\left(ic+idx\cdots\right.\right.}{4d}}{b}\right.}{b}$$

$\downarrow$ 3042

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx \cdots}{1000d^3}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{f^2\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\int\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx\right)}{8d^2}+\frac{3}{4}\int(e+fx)^2\sin\left(ic+\cdots\right.\cdots\right)}{4d}}{b}\cdots\right.}{b}$$

$\Big\downarrow$ 3115

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx \cdots}{1000d^3}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{f^2\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{8d^2}+\frac{3}{4}\int(e+\cdots\right.}{\cdots}}{b}\cdots\right.}{b}$$

$\Big\downarrow$ 24

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx \cdots}{1000d^3}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx-\frac{f(e+fx)\cosh^4(c+dx)}{8d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh \cdots}{4d}\right.\cdots}{\cdots}\right.}{4 \cdots}}{b}\cdots\right.}{b}$$

$\Big\downarrow$ 3792

$$\frac{\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}}{a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(\frac{f^2\int\cosh^2(c+dx)dx}{2d^2}+\frac{1}{2}\int(e+fx)^2dx-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}\right)-\frac{f(e+fx)\cosh^4(c+dx)}{8d^2}+\right.}{4d}}{b}\right.}{b}$$

$$\downarrow 17$$

$$\frac{\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}}{a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(\frac{f^2\int\cosh^2(c+dx)dx}{2d^2}-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4(c+dx)}{8d^2}+\frac{f^2\left(\right.}{}\right.}{4d}}{b}\right.}{b}$$

$$\downarrow 3042$$

$$\frac{\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}}{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(\frac{f^2\int\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx}{2d^2}-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cos}{2d}\right.\right.}{}}{}\right.}{b}$$

$$\downarrow 3115$$

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx}{1000d^3}$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(\frac{f^2\left(\frac{\int 1dx}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cos}{8d}\right.}{4d}}{b}\right.$$

$\downarrow$ 24

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx}{1000d^3}$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4}{8d^2}\right.}{4d}}{b}\right.$$

$\downarrow$ 6113

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx}{1000d^3}$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4}{8d^2}\right.}{4d}}{b}\right.$$

$\downarrow$ 3042

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4}{8d^2}\right.}{4d}}{b}\right.$$

$$\downarrow \text{3792}$$

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4}{8d^2}\right.}{4d}}{b}\right.$$

$$\downarrow \text{3042}$$

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4}{8d^2}\right.}{4d}}{b}\right.$$

$$\downarrow \text{3777}$$

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx}{1000d^3}$$

$$a\left(\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4}{8d^2}\right.}{4d}}{b}\right.$$

$\downarrow$ 26

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx}{1000d^3}$$

$$a\left(\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4}{8d^2}\right.}{4d}}{b}\right.$$

$\downarrow$ 3042

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx}{1000d^3}$$

$$a\left(\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4}{8d^2}\right.}{4d}}{b}\right.$$

$\downarrow$ 26

$$\frac{\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx}{1000d^3}}{a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4}{8d^2}}{4d}}{b}\right.}$$

$\downarrow$ 3777

$$\frac{\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx}{1000d^3}}{a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4}{8d^2}}{4d}}{b}\right.}$$

$\downarrow$ 3042

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx}{1000d^3}$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4}{8d^2}}{4d}}{b}\right.$$

$\downarrow$ 3777

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx}{1000d^3}$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4}{8d^2}}{4d}}{b}\right.$$

$\downarrow$ 26

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx}{1000d^3}$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4}{8d^2}\right.}{4d}}{b}\right.$$

$\downarrow$ 3042

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx}{1000d^3}$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4}{8d^2}\right.}{4d}}{b}\right.$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^3 \cosh(dx+c)^3 \sinh(dx+c)^3}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18801 vs. 2(1347) = 2694.

Time = 0.46 (sec) , antiderivative size = 18801, normalized size of antiderivative = 13.03

$$\int \frac{(e+fx)^3 \cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algor
ithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)**3*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)^3\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algor
ithm="maxima")
```

output
```
-1/960*e^3*((15*a*b^3*e^(-d*x - c) - 6*b^4 - 10*(4*a^2*b^2 + b^4)*e^(-2*d*
x - 2*c) + 60*(2*a^3*b + a*b^3)*e^(-3*d*x - 3*c) - 60*(8*a^4 + 6*a^2*b^2 -
 b^4)*e^(-4*d*x - 4*c))*e^(5*d*x + 5*c)/(b^5*d) + 960*(a^5 + a^3*b^2)*(d*x
 + c)/(b^6*d) + (15*a*b^3*e^(-4*d*x - 4*c) + 6*b^4*e^(-5*d*x - 5*c) + 60*(
8*a^4 + 6*a^2*b^2 - b^4)*e^(-d*x - c) + 60*(2*a^3*b + a*b^3)*e^(-2*d*x - 2
*c) + 10*(4*a^2*b^2 + b^4)*e^(-3*d*x - 3*c))/(b^5*d) + 960*(a^5 + a^3*b^2)
*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^6*d)) - 1/34560000*(86
40000*(a^5*d^4*f^3*e^(5*c) + a^3*b^2*d^4*f^3*e^(5*c))*x^4 + 34560000*(a^5*
d^4*e*f^2*e^(5*c) + a^3*b^2*d^4*e*f^2*e^(5*c))*x^3 + 51840000*(a^5*d^4*e^2
*f*e^(5*c) + a^3*b^2*d^4*e^2*f*e^(5*c))*x^2 - 1728*(125*b^5*d^3*f^3*x^3*e^
(10*c) + 75*(5*d^3*e*f^2 - d^2*f^3)*b^5*x^2*e^(10*c) + 15*(25*d^3*e^2*f -
10*d^2*e*f^2 + 2*d*f^3)*b^5*x*e^(10*c) - 3*(25*d^2*e^2*f - 10*d*e*f^2 + 2*
f^3)*b^5*e^(10*c))*e^(5*d*x) + 16875*(32*a*b^4*d^3*f^3*x^3*e^(9*c) + 24*(4
*d^3*e*f^2 - d^2*f^3)*a*b^4*x^2*e^(9*c) + 12*(8*d^3*e^2*f - 4*d^2*e*f^2 +
d*f^3)*a*b^4*x*e^(9*c) - 3*(8*d^2*e^2*f - 4*d*e*f^2 + f^3)*a*b^4*e^(9*c))*
e^(4*d*x) + 40000*(4*(9*d^2*e^2*f - 6*d*e*f^2 + 2*f^3)*a^2*b^3*e^(8*c) + (
9*d^2*e^2*f - 6*d*e*f^2 + 2*f^3)*b^5*e^(8*c) - 9*(4*a^2*b^3*d^3*f^3*e^(8*c
) + b^5*d^3*f^3*e^(8*c))*x^3 - 9*(4*(3*d^3*e*f^2 - d^2*f^3)*a^2*b^3*e^(8*c
) + (3*d^3*e*f^2 - d^2*f^3)*b^5*e^(8*c))*x^2 - 3*(4*(9*d^3*e^2*f - 6*d^2*e
*f^2 + 2*d*f^3)*a^2*b^3*e^(8*c) + (9*d^3*e^2*f - 6*d^2*e*f^2 + 2*d*f^3)...
```

Giac [F]

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh{(dx+c)}^3\sinh{(dx+c)}^3}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algor
ithm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)^3*sinh(d*x + c)^3/(b*sinh(d*x + c) + a
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \cosh^3(c+dx) \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{{\cosh\left(c+d\,x\right)}^3\,{\sinh\left(c+d\,x\right)}^3\,{\left(e+f\,x\right)}^3}{a+b\,\sinh\left(c+d\,x\right)} \,d x$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^3*sinh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)), x
)
```

Reduce [F]

$$\int \frac{(e+fx)^3 \cosh^3(c+dx) \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\left(f x +e \right)^{3} \cosh \left(d x +c \right)^{3} \sinh \left(d x +c \right)^{3}}{a +b \sinh \left(d x +c \right)}d x$$

input
```
int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

3.402 $\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 1021

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output
```
14/9*a^2*f^2*sinh(d*x+c)/b^3/d^3+2*a^3*(a^2+b^2)*f^2*polylog(3,-b*exp(d*x+
c)/(a+(a^2+b^2)^(1/2)))/b^6/d^3+2*a^3*(a^2+b^2)*f^2*polylog(3,-b*exp(d*x+c
)/(a-(a^2+b^2)^(1/2)))/b^6/d^3-a^3*(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(
a+(a^2+b^2)^(1/2)))/b^6/d-a^3*(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^
2+b^2)^(1/2)))/b^6/d-2*a^3*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+
(a^2+b^2)^(1/2)))/b^6/d^2-2*a^3*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c
)/(a-(a^2+b^2)^(1/2)))/b^6/d^2+2*a^4*f^2*sinh(d*x+c)/b^5/d^3-1/4*a^3*f^2*s
inh(d*x+c)^2/b^4/d^3-1/2*a^3*(f*x+e)^2*sinh(d*x+c)^2/b^4/d+2/27*a^2*f^2*si
nh(d*x+c)^3/b^3/d^3-1/200*f*(f*x+e)*cosh(5*d*x+5*c)/b/d^2-1/72*f*(f*x+e)*c
osh(3*d*x+3*c)/b/d^2-3/32*a*f^2*cosh(d*x+c)^2/b^2/d^3-1/32*a*f^2*cosh(d*x+
c)^4/b^2/d^3-1/4*a*(f*x+e)^2*cosh(d*x+c)^4/b^2/d+1/3*a^3*(a^2+b^2)*(f*x+e)
^3/b^6/f+2/3*a^2*(f*x+e)^2*sinh(d*x+c)/b^3/d-2*a^4*f*(f*x+e)*cosh(d*x+c)/b
^5/d^2-2/9*a^2*f*(f*x+e)*cosh(d*x+c)^3/b^3/d^2+1/3*a^2*(f*x+e)^2*cosh(d*x+
c)^2*sinh(d*x+c)/b^3/d-4/3*a^2*f*(f*x+e)*cosh(d*x+c)/b^3/d^2+1/2*a^3*f*(f*
x+e)*cosh(d*x+c)*sinh(d*x+c)/b^4/d^2+1/8*a*f*(f*x+e)*cosh(d*x+c)^3*sinh(d*
x+c)/b^2/d^2+1/1000*f^2*sinh(5*d*x+5*c)/b/d^3+1/80*(f*x+e)^2*sinh(5*d*x+5*
c)/b/d+1/216*f^2*sinh(3*d*x+3*c)/b/d^3+1/48*(f*x+e)^2*sinh(3*d*x+3*c)/b/d-
1/4*a^3*(f*x+e)^2/b^4/d-1/4*f^2*sinh(d*x+c)/b/d^3+3/32*a*(f*x+e)^2/b^2/d+a
^4*(f*x+e)^2*sinh(d*x+c)/b^5/d-1/8*(f*x+e)^2*sinh(d*x+c)/b/d+1/4*f*(f*x+e)
*cosh(d*x+c)/b/d^2+3/16*a*f*(f*x+e)*cosh(d*x+c)*sinh(d*x+c)/b^2/d^2
```

Mathematica [A] (verified)

Time = 9.12 (sec) , antiderivative size = 1811, normalized size of antiderivative = 1.77

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*
x]),x]
```

output
```
((-8*a^3*(a^2 + b^2)*e^2*x*Coth[c])/b^6 - (8*a^3*(a^2 + b^2)*e*f*x^2*Coth[
c])/b^6 - (8*a^3*(a^2 + b^2)*f^2*x^3*Coth[c])/(3*b^6) + (8*a^3*(a^2 + b^2)
*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2
+ b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2
)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))
/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*
ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*
a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 +
b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*
(c + d*x)))])/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d
*x)))])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E
^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a
^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sq
rt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x)
)/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d
*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b
*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1
 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*
f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d
- (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c -...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sinh^3(c+dx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6113}$$

$$\frac{\int (e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{5971}$$

$$\frac{\int \left(-\frac{1}{8}\cosh(c+dx)(e+fx)^2+\frac{1}{16}\cosh(3c+3dx)(e+fx)^2+\frac{1}{16}\cosh(5c+5dx)(e+fx)^2\right)dx}{b}-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-\cdots}{b}$$

$$\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 6113$$

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-\cdots}{b}$$

$$\frac{a\left(\frac{\int(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 5970$$

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-\cdots}{b}$$

$$\frac{a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\int(e+fx)\cosh^4(c+dx)dx}{2d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-\cdots}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^4dx}{2d}}{b}\right)}{b}$$

$$\Big\downarrow \; 3791$$

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-\cdots}{b}$$

$$\frac{a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\int(e+fx)\cosh^2(c+dx)dx-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+d}{4d}\right.}{2d}}{b}\right.}{b}$$

$$\Big\downarrow \; 3791$$

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-}{b}$$

$$\frac{a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(\frac{1}{2}\int(e+fx)dx-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a}{}\right.}{b}$$

$$\Big\downarrow \; 17$$

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-}{b}$$

$$\frac{a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\int}{}\right.}{b}$$

$$\Big\downarrow \; 6113$$

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-}{b}$$

$$\frac{a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(\right.}{}\right.}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-\cdots$$
$$b$$
$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(\cdots\right.}{\cdots}\right.$$
$$b$$

$\downarrow$ 3792

$$-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-\cdots$$
$$b$$
$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(\cdots\right.}{\cdots}\right.$$
$$b$$

$\downarrow$ 3042

$$-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-\cdots$$
$$b$$
$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(\cdots\right.}{\cdots}\right.$$
$$b$$

$\downarrow$ 3113

$$-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-\cdots$$
$$b$$
$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(\cdots\right.}{\cdots}\right.$$

$\downarrow$ 2009

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-}{b}$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\Bigg(}{}\right.$$

$\downarrow$ 3777

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-}{b}$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\Bigg(}{}\right.$$

$\downarrow$ 26

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-}{b}$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\Bigg(}{}\right.$$

$\downarrow$ 3042

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-}{b}$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\Bigg(}{}\right.$$

$\Big\downarrow$ 26

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-}{b}$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\Big(}{}\right.$$

$\Big\downarrow$ 3777

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-}{b}$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\Big(}{}\right.$$

$\Big\downarrow$ 3042

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-}{b}$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\Big(}{}\right.$$

$$\Big\downarrow \text{3117}$$

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-}{b}$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\Big(}{}\right.$$

$$\Big\downarrow \text{6099}$$

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-}{b}$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\Big(}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-}{b}$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\Big(}{}\right.$$

$\Big\downarrow$ 3777

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-}{b}$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\Bigg(}{}\right.$$

$\Big\downarrow$ 26

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-}{b}$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\Bigg(}{}\right.$$

$\Big\downarrow$ 3042

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-}{b}$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(\right.}{}\right.$$

$\downarrow$ 26

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-}{b}$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(\right.}{}\right.$$

$\downarrow$ 3777

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-}{b}$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(\right.}{}\right.$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^2\cosh(dx+c)^3\sinh(dx+c)^3}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11318 vs. 2(949) = 1898.

Time = 0.25 (sec) , antiderivative size = 11318, normalized size of antiderivative = 11.09

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algor
ithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)**3*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\cosh\left(dx+c\right)^3\sinh\left(dx+c\right)^3}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algor
ithm="maxima")
```

output
```
-1/960*e^2*((15*a*b^3*e^(-d*x - c) - 6*b^4 - 10*(4*a^2*b^2 + b^4)*e^(-2*d*
x - 2*c) + 60*(2*a^3*b + a*b^3)*e^(-3*d*x - 3*c) - 60*(8*a^4 + 6*a^2*b^2 -
 b^4)*e^(-4*d*x - 4*c))*e^(5*d*x + 5*c)/(b^5*d) + 960*(a^5 + a^3*b^2)*(d*x
 + c)/(b^6*d) + (15*a*b^3*e^(-4*d*x - 4*c) + 6*b^4*e^(-5*d*x - 5*c) + 60*(
8*a^4 + 6*a^2*b^2 - b^4)*e^(-d*x - c) + 60*(2*a^3*b + a*b^3)*e^(-2*d*x - 2
*c) + 10*(4*a^2*b^2 + b^4)*e^(-3*d*x - 3*c))/(b^5*d) + 960*(a^5 + a^3*b^2)
*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^6*d)) - 1/1728000*(576
000*(a^5*d^3*f^2*e^(5*c) + a^3*b^2*d^3*f^2*e^(5*c))*x^3 + 1728000*(a^5*d^3
*e*f*e^(5*c) + a^3*b^2*d^3*e*f*e^(5*c))*x^2 - 432*(25*b^5*d^2*f^2*x^2*e^(1
0*c) + 10*(5*d^2*e*f - d*f^2)*b^5*x*e^(10*c) - 2*(5*d*e*f - f^2)*b^5*e^(10
*c))*e^(5*d*x) + 3375*(8*a*b^4*d^2*f^2*x^2*e^(9*c) + 4*(4*d^2*e*f - d*f^2)
*a*b^4*x*e^(9*c) - (4*d*e*f - f^2)*a*b^4*e^(9*c))*e^(4*d*x) + 2000*(8*(3*d
*e*f - f^2)*a^2*b^3*e^(8*c) + 2*(3*d*e*f - f^2)*b^5*e^(8*c) - 9*(4*a^2*b^3
*d^2*f^2*e^(8*c) + b^5*d^2*f^2*e^(8*c))*x^2 - 6*(4*(3*d^2*e*f - d*f^2)*a^2
*b^3*e^(8*c) + (3*d^2*e*f - d*f^2)*b^5*e^(8*c))*x)*e^(3*d*x) - 54000*(2*(2
*d*e*f - f^2)*a^3*b^2*e^(7*c) + (2*d*e*f - f^2)*a*b^4*e^(7*c) - 2*(2*a^3*b
^2*d^2*f^2*e^(7*c) + a*b^4*d^2*f^2*e^(7*c))*x^2 - 2*(2*(2*d^2*e*f - d*f^2)
*a^3*b^2*e^(7*c) + (2*d^2*e*f - d*f^2)*a*b^4*e^(7*c))*x)*e^(2*d*x) + 10800
0*(16*(d*e*f - f^2)*a^4*b*e^(6*c) + 12*(d*e*f - f^2)*a^2*b^3*e^(6*c) - 2*(
d*e*f - f^2)*b^5*e^(6*c) - (8*a^4*b*d^2*f^2*e^(6*c) + 6*a^2*b^3*d^2*f^2...
```

Giac [F]

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh\left(dx+c\right)^3\sinh\left(dx+c\right)^3}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algor
ithm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)^3*sinh(d*x + c)^3/(b*sinh(d*x + c) + a
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^3\,\sinh(c+d\,x)^3\,(e+f\,x)^2}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^3*sinh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)), x
)
```

Reduce [F]

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)^3\sinh(dx+c)^3}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

3.403 $\int \frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 641

$$\begin{aligned}
&\int \frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= -\frac{a^3fx}{4b^4d}+\frac{3afx}{32b^2d}+\frac{a^3(a^2+b^2)(e+fx)^2}{2b^6f}-\frac{a^4f\cosh(c+dx)}{b^5d^2}-\frac{2a^2f\cosh(c+dx)}{3b^3d^2} \\
&\quad+\frac{f\cosh(c+dx)}{8bd^2}-\frac{a^2f\cosh^3(c+dx)}{9b^3d^2}-\frac{a(e+fx)\cosh^4(c+dx)}{4b^2d} \\
&\quad-\frac{f\cosh(3c+3dx)}{144bd^2}-\frac{f\cosh(5c+5dx)}{400bd^2}-\frac{a^3(a^2+b^2)(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^6d} \\
&\quad-\frac{a^3(a^2+b^2)(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^6d}-\frac{a^3(a^2+b^2)f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^6d^2} \\
&\quad-\frac{a^3(a^2+b^2)f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^6d^2}+\frac{a^4(e+fx)\sinh(c+dx)}{b^5d} \\
&\quad+\frac{2a^2(e+fx)\sinh(c+dx)}{3b^3d}-\frac{(e+fx)\sinh(c+dx)}{8bd} \\
&\quad+\frac{a^3f\cosh(c+dx)\sinh(c+dx)}{4b^4d^2}+\frac{3af\cosh(c+dx)\sinh(c+dx)}{32b^2d^2} \\
&\quad+\frac{a^2(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{3b^3d}+\frac{af\cosh^3(c+dx)\sinh(c+dx)}{16b^2d^2} \\
&\quad-\frac{a^3(e+fx)\sinh^2(c+dx)}{2b^4d}+\frac{(e+fx)\sinh(3c+3dx)}{48bd}+\frac{(e+fx)\sinh(5c+5dx)}{80bd}
\end{aligned}$$

output
```
-1/4*a^3*f*x/b^4/d+3/32*a*f*x/b^2/d+1/2*a^3*(a^2+b^2)*(f*x+e)^2/b^6/f-a^4*
f*cosh(d*x+c)/b^5/d^2-2/3*a^2*f*cosh(d*x+c)/b^3/d^2+1/8*f*cosh(d*x+c)/b/d^
2-1/9*a^2*f*cosh(d*x+c)^3/b^3/d^2-1/4*a*(f*x+e)*cosh(d*x+c)^4/b^2/d-1/144*
f*cosh(3*d*x+3*c)/b/d^2-1/400*f*cosh(5*d*x+5*c)/b/d^2-a^3*(a^2+b^2)*(f*x+e
)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^6/d-a^3*(a^2+b^2)*(f*x+e)*ln(1+
b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^6/d-a^3*(a^2+b^2)*f*polylog(2,-b*exp(d
*x+c)/(a-(a^2+b^2)^(1/2)))/b^6/d^2-a^3*(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)
/(a+(a^2+b^2)^(1/2)))/b^6/d^2+a^4*(f*x+e)*sinh(d*x+c)/b^5/d+2/3*a^2*(f*x+e
)*sinh(d*x+c)/b^3/d-1/8*(f*x+e)*sinh(d*x+c)/b/d+1/4*a^3*f*cosh(d*x+c)*sinh
(d*x+c)/b^4/d^2+3/32*a*f*cosh(d*x+c)*sinh(d*x+c)/b^2/d^2+1/3*a^2*(f*x+e)*c
osh(d*x+c)^2*sinh(d*x+c)/b^3/d+1/16*a*f*cosh(d*x+c)^3*sinh(d*x+c)/b^2/d^2-
1/2*a^3*(f*x+e)*sinh(d*x+c)^2/b^4/d+1/48*(f*x+e)*sinh(3*d*x+3*c)/b/d+1/80*
(f*x+e)*sinh(5*d*x+5*c)/b/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2704 vs. $2(641) = 1282$.

Time = 8.32 (sec) , antiderivative size = 2704, normalized size of antiderivative = 4.22

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]
),x]
```

output
```
((-4*a^3*(a^2 + b^2)*(-2*d*e*(c + d*x) + 2*c*f*(c + d*x) - f*(c + d*x)^2 +
 (4*a*Sqrt[a^2 + b^2]*d*e*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sq
rt[-(a^2 + b^2)^2] - (4*a*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d
*x))/Sqrt[a^2 + b^2]])/(-a^2 - b^2)^(3/2) + 2*f*(c + d*x)*Log[1 + (b*E^(c
+ d*x))/(a - Sqrt[a^2 + b^2])] + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])] - 2*c*f*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] +
 2*d*e*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] + 2*f*PolyLog[2, (b
*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))]))/(b^6*d^2) + ((Cosh[5*(c + d*x)]/(7200*b^5*d) - Si
nh[5*(c + d*x)]/(7200*b^5*d))*(-360*b^4*d*e - 72*b^4*f + 360*b^4*c*f - 360
*b^4*f*(c + d*x) - 900*a*b^3*d*e*Cosh[c + d*x] - 225*a*b^3*f*Cosh[c + d*x]
 + 900*a*b^3*c*f*Cosh[c + d*x] - 900*a*b^3*f*(c + d*x)*Cosh[c + d*x] - 240
0*a^2*b^2*d*e*Cosh[2*(c + d*x)] - 600*b^4*d*e*Cosh[2*(c + d*x)] - 800*a^2*
b^2*f*Cosh[2*(c + d*x)] - 200*b^4*f*Cosh[2*(c + d*x)] + 2400*a^2*b^2*c*f*C
osh[2*(c + d*x)] + 600*b^4*c*f*Cosh[2*(c + d*x)] - 2400*a^2*b^2*f*(c + d*x
)*Cosh[2*(c + d*x)] - 600*b^4*f*(c + d*x)*Cosh[2*(c + d*x)] - 7200*a^3*b*d
*e*Cosh[3*(c + d*x)] - 3600*a*b^3*d*e*Cosh[3*(c + d*x)] - 3600*a^3*b*f*Cos
h[3*(c + d*x)] - 1800*a*b^3*f*Cosh[3*(c + d*x)] + 7200*a^3*b*c*f*Cosh[3*(c
 + d*x)] + 3600*a*b^3*c*f*Cosh[3*(c + d*x)] - 7200*a^3*b*f*(c + d*x)*Cosh[
3*(c + d*x)] - 3600*a*b^3*f*(c + d*x)*Cosh[3*(c + d*x)] - 28800*a^4*d*e...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh^3(c+dx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6113}$$

$$\frac{\int (e+fx)\cosh^3(c+dx)\sinh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{\int \left(-\frac{1}{8}(e+fx)\cosh(c+dx)+\frac{1}{16}(e+fx)\cosh(3c+3dx)+\frac{1}{16}(e+fx)\cosh(5c+5dx)\right)dx}{b}-\frac{a\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{6113}$$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-\frac{a\left(\frac{\int(e+fx)\cosh^3(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{5970}$$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\int\cosh^4(c+dx)dx}{4d}}{b}-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\int\sin\left(ic+idx+\frac{\pi}{2}\right)^4dx}{4d}}{b}\right)}{b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\int\cosh^2(c+dx)dx+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{4d}}{b}-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\int\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx\right)}{4d}}{b}\right)}{b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{4d}}{b}-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{6113}$$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\int(e+fx)\cosh^3(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3d\cdots}{b}\right.\cdots}{b}\right.\cdots}{b}$$

$\downarrow$ 3791

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\int(e+fx)\cosh(c+dx)dx-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+\cdots}{3\cdots}}{b}\right.\cdots}{b}\right.\cdots}{b}$$

$\downarrow$ 3042

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)\cdots}{b}\right.\cdots}{b}\right.\cdots}{b}$$

$\downarrow$ 3777

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\cdots}{\cdots}\right.\cdots}{\cdots}\right.\cdots}{\cdots}\right.\cdots}{b}$$

$\downarrow$ 26

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}\cdots}{b}\cdots\right.}{\cdots}\cdots\right.}{b}$$

$\big\downarrow$ 3042

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\cdots}{\cdots}\right.\cdots}{\cdots}\right.}{\cdots}\cdots\right.}{b}$$

$\big\downarrow$ 26

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\cdots}{\cdots}\right.\cdots}{\cdots}\right.}{\cdots}\cdots\right.}{b}$$

$\big\downarrow$ 3118

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\cdots}{b}\cdots\right.}{\cdots}\cdots\right.}{b}$$

$\big\downarrow$ 6099

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\cdots}{b}\cdots\right.}{\cdots}\right.}{b}$$

$\big\downarrow$ 3042

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\cdots}{b}\cdots\right.}{\cdots}\right.}{b}$$

$\big\downarrow$ 3777

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\cdots}{b}\cdots\right.}{\cdots}\right.}{\cdots}$$

$\big\downarrow$ 26

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-$$

$$a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\cdots}{b}\right.}{\cdots}\right.$$

↓ 3042

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-$$

$$a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\cdots}{b}\right.}{\cdots}\right.$$

↓ 26

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-$$

$$a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\cdots}{b}\right.}{\cdots}\right.$$

↓ 3118

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\ldots}{b}\ldots\right.}{\ldots}\ldots\right.}{b}$$

$\downarrow$ 5969

$$\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d} \over b$$
$$-\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\ldots}{b}\ldots\right.}{\ldots}\ldots\right.}{\ldots}$$

$\downarrow$ 3042

$$\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d} \over b$$
$$-\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\ldots}{b}\ldots\right.}{\ldots}\ldots\right.}{\ldots}$$

$\downarrow$ 25

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-$$

$$a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+}{b}\right.}{}\right.$$

$$\Big\downarrow \ 3115$$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-$$

$$a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+}{b}\right.}{}\right.$$

input
```
Int[((e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1362 vs. 2(593) = 1186.

Time = 284.30 (sec) , antiderivative size = 1363, normalized size of antiderivative = 2.13

method	result	size
risch	Expression too large to display	1363

input
```
int((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURN
VERBOSE)
```

output
```
2/d*a^5/b^6*f*c*x-1/32*a*(2*a^2+b^2)*(2*d*f*x+2*d*e+f)/b^4/d^2*exp(-2*d*x-
2*c)-1/16*(8*a^4+6*a^2*b^2-b^4)*(d*f*x+d*e+f)/b^5/d^2*exp(-d*x-c)-1/288*(4
*a^2+b^2)*(3*d*f*x+3*d*e+f)/b^3/d^2*exp(-3*d*x-3*c)-1/256*a*(4*d*f*x+4*d*e
+f)/b^2/d^2*exp(-4*d*x-4*c)+1/d^2*a^5/b^6*f*c^2-1/d^2*a^5/b^6*f*dilog((-b*
exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2*a^5/b^6*f*dilog(
(b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/d*a^5/b^6*e*ln(exp
(d*x+c))-1/d*a^5/b^6*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/d^2*a^3/b^4
*f*c^2+2/d*a^3/b^4*e*ln(exp(d*x+c))-1/d^2*a^3/b^4*f*dilog((-b*exp(d*x+c)+(
a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2*a^3/b^4*f*dilog((b*exp(d*x+c
)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d*a^3/b^4*e*ln(b*exp(2*d*x+2*c
)+2*a*exp(d*x+c)-b)+1/2*a^3/b^4*f*x^2-a^3/b^4*e*x+1/2*a^5/b^6*f*x^2-a^5/b^
6*e*x+2/d*a^3/b^4*f*c*x-1/d*a^3/b^4*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)
/(-a+(a^2+b^2)^(1/2)))*x-1/d*a^3/b^4*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)
/(a+(a^2+b^2)^(1/2)))*x-1/d^2*a^3/b^4*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-
a)/(-a+(a^2+b^2)^(1/2)))*c-1/d^2*a^3/b^4*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2
)+a)/(a+(a^2+b^2)^(1/2)))*c+1/d^2*a^3/b^4*c*f*ln(b*exp(2*d*x+2*c)+2*a*exp(
d*x+c)-b)-2/d^2*a^3/b^4*c*f*ln(exp(d*x+c))+1/288*(12*a^2*d*f*x+3*b^2*d*f*x
+12*a^2*d*e+3*b^2*d*e-4*a^2*f-b^2*f)/b^3/d^2*exp(3*d*x+3*c)-1/800*(5*d*f*x
+5*d*e+f)/b/d^2*exp(-5*d*x-5*c)+1/800*(5*d*f*x+5*d*e-f)/b/d^2*exp(5*d*x+5*
c)+1/16*(8*a^4*d*f*x+6*a^2*b^2*d*f*x-b^4*d*f*x+8*a^4*d*e+6*a^2*b^2*d*e-...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5548 vs. 2(591) = 1182.

Time = 0.17 (sec) , antiderivative size = 5548, normalized size of antiderivative = 8.66

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)**3*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)^3\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/960*e*((15*a*b^3*e^(-d*x - c) - 6*b^4 - 10*(4*a^2*b^2 + b^4)*e^(-2*d*x
- 2*c) + 60*(2*a^3*b + a*b^3)*e^(-3*d*x - 3*c) - 60*(8*a^4 + 6*a^2*b^2 - b
^4)*e^(-4*d*x - 4*c))*e^(5*d*x + 5*c)/(b^5*d) + 960*(a^5 + a^3*b^2)*(d*x +
 c)/(b^6*d) + (15*a*b^3*e^(-4*d*x - 4*c) + 6*b^4*e^(-5*d*x - 5*c) + 60*(8*
a^4 + 6*a^2*b^2 - b^4)*e^(-d*x - c) + 60*(2*a^3*b + a*b^3)*e^(-2*d*x - 2*c
) + 10*(4*a^2*b^2 + b^4)*e^(-3*d*x - 3*c))/(b^5*d) + 960*(a^5 + a^3*b^2)*l
og(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^6*d)) - 1/57600*f*((2880
0*(a^5*d^2*e^(5*c) + a^3*b^2*d^2*e^(5*c))*x^2 - 72*(5*b^5*d*x*e^(10*c) - b
^5*e^(10*c))*e^(5*d*x) + 225*(4*a*b^4*d*x*e^(9*c) - a*b^4*e^(9*c))*e^(4*d*
x) + 200*(4*a^2*b^3*e^(8*c) + b^5*e^(8*c) - 3*(4*a^2*b^3*d*e^(8*c) + b^5*d
*e^(8*c))*x)*e^(3*d*x) - 1800*(2*a^3*b^2*e^(7*c) + a*b^4*e^(7*c) - 2*(2*a^
3*b^2*d*e^(7*c) + a*b^4*d*e^(7*c))*x)*e^(2*d*x) + 3600*(8*a^4*b*e^(6*c) +
6*a^2*b^3*e^(6*c) - b^5*e^(6*c) - (8*a^4*b*d*e^(6*c) + 6*a^2*b^3*d*e^(6*c)
 - b^5*d*e^(6*c))*x)*e^(d*x) + 3600*(8*a^4*b*e^(4*c) + 6*a^2*b^3*e^(4*c) -
 b^5*e^(4*c) + (8*a^4*b*d*e^(4*c) + 6*a^2*b^3*d*e^(4*c) - b^5*d*e^(4*c))*x
)*e^(-d*x) + 1800*(2*a^3*b^2*e^(3*c) + a*b^4*e^(3*c) + 2*(2*a^3*b^2*d*e^(3
*c) + a*b^4*d*e^(3*c))*x)*e^(-2*d*x) + 200*(4*a^2*b^3*e^(2*c) + b^5*e^(2*c
) + 3*(4*a^2*b^3*d*e^(2*c) + b^5*d*e^(2*c))*x)*e^(-3*d*x) + 225*(4*a*b^4*d
*x*e^c + a*b^4*e^c)*e^(-4*d*x) + 72*(5*b^5*d*x + b^5)*e^(-5*d*x))*e^(-5*c)
/(b^6*d^2) - 900*integrate(128*((a^6*e^c + a^4*b^2*e^c)*x*e^(d*x) - (a^...
```

Giac [F]

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)\cosh{(dx+c)}^3\sinh{(dx+c)}^3}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)^3*sinh(d*x + c)^3/(b*sinh(d*x + c) + a),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^3\sinh(c+d\,x)^3\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^3*sinh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output

```
(360*e**(10*c + 10*d*x)*b**10*d*e + 360*e**(10*c + 10*d*x)*b**10*d*f*x - 7
2*e**(10*c + 10*d*x)*b**10*f - 900*e**(9*c + 9*d*x)*a*b**9*d*e - 900*e**(9
*c + 9*d*x)*a*b**9*d*f*x + 225*e**(9*c + 9*d*x)*a*b**9*f + 2400*e**(8*c +
8*d*x)*a**2*b**8*d*e + 2400*e**(8*c + 8*d*x)*a**2*b**8*d*f*x - 800*e**(8*c
 + 8*d*x)*a**2*b**8*f + 600*e**(8*c + 8*d*x)*b**10*d*e + 600*e**(8*c + 8*d
*x)*b**10*d*f*x - 200*e**(8*c + 8*d*x)*b**10*f - 7200*e**(7*c + 7*d*x)*a**
3*b**7*d*e - 7200*e**(7*c + 7*d*x)*a**3*b**7*d*f*x + 3600*e**(7*c + 7*d*x)
*a**3*b**7*f - 3600*e**(7*c + 7*d*x)*a*b**9*d*e - 3600*e**(7*c + 7*d*x)*a*
b**9*d*f*x + 1800*e**(7*c + 7*d*x)*a*b**9*f + 28800*e**(6*c + 6*d*x)*a**4*
b**6*d*e + 28800*e**(6*c + 6*d*x)*a**4*b**6*d*f*x - 28800*e**(6*c + 6*d*x)
*a**4*b**6*f + 21600*e**(6*c + 6*d*x)*a**2*b**8*d*e + 21600*e**(6*c + 6*d*
x)*a**2*b**8*d*f*x - 21600*e**(6*c + 6*d*x)*a**2*b**8*f - 3600*e**(6*c + 6
*d*x)*b**10*d*e - 3600*e**(6*c + 6*d*x)*b**10*d*f*x + 3600*e**(6*c + 6*d*x
)*b**10*f + 1843200*e**(5*c + 5*d*x)*int(x/(e**(7*c + 7*d*x)*b + 2*e**(6*c
 + 6*d*x)*a - e**(5*c + 5*d*x)*b),x)*a**10*b*d**2*f + 4147200*e**(5*c + 5*
d*x)*int(x/(e**(7*c + 7*d*x)*b + 2*e**(6*c + 6*d*x)*a - e**(5*c + 5*d*x)*b
),x)*a**8*b**3*d**2*f + 2880000*e**(5*c + 5*d*x)*int(x/(e**(7*c + 7*d*x)*b
 + 2*e**(6*c + 6*d*x)*a - e**(5*c + 5*d*x)*b),x)*a**6*b**5*d**2*f + 576000
*e**(5*c + 5*d*x)*int(x/(e**(7*c + 7*d*x)*b + 2*e**(6*c + 6*d*x)*a - e**(5
*c + 5*d*x)*b),x)*a**4*b**7*d**2*f - 57600*e**(5*c + 5*d*x)*log(e**(2*c...
```

3.404 $\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 141

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a^3(a^2+b^2)\log(a+b\sinh(c+dx))}{b^6d} + \frac{a^2(a^2+b^2)\sinh(c+dx)}{b^5d} - \frac{a(a^2+b^2)\sinh^2(c+dx)}{2b^4d} + \frac{(a^2+b^2)\sinh^3(c+dx)}{3b^3d} - \frac{a\sinh^4(c+dx)}{4b^2d} + \frac{\sinh^5(c+dx)}{5bd}$$

output
```
-a^3*(a^2+b^2)*ln(a+b*sinh(d*x+c))/b^6/d+a^2*(a^2+b^2)*sinh(d*x+c)/b^5/d-1
/2*a*(a^2+b^2)*sinh(d*x+c)^2/b^4/d+1/3*(a^2+b^2)*sinh(d*x+c)^3/b^3/d-1/4*a
*sinh(d*x+c)^4/b^2/d+1/5*sinh(d*x+c)^5/b/d
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.87

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{60a^3(a^2+b^2)\log(a+b\sinh(c+dx))}{b^6}-\frac{60a^2(a^2+b^2)\sinh(c+dx)}{b^5}+\frac{30a(a^2+b^2)\sinh^2(c+dx)}{b^4}-\frac{20(a^2+b^2)\sinh^3(c+dx)}{b^3}+\frac{15a\sinh^4(c\cdots}{b^2}}{60d}$$

input
```
Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/60*((60*a^3*(a^2 + b^2)*Log[a + b*Sinh[c + d*x]])/b^6 - (60*a^2*(a^2 +
b^2)*Sinh[c + d*x])/b^5 + (30*a*(a^2 + b^2)*Sinh[c + d*x]^2)/b^4 - (20*(a^
2 + b^2)*Sinh[c + d*x]^3)/b^3 + (15*a*Sinh[c + d*x]^4)/b^2 - (12*Sinh[c +
d*x]^5)/b)/d
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {3042, 26, 3316, 26, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i\sin(ic+idx)^3\cos(ic+idx)^3}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{\cos(ic+idx)^3\sin(ic+idx)^3}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{ 3316}$$

$$-\frac{i\int \frac{i\sinh^3(c+dx)\left(\sinh^2(c+dx)b^2+b^2\right)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^3d}$$

$\Big\downarrow$ 26

$$\frac{\int \frac{\sinh^3(c+dx)\left(\sinh^2(c+dx)b^2+b^2\right)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^3d}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{b^3\sinh^3(c+dx)\left(\sinh^2(c+dx)b^2+b^2\right)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^6d}$$

$\Big\downarrow$ 522

$$\frac{\int \left(b^4\sinh^4(c+dx)-ab^3\sinh^3(c+dx)+b^2\left(a^2+b^2\right)\sinh^2(c+dx)-ab\left(a^2+b^2\right)\sinh(c+dx)+a^2\left(a^2+b^2\right)-\right.}{b^6d}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{1}{2}ab^2\left(a^2+b^2\right)\sinh^2(c+dx)+a^2b\left(a^2+b^2\right)\sinh(c+dx)+\frac{1}{3}b^3\left(a^2+b^2\right)\sinh^3(c+dx)-a^3\left(a^2+b^2\right)\log(a+b}{b^6d}$$

input
```
Int[(Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-(a^3*(a^2 + b^2)*Log[a + b*Sinh[c + d*x]]) + a^2*b*(a^2 + b^2)*Sinh[c +
d*x] - (a*b^2*(a^2 + b^2)*Sinh[c + d*x]^2)/2 + (b^3*(a^2 + b^2)*Sinh[c + d
*x]^3)/3 - (a*b^4*Sinh[c + d*x]^4)/4 + (b^5*Sinh[c + d*x]^5)/5)/(b^6*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 152.48 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{\frac{\frac{\sinh(dx+c)^5 b^4}{5}-\frac{a\sinh(dx+c)^4 b^3}{4}+\frac{a^2b^2\sinh(dx+c)^3}{3}+\frac{b^4\sinh(dx+c)^3}{3}-\frac{a^3b\sinh(dx+c)^2}{2}-\frac{a\,b^3\sinh(dx+c)^2}{2}+a^4\sinh(dx+c)+a^2}{b^5}}{d}$
default	$\frac{\frac{\frac{\sinh(dx+c)^5 b^4}{5}-\frac{a\sinh(dx+c)^4 b^3}{4}+\frac{a^2b^2\sinh(dx+c)^3}{3}+\frac{b^4\sinh(dx+c)^3}{3}-\frac{a^3b\sinh(dx+c)^2}{2}-\frac{a\,b^3\sinh(dx+c)^2}{2}+a^4\sinh(dx+c)+a^2}{b^5}}{d}$
risch	$-\frac{e^{dx+c}}{16bd}+\frac{e^{-dx-c}}{16bd}+\frac{e^{5dx+5c}}{160bd}+\frac{a^3x}{b^4}-\frac{e^{-3dx-3c}}{96bd}-\frac{a\,e^{-4dx-4c}}{64b^2d}-\frac{a\,e^{4dx+4c}}{64b^2d}+\frac{3\,e^{dx+c}a^2}{8b^3d}-\frac{3\,e^{-dx-c}a^2}{8b^3d}$

input
```
int(cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^5*(1/5*sinh(d*x+c)^5*b^4-1/4*a*sinh(d*x+c)^4*b^3+1/3*a^2*b^2*sinh
(d*x+c)^3+1/3*b^4*sinh(d*x+c)^3-1/2*a^3*b*sinh(d*x+c)^2-1/2*a*b^3*sinh(d*x
+c)^2+a^4*sinh(d*x+c)+a^2*b^2*sinh(d*x+c))-a^3*(a^2+b^2)/b^6*ln(a+b*sinh(d
*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1660 vs. $2(133) = 266$.

Time = 0.11 (sec) , antiderivative size = 1660, normalized size of antiderivative = 11.77

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fric
as")
```

output
```
1/960*(6*b^5*cosh(d*x + c)^10 + 6*b^5*sinh(d*x + c)^10 - 15*a*b^4*cosh(d*x
 + c)^9 + 15*(4*b^5*cosh(d*x + c) - a*b^4)*sinh(d*x + c)^9 + 10*(4*a^2*b^3
 + b^5)*cosh(d*x + c)^8 + 5*(54*b^5*cosh(d*x + c)^2 - 27*a*b^4*cosh(d*x +
c) + 8*a^2*b^3 + 2*b^5)*sinh(d*x + c)^8 + 960*(a^5 + a^3*b^2)*d*x*cosh(d*x
 + c)^5 - 60*(2*a^3*b^2 + a*b^4)*cosh(d*x + c)^7 + 20*(36*b^5*cosh(d*x + c
)^3 - 27*a*b^4*cosh(d*x + c)^2 - 6*a^3*b^2 - 3*a*b^4 + 4*(4*a^2*b^3 + b^5)
*cosh(d*x + c))*sinh(d*x + c)^7 + 60*(8*a^4*b + 6*a^2*b^3 - b^5)*cosh(d*x
+ c)^6 + 20*(63*b^5*cosh(d*x + c)^4 - 63*a*b^4*cosh(d*x + c)^3 + 24*a^4*b
+ 18*a^2*b^3 - 3*b^5 + 14*(4*a^2*b^3 + b^5)*cosh(d*x + c)^2 - 21*(2*a^3*b^
2 + a*b^4)*cosh(d*x + c))*sinh(d*x + c)^6 - 15*a*b^4*cosh(d*x + c) + 2*(75
6*b^5*cosh(d*x + c)^5 - 945*a*b^4*cosh(d*x + c)^4 + 280*(4*a^2*b^3 + b^5)*
cosh(d*x + c)^3 + 480*(a^5 + a^3*b^2)*d*x - 630*(2*a^3*b^2 + a*b^4)*cosh(d
*x + c)^2 + 180*(8*a^4*b + 6*a^2*b^3 - b^5)*cosh(d*x + c))*sinh(d*x + c)^5
 - 6*b^5 - 60*(8*a^4*b + 6*a^2*b^3 - b^5)*cosh(d*x + c)^4 + 10*(126*b^5*co
sh(d*x + c)^6 - 189*a*b^4*cosh(d*x + c)^5 - 48*a^4*b - 36*a^2*b^3 + 6*b^5
+ 70*(4*a^2*b^3 + b^5)*cosh(d*x + c)^4 + 480*(a^5 + a^3*b^2)*d*x*cosh(d*x
+ c) - 210*(2*a^3*b^2 + a*b^4)*cosh(d*x + c)^3 + 90*(8*a^4*b + 6*a^2*b^3 -
 b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 60*(2*a^3*b^2 + a*b^4)*cosh(d*x +
 c)^3 + 20*(36*b^5*cosh(d*x + c)^7 - 63*a*b^4*cosh(d*x + c)^6 + 28*(4*a^2*
b^3 + b^5)*cosh(d*x + c)^5 - 6*a^3*b^2 - 3*a*b^4 + 480*(a^5 + a^3*b^2)*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**3*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(133) = 266.

Time = 0.05 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.13

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\left(15\,ab^3e^{(-dx-c)}-6\,b^4-10\,(4\,a^2b^2+b^4)e^{(-2\,dx-2\,c)}+60\,(2\,a^3b+ab^3)e^{(-3\,dx-3\,c)}-60\,(8\,a^4+6\,a^2b^2-b\right.}{960\,b^5d} - \frac{(a^5+a^3b^2)(dx+c)}{b^6d} - \frac{15\,ab^3e^{(-4\,dx-4\,c)}+6\,b^4e^{(-5\,dx-5\,c)}+60\,(8\,a^4+6\,a^2b^2-b^4)e^{(-dx-c)}+60\,(2\,a^3b+ab^3)e^{(-2\,dx-2\,c)}+10\,(4}{960\,b^5d} - \frac{(a^5+a^3b^2)\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{b^6d}$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxi
ma")
```

output
```
-1/960*(15*a*b^3*e^(-d*x - c) - 6*b^4 - 10*(4*a^2*b^2 + b^4)*e^(-2*d*x - 2
*c) + 60*(2*a^3*b + a*b^3)*e^(-3*d*x - 3*c) - 60*(8*a^4 + 6*a^2*b^2 - b^4)
*e^(-4*d*x - 4*c))*e^(5*d*x + 5*c)/(b^5*d) - (a^5 + a^3*b^2)*(d*x + c)/(b^
6*d) - 1/960*(15*a*b^3*e^(-4*d*x - 4*c) + 6*b^4*e^(-5*d*x - 5*c) + 60*(8*a
^4 + 6*a^2*b^2 - b^4)*e^(-d*x - c) + 60*(2*a^3*b + a*b^3)*e^(-2*d*x - 2*c)
 + 10*(4*a^2*b^2 + b^4)*e^(-3*d*x - 3*c))/(b^5*d) - (a^5 + a^3*b^2)*log(-2
*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^6*d)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.83

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{\frac{6\,b^4\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5-15\,ab^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^4+40\,a^2b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+40\,b^4\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3-120\,a^3b\left(e^{(dx+c)}-e^{(-dx-c}\right.}{b^5}}{9}$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac
")
```

output
```
1/960*((6*b^4*(e^(d*x + c) - e^(-d*x - c))^5 - 15*a*b^3*(e^(d*x + c) - e^(
-d*x - c))^4 + 40*a^2*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 40*b^4*(e^(d*x
+ c) - e^(-d*x - c))^3 - 120*a^3*b*(e^(d*x + c) - e^(-d*x - c))^2 - 120*a*
b^3*(e^(d*x + c) - e^(-d*x - c))^2 + 480*a^4*(e^(d*x + c) - e^(-d*x - c))
+ 480*a^2*b^2*(e^(d*x + c) - e^(-d*x - c)))/b^5 - 960*(a^5 + a^3*b^2)*log(
abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/b^6)/d
```

Mupad [B] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.18

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{e^{5\,c+5\,d\,x}}{160\,b\,d} - \frac{e^{-5\,c-5\,d\,x}}{160\,b\,d} - \frac{a\,e^{-4\,c-4\,d\,x}}{64\,b^2\,d} - \frac{a\,e^{4\,c+4\,d\,x}}{64\,b^2\,d} - \frac{\ln\left(2\,a\,e^{d\,x}\,e^{c}-b+b\,e^{2\,c}\,e^{2\,d\,x}\right)\,\left(a^5+a^3\,b^2\right)}{b^6\,d} - \frac{e^{-c-d\,x}\,\left(8\,a^4+6\,a^2\,b^2-b^4\right)}{16\,b^5\,d} + \frac{a^3\,x\,\left(a^2+b^2\right)}{b^6} - \frac{e^{-2\,c-2\,d\,x}\,\left(2\,a^3+a\,b^2\right)}{16\,b^4\,d} - \frac{e^{2\,c+2\,d\,x}\,\left(2\,a^3+a\,b^2\right)}{16\,b^4\,d} + \frac{e^{c+d\,x}\,\left(8\,a^4+6\,a^2\,b^2-b^4\right)}{16\,b^5\,d} - \frac{e^{-3\,c-3\,d\,x}\,\left(4\,a^2+b^2\right)}{96\,b^3\,d} + \frac{e^{3\,c+3\,d\,x}\,\left(4\,a^2+b^2\right)}{96\,b^3\,d}$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x)^3)/(a + b*sinh(c + d*x)),x)
```

output

```
exp(5*c + 5*d*x)/(160*b*d) - exp(- 5*c - 5*d*x)/(160*b*d) - (a*exp(- 4*c -
 4*d*x))/(64*b^2*d) - (a*exp(4*c + 4*d*x))/(64*b^2*d) - (log(2*a*exp(d*x)*
exp(c) - b + b*exp(2*c)*exp(2*d*x))*(a^5 + a^3*b^2))/(b^6*d) - (exp(- c -
d*x)*(8*a^4 - b^4 + 6*a^2*b^2))/(16*b^5*d) + (a^3*x*(a^2 + b^2))/b^6 - (ex
p(- 2*c - 2*d*x)*(a*b^2 + 2*a^3))/(16*b^4*d) - (exp(2*c + 2*d*x)*(a*b^2 +
2*a^3))/(16*b^4*d) + (exp(c + d*x)*(8*a^4 - b^4 + 6*a^2*b^2))/(16*b^5*d) -
 (exp(- 3*c - 3*d*x)*(4*a^2 + b^2))/(96*b^3*d) + (exp(3*c + 3*d*x)*(4*a^2
+ b^2))/(96*b^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 426, normalized size of antiderivative = 3.02

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{6e^{10dx+10c}b^5 - 15e^{9dx+9c}a\,b^4 + 40e^{8dx+8c}a^2b^3 + 10e^{8dx+8c}b^5 - 120e^{7dx+7c}a^3b^2 - 60e^{7dx+7c}a\,b^4 + 480e^{6dx+6c}\cdots}{}$$

input

```
int(cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output

```
(6*e**(10*c + 10*d*x)*b**5 - 15*e**(9*c + 9*d*x)*a*b**4 + 40*e**(8*c + 8*d
*x)*a**2*b**3 + 10*e**(8*c + 8*d*x)*b**5 - 120*e**(7*c + 7*d*x)*a**3*b**2
- 60*e**(7*c + 7*d*x)*a*b**4 + 480*e**(6*c + 6*d*x)*a**4*b + 360*e**(6*c +
 6*d*x)*a**2*b**3 - 60*e**(6*c + 6*d*x)*b**5 - 960*e**(5*c + 5*d*x)*log(e*
*(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*a**5 - 960*e**(5*c + 5*d*x)*log(e
**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*a**3*b**2 + 960*e**(5*c + 5*d*x)
*a**5*d*x + 960*e**(5*c + 5*d*x)*a**3*b**2*d*x - 480*e**(4*c + 4*d*x)*a**4
*b - 360*e**(4*c + 4*d*x)*a**2*b**3 + 60*e**(4*c + 4*d*x)*b**5 - 120*e**(3
*c + 3*d*x)*a**3*b**2 - 60*e**(3*c + 3*d*x)*a*b**4 - 40*e**(2*c + 2*d*x)*a
**2*b**3 - 10*e**(2*c + 2*d*x)*b**5 - 15*e**(c + d*x)*a*b**4 - 6*b**5)/(96
0*e**(5*c + 5*d*x)*b**6*d)
```

3.405 $\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 36

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(cosh(d*x+c)^3*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 30.59 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x
])),x]
```

output
```
Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

$$\Big\downarrow \text{ 6125}$$

$$\int \frac{\sinh^3(c+dx)\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Cosh[c + d*x]^3*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(dx+c)^3 \sinh(dx+c)^3}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(cosh(d*x+c)^3*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)^3*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\cosh\left(dx+c\right)^3\sinh\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
integral(cosh(d*x + c)^3*sinh(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh
(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\text{Timed out}$$

input
```
integrate(cosh(d*x+c)**3*sinh(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 551, normalized size of antiderivative = 15.31

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\cosh\left(dx+c\right)^3\sinh\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/32*e^(-5*c + 5*d*e/f)*exp_integral_e(1, 5*(f*x + e)*d/f)/(b*f) - 1/16*a
*e^(-4*c + 4*d*e/f)*exp_integral_e(1, 4*(f*x + e)*d/f)/(b^2*f) + 1/16*a*e^
(4*c - 4*d*e/f)*exp_integral_e(1, -4*(f*x + e)*d/f)/(b^2*f) - 1/32*e^(5*c
- 5*d*e/f)*exp_integral_e(1, -5*(f*x + e)*d/f)/(b*f) - 1/32*(4*a^2 + b^2)*
e^(-3*c + 3*d*e/f)*exp_integral_e(1, 3*(f*x + e)*d/f)/(b^3*f) - 1/32*(4*a^
2*e^(3*c) + b^2*e^(3*c))*e^(-3*d*e/f)*exp_integral_e(1, -3*(f*x + e)*d/f)/
(b^3*f) - 1/8*(2*a^3 + a*b^2)*e^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x
+ e)*d/f)/(b^4*f) + 1/8*(2*a^3*e^(2*c) + a*b^2*e^(2*c))*e^(-2*d*e/f)*exp_i
ntegral_e(1, -2*(f*x + e)*d/f)/(b^4*f) - 1/16*(8*a^4 + 6*a^2*b^2 - b^4)*e^
(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^5*f) - 1/16*(8*a^4*e^c +
6*a^2*b^2*e^c - b^4*e^c)*e^(-d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b^5
*f) - (a^5 + a^3*b^2)*log(f*x + e)/(b^6*f) + 1/64*integrate(128*(a^5*b + a
^3*b^3 - (a^6*e^c + a^4*b^2*e^c)*e^(d*x))/(b^7*f*x + b^7*e - (b^7*f*x*e^(2
*c) + b^7*e*e^(2*c))*e^(2*d*x) - 2*(a*b^6*f*x*e^c + a*b^6*e*e^c)*e^(d*x)),
 x)
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)^3\sinh\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate(cosh(d*x + c)^3*sinh(d*x + c)^3/((f*x + e)*(b*sinh(d*x + c) + a)
), x)
```

Mupad [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)^3\sinh(c+d\,x)^3}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x)^3)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((cosh(c + d*x)^3*sinh(c + d*x)^3)/((e + f*x)*(a + b*sinh(c + d*x))), x
)
```

Reduce [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 336, normalized size of antiderivative = 9.33

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$= \frac{e^{8c}\left(\int \frac{e^{7dx}}{e^{2dx+2c}be+e^{2dx+2c}bfx+2e^{dx+c}ae+2e^{dx+c}afx-be-bfx}dx\right) - 3e^{4c}\left(\int \frac{e^{3dx}}{e^{2dx+2c}be+e^{2dx+2c}bfx+2e^{dx+c}ae+2e^{dx+c}afx-be-bf}\right.}{}$$

input
```
int(cosh(d*x+c)^3*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
(e**(8*c)*int(e**(7*d*x)/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x +
2*e**(c + d*x)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x) - 3*e**(4*c)*i
nt(e**(3*d*x)/(e**(2*c + 2*d*x)*b*e + e**(2*c + 2*d*x)*b*f*x + 2*e**(c + d
*x)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x) - e**c*int(1/(e**(7*c + 7
*d*x)*b*e + e**(7*c + 7*d*x)*b*f*x + 2*e**(6*c + 6*d*x)*a*e + 2*e**(6*c +
6*d*x)*a*f*x - e**(5*c + 5*d*x)*b*e - e**(5*c + 5*d*x)*b*f*x),x) + 3*int(1
/(e**(2*c + 3*d*x)*b*e + e**(2*c + 3*d*x)*b*f*x + 2*e**(c + 2*d*x)*a*e + 2
*e**(c + 2*d*x)*a*f*x - e**(d*x)*b*e - e**(d*x)*b*f*x),x))/(32*e**c)
```

3.406 $\int \frac{(e+fx)^3\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 1519

$$\int \frac{(e+fx)^3\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output
```
-6*a^3*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d^4-
6*a^3*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d^4-a
^3*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d-a^3*(f
*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d-3*a^3*f*(f*
x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d^2+6*a^
3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d
^3+6*a^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/(a^2
+b^2)/d^3-3*a^3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b
^2/(a^2+b^2)/d^2+6*I*a^4*f^3*polylog(4,-I*exp(d*x+c))/b^3/(a^2+b^2)/d^4+6*
I*a^2*f^2*(f*x+e)*polylog(3,-I*exp(d*x+c))/b^3/d^3+6*I*a^4*f^2*(f*x+e)*pol
ylog(3,I*exp(d*x+c))/b^3/(a^2+b^2)/d^3+3*I*a^4*f*(f*x+e)^2*polylog(2,-I*ex
p(d*x+c))/b^3/(a^2+b^2)/d^2+6*I*a^2*f^3*polylog(4,I*exp(d*x+c))/b^3/d^4-3/
4*a*f^3*polylog(4,-exp(2*d*x+2*c))/b^2/d^4+2*a^2*(f*x+e)^3*arctan(exp(d*x+
c))/b^3/d-6*I*f^3*polylog(4,I*exp(d*x+c))/b/d^4+6*I*f^3*polylog(4,-I*exp(d
*x+c))/b/d^4+3/2*a*f^2*(f*x+e)*polylog(3,-exp(2*d*x+2*c))/b^2/d^3-3/2*a*f*
(f*x+e)^2*polylog(2,-exp(2*d*x+2*c))/b^2/d^2-2*a^4*(f*x+e)^3*arctan(exp(d*
x+c))/b^3/(a^2+b^2)/d+3/4*a^3*f^3*polylog(4,-exp(2*d*x+2*c))/b^2/(a^2+b^2)
/d^4-6*I*a^2*f^3*polylog(4,-I*exp(d*x+c))/b^3/d^4-6*I*f^2*(f*x+e)*polylog(
3,-I*exp(d*x+c))/b/d^3-3*I*f*(f*x+e)^2*polylog(2,I*exp(d*x+c))/b/d^2+6*I*f
^2*(f*x+e)*polylog(3,I*exp(d*x+c))/b/d^3+3*I*f*(f*x+e)^2*polylog(2,-I*e...
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4239 vs. 2(1519) = 3038.

Time = 11.06 (sec) , antiderivative size = 4239, normalized size of antiderivative = 2.79

$$\int \frac{(e+fx)^3 \sinh^2(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
-1/4*(-8*a*d^4*e^3*E^(2*c)*x - 12*a*d^4*e^2*E^(2*c)*f*x^2 - 8*a*d^4*e*E^(2
*c)*f^2*x^3 - 2*a*d^4*E^(2*c)*f^3*x^4 + 8*b*d^3*e^3*ArcTan[E^(c + d*x)] +
8*b*d^3*e^3*E^(2*c)*ArcTan[E^(c + d*x)] + (12*I)*b*d^3*e^2*f*x*Log[1 - I*E
^(c + d*x)] + (12*I)*b*d^3*e^2*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] + (12*I)
*b*d^3*e*f^2*x^2*Log[1 - I*E^(c + d*x)] + (12*I)*b*d^3*e*E^(2*c)*f^2*x^2*L
og[1 - I*E^(c + d*x)] + (4*I)*b*d^3*f^3*x^3*Log[1 - I*E^(c + d*x)] + (4*I)
*b*d^3*E^(2*c)*f^3*x^3*Log[1 - I*E^(c + d*x)] - (12*I)*b*d^3*e^2*f*x*Log[1
 + I*E^(c + d*x)] - (12*I)*b*d^3*e^2*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] -
(12*I)*b*d^3*e*f^2*x^2*Log[1 + I*E^(c + d*x)] - (12*I)*b*d^3*e*E^(2*c)*f^2
*x^2*Log[1 + I*E^(c + d*x)] - (4*I)*b*d^3*f^3*x^3*Log[1 + I*E^(c + d*x)] -
 (4*I)*b*d^3*E^(2*c)*f^3*x^3*Log[1 + I*E^(c + d*x)] + 4*a*d^3*e^3*Log[1 +
E^(2*(c + d*x))] + 4*a*d^3*e^3*E^(2*c)*Log[1 + E^(2*(c + d*x))] + 12*a*d^3
*e^2*f*x*Log[1 + E^(2*(c + d*x))] + 12*a*d^3*e^2*E^(2*c)*f*x*Log[1 + E^(2*
(c + d*x))] + 12*a*d^3*e*f^2*x^2*Log[1 + E^(2*(c + d*x))] + 12*a*d^3*e*E^(
2*c)*f^2*x^2*Log[1 + E^(2*(c + d*x))] + 4*a*d^3*f^3*x^3*Log[1 + E^(2*(c +
d*x))] + 4*a*d^3*E^(2*c)*f^3*x^3*Log[1 + E^(2*(c + d*x))] - (12*I)*b*d^2*(
1 + E^(2*c))*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)] + (12*I)*b*d^2*(1
+ E^(2*c))*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)] + 6*a*d^2*e^2*f*PolyLog
[2, -E^(2*(c + d*x))] + 6*a*d^2*e^2*E^(2*c)*f*PolyLog[2, -E^(2*(c + d*x))]
 + 12*a*d^2*e*f^2*x*PolyLog[2, -E^(2*(c + d*x))] + 12*a*d^2*e*E^(2*c)*f...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6115}$$

$$\frac{\int (e+fx)^3\sinh(c+dx)\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{5972}$$

$$\frac{\int (e+fx)^3\cosh(c+dx)dx-\int (e+fx)^3\text{sech}(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^3\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{b}$$

$\Big\downarrow$ 3777

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{3if\int -i(e+fx)^2\sinh(c+dx)dx}{d}+\frac{(e+fx)^3\sinh(c+dx)}{d}}{b}$$

$\Big\downarrow$ 26

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{3f\int (e+fx)^2\sinh(c+dx)dx}{d}+\frac{(e+fx)^3\sinh(c+dx)}{d}}{b}$$

$\Big\downarrow$ 3042

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{3f\int -i(e+fx)^2\sin(ic+idx)dx}{d}-\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{(e+fx)^3\sinh(c+dx)}{d}}{b}$$

$\Big\downarrow$ 26

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{3if\int (e+fx)^2\sin(ic+idx)dx}{d}-\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{(e+fx)^3\sinh(c+dx)}{d}}{b}$$

$\Big\downarrow$ 3777

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\cosh(c+dx)dx}{d}\right)}{d}+\frac{(e+fx)^3\sinh(c+dx)}{d}}{b}$$

$\Big\downarrow$ 3042

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}+\frac{(e+fx)^3\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \; 3777$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{d}\right)}{d}+\frac{(e+fx)^3\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)}{d}\right)}{d}+\frac{(e+fx)^3\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}\right)}{d}+\frac{(e+fx)^3\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}\right)}{d}\right)}{d}+\frac{(e+fx)^3\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \; 3118$$

$$\frac{-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+}{-\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}+\frac{(e+fx)^3\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \text{4668}$$

$$\frac{-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+}{\frac{3if\int (e+fx)^2\log(1-ie^{c+dx})dx}{d}-\frac{3if\int (e+fx)^2\log(1+ie^{c+dx})dx}{d}-\frac{2(e+fx)^3\arctan(e^{c+dx})}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh}{d}\right.}{}\right.}{d}}{b}$$

$$\Big\downarrow \text{3011}$$

$$\frac{-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+}{-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}(2,-ie^{c+dx})dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,-ie^{c+dx})}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}(2,ie^{c+dx})dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,ie^{c+dx})}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{6115}$$

$$\frac{-\frac{a\left(\frac{\int (e+fx)^3\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+}{-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}(2,-ie^{c+dx})dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,-ie^{c+dx})}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}(2,ie^{c+dx})dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,ie^{c+dx})}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}(2,-ie^{c+dx})dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,-ie^{c+dx})}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}(2,ie^{c+dx})dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,ie^{c+dx})}{d}\right)}{d}}{b}$$

$$\frac{a\left(-\frac{a\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -i(e+fx)^3\tan(ic+idx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\int(e+fx)^3\tan(ic+idx)dx}{b}\right)}{b}$$

$\downarrow$ 4201

$$\frac{-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\int\frac{e^{2(c+dx)}(e+fx)^3}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^4}{4f}\right)}{b}\right)}{b}$$

$\downarrow$ 2620

$$\frac{-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\int(e+fx)^2\log\left(1+e^{2(c+dx)}\right)dx}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{b}\right)}{b}$$

$\downarrow$ 3011

$$\frac{-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)-\right.}{b}\right.}{b}$$

$\downarrow$ 6101

$$-\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}\cdots$$

$$\frac{a\left(-\frac{a\left(\frac{\int (e+fx)^3\text{sech}(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\int (e+fx)\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{d}-\frac{(e+\cdots}{\cdots}\right.}{2d}\cdots\right.\right.}{b}\cdots\right.}{b}$$

$\downarrow$ 3042

$$-\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}\cdots$$

$$\frac{a\left(-\frac{a\left(-\frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{b}\right)}{b}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\int (e+fx)\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{d}\cdots\right.}{2d}\cdots\right.\right.}{b}\cdots\right.}{b}$$

$\downarrow$ 4668

$$-\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}\cdots$$

$$\frac{a\left(-\frac{a\left(-\frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{3if\int (e+fx)^2\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{3if\int (e+fx)^2\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)^3\arctan\left(e^{c+dx}\right)}{d}}{b}\right)}{b}-\frac{i\left(2i\left(\frac{(e+\cdots}{\cdots}\right.\right.}{\cdots}\cdots\right.}{b}$$

$\downarrow$ 3011

$$-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}$$

$$b$$

$$a\left(-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right.}{d}\right.}{b}\right.}{b}\right.$$

$$\downarrow 6107$$

$$-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}$$

$$b$$

$$a\left(-\frac{a\left(-\frac{a\left(\frac{b^2\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,\right.}{d}\right.}{d}\right.}{b}\right.$$

$$\downarrow 6095$$

$$-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}$$

$$b$$

$$a\left(-\frac{a\left(-\frac{a\left(\frac{b^2\left(\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^4}{4bf}\right)}{a^2+b^2}+\frac{\int(e+fx)^3\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyL}\ldots}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.$$

$\downarrow$ 2620

$$-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}$$

$$b$$

$$a\left(-\frac{a\left(-\frac{a\left(\frac{b^2\left(-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\ldots\right.}{bd}\ldots\right.}{a^2+b^2}\ldots\right.}{b}\ldots\right.}{\ldots}\right.$$

$\downarrow$ 3011

$$-\frac{2\arctan\left(e^{c+dx}\right)(e+fx)^3}{d}+\frac{\sinh(c+dx)(e+fx)^3}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\,\mathrm{P}\ldots}{\ldots}\right.}{\ldots}$$

$$b$$

$$a\left(-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2(c+dx)}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{b}-\frac{a\left(\frac{2\arctan\left(e\ldots\right.}{\ldots}\right.}{\ldots}\right.$$

input

```
Int[((e + f*x)^3*Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^3 \sinh (dx+c)^2 \tanh (dx+c)}{a+b\sinh (dx+c)}dx$$

input
```
int((f*x+e)^3*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4522 vs. $2(1378) = 2756$.

Time = 0.29 (sec) , antiderivative size = 4522, normalized size of antiderivative = 2.98

$$\int \frac{(e+fx)^3 \sinh^2(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)}\, dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^3 \sinh^2(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)}\, dx = \int \frac{(e+fx)^3 \sinh^2{(c+dx)} \tanh{(c+dx)}}{a+b\sinh{(c+dx)}}\, dx$$

input
```
integrate((f*x+e)**3*sinh(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*sinh(c + d*x)**2*tanh(c + d*x)/(a + b*sinh(c + d*x))
, x)
```

Maxima [F]

$$\int \frac{(e+fx)^3\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\int \frac{(fx+e)^3\sinh\left(dx+c\right)^2\tanh\left(dx+c\right)}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/2*(2*a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b^2 + b^
4)*d) - 4*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + 2*a*log(e^(-2*d*x - 2*c
) + 1)/((a^2 + b^2)*d) + 2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d
*x - c)/(b*d))*e^3 - 1/4*(a*d^4*f^3*x^4*e^c + 4*a*d^4*e*f^2*x^3*e^c + 6*a*
d^4*e^2*f*x^2*e^c - 2*(b*d^3*f^3*x^3*e^(2*c) + 3*(d^3*e*f^2 - d^2*f^3)*b*x
^2*e^(2*c) + 3*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*b*x*e^(2*c) - 3*(d^2*e^
2*f - 2*d*e*f^2 + 2*f^3)*b*e^(2*c))*e^(d*x) + 2*(b*d^3*f^3*x^3 + 3*(d^3*e*
f^2 + d^2*f^3)*b*x^2 + 3*(d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)*b*x + 3*(d^2*
e^2*f + 2*d*e*f^2 + 2*f^3)*b)*e^(-d*x))*e^(-c)/(b^2*d^4) + integrate(2*(a^
3*b*f^3*x^3 + 3*a^3*b*e*f^2*x^2 + 3*a^3*b*e^2*f*x - (a^4*f^3*x^3*e^c + 3*a
^4*e*f^2*x^2*e^c + 3*a^4*e^2*f*x*e^c)*e^(d*x))/(a^2*b^3 + b^5 - (a^2*b^3*e
^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^3*b^2*e^c + a*b^4*e^c)*e^(d*x)), x)
 - integrate(-2*(a*f^3*x^3 + 3*a*e*f^2*x^2 + 3*a*e^2*f*x - (b*f^3*x^3*e^c
+ 3*b*e*f^2*x^2*e^c + 3*b*e^2*f*x*e^c)*e^(d*x))/(a^2 + b^2 + (a^2*e^(2*c)
+ b^2*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)^2\tanh(c+d\,x)\,(e+f\,x)^3}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((sinh(c + d*x)^2*tanh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)^2*tanh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^3*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 8*e**(c + d*x)*atan(e**(c + d*x))*b**4*d**3*e**3 + 2*e**(2*c + 2*d*x)*
a**2*b**2*d**3*e**3 + 6*e**(2*c + 2*d*x)*a**2*b**2*d**3*e**2*f*x + 6*e**(2
*c + 2*d*x)*a**2*b**2*d**3*e*f**2*x**2 + 2*e**(2*c + 2*d*x)*a**2*b**2*d**3
*f**3*x**3 - 6*e**(2*c + 2*d*x)*a**2*b**2*d**2*e**2*f - 12*e**(2*c + 2*d*x
)*a**2*b**2*d**2*e*f**2*x - 6*e**(2*c + 2*d*x)*a**2*b**2*d**2*f**3*x**2 +
12*e**(2*c + 2*d*x)*a**2*b**2*d*e*f**2 + 12*e**(2*c + 2*d*x)*a**2*b**2*d*f
**3*x - 12*e**(2*c + 2*d*x)*a**2*b**2*f**3 + 2*e**(2*c + 2*d*x)*b**4*d**3*
e**3 + 6*e**(2*c + 2*d*x)*b**4*d**3*e**2*f*x + 6*e**(2*c + 2*d*x)*b**4*d**
3*e*f**2*x**2 + 2*e**(2*c + 2*d*x)*b**4*d**3*f**3*x**3 - 6*e**(2*c + 2*d*x
)*b**4*d**2*e**2*f - 12*e**(2*c + 2*d*x)*b**4*d**2*e*f**2*x - 6*e**(2*c +
2*d*x)*b**4*d**2*f**3*x**2 + 12*e**(2*c + 2*d*x)*b**4*d*e*f**2 + 12*e**(2*
c + 2*d*x)*b**4*d*f**3*x - 12*e**(2*c + 2*d*x)*b**4*f**3 - 16*e**(3*c + d*
x)*int((e**(2*d*x)*x**3)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e*
*(c + d*x)*a - b),x)*a**5*d**4*f**3 - 8*e**(3*c + d*x)*int((e**(2*d*x)*x**
3)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a
**3*b**2*d**4*f**3 + 8*e**(3*c + d*x)*int((e**(2*d*x)*x**3)/(e**(4*c + 4*d
*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a*b**4*d**4*f**3 -
 48*e**(3*c + d*x)*int((e**(2*d*x)*x**2)/(e**(4*c + 4*d*x)*b + 2*e**(3*c +
 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a**5*d**4*e*f**2 - 24*e**(3*c + d*x)*
int((e**(2*d*x)*x**2)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e*...
```

3.407 $\int \frac{(e+fx)^2 \sinh^2(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} \, dx$

Optimal result

Integrand size = 34, antiderivative size = 1067

$$\int \frac{(e+fx)^2 \sinh^2(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} \, dx = \text{Too large to display}$$

output
```
2*a^3*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d^3+2
*a^3*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d^3-a^
3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d-a^3*(f*
x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d+2*I*f*(f*x+e
)*polylog(2,-I*exp(d*x+c))/b/d^2-2*a^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(
a+(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d^2-2*a^3*f*(f*x+e)*polylog(2,-b*exp(d*x
+c)/(a-(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d^2-2*I*a^4*f^2*polylog(3,-I*exp(d*
x+c))/b^3/(a^2+b^2)/d^3-2*I*a^2*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/b^3/d^2
+2*I*f^2*polylog(3,I*exp(d*x+c))/b/d^3+2*I*a^4*f*(f*x+e)*polylog(2,-I*exp(
d*x+c))/b^3/(a^2+b^2)/d^2+a^3*f*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/b^2/(a^
2+b^2)/d^2+2*I*a^2*f^2*polylog(3,-I*exp(d*x+c))/b^3/d^3+2*I*a^4*f^2*polylo
g(3,I*exp(d*x+c))/b^3/(a^2+b^2)/d^3+2*I*a^2*f*(f*x+e)*polylog(2,I*exp(d*x+
c))/b^3/d^2+1/2*a*f^2*polylog(3,-exp(2*d*x+2*c))/b^2/d^3+2*a^2*(f*x+e)^2*a
rctan(exp(d*x+c))/b^3/d-2*I*f^2*polylog(3,-I*exp(d*x+c))/b/d^3-2*I*a^4*f*(
f*x+e)*polylog(2,I*exp(d*x+c))/b^3/(a^2+b^2)/d^2+1/3*a*(f*x+e)^3/b^2/f+2*f
^2*sinh(d*x+c)/b/d^3-2*(f*x+e)^2*arctan(exp(d*x+c))/b/d-2*a^4*(f*x+e)^2*ar
ctan(exp(d*x+c))/b^3/(a^2+b^2)/d-1/2*a^3*f^2*polylog(3,-exp(2*d*x+2*c))/b^
2/(a^2+b^2)/d^3-2*I*a^2*f^2*polylog(3,I*exp(d*x+c))/b^3/d^3-2*I*f*(f*x+e)*
polylog(2,I*exp(d*x+c))/b/d^2+(f*x+e)^2*sinh(d*x+c)/b/d-a*(f*x+e)^2*ln(1+e
xp(2*d*x+2*c))/b^2/d-a*f*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/b^2/d^2+a^3...
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3418 vs. 2(1067) = 2134.

Time = 10.57 (sec) , antiderivative size = 3418, normalized size of antiderivative = 3.20

$$\int \frac{(e+fx)^2 \sinh^2(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)} \, dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
-1/6*(-12*a*d^3*e^2*E^(2*c)*x + 12*a*d^3*e^2*(1 + E^(2*c))*x + 12*a*d^3*e*
f*x^2 + 4*a*d^3*f^2*x^3 + 12*b*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] -
 6*a*d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*b*d
*e*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)])
- PolyLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) - 6*a*d*e*(1 +
 E^(2*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c
+ d*x))]) + (6*I)*b*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^
2*x^2*Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*
PolyLog[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3,
I*E^(c + d*x)]) - a*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(
c + d*x))]) - 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c +
 d*x))]))/((a^2 + b^2)*d^3*(1 + E^(2*c))) + (a^3*(6*e^2*E^(2*c)*x + 6*e*E^
(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E
^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2
+ b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2
+ b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x)
)/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2
*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)
*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d - (3*e^2*E
^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e*f*x*Lo...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6115}$$

$$\frac{\int (e+fx)^2\sinh(c+dx)\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 5972}$$

$$\frac{\int (e+fx)^2\cosh(c+dx)dx-\int (e+fx)^2\text{sech}(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (e+fx)^2 \sin\left(ic+idx+\frac{\pi}{2}\right)dx-\int (e+fx)^2 \csc\left(ic+idx+\frac{\pi}{2}\right)dx}{b}-\frac{a\int \frac{(e+fx)^2 \sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

3777

$$\frac{-\int (e+fx)^2 \csc\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}+\frac{(e+fx)^2\sinh(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)^2 \sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

26

$$\frac{-\int (e+fx)^2 \csc\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{2f\int (e+fx)\sinh(c+dx)dx}{d}+\frac{(e+fx)^2\sinh(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)^2 \sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

3042

$$\frac{-\frac{2f\int -i(e+fx)\sin(ic+idx)dx}{d}-\int (e+fx)^2 \csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{(e+fx)^2\sinh(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)^2 \sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

26

$$\frac{\frac{2if\int (e+fx)\sin(ic+idx)dx}{d}-\int (e+fx)^2 \csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{(e+fx)^2\sinh(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)^2 \sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

3777

$$\frac{-\int (e+fx)^2 \csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}\right)}{d}+\frac{(e+fx)^2\sinh(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)^2 \sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

3042

$$\frac{-\int (e+fx)^2 \csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}+\frac{(e+fx)^2\sinh(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)^2 \sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 3117$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \; 4668$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2if\int (e+fx)\log(1-ie^{c+dx})dx}{d}-\frac{2if\int (e+fx)\log(1+ie^{c+dx})dx}{d}-\frac{2(e+fx)^2\arctan(e^{c+dx})}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+(}{b}$$

$$\Big\downarrow \; 3011$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}-\frac{2(e+fx)^2\text{arc}}{d}}{b}$$

$$\Big\downarrow \; 2720$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c}\right.}{d}\right.}{d}}{b}$$

$$\Big\downarrow \; 6115$$

$$-\frac{a\left(\frac{\int (e+fx)^2\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c}\right.}{d}\right.}{d}}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c}\right.}{d}\right.}{d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -i(e+fx)^2\tan(ic+idx)dx}{b}\right)}{b}$$

$\downarrow$ 26

$$\frac{-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c}\right.}{d}\right.}{d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\int(e+fx)^2\tan(ic+idx)dx}{b}\right)}{b}$$

$\downarrow$ 4201

$$\frac{-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c}\right.}{d}\right.}{d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\int\frac{e^{2(c+dx)}(e+fx)^2}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^3}{3f}\right)}{b}\right)}{b}$$

$\downarrow$ 2620

$$\frac{-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c}\right.}{d}\right.}{d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int(e+fx)\log\left(1+e^{2(c+dx)}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}\right)}{b}$$

$\downarrow$ 3011

$$\frac{-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c}\right.}{d}\right.}{d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int\text{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{2d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)}{3f}\right.}{b}\right.}{b}$$

$\downarrow$ 2720

$$\frac{-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c}\right.}{d}\right.}{d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+\right.}}{2d}\right.}{d}\right.\right.}{b}\right.}{b}$$

$\downarrow$ 6101

$$\frac{-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c}\right.}{d}\right.}{d}}{b}$$

$$\frac{a\left(-\frac{a\left(\frac{\int(e+fx)^2\text{sech}(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c\right.}}{4d^2}\right.}{d}\right.\right.}{b}\right.}{b}$$

$\downarrow$ 3042

$$a\left(-\frac{a\left(-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{b}\right)}{b}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}\right.}{}\right.\right.}{b}\right.$$

$$-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c}\right.}{d}\right.}{d}$$

$$\downarrow \text{4668}$$

$$a\left(-\frac{a\left(-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{2if\int (e+fx)\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{2if\int (e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}}{b}\right)}{b}-\frac{i\left(2i\left(\frac{(e+f\ldots}{}\right.\right.}{}\right.$$

$$-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c}\right.}{d}\right.}{d}$$

$$\downarrow \text{3011}$$

$$a\left(-\frac{a\left(-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{Poly}\ldots}{}\right.}{d}}{b}\right.}{b}\right.$$

$$-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c}\right.}{d}\right.}{d}$$

$$\downarrow \text{2720}$$

$$-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,ie^{c\cdots}\right.}{d}\right.}{d}$$

$$b$$

$$a\left(-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c\cdots}\right.}{d^2}\right.}{b}\right.}{b}\right.$$

$\downarrow$ 6107

$$-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,ie^{c\cdots}\right.}{d}\right.}{d}$$

$$b$$

$$a\left(-\frac{a\left(-\frac{a\left(\frac{b^2\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLo}\cdots}{d}\right.}{d}\right.}{b}\right.$$

$\downarrow$ 6095

$$-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c}\right.}{d}\right.}{d}$$

$$\frac{b}{a\left(-\frac{a\left(-\frac{a\left(\frac{b^2\left(\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{a^2+b^2}+\frac{\int(e+fx)^2\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{Poly}\ldots}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.}$$

$\Big\downarrow$ 2620

$$-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c}\right.}{d}\right.}{d}$$

$$\frac{b}{a\left(-\frac{a\left(-\frac{a\left(\frac{b^2\left(-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+\ldots\right.}{bd}\right.}{a^2+b^2}\right.}{b}\right.}{\ldots}\right.}$$

$\Big\downarrow$ 3011

$$\frac{-\frac{2\arctan\left(e^{c+dx}\right)(e+fx)^2}{d}+\frac{\sinh(c+dx)(e+fx)^2}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\right.}{d}}{b}$$

$$a\left(-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}-\frac{a\left(\right.}{}\right.$$

$\downarrow$ 2720

$$-\frac{2\arctan(e^{c+dx})(e+fx)^2}{d}+\frac{\sinh(c+dx)(e+fx)^2}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\right.}{d}$$

$$b$$

$$a\left(-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}-\ a\left(\cdots\right.\right.$$

input
```
Int[((e + f*x)^2*Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^2 \sinh(dx+c)^2 \tanh(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2775 vs. $2(970) = 1940$.

Time = 0.17 (sec) , antiderivative size = 2775, normalized size of antiderivative = 2.60

$$\int \frac{(e+fx)^2 \sinh^2(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)}\, dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
-1/6*(3*(a^2*b + b^3)*d^2*f^2*x^2 + 3*(a^2*b + b^3)*d^2*e^2 + 6*(a^2*b + b
^3)*d*e*f + 6*(a^2*b + b^3)*f^2 - 3*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b +
b^3)*d^2*e^2 - 2*(a^2*b + b^3)*d*e*f + 2*(a^2*b + b^3)*f^2 + 2*((a^2*b + b
^3)*d^2*e*f - (a^2*b + b^3)*d*f^2)*x)*cosh(d*x + c)^2 - 3*((a^2*b + b^3)*d
^2*f^2*x^2 + (a^2*b + b^3)*d^2*e^2 - 2*(a^2*b + b^3)*d*e*f + 2*(a^2*b + b^
3)*f^2 + 2*((a^2*b + b^3)*d^2*e*f - (a^2*b + b^3)*d*f^2)*x)*sinh(d*x + c)^
2 + 6*((a^2*b + b^3)*d^2*e*f + (a^2*b + b^3)*d*f^2)*x - 2*((a^3 + a*b^2)*d
^3*f^2*x^3 + 3*(a^3 + a*b^2)*d^3*e*f*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*x + 6*(
a^3 + a*b^2)*c*d^2*e^2 - 6*(a^3 + a*b^2)*c^2*d*e*f + 2*(a^3 + a*b^2)*c^3*f
^2)*cosh(d*x + c) + 12*((a^3*d*f^2*x + a^3*d*e*f)*cosh(d*x + c) + (a^3*d*f
^2*x + a^3*d*e*f)*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c)
+ (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) +
12*((a^3*d*f^2*x + a^3*d*e*f)*cosh(d*x + c) + (a^3*d*f^2*x + a^3*d*e*f)*si
nh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 12*((a*b^2*d*f^2*x +
 I*b^3*d*f^2*x + a*b^2*d*e*f + I*b^3*d*e*f)*cosh(d*x + c) + (a*b^2*d*f^2*x
 + I*b^3*d*f^2*x + a*b^2*d*e*f + I*b^3*d*e*f)*sinh(d*x + c))*dilog(I*cosh(
d*x + c) + I*sinh(d*x + c)) + 12*((a*b^2*d*f^2*x - I*b^3*d*f^2*x + a*b^2*d
*e*f - I*b^3*d*e*f)*cosh(d*x + c) + (a*b^2*d*f^2*x - I*b^3*d*f^2*x + a*b^2
*d*e*f - I*b^3*d*e*f)*sinh(d*x + c))*dilog(-I*cosh(d*x + c) - I*sinh(d*...
```

Sympy [F]

$$\int \frac{(e+fx)^2\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2\sinh^2\left(c+dx\right)\tanh\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate((f*x+e)**2*sinh(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*sinh(c + d*x)**2*tanh(c + d*x)/(a + b*sinh(c + d*x))
, x)
```

Maxima [F]

$$\int \frac{(e+fx)^2\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\sinh(dx+c)^2\tanh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/2*(2*a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b^2 + b^
4)*d) - 4*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + 2*a*log(e^(-2*d*x - 2*c
) + 1)/((a^2 + b^2)*d) + 2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d
*x - c)/(b*d))*e^2 - 1/6*(2*a*d^3*f^2*x^3*e^c + 6*a*d^3*e*f*x^2*e^c - 3*(b
*d^2*f^2*x^2*e^(2*c) + 2*(d^2*e*f - d*f^2)*b*x*e^(2*c) - 2*(d*e*f - f^2)*b
*e^(2*c))*e^(d*x) + 3*(b*d^2*f^2*x^2 + 2*(d^2*e*f + d*f^2)*b*x + 2*(d*e*f
+ f^2)*b)*e^(-d*x))*e^(-c)/(b^2*d^3) + integrate(2*(a^3*b*f^2*x^2 + 2*a^3*
b*e*f*x - (a^4*f^2*x^2*e^c + 2*a^4*e*f*x*e^c)*e^(d*x))/(a^2*b^3 + b^5 - (a
^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^3*b^2*e^c + a*b^4*e^c)*e^(d
*x)), x) - integrate(-2*(a*f^2*x^2 + 2*a*e*f*x - (b*f^2*x^2*e^c + 2*b*e*f*
x*e^c)*e^(d*x))/(a^2 + b^2 + (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)^2\tanh(c+d\,x)\,\left(e+f\,x\right)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((sinh(c + d*x)^2*tanh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)^2*tanh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^2*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 12*e**(c + d*x)*atan(e**(c + d*x))*b**4*d**2*e**2 + 3*e**(2*c + 2*d*x)
*a**2*b**2*d**2*e**2 + 6*e**(2*c + 2*d*x)*a**2*b**2*d**2*e*f*x + 3*e**(2*c
 + 2*d*x)*a**2*b**2*d**2*f**2*x**2 - 6*e**(2*c + 2*d*x)*a**2*b**2*d*e*f -
6*e**(2*c + 2*d*x)*a**2*b**2*d*f**2*x + 6*e**(2*c + 2*d*x)*a**2*b**2*f**2
+ 3*e**(2*c + 2*d*x)*b**4*d**2*e**2 + 6*e**(2*c + 2*d*x)*b**4*d**2*e*f*x +
 3*e**(2*c + 2*d*x)*b**4*d**2*f**2*x**2 - 6*e**(2*c + 2*d*x)*b**4*d*e*f -
6*e**(2*c + 2*d*x)*b**4*d*f**2*x + 6*e**(2*c + 2*d*x)*b**4*f**2 - 24*e**(3
*c + d*x)*int((e**(2*d*x)*x**2)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a
 + 2*e**(c + d*x)*a - b),x)*a**5*d**3*f**2 - 12*e**(3*c + d*x)*int((e**(2*
d*x)*x**2)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a -
 b),x)*a**3*b**2*d**3*f**2 + 12*e**(3*c + d*x)*int((e**(2*d*x)*x**2)/(e**(
4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a*b**4*d*
*3*f**2 - 48*e**(3*c + d*x)*int((e**(2*d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**
(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a**5*d**3*e*f - 24*e**(3*c + d*
x)*int((e**(2*d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c
 + d*x)*a - b),x)*a**3*b**2*d**3*e*f + 24*e**(3*c + d*x)*int((e**(2*d*x)*x
)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a*
b**4*d**3*e*f + 12*e**(2*c + d*x)*int((e**(d*x)*x**2)/(e**(4*c + 4*d*x)*b
+ 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a**4*b*d**3*f**2 + 24*e*
*(2*c + d*x)*int((e**(d*x)*x**2)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*...
```

3.408 $\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 631

$$\begin{aligned}
&\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= \frac{a(e+fx)^2}{2b^2f} + \frac{2a^2(e+fx)\arctan\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{bd} \\
&\quad - \frac{2a^4(e+fx)\arctan\left(e^{c+dx}\right)}{b^3(a^2+b^2)d} - \frac{f\cosh(c+dx)}{bd^2} - \frac{a^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2(a^2+b^2)d} \\
&\quad - \frac{a^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2(a^2+b^2)d} - \frac{a(e+fx)\log\left(1+e^{2(c+dx)}\right)}{b^2d} \\
&\quad + \frac{a^3(e+fx)\log\left(1+e^{2(c+dx)}\right)}{b^2(a^2+b^2)d} - \frac{ia^2f\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{b^3d^2} \\
&\quad + \frac{if\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{bd^2} + \frac{ia^4f\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{b^3(a^2+b^2)d^2} \\
&\quad + \frac{ia^2f\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{b^3d^2} - \frac{if\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{bd^2} - \frac{ia^4f\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{b^3(a^2+b^2)d^2} \\
&\quad - \frac{a^3f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2(a^2+b^2)d^2} - \frac{a^3f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2(a^2+b^2)d^2} \\
&\quad - \frac{af\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2b^2d^2} + \frac{a^3f\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2b^2(a^2+b^2)d^2} + \frac{(e+fx)\sinh(c+dx)}{bd}
\end{aligned}$$

output
```
1/2*a*(f*x+e)^2/b^2/f+2*a^2*(f*x+e)*arctan(exp(d*x+c))/b^3/d-2*(f*x+e)*arc
tan(exp(d*x+c))/b/d-2*a^4*(f*x+e)*arctan(exp(d*x+c))/b^3/(a^2+b^2)/d-f*cos
h(d*x+c)/b/d^2-a^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/(a^2
+b^2)/d-a^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d
-a*(f*x+e)*ln(1+exp(2*d*x+2*c))/b^2/d+a^3*(f*x+e)*ln(1+exp(2*d*x+2*c))/b^2
/(a^2+b^2)/d-I*a^2*f*polylog(2,-I*exp(d*x+c))/b^3/d^2+I*f*polylog(2,-I*exp
(d*x+c))/b/d^2+I*a^4*f*polylog(2,-I*exp(d*x+c))/b^3/(a^2+b^2)/d^2-I*f*poly
log(2,I*exp(d*x+c))/b/d^2+I*a^2*f*polylog(2,I*exp(d*x+c))/b^3/d^2-I*a^4*f*
polylog(2,I*exp(d*x+c))/b^3/(a^2+b^2)/d^2-a^3*f*polylog(2,-b*exp(d*x+c)/(a
-(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d^2-a^3*f*polylog(2,-b*exp(d*x+c)/(a+(a^2
+b^2)^(1/2)))/b^2/(a^2+b^2)/d^2-1/2*a*f*polylog(2,-exp(2*d*x+2*c))/b^2/d^2
+1/2*a^3*f*polylog(2,-exp(2*d*x+2*c))/b^2/(a^2+b^2)/d^2+(f*x+e)*sinh(d*x+c
)/b/d
```

Mathematica [A] (warning: unable to verify)

Time = 8.41 (sec) , antiderivative size = 621, normalized size of antiderivative = 0.98

$$\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{\frac{2f\cosh(c+dx)}{b}+\frac{a^3\left(-2de(c+dx)+2cf(c+dx)-f(c+dx)^2+\frac{4a\sqrt{a^2+b^2}de\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-\left(a^2+b^2\right)^2}}-\frac{4a\sqrt{-\left(a^2+b^2\right)^2}de\,\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)}{\left(-a^2-b^2\right)^{3/2}}+\right.}{}}{}$$

input
```
Integrate[((e + f*x)*Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
-1/2*((2*f*Cosh[c + d*x])/b + (a^3*(-2*d*e*(c + d*x) + 2*c*f*(c + d*x) - f
*(c + d*x)^2 + (4*a*Sqrt[a^2 + b^2]*d*e*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a
^2 - b^2]])/Sqrt[-(a^2 + b^2)^2] - (4*a*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh[(
a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/(-a^2 - b^2)^(3/2) + 2*f*(c + d*x)*Lo
g[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*f*(c + d*x)*Log[1 + (b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*f*Log[b - 2*a*E^(c + d*x) - b*E^(2
*(c + d*x))] + 2*d*e*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] + 2*f
*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f*PolyLog[2, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(b^2*(a^2 + b^2)) + (2*(-(a*d*e*(c
+ d*x)) + a*c*f*(c + d*x) - (a*f*(c + d*x)^2)/2 + 2*b*d*e*ArcTan[E^(c + d*
x)] - 2*b*c*f*ArcTan[E^(c + d*x)] + I*b*f*(c + d*x)*Log[1 - I*E^(c + d*x)]
 - I*b*f*(c + d*x)*Log[1 + I*E^(c + d*x)] + a*d*e*Log[1 + E^(2*(c + d*x))]
 - a*c*f*Log[1 + E^(2*(c + d*x))] + a*f*(c + d*x)*Log[1 + E^(2*(c + d*x))]
 - I*b*f*PolyLog[2, (-I)*E^(c + d*x)] + I*b*f*PolyLog[2, I*E^(c + d*x)] +
(a*f*PolyLog[2, -E^(2*(c + d*x))])/2))/(a^2 + b^2) - (2*d*(e + f*x)*Sinh[c
 + d*x])/b)/d^2
```

Rubi [A] (verified)

Time = 3.74 (sec) , antiderivative size = 570, normalized size of antiderivative = 0.90, number of steps used = 31, number of rules used = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.938$, Rules used = {6115, 5972, 3042, 3777, 26, 3042, 26, 3118, 4668, 2715, 2838, 6115, 3042, 26, 4201, 2620, 2715, 2838, 6101, 3042, 4668, 2715, 2838, 6107, 6095, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6115}$$

$$\frac{\int (e+fx)\sinh(c+dx)\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{5972}$$

$$\frac{\int (e+fx)\cosh(c+dx)dx-\int (e+fx)\text{sech}(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{b}$$

$$\Big\downarrow \ 3777$$

$$-\frac{a\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{if\int -i\sinh(c+dx)dx}{d}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \ 26$$

$$-\frac{a\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f\int \sinh(c+dx)dx}{d}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f\int -i\sin(ic+idx)dx}{d}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \ 26$$

$$-\frac{a\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{if\int \sin(ic+idx)dx}{d}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \ 3118$$

$$-\frac{a\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \ 4668$$

$$-\frac{a\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{if\int \log\left(1-ie^{c+dx}\right)dx}{d}-\frac{if\int \log\left(1+ie^{c+dx}\right)dx}{d}-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \text{ 2715}$$

$$-\frac{a\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \text{ 2838}$$

$$-\frac{a\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \text{ 6115}$$

$$-\frac{a\left(\frac{\int(e+fx)\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -i(e+fx)\tan(ic+idx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\int(e+fx)\tan(ic+idx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 4201}$$

$$\frac{-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\int\frac{e^{2(c+dx)}(e+fx)}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^2}{2f}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int\log\left(1+e^{2(c+dx)}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int e^{-2(c+dx)}\log\left(1+e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 6101}$$

$$\frac{-\frac{2(e+fx)\arctan(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,ie^{c+dx})}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)\text{sech}(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{2(e+fx)\arctan(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,ie^{c+dx})}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\left(-\frac{a\int\frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{b}\right)}{b}-\frac{i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 4668$$

$$\frac{-\frac{2(e+fx)\arctan(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,ie^{c+dx})}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\left(-\frac{a\int\frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{if\int\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}}{b}\right)}{b}-\frac{i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}\right.\right.}{}\right.}{b}$$

$$\Big\downarrow \; 2715$$

$$\frac{-\frac{2(e+fx)\arctan(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,ie^{c+dx})}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\left(-\frac{a\int\frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}}{b}\right)}{b}-\frac{i\left(2i\right.}{}\right.}{b}$$

$$\Big\downarrow \; 2838$$

$$\frac{-\frac{2(e+fx)\arctan(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,ie^{c+dx})}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-$$

$$\frac{a\left(-\frac{a\left(-\frac{a\int\frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}}{b}\right)}{b}-\frac{i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}\right.\right.}{}\right.}{b}$$

$\downarrow$ 6107

$$\frac{-\frac{2(e+fx)\arctan(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,ie^{c+dx})}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-$$

$$\frac{a\left(-\frac{a\left(-\frac{a\left(\frac{b^2\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,\right.}{d^2}}{b}\right.}{b}\right.}{b}$$

$\downarrow$ 6095

$$\frac{-\frac{2(e+fx)\arctan(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,ie^{c+dx})}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-$$

$$\frac{a\left(-\frac{a\left(-\frac{a\left(\frac{b^2\left(\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int(e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}}{}\right.}{b}\right.}{b}$$

$\downarrow$ 2620

$$\frac{-\frac{2(e+fx)\arctan(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,ie^{c+dx})}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-$$

$$\left(a\left(a-\frac{\left(a\left(-\frac{\left(a\left(\frac{b^2\left(-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\int\right.\right.}{b}\right.\right.}{b}\right.\right.$$

$$\Big\downarrow \text{2715}$$

$$\frac{-\frac{2(e+fx)\arctan(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,ie^{c+dx})}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-$$

$$\left(a\left(a-\frac{\left(a\left(-\frac{\left(a\left(\frac{b^2\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+}}\right.}{bd}\right.}{a^2+b^2}\right.\right.}{b}\right.\right.}{b}\right.\right.$$

$\downarrow$ 2838

$$\frac{-\frac{2(e+fx)\arctan(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,ie^{c+dx})}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-$$

$$a\left(-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\cdots\right)}{a^2+b^2}\right)}{b}\cdots\right)}{b}\cdots\right)$$

$\downarrow$ 7293

$$\frac{-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-$$

$$a\left(-\frac{a\left(-\frac{a\left(\frac{\int(a(e+fx)\mathrm{sech}(c+dx)-b(e+fx)\tanh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right.}{bd}\right.}{a^2+b^2}\right.}{b}\right.}{b}\right.$$

$$\downarrow \text{2009}$$

$$\frac{-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-$$

$$a\left(-\frac{a\left(\frac{\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}}{b}-\frac{a\left(\frac{b^2\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\right.}{}\right.}{}\right.}{}\right.$$

input
```
Int[((e + f*x)*Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-((a*(((-I)*(((-1/2*I)*(e + f*x)^2)/f + (2*I)*(((e + f*x)*Log[1 + E^(2*(c
+ d*x))])/(2*d) + (f*PolyLog[2, -E^(2*(c + d*x))])/(4*d^2))))/b - (a*(((2*
(e + f*x)*ArcTan[E^(c + d*x)])/d - (I*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2
+ (I*f*PolyLog[2, I*E^(c + d*x)])/d^2)/b - (a*((b^2*(-1/2*(e + f*x)^2/(b*f
) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e
 + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog
[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (f*PolyLog[2, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(a^2 + b^2) + ((b*(e + f
*x)^2)/(2*f) + (2*a*(e + f*x)*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)*Log[1
+ E^(2*(c + d*x))])/d - (I*a*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + (I*a*f*
PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*PolyLog[2, -E^(2*(c + d*x))])/(2*d^2
))/(a^2 + b^2)))/b))/b))/b) + ((-2*(e + f*x)*ArcTan[E^(c + d*x)])/d - (f*C
osh[c + d*x])/d^2 + (I*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2 - (I*f*PolyLog[
2, I*E^(c + d*x)])/d^2 + ((e + f*x)*Sinh[c + d*x])/d)/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5972
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6101
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sech[
c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*Sech[c
+ d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6115
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - S
imp[a/b   Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*(Tanh[c + d*x]^n/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4065 vs. 2(592) = 1184.

Time = 1.86 (sec) , antiderivative size = 4066, normalized size of antiderivative = 6.44

method	result	size
risch	Expression too large to display	4066

input
```
int((f*x+e)*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
-1/d^2/b^2*a^4*f/(a^2+b^2)^(3/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+
(a^2+b^2)^(1/2)))*c-2*I/d^2*b*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*c+2*I/d*b
*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*x-2*I/d*b*f/(2*a^2+2*b^2)*ln(1-I*exp(d
*x+c))*x+2*I/d^2*b*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*c-2*b^2/d^2/(a^2+b^2
)^(1/2)*c*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2)
)+2/d^2/b^2*c*a^4*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x
+c)+2*a)/(a^2+b^2)^(1/2))-2/d/b^2*a^4*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*ln((
b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+2/d^2/b^2*a^4*f/(2*
a^2+2*b^2)/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b
^2)^(1/2)))*c-2/d^2/b^2*a^4*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*ln((b*exp(d*x+
c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2/d/b^2*a^4*f/(2*a^2+2*b^2)/(
a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*
x+2/d^2/b^2*c*a^2*f/(2*a^2+2*b^2)*(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x
+c)+2*a)/(a^2+b^2)^(1/2))-2/d*a^2*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*ln((b*ex
p(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+2/d^2*a^2*f/(2*a^2+2*b^
2)/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2
)))*c-2/d^2*a^2*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)
^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d/b^2*a^3*f/(a^2+b^2)*ln((-b*exp(d*x+c)
+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d^2/b^2*a^3*f/(a^2+b^2)*ln((
-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2/d^2*a/b^2*c*...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1410 vs. $2(570) = 1140$.

Time = 0.14 (sec) , antiderivative size = 1410, normalized size of antiderivative = 2.23

$$\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
-1/2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e - ((a^2*b + b^3)*d*f*x + (a^
2*b + b^3)*d*e - (a^2*b + b^3)*f)*cosh(d*x + c)^2 - ((a^2*b + b^3)*d*f*x +
 (a^2*b + b^3)*d*e - (a^2*b + b^3)*f)*sinh(d*x + c)^2 + (a^2*b + b^3)*f -
((a^3 + a*b^2)*d^2*f*x^2 + 2*(a^3 + a*b^2)*d^2*e*x + 4*(a^3 + a*b^2)*c*d*e
 - 2*(a^3 + a*b^2)*c^2*f)*cosh(d*x + c) + 2*(a^3*f*cosh(d*x + c) + a^3*f*s
inh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(a^3*f*cosh(d*x +
 c) + a^3*f*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*((a*
b^2*f + I*b^3*f)*cosh(d*x + c) + (a*b^2*f + I*b^3*f)*sinh(d*x + c))*dilog(
I*cosh(d*x + c) + I*sinh(d*x + c)) + 2*((a*b^2*f - I*b^3*f)*cosh(d*x + c)
+ (a*b^2*f - I*b^3*f)*sinh(d*x + c))*dilog(-I*cosh(d*x + c) - I*sinh(d*x +
 c)) + 2*((a^3*d*e - a^3*c*f)*cosh(d*x + c) + (a^3*d*e - a^3*c*f)*sinh(d*x
 + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^
2) + 2*a) + 2*((a^3*d*e - a^3*c*f)*cosh(d*x + c) + (a^3*d*e - a^3*c*f)*sin
h(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^
2)/b^2) + 2*a) + 2*((a^3*d*f*x + a^3*c*f)*cosh(d*x + c) + (a^3*d*f*x + a^3
*c*f)*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
 + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*((a^3*d*f*x + a
^3*c*f)*cosh(d*x + c) + (a^3*d*f*x + a^3*c*f)*sinh(d*x + c))*log(-(a*co...
```

Sympy [F]

$$\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)\sinh^2{(c+dx)}\tanh{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*sinh(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*sinh(c + d*x)**2*tanh(c + d*x)/(a + b*sinh(c + d*x)), x
)
```

Maxima [F]

$$\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)\sinh{(dx+c)}^2\tanh{(dx+c)}}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-1/2*(2*a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b^2 + b^
4)*d) - 4*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + 2*a*log(e^(-2*d*x - 2*c
) + 1)/((a^2 + b^2)*d) + 2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d
*x - c)/(b*d))*e - 1/4*f*(2*(a*d^2*x^2*e^c - (b*d*x*e^(2*c) - b*e^(2*c))*e
^(d*x) + (b*d*x + b)*e^(-d*x))*e^(-c)/(b^2*d^2) - integrate(-8*(a^4*x*e^(d
*x + c) - a^3*b*x)/(a^2*b^3 + b^5 - (a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d
*x) - 2*(a^3*b^2*e^c + a*b^4*e^c)*e^(d*x)), x) + integrate(8*(b*x*e^(d*x +
 c) - a*x)/(a^2 + b^2 + (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x))
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)^2\tanh(c+d\,x)\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((sinh(c + d*x)^2*tanh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)^2*tanh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output

```
( - 4*e**(c + d*x)*atan(e**(c + d*x))*b**4*d*e + e**(2*c + 2*d*x)*a**2*b**
2*d*e + e**(2*c + 2*d*x)*a**2*b**2*d*f*x - e**(2*c + 2*d*x)*a**2*b**2*f +
e**(2*c + 2*d*x)*b**4*d*e + e**(2*c + 2*d*x)*b**4*d*f*x - e**(2*c + 2*d*x)
*b**4*f - 8*e**(3*c + d*x)*int((e**(2*d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(
3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a**5*d**2*f - 4*e**(3*c + d*x)*i
nt((e**(2*d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d
*x)*a - b),x)*a**3*b**2*d**2*f + 4*e**(3*c + d*x)*int((e**(2*d*x)*x)/(e**(
4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a*b**4*d*
*2*f + 4*e**(2*c + d*x)*int((e**(d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c +
 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a**4*b*d**2*f + 8*e**(2*c + d*x)*int(
(e**(d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a
 - b),x)*a**2*b**3*d**2*f + 4*e**(2*c + d*x)*int((e**(d*x)*x)/(e**(4*c + 4
*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*b**5*d**2*f - 8*
e**(c + d*x)*int(x/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c +
d*x)*a - b),x)*a**5*d**2*f - 4*e**(c + d*x)*int(x/(e**(4*c + 4*d*x)*b + 2*
e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a**3*b**2*d**2*f + 4*e**(c +
 d*x)*int(x/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a
- b),x)*a*b**4*d**2*f - 2*e**(c + d*x)*log(e**(2*c + 2*d*x) + 1)*a*b**3*d*
e - 2*e**(c + d*x)*log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*a**3*b*d
*e + 2*e**(c + d*x)*a**3*b*d**2*e*x - e**(c + d*x)*a**3*b*d**2*f*x**2 +...
```

3.409 $\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 89

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b\arctan(\sinh(c+dx))}{(a^2+b^2)\,d} - \frac{a\log(\cosh(c+dx))}{(a^2+b^2)\,d} - \frac{a^3\log(a+b\sinh(c+dx))}{b^2\,(a^2+b^2)\,d} + \frac{\sinh(c+dx)}{bd}$$

output
```
-b*arctan(sinh(d*x+c))/(a^2+b^2)/d-a*ln(cosh(d*x+c))/(a^2+b^2)/d-a^3*ln(a+
b*sinh(d*x+c))/b^2/(a^2+b^2)/d+sinh(d*x+c)/b/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{\log(i-\sinh(c+dx))}{a+ib} + \frac{\log(i+\sinh(c+dx))}{a-ib} + \frac{2a^3\log(a+b\sinh(c+dx))}{b^2(a^2+b^2)} - \frac{2\sinh(c+dx)}{b}}{2d}$$

input
```
Integrate[(Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*(Log[I - Sinh[c + d*x]]/(a + I*b) + Log[I + Sinh[c + d*x]]/(a - I*b)
+ (2*a^3*Log[a + b*Sinh[c + d*x]])/(b^2*(a^2 + b^2)) - (2*Sinh[c + d*x])/b
)/d
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 3316, 26, 27, 604, 25, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ic+idx)^3}{\cos(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ic+idx)^3}{\cos(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3316}$$

$$-\frac{ib\int \frac{i\sinh^3(c+dx)}{(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{b\int \frac{\sinh^3(c+dx)}{(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b^3\sinh^3(c+dx)}{(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))}{b^2d}$$

$$\Big\downarrow \text{604}$$

$$\frac{\int -\frac{\sinh(c+dx)b^3+a\sinh^2(c+dx)b^2+ab^2}{(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))+a+b\sinh(c+dx)}{b^2d}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\int \frac{\sinh(c+dx)b^3+a\sinh^2(c+dx)b^2+ab^2}{(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))+a+b\sinh(c+dx)}{b^2d}$$

$$\Big\downarrow \text{2160}$$

$$\frac{-\int \left(\frac{a^3}{(a^2+b^2)(a+b\sinh(c+dx))}+\frac{b^4+a\sinh(c+dx)b^3}{(a^2+b^2)(\sinh^2(c+dx)b^2+b^2)}\right)d(b\sinh(c+dx))+a+b\sinh(c+dx)}{b^2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^3\arctan(\sinh(c+dx))}{a^2+b^2}-\frac{ab^2\log(b^2\sinh^2(c+dx)+b^2)}{2(a^2+b^2)}-\frac{a^3\log(a+b\sinh(c+dx))}{a^2+b^2}+a+b\sinh(c+dx)}{b^2d}$$

input
```
Int[(Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(a - (b^3*ArcTan[Sinh[c + d*x]])/(a^2 + b^2) - (a^3*Log[a + b*Sinh[c + d*x
]])/(a^2 + b^2) - (a*b^2*Log[b^2 + b^2*Sinh[c + d*x]^2])/(2*(a^2 + b^2)) +
 b*Sinh[c + d*x])/(b^2*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 604
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(b*d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*
x^2)^p*ExpandToSum[b*d^m*(m + n + 2*p + 1)*x^m - b*(m + n + 2*p + 1)*(c + d
*x)^m - (c + d*x)^(m - 2)*(a*d^2*(m + n - 1) - b*c^2*(m + n + 2*p + 1) - 2*
b*c*d*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && IGtQ[m,
 1] && NeQ[m + n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.90

method	result
derivativedivides	$\frac{-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^{2}}-\frac{a^{3}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{b^{2}\left(a^{2}+b^{2}\right)}-\frac{1}{b\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{a\ln\left(1+\right.}{}}{d}$
default	$\frac{-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^{2}}-\frac{a^{3}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{b^{2}\left(a^{2}+b^{2}\right)}-\frac{1}{b\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{a\ln\left(1+\right.}{}}{d}$
risch	$-\frac{ax}{b^{2}}+\frac{e^{dx+c}}{2bd}-\frac{e^{-dx-c}}{2bd}+\frac{2d^{2}ax}{a^{2}d^{2}+b^{2}d^{2}}+\frac{2dac}{a^{2}d^{2}+b^{2}d^{2}}+\frac{2a^{3}x}{b^{2}(a^{2}+b^{2})}+\frac{2a^{3}c}{b^{2}d(a^{2}+b^{2})}+\frac{i\ln(e^{dx+c}-i)b}{(a^{2}+b^{2})d}-\ln$

input
```
int(sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b/(tanh(1/2*d*x+1/2*c)-1)+a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)-a^3/b^2/
(a^2+b^2)*ln(tanh(1/2*d*x+1/2*c)^2*a-2*b*tanh(1/2*d*x+1/2*c)-a)-1/b/(1+tan
h(1/2*d*x+1/2*c))+a/b^2*ln(1+tanh(1/2*d*x+1/2*c))+16/(8*a^2+8*b^2)*(-1/2*a
*ln(1+tanh(1/2*d*x+1/2*c)^2)-b*arctan(tanh(1/2*d*x+1/2*c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(89) = 178$.

Time = 0.10 (sec) , antiderivative size = 288, normalized size of antiderivative = 3.24

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{2\left(a^3+ab^2\right)dx\cosh\left(dx+c\right)-a^2b-b^3+\left(a^2b+b^3\right)\cosh\left(dx+c\right)^2+\left(a^2b+b^3\right)\sinh\left(dx+c\right)^2-4\left(b^3\cos\right.}{}$$

input
```
integrate(sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/2*(2*(a^3 + a*b^2)*d*x*cosh(d*x + c) - a^2*b - b^3 + (a^2*b + b^3)*cosh(
d*x + c)^2 + (a^2*b + b^3)*sinh(d*x + c)^2 - 4*(b^3*cosh(d*x + c) + b^3*si
nh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 2*(a^3*cosh(d*x + c)
+ a^3*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x
 + c))) - 2*(a*b^2*cosh(d*x + c) + a*b^2*sinh(d*x + c))*log(2*cosh(d*x + c
)/(cosh(d*x + c) - sinh(d*x + c))) + 2*((a^3 + a*b^2)*d*x + (a^2*b + b^3)*
cosh(d*x + c))*sinh(d*x + c))/((a^2*b^2 + b^4)*d*cosh(d*x + c) + (a^2*b^2
+ b^4)*d*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh^2{(c+dx)}\tanh{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate(sinh(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(sinh(c + d*x)**2*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.65

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a^3\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^2b^2+b^4)d} + \frac{2\,b\arctan\left(e^{(-dx-c)}\right)}{(a^2+b^2)d} - \frac{a\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^2+b^2)d} - \frac{(dx+c)a}{b^2d} + \frac{e^{(dx+c)}}{2\,bd} - \frac{e^{(-dx-c)}}{2\,bd}$$

input
```
integrate(sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b^2 + b^4)*d) +
 2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/((
a^2 + b^2)*d) - (d*x + c)*a/(b^2*d) + 1/2*e^(d*x + c)/(b*d) - 1/2*e^(-d*x
- c)/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.63

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{2a^3\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2a\right|\right)}{a^2b^2+b^4}+\frac{\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)}-1\right)e^{(-dx-c)}\right)\right)b}{a^2+b^2}+\frac{a\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^2+b^2}-\frac{e^{(dx+c)}-e^{(-dx-c)}}{b}}{2d}$$

input
```
integrate(sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(2*a^3*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^2*b^2 + b^4)
 + (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*b/(a^2 + b^2) +
 a*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^2 + b^2) - (e^(d*x + c) - e^
(-d*x - c))/b)/d
```

Mupad [B] (verification not implemented)

Time = 2.70 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.80

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{e^{c+dx}}{2bd}-\frac{\ln\left(e^{c+dx}+1i\right)}{ad-bd\,1i} - \frac{a^3\ln\left(2a^4b^3-b^7-a^2b^5-a^6b+2a^7e^{dx}e^c+b^7e^{2c}e^{2dx}+a^6be^{2c}e^{2dx}+2a^3b^4e^{dx}e^c-4a^5b^2e^{dx}e^c\cdots\right)}{da^2b^2+db^4} - \frac{e^{-c-dx}}{2bd}+\frac{ax}{b^2}-\frac{\ln\left(1+e^{c+dx}\,1i\right)\,1i}{-bd+ad\,1i}$$

input
```
int((sinh(c + d*x)^2*tanh(c + d*x))/(a + b*sinh(c + d*x)),x)
```

output
```
exp(c + d*x)/(2*b*d) - (log(exp(c + d*x)*1i + 1)*1i)/(a*d*1i - b*d) - log(
exp(c + d*x) + 1i)/(a*d - b*d*1i) - (a^3*log(2*a^4*b^3 - b^7 - a^2*b^5 - a
^6*b + 2*a^7*exp(d*x)*exp(c) + b^7*exp(2*c)*exp(2*d*x) + a^6*b*exp(2*c)*ex
p(2*d*x) + 2*a^3*b^4*exp(d*x)*exp(c) - 4*a^5*b^2*exp(d*x)*exp(c) + a^2*b^5
*exp(2*c)*exp(2*d*x) - 2*a^4*b^3*exp(2*c)*exp(2*d*x) + 2*a*b^6*exp(d*x)*ex
p(c)))/(b^4*d + a^2*b^2*d) - exp(- c - d*x)/(2*b*d) + (a*x)/b^2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.03

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-4e^{dx+c}atan\left(e^{dx+c}\right)b^3+e^{2dx+2c}a^2b+e^{2dx+2c}b^3-2e^{dx+c}\log\left(e^{2dx+2c}+1\right)a\,b^2-2e^{dx+c}\log\left(e^{2dx+2c}b+2e^{d}\right.}{2e^{dx+c}b^2d\left(a^2+b^2\right)}$$

input
```
int(sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 4*e**(c + d*x)*atan(e**(c + d*x))*b**3 + e**(2*c + 2*d*x)*a**2*b + e**
(2*c + 2*d*x)*b**3 - 2*e**(c + d*x)*log(e**(2*c + 2*d*x) + 1)*a*b**2 - 2*e
**(c + d*x)*log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*a**3 + 2*e**(c
+ d*x)*a**3*d*x + 2*e**(c + d*x)*a*b**2*d*x - a**2*b - b**3)/(2*e**(c + d*
x)*b**2*d*(a**2 + b**2))
```

3.410 $\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(sinh(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 38.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Sinh[c + d*x]^2*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Sinh[c + d*x]^2*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \ 6125$$
$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Sinh[c + d*x]^2*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(dx+c)^2\tanh(dx+c)}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(sinh(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)^2\tanh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(sinh(d*x + c)^2*tanh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 2.45 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

input
```
integrate(sinh(d*x+c)**2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(sinh(c + d*x)**2*tanh(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)),
 x)
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 322, normalized size of antiderivative = 9.47

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)^2\tanh\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-1/2*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b*f) - 1/2*e^(c - d*
e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b*f) - a*log(f*x + e)/(b^2*f) + 1/
4*integrate(-8*(a^4*e^(d*x + c) - a^3*b)/(a^2*b^3*e + b^5*e + (a^2*b^3*f +
 b^5*f)*x - (a^2*b^3*e*e^(2*c) + b^5*e*e^(2*c) + (a^2*b^3*f*e^(2*c) + b^5*
f*e^(2*c))*x)*e^(2*d*x) - 2*(a^3*b^2*e*e^c + a*b^4*e*e^c + (a^3*b^2*f*e^c
+ a*b^4*f*e^c)*x)*e^(d*x)), x) - 1/4*integrate(8*(b*e^(d*x + c) - a)/(a^2*
e + b^2*e + (a^2*f + b^2*f)*x + (a^2*e*e^(2*c) + b^2*e*e^(2*c) + (a^2*f*e^
(2*c) + b^2*f*e^(2*c))*x)*e^(2*d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 2.38 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\sinh(c+d\,x)^2\tanh(c+d\,x)}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((sinh(c + d*x)^2*tanh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((sinh(c + d*x)^2*tanh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)^2\tanh\left(dx+c\right)}{\sinh\left(dx+c\right)be+\sinh\left(dx+c\right)bfx+ae+afx}dx$$

input
```
int(sinh(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int((sinh(c + d*x)**2*tanh(c + d*x))/(sinh(c + d*x)*b*e + sinh(c + d*x)*b*
f*x + a*e + a*f*x),x)
```

3.411 $\int \frac{(e+fx)^3 \sinh(c+dx) \tanh^2(c+dx)}{a+b \sinh(c+dx)} \, dx$

Optimal result

Integrand size = 34, antiderivative size = 1294

$$\int \frac{(e+fx)^3 \sinh(c+dx) \tanh^2(c+dx)}{a+b \sinh(c+dx)} \, dx = \text{Too large to display}$$

output
```
a^3*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d-(
f*x+e)^3*tanh(d*x+c)/b/d+6*I*a*f^3*polylog(3,I*exp(d*x+c))/b^2/d^4-6*a^3*f
^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/
d^3+6*a^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+
b^2)^(3/2)/d^3+3*a^3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2
)))/b/(a^2+b^2)^(3/2)/d^2-3*a^3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^
2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^2-a^4*(f*x+e)^3*tanh(d*x+c)/b^3/(a^2+b^
2)/d-a^3*(f*x+e)^3*sech(d*x+c)/b^2/(a^2+b^2)/d-a^4*(f*x+e)^3/b^3/(a^2+b^2)
/d+a^2*(f*x+e)^3*tanh(d*x+c)/b^3/d+a*(f*x+e)^3*sech(d*x+c)/b^2/d+3*a^4*f^2
*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/b^3/(a^2+b^2)/d^3+3*a^4*f*(f*x+e)^2*ln
(1+exp(2*d*x+2*c))/b^3/(a^2+b^2)/d^2+6*a^3*f*(f*x+e)^2*arctan(exp(d*x+c))/
b^2/(a^2+b^2)/d^2-6*I*a^3*f^3*polylog(3,I*exp(d*x+c))/b^2/(a^2+b^2)/d^4-6*
I*a*f^2*(f*x+e)*polylog(2,I*exp(d*x+c))/b^2/d^3+a^2*(f*x+e)^3/b^3/d-6*I*a^
3*f^2*(f*x+e)*polylog(2,-I*exp(d*x+c))/b^2/(a^2+b^2)/d^3+3*f*(f*x+e)^2*ln(
1+exp(2*d*x+2*c))/b/d^2+3*f^2*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/b/d^3+6*I
*a^3*f^3*polylog(3,-I*exp(d*x+c))/b^2/(a^2+b^2)/d^4+6*I*a*f^2*(f*x+e)*poly
log(2,-I*exp(d*x+c))/b^2/d^3-6*a*f*(f*x+e)^2*arctan(exp(d*x+c))/b^2/d^2-3/
2*a^4*f^3*polylog(3,-exp(2*d*x+2*c))/b^3/(a^2+b^2)/d^4-3*a^2*f^2*(f*x+e)*p
olylog(2,-exp(2*d*x+2*c))/b^3/d^3-3*a^2*f*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/b
^3/d^2-6*I*a*f^3*polylog(3,-I*exp(d*x+c))/b^2/d^4+1/4*(f*x+e)^4/b/f-3/2...
```

Mathematica [A] (warning: unable to verify)

Time = 4.78 (sec) , antiderivative size = 1111, normalized size of antiderivative = 0.86

$$\int \frac{(e+fx)^3 \sinh(c+dx) \tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
(x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))/(4*b) - (f*(12*b*d^3*e^2*E
^(2*c)*x - 12*b*d^3*e^2*(1 + E^(2*c))*x - 12*b*d^3*e*f*x^2 - 4*b*d^3*f^2*x
^3 + 12*a*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] + 6*b*d^2*e^2*(1 + E^(
2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*a*d*e*(1 + E^(2*c))*f*(d
*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - PolyLog[2, (-I)*E^(
c + d*x)] + PolyLog[2, I*E^(c + d*x)]) + 6*b*d*e*(1 + E^(2*c))*f*(2*d*x*(d
*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x))]) + (6*I)*a*(
1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*x^2*Log[1 + I*E^(c
+ d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*PolyLog[2, I*E^(c + d
*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I*E^(c + d*x)]) + b*(
1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c + d*x))]) - 6*d*x*P
olyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d*x))])))/(2*(a^2 +
b^2)*d^4*(1 + E^(2*c))) + (a^3*(2*d^3*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt
[a^2 + b^2]] - 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
] - 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^3*f
^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 3*d^3*e^2*f*x*Log[
1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 3*d^3*e*f^2*x^2*Log[1 + (b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2])] + d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2])] - 3*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a
+ Sqrt[a^2 + b^2])] + 3*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6115}$$

$$\frac{\int (e+fx)^3\tanh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -(e+fx)^3\tan(ic+idx)^2dx}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\int (e+fx)^3\tan(ic+idx)^2dx}{b}$$

$\big\downarrow$ 4203

$$-\frac{a\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{3if\int i(e+fx)^2\tanh(c+dx)dx}{d}-\int (e+fx)^3dx+\frac{(e+fx)^3\tanh(c+dx)}{d}}{b}$$

$\big\downarrow$ 17

$$-\frac{a\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{3if\int i(e+fx)^2\tanh(c+dx)dx}{d}+\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{b}$$

$\big\downarrow$ 26

$$-\frac{a\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{-\frac{3f\int (e+fx)^2\tanh(c+dx)dx}{d}+\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{b}$$

$\big\downarrow$ 3042

$$-\frac{a\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{-\frac{3f\int -i(e+fx)^2\tan(ic+idx)dx}{d}+\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{b}$$

$\big\downarrow$ 26

$$-\frac{a\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{3if\int (e+fx)^2\tan(ic+idx)dx}{d}+\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{b}$$

$\big\downarrow$ 4201

$$-\frac{a\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{3if\left(2i\int \frac{e^{2(c+dx)}(e+fx)^2}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{b}$$

$\big\downarrow$ 2620

$$-\frac{a\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int (e+fx)\log\left(1+e^{2(c+dx)}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{b}$$

$\big\downarrow$ 3011

$$\frac{-\frac{a\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int \text{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{2d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\tanh(c+dx)}{d}-}{b}$$

$$\Big\downarrow \text{2720}$$

$$\frac{-\frac{a\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3}{}}{b}$$

$$\Big\downarrow \text{6101}$$

$$\frac{-\frac{a\left(\frac{\int (e+fx)^3\text{sech}(c+dx)\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3}{}}{b}$$

$$\Big\downarrow \text{5974}$$

$$\frac{-\frac{a\left(\frac{\frac{3f\int (e+fx)^2\text{sech}(c+dx)dx}{d}-\frac{(e+fx)^3\text{sech}(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3}{}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\left(-\frac{a\int \frac{(e+fx)^3\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{b}\right)}{b}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3}{\ldots}}{b}$$

$$\Big\downarrow \text{4668}$$

$$-\frac{a\left(-\frac{a\int \frac{(e+fx)^3\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(-\frac{2if\int (e+fx)\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{2if\int (e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+\ldots}{\ldots}\right.}{d}}{b}\ldots\right.}{b}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3}{\ldots}}{b}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{a\left(-\frac{a\int \frac{(e+fx)^3\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int \operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\ldots}{\ldots}\right.}{\ldots}}{b}\ldots\right.}{b}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3}{\ldots}}{b}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}(2,-ie^{c+dx})de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}(2,-ie^{c}\cdots}{d}\cdots\right)}{d}\cdots\right)}{\cdots}}{\cdots}\cdots\right)}{b}$$

$$\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\cdots}{\cdots}}{b}$$

$\downarrow$ 6117

$$-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)^3\operatorname{sech}^2(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}(2,-ie^{c+dx})de^{c+dx}}{d^2}\cdots\right)}{d}\cdots\right)}{\cdots}}{\cdots}\cdots\right)}{b}$$

$$\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\cdots}{\cdots}}{b}$$

$\downarrow$ 3042

$$-\frac{a\left(\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)\cdots}{d^2}\cdots\right)}{\cdots}\cdots\right)}{d}\cdots}{b}\cdots\right)}{b}\cdots$$

$$\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\cdots}{\cdots}}{b}$$

$\downarrow$ 4672

$$-\frac{a\left(\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)\cdots}{d^2}\cdots\right)}{\cdots}\cdots\right)}{d}\cdots}{b}\cdots\right)}{b}\cdots$$

$$\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\cdots}{\cdots}}{b}$$

$\downarrow$ 26

$$-\frac{a\left(-\frac{a\left(\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{3f\int(e+fx)^2\tanh(c+dx)dx}{d}}{b}-\frac{a\int\frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLo}}{}\right.}{}\right.}{}}{}\right.}{}$$

$$\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3}{}}{b}$$

$\Big\downarrow$ 3042

$$-\frac{a\left(\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right.}{}\right.}{d}}{b}\right.}{}$$

$$\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3}{}}{b}$$

$\Big\downarrow$ 26

$$-\frac{a\left(\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)\ldots}{d^2}\ldots\right.}{\ldots}\ldots\right.}{d}\ldots}{b}\ldots\right.}{\ldots}$$

$$\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\ldots}{\ldots}\ldots}{b}$$

$\downarrow$ 4201

$$-\frac{a\left(\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)\ldots}{d^2}\ldots\right.}{\ldots}\ldots\right.}{d}\ldots}{b}\ldots\right.}{\ldots}$$

$$\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\ldots}{\ldots}\ldots}{b}$$

$\downarrow$ 2620

$$-\frac{a\left(\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)\cdots}{d^2}\cdots\right.}{\cdots}\right.}{d}}{b}\cdots\right.}{\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\cdots}{\cdots}}{b}\cdots}$$

$\downarrow$ 3011

$$-\frac{a\left(\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)\cdots}{d^2}\cdots\right.}{\cdots}\right.}{d}}{b}\cdots\right.}{\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\cdots}{\cdots}}{b}\cdots}$$

$\downarrow$ 2720

$$-\frac{a\left(\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)\cdots}{d^2}\cdots\right.}{\cdots}\right.}{d}}{b}\right.}{\cdots}$$

$$\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\cdots}{\cdots}}{b}$$

$\downarrow$ 6107

$$-\frac{a\left(\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)\cdots}{d^2}\cdots\right.}{\cdots}\right.}{d}}{b}\right.}{\cdots}$$

$$\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\cdots}{\cdots}}{b}$$

$\Big\downarrow$ 3042

$$-\frac{a\left(-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)\cdots}{d^2}\right.}{}\cdots\right)}{d}}{b}\cdots}{\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3}{\cdots}}{b}}$$

input
```
Int[((e + f*x)^3*Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^3 \sinh(dx+c)\tanh(dx+c)^2}{a+b\sinh(dx+c)}\,dx$$

input
```
int((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 7331 vs. $2(1195) = 2390$.

Time = 0.30 (sec) , antiderivative size = 7331, normalized size of antiderivative = 5.67

$$\int \frac{(e+fx)^3 \sinh(c+dx) \tanh^2(c+dx)}{a+b\sinh(c+dx)} \, dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^3 \sinh(c+dx) \tanh^2(c+dx)}{a+b\sinh(c+dx)} \, dx = \int \frac{(e+fx)^3 \sinh(c+dx) \tanh^2(c+dx)}{a+b\sinh(c+dx)} \, dx$$

input
```
integrate((f*x+e)**3*sinh(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*sinh(c + d*x)*tanh(c + d*x)**2/(a + b*sinh(c + d*x))
, x)
```

Maxima [F]

$$\int \frac{(e+fx)^3\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\sinh(dx+c)\tanh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-3*b*e^2*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2
 + b^2)*d^2)) - 6*a*f^3*integrate(x^2*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) +
 b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 6*b*f^3*integrate(x^2/(a^2*d
*e^(2*d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 12*a*e*f^2
*integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) +
a^2*d + b^2*d), x) - 12*b*e*f^2*integrate(x/(a^2*d*e^(2*d*x + 2*c) + b^2*d
*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - (a^3*log((b*e^(-d*x - c) - a - sqr
t(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^2*b + b^3)*sqrt(
a^2 + b^2)*d) - 2*(a*e^(-d*x - c) - b)/((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x
 - 2*c))*d) - (d*x + c)/(b*d))*e^3 - 6*a*e^2*f*arctan(e^(d*x + c))/((a^2 +
 b^2)*d^2) + 1/4*(24*b^2*e^2*f*x + (a^2*d*f^3 + b^2*d*f^3)*x^4 + 4*(a^2*d*
e*f^2 + (d*e*f^2 + 2*f^3)*b^2)*x^3 + 6*(a^2*d*e^2*f + (d*e^2*f + 4*e*f^2)*
b^2)*x^2 + ((a^2*d*f^3*e^(2*c) + b^2*d*f^3*e^(2*c))*x^4 + 4*(a^2*d*e*f^2*e
^(2*c) + b^2*d*e*f^2*e^(2*c))*x^3 + 6*(a^2*d*e^2*f*e^(2*c) + b^2*d*e^2*f*e
^(2*c))*x^2)*e^(2*d*x) + 8*(a*b*f^3*x^3*e^c + 3*a*b*e*f^2*x^2*e^c + 3*a*b*
e^2*f*x*e^c)*e^(d*x))/(a^2*b*d + b^3*d + (a^2*b*d*e^(2*c) + b^3*d*e^(2*c))
*e^(2*d*x)) - integrate(-2*(a^3*f^3*x^3*e^c + 3*a^3*e*f^2*x^2*e^c + 3*a^3*
e^2*f*x*e^c)*e^(d*x)/(a^2*b^2 + b^4 - (a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2
*d*x) - 2*(a^3*b*e^c + a*b^3*e^c)*e^(d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\sinh(c+d\,x)\,\tanh(c+d\,x)^2\,(e+f\,x)^3}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((sinh(c + d*x)*tanh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)*tanh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{too large to display}$$

input
```
int((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output

```
( - 24*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**5*b*d**2*e**2*f - 48*e**(2*c
 + 2*d*x)*atan(e**(c + d*x))*a**5*b*d*e*f**2 - 48*e**(2*c + 2*d*x)*atan(e*
*(c + d*x))*a**5*b*f**3 - 48*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**3*b**3
*d**2*e**2*f - 96*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**3*b**3*d*e*f**2 -
 96*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**3*b**3*f**3 - 24*e**(2*c + 2*d*
x)*atan(e**(c + d*x))*a*b**5*d**2*e**2*f - 48*e**(2*c + 2*d*x)*atan(e**(c
+ d*x))*a*b**5*d*e*f**2 - 48*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a*b**5*f*
*3 - 24*atan(e**(c + d*x))*a**5*b*d**2*e**2*f - 48*atan(e**(c + d*x))*a**5
*b*d*e*f**2 - 48*atan(e**(c + d*x))*a**5*b*f**3 - 48*atan(e**(c + d*x))*a*
*3*b**3*d**2*e**2*f - 96*atan(e**(c + d*x))*a**3*b**3*d*e*f**2 - 96*atan(e
**(c + d*x))*a**3*b**3*f**3 - 24*atan(e**(c + d*x))*a*b**5*d**2*e**2*f - 4
8*atan(e**(c + d*x))*a*b**5*d*e*f**2 - 48*atan(e**(c + d*x))*a*b**5*f**3 -
 8*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a
**2 + b**2))*a**3*b**2*d**3*e**3*i - 8*sqrt(a**2 + b**2)*atan((e**(c + d*x
)*b*i + a*i)/sqrt(a**2 + b**2))*a**3*b**2*d**3*e**3*i - 32*e**(5*c + 2*d*x
)*int((e**(3*d*x)*x**3)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4
*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b + 2*e**(c + d*x)
*a - b),x)*a**7*d**4*f**3 - 64*e**(5*c + 2*d*x)*int((e**(3*d*x)*x**3)/(e**
(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*c +
3*d*x)*a - e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**5*b**2*d**4...
```

3.412 $\int \frac{(e+fx)^2\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 904

$$\int \frac{(e+fx)^2\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output

```
f^2*polylog(2,-exp(2*d*x+2*c))/b/d^3-(f*x+e)^2*tanh(d*x+c)/b/d+2*I*a^3*f^2
*polylog(2,I*exp(d*x+c))/b^2/(a^2+b^2)/d^3+2*a^3*f*(f*x+e)*polylog(2,-b*ex
p(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^2-2*a^3*f*(f*x+e)*polylo
g(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^2+a^4*f^2*polyl
og(2,-exp(2*d*x+2*c))/b^3/(a^2+b^2)/d^3-a^4*(f*x+e)^2*tanh(d*x+c)/b^3/(a^2
+b^2)/d-a^3*(f*x+e)^2*sech(d*x+c)/b^2/(a^2+b^2)/d-a^2*f^2*polylog(2,-exp(2
*d*x+2*c))/b^3/d^3+a^2*(f*x+e)^2*tanh(d*x+c)/b^3/d+a*(f*x+e)^2*sech(d*x+c)
/b^2/d-a^4*(f*x+e)^2/b^3/(a^2+b^2)/d+2*a^4*f*(f*x+e)*ln(1+exp(2*d*x+2*c))/
b^3/(a^2+b^2)/d^2+4*a^3*f*(f*x+e)*arctan(exp(d*x+c))/b^2/(a^2+b^2)/d^2-2*I
*a^3*f^2*polylog(2,-I*exp(d*x+c))/b^2/(a^2+b^2)/d^3+a^2*(f*x+e)^2/b^3/d+2*
f*(f*x+e)*ln(1+exp(2*d*x+2*c))/b/d^2-2*a^2*f*(f*x+e)*ln(1+exp(2*d*x+2*c))/
b^3/d^2-4*a*f*(f*x+e)*arctan(exp(d*x+c))/b^2/d^2-2*I*a*f^2*polylog(2,I*exp
(d*x+c))/b^2/d^3+1/3*(f*x+e)^3/b/f-(f*x+e)^2/b/d+a^3*(f*x+e)^2*ln(1+b*exp(
d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d-a^3*(f*x+e)^2*ln(1+b*exp(d
*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d-2*a^3*f^2*polylog(3,-b*exp(
d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^3+2*a^3*f^2*polylog(3,-b*e
xp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^3+2*I*a*f^2*polylog(2,-
I*exp(d*x+c))/b^2/d^3
```

Mathematica [A] (warning: unable to verify)

Time = 2.89 (sec) , antiderivative size = 665, normalized size of antiderivative = 0.74

$$\int \frac{(e+fx)^2\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{x(3e^2+3efx+f^2x^2)}{3b}$$
$$-\frac{f\left(4bd^2ee^{2c}x-4bd^2e(1+e^{2c})\,x+2bd^2e^{2c}fx^2-2bd^2(1+e^{2c})\,fx^2+4ade(1+e^{2c})\arctan\left(e^{c+dx}\right)+2b\right.}{}$$
$$+\frac{a^3\left(2d^2e^2\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)-2d^2efx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-d^2f^2x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+2d^2efx\log\left(\right.\right.}{}$$
$$+\frac{(e+fx)^2\text{sech}(c+dx)(a-b\text{sech}(c)\sinh(dx))}{(a^2+b^2)\,d}$$

input
```
Integrate[((e + f*x)^2*Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
(x*(3*e^2 + 3*e*f*x + f^2*x^2))/(3*b) - (f*(4*b*d^2*e*E^(2*c)*x - 4*b*d^2*
e*(1 + E^(2*c))*x + 2*b*d^2*E^(2*c)*f*x^2 - 2*b*d^2*(1 + E^(2*c))*f*x^2 +
4*a*d*e*(1 + E^(2*c))*ArcTan[E^(c + d*x)] + 2*b*d*e*(1 + E^(2*c))*(2*d*x -
 Log[1 + E^(2*(c + d*x))]) + (2*I)*a*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c
+ d*x)] - Log[1 + I*E^(c + d*x)]) - PolyLog[2, (-I)*E^(c + d*x)] + PolyLog
[2, I*E^(c + d*x)]) + b*(1 + E^(2*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*
x))]) - PolyLog[2, -E^(2*(c + d*x))])))/((a^2 + b^2)*d^3*(1 + E^(2*c))) +
(a^3*(2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 2*d^2*e*f*x
*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^2*f^2*x^2*Log[1 + (b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2])] - 2*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2
])] + 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
 + 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*f^2*PolyLo
g[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(b*(a^2 + b^2)^(3/2)*d^3)
 + ((e + f*x)^2*Sech[c + d*x]*(a - b*Sech[c]*Sinh[d*x]))/((a^2 + b^2)*d)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6115}$$

$$\frac{\int (e+fx)^2\tanh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -(e+fx)^2\tan(ic+idx)^2dx}{b}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{a\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\int (e+fx)^2\tan(ic+idx)^2dx}{b}$$

$$\Big\downarrow \text{ 4203}$$

$$-\frac{a\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{2if\int i(e+fx)\tanh(c+dx)dx}{d}-\int (e+fx)^2dx+\frac{(e+fx)^2\tanh(c+dx)}{d}}{b}$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{a\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{2if\int i(e+fx)\tanh(c+dx)dx}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{a\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{-\frac{2f\int (e+fx)\tanh(c+dx)dx}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{-\frac{2f\int -i(e+fx)\tan(ic+idx)dx}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{a\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{2if\int (e+fx)\tan(ic+idx)dx}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{a\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{2if\left(2i\int \frac{e^{2(c+dx)}(e+fx)}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{a\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{2if\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int \log\left(1+e^{2(c+dx)}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{a\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{2if\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int e^{-2(c+dx)}\log\left(1+e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{a\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \text{6101}$$

$$-\frac{a\left(\frac{\int (e+fx)^2\text{sech}(c+dx)\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \text{5974}$$

$$-\frac{a\left(\frac{\frac{2f\int (e+fx)\text{sech}(c+dx)dx}{d}-\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$\Big\downarrow$ 3042

$$-\frac{a\left(-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{b}\right)}{b}-\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$\Big\downarrow$ 4668

$$-\frac{a\left(-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(-\frac{if\int \log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int \log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c+\cdots}\right.}{d}\cdots\right)}{d}}{b}\cdots\right)}{b}\cdots$$

$$\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$\Big\downarrow$ 2715

$$-\frac{a\left(-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c\cdots}}{d^2}\cdots\right)}{d}}{b}\cdots\right)}{b}\cdots$$

$$\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$\Big\downarrow$ 2838

$$-\frac{a\left(-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+d}\right.}{d^2}\right.}{d}}{b}\right.}{b}$$

$$\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$\downarrow$ 6117

$$-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+d}\right.}{d^2}\right.}{d}}{b}\right.}{b}$$

$$\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$\downarrow$ 3042

$$-\frac{a\left(\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\right.}{b}\right.}{b}$$

$$\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$\downarrow$ 4672

$$-\frac{a\left(\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\right.}{}\right.}{b}$$

$$\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$\downarrow$ 26

$$-\frac{a\left(-\frac{a\left(\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2f\int(e+fx)\tanh(c+dx)dx}{d}}{b}-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right.}{}}{b}\right.}{b}$$

$$\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$\downarrow$ 3042

$$-\frac{a\left(\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\right.}{}\right.}{b}$$

$$\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$\downarrow$ 26

$$-\frac{a\left(\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\cdots\right.}{\cdots}\right.}{b}$$

$$\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \; 4201$$

$$-\frac{a\left(\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\cdots\right.}{\cdots}\right.}{b}$$

$$\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \; 2620$$

$$-\frac{a\left(\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\cdots\right.}{\cdots}\right.}{b}$$

$$\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \; 2715$$

$$-\frac{a\left(\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\cdots\right.}{\cdots}\right.}{b}\cdots$$

$$\frac{\frac{2if\left(2i\left(\frac{f\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$\downarrow$ 2838

$$-\frac{a\left(\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\cdots\right.}{\cdots}\right.}{b}\cdots$$

$$\frac{\frac{2if\left(2i\left(\frac{f\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$\downarrow$ 6107

$$-\frac{a\left(\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\left(\frac{b^2\int\frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\int(e\cdots\right.}{\cdots}\right.}{\cdots}\right.}{b}\cdots$$

$$\frac{\frac{2if\left(2i\left(\frac{f\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\left(\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f}{\cdots}\cdots\right.\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{b}$$

$$\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

input
```
Int[((e + f*x)^2*Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^2\sinh(dx+c)\tanh(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4196 vs. $2(841) = 1682$.

Time = 0.18 (sec) , antiderivative size = 4196, normalized size of antiderivative = 4.64

$$\int \frac{(e+fx)^2\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^2\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*sinh(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*sinh(c + d*x)*tanh(c + d*x)**2/(a + b*sinh(c + d*x))
, x)
```

Maxima [F]

$$\int \frac{(e+fx)^2\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\sinh(dx+c)\tanh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-2*b*e*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2 +
 b^2)*d^2)) - 4*a*f^2*integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2
*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 4*b*f^2*integrate(x/(a^2*d*e^(2*
d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - (a^3*log((b*e^(-
d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((
a^2*b + b^3)*sqrt(a^2 + b^2)*d) - 2*(a*e^(-d*x - c) - b)/((a^2 + b^2 + (a^
2 + b^2)*e^(-2*d*x - 2*c))*d) - (d*x + c)/(b*d))*e^2 - 4*a*e*f*arctan(e^(d
*x + c))/((a^2 + b^2)*d^2) + 1/3*(12*b^2*e*f*x + (a^2*d*f^2 + b^2*d*f^2)*x
^3 + 3*(a^2*d*e*f + (d*e*f + 2*f^2)*b^2)*x^2 + ((a^2*d*f^2*e^(2*c) + b^2*d
*f^2*e^(2*c))*x^3 + 3*(a^2*d*e*f*e^(2*c) + b^2*d*e*f*e^(2*c))*x^2)*e^(2*d*
x) + 6*(a*b*f^2*x^2*e^c + 2*a*b*e*f*x*e^c)*e^(d*x))/(a^2*b*d + b^3*d + (a^
2*b*d*e^(2*c) + b^3*d*e^(2*c))*e^(2*d*x)) - integrate(-2*(a^3*f^2*x^2*e^c
+ 2*a^3*e*f*x*e^c)*e^(d*x)/(a^2*b^2 + b^4 - (a^2*b^2*e^(2*c) + b^4*e^(2*c)
)*e^(2*d*x) - 2*(a^3*b*e^c + a*b^3*e^c)*e^(d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)\,\tanh(c+d\,x)^2\,(e+f\,x)^2}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((sinh(c + d*x)*tanh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)*tanh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2 \sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
( - 12*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**5*b*d*e*f - 12*e**(2*c + 2*d
*x)*atan(e**(c + d*x))*a**5*b*f**2 - 24*e**(2*c + 2*d*x)*atan(e**(c + d*x)
)*a**3*b**3*d*e*f - 24*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**3*b**3*f**2
- 12*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a*b**5*d*e*f - 12*e**(2*c + 2*d*x
)*atan(e**(c + d*x))*a*b**5*f**2 - 12*atan(e**(c + d*x))*a**5*b*d*e*f - 12
*atan(e**(c + d*x))*a**5*b*f**2 - 24*atan(e**(c + d*x))*a**3*b**3*d*e*f -
24*atan(e**(c + d*x))*a**3*b**3*f**2 - 12*atan(e**(c + d*x))*a*b**5*d*e*f
- 12*atan(e**(c + d*x))*a*b**5*f**2 - 6*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)
*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**3*b**2*d**2*e**2*i -
6*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**3*
b**2*d**2*e**2*i - 24*e**(5*c + 2*d*x)*int((e**(3*d*x)*x**2)/(e**(6*c + 6*
d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a
- e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**7*d**3*f**2 - 48*e**(5*
c + 2*d*x)*int((e**(3*d*x)*x**2)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*
a + e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b + 2*e**
(c + d*x)*a - b),x)*a**5*b**2*d**3*f**2 - 24*e**(5*c + 2*d*x)*int((e**(3*d
*x)*x**2)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)*b
+ 4*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**
3*b**4*d**3*f**2 - 48*e**(5*c + 2*d*x)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x
)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a ...
```

3.413 $\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 454

$$
\begin{aligned}
&\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= \frac{(e+fx)^2}{2bf} - \frac{af\arctan(\sinh(c+dx))}{b^2d^2} + \frac{a^3f\arctan(\sinh(c+dx))}{b^2(a^2+b^2)d^2} \\
&\quad - \frac{a^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d} + \frac{a^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d} \\
&\quad - \frac{a^2f\log(\cosh(c+dx))}{b^3d^2} + \frac{f\log(\cosh(c+dx))}{bd^2} + \frac{a^4f\log(\cosh(c+dx))}{b^3(a^2+b^2)d^2} \\
&\quad - \frac{a^3f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d^2} + \frac{a^3f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d^2} \\
&\quad + \frac{a(e+fx)\mathrm{sech}(c+dx)}{b^2d} - \frac{a^3(e+fx)\mathrm{sech}(c+dx)}{b^2(a^2+b^2)d} + \frac{a^2(e+fx)\tanh(c+dx)}{b^3d} \\
&\quad - \frac{(e+fx)\tanh(c+dx)}{bd} - \frac{a^4(e+fx)\tanh(c+dx)}{b^3(a^2+b^2)d}
\end{aligned}
$$

output
```
1/2*(f*x+e)^2/b/f-a*f*arctan(sinh(d*x+c))/b^2/d^2+a^3*f*arctan(sinh(d*x+c)
)/b^2/(a^2+b^2)/d^2-a^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(
a^2+b^2)^(3/2)/d+a^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2
+b^2)^(3/2)/d-a^2*f*ln(cosh(d*x+c))/b^3/d^2+f*ln(cosh(d*x+c))/b/d^2+a^4*f*
ln(cosh(d*x+c))/b^3/(a^2+b^2)/d^2-a^3*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^
2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^2+a^3*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2
)^(1/2)))/b/(a^2+b^2)^(3/2)/d^2+a*(f*x+e)*sech(d*x+c)/b^2/d-a^3*(f*x+e)*se
ch(d*x+c)/b^2/(a^2+b^2)/d+a^2*(f*x+e)*tanh(d*x+c)/b^3/d-(f*x+e)*tanh(d*x+c
)/b/d-a^4*(f*x+e)*tanh(d*x+c)/b^3/(a^2+b^2)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.45 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.79

$$\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{\frac{(c+dx)(cf-d(2e+fx))}{b}+\frac{2f\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a-ib}+\frac{2f\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a+ib}+\frac{f\log(\cosh(c+dx))}{ia-b}-\frac{f\log(\cosh(c+dx))}{ia+b}+\cdots}{\cdots}$$

input
```
Integrate[((e + f*x)*Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),
x]
```

output
```
-1/2*(((c + d*x)*(c*f - d*(2*e + f*x)))/b + (2*f*ArcTan[Tanh[(c + d*x)/2]]
)/(a - I*b) + (2*f*ArcTan[Tanh[(c + d*x)/2]])/(a + I*b) + (f*Log[Cosh[c +
d*x]])/(I*a - b) - (f*Log[Cosh[c + d*x]])/(I*a + b) + (2*a^3*(-2*d*e*ArcTa
nh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*c*f*ArcTanh[(a + b*E^(c + d*x)
)/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2])] - f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + f*Pol
yLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - f*PolyLog[2, -((b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2]))]))/(b*(a^2 + b^2)^(3/2)) + (2*d*(e + f*x)*Se
ch[c + d*x]*(-a + b*Sinh[c + d*x]))/(a^2 + b^2))/d^2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6115}$$

$$\frac{\int (e+fx)\tanh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -\left((e+fx)\tan(ic+idx)^2\right)dx}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\int (e+fx)\tan(ic+idx)^2dx}{b}$$

$$\Big\downarrow \text{4203}$$

$$-\frac{a\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{if\int i\tanh(c+dx)dx}{d}-\int (e+fx)dx+\frac{(e+fx)\tanh(c+dx)}{d}}{b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{a\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{if\int i\tanh(c+dx)dx}{d}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{-\frac{f\int \tanh(c+dx)dx}{d}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{-\frac{f\int -i\tan(ic+idx)dx}{d}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{if\int \tan(ic+idx)dx}{d}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$$\Big\downarrow \; 3956$$

$$-\frac{a\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$$\Big\downarrow \; 6101$$

$$-\frac{a\left(\frac{\int (e+fx)\text{sech}(c+dx)\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$$\Big\downarrow \; 5974$$

$$-\frac{a\left(\frac{\frac{f\int \text{sech}(c+dx)dx}{d}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)\text{sech}(c+dx)}{d}+\frac{f\int \csc\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \; 4257$$

$$-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$$\Big\downarrow \; 6117$$

$$-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\int (e+fx)\text{sech}^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}\right)}{b}-\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$\Big\downarrow$ 3042

$$-\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{b}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 4672

$$-\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{if\int -i\tanh(c+dx)dx}{d}}{b}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 26

$$-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int \tanh(c+dx)dx}{d}}{b}-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}\right)}{b}-\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$\Big\downarrow$ 3042

$$-\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int-i\tan(ic+idx)dx}{d}}{b}\right)}{b}\right)}{b}$$

$\downarrow$ 26

$$-\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\tanh(c+dx)}{d}+\frac{if\int\tan(ic+idx)dx}{d}}{b}\right)}{b}\right)}{b}$$

$\downarrow$ 3956

$$-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\int\frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}\right)}{b}-\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$\downarrow$ 6107

$$-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{b^2\int\frac{e+fx}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)\text{sech}^2(c+dx)(a-}{a^2+b^2}\right.}{b}\right.}{b}}{b}$$

$$\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$\downarrow$ 3042

$$-\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}-$$

$$\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int(e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int\frac{}{a-ib}}{a}\right.}{b}}{b}\right.}{b}$$

$\downarrow$ 3803

$$-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{2b^2\int-\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{\int(e+fx)\text{sech}\right.}{b}}{b}\right.}{b}$$

$$\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$\downarrow$ 25

$$-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int(e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\int -}{}\right.}{b}}{b}\right.}{b}$$

$$\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$\downarrow$ 2694

$$-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int(e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\ldots}{\ldots}\ldots\right)}{\ldots}\right)}{\ldots}\right)}{b}\ldots\right)}{b}\ldots$$

$$\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$\Big\downarrow$ 27

$$-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int(e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b}{\cdots}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{b}\right.}{b}$$

$$\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$$\downarrow \text{2620}$$

$$-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int(e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(b \; - \; \cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{b}$$

$$\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$\Big\downarrow$ 2715

$$-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\operatorname{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int(e+fx)\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(b \; - \cdots\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}$$

$$\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$$\downarrow 2838$$

$$-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\operatorname{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int(e+fx)\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(b\ \cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{b\ \cdots}$$

$$\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$\downarrow$ 7293

$$-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\operatorname{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int\left(a(e+fx)\operatorname{sech}^2(c+dx)-b(e+fx)\operatorname{sech}(c+dx)\tan\right.}{a^2+b^2}\right.}{}\right.}{}\right)}{}$$

$$\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

input
```
Int[((e + f*x)*Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1896 vs. $2(432) = 864$.

Time = 1.25 (sec) , antiderivative size = 1897, normalized size of antiderivative = 4.18

method	result	size
risch	Expression too large to display	1897

input
```
int((f*x+e)*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
-2/d^2*a^3/(a^2+b^2)^(3/2)*c*b*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)
+2*a)/(a^2+b^2)^(1/2))+2/b/(a^2+b^2)^(3/2)/d^2*a^5*f/(2*a^2+2*b^2)*dilog((
b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-2/b/(a^2+b^2)^(3/2)/d
^2*a^5*f/(2*a^2+2*b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^
2)^(1/2)))+2*b/(a^2+b^2)^(1/2)/d^2*a*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(
d*x+c)+2*a)/(a^2+b^2)^(1/2))+2*b^3/(a^2+b^2)^(3/2)/d^2*a*f/(2*a^2+2*b^2)*a
rctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2*b/(a^2+b^2)^(3/2)/d^2*a
^3*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/b/(
a^2+b^2)^(3/2)/d*a^5*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2
+b^2)^(1/2))+2*b/(a^2+b^2)^(3/2)/d^2*a^3*f/(2*a^2+2*b^2)*dilog((b*exp(d*x+
c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-2*b/(a^2+b^2)^(3/2)/d^2*a^3*f/(
2*a^2+2*b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))
+2*b^3/(a^2+b^2)/d^2*f/(2*a^2+2*b^2)*ln(1+exp(2*d*x+2*c))-b^3/(a^2+b^2)/d^
2*f/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+b/(a^2+b^2)^2/d^2*
f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)*a^2-4/(a^2+b^2)/d^2*a^3*f/(2*a^2+2
*b^2)*arctan(exp(d*x+c))+2*b/(a^2+b^2)/d^2*a^2*f/(2*a^2+2*b^2)*ln(1+exp(2*
d*x+2*c))-2*b/(a^2+b^2)/d^2*a^2*f/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*ex
p(d*x+c)-b)-4*b^2/(a^2+b^2)/d^2*f/(2*a^2+2*b^2)*a*arctan(exp(d*x+c))-2*b/(
a^2+b^2)^(5/2)/d^2*f*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^3
-2*b^3/(a^2+b^2)^(5/2)/d^2*f*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1571 vs. $2(430) = 860$.

Time = 0.14 (sec) , antiderivative size = 1571, normalized size of antiderivative = 3.46

$$\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input

```
integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output

```
1/2*((a^4 + 2*a^2*b^2 + b^4)*d^2*f*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*e*x
 + 4*(a^2*b^2 + b^4)*d*e + ((a^4 + 2*a^2*b^2 + b^4)*d^2*f*x^2 + 2*((a^4 +
2*a^2*b^2 + b^4)*d^2*e - 2*(a^2*b^2 + b^4)*d*f)*x)*cosh(d*x + c)^2 + ((a^4
 + 2*a^2*b^2 + b^4)*d^2*f*x^2 + 2*((a^4 + 2*a^2*b^2 + b^4)*d^2*e - 2*(a^2*
b^2 + b^4)*d*f)*x)*sinh(d*x + c)^2 - 2*(a^3*b*f*cosh(d*x + c)^2 + 2*a^3*b*
f*cosh(d*x + c)*sinh(d*x + c) + a^3*b*f*sinh(d*x + c)^2 + a^3*b*f)*sqrt((a
^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
 + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(a^3*b*f*cosh(d*
x + c)^2 + 2*a^3*b*f*cosh(d*x + c)*sinh(d*x + c) + a^3*b*f*sinh(d*x + c)^2
 + a^3*b*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c)
 - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) +
 2*(a^3*b*d*e - a^3*b*c*f + (a^3*b*d*e - a^3*b*c*f)*cosh(d*x + c)^2 + 2*(a
^3*b*d*e - a^3*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d*e - a^3*b*c*f
)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(
d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(a^3*b*d*e - a^3*b*c*f + (
a^3*b*d*e - a^3*b*c*f)*cosh(d*x + c)^2 + 2*(a^3*b*d*e - a^3*b*c*f)*cosh(d*
x + c)*sinh(d*x + c) + (a^3*b*d*e - a^3*b*c*f)*sinh(d*x + c)^2)*sqrt((a^2
+ b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^
2)/b^2) + 2*a) - 2*(a^3*b*d*f*x + a^3*b*c*f + (a^3*b*d*f*x + a^3*b*c*f)*co
sh(d*x + c)^2 + 2*(a^3*b*d*f*x + a^3*b*c*f)*cosh(d*x + c)*sinh(d*x + c)...
```

Sympy [F]

$$\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*sinh(c + d*x)*tanh(c + d*x)**2/(a + b*sinh(c + d*x)), x
)
```

Maxima [F]

$$\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\sinh(dx+c)\tanh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-(a^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqr
t(a^2 + b^2)))/((a^2*b + b^3)*sqrt(a^2 + b^2)*d) - 2*(a*e^(-d*x - c) - b)/
((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c))*d) - (d*x + c)/(b*d))*e - 1/2*
(4*a^3*integrate(-x*e^(d*x + c)/(a^2*b^2 + b^4 - (a^2*b^2*e^(2*c) + b^4*e^
(2*c))*e^(2*d*x) - 2*(a^3*b*e^c + a*b^3*e^c)*e^(d*x)), x) - ((a^2*d*e^(2*c
) + b^2*d*e^(2*c))*x^2*e^(2*d*x) + 4*a*b*x*e^(d*x + c) + 4*b^2*x + (a^2*d
+ b^2*d)*x^2)/(a^2*b*d + b^3*d + (a^2*b*d*e^(2*c) + b^3*d*e^(2*c))*e^(2*d*
x)) + 4*b*x/((a^2 + b^2)*d) + 4*a*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) -
2*b*log(e^(2*d*x + 2*c) + 1)/((a^2 + b^2)*d^2))*f
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)\,\tanh(c+d\,x)^2\,(e+f\,x)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((sinh(c + d*x)*tanh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)*tanh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
( - 4*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**5*b*f - 8*e**(2*c + 2*d*x)*at
an(e**(c + d*x))*a**3*b**3*f - 4*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a*b**
5*f - 4*atan(e**(c + d*x))*a**5*b*f - 8*atan(e**(c + d*x))*a**3*b**3*f - 4
*atan(e**(c + d*x))*a*b**5*f - 4*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((
e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**3*b**2*d*e*i - 4*sqrt(a**2 +
 b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**3*b**2*d*e*i -
16*e**(5*c + 2*d*x)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5
*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b +
 2*e**(c + d*x)*a - b),x)*a**7*d**2*f - 32*e**(5*c + 2*d*x)*int((e**(3*d*x
)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e
**(3*c + 3*d*x)*a - e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**5*b**
2*d**2*f - 16*e**(5*c + 2*d*x)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*
e**(5*c + 5*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - e**(2*c +
 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**3*b**4*d**2*f - 2*e**(2*c + 2*d*x)
*log(e**(2*c + 2*d*x) + 1)*a**6*f - 2*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x
) + 1)*a**4*b**2*f + 2*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2*b**
4*f + 2*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x) + 1)*b**6*f + 4*e**(2*c + 2*
d*x)*a**6*d*f*x + 2*e**(2*c + 2*d*x)*a**4*b**2*d**2*e*x + e**(2*c + 2*d*x)
*a**4*b**2*d**2*f*x**2 + 4*e**(2*c + 2*d*x)*a**4*b**2*d*f*x + 4*e**(2*c +
2*d*x)*a**2*b**4*d**2*e*x + 2*e**(2*c + 2*d*x)*a**2*b**4*d**2*f*x**2 - ...
```

3.414 $\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 121

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\frac{a^2x}{b(a^2+b^2)}+\frac{bx}{a^2+b^2}+\frac{2a^3\text{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d}+\frac{a\text{sech}(c+dx)}{(a^2+b^2)d}-\frac{b\tanh(c+dx)}{(a^2+b^2)d}$$

output
```
a^2*x/b/(a^2+b^2)+b*x/(a^2+b^2)+2*a^3*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a
^2+b^2)^(1/2))/b/(a^2+b^2)^(3/2)/d+a*sech(d*x+c)/(a^2+b^2)/d-b*tanh(d*x+c)
/(a^2+b^2)/d
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\frac{\frac{c+dx}{b}+\frac{2a^3\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{b(-a^2-b^2)^{3/2}}+\frac{\text{sech}(c+dx)(a-b\sinh(c+dx))}{a^2+b^2}}{d}$$

input
```
Integrate[(Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
((c + d*x)/b + (2*a^3*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/
(b*(-a^2 - b^2)^(3/2)) + (Sech[c + d*x]*(a - b*Sinh[c + d*x]))/(a^2 + b^2)
)/d
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.05, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.630$, Rules used = {3042, 26, 3381, 25, 26, 3042, 25, 26, 3086, 24, 3214, 3042, 3139, 1083, 217, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ic+idx)^3}{\cos(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ic+idx)^3}{\cos(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3381}$$

$$i\left(-\frac{a^2\int \frac{i\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{ib\int -\tanh^2(c+dx)dx}{a^2+b^2}+\frac{a\int i\text{sech}(c+dx)\tanh(c+dx)dx}{a^2+b^2}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(-\frac{a^2\int \frac{i\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{ib\int \tanh^2(c+dx)dx}{a^2+b^2}+\frac{a\int i\text{sech}(c+dx)\tanh(c+dx)dx}{a^2+b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{ia^2\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{ib\int \tanh^2(c+dx)dx}{a^2+b^2}+\frac{ia\int \operatorname{sech}(c+dx)\tanh(c+dx)dx}{a^2+b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(-\frac{ia^2\int -\frac{i\sin(ic+idx)}{a-ib\sin(ic+idx)}dx}{a^2+b^2}-\frac{ib\int -\tan(ic+idx)^2dx}{a^2+b^2}+\frac{ia\int -i\sec(ic+idx)\tan(ic+idx)dx}{a^2+b^2}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(-\frac{ia^2\int -\frac{i\sin(ic+idx)}{a-ib\sin(ic+idx)}dx}{a^2+b^2}+\frac{ib\int \tan(ic+idx)^2dx}{a^2+b^2}+\frac{ia\int -i\sec(ic+idx)\tan(ic+idx)dx}{a^2+b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{a^2\int \frac{\sin(ic+idx)}{a-ib\sin(ic+idx)}dx}{a^2+b^2}+\frac{ib\int \tan(ic+idx)^2dx}{a^2+b^2}+\frac{a\int \sec(ic+idx)\tan(ic+idx)dx}{a^2+b^2}\right)$$

$$\Big\downarrow \text{3086}$$

$$i\left(-\frac{a^2\int \frac{\sin(ic+idx)}{a-ib\sin(ic+idx)}dx}{a^2+b^2}+\frac{ib\int \tan(ic+idx)^2dx}{a^2+b^2}-\frac{ia\int 1d\operatorname{sech}(c+dx)}{d(a^2+b^2)}\right)$$

$$\Big\downarrow \text{24}$$

$$i\left(-\frac{a^2\int \frac{\sin(ic+idx)}{a-ib\sin(ic+idx)}dx}{a^2+b^2}+\frac{ib\int \tan(ic+idx)^2dx}{a^2+b^2}-\frac{ia\operatorname{sech}(c+dx)}{d(a^2+b^2)}\right)$$

$$\Big\downarrow \text{3214}$$

$$i\left(\frac{ib\int \tan(ic+idx)^2dx}{a^2+b^2}-\frac{a^2\left(\frac{ix}{b}-\frac{ia\int \frac{1}{a+b\sinh(c+dx)}dx}{b}\right)}{a^2+b^2}-\frac{ia\operatorname{sech}(c+dx)}{d(a^2+b^2)}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(-\frac{a^2\left(\frac{ix}{b}-\frac{ia\int \frac{1}{a-ib\sin(ic+idx)}dx}{b}\right)}{a^2+b^2}+\frac{ib\int \tan(ic+idx)^2dx}{a^2+b^2}-\frac{ia\operatorname{sech}(c+dx)}{d(a^2+b^2)}\right)$$

$$\Big\downarrow \text{3139}$$

$$i\left(\frac{ib\int \tan(ic+idx)^2dx}{a^2+b^2}-\frac{a^2\left(\frac{ix}{b}-\frac{2a\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{bd}\right)}{a^2+b^2}-\frac{ia\text{sech}(c+dx)}{d\left(a^2+b^2\right)}\right)$$

$$\Big\downarrow \text{1083}$$

$$i\left(\frac{ib\int \tan(ic+idx)^2dx}{a^2+b^2}-\frac{a^2\left(\frac{4a\int \frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4\left(a^2+b^2\right)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{bd}+\frac{ix}{b}\right)}{a^2+b^2}-\frac{ia\text{sech}(c+dx)}{d\left(a^2+b^2\right)}\right)$$

$$\Big\downarrow \text{217}$$

$$i\left(\frac{ib\int \tan(ic+idx)^2dx}{a^2+b^2}-\frac{a^2\left(\frac{ix}{b}-\frac{2ia\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{bd\sqrt{a^2+b^2}}\right)}{a^2+b^2}-\frac{ia\text{sech}(c+dx)}{d\left(a^2+b^2\right)}\right)$$

$$\Big\downarrow \text{3954}$$

$$i\left(\frac{ib\left(\frac{\tanh(c+dx)}{d}-\int 1dx\right)}{a^2+b^2}-\frac{a^2\left(\frac{ix}{b}-\frac{2ia\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{bd\sqrt{a^2+b^2}}\right)}{a^2+b^2}-\frac{ia\text{sech}(c+dx)}{d\left(a^2+b^2\right)}\right)$$

$$\Big\downarrow \text{24}$$

$$i\left(-\frac{a^2\left(\frac{ix}{b}-\frac{2ia\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{bd\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{ib\left(\frac{\tanh(c+dx)}{d}-x\right)}{a^2+b^2}-\frac{ia\text{sech}(c+dx)}{d\left(a^2+b^2\right)}\right)$$

input
```
Int[(Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
I*(-((a^2*((I*x)/b - ((2*I)*a*ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b^2]
)])/(b*Sqrt[a^2 + b^2]*d)))/(a^2 + b^2)) - (I*a*Sech[c + d*x])/((a^2 + b^2
)*d) + (I*b*(-x + Tanh[c + d*x]/d))/(a^2 + b^2))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3381
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a*(d^2/(a^2
- b^2))   Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Simp[
b*(d/(a^2 - b^2))   Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x]
 - Simp[a^2*(d^2/(g^2*(a^2 - b^2)))   Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[
e + f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g
}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1
]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{-\frac{2\left(b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{\left(a^2+b^2\right)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}-\frac{2a^3\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{\frac{3}{2}}}+\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}}{d}$
default	$\frac{-\frac{2\left(b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{\left(a^2+b^2\right)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}-\frac{2a^3\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{\frac{3}{2}}}+\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}}{d}$
risch	$\frac{x}{b}+\frac{2a\,\mathrm{e}^{dx+c}+2b}{d(a^2+b^2)\left(1+\mathrm{e}^{2dx+2c}\right)}+\frac{a^3\ln\left(\mathrm{e}^{dx+c}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}db}-\frac{a^3\ln\left(\mathrm{e}^{dx+c}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}-a^4-2a^2b^2}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}db}$

input
```
int(sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/(a^2+b^2)*(b*tanh(1/2*d*x+1/2*c)-a)/(1+tanh(1/2*d*x+1/2*c)^2)-2/b*
a^3/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1
/2))+1/b*ln(1+tanh(1/2*d*x+1/2*c))-1/b*ln(tanh(1/2*d*x+1/2*c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(118) = 236$.

Time = 0.09 (sec) , antiderivative size = 459, normalized size of antiderivative = 3.79

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{(a^4+2\,a^2b^2+b^4)dx\cosh\left(dx+c\right)^2+(a^4+2\,a^2b^2+b^4)dx\sinh\left(dx+c\right)^2+2\,a^2b^2+2\,b^4+(a^4+2\,a^2b^2+}{}$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
((a^4 + 2*a^2*b^2 + b^4)*d*x*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d*x
*sinh(d*x + c)^2 + 2*a^2*b^2 + 2*b^4 + (a^4 + 2*a^2*b^2 + b^4)*d*x + (a^3*
cosh(d*x + c)^2 + 2*a^3*cosh(d*x + c)*sinh(d*x + c) + a^3*sinh(d*x + c)^2
+ a^3)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*
a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c
) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x
 + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*
sinh(d*x + c) - b)) + 2*(a^3*b + a*b^3)*cosh(d*x + c) + 2*(a^3*b + a*b^3 +
 (a^4 + 2*a^2*b^2 + b^4)*d*x*cosh(d*x + c))*sinh(d*x + c))/((a^4*b + 2*a^2
*b^3 + b^5)*d*cosh(d*x + c)^2 + 2*(a^4*b + 2*a^2*b^3 + b^5)*d*cosh(d*x + c
)*sinh(d*x + c) + (a^4*b + 2*a^2*b^3 + b^5)*d*sinh(d*x + c)^2 + (a^4*b + 2
*a^2*b^3 + b^5)*d)
```

Sympy [F]

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh{(c+dx)}\tanh^2{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(sinh(c + d*x)*tanh(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.17

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a^3\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{(a^2b+b^3)\sqrt{a^2+b^2}d} + \frac{2\left(ae^{(-dx-c)}-b\right)}{(a^2+b^2+(a^2+b^2)e^{(-2dx-2c)})d} + \frac{dx+c}{bd}$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-a^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt
(a^2 + b^2)))/((a^2*b + b^3)*sqrt(a^2 + b^2)*d) + 2*(a*e^(-d*x - c) - b)/(
(a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c))*d) + (d*x + c)/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.06

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{a^3\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2b+b^3)\sqrt{a^2+b^2}}-\frac{dx+c}{b}-\frac{2\,(ae^{(dx+c)}+b)}{(a^2+b^2)\left(e^{(2\,dx+2\,c)}+1\right)}}{d}$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-(a^3*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x +
c) + 2*a + 2*sqrt(a^2 + b^2)))/((a^2*b + b^3)*sqrt(a^2 + b^2)) - (d*x + c)
/b - 2*(a*e^(d*x + c) + b)/((a^2 + b^2)*(e^(2*d*x + 2*c) + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 3.15 (sec) , antiderivative size = 468, normalized size of antiderivative = 3.87

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{2\,b}{d\,(a^2+b^2)}+\frac{2\,a\,e^{c+d\,x}}{d\,(a^2+b^2)}}{e^{2\,c+2\,d\,x}+1}+\frac{x}{b} + \frac{2\,\mathrm{atan}\left(\left(e^{d\,x}\,e^{c}\left(\frac{2\,a^3}{b^3\,d\,(a^2\,b+b^3)\,\sqrt{a^6}\,(a^2+b^2)}+\frac{2\,\left(a\,b^3\,d\,\sqrt{a^6}+a^3\,b\,d\,\sqrt{a^6}\right)}{a^2\,b^2\,(a^2\,b+b^3)\,\sqrt{-b^2\,d^2\,(a^2+b^2)^3}\,\sqrt{-a^6\,b^2\,d^2-3\,a^4\,b^4\,d^2-3\,a^2\,b^6\,d^2-b^8\,d^2}}\right)-\cdots\right.\right.}{\sqrt{-\cdots}}$$

input
```
int((sinh(c + d*x)*tanh(c + d*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
((2*b)/(d*(a^2 + b^2)) + (2*a*exp(c + d*x))/(d*(a^2 + b^2)))/(exp(2*c + 2*
d*x) + 1) + x/b + (2*atan((exp(d*x)*exp(c)*((2*a^3)/(b^3*d*(a^2*b + b^3)*(
a^6)^(1/2)*(a^2 + b^2)) + (2*(a*b^3*d*(a^6)^(1/2) + a^3*b*d*(a^6)^(1/2)))/
(a^2*b^2*(a^2*b + b^3)*(-b^2*d^2*(a^2 + b^2)^3)^(1/2)*(- b^8*d^2 - 3*a^2*b
^6*d^2 - 3*a^4*b^4*d^2 - a^6*b^2*d^2)^(1/2))) - (2*(b^4*d*(a^6)^(1/2) + a^
2*b^2*d*(a^6)^(1/2)))/(a^2*b^2*(a^2*b + b^3)*(-b^2*d^2*(a^2 + b^2)^3)^(1/2
)*(- b^8*d^2 - 3*a^2*b^6*d^2 - 3*a^4*b^4*d^2 - a^6*b^2*d^2)^(1/2)))*((b^4*
(- b^8*d^2 - 3*a^2*b^6*d^2 - 3*a^4*b^4*d^2 - a^6*b^2*d^2)^(1/2))/2 + (a^2*
b^2*(- b^8*d^2 - 3*a^2*b^6*d^2 - 3*a^4*b^4*d^2 - a^6*b^2*d^2)^(1/2))/2))*(
a^6)^(1/2))/(- b^8*d^2 - 3*a^2*b^6*d^2 - 3*a^4*b^4*d^2 - a^6*b^2*d^2)^(1/2
)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 396, normalized size of antiderivative = 3.27

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-2e^{2dx+2c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)a^3i-2\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)a^3i-e^{2dx+2c}\tanh(dx+c)\,a^4-2e}{}$$

input
```
int(sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqr
t(a**2 + b**2))*a**3*i - 2*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)
/sqrt(a**2 + b**2))*a**3*i - e**(2*c + 2*d*x)*tanh(c + d*x)*a**4 - 2*e**(2
*c + 2*d*x)*tanh(c + d*x)*a**2*b**2 - e**(2*c + 2*d*x)*tanh(c + d*x)*b**4
+ e**(2*c + 2*d*x)*a**4*d*x + 2*e**(2*c + 2*d*x)*a**4 + 2*e**(2*c + 2*d*x)
*a**2*b**2*d*x + 2*e**(2*c + 2*d*x)*a**2*b**2 + e**(2*c + 2*d*x)*b**4*d*x
+ 2*e**(c + d*x)*a**3*b + 2*e**(c + d*x)*a*b**3 - tanh(c + d*x)*a**4 - 2*t
anh(c + d*x)*a**2*b**2 - tanh(c + d*x)*b**4 + a**4*d*x + 2*a**2*b**2*d*x +
 b**4*d*x)/(b*d*(e**(2*c + 2*d*x)*a**4 + 2*e**(2*c + 2*d*x)*a**2*b**2 + e*
*(2*c + 2*d*x)*b**4 + a**4 + 2*a**2*b**2 + b**4))
```

3.415 $\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(sinh(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 36.68 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Sinh[c + d*x]*Tanh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Sinh[c + d*x]*Tanh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{ 6125}$$
$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Sinh[c + d*x]*Tanh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(dx+c)\tanh(dx+c)^2}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(sinh(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)\tanh\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(sinh(d*x + c)*tanh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 2.54 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh\left(c+dx\right)\tanh^2\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(sinh(c + d*x)*tanh(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)),
 x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 411, normalized size of antiderivative = 12.09

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)\tanh(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-2*a^3*integrate(-e^(d*x + c)/(a^2*b^2*e + b^4*e + (a^2*b^2*f + b^4*f)*x -
 (a^2*b^2*e*e^(2*c) + b^4*e*e^(2*c) + (a^2*b^2*f*e^(2*c) + b^4*f*e^(2*c))*
x)*e^(2*d*x) - 2*(a^3*b*e*e^c + a*b^3*e*e^c + (a^3*b*f*e^c + a*b^3*f*e^c)*
x)*e^(d*x)), x) + 2*(a*e^(d*x + c) + b)/(a^2*d*e + b^2*d*e + (a^2*d*f + b^
2*d*f)*x + (a^2*d*e*e^(2*c) + b^2*d*e*e^(2*c) + (a^2*d*f*e^(2*c) + b^2*d*f
*e^(2*c))*x)*e^(2*d*x)) + log(f*x + e)/(b*f) + 1/2*integrate(4*(a*f*e^(d*x
 + c) + b*f)/(a^2*d*e^2 + b^2*d*e^2 + (a^2*d*f^2 + b^2*d*f^2)*x^2 + 2*(a^2
*d*e*f + b^2*d*e*f)*x + (a^2*d*e^2*e^(2*c) + b^2*d*e^2*e^(2*c) + (a^2*d*f^
2*e^(2*c) + b^2*d*f^2*e^(2*c))*x^2 + 2*(a^2*d*e*f*e^(2*c) + b^2*d*e*f*e^(2
*c))*x)*e^(2*d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 3.87 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(c+d\,x)\,\tanh(c+d\,x)^2}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((sinh(c + d*x)*tanh(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((sinh(c + d*x)*tanh(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 981, normalized size of antiderivative = 28.85

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$= \frac{-2e^{5c}\left(\int \frac{e^{5dx}}{e^{6dx+6c}be+e^{6dx+6c}bfx+2e^{5dx+5c}ae+2e^{5dx+5c}afx+e^{4dx+4c}be+e^{4dx+4c}bfx+4e^{3dx+3c}ae+4e^{3dx+3c}afx-e^{2dx+2c}be-e^{2dx+2c}bf}\right.}{}$$

input
```
int(sinh(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*e**(5*c)*int(e**(5*d*x)/(e**(6*c + 6*d*x)*b*e + e**(6*c + 6*d*x)*b*f
*x + 2*e**(5*c + 5*d*x)*a*e + 2*e**(5*c + 5*d*x)*a*f*x + e**(4*c + 4*d*x)*
b*e + e**(4*c + 4*d*x)*b*f*x + 4*e**(3*c + 3*d*x)*a*e + 4*e**(3*c + 3*d*x)
*a*f*x - e**(2*c + 2*d*x)*b*e - e**(2*c + 2*d*x)*b*f*x + 2*e**(c + d*x)*a*
e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x)*a*f - 4*e**(4*c)*int(e**(4*d*x)
/(e**(6*c + 6*d*x)*b*e + e**(6*c + 6*d*x)*b*f*x + 2*e**(5*c + 5*d*x)*a*e +
 2*e**(5*c + 5*d*x)*a*f*x + e**(4*c + 4*d*x)*b*e + e**(4*c + 4*d*x)*b*f*x
+ 4*e**(3*c + 3*d*x)*a*e + 4*e**(3*c + 3*d*x)*a*f*x - e**(2*c + 2*d*x)*b*e
 - e**(2*c + 2*d*x)*b*f*x + 2*e**(c + d*x)*a*e + 2*e**(c + d*x)*a*f*x - b*
e - b*f*x),x)*b*f - 4*e**(3*c)*int(e**(3*d*x)/(e**(6*c + 6*d*x)*b*e + e**(
6*c + 6*d*x)*b*f*x + 2*e**(5*c + 5*d*x)*a*e + 2*e**(5*c + 5*d*x)*a*f*x + e
**(4*c + 4*d*x)*b*e + e**(4*c + 4*d*x)*b*f*x + 4*e**(3*c + 3*d*x)*a*e + 4*
e**(3*c + 3*d*x)*a*f*x - e**(2*c + 2*d*x)*b*e - e**(2*c + 2*d*x)*b*f*x + 2
*e**(c + d*x)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x)*a*f + 4*e**(2*c
)*int(e**(2*d*x)/(e**(6*c + 6*d*x)*b*e + e**(6*c + 6*d*x)*b*f*x + 2*e**(5*
c + 5*d*x)*a*e + 2*e**(5*c + 5*d*x)*a*f*x + e**(4*c + 4*d*x)*b*e + e**(4*c
 + 4*d*x)*b*f*x + 4*e**(3*c + 3*d*x)*a*e + 4*e**(3*c + 3*d*x)*a*f*x - e**(
2*c + 2*d*x)*b*e - e**(2*c + 2*d*x)*b*f*x + 2*e**(c + d*x)*a*e + 2*e**(c +
 d*x)*a*f*x - b*e - b*f*x),x)*b*f - 2*e**c*int(e**(d*x)/(e**(6*c + 6*d*x)*
b*e + e**(6*c + 6*d*x)*b*f*x + 2*e**(5*c + 5*d*x)*a*e + 2*e**(5*c + 5*d...
```

3.416 $\int \frac{(e+fx)^2 \tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 1479

$$\int \frac{(e+fx)^2 \tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output
```
I*f*(f*x+e)*polylog(2,I*exp(d*x+c))/b/d^2+I*f^2*polylog(3,-I*exp(d*x+c))/b
/d^3-f*(f*x+e)*sech(d*x+c)/b/d^2+f^2*arctan(sinh(d*x+c))/b/d^3+a*f^2*ln(co
sh(d*x+c))/b^2/d^3+a^3*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/(a^2+b^2)^2/d-a^2*f^
2*arctan(sinh(d*x+c))/b^3/d^3+a^3*f*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/(a^
2+b^2)^2/d^2+a^4*f^2*arctan(sinh(d*x+c))/b^3/(a^2+b^2)/d^3-a^3*f^2*ln(cosh
(d*x+c))/b^2/(a^2+b^2)/d^3-a*f*(f*x+e)*tanh(d*x+c)/b^2/d^2+a^2*f*(f*x+e)*s
ech(d*x+c)/b^3/d^2+I*a^2*f^2*polylog(3,-I*exp(d*x+c))/b^3/d^3-1/2*a^4*(f*x
+e)^2*sech(d*x+c)*tanh(d*x+c)/b^3/(a^2+b^2)/d-2*I*a^4*f^2*polylog(3,-I*exp
(d*x+c))/b/(a^2+b^2)^2/d^3-I*a^4*f^2*polylog(3,-I*exp(d*x+c))/b^3/(a^2+b^2
)/d^3-I*a^2*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/b^3/d^2+a^2*(f*x+e)^2*arcta
n(exp(d*x+c))/b^3/d-2*I*a^4*f*(f*x+e)*polylog(2,I*exp(d*x+c))/b/(a^2+b^2)^
2/d^2-I*a^4*f*(f*x+e)*polylog(2,I*exp(d*x+c))/b^3/(a^2+b^2)/d^2-a^3*(f*x+e
)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d-a^3*(f*x+e)^2*ln(
1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d+2*a^3*f^2*polylog(3,-b*e
xp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^3+2*a^3*f^2*polylog(3,-b*exp(
d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^3+(f*x+e)^2*arctan(exp(d*x+c))/b
/d-1/2*(f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/b/d-I*f^2*polylog(3,I*exp(d*x+c))
/b/d^3-a^4*(f*x+e)^2*arctan(exp(d*x+c))/b^3/(a^2+b^2)/d-2*a^4*(f*x+e)^2*ar
ctan(exp(d*x+c))/b/(a^2+b^2)^2/d-1/2*a^3*(f*x+e)^2*sech(d*x+c)^2/b^2/(a^2+
b^2)/d+1/2*a^2*(f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/b^3/d-I*a^2*f^2*polylo...
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3368 vs. 2(1479) = 2958.

Time = 11.47 (sec) , antiderivative size = 3368, normalized size of antiderivative = 2.28

$$\int \frac{(e+fx)^2\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Tanh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-12*a^3*d^3*e^2*E^(2*c)*x - 12*a^3*d*E^(2*c)*f^2*x - 12*a*b^2*d*E^(2*c)*f
^2*x - 12*a^3*d^3*e*E^(2*c)*f*x^2 - 4*a^3*d^3*E^(2*c)*f^2*x^3 + 18*a^2*b*d
^2*e^2*ArcTan[E^(c + d*x)] + 6*b^3*d^2*e^2*ArcTan[E^(c + d*x)] + 18*a^2*b*
d^2*e^2*E^(2*c)*ArcTan[E^(c + d*x)] + 6*b^3*d^2*e^2*E^(2*c)*ArcTan[E^(c +
d*x)] + 12*a^2*b*f^2*ArcTan[E^(c + d*x)] + 12*b^3*f^2*ArcTan[E^(c + d*x)]
+ 12*a^2*b*E^(2*c)*f^2*ArcTan[E^(c + d*x)] + 12*b^3*E^(2*c)*f^2*ArcTan[E^(
c + d*x)] + (18*I)*a^2*b*d^2*e*f*x*Log[1 - I*E^(c + d*x)] + (6*I)*b^3*d^2*
e*f*x*Log[1 - I*E^(c + d*x)] + (18*I)*a^2*b*d^2*e*E^(2*c)*f*x*Log[1 - I*E^
(c + d*x)] + (6*I)*b^3*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] + (9*I)*a^
2*b*d^2*f^2*x^2*Log[1 - I*E^(c + d*x)] + (3*I)*b^3*d^2*f^2*x^2*Log[1 - I*E
^(c + d*x)] + (9*I)*a^2*b*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] + (3*
I)*b^3*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] - (18*I)*a^2*b*d^2*e*f*x
*Log[1 + I*E^(c + d*x)] - (6*I)*b^3*d^2*e*f*x*Log[1 + I*E^(c + d*x)] - (18
*I)*a^2*b*d^2*e*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] - (6*I)*b^3*d^2*e*E^(2*
c)*f*x*Log[1 + I*E^(c + d*x)] - (9*I)*a^2*b*d^2*f^2*x^2*Log[1 + I*E^(c + d
*x)] - (3*I)*b^3*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] - (9*I)*a^2*b*d^2*E^(2
*c)*f^2*x^2*Log[1 + I*E^(c + d*x)] - (3*I)*b^3*d^2*E^(2*c)*f^2*x^2*Log[1 +
 I*E^(c + d*x)] + 6*a^3*d^2*e^2*Log[1 + E^(2*(c + d*x))] + 6*a^3*d^2*e^2*E
^(2*c)*Log[1 + E^(2*(c + d*x))] + 6*a^3*f^2*Log[1 + E^(2*(c + d*x))] + 6*a
*b^2*f^2*Log[1 + E^(2*(c + d*x))] + 6*a^3*E^(2*c)*f^2*Log[1 + E^(2*(c +...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\tanh^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6101}$$

$$\frac{\int (e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 5978}$$

$$\frac{\int (e+fx)^2\text{sech}(c+dx)dx-\int (e+fx)^2\text{sech}^3(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int\frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx-\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{b}$$

$\downarrow$ 4668

$$-\frac{a\int\frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{2if\int(e+fx)\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{2if\int(e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}-\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx+\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}}{b}$$

$\downarrow$ 3011

$$-\frac{a\int\frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2if\left(\frac{f\int\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}-\int(e+fx)^2c}{b}$$

$\downarrow$ 2720

$$-\frac{a\int\frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c+d}}{d}\right.}{d}}{b}$$

$\downarrow$ 4674

$$-\frac{a\int\frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c+d}}{d}\right.}{d}$$

$\downarrow$ 3042

$$-\frac{a\int\frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c+d}}{d}\right.}{d}$$

$\downarrow$ 4257

$$-\frac{a\int\frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,ie^{c+d}\right.}{d}\right.}{d}}{}$$

$$\Big\downarrow \text{4668}$$

$$-\frac{a\int\frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(\frac{2if\int(e+fx)\log\left(1-ie^{c+dx}\right)dx}{d}-\frac{2if\int(e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}-\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c}}{d^2}\right.}{d}}{}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{a\int\frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(-\frac{2if\left(\frac{f\int\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}-\frac{2(e+fx)^2}{}\right.}{}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{a\int\frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}(}{d}\right.}{d}\right.}{}$$

$$\Big\downarrow \text{6117}$$

$$-\frac{a\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}(}{d}\right.}{d}\right.}{}$$

$$\Big\downarrow \text{5974}$$

$$-\frac{a\left(\frac{\frac{f\int (e+fx)\text{sech}^2(c+dx)dx}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+$$

$$\frac{\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}(}{d}\right.}{d}\right.}{}$$

$\downarrow$ 3042

$$\frac{\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}(}{d}\right.}{d}\right.}{}$$

$$\frac{a\left(-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}+\frac{f\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{d}}{b}\right)}{b}$$

$\downarrow$ 4672

$$\frac{\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}(}{d}\right.}{d}\right.}{}$$

$$\frac{a\left(-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}+\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{if\int -i\tanh(c+dx)dx}{d}\right)}{d}}{b}\right)}{b}$$

$\downarrow$ 26

$$-\frac{a\left(\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int \tanh(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+$$

$$\frac{\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}(}{d}\right.}{d}\right.}{}$$

$\downarrow$ 3042

$$\frac{\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}(}{d}\right.}{d}\right.}{\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}+\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int -i\tan(ic+idx)dx}{d}\right)}{d}}{b}\right)}{b}}$$

$$\downarrow 26$$

$$\frac{\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}(}{d}\right.}{d}\right.}{\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}+\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}+\frac{if\int\tan(ic+idx)dx}{d}\right)}{d}}{b}\right)}{b}}$$

$$\downarrow 3956$$

$$-\frac{a\left(\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+$$

$$\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}(}{d}\right.}{d}\right.$$

$$\downarrow 6117$$

$$-\frac{a\left(\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{\int(e+fx)^2\text{sech}^3(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}\right)}{b}+$$

$$\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}(}{d}\right.}{d}\right.$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{2}\left(-\frac{2 i f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2 i f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(\right.}{d}\right.}{d}\right.}{a\left(\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{b}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 4674

$$-\frac{a\left(\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{-\frac{f^2\int\text{sech}(c+dx)dx}{d^2}+\frac{1}{2}\int(e+fx)^2\text{sech}(c+dx)dx+\frac{f(e+fx)\text{sech}(c+}{d^2}}{b}\right.}{b}\right.}{b}$$

$$\frac{1}{2}\left(-\frac{2 i f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2 i f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(\right.}{d}\right.}{d}\right.$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{2}\left(-\frac{2 i f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2 i f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(\right.}{d}\right.}{d}\right.}{a\left(\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{f^2\int\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{d^2}+\frac{1}{2}\int(e+fx)^2 c\right.}{}\right.}{b}\right.}$$

$\Big\downarrow$ 4257

$$\frac{\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(\cdots\right.}{d}\cdots\right.}{d}\cdots\right.}{\cdots}$$

$$\frac{a\left(\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{1}{2}\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f^2\operatorname{arcta}\cdots}{\cdots}\right.}{b}\cdots\right.}{b}$$

$\downarrow$ 4668

$$\frac{\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(\cdots\right.}{d}\cdots\right.}{d}\cdots\right.}{\cdots}$$

$$\frac{a\left(\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(-\frac{2if\int(e+fx)\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{2i\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\cdots\right.}{b}$$

$\downarrow$ 3011

$$\frac{\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(\cdots\right.}{d}\right.}{d}\right.}{a\left(\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\cdots\right.}{d}\right.}{\cdots}\right.}{\cdots}\right.}$$

$\downarrow$ 2720

$$\frac{\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(\cdots\right.}{d}\right.}{d}\right.}{a\left(\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+d\cdots}\right.}{d^2}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}$$

$\downarrow$ 6107

$$\frac{\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(\right.}{d}\right.}{d}\right.}{a\left(\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int\frac{(e+fx)^2\text{sech}(c+}{a+b\sinh(c+dx)}}{a^2+b^2}\right.}{b}\right.}{}\right.\right.}$$

$$\Big\downarrow \ 6107$$

$$\frac{\frac{\arctan(\sinh(c+dx))f^2}{d^3}-\frac{(e+fx)\text{sech}(c+dx)f}{d^2}+\frac{2i\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)f}{d}-\frac{2i\left(\frac{f\int e^{-c-}}{}\right.}{}}{a\left(\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{-\frac{\arctan(\sinh(c+dx))f^2}{d^3}+\frac{(e+fx)\text{sech}(c+dx)f}{d^2}+\frac{(e+fx)^2\text{sech}(c+dx)\tan}{2d}}{}\right.}{}\right.}$$

input
```
Int[((e + f*x)^2*Tanh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^2 \tanh(dx+c)^3}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 10574 vs. 2(1354) = 2708.

Time = 0.30 (sec) , antiderivative size = 10574, normalized size of antiderivative = 7.15

$$\int \frac{(e+fx)^2 \tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^2\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*tanh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*tanh(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)^2\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\tanh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
3*a^2*b*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2
*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d
^2 + b^4*d^2), x) + b^3*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*
x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d
^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - 2*a^3*d^2*f^2*integrate(x^2/(a^4*d^2*e
^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) +
 a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 6*a^2*b*d^2*e*f*integrate(x*e^(d
*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2
*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 2*b^3*d^2*e*f*
integrate(x*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x
+ 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x)
- 4*a^3*d^2*e*f*integrate(x/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*
d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2),
 x) - a^3*f^2*(2*(d*x + c)/((a^4 + 2*a^2*b^2 + b^4)*d^3) - log(e^(2*d*x +
2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d^3)) - a*b^2*f^2*(2*(d*x + c)/((a^4 +
2*a^2*b^2 + b^4)*d^3) - log(e^(2*d*x + 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*
d^3)) - (a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2
*b^2 + b^4)*d) - a^3*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d)
 + (3*a^2*b + b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) + (b*e
^(-d*x - c) - 2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^2 + ...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\tanh(c+d\,x)^3\,(e+f\,x)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((tanh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((tanh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{too large to display}$$

input
```
int((f*x+e)^2*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
(48*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**6*b*d*e*f + 16*e**(4*c + 4*d*x)
*atan(e**(c + d*x))*a**6*b*f**2 + 144*e**(4*c + 4*d*x)*atan(e**(c + d*x))*
a**4*b**3*d*e*f + 120*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**4*b**3*f**2 +
 54*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**2*b**5*d**2*e**2 + 144*e**(4*c
+ 4*d*x)*atan(e**(c + d*x))*a**2*b**5*d*e*f + 192*e**(4*c + 4*d*x)*atan(e*
*(c + d*x))*a**2*b**5*f**2 + 18*e**(4*c + 4*d*x)*atan(e**(c + d*x))*b**7*d
**2*e**2 + 48*e**(4*c + 4*d*x)*atan(e**(c + d*x))*b**7*d*e*f + 88*e**(4*c
+ 4*d*x)*atan(e**(c + d*x))*b**7*f**2 + 96*e**(2*c + 2*d*x)*atan(e**(c + d
*x))*a**6*b*d*e*f + 32*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**6*b*f**2 + 2
88*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**4*b**3*d*e*f + 240*e**(2*c + 2*d
*x)*atan(e**(c + d*x))*a**4*b**3*f**2 + 108*e**(2*c + 2*d*x)*atan(e**(c +
d*x))*a**2*b**5*d**2*e**2 + 288*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**2*b
**5*d*e*f + 384*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**2*b**5*f**2 + 36*e*
*(2*c + 2*d*x)*atan(e**(c + d*x))*b**7*d**2*e**2 + 96*e**(2*c + 2*d*x)*ata
n(e**(c + d*x))*b**7*d*e*f + 176*e**(2*c + 2*d*x)*atan(e**(c + d*x))*b**7*
f**2 + 48*atan(e**(c + d*x))*a**6*b*d*e*f + 16*atan(e**(c + d*x))*a**6*b*f
**2 + 144*atan(e**(c + d*x))*a**4*b**3*d*e*f + 120*atan(e**(c + d*x))*a**4
*b**3*f**2 + 54*atan(e**(c + d*x))*a**2*b**5*d**2*e**2 + 144*atan(e**(c +
d*x))*a**2*b**5*d*e*f + 192*atan(e**(c + d*x))*a**2*b**5*f**2 + 18*atan(e*
*(c + d*x))*b**7*d**2*e**2 + 48*atan(e**(c + d*x))*b**7*d*e*f + 88*atan...
```

3.417 $\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 894

$$\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output
```
1/2*a^3*f*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)^2/d^2+a^3*(f*x+e)*ln(1+exp(
2*d*x+2*c))/(a^2+b^2)^2/d+a^2*(f*x+e)*arctan(exp(d*x+c))/b^3/d-a^3*f*polyl
og(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2-a^3*f*polylog(2,-b
*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2-a^3*(f*x+e)*ln(1+b*exp(d*
x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d-a^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(
a^2+b^2)^(1/2)))/(a^2+b^2)^2/d-1/2*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/b/d-1/2
*I*f*polylog(2,-I*exp(d*x+c))/b/d^2-a^4*(f*x+e)*arctan(exp(d*x+c))/b^3/(a^
2+b^2)/d-2*a^4*(f*x+e)*arctan(exp(d*x+c))/b/(a^2+b^2)^2/d+1/2*a^3*f*tanh(d
*x+c)/b^2/(a^2+b^2)/d^2+1/2*a^2*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/b^3/d-1/2*
a^4*f*sech(d*x+c)/b^3/(a^2+b^2)/d^2-1/2*a^3*(f*x+e)*sech(d*x+c)^2/b^2/(a^2
+b^2)/d-1/2*I*a^2*f*polylog(2,-I*exp(d*x+c))/b^3/d^2+1/2*I*f*polylog(2,I*e
xp(d*x+c))/b/d^2+(f*x+e)*arctan(exp(d*x+c))/b/d-1/2*f*sech(d*x+c)/b/d^2-1/
2*a*f*tanh(d*x+c)/b^2/d^2+1/2*a^2*f*sech(d*x+c)/b^3/d^2+1/2*a*(f*x+e)*sech
(d*x+c)^2/b^2/d+I*a^4*f*polylog(2,-I*exp(d*x+c))/b/(a^2+b^2)^2/d^2+1/2*I*a
^4*f*polylog(2,-I*exp(d*x+c))/b^3/(a^2+b^2)/d^2+1/2*I*a^2*f*polylog(2,I*ex
p(d*x+c))/b^3/d^2-1/2*a^4*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/b^3/(a^2+b^2)/d-
I*a^4*f*polylog(2,I*exp(d*x+c))/b/(a^2+b^2)^2/d^2-1/2*I*a^4*f*polylog(2,I*
exp(d*x+c))/b^3/(a^2+b^2)/d^2
```

Mathematica [A] (warning: unable to verify)

Time = 8.95 (sec) , antiderivative size = 834, normalized size of antiderivative = 0.93

$$\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{a^3\left(-2de(c+dx)+2cf(c+dx)-f(c+dx)^2+\frac{4a\sqrt{a^2+b^2}de\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-(a^2+b^2)^2}}-\frac{4a\sqrt{-(a^2+b^2)^2}de\,\text{arctanh}\left(\frac{a\cdots}{\cdots}\right)}{(-a^2-b^2)^{3/2}}\cdots\right.}{\cdots}$$

$$+\frac{-2a^3de(c+dx)+2a^3cf(c+dx)-a^3f(c+dx)^2+6a^2bde\arctan\left(e^{c+dx}\right)+2b^3de\arctan\left(e^{c+dx}\right)-6a\cdots}{\cdots}$$

$$+\frac{\text{sech}(c+dx)(-bf-af\sinh(c+dx))}{2\left(a^2+b^2\right)d^2}$$

$$+\frac{\text{sech}^2(c+dx)(ade-acf+af(c+dx)-bde\sinh(c+dx)+bcf\sinh(c+dx)-bf(c+dx)\sinh(c+d\cdots}{2\left(a^2+b^2\right)d^2}$$

input
```
Integrate[((e + f*x)*Tanh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*(a^3*(-2*d*e*(c + d*x) + 2*c*f*(c + d*x) - f*(c + d*x)^2 + (4*a*Sqrt[
a^2 + b^2]*d*e*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 +
b^2)^2] - (4*a*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a
^2 + b^2]])/(-a^2 - b^2)^(3/2) + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2])] + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2])] - 2*c*f*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*d*e*Log[
2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] + 2*f*PolyLog[2, (b*E^(c + d*x
))/(-a + Sqrt[a^2 + b^2])] + 2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))]))/((a^2 + b^2)^2*d^2) + (-2*a^3*d*e*(c + d*x) + 2*a^3*c*f*(c +
 d*x) - a^3*f*(c + d*x)^2 + 6*a^2*b*d*e*ArcTan[E^(c + d*x)] + 2*b^3*d*e*Ar
cTan[E^(c + d*x)] - 6*a^2*b*c*f*ArcTan[E^(c + d*x)] - 2*b^3*c*f*ArcTan[E^(
c + d*x)] + (3*I)*a^2*b*f*(c + d*x)*Log[1 - I*E^(c + d*x)] + I*b^3*f*(c +
d*x)*Log[1 - I*E^(c + d*x)] - (3*I)*a^2*b*f*(c + d*x)*Log[1 + I*E^(c + d*x
)] - I*b^3*f*(c + d*x)*Log[1 + I*E^(c + d*x)] + 2*a^3*d*e*Log[1 + E^(2*(c
+ d*x))] - 2*a^3*c*f*Log[1 + E^(2*(c + d*x))] + 2*a^3*f*(c + d*x)*Log[1 +
E^(2*(c + d*x))] - I*b*(3*a^2 + b^2)*f*PolyLog[2, (-I)*E^(c + d*x)] + I*b*
(3*a^2 + b^2)*f*PolyLog[2, I*E^(c + d*x)] + a^3*f*PolyLog[2, -E^(2*(c + d*
x))])/(2*(a^2 + b^2)^2*d^2) + (Sech[c + d*x]*(-(b*f) - a*f*Sinh[c + d*x]))
/(2*(a^2 + b^2)*d^2) + (Sech[c + d*x]^2*(a*d*e - a*c*f + a*f*(c + d*x) - b
*d*e*Sinh[c + d*x] + b*c*f*Sinh[c + d*x] - b*f*(c + d*x)*Sinh[c + d*x])...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6101}$$

$$\frac{\int (e+fx)\text{sech}(c+dx)\tanh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 5978}$$

$$\frac{\int (e+fx)\text{sech}(c+dx)dx-\int (e+fx)\text{sech}^3(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx-\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{b}$$

$$\Big\downarrow \text{ 4668}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx-\frac{if\int \log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int \log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}}{b}$$

$$\Big\downarrow \text{ 2715}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}-\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx+\frac{2(e+fx)\arctan(e}{d}}{b}$$

$$\Big\downarrow \text{ 2838}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}}{b}$$

$$\Big\downarrow \; 4673$$

$$-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{1}{2}\int (e+fx)\text{sech}(c+dx)dx+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}-\frac{f\text{sech}(c+dx)}{2d^2}-\frac{(e\cdots}{}}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{1}{2}\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}-\frac{f\text{sech}(c+d\cdots}{2d^2}}{b}$$

$$\Big\downarrow \; 4668$$

$$-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(\frac{if\int \log\left(1-ie^{c+dx}\right)dx}{d}-\frac{if\int \log\left(1+ie^{c+dx}\right)dx}{d}-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\cdots}{b}$$

$$\Big\downarrow \; 2715$$

$$-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{P}\cdots}{}}{b}$$

$$\Big\downarrow \; 2838$$

$$-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\cdots}{b}$$

$$\Big\downarrow \; 6117$$

$$-\frac{a\left(\frac{\int (e+fx)\text{sech}^2(c+dx)\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\cdots}{b}$$

$$\Big\downarrow \; 5974$$

$$-\frac{a\left(\frac{\frac{f\int \operatorname{sech}^2(c+dx)dx}{2d}-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+}{b}$$
$$\frac{a\left(-\frac{a\int \frac{(e+fx)\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}+\frac{f\int \csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{2d}}{b}\right)}{b}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+}{b}$$
$$\frac{a\left(-\frac{a\int \frac{(e+fx)\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}+\frac{if\int 1d(-i\tanh(c+dx))}{2d^2}}{b}\right)}{b}$$

$$\Big\downarrow \text{24}$$

$$-\frac{a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+}{b}$$

$$\Big\downarrow \text{6117}$$

$$-\frac{a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{\int (e+fx)\operatorname{sech}^3(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}\right)}{b}+\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+}{b}$$

$\downarrow$ 3042

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+}{b}$$

$$\frac{a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{b}\right)}{b}\right)}{b}$$

$\downarrow$ 4673

$$-\frac{a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{\frac{1}{2}\int(e+fx)\text{sech}(c+dx)dx+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)\text{sech}^3(c+dx}{a+b\sinh(c+dx)}}{b}\right.}{b}\right.}{b}$$

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+}{b}$$

$\downarrow$ 3042

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+}{b}$$

$$\frac{a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\int(e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(}{2d}}{b}\right.}{b}\right.}{b}$$

$\downarrow$ 4668

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+}{b}$$

$$\frac{a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(-\frac{if\int\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c-}\right.}{d}\right.}{b}\right.}{b}\right.}{b}$$

$\downarrow$ 2715

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+}{b}$$

$$\frac{a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)d}{d^2}\right.}{}\right.}{b}\right.}{b}$$

$\downarrow$ 2838

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+}{b}$$

$$\frac{a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\operatorname{PolyLog}\left(2,ie^{c+}\right.}{d^2}\right.}{b}\right.}{b}\right.}{b}$$

$\downarrow$ 6107

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+}{b}$$

$$a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int\frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}\right)}{b}+\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+}\right.}{d}\right.}{}\right)}{b}\right.$$

$$\overline{\qquad b \qquad}$$

$\downarrow$ 6107

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+}{b}$$

$$a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)\text{sech}(c+dx)(a}{a^2+b^2}\right.}{a^2+b^2}\right.}{b}\right.}{}\right.$$

$\downarrow$ 6095

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\cdots}{b}$$

$$a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)\operatorname{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+\cdots}}\cdots\right)}{a^2+b^2}\cdots\right)}{\cdots}\right)}{b}\cdots\right)}{\cdots}\cdots\right)$$

$\downarrow$ 2620

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,ie^{c+dx})}{d^2}\right)+\frac{2(e+fx)\arctan(e^{c+dx})}{d}-\frac{if\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}+}{a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(b^2\left(-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}}{a+\sqrt{a^2-}}\right.}{bd}\right.\right.}{}\right)}{}\right.}{}\right.}$$

↓ 2715

$$\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\cdots$$

$$b$$

$$a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\left(\frac{b^2\left(\frac{b^2\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+\cdots}{\cdots}\cdots\right)}{a^2+b^2}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right)$$

↓ 2838

$$\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+$$

$$b$$

$$a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\left(\frac{b^2\left(\frac{\int(e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{b}{a+}\right.}{bd^2}\right.}{}\right.}{a^2+b^2}\right.}{}\right.}{}\right.$$

$\downarrow$ 7293

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\cdots}{b}$$

$$a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(a-\frac{a\left(\frac{b^2\left(\frac{\int(a(e+fx)\operatorname{sech}(c+dx)-b(e+fx)\tanh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\operatorname{PolyLog}(\cdots}{\cdots}\cdots\right.}{\cdots}\right.}{a^2+b^2\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.$$

input
```
Int[((e + f*x)*Tanh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2283 vs. $2(820) = 1640$.

Time = 2.47 (sec) , antiderivative size = 2284, normalized size of antiderivative = 2.55

method	result	size
risch	Expression too large to display	2284

input
```
int((f*x+e)*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(-b*d*f*x*exp(3*d*x+3*c)+2*a*d*f*x*exp(2*d*x+2*c)-b*d*e*exp(3*d*x+3*c)+2*a
*d*e*exp(2*d*x+2*c)+b*d*f*x*exp(d*x+c)-b*f*exp(3*d*x+3*c)+a*f*exp(2*d*x+2*
c)+b*d*e*exp(d*x+c)-f*b*exp(d*x+c)+a*f)/d^2/(a^2+b^2)/(1+exp(2*d*x+2*c))^2
+2/d^2/(a^2+b^2)^(3/2)*c*a^4*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2
*a)/(a^2+b^2)^(1/2))-2/d^2/(a^2+b^2)^(1/2)*c*a^2*f/(2*a^2+2*b^2)*arctanh(1
/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-3*I*b/d^2/(a^2+b^2)*a^2*f/(2*a^2+
2*b^2)*dilog(1+I*exp(d*x+c))+3*I/d^2/(a^2+b^2)*a^2*f/(2*a^2+2*b^2)*dilog(1
-I*exp(d*x+c))*b+3*I/d/(a^2+b^2)*a^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*b*
x-3*I*b/d^2/(a^2+b^2)*a^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*c+3*I/d^2/(a^
2+b^2)*a^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*b*c-3*I*b/d/(a^2+b^2)*a^2*f/
(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*x+b^4/d^2/(a^2+b^2)^(3/2)*c*f/(2*a^2+2*b^
2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-b^2/d^2/(a^2+b^2)^(1/
2)*c*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-3*b
^2/d/(a^2+b^2)^(3/2)*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2
+b^2)^(1/2))*a^2-6*b/d^2/(a^2+b^2)*c*a^2*f/(2*a^2+2*b^2)*arctan(exp(d*x+c)
)+I*b^3/d^2/(a^2+b^2)*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*c-I*b^3/d/(a^2+b^
2)*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*x-I*b^3/d^2/(a^2+b^2)*f/(2*a^2+2*b^2
)*ln(1+I*exp(d*x+c))*c+2/d^2/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)*dilog(1-I*exp(d
*x+c))-2/d^2/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^
(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-2/d^2/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)*dilo...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4729 vs. 2(793) = 1586.

Time = 0.22 (sec) , antiderivative size = 4729, normalized size of antiderivative = 5.29

$$\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\tanh^3{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*tanh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*tanh(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\tanh{(dx+c)}^3}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-(a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 +
b^4)*d) - a^3*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) + (3*a
^2*b + b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) + (b*e^(-d*x
- c) - 2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b
^2)*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d))*e - f*(((b*d*x*e^
(3*c) + b*e^(3*c))*e^(3*d*x) - (2*a*d*x*e^(2*c) + a*e^(2*c))*e^(2*d*x) - (
b*d*x*e^c - b*e^c)*e^(d*x) - a)/(a^2*d^2 + b^2*d^2 + (a^2*d^2*e^(4*c) + b^
2*d^2*e^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*e^(2*d*x)
) - integrate(-2*(a^4*x*e^(d*x + c) - a^3*b*x)/(a^4*b + 2*a^2*b^3 + b^5 -
(a^4*b*e^(2*c) + 2*a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^5*e^c +
 2*a^3*b^2*e^c + a*b^4*e^c)*e^(d*x)), x) - integrate(-(2*a^3*x - (3*a^2*b*
e^c + b^3*e^c)*x*e^(d*x))/(a^4 + 2*a^2*b^2 + b^4 + (a^4*e^(2*c) + 2*a^2*b^
2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x))
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh(c+d\,x)^3\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((tanh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((tanh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
(8*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**6*b*f + 24*e**(4*c + 4*d*x)*atan
(e**(c + d*x))*a**4*b**3*f + 18*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**2*b
**5*d*e + 24*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**2*b**5*f + 6*e**(4*c +
 4*d*x)*atan(e**(c + d*x))*b**7*d*e + 8*e**(4*c + 4*d*x)*atan(e**(c + d*x)
)*b**7*f + 16*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**6*b*f + 48*e**(2*c +
2*d*x)*atan(e**(c + d*x))*a**4*b**3*f + 36*e**(2*c + 2*d*x)*atan(e**(c + d
*x))*a**2*b**5*d*e + 48*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**2*b**5*f +
12*e**(2*c + 2*d*x)*atan(e**(c + d*x))*b**7*d*e + 16*e**(2*c + 2*d*x)*atan
(e**(c + d*x))*b**7*f + 8*atan(e**(c + d*x))*a**6*b*f + 24*atan(e**(c + d*
x))*a**4*b**3*f + 18*atan(e**(c + d*x))*a**2*b**5*d*e + 24*atan(e**(c + d*
x))*a**2*b**5*f + 6*atan(e**(c + d*x))*b**7*d*e + 8*atan(e**(c + d*x))*b**
7*f + 192*e**(7*c + 4*d*x)*int((e**(3*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(
7*c + 7*d*x)*a + 2*e**(6*c + 6*d*x)*b + 6*e**(5*c + 5*d*x)*a + 6*e**(3*c +
 3*d*x)*a - 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**8*d**2*f +
400*e**(7*c + 4*d*x)*int((e**(3*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c +
7*d*x)*a + 2*e**(6*c + 6*d*x)*b + 6*e**(5*c + 5*d*x)*a + 6*e**(3*c + 3*d*x
)*a - 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**6*b**2*d**2*f + 2
40*e**(7*c + 4*d*x)*int((e**(3*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7
*d*x)*a + 2*e**(6*c + 6*d*x)*b + 6*e**(5*c + 5*d*x)*a + 6*e**(3*c + 3*d*x)
*a - 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*b**4*d**2*f +...
```

3.418 $\int \frac{\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 120

$$\int \frac{\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b(3a^2+b^2)\arctan(\sinh(c+dx))}{2(a^2+b^2)^2 d} + \frac{a^3\log(\cosh(c+dx))}{(a^2+b^2)^2 d} - \frac{a^3\log(a+b\sinh(c+dx))}{(a^2+b^2)^2 d} + \frac{\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))}{2(a^2+b^2) d}$$

output
```
1/2*b*(3*a^2+b^2)*arctan(sinh(d*x+c))/(a^2+b^2)^2/d+a^3*ln(cosh(d*x+c))/(a
^2+b^2)^2/d-a^3*ln(a+b*sinh(d*x+c))/(a^2+b^2)^2/d+1/2*sech(d*x+c)^2*(a-b*s
inh(d*x+c))/(a^2+b^2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.27

$$\int \frac{\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b(a^2+b^2)\arctan(\sinh(c+dx)) - (a^3-i(2a^2b+b^3))\log(i-\sinh(c+dx)) - (a^3+i(2a^2b+b^3))\log(\ldots}{2\ldots}$$

input
```
Integrate[Tanh[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*(b*(a^2 + b^2)*ArcTan[Sinh[c + d*x]] - (a^3 - I*(2*a^2*b + b^3))*Log[
I - Sinh[c + d*x]] - (a^3 + I*(2*a^2*b + b^3))*Log[I + Sinh[c + d*x]] + 2*
a^3*Log[a + b*Sinh[c + d*x]] - a*(a^2 + b^2)*Sech[c + d*x]^2 + b*(a^2 + b^
2)*Sech[c + d*x]*Tanh[c + d*x])/((a^2 + b^2)^2*d)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 26, 3200, 601, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\tan(ic+idx)^3}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\tan(ic+idx)^3}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3200}$$

$$\frac{\int \frac{b^3\sinh^3(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{601}$$

$$\frac{\frac{b^2(a-b\sinh(c+dx))}{2(a^2+b^2)\left(b^2\sinh^2(c+dx)+b^2\right)}-\frac{\int -\frac{b^2\left(ab^2+\left(2a^2+b^2\right)\sinh(c+dx)b\right)}{\left(a^2+b^2\right)(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)}d(b\sinh(c+dx))}{2b^2}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{b^2\left(ab^2+\left(2a^2+b^2\right)\sinh(c+dx)b\right)}{\left(a^2+b^2\right)(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)}d(b\sinh(c+dx))}{2b^2}+\frac{b^2(a-b\sinh(c+dx))}{2(a^2+b^2)(b^2\sinh^2(c+dx)+b^2)}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{ab^2+\left(2a^2+b^2\right)\sinh(c+dx)b}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)}d(b\sinh(c+dx))}{2(a^2+b^2)}+\frac{b^2(a-b\sinh(c+dx))}{2(a^2+b^2)(b^2\sinh^2(c+dx)+b^2)}}{d}$$

$$\Big\downarrow \text{657}$$

$$\frac{\frac{\int \left(\frac{b^4+3a^2b^2+2a^3\sinh(c+dx)b}{\left(a^2+b^2\right)\left(\sinh^2(c+dx)b^2+b^2\right)}-\frac{2a^3}{\left(a^2+b^2\right)(a+b\sinh(c+dx))}\right)d(b\sinh(c+dx))}{2(a^2+b^2)}+\frac{b^2(a-b\sinh(c+dx))}{2(a^2+b^2)(b^2\sinh^2(c+dx)+b^2)}}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^2(a-b\sinh(c+dx))}{2(a^2+b^2)(b^2\sinh^2(c+dx)+b^2)}+\frac{\frac{b\left(3a^2+b^2\right)\arctan(\sinh(c+dx))}{a^2+b^2}+\frac{a^3\log\left(b^2\sinh^2(c+dx)+b^2\right)}{a^2+b^2}-\frac{2a^3\log(a+b\sinh(c+dx))}{a^2+b^2}}{2(a^2+b^2)}}{d}$$

input
```
Int[Tanh[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

output
```
(((b*(3*a^2 + b^2)*ArcTan[Sinh[c + d*x]])/(a^2 + b^2) - (2*a^3*Log[a + b*S
inh[c + d*x]])/(a^2 + b^2) + (a^3*Log[b^2 + b^2*Sinh[c + d*x]^2])/(a^2 + b
^2))/(2*(a^2 + b^2)) + (b^2*(a - b*Sinh[c + d*x]))/(2*(a^2 + b^2)*(b^2 + b
^2*Sinh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.75

method	result
derivativedivides	$\frac{\frac{\frac{2\left(\left(\frac{1}{2}a^2b+\frac{1}{2}b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-a^3-a\,b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(-\frac{1}{2}a^2b-\frac{1}{2}b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+a^3\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+\left(3a^2\right.}{a^4+2a^2b^2+b^4}}{d}$
default	$\frac{\frac{\frac{2\left(\left(\frac{1}{2}a^2b+\frac{1}{2}b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-a^3-a\,b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(-\frac{1}{2}a^2b-\frac{1}{2}b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+a^3\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+\left(3a^2\right.}{a^4+2a^2b^2+b^4}}{d}$
risch	$-\frac{2a^3d^2x}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{2a^3dc}{a^4d^2+2a^2b^2d^2+b^4d^2}+\frac{2a^3x}{a^4+2a^2b^2+b^4}+\frac{2a^3c}{d(a^4+2a^2b^2+b^4)}+\frac{e^{dx+c}(-b\,e^{2dx+2c}+2a}{d(a^2+b^2)(1+e^{2dx}}$

input
```
int(tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/(a^4+2*a^2*b^2+b^4)*(((1/2*a^2*b+1/2*b^3)*tanh(1/2*d*x+1/2*c)^3+(-a
^3-a*b^2)*tanh(1/2*d*x+1/2*c)^2+(-1/2*a^2*b-1/2*b^3)*tanh(1/2*d*x+1/2*c))/
(1+tanh(1/2*d*x+1/2*c)^2)^2+1/2*a^3*ln(1+tanh(1/2*d*x+1/2*c)^2)+1/2*(3*a^2
*b+b^3)*arctan(tanh(1/2*d*x+1/2*c)))-8*a^3/(8*a^4+16*a^2*b^2+8*b^4)*ln(tan
h(1/2*d*x+1/2*c)^2*a-2*b*tanh(1/2*d*x+1/2*c)-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 896 vs. $2(117) = 234$.

Time = 0.13 (sec) , antiderivative size = 896, normalized size of antiderivative = 7.47

$$\int \frac{\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-((a^2*b + b^3)*cosh(d*x + c)^3 + (a^2*b + b^3)*sinh(d*x + c)^3 - 2*(a^3 +
 a*b^2)*cosh(d*x + c)^2 - (2*a^3 + 2*a*b^2 - 3*(a^2*b + b^3)*cosh(d*x + c)
)*sinh(d*x + c)^2 - ((3*a^2*b + b^3)*cosh(d*x + c)^4 + 4*(3*a^2*b + b^3)*c
osh(d*x + c)*sinh(d*x + c)^3 + (3*a^2*b + b^3)*sinh(d*x + c)^4 + 3*a^2*b +
 b^3 + 2*(3*a^2*b + b^3)*cosh(d*x + c)^2 + 2*(3*a^2*b + b^3 + 3*(3*a^2*b +
 b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((3*a^2*b + b^3)*cosh(d*x + c)^
3 + (3*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + s
inh(d*x + c)) - (a^2*b + b^3)*cosh(d*x + c) + (a^3*cosh(d*x + c)^4 + 4*a^3
*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*sinh(d*x + c)^4 + 2*a^3*cosh(d*x + c)
^2 + a^3 + 2*(3*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^2 + 4*(a^3*cosh(d
*x + c)^3 + a^3*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/
(cosh(d*x + c) - sinh(d*x + c))) - (a^3*cosh(d*x + c)^4 + 4*a^3*cosh(d*x +
 c)*sinh(d*x + c)^3 + a^3*sinh(d*x + c)^4 + 2*a^3*cosh(d*x + c)^2 + a^3 +
2*(3*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^2 + 4*(a^3*cosh(d*x + c)^3 +
 a^3*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - si
nh(d*x + c))) - (a^2*b + b^3 - 3*(a^2*b + b^3)*cosh(d*x + c)^2 + 4*(a^3 +
a*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x +
 c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 +
 2*a^2*b^2 + b^4)*d*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x
 + c)^2 + 2*(3*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 + (a^4 + 2*a^2...
```

Sympy [F]

$$\int \frac{\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh^3{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate(tanh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(tanh(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.81

$$\int \frac{\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a^3\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^4+2\,a^2b^2+b^4)d} + \frac{a^3\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^4+2\,a^2b^2+b^4)d} - \frac{(3\,a^2b+b^3)\arctan\left(e^{(-dx-c)}\right)}{(a^4+2\,a^2b^2+b^4)d} - \frac{be^{(-dx-c)}-2\,ae^{(-2\,dx-2\,c)}-be^{(-3\,dx-3\,c)}}{(a^2+b^2+2\,(a^2+b^2)e^{(-2\,dx-2\,c)}+(a^2+b^2)e^{(-4\,dx-4\,c)})d}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 + b
^4)*d) + a^3*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (3*a^
2*b + b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) - (b*e^(-d*x -
 c) - 2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^
2)*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(117) = 234.

Time = 0.18 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.32

$$\int \frac{\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{4\,a^3b\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^4b+2\,a^2b^3+b^5} - \frac{2\,a^3\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^4+2\,a^2b^2+b^4} - \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(3\,a^2b+b^3\right)}{a^4+2\,a^2b^2+b^4} - \cdots}{4\,d}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*(4*a^3*b*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^4*b + 2*a^
2*b^3 + b^5) - 2*a^3*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^4 + 2*a^2*
b^2 + b^4) - (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(3*a^
2*b + b^3)/(a^4 + 2*a^2*b^2 + b^4) + 2*(a^3*(e^(d*x + c) - e^(-d*x - c))^2
 + 2*a^2*b*(e^(d*x + c) - e^(-d*x - c)) + 2*b^3*(e^(d*x + c) - e^(-d*x - c
)) - 4*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*((e^(d*x + c) - e^(-d*x - c))^2 + 4
)))/d
```

Mupad [B] (verification not implemented)

Time = 3.54 (sec) , antiderivative size = 381, normalized size of antiderivative = 3.18

$$\int \frac{\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{2\,(a^3+a\,b^2)}{d\,(a^2+b^2)^2}-\frac{e^{c+d\,x}\,(a^2\,b+b^3)}{d\,(a^2+b^2)^2}}{e^{2\,c+2\,d\,x}+1}-\frac{\frac{2\,a}{d\,(a^2+b^2)}-\frac{2\,b\,e^{c+d\,x}}{d\,(a^2+b^2)}}{2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1}+\frac{\ln\left(1+e^{c+d\,x}\,1\mathrm{i}\right)\,(2\,a+b\,1\mathrm{i})}{2\,(d\,a^2+2\mathrm{i}\,d\,a\,b-d\,b^2)}+\frac{\ln\left(e^{c+d\,x}+1\mathrm{i}\right)\,(b+a\,2\mathrm{i})}{2\,(1\mathrm{i}\,d\,a^2+2\,d\,a\,b-1\mathrm{i}\,d\,b^2)}-\frac{a^3\,\ln\left(32\,a^7\,e^{d\,x}\,e^{c}-b^7-6\,a^2\,b^5-9\,a^4\,b^3-16\,a^6\,b+b^7\,e^{2\,c}\,e^{2\,d\,x}+16\,a^6\,b\,e^{2\,c}\,e^{2\,d\,x}+12\,a^3\,b^4\,e^{d\,x}\,e^{c}+18\right.}{d\,a^4+2\,d\,a^2\,b^2+d\,b^4}$$

input
```
int(tanh(c + d*x)^3/(a + b*sinh(c + d*x)),x)
```

output
```
((2*(a*b^2 + a^3))/(d*(a^2 + b^2)^2) - (exp(c + d*x)*(a^2*b + b^3))/(d*(a^
2 + b^2)^2))/(exp(2*c + 2*d*x) + 1) - ((2*a)/(d*(a^2 + b^2)) - (2*b*exp(c
+ d*x))/(d*(a^2 + b^2)))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + (lo
g(exp(c + d*x)*1i + 1)*(2*a + b*1i))/(2*(a^2*d - b^2*d + a*b*d*2i)) + (log
(exp(c + d*x) + 1i)*(a*2i + b))/(2*(a^2*d*1i - b^2*d*1i + 2*a*b*d)) - (a^3
*log(32*a^7*exp(d*x)*exp(c) - b^7 - 6*a^2*b^5 - 9*a^4*b^3 - 16*a^6*b + b^7
*exp(2*c)*exp(2*d*x) + 16*a^6*b*exp(2*c)*exp(2*d*x) + 12*a^3*b^4*exp(d*x)*
exp(c) + 18*a^5*b^2*exp(d*x)*exp(c) + 6*a^2*b^5*exp(2*c)*exp(2*d*x) + 9*a^
4*b^3*exp(2*c)*exp(2*d*x) + 2*a*b^6*exp(d*x)*exp(c)))/(a^4*d + b^4*d + 2*a
^2*b^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 519, normalized size of antiderivative = 4.32

$$\int \frac{\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{3e^{4dx+4c}atan\left(e^{dx+c}\right)a^2b+e^{4dx+4c}atan\left(e^{dx+c}\right)b^3+6e^{2dx+2c}atan\left(e^{dx+c}\right)a^2b+2e^{2dx+2c}atan\left(e^{dx+c}\right)b^3+3}{}$$

input
```
int(tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
(3*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**2*b + e**(4*c + 4*d*x)*atan(e**(
c + d*x))*b**3 + 6*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**2*b + 2*e**(2*c
+ 2*d*x)*atan(e**(c + d*x))*b**3 + 3*atan(e**(c + d*x))*a**2*b + atan(e**(
c + d*x))*b**3 + e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x) + 1)*a**3 - e**(4*c
 + 4*d*x)*log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*a**3 - e**(4*c +
4*d*x)*a**3 - e**(4*c + 4*d*x)*a*b**2 - e**(3*c + 3*d*x)*a**2*b - e**(3*c
+ 3*d*x)*b**3 + 2*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x) + 1)*a**3 - 2*e**(
2*c + 2*d*x)*log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*a**3 + e**(c +
 d*x)*a**2*b + e**(c + d*x)*b**3 + log(e**(2*c + 2*d*x) + 1)*a**3 - log(e*
*(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*a**3 - a**3 - a*b**2)/(d*(e**(4*c
 + 4*d*x)*a**4 + 2*e**(4*c + 4*d*x)*a**2*b**2 + e**(4*c + 4*d*x)*b**4 + 2*
e**(2*c + 2*d*x)*a**4 + 4*e**(2*c + 2*d*x)*a**2*b**2 + 2*e**(2*c + 2*d*x)*
b**4 + a**4 + 2*a**2*b**2 + b**4))
```

3.419 $\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\text{Int}\left(\frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(tanh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 58.90 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[Tanh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Tanh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$\Big\downarrow \text{6111}$$
$$\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Tanh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\tanh\left(dx+c\right)^{3}}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(tanh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(tanh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 4.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(tanh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(tanh(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 2.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh^3\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

input
```
integrate(tanh(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(tanh(c + d*x)**3/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Maxima [N/A]

Not integrable

Time = 1.76 (sec) , antiderivative size = 1095, normalized size of antiderivative = 39.11

$$\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(tanh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-(a*f + (b*d*f*x*e^(3*c) + (d*e - f)*b*e^(3*c))*e^(3*d*x) - (2*a*d*f*x*e^(
2*c) + (2*d*e - f)*a*e^(2*c))*e^(2*d*x) - (b*d*f*x*e^c + (d*e + f)*b*e^c)*
e^(d*x))/(a^2*d^2*e^2 + b^2*d^2*e^2 + (a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + 2*
(a^2*d^2*e*f + b^2*d^2*e*f)*x + (a^2*d^2*e^2*e^(4*c) + b^2*d^2*e^2*e^(4*c)
 + (a^2*d^2*f^2*e^(4*c) + b^2*d^2*f^2*e^(4*c))*x^2 + 2*(a^2*d^2*e*f*e^(4*c
) + b^2*d^2*e*f*e^(4*c))*x)*e^(4*d*x) + 2*(a^2*d^2*e^2*e^(2*c) + b^2*d^2*e
^2*e^(2*c) + (a^2*d^2*f^2*e^(2*c) + b^2*d^2*f^2*e^(2*c))*x^2 + 2*(a^2*d^2*
e*f*e^(2*c) + b^2*d^2*e*f*e^(2*c))*x)*e^(2*d*x)) + integrate(-(2*a^3*d^2*f
^2*x^2 + 4*a^3*d^2*e*f*x + 2*a*b^2*f^2 + 2*(d^2*e^2 + f^2)*a^3 - ((3*d^2*e
^2 + 2*f^2)*a^2*b*e^c + (d^2*e^2 + 2*f^2)*b^3*e^c + (3*a^2*b*d^2*f^2*e^c +
 b^3*d^2*f^2*e^c)*x^2 + 2*(3*a^2*b*d^2*e*f*e^c + b^3*d^2*e*f*e^c)*x)*e^(d*
x))/(a^4*d^2*e^3 + 2*a^2*b^2*d^2*e^3 + b^4*d^2*e^3 + (a^4*d^2*f^3 + 2*a^2*
b^2*d^2*f^3 + b^4*d^2*f^3)*x^3 + 3*(a^4*d^2*e*f^2 + 2*a^2*b^2*d^2*e*f^2 +
b^4*d^2*e*f^2)*x^2 + 3*(a^4*d^2*e^2*f + 2*a^2*b^2*d^2*e^2*f + b^4*d^2*e^2*
f)*x + (a^4*d^2*e^3*e^(2*c) + 2*a^2*b^2*d^2*e^3*e^(2*c) + b^4*d^2*e^3*e^(2
*c) + (a^4*d^2*f^3*e^(2*c) + 2*a^2*b^2*d^2*f^3*e^(2*c) + b^4*d^2*f^3*e^(2*
c))*x^3 + 3*(a^4*d^2*e*f^2*e^(2*c) + 2*a^2*b^2*d^2*e*f^2*e^(2*c) + b^4*d^2
*e*f^2*e^(2*c))*x^2 + 3*(a^4*d^2*e^2*f*e^(2*c) + 2*a^2*b^2*d^2*e^2*f*e^(2*
c) + b^4*d^2*e^2*f*e^(2*c))*x)*e^(2*d*x)), x) + integrate(-2*(a^4*e^(d*x +
 c) - a^3*b)/(a^4*b*e + 2*a^2*b^3*e + b^5*e + (a^4*b*f + 2*a^2*b^3*f + ...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(tanh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 4.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh(c+d\,x)^3}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(tanh(c + d*x)^3/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(tanh(c + d*x)^3/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 2848, normalized size of antiderivative = 101.71

$$\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Too large to display}$$

input
```
int(tanh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output

```
(4*e**(11*c + 4*d*x)*int(e**(7*d*x)/(e**(8*c + 8*d*x)*b*e + e**(8*c + 8*d*
x)*b*f*x + 2*e**(7*c + 7*d*x)*a*e + 2*e**(7*c + 7*d*x)*a*f*x + 2*e**(6*c +
 6*d*x)*b*e + 2*e**(6*c + 6*d*x)*b*f*x + 6*e**(5*c + 5*d*x)*a*e + 6*e**(5*
c + 5*d*x)*a*f*x + 6*e**(3*c + 3*d*x)*a*e + 6*e**(3*c + 3*d*x)*a*f*x - 2*e
**(2*c + 2*d*x)*b*e - 2*e**(2*c + 2*d*x)*b*f*x + 2*e**(c + d*x)*a*e + 2*e*
*(c + d*x)*a*f*x - b*e - b*f*x),x)*a*d*e - 12*e**(9*c + 4*d*x)*int(e**(5*d
*x)/(e**(8*c + 8*d*x)*b*e + e**(8*c + 8*d*x)*b*f*x + 2*e**(7*c + 7*d*x)*a*
e + 2*e**(7*c + 7*d*x)*a*f*x + 2*e**(6*c + 6*d*x)*b*e + 2*e**(6*c + 6*d*x)
*b*f*x + 6*e**(5*c + 5*d*x)*a*e + 6*e**(5*c + 5*d*x)*a*f*x + 6*e**(3*c + 3
*d*x)*a*e + 6*e**(3*c + 3*d*x)*a*f*x - 2*e**(2*c + 2*d*x)*b*e - 2*e**(2*c
+ 2*d*x)*b*f*x + 2*e**(c + d*x)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),
x)*a*d*e + 12*e**(7*c + 4*d*x)*int(e**(3*d*x)/(e**(8*c + 8*d*x)*b*e + e**(
8*c + 8*d*x)*b*f*x + 2*e**(7*c + 7*d*x)*a*e + 2*e**(7*c + 7*d*x)*a*f*x + 2
*e**(6*c + 6*d*x)*b*e + 2*e**(6*c + 6*d*x)*b*f*x + 6*e**(5*c + 5*d*x)*a*e
+ 6*e**(5*c + 5*d*x)*a*f*x + 6*e**(3*c + 3*d*x)*a*e + 6*e**(3*c + 3*d*x)*a
*f*x - 2*e**(2*c + 2*d*x)*b*e - 2*e**(2*c + 2*d*x)*b*f*x + 2*e**(c + d*x)*
a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x)*a*d*e - 4*e**(5*c + 4*d*x)*in
t(e**(d*x)/(e**(8*c + 8*d*x)*b*e + e**(8*c + 8*d*x)*b*f*x + 2*e**(7*c + 7*
d*x)*a*e + 2*e**(7*c + 7*d*x)*a*f*x + 2*e**(6*c + 6*d*x)*b*e + 2*e**(6*c +
 6*d*x)*b*f*x + 6*e**(5*c + 5*d*x)*a*e + 6*e**(5*c + 5*d*x)*a*f*x + 6*e...
```

3.420 $\int \frac{(e+fx)^3 \coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 451

$$\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} + \frac{(e+fx)^3\log\left(1-e^{2(c+dx)}\right)}{ad} - \frac{3f(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2} - \frac{3f(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^2} + \frac{3f(e+fx)^2\,\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{2ad^2} + \frac{6f^2(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^3} + \frac{6f^2(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^3} - \frac{3f^2(e+fx)\,\text{PolyLog}\left(3,e^{2(c+dx)}\right)}{2ad^3} - \frac{6f^3\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^4} - \frac{6f^3\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^4} + \frac{3f^3\,\text{PolyLog}\left(4,e^{2(c+dx)}\right)}{4ad^4}$$

output
```
-(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/d-(f*x+e)^3*ln(1+b*exp
(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/d+(f*x+e)^3*ln(1-exp(2*d*x+2*c))/a/d-3*f*(f
*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/d^2-3*f*(f*x+e)^2*p
olylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/d^2+3/2*f*(f*x+e)^2*polylog(
2,exp(2*d*x+2*c))/a/d^2+6*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)
^(1/2)))/a/d^3+6*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/
a/d^3-3/2*f^2*(f*x+e)*polylog(3,exp(2*d*x+2*c))/a/d^3-6*f^3*polylog(4,-b*e
xp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/d^4-6*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2
+b^2)^(1/2)))/a/d^4+3/4*f^3*polylog(4,exp(2*d*x+2*c))/a/d^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1914 vs. $2(451) = 902$.

Time = 9.46 (sec) , antiderivative size = 1914, normalized size of antiderivative = 4.24

$$\int \frac{(e+fx)^3 \coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output

```
-1/2*(E^(2*c)*((e + f*x)^4/(E^(2*c)*f) - (2*(1 - E^(-2*c))*(e + f*x)^3*Log
[1 - E^(-c - d*x)])/d - (2*(1 - E^(-2*c))*(e + f*x)^3*Log[1 + E^(-c - d*x)
])/d + (6*(-1 + E^(2*c))*f*(d^2*(e + f*x)^2*PolyLog[2, -E^(-c - d*x)] + 2*
f*(d*(e + f*x)*PolyLog[3, -E^(-c - d*x)] + f*PolyLog[4, -E^(-c - d*x)])))/
(d^4*E^(2*c)) + (6*(-1 + E^(2*c))*f*(d^2*(e + f*x)^2*PolyLog[2, E^(-c - d*
x)] + 2*f*(d*(e + f*x)*PolyLog[3, E^(-c - d*x)] + f*PolyLog[4, E^(-c - d*x
)])))/(d^4*E^(2*c))))/(a*(-1 + E^(2*c))) + (4*e^3*E^(2*c)*x + 6*e^2*E^(2*c
)*f*x^2 + 4*e*E^(2*c)*f^2*x^3 + E^(2*c)*f^3*x^4 + (4*a*Sqrt[a^2 + b^2]*e^3
*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) +
(4*a*Sqrt[-a^2 - b^2]*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/(S
qrt[-(a^2 + b^2)^2]*d) - (2*e^3*E^(2*c)*Log[b - 2*a*E^(c + d*x) - b*E^(2*(
c + d*x))])/d + (2*e^3*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d
+ (6*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])
])/d - (6*e^2*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b
^2)*E^(2*c)])])/d + (6*e*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(
a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/
(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (2*f^3*x^3*Log[1 + (b*E^(2*c + d
*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (2*E^(2*c)*f^3*x^3*Log[1 +
(b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e^2*f*x*Log
[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e^2...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.50 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.18, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {6103, 3042, 26, 4201, 2620, 3011, 6095, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6103}$$

$$\frac{\int (e+fx)^3\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i(e+fx)^3\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\int (e+fx)^3\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{a}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\int \frac{e^{2c+2dx-i\pi}(e+fx)^3}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^4}{4f}\right)}{a}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\int (e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{a}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int (e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{a}$$

$$\Big\downarrow \text{6095}$$

$$-\frac{b\left(\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^4}{4bf}\right)}{a}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int (e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{a}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{b\left(-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+\cdots}}\right.}{bd}\right.}{a}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}(2,-e^{2c+2dx-i\pi})dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,-e^{2c+2dx-i\pi})}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{a}$$

$\downarrow$ 3011

$$-\frac{b\left(-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\cdots}{}\right.}{bd}\right.}{a}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}(2,-e^{2c+2dx-i\pi})dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,-e^{2c+2dx-i\pi})}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{a}$$

$\downarrow$ 7163

$$-\frac{b\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{Pol}\cdots}{}\right.}{}\right.}{}\right.}{}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}(3,-e^{2c+2dx-i\pi})}{2d}-\frac{f\int\operatorname{PolyLog}(3,-e^{2c+2dx-i\pi})dx}{2d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,-e^{2c+2d\cdots})}{2d}\right.}{2d}\right.\right.}{a}$$

$\downarrow$ 2720

$$-\frac{b\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2\ldots}{\ldots}\right.}{\ldots}\right.}{\ldots}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(3,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}\right)}{d}-\frac{(e\ldots}{\ldots}\right.}{2d}\right.\right.}{a}$$

$\downarrow$ 7143

$$-\frac{b\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\ldots}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\operatorname{PolyLog}\left(4,-e^{2c+2dx-i\pi}\right)}{4d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\ldots}\right.}{2d}\right.}{2d}\right.\right.}{a}$$

input
```
Int[((e + f*x)^3*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-((b*(-1/4*(e + f*x)^4/(b*f) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 + b^2])])/(b*d) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2]))])/d^2))/d))/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/a) - (I*(((-1/4*I)*(e + f*x)^4)/f + (
2*I)*(((e + f*x)^3*Log[1 + E^(2*c - I*Pi + 2*d*x)])/(2*d) - (3*f*(-1/2*((e
 + f*x)^2*PolyLog[2, -E^(2*c - I*Pi + 2*d*x)])/d + (f*(((e + f*x)*PolyLog[
3, -E^(2*c - I*Pi + 2*d*x)])/(2*d) - (f*PolyLog[4, -E^(2*c - I*Pi + 2*d*x)
])/(4*d^2)))/d))/(2*d))))/a
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6103
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Coth[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x
]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(fx+e)^3 \coth(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1228 vs. $2(418) = 836$.

Time = 0.13 (sec) , antiderivative size = 1228, normalized size of antiderivative = 2.72

$$\int \frac{(e+fx)^3 \coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(6*f^3*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*f^3*polylog(4, (a*cosh(d*x
 + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2))/b) - 6*f^3*polylog(4, cosh(d*x + c) + sinh(d*x + c)) - 6*f^3*po
lylog(4, -cosh(d*x + c) - sinh(d*x + c)) + 3*(d^2*f^3*x^2 + 2*d^2*e*f^2*x
+ d^2*e^2*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 3*(d^2*f^3*x^2 + 2*d
^2*e*f^2*x + d^2*e^2*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*(d^2*f^
3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f)*dilog(cosh(d*x + c) + sinh(d*x + c)) -
3*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f)*dilog(-cosh(d*x + c) - sinh(d*
x + c)) + (d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*log(2*b*cosh
(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (d^3*e^
3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*log(2*b*cosh(d*x + c) + 2*b*s
inh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (d^3*f^3*x^3 + 3*d^3*e*f
^2*x^2 + 3*d^3*e^2*f*x + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f^3)*log(-(a*
cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2) - b)/b) + (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x
 + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f^3)*log(-(a*cosh(d*x + c) + a*sinh
(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) -...
```

Sympy [F]

$$\int \frac{(e+fx)^3 \coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^3 \coth{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**3*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)^3 \coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3 \coth(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e^3*(log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*d) - log(e^(-d*x
- c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d)) + 3*(d*x*log(e^(d*x + c) +
1) + dilog(-e^(d*x + c)))*e^2*f/(a*d^2) + 3*(d*x*log(-e^(d*x + c) + 1) + d
ilog(e^(d*x + c)))*e^2*f/(a*d^2) + 3*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x
*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*e*f^2/(a*d^3) + 3*(d^2*
x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x
 + c)))*e*f^2/(a*d^3) + (d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e
^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))
*f^3/(a*d^4) + (d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c
)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))*f^3/(a*d^4
) - 1/2*(d^4*f^3*x^4 + 4*d^4*e*f^2*x^3 + 6*d^4*e^2*f*x^2)/(a*d^4) + integr
ate(-2*(b*f^3*x^3 + 3*b*e*f^2*x^2 + 3*b*e^2*f*x - (a*f^3*x^3*e^c + 3*a*e*f
^2*x^2*e^c + 3*a*e^2*f*x*e^c)*e^(d*x))/(a*b*e^(2*d*x + 2*c) + 2*a^2*e^(d*x
 + c) - a*b), x)
```

Giac [F]

$$\int \frac{(e+fx)^3 \coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3 \coth(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*coth(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+d\,x)\,\left(e+f\,x\right)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((coth(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((coth(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3 \coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2e^{3c}\left(\int \frac{e^{3dx}x^3}{e^{4dx+4c}b+2e^{3dx+3c}a-2e^{2dx+2c}b-2e^{dx+c}a+b}dx\right)ad\,f^3+6e^{3c}\left(\int \frac{e^{3dx}x^2}{e^{4dx+4c}b+2e^{3dx+3c}a-2e^{2dx+2c}b-2e^{dx+c}a+b}dx\right)ade}{}$$

input
```
int((f*x+e)^3*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
(2*e**(3*c)*int((e**(3*d*x)*x**3)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)
*a - 2*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a*d*f**3 + 6*e**(3*c)
*int((e**(3*d*x)*x**2)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a - 2*e**(
2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a*d*e*f**2 + 6*e**(3*c)*int((e**
(3*d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a - 2*e**(2*c + 2*d*x)
*b - 2*e**(c + d*x)*a + b),x)*a*d*e**2*f + 2*e**c*int((e**(d*x)*x**3)/(e**
(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a - 2*e**(2*c + 2*d*x)*b - 2*e**(c +
d*x)*a + b),x)*a*d*f**3 + 6*e**c*int((e**(d*x)*x**2)/(e**(4*c + 4*d*x)*b +
 2*e**(3*c + 3*d*x)*a - 2*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a*
d*e*f**2 + 6*e**c*int((e**(d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x
)*a - 2*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a*d*e**2*f + log(e**
(c + d*x) - 1)*e**3 + log(e**(c + d*x) + 1)*e**3 - log(e**(2*c + 2*d*x)*b
+ 2*e**(c + d*x)*a - b)*e**3)/(a*d)
```

3.421 $\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 325

$$\begin{aligned}\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}\,dx =& -\frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} \\ &-\frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} \\ &+\frac{(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{ad} \\ &-\frac{2f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2} \\ &-\frac{2f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^2} \\ &+\frac{f(e+fx)\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{ad^2} \\ &+\frac{2f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^3} \\ &+\frac{2f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^3} \\ &-\frac{f^2\operatorname{PolyLog}\left(3,e^{2(c+dx)}\right)}{2ad^3}\end{aligned}$$

output
```
-(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/d-(f*x+e)^2*ln(1+b*exp
(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/d+(f*x+e)^2*ln(1-exp(2*d*x+2*c))/a/d-2*f*(f
*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/d^2-2*f*(f*x+e)*polyl
og(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/d^2+f*(f*x+e)*polylog(2,exp(2*d*
x+2*c))/a/d^2+2*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/d^3+2*f
^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/d^3-1/2*f^2*polylog(3,ex
p(2*d*x+2*c))/a/d^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1296 vs. $2(325) = 650$.

Time = 5.29 (sec) , antiderivative size = 1296, normalized size of antiderivative = 3.99

$$\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 - (2*(e + f*x)^3)
/f + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]]
)/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[
(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-
(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 -
b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c
+ d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*(-1 + E^(2*c))*(e +
f*x)^2*Log[1 - E^(-c - d*x)])/d + (3*(-1 + E^(2*c))*(e + f*x)^2*Log[1 + E^
(-c - d*x)])/d + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d
 - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*
e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d -
(6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*
c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*
E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt
[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + S
qrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))
/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c +
d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 +
 (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*(-1 + E^(2
*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.09 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.22, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {6103, 3042, 26, 4201, 2620, 3011, 2720, 6095, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6103}$$

$$\frac{\int (e+fx)^2\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{a}$$

$\downarrow$ 26

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\int (e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{a}$$

$\downarrow$ 4201

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\int \frac{e^{2c+2dx-i\pi}(e+fx)^2}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^3}{3f}\right)}{a}$$

$\downarrow$ 2620

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int (e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{a}$$

$\downarrow$ 3011

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int \text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{2d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{a}$$

$\downarrow$ 2720

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-i(\ldots\right)}{a}$$

$\downarrow$ 6095

$$-\frac{b\left(\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{a}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-i(\ldots\right)}{a}$$

$\Big\downarrow$ 2620

$$-\frac{b\left(-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+d}}{\sqrt{a^2+b^2}}\right.}{bd}\right.}{a}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^2\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(}{}\right.}{a}$$

$\Big\downarrow$ 3011

$$-\frac{b\left(-\frac{2f\left(\frac{f\int\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be}{a+\sqrt{}}\right.}{d}\right.}{bd}\right.}{a}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^2\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(}{}\right.}{a}$$

$\Big\downarrow$ 2720

$$-\frac{b\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right.}{bd}\right.}{a}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^2\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(}{}\right.}{a}$$

$\Big\downarrow$ 7143

$$-\frac{b\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right.}{a}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\,\text{PolyLog}\left(3,-e^{2c+2dx-i\pi}\right)}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{a}$$

input
```
Int[((e + f*x)^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-((b*(-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2]))])/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2]))])/d^2))/(b*d)))/a) - (I*(((-1/3*I)*(e + f*x)^3)/f + (2*I)*(((e + f*
x)^2*Log[1 + E^(2*c - I*Pi + 2*d*x)])/(2*d) - (f*(-1/2*((e + f*x)*PolyLog[
2, -E^(2*c - I*Pi + 2*d*x)])/d + (f*PolyLog[3, -E^(2*c - I*Pi + 2*d*x)])/(
4*d^2)))/d)))/a
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6103
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Coth[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x
]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(fx+e)^2 \coth(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 813 vs. $2(302) = 604$.

Time = 0.11 (sec) , antiderivative size = 813, normalized size of antiderivative = 2.50

$$\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(2*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 2*f^2*polylog(3, (a*cosh(d*x
+ c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b
^2)/b^2))/b) - 2*f^2*polylog(3, cosh(d*x + c) + sinh(d*x + c)) - 2*f^2*pol
ylog(3, -cosh(d*x + c) - sinh(d*x + c)) - 2*(d*f^2*x + d*e*f)*dilog((a*cos
h(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2) - b)/b + 1) - 2*(d*f^2*x + d*e*f)*dilog((a*cosh(d*x + c) +
a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2
) - b)/b + 1) + 2*(d*f^2*x + d*e*f)*dilog(cosh(d*x + c) + sinh(d*x + c)) +
 2*(d*f^2*x + d*e*f)*dilog(-cosh(d*x + c) - sinh(d*x + c)) - (d^2*e^2 - 2*
c*d*e*f + c^2*f^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a
^2 + b^2)/b^2) + 2*a) - (d^2*e^2 - 2*c*d*e*f + c^2*f^2)*log(2*b*cosh(d*x +
 c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (d^2*f^2*x^2
+ 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x +
c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (
d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*log(-(a*cosh(d*x + c) + a
*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
 - b)/b) + (d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2)*log(cosh(d*x + c) + sinh(
d*x + c) + 1) + (d^2*e^2 - 2*c*d*e*f + c^2*f^2)*log(cosh(d*x + c) + sinh(d
*x + c) - 1) + (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*log(-c...
```

Sympy [F]

$$\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)^2\coth{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\coth\left(dx+c\right)}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e^2*(log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*d) - log(e^(-d*x
- c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d)) + 2*(d*x*log(e^(d*x + c) +
1) + dilog(-e^(d*x + c)))*e*f/(a*d^2) + 2*(d*x*log(-e^(d*x + c) + 1) + dil
og(e^(d*x + c)))*e*f/(a*d^2) + (d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog
(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*f^2/(a*d^3) + (d^2*x^2*log(-e
^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*f^
2/(a*d^3) - 2/3*(d^3*f^2*x^3 + 3*d^3*e*f*x^2)/(a*d^3) + integrate(-2*(b*f^
2*x^2 + 2*b*e*f*x - (a*f^2*x^2*e^c + 2*a*e*f*x*e^c)*e^(d*x))/(a*b*e^(2*d*x
 + 2*c) + 2*a^2*e^(d*x + c) - a*b), x)
```

Giac [F]

$$\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\coth\left(dx+c\right)}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*coth(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+d\,x)\,\left(e+f\,x\right)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((coth(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((coth(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2e^{3c}\left(\int \frac{e^{3dx}x^2}{e^{4dx+4c}b+2e^{3dx+3c}a-2e^{2dx+2c}b-2e^{dx+c}a+b}dx\right)ad\,f^2+4e^{3c}\left(\int \frac{e^{3dx}x}{e^{4dx+4c}b+2e^{3dx+3c}a-2e^{2dx+2c}b-2e^{dx+c}a+b}dx\right)ade}{}$$

input
```
int((f*x+e)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
(2*e**(3*c)*int((e**(3*d*x)*x**2)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)
*a - 2*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a*d*f**2 + 4*e**(3*c)
*int((e**(3*d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a - 2*e**(2*c
 + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a*d*e*f + 2*e**c*int((e**(d*x)*x**2
)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a - 2*e**(2*c + 2*d*x)*b - 2*e*
*(c + d*x)*a + b),x)*a*d*f**2 + 4*e**c*int((e**(d*x)*x)/(e**(4*c + 4*d*x)*
b + 2*e**(3*c + 3*d*x)*a - 2*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)
*a*d*e*f + log(e**(c + d*x) - 1)*e**2 + log(e**(c + d*x) + 1)*e**2 - log(e
**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*e**2)/(a*d)
```

3.422 $\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 205

$$\begin{aligned}\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx =& -\frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} \\ & -\frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} \\ & +\frac{(e+fx)\log\left(1-e^{2(c+dx)}\right)}{ad} \\ & -\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2} \\ & -\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^2}+\frac{f\,\mathrm{PolyLog}\left(2,e^{2(c+dx)}\right)}{2ad^2}\end{aligned}$$

output

```
-(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/d-(f*x+e)*ln(1+b*exp(d*x
+c)/(a+(a^2+b^2)^(1/2)))/a/d+(f*x+e)*ln(1-exp(2*d*x+2*c))/a/d-f*polylog(2,
-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/d^2-f*polylog(2,-b*exp(d*x+c)/(a+(a^2
+b^2)^(1/2)))/a/d^2+1/2*f*polylog(2,exp(2*d*x+2*c))/a/d^2
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 431 vs. $2(205) = 410$.

Time = 2.05 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.10

$$\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{d^2fx^2 + 4de(c+dx) - 2cf(c+dx) + f(c+dx)^2 + \frac{4a(a^2+b^2)^{5/2}de\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\left(-(a^2+b^2)^2\right)^{3/2}} + \frac{4a\sqrt{-(a^2+b^2)^2}de\,\mathrm{arctanh}}{(-a^2-b^2)^{3/2}}}{}$$

input
```
Integrate[((e + f*x)*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(d^2*f*x^2 + 4*d*e*(c + d*x) - 2*c*f*(c + d*x) + f*(c + d*x)^2 + (4*a*(a^2
 + b^2)^(5/2)*d*e*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(-(a^2 + b
^2)^2)^(3/2) + (4*a*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/S
qrt[a^2 + b^2]])/(-a^2 - b^2)^(3/2) + 2*d*(e + f*x)*Log[1 - E^(-c - d*x)]
+ 2*d*(e + f*x)*Log[1 + E^(-c - d*x)] - 2*f*(c + d*x)*Log[1 + (b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2])] - 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2])] + 2*c*f*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] - 2*d
*e*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] - 2*f*PolyLog[2, -E^(-c
 - d*x)] - 2*f*PolyLog[2, E^(-c - d*x)] - 2*f*PolyLog[2, (b*E^(c + d*x))/(
-a + Sqrt[a^2 + b^2])] - 2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2]))])/(2*a*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.28, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {6103, 3042, 26, 4201, 2620, 2715, 2838, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \ 6103$$

$$\frac{\int (e+fx)\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \ 3042$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \ 26$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{a}$$

$$\Big\downarrow \ 4201$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\int \frac{e^{2c+2dx-i\pi}(e+fx)}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^2}{2f}\right)}{a}$$

$$\Big\downarrow \ 2620$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int \log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{a}$$

$$\Big\downarrow \ 2715$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\log\left(1+e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{a}$$

$$\Big\downarrow \ 2838$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{a}$$

$$\Big\downarrow \ 6095$$

$$-\frac{b\left(\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{a}-\frac{i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{a}$$

$$\Big\downarrow \ 2620$$

$$-\frac{b\left(-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)}{2bf}\right.}{a} \\ -\frac{i\left(2i\left(\frac{f\,\text{PolyLog}(2,-e^{2c+2dx-i\pi})}{4d^2}+\frac{(e+fx)\log(1+e^{2c+2dx-i\pi})}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{a}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{b\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\right.}{b}\right.}{a} \\ -\frac{i\left(2i\left(\frac{f\,\text{PolyLog}(2,-e^{2c+2dx-i\pi})}{4d^2}+\frac{(e+fx)\log(1+e^{2c+2dx-i\pi})}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{a}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)}{2bf}\right.}{a} \\ -\frac{i\left(2i\left(\frac{f\,\text{PolyLog}(2,-e^{2c+2dx-i\pi})}{4d^2}+\frac{(e+fx)\log(1+e^{2c+2dx-i\pi})}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{a}$$

input
```
Int[((e + f*x)*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-((b*(-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/
(b*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)
))/a) - (I*(((-1/2*I)*(e + f*x)^2)/f + (2*I)*(((e + f*x)*Log[1 + E^(2*c -
I*Pi + 2*d*x)])/(2*d) + (f*PolyLog[2, -E^(2*c - I*Pi + 2*d*x)])/(4*d^2)))
/a
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6103
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Coth[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x
]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(191) = 382$.

Time = 0.94 (sec) , antiderivative size = 451, normalized size of antiderivative = 2.20

method	result
risch	$-\frac{cf\ln(\mathrm{e}^{dx+c}-1)}{d^2a}+\frac{cf\ln(b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}^{dx+c}-b)}{d^2a}-\frac{f\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)c}{d^2a}-\frac{f\ln\left(\frac{b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}+a}{a+\sqrt{a^2+b^2}}\right)c}{d^2a}+\frac{e\ln(\mathrm{e}$

input
```
int((f*x+e)*coth(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/d^2*c*f/a*ln(exp(d*x+c)-1)+1/d^2*c*f/a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+
c)-b)-1/d^2*f/a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))
*c-1/d^2*f/a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/
d*e/a*ln(exp(d*x+c)-1)-1/d*e/a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/d*e
/a*ln(exp(d*x+c)+1)+1/d*f/a*ln(exp(d*x+c)+1)*x+1/d^2*f/a*dilog(exp(d*x+c)+
1)-1/d*f/a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/
d*f/a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2*f/a
*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2*f/a*d
ilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d^2*f*dilog(e
xp(d*x+c))/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(188) = 376$.

Time = 0.11 (sec) , antiderivative size = 476, normalized size of antiderivative = 2.32

$$\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{f\text{Li}_2\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2+b^2}{b^2}}-b}{b}+1\right)+f\text{Li}_2\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)-(b\cosh(dx+c)}{b}\right.}{}$$

input
```
integrate((f*x+e)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(f*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + f*dilog((a*cosh(d*x + c) + a*
sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
- b)/b + 1) - f*dilog(cosh(d*x + c) + sinh(d*x + c)) - f*dilog(-cosh(d*x +
 c) - sinh(d*x + c)) + (d*e - c*f)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x +
c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (d*e - c*f)*log(2*b*cosh(d*x + c)
+ 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (d*f*x + c*f)*log
(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))
*sqrt((a^2 + b^2)/b^2) - b)/b) + (d*f*x + c*f)*log(-(a*cosh(d*x + c) + a*s
inh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) -
 b)/b) - (d*f*x + d*e)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (d*e - c*f
)*log(cosh(d*x + c) + sinh(d*x + c) - 1) - (d*f*x + c*f)*log(-cosh(d*x + c
) - sinh(d*x + c) + 1))/(a*d^2)
```

Sympy [F]

$$\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\coth{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)\coth(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e*(log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*d) - log(e^(-d*x -
c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d)) + f*integrate(2*x*(e^(d*x + c
) + e^(-d*x - c))/((b*(e^(d*x + c) - e^(-d*x - c)) + 2*a)*(e^(d*x + c) - e
^(-d*x - c))), x)
```

Giac [F]

$$\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)\coth(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*coth(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\coth(c+d\,x)\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((coth(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((coth(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2e^{3c}\left(\int \frac{e^{3dx}x}{e^{4dx+4c}b+2e^{3dx+3c}a-2e^{2dx+2c}b-2e^{dx+c}a+b}dx\right)adf+2e^{c}\left(\int \frac{e^{dx}x}{e^{4dx+4c}b+2e^{3dx+3c}a-2e^{2dx+2c}b-2e^{dx+c}a+b}dx\right)adf+}{ad}$$

input
```
int((f*x+e)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
(2*e**(3*c)*int((e**(3*d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a
- 2*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a*d*f + 2*e**c*int((e**(
d*x)*x)/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a - 2*e**(2*c + 2*d*x)*b
- 2*e**(c + d*x)*a + b),x)*a*d*f + log(e**(c + d*x) - 1)*e + log(e**(c + d
*x) + 1)*e - log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*e)/(a*d)
```

3.423 $\int \frac{\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 34

$$\int \frac{\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\log(\sinh(c+dx))}{ad} - \frac{\log(a+b\sinh(c+dx))}{ad}$$

output
```
ln(sinh(d*x+c))/a/d-ln(a+b*sinh(d*x+c))/a/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\log(\sinh(c+dx)) - \log(a+b\sinh(c+dx))}{ad}$$

input
```
Integrate[Coth[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

output
```
(Log[Sinh[c + d*x]] - Log[a + b*Sinh[c + d*x]])/(a*d)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 26, 3200, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\tan(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{(a-ib\sin(ic+idx))\tan(ic+idx)}dx$$

$$\Big\downarrow \text{3200}$$

$$\frac{\int \frac{\text{csch}(c+dx)}{b(a+b\sinh(c+dx))}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{47}$$

$$\frac{\frac{\int \frac{\text{csch}(c+dx)}{b}d(b\sinh(c+dx))}{a}-\frac{\int \frac{1}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{a}}{d}$$

$$\Big\downarrow \text{14}$$

$$\frac{\frac{\log(b\sinh(c+dx))}{a}-\frac{\int \frac{1}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{a}}{d}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{\log(b\sinh(c+dx))}{a}-\frac{\log(a+b\sinh(c+dx))}{a}}{d}$$

input
```
Int[Coth[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

output
```
(Log[b*Sinh[c + d*x]]/a - Log[a + b*Sinh[c + d*x]]/a)/d
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

method	result	size
risch	$\frac{\ln\left(e^{2dx+2c}-1\right)}{da}-\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{b}-1\right)}{da}$	53
derivativedivides	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{a}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$	55
default	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{a}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$	55

input
```
int(coth(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*ln(exp(2*d*x+2*c)-1)-1/d/a*ln(exp(2*d*x+2*c)+2/b*a*exp(d*x+c)-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.97

$$\int \frac{\coth(c+dx)}{a+b\sinh(c+dx)}\,dx=-\frac{\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)-\log\left(\frac{2\,\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{ad}$$

input
```
integrate(coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) - log(2*sin
h(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/(a*d)
```

Sympy [F]

$$\int \frac{\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate(coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(34) = 68.

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.21

$$\int \frac{\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{ad} + \frac{\log\left(e^{(-dx-c)}+1\right)}{ad} + \frac{\log\left(e^{(-dx-c)}-1\right)}{ad}$$

input
```
integrate(coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*d) + log(e^(-d*x - c)
+ 1)/(a*d) + log(e^(-d*x - c) - 1)/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int \frac{\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a} - \frac{\log\left(\left|e^{(dx+c)}-e^{(-dx-c)}\right|\right)}{a}}{d}$$

input
```
integrate(coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-(log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/a - log(abs(e^(d*x + c) -
 e^(-d*x - c)))/a)/d
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 254, normalized size of antiderivative = 7.47

$$\int \frac{\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{a\sqrt{-a^2\,d^2}+b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-a^2\,d^2}-2\,a\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\sqrt{-a^2\,d^2}-b\,\mathrm{e}^{3\,c}\,\mathrm{e}^{3\,d\,x}\,\sqrt{-a^2\,d^2}}{a^2\,d}\right)}{\sqrt{-a^2\,d^2}} - \frac{2\,\mathrm{atan}\left(\left(4\,a^4\,b\,d\,\sqrt{-a^2\,d^2}+4\,a^2\,b^3\,d\,\sqrt{-a^2\,d^2}\right)\,\left(\frac{1}{8\,a\,b\,d^2\,(a^2+b^2)^2}-\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(\frac{1}{16\,b^2\,d^2\,(a^2+b^2)^2}-\frac{(a^2+2\,b^2)^2}{16\,a^4\,b^2\,d^2\,(a^2+b^2)^2}\right.\right.\right.}{\sqrt{-a^2\,d^2}}$$

input
```
int(coth(c + d*x)/(a + b*sinh(c + d*x)),x)
```

output
```
(2*atan((a*(-a^2*d^2)^(1/2) + b*exp(d*x)*exp(c)*(-a^2*d^2)^(1/2) - 2*a*exp
(2*c)*exp(2*d*x)*(-a^2*d^2)^(1/2) - b*exp(3*c)*exp(3*d*x)*(-a^2*d^2)^(1/2)
)/(a^2*d)))/(-a^2*d^2)^(1/2) - (2*atan((4*a^4*b*d*(-a^2*d^2)^(1/2) + 4*a^2
*b^3*d*(-a^2*d^2)^(1/2))*(1/(8*a*b*d^2*(a^2 + b^2)^2) - exp(d*x)*exp(c)*(1
/(16*b^2*d^2*(a^2 + b^2)^2) - (a^2 + 2*b^2)^2/(16*a^4*b^2*d^2*(a^2 + b^2)^
2)) + (a^2 + 2*b^2)/(8*a^3*b*d^2*(a^2 + b^2)^2))))/(-a^2*d^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.68

$$\int \frac{\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\log\left(e^{dx+c}-1\right)+\log\left(e^{dx+c}+1\right)-\log\left(e^{2dx+2c}b+2e^{dx+c}a-b\right)}{ad}$$

input
```
int(coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
(log(e**(c + d*x) - 1) + log(e**(c + d*x) + 1) - log(e**(2*c + 2*d*x)*b +
2*e**(c + d*x)*a - b))/(a*d)
```

3.424 $\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \operatorname{Int}\left(\frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}, x\right)$$

output
```
Defer(Int)(coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 9.67 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

input
```
Integrate[Coth[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Coth[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$\Big\downarrow \ 6111$$
$$\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Coth[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\coth (dx+c)}{(fx+e)(a+b\sinh (dx+c))}dx$$

input
```
int(coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(coth(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 3.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

input
```
integrate(coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(coth(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(coth(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(coth(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(c+d\,x)}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(coth(c + d*x)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(coth(c + d*x)/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(dx+c)}{\sinh(dx+c)\,be+\sinh(dx+c)\,bfx+ae+afx}dx$$

input
```
int(coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(coth(c + d*x)/(sinh(c + d*x)*b*e + sinh(c + d*x)*b*f*x + a*e + a*f*x),
x)
```

3.425 $\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 638

$$\begin{aligned}
&\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= \frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\operatorname{arctanh}\left(e^{c+dx}\right)}{ad} - \frac{\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd} \\
&\quad + \frac{\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{abd} - \frac{3f(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2} \\
&\quad + \frac{3f(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{ad^2} - \frac{3\sqrt{a^2+b^2}f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^2} \\
&\quad + \frac{3\sqrt{a^2+b^2}f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{abd^2} + \frac{6f^2(e+fx)\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3} \\
&\quad - \frac{6f^2(e+fx)\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{ad^3} + \frac{6\sqrt{a^2+b^2}f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^3} \\
&\quad - \frac{6\sqrt{a^2+b^2}f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{abd^3} \\
&\quad - \frac{6f^3\operatorname{PolyLog}\left(4,-e^{c+dx}\right)}{ad^4} + \frac{6f^3\operatorname{PolyLog}\left(4,e^{c+dx}\right)}{ad^4} \\
&\quad - \frac{6\sqrt{a^2+b^2}f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^4} + \frac{6\sqrt{a^2+b^2}f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{abd^4}
\end{aligned}$$

output
```
1/4*(f*x+e)^4/b/f-2*(f*x+e)^3*arctanh(exp(d*x+c))/a/d-(a^2+b^2)^(1/2)*(f*x
+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/b/d+(a^2+b^2)^(1/2)*(f*x+e)
^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/b/d-3*f*(f*x+e)^2*polylog(2,-e
xp(d*x+c))/a/d^2+3*f*(f*x+e)^2*polylog(2,exp(d*x+c))/a/d^2-3*(a^2+b^2)^(1/
2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/b/d^2+3*(a^2
+b^2)^(1/2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/b/d
^2+6*f^2*(f*x+e)*polylog(3,-exp(d*x+c))/a/d^3-6*f^2*(f*x+e)*polylog(3,exp(
d*x+c))/a/d^3+6*(a^2+b^2)^(1/2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^
2+b^2)^(1/2)))/a/b/d^3-6*(a^2+b^2)^(1/2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+
c)/(a+(a^2+b^2)^(1/2)))/a/b/d^3-6*f^3*polylog(4,-exp(d*x+c))/a/d^4+6*f^3*p
olylog(4,exp(d*x+c))/a/d^4-6*(a^2+b^2)^(1/2)*f^3*polylog(4,-b*exp(d*x+c)/(
a-(a^2+b^2)^(1/2)))/a/b/d^4+6*(a^2+b^2)^(1/2)*f^3*polylog(4,-b*exp(d*x+c)/
(a+(a^2+b^2)^(1/2)))/a/b/d^4
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 781, normalized size of antiderivative = 1.22

$$\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{x(4e^3+6e^2fx+4ef^2x^2+f^3x^3)}{4b}$$
$$+\frac{(e+fx)^3\log\left(1-e^{c+dx}\right)-(e+fx)^3\log\left(1+e^{c+dx}\right)-\frac{3f\left(d^2(e+fx)^2\operatorname{PolyLog}(2,-e^{c+dx})-2df(e+fx)\operatorname{PolyLog}(3,-e\right.}{d^3}}{ad}$$
$$-\frac{\sqrt{a^2+b^2}\left(-2d^3e^3\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+3d^3e^2fx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+3d^3ef^2x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+\right.}{}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
(x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))/(4*b) + ((e + f*x)^3*Log[1
 - E^(c + d*x)] - (e + f*x)^3*Log[1 + E^(c + d*x)] - (3*f*(d^2*(e + f*x)^2
*PolyLog[2, -E^(c + d*x)] - 2*d*f*(e + f*x)*PolyLog[3, -E^(c + d*x)] + 2*f
^2*PolyLog[4, -E^(c + d*x)]))/d^3 + (3*f*(d^2*(e + f*x)^2*PolyLog[2, E^(c
+ d*x)] - 2*d*f*(e + f*x)*PolyLog[3, E^(c + d*x)] + 2*f^2*PolyLog[4, E^(c
+ d*x)]))/d^3)/(a*d) - (Sqrt[a^2 + b^2]*(-2*d^3*e^3*ArcTanh[(a + b*E^(c +
d*x))/Sqrt[a^2 + b^2]] + 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2])] + 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]
)] + d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 3*d^3*e^
2*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 3*d^3*e*f^2*x^2*Log
[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - d^3*f^3*x^3*Log[1 + (b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2])] + 3*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(c +
d*x))/(-a + Sqrt[a^2 + b^2])] - 3*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2]))] - 6*d*e*f^2*PolyLog[3, (b*E^(c + d*x))/(-a
+ Sqrt[a^2 + b^2])] - 6*d*f^3*x*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2
+ b^2])] + 6*d*e*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
+ 6*d*f^3*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 6*f^3*P
olyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 6*f^3*PolyLog[4, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a*b*d^4)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6119}$$

$$\frac{\int (e+fx)^3\cosh(c+dx)\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 5973}$$

$$\frac{\int (e+fx)^3\sinh(c+dx)dx+\int (e+fx)^3\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i(e+fx)^3\sin(ic+idx)dx+\int i(e+fx)^3\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\int (e+fx)^3\sin(ic+idx)dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\cosh(c+dx)dx}{d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{a}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int (e+fx)\sinh(c+dx)dx}{d}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int -i(e+fx)\sin(ic+idx)dx}{d}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int (e+fx)\sin(ic+idx)dx}{d}\right)}{d}\right)}{a}}{}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}\right)}{d}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{3117}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$\frac{i\left(\frac{3if\int (e+fx)^2\log(1-e^{c+dx})dx}{d}-\frac{3if\int (e+fx)^2\log(1+e^{c+dx})dx}{d}+\frac{2i(e+fx)^3\text{arctanh}(e^{c+dx})}{d}\right)-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\right.}{}\right.}{a}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\Big\downarrow \text{6099}$$

$$-\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)^3dx}{b^2}+\frac{\int (e+fx)^3\sinh(c+dx)dx}{b}\right)}{a}+$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\Big\downarrow \text{17}$$

$$-\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int (e+fx)^3\sinh(c+dx)dx}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{a}+$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}+\frac{\int -i(e+fx)^3\sin(ic+idx)dx}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{a}$$

$\downarrow$ 26

$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\int (e+fx)^3\sin(ic+idx)dx}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{a}$$

$\downarrow$ 3777

$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\cosh(c+dx)dx}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{a}$$

$\downarrow$ 3042

$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{a}$$

$\Big\downarrow$ 3777

$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{a}$$

$\Big\downarrow$ 26

$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int (e+fx)\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{a}$$

$\Big\downarrow$ 3042

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int-i(e+fx)\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{a}$$

$$\Big\downarrow \ 26$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int(e+fx)\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{a}$$

$$\Big\downarrow \ 3777$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{d}\right)}{d}\right)}{b}-\frac{a(e+}{4b^2}\right.}{a}$$

$\downarrow$ 3042

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{d}\right)}{b}-\right.}{a}$$

$\downarrow$ 3117

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}\right.}{a}$$

$\downarrow$ 3803

$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{b\left(\frac{2(a^2+b^2)\int -\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(}{d^2}\right.}{d}\right.}{d}\right.}{b}\right.}{a}$$

$\downarrow$ 25

$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{b\left(-\frac{2(a^2+b^2)\int \frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(}{d}\right.}{d}\right.}{d}\right.}{b}\right.}{a}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^3\cosh(dx+c)\coth(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1470 vs. $2(585) = 1170$.

Time = 0.16 (sec) , antiderivative size = 1470, normalized size of antiderivative = 2.30

$$\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
1/4*(a*d^4*f^3*x^4 + 4*a*d^4*e*f^2*x^3 + 6*a*d^4*e^2*f*x^2 + 4*a*d^4*e^3*x
 - 24*b*f^3*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x
 + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 24
*b*f^3*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c)
 - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 24*b*f^
3*polylog(4, cosh(d*x + c) + sinh(d*x + c)) - 24*b*f^3*polylog(4, -cosh(d*
x + c) - sinh(d*x + c)) - 12*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*
f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cos
h(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 12*(b*d^
2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*
cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2) - b)/b + 1) + 4*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d
*e*f^2 - b*c^3*f^3)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh
(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 4*(b*d^3*e^3 - 3*b*c*d^2*e^
2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x
+ c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 4*(b*d^3*f^3
*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e
*f^2 + b*c^3*f^3)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x
 + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b)
+ 4*(b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^...
```

Sympy [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)^3\cosh{(c+dx)}\coth{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*cosh(c + d*x)*coth(c + d*x)/(a + b*sinh(c + d*x)), x
)
```

Maxima [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)\coth(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
e^3*((d*x + c)/(b*d) - log(e^(-d*x - c) + 1)/(a*d) + log(e^(-d*x - c) - 1)
/(a*d) - sqrt(a^2 + b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(
-d*x - c) - a + sqrt(a^2 + b^2)))/(a*b*d)) - 3*(d*x*log(e^(d*x + c) + 1) +
 dilog(-e^(d*x + c)))*e^2*f/(a*d^2) + 3*(d*x*log(-e^(d*x + c) + 1) + dilog
(e^(d*x + c)))*e^2*f/(a*d^2) + 1/4*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2)/b
 - 3*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog
(3, -e^(d*x + c)))*e*f^2/(a*d^3) + 3*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*
x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*e*f^2/(a*d^3) - (d^3*x^3
*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -
e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*f^3/(a*d^4) + (d^3*x^3*log(-e^(
d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c
)) + 6*polylog(4, e^(d*x + c)))*f^3/(a*d^4) - integrate(2*((a^2*f^3*e^c +
b^2*f^3*e^c)*x^3 + 3*(a^2*e*f^2*e^c + b^2*e*f^2*e^c)*x^2 + 3*(a^2*e^2*f*e^
c + b^2*e^2*f*e^c)*x)*e^(d*x)/(a*b^2*e^(2*d*x + 2*c) + 2*a^2*b*e^(d*x + c)
 - a*b^2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\cosh(c+d\,x)\,\coth(c+d\,x)\,\left(e+f\,x\right)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)*coth(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*coth(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\cosh\left(dx+c\right)\coth\left(dx+c\right)}{a+b\sinh\left(dx+c\right)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.426 $\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 462

$$\begin{aligned}
\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx
&= \frac{(e+fx)^3}{3bf} - \frac{2(e+fx)^2\operatorname{arctanh}\left(e^{c+dx}\right)}{ad} - \frac{\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd} \\
&\quad + \frac{\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{abd} - \frac{2f(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2} \\
&\quad + \frac{2f(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{ad^2} - \frac{2\sqrt{a^2+b^2}f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^2} \\
&\quad + \frac{2\sqrt{a^2+b^2}f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{abd^2} \\
&\quad + \frac{2f^2\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3} - \frac{2f^2\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{ad^3} \\
&\quad + \frac{2\sqrt{a^2+b^2}f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^3} - \frac{2\sqrt{a^2+b^2}f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{abd^3}
\end{aligned}$$

output
```
1/3*(f*x+e)^3/b/f-2*(f*x+e)^2*arctanh(exp(d*x+c))/a/d-(a^2+b^2)^(1/2)*(f*x
+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/b/d+(a^2+b^2)^(1/2)*(f*x+e)
^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/b/d-2*f*(f*x+e)*polylog(2,-exp
(d*x+c))/a/d^2+2*f*(f*x+e)*polylog(2,exp(d*x+c))/a/d^2-2*(a^2+b^2)^(1/2)*f
*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/b/d^2+2*(a^2+b^2)^
(1/2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/b/d^2+2*f^2
*polylog(3,-exp(d*x+c))/a/d^3-2*f^2*polylog(3,exp(d*x+c))/a/d^3+2*(a^2+b^2
)^(1/2)*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/b/d^3-2*(a^2+b^
2)^(1/2)*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/b/d^3
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.06

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{x(3e^2+3efx+f^2x^2)}{3b} + \frac{(e+fx)^2\log\left(1-e^{c+dx}\right)-(e+fx)^2\log\left(1+e^{c+dx}\right)-\frac{2f\left(d(e+fx)\operatorname{PolyLog}(2,-e^{c+dx})-f\operatorname{PolyLog}(3,-e^{c+dx})\right)}{d^2}+ \cdots}{ad} - \frac{\sqrt{a^2+b^2}\left(-2d^2e^2\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+2d^2efx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+d^2f^2x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-2d^2\cdots\right)}{\cdots}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
(x*(3*e^2 + 3*e*f*x + f^2*x^2))/(3*b) + ((e + f*x)^2*Log[1 - E^(c + d*x)]
- (e + f*x)^2*Log[1 + E^(c + d*x)] - (2*f*(d*(e + f*x)*PolyLog[2, -E^(c +
d*x)] - f*PolyLog[3, -E^(c + d*x)]))/d^2 + (2*f*(d*(e + f*x)*PolyLog[2, E^
(c + d*x)] - f*PolyLog[3, E^(c + d*x)]))/d^2)/(a*d) - (Sqrt[a^2 + b^2]*(-2
*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*d^2*e*f*x*Log[1
+ (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2])] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
] + 2*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2
*d*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2*f^
2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f^2*PolyLog[3, -(
(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a*b*d^3)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6119}$$

$$\frac{\int (e+fx)^2\cosh(c+dx)\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5973}$$

$$\frac{\int (e+fx)^2\sinh(c+dx)dx+\int (e+fx)^2\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i(e+fx)^2\sin(ic+idx)dx+\int i(e+fx)^2\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx-i\int (e+fx)^2\sin(ic+idx)dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\cosh(c+dx)dx}{d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+i\int (e+fx)^2\csc(ic+idx)dx-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{d}\right)}{a}$$

$$\Big\downarrow \ 26$$

$$\frac{-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+i\int (e+fx)^2\csc(ic+idx)dx-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)}{d}\right)}{a}$$

$$\Big\downarrow \ 3042$$

$$\frac{-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+i\int (e+fx)^2\csc(ic+idx)dx-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}\right)}{a}$$

$$\Big\downarrow \ 26$$

$$\frac{-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+i\int (e+fx)^2\csc(ic+idx)dx-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}\right)}{d}\right)}{a}$$

$$\Big\downarrow \ 3118$$

$$\frac{-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+i\int (e+fx)^2\csc(ic+idx)dx-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{a}$$

$$\Big\downarrow \ 4670$$

$$\frac{-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+i\left(\frac{2if\int (e+fx)\log\left(1-e^{c+dx}\right)dx}{d}-\frac{2if\int (e+fx)\log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)^2\text{arctanh}\left(e^{c+dx}\right)}{d}\right)-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left((e\cdots\right.}{}\right.}{a}$$

$$\Big\downarrow \ 3011$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}+\frac{2i(e+fx)^2\text{ar}}{}\right.}{a}$$

$$\Big\downarrow \ 2720$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{\cdot}\right.}{d}\right.}{d}\right.}{a}$$

$$\Big\downarrow \ 6099$$

$$-\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)^2dx}{b^2}+\frac{\int (e+fx)^2\sinh(c+dx)dx}{b}\right)}{a}+\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{\cdot}\right.}{d}\right.}{d}\right.}{a}$$

$$\Big\downarrow \ 17$$

$$-\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int (e+fx)^2\sinh(c+dx)dx}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{a}+\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{\cdot}\right.}{d}\right.}{d}\right.}{a}$$

$$\Big\downarrow \ 3042$$

$$\frac{i\left(-\frac{2 i f\left(\frac{f \int e^{-c-dx} \operatorname{PolyLog}\left(2,-e^{c+dx}\right) d e^{c+dx}}{d^2}-\frac{(e+fx) \operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2 i f\left(\frac{f \int e^{-c-dx} \operatorname{PolyLog}\left(2,e^{c+dx}\right) d e^{c+dx}}{d^2}-\frac{(e+fx) \operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right.}{d}\right.}{a}$$

$$\frac{b\left(\frac{\left(a^2+b^2\right) \int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}+\frac{\int -i(e+fx)^2 \sin(ic+idx)dx}{b}-\frac{a(e+fx)^3}{3b^2 f}\right)}{a}$$

$\Big\downarrow$ 26

$$\frac{i\left(-\frac{2 i f\left(\frac{f \int e^{-c-dx} \operatorname{PolyLog}\left(2,-e^{c+dx}\right) d e^{c+dx}}{d^2}-\frac{(e+fx) \operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2 i f\left(\frac{f \int e^{-c-dx} \operatorname{PolyLog}\left(2,e^{c+dx}\right) d e^{c+dx}}{d^2}-\frac{(e+fx) \operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right.}{d}\right.}{a}$$

$$\frac{b\left(\frac{\left(a^2+b^2\right) \int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\int (e+fx)^2 \sin(ic+idx)dx}{b}-\frac{a(e+fx)^3}{3b^2 f}\right)}{a}$$

$\Big\downarrow$ 3777

$$\frac{i\left(-\frac{2 i f\left(\frac{f \int e^{-c-dx} \operatorname{PolyLog}\left(2,-e^{c+dx}\right) d e^{c+dx}}{d^2}-\frac{(e+fx) \operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2 i f\left(\frac{f \int e^{-c-dx} \operatorname{PolyLog}\left(2,e^{c+dx}\right) d e^{c+dx}}{d^2}-\frac{(e+fx) \operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right.}{d}\right.}{a}$$

$$\frac{b\left(\frac{\left(a^2+b^2\right) \int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\cosh(c+dx)dx}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2 f}\right)}{a}$$

$\Big\downarrow$ 3042

$$\frac{i\left(-\frac{2 i f\left(\frac{f \int e^{-c-dx} \operatorname{PolyLog}\left(2,-e^{c+dx}\right) d e^{c+dx}}{d^2}-\frac{(e+fx) \operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2 i f\left(\frac{f \int e^{-c-dx} \operatorname{PolyLog}\left(2,e^{c+dx}\right) d e^{c+dx}}{d^2}-\frac{(e+fx) \operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right.}{d}\right.}{a}$$

$$\frac{b\left(\frac{\left(a^2+b^2\right) \int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2 f}\right)}{a}$$

$\Big\downarrow$ 3777

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right.}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{a}$$

$\downarrow$ 26

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right.}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{a}$$

$\downarrow$ 3042

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right.}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{a}$$

$\downarrow$ 26

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{\cdots}\right.}{d}\right.}{d}\cdots}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{a}$$

$$\Big\downarrow \text{3118}$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{\cdots}\right.}{d}\right.}{d}\cdots}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}\right)}{a}$$

$$\Big\downarrow \text{3803}$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{\cdots}\right.}{d}\right.}{d}\cdots}{a}$$

$$\frac{b\left(\frac{2(a^2+b^2)\int-\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}\right)}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\cdots\right.}{d}\cdots\right.}{a}$$

$$\frac{b\left(-\frac{2(a^2+b^2)\int\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}\right)}{a}$$

$\downarrow$ 2694

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\cdots\right.}{d}\cdots\right.}{a}$$

$$\frac{b\left(-\frac{2(a^2+b^2)\left(\frac{b\int-\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int-\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+\cdots}{d}\cdots\right.}{\cdots}\right.}{b}\cdots\right.}{a}$$

$\downarrow$ 27

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\cdots\right.}{d}\cdots\right.}{a}$$

$$\frac{b\left(-\frac{2(a^2+b^2)\left(\frac{b\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh\cdots}{\cdots}\right.}{d}\cdots\right.}{b}\cdots\right.}{a}$$

$\downarrow$ 2620

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right.}{d}}{a}$$

$$\frac{b\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b}}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{b^2}\right.}{a}$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^2\cosh(dx+c)\coth(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. $2(421) = 842$.

Time = 0.12 (sec) , antiderivative size = 992, normalized size of antiderivative = 2.15

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input

```
integrate((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output

```
1/3*(a*d^3*f^2*x^3 + 3*a*d^3*e*f*x^2 + 3*a*d^3*e^2*x + 6*b*f^2*sqrt((a^2 +
 b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c
) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*b*f^2*sqrt((a^2 + b^2)/
b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*
sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*b*f^2*polylog(3, cosh(d*x + c
) + sinh(d*x + c)) + 6*b*f^2*polylog(3, -cosh(d*x + c) - sinh(d*x + c)) -
6*(b*d*f^2*x + b*d*e*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*s
inh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) -
 b)/b + 1) + 6*(b*d*f^2*x + b*d*e*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d
*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
+ b^2)/b^2) - b)/b + 1) + 3*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sqrt((a^
2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 +
b^2)/b^2) + 2*a) - 3*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sqrt((a^2 + b^2
)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^
2) + 2*a) - 3*(b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*sq
rt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 3*(b*d^2*f^2*x^2 +
 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*co
sh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((
a^2 + b^2)/b^2) - b)/b) + 6*(b*d*f^2*x + b*d*e*f)*dilog(cosh(d*x + c) +...
```

Sympy [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*cosh(c + d*x)*coth(c + d*x)/(a + b*sinh(c + d*x)), x
)
```

Maxima [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)\coth(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
e^2*((d*x + c)/(b*d) - log(e^(-d*x - c) + 1)/(a*d) + log(e^(-d*x - c) - 1)
/(a*d) - sqrt(a^2 + b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(
-d*x - c) - a + sqrt(a^2 + b^2)))/(a*b*d)) + 1/3*(f^2*x^3 + 3*e*f*x^2)/b -
 2*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))*e*f/(a*d^2) + 2*(d*x*l
og(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))*e*f/(a*d^2) - (d^2*x^2*log(e^(d
*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*f^2
/(a*d^3) + (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*p
olylog(3, e^(d*x + c)))*f^2/(a*d^3) - integrate(2*((a^2*f^2*e^c + b^2*f^2*
e^c)*x^2 + 2*(a^2*e*f*e^c + b^2*e*f*e^c)*x)*e^(d*x)/(a*b^2*e^(2*d*x + 2*c)
 + 2*a^2*b*e^(d*x + c) - a*b^2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,\coth(c+d\,x)\,\left(e+f\,x\right)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)*coth(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*coth(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh\left(dx+c\right)\coth\left(dx+c\right)}{a+b\sinh\left(dx+c\right)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.427 $\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 286

$$\begin{aligned}\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{(e+fx)^2}{2bf}-\frac{2(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{ad}\\
&-\frac{\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd}\\
&+\frac{\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{abd}\\
&-\frac{f\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}\\
&+\frac{f\,\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2}\\
&-\frac{\sqrt{a^2+b^2}f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^2}\\
&+\frac{\sqrt{a^2+b^2}f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{abd^2}\end{aligned}$$

output
```
1/2*(f*x+e)^2/b/f-2*(f*x+e)*arctanh(exp(d*x+c))/a/d-(a^2+b^2)^(1/2)*(f*x+e
)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/b/d+(a^2+b^2)^(1/2)*(f*x+e)*ln(
1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/b/d-f*polylog(2,-exp(d*x+c))/a/d^2+f
*polylog(2,exp(d*x+c))/a/d^2-(a^2+b^2)^(1/2)*f*polylog(2,-b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))/a/b/d^2+(a^2+b^2)^(1/2)*f*polylog(2,-b*exp(d*x+c)/(a+(a^
2+b^2)^(1/2)))/a/b/d^2
```

Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.04

$$\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-a(c+dx)(cf-d(2e+fx))+2b\big(d(e+fx)\left(\log\left(1-e^{c+dx}\right)-\log\left(1+e^{c+dx}\right)\right)-f\,\mathrm{PolyLog}\left(2,-e^{c+c}\right.}{}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-(a*(c + d*x)*(c*f - d*(2*e + f*x))) + 2*b*(d*(e + f*x)*(Log[1 - E^(c + d
*x)] - Log[1 + E^(c + d*x)]) - f*PolyLog[2, -E^(c + d*x)] + f*PolyLog[2, E
^(c + d*x)]) + 2*Sqrt[a^2 + b^2]*(2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a
^2 + b^2]] - 2*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - f*(c + d
*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - f*PolyLog[2, (b*E^(c + d*x))/(-a +
 Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
]))/(2*a*b*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.35 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.29, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {6119, 5973, 3042, 26, 3777, 3042, 3117, 4670, 2715, 2838, 6099, 17, 3042, 26, 3777, 3042, 3117, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6119}$$

$$\frac{\int (e+fx)\cosh(c+dx)\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 5973}$$

$$\frac{\int (e+fx)\sinh(c+dx)dx+\int (e+fx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i(e+fx)\sin(ic+idx)dx+\int i(e+fx)\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx-i\int (e+fx)\sin(ic+idx)dx}{a}$$

$$\Big\downarrow \text{ 3777}$$

$$-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}\right)}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{a}$$

$$\Big\downarrow \text{ 3117}$$

$$-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}$$

$$\Big\downarrow \text{ 4670}$$

$$-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{if\int \log\left(1-e^{c+dx}\right)dx}{d}-\frac{if\int \log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}$$

$$\Big\downarrow \; 2715$$

$$-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{if\int e^{-c-dx}\log\left(1-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log\left(1+e^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-if\right.}{a}$$

$$\Big\downarrow \; 2838$$

$$-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}$$

$$\Big\downarrow \; 6099$$

$$-\frac{b\left(\frac{\left(a^2+b^2\right)\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)dx}{b^2}+\frac{\int (e+fx)\sinh(c+dx)dx}{b}\right)}{a}+\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}$$

$$\Big\downarrow \; 17$$

$$-\frac{b\left(\frac{\left(a^2+b^2\right)\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int (e+fx)\sinh(c+dx)dx}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{a}+\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(\frac{\left(a^2+b^2\right)\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}+\frac{\int -i(e+fx)\sin(ic+idx)dx}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(\frac{(a^2+b^2)\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\int(e+fx)\sin(ic+idx)dx}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{a}$$

$$\Big\downarrow \; 3777$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(\frac{(a^2+b^2)\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(\frac{(a^2+b^2)\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{a}$$

$$\Big\downarrow \; 3117$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(\frac{(a^2+b^2)\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{a}$$

$$\Big\downarrow \; 3803$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(\frac{2(a^2+b^2)\int-\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{a}$$

$$\Big\downarrow \ 25$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(-\frac{2(a^2+b^2)\int\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{a}$$

$$\Big\downarrow \ 2694$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(-\frac{2(a^2+b^2)\left(\frac{b\int-\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int-\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{a}$$

$$\Big\downarrow \ 27$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(-\frac{2(a^2+b^2)\left(\frac{b\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{a}$$

$$\Big\downarrow \ 2620$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{b^2}-\cdots\right)}{a}$$

$\Big\downarrow$ 2715

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}\right)}{b^2}\cdots\right)}{a}$$

$\Big\downarrow$ 2838

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{b^2}-\cdots\right)}{a}$$

input
```
Int[((e + f*x)*Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(I*(((2*I)*(e + f*x)*ArcTanh[E^(c + d*x)])/d + (I*f*PolyLog[2, -E^(c + d*x
)])/d^2 - (I*f*PolyLog[2, E^(c + d*x)])/d^2) - I*((I*(e + f*x)*Cosh[c + d*
x])/d - (I*f*Sinh[c + d*x])/d^2))/a - (b*(-1/2*(a*(e + f*x)^2)/(b^2*f) - (
2*(a^2 + b^2)*(-1/2*(b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/
(b*d^2)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2]))])/(b*d^2)))/(2*Sqrt[a^2 + b^2])))/b^2 - (I*((I*(e + f*x)*Cosh[c + d*
x])/d - (I*f*Sinh[c + d*x])/d^2))/b))/a
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 6119
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(261) = 522$.

Time = 1.23 (sec) , antiderivative size = 970, normalized size of antiderivative = 3.39

method	result
risch	$\frac{f\,x^2}{2b}+\frac{ex}{b}-\frac{af\ln\left(\frac{-b\,e^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)c}{b\,d^2\sqrt{a^2+b^2}}-\frac{af\ln\left(\frac{-b\,e^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)x}{bd\sqrt{a^2+b^2}}+\frac{af\ln\left(\frac{b\,e^{dx+c}+\sqrt{a^2+b^2}+a}{a+\sqrt{a^2+b^2}}\right)x}{bd\sqrt{a^2+b^2}}-\frac{af\,\mathrm{dilog}}{}$

input
```
int((f*x+e)*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERB
OSE)
```

output
```
1/2/b*f*x^2+1/b*e*x-1/b/d^2*a*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2
)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/b/d*a*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*
x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/b/d*a*f/(a^2+b^2)^(1/2)*
ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/b/d^2*a*f/(a^
2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))
+1/b/d^2*a*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^
2+b^2)^(1/2)))-1/d*f/a*ln(exp(d*x+c)+1)*x-1/d^2*f/a*dilog(exp(d*x+c)+1)-1/
d^2*f*dilog(exp(d*x+c))/a+2/b/d*a*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d
*x+c)+2*a)/(a^2+b^2)^(1/2))+2*b/d*e/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp
(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2*b/d^2*c*f/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2
*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-b/d^2*f/a/(a^2+b^2)^(1/2)*ln((-b*exp(d
*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+b/d^2*f/a/(a^2+b^2)^(1/2)
*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/b/d^2*a*f/(a
^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1
/d^2*c*f/a*ln(exp(d*x+c)-1)-b/d*f/a/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2
+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+b/d*f/a/(a^2+b^2)^(1/2)*ln((b*exp(d
*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-b/d^2*f/a/(a^2+b^2)^(1/2)*
dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+b/d^2*f/a/(a
^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-
1/d*e/a*ln(exp(d*x+c)+1)+1/d*e/a*ln(exp(d*x+c)-1)-2/b/d^2*c*a*f/(a^2+b^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(257) = 514$.

Time = 0.14 (sec) , antiderivative size = 598, normalized size of antiderivative = 2.09

$$\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
fricas")
```

output
```
1/2*(a*d^2*f*x^2 + 2*a*d^2*e*x - 2*b*f*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh
(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2) - b)/b + 1) + 2*b*f*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x
+ c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b
^2)/b^2) - b)/b + 1) + 2*b*f*dilog(cosh(d*x + c) + sinh(d*x + c)) - 2*b*f*
dilog(-cosh(d*x + c) - sinh(d*x + c)) + 2*(b*d*e - b*c*f)*sqrt((a^2 + b^2)
/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2
) + 2*a) - 2*(b*d*e - b*c*f)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) +
 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(b*d*f*x + b*c*f
)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(
d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*(b*d*f*x + b
*c*f)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*(b*d*f*x
 + b*d*e)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 2*(b*d*e - b*c*f)*log(c
osh(d*x + c) + sinh(d*x + c) - 1) + 2*(b*d*f*x + b*c*f)*log(-cosh(d*x + c)
 - sinh(d*x + c) + 1))/(a*b*d^2)
```

Sympy [F]

$$\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\cosh{(c+dx)}\coth{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*cosh(c + d*x)*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh{(dx+c)}\coth{(dx+c)}}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
maxima")
```

output
```
-1/2*(4*(a^2*e^c + b^2*e^c)*integrate(x*e^(d*x)/(a*b^2*e^(2*d*x + 2*c) + 2
*a^2*b*e^(d*x + c) - a*b^2), x) - x^2/b - 2*integrate(x/(a*e^(d*x + c) + a
), x) - 2*integrate(x/(a*e^(d*x + c) - a), x))*f + e*((d*x + c)/(b*d) - lo
g(e^(-d*x - c) + 1)/(a*d) + log(e^(-d*x - c) - 1)/(a*d) - sqrt(a^2 + b^2)*
log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2
+ b^2)))/(a*b*d))
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,\coth(c+d\,x)\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)*coth(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*coth(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)\coth(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.428 $\int \frac{\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 71

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{x}{b}-\frac{\operatorname{arctanh}(\cosh(c+dx))}{ad}+\frac{2\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{abd}$$

output
```
x/b-arctanh(cosh(d*x+c))/a/d+2*(a^2+b^2)^(1/2)*arctanh((b-a*tanh(1/2*d*x+1
/2*c))/(a^2+b^2)^(1/2))/a/b/d
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.32

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{ac+adx+2\sqrt{-a^2-b^2}\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)-b\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)+b\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{abd}$$

input
```
Integrate[(Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(a*c + a*d*x + 2*Sqrt[-a^2 - b^2]*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a
^2 - b^2]] - b*Log[Cosh[(c + d*x)/2]] + b*Log[Sinh[(c + d*x)/2]])/(a*b*d)
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 26, 3368, 26, 3042, 26, 3537, 26, 3042, 26, 3139, 1083, 217, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cos(ic+idx)^2}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cos(ic+idx)^2}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3368}$$

$$i\int -\frac{i\text{csch}(c+dx)\left(\sinh^2(c+dx)+1\right)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\left(\sinh^2(c+dx)+1\right)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(1-\sin(ic+idx)^2\right)}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1-\sin(ic+idx)^2}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3537}$$

$$i\left(\frac{i(a^2+b^2)\int \frac{1}{a+b\sinh(c+dx)}dx}{ab}+\frac{\int -i\text{csch}(c+dx)dx}{a}-\frac{ix}{b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i(a^2+b^2)\int \frac{1}{a+b\sinh(c+dx)}dx}{ab}-\frac{i\int \text{csch}(c+dx)dx}{a}-\frac{ix}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{i(a^2+b^2)\int \frac{1}{a-ib\sin(ic+idx)}dx}{ab}-\frac{i\int i\csc(ic+idx)dx}{a}-\frac{ix}{b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i(a^2+b^2)\int \frac{1}{a-ib\sin(ic+idx)}dx}{ab}+\frac{\int \csc(ic+idx)dx}{a}-\frac{ix}{b}\right)$$

$$\Big\downarrow \text{3139}$$

$$i\left(\frac{2(a^2+b^2)\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{abd}+\frac{\int \csc(ic+idx)dx}{a}-\frac{ix}{b}\right)$$

$$\Big\downarrow \text{1083}$$

$$i\left(-\frac{4(a^2+b^2)\int \frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4(a^2+b^2)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{abd}+\frac{\int \csc(ic+idx)dx}{a}-\frac{ix}{b}\right)$$

$$\Big\downarrow \text{217}$$

$$i\left(\frac{\int \csc(ic+idx)dx}{a}+\frac{2i\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{abd}-\frac{ix}{b}\right)$$

$$\Big\downarrow \text{4257}$$

$$i\left(\frac{2i\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{abd}+\frac{i\text{arctanh}(\cosh(c+dx))}{ad}-\frac{ix}{b}\right)$$

input
```
Int[(Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
I*(((-I)*x)/b + (I*ArcTanh[Cosh[c + d*x]])/(a*d) + ((2*I)*Sqrt[a^2 + b^2]*
ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b^2])])/(a*b*d))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3368
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

rule 3537
```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[C*(x
/(b*d)), x] + (Simp[(A*b^2 + a^2*C)/(b*(b*c - a*d))   Int[1/(a + b*Sin[e +
f*x]), x], x] - Simp[(c^2*C + A*d^2)/(d*(b*c - a*d))   Int[1/(c + d*Sin[e +
 f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.54

method	result
derivativedivides	$\frac{\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}-\frac{\left(2a^2+2b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{ab\sqrt{a^2+b^2}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
default	$\frac{\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}-\frac{\left(2a^2+2b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{ab\sqrt{a^2+b^2}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
risch	$\frac{x}{b}+\frac{\ln\left(e^{dx+c}-1\right)}{da}-\frac{\ln\left(e^{dx+c}+1\right)}{da}+\frac{\sqrt{a^2+b^2}\,\ln\left(e^{dx+c}+\frac{a+\sqrt{a^2+b^2}}{b}\right)}{dba}-\frac{\sqrt{a^2+b^2}\,\ln\left(e^{dx+c}-\frac{-a+\sqrt{a^2+b^2}}{b}\right)}{dba}$

input
```
int(cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b*ln(1+tanh(1/2*d*x+1/2*c))-1/b*ln(tanh(1/2*d*x+1/2*c)-1)-(2*a^2+2*
b^2)/a/b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^
2)^(1/2))+1/a*ln(tanh(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(68) = 136$.

Time = 0.10 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.94

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{adx - b\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)+1\right)+b\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)-1\right)+\sqrt{a^2+b^2}\,\mathrm{lo}\cdots}{\cdots}$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(a*d*x - b*log(cosh(d*x + c) + sinh(d*x + c) + 1) + b*log(cosh(d*x + c) +
sinh(d*x + c) - 1) + sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d
*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b
)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a
))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(
d*x + c) + a)*sinh(d*x + c) - b)))/(a*b*d)
```

Sympy [F]

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh\left(c+dx\right)\coth\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(cosh(c + d*x)*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.77

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{dx+c}{bd} - \frac{\log\left(e^{(-dx-c)}+1\right)}{ad} + \frac{\log\left(e^{(-dx-c)}-1\right)}{ad} - \frac{\sqrt{a^2+b^2}\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{abd}$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
(d*x + c)/(b*d) - log(e^(-d*x - c) + 1)/(a*d) + log(e^(-d*x - c) - 1)/(a*d
) - sqrt(a^2 + b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x
- c) - a + sqrt(a^2 + b^2)))/(a*b*d)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.59

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{dx+c}{b} - \frac{\log\left(e^{(dx+c)}+1\right)}{a} + \frac{\log\left(\left|e^{(dx+c)}-1\right|\right)}{a} - \frac{\sqrt{a^2+b^2}\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{ab}}{d}$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
((d*x + c)/b - log(e^(d*x + c) + 1)/a + log(abs(e^(d*x + c) - 1))/a - sqrt
(a^2 + b^2)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(
d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/(a*b))/d
```

Mupad [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 384, normalized size of antiderivative = 5.41

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{x}{b} + \frac{\ln\left(32\,a\,b^2+32\,a^3-32\,a^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-32\,a\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\right)}{a\,d}$$
$$-\frac{\ln\left(32\,a\,b^2+32\,a^3+32\,a^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+32\,a\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\right)}{a\,d}$$
$$-\frac{\ln\left(128\,a^5\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-64\,a^2\,b^3-64\,a^4\,b-32\,a\,b^3\,\sqrt{a^2+b^2}-64\,a^3\,b\,\sqrt{a^2+b^2}+160\,a^3\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+128\,a^4\,\mathrm{e}^{\cdots}}{a\,b\,d}$$
$$+\frac{\ln\left(128\,a^5\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-64\,a^2\,b^3-64\,a^4\,b+32\,a\,b^3\,\sqrt{a^2+b^2}+64\,a^3\,b\,\sqrt{a^2+b^2}+160\,a^3\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-128\,a^4\,\mathrm{e}^{\cdots}}{a\,b\,d}$$

input
```
int((cosh(c + d*x)*coth(c + d*x))/(a + b*sinh(c + d*x)),x)
```

output
```
x/b + log(32*a*b^2 + 32*a^3 - 32*a^3*exp(d*x)*exp(c) - 32*a*b^2*exp(d*x)*e
xp(c))/(a*d) - log(32*a*b^2 + 32*a^3 + 32*a^3*exp(d*x)*exp(c) + 32*a*b^2*e
xp(d*x)*exp(c))/(a*d) - (log(128*a^5*exp(d*x)*exp(c) - 64*a^2*b^3 - 64*a^4
*b - 32*a*b^3*(a^2 + b^2)^(1/2) - 64*a^3*b*(a^2 + b^2)^(1/2) + 160*a^3*b^2
*exp(d*x)*exp(c) + 128*a^4*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2) + 32*a*b^4*ex
p(d*x)*exp(c) + 96*a^2*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^
(1/2))/(a*b*d) + (log(128*a^5*exp(d*x)*exp(c) - 64*a^2*b^3 - 64*a^4*b + 32
*a*b^3*(a^2 + b^2)^(1/2) + 64*a^3*b*(a^2 + b^2)^(1/2) + 160*a^3*b^2*exp(d*
x)*exp(c) - 128*a^4*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2) + 32*a*b^4*exp(d*x)*
exp(c) - 96*a^2*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/
(a*b*d)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{-2\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)i+\log\left(e^{dx+c}-1\right)b-\log\left(e^{dx+c}+1\right)b+adx}{abd}$$

input
```
int(cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output

```
( - 2*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*i
 + log(e**(c + d*x) - 1)*b - log(e**(c + d*x) + 1)*b + a*d*x)/(a*b*d)
```

3.429 $\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 7.86 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Cosh[c + d*x]*Coth[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
x]
```

output
```
Integrate[(Cosh[c + d*x]*Coth[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
 x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

$$\Big\downarrow \text{ 6125}$$

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Cosh[c + d*x]*Coth[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(dx+c)\coth(dx+c)}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)\coth(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
fricas")
```

output
```
integral(cosh(d*x + c)*coth(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x
 + c)), x)
```

Sympy [N/A]

Not integrable

Time = 4.75 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)\coth(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(cosh(c + d*x)*coth(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 5.22

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)\coth(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
maxima")
```

output
```
-2*(a^2*e^c + b^2*e^c)*integrate(-e^(d*x)/(a*b^2*f*x + a*b^2*e - (a*b^2*f*
x*e^(2*c) + a*b^2*e*e^(2*c))*e^(2*d*x) - 2*(a^2*b*f*x*e^c + a^2*b*e*e^c)*e
^(d*x)), x) + log(f*x + e)/(b*f) + integrate(1/(a*f*x + a*e + (a*f*x*e^c +
 a*e*e^c)*e^(d*x)), x) + integrate(-1/(a*f*x + a*e - (a*f*x*e^c + a*e*e^c)
*e^(d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)\,\coth(c+d\,x)}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((cosh(c + d*x)*coth(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((cosh(c + d*x)*coth(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 419, normalized size of antiderivative = 13.09

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$= \frac{-2e^{3c}\left(\int \frac{e^{3dx}}{e^{4dx+4c}be+e^{4dx+4c}bfx+2e^{3dx+3c}ae+2e^{3dx+3c}afx-2e^{2dx+2c}be-2e^{2dx+2c}bfx-2e^{dx+c}ae-2e^{dx+c}afx+be+bfx}dx\right)af+4e}{}$$

input
```
int(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*e**(3*c)*int(e**(3*d*x)/(e**(4*c + 4*d*x)*b*e + e**(4*c + 4*d*x)*b*f
*x + 2*e**(3*c + 3*d*x)*a*e + 2*e**(3*c + 3*d*x)*a*f*x - 2*e**(2*c + 2*d*x
)*b*e - 2*e**(2*c + 2*d*x)*b*f*x - 2*e**(c + d*x)*a*e - 2*e**(c + d*x)*a*f
*x + b*e + b*f*x),x)*a*f + 4*e**(2*c)*int(e**(2*d*x)/(e**(4*c + 4*d*x)*b*e
 + e**(4*c + 4*d*x)*b*f*x + 2*e**(3*c + 3*d*x)*a*e + 2*e**(3*c + 3*d*x)*a*
f*x - 2*e**(2*c + 2*d*x)*b*e - 2*e**(2*c + 2*d*x)*b*f*x - 2*e**(c + d*x)*a
*e - 2*e**(c + d*x)*a*f*x + b*e + b*f*x),x)*b*f + 2*e**c*int(e**(d*x)/(e**
(4*c + 4*d*x)*b*e + e**(4*c + 4*d*x)*b*f*x + 2*e**(3*c + 3*d*x)*a*e + 2*e*
*(3*c + 3*d*x)*a*f*x - 2*e**(2*c + 2*d*x)*b*e - 2*e**(2*c + 2*d*x)*b*f*x -
 2*e**(c + d*x)*a*e - 2*e**(c + d*x)*a*f*x + b*e + b*f*x),x)*a*f + log(e +
 f*x))/(b*f)
```

3.430 $\int \frac{(e+fx)^3 \cosh^2(c+dx) \coth(c+dx)}{a+b \sinh(c+dx)} \, dx$

Optimal result

Integrand size = 34, antiderivative size = 656

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= -\frac{(e+fx)^4}{4af}+\frac{(a^2+b^2)(e+fx)^4}{4ab^2f}-\frac{6f^3\cosh(c+dx)}{bd^4}-\frac{3f(e+fx)^2\cosh(c+dx)}{bd^2}$$
$$-\frac{(a^2+b^2)(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d}-\frac{(a^2+b^2)(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ab^2d}$$
$$+\frac{(e+fx)^3\log\left(1-e^{2(c+dx)}\right)}{ad}-\frac{3(a^2+b^2)f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^2}$$
$$-\frac{3(a^2+b^2)f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ab^2d^2}$$
$$+\frac{3f(e+fx)^2\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{2ad^2}$$
$$+\frac{6(a^2+b^2)f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^3}$$
$$+\frac{6(a^2+b^2)f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ab^2d^3}$$
$$-\frac{3f^2(e+fx)\operatorname{PolyLog}\left(3,e^{2(c+dx)}\right)}{2ad^3}-\frac{6(a^2+b^2)f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^4}$$
$$-\frac{6(a^2+b^2)f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ab^2d^4}+\frac{3f^3\operatorname{PolyLog}\left(4,e^{2(c+dx)}\right)}{4ad^4}$$
$$+\frac{6f^2(e+fx)\sinh(c+dx)}{bd^3}+\frac{(e+fx)^3\sinh(c+dx)}{bd}$$

output
```
-1/4*(f*x+e)^4/a/f+1/4*(a^2+b^2)*(f*x+e)^4/a/b^2/f-6*f^3*cosh(d*x+c)/b/d^4
-3*f*(f*x+e)^2*cosh(d*x+c)/b/d^2-(a^2+b^2)*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))/a/b^2/d-(a^2+b^2)*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/a/b^2/d+(f*x+e)^3*ln(1-exp(2*d*x+2*c))/a/d-3*(a^2+b^2)*f*(f*x+e
)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/b^2/d^2-3*(a^2+b^2)*f*(
f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/b^2/d^2+3/2*f*(f*x
+e)^2*polylog(2,exp(2*d*x+2*c))/a/d^2+6*(a^2+b^2)*f^2*(f*x+e)*polylog(3,-b
*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/b^2/d^3+6*(a^2+b^2)*f^2*(f*x+e)*polylog
(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/b^2/d^3-3/2*f^2*(f*x+e)*polylog(3,
exp(2*d*x+2*c))/a/d^3-6*(a^2+b^2)*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)
^(1/2)))/a/b^2/d^4-6*(a^2+b^2)*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1
/2)))/a/b^2/d^4+3/4*f^3*polylog(4,exp(2*d*x+2*c))/a/d^4+6*f^2*(f*x+e)*sinh
(d*x+c)/b/d^3+(f*x+e)^3*sinh(d*x+c)/b/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3089 vs. $2(656) = 1312$.

Time = 9.94 (sec) , antiderivative size = 3089, normalized size of antiderivative = 4.71

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
-1/2*(E^(2*c)*((e + f*x)^4/(E^(2*c)*f) - (2*(1 - E^(-2*c))*(e + f*x)^3*Log
[1 - E^(-c - d*x)])/d - (2*(1 - E^(-2*c))*(e + f*x)^3*Log[1 + E^(-c - d*x)
])/d + (6*(-1 + E^(2*c))*f*(d^2*(e + f*x)^2*PolyLog[2, -E^(-c - d*x)] + 2*
f*(d*(e + f*x)*PolyLog[3, -E^(-c - d*x)] + f*PolyLog[4, -E^(-c - d*x)])))/
(d^4*E^(2*c)) + (6*(-1 + E^(2*c))*f*(d^2*(e + f*x)^2*PolyLog[2, E^(-c - d*
x)] + 2*f*(d*(e + f*x)*PolyLog[3, E^(-c - d*x)] + f*PolyLog[4, E^(-c - d*x
)])))/(d^4*E^(2*c))))/(a*(-1 + E^(2*c))) + ((a^2 + b^2)*(4*e^3*E^(2*c)*x +
 6*e^2*E^(2*c)*f*x^2 + 4*e*E^(2*c)*f^2*x^3 + E^(2*c)*f^3*x^4 + (4*a*Sqrt[a
^2 + b^2]*e^3*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 +
b^2)^2]*d) + (4*a*Sqrt[-a^2 - b^2]*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^
2 + b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) - (2*e^3*E^(2*c)*Log[b - 2*a*E^(c + d*
x) - b*E^(2*(c + d*x))])/d + (2*e^3*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c
+ d*x)))])/d + (6*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b
^2)*E^(2*c)])])/d - (6*e^2*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c -
Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a
*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f^2*x^2*Log[1 + (b*E^
(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (2*f^3*x^3*Log[1 +
(b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (2*E^(2*c)*f^3
*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (
6*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6119}$$

$$\frac{\int (e+fx)^3\cosh^2(c+dx)\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5973}$$

$$\frac{\int (e+fx)^3\coth(c+dx)dx+\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx+\int-i(e+fx)^3\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx-i\int(e+fx)^3\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{a}$$

$$\Big\downarrow \; 4201$$

$$-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\int\frac{e^{2c+2dx-i\pi}(e+fx)^3}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^4}{4f}\right)}{a}$$

$$\Big\downarrow \; 2620$$

$$-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\int(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)-\frac{i(e+f}{4f}\right.}{a}$$

$$\Big\downarrow \; 3011$$

$$-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+}{}\right.}{2d}\right.\right.}{a}$$

$$\Big\downarrow \; 5969$$

$$-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)-}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)-}{a}$$

$$\Big\downarrow \; 25$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+}{a}$$

$$\Big\downarrow \; 3792$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{3f\left(\frac{f^2\int-\sinh^2(c+dx)dx}{2d^2}+\frac{1}{2}\int(e+fx)^2dx+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}-i\left(2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f}{}\right.\right.}{a}$$

$$\Big\downarrow \; 17$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{3f\left(\frac{f^2\int-\sinh^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}-i\left(2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(}{}\right.}{}\right.\right.}{a}$$

$$\Big\downarrow \; 25$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{3f\left(-\frac{f^2\int\sinh^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}-i\left(2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(}{}\right.}{}\right.\right.}{a}$$

$\downarrow$ 3042

$$-\frac{b\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{3f\left(-\frac{f^2\int -\sin(ic+idx)^2dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}-i\left(2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f}{\cdots}\right.}{\cdots}\right.\right.}{a}$$

$\downarrow$ 25

$$-\frac{b\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{3f\left(\frac{f^2\int \sin(ic+idx)^2dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}-i\left(2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(e-\cdots}{\cdots}\right.}{\cdots}\right.\right.}{a}$$

$\downarrow$ 3115

$$-\frac{b\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{3f\left(\frac{f^2\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}-i\left(2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi\cdots}}{2d}\right.\right.}{a}$$

$\downarrow$ 24

$$-\frac{b\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+\cdots}{a}$$

$\downarrow$ 6099

$$-\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)^3\cosh(c+dx)dx}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+}{a}$$

$\downarrow$ 3042

$$\frac{-i\left(2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+}{a}$$
$$\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)^3\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}$$

$\downarrow$ 3777

$$\frac{-i\left(2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+}{a}$$
$$\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3if\int -i(e+fx)^2\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}$$

$\downarrow$ 26

$$-\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int(e+fx)^2\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+}{a}$$

$\downarrow$ 3042

$$\frac{-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int-i(e+fx)^2\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\int(e+fx)^2\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}$$

$$\Big\downarrow \; 3777$$

$$\frac{-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\cosh(c+dx)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx}{b}\right.}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(\right.}{a}$$

$$\downarrow 3777$$

$$\frac{-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int-i\sinh(c+dx)dx}{d}\right)}{d}\right)}{d}\right)}{b^2}+\right.}{a}$$

$$\downarrow 26$$

$$\frac{-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}\right)}{d}\right)}{d}\right)}{b^2}+\frac{\int(e\ldots}{}\right.}{a}$$

$\downarrow$ 3042

$$\frac{-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int-i\sin(ic+idx)dx}{d}\right)}{d}\right)}{d}\right)}{b^2}+\ldots\right.}{a}$$

$\downarrow$ 26

$$\frac{-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+ \cdots}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int\sin(ic+idx)dx}{d}\right)}{d}\right)}{d}\right)}{b^2}+\frac{\int(\cdots}{\cdots}\right)}{a}$$

$\downarrow$ 3118

$$\frac{-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+ \cdots}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)s \cdots}{\cdots}\right)}{\cdots}\right)}{d}\right)}{b^2} \cdots\right)}{a}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^3 \cosh(dx+c)^2 \coth(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3344 vs. $2(619) = 1238$.

Time = 0.18 (sec) , antiderivative size = 3344, normalized size of antiderivative = 5.10

$$\int \frac{(e+fx)^3 \cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^3 \cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^3 \cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)**2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*cosh(c + d*x)**2*coth(c + d*x)/(a + b*sinh(c + d*x))
, x)
```

Maxima [F]

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)^2\coth(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/2*e^3*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d) -
 2*log(e^(-d*x - c) + 1)/(a*d) - 2*log(e^(-d*x - c) - 1)/(a*d) + 2*(a^2 +
b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*b^2*d)) + 3*(d*x*l
og(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))*e^2*f/(a*d^2) + 3*(d*x*log(-e^(
d*x + c) + 1) + dilog(e^(d*x + c)))*e^2*f/(a*d^2) + 3*(d^2*x^2*log(e^(d*x
+ c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*e*f^2/
(a*d^3) + 3*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*
polylog(3, e^(d*x + c)))*e*f^2/(a*d^3) + (d^3*x^3*log(e^(d*x + c) + 1) + 3
*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) + 6*polylog(
4, -e^(d*x + c)))*f^3/(a*d^4) + (d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2
*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x
+ c)))*f^3/(a*d^4) - 1/2*(d^4*f^3*x^4 + 4*d^4*e*f^2*x^3 + 6*d^4*e^2*f*x^2)
/(a*d^4) - 1/4*(a*d^4*f^3*x^4*e^c + 4*a*d^4*e*f^2*x^3*e^c + 6*a*d^4*e^2*f*
x^2*e^c - 2*(b*d^3*f^3*x^3*e^(2*c) + 3*(d^3*e*f^2 - d^2*f^3)*b*x^2*e^(2*c)
 + 3*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*b*x*e^(2*c) - 3*(d^2*e^2*f - 2*d*
e*f^2 + 2*f^3)*b*e^(2*c))*e^(d*x) + 2*(b*d^3*f^3*x^3 + 3*(d^3*e*f^2 + d^2*
f^3)*b*x^2 + 3*(d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)*b*x + 3*(d^2*e^2*f + 2*
d*e*f^2 + 2*f^3)*b)*e^(-d*x))*e^(-c)/(b^2*d^4) + integrate(-2*((a^2*b*f^3
+ b^3*f^3)*x^3 + 3*(a^2*b*e*f^2 + b^3*e*f^2)*x^2 + 3*(a^2*b*e^2*f + b^3*e^
2*f)*x - ((a^3*f^3*e^c + a*b^2*f^3*e^c)*x^3 + 3*(a^3*e*f^2*e^c + a*b^2*...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^2\coth(c+d\,x)\,\left(e+f\,x\right)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)^2*coth(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*coth(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh\left(dx+c\right)^2\coth\left(dx+c\right)}{a+b\sinh\left(dx+c\right)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.431 $\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 486

$$\begin{aligned}
&\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= -\frac{(e+fx)^3}{3af}+\frac{(a^2+b^2)(e+fx)^3}{3ab^2f}-\frac{2f(e+fx)\cosh(c+dx)}{bd^2} \\
&\quad -\frac{(a^2+b^2)(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d}-\frac{(a^2+b^2)(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ab^2d} \\
&\quad +\frac{(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{ad}-\frac{2(a^2+b^2)f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^2} \\
&\quad -\frac{2(a^2+b^2)f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ab^2d^2}+\frac{f(e+fx)\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{ad^2} \\
&\quad +\frac{2(a^2+b^2)f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^3}+\frac{2(a^2+b^2)f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ab^2d^3} \\
&\quad -\frac{f^2\operatorname{PolyLog}\left(3,e^{2(c+dx)}\right)}{2ad^3}+\frac{2f^2\sinh(c+dx)}{bd^3}+\frac{(e+fx)^2\sinh(c+dx)}{bd}
\end{aligned}$$

output
```
-1/3*(f*x+e)^3/a/f+1/3*(a^2+b^2)*(f*x+e)^3/a/b^2/f-2*f*(f*x+e)*cosh(d*x+c)
/b/d^2-(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/b^2/d-
(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/b^2/d+(f*x+e)
^2*ln(1-exp(2*d*x+2*c))/a/d-2*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/
(a-(a^2+b^2)^(1/2)))/a/b^2/d^2-2*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+
c)/(a+(a^2+b^2)^(1/2)))/a/b^2/d^2+f*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a/d^
2+2*(a^2+b^2)*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/b^2/d^3+2
*(a^2+b^2)*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/b^2/d^3-1/2*
f^2*polylog(3,exp(2*d*x+2*c))/a/d^3+2*f^2*sinh(d*x+c)/b/d^3+(f*x+e)^2*sinh
(d*x+c)/b/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1462 vs. $2(486) = 972$.

Time = 7.72 (sec) , antiderivative size = 1462, normalized size of antiderivative = 3.01

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
(-((a*x*(3*e^2 + 3*e*f*x + f^2*x^2)*Coth[c])/b^2) - (E^(2*c)*((2*(e + f*x)
^3)/(E^(2*c)*f) - (3*(1 - E^(-2*c))*(e + f*x)^2*Log[1 - E^(-c - d*x)])/d -
 (3*(1 - E^(-2*c))*(e + f*x)^2*Log[1 + E^(-c - d*x)])/d + (6*(-1 + E^(2*c)
)*f*(d*(e + f*x)*PolyLog[2, -E^(-c - d*x)] + f*PolyLog[3, -E^(-c - d*x)]))
/(d^3*E^(2*c)) + (6*(-1 + E^(2*c))*f*(d*(e + f*x)*PolyLog[2, E^(-c - d*x)]
 + f*PolyLog[3, E^(-c - d*x)]))/(d^3*E^(2*c))))/(a*(-1 + E^(2*c))) + ((a^2
 + b^2)*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sq
rt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^
2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c
+ d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^
2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d
) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt
[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1
+ E^(2*(c + d*x)))])/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2
*(c + d*x)))])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 +
 b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c -
Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E
^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c
 + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f*x*Log[1 + (b*E^(
2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6119}$$

$$\frac{\int (e+fx)^2\cosh^2(c+dx)\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5973}$$

$$\frac{\int (e+fx)^2\coth(c+dx)dx+\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{b\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx+\int -i(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{a}}{}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx-i\int (e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{a}$$

$$\Big\downarrow \; 4201$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\int \frac{e^{2c+2dx-i\pi}(e+fx)^2}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^3}{3f}\right)}{a}$$

$$\Big\downarrow \; 2620$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int (e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{a}$$

$$\Big\downarrow \; 3011$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int \text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{2d}-\frac{(e+fx)\,\text{Poly}}{}\right.}{d}\right.\right.}{a}$$

$$\Big\downarrow \; 2720$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{}}{4d^2}\right.}{d}\right.\right.}{a}$$

$$\Big\downarrow \text{5969}$$

$$-\frac{-\frac{b\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+}{}$$

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\right.}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{b\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+}{}$$

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\right.}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-\frac{b\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+}{}$$

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\right.}{a}$$

$$\Big\downarrow \text{3791}$$

$$-\frac{-\frac{b\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+}{}$$

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\right.}{a}$$

$$\Big\downarrow \text{17}$$

$$-\frac{-\frac{b\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+}{}$$

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\right.}{a}$$

$$\Big\downarrow \; 6099$$

$$-\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)^2\cosh(c+dx)dx}{b^2}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\right.}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b^2}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}$$

$$\Big\downarrow \; 3777$$

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int (e+fx)\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\right.}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)-}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int-i(e+fx)\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}$$

$\downarrow$ 26

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)-}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int(e+fx)\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}$$

$\downarrow$ 3777

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)-}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c-}{b}\right.}{a}$$

$\downarrow$ 3042

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)-}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sin}{b}\right.}{a}$$

$\downarrow$ 3117

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)-}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right.}{b^2}\right.}{a}$$

$\downarrow$ 5969

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)-}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}-\frac{f\int(e+fx)\sinh^2(c+dx)dx}{d}}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sin}{}\right.}{d}\right.}{b^2}\right.}{a}$$

$\downarrow$ 3042

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)-}{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}-\frac{f\int-\left((e+fx)\sin(ic+idx)^2\right)dx}{d}}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\right.}{d}\right.}{b^2}\right.}\bigg/ a$$

$\downarrow$ 25

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)-}{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}+\frac{f\int(e+fx)\sin(ic+idx)^2dx}{d}}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sin}{}\right.}{d}\right.}{b^2}\right.}\bigg/ a$$

$\downarrow$ 3791

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)-}{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{f\left(\frac{1}{2}\int(e+fx)dx+\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(}{d}\right.}{}\right.}\bigg/ a$$

$\downarrow$ 17

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)-}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b^2}+\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx}{2d}\right.}{d}\right.}{a}$$

$\downarrow$ 6095

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)-}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\left(\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right.}{b^2}\right.}{a}$$

$\downarrow$ 2620

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)-}{a}$$

$$b\left(\frac{(a^2+b^2)\left(-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\right.}{b^2}\right.$$

$\downarrow$ 3011

$$-i\left(2i\left(\frac{(e+fx)^2\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)-$$

$$b\left(\frac{(a^2+b^2)\left(-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\right.}{d}\right.}{bd}\right.}{b^2}\right.$$

$$a$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^2\cosh(dx+c)^2\coth(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2101 vs. $2(459) = 918$.

Time = 0.15 (sec) , antiderivative size = 2101, normalized size of antiderivative = 4.32

$$\int \frac{(e+fx)^2 \cosh^2(c+dx) \coth(c+dx)}{a+b\sinh(c+dx)} \, dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
-1/6*(3*a*b*d^2*f^2*x^2 + 3*a*b*d^2*e^2 + 6*a*b*d*e*f + 6*a*b*f^2 - 3*(a*b
*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f - a*
b*d*f^2)*x)*cosh(d*x + c)^2 - 3*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d*e
*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f - a*b*d*f^2)*x)*sinh(d*x + c)^2 + 6*(a*b*d
^2*e*f + a*b*d*f^2)*x - 2*(a^2*d^3*f^2*x^3 + 3*a^2*d^3*e*f*x^2 + 3*a^2*d^3
*e^2*x + 6*a^2*c*d^2*e^2 - 6*a^2*c^2*d*e*f + 2*a^2*c^3*f^2)*cosh(d*x + c)
+ 12*(((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*cosh(d*x + c) + ((a^2 + b^
2)*d*f^2*x + (a^2 + b^2)*d*e*f)*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*
sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
- b)/b + 1) + 12*(((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*cosh(d*x + c)
+ ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*sinh(d*x + c))*dilog((a*cosh(d
*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
+ b^2)/b^2) - b)/b + 1) - 12*((b^2*d*f^2*x + b^2*d*e*f)*cosh(d*x + c) + (b
^2*d*f^2*x + b^2*d*e*f)*sinh(d*x + c))*dilog(cosh(d*x + c) + sinh(d*x + c)
) - 12*((b^2*d*f^2*x + b^2*d*e*f)*cosh(d*x + c) + (b^2*d*f^2*x + b^2*d*e*f
)*sinh(d*x + c))*dilog(-cosh(d*x + c) - sinh(d*x + c)) + 6*(((a^2 + b^2)*d
^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*cosh(d*x + c) + ((a^
2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*sinh(d*x +
 c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2)
 + 2*a) + 6*(((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)...
```

Sympy [F]

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)**2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*cosh(c + d*x)**2*coth(c + d*x)/(a + b*sinh(c + d*x))
, x)
```

Maxima [F]

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)^2\coth(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/2*e^2*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d) -
 2*log(e^(-d*x - c) + 1)/(a*d) - 2*log(e^(-d*x - c) - 1)/(a*d) + 2*(a^2 +
b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*b^2*d)) + 2*(d*x*l
og(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))*e*f/(a*d^2) + 2*(d*x*log(-e^(d*
x + c) + 1) + dilog(e^(d*x + c)))*e*f/(a*d^2) + (d^2*x^2*log(e^(d*x + c) +
 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*f^2/(a*d^3)
+ (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3,
 e^(d*x + c)))*f^2/(a*d^3) - 2/3*(d^3*f^2*x^3 + 3*d^3*e*f*x^2)/(a*d^3) - 1
/6*(2*a*d^3*f^2*x^3*e^c + 6*a*d^3*e*f*x^2*e^c - 3*(b*d^2*f^2*x^2*e^(2*c) +
 2*(d^2*e*f - d*f^2)*b*x*e^(2*c) - 2*(d*e*f - f^2)*b*e^(2*c))*e^(d*x) + 3*
(b*d^2*f^2*x^2 + 2*(d^2*e*f + d*f^2)*b*x + 2*(d*e*f + f^2)*b)*e^(-d*x))*e^
(-c)/(b^2*d^3) + integrate(-2*((a^2*b*f^2 + b^3*f^2)*x^2 + 2*(a^2*b*e*f +
b^3*e*f)*x - ((a^3*f^2*e^c + a*b^2*f^2*e^c)*x^2 + 2*(a^3*e*f*e^c + a*b^2*e
*f*e^c)*x)*e^(d*x))/(a*b^3*e^(2*d*x + 2*c) + 2*a^2*b^2*e^(d*x + c) - a*b^3
), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^2\coth(c+d\,x)\,\left(e+f\,x\right)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)^2*coth(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*coth(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)^2\coth(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.432 $\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 322

$$\begin{aligned}
&\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= -\frac{(e+fx)^2}{2af}+\frac{(a^2+b^2)(e+fx)^2}{2ab^2f}-\frac{f\cosh(c+dx)}{bd^2} \\
&\quad -\frac{(a^2+b^2)(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d} \\
&\quad -\frac{(a^2+b^2)(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ab^2d}+\frac{(e+fx)\log\left(1-e^{2(c+dx)}\right)}{ad} \\
&\quad -\frac{(a^2+b^2)f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^2}-\frac{(a^2+b^2)f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ab^2d^2} \\
&\quad +\frac{f\,\mathrm{PolyLog}\left(2,e^{2(c+dx)}\right)}{2ad^2}+\frac{(e+fx)\sinh(c+dx)}{bd}
\end{aligned}$$

output
```
-1/2*(f*x+e)^2/a/f+1/2*(a^2+b^2)*(f*x+e)^2/a/b^2/f-f*cosh(d*x+c)/b/d^2-(a^
2+b^2)*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/b^2/d-(a^2+b^2)*(f
*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/b^2/d+(f*x+e)*ln(1-exp(2*d*
x+2*c))/a/d-(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/b^2
/d^2-(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/b^2/d^2+1/
2*f*polylog(2,exp(2*d*x+2*c))/a/d^2+(f*x+e)*sinh(d*x+c)/b/d
```

Mathematica [A] (warning: unable to verify)

Time = 3.56 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.51

$$\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{-\frac{2f\cosh(c+dx)}{b}+\frac{d(2ce+2dex+dfx^2+2(e+fx)\log(1-e^{-c-dx})+2(e+fx)\log(1+e^{-c-dx}))-2f\,\mathrm{PolyLog}(2,-e^{-c-dx})-2f\,\mathrm{PolyLog}(2,e^{-c-}}{a}}{}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
((-2*f*Cosh[c + d*x])/b + (d*(2*c*e + 2*d*e*x + d*f*x^2 + 2*(e + f*x)*Log[
1 - E^(-c - d*x)] + 2*(e + f*x)*Log[1 + E^(-c - d*x)]) - 2*f*PolyLog[2, -E
^(-c - d*x)] - 2*f*PolyLog[2, E^(-c - d*x)])/a - ((a^2 + b^2)*(-2*d*e*(c +
 d*x) + 2*c*f*(c + d*x) - f*(c + d*x)^2 + (4*a*Sqrt[a^2 + b^2]*d*e*ArcTan[
(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b^2)^2] - (4*a*Sqrt[-(
a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/(-a^2 - b^
2)^(3/2) + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] +
2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*f*Log[b
 - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*d*e*Log[2*a*E^(c + d*x) + b*(-
1 + E^(2*(c + d*x)))] + 2*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^
2])] + 2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a*b^2)
+ (2*d*(e + f*x)*Sinh[c + d*x])/b)/(2*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.97 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.31, number of steps used = 30, number of rules used = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.906$, Rules used = {6119, 5973, 3042, 26, 4201, 2620, 2715, 2838, 5969, 3042, 25, 3115, 24, 6099, 3042, 3777, 26, 3042, 26, 3118, 5969, 3042, 25, 3115, 24, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \; 6119$$

$$\frac{\int (e+fx)\cosh^2(c+dx)\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 5973$$

$$\frac{\int (e+fx)\coth(c+dx)dx+\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx+\int -i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx-i\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{a}$$

$$\Big\downarrow \; 4201$$

$$-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\int \frac{e^{2c+2dx-i\pi}(e+fx)}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^2}{2f}\right)}{a}$$

$$\Big\downarrow \; 2620$$

$$-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int \log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{a}$$

$$\Big\downarrow \; 2715$$

$$-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\log\left(1+e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}\right.\right.}{a}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{a}$$

$$\Big\downarrow \text{5969}$$

$$-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{f\int \sinh^2(c+dx)dx}{2d}-i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{f\int -\sin(ic+idx)^2dx}{2d}-i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{f\int \sin(ic+idx)^2dx}{2d}-i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}}{a}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{f\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}-i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}}{a}$$

$$\Big\downarrow \text{24}$$

$$-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}$$

$$\Big\downarrow \text{6099}$$

$$-\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)\cosh(c+dx)dx}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}+\frac{-i\left(2i\left(\frac{f\,\text{PolyLog}(2,-e^{2c+2dx-i\pi})}{4d^2}+\frac{(e+fx)\log(1+e^{2c+2dx-i\pi})}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}$$

$\downarrow$ 3042

$$\frac{-i\left(2i\left(\frac{f\,\text{PolyLog}(2,-e^{2c+2dx-i\pi})}{4d^2}+\frac{(e+fx)\log(1+e^{2c+2dx-i\pi})}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}$$
$$\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}$$

$\downarrow$ 3777

$$\frac{-i\left(2i\left(\frac{f\,\text{PolyLog}(2,-e^{2c+2dx-i\pi})}{4d^2}+\frac{(e+fx)\log(1+e^{2c+2dx-i\pi})}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}$$
$$\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}$$

$\downarrow$ 26

$$-\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}+\frac{-i\left(2i\left(\frac{f\,\text{PolyLog}(2,-e^{2c+2dx-i\pi})}{4d^2}+\frac{(e+fx)\log(1+e^{2c+2dx-i\pi})}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}$$

$\downarrow$ 3042

$$\frac{-i\left(2i\left(\frac{f\,\text{PolyLog}(2,-e^{2c+2dx-i\pi})}{4d^2}+\frac{(e+fx)\log(1+e^{2c+2dx-i\pi})}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}$$
$$\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}$$

$\downarrow$ 26

$$\frac{-i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}$$

$$\Big\downarrow \; 3118$$

$$-\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}\right)}{a}+$$

$$\frac{-i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}$$

$$\Big\downarrow \; 5969$$

$$-\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}-\frac{f\int\sinh^2(c+dx)dx}{2d}}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}\right)}{a}+$$

$$\frac{-i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{-i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}-\frac{f\int-\sin(ic+idx)^2dx}{2d}}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}\right)}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{-i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\int\sin(ic+idx)^2dx}{2d}}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}\right)}{a}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{f\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}+\frac{(e+fx)\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}\right)}{a}+$$

$$\frac{-i\left(2i\left(\frac{f\,\text{PolyLog}(2,-e^{2c+2dx-i\pi})}{4d^2}+\frac{(e+fx)\log(1+e^{2c+2dx-i\pi})}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}$$

$$\Big\downarrow \text{24}$$

$$-\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}\right)}{a}+$$

$$\frac{-i\left(2i\left(\frac{f\,\text{PolyLog}(2,-e^{2c+2dx-i\pi})}{4d^2}+\frac{(e+fx)\log(1+e^{2c+2dx-i\pi})}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}$$

$$\Big\downarrow \text{6095}$$

$$-\frac{b\left(\frac{(a^2+b^2)\left(\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\cdots}{\cdots}\right)}{a}$$

$$\frac{-i\left(2i\left(\frac{f\,\text{PolyLog}(2,-e^{2c+2dx-i\pi})}{4d^2}+\frac{(e+fx)\log(1+e^{2c+2dx-i\pi})}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{b\left(\frac{(a^2+b^2)\left(-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{b^2}-\cdots\right)}{a}$$

$$\frac{-i\left(2i\left(\frac{f\,\text{PolyLog}(2,-e^{2c+2dx-i\pi})}{4d^2}+\frac{(e+fx)\log(1+e^{2c+2dx-i\pi})}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{b\left(\frac{(a^2+b^2)\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+}}{\sqrt{a^2+}}\right.}{bd}}{b^2}\right.}{a}$$

$$\frac{-i\left(2i\left(\frac{f\,\mathrm{PolyLog}(2,-e^{2c+2dx-i\pi})}{4d^2}+\frac{(e+fx)\log(1+e^{2c+2dx-i\pi})}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right.}{2d}}{a}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{b\left(\frac{(a^2+b^2)\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{b^2}-\right.}{a}$$

$$\frac{-i\left(2i\left(\frac{f\,\mathrm{PolyLog}(2,-e^{2c+2dx-i\pi})}{4d^2}+\frac{(e+fx)\log(1+e^{2c+2dx-i\pi})}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right.}{2d}}{a}$$

input
```
Int[((e + f*x)*Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
((-I)*(((-1/2*I)*(e + f*x)^2)/f + (2*I)*(((e + f*x)*Log[1 + E^(2*c - I*Pi
+ 2*d*x)])/(2*d) + (f*PolyLog[2, -E^(2*c - I*Pi + 2*d*x)])/(4*d^2))) + ((e
 + f*x)*Sinh[c + d*x]^2)/(2*d) + (f*(x/2 - (Cosh[c + d*x]*Sinh[c + d*x])/(
2*d)))/(2*d))/a - (b*(((a^2 + b^2)*(-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Lo
g[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)*Log[1 + (
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2]))])/(b*d^2)))/b^2 - (a*(-((f*Cosh[c + d*x])/d^2) + ((e
+ f*x)*Sinh[c + d*x])/d))/b^2 + (((e + f*x)*Sinh[c + d*x]^2)/(2*d) + (f*(x
/2 - (Cosh[c + d*x]*Sinh[c + d*x])/(2*d)))/(2*d))/b))/a
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 6119
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 931 vs. $2(304) = 608$.

Time = 2.97 (sec) , antiderivative size = 932, normalized size of antiderivative = 2.89

method	result
risch	$-\frac{af\ln\left(\frac{-b\,e^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)x}{d\,b^2}-\frac{2acf\ln\left(e^{dx+c}\right)}{d^2b^2}-\frac{af\,\mathrm{dilog}\left(\frac{-b\,e^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)}{d^2b^2}-\frac{cf\ln\left(e^{dx+c}-1\right)}{d^2a}+\frac{cf\ln\left(b\,e^{2dx+}\right.}{}$

input
```
int((f*x+e)*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
-1/d*a/b^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-
2/d^2*a/b^2*c*f*ln(exp(d*x+c))-1/d^2*a/b^2*f*dilog((-b*exp(d*x+c)+(a^2+b^2
)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2*c*f/a*ln(exp(d*x+c)-1)+1/d^2*c*f/a*
ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/d^2*f/a*ln((-b*exp(d*x+c)+(a^2+b^2
)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d^2*f/a*ln((b*exp(d*x+c)+(a^2+b^2)^(1
/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/d*f/a*ln(exp(d*x+c)+1)*x-1/d*f/a*ln((-b*ex
p(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d*f/a*ln((b*exp(d*x+
c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/2*a/b^2*f*x^2-a/b^2*e*x-1/d
^2/b^2*a*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/d
/b^2*a*e*ln(exp(d*x+c))-1/d/b^2*a*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+
1/d^2/b^2*a*f*c^2+1/d*e/a*ln(exp(d*x+c)-1)-1/d*e/a*ln(b*exp(2*d*x+2*c)+2*a
*exp(d*x+c)-b)+1/d*e/a*ln(exp(d*x+c)+1)+1/d^2*f/a*dilog(exp(d*x+c)+1)-1/d^
2*f/a*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2*
f/a*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d^2*f*di
log(exp(d*x+c))/a+1/2*(d*f*x+d*e-f)/b/d^2*exp(d*x+c)-1/2*(d*f*x+d*e+f)/b/d
^2*exp(-d*x-c)+2/d/b^2*a*f*c*x-1/d^2/b^2*a*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(
1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d^2/b^2*a*f*ln((b*exp(d*x+c)+(a^2+b^2)^(
1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/d^2/b^2*c*a*f*ln(b*exp(2*d*x+2*c)+2*a*exp
(d*x+c)-b)-1/d/b^2*a*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1
/2)))*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1108 vs. $2(301) = 602$.

Time = 0.12 (sec) , antiderivative size = 1108, normalized size of antiderivative = 3.44

$$\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
-1/2*(a*b*d*f*x + a*b*d*e + a*b*f - (a*b*d*f*x + a*b*d*e - a*b*f)*cosh(d*x
 + c)^2 - (a*b*d*f*x + a*b*d*e - a*b*f)*sinh(d*x + c)^2 - (a^2*d^2*f*x^2 +
 2*a^2*d^2*e*x + 4*a^2*c*d*e - 2*a^2*c^2*f)*cosh(d*x + c) + 2*((a^2 + b^2)
*f*cosh(d*x + c) + (a^2 + b^2)*f*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a
*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
 - b)/b + 1) + 2*((a^2 + b^2)*f*cosh(d*x + c) + (a^2 + b^2)*f*sinh(d*x + c
))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(b^2*f*cosh(d*x + c) + b^2*f
*sinh(d*x + c))*dilog(cosh(d*x + c) + sinh(d*x + c)) - 2*(b^2*f*cosh(d*x +
 c) + b^2*f*sinh(d*x + c))*dilog(-cosh(d*x + c) - sinh(d*x + c)) + 2*(((a^
2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c) + ((a^2 + b^2)*d*e - (a^2 +
b^2)*c*f)*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*s
qrt((a^2 + b^2)/b^2) + 2*a) + 2*(((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(
d*x + c) + ((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*sinh(d*x + c))*log(2*b*cosh
(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*(((a^
2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c) + ((a^2 + b^2)*d*f*x + (a^
2 + b^2)*c*f)*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*(((a^2
+ b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c) + ((a^2 + b^2)*d*f*x + (a^2
+ b^2)*c*f)*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b...
```

Sympy [F]

$$\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\cosh^2\left(c+dx\right)\coth\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)**2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*cosh(c + d*x)**2*coth(c + d*x)/(a + b*sinh(c + d*x)), x
)
```

Maxima [F]

$$\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)^2\coth(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-1/2*e*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d) - 2
*log(e^(-d*x - c) + 1)/(a*d) - 2*log(e^(-d*x - c) - 1)/(a*d) + 2*(a^2 + b^
2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*b^2*d)) - 1/4*f*(2*(
a*d^2*x^2*e^c - (b*d*x*e^(2*c) - b*e^(2*c))*e^(d*x) + (b*d*x + b)*e^(-d*x)
)*e^(-c)/(b^2*d^2) - integrate(8*((a^3*e^c + a*b^2*e^c)*x*e^(d*x) - (a^2*b
 + b^3)*x)/(a*b^3*e^(2*d*x + 2*c) + 2*a^2*b^2*e^(d*x + c) - a*b^3), x) + 4
*integrate(x/(a*e^(d*x + c) + a), x) - 4*integrate(x/(a*e^(d*x + c) - a),
x))
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^2\coth(c+d\,x)\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)^2*coth(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*coth(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)^2\coth(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.433 $\int \frac{\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 57

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\log(\sinh(c+dx))}{ad} - \frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{ab^2d} + \frac{\sinh(c+dx)}{bd}$$

output
```
ln(sinh(d*x+c))/a/d-(a^2+b^2)*ln(a+b*sinh(d*x+c))/a/b^2/d+sinh(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{\log(\sinh(c+dx))}{a} - \left(\frac{1}{a}+\frac{a}{b^2}\right)\log(a+b\sinh(c+dx)) + \frac{\sinh(c+dx)}{b}}{d}$$

input
```
Integrate[(Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(Log[Sinh[c + d*x]]/a - (a^(-1) + a/b^2)*Log[a + b*Sinh[c + d*x]] + Sinh[c
 + d*x]/b)/d
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3042, 26, 3316, 26, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cos(ic+idx)^3}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cos(ic+idx)^3}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3316}$$

$$-\frac{i\int \frac{i\text{csch}(c+dx)\left(\sinh^2(c+dx)b^2+b^2\right)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^3d}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\text{csch}(c+dx)\left(\sinh^2(c+dx)b^2+b^2\right)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\text{csch}(c+dx)\left(\sinh^2(c+dx)b^2+b^2\right)}{b(a+b\sinh(c+dx))}d(b\sinh(c+dx))}{b^2d}$$

$$\Big\downarrow \text{522}$$

$$\frac{\int \left(\frac{-a^2-b^2}{a(a+b\sinh(c+dx))}+\frac{b\text{csch}(c+dx)}{a}+1\right)d(b\sinh(c+dx))}{b^2d}$$

$$\downarrow \text{2009}$$

$$\frac{-\frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{a}+\frac{b^2\log(b\sinh(c+dx))}{a}+b\sinh(c+dx)}{b^2d}$$

input
```
Int[(Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
((b^2*Log[b*Sinh[c + d*x]])/a - ((a^2 + b^2)*Log[a + b*Sinh[c + d*x]])/a +
 b*Sinh[c + d*x])/(b^2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(57) = 114$.

Time = 2.94 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.33

method	result
risch	$\frac{ax}{b^2}+\frac{e^{dx+c}}{2bd}-\frac{e^{-dx-c}}{2bd}+\frac{2ac}{b^2d}+\frac{\ln(e^{2dx+2c}-1)}{da}-\frac{a\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{b}-1\right)}{b^2d}-\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{b}-1\right)}{ad}$
derivativedivides	$\frac{-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{\left(-a^2-b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{a\,b^2}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-}{d}$
default	$\frac{-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{\left(-a^2-b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{a\,b^2}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-}{d}$

input
```
int(cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
a*x/b^2+1/2/b/d*exp(d*x+c)-1/2/b/d*exp(-d*x-c)+2*a/b^2/d*c+1/d/a*ln(exp(2*
d*x+2*c)-1)-a/b^2/d*ln(exp(2*d*x+2*c)+2/b*a*exp(d*x+c)-1)-1/a/d*ln(exp(2*d
*x+2*c)+2/b*a*exp(d*x+c)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(57) = 114$.

Time = 0.11 (sec) , antiderivative size = 203, normalized size of antiderivative = 3.56

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\frac{2\,a^2dx\cosh(dx+c)+ab\cosh(dx+c)^2+ab\sinh(dx+c)^2-ab-2\left((a^2+b^2)\cosh(dx+c)+(a^2+b^2\right)}{}$$

input
```
integrate(cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/2*(2*a^2*d*x*cosh(d*x + c) + a*b*cosh(d*x + c)^2 + a*b*sinh(d*x + c)^2 -
 a*b - 2*((a^2 + b^2)*cosh(d*x + c) + (a^2 + b^2)*sinh(d*x + c))*log(2*(b*
sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + 2*(b^2*cosh(d*x + c)
 + b^2*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c)))
 + 2*(a^2*d*x + a*b*cosh(d*x + c))*sinh(d*x + c))/(a*b^2*d*cosh(d*x + c) +
 a*b^2*d*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate(cosh(d*x+c)**2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(cosh(c + d*x)**2*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(57) = 114$.

Time = 0.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.28

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{(dx+c)a}{b^2d} + \frac{e^{(dx+c)}}{2\,bd} - \frac{e^{(-dx-c)}}{2\,bd} + \frac{\log\left(e^{(-dx-c)}+1\right)}{ad} + \frac{\log\left(e^{(-dx-c)}-1\right)}{ad} - \frac{(a^2+b^2)\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{ab^2d}$$

input
```
integrate(cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-(d*x + c)*a/(b^2*d) + 1/2*e^(d*x + c)/(b*d) - 1/2*e^(-d*x - c)/(b*d) + lo
g(e^(-d*x - c) + 1)/(a*d) + log(e^(-d*x - c) - 1)/(a*d) - (a^2 + b^2)*log(
-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*b^2*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.65

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{e^{(dx+c)}-e^{(-dx-c)}}{b}+\frac{2\log\left(\left|e^{(dx+c)}-e^{(-dx-c)}\right|\right)}{a}-\frac{2\,(a^2+b^2)\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{ab^2}}{2\,d}$$

input
```
integrate(cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*((e^(d*x + c) - e^(-d*x - c))/b + 2*log(abs(e^(d*x + c) - e^(-d*x - c)
))/a - 2*(a^2 + b^2)*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a*b^2
))/d
```

Mupad [B] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 360, normalized size of antiderivative = 6.32

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{e^{c+d\,x}}{2\,b\,d}-\frac{e^{-c-d\,x}}{2\,b\,d}$$
$$-\frac{\ln\left(8\,a^5\,e^{d\,x}\,e^c-16\,b^5-16\,a^2\,b^3-4\,a^4\,b+16\,b^5\,e^{2\,c}\,e^{2\,d\,x}+4\,a^4\,b\,e^{2\,c}\,e^{2\,d\,x}+32\,a^3\,b^2\,e^{d\,x}\,e^c+16\,a^2\,b^3\,e^{2\,c}\right.}{a\,d}$$
$$+\frac{\ln\left(4\,a^6+16\,b^6+32\,a^2\,b^4+20\,a^4\,b^2-4\,a^6\,e^{2\,c}\,e^{2\,d\,x}-16\,b^6\,e^{2\,c}\,e^{2\,d\,x}-32\,a^2\,b^4\,e^{2\,c}\,e^{2\,d\,x}-20\,a^4\,b^2\,e^{2\,c}\,e^{2\,c}\right.}{a\,d}$$
$$+\frac{a\,x}{b^2}$$
$$-\frac{a\,\ln\left(8\,a^5\,e^{d\,x}\,e^c-16\,b^5-16\,a^2\,b^3-4\,a^4\,b+16\,b^5\,e^{2\,c}\,e^{2\,d\,x}+4\,a^4\,b\,e^{2\,c}\,e^{2\,d\,x}+32\,a^3\,b^2\,e^{d\,x}\,e^c+16\,a^2\,b^3\,e\right.}{b^2\,d}$$

input
```
int((cosh(c + d*x)^2*coth(c + d*x))/(a + b*sinh(c + d*x)),x)
```

output
```
exp(c + d*x)/(2*b*d) - exp(- c - d*x)/(2*b*d) - log(8*a^5*exp(d*x)*exp(c)
- 16*b^5 - 16*a^2*b^3 - 4*a^4*b + 16*b^5*exp(2*c)*exp(2*d*x) + 4*a^4*b*exp
(2*c)*exp(2*d*x) + 32*a^3*b^2*exp(d*x)*exp(c) + 16*a^2*b^3*exp(2*c)*exp(2*
d*x) + 32*a*b^4*exp(d*x)*exp(c))/(a*d) + log(4*a^6 + 16*b^6 + 32*a^2*b^4 +
 20*a^4*b^2 - 4*a^6*exp(2*c)*exp(2*d*x) - 16*b^6*exp(2*c)*exp(2*d*x) - 32*
a^2*b^4*exp(2*c)*exp(2*d*x) - 20*a^4*b^2*exp(2*c)*exp(2*d*x))/(a*d) + (a*x
)/b^2 - (a*log(8*a^5*exp(d*x)*exp(c) - 16*b^5 - 16*a^2*b^3 - 4*a^4*b + 16*
b^5*exp(2*c)*exp(2*d*x) + 4*a^4*b*exp(2*c)*exp(2*d*x) + 32*a^3*b^2*exp(d*x
)*exp(c) + 16*a^2*b^3*exp(2*c)*exp(2*d*x) + 32*a*b^4*exp(d*x)*exp(c)))/(b^
2*d)
```

Reduce [B] (verification not implemented)

Time = 4.62 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.05

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{e^{2dx+2c}ab + 2e^{dx+c}\log\left(e^{dx+c}-1\right)b^2 + 2e^{dx+c}\log\left(e^{dx+c}+1\right)b^2 - 2e^{dx+c}\log\left(e^{2dx+2c}b + 2e^{dx+c}a - b\right)a^2 - \cdots}{2e^{dx+c}a\,b^2d}$$

input
```
int(cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
(e**(2*c + 2*d*x)*a*b + 2*e**(c + d*x)*log(e**(c + d*x) - 1)*b**2 + 2*e**(
c + d*x)*log(e**(c + d*x) + 1)*b**2 - 2*e**(c + d*x)*log(e**(2*c + 2*d*x)*
b + 2*e**(c + d*x)*a - b)*a**2 - 2*e**(c + d*x)*log(e**(2*c + 2*d*x)*b + 2
*e**(c + d*x)*a - b)*b**2 + 2*e**(c + d*x)*a**2*d*x - a*b)/(2*e**(c + d*x)
*a*b**2*d)
```

3.434 $\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\text{Int}\left(\frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(cosh(d*x+c)^2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 42.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Cosh[c + d*x]^2*Coth[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Cosh[c + d*x]^2*Coth[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{6125}$$
$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Cosh[c + d*x]^2*Coth[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(dx+c\right)^{2}\coth\left(dx+c\right)}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(cosh(d*x+c)^2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)^2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)^2\coth\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(cosh(d*x + c)^2*coth(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 7.83 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh^2\left(c+dx\right)\coth\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

input
```
integrate(cosh(d*x+c)**2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(cosh(c + d*x)**2*coth(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)),
 x)
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 249, normalized size of antiderivative = 7.32

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)^2\coth(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-1/2*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b*f) - 1/2*e^(c - d*
e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b*f) - a*log(f*x + e)/(b^2*f) + 1/
4*integrate(8*(a^2*b + b^3 - (a^3*e^c + a*b^2*e^c)*e^(d*x))/(a*b^3*f*x + a
*b^3*e - (a*b^3*f*x*e^(2*c) + a*b^3*e*e^(2*c))*e^(2*d*x) - 2*(a^2*b^2*f*x*
e^c + a^2*b^2*e*e^c)*e^(d*x)), x) - integrate(1/(a*f*x + a*e + (a*f*x*e^c
+ a*e*e^c)*e^(d*x)), x) + integrate(-1/(a*f*x + a*e - (a*f*x*e^c + a*e*e^c
)*e^(d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)^2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 2.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{{\mathrm{cosh}\left(c+d\,x\right)}^2\,\mathrm{coth}\left(c+d\,x\right)}{\left(e+f\,x\right)\,\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)} \,d x$$

input
```
int((cosh(c + d*x)^2*coth(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((cosh(c + d*x)^2*coth(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 200.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh \left(d x +c \right)^{2} \coth \left(d x +c \right)}{\left(f x +e \right) \left(a +b \sinh \left(d x +c \right)\right)}d x$$

input
```
int(cosh(d*x+c)^2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)^2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.435 $\int \frac{(e+fx)^3 \operatorname{csch}(c+dx) \operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 1049

$$\int \frac{(e+fx)^3 \operatorname{csch}(c+dx) \operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output
```
b^2*(f*x+e)^3*ln(1+exp(2*d*x+2*c))/a/(a^2+b^2)/d+6*b^2*f^2*(f*x+e)*polylog
(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d^3+6*b^2*f^2*(f*x+e)*po
lylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d^3-3*b^2*f*(f*x+e)
^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d^2-3*b^2*f*(f
*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d^2+3/2*f
*(f*x+e)^2*polylog(2,exp(2*d*x+2*c))/a/d^2-3/2*f^2*(f*x+e)*polylog(3,exp(2
*d*x+2*c))/a/d^3+3/2*f^2*(f*x+e)*polylog(3,-exp(2*d*x+2*c))/a/d^3-3/2*f*(f
*x+e)^2*polylog(2,-exp(2*d*x+2*c))/a/d^2-2*b*(f*x+e)^3*arctan(exp(d*x+c))/
(a^2+b^2)/d+3/4*f^3*polylog(4,exp(2*d*x+2*c))/a/d^4-3/4*f^3*polylog(4,-exp
(2*d*x+2*c))/a/d^4-2*(f*x+e)^3*arctanh(exp(2*d*x+2*c))/a/d-b^2*(f*x+e)^3*l
n(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d-b^2*(f*x+e)^3*ln(1+b*e
xp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d-6*b^2*f^3*polylog(4,-b*exp(d*
x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d^4-6*b^2*f^3*polylog(4,-b*exp(d*x+c
)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d^4+6*I*b*f^2*(f*x+e)*polylog(3,I*exp(d
*x+c))/(a^2+b^2)/d^3+6*I*b*f^3*polylog(4,-I*exp(d*x+c))/(a^2+b^2)/d^4+3*I*
b*f*(f*x+e)^2*polylog(2,-I*exp(d*x+c))/(a^2+b^2)/d^2+3/4*b^2*f^3*polylog(4
,-exp(2*d*x+2*c))/a/(a^2+b^2)/d^4-6*I*b*f^3*polylog(4,I*exp(d*x+c))/(a^2+b
^2)/d^4-3/2*b^2*f^2*(f*x+e)*polylog(3,-exp(2*d*x+2*c))/a/(a^2+b^2)/d^3+3/2
*b^2*f*(f*x+e)^2*polylog(2,-exp(2*d*x+2*c))/a/(a^2+b^2)/d^2-6*I*b*f^2*(f*x
+e)*polylog(3,-I*exp(d*x+c))/(a^2+b^2)/d^3-3*I*b*f*(f*x+e)^2*polylog(2,...
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3572 vs. 2(1049) = 2098.

Time = 12.45 (sec) , antiderivative size = 3572, normalized size of antiderivative = 3.41

$$\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
2*((E^c*((e + f*x)^4/(4*E^c*f) + ((1 + E^(-c))*(e + f*x)^3*Log[1 + E^(-c -
 d*x)])/d - (3*(1 + E^c)*f*(d^2*(e + f*x)^2*PolyLog[2, -E^(-c - d*x)] + 2*
f*(d*(e + f*x)*PolyLog[3, -E^(-c - d*x)] + f*PolyLog[4, -E^(-c - d*x)])))/
(d^4*E^c)))/(2*a*(1 + E^c)) - ((e + f*x)^4/f - (4*(-1 + E^c)*(e + f*x)^3*L
og[1 - E^(-c - d*x)])/d + (12*(-1 + E^c)*f*(d^2*(e + f*x)^2*PolyLog[2, E^(
-c - d*x)] + 2*f*(d*(e + f*x)*PolyLog[3, E^(-c - d*x)] + f*PolyLog[4, E^(-
c - d*x)])))/d^4)/(8*a*(-1 + E^c)) - (-8*a*d^4*e^3*E^(2*c)*x - 12*a*d^4*e^
2*E^(2*c)*f*x^2 - 8*a*d^4*e*E^(2*c)*f^2*x^3 - 2*a*d^4*E^(2*c)*f^3*x^4 + 8*
b*d^3*e^3*ArcTan[E^(c + d*x)] + 8*b*d^3*e^3*E^(2*c)*ArcTan[E^(c + d*x)] +
(12*I)*b*d^3*e^2*f*x*Log[1 - I*E^(c + d*x)] + (12*I)*b*d^3*e^2*E^(2*c)*f*x
*Log[1 - I*E^(c + d*x)] + (12*I)*b*d^3*e*f^2*x^2*Log[1 - I*E^(c + d*x)] +
(12*I)*b*d^3*e*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] + (4*I)*b*d^3*f^3*x^
3*Log[1 - I*E^(c + d*x)] + (4*I)*b*d^3*E^(2*c)*f^3*x^3*Log[1 - I*E^(c + d*
x)] - (12*I)*b*d^3*e^2*f*x*Log[1 + I*E^(c + d*x)] - (12*I)*b*d^3*e^2*E^(2*
c)*f*x*Log[1 + I*E^(c + d*x)] - (12*I)*b*d^3*e*f^2*x^2*Log[1 + I*E^(c + d*
x)] - (12*I)*b*d^3*e*E^(2*c)*f^2*x^2*Log[1 + I*E^(c + d*x)] - (4*I)*b*d^3*
f^3*x^3*Log[1 + I*E^(c + d*x)] - (4*I)*b*d^3*E^(2*c)*f^3*x^3*Log[1 + I*E^(
c + d*x)] + 4*a*d^3*e^3*Log[1 + E^(2*(c + d*x))] + 4*a*d^3*e^3*E^(2*c)*Log
[1 + E^(2*(c + d*x))] + 12*a*d^3*e^2*f*x*Log[1 + E^(2*(c + d*x))] + 12*a*d
^3*e^2*E^(2*c)*f*x*Log[1 + E^(2*(c + d*x))] + 12*a*d^3*e*f^2*x^2*Log[1 ...
```

Rubi [A] (verified)

Time = 4.58 (sec) , antiderivative size = 910, normalized size of antiderivative = 0.87, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.469$, Rules used = {6123, 5984, 3042, 26, 4670, 3011, 6107, 6095, 2620, 3011, 7163, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5984}$$

$$\frac{2\int (e+fx)^3\text{csch}(2c+2dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \ 3042$$

$$-\frac{b\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2\int i(e+fx)^3\csc(2ic+2idx)dx}{a}$$

$$\Big\downarrow \ 26$$

$$-\frac{b\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\int (e+fx)^3\csc(2ic+2idx)dx}{a}$$

$$\Big\downarrow \ 4670$$

$$-\frac{b\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(\frac{3if\int (e+fx)^2\log\left(1-e^{2c+2dx}\right)dx}{2d}-\frac{3if\int (e+fx)^2\log\left(1+e^{2c+2dx}\right)dx}{2d}+\frac{i(e+fx)^3\text{arctanh}\left(e^{2c+2dx}\right)}{d}\right)}{a}$$

$$\Big\downarrow \ 3011$$

$$-\frac{b\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(-\frac{3if\left(\frac{f\int (e+fx)\text{PolyLog}\left(2,-e^{2c+2dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{2d}+\frac{3if\left(\frac{f\int (e+fx)\text{PolyLog}\left(2,e^{2c+2dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,e^{2c-}\right.}{2d}\right.}{2d}\right.}{a}$$

$$\Big\downarrow \ 6107$$

$$-\frac{b\left(\frac{b^2\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}+\frac{2i\left(-\frac{3if\left(\frac{f\int (e+fx)\text{PolyLog}\left(2,-e^{2c+2dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{2d}+\frac{3if\left(\frac{f\int (e+fx)\text{PolyLog}\left(2,e^{2c+2dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,e^{2c-}\right.}{2d}\right.}{2d}\right.}{a}$$

$$\Big\downarrow \ 6095$$

$$-\frac{b\left(\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^4}{4bf}\right)}{a^2+b^2}+\frac{\int (e+fx)^3\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}+\frac{2i\left(-\frac{3if\left(\frac{f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{2d}+\frac{3if\left(\frac{f\int (e+fx)\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{2c\cdots}\right)}{2d}\cdots\right)}{2d}\cdots\right)}{a}$$

$\downarrow$ 2620

$$-\frac{b\left(\frac{b^2\left(-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\cdots\right)}{a^2+b^2}\cdots\right)}{a}+\frac{2i\left(-\frac{3if\left(\frac{f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{2d}+\frac{3if\left(\frac{f\int (e+fx)\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{2c\cdots}\right)}{2d}\cdots\right)}{2d}\cdots\right)}{a}$$

$\downarrow$ 3011

$$-\frac{b\left(\frac{b^2\left(-\frac{3f\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\cdots}{\cdots}\right)}{bd}\cdots\right)}{a^2+b^2}\cdots\right)}{a}+\frac{2i\left(-\frac{3if\left(\frac{f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{2d}+\frac{3if\left(\frac{f\int (e+fx)\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{2c\cdots}\right)}{2d}\cdots\right)}{2d}\cdots\right)}{a}$$

$\downarrow$ 7163

$$-\frac{b\left(\frac{b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int \operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{Pol}}{}\right.}{}\right.}{}\right.}{}\right.}{}}{}\right.}{}$$

$$\frac{2i\left(-\frac{3if\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{2c+2dx}\right)}{2d}-\frac{f\int \operatorname{PolyLog}\left(3,-e^{2c+2dx}\right)dx}{2d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{2d}+\frac{3if\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,e^{2c+2}\right.}{2d}\right.}{}\right.}{}\right.}{a}$$

$\downarrow$ 2720

$$-\frac{b\left(\frac{b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(2\ldots\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}$$

$$2i\left(-\frac{3if\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{2c+2dx}\right)}{2d}-\frac{f\int e^{-2c-2dx}\operatorname{PolyLog}\left(3,-e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{2d}+\frac{3if\left(\frac{f\left(\frac{(e+fx)\ldots}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.$$

a

↓ 7143

$$-\frac{b\left(\frac{\int (e+fx)^3\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{Poly}\cdots}{\cdots}\cdots\right)}{bd}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

$$\frac{2i\left(\frac{i(e+fx)^3\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{3if\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{2c+2dx}\right)}{2d}-\frac{f\operatorname{PolyLog}\left(4,-e^{2c+2dx}\right)}{4d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{2d}+\frac{3if\cdots}{\cdots}\cdots\right)}{a}$$

$\downarrow$ 7293

$$-\frac{b\left(\frac{\int\left(a(e+fx)^3\operatorname{sech}(c+dx)-b(e+fx)^3\tanh(c+dx)\right)dx}{a^2+b^2}+\frac{b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e\cdots}{\cdots}\right)}{bd}\cdots}{\cdots}\right)}{\cdots}$$

$$\frac{2i\left(\frac{i(e+fx)^3\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{3if\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{2c+2dx}\right)}{2d}-\frac{f\operatorname{PolyLog}\left(4,-e^{2c+2dx}\right)}{4d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{2d}+\frac{3if\cdots}{\cdots}\right)}{a}$$

↓ 2009

$$2i\left(\frac{i\operatorname{arctanh}(e^{2c+2dx})(e+fx)^3}{d}-\frac{3if\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}(3,-e^{2c+2dx})}{2d}-\frac{f\operatorname{PolyLog}(4,-e^{2c+2dx})}{4d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,-e^{2c+2dx})}{2d}\right)}{2d}+\frac{3if}{}\right.$$

$$a$$

$$b\left(\left(-\frac{(e+fx)^4}{4bf}+\frac{\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)(e+fx)^3}{bd}+\frac{\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)(e+fx)^3}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}\right.}{bd}\right.\right.$$

input
```
Int[((e + f*x)^3*Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-((b*((b^2*(-1/4*(e + f*x)^4/(b*f) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x
))/(a - Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2]))])/d^2))/d))/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/(a^2 + b^2) + ((b*(e + f*x)^4)/
(4*f) + (2*a*(e + f*x)^3*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)^3*Log[1 + E
^(2*(c + d*x))])/d - ((3*I)*a*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/
d^2 + ((3*I)*a*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/d^2 - (3*b*f*(e +
f*x)^2*PolyLog[2, -E^(2*(c + d*x))])/(2*d^2) + ((6*I)*a*f^2*(e + f*x)*Poly
Log[3, (-I)*E^(c + d*x)])/d^3 - ((6*I)*a*f^2*(e + f*x)*PolyLog[3, I*E^(c +
 d*x)])/d^3 + (3*b*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*d^3) - (
(6*I)*a*f^3*PolyLog[4, (-I)*E^(c + d*x)])/d^4 + ((6*I)*a*f^3*PolyLog[4, I*
E^(c + d*x)])/d^4 - (3*b*f^3*PolyLog[4, -E^(2*(c + d*x))])/(4*d^4))/(a^2 +
 b^2)))/a) + ((2*I)*((I*(e + f*x)^3*ArcTanh[E^(2*c + 2*d*x)])/d - (((3*I)/
2)*f*(-1/2*((e + f*x)^2*PolyLog[2, -E^(2*c + 2*d*x)])/d + (f*(((e + f*x)*P
olyLog[3, -E^(2*c + 2*d*x)])/(2*d) - (f*PolyLog[4, -E^(2*c + 2*d*x)])/(...
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^3 \operatorname{csch}(dx+c) \operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}\,dx$$

input
```
int((f*x+e)^3*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2448 vs. $2(962) = 1924$.

Time = 0.18 (sec) , antiderivative size = 2448, normalized size of antiderivative = 2.33

$$\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
-(6*b^2*f^3*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*b^2*f^3*polylog(4, (a*
cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2))/b) - 6*(a^2 + b^2)*f^3*polylog(4, cosh(d*x + c) + sinh(
d*x + c)) - 6*(a^2 + b^2)*f^3*polylog(4, -cosh(d*x + c) - sinh(d*x + c)) +
 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*dilog((a*cosh(d*x
 + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2) - b)/b + 1) + 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e
^2*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*((a^2 + b^2)*d^2*f^3*x^2
+ 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*dilog(cosh(d*x + c) +
 sinh(d*x + c)) + 3*(a^2*d^2*f^3*x^2 + I*a*b*d^2*f^3*x^2 + 2*a^2*d^2*e*f^2
*x + 2*I*a*b*d^2*e*f^2*x + a^2*d^2*e^2*f + I*a*b*d^2*e^2*f)*dilog(I*cosh(d
*x + c) + I*sinh(d*x + c)) + 3*(a^2*d^2*f^3*x^2 - I*a*b*d^2*f^3*x^2 + 2*a^
2*d^2*e*f^2*x - 2*I*a*b*d^2*e*f^2*x + a^2*d^2*e^2*f - I*a*b*d^2*e^2*f)*dil
og(-I*cosh(d*x + c) - I*sinh(d*x + c)) - 3*((a^2 + b^2)*d^2*f^3*x^2 + 2*(a
^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*dilog(-cosh(d*x + c) - sinh
(d*x + c)) + (b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^
3*f^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^
2) + 2*a) + (b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^3\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-e^3*(b^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^3 + a*b^2)*d
) - 2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + a*log(e^(-2*d*x - 2*c) + 1)
/((a^2 + b^2)*d) - log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*
d)) + 3*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))*e^2*f/(a*d^2) + 3
*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))*e^2*f/(a*d^2) + 3*(d^2*x
^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x
 + c)))*e*f^2/(a*d^3) + 3*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(
d*x + c)) - 2*polylog(3, e^(d*x + c)))*e*f^2/(a*d^3) + (d^3*x^3*log(e^(d*x
 + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)
) + 6*polylog(4, -e^(d*x + c)))*f^3/(a*d^4) + (d^3*x^3*log(-e^(d*x + c) +
1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*poly
log(4, e^(d*x + c)))*f^3/(a*d^4) - 1/2*(d^4*f^3*x^4 + 4*d^4*e*f^2*x^3 + 6*
d^4*e^2*f*x^2)/(a*d^4) + integrate(2*(b^3*f^3*x^3 + 3*b^3*e*f^2*x^2 + 3*b^
3*e^2*f*x - (a*b^2*f^3*x^3*e^c + 3*a*b^2*e*f^2*x^2*e^c + 3*a*b^2*e^2*f*x*e
^c)*e^(d*x))/(a^3*b + a*b^3 - (a^3*b*e^(2*c) + a*b^3*e^(2*c))*e^(2*d*x) -
2*(a^4*e^c + a^2*b^2*e^c)*e^(d*x)), x) - integrate(-2*(a*f^3*x^3 + 3*a*e*f
^2*x^2 + 3*a*e^2*f*x - (b*f^3*x^3*e^c + 3*b*e*f^2*x^2*e^c + 3*b*e^2*f*x*e^
c)*e^(d*x))/(a^2 + b^2 + (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\,\text{csch}\,(dx+c)\,\text{sech}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
integrate((f*x + e)^3*csch(d*x + c)*sech(d*x + c)/(b*sinh(d*x + c) + a), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\int \frac{(e+f\,x)^3}{\cosh\,(c+d\,x)\,\sinh\,(c+d\,x)\,(a+b\sinh\,(c+d\,x))}\,dx$$

input
```
int((e + f*x)^3/(cosh(c + d*x)*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^3/(cosh(c + d*x)*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\frac{-2atan\left(e^{dx+c}\right)ab\,e^3+8e^{3c}\left(\int \frac{e^{3dx}x^3}{e^{6dx+6c}b+2e^{5dx+5c}a-e^{4dx+4c}b-e^{2dx+2c}b-2e^{dx+c}a+b}dx\right)a^3d\,f^3+8e^{3c}\left(\int \frac{}{e^{6dx+6c}b+2e^{5d}}\right.}{}$$

input
```
int((f*x+e)^3*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*atan(e**(c + d*x))*a*b*e**3 + 8*e**(3*c)*int((e**(3*d*x)*x**3)/(e**(
6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - e**(4*c + 4*d*x)*b - e**(2*c + 2*d
*x)*b - 2*e**(c + d*x)*a + b),x)*a**3*d*f**3 + 8*e**(3*c)*int((e**(3*d*x)*
x**3)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - e**(4*c + 4*d*x)*b - e*
*(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a*b**2*d*f**3 + 24*e**(3*c)*in
t((e**(3*d*x)*x**2)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - e**(4*c +
 4*d*x)*b - e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**3*d*e*f**2 +
24*e**(3*c)*int((e**(3*d*x)*x**2)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)
*a - e**(4*c + 4*d*x)*b - e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a*
b**2*d*e*f**2 + 24*e**(3*c)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**
(5*c + 5*d*x)*a - e**(4*c + 4*d*x)*b - e**(2*c + 2*d*x)*b - 2*e**(c + d*x)
*a + b),x)*a**3*d*e**2*f + 24*e**(3*c)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x
)*b + 2*e**(5*c + 5*d*x)*a - e**(4*c + 4*d*x)*b - e**(2*c + 2*d*x)*b - 2*e
**(c + d*x)*a + b),x)*a*b**2*d*e**2*f - log(e**(2*c + 2*d*x) + 1)*a**2*e**
3 + log(e**(c + d*x) - 1)*a**2*e**3 + log(e**(c + d*x) - 1)*b**2*e**3 + lo
g(e**(c + d*x) + 1)*a**2*e**3 + log(e**(c + d*x) + 1)*b**2*e**3 - log(e**(
2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*b**2*e**3)/(a*d*(a**2 + b**2))
```

3.436 $\int \frac{(e+fx)^2 \text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 734

$$\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= -\frac{2b(e+fx)^2\arctan\left(e^{c+dx}\right)}{(a^2+b^2)\,d} - \frac{2(e+fx)^2\operatorname{arctanh}\left(e^{2c+2dx}\right)}{ad}$$
$$- \frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)\,d} - \frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)\,d}$$
$$+ \frac{b^2(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{a\,(a^2+b^2)\,d} + \frac{2ibf(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{(a^2+b^2)\,d^2}$$
$$- \frac{2ibf(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{(a^2+b^2)\,d^2} - \frac{2b^2f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)\,d^2}$$
$$- \frac{2b^2f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)\,d^2} + \frac{b^2f(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{a\,(a^2+b^2)\,d^2}$$
$$- \frac{f(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{ad^2} + \frac{f(e+fx)\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{ad^2}$$
$$- \frac{2ibf^2\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{(a^2+b^2)\,d^3} + \frac{2ibf^2\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{(a^2+b^2)\,d^3}$$
$$+ \frac{2b^2f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)\,d^3} + \frac{2b^2f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)\,d^3}$$
$$- \frac{b^2f^2\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2a\,(a^2+b^2)\,d^3} + \frac{f^2\operatorname{PolyLog}\left(3,-e^{2c+2dx}\right)}{2ad^3} - \frac{f^2\operatorname{PolyLog}\left(3,e^{2c+2dx}\right)}{2ad^3}$$

output
```
-2*b*(f*x+e)^2*arctan(exp(d*x+c))/(a^2+b^2)/d-2*(f*x+e)^2*arctanh(exp(2*d*
x+2*c))/a/d-b^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^
2)/d-b^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d+b^
2*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/a/(a^2+b^2)/d+2*I*b*f^2*polylog(3,I*exp(d
*x+c))/(a^2+b^2)/d^3-2*I*b*f*(f*x+e)*polylog(2,I*exp(d*x+c))/(a^2+b^2)/d^2
-2*b^2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)/
d^2-2*b^2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^
2)/d^2+b^2*f*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/a/(a^2+b^2)/d^2-f*(f*x+e)*
polylog(2,-exp(2*d*x+2*c))/a/d^2+f*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a/d^2
-2*I*b*f^2*polylog(3,-I*exp(d*x+c))/(a^2+b^2)/d^3+2*I*b*f*(f*x+e)*polylog(
2,-I*exp(d*x+c))/(a^2+b^2)/d^2+2*b^2*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b
^2)^(1/2)))/a/(a^2+b^2)/d^3+2*b^2*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)
^(1/2)))/a/(a^2+b^2)/d^3-1/2*b^2*f^2*polylog(3,-exp(2*d*x+2*c))/a/(a^2+b^2
)/d^3+1/2*f^2*polylog(3,-exp(2*d*x+2*c))/a/d^3-1/2*f^2*polylog(3,exp(2*d*x
+2*c))/a/d^3
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2019 vs. $2(734) = 1468$.

Time = 12.42 (sec) , antiderivative size = 2019, normalized size of antiderivative = 2.75

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
2*((E^c*((e + f*x)^3/(3*E^c*f) + ((1 + E^(-c))*(e + f*x)^2*Log[1 + E^(-c -
 d*x)])/d - (2*(1 + E^c)*f*(d*(e + f*x)*PolyLog[2, -E^(-c - d*x)] + f*Poly
Log[3, -E^(-c - d*x)]))/(d^3*E^c)))/(2*a*(1 + E^c)) - ((e + f*x)^3/f - (3*
(-1 + E^c)*(e + f*x)^2*Log[1 - E^(-c - d*x)])/d + (6*(-1 + E^c)*f*(d*(e +
f*x)*PolyLog[2, E^(-c - d*x)] + f*PolyLog[3, E^(-c - d*x)]))/d^3)/(6*a*(-1
 + E^c)) - (-12*a*d^3*e^2*E^(2*c)*x + 12*a*d^3*e^2*(1 + E^(2*c))*x + 12*a*
d^3*e*f*x^2 + 4*a*d^3*f^2*x^3 + 12*b*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d
*x)] - 6*a*d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*
I)*b*d*e*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d
*x)]) - PolyLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) - 6*a*d*
e*(1 + E^(2*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^
(2*(c + d*x))]) + (6*I)*b*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)
] - d^2*x^2*Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] +
2*d*x*PolyLog[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyL
og[3, I*E^(c + d*x)]) - a*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 +
E^(2*(c + d*x))]) - 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(
2*(c + d*x))]))/(12*(a^2 + b^2)*d^3*(1 + E^(2*c))) + (b^2*(6*e^2*E^(2*c)*x
 + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan
[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sq
rt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b...
```

Rubi [A] (verified)

Time = 3.27 (sec) , antiderivative size = 651, normalized size of antiderivative = 0.89, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.469$, Rules used = {6123, 5984, 3042, 26, 4670, 3011, 2720, 6107, 6095, 2620, 3011, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5984}$$

$$\frac{2\int (e+fx)^2\text{csch}(2c+2dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2\int i(e+fx)^2\csc(2ic+2idx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\int (e+fx)^2\csc(2ic+2idx)dx}{a}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{b\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(\frac{if\int (e+fx)\log\left(1-e^{2c+2dx}\right)dx}{d}-\frac{if\int (e+fx)\log\left(1+e^{2c+2dx}\right)dx}{d}+\frac{i(e+fx)^2\text{arctanh}\left(e^{2c+2dx}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{b\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(-\frac{if\left(\frac{f\int \text{PolyLog}\left(2,-e^{2c+2dx}\right)dx}{2d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{d}+\frac{if\left(\frac{f\int \text{PolyLog}\left(2,e^{2c+2dx}\right)dx}{2d}-\frac{(e+fx)\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d}\right)}{d}+\frac{i(e+f\cdots}{\cdots}\right)}{a}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{b\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(-\frac{if\left(\frac{f\int e^{-2c-2dx}\text{PolyLog}\left(2,-e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{d}+\frac{if\left(\frac{f\int e^{-2c-2dx}\text{PolyLog}\left(2,e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{(e+f\cdots}{\cdots}\right)}{d}\cdots\right)}{a}$$

$$\Big\downarrow \text{6107}$$

$$-\frac{b\left(\frac{b^2\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}+\frac{2i\left(-\frac{if\left(\frac{f\int e^{-2c-2dx}\text{PolyLog}\left(2,-e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{d}+\frac{if\left(\frac{f\int e^{-2c-2dx}\text{PolyLog}\left(2,e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{(e+f\cdots}{\cdots}\right)}{d}\cdots\right)}{a}$$

$\downarrow$ 6095

$$-\frac{b\left(\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}+\frac{2i\left(-\frac{if\left(\frac{f\int e^{-2c-2dx}\text{PolyLog}\left(2,-e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{d}+\frac{if\left(\frac{f\int e^{-2c-2dx}\text{PolyLog}\left(2,e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{(e+f\cdots}{\cdots}\right)}{d}\cdots\right)}{a}$$

$\downarrow$ 2620

$$-\frac{b\left(\frac{b^2\left(-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+\cdots}{3\cdots}\cdots\right)}{a^2+b^2}\cdots\right)}{a}$$

$$\frac{2i\left(-\frac{if\left(\frac{f\int e^{-2c-2dx}\text{PolyLog}\left(2,-e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{d}+\frac{if\left(\frac{f\int e^{-2c-2dx}\text{PolyLog}\left(2,e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{(e+f\cdots}{\cdots}\right)}{d}\cdots\right)}{a}$$

$\downarrow$ 3011

$$-\frac{b\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be}{a+\sqrt{}}\right.}{d}\right.}{bd}\right.}{a^2+b^2}\right.}{a}$$

$$\frac{2i\left(-\frac{if\left(\frac{f\int e^{-2c-2dx}\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{d}+\frac{if\left(\frac{f\int e^{-2c-2dx}\,\text{PolyLog}\left(2,e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{(e+f}{}\right.}{d}\right.}{a}$$

$\downarrow$ 2720

$$-\frac{b\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right.}{bd}\right.}{a^2+b^2}\right.}{}$$

$$\frac{2i\left(-\frac{if\left(\frac{f\int e^{-2c-2dx}\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{d}+\frac{if\left(\frac{f\int e^{-2c-2dx}\,\text{PolyLog}\left(2,e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{(e+f}{}\right.}{d}\right.}{a}$$

$\downarrow$ 7143

$$-\frac{b\left(\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{a\cdots}\right.}{\cdots}\cdots$$

$$\frac{2i\left(\frac{i(e+fx)^2\text{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{if\left(\frac{f\,\text{PolyLog}\left(3,-e^{2c+2dx}\right)}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{d}+\frac{if\left(\frac{f\,\text{PolyLog}\left(3,e^{2c+2dx}\right)}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\cdots}{2d}\cdots\right.}{d}\cdots\right.}{a}\cdots$$

$\downarrow$ 7293

$$-\frac{b\left(\frac{\int \left(a(e+fx)^2\text{sech}(c+dx)-b(e+fx)^2\tanh(c+dx)\right)dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\cdots}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{a\cdots}\right.}{\cdots}\cdots$$

$$\frac{2i\left(\frac{i(e+fx)^2\text{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{if\left(\frac{f\,\text{PolyLog}\left(3,-e^{2c+2dx}\right)}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{d}+\frac{if\left(\frac{f\,\text{PolyLog}\left(3,e^{2c+2dx}\right)}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\cdots}{2d}\cdots\right.}{d}\cdots\right.}{a}\cdots$$

$\downarrow$ 2009

$$\frac{2i\left(\frac{i(e+fx)^2\text{arctanh}(e^{2c+2dx})}{d}-\frac{if\left(\frac{f\,\text{PolyLog}\left(3,-e^{2c+2dx}\right)}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{d}+\frac{if\left(\frac{f\,\text{PolyLog}\left(3,e^{2c+2dx}\right)}{4d^2}-\frac{(e+fx)\,\text{PolyLog}}{2d}\right.}{d}\right.}{a}$$

$$b\left(\frac{b^2\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right.}{a^2+b^2}\right.$$

input
```
Int[((e + f*x)^2*Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-((b*((b^2*(-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[
a^2 + b^2]))])/d^2))/(b*d)))/(a^2 + b^2) + ((b*(e + f*x)^3)/(3*f) + (2*a*(
e + f*x)^2*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)^2*Log[1 + E^(2*(c + d*x))
])/d - ((2*I)*a*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + ((2*I)*a*f
*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*(e + f*x)*PolyLog[2, -E^(
2*(c + d*x))])/d^2 + ((2*I)*a*f^2*PolyLog[3, (-I)*E^(c + d*x)])/d^3 - ((2*
I)*a*f^2*PolyLog[3, I*E^(c + d*x)])/d^3 + (b*f^2*PolyLog[3, -E^(2*(c + d*x
))])/(2*d^3))/(a^2 + b^2)))/a) + ((2*I)*((I*(e + f*x)^2*ArcTanh[E^(2*c + 2
*d*x)])/d - (I*f*(-1/2*((e + f*x)*PolyLog[2, -E^(2*c + 2*d*x)])/d + (f*Pol
yLog[3, -E^(2*c + 2*d*x)])/(4*d^2)))/d + (I*f*(-1/2*((e + f*x)*PolyLog[2,
E^(2*c + 2*d*x)])/d + (f*PolyLog[3, E^(2*c + 2*d*x)])/(4*d^2)))/d))/a
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c) \operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1535 vs. $2(677) = 1354$.

Time = 0.16 (sec) , antiderivative size = 1535, normalized size of antiderivative = 2.09

$$\int \frac{(e+fx)^2 \operatorname{csch}(c+dx) \operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
(2*b^2*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c
) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 2*b^2*f^2*polylog(3, (a*c
osh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2))/b) - 2*(a^2 + b^2)*f^2*polylog(3, cosh(d*x + c) + sinh(d
*x + c)) - 2*(a^2 + b^2)*f^2*polylog(3, -cosh(d*x + c) - sinh(d*x + c)) -
2*(b^2*d*f^2*x + b^2*d*e*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(b^
2*d*f^2*x + b^2*d*e*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(
d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*((a^2 +
b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*dilog(cosh(d*x + c) + sinh(d*x + c)) - 2
*(a^2*d*f^2*x + I*a*b*d*f^2*x + a^2*d*e*f + I*a*b*d*e*f)*dilog(I*cosh(d*x
+ c) + I*sinh(d*x + c)) - 2*(a^2*d*f^2*x - I*a*b*d*f^2*x + a^2*d*e*f - I*a
*b*d*e*f)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) + 2*((a^2 + b^2)*d*f^2
*x + (a^2 + b^2)*d*e*f)*dilog(-cosh(d*x + c) - sinh(d*x + c)) - (b^2*d^2*e
^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c
) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*
c^2*f^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/
b^2) + 2*a) - (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2
*f^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b^2*d^2*f^2*x^2 + 2*b^2*d^2*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\,\text{csch}\,(dx+c)\,\text{sech}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-e^2*(b^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^3 + a*b^2)*d
) - 2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + a*log(e^(-2*d*x - 2*c) + 1)
/((a^2 + b^2)*d) - log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*
d)) + 2*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))*e*f/(a*d^2) + 2*(
d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))*e*f/(a*d^2) + (d^2*x^2*log
(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c))
)*f^2/(a*d^3) + (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c))
- 2*polylog(3, e^(d*x + c)))*f^2/(a*d^3) - 2/3*(d^3*f^2*x^3 + 3*d^3*e*f*x^
2)/(a*d^3) + integrate(2*(b^3*f^2*x^2 + 2*b^3*e*f*x - (a*b^2*f^2*x^2*e^c +
 2*a*b^2*e*f*x*e^c)*e^(d*x))/(a^3*b + a*b^3 - (a^3*b*e^(2*c) + a*b^3*e^(2*
c))*e^(2*d*x) - 2*(a^4*e^c + a^2*b^2*e^c)*e^(d*x)), x) - integrate(-2*(a*f
^2*x^2 + 2*a*e*f*x - (b*f^2*x^2*e^c + 2*b*e*f*x*e^c)*e^(d*x))/(a^2 + b^2 +
 (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\,\text{csch}\,(dx+c)\,\text{sech}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
integrate((f*x + e)^2*csch(d*x + c)*sech(d*x + c)/(b*sinh(d*x + c) + a), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^2}{\cosh(c+d\,x)\,\sinh(c+d\,x)\,(a+b\sinh(c+d\,x))}\,dx$$

input
```
int((e + f*x)^2/(cosh(c + d*x)*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^2/(cosh(c + d*x)*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{-2atan\left(e^{dx+c}\right)ab\,e^2+8e^{3c}\left(\int \frac{e^{3dx}x^2}{e^{6dx+6c}b+2e^{5dx+5c}a-e^{4dx+4c}b-e^{2dx+2c}b-2e^{dx+c}a+b}dx\right)a^3d\,f^2+8e^{3c}\left(\int \frac{}{e^{6dx+6c}b+2e^{5d}}\right.}{}$$

input
```
int((f*x+e)^2*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*atan(e**(c + d*x))*a*b*e**2 + 8*e**(3*c)*int((e**(3*d*x)*x**2)/(e**(
6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - e**(4*c + 4*d*x)*b - e**(2*c + 2*d
*x)*b - 2*e**(c + d*x)*a + b),x)*a**3*d*f**2 + 8*e**(3*c)*int((e**(3*d*x)*
x**2)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - e**(4*c + 4*d*x)*b - e*
*(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a*b**2*d*f**2 + 16*e**(3*c)*in
t((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - e**(4*c + 4*
d*x)*b - e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**3*d*e*f + 16*e**
(3*c)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - e**(
4*c + 4*d*x)*b - e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a*b**2*d*e*
f - log(e**(2*c + 2*d*x) + 1)*a**2*e**2 + log(e**(c + d*x) - 1)*a**2*e**2
 + log(e**(c + d*x) - 1)*b**2*e**2 + log(e**(c + d*x) + 1)*a**2*e**2 + log(
e**(c + d*x) + 1)*b**2*e**2 - log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a -
b)*b**2*e**2)/(a*d*(a**2 + b**2))
```

3.437 $\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 439

$$\begin{aligned}
&\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= -\frac{2b(e+fx)\arctan\left(e^{c+dx}\right)}{(a^2+b^2)\,d} - \frac{2(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{ad} \\
&\quad - \frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)\,d} - \frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)\,d} \\
&\quad + \frac{b^2(e+fx)\log\left(1+e^{2(c+dx)}\right)}{a\,(a^2+b^2)\,d} + \frac{ibf\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{(a^2+b^2)\,d^2} - \frac{ibf\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{(a^2+b^2)\,d^2} \\
&\quad - \frac{b^2f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)\,d^2} - \frac{b^2f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)\,d^2} \\
&\quad + \frac{b^2f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2a\,(a^2+b^2)\,d^2} - \frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2ad^2} + \frac{f\,\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2ad^2}
\end{aligned}$$

output
```
-2*b*(f*x+e)*arctan(exp(d*x+c))/(a^2+b^2)/d-2*(f*x+e)*arctanh(exp(2*d*x+2*
c))/a/d-b^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d-b
^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d+b^2*(f*x+e
)*ln(1+exp(2*d*x+2*c))/a/(a^2+b^2)/d+I*b*f*polylog(2,-I*exp(d*x+c))/(a^2+b
^2)/d^2-I*b*f*polylog(2,I*exp(d*x+c))/(a^2+b^2)/d^2-b^2*f*polylog(2,-b*exp
(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d^2-b^2*f*polylog(2,-b*exp(d*x+c)
/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d^2+1/2*b^2*f*polylog(2,-exp(2*d*x+2*c))
/a/(a^2+b^2)/d^2-1/2*f*polylog(2,-exp(2*d*x+2*c))/a/d^2+1/2*f*polylog(2,ex
p(2*d*x+2*c))/a/d^2
```

Mathematica [A] (verified)

Time = 5.14 (sec) , antiderivative size = 784, normalized size of antiderivative = 1.79

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{\frac{\frac{1}{2}d^2fx^2+de(c+dx)-2(de-cf)(c+dx)+2f(c+dx)\log\left(1+e^{-c-dx}\right)+2(de-cf)\log\left(1+e^{c+dx}\right)-2f\,\text{PolyLog}\left(2,-e^{-c-dx}\right)}{a}+\frac{\frac{1}{2}d^2fx^2-de(c+dx)\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[((e + f*x)*Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(((d^2*f*x^2)/2 + d*e*(c + d*x) - 2*(d*e - c*f)*(c + d*x) + 2*f*(c + d*x)*
Log[1 + E^(-c - d*x)] + 2*(d*e - c*f)*Log[1 + E^(c + d*x)] - 2*f*PolyLog[2
, -E^(-c - d*x)])/a + ((d^2*f*x^2)/2 - d*e*(c + d*x) + 2*c*f*(c + d*x) + 2
*f*(c + d*x)*Log[1 - E^(-c - d*x)] + 2*(d*e - c*f)*Log[1 - E^(c + d*x)] -
2*f*PolyLog[2, E^(-c - d*x)])/a - (b^2*(-2*d*e*(c + d*x) + 2*c*f*(c + d*x)
 - f*(c + d*x)^2 + (4*a*Sqrt[a^2 + b^2]*d*e*ArcTan[(a + b*E^(c + d*x))/Sqr
t[-a^2 - b^2]])/Sqrt[-(a^2 + b^2)^2] - (4*a*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTa
nh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/(-a^2 - b^2)^(3/2) + 2*f*(c + d*x
)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*f*(c + d*x)*Log[1 + (
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*f*Log[b - 2*a*E^(c + d*x) - b*
E^(2*(c + d*x))] + 2*d*e*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] +
 2*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f*PolyLog[2, -
((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a*(a^2 + b^2)) - (2*(-(a*d*e*(
c + d*x)) + a*c*f*(c + d*x) - (a*f*(c + d*x)^2)/2 + 2*b*d*e*ArcTan[E^(c +
d*x)] - 2*b*c*f*ArcTan[E^(c + d*x)] + I*b*f*(c + d*x)*Log[1 - I*E^(c + d*x
)] - I*b*f*(c + d*x)*Log[1 + I*E^(c + d*x)] + a*d*e*Log[1 + E^(2*(c + d*x)
)] - a*c*f*Log[1 + E^(2*(c + d*x))] + a*f*(c + d*x)*Log[1 + E^(2*(c + d*x)
)] - I*b*f*PolyLog[2, (-I)*E^(c + d*x)] + I*b*f*PolyLog[2, I*E^(c + d*x)]
+ (a*f*PolyLog[2, -E^(2*(c + d*x))])/2))/(a^2 + b^2))/(2*d^2)
```

Rubi [A] (verified)

Time = 2.00 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.92, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {6123, 5984, 3042, 26, 4670, 2715, 2838, 6107, 6095, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)\text{csch}(c+dx)\text{sech}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5984}$$

$$\frac{2\int (e+fx)\text{csch}(2c+2dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2\int i(e+fx)\csc(2ic+2idx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\int (e+fx)\csc(2ic+2idx)dx}{a}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{b\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(\frac{if\int \log(1-e^{2c+2dx})dx}{2d}-\frac{if\int \log(1+e^{2c+2dx})dx}{2d}+\frac{i(e+fx)\text{arctanh}(e^{2c+2dx})}{d}\right)}{a}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{b\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(\frac{if\int e^{-2c-2dx}\log(1-e^{2c+2dx})de^{2c+2dx}}{4d^2}-\frac{if\int e^{-2c-2dx}\log(1+e^{2c+2dx})de^{2c+2dx}}{4d^2}+\frac{i(e+fx)\text{arctanh}(e^{2c+2dx})}{d}\right)}{a}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{b\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(\frac{i(e+fx)\text{arctanh}(e^{2c+2dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{2c+2dx})}{4d^2}-\frac{if\,\text{PolyLog}(2,e^{2c+2dx})}{4d^2}\right)}{a}$$

$$\Big\downarrow \text{6107}$$

$$-\frac{b\left(\frac{b^2\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}+\frac{2i\left(\frac{i(e+fx)\text{arctanh}(e^{2c+2dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{2c+2dx})}{4d^2}-\frac{if\,\text{PolyLog}(2,e^{2c+2dx})}{4d^2}\right)}{a}$$

$$\Big\downarrow \text{6095}$$

$$-\frac{b\left(\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}+\frac{2i\left(\frac{i(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\,\text{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)}{a}$$

$\downarrow$ 2620

$$-\frac{b\left(\frac{b^2\left(-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int (e+ \cdots}{\cdots}\right)}{a}$$

$$\frac{2i\left(\frac{i(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\,\text{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)}{a}$$

$\downarrow$ 2715

$$-\frac{b\left(\frac{b^2\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+ \cdots\right)}{bd}\cdots\right)}{a^2+b^2}\cdots\right)}{a}$$

$$\frac{2i\left(\frac{i(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\,\text{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)}{a}$$

$\downarrow$ 2838

$$-\frac{b\left(\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\cdots\right)}{a^2+b^2}\cdots\right)}{a}$$

$$\frac{2i\left(\frac{i(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\,\text{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)}{a}$$

$\downarrow$ 7293

$$-\frac{b\left(\frac{\int (a(e+fx)\text{sech}(c+dx)-b(e+fx)\tanh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right.}{bd}\right.}{a^2+b^2}\right.}{a}$$

$$\frac{2i\left(\frac{i(e+fx)\text{arctanh}(e^{2c+2dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{2c+2dx})}{4d^2}-\frac{if\,\text{PolyLog}(2,e^{2c+2dx})}{4d^2}\right)}{a}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{2i\left(\frac{i(e+fx)\text{arctanh}(e^{2c+2dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{2c+2dx})}{4d^2}-\frac{if\,\text{PolyLog}(2,e^{2c+2dx})}{4d^2}\right)}{a}-$$

$$\frac{b\left(\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{2a(e+f\right.}{a}$$

input
```
Int[((e + f*x)*Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-((b*((b^2*(-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
]))])/(b*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(
b*d^2)))/(a^2 + b^2) + ((b*(e + f*x)^2)/(2*f) + (2*a*(e + f*x)*ArcTan[E^(c
 + d*x)])/d - (b*(e + f*x)*Log[1 + E^(2*(c + d*x))])/d - (I*a*f*PolyLog[2,
 (-I)*E^(c + d*x)])/d^2 + (I*a*f*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*Pol
yLog[2, -E^(2*(c + d*x))])/(2*d^2))/(a^2 + b^2)))/a) + ((2*I)*((I*(e + f*x
)*ArcTanh[E^(2*c + 2*d*x)])/d + ((I/4)*f*PolyLog[2, -E^(2*c + 2*d*x)])/d^2
 - ((I/4)*f*PolyLog[2, E^(2*c + 2*d*x)])/d^2))/a
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1064 vs. $2(411) = 822$.

Time = 4.75 (sec) , antiderivative size = 1065, normalized size of antiderivative = 2.43

method	result	size
risch	Expression too large to display	1065

input
```
int((f*x+e)*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERB
OSE)
```

output
```
1/d^2*c*f*b^2/(a^2+b^2)/a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/d*f*b^2/
(a^2+b^2)/a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1
/d^2*f*b^2/(a^2+b^2)/a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^
(1/2)))*c-1/d*f*b^2/(a^2+b^2)/a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^
2+b^2)^(1/2)))*x-1/d^2*f*b^2/(a^2+b^2)/a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+
a)/(a+(a^2+b^2)^(1/2)))*c-4*I/d^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*b*c+4
*I/d^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*b*c-4*I/d*f/(4*a^2+4*b^2)*ln(1-I
*exp(d*x+c))*b*x+4*I/d*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*b*x-1/d^2*c*f/a*
ln(exp(d*x+c)-1)+1/d*f/a*ln(exp(d*x+c)+1)*x-1/d^2*f*b^2/(a^2+b^2)/a*dilog(
(-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2*f*b^2/(a^2+b
^2)/a*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-4/d*f/(4
*a^2+4*b^2)*ln(1+I*exp(d*x+c))*a*x-4/d^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c)
)*a*c-4/d*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*a*x-4/d^2*f/(4*a^2+4*b^2)*ln(
1-I*exp(d*x+c))*a*c+1/d*e/a*ln(exp(d*x+c)-1)+1/d*e/a*ln(exp(d*x+c)+1)+1/d^
2*f/a*dilog(exp(d*x+c)+1)-1/d^2*f*dilog(exp(d*x+c))/a-4/d*e/(4*a^2+4*b^2)*
a*ln(1+exp(2*d*x+2*c))-8/d*e/(4*a^2+4*b^2)*b*arctan(exp(d*x+c))-4/d^2*f/(4
*a^2+4*b^2)*dilog(1+I*exp(d*x+c))*a-4/d^2*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*
x+c))*a+4/d^2*c*f/(4*a^2+4*b^2)*a*ln(1+exp(2*d*x+2*c))+8/d^2*c*f/(4*a^2+4*
b^2)*b*arctan(exp(d*x+c))-1/d*e*b^2/(a^2+b^2)/a*ln(b*exp(2*d*x+2*c)+2*a*ex
p(d*x+c)-b)-4*I/d^2*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))*b+4*I/d^2*f/(...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(400) = 800$.

Time = 0.13 (sec) , antiderivative size = 808, normalized size of antiderivative = 1.84

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
fricas")
```

output
```
-(b^2*f*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + b^2*f*dilog((a*cosh(d*x +
 c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^
2)/b^2) - b)/b + 1) - (a^2 + b^2)*f*dilog(cosh(d*x + c) + sinh(d*x + c)) -
 (a^2 + b^2)*f*dilog(-cosh(d*x + c) - sinh(d*x + c)) + (a^2*f + I*a*b*f)*d
ilog(I*cosh(d*x + c) + I*sinh(d*x + c)) + (a^2*f - I*a*b*f)*dilog(-I*cosh(
d*x + c) - I*sinh(d*x + c)) + (b^2*d*e - b^2*c*f)*log(2*b*cosh(d*x + c) +
2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^2*d*e - b^2*c*f)
*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2
*a) + (b^2*d*f*x + b^2*c*f)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (b^2*d*f*x
 + b^2*c*f)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - ((a^2 + b^2)*d*f*x + (a^2
+ b^2)*d*e)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + (a^2*d*e + I*a*b*d*e
- a^2*c*f - I*a*b*c*f)*log(cosh(d*x + c) + sinh(d*x + c) + I) + (a^2*d*e -
 I*a*b*d*e - a^2*c*f + I*a*b*c*f)*log(cosh(d*x + c) + sinh(d*x + c) - I) -
 ((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*log(cosh(d*x + c) + sinh(d*x + c) - 1
) + (a^2*d*f*x - I*a*b*d*f*x + a^2*c*f - I*a*b*c*f)*log(I*cosh(d*x + c) +
I*sinh(d*x + c) + 1) + (a^2*d*f*x + I*a*b*d*f*x + a^2*c*f + I*a*b*c*f)*log
(-I*cosh(d*x + c) - I*sinh(d*x + c) + 1) - ((a^2 + b^2)*d*f*x + (a^2 + ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{csch}\,(dx+c)\,\text{sech}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
maxima")
```

output
```
-e*(b^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^3 + a*b^2)*d)
- 2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + a*log(e^(-2*d*x - 2*c) + 1)/(
(a^2 + b^2)*d) - log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d)
) + 4*f*integrate(2*x/((b*(e^(d*x + c) - e^(-d*x - c)) + 2*a)*(e^(d*x + c)
 + e^(-d*x - c))*(e^(d*x + c) - e^(-d*x - c))), x)
```

Giac [F]

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{csch}\,(dx+c)\,\text{sech}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
giac")
```

output
```
integrate((f*x + e)*csch(d*x + c)*sech(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{e+f\,x}{\cosh\,(c+d\,x)\,\sinh\,(c+d\,x)\,\left(a+b\sinh\,(c+d\,x)\right)}\,dx$$

input
```
int((e + f*x)/(cosh(c + d*x)*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(cosh(c + d*x)*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-2atan\left(e^{dx+c}\right)abe+8e^{3c}\left(\int \frac{e^{3dx}x}{e^{6dx+6c}b+2e^{5dx+5c}a-e^{4dx+4c}b-e^{2dx+2c}b-2e^{dx+c}a+b}dx\right)a^3df+8e^{3c}\left(\int \frac{}{e^{6dx+6c}b+2e^{5dx+5c}}\right.}{}$$

input
```
int((f*x+e)*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*atan(e**(c + d*x))*a*b*e + 8*e**(3*c)*int((e**(3*d*x)*x)/(e**(6*c +
6*d*x)*b + 2*e**(5*c + 5*d*x)*a - e**(4*c + 4*d*x)*b - e**(2*c + 2*d*x)*b
- 2*e**(c + d*x)*a + b),x)*a**3*d*f + 8*e**(3*c)*int((e**(3*d*x)*x)/(e**(6
*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - e**(4*c + 4*d*x)*b - e**(2*c + 2*d*
x)*b - 2*e**(c + d*x)*a + b),x)*a*b**2*d*f - log(e**(2*c + 2*d*x) + 1)*a**
2*e + log(e**(c + d*x) - 1)*a**2*e + log(e**(c + d*x) - 1)*b**2*e + log(e*
*(c + d*x) + 1)*a**2*e + log(e**(c + d*x) + 1)*b**2*e - log(e**(2*c + 2*d*
x)*b + 2*e**(c + d*x)*a - b)*b**2*e)/(a*d*(a**2 + b**2))
```

3.438 $\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 90

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b\arctan(\sinh(c+dx))}{(a^2+b^2)\,d} - \frac{a\log(\cosh(c+dx))}{(a^2+b^2)\,d} + \frac{\log(\sinh(c+dx))}{ad} - \frac{b^2\log(a+b\sinh(c+dx))}{a\,(a^2+b^2)\,d}$$

output
```
-b*arctan(sinh(d*x+c))/(a^2+b^2)/d-a*ln(cosh(d*x+c))/(a^2+b^2)/d+ln(sinh(d
*x+c))/a/d-b^2*ln(a+b*sinh(d*x+c))/a/(a^2+b^2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{\log(i-\sinh(c+dx))}{a+ib} - \frac{2\log(\sinh(c+dx))}{a} + \frac{\log(i+\sinh(c+dx))}{a-ib} + \frac{2b^2\log(a+b\sinh(c+dx))}{a(a^2+b^2)}}{2d}$$

input
```
Integrate[(Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*(Log[I - Sinh[c + d*x]]/(a + I*b) - (2*Log[Sinh[c + d*x]])/a + Log[I
+ Sinh[c + d*x]]/(a - I*b) + (2*b^2*Log[a + b*Sinh[c + d*x]])/(a*(a^2 + b^
2)))/d
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 26, 3316, 26, 27, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\sin(ic+idx)\cos(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\cos(ic+idx)\sin(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3316}$$

$$-\frac{ib\int \frac{i\operatorname{csch}(c+dx)}{(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{b\int \frac{\operatorname{csch}(c+dx)}{(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^2\int \frac{\operatorname{csch}(c+dx)}{b(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{615}$$

$$\frac{b^2\int \left(\frac{\operatorname{csch}(c+dx)}{ab^3}-\frac{1}{a(a^2+b^2)(a+b\sinh(c+dx))}+\frac{-b^2-a\sinh(c+dx)b}{b^2(a^2+b^2)(\sinh^2(c+dx)b^2+b^2)}\right)d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2\left(-\frac{\arctan(\sinh(c+dx))}{b(a^2+b^2)}-\frac{a\log(b^2\sinh^2(c+dx)+b^2)}{2b^2(a^2+b^2)}-\frac{\log(a+b\sinh(c+dx))}{a(a^2+b^2)}+\frac{\log(b\sinh(c+dx))}{ab^2}\right)}{d}$$

input
```
Int[(Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(b^2*(-(ArcTan[Sinh[c + d*x]]/(b*(a^2 + b^2))) + Log[b*Sinh[c + d*x]]/(a*b
^2) - Log[a + b*Sinh[c + d*x]]/(a*(a^2 + b^2)) - (a*Log[b^2 + b^2*Sinh[c +
 d*x]^2])/(2*b^2*(a^2 + b^2))))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{-a\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)-2b\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^{2}+b^{2}}-\frac{b^{2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{\left(a^{2}+b^{2}\right)a}}{d}$
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{-a\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)-2b\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^{2}+b^{2}}-\frac{b^{2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{\left(a^{2}+b^{2}\right)a}}{d}$
risch	$\frac{2d^{2}ax}{a^{2}d^{2}+b^{2}d^{2}}+\frac{2dac}{a^{2}d^{2}+b^{2}d^{2}}-\frac{2x}{a}-\frac{2c}{da}+\frac{2b^{2}x}{a(a^{2}+b^{2})}+\frac{2b^{2}c}{da(a^{2}+b^{2})}+\frac{i\ln(e^{dx+c}-i)b}{(a^{2}+b^{2})d}-\frac{\ln(e^{dx+c}-i)a}{(a^{2}+b^{2})d}-\frac{i\ln}{(}$

input
```
int(csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a*ln(tanh(1/2*d*x+1/2*c))+1/(a^2+b^2)*(-a*ln(1+tanh(1/2*d*x+1/2*c)^
2)-2*b*arctan(tanh(1/2*d*x+1/2*c)))-b^2/(a^2+b^2)/a*ln(tanh(1/2*d*x+1/2*c)
^2*a-2*b*tanh(1/2*d*x+1/2*c)-a))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.49

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{2\,ab\arctan\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)+b^{2}\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)+a^{2}\log\left(\frac{2\,\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right.}{(a^{3}+ab^{2})d}$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(2*a*b*arctan(cosh(d*x + c) + sinh(d*x + c)) + b^2*log(2*(b*sinh(d*x + c)
 + a)/(cosh(d*x + c) - sinh(d*x + c))) + a^2*log(2*cosh(d*x + c)/(cosh(d*x
 + c) - sinh(d*x + c))) - (a^2 + b^2)*log(2*sinh(d*x + c)/(cosh(d*x + c) -
 sinh(d*x + c))))/((a^3 + a*b^2)*d)
```

Sympy [F]

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\operatorname{csch}\left(c+dx\right)\operatorname{sech}\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(csch(c + d*x)*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.53

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b^2\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^3+ab^2)d} + \frac{2\,b\arctan\left(e^{(-dx-c)}\right)}{(a^2+b^2)d} - \frac{a\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^2+b^2)d} + \frac{\log\left(e^{(-dx-c)}+1\right)}{ad} + \frac{\log\left(e^{(-dx-c)}-1\right)}{ad}$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-b^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^3 + a*b^2)*d) + 2
*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/((a^
2 + b^2)*d) + log(e^(-d*x - c) + 1)/(a*d) + log(e^(-d*x - c) - 1)/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.63

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{\frac{2\,b^3\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^3b+ab^3}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)b}{a^2+b^2}+\frac{a\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^2+b^2}-\frac{2\,\log\left(\left|e^{(dx-\cdots}\right.\right)}{\cdots}}{2\,d}$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(2*b^3*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^3*b + a*b^3)
 + (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*b/(a^2 + b^2) +
 a*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^2 + b^2) - 2*log(abs(e^(d*x
+ c) - e^(-d*x - c)))/a)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(cosh(c + d*x)*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.50

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-2atan\left(e^{dx+c}\right)ab-\log\left(e^{2dx+2c}+1\right)a^2+\log\left(e^{dx+c}-1\right)a^2+\log\left(e^{dx+c}-1\right)b^2+\log\left(e^{dx+c}+1\right)a^2+\mathrm{lo}}{ad\left(a^2+b^2\right)}$$

input
```
int(csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*atan(e**(c + d*x))*a*b - log(e**(2*c + 2*d*x) + 1)*a**2 + log(e**(c
+ d*x) - 1)*a**2 + log(e**(c + d*x) - 1)*b**2 + log(e**(c + d*x) + 1)*a**2
 + log(e**(c + d*x) + 1)*b**2 - log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a
- b)*b**2)/(a*d*(a**2 + b**2))
```

3.439 $\int \frac{\text{csch}(c+dx)\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{\text{csch}(c+dx)\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\text{csch}(c+dx)\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 18.62 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\text{csch}(c+dx)\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\text{csch}(c+dx)\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Csch[c + d*x]*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
x]
```

output
```
Integrate[(Csch[c + d*x]*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
 x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

$$\Big\downarrow \text{6125}$$

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Csch[c + d*x]*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)\operatorname{sech}\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
fricas")
```

output
```
integral(csch(d*x + c)*sech(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x
 + c)), x)
```

Sympy [N/A]

Not integrable

Time = 91.86 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}\left(c+dx\right)\operatorname{sech}\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(csch(c + d*x)*sech(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)\operatorname{sech}\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
maxima")
```

output
```
integrate(csch(d*x + c)*sech(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x
)
```

Giac [N/A]

Not integrable

Time = 9.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
giac")
```

output
```
integrate(csch(d*x + c)*sech(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x
)
```

Mupad [N/A]

Not integrable

Time = 1.67 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{1}{\cosh(c+d\,x)\,\sinh(c+d\,x)\,(e+f\,x)\,(a+b\sinh(c+d\,x))}\,dx$$

input
```
int(1/(cosh(c + d*x)*sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(cosh(c + d*x)*sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.38

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$=\int \frac{\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)}{\sinh(dx+c)\,be+\sinh(dx+c)\,bfx+ae+afx}dx$$

input
```
int(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int((csch(c + d*x)*sech(c + d*x))/(sinh(c + d*x)*b*e + sinh(c + d*x)*b*f*x
 + a*e + a*f*x),x)
```

3.440 $\int \frac{(e+fx)^3 \operatorname{csch}(c+dx) \operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 1164

$$\int \frac{(e+fx)^3 \operatorname{csch}(c+dx) \operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output
```
3*b*f^2*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)/d^3+3*b*f*(f*x+e)^2*l
n(1+exp(2*d*x+2*c))/(a^2+b^2)/d^2-6*I*f^2*(f*x+e)*polylog(2,I*exp(d*x+c))/
a/d^3+6*b^2*f*(f*x+e)^2*arctan(exp(d*x+c))/a/(a^2+b^2)/d^2-6*I*b^2*f^3*pol
ylog(3,I*exp(d*x+c))/a/(a^2+b^2)/d^4-3*b^3*f*(f*x+e)^2*polylog(2,-b*exp(d*
x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d^2-6*b^3*f^2*(f*x+e)*polylog(
3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d^3+6*b^3*f^2*(f*x+
e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d^3+3*b^
3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/
2)/d^2-2*(f*x+e)^3*arctanh(exp(d*x+c))/a/d-6*f^3*polylog(4,-exp(d*x+c))/a/
d^4+6*f^3*polylog(4,exp(d*x+c))/a/d^4-b*(f*x+e)^3/(a^2+b^2)/d+b^3*(f*x+e)^
3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d-b^3*(f*x+e)^3
*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d+6*b^3*f^3*poly
log(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d^4-6*b^3*f^3*p
olylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d^4-3/2*b*f^
3*polylog(3,-exp(2*d*x+2*c))/(a^2+b^2)/d^4-6*I*f^3*polylog(3,-I*exp(d*x+c)
)/a/d^4+6*I*b^2*f^2*(f*x+e)*polylog(2,I*exp(d*x+c))/a/(a^2+b^2)/d^3+6*I*b^
2*f^3*polylog(3,-I*exp(d*x+c))/a/(a^2+b^2)/d^4+6*I*f^2*(f*x+e)*polylog(2,-
I*exp(d*x+c))/a/d^3-6*f*(f*x+e)^2*arctan(exp(d*x+c))/a/d^2+(f*x+e)^3*sech(
d*x+c)/a/d-6*I*b^2*f^2*(f*x+e)*polylog(2,-I*exp(d*x+c))/a/(a^2+b^2)/d^3-3*
f*(f*x+e)^2*polylog(2,-exp(d*x+c))/a/d^2+3*f*(f*x+e)^2*polylog(2,exp(d*...
```

Mathematica [A] (warning: unable to verify)

Time = 8.93 (sec) , antiderivative size = 1441, normalized size of antiderivative = 1.24

$$\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
4*(-1/8*(f*Csch[c + d*x]*(12*b*d^3*e^2*E^(2*c)*x - 12*b*d^3*e^2*(1 + E^(2*
c))*x - 12*b*d^3*e*f*x^2 - 4*b*d^3*f^2*x^3 + 12*a*d^2*e^2*(1 + E^(2*c))*Ar
cTan[E^(c + d*x)] + 6*b*d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d
*x))]) + (12*I)*a*d*e*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1
 + I*E^(c + d*x)]) - PolyLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*
x)]) + 6*b*d*e*(1 + E^(2*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - P
olyLog[2, -E^(2*(c + d*x))]) + (6*I)*a*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 -
I*E^(c + d*x)] - d^2*x^2*Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^
(c + d*x)] + 2*d*x*PolyLog[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*
x)] - 2*PolyLog[3, I*E^(c + d*x)]) + b*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x
 - 3*Log[1 + E^(2*(c + d*x))]) - 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*Po
lyLog[3, -E^(2*(c + d*x))]))*(a + b*Sinh[c + d*x]))/((a^2 + b^2)*d^4*(1 +
E^(2*c))*(b + a*Csch[c + d*x])) + (Csch[c + d*x]*((e + f*x)^3*Log[1 - E^(c
 + d*x)] - (e + f*x)^3*Log[1 + E^(c + d*x)] - (3*f*(d^2*(e + f*x)^2*PolyLo
g[2, -E^(c + d*x)] - 2*d*f*(e + f*x)*PolyLog[3, -E^(c + d*x)] + 2*f^2*Poly
Log[4, -E^(c + d*x)]))/d^3 + (3*f*(d^2*(e + f*x)^2*PolyLog[2, E^(c + d*x)]
 - 2*d*f*(e + f*x)*PolyLog[3, E^(c + d*x)] + 2*f^2*PolyLog[4, E^(c + d*x)]
))/d^3)*(a + b*Sinh[c + d*x]))/(4*a*d*(b + a*Csch[c + d*x])) - (b^3*Csch[c
 + d*x]*(-2*d^3*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 3*d^3*e
^2*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 3*d^3*e*f^2*x^2...
```

Rubi [A] (verified)

Time = 5.56 (sec) , antiderivative size = 991, normalized size of antiderivative = 0.85, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {6123, 5985, 25, 6107, 3042, 3803, 25, 2694, 27, 2620, 3011, 7163, 2720, 7143, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)^3\text{csch}(c+dx)\text{sech}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5985}$$

$$\frac{-3f\int -(e+fx)^2\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{3f\int (e+fx)^2\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{6107}$$

$$\frac{3f\int (e+fx)^2\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{b^2\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3f\int (e+fx)^2\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \text{3803}$$

$$\frac{3f\int (e+fx)^2\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{2b^2\int -\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{\int (e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{3f\int (e+fx)^2\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\int \frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \ 2694$$

$$\frac{3f\int (e+fx)^2\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \ 27$$

$$\frac{3f\int (e+fx)^2\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \ 2620$$

$$\frac{3f\int (e+fx)^2\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log(\cdots}{\cdots}\right.}{\cdots}\right)}{a^2+b^2}\right)}{a}$$

$\downarrow$ 3011

$$\frac{3f\int (e+fx)^2\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\text{sech}(c+dx)}{d}}{a}-$$

$$b\left(\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\text{P}}{}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}}{}\right.$$

$\downarrow$ 7163

$$\frac{3f\int(e+fx)^2\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}}{a}-$$

$$b\left(\frac{\int(e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int\operatorname{PolyLog}(\cdots}{\cdots}\right)}{d}\cdots\right)}{\cdots}\right)}{2\sqrt{a^2+b^2}}\cdots}{\cdots}\right)$$

↓ 2720

$$\frac{3f\int(e+fx)^2\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}}{a}-$$

$$b\left(\frac{\int(e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\ldots}{\ldots}\right.}{d}\ldots}{\ldots}\right)}{2\sqrt{a^2+b^2}}\ldots}{\ldots}\right.$$

↓ 7143

$$\frac{3f\int(e+fx)^2\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}}{a}-$$

$$b\left(\frac{\int(e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}(4\ldots}{\ldots}\right)}{d}\ldots\right)}{b\ldots}\right)}{2\sqrt{a^2+b^2}\ldots}\ldots\right)}{\ldots}\right)$$

↓ 7292

$$\frac{3f\int\frac{(e+fx)^2(\operatorname{arctanh}(\cosh(c+dx))-\operatorname{sech}(c+dx))}{d}dx-\frac{(e+fx)^3\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}}{b\left(\frac{\int(e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{a\left(b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4\right.}{}\right.}{d}\right.}{b}\right.\right.}{2\sqrt{a^2+b^2}}\right.}{}\right.}-$$

↓ 27

$$\frac{3f\int(e+fx)^2(\text{arctanh}(\cosh(c+dx))-\text{sech}(c+dx))dx}{d}-\frac{(e+fx)^3\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\text{sech}(c+dx)}{d}-$$

$$b\left(\frac{\int(e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(a\frac{\left(b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\text{PolyLog}\left(4\right.}{}\right.}{d}\right.}{b}\right.}{2\sqrt{a^2+b^2}}\right.}{}\right.$$

$\downarrow$ 7293

$$\frac{\frac{3f\int\left((e+fx)^2\operatorname{arctanh}(\cosh(c+dx))-(e+fx)^2\operatorname{sech}(c+dx)\right)dx}{d}-\frac{(e+fx)^3\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}}{a}-$$

$$b\left(\frac{\int\left(a(e+fx)^3\operatorname{sech}^2(c+dx)-b(e+fx)^3\operatorname{sech}(c+dx)\tanh(c+dx)\right)dx}{a^2+b^2}-\frac{2b^2\left(b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{b}{a+\cdots}\right)}{d}\cdots\right)}{\cdots}\right)}{\cdots}\right)\cdots\right)}{\cdots}\cdots\right)$$

↓ 2009

$$-\frac{\operatorname{arctanh}(\cosh(c+dx))(e+fx)^3}{d}+\frac{\operatorname{sech}(c+dx)(e+fx)^3}{d}+\frac{3f\left(-\frac{2\operatorname{arctanh}\left(e^{c+dx}\right)(e+fx)^3}{3f}+\frac{\operatorname{arctanh}(\cosh(c+dx))(e+fx)^3}{3f}-\frac{2\arctan\left(e^{c+\ldots}\right.}{\ldots}\right.}{\ldots}$$

$$b\left(-\frac{6ib\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)f^3}{d^4}+\frac{6ib\operatorname{PolyLog}\left(3,ie^{c+dx}\right)f^3}{d^4}+\frac{3a\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)f^3}{2d^4}+\frac{6ib(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)f^2}{d^3}-\frac{6ib(e+fx)\operatorname{PolyLog}\left(2,ie^{c\ldots}\right.}{d^3}\right.$$

input
```
Int[((e + f*x)^3*Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-(((e + f*x)^3*ArcTanh[Cosh[c + d*x]])/d) + (3*f*((-2*(e + f*x)^2*ArcTan[
E^(c + d*x)])/d - (2*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(3*f) + ((e + f*x)^
3*ArcTanh[Cosh[c + d*x]])/(3*f) - ((e + f*x)^2*PolyLog[2, -E^(c + d*x)])/d
 + ((2*I)*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d^2 - ((2*I)*f*(e + f*
x)*PolyLog[2, I*E^(c + d*x)])/d^2 + ((e + f*x)^2*PolyLog[2, E^(c + d*x)])/
d + (2*f*(e + f*x)*PolyLog[3, -E^(c + d*x)])/d^2 - ((2*I)*f^2*PolyLog[3, (
-I)*E^(c + d*x)])/d^3 + ((2*I)*f^2*PolyLog[3, I*E^(c + d*x)])/d^3 - (2*f*(
e + f*x)*PolyLog[3, E^(c + d*x)])/d^2 - (2*f^2*PolyLog[4, -E^(c + d*x)])/d
^3 + (2*f^2*PolyLog[4, E^(c + d*x)])/d^3))/d + ((e + f*x)^3*Sech[c + d*x])
/d)/a - (b*((-2*b^2*(-1/2*(b*(((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x
))/(a - Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[
a^2 + b^2]))])/d^2))/d))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^3*Log[1
+ (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*Pol
yLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*P
olyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/(2*Sqrt[a^2 + b^
2])))/(a^2 + b^2) + ((a*(e + f*x)^3)/d - (6*b*f*(e + f*x)^2*ArcTan[E^(c +
d*x)])/d^2 - (3*a*f*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/d^2 + ((6*I)*...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^3 \operatorname{csch}(dx+c) \operatorname{sech}(dx+c)^2}{a+b\sinh(dx+c)} dx$$

input
```
int((f*x+e)^3*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 9707 vs. 2(1064) = 2128.

Time = 0.35 (sec) , antiderivative size = 9707, normalized size of antiderivative = 8.34

$$\int \frac{(e+fx)^3 \operatorname{csch}(c+dx) \operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \operatorname{csch}(c+dx) \operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*csch(d*x+c)*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\,\text{csch}\,(dx+c)\,\text{sech}\,(dx+c)^2}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-3*b*e^2*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2
 + b^2)*d^2)) - 6*a*f^3*integrate(x^2*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) +
 b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 6*b*f^3*integrate(x^2/(a^2*d
*e^(2*d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 12*a*e*f^2
*integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) +
a^2*d + b^2*d), x) - 12*b*e*f^2*integrate(x/(a^2*d*e^(2*d*x + 2*c) + b^2*d
*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - (b^3*log((b*e^(-d*x - c) - a - sqr
t(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^3 + a*b^2)*sqrt(
a^2 + b^2)*d) - 2*(a*e^(-d*x - c) - b)/((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x
 - 2*c))*d) + log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d))*e
^3 - 6*a*e^2*f*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) - 3*(d*x*log(e^(d*x +
 c) + 1) + dilog(-e^(d*x + c)))*e^2*f/(a*d^2) + 3*(d*x*log(-e^(d*x + c) +
1) + dilog(e^(d*x + c)))*e^2*f/(a*d^2) + 2*(b*f^3*x^3 + 3*b*e*f^2*x^2 + 3*
b*e^2*f*x + (a*f^3*x^3*e^c + 3*a*e*f^2*x^2*e^c + 3*a*e^2*f*x*e^c)*e^(d*x))
/(a^2*d + b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)) - 3*(d^2*x^2*
log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x +
c)))*e*f^2/(a*d^3) + 3*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x
 + c)) - 2*polylog(3, e^(d*x + c)))*e*f^2/(a*d^3) - (d^3*x^3*log(e^(d*x +
c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) +
 6*polylog(4, -e^(d*x + c)))*f^3/(a*d^4) + (d^3*x^3*log(-e^(d*x + c) + ...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^3}{\cosh(c+d\,x)^2\sinh(c+d\,x)\,\left(a+b\sinh(c+d\,x)\right)}\,dx$$

input
```
int((e + f*x)^3/(cosh(c + d*x)^2*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^3/(cosh(c + d*x)^2*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3 \operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^3*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqr
t(a**2 + b**2))*b**3*e**3*i - 2*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i +
 a*i)/sqrt(a**2 + b**2))*b**3*e**3*i + 16*e**(6*c + 2*d*x)*int((e**(4*d*x)
*x**3)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a + 2*e**(5*c + 5*d*x)*a -
 2*e**(4*c + 4*d*x)*b - 2*e**(3*c + 3*d*x)*a - 2*e**(c + d*x)*a + b),x)*a*
*5*d*f**3 + 32*e**(6*c + 2*d*x)*int((e**(4*d*x)*x**3)/(e**(8*c + 8*d*x)*b
+ 2*e**(7*c + 7*d*x)*a + 2*e**(5*c + 5*d*x)*a - 2*e**(4*c + 4*d*x)*b - 2*e
**(3*c + 3*d*x)*a - 2*e**(c + d*x)*a + b),x)*a**3*b**2*d*f**3 + 16*e**(6*c
 + 2*d*x)*int((e**(4*d*x)*x**3)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a
 + 2*e**(5*c + 5*d*x)*a - 2*e**(4*c + 4*d*x)*b - 2*e**(3*c + 3*d*x)*a - 2*
e**(c + d*x)*a + b),x)*a*b**4*d*f**3 + 48*e**(6*c + 2*d*x)*int((e**(4*d*x)
*x**2)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a + 2*e**(5*c + 5*d*x)*a -
 2*e**(4*c + 4*d*x)*b - 2*e**(3*c + 3*d*x)*a - 2*e**(c + d*x)*a + b),x)*a*
*5*d*e*f**2 + 96*e**(6*c + 2*d*x)*int((e**(4*d*x)*x**2)/(e**(8*c + 8*d*x)*
b + 2*e**(7*c + 7*d*x)*a + 2*e**(5*c + 5*d*x)*a - 2*e**(4*c + 4*d*x)*b - 2
*e**(3*c + 3*d*x)*a - 2*e**(c + d*x)*a + b),x)*a**3*b**2*d*e*f**2 + 48*e**
(6*c + 2*d*x)*int((e**(4*d*x)*x**2)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*
x)*a + 2*e**(5*c + 5*d*x)*a - 2*e**(4*c + 4*d*x)*b - 2*e**(3*c + 3*d*x)*a
- 2*e**(c + d*x)*a + b),x)*a*b**4*d*e*f**2 + 48*e**(6*c + 2*d*x)*int((e**(
4*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a + 2*e**(5*c + 5*d*...
```

3.441 $\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 795

$$\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= -\frac{b(e+fx)^2}{(a^2+b^2)\,d} - \frac{4f(e+fx)\arctan\left(e^{c+dx}\right)}{ad^2} + \frac{4b^2f(e+fx)\arctan\left(e^{c+dx}\right)}{a\,(a^2+b^2)\,d^2} - \frac{2(e+fx)^2\operatorname{arctanh}\left(e^{c+dx}\right)}{ad} - \frac{b^3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^{3/2}\,d} + \frac{b^3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^{3/2}\,d} + \frac{2bf(e+fx)\log\left(1+e^{2(c+dx)}\right)}{(a^2+b^2)\,d^2} - \frac{2f(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2} + \frac{2if^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3} - \frac{2ib^2f^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{a\,(a^2+b^2)\,d^3} - \frac{2if^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{ad^3} + \frac{2ib^2f^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{a\,(a^2+b^2)\,d^3} + \frac{2f(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{ad^2} - \frac{2b^3f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^{3/2}\,d^2} + \frac{2b^3f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^{3/2}\,d^2} + \frac{bf^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{(a^2+b^2)\,d^3} + \frac{2f^2\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3} - \frac{2f^2\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{ad^3} + \frac{2b^3f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^{3/2}\,d^3} - \frac{2b^3f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^{3/2}\,d^3} + \frac{(e+fx)^2\operatorname{sech}(c+dx)}{ad} - \frac{b^2(e+fx)^2\operatorname{sech}(c+dx)}{a\,(a^2+b^2)\,d} - \frac{b(e+fx)^2\tanh(c+dx)}{(a^2+b^2)\,d}$$

output
```
-b*(f*x+e)^2/(a^2+b^2)/d-4*f*(f*x+e)*arctan(exp(d*x+c))/a/d^2+4*b^2*f*(f*x
+e)*arctan(exp(d*x+c))/a/(a^2+b^2)/d^2-2*(f*x+e)^2*arctanh(exp(d*x+c))/a/d
-b^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d+
b^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d+2
*b*f*(f*x+e)*ln(1+exp(2*d*x+2*c))/(a^2+b^2)/d^2-2*f*(f*x+e)*polylog(2,-exp
(d*x+c))/a/d^2-2*I*b^2*f^2*polylog(2,-I*exp(d*x+c))/a/(a^2+b^2)/d^3+2*I*b^
2*f^2*polylog(2,I*exp(d*x+c))/a/(a^2+b^2)/d^3+2*I*f^2*polylog(2,-I*exp(d*x
+c))/a/d^3-2*I*f^2*polylog(2,I*exp(d*x+c))/a/d^3+2*f*(f*x+e)*polylog(2,exp
(d*x+c))/a/d^2-2*b^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2))
)/a/(a^2+b^2)^(3/2)/d^2+2*b^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/a/(a^2+b^2)^(3/2)/d^2+b*f^2*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2
)/d^3+2*f^2*polylog(3,-exp(d*x+c))/a/d^3-2*f^2*polylog(3,exp(d*x+c))/a/d^3
+2*b^3*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/
d^3-2*b^3*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/
2)/d^3+(f*x+e)^2*sech(d*x+c)/a/d-b^2*(f*x+e)^2*sech(d*x+c)/a/(a^2+b^2)/d-b
*(f*x+e)^2*tanh(d*x+c)/(a^2+b^2)/d
```

Mathematica [A] (warning: unable to verify)

Time = 8.08 (sec) , antiderivative size = 928, normalized size of antiderivative = 1.17

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
4*(-1/4*(f*Csch[c + d*x]*(4*b*d^2*e*E^(2*c)*x - 4*b*d^2*e*(1 + E^(2*c))*x
+ 2*b*d^2*E^(2*c)*f*x^2 - 2*b*d^2*(1 + E^(2*c))*f*x^2 + 4*a*d*e*(1 + E^(2*
c))*ArcTan[E^(c + d*x)] + 2*b*d*e*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c +
 d*x))]) + (2*I)*a*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 +
I*E^(c + d*x)]) - PolyLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]
) + b*(1 + E^(2*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2,
 -E^(2*(c + d*x))]))*(a + b*Sinh[c + d*x]))/((a^2 + b^2)*d^3*(1 + E^(2*c))
*(b + a*Csch[c + d*x])) + (Csch[c + d*x]*((e + f*x)^2*Log[1 - E^(c + d*x)]
 - (e + f*x)^2*Log[1 + E^(c + d*x)] - (2*f*(d*(e + f*x)*PolyLog[2, -E^(c +
 d*x)] - f*PolyLog[3, -E^(c + d*x)]))/d^2 + (2*f*(d*(e + f*x)*PolyLog[2, E
^(c + d*x)] - f*PolyLog[3, E^(c + d*x)]))/d^2)*(a + b*Sinh[c + d*x]))/(4*a
*d*(b + a*Csch[c + d*x])) - (b^3*Csch[c + d*x]*(-2*d^2*e^2*ArcTanh[(a + b*
E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
])] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - d^2*f^2
*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*f*(e + f*x)*Poly
Log[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*f*(e + f*x)*PolyLog[2
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2*f^2*PolyLog[3, (b*E^(c + d
*x))/(-a + Sqrt[a^2 + b^2])] + 2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2]))])*(a + b*Sinh[c + d*x]))/(4*a*(a^2 + b^2)^(3/2)*d^3*(b +...
```

Rubi [A] (verified)

Time = 3.93 (sec) , antiderivative size = 695, normalized size of antiderivative = 0.87, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6123, 5985, 25, 6107, 3042, 3803, 25, 2694, 27, 2620, 3011, 2720, 7143, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5985}$$

$$\frac{-2f\int -\left((e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{2f\int (e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{6107}$$

$$\frac{2f\int (e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{b^2\int\frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2f\int (e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \text{3803}$$

$$\frac{2f\int (e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{2b^2\int -\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{2f\int (e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\int \frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \text{ 2694}$$

$$\frac{2f\int (e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2f\int (e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{2f\int (e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\cdots\right.}{bd}\cdots\right.}{\cdots}\cdots\right.}{a^2+b^2}\cdots\right.}{a}\cdots$$

$\downarrow$ 3011

$$\frac{2f\int (e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-$$

$$\frac{b\left(\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,\right.}{d}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{}\right.}{a}$$

$\downarrow$ 2720

$$\frac{2f\int(e+fx)\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}}{a}-$$

$$b\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+f\cdots}{\cdots}\right.}{bd}\cdots\right.}{2\sqrt{a^2+b^2}}\cdots\right.}{\cdots}\cdots\right.$$

$$\downarrow 7143$$

$$\frac{2f\int(e+fx)\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}}{a}-$$

$$b\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{\cdots}{a\cdots}\right.}{d}\cdots\right.}{bd}\cdots\right.}{2\sqrt{a^2+b^2}}\cdots\right.}{\cdots}\cdots\right.$$

$$a$$

$\downarrow$ 7292

$$\frac{2f\int\frac{(e+fx)(\text{arctanh}(\cosh(c+dx))-\text{sech}(c+dx))}{d}dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)\ldots}{2\sqrt{a^2+b^2}}\ldots}{\ldots}\right)}{a\ldots}$$

$\downarrow$ 27

$$\frac{\frac{2f\int(e+fx)(\text{arctanh}(\cosh(c+dx))-\text{sech}(c+dx))dx}{d}-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-$$

$$\frac{b\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{}{a}\right.}{d}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{}\right.}{a}$$

$\downarrow$ 7293

$$\frac{\frac{2f\int((e+fx)\text{arctanh}(\cosh(c+dx))-(e+fx)\text{sech}(c+dx))dx}{d}-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-$$

$$\frac{b\left(\frac{\int\left(a(e+fx)^2\text{sech}^2(c+dx)-b(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)\right)dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}\right.}{}\right.}{2\sqrt{a^2+b^2}}\right.}{}\right.}{}$$

$\Big\downarrow$ 2009

$$\frac{2f\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{(e+fx)^2\text{arctanh}\left(e^{c+dx}\right)}{f}+\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{2f}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}+\frac{f\,\text{PolyL}}{}\right.}{d}$$

$$b\left(-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right.}{}}{a^2+b^2}\right.$$

input
```
Int[((e + f*x)^2*Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-(((e + f*x)^2*ArcTanh[Cosh[c + d*x]])/d) + (2*f*((-2*(e + f*x)*ArcTan[E^
(c + d*x)])/d - ((e + f*x)^2*ArcTanh[E^(c + d*x)])/f + ((e + f*x)^2*ArcTan
h[Cosh[c + d*x]])/(2*f) - ((e + f*x)*PolyLog[2, -E^(c + d*x)])/d + (I*f*Po
lyLog[2, (-I)*E^(c + d*x)])/d^2 - (I*f*PolyLog[2, I*E^(c + d*x)])/d^2 + ((
e + f*x)*PolyLog[2, E^(c + d*x)])/d + (f*PolyLog[3, -E^(c + d*x)])/d^2 - (
f*PolyLog[3, E^(c + d*x)])/d^2))/d + ((e + f*x)^2*Sech[c + d*x])/d)/a - (b
*((-2*b^2*(-1/2*(b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]
)/d^2))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/(2*Sqrt[a^2 + b^2])))/(a^2 + b^2) + ((a*
(e + f*x)^2)/d - (4*b*f*(e + f*x)*ArcTan[E^(c + d*x)])/d^2 - (2*a*f*(e + f
*x)*Log[1 + E^(2*(c + d*x))])/d^2 + ((2*I)*b*f^2*PolyLog[2, (-I)*E^(c + d*
x)])/d^3 - ((2*I)*b*f^2*PolyLog[2, I*E^(c + d*x)])/d^3 - (a*f^2*PolyLog[2,
 -E^(2*(c + d*x))])/d^3 + (b*(e + f*x)^2*Sech[c + d*x])/d + (a*(e + f*x)^2
*Tanh[c + d*x])/d)/(a^2 + b^2)))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c) \operatorname{sech}(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5562 vs. 2(729) = 1458.

Time = 0.22 (sec) , antiderivative size = 5562, normalized size of antiderivative = 7.00

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*csch(d*x+c)*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\,\text{csch}\,(dx+c)\,\text{sech}\,(dx+c)^2}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-2*b*e*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2 +
 b^2)*d^2)) - 4*a*f^2*integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2
*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 4*b*f^2*integrate(x/(a^2*d*e^(2*
d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - (b^3*log((b*e^(-
d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((
a^3 + a*b^2)*sqrt(a^2 + b^2)*d) - 2*(a*e^(-d*x - c) - b)/((a^2 + b^2 + (a^
2 + b^2)*e^(-2*d*x - 2*c))*d) + log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x
- c) - 1)/(a*d))*e^2 - 4*a*e*f*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) + 2*(
b*f^2*x^2 + 2*b*e*f*x + (a*f^2*x^2*e^c + 2*a*e*f*x*e^c)*e^(d*x))/(a^2*d +
b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)) - 2*(d*x*log(e^(d*x + c
) + 1) + dilog(-e^(d*x + c)))*e*f/(a*d^2) + 2*(d*x*log(-e^(d*x + c) + 1) +
 dilog(e^(d*x + c)))*e*f/(a*d^2) - (d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*d
ilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*f^2/(a*d^3) + (d^2*x^2*lo
g(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c))
)*f^2/(a*d^3) - integrate(-2*(b^3*f^2*x^2*e^c + 2*b^3*e*f*x*e^c)*e^(d*x)/(
a^3*b + a*b^3 - (a^3*b*e^(2*c) + a*b^3*e^(2*c))*e^(2*d*x) - 2*(a^4*e^c + a
^2*b^2*e^c)*e^(d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^2}{\cosh\left(c+d\,x\right)^2\sinh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^2/(cosh(c + d*x)^2*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^2/(cosh(c + d*x)^2*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqr
t(a**2 + b**2))*b**3*e**2*i - 2*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i +
 a*i)/sqrt(a**2 + b**2))*b**3*e**2*i + 16*e**(6*c + 2*d*x)*int((e**(4*d*x)
*x**2)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a + 2*e**(5*c + 5*d*x)*a -
 2*e**(4*c + 4*d*x)*b - 2*e**(3*c + 3*d*x)*a - 2*e**(c + d*x)*a + b),x)*a*
*5*d*f**2 + 32*e**(6*c + 2*d*x)*int((e**(4*d*x)*x**2)/(e**(8*c + 8*d*x)*b
+ 2*e**(7*c + 7*d*x)*a + 2*e**(5*c + 5*d*x)*a - 2*e**(4*c + 4*d*x)*b - 2*e
**(3*c + 3*d*x)*a - 2*e**(c + d*x)*a + b),x)*a**3*b**2*d*f**2 + 16*e**(6*c
 + 2*d*x)*int((e**(4*d*x)*x**2)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a
 + 2*e**(5*c + 5*d*x)*a - 2*e**(4*c + 4*d*x)*b - 2*e**(3*c + 3*d*x)*a - 2*
e**(c + d*x)*a + b),x)*a*b**4*d*f**2 + 32*e**(6*c + 2*d*x)*int((e**(4*d*x)
*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a + 2*e**(5*c + 5*d*x)*a - 2*
e**(4*c + 4*d*x)*b - 2*e**(3*c + 3*d*x)*a - 2*e**(c + d*x)*a + b),x)*a**5*
d*e*f + 64*e**(6*c + 2*d*x)*int((e**(4*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**
(7*c + 7*d*x)*a + 2*e**(5*c + 5*d*x)*a - 2*e**(4*c + 4*d*x)*b - 2*e**(3*c
+ 3*d*x)*a - 2*e**(c + d*x)*a + b),x)*a**3*b**2*d*e*f + 32*e**(6*c + 2*d*x
)*int((e**(4*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a + 2*e**(5*
c + 5*d*x)*a - 2*e**(4*c + 4*d*x)*b - 2*e**(3*c + 3*d*x)*a - 2*e**(c + d*x
)*a + b),x)*a*b**4*d*e*f + e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a**4*e**
2 + 2*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a**2*b**2*e**2 + e**(2*c +...
```

3.442 $\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 442

$$\begin{aligned}
&\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= -\frac{f\arctan(\sinh(c+dx))}{ad^2}+\frac{b^2f\arctan(\sinh(c+dx))}{a(a^2+b^2)d^2}-\frac{2fx\text{arctanh}(e^{c+dx})}{ad} \\
&\quad+\frac{fx\text{arctanh}(\cosh(c+dx))}{ad}-\frac{(e+fx)\text{arctanh}(\cosh(c+dx))}{ad} \\
&\quad-\frac{b^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^{3/2}d}+\frac{b^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^{3/2}d} \\
&\quad+\frac{bf\log(\cosh(c+dx))}{(a^2+b^2)d^2}-\frac{f\,\text{PolyLog}(2,-e^{c+dx})}{ad^2}+\frac{f\,\text{PolyLog}(2,e^{c+dx})}{ad^2} \\
&\quad-\frac{b^3f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^{3/2}d^2}+\frac{b^3f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^{3/2}d^2} \\
&\quad+\frac{(e+fx)\text{sech}(c+dx)}{ad}-\frac{b^2(e+fx)\text{sech}(c+dx)}{a(a^2+b^2)d}-\frac{b(e+fx)\tanh(c+dx)}{(a^2+b^2)d}
\end{aligned}$$

output
```
-f*arctan(sinh(d*x+c))/a/d^2+b^2*f*arctan(sinh(d*x+c))/a/(a^2+b^2)/d^2-2*f
*x*arctanh(exp(d*x+c))/a/d+f*x*arctanh(cosh(d*x+c))/a/d-(f*x+e)*arctanh(co
sh(d*x+c))/a/d-b^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b
^2)^(3/2)/d+b^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)
^(3/2)/d+b*f*ln(cosh(d*x+c))/(a^2+b^2)/d^2-f*polylog(2,-exp(d*x+c))/a/d^2+
f*polylog(2,exp(d*x+c))/a/d^2-b^3*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(
1/2)))/a/(a^2+b^2)^(3/2)/d^2+b^3*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1
/2)))/a/(a^2+b^2)^(3/2)/d^2+(f*x+e)*sech(d*x+c)/a/d-b^2*(f*x+e)*sech(d*x+c
)/a/(a^2+b^2)/d-b*(f*x+e)*tanh(d*x+c)/(a^2+b^2)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.45 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.97

$$\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{\operatorname{csch}(c+dx)(a+b\sinh(c+dx))\left(-\frac{2f\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a-ib}-\frac{2f\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a+ib}-\frac{if\log(\cosh(c+dx))}{a-ib}+\frac{if\log\right.}{}$$

input
```
Integrate[((e + f*x)*Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),
x]
```

output
```
(Csch[c + d*x]*(a + b*Sinh[c + d*x])*((-2*f*ArcTan[Tanh[(c + d*x)/2]])/(a
- I*b) - (2*f*ArcTan[Tanh[(c + d*x)/2]])/(a + I*b) - (I*f*Log[Cosh[c + d*x
]])/(a - I*b) + (I*f*Log[Cosh[c + d*x]])/(a + I*b) + (2*(d*(e + f*x)*(Log[
1 - E^(c + d*x)] - Log[1 + E^(c + d*x)]) - f*PolyLog[2, -E^(c + d*x)] + f*
PolyLog[2, E^(c + d*x)]))/a - (2*b^3*(-2*d*e*ArcTanh[(a + b*E^(c + d*x))/S
qrt[a^2 + b^2]] + 2*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + f*(
c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[
1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + f*PolyLog[2, (b*E^(c + d*x))/
(-a + Sqrt[a^2 + b^2])] - f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2]))]))/(a*(a^2 + b^2)^(3/2)) + (2*d*(e + f*x)*Sech[c + d*x]*(a - b*Sinh[
c + d*x]))/(a^2 + b^2)))/(2*d^2*(b + a*Csch[c + d*x]))
```

Rubi [A] (verified)

Time = 2.45 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.88, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6123, 5985, 2009, 6107, 3042, 3803, 25, 2694, 27, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5985}$$

$$\frac{-f\int \left(\frac{\text{sech}(c+dx)}{d}-\frac{\text{arctanh}(\cosh(c+dx))}{d}\right)dx-\frac{(e+fx)\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)\text{sech}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}(e^{c+dx})}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{(e+fx)\text{a}}{}}{a}$$
$$-\frac{b\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{6107}$$

$$\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}(e^{c+dx})}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{(e+fx)\text{a}}{}}{a}$$
$$\frac{b\left(\frac{b^2\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{a}}{}}{a}$$

$$\frac{b\left(\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \textbf{3803}$$

$$\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{a}}{}}{a}$$

$$\frac{b\left(\frac{2b^2\int -\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{a}}{}}{a}$$

$$\frac{b\left(\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\int \frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \textbf{2694}$$

$$\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{a}}{}}{a}$$

$$\frac{b\left(\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{a}}{}}{a}$$

$$\frac{b\left(\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{a}$$

$\Big\downarrow$ 2620

$$\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{a}}{}}{a}$$

$$\frac{b\left(\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+\cdots\right.}{bd}\right.}{2\sqrt{\cdots}}\right.}{a^2+b^2}\right.}{a}$$

$\Big\downarrow$ 2715

$$\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{a}}{}}{a}$$

$$\frac{b\left(\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\cdots\right.}{}\right.}{}\right.}{a^2+b^2}\right.}{a}$$

$\Big\downarrow$ 2838

$$\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{a}}{}}{a}$$

$$\frac{b\left(\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+}}\right)}{bd^2}\right.}{}\right.}{a^2+b^2}\right.}{a}$$

$\Big\downarrow$ 7293

$$\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{a}}{}}{a}$$

$$\frac{b\left(\frac{\int \left(a(e+fx)\text{sech}^2(c+dx)-b(e+fx)\text{sech}(c+dx)\tanh(c+dx)\right)dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\right.}{}\right.}{a^2+b^2}\right.}{a}$$

$\Big\downarrow$ 2009

$$\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{a}}{}}{a}$$

$$\frac{b\left(\frac{-\frac{af\log(\cosh(c+dx))}{d^2}+\frac{a(e+fx)\tanh(c+dx)}{d}-\frac{bf\arctan(\sinh(c+dx))}{d^2}+\frac{b(e+fx)\text{sech}(c+dx)}{d}}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be}{\sqrt{a}}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{}\right.}{a}$$

input
```
Int[((e + f*x)*Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-(((e + f*x)*ArcTanh[Cosh[c + d*x]])/d) - f*(ArcTan[Sinh[c + d*x]]/d^2 +
(2*x*ArcTanh[E^(c + d*x)])/d - (x*ArcTanh[Cosh[c + d*x]])/d + PolyLog[2, -
E^(c + d*x)]/d^2 - PolyLog[2, E^(c + d*x)]/d^2) + ((e + f*x)*Sech[c + d*x]
)/d)/a - (b*((-2*b^2*(-1/2*(b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2]))])/(b*d^2)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[
a^2 + b^2]))])/(b*d^2)))/(2*Sqrt[a^2 + b^2])))/(a^2 + b^2) + (-((b*f*ArcTa
n[Sinh[c + d*x]])/d^2) - (a*f*Log[Cosh[c + d*x]])/d^2 + (b*(e + f*x)*Sech[
c + d*x])/d + (a*(e + f*x)*Tanh[c + d*x])/d)/(a^2 + b^2)))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1814 vs. 2(419) = 838.

Time = 8.78 (sec) , antiderivative size = 1815, normalized size of antiderivative = 4.11

method	result	size
risch	Expression too large to display	1815

input
```
int((f*x+e)*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
b/d*e/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a-
1/(a^2+b^2)^(5/2)/d*b^5*f/a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+
b^2)^(1/2)))*x-b/d^2*c*f/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/
(a^2+b^2)^(1/2))*a-b/(a^2+b^2)^(5/2)/d^2*f*arctanh(1/2*(2*b*exp(d*x+c)+2*a
)/(a^2+b^2)^(1/2))*a^3-2*b^3/(a^2+b^2)^(5/2)/d^2*f*arctanh(1/2*(2*b*exp(d*
x+c)+2*a)/(a^2+b^2)^(1/2))*a+1/(a^2+b^2)^(5/2)/d^2*a*b^3*f*dilog((b*exp(d*
x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/(a^2+b^2)^(5/2)/d^2*a*b^3*f
*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+4/(a^2+b^2)
/d^2*b^3*f/(4*a^2+4*b^2)*ln(1+exp(2*d*x+2*c))-1/(a^2+b^2)/d*ln(exp(d*x+c)+
1)*a*f*x-1/(a^2+b^2)/d^2*a*f*dilog(exp(d*x+c)+1)+1/(a^2+b^2)/d*a*e*ln(exp(
d*x+c)-1)-1/(a^2+b^2)/d*a*e*ln(exp(d*x+c)+1)-1/(a^2+b^2)/d^2*a*f*dilog(exp
(d*x+c))-2*b/(a^2+b^2)/d^2*f*ln(exp(d*x+c))+1/(a^2+b^2)/d*b^2*e/a*ln(exp(d
*x+c)-1)-1/(a^2+b^2)/d*b^2*e/a*ln(exp(d*x+c)+1)-1/(a^2+b^2)/d^2*c*a*f*ln(e
xp(d*x+c)-1)-8/(a^2+b^2)/d^2*a^3*f/(4*a^2+4*b^2)*arctan(exp(d*x+c))-1/(a^2
+b^2)/d^2*b^2*f*dilog(exp(d*x+c))/a-1/(a^2+b^2)/d^2*b^2*f/a*dilog(exp(d*x+
c)+1)+1/(a^2+b^2)^(5/2)/d*a*b^3*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(
a^2+b^2)^(1/2)))*x+1/(a^2+b^2)^(5/2)/d*b^5*f/a*ln((b*exp(d*x+c)+(a^2+b^2)^
(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/(a^2+b^2)^(5/2)/d*a*b^3*f*ln((-b*exp(d*x
+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/(a^2+b^2)^(5/2)/d^2*f*b^5
*c/a*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/(a^2+b^2)^(3/2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2176 vs. $2(415) = 830$.

Time = 0.19 (sec) , antiderivative size = 2176, normalized size of antiderivative = 4.92

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input

```
integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output

```
-(2*(a^3*b + a*b^3)*d*f*x*cosh(d*x + c)^2 + 2*(a^3*b + a*b^3)*d*f*x*sinh(d
*x + c)^2 - 2*(a^3*b + a*b^3)*d*e + (b^4*f*cosh(d*x + c)^2 + 2*b^4*f*cosh(
d*x + c)*sinh(d*x + c) + b^4*f*sinh(d*x + c)^2 + b^4*f)*sqrt((a^2 + b^2)/b
^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^4*f*cosh(d*x + c)^2 + 2*b^
4*f*cosh(d*x + c)*sinh(d*x + c) + b^4*f*sinh(d*x + c)^2 + b^4*f)*sqrt((a^2
 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^4*d*e - b^4*c*f +
 (b^4*d*e - b^4*c*f)*cosh(d*x + c)^2 + 2*(b^4*d*e - b^4*c*f)*cosh(d*x + c)
*sinh(d*x + c) + (b^4*d*e - b^4*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2
)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) +
2*a) + (b^4*d*e - b^4*c*f + (b^4*d*e - b^4*c*f)*cosh(d*x + c)^2 + 2*(b^4*d
*e - b^4*c*f)*cosh(d*x + c)*sinh(d*x + c) + (b^4*d*e - b^4*c*f)*sinh(d*x +
 c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2
*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^4*d*f*x + b^4*c*f + (b^4*d*f*x + b^4*
c*f)*cosh(d*x + c)^2 + 2*(b^4*d*f*x + b^4*c*f)*cosh(d*x + c)*sinh(d*x + c)
 + (b^4*d*f*x + b^4*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*co
sh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((
a^2 + b^2)/b^2) - b)/b) - (b^4*d*f*x + b^4*c*f + (b^4*d*f*x + b^4*c*f)*cos
h(d*x + c)^2 + 2*(b^4*d*f*x + b^4*c*f)*cosh(d*x + c)*sinh(d*x + c) + (b...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{csch}\,(dx+c)\,\text{sech}\,(dx+c)^2}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-(b^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqr
t(a^2 + b^2)))/((a^3 + a*b^2)*sqrt(a^2 + b^2)*d) - 2*(a*e^(-d*x - c) - b)/
((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c))*d) + log(e^(-d*x - c) + 1)/(a*
d) - log(e^(-d*x - c) - 1)/(a*d))*e - (8*b^3*integrate(-1/4*x*e^(d*x + c)/
(a^3*b + a*b^3 - (a^3*b*e^(2*c) + a*b^3*e^(2*c))*e^(2*d*x) - 2*(a^4*e^c +
a^2*b^2*e^c)*e^(d*x)), x) - 2*(a*x*e^(d*x + c) + b*x)/(a^2*d + b^2*d + (a^
2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)) + 2*b*x/((a^2 + b^2)*d) + 2*a*arct
an(e^(d*x + c))/((a^2 + b^2)*d^2) - b*log(e^(2*d*x + 2*c) + 1)/((a^2 + b^2
)*d^2) - 8*integrate(1/8*x/(a*e^(d*x + c) + a), x) - 8*integrate(1/8*x/(a*
e^(d*x + c) - a), x))*f
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{e+f\,x}{\cosh\left(c+d\,x\right)^2\sinh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)/(cosh(c + d*x)^2*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(cosh(c + d*x)^2*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqr
t(a**2 + b**2))*b**3*e*i - 2*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*
i)/sqrt(a**2 + b**2))*b**3*e*i + 16*e**(6*c + 2*d*x)*int((e**(4*d*x)*x)/(e
**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a + 2*e**(5*c + 5*d*x)*a - 2*e**(4*
c + 4*d*x)*b - 2*e**(3*c + 3*d*x)*a - 2*e**(c + d*x)*a + b),x)*a**5*d*f +
32*e**(6*c + 2*d*x)*int((e**(4*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7
*d*x)*a + 2*e**(5*c + 5*d*x)*a - 2*e**(4*c + 4*d*x)*b - 2*e**(3*c + 3*d*x)
*a - 2*e**(c + d*x)*a + b),x)*a**3*b**2*d*f + 16*e**(6*c + 2*d*x)*int((e**
(4*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a + 2*e**(5*c + 5*d*x)
*a - 2*e**(4*c + 4*d*x)*b - 2*e**(3*c + 3*d*x)*a - 2*e**(c + d*x)*a + b),x
)*a*b**4*d*f + e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a**4*e + 2*e**(2*c +
 2*d*x)*log(e**(c + d*x) - 1)*a**2*b**2*e + e**(2*c + 2*d*x)*log(e**(c + d
*x) - 1)*b**4*e - e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*a**4*e - 2*e**(2*
c + 2*d*x)*log(e**(c + d*x) + 1)*a**2*b**2*e - e**(2*c + 2*d*x)*log(e**(c
+ d*x) + 1)*b**4*e - 2*e**(2*c + 2*d*x)*a**3*b*e - 2*e**(2*c + 2*d*x)*a*b*
*3*e + 2*e**(c + d*x)*a**4*e + 2*e**(c + d*x)*a**2*b**2*e + 16*e**(4*c)*in
t((e**(4*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a + 2*e**(5*c +
5*d*x)*a - 2*e**(4*c + 4*d*x)*b - 2*e**(3*c + 3*d*x)*a - 2*e**(c + d*x)*a
+ b),x)*a**5*d*f + 32*e**(4*c)*int((e**(4*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*
e**(7*c + 7*d*x)*a + 2*e**(5*c + 5*d*x)*a - 2*e**(4*c + 4*d*x)*b - 2*e*...
```

3.443 $\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 113

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\operatorname{arctanh}(\cosh(c+dx))}{ad} + \frac{2b^3\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^{3/2}\,d} + \frac{\operatorname{sech}(c+dx)}{ad} - \frac{b\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{a\,(a^2+b^2)\,d}$$

output
```
-arctanh(cosh(d*x+c))/a/d+2*b^3*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2
)^(1/2))/a/(a^2+b^2)^(3/2)/d+sech(d*x+c)/a/d-b*sech(d*x+c)*(b+a*sinh(d*x+c
))/a/(a^2+b^2)/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 233 vs. $2(113) = 226$.

Time = 0.97 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.06

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{-2b^3\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)-a^2\sqrt{-a^2-b^2}\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)-b^2\sqrt{-a^2-b^2}\log\left(\cosh\left(\frac{1}{2}(c+\right.\right.}{}$$

input
```
Integrate[(Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-((-2*b^3*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]] - a^2*Sqrt[-a
^2 - b^2]*Log[Cosh[(c + d*x)/2]] - b^2*Sqrt[-a^2 - b^2]*Log[Cosh[(c + d*x)
/2]] + a^2*Sqrt[-a^2 - b^2]*Log[Sinh[(c + d*x)/2]] + b^2*Sqrt[-a^2 - b^2]*
Log[Sinh[(c + d*x)/2]] + a^2*Sqrt[-a^2 - b^2]*Sech[c + d*x] - a*b*Sqrt[-a^
2 - b^2]*Tanh[c + d*x])/(a*(-a^2 - b^2)^(3/2)*d))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3042, 26, 3377, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i}{\sin(ic+idx)\cos(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{ 26}$$

$$i \int \frac{1}{\cos(ic+idx)^2 \sin(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3377}$$

$$i \int \left(\frac{ib\text{sech}^2(c+dx)}{a(a+b\sinh(c+dx))}-\frac{i\text{csch}(c+dx)\text{sech}^2(c+dx)}{a}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$i\left(-\frac{2ib^3\text{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{ad\left(a^2+b^2\right)^{3/2}}+\frac{ib\text{sech}(c+dx)(a\sinh(c+dx)+b)}{ad\left(a^2+b^2\right)}+\frac{i\text{arctanh}(\cosh(c+dx))}{ad}-\frac{i\text{sech}(c+}{ad}\right.$$

input
```
Int[(Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
I*((I*ArcTanh[Cosh[c + d*x]])/(a*d) - ((2*I)*b^3*ArcTanh[(b - a*Tanh[(c +
d*x)/2])/Sqrt[a^2 + b^2]])/(a*(a^2 + b^2)^(3/2)*d) - (I*Sech[c + d*x])/(a*
d) + (I*b*Sech[c + d*x]*(b + a*Sinh[c + d*x]))/(a*(a^2 + b^2)*d))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3377
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Maple [A] (verified)

Time = 4.47 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{-\frac{2b^3 \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{\left(a^2+b^2\right)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
default	$\frac{-\frac{2b^3 \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{\left(a^2+b^2\right)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
risch	$\frac{2a\,e^{dx+c}+2b}{d(a^2+b^2)(1+e^{2dx+2c})}+\frac{\ln(e^{dx+c}-1)}{da}-\frac{\ln(e^{dx+c}+1)}{da}+\frac{b^3\ln\left(e^{dx+c}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}da}-\frac{b^3\ln\left(e\right.}{}$

input
```
int(csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/a*b^3/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a
^2+b^2)^(1/2))-2/(a^2+b^2)*(b*tanh(1/2*d*x+1/2*c)-a)/(1+tanh(1/2*d*x+1/2*c
)^2)+1/a*ln(tanh(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. 2(110) = 220.

Time = 0.18 (sec) , antiderivative size = 581, normalized size of antiderivative = 5.14

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
(2*a^3*b + 2*a*b^3 + (b^3*cosh(d*x + c)^2 + 2*b^3*cosh(d*x + c)*sinh(d*x +
 c) + b^3*sinh(d*x + c)^2 + b^3)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2
+ b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*
x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(
d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c)
+ 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 2*(a^4 + a^2*b^2)*cosh(d*x
 + c) - (a^4 + 2*a^2*b^2 + b^4 + (a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c)^2 +
 2*(a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2
+ b^4)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + (a^4 + 2*
a^2*b^2 + b^4 + (a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b
^2 + b^4)*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*sinh(d*x +
 c)^2)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(a^4 + a^2*b^2)*sinh(d*x
 + c))/((a^5 + 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c)^2 + 2*(a^5 + 2*a^3*b^2 +
 a*b^4)*d*cosh(d*x + c)*sinh(d*x + c) + (a^5 + 2*a^3*b^2 + a*b^4)*d*sinh(d
*x + c)^2 + (a^5 + 2*a^3*b^2 + a*b^4)*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.49

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b^3\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{(a^3+ab^2)\sqrt{a^2+b^2}d} + \frac{2\left(ae^{(-dx-c)}-b\right)}{(a^2+b^2+(a^2+b^2)e^{(-2dx-2c)})d} - \frac{\log\left(e^{(-dx-c)}+1\right)}{ad} + \frac{\log\left(e^{(-dx-c)}-1\right)}{ad}$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-b^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt
(a^2 + b^2)))/((a^3 + a*b^2)*sqrt(a^2 + b^2)*d) + 2*(a*e^(-d*x - c) - b)/(
(a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c))*d) - log(e^(-d*x - c) + 1)/(a*d
) + log(e^(-d*x - c) - 1)/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.29

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{b^3\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^3+ab^2)\sqrt{a^2+b^2}}+\frac{\log\left(e^{(dx+c)}+1\right)}{a}-\frac{\log\left(\left|e^{(dx+c)}-1\right|\right)}{a}-\frac{2\,\left(ae^{(dx+c)}+b\right)}{(a^2+b^2)\left(e^{(2\,dx+2\,c)}+1\right)}}{d}$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-(b^3*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x +
c) + 2*a + 2*sqrt(a^2 + b^2)))/((a^3 + a*b^2)*sqrt(a^2 + b^2)) + log(e^(d*
x + c) + 1)/a - log(abs(e^(d*x + c) - 1))/a - 2*(a*e^(d*x + c) + b)/((a^2
+ b^2)*(e^(2*d*x + 2*c) + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 6.26 (sec) , antiderivative size = 668, normalized size of antiderivative = 5.91

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{2\,b}{d\,(a^2+b^2)}+\frac{2\,a\,e^{c+d\,x}}{d\,(a^2+b^2)}}{e^{2\,c+2\,d\,x}+1}+\frac{\ln\left(e^{c+d\,x}-1\right)}{a\,d}-\frac{\ln\left(e^{c+d\,x}+1\right)}{a\,d}$$

$$-\frac{b^3\,\ln\left(\frac{32\,\left(-4\,e^{c+d\,x}\,a^3+2\,a^2\,b-5\,e^{c+d\,x}\,a\,b^2+2\,b^3\right)}{b^2\,\left(a^2+b^2\right)^2}-\frac{128\,a^{10}\,e^{c+d\,x}-64\,a^9\,b-96\,a\,b^9+64\,b^7\,\sqrt{\left(a^2+b^2\right)^3}-384\,a^3\,b^7-512\,a^5\,b^5-288\,a^7\,b\cdots}{\cdots}\right.}{d\,a^7+3\,d\,a^5\,b\cdots}$$

$$+\frac{b^3\,\ln\left(\frac{32\,\left(-4\,e^{c+d\,x}\,a^3+2\,a^2\,b-5\,e^{c+d\,x}\,a\,b^2+2\,b^3\right)}{b^2\,\left(a^2+b^2\right)^2}-\frac{96\,a\,b^9+64\,a^9\,b-128\,a^{10}\,e^{c+d\,x}+64\,b^7\,\sqrt{\left(a^2+b^2\right)^3}+384\,a^3\,b^7+512\,a^5\,b^5+288\,a^7\,b\cdots}{\cdots}\right.}{d\,a^7+3\,d\,a^5\,b\cdots}$$

input
```
int(1/(cosh(c + d*x)^2*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
((2*b)/(d*(a^2 + b^2)) + (2*a*exp(c + d*x))/(d*(a^2 + b^2)))/(exp(2*c + 2*
d*x) + 1) + log(exp(c + d*x) - 1)/(a*d) - log(exp(c + d*x) + 1)/(a*d) - (b
^3*log((32*(2*a^2*b - 4*a^3*exp(c + d*x) + 2*b^3 - 5*a*b^2*exp(c + d*x)))/
(b^2*(a^2 + b^2)^2) - (128*a^10*exp(c + d*x) - 64*a^9*b - 96*a*b^9 + 64*b^
7*((a^2 + b^2)^3)^(1/2) - 384*a^3*b^7 - 512*a^5*b^5 - 288*a^7*b^3 + 288*a^
2*b^8*exp(c + d*x) + 960*a^4*b^6*exp(c + d*x) + 1152*a^6*b^4*exp(c + d*x)
+ 608*a^8*b^2*exp(c + d*x) - 64*a*b^6*exp(c + d*x)*((a^2 + b^2)^3)^(1/2) +
 32*a^3*b^4*exp(c + d*x)*((a^2 + b^2)^3)^(1/2))/(b^2*((a^2 + b^2)^3)^(3/2)
*(a^2 + b^2)))*((a^2 + b^2)^3)^(1/2))/(a^7*d + 3*a^3*b^4*d + 3*a^5*b^2*d +
 a*b^6*d) + (b^3*log((32*(2*a^2*b - 4*a^3*exp(c + d*x) + 2*b^3 - 5*a*b^2*e
xp(c + d*x)))/(b^2*(a^2 + b^2)^2) - (96*a*b^9 + 64*a^9*b - 128*a^10*exp(c
+ d*x) + 64*b^7*((a^2 + b^2)^3)^(1/2) + 384*a^3*b^7 + 512*a^5*b^5 + 288*a^
7*b^3 - 288*a^2*b^8*exp(c + d*x) - 960*a^4*b^6*exp(c + d*x) - 1152*a^6*b^4
*exp(c + d*x) - 608*a^8*b^2*exp(c + d*x) - 64*a*b^6*exp(c + d*x)*((a^2 + b
^2)^3)^(1/2) + 32*a^3*b^4*exp(c + d*x)*((a^2 + b^2)^3)^(1/2))/(b^2*((a^2 +
 b^2)^3)^(3/2)*(a^2 + b^2)))*((a^2 + b^2)^3)^(1/2))/(a^7*d + 3*a^3*b^4*d +
 3*a^5*b^2*d + a*b^6*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 468, normalized size of antiderivative = 4.14

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-2e^{2dx+2c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)b^3i-2\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)b^3i+e^{2dx+2c}\log\left(e^{dx+c}-1\right)a^4+2e}{}$$

input
```
int(csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqr
t(a**2 + b**2))*b**3*i - 2*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)
/sqrt(a**2 + b**2))*b**3*i + e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a**4 +
 2*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a**2*b**2 + e**(2*c + 2*d*x)*log
(e**(c + d*x) - 1)*b**4 - e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*a**4 - 2*
e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*a**2*b**2 - e**(2*c + 2*d*x)*log(e*
*(c + d*x) + 1)*b**4 - 2*e**(2*c + 2*d*x)*a**3*b - 2*e**(2*c + 2*d*x)*a*b*
*3 + 2*e**(c + d*x)*a**4 + 2*e**(c + d*x)*a**2*b**2 + log(e**(c + d*x) - 1
)*a**4 + 2*log(e**(c + d*x) - 1)*a**2*b**2 + log(e**(c + d*x) - 1)*b**4 -
log(e**(c + d*x) + 1)*a**4 - 2*log(e**(c + d*x) + 1)*a**2*b**2 - log(e**(c
 + d*x) + 1)*b**4)/(a*d*(e**(2*c + 2*d*x)*a**4 + 2*e**(2*c + 2*d*x)*a**2*b
**2 + e**(2*c + 2*d*x)*b**4 + a**4 + 2*a**2*b**2 + b**4))
```

3.444 $\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\operatorname{Int}\left(\frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(csch(d*x+c)*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 48.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Csch[c + d*x]*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Csch[c + d*x]*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

$$\Big\downarrow \text{6125}$$

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Csch[c + d*x]*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)^2}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(csch(d*x+c)*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)\operatorname{sech}\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(csch(d*x + c)*sech(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 463, normalized size of antiderivative = 13.62

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)\operatorname{sech}\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-8*b^3*integrate(-1/4*e^(d*x + c)/(a^3*b*e + a*b^3*e + (a^3*b*f + a*b^3*f)
*x - (a^3*b*e*e^(2*c) + a*b^3*e*e^(2*c) + (a^3*b*f*e^(2*c) + a*b^3*f*e^(2*
c))*x)*e^(2*d*x) - 2*(a^4*e*e^c + a^2*b^2*e*e^c + (a^4*f*e^c + a^2*b^2*f*e
^c)*x)*e^(d*x)), x) + 2*(a*e^(d*x + c) + b)/(a^2*d*e + b^2*d*e + (a^2*d*f
+ b^2*d*f)*x + (a^2*d*e*e^(2*c) + b^2*d*e*e^(2*c) + (a^2*d*f*e^(2*c) + b^2
*d*f*e^(2*c))*x)*e^(2*d*x)) + 8*integrate(1/4*(a*f*e^(d*x + c) + b*f)/(a^2
*d*e^2 + b^2*d*e^2 + (a^2*d*f^2 + b^2*d*f^2)*x^2 + 2*(a^2*d*e*f + b^2*d*e*
f)*x + (a^2*d*e^2*e^(2*c) + b^2*d*e^2*e^(2*c) + (a^2*d*f^2*e^(2*c) + b^2*d
*f^2*e^(2*c))*x^2 + 2*(a^2*d*e*f*e^(2*c) + b^2*d*e*f*e^(2*c))*x)*e^(2*d*x)
), x) + 8*integrate(1/8/(a*f*x + a*e + (a*f*x*e^c + a*e*e^c)*e^(d*x)), x)
+ 8*integrate(-1/8/(a*f*x + a*e - (a*f*x*e^c + a*e*e^c)*e^(d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 5.43 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^2\sinh\left(c+d\,x\right)\,\left(e+f\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)^2*sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(cosh(c + d*x)^2*sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$=\int \frac{\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)^2}{\sinh(dx+c)\,be+\sinh(dx+c)\,bfx+ae+afx}dx$$

input
```
int(csch(d*x+c)*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int((csch(c + d*x)*sech(c + d*x)**2)/(sinh(c + d*x)*b*e + sinh(c + d*x)*b*
f*x + a*e + a*f*x),x)
```

3.445 $\int \frac{(e+fx)^2\mathrm{csch}(c+dx)\mathrm{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 1176

$$\int \frac{(e+fx)^2\mathrm{csch}(c+dx)\mathrm{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output
```
-1/2*b^4*f^2*polylog(3,-exp(2*d*x+2*c))/a/(a^2+b^2)^2/d^3-1/2*b^2*(f*x+e)^
2*sech(d*x+c)^2/a/(a^2+b^2)/d-1/2*b*(f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/(a^2
+b^2)/d-2*I*b^3*f^2*polylog(3,-I*exp(d*x+c))/(a^2+b^2)^2/d^3-I*b*f^2*polyl
og(3,-I*exp(d*x+c))/(a^2+b^2)/d^3-2*I*b^3*f*(f*x+e)*polylog(2,I*exp(d*x+c)
)/(a^2+b^2)^2/d^2-I*b*f*(f*x+e)*polylog(2,I*exp(d*x+c))/(a^2+b^2)/d^2+I*b*
f^2*polylog(3,I*exp(d*x+c))/(a^2+b^2)/d^3+b^4*(f*x+e)^2*ln(1+exp(2*d*x+2*c
))/a/(a^2+b^2)^2/d-2*b^4*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1
/2)))/a/(a^2+b^2)^2/d^2-2*b^4*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^
2)^(1/2)))/a/(a^2+b^2)^2/d^2-b^2*f^2*ln(cosh(d*x+c))/a/(a^2+b^2)/d^3-b*f*(
f*x+e)*sech(d*x+c)/(a^2+b^2)/d^2+b*f^2*arctan(sinh(d*x+c))/(a^2+b^2)/d^3+f
^2*ln(cosh(d*x+c))/a/d^3-1/2*f^2*polylog(3,exp(2*d*x+2*c))/a/d^3-2*(f*x+e)
^2*arctanh(exp(2*d*x+2*c))/a/d+1/2*f^2*polylog(3,-exp(2*d*x+2*c))/a/d^3-f*
(f*x+e)*tanh(d*x+c)/a/d^2+2*b^4*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(
1/2)))/a/(a^2+b^2)^2/d^3+2*b^4*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1
/2)))/a/(a^2+b^2)^2/d^3-b^4*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)
))/a/(a^2+b^2)^2/d-b^4*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/
(a^2+b^2)^2/d-b*(f*x+e)^2*arctan(exp(d*x+c))/(a^2+b^2)/d-2*b^3*(f*x+e)^2*a
rctan(exp(d*x+c))/(a^2+b^2)^2/d+f*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a/d^2+
2*I*b^3*f^2*polylog(3,I*exp(d*x+c))/(a^2+b^2)^2/d^3+2*I*b^3*f*(f*x+e)*poly
log(2,-I*exp(d*x+c))/(a^2+b^2)^2/d^2+I*b*f*(f*x+e)*polylog(2,-I*exp(d*x...
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4072 vs. $2(1176) = 2352$.

Time = 12.57 (sec) , antiderivative size = 4072, normalized size of antiderivative = 3.46

$$\int \frac{(e+fx)^2 \operatorname{csch}(c+dx) \operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Csch[c + d*x]*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]
),x]
```

output
```
-1/3*(E^(2*c)*((2*(e + f*x)^3)/(E^(2*c)*f) - (3*(1 - E^(-2*c))*(e + f*x)^2
*Log[1 - E^(-c - d*x)])/d - (3*(1 - E^(-2*c))*(e + f*x)^2*Log[1 + E^(-c -
d*x)])/d + (6*(-1 + E^(2*c))*f*(d*(e + f*x)*PolyLog[2, -E^(-c - d*x)] + f*
PolyLog[3, -E^(-c - d*x)]))/(d^3*E^(2*c)) + (6*(-1 + E^(2*c))*f*(d*(e + f*
x)*PolyLog[2, E^(-c - d*x)] + f*PolyLog[3, E^(-c - d*x)]))/(d^3*E^(2*c))))
/(a*(-1 + E^(2*c))) - (-12*a^3*d^3*e^2*E^(2*c)*x - 24*a*b^2*d^3*e^2*E^(2*c
)*x + 12*a^3*d*E^(2*c)*f^2*x + 12*a*b^2*d*E^(2*c)*f^2*x - 12*a^3*d^3*e*E^(
2*c)*f*x^2 - 24*a*b^2*d^3*e*E^(2*c)*f*x^2 - 4*a^3*d^3*E^(2*c)*f^2*x^3 - 8*
a*b^2*d^3*E^(2*c)*f^2*x^3 + 6*a^2*b*d^2*e^2*ArcTan[E^(c + d*x)] + 18*b^3*d
^2*e^2*ArcTan[E^(c + d*x)] + 6*a^2*b*d^2*e^2*E^(2*c)*ArcTan[E^(c + d*x)] +
 18*b^3*d^2*e^2*E^(2*c)*ArcTan[E^(c + d*x)] - 12*a^2*b*f^2*ArcTan[E^(c + d
*x)] - 12*b^3*f^2*ArcTan[E^(c + d*x)] - 12*a^2*b*E^(2*c)*f^2*ArcTan[E^(c +
 d*x)] - 12*b^3*E^(2*c)*f^2*ArcTan[E^(c + d*x)] + (6*I)*a^2*b*d^2*e*f*x*Lo
g[1 - I*E^(c + d*x)] + (18*I)*b^3*d^2*e*f*x*Log[1 - I*E^(c + d*x)] + (6*I)
*a^2*b*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] + (18*I)*b^3*d^2*e*E^(2*c)
*f*x*Log[1 - I*E^(c + d*x)] + (3*I)*a^2*b*d^2*f^2*x^2*Log[1 - I*E^(c + d*x
)] + (9*I)*b^3*d^2*f^2*x^2*Log[1 - I*E^(c + d*x)] + (3*I)*a^2*b*d^2*E^(2*c
)*f^2*x^2*Log[1 - I*E^(c + d*x)] + (9*I)*b^3*d^2*E^(2*c)*f^2*x^2*Log[1 - I
*E^(c + d*x)] - (6*I)*a^2*b*d^2*e*f*x*Log[1 + I*E^(c + d*x)] - (18*I)*b^3*
d^2*e*f*x*Log[1 + I*E^(c + d*x)] - (6*I)*a^2*b*d^2*e*E^(2*c)*f*x*Log[1 ...
```

Rubi [A] (verified)

Time = 4.77 (sec) , antiderivative size = 993, normalized size of antiderivative = 0.84, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {6123, 5985, 27, 6107, 6107, 6095, 2620, 3011, 2720, 7143, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)^2\text{csch}(c+dx)\text{sech}^3(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5985}$$

$$\frac{-2f\int \frac{1}{2}(e+fx)\left(\frac{2\log(\tanh(c+dx))}{d}-\frac{\tanh^2(c+dx)}{d}\right)dx-\frac{(e+fx)^2\tanh^2(c+dx)}{2d}+\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-\frac{b\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$\Big\downarrow$ 27

$$\frac{-f\int (e+fx)\left(\frac{2\log(\tanh(c+dx))}{d}-\frac{\tanh^2(c+dx)}{d}\right)dx-\frac{(e+fx)^2\tanh^2(c+dx)}{2d}+\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-\frac{b\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$\Big\downarrow$ 6107

$$\frac{-f\int (e+fx)\left(\frac{2\log(\tanh(c+dx))}{d}-\frac{\tanh^2(c+dx)}{d}\right)dx-\frac{(e+fx)^2\tanh^2(c+dx)}{2d}+\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}\right)}{a}$$

$\Big\downarrow$ 6107

$$\frac{-f\int (e+fx)\left(\frac{2\log(\tanh(c+dx))}{d}-\frac{\tanh^2(c+dx)}{d}\right)dx-\frac{(e+fx)^2\tanh^2(c+dx)}{2d}+\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a^2+b^2}\right)}{a}$$

$\Big\downarrow$ 6095

$$\frac{-f\int (e+fx)\left(\frac{2\log(\tanh(c+dx))}{d}-\frac{\tanh^2(c+dx)}{d}\right)dx-\frac{(e+fx)^2\tanh^2(c+dx)}{2d}+\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}(c+dx)\cdots}{a^2+b\cdots}\right)}{a^2+b^2}\right)}{a}$$

$\Big\downarrow$ 2620

$$\frac{-f\int(e+fx)\left(\frac{2\log(\tanh(c+dx))}{d}-\frac{\tanh^2(c+dx)}{d}\right)dx-\frac{(e+fx)^2\tanh^2(c+dx)}{2d}+\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$\frac{b\left(\frac{\int(e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\cdots\right.}{\cdots}\right.}{a^2+b^2}\right.}{\cdots}\right.}{a}$$

$\downarrow$ 3011

$$\frac{-f\int(e+fx)\left(\frac{2\log(\tanh(c+dx))}{d}-\frac{\tanh^2(c+dx)}{d}\right)dx-\frac{(e+fx)^2\tanh^2(c+dx)}{2d}+\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{b^2\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{b\cdots}{a+\cdots}\right.}{d}\right.}{bd}\right.}{a^2+b^2}\right.}{a^2+b^2}\right.$$

$\downarrow$ 2720

$$\frac{-f\int(e+fx)\left(\frac{2\log(\tanh(c+dx))}{d}-\frac{\tanh^2(c+dx)}{d}\right)dx-\frac{(e+fx)^2\tanh^2(c+dx)}{2d}+\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{b^2\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+d\cdots}}{d^2}\cdots\right)}{bd}\cdots\right)}{a^2+b^2}\cdots}{\cdots}\cdots\right)$$

↓ 7143

$$\frac{-f\int(e+fx)\left(\frac{2\log(\tanh(c+dx))}{d}-\frac{\tanh^2(c+dx)}{d}\right)dx-\frac{(e+fx)^2\tanh^2(c+dx)}{2d}+\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{b^2\left(\frac{\int(e+fx)^2\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{b\cdots}{a+\cdots}\right)}{d^2}\cdots\right)}{\cdots}\cdots\right)}{a^2+b^2}\right)}{\cdots}\cdots\right)$$

↓ 7292

$$\frac{-f\int \frac{(e+fx)(2\log(\tanh(c+dx))-\tanh^2(c+dx))}{d}dx-\frac{(e+fx)^2\tanh^2(c+dx)}{2d}+\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a} -$$

$$b\left(\frac{b^2\left(\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{b}{a+}\right.}{d^2}\right.}{}\right.}{}\right)}{a^2+b^2}\right.$$

$\downarrow$ 27

$$\frac{-\frac{f\int (e+fx)(2\log(\tanh(c+dx))-\tanh^2(c+dx))dx}{d}-\frac{(e+fx)^2\tanh^2(c+dx)}{2d}+\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a} -$$

$$b\left(\frac{b^2\left(\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{b}{a+}\right.}{d^2}\right.}{}\right.}{}\right)}{a^2+b^2}\right.$$

$\downarrow$ 7293

$$-\frac{\frac{f\int(2(e+fx)\log(\tanh(c+dx))-(e+fx)\tanh^2(c+dx))dx}{d}-\frac{(e+fx)^2\tanh^2(c+dx)}{2d}+\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{b^2\left(\frac{\int\left(a(e+fx)^2\text{sech}(c+dx)-b(e+fx)^2\tanh(c+dx)\right)dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLo}}{}\right.}{}\right.}{}\right)}{a^2+b^2}\right.$$

↓ 2009

$$-\frac{\tanh^2(c+dx)(e+fx)^2}{2d}+\frac{\log(\tanh(c+dx))(e+fx)^2}{d}-\frac{f\left(\frac{2\text{arctanh}\left(e^{2c+2dx}\right)(e+fx)^2}{f}+\frac{\log(\tanh(c+dx))(e+fx)^2}{f}-\frac{(e+fx)^2}{2f}+\frac{\text{PolyLog}\left(2,-e^{2c+}\right.}{d}\right.}{a}$$

$$b\left(\frac{\left(\frac{-\frac{(e+fx)^3}{3bf}+\frac{\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)(e+fx)^2}{bd}+\frac{\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)(e+fx)^2}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}}{a^2+b^2}\right.}{}\right.$$

input
```
Int[((e + f*x)^2*Csch[c + d*x]*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(((e + f*x)^2*Log[Tanh[c + d*x]])/d - ((e + f*x)^2*Tanh[c + d*x]^2)/(2*d)
- (f*(-1/2*(e + f*x)^2/f + (2*(e + f*x)^2*ArcTanh[E^(2*c + 2*d*x)])/f - (f
*Log[Cosh[c + d*x]])/d^2 + ((e + f*x)^2*Log[Tanh[c + d*x]])/f + ((e + f*x)
*PolyLog[2, -E^(2*c + 2*d*x)])/d - ((e + f*x)*PolyLog[2, E^(2*c + 2*d*x)])
/d - (f*PolyLog[3, -E^(2*c + 2*d*x)])/(2*d^2) + (f*PolyLog[3, E^(2*c + 2*d
*x)])/(2*d^2) + ((e + f*x)*Tanh[c + d*x])/d))/d)/a - (b*((b^2*((b^2*(-1/3*
(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]
)/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d^2
))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^
2))/(b*d)))/(a^2 + b^2) + ((b*(e + f*x)^3)/(3*f) + (2*a*(e + f*x)^2*ArcTan
[E^(c + d*x)])/d - (b*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/d - ((2*I)*a*f
*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + ((2*I)*a*f*(e + f*x)*PolyLo
g[2, I*E^(c + d*x)])/d^2 - (b*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/d^
2 + ((2*I)*a*f^2*PolyLog[3, (-I)*E^(c + d*x)])/d^3 - ((2*I)*a*f^2*PolyLog[
3, I*E^(c + d*x)])/d^3 + (b*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*d^3))/(a^
2 + b^2)))/(a^2 + b^2) + ((a*(e + f*x)^2*ArcTan[E^(c + d*x)])/d - (a*f^2*A
rcTan[Sinh[c + d*x]])/d^3 + (b*f^2*Log[Cosh[c + d*x]])/d^3 - (I*a*f*(e ...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c) \operatorname{sech}(dx+c)^3}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 16670 vs. 2(1088) = 2176.

Time = 0.54 (sec) , antiderivative size = 16670, normalized size of antiderivative = 14.18

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Too large to display}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)**2*csch(d*x+c)*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\,\text{csch}\,(dx+c)\,\text{sech}\,(dx+c)^3}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-a^2*b*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*
b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^
2 + b^4*d^2), x) - 3*b^3*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d
*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*
d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 2*a^3*d^2*f^2*integrate(x^2/(a^4*d^2*
e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c)
+ a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 4*a*b^2*d^2*f^2*integrate(x^2/(
a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x
 + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - 2*a^2*b*d^2*e*f*integra
te(x*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c)
+ b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - 6*b^3
*d^2*e*f*integrate(x*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*
e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*
d^2), x) + 4*a^3*d^2*e*f*integrate(x/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*
d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 +
b^4*d^2), x) + 8*a*b^2*d^2*e*f*integrate(x/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^
2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*
d^2 + b^4*d^2), x) - a^3*f^2*(2*(d*x + c)/((a^4 + 2*a^2*b^2 + b^4)*d^3) -
log(e^(2*d*x + 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d^3)) - a*b^2*f^2*(2*(d*
x + c)/((a^4 + 2*a^2*b^2 + b^4)*d^3) - log(e^(2*d*x + 2*c) + 1)/((a^4 +...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^2}{\cosh\left(c+d\,x\right)^3\sinh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^2/(cosh(c + d*x)^3*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^2/(cosh(c + d*x)^3*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
( - e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**3*b*e**2 - 3*e**(4*c + 4*d*x)*a
tan(e**(c + d*x))*a*b**3*e**2 - 2*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**3
*b*e**2 - 6*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a*b**3*e**2 - atan(e**(c +
 d*x))*a**3*b*e**2 - 3*atan(e**(c + d*x))*a*b**3*e**2 + 32*e**(9*c + 4*d*x
)*int((e**(5*d*x)*x**2)/(e**(10*c + 10*d*x)*b + 2*e**(9*c + 9*d*x)*a + e**
(8*c + 8*d*x)*b + 4*e**(7*c + 7*d*x)*a - 2*e**(6*c + 6*d*x)*b - 2*e**(4*c
+ 4*d*x)*b - 4*e**(3*c + 3*d*x)*a + e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a
+ b),x)*a**5*d*f**2 + 64*e**(9*c + 4*d*x)*int((e**(5*d*x)*x**2)/(e**(10*c
+ 10*d*x)*b + 2*e**(9*c + 9*d*x)*a + e**(8*c + 8*d*x)*b + 4*e**(7*c + 7*d*
x)*a - 2*e**(6*c + 6*d*x)*b - 2*e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a
+ e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**3*b**2*d*f**2 + 32*e**(
9*c + 4*d*x)*int((e**(5*d*x)*x**2)/(e**(10*c + 10*d*x)*b + 2*e**(9*c + 9*d
*x)*a + e**(8*c + 8*d*x)*b + 4*e**(7*c + 7*d*x)*a - 2*e**(6*c + 6*d*x)*b -
 2*e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a + e**(2*c + 2*d*x)*b - 2*e**(
c + d*x)*a + b),x)*a*b**4*d*f**2 + 64*e**(9*c + 4*d*x)*int((e**(5*d*x)*x)/
(e**(10*c + 10*d*x)*b + 2*e**(9*c + 9*d*x)*a + e**(8*c + 8*d*x)*b + 4*e**(
7*c + 7*d*x)*a - 2*e**(6*c + 6*d*x)*b - 2*e**(4*c + 4*d*x)*b - 4*e**(3*c +
 3*d*x)*a + e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**5*d*e*f + 128
*e**(9*c + 4*d*x)*int((e**(5*d*x)*x)/(e**(10*c + 10*d*x)*b + 2*e**(9*c + 9
*d*x)*a + e**(8*c + 8*d*x)*b + 4*e**(7*c + 7*d*x)*a - 2*e**(6*c + 6*d*x...
```

3.446 $\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 746

$$\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{fx}{2ad} - \frac{2b^3(e+fx)\arctan\left(e^{c+dx}\right)}{(a^2+b^2)^2 d} - \frac{b(e+fx)\arctan\left(e^{c+dx}\right)}{(a^2+b^2)\,d} - \frac{2fx\operatorname{arctanh}\left(e^{2c+2dx}\right)}{ad} - \frac{b^4(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^2 d} - \frac{b^4(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^2 d} + \frac{b^4(e+fx)\log\left(1+e^{2(c+dx)}\right)}{a\,(a^2+b^2)^2 d} - \frac{fx\log(\tanh(c+dx))}{ad} + \frac{(e+fx)\log(\tanh(c+dx))}{ad} + \frac{ib^3f\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{(a^2+b^2)^2 d^2} + \frac{ibf\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{2\,(a^2+b^2)\,d^2} - \frac{ib^3f\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{(a^2+b^2)^2 d^2} - \frac{ibf\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{2\,(a^2+b^2)\,d^2} - \frac{b^4f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^2 d^2} - \frac{b^4f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^2 d^2} + \frac{b^4f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2a\,(a^2+b^2)^2 d^2} - \frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2ad^2} + \frac{f\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2ad^2} - \frac{bf\operatorname{sech}(c+dx)}{2\,(a^2+b^2)\,d^2} - \frac{b^2(e+fx)\operatorname{sech}^2(c+dx)}{2a\,(a^2+b^2)\,d} - \frac{f\tanh(c+dx)}{2ad^2} + \frac{b^2f\tanh(c+dx)}{2a\,(a^2+b^2)\,d^2} - \frac{b(e+fx)\operatorname{sech}(c+dx)\tanh(c+dx)}{2\,(a^2+b^2)\,d} - \frac{(e+fx)\tanh^2(c+dx)}{2ad}$$

output
```
1/2*f*x/a/d-2*b^3*(f*x+e)*arctan(exp(d*x+c))/(a^2+b^2)^2/d-b*(f*x+e)*arcta
n(exp(d*x+c))/(a^2+b^2)/d-2*f*x*arctanh(exp(2*d*x+2*c))/a/d-b^4*(f*x+e)*ln
(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)^2/d-b^4*(f*x+e)*ln(1+b*ex
p(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)^2/d+b^4*(f*x+e)*ln(1+exp(2*d*x+2
*c))/a/(a^2+b^2)^2/d-f*x*ln(tanh(d*x+c))/a/d+(f*x+e)*ln(tanh(d*x+c))/a/d+1
/2*I*b*f*polylog(2,-I*exp(d*x+c))/(a^2+b^2)/d^2-I*b^3*f*polylog(2,I*exp(d*
x+c))/(a^2+b^2)^2/d^2-1/2*I*b*f*polylog(2,I*exp(d*x+c))/(a^2+b^2)/d^2+I*b^
3*f*polylog(2,-I*exp(d*x+c))/(a^2+b^2)^2/d^2-b^4*f*polylog(2,-b*exp(d*x+c)
/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)^2/d^2-b^4*f*polylog(2,-b*exp(d*x+c)/(a+(
a^2+b^2)^(1/2)))/a/(a^2+b^2)^2/d^2+1/2*b^4*f*polylog(2,-exp(2*d*x+2*c))/a/
(a^2+b^2)^2/d^2-1/2*f*polylog(2,-exp(2*d*x+2*c))/a/d^2+1/2*f*polylog(2,exp
(2*d*x+2*c))/a/d^2-1/2*b*f*sech(d*x+c)/(a^2+b^2)/d^2-1/2*b^2*(f*x+e)*sech(
d*x+c)^2/a/(a^2+b^2)/d-1/2*f*tanh(d*x+c)/a/d^2+1/2*b^2*f*tanh(d*x+c)/a/(a^
2+b^2)/d^2-1/2*b*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a^2+b^2)/d-1/2*(f*x+e)*t
anh(d*x+c)^2/a/d
```

Mathematica [A] (warning: unable to verify)

Time = 10.44 (sec) , antiderivative size = 1080, normalized size of antiderivative = 1.45

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)*Csch[c + d*x]*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),
x]
```

output
```
((d^2*f*x^2)/2 + d*e*(c + d*x) + (d*e - c*f + f*(c + d*x))*Log[1 - E^(-c -
 d*x)] + (d*e - c*f + f*(c + d*x))*Log[1 + E^(-c - d*x)] - f*PolyLog[2, -E
^(-c - d*x)] - f*PolyLog[2, E^(-c - d*x)])/(a*d^2) - (b^4*(-2*d*e*(c + d*x
) + 2*c*f*(c + d*x) - f*(c + d*x)^2 + (4*a*Sqrt[a^2 + b^2]*d*e*ArcTan[(a +
 b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b^2)^2] - (4*a*Sqrt[-(a^2
+ b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/(-a^2 - b^2)^(
3/2) + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*f*
(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*f*Log[b - 2
*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*d*e*Log[2*a*E^(c + d*x) + b*(-1 +
E^(2*(c + d*x)))] + 2*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
 + 2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(2*a*(a^2 +
b^2)^2*d^2) - (-2*a^3*d*e*(c + d*x) - 4*a*b^2*d*e*(c + d*x) + 2*a^3*c*f*(c
 + d*x) + 4*a*b^2*c*f*(c + d*x) - a^3*f*(c + d*x)^2 - 2*a*b^2*f*(c + d*x)^
2 + 2*a^2*b*d*e*ArcTan[E^(c + d*x)] + 6*b^3*d*e*ArcTan[E^(c + d*x)] - 2*a^
2*b*c*f*ArcTan[E^(c + d*x)] - 6*b^3*c*f*ArcTan[E^(c + d*x)] + I*a^2*b*f*(c
 + d*x)*Log[1 - I*E^(c + d*x)] + (3*I)*b^3*f*(c + d*x)*Log[1 - I*E^(c + d*
x)] - I*a^2*b*f*(c + d*x)*Log[1 + I*E^(c + d*x)] - (3*I)*b^3*f*(c + d*x)*L
og[1 + I*E^(c + d*x)] + 2*a^3*d*e*Log[1 + E^(2*(c + d*x))] + 4*a*b^2*d*e*L
og[1 + E^(2*(c + d*x))] - 2*a^3*c*f*Log[1 + E^(2*(c + d*x))] - 4*a*b^2*c*f
*Log[1 + E^(2*(c + d*x))] + 2*a^3*f*(c + d*x)*Log[1 + E^(2*(c + d*x))] ...
```

Rubi [A] (verified)

Time = 2.77 (sec) , antiderivative size = 627, normalized size of antiderivative = 0.84, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {6123, 5985, 2009, 6107, 6107, 6095, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5985}$$

$$\frac{-f\int\left(\frac{\log(\tanh(c+dx))}{d}-\frac{\tanh^2(c+dx)}{2d}\right)dx-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e+fx)\log(\tanh(c+dx))}{d}}{a}-\frac{b\int\frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$\downarrow$ 2009

$$\frac{-f\left(\frac{2x\text{arctanh}(e^{2c+2dx})}{d}+\frac{\text{PolyLog}(2,-e^{2c+2dx})}{2d^2}-\frac{\text{PolyLog}(2,e^{2c+2dx})}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tan}{2}}{a}$$
$$\frac{b\int\frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$\downarrow$ 6107

$$\frac{-f\left(\frac{2x\text{arctanh}(e^{2c+2dx})}{d}+\frac{\text{PolyLog}(2,-e^{2c+2dx})}{2d^2}-\frac{\text{PolyLog}(2,e^{2c+2dx})}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tan}{2}}{a}$$
$$\frac{b\left(\frac{\int(e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int\frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}\right)}{a}$$

$\downarrow$ 6107

$$\frac{-f\left(\frac{2x\text{arctanh}(e^{2c+2dx})}{d}+\frac{\text{PolyLog}(2,-e^{2c+2dx})}{2d^2}-\frac{\text{PolyLog}(2,e^{2c+2dx})}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tan}{2}}{a}$$
$$\frac{b\left(\frac{\int(e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a^2+b^2}\right)}{a}$$

$\downarrow$ 6095

$$\frac{-f\left(\frac{2x\text{arctanh}(e^{2c+2dx})}{d}+\frac{\text{PolyLog}(2,-e^{2c+2dx})}{2d^2}-\frac{\text{PolyLog}(2,e^{2c+2dx})}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tan}{2}}{a}$$

$$\frac{b\left(\frac{\int(e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int(e+fx)\text{sech}(c+dx)(a-}{a^2+b^2}\right.}{a^2+b^2}\right.}{a}$$

$$\Big\downarrow \; 2620$$

$$\frac{-f\left(\frac{2x\text{arctanh}(e^{2c+2dx})}{d}+\frac{\text{PolyLog}(2,-e^{2c+2dx})}{2d^2}-\frac{\text{PolyLog}(2,e^{2c+2dx})}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tan}{2}}{a}$$

$$\frac{b\left(\frac{\int(e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\right.}{a^2+b^2}\right.}{a^2+b^2}\right.}{a}$$

$$\Big\downarrow \; 2715$$

$$\frac{-f\left(\frac{2x\text{arctanh}(e^{2c+2dx})}{d}+\frac{\text{PolyLog}(2,-e^{2c+2dx})}{2d^2}-\frac{\text{PolyLog}(2,e^{2c+2dx})}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tan}{2}}{a}$$

$$\frac{b\left(\frac{b^2\left(\frac{b^2\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+\right.}{bd}\right.}{a^2+b^2}\right.}{a^2+b^2}\right.}{a}$$

$$\Big\downarrow \; 2838$$

$$\frac{-f\left(\frac{2x\text{arctanh}(e^{2c+2dx})}{d}+\frac{\text{PolyLog}(2,-e^{2c+2dx})}{2d^2}-\frac{\text{PolyLog}(2,e^{2c+2dx})}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tan}{2}}{a}$$

$$\frac{b\left(\frac{b^2\left(\frac{\int(e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)}{}\right.}{a^2+b^2}\right.}{a^2+b^2}\right.}{a}$$

$\Big\downarrow$ 7293

$$\frac{-f\left(\frac{2x\text{arctanh}(e^{2c+2dx})}{d}+\frac{\text{PolyLog}(2,-e^{2c+2dx})}{2d^2}-\frac{\text{PolyLog}(2,e^{2c+2dx})}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tan}{2}}{a}$$

$$\frac{b\left(\frac{b^2\left(\frac{\int(a(e+fx)\text{sech}(c+dx)-b(e+fx)\tanh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\right.}{a^2+b^2}\right.}{a^2+b^2}\right.}{a}$$

$\Big\downarrow$ 2009

$$\frac{-f\left(\frac{2x\text{arctanh}(e^{2c+2dx})}{d}+\frac{\text{PolyLog}(2,-e^{2c+2dx})}{2d^2}-\frac{\text{PolyLog}(2,e^{2c+2dx})}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tan}{2}}{a}$$

$$b\left(\frac{b^2\left(\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{2a(e+}{}\right.}{a^2+b^2}\right.$$

input
```
Int[((e + f*x)*Csch[c + d*x]*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(((e + f*x)*Log[Tanh[c + d*x]])/d - ((e + f*x)*Tanh[c + d*x]^2)/(2*d) - f*
(-1/2*x/d + (2*x*ArcTanh[E^(2*c + 2*d*x)])/d + (x*Log[Tanh[c + d*x]])/d +
PolyLog[2, -E^(2*c + 2*d*x)]/(2*d^2) - PolyLog[2, E^(2*c + 2*d*x)]/(2*d^2)
 + Tanh[c + d*x]/(2*d^2)))/a - (b*((b^2*((b^2*(-1/2*(e + f*x)^2/(b*f) + ((
e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x
)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -(
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (f*PolyLog[2, -((b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(a^2 + b^2) + ((b*(e + f*x)^2)
/(2*f) + (2*a*(e + f*x)*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)*Log[1 + E^(2
*(c + d*x))])/d - (I*a*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + (I*a*f*PolyLo
g[2, I*E^(c + d*x)])/d^2 - (b*f*PolyLog[2, -E^(2*(c + d*x))])/(2*d^2))/(a^
2 + b^2)))/(a^2 + b^2) + ((a*(e + f*x)*ArcTan[E^(c + d*x)])/d - ((I/2)*a*f
*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + ((I/2)*a*f*PolyLog[2, I*E^(c + d*x)])
/d^2 + (a*f*Sech[c + d*x])/(2*d^2) + (b*(e + f*x)*Sech[c + d*x]^2)/(2*d) -
 (b*f*Tanh[c + d*x])/(2*d^2) + (a*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(
2*d))/(a^2 + b^2)))/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2579 vs. 2(693) = 1386.

Time = 24.97 (sec) , antiderivative size = 2580, normalized size of antiderivative = 3.46

method	result	size
risch	Expression too large to display	2580

input
```
int((f*x+e)*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
4/d^2/(a^2+b^2)*c*a^2*f/(4*a^2+4*b^2)*b*arctan(exp(d*x+c))-8/d/(a^2+b^2)*b
^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*a*x-8/d^2/(a^2+b^2)*b^2*f/(4*a^2+4*b
^2)*ln(1+I*exp(d*x+c))*a*c-8/d/(a^2+b^2)*b^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*
x+c))*a*x-8/d^2/(a^2+b^2)*b^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*a*c+(-b*d
*f*x*exp(3*d*x+3*c)+2*a*d*f*x*exp(2*d*x+2*c)-b*d*e*exp(3*d*x+3*c)+2*a*d*e*
exp(2*d*x+2*c)+b*d*f*x*exp(d*x+c)-b*f*exp(3*d*x+3*c)+a*f*exp(2*d*x+2*c)+b*
d*e*exp(d*x+c)-f*b*exp(d*x+c)+a*f)/d^2/(a^2+b^2)/(1+exp(2*d*x+2*c))^2-1/d/
(a^2+b^2)^2*b^4*e/a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/2/d/(a^2+b^2)^
(3/2)*b^2*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d^2/(a^2+b
^2)^2*b^4*f/a*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2))
)-1/d^2/(a^2+b^2)^2*b^4*f/a*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2
+b^2)^(1/2)))-12/d/(a^2+b^2)*b^3*e/(4*a^2+4*b^2)*arctan(exp(d*x+c))+1/2/d/
(a^2+b^2)^(5/2)*b^4*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-4/
d/(a^2+b^2)*a^3*e/(4*a^2+4*b^2)*ln(1+exp(2*d*x+2*c))-4/d^2/(a^2+b^2)*a^3*f
/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+c))-4/d^2/(a^2+b^2)*a^3*f/(4*a^2+4*b^2)*d
ilog(1-I*exp(d*x+c))-6*I/d^2/(a^2+b^2)*b^3*f/(4*a^2+4*b^2)*dilog(1-I*exp(d
*x+c))+1/(a^2+b^2)/d*ln(exp(d*x+c)+1)*a*f*x-1/2/d^2/(a^2+b^2)^(5/2)*c*b^4*
f*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+12/d^2/(a^2+b^2)*c*b^3
*f/(4*a^2+4*b^2)*arctan(exp(d*x+c))-8/d/(a^2+b^2)*b^2*e/(4*a^2+4*b^2)*a*ln
(1+exp(2*d*x+2*c))+1/2/d/(a^2+b^2)^(5/2)*b^2*e*a^2*arctanh(1/2*(2*b*exp...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 7645 vs. $2(676) = 1352$.

Time = 0.36 (sec) , antiderivative size = 7645, normalized size of antiderivative = 10.25

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{csch}\,(dx+c)\,\text{sech}\,(dx+c)^3}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-(b^4*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^5 + 2*a^3*b^2 +
a*b^4)*d) - (a^2*b + 3*b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*
d) + (a^3 + 2*a*b^2)*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d)
 + (b*e^(-d*x - c) - 2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^
2 + 2*(a^2 + b^2)*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d) - lo
g(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d))*e - f*(((b*d*x*e^
(3*c) + b*e^(3*c))*e^(3*d*x) - (2*a*d*x*e^(2*c) + a*e^(2*c))*e^(2*d*x) - (
b*d*x*e^c - b*e^c)*e^(d*x) - a)/(a^2*d^2 + b^2*d^2 + (a^2*d^2*e^(4*c) + b^
2*d^2*e^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*e^(2*d*x)
) - 16*integrate(-1/8*(a*b^4*x*e^(d*x + c) - b^5*x)/(a^5*b + 2*a^3*b^3 + a
*b^5 - (a^5*b*e^(2*c) + 2*a^3*b^3*e^(2*c) + a*b^5*e^(2*c))*e^(2*d*x) - 2*(
a^6*e^c + 2*a^4*b^2*e^c + a^2*b^4*e^c)*e^(d*x)), x) + 16*integrate(1/16*((
a^2*b*e^c + 3*b^3*e^c)*x*e^(d*x) - 2*(a^3 + 2*a*b^2)*x)/(a^4 + 2*a^2*b^2 +
 b^4 + (a^4*e^(2*c) + 2*a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x) + 16
*integrate(1/16*x/(a*e^(d*x + c) + a), x) - 16*integrate(1/16*x/(a*e^(d*x
+ c) - a), x))
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\int \frac{e+f\,x}{\cosh\left(c+d\,x\right)^3\,\sinh\left(c+d\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)/(cosh(c + d*x)^3*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(cosh(c + d*x)^3*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
( - e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**3*b*e - 3*e**(4*c + 4*d*x)*atan
(e**(c + d*x))*a*b**3*e - 2*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**3*b*e -
 6*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a*b**3*e - atan(e**(c + d*x))*a**3*
b*e - 3*atan(e**(c + d*x))*a*b**3*e + 32*e**(9*c + 4*d*x)*int((e**(5*d*x)*
x)/(e**(10*c + 10*d*x)*b + 2*e**(9*c + 9*d*x)*a + e**(8*c + 8*d*x)*b + 4*e
**(7*c + 7*d*x)*a - 2*e**(6*c + 6*d*x)*b - 2*e**(4*c + 4*d*x)*b - 4*e**(3*
c + 3*d*x)*a + e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**5*d*f + 64
*e**(9*c + 4*d*x)*int((e**(5*d*x)*x)/(e**(10*c + 10*d*x)*b + 2*e**(9*c + 9
*d*x)*a + e**(8*c + 8*d*x)*b + 4*e**(7*c + 7*d*x)*a - 2*e**(6*c + 6*d*x)*b
 - 2*e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a + e**(2*c + 2*d*x)*b - 2*e*
*(c + d*x)*a + b),x)*a**3*b**2*d*f + 32*e**(9*c + 4*d*x)*int((e**(5*d*x)*x
)/(e**(10*c + 10*d*x)*b + 2*e**(9*c + 9*d*x)*a + e**(8*c + 8*d*x)*b + 4*e*
*(7*c + 7*d*x)*a - 2*e**(6*c + 6*d*x)*b - 2*e**(4*c + 4*d*x)*b - 4*e**(3*c
 + 3*d*x)*a + e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a*b**4*d*f - e
**(4*c + 4*d*x)*log(e**(2*c + 2*d*x) + 1)*a**4*e - 2*e**(4*c + 4*d*x)*log(
e**(2*c + 2*d*x) + 1)*a**2*b**2*e + e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)
*a**4*e + 2*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**2*b**2*e + e**(4*c +
 4*d*x)*log(e**(c + d*x) - 1)*b**4*e + e**(4*c + 4*d*x)*log(e**(c + d*x) +
 1)*a**4*e + 2*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a**2*b**2*e + e**(4*
c + 4*d*x)*log(e**(c + d*x) + 1)*b**4*e - e**(4*c + 4*d*x)*log(e**(2*c ...
```

3.447 $\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 160

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b^3\arctan(\sinh(c+dx))}{(a^2+b^2)^2 d} - \frac{b\arctan(\sinh(c+dx))}{2(a^2+b^2)d} - \frac{a(a^2+2b^2)\log(\cosh(c+dx))}{(a^2+b^2)^2 d} + \frac{\log(\sinh(c+dx))}{ad} - \frac{b^4\log(a+b\sinh(c+dx))}{a(a^2+b^2)^2 d} + \frac{\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))}{2(a^2+b^2)d}$$

output
```
-b^3*arctan(sinh(d*x+c))/(a^2+b^2)^2/d-1/2*b*arctan(sinh(d*x+c))/(a^2+b^2)
/d-a*(a^2+2*b^2)*ln(cosh(d*x+c))/(a^2+b^2)^2/d+ln(sinh(d*x+c))/a/d-b^4*ln(
a+b*sinh(d*x+c))/a/(a^2+b^2)^2/d+1/2*sech(d*x+c)^2*(a-b*sinh(d*x+c))/(a^2+
b^2)/d
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{ab(a^2+b^2)\arctan(\sinh(c+dx)) - 2(a^2+b^2)^2\log(\sinh(c+dx)) + a\left(a^3+2ab^2+(-b^2)^{3/2}\right)\log\left(\sqrt{-\right.}$$

input
```
Integrate[(Csch[c + d*x]*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*(a*b*(a^2 + b^2)*ArcTan[Sinh[c + d*x]] - 2*(a^2 + b^2)^2*Log[Sinh[c +
 d*x]] + a*(a^3 + 2*a*b^2 + (-b^2)^(3/2))*Log[Sqrt[-b^2] - b*Sinh[c + d*x]
] + 2*b^4*Log[a + b*Sinh[c + d*x]] + a*(a^3 + 2*a*b^2 - (-b^2)^(3/2))*Log[
Sqrt[-b^2] + b*Sinh[c + d*x]] - a^2*(a^2 + b^2)*Sech[c + d*x]^2 + a*b*(a^2
 + b^2)*Sech[c + d*x]*Tanh[c + d*x])/(a*(a^2 + b^2)^2*d)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3042, 26, 3316, 26, 27, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i}{\sin(ic+idx)\cos(ic+idx)^3(a-ib\sin(ic+idx))}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{1}{\cos(ic+idx)^3\sin(ic+idx)(a-ib\sin(ic+idx))}dx$$
$$\Big\downarrow \text{3316}$$

$$\frac{ib^3 \int -\frac{i\text{csch}(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$\Big\downarrow$ 26

$$\frac{b^3 \int \frac{\text{csch}(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$\Big\downarrow$ 27

$$\frac{b^4 \int \frac{\text{csch}(c+dx)}{b(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$\Big\downarrow$ 615

$$\frac{b^4 \int \left(\frac{\text{csch}(c+dx)}{ab^5}-\frac{1}{a(a^2+b^2)^2(a+b\sinh(c+dx))}+\frac{-b^4-a(a^2+2b^2)\sinh(c+dx)b}{b^4(a^2+b^2)^2\left(\sinh^2(c+dx)b^2+b^2\right)}+\frac{-b^2-a\sinh(c+dx)b}{b^2(a^2+b^2)\left(\sinh^2(c+dx)b^2+b^2\right)^2}\right)d(b\sinh(c}{d}$$

$\Big\downarrow$ 2009

$$\frac{b^4\left(-\frac{\arctan(\sinh(c+dx))}{b(a^2+b^2)^2}-\frac{\arctan(\sinh(c+dx))}{2b^3(a^2+b^2)}+\frac{a-b\sinh(c+dx)}{2b^2(a^2+b^2)\left(b^2\sinh^2(c+dx)+b^2\right)}-\frac{\log(a+b\sinh(c+dx))}{a(a^2+b^2)^2}-\frac{a(a^2+2b^2)\log(b^2\sinh^2(c}{2b^4(a^2+b^2)^2}\right.}{d}$$

input
```
Int[(Csch[c + d*x]*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(b^4*(-(ArcTan[Sinh[c + d*x]]/(b*(a^2 + b^2)^2)) - ArcTan[Sinh[c + d*x]]/(
2*b^3*(a^2 + b^2)) + Log[b*Sinh[c + d*x]]/(a*b^4) - Log[a + b*Sinh[c + d*x
]]/(a*(a^2 + b^2)^2) - (a*(a^2 + 2*b^2)*Log[b^2 + b^2*Sinh[c + d*x]^2])/(2
*b^4*(a^2 + b^2)^2) + (a - b*Sinh[c + d*x])/(2*b^2*(a^2 + b^2)*(b^2 + b^2*
Sinh[c + d*x]^2))))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 12.77 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.44

method	result
derivativedivides	$-\frac{\frac{2\left(\frac{\left(-\frac{1}{2}a^2b-\frac{1}{2}b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(a^3+a\,b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(\frac{1}{2}a^2b+\frac{1}{2}b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{\left(2a^3+4a\,b^2\right)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{4}}{a^4+2a^2b^2+b^4}}{d}$
default	$-\frac{\frac{2\left(\frac{\left(-\frac{1}{2}a^2b-\frac{1}{2}b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(a^3+a\,b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(\frac{1}{2}a^2b+\frac{1}{2}b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{\left(2a^3+4a\,b^2\right)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{4}}{a^4+2a^2b^2+b^4}}{d}$
risch	$\frac{2a^3d^2x}{a^4d^2+2a^2b^2d^2+b^4d^2}+\frac{2a^3dc}{a^4d^2+2a^2b^2d^2+b^4d^2}+\frac{4a\,b^2d^2x}{a^4d^2+2a^2b^2d^2+b^4d^2}+\frac{4a\,b^2dc}{a^4d^2+2a^2b^2d^2+b^4d^2}+\frac{2b^4x}{a(a^4+2a^2b^2+}$

input
```
int(csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/(a^4+2*a^2*b^2+b^4)*(((-1/2*a^2*b-1/2*b^3)*tanh(1/2*d*x+1/2*c)^3+(
a^3+a*b^2)*tanh(1/2*d*x+1/2*c)^2+(1/2*a^2*b+1/2*b^3)*tanh(1/2*d*x+1/2*c))/
(1+tanh(1/2*d*x+1/2*c)^2)^2+1/4*(2*a^3+4*a*b^2)*ln(1+tanh(1/2*d*x+1/2*c)^2
)+1/2*(a^2*b+3*b^3)*arctan(tanh(1/2*d*x+1/2*c)))+1/a*ln(tanh(1/2*d*x+1/2*c
))-b^4/a/(a^4+2*a^2*b^2+b^4)*ln(tanh(1/2*d*x+1/2*c)^2*a-2*b*tanh(1/2*d*x+1
/2*c)-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1279 vs. $2(157) = 314$.

Time = 0.26 (sec) , antiderivative size = 1279, normalized size of antiderivative = 7.99

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
-((a^3*b + a*b^3)*cosh(d*x + c)^3 + (a^3*b + a*b^3)*sinh(d*x + c)^3 - 2*(a
^4 + a^2*b^2)*cosh(d*x + c)^2 - (2*a^4 + 2*a^2*b^2 - 3*(a^3*b + a*b^3)*cos
h(d*x + c))*sinh(d*x + c)^2 + ((a^3*b + 3*a*b^3)*cosh(d*x + c)^4 + 4*(a^3*
b + 3*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b + 3*a*b^3)*sinh(d*x +
c)^4 + a^3*b + 3*a*b^3 + 2*(a^3*b + 3*a*b^3)*cosh(d*x + c)^2 + 2*(a^3*b +
3*a*b^3 + 3*(a^3*b + 3*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3*b
 + 3*a*b^3)*cosh(d*x + c)^3 + (a^3*b + 3*a*b^3)*cosh(d*x + c))*sinh(d*x +
c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - (a^3*b + a*b^3)*cosh(d*x + c)
+ (b^4*cosh(d*x + c)^4 + 4*b^4*cosh(d*x + c)*sinh(d*x + c)^3 + b^4*sinh(d*
x + c)^4 + 2*b^4*cosh(d*x + c)^2 + b^4 + 2*(3*b^4*cosh(d*x + c)^2 + b^4)*s
inh(d*x + c)^2 + 4*(b^4*cosh(d*x + c)^3 + b^4*cosh(d*x + c))*sinh(d*x + c)
)*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + ((a^4 + 2
*a^2*b^2)*cosh(d*x + c)^4 + 4*(a^4 + 2*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c
)^3 + (a^4 + 2*a^2*b^2)*sinh(d*x + c)^4 + a^4 + 2*a^2*b^2 + 2*(a^4 + 2*a^2
*b^2)*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + 3*(a^4 + 2*a^2*b^2)*cosh(d*x
+ c)^2)*sinh(d*x + c)^2 + 4*((a^4 + 2*a^2*b^2)*cosh(d*x + c)^3 + (a^4 + 2*
a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c)
- sinh(d*x + c))) - ((a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c)^4 + 4*(a^4 + 2*
a^2*b^2 + b^4)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*sin
h(d*x + c)^4 + a^4 + 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*cosh(d...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.66

$$
\begin{aligned}
\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx
&= -\frac{b^4\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^5+2\,a^3b^2+ab^4)d} \\
&\quad+\frac{(a^2b+3\,b^3)\arctan\left(e^{(-dx-c)}\right)}{(a^4+2\,a^2b^2+b^4)d}-\frac{(a^3+2\,ab^2)\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^4+2\,a^2b^2+b^4)d} \\
&\quad-\frac{be^{(-dx-c)}-2\,ae^{(-2\,dx-2\,c)}-be^{(-3\,dx-3\,c)}}{(a^2+b^2+2\,(a^2+b^2)e^{(-2\,dx-2\,c)}+(a^2+b^2)e^{(-4\,dx-4\,c)})d} \\
&\quad+\frac{\log\left(e^{(-dx-c)}+1\right)}{ad}+\frac{\log\left(e^{(-dx-c)}-1\right)}{ad}
\end{aligned}
$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-b^4*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^5 + 2*a^3*b^2 + a
*b^4)*d) + (a^2*b + 3*b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d
) - (a^3 + 2*a*b^2)*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d)
- (b*e^(-d*x - c) - 2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^2
 + 2*(a^2 + b^2)*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d) + log
(e^(-d*x - c) + 1)/(a*d) + log(e^(-d*x - c) - 1)/(a*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(157) = 314.

Time = 0.14 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.14

$$
\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{4\,b^5\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^5b+2\,a^3b^3+ab^5}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(a^2b+3\,b^3\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\,\left(a^3+2\,ab^2\right)\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)\right.}{a^4+2\,a^2b^2+b^4}}{}
$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*(4*b^5*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^5*b + 2*a^3*
b^3 + a*b^5) + (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(a^
2*b + 3*b^3)/(a^4 + 2*a^2*b^2 + b^4) + 2*(a^3 + 2*a*b^2)*log((e^(d*x + c)
- e^(-d*x - c))^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) - 4*log(abs(e^(d*x + c) - e
^(-d*x - c)))/a - 2*(a^3*(e^(d*x + c) - e^(-d*x - c))^2 + 2*a*b^2*(e^(d*x
+ c) - e^(-d*x - c))^2 - 2*a^2*b*(e^(d*x + c) - e^(-d*x - c)) - 2*b^3*(e^(
d*x + c) - e^(-d*x - c)) + 8*a^3 + 12*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*((e^
(d*x + c) - e^(-d*x - c))^2 + 4)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\mathrm{csch}(c+dx)\mathrm{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^3\,\sinh\left(c+d\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)^3*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(cosh(c + d*x)^3*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1018, normalized size of antiderivative = 6.36

$$\int \frac{\mathrm{csch}(c+dx)\mathrm{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
( - e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**3*b - 3*e**(4*c + 4*d*x)*atan(e
**(c + d*x))*a*b**3 - 2*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**3*b - 6*e**
(2*c + 2*d*x)*atan(e**(c + d*x))*a*b**3 - atan(e**(c + d*x))*a**3*b - 3*at
an(e**(c + d*x))*a*b**3 - e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x) + 1)*a**4
- 2*e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2*b**2 + e**(4*c + 4*d*x
)*log(e**(c + d*x) - 1)*a**4 + 2*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a*
*2*b**2 + e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*b**4 + e**(4*c + 4*d*x)*l
og(e**(c + d*x) + 1)*a**4 + 2*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a**2*
b**2 + e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*b**4 - e**(4*c + 4*d*x)*log(
e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*b**4 - e**(4*c + 4*d*x)*a**4 -
e**(4*c + 4*d*x)*a**2*b**2 - e**(3*c + 3*d*x)*a**3*b - e**(3*c + 3*d*x)*a*
b**3 - 2*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x) + 1)*a**4 - 4*e**(2*c + 2*d
*x)*log(e**(2*c + 2*d*x) + 1)*a**2*b**2 + 2*e**(2*c + 2*d*x)*log(e**(c + d
*x) - 1)*a**4 + 4*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a**2*b**2 + 2*e**
(2*c + 2*d*x)*log(e**(c + d*x) - 1)*b**4 + 2*e**(2*c + 2*d*x)*log(e**(c +
d*x) + 1)*a**4 + 4*e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*a**2*b**2 + 2*e*
*(2*c + 2*d*x)*log(e**(c + d*x) + 1)*b**4 - 2*e**(2*c + 2*d*x)*log(e**(2*c
 + 2*d*x)*b + 2*e**(c + d*x)*a - b)*b**4 + e**(c + d*x)*a**3*b + e**(c + d
*x)*a*b**3 - log(e**(2*c + 2*d*x) + 1)*a**4 - 2*log(e**(2*c + 2*d*x) + 1)*
a**2*b**2 + log(e**(c + d*x) - 1)*a**4 + 2*log(e**(c + d*x) - 1)*a**2*b...
```

3.448 $\int \frac{\text{csch}(c+dx)\text{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\text{csch}(c+dx)\text{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\text{csch}(c+dx)\text{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(csch(d*x+c)*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 96.79 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\text{csch}(c+dx)\text{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\text{csch}(c+dx)\text{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Csch[c + d*x]*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Csch[c + d*x]*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

$$\Big\downarrow \; 6125$$

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Csch[c + d*x]*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)^3}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(csch(d*x+c)*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 19.73 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)^3}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(csch(d*x + c)*sech(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 2.40 (sec) , antiderivative size = 1214, normalized size of antiderivative = 35.71

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)^3}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-(a*f + (b*d*f*x*e^(3*c) + (d*e - f)*b*e^(3*c))*e^(3*d*x) - (2*a*d*f*x*e^(
2*c) + (2*d*e - f)*a*e^(2*c))*e^(2*d*x) - (b*d*f*x*e^c + (d*e + f)*b*e^c)*
e^(d*x))/(a^2*d^2*e^2 + b^2*d^2*e^2 + (a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + 2*
(a^2*d^2*e*f + b^2*d^2*e*f)*x + (a^2*d^2*e^2*e^(4*c) + b^2*d^2*e^2*e^(4*c)
 + (a^2*d^2*f^2*e^(4*c) + b^2*d^2*f^2*e^(4*c))*x^2 + 2*(a^2*d^2*e*f*e^(4*c
) + b^2*d^2*e*f*e^(4*c))*x)*e^(4*d*x) + 2*(a^2*d^2*e^2*e^(2*c) + b^2*d^2*e
^2*e^(2*c) + (a^2*d^2*f^2*e^(2*c) + b^2*d^2*f^2*e^(2*c))*x^2 + 2*(a^2*d^2*
e*f*e^(2*c) + b^2*d^2*e*f*e^(2*c))*x)*e^(2*d*x)) + 16*integrate(-1/8*(a*b^
4*e^(d*x + c) - b^5)/(a^5*b*e + 2*a^3*b^3*e + a*b^5*e + (a^5*b*f + 2*a^3*b
^3*f + a*b^5*f)*x - (a^5*b*e*e^(2*c) + 2*a^3*b^3*e*e^(2*c) + a*b^5*e*e^(2*
c) + (a^5*b*f*e^(2*c) + 2*a^3*b^3*f*e^(2*c) + a*b^5*f*e^(2*c))*x)*e^(2*d*x
) - 2*(a^6*e*e^c + 2*a^4*b^2*e*e^c + a^2*b^4*e*e^c + (a^6*f*e^c + 2*a^4*b^
2*f*e^c + a^2*b^4*f*e^c)*x)*e^(d*x)), x) - 16*integrate(-1/16*(2*(d^2*e^2
- f^2)*a^3 + 2*(2*d^2*e^2 - f^2)*a*b^2 + 2*(a^3*d^2*f^2 + 2*a*b^2*d^2*f^2)
*x^2 + 4*(a^3*d^2*e*f + 2*a*b^2*d^2*e*f)*x - ((d^2*e^2 - 2*f^2)*a^2*b*e^c
+ (3*d^2*e^2 - 2*f^2)*b^3*e^c + (a^2*b*d^2*f^2*e^c + 3*b^3*d^2*f^2*e^c)*x^
2 + 2*(a^2*b*d^2*e*f*e^c + 3*b^3*d^2*e*f*e^c)*x)*e^(d*x))/(a^4*d^2*e^3 + 2
*a^2*b^2*d^2*e^3 + b^4*d^2*e^3 + (a^4*d^2*f^3 + 2*a^2*b^2*d^2*f^3 + b^4*d^
2*f^3)*x^3 + 3*(a^4*d^2*e*f^2 + 2*a^2*b^2*d^2*e*f^2 + b^4*d^2*e*f^2)*x^2 +
 3*(a^4*d^2*e^2*f + 2*a^2*b^2*d^2*e^2*f + b^4*d^2*e^2*f)*x + (a^4*d^2*e...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 11.46 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$=\int \frac{1}{\cosh\left(c+d\,x\right)^3\sinh\left(c+d\,x\right)\,\left(e+f\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)^3*sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(cosh(c + d*x)^3*sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$=\int \frac{\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)^3}{\sinh\left(dx+c\right)be+\sinh\left(dx+c\right)bfx+ae+afx}dx$$

input
```
int(csch(d*x+c)*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int((csch(c + d*x)*sech(c + d*x)**3)/(sinh(c + d*x)*b*e + sinh(c + d*x)*b*
f*x + a*e + a*f*x),x)
```

3.449 $\int \frac{(e+fx)^3 \coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 601

$$\int \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= -\frac{6f(e+fx)^2\text{arctanh}(e^{c+dx})}{ad^2} - \frac{(e+fx)^3\text{csch}(c+dx)}{ad} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d} - \frac{b(e+fx)^3\log\left(1-e^{2(c+dx)}\right)}{a^2d} - \frac{6f^2(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^3} + \frac{6f^2(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^3} + \frac{3bf(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2} + \frac{3bf(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^2} - \frac{3bf(e+fx)^2\,\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{2a^2d^2} + \frac{6f^3\,\text{PolyLog}\left(3,-e^{c+dx}\right)}{ad^4} - \frac{6f^3\,\text{PolyLog}\left(3,e^{c+dx}\right)}{ad^4} - \frac{6bf^2(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3} - \frac{6bf^2(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^3} + \frac{3bf^2(e+fx)\,\text{PolyLog}\left(3,e^{2(c+dx)}\right)}{2a^2d^3} + \frac{6bf^3\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^4} + \frac{6bf^3\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^4} - \frac{3bf^3\,\text{PolyLog}\left(4,e^{2(c+dx)}\right)}{4a^2d^4}$$

output
```
-6*f*(f*x+e)^2*arctanh(exp(d*x+c))/a/d^2-(f*x+e)^3*csch(d*x+c)/a/d+b*(f*x+
e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d+b*(f*x+e)^3*ln(1+b*exp(d
*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d-b*(f*x+e)^3*ln(1-exp(2*d*x+2*c))/a^2/d-6*
f^2*(f*x+e)*polylog(2,-exp(d*x+c))/a/d^3+6*f^2*(f*x+e)*polylog(2,exp(d*x+c
))/a/d^3+3*b*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/
d^2+3*b*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d^2-3
/2*b*f*(f*x+e)^2*polylog(2,exp(2*d*x+2*c))/a^2/d^2+6*f^3*polylog(3,-exp(d*
x+c))/a/d^4-6*f^3*polylog(3,exp(d*x+c))/a/d^4-6*b*f^2*(f*x+e)*polylog(3,-b
*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d^3-6*b*f^2*(f*x+e)*polylog(3,-b*exp(
d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d^3+3/2*b*f^2*(f*x+e)*polylog(3,exp(2*d*x+
2*c))/a^2/d^3+6*b*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d^4
+6*b*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d^4-3/4*b*f^3*po
lylog(4,exp(2*d*x+2*c))/a^2/d^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2598 vs. 2(601) = 1202.

Time = 10.60 (sec) , antiderivative size = 2598, normalized size of antiderivative = 4.32

$$\int \frac{(e+fx)^3 \coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
-(((e + f*x)^3*Csch[c])/(a*d)) + (2*d^3*e^2*(-1 + E^(2*c))*f*(b*d*e - 3*a*
f)*x + 2*d^3*e^2*(-1 + E^(2*c))*f*(b*d*e + 3*a*f)*x + b*d^4*(e + f*x)^4 -
6*d^2*e*(-1 + E^(2*c))*f^2*(b*d*e - 2*a*f)*x*Log[1 - E^(-c - d*x)] - 6*d^2
*(-1 + E^(2*c))*f^3*(b*d*e - a*f)*x^2*Log[1 - E^(-c - d*x)] - 2*b*d^3*(-1
+ E^(2*c))*f^4*x^3*Log[1 - E^(-c - d*x)] - 6*d^2*e*(-1 + E^(2*c))*f^2*(b*d
*e + 2*a*f)*x*Log[1 + E^(-c - d*x)] - 6*d^2*(-1 + E^(2*c))*f^3*(b*d*e + a*
f)*x^2*Log[1 + E^(-c - d*x)] - 2*b*d^3*(-1 + E^(2*c))*f^4*x^3*Log[1 + E^(-
c - d*x)] - 2*d^2*e^2*(-1 + E^(2*c))*f*(b*d*e - 3*a*f)*Log[1 - E^(c + d*x)
] - 2*d^2*e^2*(-1 + E^(2*c))*f*(b*d*e + 3*a*f)*Log[1 + E^(c + d*x)] + 6*d*
e*(-1 + E^(2*c))*f^2*(b*d*e + 2*a*f)*PolyLog[2, -E^(-c - d*x)] + 12*d*(-1
+ E^(2*c))*f^3*(b*d*e + a*f)*x*PolyLog[2, -E^(-c - d*x)] + 6*b*d^2*(-1 + E
^(2*c))*f^4*x^2*PolyLog[2, -E^(-c - d*x)] + 6*d*e*(-1 + E^(2*c))*f^2*(b*d*
e - 2*a*f)*PolyLog[2, E^(-c - d*x)] + 12*d*(-1 + E^(2*c))*f^3*(b*d*e - a*f
)*x*PolyLog[2, E^(-c - d*x)] + 6*b*d^2*(-1 + E^(2*c))*f^4*x^2*PolyLog[2, E
^(-c - d*x)] + 12*(-1 + E^(2*c))*f^3*(b*d*e + a*f)*PolyLog[3, -E^(-c - d*x
)] + 12*b*d*(-1 + E^(2*c))*f^4*x*PolyLog[3, -E^(-c - d*x)] - 12*(-1 + E^(2
*c))*f^3*(-(b*d*e) + a*f)*PolyLog[3, E^(-c - d*x)] + 12*b*d*(-1 + E^(2*c))
*f^4*x*PolyLog[3, E^(-c - d*x)] + 12*b*(-1 + E^(2*c))*f^4*PolyLog[4, -E^(-
c - d*x)] + 12*b*(-1 + E^(2*c))*f^4*PolyLog[4, E^(-c - d*x)])/(2*a^2*d^4*(
-1 + E^(2*c))*f) - (b*(4*e^3*E^(2*c)*x + 6*e^2*E^(2*c)*f*x^2 + 4*e*E^(2...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 4.45 (sec) , antiderivative size = 681, normalized size of antiderivative = 1.13, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6121, 5975, 3042, 26, 4670, 3011, 2720, 6103, 3042, 26, 4201, 2620, 3011, 6095, 2620, 3011, 7143, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6121}$$

$$\frac{\int (e+fx)^3\coth(c+dx)\operatorname{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5975}$$

$$\frac{\frac{3f\int (e+fx)^2\operatorname{csch}(c+dx)dx}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3f\int i(e+fx)^2\csc(ic+idx)dx}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\int (e+fx)^2\csc(ic+idx)dx}{d}}{a}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{b\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\left(\frac{2if\int (e+fx)\log\left(1-e^{c+dx}\right)dx}{d}-\frac{2if\int (e+fx)\log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)^2\operatorname{arctanh}\left(e^{c+dx}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{3011}$$

$$\frac{-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,e\right.}{d}\right.}{d}}{d}}{a} - \frac{b\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$\downarrow$ 2720

$$\frac{-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de\right.}{d^2}}{d}}{d}}{a} - \frac{b\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$\downarrow$ 6103

$$-\frac{b\left(\frac{\int (e+fx)^3\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+$$

$$\frac{-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de\right.}{d^2}}{d}}{d}}{a}$$

$\downarrow$ 3042

$$\frac{-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de\right.}{d^2}}{d}}{d}}{a}$$

$$\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i(e+fx)^3\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{a}\right)}{a}$$

$\downarrow$ 26

$$-\frac{\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}\cdots\right.}{d}\right.}{d}}{a}}{}\quad
\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\int(e+fx)^3\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{a}\right)}{a}$$

$\downarrow$ 4201

$$-\frac{\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}\cdots\right.}{d}\right.}{d}}{a}}{}\quad
\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\int\frac{e^{2c+2dx-i\pi}(e+fx)^3}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^4}{4f}\right)}{a}\right)}{a}$$

$\downarrow$ 2620

$$-\frac{\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}\cdots\right.}{d}\right.}{d}}{a}}{}\quad
\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\int(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{a}\right)}{a}$$

$\downarrow$ 3011

$$\frac{-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}\cdots\right.}{d}\cdots\right.}{d}\cdots}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{2c+2dx-\cdots}\right)}{2d}\cdots\right.}{2d}\cdots\right.\right.}{a}\cdots\right.}{a}$$

$$\Big\downarrow \text{6095}$$

$$\frac{-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}\cdots\right.}{d}\cdots\right.}{d}\cdots}{a}$$

$$\frac{b\left(-\frac{b\left(\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^4}{4bf}\right)}{a}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\text{PolyLog}\left(2,-e^{2c+2d\cdots}\right.}{d}\cdots\right.}{\cdots}\cdots\right.\right.}{a}\cdots\right.}{a}$$

$$\Big\downarrow \text{2620}$$

$$\frac{-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}\cdots\right.}{d}\cdots\right.}{d}\cdots}{a}$$

$$\frac{b\left(-\frac{b\left(-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e\cdots}{\cdots}\right.}{a}\cdots\right.}{a}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}(2,-e^{c+dx})de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}(2,-e^{c+dx})}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}(2,e^{c+dx})de^{c+dx}}{d^2}\cdots\right.}{d}\cdots\right.}{d}}{a}$$

$$\frac{b\left(-\frac{b\left(-\frac{3f\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\cdots}{}\right.}{bd}\cdots\right)}{a}\cdots\right)}{}$$

$\downarrow$ 7143

$$-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3if\left(\frac{2i(e+fx)^2\text{arctanh}(e^{c+dx})}{d}-\frac{2if\left(\frac{f\text{PolyLog}(3,-e^{c+dx})}{d^2}-\frac{(e+fx)\text{PolyLog}(2,-e^{c+dx})}{d}\right)}{d}+\frac{2if\left(\frac{f\text{PolyLog}(3,e^{c+dx})}{d^2}\cdots\right.}{}\cdots\right.}{d}}{a}$$

$$\frac{b\left(-\frac{b\left(-\frac{3f\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\cdots}{}\right.}{bd}\cdots\right)}{a}\cdots\right)}{}$$

$\downarrow$ 7163

$$\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\left(\frac{2i(e+fx)^2\operatorname{arctanh}\left(e^{c+dx}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{d^2}\right.}{}\right.}{d}}{a}$$

$$b\left(-\frac{b\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{Poly}}{}\right.}{}\right.}{}\right.}{}\right.$$

$\downarrow$ 2720

$$-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\left(\frac{2i(e+fx)^2\operatorname{arctanh}\left(e^{c+dx}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{d^2}\right.}{}\right.}{d}$$

$$a$$

$$\frac{b\left(-\frac{b\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f}{}\right.}{}\right.}{}\right.}{}$$

$\downarrow$ 7143

$$\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\left(\frac{2i(e+fx)^2\operatorname{arctanh}\left(e^{c+dx}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{d^2}\cdots\right.}{\cdots}\right.}{d}}{a}$$

$$b\left(-\frac{b\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Int[((e + f*x)^3*Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-(((e + f*x)^3*Csch[c + d*x])/d) + ((3*I)*f*(((2*I)*(e + f*x)^2*ArcTanh[E
^(c + d*x)])/d - ((2*I)*f*(-(((e + f*x)*PolyLog[2, -E^(c + d*x)])/d) + (f*
PolyLog[3, -E^(c + d*x)])/d^2))/d + ((2*I)*f*(-(((e + f*x)*PolyLog[2, E^(c
 + d*x)])/d) + (f*PolyLog[3, E^(c + d*x)])/d^2))/d))/d)/a - (b*(-((b*(-1/4
*(e + f*x)^4/(b*f) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2])])/(b*d) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2
 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])
/d^2))/d))/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2]))])/d^2))/d))/(b*d)))/a) - (I*(((-1/4*I)*(e + f*x)^4)/f + (2*I)*(((e
+ f*x)^3*Log[1 + E^(2*c - I*Pi + 2*d*x)])/(2*d) - (3*f*(-1/2*((e + f*x)^2*
PolyLog[2, -E^(2*c - I*Pi + 2*d*x)])/d + (f*(((e + f*x)*PolyLog[3, -E^(2*c
 - I*Pi + 2*d*x)])/(2*d) - (f*PolyLog[4, -E^(2*c - I*Pi + 2*d*x)])/(4*d^2)
))/d))/(2*d))))/a))/a
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5975
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6103
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Coth[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x
]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6121
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Csch[c + d*x]^(p - 1)*(Coth[c + d*x]^n/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(fx+e)^3 \coth(dx+c) \operatorname{csch}(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4313 vs. 2(561) = 1122.

Time = 0.17 (sec) , antiderivative size = 4313, normalized size of antiderivative = 7.18

$$\int \frac{(e+fx)^3 \coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^3 \coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3 \coth(dx+c)\operatorname{csch}(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
e^3*(2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) + b*log(-2*a*e^(-d*x - c)
 + b*e^(-2*d*x - 2*c) - b)/(a^2*d) - b*log(e^(-d*x - c) + 1)/(a^2*d) - b*l
og(e^(-d*x - c) - 1)/(a^2*d)) - 2*(f^3*x^3*e^c + 3*e*f^2*x^2*e^c + 3*e^2*f
*x*e^c)*e^(d*x)/(a*d*e^(2*d*x + 2*c) - a*d) - 3*e^2*f*log(e^(d*x + c) + 1)
/(a*d^2) + 3*e^2*f*log(e^(d*x + c) - 1)/(a*d^2) - (d^3*x^3*log(e^(d*x + c)
 + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) + 6
*polylog(4, -e^(d*x + c)))*b*f^3/(a^2*d^4) - (d^3*x^3*log(-e^(d*x + c) + 1
) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polyl
og(4, e^(d*x + c)))*b*f^3/(a^2*d^4) - 3*(b*d*e^2*f + 2*a*e*f^2)*(d*x*log(e
^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^2*d^3) - 3*(b*d*e^2*f - 2*a*e*f^
2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^2*d^3) - 3*(b*d*e*f
^2 + a*f^3)*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*
polylog(3, -e^(d*x + c)))/(a^2*d^4) - 3*(b*d*e*f^2 - a*f^3)*(d^2*x^2*log(-
e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))/(
a^2*d^4) + 1/4*(b*d^4*f^3*x^4 + 4*(b*d*e*f^2 + a*f^3)*d^3*x^3 + 6*(b*d^2*e
^2*f + 2*a*d*e*f^2)*d^2*x^2)/(a^2*d^4) + 1/4*(b*d^4*f^3*x^4 + 4*(b*d*e*f^2
 - a*f^3)*d^3*x^3 + 6*(b*d^2*e^2*f - 2*a*d*e*f^2)*d^2*x^2)/(a^2*d^4) - int
egrate(-2*(b^2*f^3*x^3 + 3*b^2*e*f^2*x^2 + 3*b^2*e^2*f*x - (a*b*f^3*x^3*e^
c + 3*a*b*e*f^2*x^2*e^c + 3*a*b*e^2*f*x*e^c)*e^(d*x))/(a^2*b*e^(2*d*x + 2*
c) + 2*a^3*e^(d*x + c) - a^2*b), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+d\,x)\,\left(e+f\,x\right)^3}{\sinh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((coth(c + d*x)*(e + f*x)^3)/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((coth(c + d*x)*(e + f*x)^3)/(sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 16*e**(5*c + 2*d*x)*int((e**(3*d*x)*x**3)/(e**(6*c + 6*d*x)*b + 2*e**(
5*c + 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*c +
 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**3*d**4*f**3 - 48*e**(5*c + 2*d*x)*
int((e**(3*d*x)*x**2)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4
*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b + 2*e**(c + d*
x)*a - b),x)*a**3*d**4*e*f**2 - 48*e**(5*c + 2*d*x)*int((e**(3*d*x)*x)/(e*
*(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e**(3*c
 + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**3*d**4*e*
*2*f + 16*e**(4*c + 2*d*x)*int((e**(2*d*x)*x**3)/(e**(6*c + 6*d*x)*b + 2*e
**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*
c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b*d**4*f**3 + 48*e**(4*c + 2*
d*x)*int((e**(2*d*x)*x**2)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*
e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b + 2*e**(c
 + d*x)*a - b),x)*a**2*b*d**4*e*f**2 - 12*e**(4*c + 2*d*x)*int((e**(2*d*x)
*x**2)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d*x)*b -
 4*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a*
*2*b*d**3*f**3 + 48*e**(4*c + 2*d*x)*int((e**(2*d*x)*x)/(e**(6*c + 6*d*x)*
b + 2*e**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a + 3
*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b*d**4*e**2*f - 24*e**
(4*c + 2*d*x)*int((e**(2*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*...
```

3.450 $\int \frac{(e+fx)^2 \coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 419

$$\begin{aligned}
&\int \frac{(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= -\frac{4f(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{ad^2} - \frac{(e+fx)^2\operatorname{csch}(c+dx)}{ad} \\
&\quad + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d} + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d} \\
&\quad - \frac{b(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{a^2d} - \frac{2f^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{ad^3} + \frac{2f^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{ad^3} \\
&\quad + \frac{2bf(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2} + \frac{2bf(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^2} \\
&\quad - \frac{bf(e+fx)\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{a^2d^2} - \frac{2bf^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3} \\
&\quad - \frac{2bf^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^3} + \frac{bf^2\operatorname{PolyLog}\left(3,e^{2(c+dx)}\right)}{2a^2d^3}
\end{aligned}$$

output
```
-4*f*(f*x+e)*arctanh(exp(d*x+c))/a/d^2-(f*x+e)^2*csch(d*x+c)/a/d+b*(f*x+e)
^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d+b*(f*x+e)^2*ln(1+b*exp(d*x
+c)/(a+(a^2+b^2)^(1/2)))/a^2/d-b*(f*x+e)^2*ln(1-exp(2*d*x+2*c))/a^2/d-2*f^
2*polylog(2,-exp(d*x+c))/a/d^3+2*f^2*polylog(2,exp(d*x+c))/a/d^3+2*b*f*(f*
x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d^2+2*b*f*(f*x+e)*po
lylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d^2-b*f*(f*x+e)*polylog(2,e
xp(2*d*x+2*c))/a^2/d^2-2*b*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2))
)/a^2/d^3-2*b*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d^3+1/2
*b*f^2*polylog(3,exp(2*d*x+2*c))/a^2/d^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1735 vs. $2(419) = 838$.

Time = 10.06 (sec) , antiderivative size = 1735, normalized size of antiderivative = 4.14

$$\int \frac{(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
-(((e + f*x)^2*Csch[c])/(a*d)) + (3*d^2*e*(-1 + E^(2*c))*f*(b*d*e - 2*a*f)
*x + 3*d^2*e*(-1 + E^(2*c))*f*(b*d*e + 2*a*f)*x + 2*b*d^3*(e + f*x)^3 - 6*
d*(-1 + E^(2*c))*f^2*(b*d*e - a*f)*x*Log[1 - E^(-c - d*x)] - 3*b*d^2*(-1 +
 E^(2*c))*f^3*x^2*Log[1 - E^(-c - d*x)] - 6*d*(-1 + E^(2*c))*f^2*(b*d*e +
a*f)*x*Log[1 + E^(-c - d*x)] - 3*b*d^2*(-1 + E^(2*c))*f^3*x^2*Log[1 + E^(-
c - d*x)] - 3*d*e*(-1 + E^(2*c))*f*(b*d*e - 2*a*f)*Log[1 - E^(c + d*x)] -
3*d*e*(-1 + E^(2*c))*f*(b*d*e + 2*a*f)*Log[1 + E^(c + d*x)] + 6*(-1 + E^(2
*c))*f^2*(b*d*e + a*f)*PolyLog[2, -E^(-c - d*x)] + 6*b*d*(-1 + E^(2*c))*f^
3*x*PolyLog[2, -E^(-c - d*x)] - 6*(-1 + E^(2*c))*f^2*(-(b*d*e) + a*f)*Poly
Log[2, E^(-c - d*x)] + 6*b*d*(-1 + E^(2*c))*f^3*x*PolyLog[2, E^(-c - d*x)]
 + 6*b*(-1 + E^(2*c))*f^3*PolyLog[3, -E^(-c - d*x)] + 6*b*(-1 + E^(2*c))*f
^3*PolyLog[3, E^(-c - d*x)])/(3*a^2*d^3*(-1 + E^(2*c))*f) - (b*(6*e^2*E^(2
*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*A
rcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6
*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 -
 b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a +
 b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2
 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^
2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))]
)/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.81 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.17, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.594$, Rules used = {6121, 5975, 3042, 26, 4670, 2715, 2838, 6103, 3042, 26, 4201, 2620, 3011, 2720, 6095, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6121}$$

$$\frac{\int (e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5975}$$

$$\frac{\frac{2f\int (e+fx)\text{csch}(c+dx)dx}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2f\int i(e+fx)\csc(ic+idx)dx}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\int (e+fx)\csc(ic+idx)dx}{d}}{a}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{b\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{if\int \log\left(1-e^{c+dx}\right)dx}{d}-\frac{if\int \log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{b\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{if\int e^{-c-dx}\log\left(1-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log\left(1+e^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{b\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \text{6103}$$

$$-\frac{b\left(\frac{\int (e+fx)^2\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\text{ PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\text{ PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int-i(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{a}\right)}{a}$$

$\Big\downarrow$ 26

$$\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\text{ PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\text{ PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\int(e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{a}\right)}{a}$$

$\Big\downarrow$ 4201

$$\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\text{ PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\text{ PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\int\frac{e^{2c+2dx-i\pi}(e+fx)^2}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^3}{3f}\right)}{a}\right)}{a}$$

$\Big\downarrow$ 2620

$$\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\text{ PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\text{ PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{a}\right)}{a}$$

$\Big\downarrow$ 3011

$$\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{2d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\right.}{a}\right.}{a}$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLo}}{}\right.}{d}\right.\right.}{a}\right.}{a}$$

$$\Big\downarrow \text{ 6095}$$

$$\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\left(\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{a}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e\right.}{4d^2}\right.}{}\right.\right.}{}\right.}{a}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-$$

$$b\left(-\frac{b\left(-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+f}{3b}\right.}{a}\right.$$

$\downarrow$ 3011

$$\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-$$

$$b\left(-\frac{b\left(-\frac{2f\left(\frac{f\int\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c}}{a+\sqrt{}}\right.}{d}\right.}{bd}\right.}{a}\right.$$

$\downarrow$ 2720

$$\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-$$

$$b\left(-\frac{b\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\right.}{bd}\right.}{a}\right.$$

$\downarrow$ 7143

$$\frac{-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-$$

$$b\left(-\frac{b\left(-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right.}{bd}\right.}{a}\right.$$

input
```
Int[((e + f*x)^2*Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-(((e + f*x)^2*Csch[c + d*x])/d) + ((2*I)*f*(((2*I)*(e + f*x)*ArcTanh[E^(
c + d*x)])/d + (I*f*PolyLog[2, -E^(c + d*x)])/d^2 - (I*f*PolyLog[2, E^(c +
 d*x)])/d^2))/d)/a - (b*(-((b*(-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1
 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2
, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c
+ d*x))/(a - Sqrt[a^2 + b^2]))])/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[
2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/a) - (I*(((-1/3*I)*(e + f*
x)^3)/f + (2*I)*(((e + f*x)^2*Log[1 + E^(2*c - I*Pi + 2*d*x)])/(2*d) - (f*
(-1/2*((e + f*x)*PolyLog[2, -E^(2*c - I*Pi + 2*d*x)])/d + (f*PolyLog[3, -E
^(2*c - I*Pi + 2*d*x)])/(4*d^2)))/d)))/a))/a
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5975
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6103
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Coth[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x
]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6121
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Csch[c + d*x]^(p - 1)*(Coth[c + d*x]^n/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(fx+e)^2 \coth(dx+c) \operatorname{csch}(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2528 vs. $2(391) = 782$.

Time = 0.13 (sec) , antiderivative size = 2528, normalized size of antiderivative = 6.03

$$\int \frac{(e+fx)^2 \coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
-(2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cosh(d*x + c) + 2*(b*d*f^2
*x + b*d*e*f - (b*d*f^2*x + b*d*e*f)*cosh(d*x + c)^2 - 2*(b*d*f^2*x + b*d*
e*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f^2*x + b*d*e*f)*sinh(d*x + c)^2)*
dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x +
 c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(b*d*f^2*x + b*d*e*f - (b*d*f^2
*x + b*d*e*f)*cosh(d*x + c)^2 - 2*(b*d*f^2*x + b*d*e*f)*cosh(d*x + c)*sinh
(d*x + c) - (b*d*f^2*x + b*d*e*f)*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c)
+ a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2) - b)/b + 1) - 2*(b*d*f^2*x + b*d*e*f - a*f^2 - (b*d*f^2*x + b*d*e*f -
a*f^2)*cosh(d*x + c)^2 - 2*(b*d*f^2*x + b*d*e*f - a*f^2)*cosh(d*x + c)*sin
h(d*x + c) - (b*d*f^2*x + b*d*e*f - a*f^2)*sinh(d*x + c)^2)*dilog(cosh(d*x
 + c) + sinh(d*x + c)) - 2*(b*d*f^2*x + b*d*e*f + a*f^2 - (b*d*f^2*x + b*d
*e*f + a*f^2)*cosh(d*x + c)^2 - 2*(b*d*f^2*x + b*d*e*f + a*f^2)*cosh(d*x +
 c)*sinh(d*x + c) - (b*d*f^2*x + b*d*e*f + a*f^2)*sinh(d*x + c)^2)*dilog(-
cosh(d*x + c) - sinh(d*x + c)) + (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2 - (b
*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*cosh(d*x + c)^2 - 2*(b*d^2*e^2 - 2*b*c
*d*e*f + b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) - (b*d^2*e^2 - 2*b*c*d*e*f
 + b*c^2*f^2)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) +
 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2 -
 (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*cosh(d*x + c)^2 - 2*(b*d^2*e^2 -...
```

Sympy [F]

$$\int \frac{(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*coth(c + d*x)*csch(c + d*x)/(a + b*sinh(c + d*x)), x
)
```

Maxima [F]

$$\int \frac{(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\coth(dx+c)\,\text{csch}(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
e^2*(2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) + b*log(-2*a*e^(-d*x - c)
 + b*e^(-2*d*x - 2*c) - b)/(a^2*d) - b*log(e^(-d*x - c) + 1)/(a^2*d) - b*l
og(e^(-d*x - c) - 1)/(a^2*d)) - 2*(f^2*x^2*e^c + 2*e*f*x*e^c)*e^(d*x)/(a*d
*e^(2*d*x + 2*c) - a*d) - 2*e*f*log(e^(d*x + c) + 1)/(a*d^2) + 2*e*f*log(e
^(d*x + c) - 1)/(a*d^2) - (d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(
d*x + c)) - 2*polylog(3, -e^(d*x + c)))*b*f^2/(a^2*d^3) - (d^2*x^2*log(-e^
(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*b*f
^2/(a^2*d^3) - 2*(b*d*e*f + a*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d
*x + c)))/(a^2*d^3) - 2*(b*d*e*f - a*f^2)*(d*x*log(-e^(d*x + c) + 1) + dil
og(e^(d*x + c)))/(a^2*d^3) + 1/3*(b*d^3*f^2*x^3 + 3*(b*d*e*f + a*f^2)*d^2*
x^2)/(a^2*d^3) + 1/3*(b*d^3*f^2*x^3 + 3*(b*d*e*f - a*f^2)*d^2*x^2)/(a^2*d^
3) - integrate(-2*(b^2*f^2*x^2 + 2*b^2*e*f*x - (a*b*f^2*x^2*e^c + 2*a*b*e*
f*x*e^c)*e^(d*x))/(a^2*b*e^(2*d*x + 2*c) + 2*a^3*e^(d*x + c) - a^2*b), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+d\,x)\,\left(e+f\,x\right)^2}{\sinh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((coth(c + d*x)*(e + f*x)^2)/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((coth(c + d*x)*(e + f*x)^2)/(sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^2*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 8*e**(5*c + 2*d*x)*int((e**(3*d*x)*x**2)/(e**(6*c + 6*d*x)*b + 2*e**(5
*c + 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*c +
2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**3*d**3*f**2 - 16*e**(5*c + 2*d*x)*i
nt((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4*c +
 4*d*x)*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a
 - b),x)*a**3*d**3*e*f + 8*e**(4*c + 2*d*x)*int((e**(2*d*x)*x**2)/(e**(6*c
 + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*
d*x)*a + 3*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b*d**3*f**2
+ 16*e**(4*c + 2*d*x)*int((e**(2*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c +
 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*
x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b*d**3*e*f - 4*e**(4*c + 2*d*x)*int((
e**(2*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d
*x)*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b
),x)*a**2*b*d**2*f**2 - 8*e**(3*c + 2*d*x)*int((e**(d*x)*x)/(e**(6*c + 6*d
*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a
 + 3*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**3*d**2*f**2 + 4*e**(
2*c + 2*d*x)*int(x/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4*c
+ 4*d*x)*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*
a - b),x)*a**2*b*d**2*f**2 + 2*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a**2
*d*e*f + e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a**2*f**2 - e**(2*c + 2...
```

3.451 $\int \frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 243

$$\int \frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{f\,\text{arctanh}(\cosh(c+dx))}{ad^2} - \frac{(e+fx)\text{csch}(c+dx)}{ad} + \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d} + \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d} - \frac{b(e+fx)\log\left(1-e^{2(c+dx)}\right)}{a^2d} + \frac{bf\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2} + \frac{bf\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^2} - \frac{bf\,\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{2a^2d^2}$$

output
```
-f*arctanh(cosh(d*x+c))/a/d^2-(f*x+e)*csch(d*x+c)/a/d+b*(f*x+e)*ln(1+b*exp
(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d+b*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2
)^(1/2)))/a^2/d-b*(f*x+e)*ln(1-exp(2*d*x+2*c))/a^2/d+b*f*polylog(2,-b*exp(
d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d^2+b*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/a^2/d^2-1/2*b*f*polylog(2,exp(2*d*x+2*c))/a^2/d^2
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 561 vs. $2(243) = 486$.

Time = 7.89 (sec) , antiderivative size = 561, normalized size of antiderivative = 2.31

$$\int \frac{(e+fx)\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-2bde(c+dx)+2bcf(c+dx)-bf(c+dx)^2-\frac{bd^2(e+fx)^2}{f}+\frac{4ab\sqrt{a^2+b^2}de\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-(a^2+b^2)^2}}-\frac{4ab\sqrt{-(a^2+b^2)^2}de\cdots}{(-a^2\cdots}}{}$$

input
```
Integrate[((e + f*x)*Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-2*b*d*e*(c + d*x) + 2*b*c*f*(c + d*x) - b*f*(c + d*x)^2 - (b*d^2*(e + f*
x)^2)/f + (4*a*b*Sqrt[a^2 + b^2]*d*e*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2
- b^2]])/Sqrt[-(a^2 + b^2)^2] - (4*a*b*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh[(a
 + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/(-a^2 - b^2)^(3/2) - a*d*e*Coth[(c + d
*x)/2] + a*c*f*Coth[(c + d*x)/2] - a*f*(c + d*x)*Coth[(c + d*x)/2] - 2*(-(
a*f) + b*d*(e + f*x))*Log[1 - E^(-c - d*x)] - 2*(a*f + b*d*(e + f*x))*Log[
1 + E^(-c - d*x)] + 2*b*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2])] + 2*b*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]
- 2*b*c*f*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*b*d*e*Log[2*a*E
^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] + 2*b*f*PolyLog[2, -E^(-c - d*x)] +
 2*b*f*PolyLog[2, E^(-c - d*x)] + 2*b*f*PolyLog[2, (b*E^(c + d*x))/(-a + S
qrt[a^2 + b^2])] + 2*b*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
))] + a*d*e*Tanh[(c + d*x)/2] - a*c*f*Tanh[(c + d*x)/2] + a*f*(c + d*x)*Ta
nh[(c + d*x)/2])/(2*a^2*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.25, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {6121, 5975, 3042, 26, 4257, 6103, 3042, 26, 4201, 2620, 2715, 2838, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6121}$$

$$\frac{\int (e+fx)\coth(c+dx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5975}$$

$$\frac{\frac{f\int \text{csch}(c+dx)dx}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)\text{csch}(c+dx)}{d}+\frac{f\int i\csc(ic+idx)dx}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)\text{csch}(c+dx)}{d}+\frac{if\int \csc(ic+idx)dx}{d}}{a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{6103}$$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{ 4201}$$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\int \frac{e^{2c+2dx-i\pi}(e+fx)}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^2}{2f}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int \log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\log\left(1+e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{-\frac{f\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{a}\right)}{a}$$

$\downarrow$ 6095

$$\frac{-\frac{f\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{a}-\frac{i\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+
$$

$\downarrow$ 2620

$$\frac{-\frac{f\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a}-\frac{i\left(2i\left(
$$

$\downarrow$ 2715

$$\frac{-\frac{f\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1
$$

$\downarrow$ 2838

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a}-\frac{i\left(2i\left(\right.\right.}{}\right.}{a}$$

input
```
Int[((e + f*x)*Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-((f*ArcTanh[Cosh[c + d*x]])/d^2) - ((e + f*x)*Csch[c + d*x])/d)/a - (b*(
-((b*(-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/
(b*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)
))/a) - (I*(((-1/2*I)*(e + f*x)^2)/f + (2*I)*(((e + f*x)*Log[1 + E^(2*c -
I*Pi + 2*d*x)])/(2*d) + (f*PolyLog[2, -E^(2*c - I*Pi + 2*d*x)])/(4*d^2))))
/a))/a
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
 :> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5975
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6103
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Coth[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x
]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6121
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Csch[c + d*x]^(p - 1)*(Coth[c + d*x]^n/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(229) = 458$.

Time = 0.77 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.17

method	result
risch	$-\frac{2(fx+e)\mathrm{e}^{dx+c}}{da\left(\mathrm{e}^{2dx+2c}-1\right)}-\frac{bf\ln\left(\mathrm{e}^{dx+c}+1\right)x}{a^2d}+\frac{bf\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)x}{a^2d}+\frac{bf\ln\left(\frac{b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}+a}{a+\sqrt{a^2+b^2}}\right)x}{a^2d}+\frac{f\ln\left(\mathrm{e}^{dx+c}-1\right)}{a\,d^2}$

input
```
int((f*x+e)*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERB
OSE)
```

output
```
-2/d*(f*x+e)/a*exp(d*x+c)/(exp(2*d*x+2*c)-1)-1/a^2/d*b*f*ln(exp(d*x+c)+1)*
x+1/a^2/d*b*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x
+1/a^2/d*b*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/
a/d^2*f*ln(exp(d*x+c)-1)-1/a/d^2*f*ln(exp(d*x+c)+1)-1/a^2/d^2*b*f*dilog(ex
p(d*x+c)+1)+1/a^2/d^2*b*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2
+b^2)^(1/2)))+1/a^2/d^2*b*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2
+b^2)^(1/2)))+1/a^2/d^2*b*f*dilog(exp(d*x+c))-1/a^2/d*b*e*ln(exp(d*x+c)-1)
+1/a^2/d*b*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/a^2/d*b*e*ln(exp(d*x+
c)+1)+1/a^2/d^2*b*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/
2)))*c+1/a^2/d^2*b*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2
)))*c+1/a^2/d^2*c*b*f*ln(exp(d*x+c)-1)-1/a^2/d^2*c*b*f*ln(b*exp(2*d*x+2*c)
+2*a*exp(d*x+c)-b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1221 vs. $2(226) = 452$.

Time = 0.12 (sec) , antiderivative size = 1221, normalized size of antiderivative = 5.02

$$\int \frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
fricas")
```

output
```
-(2*(a*d*f*x + a*d*e)*cosh(d*x + c) - (b*f*cosh(d*x + c)^2 + 2*b*f*cosh(d*
x + c)*sinh(d*x + c) + b*f*sinh(d*x + c)^2 - b*f)*dilog((a*cosh(d*x + c) +
 a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^
2) - b)/b + 1) - (b*f*cosh(d*x + c)^2 + 2*b*f*cosh(d*x + c)*sinh(d*x + c)
+ b*f*sinh(d*x + c)^2 - b*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b
*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b*f
*cosh(d*x + c)^2 + 2*b*f*cosh(d*x + c)*sinh(d*x + c) + b*f*sinh(d*x + c)^2
 - b*f)*dilog(cosh(d*x + c) + sinh(d*x + c)) + (b*f*cosh(d*x + c)^2 + 2*b*
f*cosh(d*x + c)*sinh(d*x + c) + b*f*sinh(d*x + c)^2 - b*f)*dilog(-cosh(d*x
 + c) - sinh(d*x + c)) + (b*d*e - b*c*f - (b*d*e - b*c*f)*cosh(d*x + c)^2
- 2*(b*d*e - b*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*e - b*c*f)*sinh(d*x
 + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/
b^2) + 2*a) + (b*d*e - b*c*f - (b*d*e - b*c*f)*cosh(d*x + c)^2 - 2*(b*d*e
- b*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*e - b*c*f)*sinh(d*x + c)^2)*lo
g(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
 + (b*d*f*x + b*c*f - (b*d*f*x + b*c*f)*cosh(d*x + c)^2 - 2*(b*d*f*x + b*c
*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f*x + b*c*f)*sinh(d*x + c)^2)*log(-
(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*s
qrt((a^2 + b^2)/b^2) - b)/b) + (b*d*f*x + b*c*f - (b*d*f*x + b*c*f)*cosh(d
*x + c)^2 - 2*(b*d*f*x + b*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f*x ...
```

Sympy [F]

$$\int \frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*coth(c + d*x)*csch(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\coth(dx+c)\operatorname{csch}(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
maxima")
```

output
```
(2*b*d*integrate(1/2*x/(a^2*d*e^(d*x + c) + a^2*d), x) - 2*b*d*integrate(1
/2*x/(a^2*d*e^(d*x + c) - a^2*d), x) + a*((d*x + c)/(a^2*d^2) - log(e^(d*x
 + c) + 1)/(a^2*d^2)) - a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) - 1)/(a^2
*d^2)) - 2*x*e^(d*x + c)/(a*d*e^(2*d*x + 2*c) - a*d) - 2*integrate((a*b*x*
e^(d*x + c) - b^2*x)/(a^2*b*e^(2*d*x + 2*c) + 2*a^3*e^(d*x + c) - a^2*b),
x))*f + e*(2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) + b*log(-2*a*e^(-d*
x - c) + b*e^(-2*d*x - 2*c) - b)/(a^2*d) - b*log(e^(-d*x - c) + 1)/(a^2*d)
 - b*log(e^(-d*x - c) - 1)/(a^2*d))
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+d\,x)\,(e+f\,x)}{\sinh(c+d\,x)\,(a+b\sinh(c+d\,x))}\,dx$$

input
```
int((coth(c + d*x)*(e + f*x))/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((coth(c + d*x)*(e + f*x))/(sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-8e^{2dx+5c}\left(\int \frac{e^{3dx}x}{e^{6dx+6c}b+2e^{5dx+5c}a-3e^{4dx+4c}b-4e^{3dx+3c}a+3e^{2dx+2c}b+2e^{dx+c}a-b}dx\right)a^3d^2f+8e^{2dx+4c}\left(\int \frac{}{e^{6dx+6c}b+2e^{5dx+5}}\right.}{}$$

input
```
int((f*x+e)*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 8*e**(5*c + 2*d*x)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c
+ 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d
*x)*b + 2*e**(c + d*x)*a - b),x)*a**3*d**2*f + 8*e**(4*c + 2*d*x)*int((e**
(2*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d*x)
*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x
)*a**2*b*d**2*f + e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a**2*f - e**(2*c
+ 2*d*x)*log(e**(c + d*x) - 1)*b**2*d*e + e**(2*c + 2*d*x)*log(e**(c + d*x
) + 1)*a**2*f - e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*b**2*d*e + e**(2*c
+ 2*d*x)*log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*b**2*d*e - 2*e**(2
*c + 2*d*x)*a**2*d*f*x - 2*e**(c + d*x)*a*b*d*e + 8*e**(3*c)*int((e**(3*d*
x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d*x)*b -
4*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**
3*d**2*f - 8*e**(2*c)*int((e**(2*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c +
 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*
x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b*d**2*f - log(e**(c + d*x) - 1)*a**2
*f + log(e**(c + d*x) - 1)*b**2*d*e - log(e**(c + d*x) + 1)*a**2*f + log(e
**(c + d*x) + 1)*b**2*d*e - log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)
*b**2*d*e)/(a**2*b*d**2*(e**(2*c + 2*d*x) - 1))
```

3.452 $\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 50

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\operatorname{csch}(c+dx)}{ad} - \frac{b\log(\sinh(c+dx))}{a^2d} + \frac{b\log(a+b\sinh(c+dx))}{a^2d}$$

output
```
-csch(d*x+c)/a/d-b*ln(sinh(d*x+c))/a^2/d+b*ln(a+b*sinh(d*x+c))/a^2/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\operatorname{csch}(c+dx)}{ad} - \frac{b\log(\sinh(c+dx))}{a^2d} + \frac{b\log(a+b\sinh(c+dx))}{a^2d}$$

input
```
Integrate[(Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-(Csch[c + d*x]/(a*d)) - (b*Log[Sinh[c + d*x]])/(a^2*d) + (b*Log[a + b*Sin
h[c + d*x]])/(a^2*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 25, 3312, 25, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cos(ic+idx)}{\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos(ic+idx)}{\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3312}$$

$$-\frac{\int -\frac{\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{bd}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{\text{csch}^2(c+dx)}{b^2(a+b\sinh(c+dx))}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$\frac{b\int \left(\frac{\text{csch}^2(c+dx)}{ab^2}-\frac{\text{csch}(c+dx)}{a^2b}+\frac{1}{a^2(a+b\sinh(c+dx))}\right)d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{\log(b\sinh(c+dx))}{a^2}+\frac{\log(a+b\sinh(c+dx))}{a^2}-\frac{\operatorname{csch}(c+dx)}{ab}\right)}{d}$$

input
```
Int[(Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(b*(-(Csch[c + d*x]/(a*b)) - Log[b*Sinh[c + d*x]]/a^2 + Log[a + b*Sinh[c +
 d*x]]/a^2))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$-\frac{\operatorname{csch}(dx+c)}{ad}+\frac{b\ln(a\ \operatorname{csch}(dx+c)+b)}{d\,a^2}$	35
default	$-\frac{\operatorname{csch}(dx+c)}{ad}+\frac{b\ln(a\ \operatorname{csch}(dx+c)+b)}{d\,a^2}$	35
risch	$-\frac{2\,\mathrm{e}^{dx+c}}{da\left(\mathrm{e}^{2dx+2c}-1\right)}-\frac{b\ln\left(\mathrm{e}^{2dx+2c}-1\right)}{a^2d}+\frac{b\ln\left(\mathrm{e}^{2dx+2c}+\frac{2a\,\mathrm{e}^{dx+c}}{b}-1\right)}{a^2d}$	82

input
```
int(coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-csch(d*x+c)/a/d+1/d*b/a^2*ln(a*csch(d*x+c)+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(50) = 100$.

Time = 0.09 (sec) , antiderivative size = 211, normalized size of antiderivative = 4.22

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{2\,a\cosh\left(dx+c\right)-\left(b\cosh\left(dx+c\right)^2+2\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b\sinh\left(dx+c\right)^2-b\right)\log\left(\frac{2(}{\cosh(}\right.}{a^2d\cosh\left(dx+c\right)^2+2}$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(2*a*cosh(d*x + c) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c)
 + b*sinh(d*x + c)^2 - b)*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sin
h(d*x + c))) + (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*si
nh(d*x + c)^2 - b)*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) +
2*a*sinh(d*x + c))/(a^2*d*cosh(d*x + c)^2 + 2*a^2*d*cosh(d*x + c)*sinh(d*x
 + c) + a^2*d*sinh(d*x + c)^2 - a^2*d)
```

Sympy [F]

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(coth(c + d*x)*csch(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(50) = 100$.

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.20

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2\,e^{(-dx-c)}}{(ae^{(-2\,dx-2\,c)}-a)d} + \frac{b\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{a^2d} - \frac{b\log\left(e^{(-dx-c)}+1\right)}{a^2d} - \frac{b\log\left(e^{(-dx-c)}-1\right)}{a^2d}$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) + b*log(-2*a*e^(-d*x - c) + b*
e^(-2*d*x - 2*c) - b)/(a^2*d) - b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^
(-d*x - c) - 1)/(a^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(50) = 100$.

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.20

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{b\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2a\right|\right)}{a^2} - \frac{b\log\left(\left|e^{(dx+c)}-e^{(-dx-c)}\right|\right)}{a^2} + \frac{b\left(e^{(dx+c)}-e^{(-dx-c)}\right)-2a}{a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)}}{d}$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
(b*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/a^2 - b*log(abs(e^(d*x +
 c) - e^(-d*x - c)))/a^2 + (b*(e^(d*x + c) - e^(-d*x - c)) - 2*a)/(a^2*(e^
(d*x + c) - e^(-d*x - c))))/d
```

Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 409, normalized size of antiderivative = 8.18

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\left(2\,\mathrm{atan}\left(\left(\left(4\,a^3\,b\,d\,{\left(b^2\right)}^{5/2}\,\sqrt{-a^4\,d^2}+4\,a^5\,b\,d\,{\left(b^2\right)}^{3/2}\,\sqrt{-a^4\,d^2}\right)\,\left(\frac{1}{8\,a^3\,b^4\,d^2\,{\left(a^2+b^2\right)}^2}-\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\left(\frac{1}{16\,a^2\,b^5\,d^2\,{\left(a^2+b^2\right)}^2}\right.\right.\right.\right.\right.}{} - \frac{1}{a\,d\,\sinh\left(c+d\,x\right)}$$

input
```
int(coth(c + d*x)/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
((2*atan((4*a^3*b*d*(b^2)^(5/2)*(-a^4*d^2)^(1/2) + 4*a^5*b*d*(b^2)^(3/2)*(
-a^4*d^2)^(1/2))*(1/(8*a^3*b^4*d^2*(a^2 + b^2)^2) - exp(d*x)*exp(c)*(1/(16
*a^2*b^5*d^2*(a^2 + b^2)^2) - (a^2 + 2*b^2)^2/(16*a^6*b^5*d^2*(a^2 + b^2)^
2)) + (a^2 + 2*b^2)/(8*a^5*b^4*d^2*(a^2 + b^2)^2))) + 2*atan(-(4*a^3*b^5*(
-a^4*d^2)^(1/2) + 4*a*b^7*(-a^4*d^2)^(1/2) - 4*b^8*exp(3*c)*exp(3*d*x)*(-a
^4*d^2)^(1/2) + 4*b^8*exp(d*x)*exp(c)*(-a^4*d^2)^(1/2) - 8*a*b^7*exp(2*c)*
exp(2*d*x)*(-a^4*d^2)^(1/2) + 4*a^2*b^6*exp(d*x)*exp(c)*(-a^4*d^2)^(1/2) -
 8*a^3*b^5*exp(2*c)*exp(2*d*x)*(-a^4*d^2)^(1/2) - 4*a^2*b^6*exp(3*c)*exp(3
*d*x)*(-a^4*d^2)^(1/2))/(b^4*(4*a^3*d*(b^2)^(3/2) + 4*a^5*d*(b^2)^(1/2)))
)*(b^2)^(1/2))/(-a^4*d^2)^(1/2) - 1/(a*d*sinh(c + d*x))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 171, normalized size of antiderivative = 3.42

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-e^{2dx+2c}\log\left(e^{dx+c}-1\right)b-e^{2dx+2c}\log\left(e^{dx+c}+1\right)b+e^{2dx+2c}\log\left(e^{2dx+2c}b+2e^{dx+c}a-b\right)b-2e^{dx+c}a+\mathrm{lo}}{a^2d\left(e^{2dx+2c}-1\right)}$$

input
```
int(coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*b - e**(2*c + 2*d*x)*log(e**(c
+ d*x) + 1)*b + e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a
 - b)*b - 2*e**(c + d*x)*a + log(e**(c + d*x) - 1)*b + log(e**(c + d*x) +
1)*b - log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*b)/(a**2*d*(e**(2*c
+ 2*d*x) - 1))
```

3.453 $\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 72.59 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Coth[c + d*x]*Csch[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
x]
```

output
```
Integrate[(Coth[c + d*x]*Csch[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
 x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(c+dx)\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{ 6125}$$
$$\int \frac{\coth(c+dx)\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Coth[c + d*x]*Csch[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\coth(dx+c)\,\text{csch}(dx+c)}{(fx+e)\,(a+b\sinh(dx+c))}dx$$

input
```
int(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{\coth(c+dx)\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth\left(dx+c\right)\text{csch}\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
fricas")
```

output
```
integral(coth(d*x + c)*csch(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x
 + c)), x)
```

Sympy [N/A]

Not integrable

Time = 7.48 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{\coth(c+dx)\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth\left(c+dx\right)\text{csch}\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(coth(c + d*x)*csch(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 316, normalized size of antiderivative = 9.88

$$\int \frac{\coth(c+dx)\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth\left(dx+c\right)\text{csch}\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
maxima")
```

output
```
2*e^(d*x + c)/(a*d*f*x + a*d*e - (a*d*f*x*e^(2*c) + a*d*e*e^(2*c))*e^(2*d*
x)) - 2*integrate(-1/2*(b*d*f*x + b*d*e + a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*
f*x + a^2*d*e^2 - (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*
e^(d*x)), x) + 2*integrate(1/2*(b*d*f*x + b*d*e - a*f)/(a^2*d*f^2*x^2 + 2*
a^2*d*e*f*x + a^2*d*e^2 + (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e
^2*e^c)*e^(d*x)), x) - 2*integrate(-(a*b*e^(d*x + c) - b^2)/(a^2*b*f*x + a
^2*b*e - (a^2*b*f*x*e^(2*c) + a^2*b*e*e^(2*c))*e^(2*d*x) - 2*(a^3*f*x*e^c
+ a^3*e*e^c)*e^(d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(c+d\,x)}{\sinh(c+d\,x)\,(e+f\,x)\,(a+b\sinh(c+d\,x))}\,dx$$

input
```
int(coth(c + d*x)/(sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(coth(c + d*x)/(sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.38

$$\int \frac{\coth(c+dx)\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(dx+c)\,\text{csch}(dx+c)}{\sinh(dx+c)\,be+\sinh(dx+c)\,bfx+ae+afx}dx$$

input
```
int(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int((coth(c + d*x)*csch(c + d*x))/(sinh(c + d*x)*b*e + sinh(c + d*x)*b*f*x
 + a*e + a*f*x),x)
```

3.454 $\int \frac{(e+fx)^3 \coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 721

$$\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{(e+fx)^3}{ad}+\frac{2b(e+fx)^3\operatorname{arctanh}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d} - \frac{\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d} + \frac{3f(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{ad^2} + \frac{3bf(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{a^2d^2} - \frac{3bf(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{a^2d^2} + \frac{3\sqrt{a^2+b^2}f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2} - \frac{3\sqrt{a^2+b^2}f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^2} + \frac{3f^2(e+fx)\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{ad^3} - \frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{a^2d^3} + \frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{a^2d^3} - \frac{6\sqrt{a^2+b^2}f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3} + \frac{6\sqrt{a^2+b^2}f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^3} - \frac{3f^3\operatorname{PolyLog}\left(3,e^{2(c+dx)}\right)}{2ad^4} + \frac{6bf^3\operatorname{PolyLog}\left(4,-e^{c+dx}\right)}{a^2d^4} - \frac{6bf^3\operatorname{PolyLog}\left(4,e^{c+dx}\right)}{a^2d^4} + \frac{6\sqrt{a^2+b^2}f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^4} - \frac{6\sqrt{a^2+b^2}f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^4}$$

output
```
-(f*x+e)^3/a/d+2*b*(f*x+e)^3*arctanh(exp(d*x+c))/a^2/d-(f*x+e)^3*coth(d*x+
c)/a/d+(a^2+b^2)^(1/2)*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^
2/d-(a^2+b^2)^(1/2)*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d
+3*f*(f*x+e)^2*ln(1-exp(2*d*x+2*c))/a/d^2+3*b*f*(f*x+e)^2*polylog(2,-exp(d
*x+c))/a^2/d^2-3*b*f*(f*x+e)^2*polylog(2,exp(d*x+c))/a^2/d^2+3*(a^2+b^2)^(
1/2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d^2-3*(a
^2+b^2)^(1/2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2
/d^2+3*f^2*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a/d^3-6*b*f^2*(f*x+e)*polylog
(3,-exp(d*x+c))/a^2/d^3+6*b*f^2*(f*x+e)*polylog(3,exp(d*x+c))/a^2/d^3-6*(a
^2+b^2)^(1/2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2
/d^3+6*(a^2+b^2)^(1/2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1
/2)))/a^2/d^3-3/2*f^3*polylog(3,exp(2*d*x+2*c))/a/d^4+6*b*f^3*polylog(4,-e
xp(d*x+c))/a^2/d^4-6*b*f^3*polylog(4,exp(d*x+c))/a^2/d^4+6*(a^2+b^2)^(1/2)
*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d^4-6*(a^2+b^2)^(1/2
)*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d^4
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1490 vs. $2(721) = 1442$.

Time = 7.03 (sec) , antiderivative size = 1490, normalized size of antiderivative = 2.07

$$\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-((-(d^3*e^2*(-1 + E^(2*c))*(b*d*e - 3*a*f)*x) + d^3*e^2*(-1 + E^(2*c))*(b
*d*e + 3*a*f)*x + 2*a*d^3*(e + f*x)^3 + 3*d^2*e*(-1 + E^(2*c))*f*(b*d*e -
2*a*f)*x*Log[1 - E^(-c - d*x)] + 3*d^2*(-1 + E^(2*c))*f^2*(b*d*e - a*f)*x^
2*Log[1 - E^(-c - d*x)] + b*d^3*(-1 + E^(2*c))*f^3*x^3*Log[1 - E^(-c - d*x
)] - 3*d^2*e*(-1 + E^(2*c))*f*(b*d*e + 2*a*f)*x*Log[1 + E^(-c - d*x)] - 3*
d^2*(-1 + E^(2*c))*f^2*(b*d*e + a*f)*x^2*Log[1 + E^(-c - d*x)] - b*d^3*(-1
 + E^(2*c))*f^3*x^3*Log[1 + E^(-c - d*x)] + d^2*e^2*(-1 + E^(2*c))*(b*d*e
- 3*a*f)*Log[1 - E^(c + d*x)] - d^2*e^2*(-1 + E^(2*c))*(b*d*e + 3*a*f)*Log
[1 + E^(c + d*x)] + 3*d*e*(-1 + E^(2*c))*f*(b*d*e + 2*a*f)*PolyLog[2, -E^(
-c - d*x)] + 6*d*(-1 + E^(2*c))*f^2*(b*d*e + a*f)*x*PolyLog[2, -E^(-c - d*
x)] + 3*b*d^2*(-1 + E^(2*c))*f^3*x^2*PolyLog[2, -E^(-c - d*x)] - 3*d*e*(-1
 + E^(2*c))*f*(b*d*e - 2*a*f)*PolyLog[2, E^(-c - d*x)] - 6*d*(-1 + E^(2*c)
)*f^2*(b*d*e - a*f)*x*PolyLog[2, E^(-c - d*x)] - 3*b*d^2*(-1 + E^(2*c))*f^
3*x^2*PolyLog[2, E^(-c - d*x)] + 6*(-1 + E^(2*c))*f^2*(b*d*e + a*f)*PolyLo
g[3, -E^(-c - d*x)] + 6*b*d*(-1 + E^(2*c))*f^3*x*PolyLog[3, -E^(-c - d*x)]
 + 6*(-1 + E^(2*c))*f^2*(-(b*d*e) + a*f)*PolyLog[3, E^(-c - d*x)] - 6*b*d*
(-1 + E^(2*c))*f^3*x*PolyLog[3, E^(-c - d*x)] + 6*b*(-1 + E^(2*c))*f^3*Pol
yLog[4, -E^(-c - d*x)] - 6*b*(-1 + E^(2*c))*f^3*PolyLog[4, E^(-c - d*x)])/
(a^2*d^4*(-1 + E^(2*c)))) + (Sqrt[a^2 + b^2]*(-2*d^3*e^3*ArcTanh[(a + b*E^
(c + d*x))/Sqrt[a^2 + b^2]] + 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a ...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3 \coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6103}$$

$$\frac{\int (e+fx)^3\coth^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -(e+fx)^3\tan\left(ic+idx+\frac{\pi}{2}\right)^2dx}{a}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\int (e+fx)^3\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{a}$$

$\Big\downarrow$ 4203

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{3if\int i(e+fx)^2\coth(c+dx)dx}{d}-\int (e+fx)^3dx+\frac{(e+fx)^3\coth(c+dx)}{d}}{a}$$

$\Big\downarrow$ 17

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{3if\int i(e+fx)^2\coth(c+dx)dx}{d}+\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{a}$$

$\Big\downarrow$ 26

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{-\frac{3f\int (e+fx)^2\coth(c+dx)dx}{d}+\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{a}$$

$\Big\downarrow$ 3042

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{-\frac{3f\int -i(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}+\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{a}$$

$\Big\downarrow$ 26

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{3if\int (e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}+\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{a}$$

$\Big\downarrow$ 4201

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{3if\left(2i\int \frac{e^{2c+2dx-i\pi}(e+fx)^2}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{a}$$

$\Big\downarrow$ 2620

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{a}$$

$$\Big\downarrow \; 3011$$

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int \text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{2d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\coth(c+}{d}}{a}$$

$$\Big\downarrow \; 2720$$

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}}{d}}{a}$$

$$\Big\downarrow \; 6119$$

$$-\frac{b\left(\frac{\int(e+fx)^3\cosh(c+dx)\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}}{d}}{a}$$

$$\Big\downarrow \; 5973$$

$$-\frac{b\left(\frac{\int(e+fx)^3\sinh(c+dx)dx+\int(e+fx)^3\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i(e+fx)^3\sin(ic+idx)dx+\int i(e+fx)^3\csc(ic+idx)dx}{a}\right)}{a}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int(e+fx)^3\csc(ic+idx)dx-i\int(e+fx)^3\sin(ic+idx)dx}{a}\right)}{a}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}}{d}}{a}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int(e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int(e+fx)^2\cosh(c+dx)dx}{d}\right)}{a}\right)}{a}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int(e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{a}\right)}{a}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}}{d}}{a}$$

$\downarrow$ 3777

$$-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int(e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int-i(e+fx)\sinh(c+dx)dx}{d}\right.}{d}\right.}{a}\right.}{a}$$

$$\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}}{d}$$

$$a$$

$\downarrow$ 26

$$-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int(e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int(e+fx)\sinh(c+dx)dx}{d}\right)}{d}\right)}{a}\right.}{a}$$

$$\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}}{d}$$

$$a$$

$\downarrow$ 3042

$$-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int(e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int-i(e+fx)\sin(ic+idx)dx}{d}\right)}{d}\right.}{a}\right.}{a}$$

$$\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}}{d}$$

$$a$$

$\downarrow$ 26

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int (e+fx)\sin(ic+idx)dx}{d}\right)}{d}\right)}{a}\right)}{a}$$

$$\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)-\frac{i(e+fx)^3}{3f}}{d}}{a}$$

$\big\downarrow$ 3777

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if}{}\right)}{d}\right)}{d}\right)}{a}\right)}{a}$$

$$\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)-\frac{i(e+fx)^3}{3f}}{d}}{a}$$

$\big\downarrow$ 3042

$$-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int(e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if}{}\cdots\right.}{d}\right.}{d}\right.}{a}\right.}{a}$$

$$\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\cdots\right.}{d}}{a}$$

$\Big\downarrow$ 3117

$$-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int(e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\cdots}{}\right.}{d}\right.}{d}\right.}{a}\right.}{a}$$

$$\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\cdots\right.}{d}}{a}$$

$\Big\downarrow$ 4670

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{3if\int (e+fx)^2\log\left(1-e^{c+dx}\right)dx}{d}-\frac{3if\int (e+fx)^2\log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)^3\text{arctanh}\left(e^{c+dx}\right)}{d}\right)-i\left(\frac{i(e+fx}{}\right.}{a}\right.}{a}$$

$$\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right.}{d}$$

$$\overline{\qquad a \qquad}$$

$\downarrow$ 3011

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,e^{c+d}\right.}{d}\right.}{}\right.}{}\right.}{}$$

$$\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right.}{d}$$

$$\overline{\qquad a \qquad}$$

$\downarrow$ 6099

$$-\frac{b\left(-\frac{b\left(\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)^3dx}{b^2}+\frac{\int (e+fx)^3\sinh(c+dx)dx}{b}\right)}{a}+\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}}{d}\right.}{d}\right.}{}\right.}{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)-\frac{i(e+fx)^3}{3f}}{d}}{a}$$

$\downarrow$ 17

$$-\frac{b\left(-\frac{b\left(\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int (e+fx)^3\sinh(c+dx)dx}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{a}+\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\right.}{d}\right.}{d}\right.}{}\right.}{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)-\frac{i(e+fx)^3}{3f}}{d}}{a}$$

$\downarrow$ 3042

$$-\frac{b\left(\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right.}{d}}{a}\right.}{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right.}{d}}{a}$$

input
```
Int[((e + f*x)^3*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^3\coth(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4612 vs. $2(667) = 1334$.

Time = 0.23 (sec) , antiderivative size = 4612, normalized size of antiderivative = 6.40

$$\int \frac{(e+fx)^3 \coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^3 \coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^3 \coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**3*coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*coth(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)^3 \coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3 \coth(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
e^3*(b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d) + s
qrt(a^2 + b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c)
- a + sqrt(a^2 + b^2)))/(a^2*d) + 2/((a*e^(-2*d*x - 2*c) - a)*d)) - 6*e^2*
f*x/(a*d) + 3*e^2*f*log(e^(d*x + c) + 1)/(a*d^2) + 3*e^2*f*log(e^(d*x + c)
 - 1)/(a*d^2) - 2*(f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x)/(a*d*e^(2*d*x + 2*c)
 - a*d) + (d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) -
6*d*x*polylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*b*f^3/(a^2*d^
4) - (d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x
*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))*b*f^3/(a^2*d^4) + 3*
(b*d*e^2*f + 2*a*e*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(
a^2*d^3) - 3*(b*d*e^2*f - 2*a*e*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^
(d*x + c)))/(a^2*d^3) + 3*(b*d*e*f^2 + a*f^3)*(d^2*x^2*log(e^(d*x + c) + 1
) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))/(a^2*d^4) - 3*
(b*d*e*f^2 - a*f^3)*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x +
c)) - 2*polylog(3, e^(d*x + c)))/(a^2*d^4) - 1/4*(b*d^4*f^3*x^4 + 4*(b*d*e
*f^2 + a*f^3)*d^3*x^3 + 6*(b*d^2*e^2*f + 2*a*d*e*f^2)*d^2*x^2)/(a^2*d^4) +
 1/4*(b*d^4*f^3*x^4 + 4*(b*d*e*f^2 - a*f^3)*d^3*x^3 + 6*(b*d^2*e^2*f - 2*a
*d*e*f^2)*d^2*x^2)/(a^2*d^4) + integrate(2*((a^2*f^3*e^c + b^2*f^3*e^c)*x^
3 + 3*(a^2*e*f^2*e^c + b^2*e*f^2*e^c)*x^2 + 3*(a^2*e^2*f*e^c + b^2*e^2*f*e
^c)*x)*e^(d*x)/(a^2*b*e^(2*d*x + 2*c) + 2*a^3*e^(d*x + c) - a^2*b), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)^3*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+d\,x)^2\,(e+f\,x)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((coth(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((coth(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^3*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(4*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a
**2 + b**2))*b**2*d**3*e**3*i - 4*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i
 + a*i)/sqrt(a**2 + b**2))*b**2*d**3*e**3*i + 16*e**(5*c + 2*d*x)*int((e**
(3*d*x)*x**3)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d
*x)*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b
),x)*a**4*d**4*f**3 + 8*e**(5*c + 2*d*x)*int((e**(3*d*x)*x**3)/(e**(6*c +
6*d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x
)*a + 3*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b**2*d**4*f**3
+ 48*e**(5*c + 2*d*x)*int((e**(3*d*x)*x**2)/(e**(6*c + 6*d*x)*b + 2*e**(5*
c + 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2
*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*d**4*e*f**2 + 24*e**(5*c + 2*d*x)*
int((e**(3*d*x)*x**2)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4
*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b + 2*e**(c + d*
x)*a - b),x)*a**2*b**2*d**4*e*f**2 - 24*e**(5*c + 2*d*x)*int((e**(3*d*x)*x
**2)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4
*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2
*b**2*d**3*f**3 + 48*e**(5*c + 2*d*x)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x)
*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a +
3*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*d**4*e**2*f + 24*e**(
5*c + 2*d*x)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x...
```

3.455 $\int \frac{(e+fx)^2 \coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 517

$$\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{(e+fx)^2}{ad}+\frac{2b(e+fx)^2\operatorname{arctanh}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^2\coth(c+dx)}{ad} + \frac{\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d} - \frac{\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d} + \frac{2f(e+fx)\log\left(1-e^{2(c+dx)}\right)}{ad^2} + \frac{2bf(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{a^2d^2} - \frac{2bf(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{a^2d^2} + \frac{2\sqrt{a^2+b^2}f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2} - \frac{2\sqrt{a^2+b^2}f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^2} + \frac{f^2\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{ad^3} - \frac{2bf^2\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{a^2d^3} + \frac{2bf^2\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{a^2d^3} - \frac{2\sqrt{a^2+b^2}f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3} + \frac{2\sqrt{a^2+b^2}f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^3}$$

output
```
-(f*x+e)^2/a/d+2*b*(f*x+e)^2*arctanh(exp(d*x+c))/a^2/d-(f*x+e)^2*coth(d*x+
c)/a/d+(a^2+b^2)^(1/2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^
2/d-(a^2+b^2)^(1/2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d
+2*f*(f*x+e)*ln(1-exp(2*d*x+2*c))/a/d^2+2*b*f*(f*x+e)*polylog(2,-exp(d*x+c
))/a^2/d^2-2*b*f*(f*x+e)*polylog(2,exp(d*x+c))/a^2/d^2+2*(a^2+b^2)^(1/2)*f
*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d^2-2*(a^2+b^2)^
(1/2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d^2+f^2*p
olylog(2,exp(2*d*x+2*c))/a/d^3-2*b*f^2*polylog(3,-exp(d*x+c))/a^2/d^3+2*b*
f^2*polylog(3,exp(d*x+c))/a^2/d^3-2*(a^2+b^2)^(1/2)*f^2*polylog(3,-b*exp(d
*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d^3+2*(a^2+b^2)^(1/2)*f^2*polylog(3,-b*exp(
d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d^3
```

Mathematica [A] (warning: unable to verify)

Time = 6.82 (sec) , antiderivative size = 917, normalized size of antiderivative = 1.77

$$\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-((-(d^2*e*(-1 + E^(2*c))*(b*d*e - 2*a*f)*x) + d^2*e*(-1 + E^(2*c))*(b*d*e
 + 2*a*f)*x + 2*a*d^2*(e + f*x)^2 + 2*d*(-1 + E^(2*c))*f*(b*d*e - a*f)*x*L
og[1 - E^(-c - d*x)] + b*d^2*(-1 + E^(2*c))*f^2*x^2*Log[1 - E^(-c - d*x)]
- 2*d*(-1 + E^(2*c))*f*(b*d*e + a*f)*x*Log[1 + E^(-c - d*x)] - b*d^2*(-1 +
 E^(2*c))*f^2*x^2*Log[1 + E^(-c - d*x)] + d*e*(-1 + E^(2*c))*(b*d*e - 2*a*
f)*Log[1 - E^(c + d*x)] - d*e*(-1 + E^(2*c))*(b*d*e + 2*a*f)*Log[1 + E^(c
+ d*x)] + 2*(-1 + E^(2*c))*f*(b*d*e + a*f)*PolyLog[2, -E^(-c - d*x)] + 2*b
*d*(-1 + E^(2*c))*f^2*x*PolyLog[2, -E^(-c - d*x)] + 2*(-1 + E^(2*c))*f*(-(
b*d*e) + a*f)*PolyLog[2, E^(-c - d*x)] - 2*b*d*(-1 + E^(2*c))*f^2*x*PolyLo
g[2, E^(-c - d*x)] + 2*b*(-1 + E^(2*c))*f^2*PolyLog[3, -E^(-c - d*x)] - 2*
b*(-1 + E^(2*c))*f^2*PolyLog[3, E^(-c - d*x)])/(a^2*d^3*(-1 + E^(2*c)))) +
 (Sqrt[a^2 + b^2]*(-2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]]
 + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + d^2*f^2*x^
2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*d^2*e*f*x*Log[1 + (b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2])] + 2*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a +
Sqrt[a^2 + b^2])] - 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2]))] - 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
+ 2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^2*d^3) +
 (Sech[c/2]*Sech[c/2 + (d*x)/2]*(-(e^2*Sinh[(d*x)/2]) - 2*e*f*x*Sinh[(d...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6103}$$

$$\frac{\int (e+fx)^2\coth^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)^2dx}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\int (e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{a}$$

$\Big\downarrow$ 4203

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{2if\int i(e+fx)\coth(c+dx)dx}{d}-\int (e+fx)^2dx+\frac{(e+fx)^2\coth(c+dx)}{d}}{a}$$

$\Big\downarrow$ 17

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{2if\int i(e+fx)\coth(c+dx)dx}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$\Big\downarrow$ 26

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{-\frac{2f\int (e+fx)\coth(c+dx)dx}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$\Big\downarrow$ 3042

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{-\frac{2f\int -i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$\Big\downarrow$ 26

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{2if\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$\Big\downarrow$ 4201

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{2if\left(2i\int \frac{e^{2c+2dx-i\pi}(e+fx)}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$\Big\downarrow$ 2620

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int \log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\log\left(1+e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \text{6119}$$

$$-\frac{b\left(\frac{\int (e+fx)^2\cosh(c+dx)\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \text{5973}$$

$$-\frac{b\left(\frac{\int (e+fx)^2\sinh(c+dx)dx+\int (e+fx)^2\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i(e+fx)^2\sin(ic+idx)dx+\int i(e+fx)^2\csc(ic+idx)dx}{a}\right)}{a}-\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx-i\int (e+fx)^2\sin(ic+idx)dx}{a}\right)}{a}-\frac{\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\cosh(c+dx)dx}{d}\right)}{a}\right)}{a}-\frac{\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{a}\right)}{a}-\frac{\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{d}\right)}{a}\right)}{a}-\frac{\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)}{d}\right)}{a}\right)}{a}-\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$\downarrow$ 3042

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}\right)}{a}\right)}{a}-\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$\downarrow$ 26

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}\right)}{d}\right)}{a}\right)}{a}-\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$\downarrow$ 3118

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{a}\right)}{a}-\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{2if\int (e+fx)\log\left(1-e^{c+dx}\right)dx}{d}-\frac{2if\int (e+fx)\log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)^2\text{arctanh}\left(e^{c+dx}\right)}{d}\right)-i\left(\frac{i(e+fx)^2}{\cdots}\right.}{a}\right.}{a}$$

$$\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{Po}\cdots}{\cdots}\right)}{d}\right.}{\cdots}\right.}{a}$$

$$\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,\cdots\right.}{d^2}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \text{6099}$$

$$-\frac{b\left(-\frac{b\left(\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)^2dx}{b^2}+\frac{\int (e+fx)^2\sinh(c+dx)dx}{b}\right)}{a}+\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{Poly}}{}\right.}{d}\right.}{}\right.}{\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}$$

$$a$$

$\downarrow$ 17

$$-\frac{b\left(-\frac{b\left(\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int (e+fx)^2\sinh(c+dx)dx}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{a}+\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}}{d}\right.}{d}\right.}{}\right.}{\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}$$

$$a$$

$\downarrow$ 3042

$$-\frac{b\left(\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,\right.}{d}\right.}{d}\right.}{a}\right.}{\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}$$

$$a$$

$\downarrow$ 26

$$-\frac{b\left(\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,\right.}{d}\right.}{d}\right.}{a}\right.}{}$$

$$\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

input
```
Int[((e + f*x)^2*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^2\coth(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2729 vs. $2(477) = 954$.

Time = 0.15 (sec) , antiderivative size = 2729, normalized size of antiderivative = 5.28

$$\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(2*a*d^2*e^2 - 4*a*c*d*e*f + 2*a*c^2*f^2 + 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f
*x + 2*a*c*d*e*f - a*c^2*f^2)*cosh(d*x + c)^2 + 4*(a*d^2*f^2*x^2 + 2*a*d^2
*e*f*x + 2*a*c*d*e*f - a*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + 2*(a*d^2*f
^2*x^2 + 2*a*d^2*e*f*x + 2*a*c*d*e*f - a*c^2*f^2)*sinh(d*x + c)^2 + 2*(b*d
*f^2*x + b*d*e*f - (b*d*f^2*x + b*d*e*f)*cosh(d*x + c)^2 - 2*(b*d*f^2*x +
b*d*e*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f^2*x + b*d*e*f)*sinh(d*x + c)
^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*co
sh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(b*d*
f^2*x + b*d*e*f - (b*d*f^2*x + b*d*e*f)*cosh(d*x + c)^2 - 2*(b*d*f^2*x + b
*d*e*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f^2*x + b*d*e*f)*sinh(d*x + c)^
2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cos
h(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b*d^2*e
^2 - 2*b*c*d*e*f + b*c^2*f^2 - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*cosh(
d*x + c)^2 - 2*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*cosh(d*x + c)*sinh(d*
x + c) - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2
+ b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^
2)/b^2) + 2*a) + (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2 - (b*d^2*e^2 - 2*b*c
*d*e*f + b*c^2*f^2)*cosh(d*x + c)^2 - 2*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f
^2)*cosh(d*x + c)*sinh(d*x + c) - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*si
nh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d...
```

Sympy [F]

$$\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)^2\coth^2{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**2*coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*coth(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)^2 \coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2 \coth{(dx+c)}^2}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^2*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
e^2*(b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d) + s
qrt(a^2 + b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c)
- a + sqrt(a^2 + b^2)))/(a^2*d) + 2/((a*e^(-2*d*x - 2*c) - a)*d)) - 4*e*f*
x/(a*d) - 2*(f^2*x^2 + 2*e*f*x)/(a*d*e^(2*d*x + 2*c) - a*d) + 2*e*f*log(e^
(d*x + c) + 1)/(a*d^2) + 2*e*f*log(e^(d*x + c) - 1)/(a*d^2) + (d^2*x^2*log
(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c))
)*b*f^2/(a^2*d^3) - (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x +
c)) - 2*polylog(3, e^(d*x + c)))*b*f^2/(a^2*d^3) + 2*(b*d*e*f + a*f^2)*(d*
x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^2*d^3) - 2*(b*d*e*f - a*f
^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^2*d^3) - 1/3*(b*d^
3*f^2*x^3 + 3*(b*d*e*f + a*f^2)*d^2*x^2)/(a^2*d^3) + 1/3*(b*d^3*f^2*x^3 +
3*(b*d*e*f - a*f^2)*d^2*x^2)/(a^2*d^3) + integrate(2*((a^2*f^2*e^c + b^2*f
^2*e^c)*x^2 + 2*(a^2*e*f*e^c + b^2*e*f*e^c)*x)*e^(d*x)/(a^2*b*e^(2*d*x + 2
*c) + 2*a^3*e^(d*x + c) - a^2*b), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+d\,x)^2\,(e+f\,x)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((coth(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((coth(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^2*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(2*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a
**2 + b**2))*b**2*d**2*e**2*i - 2*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i
 + a*i)/sqrt(a**2 + b**2))*b**2*d**2*e**2*i + 8*e**(5*c + 2*d*x)*int((e**(
3*d*x)*x**2)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d*
x)*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)
,x)*a**4*d**3*f**2 + 4*e**(5*c + 2*d*x)*int((e**(3*d*x)*x**2)/(e**(6*c + 6
*d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)
*a + 3*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b**2*d**3*f**2 +
 16*e**(5*c + 2*d*x)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c +
5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x
)*b + 2*e**(c + d*x)*a - b),x)*a**4*d**3*e*f + 8*e**(5*c + 2*d*x)*int((e**
(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d*x)
*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x
)*a**2*b**2*d**3*e*f - 8*e**(5*c + 2*d*x)*int((e**(3*d*x)*x)/(e**(6*c + 6*
d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*
a + 3*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b**2*d**2*f**2 -
8*e**(4*c + 2*d*x)*int((e**(2*d*x)*x**2)/(e**(6*c + 6*d*x)*b + 2*e**(5*c +
 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*
x)*b + 2*e**(c + d*x)*a - b),x)*a**3*b*d**3*f**2 - 16*e**(4*c + 2*d*x)*int
((e**(2*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4*c ...
```

3.456 $\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 294

$$\begin{aligned}\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{2b(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{a^2d}-\frac{(e+fx)\coth(c+dx)}{ad}\\ &+\frac{\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d}\\ &-\frac{\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d}\\ &+\frac{f\log(\sinh(c+dx))}{ad^2}+\frac{bf\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{a^2d^2}\\ &-\frac{bf\,\text{PolyLog}\left(2,e^{c+dx}\right)}{a^2d^2}\\ &+\frac{\sqrt{a^2+b^2}f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2}\\ &-\frac{\sqrt{a^2+b^2}f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^2}\end{aligned}$$

output
```
2*b*(f*x+e)*arctanh(exp(d*x+c))/a^2/d-(f*x+e)*coth(d*x+c)/a/d+(a^2+b^2)^(1
/2)*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d-(a^2+b^2)^(1/2)*(
f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d+f*ln(sinh(d*x+c))/a/d^
2+b*f*polylog(2,-exp(d*x+c))/a^2/d^2-b*f*polylog(2,exp(d*x+c))/a^2/d^2+(a^
2+b^2)^(1/2)*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d^2-(a^2+b
^2)^(1/2)*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d^2
```

Mathematica [A] (verified)

Time = 3.11 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.18

$$\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{ad(e+fx)\coth\left(\frac{1}{2}(c+dx)\right)-2\left(af(c+dx)+(af-bd(e+fx))\log\left(1-e^{-c-dx}\right)+(af+bd(e+fx\cdots\right.}{\cdots}$$

input
```
Integrate[((e + f*x)*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*(a*d*(e + f*x)*Coth[(c + d*x)/2] - 2*(a*f*(c + d*x) + (a*f - b*d*(e +
 f*x))*Log[1 - E^(-c - d*x)] + (a*f + b*d*(e + f*x))*Log[1 + E^(-c - d*x)]
 - b*f*PolyLog[2, -E^(-c - d*x)] + b*f*PolyLog[2, E^(-c - d*x)]) - 2*Sqrt[
a^2 + b^2]*(-2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*c*f*Ar
cTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log[1 + (b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + S
qrt[a^2 + b^2])] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] -
f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) + a*d*(e + f*x)*Ta
nh[(c + d*x)/2])/(a^2*d^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \; 6103$$

$$\frac{\int (e+fx)\coth^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -\left((e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)^2\right)dx}{a}$$

$$\Big\downarrow \; 25$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{a}$$

$$\Big\downarrow \; 4203$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{if\int i\coth(c+dx)dx}{d}-\int (e+fx)dx+\frac{(e+fx)\coth(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 17$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{if\int i\coth(c+dx)dx}{d}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{-\frac{f\int \coth(c+dx)dx}{d}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{-\frac{f\int -i\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{if\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \; 3956$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \; 6119$$

$$-\frac{b\left(\frac{\int (e+fx)\cosh(c+dx)\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{5973}$$

$$-\frac{b\left(\frac{\int (e+fx)\sinh(c+dx)dx+\int (e+fx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i(e+fx)\sin(ic+idx)dx+\int i(e+fx)\csc(ic+idx)dx}{a}\right)}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx-i\int (e+fx)\sin(ic+idx)dx}{a}\right)}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}\right)}{a}\right)}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{a}\right)}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$\downarrow$ 3117

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}\right)}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$\downarrow$ 4670

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{if\int \log\left(1-e^{c+dx}\right)dx}{d}-\frac{if\int \log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh\cdots}{\cdots}\right.}{a}\cdots}{a}\cdots$$
$$\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$\downarrow$ 2715

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{if\int e^{-c-dx}\log\left(1-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log\left(1+e^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)-i\left(\cdots\right.}{a}\cdots}{a}\cdots$$
$$\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$\downarrow$ 2838

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sin\cdots}{\cdots}\right.}{a}\cdots}{a}\cdots$$
$$\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$\downarrow$ 6099

$$-\frac{b\left(-\frac{b\left(\frac{(a^2+b^2)\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)dx}{b^2}+\frac{\int (e+fx)\sinh(c+dx)dx}{b}\right)}{a}+\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{Poly}}{}\right.}{a}\right.}{a}$$

$$\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$\big\downarrow$ 17

$$-\frac{b\left(-\frac{b\left(\frac{(a^2+b^2)\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int (e+fx)\sinh(c+dx)dx}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{a}+\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}}{}\right.}{a}\right.}{a}$$

$$\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$\big\downarrow$ 3042

$$-\frac{b\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(\frac{(a^2+b^2)\int \frac{}{a-ib}}{b^2}\right.}{}\right.}{a}$$

$$\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$\big\downarrow$ 26

$$-\frac{b\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(\frac{(a^2+b^2)\int \frac{}{a-ib}}{b^2}\right.}{}\right.}{a}$$

$$\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$\big\downarrow$ 3777

$$-\frac{b\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(\frac{\left(a^2+b^2\right)\int\frac{}{a-i}}{b^2}\right.}{}\right.}{a}$$

$$\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$\Big\downarrow$ 3042

$$-\frac{b\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(\frac{\left(a^2+b^2\right)\int\frac{}{a-i}}{b^2}\right.}{}\right.}{a}$$

$$\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$\Big\downarrow$ 3117

$$-\frac{b\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(\frac{\left(a^2+b^2\right)\int\frac{}{a-i}}{b^2}\right.}{}\right.}{a}$$

$$\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$\Big\downarrow$ 3803

$$-\frac{b\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(\frac{2\left(a^2+b^2\right)\int-\frac{}{}}{}\right.}{}\right.}{a}$$

$$\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$\Big\downarrow$ 25

$$-\frac{b\left(\frac{i\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(-\frac{2\left(a^2+b^2\right)\int -\ldots}{\ldots}\right.}{\ldots}\right.}{a}$$

$$\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$\downarrow$ 2694

$$-\frac{b\left(\frac{i\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(-\frac{2\left(a^2+b^2\right)\left(\frac{b}{\ldots}-\ldots\right.}{\ldots}\right.}{\ldots}\right.}{a}$$

$$\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$\downarrow$ 27

$$-\frac{b\left(\frac{i\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(-\frac{2\left(a^2+b^2\right)\left(\frac{b}{\ldots}-\ldots\right.}{\ldots}\right.}{\ldots}\right.}{a}$$

$$\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

input
```
Int[((e + f*x)*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1016 vs. $2(271) = 542$.

Time = 0.72 (sec) , antiderivative size = 1017, normalized size of antiderivative = 3.46

method	result	size
risch	Expression too large to display	1017

input
```
int((f*x+e)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/a^2/d*b^2*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^
(1/2))+1/a^2/d^2*b^2*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2
)-a)/(-a+(a^2+b^2)^(1/2)))-1/a^2/d^2*b^2*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*
x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/d^2*c*f/(a^2+b^2)^(1/2)*arc
tanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d*f/(a^2+b^2)^(1/2)*ln((b
*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2*f/(a^2+b^2)^(1
/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d^2*f/(
a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-
2/d*(f*x+e)/a/(exp(2*d*x+2*c)-1)+1/a^2/d^2*b^2*f/(a^2+b^2)^(1/2)*ln((-b*ex
p(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/a^2/d^2*b^2*f/(a^2+b
^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/a^2
/d*b^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2
)^(1/2)))*x+1/a/d^2*f*ln(exp(d*x+c)-1)+1/a/d^2*f*ln(exp(d*x+c)+1)+1/a^2/d*
b*f*ln(exp(d*x+c)+1)*x+1/a^2/d^2*c*b*f*ln(exp(d*x+c)-1)-1/a^2/d*b^2*f/(a^2
+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+2/a
^2/d^2*c*b^2*f/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^
(1/2))-2/d*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1
/2))-1/d^2*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^
2+b^2)^(1/2)))+1/d^2*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2
)-a)/(-a+(a^2+b^2)^(1/2)))-2/a/d^2*f*ln(exp(d*x+c))+1/a^2/d^2*b*f*dilog...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1338 vs. $2(267) = 534$.

Time = 0.13 (sec) , antiderivative size = 1338, normalized size of antiderivative = 4.55

$$\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(2*a*d*e - 2*a*c*f + 2*(a*d*f*x + a*c*f)*cosh(d*x + c)^2 + 4*(a*d*f*x + a
*c*f)*cosh(d*x + c)*sinh(d*x + c) + 2*(a*d*f*x + a*c*f)*sinh(d*x + c)^2 -
(b*f*cosh(d*x + c)^2 + 2*b*f*cosh(d*x + c)*sinh(d*x + c) + b*f*sinh(d*x +
c)^2 - b*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c)
 + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) +
 (b*f*cosh(d*x + c)^2 + 2*b*f*cosh(d*x + c)*sinh(d*x + c) + b*f*sinh(d*x +
 c)^2 - b*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c
) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1)
- (b*d*e - b*c*f - (b*d*e - b*c*f)*cosh(d*x + c)^2 - 2*(b*d*e - b*c*f)*cos
h(d*x + c)*sinh(d*x + c) - (b*d*e - b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^
2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b
^2) + 2*a) + (b*d*e - b*c*f - (b*d*e - b*c*f)*cosh(d*x + c)^2 - 2*(b*d*e -
 b*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*e - b*c*f)*sinh(d*x + c)^2)*sqr
t((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a
^2 + b^2)/b^2) + 2*a) + (b*d*f*x + b*c*f - (b*d*f*x + b*c*f)*cosh(d*x + c)
^2 - 2*(b*d*f*x + b*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f*x + b*c*f)*s
inh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x +
 c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) -
(b*d*f*x + b*c*f - (b*d*f*x + b*c*f)*cosh(d*x + c)^2 - 2*(b*d*f*x + b*c*f)
*cosh(d*x + c)*sinh(d*x + c) - (b*d*f*x + b*c*f)*sinh(d*x + c)^2)*sqrt(...
```

Sympy [F]

$$\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)\coth^2{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*coth(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)\coth{(dx+c)}^2}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-(b*d*integrate(x/(a^2*d*e^(d*x + c) + a^2*d), x) + b*d*integrate(x/(a^2*d
*e^(d*x + c) - a^2*d), x) + a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) + 1)/
(a^2*d^2)) + a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) - 1)/(a^2*d^2)) - 2*
(a^2*e^c + b^2*e^c)*integrate(x*e^(d*x)/(a^2*b*e^(2*d*x + 2*c) + 2*a^3*e^(
d*x + c) - a^2*b), x) + 2*x/(a*d*e^(2*d*x + 2*c) - a*d))*f + e*(b*log(e^(-
d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d) + sqrt(a^2 + b^2)*
log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2
+ b^2)))/(a^2*d) + 2/((a*e^(-2*d*x - 2*c) - a)*d))
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+d\,x)^2\,(e+f\,x)}{a+b\sinh\,(c+d\,x)}\,dx$$

input
```
int((coth(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((coth(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output

```
(2*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a
**2 + b**2))*b**2*d*e*i - 2*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i
)/sqrt(a**2 + b**2))*b**2*d*e*i + 8*e**(5*c + 2*d*x)*int((e**(3*d*x)*x)/(e
**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e**(3*
c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*d**2*f
 + 4*e**(5*c + 2*d*x)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c +
 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*
x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b**2*d**2*f - 8*e**(4*c + 2*d*x)*int(
(e**(2*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4*c + 4*
d*x)*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a -
b),x)*a**3*b*d**2*f + 4*e**(3*c + 2*d*x)*int((e**(d*x)*x)/(e**(6*c + 6*d*x
)*b + 2*e**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a +
 3*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b**2*d**2*f - e**(2*
c + 2*d*x)*log(e**(c + d*x) - 1)*a**3*f + e**(2*c + 2*d*x)*log(e**(c + d*x
) - 1)*a**2*b*f - e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*b**3*d*e - e**(2*
c + 2*d*x)*log(e**(c + d*x) + 1)*a**3*f - e**(2*c + 2*d*x)*log(e**(c + d*x
) + 1)*a**2*b*f + e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*b**3*d*e + 2*e**(
2*c + 2*d*x)*a**3*d*f*x - 2*e**(2*c + 2*d*x)*a*b**2*d*e - 2*e**(c + d*x)*a
**2*b*d*f*x - 8*e**(3*c)*int((e**(3*d*x)*x)/(e**(6*c + 6*d*x)*b + 2*e**(5*
c + 5*d*x)*a - 3*e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a + 3*e**(2*c ...
```

3.457 $\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b\text{arctanh}(\cosh(c+dx))}{a^2d} - \frac{2\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2d} - \frac{\coth(c+dx)}{ad}$$

output
```
b*arctanh(cosh(d*x+c))/a^2/d-2*(a^2+b^2)^(1/2)*arctanh((b-a*tanh(1/2*d*x+1
/2*c))/(a^2+b^2)^(1/2))/a^2/d-coth(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.45

$$\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{4\sqrt{-a^2-b^2}\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right) + a\coth\left(\frac{1}{2}(c+dx)\right) - 2b\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right) + 2b\log\left(\sinh\right.}{2a^2d}$$

input
```
Integrate[Coth[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*(4*Sqrt[-a^2 - b^2]*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]
] + a*Coth[(c + d*x)/2] - 2*b*Log[Cosh[(c + d*x)/2]] + 2*b*Log[Sinh[(c + d
*x)/2]] + a*Tanh[(c + d*x)/2])/(a^2*d)
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.14, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.810$, Rules used = {3042, 25, 3202, 25, 3042, 25, 3535, 3042, 26, 3480, 26, 3042, 26, 3139, 1083, 217, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{1}{\tan(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{1}{(a-ib\sin(ic+idx))\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{ 3202}$$

$$-\int -\frac{\operatorname{csch}^2(c+dx)\left(\sinh^2(c+dx)+1\right)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{\left(\sinh^2(c+dx)+1\right)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{1-\sin(ic+idx)^2}{\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{1-\sin(ic+idx)^2}{\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3535}$$

$$-\frac{\int \frac{\text{csch}(c+dx)(b-a\sinh(c+dx))}{a+b\sinh(c+dx)}dx}{a}-\frac{\coth(c+dx)}{ad}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{\int \frac{i(b+ia\sin(ic+idx))}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{i\int \frac{b+ia\sin(ic+idx)}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx}{a}$$

$$\Big\downarrow \text{3480}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{i\left(\frac{i(a^2+b^2)\int \frac{1}{a+b\sinh(c+dx)}dx}{a}+\frac{b\int -i\text{csch}(c+dx)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{i\left(\frac{i(a^2+b^2)\int \frac{1}{a+b\sinh(c+dx)}dx}{a}-\frac{ib\int \text{csch}(c+dx)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{i\left(\frac{i(a^2+b^2)\int \frac{1}{a-ib\sin(ic+idx)}dx}{a}-\frac{ib\int i\csc(ic+idx)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{i\left(\frac{i(a^2+b^2)\int \frac{1}{a-ib\sin(ic+idx)}dx}{a}+\frac{b\int \csc(ic+idx)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3139}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{i\left(\frac{2(a^2+b^2)\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{ad}+\frac{b\int \csc(ic+idx)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{i\left(\frac{b\int\csc(ic+idx)dx}{a}-\frac{4(a^2+b^2)\int\frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4(a^2+b^2)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{ad}\right)}{a}$$

$$\Big\downarrow \text{ 217}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{i\left(\frac{b\int\csc(ic+idx)dx}{a}+\frac{2i\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{ad}\right)}{a}$$

$$\Big\downarrow \text{ 4257}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{i\left(\frac{2i\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{ad}+\frac{ib\text{arctanh}(\cosh(c+dx))}{ad}\right)}{a}$$

input
```
Int[Coth[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

output
```
((-I)*((I*b*ArcTanh[Cosh[c + d*x]])/(a*d) + ((2*I)*Sqrt[a^2 + b^2]*ArcTanh
[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b^2])])/(a*d)))/a - Coth[c + d*x]/(a*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3202
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((1 - Sin[e + f*x]^2)/Sin[e + f*x]^
2), x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3535
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n
+ 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d
*(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
 !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{\left(-4a^2-4b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{2a^2\sqrt{a^2+b^2}}}{d}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{\left(-4a^2-4b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{2a^2\sqrt{a^2+b^2}}}{d}$
risch	$-\frac{2}{da\left(e^{2dx+2c}-1\right)}+\frac{b\ln\left(e^{dx+c}+1\right)}{d\,a^2}-\frac{b\ln\left(e^{dx+c}-1\right)}{d\,a^2}+\frac{\sqrt{a^2+b^2}\,\ln\left(e^{dx+c}-\frac{-a+\sqrt{a^2+b^2}}{b}\right)}{d\,a^2}-\frac{\sqrt{a^2+b^2}\,\ln\left(e^{dx}\right.}{d}$

input
```
int(coth(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/a*tanh(1/2*d*x+1/2*c)-1/2/a/tanh(1/2*d*x+1/2*c)-1/a^2*b*ln(tanh(
1/2*d*x+1/2*c))-1/2/a^2*(-4*a^2-4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*ta
nh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(74) = 148.

Time = 0.11 (sec) , antiderivative size = 360, normalized size of antiderivative = 4.68

$$\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\frac{\sqrt{a^2+b^2}\left(\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2-1\right)\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx\right.}{}}{}$$

input
```
integrate(coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(sqrt(a^2 + b^2)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d
*x + c)^2 - 1)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh
(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sq
rt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2
+ b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x
 + c) - b)) + (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sin
h(d*x + c)^2 - b)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (b*cosh(d*x + c
)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*log(cosh(d*
x + c) + sinh(d*x + c) - 1) - 2*a)/(a^2*d*cosh(d*x + c)^2 + 2*a^2*d*cosh(d
*x + c)*sinh(d*x + c) + a^2*d*sinh(d*x + c)^2 - a^2*d)
```

Sympy [F]

$$\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate(coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(coth(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.74

$$\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b\log\left(e^{(-dx-c)}+1\right)}{a^2d} - \frac{b\log\left(e^{(-dx-c)}-1\right)}{a^2d} + \frac{\sqrt{a^2+b^2}\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{a^2d} + \frac{2}{(ae^{(-2\,dx-2\,c)}-a)d}$$

input
```
integrate(coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d) + sqrt(a
^2 + b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a +
 sqrt(a^2 + b^2)))/(a^2*d) + 2/((a*e^(-2*d*x - 2*c) - a)*d)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.56

$$\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{b\log\left(e^{(dx+c)}+1\right)}{a^2} - \frac{b\log\left(\left|e^{(dx+c)}-1\right|\right)}{a^2} + \frac{\sqrt{a^2+b^2}\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{a^2} - \frac{2}{a\left(e^{(2\,dx+2\,c)}-1\right)}}{d}$$

input
```
integrate(coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
(b*log(e^(d*x + c) + 1)/a^2 - b*log(abs(e^(d*x + c) - 1))/a^2 + sqrt(a^2 +
 b^2)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x +
c) + 2*a + 2*sqrt(a^2 + b^2)))/a^2 - 2/(a*(e^(2*d*x + 2*c) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 380, normalized size of antiderivative = 4.94

$$\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2}{a\,d-a\,d\,\mathrm{e}^{2\,c+2\,d\,x}} - \frac{b\,\ln\left(32\,a^2+32\,b^2-32\,a^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-32\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\right)}{a^2\,d}$$
$$+ \frac{b\,\ln\left(32\,a^2+32\,b^2+32\,a^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+32\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\right)}{a^2\,d}$$
$$+ \frac{\ln\left(128\,a^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-64\,a\,b^3-64\,a^3\,b-32\,b^3\,\sqrt{a^2+b^2}+32\,b^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-64\,a^2\,b\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}}{a^2\,d}$$
$$- \frac{\ln\left(32\,b^3\,\sqrt{a^2+b^2}-64\,a\,b^3-64\,a^3\,b+128\,a^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+32\,b^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+64\,a^2\,b\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}}{a^2\,d}$$

input
```
int(coth(c + d*x)^2/(a + b*sinh(c + d*x)),x)
```

output
```
2/(a*d - a*d*exp(2*c + 2*d*x)) - (b*log(32*a^2 + 32*b^2 - 32*a^2*exp(d*x)*
exp(c) - 32*b^2*exp(d*x)*exp(c)))/(a^2*d) + (b*log(32*a^2 + 32*b^2 + 32*a^
2*exp(d*x)*exp(c) + 32*b^2*exp(d*x)*exp(c)))/(a^2*d) + (log(128*a^4*exp(d*
x)*exp(c) - 64*a*b^3 - 64*a^3*b - 32*b^3*(a^2 + b^2)^(1/2) + 32*b^4*exp(d*
x)*exp(c) - 64*a^2*b*(a^2 + b^2)^(1/2) + 160*a^2*b^2*exp(d*x)*exp(c) + 128
*a^3*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2) + 96*a*b^2*exp(d*x)*exp(c)*(a^2 + b
^2)^(1/2))*(a^2 + b^2)^(1/2))/(a^2*d) - (log(32*b^3*(a^2 + b^2)^(1/2) - 64
*a*b^3 - 64*a^3*b + 128*a^4*exp(d*x)*exp(c) + 32*b^4*exp(d*x)*exp(c) + 64*
a^2*b*(a^2 + b^2)^(1/2) + 160*a^2*b^2*exp(d*x)*exp(c) - 128*a^3*exp(d*x)*e
xp(c)*(a^2 + b^2)^(1/2) - 96*a*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2))*(a^2
 + b^2)^(1/2))/(a^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.45

$$\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2e^{2dx+2c}\sqrt{a^2+b^2}\, atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right) i-2\sqrt{a^2+b^2}\, atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right) i-e^{2dx+2c}\log\left(e^{dx+c}-1\right) b+e^{2dx+2c}\mathrm{lo}}{a^2 d\left(e^{2dx+2c}-1\right)}$$

input
```
int(coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(2*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a
**2 + b**2))*i - 2*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a*
*2 + b**2))*i - e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*b + e**(2*c + 2*d*x
)*log(e**(c + d*x) + 1)*b - 2*e**(2*c + 2*d*x)*a + log(e**(c + d*x) - 1)*b
 - log(e**(c + d*x) + 1)*b)/(a**2*d*(e**(2*c + 2*d*x) - 1))
```

3.458 $\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx = \text{Int}\left(\frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}, x\right)$$

output
```
Defer(Int)(coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 64.72 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx = \int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx$$

input
```
Integrate[Coth[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Coth[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

$$\Big\downarrow \text{ 6111}$$

$$\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Coth[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\coth(dx+c)^2}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(coth(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 3.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth^2\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

input
```
integrate(coth(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(coth(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 311, normalized size of antiderivative = 11.11

$$\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
2*(a^2*e^c + b^2*e^c)*integrate(-e^(d*x)/(a^2*b*f*x + a^2*b*e - (a^2*b*f*x
*e^(2*c) + a^2*b*e*e^(2*c))*e^(2*d*x) - 2*(a^3*f*x*e^c + a^3*e*e^c)*e^(d*x
)), x) + 2/(a*d*f*x + a*d*e - (a*d*f*x*e^(2*c) + a*d*e*e^(2*c))*e^(2*d*x))
 - integrate(-(b*d*f*x + b*d*e + a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2
*d*e^2 - (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)),
 x) - integrate((b*d*f*x + b*d*e - a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a
^2*d*e^2 + (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)
), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(c+d\,x)^2}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(coth(c + d*x)^2/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(coth(c + d*x)^2/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.43

$$\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$=\int \frac{\coth(dx+c)^2}{\sinh(dx+c)\,be+\sinh(dx+c)\,bfx+ae+afx}dx$$

input
```
int(coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(coth(c + d*x)**2/(sinh(c + d*x)*b*e + sinh(c + d*x)*b*f*x + a*e + a*f*
x),x)
```

3.459 $\int \frac{(e+fx)^3 \cosh(c+dx) \coth^2(c+dx)}{a+b \sinh(c+dx)} \, dx$

Optimal result

Integrand size = 34, antiderivative size = 718

$$\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{b(e+fx)^4}{4a^2f} - \frac{(a^2+b^2)(e+fx)^4}{4a^2bf} - \frac{6f(e+fx)^2\text{arctanh}(e^{c+dx})}{ad^2} - \frac{(e+fx)^3\text{csch}(c+dx)}{ad} + \frac{(a^2+b^2)(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd} + \frac{(a^2+b^2)(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2bd} - \frac{b(e+fx)^3\log\left(1-e^{2(c+dx)}\right)}{a^2d} - \frac{6f^2(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^3} + \frac{6f^2(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^3} + \frac{3(a^2+b^2)f(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd^2} + \frac{3(a^2+b^2)f(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2bd^2} - \frac{3bf(e+fx)^2\,\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{2a^2d^2} + \frac{6f^3\,\text{PolyLog}\left(3,-e^{c+dx}\right)}{ad^4} - \frac{6f^3\,\text{PolyLog}\left(3,e^{c+dx}\right)}{ad^4} - \frac{6(a^2+b^2)f^2(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd^3} - \frac{6(a^2+b^2)f^2(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2bd^3} + \frac{3bf^2(e+fx)\,\text{PolyLog}\left(3,e^{2(c+dx)}\right)}{2a^2d^3} + \frac{6(a^2+b^2)f^3\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd^4} + \frac{6(a^2+b^2)f^3\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2bd^4} - \frac{3bf^3\,\text{PolyLog}\left(4,e^{2(c+dx)}\right)}{4a^2d^4}$$

output
```
1/4*b*(f*x+e)^4/a^2/f-1/4*(a^2+b^2)*(f*x+e)^4/a^2/b/f-6*f*(f*x+e)^2*arctan
h(exp(d*x+c))/a/d^2-(f*x+e)^3*csch(d*x+c)/a/d+(a^2+b^2)*(f*x+e)^3*ln(1+b*e
xp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/b/d+(a^2+b^2)*(f*x+e)^3*ln(1+b*exp(d*x+
c)/(a+(a^2+b^2)^(1/2)))/a^2/b/d-b*(f*x+e)^3*ln(1-exp(2*d*x+2*c))/a^2/d-6*f
^2*(f*x+e)*polylog(2,-exp(d*x+c))/a/d^3+6*f^2*(f*x+e)*polylog(2,exp(d*x+c)
)/a/d^3+3*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)
))/a^2/b/d^2+3*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^
(1/2)))/a^2/b/d^2-3/2*b*f*(f*x+e)^2*polylog(2,exp(2*d*x+2*c))/a^2/d^2+6*f^
3*polylog(3,-exp(d*x+c))/a/d^4-6*f^3*polylog(3,exp(d*x+c))/a/d^4-6*(a^2+b^
2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/b/d^3-6*(a
^2+b^2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/b/d^3
+3/2*b*f^2*(f*x+e)*polylog(3,exp(2*d*x+2*c))/a^2/d^3+6*(a^2+b^2)*f^3*polyl
og(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/b/d^4+6*(a^2+b^2)*f^3*polylog(
4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/b/d^4-3/4*b*f^3*polylog(4,exp(2*d
*x+2*c))/a^2/d^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2696 vs. $2(718) = 1436$.

Time = 10.18 (sec) , antiderivative size = 2696, normalized size of antiderivative = 3.75

$$\int \frac{(e+fx)^3 \cosh(c+dx) \coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
(2*d^3*e^2*(-1 + E^(2*c))*f*(b*d*e - 3*a*f)*x + 2*d^3*e^2*(-1 + E^(2*c))*f
*(b*d*e + 3*a*f)*x + b*d^4*(e + f*x)^4 - 6*d^2*e*(-1 + E^(2*c))*f^2*(b*d*e
 - 2*a*f)*x*Log[1 - E^(-c - d*x)] - 6*d^2*(-1 + E^(2*c))*f^3*(b*d*e - a*f)
*x^2*Log[1 - E^(-c - d*x)] - 2*b*d^3*(-1 + E^(2*c))*f^4*x^3*Log[1 - E^(-c
- d*x)] - 6*d^2*e*(-1 + E^(2*c))*f^2*(b*d*e + 2*a*f)*x*Log[1 + E^(-c - d*x
)] - 6*d^2*(-1 + E^(2*c))*f^3*(b*d*e + a*f)*x^2*Log[1 + E^(-c - d*x)] - 2*
b*d^3*(-1 + E^(2*c))*f^4*x^3*Log[1 + E^(-c - d*x)] - 2*d^2*e^2*(-1 + E^(2*
c))*f*(b*d*e - 3*a*f)*Log[1 - E^(c + d*x)] - 2*d^2*e^2*(-1 + E^(2*c))*f*(b
*d*e + 3*a*f)*Log[1 + E^(c + d*x)] + 6*d*e*(-1 + E^(2*c))*f^2*(b*d*e + 2*a
*f)*PolyLog[2, -E^(-c - d*x)] + 12*d*(-1 + E^(2*c))*f^3*(b*d*e + a*f)*x*Po
lyLog[2, -E^(-c - d*x)] + 6*b*d^2*(-1 + E^(2*c))*f^4*x^2*PolyLog[2, -E^(-c
 - d*x)] + 6*d*e*(-1 + E^(2*c))*f^2*(b*d*e - 2*a*f)*PolyLog[2, E^(-c - d*x
)] + 12*d*(-1 + E^(2*c))*f^3*(b*d*e - a*f)*x*PolyLog[2, E^(-c - d*x)] + 6*
b*d^2*(-1 + E^(2*c))*f^4*x^2*PolyLog[2, E^(-c - d*x)] + 12*(-1 + E^(2*c))*
f^3*(b*d*e + a*f)*PolyLog[3, -E^(-c - d*x)] + 12*b*d*(-1 + E^(2*c))*f^4*x*
PolyLog[3, -E^(-c - d*x)] - 12*(-1 + E^(2*c))*f^3*(-(b*d*e) + a*f)*PolyLog
[3, E^(-c - d*x)] + 12*b*d*(-1 + E^(2*c))*f^4*x*PolyLog[3, E^(-c - d*x)] +
 12*b*(-1 + E^(2*c))*f^4*PolyLog[4, -E^(-c - d*x)] + 12*b*(-1 + E^(2*c))*f
^4*PolyLog[4, E^(-c - d*x)])/(2*a^2*d^4*(-1 + E^(2*c))*f) - ((a^2 + b^2)*(
4*e^3*E^(2*c)*x + 6*e^2*E^(2*c)*f*x^2 + 4*e*E^(2*c)*f^2*x^3 + E^(2*c)*f...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6119}$$

$$\frac{\int (e+fx)^3\cosh(c+dx)\coth^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5973}$$

$$\frac{\int (e+fx)^3\cosh(c+dx)dx+\int (e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\int (e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx+\int (e+fx)^3\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \; 3777$$

$$\frac{-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+-\frac{3if\int -i(e+fx)^2\sinh(c+dx)dx}{d}+\int (e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^3\sinh(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{-\frac{3f\int (e+fx)^2\sinh(c+dx)dx}{d}+\int (e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^3\sinh(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+-\frac{3f\int -i(e+fx)^2\sin(ic+idx)dx}{d}+\int (e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^3\sinh(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{3if\int (e+fx)^2\sin(ic+idx)dx}{d}+\int (e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^3\sinh(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 3777$$

$$\frac{-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\cosh(c+dx)dx}{d}\right)}{d}+\int (e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^3\sinh(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}+\int(e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^3\sinh(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$\downarrow$ 3777

$$\frac{\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int-i\sinh(c+dx)dx}{d}\right)}{d}\right)}{d}+\int(e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^3\sinh(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$\downarrow$ 26

$$\frac{\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}\right)}{d}\right)}{d}+\int(e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^3\sinh(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$\downarrow$ 3042

$$\frac{\int(e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int-i\sin(ic+idx)dx}{d}\right)}{d}\right)}{d}+\frac{(e+fx)^3\sinh(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$\downarrow$ 26

$$\frac{\int(e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int\sin(ic+idx)dx}{d}\right)}{d}\right)}{d}+\frac{(e+fx)^3\sinh(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$\downarrow$ 3118

$$\frac{\int (e+fx)^3 \coth(c+dx)\operatorname{csch}(c+dx)dx + \frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d} + \frac{(e+fx)^3\sinh(c+d}{d}}{a} - \frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a} +$$

$\downarrow$ 5975

$$\frac{\frac{3f\int (e+fx)^2\operatorname{csch}(c+dx)dx}{d} + \frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d} + \frac{(e+fx)^3\sinh(c+dx)}{d} - \frac{(e+fx)^3\operatorname{csch}(}{d}}{a} - \frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a} +$$

$\downarrow$ 3042

$$\frac{\frac{3f\int i(e+fx)^2\csc(ic+idx)dx}{d} + \frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d} + \frac{(e+fx)^3\sinh(c+dx)}{d} - \frac{(e+fx)^3\operatorname{csch}(}{d}}{a} - \frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a} +$$

$\downarrow$ 26

$$\frac{\frac{3if\int (e+fx)^2\csc(ic+idx)dx}{d} + \frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d} + \frac{(e+fx)^3\sinh(c+dx)}{d} - \frac{(e+fx)^3\operatorname{csch}(}{d}}{a} - \frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a} +$$

$\downarrow$ 4670

$$\frac{\frac{3if\left(\frac{2if\int (e+fx)\log\left(1-e^{c+dx}\right)dx}{d}-\frac{2if\int (e+fx)\log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)^2\operatorname{arctanh}\left(e^{c+dx}\right)}{d}\right)}{d} + \frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+}{d}\right.}{}\right.}{d}}{a} - \frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a} +$$

$\downarrow$ 3011

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{3if\left(-\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}+\frac{2i(e+fx)^2\text{arctan}}{d}\right.}{d}}{a}$$

$\downarrow$ 2720

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+}\right.}{d}\right.}{d}\right.}{d}$$

$\downarrow$ 6119

$$-\frac{b\left(\frac{\int (e+fx)^3\cosh^2(c+dx)\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+}\right.}{d}\right.}{d}\right.}{d}$$

$\downarrow$ 5973

$$-\frac{b\left(\frac{\int (e+fx)^3\coth(c+dx)dx+\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+}\right.}{d}\right.}{d}\right.}{d}$$

$\downarrow$ 3042

$$\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{c+}\right.}{d}\right.}{d}\right.}{d}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx+\int -i(e+fx)^3\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{c+}\right.}{d}\right.}{d}\right.}{d}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx-i\int(e+fx)^3\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{4201}$$

$$\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{c+}\right.}{d}\right.}{d}\right.}{d}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\int\frac{e^{2c+2dx-i\pi}(e+fx)^3}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^4}{4f}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{2620}$$

$$\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{c+}\right.}{d}\right.}{d}\right.}{d}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\int(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)da}{2d}\right.\right.}{a}\right.}{a}$$

$\Big\downarrow$ 3011

$$\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{c+\cdots}\right.}{d}\cdots\right.}{d}\cdots\right.}{d}\cdots$$

$$b\left(-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2d\cdots}\right.}{d}\cdots\right.}{\cdots}\right.\right.}{a}\cdots\right.$$

$$\overline{\qquad\qquad\qquad\qquad a\qquad\qquad\qquad\qquad}$$

$\Big\downarrow$ 5969

$$\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{c+\cdots}\right.}{d}\cdots\right.}{d}\cdots\right.}{d}\cdots$$

$$b\left(-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{2c+2d\cdots}\right.}{2d}\cdots\right.}{2d}\cdots\right.\right.}{a}\cdots\right.$$

$$\overline{\qquad\qquad\qquad\qquad a\qquad\qquad\qquad\qquad}$$

$\Big\downarrow$ 3042

$$\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{c+\cdots}\right.}{d}\cdots\right.}{d}\cdots\right.}{d}\cdots$$

$$b\left(-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{2c+2d\cdots}\right.}{2d}\cdots\right.}{2d}\cdots\right.\right.}{a}\cdots\right.$$

$$\overline{\qquad\qquad\qquad\qquad a\qquad\qquad\qquad\qquad}$$

$\downarrow$ 25

$$\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{c+\right.}}{d}\right.}{d}\right.}{d}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{2c+2d\right.}}{2d}\right.}{2d}\right.\right.}{a}\right.}{a}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^3\cosh(dx+c)\coth(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5829 vs. 2(674) = 1348.

Time = 0.22 (sec) , antiderivative size = 5829, normalized size of antiderivative = 8.12

$$\int \frac{(e+fx)^3 \cosh(c+dx) \coth^2(c+dx)}{a+b\sinh(c+dx)} \, dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^3 \cosh(c+dx) \coth^2(c+dx)}{a+b\sinh(c+dx)} \, dx = \int \frac{(e+fx)^3 \cosh(c+dx) \coth^2(c+dx)}{a+b\sinh(c+dx)} \, dx$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)*coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*cosh(c + d*x)*coth(c + d*x)**2/(a + b*sinh(c + d*x))
, x)
```

Maxima [F]

$$\int \frac{(e+fx)^3 \cosh(c+dx) \coth^2(c+dx)}{a+b\sinh(c+dx)} \, dx = \int \frac{(fx+e)^3 \cosh(dx+c) \coth(dx+c)^2}{b\sinh(dx+c)+a} \, dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
e^3*((d*x + c)/(b*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log
(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d) + (a^2 + b^2)
*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^2*b*d)) - 3*e^2*f*log(
e^(d*x + c) + 1)/(a*d^2) + 3*e^2*f*log(e^(d*x + c) - 1)/(a*d^2) - 1/4*(a*d
*f^3*x^4 + 4*a*d*e*f^2*x^3 + 6*a*d*e^2*f*x^2 - (a*d*f^3*x^4*e^(2*c) + 4*a*
d*e*f^2*x^3*e^(2*c) + 6*a*d*e^2*f*x^2*e^(2*c))*e^(2*d*x) + 8*(b*f^3*x^3*e^
c + 3*b*e*f^2*x^2*e^c + 3*b*e^2*f*x*e^c)*e^(d*x))/(a*b*d*e^(2*d*x + 2*c) -
 a*b*d) - (d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) -
6*d*x*polylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*b*f^3/(a^2*d^
4) - (d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x
*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))*b*f^3/(a^2*d^4) - 3*
(b*d*e^2*f + 2*a*e*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(
a^2*d^3) - 3*(b*d*e^2*f - 2*a*e*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^
(d*x + c)))/(a^2*d^3) - 3*(b*d*e*f^2 + a*f^3)*(d^2*x^2*log(e^(d*x + c) + 1
) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))/(a^2*d^4) - 3*
(b*d*e*f^2 - a*f^3)*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x +
c)) - 2*polylog(3, e^(d*x + c)))/(a^2*d^4) + 1/4*(b*d^4*f^3*x^4 + 4*(b*d*e
*f^2 + a*f^3)*d^3*x^3 + 6*(b*d^2*e^2*f + 2*a*d*e*f^2)*d^2*x^2)/(a^2*d^4) +
 1/4*(b*d^4*f^3*x^4 + 4*(b*d*e*f^2 - a*f^3)*d^3*x^3 + 6*(b*d^2*e^2*f - 2*a
*d*e*f^2)*d^2*x^2)/(a^2*d^4) - integrate(-2*((a^2*b*f^3 + b^3*f^3)*x^3 ...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,\coth(c+d\,x)^2\,(e+f\,x)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)*coth(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*coth(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh\left(dx+c\right)\coth\left(dx+c\right)^2}{a+b\sinh\left(dx+c\right)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.460 $\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 518

$$\begin{aligned}
&\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= \frac{b(e+fx)^3}{3a^2f} - \frac{(a^2+b^2)(e+fx)^3}{3a^2bf} - \frac{4f(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{ad^2} \\
&\quad - \frac{(e+fx)^2\text{csch}(c+dx)}{ad} + \frac{(a^2+b^2)(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd} \\
&\quad + \frac{(a^2+b^2)(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2bd} - \frac{b(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{a^2d} \\
&\quad - \frac{2f^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^3} + \frac{2f^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^3} \\
&\quad + \frac{2(a^2+b^2)f(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd^2} \\
&\quad + \frac{2(a^2+b^2)f(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2bd^2} \\
&\quad - \frac{bf(e+fx)\,\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{a^2d^2} - \frac{2(a^2+b^2)f^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd^3} \\
&\quad - \frac{2(a^2+b^2)f^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2bd^3} + \frac{bf^2\,\text{PolyLog}\left(3,e^{2(c+dx)}\right)}{2a^2d^3}
\end{aligned}$$

output
```
1/3*b*(f*x+e)^3/a^2/f-1/3*(a^2+b^2)*(f*x+e)^3/a^2/b/f-4*f*(f*x+e)*arctanh(
exp(d*x+c))/a/d^2-(f*x+e)^2*csch(d*x+c)/a/d+(a^2+b^2)*(f*x+e)^2*ln(1+b*exp
(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/b/d+(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)
/(a+(a^2+b^2)^(1/2)))/a^2/b/d-b*(f*x+e)^2*ln(1-exp(2*d*x+2*c))/a^2/d-2*f^2
*polylog(2,-exp(d*x+c))/a/d^3+2*f^2*polylog(2,exp(d*x+c))/a/d^3+2*(a^2+b^2
)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/b/d^2+2*(a^2+
b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/b/d^2-b*f*
(f*x+e)*polylog(2,exp(2*d*x+2*c))/a^2/d^2-2*(a^2+b^2)*f^2*polylog(3,-b*exp
(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/b/d^3-2*(a^2+b^2)*f^2*polylog(3,-b*exp(d*
x+c)/(a+(a^2+b^2)^(1/2)))/a^2/b/d^3+1/2*b*f^2*polylog(3,exp(2*d*x+2*c))/a^
2/d^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1806 vs. $2(518) = 1036$.

Time = 9.80 (sec) , antiderivative size = 1806, normalized size of antiderivative = 3.49

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
(3*d^2*e*(-1 + E^(2*c))*f*(b*d*e - 2*a*f)*x + 3*d^2*e*(-1 + E^(2*c))*f*(b*
d*e + 2*a*f)*x + 2*b*d^3*(e + f*x)^3 - 6*d*(-1 + E^(2*c))*f^2*(b*d*e - a*f
)*x*Log[1 - E^(-c - d*x)] - 3*b*d^2*(-1 + E^(2*c))*f^3*x^2*Log[1 - E^(-c -
 d*x)] - 6*d*(-1 + E^(2*c))*f^2*(b*d*e + a*f)*x*Log[1 + E^(-c - d*x)] - 3*
b*d^2*(-1 + E^(2*c))*f^3*x^2*Log[1 + E^(-c - d*x)] - 3*d*e*(-1 + E^(2*c))*
f*(b*d*e - 2*a*f)*Log[1 - E^(c + d*x)] - 3*d*e*(-1 + E^(2*c))*f*(b*d*e + 2
*a*f)*Log[1 + E^(c + d*x)] + 6*(-1 + E^(2*c))*f^2*(b*d*e + a*f)*PolyLog[2,
 -E^(-c - d*x)] + 6*b*d*(-1 + E^(2*c))*f^3*x*PolyLog[2, -E^(-c - d*x)] - 6
*(-1 + E^(2*c))*f^2*(-(b*d*e) + a*f)*PolyLog[2, E^(-c - d*x)] + 6*b*d*(-1
+ E^(2*c))*f^3*x*PolyLog[2, E^(-c - d*x)] + 6*b*(-1 + E^(2*c))*f^3*PolyLog
[3, -E^(-c - d*x)] + 6*b*(-1 + E^(2*c))*f^3*PolyLog[3, E^(-c - d*x)])/(3*a
^2*d^3*(-1 + E^(2*c))*f) - ((a^2 + b^2)*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x
^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x
))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]
*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3
/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^
2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*
ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*
e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d - (3*e^2*E^(2*c)*Lo
g[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e*f*x*Log[1 + (b*...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6119}$$

$$\frac{\int (e+fx)^2\cosh(c+dx)\coth^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5973}$$

$$\frac{\int (e+fx)^2\cosh(c+dx)dx+\int (e+fx)^2\coth(c+dx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\int (e+fx)^2\coth(c+dx)\text{csch}(c+dx)dx+\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}+\int (e+fx)^2\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^2\sinh(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{2f\int (e+fx)\sinh(c+dx)dx}{d}+\int (e+fx)^2\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^2\sinh(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+-\frac{2f\int -i(e+fx)\sin(ic+idx)dx}{d}+\int (e+fx)^2\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^2\sinh(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2if\int (e+fx)\sin(ic+idx)dx}{d}+\int (e+fx)^2\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^2\sinh(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}\right)}{d}+\int (e+fx)^2\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^2\sinh(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}+\int(e+fx)^2\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^2\sinh(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$\big\downarrow$ 3117

$$\frac{\int(e+fx)^2\coth(c+dx)\text{csch}(c+dx)dx+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$\big\downarrow$ 5975

$$\frac{\frac{2f\int(e+fx)\text{csch}(c+dx)dx}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$\big\downarrow$ 3042

$$\frac{\frac{2f\int i(e+fx)\csc(ic+idx)dx}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$\big\downarrow$ 26

$$\frac{\frac{2if\int(e+fx)\csc(ic+idx)dx}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$\big\downarrow$ 4670

$$\frac{\frac{2if\left(\frac{if\int\log\left(1-e^{c+dx}\right)dx}{d}-\frac{if\int\log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+\text{…}}{d}}{a}-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$\big\downarrow$ 2715

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{2if\left(\frac{if\int e^{-c-dx}\log\left(1-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log\left(1+e^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\cdots}{a}$$

$\downarrow$ 2838

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+\cdots}{d}}{a}$$

$\downarrow$ 6119

$$-\frac{b\left(\frac{\int(e+fx)^2\cosh^2(c+dx)\coth(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+\cdots}{d}}{a}$$

$\downarrow$ 5973

$$-\frac{b\left(\frac{\int(e+fx)^2\coth(c+dx)dx+\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+\cdots}{d}}{a}$$

$\downarrow$ 3042

$$\frac{\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+\cdots}{d}}{a}$$
$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx+\int-i(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{a}\right)}{a}$$

$\downarrow$ 26

$$\frac{\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+}{d}}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx-i\int(e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{a}\right)}{a}$$

$\downarrow$ 4201

$$\frac{\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+}{d}}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\int\frac{e^{2c+2dx-i\pi}(e+fx)^2}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^3}{3f}\right)}{a}\right)}{a}$$

$\downarrow$ 2620

$$\frac{\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+}{d}}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)dx}{d}\right)\right.}{a}\right.}{a}$$

$\downarrow$ 3011

$$\frac{\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+}{d}}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{2d}\right.}{}\right.\right.}{a}\right.}{a}$$

$\downarrow$ 2720

$$\frac{\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+\cdots}{d}}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e\cdots\right.}{4d^2}\right.}{\cdots}\right.\right.}{a}\right.}{a}$$

$\downarrow$ 5969

$$\frac{\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+\cdots}{d}}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{Pol}\cdots}{\cdots}\right.}{d}\right.\right.}{a}\right.}{a}$$

$\downarrow$ 3042

$$\frac{\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+\cdots}{d}}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{Pol}\cdots}{\cdots}\right.}{d}\right.\right.}{a}\right.}{a}$$

$\downarrow$ 25

$$\frac{\frac{\frac{2if\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+}{d}}{a}}{}$$

$$b\left(-\frac{b\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{Pol}}{}\right.}{d}\right.\right.}{a}\right.$$

$$\overline{\qquad a \qquad}$$

$\downarrow$ 3791

$$\frac{\frac{\frac{2if\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+}{d}}{a}}{}$$

$$b\left(-\frac{b\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{Pol}}{}\right.}{d}\right.\right.}{}\right.$$

$$\overline{\qquad a \qquad}$$

$\downarrow$ 17

$$\frac{\frac{\frac{2if\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+}{d}}{a}}{}$$

$$b\left(-\frac{b\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{Pol}}{}\right.}{d}\right.\right.}{}\right.$$

$$\overline{\qquad a \qquad}$$

$\downarrow$ 6099

$$\frac{\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+}{d}}{a}$$

$$b\left(-\frac{b\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)^2\cosh(c+dx)dx}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right.\right.}{}\right.$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^2\cosh(dx+c)\coth(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3506 vs. $2(486) = 972$.

Time = 0.18 (sec) , antiderivative size = 3506, normalized size of antiderivative = 6.77

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2\cosh{(c+dx)}\coth^2{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)*coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*cosh(c + d*x)*coth(c + d*x)**2/(a + b*sinh(c + d*x))
, x)
```

Maxima [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh{(dx+c)}\coth{(dx+c)}^2}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
e^2*((d*x + c)/(b*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log
(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d) + (a^2 + b^2)
*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^2*b*d)) - 1/3*(a*d*f^2
*x^3 + 3*a*d*e*f*x^2 - (a*d*f^2*x^3*e^(2*c) + 3*a*d*e*f*x^2*e^(2*c))*e^(2*
d*x) + 6*(b*f^2*x^2*e^c + 2*b*e*f*x*e^c)*e^(d*x))/(a*b*d*e^(2*d*x + 2*c) -
 a*b*d) - 2*e*f*log(e^(d*x + c) + 1)/(a*d^2) + 2*e*f*log(e^(d*x + c) - 1)/
(a*d^2) - (d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*po
lylog(3, -e^(d*x + c)))*b*f^2/(a^2*d^3) - (d^2*x^2*log(-e^(d*x + c) + 1) +
 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*b*f^2/(a^2*d^3) - 2
*(b*d*e*f + a*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^2*d
^3) - 2*(b*d*e*f - a*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))
/(a^2*d^3) + 1/3*(b*d^3*f^2*x^3 + 3*(b*d*e*f + a*f^2)*d^2*x^2)/(a^2*d^3) +
 1/3*(b*d^3*f^2*x^3 + 3*(b*d*e*f - a*f^2)*d^2*x^2)/(a^2*d^3) - integrate(-
2*((a^2*b*f^2 + b^3*f^2)*x^2 + 2*(a^2*b*e*f + b^3*e*f)*x - ((a^3*f^2*e^c +
 a*b^2*f^2*e^c)*x^2 + 2*(a^3*e*f*e^c + a*b^2*e*f*e^c)*x)*e^(d*x))/(a^2*b^2
*e^(2*d*x + 2*c) + 2*a^3*b*e^(d*x + c) - a^2*b^2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,\coth(c+d\,x)^2\,(e+f\,x)^2}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)*coth(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*coth(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\cosh(dx+c)\coth(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.461 $\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 324

$$\begin{aligned}\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{b(e+fx)^2}{2a^2f}-\frac{(a^2+b^2)(e+fx)^2}{2a^2bf}-\frac{f\operatorname{arctanh}(\cosh(c+dx))}{ad^2}-\frac{(e+fx)\operatorname{csch}(c+dx)}{ad}\\ &+\frac{(a^2+b^2)(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd}+\frac{(a^2+b^2)(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2bd}\\ &-\frac{b(e+fx)\log\left(1-e^{2(c+dx)}\right)}{a^2d}+\frac{(a^2+b^2)f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd^2}\\ &+\frac{(a^2+b^2)f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2bd^2}-\frac{bf\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{2a^2d^2}\end{aligned}$$

output
```
1/2*b*(f*x+e)^2/a^2/f-1/2*(a^2+b^2)*(f*x+e)^2/a^2/b/f-f*arctanh(cosh(d*x+c
))/a/d^2-(f*x+e)*csch(d*x+c)/a/d+(a^2+b^2)*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a
^2+b^2)^(1/2)))/a^2/b/d+(a^2+b^2)*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(
1/2)))/a^2/b/d-b*(f*x+e)*ln(1-exp(2*d*x+2*c))/a^2/d+(a^2+b^2)*f*polylog(2,
-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/b/d^2+(a^2+b^2)*f*polylog(2,-b*exp(
d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/b/d^2-1/2*b*f*polylog(2,exp(2*d*x+2*c))/a^
2/d^2
```

Mathematica [A] (warning: unable to verify)

Time = 6.68 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.56

$$\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{-\frac{bd^2(e+fx)^2}{f} - ad(e+fx)\coth\left(\frac{1}{2}(c+dx)\right) - 2(-af+bd(e+fx))\log\left(1-e^{-c-dx}\right) - 2(af+bd(e+fx$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),
x]
```

output
```
(-((b*d^2*(e + f*x)^2)/f) - a*d*(e + f*x)*Coth[(c + d*x)/2] - 2*(-(a*f) +
b*d*(e + f*x))*Log[1 - E^(-c - d*x)] - 2*(a*f + b*d*(e + f*x))*Log[1 + E^(
-c - d*x)] + 2*b*f*PolyLog[2, -E^(-c - d*x)] + 2*b*f*PolyLog[2, E^(-c - d*
x)] + ((a^2 + b^2)*(-2*d*e*(c + d*x) + 2*c*f*(c + d*x) - f*(c + d*x)^2 + (
4*a*Sqrt[a^2 + b^2]*d*e*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt
[-(a^2 + b^2)^2] - (4*a*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x
))/Sqrt[a^2 + b^2]])/(-a^2 - b^2)^(3/2) + 2*f*(c + d*x)*Log[1 + (b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2])] + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2])] - 2*c*f*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2
*d*e*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] + 2*f*PolyLog[2, (b*E
^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f*PolyLog[2, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2]))]))/b + a*d*(e + f*x)*Tanh[(c + d*x)/2])/(2*a^2*d^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \; 6119$$

$$\frac{\int (e+fx)\cosh(c+dx)\coth^2(c+dx)dx}{a} - \frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5973}$$

$$\frac{\int (e+fx)\cosh(c+dx)dx+\int (e+fx)\coth(c+dx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)\coth(c+dx)\text{csch}(c+dx)dx+\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)\coth(c+dx)\text{csch}(c+dx)dx-\frac{if\int -i\sinh(c+dx)dx}{d}+\frac{(e+fx)\sinh(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int (e+fx)\coth(c+dx)\text{csch}(c+dx)dx-\frac{f\int \sinh(c+dx)dx}{d}+\frac{(e+fx)\sinh(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)\coth(c+dx)\text{csch}(c+dx)dx-\frac{f\int -i\sin(ic+idx)dx}{d}+\frac{(e+fx)\sinh(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)\coth(c+dx)\text{csch}(c+dx)dx+\frac{if\int \sin(ic+idx)dx}{d}+\frac{(e+fx)\sinh(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{3118}$$

$$\frac{\int (e+fx)\coth(c+dx)\text{csch}(c+dx)dx-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5975}$$

$$\frac{\frac{f\int \text{csch}(c+dx)dx}{d}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{f\int i\csc(ic+idx)dx}{d}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{if\int \csc(ic+idx)dx}{d}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{6119}$$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)\cosh^2(c+dx)\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{5973}$$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)\coth(c+dx)dx+\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx+\int -i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx-i\int(e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{ 4201}$$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\int\frac{e^{2c+2dx-i\pi}(e+fx)}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^2}{2f}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int\log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)-\frac{i(e+fx)}{2f}\right.}{a}}{a}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\log\left(1+e^{2c+2dx-i\pi}\right)}{4d^2}\right.\right.}{a}}{a}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx}{2f}\right.}{a}\right.}{a}$$

$$\Big\downarrow \text{5969}$$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{f\int\sinh^2(c+dx)dx}{2d}-i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh}{2d}}{a}\right.}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{f\int-\sin(ic+idx)^2dx}{2d}-i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)s}{}}{a}\right.}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{f\int\sin(ic+idx)^2dx}{2d}-i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2}{2d}}{a}\right.}{a}$$

$$\Big\downarrow \text{3115}$$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{f\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}-i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right.}{a}\right.}{a}$$

$$\Big\downarrow \text{24}$$

$$\frac{-\frac{f\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\operatorname{csch}(c+dx)}{d}}{a}-$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-i\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sin}{}\right.}{}}{a}\right.}{a}$$

$\downarrow$ 6099

$$\frac{-\frac{f\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\operatorname{csch}(c+dx)}{d}}{a}-$$

$$\frac{b\left(-\frac{b\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)\cosh(c+dx)dx}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}+\frac{-i\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e-}{}\right.\right.}{}\right.}{a}$$

$\downarrow$ 3042

$$\frac{-\frac{f\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\operatorname{csch}(c+dx)}{d}}{a}-$$

$$\frac{b\left(\frac{-i\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}-\frac{b\left(\left(a^2\right.\right.}{}\right.}{a}$$

$\downarrow$ 3777

$$\frac{-\frac{f\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\operatorname{csch}(c+dx)}{d}}{a}-$$

$$\frac{b\left(\frac{-i\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}-\frac{b\left(\left(a^2\right.\right.}{}\right.}{a}$$

$\downarrow$ 26

$$\frac{\frac{-\frac{f\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}+\frac{-i\left(2i\left(\frac{f\operatorname{PolyLog}(2,-e}{4d^2}\right.\right.}{}\right.}{a}}{}$$

$\downarrow$ 3042

$$\frac{\frac{-\frac{f\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{-i\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}-\frac{b\left(\left(a^2\right.\right.}{}\right.}{a}}{}$$

input
```
Int[((e + f*x)*Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 937 vs. 2(306) = 612.

Time = 1.22 (sec) , antiderivative size = 938, normalized size of antiderivative = 2.90

method	result
risch	$-\frac{f\,c^2}{d^2 b}+\frac{e\ln\left(b\,e^{2dx+2c}+2a\,e^{dx+c}-b\right)}{db}-\frac{2e\ln\left(e^{dx+c}\right)}{db}-\frac{2fcx}{db}+\frac{2cf\ln\left(e^{dx+c}\right)}{d^2 b}+\frac{f\ln\left(e^{dx+c}-1\right)}{a\,d^2}-\frac{f\ln\left(e^{dx+c}+1\right)}{a\,d^2}-$

input
```
int((f*x+e)*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
-1/d^2/b*f*c^2+1/d/b*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d/b*e*ln(ex
p(d*x+c))-2/d/b*f*c*x+2/d^2/b*c*f*ln(exp(d*x+c))+1/a/d^2*f*ln(exp(d*x+c)-1
)-1/a/d^2*f*ln(exp(d*x+c)+1)-1/a^2/d*b*f*ln(exp(d*x+c)+1)*x+1/a^2/d*b*f*ln
((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/a^2/d*b*f*ln(
(b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/a^2/d^2*b*f*ln((
-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/a^2/d^2*b*f*ln(
(b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/a^2/d^2*c*b*f*ln
(exp(d*x+c)-1)-1/a^2/d^2*c*b*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/2/b
*f*x^2-1/d^2/b*c*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/d^2/b*f*ln((-b*
exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d^2/b*f*ln((b*exp(
d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/d/b*f*ln((-b*exp(d*x+c)
+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d/b*f*ln((b*exp(d*x+c)+(a^2+
b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/b*e*x+1/d^2/b*f*dilog((-b*exp(d*x+c
)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/d^2/b*f*dilog((b*exp(d*x+c)+(
a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/a^2/d^2*b*f*dilog(exp(d*x+c)+1)+1
/a^2/d^2*b*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))
+1/a^2/d^2*b*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))
+1/a^2/d^2*b*f*dilog(exp(d*x+c))-1/a^2/d*b*e*ln(exp(d*x+c)-1)+1/a^2/d*b*e*
ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/a^2/d*b*e*ln(exp(d*x+c)+1)-2/d*(f*
x+e)/a*exp(d*x+c)/(exp(2*d*x+2*c)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1735 vs. $2(303) = 606$.

Time = 0.12 (sec) , antiderivative size = 1735, normalized size of antiderivative = 5.35

$$\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
1/2*(a^2*d^2*f*x^2 + 2*a^2*d^2*e*x + 4*a^2*c*d*e - 2*a^2*c^2*f - (a^2*d^2*
f*x^2 + 2*a^2*d^2*e*x + 4*a^2*c*d*e - 2*a^2*c^2*f)*cosh(d*x + c)^2 - (a^2*
d^2*f*x^2 + 2*a^2*d^2*e*x + 4*a^2*c*d*e - 2*a^2*c^2*f)*sinh(d*x + c)^2 - 4
*(a*b*d*f*x + a*b*d*e)*cosh(d*x + c) + 2*((a^2 + b^2)*f*cosh(d*x + c)^2 +
2*(a^2 + b^2)*f*cosh(d*x + c)*sinh(d*x + c) + (a^2 + b^2)*f*sinh(d*x + c)^
2 - (a^2 + b^2)*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*((a^2 + b^2)
*f*cosh(d*x + c)^2 + 2*(a^2 + b^2)*f*cosh(d*x + c)*sinh(d*x + c) + (a^2 +
b^2)*f*sinh(d*x + c)^2 - (a^2 + b^2)*f)*dilog((a*cosh(d*x + c) + a*sinh(d*
x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b
+ 1) - 2*(b^2*f*cosh(d*x + c)^2 + 2*b^2*f*cosh(d*x + c)*sinh(d*x + c) + b^
2*f*sinh(d*x + c)^2 - b^2*f)*dilog(cosh(d*x + c) + sinh(d*x + c)) - 2*(b^2
*f*cosh(d*x + c)^2 + 2*b^2*f*cosh(d*x + c)*sinh(d*x + c) + b^2*f*sinh(d*x
+ c)^2 - b^2*f)*dilog(-cosh(d*x + c) - sinh(d*x + c)) - 2*((a^2 + b^2)*d*e
 - (a^2 + b^2)*c*f - ((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c)^2 -
 2*((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c) - ((a^2
 + b^2)*d*e - (a^2 + b^2)*c*f)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*
b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*((a^2 + b^2)*d*e -
(a^2 + b^2)*c*f - ((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c)^2 - 2*
((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c) - ((a^2...
```

Sympy [F]

$$\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*cosh(c + d*x)*coth(c + d*x)**2/(a + b*sinh(c + d*x)), x
)
```

Maxima [F]

$$\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh\left(dx+c\right)\coth\left(dx+c\right)^2}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
1/2*(2*b*d*integrate(x/(a^2*d*e^(d*x + c) + a^2*d), x) - 2*b*d*integrate(x
/(a^2*d*e^(d*x + c) - a^2*d), x) + 2*a*((d*x + c)/(a^2*d^2) - log(e^(d*x +
 c) + 1)/(a^2*d^2)) - 2*a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) - 1)/(a^2
*d^2)) + (a*d*x^2*e^(2*d*x + 2*c) - a*d*x^2 - 4*b*x*e^(d*x + c))/(a*b*d*e^
(2*d*x + 2*c) - a*b*d) - integrate(4*((a^3*e^c + a*b^2*e^c)*x*e^(d*x) - (a
^2*b + b^3)*x)/(a^2*b^2*e^(2*d*x + 2*c) + 2*a^3*b*e^(d*x + c) - a^2*b^2),
x))*f + e*((d*x + c)/(b*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) -
 b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d) + (a^2
+ b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^2*b*d))
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\cosh(c+d\,x)\,\coth(c+d\,x)^2\,(e+f\,x)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)*coth(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*coth(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)\cosh\left(dx+c\right)\coth\left(dx+c\right)^2}{a+b\sinh\left(dx+c\right)}dx$$

input
```
int((f*x+e)*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.462 $\int \frac{\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 59

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\operatorname{csch}(c+dx)}{ad} - \frac{b\log(\sinh(c+dx))}{a^2d} + \frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{a^2bd}$$

output
```
-csch(d*x+c)/a/d-b*ln(sinh(d*x+c))/a^2/d+(a^2+b^2)*ln(a+b*sinh(d*x+c))/a^2
/b/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{-ab\operatorname{csch}(c+dx) - b^2\log(\sinh(c+dx)) + (a^2+b^2)\log(a+b\sinh(c+dx))}{a^2bd}$$

input
```
Integrate[(Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-(a*b*Csch[c + d*x]) - b^2*Log[Sinh[c + d*x]] + (a^2 + b^2)*Log[a + b*Sin
h[c + d*x]])/(a^2*b*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3042, 25, 3316, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cos(ic+idx)^3}{\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos(ic+idx)^3}{\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3316}$$

$$\frac{\int \frac{\text{csch}^2(c+dx)\left(\sinh^2(c+dx)b^2+b^2\right)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\text{csch}^2(c+dx)\left(\sinh^2(c+dx)b^2+b^2\right)}{b^2(a+b\sinh(c+dx))}d(b\sinh(c+dx))}{bd}$$

$$\Big\downarrow \text{522}$$

$$\frac{\int \left(\frac{\text{csch}^2(c+dx)}{a}-\frac{b\text{csch}(c+dx)}{a^2}+\frac{a^2+b^2}{a^2(a+b\sinh(c+dx))}\right)d(b\sinh(c+dx))}{bd}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^2\log(b\sinh(c+dx))}{a^2}+\frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{a^2}-\frac{b\text{csch}(c+dx)}{a}}{bd}$$

input
```
Int[(Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-((b*Csch[c + d*x])/a) - (b^2*Log[b*Sinh[c + d*x]])/a^2 + ((a^2 + b^2)*Lo
g[a + b*Sinh[c + d*x]])/a^2)/(b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(59) = 118$.

Time = 0.81 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.17

method	result
risch	$-\frac{x}{b}-\frac{2c}{bd}-\frac{2\,\mathrm{e}^{dx+c}}{da\left(\mathrm{e}^{2dx+2c}-1\right)}-\frac{b\ln\left(\mathrm{e}^{2dx+2c}-1\right)}{a^2d}+\frac{\ln\left(\mathrm{e}^{2dx+2c}+\frac{2a\,\mathrm{e}^{dx+c}}{b}-1\right)}{bd}+\frac{b\ln\left(\mathrm{e}^{2dx+2c}+\frac{2a\,\mathrm{e}^{dx+c}}{b}-1\right)}{a^2d}$
derivativedivides	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{\left(2a^2+2b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tanh\left(\frac{dx}{2}+\right.\right.}{2b\,a^2}}{d}$
default	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{\left(2a^2+2b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tanh\left(\frac{dx}{2}+\right.\right.}{2b\,a^2}}{d}$

input
```
int(cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-x/b-2/b/d*c-2/d/a*exp(d*x+c)/(exp(2*d*x+2*c)-1)-b/a^2/d*ln(exp(2*d*x+2*c)
-1)+1/b/d*ln(exp(2*d*x+2*c)+2/b*a*exp(d*x+c)-1)+b/a^2/d*ln(exp(2*d*x+2*c)+
2/b*a*exp(d*x+c)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(59) = 118$.

Time = 0.10 (sec) , antiderivative size = 299, normalized size of antiderivative = 5.07

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{a^2dx\cosh\left(dx+c\right)^2+a^2dx\sinh\left(dx+c\right)^2-a^2dx+2\,ab\cosh\left(dx+c\right)-\left(\left(a^2+b^2\right)\cosh\left(dx+c\right)^2+2\right.}{}$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
-(a^2*d*x*cosh(d*x + c)^2 + a^2*d*x*sinh(d*x + c)^2 - a^2*d*x + 2*a*b*cosh
(d*x + c) - ((a^2 + b^2)*cosh(d*x + c)^2 + 2*(a^2 + b^2)*cosh(d*x + c)*sin
h(d*x + c) + (a^2 + b^2)*sinh(d*x + c)^2 - a^2 - b^2)*log(2*(b*sinh(d*x +
c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + (b^2*cosh(d*x + c)^2 + 2*b^2*co
sh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 - b^2)*log(2*sinh(d*x + c)
/(cosh(d*x + c) - sinh(d*x + c))) + 2*(a^2*d*x*cosh(d*x + c) + a*b)*sinh(d
*x + c))/(a^2*b*d*cosh(d*x + c)^2 + 2*a^2*b*d*cosh(d*x + c)*sinh(d*x + c)
+ a^2*b*d*sinh(d*x + c)^2 - a^2*b*d)
```

Sympy [F]

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh\left(c+dx\right)\coth^2\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(cosh(c + d*x)*coth(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(59) = 118$.

Time = 0.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.22

$$\begin{aligned}\int \frac{\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{dx+c}{bd} + \frac{2\,e^{(-dx-c)}}{\left(ae^{(-2\,dx-2\,c)}-a\right)d} \\ &\quad - \frac{b\log\left(e^{(-dx-c)}+1\right)}{a^2d} - \frac{b\log\left(e^{(-dx-c)}-1\right)}{a^2d} \\ &\quad + \frac{\left(a^2+b^2\right)\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{a^2bd}\end{aligned}$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
(d*x + c)/(b*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log(e^(-
d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d) + (a^2 + b^2)*log(
-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^2*b*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(59) = 118.

Time = 0.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.05

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{b\log\left(\left|e^{(dx+c)}-e^{(-dx-c)}\right|\right)}{a^2}-\frac{(a^2+b^2)\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2a\right|\right)}{a^2b}-\frac{b\left(e^{(dx+c)}-e^{(-dx-c)}\right)-2a}{a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)}}{d}$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-(b*log(abs(e^(d*x + c) - e^(-d*x - c)))/a^2 - (a^2 + b^2)*log(abs(b*(e^(d
*x + c) - e^(-d*x - c)) + 2*a))/(a^2*b) - (b*(e^(d*x + c) - e^(-d*x - c))
- 2*a)/(a^2*(e^(d*x + c) - e^(-d*x - c))))/d
```

Mupad [B] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 356, normalized size of antiderivative = 6.03

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2\,e^{c+d\,x}}{a\,d-a\,d\,e^{2\,c+2\,d\,x}}-\frac{x}{b}$$
$$+\frac{\ln\left(8\,a^5\,e^{d\,x}\,e^{c}-16\,b^5-16\,a^2\,b^3-4\,a^4\,b+16\,b^5\,e^{2\,c}\,e^{2\,d\,x}+4\,a^4\,b\,e^{2\,c}\,e^{2\,d\,x}+32\,a^3\,b^2\,e^{d\,x}\,e^{c}+16\,a^2\,b^3\,e^{2\,c}\right.}{b\,d}$$
$$+\frac{b\,\ln\left(8\,a^5\,e^{d\,x}\,e^{c}-16\,b^5-16\,a^2\,b^3-4\,a^4\,b+16\,b^5\,e^{2\,c}\,e^{2\,d\,x}+4\,a^4\,b\,e^{2\,c}\,e^{2\,d\,x}+32\,a^3\,b^2\,e^{d\,x}\,e^{c}+16\,a^2\,b^3\,e\right.}{a^2\,d}$$
$$-\frac{b\,\ln\left(4\,a^6+16\,b^6+32\,a^2\,b^4+20\,a^4\,b^2-4\,a^6\,e^{2\,c}\,e^{2\,d\,x}-16\,b^6\,e^{2\,c}\,e^{2\,d\,x}-32\,a^2\,b^4\,e^{2\,c}\,e^{2\,d\,x}-20\,a^4\,b^2\,e^{2\,c}\,e\right.}{a^2\,d}$$

input
```
int((cosh(c + d*x)*coth(c + d*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
(2*exp(c + d*x))/(a*d - a*d*exp(2*c + 2*d*x)) - x/b + log(8*a^5*exp(d*x)*e
xp(c) - 16*b^5 - 16*a^2*b^3 - 4*a^4*b + 16*b^5*exp(2*c)*exp(2*d*x) + 4*a^4
*b*exp(2*c)*exp(2*d*x) + 32*a^3*b^2*exp(d*x)*exp(c) + 16*a^2*b^3*exp(2*c)*
exp(2*d*x) + 32*a*b^4*exp(d*x)*exp(c))/(b*d) + (b*log(8*a^5*exp(d*x)*exp(c
) - 16*b^5 - 16*a^2*b^3 - 4*a^4*b + 16*b^5*exp(2*c)*exp(2*d*x) + 4*a^4*b*e
xp(2*c)*exp(2*d*x) + 32*a^3*b^2*exp(d*x)*exp(c) + 16*a^2*b^3*exp(2*c)*exp(
2*d*x) + 32*a*b^4*exp(d*x)*exp(c)))/(a^2*d) - (b*log(4*a^6 + 16*b^6 + 32*a
^2*b^4 + 20*a^4*b^2 - 4*a^6*exp(2*c)*exp(2*d*x) - 16*b^6*exp(2*c)*exp(2*d*
x) - 32*a^2*b^4*exp(2*c)*exp(2*d*x) - 20*a^4*b^2*exp(2*c)*exp(2*d*x)))/(a^
2*d)
```

Reduce [B] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 283, normalized size of antiderivative = 4.80

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{-e^{2dx+2c}\log\left(e^{dx+c}-1\right)b^2-e^{2dx+2c}\log\left(e^{dx+c}+1\right)b^2+e^{2dx+2c}\log\left(e^{2dx+2c}b+2e^{dx+c}a-b\right)a^2+e^{2dx+2c}\mathrm{lo}}{}$$

input
```
int(cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
( - e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*b**2 - e**(2*c + 2*d*x)*log(e**
(c + d*x) + 1)*b**2 + e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x)*b + 2*e**(c +
d*x)*a - b)*a**2 + e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x
)*a - b)*b**2 - e**(2*c + 2*d*x)*a**2*d*x - 2*e**(c + d*x)*a*b + log(e**(c
 + d*x) - 1)*b**2 + log(e**(c + d*x) + 1)*b**2 - log(e**(2*c + 2*d*x)*b +
2*e**(c + d*x)*a - b)*a**2 - log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b
)*b**2 + a**2*d*x)/(a**2*b*d*(e**(2*c + 2*d*x) - 1))
```

3.463 $\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(cosh(d*x+c)*coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 51.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Cosh[c + d*x]*Coth[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Cosh[c + d*x]*Coth[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{ 6125}$$
$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Cosh[c + d*x]*Coth[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(dx+c\right)\coth\left(dx+c\right)^{2}}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(cosh(d*x+c)*coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)*coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)\coth\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(cosh(d*x + c)*coth(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 6.39 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh\left(c+dx\right)\coth^2\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(cosh(c + d*x)*coth(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)),
 x)
```

Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 353, normalized size of antiderivative = 10.38

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)\coth(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
2*e^(d*x + c)/(a*d*f*x + a*d*e - (a*d*f*x*e^(2*c) + a*d*e*e^(2*c))*e^(2*d*
x)) + log(f*x + e)/(b*f) - 1/2*integrate(-2*(b*d*f*x + b*d*e + a*f)/(a^2*d
*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2 - (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*
e^c + a^2*d*e^2*e^c)*e^(d*x)), x) + 1/2*integrate(2*(b*d*f*x + b*d*e - a*f
)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2 + (a^2*d*f^2*x^2*e^c + 2*a^2*
d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)), x) - 1/2*integrate(4*(a^2*b + b^3 -
 (a^3*e^c + a*b^2*e^c)*e^(d*x))/(a^2*b^2*f*x + a^2*b^2*e - (a^2*b^2*f*x*e^
(2*c) + a^2*b^2*e*e^(2*c))*e^(2*d*x) - 2*(a^3*b*f*x*e^c + a^3*b*e*e^c)*e^(
d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 3.89 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)\,\coth(c+d\,x)^2}{(e+f\,x)\,(a+b\sinh(c+d\,x))}\,dx$$

input
```
int((cosh(c + d*x)*coth(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((cosh(c + d*x)*coth(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 979, normalized size of antiderivative = 28.79

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$= \frac{-2e^{5c}\left(\int \frac{e^{5dx}}{e^{6dx+6c}be+e^{6dx+6c}bfx+2e^{5dx+5c}ae+2e^{5dx+5c}afx-3e^{4dx+4c}be-3e^{4dx+4c}bfx-4e^{3dx+3c}ae-4e^{3dx+3c}afx+3e^{2dx+2c}be+3e^{2dx+}}\right.}{}$$

input
```
int(cosh(d*x+c)*coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*e**(5*c)*int(e**(5*d*x)/(e**(6*c + 6*d*x)*b*e + e**(6*c + 6*d*x)*b*f
*x + 2*e**(5*c + 5*d*x)*a*e + 2*e**(5*c + 5*d*x)*a*f*x - 3*e**(4*c + 4*d*x
)*b*e - 3*e**(4*c + 4*d*x)*b*f*x - 4*e**(3*c + 3*d*x)*a*e - 4*e**(3*c + 3*
d*x)*a*f*x + 3*e**(2*c + 2*d*x)*b*e + 3*e**(2*c + 2*d*x)*b*f*x + 2*e**(c +
 d*x)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x)*a*f + 6*e**(4*c)*int(e*
*(4*d*x)/(e**(6*c + 6*d*x)*b*e + e**(6*c + 6*d*x)*b*f*x + 2*e**(5*c + 5*d*
x)*a*e + 2*e**(5*c + 5*d*x)*a*f*x - 3*e**(4*c + 4*d*x)*b*e - 3*e**(4*c + 4
*d*x)*b*f*x - 4*e**(3*c + 3*d*x)*a*e - 4*e**(3*c + 3*d*x)*a*f*x + 3*e**(2*
c + 2*d*x)*b*e + 3*e**(2*c + 2*d*x)*b*f*x + 2*e**(c + d*x)*a*e + 2*e**(c +
 d*x)*a*f*x - b*e - b*f*x),x)*b*f + 4*e**(3*c)*int(e**(3*d*x)/(e**(6*c + 6
*d*x)*b*e + e**(6*c + 6*d*x)*b*f*x + 2*e**(5*c + 5*d*x)*a*e + 2*e**(5*c +
5*d*x)*a*f*x - 3*e**(4*c + 4*d*x)*b*e - 3*e**(4*c + 4*d*x)*b*f*x - 4*e**(3
*c + 3*d*x)*a*e - 4*e**(3*c + 3*d*x)*a*f*x + 3*e**(2*c + 2*d*x)*b*e + 3*e*
*(2*c + 2*d*x)*b*f*x + 2*e**(c + d*x)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b
*f*x),x)*a*f - 2*e**c*int(e**(d*x)/(e**(6*c + 6*d*x)*b*e + e**(6*c + 6*d*x
)*b*f*x + 2*e**(5*c + 5*d*x)*a*e + 2*e**(5*c + 5*d*x)*a*f*x - 3*e**(4*c +
4*d*x)*b*e - 3*e**(4*c + 4*d*x)*b*f*x - 4*e**(3*c + 3*d*x)*a*e - 4*e**(3*c
 + 3*d*x)*a*f*x + 3*e**(2*c + 2*d*x)*b*e + 3*e**(2*c + 2*d*x)*b*f*x + 2*e*
*(c + d*x)*a*e + 2*e**(c + d*x)*a*f*x - b*e - b*f*x),x)*a*f + 2*int(1/(e**
(6*c + 6*d*x)*b*e + e**(6*c + 6*d*x)*b*f*x + 2*e**(5*c + 5*d*x)*a*e + 2...
```

3.464 $\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 1428

$$\int \frac{(e+fx)^3\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output
```
3/2*b*f^2*(f*x+e)*polylog(3,exp(2*d*x+2*c))/a^2/d^3-3/4*b^3*f^3*polylog(4,
-exp(2*d*x+2*c))/a^2/(a^2+b^2)/d^4-3/2*b*f^2*(f*x+e)*polylog(3,-exp(2*d*x+
2*c))/a^2/d^3+3/2*b*f*(f*x+e)^2*polylog(2,-exp(2*d*x+2*c))/a^2/d^2+2*b^2*(
f*x+e)^3*arctan(exp(d*x+c))/a/(a^2+b^2)/d-6*I*f^2*(f*x+e)*polylog(3,-I*exp
(d*x+c))/a/d^3-3*I*f*(f*x+e)^2*polylog(2,I*exp(d*x+c))/a/d^2-2*(f*x+e)^3*a
rctan(exp(d*x+c))/a/d-3/2*b*f*(f*x+e)^2*polylog(2,exp(2*d*x+2*c))/a^2/d^2+
3/2*b^3*f^2*(f*x+e)*polylog(3,-exp(2*d*x+2*c))/a^2/(a^2+b^2)/d^3-3/2*b^3*f
*(f*x+e)^2*polylog(2,-exp(2*d*x+2*c))/a^2/(a^2+b^2)/d^2-6*I*b^2*f^3*polylo
g(4,-I*exp(d*x+c))/a/(a^2+b^2)/d^4+6*I*b^2*f^3*polylog(4,I*exp(d*x+c))/a/(
a^2+b^2)/d^4+6*b^3*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a
^2+b^2)/d^4+6*b^3*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^
2+b^2)/d^4+b^3*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b
^2)/d+b^3*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d
-(f*x+e)^3*csch(d*x+c)/a/d-6*I*b^2*f^2*(f*x+e)*polylog(3,I*exp(d*x+c))/a/(
a^2+b^2)/d^3-3*I*b^2*f*(f*x+e)^2*polylog(2,-I*exp(d*x+c))/a/(a^2+b^2)/d^2+
6*f^3*polylog(3,-exp(d*x+c))/a/d^4-6*f^3*polylog(3,exp(d*x+c))/a/d^4-6*f*(
f*x+e)^2*arctanh(exp(d*x+c))/a/d^2-6*f^2*(f*x+e)*polylog(2,-exp(d*x+c))/a/
d^3+6*f^2*(f*x+e)*polylog(2,exp(d*x+c))/a/d^3-3/4*b*f^3*polylog(4,exp(2*d*
x+2*c))/a^2/d^4+2*b*(f*x+e)^3*arctanh(exp(2*d*x+2*c))/a^2/d+3/4*b*f^3*poly
log(4,-exp(2*d*x+2*c))/a^2/d^4-6*I*f^3*polylog(4,I*exp(d*x+c))/a/d^4-b^...
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4139 vs. 2(1428) = 2856.

Time = 10.72 (sec) , antiderivative size = 4139, normalized size of antiderivative = 2.90

$$\int \frac{(e+fx)^3\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
(2*d^3*e^2*(-1 + E^(2*c))*f*(b*d*e - 3*a*f)*x + 2*d^3*e^2*(-1 + E^(2*c))*f
*(b*d*e + 3*a*f)*x + b*d^4*(e + f*x)^4 - 6*d^2*e*(-1 + E^(2*c))*f^2*(b*d*e
 - 2*a*f)*x*Log[1 - E^(-c - d*x)] - 6*d^2*(-1 + E^(2*c))*f^3*(b*d*e - a*f)
*x^2*Log[1 - E^(-c - d*x)] - 2*b*d^3*(-1 + E^(2*c))*f^4*x^3*Log[1 - E^(-c
- d*x)] - 6*d^2*e*(-1 + E^(2*c))*f^2*(b*d*e + 2*a*f)*x*Log[1 + E^(-c - d*x
)] - 6*d^2*(-1 + E^(2*c))*f^3*(b*d*e + a*f)*x^2*Log[1 + E^(-c - d*x)] - 2*
b*d^3*(-1 + E^(2*c))*f^4*x^3*Log[1 + E^(-c - d*x)] - 2*d^2*e^2*(-1 + E^(2*
c))*f*(b*d*e - 3*a*f)*Log[1 - E^(c + d*x)] - 2*d^2*e^2*(-1 + E^(2*c))*f*(b
*d*e + 3*a*f)*Log[1 + E^(c + d*x)] + 6*d*e*(-1 + E^(2*c))*f^2*(b*d*e + 2*a
*f)*PolyLog[2, -E^(-c - d*x)] + 12*d*(-1 + E^(2*c))*f^3*(b*d*e + a*f)*x*Po
lyLog[2, -E^(-c - d*x)] + 6*b*d^2*(-1 + E^(2*c))*f^4*x^2*PolyLog[2, -E^(-c
 - d*x)] + 6*d*e*(-1 + E^(2*c))*f^2*(b*d*e - 2*a*f)*PolyLog[2, E^(-c - d*x
)] + 12*d*(-1 + E^(2*c))*f^3*(b*d*e - a*f)*x*PolyLog[2, E^(-c - d*x)] + 6*
b*d^2*(-1 + E^(2*c))*f^4*x^2*PolyLog[2, E^(-c - d*x)] + 12*(-1 + E^(2*c))*
f^3*(b*d*e + a*f)*PolyLog[3, -E^(-c - d*x)] + 12*b*d*(-1 + E^(2*c))*f^4*x*
PolyLog[3, -E^(-c - d*x)] - 12*(-1 + E^(2*c))*f^3*(-(b*d*e) + a*f)*PolyLog
[3, E^(-c - d*x)] + 12*b*d*(-1 + E^(2*c))*f^4*x*PolyLog[3, E^(-c - d*x)] +
 12*b*(-1 + E^(2*c))*f^4*PolyLog[4, -E^(-c - d*x)] + 12*b*(-1 + E^(2*c))*f
^4*PolyLog[4, E^(-c - d*x)])/(2*a^2*d^4*(-1 + E^(2*c))*f) - (8*b*d^4*e^3*E
^(2*c)*x + 12*b*d^4*e^2*E^(2*c)*f*x^2 + 8*b*d^4*e*E^(2*c)*f^2*x^3 + 2*b...
```

Rubi [A] (verified)

Time = 6.58 (sec) , antiderivative size = 1259, normalized size of antiderivative = 0.88, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {6123, 5985, 25, 6123, 5984, 3042, 26, 4670, 3011, 6107, 6095, 2620, 3011, 7163, 2720, 7143, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)^3\text{csch}^2(c+dx)\text{sech}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5985}$$

$$\frac{-3f\int -(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^3\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$\downarrow$ 25

$$\frac{3f\int (e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^3\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$\downarrow$ 6123

$$\frac{3f\int (e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^3\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$\downarrow$ 5984

$$\frac{3f\int (e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{2\int(e+fx)^3\operatorname{csch}(2c+2dx)dx}{a}-\frac{b\int\frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$\downarrow$ 3042

$$\frac{3f\int (e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2\int i(e+fx)^3\csc(2ic+2idx)dx}{a}\right)}{a}$$

$\downarrow$ 26

$$\frac{3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\int(e+fx)^3\csc(2ic+2idx)dx}{a}\right)}{a}$$

$\Big\downarrow$ 4670

$$\frac{3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(\frac{3if\int(e+fx)^2\log\left(1-e^{2c+2dx}\right)dx}{2d}-\frac{3if\int(e+fx)^2\log\left(1+e^{2c+2dx}\right)dx}{2d}+\frac{i(e+fx)^3\text{arctanh}\left(e^{2c+2dx}\right)}{d}\right)}{a}\right)}{a}$$

$\Big\downarrow$ 3011

$$\frac{3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(-\frac{3if\left(\frac{f\int(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{2d}+\frac{3if\left(\frac{f\int(e+fx)\text{PolyLog}\left(2,e^{2c\cdots}\right.}{d}\right.}{\cdots}\right.}{a}\cdots\right)}{a}$$

$\Big\downarrow$ 6107

$$\frac{3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(\frac{b^2\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}+\frac{2i\left(-\frac{3if\left(\frac{f\int(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\cdots}{2d}\right.}{2d}\cdots\right.}{\cdots}\right)}{a}$$

$\Big\downarrow$ 6095

$$\frac{3f\int (e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^4}{4bf}\right)}{a^2+b^2}+\frac{\int (e+fx)^3\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}+\frac{2i\left(-\frac{3if\left(\frac{f\int (e+fx)\operatorname{Po}\ldots}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}$$

$$\downarrow 2620$$

$$\frac{3f\int (e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(\frac{b^2\left(-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\ldots\right.}{a^2+b^2}\ldots\right.}{a}\ldots\right.}{\ldots}$$

$$\downarrow 3011$$

$$\frac{3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-$$

$$b\left(-\frac{b\left(\frac{b^2\left(-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+f}{}\right.}{bd}\right.}{a^2+b^2}\right.}{}\right.$$

$\downarrow$ 7163

$$\frac{3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-$$

$$b\left(-\frac{b\left(\frac{b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\mathrm{I}}{}\right.}{}\right.}{}\right.}{}\right.}{}\right.$$

↓ 2720

$$\frac{-\frac{\arctan(\sinh(c+dx))(e+fx)^3}{d}-\frac{\operatorname{csch}(c+dx)(e+fx)^3}{d}+3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx}{a}-$$

$$b\left(\frac{2i\left(\frac{i\operatorname{arctanh}\left(e^{2c+2dx}\right)(e+fx)^3}{d}-\frac{3if\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{2c+2dx}\right)}{2d}-\frac{f\int e^{-2c-2dx}\operatorname{PolyLog}\left(3,-e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,-}{2d}}{2d}\right.}{a}\right.$$

↓ 7143

$$\frac{3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-$$

$$b\left(-\frac{b\left(\frac{\int(e+fx)^3\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\cdots}{\cdots}\right)}{bd}\cdots\right)}{\cdots}}{\cdots}\right)}{\cdots}$$

↓ 7292

$$\frac{3f\int \frac{(e+fx)^2(\arctan(\sinh(c+dx))+\operatorname{csch}(c+dx))}{d}dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-$$

$$b\left(-\frac{b\left(\frac{\int (e+fx)^3\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\ldots}{\ldots}\right)}{bd}\ldots\right)}{\ldots}\right)}{\ldots}\ldots\right)$$

$\downarrow$ 27

$$\frac{\frac{3f\int(e+fx)^2(\arctan(\sinh(c+dx))+\operatorname{csch}(c+dx))dx}{d}-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-$$

$$b\left(-\frac{b\left(\frac{\int(e+fx)^3\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}}{}\right.}{bd}\right.}{}\right.}{}\right.$$

$\downarrow$ 7293

$$\frac{\frac{3f\int\left(\arctan(\sinh(c+dx))(e+fx)^2+\operatorname{csch}(c+dx)(e+fx)^2\right)dx}{d}-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-$$

$$b\left(-\frac{b\left(\frac{\int\left(a(e+fx)^3\operatorname{sech}(c+dx)-b(e+fx)^3\tanh(c+dx)\right)dx}{a^2+b^2}+\frac{b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)}{}\right.}{bd}\right.}{}\right.}{}\right.$$

$\downarrow$ 2009

$$-\frac{\arctan(\sinh(c+dx))(e+fx)^3}{d}-\frac{\operatorname{csch}(c+dx)(e+fx)^3}{d}+\frac{3f\left(-\frac{2\arctan\left(e^{c+dx}\right)(e+fx)^3}{3f}+\frac{\arctan(\sinh(c+dx))(e+fx)^3}{3f}-\frac{2\operatorname{arctanh}\left(e^{c+dx}\right)(e+f}{d}\right.}{}$$

$$b\left(\frac{2i\left(\frac{i\operatorname{arctanh}\left(e^{2c+2dx}\right)(e+fx)^3}{d}-\frac{3if\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{2c+2dx}\right)}{2d}-\frac{f\operatorname{PolyLog}\left(4,-e^{2c+2dx}\right)}{4d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{2d}+\frac{3if\left(\frac{f(}{}\right.}{}\right.}{a}\right.$$

input
```
Int[((e + f*x)^3*Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-(((e + f*x)^3*ArcTan[Sinh[c + d*x]])/d) - ((e + f*x)^3*Csch[c + d*x])/d
+ (3*f*((-2*(e + f*x)^3*ArcTan[E^(c + d*x)])/(3*f) + ((e + f*x)^3*ArcTan[S
inh[c + d*x]])/(3*f) - (2*(e + f*x)^2*ArcTanh[E^(c + d*x)])/d - (2*f*(e +
f*x)*PolyLog[2, -E^(c + d*x)])/d^2 + (I*(e + f*x)^2*PolyLog[2, (-I)*E^(c +
 d*x)])/d - (I*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/d + (2*f*(e + f*x)*P
olyLog[2, E^(c + d*x)])/d^2 + (2*f^2*PolyLog[3, -E^(c + d*x)])/d^3 - ((2*I
)*f*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/d^2 + ((2*I)*f*(e + f*x)*PolyL
og[3, I*E^(c + d*x)])/d^2 - (2*f^2*PolyLog[3, E^(c + d*x)])/d^3 + ((2*I)*f
^2*PolyLog[4, (-I)*E^(c + d*x)])/d^3 - ((2*I)*f^2*PolyLog[4, I*E^(c + d*x)
])/d^3))/d)/a - (b*(-((b*((b^2*(-1/4*(e + f*x)^4/(b*f) + ((e + f*x)^3*Log[
1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)^3*Log[1 + (
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLo
g[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*Poly
Log[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d) - (3*f*(-(((e + f*x)^
2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f
*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/(a^2 + b^2)
 + ((b*(e + f*x)^4)/(4*f) + (2*a*(e + f*x)^3*ArcTan[E^(c + d*x)])/d - (b*(
e + f*x)^3*Log[1 + E^(2*(c + d*x))])/d - ((3*I)*a*f*(e + f*x)^2*PolyLog...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^3 \operatorname{csch}(dx+c)^2 \operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 9763 vs. $2(1301) = 2602$.

Time = 0.34 (sec) , antiderivative size = 9763, normalized size of antiderivative = 6.84

$$\int \frac{(e+fx)^3 \operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)**3*csch(d*x+c)**2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^3\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\,\text{csch}\,(dx+c)^2\,\text{sech}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
(b^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + a^2*b^2)*d) +
 2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((
a^2 + b^2)*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log(e^(-d*
x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d))*e^3 - 2*(f^3*x^3*e^
c + 3*e*f^2*x^2*e^c + 3*e^2*f*x*e^c)*e^(d*x)/(a*d*e^(2*d*x + 2*c) - a*d) -
 3*e^2*f*log(e^(d*x + c) + 1)/(a*d^2) + 3*e^2*f*log(e^(d*x + c) - 1)/(a*d^
2) - (d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x
*polylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*b*f^3/(a^2*d^4) -
(d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*poly
log(3, e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))*b*f^3/(a^2*d^4) - 3*(b*d*
e^2*f + 2*a*e*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^2*d
^3) - 3*(b*d*e^2*f - 2*a*e*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x
+ c)))/(a^2*d^3) - 3*(b*d*e*f^2 + a*f^3)*(d^2*x^2*log(e^(d*x + c) + 1) + 2
*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))/(a^2*d^4) - 3*(b*d*
e*f^2 - a*f^3)*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) -
 2*polylog(3, e^(d*x + c)))/(a^2*d^4) + 1/4*(b*d^4*f^3*x^4 + 4*(b*d*e*f^2
+ a*f^3)*d^3*x^3 + 6*(b*d^2*e^2*f + 2*a*d*e*f^2)*d^2*x^2)/(a^2*d^4) + 1/4*
(b*d^4*f^3*x^4 + 4*(b*d*e*f^2 - a*f^3)*d^3*x^3 + 6*(b*d^2*e^2*f - 2*a*d*e*
f^2)*d^2*x^2)/(a^2*d^4) - integrate(2*(b^4*f^3*x^3 + 3*b^4*e*f^2*x^2 + 3*b
^4*e^2*f*x - (a*b^3*f^3*x^3*e^c + 3*a*b^3*e*f^2*x^2*e^c + 3*a*b^3*e^2*f...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^3}{\cosh(c+d\,x)\,\sinh(c+d\,x)^2\,(a+b\sinh(c+d\,x))}\,dx$$

input
```
int((e + f*x)^3/(cosh(c + d*x)*sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^3/(cosh(c + d*x)*sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^3*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**3*e**3 + 2*atan(e**(c + d*x))
*a**3*e**3 + 16*e**(6*c + 2*d*x)*int((e**(4*d*x)*x**3)/(e**(8*c + 8*d*x)*b
 + 2*e**(7*c + 7*d*x)*a - 2*e**(6*c + 6*d*x)*b - 2*e**(5*c + 5*d*x)*a - 2*
e**(3*c + 3*d*x)*a + 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*
d*f**3 + 16*e**(6*c + 2*d*x)*int((e**(4*d*x)*x**3)/(e**(8*c + 8*d*x)*b + 2
*e**(7*c + 7*d*x)*a - 2*e**(6*c + 6*d*x)*b - 2*e**(5*c + 5*d*x)*a - 2*e**(
3*c + 3*d*x)*a + 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b**2
*d*f**3 + 48*e**(6*c + 2*d*x)*int((e**(4*d*x)*x**2)/(e**(8*c + 8*d*x)*b +
2*e**(7*c + 7*d*x)*a - 2*e**(6*c + 6*d*x)*b - 2*e**(5*c + 5*d*x)*a - 2*e**
(3*c + 3*d*x)*a + 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*d*e
*f**2 + 48*e**(6*c + 2*d*x)*int((e**(4*d*x)*x**2)/(e**(8*c + 8*d*x)*b + 2*
e**(7*c + 7*d*x)*a - 2*e**(6*c + 6*d*x)*b - 2*e**(5*c + 5*d*x)*a - 2*e**(3
*c + 3*d*x)*a + 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b**2*
d*e*f**2 + 48*e**(6*c + 2*d*x)*int((e**(4*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*
e**(7*c + 7*d*x)*a - 2*e**(6*c + 6*d*x)*b - 2*e**(5*c + 5*d*x)*a - 2*e**(3
*c + 3*d*x)*a + 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*d*e**
2*f + 48*e**(6*c + 2*d*x)*int((e**(4*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7
*c + 7*d*x)*a - 2*e**(6*c + 6*d*x)*b - 2*e**(5*c + 5*d*x)*a - 2*e**(3*c +
3*d*x)*a + 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b**2*d*e**
2*f + e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2*b*e**3 - e**(2*c ...
```

3.465 $\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 982

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output
```
-2*(f*x+e)^2*arctan(exp(d*x+c))/a/d+2*I*f*(f*x+e)*polylog(2,-I*exp(d*x+c))
/a/d^2+2*I*f^2*polylog(3,I*exp(d*x+c))/a/d^3+b*f*(f*x+e)*polylog(2,-exp(2*
d*x+2*c))/a^2/d^2-b^3*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/a^2/(a^2+b^2)/d+2*I*b
^2*f*(f*x+e)*polylog(2,I*exp(d*x+c))/a/(a^2+b^2)/d^2+2*I*b^2*f^2*polylog(3
,-I*exp(d*x+c))/a/(a^2+b^2)/d^3-b^3*f*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/a
^2/(a^2+b^2)/d^2+b^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/
(a^2+b^2)/d+b^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+
b^2)/d-2*b^3*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2
)/d^3-2*b^3*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)
/d^3-(f*x+e)^2*csch(d*x+c)/a/d-2*I*b^2*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/
a/(a^2+b^2)/d^2-2*f^2*polylog(2,-exp(d*x+c))/a/d^3+2*f^2*polylog(2,exp(d*x
+c))/a/d^3-2*I*b^2*f^2*polylog(3,I*exp(d*x+c))/a/(a^2+b^2)/d^3-4*f*(f*x+e)
*arctanh(exp(d*x+c))/a/d^2+1/2*b*f^2*polylog(3,exp(2*d*x+2*c))/a^2/d^3-b*f
*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a^2/d^2-1/2*b*f^2*polylog(3,-exp(2*d*x+
2*c))/a^2/d^3+2*b*(f*x+e)^2*arctanh(exp(2*d*x+2*c))/a^2/d-2*I*f^2*polylog(
3,-I*exp(d*x+c))/a/d^3+2*b^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2
)^(1/2)))/a^2/(a^2+b^2)/d^2+2*b^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^
2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^2+1/2*b^3*f^2*polylog(3,-exp(2*d*x+2*c))/a^
2/(a^2+b^2)/d^3+2*b^2*(f*x+e)^2*arctan(exp(d*x+c))/a/(a^2+b^2)/d-2*I*f*(f*
x+e)*polylog(2,I*exp(d*x+c))/a/d^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2323 vs. $2(982) = 1964$.

Time = 10.30 (sec) , antiderivative size = 2323, normalized size of antiderivative = 2.37

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
(3*d^2*e*(-1 + E^(2*c))*f*(b*d*e - 2*a*f)*x + 3*d^2*e*(-1 + E^(2*c))*f*(b*
d*e + 2*a*f)*x + 2*b*d^3*(e + f*x)^3 - 6*d*(-1 + E^(2*c))*f^2*(b*d*e - a*f
)*x*Log[1 - E^(-c - d*x)] - 3*b*d^2*(-1 + E^(2*c))*f^3*x^2*Log[1 - E^(-c -
 d*x)] - 6*d*(-1 + E^(2*c))*f^2*(b*d*e + a*f)*x*Log[1 + E^(-c - d*x)] - 3*
b*d^2*(-1 + E^(2*c))*f^3*x^2*Log[1 + E^(-c - d*x)] - 3*d*e*(-1 + E^(2*c))*
f*(b*d*e - 2*a*f)*Log[1 - E^(c + d*x)] - 3*d*e*(-1 + E^(2*c))*f*(b*d*e + 2
*a*f)*Log[1 + E^(c + d*x)] + 6*(-1 + E^(2*c))*f^2*(b*d*e + a*f)*PolyLog[2,
 -E^(-c - d*x)] + 6*b*d*(-1 + E^(2*c))*f^3*x*PolyLog[2, -E^(-c - d*x)] - 6
*(-1 + E^(2*c))*f^2*(-(b*d*e) + a*f)*PolyLog[2, E^(-c - d*x)] + 6*b*d*(-1
+ E^(2*c))*f^3*x*PolyLog[2, E^(-c - d*x)] + 6*b*(-1 + E^(2*c))*f^3*PolyLog
[3, -E^(-c - d*x)] + 6*b*(-1 + E^(2*c))*f^3*PolyLog[3, E^(-c - d*x)])/(3*a
^2*d^3*(-1 + E^(2*c))*f) - (12*b*d^3*e^2*E^(2*c)*x - 12*b*d^3*e^2*(1 + E^(
2*c))*x - 12*b*d^3*e*f*x^2 - 4*b*d^3*f^2*x^3 + 12*a*d^2*e^2*(1 + E^(2*c))*
ArcTan[E^(c + d*x)] + 6*b*d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c +
 d*x))]) + (12*I)*a*d*e*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log
[1 + I*E^(c + d*x)]) - PolyLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c +
d*x)]) + 6*b*d*e*(1 + E^(2*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) -
 PolyLog[2, -E^(2*(c + d*x))]) + (6*I)*a*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1
- I*E^(c + d*x)] - d^2*x^2*Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*
E^(c + d*x)] + 2*d*x*PolyLog[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c...
```

Rubi [A] (verified)

Time = 4.87 (sec) , antiderivative size = 889, normalized size of antiderivative = 0.91, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {6123, 5985, 25, 6123, 5984, 3042, 26, 4670, 3011, 2720, 6107, 6095, 2620, 3011, 2720, 7143, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)^2\text{csch}^2(c+dx)\text{sech}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5985}$$

$$\frac{-2f\int -\left((e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$\downarrow$ 25

$$\frac{2f\int (e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$\downarrow$ 6123

$$\frac{2f\int (e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$\downarrow$ 5984

$$\frac{2f\int (e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{2\int (e+fx)^2\text{csch}(2c+2dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$\downarrow$ 3042

$$\frac{2f\int (e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2\int i(e+fx)^2\csc(2ic+2idx)dx}{a}\right)}{a}$$

$\downarrow$ 26

$$\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\int(e+fx)^2\csc(2ic+2idx)dx}{a}\right)}{a}$$

$\Big\downarrow$ 4670

$$\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(\frac{if\int(e+fx)\log\left(1-e^{2c+2dx}\right)dx}{d}-\frac{if\int(e+fx)\log\left(1+e^{2c+2dx}\right)dx}{d}+\frac{i(e+fx)^2\text{arctanh}\left(e^{2c+2dx}\right)}{d}\right)}{a}\right)}{a}$$

$\Big\downarrow$ 3011

$$\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(-\frac{if\left(\frac{f\int\text{PolyLog}\left(2,-e^{2c+2dx}\right)dx}{2d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{d}+\frac{if\left(\frac{f\int\text{PolyLog}\left(2,e^{2c+2dx}\right)dx}{2d}-\frac{(e+fx)\cdots}{}\right)}{d}\cdots\right)}{a}\right)}{a}$$

$\Big\downarrow$ 2720

$$\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(-\frac{if\left(\frac{f\int e^{-2c-2dx}\text{PolyLog}\left(2,-e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{d}+\frac{if\left(\frac{f\int e^{-2c-2dx}\text{Pol}\cdots}{}\right)}{}\cdots\right)}{a}\right)}{a}$$

$\Big\downarrow$ 6107

$$\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(\frac{b^2\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)^2\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}+\frac{2i\left(-\frac{if\left(\frac{f\int e^{-2c-2dx}\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{(e+fx)}{}\right.}{d}\right.}{}\right.}{a}$$

$\downarrow$ 6095

$$\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(\frac{b^2\left(\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{a^2+b^2}+\frac{\int(e+fx)^2\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}+\frac{2i\left(-\frac{if\left(f\int e^{-2c-2dx}\right.}{}\right.}{}\right.}{}$$

$\downarrow$ 2620

$$\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(\frac{b^2\left(-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e}{}\right.}{a^2+b^2}\right.}{a}\right.}{}$$

$\downarrow$ 3011

$$\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-$$

$$\frac{b\left(-\frac{b\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{}{a}\right.}{d}\right.}{bd}\right.}{a^2+b^2}}{a}\right.}{}$$

$$\Big\downarrow \; 2720$$

$$\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-$$

$$\frac{b\left(-\frac{b\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+}}{d^2}\right.}{bd}\right.}{a^2+b^2}}{}\right.}{}$$

$$\Big\downarrow \text{7143}$$

$$\frac{2f\int (e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-$$

$$b\left(-\frac{b\left(\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{}{a}\right.}{d^2}\right.}{}\right.}{}\right.}{a}\right.$$

$$\Big\downarrow \text{7292}$$

$$\frac{2f\int \frac{(e+fx)(\arctan(\sinh(c+dx))+\operatorname{csch}(c+dx))}{d}dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-$$

$$\frac{b\left(-\frac{b\left(\frac{\int (e+fx)^2\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\cdots\right)}{d^2}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\right.}{a}\cdots\right.}{\cdots}$$

$\downarrow$ 27

$$\frac{\frac{2f\int (e+fx)(\arctan(\sinh(c+dx))+\operatorname{csch}(c+dx))dx}{d}-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-$$

$$\frac{b\left(-\frac{b\left(\frac{\int (e+fx)^2\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\cdots\right)}{d^2}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\right.}{a}\cdots\right.}{\cdots}$$

$\downarrow$ 7293

$$\frac{\frac{2f\int((e+fx)\arctan(\sinh(c+dx))+(e+fx)\operatorname{csch}(c+dx))dx}{d}-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-$$

$$b\left(-\frac{b\left(\frac{\int\left(a(e+fx)^2\operatorname{sech}(c+dx)-b(e+fx)^2\tanh(c+dx)\right)dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\operatorname{PolyL}}{}\right.}{}\right.}{}\right.}{a}\right.$$

$\downarrow$ 2009

$$-\frac{\arctan(\sinh(c+dx))(e+fx)^2}{d}-\frac{\operatorname{csch}(c+dx)(e+fx)^2}{d}+\frac{2f\left(-\frac{\arctan\left(e^{c+dx}\right)(e+fx)^2}{f}+\frac{\arctan(\sinh(c+dx))(e+fx)^2}{2f}-\frac{2\operatorname{arctanh}\left(e^{c+dx}\right)(e+fx}{d}\right.}{}$$

$$b\left(\frac{2i\left(\frac{i\operatorname{arctanh}\left(e^{2c+2dx}\right)(e+fx)^2}{d}-\frac{if\left(\frac{f\operatorname{PolyLog}\left(3,-e^{2c+2dx}\right)}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{d}+\frac{if\left(\frac{f\operatorname{PolyLog}\left(3,e^{2c+2dx}\right)}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e\right.}{2d}\right.}{d}\right.}{a}\right.$$

input
```
Int[((e + f*x)^2*Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-(((e + f*x)^2*ArcTan[Sinh[c + d*x]])/d) - ((e + f*x)^2*Csch[c + d*x])/d
+ (2*f*(-(((e + f*x)^2*ArcTan[E^(c + d*x)])/f) + ((e + f*x)^2*ArcTan[Sinh[
c + d*x]])/(2*f) - (2*(e + f*x)*ArcTanh[E^(c + d*x)])/d - (f*PolyLog[2, -E
^(c + d*x)])/d^2 + (I*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d - (I*(e +
f*x)*PolyLog[2, I*E^(c + d*x)])/d + (f*PolyLog[2, E^(c + d*x)])/d^2 - (I*f
*PolyLog[3, (-I)*E^(c + d*x)])/d^2 + (I*f*PolyLog[3, I*E^(c + d*x)])/d^2))
/d)/a - (b*(-((b*((b^2*(-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 + (b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2]))])/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/(a^2 + b^2) + ((b*(e + f*x)^3)/(3
*f) + (2*a*(e + f*x)^2*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)^2*Log[1 + E^(
2*(c + d*x))])/d - ((2*I)*a*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d^2
+ ((2*I)*a*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*(e + f*x)*Pol
yLog[2, -E^(2*(c + d*x))])/d^2 + ((2*I)*a*f^2*PolyLog[3, (-I)*E^(c + d*x)]
)/d^3 - ((2*I)*a*f^2*PolyLog[3, I*E^(c + d*x)])/d^3 + (b*f^2*PolyLog[3, -E
^(2*(c + d*x))])/(2*d^3))/(a^2 + b^2)))/a) + ((2*I)*((I*(e + f*x)^2*ArcTan
h[E^(2*c + 2*d*x)])/d - (I*f*(-1/2*((e + f*x)*PolyLog[2, -E^(2*c + 2*d*...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c)^2 \operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5664 vs. 2(897) = 1794.

Time = 0.22 (sec) , antiderivative size = 5664, normalized size of antiderivative = 5.77

$$\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*csch(d*x+c)**2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^2 \text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2 \operatorname{csch}(dx+c)^2 \operatorname{sech}(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
(b^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + a^2*b^2)*d) +
 2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((
a^2 + b^2)*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log(e^(-d*
x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d))*e^2 - 2*(f^2*x^2*e^
c + 2*e*f*x*e^c)*e^(d*x)/(a*d*e^(2*d*x + 2*c) - a*d) - 2*e*f*log(e^(d*x +
c) + 1)/(a*d^2) + 2*e*f*log(e^(d*x + c) - 1)/(a*d^2) - (d^2*x^2*log(e^(d*x
 + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*b*f^2
/(a^2*d^3) - (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2
*polylog(3, e^(d*x + c)))*b*f^2/(a^2*d^3) - 2*(b*d*e*f + a*f^2)*(d*x*log(e
^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^2*d^3) - 2*(b*d*e*f - a*f^2)*(d*
x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^2*d^3) + 1/3*(b*d^3*f^2*x
^3 + 3*(b*d*e*f + a*f^2)*d^2*x^2)/(a^2*d^3) + 1/3*(b*d^3*f^2*x^3 + 3*(b*d*
e*f - a*f^2)*d^2*x^2)/(a^2*d^3) - integrate(2*(b^4*f^2*x^2 + 2*b^4*e*f*x -
 (a*b^3*f^2*x^2*e^c + 2*a*b^3*e*f*x*e^c)*e^(d*x))/(a^4*b + a^2*b^3 - (a^4*
b*e^(2*c) + a^2*b^3*e^(2*c))*e^(2*d*x) - 2*(a^5*e^c + a^3*b^2*e^c)*e^(d*x)
), x) - integrate(2*(b*f^2*x^2 + 2*b*e*f*x + (a*f^2*x^2*e^c + 2*a*e*f*x*e^
c)*e^(d*x))/(a^2 + b^2 + (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\int \frac{(e+f\,x)^2}{\cosh(c+d\,x)\,\sinh(c+d\,x)^2\,(a+b\sinh(c+d\,x))}\,dx$$

input
```
int((e + f*x)^2/(cosh(c + d*x)*sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^2/(cosh(c + d*x)*sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Too large to display}$$

input
```
int((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**3*e**2 + 2*atan(e**(c + d*x))
*a**3*e**2 + 16*e**(6*c + 2*d*x)*int((e**(4*d*x)*x**2)/(e**(8*c + 8*d*x)*b
 + 2*e**(7*c + 7*d*x)*a - 2*e**(6*c + 6*d*x)*b - 2*e**(5*c + 5*d*x)*a - 2*
e**(3*c + 3*d*x)*a + 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*
d*f**2 + 16*e**(6*c + 2*d*x)*int((e**(4*d*x)*x**2)/(e**(8*c + 8*d*x)*b + 2
*e**(7*c + 7*d*x)*a - 2*e**(6*c + 6*d*x)*b - 2*e**(5*c + 5*d*x)*a - 2*e**(
3*c + 3*d*x)*a + 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b**2
*d*f**2 + 32*e**(6*c + 2*d*x)*int((e**(4*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e
**(7*c + 7*d*x)*a - 2*e**(6*c + 6*d*x)*b - 2*e**(5*c + 5*d*x)*a - 2*e**(3*
c + 3*d*x)*a + 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*d*e*f
+ 32*e**(6*c + 2*d*x)*int((e**(4*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c +
 7*d*x)*a - 2*e**(6*c + 6*d*x)*b - 2*e**(5*c + 5*d*x)*a - 2*e**(3*c + 3*d*
x)*a + 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b**2*d*e*f + e
**(2*c + 2*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2*b*e**2 - e**(2*c + 2*d*x)*l
og(e**(c + d*x) - 1)*a**2*b*e**2 - e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*
b**3*e**2 - e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*a**2*b*e**2 - e**(2*c +
 2*d*x)*log(e**(c + d*x) + 1)*b**3*e**2 + e**(2*c + 2*d*x)*log(e**(2*c + 2
*d*x)*b + 2*e**(c + d*x)*a - b)*b**3*e**2 - 2*e**(c + d*x)*a**3*e**2 - 2*e
**(c + d*x)*a*b**2*e**2 - 16*e**(4*c)*int((e**(4*d*x)*x**2)/(e**(8*c + 8*d
*x)*b + 2*e**(7*c + 7*d*x)*a - 2*e**(6*c + 6*d*x)*b - 2*e**(5*c + 5*d*x...
```

3.466 $\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 591

$$\begin{aligned}
&\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= -\frac{2fx\arctan\left(e^{c+dx}\right)}{ad}+\frac{2b^2(e+fx)\arctan\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d}+\frac{fx\arctan(\sinh(c+dx))}{ad} \\
&\quad -\frac{(e+fx)\arctan(\sinh(c+dx))}{ad}+\frac{2b(e+fx)\operatorname{arctanh}\left(e^{2c+2dx}\right)}{a^2d} \\
&\quad -\frac{f\operatorname{arctanh}(\cosh(c+dx))}{ad^2}-\frac{(e+fx)\operatorname{csch}(c+dx)}{ad}+\frac{b^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d} \\
&\quad +\frac{b^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d}-\frac{b^3(e+fx)\log\left(1+e^{2(c+dx)}\right)}{a^2\left(a^2+b^2\right)d} \\
&\quad +\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^2}-\frac{ib^2f\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{a\left(a^2+b^2\right)d^2}-\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{ad^2} \\
&\quad +\frac{ib^2f\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{a\left(a^2+b^2\right)d^2}+\frac{b^3f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^2} \\
&\quad +\frac{b^3f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^2}-\frac{b^3f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2a^2\left(a^2+b^2\right)d^2} \\
&\quad +\frac{bf\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2a^2d^2}-\frac{bf\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2a^2d^2}
\end{aligned}$$

output
```
-2*f*x*arctan(exp(d*x+c))/a/d+2*b^2*(f*x+e)*arctan(exp(d*x+c))/a/(a^2+b^2)
/d+f*x*arctan(sinh(d*x+c))/a/d-(f*x+e)*arctan(sinh(d*x+c))/a/d+2*b*(f*x+e)
*arctanh(exp(2*d*x+2*c))/a^2/d-f*arctanh(cosh(d*x+c))/a/d^2-(f*x+e)*csch(d
*x+c)/a/d+b^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)
/d+b^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d-b^3*
(f*x+e)*ln(1+exp(2*d*x+2*c))/a^2/(a^2+b^2)/d-I*f*polylog(2,I*exp(d*x+c))/a
/d^2-I*b^2*f*polylog(2,-I*exp(d*x+c))/a/(a^2+b^2)/d^2+I*b^2*f*polylog(2,I*
exp(d*x+c))/a/(a^2+b^2)/d^2+I*f*polylog(2,-I*exp(d*x+c))/a/d^2+b^3*f*polyl
og(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^2+b^3*f*polylog(2,
-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^2-1/2*b^3*f*polylog(2,-
exp(2*d*x+2*c))/a^2/(a^2+b^2)/d^2+1/2*b*f*polylog(2,-exp(2*d*x+2*c))/a^2/d
^2-1/2*b*f*polylog(2,exp(2*d*x+2*c))/a^2/d^2
```

Mathematica [A] (warning: unable to verify)

Time = 8.75 (sec) , antiderivative size = 864, normalized size of antiderivative = 1.46

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
((-(d*e*Cosh[(c + d*x)/2]) + c*f*Cosh[(c + d*x)/2] - f*(c + d*x)*Cosh[(c +
 d*x)/2])*Csch[(c + d*x)/2])/(2*a*d^2) + (-1/2*(b*(d*e - c*f + f*(c + d*x)
)^2)/f + (-(b*d*e) + a*f + b*c*f - b*f*(c + d*x))*Log[1 - E^(-c - d*x)] +
(-(b*d*e) - a*f + b*c*f - b*f*(c + d*x))*Log[1 + E^(-c - d*x)] + b*f*PolyL
og[2, -E^(-c - d*x)] + b*f*PolyLog[2, E^(-c - d*x)])/(a^2*d^2) + (b^3*(-2*
d*e*(c + d*x) + 2*c*f*(c + d*x) - f*(c + d*x)^2 + (4*a*Sqrt[a^2 + b^2]*d*e
*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b^2)^2] - (4*a
*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/(-
a^2 - b^2)^(3/2) + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2])] + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c
*f*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*d*e*Log[2*a*E^(c + d*x
) + b*(-1 + E^(2*(c + d*x)))] + 2*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[
a^2 + b^2])] + 2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/
(2*a^2*(a^2 + b^2)*d^2) + (-(b*d*e*(c + d*x)) + b*c*f*(c + d*x) - (b*f*(c
+ d*x)^2)/2 - 2*a*d*e*ArcTan[E^(c + d*x)] + 2*a*c*f*ArcTan[E^(c + d*x)] -
I*a*f*(c + d*x)*Log[1 - I*E^(c + d*x)] + I*a*f*(c + d*x)*Log[1 + I*E^(c +
d*x)] + b*d*e*Log[1 + E^(2*(c + d*x))] - b*c*f*Log[1 + E^(2*(c + d*x))] +
b*f*(c + d*x)*Log[1 + E^(2*(c + d*x))] + I*a*f*PolyLog[2, (-I)*E^(c + d*x)
] - I*a*f*PolyLog[2, I*E^(c + d*x)] + (b*f*PolyLog[2, -E^(2*(c + d*x))])/2
)/((a^2 + b^2)*d^2) + (Sech[(c + d*x)/2]*(d*e*Sinh[(c + d*x)/2] - c*f*S...
```

Rubi [A] (verified)

Time = 3.13 (sec) , antiderivative size = 529, normalized size of antiderivative = 0.90, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.531$, Rules used = {6123, 5985, 2009, 6123, 5984, 3042, 26, 4670, 2715, 2838, 6107, 6095, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6123}$$

$$\frac{\int (e+fx)\text{csch}^2(c+dx)\text{sech}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 5985}$$

$$\frac{-f\int\left(-\frac{\arctan(\sinh(c+dx))}{d}-\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$\downarrow$ 2009

$$-\frac{b\int\frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-f\left(\frac{2x\arctan(e^{c+dx})}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\operatorname{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{i\operatorname{PolyLog}(2,ie^{c+dx})}{d^2}\right)-\frac{(e+fx)\text{ a}}{}}{a}$$

$\downarrow$ 6123

$$-\frac{b\left(\frac{\int(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{-f\left(\frac{2x\arctan(e^{c+dx})}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\operatorname{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{i\operatorname{PolyLog}(2,ie^{c+dx})}{d^2}\right)-\frac{(e+fx)\text{ a}}{}}{a}$$

$\downarrow$ 5984

$$-\frac{b\left(\frac{2\int(e+fx)\operatorname{csch}(2c+2dx)dx}{a}-\frac{b\int\frac{(e+fx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{-f\left(\frac{2x\arctan(e^{c+dx})}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\operatorname{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{i\operatorname{PolyLog}(2,ie^{c+dx})}{d^2}\right)-\frac{(e+fx)\text{ a}}{}}{a}$$

$\downarrow$ 3042

$$\frac{-f\left(\frac{2x\arctan(e^{c+dx})}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\operatorname{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{i\operatorname{PolyLog}(2,ie^{c+dx})}{d^2}\right)-\frac{(e+fx)\text{ a}}{}}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2\int i(e+fx)\csc(2ic+2idx)dx}{a}\right)}{a}$$

$\downarrow$ 26

$$\frac{-f\left(\frac{2x\arctan(e^{c+dx})}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\operatorname{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{i\operatorname{PolyLog}(2,ie^{c+dx})}{d^2}\right)-\frac{(e+fx)\text{ a}}{}}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\int(e+fx)\csc(2ic+2idx)dx}{a}\right)}{a}$$

$$\Big\downarrow \; 4670$$

$$\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\,a}{}}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(\frac{if\int\log\left(1-e^{2c+2dx}\right)dx}{2d}-\frac{if\int\log\left(1+e^{2c+2dx}\right)dx}{2d}+\frac{i(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{d}\right)}{a}\right)}{a}$$

$$\Big\downarrow \; 2715$$

$$\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\,a}{}}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(\frac{if\int e^{-2c-2dx}\log\left(1-e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{if\int e^{-2c-2dx}\log\left(1+e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}+\frac{i(e+fx)\text{arctanh}\left(e^{2c+}\right.}{d}\right.}{a}\right.}{a}$$

$$\Big\downarrow \; 2838$$

$$\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\,a}{}}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(\frac{i(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\,\text{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)}{a}\right)}{a}$$

$$\Big\downarrow \; 6107$$

$$\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\,a}{}}{a}$$

$$\frac{b\left(-\frac{b\left(\frac{b^2\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}+\frac{2i\left(\frac{i(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\,\text{Po}}{}\right.}{a}\right.}{a}$$

$$\Big\downarrow \; 6095$$

$$\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\,a\cdots}{\cdots}}{a}$$

$$\frac{b\left(-\frac{b\left(\frac{b^2\left(\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int(e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}+\frac{2i\left(\frac{i(e+fx)\text{arctanh}\left(e^{2\cdots}\right)}{d}\cdots\right.}{\cdots}\cdots\right)}{a}$$

$$\Big\downarrow \ \color{blue}{2620}$$

$$\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\,a\cdots}{\cdots}}{a}$$

$$\frac{b\left(-\frac{b\left(\frac{b^2\left(-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int(e+fx\cdots}{\cdots}\cdots\right)}{a}\cdots\right)}{a}$$

$$\Big\downarrow \ \color{blue}{2715}$$

$$\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\,a\cdots}{\cdots}}{a}$$

$$\frac{b\left(-\frac{b\left(\frac{b^2\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a\cdots}\right)}{bd}\cdots\right)}{a^2+b^2}\cdots\right)}{a}\cdots\right)}{\cdots}$$

$$\Big\downarrow \ \color{blue}{2838}$$

$$-f\left(\frac{2x\arctan(e^{c+dx})}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{i\,\text{PolyLog}(2,ie^{c+dx})}{d^2}\right)-\frac{(e+fx)\,\text{a}}{\ }$$

$$a$$

$$b\left(-\frac{b\left(\frac{\int(e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx}{\ }\right.}{a^2+b^2}\right.}{a}\right.$$

$$a$$

$\downarrow$ 7293

$$-f\left(\frac{2x\arctan(e^{c+dx})}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{i\,\text{PolyLog}(2,ie^{c+dx})}{d^2}\right)-\frac{(e+fx)\,\text{a}}{\ }$$

$$a$$

$$b\left(-\frac{b\left(\frac{\int(a(e+fx)\text{sech}(c+dx)-b(e+fx)\tanh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right.}{a^2+b^2}\right.}{a}\right.$$

$$a$$

$\downarrow$ 2009

$$-f\left(\frac{2x\arctan(e^{c+dx})}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{i\,\text{PolyLog}(2,ie^{c+dx})}{d^2}\right)-\frac{(e+fx)\,\text{a}}{\ }$$

$$a$$

$$b\left(\frac{2i\left(\frac{i(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\,\text{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)}{a}-\frac{b\left(\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{b}{a+}\right.}{bd^2}\right.}{\ }\right.}{\ }\right.$$

input
```
Int[((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-(((e + f*x)*ArcTan[Sinh[c + d*x]])/d) - ((e + f*x)*Csch[c + d*x])/d - f*
((2*x*ArcTan[E^(c + d*x)])/d - (x*ArcTan[Sinh[c + d*x]])/d + ArcTanh[Cosh[
c + d*x]]/d^2 - (I*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + (I*PolyLog[2, I*E^(
c + d*x)])/d^2))/a - (b*(-((b*((b^2*(-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*L
og[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)*Log[1 +
(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(a^2 + b^2) + ((b*(e + f*x)^2)/(2*f) + (
2*a*(e + f*x)*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)*Log[1 + E^(2*(c + d*x)
)])/d - (I*a*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + (I*a*f*PolyLog[2, I*E^(
c + d*x)])/d^2 - (b*f*PolyLog[2, -E^(2*(c + d*x))])/(2*d^2))/(a^2 + b^2)))
/a) + ((2*I)*((I*(e + f*x)*ArcTanh[E^(2*c + 2*d*x)])/d + ((I/4)*f*PolyLog[
2, -E^(2*c + 2*d*x)])/d^2 - ((I/4)*f*PolyLog[2, E^(2*c + 2*d*x)])/d^2))/a)
)/a
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1528 vs. $2(556) = 1112$.

Time = 6.15 (sec) , antiderivative size = 1529, normalized size of antiderivative = 2.59

method	result	size
risch	Expression too large to display	1529

input
```
int((f*x+e)*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
-b/d*e/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a
+b/d^2*c*f/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2
))*a+1/a^2/d^2*b^3*f/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a
^2+b^2)^(1/2)))*c+1/a^2/d^2*b^3*f/(a^2+b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/
2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/a^2/d*b^3*f/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^
2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/a^2/d*b^3*f/(a^2+b^2)*ln((b*exp(
d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/a/d^2*c*f*b/(a^2+b^2)^(
1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/a^2/d^2*c*b^3*f/(
a^2+b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+4*I*a/d^2*f/(4*a^2+4*b^2)*l
n(1+I*exp(d*x+c))*c-4*I*a/d^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*c+4*I*a/d
*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*x-4*I*a/d*f/(4*a^2+4*b^2)*ln(1-I*exp(d
*x+c))*x-8*a/d*e/(4*a^2+4*b^2)*arctan(exp(d*x+c))+4/d^2*b*f/(4*a^2+4*b^2)*
dilog(1+I*exp(d*x+c))+4/d^2*b*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))+4/d*b*
e/(4*a^2+4*b^2)*ln(1+exp(2*d*x+2*c))+1/a/d^2*f*ln(exp(d*x+c)-1)-1/a/d^2*f*
ln(exp(d*x+c)+1)-1/a^2/d*b*f*ln(exp(d*x+c)+1)*x+1/a^2/d^2*c*b*f*ln(exp(d*x
+c)-1)+4/d^2*b*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*c+4/d^2*b*f/(4*a^2+4*b^2
)*ln(1-I*exp(d*x+c))*c+1/a/d^2*f*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*
x+c)+2*a)/(a^2+b^2)^(1/2))+8*a/d^2*c*f/(4*a^2+4*b^2)*arctan(exp(d*x+c))+1/
a/d*e*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+
1/a^2/d^2*b^3*f/(a^2+b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2593 vs. $2(539) = 1078$.

Time = 0.16 (sec) , antiderivative size = 2593, normalized size of antiderivative = 4.39

$$\int \frac{(e + fx)\text{csch}^2(c + dx)\text{sech}(c + dx)}{a + b\sinh(c + dx)} \, dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
-(2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e)*cosh(d*x + c) - (b^3*f*cosh(
d*x + c)^2 + 2*b^3*f*cosh(d*x + c)*sinh(d*x + c) + b^3*f*sinh(d*x + c)^2 -
 b^3*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^3*f*cosh(d*x + c)^2 +
2*b^3*f*cosh(d*x + c)*sinh(d*x + c) + b^3*f*sinh(d*x + c)^2 - b^3*f)*dilog
((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*
sqrt((a^2 + b^2)/b^2) - b)/b + 1) + ((a^2*b + b^3)*f*cosh(d*x + c)^2 + 2*(
a^2*b + b^3)*f*cosh(d*x + c)*sinh(d*x + c) + (a^2*b + b^3)*f*sinh(d*x + c)
^2 - (a^2*b + b^3)*f)*dilog(cosh(d*x + c) + sinh(d*x + c)) - (I*a^3*f - a^
2*b*f + (-I*a^3*f + a^2*b*f)*cosh(d*x + c)^2 - 2*(I*a^3*f - a^2*b*f)*cosh(
d*x + c)*sinh(d*x + c) + (-I*a^3*f + a^2*b*f)*sinh(d*x + c)^2)*dilog(I*cos
h(d*x + c) + I*sinh(d*x + c)) - (-I*a^3*f - a^2*b*f + (I*a^3*f + a^2*b*f)*
cosh(d*x + c)^2 - 2*(-I*a^3*f - a^2*b*f)*cosh(d*x + c)*sinh(d*x + c) + (I*
a^3*f + a^2*b*f)*sinh(d*x + c)^2)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)
) + ((a^2*b + b^3)*f*cosh(d*x + c)^2 + 2*(a^2*b + b^3)*f*cosh(d*x + c)*sin
h(d*x + c) + (a^2*b + b^3)*f*sinh(d*x + c)^2 - (a^2*b + b^3)*f)*dilog(-cos
h(d*x + c) - sinh(d*x + c)) + (b^3*d*e - b^3*c*f - (b^3*d*e - b^3*c*f)*cos
h(d*x + c)^2 - 2*(b^3*d*e - b^3*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d*
e - b^3*c*f)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) +
2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^3*d*e - b^3*c*f - (b^3*d*e - b^3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)**2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\operatorname{csch}(dx+c)^2\operatorname{sech}(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
(b^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + a^2*b^2)*d) +
 2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((
a^2 + b^2)*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log(e^(-d*
x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d))*e + (8*b*d*integrat
e(1/8*x/(a^2*d*e^(d*x + c) + a^2*d), x) - 8*b*d*integrate(1/8*x/(a^2*d*e^(
d*x + c) - a^2*d), x) + a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) + 1)/(a^2
*d^2)) - a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) - 1)/(a^2*d^2)) - 2*x*e^
(d*x + c)/(a*d*e^(2*d*x + 2*c) - a*d) - 8*integrate(-1/4*(a*b^3*x*e^(d*x +
 c) - b^4*x)/(a^4*b + a^2*b^3 - (a^4*b*e^(2*c) + a^2*b^3*e^(2*c))*e^(2*d*x
) - 2*(a^5*e^c + a^3*b^2*e^c)*e^(d*x)), x) - 8*integrate(1/4*(a*x*e^(d*x +
 c) + b*x)/(a^2 + b^2 + (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x))*f
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{e+f\,x}{\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)^2\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)/(cosh(c + d*x)*sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(cosh(c + d*x)*sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{-2e^{2dx+2c}atan\left(e^{dx+c}\right)a^3e+2atan\left(e^{dx+c}\right)a^3e+16e^{2dx+6c}\left(\int \frac{e^{4dx}x}{e^{8dx+8c}b+2e^{7dx+7c}a-2e^{6dx+6c}b-2e^{5dx+5c}a-2e^{3dx+3c}a+}\right.}{}$$

input
```
int((f*x+e)*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**3*e + 2*atan(e**(c + d*x))*a*
*3*e + 16*e**(6*c + 2*d*x)*int((e**(4*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(
7*c + 7*d*x)*a - 2*e**(6*c + 6*d*x)*b - 2*e**(5*c + 5*d*x)*a - 2*e**(3*c +
 3*d*x)*a + 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*d*f + 16*
e**(6*c + 2*d*x)*int((e**(4*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*
x)*a - 2*e**(6*c + 6*d*x)*b - 2*e**(5*c + 5*d*x)*a - 2*e**(3*c + 3*d*x)*a
+ 2*e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b**2*d*f + e**(2*c
+ 2*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2*b*e - e**(2*c + 2*d*x)*log(e**(c +
 d*x) - 1)*a**2*b*e - e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*b**3*e - e**(
2*c + 2*d*x)*log(e**(c + d*x) + 1)*a**2*b*e - e**(2*c + 2*d*x)*log(e**(c +
 d*x) + 1)*b**3*e + e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x)*b + 2*e**(c + d*
x)*a - b)*b**3*e - 2*e**(c + d*x)*a**3*e - 2*e**(c + d*x)*a*b**2*e - 16*e*
*(4*c)*int((e**(4*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 2*e
**(6*c + 6*d*x)*b - 2*e**(5*c + 5*d*x)*a - 2*e**(3*c + 3*d*x)*a + 2*e**(2*
c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*d*f - 16*e**(4*c)*int((e**(4*
d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 2*e**(6*c + 6*d*x)*b
- 2*e**(5*c + 5*d*x)*a - 2*e**(3*c + 3*d*x)*a + 2*e**(2*c + 2*d*x)*b + 2*e
**(c + d*x)*a - b),x)*a**2*b**2*d*f - log(e**(2*c + 2*d*x) + 1)*a**2*b*e +
 log(e**(c + d*x) - 1)*a**2*b*e + log(e**(c + d*x) - 1)*b**3*e + log(e**(c
 + d*x) + 1)*a**2*b*e + log(e**(c + d*x) + 1)*b**3*e - log(e**(2*c + 2*...
```

3.467 $\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 104

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a\arctan(\sinh(c+dx))}{(a^2+b^2)\,d}-\frac{\operatorname{csch}(c+dx)}{ad}+\frac{b\log(\cosh(c+dx))}{(a^2+b^2)\,d}-\frac{b\log(\sinh(c+dx))}{a^2d}+\frac{b^3\log(a+b\sinh(c+dx))}{a^2\,(a^2+b^2)\,d}$$

output
```
-a*arctan(sinh(d*x+c))/(a^2+b^2)/d-csch(d*x+c)/a/d+b*ln(cosh(d*x+c))/(a^2+
b^2)/d-b*ln(sinh(d*x+c))/a^2/d+b^3*ln(a+b*sinh(d*x+c))/a^2/(a^2+b^2)/d
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.54

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b^3\left(\frac{\operatorname{csch}(c+dx)}{ab^3}+\frac{\log(\sinh(c+dx))}{a^2b^2}-\frac{\left(b^2+a\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}-b\sinh(c+dx)\right)}{2b^4(a^2+b^2)}-\frac{\log(a+b\sinh(c+dx))}{a^2(a^2+b^2)}-\frac{\left(1+\frac{a}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}\right.}{2b^2(a^2+b}\right.}{d}$$

input
```
Integrate[(Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-((b^3*(Csch[c + d*x]/(a*b^3) + Log[Sinh[c + d*x]]/(a^2*b^2) - ((b^2 + a*S
qrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[c + d*x]])/(2*b^4*(a^2 + b^2)) - Log[a
+ b*Sinh[c + d*x]]/(a^2*(a^2 + b^2)) - ((1 + a/Sqrt[-b^2])*Log[Sqrt[-b^2]
+ b*Sinh[c + d*x]])/(2*b^2*(a^2 + b^2))))/d)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 25, 3316, 27, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\sin(ic+idx)^2\cos(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\cos(ic+idx)\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3316}$$

$$\frac{b\int \frac{\text{csch}^2(c+dx)}{(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^3\int \frac{\text{csch}^2(c+dx)}{b^2(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{615}$$

$$\frac{b^3 \int \left(\frac{\operatorname{csch}^2(c+dx)}{ab^4} - \frac{\operatorname{csch}(c+dx)}{a^2b^3} + \frac{1}{a^2(a^2+b^2)(a+b\sinh(c+dx))} + \frac{b\sinh(c+dx)-a}{b^2(a^2+b^2)(\sinh^2(c+dx)b^2+b^2)}\right)d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^3\left(-\frac{a\arctan(\sinh(c+dx))}{b^3(a^2+b^2)} + \frac{\log(b^2\sinh^2(c+dx)+b^2)}{2b^2(a^2+b^2)} - \frac{\log(b\sinh(c+dx))}{a^2b^2} + \frac{\log(a+b\sinh(c+dx))}{a^2(a^2+b^2)} - \frac{\operatorname{csch}(c+dx)}{ab^3}\right)}{d}$$

input
```
Int[(Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(b^3*(-((a*ArcTan[Sinh[c + d*x]])/(b^3*(a^2 + b^2))) - Csch[c + d*x]/(a*b^
3) - Log[b*Sinh[c + d*x]]/(a^2*b^2) + Log[a + b*Sinh[c + d*x]]/(a^2*(a^2 +
 b^2)) + Log[b^2 + b^2*Sinh[c + d*x]^2]/(2*b^2*(a^2 + b^2))))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.35

method	result
derivativedivides	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{2b\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-4a\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2+2b^2}+\frac{b^3\ln\left(\tanh\left(\frac{dx}{2}\right.\right.}{}}{d}$
default	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{2b\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-4a\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2+2b^2}+\frac{b^3\ln\left(\tanh\left(\frac{dx}{2}\right.\right.}{}}{d}$
risch	$-\frac{2bd^2x}{a^2d^2+b^2d^2}-\frac{2bdc}{a^2d^2+b^2d^2}-\frac{2b^3x}{a^2(a^2+b^2)}-\frac{2b^3c}{a^2d(a^2+b^2)}+\frac{2bx}{a^2}+\frac{2bc}{a^2d}-\frac{2\,e^{dx+c}}{da(e^{2dx+2c}-1)}+\frac{i\ln(e^{dx+c}-i)}{(a^2+b^2)d}$

input
```
int(csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/a*tanh(1/2*d*x+1/2*c)-1/2/a/tanh(1/2*d*x+1/2*c)-1/a^2*b*ln(tanh(1
/2*d*x+1/2*c))+1/2/(a^2+b^2)*(2*b*ln(1+tanh(1/2*d*x+1/2*c)^2)-4*a*arctan(t
anh(1/2*d*x+1/2*c)))+b^3/a^2/(a^2+b^2)*ln(tanh(1/2*d*x+1/2*c)^2*a-2*b*tanh
(1/2*d*x+1/2*c)-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(104) = 208$.

Time = 0.14 (sec) , antiderivative size = 441, normalized size of antiderivative = 4.24

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{2\left(a^3\cosh\left(dx+c\right)^2+2\,a^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a^3\sinh\left(dx+c\right)^2-a^3\right)\arctan\left(\cosh\left(dx+c\right)\right.}{}$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
-(2*(a^3*cosh(d*x + c)^2 + 2*a^3*cosh(d*x + c)*sinh(d*x + c) + a^3*sinh(d*
x + c)^2 - a^3)*arctan(cosh(d*x + c) + sinh(d*x + c)) + 2*(a^3 + a*b^2)*co
sh(d*x + c) - (b^3*cosh(d*x + c)^2 + 2*b^3*cosh(d*x + c)*sinh(d*x + c) + b
^3*sinh(d*x + c)^2 - b^3)*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sin
h(d*x + c))) - (a^2*b*cosh(d*x + c)^2 + 2*a^2*b*cosh(d*x + c)*sinh(d*x + c
) + a^2*b*sinh(d*x + c)^2 - a^2*b)*log(2*cosh(d*x + c)/(cosh(d*x + c) - si
nh(d*x + c))) - (a^2*b + b^3 - (a^2*b + b^3)*cosh(d*x + c)^2 - 2*(a^2*b +
b^3)*cosh(d*x + c)*sinh(d*x + c) - (a^2*b + b^3)*sinh(d*x + c)^2)*log(2*si
nh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 2*(a^3 + a*b^2)*sinh(d*x +
c))/((a^4 + a^2*b^2)*d*cosh(d*x + c)^2 + 2*(a^4 + a^2*b^2)*d*cosh(d*x + c)
*sinh(d*x + c) + (a^4 + a^2*b^2)*d*sinh(d*x + c)^2 - (a^4 + a^2*b^2)*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)**2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.66

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b^3\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^4+a^2b^2)d} + \frac{2\,a\arctan\left(e^{(-dx-c)}\right)}{(a^2+b^2)d} + \frac{b\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^2+b^2)d} + \frac{2\,e^{(-dx-c)}}{(ae^{(-2\,dx-2\,c)}-a)d} - \frac{b\log\left(e^{(-dx-c)}+1\right)}{a^2d} - \frac{b\log\left(e^{(-dx-c)}-1\right)}{a^2d}$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
b^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + a^2*b^2)*d) +
2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a
^2 + b^2)*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log(e^(-d*x
 - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.92

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{2\,b^4\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^4b+a^2b^3} - \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)a}{a^2+b^2} + \frac{b\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^2+b^2} - \frac{2\,b\log\left(\left|e^{(dx+c)}\right.\right.}{a^2}}{2\,d}$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*b^4*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^4*b + a^2*b^3
) - (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*a/(a^2 + b^2)
+ b*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^2 + b^2) - 2*b*log(abs(e^(d
*x + c) - e^(-d*x - c)))/a^2 + 2*(b*(e^(d*x + c) - e^(-d*x - c)) - 2*a)/(a
^2*(e^(d*x + c) - e^(-d*x - c))))/d
```

Mupad [B] (verification not implemented)

Time = 4.26 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.37

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\ln\left(e^{c+d\,x}+1i\right)}{b\,d+a\,d\,1i}+\frac{b^3\,\ln\left(2\,a\,e^{c+d\,x}-b+b\,e^{2\,c+2\,d\,x}\right)}{d\,a^4+d\,a^2\,b^2}-\frac{2\,e^{c+d\,x}}{a\,d\,\left(e^{2\,c+2\,d\,x}-1\right)}-\frac{b\,\ln\left(e^{2\,c+2\,d\,x}-1\right)}{a^2\,d}+\frac{\ln\left(1+e^{c+d\,x}\,1i\right)\,1i}{a\,d+b\,d\,1i}$$

input
```
int(1/(cosh(c + d*x)*sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
log(exp(c + d*x) + 1i)/(a*d*1i + b*d) + (log(exp(c + d*x)*1i + 1)*1i)/(a*d
 + b*d*1i) + (b^3*log(2*a*exp(c + d*x) - b + b*exp(2*c + 2*d*x)))/(a^4*d +
 a^2*b^2*d) - (2*exp(c + d*x))/(a*d*(exp(2*c + 2*d*x) - 1)) - (b*log(exp(2
*c + 2*d*x) - 1))/(a^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 390, normalized size of antiderivative = 3.75

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{-2e^{2dx+2c}atan\left(e^{dx+c}\right)a^3+2atan\left(e^{dx+c}\right)a^3+e^{2dx+2c}\log\left(e^{2dx+2c}+1\right)a^2b-e^{2dx+2c}\log\left(e^{dx+c}-1\right)a^2b-\cdots}{}$$

input
```
int(csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 2*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**3 + 2*atan(e**(c + d*x))*a**3
 + e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2*b - e**(2*c + 2*d*x)*lo
g(e**(c + d*x) - 1)*a**2*b - e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*b**3 -
 e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*a**2*b - e**(2*c + 2*d*x)*log(e**(
c + d*x) + 1)*b**3 + e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x)*b + 2*e**(c + d
*x)*a - b)*b**3 - 2*e**(c + d*x)*a**3 - 2*e**(c + d*x)*a*b**2 - log(e**(2*
c + 2*d*x) + 1)*a**2*b + log(e**(c + d*x) - 1)*a**2*b + log(e**(c + d*x) -
 1)*b**3 + log(e**(c + d*x) + 1)*a**2*b + log(e**(c + d*x) + 1)*b**3 - log
(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*b**3)/(a**2*d*(e**(2*c + 2*d*x
)*a**2 + e**(2*c + 2*d*x)*b**2 - a**2 - b**2))
```

3.468 $\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\operatorname{Int}\left(\frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(csch(d*x+c)^2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 40.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Csch[c + d*x]^2*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Csch[c + d*x]^2*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^2(c+dx)\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{6125}$$
$$\int \frac{\text{csch}^2(c+dx)\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Csch[c + d*x]^2*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\text{csch}\left(dx+c\right)^2\text{sech}\left(dx+c\right)}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(csch(d*x+c)^2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)^2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 6.69 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)^2\operatorname{sech}\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(csch(d*x + c)^2*sech(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)**2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 472, normalized size of antiderivative = 13.88

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)^2\operatorname{sech}\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
2*e^(d*x + c)/(a*d*f*x + a*d*e - (a*d*f*x*e^(2*c) + a*d*e*e^(2*c))*e^(2*d*
x)) - 8*integrate(-1/4*(a*b^3*e^(d*x + c) - b^4)/(a^4*b*e + a^2*b^3*e + (a
^4*b*f + a^2*b^3*f)*x - (a^4*b*e*e^(2*c) + a^2*b^3*e*e^(2*c) + (a^4*b*f*e^
(2*c) + a^2*b^3*f*e^(2*c))*x)*e^(2*d*x) - 2*(a^5*e*e^c + a^3*b^2*e*e^c + (
a^5*f*e^c + a^3*b^2*f*e^c)*x)*e^(d*x)), x) - 8*integrate(-1/8*(b*d*f*x + b
*d*e + a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2 - (a^2*d*f^2*x^2*e^
c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)), x) + 8*integrate(1/8*(b*d
*f*x + b*d*e - a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2 + (a^2*d*f^
2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)), x) - 8*integrate(
1/4*(a*e^(d*x + c) + b)/(a^2*e + b^2*e + (a^2*f + b^2*f)*x + (a^2*e*e^(2*c
) + b^2*e*e^(2*c) + (a^2*f*e^(2*c) + b^2*f*e^(2*c))*x)*e^(2*d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 4.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{1}{\cosh(c+d\,x)\,\sinh(c+d\,x)^2\,(e+f\,x)\,(a+b\sinh(c+d\,x))}\,dx$$

input
```
int(1/(cosh(c + d*x)*sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(cosh(c + d*x)*sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$=\int \frac{\operatorname{csch}(dx+c)^2\operatorname{sech}(dx+c)}{\sinh(dx+c)\,be+\sinh(dx+c)\,bfx+ae+afx}dx$$

input
```
int(csch(d*x+c)^2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int((csch(c + d*x)**2*sech(c + d*x))/(sinh(c + d*x)*b*e + sinh(c + d*x)*b*
f*x + a*e + a*f*x),x)
```

3.469 $\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 914

$$\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output
```
b^2*(f*x+e)^2/a/(a^2+b^2)/d+2*b*(f*x+e)^2*arctanh(exp(d*x+c))/a^2/d-2*b*f^
2*polylog(3,-exp(d*x+c))/a^2/d^3+2*b*f^2*polylog(3,exp(d*x+c))/a^2/d^3-b^2
*f^2*polylog(2,-exp(2*d*x+2*c))/a/(a^2+b^2)/d^3+b^2*(f*x+e)^2*tanh(d*x+c)/
a/(a^2+b^2)/d+b^3*(f*x+e)^2*sech(d*x+c)/a^2/(a^2+b^2)/d-b*(f*x+e)^2*sech(d
*x+c)/a^2/d+2*I*b^3*f^2*polylog(2,-I*exp(d*x+c))/a^2/(a^2+b^2)/d^3+2*I*b*f
^2*polylog(2,I*exp(d*x+c))/a^2/d^3+2*b^4*f^2*polylog(3,-b*exp(d*x+c)/(a+(a
^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^3-2*b^4*f^2*polylog(3,-b*exp(d*x+c)/
(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^3-2*b^2*f*(f*x+e)*ln(1+exp(2*d*
x+2*c))/a/(a^2+b^2)/d^2-4*b^3*f*(f*x+e)*arctan(exp(d*x+c))/a^2/(a^2+b^2)/d
^2-2*I*b^3*f^2*polylog(2,I*exp(d*x+c))/a^2/(a^2+b^2)/d^3-b^4*(f*x+e)^2*ln(
1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d+b^4*(f*x+e)^2*ln
(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d+2*b*f*(f*x+e)*p
olylog(2,-exp(d*x+c))/a^2/d^2-2*b*f*(f*x+e)*polylog(2,exp(d*x+c))/a^2/d^2+
2*f*(f*x+e)*ln(1-exp(4*d*x+4*c))/a/d^2+1/2*f^2*polylog(2,exp(4*d*x+4*c))/a
/d^3-2*(f*x+e)^2*coth(2*d*x+2*c)/a/d-2*(f*x+e)^2/a/d-2*b^4*f*(f*x+e)*polyl
og(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^2+2*b^4*f*(f
*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^2
+4*b*f*(f*x+e)*arctan(exp(d*x+c))/a^2/d^2-2*I*b*f^2*polylog(2,-I*exp(d*x+c
))/a^2/d^3
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1829 vs. $2(914) = 1828$.

Time = 9.45 (sec) , antiderivative size = 1829, normalized size of antiderivative = 2.00

$$\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Csch[c + d*x]^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*
x]),x]
```

output

```
4*(-1/4*(f*(4*a*d^2*e*E^(2*c)*x + 2*a*d^2*E^(2*c)*f*x^2 - 4*b*d*e*ArcTan[E
^(c + d*x)] - 4*b*d*e*E^(2*c)*ArcTan[E^(c + d*x)] - (2*I)*b*d*f*x*Log[1 -
I*E^(c + d*x)] - (2*I)*b*d*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] + (2*I)*b*d*
f*x*Log[1 + I*E^(c + d*x)] + (2*I)*b*d*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)]
- 2*a*d*e*Log[1 + E^(2*(c + d*x))] - 2*a*d*e*E^(2*c)*Log[1 + E^(2*(c + d*x
))] - 2*a*d*f*x*Log[1 + E^(2*(c + d*x))] - 2*a*d*E^(2*c)*f*x*Log[1 + E^(2*
(c + d*x))] + (2*I)*b*(1 + E^(2*c))*f*PolyLog[2, (-I)*E^(c + d*x)] - (2*I)
*b*(1 + E^(2*c))*f*PolyLog[2, I*E^(c + d*x)] - a*f*PolyLog[2, -E^(2*(c + d
*x))] - a*E^(2*c)*f*PolyLog[2, -E^(2*(c + d*x))]))/((a^2 + b^2)*d^3*(1 + E
^(2*c))) + (d*(d*x*(3*a*f*(-2*e*E^c + f*x) + b*d*(-3*e^2*E^c + 3*e*f*x + f
^2*x^2)) + 3*(1 + E^c)*f*x*(2*a*f + b*d*(2*e + f*x))*Log[1 + E^(-c - d*x)]
 + 3*e*(1 + E^c)*(b*d*e + 2*a*f)*Log[1 + E^(c + d*x)]) - 6*(1 + E^c)*f*(a*
f + b*d*(e + f*x))*PolyLog[2, -E^(-c - d*x)] - 6*b*(1 + E^c)*f^2*PolyLog[3
, -E^(-c - d*x)])/(12*a^2*d^3*(1 + E^c)) + (d*(d*x*(-3*a*f*(2*e*E^c + f*x)
 + b*d*(3*e^2*E^c + 3*e*f*x + f^2*x^2)) - 3*(-1 + E^c)*f*x*(-2*a*f + b*d*(
2*e + f*x))*Log[1 - E^(-c - d*x)] - 3*e*(-1 + E^c)*(b*d*e - 2*a*f)*Log[1 -
 E^(c + d*x)]) + 6*(-1 + E^c)*f*(-(a*f) + b*d*(e + f*x))*PolyLog[2, E^(-c
- d*x)] + 6*b*(-1 + E^c)*f^2*PolyLog[3, E^(-c - d*x)])/(12*a^2*d^3*(-1 + E
^c)) + (b^4*(-2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*d
^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + d^2*f^2*x^2*L...
```

Rubi [A] (verified)

Time = 6.36 (sec) , antiderivative size = 818, normalized size of antiderivative = 0.89, number of steps used = 30, number of rules used = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.806$, Rules used = {6123, 5984, 3042, 25, 4672, 26, 3042, 26, 4201, 2620, 2715, 2838, 6123, 5985, 25, 6107, 3042, 3803, 25, 2694, 27, 2620, 3011, 2720, 7143, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6123}$$

$$\frac{\int (e+fx)^2\text{csch}^2(c+dx)\text{sech}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 5984}$$

$$\frac{4\int (e+fx)^2\text{csch}^2(2c+2dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{4\int -(e+fx)^2\csc(2ic+2idx)^2dx}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\int (e+fx)^2\csc(2ic+2idx)^2dx}{a}$$

$$\Big\downarrow \text{4672}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}-\frac{if\int -i(e+fx)\coth(2c+2dx)dx}{d}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}-\frac{f\int (e+fx)\coth(2c+2dx)dx}{d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}-\frac{f\int -i(e+fx)\tan\left(2ic+2idx+\frac{\pi}{2}\right)dx}{d}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\int (e+fx)\tan\left(\frac{1}{2}(4ic+\pi)+2idx\right)dx}{d}\right)}{a}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\int \frac{e^{4c+4dx-i\pi}(e+fx)}{1+e^{4c+4dx-i\pi}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{2620}$$

$$\frac{-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}-\frac{f\int \log\left(1+e^{4c+4dx-i\pi}\right)dx}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{2715}$$

$$\frac{-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}-\frac{f\int e^{-4c-4dx+i\pi}\log\left(1+e^{4c+4dx-i\pi}\right)de^{4c+4dx-i\pi}}{16d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{2838}$$

$$\frac{-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{6123}$$

$$\frac{-\frac{b\left(\frac{\int (e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{5985}$$

$$-\frac{b\left(\frac{-2f\int -\left((e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^2\text{sech}}{a+b\sinh(c}}{a}\right.}{a}$$

$$\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$\downarrow$ 25

$$-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}}{a}\right.}{a}$$

$$\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$\downarrow$ 6107

$$-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{b^2\int\frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{a^2+b^2}\right.}{}\right.}{a}$$

$$\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$\downarrow$ 3042

$$-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx}{a^2+\cdots}\right.}{}\right.}{a}$$

$$\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$\big\downarrow$ 3803

$$-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{2b^2\int-\frac{e^{c+dx}(e+\cdots}{-2e^{c+dx}a-be^{2\cdots}}}{a^2+b^2}\right.}{}\right.}{a}$$

$$\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$\big\downarrow$ 25

$$-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx}{a^2+\cdots}\right.}{}\right.}{a}$$

$$\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$\big\downarrow$ 2694

$$-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx}{a^2+\cdots}\right.}{a}\right.}{\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}}$$

$$\downarrow 27$$

$$-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx}{a^2+\cdots}\right.}{\cdots}\right.}{\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}}$$

$$\downarrow 2620$$

$$-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx \ldots}{a^2+\ldots}\right.}{\ldots}\right.}{\ldots}$$

$$\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$\downarrow$ 3011

$$-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)}{a^2+\cdots}\cdots\right)}{\cdots}\right)}{\cdots}$$

$$\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$\downarrow$ 2720

$$-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx\cdots}{a^2+\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

$$\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$\downarrow$ 7143

$$-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx}{a^2+}\right.}{}\right.}{}$$

$$\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$\downarrow$ 7292

$$-\frac{b\left(\frac{2f\int\frac{(e+fx)(\operatorname{arctanh}(\cosh(c+dx))-\operatorname{sech}(c+dx))}{d}dx-\frac{(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)(a\cdots}{a^2+b^2}\cdots\right.}{\cdots}\right.}{\cdots}$$

$$\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$\Big\downarrow$ 27

$$-\frac{b\left(\frac{\frac{2f\int(e+fx)(\operatorname{arctanh}(\cosh(c+dx))-\operatorname{sech}(c+dx))dx}{d}-\frac{(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)(a-i}{a^2+b^2}\right.}{}\right.}{\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}}$$

$\downarrow$ 7293

$$-\frac{b\left(\frac{\frac{2f\int((e+fx)\operatorname{arctanh}(\cosh(c+dx))-(e+fx)\operatorname{sech}(c+dx))dx}{d}-\frac{(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int\left(a(e+fx)^2\operatorname{sech}^2(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$\downarrow$ 2009

$$-\frac{4\left(\frac{\coth(2c+2dx)(e+fx)^2}{2d}+\frac{if\left(2i\left(\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}+\frac{f\,\text{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}-$$

$$b\left(\frac{-\frac{\text{arctanh}(\cosh(c+dx))(e+fx)^2}{d}+\frac{\text{sech}(c+dx)(e+fx)^2}{d}+\frac{2f\left(-\frac{\text{arctanh}\left(e^{c+dx}\right)(e+fx)^2}{f}+\frac{\text{arctanh}(\cosh(c+dx))(e+fx)^2}{2f}-\frac{2\arctan\left(e^{c+dx}\right)(e+}{d}\right.}{}}{}\right.$$

input
```
Int[((e + f*x)^2*Csch[c + d*x]^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-4*(((e + f*x)^2*Coth[2*c + 2*d*x])/(2*d) + (I*f*(((-1/2*I)*(e + f*x)^2)/
f + (2*I)*(((e + f*x)*Log[1 + E^(4*c - I*Pi + 4*d*x)])/(4*d) + (f*PolyLog[
2, -E^(4*c - I*Pi + 4*d*x)])/(16*d^2))))/d))/a - (b*((-(((e + f*x)^2*ArcTa
nh[Cosh[c + d*x]])/d) + (2*f*((-2*(e + f*x)*ArcTan[E^(c + d*x)])/d - ((e +
 f*x)^2*ArcTanh[E^(c + d*x)])/f + ((e + f*x)^2*ArcTanh[Cosh[c + d*x]])/(2*
f) - ((e + f*x)*PolyLog[2, -E^(c + d*x)])/d + (I*f*PolyLog[2, (-I)*E^(c +
d*x)])/d^2 - (I*f*PolyLog[2, I*E^(c + d*x)])/d^2 + ((e + f*x)*PolyLog[2, E
^(c + d*x)])/d + (f*PolyLog[3, -E^(c + d*x)])/d^2 - (f*PolyLog[3, E^(c + d
*x)])/d^2))/d + ((e + f*x)^2*Sech[c + d*x])/d)/a - (b*((-2*b^2*(-1/2*(b*((
(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) - (2*f*(
-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (f
*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/Sqrt[
a^2 + b^2] + (b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
)])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d
^2))/(b*d)))/(2*Sqrt[a^2 + b^2])))/(a^2 + b^2) + ((a*(e + f*x)^2)/d - (4*b
*f*(e + f*x)*ArcTan[E^(c + d*x)])/d^2 - (2*a*f*(e + f*x)*Log[1 + E^(2*(c +
 d*x))])/d^2 + ((2*I)*b*f^2*PolyLog[2, (-I)*E^(c + d*x)])/d^3 - ((2*I)*b*f
^2*PolyLog[2, I*E^(c + d*x)])/d^3 - (a*f^2*PolyLog[2, -E^(2*(c + d*x))])/d
^3 + (b*(e + f*x)^2*Sech[c + d*x])/d + (a*(e + f*x)^2*Tanh[c + d*x])/d)...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c)^2 \operatorname{sech}(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 10432 vs. 2(845) = 1690.

Time = 0.33 (sec) , antiderivative size = 10432, normalized size of antiderivative = 11.41

$$\int \frac{(e+fx)^2 \operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*csch(d*x+c)**2*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^2 \operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2 \operatorname{csch}(dx+c)^2 \operatorname{sech}(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="maxima")
```

output
```
-2*a*e*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2 +
 b^2)*d^2)) + 4*b*f^2*integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2
*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 4*a*f^2*integrate(x/(a^2*d*e^(2*
d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + (b^4*log((b*e^(-
d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((
a^4 + a^2*b^2)*sqrt(a^2 + b^2)*d) - 2*(a*b*e^(-d*x - c) + b^2*e^(-2*d*x -
2*c) - a*b*e^(-3*d*x - 3*c) + 2*a^2 + b^2)/((a^3 + a*b^2 - (a^3 + a*b^2)*e
^(-4*d*x - 4*c))*d) + b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c)
 - 1)/(a^2*d))*e^2 - 4*e*f*x/(a*d) + 4*b*e*f*arctan(e^(d*x + c))/((a^2 + b
^2)*d^2) + 2*((2*a^2*f^2 + b^2*f^2)*x^2 + 2*(2*a^2*e*f + b^2*e*f)*x + (a*b
*f^2*x^2*e^(3*c) + 2*a*b*e*f*x*e^(3*c))*e^(3*d*x) + (b^2*f^2*x^2*e^(2*c) +
 2*b^2*e*f*x*e^(2*c))*e^(2*d*x) - (a*b*f^2*x^2*e^c + 2*a*b*e*f*x*e^c)*e^(d
*x))/(a^3*d + a*b^2*d - (a^3*d*e^(4*c) + a*b^2*d*e^(4*c))*e^(4*d*x)) + 2*e
*f*log(e^(d*x + c) + 1)/(a*d^2) + 2*e*f*log(e^(d*x + c) - 1)/(a*d^2) + (d^
2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(
d*x + c)))*b*f^2/(a^2*d^3) - (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(
e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*b*f^2/(a^2*d^3) + 2*(b*d*e*f + a
*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^2*d^3) - 2*(b*d*
e*f - a*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^2*d^3) -
1/3*(b*d^3*f^2*x^3 + 3*(b*d*e*f + a*f^2)*d^2*x^2)/(a^2*d^3) + 1/3*(b*d^...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\int \frac{(e+f\,x)^2}{\cosh\left(c+d\,x\right)^2\sinh\left(c+d\,x\right)^2\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^2/(cosh(c + d*x)^2*sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^2/(cosh(c + d*x)^2*sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x
)
```

Reduce [F]

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(2*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a
**2 + b**2))*b**4*e**2*i - 2*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*
i)/sqrt(a**2 + b**2))*b**4*e**2*i + 32*e**(9*c + 4*d*x)*int((e**(5*d*x)*x*
*2)/(e**(10*c + 10*d*x)*b + 2*e**(9*c + 9*d*x)*a - e**(8*c + 8*d*x)*b - 2*
e**(6*c + 6*d*x)*b - 4*e**(5*c + 5*d*x)*a + 2*e**(4*c + 4*d*x)*b + e**(2*c
 + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**6*d*f**2 + 64*e**(9*c + 4*d*x)*i
nt((e**(5*d*x)*x**2)/(e**(10*c + 10*d*x)*b + 2*e**(9*c + 9*d*x)*a - e**(8*
c + 8*d*x)*b - 2*e**(6*c + 6*d*x)*b - 4*e**(5*c + 5*d*x)*a + 2*e**(4*c + 4
*d*x)*b + e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*b**2*d*f**2 +
 32*e**(9*c + 4*d*x)*int((e**(5*d*x)*x**2)/(e**(10*c + 10*d*x)*b + 2*e**(9
*c + 9*d*x)*a - e**(8*c + 8*d*x)*b - 2*e**(6*c + 6*d*x)*b - 4*e**(5*c + 5*
d*x)*a + 2*e**(4*c + 4*d*x)*b + e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)
,x)*a**2*b**4*d*f**2 + 64*e**(9*c + 4*d*x)*int((e**(5*d*x)*x)/(e**(10*c +
10*d*x)*b + 2*e**(9*c + 9*d*x)*a - e**(8*c + 8*d*x)*b - 2*e**(6*c + 6*d*x)
*b - 4*e**(5*c + 5*d*x)*a + 2*e**(4*c + 4*d*x)*b + e**(2*c + 2*d*x)*b + 2*
e**(c + d*x)*a - b),x)*a**6*d*e*f + 128*e**(9*c + 4*d*x)*int((e**(5*d*x)*x
)/(e**(10*c + 10*d*x)*b + 2*e**(9*c + 9*d*x)*a - e**(8*c + 8*d*x)*b - 2*e*
*(6*c + 6*d*x)*b - 4*e**(5*c + 5*d*x)*a + 2*e**(4*c + 4*d*x)*b + e**(2*c +
 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*b**2*d*e*f + 64*e**(9*c + 4*d*x)
*int((e**(5*d*x)*x)/(e**(10*c + 10*d*x)*b + 2*e**(9*c + 9*d*x)*a - e**(...
```

3.470 $\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 499

$$
\begin{aligned}
\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx
&= \frac{bf\arctan(\sinh(c+dx))}{a^2d^2} - \frac{b^3f\arctan(\sinh(c+dx))}{a^2(a^2+b^2)\,d^2} + \frac{2bfx\text{arctanh}\left(e^{c+dx}\right)}{a^2d} \\
&\quad - \frac{bfx\text{arctanh}(\cosh(c+dx))}{a^2d} + \frac{b(e+fx)\text{arctanh}(\cosh(c+dx))}{a^2d} \\
&\quad - \frac{2(e+fx)\coth(2c+2dx)}{ad} + \frac{b^4(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d} \\
&\quad - \frac{b^4(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d} - \frac{b^2f\log(\cosh(c+dx))}{a(a^2+b^2)\,d^2} \\
&\quad + \frac{f\log(\sinh(2c+2dx))}{ad^2} + \frac{bf\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{a^2d^2} - \frac{bf\,\text{PolyLog}\left(2,e^{c+dx}\right)}{a^2d^2} \\
&\quad + \frac{b^4f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^2} - \frac{b^4f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^2} \\
&\quad - \frac{b(e+fx)\text{sech}(c+dx)}{a^2d} + \frac{b^3(e+fx)\text{sech}(c+dx)}{a^2(a^2+b^2)\,d} + \frac{b^2(e+fx)\tanh(c+dx)}{a(a^2+b^2)\,d}
\end{aligned}
$$

output
```
b*f*arctan(sinh(d*x+c))/a^2/d^2-b^3*f*arctan(sinh(d*x+c))/a^2/(a^2+b^2)/d^
2+2*b*f*x*arctanh(exp(d*x+c))/a^2/d-b*f*x*arctanh(cosh(d*x+c))/a^2/d+b*(f*
x+e)*arctanh(cosh(d*x+c))/a^2/d-2*(f*x+e)*coth(2*d*x+2*c)/a/d+b^4*(f*x+e)*
ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d-b^4*(f*x+e)*l
n(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d-b^2*f*ln(cosh(
d*x+c))/a/(a^2+b^2)/d^2+f*ln(sinh(2*d*x+2*c))/a/d^2+b*f*polylog(2,-exp(d*x
+c))/a^2/d^2-b*f*polylog(2,exp(d*x+c))/a^2/d^2+b^4*f*polylog(2,-b*exp(d*x+
c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^2-b^4*f*polylog(2,-b*exp(d*x
+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^2-b*(f*x+e)*sech(d*x+c)/a^2
/d+b^3*(f*x+e)*sech(d*x+c)/a^2/(a^2+b^2)/d+b^2*(f*x+e)*tanh(d*x+c)/a/(a^2+
b^2)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.25 (sec) , antiderivative size = 780, normalized size of antiderivative = 1.56

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= 4\left(-\frac{if\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{4(a-ib)d^2}+\frac{if\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{4(a+ib)d^2}\right.$$
$$+\frac{\left(-de\cosh\left(\frac{1}{2}(c+dx)\right)+cf\cosh\left(\frac{1}{2}(c+dx)\right)-f(c+dx)\cosh\left(\frac{1}{2}(c+dx)\right)\right)\text{csch}\left(\frac{1}{2}(c+dx)\right)}{8ad^2}$$
$$+\frac{f\log(\cosh(c+dx))}{8(a-ib)d^2}+\frac{f\log(\cosh(c+dx))}{8(a+ib)d^2}$$
$$+\frac{\frac{1}{2}bd^2fx^2+(bde+af)(c+dx)-2(bde+af-bcf)(c+dx)+2bf(c+dx)\log\left(1+e^{-c-dx}\right)+2(bde+}{8a^2d^2}$$
$$+\frac{af(c+dx)+2(bde-af-bcf)(c+dx)-\frac{b(de+dfx)^2}{2f}-2bf(c+dx)\log\left(1-e^{-c-dx}\right)-2(bde-af-}{8a^2d^2}$$
$$+\frac{b^4\left(-2de\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+2cf\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+f(c+dx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-f(c+dx)\log\right.}{4a^2\left(a^2+b^2\right)^{3/2}d^2}$$
$$+\frac{\text{sech}\left(\frac{1}{2}(c+dx)\right)\left(-de\sinh\left(\frac{1}{2}(c+dx)\right)+cf\sinh\left(\frac{1}{2}(c+dx)\right)-f(c+dx)\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{8ad^2}$$
$$+\frac{\text{sech}(c+dx)(-bde+bcf-bf(c+dx)-ade\sinh(c+dx)+acf\sinh(c+dx)-af(c+dx)\sinh(c+}{4\left(a^2+b^2\right)d^2}$$

input
```
Integrate[((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
4*(((-1/4*I)*f*ArcTan[Tanh[(c + d*x)/2]])/((a - I*b)*d^2) + ((I/4)*f*ArcTa
n[Tanh[(c + d*x)/2]])/((a + I*b)*d^2) + ((-(d*e*Cosh[(c + d*x)/2]) + c*f*C
osh[(c + d*x)/2] - f*(c + d*x)*Cosh[(c + d*x)/2])*Csch[(c + d*x)/2])/(8*a*
d^2) + (f*Log[Cosh[c + d*x]])/(8*(a - I*b)*d^2) + (f*Log[Cosh[c + d*x]])/(
8*(a + I*b)*d^2) + ((b*d^2*f*x^2)/2 + (b*d*e + a*f)*(c + d*x) - 2*(b*d*e +
 a*f - b*c*f)*(c + d*x) + 2*b*f*(c + d*x)*Log[1 + E^(-c - d*x)] + 2*(b*d*e
 + a*f - b*c*f)*Log[1 + E^(c + d*x)] - 2*b*f*PolyLog[2, -E^(-c - d*x)])/(8
*a^2*d^2) + (a*f*(c + d*x) + 2*(b*d*e - a*f - b*c*f)*(c + d*x) - (b*(d*e +
 d*f*x)^2)/(2*f) - 2*b*f*(c + d*x)*Log[1 - E^(-c - d*x)] - 2*(b*d*e - a*f
- b*c*f)*Log[1 - E^(c + d*x)] + 2*b*f*PolyLog[2, E^(-c - d*x)])/(8*a^2*d^2
) + (b^4*(-2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*c*f*ArcT
anh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log[1 + (b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2])] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - f*
PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(4*a^2*(a^2 + b^2)^
(3/2)*d^2) + (Sech[(c + d*x)/2]*(-(d*e*Sinh[(c + d*x)/2]) + c*f*Sinh[(c +
d*x)/2] - f*(c + d*x)*Sinh[(c + d*x)/2]))/(8*a*d^2) + (Sech[c + d*x]*(-(b*
d*e) + b*c*f - b*f*(c + d*x) - a*d*e*Sinh[c + d*x] + a*c*f*Sinh[c + d*x] -
 a*f*(c + d*x)*Sinh[c + d*x]))/(4*(a^2 + b^2)*d^2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 3.08 (sec) , antiderivative size = 444, normalized size of antiderivative = 0.89, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.676$, Rules used = {6123, 5984, 3042, 25, 4672, 26, 3042, 26, 3956, 6123, 5985, 2009, 6107, 3042, 3803, 25, 2694, 27, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)\text{csch}^2(c+dx)\text{sech}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5984}$$

$$\frac{4\int (e+fx)\text{csch}^2(2c+2dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{4\int -\left((e+fx)\csc(2ic+2idx)^2\right)dx}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\int (e+fx)\csc(2ic+2idx)^2dx}{a}$$

$$\Big\downarrow \text{4672}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{if\int -i\coth(2c+2dx)dx}{2d}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\int \coth(2c+2dx)dx}{2d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\int -i\tan\left(2ic+2idx+\frac{\pi}{2}\right)dx}{2d}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}+\frac{if\int \tan\left(\frac{1}{2}(4ic+\pi)+2idx\right)dx}{2d}\right)}{a}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}$$

$$\Big\downarrow \text{6123}$$

$$-\frac{b\left(\frac{\int (e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}$$

$\downarrow$ 5985

$$-\frac{b\left(\frac{-f\int\left(\frac{\text{sech}(c+dx)}{d}-\frac{\text{arctanh}(\cosh(c+dx))}{d}\right)dx-\frac{(e+fx)\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)\text{sech}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}$$

$\downarrow$ 2009

$$-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}(e^{c+dx})}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{(e+fx)\text{arctanh}(\cosh}{d}}{a}\right.}{a}$$

$$\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}$$

$\downarrow$ 6107

$$-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}(e^{c+dx})}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{(e+fx)\text{arctanh}(\cosh}{d}}{a}\right.}{a}$$

$$\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}$$

$\downarrow$ 3042

$$-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}(e^{c+dx})}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{(e+fx)\text{arctanh}(\cosh}{d}}{a}\right.}{a}$$

$$\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}$$

$\downarrow$ 3803

$$-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{arctanh}(\cosh}{d}}{a}\right.}{a}$$

$$\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}$$

$\downarrow$ 25

$$-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{arctanh}(\cosh}{d}}{a}\right.}{a}$$

$$\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}$$

$\downarrow$ 2694

$$-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{arctanh}(\cosh}{d}}{a}\right.}{}$$

$$\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}$$

$\downarrow$ 27

$$-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\operatorname{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\operatorname{arctanh}(\cosh}{d}}{a}\right.}{a}$$

$$\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}$$

$\downarrow$ 2620

$$-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\operatorname{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\operatorname{arctanh}(\cosh}{d}}{a}\right.}{}$$

$$\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}$$

$\downarrow$ 2715

$$-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\operatorname{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\operatorname{arctanh}(\cosh}{d}}{a}\right.}{\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\operatorname{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\operatorname{arctanh}(\cosh}{d}}{a}\right.}{\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{arctanh}(\cosh}{d}}{a}\right.}{\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{arctanh}(\cosh}{d}}{a}\right.}{\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}}$$

input
```
Int[((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-4*(((e + f*x)*Coth[2*c + 2*d*x])/(2*d) - (f*Log[(-I)*Sinh[2*c + 2*d*x]])
/(4*d^2)))/a - (b*((-(((e + f*x)*ArcTanh[Cosh[c + d*x]])/d) - f*(ArcTan[Si
nh[c + d*x]]/d^2 + (2*x*ArcTanh[E^(c + d*x)])/d - (x*ArcTanh[Cosh[c + d*x]
])/d + PolyLog[2, -E^(c + d*x)]/d^2 - PolyLog[2, E^(c + d*x)]/d^2) + ((e +
 f*x)*Sech[c + d*x])/d)/a - (b*((-2*b^2*(-1/2*(b*(((e + f*x)*Log[1 + (b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2]))])/(b*d^2)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[1
 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(2*Sqrt[a^2 + b^2])))/(a^2 + b
^2) + (-((b*f*ArcTan[Sinh[c + d*x]])/d^2) - (a*f*Log[Cosh[c + d*x]])/d^2 +
 (b*(e + f*x)*Sech[c + d*x])/d + (a*(e + f*x)*Tanh[c + d*x])/d)/(a^2 + b^2
)))/a))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3716 vs. $2(476) = 952$.

Time = 18.05 (sec) , antiderivative size = 3717, normalized size of antiderivative = 7.45

method	result	size
risch	Expression too large to display	3717

input
```
int((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURN
VERBOSE)
```

output
```
1/2*(-2*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*(a^2+b^2)*a^2*b^
2*d*e-2*(a^2+b^2)^(3/2)*ln(exp(d*x+c)+1)*a^2*b*d*e-2*(a^2+b^2)^(3/2)*ln(ex
p(d*x+c)-1)*a^2*b*c*f+2*(a^2+b^2)^(3/2)*ln(exp(d*x+c)-1)*a^2*b*d*e-14*exp(
4*d*x+4*c)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^4*b^2*f-7*e
xp(4*d*x+4*c)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^2*b^4*f+
2*exp(4*d*x+4*c)*(a^2+b^2)^(3/2)*ln(exp(d*x+c)+1)*a^3*f+2*exp(4*d*x+4*c)*(
a^2+b^2)^(3/2)*ln(exp(d*x+c)-1)*a^3*f+2*dilog(exp(d*x+c))*exp(4*d*x+4*c)*(
a^2+b^2)^(3/2)*b^3*f+2*dilog(exp(d*x+c)+1)*exp(4*d*x+4*c)*(a^2+b^2)^(3/2)*
b^3*f+2*ln(1+exp(2*d*x+2*c))*exp(4*d*x+4*c)*(a^2+b^2)^(3/2)*a^3*f-8*exp(4*
d*x+4*c)*ln(exp(d*x+c))*(a^2+b^2)^(3/2)*a^3*f+8*exp(4*d*x+4*c)*arctanh(1/2
*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*(a^2+b^2)*a^4*f+exp(4*d*x+4*c)*arct
anh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*(a^2+b^2)*b^4*f-4*(a^2+b^2)^
(3/2)*a*b^2*d*e-8*(a^2+b^2)^(3/2)*a^3*d*f*x-2*(a^2+b^2)^(3/2)*ln(exp(d*x+c
)+1)*a^2*b*d*f*x+2*exp(4*d*x+4*c)*(a^2+b^2)^(3/2)*ln(exp(d*x+c)-1)*b^3*c*f
-2*exp(4*d*x+4*c)*(a^2+b^2)^(3/2)*ln(exp(d*x+c)-1)*b^3*d*e+2*dilog(exp(d*x
+c))*exp(4*d*x+4*c)*(a^2+b^2)^(3/2)*a^2*b*f+2*dilog(exp(d*x+c)+1)*exp(4*d*
x+4*c)*(a^2+b^2)^(3/2)*a^2*b*f+4*arctan(exp(d*x+c))*exp(4*d*x+4*c)*(a^2+b^
2)^(3/2)*a^2*b*f-4*exp(4*d*x+4*c)*ln(exp(d*x+c))*(a^2+b^2)^(3/2)*a*b^2*f+6
*exp(4*d*x+4*c)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*(a^2+b^2
)*a^2*b^2*f+2*exp(4*d*x+4*c)*(a^2+b^2)^(3/2)*ln(exp(d*x+c)+1)*a*b^2*f+2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4086 vs. $2(472) = 944$.

Time = 0.22 (sec) , antiderivative size = 4086, normalized size of antiderivative = 8.19

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)**2*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{csch}\,(dx+c)^2\,\text{sech}\,(dx+c)^2}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
(b^4*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt
(a^2 + b^2)))/((a^4 + a^2*b^2)*sqrt(a^2 + b^2)*d) - 2*(a*b*e^(-d*x - c) +
b^2*e^(-2*d*x - 2*c) - a*b*e^(-3*d*x - 3*c) + 2*a^2 + b^2)/((a^3 + a*b^2 -
 (a^3 + a*b^2)*e^(-4*d*x - 4*c))*d) + b*log(e^(-d*x - c) + 1)/(a^2*d) - b*
log(e^(-d*x - c) - 1)/(a^2*d))*e + (16*b^4*integrate(-1/8*x*e^(d*x + c)/(a
^4*b + a^2*b^3 - (a^4*b*e^(2*c) + a^2*b^3*e^(2*c))*e^(2*d*x) - 2*(a^5*e^c
+ a^3*b^2*e^c)*e^(d*x)), x) - 16*b*d*integrate(1/16*x/(a^2*d*e^(d*x + c) +
 a^2*d), x) - 16*b*d*integrate(1/16*x/(a^2*d*e^(d*x + c) - a^2*d), x) - a*
((d*x + c)/(a^2*d^2) - log(e^(d*x + c) + 1)/(a^2*d^2)) - a*((d*x + c)/(a^2
*d^2) - log(e^(d*x + c) - 1)/(a^2*d^2)) + 2*(a*b*x*e^(3*d*x + 3*c) + b^2*x
*e^(2*d*x + 2*c) - a*b*x*e^(d*x + c) + (2*a^2 + b^2)*x)/(a^3*d + a*b^2*d -
 (a^3*d*e^(4*c) + a*b^2*d*e^(4*c))*e^(4*d*x)) - 2*a*x/((a^2 + b^2)*d) + 2*
b*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) + a*log(e^(2*d*x + 2*c) + 1)/((a^2
 + b^2)*d^2))*f
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{e+f\,x}{\cosh\left(c+d\,x\right)^2\,\sinh\left(c+d\,x\right)^2\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)/(cosh(c + d*x)^2*sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(cosh(c + d*x)^2*sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Too large to display}$$

input
```
int((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output

```
(2*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a
**2 + b**2))*b**4*e*i - 2*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/
sqrt(a**2 + b**2))*b**4*e*i + 32*e**(9*c + 4*d*x)*int((e**(5*d*x)*x)/(e**(
10*c + 10*d*x)*b + 2*e**(9*c + 9*d*x)*a - e**(8*c + 8*d*x)*b - 2*e**(6*c +
 6*d*x)*b - 4*e**(5*c + 5*d*x)*a + 2*e**(4*c + 4*d*x)*b + e**(2*c + 2*d*x)
*b + 2*e**(c + d*x)*a - b),x)*a**6*d*f + 64*e**(9*c + 4*d*x)*int((e**(5*d*
x)*x)/(e**(10*c + 10*d*x)*b + 2*e**(9*c + 9*d*x)*a - e**(8*c + 8*d*x)*b -
2*e**(6*c + 6*d*x)*b - 4*e**(5*c + 5*d*x)*a + 2*e**(4*c + 4*d*x)*b + e**(2
*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**4*b**2*d*f + 32*e**(9*c + 4*d*
x)*int((e**(5*d*x)*x)/(e**(10*c + 10*d*x)*b + 2*e**(9*c + 9*d*x)*a - e**(8
*c + 8*d*x)*b - 2*e**(6*c + 6*d*x)*b - 4*e**(5*c + 5*d*x)*a + 2*e**(4*c +
4*d*x)*b + e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a**2*b**4*d*f - e
**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**4*b*e - 2*e**(4*c + 4*d*x)*log(e*
*(c + d*x) - 1)*a**2*b**3*e - e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*b**5*
e + e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a**4*b*e + 2*e**(4*c + 4*d*x)*l
og(e**(c + d*x) + 1)*a**2*b**3*e + e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*
b**5*e - 4*e**(4*c + 4*d*x)*a**5*e - 6*e**(4*c + 4*d*x)*a**3*b**2*e - 2*e*
*(4*c + 4*d*x)*a*b**4*e - 2*e**(3*c + 3*d*x)*a**4*b*e - 2*e**(3*c + 3*d*x)
*a**2*b**3*e - 2*e**(2*c + 2*d*x)*a**3*b**2*e - 2*e**(2*c + 2*d*x)*a*b**4*
e + 2*e**(c + d*x)*a**4*b*e + 2*e**(c + d*x)*a**2*b**3*e - 32*e**(5*c)*...
```

3.471 $\int \frac{\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 144

$$\int \frac{\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b\,\text{arctanh}(\cosh(c+dx))}{a^2d} - \frac{2b^4\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d} - \frac{\coth(c+dx)}{ad} - \frac{b\,\text{sech}(c+dx)}{a^2d} + \frac{b^2\,\text{sech}(c+dx)(b+a\sinh(c+dx))}{a^2\left(a^2+b^2\right)d} - \frac{\tanh(c+dx)}{ad}$$

output
```
b*arctanh(cosh(d*x+c))/a^2/d-2*b^4*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+
b^2)^(1/2))/a^2/(a^2+b^2)^(3/2)/d-coth(d*x+c)/a/d-b*sech(d*x+c)/a^2/d+b^2*
sech(d*x+c)*(b+a*sinh(d*x+c))/a^2/(a^2+b^2)/d-tanh(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{\frac{4b^4\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{a^2(-a^2-b^2)^{3/2}}+\frac{\coth\left(\frac{1}{2}(c+dx)\right)}{a}-\frac{2b\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{a^2}+\frac{2b\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{a^2}+\frac{2\operatorname{sech}(c+dx)(b+a\sinh(c}{a^2+b^2}}{2d}$$

input
```
Integrate[(Csch[c + d*x]^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*((4*b^4*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(a^2*(-a^
2 - b^2)^(3/2)) + Coth[(c + d*x)/2]/a - (2*b*Log[Cosh[(c + d*x)/2]])/a^2 +
 (2*b*Log[Sinh[(c + d*x)/2]])/a^2 + (2*Sech[c + d*x]*(b + a*Sinh[c + d*x])
)/(a^2 + b^2) + Tanh[(c + d*x)/2]/a)/d
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {3042, 25, 3377, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{1}{\sin(ic+idx)^2\cos(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{1}{\cos(ic+idx)^2\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \; 3377$$

$$-\int \left(-\frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a}+\frac{b\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a^2}-\frac{b^2\operatorname{sech}^2(c+dx)}{a^2(a+b\sinh(c+dx))}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2b^4\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2d\,(a^2+b^2)^{3/2}}+\frac{b\operatorname{arctanh}(\cosh(c+dx))}{a^2d}+\frac{b^2\operatorname{sech}(c+dx)(a\sinh(c+dx)+b)}{a^2d\,(a^2+b^2)}-\frac{b\operatorname{sech}(c+dx)}{a^2d}-\frac{\tanh(c+dx)}{ad}-\frac{\coth(c+dx)}{ad}$$

input
```
Int[(Csch[c + d*x]^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(b*ArcTanh[Cosh[c + d*x]])/(a^2*d) - (2*b^4*ArcTanh[(b - a*Tanh[(c + d*x)/
2])/Sqrt[a^2 + b^2]])/(a^2*(a^2 + b^2)^(3/2)*d) - Coth[c + d*x]/(a*d) - (b
*Sech[c + d*x])/(a^2*d) + (b^2*Sech[c + d*x]*(b + a*Sinh[c + d*x]))/(a^2*(
a^2 + b^2)*d) - Tanh[c + d*x]/(a*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3377
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Maple [A] (verified)

Time = 8.79 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{2b^4\,\text{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{\frac{3}{2}}}+\frac{-2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{\left(a^2+b^2\right)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}}{d}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{2b^4\,\text{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{\frac{3}{2}}}+\frac{-2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{\left(a^2+b^2\right)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}}{d}$
risch	$-\frac{2\left(e^{3dx+3c}ab+e^{2dx+2c}b^2-b\,e^{dx+c}a+2a^2+b^2\right)}{d(a^2+b^2)(1+e^{2dx+2c})a(e^{2dx+2c}-1)}+\frac{b^4\ln\left(e^{dx+c}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}-a^4-2a^2b^2-b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}\,d\,a^2}-\frac{b^4\ln\left(e^{dx+c}+\frac{a(}{}\right.}{(a^2}$

input
```
int(csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/a*tanh(1/2*d*x+1/2*c)-1/2/a/tanh(1/2*d*x+1/2*c)-1/a^2*b*ln(tanh(
1/2*d*x+1/2*c))+2/a^2*b^4/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/
2*c)-2*b)/(a^2+b^2)^(1/2))+2/(a^2+b^2)*(-a*tanh(1/2*d*x+1/2*c)-b)/(1+tanh(
1/2*d*x+1/2*c)^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1040 vs. $2(141) = 282$.

Time = 0.17 (sec) , antiderivative size = 1040, normalized size of antiderivative = 7.22

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fric
as")
```

output
```
-(4*a^5 + 6*a^3*b^2 + 2*a*b^4 + 2*(a^4*b + a^2*b^3)*cosh(d*x + c)^3 + 2*(a
^4*b + a^2*b^3)*sinh(d*x + c)^3 + 2*(a^3*b^2 + a*b^4)*cosh(d*x + c)^2 + 2*
(a^3*b^2 + a*b^4 + 3*(a^4*b + a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - (b
^4*cosh(d*x + c)^4 + 4*b^4*cosh(d*x + c)^3*sinh(d*x + c) + 6*b^4*cosh(d*x
+ c)^2*sinh(d*x + c)^2 + 4*b^4*cosh(d*x + c)*sinh(d*x + c)^3 + b^4*sinh(d*
x + c)^4 - b^4)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x +
c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sin
h(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b
*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x +
 c) + a)*sinh(d*x + c) - b)) - 2*(a^4*b + a^2*b^3)*cosh(d*x + c) + (a^4*b
+ 2*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^4 - 4*(a^4*b +
 2*a^2*b^3 + b^5)*cosh(d*x + c)^3*sinh(d*x + c) - 6*(a^4*b + 2*a^2*b^3 + b
^5)*cosh(d*x + c)^2*sinh(d*x + c)^2 - 4*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x
 + c)*sinh(d*x + c)^3 - (a^4*b + 2*a^2*b^3 + b^5)*sinh(d*x + c)^4)*log(cos
h(d*x + c) + sinh(d*x + c) + 1) - (a^4*b + 2*a^2*b^3 + b^5 - (a^4*b + 2*a^
2*b^3 + b^5)*cosh(d*x + c)^4 - 4*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^3
*sinh(d*x + c) - 6*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^2*sinh(d*x + c)
^2 - 4*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)*sinh(d*x + c)^3 - (a^4*b +
2*a^2*b^3 + b^5)*sinh(d*x + c)^4)*log(cosh(d*x + c) + sinh(d*x + c) - 1) -
 2*(a^4*b + a^2*b^3 - 3*(a^4*b + a^2*b^3)*cosh(d*x + c)^2 - 2*(a^3*b^2 ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**2*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.44

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{b^4\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{(a^4+a^2b^2)\sqrt{a^2+b^2}d} - \frac{2\left(abe^{(-dx-c)}+b^2e^{(-2\,dx-2\,c)}-abe^{(-3\,dx-3\,c)}+2\,a^2+b^2\right)}{(a^3+ab^2-(a^3+ab^2)e^{(-4\,dx-4\,c)})d} + \frac{b\log\left(e^{(-dx-c)}+1\right)}{a^2d} - \frac{b\log\left(e^{(-dx-c)}-1\right)}{a^2d}$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxi
ma")
```

output
```
b^4*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(
a^2 + b^2)))/((a^4 + a^2*b^2)*sqrt(a^2 + b^2)*d) - 2*(a*b*e^(-d*x - c) + b
^2*e^(-2*d*x - 2*c) - a*b*e^(-3*d*x - 3*c) + 2*a^2 + b^2)/((a^3 + a*b^2 -
(a^3 + a*b^2)*e^(-4*d*x - 4*c))*d) + b*log(e^(-d*x - c) + 1)/(a^2*d) - b*l
og(e^(-d*x - c) - 1)/(a^2*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.28

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{\frac{b^4\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+a^2b^2)\sqrt{a^2+b^2}} + \frac{b\log\left(e^{(dx+c)}+1\right)}{a^2} - \frac{b\log\left(\left|e^{(dx+c)}-1\right|\right)}{a^2} - \frac{2\left(abe^{(3\,dx+3\,c)}+b^2e^{(2\,dx+2\,c)}-abe^{(dx+c)}+2\,a^2+b^2\right)}{(a^3+ab^2)\left(e^{(4\,dx+4\,c)}-1\right)}}{d}$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac
")
```

output
```
(b^4*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c
) + 2*a + 2*sqrt(a^2 + b^2)))/((a^4 + a^2*b^2)*sqrt(a^2 + b^2)) + b*log(e^
(d*x + c) + 1)/a^2 - b*log(abs(e^(d*x + c) - 1))/a^2 - 2*(a*b*e^(3*d*x + 3
*c) + b^2*e^(2*d*x + 2*c) - a*b*e^(d*x + c) + 2*a^2 + b^2)/((a^3 + a*b^2)*
(e^(4*d*x + 4*c) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 7.23 (sec) , antiderivative size = 768, normalized size of antiderivative = 5.33

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{b^4\,\ln\left(\frac{64\,b^8\,\sqrt{(a^2+b^2)^3}-96\,a\,b^{10}-384\,a^3\,b^8-512\,a^5\,b^6-288\,a^7\,b^4-64\,a^9\,b^2+288\,a^2\,b^9\,e^{c+d\,x}+960\,a^4\,b^7\,e^{c+d\,x}+1152\,a^6\,b^5\,e^{c+d\,x}+608\,a^8\,b^3\,\ldots}{a^3\,\left((a^2+b^2)^3\right)^{3/2}\,(a^2+b^2)}\ldots\right.}{d\,a^8+3\,d\,a^6\,b^2\ldots}$$

$$-\frac{\frac{2\,b^4\,e^{3\,c+3\,d\,x}}{d\,(a^2\,b^3+b^5)}-\frac{2\,b^4\,e^{c+d\,x}}{d\,(a^2\,b^3+b^5)}+\frac{2\,b^3\,(2\,a^2+b^2)}{a\,d\,(a^2\,b^3+b^5)}+\frac{2\,b^5\,e^{2\,c+2\,d\,x}}{a\,d\,(a^2\,b^3+b^5)}}{e^{4\,c+4\,d\,x}-1}$$

$$-\frac{b^4\,\ln\left(\frac{96\,a\,b^{10}+64\,b^8\,\sqrt{(a^2+b^2)^3}+384\,a^3\,b^8+512\,a^5\,b^6+288\,a^7\,b^4+64\,a^9\,b^2-288\,a^2\,b^9\,e^{c+d\,x}-960\,a^4\,b^7\,e^{c+d\,x}-1152\,a^6\,b^5\,e^{c+d\,x}-608\,a^8\ldots}{a^3\,\left((a^2+b^2)^3\right)^{3/2}\,(a^2+b^2)}\ldots\right.}{d\,a^8+3\,d\,a^6\ldots}$$

$$-\frac{b\,\ln\left(e^{c+d\,x}-1\right)}{a^2\,d}+\frac{b\,\ln\left(e^{c+d\,x}+1\right)}{a^2\,d}$$

input
```
int(1/(cosh(c + d*x)^2*sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
(b^4*log((64*b^8*((a^2 + b^2)^3)^(1/2) - 96*a*b^10 - 384*a^3*b^8 - 512*a^5
*b^6 - 288*a^7*b^4 - 64*a^9*b^2 + 288*a^2*b^9*exp(c + d*x) + 960*a^4*b^7*e
xp(c + d*x) + 1152*a^6*b^5*exp(c + d*x) + 608*a^8*b^3*exp(c + d*x) + 128*a
^10*b*exp(c + d*x) - 64*a*b^7*exp(c + d*x)*((a^2 + b^2)^3)^(1/2) + 32*a^3*
b^5*exp(c + d*x)*((a^2 + b^2)^3)^(1/2))/(a^3*((a^2 + b^2)^3)^(3/2)*(a^2 +
b^2)) - (32*b*(2*a^2*b - 4*a^3*exp(c + d*x) + 2*b^3 - 5*a*b^2*exp(c + d*x)
))/(a^3*(a^2 + b^2)^2))*((a^2 + b^2)^3)^(1/2))/(a^8*d + a^2*b^6*d + 3*a^4*
b^4*d + 3*a^6*b^2*d) - ((2*b^4*exp(3*c + 3*d*x))/(d*(b^5 + a^2*b^3)) - (2*
b^4*exp(c + d*x))/(d*(b^5 + a^2*b^3)) + (2*b^3*(2*a^2 + b^2))/(a*d*(b^5 +
a^2*b^3)) + (2*b^5*exp(2*c + 2*d*x))/(a*d*(b^5 + a^2*b^3)))/(exp(4*c + 4*d
*x) - 1) - (b^4*log((96*a*b^10 + 64*b^8*((a^2 + b^2)^3)^(1/2) + 384*a^3*b^
8 + 512*a^5*b^6 + 288*a^7*b^4 + 64*a^9*b^2 - 288*a^2*b^9*exp(c + d*x) - 96
0*a^4*b^7*exp(c + d*x) - 1152*a^6*b^5*exp(c + d*x) - 608*a^8*b^3*exp(c + d
*x) - 128*a^10*b*exp(c + d*x) - 64*a*b^7*exp(c + d*x)*((a^2 + b^2)^3)^(1/2
) + 32*a^3*b^5*exp(c + d*x)*((a^2 + b^2)^3)^(1/2))/(a^3*((a^2 + b^2)^3)^(3
/2)*(a^2 + b^2)) - (32*b*(2*a^2*b - 4*a^3*exp(c + d*x) + 2*b^3 - 5*a*b^2*e
xp(c + d*x)))/(a^3*(a^2 + b^2)^2))*((a^2 + b^2)^3)^(1/2))/(a^8*d + a^2*b^6
*d + 3*a^4*b^4*d + 3*a^6*b^2*d) - (b*log(exp(c + d*x) - 1))/(a^2*d) + (b*l
og(exp(c + d*x) + 1))/(a^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 562, normalized size of antiderivative = 3.90

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{2e^{4dx+4c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)b^4i - 2\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)b^4i - e^{4dx+4c}\log\left(e^{dx+c}-1\right)a^4b - 2e^{\cdots}}{}$$

input
```
int(csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(2*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a
**2 + b**2))*b**4*i - 2*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sq
rt(a**2 + b**2))*b**4*i - e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**4*b -
2*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**2*b**3 - e**(4*c + 4*d*x)*log(
e**(c + d*x) - 1)*b**5 + e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a**4*b + 2
*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a**2*b**3 + e**(4*c + 4*d*x)*log(e
**(c + d*x) + 1)*b**5 - 4*e**(4*c + 4*d*x)*a**5 - 6*e**(4*c + 4*d*x)*a**3*
b**2 - 2*e**(4*c + 4*d*x)*a*b**4 - 2*e**(3*c + 3*d*x)*a**4*b - 2*e**(3*c +
 3*d*x)*a**2*b**3 - 2*e**(2*c + 2*d*x)*a**3*b**2 - 2*e**(2*c + 2*d*x)*a*b*
*4 + 2*e**(c + d*x)*a**4*b + 2*e**(c + d*x)*a**2*b**3 + log(e**(c + d*x) -
 1)*a**4*b + 2*log(e**(c + d*x) - 1)*a**2*b**3 + log(e**(c + d*x) - 1)*b**
5 - log(e**(c + d*x) + 1)*a**4*b - 2*log(e**(c + d*x) + 1)*a**2*b**3 - log
(e**(c + d*x) + 1)*b**5)/(a**2*d*(e**(4*c + 4*d*x)*a**4 + 2*e**(4*c + 4*d*
x)*a**2*b**2 + e**(4*c + 4*d*x)*b**4 - a**4 - 2*a**2*b**2 - b**4))
```

3.472 $\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 36

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(csch(d*x+c)^2*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 50.59 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Csch[c + d*x]^2*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x
])),x]
```

output
```
Integrate[(Csch[c + d*x]^2*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

$$\Big\downarrow \ 6125$$

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Csch[c + d*x]^2*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(dx+c\right)^{2}\operatorname{sech}\left(dx+c\right)^{2}}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(csch(d*x+c)^2*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)^2*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^2\operatorname{sech}\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
integral(csch(d*x + c)^2*sech(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh
(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**2*sech(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 626, normalized size of antiderivative = 17.39

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^2\operatorname{sech}\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
16*b^4*integrate(-1/8*e^(d*x + c)/(a^4*b*e + a^2*b^3*e + (a^4*b*f + a^2*b^
3*f)*x - (a^4*b*e*e^(2*c) + a^2*b^3*e*e^(2*c) + (a^4*b*f*e^(2*c) + a^2*b^3
*f*e^(2*c))*x)*e^(2*d*x) - 2*(a^5*e*e^c + a^3*b^2*e*e^c + (a^5*f*e^c + a^3
*b^2*f*e^c)*x)*e^(d*x)), x) + 2*(a*b*e^(3*d*x + 3*c) + b^2*e^(2*d*x + 2*c)
 - a*b*e^(d*x + c) + 2*a^2 + b^2)/(a^3*d*e + a*b^2*d*e + (a^3*d*f + a*b^2*
d*f)*x - (a^3*d*e*e^(4*c) + a*b^2*d*e*e^(4*c) + (a^3*d*f*e^(4*c) + a*b^2*d
*f*e^(4*c))*x)*e^(4*d*x)) - 16*integrate(-1/16*(b*d*f*x + b*d*e + a*f)/(a^
2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2 - (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f
*x*e^c + a^2*d*e^2*e^c)*e^(d*x)), x) - 16*integrate(1/16*(b*d*f*x + b*d*e
- a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2 + (a^2*d*f^2*x^2*e^c + 2
*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)), x) - 16*integrate(1/8*(b*f*e^(
d*x + c) - a*f)/(a^2*d*e^2 + b^2*d*e^2 + (a^2*d*f^2 + b^2*d*f^2)*x^2 + 2*(
a^2*d*e*f + b^2*d*e*f)*x + (a^2*d*e^2*e^(2*c) + b^2*d*e^2*e^(2*c) + (a^2*d
*f^2*e^(2*c) + b^2*d*f^2*e^(2*c))*x^2 + 2*(a^2*d*e*f*e^(2*c) + b^2*d*e*f*e
^(2*c))*x)*e^(2*d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 7.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{1}{\cosh(c+d\,x)^2\,\sinh(c+d\,x)^2\,(e+f\,x)\,(a+b\sinh(c+d\,x))}\,dx$$

input
```
int(1/(cosh(c + d*x)^2*sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(cosh(c + d*x)^2*sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))), x
)
```

Reduce [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)^2\operatorname{sech}(dx+c)^2}{\sinh(dx+c)\,be+\sinh(dx+c)\,bfx+ae+afx}dx$$

input
```
int(csch(d*x+c)^2*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int((csch(c + d*x)**2*sech(c + d*x)**2)/(sinh(c + d*x)*b*e + sinh(c + d*x)
*b*f*x + a*e + a*f*x),x)
```

3.473 $\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 974

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output
```
-b^5*(f*x+e)*ln(1+exp(2*d*x+2*c))/a^2/(a^2+b^2)^2/d-b*(f*x+e)*ln(tanh(d*x+
c))/a^2/d+b*f*x*ln(tanh(d*x+c))/a^2/d-3/2*(f*x+e)*arctan(sinh(d*x+c))/a/d+
3/2*f*x*arctan(sinh(d*x+c))/a/d+3/2*I*f*polylog(2,-I*exp(d*x+c))/a/d^2+1/2
*I*b^2*f*polylog(2,I*exp(d*x+c))/a/(a^2+b^2)/d^2+I*b^4*f*polylog(2,I*exp(d
*x+c))/a/(a^2+b^2)^2/d^2-f*arctanh(cosh(d*x+c))/a/d^2-(f*x+e)*csch(d*x+c)/
a/d-1/2*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/a/d+b^5*f*polylog(2,-b*exp(d*x+c)/
(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^2/d^2+1/2*b*f*tanh(d*x+c)/a^2/d^2+1/2*b
*(f*x+e)*tanh(d*x+c)^2/a^2/d-1/2*b*f*x/a^2/d-3/2*I*f*polylog(2,I*exp(d*x+c
))/a/d^2+1/2*b^2*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/a/(a^2+b^2)/d-I*b^4*f*pol
ylog(2,-I*exp(d*x+c))/a/(a^2+b^2)^2/d^2-1/2*I*b^2*f*polylog(2,-I*exp(d*x+c
))/a/(a^2+b^2)/d^2-1/2*f*sech(d*x+c)/a/d^2-1/2*b*f*polylog(2,exp(2*d*x+2*c
))/a^2/d^2-3*f*x*arctan(exp(d*x+c))/a/d+1/2*b*f*polylog(2,-exp(2*d*x+2*c))
/a^2/d^2+b^2*(f*x+e)*arctan(exp(d*x+c))/a/(a^2+b^2)/d-1/2*b^5*f*polylog(2,
-exp(2*d*x+2*c))/a^2/(a^2+b^2)^2/d^2+2*b*f*x*arctanh(exp(2*d*x+2*c))/a^2/d
+2*b^4*(f*x+e)*arctan(exp(d*x+c))/a/(a^2+b^2)^2/d-1/2*b^3*f*tanh(d*x+c)/a^
2/(a^2+b^2)/d^2+1/2*b^2*f*sech(d*x+c)/a/(a^2+b^2)/d^2+1/2*b^3*(f*x+e)*sech
(d*x+c)^2/a^2/(a^2+b^2)/d+b^5*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)
))/a^2/(a^2+b^2)^2/d^2+b^5*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/
a^2/(a^2+b^2)^2/d+b^5*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(
a^2+b^2)^2/d
```

Mathematica [A] (warning: unable to verify)

Time = 11.17 (sec) , antiderivative size = 1437, normalized size of antiderivative = 1.48

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]
),x]
```

output
```
8*(((-(d*e*Cosh[(c + d*x)/2]) + c*f*Cosh[(c + d*x)/2] - f*(c + d*x)*Cosh[(
c + d*x)/2])*Csch[(c + d*x)/2]*Csch[c + d*x]*(a + b*Sinh[c + d*x]))/(16*a*
d^2*(b + a*Csch[c + d*x])) + (Csch[c + d*x]*(-1/2*(b*(d*e - c*f + f*(c + d
*x))^2)/f + (-(b*d*e) + a*f + b*c*f - b*f*(c + d*x))*Log[1 - E^(-c - d*x)]
 + (-(b*d*e) - a*f + b*c*f - b*f*(c + d*x))*Log[1 + E^(-c - d*x)] + b*f*Po
lyLog[2, -E^(-c - d*x)] + b*f*PolyLog[2, E^(-c - d*x)])*(a + b*Sinh[c + d*
x]))/(8*a^2*d^2*(b + a*Csch[c + d*x])) + (b^5*Csch[c + d*x]*(-2*d*e*(c + d
*x) + 2*c*f*(c + d*x) - f*(c + d*x)^2 + (4*a*Sqrt[a^2 + b^2]*d*e*ArcTan[(a
 + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b^2)^2] - (4*a*Sqrt[-(a^
2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/(-a^2 - b^2)
^(3/2) + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*
f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*f*Log[b -
 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*d*e*Log[2*a*E^(c + d*x) + b*(-1
+ E^(2*(c + d*x)))] + 2*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2]
)] + 2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])*(a + b*Sinh
[c + d*x]))/(16*a^2*(a^2 + b^2)^2*d^2*(b + a*Csch[c + d*x])) + (Csch[c + d
*x]*(-2*a^2*b*d*e*(c + d*x) - 4*b^3*d*e*(c + d*x) + 2*a^2*b*c*f*(c + d*x)
+ 4*b^3*c*f*(c + d*x) - a^2*b*f*(c + d*x)^2 - 2*b^3*f*(c + d*x)^2 - 6*a^3*
d*e*ArcTan[E^(c + d*x)] - 10*a*b^2*d*e*ArcTan[E^(c + d*x)] + 6*a^3*c*f*Arc
Tan[E^(c + d*x)] + 10*a*b^2*c*f*ArcTan[E^(c + d*x)] - (3*I)*a^3*f*(c + ...
```

Rubi [A] (verified)

Time = 4.00 (sec) , antiderivative size = 803, normalized size of antiderivative = 0.82, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {6123, 5985, 2009, 6123, 5985, 2009, 6107, 6107, 6095, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5985}$$

$$\frac{-f\int\left(\frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{2d}-\frac{3\arctan(\sinh(c+dx))}{2d}-\frac{3\operatorname{csch}(c+dx)}{2d}\right)dx-\frac{3(e+fx)\arctan(\sinh(c+dx))}{2d}-\frac{3(e+fx)\operatorname{csch}(c+dx}{2d}}{a}$$

$$\frac{b\int\frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$\downarrow$ 2009

$$-\frac{b\int\frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-f\left(\frac{3x\arctan(e^{c+dx})}{d}-\frac{3x\arctan(\sinh(c+dx))}{2d}+\frac{\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{3i\operatorname{PolyLog}(2,-ie^{c+dx})}{2d^2}+\frac{3i\operatorname{PolyLog}(2,ie^{c+dx})}{2d^2}+\frac{\operatorname{sech}(c}{2d}\right.}{a}$$

$\downarrow$ 6123

$$-\frac{b\left(\frac{\int(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{-f\left(\frac{3x\arctan(e^{c+dx})}{d}-\frac{3x\arctan(\sinh(c+dx))}{2d}+\frac{\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{3i\operatorname{PolyLog}(2,-ie^{c+dx})}{2d^2}+\frac{3i\operatorname{PolyLog}(2,ie^{c+dx})}{2d^2}+\frac{\operatorname{sech}(c}{2d}\right.}{a}$$

$\downarrow$ 5985

$$-\frac{b\left(\frac{-f\int\left(\frac{\log(\tanh(c+dx))}{d}-\frac{\tanh^2(c+dx)}{2d}\right)dx-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e+fx)\log(\tanh(c+dx))}{d}}{a}-\frac{b\int\frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{-f\left(\frac{3x\arctan(e^{c+dx})}{d}-\frac{3x\arctan(\sinh(c+dx))}{2d}+\frac{\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{3i\operatorname{PolyLog}(2,-ie^{c+dx})}{2d^2}+\frac{3i\operatorname{PolyLog}(2,ie^{c+dx})}{2d^2}+\frac{\operatorname{sech}(c}{2d}\right.}{a}$$

$\downarrow$ 2009

$$-\frac{b\left(\frac{-f\left(\frac{2x\operatorname{arctanh}(e^{2c+2dx})}{d}+\frac{\operatorname{PolyLog}(2,-e^{2c+2dx})}{2d^2}-\frac{\operatorname{PolyLog}(2,e^{2c+2dx})}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e+}{}}{a}\right.}{a}$$

$$\frac{-f\left(\frac{3x\arctan(e^{c+dx})}{d}-\frac{3x\arctan(\sinh(c+dx))}{2d}+\frac{\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{3i\operatorname{PolyLog}(2,-ie^{c+dx})}{2d^2}+\frac{3i\operatorname{PolyLog}(2,ie^{c+dx})}{2d^2}+\frac{\operatorname{sech}(c}{2d}\right.}{a}$$

$\downarrow$ 6107

$$-\frac{b\left(\frac{-f\left(\frac{2x\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e+}{}}{a}}{a}$$

$$\frac{-f\left(\frac{3x\arctan\left(e^{c+dx}\right)}{d}-\frac{3x\arctan(\sinh(c+dx))}{2d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{3i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{2d^2}+\frac{3i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{2d^2}+\frac{\text{sech}(c}{2d}\right.}{a}$$

$\downarrow$ 6107

$$-\frac{b\left(\frac{-f\left(\frac{2x\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e+}{}}{a}}{a}$$

$$\frac{-f\left(\frac{3x\arctan\left(e^{c+dx}\right)}{d}-\frac{3x\arctan(\sinh(c+dx))}{2d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{3i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{2d^2}+\frac{3i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{2d^2}+\frac{\text{sech}(c}{2d}\right.}{a}$$

$\downarrow$ 6095

$$-\frac{b\left(\frac{-f\left(\frac{2x\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e+}{}}{a}}{}$$

$$\frac{-f\left(\frac{3x\arctan\left(e^{c+dx}\right)}{d}-\frac{3x\arctan(\sinh(c+dx))}{2d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{3i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{2d^2}+\frac{3i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{2d^2}+\frac{\text{sech}(c}{2d}\right.}{a}$$

$\downarrow$ 2620

$$-\frac{b\left(\frac{-f\left(\frac{2x\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e+}{}}{a}\right.}{-f\left(\frac{3x\arctan\left(e^{c+dx}\right)}{d}-\frac{3x\arctan(\sinh(c+dx))}{2d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{3i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{2d^2}+\frac{3i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{2d^2}+\frac{\text{sech}(c}{2d}\right.}{a}$$

$\downarrow$ 2715

$$-\frac{b\left(\frac{-f\left(\frac{2x\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e+}{}}{a}\right.}{-f\left(\frac{3x\arctan\left(e^{c+dx}\right)}{d}-\frac{3x\arctan(\sinh(c+dx))}{2d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{3i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{2d^2}+\frac{3i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{2d^2}+\frac{\text{sech}(c}{2d}\right.}{a}$$

$\downarrow$ 2838

$$-\frac{b\left(\frac{-f\left(\frac{2x\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e+}{}}{a}}{-f\left(\frac{3x\arctan\left(e^{c+dx}\right)}{d}-\frac{3x\arctan(\sinh(c+dx))}{2d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{3i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{2d^2}+\frac{3i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{2d^2}+\frac{\text{sech}(c}{2d}}{a}}$$

$\downarrow$ 7293

$$-\frac{b\left(\frac{-f\left(\frac{2x\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e+}{}}{a}\right)}{\frac{-f\left(\frac{3x\arctan\left(e^{c+dx}\right)}{d}-\frac{3x\arctan(\sinh(c+dx))}{2d}+\frac{\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{3i\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{2d^2}+\frac{3i\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{2d^2}+\frac{\operatorname{sech}(c}{2d}\right.}{a}}$$

$\downarrow$ 2009

$$\frac{\frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{2d}-\frac{3(e+fx)\arctan(\sinh(c+dx))}{2d}-\frac{3(e+fx)\operatorname{csch}(c+dx)}{2d}-f\left(\frac{3x\arctan\left(e^{c+dx}\right)}{d}-\frac{3x\arctan(\sinh(c+dx}{2d}\right.}{a}$$

$$b\left(\frac{-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e+fx)\log(\tanh(c+dx))}{d}-f\left(\frac{2\operatorname{arctanh}\left(e^{2c+2dx}\right)x}{d}+\frac{\log(\tanh(c+dx))x}{d}-\frac{x}{2d}+\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}\right.}{a}\right.$$

input
```
Int[((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
((-3*(e + f*x)*ArcTan[Sinh[c + d*x]])/(2*d) - (3*(e + f*x)*Csch[c + d*x])/
(2*d) + ((e + f*x)*Csch[c + d*x]*Sech[c + d*x]^2)/(2*d) - f*((3*x*ArcTan[E
^(c + d*x)])/d - (3*x*ArcTan[Sinh[c + d*x]])/(2*d) + ArcTanh[Cosh[c + d*x]
]/d^2 - (((3*I)/2)*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + (((3*I)/2)*PolyLog[
2, I*E^(c + d*x)])/d^2 + Sech[c + d*x]/(2*d^2)))/a - (b*((((e + f*x)*Log[T
anh[c + d*x]])/d - ((e + f*x)*Tanh[c + d*x]^2)/(2*d) - f*(-1/2*x/d + (2*x*
ArcTanh[E^(2*c + 2*d*x)])/d + (x*Log[Tanh[c + d*x]])/d + PolyLog[2, -E^(2*
c + 2*d*x)]/(2*d^2) - PolyLog[2, E^(2*c + 2*d*x)]/(2*d^2) + Tanh[c + d*x]/
(2*d^2)))/a - (b*((b^2*((b^2*(-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Log[1 +
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2]))])/(b*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2]))])/(b*d^2)))/(a^2 + b^2) + ((b*(e + f*x)^2)/(2*f) + (2*a*(e
+ f*x)*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)*Log[1 + E^(2*(c + d*x))])/d -
 (I*a*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + (I*a*f*PolyLog[2, I*E^(c + d*x
)])/d^2 - (b*f*PolyLog[2, -E^(2*(c + d*x))])/(2*d^2))/(a^2 + b^2)))/(a^2 +
 b^2) + ((a*(e + f*x)*ArcTan[E^(c + d*x)])/d - ((I/2)*a*f*PolyLog[2, (-I)*
E^(c + d*x)])/d^2 + ((I/2)*a*f*PolyLog[2, I*E^(c + d*x)])/d^2 + (a*f*Sech[
c + d*x])/(2*d^2) + (b*(e + f*x)*Sech[c + d*x]^2)/(2*d) - (b*f*Tanh[c + d*
x])/(2*d^2) + (a*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*d))/(a^2 + b...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3279 vs. $2(902) = 1804$.

Time = 46.24 (sec) , antiderivative size = 3280, normalized size of antiderivative = 3.37

method	result	size
risch	Expression too large to display	3280

input
```
int((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURN
VERBOSE)
```

output
```
-10*I/(a^2+b^2)/d^2*a*b^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*c+10*I/(a^2+b
^2)/d^2*a*b^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*c-10*I/(a^2+b^2)/d*a*b^2*
f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*x+10*I/(a^2+b^2)/d*a*b^2*f/(4*a^2+4*b^2
)*ln(1+I*exp(d*x+c))*x+6*I/(a^2+b^2)/d^2*a^3*f/(4*a^2+4*b^2)*dilog(1+I*exp
(d*x+c))-6*I/(a^2+b^2)/d^2*a^3*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))+4/(a^
2+b^2)/d*a^2*b*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*x+4/(a^2+b^2)/d*a^2*b*f/
(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*x+4/(a^2+b^2)/d^2*a^2*b*f/(4*a^2+4*b^2)*l
n(1+I*exp(d*x+c))*c+4/(a^2+b^2)/d^2*a^2*b*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c
))*c+7/2/(a^2+b^2)^(5/2)/d^2*a*c*b^3*f*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a
^2+b^2)^(1/2))-4/(a^2+b^2)/d^2*a^2*c*b*f/(4*a^2+4*b^2)*ln(1+exp(2*d*x+2*c)
)+6*I/(a^2+b^2)/d^2*a^3*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*c-6*I/(a^2+b^2)
/d^2*a^3*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*c+10*I/(a^2+b^2)/d^2*a*b^2*f/(
4*a^2+4*b^2)*dilog(1+I*exp(d*x+c))-10*I/(a^2+b^2)/d^2*a*b^2*f/(4*a^2+4*b^2
)*dilog(1-I*exp(d*x+c))+6*I/(a^2+b^2)/d*a^3*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x
+c))*x-6*I/(a^2+b^2)/d*a^3*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*x+1/(a^2+b^2
)^2/d^2/a^2*b^5*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2
)))+1/(a^2+b^2)^2/d^2/a^2*b^5*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-
a+(a^2+b^2)^(1/2)))-1/(a^2+b^2)/d/a^2*b^3*e*ln(exp(d*x+c)+1)-1/(a^2+b^2)/d
/a^2*b^3*e*ln(exp(d*x+c)-1)-12/(a^2+b^2)/d*a^3*e/(4*a^2+4*b^2)*arctan(exp(
d*x+c))+1/(a^2+b^2)/d^2/a^2*b^3*f*dilog(exp(d*x+c))-1/(a^2+b^2)/d^2/a^2...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 15223 vs. 2(879) = 1758.

Time = 0.52 (sec) , antiderivative size = 15223, normalized size of antiderivative = 15.63

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)**2*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{csch}\,(dx+c)^2\,\text{sech}\,(dx+c)^3}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
(b^5*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^6 + 2*a^4*b^2 + a
^2*b^4)*d) + (3*a^3 + 5*a*b^2)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^
4)*d) + (a^2*b + 2*b^3)*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)
*d) - (2*a*b*e^(-2*d*x - 2*c) - 2*a*b*e^(-4*d*x - 4*c) + (3*a^2 + 2*b^2)*e
^(-d*x - c) + 2*(a^2 + 2*b^2)*e^(-3*d*x - 3*c) + (3*a^2 + 2*b^2)*e^(-5*d*x
 - 5*c))/((a^3 + a*b^2 + (a^3 + a*b^2)*e^(-2*d*x - 2*c) - (a^3 + a*b^2)*e^
(-4*d*x - 4*c) - (a^3 + a*b^2)*e^(-6*d*x - 6*c))*d) - b*log(e^(-d*x - c) +
 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d))*e + (32*b*d*integrate(1/32*
x/(a^2*d*e^(d*x + c) + a^2*d), x) - 32*b*d*integrate(1/32*x/(a^2*d*e^(d*x
+ c) - a^2*d), x) + a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) + 1)/(a^2*d^2
)) - a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) - 1)/(a^2*d^2)) - (2*a*b*d*x
*e^(2*d*x + 2*c) - 2*(a^2*d*e^(3*c) + 2*b^2*d*e^(3*c))*x*e^(3*d*x) + a*b -
 (a^2*e^(5*c) + (3*a^2*d*e^(5*c) + 2*b^2*d*e^(5*c))*x)*e^(5*d*x) - (2*a*b*
d*x*e^(4*c) + a*b*e^(4*c))*e^(4*d*x) + (a^2*e^c - (3*a^2*d*e^c + 2*b^2*d*e
^c)*x)*e^(d*x))/(a^3*d^2 + a*b^2*d^2 - (a^3*d^2*e^(6*c) + a*b^2*d^2*e^(6*c
))*e^(6*d*x) - (a^3*d^2*e^(4*c) + a*b^2*d^2*e^(4*c))*e^(4*d*x) + (a^3*d^2*
e^(2*c) + a*b^2*d^2*e^(2*c))*e^(2*d*x)) - 32*integrate(-1/16*(a*b^5*x*e^(d
*x + c) - b^6*x)/(a^6*b + 2*a^4*b^3 + a^2*b^5 - (a^6*b*e^(2*c) + 2*a^4*b^3
*e^(2*c) + a^2*b^5*e^(2*c))*e^(2*d*x) - 2*(a^7*e^c + 2*a^5*b^2*e^c + a^3*b
^4*e^c)*e^(d*x)), x) - 32*integrate(1/32*((3*a^3*e^c + 5*a*b^2*e^c)*x*e...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\mathrm{csch}^2(c+dx)\mathrm{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{e+f\,x}{\cosh\left(c+d\,x\right)^3\,\sinh\left(c+d\,x\right)^2\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)/(cosh(c + d*x)^3*sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(cosh(c + d*x)^3*sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output

```
( - 9*e**(6*c + 6*d*x)*atan(e**(c + d*x))*a**5*b*d*e - 15*e**(6*c + 6*d*x)
*atan(e**(c + d*x))*a**3*b**3*d*e - 9*e**(4*c + 4*d*x)*atan(e**(c + d*x))*
a**5*b*d*e - 15*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**3*b**3*d*e + 9*e**(
2*c + 2*d*x)*atan(e**(c + d*x))*a**5*b*d*e + 15*e**(2*c + 2*d*x)*atan(e**(
c + d*x))*a**3*b**3*d*e + 9*atan(e**(c + d*x))*a**5*b*d*e + 15*atan(e**(c
+ d*x))*a**3*b**3*d*e - 384*e**(11*c + 6*d*x)*int((e**(5*d*x)*x)/(e**(12*c
 + 12*d*x)*b + 2*e**(11*c + 11*d*x)*a + 2*e**(9*c + 9*d*x)*a - 3*e**(8*c +
 8*d*x)*b - 4*e**(7*c + 7*d*x)*a - 4*e**(5*c + 5*d*x)*a + 3*e**(4*c + 4*d*
x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a**7*d**2*f - 768*e
**(11*c + 6*d*x)*int((e**(5*d*x)*x)/(e**(12*c + 12*d*x)*b + 2*e**(11*c + 1
1*d*x)*a + 2*e**(9*c + 9*d*x)*a - 3*e**(8*c + 8*d*x)*b - 4*e**(7*c + 7*d*x
)*a - 4*e**(5*c + 5*d*x)*a + 3*e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a +
 2*e**(c + d*x)*a - b),x)*a**5*b**2*d**2*f - 384*e**(11*c + 6*d*x)*int((e*
*(5*d*x)*x)/(e**(12*c + 12*d*x)*b + 2*e**(11*c + 11*d*x)*a + 2*e**(9*c + 9
*d*x)*a - 3*e**(8*c + 8*d*x)*b - 4*e**(7*c + 7*d*x)*a - 4*e**(5*c + 5*d*x)
*a + 3*e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x
)*a**3*b**4*d**2*f + 256*e**(10*c + 6*d*x)*int((e**(4*d*x)*x)/(e**(12*c +
12*d*x)*b + 2*e**(11*c + 11*d*x)*a + 2*e**(9*c + 9*d*x)*a - 3*e**(8*c + 8*
d*x)*b - 4*e**(7*c + 7*d*x)*a - 4*e**(5*c + 5*d*x)*a + 3*e**(4*c + 4*d*x)*
b + 2*e**(3*c + 3*d*x)*a + 2*e**(c + d*x)*a - b),x)*a**6*b*d**2*f + 512...
```

3.474 $\int \frac{\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 180

$$\begin{aligned}\int \frac{\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = &-\frac{a\arctan(\sinh(c+dx))}{2(a^2+b^2)\,d} \\ &-\frac{a(a^2+2b^2)\arctan(\sinh(c+dx))}{(a^2+b^2)^2\,d} \\ &-\frac{\text{csch}(c+dx)}{ad}+\frac{b(a^2+2b^2)\log(\cosh(c+dx))}{(a^2+b^2)^2\,d} \\ &-\frac{b\log(\sinh(c+dx))}{a^2d}+\frac{b^5\log(a+b\sinh(c+dx))}{a^2(a^2+b^2)^2\,d} \\ &-\frac{\text{sech}^2(c+dx)(b+a\sinh(c+dx))}{2(a^2+b^2)\,d}\end{aligned}$$

output
```
-1/2*a*arctan(sinh(d*x+c))/(a^2+b^2)/d-a*(a^2+2*b^2)*arctan(sinh(d*x+c))/(
a^2+b^2)^2/d-csch(d*x+c)/a/d+b*(a^2+2*b^2)*ln(cosh(d*x+c))/(a^2+b^2)^2/d-b
*ln(sinh(d*x+c))/a^2/d+b^5*ln(a+b*sinh(d*x+c))/a^2/(a^2+b^2)^2/d-1/2*sech(
d*x+c)^2*(b+a*sinh(d*x+c))/(a^2+b^2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.26

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\operatorname{csch}(c+dx)(a+b\sinh(c+dx))\left(\frac{a\arctan(\sinh(c+dx))}{a^2+b^2}+\frac{2\operatorname{csch}(c+dx)}{a}-\frac{(ia+b)(a^2+2b^2)\log(i-\sinh(c+dx))}{(a^2+b^2)^2}+\frac{2b\log}{}\right.}{2d(b+a\operatorname{csc}}$$

input
```
Integrate[(Csch[c + d*x]^2*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*(Csch[c + d*x]*(a + b*Sinh[c + d*x])*((a*ArcTan[Sinh[c + d*x]])/(a^2
+ b^2) + (2*Csch[c + d*x])/a - ((I*a + b)*(a^2 + 2*b^2)*Log[I - Sinh[c + d
*x]])/(a^2 + b^2)^2 + (2*b*Log[Sinh[c + d*x]])/a^2 + ((I*a - b)*(a^2 + 2*b
^2)*Log[I + Sinh[c + d*x]])/(a^2 + b^2)^2 - (2*b^5*Log[a + b*Sinh[c + d*x]
])/(a^2*(a^2 + b^2)^2) + (b*Sech[c + d*x]^2)/(a^2 + b^2) + (a*Sech[c + d*x
]*Tanh[c + d*x])/(a^2 + b^2)))/(d*(b + a*Csch[c + d*x]))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {3042, 25, 3316, 25, 27, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\sin(ic+idx)^2\cos(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\cos(ic+idx)^3\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$\Big\downarrow$ 3316

$$-\frac{b^3\int -\frac{\text{csch}^2(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$\Big\downarrow$ 25

$$\frac{b^3\int \frac{\text{csch}^2(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$\Big\downarrow$ 27

$$\frac{b^5\int \frac{\text{csch}^2(c+dx)}{b^2(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$\Big\downarrow$ 615

$$\frac{b^5\int \left(\frac{\text{csch}^2(c+dx)}{ab^6}-\frac{\text{csch}(c+dx)}{a^2b^5}+\frac{1}{a^2(a^2+b^2)^2(a+b\sinh(c+dx))}-\frac{(a^2+2b^2)(a-b\sinh(c+dx))}{b^4(a^2+b^2)^2(\sinh^2(c+dx)b^2+b^2)}+\frac{b\sinh(c+dx)-a}{b^2(a^2+b^2)(\sinh^2(c+dx)b^2+b^2}\right.}{d}$$

$\Big\downarrow$ 2009

$$\frac{b^5\left(-\frac{a(a^2+2b^2)\arctan(\sinh(c+dx))}{b^5(a^2+b^2)^2}-\frac{a\arctan(\sinh(c+dx))}{2b^5(a^2+b^2)}-\frac{\log(b\sinh(c+dx))}{a^2b^4}+\frac{\log(a+b\sinh(c+dx))}{a^2(a^2+b^2)^2}-\frac{ab\sinh(c+dx)+b^2}{2b^4(a^2+b^2)(b^2\sinh^2(c+dx)-\right.}{d}$$

input
```
Int[(Csch[c + d*x]^2*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(b^5*(-1/2*(a*ArcTan[Sinh[c + d*x]])/(b^5*(a^2 + b^2)) - (a*(a^2 + 2*b^2)*
ArcTan[Sinh[c + d*x]])/(b^5*(a^2 + b^2)^2) - Csch[c + d*x]/(a*b^5) - Log[b
*Sinh[c + d*x]]/(a^2*b^4) + Log[a + b*Sinh[c + d*x]]/(a^2*(a^2 + b^2)^2) +
 ((a^2 + 2*b^2)*Log[b^2 + b^2*Sinh[c + d*x]^2])/(2*b^4*(a^2 + b^2)^2) - (b
^2 + a*b*Sinh[c + d*x])/(2*b^4*(a^2 + b^2)*(b^2 + b^2*Sinh[c + d*x]^2))))/
d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 24.44 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.38

method	result
derivativedivides	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+\frac{b^5\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{\left(a^2+b^2\right)^2a^2}-\frac{2\left(\frac{\left(-\frac{1}{2}a^3-\frac{1}{2}ab^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-a^2b-b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}\right.}{}}{}$
default	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+\frac{b^5\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{\left(a^2+b^2\right)^2a^2}-\frac{2\left(\frac{\left(-\frac{1}{2}a^3-\frac{1}{2}ab^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-a^2b-b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}\right.}{}}{}$
risch	$-\frac{2a^2b\,d^2x}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{2a^2bdc}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{4b^3d^2x}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{4b^3dc}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{2b^5x}{a^2(a^4+2a^2}$

input
```
int(csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/a*tanh(1/2*d*x+1/2*c)+b^5/(a^2+b^2)^2/a^2*ln(tanh(1/2*d*x+1/2*c)^
2*a-2*b*tanh(1/2*d*x+1/2*c)-a)-2/(a^2+b^2)^2*(((-1/2*a^3-1/2*a*b^2)*tanh(1
/2*d*x+1/2*c)^3+(-a^2*b-b^3)*tanh(1/2*d*x+1/2*c)^2+(1/2*a^3+1/2*a*b^2)*tan
h(1/2*d*x+1/2*c))/(1+tanh(1/2*d*x+1/2*c)^2)^2+1/4*(-2*a^2*b-4*b^3)*ln(1+ta
nh(1/2*d*x+1/2*c)^2)+1/2*(3*a^3+5*a*b^2)*arctan(tanh(1/2*d*x+1/2*c)))-1/2/
a/tanh(1/2*d*x+1/2*c)-1/a^2*b*ln(tanh(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2568 vs. $2(176) = 352$.

Time = 0.36 (sec) , antiderivative size = 2568, normalized size of antiderivative = 14.27

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fric
as")
```

output
```
-((3*a^5 + 5*a^3*b^2 + 2*a*b^4)*cosh(d*x + c)^5 + (3*a^5 + 5*a^3*b^2 + 2*a
*b^4)*sinh(d*x + c)^5 + 2*(a^4*b + a^2*b^3)*cosh(d*x + c)^4 + (2*a^4*b + 2
*a^2*b^3 + 5*(3*a^5 + 5*a^3*b^2 + 2*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^4
+ 2*(a^5 + 3*a^3*b^2 + 2*a*b^4)*cosh(d*x + c)^3 + 2*(a^5 + 3*a^3*b^2 + 2*a
*b^4 + 5*(3*a^5 + 5*a^3*b^2 + 2*a*b^4)*cosh(d*x + c)^2 + 4*(a^4*b + a^2*b^
3)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(a^4*b + a^2*b^3)*cosh(d*x + c)^2 -
2*(a^4*b + a^2*b^3 - 5*(3*a^5 + 5*a^3*b^2 + 2*a*b^4)*cosh(d*x + c)^3 - 6*(
a^4*b + a^2*b^3)*cosh(d*x + c)^2 - 3*(a^5 + 3*a^3*b^2 + 2*a*b^4)*cosh(d*x
+ c))*sinh(d*x + c)^2 + ((3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^6 + 6*(3*a^5 +
5*a^3*b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (3*a^5 + 5*a^3*b^2)*sinh(d*x +
c)^6 - 3*a^5 - 5*a^3*b^2 + (3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^4 + (3*a^5 +
5*a^3*b^2 + 15*(3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5
*(3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^3 + (3*a^5 + 5*a^3*b^2)*cosh(d*x + c))*
sinh(d*x + c)^3 - (3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^2 - (3*a^5 + 5*a^3*b^2
 - 15*(3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^4 - 6*(3*a^5 + 5*a^3*b^2)*cosh(d*x
 + c)^2)*sinh(d*x + c)^2 + 2*(3*(3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^5 + 2*(3
*a^5 + 5*a^3*b^2)*cosh(d*x + c)^3 - (3*a^5 + 5*a^3*b^2)*cosh(d*x + c))*sin
h(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) + (3*a^5 + 5*a^3*b^2 + 2
*a*b^4)*cosh(d*x + c) - (b^5*cosh(d*x + c)^6 + 6*b^5*cosh(d*x + c)*sinh(d*
x + c)^5 + b^5*sinh(d*x + c)^6 + b^5*cosh(d*x + c)^4 - b^5*cosh(d*x + c...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**2*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.94

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b^5\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^6+2\,a^4b^2+a^2b^4)d} + \frac{(3\,a^3+5\,ab^2)\arctan\left(e^{(-dx-c)}\right)}{(a^4+2\,a^2b^2+b^4)d} + \frac{(a^2b+2\,b^3)\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^4+2\,a^2b^2+b^4)d} - \frac{2\,abe^{(-2\,dx-2\,c)}-2\,abe^{(-4\,dx-4\,c)}+(3\,a^2+2\,b^2)e^{(-dx-c)}+2\,(a^2+2\,b^2)e^{(-3\,dx-3\,c)}+(3\,a^2+2\,b^2)e^{(-5\,dx-5}}{(a^3+ab^2+(a^3+ab^2)e^{(-2\,dx-2\,c)}-(a^3+ab^2)e^{(-4\,dx-4\,c)}-(a^3+ab^2)e^{(-6\,dx-6\,c)})d} - \frac{b\log\left(e^{(-dx-c)}+1\right)}{a^2d} - \frac{b\log\left(e^{(-dx-c)}-1\right)}{a^2d}$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxi
ma")
```

output
```
b^5*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^6 + 2*a^4*b^2 + a^
2*b^4)*d) + (3*a^3 + 5*a*b^2)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4
)*d) + (a^2*b + 2*b^3)*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*
d) - (2*a*b*e^(-2*d*x - 2*c) - 2*a*b*e^(-4*d*x - 4*c) + (3*a^2 + 2*b^2)*e^
(-d*x - c) + 2*(a^2 + 2*b^2)*e^(-3*d*x - 3*c) + (3*a^2 + 2*b^2)*e^(-5*d*x
- 5*c))/((a^3 + a*b^2 + (a^3 + a*b^2)*e^(-2*d*x - 2*c) - (a^3 + a*b^2)*e^(
-4*d*x - 4*c) - (a^3 + a*b^2)*e^(-6*d*x - 6*c))*d) - b*log(e^(-d*x - c) +
1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(176) = 352$.

Time = 0.13 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.54

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{12\,b^6\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^6b+2\,a^4b^3+a^2b^5} - \frac{3\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)(3\,a^3+5\,ab^2)}{a^4+2\,a^2b^2+b^4} + \frac{6\left(a^2b+2\,b^3\right)\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right.\right.}{a^4+2\,a^2b^2+b^4}}{}$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac
")
```

output
```
1/12*(12*b^6*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^6*b + 2*a^4
*b^3 + a^2*b^5) - 3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c))
)*(3*a^3 + 5*a*b^2)/(a^4 + 2*a^2*b^2 + b^4) + 6*(a^2*b + 2*b^3)*log((e^(d*
x + c) - e^(-d*x - c))^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) - 12*b*log(abs(e^(d*
x + c) - e^(-d*x - c)))/a^2 + 4*(b^5*(e^(d*x + c) - e^(-d*x - c))^3 - 9*a^
5*(e^(d*x + c) - e^(-d*x - c))^2 - 15*a^3*b^2*(e^(d*x + c) - e^(-d*x - c))
^2 - 6*a*b^4*(e^(d*x + c) - e^(-d*x - c))^2 - 6*a^4*b*(e^(d*x + c) - e^(-d
*x - c)) - 6*a^2*b^3*(e^(d*x + c) - e^(-d*x - c)) + 4*b^5*(e^(d*x + c) - e
^(-d*x - c)) - 24*a^5 - 48*a^3*b^2 - 24*a*b^4)/((a^6 + 2*a^4*b^2 + a^2*b^4
)*((e^(d*x + c) - e^(-d*x - c))^3 + 4*e^(d*x + c) - 4*e^(-d*x - c))))/d
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.21

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2\,b}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)^2\,\left(a^2+b^2\right)} - \frac{b\,\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-1\right)}{a^2\,d} + \frac{2\,b\,\ln\left(1+\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,1\mathrm{i}\right)}{d\,{\left(-b+a\,1\mathrm{i}\right)}^2} - \frac{2\,b^3}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)\,{\left(a^2+b^2\right)}^2} - \frac{2\,\mathrm{e}^{c+d\,x}}{a\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)} + \frac{2\,b\,\ln\left(\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+1\mathrm{i}\right)}{d\,{\left(b+a\,1\mathrm{i}\right)}^2} - \frac{2\,a^2\,b}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)\,{\left(a^2+b^2\right)}^2} - \frac{a^3\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)\,{\left(a^2+b^2\right)}^2} + \frac{b^5\,\ln\left(2\,a\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-b+b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\right)}{a^2\,d\,{\left(a^2+b^2\right)}^2} + \frac{2\,a\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)^2\,\left(a^2+b^2\right)} - \frac{a\,b^2\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)\,{\left(a^2+b^2\right)}^2} - \frac{a\,\ln\left(1+\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,1\mathrm{i}\right)\,3\mathrm{i}}{2\,d\,{\left(-b+a\,1\mathrm{i}\right)}^2} + \frac{a\,\ln\left(\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+1\mathrm{i}\right)\,3\mathrm{i}}{2\,d\,{\left(b+a\,1\mathrm{i}\right)}^2}$$

input
```
int(1/(cosh(c + d*x)^3*sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
(2*b)/(d*(exp(2*c + 2*d*x) + 1)^2*(a^2 + b^2)) - (b*log(exp(2*c)*exp(2*d*x
) - 1))/(a^2*d) - (a*log(exp(d*x)*exp(c)*1i + 1)*3i)/(2*d*(a*1i - b)^2) +
(2*b*log(exp(d*x)*exp(c)*1i + 1))/(d*(a*1i - b)^2) - (2*b^3)/(d*(exp(2*c +
 2*d*x) + 1)*(a^2 + b^2)^2) - (2*exp(c + d*x))/(a*d*(exp(2*c + 2*d*x) - 1)
) + (a*log(exp(d*x)*exp(c) + 1i)*3i)/(2*d*(a*1i + b)^2) + (2*b*log(exp(d*x
)*exp(c) + 1i))/(d*(a*1i + b)^2) - (2*a^2*b)/(d*(exp(2*c + 2*d*x) + 1)*(a^
2 + b^2)^2) - (a^3*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1)*(a^2 + b^2)^2)
+ (b^5*log(2*a*exp(d*x)*exp(c) - b + b*exp(2*c)*exp(2*d*x)))/(a^2*d*(a^2 +
 b^2)^2) + (2*a*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1)^2*(a^2 + b^2)) - (
a*b^2*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1)*(a^2 + b^2)^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1469, normalized size of antiderivative = 8.16

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
( - 3*e**(6*c + 6*d*x)*atan(e**(c + d*x))*a**5 - 5*e**(6*c + 6*d*x)*atan(e
**(c + d*x))*a**3*b**2 - 3*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**5 - 5*e*
*(4*c + 4*d*x)*atan(e**(c + d*x))*a**3*b**2 + 3*e**(2*c + 2*d*x)*atan(e**(
c + d*x))*a**5 + 5*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**3*b**2 + 3*atan(
e**(c + d*x))*a**5 + 5*atan(e**(c + d*x))*a**3*b**2 + e**(6*c + 6*d*x)*log
(e**(2*c + 2*d*x) + 1)*a**4*b + 2*e**(6*c + 6*d*x)*log(e**(2*c + 2*d*x) +
1)*a**2*b**3 - e**(6*c + 6*d*x)*log(e**(c + d*x) - 1)*a**4*b - 2*e**(6*c +
 6*d*x)*log(e**(c + d*x) - 1)*a**2*b**3 - e**(6*c + 6*d*x)*log(e**(c + d*x
) - 1)*b**5 - e**(6*c + 6*d*x)*log(e**(c + d*x) + 1)*a**4*b - 2*e**(6*c +
6*d*x)*log(e**(c + d*x) + 1)*a**2*b**3 - e**(6*c + 6*d*x)*log(e**(c + d*x)
 + 1)*b**5 + e**(6*c + 6*d*x)*log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a -
b)*b**5 + 2*e**(6*c + 6*d*x)*a**4*b + 2*e**(6*c + 6*d*x)*a**2*b**3 - 3*e**
(5*c + 5*d*x)*a**5 - 5*e**(5*c + 5*d*x)*a**3*b**2 - 2*e**(5*c + 5*d*x)*a*b
**4 + e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x) + 1)*a**4*b + 2*e**(4*c + 4*d*
x)*log(e**(2*c + 2*d*x) + 1)*a**2*b**3 - e**(4*c + 4*d*x)*log(e**(c + d*x)
 - 1)*a**4*b - 2*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**2*b**3 - e**(4*
c + 4*d*x)*log(e**(c + d*x) - 1)*b**5 - e**(4*c + 4*d*x)*log(e**(c + d*x)
+ 1)*a**4*b - 2*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a**2*b**3 - e**(4*c
 + 4*d*x)*log(e**(c + d*x) + 1)*b**5 + e**(4*c + 4*d*x)*log(e**(2*c + 2*d*
x)*b + 2*e**(c + d*x)*a - b)*b**5 - 2*e**(3*c + 3*d*x)*a**5 - 6*e**(3*c...
```

3.475 $\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 36

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(csch(d*x+c)^2*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 104.44 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Csch[c + d*x]^2*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x
])),x]
```

output
```
Integrate[(Csch[c + d*x]^2*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{6125}$$
$$\int \frac{\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Csch[c + d*x]^2*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\text{csch}\,(dx+c)^2\,\text{sech}\,(dx+c)^3}{(fx+e)\,(a+b\sinh\,(dx+c))}dx$$

input
```
int(csch(d*x+c)^2*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)^2*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 147.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^2\operatorname{sech}\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
integral(csch(d*x + c)^2*sech(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh
(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**2*sech(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 2.65 (sec) , antiderivative size = 1586, normalized size of antiderivative = 44.06

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^2\operatorname{sech}\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
(a*b*f + (2*b^2*d*e*e^(5*c) + (3*d*e - f)*a^2*e^(5*c) + (3*a^2*d*f*e^(5*c)
 + 2*b^2*d*f*e^(5*c))*x)*e^(5*d*x) + (2*a*b*d*f*x*e^(4*c) + (2*d*e - f)*a*
b*e^(4*c))*e^(4*d*x) + 2*(a^2*d*e*e^(3*c) + 2*b^2*d*e*e^(3*c) + (a^2*d*f*e
^(3*c) + 2*b^2*d*f*e^(3*c))*x)*e^(3*d*x) - 2*(a*b*d*f*x*e^(2*c) + a*b*d*e*
e^(2*c))*e^(2*d*x) + (2*b^2*d*e*e^c + (3*d*e + f)*a^2*e^c + (3*a^2*d*f*e^c
 + 2*b^2*d*f*e^c)*x)*e^(d*x))/(a^3*d^2*e^2 + a*b^2*d^2*e^2 + (a^3*d^2*f^2
+ a*b^2*d^2*f^2)*x^2 + 2*(a^3*d^2*e*f + a*b^2*d^2*e*f)*x - (a^3*d^2*e^2*e^
(6*c) + a*b^2*d^2*e^2*e^(6*c) + (a^3*d^2*f^2*e^(6*c) + a*b^2*d^2*f^2*e^(6*
c))*x^2 + 2*(a^3*d^2*e*f*e^(6*c) + a*b^2*d^2*e*f*e^(6*c))*x)*e^(6*d*x) - (
a^3*d^2*e^2*e^(4*c) + a*b^2*d^2*e^2*e^(4*c) + (a^3*d^2*f^2*e^(4*c) + a*b^2
*d^2*f^2*e^(4*c))*x^2 + 2*(a^3*d^2*e*f*e^(4*c) + a*b^2*d^2*e*f*e^(4*c))*x)
*e^(4*d*x) + (a^3*d^2*e^2*e^(2*c) + a*b^2*d^2*e^2*e^(2*c) + (a^3*d^2*f^2*e
^(2*c) + a*b^2*d^2*f^2*e^(2*c))*x^2 + 2*(a^3*d^2*e*f*e^(2*c) + a*b^2*d^2*e
*f*e^(2*c))*x)*e^(2*d*x)) - 32*integrate(-1/16*(a*b^5*e^(d*x + c) - b^6)/(
a^6*b*e + 2*a^4*b^3*e + a^2*b^5*e + (a^6*b*f + 2*a^4*b^3*f + a^2*b^5*f)*x
- (a^6*b*e*e^(2*c) + 2*a^4*b^3*e*e^(2*c) + a^2*b^5*e*e^(2*c) + (a^6*b*f*e^
(2*c) + 2*a^4*b^3*f*e^(2*c) + a^2*b^5*f*e^(2*c))*x)*e^(2*d*x) - 2*(a^7*e*e
^c + 2*a^5*b^2*e*e^c + a^3*b^4*e*e^c + (a^7*f*e^c + 2*a^5*b^2*f*e^c + a^3*
b^4*f*e^c)*x)*e^(d*x)), x) - 32*integrate(1/32*(2*(d^2*e^2 - f^2)*a^2*b +
2*(2*d^2*e^2 - f^2)*b^3 + 2*(a^2*b*d^2*f^2 + 2*b^3*d^2*f^2)*x^2 + 4*(a^...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 21.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^3\,\sinh\left(c+d\,x\right)^2\,\left(e+f\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)^3*sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(cosh(c + d*x)^3*sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))), x
)
```

Reduce [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)^2\operatorname{sech}(dx+c)^3}{\sinh\left(dx+c\right)be+\sinh\left(dx+c\right)bfx+ae+afx}dx$$

input
```
int(csch(d*x+c)^2*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int((csch(c + d*x)**2*sech(c + d*x)**3)/(sinh(c + d*x)*b*e + sinh(c + d*x)
*b*f*x + a*e + a*f*x),x)
```

3.476 $\int \frac{(e+fx)^3 \coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 752

$$\int \frac{(e+fx)^3\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= -\frac{3f(e+fx)^2}{2ad^2}+\frac{6bf(e+fx)^2\operatorname{arctanh}\left(e^{c+dx}\right)}{a^2d^2}-\frac{3f(e+fx)^2\coth(c+dx)}{2ad^2}$$
$$+\frac{b(e+fx)^3\operatorname{csch}(c+dx)}{a^2d}-\frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{2ad}$$
$$-\frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d}-\frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d}$$
$$+\frac{3f^2(e+fx)\log\left(1-e^{2(c+dx)}\right)}{ad^3}+\frac{b^2(e+fx)^3\log\left(1-e^{2(c+dx)}\right)}{a^3d}$$
$$+\frac{6bf^2(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{a^2d^3}-\frac{6bf^2(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{a^2d^3}$$
$$-\frac{3b^2f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2}-\frac{3b^2f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^2}$$
$$+\frac{3f^3\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{2ad^4}+\frac{3b^2f(e+fx)^2\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{2a^3d^2}$$
$$-\frac{6bf^3\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{a^2d^4}+\frac{6bf^3\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{a^2d^4}$$
$$+\frac{6b^2f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^3}+\frac{6b^2f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^3}$$
$$-\frac{3b^2f^2(e+fx)\operatorname{PolyLog}\left(3,e^{2(c+dx)}\right)}{2a^3d^3}-\frac{6b^2f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^4}$$
$$-\frac{6b^2f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^4}+\frac{3b^2f^3\operatorname{PolyLog}\left(4,e^{2(c+dx)}\right)}{4a^3d^4}$$

output
```
-3/2*f*(f*x+e)^2/a/d^2+6*b*f*(f*x+e)^2*arctanh(exp(d*x+c))/a^2/d^2-3/2*f*(
f*x+e)^2*coth(d*x+c)/a/d^2+b*(f*x+e)^3*csch(d*x+c)/a^2/d-1/2*(f*x+e)^3*csc
h(d*x+c)^2/a/d-b^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d-
b^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d+3*f^2*(f*x+e)*l
n(1-exp(2*d*x+2*c))/a/d^3+b^2*(f*x+e)^3*ln(1-exp(2*d*x+2*c))/a^3/d+6*b*f^2
*(f*x+e)*polylog(2,-exp(d*x+c))/a^2/d^3-6*b*f^2*(f*x+e)*polylog(2,exp(d*x+
c))/a^2/d^3-3*b^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))
/a^3/d^2-3*b^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^
3/d^2+3/2*f^3*polylog(2,exp(2*d*x+2*c))/a/d^4+3/2*b^2*f*(f*x+e)^2*polylog(
2,exp(2*d*x+2*c))/a^3/d^2-6*b*f^3*polylog(3,-exp(d*x+c))/a^2/d^4+6*b*f^3*p
olylog(3,exp(d*x+c))/a^2/d^4+6*b^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))/a^3/d^3+6*b^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^
2+b^2)^(1/2)))/a^3/d^3-3/2*b^2*f^2*(f*x+e)*polylog(3,exp(2*d*x+2*c))/a^3/d
^3-6*b^2*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d^4-6*b^2*f^
3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d^4+3/4*b^2*f^3*polylog
(4,exp(2*d*x+2*c))/a^3/d^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3254 vs. 2(752) = 1504.

Time = 11.18 (sec) , antiderivative size = 3254, normalized size of antiderivative = 4.33

$$\int \frac{(e+fx)^3 \coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
(b*(e + f*x)^3*Csch[c])/(a^2*d) + ((-e^3 - 3*e^2*f*x - 3*e*f^2*x^2 - f^3*x
^3)*Csch[c/2 + (d*x)/2]^2)/(8*a*d) - (8*b^2*d^4*e^3*E^(2*c)*x + 24*a^2*d^2
*e*E^(2*c)*f^2*x + 12*b^2*d^4*e^2*E^(2*c)*f*x^2 + 12*a^2*d^2*E^(2*c)*f^3*x
^2 + 8*b^2*d^4*e*E^(2*c)*f^2*x^3 + 2*b^2*d^4*E^(2*c)*f^3*x^4 + 24*a*b*d^2*
e^2*f*ArcTanh[E^(c + d*x)] - 24*a*b*d^2*e^2*E^(2*c)*f*ArcTanh[E^(c + d*x)]
 - 24*a*b*d^2*e*f^2*x*Log[1 - E^(c + d*x)] + 24*a*b*d^2*e*E^(2*c)*f^2*x*Lo
g[1 - E^(c + d*x)] - 12*a*b*d^2*f^3*x^2*Log[1 - E^(c + d*x)] + 12*a*b*d^2*
E^(2*c)*f^3*x^2*Log[1 - E^(c + d*x)] + 24*a*b*d^2*e*f^2*x*Log[1 + E^(c + d
*x)] - 24*a*b*d^2*e*E^(2*c)*f^2*x*Log[1 + E^(c + d*x)] + 12*a*b*d^2*f^3*x^
2*Log[1 + E^(c + d*x)] - 12*a*b*d^2*E^(2*c)*f^3*x^2*Log[1 + E^(c + d*x)] +
 4*b^2*d^3*e^3*Log[1 - E^(2*(c + d*x))] - 4*b^2*d^3*e^3*E^(2*c)*Log[1 - E^
(2*(c + d*x))] + 12*a^2*d*e*f^2*Log[1 - E^(2*(c + d*x))] - 12*a^2*d*e*E^(2
*c)*f^2*Log[1 - E^(2*(c + d*x))] + 12*b^2*d^3*e^2*f*x*Log[1 - E^(2*(c + d*
x))] - 12*b^2*d^3*e^2*E^(2*c)*f*x*Log[1 - E^(2*(c + d*x))] + 12*a^2*d*f^3*
x*Log[1 - E^(2*(c + d*x))] - 12*a^2*d*E^(2*c)*f^3*x*Log[1 - E^(2*(c + d*x)
)] + 12*b^2*d^3*e*f^2*x^2*Log[1 - E^(2*(c + d*x))] - 12*b^2*d^3*e*E^(2*c)*
f^2*x^2*Log[1 - E^(2*(c + d*x))] + 4*b^2*d^3*f^3*x^3*Log[1 - E^(2*(c + d*x
))] - 4*b^2*d^3*E^(2*c)*f^3*x^3*Log[1 - E^(2*(c + d*x))] - 24*a*b*d*(-1 +
E^(2*c))*f^2*(e + f*x)*PolyLog[2, -E^(c + d*x)] + 24*a*b*d*(-1 + E^(2*c))*
f^2*(e + f*x)*PolyLog[2, E^(c + d*x)] + 6*b^2*d^2*e^2*f*PolyLog[2, E^(2...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6121}$$

$$\frac{\int (e+fx)^3\coth(c+dx)\text{csch}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 5975}$$

$$\frac{\frac{3f\int (e+fx)^2\text{csch}^2(c+dx)dx}{2d}-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\int \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{b\int \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}+\frac{3f\int -(e+fx)^2\csc(ic+idx)^2dx}{2d}}{a}$$

$\downarrow$ 25

$$-\frac{b\int \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}-\frac{3f\int (e+fx)^2\csc(ic+idx)^2dx}{2d}}{a}$$

$\downarrow$ 4672

$$-\frac{b\int \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2if\int -i(e+fx)\coth(c+dx)dx}{d}\right)}{2d}}{a}$$

$\downarrow$ 26

$$\frac{-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2f\int (e+fx)\coth(c+dx)dx}{d}\right)}{2d}-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\int \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$\downarrow$ 3042

$$-\frac{b\int \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2f\int -i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{2d}}{a}$$

$\downarrow$ 26

$$-\frac{b\int \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}\right)}{2d}}{a}$$

$\downarrow$ 4201

$$-\frac{b\int \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\int \frac{e^{2c+2dx-i\pi}(e+fx)}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}$$

$\downarrow$ 2620

$$-\frac{b\int \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int \log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}$$

$\downarrow$ 2715

$$-\frac{b\int \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\log\left(1+e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}\right)-i}{d}}{2d}}{a}$$

$\downarrow$ 2838

$$-\frac{b\int \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}$$

$\downarrow$ 6121

$$-\frac{b\left(\frac{\int (e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}$$

$$\downarrow \text{5975}$$

$$-\frac{b\left(\frac{\frac{3f\int (e+fx)^2\text{csch}(c+dx)dx}{d}-\frac{(e+fx)^3\text{csch}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\text{ PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}$$

$$\downarrow \text{3042}$$

$$\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\text{ PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3f\int i(e+fx)^2\csc(ic+idx)dx}{d}}{a}\right)}{a}$$

$$\downarrow \text{26}$$

$$\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\text{ PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3if\int (e+fx)^2\csc(ic+idx)dx}{d}}{a}\right)}{a}$$

$$\downarrow \text{4670}$$

$$\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\text{ PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-$$

$$\frac{b\left(-\frac{b\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3if\left(\frac{2if\int (e+fx)\log\left(1-e^{c+dx}\right)dx}{d}-\frac{2if\int (e+fx)\log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)^2\text{arctanh}(\ldots}{d}\right.}{d}}{a}\right.}{a}$$

$\downarrow$ 3011

$$\frac{-\frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int\operatorname{PolyLog}\left(\right.}{d}\right.}{}\right.}{d}}{a}\right.}{a}$$

$\downarrow$ 2720

$$\frac{-\frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\right.}{}\right.}{}}{a}\right.}{a}$$

$\downarrow$ 6103

$$\frac{-\frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-$$

$$\frac{b\left(-\frac{b\left(\frac{\int(e+fx)^3\coth(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+f\cdots}{}\right)}{d}\right.}{}}{}\right.}{a}$$

$\downarrow$ 3042

$$\frac{-\frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-$$

$$\frac{b\left(\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c\cdots}}{d^2}\right.}{d}\right.}{d}}{a}\right.}{a}$$

$\downarrow$ 26

$$\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-\frac{b\left(\frac{-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c}}{d^2}\right.}{d}\right.}{d}}{a}\right.}{a}$$

$$\Big\downarrow \text{4201}$$

$$\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-\frac{b\left(\frac{-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c}}{d^2}\right.}{d}\right.}{d}}{a}\right.}{}$$

$$\Big\downarrow \text{2620}$$

$$\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-$$

$$b\left(\frac{-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c}}{d^2}\right.}{d}\right.}{d}}{a}\right.$$

$\downarrow$ 3011

$$\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-$$

$$b\left(\frac{-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c}}{d^2}\right.}{d}\right.}{d}}{a}\right.$$

$\downarrow$ 6095

$$\frac{-\frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-$$

$$\frac{b\left(\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c}}{d^2}\right.}{d}\right.}{d}}{a}\right.}{}$$

$\downarrow$ 2620

$$\frac{-\frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-$$

$$\frac{b\left(\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c}}{d^2}\right.}{d}\right.}{d}}{a}\right.}{}$$

$\downarrow$ 3011

$$\frac{-\frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-$$

$$b\left(\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}\cdots\right.}{d}\cdots\right.}{d}}{a}\cdots\right.$$

↓ 7143

$$\frac{-\frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-$$

$$b\left(\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\left(\frac{2i(e+fx)^2\operatorname{arctanh}\left(e^{c+dx}\right)}{d}-\frac{2if\left(\frac{f\,\text{PolyLog}\left(3,-e^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\,\text{PolyLog}\left(3,e^{c+dx}\right)}{d^2}\right.}{}\right.}{d}}{a}\right.$$

$\downarrow$ 7163

$$-\frac{\frac{\operatorname{csch}^2(c+dx)(e+fx)^3}{2d} - \frac{3f\left(\frac{\coth(c+dx)(e+fx)^2}{d} + \frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d} + \frac{f\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}\right) - \frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a} -$$

$$b\left(\frac{\frac{3if\left(\frac{2i\operatorname{arctanh}\left(e^{c+dx}\right)(e+fx)^2}{d} - \frac{2if\left(\frac{f\,\mathrm{PolyLog}\left(3,-e^{c+dx}\right)}{d^2} - \frac{(e+fx)\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d} + \frac{2if\left(\frac{f\,\mathrm{PolyLog}\left(3,e^{c+dx}\right)}{d^2} - \frac{(e+fx)\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)}{d}\right.}{d}\right.}{d}}{a}\right.$$

input
```
Int[((e + f*x)^3*Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^3 \coth(dx+c)\operatorname{csch}(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11595 vs. 2(703) = 1406.

Time = 0.30 (sec) , antiderivative size = 11595, normalized size of antiderivative = 15.42

$$\int \frac{(e+fx)^3 \coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to display
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*coth(d*x+c)*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^3 \coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3 \coth(dx+c)\operatorname{csch}(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-e^3*(2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2
*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + b^2*log(-2*a*e^(-d*x
- c) + b*e^(-2*d*x - 2*c) - b)/(a^3*d) - b^2*log(e^(-d*x - c) + 1)/(a^3*d)
 - b^2*log(e^(-d*x - c) - 1)/(a^3*d)) + (3*a*f^3*x^2 + 6*a*e*f^2*x + 3*a*e
^2*f + 2*(b*d*f^3*x^3*e^(3*c) + 3*b*d*e*f^2*x^2*e^(3*c) + 3*b*d*e^2*f*x*e^
(3*c))*e^(3*d*x) - (2*a*d*f^3*x^3*e^(2*c) + 3*a*e^2*f*e^(2*c) + 3*(2*d*e*f
^2 + f^3)*a*x^2*e^(2*c) + 6*(d*e^2*f + e*f^2)*a*x*e^(2*c))*e^(2*d*x) - 2*(
b*d*f^3*x^3*e^c + 3*b*d*e*f^2*x^2*e^c + 3*b*d*e^2*f*x*e^c)*e^(d*x))/(a^2*d
^2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2) + (d^3*x^3*log(e
^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x
 + c)) + 6*polylog(4, -e^(d*x + c)))*b^2*f^3/(a^3*d^4) + (d^3*x^3*log(-e^(
d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c
)) + 6*polylog(4, e^(d*x + c)))*b^2*f^3/(a^3*d^4) - 3*(b*d*e^2*f + a*e*f^2
)*x/(a^2*d^2) + 3*(b*d*e^2*f - a*e*f^2)*x/(a^2*d^2) + 3*(b*d*e^2*f + a*e*f
^2)*log(e^(d*x + c) + 1)/(a^2*d^3) - 3*(b*d*e^2*f - a*e*f^2)*log(e^(d*x +
c) - 1)/(a^2*d^3) + 3*(b^2*d*e*f^2 + a*b*f^3)*(d^2*x^2*log(e^(d*x + c) + 1
) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))/(a^3*d^4) + 3*
(b^2*d*e*f^2 - a*b*f^3)*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*
x + c)) - 2*polylog(3, e^(d*x + c)))/(a^3*d^4) + 3*(b^2*d^2*e^2*f + 2*a*b*
d*e*f^2 + a^2*f^3)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\coth(c+d\,x)\,\left(e+f\,x\right)^3}{\sinh\left(c+d\,x\right)^2\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((coth(c + d*x)*(e + f*x)^3)/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((coth(c + d*x)*(e + f*x)^3)/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x
)
```

Reduce [F]

$$\int \frac{(e+fx)^3\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{too large to display}$$

input
```
int((f*x+e)^3*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(2304*e**(7*c + 4*d*x)*int((e**(3*d*x)*x**3)/(e**(8*c + 8*d*x)*b + 2*e**(7
*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c +
4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a
+ b),x)*a**5*d**4*f**3 + 960*e**(7*c + 4*d*x)*int((e**(3*d*x)*x**3)/(e**(8
*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c +
5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x
)*b - 2*e**(c + d*x)*a + b),x)*a**3*b**2*d**4*f**3 + 6912*e**(7*c + 4*d*x)
*int((e**(3*d*x)*x**2)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(
6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c +
 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**5*d**4*e*f*
*2 + 2880*e**(7*c + 4*d*x)*int((e**(3*d*x)*x**2)/(e**(8*c + 8*d*x)*b + 2*e
**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*
c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x
)*a + b),x)*a**3*b**2*d**4*e*f**2 - 768*e**(7*c + 4*d*x)*int((e**(3*d*x)*x
**2)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6
*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(
2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**3*b**2*d**3*f**3 + 6912*e**(7
*c + 4*d*x)*int((e**(3*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a
- 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e
**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**...
```

3.477 $\int \frac{(e+fx)^2\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 502

$$\begin{aligned}
&\int \frac{(e+fx)^2\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= \frac{4bf(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{a^2d^2} - \frac{f(e+fx)\coth(c+dx)}{ad^2} + \frac{b(e+fx)^2\operatorname{csch}(c+dx)}{a^2d} \\
&\quad - \frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2ad} - \frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d} \\
&\quad - \frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d} + \frac{b^2(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{a^3d} \\
&\quad + \frac{f^2\log(\sinh(c+dx))}{ad^3} + \frac{2bf^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{a^2d^3} - \frac{2bf^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{a^2d^3} \\
&\quad - \frac{2b^2f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2} - \frac{2b^2f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^2} \\
&\quad + \frac{b^2f(e+fx)\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{a^3d^2} + \frac{2b^2f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^3} \\
&\quad + \frac{2b^2f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^3} - \frac{b^2f^2\operatorname{PolyLog}\left(3,e^{2(c+dx)}\right)}{2a^3d^3}
\end{aligned}$$

output
```
4*b*f*(f*x+e)*arctanh(exp(d*x+c))/a^2/d^2-f*(f*x+e)*coth(d*x+c)/a/d^2+b*(f
*x+e)^2*csch(d*x+c)/a^2/d-1/2*(f*x+e)^2*csch(d*x+c)^2/a/d-b^2*(f*x+e)^2*ln
(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d-b^2*(f*x+e)^2*ln(1+b*exp(d*x+c)
/(a+(a^2+b^2)^(1/2)))/a^3/d+b^2*(f*x+e)^2*ln(1-exp(2*d*x+2*c))/a^3/d+f^2*l
n(sinh(d*x+c))/a/d^3+2*b*f^2*polylog(2,-exp(d*x+c))/a^2/d^3-2*b*f^2*polylo
g(2,exp(d*x+c))/a^2/d^3-2*b^2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^
2)^(1/2)))/a^3/d^2-2*b^2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1
/2)))/a^3/d^2+b^2*f*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a^3/d^2+2*b^2*f^2*po
lylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d^3+2*b^2*f^2*polylog(3,-b*
exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d^3-1/2*b^2*f^2*polylog(3,exp(2*d*x+2*
c))/a^3/d^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1816 vs. $2(502) = 1004$.

Time = 10.45 (sec) , antiderivative size = 1816, normalized size of antiderivative = 3.62

$$\int \frac{(e+fx)^2 \coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
(b*(e + f*x)^2*Csch[c])/(a^2*d) + ((-e^2 - 2*e*f*x - f^2*x^2)*Csch[c/2 + (
d*x)/2]^2)/(8*a*d) - (12*d*E^(2*c)*(b^2*d^2*e^2 + a^2*f^2)*x - 12*d*(-1 +
E^(2*c))*(b^2*d^2*e^2 + a^2*f^2)*x + 12*b^2*d^3*e*f*x^2 + 4*b^2*d^3*f^2*x^
3 - 24*a*b*d*e*(-1 + E^(2*c))*f*ArcTanh[E^(c + d*x)] + 6*b^2*d^2*e^2*(-1 +
 E^(2*c))*(2*d*x - Log[1 - E^(2*(c + d*x))]) + 6*a^2*(-1 + E^(2*c))*f^2*(2
*d*x - Log[1 - E^(2*(c + d*x))]) + 12*a*b*(-1 + E^(2*c))*f^2*(d*x*(Log[1 -
 E^(c + d*x)] - Log[1 + E^(c + d*x)]) - PolyLog[2, -E^(c + d*x)] + PolyLog
[2, E^(c + d*x)]) + 6*b^2*d*e*(-1 + E^(2*c))*f*(2*d*x*(d*x - Log[1 - E^(2*
(c + d*x))]) - PolyLog[2, E^(2*(c + d*x))]) + b^2*(-1 + E^(2*c))*f^2*(2*d^
2*x^2*(2*d*x - 3*Log[1 - E^(2*(c + d*x))]) - 6*d*x*PolyLog[2, E^(2*(c + d*
x))] + 3*PolyLog[3, E^(2*(c + d*x))]))/(6*a^3*d^3*(-1 + E^(2*c))) + (b^2*(
6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 +
b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^
2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/S
qrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*Ar
cTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*
Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^
2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c
 + d*x)))])/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x
)))])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 4.14 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.13, number of steps used = 29, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.824$, Rules used = {6121, 5975, 3042, 25, 4672, 26, 3042, 26, 3956, 6121, 5975, 3042, 26, 4670, 2715, 2838, 6103, 3042, 26, 4201, 2620, 3011, 2720, 6095, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6121}$$

$$\frac{\int (e+fx)^2\coth(c+dx)\text{csch}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5975}$$

$$\frac{\frac{f\int(e+fx)\text{csch}^2(c+dx)dx}{d}-\frac{(e+fx)^2\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\int\frac{(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int\frac{(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\text{csch}^2(c+dx)}{2d}+\frac{f\int-\left((e+fx)\csc(ic+idx)^2\right)dx}{d}}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\int\frac{(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\text{csch}^2(c+dx)}{2d}-\frac{f\int(e+fx)\csc(ic+idx)^2dx}{d}}{a}$$

$$\Big\downarrow \text{4672}$$

$$-\frac{b\int\frac{(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\text{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{if\int-i\coth(c+dx)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\int\coth(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\int\frac{(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int\frac{(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\text{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\int-i\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int\frac{(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\text{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}+\frac{if\int\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{b\int\frac{(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\text{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \text{6121}$$

$$-\frac{b\left(\frac{\int (e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{-\frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}$$

$\Big\downarrow$ 5975

$$-\frac{b\left(\frac{\frac{2f\int (e+fx)\operatorname{csch}(c+dx)dx}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{-\frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}+\frac{2f\int i(e+fx)\csc(ic+idx)dx}{d}}{a}\right)}{a}$$

$\Big\downarrow$ 26

$$\frac{-\frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}+\frac{2if\int (e+fx)\csc(ic+idx)dx}{d}}{a}\right)}{a}$$

$\Big\downarrow$ 4670

$$\frac{-\frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}+\frac{2if\left(\frac{if\int \log\left(1-e^{c+dx}\right)dx}{d}-\frac{if\int \log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}\right)}{d}}{a}\right)}{a}$$

$\Big\downarrow$ 2715

$$\frac{-\frac{(e+fx)^2\text{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{if\int e^{-c-dx}\log\left(1-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log\left(1+e^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2i(e+fx)\text{ar}}{}\right.}{d}}{a}\right.}{a}$$

$\downarrow$ 2838

$$\frac{-\frac{(e+fx)^2\text{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}\right)}{a}$$

$\downarrow$ 6103

$$\frac{-\frac{(e+fx)^2\text{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\left(\frac{\int(e+fx)^2\coth(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}\right.}{d}}{a}\right.}{a}$$

$\downarrow$ 3042

$$\frac{-\frac{(e+fx)^2\text{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-\frac{b\left(\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\int\right.}{a}\right.}{a}$$

$\downarrow$ 26

$$\frac{-\frac{(e+fx)^2\text{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-$$

$$b\left(\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{\frac{b\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i}{}}{}\right.}{a}\right.$$

$$\overline{\qquad a \qquad}$$

$\downarrow$ 4201

$$\frac{-\frac{(e+fx)^2\text{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-$$

$$b\left(\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{\frac{b\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i(}{}}{}\right.}{}\right.$$

$$\overline{\qquad a \qquad}$$

$\downarrow$ 2620

$$\frac{-\frac{(e+fx)^2\text{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-$$

$$b\left(\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{\frac{b\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i}{}}{}\right.}{}\right.$$

$$\overline{\qquad a \qquad}$$

$\downarrow$ 3011

$$\frac{-\frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-$$

$$\frac{b\left(\frac{-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{\frac{b\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\ldots i}{}\right.}{}\right.}{a}$$

$\downarrow$ 2720

$$\frac{-\frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-$$

$$\frac{b\left(\frac{-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{\frac{b\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\ldots i}{}\right.}{}\right.}{}$$

$\downarrow$ 6095

$$\frac{-\frac{(e+fx)^2\text{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-$$

$$b\left(\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\left(\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\ldots\right.}{}\right.}{}\right.$$

$\downarrow$ 2620

$$\frac{-\frac{(e+fx)^2\text{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-$$

$$b\left(\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\left(-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+\ldots}}\right.}{bd}\right.}{}\right.}{}\right.$$

$\downarrow$ 3011

$$\frac{-\frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-$$

$$b\left(\frac{-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\left(-\frac{2f\left(\frac{f\int\operatorname{PolyLog}\left(2,-\frac{be}{a-\sqrt{}}\right.}{d}\right.}{}\right.}{}\right.}{}\right.$$

$\Big\downarrow$ 2720

$$\frac{-\frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-$$

$$b\left(\frac{-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(\right.}{}\right.}{}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \; 7143$$

$$\frac{-\frac{(e+fx)^2\text{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-$$

$$b\left(\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c\cdots}}{a-\sqrt{\cdots}}\right)}{d^2}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.$$

input
```
Int[((e + f*x)^2*Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-1/2*((e + f*x)^2*Csch[c + d*x]^2)/d - (f*(((e + f*x)*Coth[c + d*x])/d -
(f*Log[(-I)*Sinh[c + d*x]])/d^2))/d)/a - (b*((-(((e + f*x)^2*Csch[c + d*x]
)/d) + ((2*I)*f*(((2*I)*(e + f*x)*ArcTanh[E^(c + d*x)])/d + (I*f*PolyLog[2
, -E^(c + d*x)])/d^2 - (I*f*PolyLog[2, E^(c + d*x)])/d^2))/d)/a - (b*(-((b
*(-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[
a^2 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt
[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
)])/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
))])/d^2))/(b*d)))/a) - (I*(((-1/3*I)*(e + f*x)^3)/f + (2*I)*(((e + f*x)^2
*Log[1 + E^(2*c - I*Pi + 2*d*x)])/(2*d) - (f*(-1/2*((e + f*x)*PolyLog[2, -
E^(2*c - I*Pi + 2*d*x)])/d + (f*PolyLog[3, -E^(2*c - I*Pi + 2*d*x)])/(4*d^
2)))/d)))/a))/a))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5975
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6103
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Coth[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x
]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6121
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Csch[c + d*x]^(p - 1)*(Coth[c + d*x]^n/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(fx+e)^2 \coth(dx+c)\operatorname{csch}(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6479 vs. $2(472) = 944$.

Time = 0.18 (sec) , antiderivative size = 6479, normalized size of antiderivative = 12.91

$$\int \frac{(e+fx)^2 \coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*coth(d*x+c)*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^2\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\coth(dx+c)\,\text{csch}\,(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-e^2*(2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2
*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + b^2*log(-2*a*e^(-d*x
- c) + b*e^(-2*d*x - 2*c) - b)/(a^3*d) - b^2*log(e^(-d*x - c) + 1)/(a^3*d)
 - b^2*log(e^(-d*x - c) - 1)/(a^3*d)) + 2*(a*f^2*x + a*e*f + (b*d*f^2*x^2*
e^(3*c) + 2*b*d*e*f*x*e^(3*c))*e^(3*d*x) - (a*d*f^2*x^2*e^(2*c) + a*e*f*e^
(2*c) + (2*d*e*f + f^2)*a*x*e^(2*c))*e^(2*d*x) - (b*d*f^2*x^2*e^c + 2*b*d*
e*f*x*e^c)*e^(d*x))/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) +
 a^2*d^2) + (d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*
polylog(3, -e^(d*x + c)))*b^2*f^2/(a^3*d^3) + (d^2*x^2*log(-e^(d*x + c) +
1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*b^2*f^2/(a^3*d^
3) - (2*b*d*e*f + a*f^2)*x/(a^2*d^2) + (2*b*d*e*f - a*f^2)*x/(a^2*d^2) + (
2*b*d*e*f + a*f^2)*log(e^(d*x + c) + 1)/(a^2*d^3) - (2*b*d*e*f - a*f^2)*lo
g(e^(d*x + c) - 1)/(a^2*d^3) + 2*(b^2*d*e*f + a*b*f^2)*(d*x*log(e^(d*x + c
) + 1) + dilog(-e^(d*x + c)))/(a^3*d^3) + 2*(b^2*d*e*f - a*b*f^2)*(d*x*log
(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^3*d^3) - 1/3*(b^2*d^3*f^2*x^3
+ 3*(b^2*d*e*f + a*b*f^2)*d^2*x^2)/(a^3*d^3) - 1/3*(b^2*d^3*f^2*x^3 + 3*(b
^2*d*e*f - a*b*f^2)*d^2*x^2)/(a^3*d^3) + integrate(-2*(b^3*f^2*x^2 + 2*b^3
*e*f*x - (a*b^2*f^2*x^2*e^c + 2*a*b^2*e*f*x*e^c)*e^(d*x))/(a^3*b*e^(2*d*x
+ 2*c) + 2*a^4*e^(d*x + c) - a^3*b), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+d\,x)\,\left(e+f\,x\right)^2}{\sinh\left(c+d\,x\right)^2\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((coth(c + d*x)*(e + f*x)^2)/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((coth(c + d*x)*(e + f*x)^2)/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x
)
```

Reduce [F]

$$\int \frac{(e+fx)^2\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^2*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output

```
(576*e**(7*c + 4*d*x)*int((e**(3*d*x)*x**2)/(e**(8*c + 8*d*x)*b + 2*e**(7*
c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4
*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a +
 b),x)*a**5*d**3*f**2 + 240*e**(7*c + 4*d*x)*int((e**(3*d*x)*x**2)/(e**(8*
c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5
*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)
*b - 2*e**(c + d*x)*a + b),x)*a**3*b**2*d**3*f**2 + 1152*e**(7*c + 4*d*x)*
int((e**(3*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c
+ 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d
*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**5*d**3*e*f + 48
0*e**(7*c + 4*d*x)*int((e**(3*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*
d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*
b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)
*a**3*b**2*d**3*e*f - 128*e**(7*c + 4*d*x)*int((e**(3*d*x)*x)/(e**(8*c + 8
*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)
*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b -
2*e**(c + d*x)*a + b),x)*a**3*b**2*d**2*f**2 - 384*e**(6*c + 4*d*x)*int((e
**(2*d*x)*x**2)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6
*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)
*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**4*b*d**3*f**2 -...
```

3.478 $\int \frac{(e+fx)\coth(c+dx)\mathrm{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 298

$$\begin{aligned}\int \frac{(e+fx)\coth(c+dx)\mathrm{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{bf\,\mathrm{arctanh}(\cosh(c+dx))}{a^2d^2} - \frac{f\coth(c+dx)}{2ad^2} \\ &+ \frac{b(e+fx)\mathrm{csch}(c+dx)}{a^2d} - \frac{(e+fx)\mathrm{csch}^2(c+dx)}{2ad} \\ &- \frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d} - \frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d} \\ &+ \frac{b^2(e+fx)\log\left(1-e^{2(c+dx)}\right)}{a^3d} - \frac{b^2f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2} \\ &- \frac{b^2f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^2} + \frac{b^2f\,\mathrm{PolyLog}\left(2,e^{2(c+dx)}\right)}{2a^3d^2}\end{aligned}$$

output
```
b*f*arctanh(cosh(d*x+c))/a^2/d^2-1/2*f*coth(d*x+c)/a/d^2+b*(f*x+e)*csch(d*
x+c)/a^2/d-1/2*(f*x+e)*csch(d*x+c)^2/a/d-b^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))/a^3/d-b^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))
/a^3/d+b^2*(f*x+e)*ln(1-exp(2*d*x+2*c))/a^3/d-b^2*f*polylog(2,-b*exp(d*x+c
)/(a-(a^2+b^2)^(1/2)))/a^3/d^2-b^2*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^
(1/2)))/a^3/d^2+1/2*b^2*f*polylog(2,exp(2*d*x+2*c))/a^3/d^2
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 713 vs. $2(298) = 596$.

Time = 8.22 (sec) , antiderivative size = 713, normalized size of antiderivative = 2.39

$$\int \frac{(e+fx)\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{\left(2bde\cosh\left(\frac{1}{2}(c+dx)\right)-af\cosh\left(\frac{1}{2}(c+dx)\right)-2bcf\cosh\left(\frac{1}{2}(c+dx)\right)+2bf(c+dx)\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{4a^2d^2}$$
$$+\frac{(-de+cf-f(c+dx))\text{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8ad^2}$$
$$+\frac{b\left(\frac{b(de+dfx)^2}{2f}+(bde-af+bdfx)\log\left(1-e^{-c-dx}\right)+(bde+af+bdfx)\log\left(1+e^{-c-dx}\right)-bf\,\text{PolyLog}\right.}{a^3d^2}$$
$$-\frac{b^2\left(-2de(c+dx)+2cf(c+dx)-f(c+dx)^2+\frac{4a\sqrt{a^2+b^2}de\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-(a^2+b^2)^2}}-\frac{4a\sqrt{-(a^2+b^2)^2}de\text{arctanh}\left(\frac{a+}{\sqrt{}}\right.}{(-a^2-b^2)^{3/2}}\right.}{}$$
$$+\frac{(de-cf+f(c+dx))\text{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8ad^2}$$
$$+\frac{\text{sech}\left(\frac{1}{2}(c+dx)\right)\left(-2bde\sinh\left(\frac{1}{2}(c+dx)\right)-af\sinh\left(\frac{1}{2}(c+dx)\right)+2bcf\sinh\left(\frac{1}{2}(c+dx)\right)-2bf(c+dx\right.}{4a^2d^2}$$

input
```
Integrate[((e + f*x)*Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),
x]
```

output
```
((2*b*d*e*Cosh[(c + d*x)/2] - a*f*Cosh[(c + d*x)/2] - 2*b*c*f*Cosh[(c + d*
x)/2] + 2*b*f*(c + d*x)*Cosh[(c + d*x)/2])*Csch[(c + d*x)/2])/(4*a^2*d^2)
+ ((-(d*e) + c*f - f*(c + d*x))*Csch[(c + d*x)/2]^2)/(8*a*d^2) + (b*((b*(d
*e + d*f*x)^2)/(2*f) + (b*d*e - a*f + b*d*f*x)*Log[1 - E^(-c - d*x)] + (b*
d*e + a*f + b*d*f*x)*Log[1 + E^(-c - d*x)] - b*f*PolyLog[2, -E^(-c - d*x)]
 - b*f*PolyLog[2, E^(-c - d*x)]))/(a^3*d^2) - (b^2*(-2*d*e*(c + d*x) + 2*c
*f*(c + d*x) - f*(c + d*x)^2 + (4*a*Sqrt[a^2 + b^2]*d*e*ArcTan[(a + b*E^(c
 + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b^2)^2] - (4*a*Sqrt[-(a^2 + b^2)^
2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/(-a^2 - b^2)^(3/2) +
2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*f*(c + d*
x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*f*Log[b - 2*a*E^(c
 + d*x) - b*E^(2*(c + d*x))] + 2*d*e*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c
 + d*x)))] + 2*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f*
PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(2*a^3*d^2) + ((d*e
 - c*f + f*(c + d*x))*Sech[(c + d*x)/2]^2)/(8*a*d^2) + (Sech[(c + d*x)/2]*
(-2*b*d*e*Sinh[(c + d*x)/2] - a*f*Sinh[(c + d*x)/2] + 2*b*c*f*Sinh[(c + d*
x)/2] - 2*b*f*(c + d*x)*Sinh[(c + d*x)/2]))/(4*a^2*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.33 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.17, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {6121, 5975, 3042, 25, 4254, 24, 6121, 5975, 3042, 26, 4257, 6103, 3042, 26, 4201, 2620, 2715, 2838, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6121}$$

$$\frac{\int (e+fx)\coth(c+dx)\text{csch}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 5975}$$

$$\frac{\frac{f\int \text{csch}^2(c+dx)dx}{2d}-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\int \frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{b\int \frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}+\frac{f\int -\csc(ic+idx)^2dx}{2d}}{a}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{b\int \frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}-\frac{f\int \csc(ic+idx)^2dx}{2d}}{a}$$

$$\Big\downarrow \text{ 4254}$$

$$-\frac{b\int \frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}-\frac{if\int 1d(-i\coth(c+dx))}{2d^2}}{a}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\int \frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 6121}$$

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\left(\frac{\int (e+fx)\coth(c+dx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{ 5975}$$

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\left(\frac{\frac{f\int \text{csch}(c+dx)dx}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)\text{csch}(c+dx)}{d}+\frac{f\int i\csc(ic+idx)dx}{d}}{a}\right)}{a}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)\text{csch}(c+dx)}{d}+\frac{if\int\csc(ic+idx)dx}{d}}{a}\right)}{a}$$

$\Big\downarrow$ 4257

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\left(\frac{-\frac{f\,\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$\Big\downarrow$ 6103

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\left(\frac{-\frac{f\,\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)\coth(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}\right)}{a}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\left(\frac{-\frac{f\,\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int-i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{a}\right)}{a}\right)}{a}$$

$\Big\downarrow$ 26

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\left(\frac{-\frac{f\,\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\int(e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{a}\right)}{a}\right)}{a}$$

$\Big\downarrow$ 4201

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\left(\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\int\frac{e^{2c+2dx-i\pi}(e+fx)}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^2}{2f}\right)}{a}\right)}{a}\right)}{a}$$

$\Big\downarrow$ 2620

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\left(\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int\log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\cdots\right.\right.}{a}\cdots\right.}{a}\cdots\right.}{a}$$

$\Big\downarrow$ 2715

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\left(\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\log\left(1+e^{2\cdots}\right.}{4d\cdots}\cdots\right.\right.}{a}\cdots\right.}{a}\cdots\right.}{a}$$

$\Big\downarrow$ 2838

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\left(\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\cdots\right.}{2d}\cdots\right.\right.}{a}\cdots\right.}{a}\cdots\right.}{a}$$

$\downarrow$ 6095

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\left(\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{a}-\frac{i\left(2i\left(\frac{f\,\text{PolyLog}(2,-}{4d}\right.\right.}{}\right)}{a}\right)}{a}$$

$\downarrow$ 2620

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\left(\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right.}{a}\right.}{}\right)}{a}$$

$\downarrow$ 2715

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\left(\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\right.}{a}\right.}{}\right)}{}$$

$\downarrow$ 2838

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\left(\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(\frac{f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\cdots\right.}{a}\cdots\right.}{\cdots}\right.}{a}$$

input
```
Int[((e + f*x)*Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-1/2*(f*Coth[c + d*x])/d^2 - ((e + f*x)*Csch[c + d*x]^2)/(2*d))/a - (b*((
-((f*ArcTanh[Cosh[c + d*x]])/d^2) - ((e + f*x)*Csch[c + d*x])/d)/a - (b*(-
((b*(-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt
[a^2 + b^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(
b*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2))
)/a) - (I*(((-1/2*I)*(e + f*x)^2)/f + (2*I)*(((e + f*x)*Log[1 + E^(2*c - I
*Pi + 2*d*x)])/(2*d) + (f*PolyLog[2, -E^(2*c - I*Pi + 2*d*x)])/(4*d^2))))/
a))/a))/a
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5975
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6103
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Coth[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x
]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6121
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Csch[c + d*x]^(p - 1)*(Coth[c + d*x]^n/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(280) = 560$.

Time = 1.33 (sec) , antiderivative size = 649, normalized size of antiderivative = 2.18

method	result
risch	$-\frac{-2bdfx\,\mathrm{e}^{3dx+3c}+2adfx\,\mathrm{e}^{2dx+2c}-2bde\,\mathrm{e}^{3dx+3c}+2ade\,\mathrm{e}^{2dx+2c}+2bdfx\,\mathrm{e}^{dx+c}+af\,\mathrm{e}^{2dx+2c}+2bde\,\mathrm{e}^{dx+c}-af}{d^2a^2\left(\mathrm{e}^{2dx+2c}-1\right)^2}-\frac{b^2f\,\mathrm{dilog}\left(\mathrm{e}^{dx+c}\right)}{d^2a^3}$

input
```
int((f*x+e)*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
-(-2*b*d*f*x*exp(3*d*x+3*c)+2*a*d*f*x*exp(2*d*x+2*c)-2*b*d*e*exp(3*d*x+3*c
)+2*a*d*e*exp(2*d*x+2*c)+2*b*d*f*x*exp(d*x+c)+a*f*exp(2*d*x+2*c)+2*b*d*e*e
xp(d*x+c)-a*f)/d^2/a^2/(exp(2*d*x+2*c)-1)^2-1/d^2*b^2/a^3*f*dilog(exp(d*x+
c))+1/d^2*b^2/a^3*f*dilog(exp(d*x+c)+1)+1/d*b^2/a^3*e*ln(exp(d*x+c)-1)-1/d
*b^2/a^3*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/d*b^2/a^3*e*ln(exp(d*x+
c)+1)-1/d^2*b^2/a^3*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(
1/2)))*c-1/d^2*b^2/a^3*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^
(1/2)))*c-1/d^2*b/a^2*f*ln(exp(d*x+c)-1)+1/d^2*b/a^2*f*ln(exp(d*x+c)+1)-1/
d^2*b^2/a^3*c*f*ln(exp(d*x+c)-1)+1/d^2*b^2/a^3*c*f*ln(b*exp(2*d*x+2*c)+2*a
*exp(d*x+c)-b)-1/d^2*b^2/a^3*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a
+(a^2+b^2)^(1/2)))-1/d^2*b^2/a^3*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/
(a+(a^2+b^2)^(1/2)))+1/d*b^2/a^3*f*ln(exp(d*x+c)+1)*x-1/d*b^2/a^3*f*ln((-b
*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d*b^2/a^3*f*ln((b
*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2899 vs. $2(277) = 554$.

Time = 0.19 (sec) , antiderivative size = 2899, normalized size of antiderivative = 9.73

$$\int \frac{(e+fx)\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
(2*(a*b*d*f*x + a*b*d*e)*cosh(d*x + c)^3 + 2*(a*b*d*f*x + a*b*d*e)*sinh(d*
x + c)^3 + a^2*f - (2*a^2*d*f*x + 2*a^2*d*e + a^2*f)*cosh(d*x + c)^2 - (2*
a^2*d*f*x + 2*a^2*d*e + a^2*f - 6*(a*b*d*f*x + a*b*d*e)*cosh(d*x + c))*sin
h(d*x + c)^2 - 2*(a*b*d*f*x + a*b*d*e)*cosh(d*x + c) - (b^2*f*cosh(d*x + c
)^4 + 4*b^2*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*f*sinh(d*x + c)^4 - 2*b^
2*f*cosh(d*x + c)^2 + b^2*f + 2*(3*b^2*f*cosh(d*x + c)^2 - b^2*f)*sinh(d*x
 + c)^2 + 4*(b^2*f*cosh(d*x + c)^3 - b^2*f*cosh(d*x + c))*sinh(d*x + c))*d
ilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^2*f*cosh(d*x + c)^4 + 4*b^2*f*c
osh(d*x + c)*sinh(d*x + c)^3 + b^2*f*sinh(d*x + c)^4 - 2*b^2*f*cosh(d*x +
c)^2 + b^2*f + 2*(3*b^2*f*cosh(d*x + c)^2 - b^2*f)*sinh(d*x + c)^2 + 4*(b^
2*f*cosh(d*x + c)^3 - b^2*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*
x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
 b^2)/b^2) - b)/b + 1) + (b^2*f*cosh(d*x + c)^4 + 4*b^2*f*cosh(d*x + c)*si
nh(d*x + c)^3 + b^2*f*sinh(d*x + c)^4 - 2*b^2*f*cosh(d*x + c)^2 + b^2*f +
2*(3*b^2*f*cosh(d*x + c)^2 - b^2*f)*sinh(d*x + c)^2 + 4*(b^2*f*cosh(d*x +
c)^3 - b^2*f*cosh(d*x + c))*sinh(d*x + c))*dilog(cosh(d*x + c) + sinh(d*x
+ c)) + (b^2*f*cosh(d*x + c)^4 + 4*b^2*f*cosh(d*x + c)*sinh(d*x + c)^3 + b
^2*f*sinh(d*x + c)^4 - 2*b^2*f*cosh(d*x + c)^2 + b^2*f + 2*(3*b^2*f*cosh(d
*x + c)^2 - b^2*f)*sinh(d*x + c)^2 + 4*(b^2*f*cosh(d*x + c)^3 - b^2*f*c...
```

Sympy [F]

$$\int \frac{(e+fx)\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)\coth\left(c+dx\right)\text{csch}^2\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*coth(c + d*x)*csch(c + d*x)**2/(a + b*sinh(c + d*x)), x
)
```

Maxima [F]

$$\int \frac{(e+fx)\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\coth\left(dx+c\right)\text{csch}\left(dx+c\right)^2}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-(4*b^2*d*integrate(1/4*x/(a^3*d*e^(d*x + c) + a^3*d), x) - 4*b^2*d*integr
ate(1/4*x/(a^3*d*e^(d*x + c) - a^3*d), x) + a*b*((d*x + c)/(a^3*d^2) - log
(e^(d*x + c) + 1)/(a^3*d^2)) - a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c)
- 1)/(a^3*d^2)) - (2*b*d*x*e^(3*d*x + 3*c) - 2*b*d*x*e^(d*x + c) - (2*a*d*
x*e^(2*c) + a*e^(2*c))*e^(2*d*x) + a)/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2
*e^(2*d*x + 2*c) + a^2*d^2) - 4*integrate(1/2*(a*b^2*x*e^(d*x + c) - b^3*x
)/(a^3*b*e^(2*d*x + 2*c) + 2*a^4*e^(d*x + c) - a^3*b), x))*f - e*(2*(b*e^(
-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2*e^(-2*d*x - 2
*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + b^2*log(-2*a*e^(-d*x - c) + b*e^(-2
*d*x - 2*c) - b)/(a^3*d) - b^2*log(e^(-d*x - c) + 1)/(a^3*d) - b^2*log(e^(
-d*x - c) - 1)/(a^3*d))
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+d\,x)\,\left(e+f\,x\right)}{\sinh\left(c+d\,x\right)^2\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((coth(c + d*x)*(e + f*x))/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((coth(c + d*x)*(e + f*x))/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(96*e**(7*c + 4*d*x)*int((e**(3*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c +
7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x
)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),
x)*a**5*d**2*f + 40*e**(7*c + 4*d*x)*int((e**(3*d*x)*x)/(e**(8*c + 8*d*x)*
b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6
*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(
c + d*x)*a + b),x)*a**3*b**2*d**2*f - 64*e**(6*c + 4*d*x)*int((e**(2*d*x)*
x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e
**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*
c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**4*b*d**2*f + 8*e**(5*c + 4*d*x)
*int((e**(d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c +
 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*
x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**3*b**2*d**2*f +
6*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**4*f - 2*e**(4*c + 4*d*x)*log(e
**(c + d*x) - 1)*a**3*b*f + 3*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*b**4*
d*e + 6*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a**4*f + 2*e**(4*c + 4*d*x)
*log(e**(c + d*x) + 1)*a**3*b*f + 3*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)
*b**4*d*e - 3*e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a -
 b)*b**4*d*e - 12*e**(4*c + 4*d*x)*a**4*d*f*x + 3*e**(4*c + 4*d*x)*a**4*f
- 3*e**(4*c + 4*d*x)*a**2*b**2*d*e - 4*e**(3*c + 3*d*x)*a**3*b*f + 6*e*...
```

3.479 $\int \frac{\coth(c+dx)\mathbf{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 72

$$\int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b\text{csch}(c+dx)}{a^2d} - \frac{\text{csch}^2(c+dx)}{2ad} + \frac{b^2\log(\sinh(c+dx))}{a^3d} - \frac{b^2\log(a+b\sinh(c+dx))}{a^3d}$$

output
```
b*csch(d*x+c)/a^2/d-1/2*csch(d*x+c)^2/a/d+b^2*ln(sinh(d*x+c))/a^3/d-b^2*ln
(a+b*sinh(d*x+c))/a^3/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2ab\text{csch}(c+dx) - a^2\text{csch}^2(c+dx) + 2b^2(\log(\sinh(c+dx)) - \log(a+b\sinh(c+dx)))}{2a^3d}$$

input
```
Integrate[(Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(2*a*b*Csch[c + d*x] - a^2*Csch[c + d*x]^2 + 2*b^2*(Log[Sinh[c + d*x]] - L
og[a + b*Sinh[c + d*x]]))/(2*a^3*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3042, 26, 3312, 26, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\cos(ic+idx)}{\sin(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos(ic+idx)}{\sin(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3312}$$

$$-\frac{i\int \frac{i\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{bd}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^2\int \frac{\text{csch}^3(c+dx)}{b^3(a+b\sinh(c+dx))}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$\frac{b^2\int \left(\frac{\text{csch}^3(c+dx)}{ab^3}-\frac{\text{csch}^2(c+dx)}{a^2b^2}+\frac{\text{csch}(c+dx)}{a^3b}-\frac{1}{a^3(a+b\sinh(c+dx))}\right)d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2\left(\frac{\log(b\sinh(c+dx))}{a^3}-\frac{\log(a+b\sinh(c+dx))}{a^3}+\frac{\operatorname{csch}(c+dx)}{a^2b}-\frac{\operatorname{csch}^2(c+dx)}{2ab^2}\right)}{d}$$

input
```
Int[(Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(b^2*(Csch[c + d*x]/(a^2*b) - Csch[c + d*x]^2/(2*a*b^2) + Log[b*Sinh[c + d
*x]]/a^3 - Log[a + b*Sinh[c + d*x]]/a^3))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$-\frac{\operatorname{csch}(dx+c)^{2}}{2ad}+\frac{b\operatorname{csch}(dx+c)}{a^{2}d}-\frac{b^{2}\ln(a\operatorname{csch}(dx+c)+b)}{d\,a^{3}}$	54
default	$-\frac{\operatorname{csch}(dx+c)^{2}}{2ad}+\frac{b\operatorname{csch}(dx+c)}{a^{2}d}-\frac{b^{2}\ln(a\operatorname{csch}(dx+c)+b)}{d\,a^{3}}$	54
risch	$-\frac{2\,\mathrm{e}^{dx+c}\left(-b\,\mathrm{e}^{2dx+2c}+a\,\mathrm{e}^{dx+c}+b\right)}{a^{2}d\left(\mathrm{e}^{2dx+2c}-1\right)^{2}}+\frac{b^{2}\ln\left(\mathrm{e}^{2dx+2c}-1\right)}{a^{3}d}-\frac{b^{2}\ln\left(\mathrm{e}^{2dx+2c}+\frac{2a\,\mathrm{e}^{dx+c}}{b}-1\right)}{a^{3}d}$	108

input
```
int(coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*csch(d*x+c)^2/a/d+b*csch(d*x+c)/a^2/d-1/d*b^2/a^3*ln(a*csch(d*x+c)+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 545 vs. $2(70) = 140$.

Time = 0.11 (sec) , antiderivative size = 545, normalized size of antiderivative = 7.57

$$\int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{2\,ab\cosh\left(dx+c\right)^{3}+2\,ab\sinh\left(dx+c\right)^{3}-2\,a^{2}\cosh\left(dx+c\right)^{2}-2\,ab\cosh\left(dx+c\right)+2\left(3\,ab\cosh\left(dx+c\right)\right.}{}$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
(2*a*b*cosh(d*x + c)^3 + 2*a*b*sinh(d*x + c)^3 - 2*a^2*cosh(d*x + c)^2 - 2
*a*b*cosh(d*x + c) + 2*(3*a*b*cosh(d*x + c) - a^2)*sinh(d*x + c)^2 - (b^2*
cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^
4 - 2*b^2*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^
2 + b^2 + 4*(b^2*cosh(d*x + c)^3 - b^2*cosh(d*x + c))*sinh(d*x + c))*log(2
*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + (b^2*cosh(d*x +
c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 - 2*b^2*c
osh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^2 + b^2 + 4
*(b^2*cosh(d*x + c)^3 - b^2*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x +
 c)/(cosh(d*x + c) - sinh(d*x + c))) + 2*(3*a*b*cosh(d*x + c)^2 - 2*a^2*co
sh(d*x + c) - a*b)*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*cosh(d*
x + c)*sinh(d*x + c)^3 + a^3*d*sinh(d*x + c)^4 - 2*a^3*d*cosh(d*x + c)^2 +
 a^3*d + 2*(3*a^3*d*cosh(d*x + c)^2 - a^3*d)*sinh(d*x + c)^2 + 4*(a^3*d*co
sh(d*x + c)^3 - a^3*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(coth(c + d*x)*csch(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(70) = 140$.

Time = 0.04 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.24

$$\int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{2\left(be^{(-dx-c)} - ae^{(-2\,dx-2\,c)} - be^{(-3\,dx-3\,c)}\right)}{(2\,a^2e^{(-2\,dx-2\,c)} - a^2e^{(-4\,dx-4\,c)} - a^2)d} - \frac{b^2\log\left(-2\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)} - b\right)}{a^3d} + \frac{b^2\log\left(e^{(-dx-c)} + 1\right)}{a^3d} + \frac{b^2\log\left(e^{(-dx-c)} - 1\right)}{a^3d}$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2*e^(-
2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) - b^2*log(-2*a*e^(-d*x - c)
+ b*e^(-2*d*x - 2*c) - b)/(a^3*d) + b^2*log(e^(-d*x - c) + 1)/(a^3*d) + b^
2*log(e^(-d*x - c) - 1)/(a^3*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(70) = 140$.

Time = 0.15 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.01

$$\int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{2\,b^2\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^3}-\frac{2\,b^2\log\left(\left|e^{(dx+c)}-e^{(-dx-c)}\right|\right)}{a^3}+\frac{3\,b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2-4\,ab\left(e^{(dx+c)}-e^{(-dx-c)}\right)+4\,a^2}{a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2}}{2\,d}$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(2*b^2*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/a^3 - 2*b^2*log
(abs(e^(d*x + c) - e^(-d*x - c)))/a^3 + (3*b^2*(e^(d*x + c) - e^(-d*x - c)
)^2 - 4*a*b*(e^(d*x + c) - e^(-d*x - c)) + 4*a^2)/(a^3*(e^(d*x + c) - e^(-
d*x - c))^2))/d
```

Mupad [B] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 470, normalized size of antiderivative = 6.53

$$\int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{2}{a\,d}-\frac{2\,b\,\mathrm{e}^{c+d\,x}}{a^2\,d}}{\mathrm{e}^{2\,c+2\,d\,x}-1} - \frac{\left(2\,\mathrm{atan}\left(-\frac{4\,a^3\,b^5\,\sqrt{-a^6\,d^2}+4\,a\,b^7\,\sqrt{-a^6\,d^2}-4\,b^8\,\mathrm{e}^{3\,c}\,\mathrm{e}^{3\,d\,x}\,\sqrt{-a^6\,d^2}+4\,b^8\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-a^6\,d^2}-8\,a\,b^7\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\sqrt{-a^6\,d^2}+4\,a^2\,b^6\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-a}}{4\,a^4\,b\,d\,\left(b^4\right)^{3/2}+4\,a^6\,b^3\,d\,\sqrt{b^4}}\right.\right.}{} - \frac{2}{a\,d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

input
```
int(coth(c + d*x)/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
- (2/(a*d) - (2*b*exp(c + d*x))/(a^2*d))/(exp(2*c + 2*d*x) - 1) - ((2*atan
(-(4*a^3*b^5*(-a^6*d^2)^(1/2) + 4*a*b^7*(-a^6*d^2)^(1/2) - 4*b^8*exp(3*c)*
exp(3*d*x)*(-a^6*d^2)^(1/2) + 4*b^8*exp(d*x)*exp(c)*(-a^6*d^2)^(1/2) - 8*a
*b^7*exp(2*c)*exp(2*d*x)*(-a^6*d^2)^(1/2) + 4*a^2*b^6*exp(d*x)*exp(c)*(-a^
6*d^2)^(1/2) - 8*a^3*b^5*exp(2*c)*exp(2*d*x)*(-a^6*d^2)^(1/2) - 4*a^2*b^6*
exp(3*c)*exp(3*d*x)*(-a^6*d^2)^(1/2))/(4*a^4*b*d*(b^4)^(3/2) + 4*a^6*b^3*d
*(b^4)^(1/2))) + 2*atan((4*a^4*b^5*d*(b^4)^(1/2)*(-a^6*d^2)^(1/2) + 4*a^6*
b^3*d*(b^4)^(1/2)*(-a^6*d^2)^(1/2))*(1/(8*a^5*b^5*d^2*(a^2 + b^2)^2) - exp
(d*x)*exp(c)*(1/(16*a^4*b^6*d^2*(a^2 + b^2)^2) - (a^2 + 2*b^2)^2/(16*a^8*b
^6*d^2*(a^2 + b^2)^2)) + (a^2 + 2*b^2)/(8*a^7*b^5*d^2*(a^2 + b^2)^2))))*(b
^4)^(1/2))/(-a^6*d^2)^(1/2) - 2/(a*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x
) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 321, normalized size of antiderivative = 4.46

$$\int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{e^{4dx+4c}\log\left(e^{dx+c}-1\right)b^2+e^{4dx+4c}\log\left(e^{dx+c}+1\right)b^2-e^{4dx+4c}\log\left(e^{2dx+2c}b+2e^{dx+c}a-b\right)b^2-e^{4dx+4c}a^2+}{}$$

input
```
int(coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
(e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*b**2 + e**(4*c + 4*d*x)*log(e**(c
+ d*x) + 1)*b**2 - e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x
)*a - b)*b**2 - e**(4*c + 4*d*x)*a**2 + 2*e**(3*c + 3*d*x)*a*b - 2*e**(2*c
 + 2*d*x)*log(e**(c + d*x) - 1)*b**2 - 2*e**(2*c + 2*d*x)*log(e**(c + d*x)
 + 1)*b**2 + 2*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a
- b)*b**2 - 2*e**(c + d*x)*a*b + log(e**(c + d*x) - 1)*b**2 + log(e**(c +
d*x) + 1)*b**2 - log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*b**2 - a**
2)/(a**3*d*(e**(4*c + 4*d*x) - 2*e**(2*c + 2*d*x) + 1))
```

3.480 $\int \frac{\coth(c+dx)\mathbf{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(coth(d*x+c)*csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 134.92 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Coth[c + d*x]*Csch[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Coth[c + d*x]*Csch[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{ 6125}$$
$$\int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Coth[c + d*x]*Csch[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\coth(dx+c)\,\text{csch}(dx+c)^2}{(fx+e)\,(a+b\sinh(dx+c))}dx$$

input
```
int(coth(d*x+c)*csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(coth(d*x+c)*csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(dx+c)\,\text{csch}\,(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(coth(d*x + c)*csch(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 21.63 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(c+dx)\,\text{csch}^2(c+dx)}{(a+b\sinh(c+dx))\,(e+fx)}\,dx$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(coth(c + d*x)*csch(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)),
 x)
```

Maxima [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 684, normalized size of antiderivative = 20.12

$$\int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(dx+c)\operatorname{csch}(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-(a*f - 2*(b*d*f*x*e^(3*c) + b*d*e*e^(3*c))*e^(3*d*x) + (2*a*d*f*x*e^(2*c)
 + (2*d*e - f)*a*e^(2*c))*e^(2*d*x) + 2*(b*d*f*x*e^c + b*d*e*e^c)*e^(d*x))
/(a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + a^2*d^2*e^2 + (a^2*d^2*f^2*x^2*e^(4*
c) + 2*a^2*d^2*e*f*x*e^(4*c) + a^2*d^2*e^2*e^(4*c))*e^(4*d*x) - 2*(a^2*d^2
*f^2*x^2*e^(2*c) + 2*a^2*d^2*e*f*x*e^(2*c) + a^2*d^2*e^2*e^(2*c))*e^(2*d*x
)) + 4*integrate(-1/4*(b^2*d^2*f^2*x^2 + b^2*d^2*e^2 + a*b*d*e*f + a^2*f^2
 + (2*b^2*d^2*e*f + a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 +
 3*a^3*d^2*e^2*f*x + a^3*d^2*e^3 - (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*
x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)*e^(d*x)), x) - 4*integr
ate(1/4*(b^2*d^2*f^2*x^2 + b^2*d^2*e^2 - a*b*d*e*f + a^2*f^2 + (2*b^2*d^2*
e*f - a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2
*f*x + a^3*d^2*e^3 + (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^
3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)*e^(d*x)), x) + 4*integrate(-1/2*(a*b^
2*e^(d*x + c) - b^3)/(a^3*b*f*x + a^3*b*e - (a^3*b*f*x*e^(2*c) + a^3*b*e*e
^(2*c))*e^(2*d*x) - 2*(a^4*f*x*e^c + a^4*e*e^c)*e^(d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 2.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\coth(c+d\,x)}{{\sinh\left(c+d\,x\right)}^2\,\left(e+f\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(coth(c + d*x)/(sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(coth(c + d*x)/(sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$=\int \frac{\coth\left(dx+c\right)\text{csch}(dx+c)^2}{\sinh\left(dx+c\right)be+\sinh\left(dx+c\right)bfx+ae+afx}dx$$

input
```
int(coth(d*x+c)*csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int((coth(c + d*x)*csch(c + d*x)**2)/(sinh(c + d*x)*b*e + sinh(c + d*x)*b*
f*x + a*e + a*f*x),x)
```

3.481 $\int \frac{(e+fx)^3 \coth^2(c+dx)\mathrm{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 1038

$$\int \frac{(e+fx)^3 \coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output
```
-6*b^2*f^3*polylog(4,-exp(d*x+c))/a^3/d^4+6*b^2*f^3*polylog(4,exp(d*x+c))/
a^3/d^4+3/2*b*f^3*polylog(3,exp(2*d*x+2*c))/a^2/d^4-2*b^2*(f*x+e)^3*arctan
h(exp(d*x+c))/a^3/d-(f*x+e)^3*arctanh(exp(d*x+c))/a/d-3*f^3*polylog(4,-exp
(d*x+c))/a/d^4+3*f^3*polylog(4,exp(d*x+c))/a/d^4+6*b^2*f^2*(f*x+e)*polylog
(3,-exp(d*x+c))/a^3/d^3-3*b^2*f*(f*x+e)^2*polylog(2,-exp(d*x+c))/a^3/d^2-3
*b*f*(f*x+e)^2*ln(1-exp(2*d*x+2*c))/a^2/d^2-6*b^2*f^2*(f*x+e)*polylog(3,ex
p(d*x+c))/a^3/d^3-3*b*f^2*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a^2/d^3+3*b^2*
f*(f*x+e)^2*polylog(2,exp(d*x+c))/a^3/d^2+b*(f*x+e)^3/a^2/d-6*f^2*(f*x+e)*
arctanh(exp(d*x+c))/a/d^3-3/2*f*(f*x+e)^2*csch(d*x+c)/a/d^2-1/2*(f*x+e)^3*
coth(d*x+c)*csch(d*x+c)/a/d-3/2*f*(f*x+e)^2*polylog(2,-exp(d*x+c))/a/d^2+3
/2*f*(f*x+e)^2*polylog(2,exp(d*x+c))/a/d^2+3*f^2*(f*x+e)*polylog(3,-exp(d*
x+c))/a/d^3-3*f^2*(f*x+e)*polylog(3,exp(d*x+c))/a/d^3+b*(f*x+e)^3*coth(d*x
+c)/a^2/d-3*f^3*polylog(2,-exp(d*x+c))/a/d^4+3*f^3*polylog(2,exp(d*x+c))/a
/d^4+6*b*(a^2+b^2)^(1/2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^
(1/2)))/a^3/d^3+3*b*(a^2+b^2)^(1/2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a
+(a^2+b^2)^(1/2)))/a^3/d^2-3*b*(a^2+b^2)^(1/2)*f*(f*x+e)^2*polylog(2,-b*ex
p(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d^2-6*b*(a^2+b^2)^(1/2)*f^2*(f*x+e)*poly
log(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d^3+b*(a^2+b^2)^(1/2)*(f*x+e)
^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d-b*(a^2+b^2)^(1/2)*(f*x+e)^
3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d+6*b*(a^2+b^2)^(1/2)*f^3*...
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2799 vs. $2(1038) = 2076$.

Time = 9.18 (sec) , antiderivative size = 2799, normalized size of antiderivative = 2.70

$$\int \frac{(e+fx)^3\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
(12*a*b*d^3*e^2*E^(2*c)*f*x + 12*a*b*d^3*e*E^(2*c)*f^2*x^2 + 4*a*b*d^3*E^(
2*c)*f^3*x^3 + 2*a^2*d^3*e^3*ArcTanh[E^(c + d*x)] + 4*b^2*d^3*e^3*ArcTanh[
E^(c + d*x)] - 2*a^2*d^3*e^3*E^(2*c)*ArcTanh[E^(c + d*x)] - 4*b^2*d^3*e^3*
E^(2*c)*ArcTanh[E^(c + d*x)] + 12*a^2*d*e*f^2*ArcTanh[E^(c + d*x)] - 12*a^
2*d*e*E^(2*c)*f^2*ArcTanh[E^(c + d*x)] - 3*a^2*d^3*e^2*f*x*Log[1 - E^(c +
d*x)] - 6*b^2*d^3*e^2*f*x*Log[1 - E^(c + d*x)] + 3*a^2*d^3*e^2*E^(2*c)*f*x
*Log[1 - E^(c + d*x)] + 6*b^2*d^3*e^2*E^(2*c)*f*x*Log[1 - E^(c + d*x)] - 6
*a^2*d*f^3*x*Log[1 - E^(c + d*x)] + 6*a^2*d*E^(2*c)*f^3*x*Log[1 - E^(c + d
*x)] - 3*a^2*d^3*e*f^2*x^2*Log[1 - E^(c + d*x)] - 6*b^2*d^3*e*f^2*x^2*Log[
1 - E^(c + d*x)] + 3*a^2*d^3*e*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] + 6*b^
2*d^3*e*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] - a^2*d^3*f^3*x^3*Log[1 - E^(
c + d*x)] - 2*b^2*d^3*f^3*x^3*Log[1 - E^(c + d*x)] + a^2*d^3*E^(2*c)*f^3*x
^3*Log[1 - E^(c + d*x)] + 2*b^2*d^3*E^(2*c)*f^3*x^3*Log[1 - E^(c + d*x)] +
 3*a^2*d^3*e^2*f*x*Log[1 + E^(c + d*x)] + 6*b^2*d^3*e^2*f*x*Log[1 + E^(c +
 d*x)] - 3*a^2*d^3*e^2*E^(2*c)*f*x*Log[1 + E^(c + d*x)] - 6*b^2*d^3*e^2*E^
(2*c)*f*x*Log[1 + E^(c + d*x)] + 6*a^2*d*f^3*x*Log[1 + E^(c + d*x)] - 6*a^
2*d*E^(2*c)*f^3*x*Log[1 + E^(c + d*x)] + 3*a^2*d^3*e*f^2*x^2*Log[1 + E^(c
+ d*x)] + 6*b^2*d^3*e*f^2*x^2*Log[1 + E^(c + d*x)] - 3*a^2*d^3*e*E^(2*c)*f
^2*x^2*Log[1 + E^(c + d*x)] - 6*b^2*d^3*e*E^(2*c)*f^2*x^2*Log[1 + E^(c + d
*x)] + a^2*d^3*f^3*x^3*Log[1 + E^(c + d*x)] + 2*b^2*d^3*f^3*x^3*Log[1 +...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3 \coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6121}$$

$$\frac{\int (e+fx)^3\coth^2(c+dx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5980}$$

$$\frac{\int (e+fx)^3\text{csch}^3(c+dx)dx+\int (e+fx)^3\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int i(e+fx)^3\csc(ic+idx)dx+\int -i(e+fx)^3\csc(ic+idx)^3dx}{a}$$

$\downarrow$ 26

$$-\frac{b\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\int (e+fx)^3\csc(ic+idx)^3dx}{a}$$

$\downarrow$ 4670

$$-\frac{b\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{3if\int (e+fx)^2\log(1-e^{c+dx})dx}{d}-\frac{3if\int (e+fx)^2\log(1+e^{c+dx})dx}{d}+\frac{2i(e+fx)^3\text{arctanh}(e^{c+dx})}{d}\right)-i\int (e+fx)^3\csc(ic+idx)^3 d}{a}$$

$\downarrow$ 3011

$$-\frac{b\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}(2,-e^{c+dx})dx}{d}-\frac{(e+fx)^2\text{PolyLog}(2,-e^{c+dx})}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}(2,e^{c+dx})dx}{d}-\frac{(e+fx)^2\text{PolyLog}(2,e^{c+dx})}{d}\right)}{d}\right.}{a}$$

$\downarrow$ 4674

$$-\frac{b\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}(2,-e^{c+dx})dx}{d}-\frac{(e+fx)^2\text{PolyLog}(2,-e^{c+dx})}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}(2,e^{c+dx})dx}{d}-\frac{(e+fx)^2\text{PolyLog}(2,e^{c+dx})}{d}\right)}{d}\right.}{}$$

$\downarrow$ 26

$$-\frac{b\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}(2,-e^{c+dx})dx}{d}-\frac{(e+fx)^2\text{PolyLog}(2,-e^{c+dx})}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}(2,e^{c+dx})dx}{d}-\frac{(e+fx)^2\text{PolyLog}(2,e^{c+dx})}{d}\right)}{d}\right.}{}$$

$\downarrow$ 3042

$$-\frac{b\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$
$$i\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.$$

$\downarrow$ 26

$$-\frac{b\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$
$$i\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.$$

$\downarrow$ 4670

$$-\frac{b\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$
$$i\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.$$

$\downarrow$ 2715

$$-\frac{b\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$
$$i\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.$$

$\downarrow$ 2838

$$-\frac{b\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$
$$i\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.$$

$\downarrow$ 3011

$$-\frac{b\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{}$$

$\downarrow$ 6103

$$-\frac{b\left(\frac{\int (e+fx)^3\coth^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{}$$

$\downarrow$ 3042

$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{}$$

$$\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -(e+fx)^3\tan\left(ic+idx+\frac{\pi}{2}\right)^2dx}{a}\right)}{a}$$

$\downarrow$ 25

$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{}$$

$$\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\int (e+fx)^3\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{a}\right)}{a}$$

$\downarrow$ 4203

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{3if\int i(e+fx)^2\coth(c+dx)dx}{d}-\int(e+fx)^3dx+\frac{(e+fx)^3\coth(c+dx)}{d}}{a}\right)}{a}}$$

$$\Big\downarrow \text{17}$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{3if\int i(e+fx)^2\coth(c+dx)dx}{d}+\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{a}\right)}{a}}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{-\frac{3f\int(e+fx)^2\coth(c+dx)dx}{d}+\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{a}\right)}{a}+$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{-\frac{3f\int -i(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}+\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{a}\right)}{a}}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{3if\int(e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}+\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{a}\right)}{a}}$$

$\downarrow$ 4201

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{3if\left(2i\int\frac{e^{2c+2dx-i\pi}(e+fx)^2}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{a}\right)}{a}}$$

$\downarrow$ 2620

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\coth(}{d}}{a}\right.}{a}}$$

$\downarrow$ 3011

$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\cdots\right.}{b\left(-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int \text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{2d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2d\cdots}\right)}{2d}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{a}\cdots\right.}\cdots$$

$$\Big\downarrow \textbf{2720}$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\cdots\right.}{b\left(-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{a}\cdots\right.}\cdots$$

$$\Big\downarrow \textbf{6119}$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\cdots\right.}{b\left(-\frac{b\left(\frac{\int (e+fx)^3\cosh(c+dx)\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,\cdots\right.}{4\cdots}\cdots\right.}{\cdots}\cdots\right.\right.}{\cdots}\cdots\right.}\cdots$$

$$\Big\downarrow 5973$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{}$$

$$\frac{b\left(-\frac{b\left(\frac{\int(e+fx)^3\sinh(c+dx)dx+\int(e+fx)^3\text{csch}(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx}}{}\right.}{}\right.\right.}{}\right.}{a}$$

$$\Big\downarrow 3042$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{}$$

$$\frac{b\left(-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int-i(e+fx)^3\sin(ic+idx)dx+\int i(e+fx)^3\csc(ic+idx)dx}{a}\right)}{a}-\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c}}{}\right.}{}\right.\right.}{}\right.}{a}$$

$$\Big\downarrow 26$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{b\left(-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int(e+fx)^3\csc(ic+idx)dx-i\int(e+fx)^3\sin(ic+idx)dx}{a}\right)}{a}-\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-}}{}\right.}{}\right.\right.}{}\right.}{a}$$

input
```
Int[((e + f*x)^3*Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^3 \coth(dx+c)^2 \operatorname{csch}(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13504 vs. 2(962) = 1924.

Time = 0.34 (sec) , antiderivative size = 13504, normalized size of antiderivative = 13.01

$$\int \frac{(e+fx)^3 \coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\coth^2(c+dx)\mathrm{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*coth(d*x+c)**2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^3\coth^2(c+dx)\mathrm{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\coth{(dx+c)}^2\,\mathrm{csch}\,(dx+c)}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^3*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
1/2*e^3*(2*(a*e^(-d*x - c) + 2*b*e^(-2*d*x - 2*c) + a*e^(-3*d*x - 3*c) - 2
*b)/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) - (a^2 + 2*b
^2)*log(e^(-d*x - c) + 1)/(a^3*d) + (a^2 + 2*b^2)*log(e^(-d*x - c) - 1)/(a
^3*d) - 2*(a^2*b + b^3)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-
d*x - c) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3*d)) - (2*b*d*f^3*x^3
 + 6*b*d*e*f^2*x^2 + 6*b*d*e^2*f*x + (a*d*f^3*x^3*e^(3*c) + 3*a*e^2*f*e^(3
*c) + 3*(d*e*f^2 + f^3)*a*x^2*e^(3*c) + 3*(d*e^2*f + 2*e*f^2)*a*x*e^(3*c))
*e^(3*d*x) - 2*(b*d*f^3*x^3*e^(2*c) + 3*b*d*e*f^2*x^2*e^(2*c) + 3*b*d*e^2*
f*x*e^(2*c))*e^(2*d*x) + (a*d*f^3*x^3*e^c - 3*a*e^2*f*e^c + 3*(d*e*f^2 - f
^3)*a*x^2*e^c + 3*(d*e^2*f - 2*e*f^2)*a*x*e^c)*e^(d*x))/(a^2*d^2*e^(4*d*x
+ 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2) + 3*(b*d*e^2*f + a*e*f^2)*x/
(a^2*d^2) + 3*(b*d*e^2*f - a*e*f^2)*x/(a^2*d^2) - 3*(b*d*e^2*f + a*e*f^2)*
log(e^(d*x + c) + 1)/(a^2*d^3) - 3*(b*d*e^2*f - a*e*f^2)*log(e^(d*x + c) -
 1)/(a^2*d^3) - 1/2*(d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*
x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*(a^
2*f^3 + 2*b^2*f^3)/(a^3*d^4) + 1/2*(d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*
x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d
*x + c)))*(a^2*f^3 + 2*b^2*f^3)/(a^3*d^4) - 3/2*(a^2*d*e*f^2 + 2*b^2*d*e*f
^2 + 2*a*b*f^3)*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c))
- 2*polylog(3, -e^(d*x + c)))/(a^3*d^4) + 3/2*(a^2*d*e*f^2 + 2*b^2*d*e*...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+d\,x)^2\,(e+f\,x)^3}{\sinh(c+d\,x)\,\left(a+b\sinh(c+d\,x)\right)}\,dx$$

input
```
int((coth(c + d*x)^2*(e + f*x)^3)/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((coth(c + d*x)^2*(e + f*x)^3)/(sinh(c + d*x)*(a + b*sinh(c + d*x))), x
)
```

Reduce [F]

$$\int \frac{(e+fx)^3 \coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^3*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 288*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/s
qrt(a**2 + b**2))*b**4*d**3*e**3*i + 576*e**(2*c + 2*d*x)*sqrt(a**2 + b**2
)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*b**4*d**3*e**3*i - 288*
sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*b**4*d*
*3*e**3*i - 4608*e**(7*c + 4*d*x)*int((e**(3*d*x)*x**3)/(e**(8*c + 8*d*x)*
b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6
*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(
c + d*x)*a + b),x)*a**6*d**4*f**3 - 4224*e**(7*c + 4*d*x)*int((e**(3*d*x)*
x**3)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b -
6*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**
(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**4*b**2*d**4*f**3 - 13824*e**
(7*c + 4*d*x)*int((e**(3*d*x)*x**2)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*
x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*b
+ 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a
**6*d**4*e*f**2 - 12672*e**(7*c + 4*d*x)*int((e**(3*d*x)*x**2)/(e**(8*c +
8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x
)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b -
 2*e**(c + d*x)*a + b),x)*a**4*b**2*d**4*e*f**2 + 1536*e**(7*c + 4*d*x)*in
t((e**(3*d*x)*x**2)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c
 + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c +...
```

3.482 $\int \frac{(e+fx)^2 \coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 714

$$\int \frac{(e+fx)^2\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{b(e+fx)^2}{a^2d} - \frac{(e+fx)^2\operatorname{arctanh}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^2\operatorname{arctanh}(e^{c+dx})}{a^3d} - \frac{f^2\operatorname{arctanh}(\cosh(c+dx))}{ad^3} + \frac{b(e+fx)^2\coth(c+dx)}{a^2d} - \frac{f(e+fx)\operatorname{csch}(c+dx)}{ad^2} - \frac{(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{b\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d} + \frac{b\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d} - \frac{2bf(e+fx)\log\left(1-e^{2(c+dx)}\right)}{a^2d^2} - \frac{f(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2} - \frac{2b^2f(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{a^3d^2} + \frac{f(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{ad^2} + \frac{2b^2f(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{a^3d^2} - \frac{2b\sqrt{a^2+b^2}f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2} + \frac{2b\sqrt{a^2+b^2}f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^2} - \frac{bf^2\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{a^2d^3} + \frac{f^2\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3} + \frac{2b^2f^2\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{a^3d^3} - \frac{f^2\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{ad^3} - \frac{2b^2f^2\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{a^3d^3} + \frac{2b\sqrt{a^2+b^2}f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^3} - \frac{2b\sqrt{a^2+b^2}f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^3}$$

output
```
b*(f*x+e)^2/a^2/d-(f*x+e)^2*arctanh(exp(d*x+c))/a/d-2*b^2*(f*x+e)^2*arctan
h(exp(d*x+c))/a^3/d-f^2*arctanh(cosh(d*x+c))/a/d^3+b*(f*x+e)^2*coth(d*x+c)
/a^2/d-f*(f*x+e)*csch(d*x+c)/a/d^2-1/2*(f*x+e)^2*coth(d*x+c)*csch(d*x+c)/a
/d-b*(a^2+b^2)^(1/2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/
d+b*(a^2+b^2)^(1/2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d
-2*b*f*(f*x+e)*ln(1-exp(2*d*x+2*c))/a^2/d^2-f*(f*x+e)*polylog(2,-exp(d*x+c
))/a/d^2-2*b^2*f*(f*x+e)*polylog(2,-exp(d*x+c))/a^3/d^2+f*(f*x+e)*polylog(
2,exp(d*x+c))/a/d^2+2*b^2*f*(f*x+e)*polylog(2,exp(d*x+c))/a^3/d^2-2*b*(a^2
+b^2)^(1/2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d^2
+2*b*(a^2+b^2)^(1/2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2))
)/a^3/d^2-b*f^2*polylog(2,exp(2*d*x+2*c))/a^2/d^3+f^2*polylog(3,-exp(d*x+c
))/a/d^3+2*b^2*f^2*polylog(3,-exp(d*x+c))/a^3/d^3-f^2*polylog(3,exp(d*x+c)
)/a/d^3-2*b^2*f^2*polylog(3,exp(d*x+c))/a^3/d^3+2*b*(a^2+b^2)^(1/2)*f^2*po
lylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d^3-2*b*(a^2+b^2)^(1/2)*f^2
*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d^3
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1530 vs. $2(714) = 1428$.

Time = 8.20 (sec) , antiderivative size = 1530, normalized size of antiderivative = 2.14

$$\int \frac{(e+fx)^2 \coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
(8*a*b*d^2*e*E^(2*c)*f*x + 4*a*b*d^2*E^(2*c)*f^2*x^2 + 2*a^2*d^2*e^2*ArcTa
nh[E^(c + d*x)] + 4*b^2*d^2*e^2*ArcTanh[E^(c + d*x)] - 2*a^2*d^2*e^2*E^(2*
c)*ArcTanh[E^(c + d*x)] - 4*b^2*d^2*e^2*E^(2*c)*ArcTanh[E^(c + d*x)] + 4*a
^2*f^2*ArcTanh[E^(c + d*x)] - 4*a^2*E^(2*c)*f^2*ArcTanh[E^(c + d*x)] - 2*a
^2*d^2*e*f*x*Log[1 - E^(c + d*x)] - 4*b^2*d^2*e*f*x*Log[1 - E^(c + d*x)] +
 2*a^2*d^2*e*E^(2*c)*f*x*Log[1 - E^(c + d*x)] + 4*b^2*d^2*e*E^(2*c)*f*x*Lo
g[1 - E^(c + d*x)] - a^2*d^2*f^2*x^2*Log[1 - E^(c + d*x)] - 2*b^2*d^2*f^2*
x^2*Log[1 - E^(c + d*x)] + a^2*d^2*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] +
2*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] + 2*a^2*d^2*e*f*x*Log[1 + E
^(c + d*x)] + 4*b^2*d^2*e*f*x*Log[1 + E^(c + d*x)] - 2*a^2*d^2*e*E^(2*c)*f
*x*Log[1 + E^(c + d*x)] - 4*b^2*d^2*e*E^(2*c)*f*x*Log[1 + E^(c + d*x)] + a
^2*d^2*f^2*x^2*Log[1 + E^(c + d*x)] + 2*b^2*d^2*f^2*x^2*Log[1 + E^(c + d*x
)] - a^2*d^2*E^(2*c)*f^2*x^2*Log[1 + E^(c + d*x)] - 2*b^2*d^2*E^(2*c)*f^2*
x^2*Log[1 + E^(c + d*x)] + 4*a*b*d*e*f*Log[1 - E^(2*(c + d*x))] - 4*a*b*d*
e*E^(2*c)*f*Log[1 - E^(2*(c + d*x))] + 4*a*b*d*f^2*x*Log[1 - E^(2*(c + d*x
))] - 4*a*b*d*E^(2*c)*f^2*x*Log[1 - E^(2*(c + d*x))] - 2*(a^2 + 2*b^2)*d*(
-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -E^(c + d*x)] + 2*(a^2 + 2*b^2)*d*(-1
 + E^(2*c))*f*(e + f*x)*PolyLog[2, E^(c + d*x)] + 2*a*b*f^2*PolyLog[2, E^(
2*(c + d*x))] - 2*a*b*E^(2*c)*f^2*PolyLog[2, E^(2*(c + d*x))] - 2*a^2*f^2*
PolyLog[3, -E^(c + d*x)] - 4*b^2*f^2*PolyLog[3, -E^(c + d*x)] + 2*a^2*E...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6121}$$

$$\frac{\int (e+fx)^2\coth^2(c+dx)\operatorname{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5980}$$

$$\frac{\int (e+fx)^2\operatorname{csch}^3(c+dx)dx+\int (e+fx)^2\operatorname{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int i(e+fx)^2\csc(ic+idx)dx+\int -i(e+fx)^2\csc(ic+idx)^3dx}{a}$$

$\downarrow$ 26

$$-\frac{b\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx-i\int (e+fx)^2\csc(ic+idx)^3dx}{a}$$

$\downarrow$ 4670

$$-\frac{b\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{2if\int (e+fx)\log(1-e^{c+dx})dx}{d}-\frac{2if\int (e+fx)\log(1+e^{c+dx})dx}{d}+\frac{2i(e+fx)^2\text{arctanh}(e^{c+dx})}{d}\right)-i\int (e+fx)^2\csc(ic+idx)^3dx}{a}$$

$\downarrow$ 3011

$$-\frac{b\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\frac{2if\left(\frac{f\int \text{PolyLog}(2,-e^{c+dx})dx}{d}-\frac{(e+fx)\text{PolyLog}(2,-e^{c+dx})}{d}\right)}{d}+\frac{2if\left(\frac{f\int \text{PolyLog}(2,e^{c+dx})dx}{d}-\frac{(e+fx)\text{PolyLog}(2,e^{c+dx})}{d}\right)}{d}+\frac{2i(e+fx)^2\text{ar}}{}\right.}{a}$$

$\downarrow$ 2720

$$-\frac{b\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}(2,-e^{c+dx})de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}(2,-e^{c+dx})}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}(2,e^{c+dx})de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}(2,e^{}}{d}\right.}{d}\right.}{a}$$

$\downarrow$ 4674

$$-\frac{b\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}(2,-e^{c+dx})de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}(2,-e^{c+dx})}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}(2,e^{c+dx})de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}(2,e^{}}{d}\right.}{d}\right.}{}$$

$\downarrow$ 26

$$-\frac{b\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+ i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{\cdots}\right.}{d}\right.}{d}\right.$$

$\downarrow$ 3042

$$-\frac{b\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+ i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{\cdots}\right.}{d}\right.}{d}\right.$$

$\downarrow$ 26

$$-\frac{b\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+ i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{\cdots}\right.}{d}\right.}{d}\right.$$

$\downarrow$ 4257

$$-\frac{b\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+ i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{\cdots}\right.}{d}\right.}{d}\right.$$

$\downarrow$ 4670

$$-\frac{b\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+ i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{\cdots}\right.}{d}\right.}{d}\right.$$

$\downarrow$ 3011

$$-\frac{b\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$
$$i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{\cdots}\right)}{d}\right.}{d}\right.$$

$\downarrow$ 2720

$$-\frac{b\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$
$$i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{\cdots}\right)}{d}\right.}{d}\right.$$

$\downarrow$ 6103

$$-\frac{b\left(\frac{\int (e+fx)^2\coth^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+$$
$$i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{\cdots}\right)}{d}\right.}{d}\right.$$

$\downarrow$ 3042

$$i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{\cdots}\right)}{d}\right.}{d}\right.$$
$$\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)^2dx}{a}\right)}{a}$$

$\downarrow$ 25

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{\cdots}\right)}{d}\right.}{d}\cdots}{\cdots}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\int(e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{a}\right)}{a}$$

$\downarrow$ 4203

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{\cdots}\right)}{d}\right.}{d}\cdots}{\cdots}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{2if\int i(e+fx)\coth(c+dx)dx}{d}-\int(e+fx)^2dx+\frac{(e+fx)^2\coth(c+dx)}{d}}{a}\right)}{a}$$

$\downarrow$ 17

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{\cdots}\right)}{d}\right.}{d}\cdots}{\cdots}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{2if\int i(e+fx)\coth(c+dx)dx}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}\right)}{a}$$

$\downarrow$ 26

$$-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{-\frac{2f\int(e+fx)\coth(c+dx)dx}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}\right)}{a}+$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{\cdots}\right)}{d}\right.}{d}\cdots}{\cdots}$$

$\downarrow$ 3042

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right.}{d}\right.}{}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{-\frac{2f\int -i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}\right)}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right.}{d}\right.}{}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{2if\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}\right)}{a}$$

$$\Big\downarrow \; 4201$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right.}{d}\right.}{}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{2if\left(2i\int\frac{e^{2c+2dx-i\pi}(e+fx)}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}\right)}{a}$$

$$\Big\downarrow \; 2620$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right.}{d}\right.}{}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int\log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\ldots}{a}\right.}{a}$$

$\downarrow$ 2715

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{\cdots}\right)}{d}\cdots\right)}{d}\cdots\right.}{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\log\left(1+e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\cdots\right)}{d}}{a}\cdots\right.}$$
$$a$$

$\downarrow$ 2838

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{\cdots}\right)}{d}\cdots\right)}{d}\cdots\right.}{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}\cdots}{a}\cdots\right.}$$
$$a$$

$\downarrow$ 6119

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{\cdots}\right)}{d}\cdots\right)}{d}\cdots\right.}{b\left(-\frac{b\left(\frac{\int(e+fx)^2\cosh(c+dx)\coth(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\cdots\right)}{d}}{a}\cdots\right.}$$
$$a$$

$\downarrow$ 5973

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{\cdots}\right)}{d}\right.}{d}\cdots}{b\left(-\frac{b\left(\frac{\int(e+fx)^2\sinh(c+dx)dx+\int(e+fx)^2\operatorname{csch}(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+\cdots}\right)}{2d}\cdots\right.\right.}{d}\cdots\right.}{a}$$

$\downarrow$ 3042

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{\cdots}\right)}{d}\right.}{d}\cdots}{b\left(-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int-i(e+fx)^2\sin(ic+idx)dx+\int i(e+fx)^2\csc(ic+idx)dx}{a}\right)}{a}-\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1-\cdots\right)}{\cdots}\right.\right.}{d}\cdots\right.}{a}$$

input
```
Int[((e + f*x)^2*Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int\frac{(fx+e)^2\coth(dx+c)^2\operatorname{csch}(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7726 vs. 2(665) = 1330.

Time = 0.23 (sec) , antiderivative size = 7726, normalized size of antiderivative = 10.82

$$\int \frac{(e+fx)^2\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*coth(d*x+c)**2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^2\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\coth(dx+c)^2\text{csch}(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
1/2*e^2*(2*(a*e^(-d*x - c) + 2*b*e^(-2*d*x - 2*c) + a*e^(-3*d*x - 3*c) - 2
*b)/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) - (a^2 + 2*b
^2)*log(e^(-d*x - c) + 1)/(a^3*d) + (a^2 + 2*b^2)*log(e^(-d*x - c) - 1)/(a
^3*d) - 2*(a^2*b + b^3)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-
d*x - c) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3*d)) - (2*b*d*f^2*x^2
 + 4*b*d*e*f*x + (a*d*f^2*x^2*e^(3*c) + 2*a*e*f*e^(3*c) + 2*(d*e*f + f^2)*
a*x*e^(3*c))*e^(3*d*x) - 2*(b*d*f^2*x^2*e^(2*c) + 2*b*d*e*f*x*e^(2*c))*e^(
2*d*x) + (a*d*f^2*x^2*e^c - 2*a*e*f*e^c + 2*(d*e*f - f^2)*a*x*e^c)*e^(d*x)
)/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2) + (2*b*d
*e*f + a*f^2)*x/(a^2*d^2) + (2*b*d*e*f - a*f^2)*x/(a^2*d^2) - (2*b*d*e*f +
 a*f^2)*log(e^(d*x + c) + 1)/(a^2*d^3) - (2*b*d*e*f - a*f^2)*log(e^(d*x +
c) - 1)/(a^2*d^3) - 1/2*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*
x + c)) - 2*polylog(3, -e^(d*x + c)))*(a^2*f^2 + 2*b^2*f^2)/(a^3*d^3) + 1/
2*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3,
 e^(d*x + c)))*(a^2*f^2 + 2*b^2*f^2)/(a^3*d^3) - (a^2*d*e*f + 2*b^2*d*e*f
+ 2*a*b*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^3*d^3) +
(a^2*d*e*f + 2*b^2*d*e*f - 2*a*b*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e
^(d*x + c)))/(a^3*d^3) + 1/6*((a^2*f^2 + 2*b^2*f^2)*d^3*x^3 + 3*(a^2*d*e*f
 + 2*b^2*d*e*f + 2*a*b*f^2)*d^2*x^2)/(a^3*d^3) - 1/6*((a^2*f^2 + 2*b^2*f^2
)*d^3*x^3 + 3*(a^2*d*e*f + 2*b^2*d*e*f - 2*a*b*f^2)*d^2*x^2)/(a^3*d^3) ...
```

Giac [F]

$$\int \frac{(e+fx)^2\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\coth\left(dx+c\right)^2\text{csch}\left(dx+c\right)}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^2*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+d\,x)^2\,(e+f\,x)^2}{\sinh(c+d\,x)\,(a+b\sinh(c+d\,x))}\,dx$$

input
```
int((coth(c + d*x)^2*(e + f*x)^2)/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((coth(c + d*x)^2*(e + f*x)^2)/(sinh(c + d*x)*(a + b*sinh(c + d*x))), x
)
```

Reduce [F]

$$\int \frac{(e+fx)^2 \coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^2*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output

```
( - 72*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sq
rt(a**2 + b**2))*b**4*d**2*e**2*i + 144*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)
*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*b**4*d**2*e**2*i - 72*sq
rt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*b**4*d**2
*e**2*i - 1152*e**(7*c + 4*d*x)*int((e**(3*d*x)*x**2)/(e**(8*c + 8*d*x)*b
+ 2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e
**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c
+ d*x)*a + b),x)*a**6*d**3*f**2 - 1056*e**(7*c + 4*d*x)*int((e**(3*d*x)*x*
*2)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*
e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2
*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**4*b**2*d**3*f**2 - 2304*e**(7*
c + 4*d*x)*int((e**(3*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a -
 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e*
*(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**6*d*
*3*e*f - 2112*e**(7*c + 4*d*x)*int((e**(3*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*
e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4
*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*
x)*a + b),x)*a**4*b**2*d**3*e*f + 256*e**(7*c + 4*d*x)*int((e**(3*d*x)*x)/
(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(
5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*...
```

3.483 $\int \frac{(e+fx)\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 413

$$\begin{aligned}
&\int \frac{(e+fx)\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= -\frac{(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{ad}-\frac{2b^2(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{a^3d} \\
&\quad+\frac{b(e+fx)\coth(c+dx)}{a^2d}-\frac{f\operatorname{csch}(c+dx)}{2ad^2}-\frac{(e+fx)\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} \\
&\quad-\frac{b\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d}+\frac{b\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d} \\
&\quad-\frac{bf\log(\sinh(c+dx))}{a^2d^2}-\frac{f\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{2ad^2} \\
&\quad-\frac{b^2f\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{a^3d^2}+\frac{f\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{2ad^2}+\frac{b^2f\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{a^3d^2} \\
&\quad-\frac{b\sqrt{a^2+b^2}f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2}+\frac{b\sqrt{a^2+b^2}f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^2}
\end{aligned}$$

output
```
-(f*x+e)*arctanh(exp(d*x+c))/a/d-2*b^2*(f*x+e)*arctanh(exp(d*x+c))/a^3/d+b
*(f*x+e)*coth(d*x+c)/a^2/d-1/2*f*csch(d*x+c)/a/d^2-1/2*(f*x+e)*coth(d*x+c)
*csch(d*x+c)/a/d-b*(a^2+b^2)^(1/2)*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^
(1/2)))/a^3/d+b*(a^2+b^2)^(1/2)*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/
2)))/a^3/d-b*f*ln(sinh(d*x+c))/a^2/d^2-1/2*f*polylog(2,-exp(d*x+c))/a/d^2-
b^2*f*polylog(2,-exp(d*x+c))/a^3/d^2+1/2*f*polylog(2,exp(d*x+c))/a/d^2+b^2
*f*polylog(2,exp(d*x+c))/a^3/d^2-b*(a^2+b^2)^(1/2)*f*polylog(2,-b*exp(d*x+
c)/(a-(a^2+b^2)^(1/2)))/a^3/d^2+b*(a^2+b^2)^(1/2)*f*polylog(2,-b*exp(d*x+c
)/(a+(a^2+b^2)^(1/2)))/a^3/d^2
```

Mathematica [A] (warning: unable to verify)

Time = 8.64 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.49

$$\int \frac{(e+fx)\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{\left(2bde\cosh\left(\frac{1}{2}(c+dx)\right)-af\cosh\left(\frac{1}{2}(c+dx)\right)-2bcf\cosh\left(\frac{1}{2}(c+dx)\right)+2bf(c+dx)\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{4a^2d^2}$$
$$+\frac{(-de+cf-f(c+dx))\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8ad^2}$$
$$+\frac{-2abf(c+dx)+(-2abf+a^2(de+dfx)+2b^2(de+dfx))\log\left(1-e^{-c-dx}\right)-\left(2abf+a^2(de+dfx)\right.}{2}$$
$$-\frac{b\sqrt{a^2+b^2}\left(-2de\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+2cf\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+f(c+dx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-f(c+\right.}{a^3d^2}$$
$$+\frac{(-de+cf-f(c+dx))\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8ad^2}$$
$$+\frac{\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\left(2bde\sinh\left(\frac{1}{2}(c+dx)\right)+af\sinh\left(\frac{1}{2}(c+dx)\right)-2bcf\sinh\left(\frac{1}{2}(c+dx)\right)+2bf(c+dx)\right.}{4a^2d^2}$$

input
```
Integrate[((e + f*x)*Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
((2*b*d*e*Cosh[(c + d*x)/2] - a*f*Cosh[(c + d*x)/2] - 2*b*c*f*Cosh[(c + d*
x)/2] + 2*b*f*(c + d*x)*Cosh[(c + d*x)/2])*Csch[(c + d*x)/2])/(4*a^2*d^2)
+ ((-(d*e) + c*f - f*(c + d*x))*Csch[(c + d*x)/2]^2)/(8*a*d^2) + (-2*a*b*f
*(c + d*x) + (-2*a*b*f + a^2*(d*e + d*f*x) + 2*b^2*(d*e + d*f*x))*Log[1 -
E^(-c - d*x)] - (2*a*b*f + a^2*(d*e + d*f*x) + 2*b^2*(d*e + d*f*x))*Log[1
+ E^(-c - d*x)] + (a^2 + 2*b^2)*f*PolyLog[2, -E^(-c - d*x)] - (a^2 + 2*b^2
)*f*PolyLog[2, E^(-c - d*x)])/(2*a^3*d^2) - (b*Sqrt[a^2 + b^2]*(-2*d*e*Arc
Tanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*c*f*ArcTanh[(a + b*E^(c + d*
x))/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])] - f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + f*P
olyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - f*PolyLog[2, -((b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^3*d^2) + ((-(d*e) + c*f - f*(c + d*x
))*Sech[(c + d*x)/2]^2)/(8*a*d^2) + (Sech[(c + d*x)/2]*(2*b*d*e*Sinh[(c +
d*x)/2] + a*f*Sinh[(c + d*x)/2] - 2*b*c*f*Sinh[(c + d*x)/2] + 2*b*f*(c + d
*x)*Sinh[(c + d*x)/2]))/(4*a^2*d^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6121}$$

$$\frac{\int (e+fx)\coth^2(c+dx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5980}$$

$$\frac{\int (e+fx)\text{csch}^3(c+dx)dx+\int (e+fx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int i(e+fx)\csc(ic+idx)dx+\int -i(e+fx)\csc(ic+idx)^3dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx-i\int (e+fx)\csc(ic+idx)^3dx}{a}$$

$$\Big\downarrow \ 4670$$

$$\frac{-\frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+i\left(\frac{if\int \log(1-e^{c+dx})dx}{d}-\frac{if\int \log(1+e^{c+dx})dx}{d}+\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}\right)-i\int(e+fx)\csc(ic+idx)^3dx}{a}$$

$$\Big\downarrow \ 2715$$

$$\frac{-\frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+i\left(\frac{if\int e^{-c-dx}\log(1-e^{c+dx})de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log(1+e^{c+dx})de^{c+dx}}{d^2}+\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}\right)-i\int(e+fx)\csc(ic+idx}{a}$$

$$\Big\downarrow \ 2838$$

$$\frac{-\frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-i\int(e+fx)\csc(ic+idx)^3dx}{a}$$

$$\Big\downarrow \ 4673$$

$$\frac{-\frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-i\left(\frac{1}{2}\int -i(e+fx)\text{csch}(c+dx)dx-\frac{if\text{csch}(}{2d^2}\right.}{a}$$

$$\Big\downarrow \ 26$$

$$\frac{-\frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-i\left(-\frac{1}{2}i\int(e+fx)\text{csch}(c+dx)dx-\frac{if\text{csch}(}{2d^2}\right.}{a}$$

$$\Big\downarrow \ 3042$$

$$\frac{-\frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-i\left(-\frac{1}{2}i\int i(e+fx)\csc(ic+idx)dx-\frac{if\text{csc}}{2}\right.}{a}$$

$\downarrow$ 26

$$-\frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-i\left(\frac{1}{2}\int (e+fx)\csc(ic+idx)dx-\frac{if\text{csch}(c+}{2d^2}\right.}{a}$$

$\downarrow$ 4670

$$-\frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{if\int\log(1-e^{c+dx})dx}{d}-\frac{if\int\log(1+e^{c+dx})dx}{d}\right.\right.}{a}$$

$\downarrow$ 2715

$$-\frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{if\int e^{-c-dx}\log(1-e^{c+dx})de^{c+dx}}{d^2}-\frac{if\int e^{-c}}{}\right.\right.}{a}$$

$\downarrow$ 2838

$$-\frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,}{d^2}\right.\right.}{a}$$

$\downarrow$ 6103

$$-\frac{b\left(\frac{\int(e+fx)\coth^2(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,}{d^2}\right.\right.}{a}$$

$\downarrow$ 3042

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}(2,\cdots}{d^2}\right.\right.}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int-\left((e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)^2\right)dx}{a}\right)}{a}$$

$$\Big\downarrow 25$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}(2,\cdots}{d^2}\right.\right.}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\int(e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{a}\right)}{a}$$

$$\Big\downarrow 4203$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}(2,\cdots}{d^2}\right.\right.}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{if\int i\coth(c+dx)dx}{d}-\int(e+fx)dx+\frac{(e+fx)\coth(c+dx)}{d}}{a}\right)}{a}$$

$$\Big\downarrow 17$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}(2,\cdots}{d^2}\right.\right.}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{if\int i\coth(c+dx)dx}{d}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}\right)}{a}$$

$$\Big\downarrow 26$$

$$-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{-\frac{f\int\coth(c+dx)dx}{d}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}\right)}{a}+$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}(2,\cdots}{d^2}\right.\right.}{a}$$

$$\Big\downarrow 3042$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}(2,}{d^2}\right.\right.}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{-\frac{f\int -i\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}\right)}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}(2,}{d^2}\right.\right.}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{if\int\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}\right)}{a}$$

$$\Big\downarrow \; 3956$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}(2,}{d^2}\right.\right.}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}\right)}{a}$$

$$\Big\downarrow \; 6119$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}(2,}{d^2}\right.\right.}{a}$$

$$\frac{b\left(-\frac{b\left(\frac{\int(e+fx)\cosh(c+dx)\coth(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}\right)}{a}$$

$$\Big\downarrow \; 5973$$

$$\frac{\frac{i\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}(2,-}{d^2}\right.\right.}{a}}{}$$

$$\frac{b\left(-\frac{b\left(\frac{\int(e+fx)\sinh(c+dx)dx+\int(e+fx)\operatorname{csch}(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}\right)}{a}$$

$\downarrow$ 3042

$$\frac{\frac{i\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}(2,-}{d^2}\right.\right.}{a}}{}$$

$$\frac{b\left(-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int-i(e+fx)\sin(ic+idx)dx+\int i(e+fx)\csc(ic+idx)dx}{a}\right)}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}\right)}{a}$$

$\downarrow$ 26

$$\frac{\frac{i\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}(2,-}{d^2}\right.\right.}{a}}{}$$

$$\frac{b\left(-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int(e+fx)\csc(ic+idx)dx-i\int(e+fx)\sin(ic+idx)dx}{a}\right)}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}\right)}{a}$$

$\downarrow$ 3777

$$\frac{\frac{i\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}(2,-}{d^2}\right.\right.}{a}}{}$$

$$\frac{b\left(-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int(e+fx)\csc(ic+idx)dx-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{a}\right)}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c}{d}}{a}\right.}{a}$$

$\downarrow$ 3042

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,\cdot}{d^2}\right.\right.}{a}$$

$$\frac{b\left(-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int(e+fx)\csc(ic+idx)dx-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{a}\right)}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\cdot}{}}{a}\right.}{a}$$

$$\Big\downarrow \text{3117}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,\cdot}{d^2}\right.\right.}{a}$$

$$\frac{b\left(-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int(e+fx)\csc(ic+idx)dx-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}\right)}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx}{d}}{a}\right.}{a}$$

input
```
Int[((e + f*x)*Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1283 vs. $2(379) = 758$.

Time = 0.91 (sec) , antiderivative size = 1284, normalized size of antiderivative = 3.11

method	result	size
risch	Expression too large to display	1284

input
```
int((f*x+e)*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
1/d^2/a*b*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2
+b^2)^(1/2)))+2/d/a^3*b^3*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*
a)/(a^2+b^2)^(1/2))-1/d^2*b^2/a^3*f*dilog(exp(d*x+c)+1)+1/d*b^2/a^3*e*ln(e
xp(d*x+c)-1)-1/d*b^2/a^3*e*ln(exp(d*x+c)+1)-1/d^2*b/a^2*f*ln(exp(d*x+c)-1)
-1/d^2*b/a^2*f*ln(exp(d*x+c)+1)-1/d^2*b^2/a^3*f*dilog(exp(d*x+c))+1/d^2/a*
b*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)
))*c-2/d^2/a^3*c*b^3*f/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a
^2+b^2)^(1/2))-1/d^2/a^3*b^3*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)
^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d^2/a^3*b^3*f/(a^2+b^2)^(1/2)*ln((b*ex
p(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d/a*b*f/(a^2+b^2)^(1/
2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d/a*b*f/
(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x
-1/d/a^3*b^3*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a
^2+b^2)^(1/2)))*x+1/d/a^3*b^3*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)
^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2/a*b*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*
x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2/a/d^2*c*f*b/(a^2+b^2)^(1
/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/2/d^2*c*f/a*ln(exp
(d*x+c)-1)-1/2/d^2/a*f*dilog(exp(d*x+c))-1/2/d/a*e*ln(exp(d*x+c)+1)+1/2/d/
a*e*ln(exp(d*x+c)-1)-1/d^2*b^2/a^3*c*f*ln(exp(d*x+c)-1)-1/d*b^2/a^3*f*ln(e
xp(d*x+c)+1)*x+2/d^2/a^2*b*f*ln(exp(d*x+c))-1/2/d/a*f*ln(exp(d*x+c)+1)*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3585 vs. $2(373) = 746$.

Time = 0.22 (sec) , antiderivative size = 3585, normalized size of antiderivative = 8.68

$$\int \frac{(e+fx)\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)\coth^2{(c+dx)}\,\text{csch}\,(c+dx)}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*coth(d*x+c)**2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*coth(c + d*x)**2*csch(c + d*x)/(a + b*sinh(c + d*x)), x
)
```

Maxima [F]

$$\int \frac{(e+fx)\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)\coth{(dx+c)}^2\,\text{csch}\,(dx+c)}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
(2*a^2*d*integrate(1/4*x/(a^3*d*e^(d*x + c) + a^3*d), x) + 4*b^2*d*integra
te(1/4*x/(a^3*d*e^(d*x + c) + a^3*d), x) + 2*a^2*d*integrate(1/4*x/(a^3*d*
e^(d*x + c) - a^3*d), x) + 4*b^2*d*integrate(1/4*x/(a^3*d*e^(d*x + c) - a^
3*d), x) + a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) + 1)/(a^3*d^2)) + a*
b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) - 1)/(a^3*d^2)) - 2*(a^2*b*e^c +
b^3*e^c)*integrate(x*e^(d*x)/(a^3*b*e^(2*d*x + 2*c) + 2*a^4*e^(d*x + c) -
a^3*b), x) + (2*b*d*x*e^(2*d*x + 2*c) - 2*b*d*x - (a*d*x*e^(3*c) + a*e^(3*
c))*e^(3*d*x) - (a*d*x*e^c - a*e^c)*e^(d*x))/(a^2*d^2*e^(4*d*x + 4*c) - 2*
a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2))*f + 1/2*e*(2*(a*e^(-d*x - c) + 2*b*e^(
-2*d*x - 2*c) + a*e^(-3*d*x - 3*c) - 2*b)/((2*a^2*e^(-2*d*x - 2*c) - a^2*e
^(-4*d*x - 4*c) - a^2)*d) - (a^2 + 2*b^2)*log(e^(-d*x - c) + 1)/(a^3*d) +
(a^2 + 2*b^2)*log(e^(-d*x - c) - 1)/(a^3*d) - 2*(a^2*b + b^3)*log((b*e^(-d
*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(sq
rt(a^2 + b^2)*a^3*d))
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+d\,x)^2\,(e+f\,x)}{\sinh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((coth(c + d*x)^2*(e + f*x))/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((coth(c + d*x)^2*(e + f*x))/(sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 12*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sq
rt(a**2 + b**2))*b**4*d*e*i + 24*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((
e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*b**4*d*e*i - 12*sqrt(a**2 + b**
2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*b**4*d*e*i - 192*e**(7
*c + 4*d*x)*int((e**(3*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a
 - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e
**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**6*d
**2*f - 176*e**(7*c + 4*d*x)*int((e**(3*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e*
*(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c
 + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)
*a + b),x)*a**4*b**2*d**2*f + 128*e**(6*c + 4*d*x)*int((e**(2*d*x)*x)/(e**
(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c
+ 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d
*x)*b - 2*e**(c + d*x)*a + b),x)*a**5*b*d**2*f + 96*e**(6*c + 4*d*x)*int((
e**(2*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d
*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a
 - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**3*b**3*d**2*f - 16*e
**(5*c + 4*d*x)*int((e**(d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*
a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6
*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a...
```

3.484 $\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 111

$$\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{(a^2+2b^2)\operatorname{arctanh}(\cosh(c+dx))}{2a^3d} + \frac{2b\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3d} + \frac{b\coth(c+dx)}{a^2d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}$$

output
```
-1/2*(a^2+2*b^2)*arctanh(cosh(d*x+c))/a^3/d+2*b*(a^2+b^2)^(1/2)*arctanh((b
-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/a^3/d+b*coth(d*x+c)/a^2/d-1/2*cot
h(d*x+c)*csch(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 1.92 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.50

$$\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{16b\sqrt{-a^2-b^2}\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right) + 4ab\coth\left(\frac{1}{2}(c+dx)\right) - a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right) - 4(a^2+2b^2)\log}{8a^3}$$

input
```
Integrate[(Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(16*b*Sqrt[-a^2 - b^2]*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]]
+ 4*a*b*Coth[(c + d*x)/2] - a^2*Csch[(c + d*x)/2]^2 - 4*(a^2 + 2*b^2)*Log[
Cosh[(c + d*x)/2]] + 4*(a^2 + 2*b^2)*Log[Sinh[(c + d*x)/2]] - a^2*Sech[(c
+ d*x)/2]^2 + 4*a*b*Tanh[(c + d*x)/2])/(8*a^3*d)
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.23, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {3042, 26, 3368, 26, 3042, 26, 3535, 26, 3042, 25, 3534, 3042, 26, 3480, 26, 3042, 26, 3139, 1083, 217, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\cos(ic+idx)^2}{\sin(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos(ic+idx)^2}{\sin(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3368}$$

$$-i\int \frac{i\text{csch}^3(c+dx)\left(\sinh^2(c+dx)+1\right)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\left(\sinh^2(c+dx)+1\right)\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i(1-\sin(ic+idx)^2)}{\sin(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1-\sin(ic+idx)^2}{\sin(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3535}$$

$$-i\left(\frac{\int -\frac{i\text{csch}^2(c+dx)(b\sinh^2(c+dx)-a\sinh(c+dx)+2b)}{a+b\sinh(c+dx)}dx}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{i\int \frac{\text{csch}^2(c+dx)(b\sinh^2(c+dx)-a\sinh(c+dx)+2b)}{a+b\sinh(c+dx)}dx}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{i\int -\frac{-b\sin(ic+idx)^2+ia\sin(ic+idx)+2b}{\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{25}$$

$$-i\left(\frac{i\int \frac{-b\sin(ic+idx)^2+ia\sin(ic+idx)+2b}{\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{3534}$$

$$-i\left(\frac{i\left(\frac{\int \frac{\text{csch}(c+dx)\left(a^2-b\sinh(c+dx)a+2b^2\right)}{a+b\sinh(c+dx)}dx}{a}+\frac{2b\coth(c+dx)}{ad}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}+\frac{\int \frac{i\left(a^2+ib\sin(ic+idx)a+2b^2\right)}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \; 26$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}+\frac{i\int \frac{a^2+ib\sin(ic+idx)a+2b^2}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \; 3480$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}+\frac{i\left(\frac{2ib(a^2+b^2)\int \frac{1}{a+b\sinh(c+dx)}dx}{a}+\frac{(a^2+2b^2)\int -i\text{csch}(c+dx)dx}{a}\right)}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \; 26$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}+\frac{i\left(\frac{2ib(a^2+b^2)\int \frac{1}{a+b\sinh(c+dx)}dx}{a}-\frac{i(a^2+2b^2)\int \text{csch}(c+dx)dx}{a}\right)}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \; 3042$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}+\frac{i\left(\frac{2ib(a^2+b^2)\int \frac{1}{a-ib\sin(ic+idx)}dx}{a}-\frac{i(a^2+2b^2)\int i\csc(ic+idx)dx}{a}\right)}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \; 26$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}+\frac{i\left(\frac{2ib\left(a^2+b^2\right)\int\frac{1}{a-ib\sin(ic+idx)}dx}{a}+\frac{\left(a^2+2b^2\right)\int\csc(ic+idx)dx}{a}\right)}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$\big\downarrow$ 3139

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}+\frac{i\left(\frac{\left(a^2+2b^2\right)\int\csc(ic+idx)dx}{a}+\frac{4b\left(a^2+b^2\right)\int\frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{ad}\right)}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$\big\downarrow$ 1083

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}+\frac{i\left(\frac{\left(a^2+2b^2\right)\int\csc(ic+idx)dx}{a}-\frac{8b\left(a^2+b^2\right)\int\frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4\left(a^2+b^2\right)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{ad}\right)}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$\big\downarrow$ 217

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}+\frac{i\left(\frac{\left(a^2+2b^2\right)\int \csc(ic+idx)dx}{a}+\frac{4ib\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{ad}\right)}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$\downarrow$ 4257

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}+\frac{i\left(\frac{i\left(a^2+2b^2\right)\text{arctanh}(\cosh(c+dx))}{ad}+\frac{4ib\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{ad}\right)}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+
}{2ad}$$

input
```
Int[(Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-I)*(((I/2)*((I*((I*(a^2 + 2*b^2)*ArcTanh[Cosh[c + d*x]])/(a*d) + ((4*I)*
b*Sqrt[a^2 + b^2]*ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b^2])])/(a*d)))/
a + (2*b*Coth[c + d*x])/(a*d)))/a - ((I/2)*Coth[c + d*x]*Csch[c + d*x])/(a
*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3368
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3535
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n
+ 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d
*(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
 !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^{2}}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+\frac{\left(2a^{2}+4b^{2}\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^{3}}+\frac{b}{2a^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\sqrt{a^{2}+b^{2}}\ \mathrm{arctan}}{}}{d}$
default	$\frac{\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^{2}}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+\frac{\left(2a^{2}+4b^{2}\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^{3}}+\frac{b}{2a^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\sqrt{a^{2}+b^{2}}\ \mathrm{arctan}}{}}{d}$
risch	$-\frac{e^{3dx+3c}a-2b\,e^{2dx+2c}+a\,e^{dx+c}+2b}{d\,a^{2}\left(e^{2dx+2c}-1\right)^{2}}+\frac{\ln\left(e^{dx+c}-1\right)}{2da}+\frac{\ln\left(e^{dx+c}-1\right)b^{2}}{a^{3}d}-\frac{\ln\left(e^{dx+c}+1\right)}{2da}-\frac{\ln\left(e^{dx+c}+1\right)b^{2}}{a^{3}d}+$

input
```
int(coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4/a^2*(1/2*tanh(1/2*d*x+1/2*c)^2*a+2*b*tanh(1/2*d*x+1/2*c))-1/8/a/t
anh(1/2*d*x+1/2*c)^2+1/4/a^3*(2*a^2+4*b^2)*ln(tanh(1/2*d*x+1/2*c))+1/2*b/a
^2/tanh(1/2*d*x+1/2*c)-2*b*(a^2+b^2)^(1/2)/a^3*arctanh(1/2*(2*a*tanh(1/2*d
*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 892 vs. 2(104) = 208.

Time = 0.12 (sec) , antiderivative size = 892, normalized size of antiderivative = 8.04

$$\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
-1/2*(2*a^2*cosh(d*x + c)^3 + 2*a^2*sinh(d*x + c)^3 - 4*a*b*cosh(d*x + c)^
2 + 2*a^2*cosh(d*x + c) + 2*(3*a^2*cosh(d*x + c) - 2*a*b)*sinh(d*x + c)^2
- 2*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x +
c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 +
 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(a^2 + b^2
)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2
*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)
*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x
+ c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) +
 4*a*b + ((a^2 + 2*b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*b^2)*cosh(d*x + c)*si
nh(d*x + c)^3 + (a^2 + 2*b^2)*sinh(d*x + c)^4 - 2*(a^2 + 2*b^2)*cosh(d*x +
 c)^2 + 2*(3*(a^2 + 2*b^2)*cosh(d*x + c)^2 - a^2 - 2*b^2)*sinh(d*x + c)^2
+ a^2 + 2*b^2 + 4*((a^2 + 2*b^2)*cosh(d*x + c)^3 - (a^2 + 2*b^2)*cosh(d*x
+ c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - ((a^2 + 2*b^
2)*cosh(d*x + c)^4 + 4*(a^2 + 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2
+ 2*b^2)*sinh(d*x + c)^4 - 2*(a^2 + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2
*b^2)*cosh(d*x + c)^2 - a^2 - 2*b^2)*sinh(d*x + c)^2 + a^2 + 2*b^2 + 4*((a
^2 + 2*b^2)*cosh(d*x + c)^3 - (a^2 + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*
log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(3*a^2*cosh(d*x + c)^2 - 4*a*b*
cosh(d*x + c) + a^2)*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*co...
```

Sympy [F]

$$\int \frac{\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\coth^2{(c+dx)}\,\text{csch}\,(c+dx)}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate(coth(d*x+c)**2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(coth(c + d*x)**2*csch(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(104) = 208$.

Time = 0.13 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.95

$$\int \frac{\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{ae^{(-dx-c)}+2\,be^{(-2\,dx-2\,c)}+ae^{(-3\,dx-3\,c)}-2\,b}{(2\,a^2e^{(-2\,dx-2\,c)}-a^2e^{(-4\,dx-4\,c)}-a^2)d} - \frac{(a^2+2\,b^2)\log\left(e^{(-dx-c)}+1\right)}{2\,a^3d} + \frac{(a^2+2\,b^2)\log\left(e^{(-dx-c)}-1\right)}{2\,a^3d} - \frac{(a^2b+b^3)\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a^3d}$$

input
```
integrate(coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
(a*e^(-d*x - c) + 2*b*e^(-2*d*x - 2*c) + a*e^(-3*d*x - 3*c) - 2*b)/((2*a^2
*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) - 1/2*(a^2 + 2*b^2)*log
(e^(-d*x - c) + 1)/(a^3*d) + 1/2*(a^2 + 2*b^2)*log(e^(-d*x - c) - 1)/(a^3*
d) - (a^2*b + b^3)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x -
 c) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3*d)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.64

$$\int \frac{\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{(a^2+2\,b^2)\log(e^{(dx+c)}+1)}{a^3} - \frac{(a^2+2\,b^2)\log(|e^{(dx+c)}-1|)}{a^3} + \frac{2\,(a^2b+b^3)\log\left(\frac{|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}|}{|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}|}\right)}{\sqrt{a^2+b^2}a^3} + \frac{2\,(ae^{(3\,dx+3\,c)}-2\,be^{(2\,dx+2}}{a^2\,(e^{(2\,dx+2\,c)}}}{2\,d}$$

input
```
integrate(coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*((a^2 + 2*b^2)*log(e^(d*x + c) + 1)/a^3 - (a^2 + 2*b^2)*log(abs(e^(d*
x + c) - 1))/a^3 + 2*(a^2*b + b^3)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(
a^2 + b^2))/abs(2*b*e^(d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^
2)*a^3) + 2*(a*e^(3*d*x + 3*c) - 2*b*e^(2*d*x + 2*c) + a*e^(d*x + c) + 2*b
)/(a^2*(e^(2*d*x + 2*c) - 1)^2))/d
```

Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 628, normalized size of antiderivative = 5.66

$$\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{e^{c+dx}}{a\,d-a\,d\,e^{2\,c+2\,d\,x}} - \frac{2\,e^{c+dx}}{a\,d-2\,a\,d\,e^{2\,c+2\,d\,x}+a\,d\,e^{4\,c+4\,d\,x}} - \frac{2\,b}{a^2\,d-a^2\,d\,e^{2\,c+2\,d\,x}}$$
$$+ \frac{\ln\left(4\,a^4+8\,b^4+12\,a^2\,b^2-4\,a^4\,e^{d\,x}\,e^c-8\,b^4\,e^{d\,x}\,e^c-12\,a^2\,b^2\,e^{d\,x}\,e^c\right)}{2\,a\,d}$$
$$- \frac{\ln\left(4\,a^4+8\,b^4+12\,a^2\,b^2+4\,a^4\,e^{d\,x}\,e^c+8\,b^4\,e^{d\,x}\,e^c+12\,a^2\,b^2\,e^{d\,x}\,e^c\right)}{2\,a\,d}$$
$$+ \frac{b^2\,\ln\left(4\,a^4+8\,b^4+12\,a^2\,b^2-4\,a^4\,e^{d\,x}\,e^c-8\,b^4\,e^{d\,x}\,e^c-12\,a^2\,b^2\,e^{d\,x}\,e^c\right)}{a^3\,d}$$
$$- \frac{b^2\,\ln\left(4\,a^4+8\,b^4+12\,a^2\,b^2+4\,a^4\,e^{d\,x}\,e^c+8\,b^4\,e^{d\,x}\,e^c+12\,a^2\,b^2\,e^{d\,x}\,e^c\right)}{a^3\,d}$$
$$- \frac{b\,\ln\left(32\,a^4\,e^{d\,x}\,e^c-16\,a\,b^3-16\,a^3\,b-8\,b^3\,\sqrt{a^2+b^2}+8\,b^4\,e^{d\,x}\,e^c-16\,a^2\,b\,\sqrt{a^2+b^2}+40\,a^2\,b^2\,e^{d\,x}\,e^c+\right.}{a^3\,d}$$
$$+ \frac{b\,\ln\left(8\,b^3\,\sqrt{a^2+b^2}-16\,a\,b^3-16\,a^3\,b+32\,a^4\,e^{d\,x}\,e^c+8\,b^4\,e^{d\,x}\,e^c+16\,a^2\,b\,\sqrt{a^2+b^2}+40\,a^2\,b^2\,e^{d\,x}\,e^c-\right.}{a^3\,d}$$

input
```
int(coth(c + d*x)^2/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
exp(c + d*x)/(a*d - a*d*exp(2*c + 2*d*x)) - (2*exp(c + d*x))/(a*d - 2*a*d*
exp(2*c + 2*d*x) + a*d*exp(4*c + 4*d*x)) - (2*b)/(a^2*d - a^2*d*exp(2*c +
2*d*x)) + log(4*a^4 + 8*b^4 + 12*a^2*b^2 - 4*a^4*exp(d*x)*exp(c) - 8*b^4*e
xp(d*x)*exp(c) - 12*a^2*b^2*exp(d*x)*exp(c))/(2*a*d) - log(4*a^4 + 8*b^4 +
 12*a^2*b^2 + 4*a^4*exp(d*x)*exp(c) + 8*b^4*exp(d*x)*exp(c) + 12*a^2*b^2*e
xp(d*x)*exp(c))/(2*a*d) + (b^2*log(4*a^4 + 8*b^4 + 12*a^2*b^2 - 4*a^4*exp(
d*x)*exp(c) - 8*b^4*exp(d*x)*exp(c) - 12*a^2*b^2*exp(d*x)*exp(c)))/(a^3*d)
 - (b^2*log(4*a^4 + 8*b^4 + 12*a^2*b^2 + 4*a^4*exp(d*x)*exp(c) + 8*b^4*exp
(d*x)*exp(c) + 12*a^2*b^2*exp(d*x)*exp(c)))/(a^3*d) - (b*log(32*a^4*exp(d*
x)*exp(c) - 16*a*b^3 - 16*a^3*b - 8*b^3*(a^2 + b^2)^(1/2) + 8*b^4*exp(d*x)
*exp(c) - 16*a^2*b*(a^2 + b^2)^(1/2) + 40*a^2*b^2*exp(d*x)*exp(c) + 32*a^3
*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2) + 24*a*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^
(1/2))*(a^2 + b^2)^(1/2))/(a^3*d) + (b*log(8*b^3*(a^2 + b^2)^(1/2) - 16*a*
b^3 - 16*a^3*b + 32*a^4*exp(d*x)*exp(c) + 8*b^4*exp(d*x)*exp(c) + 16*a^2*b
*(a^2 + b^2)^(1/2) + 40*a^2*b^2*exp(d*x)*exp(c) - 32*a^3*exp(d*x)*exp(c)*(
a^2 + b^2)^(1/2) - 24*a*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2))*(a^2 + b^2)
^(1/2))/(a^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 472, normalized size of antiderivative = 4.25

$$\int \frac{\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-4e^{4dx+4c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)bi+8e^{2dx+2c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)bi-4\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+}{\sqrt{a^2+b^2}}\right.}{}$$

input
```
int(coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
( - 4*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqr
t(a**2 + b**2))*b*i + 8*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d
*x)*b*i + a*i)/sqrt(a**2 + b**2))*b*i - 4*sqrt(a**2 + b**2)*atan((e**(c +
d*x)*b*i + a*i)/sqrt(a**2 + b**2))*b*i + e**(4*c + 4*d*x)*log(e**(c + d*x)
 - 1)*a**2 + 2*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*b**2 - e**(4*c + 4*d
*x)*log(e**(c + d*x) + 1)*a**2 - 2*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*
b**2 + 2*e**(4*c + 4*d*x)*a*b - 2*e**(3*c + 3*d*x)*a**2 - 2*e**(2*c + 2*d*
x)*log(e**(c + d*x) - 1)*a**2 - 4*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*b
**2 + 2*e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*a**2 + 4*e**(2*c + 2*d*x)*l
og(e**(c + d*x) + 1)*b**2 - 2*e**(c + d*x)*a**2 + log(e**(c + d*x) - 1)*a*
*2 + 2*log(e**(c + d*x) - 1)*b**2 - log(e**(c + d*x) + 1)*a**2 - 2*log(e**
(c + d*x) + 1)*b**2 - 2*a*b)/(2*a**3*d*(e**(4*c + 4*d*x) - 2*e**(2*c + 2*d
*x) + 1))
```

3.485 $\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(coth(d*x+c)^2*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 67.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Coth[c + d*x]^2*Csch[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Coth[c + d*x]^2*Csch[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(c+dx)\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$\Big\downarrow \; 6125$$
$$\int \frac{\coth^2(c+dx)\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Coth[c + d*x]^2*Csch[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\coth\left(dx+c\right)^{2} \operatorname{csch}\left(dx+c\right)}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(coth(d*x+c)^2*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(coth(d*x+c)^2*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\coth^2(c+dx)\mathrm{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth\left(dx+c\right)^2\mathrm{csch}\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(coth(d*x+c)^2*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(coth(d*x + c)^2*csch(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 19.43 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\coth^2(c+dx)\mathrm{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth^2\left(c+dx\right)\mathrm{csch}\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

input
```
integrate(coth(d*x+c)**2*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(coth(c + d*x)**2*csch(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)),
 x)
```

Maxima [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 762, normalized size of antiderivative = 22.41

$$\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(dx+c)^2\operatorname{csch}(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(coth(d*x+c)^2*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-2*(a^2*b*e^c + b^3*e^c)*integrate(-e^(d*x)/(a^3*b*f*x + a^3*b*e - (a^3*b*
f*x*e^(2*c) + a^3*b*e*e^(2*c))*e^(2*d*x) - 2*(a^4*f*x*e^c + a^4*e*e^c)*e^(
d*x)), x) - (2*b*d*f*x + 2*b*d*e + (a*d*f*x*e^(3*c) + (d*e - f)*a*e^(3*c))
*e^(3*d*x) - 2*(b*d*f*x*e^(2*c) + b*d*e*e^(2*c))*e^(2*d*x) + (a*d*f*x*e^c
+ (d*e + f)*a*e^c)*e^(d*x))/(a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + a^2*d^2*e
^2 + (a^2*d^2*f^2*x^2*e^(4*c) + 2*a^2*d^2*e*f*x*e^(4*c) + a^2*d^2*e^2*e^(4
*c))*e^(4*d*x) - 2*(a^2*d^2*f^2*x^2*e^(2*c) + 2*a^2*d^2*e*f*x*e^(2*c) + a^
2*d^2*e^2*e^(2*c))*e^(2*d*x)) + 2*integrate(-1/4*(2*b^2*d^2*e^2 + 2*a*b*d*
e*f + (d^2*e^2 + 2*f^2)*a^2 + (a^2*d^2*f^2 + 2*b^2*d^2*f^2)*x^2 + 2*(a^2*d
^2*e*f + 2*b^2*d^2*e*f + a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*
x^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*e^3 - (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e
*f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)*e^(d*x)), x) + 2*i
ntegrate(1/4*(2*b^2*d^2*e^2 - 2*a*b*d*e*f + (d^2*e^2 + 2*f^2)*a^2 + (a^2*d
^2*f^2 + 2*b^2*d^2*f^2)*x^2 + 2*(a^2*d^2*e*f + 2*b^2*d^2*e*f - a*b*d*f^2)*
x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*e^
3 + (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c
 + a^3*d^2*e^3*e^c)*e^(d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(coth(d*x+c)^2*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 2.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\coth(c+d\,x)^2}{\sinh\left(c+d\,x\right)\,\left(e+f\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(coth(c + d*x)^2/(sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(coth(c + d*x)^2/(sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$=\int \frac{\coth\left(dx+c\right)^2\operatorname{csch}(dx+c)}{\sinh\left(dx+c\right)be+\sinh\left(dx+c\right)bfx+ae+afx}dx$$

input
```
int(coth(d*x+c)^2*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int((coth(c + d*x)**2*csch(c + d*x))/(sinh(c + d*x)*b*e + sinh(c + d*x)*b*
f*x + a*e + a*f*x),x)
```

3.486 $\int \frac{(e+fx)^3 \coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 972

$$\int \frac{(e+fx)^3 \coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output
```
3/2*f*(f*x+e)^2*polylog(2,exp(2*d*x+2*c))/a/d^2-3/2*f^2*(f*x+e)*polylog(3,
exp(2*d*x+2*c))/a/d^3+3/4*f^3*polylog(4,exp(2*d*x+2*c))/a/d^4-6*(a^2+b^2)*
f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d^4-(a^2+b^2)*(f*x+e)
^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d-(a^2+b^2)*(f*x+e)^3*ln(1+b
*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d-6*(a^2+b^2)*f^3*polylog(4,-b*exp(d*
x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d^4+6*b*f*(f*x+e)^2*arctanh(exp(d*x+c))/a^2/
d^2+6*b*f^2*(f*x+e)*polylog(2,-exp(d*x+c))/a^2/d^3-6*b*f^2*(f*x+e)*polylog
(2,exp(d*x+c))/a^2/d^3+3/2*b^2*f*(f*x+e)^2*polylog(2,exp(2*d*x+2*c))/a^3/d
^2-3/2*b^2*f^2*(f*x+e)*polylog(3,exp(2*d*x+2*c))/a^3/d^3-3/2*f*(f*x+e)^2*c
oth(d*x+c)/a/d^2+3*f^2*(f*x+e)*ln(1-exp(2*d*x+2*c))/a/d^3-6*b*f^3*polylog(
3,-exp(d*x+c))/a^2/d^4+6*b*f^3*polylog(3,exp(d*x+c))/a^2/d^4+3/4*b^2*f^3*p
olylog(4,exp(2*d*x+2*c))/a^3/d^4-3/2*f*(f*x+e)^2/a/d^2+(f*x+e)^3*ln(1-exp(
2*d*x+2*c))/a/d+b*(f*x+e)^3*csch(d*x+c)/a^2/d+b^2*(f*x+e)^3*ln(1-exp(2*d*x
+2*c))/a^3/d+3/2*f^3*polylog(2,exp(2*d*x+2*c))/a/d^4-1/4*b^2*(f*x+e)^4/a^3
/f+1/4*(a^2+b^2)*(f*x+e)^4/a^3/f-1/2*(f*x+e)^3*coth(d*x+c)^2/a/d+1/2*(f*x+
e)^3/a/d-1/4*(f*x+e)^4/a/f-3*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)
/(a+(a^2+b^2)^(1/2)))/a^3/d^2-3*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x
+c)/(a-(a^2+b^2)^(1/2)))/a^3/d^2+6*(a^2+b^2)*f^2*(f*x+e)*polylog(3,-b*exp(
d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d^3+6*(a^2+b^2)*f^2*(f*x+e)*polylog(3,-b*e
xp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3868 vs. $2(972) = 1944$.

Time = 12.17 (sec) , antiderivative size = 3868, normalized size of antiderivative = 3.98

$$\int \frac{(e+fx)^3 \coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Coth[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(b*(e + f*x)^3*Csch[c])/(a^2*d) + ((-e^3 - 3*e^2*f*x - 3*e*f^2*x^2 - f^3*x
^3)*Csch[c/2 + (d*x)/2]^2)/(8*a*d) - (8*a^2*d^4*e^3*E^(2*c)*x + 8*b^2*d^4*
e^3*E^(2*c)*x + 24*a^2*d^2*e*E^(2*c)*f^2*x + 12*a^2*d^4*e^2*E^(2*c)*f*x^2
+ 12*b^2*d^4*e^2*E^(2*c)*f*x^2 + 12*a^2*d^2*E^(2*c)*f^3*x^2 + 8*a^2*d^4*e*
E^(2*c)*f^2*x^3 + 8*b^2*d^4*e*E^(2*c)*f^2*x^3 + 2*a^2*d^4*E^(2*c)*f^3*x^4
+ 2*b^2*d^4*E^(2*c)*f^3*x^4 + 24*a*b*d^2*e^2*f*ArcTanh[E^(c + d*x)] - 24*a
*b*d^2*e^2*E^(2*c)*f*ArcTanh[E^(c + d*x)] - 24*a*b*d^2*e*f^2*x*Log[1 - E^(
c + d*x)] + 24*a*b*d^2*e*E^(2*c)*f^2*x*Log[1 - E^(c + d*x)] - 12*a*b*d^2*f
^3*x^2*Log[1 - E^(c + d*x)] + 12*a*b*d^2*E^(2*c)*f^3*x^2*Log[1 - E^(c + d*
x)] + 24*a*b*d^2*e*f^2*x*Log[1 + E^(c + d*x)] - 24*a*b*d^2*e*E^(2*c)*f^2*x
*Log[1 + E^(c + d*x)] + 12*a*b*d^2*f^3*x^2*Log[1 + E^(c + d*x)] - 12*a*b*d
^2*E^(2*c)*f^3*x^2*Log[1 + E^(c + d*x)] + 4*a^2*d^3*e^3*Log[1 - E^(2*(c +
d*x))] + 4*b^2*d^3*e^3*Log[1 - E^(2*(c + d*x))] - 4*a^2*d^3*e^3*E^(2*c)*Lo
g[1 - E^(2*(c + d*x))] - 4*b^2*d^3*e^3*E^(2*c)*Log[1 - E^(2*(c + d*x))] +
12*a^2*d*e*f^2*Log[1 - E^(2*(c + d*x))] - 12*a^2*d*e*E^(2*c)*f^2*Log[1 - E
^(2*(c + d*x))] + 12*a^2*d^3*e^2*f*x*Log[1 - E^(2*(c + d*x))] + 12*b^2*d^3
*e^2*f*x*Log[1 - E^(2*(c + d*x))] - 12*a^2*d^3*e^2*E^(2*c)*f*x*Log[1 - E^(
2*(c + d*x))] - 12*b^2*d^3*e^2*E^(2*c)*f*x*Log[1 - E^(2*(c + d*x))] + 12*a
^2*d*f^3*x*Log[1 - E^(2*(c + d*x))] - 12*a^2*d*E^(2*c)*f^3*x*Log[1 - E^(2*
(c + d*x))] + 12*a^2*d^3*e*f^2*x^2*Log[1 - E^(2*(c + d*x))] + 12*b^2*d^...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6103}$$

$$\frac{\int (e+fx)^3\coth^3(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int i(e+fx)^3\tan\left(ic+idx+\frac{\pi}{2}\right)^3dx}{a}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^3dx}{a}$$

$\big\downarrow$ 4203

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{3if\int -(e+fx)^2\coth^2(c+dx)dx}{2d}-\int i(e+fx)^3\coth(c+dx)dx+\frac{i(e+fx)^3\coth^2(c+dx)}{2d}\right)}{a}$$

$\big\downarrow$ 25

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\frac{3if\int (e+fx)^2\coth^2(c+dx)dx}{2d}-\int i(e+fx)^3\coth(c+dx)dx+\frac{i(e+fx)^3\coth^2(c+dx)}{2d}\right)}{a}$$

$\big\downarrow$ 26

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\frac{3if\int (e+fx)^2\coth^2(c+dx)dx}{2d}-i\int (e+fx)^3\coth(c+dx)dx+\frac{i(e+fx)^3\coth^2(c+dx)}{2d}\right)}{a}$$

$\big\downarrow$ 3042

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-i\int -i(e+fx)^3\tan\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{3if\int -(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)^2dx}{2d}+\frac{i(e+fx)^3\coth^2(c+dx)}{2d}\right)}{a}$$

$\big\downarrow$ 25

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-i\int -i(e+fx)^3\tan\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{3if\int (e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{2d}+\frac{i(e+fx)^3\coth^2(c+dx)}{2d}\right)}{a}$$

$\big\downarrow$ 26

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\int (e+fx)^3\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx+\frac{3if\int (e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{2d}+\frac{i(e+fx)^3\coth^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \; 4201$$

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-2i\int \frac{e^{2c+2dx-i\pi}(e+fx)^3}{1+e^{2c+2dx-i\pi}}dx+\frac{3if\int (e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{2d}+\frac{i(e+fx)^3\coth^2(c+dx)}{2d}+\frac{i(e+fx)^4}{4f}\right)}{a}$$

$$\Big\downarrow \; 2620$$

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\int (e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)+\frac{3if\int (e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{2d}+\frac{i(e+fx)^3\coth^2(}{2d}\right.}{a}$$

$$\Big\downarrow \; 3011$$

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\int (e+fx)^2\right.}{a}$$

$$\Big\downarrow \; 4203$$

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(\frac{2if\int i(e+\right.}{}\right.}{a}$$

$$\Big\downarrow \; 17$$

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(\frac{2if\int i(e+\right.}{}\right.}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$
$$\frac{i\left(-2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(-\frac{2f\int(e+\cdots}{\cdots}\right.}{\cdots}}{a}$$

$\downarrow$ 3042

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$
$$\frac{i\left(-2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(-\frac{2f\int -i(\cdots}{\cdots}\right.}{\cdots}}{a}$$

$\downarrow$ 26

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$
$$\frac{i\left(-2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(\frac{2if\int(e+f\cdots}{\cdots}\right.}{\cdots}}{a}$$

$\downarrow$ 4201

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$
$$\frac{i\left(-2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(\frac{2if\left(2i\int\cdots\right.}{\cdots}\right.}{\cdots}}{a}$$

$\downarrow$ 2620

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$i\left(-2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(\frac{2if\left(2i\left(\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 2715

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$i\left(\frac{3if\left(\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\log\left(1+e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}\right)}{2d}-\cdots\right.$$

$\downarrow$ 2838

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$i\left(-2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(\frac{2if\left(2i\left(\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 6119

$$-\frac{b\left(\frac{\int (e+fx)^3\cosh(c+dx)\coth^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+$$

$$\frac{i\left(-2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(\frac{2if\left(2i\left(\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.}{a\cdots}$$

$\downarrow$ 5973

$$-\frac{b\left(\frac{\int (e+fx)^3\cosh(c+dx)dx+\int (e+fx)^3\coth(c+dx)\operatorname{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+$$

$$\frac{i\left(-2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(\frac{2if\left(2i\left(\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.}{a\cdots}$$

$\downarrow$ 3042

$$\frac{i\left(-2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(\frac{2if\left(2i\left(\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.}{a\cdots}$$

$$\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)^3\coth(c+dx)\operatorname{csch}(c+dx)dx+\int (e+fx)^3\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{a}\right)}{a}$$

$\downarrow$ 3777

$$\frac{i\left(-2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(\frac{2if\left(2i\left(\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.}{c\cdots}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{3if\int-i(e+fx)^2\sinh(c+dx)dx}{d}+\int(e+fx)^3\coth(c+dx)\operatorname{csch}(c+dx)dx+\frac{(e+fx)^3\sinh(c+dx)}{d}}{a}\right)}{a}$$

$\downarrow$ 26

$$-\frac{b\left(\frac{-\frac{3f\int(e+fx)^2\sinh(c+dx)dx}{d}+\int(e+fx)^3\coth(c+dx)\operatorname{csch}(c+dx)dx+\frac{(e+fx)^3\sinh(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+$$

$$\frac{i\left(-2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(\frac{2if\left(2i\left(\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.}{c\cdots}$$

$\downarrow$ 3042

$$\frac{i\left(-2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(\frac{2if\left(2i\left(\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.}{c\cdots}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{3f\int-i(e+fx)^2\sin(ic+idx)dx}{d}+\int(e+fx)^3\coth(c+dx)\operatorname{csch}(c+dx)dx+\frac{(e+fx)^3\sinh(c+dx)}{d}}{a}\right)}{a}$$

$\downarrow$ 26

$$\frac{i\left(-2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(\frac{2if\left(2i\left(\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{3if\int(e+fx)^2\sin(ic+idx)dx}{d}+\int(e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^3\sinh(c+dx)}{d}}{a}\right)}{a}$$

$\downarrow$ 3777

$$\frac{i\left(-2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(\frac{2if\left(2i\left(\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\cosh(c+dx)dx}{d}\right)}{d}+\int(e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+\cdots}{\cdots}}{a}\right.}{a}$$

$\downarrow$ 3042

$$\frac{i\left(-2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(\frac{2if\left(2i\left(\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}+\int(e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx\cdots}{a}\right.}{a}$$

input
```
Int[((e + f*x)^3*Coth[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^3 \coth(dx+c)^3}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13683 vs. 2(908) = 1816.

Time = 0.36 (sec) , antiderivative size = 13683, normalized size of antiderivative = 14.08

$$\int \frac{(e+fx)^3\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^3\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^3\coth^3{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**3*coth(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*coth(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)^3\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\coth{(dx+c)}^3}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^3*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e^3*(2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2
*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + (a^2 + b^2)*log(-2*a*
e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^3*d) - (a^2 + b^2)*log(e^(-d*x -
 c) + 1)/(a^3*d) - (a^2 + b^2)*log(e^(-d*x - c) - 1)/(a^3*d)) + (3*a*f^3*x
^2 + 6*a*e*f^2*x + 3*a*e^2*f + 2*(b*d*f^3*x^3*e^(3*c) + 3*b*d*e*f^2*x^2*e^
(3*c) + 3*b*d*e^2*f*x*e^(3*c))*e^(3*d*x) - (2*a*d*f^3*x^3*e^(2*c) + 3*a*e^
2*f*e^(2*c) + 3*(2*d*e*f^2 + f^3)*a*x^2*e^(2*c) + 6*(d*e^2*f + e*f^2)*a*x*
e^(2*c))*e^(2*d*x) - 2*(b*d*f^3*x^3*e^c + 3*b*d*e*f^2*x^2*e^c + 3*b*d*e^2*
f*x*e^c)*e^(d*x))/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) + a
^2*d^2) - 3*(b*d*e^2*f + a*e*f^2)*x/(a^2*d^2) + 3*(b*d*e^2*f - a*e*f^2)*x/
(a^2*d^2) + 3*(b*d*e^2*f + a*e*f^2)*log(e^(d*x + c) + 1)/(a^2*d^3) - 3*(b*
d*e^2*f - a*e*f^2)*log(e^(d*x + c) - 1)/(a^2*d^3) + (d^3*x^3*log(e^(d*x +
c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) +
 6*polylog(4, -e^(d*x + c)))*(a^2*f^3 + b^2*f^3)/(a^3*d^4) + (d^3*x^3*log(
-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x
 + c)) + 6*polylog(4, e^(d*x + c)))*(a^2*f^3 + b^2*f^3)/(a^3*d^4) + 3*(a^2
*d*e*f^2 + b^2*d*e*f^2 + a*b*f^3)*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*di
log(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))/(a^3*d^4) + 3*(a^2*d*e*f^2
 + b^2*d*e*f^2 - a*b*f^3)*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(
d*x + c)) - 2*polylog(3, e^(d*x + c)))/(a^3*d^4) + 3*(b^2*d^2*e^2*f + 2...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\coth(c+d\,x)^3\,(e+f\,x)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((coth(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((coth(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{too large to display}$$

input
```
int((f*x+e)^3*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output

```
(2304*e**(7*c + 4*d*x)*int((e**(3*d*x)*x**3)/(e**(8*c + 8*d*x)*b + 2*e**(7
*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c +
4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a
+ b),x)*a**7*d**4*f**3 + 3264*e**(7*c + 4*d*x)*int((e**(3*d*x)*x**3)/(e**(
8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c +
 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*
x)*b - 2*e**(c + d*x)*a + b),x)*a**5*b**2*d**4*f**3 + 960*e**(7*c + 4*d*x)
*int((e**(3*d*x)*x**3)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(
6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c +
 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**3*b**4*d**4
*f**3 + 6912*e**(7*c + 4*d*x)*int((e**(3*d*x)*x**2)/(e**(8*c + 8*d*x)*b +
2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**
(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c +
d*x)*a + b),x)*a**7*d**4*e*f**2 + 9792*e**(7*c + 4*d*x)*int((e**(3*d*x)*x*
*2)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*
e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2
*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**5*b**2*d**4*e*f**2 - 768*e**(7
*c + 4*d*x)*int((e**(3*d*x)*x**2)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)
*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*b +
6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*...
```

3.487 $\int \frac{(e+fx)^2 \coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 680

$$\int \frac{(e+fx)^2\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{(e+fx)^2}{2ad}-\frac{(e+fx)^3}{3af}-\frac{b^2(e+fx)^3}{3a^3f}+\frac{(a^2+b^2)(e+fx)^3}{3a^3f}+\frac{4bf(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{a^2d^2}-\frac{f(e+fx)\coth(c+dx)}{ad^2}-\frac{(e+fx)^2\coth^2(c+dx)}{2ad}+\frac{b(e+fx)^2\text{csch}(c+dx)}{a^2d}-\frac{(a^2+b^2)(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d}-\frac{(a^2+b^2)(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d}+\frac{(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{ad}+\frac{b^2(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{a^3d}+\frac{f^2\log(\sinh(c+dx))}{ad^3}+\frac{2bf^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{a^2d^3}-\frac{2bf^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{a^2d^3}-\frac{2(a^2+b^2)f(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2}-\frac{2(a^2+b^2)f(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^2}+\frac{f(e+fx)\,\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{ad^2}+\frac{b^2f(e+fx)\,\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{a^3d^2}+\frac{2(a^2+b^2)f^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^3}+\frac{2(a^2+b^2)f^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^3}-\frac{f^2\,\text{PolyLog}\left(3,e^{2(c+dx)}\right)}{2ad^3}-\frac{b^2f^2\,\text{PolyLog}\left(3,e^{2(c+dx)}\right)}{2a^3d^3}$$

output
```
1/2*(f*x+e)^2/a/d-1/3*(f*x+e)^3/a/f-1/3*b^2*(f*x+e)^3/a^3/f+1/3*(a^2+b^2)*
(f*x+e)^3/a^3/f+4*b*f*(f*x+e)*arctanh(exp(d*x+c))/a^2/d^2-f*(f*x+e)*coth(d
*x+c)/a/d^2-1/2*(f*x+e)^2*coth(d*x+c)^2/a/d+b*(f*x+e)^2*csch(d*x+c)/a^2/d-
(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d-(a^2+b^2)
*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d+(f*x+e)^2*ln(1-exp
(2*d*x+2*c))/a/d+b^2*(f*x+e)^2*ln(1-exp(2*d*x+2*c))/a^3/d+f^2*ln(sinh(d*x+
c))/a/d^3+2*b*f^2*polylog(2,-exp(d*x+c))/a^2/d^3-2*b*f^2*polylog(2,exp(d*x
+c))/a^2/d^3-2*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1
/2)))/a^3/d^2-2*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(
1/2)))/a^3/d^2+f*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a/d^2+b^2*f*(f*x+e)*pol
ylog(2,exp(2*d*x+2*c))/a^3/d^2+2*(a^2+b^2)*f^2*polylog(3,-b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))/a^3/d^3+2*(a^2+b^2)*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+
b^2)^(1/2)))/a^3/d^3-1/2*f^2*polylog(3,exp(2*d*x+2*c))/a/d^3-1/2*b^2*f^2*p
olylog(3,exp(2*d*x+2*c))/a^3/d^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2403 vs. $2(680) = 1360$.

Time = 10.75 (sec) , antiderivative size = 2403, normalized size of antiderivative = 3.53

$$\int \frac{(e+fx)^2\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Coth[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(b*(e + f*x)^2*Csch[c])/(a^2*d) + ((-e^2 - 2*e*f*x - f^2*x^2)*Csch[c/2 + (
d*x)/2]^2)/(8*a*d) - (12*a^2*d^3*e^2*E^(2*c)*x + 12*b^2*d^3*e^2*E^(2*c)*x
+ 12*a^2*d*E^(2*c)*f^2*x + 12*a^2*d^3*e*E^(2*c)*f*x^2 + 12*b^2*d^3*e*E^(2*
c)*f*x^2 + 4*a^2*d^3*E^(2*c)*f^2*x^3 + 4*b^2*d^3*E^(2*c)*f^2*x^3 + 24*a*b*
d*e*f*ArcTanh[E^(c + d*x)] - 24*a*b*d*e*E^(2*c)*f*ArcTanh[E^(c + d*x)] - 1
2*a*b*d*f^2*x*Log[1 - E^(c + d*x)] + 12*a*b*d*E^(2*c)*f^2*x*Log[1 - E^(c +
 d*x)] + 12*a*b*d*f^2*x*Log[1 + E^(c + d*x)] - 12*a*b*d*E^(2*c)*f^2*x*Log[
1 + E^(c + d*x)] + 6*a^2*d^2*e^2*Log[1 - E^(2*(c + d*x))] + 6*b^2*d^2*e^2*
Log[1 - E^(2*(c + d*x))] - 6*a^2*d^2*e^2*E^(2*c)*Log[1 - E^(2*(c + d*x))]
- 6*b^2*d^2*e^2*E^(2*c)*Log[1 - E^(2*(c + d*x))] + 6*a^2*f^2*Log[1 - E^(2*
(c + d*x))] - 6*a^2*E^(2*c)*f^2*Log[1 - E^(2*(c + d*x))] + 12*a^2*d^2*e*f*
x*Log[1 - E^(2*(c + d*x))] + 12*b^2*d^2*e*f*x*Log[1 - E^(2*(c + d*x))] - 1
2*a^2*d^2*e*E^(2*c)*f*x*Log[1 - E^(2*(c + d*x))] - 12*b^2*d^2*e*E^(2*c)*f*
x*Log[1 - E^(2*(c + d*x))] + 6*a^2*d^2*f^2*x^2*Log[1 - E^(2*(c + d*x))] +
6*b^2*d^2*f^2*x^2*Log[1 - E^(2*(c + d*x))] - 6*a^2*d^2*E^(2*c)*f^2*x^2*Log
[1 - E^(2*(c + d*x))] - 6*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 - E^(2*(c + d*x))]
 - 12*a*b*(-1 + E^(2*c))*f^2*PolyLog[2, -E^(c + d*x)] + 12*a*b*(-1 + E^(2*
c))*f^2*PolyLog[2, E^(c + d*x)] + 6*a^2*d*e*f*PolyLog[2, E^(2*(c + d*x))]
+ 6*b^2*d*e*f*PolyLog[2, E^(2*(c + d*x))] - 6*a^2*d*e*E^(2*c)*f*PolyLog[2,
 E^(2*(c + d*x))] - 6*b^2*d*e*E^(2*c)*f*PolyLog[2, E^(2*(c + d*x))] + 6...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \; 6103$$

$$\frac{\int (e+fx)^2\coth^3(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int i(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)^3dx}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^3dx}{a}$$

$$\Big\downarrow \text{4203}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{if\int -\left((e+fx)\coth^2(c+dx)\right)dx}{d}-\int i(e+fx)^2\coth(c+dx)dx+\frac{i(e+fx)^2\coth^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\frac{if\int (e+fx)\coth^2(c+dx)dx}{d}-\int i(e+fx)^2\coth(c+dx)dx+\frac{i(e+fx)^2\coth^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\frac{if\int (e+fx)\coth^2(c+dx)dx}{d}-i\int (e+fx)^2\coth(c+dx)dx+\frac{i(e+fx)^2\coth^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-i\int -i(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{if\int -\left((e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)^2\right)dx}{d}+\frac{i(e+fx)^2\coth^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-i\int -i(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{if\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{d}+\frac{i(e+fx)^2\coth^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+}{\frac{i\left(-\int(e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx+\frac{if\int(e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{d}+\frac{i(e+fx)^2\coth^2(c+dx)}{2d}\right)}{a}}$$

$$\Big\downarrow \text{4201}$$

$$\frac{-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+}{\frac{i\left(-2i\int\frac{e^{2c+2dx-i\pi}(e+fx)^2}{1+e^{2c+2dx-i\pi}}dx+\frac{if\int(e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{d}+\frac{i(e+fx)^2\coth^2(c+dx)}{2d}+\frac{i(e+fx)^3}{3f}\right)}{a}}$$

$$\Big\downarrow \text{2620}$$

$$\frac{-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+}{\frac{i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)dx}{d}\right)+\frac{if\int(e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{d}+\frac{i(e+fx)^2\coth^2(c+d}{2d}\right.}{a}}$$

$$\Big\downarrow \text{3011}$$

$$\frac{-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+}{\frac{i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{2d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)+\frac{if\int(e+fx)\tan\left(\frac{1}{2}(2ic+\right.}{d}\right.}{a}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+}{\frac{i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)+\right.}{a}}$$

$$\Big\downarrow \text{4203}$$

$$\frac{-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)+}{a}$$

$\downarrow$ 17

$$\frac{-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)+}{a}$$

$\downarrow$ 26

$$\frac{-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)+}{a}$$

$\downarrow$ 3042

$$\frac{-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)+}{a}$$

$\downarrow$ 26

$$\frac{-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)+}{a}$$

$\downarrow$ 3956

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)+}{a}$$

$$\Big\downarrow \textbf{6119}$$

$$-\frac{b\left(\frac{\int (e+fx)^2\cosh(c+dx)\coth^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)+}{a}$$

$$\Big\downarrow \textbf{5973}$$

$$-\frac{b\left(\frac{\int (e+fx)^2\cosh(c+dx)dx+\int (e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)+}{a}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)+}{a}$$

$$\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)dx+\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \textbf{3777}$$

$$\frac{i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)+\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}+\int(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)dx+\frac{(e+fx)^2\sinh(c+dx)}{d}}{a}\right)}{a}}{a}$$

$\downarrow$ 26

$$-\frac{b\left(\frac{-\frac{2f\int(e+fx)\sinh(c+dx)dx}{d}+\int(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)dx+\frac{(e+fx)^2\sinh(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)+}{a}$$

$\downarrow$ 3042

$$\frac{i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)+\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{2f\int -i(e+fx)\sin(ic+idx)dx}{d}+\int(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)dx+\frac{(e+fx)^2\sinh(c+dx)}{d}}{a}\right)}{a}}{a}$$

$\downarrow$ 26

$$\frac{i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)+\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{2if\int(e+fx)\sin(ic+idx)dx}{d}+\int(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)dx+\frac{(e+fx)^2\sinh(c+dx)}{d}}{a}\right)}{a}}{a}$$

$\downarrow$ 3777

$$i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)+$$

$$a$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{d}+\int(e+fx)^2\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^2\sinh}{d}}{a}\right.}{a}$$

$$\Big\downarrow \; 3042$$

$$i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)+$$

$$a$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}+\int(e+fx)^2\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^2}{}}{a}\right.}{a}$$

$$\Big\downarrow \; 3117$$

$$i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)+$$

$$a$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^2\coth(c+dx)\text{csch}(c+dx)dx+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+d}{d}}{a}\right.}{a}$$

$$\Big\downarrow \; 5975$$

$$i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)+$$

$$a$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{2f\int(e+fx)\text{csch}(c+dx)dx}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{(e+fx)^2}{}}{a}\right.}{a}$$

input
```
Int[((e + f*x)^2*Coth[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^2 \coth(dx+c)^3}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7775 vs. $2(639) = 1278$.

Time = 0.21 (sec) , antiderivative size = 7775, normalized size of antiderivative = 11.43

$$\int \frac{(e+fx)^2\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^2\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)^2\coth^3{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**2*coth(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*coth(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)^2\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\coth{(dx+c)}^3}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^2*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e^2*(2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2
*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + (a^2 + b^2)*log(-2*a*
e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^3*d) - (a^2 + b^2)*log(e^(-d*x -
 c) + 1)/(a^3*d) - (a^2 + b^2)*log(e^(-d*x - c) - 1)/(a^3*d)) + 2*(a*f^2*x
 + a*e*f + (b*d*f^2*x^2*e^(3*c) + 2*b*d*e*f*x*e^(3*c))*e^(3*d*x) - (a*d*f^
2*x^2*e^(2*c) + a*e*f*e^(2*c) + (2*d*e*f + f^2)*a*x*e^(2*c))*e^(2*d*x) - (
b*d*f^2*x^2*e^c + 2*b*d*e*f*x*e^c)*e^(d*x))/(a^2*d^2*e^(4*d*x + 4*c) - 2*a
^2*d^2*e^(2*d*x + 2*c) + a^2*d^2) - (2*b*d*e*f + a*f^2)*x/(a^2*d^2) + (2*b
*d*e*f - a*f^2)*x/(a^2*d^2) + (2*b*d*e*f + a*f^2)*log(e^(d*x + c) + 1)/(a^
2*d^3) - (2*b*d*e*f - a*f^2)*log(e^(d*x + c) - 1)/(a^2*d^3) + (d^2*x^2*log
(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c))
)*(a^2*f^2 + b^2*f^2)/(a^3*d^3) + (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*d
ilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*(a^2*f^2 + b^2*f^2)/(a^3*d^
3) + 2*(a^2*d*e*f + b^2*d*e*f + a*b*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog
(-e^(d*x + c)))/(a^3*d^3) + 2*(a^2*d*e*f + b^2*d*e*f - a*b*f^2)*(d*x*log(-
e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^3*d^3) - 1/3*((a^2*f^2 + b^2*f^2
)*d^3*x^3 + 3*(a^2*d*e*f + b^2*d*e*f + a*b*f^2)*d^2*x^2)/(a^3*d^3) - 1/3*(
(a^2*f^2 + b^2*f^2)*d^3*x^3 + 3*(a^2*d*e*f + b^2*d*e*f - a*b*f^2)*d^2*x^2)
/(a^3*d^3) + integrate(-2*((a^2*b*f^2 + b^3*f^2)*x^2 + 2*(a^2*b*e*f + b^3*
e*f)*x - ((a^3*f^2*e^c + a*b^2*f^2*e^c)*x^2 + 2*(a^3*e*f*e^c + a*b^2*e*...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+d\,x)^3\,(e+f\,x)^2}{a+b\sinh\,(c+d\,x)}\,dx$$

input
```
int((coth(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((coth(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2 \coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^2*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output

```
(576*e**(7*c + 4*d*x)*int((e**(3*d*x)*x**2)/(e**(8*c + 8*d*x)*b + 2*e**(7*
c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4
*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a +
 b),x)*a**7*d**3*f**2 + 816*e**(7*c + 4*d*x)*int((e**(3*d*x)*x**2)/(e**(8*
c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5
*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)
*b - 2*e**(c + d*x)*a + b),x)*a**5*b**2*d**3*f**2 + 240*e**(7*c + 4*d*x)*i
nt((e**(3*d*x)*x**2)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*
c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3
*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**3*b**4*d**3*f
**2 + 1152*e**(7*c + 4*d*x)*int((e**(3*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**
(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c
+ 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*
a + b),x)*a**7*d**3*e*f + 1632*e**(7*c + 4*d*x)*int((e**(3*d*x)*x)/(e**(8*
c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5
*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)
*b - 2*e**(c + d*x)*a + b),x)*a**5*b**2*d**3*e*f - 128*e**(7*c + 4*d*x)*in
t((e**(3*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c +
6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x
)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**5*b**2*d**2*f*...
```

3.488 $\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 435

$$\begin{aligned}\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{fx}{2ad}-\frac{(e+fx)^2}{2af}-\frac{b^2(e+fx)^2}{2a^3f}+\frac{(a^2+b^2)(e+fx)^2}{2a^3f}\\
&\quad+\frac{bf\operatorname{arctanh}(\cosh(c+dx))}{a^2d^2}-\frac{f\coth(c+dx)}{2ad^2}\\
&\quad-\frac{(e+fx)\coth^2(c+dx)}{2ad}+\frac{b(e+fx)\operatorname{csch}(c+dx)}{a^2d}\\
&\quad-\frac{(a^2+b^2)(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d}\\
&\quad-\frac{(a^2+b^2)(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d}\\
&\quad+\frac{(e+fx)\log\left(1-e^{2(c+dx)}\right)}{ad}\\
&\quad+\frac{b^2(e+fx)\log\left(1-e^{2(c+dx)}\right)}{a^3d}\\
&\quad-\frac{(a^2+b^2)f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2}\\
&\quad-\frac{(a^2+b^2)f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^2}\\
&\quad+\frac{f\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{2ad^2}+\frac{b^2f\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{2a^3d^2}\end{aligned}$$

output
```
1/2*f*x/a/d-1/2*(f*x+e)^2/a/f-1/2*b^2*(f*x+e)^2/a^3/f+1/2*(a^2+b^2)*(f*x+e
)^2/a^3/f+b*f*arctanh(cosh(d*x+c))/a^2/d^2-1/2*f*coth(d*x+c)/a/d^2-1/2*(f*
x+e)*coth(d*x+c)^2/a/d+b*(f*x+e)*csch(d*x+c)/a^2/d-(a^2+b^2)*(f*x+e)*ln(1+
b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d-(a^2+b^2)*(f*x+e)*ln(1+b*exp(d*x+c
)/(a+(a^2+b^2)^(1/2)))/a^3/d+(f*x+e)*ln(1-exp(2*d*x+2*c))/a/d+b^2*(f*x+e)*
ln(1-exp(2*d*x+2*c))/a^3/d-(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2
)^(1/2)))/a^3/d^2-(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))
/a^3/d^2+1/2*f*polylog(2,exp(2*d*x+2*c))/a/d^2+1/2*b^2*f*polylog(2,exp(2*d
*x+2*c))/a^3/d^2
```

Mathematica [A] (warning: unable to verify)

Time = 8.41 (sec) , antiderivative size = 766, normalized size of antiderivative = 1.76

$$\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\left(2bde\cosh\left(\frac{1}{2}(c+dx)\right)-af\cosh\left(\frac{1}{2}(c+dx)\right)-2bcf\cosh\left(\frac{1}{2}(c+dx)\right)+2bf(c+dx)\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{4a^2d^2}$$
$$+\frac{(-de+cf-f(c+dx))\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8ad^2}$$
$$+\frac{\frac{(a^2+b^2)(de+dfx)^2}{2f}+\left(-abf+a^2(de+dfx)+b^2(de+dfx)\right)\log\left(1-e^{-c-dx}\right)+\left(abf+a^2(de+dfx)+b^2(\right.}{a^3d^2}$$
$$-\frac{(a^2+b^2)\left(-2de(c+dx)+2cf(c+dx)-f(c+dx)^2+\frac{4a\sqrt{a^2+b^2}de\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-(a^2+b^2)^2}}-\frac{4a\sqrt{-(a^2+b^2)^2}de\operatorname{arct}}{(-a^2-b^2}\right.}{}$$
$$+\frac{(de-cf+f(c+dx))\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8ad^2}$$
$$+\frac{\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\left(-2bde\sinh\left(\frac{1}{2}(c+dx)\right)-af\sinh\left(\frac{1}{2}(c+dx)\right)+2bcf\sinh\left(\frac{1}{2}(c+dx)\right)-2bf(c+dx\right.}{4a^2d^2}$$

input
```
Integrate[((e + f*x)*Coth[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
((2*b*d*e*Cosh[(c + d*x)/2] - a*f*Cosh[(c + d*x)/2] - 2*b*c*f*Cosh[(c + d*
x)/2] + 2*b*f*(c + d*x)*Cosh[(c + d*x)/2])*Csch[(c + d*x)/2])/(4*a^2*d^2)
+ ((-(d*e) + c*f - f*(c + d*x))*Csch[(c + d*x)/2]^2)/(8*a*d^2) + (((a^2 +
b^2)*(d*e + d*f*x)^2)/(2*f) + (-(a*b*f) + a^2*(d*e + d*f*x) + b^2*(d*e + d
*f*x))*Log[1 - E^(-c - d*x)] + (a*b*f + a^2*(d*e + d*f*x) + b^2*(d*e + d*f
*x))*Log[1 + E^(-c - d*x)] - (a^2 + b^2)*f*PolyLog[2, -E^(-c - d*x)] - (a^
2 + b^2)*f*PolyLog[2, E^(-c - d*x)])/(a^3*d^2) - ((a^2 + b^2)*(-2*d*e*(c +
 d*x) + 2*c*f*(c + d*x) - f*(c + d*x)^2 + (4*a*Sqrt[a^2 + b^2]*d*e*ArcTan[
(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b^2)^2] - (4*a*Sqrt[-(
a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/(-a^2 - b^
2)^(3/2) + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] +
2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*f*Log[b
 - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*d*e*Log[2*a*E^(c + d*x) + b*(-
1 + E^(2*(c + d*x)))] + 2*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^
2])] + 2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(2*a^3*d
^2) + ((d*e - c*f + f*(c + d*x))*Sech[(c + d*x)/2]^2)/(8*a*d^2) + (Sech[(c
 + d*x)/2]*(-2*b*d*e*Sinh[(c + d*x)/2] - a*f*Sinh[(c + d*x)/2] + 2*b*c*f*S
inh[(c + d*x)/2] - 2*b*f*(c + d*x)*Sinh[(c + d*x)/2]))/(4*a^2*d^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6103}$$

$$\frac{\int (e+fx)\coth^3(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)^3dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^3dx}{a}$$

$$\Big\downarrow \text{4203}$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\int i(e+fx)\coth(c+dx)dx+\frac{if\int -\coth^2(c+dx)dx}{2d}+\frac{i(e+fx)\coth^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\int i(e+fx)\coth(c+dx)dx-\frac{if\int \coth^2(c+dx)dx}{2d}+\frac{i(e+fx)\coth^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-i\int (e+fx)\coth(c+dx)dx-\frac{if\int \coth^2(c+dx)dx}{2d}+\frac{i(e+fx)\coth^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-i\int -i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{if\int -\tan\left(ic+idx+\frac{\pi}{2}\right)^2dx}{2d}+\frac{i(e+fx)\coth^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-i\int -i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{if\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{2d}+\frac{i(e+fx)\coth^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx+\frac{if\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{2d}+\frac{i(e+fx)\coth^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{3954}$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx+\frac{if\left(\frac{\coth(c+dx)}{d}-\int 1dx\right)}{2d}+\frac{i(e+fx)\coth^2(c+dx)}{2d}\right)}{a}$$

$\Big\downarrow$ 24

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}\right)}{a}$$

$\Big\downarrow$ 4201

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-2i\int \frac{e^{2c+2dx-i\pi}(e+fx)}{1+e^{2c+2dx-i\pi}}dx+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}$$

$\Big\downarrow$ 2620

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int \log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}$$

$\Big\downarrow$ 2715

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\log\left(1+e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}\right.}{a}$$

$\Big\downarrow$ 2838

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}$$

$$\Big\downarrow \text{6119}$$

$$-\frac{b\left(\frac{\int (e+fx)\cosh(c+dx)\coth^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{i\left(-2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}$$

$$\Big\downarrow \text{5973}$$

$$-\frac{b\left(\frac{\int (e+fx)\cosh(c+dx)dx+\int (e+fx)\coth(c+dx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{i\left(-2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\left(-2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)\coth(c+dx)\text{csch}(c+dx)dx+\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{i\left(-2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)\coth(c+dx)\text{csch}(c+dx)dx-\frac{if\int -i\sinh(c+dx)dx}{d}+\frac{(e+fx)\sinh(c+dx)}{d}}{a}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\left(\frac{\int (e+fx)\coth(c+dx)\text{csch}(c+dx)dx-\frac{f\int \sinh(c+dx)dx}{d}+\frac{(e+fx)\sinh(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{i\left(-2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}$$

$\downarrow$ 3042

$$\frac{i\left(-2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)\coth(c+dx)\text{csch}(c+dx)dx-\frac{f\int -i\sin(ic+idx)dx}{d}+\frac{(e+fx)\sinh(c+dx)}{d}}{a}\right)}{a}$$

$\downarrow$ 26

$$\frac{i\left(-2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)\coth(c+dx)\text{csch}(c+dx)dx+\frac{if\int \sin(ic+idx)dx}{d}+\frac{(e+fx)\sinh(c+dx)}{d}}{a}\right)}{a}$$

$\downarrow$ 3118

$$-\frac{b\left(\frac{\int (e+fx)\coth(c+dx)\text{csch}(c+dx)dx-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{i\left(-2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}$$

$\downarrow$ 5975

$$-\frac{b\left(\frac{\frac{f\int \text{csch}(c+dx)dx}{d}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{i\left(-2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\left(-2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{f\int i\csc(ic+idx)dx}{d}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\left(-2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{if\int \csc(ic+idx)dx}{d}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}\right)}{a}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{b\left(\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{i\left(-2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}$$

$$\Big\downarrow \text{6119}$$

$$-\frac{b\left(\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)\cosh^2(c+dx)\coth(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\cosh^3}{a+b\sinh(c}}{a}\right.}{a}\right.}{a}$$

$$\frac{i\left(-2i\left(\frac{f\,\text{PolyLog}(2,-e^{2c+2dx-i\pi})}{4d^2}+\frac{(e+fx)\log(1+e^{2c+2dx-i\pi})}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}$$

$$\Big\downarrow \text{5973}$$

$$-\frac{b\left(\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)\coth(c+dx)dx+\int(e+fx)\cosh(c+dx)\sinh(c+dx}{a}\right.}{a}\right.}{a}$$

$$\frac{i\left(-2i\left(\frac{f\,\text{PolyLog}(2,-e^{2c+2dx-i\pi})}{4d^2}+\frac{(e+fx)\log(1+e^{2c+2dx-i\pi})}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\left(-2i\left(\frac{f\,\text{PolyLog}(2,-e^{2c+2dx-i\pi})}{4d^2}+\frac{(e+fx)\log(1+e^{2c+2dx-i\pi})}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}-$$

$$\frac{b\left(\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c}{}\right.}{a}\right.}{a}$$

input
```
Int[((e + f*x)*Coth[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1097 vs. 2(407) = 814.

Time = 1.02 (sec) , antiderivative size = 1098, normalized size of antiderivative = 2.52

method	result	size
risch	Expression too large to display	1098

input
```
int((f*x+e)*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/d/a*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d/a*
f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d^2/a*c*f
*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/d^2*b^2/a^3*f*dilog(exp(d*x+c)+1)
+1/d*b^2/a^3*e*ln(exp(d*x+c)-1)-1/d*b^2/a^3*e*ln(b*exp(2*d*x+2*c)+2*a*exp(
d*x+c)-b)+1/d*b^2/a^3*e*ln(exp(d*x+c)+1)-1/d^2*b/a^2*f*ln(exp(d*x+c)-1)+1/
d^2*b/a^2*f*ln(exp(d*x+c)+1)-1/d^2*b^2/a^3*f*dilog((-b*exp(d*x+c)+(a^2+b^2
)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2*b^2/a^3*f*dilog((b*exp(d*x+c)+(a^2+
b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d^2*b^2/a^3*f*dilog(exp(d*x+c))-1/d^2
/a*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d^2*c*
f/a*ln(exp(d*x+c)-1)-1/d^2*f/a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2
+b^2)^(1/2)))*c-1/d^2/a*f*dilog(exp(d*x+c))+1/d/a*e*ln(exp(d*x+c)+1)+1/d/a
*e*ln(exp(d*x+c)-1)-1/d^2*b^2/a^3*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(
-a+(a^2+b^2)^(1/2)))*c-1/d^2*b^2/a^3*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)
/(a+(a^2+b^2)^(1/2)))*c-1/d^2*b^2/a^3*c*f*ln(exp(d*x+c)-1)+1/d^2*b^2/a^3*c
*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/d*b^2/a^3*f*ln(exp(d*x+c)+1)*x-
1/d*b^2/a^3*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x
-1/d*b^2/a^3*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+
1/d/a*f*ln(exp(d*x+c)+1)*x+1/d^2*f/a*dilog(exp(d*x+c)+1)-1/d^2*f/a*dilog((
-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2/a*f*dilog((b*
exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d/a*e*ln(b*exp(2*d...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3547 vs. $2(403) = 806$.

Time = 0.15 (sec) , antiderivative size = 3547, normalized size of antiderivative = 8.15

$$\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)*coth(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*coth(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\coth(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-(a^2*d*integrate(x/(a^3*d*e^(d*x + c) + a^3*d), x) + b^2*d*integrate(x/(a
^3*d*e^(d*x + c) + a^3*d), x) - a^2*d*integrate(x/(a^3*d*e^(d*x + c) - a^3
*d), x) - b^2*d*integrate(x/(a^3*d*e^(d*x + c) - a^3*d), x) + a*b*((d*x +
c)/(a^3*d^2) - log(e^(d*x + c) + 1)/(a^3*d^2)) - a*b*((d*x + c)/(a^3*d^2)
- log(e^(d*x + c) - 1)/(a^3*d^2)) - (2*b*d*x*e^(3*d*x + 3*c) - 2*b*d*x*e^(
d*x + c) - (2*a*d*x*e^(2*c) + a*e^(2*c))*e^(2*d*x) + a)/(a^2*d^2*e^(4*d*x
+ 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2) - integrate(2*((a^3*e^c + a*
b^2*e^c)*x*e^(d*x) - (a^2*b + b^3)*x)/(a^3*b*e^(2*d*x + 2*c) + 2*a^4*e^(d*
x + c) - a^3*b), x))*f - e*(2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(
-3*d*x - 3*c))/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) +
 (a^2 + b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^3*d) - (a^
2 + b^2)*log(e^(-d*x - c) + 1)/(a^3*d) - (a^2 + b^2)*log(e^(-d*x - c) - 1)
/(a^3*d))
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+d\,x)^3\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((coth(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((coth(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
(192*e**(7*c + 4*d*x)*int((e**(3*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c +
 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*
x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b)
,x)*a**7*d**2*f + 272*e**(7*c + 4*d*x)*int((e**(3*d*x)*x)/(e**(8*c + 8*d*x
)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a +
 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e*
*(c + d*x)*a + b),x)*a**5*b**2*d**2*f + 80*e**(7*c + 4*d*x)*int((e**(3*d*x
)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6
*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(
2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**3*b**4*d**2*f - 128*e**(6*c +
 4*d*x)*int((e**(2*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*
e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3
*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**6*b*d**
2*f - 128*e**(6*c + 4*d*x)*int((e**(2*d*x)*x)/(e**(8*c + 8*d*x)*b + 2*e**(
7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d*x)*a + 6*e**(4*c +
 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a
 + b),x)*a**4*b**3*d**2*f + 16*e**(5*c + 4*d*x)*int((e**(d*x)*x)/(e**(8*c
+ 8*d*x)*b + 2*e**(7*c + 7*d*x)*a - 4*e**(6*c + 6*d*x)*b - 6*e**(5*c + 5*d
*x)*a + 6*e**(4*c + 4*d*x)*b + 6*e**(3*c + 3*d*x)*a - 4*e**(2*c + 2*d*x)*b
 - 2*e**(c + d*x)*a + b),x)*a**5*b**2*d**2*f + 16*e**(5*c + 4*d*x)*int(...
```

3.489 $\int \frac{\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b\text{csch}(c+dx)}{a^2d} - \frac{\text{csch}^2(c+dx)}{2ad} + \frac{(a^2+b^2)\log(\sinh(c+dx))}{a^3d} - \frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{a^3d}$$

output
```
b*csch(d*x+c)/a^2/d-1/2*csch(d*x+c)^2/a/d+(a^2+b^2)*ln(sinh(d*x+c))/a^3/d-
(a^2+b^2)*ln(a+b*sinh(d*x+c))/a^3/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2ab\text{csch}(c+dx) - a^2\text{csch}^2(c+dx) + 2(a^2+b^2)\left(\log(\sinh(c+dx)) - \log(a+b\sinh(c+dx))\right)}{2a^3d}$$

input
```
Integrate[Coth[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

output
```
(2*a*b*Csch[c + d*x] - a^2*Csch[c + d*x]^2 + 2*(a^2 + b^2)*(Log[Sinh[c + d
*x]] - Log[a + b*Sinh[c + d*x]]))/(2*a^3*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 26, 3200, 25, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\tan(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{(a-ib\sin(ic+idx))\tan(ic+idx)^3}dx$$

$$\Big\downarrow \text{3200}$$

$$-\frac{\int -\frac{\operatorname{csch}^3(c+dx)\left(\sinh^2(c+dx)b^2+b^2\right)}{b^3(a+b\sinh(c+dx))}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\operatorname{csch}^3(c+dx)\left(\sinh^2(c+dx)b^2+b^2\right)}{b^3(a+b\sinh(c+dx))}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{522}$$

$$\frac{\int \left(\frac{\operatorname{csch}^3(c+dx)}{ab}-\frac{\operatorname{csch}^2(c+dx)}{a^2}+\frac{(a^2+b^2)\operatorname{csch}(c+dx)}{a^3b}+\frac{-a^2-b^2}{a^3(a+b\sinh(c+dx))}\right)d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{b\operatorname{csch}(c+dx)}{a^2}-\frac{(a^2+b^2)\log(b\sinh(c+dx))}{a^3}+\frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{a^3}+\frac{\operatorname{csch}^2(c+dx)}{2a}}{d}$$

input
```
Int[Coth[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

output
```
-((-((b*Csch[c + d*x])/a^2) + Csch[c + d*x]^2/(2*a) - ((a^2 + b^2)*Log[b*S
inh[c + d*x]])/a^3 + ((a^2 + b^2)*Log[a + b*Sinh[c + d*x]])/a^3)/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.79

method	result
derivativedivides	$\frac{-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(4a^2+4b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}+\frac{b}{2a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{\left(-4a^2-4b^2\right)\ln}{}}{d}$
default	$\frac{-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(4a^2+4b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}+\frac{b}{2a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{\left(-4a^2-4b^2\right)\ln}{}}{d}$
risch	$-\frac{2\,e^{dx+c}\left(-b\,e^{2dx+2c}+a\,e^{dx+c}+b\right)}{a^2 d\left(e^{2dx+2c}-1\right)^2}+\frac{\ln\left(e^{2dx+2c}-1\right)}{ad}+\frac{\ln\left(e^{2dx+2c}-1\right)b^2}{a^3 d}-\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{b}-1\right)}{ad}-\frac{b^2\ln}{}$

input
```
int(coth(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4/a^2*(1/2*tanh(1/2*d*x+1/2*c)^2*a+2*b*tanh(1/2*d*x+1/2*c))-1/8/a/
tanh(1/2*d*x+1/2*c)^2+1/4/a^3*(4*a^2+4*b^2)*ln(tanh(1/2*d*x+1/2*c))+1/2*b/
a^2/tanh(1/2*d*x+1/2*c)+1/4/a^3*(-4*a^2-4*b^2)*ln(tanh(1/2*d*x+1/2*c)^2*a-
2*b*tanh(1/2*d*x+1/2*c)-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. 2(78) = 156.

Time = 0.13 (sec) , antiderivative size = 617, normalized size of antiderivative = 7.71

$$\int \frac{\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(2*a*b*cosh(d*x + c)^3 + 2*a*b*sinh(d*x + c)^3 - 2*a^2*cosh(d*x + c)^2 - 2
*a*b*cosh(d*x + c) + 2*(3*a*b*cosh(d*x + c) - a^2)*sinh(d*x + c)^2 - ((a^2
 + b^2)*cosh(d*x + c)^4 + 4*(a^2 + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a
^2 + b^2)*sinh(d*x + c)^4 - 2*(a^2 + b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + b^
2)*cosh(d*x + c)^2 - a^2 - b^2)*sinh(d*x + c)^2 + a^2 + b^2 + 4*((a^2 + b^
2)*cosh(d*x + c)^3 - (a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*si
nh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + ((a^2 + b^2)*cosh(d*x
+ c)^4 + 4*(a^2 + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + b^2)*sinh(d*
x + c)^4 - 2*(a^2 + b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + b^2)*cosh(d*x + c)^
2 - a^2 - b^2)*sinh(d*x + c)^2 + a^2 + b^2 + 4*((a^2 + b^2)*cosh(d*x + c)^
3 - (a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*
x + c) - sinh(d*x + c))) + 2*(3*a*b*cosh(d*x + c)^2 - 2*a^2*cosh(d*x + c)
- a*b)*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*cosh(d*x + c)*sinh(
d*x + c)^3 + a^3*d*sinh(d*x + c)^4 - 2*a^3*d*cosh(d*x + c)^2 + a^3*d + 2*(
3*a^3*d*cosh(d*x + c)^2 - a^3*d)*sinh(d*x + c)^2 + 4*(a^3*d*cosh(d*x + c)^
3 - a^3*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth^3{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate(coth(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(coth(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(78) = 156$.

Time = 0.04 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.16

$$\int \frac{\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{2\left(be^{(-dx-c)} - ae^{(-2\,dx-2\,c)} - be^{(-3\,dx-3\,c)}\right)}{\left(2\,a^2e^{(-2\,dx-2\,c)} - a^2e^{(-4\,dx-4\,c)} - a^2\right)d} - \frac{\left(a^2+b^2\right)\log\left(-2\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)} - b\right)}{a^3d} + \frac{\left(a^2+b^2\right)\log\left(e^{(-dx-c)}+1\right)}{a^3d} + \frac{\left(a^2+b^2\right)\log\left(e^{(-dx-c)}-1\right)}{a^3d}$$

input
```
integrate(coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2*e^(-
2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) - (a^2 + b^2)*log(-2*a*e^(-d
*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^3*d) + (a^2 + b^2)*log(e^(-d*x - c) +
 1)/(a^3*d) + (a^2 + b^2)*log(e^(-d*x - c) - 1)/(a^3*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(78) = 156$.

Time = 0.17 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.30

$$\int \frac{\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{2\left(a^2+b^2\right)\log\left(\left|e^{(dx+c)}-e^{(-dx-c)}\right|\right)}{a^3} - \frac{2\left(a^2b+b^3\right)\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^3b} - \frac{3\,a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+3\,b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2}{a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right.}}{2\,d}$$

input
```
integrate(coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*(a^2 + b^2)*log(abs(e^(d*x + c) - e^(-d*x - c)))/a^3 - 2*(a^2*b + b
^3)*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^3*b) - (3*a^2*(e^(d*
x + c) - e^(-d*x - c))^2 + 3*b^2*(e^(d*x + c) - e^(-d*x - c))^2 - 4*a*b*(e
^(d*x + c) - e^(-d*x - c)) + 4*a^2)/(a^3*(e^(d*x + c) - e^(-d*x - c))^2))/
d
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 1329, normalized size of antiderivative = 16.61

$$\int \frac{\coth^3(c + dx)}{a + b\sinh(c + dx)}\, dx = \text{Too large to display}$$

input
```
int(coth(c + d*x)^3/(a + b*sinh(c + d*x)),x)
```

output
```
((2*atan((a^2*(-a^6*d^2)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2) + 2*b^2*(-a^6
*d^2)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(2*a^3*d*(a^2 + b^2)^2) + ((a^7
*d + a^5*b^2*d)*(-a^6*d^2)^(1/2))/(2*a^6*d^2*((a^2 + b^2)^2)^(1/2)*(a^2 +
b^2)) - (a^6*b^2*exp(2*c)*exp(2*d*x)*(-a^6*d^2)^(1/2)*((4*(a^2 + 2*b^2)*(a
^4 + b^4 + 2*a^2*b^2))/(a^9*b^2*d*(a^2 + b^2)^2) + (2*(2*a^4*b^3*d + 2*a^6
*b*d)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^11*b^3*d^2*((a^2 + b^2)^2)^(1/2)*(
a^2 + b^2)) + (4*(a^2*(-a^6*d^2)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2) + 2*b
^2*(-a^6*d^2)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))*(a^4 + b^4 + 2*a^2*b^2)
^(1/2))/(a^9*b^2*d*(a^2 + b^2)^2*(-a^6*d^2)^(1/2)) + (4*(a^7*d + a^5*b^2*d
)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^12*b^2*d^2*((a^2 + b^2)^2)^(1/2)*(a^2
+ b^2))))/(8*(a^4 + b^4 + 2*a^2*b^2)^(1/2)) + (a^6*b^2*exp(3*c)*exp(3*d*x)
*((2*(a^7*d + a^5*b^2*d)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^11*b^3*d^2*((a^
2 + b^2)^2)^(1/2)*(a^2 + b^2)) - (2*(a^2 + 2*b^2)*(a^2*(-a^6*d^2)^(1/2)*(a
^4 + b^4 + 2*a^2*b^2)^(1/2) + 2*b^2*(-a^6*d^2)^(1/2)*(a^4 + b^4 + 2*a^2*b^
2)^(1/2))*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^10*b^3*d*(a^2 + b^2)^2*(-a^6*d
^2)^(1/2)))*(-a^6*d^2)^(1/2))/(8*(a^4 + b^4 + 2*a^2*b^2)^(1/2)) - (a^6*b^2
*exp(d*x)*exp(c)*(-a^6*d^2)^(1/2)*((8*(a^4 + b^4 + 2*a^2*b^2))/(a^8*b*d*(a
^2 + b^2)^2) - (4*(2*a^4*b^3*d + 2*a^6*b*d)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))
/(a^12*b^2*d^2*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)) + (2*(a^7*d + a^5*b^2*d)
*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^11*b^3*d^2*((a^2 + b^2)^2)^(1/2)*(a^...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 563, normalized size of antiderivative = 7.04

$$\int \frac{\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{e^{4dx+4c}\log\left(e^{dx+c}-1\right)a^2+e^{4dx+4c}\log\left(e^{dx+c}-1\right)b^2+e^{4dx+4c}\log\left(e^{dx+c}+1\right)a^2+e^{4dx+4c}\log\left(e^{dx+c}+1\right)b^2}{}$$

input
```
int(coth(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
(e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**2 + e**(4*c + 4*d*x)*log(e**(c
+ d*x) - 1)*b**2 + e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a**2 + e**(4*c +
 4*d*x)*log(e**(c + d*x) + 1)*b**2 - e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)
*b + 2*e**(c + d*x)*a - b)*a**2 - e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*b
+ 2*e**(c + d*x)*a - b)*b**2 - e**(4*c + 4*d*x)*a**2 + 2*e**(3*c + 3*d*x)*
a*b - 2*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a**2 - 2*e**(2*c + 2*d*x)*l
og(e**(c + d*x) - 1)*b**2 - 2*e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*a**2
- 2*e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*b**2 + 2*e**(2*c + 2*d*x)*log(e
**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*a**2 + 2*e**(2*c + 2*d*x)*log(e*
*(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*b**2 - 2*e**(c + d*x)*a*b + log(e
**(c + d*x) - 1)*a**2 + log(e**(c + d*x) - 1)*b**2 + log(e**(c + d*x) + 1)
*a**2 + log(e**(c + d*x) + 1)*b**2 - log(e**(2*c + 2*d*x)*b + 2*e**(c + d*
x)*a - b)*a**2 - log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*b**2 - a**
2)/(a**3*d*(e**(4*c + 4*d*x) - 2*e**(2*c + 2*d*x) + 1))
```

3.490 $\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \text{Int}\left(\frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(coth(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 133.72 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

input
```
Integrate[Coth[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Coth[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$\Big\downarrow \ 6111$$
$$\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Coth[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\coth\left(dx+c\right)^{3}}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(coth(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(coth(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(coth(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(coth(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 6.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth^3\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

input
```
integrate(coth(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(coth(c + d*x)**3/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 751, normalized size of antiderivative = 26.82

$$\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(coth(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-(a*f - 2*(b*d*f*x*e^(3*c) + b*d*e*e^(3*c))*e^(3*d*x) + (2*a*d*f*x*e^(2*c)
 + (2*d*e - f)*a*e^(2*c))*e^(2*d*x) + 2*(b*d*f*x*e^c + b*d*e*e^c)*e^(d*x))
/(a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + a^2*d^2*e^2 + (a^2*d^2*f^2*x^2*e^(4*
c) + 2*a^2*d^2*e*f*x*e^(4*c) + a^2*d^2*e^2*e^(4*c))*e^(4*d*x) - 2*(a^2*d^2
*f^2*x^2*e^(2*c) + 2*a^2*d^2*e*f*x*e^(2*c) + a^2*d^2*e^2*e^(2*c))*e^(2*d*x
)) + integrate(-(b^2*d^2*e^2 + a*b*d*e*f + (d^2*e^2 + f^2)*a^2 + (a^2*d^2*
f^2 + b^2*d^2*f^2)*x^2 + (2*a^2*d^2*e*f + 2*b^2*d^2*e*f + a*b*d*f^2)*x)/(a
^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*e^3 - (
a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c + a^
3*d^2*e^3*e^c)*e^(d*x)), x) - integrate((b^2*d^2*e^2 - a*b*d*e*f + (d^2*e^
2 + f^2)*a^2 + (a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + (2*a^2*d^2*e*f + 2*b^2*d^
2*e*f - a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e
^2*f*x + a^3*d^2*e^3 + (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*
a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)*e^(d*x)), x) + integrate(2*(a^2*b +
 b^3 - (a^3*e^c + a*b^2*e^c)*e^(d*x))/(a^3*b*f*x + a^3*b*e - (a^3*b*f*x*e^
(2*c) + a^3*b*e*e^(2*c))*e^(2*d*x) - 2*(a^4*f*x*e^c + a^4*e*e^c)*e^(d*x)),
 x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(coth(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 2.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(c+d\,x)^3}{(e+f\,x)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(coth(c + d*x)^3/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(coth(c + d*x)^3/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.43

$$\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$=\int \frac{\coth(dx+c)^3}{\sinh(dx+c)\,be+\sinh(dx+c)\,bfx+ae+afx}dx$$

input
```
int(coth(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(coth(c + d*x)**3/(sinh(c + d*x)*b*e + sinh(c + d*x)*b*f*x + a*e + a*f*
x),x)
```

3.491 $\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 1795

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output
```
-3/4*b^2*f^3*polylog(4,-exp(2*d*x+2*c))/a^3/d^4-2*b^2*(f*x+e)^3*arctanh(ex
p(2*d*x+2*c))/a^3/d+2*b*(f*x+e)^3*arctan(exp(d*x+c))/a^2/d-3/2*f*(f*x+e)^2
*polylog(2,exp(2*d*x+2*c))/a/d^2+3/2*f^2*(f*x+e)*polylog(3,exp(2*d*x+2*c))
/a/d^3-3/2*f^2*(f*x+e)*polylog(3,-exp(2*d*x+2*c))/a/d^3+3/2*f*(f*x+e)^2*po
lylog(2,-exp(2*d*x+2*c))/a/d^2-3/4*f^3*polylog(4,exp(2*d*x+2*c))/a/d^4+3/4
*f^3*polylog(4,-exp(2*d*x+2*c))/a/d^4+2*(f*x+e)^3*arctanh(exp(2*d*x+2*c))/
a/d-3*I*b^3*f*(f*x+e)^2*polylog(2,I*exp(d*x+c))/a^2/(a^2+b^2)/d^2-6*I*b^3*
f^2*(f*x+e)*polylog(3,-I*exp(d*x+c))/a^2/(a^2+b^2)/d^3+3*I*b^3*f*(f*x+e)^2
*polylog(2,-I*exp(d*x+c))/a^2/(a^2+b^2)/d^2+6*I*b^3*f^2*(f*x+e)*polylog(3,
I*exp(d*x+c))/a^2/(a^2+b^2)/d^3+6*I*b^3*f^3*polylog(4,-I*exp(d*x+c))/a^2/(
a^2+b^2)/d^4+6*I*b*f^2*(f*x+e)*polylog(3,-I*exp(d*x+c))/a^2/d^3+6*b*f*(f*x
+e)^2*arctanh(exp(d*x+c))/a^2/d^2+6*b*f^2*(f*x+e)*polylog(2,-exp(d*x+c))/a
^2/d^3-6*b*f^2*(f*x+e)*polylog(2,exp(d*x+c))/a^2/d^3+3/2*b^2*f*(f*x+e)^2*p
olylog(2,exp(2*d*x+2*c))/a^3/d^2-3/2*b^2*f^2*(f*x+e)*polylog(3,exp(2*d*x+2
*c))/a^3/d^3-3/2*f*(f*x+e)^2*coth(d*x+c)/a/d^2+3*f^2*(f*x+e)*ln(1-exp(2*d*
x+2*c))/a/d^3-6*b*f^3*polylog(3,-exp(d*x+c))/a^2/d^4+6*b*f^3*polylog(3,exp
(d*x+c))/a^2/d^4+3/4*b^2*f^3*polylog(4,exp(2*d*x+2*c))/a^3/d^4-3/2*b^4*f^2
*(f*x+e)*polylog(3,-exp(2*d*x+2*c))/a^3/(a^2+b^2)/d^3+3/2*b^4*f*(f*x+e)^2*
polylog(2,-exp(2*d*x+2*c))/a^3/(a^2+b^2)/d^2-6*I*b^3*f^3*polylog(4,I*exp(d
*x+c))/a^2/(a^2+b^2)/d^4-6*I*b*f^2*(f*x+e)*polylog(3,I*exp(d*x+c))/a^2/...
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 5813 vs. $2(1795) = 3590$.

Time = 12.24 (sec) , antiderivative size = 5813, normalized size of antiderivative = 3.24

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 8.58 (sec) , antiderivative size = 1596, normalized size of antiderivative = 0.89, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.676, Rules used = {6123, 5985, 27, 6123, 5985, 25, 6123, 5984, 3042, 26, 4670, 3011, 6107, 6095, 2620, 3011, 7163, 2720, 7143, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)^3\text{csch}^3(c+dx)\text{sech}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5985}$$

$$\frac{-3f\int -\frac{1}{2}(e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-\frac{b\int \frac{(e+fx)^3\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3}{2}f\int (e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-\frac{b\int \frac{(e+fx)^3\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{6123}$$

$$\frac{\frac{3}{2}f\int (e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^3\text{csch}^2(c+dx)\text{sech}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{5985}$$

$$\frac{\frac{3}{2}f\int(e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-\frac{b\left(\frac{-3f\int-(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\text{csch}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^3\text{csch}(c+dx)\text{sech}(c}{a+b\sinh(c+dx)}}{a}\right.}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{3}{2}f\int(e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-\frac{b\left(\frac{3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\text{csch}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}}{a}\right.}{a}$$

$$\Big\downarrow \text{6123}$$

$$\frac{\frac{3}{2}f\int(e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-\frac{b\left(\frac{3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\text{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx}{a}\right.}{a}\right.}{a}$$

$$\Big\downarrow \text{5984}$$

$$\frac{\frac{3}{2}f\int(e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-\frac{b\left(\frac{3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\text{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{2\int(e+fx)^3\text{csch}(2c+2dx)dx}{a}-\frac{b\int}{}\right.}{a}\right.}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{2}f\int(e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-$$

$$\frac{b\left(\frac{3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2}{}\right.}{a}\right.}{a}$$

$\downarrow$ 26

$$\frac{\frac{3}{2}f\int(e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-$$

$$\frac{b\left(\frac{3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+2i\right.}{a}\right.}{a}$$

$\downarrow$ 4670

$$\frac{\frac{3}{2}f\int(e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-$$

$$\frac{b\left(\frac{3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i}{}\right.}{}\right.}{a}$$

$\downarrow$ 3011

$$\frac{\frac{3}{2}f\int (e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{3f\int (e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\cdots\right.}{\cdots}\right.$$

$\downarrow$ 6107

$$\frac{\frac{3}{2}f\int (e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{3f\int (e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(\frac{b^2\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 6095

$$\frac{\frac{3}{2}f\int(e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(b^2\left(\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx\right.\right.}{}\right.}{}\right.$$

$\downarrow$ 2620

$$\frac{\frac{3}{2}f\int(e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(b^2\left(-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+}}{a-\sqrt{}}\right.}{bd}\right.\right.}{}\right.}{}\right.$$

$\downarrow$ 3011

$$\frac{\frac{3}{2}f\int(e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(b^2\left(-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLo}\ldots}{\ldots}\right)}{\ldots}\right)\ldots\right)}{\ldots}\right)}{\ldots}\right)$$

↓ 7163

$$\frac{-\frac{\coth^2(c+dx)(e+fx)^3}{2d}-\frac{\log(\tanh(c+dx))(e+fx)^3}{d}+\frac{3}{2}f\int(e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx}{a}-$$

$$b\left(\frac{-\frac{\arctan(\sinh(c+dx))(e+fx)^3}{d}-\frac{\operatorname{csch}(c+dx)(e+fx)^3}{d}+3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx}{a}-\frac{b\left(2i\left(\frac{i\operatorname{arctanh}\left(e^{2c+2dx}\right)(e+f\ldots}{d}\right.\right.}{\ldots}\right.$$

↓ 2720

$$\frac{-\frac{\coth^2(c+dx)(e+fx)^3}{2d}-\frac{\log(\tanh(c+dx))(e+fx)^3}{d}+\frac{3}{2}f\int(e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx}{a}-$$

$$b\left(\frac{-\frac{\arctan(\sinh(c+dx))(e+fx)^3}{d}-\frac{\operatorname{csch}(c+dx)(e+fx)^3}{d}+3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx}{a}-\frac{b\left(\frac{2i\left(\frac{i\operatorname{arctanh}\left(e^{2c+2dx}\right)(e+f\ldots}{d}\right.}{\ldots}\right.}{\ldots}\right.$$

↓ 7143

$$\frac{-\frac{\coth^2(c+dx)(e+fx)^3}{2d} - \frac{\log(\tanh(c+dx))(e+fx)^3}{d} + \frac{3}{2}f\int(e+fx)^2\left(\frac{\coth^2(c+dx)}{d} + \frac{2\log(\tanh(c+dx))}{d}\right)dx}{a} -$$

$$b\left(\frac{-\frac{\arctan(\sinh(c+dx))(e+fx)^3}{d} - \frac{\operatorname{csch}(c+dx)(e+fx)^3}{d} + 3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d} + \frac{\operatorname{csch}(c+dx)}{d}\right)dx}{a} - \frac{b\left(2i\left(\frac{i\operatorname{arctanh}\left(e^{2c+2dx}\right)(e+f\ldots}{d}\right.\right.}{\ldots}\right.$$

↓ 7292

$$\frac{-\frac{\coth^2(c+dx)(e+fx)^3}{2d}-\frac{\log(\tanh(c+dx))(e+fx)^3}{d}+\frac{3}{2}f\int\frac{(e+fx)^2\left(\coth^2(c+dx)+2\log(\tanh(c+dx))\right)}{d}dx}{a}-$$

$$b\left(\frac{-\frac{\arctan(\sinh(c+dx))(e+fx)^3}{d}-\frac{\operatorname{csch}(c+dx)(e+fx)^3}{d}+3f\int\frac{(e+fx)^2(\arctan(\sinh(c+dx))+\operatorname{csch}(c+dx))}{d}dx}{a}-\frac{b\left(2i\left(\frac{i\operatorname{arctanh}\left(e^{2c+2dx}\right)(e+fx)^3}{d}\right.\right.}{}\right.$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{\coth^2(c+dx)(e+fx)^3}{2d}-\frac{\log(\tanh(c+dx))(e+fx)^3}{d}+\frac{3f\int(e+fx)^2\left(\coth^2(c+dx)+2\log(\tanh(c+dx))\right)dx}{2d}}{a}-$$

$$b\left(\frac{-\frac{\arctan(\sinh(c+dx))(e+fx)^3}{d}-\frac{\operatorname{csch}(c+dx)(e+fx)^3}{d}+\frac{3f\int(e+fx)^2(\arctan(\sinh(c+dx))+\operatorname{csch}(c+dx))dx}{d}}{a}-\frac{b\left(\frac{2i\left(\frac{i\operatorname{arctanh}\left(e^{2c+2dx}\right)(e+fx)^3}{d}-\right.}{}\right.}{}\right.$$

↓ 7293

$$\frac{-\frac{\coth^2(c+dx)(e+fx)^3}{2d}-\frac{\log(\tanh(c+dx))(e+fx)^3}{d}+\frac{3f\int\left(\coth^2(c+dx)(e+fx)^2+2\log(\tanh(c+dx))(e+fx)^2\right)dx}{2d}}{a}-$$

$$b\left(\frac{-\frac{\arctan(\sinh(c+dx))(e+fx)^3}{d}-\frac{\operatorname{csch}(c+dx)(e+fx)^3}{d}+\frac{3f\int\left(\arctan(\sinh(c+dx))(e+fx)^2+\operatorname{csch}(c+dx)(e+fx)^2\right)dx}{d}}{a}-\frac{b\left(2i\left(\frac{i\operatorname{arctanh}\left(e^{2c+2dx}\right)(}{d}\right.\right.}{}\right.$$

↓ 2009

$$-\frac{\coth^2(c+dx)(e+fx)^3}{2d}-\frac{\log(\tanh(c+dx))(e+fx)^3}{d}+\frac{3f\left(\frac{4\operatorname{arctanh}\left(e^{2c+2dx}\right)(e+fx)^3}{3f}+\frac{2\log(\tanh(c+dx))(e+fx)^3}{3f}+\frac{(e+fx)^3}{3f}-\frac{\coth(c+dx)(e+\cdots}{d}\right.}{}$$

$$b\left(-\frac{\arctan(\sinh(c+dx))(e+fx)^3}{d}-\frac{\operatorname{csch}(c+dx)(e+fx)^3}{d}+\frac{3f\left(-\frac{2\arctan\left(e^{c+dx}\right)(e+fx)^3}{3f}+\frac{\arctan(\sinh(c+dx))(e+fx)^3}{3f}-\frac{2\operatorname{arctanh}\left(e^{c+dx}\right)(e+fx)^2}{d}+\frac{i\cdots}{}\right.}{}\right.$$

input
```
Int[((e + f*x)^3*Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-1/2*((e + f*x)^3*Coth[c + d*x]^2)/d - ((e + f*x)^3*Log[Tanh[c + d*x]])/d
 + (3*f*(-((e + f*x)^2/d) + (e + f*x)^3/(3*f) + (4*(e + f*x)^3*ArcTanh[E^(
2*c + 2*d*x)])/(3*f) - ((e + f*x)^2*Coth[c + d*x])/d + (2*f*(e + f*x)*Log[
1 - E^(2*(c + d*x))])/d^2 + (2*(e + f*x)^3*Log[Tanh[c + d*x]])/(3*f) + (f^
2*PolyLog[2, E^(2*(c + d*x))])/d^3 + ((e + f*x)^2*PolyLog[2, -E^(2*c + 2*d
*x)])/d - ((e + f*x)^2*PolyLog[2, E^(2*c + 2*d*x)])/d - (f*(e + f*x)*PolyL
og[3, -E^(2*c + 2*d*x)])/d^2 + (f*(e + f*x)*PolyLog[3, E^(2*c + 2*d*x)])/d
^2 + (f^2*PolyLog[4, -E^(2*c + 2*d*x)])/(2*d^3) - (f^2*PolyLog[4, E^(2*c +
 2*d*x)])/(2*d^3)))/(2*d))/a - (b*((-(((e + f*x)^3*ArcTan[Sinh[c + d*x]])/
d) - ((e + f*x)^3*Csch[c + d*x])/d + (3*f*((-2*(e + f*x)^3*ArcTan[E^(c + d
*x)])/(3*f) + ((e + f*x)^3*ArcTan[Sinh[c + d*x]])/(3*f) - (2*(e + f*x)^2*A
rcTanh[E^(c + d*x)])/d - (2*f*(e + f*x)*PolyLog[2, -E^(c + d*x)])/d^2 + (I
*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/d - (I*(e + f*x)^2*PolyLog[2, I
*E^(c + d*x)])/d + (2*f*(e + f*x)*PolyLog[2, E^(c + d*x)])/d^2 + (2*f^2*Po
lyLog[3, -E^(c + d*x)])/d^3 - ((2*I)*f*(e + f*x)*PolyLog[3, (-I)*E^(c + d*
x)])/d^2 + ((2*I)*f*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/d^2 - (2*f^2*Poly
Log[3, E^(c + d*x)])/d^3 + ((2*I)*f^2*PolyLog[4, (-I)*E^(c + d*x)])/d^3 -
((2*I)*f^2*PolyLog[4, I*E^(c + d*x)])/d^3))/d)/a - (b*(-((b*((b^2*(-1/4*(e
 + f*x)^4/(b*f) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
])])/(b*d) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^3 \operatorname{csch}(dx+c)^3 \operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 23903 vs. $2(1636) = 3272$.

Time = 0.66 (sec) , antiderivative size = 23903, normalized size of antiderivative = 13.32

$$\int \frac{(e+fx)^3 \operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)**3*csch(d*x+c)**3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\,\text{csch}\,(dx+c)^3\,\text{sech}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-(b^4*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^5 + a^3*b^2)*d)
+ 2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/(
(a^2 + b^2)*d) + 2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*
c))/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + (a^2 - b^2
)*log(e^(-d*x - c) + 1)/(a^3*d) + (a^2 - b^2)*log(e^(-d*x - c) - 1)/(a^3*d
))*e^3 + (3*a*f^3*x^2 + 6*a*e*f^2*x + 3*a*e^2*f + 2*(b*d*f^3*x^3*e^(3*c) +
 3*b*d*e*f^2*x^2*e^(3*c) + 3*b*d*e^2*f*x*e^(3*c))*e^(3*d*x) - (2*a*d*f^3*x
^3*e^(2*c) + 3*a*e^2*f*e^(2*c) + 3*(2*d*e*f^2 + f^3)*a*x^2*e^(2*c) + 6*(d*
e^2*f + e*f^2)*a*x*e^(2*c))*e^(2*d*x) - 2*(b*d*f^3*x^3*e^c + 3*b*d*e*f^2*x
^2*e^c + 3*b*d*e^2*f*x*e^c)*e^(d*x))/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*
e^(2*d*x + 2*c) + a^2*d^2) - 3*(b*d*e^2*f + a*e*f^2)*x/(a^2*d^2) + 3*(b*d*
e^2*f - a*e*f^2)*x/(a^2*d^2) + 3*(b*d*e^2*f + a*e*f^2)*log(e^(d*x + c) + 1
)/(a^2*d^3) - 3*(b*d*e^2*f - a*e*f^2)*log(e^(d*x + c) - 1)/(a^2*d^3) - (d^
3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog
(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*(a^2*f^3 - b^2*f^3)/(a^3*d
^4) - (d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*
x*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))*(a^2*f^3 - b^2*f^3)
/(a^3*d^4) - 3*(a^2*d*e*f^2 - b^2*d*e*f^2 - a*b*f^3)*(d^2*x^2*log(e^(d*x +
 c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))/(a^3*d^
4) - 3*(a^2*d*e*f^2 - b^2*d*e*f^2 + a*b*f^3)*(d^2*x^2*log(-e^(d*x + c) ...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^3}{\cosh(c+d\,x)\,\sinh(c+d\,x)^3\,(a+b\sinh(c+d\,x))}\,dx$$

input
```
int((e + f*x)^3/(cosh(c + d*x)*sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^3/(cosh(c + d*x)*sinh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^3*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
(2*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**3*b*e**3 - 4*e**(2*c + 2*d*x)*at
an(e**(c + d*x))*a**3*b*e**3 + 2*atan(e**(c + d*x))*a**3*b*e**3 + 32*e**(9
*c + 4*d*x)*int((e**(5*d*x)*x**3)/(e**(10*c + 10*d*x)*b + 2*e**(9*c + 9*d*
x)*a - 3*e**(8*c + 8*d*x)*b - 4*e**(7*c + 7*d*x)*a + 2*e**(6*c + 6*d*x)*b
+ 2*e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - 3*e**(2*c + 2*d*x)*b - 2*e
**(c + d*x)*a + b),x)*a**5*d*f**3 + 32*e**(9*c + 4*d*x)*int((e**(5*d*x)*x*
*3)/(e**(10*c + 10*d*x)*b + 2*e**(9*c + 9*d*x)*a - 3*e**(8*c + 8*d*x)*b -
4*e**(7*c + 7*d*x)*a + 2*e**(6*c + 6*d*x)*b + 2*e**(4*c + 4*d*x)*b + 4*e**
(3*c + 3*d*x)*a - 3*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**3*b**
2*d*f**3 + 96*e**(9*c + 4*d*x)*int((e**(5*d*x)*x**2)/(e**(10*c + 10*d*x)*b
 + 2*e**(9*c + 9*d*x)*a - 3*e**(8*c + 8*d*x)*b - 4*e**(7*c + 7*d*x)*a + 2*
e**(6*c + 6*d*x)*b + 2*e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - 3*e**(2
*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**5*d*e*f**2 + 96*e**(9*c + 4*d*
x)*int((e**(5*d*x)*x**2)/(e**(10*c + 10*d*x)*b + 2*e**(9*c + 9*d*x)*a - 3*
e**(8*c + 8*d*x)*b - 4*e**(7*c + 7*d*x)*a + 2*e**(6*c + 6*d*x)*b + 2*e**(4
*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - 3*e**(2*c + 2*d*x)*b - 2*e**(c + d*
x)*a + b),x)*a**3*b**2*d*e*f**2 + 96*e**(9*c + 4*d*x)*int((e**(5*d*x)*x)/(
e**(10*c + 10*d*x)*b + 2*e**(9*c + 9*d*x)*a - 3*e**(8*c + 8*d*x)*b - 4*e**
(7*c + 7*d*x)*a + 2*e**(6*c + 6*d*x)*b + 2*e**(4*c + 4*d*x)*b + 4*e**(3*c
+ 3*d*x)*a - 3*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**5*d*e**...
```

3.492 $\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 1210

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output
```
-b^2*f*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/a^3/d^2+b^4*(f*x+e)^2*ln(1+exp(2
*d*x+2*c))/a^3/(a^2+b^2)/d+1/2*b^2*f^2*polylog(3,-exp(2*d*x+2*c))/a^3/d^3-
2*b^2*(f*x+e)^2*arctanh(exp(2*d*x+2*c))/a^3/d+2*b*(f*x+e)^2*arctan(exp(d*x
+c))/a^2/d+1/2*f^2*polylog(3,exp(2*d*x+2*c))/a/d^3+2*(f*x+e)^2*arctanh(exp
(2*d*x+2*c))/a/d-1/2*f^2*polylog(3,-exp(2*d*x+2*c))/a/d^3+2*I*b^3*f*(f*x+e
)*polylog(2,-I*exp(d*x+c))/a^2/(a^2+b^2)/d^2+2*I*b*f^2*polylog(3,-I*exp(d*
x+c))/a^2/d^3+2*I*b^3*f^2*polylog(3,I*exp(d*x+c))/a^2/(a^2+b^2)/d^3+2*I*b*
f*(f*x+e)*polylog(2,I*exp(d*x+c))/a^2/d^2+b^4*f*(f*x+e)*polylog(2,-exp(2*d
*x+2*c))/a^3/(a^2+b^2)/d^2-f*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a/d^2-2*I*b
^3*f*(f*x+e)*polylog(2,I*exp(d*x+c))/a^2/(a^2+b^2)/d^2+4*b*f*(f*x+e)*arcta
nh(exp(d*x+c))/a^2/d^2+2*b*f^2*polylog(2,-exp(d*x+c))/a^2/d^3-2*b*f^2*poly
log(2,exp(d*x+c))/a^2/d^3-1/2*b^2*f^2*polylog(3,exp(2*d*x+2*c))/a^3/d^3+f^
2*ln(sinh(d*x+c))/a/d^3-1/2*b^4*f^2*polylog(3,-exp(2*d*x+2*c))/a^3/(a^2+b^
2)/d^3-2*I*b*f^2*polylog(3,I*exp(d*x+c))/a^2/d^3+f*(f*x+e)*polylog(2,-exp(
2*d*x+2*c))/a/d^2-f*(f*x+e)*coth(d*x+c)/a/d^2+b*(f*x+e)^2*csch(d*x+c)/a^2/
d+b^2*f*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a^3/d^2-1/2*(f*x+e)^2*coth(d*x+c
)^2/a/d-2*b^3*(f*x+e)^2*arctan(exp(d*x+c))/a^2/(a^2+b^2)/d-2*I*b^3*f^2*pol
ylog(3,-I*exp(d*x+c))/a^2/(a^2+b^2)/d^3-2*I*b*f*(f*x+e)*polylog(2,-I*exp(d
*x+c))/a^2/d^2+1/2*(f*x+e)^2/a/d-2*b^4*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(
a+(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)/d^2-2*b^4*f*(f*x+e)*polylog(2,-b*exp(...
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2784 vs. 2(1210) = 2420.

Time = 10.20 (sec) , antiderivative size = 2784, normalized size of antiderivative = 2.30

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
(b*(e + f*x)^2*Csch[c])/(a^2*d) - ((e + f*x)^2*Csch[(c + d*x)/2]^2)/(8*a*d
) + (-12*a*d^3*e^2*E^(2*c)*x + 12*a*d^3*e^2*(1 + E^(2*c))*x + 12*a*d^3*e*f
*x^2 + 4*a*d^3*f^2*x^3 + 12*b*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] -
6*a*d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*b*d*
e*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) -
 PolyLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) - 6*a*d*e*(1 +
E^(2*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c +
 d*x))]) + (6*I)*b*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2
*x^2*Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*P
olyLog[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I
*E^(c + d*x)]) - a*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c
 + d*x))]) - 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c +
d*x))]))/(6*(a^2 + b^2)*d^3*(1 + E^(2*c))) + (12*a^2*d^3*e^2*E^(2*c)*x - 1
2*b^2*d^3*e^2*E^(2*c)*x - 12*a^2*d*E^(2*c)*f^2*x + 12*a^2*d^3*e*E^(2*c)*f*
x^2 - 12*b^2*d^3*e*E^(2*c)*f*x^2 + 4*a^2*d^3*E^(2*c)*f^2*x^3 - 4*b^2*d^3*E
^(2*c)*f^2*x^3 - 24*a*b*d*e*f*ArcTanh[E^(c + d*x)] + 24*a*b*d*e*E^(2*c)*f*
ArcTanh[E^(c + d*x)] + 12*a*b*d*f^2*x*Log[1 - E^(c + d*x)] - 12*a*b*d*E^(2
*c)*f^2*x*Log[1 - E^(c + d*x)] - 12*a*b*d*f^2*x*Log[1 + E^(c + d*x)] + 12*
a*b*d*E^(2*c)*f^2*x*Log[1 + E^(c + d*x)] + 6*a^2*d^2*e^2*Log[1 - E^(2*(c +
 d*x))] - 6*b^2*d^2*e^2*Log[1 - E^(2*(c + d*x))] - 6*a^2*d^2*e^2*E^(2*c...
```

Rubi [A] (verified)

Time = 6.34 (sec) , antiderivative size = 1118, normalized size of antiderivative = 0.92, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.676$, Rules used = {6123, 5985, 27, 6123, 5985, 25, 6123, 5984, 3042, 26, 4670, 3011, 2720, 6107, 6095, 2620, 3011, 2720, 7143, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)^2\text{csch}^3(c+dx)\text{sech}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5985}$$

$$\frac{-2f\int -\frac{1}{2}(e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-\frac{b\int\frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$\Big\downarrow$ 27

$$\frac{f\int(e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-\frac{b\int\frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$\Big\downarrow$ 6123

$$\frac{f\int(e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\text{csch}^2(c+dx)\text{sech}(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$\Big\downarrow$ 5985

$$\frac{f\int(e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-\frac{b\left(\frac{-2f\int-\left((e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$\Big\downarrow$ 25

$$\frac{f\int(e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$\Big\downarrow$ 6123

$$\frac{f\int(e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)}{a}\right.}{a}\right.}{a}$$

$\downarrow$ 5984

$$\frac{f\int(e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{2\int(e+fx)^2\text{csch}(2c+2dx)dx}{a}-\frac{b\int}{}\right.}{a}\right.}{a}$$

$\downarrow$ 3042

$$\frac{f\int(e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+2\int\right.}{a}\right.}{a}$$

$\downarrow$ 26

$$\frac{f\int(e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+2i\int\right.}{a}\right.}{a}$$

$\downarrow$ 4670

$$\frac{f\int (e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$\frac{b\left(\frac{2f\int (e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(\right.}{}\right.}{}\right.}{a}$$

$\downarrow$ 3011

$$\frac{f\int (e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$\frac{b\left(\frac{2f\int (e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(\right.}{}\right.}{}\right.}{}$$

$\downarrow$ 2720

$$\frac{f\int(e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(\right.}{}\right.}{}\right.$$

$\downarrow$ 6107

$$\frac{f\int(e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(\frac{b^2\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\right.}{}\right.}{}\right.$$

$\downarrow$ 6095

$$\frac{f\int (e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{2f\int (e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(b^2\left(\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx-\right.\right.}{}\right.}{}\right.$$

$$\Big\downarrow \text{2620}$$

$$\frac{f\int (e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{2f\int (e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(b^2\left(-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}}{a-\sqrt{a^2}}\right.}{bd}\right.\right.}{}\right.}{}\right.$$

$$\Big\downarrow \text{3011}$$

$$\frac{f\int(e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(b-\frac{b\left(b^2\left(-\frac{2f\left(\frac{f\int \operatorname{PolyLog}\left(2,-\frac{b}{a-\ldots}\right.}{d}\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.$$

$\downarrow$ 2720

$$\frac{f\int(e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(b^2\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}}{}\right.}{}\right.}{}\right.}{}\right.}{}\right.$$

↓ 7143

$$\frac{f\int(e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\left(b-\frac{b\left(\frac{\int(e+fx)^2\text{sech}(c+dx)(a-b\sin\ldots}{a^2+b^2}\right.}{\ldots}\right.}{\ldots}\right.$$

↓ 7292

$$\frac{f\int \frac{(e+fx)\left(\coth^2(c+dx)+2\log(\tanh(c+dx))\right)}{d}dx - \frac{(e+fx)^2\coth^2(c+dx)}{2d} - \frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a} -$$

$$b\left(\frac{2f\int \frac{(e+fx)(\arctan(\sinh(c+dx))+\operatorname{csch}(c+dx))}{d}dx - \frac{(e+fx)^2\arctan(\sinh(c+dx))}{d} - \frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a} - \frac{b\left(-\frac{b\left(\frac{\int (e+fx)^2\operatorname{sech}(c+dx)(a-b\sinh(c}{a^2+b^2}\right.}{}\right.}{}\right.$$

$\downarrow$ 27

$$\frac{\frac{f\int(e+fx)\left(\coth^2(c+dx)+2\log(\tanh(c+dx))\right)dx}{d}-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{\frac{2f\int(e+fx)(\arctan(\sinh(c+dx))+\text{csch}(c+dx))dx}{d}-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(\frac{\int(e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+\cdots}{a^2+b^2}\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 7293

$$\frac{\frac{f\int\left((e+fx)\coth^2(c+dx)+2(e+fx)\log(\tanh(c+dx))\right)dx}{d}-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{\frac{2f\int((e+fx)\arctan(\sinh(c+dx))+(e+fx)\operatorname{csch}(c+dx))dx}{d}-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(\frac{\int\left(a(e+fx)^2\operatorname{sech}(c+dx)-b\right.}{a^2+b}\right.}{}\right.}{}\right.$$

$\downarrow$ 2009

$$\frac{-\frac{\coth^2(c+dx)(e+fx)^2}{2d} - \frac{\log(\tanh(c+dx))(e+fx)^2}{d} + \frac{f\left(\frac{2\operatorname{arctanh}\left(e^{2c+2dx}\right)(e+fx)^2}{f} + \frac{\log(\tanh(c+dx))(e+fx)^2}{f} + \frac{(e+fx)^2}{2f} - \frac{\coth(c+dx)(e+fx}{d}\right.}{}}{a}$$

$$b\left(\frac{-\frac{\arctan(\sinh(c+dx))(e+fx)^2}{d} - \frac{\operatorname{csch}(c+dx)(e+fx)^2}{d} + \frac{2f\left(-\frac{\arctan\left(e^{c+dx}\right)(e+fx)^2}{f} + \frac{\arctan(\sinh(c+dx))(e+fx)^2}{2f} - \frac{2\operatorname{arctanh}\left(e^{c+dx}\right)(e+fx)}{d} + \frac{i\,\mathrm{Pol}}{}\right.}{}}{}\right.$$

input
```
Int[((e + f*x)^2*Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-1/2*((e + f*x)^2*Coth[c + d*x]^2)/d - ((e + f*x)^2*Log[Tanh[c + d*x]])/d
 + (f*((e + f*x)^2/(2*f) + (2*(e + f*x)^2*ArcTanh[E^(2*c + 2*d*x)])/f - ((
e + f*x)*Coth[c + d*x])/d + (f*Log[Sinh[c + d*x]])/d^2 + ((e + f*x)^2*Log[
Tanh[c + d*x]])/f + ((e + f*x)*PolyLog[2, -E^(2*c + 2*d*x)])/d - ((e + f*x
)*PolyLog[2, E^(2*c + 2*d*x)])/d - (f*PolyLog[3, -E^(2*c + 2*d*x)])/(2*d^2
) + (f*PolyLog[3, E^(2*c + 2*d*x)])/(2*d^2)))/d)/a - (b*((-(((e + f*x)^2*A
rcTan[Sinh[c + d*x]])/d) - ((e + f*x)^2*Csch[c + d*x])/d + (2*f*(-(((e + f
*x)^2*ArcTan[E^(c + d*x)])/f) + ((e + f*x)^2*ArcTan[Sinh[c + d*x]])/(2*f)
- (2*(e + f*x)*ArcTanh[E^(c + d*x)])/d - (f*PolyLog[2, -E^(c + d*x)])/d^2
+ (I*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d - (I*(e + f*x)*PolyLog[2, I
*E^(c + d*x)])/d + (f*PolyLog[2, E^(c + d*x)])/d^2 - (I*f*PolyLog[3, (-I)*
E^(c + d*x)])/d^2 + (I*f*PolyLog[3, I*E^(c + d*x)])/d^2))/d)/a - (b*(-((b*
((b^2*(-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[
a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2]))])/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))])/d^2))/(b*d)))/(a^2 + b^2) + ((b*(e + f*x)^3)/(3*f) + (2*a*(e + f
*x)^2*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c)^3 \operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 13309 vs. 2(1110) = 2220.

Time = 0.43 (sec) , antiderivative size = 13309, normalized size of antiderivative = 11.00

$$\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*csch(d*x+c)**3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\,\text{csch}\,(dx+c)^3\,\text{sech}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-(b^4*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^5 + a^3*b^2)*d)
+ 2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/(
(a^2 + b^2)*d) + 2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*
c))/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + (a^2 - b^2
)*log(e^(-d*x - c) + 1)/(a^3*d) + (a^2 - b^2)*log(e^(-d*x - c) - 1)/(a^3*d
))*e^2 + 2*(a*f^2*x + a*e*f + (b*d*f^2*x^2*e^(3*c) + 2*b*d*e*f*x*e^(3*c))*
e^(3*d*x) - (a*d*f^2*x^2*e^(2*c) + a*e*f*e^(2*c) + (2*d*e*f + f^2)*a*x*e^(
2*c))*e^(2*d*x) - (b*d*f^2*x^2*e^c + 2*b*d*e*f*x*e^c)*e^(d*x))/(a^2*d^2*e^
(4*d*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2) - (2*b*d*e*f + a*f^2)
*x/(a^2*d^2) + (2*b*d*e*f - a*f^2)*x/(a^2*d^2) + (2*b*d*e*f + a*f^2)*log(e
^(d*x + c) + 1)/(a^2*d^3) - (2*b*d*e*f - a*f^2)*log(e^(d*x + c) - 1)/(a^2*
d^3) - (d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polyl
og(3, -e^(d*x + c)))*(a^2*f^2 - b^2*f^2)/(a^3*d^3) - (d^2*x^2*log(-e^(d*x
+ c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*(a^2*f^2
 - b^2*f^2)/(a^3*d^3) - 2*(a^2*d*e*f - b^2*d*e*f - a*b*f^2)*(d*x*log(e^(d*
x + c) + 1) + dilog(-e^(d*x + c)))/(a^3*d^3) - 2*(a^2*d*e*f - b^2*d*e*f +
a*b*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^3*d^3) + 1/3*
((a^2*f^2 - b^2*f^2)*d^3*x^3 + 3*(a^2*d*e*f - b^2*d*e*f + a*b*f^2)*d^2*x^2
)/(a^3*d^3) + 1/3*((a^2*f^2 - b^2*f^2)*d^3*x^3 + 3*(a^2*d*e*f - b^2*d*e*f
- a*b*f^2)*d^2*x^2)/(a^3*d^3) + integrate(2*(b^5*f^2*x^2 + 2*b^5*e*f*x ...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\int \frac{(e+f\,x)^2}{\cosh(c+d\,x)\,\sinh(c+d\,x)^3\,(a+b\sinh(c+d\,x))}\,dx$$

input
```
int((e + f*x)^2/(cosh(c + d*x)*sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^2/(cosh(c + d*x)*sinh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
(2*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**3*b*e**2 - 4*e**(2*c + 2*d*x)*at
an(e**(c + d*x))*a**3*b*e**2 + 2*atan(e**(c + d*x))*a**3*b*e**2 + 32*e**(9
*c + 4*d*x)*int((e**(5*d*x)*x**2)/(e**(10*c + 10*d*x)*b + 2*e**(9*c + 9*d*
x)*a - 3*e**(8*c + 8*d*x)*b - 4*e**(7*c + 7*d*x)*a + 2*e**(6*c + 6*d*x)*b
+ 2*e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - 3*e**(2*c + 2*d*x)*b - 2*e
**(c + d*x)*a + b),x)*a**5*d*f**2 + 32*e**(9*c + 4*d*x)*int((e**(5*d*x)*x*
*2)/(e**(10*c + 10*d*x)*b + 2*e**(9*c + 9*d*x)*a - 3*e**(8*c + 8*d*x)*b -
4*e**(7*c + 7*d*x)*a + 2*e**(6*c + 6*d*x)*b + 2*e**(4*c + 4*d*x)*b + 4*e**
(3*c + 3*d*x)*a - 3*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**3*b**
2*d*f**2 + 64*e**(9*c + 4*d*x)*int((e**(5*d*x)*x)/(e**(10*c + 10*d*x)*b +
2*e**(9*c + 9*d*x)*a - 3*e**(8*c + 8*d*x)*b - 4*e**(7*c + 7*d*x)*a + 2*e**
(6*c + 6*d*x)*b + 2*e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - 3*e**(2*c
+ 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**5*d*e*f + 64*e**(9*c + 4*d*x)*int
((e**(5*d*x)*x)/(e**(10*c + 10*d*x)*b + 2*e**(9*c + 9*d*x)*a - 3*e**(8*c +
 8*d*x)*b - 4*e**(7*c + 7*d*x)*a + 2*e**(6*c + 6*d*x)*b + 2*e**(4*c + 4*d*
x)*b + 4*e**(3*c + 3*d*x)*a - 3*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b)
,x)*a**3*b**2*d*e*f + e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x) + 1)*a**4*e**2
 - e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**4*e**2 + e**(4*c + 4*d*x)*log
(e**(c + d*x) - 1)*b**4*e**2 - e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a**4
*e**2 + e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*b**4*e**2 - e**(4*c + 4*...
```

3.493 $\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 762

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{fx}{2ad}+\frac{2bfx\arctan\left(e^{c+dx}\right)}{a^2d}-\frac{2b^3(e+fx)\arctan\left(e^{c+dx}\right)}{a^2(a^2+b^2)d}-\frac{bfx\arctan(\sinh(c+dx))}{a^2d}$$
$$+\frac{b(e+fx)\arctan(\sinh(c+dx))}{a^2d}+\frac{2fx\operatorname{arctanh}\left(e^{2c+2dx}\right)}{ad}$$
$$-\frac{2b^2(e+fx)\operatorname{arctanh}\left(e^{2c+2dx}\right)}{a^3d}+\frac{bf\operatorname{arctanh}(\cosh(c+dx))}{a^2d^2}-\frac{f\coth(c+dx)}{2ad^2}$$
$$-\frac{(e+fx)\coth^2(c+dx)}{2ad}+\frac{b(e+fx)\operatorname{csch}(c+dx)}{a^2d}-\frac{b^4(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3(a^2+b^2)d}$$
$$-\frac{b^4(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3(a^2+b^2)d}+\frac{b^4(e+fx)\log\left(1+e^{2(c+dx)}\right)}{a^3(a^2+b^2)d}$$
$$+\frac{fx\log(\tanh(c+dx))}{ad}-\frac{(e+fx)\log(\tanh(c+dx))}{ad}-\frac{ibf\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{a^2d^2}$$
$$+\frac{ib^3f\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{a^2(a^2+b^2)d^2}+\frac{ibf\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{a^2d^2}-\frac{ib^3f\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{a^2(a^2+b^2)d^2}$$
$$-\frac{b^4f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3(a^2+b^2)d^2}-\frac{b^4f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3(a^2+b^2)d^2}$$
$$+\frac{b^4f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2a^3(a^2+b^2)d^2}+\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2ad^2}$$
$$-\frac{b^2f\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2a^3d^2}-\frac{f\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2ad^2}+\frac{b^2f\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2a^3d^2}$$

output
```
I*b*f*polylog(2,I*exp(d*x+c))/a^2/d^2+b^4*(f*x+e)*ln(1+exp(2*d*x+2*c))/a^3
/(a^2+b^2)/d-b*f*x*arctan(sinh(d*x+c))/a^2/d+b*(f*x+e)*arctan(sinh(d*x+c))
/a^2/d+f*x*ln(tanh(d*x+c))/a/d-1/2*f*polylog(2,exp(2*d*x+2*c))/a/d^2+1/2*f
*polylog(2,-exp(2*d*x+2*c))/a/d^2+I*b^3*f*polylog(2,-I*exp(d*x+c))/a^2/(a^
2+b^2)/d^2-(f*x+e)*ln(tanh(d*x+c))/a/d+2*f*x*arctanh(exp(2*d*x+2*c))/a/d+1
/2*b^2*f*polylog(2,exp(2*d*x+2*c))/a^3/d^2+1/2*f*x/a/d+1/2*b^4*f*polylog(2
,-exp(2*d*x+2*c))/a^3/(a^2+b^2)/d^2+2*b*f*x*arctan(exp(d*x+c))/a^2/d-2*b^3
*(f*x+e)*arctan(exp(d*x+c))/a^2/(a^2+b^2)/d-I*b*f*polylog(2,-I*exp(d*x+c))
/a^2/d^2-1/2*b^2*f*polylog(2,-exp(2*d*x+2*c))/a^3/d^2-2*b^2*(f*x+e)*arctan
h(exp(2*d*x+2*c))/a^3/d+b*f*arctanh(cosh(d*x+c))/a^2/d^2+b*(f*x+e)*csch(d*
x+c)/a^2/d-1/2*f*coth(d*x+c)/a/d^2-1/2*(f*x+e)*coth(d*x+c)^2/a/d-I*b^3*f*p
olylog(2,I*exp(d*x+c))/a^2/(a^2+b^2)/d^2-b^4*f*polylog(2,-b*exp(d*x+c)/(a+
(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)/d^2-b^4*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+
b^2)^(1/2)))/a^3/(a^2+b^2)/d^2-b^4*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^
(1/2)))/a^3/(a^2+b^2)/d-b^4*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))
/a^3/(a^2+b^2)/d
```

Mathematica [A] (warning: unable to verify)

Time = 9.15 (sec) , antiderivative size = 1009, normalized size of antiderivative = 1.32

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)*Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
((2*b*d*e*Cosh[(c + d*x)/2] - a*f*Cosh[(c + d*x)/2] - 2*b*c*f*Cosh[(c + d*
x)/2] + 2*b*f*(c + d*x)*Cosh[(c + d*x)/2])*Csch[(c + d*x)/2])/(4*a^2*d^2)
+ ((-(d*e) + c*f - f*(c + d*x))*Csch[(c + d*x)/2]^2)/(8*a*d^2) + (-1/2*((a
^2 - b^2)*(d*e + d*f*x)^2)/f - (a*b*f + a^2*(d*e + d*f*x) - b^2*(d*e + d*f
*x))*Log[1 - E^(-c - d*x)] + (a*b*f - a^2*(d*e + d*f*x) + b^2*(d*e + d*f*x
))*Log[1 + E^(-c - d*x)] + (a^2 - b^2)*f*PolyLog[2, -E^(-c - d*x)] + (a^2
- b^2)*f*PolyLog[2, E^(-c - d*x)])/(a^3*d^2) - (b^4*(-2*d*e*(c + d*x) + 2*
c*f*(c + d*x) - f*(c + d*x)^2 + (4*a*Sqrt[a^2 + b^2]*d*e*ArcTan[(a + b*E^(
c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b^2)^2] - (4*a*Sqrt[-(a^2 + b^2)
^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/(-a^2 - b^2)^(3/2) +
 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*f*(c + d
*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*f*Log[b - 2*a*E^(
c + d*x) - b*E^(2*(c + d*x))] + 2*d*e*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(
c + d*x)))] + 2*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f
*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(2*a^3*(a^2 + b^2)
*d^2) + (-(a*d*e*(c + d*x)) + a*c*f*(c + d*x) - (a*f*(c + d*x)^2)/2 + 2*b*
d*e*ArcTan[E^(c + d*x)] - 2*b*c*f*ArcTan[E^(c + d*x)] + I*b*f*(c + d*x)*Lo
g[1 - I*E^(c + d*x)] - I*b*f*(c + d*x)*Log[1 + I*E^(c + d*x)] + a*d*e*Log[
1 + E^(2*(c + d*x))] - a*c*f*Log[1 + E^(2*(c + d*x))] + a*f*(c + d*x)*Log[
1 + E^(2*(c + d*x))] - I*b*f*PolyLog[2, (-I)*E^(c + d*x)] + I*b*f*PolyL...
```

Rubi [A] (verified)

Time = 4.26 (sec) , antiderivative size = 673, normalized size of antiderivative = 0.88, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6123, 5985, 2009, 6123, 5985, 2009, 6123, 5984, 3042, 26, 4670, 2715, 2838, 6107, 6095, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)\text{csch}^3(c+dx)\text{sech}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5985}$$

$$\frac{-f\int\left(-\frac{\coth^2(c+dx)}{2d}-\frac{\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)\coth^2(c+dx)}{2d}-\frac{(e+fx)\log(\tanh(c+dx))}{d}}{a}-\frac{b\int\frac{(e+fx)\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$\big\downarrow$ 2009

$$\frac{-f\left(-\frac{2x\text{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\,c}{}}{a}$$
$$\frac{b\int\frac{(e+fx)\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$\big\downarrow$ 6123

$$\frac{-f\left(-\frac{2x\text{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\,c}{}}{a}$$
$$\frac{b\left(\frac{\int(e+fx)\text{csch}^2(c+dx)\text{sech}(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$\big\downarrow$ 5985

$$\frac{-f\left(-\frac{2x\text{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\,c}{}}{a}$$
$$\frac{b\left(\frac{-f\int\left(-\frac{\arctan(\sinh(c+dx))}{d}-\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$\big\downarrow$ 2009

$$\frac{-f\left(-\frac{2x\text{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\,c}{}}{a}$$
$$\frac{b\left(-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\,\text{Pol}}{}\right.}{a}\right.}{a}$$

$\big\downarrow$ 6123

$$\frac{-f\left(-\frac{2x\text{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\,\text{c}\cdots}{\cdots}}{a}$$

$$\frac{b\left(-\frac{b\left(\frac{\int(e+fx)\text{csch}(c+dx)\text{sech}(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}\cdots\right.}{\cdots}\right.}{a}$$

$\Big\downarrow$ 5984

$$\frac{-f\left(-\frac{2x\text{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\,\text{c}\cdots}{\cdots}}{a}$$

$$\frac{b\left(-\frac{b\left(\frac{2\int(e+fx)\text{csch}(2c+2dx)dx}{a}-\frac{b\int\frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyL}\cdots}{\cdots}\right.}{\cdots}\right.}{a}$$

$\Big\downarrow$ 3042

$$\frac{-f\left(-\frac{2x\text{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\,\text{c}\cdots}{\cdots}}{a}$$

$$\frac{b\left(\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\arctan(\sinh(c+dx))}{d}\cdots}{a}\cdots\right.}{a}$$

$\Big\downarrow$ 26

$$\frac{-f\left(-\frac{2x\text{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\,\text{c}\cdots}{\cdots}}{a}$$

$$\frac{b\left(\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\arctan(\sinh(c+dx))}{d}\cdots}{a}\cdots\right.}{a}$$

$$\Big\downarrow \text{4670}$$

$$\frac{-f\left(-\frac{2x\text{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\,c}{}}{a}$$

$$\frac{b\left(\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\arctan(\sinh(c+dx))}{d}}{a}\right.}{a}$$

$$\Big\downarrow \text{2715}$$

$$\frac{-f\left(-\frac{2x\text{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\,c}{}}{a}$$

$$b\left(\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\arctan(\sinh(c+dx))}{d}}{a}\right.$$

$$\Big\downarrow \text{2838}$$

$$\frac{-f\left(-\frac{2x\text{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\,c}{}}{a}$$

$$b\left(\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\arctan(\sinh(c+dx))}{d}}{a}\right.$$

$$\Big\downarrow \text{6107}$$

$$\frac{-f\left(-\frac{2x\text{arctanh}(e^{2c+2dx})}{d}-\frac{\text{PolyLog}(2,-e^{2c+2dx})}{2d^2}+\frac{\text{PolyLog}(2,e^{2c+2dx})}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\,c}{}}{a}$$

$$b\left(\frac{-f\left(\frac{2x\arctan(e^{c+dx})}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{i\,\text{PolyLog}(2,ie^{c+dx})}{d^2}\right)-\frac{(e+fx)\arctan(\sinh(c+dx))}{d}}{a}\right.$$

$$\downarrow \text{6095}$$

$$\frac{-f\left(-\frac{2x\text{arctanh}(e^{2c+2dx})}{d}-\frac{\text{PolyLog}(2,-e^{2c+2dx})}{2d^2}+\frac{\text{PolyLog}(2,e^{2c+2dx})}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\,c}{}}{a}$$

$$b\left(\frac{-f\left(\frac{2x\arctan(e^{c+dx})}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{i\,\text{PolyLog}(2,ie^{c+dx})}{d^2}\right)-\frac{(e+fx)\arctan(\sinh(c+dx))}{d}}{a}\right.$$

$$\downarrow \text{2620}$$

$$\frac{-f\left(-\frac{2x\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\,c}{}}{a}$$

$$b\left(\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\arctan(\sinh(c+dx))}{d}}{a}\right.$$

$$\Big\downarrow \text{2715}$$

$$\frac{-f\left(-\frac{2x\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\,\mathrm{c}}{}}{a}$$

$$b\left(\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\arctan(\sinh(c+dx))}{d}}{a}\right.$$

$$\downarrow \; 2838$$

$$\frac{-f\left(-\frac{2x\text{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\,\text{c}}{}}{a}$$

$$b\left(\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\arctan(\sinh(c+dx))}{d}}{a}\right.$$

$$\Big\downarrow \; 7293$$

$$\frac{-f\left(-\frac{2x\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\,\mathrm{c}}{}}{a}$$

$$b\left(\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\arctan(\sinh(c+dx))}{d}}{a}\right.$$

$\downarrow$ 2009

$$\frac{-f\left(-\frac{2x\text{arctanh}(e^{2c+2dx})}{d}-\frac{\text{PolyLog}(2,-e^{2c+2dx})}{2d^2}+\frac{\text{PolyLog}(2,e^{2c+2dx})}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\,c}{}}{a}$$

$$b\left(\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\arctan(\sinh(c+dx))}{d}}{a}\right.$$

input
```
Int[((e + f*x)*Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-1/2*((e + f*x)*Coth[c + d*x]^2)/d - ((e + f*x)*Log[Tanh[c + d*x]])/d - f
*(-1/2*x/d - (2*x*ArcTanh[E^(2*c + 2*d*x)])/d + Coth[c + d*x]/(2*d^2) - (x
*Log[Tanh[c + d*x]])/d - PolyLog[2, -E^(2*c + 2*d*x)]/(2*d^2) + PolyLog[2,
 E^(2*c + 2*d*x)]/(2*d^2)))/a - (b*((-(((e + f*x)*ArcTan[Sinh[c + d*x]])/d
) - ((e + f*x)*Csch[c + d*x])/d - f*((2*x*ArcTan[E^(c + d*x)])/d - (x*ArcT
an[Sinh[c + d*x]])/d + ArcTanh[Cosh[c + d*x]]/d^2 - (I*PolyLog[2, (-I)*E^(
c + d*x)])/d^2 + (I*PolyLog[2, I*E^(c + d*x)])/d^2))/a - (b*(-((b*((b^2*(-
1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]
)/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2)
 + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(a^2
 + b^2) + ((b*(e + f*x)^2)/(2*f) + (2*a*(e + f*x)*ArcTan[E^(c + d*x)])/d -
 (b*(e + f*x)*Log[1 + E^(2*(c + d*x))])/d - (I*a*f*PolyLog[2, (-I)*E^(c +
d*x)])/d^2 + (I*a*f*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*PolyLog[2, -E^(2
*(c + d*x))])/(2*d^2))/(a^2 + b^2)))/a) + ((2*I)*((I*(e + f*x)*ArcTanh[E^(
2*c + 2*d*x)])/d + ((I/4)*f*PolyLog[2, -E^(2*c + 2*d*x)])/d^2 - ((I/4)*f*P
olyLog[2, E^(2*c + 2*d*x)])/d^2))/a))/a))/a
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1477 vs. $2(714) = 1428$.

Time = 13.31 (sec) , antiderivative size = 1478, normalized size of antiderivative = 1.94

method	result	size
risch	Expression too large to display	1478

input
```
int((f*x+e)*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
8/d*e/(4*a^2+4*b^2)*b*arctan(exp(d*x+c))+4/d*a*e/(4*a^2+4*b^2)*ln(1+exp(2*
d*x+2*c))+4/d^2*a*f/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+c))+4/d^2*a*f/(4*a^2+4
*b^2)*dilog(1-I*exp(d*x+c))+1/d^2*b^2*f/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*e
xp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/d^2*b^2/a^3*f*dilog(exp(d*x+c)+1)+1/d*b^
2/a^3*e*ln(exp(d*x+c)-1)+1/d*b^2/a^3*e*ln(exp(d*x+c)+1)-1/d^2*b/a^2*f*ln(e
xp(d*x+c)-1)+1/d^2*b/a^2*f*ln(exp(d*x+c)+1)-1/d^2*b^2/a^3*f*dilog(exp(d*x+
c))-1/d^2/a^3*b^4*f/(a^2+b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+
(a^2+b^2)^(1/2)))-1/d^2/a^3*b^4*f/(a^2+b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^
(1/2)+a)/(a+(a^2+b^2)^(1/2)))+4/d*a*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*x+4
/d*a*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*x-8/d^2*c*f/(4*a^2+4*b^2)*b*arctan
(exp(d*x+c))+4*I/d^2*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))*b-4*I/d^2*f/(4*
a^2+4*b^2)*dilog(1+I*exp(d*x+c))*b-4/d^2*a*c*f/(4*a^2+4*b^2)*ln(1+exp(2*d*
x+2*c))+1/d^2/a^2*b^4*f/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(
a^2+b^2)^(1/2))-1/d/a^3*b^4*e/(a^2+b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)
-b)+4/d^2*a*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*c+4/d^2*a*f/(4*a^2+4*b^2)*l
n(1-I*exp(d*x+c))*c-1/d^2/a^2*b^2*f/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d
*x+c)+2*a)/(a^2+b^2)^(1/2))+1/d^2*c*f/a*ln(exp(d*x+c)-1)+1/d^2/a*f*dilog(e
xp(d*x+c))-1/d/a*e*ln(exp(d*x+c)+1)-1/d/a*e*ln(exp(d*x+c)-1)-1/d^2*b^2/a^3
*c*f*ln(exp(d*x+c)-1)+1/d*b^2/a^3*f*ln(exp(d*x+c)+1)*x-1/d/a*f*ln(exp(d*x+
c)+1)*x-1/d^2*f/a*dilog(exp(d*x+c)+1)-(-2*b*d*f*x*exp(3*d*x+3*c)+2*a*d*...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5731 vs. $2(695) = 1390$.

Time = 0.27 (sec) , antiderivative size = 5731, normalized size of antiderivative = 7.52

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)**3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{csch}\,(dx+c)^3\,\text{sech}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-(b^4*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^5 + a^3*b^2)*d)
+ 2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/(
(a^2 + b^2)*d) + 2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*
c))/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + (a^2 - b^2
)*log(e^(-d*x - c) + 1)/(a^3*d) + (a^2 - b^2)*log(e^(-d*x - c) - 1)/(a^3*d
))*e + (16*a^2*d*integrate(1/16*x/(a^3*d*e^(d*x + c) + a^3*d), x) - 16*b^2
*d*integrate(1/16*x/(a^3*d*e^(d*x + c) + a^3*d), x) - 16*a^2*d*integrate(1
/16*x/(a^3*d*e^(d*x + c) - a^3*d), x) + 16*b^2*d*integrate(1/16*x/(a^3*d*e
^(d*x + c) - a^3*d), x) - a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) + 1)/
(a^3*d^2)) + a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) - 1)/(a^3*d^2)) +
(2*b*d*x*e^(3*d*x + 3*c) - 2*b*d*x*e^(d*x + c) - (2*a*d*x*e^(2*c) + a*e^(2
*c))*e^(2*d*x) + a)/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) +
 a^2*d^2) + 16*integrate(-1/8*(a*b^4*x*e^(d*x + c) - b^5*x)/(a^5*b + a^3*b
^3 - (a^5*b*e^(2*c) + a^3*b^3*e^(2*c))*e^(2*d*x) - 2*(a^6*e^c + a^4*b^2*e^
c)*e^(d*x)), x) + 16*integrate(1/8*(b*x*e^(d*x + c) - a*x)/(a^2 + b^2 + (a
^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x))*f
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{e+f\,x}{\cosh\left(c+d\,x\right)\,{\sinh\left(c+d\,x\right)}^3\,\left(a+b\,\sinh\left(c+d\,x\right)\right)} \,d x$$

input
```
int((e + f*x)/(cosh(c + d*x)*sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(cosh(c + d*x)*sinh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
(2*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**3*b*e - 4*e**(2*c + 2*d*x)*atan(
e**(c + d*x))*a**3*b*e + 2*atan(e**(c + d*x))*a**3*b*e + 32*e**(9*c + 4*d*
x)*int((e**(5*d*x)*x)/(e**(10*c + 10*d*x)*b + 2*e**(9*c + 9*d*x)*a - 3*e**
(8*c + 8*d*x)*b - 4*e**(7*c + 7*d*x)*a + 2*e**(6*c + 6*d*x)*b + 2*e**(4*c
+ 4*d*x)*b + 4*e**(3*c + 3*d*x)*a - 3*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*
a + b),x)*a**5*d*f + 32*e**(9*c + 4*d*x)*int((e**(5*d*x)*x)/(e**(10*c + 10
*d*x)*b + 2*e**(9*c + 9*d*x)*a - 3*e**(8*c + 8*d*x)*b - 4*e**(7*c + 7*d*x)
*a + 2*e**(6*c + 6*d*x)*b + 2*e**(4*c + 4*d*x)*b + 4*e**(3*c + 3*d*x)*a -
3*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**3*b**2*d*f + e**(4*c +
4*d*x)*log(e**(2*c + 2*d*x) + 1)*a**4*e - e**(4*c + 4*d*x)*log(e**(c + d*x
) - 1)*a**4*e + e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*b**4*e - e**(4*c +
4*d*x)*log(e**(c + d*x) + 1)*a**4*e + e**(4*c + 4*d*x)*log(e**(c + d*x) +
1)*b**4*e - e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b
)*b**4*e - e**(4*c + 4*d*x)*a**4*e - e**(4*c + 4*d*x)*a**2*b**2*e + 2*e**(
3*c + 3*d*x)*a**3*b*e + 2*e**(3*c + 3*d*x)*a*b**3*e - 64*e**(7*c + 2*d*x)*
int((e**(5*d*x)*x)/(e**(10*c + 10*d*x)*b + 2*e**(9*c + 9*d*x)*a - 3*e**(8*
c + 8*d*x)*b - 4*e**(7*c + 7*d*x)*a + 2*e**(6*c + 6*d*x)*b + 2*e**(4*c + 4
*d*x)*b + 4*e**(3*c + 3*d*x)*a - 3*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a +
 b),x)*a**5*d*f - 64*e**(7*c + 2*d*x)*int((e**(5*d*x)*x)/(e**(10*c + 10*d*
x)*b + 2*e**(9*c + 9*d*x)*a - 3*e**(8*c + 8*d*x)*b - 4*e**(7*c + 7*d*x)...
```

3.494 $\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 130

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b\arctan(\sinh(c+dx))}{(a^2+b^2)\,d} + \frac{b\operatorname{csch}(c+dx)}{a^2d} - \frac{\operatorname{csch}^2(c+dx)}{2ad} + \frac{a\log(\cosh(c+dx))}{(a^2+b^2)\,d} - \frac{(a^2-b^2)\log(\sinh(c+dx))}{a^3d} - \frac{b^4\log(a+b\sinh(c+dx))}{a^3(a^2+b^2)\,d}$$

output
```
b*arctan(sinh(d*x+c))/(a^2+b^2)/d+b*csch(d*x+c)/a^2/d-1/2*csch(d*x+c)^2/a/
d+a*ln(cosh(d*x+c))/(a^2+b^2)/d-(a^2-b^2)*ln(sinh(d*x+c))/a^3/d-b^4*ln(a+b
*sinh(d*x+c))/a^3/(a^2+b^2)/d
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.26

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{2b\operatorname{csch}(c+dx)}{a^2}-\frac{\operatorname{csch}^2(c+dx)}{a}-\frac{2(a-b)(a+b)\log(\sinh(c+dx))}{a^3}+\frac{\left(a-\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}-b\sinh(c+dx)\right)}{a^2+b^2}-\frac{2b^4\log(a+b\sinh(c+dx))}{a^3(a^2+b^2)}}{2d}$$

input
```
Integrate[(Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
((2*b*Csch[c + d*x])/a^2 - Csch[c + d*x]^2/a - (2*(a - b)*(a + b)*Log[Sinh
[c + d*x]])/a^3 + ((a - Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[c + d*x]])/(a^
2 + b^2) - (2*b^4*Log[a + b*Sinh[c + d*x]])/(a^3*(a^2 + b^2)) + ((a + Sqrt
[-b^2])*Log[Sqrt[-b^2] + b*Sinh[c + d*x]])/(a^2 + b^2))/(2*d)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3042, 26, 3316, 26, 27, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\sin(ic+idx)^3\cos(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\cos(ic+idx)\sin(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3316}$$

$$\frac{ib\int -\frac{i\text{csch}^3(c+dx)}{(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{b\int \frac{\text{csch}^3(c+dx)}{(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^4\int \frac{\text{csch}^3(c+dx)}{b^3(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{615}$$

$$\frac{b^4\int \left(\frac{\text{csch}^3(c+dx)}{ab^5}-\frac{\text{csch}^2(c+dx)}{a^2b^4}+\frac{(b^2-a^2)\text{csch}(c+dx)}{a^3b^5}-\frac{1}{a^3(a^2+b^2)(a+b\sinh(c+dx))}+\frac{b^2+a\sinh(c+dx)b}{b^4(a^2+b^2)(\sinh^2(c+dx)b^2+b^2)}\right)d(b\sinh}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^4\left(\frac{\arctan(\sinh(c+dx))}{b^3(a^2+b^2)}+\frac{\text{csch}(c+dx)}{a^2b^3}+\frac{a\log(b^2\sinh^2(c+dx)+b^2)}{2b^4(a^2+b^2)}-\frac{\log(a+b\sinh(c+dx))}{a^3(a^2+b^2)}-\frac{(a^2-b^2)\log(b\sinh(c+dx))}{a^3b^4}-\frac{\text{csch}^2(c+}{2ab^4}\right.}{d}$$

input
```
Int[(Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(b^4*(ArcTan[Sinh[c + d*x]]/(b^3*(a^2 + b^2)) + Csch[c + d*x]/(a^2*b^3) -
Csch[c + d*x]^2/(2*a*b^4) - ((a^2 - b^2)*Log[b*Sinh[c + d*x]])/(a^3*b^4) -
 Log[a + b*Sinh[c + d*x]]/(a^3*(a^2 + b^2)) + (a*Log[b^2 + b^2*Sinh[c + d*
x]^2])/(2*b^4*(a^2 + b^2))))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 6.30 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.43

method	result
derivativedivides	$\frac{-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(-4a^2+4b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}+\frac{b}{2a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+ 4a\ln\left(1+\tanh\right.}{d}$
default	$\frac{-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(-4a^2+4b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}+\frac{b}{2a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+ 4a\ln\left(1+\tanh\right.}{d}$
risch	$-\frac{2d^2ax}{a^2d^2+b^2d^2}-\frac{2dac}{a^2d^2+b^2d^2}+\frac{2x}{a}+\frac{2c}{da}-\frac{2b^2x}{a^3}-\frac{2b^2c}{a^3d}+\frac{2b^4x}{a^3(a^2+b^2)}+\frac{2b^4c}{a^3d(a^2+b^2)}-\frac{2\,\mathrm{e}^{dx+c}\left(-b\,\mathrm{e}^{2dx+\cdots\right.}{a^2d\left(\mathrm{e}^{2dx+\cdots\right.}$

input
```
int(csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4/a^2*(1/2*tanh(1/2*d*x+1/2*c)^2*a+2*b*tanh(1/2*d*x+1/2*c))-1/8/a/
tanh(1/2*d*x+1/2*c)^2+1/4/a^3*(-4*a^2+4*b^2)*ln(tanh(1/2*d*x+1/2*c))+1/2*b
/a^2/tanh(1/2*d*x+1/2*c)+1/4/(a^2+b^2)*(4*a*ln(1+tanh(1/2*d*x+1/2*c)^2)+8*
b*arctan(tanh(1/2*d*x+1/2*c)))-b^4/a^3/(a^2+b^2)*ln(tanh(1/2*d*x+1/2*c)^2*
a-2*b*tanh(1/2*d*x+1/2*c)-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1035 vs. $2(128) = 256$.

Time = 0.23 (sec) , antiderivative size = 1035, normalized size of antiderivative = 7.96

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
(2*(a^3*b + a*b^3)*cosh(d*x + c)^3 + 2*(a^3*b + a*b^3)*sinh(d*x + c)^3 - 2
*(a^4 + a^2*b^2)*cosh(d*x + c)^2 - 2*(a^4 + a^2*b^2 - 3*(a^3*b + a*b^3)*co
sh(d*x + c))*sinh(d*x + c)^2 + 2*(a^3*b*cosh(d*x + c)^4 + 4*a^3*b*cosh(d*x
 + c)*sinh(d*x + c)^3 + a^3*b*sinh(d*x + c)^4 - 2*a^3*b*cosh(d*x + c)^2 +
a^3*b + 2*(3*a^3*b*cosh(d*x + c)^2 - a^3*b)*sinh(d*x + c)^2 + 4*(a^3*b*cos
h(d*x + c)^3 - a^3*b*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) +
sinh(d*x + c)) - 2*(a^3*b + a*b^3)*cosh(d*x + c) - (b^4*cosh(d*x + c)^4 +
4*b^4*cosh(d*x + c)*sinh(d*x + c)^3 + b^4*sinh(d*x + c)^4 - 2*b^4*cosh(d*x
 + c)^2 + b^4 + 2*(3*b^4*cosh(d*x + c)^2 - b^4)*sinh(d*x + c)^2 + 4*(b^4*c
osh(d*x + c)^3 - b^4*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*sinh(d*x + c)
+ a)/(cosh(d*x + c) - sinh(d*x + c))) + (a^4*cosh(d*x + c)^4 + 4*a^4*cosh(
d*x + c)*sinh(d*x + c)^3 + a^4*sinh(d*x + c)^4 - 2*a^4*cosh(d*x + c)^2 + a
^4 + 2*(3*a^4*cosh(d*x + c)^2 - a^4)*sinh(d*x + c)^2 + 4*(a^4*cosh(d*x + c
)^3 - a^4*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c)
 - sinh(d*x + c))) - ((a^4 - b^4)*cosh(d*x + c)^4 + 4*(a^4 - b^4)*cosh(d*x
 + c)*sinh(d*x + c)^3 + (a^4 - b^4)*sinh(d*x + c)^4 + a^4 - b^4 - 2*(a^4 -
 b^4)*cosh(d*x + c)^2 - 2*(a^4 - b^4 - 3*(a^4 - b^4)*cosh(d*x + c)^2)*sinh
(d*x + c)^2 + 4*((a^4 - b^4)*cosh(d*x + c)^3 - (a^4 - b^4)*cosh(d*x + c))*
sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - 2*(a
^3*b + a*b^3 - 3*(a^3*b + a*b^3)*cosh(d*x + c)^2 + 2*(a^4 + a^2*b^2)*co...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.82

$$\begin{aligned}\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx &= -\frac{b^4\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^5+a^3b^2)d} \\ &\quad -\frac{2\,b\arctan\left(e^{(-dx-c)}\right)}{(a^2+b^2)d}+\frac{a\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^2+b^2)d} \\ &\quad -\frac{2\left(be^{(-dx-c)}-ae^{(-2\,dx-2\,c)}-be^{(-3\,dx-3\,c)}\right)}{(2\,a^2e^{(-2\,dx-2\,c)}-a^2e^{(-4\,dx-4\,c)}-a^2)d} \\ &\quad -\frac{(a^2-b^2)\log\left(e^{(-dx-c)}+1\right)}{a^3d} \\ &\quad -\frac{(a^2-b^2)\log\left(e^{(-dx-c)}-1\right)}{a^3d}\end{aligned}$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-b^4*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^5 + a^3*b^2)*d) -
 2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + a*log(e^(-2*d*x - 2*c) + 1)/((
a^2 + b^2)*d) - 2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c
))/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) - (a^2 - b^2)
*log(e^(-d*x - c) + 1)/(a^3*d) - (a^2 - b^2)*log(e^(-d*x - c) - 1)/(a^3*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(128) = 256$.

Time = 0.17 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.02

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{2\,b^5\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^5b+a^3b^3} - \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)b}{a^2+b^2} - \frac{a\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^2+b^2} + \frac{2\left(a^2-b^2\right)\mathrm{l}}{}}{2\,d}$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(2*b^5*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^5*b + a^3*b^
3) - (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*b/(a^2 + b^2)
 - a*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^2 + b^2) + 2*(a^2 - b^2)*l
og(abs(e^(d*x + c) - e^(-d*x - c)))/a^3 - (3*a^2*(e^(d*x + c) - e^(-d*x -
c))^2 - 3*b^2*(e^(d*x + c) - e^(-d*x - c))^2 + 4*a*b*(e^(d*x + c) - e^(-d*
x - c)) - 4*a^2)/(a^3*(e^(d*x + c) - e^(-d*x - c))^2))/d
```

Mupad [B] (verification not implemented)

Time = 4.83 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.51

$$\begin{aligned}\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{\ln\left(e^{c+d\,x}+1\mathrm{i}\right)}{a\,d-b\,d\,1\mathrm{i}} - \frac{\frac{2}{a\,d}-\frac{2\,b\,e^{c+d\,x}}{a^2\,d}}{e^{2\,c+2\,d\,x}-1} \\ &\quad - \frac{2}{a\,d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)} \\ &\quad - \frac{b^4\,\ln\left(2\,a\,e^{c+d\,x}-b+b\,e^{2\,c+2\,d\,x}\right)}{d\,a^5+d\,a^3\,b^2} \\ &\quad - \frac{\ln\left(e^{2\,c+2\,d\,x}-1\right)\,\left(a^2-b^2\right)}{a^3\,d} + \frac{\ln\left(1+e^{c+d\,x}\,1\mathrm{i}\right)\,1\mathrm{i}}{-b\,d+a\,d\,1\mathrm{i}}\end{aligned}$$

input
```
int(1/(cosh(c + d*x)*sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
log(exp(c + d*x) + 1i)/(a*d - b*d*1i) - (2/(a*d) - (2*b*exp(c + d*x))/(a^2
*d))/(exp(2*c + 2*d*x) - 1) + (log(exp(c + d*x)*1i + 1)*1i)/(a*d*1i - b*d)
 - 2/(a*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (b^4*log(2*a*exp(
c + d*x) - b + b*exp(2*c + 2*d*x)))/(a^5*d + a^3*b^2*d) - (log(exp(2*c + 2
*d*x) - 1)*(a^2 - b^2))/(a^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 685, normalized size of antiderivative = 5.27

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-2e^{2dx+2c}\log\left(e^{2dx+2c}+1\right)a^4+2e^{2dx+2c}\log\left(e^{2dx+2c}b+2e^{dx+c}a-b\right)b^4-e^{4dx+4c}\log\left(e^{dx+c}-1\right)a^4-e^{4dx+}}{}$$

input
```
int(csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
(2*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**3*b - 4*e**(2*c + 2*d*x)*atan(e*
*(c + d*x))*a**3*b + 2*atan(e**(c + d*x))*a**3*b + e**(4*c + 4*d*x)*log(e*
*(2*c + 2*d*x) + 1)*a**4 - e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**4 + e
**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*b**4 - e**(4*c + 4*d*x)*log(e**(c +
d*x) + 1)*a**4 + e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*b**4 - e**(4*c + 4
*d*x)*log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*b**4 - e**(4*c + 4*d*
x)*a**4 - e**(4*c + 4*d*x)*a**2*b**2 + 2*e**(3*c + 3*d*x)*a**3*b + 2*e**(3
*c + 3*d*x)*a*b**3 - 2*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x) + 1)*a**4 + 2
*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a**4 - 2*e**(2*c + 2*d*x)*log(e**(
c + d*x) - 1)*b**4 + 2*e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*a**4 - 2*e**
(2*c + 2*d*x)*log(e**(c + d*x) + 1)*b**4 + 2*e**(2*c + 2*d*x)*log(e**(2*c
+ 2*d*x)*b + 2*e**(c + d*x)*a - b)*b**4 - 2*e**(c + d*x)*a**3*b - 2*e**(c
+ d*x)*a*b**3 + log(e**(2*c + 2*d*x) + 1)*a**4 - log(e**(c + d*x) - 1)*a**
4 + log(e**(c + d*x) - 1)*b**4 - log(e**(c + d*x) + 1)*a**4 + log(e**(c +
d*x) + 1)*b**4 - log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*b**4 - a**
4 - a**2*b**2)/(a**3*d*(e**(4*c + 4*d*x)*a**2 + e**(4*c + 4*d*x)*b**2 - 2*
e**(2*c + 2*d*x)*a**2 - 2*e**(2*c + 2*d*x)*b**2 + a**2 + b**2))
```

3.495 $\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\operatorname{Int}\left(\frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(csch(d*x+c)^3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 104.52 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Csch[c + d*x]^3*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Csch[c + d*x]^3*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{6125}$$
$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Csch[c + d*x]^3*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(dx+c)^3 \operatorname{sech}(dx+c)}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(csch(d*x+c)^3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)^3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 38.49 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)^3\operatorname{sech}\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(csch(d*x + c)^3*sech(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)**3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 912, normalized size of antiderivative = 26.82

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)^3\operatorname{sech}\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-(a*f - 2*(b*d*f*x*e^(3*c) + b*d*e*e^(3*c))*e^(3*d*x) + (2*a*d*f*x*e^(2*c)
 + (2*d*e - f)*a*e^(2*c))*e^(2*d*x) + 2*(b*d*f*x*e^c + b*d*e*e^c)*e^(d*x))
/(a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + a^2*d^2*e^2 + (a^2*d^2*f^2*x^2*e^(4*
c) + 2*a^2*d^2*e*f*x*e^(4*c) + a^2*d^2*e^2*e^(4*c))*e^(4*d*x) - 2*(a^2*d^2
*f^2*x^2*e^(2*c) + 2*a^2*d^2*e*f*x*e^(2*c) + a^2*d^2*e^2*e^(2*c))*e^(2*d*x
)) - 16*integrate(1/16*(b^2*d^2*e^2 + a*b*d*e*f - (d^2*e^2 - f^2)*a^2 - (a
^2*d^2*f^2 - b^2*d^2*f^2)*x^2 - (2*a^2*d^2*e*f - 2*b^2*d^2*e*f - a*b*d*f^2
)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*
e^3 - (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e
^c + a^3*d^2*e^3*e^c)*e^(d*x)), x) + 16*integrate(-1/16*(b^2*d^2*e^2 - a*b
*d*e*f - (d^2*e^2 - f^2)*a^2 - (a^2*d^2*f^2 - b^2*d^2*f^2)*x^2 - (2*a^2*d^
2*e*f - 2*b^2*d^2*e*f + a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x
^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*e^3 + (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*
f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)*e^(d*x)), x) + 16*i
ntegrate(-1/8*(a*b^4*e^(d*x + c) - b^5)/(a^5*b*e + a^3*b^3*e + (a^5*b*f +
a^3*b^3*f)*x - (a^5*b*e*e^(2*c) + a^3*b^3*e*e^(2*c) + (a^5*b*f*e^(2*c) + a
^3*b^3*f*e^(2*c))*x)*e^(2*d*x) - 2*(a^6*e*e^c + a^4*b^2*e*e^c + (a^6*f*e^c
 + a^4*b^2*f*e^c)*x)*e^(d*x)), x) + 16*integrate(1/8*(b*e^(d*x + c) - a)/(
a^2*e + b^2*e + (a^2*f + b^2*f)*x + (a^2*e*e^(2*c) + b^2*e*e^(2*c) + (a^2*
f*e^(2*c) + b^2*f*e^(2*c))*x)*e^(2*d*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 11.82 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{1}{\cosh(c+d\,x)\,\sinh(c+d\,x)^3\,(e+f\,x)\,(a+b\sinh(c+d\,x))}\,dx$$

input
```
int(1/(cosh(c + d*x)*sinh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(cosh(c + d*x)*sinh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)^3\operatorname{sech}(dx+c)}{\sinh(dx+c)\,be+\sinh(dx+c)\,bfx+ae+afx}dx$$

input
```
int(csch(d*x+c)^3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int((csch(c + d*x)**3*sech(c + d*x))/(sinh(c + d*x)*b*e + sinh(c + d*x)*b*
f*x + a*e + a*f*x),x)
```

3.496 $\int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 1212

$$\int \frac{(e+fx)^2 \text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output
```
-1/2*(f*x+e)^2*coth(d*x+c)*csch(d*x+c)/a/d-2*b^2*(f*x+e)^2*arctanh(exp(d*x
+c))/a^3/d+2*b^2*f^2*polylog(3,-exp(d*x+c))/a^3/d^3+2*I*f^2*polylog(2,I*ex
p(d*x+c))/a/d^3-3*f^2*polylog(3,-exp(d*x+c))/a/d^3+3*f^2*polylog(3,exp(d*x
+c))/a/d^3+3*(f*x+e)^2*arctanh(exp(d*x+c))/a/d+3*f*(f*x+e)*polylog(2,-exp(
d*x+c))/a/d^2-3*f*(f*x+e)*polylog(2,exp(d*x+c))/a/d^2-2*b^2*f^2*polylog(3,
exp(d*x+c))/a^3/d^3-2*b^2*f*(f*x+e)*polylog(2,-exp(d*x+c))/a^3/d^2+2*b^2*f
*(f*x+e)*polylog(2,exp(d*x+c))/a^3/d^2-b^3*(f*x+e)^2/a^2/(a^2+b^2)/d+b^3*f
^2*polylog(2,-exp(2*d*x+2*c))/a^2/(a^2+b^2)/d^3+b^2*(f*x+e)^2*sech(d*x+c)/
a^3/d-b^3*(f*x+e)^2*tanh(d*x+c)/a^2/(a^2+b^2)/d-b^4*(f*x+e)^2*sech(d*x+c)/
a^3/(a^2+b^2)/d+4*f*(f*x+e)*arctan(exp(d*x+c))/a/d^2+2*b*(f*x+e)^2/a^2/d-(
f*x+e)^2*sech(d*x+c)/a/d+2*I*b^4*f^2*polylog(2,I*exp(d*x+c))/a^3/(a^2+b^2)
/d^3-f*(f*x+e)*csch(d*x+c)/a/d^2-2*b*f*(f*x+e)*ln(1-exp(4*d*x+4*c))/a^2/d^
2-4*b^2*f*(f*x+e)*arctan(exp(d*x+c))/a^3/d^2-2*I*b^2*f^2*polylog(2,I*exp(d
*x+c))/a^3/d^3-1/2*b*f^2*polylog(2,exp(4*d*x+4*c))/a^2/d^3+2*b*(f*x+e)^2*c
oth(2*d*x+2*c)/a^2/d-2*I*f^2*polylog(2,-I*exp(d*x+c))/a/d^3-f^2*arctanh(co
sh(d*x+c))/a/d^3+b^5*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/
(a^2+b^2)^(3/2)/d-b^5*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3
/(a^2+b^2)^(3/2)/d-2*b^5*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/
a^3/(a^2+b^2)^(3/2)/d^3+2*b^5*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/
2)))/a^3/(a^2+b^2)^(3/2)/d^3+2*b^3*f*(f*x+e)*ln(1+exp(2*d*x+2*c))/a^2/(...
```

Mathematica [A] (warning: unable to verify)

Time = 9.43 (sec) , antiderivative size = 2346, normalized size of antiderivative = 1.94

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Csch[c + d*x]^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*
x]),x]
```

output
```
(f*(4*b*d^2*e*E^(2*c)*x - 4*b*d^2*e*(1 + E^(2*c))*x + 2*b*d^2*E^(2*c)*f*x^
2 - 2*b*d^2*(1 + E^(2*c))*f*x^2 + 4*a*d*e*(1 + E^(2*c))*ArcTan[E^(c + d*x)
] + 2*b*d*e*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (2*I)*a*(1
+ E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - Poly
Log[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) + b*(1 + E^(2*c))*f*
(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x))])))/
((a^2 + b^2)*d^3*(1 + E^(2*c))) + (8*a*b*d^2*e*E^(2*c)*f*x + 4*a*b*d^2*E^(
2*c)*f^2*x^2 - 6*a^2*d^2*e^2*ArcTanh[E^(c + d*x)] + 4*b^2*d^2*e^2*ArcTanh[
E^(c + d*x)] + 6*a^2*d^2*e^2*E^(2*c)*ArcTanh[E^(c + d*x)] - 4*b^2*d^2*e^2*
E^(2*c)*ArcTanh[E^(c + d*x)] + 4*a^2*f^2*ArcTanh[E^(c + d*x)] - 4*a^2*E^(2
*c)*f^2*ArcTanh[E^(c + d*x)] + 6*a^2*d^2*e*f*x*Log[1 - E^(c + d*x)] - 4*b^
2*d^2*e*f*x*Log[1 - E^(c + d*x)] - 6*a^2*d^2*e*E^(2*c)*f*x*Log[1 - E^(c +
d*x)] + 4*b^2*d^2*e*E^(2*c)*f*x*Log[1 - E^(c + d*x)] + 3*a^2*d^2*f^2*x^2*L
og[1 - E^(c + d*x)] - 2*b^2*d^2*f^2*x^2*Log[1 - E^(c + d*x)] - 3*a^2*d^2*E
^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] + 2*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 - E^
(c + d*x)] - 6*a^2*d^2*e*f*x*Log[1 + E^(c + d*x)] + 4*b^2*d^2*e*f*x*Log[1
+ E^(c + d*x)] + 6*a^2*d^2*e*E^(2*c)*f*x*Log[1 + E^(c + d*x)] - 4*b^2*d^2*
e*E^(2*c)*f*x*Log[1 + E^(c + d*x)] - 3*a^2*d^2*f^2*x^2*Log[1 + E^(c + d*x)
] + 2*b^2*d^2*f^2*x^2*Log[1 + E^(c + d*x)] + 3*a^2*d^2*E^(2*c)*f^2*x^2*Log
[1 + E^(c + d*x)] - 2*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 + E^(c + d*x)] + 4*...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)^2\text{csch}^3(c+dx)\text{sech}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5985}$$

$$\frac{-2f\int \frac{1}{2}(e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}}{a}$$

$$\frac{b\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$\big\downarrow$ 27

$$\frac{-f\int (e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-}{a}$$

$$\frac{b\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$\big\downarrow$ 6123

$$\frac{-f\int (e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-}{a}$$

$$\frac{b\left(\frac{\int (e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$\big\downarrow$ 5984

$$\frac{-f\int (e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-}{a}$$

$$\frac{b\left(\frac{4\int (e+fx)^2\operatorname{csch}^2(2c+2dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$\big\downarrow$ 3042

$$\frac{-f\int (e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-}{a}$$

$$\frac{b\left(-\frac{b\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{4\int -(e+fx)^2\csc(2ic+2idx)^2dx}{a}\right)}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{-f\int(e+fx)\left(-\frac{\text{sech}(c+dx)\text{csch}^2(c+dx)}{d}+\frac{3\text{arctanh}(\cosh(c+dx))}{d}-\frac{3\text{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\text{arctanh}(\cosh(c+dx))}{2d}-}{a}$$
$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\int(e+fx)^2\csc(2ic+2idx)^2dx}{a}\right)}{a}$$

$$\Big\downarrow \; 4672$$

$$\frac{-f\int(e+fx)\left(-\frac{\text{sech}(c+dx)\text{csch}^2(c+dx)}{d}+\frac{3\text{arctanh}(\cosh(c+dx))}{d}-\frac{3\text{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\text{arctanh}(\cosh(c+dx))}{2d}-}{a}$$
$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}-\frac{if\int-i(e+fx)\coth(2c+2dx)dx}{d}\right)}{a}\right)}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{-f\int(e+fx)\left(-\frac{\text{sech}(c+dx)\text{csch}^2(c+dx)}{d}+\frac{3\text{arctanh}(\cosh(c+dx))}{d}-\frac{3\text{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\text{arctanh}(\cosh(c+dx))}{2d}-}{a}$$
$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}-\frac{f\int(e+fx)\coth(2c+2dx)dx}{d}\right)}{a}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{-f\int(e+fx)\left(-\frac{\text{sech}(c+dx)\text{csch}^2(c+dx)}{d}+\frac{3\text{arctanh}(\cosh(c+dx))}{d}-\frac{3\text{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\text{arctanh}(\cosh(c+dx))}{2d}-}{a}$$
$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}-\frac{f\int-i(e+fx)\tan\left(2ic+2idx+\frac{\pi}{2}\right)dx}{d}\right)}{a}\right)}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{-f\int(e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-}{a}$$
$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\int(e+fx)\tan\left(\frac{1}{2}(4ic+\pi)+2idx\right)dx}{d}\right)}{a}\right)}{a}$$

$\downarrow$ 4201

$$\frac{-f\int(e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-}{a}$$
$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\int\frac{e^{4c+4dx-i\pi}(e+fx)}{1+e^{4c+4dx-i\pi}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}\right)}{a}$$

$\downarrow$ 2620

$$\frac{-f\int(e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-}{a}$$
$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}-\frac{f\int\log\left(1+e^{4c+4dx-i\pi}\right)dx}{4d}\right)-\frac{i(e+}{2}\right.}{d}\right.}{a}\right.}{a}$$

$\downarrow$ 2715

$$\frac{-f\int(e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}-\frac{f\int e^{-4c-4dx+i\pi}\log\left(1+e^{4c+4dx-\right.}}{16d^2}\right.\right.}{d}\right.}{a}\right.}{a}$$

↓ 2838

$$\frac{-f\int(e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e}{}\right.}{d}\right.}{a}\right.}{a}$$

↓ 6123

$$\frac{-f\int(e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-}{a}$$

$$\frac{b\left(-\frac{b\left(\frac{\int(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(}{}\right.\right.}{}\right.}{a}\right.}{a}$$

↓ 5985

$$\frac{-f\int(e+fx)\left(-\frac{\text{sech}(c+dx)\text{csch}^2(c+dx)}{d}+\frac{3\text{arctanh}(\cosh(c+dx))}{d}-\frac{3\text{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\text{arctanh}(\cosh(c+dx))}{2d}-}{a}$$

$$\frac{b\left(-\frac{b\left(\frac{-2f\int-\left((e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^2\text{sech}^2(c}{a+b\sinh(c+dx}}{a}\right.}{a}\right.}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{-f\int(e+fx)\left(-\frac{\text{sech}(c+dx)\text{csch}^2(c+dx)}{d}+\frac{3\text{arctanh}(\cosh(c+dx))}{d}-\frac{3\text{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\text{arctanh}(\cosh(c+dx))}{2d}-}{a}$$

$$\frac{b\left(-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right.}{a}\right.}{a}$$

$$\Big\downarrow \text{6107}$$

$$\frac{-f\int(e+fx)\left(-\frac{\text{sech}(c+dx)\text{csch}^2(c+dx)}{d}+\frac{3\text{arctanh}(\cosh(c+dx))}{d}-\frac{3\text{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\text{arctanh}(\cosh(c+dx))}{2d}-}{a}$$

$$b\left(-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{b^2\int\frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\right.}{}\right.}{a}\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{-f\int(e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-}{a}$$

$$b\left(-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)(\cdots}{a^2+b^2}\right.}{}\right.}{a}\right.$$

$\downarrow$ 3803

$$\frac{-f\int(e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-}{a}$$

$$b\left(-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{2b^2\int-\frac{e^{c+dx}(e+fx\cdots}{-2e^{c+dx}a-be^{2(c\cdots}}}{a^2+b^2}\right.}{}\right.}{a}\right.$$

$\downarrow$ 25

$$\frac{-f\int(e+fx)\left(-\frac{\text{sech}(c+dx)\text{csch}^2(c+dx)}{d}+\frac{3\text{arctanh}(\cosh(c+dx))}{d}-\frac{3\text{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\text{arctanh}(\cosh(c+dx))}{2d}-}{a}$$

$$b\left(-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(}{a^2+b^2}\right.}{}\right.}{a}\right.$$

$\downarrow$ 2694

$$\frac{-f\int(e+fx)\left(-\frac{\text{sech}(c+dx)\text{csch}^2(c+dx)}{d}+\frac{3\text{arctanh}(\cosh(c+dx))}{d}-\frac{3\text{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\text{arctanh}(\cosh(c+dx))}{2d}-}{a}$$

$$b\left(-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(}{a^2+b^2}\right.}{}\right.}{a}\right.$$

$$\downarrow 27$$

$$\frac{-f\int(e+fx)\left(-\frac{\text{sech}(c+dx)\text{csch}^2(c+dx)}{d}+\frac{3\text{arctanh}(\cosh(c+dx))}{d}-\frac{3\text{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\text{arctanh}(\cosh(c+dx))}{2d}-}{a}$$

$$b\left(-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(\cdots}{a^2+b^2}\right.}{}\right)}{a}\right.$$

$$\downarrow 2620$$

$$\frac{-f\int(e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-}{a}$$

$$b\left(-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)(c}{a^2+b^2}\right.}{}\right.}{}\right.$$

$\downarrow$ 3011

$$\frac{-f\int(e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-}{a}$$

$$b\left(-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)(\cdots}{a^2+b^2}\right.}{\cdots}\right.}{\cdots}\right.$$

↓ 2720

$$\frac{-f\int(e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-}{a}$$

$$b\left(-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)(\cdots}{a^2+b^2}\right.}{}\right.}{}\right.$$

↓ 7143

$$\frac{-f\int(e+fx)\left(-\frac{\text{sech}(c+dx)\text{csch}^2(c+dx)}{d}+\frac{3\text{arctanh}(\cosh(c+dx))}{d}-\frac{3\text{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\text{arctanh}(\cosh(c+dx))}{2d}-}{a}$$

$$b\left(-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(\cdots}{a^2+b^2}\right.}{}\right.}{}\right.$$

↓ 7292

$$\frac{-f\int \frac{(e+fx)\left(-\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)+3\operatorname{arctanh}(\cosh(c+dx))-3\operatorname{sech}(c+dx)\right)}{d}dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{3(e+fx)^2\mathrm{s}}{}}{a}$$

$$b\left(-\frac{b\left(\frac{2f\int \frac{(e+fx)(\operatorname{arctanh}(\cosh(c+dx))-\operatorname{sech}(c+dx))}{d}dx-\frac{(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^2\operatorname{sech}^2(c+dx)(a-}{a^2+b^2}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \; 27$$

$$-\frac{f\int(e+fx)\left(-\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)+3\operatorname{arctanh}(\cosh(c+dx))-3\operatorname{sech}(c+dx)\right)dx}{d}+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{3(e+fx)^2\operatorname{se}\cdots}{2d}$$

$$a$$

$$b\left(\cdots\right.$$

$$b\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)(a-\cdots}{a^2+b^2}\right.$$

$$b\left(\frac{\frac{2f\int(e+fx)(\operatorname{arctanh}(\cosh(c+dx))-\operatorname{sech}(c+dx))dx}{d}-\frac{(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}}{a}-\frac{\cdots}{\cdots}\right.$$

$$b\;-\frac{\left(\cdots\right)}{\cdots}$$

input
```
Int[((e + f*x)^2*Csch[c + d*x]^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c)^3 \operatorname{sech}(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 29722 vs. 2(1121) = 2242.

Time = 0.65 (sec) , antiderivative size = 29722, normalized size of antiderivative = 24.52

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*csch(d*x+c)**3*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2 \operatorname{csch}(dx+c)^3 \operatorname{sech}(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="maxima")
```

output
```
2*b*e*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2 +
b^2)*d^2)) + 4*a*f^2*integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2*
d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + 4*b*f^2*integrate(x/(a^2*d*e^(2*d
*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 1/2*(2*b^5*log((b
*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)
))/((a^5 + a^3*b^2)*sqrt(a^2 + b^2)*d) + 2*(4*a^2*b*e^(-2*d*x - 2*c) + 2*b
^3*e^(-4*d*x - 4*c) - 4*a^2*b - 2*b^3 + (3*a^3 + a*b^2)*e^(-d*x - c) - 2*(
a^3 - a*b^2)*e^(-3*d*x - 3*c) + (3*a^3 + a*b^2)*e^(-5*d*x - 5*c))/((a^4 +
a^2*b^2 - (a^4 + a^2*b^2)*e^(-2*d*x - 2*c) - (a^4 + a^2*b^2)*e^(-4*d*x - 4
*c) + (a^4 + a^2*b^2)*e^(-6*d*x - 6*c))*d) - (3*a^2 - 2*b^2)*log(e^(-d*x -
 c) + 1)/(a^3*d) + (3*a^2 - 2*b^2)*log(e^(-d*x - c) - 1)/(a^3*d))*e^2 + 4*
a*e*f*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) - (2*(2*a^2*b*d*f^2 + b^3*d*f^
2)*x^2 + 4*(2*a^2*b*d*e*f + b^3*d*e*f)*x + (2*a^3*e*f*e^(5*c) + 2*a*b^2*e*
f*e^(5*c) + (3*a^3*d*f^2*e^(5*c) + a*b^2*d*f^2*e^(5*c))*x^2 + 2*((3*d*e*f
+ f^2)*a^3*e^(5*c) + (d*e*f + f^2)*a*b^2*e^(5*c))*x)*e^(5*d*x) - 2*(b^3*d*
f^2*x^2*e^(4*c) + 2*b^3*d*e*f*x*e^(4*c))*e^(4*d*x) - 2*((a^3*d*f^2*e^(3*c)
 - a*b^2*d*f^2*e^(3*c))*x^2 + 2*(a^3*d*e*f*e^(3*c) - a*b^2*d*e*f*e^(3*c))*
x)*e^(3*d*x) - 4*(a^2*b*d*f^2*x^2*e^(2*c) + 2*a^2*b*d*e*f*x*e^(2*c))*e^(2*
d*x) - (2*a^3*e*f*e^c + 2*a*b^2*e*f*e^c - (3*a^3*d*f^2*e^c + a*b^2*d*f^2*e
^c)*x^2 - 2*((3*d*e*f - f^2)*a^3*e^c + (d*e*f - f^2)*a*b^2*e^c)*x)*e^(d...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Not invertible Error: Bad Argument
Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^2}{\cosh(c+d\,x)^2\sinh(c+d\,x)^3\,(a+b\sinh(c+d\,x))}\,dx$$

input
```
int((e + f*x)^2/(cosh(c + d*x)^2*sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^2/(cosh(c + d*x)^2*sinh(c + d*x)^3*(a + b*sinh(c + d*x))), x
)
```

Reduce [F]

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
( - 36*e**(6*c + 6*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sq
rt(a**2 + b**2))*b**6*d**2*e**2*i + 36*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*
atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*b**6*d**2*e**2*i + 36*e**
(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 +
b**2))*b**6*d**2*e**2*i - 36*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*
i)/sqrt(a**2 + b**2))*b**6*d**2*e**2*i - 2304*e**(11*c + 6*d*x)*int((e**(5
*d*x)*x**2)/(e**(12*c + 12*d*x)*b + 2*e**(11*c + 11*d*x)*a - 2*e**(10*c +
10*d*x)*b - 2*e**(9*c + 9*d*x)*a - e**(8*c + 8*d*x)*b - 4*e**(7*c + 7*d*x)
*a + 4*e**(6*c + 6*d*x)*b + 4*e**(5*c + 5*d*x)*a - e**(4*c + 4*d*x)*b + 2*
e**(3*c + 3*d*x)*a - 2*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**8*
d**3*f**2 - 4608*e**(11*c + 6*d*x)*int((e**(5*d*x)*x**2)/(e**(12*c + 12*d*
x)*b + 2*e**(11*c + 11*d*x)*a - 2*e**(10*c + 10*d*x)*b - 2*e**(9*c + 9*d*x
)*a - e**(8*c + 8*d*x)*b - 4*e**(7*c + 7*d*x)*a + 4*e**(6*c + 6*d*x)*b + 4
*e**(5*c + 5*d*x)*a - e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a - 2*e**(2*
c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**6*b**2*d**3*f**2 - 2304*e**(11*
c + 6*d*x)*int((e**(5*d*x)*x**2)/(e**(12*c + 12*d*x)*b + 2*e**(11*c + 11*d
*x)*a - 2*e**(10*c + 10*d*x)*b - 2*e**(9*c + 9*d*x)*a - e**(8*c + 8*d*x)*b
 - 4*e**(7*c + 7*d*x)*a + 4*e**(6*c + 6*d*x)*b + 4*e**(5*c + 5*d*x)*a - e*
*(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a - 2*e**(2*c + 2*d*x)*b - 2*e**(c +
 d*x)*a + b),x)*a**4*b**4*d**3*f**2 - 4608*e**(11*c + 6*d*x)*int((e**(5...
```

3.497 $\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 695

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{f\arctan(\sinh(c+dx))}{ad^2} - \frac{b^2 f\arctan(\sinh(c+dx))}{a^3d^2} + \frac{b^4 f\arctan(\sinh(c+dx))}{a^3(a^2+b^2)d^2} + \frac{3fx\operatorname{arctanh}(e^{c+dx})}{ad} - \frac{2b^2fx\operatorname{arctanh}(e^{c+dx})}{a^3d} - \frac{3fx\operatorname{arctanh}(\cosh(c+dx))}{2ad} + \frac{b^2fx\operatorname{arctanh}(\cosh(c+dx))}{a^3d} + \frac{3(e+fx)\operatorname{arctanh}(\cosh(c+dx))}{2ad} - \frac{b^2(e+fx)\operatorname{arctanh}(\cosh(c+dx))}{a^3d} + \frac{2b(e+fx)\coth(2c+2dx)}{a^2d} - \frac{f\operatorname{csch}(c+dx)}{2ad^2} - \frac{(e+fx)\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{b^5(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3(a^2+b^2)^{3/2}d} + \frac{b^5(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3(a^2+b^2)^{3/2}d} + \frac{b^3f\log(\cosh(c+dx))}{a^2(a^2+b^2)d^2} - \frac{bf\log(\sinh(2c+2dx))}{a^2d^2} + \frac{3f\operatorname{PolyLog}(2,-e^{c+dx})}{2ad^2} - \frac{b^2f\operatorname{PolyLog}(2,-e^{c+dx})}{a^3d^2} - \frac{3f\operatorname{PolyLog}(2,e^{c+dx})}{2ad^2} + \frac{b^2f\operatorname{PolyLog}(2,e^{c+dx})}{a^3d^2} - \frac{b^5f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3(a^2+b^2)^{3/2}d^2} + \frac{b^5f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3(a^2+b^2)^{3/2}d^2} - \frac{(e+fx)\operatorname{sech}(c+dx)}{ad} + \frac{b^2(e+fx)\operatorname{sech}(c+dx)}{a^3d} - \frac{b^4(e+fx)\operatorname{sech}(c+dx)}{a^3(a^2+b^2)d} - \frac{b^3(e+fx)\tanh(c+dx)}{a^2(a^2+b^2)d}$$

output
```
-1/2*(f*x+e)*coth(d*x+c)*csch(d*x+c)/a/d-3/2*f*x*arctanh(cosh(d*x+c))/a/d+
f*arctan(sinh(d*x+c))/a/d^2-b^2*f*arctan(sinh(d*x+c))/a^3/d^2-b*f*ln(sinh(
2*d*x+2*c))/a^2/d^2+b^2*(f*x+e)*sech(d*x+c)/a^3/d+b^4*f*arctan(sinh(d*x+c)
)/a^3/(a^2+b^2)/d^2+b^2*f*x*arctanh(cosh(d*x+c))/a^3/d+b^3*f*ln(cosh(d*x+c
))/a^2/(a^2+b^2)/d^2-b^3*(f*x+e)*tanh(d*x+c)/a^2/(a^2+b^2)/d-b^4*(f*x+e)*s
ech(d*x+c)/a^3/(a^2+b^2)/d-b^2*(f*x+e)*arctanh(cosh(d*x+c))/a^3/d+3*f*x*ar
ctanh(exp(d*x+c))/a/d+3/2*(f*x+e)*arctanh(cosh(d*x+c))/a/d-(f*x+e)*sech(d*
x+c)/a/d-1/2*f*csch(d*x+c)/a/d^2-2*b^2*f*x*arctanh(exp(d*x+c))/a^3/d-b^2*f
*polylog(2,-exp(d*x+c))/a^3/d^2+b^2*f*polylog(2,exp(d*x+c))/a^3/d^2+2*b*(f
*x+e)*coth(2*d*x+2*c)/a^2/d+3/2*f*polylog(2,-exp(d*x+c))/a/d^2-3/2*f*polyl
og(2,exp(d*x+c))/a/d^2+b^5*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/
a^3/(a^2+b^2)^(3/2)/d^2-b^5*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))
/a^3/(a^2+b^2)^(3/2)/d^2+b^5*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2))
)/a^3/(a^2+b^2)^(3/2)/d-b^5*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))
/a^3/(a^2+b^2)^(3/2)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.13 (sec) , antiderivative size = 798, normalized size of antiderivative = 1.15

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{f\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{(a-ib)d^2} + \frac{f\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{(a+ib)d^2}$$
$$+ \frac{\left(2bde\cosh\left(\frac{1}{2}(c+dx)\right) - af\cosh\left(\frac{1}{2}(c+dx)\right) - 2bcf\cosh\left(\frac{1}{2}(c+dx)\right) + 2bf(c+dx)\cosh\left(\frac{1}{2}(c+dx)\right)\right.}{4a^2d^2}$$
$$+ \frac{(-de+cf-f(c+dx))\text{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8ad^2}$$
$$+ \frac{if\log(\cosh(c+dx))}{2(a-ib)d^2} - \frac{if\log(\cosh(c+dx))}{2(a+ib)d^2}$$
$$+ \frac{-2abf(c+dx) - (2abf+3a^2(de+dfx)-2b^2(de+dfx))\log\left(1-e^{-c-dx}\right) + (-2abf+3a^2(de+dfx}{}$$
$$- \frac{b^5\left(-2de\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) + 2cf\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) + f(c+dx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right) - f(c+dx)\log\right.}{a^3\left(a^2+b^2\right)^{3/2}d^2}$$
$$+ \frac{(-de+cf-f(c+dx))\text{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8ad^2}$$
$$+ \frac{\text{sech}\left(\frac{1}{2}(c+dx)\right)\left(2bde\sinh\left(\frac{1}{2}(c+dx)\right) + af\sinh\left(\frac{1}{2}(c+dx)\right) - 2bcf\sinh\left(\frac{1}{2}(c+dx)\right) + 2bf(c+dx)\right.}{4a^2d^2}$$
$$+ \frac{\text{sech}(c+dx)(-ade+acf-af(c+dx)+bde\sinh(c+dx)-bcf\sinh(c+dx)+bf(c+dx)\sinh(c+}{\left(a^2+b^2\right)d^2}$$

input
```
Integrate[((e + f*x)*Csch[c + d*x]^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
(f*ArcTan[Tanh[(c + d*x)/2]])/((a - I*b)*d^2) + (f*ArcTan[Tanh[(c + d*x)/2
]])/((a + I*b)*d^2) + ((2*b*d*e*Cosh[(c + d*x)/2] - a*f*Cosh[(c + d*x)/2]
- 2*b*c*f*Cosh[(c + d*x)/2] + 2*b*f*(c + d*x)*Cosh[(c + d*x)/2])*Csch[(c +
 d*x)/2])/(4*a^2*d^2) + ((-(d*e) + c*f - f*(c + d*x))*Csch[(c + d*x)/2]^2)
/(8*a*d^2) + ((I/2)*f*Log[Cosh[c + d*x]])/((a - I*b)*d^2) - ((I/2)*f*Log[C
osh[c + d*x]])/((a + I*b)*d^2) + (-2*a*b*f*(c + d*x) - (2*a*b*f + 3*a^2*(d
*e + d*f*x) - 2*b^2*(d*e + d*f*x))*Log[1 - E^(-c - d*x)] + (-2*a*b*f + 3*a
^2*(d*e + d*f*x) - 2*b^2*(d*e + d*f*x))*Log[1 + E^(-c - d*x)] - (3*a^2 - 2
*b^2)*f*PolyLog[2, -E^(-c - d*x)] + (3*a^2 - 2*b^2)*f*PolyLog[2, E^(-c - d
*x)])/(2*a^3*d^2) - (b^5*(-2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^
2]] + 2*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log
[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1 + (b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2])] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a
^2 + b^2])] - f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^
3*(a^2 + b^2)^(3/2)*d^2) + ((-(d*e) + c*f - f*(c + d*x))*Sech[(c + d*x)/2]
^2)/(8*a*d^2) + (Sech[(c + d*x)/2]*(2*b*d*e*Sinh[(c + d*x)/2] + a*f*Sinh[(
c + d*x)/2] - 2*b*c*f*Sinh[(c + d*x)/2] + 2*b*f*(c + d*x)*Sinh[(c + d*x)/2
]))/(4*a^2*d^2) + (Sech[c + d*x]*(-(a*d*e) + a*c*f - a*f*(c + d*x) + b*d*e
*Sinh[c + d*x] - b*c*f*Sinh[c + d*x] + b*f*(c + d*x)*Sinh[c + d*x]))/((a^2
 + b^2)*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 4.89 (sec) , antiderivative size = 611, normalized size of antiderivative = 0.88, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.765$, Rules used = {6123, 5985, 2009, 6123, 5984, 3042, 25, 4672, 26, 3042, 26, 3956, 6123, 5985, 2009, 6107, 3042, 3803, 25, 2694, 27, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)\text{csch}^3(c+dx)\text{sech}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \ 5985$$

$$\frac{-f\int\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{2d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{3\operatorname{sech}(c+dx)}{2d}\right)dx+\frac{3(e+fx)\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{3(e+fx)\operatorname{se}}{2}}{a}$$
$$\frac{b\int\frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \ 2009$$

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\operatorname{arctanh}(e^{c+dx})}{d}+\frac{3x\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{3\operatorname{PolyLog}(2,-e^{c+dx})}{2d^2}+\frac{3\operatorname{PolyLog}(2,e^{c+dx})}{2d^2}+\operatorname{csch}\right.}{a}$$
$$\frac{b\int\frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \ 6123$$

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\operatorname{arctanh}(e^{c+dx})}{d}+\frac{3x\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{3\operatorname{PolyLog}(2,-e^{c+dx})}{2d^2}+\frac{3\operatorname{PolyLog}(2,e^{c+dx})}{2d^2}+\operatorname{csch}\right.}{a}$$
$$\frac{b\left(\frac{\int(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \ 5984$$

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\operatorname{arctanh}(e^{c+dx})}{d}+\frac{3x\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{3\operatorname{PolyLog}(2,-e^{c+dx})}{2d^2}+\frac{3\operatorname{PolyLog}(2,e^{c+dx})}{2d^2}+\operatorname{csch}\right.}{a}$$
$$\frac{b\left(\frac{4\int(e+fx)\operatorname{csch}^2(2c+2dx)dx}{a}-\frac{b\int\frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \ 3042$$

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\,\text{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\text{csch}\right.}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{4\int-\left((e+fx)\csc(2ic+2idx)^2\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\,\text{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\text{csch}\right.}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\int(e+fx)\csc(2ic+2idx)^2dx}{a}\right)}{a}$$

$$\Big\downarrow \text{4672}$$

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\,\text{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\text{csch}\right.}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{if\int-i\coth(2c+2dx)dx}{2d}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\,\text{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\text{csch}\right.}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\int\coth(2c+2dx)dx}{2d}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\,\text{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\text{csch}\right.}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\int-i\tan\left(2ic+2idx+\frac{\pi}{2}\right)dx}{2d}\right)}{a}\right)}{a}$$

$\downarrow$ 26

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\,\text{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\text{csch}\right.}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}+\frac{if\int\tan\left(\frac{1}{2}(4ic+\pi)+2idx\right)dx}{2d}\right)}{a}\right)}{a}$$

$\downarrow$ 3956

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\,\text{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\text{csch}\right.}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}\right)}{a}$$

$\downarrow$ 6123

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\,\text{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\text{csch}\right.}{a}$$

$$\frac{b\left(-\frac{b\left(\frac{\int(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}\right)}{a}$$

$\downarrow$ 5985

$$-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\,\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\,\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\,\text{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\text{csch}\ldots\right.$$

$$a$$

$$b\left(-\frac{b\left(\frac{-f\int\left(\frac{\text{sech}(c+dx)}{d}-\frac{\text{arctanh}(\cosh(c+dx))}{d}\right)dx-\frac{(e+fx)\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)\text{sech}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{4(\ldots}{\ldots}\right.$$

$$a$$

$\Big\downarrow$ 2009

$$-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\,\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\,\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\,\text{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\text{csch}\ldots\right.$$

$$a$$

$$b\left(-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\,\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\,\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{arctanh}(\cos\ldots}{d}}{a}\ldots\right.}{a}\ldots\right.$$

$$a$$

$\Big\downarrow$ 6107

$$-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\,\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\,\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\,\text{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\text{csch}\ldots\right.$$

$$a$$

$$b\left(-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\,\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\,\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{arctanh}(\cos\ldots}{d}}{a}\ldots\right.}{a}\ldots\right.$$

$\Big\downarrow$ 3042

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\,\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\,\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\,\text{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\text{csch}\cdots}{a}$$

$$b\left(-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\,\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\,\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\,\text{arctanh}(\cos\cdots}{d}\cdots}{a}\cdots\right)}{a}\cdots\right.$$

$\big\downarrow$ 3803

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\,\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\,\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\,\text{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\text{csch}\cdots}{a}$$

$$b\left(-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\,\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\,\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\,\text{arctanh}(\cos\cdots}{d}\cdots}{a}\cdots\right)}{a}\cdots\right.$$

$\big\downarrow$ 25

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\,\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\,\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\,\text{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\text{csch}\cdots}{a}$$

$$b\left(-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\,\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\,\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\,\text{arctanh}(\cos\cdots}{d}\cdots}{a}\cdots\right)}{a}\cdots\right.$$

$\downarrow$ 2694

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{3\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\operatorname{csch}\right.}{a}$$

$$b\left(-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\operatorname{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\operatorname{arctanh}(\cos}{d}}{a}\right.}{}\right.$$

$\downarrow$ 27

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\,\text{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\frac{\text{csch}}{}\right.}{a}$$

$$b\left(-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{arctanh}(\cos}{d}}{a}\right.}{}\right.$$

$\downarrow$ 2620

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\operatorname{arctanh}(e^{c+dx})}{d}+\frac{3x\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{3\operatorname{PolyLog}(2,-e^{c+dx})}{2d^2}+\frac{3\operatorname{PolyLog}(2,e^{c+dx})}{2d^2}+\operatorname{csch}\right.}{a}$$

$$b\left(-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\operatorname{arctanh}(e^{c+dx})}{d}-\frac{x\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{\operatorname{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{\operatorname{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{(e+fx)\operatorname{arctanh}(\cos}{d}}{a}\right.}{}\right.$$

↓ 2715

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\operatorname{arctanh}(e^{c+dx})}{d}+\frac{3x\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{3\operatorname{PolyLog}(2,-e^{c+dx})}{2d^2}+\frac{3\operatorname{PolyLog}(2,e^{c+dx})}{2d^2}+\frac{\operatorname{csch}}{}\right.}{a}$$

$$b\left(-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\operatorname{arctanh}(e^{c+dx})}{d}-\frac{x\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{\operatorname{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{\operatorname{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{(e+fx)\operatorname{arctanh}(\cos}{d}}{a}\right.}{}\right.$$

↓ 2838

$$-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\operatorname{arctanh}(e^{c+dx})}{d}+\frac{3x\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{3\,\text{PolyLog}(2,-e^{c+dx})}{2d^2}+\frac{3\,\text{PolyLog}(2,e^{c+dx})}{2d^2}+\frac{\text{csch}}{}\right.$$

$$a$$

$$b\left(-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\operatorname{arctanh}(e^{c+dx})}{d}-\frac{x\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{(e+fx)\operatorname{arctanh}(\cos}{d}}{a}\right.}{}\right.$$

↓ 7293

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\,\text{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\frac{\text{csch}}{}\right.}{a}$$

$$b\left(-\frac{\left(b\,\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{arctanh}(\cos}{d}}{a}\right.}{}\right.$$

↓ 2009

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\operatorname{arctanh}(e^{c+dx})}{d}+\frac{3x\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{3\operatorname{PolyLog}(2,-e^{c+dx})}{2d^2}+\frac{3\operatorname{PolyLog}(2,e^{c+dx})}{2d^2}+\operatorname{csch}\right.}{a}$$

$$b\left(b-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\operatorname{arctanh}(e^{c+dx})}{d}-\frac{x\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{\operatorname{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{\operatorname{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{(e+fx)\operatorname{arctanh}(\cos}{d}}{a}\right.}{}\right.$$

input
```
Int[((e + f*x)*Csch[c + d*x]^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
((3*(e + f*x)*ArcTanh[Cosh[c + d*x]])/(2*d) - f*(-(ArcTan[Sinh[c + d*x]]/d
^2) - (3*x*ArcTanh[E^(c + d*x)])/d + (3*x*ArcTanh[Cosh[c + d*x]])/(2*d) +
Csch[c + d*x]/(2*d^2) - (3*PolyLog[2, -E^(c + d*x)])/(2*d^2) + (3*PolyLog[
2, E^(c + d*x)])/(2*d^2)) - (3*(e + f*x)*Sech[c + d*x])/(2*d) - ((e + f*x)
*Csch[c + d*x]^2*Sech[c + d*x])/(2*d))/a - (b*((-4*(((e + f*x)*Coth[2*c +
2*d*x])/(2*d) - (f*Log[(-I)*Sinh[2*c + 2*d*x]])/(4*d^2)))/a - (b*((-(((e +
 f*x)*ArcTanh[Cosh[c + d*x]])/d) - f*(ArcTan[Sinh[c + d*x]]/d^2 + (2*x*Arc
Tanh[E^(c + d*x)])/d - (x*ArcTanh[Cosh[c + d*x]])/d + PolyLog[2, -E^(c + d
*x)]/d^2 - PolyLog[2, E^(c + d*x)]/d^2) + ((e + f*x)*Sech[c + d*x])/d)/a -
 (b*((-2*b^2*(-1/2*(b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(
b*d^2)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2]))])/(b*d^2)))/(2*Sqrt[a^2 + b^2])))/(a^2 + b^2) + (-((b*f*ArcTan[Sinh[c
 + d*x]])/d^2) - (a*f*Log[Cosh[c + d*x]])/d^2 + (b*(e + f*x)*Sech[c + d*x]
)/d + (a*(e + f*x)*Tanh[c + d*x])/d)/(a^2 + b^2)))/a))/a))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2766 vs. $2(657) = 1314$.

Time = 33.37 (sec) , antiderivative size = 2767, normalized size of antiderivative = 3.98

method	result	size
risch	Expression too large to display	2767

input
```
int((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURN
VERBOSE)
```

output
```
-1/(a^2+b^2)/d^2/a^2*b^3*f*ln(exp(d*x+c)+1)-1/(a^2+b^2)/d^2/a^2*b^3*f*ln(e
xp(d*x+c)-1)+1/(a^2+b^2)/d/a^3*b^4*e*ln(exp(d*x+c)-1)-1/(a^2+b^2)/d/a^3*b^
4*e*ln(exp(d*x+c)+1)+2/(a^2+b^2)/d^2/a^2*b^3*f*ln(exp(d*x+c))-1/(a^2+b^2)/
d^2/a^3*b^4*f*dilog(exp(d*x+c))-1/(a^2+b^2)/d^2/a^3*b^4*f*dilog(exp(d*x+c)
+1)-2/(a^2+b^2)^(5/2)/d^2*a*c*b^3*f*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+
b^2)^(1/2))-1/(a^2+b^2)^(5/2)/d/a^3*b^7*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2
)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/(a^2+b^2)^(5/2)/d^2/a^3*b^7*f*ln((b*exp(d*x
+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/(a^2+b^2)^(5/2)/d^2/a^3*b^
7*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/(a^2+b^
2)^(5/2)/d^2/a^3*f*b^7*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
-1/(a^2+b^2)^(3/2)/d^2/a^3*f*b^5*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b
^2)^(1/2))+1/(a^2+b^2)^(5/2)/d/a^3*b^7*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+
a)/(a+(a^2+b^2)^(1/2)))*x-3/2*b/d*e/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d
*x+c)+2*a)/(a^2+b^2)^(1/2))*a-1/(a^2+b^2)^(5/2)/d*b^5*f/a*ln((-b*exp(d*x+c
)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+3/2*b/d^2*c*f/(a^2+b^2)^(3/2)
*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a+b/(a^2+b^2)^(5/2)/d^2
*f*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^3+2*b^3/(a^2+b^2)^(
5/2)/d^2*f*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a-4/(a^2+b^2)
/d^2*b^3*f/(4*a^2+4*b^2)*ln(1+exp(2*d*x+2*c))+3/2/(a^2+b^2)/d*ln(exp(d*x+c
)+1)*a*f*x-(3*d*f*x*a^3*exp(5*d*x+5*c)+a*d*f*x*b^2*exp(5*d*x+5*c)+3*d*e...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11126 vs. $2(651) = 1302$.

Time = 0.40 (sec) , antiderivative size = 11126, normalized size of antiderivative = 16.01

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)**3*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\operatorname{csch}(dx+c)^3\operatorname{sech}(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/2*(2*b^5*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a
 + sqrt(a^2 + b^2)))/((a^5 + a^3*b^2)*sqrt(a^2 + b^2)*d) + 2*(4*a^2*b*e^(-
2*d*x - 2*c) + 2*b^3*e^(-4*d*x - 4*c) - 4*a^2*b - 2*b^3 + (3*a^3 + a*b^2)*
e^(-d*x - c) - 2*(a^3 - a*b^2)*e^(-3*d*x - 3*c) + (3*a^3 + a*b^2)*e^(-5*d*
x - 5*c))/((a^4 + a^2*b^2 - (a^4 + a^2*b^2)*e^(-2*d*x - 2*c) - (a^4 + a^2*
b^2)*e^(-4*d*x - 4*c) + (a^4 + a^2*b^2)*e^(-6*d*x - 6*c))*d) - (3*a^2 - 2*
b^2)*log(e^(-d*x - c) + 1)/(a^3*d) + (3*a^2 - 2*b^2)*log(e^(-d*x - c) - 1)
/(a^3*d))*e - (32*b^5*integrate(-1/16*x*e^(d*x + c)/(a^5*b + a^3*b^3 - (a^
5*b*e^(2*c) + a^3*b^3*e^(2*c))*e^(2*d*x) - 2*(a^6*e^c + a^4*b^2*e^c)*e^(d*
x)), x) + 96*a^2*d*integrate(1/64*x/(a^3*d*e^(d*x + c) + a^3*d), x) - 64*b
^2*d*integrate(1/64*x/(a^3*d*e^(d*x + c) + a^3*d), x) + 96*a^2*d*integrate
(1/64*x/(a^3*d*e^(d*x + c) - a^3*d), x) - 64*b^2*d*integrate(1/64*x/(a^3*d
*e^(d*x + c) - a^3*d), x) - a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) + 1
)/(a^3*d^2)) - a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) - 1)/(a^3*d^2))
- (2*b^3*d*x*e^(4*d*x + 4*c) + 4*a^2*b*d*x*e^(2*d*x + 2*c) + 2*(a^3*d*e^(3
*c) - a*b^2*d*e^(3*c))*x*e^(3*d*x) - 2*(2*a^2*b*d + b^3*d)*x - (a^3*e^(5*c
) + a*b^2*e^(5*c) + (3*a^3*d*e^(5*c) + a*b^2*d*e^(5*c))*x)*e^(5*d*x) + (a^
3*e^c + a*b^2*e^c - (3*a^3*d*e^c + a*b^2*d*e^c)*x)*e^(d*x))/(a^4*d^2 + a^2
*b^2*d^2 + (a^4*d^2*e^(6*c) + a^2*b^2*d^2*e^(6*c))*e^(6*d*x) - (a^4*d^2*e^
(4*c) + a^2*b^2*d^2*e^(4*c))*e^(4*d*x) - (a^4*d^2*e^(2*c) + a^2*b^2*d^2...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{e+f\,x}{\cosh\left(c+d\,x\right)^2\,\sinh\left(c+d\,x\right)^3\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)/(cosh(c + d*x)^2*sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(cosh(c + d*x)^2*sinh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output

```
( - 12*e**(6*c + 6*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sq
rt(a**2 + b**2))*b**6*d*e*i + 12*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((
e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*b**6*d*e*i + 12*e**(2*c + 2*d*x
)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*b**6*
d*e*i - 12*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**
2))*b**6*d*e*i - 768*e**(11*c + 6*d*x)*int((e**(5*d*x)*x)/(e**(12*c + 12*d
*x)*b + 2*e**(11*c + 11*d*x)*a - 2*e**(10*c + 10*d*x)*b - 2*e**(9*c + 9*d*
x)*a - e**(8*c + 8*d*x)*b - 4*e**(7*c + 7*d*x)*a + 4*e**(6*c + 6*d*x)*b +
4*e**(5*c + 5*d*x)*a - e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a - 2*e**(2
*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**8*d**2*f - 1536*e**(11*c + 6*d
*x)*int((e**(5*d*x)*x)/(e**(12*c + 12*d*x)*b + 2*e**(11*c + 11*d*x)*a - 2*
e**(10*c + 10*d*x)*b - 2*e**(9*c + 9*d*x)*a - e**(8*c + 8*d*x)*b - 4*e**(7
*c + 7*d*x)*a + 4*e**(6*c + 6*d*x)*b + 4*e**(5*c + 5*d*x)*a - e**(4*c + 4*
d*x)*b + 2*e**(3*c + 3*d*x)*a - 2*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a +
b),x)*a**6*b**2*d**2*f - 768*e**(11*c + 6*d*x)*int((e**(5*d*x)*x)/(e**(12*
c + 12*d*x)*b + 2*e**(11*c + 11*d*x)*a - 2*e**(10*c + 10*d*x)*b - 2*e**(9*
c + 9*d*x)*a - e**(8*c + 8*d*x)*b - 4*e**(7*c + 7*d*x)*a + 4*e**(6*c + 6*d
*x)*b + 4*e**(5*c + 5*d*x)*a - e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a -
 2*e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**4*b**4*d**2*f + 256*e*
*(10*c + 6*d*x)*int((e**(4*d*x)*x)/(e**(12*c + 12*d*x)*b + 2*e**(11*c +...
```

3.498 $\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 202

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{3\operatorname{arctanh}(\cosh(c+dx))}{2ad} - \frac{b^2\operatorname{arctanh}(\cosh(c+dx))}{a^3d} + \frac{2b^5\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3\left(a^2+b^2\right)^{3/2}d} + \frac{b\coth(c+dx)}{a^2d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{\operatorname{sech}(c+dx)}{ad} + \frac{b^2\operatorname{sech}(c+dx)}{a^3d} - \frac{b^3\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{a^3\left(a^2+b^2\right)d} + \frac{b\tanh(c+dx)}{a^2d}$$

output
```
3/2*arctanh(cosh(d*x+c))/a/d-b^2*arctanh(cosh(d*x+c))/a^3/d+2*b^5*arctanh(
(b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/a^3/(a^2+b^2)^(3/2)/d+b*coth(d*
x+c)/a^2/d-1/2*coth(d*x+c)*csch(d*x+c)/a/d-sech(d*x+c)/a/d+b^2*sech(d*x+c)
/a^3/d-b^3*sech(d*x+c)*(b+a*sinh(d*x+c))/a^3/(a^2+b^2)/d+b*tanh(d*x+c)/a^2
/d
```

Mathematica [A] (verified)

Time = 3.20 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.05

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{\frac{16b^5\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{a^3(-a^2-b^2)^{3/2}}+\frac{4b\coth\left(\frac{1}{2}(c+dx)\right)}{a^2}-\frac{\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{a}+\frac{4(3a^2-2b^2)\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{a^3}+\frac{4(-3a^2+2b^2)\log(\mathrm{si}}{a^3}}{8d}$$

input
```
Integrate[(Csch[c + d*x]^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
((16*b^5*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(a^3*(-a^2 -
b^2)^(3/2)) + (4*b*Coth[(c + d*x)/2])/a^2 - Csch[(c + d*x)/2]^2/a + (4*(3*
a^2 - 2*b^2)*Log[Cosh[(c + d*x)/2]])/a^3 + (4*(-3*a^2 + 2*b^2)*Log[Sinh[(c
 + d*x)/2]])/a^3 - Sech[(c + d*x)/2]^2/a + (8*Sech[c + d*x]*(-a + b*Sinh[c
 + d*x]))/(a^2 + b^2) + (4*b*Tanh[(c + d*x)/2])/a^2)/(8*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {3042, 26, 3377, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{i}{\sin(ic+idx)^3\cos(ic+idx)^2(a-ib\sin(ic+idx))}dx$$
$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\cos(ic+idx)^2\sin(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3377}$$

$$-i\int \left(-\frac{i\text{sech}^2(c+dx)b^3}{a^3(a+b\sinh(c+dx))}+\frac{i\text{csch}(c+dx)\text{sech}^2(c+dx)b^2}{a^3}-\frac{i\text{csch}^2(c+dx)\text{sech}^2(c+dx)b}{a^2}+\frac{i\text{csch}^3(c+d$$

$$\Big\downarrow \text{2009}$$

$$-i\left(-\frac{ib^2\text{arctanh}(\cosh(c+dx))}{a^3d}+\frac{ib^2\text{sech}(c+dx)}{a^3d}+\frac{ib\tanh(c+dx)}{a^2d}+\frac{ib\coth(c+dx)}{a^2d}+\frac{2ib^5\text{arctanh}\left(\frac{b-a\tanh}{\sqrt{a}}\right.}{a^3d\,(a^2+b^2)}\right.$$

input
```
Int[(Csch[c + d*x]^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-I)*((((3*I)/2)*ArcTanh[Cosh[c + d*x]])/(a*d) - (I*b^2*ArcTanh[Cosh[c + d
*x]])/(a^3*d) + ((2*I)*b^5*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^
2]])/(a^3*(a^2 + b^2)^(3/2)*d) + (I*b*Coth[c + d*x])/(a^2*d) - (((3*I)/2)*
Sech[c + d*x])/(a*d) + (I*b^2*Sech[c + d*x])/(a^3*d) - ((I/2)*Csch[c + d*x
]^2*Sech[c + d*x])/(a*d) - (I*b^3*Sech[c + d*x]*(b + a*Sinh[c + d*x]))/(a^
3*(a^2 + b^2)*d) + (I*b*Tanh[c + d*x])/(a^2*d))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3377

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Maple [A] (verified)

Time = 17.40 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\left(\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(-6a^2+4b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}+\frac{b}{2a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b\tanh\left(\frac{d\ldots}{2}\right.}{\left(a^2+b^2\right)\left(1+\mathrm{ta}\ldots\right.}\ldots\right)/d$
default	$\left(\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(-6a^2+4b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}+\frac{b}{2a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b\tanh\left(\frac{d\ldots}{2}\right.}{\left(a^2+b^2\right)\left(1+\mathrm{ta}\ldots\right.}\ldots\right)/d$
risch	$-\frac{3\,\mathrm{e}^{dx+c}a^3+\mathrm{e}^{dx+c}a\,b^2+3\,\mathrm{e}^{5dx+5c}a^3+\mathrm{e}^{5dx+5c}a\,b^2-2\,\mathrm{e}^{4dx+4c}b^3-2\,\mathrm{e}^{3dx+3c}a^3+2\,\mathrm{e}^{3dx+3c}a\,b^2-4b\,\mathrm{e}^{2dx+2c}a^2+4a^2b+\ldots}{d(a^2+b^2)\left(1+\mathrm{e}^{2dx+2c}\right)a^2\left(\mathrm{e}^{2dx+2c}-1\right)^2}$

input

```
int(csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output

```
1/d*(1/4/a^2*(1/2*tanh(1/2*d*x+1/2*c)^2*a+2*b*tanh(1/2*d*x+1/2*c))-1/8/a/t
anh(1/2*d*x+1/2*c)^2+1/4/a^3*(-6*a^2+4*b^2)*ln(tanh(1/2*d*x+1/2*c))+1/2*b/
a^2/tanh(1/2*d*x+1/2*c)+2/(a^2+b^2)*(b*tanh(1/2*d*x+1/2*c)-a)/(1+tanh(1/2*
d*x+1/2*c)^2)-2/a^3*b^5/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*
c)-2*b)/(a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2653 vs. $2(195) = 390$.

Time = 0.33 (sec) , antiderivative size = 2653, normalized size of antiderivative = 13.13

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fric
as")
```

output
```
-1/2*(8*a^5*b + 12*a^3*b^3 + 4*a*b^5 + 2*(3*a^6 + 4*a^4*b^2 + a^2*b^4)*cos
h(d*x + c)^5 + 2*(3*a^6 + 4*a^4*b^2 + a^2*b^4)*sinh(d*x + c)^5 - 4*(a^3*b^
3 + a*b^5)*cosh(d*x + c)^4 - 2*(2*a^3*b^3 + 2*a*b^5 - 5*(3*a^6 + 4*a^4*b^2
 + a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(a^6 - a^2*b^4)*cosh(d*x +
c)^3 - 4*(a^6 - a^2*b^4 - 5*(3*a^6 + 4*a^4*b^2 + a^2*b^4)*cosh(d*x + c)^2
+ 4*(a^3*b^3 + a*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 - 8*(a^5*b + a^3*b^3)
*cosh(d*x + c)^2 - 4*(2*a^5*b + 2*a^3*b^3 - 5*(3*a^6 + 4*a^4*b^2 + a^2*b^4
)*cosh(d*x + c)^3 + 6*(a^3*b^3 + a*b^5)*cosh(d*x + c)^2 + 3*(a^6 - a^2*b^4
)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(b^5*cosh(d*x + c)^6 + 6*b^5*cosh(d*x
 + c)*sinh(d*x + c)^5 + b^5*sinh(d*x + c)^6 - b^5*cosh(d*x + c)^4 - b^5*co
sh(d*x + c)^2 + b^5 + (15*b^5*cosh(d*x + c)^2 - b^5)*sinh(d*x + c)^4 + 4*(
5*b^5*cosh(d*x + c)^3 - b^5*cosh(d*x + c))*sinh(d*x + c)^3 + (15*b^5*cosh(
d*x + c)^4 - 6*b^5*cosh(d*x + c)^2 - b^5)*sinh(d*x + c)^2 + 2*(3*b^5*cosh(
d*x + c)^5 - 2*b^5*cosh(d*x + c)^3 - b^5*cosh(d*x + c))*sinh(d*x + c))*sqr
t(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d
*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt
(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 +
b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x +
 c) - b)) + 2*(3*a^6 + 4*a^4*b^2 + a^2*b^4)*cosh(d*x + c) - ((3*a^6 + 4*a^
4*b^2 - a^2*b^4 - 2*b^6)*cosh(d*x + c)^6 + 6*(3*a^6 + 4*a^4*b^2 - a^2*b...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**3*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.65

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b^5\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{(a^5+a^3b^2)\sqrt{a^2+b^2}d} - \frac{4\,a^2be^{(-2\,dx-2\,c)}+2\,b^3e^{(-4\,dx-4\,c)}-4\,a^2b-2\,b^3+(3\,a^3+ab^2)e^{(-dx-c)}-2\,(a^3-ab^2)e^{(-3\,dx-3\,c)}+(3\,a^3\cdots}{(a^4+a^2b^2-(a^4+a^2b^2)e^{(-2\,dx-2\,c)}-(a^4+a^2b^2)e^{(-4\,dx-4\,c)}+(a^4+a^2b^2)e^{(-6\,dx-6\,c)})d} + \frac{(3\,a^2-2\,b^2)\log\left(e^{(-dx-c)}+1\right)}{2\,a^3d} - \frac{(3\,a^2-2\,b^2)\log\left(e^{(-dx-c)}-1\right)}{2\,a^3d}$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxi
ma")
```

output
```
-b^5*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt
(a^2 + b^2)))/((a^5 + a^3*b^2)*sqrt(a^2 + b^2)*d) - (4*a^2*b*e^(-2*d*x - 2
*c) + 2*b^3*e^(-4*d*x - 4*c) - 4*a^2*b - 2*b^3 + (3*a^3 + a*b^2)*e^(-d*x -
 c) - 2*(a^3 - a*b^2)*e^(-3*d*x - 3*c) + (3*a^3 + a*b^2)*e^(-5*d*x - 5*c))
/((a^4 + a^2*b^2 - (a^4 + a^2*b^2)*e^(-2*d*x - 2*c) - (a^4 + a^2*b^2)*e^(-
4*d*x - 4*c) + (a^4 + a^2*b^2)*e^(-6*d*x - 6*c))*d) + 1/2*(3*a^2 - 2*b^2)*
log(e^(-d*x - c) + 1)/(a^3*d) - 1/2*(3*a^2 - 2*b^2)*log(e^(-d*x - c) - 1)/
(a^3*d)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{2\,b^5\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^5+a^3b^2)\sqrt{a^2+b^2}}+\frac{4\,(ae^{(dx+c)}+b)}{(a^2+b^2)\left(e^{(2\,dx+2\,c)}+1\right)}-\frac{(3\,a^2-2\,b^2)\log\left(e^{(dx+c)}+1\right)}{a^3}+\frac{(3\,a^2-2\,b^2)\log\left(\left|e^{(dx+c)}-1\right|\right)}{a^3}+\frac{2\,(\cdots}{}}{2\,d}$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac
")
```

output
```
-1/2*(2*b^5*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(
d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/((a^5 + a^3*b^2)*sqrt(a^2 + b^2)) + 4
*(a*e^(d*x + c) + b)/((a^2 + b^2)*(e^(2*d*x + 2*c) + 1)) - (3*a^2 - 2*b^2)
*log(e^(d*x + c) + 1)/a^3 + (3*a^2 - 2*b^2)*log(abs(e^(d*x + c) - 1))/a^3
+ 2*(a*e^(3*d*x + 3*c) - 2*b*e^(2*d*x + 2*c) + a*e^(d*x + c) + 2*b)/(a^2*(
e^(2*d*x + 2*c) - 1)^2))/d
```

Mupad [B] (verification not implemented)

Time = 4.65 (sec) , antiderivative size = 531, normalized size of antiderivative = 2.63

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{b^5\ln\left(2\,a^4\,b-4\,a^5\,e^{c+d\,x}+b^5+3\,a^2\,b^3+4\,a^2\,e^{c+d\,x}\,\sqrt{{\left(a^2+b^2\right)}^3}+b^2\,e^{c+d\,x}\,\sqrt{{\left(a^2+b^2\right)}^3}-7\,a^3\,b^2\,e^{c+d\,x}-\cdots\right.}{d\,a^9+3\,d\,a^7\,b^2+3\,d\,a^5\,b^4+d\,a^3\,b^6}$$
$$-\frac{\frac{e^{c+d\,x}}{a\,d}-\frac{2\,\left(a^2\,b+b^3\right)}{a^2\,d\,\left(a^2+b^2\right)}}{e^{2\,c+2\,d\,x}-1}-\frac{\frac{2\,b}{d\,\left(a^2+b^2\right)}+\frac{2\,a\,e^{c+d\,x}}{d\,\left(a^2+b^2\right)}}{e^{2\,c+2\,d\,x}+1}$$
$$-\frac{\ln\left(e^{c+d\,x}-1\right)\,\left(3\,a^2-2\,b^2\right)}{2\,a^3\,d}+\frac{\ln\left(e^{c+d\,x}+1\right)\,\left(3\,a^2-2\,b^2\right)}{2\,a^3\,d}$$
$$-\frac{b^5\ln\left(4\,a^5\,e^{c+d\,x}-2\,a^4\,b-b^5-3\,a^2\,b^3+4\,a^2\,e^{c+d\,x}\,\sqrt{{\left(a^2+b^2\right)}^3}+b^2\,e^{c+d\,x}\,\sqrt{{\left(a^2+b^2\right)}^3}+7\,a^3\,b^2\,e^{c+d\,x}\cdots\right.}{d\,a^9+3\,d\,a^7\,b^2+3\,d\,a^5\,b^4+d\,a^3\,b^6}$$
$$-\frac{2\,e^{c+d\,x}}{a\,d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}$$

input
```
int(1/(cosh(c + d*x)^2*sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
(b^5*log(2*a^4*b - 4*a^5*exp(c + d*x) + b^5 + 3*a^2*b^3 + 4*a^2*exp(c + d*
x)*((a^2 + b^2)^3)^(1/2) + b^2*exp(c + d*x)*((a^2 + b^2)^3)^(1/2) - 7*a^3*
b^2*exp(c + d*x) - 2*a*b*((a^2 + b^2)^3)^(1/2) - 3*a*b^4*exp(c + d*x))*((a
^2 + b^2)^3)^(1/2))/(a^9*d + a^3*b^6*d + 3*a^5*b^4*d + 3*a^7*b^2*d) - (exp
(c + d*x)/(a*d) - (2*(a^2*b + b^3))/(a^2*d*(a^2 + b^2)))/(exp(2*c + 2*d*x)
 - 1) - ((2*b)/(d*(a^2 + b^2)) + (2*a*exp(c + d*x))/(d*(a^2 + b^2)))/(exp(
2*c + 2*d*x) + 1) - (log(exp(c + d*x) - 1)*(3*a^2 - 2*b^2))/(2*a^3*d) + (l
og(exp(c + d*x) + 1)*(3*a^2 - 2*b^2))/(2*a^3*d) - (b^5*log(4*a^5*exp(c + d
*x) - 2*a^4*b - b^5 - 3*a^2*b^3 + 4*a^2*exp(c + d*x)*((a^2 + b^2)^3)^(1/2)
 + b^2*exp(c + d*x)*((a^2 + b^2)^3)^(1/2) + 7*a^3*b^2*exp(c + d*x) - 2*a*b
*((a^2 + b^2)^3)^(1/2) + 3*a*b^4*exp(c + d*x))*((a^2 + b^2)^3)^(1/2))/(a^9
*d + a^3*b^6*d + 3*a^5*b^4*d + 3*a^7*b^2*d) - (2*exp(c + d*x))/(a*d*(exp(4
*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1346, normalized size of antiderivative = 6.66

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
( - 4*e**(6*c + 6*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqr
t(a**2 + b**2))*b**5*i + 4*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c
+ d*x)*b*i + a*i)/sqrt(a**2 + b**2))*b**5*i + 4*e**(2*c + 2*d*x)*sqrt(a**2
 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*b**5*i - 4*sqrt(
a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*b**5*i - 3*e
**(6*c + 6*d*x)*log(e**(c + d*x) - 1)*a**6 - 4*e**(6*c + 6*d*x)*log(e**(c
+ d*x) - 1)*a**4*b**2 + e**(6*c + 6*d*x)*log(e**(c + d*x) - 1)*a**2*b**4 +
 2*e**(6*c + 6*d*x)*log(e**(c + d*x) - 1)*b**6 + 3*e**(6*c + 6*d*x)*log(e*
*(c + d*x) + 1)*a**6 + 4*e**(6*c + 6*d*x)*log(e**(c + d*x) + 1)*a**4*b**2
- e**(6*c + 6*d*x)*log(e**(c + d*x) + 1)*a**2*b**4 - 2*e**(6*c + 6*d*x)*lo
g(e**(c + d*x) + 1)*b**6 + 4*e**(6*c + 6*d*x)*a**3*b**3 + 4*e**(6*c + 6*d*
x)*a*b**5 - 6*e**(5*c + 5*d*x)*a**6 - 8*e**(5*c + 5*d*x)*a**4*b**2 - 2*e**
(5*c + 5*d*x)*a**2*b**4 + 3*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**6 +
4*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**4*b**2 - e**(4*c + 4*d*x)*log(
e**(c + d*x) - 1)*a**2*b**4 - 2*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*b**
6 - 3*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a**6 - 4*e**(4*c + 4*d*x)*log
(e**(c + d*x) + 1)*a**4*b**2 + e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a**2
*b**4 + 2*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*b**6 + 4*e**(3*c + 3*d*x)
*a**6 - 4*e**(3*c + 3*d*x)*a**2*b**4 + 3*e**(2*c + 2*d*x)*log(e**(c + d*x)
 - 1)*a**6 + 4*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a**4*b**2 - e**(2...
```

3.499 $\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$

Optimal result

Integrand size = 36, antiderivative size = 36

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \operatorname{Int}\left(\frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(csch(d*x+c)^3*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 75.87 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

input
```
Integrate[(Csch[c + d*x]^3*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x
])),x]
```

output
```
Integrate[(Csch[c + d*x]^3*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

$$\Big\downarrow \text{6125}$$

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Csch[c + d*x]^3*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(dx+c)^3\operatorname{sech}(dx+c)^2}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(csch(d*x+c)^3*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)^3*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 2.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)^3\operatorname{sech}\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
integral(csch(d*x + c)^3*sech(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh
(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)**3*sech(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 1426, normalized size of antiderivative = 39.61

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)^3\operatorname{sech}\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-32*b^5*integrate(-1/16*e^(d*x + c)/(a^5*b*e + a^3*b^3*e + (a^5*b*f + a^3*
b^3*f)*x - (a^5*b*e*e^(2*c) + a^3*b^3*e*e^(2*c) + (a^5*b*f*e^(2*c) + a^3*b
^3*f*e^(2*c))*x)*e^(2*d*x) - 2*(a^6*e*e^c + a^4*b^2*e*e^c + (a^6*f*e^c + a
^4*b^2*f*e^c)*x)*e^(d*x)), x) - (4*a^2*b*d*e + 2*b^3*d*e + 2*(2*a^2*b*d*f
+ b^3*d*f)*x + ((3*d*e - f)*a^3*e^(5*c) + (d*e - f)*a*b^2*e^(5*c) + (3*a^3
*d*f*e^(5*c) + a*b^2*d*f*e^(5*c))*x)*e^(5*d*x) - 2*(b^3*d*f*x*e^(4*c) + b^
3*d*e*e^(4*c))*e^(4*d*x) - 2*(a^3*d*e*e^(3*c) - a*b^2*d*e*e^(3*c) + (a^3*d
*f*e^(3*c) - a*b^2*d*f*e^(3*c))*x)*e^(3*d*x) - 4*(a^2*b*d*f*x*e^(2*c) + a^
2*b*d*e*e^(2*c))*e^(2*d*x) + ((3*d*e + f)*a^3*e^c + (d*e + f)*a*b^2*e^c +
(3*a^3*d*f*e^c + a*b^2*d*f*e^c)*x)*e^(d*x))/(a^4*d^2*e^2 + a^2*b^2*d^2*e^2
 + (a^4*d^2*f^2 + a^2*b^2*d^2*f^2)*x^2 + 2*(a^4*d^2*e*f + a^2*b^2*d^2*e*f)
*x + (a^4*d^2*e^2*e^(6*c) + a^2*b^2*d^2*e^2*e^(6*c) + (a^4*d^2*f^2*e^(6*c)
 + a^2*b^2*d^2*f^2*e^(6*c))*x^2 + 2*(a^4*d^2*e*f*e^(6*c) + a^2*b^2*d^2*e*f
*e^(6*c))*x)*e^(6*d*x) - (a^4*d^2*e^2*e^(4*c) + a^2*b^2*d^2*e^2*e^(4*c) +
(a^4*d^2*f^2*e^(4*c) + a^2*b^2*d^2*f^2*e^(4*c))*x^2 + 2*(a^4*d^2*e*f*e^(4*
c) + a^2*b^2*d^2*e*f*e^(4*c))*x)*e^(4*d*x) - (a^4*d^2*e^2*e^(2*c) + a^2*b^
2*d^2*e^2*e^(2*c) + (a^4*d^2*f^2*e^(2*c) + a^2*b^2*d^2*f^2*e^(2*c))*x^2 +
2*(a^4*d^2*e*f*e^(2*c) + a^2*b^2*d^2*e*f*e^(2*c))*x)*e^(2*d*x)) - 32*integ
rate(1/64*(2*b^2*d^2*e^2 + 2*a*b*d*e*f - (3*d^2*e^2 - 2*f^2)*a^2 - (3*a^2*
d^2*f^2 - 2*b^2*d^2*f^2)*x^2 - 2*(3*a^2*d^2*e*f - 2*b^2*d^2*e*f - a*b*d...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 19.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^2\,\sinh\left(c+d\,x\right)^3\,\left(e+f\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)^2*sinh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(cosh(c + d*x)^2*sinh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))), x
)
```

Reduce [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)^3\operatorname{sech}(dx+c)^2}{\sinh(dx+c)\,be+\sinh(dx+c)\,bfx+ae+afx}dx$$

input
```
int(csch(d*x+c)^3*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int((csch(c + d*x)**3*sech(c + d*x)**2)/(sinh(c + d*x)*b*e + sinh(c + d*x)
*b*f*x + a*e + a*f*x),x)
```

3.500 $\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 1117

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output

```
-3/2*b*f*x*arctan(sinh(d*x+c))/a^2/d+3/2*b*(f*x+e)*arctan(sinh(d*x+c))/a^2
/d+b^6*(f*x+e)*ln(1+exp(2*d*x+2*c))/a^3/(a^2+b^2)^2/d-b^2*f*x*ln(tanh(d*x+
c))/a^3/d+b^2*(f*x+e)*ln(tanh(d*x+c))/a^3/d-f*polylog(2,exp(2*d*x+2*c))/a/
d^2+4*(f*x+e)*arctanh(exp(2*d*x+2*c))/a/d+f*polylog(2,-exp(2*d*x+2*c))/a/d
^2+3/2*I*b*f*polylog(2,I*exp(d*x+c))/a^2/d^2+1/2*b^2*f*polylog(2,exp(2*d*x
+2*c))/a^3/d^2+3*b*f*x*arctan(exp(d*x+c))/a^2/d-b^3*(f*x+e)*arctan(exp(d*x
+c))/a^2/(a^2+b^2)/d+1/2*b^6*f*polylog(2,-exp(2*d*x+2*c))/a^3/(a^2+b^2)^2/
d^2-2*b^2*f*x*arctanh(exp(2*d*x+2*c))/a^3/d-2*b^5*(f*x+e)*arctan(exp(d*x+c
))/a^2/(a^2+b^2)^2/d+1/2*b^4*f*tanh(d*x+c)/a^3/(a^2+b^2)/d^2+1/2*b*(f*x+e)
*sech(d*x+c)*tanh(d*x+c)/a^2/d-f*csch(2*d*x+2*c)/a/d^2-1/2*b^2*f*polylog(2
,-exp(2*d*x+2*c))/a^3/d^2+1/2*b^2*f*x/a^3/d-1/2*b^2*f*tanh(d*x+c)/a^3/d^2-
1/2*b^2*(f*x+e)*tanh(d*x+c)^2/a^3/d+1/2*b*f*sech(d*x+c)/a^2/d^2-2*(f*x+e)*
coth(2*d*x+2*c)*csch(2*d*x+2*c)/a/d+b*f*arctanh(cosh(d*x+c))/a^2/d^2+b*(f*
x+e)*csch(d*x+c)/a^2/d-b^6*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/
a^3/(a^2+b^2)^2/d^2-b^6*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3
/(a^2+b^2)^2/d^2-b^6*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/(a
^2+b^2)^2/d-b^6*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/(a^2+b^
2)^2/d-1/2*b^3*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/a^2/(a^2+b^2)/d-I*b^5*f*pol
ylog(2,I*exp(d*x+c))/a^2/(a^2+b^2)^2/d^2-1/2*I*b^3*f*polylog(2,I*exp(d*x+c
))/a^2/(a^2+b^2)/d^2+1/2*I*b^3*f*polylog(2,-I*exp(d*x+c))/a^2/(a^2+b^2)...
```

Mathematica [A] (warning: unable to verify)

Time = 11.10 (sec) , antiderivative size = 1675, normalized size of antiderivative = 1.50

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input

```
Integrate[((e + f*x)*Csch[c + d*x]^3*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]
),x]
```

output
```
8*(-1/16*(-(a*b*f*(c + d*x)) + 2*(a*b*f + b^2*(d*e - c*f) + a^2*(-2*d*e +
2*c*f))*(c + d*x) + (a^2*(d*e + d*f*x)^2)/f - (b^2*(d*e + d*f*x)^2)/(2*f)
+ 2*(2*a^2 - b^2)*f*(c + d*x)*Log[1 + E^(-c - d*x)] - 2*(a*b*f + b^2*(d*e
- c*f) + a^2*(-2*d*e + 2*c*f))*Log[1 + E^(c + d*x)] - 2*(2*a^2 - b^2)*f*Po
lyLog[2, -E^(-c - d*x)])/(a^3*d^2) - (a*b*f*(c + d*x) - 2*(a*b*f + 2*a^2*(
d*e - c*f) + b^2*(-(d*e) + c*f))*(c + d*x) + (a^2*(d*e + d*f*x)^2)/f - (b^
2*(d*e + d*f*x)^2)/(2*f) + 2*(2*a^2 - b^2)*f*(c + d*x)*Log[1 - E^(-c - d*x
)] + 2*(a*b*f + 2*a^2*(d*e - c*f) + b^2*(-(d*e) + c*f))*Log[1 - E^(c + d*x
)] - 2*(2*a^2 - b^2)*f*PolyLog[2, E^(-c - d*x)])/(16*a^3*d^2) - (b^6*(-2*d
*e*(c + d*x) + 2*c*f*(c + d*x) - f*(c + d*x)^2 + (4*a*Sqrt[a^2 + b^2]*d*e*
ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b^2)^2] - (4*a*
Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/(-a
^2 - b^2)^(3/2) + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2])] + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*
f*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*d*e*Log[2*a*E^(c + d*x)
 + b*(-1 + E^(2*(c + d*x)))] + 2*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a
^2 + b^2])] + 2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(
16*a^3*(a^2 + b^2)^2*d^2) + (-4*a^3*d*e*(c + d*x) - 6*a*b^2*d*e*(c + d*x)
+ 4*a^3*c*f*(c + d*x) + 6*a*b^2*c*f*(c + d*x) - 2*a^3*f*(c + d*x)^2 - 3*a*
b^2*f*(c + d*x)^2 + 6*a^2*b*d*e*ArcTan[E^(c + d*x)] + 10*b^3*d*e*ArcTan...
```

Rubi [A] (verified)

Time = 6.18 (sec) , antiderivative size = 943, normalized size of antiderivative = 0.84, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.735$, Rules used = {6123, 5984, 3042, 26, 4673, 26, 3042, 26, 4670, 2715, 2838, 6123, 5985, 2009, 6123, 5985, 2009, 6107, 6107, 6095, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)\text{csch}^3(c+dx)\text{sech}^3(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5984}$$

$$\frac{8\int (e+fx)\text{csch}^3(2c+2dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$\Big\downarrow$ 3042

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{8\int -i(e+fx)\csc(2ic+2idx)^3dx}{a}$$

$\Big\downarrow$ 26

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{8i\int (e+fx)\csc(2ic+2idx)^3dx}{a}$$

$\Big\downarrow$ 4673

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{8i\left(\frac{1}{2}\int -i(e+fx)\text{csch}(2c+2dx)dx-\frac{if\text{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)\text{csch}(2c+2dx)}{4d}\right)}{a}$$

$\Big\downarrow$ 26

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{8i\left(-\frac{1}{2}i\int (e+fx)\text{csch}(2c+2dx)dx-\frac{if\text{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)\text{csch}(2c+2dx)}{4d}\right)}{a}$$

$\Big\downarrow$ 3042

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{8i\left(-\frac{1}{2}i\int i(e+fx)\csc(2ic+2idx)dx-\frac{if\text{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)\text{csch}(2c+2dx)}{4d}\right)}{a}$$

$\Big\downarrow$ 26

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{8i\left(\frac{1}{2}\int (e+fx)\csc(2ic+2idx)dx-\frac{if\text{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)\text{csch}(2c+2dx)}{4d}\right)}{a}$$

$\Big\downarrow$ 4670

$$\frac{-\frac{b\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a} - \ldots}{\ldots} \quad \frac{8i\left(\frac{1}{2}\left(\frac{if\int \log\left(1-e^{2c+2dx}\right)dx}{2d} - \frac{if\int \log\left(1+e^{2c+2dx}\right)dx}{2d} + \frac{i(e+fx)\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}\right) - \frac{if\operatorname{csch}(2c+2dx)}{8d^2} - \frac{i(e+fx)\coth(2c+2dx)\ldots}{4d}\ldots}{a}$$

$$\Big\downarrow \; 2715$$

$$\frac{-\frac{b\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a} - \ldots}{\ldots} \quad \frac{8i\left(\frac{1}{2}\left(\frac{if\int e^{-2c-2dx}\log\left(1-e^{2c+2dx}\right)de^{2c+2dx}}{4d^2} - \frac{if\int e^{-2c-2dx}\log\left(1+e^{2c+2dx}\right)de^{2c+2dx}}{4d^2} + \frac{i(e+fx)\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}\right) - \frac{if\operatorname{csch}(2c\ldots}{8d^2}\ldots}{a}$$

$$\Big\downarrow \; 2838$$

$$\frac{-\frac{b\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a} - \ldots}{\ldots} \quad \frac{8i\left(\frac{1}{2}\left(\frac{i(e+fx)\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d} + \frac{if\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2} - \frac{if\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right) - \frac{if\operatorname{csch}(2c+2dx)}{8d^2} - \frac{i(e+fx)\coth(2c+2dx\ldots}{4d}\ldots}{a}$$

$$\Big\downarrow \; 6123$$

$$-\frac{b\left(\frac{\int (e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)dx}{a} - \frac{b\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a} - \ldots \quad \frac{8i\left(\frac{1}{2}\left(\frac{i(e+fx)\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d} + \frac{if\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2} - \frac{if\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right) - \frac{if\operatorname{csch}(2c+2dx)}{8d^2} - \frac{i(e+fx)\coth(2c+2dx\ldots}{4d}\ldots}{a}$$

$$\Big\downarrow \; 5985$$

$$-\frac{b\left(\frac{-f\int\left(\frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{2d} - \frac{3\arctan(\sinh(c+dx))}{2d} - \frac{3\operatorname{csch}(c+dx)}{2d}\right)dx - \frac{3(e+fx)\arctan(\sinh(c+dx))}{2d} - \frac{3(e+fx)\operatorname{csch}(c+dx)}{2d} + \frac{(e+fx)\operatorname{csch}(\ldots}{\ldots}}{a}\ldots\right.}{a} \quad \frac{8i\left(\frac{1}{2}\left(\frac{i(e+fx)\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d} + \frac{if\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2} - \frac{if\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right) - \frac{if\operatorname{csch}(2c+2dx)}{8d^2} - \frac{i(e+fx)\coth(2c+2dx\ldots}{4d}\ldots}{a}$$

$$\Big\downarrow \; 2009$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-f\left(\frac{3x\arctan\left(e^{c+dx}\right)}{d}-\frac{3x\arctan(\sinh(c+dx))}{2d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{3i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{2d^2}\right.}{a}\right.}{8i\left(\frac{1}{2}\left(\frac{i(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\,\text{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)-\frac{if\text{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)}{4d}\right.}{a}$$

$$\Big\downarrow \; 6123$$

$$-\frac{b\left(-\frac{b\left(\frac{\int(e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{-f\left(\frac{3x\arctan\left(e^{c+dx}\right)}{d}-\frac{3x\arctan(\sinh(c+dx))}{2d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}\right.}{}\right.}{8i\left(\frac{1}{2}\left(\frac{i(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\,\text{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)-\frac{if\text{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)}{4d}\right.}{a}$$

$$\Big\downarrow \; 5985$$

$$-\frac{b\left(-\frac{b\left(\frac{-f\int\left(\frac{\log(\tanh(c+dx))}{d}-\frac{\tanh^2(c+dx)}{2d}\right)dx-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e+fx)\log(\tanh(c+dx))}{d}}{a}-\frac{b\int\frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{-f\left(\frac{3x\arctan(}{d}\right.}{}\right.}{8i\left(\frac{1}{2}\left(\frac{i(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\,\text{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)-\frac{if\text{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)}{4d}\right.}{a}$$

$$\Big\downarrow \; 2009$$

$$-\frac{b\left(-\frac{b\left(\frac{-f\left(\frac{2x\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\cdots}{a}\right.}{a}\cdots}{\cdots}$$

$$\frac{8i\left(\frac{1}{2}\left(\frac{i(e+fx)\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)-\frac{if\operatorname{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)}{4d}\cdots\right.}{a}$$

$\downarrow$ 6107

$$-\frac{b\left(-\frac{b\left(\frac{-f\left(\frac{2x\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\cdots}{a}\right.}{a}\cdots}{\cdots}$$

$$\frac{8i\left(\frac{1}{2}\left(\frac{i(e+fx)\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)-\frac{if\operatorname{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)}{4d}\cdots\right.}{a}$$

$\downarrow$ 6107

$$-\frac{b\left(b\left(-\frac{b\left(\frac{-f\left(\frac{2x\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\cdots}{a}\right.}{\cdots}\right.}{8i\left(\frac{1}{2}\left(\frac{i(e+fx)\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)-\frac{if\operatorname{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx}{4d}\cdots\right.}}{a}$$

$\downarrow$ 6095

$$-\frac{b\left(b\left(-\frac{b\left(\frac{-f\left(\frac{2x\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\cdots}{a}\right)}{\cdots}\right)\cdots\right)}{8i\left(\frac{1}{2}\left(\frac{i(e+fx)\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)-\frac{if\operatorname{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)}{4d}\cdots\right.}{a}$$

$\downarrow$ 2620

$$b\left(-\frac{b\left(\frac{-f\left(\frac{2x\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+(\cdots}{a}}{\cdots}\right.$$

$$-\frac{8i\left(\frac{1}{2}\left(\frac{i(e+fx)\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)-\frac{if\operatorname{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx}{4d}\cdots}{a}$$

$\downarrow$ 2715

$$b\left(-\frac{b\left(\frac{-f\left(\frac{2x\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+(}{a}\right.}{}\right.$$

$$-\frac{8i\left(\frac{1}{2}\left(\frac{i(e+fx)\operatorname{arctanh}(e^{2c+2dx})}{d}+\frac{if\operatorname{PolyLog}(2,-e^{2c+2dx})}{4d^2}-\frac{if\operatorname{PolyLog}(2,e^{2c+2dx})}{4d^2}\right)-\frac{if\operatorname{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx}{4d}\right.}{a}$$

$\downarrow$ 2838

$$b\left(-\frac{\left(b\left(\frac{-f\left(\frac{2x\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\left(\right.}{a}\right.\right.}{}\right.$$

$$-\frac{8i\left(\frac{1}{2}\left(\frac{i(e+fx)\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)-\frac{if\operatorname{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx}{4d}\right.}{a}$$

↓ 7293

$$
-\frac{b\left(b-\frac{b\left(\frac{-f\left(\frac{2x\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\cdots}{a}\right)}{\cdots}\right)}{\cdots}
$$

$$
\frac{8i\left(\frac{1}{2}\left(\frac{i(e+fx)\operatorname{arctanh}(e^{2c+2dx})}{d}+\frac{if\operatorname{PolyLog}(2,-e^{2c+2dx})}{4d^2}-\frac{if\operatorname{PolyLog}(2,e^{2c+2dx})}{4d^2}\right)-\frac{if\operatorname{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx}{4d}\cdots\right.}{a}
$$

$\downarrow$ 2009

$$-\frac{8i\left(-\frac{if\operatorname{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)\operatorname{csch}(2c+2dx)}{4d}+\frac{1}{2}\left(\frac{i(e+fx)\operatorname{arctanh}(e^{2c+2dx})}{d}+\frac{if\operatorname{PolyLog}(2,-e^{2c+2dx})}{4d^2}-\frac{if\operatorname{Po}}{}\right.\right.}{a}$$

$$b\left(\frac{\frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{2d}-\frac{3(e+fx)\arctan(\sinh(c+dx))}{2d}-\frac{3(e+fx)\operatorname{csch}(c+dx)}{2d}-f\left(\frac{3x\arctan\left(e^{c+dx}\right)}{d}-\frac{3x\arctan(\sinh(c+dx))}{2d}+\frac{\operatorname{arctanh}(c}{d^2}\right.}{a}\right.$$

input
```
Int[((e + f*x)*Csch[c + d*x]^3*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
((-8*I)*(((-1/8*I)*f*Csch[2*c + 2*d*x])/d^2 - ((I/4)*(e + f*x)*Coth[2*c +
2*d*x]*Csch[2*c + 2*d*x])/d + ((I*(e + f*x)*ArcTanh[E^(2*c + 2*d*x)])/d +
((I/4)*f*PolyLog[2, -E^(2*c + 2*d*x)])/d^2 - ((I/4)*f*PolyLog[2, E^(2*c +
2*d*x)])/d^2)/2))/a - (b*(((-3*(e + f*x)*ArcTan[Sinh[c + d*x]])/(2*d) - (3
*(e + f*x)*Csch[c + d*x])/(2*d) + ((e + f*x)*Csch[c + d*x]*Sech[c + d*x]^2
)/(2*d) - f*((3*x*ArcTan[E^(c + d*x)])/d - (3*x*ArcTan[Sinh[c + d*x]])/(2*
d) + ArcTanh[Cosh[c + d*x]]/d^2 - (((3*I)/2)*PolyLog[2, (-I)*E^(c + d*x)])
/d^2 + (((3*I)/2)*PolyLog[2, I*E^(c + d*x)])/d^2 + Sech[c + d*x]/(2*d^2)))
/a - (b*((((e + f*x)*Log[Tanh[c + d*x]])/d - ((e + f*x)*Tanh[c + d*x]^2)/(
2*d) - f*(-1/2*x/d + (2*x*ArcTanh[E^(2*c + 2*d*x)])/d + (x*Log[Tanh[c + d*
x]])/d + PolyLog[2, -E^(2*c + 2*d*x)]/(2*d^2) - PolyLog[2, E^(2*c + 2*d*x)
]/(2*d^2) + Tanh[c + d*x]/(2*d^2)))/a - (b*((b^2*((b^2*(-1/2*(e + f*x)^2/(
b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) +
((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*Poly
Log[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (f*PolyLog[2,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(a^2 + b^2) + ((b*(e
+ f*x)^2)/(2*f) + (2*a*(e + f*x)*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)*Log
[1 + E^(2*(c + d*x))])/d - (I*a*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + (I*a
*f*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*PolyLog[2, -E^(2*(c + d*x))])/(2*
d^2))/(a^2 + b^2)))/(a^2 + b^2) + ((a*(e + f*x)*ArcTan[E^(c + d*x)])/d ...
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3562 vs. 2(1042) = 2084.

Time = 129.46 (sec) , antiderivative size = 3563, normalized size of antiderivative = 3.19

method	result	size
risch	Expression too large to display	3563

input
```
int((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURN
VERBOSE)
```

output
```
-12/d^2/(a^2+b^2)*c*a^2*f/(4*a^2+4*b^2)*b*arctan(exp(d*x+c))+12/d/(a^2+b^2
)*b^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*a*x+12/d^2/(a^2+b^2)*b^2*f/(4*a^2
+4*b^2)*ln(1+I*exp(d*x+c))*a*c+12/d/(a^2+b^2)*b^2*f/(4*a^2+4*b^2)*ln(1-I*e
xp(d*x+c))*a*x+12/d^2/(a^2+b^2)*b^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*a*c
+1/(a^2+b^2)/d^2/a^2*b^3*f*ln(exp(d*x+c)+1)-1/(a^2+b^2)/d^2/a^2*b^3*f*ln(e
xp(d*x+c)-1)+1/(a^2+b^2)/d/a^3*b^4*e*ln(exp(d*x+c)-1)+1/(a^2+b^2)/d/a^3*b^
4*e*ln(exp(d*x+c)+1)-1/(a^2+b^2)/d^2/a^3*b^4*f*dilog(exp(d*x+c))+1/(a^2+b^
2)/d^2/a^3*b^4*f*dilog(exp(d*x+c)+1)-1/d^2*b^2*f/(a^2+b^2)^(3/2)*arctanh(1
/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-6*I/d^2/(a^2+b^2)*a^2*f/(4*a^2+4*
b^2)*dilog(1+I*exp(d*x+c))*b+6*I/d^2/(a^2+b^2)*a^2*f/(4*a^2+4*b^2)*dilog(1
-I*exp(d*x+c))*b-10*I/d/(a^2+b^2)*b^3*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*x
+10*I/d/(a^2+b^2)*b^3*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*x+10*I/d^2/(a^2+b
^2)*b^3*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*c-10*I/d^2/(a^2+b^2)*b^3*f/(4*a
^2+4*b^2)*ln(1+I*exp(d*x+c))*c+1/2/d/(a^2+b^2)^(3/2)*b^2*e*arctanh(1/2*(2*
b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+20/d/(a^2+b^2)*b^3*e/(4*a^2+4*b^2)*arct
an(exp(d*x+c))-1/2/d/(a^2+b^2)^(5/2)*b^4*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a
)/(a^2+b^2)^(1/2))+8/d/(a^2+b^2)*a^3*e/(4*a^2+4*b^2)*ln(1+exp(2*d*x+2*c))+
8/d^2/(a^2+b^2)*a^3*f/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+c))+8/d^2/(a^2+b^2)*
a^3*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))-1/d^2/a^2*b^4*f/(a^2+b^2)^(3/2)*
arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/(a^2+b^2)/d*ln(exp(...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 16848 vs. $2(1017) = 2034$.

Time = 0.55 (sec) , antiderivative size = 16848, normalized size of antiderivative = 15.08

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)**3*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{csch}\,(dx+c)^3\,\text{sech}\,(dx+c)^3}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-(b^6*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^7 + 2*a^5*b^2 +
a^3*b^4)*d) + (3*a^2*b + 5*b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b
^4)*d) - (2*a^3 + 3*a*b^2)*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b
^4)*d) + (4*a*b^2*e^(-4*d*x - 4*c) - (3*a^2*b + 2*b^3)*e^(-d*x - c) + 2*(2
*a^3 + a*b^2)*e^(-2*d*x - 2*c) + (a^2*b - 2*b^3)*e^(-3*d*x - 3*c) - (a^2*b
 - 2*b^3)*e^(-5*d*x - 5*c) + 2*(2*a^3 + a*b^2)*e^(-6*d*x - 6*c) + (3*a^2*b
 + 2*b^3)*e^(-7*d*x - 7*c))/((a^4 + a^2*b^2 - 2*(a^4 + a^2*b^2)*e^(-4*d*x
- 4*c) + (a^4 + a^2*b^2)*e^(-8*d*x - 8*c))*d) + (2*a^2 - b^2)*log(e^(-d*x
- c) + 1)/(a^3*d) + (2*a^2 - b^2)*log(e^(-d*x - c) - 1)/(a^3*d))*e + (128*
a^2*d*integrate(1/64*x/(a^3*d*e^(d*x + c) + a^3*d), x) - 64*b^2*d*integrat
e(1/64*x/(a^3*d*e^(d*x + c) + a^3*d), x) - 128*a^2*d*integrate(1/64*x/(a^3
*d*e^(d*x + c) - a^3*d), x) + 64*b^2*d*integrate(1/64*x/(a^3*d*e^(d*x + c)
 - a^3*d), x) - a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) + 1)/(a^3*d^2))
 + a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) - 1)/(a^3*d^2)) + (a*b^2 + (
a^2*b*e^(7*c) + (3*a^2*b*d*e^(7*c) + 2*b^3*d*e^(7*c))*x)*e^(7*d*x) - (2*a^
3*e^(6*c) + a*b^2*e^(6*c) + 2*(2*a^3*d*e^(6*c) + a*b^2*d*e^(6*c))*x)*e^(6*
d*x) - (a^2*b*e^(5*c) + (a^2*b*d*e^(5*c) - 2*b^3*d*e^(5*c))*x)*e^(5*d*x) -
 (4*a*b^2*d*x*e^(4*c) + a*b^2*e^(4*c))*e^(4*d*x) - (a^2*b*e^(3*c) - (a^2*b
*d*e^(3*c) - 2*b^3*d*e^(3*c))*x)*e^(3*d*x) + (2*a^3*e^(2*c) + a*b^2*e^(2*c
) - 2*(2*a^3*d*e^(2*c) + a*b^2*d*e^(2*c))*x)*e^(2*d*x) + (a^2*b*e^c - (...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{e+f\,x}{\cosh\left(c+d\,x\right)^3\,\sinh\left(c+d\,x\right)^3\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)/(cosh(c + d*x)^3*sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(cosh(c + d*x)^3*sinh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

Reduce [F]

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
(48*e**(8*c + 8*d*x)*atan(e**(c + d*x))*a**7*b*f + 21*e**(8*c + 8*d*x)*ata
n(e**(c + d*x))*a**5*b**3*d*e + 96*e**(8*c + 8*d*x)*atan(e**(c + d*x))*a**
5*b**3*f + 35*e**(8*c + 8*d*x)*atan(e**(c + d*x))*a**3*b**5*d*e + 48*e**(8
*c + 8*d*x)*atan(e**(c + d*x))*a**3*b**5*f - 96*e**(4*c + 4*d*x)*atan(e**(
c + d*x))*a**7*b*f - 42*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**5*b**3*d*e
- 192*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**5*b**3*f - 70*e**(4*c + 4*d*x
)*atan(e**(c + d*x))*a**3*b**5*d*e - 96*e**(4*c + 4*d*x)*atan(e**(c + d*x)
)*a**3*b**5*f + 48*atan(e**(c + d*x))*a**7*b*f + 21*atan(e**(c + d*x))*a**
5*b**3*d*e + 96*atan(e**(c + d*x))*a**5*b**3*f + 35*atan(e**(c + d*x))*a**
3*b**5*d*e + 48*atan(e**(c + d*x))*a**3*b**5*f + 3584*e**(13*c + 8*d*x)*in
t((e**(5*d*x)*x)/(e**(14*c + 14*d*x)*b + 2*e**(13*c + 13*d*x)*a - e**(12*c
 + 12*d*x)*b - 3*e**(10*c + 10*d*x)*b - 6*e**(9*c + 9*d*x)*a + 3*e**(8*c +
 8*d*x)*b + 3*e**(6*c + 6*d*x)*b + 6*e**(5*c + 5*d*x)*a - 3*e**(4*c + 4*d*
x)*b - e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**9*d**2*f + 8064*e*
*(13*c + 8*d*x)*int((e**(5*d*x)*x)/(e**(14*c + 14*d*x)*b + 2*e**(13*c + 13
*d*x)*a - e**(12*c + 12*d*x)*b - 3*e**(10*c + 10*d*x)*b - 6*e**(9*c + 9*d*
x)*a + 3*e**(8*c + 8*d*x)*b + 3*e**(6*c + 6*d*x)*b + 6*e**(5*c + 5*d*x)*a
- 3*e**(4*c + 4*d*x)*b - e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a + b),x)*a**
7*b**2*d**2*f + 5376*e**(13*c + 8*d*x)*int((e**(5*d*x)*x)/(e**(14*c + 14*d
*x)*b + 2*e**(13*c + 13*d*x)*a - e**(12*c + 12*d*x)*b - 3*e**(10*c + 10...
```

3.501 $\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 211

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b\arctan(\sinh(c+dx))}{2(a^2+b^2)d} + \frac{b(a^2+2b^2)\arctan(\sinh(c+dx))}{(a^2+b^2)^2 d} + \frac{b\operatorname{csch}(c+dx)}{a^2 d} - \frac{\operatorname{csch}^2(c+dx)}{2ad} + \frac{a(2a^2+3b^2)\log(\cosh(c+dx))}{(a^2+b^2)^2 d} - \frac{(2a^2-b^2)\log(\sinh(c+dx))}{a^3 d} - \frac{b^6\log(a+b\sinh(c+dx))}{a^3(a^2+b^2)^2 d} - \frac{\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))}{2(a^2+b^2)d}$$

output

```
1/2*b*arctan(sinh(d*x+c))/(a^2+b^2)/d+b*(a^2+2*b^2)*arctan(sinh(d*x+c))/(a
^2+b^2)^2/d+b*csch(d*x+c)/a^2/d-1/2*csch(d*x+c)^2/a/d+a*(2*a^2+3*b^2)*ln(c
osh(d*x+c))/(a^2+b^2)^2/d-(2*a^2-b^2)*ln(sinh(d*x+c))/a^3/d-b^6*ln(a+b*sin
h(d*x+c))/a^3/(a^2+b^2)^2/d-1/2*sech(d*x+c)^2*(a-b*sinh(d*x+c))/(a^2+b^2)/
d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{\frac{b\arctan(\sinh(c+dx))}{a^2+b^2} + \frac{2b\operatorname{csch}(c+dx)}{a^2} - \frac{\operatorname{csch}^2(c+dx)}{a} + \frac{(a-ib)(2a^2+iab+2b^2)\log(i-\sinh(c+dx))}{(a^2+b^2)^2} - \frac{2(2a^2-b^2)\log(\sinh(c+dx))}{a^3} + \cdots}{2d}$$

input
```
Integrate[(Csch[c + d*x]^3*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
((b*ArcTan[Sinh[c + d*x]])/(a^2 + b^2) + (2*b*Csch[c + d*x])/a^2 - Csch[c
+ d*x]^2/a + ((a - I*b)*(2*a^2 + I*a*b + 2*b^2)*Log[I - Sinh[c + d*x]])/(a
^2 + b^2)^2 - (2*(2*a^2 - b^2)*Log[Sinh[c + d*x]])/a^3 + ((a + I*b)*(2*a^2
 - I*a*b + 2*b^2)*Log[I + Sinh[c + d*x]])/(a^2 + b^2)^2 - (2*b^6*Log[a + b
*Sinh[c + d*x]])/(a^3*(a^2 + b^2)^2) - (a*Sech[c + d*x]^2)/(a^2 + b^2) + (
b*Sech[c + d*x]*Tanh[c + d*x])/(a^2 + b^2))/(2*d)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {3042, 26, 3316, 26, 27, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\sin(ic+idx)^3\cos(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\cos(ic+idx)^3\sin(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$\Big\downarrow$ 3316

$$-\frac{ib^3\int \frac{i\text{csch}^3(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$\Big\downarrow$ 26

$$\frac{b^3\int \frac{\text{csch}^3(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$\Big\downarrow$ 27

$$\frac{b^6\int \frac{\text{csch}^3(c+dx)}{b^3(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$\Big\downarrow$ 615

$$\frac{b^6\int \left(\frac{\text{csch}^3(c+dx)}{ab^7}-\frac{\text{csch}^2(c+dx)}{a^2b^6}+\frac{(b^2-2a^2)\text{csch}(c+dx)}{a^3b^7}-\frac{1}{a^3(a^2+b^2)^2(a+b\sinh(c+dx))}+\frac{(a^2+2b^2)b^2+a(2a^2+3b^2)\sinh(c+dx)b}{b^6(a^2+b^2)^2\left(\sinh^2(c+dx)b^2+b^2\right)}\right.}{d}$$

$\Big\downarrow$ 2009

$$\frac{b^6\left(\frac{(a^2+2b^2)\arctan(\sinh(c+dx))}{b^5(a^2+b^2)^2}+\frac{\arctan(\sinh(c+dx))}{2b^5(a^2+b^2)}+\frac{\text{csch}(c+dx)}{a^2b^5}+\frac{a(2a^2+3b^2)\log\left(b^2\sinh^2(c+dx)+b^2\right)}{2b^6(a^2+b^2)^2}-\frac{a-b\sinh(c+d}{2b^4(a^2+b^2)\left(b^2\sinh^2(c\right.}\right.}{d}$$

input

```
Int[(Csch[c + d*x]^3*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output

```
(b^6*(ArcTan[Sinh[c + d*x]]/(2*b^5*(a^2 + b^2)) + ((a^2 + 2*b^2)*ArcTan[Si
nh[c + d*x]])/(b^5*(a^2 + b^2)^2) + Csch[c + d*x]/(a^2*b^5) - Csch[c + d*x
]^2/(2*a*b^6) - ((2*a^2 - b^2)*Log[b*Sinh[c + d*x]])/(a^3*b^6) - Log[a + b
*Sinh[c + d*x]]/(a^3*(a^2 + b^2)^2) + (a*(2*a^2 + 3*b^2)*Log[b^2 + b^2*Sin
h[c + d*x]^2])/(2*b^6*(a^2 + b^2)^2) - (a - b*Sinh[c + d*x])/(2*b^4*(a^2 +
 b^2)*(b^2 + b^2*Sinh[c + d*x]^2))))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 47.01 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.38

method	result
derivativedivides	$-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2} a}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^{2}}+\frac{2\left(\frac{\left(-\frac{1}{2}a^{2}b-\frac{1}{2}b^{3}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+\left(a^{3}+a\,b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\left(\frac{1}{2}a^{2}b+\frac{1}{2}b^{3}\right)\tanh\left(\frac{dx}{2}+\right.}{\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}}}{\left(a^{2}+b^{2}\right)}$
default	$-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2} a}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^{2}}+\frac{2\left(\frac{\left(-\frac{1}{2}a^{2}b-\frac{1}{2}b^{3}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+\left(a^{3}+a\,b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\left(\frac{1}{2}a^{2}b+\frac{1}{2}b^{3}\right)\tanh\left(\frac{dx}{2}+\right.}{\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}}}{\left(a^{2}+b^{2}\right)}$
risch	$-\frac{4a^{3}d^{2}x}{a^{4}d^{2}+2a^{2}b^{2}d^{2}+b^{4}d^{2}}-\frac{4a^{3}dc}{a^{4}d^{2}+2a^{2}b^{2}d^{2}+b^{4}d^{2}}-\frac{6a\,b^{2}d^{2}x}{a^{4}d^{2}+2a^{2}b^{2}d^{2}+b^{4}d^{2}}-\frac{6a\,b^{2}dc}{a^{4}d^{2}+2a^{2}b^{2}d^{2}+b^{4}d^{2}}+\frac{4x}{a}+\frac{4c}{da}$

input
```
int(csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4/a^2*(1/2*tanh(1/2*d*x+1/2*c)^2*a+2*b*tanh(1/2*d*x+1/2*c))+2/(a^2
+b^2)^2*(((-1/2*a^2*b-1/2*b^3)*tanh(1/2*d*x+1/2*c)^3+(a^3+a*b^2)*tanh(1/2*
d*x+1/2*c)^2+(1/2*a^2*b+1/2*b^3)*tanh(1/2*d*x+1/2*c))/(1+tanh(1/2*d*x+1/2*
c)^2)^2+1/4*(4*a^3+6*a*b^2)*ln(1+tanh(1/2*d*x+1/2*c)^2)+1/2*(3*a^2*b+5*b^3
)*arctan(tanh(1/2*d*x+1/2*c)))-1/8/a/tanh(1/2*d*x+1/2*c)^2+1/4/a^3*(-8*a^2
+4*b^2)*ln(tanh(1/2*d*x+1/2*c))+1/2*b/a^2/tanh(1/2*d*x+1/2*c)-b^6/(a^2+b^2
)^2/a^3*ln(tanh(1/2*d*x+1/2*c)^2*a-2*b*tanh(1/2*d*x+1/2*c)-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3148 vs. 2(206) = 412.

Time = 0.48 (sec) , antiderivative size = 3148, normalized size of antiderivative = 14.92

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fric
as")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**3*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(206) = 412$.

Time = 0.13 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.98

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b^6\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^7+2\,a^5b^2+a^3b^4)d}$$
$$-\frac{(3\,a^2b+5\,b^3)\arctan\left(e^{(-dx-c)}\right)}{(a^4+2\,a^2b^2+b^4)d}+\frac{(2\,a^3+3\,ab^2)\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^4+2\,a^2b^2+b^4)d}$$
$$-\frac{4\,ab^2e^{(-4\,dx-4\,c)}-(3\,a^2b+2\,b^3)e^{(-dx-c)}+2\,(2\,a^3+ab^2)e^{(-2\,dx-2\,c)}+(a^2b-2\,b^3)e^{(-3\,dx-3\,c)}-(a^2b-2\,}{(a^4+a^2b^2-2\,(a^4+a^2b^2)e^{(-4\,dx-4\,c)}+(a^4+a^2b^2}$$
$$-\frac{(2\,a^2-b^2)\log\left(e^{(-dx-c)}+1\right)}{a^3d}-\frac{(2\,a^2-b^2)\log\left(e^{(-dx-c)}-1\right)}{a^3d}$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxi
ma")
```

output
```
-b^6*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^7 + 2*a^5*b^2 + a
^3*b^4)*d) - (3*a^2*b + 5*b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^
4)*d) + (2*a^3 + 3*a*b^2)*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^
4)*d) - (4*a*b^2*e^(-4*d*x - 4*c) - (3*a^2*b + 2*b^3)*e^(-d*x - c) + 2*(2*
a^3 + a*b^2)*e^(-2*d*x - 2*c) + (a^2*b - 2*b^3)*e^(-3*d*x - 3*c) - (a^2*b
- 2*b^3)*e^(-5*d*x - 5*c) + 2*(2*a^3 + a*b^2)*e^(-6*d*x - 6*c) + (3*a^2*b
+ 2*b^3)*e^(-7*d*x - 7*c))/((a^4 + a^2*b^2 - 2*(a^4 + a^2*b^2)*e^(-4*d*x -
 4*c) + (a^4 + a^2*b^2)*e^(-8*d*x - 8*c))*d) - (2*a^2 - b^2)*log(e^(-d*x -
 c) + 1)/(a^3*d) - (2*a^2 - b^2)*log(e^(-d*x - c) - 1)/(a^3*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(206) = 412$.

Time = 0.14 (sec) , antiderivative size = 464, normalized size of antiderivative = 2.20

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{\frac{4\,b^7\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^7b+2\,a^5b^3+a^3b^5}-\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(3\,a^2b+5\,b^3\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\left(2\,a^3+3\,ab^2\right)\log\left(\left(e^{(dx+c)}-e^{(-dx-}\right.\right.}{a^4+2\,a^2b^2+b^4}}{}$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac
")
```

output
```
-1/4*(4*b^7*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^7*b + 2*a^5*
b^3 + a^3*b^5) - (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(
3*a^2*b + 5*b^3)/(a^4 + 2*a^2*b^2 + b^4) - 2*(2*a^3 + 3*a*b^2)*log((e^(d*x
 + c) - e^(-d*x - c))^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) + 2*(2*a^3*(e^(d*x +
c) - e^(-d*x - c))^2 + 3*a*b^2*(e^(d*x + c) - e^(-d*x - c))^2 - 2*a^2*b*(e
^(d*x + c) - e^(-d*x - c)) - 2*b^3*(e^(d*x + c) - e^(-d*x - c)) + 12*a^3 +
 16*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*((e^(d*x + c) - e^(-d*x - c))^2 + 4))
+ 4*(2*a^2 - b^2)*log(abs(e^(d*x + c) - e^(-d*x - c)))/a^3 - 2*(6*a^2*(e^(
d*x + c) - e^(-d*x - c))^2 - 3*b^2*(e^(d*x + c) - e^(-d*x - c))^2 + 4*a*b*
(e^(d*x + c) - e^(-d*x - c)) - 4*a^2)/(a^3*(e^(d*x + c) - e^(-d*x - c))^2)
)/d
```

Mupad [B] (verification not implemented)

Time = 8.38 (sec) , antiderivative size = 554, normalized size of antiderivative = 2.63

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= -\frac{\frac{4\,b^5}{a\,d\,(a^2\,b^3+b^5)}-\frac{4\,b^4\,e^{3\,c+3\,d\,x}}{d\,(a^2\,b^3+b^5)}+\frac{4\,b^4\,e^{c+d\,x}}{d\,(a^2\,b^3+b^5)}+\frac{4\,b^3\,e^{2\,c+2\,d\,x}\,(2\,a^2+b^2)}{a\,d\,(a^2\,b^3+b^5)}}{e^{8\,c+8\,d\,x}-2\,e^{4\,c+4\,d\,x}+1}$$
$$-\frac{\frac{4\,(a^2\,b^5+b^7)}{a\,d\,(a^2\,b^3+b^5)\,(a^2+b^2)}+\frac{2\,e^{2\,c+2\,d\,x}\,(2\,a^4\,b^3+3\,a^2\,b^5+b^7)}{a\,d\,(a^2\,b^3+b^5)\,(a^2+b^2)}-\frac{e^{3\,c+3\,d\,x}\,(3\,a^4\,b^4+5\,a^2\,b^6+2\,b^8)}{a^2\,d\,(a^2\,b^3+b^5)\,(a^2+b^2)}-\frac{b^4\,e^{c+d\,x}\,(-a^4+a^2\,b^2+2\,b^4)}{a^2\,d\,(a^2\,b^3+b^5)\,(a^2+b^2)}}{e^{4\,c+4\,d\,x}-1}$$
$$+\frac{\ln\left(1+e^{c+d\,x}\,1\mathrm{i}\right)\,(4\,a+b\,5\mathrm{i})}{2\,(d\,a^2+2\mathrm{i}\,d\,a\,b-d\,b^2)}-\frac{b^6\,\ln\left(2\,a\,e^{c+d\,x}-b+b\,e^{2\,c+2\,d\,x}\right)}{d\,a^7+2\,d\,a^5\,b^2+d\,a^3\,b^4}$$
$$+\frac{\ln\left(e^{c+d\,x}+1\mathrm{i}\right)\,(5\,b+a\,4\mathrm{i})}{2\,(1\mathrm{i}\,d\,a^2+2\,d\,a\,b-1\mathrm{i}\,d\,b^2)}-\frac{\ln\left(e^{2\,c+2\,d\,x}-1\right)\,(2\,a^2-b^2)}{a^3\,d}$$

input
```
int(1/(cosh(c + d*x)^3*sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
(log(exp(c + d*x)*1i + 1)*(4*a + b*5i))/(2*(a^2*d - b^2*d + a*b*d*2i)) - (
(4*(b^7 + a^2*b^5))/(a*d*(b^5 + a^2*b^3)*(a^2 + b^2)) + (2*exp(2*c + 2*d*x
)*(b^7 + 3*a^2*b^5 + 2*a^4*b^3))/(a*d*(b^5 + a^2*b^3)*(a^2 + b^2)) - (exp(
3*c + 3*d*x)*(2*b^8 + 5*a^2*b^6 + 3*a^4*b^4))/(a^2*d*(b^5 + a^2*b^3)*(a^2
+ b^2)) - (b^4*exp(c + d*x)*(2*b^4 - a^4 + a^2*b^2))/(a^2*d*(b^5 + a^2*b^3
)*(a^2 + b^2)))/(exp(4*c + 4*d*x) - 1) - ((4*b^5)/(a*d*(b^5 + a^2*b^3)) -
(4*b^4*exp(3*c + 3*d*x))/(d*(b^5 + a^2*b^3)) + (4*b^4*exp(c + d*x))/(d*(b^
5 + a^2*b^3)) + (4*b^3*exp(2*c + 2*d*x)*(2*a^2 + b^2))/(a*d*(b^5 + a^2*b^3
)))/(exp(8*c + 8*d*x) - 2*exp(4*c + 4*d*x) + 1) - (b^6*log(2*a*exp(c + d*x
) - b + b*exp(2*c + 2*d*x)))/(a^7*d + a^3*b^4*d + 2*a^5*b^2*d) + (log(exp(
c + d*x) + 1i)*(a*4i + 5*b))/(2*(a^2*d*1i - b^2*d*1i + 2*a*b*d)) - (log(ex
p(2*c + 2*d*x) - 1)*(2*a^2 - b^2))/(a^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1269, normalized size of antiderivative = 6.01

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
(3*e**(8*c + 8*d*x)*atan(e**(c + d*x))*a**5*b + 5*e**(8*c + 8*d*x)*atan(e*
*(c + d*x))*a**3*b**3 - 6*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**5*b - 10*
e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**3*b**3 + 3*atan(e**(c + d*x))*a**5*
b + 5*atan(e**(c + d*x))*a**3*b**3 + 2*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*
x) + 1)*a**6 + 3*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x) + 1)*a**4*b**2 - 2*
e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)*a**6 - 3*e**(8*c + 8*d*x)*log(e**(c
 + d*x) - 1)*a**4*b**2 + e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)*b**6 - 2*e
**(8*c + 8*d*x)*log(e**(c + d*x) + 1)*a**6 - 3*e**(8*c + 8*d*x)*log(e**(c
+ d*x) + 1)*a**4*b**2 + e**(8*c + 8*d*x)*log(e**(c + d*x) + 1)*b**6 - e**(
8*c + 8*d*x)*log(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b)*b**6 - 2*e**(8
*c + 8*d*x)*a**4*b**2 - 2*e**(8*c + 8*d*x)*a**2*b**4 + 3*e**(7*c + 7*d*x)*
a**5*b + 5*e**(7*c + 7*d*x)*a**3*b**3 + 2*e**(7*c + 7*d*x)*a*b**5 - 4*e**(
6*c + 6*d*x)*a**6 - 6*e**(6*c + 6*d*x)*a**4*b**2 - 2*e**(6*c + 6*d*x)*a**2
*b**4 - e**(5*c + 5*d*x)*a**5*b + e**(5*c + 5*d*x)*a**3*b**3 + 2*e**(5*c +
 5*d*x)*a*b**5 - 4*e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x) + 1)*a**6 - 6*e**
(4*c + 4*d*x)*log(e**(2*c + 2*d*x) + 1)*a**4*b**2 + 4*e**(4*c + 4*d*x)*log
(e**(c + d*x) - 1)*a**6 + 6*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**4*b*
*2 - 2*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*b**6 + 4*e**(4*c + 4*d*x)*lo
g(e**(c + d*x) + 1)*a**6 + 6*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a**4*b
**2 - 2*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*b**6 + 2*e**(4*c + 4*d*x...
```

3.502 $\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 36

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Defer(Int)(csch(d*x+c)^3*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 129.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Csch[c + d*x]^3*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x
])),x]
```

output
```
Integrate[(Csch[c + d*x]^3*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^3(c+dx)\text{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{6125}$$
$$\int \frac{\text{csch}^3(c+dx)\text{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Csch[c + d*x]^3*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\text{csch}\,(dx+c)^3\,\text{sech}\,(dx+c)^3}{(fx+e)\,(a+b\sinh\,(dx+c))}dx$$

input
```
int(csch(d*x+c)^3*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)^3*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**3*sech(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 3.12 (sec) , antiderivative size = 1950, normalized size of antiderivative = 54.17

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^3\operatorname{sech}\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-(a*b^2*f - (2*b^3*d*e*e^(7*c) + (3*d*e - f)*a^2*b*e^(7*c) + (3*a^2*b*d*f*
e^(7*c) + 2*b^3*d*f*e^(7*c))*x)*e^(7*d*x) + (2*(2*d*e - f)*a^3*e^(6*c) + (
2*d*e - f)*a*b^2*e^(6*c) + 2*(2*a^3*d*f*e^(6*c) + a*b^2*d*f*e^(6*c))*x)*e^
(6*d*x) - (2*b^3*d*e*e^(5*c) - (d*e - f)*a^2*b*e^(5*c) - (a^2*b*d*f*e^(5*c
) - 2*b^3*d*f*e^(5*c))*x)*e^(5*d*x) + (4*a*b^2*d*f*x*e^(4*c) + (4*d*e - f)
*a*b^2*e^(4*c))*e^(4*d*x) + (2*b^3*d*e*e^(3*c) - (d*e + f)*a^2*b*e^(3*c) -
 (a^2*b*d*f*e^(3*c) - 2*b^3*d*f*e^(3*c))*x)*e^(3*d*x) + (2*(2*d*e + f)*a^3
*e^(2*c) + (2*d*e + f)*a*b^2*e^(2*c) + 2*(2*a^3*d*f*e^(2*c) + a*b^2*d*f*e^
(2*c))*x)*e^(2*d*x) + (2*b^3*d*e*e^c + (3*d*e + f)*a^2*b*e^c + (3*a^2*b*d*
f*e^c + 2*b^3*d*f*e^c)*x)*e^(d*x))/(a^4*d^2*e^2 + a^2*b^2*d^2*e^2 + (a^4*d
^2*f^2 + a^2*b^2*d^2*f^2)*x^2 + 2*(a^4*d^2*e*f + a^2*b^2*d^2*e*f)*x + (a^4
*d^2*e^2*e^(8*c) + a^2*b^2*d^2*e^2*e^(8*c) + (a^4*d^2*f^2*e^(8*c) + a^2*b^
2*d^2*f^2*e^(8*c))*x^2 + 2*(a^4*d^2*e*f*e^(8*c) + a^2*b^2*d^2*e*f*e^(8*c))
*x)*e^(8*d*x) - 2*(a^4*d^2*e^2*e^(4*c) + a^2*b^2*d^2*e^2*e^(4*c) + (a^4*d^
2*f^2*e^(4*c) + a^2*b^2*d^2*f^2*e^(4*c))*x^2 + 2*(a^4*d^2*e*f*e^(4*c) + a^
2*b^2*d^2*e*f*e^(4*c))*x)*e^(4*d*x)) + 64*integrate(-1/32*(a*b^6*e^(d*x +
c) - b^7)/(a^7*b*e + 2*a^5*b^3*e + a^3*b^5*e + (a^7*b*f + 2*a^5*b^3*f + a^
3*b^5*f)*x - (a^7*b*e*e^(2*c) + 2*a^5*b^3*e*e^(2*c) + a^3*b^5*e*e^(2*c) +
(a^7*b*f*e^(2*c) + 2*a^5*b^3*f*e^(2*c) + a^3*b^5*f*e^(2*c))*x)*e^(2*d*x) -
 2*(a^8*e*e^c + 2*a^6*b^2*e*e^c + a^4*b^4*e*e^c + (a^8*f*e^c + 2*a^6*b^...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 19.75 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^3\,\sinh\left(c+d\,x\right)^3\,\left(e+f\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)^3*sinh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(cosh(c + d*x)^3*sinh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))), x
)
```

Reduce [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)^3\operatorname{sech}(dx+c)^3}{\sinh(dx+c)\,be+\sinh(dx+c)\,bfx+ae+afx}dx$$

input
```
int(csch(d*x+c)^3*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int((csch(c + d*x)**3*sech(c + d*x)**3)/(sinh(c + d*x)*b*e + sinh(c + d*x)
*b*f*x + a*e + a*f*x),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file